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Linear and Nonlinear Deflection Analysis of Thick Rectangular Plates Using Finite Differences

By

Nasr-Eddine Bencharif

A thesis
presented to the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
Civil Engineering



DEPARTMENT OF CIVIL ENGINEERING
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Abstract

Variational methods are widely used for the solution of complex differential equations in mechanics for which exact solutions are not possible. The finite difference method, although well known as an efficient numerical method was applied in the past only for the solution of linear and nonlinear thin plates.

In the present study, the suitability of the method for the solution of non-linear deflection of thick plates is studied for the first time. While there is major differences between small deflection and large deflection plate theories, the former can be treated as a particular case of the latter, when the centre deflection of the plate is less than or equal to 0.2-0.25 of the thickness of the plate. The finite difference method as applied here is a modified finite difference approach to the ordinary finite difference method generally used for the solution of thin plate problems. In this thesis thin plates are treated as a particular case of the corresponding thick plate when the boundary conditions of the plates are taken into account.

The method is first applied to investigate the deflection behaviour of square clamped and simply supported square isotropic thick plates. After the validity of the method is established, it is then extended to the solution of rectangular thick plates of various aspect ratios and thicknesses.

Generally, beginning with the use of a limited number of mesh sizes for a given plate aspect ratio and boundary conditions, a general solution of the problem including the investigation of accuracy and convergence was

extended to rectangular thick plates by providing more detailed functions satisfying the rectangular mesh sizes generated automatically by the programme.

Whenever possible results of the present method are compared with the existing solutions in the technical literature obtained by much more laborious methods and close agreements are found. Significant amounts of results presented herein are not currently available in the technical literature for various plate aspect ratios and Poisson's ratios. The submatrices involved in the formation of the finite difference equations from the governing differential equations forming the general system are generated directly by the computer programme. The subroutine SOLINV from the second directed method as developed and illustrated in Chapter V takes care of the inversion of the general matrix. The subroutine developed by the author has been proven to be more efficient than the former methods known for the computation of linear simultaneous equations [61].

Simplicity in formulation and quick convergence are the obvious advantages of the finite difference formulation developed here for small and large deflection analysis of thick plate in comparison with other numerical methods requiring extensive computer facilities.

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Notations

x, y, z	rectangular co-ordinate system.
ϕ, χ, ψ	dimensionless rectangular co-ordinate system.
u, v, w	displacements of the nodes in x, y, and z direction respectively.
U, V, W	dimensionless displacements of the nodes in $\phi, \chi,$ and ψ direction respectively.
a	longitudinal dimension of the plate.
b	transverse dimension of the plate.
h	thickness of the plate.
$\frac{a}{b}$	plate aspect ratio.
$\frac{h}{a}$	plate thickness aspect ratio.
N_x	axial forces parallel to the x-axis per unit length of a section of plate perpendicular to y-axis.
N_y	axial forces parallel to the y-axis per unit length of a section of plate perpendicular to x-axis.
Q_x, Q_y	shearing forces parallel to the z-axis per unit length of a section of plate perpendicular to x-and y-axes respectively.
N_{xy}	the inplane shearing intensities per unit length.
M_{xx}, M_{yy}	bending moment intensities per unit length.
M_{xy}	twisting moment intensity per unit length.
$q = q(x, y)$	transverse external force per unit area.
$Q = \frac{q a^4}{E h^4}$	dimensionless transverse external force per unit area.
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	normal stress components in x, y, z directions respectively.
$\sigma_{xy}, \sigma_{xz}, \sigma_{yz}$	shearing stress components in rectangular co-ordinates.

$\epsilon_x, \epsilon_y, \epsilon_z$	unit elongation in x-, y-, and z- directions.
$\epsilon_{xy}, \epsilon_{xz}, \epsilon_{yz}$	shearing strain components in rectangular co-ordinates.
E	Modulus of elasticity in tension and compression.
G	modulus of elasticity in shear.
ν	Poisson's ratio 0.316.
$z = \pm h/2$	lower and upper face of a plate, respectively.
D	flexural rigidity of an isotropic plate.
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$	Laplacian operator in rectangular co-ordinates.
γ	Relaxation factor
$\alpha = \frac{W}{h}$	Dimensionless value for the central deflection.
$\bar{\alpha} = \frac{\alpha}{Q}$	Dimensionless factor for the linear central deflection.
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	Laplacian operator in space co-ordinates.
[AA]	General matrix.
[FF]	Nodal loading.
[FOR]	Force vector.
[DIS]	Linear vector displacement.
[DISN]	Nonlinear vector displacement.

Chapter 1

Introduction

1.1 General

Plates are used as components of large scale structures in both mechanical and civil engineering structures. They are plane surface structures bounded either by straight or curved lines. Plates may have free, simply-supported, or fixed boundary conditions, including elastic supports or, in some cases, point supports. The static loads carried by plates are predominantly perpendicular to the plate surface. Plates are classified either by their geometrical forms or by their physical characteristics. Thick plates are a type of plates where the thickness is considerable and the approximate theories of thin plates are no longer applicable. In such cases, the thick plate theory must be applied. This theory considers the problem of

thick plates as a three-dimensional problem of elasticity. The stress analysis becomes, consequently, much more involved and up to now, the problem of thick plates is solved only for a few simple particular cases. The difficulty of obtaining solutions for thick plates is a direct result of the complexities of the partial differential equations governing thick plate behaviour. Furthermore, the boundary conditions for thick plates also have to be satisfied in three directions and hence, three functions defining all three displacements are required. The thick plate theory finds important application in the static analysis of heavy floor slabs and the dynamic analysis of heavy floors carrying rotating machinery in industrial buildings. An analysis is often required to determine the deformation of the thick plates, under the type of loading the plate is designed to carry.

The fundamental problems in mechanics are generally governed either by deriving the differential equations or by minimum energy principles. The governing differential equations and the equations derived by minimum energy principles are usually rather complex and do not lend themselves to exact solutions. The general equations defining the large deformation theory for thick plate are very complex and their exact solutions are not available. Confronted with this problem engineers and researchers have to resort to numerical methods to effect a solution. Variational methods were often exploited by engineers and scientists as a very effective tool for solving applied mechanics problems. These methods gained popularity in recent years due to the development of high speed digital computers. Finite differences methods were also used extensively for solving such complex differential equations. This method can be considered as a means of obtaining

the approximate solutions of differential equations, where the exact solutions are not available. According to the method, the problem is solved at chosen appropriate points where the system of equations is defined. In applying the finite difference method, a mesh size must first be chosen. The mesh size covers the entire domain of the plate with n points yielding n simultaneous equations in each direction. Convergence is easily investigated by increasing the mesh size covering the plate domain. For this study, the finite difference method is applied to the three dimensional problem of thick plates where the governing differential equations of equilibrium of such plates are expressed in terms of the displacements u , v , w and the rotations ω_x , ω_y , for the non-linear analysis following the co-ordinates axes x , y and z respectively. The first objective of this thesis is to study the applicability of the finite difference method for the non-linear analysis of thick plates.

As a supporting objective of the present study, a few somewhat innovative methods for the solution of simultaneous equations are presented. Equations-solving methods can be generally categorized as either direct techniques, which yield answers in a finite, predictable number of operations, or as iterative techniques, which yield answers that become increasingly more accurate as the number of iterations becomes large. Until fairly recently, it was commonly accepted that direct methods were most suitable for small sets of equations with dense coefficient matrices, while iterative methods were best suited for large sets of equations involving sparse coefficient matrices. The current viewpoint is somewhat different. Iterative techniques are still preferred for very large sets of equations with sparse

but not banded coefficient matrices. However, it has been found that direct methods are also highly suitable for quite large sets of equations having banded coefficient matrices. These banded matrices usually result either from the application of finite difference or of finite element methods to partial differential equations. New innovative methods for solving simultaneous equations involving both the direct method and the iteration technique will be presented as one of the objectives of the thesis.

1.2 Object and Scope

The main object of this thesis is to solve the problem of large deflection non-linear analysis of thick plates using the finite difference method. To study the applicability of the method, the problem of the bending of a square clamped thick plate is first studied using a rather crude mesh size. Once the validity of the method is established, the same method is extended to analyse the corresponding problems of rectangular clamped and simply-supported thick plates. The method can be further applied to determine the buckling and vibration of thick plates by the introduction of corresponding appropriate elasticity equations.

After the large systems of simultaneous equations have been obtained from finite differences, it became quite obvious that some new mathematical subroutines must be developed to handle efficiently the solution of these equations. As a result, some rather innovative efficient subroutines have since been developed [61] to handle the solutions of simultaneous equations

resulting from the finite difference modelling of large deflection of thick plate structures.

1.3 Present Investigation

In the analysis of thin plates subjected to lateral loading, in particular, when the central deflection is equal to or greater than 25 % of the thickness of the plate, the Kirchhoff theory which neglects the membrane effect cannot yield satisfactory results. Hence, for the finite deflection analysis of plates, the Kirchhoff theory is replaced by the refined Von-Karman theory, where the involvement of the membrane forces in the equations contributes a significant relief to the plate deflection. However, the solutions of the Von-Karman equations are extremely tedious and complicated.

Linear analysis of thick plates is much more complicated than the corresponding analysis of thin plates. For thick plate analysis three equations of equilibrium are considered compared to the Kirchhoff equation where only the lateral deflection is considered. The degree of difficulty increases even higher when we consider the nonlinear terms in the three equations of equilibrium. It has since been noted that not only the equations are difficult and complicated to handle; moreover, their solution requires new subroutines in order to deal with large systems of simultaneous equations resulting from the finite difference modelling. Since no results are available in the technical literature on large deflection of thick isotropic rectangular plates and the existing results are meagre even for small deflection of thick

square plates, convergence of the problem becomes important to yield confidence to the solution. In this investigation only deflections are considered, and the stresses are not computed because they do not, in general, yield sufficiently accurate results. In order to yield both bending and membrane stresses that are of acceptable accuracy, the finite difference mesh would have to be so refined resulting in enormous additional requirements for computer storage and time far beyond the present capacity allocation by the present computer facilities at the University of Ottawa. The discussion of the difficulties in generating stresses from displacement functions for large deflections of thin plates was well documented by Wang [16] and Yang [16-a]. Chapter II will be devoted to a brief review of the large deflection of thin isotropic rectangular plates. Since one of the objectives of this thesis is to study the linear and nonlinear analysis of thick isotropic rectangular plates and the applicability of the finite difference method as a numerical solution for such plates, existing literature related to the solution of small deflection of thick plates by different numerical methods is also reviewed in this chapter. The second or supporting objective of the thesis is to develop innovative efficient subroutines to solve systems of simultaneous equations. For this part, a brief review for the most popular method is also presented in chapter II. The main analysis of thick isotropic rectangular plates for small and large deflections, formulation of the general governing equations of equilibrium, stress-strain relationship, compatibility equations, dimensionless displacements are carried out in chapter III. In chapter IV, the finite difference expressions required to analyse the deflection problem of clamped and simply-supported isotropic rectangular thick

plates are formulated. The formulation of finite difference in the case of thick plates is not an easy task. For example, in the case of thin plate theory, the problem can be reduced to one governing equation in terms of the vertical displacement w , whereas for thick plates the governing differential equations require the inclusion of two additional components u and v , thus increasing significantly the complexity of the problem. Finite difference modelling is developed for the first time to solve the problem of small and large deflection of thick rectangular plates. In the technical literature, a small amount of data is available for small deflection of square thick plates using various numerical methods. In thick plate theory, the requirement to satisfy the boundary conditions deserves careful consideration especially for boundary conditions applied at extreme surfaces of the plate. For this reason, the thick plate is divided into layers with appropriate boundary expressions for each layer. The modified finite difference method presented here may be taken as a general method applicable to both thin and thick plate structures.

The formulation of the finite difference expression for each point is presented in detail, showing how the partial derivatives are replaced by finite difference expressions, and how the resulting equations are reduced in matrix form to yield the solution of a particular problem. Chapter V is devoted to the innovative method for the solution of systems of simultaneous equations, using iterative methods and direct methods, numerical examples are included in the appendix to demonstrate the applicability and advantages of the methods in comparison with the conventional methods of analysis. Results for small and large deflection analysis of thick rectan-

gular plates obtained are summarized in chapter VI for both the clamped and simply supported boundary conditions. For small deflection analysis of thick plates, results are compared, whenever possible with the solutions previously obtained by other researchers.

Chapter 2

PREVIOUS STUDIES

The classical theory of plates has been well established and widely applied since Thomson and Tait resolved the controversy over the boundary conditions of Poisson's theory in the latter half of the nineteenth century. Due to approximations inherent in their derivations, classical theories restrict themselves in their application to plates having thickness much less than their lateral dimensions. As a result, these theories cannot be applied with acceptable accuracy to thick plates or problems where local effects predominate such as stress concentration problems. Thus, there are cases where a much more refined theory is required. This led to several significant developments in recent years in the field of analysis of thick plates. The fundamental non-linear large deflection equations for thin homogeneous plates were derived by Von-Karman in 1910. S. Way [1] solved these equations for a clamped rectangular thin plate using the Ritz method by assuming the

displacements u , v , and w as polynomials functions. S. Levy [2] considered the case of the square simply supported thin plate using double trigonometric functions. Levy also solved [3] the problem of the square clamped plate using Fourier series functions. Then Levy and Greenman [4] solved the problem for the clamped rectangular plate with the plate ratio 1.5 using the same technique. A general procedure for solving these equations for multiple boundary conditions has been worked out at National Bureau of Standards by S. Levy [5] for the NACA. E. Reissner [6] proposed a new approach to analyse finite deflection of circular plates. Due to the deformation, the thin circular plate becomes a shell of revolution. A simplified non-linear equations for a flat plate with large deflections are derived by H.M. Berger [7]. He assumed that the strain energy due to second invariant of the middle-surface strains can be neglected. This method was used to solve thin circular and rectangular plates with different boundary conditions. E. Reissner [8] obtained an exact solution of the non-linear equations for finite deflections of thin elastic rectangular plates, with twisting moments T and bending moments M simultaneously applied to two opposite edges of the plate. Three methods were discussed by E.L. Bernstein [9] for the derivation of the equilibrium equations for large deflection of plates, namely, the free body method, the direct strain energy method, and the equilibrium equation method. In all the three methods, the rotation component ω_z was neglected. It was later observed that when thickness shear and thickness-stretch deformations were included, application of the three methods produced inconsistent sets of equilibrium equations, none of which is entirely satisfactory.

M. Balachandra and S. Gopalacharyulu [10] solved the problem of clamped thin rectangular plates with large deflections using a modified Fourier series and results were compared with Timoshenko [47] using polynomials. Numerical solutions are obtained by F. Bauer et al. [11] using an iterative method for the solution of the Von-Karman equations. Numerical solutions are obtained by an iterative method which employs an acceleration parameter θ . They are numerically evaluated by approximating the Dirichlet problem using corresponding difference equations. The resulting system of linear algebraic equations which is of quasi-tridiagonal form, is solved by a factored method. Approximate solutions for the non-linear bending of thin rectangular plates were presented by K.T. Sandara Raja Iyengar and M. Martin Nagri [12] considering various boundary conditions. The solution was given by using a double series consisting of appropriate beam functions which satisfy the boundary conditions. The differential equations are satisfied by using the orthogonality properties of the series. A finite difference method was carried out by J.C. Brown and J.M. Harvey [13] to compute the finite deflection of rectangular plates subjected to uniform lateral pressure and compressive edge loading. The results are in good agreement with the some numerical solutions and some experimental values. S.S. Tezcan [14] used the framework method to calculate large deflections of plates. For his analysis he used a typical Hrennikoff cell. Each cell of the framework contains six unidimensional bars. A solution of the large deflection equations for thin rectangular clamped plates was carried out by R. Hooke [15] using the perturbation method. Wang [16] used relaxation technique to solve a square simply supported thin plate. D.W. Murray and E.L. Wilson [17]

used the finite elements to determine large deflection of cantilever and simply supported thin plates using a triangular mesh. The results are in good agreement with Timoshenko [47], and Levy [2]. R. Hooke [18] determined a non-linear function which will fit the post-elastic deflection prediction of the plates for design purposes, and the technique matched some solutions with low percentage of error. Large deflection of elastic plates under patch loading was carried out by B. Aalami [19] using two numerical methods namely finite difference and relaxation. They yielded the same result for both deflection and stresses.

The shear deformation theory of Reissner [20] derived on the basis of a variational criteria has been one of the foremost in the treatment of thick plates. This theory has been applied to various rectangular plates by many investigators. Kromm [21] demonstrated the absence of corner reactions using a theory that takes into account transverse shear deformation. This point has also been discussed in detail by Marguerre and Woernle [22]. Green [23] has pointed out that the Reissner equations can be obtained directly from the three dimensional elasticity equations without recourse to variational considerations. Donnel [24] has given a three dimensional thick plate theory in which the solution is obtained in the form of infinite series with the first term representing classical thin plate theory results. Lee [25] has applied this method to simply supported rectangular plates. Friedrichs and Dressler [26] and Goldenveiser [27] have given approximate theories which could predict boundary layer effects, by the method of asymptotic integration of the governing equations of elasticity. Lur'e [28] has given a general power series method to solve the elasticity equations. Poniotovskii

[29] employed Legendre polynomial expansion in the thickness coordinate to derive a system of two dimensional equations. A three dimensional solution for rectangular plates and laminates has been developed by Srinivas et al. [30] in which the solution for displacements is taken in the form of a double trigonometric series satisfying the equations of equilibrium in terms of displacements. Srinivas and Rao [31] solved the small deflection problem of thick plates using a three dimensional analysis by expressing the displacement functions in terms of hyperbolic functions using the techniques of collocation, orthogonalization, and orthogonalization along with collocation at corners. The generalized Levy solution [32] has been taken by David W. Cooke to solve the problem of a statically loaded, rectangular plate which is simply supported on two opposite edges and has arbitrary boundary conditions on the remaining edges. Krashnina [33] presented a method based on the expressions obtained by Berdichevskii to solve the problem of bending of thick plates. Kishida [34] carried out a three dimensional elastic stress analysis of thick plates; and solved the problem of axisymmetric bending of a clamped circular plate subjected to an annularly distributed load. An analysis of an infinite elastic thick plate subjected to loads symmetrical to the axis of revolution has been developed by Nakajima, Iamsopana and Kakuzen [35]. After they have represented the part of hyperbolic functions of the integrand of stress functions in terms of exponential functions, Maclaurin's law was used to expand the denominator and each term was then integrated separately. The approximate stress components were calculated by solving the simultaneous linear algebraic equations with four unknowns by satisfying the boundary conditions at two points

on the boundary surfaces. Kobayashi, Nagasawa, Ishikawa, and Hata [36] proposed an extension of Love's moderately thick plate theory to solve the problem of a three dimensional rectangular cantilever plate. Bending of thick plates under an arbitrary load has been carried out by Gruzdev [37] using the method proposed by Lur'e, who reduces the three dimensional plate problem to the two dimensional problem using the infinite order differential operators of the displacements and rotations of the middle plane of the plate. Deshmukh and Archer [38] proposed a new method which considerably reduces the amount of numerical computations for certain classes of plate problems as compared to the alternate methods, i.e., finite element or finite difference methods. The method proposed is based on Reissner's theory where the total unknown component is equal to the sum of particular and complimentary solutions. Mixed finite difference scheme for the analysis of simply supported thick plates was proposed by Noor [39]. His analysis is based on the linear, three dimensional theory of orthotropic elasticity and a Fourier approach is used to reduce the governing equations to six first-order ordinary differential equations in the thickness coordinate and a finite difference method is carried out to form a system of linear equations from the previous differential equations. Ryabov and Rasskazov [40] examined the bending of thick nonuniform plates through their thickness. Their assumption is based on the distribution of the shear components in the cross sections and its change over the plate thickness in accordance with a square parabola law.

