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INTEROPT

A Software System for

Interactive Function Minimization

by

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ABSTRACT

The process of locating the minimum of a nonlinear function is highly experimental in nature since the determination of an acceptable solution may require many trials involving technique and parameter changes. If this problem is handled in a batch computing environment, the user is obliged to accept significant delays between minimization experiments. This difficulty can be overcome through the use of an interactive approach.

The work described in this thesis is concerned with the development of a software package, called INTEROPT, which applies interactive/graphics techniques to the solution of nonlinear minimization problems. INTEROPT is designed for use on the IBM 360/65 computer operated by the Computing Center of the University of Ottawa. A Tektronix 4013 (or equivalent) graphics display unit is utilized as an on-line input-output device.

The thesis describes the operation and structure of INTEROPT and gives examples of typical terminal sessions.

CHAPTER 1

INTRODUCTION

1.1 Preliminary Remarks

The work described in this thesis is concerned with the development of a software package, called INTEROPT, which applies interactive/graphics techniques to the solution of non-linear minimization problems. INTEROPT is an extension of an existing package, OPTPAK [1] which is wholly batch processing oriented. INTEROPT is designed for use on the IBM 360/65 computer operated by the Computing Center of the University of Ottawa. A Tektronix 4013 (or equivalent) graphics display unit is utilized as an on-line input-output device. A collection of S/360 assembly language routines is used to control the communication between INTEROPT and the Tektronix unit.

1.2 The Optimization Problem

The general optimization problem can be stated as follows [2]:

Find the argument x^* of the scalar function $f(x)$, (x is an n-vector), which yields the minimum value for f and which satisfies the following conditions (the constraints):

$$g_j(x^*) \geq 0 \qquad j = 1,2,\ldots,m_1$$

$$h_j(x^*) = 0 \qquad j = m_1+1,\ m_1+2,\ldots,m$$

The function $f(x)$ is called the criterion or objective function.

Throughout the presentation, the minimization of a criterion function is considered since the maximization of $f(x)$ is equivalent to the minimization of $[-f(x)]$.

INTEROPT is primarily oriented toward the solution of the unconstrained problem; i.e. the case where the auxiliary conditions $g_j \geq 0$, $h_j = 0$ are absent. It is well known, however, that the more general problem with constraints can be handled as a sequence of unconstrained problems through the use of penalty function methods [3,4,5]. INTEROPT, in fact, relies on the interior penalty function approach [3] as the mechanism for handling the general problem.

In any specific application, the function being minimized is assumed to have continuity properties consistent with the requirements of the algorithmic technique (or techniques) that are applied by INTEROPT. Furthermore, the function is assumed to have at least one local minimum but convergence to the global minimum cannot be guaranteed.

The optimization problem arises in a variety of contexts [2], usually in natural manner, e.g. in engineering design [6] and in minimization of cost functions or maximization of profit functions. In some circumstances, however, it appears via a circuitous route, e.g. in the context of function approximation or as a reformulation of a two-point boundary value problem.

1.3 Software Requirements

The numerical solution of the unconstrained minimization problem has been extensively studied in recent years and a variety of good algorithms (or minimization techniques) have appeared in the literature [7]. It is generally conceded, however, that no single one of these algorithms qualifies as being best under all possible circum-

stances, i.e. "shapes" for the criterion function. Experiments with groups of algorithms have demonstrated that a given algorithm may, with respect to a particular function, perform extremely poorly (or even fail altogether) while for some other function it may out-perform the other methods being considered.

The implementation of any particular algorithm is usually associated with peripheral computational issues that must be resolved before the algorithm can be used. These matters may be relatively straight-forward, such as the specification of the values to be assigned to parameters in the algorithm. On the other hand, they may themselves represent significant computational issues; e.g. the formulation of a mechanism for solving a one-dimensional search problem that may be embedded in the algorithm. Such secondary issues can have a substantial influence on the success that can be attained with a particular algorithm applied to a particular problem [9,10].

The above suggests that total reliance on a single algorithm with pre-selected "values" for secondary parameters*, for all minimization tasks is unwise. A degree of flexibility is essential and such flexibility can be achieved through the use of a software package which places at the user's disposal several different minimization algorithms as well as flexible choice for any secondary parameter. Several batch oriented software packages have been developed based on this philosophy (e.g. [1], [8]).

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* The notion of secondary parameter is used here in a very broad sense. It includes, for example, the one-dimensional search question and questions relating to the generation of gradient information.

The range of alternatives provided by such a package, in turn, gives rise to a specification problem inasmuch as the user is obliged to make selections based principally on his insight and experience. For example, he must choose the technique he feels is best for the problem at hand, as well as the values for the secondary parameters. In addition, a choice of the "priming guess" (the initial estimate x^0 of the minimizing argument) must be provided and it is well-known that the speed with which any algorithm moves toward the solution depends greatly on the "quality" of this guess. An improper selection of any of these alternatives can result in little or no movement toward the problem solution.

Once convergence to the solution appears to have occurred some precautions still have to be taken in order to try and ensure that a global, rather than a local, minimum has been uncovered. This usually involves experiments with various different priming guesses to verify that convergence to a better point is not possible.

Thus the function minimization process is highly experimental in nature. If it is undertaken in a batch processing computing environment, the time between problem initiation and problem solution can be excessive because of the unavoidable delays that take place between job submissions. This elapsed time can, however, be greatly reduced in an interactive computing environment.

1.4 Features Provided by INTEROPT

An interactive software system which uses a graphical display device increases the computer's potential for serving as a tool in

solving the minimization problem. In an interactive environment, the user is given a very high degree of flexibility to select, and even to change within the particular run, the minimization technique together with the available secondary parameters. This is achieved in INTEROPT via a dialog that is carried out with the user at various stages of the run. The immediate availability of results from the current run or from previous runs helps the user to make appropriate decisions based on the feed-back from these results.

INTEROPT also provides optional displays of various plots that can be valuable to the user in evaluating the progress of the current run and in deciding whether or not to abandon the run because convergence appears unlikely.

The minimization methods employed in INTEROPT require the solution of one or more linear search (one-dimensional minimization) problems on each iteration. INTEROPT provides the user with an optional capability to interact with these linear searches and thereby better control the solution procedure.

1.5 General Characteristics of the Methods Provided in INTEROPT

As noted earlier, many effective function minimization algorithms have appeared in the literature in recent years. These algorithms can be grouped into various categories using any of several possible criteria. INTEROPT provides the user with eight different methods which can be divided into two classes referred to as gradient dependent and gradient independent methods.

A typical iteration with any one of the methods in INTEROPT

can be summarized as follows:

1. The minimization technique selects a "search direction" s^k (an n-vector) according to some criterion characteristic of the technique.
2. It then invokes a "linear search algorithm" to perform a one-dimensional minimization of the criterion function along this direction beginning at the current argument x^k. This results in an optimal step α^* along the direction s^k.
3. Depending on the specific technique steps 1 and 2 may be performed once or several times in the course of the iteration. In either case the result is a new argument x^{k+1} at which the criterion function has a smaller value.
4. A check is made to determine whether one of several prespecified termination conditions is satisfied. If so, the process is stopped and the final argument is accepted as an estimate of x^*. Otherwise, another iteration is started.

1.6 An Overview of INTEROPT

As noted earlier, INTEROPT makes available to the user eight different minimization algorithms. Five of these algorithms are gradient dependent (require evaluation of the function gradient) and the other three are gradient independent. These algorithms are outlined in appendix A1. In addition, the user is also provided with a convenient mechanism to incorporate any gradient dependent or gradient independent algorithm of his own choosing.

In cases where the evaluation of the function gradient is

required, INTEROPT provides two methods of calculation. The first one utilizes a subroutine provided by the user which contains a specification of the gradient of the function. The second method utilizes an approximation formula which has a control parameter that is set by the user.

With respect to the one-dimensional minimization problem, INTEROPT provides three linear searches algorithms (appendix A2). Any one of these can be coupled with any of the minimization algorithms. In this case also, INTEROPT allows the user to incorporate his own linear search method.

There is a parameter associated with the linear search to indicate the "fineness" with which the linear search is performed. This parameter is under the control of the user.

Five parameters are used in INTEROPT to control the execution (stopping and initializing) of the minimization algorithm besides the control provided by the interactive system via the dialog with the user.

In addition to the output given on the screen in the course of the run, INTEROPT produces some documentation for the user, on the line-printer. Three options are available for the user to control the amount of information outputted on the line-printer.

CHAPTER 2

INTEROPT FEATURES AND OPERATING PROCEDURES

2.1 General Features and Notation

INTEROPT is designed to handle optimization problems of dimension eight or less (i.e. $n \leq 8$). Up to 25 minimization trials (or "runs") can be executed per job submission. Each such run is initiated from a "priming guess" which represents the user's estimate of the minimizing argument x^*.

The computing session begins with the submission of a deck of cards that serves to initiate the INTEROPT package. The submitted program also contains the specification of the criterion function to be minimized and optionally the specification of its gradient.

The user interacts with INTEROPT via a group of commands which are entered at the Tektronix keyboard. The characters "line feed" (LF), "carriage return" (CR) and "blank" are ignored from any alphanumeric character string entered. The character CNT/Q (i.e. the Q key with CNT simultaneously pressed) is used to terminate all user commands.

In describing the commands, the composite characters CNT/Q and CNT/T (the T key with CNT simultaneously pressed) are represented by $\overline{Q}$ and $\overline{T}$ respectively for simplicity. Square brackets, [], are used to identify the options within the commands that may be omitted.

In all cases, after INTEROPT plots a curve, the Tektronix is placed in the "cursor mode" in which the cross-hairs of the cursor appear on the CRT screen. One character only may then be entered

at the keyboard. In this case the position of the cursor and/or the entered character serves as the user command.

Whenever it is required to enter a non-integer constant any of the usual FORTRAN format I, F, E or D may be used. INTEROPT ultimately changes the entered value to the double precision mode.

2.2 Run Control

This is the main phase of the interactive procedure during which the user is able to:

(a) check and/or alter the operating conditions for the current run,

(b) request information to be displayed,

(c) continue the current run, start a new one or terminate the whole INTEROPT job.

INTEROPT is in this phase (which is called the "Control Mode" in the sequel) whenever the following message flashes on the screen:

WHAT NOW?

This occurs at the beginning of each run and at certain points during the course of the run.

In response to this message, a command is entered by the user from the available set to specify his requirements. The commands are briefly outlined in the following table but are described in detail in the sequel.

	Command	Function	Sect.
Specification Commands	CONDITION	Displays the run conditions for the current run, then puts the system in a mode which permits these conditions to be altered if the user so wishes.	2.4
	INITIAL	Displays the initialization and stopping parameters for the current run, then puts the system in a mode which permits these parameters to be altered if the user so wishes.	2.5
	OUTPUT[,ω]	Enables the user to specify an option relating to the data to be outputted from each iteration on the line printer.	2.11
Display Commands	TABLE[/ρ][,r]	Displays a table that summarizes the results of previous iterations for the current run or a similar table for an earlier run.	2.6
	TABLE,SUB	Displays a table that summarizes the most recent 30 iterations for the current run.	2.6
	PLOT[/ITER][,r_1,r_2,...,r_k]	Displays a plot of function value against either number of function evaluations or number of iterations and/or a similar plot for some earlier run(s).	2.7
	PLOT[/ITER],SUB	Displays a plot of function value against either number of function evaluations or number of iterations using the results of the most recent 30 iterations for the current run.	2.7
	ITERATION	Displays status information about the current iteration.	2.8
Decision Commands	GO	Starts or continues the execution of the minimization procedure.	2.9
	RESTART	Restarts (i.e. initializes) the minimization algorithm.	2.9
	NEW RUN	Starts an entirely new run with a new priming guess specification.	2.3
	END JOB	Terminates the INTEROPT job completely.	

Summary of User Commands

2.3 Priming Guess Specification

Each run in INTEROPT starts with specification of the priming guess, i.e., the initial estimate of the minimizing argument of the criterion function. In this respect, the user has the following options:

(a) Initiate the run from a new estimate, i.e., the user explicitly specifies the value of the priming guess (NEW).

(b) Initiate the run from the last explicitly entered priming guess (REPEAT).

(c) Initiate the run from the final result obtained in the preceding run (CONTINUE).

Note that for the first run, only option (a) is meaningful, therefore INTEROPT starts with the message:

WHAT IS YOUR PRIMING GUESS?

as described later in this section.

A new run is started when INTEROPT is in the control mode and the user enters the command:

NEW RUN $\overline{Q}$

Then, INTEROPT responds by displaying the following:

CURRENT PRIMING GUESS IS

$$x_1^0$$
$$x_2^0$$
$$\vdots$$
$$x_n^0$$

FUNC VALUE = $f(x^0)$

CURRENT ARGUMENT IS

x_1

x_2

$\vdots$

x_n

FUNC VALUE = f(x)

REPEAT, CONTINUE OR NEW?

where $(x_1^0, x_2^0, \ldots, x_n^0)$ is the last explicitly entered priming guess,

$f(x^0)$ is the corresponding function value,

$(x_1, x_2, \ldots, x_n)$ is the final estimate of the minimizing argument obtained in the preceding run,

and f(x) is the corresponding function value.

The user now enters either NEW, REPEAT or CONTINUE. In case of the latter two responses, INTEROPT proceeds to the next phase; i.e., the specification of the run conditions (section 2.4). If the user enters NEW (or in case of the first run), the following steps take place:

(a) The following prompting message is displayed:

WHAT IS YOUR PRIMING GUESS?

(b) The user enters the components of the priming guess vector as a string of n constants separated by commas, e.g.:

$x_1^0, x_2^0, \ldots, x_n^0$ Q

where x_i^0 is the i^{th} component of the guess vector and $n \leq 8$.

(c) If all the x_i^0's are entered correctly (according to the previous specifications) and their number n agrees with the actual dimension of the argument of the submitted criterion function

INTEROPT proceeds to the next step (d); otherwise, an error message is displayed and the user re-enters the priming guess.

(d) The following is displayed as a check:

CURRENT PRIMING GUESS IS

$$\begin{matrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{matrix}$$

FUNC VALUE = f(x)

DO YOU WISH TO RESPECIFY THE PRIMING GUESS?

(e) The user may reply:

YES $\overline{Q}$

and the process is repeated starting from step (b), or

NO $\overline{Q}$

and the program proceeds to its next phase (the specification of the run conditions, section 2.4).

2.4 Run Conditions

The run conditions are a set of options specified by the user which govern various aspects of the solution procedure to be applied during the run. Two variables, MINTEC and LN$SRH, and two parameters associated with them, G$PAR and LS$PAR respectively, are used to describe the options. The options and their admissible values are described below.

(a) The Minimization Technique (MINTEC)

INTEROPT makes available to the user two groups of multidimen-

sional minimization algorithms which are characterized as gradient dependent and gradient independent.

(A) Gradient dependent algorithms

(i) An optimal-step steepest descent algorithm (GRAD) [Appendix A1.1]

(ii) The conjugate gradient algorithm proposed by Fletcher and Reeves (F-R) [Appendix A1.2]

(iii) A variation on the conjugate gradient method suggested by Polack and Ribiere (P-R) [Appendix A1.3]

(iv) Another variation on the conjugate gradient method suggested by Sorenson (SOREN) [Appendix A1.4]

(v) The Fletcher-Powell formulation on the Davidon algorithm (D-F-P) [Appendix A1.5]

An arbitrary gradient dependent technique (YA) can be added to this group by submitting a user-provided subroutine* called YOURSA.

(B) Gradient independent algorithms

(i) The method proposed by Powell (POWELL) [Appendix A1.6]

(ii) A variation on Powell's method suggested by Zangwill (ZANG) [Appendix A1.7]

(iii) A basic gradient independent method based on the classical relaxation approach, also called the extended sequential search (ESQ) [Appendix A1.8]

It is also possible to add an arbitrary gradient independent technique (YB) by submitting a user-provided subroutine called YOURSB.

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* All the user-provided subprograms are discussed later.

The mnemonics of the minimization techniques constitute the admissible values for MINTEC. They are listed below.

GRAD F-R P-R SOREN D-F-P YA

POWELL ZANG ESQ YB

(b) The Gradient Parameter (G$PAR)

This parameter is relevant only in cases where a gradient dependent algorithm is being used. INTEROPT accommodates two approaches for generating the required gradient information:

(i) An analytical gradient method (AG) in which a user-provided subroutine, called GRAD, is used to calculate the gradient of the criterion function.

(ii) A finite difference approximation method which utilizes the following relation to calculate the gradient of the criterion function at the point x^k:

$$\left.\frac{\partial f}{\partial x_i}\right|_{x^k} \approx \frac{f(x^k+\Delta e_i)-f(x^k)}{\Delta}$$

where e_i is the i^{th} column of the n×n identity matrix, and Δ is the perturbation size for the gradient approximation computed from the relation:

$\Delta = 10^{-a_1}$, a_1 is a positive integer.

The parameter G$PAR may be assigned either the value AG or a positive integer value (a_1 in the above relation) where the former case is interpreted as a request for the use of analytic gradient information.

(c) The Linear Search Algorithm (LN$SRH)

INTEROPT provides three algorithms for handling the linear search problem, namely:

(i) A quadratic polynomial fitting technique (QUADFT)

[Appendix A2.1]

(ii) A technique based on the Fibonacci Search (FBNACI)

[Appendix A2.2]

(iii) An implementation of the Golden Section Method (GOLDEN)

[Appendix A2.3]

The user may also add an arbitrary linear search algorithm (YL) by submitting a subroutine called YOURSL.

The mnemonics of the linear searches constitute the admissible values for LN$SRH. They are listed below:

QUADFT FBNACI GOLDEN YL

(d) The Linear Search Parameter (LS$PAR)

This parameter specifies the fineness with which the linear searches are carried out. This fineness increases with increasing values of LS$PAR. The value assigned to LS$PAR should be a positive integer a_2.

2.4.1 Default Values

The run conditions are initially assigned default values which are used until they are changed by the user. The default values are as follows:

MINTEC = D-F-P

$$\text{G\$PAR} = \begin{cases} \text{AG} & \text{if subroutine GRAD is submitted} \\ 9 & \text{otherwise} \end{cases}$$

LN$SRH = QUAD

LS$PAR = 2

It should be noted that the run conditions are not reset to

their default values automatically at the beginning of each run.

2.4.2 Changing the Run Conditions

Whenever INTEROPT is in the control mode, the user can enter the following command.

CONDITION $\overline{Q}$

INTEROPT responds in the following way:

(a) The current values of the run conditions (the default values in case of the first display) are displayed as shown below:

RUN CONDITIONS

MINTEC = m_1/G$PAR = a_1

LN$SRH = m_2/LS$PAR = a_2

CHANGE

where m_1 and m_2 are the mnemonics of the minimization technique and the linear search respectively,

and a_1 and a_2 are the values of the gradient parameter and the linear search parameter respectively.

(b) INTEROPT waits until the user enters the required changes according to the following format:

m_1'/a_1', m_2'/a_2' $\overline{Q}$

where m_1', m_2', a_1' and a_2' are now the new values to be assigned to the run conditions. To maintain the current value of any one of the run conditions, the corresponding m_j or a_j may be omitted. If all existing conditions are acceptable, the only response necessary is $\overline{Q}$.

(c) If the user's response does not have an acceptable syntax, an appropriate error message is displayed and the system again

awaits a proper response. If the response syntax is acceptable, the system returns to the control mode, but prior to doing so, the new run conditions, if specified by the user, are displayed.

2.5 Initialization and Stopping Parameters

With the exception of the GRAD and ESQ methods, the algorithms in INTEROPT embody a memory attribute which causes the generated search directions to be conditioned by the results of previous iterations. The stored information may however deteriorate in quality and this may lead to slow progress or even to a complete inability to make any progress. An "initialization" may be useful in such a case. By initialization it is meant that the accumulated information in the algorithm is discarded and the algorithm is restarted.

On the other hand, it may be desired to specify conditions which ensure that the algorithm stops at some point. Such conditions do exist in INTEROPT and they are specified via certain parameters that can be set by the user.

Five parameters are available in INTEROPT to control initialization and stopping. These are described below.

(a) The Automatic Restart Parameter (RESTRT)

This parameter enables the user to periodically initialize the minimization algorithm. When RESTRT is assigned the positive integer p_1, the algorithm being used will execute p_1 normal iterations, then will be automatically initialized on the $(p_1+1)^{th}$ iteration and the cycle is repeated. This feature is disabled

if RESTRT is set to zero.

(b) The Failure Parameter (FAIL)

If the algorithm being used fails to make progress on some iteration, INTEROPT will either terminate the algorithm or initialize it, depending on the specification of FAIL (STOP or INIT respectively).

(c) Maximum Number of Iterations (MXITER)

MXITER is a positive integer that specifies the number of iterations permitted before the selected algorithm is stopped. When MXITER is set to zero, the INTEROPT system proceeds without any stopping condition based on the number of iterations.

(d) Maximum Number of Function Evaluations (MXFE)

This is a positive integer that specifies a limit on the number of function evaluations (i.e. evaluations of the criterion function). The minimization algorithm is stopped if, after any iteration, the number of function evaluations to date is greater than or equal to this limit. When MXFE is set to zero, the INTEROPT system proceeds without any stopping condition based on the number of function evaluations.

(e) The Exit Parameter (E$PAR)

This is a positive integer less than 14, used to determine the "natural" termination of the minimization algorithm. A natural termination, in the case of a gradient dependent algorithm, occurs when the gradient norm is less than ϵ, where $\epsilon = 0.01 \times 10^{-p_5}$ (p_5 is the value of E$PAR); i.e. when

$$\left[\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2\right]^{\frac{1}{2}} < \epsilon$$

In case of a gradient independent algorithm, it occurs when the

relative change in the criterion function value in two successive iterations is less than ϵ', where $\epsilon' = 10^{-p_5}$; i.e. when

$$\frac{f(x^k) - f(x^{k+1})}{|f(x^k)|} < \epsilon'$$

or $$f(x^k) - f(x^{k+1}) < \epsilon' \times |f(x^k)|$$

2.5.1 Default Values

The initialization and stopping parameters are initially assigned default values which are used until they are changed by the user. These are as follows:

RESTRT = NOT APPL (not applicable, i.e. no automatic restart)

FAIL = { STOP in case of GRAD and ESQ; INIT otherwise (initialize)

MXITER = NO LIMIT

MXFE = NO LIMIT

E$PAR = 14

Note that these parameters are all automatically reset to their default values at the beginning of each run.

2.5.2 Changing the Initialization and Stopping Parameters

Whenever INTEROPT is in the control mode, the user can enter the command:

INITIAL $\overline{Q}$.

The program then proceeds as follows:

(a) The current values of the initialization and stopping parameters are displayed with the following format:

INITIALIZATION & STOPPING PARAMETERS

1 RESTRT = p_1

2 FAIL = p_2

3 MXITER = p_3

4 MXFE = p_4

5 E$PAR = p_5

CHANGE?

where p_j's are the values of the parameters such that:

p_1 is a positive integer or "NOT APPL"

p_2 is either "INIT" or "STOP"

p_3 and p_4 are positive integers or "NO LIMIT"

p_5 is a positive integer ($i \leq p_5 \leq 14$)

(b) INTEROPT waits for the user to enter a change via a command of the following format:

$j[,p_j'] \ \overline{Q}$

where j is the parameter identification number (as given in the display list) and p_j' is the new value of this parameter. The new value p_j' is not required in case of parameter number 2 since it has two values only and it is changed from INIT to STOP or vice-versa by simply entering its identification number (i.e. 2 $\overline{Q}$).

(c) If the user enters $\overline{Q}$ only (indicating no change), INTEROPT proceeds to step (d); otherwise, it returns to step (b) to receive further input.

(d) The newly assigned values of the parameters are displayed and INTEROPT returns to the control mode.

2.6 Summary Table Display

Data from each iteration, in the course of the run, is stored in a summary table which has four columns. The left most column contains the iteration number, the second column contains the iteration number of the current cycle (which will be different from the first column only if a restart has occurred), the third column contains the total number of function evaluations that have taken place and the rightmost column gives the criterion function value associated with the iteration number.

This table can accommodate the result of at most 118 iterations. It is updated by dropping some of its entries when this limit is exceeded. But, at the same time, data about the most recent iterations (maximum of 30) is stored in a sub-table to maintain complete data about the most recent past. Summary tables (but not sub-tables) of previous runs (at most 5) are stored in order to allow access to the results of earlier runs.

To display a summary table, INTEROPT must be in the control mode. The user then enters the following command:

TABLE[/ρ][,r] $\bar{Q}$

where

(a) ρ is the row number of the first row to be displayed from the table (at most 30 rows are shown in one display). If ρ is omitted, the display will start from the first row of the table.

(b) r specifies the run number for which the summary table is to be displayed. r should refer to either the current run or one of the immediately preceding five runs. The current run

is assumed if r is omitted.

To display a sub-table, the following command is used in the control mode:

TABLE,SUB

With any table or sub-table display, INTEROPT also displays a summary of the run conditions associated with the run. After the display of a table or sub-table, INTEROPT returns to the control mode when the user enters $\overline{Q}$. The screen is erased before the message 'WHAT NOW?' appears.

2.7 Function Plots

Output routines in INTEROPT enable the user to obtain a plot of the values of the criterion functions generated during the run. The user may choose the horizontal co-ordinate in such a plot to be either number of iterations or number of function evaluations. These plots give an overall picture of the effectiveness of the minimization technique in driving the function value to its minimum. Furthermore, they enable the user to decide whether to continue using the same technique, try another technique, change associated parameters or change the linear search. The user may request a complete plot that covers the whole run (the results of some intermediate iterations may, however, not be available; e.g. in case of updating the summary table) or a plot of only the most recent iterations (at most 30).

At most five of the complete plots (covering a whole run) may

be simultaneously displayed on the CRT. This feature provides a means of comparing the results of different runs having different operating conditions. These plots must be selected from the current run and the five immediately preceding runs.

These output options are accessible from the control mode via the following two commands:

PLOT[/ITER],SUB $\bar{Q}$

PLOT[/ITER][,r_1,r_2,...,r_k] $\bar{Q}$

where

(a) ITER is a request for a plot of function value against number of iterations. If ITER is omitted from the command, the horizontal co-ordinate of the plot is the number of function evaluations.

(b) SUB is a request for a plot pertaining to the most recent iterations (the last 30 or less).

(c) $r_1,r_2,\ldots,r_k$ ($k\leq 5$) are the run numbers for which the plots are required. Each r_j must be the run number of either the current run or one of the immediately preceding five runs. If no run numbers appear, then a plot for the current run is given.

For each function plot displayed, a summary of the last run conditions associated with the corresponding run is also diplayed. In addition the following information is provided on the CRT:

(a) Specification of the horizontal co-ordinate; i.e., number of function evaluations or number of iterations.

(b) The range of the function values displayed (maximum and minimum values).

(c) The range of the horizontal co-ordinate (maximum and minimum values).

Note that a logarithmic transformation may be applied to the function values prior to plotting if the rate of the function reduction is very rapid. This is decided on by the system, but the user can intervene as described below.

2.7.1 The Cursor Mode

After plotting, INTEROPT places the Tektronix in the cursor mode in which the cross-hairs of the cursor appear on the CRT (see section 2.1). Only the x-cursor (the vertical line) however, has relevance. This cursor may be positioned by the user via the manual control and by entering any character from the keyboard, the co-ordinates of the point on each displayed curve at the intersection of the curve and the vertical cursor line, are displayed (in appropriate units).

As noted above, the vertical co-ordinate of the displayed curve(s) may be subjected to a transformation under program control. If it is desired to over-ride the alternative selected by the program and to obtain the alternative opposite to the one displayed, the user simply enters $\overline{T}$.

A return to the control mode is achieved by entering $\overline{Q}$.

2.8 Status Information

The user can request the display of status information about the current iteration by entering the following command while

INTEROPT is in the control mode:

ITERATION $\overline{Q}$

The information displayed is as follows:

OF ITERATIONS = n_1

OF SUB-ITER. = n_2 (n_3)

OF FUNC EVAL. = n_4

THE ARGUMENT

x_1

x_2

$\vdots$

x_n

FUNC VALUE = f(x)

THE GRADIENT

g_1

g_2

$\vdots$

g_n

GRAD NORM = $[\sum_{i=1}^{n} g_i^2]^{\frac{1}{2}}$

where n_1 is the number of iterations executed so far,

n_2 is the number of iterations since the last restart (this line will be displayed only if a restart has occurred)

n_3 is the number of restarts that have occurred so far,

n_4 is the number of function evaluations so far,

$(x_1, x_2, \ldots, x_n)$ the current value of the argument,

f(x) is the corresponding function value,

$(g_1, g_2, \ldots, g_n)$ the current value of the gradient,

and $[\sum_{i=1}^{n} g_i^2]^{\frac{1}{2}}$ is the corresponding value of the gradient norm. Note that in a case where a gradient independent technique is being used, no gradient information is displayed.

After this display, INTEROPT returns to the control mode.

2.9 Execution of the Minimization Procedure

INTEROPT does not execute the minimization procedure continuously but rather steps at certain "decision points" and gives control back to the user (i.e. returns to the control mode). This gives the user the opportunity to examine the progress of the solution and to decide whether changes in some of the operating conditions might be worthwhile. Alternately, he may decide to stop the run.

A decision point is established prior to the execution of the minimization procedure by setting the "run control cycle" which is the number of iterations to be executed before INTEROPT returns to the control mode. The specification of the run control cycle is given in the control mode whenever the user requests the minimization procedure to be started or continued for a given run. Two commands have relevance for starting or continuing the execution; namely:

GO $\bar{Q}$

and RESTART $\bar{Q}$

where the second alternative implies an initialization of the minimization algorithm prior to continuing. The system then requests from the user the data for the run control cycle. In addition, it

requests the data for the "linear search display cycle" which specifies the linear search with which the user wishes to interact (see section 2.12).

After entering one of the commands GO or RESTART, the following is displayed:

RUN CONTROL CYCLE = c_1

LS DISPLAY CYCLE = c_2

CHANGE?

where c_1 & c_2 are current values. The user can then enter the desired changes in the following format:

c_1', c_2' $\overline{Q}$

where c_1' or c_2' or both can be omitted if current values are to be maintained. (In case of specifying a new value for c_2 only, the comma must be included in the response).

Both the run control cycle and the linear search display cycle are assigned the default value of 5 at the beginning of each run.

After receiving the user's response to the CHANGE? request, the minimization procedure is executed until one of the two cycles is completed or one of the stopping conditions is satisfied (section 2.5). When the run control cycle is completed, the current status is displayed (as described in section 2.8) and INTEROPT enters the control mode. In the case where the program stops because of a stopping condition, the current status is also displayed together with the reason for the stop. The linear search display cycle is completed, a graph of $F(\alpha)$ is displayed on the CRT screen and the system enters the interactive linear search mode (section 2.12).

2.10 Output Documentation

For documentation purposes, INTEROPT provides line-printer output for each INTEROPT run as well as for the whole INTEROPT job.

The output documentation associated with each INTEROPT run consists of the following items:

(a) The general header, which is a text specified by the user to identify the problem and the output. This text (which can be any character string) is entered via an input data card called the "problem description card". This card can be completely omitted. In this case, no general header appears in the output.

(b) A summary of the operating conditions, which consists of the run conditions and the values of the initialization and stopping parameters for the run.

(c) The initialization data which consists of the priming guess, the corresponding value of the criterion function and, if a gradient dependent minimization algorithm is being used, the gradient vector and its norm.

(d) Output from each iteration, which is specified by selecting an option related to the amount of data to be outputted from each iteration (see section 2.11).

(e) A summary of the results generated in the course of the run which is presented in the summ ry table described earlier (section 2.6). This is provided at the completion of the run.

For the whole INTEROPT job a final documentation is provided which lists each of the runs in turn and gives the following

information for each of them:

MINTEC - The minimization technique used.

G$PAR - The value of the gradient parameter.

[NA (Not Applicable), in case of the gradient independent methods].

LN$SRH - The linear search algorithm used.

LS$PAR - The value of the linear search parameter.

RESTRT - The value of the restart parameter.

[NA indicates the absence of a specification].

E$PAR - The value of the exit parameter.

Note that for all the above items, the values given are those which were in effect at the completion of the run.

TCODE - One of the following codes which serves to specify the reason for the termination (see section 2.5):

GR<E the gradient norm less than $\epsilon=0.01\ 10^{-p}$, where -p is the value specified by E$PAR.

FC<E the relative function change less than $\epsilon'=10^{-p}$.

IT=L number of iterations equals the limit value specified by MXITER.

FE=L number of function evaluations equals or exceeds the value given by MXFE

F$NR failure to make progress on an iteration and automatic restart not requested; i.e. FAIL=STOP.

F$I1 failure to make progress on the first iteration of a restart cycle.

STOP the run is terminated by the user; i.e. a new run is initiated before any of the previous termination

criteria occurs.

#ITER - The number of iterations carried out in the run. In addition, the number of restarts which occurred is given in parenthesis.

#EVAL - The number of function evaluations made during the run.

F-INIT - The initial value of the criterion function; i.e. the value corresponding to the point where the run started.

F-FINAL - The final value of the criterion function.

In addition to the above information, the priming argument used in each run is given together with the final estimate of the minimizing argument generated in the run.

2.11 Printer Output From Each Iteration

The user may wish to generate information characterizing each iteration in the course of the run. There are two options in this respect:

Option 1 With this option, the following items are given:

(i) the iteration number (with additional information if a restart has occurred),

(ii) the current value of the criterion function,

(iii) the corresponding value of the function argument,

and (iv) the number of function evaluations that have taken place thus far.

Option 2 The output begins with item (i) in Option 1, then information about the linear search (or searches) within the iteration is given followed by the other three items in Option 1. In the

case of a gradient dependent algorithm the following items are also provided:

(v) the current gradient vector,

(vi) the euclidean norm of the gradient,

(vii) the search direction used in arriving at the current estimate of the minimizing argument,

and (viii) the cosine of the angle between the current gradient vector and the search direction.

The specification of the line printer output option desired by the user takes place when INTEROPT is in the control mode, via the command:

OUTPUT[,ω] $\overline{Q}$

where ω is the option number; hence, it is 1 or 2. In fact, ω may also be 0 which implies that the output from each iteration is to be completely supressed. When ω is omitted from this command, INTEROPT displays a brief description of the options and requests a selection from the user. The display in this case is as follows:

PRINTER OUTPUT FOR EACH ITERATION

ENTER THE DESIRED OPTION IDENTIFICATION NUMBER

0 NO OUTPUT

1 ITER#, FUNC & ARG VALUES, # OF EVAL

2 SAME AS 1 PLUS GRAD & LINEAR SEARCH INFO.

Note that each INTEROPT run begins with ω set to zero.

Finally, it should be noted that when ω≠0, appropriate line printer output is produced to document user modifications to the run conditions or to the initialization and stopping parameters, during the course of the run. Also appropriate reasons are

documented to clarify the initialization of an algorithm or its stopping.

2.12 The Linear Search Display

As mentioned earlier, the minimization algorithms within INTEROPT require a mechanism for solving the following one-dimensional minimization (or linear search) problem.

Find the scalar α^* such that

$$F(\alpha^*) = \min_{\alpha} F(\alpha)$$

where $F(\alpha) = f(x+\alpha s)$

and x and s are given n-vectors

INTEROPT provides three different algorithms for handling this problem (see section 2.4).

The input to each of these algorithms is an "initial interval of uncertainty" (IIOU), namely an interval on the real line which is (more or less) guaranteed to contain α^*. The algorithms themselves then proceed to locate an estimate of α^* within this interval.

INTEROPT provides the user with various interactive capabilities relative to the solution of the linear search problem. These capabilities can be exercised at the end of each "linear search display cycle". (The number c_2 referred to in section 2.9). At the end of a linear search display cycle, (i.e. after c_2-1 linear searches have been performed completely by INTEROPT) the main minimization procedure is interrupted and control over the current linear search is passed to the user.

This activity begins with a display of $F(\alpha)$ which is generated on the screen using 10 values of α. Via the procedures described below, the user has the choice of either (a) simply examining the form of the $F(\alpha)$, curve, (b) prescribing the IIOU to be passed on to the selected linear search algorithm, or (c) actually specifying the point which he feels is the solution and by-passing the linear search algorithm altogether.

Note that A and B are used to denote the left-end and the right-end points of the interval to be considered from $F(\alpha)$. Initially, they are the limits of the first displayed interval of $F(\alpha)$. The user may change these, as described below, if he so wishes. Later, the interval determined by the final values of A and B can be used as the IIOU or to display a larger or smaller portion of $F(\alpha)$.

After the initial display of $F(\alpha)$, the Tektronix is placed in the cursor mode and the user is given various interogation/ specification capabilities by entering a single character as described below:

P - the value of α and $F(\alpha)$ corresponding to the point at the intersection of the vertical cursor line and the curve are displayed (This requires one function evaluation).

X - same effect as P, except that only the α value is displayed (hence no function evaluation is required).

A - the co-ordinate on the horizontal axis corresponding to the current position of the vertical cursor line, is interpreted as the user's specification of the left-hand end point of the IIOU.

B - analogous to A except that the specification is that of the right-hand end point of the IIOU.

M - also analogous to A except that the point is interpreted as the user's specification of α^*. In this case the current linear search problem is taken to be solved.

E - after this entry the system responds with the following message:

EXTENSION, ENTER A or B

The user is then able to enter a new value for either A or B. Since this is done mainly to extend the current interval to the left or to the right, the entered number should either be to the left of the current A value or to the right of the current B value.

D - a new display of $F(\alpha)$ is generated using the current values of A and B as the display extremities.

R - the current values of A and B are taken to define the IIOU and this interval is passed to the specified linear search algorithm which is then activated.

G - the interactive linear search activity is terminated and INTEROPT simply proceeds with its normal algorithmic procedure for conducting the linear search. The count of function evaluations made during the user's interaction with the current linear search is discarded.

The system's response to any one of character inputs, R H or G is the display of the message:

LS DISPLAY CYCLE = c_2, CHANGE?

where c_2 is the current value of the linear search display cycle.