Finite element is the most rigorous method used to solve complicated problems during the last three decades. Here, Epstein and Huttermaier [41]

made use of the finite element concept for the numerical analysis of multi-layered plates and thick plates. Guruswamy and Tang [42] have expressed a twenty four degree of freedom sector finite element for the static and dynamic analysis of thick circular plates. Sundara, Chandrashekhara and Sebastian [43] applied a method of initial functions (MIF) originally proposed by Vlassov for the analysis of thick rectangular plates. The unknowns are expanded in Maclaurin series in the thickness coordinate and hence the solution is obtained in terms of unknown initial functions on the reference plane. Ng and Bencharif [44] have programmed a solution using finite difference representation for the small deflection analysis of thick plates and the results are in good agreement with previous researchers. The applicability of the finite difference method and its ease application was well demonstrated more recently [45] by Ng and Bencharif with extensive results for the design and analysis of thick plates not previously available in the technical literature.

For the solution of systems of equations, it has been noted by Ralston [57] that sets of simultaneous linear equations can usually be put into one of two categories: either the coefficient matrix is dense (few zero elements) but the set is not large, or the matrix is sparse (many zero elements) and the set is large. The most popular methods of solving simultaneous equations with their algorithms can be found in [58] by R. W. Hornbeck, and Wang [59]. Recently, Bencharif and Ng [61] used the change of variable decomposition method to solve a system of simultaneous equations and the results show good improvement in comparison with the conventional meth-

ods Gauss-Jordan and LU decomposition methods in terms of time and space.

Chapter 3

Theoretical Derivations

3.1 General

In this chapter, formulation of the governing equilibrium and compatibility equations of the plate based on the theory of elasticity is presented. For the analysis of large deformations, it is obvious that the analysis of small deformations will be considered as a particular case. The same analogy is applied to thin plate analysis as a particular case of thick plate problem. Geometrical and material nonlinearity can arise in plate problems. In this study only geometrical nonlinearity is taken under consideration and homogeneous, isotropic material is assumed. Figure (3.1) illustrates the geometry and orientation of a plate in the cartesian coordinates system. The x and y axis delimit the longitudinal and transversal directions of the

surface of the plate respectively and the z axis the perpendicular direction to the surface of the plate.

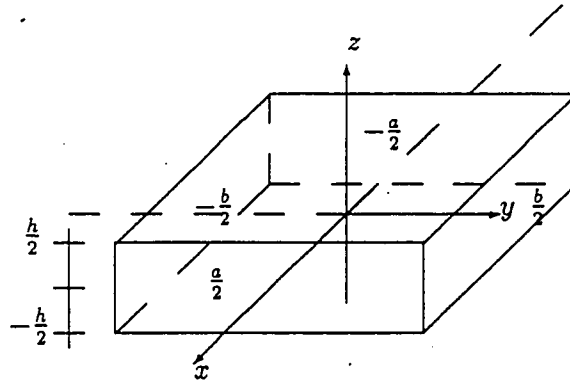


Figure (3.1) Position of The Coordinate System.

Internal forces and moments acting on the limits of plate element, as shown in Figure (3.2) and Figure (3.3), are related to the internal stresses by the equations:

$$N_x = \int_{-h/2}^{h/2} \sigma_{xx} dz \quad (3.1)$$

$$N_y = \int_{-h/2}^{h/2} \sigma_{yy} dz \quad (3.2)$$

$$N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} dz \quad (3.3)$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz} dz \quad (3.4)$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz} dz \quad (3.5)$$

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx} z dz \quad (3.6)$$

$$M_y = \int_{-h/2}^{h/2} \sigma_{yy} z dz \quad (3.7)$$

$$M_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} z dz \quad (3.8)$$

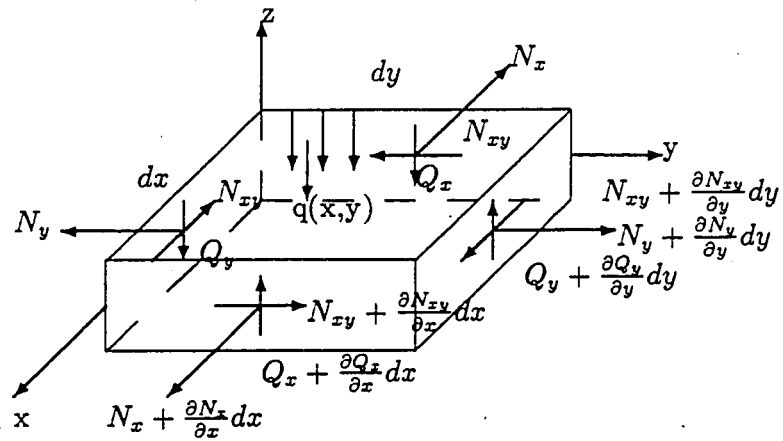


Figure (3.2) Equilibrium State of The Plate Element

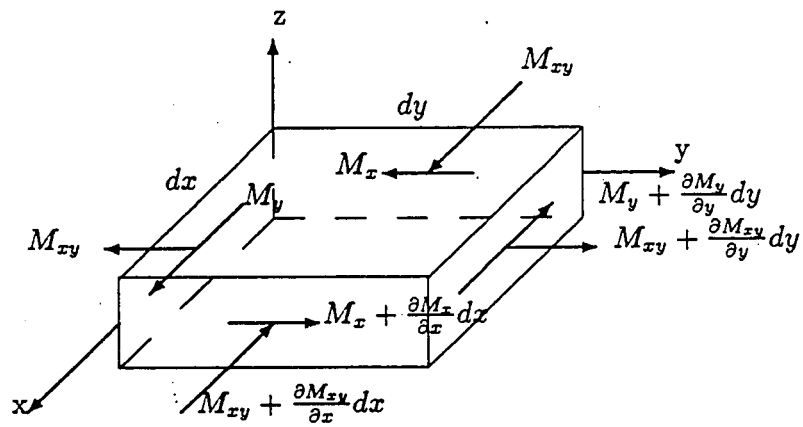


Figure (3.3) Distribution of The Moments in The Plate Element

3.1.1 Stress and Strain functions

From elasticity [48], and [49] the general Hook's law stress strain relationships are expressed by the following equations.

$$\sigma_{xx} = \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{yy} + \epsilon_{zz}) + \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\epsilon_{xx} \quad (3.9)$$

$$\sigma_{yy} = \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{xx} + \epsilon_{zz}) + \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\epsilon_{yy} \quad (3.10)$$

$$\sigma_{zz} = \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_{xx} + \epsilon_{yy}) + \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\epsilon_{zz} \quad (3.11)$$

$$\sigma_{xy} = \frac{E}{2(1+\nu)}\epsilon_{xy} \quad (3.12)$$

$$\sigma_{yz} = \frac{E}{2(1+\nu)}\epsilon_{yz} \quad (3.13)$$

$$\sigma_{zx} = \frac{E}{2(1+\nu)}\epsilon_{zx} \quad (3.14)$$

In terms of stresses the six previous equations can be defined as :

$$\epsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})] \quad (3.15)$$

$$\epsilon_{yy} = \frac{1}{E}[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})] \quad (3.16)$$

$$\epsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})] \quad (3.17)$$

$$\epsilon_{xy} = \frac{2(1+\nu)}{E}\sigma_{xy} \quad (3.18)$$

$$\epsilon_{yz} = \frac{2(1+\nu)}{E}\sigma_{yz} \quad (3.19)$$

$$\epsilon_{zx} = \frac{2(1+\nu)}{E}\sigma_{zx} \quad (3.20)$$

3.2 Nonlinear Analysis

3.2.1 Geometry of Deformations

Given the positions of the points of the body in its initial state before deformation and in its final state after deformation, the geometry of strain is determined by the change in the distance between two arbitrary infinitely near points of the body caused by its transition from the first state to the final state.

If we consider x, y, z and ξ, η, ζ as the coordinates of the initial and final state respectively, and let the points of the body undergo displacements with components u, v, w , then the final state of an arbitrary point of the body will be given by the cartesian coordinates [51]:

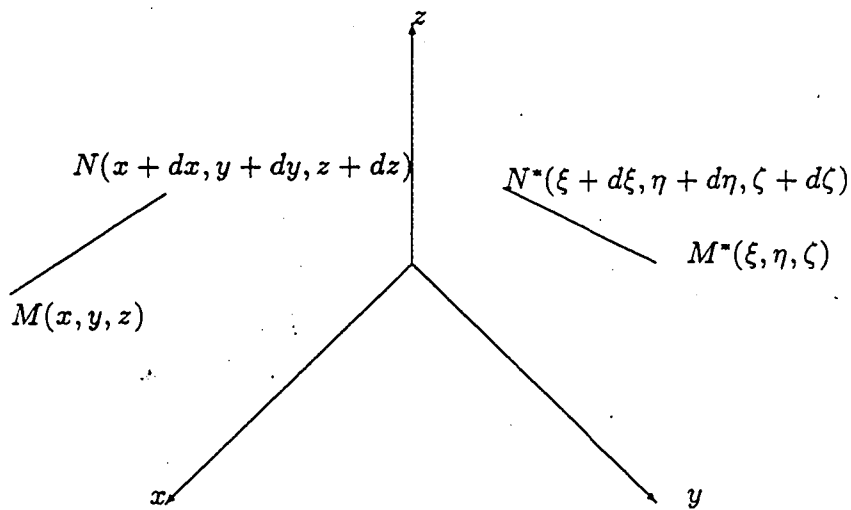


Figure (3.4) Deformation of a Line Element.

$$\xi = x + u(x, y, z) \quad (3.21)$$

$$\eta = y + v(x, y, z) \quad (3.22)$$

$$\zeta = z + w(x, y, z) \quad (3.23)$$

Where the functions u , v , w and their partial derivatives with respect to x , y , z are considered to be continuous.

From (3.21), (3.22), (3.23) above we note that the final state of the points can be determined either by the cartesian final coordinates or by the initial coordinates. The initial coordinates become curvilinear coordinates at the final state.

As a result of movement, the point $M(x,y,z)$ is displaced to the position M^* having cartesian coordinates ξ , η , ζ whereas the point $N(x+dx,y+dy,z+dz)$ infinitely near M is displaced to the position N^* having coordinates $\xi + d\xi$, $\eta + d\eta$, $\zeta + d\zeta$ as shown in Figure (3.4). The deformed vector $\vec{M^*N^*}$ (with projections $d\xi$, $d\eta$, $d\zeta$), determines the magnitude and direction of that line element of the body whose magnitude and direction just before deformation were given by the vector \vec{MN} (with projections dx , dy , dz). By applying the vector rule and using the point O as the origin of the cartesian coordinates we get:

$$\vec{M^*N^*} = \vec{ON^*} - \vec{OM^*} \quad (3.24)$$

Then the projection of the deformed vector will be expressed as shown:

$$d\xi = x + dx + u(x + dx, y + dy, z + dz) - x - u(x, y, z) \quad (3.25)$$

$$d\eta = y + dy + v(x + dx, y + dy, z + dz) - y - v(x, y, z) \quad (3.26)$$

$$d\zeta = z + dz + w(x + dx, y + dy, z + dz) - z - w(x, y, z) \quad (3.27)$$

Simplifying and then expanding the right sides in Taylor series about the point x, y, z and retaining only the first orders we get:

$$d\xi = dx + u(x, y, z) + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz - u(x, y, z) \quad (3.28)$$

$$d\eta = dy + v(x, y, z) + \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz - v(x, y, z) \quad (3.29)$$

$$d\zeta = dz + w(x, y, z) + \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz - w(x, y, z) \quad (3.30)$$

After collecting and arranging the terms we get the final expressions;

$$d\xi = (1 + \frac{\partial u}{\partial x})dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz \quad (3.31)$$

$$d\eta = \frac{\partial v}{\partial x}dx + (1 + \frac{\partial v}{\partial y})dy + \frac{\partial v}{\partial z}dz \quad (3.32)$$

$$d\zeta = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + (1 + \frac{\partial w}{\partial z})dz \quad (3.33)$$

Any arbitrary line element of the body after deformation can be expressed by the equations (3.31), (3.32), (3.33) in terms of its projections before deformation.

If we use the following notation;

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x}, & e_{yy} &= \frac{\partial v}{\partial y}, & e_{zz} &= \frac{\partial w}{\partial z} \\ e_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, & e_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, & e_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{aligned} \quad (3.34)$$

and

$$2\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad 2\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (3.35)$$

then equations (3.31), (3.32), (3.33) will be expressed in the form below:

$$d\xi = (1 + e_{xx})dx + \left(\frac{1}{2}e_{xy} - \omega_z\right)dy + \left(\frac{1}{2}e_{xz} + \omega_y\right)dz \quad (3.36)$$

$$d\eta = \left(\frac{1}{2}e_{xy} + \omega_z\right)dx + (1 + e_{yy})dy + \left(\frac{1}{2}e_{yz} - \omega_x\right)dz \quad (3.37)$$

$$d\zeta = \left(\frac{1}{2}e_{xz} - \omega_y\right)dx + \left(\frac{1}{2}e_{yz} + \omega_x\right)dy + (1 + e_{zz})dz \quad (3.38)$$

3.2.2 Strain Components

To compute the strain change between the two stages we need first to determine the square distances of the two vectors \vec{MN} and $\vec{M^*N^*}$ before and after deformation respectively.

Before deformation we have;

$$ds^2 = dx^2 + dy^2 + dz^2 \quad (3.39)$$

and after deformation we get;

$$ds_*^2 = d\xi^2 + d\eta^2 + d\zeta^2 \quad (3.40)$$

Proceeding with the difference between these two distances will give us the change due to the deformation.

$$ds_*^2 - ds^2 = d\xi^2 + d\eta^2 + d\zeta^2 - dx^2 - dy^2 - dz^2 \quad (3.41)$$

By using equations (3.31), (3.32), (3.3) for $d\xi$, $d\eta$, $d\zeta$ and substituting into (3.41) we find:

$$ds_*^2 - ds^2 = \left(1 + \frac{\partial u}{\partial x}\right)dx^2 + 2\left(1 + \frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial y}dxdy + 2\left(1 + \frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial z}dxdz$$

$$\begin{aligned}
& + \left(\frac{\partial u}{\partial y}\right)^2 dy^2 + 2\frac{\partial u}{\partial y}\frac{\partial u}{\partial z} dydz + \left(\frac{\partial u}{\partial z}\right)^2 dz^2 \\
& + \left(\frac{\partial v}{\partial x}\right)^2 dx^2 + 2\frac{\partial v}{\partial x}\left(1 + \frac{\partial v}{\partial y}\right) dx dy + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial z} dx dz \\
& + \left(1 + \frac{\partial v}{\partial y}\right)^2 dy^2 + 2\left(1 + \frac{\partial v}{\partial y}\right)\frac{\partial v}{\partial z} dy dz + \left(\frac{\partial v}{\partial z}\right)^2 dz^2 \\
& + \left(\frac{\partial w}{\partial x}\right)^2 dx^2 + 2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dx dy + 2\frac{\partial w}{\partial x}\left(1 + \frac{\partial w}{\partial z}\right) dx dz \\
& + \left(\frac{\partial w}{\partial y}\right)^2 dy^2 + 2\frac{\partial w}{\partial y}\left(1 + \frac{\partial w}{\partial z}\right) dy dz + \left(1 + \frac{\partial w}{\partial z}\right)^2 dz^2 \\
& - dx^2 - dy^2 - dz^2 \tag{3.42}
\end{aligned}$$

Simplifying the equation by arranging the terms as shown below;

$$\begin{aligned}
ds_m^2 - ds^2 = & \left[1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 - 1\right] dx^2 + \\
& \left[\left(\frac{\partial u}{\partial y}\right)^2 + 1 + 2\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 - 1\right] dy^2 + \\
& \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + 1 + 2\frac{\partial w}{\partial z} + \left(\frac{\partial w}{\partial z}\right)^2 - 1\right] dz^2 + \\
& \left[2\left(1 + \frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial x}\left(1 + \frac{\partial v}{\partial y}\right) + 2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right] dx dy + \\
& \left[2\left(1 + \frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial z} + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial z} + 2\frac{\partial w}{\partial x}\left(1 + \frac{\partial w}{\partial z}\right)\right] dx dz + \\
& \left[2\frac{\partial u}{\partial y}\frac{\partial u}{\partial z} + 2\left(1 + \frac{\partial v}{\partial y}\right)\frac{\partial v}{\partial z} + 2\frac{\partial w}{\partial y}\left(1 + \frac{\partial w}{\partial z}\right)\right] dy dz \tag{3.43}
\end{aligned}$$

we get:

$$ds_m^2 - ds^2 = 2(\epsilon_{xx}dx^2 + \epsilon_{yy}dy^2 + \epsilon_{zz}dz^2 + \epsilon_{xy}dxdy + \epsilon_{xz}dxdz + \epsilon_{yz}dydz) \tag{3.44}$$

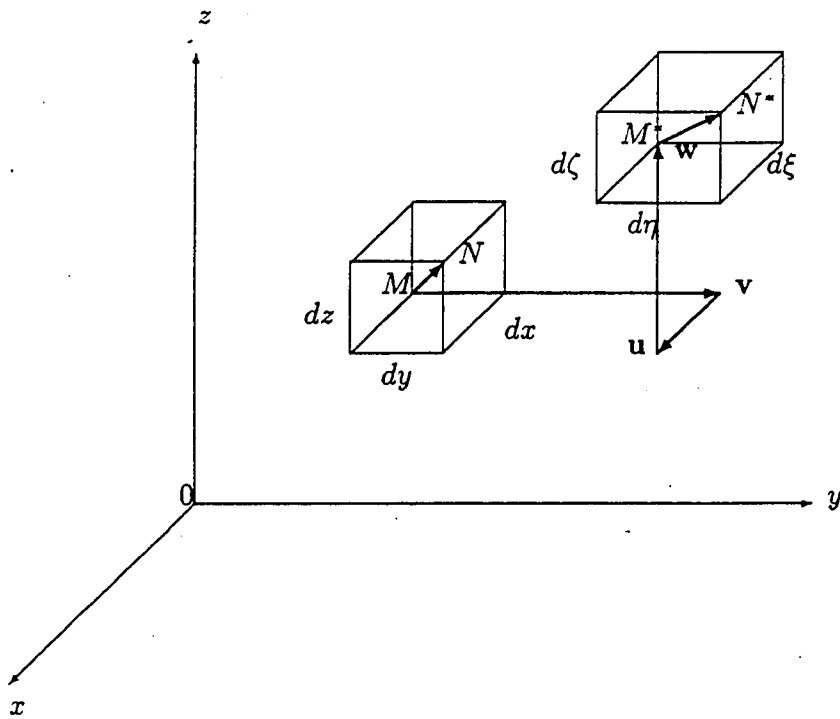


Figure (3.5) Deformation of a Body Element.

Where;

$$\epsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \quad (3.45)$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (3.46)$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \quad (3.47)$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \quad (3.48)$$

$$\epsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial z} \quad (3.49)$$

$$\epsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial w}{\partial z} \quad (3.50)$$

By using the notations given in equations (3.34), (3.35) we get the following interpretation.

$$\epsilon_{xx} = e_{xx} + \frac{1}{2} \left[e_{xx}^2 + \left(\frac{1}{2} e_{xy} + \omega_z \right)^2 + \left(\frac{1}{2} e_{xz} - \omega_y \right)^2 \right] \quad (3.51)$$

$$\epsilon_{yy} = e_{yy} + \frac{1}{2} \left[e_{yy}^2 + \left(\frac{1}{2} e_{xy} - \omega_z \right)^2 + \left(\frac{1}{2} e_{yz} + \omega_x \right)^2 \right] \quad (3.52)$$

$$\epsilon_{zz} = e_{zz} + \frac{1}{2} \left[e_{zz}^2 + \left(\frac{1}{2} e_{xz} + \omega_y \right)^2 + \left(\frac{1}{2} e_{yz} - \omega_x \right)^2 \right] \quad (3.53)$$

$$\epsilon_{xy} = e_{xy} + e_{xx} \left(\frac{1}{2} e_{xy} - \omega_z \right) + e_{yy} \left(\frac{1}{2} e_{xy} + \omega_z \right) + \left(\frac{1}{2} e_{xz} - \omega_y \right) \left(\frac{1}{2} e_{yz} + \omega_x \right) \quad (3.54)$$

$$\epsilon_{xz} = e_{xz} + e_{xx} \left(\frac{1}{2} e_{xz} + \omega_y \right) + e_{zz} \left(\frac{1}{2} e_{xz} - \omega_y \right) + \left(\frac{1}{2} e_{xy} + \omega_z \right) \left(\frac{1}{2} e_{yz} - \omega_x \right) \quad (3.55)$$

$$\epsilon_{yz} = e_{yz} + e_{yy} \left(\frac{1}{2} e_{yz} - \omega_x \right) + e_{zz} \left(\frac{1}{2} e_{yz} + \omega_x \right) + \left(\frac{1}{2} e_{xy} - \omega_z \right) \left(\frac{1}{2} e_{xz} + \omega_y \right) \quad (3.56)$$

3.2.3 Equations of Equilibrium

From a deformed body, let us isolate an elementary rectangular parallelepiped whose edges are parallel to the x , y , z axis and equal to $d\xi$, $d\eta$, $d\zeta$, respectively as shown in Figure (3.6).

Accordingly, the following surface forces act on them [51]:

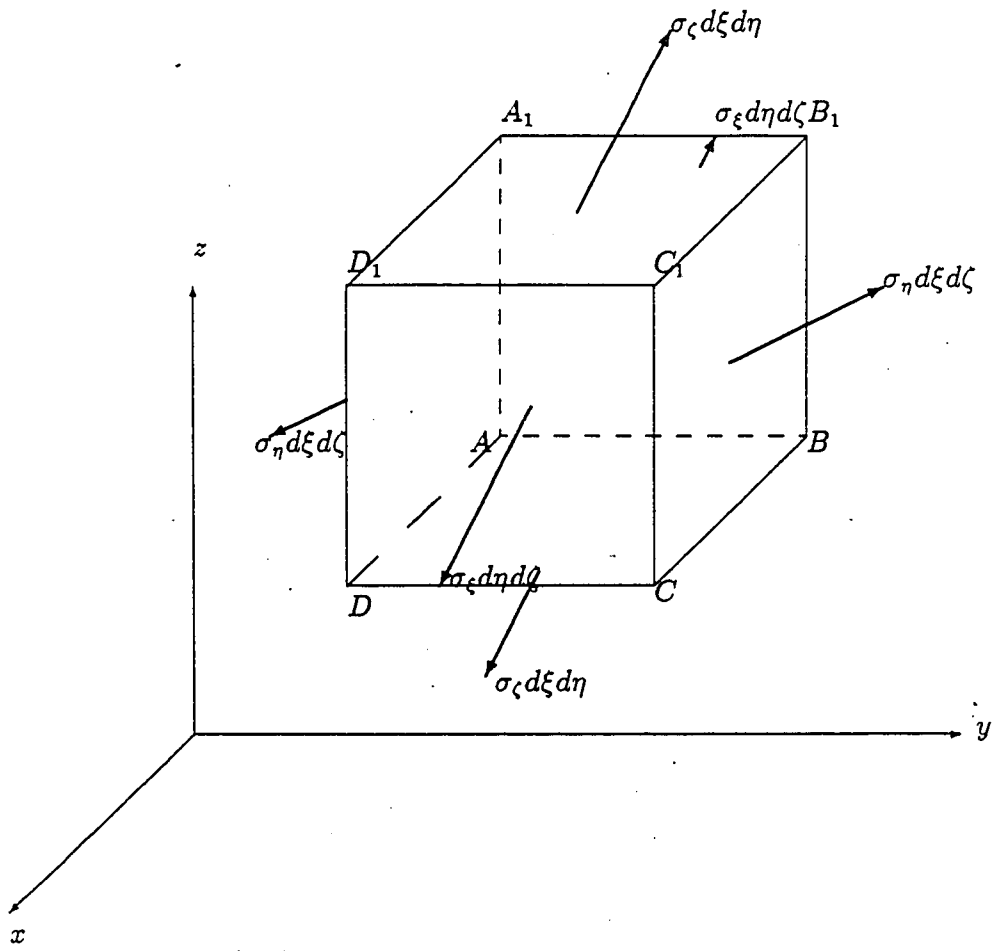


Figure (3.6) Acting Forces on a Body Element.

$$F_{AA_1B_1B} = - \left[\sigma_\xi(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial \sigma_\xi}{\partial \eta} d\eta + \frac{1}{2} \frac{\partial \sigma_\xi}{\partial \zeta} d\zeta \right] d\eta d\zeta \quad (3.57)$$

$$F_{DD_1C_1C} = \left[\sigma_\xi(\xi, \eta, \zeta) + \frac{\partial \sigma_\xi}{\partial \xi} d\xi + \frac{1}{2} \frac{\partial \sigma_\xi}{\partial \eta} d\eta + \frac{1}{2} \frac{\partial \sigma_\xi}{\partial \zeta} d\zeta \right] d\eta d\zeta \quad (3.58)$$

$$F_{DD_1A_1A} = - \left[\sigma_\eta(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial \sigma_\eta}{\partial \xi} d\xi + \frac{1}{2} \frac{\partial \sigma_\eta}{\partial \zeta} d\zeta \right] d\xi d\zeta \quad (3.59)$$

$$F_{BB_1C_1C} = \left[\sigma_\eta(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial \sigma_\eta}{\partial \xi} d\xi + \frac{\partial \sigma_\eta}{\partial \eta} d\eta + \frac{1}{2} \frac{\partial \sigma_\eta}{\partial \zeta} d\zeta \right] d\xi d\zeta \quad (3.60)$$

$$F_{ADCB} = - \left[\sigma_\zeta(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial \sigma_\zeta}{\partial \xi} d\xi + \frac{1}{2} \frac{\partial \sigma_\zeta}{\partial \eta} d\eta \right] d\xi d\eta \quad (3.61)$$

$$F_{A_1D_1C_1B_1} = \left[\sigma_\zeta(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial \sigma_\zeta}{\partial \xi} d\xi + \frac{1}{2} \frac{\partial \sigma_\zeta}{\partial \eta} d\eta + \frac{\partial \sigma_\zeta}{\partial \zeta} d\zeta \right] d\xi d\eta \quad (3.62)$$

In addition to surface forces, there will be also body forces acting on the element. Their resultant is equal to :

$$F_{body} = \left[F(\xi, \eta, \zeta) + \frac{1}{2} \frac{\partial F}{\partial \xi} d\xi + \frac{1}{2} \frac{\partial F}{\partial \eta} d\eta + \frac{1}{2} \frac{\partial F}{\partial \zeta} d\zeta \right] d\xi d\eta d\zeta \approx F^*(\xi, \eta, \zeta) d\xi d\eta d\zeta \quad (3.63)$$

Where $F^*(\xi, \eta, \zeta)$ is the specific body force at point $M^*(\xi, \eta, \zeta)$.

Summing all surface forces and body forces and setting the result equal to zero, we arrive at the following relation after eliminating the common factors:

$$\frac{\partial \sigma_\xi}{\partial \xi} + \frac{\partial \sigma_\eta}{\partial \eta} + \frac{\partial \sigma_\zeta}{\partial \zeta} + F^* = 0 \quad (3.64)$$

The vector equation (3.30) is equivalent to the three scalar equations

$$\frac{\partial \sigma_{\xi\xi}}{\partial \xi} + \frac{\partial \sigma_{\eta\xi}}{\partial \eta} + \frac{\partial \sigma_{\zeta\xi}}{\partial \zeta} + F_\xi^* = 0 \quad (3.65)$$

$$\frac{\partial \sigma_{\xi\eta}}{\partial \xi} + \frac{\partial \sigma_{\eta\eta}}{\partial \eta} + \frac{\partial \sigma_{\zeta\eta}}{\partial \zeta} + F_\eta^* = 0 \quad (3.66)$$

$$\frac{\partial \sigma_{\xi\zeta}}{\partial \xi} + \frac{\partial \sigma_{\eta\zeta}}{\partial \eta} + \frac{\partial \sigma_{\zeta\zeta}}{\partial \zeta} + F_\zeta^* = 0 \quad (3.67)$$

If we neglect the body forces and transform the governing equilibrium equations from differentiation with respect to ξ, η, ζ to the differentiation in cartesian coordinates with respect to x, y, z , we get:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[(1 + e_{xx})\sigma_{xx} + \left(\frac{1}{2}e_{xy} - \omega_z\right)\sigma_{xy} + \left(\frac{1}{2}e_{xz} + \omega_y\right)\sigma_{xz} \right] + \\ & \frac{\partial}{\partial y} \left[(1 + e_{xx})\sigma_{yx} + \left(\frac{1}{2}e_{xy} - \omega_z\right)\sigma_{yy} + \left(\frac{1}{2}e_{xz} + \omega_y\right)\sigma_{yz} \right] + \\ & \frac{\partial}{\partial z} \left[(1 + e_{xx})\sigma_{zx} + \left(\frac{1}{2}e_{xy} - \omega_z\right)\sigma_{zy} + \left(\frac{1}{2}e_{xz} + \omega_y\right)\sigma_{zz} \right] = 0 \quad (3.68) \end{aligned}$$

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\left(\frac{1}{2}e_{xy} + \omega_z\right)\sigma_{xx} + (1 + e_{yy})\sigma_{xy} + \left(\frac{1}{2}e_{yz} - \omega_x\right)\sigma_{xz} \right] + \\ & \frac{\partial}{\partial y} \left[\left(\frac{1}{2}e_{xy} + \omega_z\right)\sigma_{yx} + (1 + e_{yy})\sigma_{yy} + \left(\frac{1}{2}e_{yz} - \omega_x\right)\sigma_{yz} \right] + \\ & \frac{\partial}{\partial z} \left[\left(\frac{1}{2}e_{xy} + \omega_z\right)\sigma_{zx} + (1 + e_{yy})\sigma_{zy} + \left(\frac{1}{2}e_{yz} - \omega_x\right)\sigma_{zz} \right] = 0 \quad (3.69) \end{aligned}$$

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\left(\frac{1}{2}e_{xz} - \omega_y\right)\sigma_{xx} + \left(\frac{1}{2}e_{yz} + \omega_x\right)\sigma_{xy} + (1 + e_{zz})\sigma_{xz} \right] + \\ & \frac{\partial}{\partial y} \left[\left(\frac{1}{2}e_{xz} - \omega_y\right)\sigma_{yx} + \left(\frac{1}{2}e_{yz} + \omega_x\right)\sigma_{yy} + (1 + e_{zz})\sigma_{yz} \right] + \\ & \frac{\partial}{\partial z} \left[\left(\frac{1}{2}e_{xz} - \omega_y\right)\sigma_{zx} + \left(\frac{1}{2}e_{yz} + \omega_x\right)\sigma_{zy} + (1 + e_{zz})\sigma_{zz} \right] = 0 \quad (3.70) \end{aligned}$$

3.2.4 Compatibility Equations

In the six strain equations (3.45-50), only the three displacement unknowns u, v, w are included. Let us consider that the strains are known and we need to compute the displacement components. The equations may be regarded as a system of partial differential equations for the determination of the displacements u, v, w when the strain components $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{xy}, \epsilon_{yz}$, and ϵ_{zx} are expressed as functions of x, y, z . Since there are six equations for three

unknown functions, we cannot expect in general that these equations will possess a solution if the strain components are arbitrarily prescribed. Thus, there must be some conditions to be imposed on the strain components in order that these six equations will give a set of single-valued continuous solutions for the three displacement components. The fact that the strain components cannot be prescribed arbitrarily can be seen from the following geometrical considerations: Imagine that an elastic body is subdivided into a number of small cubic elements before deformation. Now, suppose that each element is subjected to an arbitrary deformation. After the deformation, these elements become parallelepipeds, and it may happen that it is impossible to arrange the parallelepipeds to form a continuous body, the strain components for each element must satisfy certain relations. The determination of the six components of strain at each point is completely satisfied by the three functions u , v , w defining the components of displacement. If the elongations and shears are very small compared to unity, and as a result of this simplification, we obtain the following six approximate equations:

$$\begin{aligned} \frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} - \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} &= \left(\frac{\partial \epsilon_{xx}}{\partial y}\right)^2 + \left(\frac{\partial \epsilon_{yy}}{\partial x}\right)^2 + \frac{1}{4} \left(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} - \frac{\partial \epsilon_{xy}}{\partial z}\right)^2 \\ - \left(\frac{\partial \epsilon_{xz}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial z}\right) \left(\frac{\partial \epsilon_{yz}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial z}\right) &- \frac{\partial \epsilon_{yy}}{\partial y} \left(\frac{\partial \epsilon_{xy}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial y}\right) - \frac{\partial \epsilon_{xx}}{\partial x} \left(\frac{\partial \epsilon_{xy}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial x}\right) \end{aligned} \quad (3.71)$$

$$\begin{aligned} \frac{\partial^2 \epsilon_{yy}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial y^2} - \frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} &= \left(\frac{\partial \epsilon_{yy}}{\partial z}\right)^2 + \left(\frac{\partial \epsilon_{zz}}{\partial y}\right)^2 + \frac{1}{4} \left(\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} - \frac{\partial \epsilon_{yz}}{\partial x}\right)^2 \\ - \left(\frac{\partial \epsilon_{yz}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial z}\right) \left(\frac{\partial \epsilon_{xz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial x}\right) &- \frac{\partial \epsilon_{zz}}{\partial z} \left(\frac{\partial \epsilon_{yz}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial z}\right) - \frac{\partial \epsilon_{yy}}{\partial y} \left(\frac{\partial \epsilon_{yz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial y}\right) \end{aligned} \quad (3.72)$$

$$\frac{\partial^2 \epsilon_{zz}}{\partial x^2} + \frac{\partial^2 \epsilon_{xx}}{\partial z^2} - \frac{\partial^2 \epsilon_{xz}}{\partial x \partial z} = \left(\frac{\partial \epsilon_{xx}}{\partial z}\right)^2 + \left(\frac{\partial \epsilon_{zz}}{\partial x}\right)^2 + \frac{1}{4} \left(\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{xz}}{\partial y}\right)^2$$

$$\begin{aligned}
& -\left(\frac{\partial \epsilon_{yz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial y}\right)\left(\frac{\partial \epsilon_{xy}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial y}\right) - \frac{\partial \epsilon_{xx}}{\partial x}\left(\frac{\partial \epsilon_{xz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial x}\right) - \frac{\partial \epsilon_{zz}}{\partial z}\left(\frac{\partial \epsilon_{xz}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial z}\right) \quad (3.73) \\
\frac{\partial^2 \epsilon_{xx}}{\partial y \partial z} - \frac{1}{2} \frac{\partial}{\partial x} \left(\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{xz}}{\partial y} - \frac{\partial \epsilon_{yz}}{\partial x} \right) &= \frac{\partial \epsilon_{xx}}{\partial y} \frac{\partial \epsilon_{xx}}{\partial z} + \frac{1}{2} \frac{\partial \epsilon_{yy}}{\partial x} \left(\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{xz}}{\partial y} \right) + \\
\frac{1}{2} \frac{\partial \epsilon_{zz}}{\partial x} \left(\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{xy}}{\partial z} \right) &- \frac{1}{2} \frac{\partial \epsilon_{xx}}{\partial x} \left(\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} - \frac{\partial \epsilon_{yz}}{\partial x} \right) - \frac{\partial \epsilon_{yy}}{\partial z} \left(\frac{\partial \epsilon_{xy}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial y} \right) - \\
&\frac{\partial \epsilon_{zz}}{\partial y} \left(\frac{\partial \epsilon_{xz}}{\partial x} - \frac{\partial \epsilon_{xx}}{\partial z} \right) \quad (3.74)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \epsilon_{yy}}{\partial x \partial z} - \frac{1}{2} \frac{\partial}{\partial y} \left(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xy}}{\partial z} - \frac{\partial \epsilon_{xz}}{\partial y} \right) &= \frac{\partial \epsilon_{yy}}{\partial x} \frac{\partial \epsilon_{yy}}{\partial z} + \frac{1}{2} \frac{\partial \epsilon_{zz}}{\partial y} \left(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} - \frac{\partial \epsilon_{xy}}{\partial z} \right) + \\
\frac{1}{2} \frac{\partial \epsilon_{xx}}{\partial y} \left(\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{xz}}{\partial y} - \frac{\partial \epsilon_{yz}}{\partial x} \right) &- \frac{1}{2} \frac{\partial \epsilon_{yy}}{\partial y} \left(\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{xz}}{\partial y} \right) - \frac{\partial \epsilon_{zz}}{\partial x} \left(\frac{\partial \epsilon_{yz}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial z} \right) - \\
&\frac{\partial \epsilon_{xx}}{\partial z} \left(\frac{\partial \epsilon_{xy}}{\partial y} - \frac{\partial \epsilon_{yy}}{\partial x} \right) \quad (3.75)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \epsilon_{zz}}{\partial x \partial y} - \frac{1}{2} \frac{\partial}{\partial z} \left(\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{yz}}{\partial x} - \frac{\partial \epsilon_{xy}}{\partial z} \right) &= \frac{\partial \epsilon_{zz}}{\partial x} \frac{\partial \epsilon_{zz}}{\partial y} + \frac{1}{2} \frac{\partial \epsilon_{xx}}{\partial z} \left(\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} - \frac{\partial \epsilon_{yz}}{\partial x} \right) + \\
\frac{1}{2} \frac{\partial \epsilon_{yy}}{\partial z} \left(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xy}}{\partial z} - \frac{\partial \epsilon_{xz}}{\partial y} \right) &- \frac{1}{2} \frac{\partial \epsilon_{zz}}{\partial z} \left(\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y} - \frac{\partial \epsilon_{xy}}{\partial z} \right) - \frac{\partial \epsilon_{xx}}{\partial y} \left(\frac{\partial \epsilon_{xz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial x} \right) - \\
&\frac{\partial \epsilon_{yy}}{\partial x} \left(\frac{\partial \epsilon_{yz}}{\partial z} - \frac{\partial \epsilon_{zz}}{\partial y} \right) \quad (3.76)
\end{aligned}$$