The value of c_2 can be altered by entering any positive integer. A null response (simply $\overline{Q}$) results in the system maintaining the current value of c_2. At this point INTEROPT resumes execution of the minimization procedure.

2.13 The User-Provided Subprograms

The user is required to prepare at least one FORTRAN subprogram which specifies the criterion function. Other subprograms can also be added to extend the features of INTEROPT. The different possibilities are described below.

(a) The Function Subprogram F

The user must prepare a DOUBLE PRECISION FUNCTION subprogram called F which serves to specify the criterion function to be minimized. The required structure of this subprogram is as follows:

```
      DOUBLE PRECISION FUNCTION F(X)
      IMPLICIT REAL*8  X(8)
{                      } statements specifying
{ F=                   } the criterion function
      RETURN
      END
```

where X is the argument vector.

(b) The Subroutine Subprogram GRAD

If the user wishes to use a gradient dependent minimization algorithm with analytic gradient information, then a subroutine subprogram called GRAD is required. GRAD serves to calculate the

gradient of the function from an explicit specification of the partial derivatives; i.e. $\frac{\partial f}{\partial x_i}$, i=1,2,...,n. The required structure of this subprogram is as follows:

```
      SUBROUTINE GRAD (X,G)
      IMPLICIT REAL*8 (A-H, O-Z)
      REAL*8  X(8), G(8)
      G(1)=
      G(2)=                    } statements specifying the
        :                      } components of the gradient vector.
```

where G is the gradient vector.

(c) The Subroutine Subprogram YOURSA

If the user wishes to utilize gradient dependent minimization scheme which is not available in the package, a SUBROUTINE subprogram called YOURSA is required. The algorithm coded in YOURSA would typically operate on x^k and g^k (the gradient vector at x^k; i.e., $g^k=f_x(x^k)$) to produce s^k (a search direction). The required structure of this subprogram is as follows:

```
      SUBROUTINE YOURSA (X,G,S)
      IMPLICIT REAL*8  (A-H, O-Z)
      REAL*8  X(8), G(8), S(8)
                               } statements for generating the
                               } components of the search direction.
      RETURN
      END
```

where S is the search direction vector.

(d) The Subroutine Subprogram YOURSB

If the user wishes to utilize a gradient independent minimization scheme which is not available in INTEROPT, a SUBROUTINE subprogram called YOURSB is required. The algorithm coded in YOURSB would typically generate directly a new estimate of the minimizing argument using the current estimate as input (i.e. generate x^{k+1} given x^{k}). The implementation of the procedure may require direct access to the linear search algorithms within INTEROPT since it may be necessary to solve one or more linear search problems within the context of the algorithm. This access is achieved via a cell to the subroutine SCAN (XD,SD,ALPHA) where the arguments have the following interpretation:

XD is the base point of the linear search

SD is the direction along which the linear search is conducted

ALPHA is the optimal distance from XD to SD.

Note that upon return from SCAN, XD is up-dated to contain the minimizing argument along the SD direction.

The required structure of YOURSB is as follows:

```
      SUBROUTINE YOURSB (X,FLAG)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8   X(8)
      INTEGER  FLAG
      {          } statements for updating the
      {          } value of X from x^k to x^(k+1)
      RETURN
      END
```

The argument FLAG is used as an indication to indicate whether

adequate progress has been achieved during the iteration. It should normally be set to 0, and set to 1 when a lack of progress has been detected.

(e) The Subroutine Subprogram YOURSL

The user can utilize a linear search algorithm of his own choice by appropriately coding a SUBROUTINE subprogram called YOURSL. The purpose of this algorithm is to find the value of the scalar argument α of the function:

$$F(\alpha) = f(x^k + \alpha s^k)$$

which yields the smallest value of $F(\alpha)$, (x^k & s^k are taken to be given). When coding this subroutine, the evaluation of F at some argument AA is achieved via a call to the FUNCTION subprogram FUNCS (e.g. Y=FUNCS(AA)). The required structure of YOURSL is as follows:

```
      SUBROUTINE YOURSL (ALPHA,FMIN)
      IMPLICIT REAL*8 (A-H,O-Z)
      {                          } statements required in the
      {                          } algorithm
      RETURN
      END
```

Upon exit, ALPHA should contain the estimate of the minimizing argument of $F(\alpha)$ and FMIN should contain the value of F at ALPHA.

2.14 Constrained Minimization

INTEROPT provides a very simple means for handling constrained minimization problems based on "Interior Penalty Function Method" [3].

This method is based on defining a "penalty" function of the form

$$P(x,R) = f(x) + R \sum_{i=1}^{n_c} \frac{-1}{h_i(x)}$$

where $f(x)$ is the criterion function,

$h_i(x)$ for $i=1,2,\ldots,n_c$ are the constraints which are assumed to have the form $h_i(x) \leq 0$,

and R is a positive scalar.

To find an estimate of the minimizing argument x*, several INTEROPT runs, with different values of R, are required. R is chosen arbitrarily at the first run, then it is reduced at the beginning of each successive run. The requirements for this type of jobs are (a) a special structure for the user-provided FUNCTION subprogram F, and (b) modifying R at the beginning of each run.

In this case, the user provided subprogram F will have the following general structure:

```
      DOUBLE PRECISION FUNCTION F(X)
      IMPLICIT REAL 8 (A-H,O-Z)
      REAL 8  X(8), H(nc)
      LOGICAL 1  PENFLG
      COMMON /PENALT/R,PENFLG
      PENFLG = .TRUE.
{                   } statements to compute
{ F = ...           } f(x)
{ H(1)              }
{ H(2)              } statements to compute
{  :                } the constraints hi(x)
{  :                }
```

```
      DO 10  I=1,h
         IF (H(I).GT.O.ODO) GO TO 20
10       F=F R/H(I)
      RETURN
20    F=1.0D10
      RETURN
      END
```

In the above, PENFLG is a logical flag which is set to "TRUE" to indicate that a priority function problem is being considered and statement number 20 is used to make the value of the penalty function very high in the forbidden region.

R is (initially) set to zero by the program. At the beginning of the first run as well as at the beginning of each new run, the following message is displayed:

```
R=a  CHANGE?
```

where a is the current value of R. The user, then, enters the new value of R (which must be positive). The program then proceeds to the priming guess specification as described in section 2.3. Except for the first run, the CONTINUE option would normally be selected.

2.15 Diagnostic Messages

Several diagnostic messages are provided in INTEROPT to signal improper user responses. These are described in this section.

(a) If an error occurs in specifying a numeric value, the following message is displayed:

INVALID NUMBER, PLEASE RE-ENTER

The whole user command or specification, which contains this number, should be re-entered; e.g. if when specifying the priming guess (section 2.3), the following is entered:

-3.0, -1.EE2, +5.0D1, 1 $\overline{Q}$

the above error message will be displayed. The corrected specification should then be re-entered by the user; e.g.

-3.0, -1.E2, +5.0D1, 1 $\overline{Q}$

(b) If an inadmissible value is entered for a parameter in the program, the following message is displayed:

ERROR, PLEASE RE-ENTER

INTEROPT then awaits the corrected specification to be re-entered.

(c) In case of an error within a command in the control mode, one of the following messages is displayed:

(i) ERROR, PLOT/?

(the command should have been PLOT or PLOT/ITER)

(ii) ERROR, OUTPUT,?

(the command should have been OUTPUT or OUTPUT,ω where ω is 0, 1 or 2)

(iii) INVALID COMMAND

(the entered command is not one of the admissible set)

(iv) INVALID NUMBER

(the command contains an invalid numeric value)

(v) INVALID RUN#

(for table display or function plot, the specified run number is not within the permitted range)

(vi) TABLE CONTAINS ρ' ROWS ONLY

(the user has specified a row number, ρ, (in the command TABLE/ρ,r) which is greater than the actual number of rows, ρ', in this table)

INTEROPT, in all the above cases, returns to the control mode.

(d) In the explicit specification of the priming guess, the following message may be displayed:

N=n BUT YOUR GUESS VECTOR HAS n' COMPONENTS; PLEASE RE-ENTER

This means that the number of components n', entered for the priming guess by the user, does not equal the actual dimension n, of the argument vector.

(e) In the linear search display, one of the following diagnostic messages may be displayed:

(i) INSIDE THE CURRENT INTERVAL

(the user requested the graph to be extended and gave a limit which in fact lies within the presently displayed interval; the value entered is ignored)

(ii) NEG VALUES ARE NOT ADMISSIBLE

(the user requested the graph to be extended and specified a negative value for the left-end of the interval; the value entered is ignored.

(iii) SPECIFIED IOU HAS LENGTH < .001

(the user requested a new interval of the function $F(\alpha)$ to be plotted but the length of the new interval is less than the allowed limit of .001)

In all the above cases, INTEROPT returns to the cursor mode

of the linear search display.

(f) When changing the run conditions, one of the following warnings may appear:

(i) SECOND CHANGE IS IGNORED

(the user has twice specified values for either MINTEC or LN$SRH; the first one is taken)

(ii) GRADIENT INFO. IS NOT REQUIRED

(the user has specified a value for G$PAR while the minimization technique is gradient independent; the specification is ignored)

(iii) ANALYTIC GRAD IS NOT AVAILABLE

(the user has assigned G$PAR the value AG but the subroutine GRAD has not been submitted; the current value of G$PAR is retained)

(iv) DEFAULT VALUE OF G$PAR IS USED

(the user has assigned G$PAR a negative value; G$PAR is assigned the value AG if the subroutine GRAD has been submitted or the numeric value 9 otherwise)

(v) DEFAULT VALUE OF LS$PAR IS USED

(the user has assigned LS$PAR a negative value; LS$PAR is given the default value 2)

(g) If the user attempts to assign a value to RESTRT or change FAIL while using GRAD to ESQ as the minimization technique, the following message is displayed:

NOT INIT IN CASE OF GRAD/ESQ

and the specification is ignored.

(h) While receiving an input from the Tektronix, the system may

experience an I/O error. This situation is brought to the user's attention via the message

I/O ERROR, PLEASE RE-ENTER

The user should re-enter his last command or specification.

2.16 Line Drop

Generally in the case of a telecommunication line failure, the INTEROPT job is not terminated. Instead, it detects the situation and waits until the user signs on again. It then starts from the last display which existed on the screen prior to the line drop. However, if more than three telecommunication line failures occur, then the INTEROPT job automatically terminates.

2.17 Synopsis of the Operating Procedures

The sequence of flow-charts in figures 2.1 to 2.12 provide a synopsis of the operating procedures for INTEROPT. In these flow-charts, a box drawn with solid lines is used to indicate the display of information or messages on the screen. A box drawn with a broken line indicates a stage where a user response is required. In some cases, a broken-line box is horizontally separated into segments by broken lines. This indicates that a response of the form shown in any one of the sub-boxes is admissible. A box (broken-line or solid-line) with an asterisk '*' indicates that this stop is executed only in case of a penalty function run.

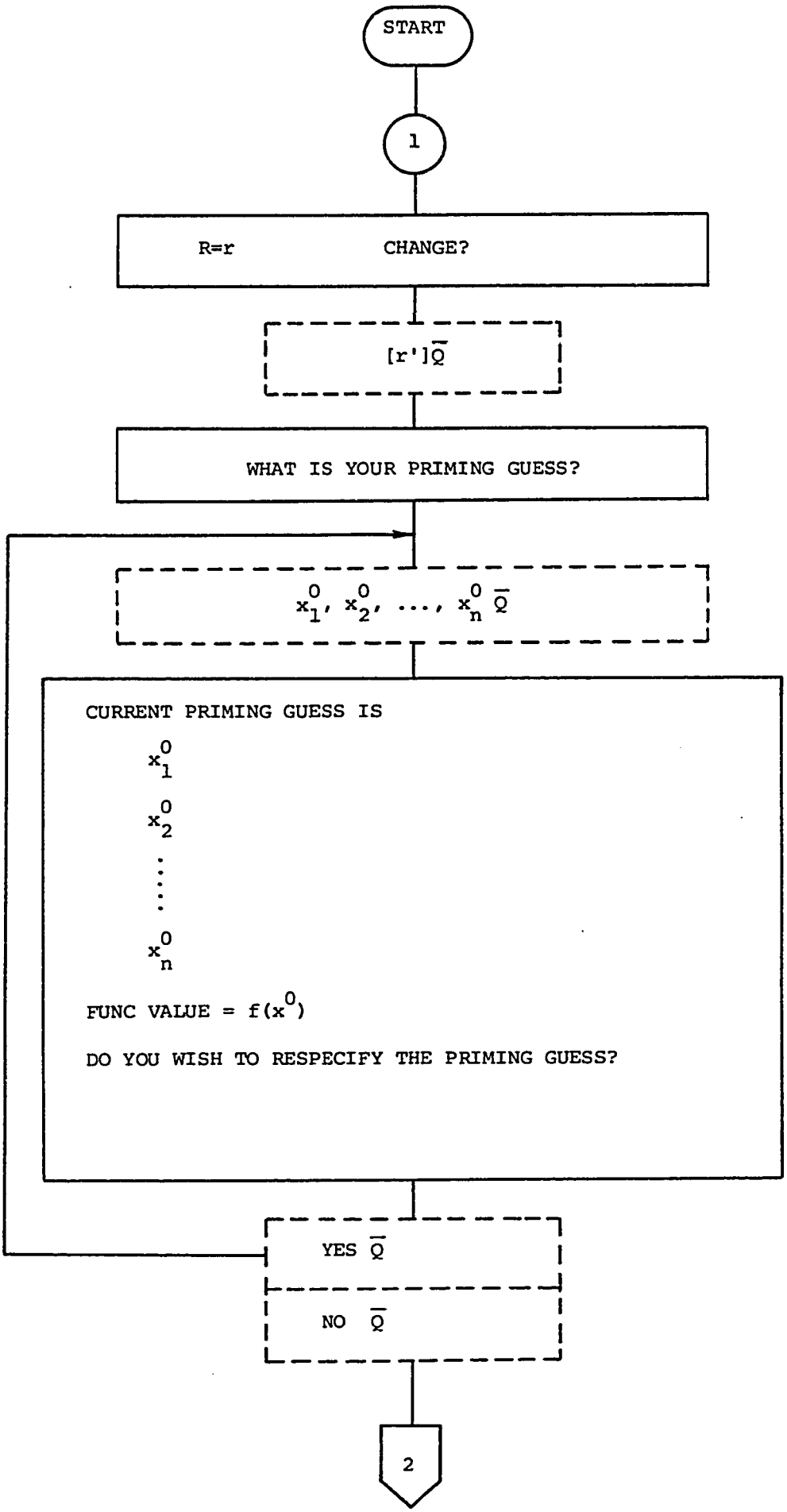
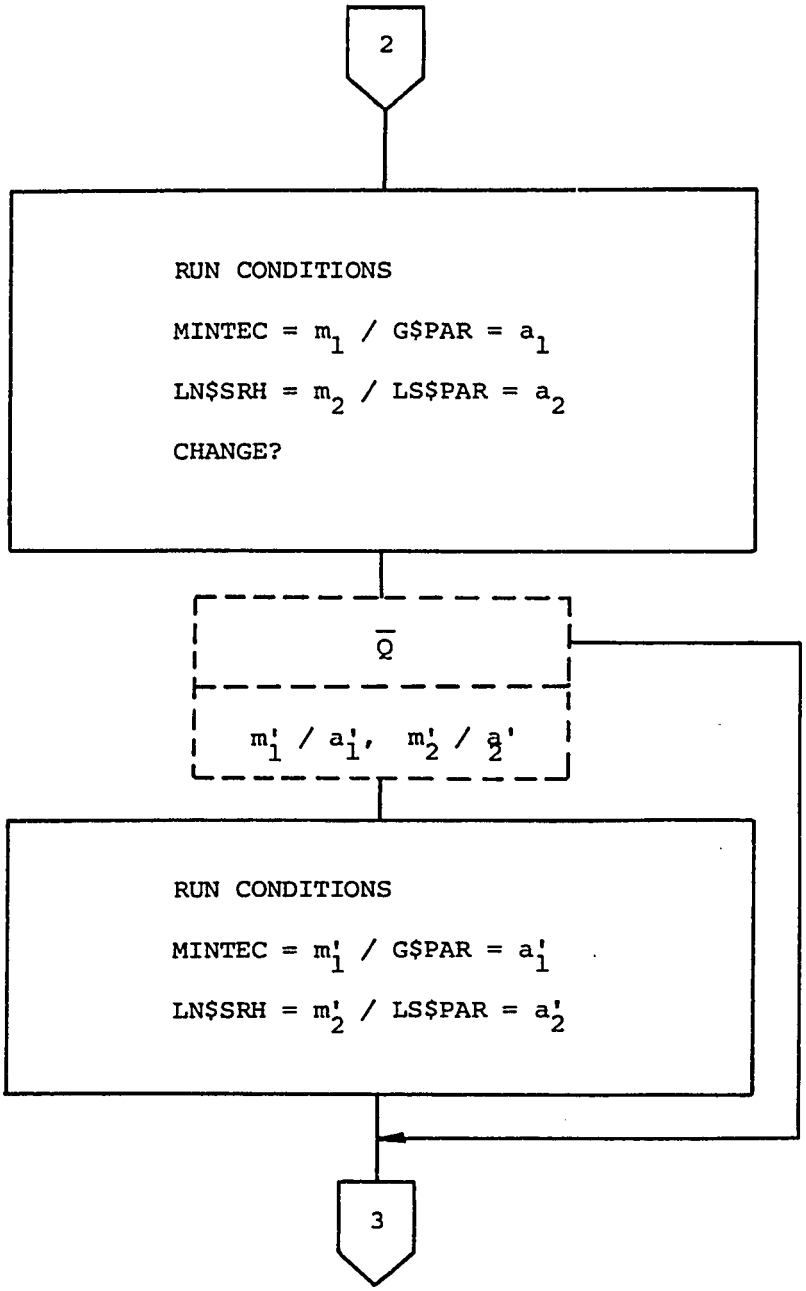

START
1
R=r CHANGE?
[r']$\bar{Q}$
WHAT IS YOUR PRIMING GUESS?
$x_1^0, x_2^0, \ldots, x_n^0$ $\bar{Q}$
CURRENT PRIMING GUESS IS
x_1^0
x_2^0
x_n^0
FUNC VALUE = $f(x^0)$
DO YOU WISH TO RESPECIFY THE PRIMING GUESS?
YES $\bar{Q}$
NO $\bar{Q}$
2

Fig.


2
RUN CONDITIONS
MINTEC = m_1 / G$PAR = a_1
LN$SRH = m_2 / LS$PAR = a_2
CHANGE?
$\overline{Q}$
m_1' / a_1', m_2' / a_2'
RUN CONDITIONS
MINTEC = m_1' / G$PAR = a_1'
LN$SRH = m_2' / LS$PAR = a_2'
3

Fig

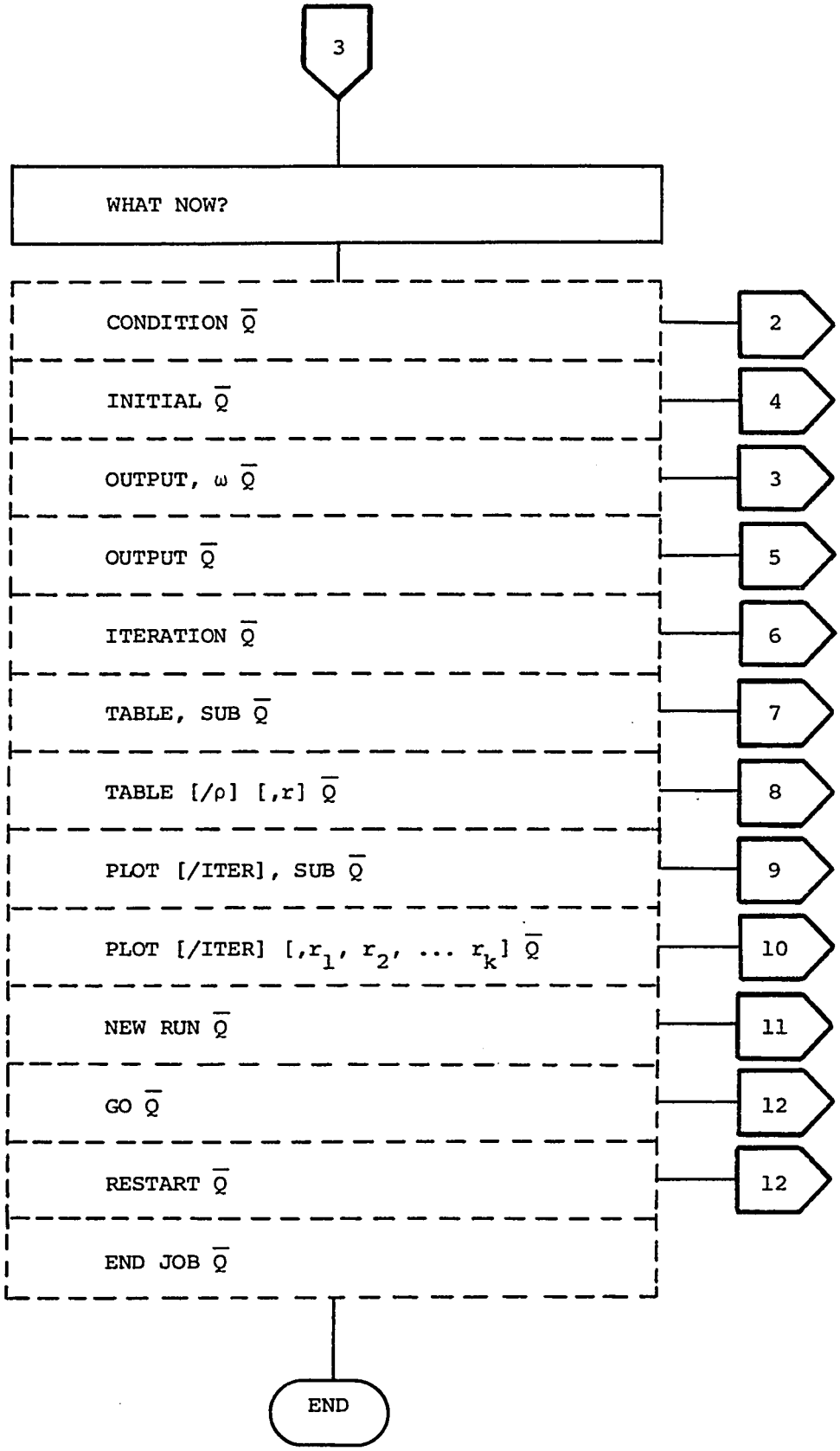

3
WHAT NOW?
CONDITION $\overline{Q}$
INITIAL $\overline{Q}$
OUTPUT, ω $\overline{Q}$
OUTPUT $\overline{Q}$
ITERATION $\overline{Q}$
TABLE, SUB $\overline{Q}$
TABLE [/ρ] [,r] $\overline{Q}$
PLOT [/ITER], SUB $\overline{Q}$
PLOT [/ITER] [,r_1, r_2, ... r_k] $\overline{Q}$
NEW RUN $\overline{Q}$
GO $\overline{Q}$
RESTART $\overline{Q}$
END JOB $\overline{Q}$
END
2
4
3
5
6
7
8
9
10
11
12
12

Fig.

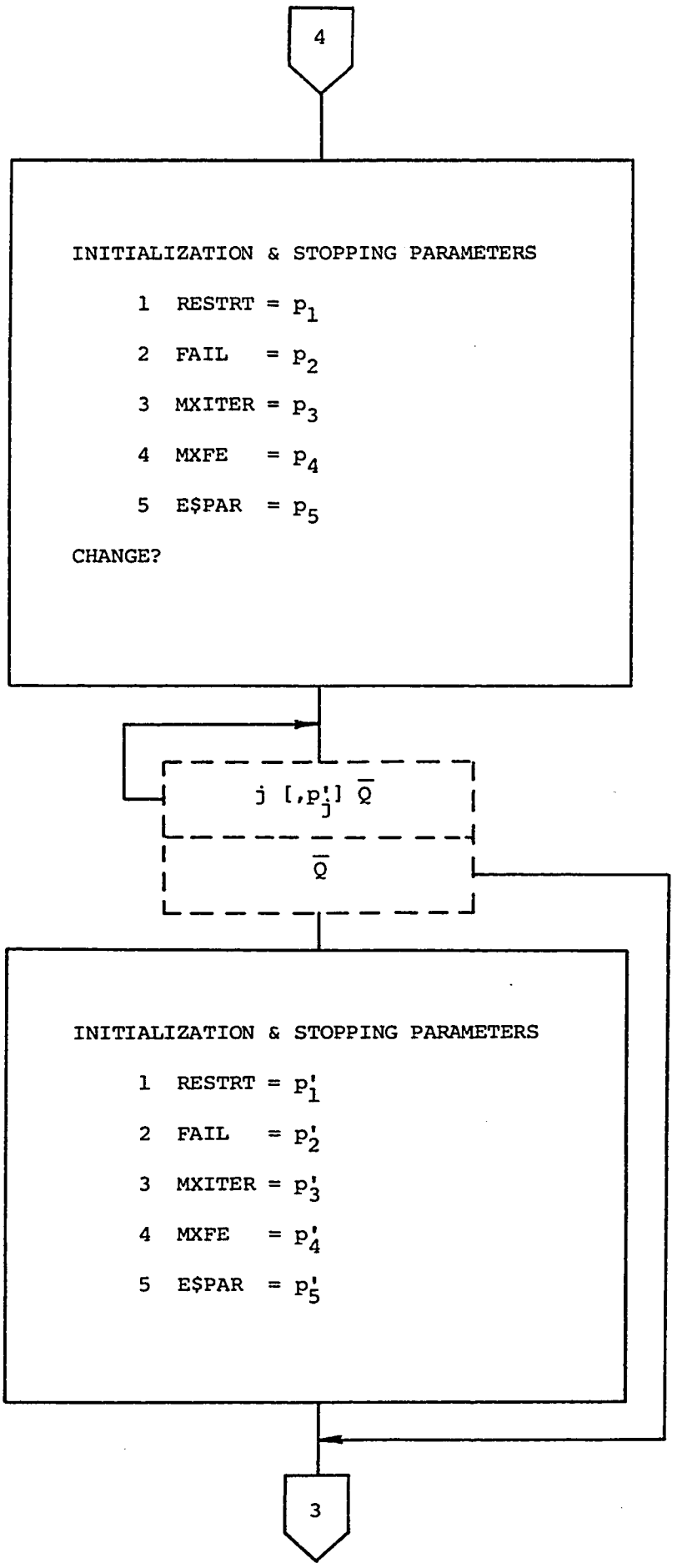

4
INITIALIZATION & STOPPING PARAMETERS
1 RESTRT = p_1
2 FAIL = p_2
3 MXITER = p_3
4 MXFE = p_4
5 E$PAR = p_5
CHANGE?
j [,p'_j] $\bar{Q}$
$\bar{Q}$
INITIALIZATION & STOPPING PARAMETERS
1 RESTRT = p'_1
2 FAIL = p'_2
3 MXITER = p'_3
4 MXFE = p'_4
5 E$PAR = p'_5
3

Fig. 2.4

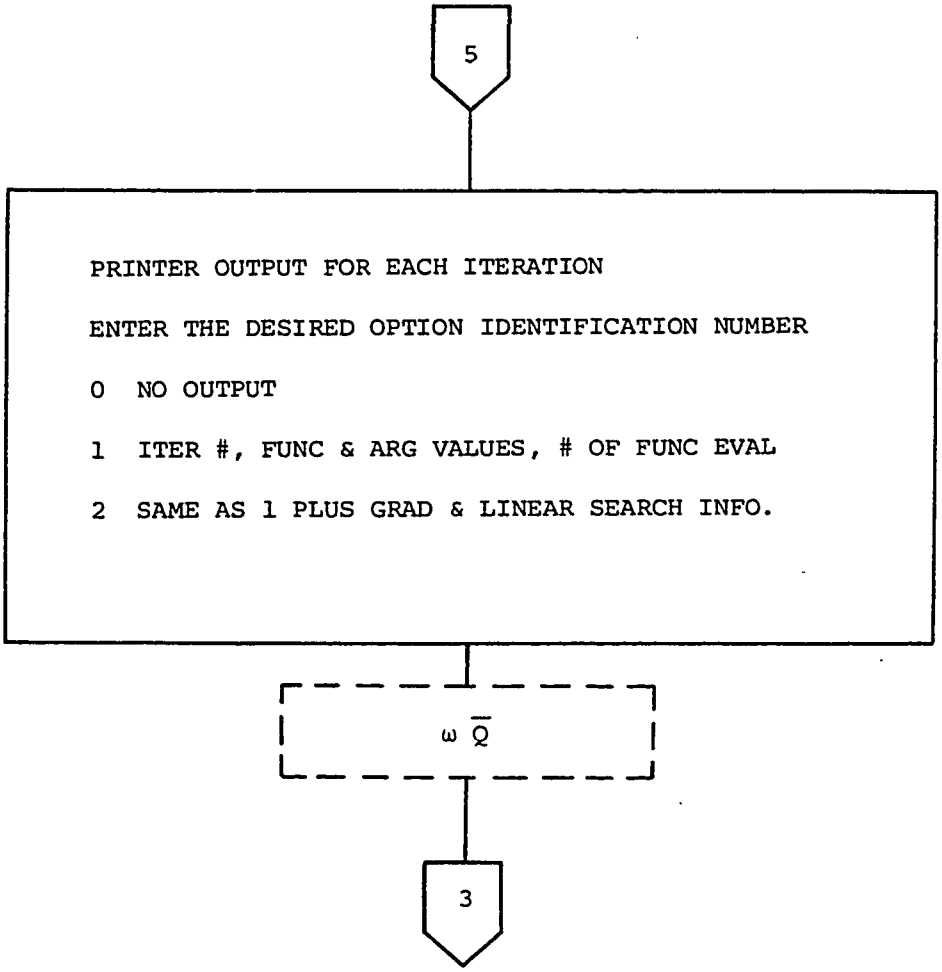

5
PRINTER OUTPUT FOR EACH ITERATION
ENTER THE DESIRED OPTION IDENTIFICATION NUMBER
0 NO OUTPUT
1 ITER #, FUNC & ARG VALUES, # OF FUNC EVAL
2 SAME AS 1 PLUS GRAD & LINEAR SEARCH INFO.
ω Q̄
3

Fig. 2.5

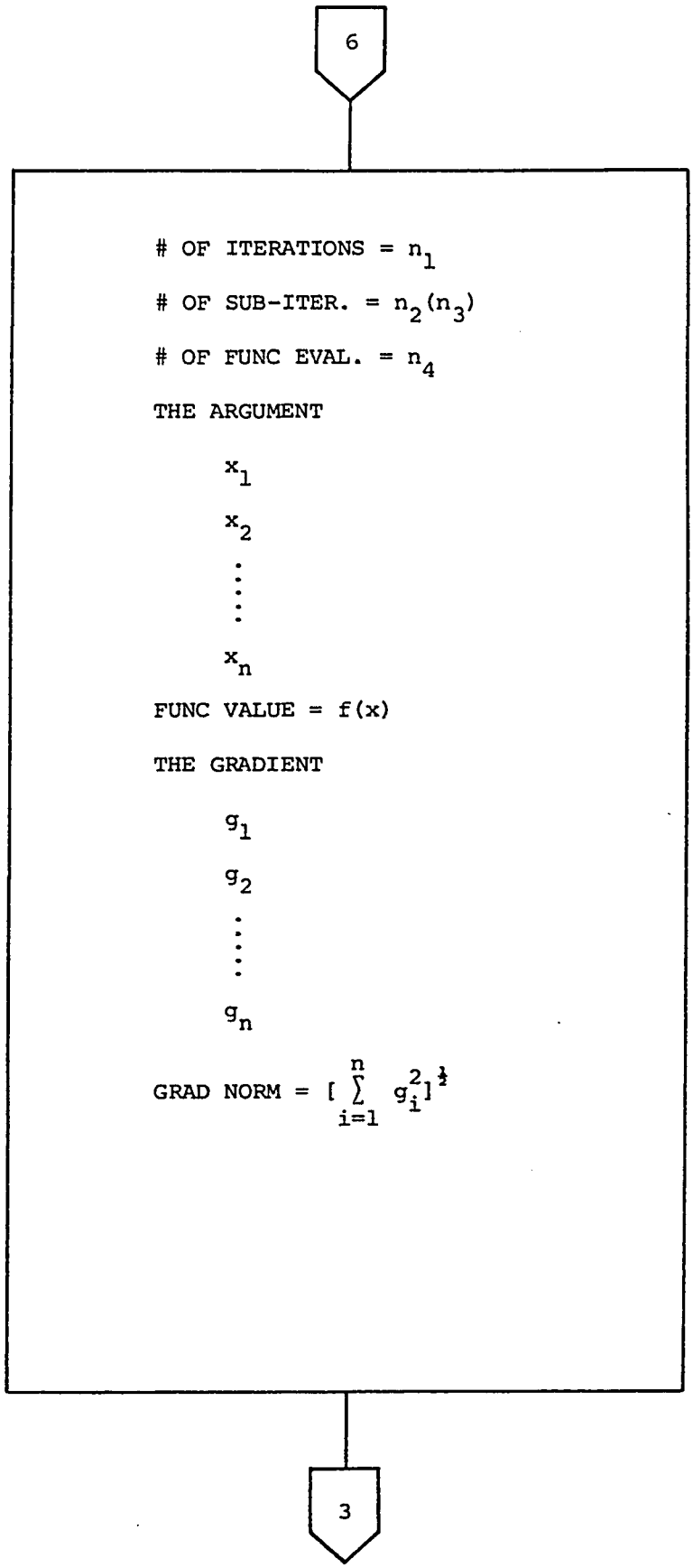

6
OF ITERATIONS = n_1
OF SUB-ITER. = $n_2(n_3)$
OF FUNC EVAL. = n_4
THE ARGUMENT
x_1
x_2
x_n
FUNC VALUE = f(x)
THE GRADIENT
g_1
g_2
g_n
GRAD NORM = $[\sum_{i=1}^{n} g_i^2]^{\frac{1}{2}}$
3

Fig. 2.6

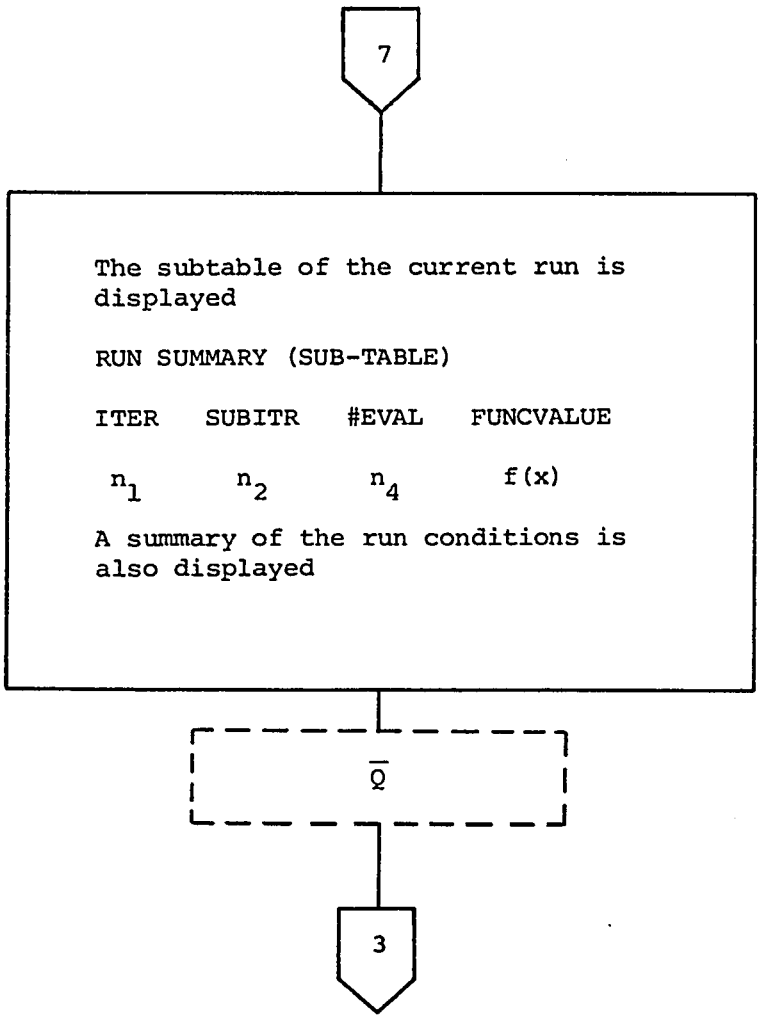

7
The subtable of the current run is displayed
RUN SUMMARY (SUB-TABLE)
ITER SUBITR #EVAL FUNCVALUE
n_1 n_2 n_4 f(x)
A summary of the run conditions is also displayed
Q
3