3.2.5 Simplification for Small Rotations

If the angles of rotation are small compared to unity, then, the linear parameters e_{ij} differ from the strain components only by quantities of the same order as the squares of the angles of rotation. This allows us to simplify the strain system (3.51-56) to:

$$\epsilon_{xx} = e_{xx} + \frac{1}{2} \left[+\omega_z^2 + \omega_y^2 \right] \quad (3.77)$$

$$\epsilon_{yy} = e_{yy} + \frac{1}{2} \left[+\omega_z^2 + \omega_x^2 \right] \quad (3.78)$$

$$\epsilon_{zz} = e_{zz} + \frac{1}{2} [\omega_y^2 + \omega_x^2] \quad (3.79)$$

$$\epsilon_{xy} = e_{xy} - \omega_y \omega_x \quad (3.80)$$

$$\epsilon_{xz} = e_{xz} - \omega_z \omega_x \quad (3.81)$$

$$\epsilon_{yz} = e_{yz} - \omega_z \omega_y \quad (3.82)$$

and the equilibrium system of equations can be written as:

$$\frac{\partial}{\partial x} [\sigma_{xx} - \omega_z \sigma_{xy} + \omega_y \sigma_{xz}] + \frac{\partial}{\partial y} [\sigma_{yx} - \omega_z \sigma_{yy} + \omega_y \sigma_{yz}] + \frac{\partial}{\partial z} [\sigma_{zx} - \omega_z \sigma_{zy} + \omega_y \sigma_{zz}] = 0 \quad (3.83)$$

$$\frac{\partial}{\partial x} [\omega_z \sigma_{xx} + \sigma_{xy} - \omega_x \sigma_{xz}] + \frac{\partial}{\partial y} [\omega_z \sigma_{yx} + \sigma_{yy} - \omega_x \sigma_{yz}] + \frac{\partial}{\partial z} [\omega_z \sigma_{zx} + \sigma_{zy} - \omega_x \sigma_{zz}] = 0 \quad (3.84)$$

$$\frac{\partial}{\partial x} [\omega_y \sigma_{xx} - \omega_x \sigma_{xy} - \sigma_{xz}] + \frac{\partial}{\partial y} [\omega_y \sigma_{yx} - \omega_x \sigma_{yy} - \sigma_{yz}] + \frac{\partial}{\partial z} [\omega_y \sigma_{zx} - \omega_x \sigma_{zy} - \sigma_{zz}] = 0 \quad (3.85)$$

If we rewrite the equations of equilibrium by taking the non-linear terms to the right side and we neglect the rotation ω_z by considering that the plate is rigid enough to undergo rotation about the z axis then we get the following equations of equilibrium. The notation used below stands for the iterative solution deployed to solve the non-linear equations of thick plates. First we solve the problem of small deflection then we start correcting the force vector in the right hand side for the large deflection analysis. In any iterative method the terms with the subscript $i+1$ are devoted to the next iteration where the terms with the subscript i are devoted to the present iteration. To ease our representation, the terms with the symbol o stands

for the achieved iteration.

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = -\frac{\partial}{\partial x} [\omega_y^o \sigma_{xz}^o] - \frac{\partial}{\partial y} [\omega_y^o \sigma_{yz}^o] - \frac{\partial}{\partial z} [\omega_y^o \sigma_{zz}^o] \quad (3.86)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \frac{\partial}{\partial x} [\omega_x^o \sigma_{xz}^o] + \frac{\partial}{\partial y} [\omega_x^o \sigma_{yz}^o] + \frac{\partial}{\partial z} [\omega_x^o \sigma_{zz}^o] \quad (3.87)$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \frac{\partial}{\partial x} [\omega_y^o \sigma_{xx}^o - \omega_x^o \sigma_{xy}^o] + \frac{\partial}{\partial y} [\omega_y^o \sigma_{yx}^o - \omega_x^o \sigma_{yy}^o] + \frac{\partial}{\partial z} [\omega_y^o \sigma_{zx}^o - \omega_x^o \sigma_{zy}^o] \quad (3.88)$$

3.3 Linear Analysis

For the small deflection the equations of equilibrium can be derived easily by just dropping the the non-linear terms from the above or by considering the right hand side of the equations equal to zero, so we get:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \quad (3.89)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \quad (3.90)$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \quad (3.91)$$

3.3.1 Formulation of the elasticity problem

If we are interested in finding only the stress components, we may reduce the system of equations to six equations with six unknown stress components. Since the displacement components are not required, the compati-

bility equations must be satisfied to ensure the existence of single valued displacements. By using the notation [48], and [49]

$$\theta = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \quad (3.92)$$

the first three strain stress relations are

$$\epsilon_{xx} = \frac{1}{E} ((1 + \nu) \sigma_{xx} - \nu \theta) \quad (3.93)$$

$$\epsilon_{yy} = \frac{1}{E} ((1 + \nu) \sigma_{yy} - \nu \theta) \quad (3.94)$$

$$\epsilon_{zz} = \frac{1}{E} ((1 + \nu) \sigma_{zz} - \nu \theta) \quad (3.95)$$

Now if we take :

$$\frac{\partial^2 \epsilon_{yy}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial y^2} = \frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} \quad (3.96)$$

and deriving the equations (3.94),(3.95),(3.19) with respect to the differential equation (3.96) above we find

$$(1 + \nu) \left(\frac{\partial^2 \sigma_{yy}}{\partial z^2} + \frac{\partial^2 \sigma_{zz}}{\partial y^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = 2(1 + \nu) \frac{\partial^2 \sigma_{yz}}{\partial y \partial z} \quad (3.97)$$

Again, if we take

$$\frac{\partial^2 \epsilon_{xx}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial x^2} = \frac{\partial^2 \epsilon_{xz}}{\partial x \partial z} \quad (3.98)$$

and deriving the equations (3.93), (3.95), (3.20) with respect to the second differential equation (3.98) we find

$$(1 + \nu) \left(\frac{\partial^2 \sigma_{xx}}{\partial z^2} + \frac{\partial^2 \sigma_{zz}}{\partial x^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial^2 \theta}{\partial x^2} \right) = 2(1 + \nu) \frac{\partial^2 \sigma_{xz}}{\partial x \partial z} \quad (3.99)$$

Similarly, using

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \quad (3.100)$$

and deriving the equations (3.93), (3.94), (3.18) with respect to the third differential equation (3.100) we find :

$$(1 + \nu) \left(\frac{\partial^2 \sigma_{xx}}{\partial y^2} + \frac{\partial^2 \sigma_{yy}}{\partial x^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = 2(1 + \nu) \frac{\partial^2 \sigma_{xy}}{\partial x \partial y} \quad (3.101)$$

From the equilibrium system, we form three couple of equations; where each couple corresponds to a simplification of the above expressions respectively.

$$\frac{\partial \sigma_{yz}}{\partial y} = -\frac{\partial \sigma_{zz}}{\partial z} - \frac{\partial \sigma_{xz}}{\partial x} - F_{zz} \quad (3.102)$$

$$\frac{\partial \sigma_{yz}}{\partial z} = -\frac{\partial \sigma_{yy}}{\partial y} - \frac{\partial \sigma_{xy}}{\partial x} - F_{yy} \quad (3.103)$$

$$\frac{\partial \sigma_{xz}}{\partial z} = -\frac{\partial \sigma_{xx}}{\partial x} - \frac{\partial \sigma_{xy}}{\partial y} - F_{xx} \quad (3.104)$$

$$\frac{\partial \sigma_{xz}}{\partial x} = -\frac{\partial \sigma_{zz}}{\partial z} - \frac{\partial \sigma_{yz}}{\partial y} - F_{zz} \quad (3.105)$$

$$\frac{\partial \sigma_{xy}}{\partial y} = -\frac{\partial \sigma_{xx}}{\partial x} - \frac{\partial \sigma_{xz}}{\partial z} - F_{xx} \quad (3.106)$$

$$\frac{\partial \sigma_{xy}}{\partial x} = -\frac{\partial \sigma_{yy}}{\partial y} - \frac{\partial \sigma_{yz}}{\partial z} - F_{yy} \quad (3.107)$$

Differentiating the first of these equations of each couple with respect to i and the second with respect to j, and adding the equations of each differentiated couple together. i.e. ,

$$\frac{\partial \sigma_{ij}}{\partial i \partial j} + \frac{\partial \sigma_{ji}}{\partial j \partial i} \quad (3.108)$$

and then

$$\frac{\partial \sigma_{ij}}{\partial i \partial j} + \frac{\partial \sigma_{ji}}{\partial j \partial i} = 2 \frac{\partial \sigma_{ij}}{\partial i \partial j} \quad (3.109)$$

the first couple gives

$$2 \frac{\partial^2 \sigma_{yz}}{\partial y \partial z} = -\frac{\partial^2 \sigma_{zz}}{\partial z^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial}{\partial x} \left(\frac{\partial \sigma_{xz}}{\partial z} + \frac{\partial \sigma_{xy}}{\partial y} \right) - \frac{\partial F_{zz}}{\partial z} - \frac{\partial F_{yy}}{\partial y} \quad (3.110)$$

using the 1st relation from equilibrium equations

$$\frac{\partial \sigma_{xx}^2}{\partial x^2} + \frac{\partial F_{xx}}{\partial x} = -\frac{\partial}{\partial x} \left(\frac{\partial \sigma_{xz}}{\partial z} + \frac{\partial \sigma_{xy}}{\partial y} \right) \quad (3.111)$$

then

$$2 \frac{\partial^2 \sigma_{yz}}{\partial y \partial z} = \frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial^2 \sigma_{zz}}{\partial z^2} + \frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{yy}}{\partial y} - \frac{\partial F_{zz}}{\partial z} \quad (3.112)$$

the second couple gives

$$2 \frac{\partial^2 \sigma_{xz}}{\partial x \partial z} = -\frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{zz}}{\partial z^2} - \frac{\partial}{\partial y} \left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} \right) - \frac{\partial F_{zz}}{\partial z} - \frac{\partial F_{xx}}{\partial x} \quad (3.113)$$

using the 2nd relation from equilibrium equation

$$\frac{\partial^2 \sigma_{yy}}{\partial y^2} + \frac{\partial F_{yy}}{\partial y} = -\frac{\partial}{\partial y} \left(\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} \right) \quad (3.114)$$

then

$$2 \frac{\partial^2 \sigma_{xz}}{\partial x \partial z} = -\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial^2 \sigma_{zz}}{\partial z^2} - \frac{\partial F_{xx}}{\partial x} + \frac{\partial F_{yy}}{\partial y} - \frac{\partial F_{zz}}{\partial z} \quad (3.115)$$

the third couple gives

$$2 \frac{\partial^2 \sigma_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial}{\partial z} \left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} \right) - \frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{yy}}{\partial y} \quad (3.116)$$

using the 3rd relation from equilibrium equation

$$\frac{\partial^2 \sigma_{zz}}{\partial z^2} + \frac{\partial F_{zz}}{\partial z} = -\frac{\partial}{\partial z} \left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} \right) \quad (3.117)$$

then

$$2 \frac{\partial^2 \sigma_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} + \frac{\partial^2 \sigma_{zz}}{\partial z^2} - \frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{yy}}{\partial y} + \frac{\partial F_{zz}}{\partial z} \quad (3.118)$$

using the notation below to simplify the writing

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (3.119)$$

substituting equation (3.118) in equation (3.101) and using the symbol ∇^2 we find :

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_{xx} - \frac{\partial^2 \theta}{\partial x^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial x^2} \right) = (1 + \nu) \left(\frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{yy}}{\partial y} - \frac{\partial F_{zz}}{\partial z} \right) \quad (3.120)$$

Two analogous equation can be obtained by substituting (3.116) in (3.99) and (3.115) in (3.97)

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_{yy} - \frac{\partial^2 \theta}{\partial y^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial y^2} \right) = (1 + \nu) \left(\frac{\partial F_{yy}}{\partial y} - \frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{zz}}{\partial z} \right) \quad (3.121)$$

and

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_{zz} - \frac{\partial^2 \theta}{\partial z^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial z^2} \right) = (1 + \nu) \left(\frac{\partial F_{zz}}{\partial z} - \frac{\partial F_{xx}}{\partial x} - \frac{\partial F_{yy}}{\partial y} \right) \quad (3.122)$$

adding together all the three equations (3.120), (3.121), (3.122) we find

$$(1 - \nu) \nabla^2 \theta = -(1 + \nu) \left(\frac{\partial F_{xx}}{\partial x} + \frac{\partial F_{yy}}{\partial y} + \frac{\partial F_{zz}}{\partial z} \right) \quad (3.123)$$

substituting this expression for $\nabla^2 \theta$ in (3.120), (3.121) and (3.122) gives

$$\nabla^2 \sigma_{xx} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial x^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_{xx}}{\partial x} + \frac{\partial F_{yy}}{\partial y} + \frac{\partial F_{zz}}{\partial z} \right) - 2 \frac{\partial F_{xx}}{\partial x} \quad (3.124)$$

$$\nabla^2 \sigma_{yy} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial y^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_{xx}}{\partial x} + \frac{\partial F_{yy}}{\partial y} + \frac{\partial F_{zz}}{\partial z} \right) - 2 \frac{\partial F_{yy}}{\partial y} \quad (3.125)$$

$$\nabla^2 \sigma_{zz} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial z^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_{xx}}{\partial x} + \frac{\partial F_{yy}}{\partial y} + \frac{\partial F_{zz}}{\partial z} \right) - 2 \frac{\partial F_{zz}}{\partial z} \quad (3.126)$$

In the same manner the remaining three conditions (3.97), (3.99), (3.101) can be transformed into equations of the following kinds,

$$\nabla^2 \sigma_{yz} + \frac{1}{1+\nu} \frac{\partial^2 \theta}{\partial y \partial z} = - \left(\frac{\partial F_{zz}}{\partial y} + \frac{\partial F_{yy}}{\partial z} \right) \quad (3.127)$$

$$\nabla^2 \sigma_{xz} + \frac{1}{1+\nu} \frac{\partial^2 \theta}{\partial x \partial z} = - \left(\frac{\partial F_{zz}}{\partial x} + \frac{\partial F_{xx}}{\partial z} \right) \quad (3.128)$$

$$\nabla^2 \sigma_{xy} + \frac{1}{1+\nu} \frac{\partial^2 \theta}{\partial x \partial y} = - \left(\frac{\partial F_{xx}}{\partial y} + \frac{\partial F_{yy}}{\partial x} \right) \quad (3.129)$$

If there are no body forces or if the body forces are constant, the (3.124), (3.125), (3.126) equations and (3.127), (3.128), (3.129) equations become

$$(1+\nu) \nabla^2 \sigma_{xx} + \frac{\partial^2 \theta}{\partial x^2} = 0 \quad (3.130)$$

$$(1+\nu) \nabla^2 \sigma_{yy} + \frac{\partial^2 \theta}{\partial y^2} = 0 \quad (3.131)$$

$$(1+\nu) \nabla^2 \sigma_{zz} + \frac{\partial^2 \theta}{\partial z^2} = 0 \quad (3.132)$$

$$(1+\nu) \nabla^2 \sigma_{yz} + \frac{\partial^2 \theta}{\partial y \partial z} = 0 \quad (3.133)$$

$$(1+\nu) \nabla^2 \sigma_{xz} + \frac{\partial^2 \theta}{\partial x \partial z} = 0 \quad (3.134)$$

$$(1+\nu) \nabla^2 \sigma_{yx} + \frac{\partial^2 \theta}{\partial x \partial y} = 0 \quad (3.135)$$

In addition to the equilibrium equation and the boundary conditions the stress components in an isotropic body must satisfy the six conditions of compatibility. Now the generalized Hook's stress components (3.9-14), expressed in terms of strain components become,

$$\sigma_{xx} = \lambda e + 2G\epsilon_{xx} \quad (3.136)$$

$$\sigma_{yy} = \lambda e + 2G\epsilon_{yy} \quad (3.137)$$

$$\sigma_{zz} = \lambda e + 2G\epsilon_{zz} \quad (3.138)$$

$$\sigma_{xy} = G\epsilon_{xy} \quad (3.139)$$

$$\sigma_{yz} = G\epsilon_{yz} \quad (3.140)$$

$$\sigma_{zx} = G\epsilon_{zx} \quad (3.141)$$

where, $e = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$, $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$, $G = \frac{E}{2(1+\nu)}$, Substituting the relation σ_{xx} , σ_{xy} , σ_{zx} into the first equation of equilibrium (3.89), (3.90), (3.91) we get

$$\lambda \frac{\partial e}{\partial x} + G \left(2 \frac{\partial \epsilon_{xx}}{\partial x} + \frac{\partial \epsilon_{xy}}{\partial y} + \frac{\partial \epsilon_{zx}}{\partial z} \right) = 0 \quad (3.142)$$

and if we substitute for the strain component the expressions (3.34) we find that (3.141) can be written in the form

$$(\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u = 0 \quad (3.143)$$

The other two equations can be transformed in a similar manner.