Fig. 2.7

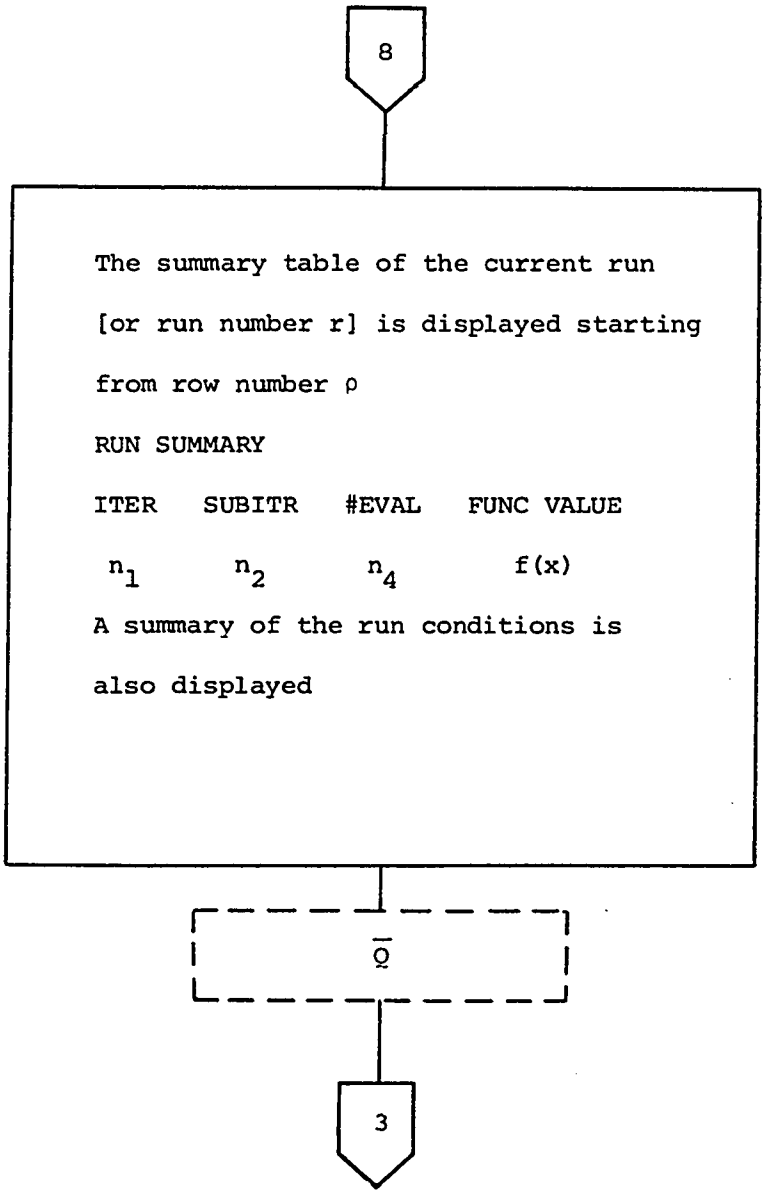

8
The summary table of the current run
[or run number r] is displayed starting
from row number ρ
RUN SUMMARY
ITER SUBITR #EVAL FUNC VALUE
n_1 n_2 n_4 f(x)
A summary of the run conditions is
also displayed
Q
3

Fig. 2.8

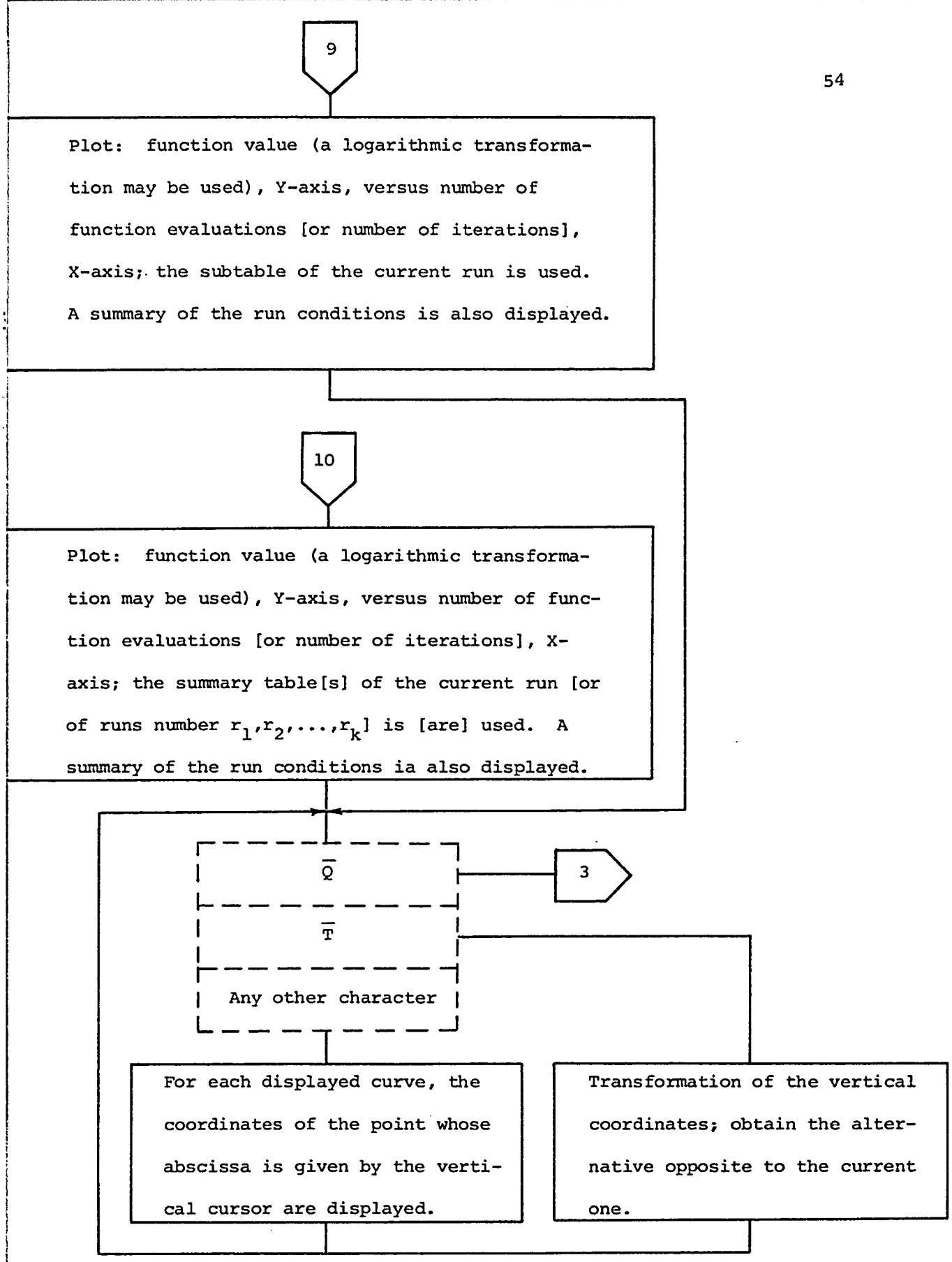

9
Plot: function value (a logarithmic transformation may be used), Y-axis, versus number of function evaluations [or number of iterations], X-axis; the subtable of the current run is used. A summary of the run conditions is also displayed.
10
Plot: function value (a logarithmic transformation may be used), Y-axis, versus number of function evaluations [or number of iterations], X-axis; the summary table[s] of the current run [or of runs number $r_1, r_2, \ldots, r_k$] is [are] used. A summary of the run conditions ia also displayed.
$\overline{Q}$
3
$\overline{T}$
Any other character
For each displayed curve, the coordinates of the point whose abscissa is given by the vertical cursor are displayed.
Transformation of the vertical coordinates; obtain the alternative opposite to the current one.

Fig. 2.9

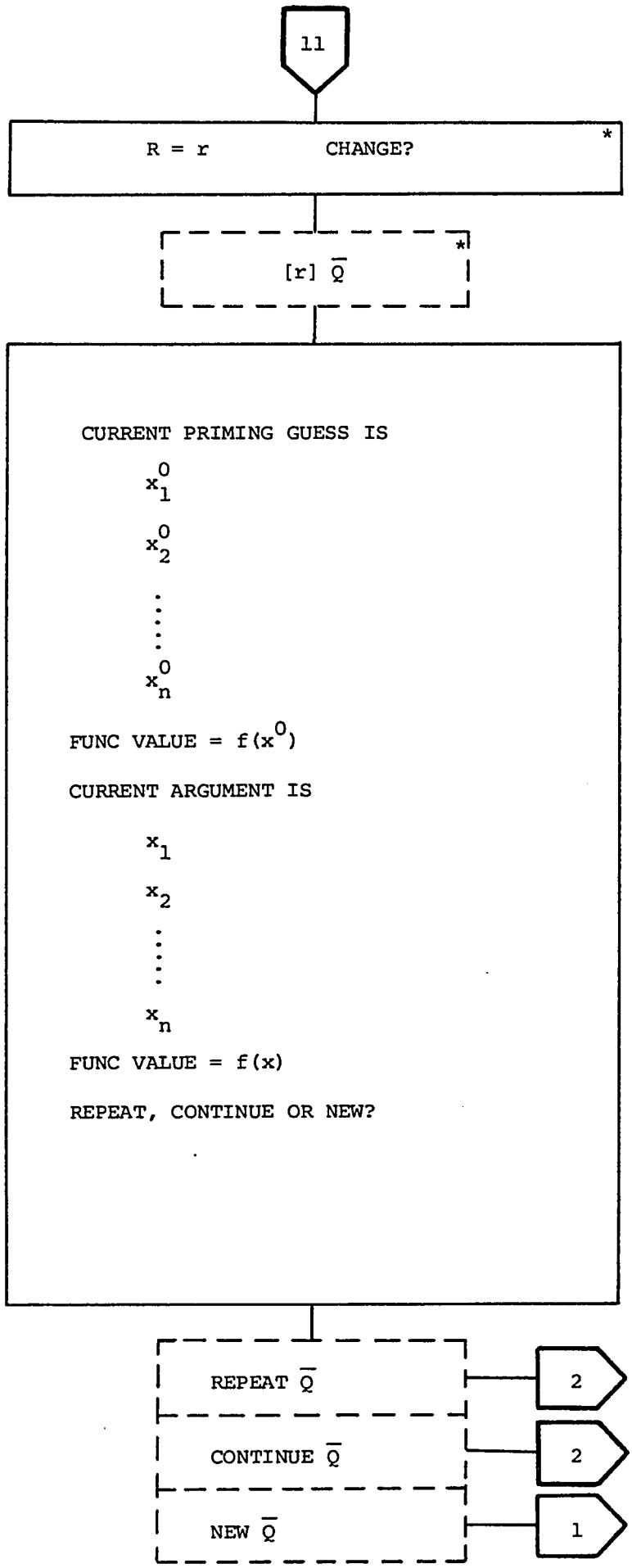

11
R = r CHANGE? *
[r] Q̄ *
CURRENT PRIMING GUESS IS
x_1^0
x_2^0
x_n^0
FUNC VALUE = $f(x^0)$
CURRENT ARGUMENT IS
x_1
x_2
x_n
FUNC VALUE = f(x)
REPEAT, CONTINUE OR NEW?
REPEAT Q̄
CONTINUE Q̄
NEW Q̄
2
2
1

Fig. 2.10

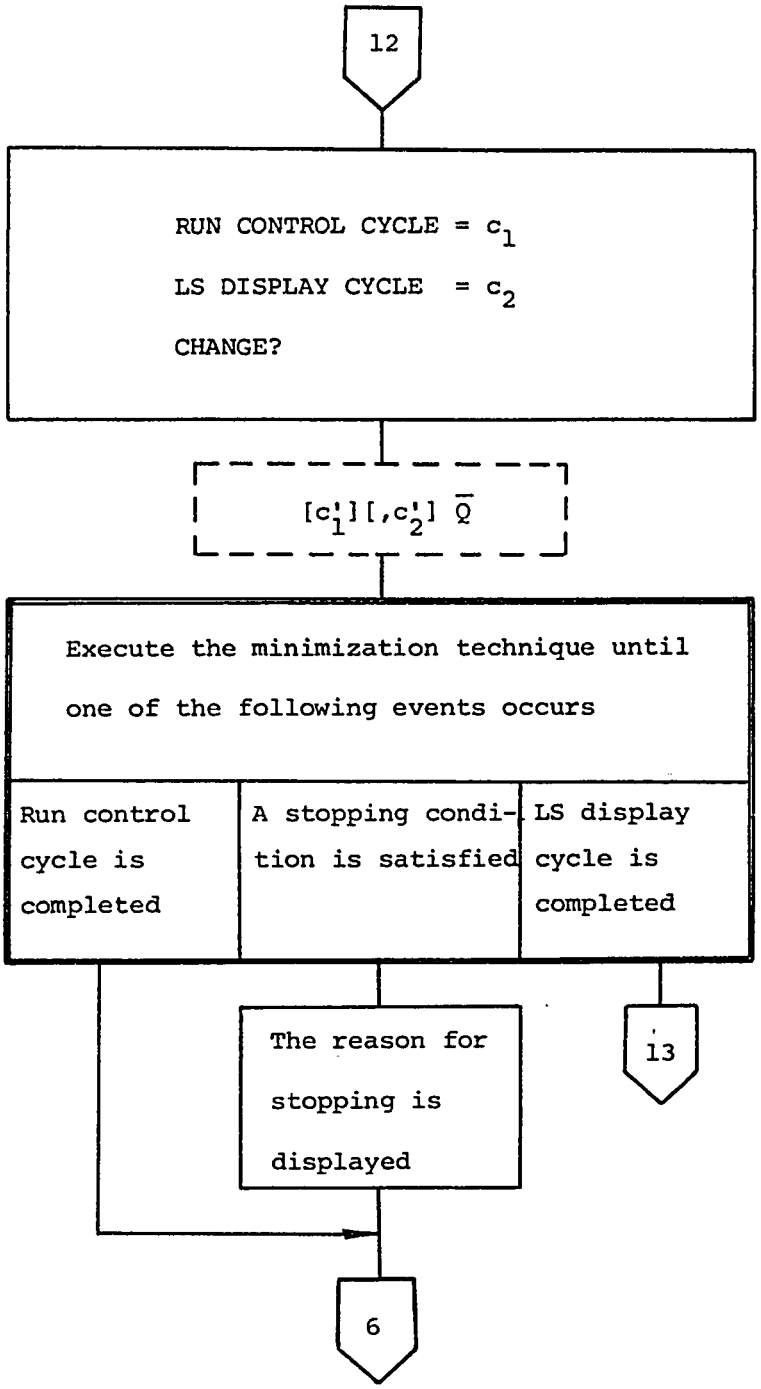

12
RUN CONTROL CYCLE = c_1
LS DISPLAY CYCLE = c_2
CHANGE?
[c_1'][,c_2'] $\bar{Q}$
Execute the minimization technique until one of the following events occurs
Run control cycle is completed
A stopping condition is satisfied
LS display cycle is completed
The reason for stopping is displayed
13
6

Fig. 2.11

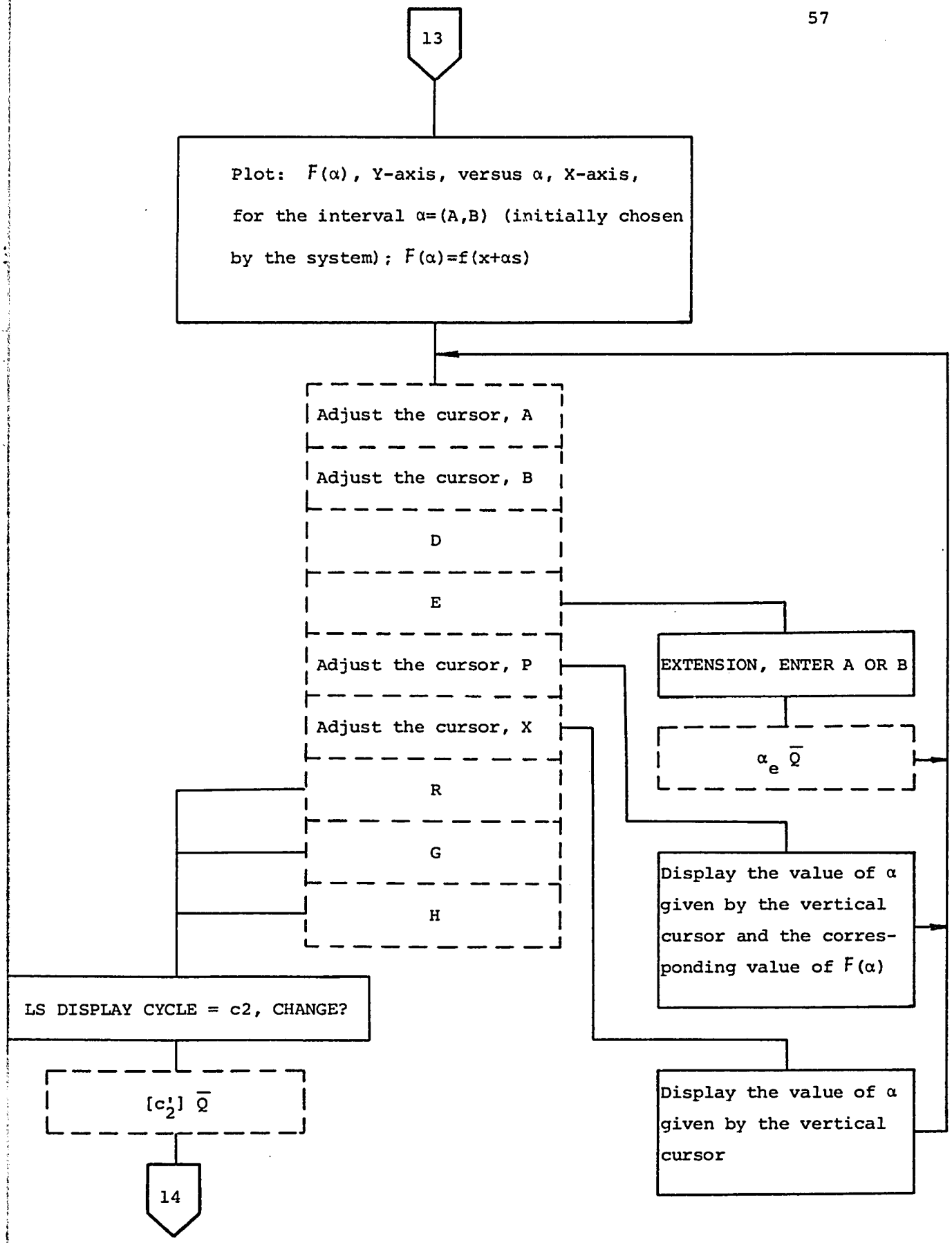

13
Plot: F(α), Y-axis, versus α, X-axis, for the interval α=(A,B) (initially chosen by the system); F(α)=f(x+αs)
Adjust the cursor, A
Adjust the cursor, B
D
E
Adjust the cursor, P
Adjust the cursor, X
R
G
H
EXTENSION, ENTER A OR B
α_e Q
Display the value of α given by the vertical cursor and the corresponding value of F(α)
Display the value of α given by the vertical cursor
LS DISPLAY CYCLE = c2, CHANGE?
[c_2'] Q
14

Fig. 2.12

CHAPTER 3

INTEROPT ORGANIZATION AND SUBROUTINES

3.1 General Structure and Flow-Charts

The INTEROPT program can be separated into the following four segments or phases:

1. The Interactive Phase
2. The Minimization Phase
3. The Single Iteration Execution Phase
4. The Linear Search Phase

Each of these phases is supervised by a "principal" subroutine: namely INTACT, MONITR, MINMIZ and SCAN respectively. The structure of each of the four phases is described below and in the flow-charts of Fig. 3.1 through Fig. 3.6. In these flow-charts, the vertical flow on the left side of the figure (Fig. 3.3 is excepted) represents the sequence of events within the subroutine under consideration, while the horizontal flow on the right side of the figure represents subroutine calls where the name of the called subroutine is given in a box to the right of the calling subroutine. The subroutines associated with the four phases are described in the next section.

3.1.1 Interactive Phase

The subroutine INTACT is called by the main program when the INTEROPT package is started (Fig. 3.1). INTACT performs four main tasks (Fig. 3.2) as described below:

Task 1 (Set-up of a new run)

The priming guess for the run is obtained via a call to NEWRUN and the run conditions are established via a call to CONCHN. Then, INTACT proceeds to Task 2.

Task 2 (Supervision of the control mode)

INTACT accepts user commands within the control mode and then calls an appropriate subroutine (Fig. 3.3) as summarized below:

User command	Subroutine called
CONDITION	CONCHN
INITIALIZATION	INTCHN
OUTPUT ...	OUTCHN
ITERATION	DISITR
TABLE ...	DISTAB
PLOT ...	FUNPLT

For each of these commands, the system returns to the control mode after the processing of the command is completed. Each of the remaining commands (NEW RUN, RESTART, GO and END JOB) directs INTACT to one of its four tasks as summarized below:

User command	Task
NEW RUN	1
RESTART or GO	3
END JOB	4

Task 3 (Starting the minimization phase)

INTACT first calls RUN to handle any output documentation and

information storage required prior to the minimization phase and to establish the values of the parameters for the linear search and control cycles (see the description of RUN, section 3.3.2( ). It then calls MONITR. When a return from MONITR is made, INTACT returns to Task 2.

Task 4 (Termination of the job)

By way of terminating the job, INTACT first calls TABLE to print the summary table of the last run and then calls SMRY to print the job summary. A return to the main program is then made, which results in the termination of the job.

3.1.2 Minimization Phase

The minimization phase is concerned with the execution of the selected minimization algorithm; hence it is the main computational part of INTEROPT. The subroutine MONITR is called from INTACT to execute a series of iterations. This series is terminated either if one of the stopping conditions is satisfied or a control cycle is completed. MONITR, as shown in Fig. 3.4, proceeds as follows:

(a) When the current iteration is the first one in a new run, the minimization algorithm is initialized, the documentation describing the run is printed, and the information to be retained for the summary table is stored via a call to the subroutine ACCESS.

(b) Prior to each iteration, MONITR performs tests to determine

if:

(i) A stopping condition is satisfied or the control cycle is completed. In this circumstance the present status is displayed via the subroutine DISITR. In the case of satisfying a stopping condition, the reason for stopping is also displayed (and may be printed if the user has requested detailed output from each iteration) via the subroutine MSSAGE.

(ii) A restart is required. Such a requirement could arise from several conditions: namely, a request for a restart has been entered by the user, a restart is necessary due to a change in MINTEC, an automatic restart is necessary because of the current value of RESTRT, or a restart is necessary due to a failure in the previous iteration. The reason for the restart is printed (via MSSAGE) together with the initialization data (via ACCESS) if detailed documentation has been requested.

In case of (i) MONITR returns to INTACT (the control mode), otherwise MONITR proceeds to step (c) below.

(c) The subroutine MINMIZ is called to execute one iteration. Then ACCESS is called to store data for the summary table and, if it has been requested by the user, ACCESS prints data from the iteration. The procedure then returns to step (b) above.

3.1.3 The Single Iteration Execution Phase

The subroutine MINMIZ (Fig. 3.5) proceeds as follows:

(a) If the minimization technique being used is gradient dependent, one of the following subroutines is called according to the value of the parameter MINTEC:

YOURSA	for the user-provided technique
CONJ	for the conjugate gradient methods (Fletcher-Reeves, Polack-Ribiere and Sorenson)
DFP	for the Davidon-Fletcher-Powell method

Each of these subroutines utilizes the gradient vector to generate a search direction (of unit length) along which MINMIZ performs a linear search via a call to SCAN. If the method being used is the steepest descent method, then the search direction is generated directly within MINMIZ by simply normalizing the available negative gradient vector.

(b) If the minimization technique being used is gradient independent, one of the following subroutines is called according to the value of the parameter MINTEC:

YOURSB	for the user-provided technique
POWELL	for both Powell's method and the extended sequential search method
ZANG	for Zangwill's method

The linear searches required by these algorithms are executed via calls to the subroutine SCAN.

3.1.4 The Linear Search Phase

The subroutine SCAN is called each time a linear search along

a given direction is required. It proceeds as follows (Fig. 3.6):

A call to the subroutine I$IOU is made. If the linear search being initiated is not the last one in a linear search display cycle, I$IOU determines an initial interval of uncertainty. This interval is passed to the selected linear search subroutine (YOURSL, QUADFT, FBNACI, GOLDEN) which establishes an approximation to the minimizing argument within the interval. The subroutine selection is controlled by the parameter LN$SRH and the basis for the respective algorithms is summarized below:

YOURSL	for the arbitrary user-provided algorithm
QUADFT	for the sequential quadratic fitting method
FBNACI	for the Fibonacci search
GOLDEN	for the Golden section search

Alternately if the linear search being initiated is the last one in the linear search display cycle, then the call to I$IOU initiates the interactive linear search procedure via a call to LS$DIS. Depending on the user's responses the initial interval of uncertainty or the minimizing argument itself, may be interactively generated. In the former situation, this initial interval of uncertainty is returned to SCAN which then calls the appropriate linear search subroutine for further processing. In the latter case, the task of SCAN is complete and a return is made.

3.2 Subroutines Associated with the Four INTEROPT Phases

As noted in the previous section, the principal subroutines perform their respective tasks via calls to secondary subroutines.

These second level subroutines and the subroutines associated with them in turn, are described briefly in this section.

3.2.1 GETN

This subroutine is used to determine the dimension, n, of the vector argument x of the submitted function F. It is called only once by the subroutine ARGMNT. The priming guess is initially assumed to contain 8 components (the maximum allowed). Each of these is perturbed in turn and the effect of the perturbation on the function value is checked. The number of the components which affect the function value is taken as the required n.

3.2.2 ARGDIS

This subroutine is called each time a display of one of the following is required:

1. The current priming guess vector (the last guess explicitly specified by the user) and the corresponding function value.
2. The current estimate of the minimizing argument and the corresponding function value.

3.2.3 ARGMNT

This subroutine is called each time a priming guess is to be explicitly specified by the user. When called for the first time, it in turn, calls GETN to determine n. It accepts the vector

entered by the user, checks the number of components in it against n, and stores it as both the current priming guess and the current estimate of the minimizing argument. The corresponding function value is evaluated and both the priming guess vector and the function value are displayed via ARGDIS. Finally, ARGMNT enquires from the user whether he wishes to respecify the priming guess.

3.2.4 NEWRUN

This subroutine is called from INTACT each time a new run is requested. It resets to their respective default values, the initialization & stopping parameters and the parameter controlling the output documentation from each iteration. Then, if the run being initiated is the first run in the INTEROPT job, it calls ARGMNT which accepts the user priming guess. Otherwise, it calls ARGDIS twice, first to display the current priming guess, then to display the current estimate of the minimizing argument. According to the user response (REPEAT, CONTINUE, or NEW) it either assigns the value of the priming guess to the new estimate of the minimizing or keeps the present estimate of the minimizing argument, or calls ARGMNT to obtain from the user a new priming guess, respectively.

3.2.5 CONDIS

This subroutine displays the current run conditions; i.e. the current values of MINTEC, G$PAR (if applicable), LN$SRH and LN$PAR.

3.2.6 EXAM

The user response for changing the run conditions (m_1/a_1, m_2/a_2) is handled in two parts (i.e. m_1/a_1 and m_2/a_2). EXAM handles one part of this response at a time. It substitutes a numeric value for the nmemonic m_j and determines the value of the associated parameter according to the value of a_j.

3.2.7 CONCHN

This subroutine displays the current values of the run conditions via a call to CONDIS, handles the user-specified changes via two calls to EXAM and displays the run conditions after modification via another call to CONDIS.

3.2.8 INTDIS

This subroutine displays the initialization and stopping parameters in effect, i.e. the current values of RESTRT, FAIL, MXITER, MXFE and E$PAR.

3.2.9 INTCHN

This subroutine is called when it is required to check and/or change the values of the initialization and stopping parameters. It calls INTDIS to display the current values of these parameters. Then, it accepts the user changes (one change at a time), examines the entered change and modifies the corresponding parameters.

Finally, it calls INTDIS to display the parameters after modification.

3.2.10 OUTCHN

This subroutine accepts the user choice of the available options for the output documentation (on the line printer) from each iteration. Unless the user enters the option number explicitly; e.g. OUTPUT, 0, OUTCHN displays a summary for the available options and, then, accepts the user's choice.

3.2.11 DISITR

This subroutine is used to display status information about the current iteration in the following cases:

(a) a stopping condition is satisfied

(b) a run control cycle is completed

(c) the user requests information via the command ITERATION

DISITR displays the number of iterations (and possibly the number of subiterations) and the number of function evaluations so far. Then it calls ARGDIS to display the current values of the argument and the criterion function. Also, it displays the gradient and the gradient norm if relevant.

3.2.12 CDIS

This subroutine displays the summary of the run conditions required each time a summary table or a function plot is displayed.

3.2.13 TABDIS

This subroutine is used to display the summary table for a given run or the sub-table of the current run. Then it calls CDIS to display the run conditions.

3.2.14 DISTAB

This subroutine examines the syntax of the commands:

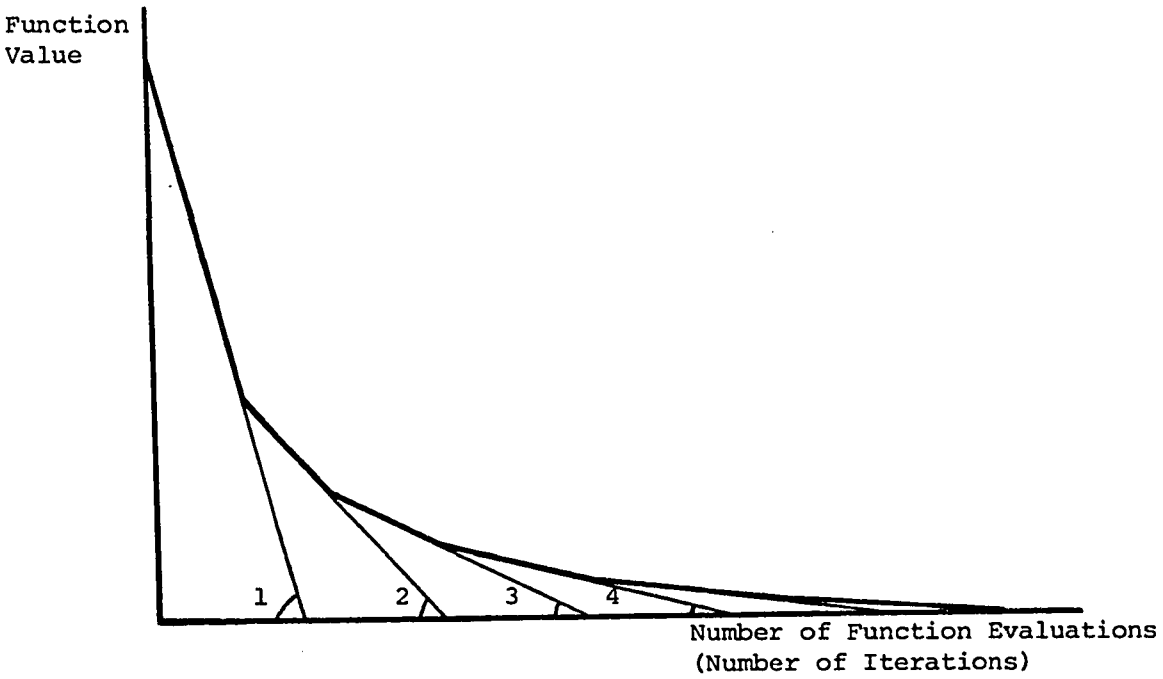
TABLE, SUB and TABLE[/ρ][,n]

It then displays the appropriate table via a call to TABDIS

3.2.15 TRANS

This subroutine determines whether a logarithmic transformation is required when five or more function values are to be plotted. The following test is performed:

The curve to be plotted has different slopes within the different segments (see below). The maximum slope is found and if more than 75% of the slopes have a value which is less than 10% of the maximum slope, then logarithmic transformation is made. This corresponds to a situation where the function value drops significantly during a few iterations but very little change occurs in most of the other iterations.



3.2.16 PREP

This subroutine applies a logarithmic transformation (base 10) to the function values where required. If the function has a negative value at some point, the most negative value attained by the function is used to bias the other values.

3.2.17 PLTMSG

This subroutine displays the heading for a function plot.

3.2.18 PLOT

This subroutine plots function value against either the number

of function evaluations or the number of iterations. It also displays the associated run conditions via a call to CDIS.

3.2.19 SRCH

This subroutine responds to a user request for co-ordinate data from the displayed curves as specified by the current vertical cursor position (section 2.7.1). If the displayed curve(s) is (are) plotted against number of iterations. Then a search is made of the stored data points to locate the one for which the number of iterations most closely corresponds to the vertical cursor position. The co-ordinates of this point (number of iterations and function value) are then displayed. An analogous operation takes place if the absicca of the plotted curves is number of function evaluations.

3.2.20 PLT

This subroutine is used to generate a plot of function value against either number of iterations or number of function evaluations, using the summary table(s) of the current run and/or of previous runs. As described in section 2.7, the user may request one function plot from a particular run as well as a group of function plots from different runs. In both cases the following steps are executed:

(a) TRANS is called once for each specified run to determine

whether a transformation is necessary for the plot for this run. If any plot requires a transformation then it is applied to all plots.

(b) PREP is called once for each plot to obtain the transformed function values if they are required.

(c) PLTMSG is called once to display the heading for the whole graph.

(d) PLOT is called as many times as required to display the various plots.

(e) The cursor mode is then entered (section 2.7.1) and according to the character entered by the user one of the following actions is taken:

(i) $\overline{Q}$ - a return is made to the calling subroutine (FUNPLT)

(ii) $\overline{T}$ - if the logarithmic transformation has been used for the current display a new display without the transformation is given and vice versa.

(iii) any other character - SRCH is called for each displayed curve to display the co-ordinates of the points on each curve which correspond to the vertical cursor position.

3.2.21 PLTSUB

This subroutine is similar to PLT but it is used for a function

plot produced from the sub-table data of the current run.

3.2.22 FUNPLT

This subroutine examines the syntax of the two commands

PLOT[/ITER],[,r_1, r_2, ..., r_k]

and PLOT[/ITER], SUB.

For the first command it calls PLT and passes all the desired options whereas for the second command it calls PLTSUB.

3.2.23 CHANGE

This subroutine prints out all the changes in the run conditions and the initialization and stopping parameters which are specified by the user in the course of the run. This is done only if the user requests a detailed printer output.

3.2.24 TABLE

This subroutine prints out the summary table of the current run when it is completed (i.e. when a new run is initiated).

3.2.25 TSTORE

The summary table of the current run is stored by this subroutine when it is completed in order to keep a history file of the most recent five runs. The tables of these runs are arranged such

that the new table replaces the "oldest" table in the list.

3.2.26 RUN

Each time the user enters either GO or RESTART, this subroutine is called to prepare for the execution of the minimization procedure and set up the run control and linear search display cycles. If a new run is being initiated, RUN initializes several pointers and calls TABLE and TSTORE to store the summary table of the previous run. Otherwise, it calls CHANGE, if required, to print out any changes that have been requested.

3.2.27 HEADER

This subroutine prints out the heading at the beginning of each run as well as the operating condition for the run.

3.2.28 UPDATE

This subroutine is used to update the stored summary table when the number of entries in this table is about to exceed the available storage for this table (119 entries). This is achieved essentially by deleting every second entry and then compacting the remaining entries.

3.2.29 ST$SUB

This subroutine stores one entry (corresponding to the results of one iteration) in the summary sub-table. Forty entries are allowed in the sub-table (only the most recent thirty rows are displayed). When this limit is exceeded, the first ten entries are deleted and the table is appropriately re-organized.

3.2.30 ACCESS

This subroutine is used to produce the appropriate print-out and store the information in the summary table. It calls HEADER at the beginning of each run. After each iteration, it prints out the results of this iteration (if required) and adds the proper entry in the summary table for this iteration. The subroutine UPDATE is called each time an update of the summary table is required. ACCESS also calls ST$SUB at the end of each iteration to create an entry for this iteration in the sub-table.

3.2.31 MSSAGE

Sometimes a message is displayed on the screen and/or on the line-printer after a certain iteration to inform the user that a stopping condition has been satisfied or the minimization algorithm is being initialized. This message is generated by the subroutine MSSAGE.

3.2.32 CONJ, DFP, POWELL and ZANG

These four subroutines contain the different minimization algorithms as described in Appendix A4.

3.2.33 I$IOU

The initial interval of uncertainty for a linear search is determined by this subroutine. This interval is passed to the selected linear search to locate the minimum value of the function within this interval. When a linear search display cycle is completed, I$IOU call LS$DIS (for the interactive linear search) and depending on the user decision (which is reflected on the type of return from LS$DIS) an appropriate action is taken.

3.2.34 QADMIN

This subroutine is used to fit a quadratic using three points. It then computes the minimum of the quadratic curve.

3.2.35 QUADFT, FBNACI & GOLDEN

The three linear search algorithms of INTEROPT are implemented within these three subroutines (see Appendix A2). QADMIN is invoked by each of these subroutines whenever a quadratic fit is required.

3.2.36 DRAW

This subroutine is used to plot the linear search display curve of the function $F(\alpha)$ (see section 2.12). Ten points, and hence ten function evaluations, are used.

3.2.37 LS$DIS

This subroutine invokes DRAW to plot the linear search display curve, places the Tektronix in the cursor mode and accepts a user command (one character) in this mode. It then takes the appropriate action according to the user command.

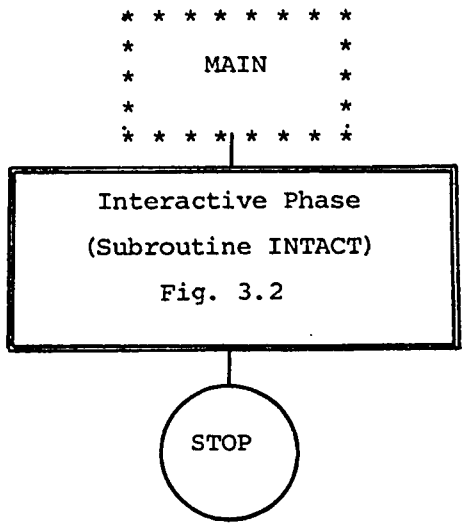


Fig. 3.1 Starting the Program

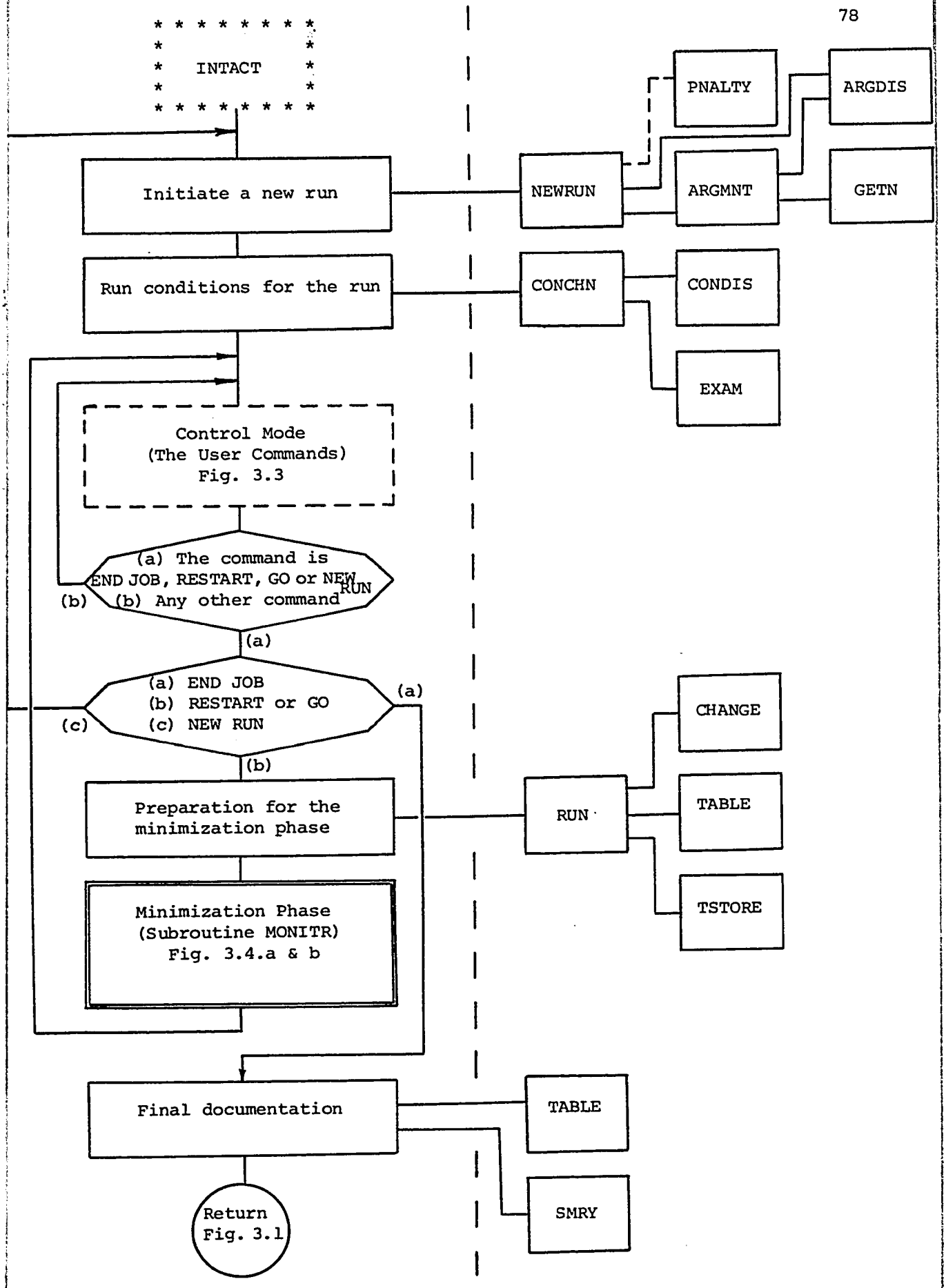


Fig. 3.2 Interactive Phase (Subroutine INTACT)

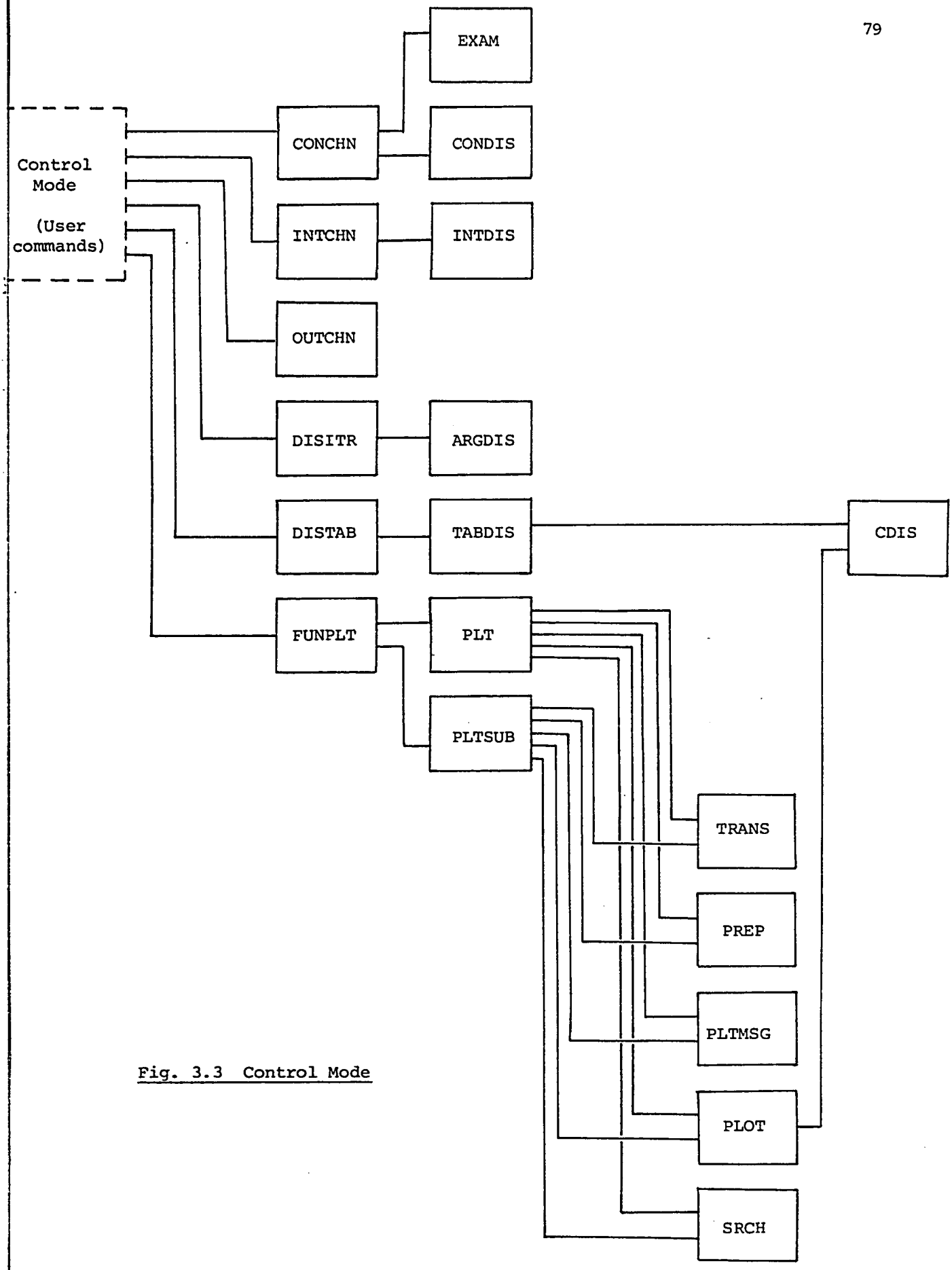


Fig. 3.3 Control Mode

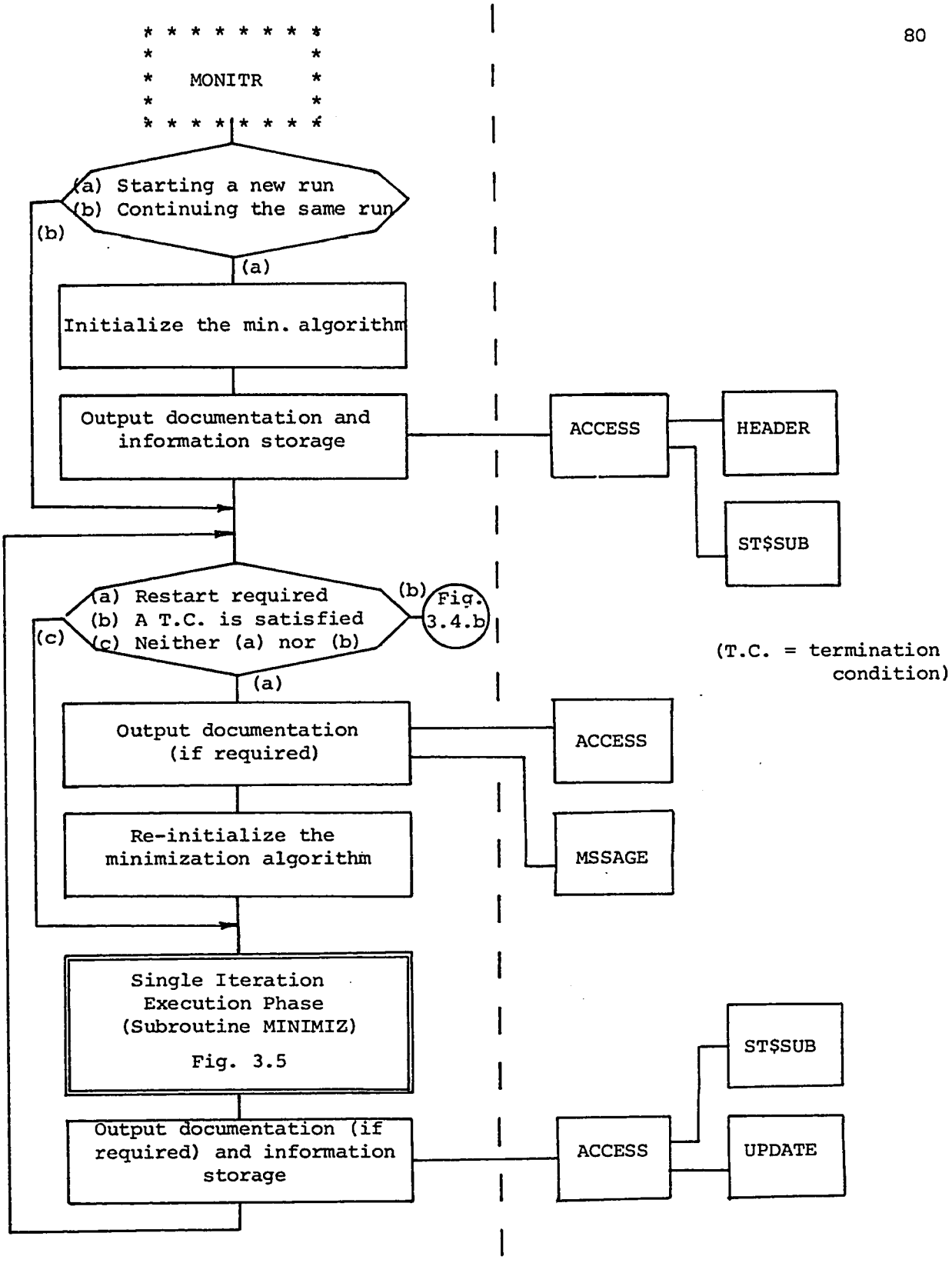


Fig. 3.4.a The Minimization Phase (Subroutine MONITR)

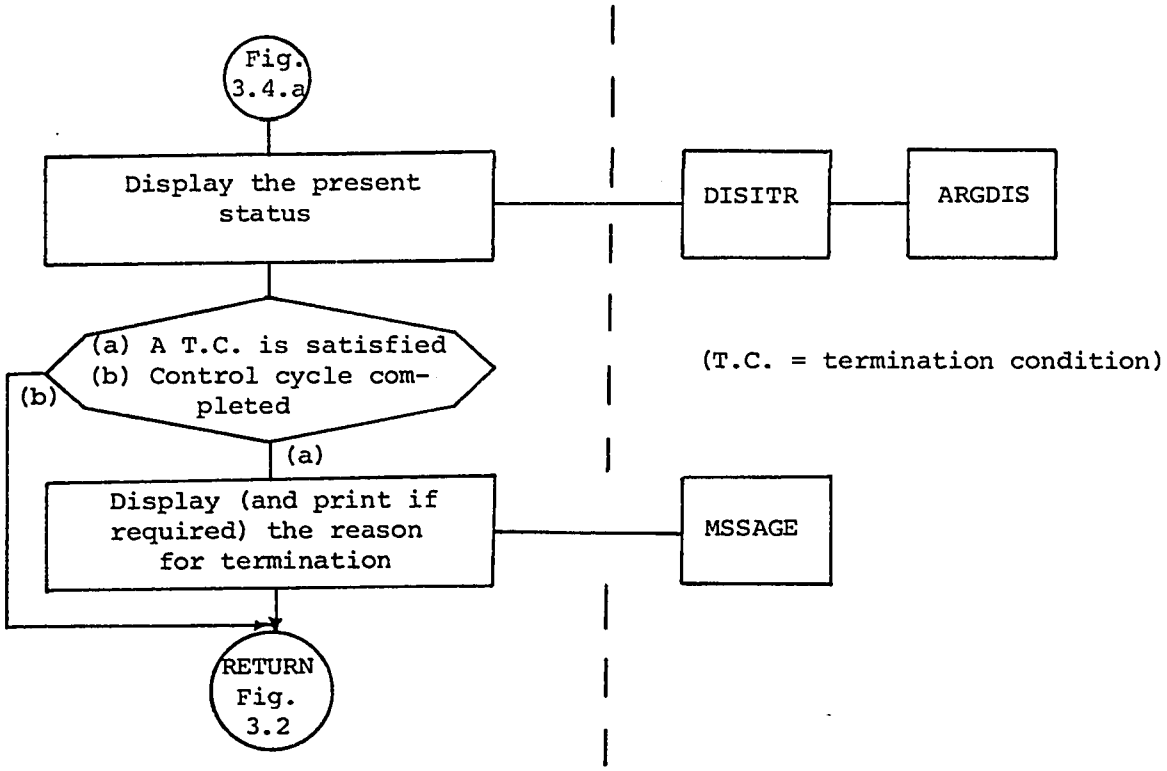


Fig. 3.4.a
Display the present status
DISITR
ARGDIS
(a) A T.C. is satisfied
(b) Control cycle completed
(b)
(a)
(T.C. = termination condition)
Display (and print if required) the reason for termination
MSSAGE
RETURN Fig. 3.2

Fig. 3.4.b

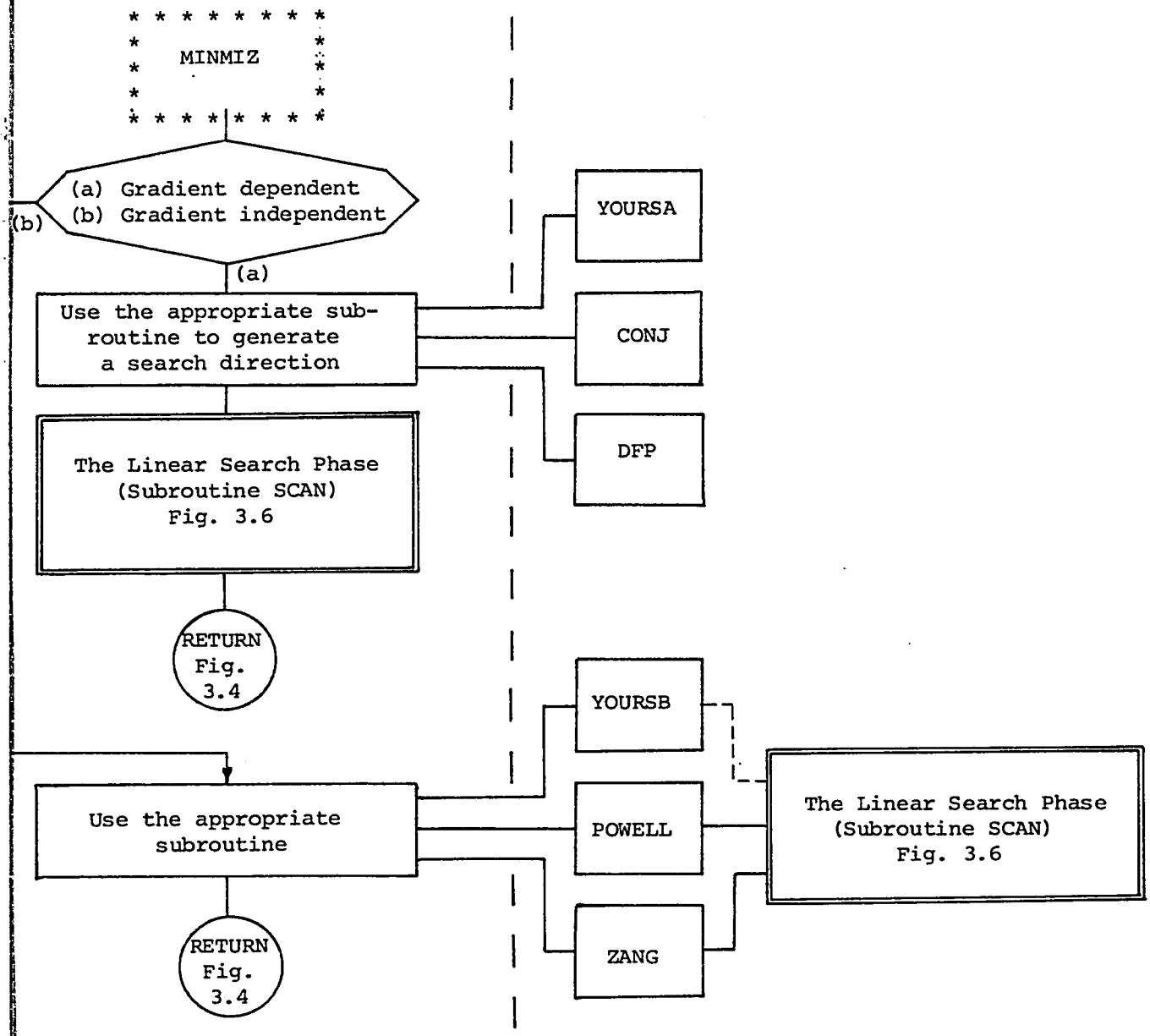


Fig. 3.5 The Single Iteration Execution Phase (Subroutine MINMIZ)

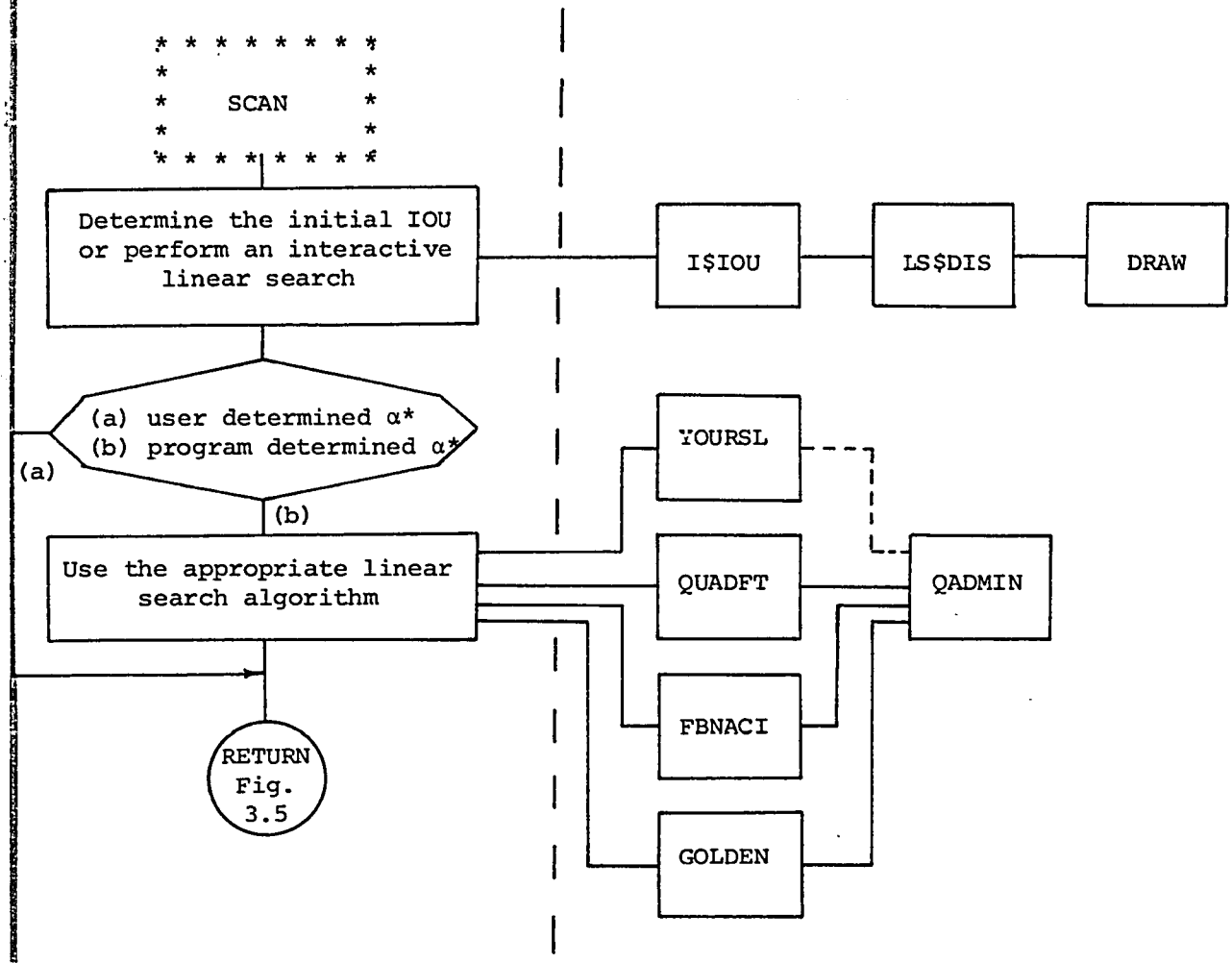


Fig. 3.6 The Linear Search Phase (Subroutine SCAN)

CHAPTER 4

EXAMPLES OF INTEROPT APPLICATIONS

4.1 Introduction

This chapter discusses the operating procedures and some applications of INTEROPT in the context of examples taken from some runs using standard test functions. These functions are described in Appendix A3. Unless stated otherwise, the standard priming guesses for these functions are used.

4.2 Examples to Illustrate the Operating Procedures of INTEROPT

Example 4.1

Several experiments with the Wood Function [Appendix A3.1] are used to illustrate some of the INTEROPT operating procedures. Fig. 4.1.1 through 4.1.53 show the dialog with INTEROPT and some comments on these figures is given in the following tabular summary. Note that the job submission included the GRAD subroutine.

Figure	Comments
	First Run
4.1.1	Priming guess specification The components are entered explicitly since this is the 1st run.
4.1.2	Run Conditions The values displayed are the default values (1st run) MINTEC is changed to F-R L$PAR is changed to 4.

Control Mode

The command GO is used to start the minimization process. The values of the LS display cycle and control cycle are not changed from their current default values.

4.1.3 Linear Search Display

This is the 5th linear search. G is entered, in the cursor mode, to permit the linear search to be executed without the user interaction. The LS display cycle is changed to 6.

4.1.4 Control Mode

INTEROPT returns to the control mode after completing a control cycle of 5 iterations. OUTPUT is entered to permit specification of an option for the printer output from each iteration

4.1.5 Printer Output for Each Iteration

The available options are displayed since no specification is given after OUTPUT in Fig. 4.1.4. Option 1 is selected.

Control Mode

GO is entered to continue the same run. The values of the LS display cycle and control cycle are not changed.

4.1.6 Control Mode

Another control cycle of 5 iterations is completed. The command PLOT is entered to obtain a plot of function value versus number of function evaluations.

4.1.7 Function Plot

$\overline{Q}$ is entered in the cursor mode to return to the control mode.

4.1.8 Control Mode

The command CONDITION is used to set the stage for subsequent changes in the run conditions.

4.1.9 Run Conditions

MINTEC is changed to D-F-P (this results in an automatic restart).

Control mode

GO is used to continue in the same run. The control cycle is changed to 10 iterations.

4.1.10 Linear Search Display

This is the 6th linear search since the previous display. G is used again, in the cursor mode (no user interaction). The LS display cycle is changed to 10.

4.1.11 Control Mode

A control cycle of 10 iterations has been completed. The command PLOT/ITER is used to obtain a plot of function value versus number of iterations.

4.1.12 Function Plot

The value of MINTEC is D-D-P (the last technique used in this run). The coordinates of a point on the curve are displayed by adjusting the x-cursor and entering any character. $\overline{Q}$ is used to return to the control mode.

4.1.13 Control

The command NEW RUN is used to initiate another run.

Second Run

4.1.14 Priming Guess Specification

REPEAT is specified, hence the run will start from the previous priming guess.

4.1.15 Run Conditions

$\overline{Q}$ is entered, i.e. no change in the run conditions.

Control Mode

OUTPUT is entered with option 2 explicitly specified.

INITIAL is used to modify the initialization and stopping parameters.

4.1.16 Initialization and Stopping Parameters

RESTRT is set to 10.

MXITER is set to 20.

Control Mode

GO is used to start the minimization phase. Both the control cycle and the LS display cycle are changed to 25.

4.1.17 Control Mode

The iteration limit (MXITER) stops the run after 20 iterations. A plot of the results for run 1 and 2 is requested. (function value versus number of iterations).

4.1.18 Function Plot

Notice the beneficial effect of using two different minimization techniques during the first run. $\overline{Q}$ is used to return to the control mode.

4.1.19 Control Mode

A display of the summary table of the first run is requested.

4.1.20 Summary Table

Notice that the run was initialized after the 10th iteration due to the change in the value of MINTEC. $\overline{Q}$ is used to return to the control mode.

4.1.21 Control Mode

NEW RUN is entered to initiate a third run.

Third run

4.1.22 Priming Guess Specification

REPEAT is specified.

4.1.23 Run Conditions

MINTEC is changed to ZANG.

LN$SRH is changed to FBNACI.

Control Mode

GO is entered and the LS display cycle is changed to 50.

4.1.24 Control Mode

5 iterations (the default value) have been completed and hence the current status is displayed on the screen.

OUTPUT is entered with option 1 specified. CONDITION is entered.

4.1.25 Run Conditions

MINTEC is changed to POWELL.

Control Mode

GO is entered and no change in either the control or the LS display cycles is made.

4.1.26 Linear Search Display

The 50th linear search since starting the run is occurring. G is entered and no change is made in the LS display cycle.

4.1.27 Control Mode

5 more iterations have been completed. OUTPUT is entered with option 0 specified (line printer output is surpressed for subsequent iterations). CONDITION is entered.

4.1.28 Run Conditions

LN$SRH is changed to QUADFT.

Control Mode

GO is entered and the run control cycle is set to 10.

4.1.29 Linear Search Display

The 100th linear search starting from the beginning of the run is occurring. G is entered in the cursor mode. No change in the display cycle is requested.

4.1.30 Control Mode

The requested control cycle (i.e., 10) has been completed. The summary table for the current run is requested.

4.1.31 Summary Table

Notice the initialization after the 5th iteration due to the change in MINTEC. $\overline{Q}$ is used to return to the control mode.

4.1.32 Control Mode

A function plot against number of iterations for the three runs is requested.

4.1.33 Function Plot

$\overline{Q}$ is used to return to the control mode.

4.1.34 Control Mode

A function plot against number of function evaluations for the second and third runs is requested.

4.1.35 Function Plot

The X-cursor is positioned and the co-ordinates at the requested point are displayed. Notice the difference in the number of function evaluations between the two runs. $\overline{Q}$ is used to return to the control mode.

4.1.36 Control Mode

A new run is requested.

Fourth Run

4.1.37 Priming Guess Specification

REPEAT is specified to initiate the run from the previous priming guess.

4.1.38 Run Conditions

MINTEC is changed to GRAD and G$PAR is set to 9.

Control Mode

INITIAL is used to enable a change in the initialization and stopping parameters.

4.1.39 Initialization and Stopping Parameters

The values displayed are the default values for the case where the analytic gradient is being used. MXFE is set to 1000 evaluations.

Control Mode

GO is specified. Run control and LS display cycles are set to 100 and 400 respectively.

4.1.40 Control Mode

The execution stops after 100 iterations. GO is specified again to continue without change.

4.1.41 Control Mode

The run stops due to the function evaluation limit. INITIAL is specified to change this limit.

4.1.42 Initialization and Stopping Parameters

MXFE is reset to its default value (NO LIMIT) and MXITER is set to 200 iterations.

Control Mode

GO is used to continue. Run control and LS display cycles are not changed.

4.1.43 Control Mode

The run stops after the 200 iterations specified by MXITER are completed. Function plot versus number

of iterations is requested.

4.1.44 Function Plot

The plot is for the whole run (200 iterations). $\overline{Q}$ is used to return to the control mode.

4.1.45 Control Mode

A plot for the last 30 iterations is requested.

4.1.46 Function Plot

(SUB-TABLE) indicates that the data for this plot is obtained from the sub-table. Note that the log scale is not used in this case. $\overline{Q}$ is used to return to the control mode. Note also the obvious linear convergence rate.

4.1.47 Control Mode

A display of the summary table is requested starting from the 70th row.

4.1.48 Table Display

30 runs of the table are displayed starting from row number 70. Notice the effect of the table updating. $\overline{Q}$ is used to return to the control mode.

4.1.49 Control Mode

A sub-table display is requested.

4.1.50 Table Display

The sub-table is displayed (all of the last 30 iterations). $\overline{Q}$ is used to return to the control mode.

4.1.51 Control Mode

A function plot against number of function evaluations is requested for runs 3 and 4.

4.1.52 Function Plot

$\overline{Q}$ is used to return to the control mode.

4.1.53 Control Mode

"END JOB" is used to terminate the job.

4.3 Comparative Studies

The plotting features provided in INTEROPT make it especially straight-forward to conduct experiments for testing and comparing either of the following:

(a) the different minimization technique and the different linear searches (note that this applies equally to user-provided algorithms)

(b) the sensitivity of any of the techniques to the auxiliary parameters: e.g. RESTRT, G$PAR, and LS$PAR.

The results of such experiments are often of great value and several studies of this type have appeared in the literature, (e.g. [10]). Some experiments of this type are given in examples 4.3(a) and 4.3(b) to illustrate this capability.

Example 4.2(a)

In this example the Powell Function [Appendix A3.2] is used. The job submission included the subroutine GRAD. Fig. 4.2.1 shows the priming guess. Fig. 4.2.2 through Fig. 4.2.4 give the results for 11 runs (in four groups), first as plots of function value against number of function evaluations and then as plots of function value against number of iterations.

The following summary provides some comments on these plots.

Figure	Comments
4.2.2	Run numbers 1, 2 and 3 - Comparison of linear searches: Each run uses one of the 3 available linear searches. The default values of all other run conditions are used (MINTEC=D-F-P, G$PAR=AG, LS$PAR=2). 30 iterations are allowed.
4.2.3	Run numbers 4, 5 and 6 - Comparison of minimization techniques: Each run uses a different MINTEC value (D-F-P, F-R and SOREN respectively). FBNACI is used and G$PAR and LS$PAR are assigned their default values (AG and 2 respectively). 30 iterations are allowed.
4.2.4	Run numbers 7 and 8 - Experiment with RESTRT: RESTRT is assigned the values 0 and 5 respectively in runs 7 and 8. F-R is used and all other run conditions are given default values. 800 function evaluations were allowed.
4.2.5	Run numbers 9, 10 and 11 - Experiment with LS$PAR: LS$PAR is assigned the values 2, 3 and 4 respectively for the runs 9, 10 and 11. POWELL is used and all other run conditions are assigned their default values. 1000 function evaluations are allowed.

Example 4.2(b)

In this example the Multi-dimensional Banana Function [Appendix A3.3] is used. The job submission included the subroutine

GRAD. Fig. 4.2.6 gives the priming guess at the beginning of the job. Fig. 4.2.7 and Fig. 4.2.8 give the results of 7 runs (in two groups) as function plots (as in example 4.2(a). A summary of these figures is given below.

Figure	Comments
4.2.7	Run numbers 1, 2, 3, and 4 - Experiment with G$PAR: G$PAR is assigned the values AG, 7, 9 and 11 respectively in these four runs. D-F-P is used with default values assigned to all other parameters. 40 iterations were allowed*.
4.2.8	Run numbers 5, 6 and 7 - Experiment with LS$PAR: LS$PAR is assigned the values 2, 3 and 4 respectively, in these three runs. SOREN is used with default values assigned to all other parameters. 500 function evaluations are allowed.

* Run number 2 was actually terminated due to successive failures in iterations 38 and 39 (i.e. iteration number 39 was an unsuccessful iteration after an initialization).

4.4 Interactive Linear Search

As outlined in section 2.12, INTEROPT permits the user to participate in (i.e. interact with) the solution of the linear search problem. The various features in this respect are demonstrated in the context of examples 4.3(a) and 4.3(b) which are described below. Example 4.3(a) corresponds to one run, while example 4.3(b) includes two distinct runs.

Example 4.3(a)

In this example, the Wood Function is used and the job submission included the subroutine GRAD. The following run conditions were used: MINTEC = D-F-P, G$PAR = AG, LIN$SRH = QUADFT, LS$PAR = 2.

Figure	Comments
4.3.1	The first linear search in the run and as noted, the IIOU is (0,2). The character 'E' is entered and the system responds with the message 'EXTENSION, ENTER A OR B'. Then the value 9 is entered indicating that the new interval to be considered is (0,9). The character 'D' is entered to draw the function over the new interval.
4.3.2	Two points are displayed by adjusting the vertical cursor and entering the character 'P' in each case. The new interval to be considered is passed to the system by moving the vertical cursor to the previous two points and entering the characters 'A' and 'B' for the left and the right ends of the interval respectively. A request for a display of the function within the new interval is

given by entering the character 'D'.

4.3.3 The same procedures as in fig. 4.3.2.

4.3.4 The same procedures as in fig. 4.3.2.

4.3.5 In this figure, it can be noted that the function has a quadratic shape near the minimum, although the Wood Function itself is not quadratic. Many algorithms for function minimization are based on this fact. Now the character 'G' is entered to allow the system to perform the entire linear search and ignore the previous steps. A new value of 3 is specified for the LS display cycle.

4.3.6 A point is displayed by adjusting the vertical cursor and entering the character 'P'. This point is also used as the left end of the interval to be considered by entering 'A'. The interval is also extended by entering 'E' and then specifying 1 as the value of the right end (B). Also a request for a new graph is given (the character 'D').

4.3.7 The interval under consideration has now been extended on the left-hand-side to 0.

4.3.8 The X-coordinates of two points are displayed by adjusting the vertical cursor at these two points and entering the character X in each case. These same two points are then specified as the limits of the interval (A,B) as described above.

4.3.9 The function is plotted within the new interval. 'G' is entered and the value of the LS display cycle is changed to 6.

4.3.10 The display is that for linear search number 10. A point is displayed (via the character 'P') and it is then used as the right-end of the next interval (via the character 'B').

4.3.11 The same procedures as in fig. 4.3.10.

4.3.12 Again, this figure, the quadratic shape of the function near the minimum is apparent. 'G' is entered and 1 is assigned to the LS display cycle.

4.3.13 For this linear search, a point is displayed and used as the limit A. The other limit, B, is moved to 0.1. A new display is then requested.

4.3.14 No change is specified in the LS display cycle (simply $\overline{Q}$).

Example 4.3(b)

Run number 2 is started from the same priming guess. The following run conditions were used: MINTEC = F-R, G$PAR = AG, LN$SRH = GOLDEN, LS$PAR = 2.

Figure	Comments
4.3.15	'P' is used to display the coordinates of a point. 'B' is entered without altering the cursor position (right-end). 'D' is entered to display the curve for the new interval.
4.3.16	'P' is entered to display the coordinates of a point. 'M' is entered without altering the cursor position, to pass this point as a solution of the linear search.

4.3.17 'E' is entered to extend the graph and 0.4 is specified for B. 'P' is entered to display the coordinates of a point. 'A' is entered without altering the cursor position (left-limit). 'D' is entered.

4.3.18 'P' is used to display the coordinates of a point. 'B' is entered without altering the cursor position. 'R' is entered to pass the interval (0.194, 0.296) to the system as the IIOU.

4.3.19 The display is for linear search number 6. The coordinates of two points are displayed (via 'P') and these two points are specified as A and B, then 'D' is entered.

4.3.20 'P' is used to display the coordinates of a point, then 'M' is entered without altering the cursor position. The displayed point then becomes the user's specification for the solution of the linear search problem.

4.3.21 The coordinates of two points are displayed (via 'P') and these two points are assigned to A and B (the limits of the IOU). 'R' is used to pass these limits to the system.

4.3.22 As before, two points are displayed and they are assigned to A and B. 'D' is entered to draw the curve in the new interval.

4.3.23 'P' is used to display the coordinates of a point and then 'M' is entered without altering the cursor position. Thus the displayed becomes the user's specification for the solution of the linear search problem.

Here, this run is stopped and a new run (run number 3) is initiated starting from the same priming guess and under the same run conditions as run number 2. In this case, however, the run is allowed to proceed without any user interaction with the linear searches. The two runs (2 and 3 respectively) are compared in fig. 4.3.24 and fig. 4.3.25 where the function value is ploted against number of function evaluations and against number of iterations respectively.

Both runs 2 and 4 proceed through 10 iterations (see fig. 4.3.25). Note however (in fig. 4.3.24) that run 3, which takes place without user interaction on the linear searches, attains a slightly lower function value in fewer function evaluations, than does run 2.

4.5 An Example of Constrained Minimization

As described in Chapter 2, INTEROPT can accommodate constrained minimization problems through the use of the Interior Penalty Function Method [3]. The example given in this section describes a penalty function run with INTEROPT where the FUNCTION subprogram F is set up as described in section 2.14. The problem under consideration (Optimal Fuel Allocation in Power Plants) is taken from [11] and has the following form:

$$\text{Minimize} \qquad C = x_3 f_1 + x_4 g_1,$$

subject to the following constraints:

(a) $18 \le x_1 \le 30$

(b) $x_2 = 50 - x_1$

(c) $14 \le x_2 \le 25$

(d) $0 \le x_3 \le 1$

(e) $0 \le x_4 \le 1$

(f) $BFG = (1-x_3)f_2 + (1-x_4)g_2 \le 10.0$

where $f_1 = 1.4609 + .15186x_1 + .00145x_1^2,$

$f_2 = 1.5742 + .1631x_1 + .001358x_1^2,$

$g_1 = .8008 + .2031x_2 + .000916x_2^2,$

and $g_2 = .7266 + .2256x_2 + .000778x_2^2$

The equality constraint (b), makes it possible to eliminate the variable x_1 from the problem formulation. Correspondingly, the inequality constraints become:

(a') $20 \le x_2 \le 25$

(b') $0 \le x_3 \le 1$

(c') $0 \le x_4 \le 1$

(d') $BFG = (1-x_3)f_2 + (1-x_4)g_2 \le 10.0$

where $f_1 = 1.4609 + .15186(50-x_2) + .00145(50-x_2)^2$

$f_2 = 1.5742 + .1631(50-x_2) + .001358(50-x_2)^2$

and g_1 and g_2 are as above.