Thus the three equations of equilibrium, expressed in terms of displacements are

$$(\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u = 0 \quad (3.144)$$

$$(\lambda + G) \frac{\partial e}{\partial y} + G \nabla^2 v = 0 \quad (3.145)$$

$$(\lambda + G) \frac{\partial e}{\partial z} + G \nabla^2 w = 0 \quad (3.146)$$

The governing differential equation can be obtained by adding the three equations of equilibrium together

$$(\lambda + G) \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} e + G \nabla^2 \{u, v, w\} = 0 \quad (3.147)$$

and by rearranging and simplifying :

$$\nabla^2 \{u, v, w\} + \frac{1}{1-2\nu} \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \quad (3.148)$$

Now, we can extend the equations of equilibrium in terms of displacement components to their explicit forms in terms of the derivatives in the case where no body forces exist. The first equation after the extension gives

$$(\lambda + G) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = 0 \quad (3.149)$$

Similarly, the other two equations can be derived and the general system of equations is transformed to the following.

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial y^2} + G \frac{\partial^2 u}{\partial z^2} + (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} + (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = 0 \quad (3.150)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} + G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial z^2} + (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = 0 \quad (3.151)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} + (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} + G \frac{\partial^2 w}{\partial x^2} + G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = 0 \quad (3.152)$$

and in matrix form the system of equation of equilibrium can be represented by :

$$\begin{bmatrix} UU & UV & UW \\ VU & VV & VW \\ WU & WV & WW \end{bmatrix}$$

The explanation of these notations will be shown at chapter IV. From the matrix notations, it should be noted that if we only consider thin plate

theory, only one element WW, is needed. This element WW takes care of the vertical deflection w. In thin plate theory all the other terms in the matrix may be neglected and their contributions are considered insignificant. However, in thick plate theory, all nine elements in the matrix are involved and all their contributions must be considered in solving the problem. We can see the increase in difficulty and complexity of the thick plate problem from this matrix interpretation. If we note by N the number of nodes, the dimension of the general matrix shown above is three times greater than the dimension of the submatrice WW which is N×N. Thus, to improve the convergence and the accuracy of the numerical technique, we need to use more nodes thus resulting in very large systems of simultaneous equations.

3.4 Dimensionless Representation

To consider the non-dimensional representation we need to interpret the equations of equilibrium for large analysis in terms of the displacements u, v, w as was shown for the small deflection analysis. As mentioned above, and by considering the iterative notation for the left hand side and the right hand side we get:

$$\begin{aligned}
 & (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial y^2} + G \frac{\partial^2 u}{\partial z^2} + (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} + (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = \\
 & -G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z} \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) - G \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 w}{\partial x^2} \right) - \\
 & G \left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) - G \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial y \partial z} - \frac{\partial^2 w}{\partial x \partial y} \right) -
 \end{aligned}$$

$$\begin{aligned} & \left((\lambda + 2G) \frac{\partial^2 w}{\partial z^2} + \lambda \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} \right) \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) - \\ & \left((\lambda + 2G) \frac{\partial w}{\partial z} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x \partial z} \right) \quad (3.153) \end{aligned}$$

$$\begin{aligned} & (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} + G \frac{\partial^2 v}{\partial x^2} (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial z^2} + (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = \\ & G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z} \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + G \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 v}{\partial x \partial z} \right) + \\ & G \left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + G \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 v}{\partial y \partial z} \right) + \\ & \left((\lambda + 2G) \frac{\partial^2 w}{\partial z^2} + \lambda \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} \right) \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + \\ & \left((\lambda + 2G) \frac{\partial w}{\partial z} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial y \partial z} - \frac{\partial^2 v}{\partial z^2} \right) \quad (3.154) \end{aligned}$$

$$\begin{aligned} & (\lambda + G) \frac{\partial^2 u}{\partial x \partial z} + (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} + G \frac{\partial^2 w}{\partial x^2} + G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = \\ & \left((\lambda + 2G) \frac{\partial^2 u}{x^2} + \lambda \left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + \\ & \left((\lambda + 2G) \frac{\partial u}{\partial x} + \lambda \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 w}{x^2} \right) - \\ & G \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - G \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 v}{\partial x \partial z} \right) + \\ & G \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + G \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial y \partial z} - \frac{\partial^2 w}{\partial x \partial y} \right) - \\ & \left((\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + \lambda \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z} \right) \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - \\ & \left((\lambda + 2G) \frac{\partial v}{\partial y} + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 v}{\partial y \partial z} \right) + \\ & G \left(\frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 u}{\partial z^2} \right) \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + G \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \frac{1}{2} \left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x \partial z} \right) - \\ & G \left(\frac{\partial^2 w}{\partial y \partial z} + \frac{\partial^2 v}{\partial z^2} \right) \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - G \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \frac{1}{2} \left(\frac{\partial^2 w}{\partial y \partial z} - \frac{\partial^2 v}{\partial z^2} \right) \quad (3.155) \end{aligned}$$

If we consider a system of dimensionless coordinate with ϕ, χ, ψ where:

$$U = \frac{au}{h^2} \quad \phi = \frac{x}{a} \quad (3.156)$$

$$V = \frac{av}{h^2} \quad \chi = \frac{y}{b} \quad (3.157)$$

$$W = \frac{w}{h} \quad \psi = \frac{z}{h} \quad (3.158)$$

$$n = \frac{a}{b} \quad m = \frac{h}{a} \quad (3.159)$$

then we get;

$$\begin{aligned} & (\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} + n^2 G \frac{\partial^2 U}{\partial \chi^2} + m^{-2} G \frac{\partial^2 U}{\partial \psi^2} + n(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} + m^{-2}(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = \\ & -G \left(\frac{\partial^2 W}{\partial \phi^2} + \frac{\partial^2 U}{\partial \phi \partial \psi} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \phi \partial \psi} - \frac{\partial^2 W}{\partial \phi^2} \right) - \\ & G \left(n \frac{\partial^2 V}{\partial \chi \partial \psi} + n^2 \frac{\partial^2 W}{\partial \chi^2} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - G \left(\frac{\partial V}{\partial \psi} + n \frac{\partial W}{\partial \chi} \right) \frac{1}{2} \left(n \frac{\partial^2 U}{\partial \chi \partial \psi} - n \frac{\partial^2 W}{\partial \phi \partial \chi} \right) - \\ & \left((\lambda + 2G) m^{-2} \frac{\partial^2 W}{\partial \psi^2} + \lambda \left(\frac{\partial^2 U}{\partial \phi \partial \psi} + n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - \\ & \left((\lambda + 2G) m^{-2} \frac{\partial W}{\partial \psi} + \lambda \left(\frac{\partial U}{\partial \phi} + n \frac{\partial V}{\partial \chi} \right) \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \psi^2} - \frac{\partial^2 W}{\partial \phi \partial \psi} \right) \end{aligned} \quad (3.160)$$

$$\begin{aligned} & n(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} + G \frac{\partial^2 V}{\partial \phi^2} + n^2(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} + m^{-2} G \frac{\partial^2 V}{\partial \psi^2} + n m^{-2}(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = \\ & G \left(\frac{\partial^2 W}{\partial \phi^2} + \frac{\partial^2 U}{\partial \phi \partial \psi} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \phi \partial \chi} - \frac{\partial^2 V}{\partial \phi \partial \psi} \right) + \\ & G \left(n \frac{\partial^2 V}{\partial \chi \partial \psi} + n^2 \frac{\partial^2 W}{\partial \chi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + G \left(\frac{\partial V}{\partial \psi} + n \frac{\partial W}{\partial \chi} \right) \frac{1}{2} \left(n^2 \frac{\partial^2 W}{\partial \chi^2} - n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) + \\ & \left((\lambda + 2G) m^{-2} \frac{\partial^2 W}{\partial \psi^2} + \lambda \left(\frac{\partial^2 U}{\partial \phi \partial \psi} + n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + \\ & \left((\lambda + 2G) m^{-2} \frac{\partial W}{\partial \psi} + \lambda \left(\frac{\partial U}{\partial \phi} + n \frac{\partial V}{\partial \chi} \right) \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} - \frac{\partial^2 V}{\partial \psi^2} \right) \end{aligned} \quad (3.161)$$

$$\begin{aligned}
& m^{-1} \left[(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} + n (\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} + G \frac{\partial^2 W}{\partial \phi^2} + n^2 G \frac{\partial^2 W}{\partial \chi^2} + m^{-2} (\lambda + 2G) \frac{\partial^2 \Pi}{\partial U^2} \right] = \\
& m^{-1} \left((\lambda + 2G) m^2 \frac{\partial^2 U}{\phi^2} + \lambda (n m^2 \frac{\partial^2 V}{\partial \phi \partial \chi} + \frac{\partial^2 W}{\partial \phi \partial \psi}) \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) + \\
& m^{-1} \left((\lambda + 2G) m^2 \frac{\partial U}{\partial \phi} + \lambda (n m^2 \frac{\partial V}{\partial \chi} + \frac{\partial W}{\partial \psi}) \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \phi \partial \psi} - \frac{\partial^2 W}{\phi^2} \right) - \\
& mG \left(n \frac{\partial^2 U}{\partial \phi \partial \chi} + \frac{\partial^2 V}{\partial \phi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - mG \left(n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \phi \partial \chi} - \frac{\partial^2 V}{\partial \phi \partial \psi} \right) + \\
& mG \left(n^2 \frac{\partial^2 U}{\partial \chi^2} + n \frac{\partial^2 V}{\partial \phi \partial \chi} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) + mG \left(n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} \right) \frac{1}{2} \left(n \frac{\partial^2 U}{\partial \chi \partial \psi} - n \frac{\partial^2 W}{\partial \phi \partial \chi} \right) - \\
& m^{-1} \left((\lambda + 2G) (nm)^2 \frac{\partial^2 V}{\chi^2} + \lambda (nm^2 \frac{\partial^2 U}{\partial \phi \partial \chi} + n \frac{\partial^2 W}{\partial \chi \partial \psi}) \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - \\
& m^{-1} \left((\lambda + 2G) n m^2 \frac{\partial V}{\partial \chi} + \lambda (m^2 \frac{\partial U}{\partial \phi} + \frac{\partial W}{\partial \psi}) \right) \frac{1}{2} \left(n^2 \frac{\partial^2 W}{\partial \chi^2} - n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) + \\
& m^{-1} G \left(\frac{\partial^2 W}{\partial \phi \partial \psi} + \frac{\partial^2 U}{\partial \psi^2} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) + m^{-1} G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \psi^2} - \frac{\partial^2 W}{\partial \phi \partial \psi} \right) - \\
& m^{-1} G \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} + \frac{\partial^2 V}{\partial \psi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - m^{-1} G \left(\frac{n \partial W}{\partial \chi} + \frac{\partial V}{\partial \psi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} - \frac{\partial^2 V}{\partial \psi^2} \right)
\end{aligned} \tag{3.162}$$

3.5 Boundary conditions

The differential equations of equilibrium which have been derived previously for stresses and displacements within the plate must also be such as to accommodate the conditions of equilibrium with respect to prescribed forces or displacements at the boundary. In thin plate theory the boundary conditions need only be satisfied at the longitudinal and transverse dimensions

but for thick plates, an additional dimension must be taken into account. In other words, for thick plates there are three boundaries to satisfy. If we consider that the origin of the plate at mid-plate, then the two boundary surfaces should be at $\psi = \pm 1/2$ with respect to the non-dimensional ψ axis upward positive. In this thesis we are considering the case of static deflection due to uniformly distributed loads acting perpendicular to the surface of the plate. For rectangular clamped and simply supported plates, the top and bottom surfaces boundaries conditions are,

$$\text{at: } \psi = \pm \frac{1}{2}, \quad \sigma_{xz} = 0, \quad \sigma_{yz} = 0, \quad \sigma_{zz} = \pm \frac{q}{2} \quad (3.163)$$

then the system of equilibrium equations reduces itself to the system of plane stress where,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad (3.164)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \quad (3.165)$$

We know from the theory of thin plates that this system can be transformed to the well known differential equation governing the small deflection of thin plates in non-dimensional coordinates: viz.,

$$\frac{\partial^4 W}{\partial \phi^4} + 2n^2 \frac{\partial^4 W}{\partial \phi^2 \partial \chi^2} + n^4 \frac{\partial^4 W}{\partial \chi^4} = \frac{q a^4}{D h} \quad (3.166)$$

For the large deflection three equations form a general solution to the problem at the boundary layers. These three equations are the result of a combination between the governing equation with bending and stretching plus the two equilibrium equations (3.160), (3.161). We represent the three

equations in terms of displacements in non-dimensional coordinates namely

U, V, W :

$$\frac{\partial^2 U}{\partial \phi^2} + n^2 \frac{1-\nu}{2} \frac{\partial^2 U}{\partial \chi^2} + \frac{1+\nu}{2} n \frac{\partial^2 V}{\partial \phi \partial \chi} = -\frac{\partial W}{\partial \phi} \left(\frac{\partial^2 W}{\partial \phi^2} + n^2 \frac{1-\nu}{2} \frac{\partial^2 W}{\partial \chi^2} \right) - n^2 \frac{1+\nu}{2} \frac{\partial \Pi}{\partial \chi} \frac{\partial^2 \Pi}{\partial \phi \partial \chi} \quad (3.167)$$

$$n^2 \frac{\partial^2 V}{\partial \chi^2} + \frac{1-\nu}{2} \frac{\partial^2 V}{\partial \phi^2} + n \frac{1+\nu}{2} \frac{\partial^2 U}{\partial \phi \partial \chi} = -n \frac{\partial W}{\partial \chi} \left(n^2 \frac{\partial^2 W}{\partial \chi^2} + \frac{1-\nu}{2} \frac{\partial^2 W}{\partial \phi^2} \right) - n \frac{1+\nu}{2} \frac{\partial \Pi}{\partial \phi} \frac{\partial^2 \Pi}{\partial \phi \partial \chi} \quad (3.168)$$

$$\begin{aligned} \frac{\partial^4 W}{\partial \phi^4} + 2n^2 \frac{\partial^4 W}{\partial \phi^2 \partial \chi^2} + n^4 \frac{\partial^4 W}{\partial \chi^4} = \frac{q a^4}{D h} &+ 12 \left(\left(\frac{\partial U}{\partial \phi} + \frac{1}{2} \left(\frac{\partial W}{\partial \phi} \right)^2 \right) \left(\frac{\partial^2 W}{\partial \phi^2} + n^2 \nu \frac{\partial^2 W}{\partial \chi^2} \right) \right. \\ &+ \left(n \frac{\partial V}{\partial \chi} + \frac{n^2}{2} \left(\frac{\partial W}{\partial \chi} \right)^2 \right) \left(n^2 \frac{\partial^2 W}{\partial \chi^2} + \nu \frac{\partial^2 W}{\partial \phi^2} \right) \\ &+ \left. (1-\nu) \left(n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} + n \frac{\partial W}{\partial \phi} \frac{\partial W}{\partial \chi} \right) n \frac{\partial^2 W}{\partial \phi \partial \chi} \right) \end{aligned} \quad (3.169)$$

3.6 Clamped Plate

For the clamped plate, the contour of the plate is fully fixed and all the displacements and rotations are equal to zero,

$$\text{at: } \phi = \pm \frac{1}{2}, \quad U = V = W = 0 \quad \omega_y = 0 \quad (3.170)$$

$$\text{at: } \chi = \pm \frac{n}{2}, \quad U = V = W = 0 \quad \omega_x = 0 \quad (3.171)$$

$$\text{at: } \psi = \pm \frac{m}{2}, \quad \sigma_{zz} = \pm \frac{q}{2}, \quad \sigma_{xx} = 0, \quad \sigma_{yz} = 0 \quad (3.172)$$

3.7 Simply Supported Plate

In the case of simply supported plates the choice is more complex and the boundary conditions considered here represent the case where the edges are immovables:

$$\text{at : } \phi = \pm \frac{1}{2}, \quad U = V = W = 0 \quad M_{xx} = 0 \quad (3.173)$$

$$\text{at : } \chi = \pm \frac{n}{2}, \quad U = V = W = 0 \quad M_{yy} = 0 \quad (3.174)$$

$$\text{at : } \psi = \pm \frac{m}{2}, \quad \sigma_{zz} = \pm \frac{q}{2}, \quad \sigma_{xz} = 0, \quad \sigma_{yz} = 0 \quad (3.175)$$

Chapter 4

Formulation of the Solution

4.1 Iterative Solution

In chapter III, the governing equations necessary for the analysis of thick plates were presented, but solving these coupled nonlinear partial differential equations numerically may prove to be extremely difficult. The finite difference technique is used here to transform the differential equations into ordinary algebraic equations in terms of the displacements U, V , and W at specified points. To solve the problem an iterative procedure is used and, in order to perform the numerical analysis efficiently in the programme, the equilibrium equations are arranged in such way so that the solutions are

represented in matrix form:

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} + n^2 G \frac{\partial^2 U}{\partial \chi^2} + m^{-2} G \frac{\partial^2 U}{\partial \psi^2} + n(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} + m^{-2}(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = RHSX \quad (4.1)$$

$$n(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} + G \frac{\partial^2 V}{\partial \phi^2} + n^2(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} + m^{-2} G \frac{\partial^2 V}{\partial \psi^2} + nm^{-2}(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = RHSY \quad (4.2)$$

$$\frac{(\lambda + G)}{m} \frac{\partial^2 U}{\partial \phi \partial \psi} + n \frac{(\lambda + G)}{m} \frac{\partial^2 V}{\partial \chi \partial \psi} + \frac{G}{m} \frac{\partial^2 W}{\partial \phi^2} + n^2 \frac{G}{m} \frac{\partial^2 W}{\partial \chi^2} + \frac{(\lambda + 2G)}{m^3} \frac{\partial^2 W}{\partial \psi^2} = RHSZ \quad (4.3)$$