The problem therefore has three independent variables, x_2, x_3 and x_4. The priming guess used in the computation was (22.5, .5, .5).

Figures 4.4.1 through 4.4.41 are displays selected from the terminal session. The following comments provide a synopsis of these figures.

1. Figures 4.4.1 through 4.4.29 provide details of the first, the second and the third runs while figures 4.4.30 through 4.4.41 provide details of the ninth and the tenth runs. The displays generated during the runs in between these are quite similar and hence are not included.
2. The value of the coefficient R associated with the "penalty term" is displayed and reduced in value at the beginning of each run in accord with the underlying concept of the penalty function approach.
3. The values 10, 1, 0.1, ..., 10^{-8} were assigned successively to R in the course of the ten runs.
4. The effect of the penalty term on the value of the penalty function in the forbidden region is clear from the linear search displays (see, fig. 4.4.3, fig. 4.4.14, fig. 4.4.15, fig. 4.4.33, fig. 4.4.35). The value of the function increases rapidly at the boundary of the feasible area and remains very high in the forbidden area.

```
 WHAT IS YOUR PRIMING GUESS?
-3.,-1.,-3.,-1
 THE NEW PRIMING GUESS IS
 -0.300000D 01
 -0.100000D 01
 -0.300000D 01
 -0.100000D 01
 FUNC VALUE =     0.161920 05

 DO YOU WISH TO RESPECIFY THE PRIMING GUESS? NO
```

Fig. 4.1.1

```
                RUN CONDITIONS

        MINTEC =   D-F-P  / GOPAR = AGRAD

        LNSSRH =  QUADFT  / LSCPAR =   8

 CHANGE?
  F-R,4

                RUN CONDITIONS

        MINTEC =     F-R  / GOPAR = AGRAD

        LNSSRH =  QUADFT  / LSCPAR =   4


 WHAT NOW?  GO

   RUN CONTROL CYCLE =    5
   LS  DISPLAY CYCLE =    5
     CHANGE?
```

Fig. 4.1.2

```
          LINEAR SRCH #      5
     FUNC=  MAX    0.7333D 02  MIN    0.3120D 02
      IOU =  FROM   0.0           TO    0.6366D 00
 LS DISPLAY CYCLE =  5, CHANGE? 6
```

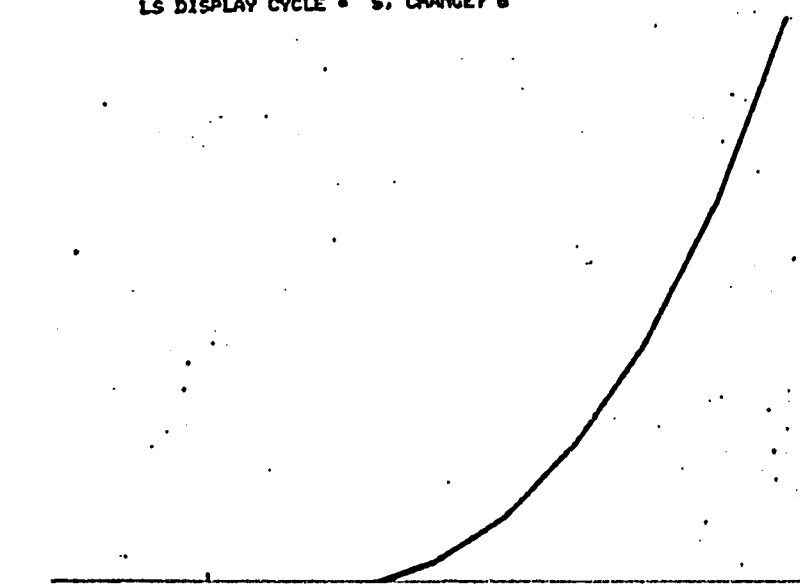


Fig. 4.1.3

```
 # OF ITERATIONS =     5
 # OF FUNC EVAL. =    66
 THE ARGUMENT
  0.402000 00
  0.150661D 00
  0.170770 00
  0.222550 00
 FUNC VALUE =     0.311670 02

 THE GRADIENT
  0.135670 02
 -0.475770 02
 -0.135470 02
  0.240590 01
 GRAD NORM  =     0.613620 02

 WHAT NOW?  OUTPUT
```

Fig. 4.1.4

```
PRINTER OUTPUT FOR EACH ITERATION

ENTER THE DESIRED OPTION IDENTIFICATION NUMBER
0 NO OUTPUT
1 ITER #, FUNC & ARG VALUES, # OF FUNC EVAL
2 SAME AS 1 PLUS GRAD & LINEAR SEARCH INFO.

          1

WHAT NOW? GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    6
    CHANGE?
    /
```

Fig. 4.1.5

```
# OF ITERATIONS =   10
# OF FUNC EVAL. =  112
THE ARGUMENT
 0.80756D 00
 0.35097D 00
 0.33062D 00
 0.23749D 00
FUNC VALUE =    0.89218D 02

THE GRADIENT
 0.93936D 02
-0.86372D 02
-0.13159D 02
-0.12772D 02
GRAD NORM =     0.12892D 03

WHAT NOW? PLOT
```

Fig. 4.1.6

```
FUNCTION VALUE / # OF FUNCTION EVALUATIONS
XLOG SCALE
   FUNC: MAX   0.1919D 05  MIN   0.8922D 02
   EVAL: FROM          6   TO          118
```

```
RUN #  1
       F-R
G3PAR = AG
      QUADFT
LSSPAR=  4
```

Fig. 4.1.7

```
WHAT NOW? CONDITION
```

Fig. 4.1.8

```
          RUN CONDITIONS

      MINTEC =    F-R  /  GSPAR = AGRAD
      LNSSRH =  QUADFT  / LSSPAR =    4

 CHANGE?
  D-F-P

          RUN CONDITIONS

      MINTEC =  D-F-P  /  GSPAR = AGRAD
      LNSSRH =  QUADFT  / LSSPAR =    4


 WHAT NOW?  GO

   RUN CONTROL CYCLE =    5
   LS  DISPLAY CYCLE =    6
     CHANGE?  10
```

Fig. 4.1.9

```
                LINEAR SRCH #    11
   FUNC:  MAX   0.1760D 03  MIN   0.1710D 02
   ITU :  FROM   0.0          TO   0.2000D 01
 LS DISPLAY CYCLE =  6. CHANGE? 10
```

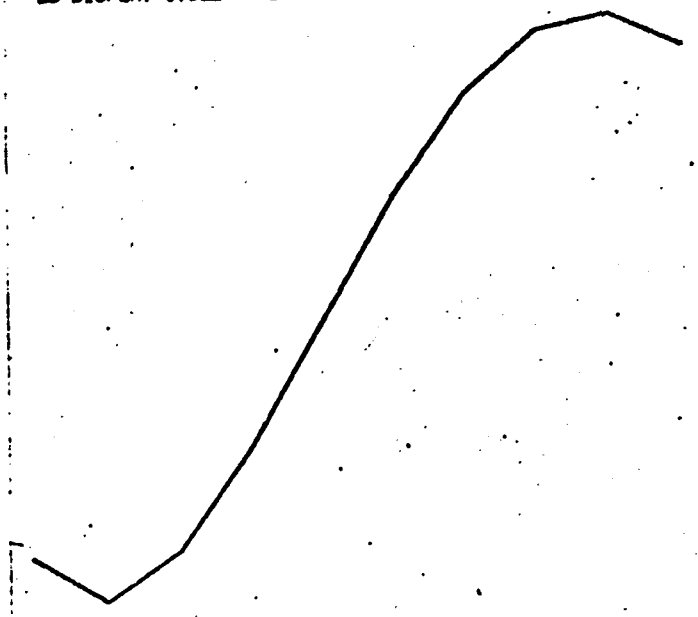


Fig. 4.1.10

```
 # OF ITERATIONS =    20
 # OF SUB-ITER.  =    10 ( 1 )
 # OF FUNC EVAL. =   232
 THE ARGUMENT
  0.100000D 01
  0.100000 01
  0.999999D 00
  0.999999D 00
 FUNC VALUE =     0.303289D-09

 THE GRADIENT
 -0.323399D-03
  0.20550D-03
 -0.228C8D-04
  0.220920-04
 GRAD NORM  =     0.380000-03

 WHAT NOW?  PLOT/ITER
```

```
 FUNCTION VALUE/# OF ITERATIONS
  #LOG SCALE
    FUNC:  MAX   0.1919D 05  MIN   0.3009D-09
    ITER:  FROM            0  TO            20
```

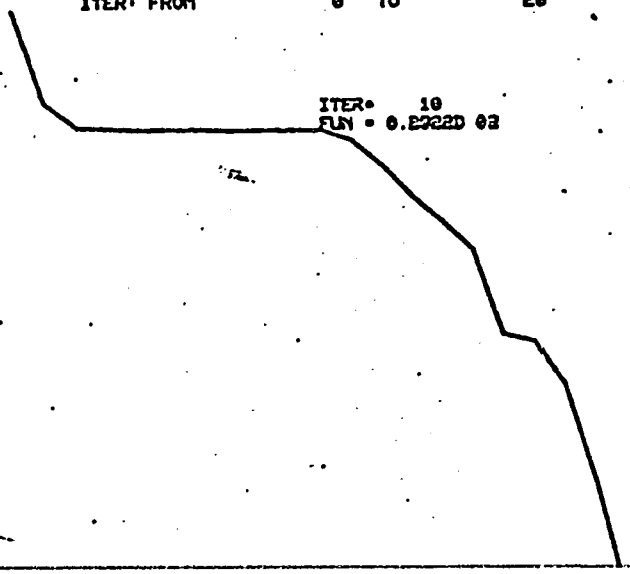


Fig. 4.1.11

```
WHAT NOW?  NEW RUN
```

Fig. 4.1.13

```
 CURRENT PRIMING GUESS IS
-0.300000D 01
-0.100000D 01
-0.300000D 01
-0.100000D 01
FUNC VALUE =    0.191920 05

 CURRENT ARGUMENT IS
 0.100000D 01
 0.100000D 01
 0.999999D 00
 0.999999D 00
FUNC VALUE =    0.300899D-09

 REPEAT, CONTINUE OR NEW?  REPEAT
```

Fig. 4.1.14

```
          RUN CONDITIONS

      MINTEC =   D-F-P  /  GSPAR = AGRAD

      LNSSRH =  QUADFT  / LSSPAR =   4

  CHANGE?

WHAT NOW?  OUTPUT,2

WHAT NOW?  INITIAL
```

```
INITIALIZATION & STOPPING PARAMETERS

        1 RESTRT = NOT APPL
        2 FAIL   =  INIT
        3 MXITER = NO LIMIT
        4 MXFE   = NO LIMIT
        5 ESPAR  =    14
 CHANGE?
          1,10
          3,20

INITIALIZATION & STOPPING PARAMETERS

        1 RESTRT =    10
        2 FAIL   =  INIT
        3 MXITER =    20
        4 MXFE   = NO LIMIT
        5 ESPAR  =    14

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  25,25
```

```
# OF ITERATIONS =   20
# OF SUB-ITER.  =   10 ( 1 )
# OF FUNC EVAL. =  258
THE ARGUMENT
 0.13528D 01
 0.18237D 01
-0.21764D 00
 0.58873D-01
FUNC VALUE =    0.20586D 01

THE GRADIENT
 0.11041D 01
-0.20424D 01
-0.15337D 01
-0.53154D 00
GRAD NORM  =    0.28329D 01

     THE REQUESTED NUMBER OF ITERATIONS HAVE NOW BEEN COMPLETED
                    (MXITER=   20)

WHAT NOW? PLOT/ITER.1.2
```

Fig. 4.1.17

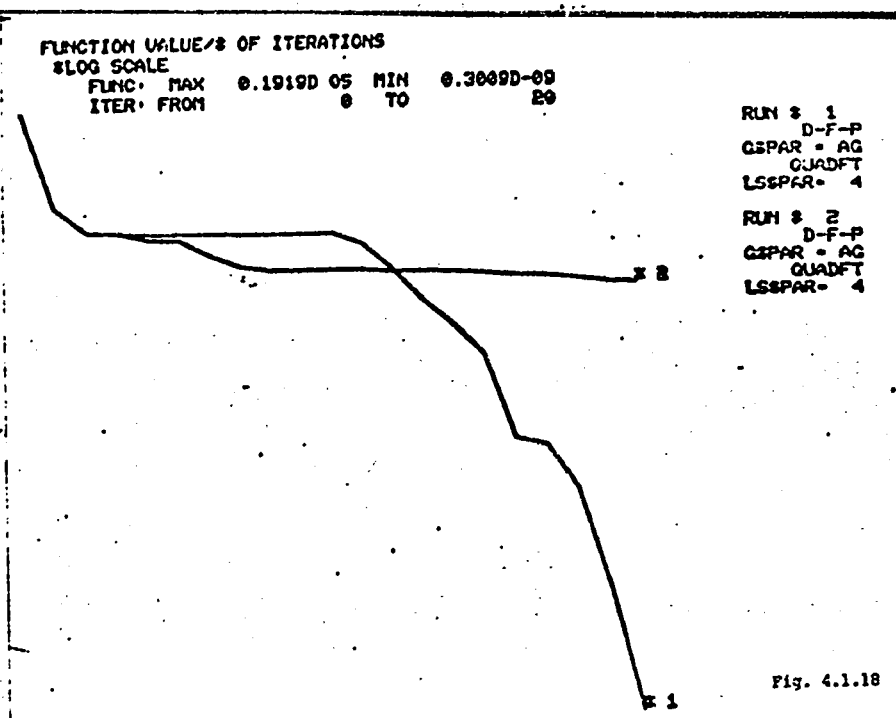


Fig. 4.1.18

```
WHAT NOW? TABLE.1
```

RUN SUMMARY ITER	SUBITR	#EVAL	FUNC VALUE
0	0	6	0.19192D 05
1	1	20	0.13425D 03
2	2	32	0.55556D 02
3	3	42	0.34435D 02
4	4	65	0.31587D 02
5	5	68	0.31167D 02
6	6	76	0.30761D 02
7	7	65	0.30237D 02
8	8	94	0.29933D 02
9	9	103	0.29931D 02
10	10	118	0.29218D 02
11	1	139	0.17059D 02
12	8	142	0.45033D 01
13	3	152	0.82734D 00
14	4	161	0.81218D 00
15	5	173	0.40559D-01
16	6	184	0.55825D-03
17	7	196	0.20742D-03
18	8	207	0.37037D-04
19	9	217	0.20764D-06
20	10	232	0.30039D-09

Fig. 4.1.19

```
RUN # 1
      D-F-P
GSPAR = AG
      QUADFT
LSSPAR= 4
```

Fig. 4.1.20

```
WHAT NOW?  NEW RUN
```

Fig. 4.1.21

```
 CURRENT PRIMING GUESS IS
-0.30000D 01
-0.10000D 01
-3.30000D 01
-0.10000D 01
FUNC VALUE =    0.19192D 06

 CURRENT ARGUMENT IS
 0.13526D 01
 0.10287D 01
-0.21764D 00
 0.50373D-01
FUNC VALUE =    0.20586D 01

 REPEAT, CONTINUE OR NEW?  REPEAT
```

Fig. 4.1.22

```
          RUN CONDITIONS

      MINTEC =   D-F-P  / G3PAR = ACRAD

      LNSRH =  QUADFT  / LSPAR =   4

CHANGE?
 ZANG,FBINACI

          RUN CONDITIONS

      MINTEC =    ZANG

      LNSRH =  FBINACI  / LSPAR =   4


WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  .50
```

```
# OF ITERATIONS =    5
# OF FUNC EVAL. =  592
THE ARGUMENT
-0.10919D 01
 0.10388D 01
-0.52482D 00
 0.33965D 00
FUNC VALUE =    0.11466D 03

WHAT NOW?  OUTPUT.1

WHAT NOW?  CONDITION
```

```
          RUN CONDITIONS

      MINTEC =    ZANO

      LNSSRH =  FRNACI  / LSSPAR =   4

  CHANGE?
   POWELL

          RUN CONDITIONS

      MINTEC =  POWELL

      LNSSRH =  FRNACI  / LSSPAR =   4


WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =   50
    CHANGE?
```

Fig. 4.1.25

```
          LINEAR SRCH #    50
    FUNC:  MAX    0.7865D 01  MIN    0.7864D 01
    IOU :  FROM   0.0           TO    0.1000D-02
LS DISPLAY CYCLE = 50, CHANGE?
```

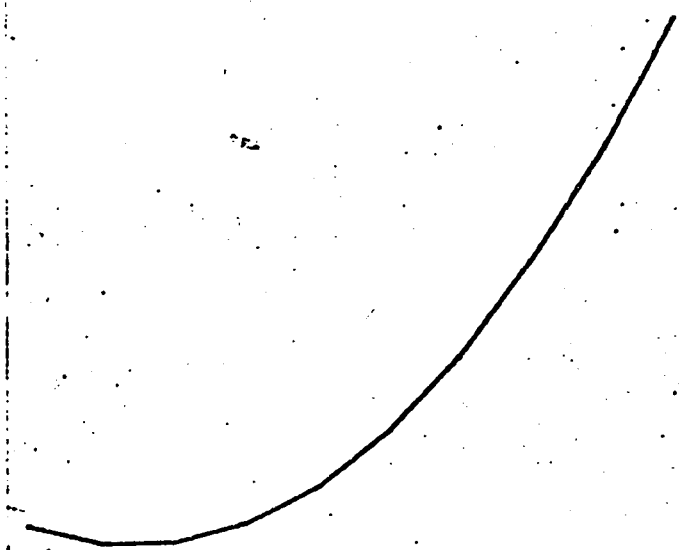


Fig. 4.1.26

```
# OF ITERATIONS =   10
# OF SUB-ITER.  =    5 ( 1 )
# OF FUNC EVAL. = 1054
THE ARGUMENT
-0.10997D 01
 0.12191D 01
-0.81489D 00
 0.67637D 03
FUNC VALUE =    0.78645D 01

WHAT NOW?  OUTPUT.0

WHAT NOW?  CONDITION
```

```
          RUN CONDITIONS

      MINTEC =  POWELL

      LNSSRH =  FRNACI  / LSSPAR =   4

  CHANGE?
   QUADFT

          RUN CONDITIONS

      MINTEC =  POWELL

      LNSSRH =  QUADFT  / LSSPAR =   4


WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =   50
    CHANGE?  10
```

```
        LINEAR SRCH #   100
   FUNC:  MAX    0.7420D 01   MIN    0.7402D 01
    IOU :  FROM    0.0            TO    0.6028D-01
LS DISPLAY CYCLE = 50. CHANGE?
```

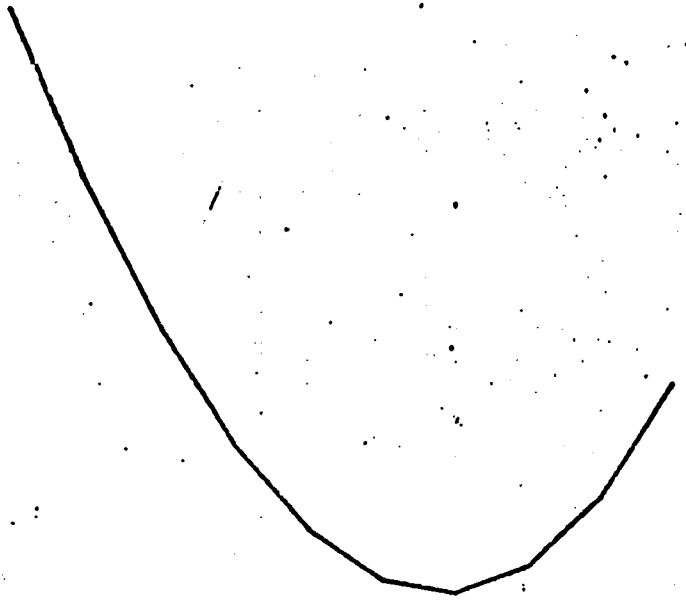


Fig. 4.1.29

```
# OF ITERATIONS =   20
# OF SUB-ITER.  =   15 ( 1 )
# OF FUNC EVAL. = 1426
THE ARGUMENT
-0.1407GD 01
 0.19644D 01
-0.90118D-01
 0.15402D-01
FUNC VALUE =    0.74023D 01

WHAT NOW? TABLE
```

Fig. 4.1.30

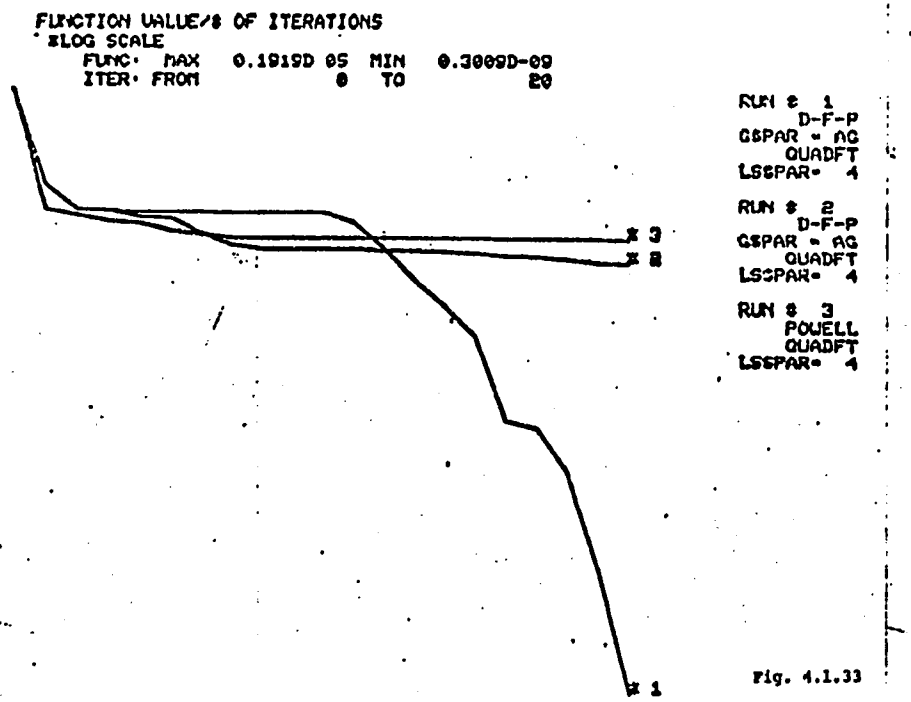
RUN SUMMARY

ITER	SUBITR	#EVAL	FUNC VALUE
0	0	1	0.10192D 05
1	1	140	0.34036D 02
2	2	256	0.26129D 02
3	3	361	0.19330D 02
4	4	455	0.17094D 02
5	5	592	0.11420D 02
6	1	716	0.85707D 01
7	2	830	0.80403D 01
8	3	927	0.76945D 01
9	4	1001	0.76845D 01
10	5	1054	0.76845D 01
11	6	1105	0.76479D 01
12	7	1147	0.76258D 01
13	8	1190	0.75241D 01
14	9	1228	0.75233D 01
15	10	1259	0.75114D 01
16	11	1298	0.77424D 01
17	12	1335	0.76374D 01
18	13	1367	0.76424D 01
19	14	1401	0.74720D 01
20	15	1426	0.74023D 01

```
RUN # 3
     POWELL
     QUADFT
LSPAR= 4
```

```
WHAT NOW? PLOT/ITER.1.8.3
```

Fig. 4.1.32



RUN # 1
D-F-P
GSPAR = AG
QUADFT
LSSPAR= 4

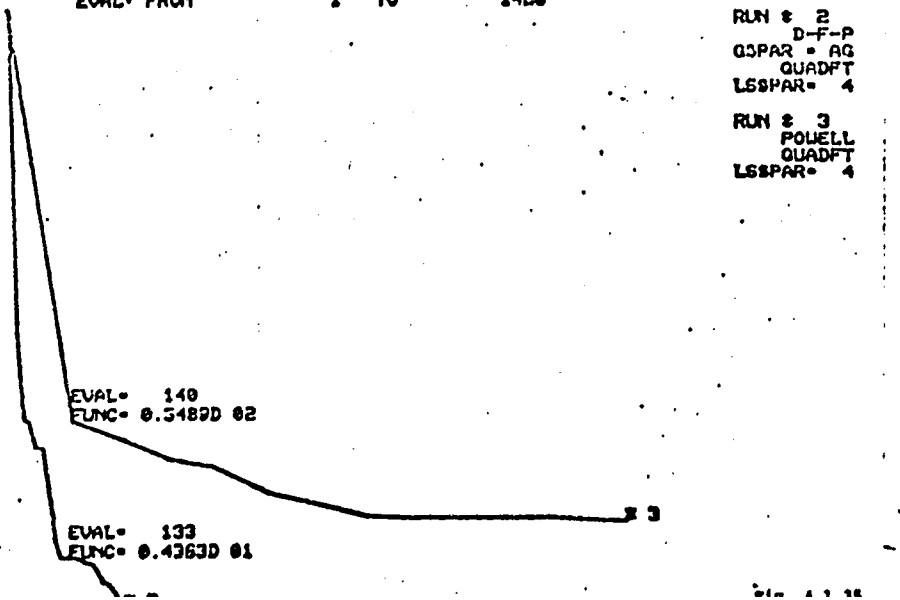
RUN # 2
D-F-P
GSPAR = AG
QUADFT
LSSPAR= 4

RUN # 3
POWELL
QUADFT
LSSPAR= 4

Fig. 4.1.33

WHAT NOW? PLOT.B.3

Fig. 4.1.34



RUN # 2
D-F-P
GSPAR = AG
QUADFT
LSSPAR= 4

RUN # 3
POWELL
QUADFT
LSSPAR= 4

WHAT NOW? NEW RUN

Fig. 4.1.35

```
 CURRENT PRIMING GUESS IS
-0.30000D 01
-0.10000D 01
-0.30000D 01
-0.10000D 01
FUNC VALUE =     0.19192D 05

 CURRENT ARGUMENT IS
-0.14076D 01
 0.19644D 01
-0.90118D-01
 0.15402D-01
FUNC VALUE =     0.74023D 01

 REPEAT, CONTINUE OR NEW?  REPEAT.
```

Fig. 4.1.37

```
          RUN CONDITIONS

      MINTEC =  POWELL

      LNSRH =  QUADFT  / LSPAR =   4

 CHANGE?
  GRAD/9

          RUN CONDITIONS

      MINTEC =    GRAD  / GPAR =   9

      LNSRH =  QUADFT  / LSPAR =   4


 WHAT NOW?  INITIAL
```

Fig. 4.1.38

```
 INITIALIZATION & STOPPING PARAMETERS

         1 RESTRT =  NOT APPL
         2 FAIL   =    STOP
         3 MXITER =  NO LIMIT
         4 MXFE   =  NO LIMIT
         5 ESPAR  =      14
 CHANGE?
          4,1000

 INITIALIZATION & STOPPING PARAMETERS

         1 RESTRT =  NOT APPL
         2 FAIL   =    STOP
         3 MXITER =  NO LIMIT
         4 MXFE   =    1000
         5 ESPAR  =      14

 WHAT NOW?  GO

   RUN CONTROL CYCLE =     5
   LS  DISPLAY CYCLE =     5
     CHANGE?  100,400
```

Fig. 4.1.39

```
 # OF ITERATIONS =   100
 # OF FUNC EVAL. =   832
 THE ARGUMENT
  0.11325D 01
  0.12827D 01
  0.84480D 00
  0.71321D 00
 FUNC VALUE =     0.74276D-01

 THE GRADIENT
  0.20008D 00
  0.29066D-01
 -0.16670D 00
 -0.28154D 00
 GRAD NORM  =      0.42444D 00

 WHAT NOW?  GO

   RUN CONTROL CYCLE =   100
   LS  DISPLAY CYCLE =   400
     CHANGE?
```

Fig. 4.1.40

```
# OF ITERATIONS =  121
# OF FUNC EVAL. = 1005
THE ARGUMENT
 0.112332D 01
 0.127642D 01
 0.850762D 00
 0.723352D 00
FUNC VALUE =    0.697812D-01

THE GRADIENT
-0.364032D 00
 0.379922D 00
-0.131282D 00
-0.215262D 00
GRAD NORM  =    0.583422D 00

   THE REQUESTED # OF FUNCTION EVALUATIONS HAVE NOW BEEN COMPLETED
                    (MXFE= 1000)

WHAT NOW?  INITIAL
```

Fig. 4.1.41

```
INITIALIZATION & STOPPING PARAMETERS

        1 RESTRT = NOT APPL
        2 FAIL   =  STOP
        3 MXITER = NO LIMIT
        4 MXFE   =  1000
        5 ESPAR  =    14
 CHANGE?
          4
          3.200

INITIALIZATION & STOPPING PARAMETERS

        1 RESTRT = NOT APPL
        2 FAIL   =  STOP
        3 MXITER =   200
        4 MXFE   = NO LIMIT
        5 ESPAR  =    14

WHAT NOW?  GO

  RUN CONTROL CYCLE =  100
  LS  DISPLAY CYCLE =  400
    CHANGE?
```

Fig. 4.1.42

```
# OF ITERATIONS =  200
# OF FUNC EVAL. = 1676
THE ARGUMENT
 0.111702D 01
 0.124752D 01
 0.862304D 00
 0.752932D 00
FUNC VALUE =    0.555842D-01

THE GRADIENT
 0.249362D 00
 0.102812D 00
-0.102142D 00
-0.181042D 00
GRAD NORM  =    0.340262D 00

    THE REQUESTED NUMBER OF ITERATIONS HAVE NOW BEEN COMPLETED
                    (MXITER=  800)

WHAT NOW?  PLOT/ITER
```

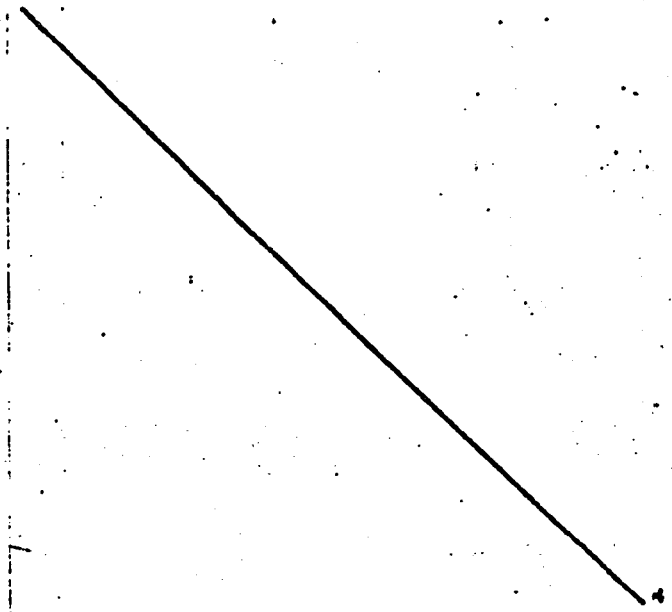
Fig. 4.1.43

```
FUNCTION VALUE/# OF ITERATIONS
 #LOG SCALE
   FUNC= MAX   0.1919D 05  MIN   0.5558D-01
   ITER= FROM           0  TO          200

RUN # 4
      GRAD
GSPAR =  9
      QUADFT
LSPAR=  4
```

```
WHAT NOW? PLOT/ITER,SUB
```

Fig. 4.1.45

```
FUNCTION VALUE/# OF ITERATIONS      (SUB-TABLE)
 FUNC:  MAX  0.6046D-01  MIN  0.6558D-01
 ITER:  FROM        171  TO        200
```




```
RUN # 4
      GRAD
GSPAR = 9
      QUADFT
LSSPAR= 4
```

Fig. 4.1.46

```
WHAT NOW? TABLE/70
```

RUN SUMMARY

ITER	SUBITR	#EVAL	FUNC VALUE
138	138	1149	0.664780-01
140	140	1166	0.662910-01
142	142	1183	0.657150-01
144	144	1200	0.653400-01
146	146	1217	0.649670-01
148	148	1234	0.645960-01
150	150	1251	0.642270-01
152	152	1268	0.638590-01
154	154	1285	0.634940-01
156	156	1302	0.631300-01
158	158	1319	0.627690-01
160	160	1336	0.624090-01
162	162	1353	0.620510-01
164	164	1370	0.616950-01
166	166	1387	0.613400-01
168	168	1404	0.609880-01
170	170	1421	0.606370-01
172	172	1438	0.602880-01
174	174	1455	0.599410-01
176	176	1472	0.595950-01
178	178	1489	0.592520-01
180	180	1506	0.589100-01
182	182	1523	0.585690-01
184	184	1540	0.582310-01
186	186	1557	0.578940-01
188	188	1574	0.575590-01
190	190	1591	0.572260-01
192	192	1608	0.568940-01
194	194	1625	0.565640-01
196	196	1642	0.562360-01

```
RUN # 4
      GRAD
GSPAR = 9
      QUADFT
LSSPAR= 4
```

Fig. 4.1.47

```
WHAT NOW? TABLE.SUB
```

Fig. 4.1.49

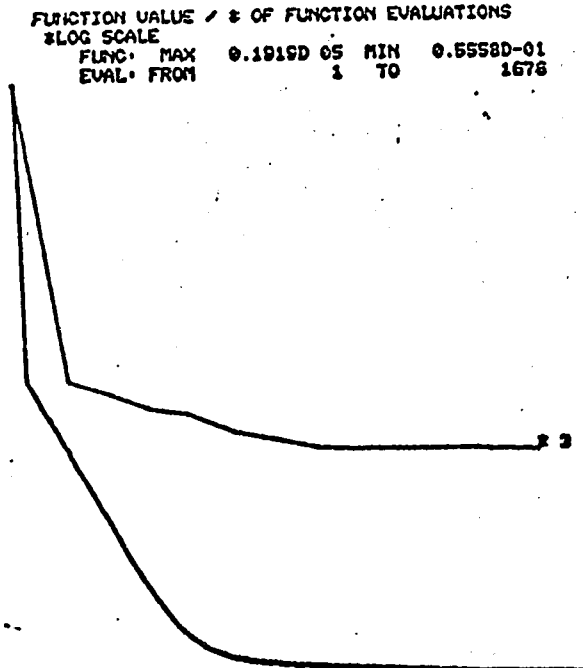
RUN SUMMARY (SUB-TABLE)

ITER	SUBITR	#EVAL	FUNC VALUE
171	171	1430	0.60462D-01
172	172	1438	0.60288D-01
173	173	1447	0.60114D-01
174	174	1455	0.59941D-01
175	175	1464	0.59768D-01
176	176	1472	0.59596D-01
177	177	1481	0.59423D-01
178	178	1489	0.59252D-01
179	179	1498	0.59080D-01
180	180	1506	0.58910D-01
181	181	1515	0.58739D-01
182	182	1523	0.58569D-01
183	183	1532	0.58402D-01
184	184	1540	0.58231D-01
185	185	1549	0.58062D-01
186	186	1557	0.57894D-01
187	187	1566	0.57726D-01
188	188	1574	0.57559D-01
189	189	1583	0.57392D-01
190	190	1591	0.57226D-01
191	191	1600	0.57060D-01
192	192	1608	0.56894D-01
193	193	1617	0.56729D-01
194	194	1625	0.56564D-01
195	195	1634	0.56400D-01
196	196	1642	0.56236D-01
197	197	1651	0.56072D-01
198	198	1659	0.55909D-01
199	199	1668	0.55747D-01
200	200	1676	0.55584D-01

```
RUN # 4
     GRAD
GSPAR = 9
     QUADFT
LSSPAR= 4
```

Fig. 4.1.50

```
WHAT NOW? PLOT.3.4
```

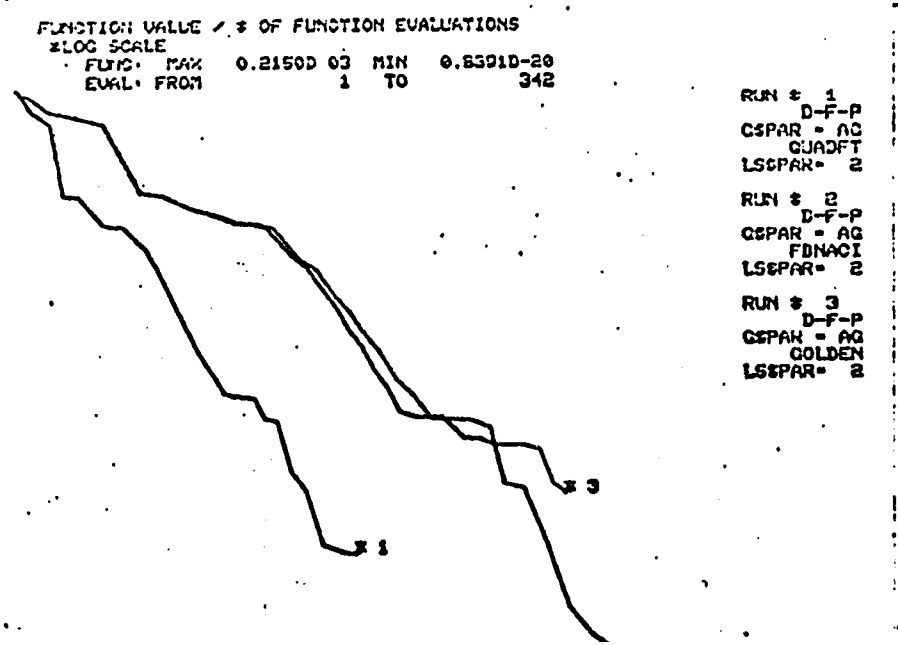


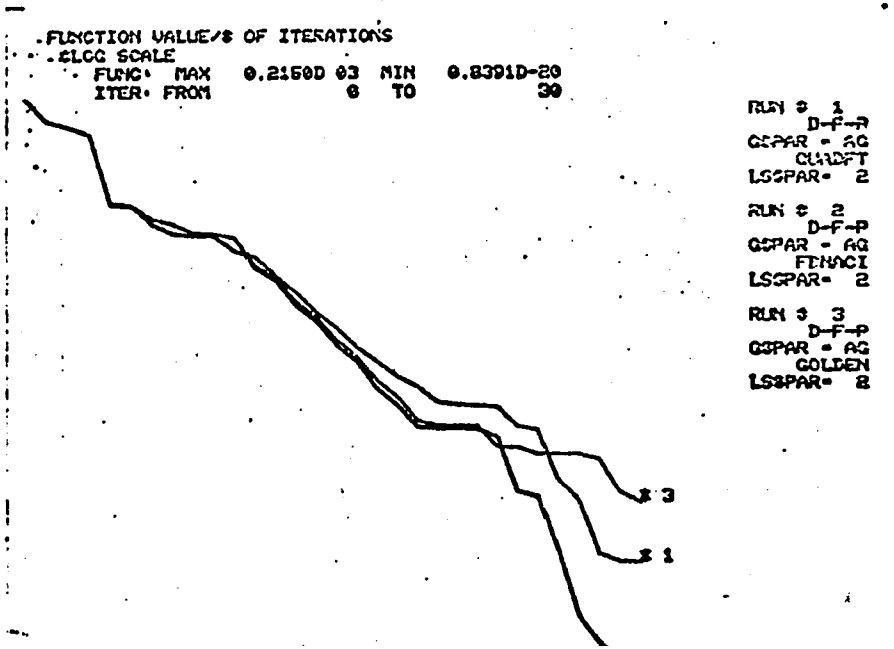
```
WHAT NOW? END JOB
```

Fig. 4.1.53

```
   WHAT IS YOUR PRIMING GUESS?
3.-1.0.1
   THE NEW PRIMING GUESS IS
    0.30000D 01
   -0.10000D 01
    0.0
    0.10000D 01
   FUNC VALUE =     0.21560D 03

   DO YOU WISH TO RESPECIFY THE PRIMING GUESS? NO
```

Fig. 4.2.1





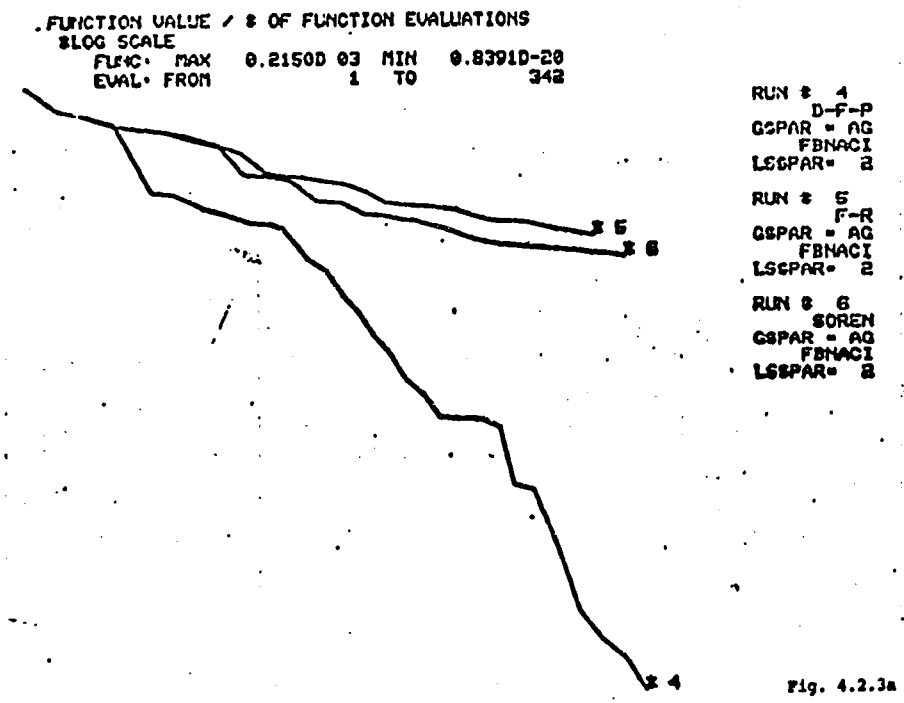


Fig. 4.2.3a

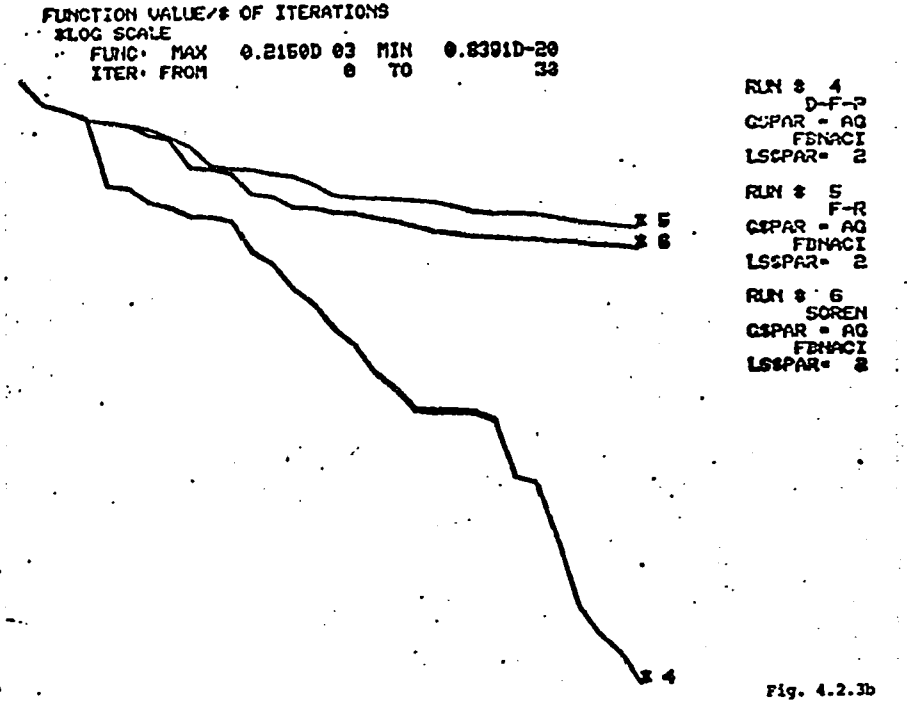
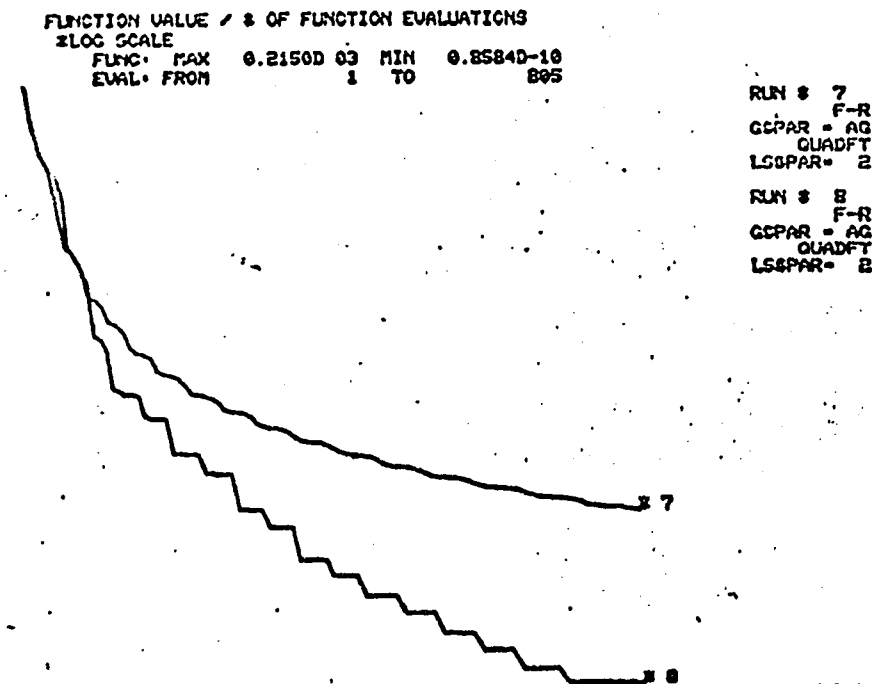
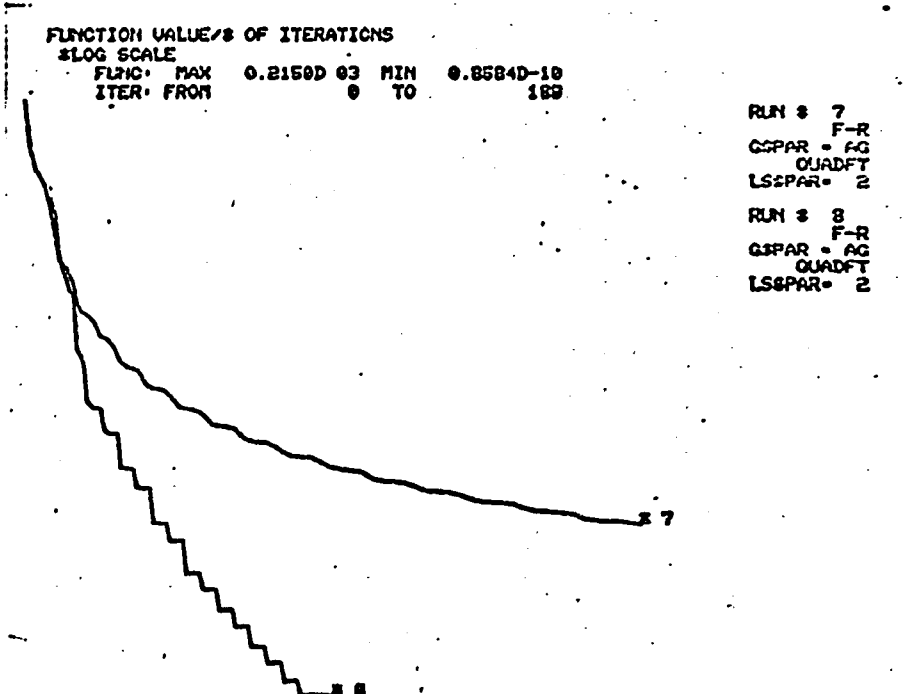


Fig. 4.2.3b





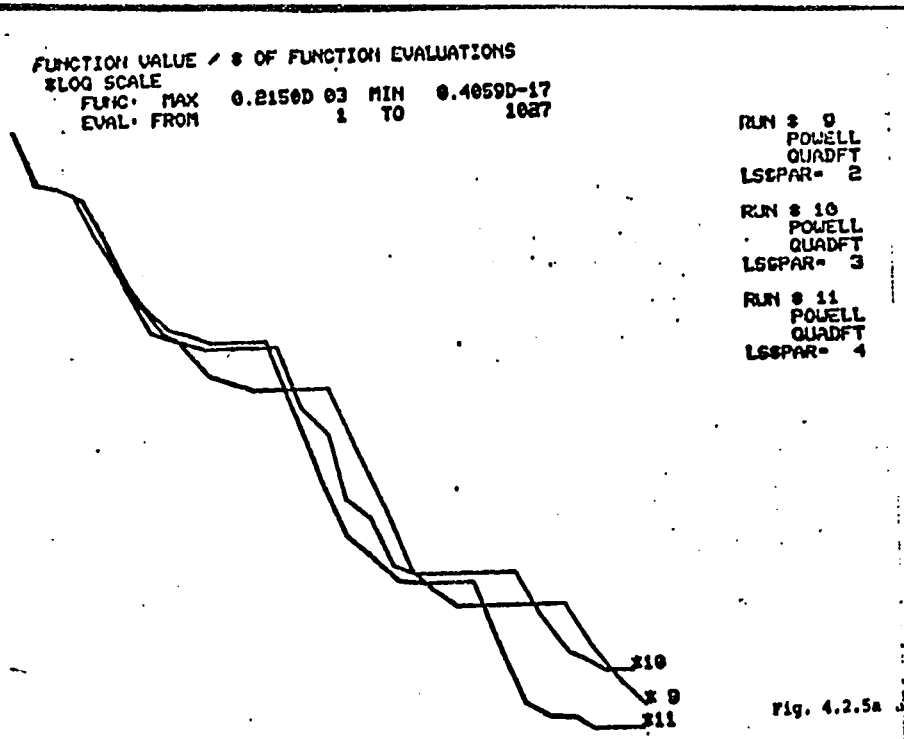


Fig. 4.2.5a

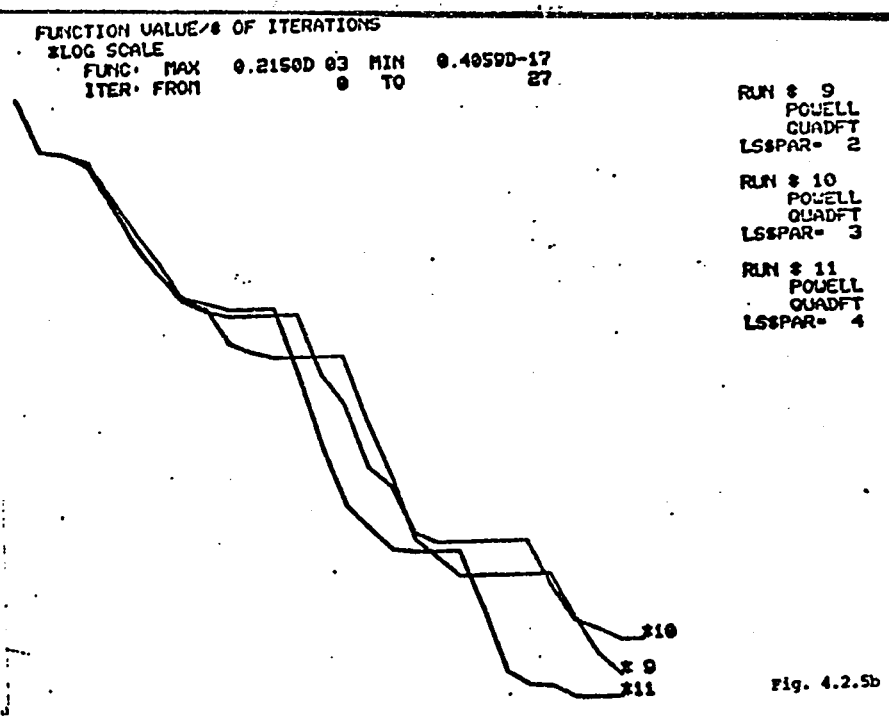


Fig. 4.2.5b

```
   WHAT IS YOUR PRIMING GUESS?
-1.2,1.,-1 2,1.,-1.2
   THE NEW PRIMING GUESS IS
   -0.12000D 01
    0.10000D 01
   -0.12000D 01
    0.10000D 01
   -0.12000D 01
   FUNC VALUE =    0.10164D 04

   DO YOU WISH TO RESPECIFY THE PRIMING GUESS? NO
```

Fig. 4.2.6

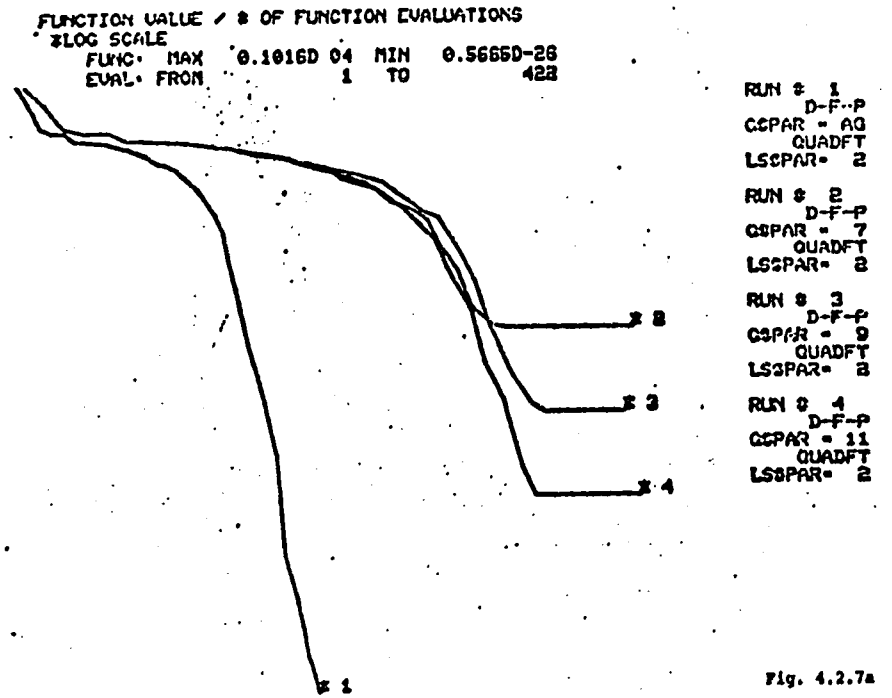


Fig. 4.2.7a

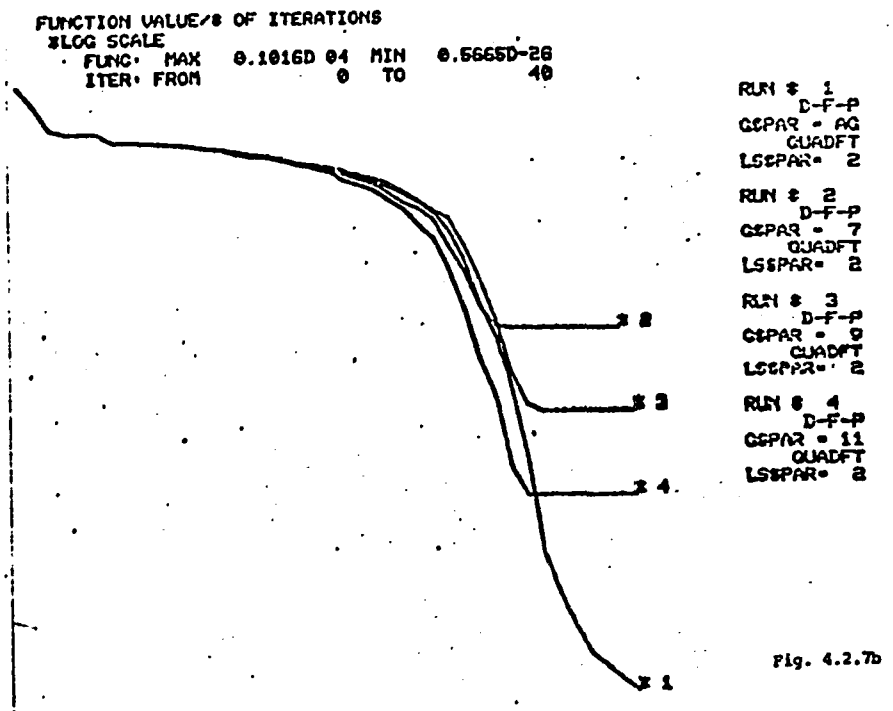


Fig. 4.2.7b

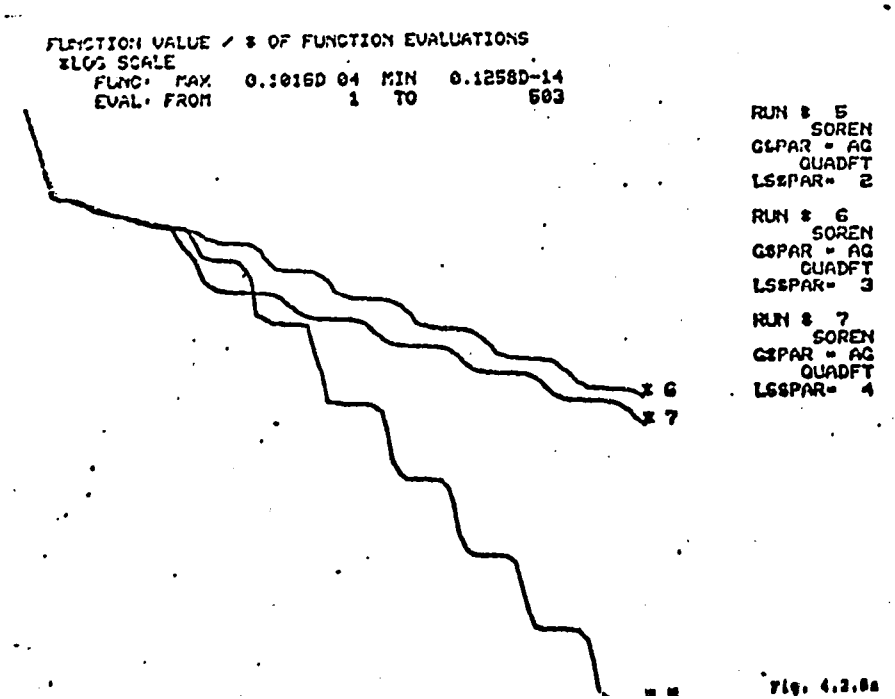
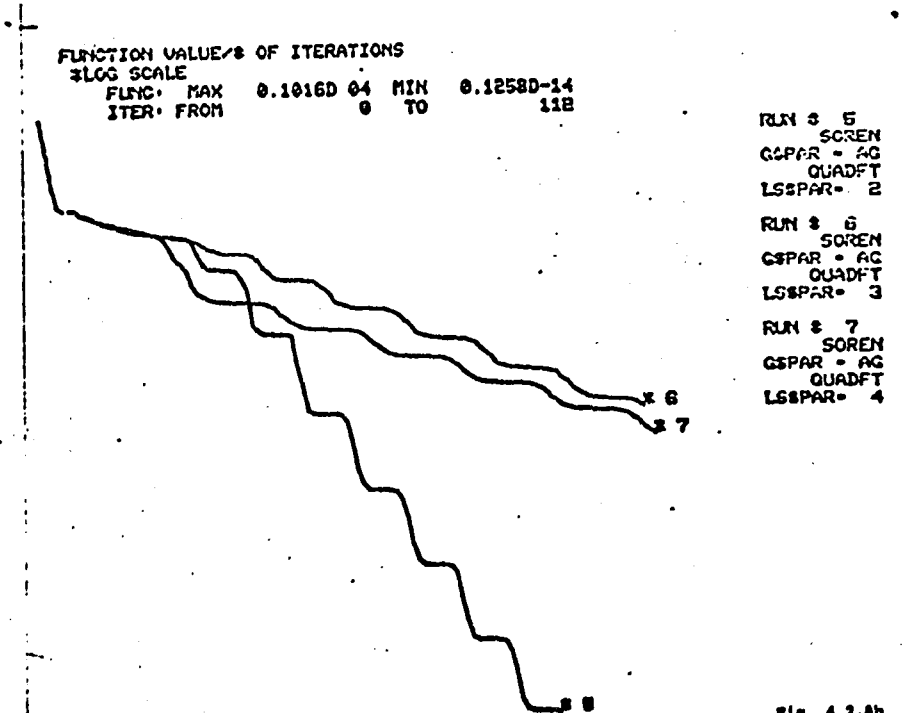


Fig. 4.2.8a



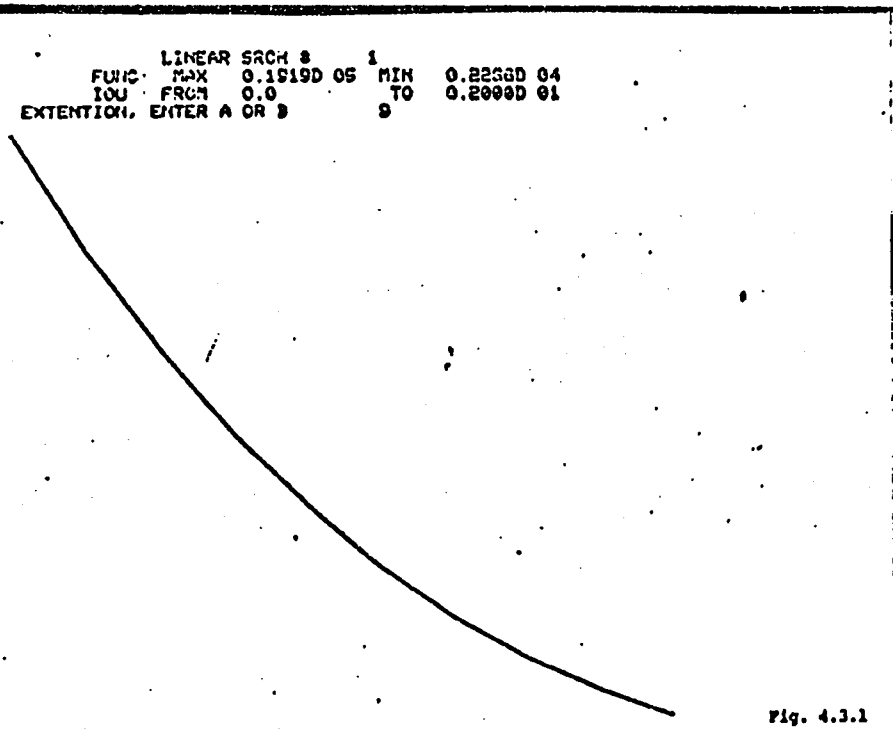


Fig. 4.3.1

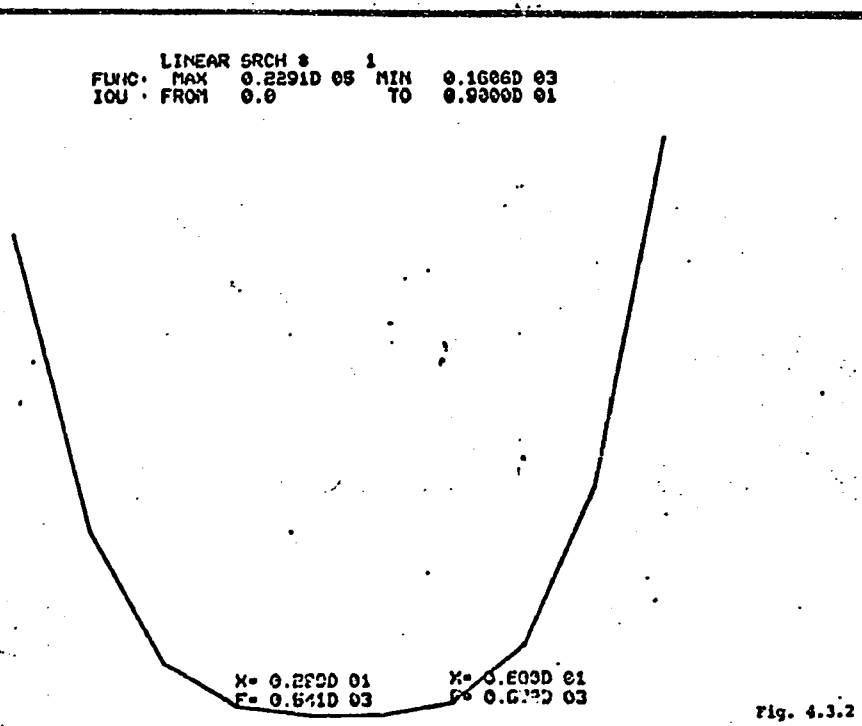


Fig. 4.3.2

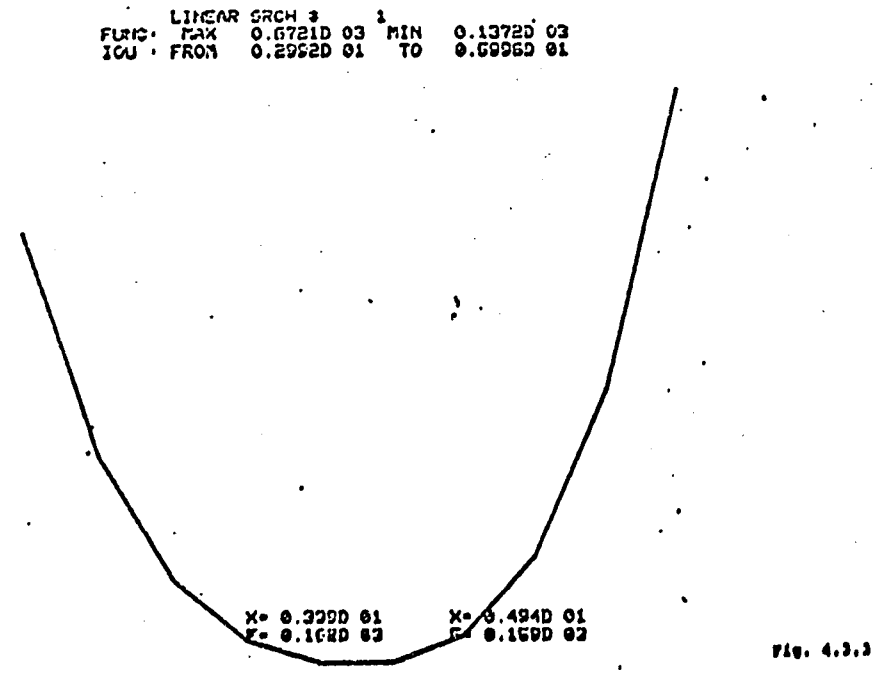


Fig. 4.3.3

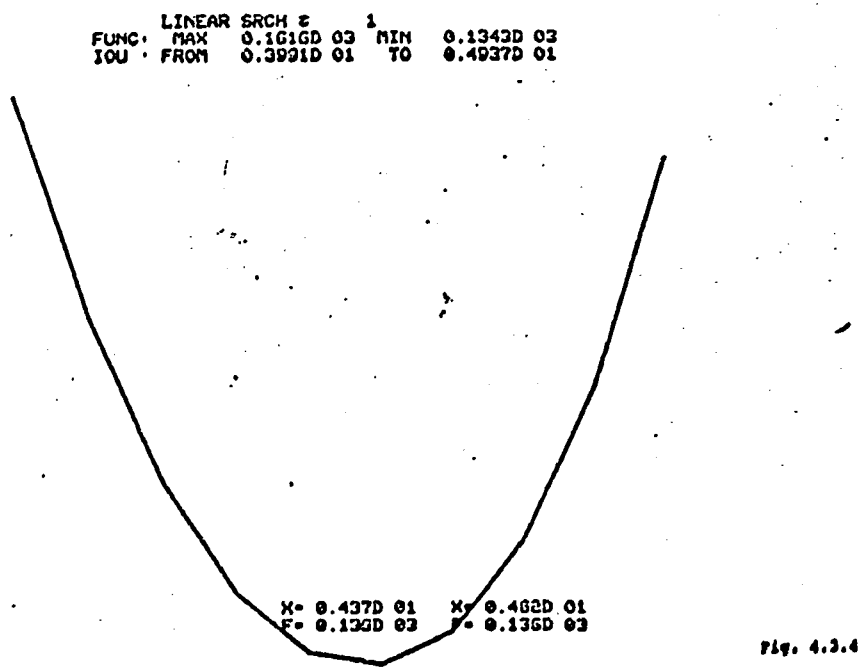


Fig. 4.3.4

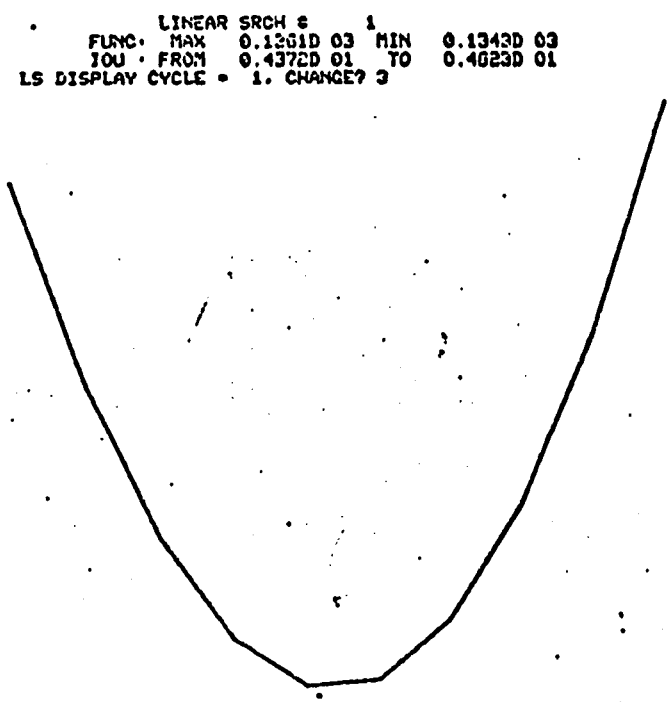


Fig. 4.3.5

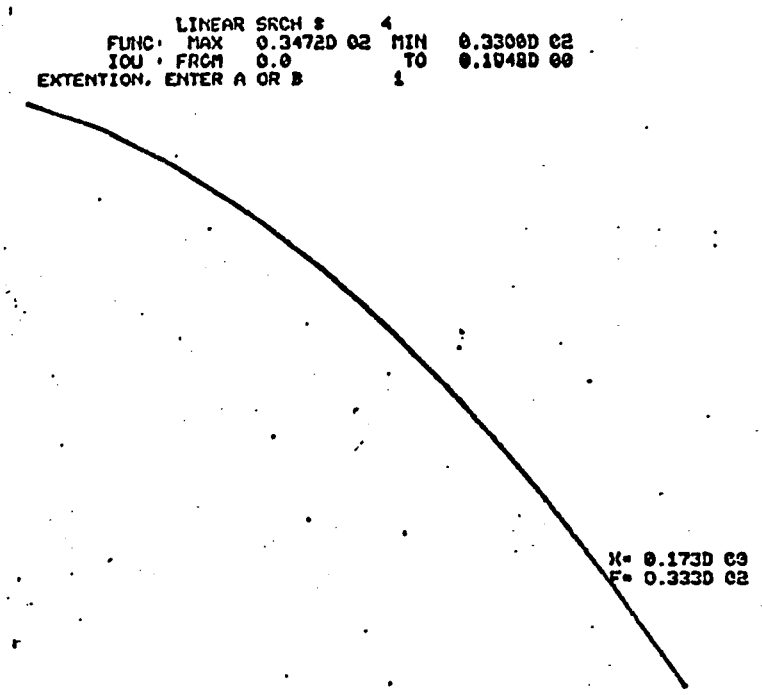
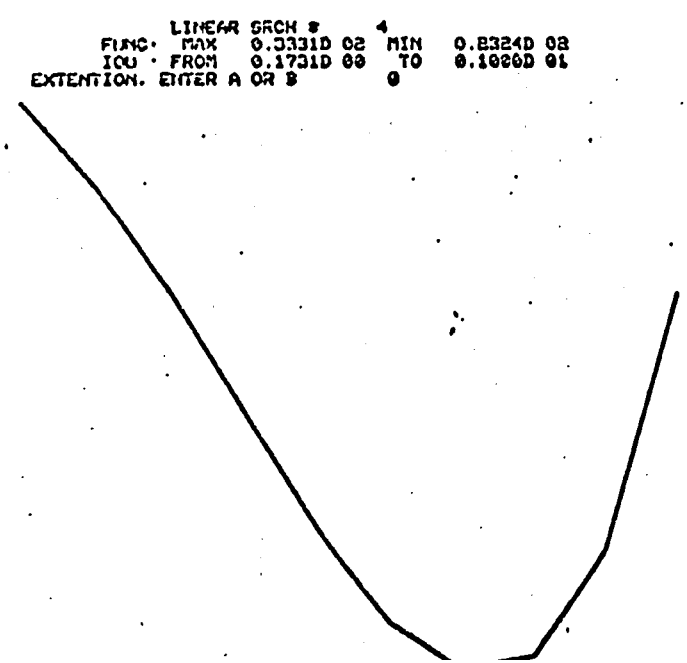
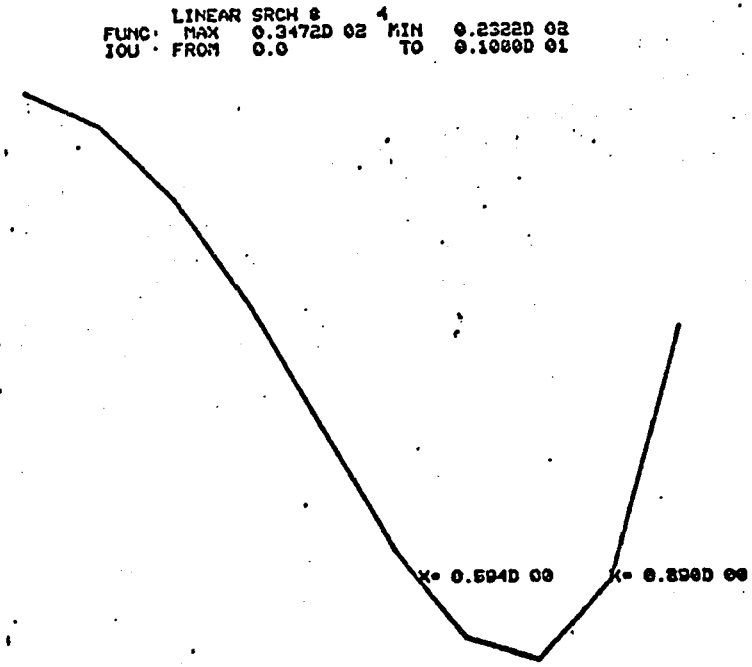