Where;

$$\begin{aligned} RHSX = & -G \left(\frac{\partial^2 W}{\partial \phi^2} + \frac{\partial^2 U}{\partial \phi \partial \psi} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \phi \partial \psi} - \frac{\partial^2 W}{\partial \phi^2} \right) - \\ & G \left(n \frac{\partial^2 V}{\partial \chi \partial \psi} + n^2 \frac{\partial^2 W}{\partial \chi^2} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - G \left(\frac{\partial V}{\partial \psi} + n \frac{\partial W}{\partial \chi} \right) \frac{1}{2} \left(n \frac{\partial^2 U}{\partial \chi \partial \psi} - n \frac{\partial^2 W}{\partial \phi \partial \chi} \right) - \\ & \left((\lambda + 2G)m^{-2} \frac{\partial^2 W}{\partial \psi^2} + \lambda \left(\frac{\partial^2 U}{\partial \phi \partial \psi} + n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) - \\ & \left((\lambda + 2G)m^{-2} \frac{\partial W}{\partial \psi} + \lambda \left(\frac{\partial U}{\partial \phi} + n \frac{\partial V}{\partial \chi} \right) \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \psi^2} - \frac{\partial^2 W}{\partial \phi \partial \psi} \right) \end{aligned} \quad (4.4)$$

$$\begin{aligned} RHSY = & G \left(\frac{\partial^2 W}{\partial \phi^2} + \frac{\partial^2 U}{\partial \phi \partial \psi} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \phi \partial \chi} - \frac{\partial^2 V}{\partial \phi \partial \psi} \right) + \\ & G \left(n \frac{\partial^2 V}{\partial \chi \partial \psi} + n^2 \frac{\partial^2 W}{\partial \chi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + G \left(\frac{\partial V}{\partial \psi} + n \frac{\partial W}{\partial \chi} \right) \frac{1}{2} \left(n^2 \frac{\partial^2 W}{\partial \chi^2} - n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) + \\ & \left((\lambda + 2G)m^{-2} \frac{\partial^2 W}{\partial \psi^2} + \lambda \left(\frac{\partial^2 U}{\partial \phi \partial \psi} + n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) + \\ & \left((\lambda + 2G)m^{-2} \frac{\partial W}{\partial \psi} + \lambda \left(\frac{\partial U}{\partial \phi} + n \frac{\partial V}{\partial \chi} \right) \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} - \frac{\partial^2 V}{\partial \psi^2} \right) \end{aligned} \quad (4.5)$$

$$RHSZ = m^{-1} \left((\lambda + 2G)m^2 \frac{\partial^2 U}{\phi^2} + \lambda(nm^2 \frac{\partial^2 V}{\partial \phi \partial \chi} + \frac{\partial^2 W}{\partial \phi \partial \psi}) \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) +$$

$$\begin{aligned}
& m^{-1} \left((\lambda + 2G)m^2 \frac{\partial U}{\partial \phi} + \lambda(nm^2 \frac{\partial V}{\partial \chi} + \frac{\partial W}{\partial \psi}) \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \phi \partial \psi} - \frac{\partial^2 W}{\phi^2} \right) - \\
& mG \left(n \frac{\partial^2 U}{\partial \phi \partial \chi} + \frac{\partial^2 V}{\partial \phi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - mG \left(n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \phi \partial \chi} - \frac{\partial^2 V}{\partial \phi \partial \psi} \right) + \\
& mG \left(n^2 \frac{\partial^2 U}{\partial \chi^2} + n \frac{\partial^2 V}{\partial \phi \partial \chi} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) + mG \left(n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} \right) \frac{1}{2} \left(n \frac{\partial^2 U}{\partial \chi \partial \psi} - n \frac{\partial^2 W}{\partial \phi \partial \chi} \right) - \\
& m^{-1} \left((\lambda + 2G)(nm)^2 \frac{\partial^2 V}{\chi^2} + \lambda(nm^2 \frac{\partial^2 U}{\partial \phi \partial \chi} + n \frac{\partial^2 W}{\partial \chi \partial \psi}) \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - \\
& m^{-1} \left((\lambda + 2G)nm^2 \frac{\partial V}{\partial \chi} + \lambda(m^2 \frac{\partial U}{\partial \phi} + \frac{\partial W}{\partial \psi}) \right) \frac{1}{2} \left(n^2 \frac{\partial^2 W}{\partial \chi^2} - n \frac{\partial^2 V}{\partial \chi \partial \psi} \right) + \\
& m^{-1} G \left(\frac{\partial^2 W}{\partial \phi \partial \psi} + \frac{\partial^2 U}{\partial \psi^2} \right) \frac{1}{2} \left(\frac{\partial U}{\partial \psi} - \frac{\partial W}{\partial \phi} \right) + m^{-1} G \left(\frac{\partial W}{\partial \phi} + \frac{\partial U}{\partial \psi} \right) \frac{1}{2} \left(\frac{\partial^2 U}{\partial \psi^2} - \frac{\partial^2 W}{\partial \phi \partial \psi} \right) - \\
& m^{-1} G \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} + \frac{\partial^2 V}{\partial \psi^2} \right) \frac{1}{2} \left(n \frac{\partial W}{\partial \chi} - \frac{\partial V}{\partial \psi} \right) - m^{-1} G \left(\frac{n \partial W}{\partial \chi} + \frac{\partial V}{\partial \psi} \right) \frac{1}{2} \left(n \frac{\partial^2 W}{\partial \chi \partial \psi} - \frac{\partial^2 V}{\partial \psi^2} \right)
\end{aligned} \tag{4.6}$$

Satisfying the equilibrium equations and boundary conditions we write:

$$[A]\{DIS\} = \{FOR\} \tag{4.7}$$

Where:

$$\{FOR\} = \{FOR\} + \gamma\{RHS\}$$

Before introducing the $\{RHS\}$ vector the problem is reduced to small deflection and the vector $\{FOR\}$ consists in representing the external forces applied on the plate only, where the output $\{DIS\}$ represent the linear displacements and will be considered as the first guess into the iterative procedure. When the $\{RHS\}$ vector is included, the displacement vector $\{DIS\}$ becomes $\{DISN\}$ a nonlinear displacement vector and we resume:

$$[A]^{-1}\{FOR\} = \begin{cases} \{DIS\} & \text{if } \{RHS\} = 0 \\ \{DISN\} & \text{if } \{RHS\} \neq 0 \end{cases} \tag{4.8}$$

4.2 Finite difference method

The derivation of the equations of equilibrium and deformation in the analysis of plates leads to a number of partial differential equations in which the unknowns are the internal forces and the deformations. The unknowns occur in differential form by considering the equilibrium and deformation of an infinitesimal plate element. The differential relations have in their turn a well defined physical meaning. They permit the local study of forces or deformations. The complete determination of these phenomena involves the integration of the equations.

In most cases, the solutions of differential or partial differential equations cannot be obtained by means of elementary functions. This arises because, in the most general case, when the radii of curvature of the middle surface and the external loads are not given explicitly, the form of the differential equation or partial differential equation is not known and the general solution cannot be obtained.

In general, solutions to the governing differential equations are difficult to obtain and numerical computations must be resorted to. The approximations in numerical calculation may be made small by suitable choice of the initial scheme for the calculation. The basis of any method of numerical computation lies in not employing infinitely small quantities, but in using very small finite quantities. For this operation however, the form of the problem is modified.

In the first case, the analytical solution leads to a continuous expression for

the unknowns at least within distinct intervals; in other words, having the expression for an unknown as a function of the independent variables, we can determine directly the value of this unknown at any point on the middle surface. On the other hand, the numerical calculation leads to the determination of the values of the unknowns only at the points of a previously established network. In Figure (4.1) we show how the network of modified finite difference is assembled with respect to the origin at the mid-plate. To make everything clear Figure (4.2) shows the three principal planes $\phi\chi$, $\chi\psi$, $\psi\phi$, which are the symmetrical planes of the plate. The same notation is used to simplify the interpretation of the partial derivatives defining the finite difference network within the plate.

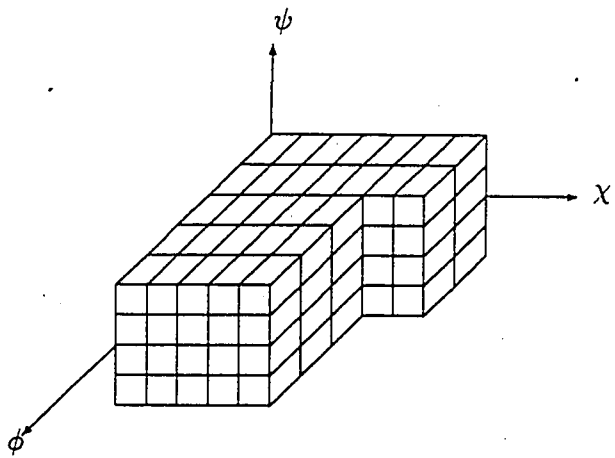


Figure (4.1) Three dimensional finite difference network

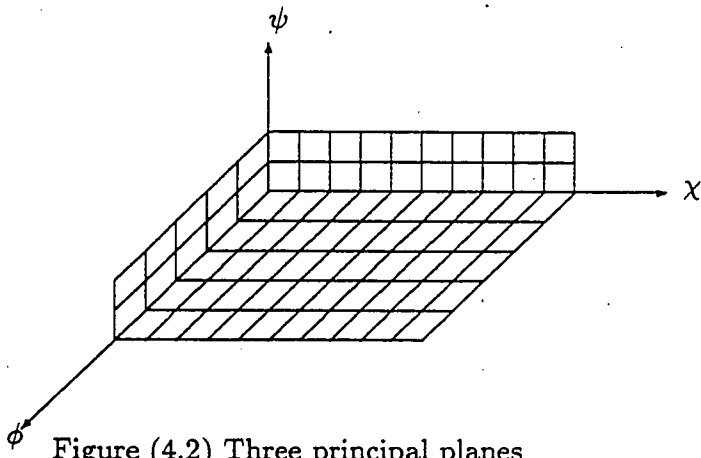
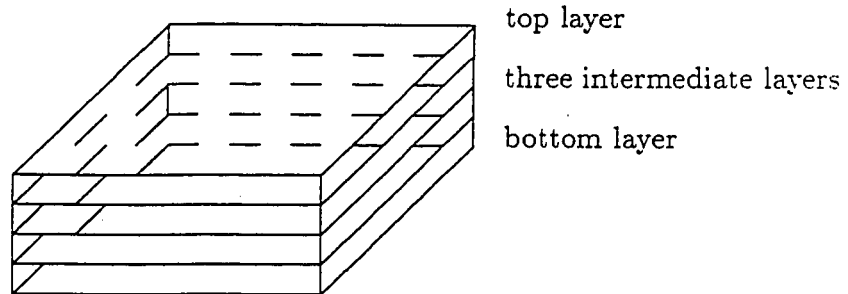


Figure (4.2) Three principal planes



Figure(4.3) Horizontal partition showing layers of the plate

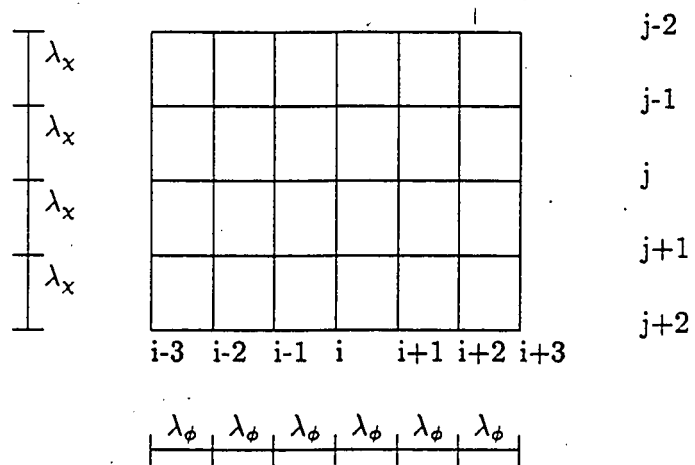


Figure (4.4) The finite difference mesh-size in the $\phi\chi$ plane

While defining the dimensionless system of notations U, V, W, for the partial derivatives in terms of finite difference equations, in the ϕ , χ and ψ directions respectively, the plate is also divided in layers as shown in Figure (4.3). The equations of equilibrium are divided into sub-matrices defining the participation of each function in one principal direction. Here,

UU : represents the participation of the U function in the ϕ direction.

UV : represents the participation of the V function in the ϕ direction.

UW : represents the participation of the W function in the ϕ direction.

VU : represents the participation of the U function in the χ direction.

VV : represents the participation of the V function in the χ direction.

VW : represents the participation of the W function in the χ direction.

WU : represents the participation of the U function in the ψ direction.

WV : represents the participation of the V function in the ψ direction.

WW : represents the participation of the W function in the ψ direction.

For the notation of our grid network, only a quarter of the plate is considered due to the symmetry in the xy plane, while in the z direction the advantage of the symmetry is not considered for the general use of the program, in case we consider subsequently a plate with variable thickness. The increment in the notation Figure (4.8) is considered in a manner to simplify the notation. For example, in the ϕ direction the increment is always unity, in the χ direction the increment is equal to the term IJ and in the ψ direction we have in the plane $\phi\psi$ the term IK, in the plane $\chi\psi$ the term JK. These two terms IK, JK are equal in the case of the plate with constant thickness.

As it was described in chapter III the top and bottom layers are determined

by equation governing thin plate theory. The diagram in Figure (4.7) shows the development of this equation and it can be employed at the boundary. As it is known for the boundary conditions of clamped plates, points are added beyond the boundary limit similar to the point inside the plate affected with a plus sign and with a minus sign for the simply supported plate, see Figures (4.5) and (4.6).

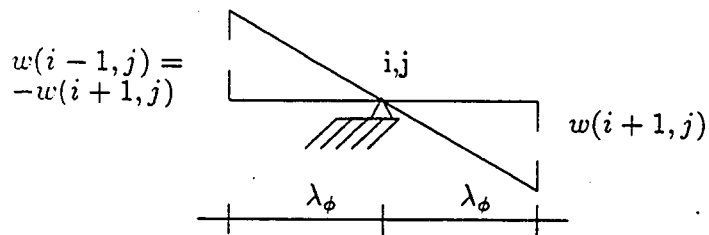


Figure (4.5) Simply Supported Edge

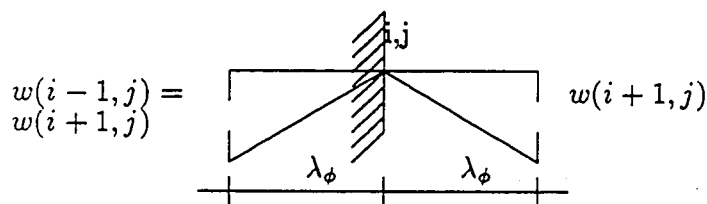


Figure (4.6) Fixed Edge

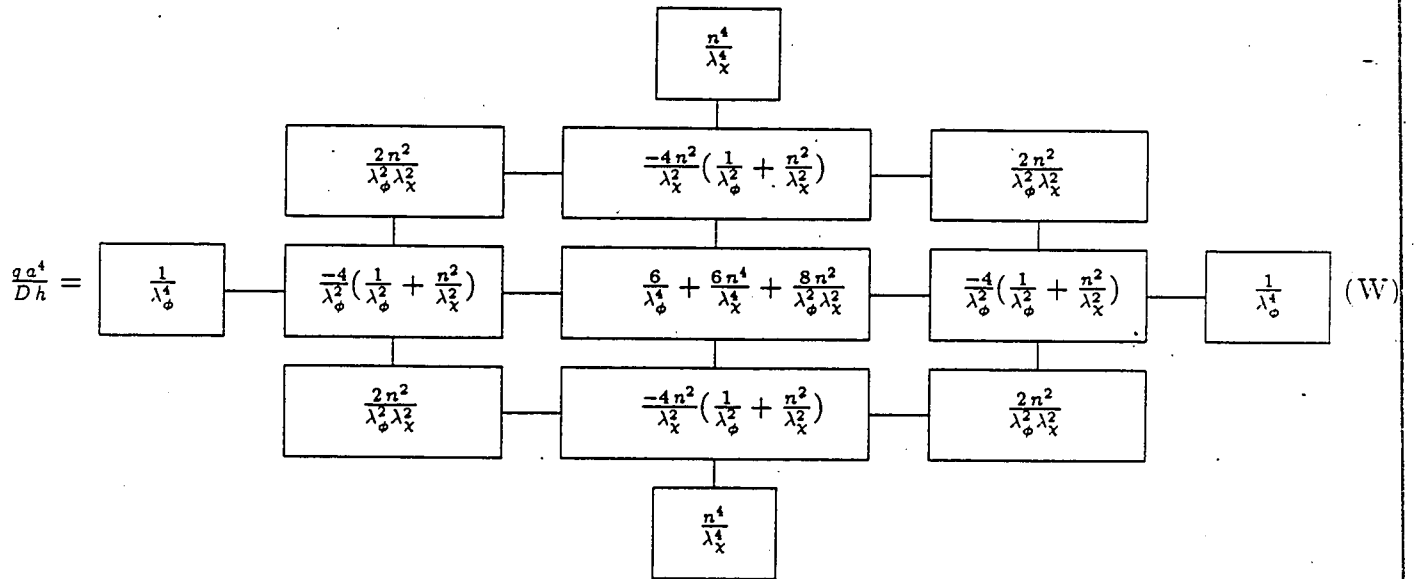
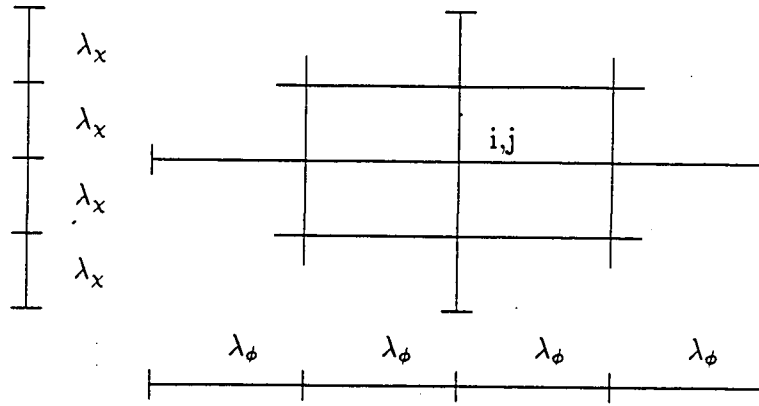
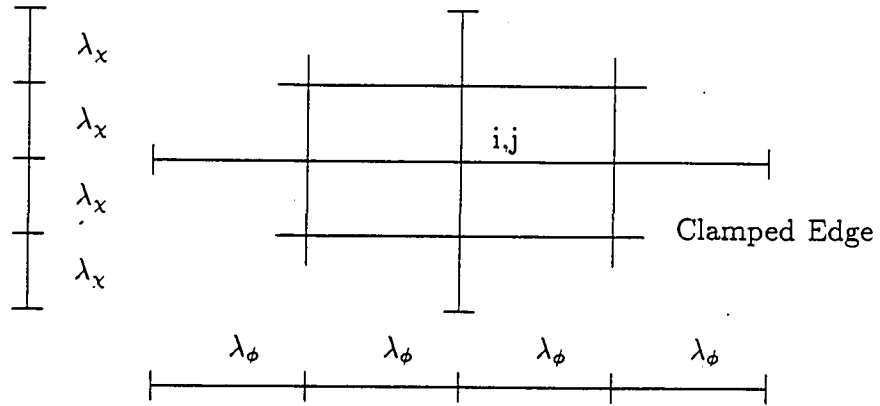
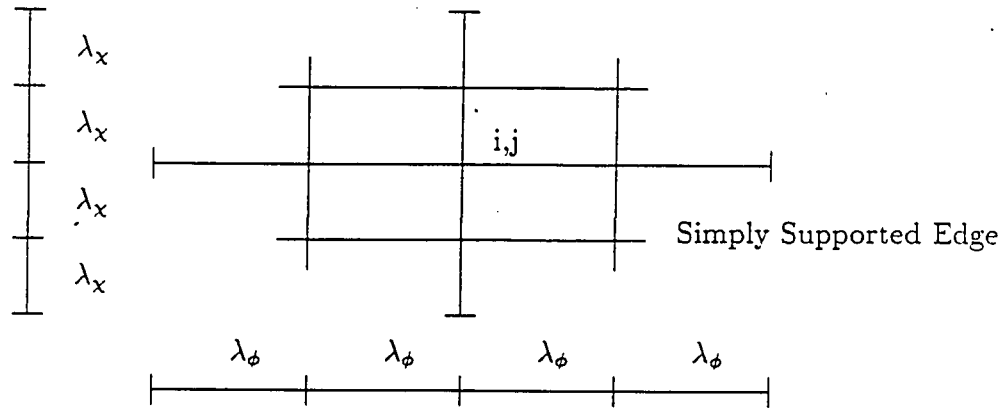