Fig. 4.3.6





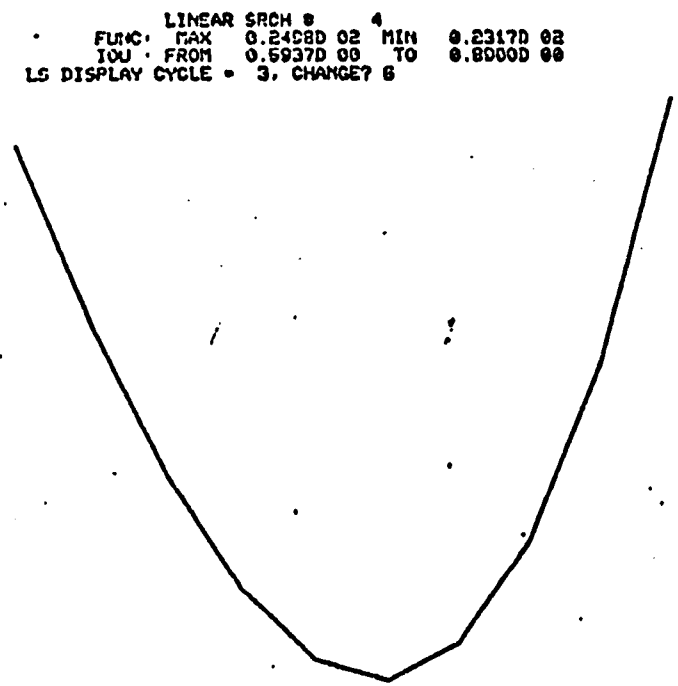


Fig. 4.3.9

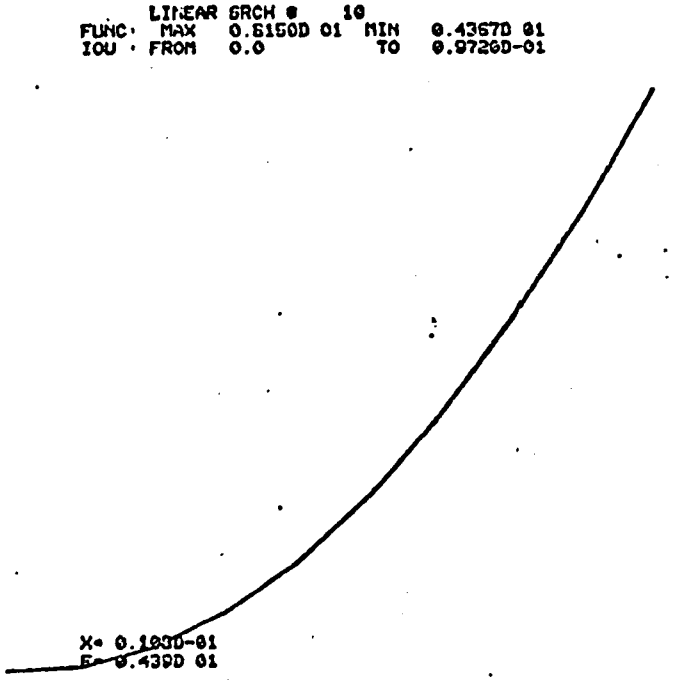


Fig. 4.3.10

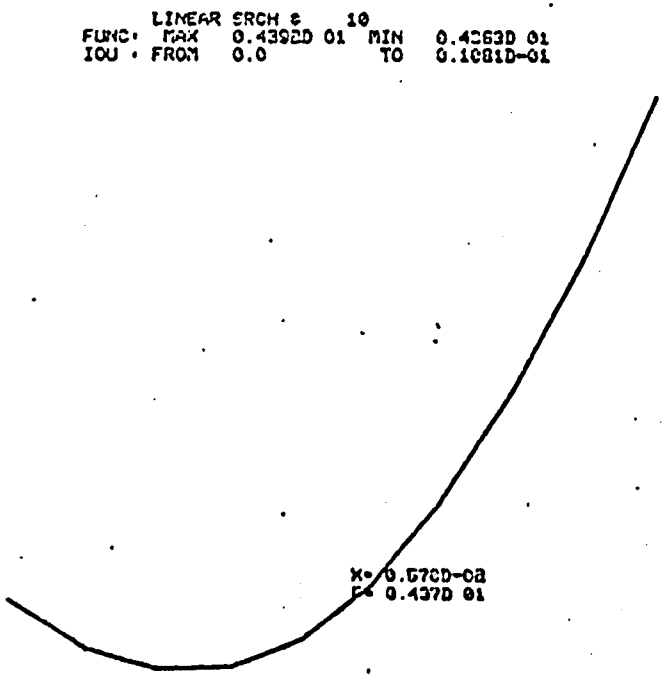


Fig. 4.3.11

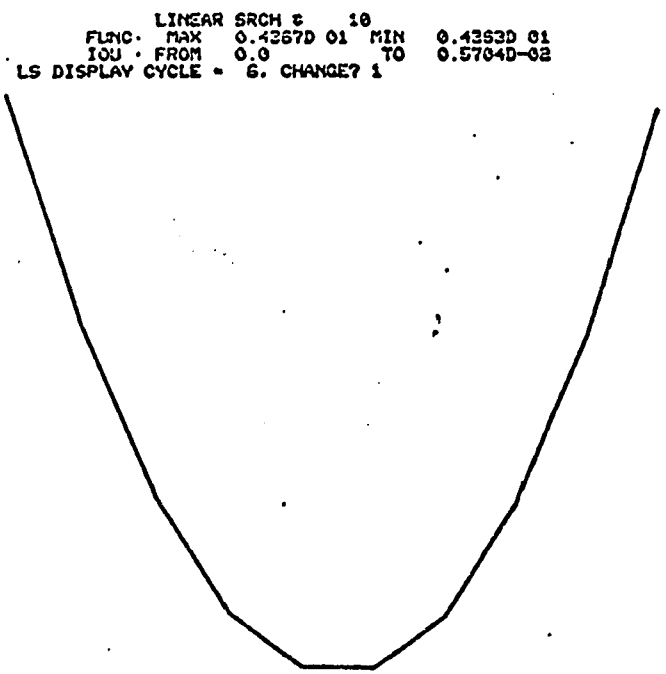


Fig. 4.3.12

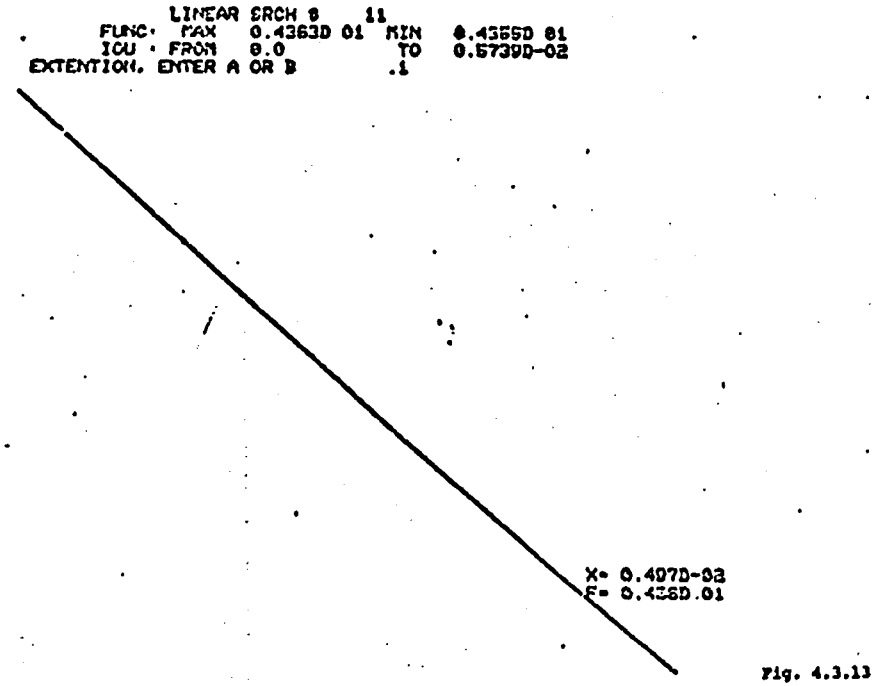


Fig. 4.3.13

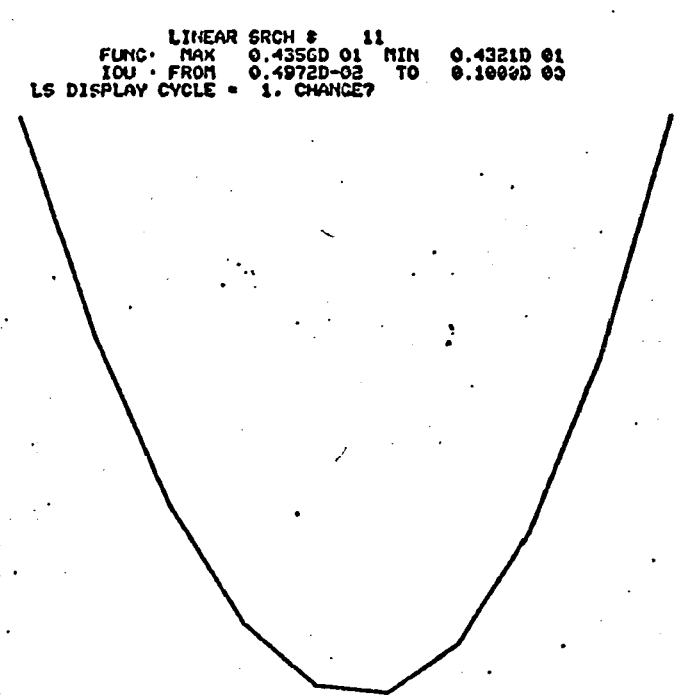


Fig. 4.3.14

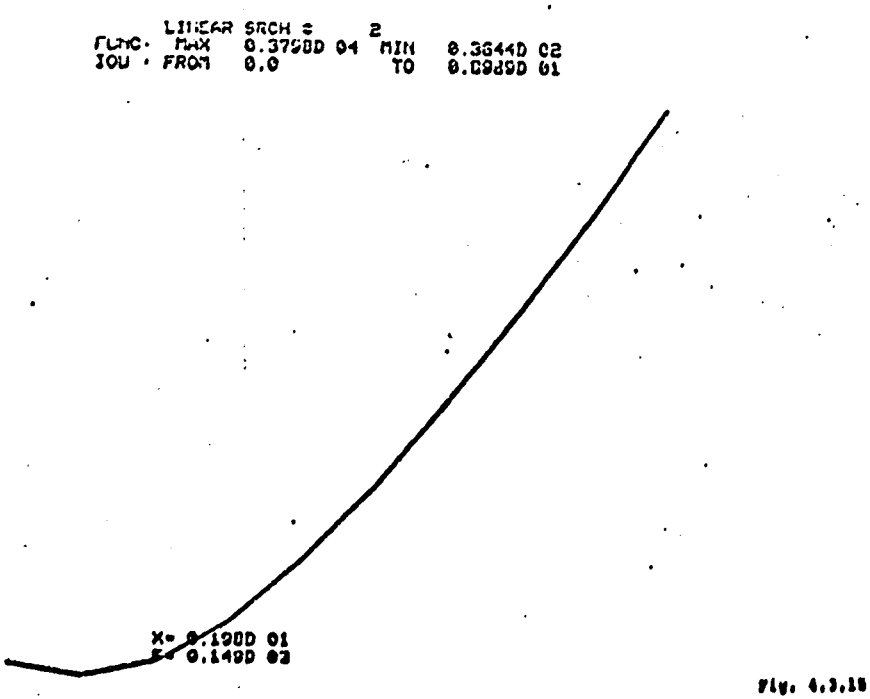


Fig. 4.3.15

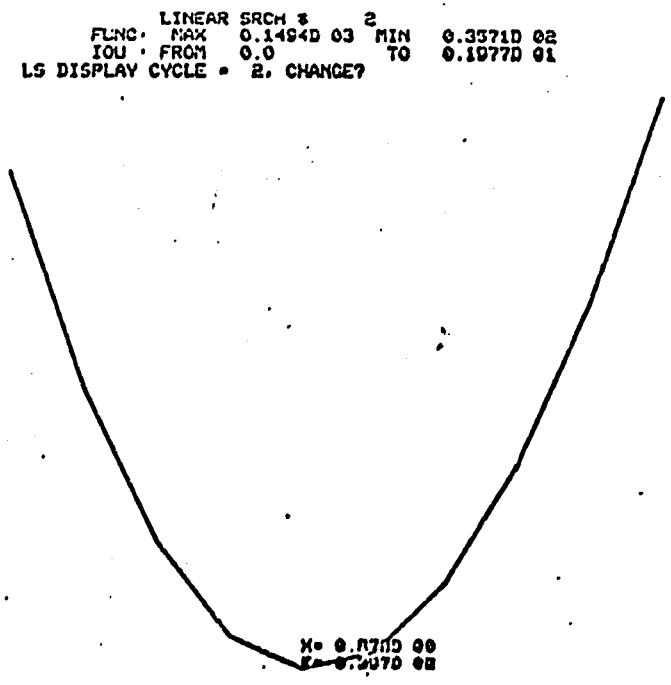


Fig. 4.3.16

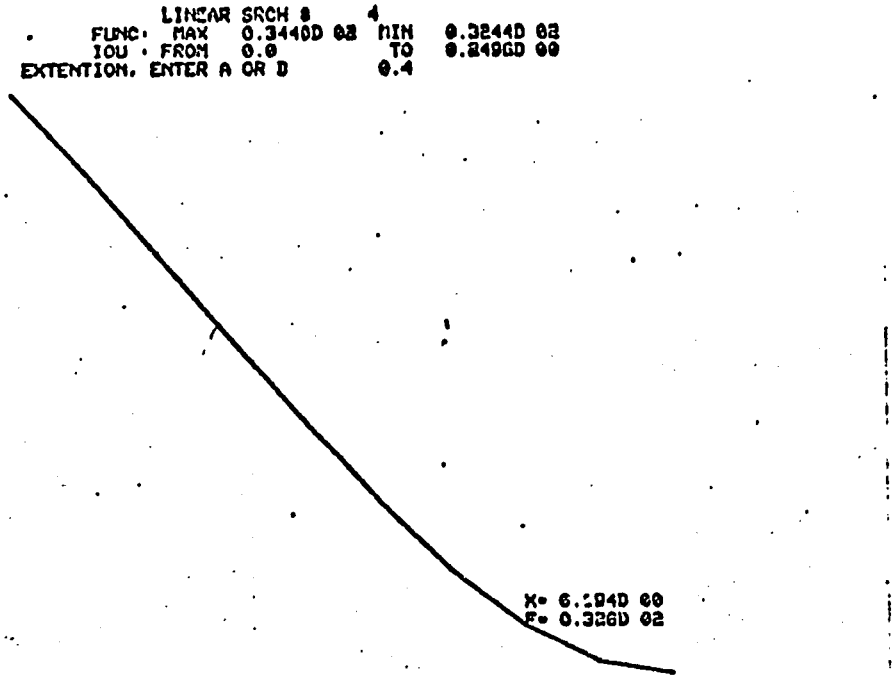


Fig. 4.3.17

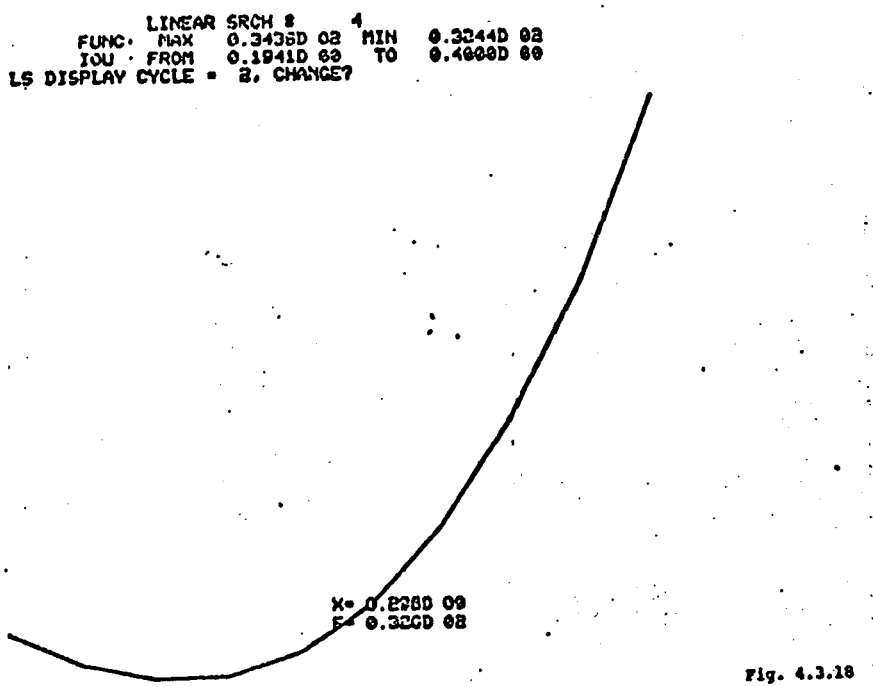


Fig. 4.3.18

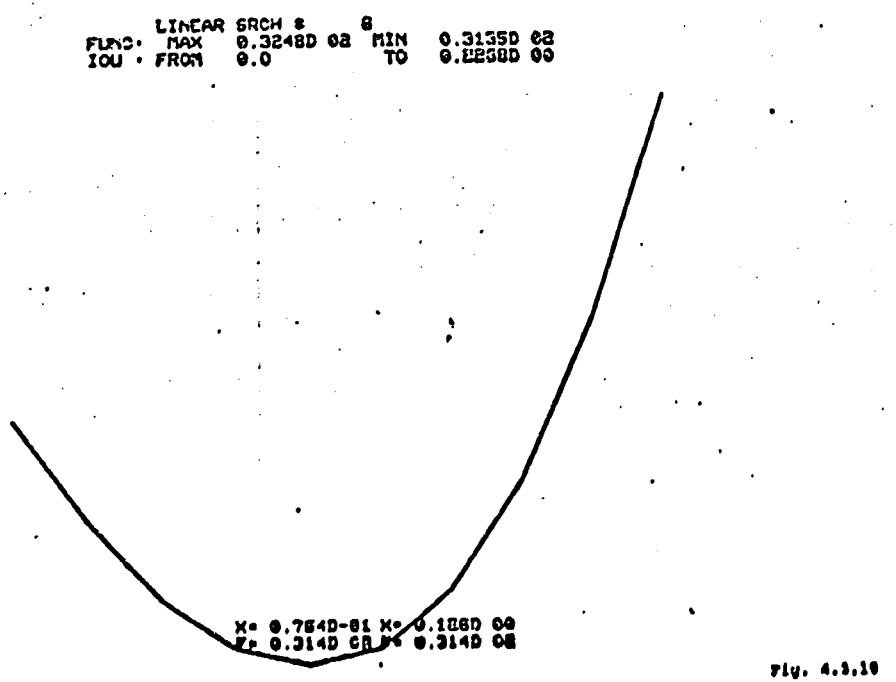


Fig. 4.3.19

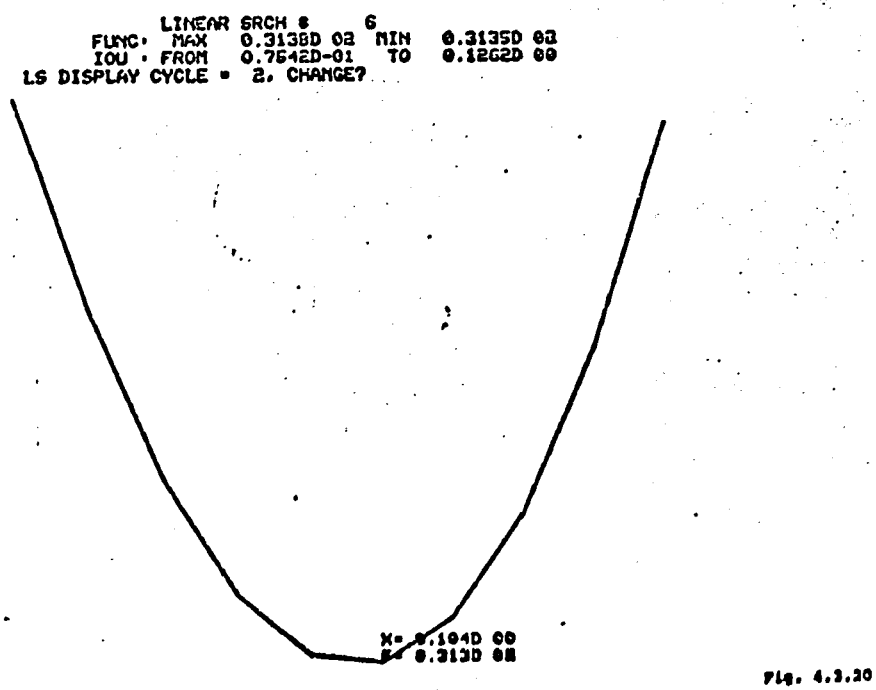


Fig. 4.3.20

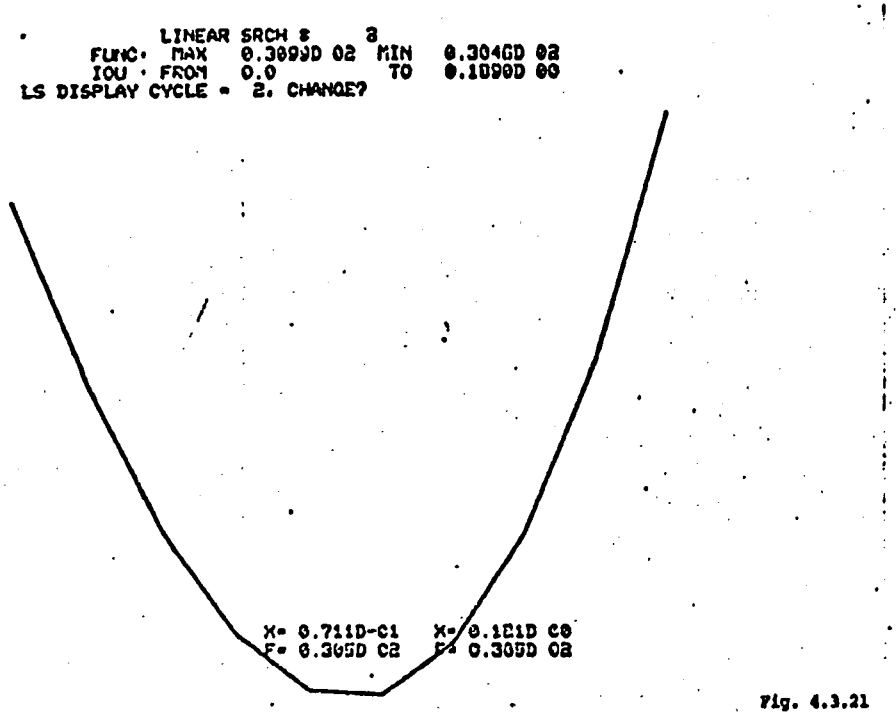


Fig. 4.3.21

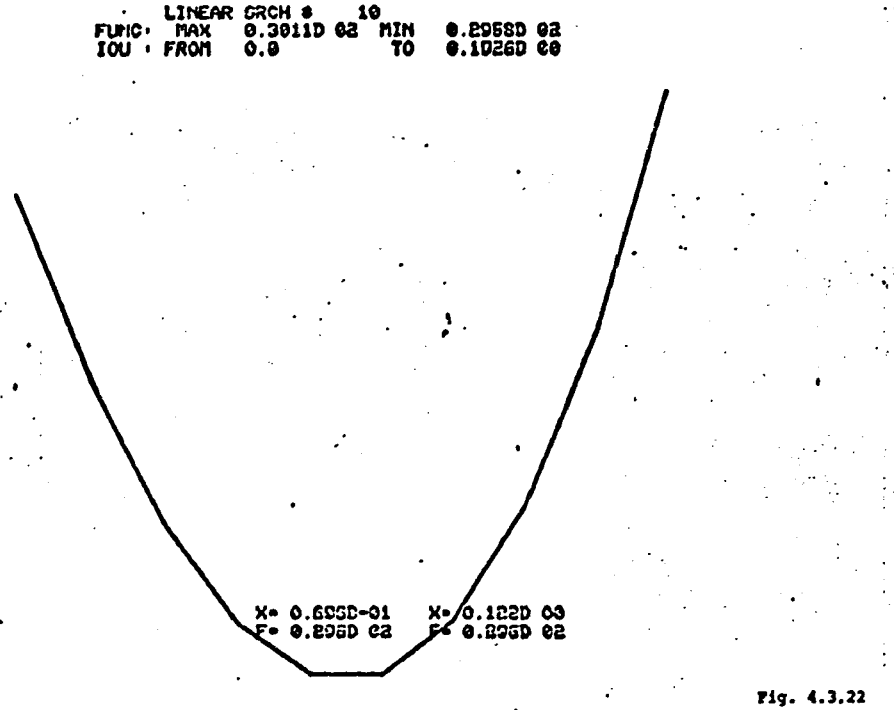


Fig. 4.3.22

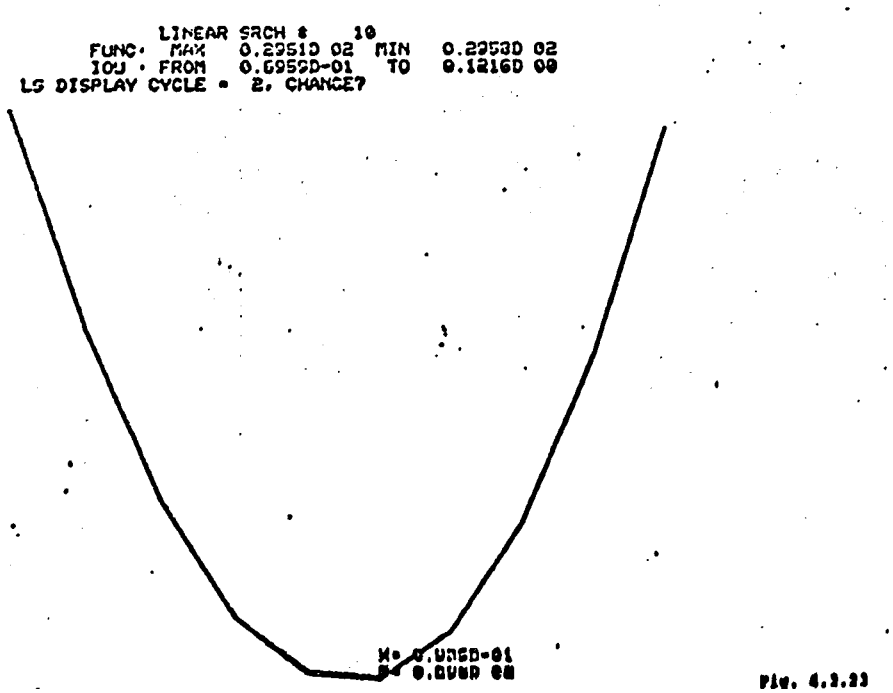
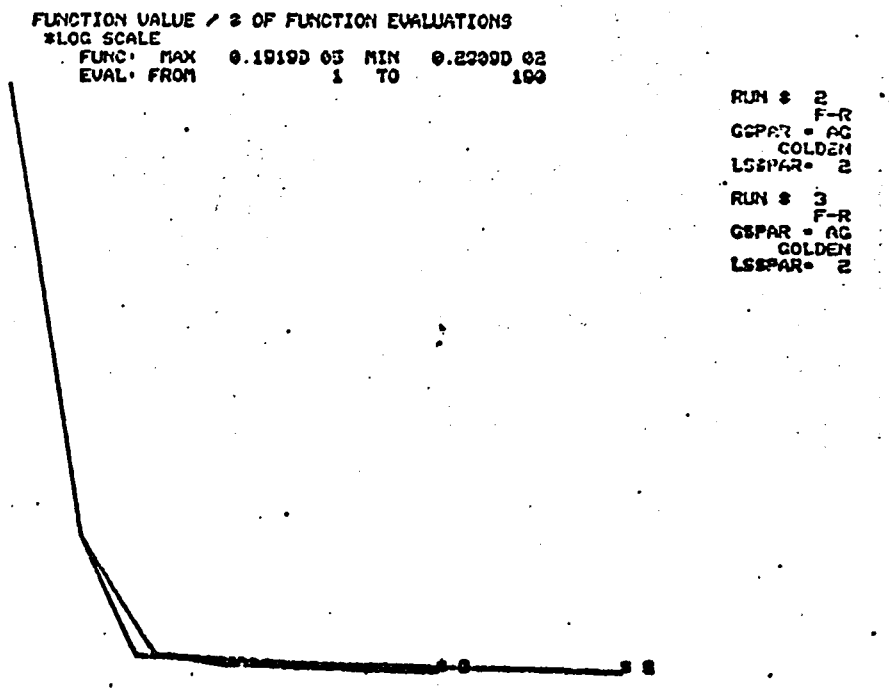


Fig. 4.3.23



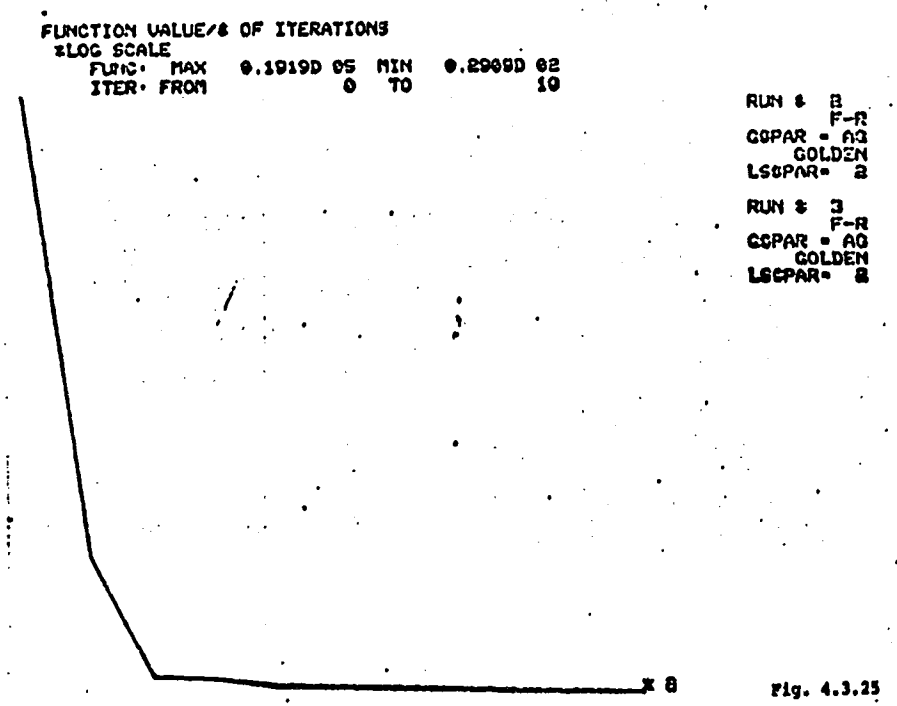

FUNCTION VALUE/# OF ITERATIONS
#LOG SCALE
FUNC. MAX 0.1919D 05 MIN 0.2969D 02
ITER. FROM 0 TO 19
RUN # 2
F-R
CGPAR = AG
GOLDEN
LSCPAR= 2
RUN # 3
F-R
CGPAR = AG
GOLDEN
LSCPAR= 2
X #

Fig. 4.3.25

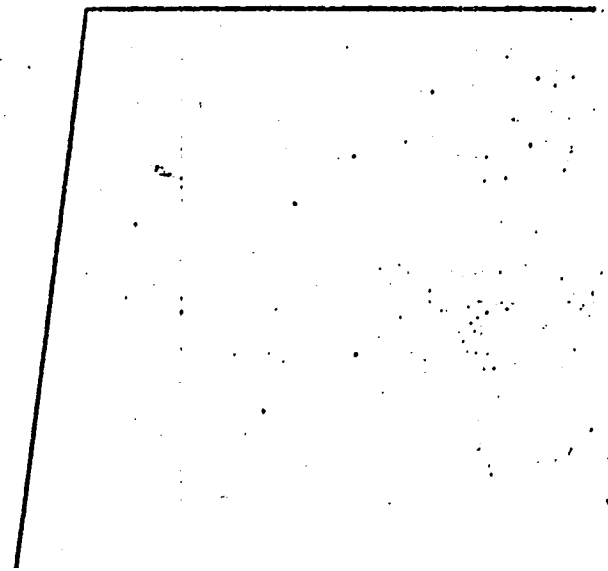
```
  R = 0.0          CHANGE?  10

  WHAT IS YOUR PRIMING GUESS?
22.5.0.5.0.5
  THE NEW PRIMING GUESS IS
   0.225000D 02
   0.500000D 00
   0.500000D 00
  FUNC VALUE =     0.97261D 02

  DO YOU WISH TO RESPECIFY THE PRIMING GUESS? NO
```

Fig. 4.4.1

```
                RUN CONDITIONS

          MINTEC =   D-F-P  / GSPAR =   8

          LNBSRH =  QUADFT  / LSSPAR =   8

  CHANGE?

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  1,1
```

Fig. 4.4.2

```
          LINEAR SRCH #     1
     FUNC:  MAX    0.1000D 05  MIN    0.9726D 02
     IOU :  FROM   0.0           TO    0.2000D 01
LS DISPLAY CYCLE =  1, CHANGE?
```



```
# OF ITERATIONS =     1
# OF FUNC EVAL. =    17
THE ARGUMENT
 0.22500D 02
 0.49877D 00
 0.49907D 00
FUNC VALUE =     0.97261D 02

THE GRADIENT
 0.16456D-01
 0.27492D-02
-0.44089D-02
GRAD NORM  =     0.17267D-01

WHAT NOW?  GO

  RUN CONTROL CYCLE =    1
  LS  DISPLAY CYCLE =    1
    CHANGE?
```

```
        LINEAR SRCH #     2
    FUNC: MAX   0.9726D 02  MIN   0.9726D 02
     IOU : FROM   0.0          TO   0.3987D-02
LS DISPLAY CYCLE =  1. CHANGE?
```

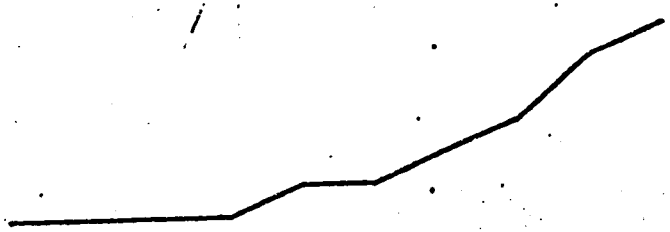


Fig. 4.4.5

```
$ OF ITERATIONS =    2
$ OF FUNC EVAL. =   24
THE ARGUMENT
 0.22499D 02
 0.49867D 00
 0.49923D 00
FUNC VALUE =     0.97261D 02

THE GRADIENT
 0.15102D-01
-0.29303D-01
 0.38092D-01
GRAD NORM =      0.50376D-01

WHAT NOW?  NEW RUN
```

Fig. 4.4.6

```
 R = 0.100D 02    CHANGE?  1

 CURRENT PRIMING GUESS IS
 0.225000 02
 0.500000 00
 0.500000 00
FUNC VALUE =     0.15382D 02

 CURRENT ARGUMENT IS
 0.22499D 02
 0.49867D 00
 0.49926D 00
FUNC VALUE =     0.15369D 02

 REPEAT, CONTINUE OR NEW?   CONTINUE
```

```
                RUN CONDITIONS

        MINTEC =   D-F-P  / CGPAR =   0
        LNSSRH =  QUADFT  / LSSPAR =  2

  CHANGE?

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  4
```

Fig. 4.4.8

```
# OF ITERATIONS =   4
# OF FUNC EVAL. =  43
THE ARGUMENT
 0.22455D 02
 0.37528D 00
 0.38984D 00
FUNC VALUE =    0.14629D 02

THE GRADIENT
 0.55809D-04
-0.22617D-02
 0.46716D-02
GRAD NORM =     0.51787D-02

WHAT NOW? TABLE
```

Fig. 4.4.9

```
RUN SUMMARY
   ITER     SUBITR     #EVAL     FUNC VALUE
      0          0         4     0.15369D 02
      1          1        13     0.14631D 02
      2          2        25     0.14631D 02
      3          3        34     0.14629D 02
      4          4        43     0.14629D 02
```

```
RUN # 2
     D-F-P
QSPAR = 9
     QUADFT
LSSPAR= 2
```

Fig. 4.4.10

```
WHAT NOW? NEW RUN
```

Fig. 4.4.11

```
 R = 0.100D 01    CHANGE? .1

CURRENT PRIMING GUESS IS
0.22500D 02
0.50000D 00
0.50000D 00
FUNC VALUE =    0.71937D 01

CURRENT ARGUMENT IS
0.22455D 02
0.37528D 00
0.38984D 00
FUNC VALUE =    0.57838D 01

REPEAT, CONTINUE OR NEW?  CONTINUE
```

Fig. 4.4.12

```
        RUN CONDITIONS

      MINTEC =   D-F-P  /  GSPAR =   9
      LNSSRH =  QUADFT  / LSSPAR =   2

  CHANGE?

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  2.8
```

Fig. 4.4.13

```
          LINEAR SRCH #     2
 FUNC:  MAX    0.1000D 05  MIN    0.4828D 01
 IOU :  FROM   0.0           TO    0.2000D 00
```

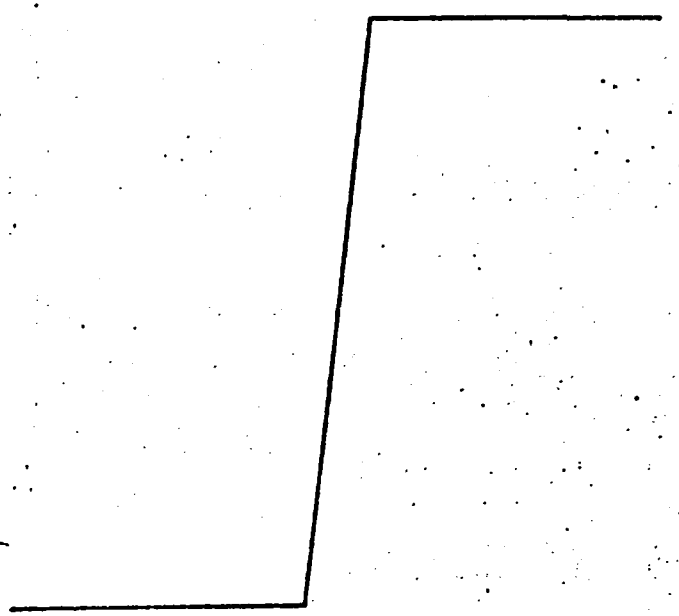


Fig. 4.4.14

```
          LINEAR SRCH #     2
      FUNC:  MAX    0.5422D 01  MIN    0.4000D 01
       IOU :  FROM   0.0           TO    0.8800D-01
LS DISPLAY CYCLE =  2, CHANGE?
```