Figure (4.7) General Finite Difference Scheme for Thin Plate



$$\frac{qa^4}{Dh} = \left[\frac{1}{\lambda_\phi^4} \right] - \left[\frac{2n^2}{\lambda_\phi^2 \lambda_x^2} \right] - \left[\frac{-4n^2}{\lambda_x^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right) \right] - \left[\frac{2n^2}{\lambda_\phi^2 \lambda_x^2} \right] + \left[\frac{n^4}{\lambda_x^4} \right] - \left[\frac{-4}{\lambda_\phi^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right) \right] - \left[\frac{6}{\lambda_\phi^4} + \frac{7n^4}{\lambda_x^4} + \frac{8n^2}{\lambda_\phi^2 \lambda_x^2} \right] - \left[\frac{-4}{\lambda_\phi^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right) \right] + \left[\frac{1}{\lambda_\phi^4} \right] \quad (W)$$

Figure (4.8) Clamped Edge Finite Difference Scheme



$$\frac{\partial^2 \alpha^4}{D h} = \begin{array}{ccccccc}
 & & \boxed{\frac{n^4}{\lambda_x^4}} & & & & \\
 & & | & & & & \\
 & \boxed{\frac{2n^2}{\lambda_\phi^2 \lambda_x^2}} & - & \boxed{\frac{-4n^2}{\lambda_x^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right)} & - & \boxed{\frac{2n^2}{\lambda_\phi^2 \lambda_x^2}} & \\
 & | & & | & & | & \\
 \boxed{\frac{1}{\lambda_\phi^4}} & - & \boxed{\frac{-4}{\lambda_\phi^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right)} & - & \boxed{\frac{6}{\lambda_\phi^4} + \frac{5n^4}{\lambda_x^4} + \frac{8n^2}{\lambda_\phi^2 \lambda_x^2}} & - & \boxed{\frac{-4}{\lambda_\phi^2} \left(\frac{1}{\lambda_\phi^2} + \frac{n^2}{\lambda_x^2} \right)} & - & \boxed{\frac{1}{\lambda_\phi^4}} & (W)
 \end{array}$$

Figure (4.9) Simply Supported Edge Finite Difference Scheme

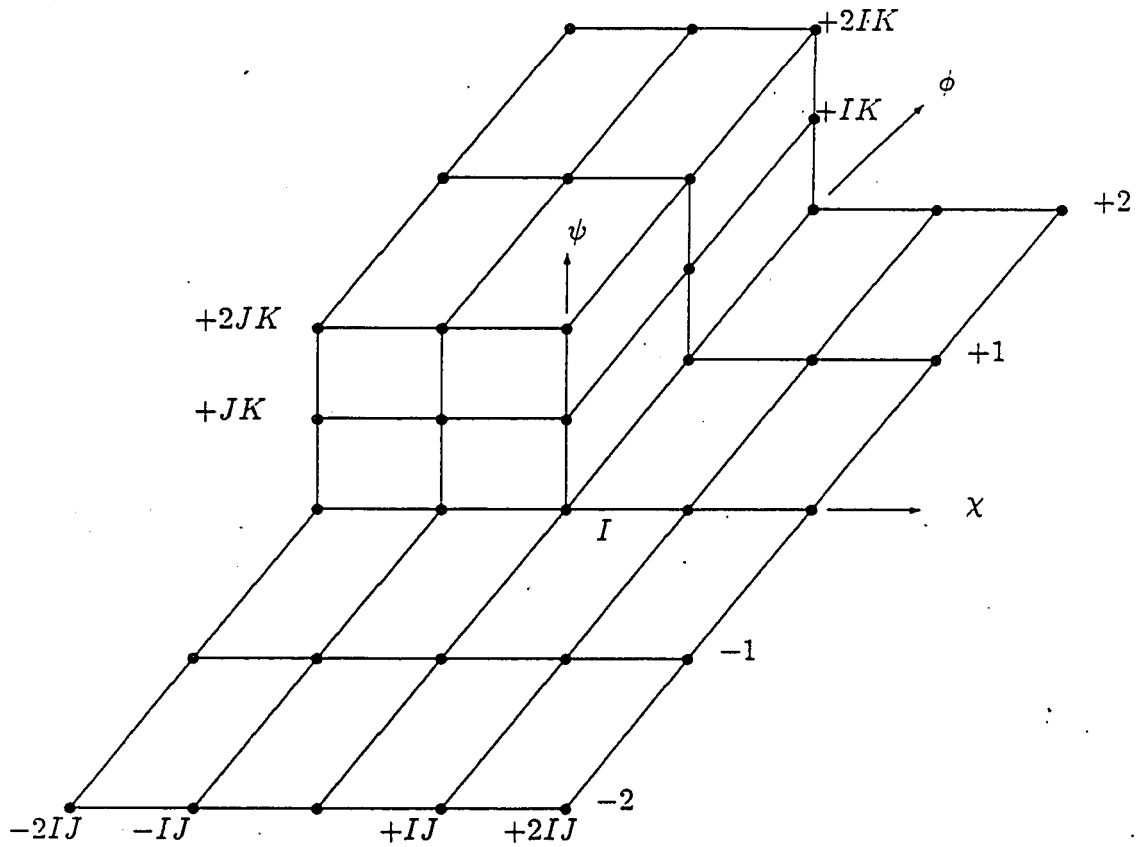


Figure (4.10) Three Dimensional Finite Difference Representation

For the general case we represent the partial derivatives determining the equilibrium equations by the following:

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 U}{\partial \psi^2} &= [U(I- IK) - 2U(I) + U(I+ IK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} &= [W(I) - W(I+1) - W(I+ IK) + W(I+ IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
G \frac{\partial^2 V}{\partial \phi^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{G}{\lambda_\phi^2} \\
(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} &= [V(I- IJ) - 2V(I) + V(I+ IJ)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 V}{\partial \psi^2} &= [V(I- IK) - 2V(I) + V(I+ IK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} &= [W(I) - W(I+ IJ) - W(I+ JK) + W(I+ IJ+ JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} &= [U(I) - U(I+1) - U(I+ IK) + U(I+ IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} &= [V(I) - V(I+ IJ) - V(I+ JK) + V(I+ IJ+ JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
G \frac{\partial^2 W}{\partial \phi^2} &= [W(I-1) - 2W(I) + W(I+1)] \frac{G}{\lambda_\phi^2} \\
G \frac{\partial^2 W}{\partial \chi^2} &= [W(I- IJ) - 2W(I) + W(I+ IJ)] \frac{G}{\lambda_x^2}
\end{aligned}$$

$$(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} = [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}$$

For the points on the χ axis

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} = [2U(I - 1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 U}{\partial \psi^2} = [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I - 1) - V(I + IJ) + V(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = [W(I) - W(I - 1) - W(I + IK) + W(I + IK - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I - 1) - U(I + IJ) + U(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [2V(I - 1) - 2V(I)] \frac{G}{\lambda_\phi^2}$$

$$(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} = [V(I - IJ) - 2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 V}{\partial \psi^2} = [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} = [U(I) - U(I - 1) - U(I + IK) + U(I + IK - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} = [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$G \frac{\partial^2 W}{\partial \phi^2} = [2W(I - 1) - 2W(I)] \frac{G}{\lambda_\phi^2}$$

$$G \frac{\partial^2 W}{\partial \chi^2} = [W(I - IJ) - 2W(I) + W(I + IJ)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} = [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}$$

For the points on the ϕ axis

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} = [U(I - 1) - 2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [2U(I - IJ) - 2U(I)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 U}{\partial \psi^2} = [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I + 1) - V(I - IJ) + V(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I + 1) - U(I - IJ) + U(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [V(I - 1) - 2V(I) + V(I + 1)] \frac{G}{\lambda_\phi^2}$$

$$(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} = [2V(I - IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 V}{\partial \psi^2} = [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = [W(I) - W(I - IJ) - U(I + JK) + U(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} = [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} = [V(I) - V(I - IJ) - V(I + IK) + V(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$G \frac{\partial^2 W}{\partial \phi^2} = [W(I-1) - 2W(I) + W(I+1)] \frac{G}{\lambda_\phi^2}$$

$$G \frac{\partial^2 W}{\partial \chi^2} = [2W(I-IJ) - 2W(I)] \frac{G}{\lambda_x^2}$$

$$(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} = [W(I-1K) - 2W(I) + W(I+1K)] \frac{(\lambda + 2G)}{\lambda_\psi^2}$$

For the central point

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} = [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 U}{\partial \psi^2} = [U(I-1K) - 2U(I) + U(I+1K)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I-1) - V(I-IJ) + V(I-IJ-1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = [W(I) - W(I-1) - W(I+1K) + W(I+1K-1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I-1) - U(I-IJ) + U(I-IJ-1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [2V(I-1) - 2V(I)] \frac{G}{\lambda_\phi^2}$$

$$(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} = [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 V}{\partial \psi^2} = [V(I-1K) - 2V(I) + V(I+1K)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = [W(I) - W(I-IJ) - U(I+JK) + U(I-IJ+JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} = [U(I) - U(I-1) - U(I+1K) + U(I+1K-1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$\begin{aligned}
(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} &= [V(I) - V(I - IJ) - V(I + JK) + V(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
G \frac{\partial^2 W}{\partial \phi^2} &= [2W(I - 1) - 2W(I)] \frac{G}{\lambda_\phi^2} \\
G \frac{\partial^2 W}{\partial \chi^2} &= [2W(I - IJ) - 2W(I)] \frac{G}{\lambda_\psi^2} \\
(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}
\end{aligned}$$

For the points on the top and bottom layers, the system of the equilibrium equations expressed in terms of the partial derivatives yields :

$$\frac{E}{1 - \nu^2} \frac{\partial^2 U}{\partial \phi^2} + n^2 G \frac{\partial^2 U}{\partial \chi^2} + \frac{E}{2(1 - \nu)} n \frac{\partial^2 V}{\partial \phi \partial \chi} = TRHSX \quad (4.9)$$

$$n \frac{E}{2(1 - \nu)} \frac{\partial^2 U}{\partial \phi \partial \chi} + G \frac{\partial^2 V}{\partial \phi^2} + n^2 \frac{E}{1 - \nu^2} \frac{\partial^2 V}{\partial \chi^2} = TRHSY \quad (4.10)$$

Where the right hand sides $TRHSX$ and $TRHSY$ are equal:

$$TRHSX = -\frac{E}{1 - \nu^2} \frac{\partial W}{\partial \phi} \frac{\partial^2 W}{\partial \phi^2} - n^2 \frac{E}{2(1 - \nu)} \frac{\partial W}{\partial \chi} \frac{\partial^2 W}{\partial \phi \partial \chi} - n^2 G \frac{\partial W}{\partial \phi} \frac{\partial^2 W}{\partial \chi^2} \quad (4.11)$$

$$TRHSY = -n^3 \frac{E}{1 - \nu^2} \frac{\partial W}{\partial \chi} \frac{\partial^2 W}{\partial \chi^2} - n \frac{E}{2(1 - \nu)} \frac{\partial W}{\partial \phi} \frac{\partial^2 W}{\partial \phi \partial \chi} - n G \frac{\partial W}{\partial \chi} \frac{\partial^2 W}{\partial \phi^2} \quad (4.12)$$

The above system of equations with three unknowns combined with the following principal differential equation governing the lateral displacement create a determined system of three equations (4.9), (4.10), (4.13) with three unknowns.

$$\frac{\partial^4 W}{\partial \phi^4} + 2n^2 \frac{\partial^4 W}{\partial \phi^2 \partial \chi^2} + n^4 \frac{\partial^4 W}{\partial \chi^4} = \frac{q a^4}{D h} + TRHSZ \quad (4.13)$$

$$\begin{aligned}
TRHSZ = & 12\left[\left(\frac{\partial U}{\partial \phi} + \frac{1}{2}\left(\frac{\partial W}{\partial \phi}\right)^2 + \nu n \frac{\partial V}{\partial \chi} + \frac{n^2 \nu}{2}\left(\frac{\partial W}{\partial \chi}\right)^2\right) \frac{\partial^2 W}{\partial \phi^2} + \right. \\
& \left. \left[\nu \frac{\partial U}{\partial \phi} + \frac{\nu}{2}\left(\frac{\partial W}{\partial \phi}\right)^2 + n \frac{\partial V}{\partial \chi} + \frac{n^2}{2}\left(\frac{\partial W}{\partial \chi}\right)^2\right] n^2 \frac{\partial^2 W}{\partial \chi^2} + \right. \\
& \left. (1 - \nu)\left[n \frac{\partial U}{\partial \chi} + \frac{\partial V}{\partial \phi} + n \frac{\partial W}{\partial \phi} \frac{\partial W}{\partial \chi}\right] n \frac{\partial^2 W}{\partial \phi \partial \chi}\right] \quad (4.14)
\end{aligned}$$

A general point on the boundary layer will be defined by the expressions below:

$$\begin{aligned}
\left(\frac{E}{1 - \nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [U(I - 1) - 2U(I) + U(I + 1)] \frac{\left(\frac{E}{1 - \nu^2}\right)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x} \\
G \frac{\partial^2 V}{\partial \phi^2} &= [V(I - 1) - 2V(I) + V(I + 1)] \frac{G}{\lambda_\phi^2} \\
\left(\frac{E}{1 - \nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [V(I - IJ) - 2V(I) + V(I + IJ)] \frac{\left(\frac{E}{1 - \nu^2}\right)}{\lambda_x^2}
\end{aligned}$$

For the case when the point is on the y axis

$$\begin{aligned}
\left(\frac{E}{1 - \nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [2U(I - 1) - 2U(I)] \frac{\left(\frac{E}{1 - \nu^2}\right)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I - 1) - V(I + IJ) + V(I + IJ - 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I - 1) - U(I + IJ) + U(I + IJ - 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x}
\end{aligned}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [2V(I-1) - 2V(I)] \frac{G}{\lambda_\phi^2}$$

$$\left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} = [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_x^2}$$

and when the point is on the x axis

$$\left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} = [U(I-1) - 2U(I) + U(I+1)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_x^2}$$

$$\left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I+1) - V(I-IJ) + V(I-IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_x}$$

$$\left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_\phi^2}$$

$$\left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} = [2V(I-IJ) - 2V(I)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_x^2}$$

and for the central point

$$\left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} = [2U(I-1) - 2U(I)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_x^2}$$

$$\left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I-1) - V(I-IJ) + V(I-IJ-1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_x}$$

$$\left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I-1) - U(I-IJ) + U(I-IJ-1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [2V(I-1) - 2V(I)] \frac{G}{\lambda_\phi^2}$$

$$\left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} = [2V(I-IJ) - 2V(I)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_x^2}$$

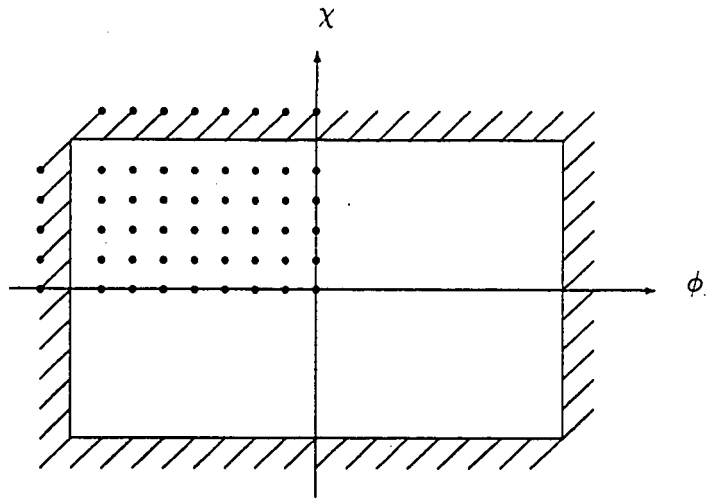
Now after describing the four cases common for all plates we now attempt to determine the particular points near the boundary. The application is limited to two cases, clamped and simply supported.

4.3 Simply supported plate

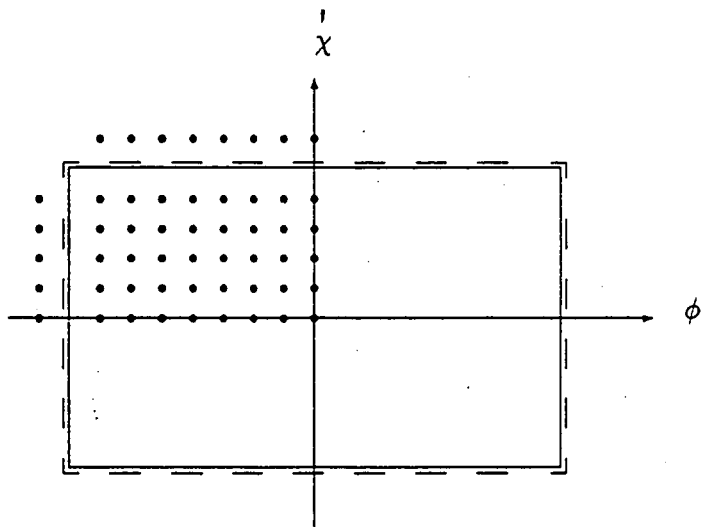
If the plate has ideal simply supported edges, it must be free to move along the supported edges in the plane of the plate, that is, the shearing stress along the edges in the plane of the plate is zero.

4.4 Clamped plate

For the clamped plate and simply supported plate with immovable edges all the components U , V , and W on the boundary are equal to zero, therefore the points near the boundary their backward finite difference components are equal to zero from the boundary conditions.