Fig. 4.4.15

```
# OF ITERATIONS =     8
# OF FUNC EVAL. =    25
THE ARGUMENT
 0.22453D 02
 0.27105D 00
 0.27942D 00
FUNC VALUE =     0.48155D 01

THE GRADIENT
 0.11676D-01
 0.16122D 01
 0.12226D 01
GRAD NORM  =      0.20234D 01

WHAT NOW?  GO

  RUN CONTROL CYCLE =    8
  LS  DISPLAY CYCLE =    8
    CHANGE?
```

```
        LINEAR SRCH #     4
FUNC:  MAX   0.4836D 01  MIN   0.4807D 01
IOU :  FROM  0.0           TO   0.5316D-01
```

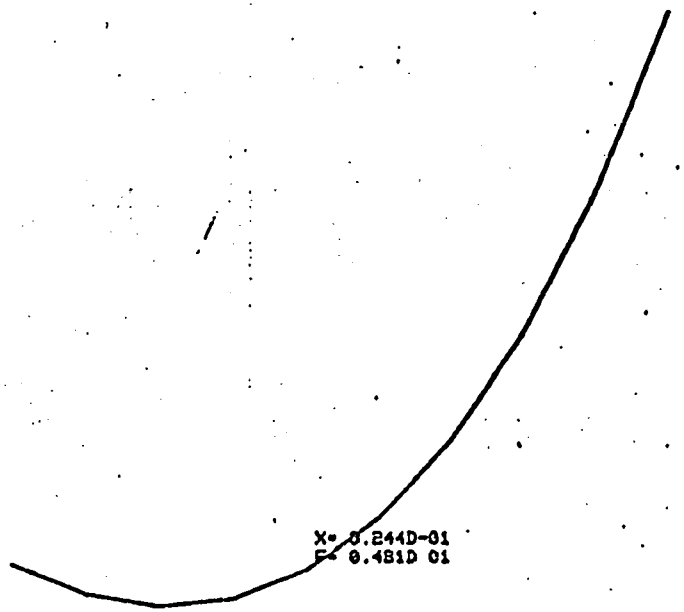


Fig. 4.4.17

```
        LINEAR SRCH #     4
FUNC:  MAX   0.4810D 01  MIN   0.4807D 01
IOU :  FROM  0.0           TO   0.2438D-01
LS DISPLAY CYCLE =  2, CHANGE?
```

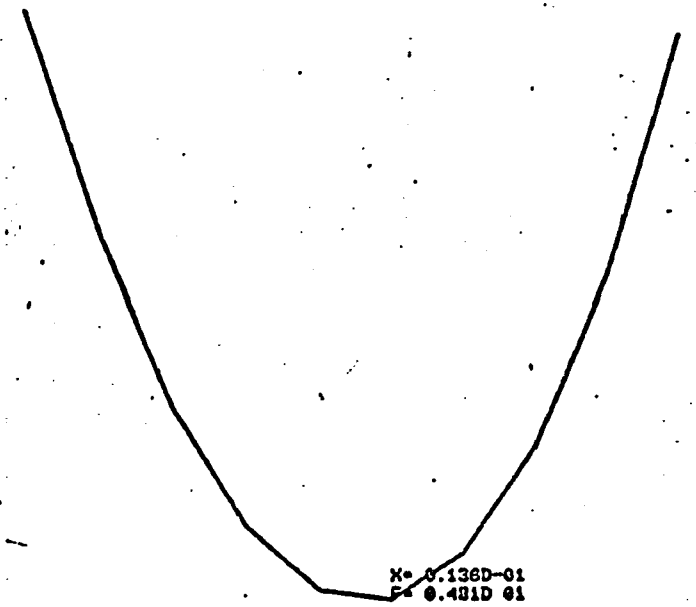


Fig. 4.4.18

```
# OF ITERATIONS =    4
# OF FUNC EVAL. =   39
THE ARGUMENT
 0.22447D 02
 0.26343D 00
 0.22610D 00
FUNC VALUE =    0.48075D 01

THE GRADIENT
 0.15251D-01
-0.61936D-01
-0.38166D-01
GRAD NORM  =     0.74433D-01

WHAT NOW?  GO

  RUN CONTROL CYCLE =    2
  LS  DISPLAY CYCLE =    8
    CHANGE?
```

```
        LINEAR SRCH #     6
FUNC:  MAX   0.4806D 01  MIN   0.4804D 01
IOU :  FROM  0.0           TO   0.2410D 00
EXTENTION, ENTER A OR B       0.5
```

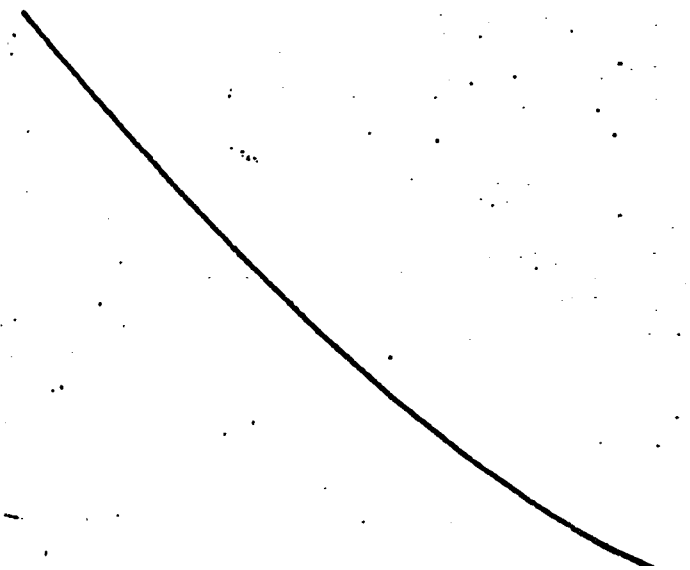


Fig. 4.4.19

```
          LINEAR SRCH #     6
    FUNC:  MAX    0.4804D 01  MIN    0.4803D 01
     IOU:  FROM   0.2407D 00   TO    0.5000D 00
LS DISPLAY CYCLE =  2, CHANGE?
```

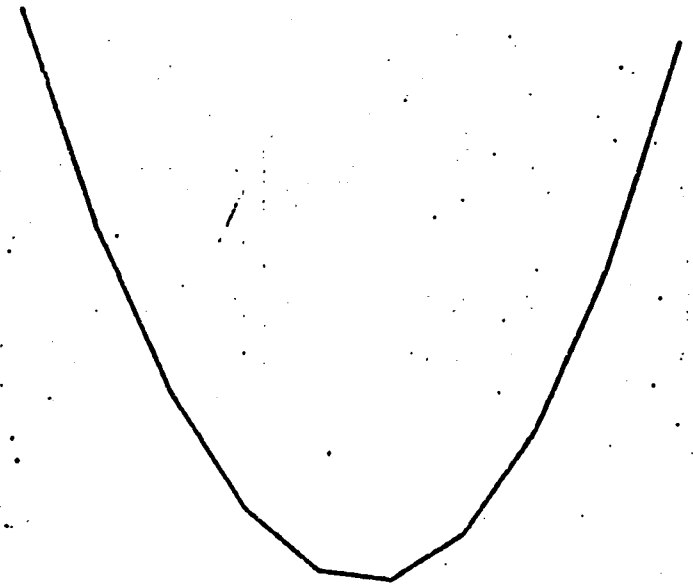


Fig. 4.4.21

```
# OF ITERATIONS =    6
# OF FUNC EVAL. =   59
THE ARGUMENT
 0.219510 02
 0.252650 00
 0.225350 00
FUNC VALUE =     0.48034D 01

THE GRADIENT
-0.11415D-03
-0.32468D-01
-0.43378D-01
GRAD NORM  =     0.54184D-01

WHAT NOW? TABLE
```

Fig. 4.4.22

```
RUN SUMMARY
   ITER      SUBITR      #EVAL      FUNC VALUE
      0           0          4     0.57638D 01
      1           1         16     0.50403D 01
      2           2         26     0.48155D 01
      3           3         32     0.48096D 01
      4           4         39     0.48078D 01
      5           5         50     0.48064D 01
      6           6         59     0.48034D 01
```

Fig. 4.4.23

```
RUN # 3
      D-F-P
QSPAR = 9
      QUADFT
LSSPAR= 8
```

```
WHAT NOW? GO

  RUN CONTROL CYCLE =    8
  LS  DISPLAY CYCLE =    8
    CHANGE? 1.1
```

Fig. 4.4.24

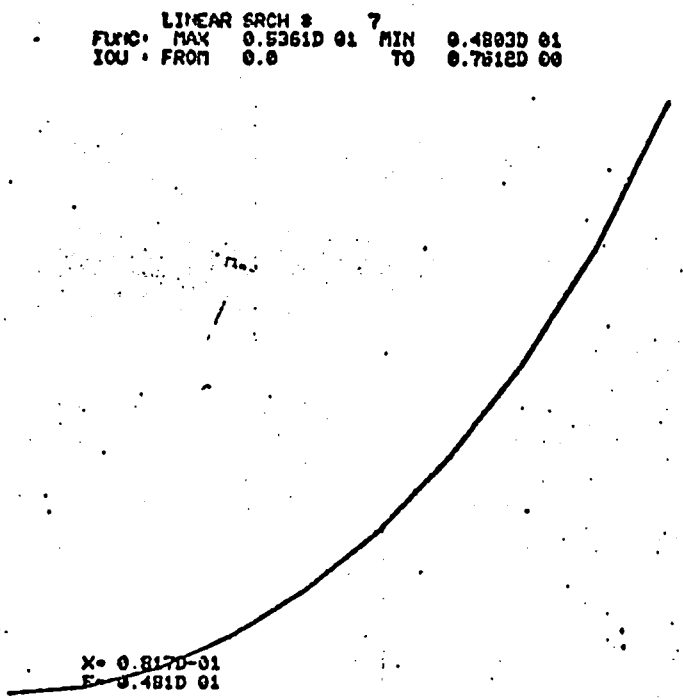


Fig. 4.4.25

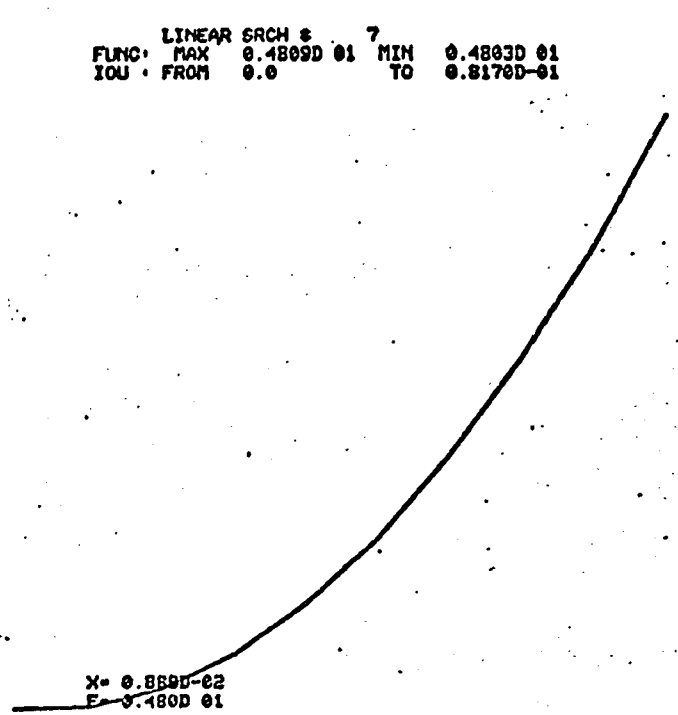


Fig. 4.4.26

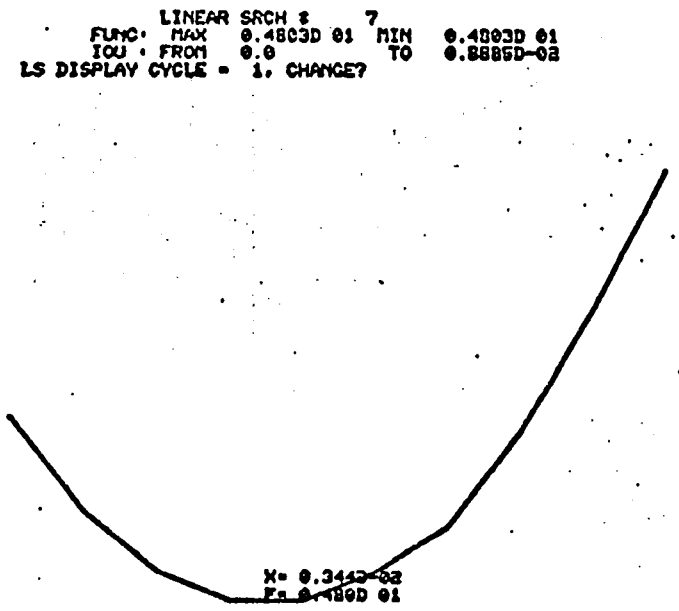


Fig. 4.4.27

```
# OF ITERATIONS =    7
# OF FUNC EVAL. =   70
THE ARGUMENT
 0.21954D 02
 0.26197D 00
 0.26737D 00
FUNC VALUE =     0.48034D 01

THE GRADIENT
 0.34373D-04
-0.69278D-03
-0.56866D-03
GRAD NORM  =     0.89896D-03

WHAT NOW?  NEW RUN
```

Fig. 4.4.28

```
 R = 0.100D 00    CHANGE?  .01

 CURRENT PRIMING GUESS IS
 0.22500D 02
 0.50000D 00
 0.50000D 00
FUNC VALUE =    0.63749D 01

 CURRENT ARGUMENT IS
 0.21954D 02
 0.26197D 00
 0.28737D 00
FUNC VALUE =    0.35724D 01

 REPEAT, CONTINUE OR NEW?   CONTINUE
```

Fig. 4.4.29

```
# OF ITERATIONS =    7
# OF FUNC EVAL. =   78
THE ARGUMENT
 0.20008D 02
 0.72038D-02
 0.57392D 00
FUNC VALUE =    0.30557D 01

THE GRADIENT
 0.99354D-01
 0.60159D 01
 0.43035D 01
GRAD NORM  =    0.73974D 01

WHAT NOW?  NEW RUN
```

Fig. 4.4.30

```
 R = 0.100D-05    CHANGE?  .1D-6

 CURRENT PRIMING GUESS IS
 0.22500D 02
 0.50000D 00
 0.50000D 00
FUNC VALUE =    0.62839D 01

 CURRENT ARGUMENT IS
 0.20008D 02
 0.72038D-02
 0.57392D 00
FUNC VALUE =    0.30551D 01

 REPEAT, CONTINUE OR NEW?   CONTINUE
```

Fig. 4.4.31

```
                RUN CONDITIONS

      MINTEC =   D-F-P  / GSPAR =   9

      LNSRH =  QUADFT  / LSPAR =   8

  CHANGE?

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  4,4
```

Fig. 4.4.32

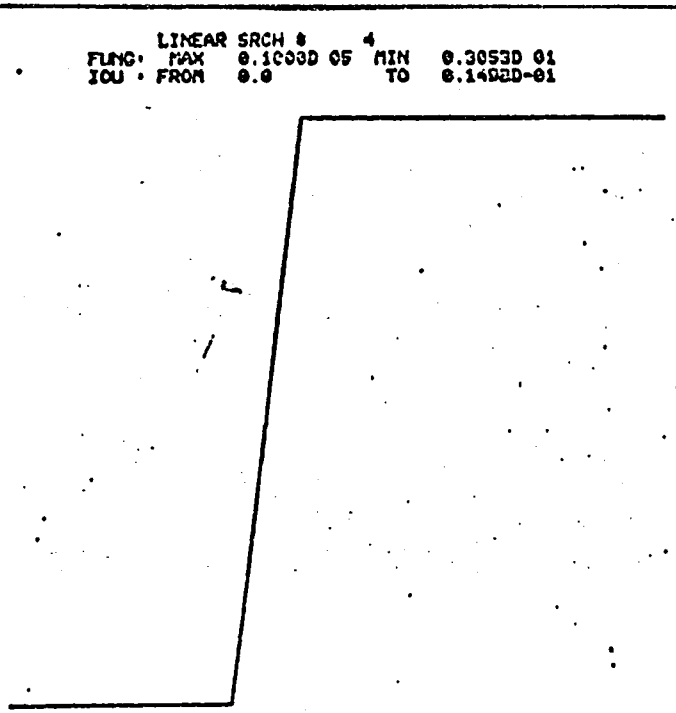


Fig. 4.4.33

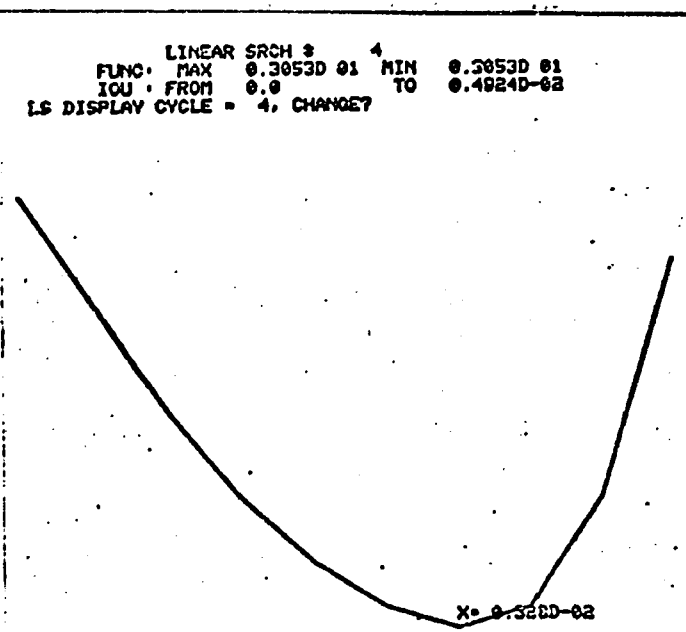


Fig. 4.4.34

```
# OF ITERATIONS =    4
# OF FUNC EVAL. =   47
THE ARGUMENT
 0.200000 02
 0.971960-03
 0.532290 00
FUNC VALUE =    0.30529D 01

THE GRADIENT
-0.84981D-02
-0.526230 03
-0.358710 00
GRAD NORM  =    0.63689D 00

WHAT NOW? GO

  RUN CONTROL CYCLE =    4
  LS  DISPLAY CYCLE =    4
    CHANGE? 1.1
```

Fig. 4.4.35

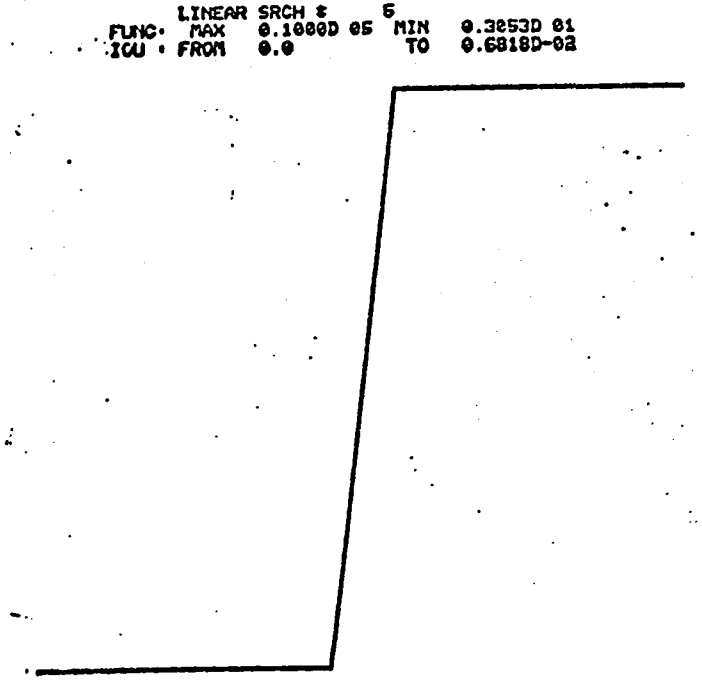


Fig. 4.4.36

```
          LINEAR SRCH #     5
    FUNC: MAX   0.3053D 01  MIN   0.3053D 01
     IOU: FROM  0.0          TO   0.1387D-02
LS DISPLAY CYCLE =  1, CHANGE?
```

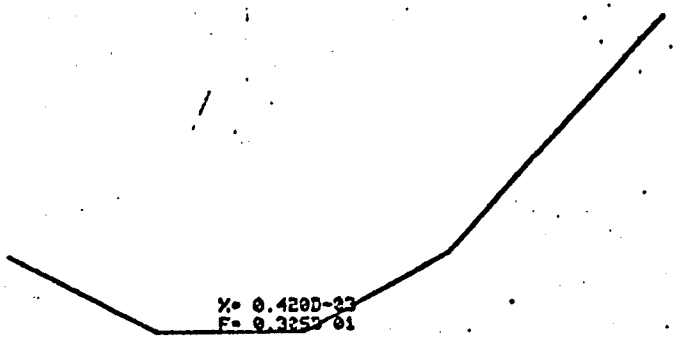


Fig. 4.4.37

```
# OF ITERATIONS =    5
# OF FUNC EVAL. =   55
THE ARGUMENT
 0.20003D 02
 0.10843D-02
 0.58215D 00
FUNC VALUE =    0.30529D 01

THE GRADIENT
-0.90617D-02
-0.36536D 00
-0.25761D 00
GRAD NORM =     0.44714D 00

WHAT NOW?  NEW RUN
```

Fig. 4.4.38

```
 R = 0.100D-06    CHANGE?  .1D-7

CURRENT PRIMING GUESS IS
0.22500D 02
0.50000D 00
0.50000D 00
FUNC VALUE =    0.62839D 01

CURRENT ARGUMENT IS
0.20003D 02
0.10843D-02
0.58215D 00
FUNC VALUE =    0.30525D 01

REPEAT, CONTINUE OR NEW?   CONTINUE
```

```
                RUN CONDITIONS

     MINTEC =   D-F-P  / GSPAR =   9

     LNSSRH =   QUADFT / LSSPAR =  2

CHANGE?

WHAT NOW?  GO

  RUN CONTROL CYCLE =    5
  LS  DISPLAY CYCLE =    5
    CHANGE?  .6
```

Fig. 4.4.40

```
# OF ITERATIONS =    5
# OF FUNC EVAL. =   60
THE ARGUMENT
 0.20201D 02
 0.36254D-03
 0.50315D 00
FUNC VALUE =     0.30623D 01

THE GRADIENT
-0.60723D-03
-0.51185D-01
-0.37355D-01
GRAD NORM  =     0.63373D-01

WHAT NOW?  END JOB
```

Fig. 4.4.41

CHAPTER 5

CONCLUSIONS

The process of locating the minimum of a nonlinear function is highly experimental in nature. Although there exist many powerful algorithms for function minimization, none is guaranteed to converge to a minimum under all circumstances. In addition, there are associated with most algorithms several secondary parameters and tasks which can significantly affect the success of their operation. Therefore, the usefulness of a software package that provides several algorithms together with various mechanisms for handling the secondary tasks, is apparent.

When such a package functions in a batch computing environment, the user is obliged to accept significant delays between minimization experiments. Since the determination of an acceptable solution may require many experiments involving technique and parameter changes, the total elapsed time in obtaining a solution may be long. This difficulty can be overcome through the use of an interactive approach in the implementation of the function minimization package. The development of the INTEROPT package was motivated by these considerations.

In addition to the basic features one would expect in a function minimization package (various techniques and alternatives for handling secondary tasks), INTEROPT also provides plotting facilities on the graphics terminal which serves as the user console (a Tektronix 4013 or equivalent). Thus numerical results can be easily presented in the more meaningful form of graphs. These, in turn,

help the user to rapidly evaluate the progress of a minimization trial.

User control over both the minimization process and the graphical output are available at the terminal during the running of the program. The response of the system to user commands rarely exceeds two or three seconds.

The package is designed to conveniently allow the testing and evaluations of new minimization algorithms as well as new linear search algorithms. This is achieved through the provision of subroutine calls to "dummy" subroutines (YOURSA, YOURSB, YOURSL) which are supplied by the user according to his own specifications.

The modular design of the package allows extensions and alterations to be made easily. This applies both to the inclusion of new algorithms as well as the addition of new interactive or graphical capabilities. Once a new algorithm has been evaluated and found to be efficient, it can be added permanently to the package with a few minor modifications. Similarly, if the need arises to extend the currently available graphical output or the present set of user commands, only a few changes in the package will be required.

In its present form, INTEROPT can accommodate functions having at most eight parameters (dimensions). In order to handle problems of higher dimension some re-dimensioning of arrays must be done in several of the subroutines in the package.

A particularly noteworthy feature of INTEROPT is its capability of directly handling the interior penalty function method for constrained minimization problems. With this approach, the constrained problem is treated as a sequence of unconstrained subproblems each

of which is concerned with minimizing a "penalty function" that is constructed from the original criterion function and the constraints. Each sub-problem must be solved to a "reasonable" degree of accuracy at which point a scalar parameter in the penalty function is altered to form a new sub-problem. Its solution begins at the last argument value determined for the preceding problem.

In a batch computing environment the user has very limited control over this procedure since he has no means of determining when the minimum of a particular sub-problem has been adequately reached. In an interactive environment, however, the user is able to determine from the results that are displayed on the screen, whether to go on or to stop the sub-program solution. Thus in an interactive environment such as provided by INTEROPT, only the amount of computing that is actually required, is committed to each sub-problem solution (as determined by the judgement of the user).

The present INTEROPT package is initiated by the submission of a deck of cards which contains the job control cards, a function sub-program for the criterion function and, optionally, a subroutine for the gradient of the function. Hence, INTEROPT is not strictly speaking, totally interactive. This is due to the fact that the IBM/360 model 65 on which INTEROPT presently operates, provides very limited interactive facilities. A significant improvement of the package would be to introduce a means of handling the whole job interactively. This could be achieved, for example, through the use of the YTS system on IBM/360.

Another enhancement would be the more extensive use of disk files to store the summary tables. This would reduce the amount of

core memory required by INTEROPT and improve the capability for storing more information about the various runs. However, this possibility should be studied carefully since it would lead to a significant increase in I/O activity which could counter any advantage that might be anticipated.

APPENDIX A1

THE MULTIDIMENSIONAL MINIMIZATION TECHNIQUES IN INTEROPT

This appendix outlines the various multidimensional minimization techniques implemented in INTEROPT. The following notation is used in the description of the algorithms:

- x^k — the argument of the criterion function at the end of the k^{th} iteration (an n-vector), (x^0 is the priming or starting point).
- $f(x^k)$ — the value of the criterion function at x^k.
- $f_x(x^k)$ — the value of the function gradient at x^k, $g_k=f_x(x^k)$.
- s_k — the current search direction (an n-vector).
- α^* — the optimal step along the direction s_k, i.e. it is the minimizing argument of the function $F(\alpha)=f(x^k+\alpha s_k)$. The notation $\alpha^*=\min_\alpha<f,x^k,s_k>$ is used to denote the scalar α^* for which $f(x^k+\alpha^* s_k)=\min_\alpha f(x^k+\alpha s_k)$

A1.1 The Steepest Descent Algorithm (GRAD)

1. Choose x^0 (the priming guess) and set k=0.
2. Let $s_k=-f_x(x^k)$.
3. Find $\alpha_k^*=\min_\alpha<f,x^k,s_k>$
4. $x^{k+1}=x^k+\alpha_k^* s_k$.
5. Replace k with k+1 and go to step 2.

A1.2 Conjugate Gradient Algorithm of Fletcher and Reeves (F-R) [12]

1. Choose x^0 (the priming guess), let $s_0=-g_0=-f_x(x^0)$ and set k=0.
2. Let $x^{k+1}=x^k+\alpha^*_k s_k$ where $\alpha^*_k=\min_\alpha<f,x^k,s_k>$.

3. Set $s_{k+1}=-g_{k+1}+\beta_k s_k$

   where $\beta_k = \dfrac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$

4. Replace k with k+1 and go to step 2.

A1.3 Conjugate Gradient Algorithm of Polack and Ribiere (P-R) [13]

Same as the Fletcher-Reeves algorithm, except that:

$$\beta_k = \frac{g_{k+1}^T y_k}{g_k^T g_k}$$

where $y_k=g_{k+1}-g_k$

A1.4 Conjugate Gradient Algorithm of Sorenson (SOREN) [14]

Same as the Fletcher-Reeves algorithm, except that:

$$\beta_k = \frac{g_{k+1}^T y_k}{s_k^T y_k}$$

where $y_k=g_{k+1}-g_k$

A1.5 The Davidon-Fletcher-Powell Algorithm (D-F-P) [15]

1. Choose x^0 (the priming guess), let $H_0=I$ (the n×n identity matrix), $s_0=-g_0=-f(x^0)$ and set k=0.
2. Let $x^{k+1}=x^k+\alpha^*_k s_k$

   where $s_k=-H_k g_k$

   and $\alpha^*_k=\min_{\alpha} \langle f,x^k,s_k\rangle$

3. Set $A_k = \dfrac{\alpha^*_k}{s_k^T y_k} \cdot s_k s_k^T$

   $$B_k = \frac{(H_k y_k)(H_k y_k)^T}{y_k^T H_k y_k}$$

$$H_{k+1} = H_k + A_k + B_k$$

4. Replace k with k+1 and go to step 2.

A1.6 Powell's Algorithm (POWELL) [16]

Let the k^{th} iteration begin at x^0

1. For j=1,2,...,n find $\alpha^*_j = \min_\alpha \langle f, x^{j-1}, \xi_j \rangle$ and let $x^j = x^{j-1} + \alpha^*_j \xi_j$.
2. For i=1,2,...,(n-1), replace ξ_i with ξ_{i+1}.
3. Let $\xi_n = (x^n - x^0) / |x^n - x^0|$
4. Find $\alpha^* = \min_\alpha \langle f, x^n, \xi_n \rangle$ and replace x^0 with $(x^n + \alpha^* \xi_n)$.

Note For the first iteration, $\xi_j = e_j$ = the j^{th} column of the n×n identity matrix

A1.7 Zangwill's Method (ZANG) [17]

First Iteration

Let e_j, j=1,2,...,n be the columns of the n×n identity matrix and let ξ_j, j=1,2,...,n be a normalized set of n linearly independent directions.

Let x^0 be the initial guess, and find

$$\alpha^* = \min_\alpha \langle f, x^0, \xi_n \rangle$$

Replace x^0 with $(x^0 + \alpha^* \xi_n)$.

Set t=1.

Subsequent Iterations

Let the iteration begin at x^0.

1. Find $\alpha^* = \min_\alpha \langle f, x^0, e_t \rangle$ and replace t with t+1 if t<n or 1 if t=n.
2. If $\alpha^* \neq 0$, replace x^0 with $(x^0 + \alpha^* e_t)$ and go to step 3. Otherwise go to step 1 (Note: if step 1 is repeated n times in succession,

then we can assume that x^0 is the optimum).

3. For j=1,2,...,n, calculate $\alpha_j^*=\min_\alpha \langle f,x^{j-1},\xi_j\rangle$ and let
$x^j=x^{j-1}+\alpha_j^*\xi_j$.

4. For j=1,2,...,(n-1), replace ξ_j with ξ_{j+1}.

5. Let $\xi_n = \dfrac{x^n-x^0}{|x^0-x_n|}$ and find $\alpha^*=\min_\alpha \langle f,x^n,\xi_n\rangle$

6. Replace x^0 with $x^n+\alpha^*\xi_n$ and go to step 1.

A1.8 The Extended Sequential Search Method (ESQ)

1. Choose x^0 (the priming guess).

2. Set j=1

3. Let $x^j=x^{j-1}+\alpha^*s_j$

where $\alpha^*=\min_\alpha \langle f,x^{j-1},s_j\rangle$

and $$s_j = \begin{cases} e_j & \text{if } j\le n \\ \dfrac{x^n-x^0}{|x^n-x^0|} & \text{if } j=n+1 \end{cases}$$

4. If j=n+1, replace x^0 with x^{n+1} and go to step 2; otherwise replace j with j+1 and go to step 3.

APPENDIX A2

THE LINEAR SEARCH ALGORITHMS

The minimization algorithms within INTEROPT require a mechanism for solving the following one-dimensional minimization (or linear search) problem.

Find the scalar α^* such that

$$F(\alpha^*) = \min_{\alpha} F(\alpha)$$

where: $F(\alpha) = f(x^k+\alpha s^k)$, and x^k and s^k are given n-vectors.

There are three algorithms for handling this problem in INTEROPT. This appendix briefly outlines their operation.

A2.1 The Quadratic Fit (QUADFT) [18]

If three points α_1, α_2 and α_3 can be selected to span the minimum value of $F(\alpha)$, the function can be approximated with the quadratic polynomial

$$p(\alpha) = a_0+a_1\alpha+a_2\alpha^2$$

The approximate position of the minimum of F is then derived by setting $\frac{dp}{d\alpha} = 0$, giving

$$\alpha_{min} = \frac{a_1}{2a_2}$$

If the function values at the three selected points are F_1, F_2 and F_3 respectively, then upon solving for a_1, a_2 and a_3 we obtain

$$\alpha_{min} = \frac{1}{2}\,\frac{(\alpha_2^2-\alpha_3^2)F_1+(\alpha_3^2-\alpha_1^2)F_2+(\alpha_1^2-\alpha_2^2)F_3}{(\alpha_2-\alpha_3)F_1+(\alpha_3-\alpha_1)F_2+(\alpha_1-\alpha_2)F_3}$$

This predicted minimum is then used with the two best of the three points α_1, α_2 and α_3 for a second quadratic approximation and the process is repeated.

In INTEROPT, the subroutine I$IOU selects the three points that span the minimum of $F(\alpha)$. Then, the subroutine QUADFT performs successive quadratic approximations from three points provided by the predicted minimum at the previous iteration and the two best of the three points used to predict it. The process stops when the distance between two predicted minima in two successive iterations is less than or equal to 10^{-m} where m is the parameter LS$PAR. The best point obtained in this procedure is taken to be approximation for the minimizing argument of $F(\alpha)$.

A2.2 The Fibonacci Search (FBNACI) [19]

The iterative steps of the Fibonacci search are implemented in the subroutine FBNACI. An initial interval of uncertainty (A_1,B_1) is passed to the subroutine. The Fibonacci search reduces this interval according to the relation:

$$|A_n-B_n| = \frac{L_1+F_{n-1}\epsilon}{F_{n+1}}, \qquad n\geq 2$$

where L_1 is the length of the initial interval of uncertainty (B_1-A_1), F_{n-1} and F_{n+1} are the $n-1^{th}$ and $n+1^{th}$ Fibonacci numbers respectively, and ϵ small positive number set to 10^{-10}. The reduction of the interval continues until one of the following occurs:

1. The interval becomes less than or equal to 10^{-m} where m is the parameter LS$PAR.
2. The relative difference between two distinct function values of a particular iteration is less than 10^{-15}.
3. A maximum of 70 steps are made before either 1 or 2 occurs.

The end points of the final interval together with the mid point of this interval are used in a quadratic fit procedure (a

call to QUADFT) and the minimizing argument of this quadratic is taken as the final result for the linear search.

A2.3 The Golden Search (GOLDEN) [18]

The Golden Search Algorithm is implemented in the subroutine GOLDEN. First, the initial interval of uncertainty, which contain the minimum of the function, is determined by the subroutine I$IOU (its left and right limits are denoted by A_1 and B_1 respectively). Then the subroutine GOLDEN applies the iterative steps of the golden section search to reduce this interval according to the relation

$$|A_n - B_n| = K^n * L_1, \qquad n \geq 1$$

where L_1 is the length of the initial interval of uncertainty $(B_1 - A_1)$, and K is the constant $= (\sqrt{5}-1)/2$.

The reduction process is stopped when the length of the interval is less than or equal to 10^{-m} where m is the parameter LS$PAR. The last two points generated by the procedure (A_n and B_n), together with their mid-points are then used in a quadratic approximation which yields one additional point; (namely, the minimum of the fitted quadratic). The best of the available points is taken to be the approximation for α^*.

APPENDIX A3

TEST PROBLEMS

The various example problems discussed in Chapter 4, use standard test problems. These are summarized in this appendix*.

A3.1 The Wood Function [20]

$$f(x) = 100(x_2-x_1^2)^2+(1-x_1)^2+90(x_4-x_3^2)^2 + (1-x_3)^2+10.1[(x_2-1)^2+(x_4-1)^2]+19.8(x_2-1)(x_4-1)$$

$$x^0 = (-3, -1, -3, -1)$$

$$x^* = (1, 1, 1, 1)$$

By specific design, this function has a non-optional stationary points which can cause inappropriate termination of minimization algorithms. These stationary points are at (-1.07, 1.16, -0.86, 0.76) and (-0.968, 0.947, -0.9695, 0.951).

A3.2 The Powell Function [21]

$$f(x) = (x_1+10x_2)^2+5(x_3-x_4)^2+(x_2-2x_3)^4+10(x_1-x_4)^4$$

$$x^0 = (3, -1, 0, 1)$$

$$x^* = (0, 0, 0, 0)$$

This function has the distinctive feature of having a Hessian matrix (matrix of second partial derivatives) which is singular at the minimizing argument, x*. This feature can undermine the effectiveness of minimization algorithms.

---

* In the discussion, x^0 is used to denote the standard starting (or priming) point and x* is used to denote the minimizing argument.

A3.3 The Multi-dimensional Banana Function [22]

$$f(x) = \sum_{k=1}^{4} 100(x_{k+1}-x_k^2)^2+(1-x_k)^2$$

$$x^0 = (-1.2,\ 1,\ -1.2,\ 1,\ -1.2)$$

$$x^* = (1,\ 1,\ 1,\ 1,\ 1)$$

This function is an extension of the Rosenbrock function [23] whose graph is distinguished by a banana-shaped valley. As such, f(x) represents a set of intersecting shallow parabolic valleys.

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