Figure(4.11) Clamped Plate



Figure(4.12) Simply Supported Plate

4.4.1 Intermediate surfaces

For the corner point,

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_\phi^2} \\
 G \frac{\partial^2 U}{\partial \chi^2} &= [-2U(I) + U(I+IJ)] \frac{G}{\lambda_x^2} \\
 G \frac{\partial^2 U}{\partial \psi^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_\psi^2} \\
 (\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
 (\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} &= [W(I) - W(I+1) - W(I+IK) + W(I+IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
 (\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
 G \frac{\partial^2 V}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{G}{\lambda_\phi^2} \\
 (\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} &= [-2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 V}{\partial \psi^2} &= [V(I - IK) - 2V(I) + V(I + IK)] \frac{G}{\lambda_\psi^2} \\
 (\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} &= [W(I) - W(I+IJ) - W(I+JK) + W(I+IJ+JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
 (\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} &= [U(I) - U(I+1) - U(I+IK) + U(I+IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
 (\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} &= [V(I) - V(I+IJ) - V(I+JK) + V(I+IJ+JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
 G \frac{\partial^2 W}{\partial \phi^2} &= [-2W(I) + W(I+1)] \frac{G}{\lambda_\phi^2} \\
 G \frac{\partial^2 W}{\partial \chi^2} &= [-2W(I) + W(I+IJ)] \frac{G}{\lambda_x^2}
 \end{aligned}$$

$$(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} = [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}$$

For points parallel to ϕ axis,

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} = [U(I - 1) - 2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [-2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 U}{\partial \psi^2} = [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [U(I - 1) - 2U(I) + U(I + 1)] \frac{G}{\lambda_\phi^2}$$

$$(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} = [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 V}{\partial \psi^2} = [V(I - IK) - 2V(I) + V(I + IK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} = [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} = [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$G \frac{\partial^2 W}{\partial \phi^2} = [W(I - 1) - 2W(I) + W(I + 1)] \frac{G}{\lambda_\phi^2}$$

$$G \frac{\partial^2 W}{\partial \chi^2} = [-2W(I) + W(I + IJ)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} = [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}$$

For points parallel to the ϕ axis and on the χ axis,

$$(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} = [2U(I - 1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_\phi^2}$$

$$G \frac{\partial^2 U}{\partial \chi^2} = [-2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 U}{\partial \psi^2} = [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} = [V(I) - V(I - 1) - V(I + IJ) + V(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} = [W(I) - W(I - 1) - W(I + IK) + W(I + IK - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} = [U(I) - U(I - 1) - U(I + IJ) + U(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x}$$

$$G \frac{\partial^2 V}{\partial \phi^2} = [2V(I - 1) - 2V(I)] \frac{G}{\lambda_\phi^2}$$

$$(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} = [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 V}{\partial \psi^2} = [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_\psi^2}$$

$$(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} = [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} = [U(I) - U(I - 1) - U(I + IK) + U(I + IK - 1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}$$

$$(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} = [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi}$$

$$\begin{aligned}
G \frac{\partial^2 W}{\partial \phi^2} &= [2W(I-1) - 2W(I)] \frac{G}{\lambda_\phi^2} \\
G \frac{\partial^2 W}{\partial \chi^2} &= [-2W(I) + W(I+IJ)] \frac{G}{\lambda_\psi^2} \\
(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} &= [W(I-IK) - 2W(I) + W(I+IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}
\end{aligned}$$

For the points parallel to the χ axis

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_\chi^2} \\
G \frac{\partial^2 U}{\partial \psi^2} &= [U(I-IK) - 2U(I) + U(I+IK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\chi} \\
(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} &= [W(I) - W(I+1) - W(I+IK) + W(I+IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\chi} \\
G \frac{\partial^2 V}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{G}{\lambda_\phi^2} \\
(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_\chi^2} \\
G \frac{\partial^2 V}{\partial \psi^2} &= [V(I-IK) - 2V(I) + V(I+IK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} &= [W(I) - W(I+IJ) - W(I+JK) + W(I+IJ+JK)] \frac{(\lambda + G)}{\lambda_\chi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} &= [U(I) - U(I+1) - U(I+IK) + U(I+IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} &= [V(I) - V(I+IJ) - V(I+JK) + V(I+IJ+JK)] \frac{(\lambda + G)}{\lambda_\chi \lambda_\psi}
\end{aligned}$$

$$\begin{aligned}
G \frac{\partial^2 W}{\partial \phi^2} &= [-2W(I) + W(I+1)] \frac{G}{\lambda_\phi^2} \\
G \frac{\partial^2 W}{\partial \chi^2} &= [W(I-IJ) - 2W(I) + W(I+IJ)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} &= [W(I- IK) - 2W(I) + W(I+ IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}
\end{aligned}$$

For a point parallel to χ axis and on the ϕ axis,

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 U}{\partial \psi^2} &= [U(I- IK) - 2U(I) + U(I+ IK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I-IJ) + V(I-IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
(\lambda + G) \frac{\partial^2 W}{\partial \phi \partial \psi} &= [W(I) - W(I+1) - W(I+ IK) + W(I+ IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_x} \\
G \frac{\partial^2 V}{\partial \phi^2} &= [-2V(I) + V(I+1)] \frac{G}{\lambda_\phi^2} \\
(\lambda + 2G) \frac{\partial^2 V}{\partial \chi^2} &= [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 V}{\partial \psi^2} &= [V(I- JK) - 2V(I) + V(I+ JK)] \frac{G}{\lambda_\psi^2} \\
(\lambda + G) \frac{\partial^2 W}{\partial \chi \partial \psi} &= [W(I) - W(I-IJ) - U(I+ JK) + U(I-IJ+ JK)] \frac{(\lambda + G)}{\lambda_x \lambda_\psi} \\
(\lambda + G) \frac{\partial^2 U}{\partial \phi \partial \psi} &= [U(I) - U(I+1) - U(I+ IK) + U(I+ IK+1)] \frac{(\lambda + G)}{\lambda_\phi \lambda_\psi}
\end{aligned}$$

$$\begin{aligned}
(\lambda + G) \frac{\partial^2 V}{\partial \chi \partial \psi} &= [V(I) - V(I - IJ) - V(I + IK) + V(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_x \lambda_w} \\
G \frac{\partial^2 W}{\partial \phi^2} &= [-2W(I) + W(I + 1)] \frac{G}{\lambda_\phi^2} \\
G \frac{\partial^2 W}{\partial \chi^2} &= [2W(I - IJ) - 2W(I)] \frac{G}{\lambda_\psi^2} \\
(\lambda + 2G) \frac{\partial^2 W}{\partial \psi^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_\psi^2}
\end{aligned}$$

4.4.2 Boundary surfaces

For the corner point,

$$\begin{aligned}
\left(\frac{E}{1 - \nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I + 1)] \frac{\left(\frac{E}{1 - \nu^2}\right)}{\lambda_\phi^2} \\
G \frac{\partial^2 U}{\partial \chi^2} &= [-2U(I) + U(I + IJ)] \frac{G}{\lambda_x^2} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x} \\
\left(\frac{E}{2(1 - \nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{\left(\frac{E}{2(1 - \nu)}\right)}{\lambda_\phi \lambda_x} \\
G \frac{\partial^2 V}{\partial \phi^2} &= [-2V(I) + V(I + 1)] \frac{G}{\lambda_\phi^2} \\
\left(\frac{E}{1 - \nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [-2V(I) + V(I + IJ)] \frac{\left(\frac{E}{1 - \nu^2}\right)}{\lambda_x^2}
\end{aligned}$$

For points parallel to the ϕ axis,

$$\begin{aligned} \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2} \\ G \frac{\partial^2 U}{\partial \chi^2} &= [-2U(I) + U(I+IJ)] \frac{G}{\lambda_\chi^2} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ G \frac{\partial^2 V}{\partial \phi^2} &= [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_\phi^2} \\ \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [-2V(I) + V(I+IJ)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\chi^2} \end{aligned}$$

For points parallel to the ϕ axis and on the χ axis,

$$\begin{aligned} \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [2U(I-1) - 2U(I)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2} \\ G \frac{\partial^2 U}{\partial \chi^2} &= [-2U(I) + U(I+IJ)] \frac{G}{\lambda_\chi^2} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I-1) - V(I+IJ) + V(I+IJ-1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I-1) - U(I+IJ) + U(I+IJ-1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ G \frac{\partial^2 V}{\partial \phi^2} &= [2V(I-1) - 2V(I)] \frac{G}{\lambda_\phi^2} \\ \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [-2V(I) + V(I+IJ)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\chi^2} \end{aligned}$$

For points parallel to the χ axis,

$$\begin{aligned} \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2} \\ G \frac{\partial^2 U}{\partial \chi^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_\chi^2} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ G \frac{\partial^2 V}{\partial \phi^2} &= [-2V(I) + V(I+1)] \frac{G}{\lambda_\phi^2} \\ \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\chi^2} \end{aligned}$$

For the point parallel to the χ axis and on the ϕ axis,

$$\begin{aligned} \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 U}{\partial \phi^2} &= [-2U(I) + U(I+1)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\phi^2} \\ G \frac{\partial^2 U}{\partial \chi^2} &= [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_\chi^2} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 V}{\partial \phi \partial \chi} &= [V(I) - V(I+1) - V(I-IJ) + V(I-IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ \left(\frac{E}{2(1-\nu)}\right) \frac{\partial^2 U}{\partial \phi \partial \chi} &= [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{\left(\frac{E}{2(1-\nu)}\right)}{\lambda_\phi \lambda_\chi} \\ G \frac{\partial^2 V}{\partial \phi^2} &= [-2V(I) + V(I+1)] \frac{G}{\lambda_\phi^2} \\ \left(\frac{E}{1-\nu^2}\right) \frac{\partial^2 V}{\partial \chi^2} &= [2V(I-IJ) - 2V(I)] \frac{\left(\frac{E}{1-\nu^2}\right)}{\lambda_\chi^2} \end{aligned}$$

Table 4.1, Expressions Used in Left Hand Side Vectors

Symbol	Partial	Finite Difference Representation
D1	$W_{,\phi}$	$[W(I+1,1)-W(I-1,1)]/2\lambda_{\phi}$
D2	$W_{,\phi\phi}$	$[W(I+1,1)+2W(I,1)-W(I-1,1)]/\lambda_{\phi^2}$
D3	$W_{,x}$	$[W(I+IJ,1)-W(I-IJ,1)]/2\lambda_x$
D4	$W_{,\phi x}$	$[W(I+1+IJ,1)-W(I+1-IJ,1)-W(I-1+IJ,1)+W(I-1-IJ,1)]/4\lambda_{\phi}\lambda_x$
D5	$W_{,xx}$	$[W(I+IJ,1)-W(I,1)+W(I-IJ,1)]/\lambda_x^2$
D6	$W_{,x\psi}$	$[W(I+IJ+JK,1)-W(I+IJ-JK,1)-W(I-IJ+JK,1)+W(I-IJ-JK,1)]/4\lambda_x\lambda_{\psi}$
D7	$W_{,\phi\psi}$	$[W(I+1+IK,1)-W(I+1-1K,1)-W(I-1+IK,1)+W(I-1-1K,1)]/4\lambda_{\phi}\lambda_{\psi}$
D8	$U_{,\phi}$	$[U(I+1,1)-U(I-1,1)]/2\lambda_{\phi}$
D9	$V_{,x}$	$[V(I+IJ,1)-V(I-IJ,1)]/2\lambda_x$
D12	$U_{,\psi}$	$[U(I+IK,1)-U(I-1K,1)]/2\lambda_{\psi}$
D13	$U_{,\phi\psi}$	$[U(I+1+IK,1)-U(I+1-1K,1)-U(I-1+IK,1)+U(I-1-1K,1)]/4\lambda_{\phi}\lambda_{\psi}$
D14	$U_{,x\psi}$	$[U(I+IJ+JK,1)-U(I+IJ-JK,1)-U(I-IJ+JK,1)+U(I-IJ-JK,1)]/4\lambda_x\lambda_{\psi}$
D15	$U_{,\psi\psi}$	$[U(I+IK,1)-U(I,1)+U(I-1K,1)]/\lambda_{\psi^2}$
D16	$V_{,\psi}$	$[V(I+JK,1)-V(I-1JK,1)]/2\lambda_{\psi}$
D17	$V_{,x\psi}$	$[V(I+IJ+JK,1)-V(I+IJ-JK,1)-V(I-IJ+JK,1)+V(I-IJ-JK,1)]/4\lambda_x\lambda_{\psi}$
D18	$V_{,\phi\psi}$	$[V(I+1+IK,1)-V(I+1-1K,1)-V(I-1+IK,1)+V(I-1-1K,1)]/4\lambda_{\phi}\lambda_{\psi}$
D19	$V_{,\psi\psi}$	$[V(I+JK,1)-V(I,1)+V(I-1JK,1)]/\lambda_{\psi^2}$
D20	$U_{,x}$	$[U(I+IJ,1)-U(I-IJ,1)]/2\lambda_x$
D21	$V_{,\phi}$	$[V(I+1,1)-V(I-1,1)]/2\lambda_{\phi}$
D22	$W_{,\psi}$	$[W(I+IK,1)-W(I-1K,1)]/2\lambda_{\psi}$
D23	$U_{,\phi\phi}$	$[U(I+1,1)-2U(I,1)+U(I-1,1)]/\lambda_{\phi^2}$
D24	$V_{,xx}$	$[V(I+IJ,1)-2V(I,1)+V(I-IJ,1)]/\lambda_x^2$
D25	$W_{,\psi\psi}$	$[W(I+IK,1)-2W(I,1)+W(I-1K,1)]/\lambda_{\psi^2}$
D26	$U_{,\phi x}$	$[U(I+1+IJ,1)-U(I+1-IJ,1)-U(I-1+IJ,1)+U(I-1-IJ,1)]/4\lambda_{\phi}\lambda_x$
D27	$V_{,\phi x}$	$[V(I+1+IJ,1)-V(I+1-IJ,1)-V(I-1+IJ,1)+V(I-1-IJ,1)]/4\lambda_{\phi}\lambda_x$
D28	$U_{,xx}$	$[U(I+IJ,1)-2U(I,1)+U(I-IJ,1)]/\lambda_x^2$
D29	$V_{,\phi\phi}$	$[V(I+IJ,1)-2V(I,1)+V(I-IJ,1)]/\lambda_{\phi^2}$

Chapter 5

Mathematical Solutions

5.1 General

Solving a nonlinear system of equations resulting from nonlinear mechanics analysis is often a difficult task. This chapter will be devoted to the applied mathematics and numerical computations. While there are few standard techniques to solve such problem, four new methods will be described, two of them are iterative solutions and two others direct solutions. These new methods are specially programmed to solve simultaneous equations, using a new approach. The approach of the methods differs from the previous iteratives and direct solutions generally available in the technical literature. For the iterative methods the accuracy depends on the nature of the system of equations, and the initial guess. The iterative methods, in general,

involves less computation than the direct methods. However, the iterative methods are much less efficient when the diagonal elements of the matrix are small in comparison with other nonzero elements of the matrix. When such situations arise, some corrections or transformations to the original system in most cases will correct this deficiency and the iterative method will still yield converging solutions instead of diverging solutions. For the direct methods the original system does not need previous transformations.

5.2 First Iterative Method

Any system of simultaneous equations can be expressed in the matrix form by:

$$[A]\{X\} = \{B\} \quad (5.1)$$

where:

A: is a square matrix.

X: is an unknown vector.

B: is a known vector.

Before we start our first iteration a simple transformation is needed. This transformation will make the sum of all row elements equal to unity. To make this transformation possible we have to divide elements in each row by the sum of the elements in that row of the original matrix. If we consider a_{ij} as the elements of the original matrix and \bar{a}_{ij} the elements of the

transformed matrix we can write:

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_{j=1}^n a_{ij}} \quad (5.2)$$

where, $\sum_{j=1}^n a_{ij} \neq 0$

If the sum of one row is zero we add to this row another row where the sum is not zero. The transformed force vector will be:

$$\bar{b}_{ij} = \frac{b_{ij}}{\sum_{j=1}^n a_{ij}} \quad (5.3)$$

Once this transformation is performed, the iterative procedure proceeds with the first initial guess. Part of this new iterative technique is the choice of the first guess. For the first guess we take $x_i = \bar{b}_i$. To clarify this choice, we develop below the main idea why we use the transformation before starting the iteration.

The transformation described before is equal to the normalization for the diagonal matrix, and the vector force transformed is the solution for the system. To explain these two rules, let us consider again a 3 by 3 system.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The transformed matrix is:

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{13} \\ \bar{a}_{21} & \bar{a}_{22} & \bar{a}_{23} \\ \bar{a}_{31} & \bar{a}_{32} & \bar{a}_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{bmatrix}$$

As we know $\sum_{j=1}^n \bar{a}_{ij} = 1$.

If the original system can be put in the diagonal form, then after normalization it will look like:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Then our first guess is the c vector, the direct solution. For the general case our guess system is:

$$\begin{bmatrix} \sum_{j=1}^3 \bar{a}_{1j} & 0 & 0 \\ 0 & \sum_{j=1}^3 \bar{a}_{2j} & 0 \\ 0 & 0 & \sum_{j=1}^3 \bar{a}_{3j} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{bmatrix}$$

Because the sum is equal to unity, the x vector will be equal to the \bar{b} vector.

Then our initial guess vector will be in general equal to the right side vector.

Using this as the initial iteration the first calculated vector is:

$$[\bar{A}]\{\bar{b}\} = \{b^1\} \quad (5.4)$$

and the next vector:

$$\begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix} = \begin{bmatrix} (\bar{b}_1 - b_1^1)/\bar{a}_{11} \\ (\bar{b}_2 - b_2^1)/\bar{a}_{22} \\ (\bar{b}_3 - b_3^1)/\bar{a}_{33} \end{bmatrix} + \begin{bmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{bmatrix}$$

In general we write;

$$\begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ x_3^{k+1} \end{bmatrix} = \begin{bmatrix} (\bar{b}_1 - b_1^k)/\bar{a}_{11} \\ (\bar{b}_2 - b_2^k)/\bar{a}_{22} \\ (\bar{b}_3 - b_3^k)/\bar{a}_{33} \end{bmatrix} + \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \end{bmatrix}$$

Where the condensed form gives;

$$x_i^{k+1} = \frac{(\bar{b}_i - b_i^k)}{\bar{a}_{ii}} + x_i^k \quad (5.5)$$

If we proceed with another iteration we will get the vector b_i^{k+1} . At this stage we check the mean value of the two last b vectors with the \bar{b} vector which is :

$$\frac{b_i^{k+1} + b_i^k}{2} = \bar{b}_i, \implies x_i = \frac{x_i^{k+1} + x_i^k}{2} \quad (5.6)$$

Because we do not know the values of the X vector, the B vector will serve as our guideline to reach the required accuracy. Accuracy using iterative solution is specified and if we limit the accuracy to $\epsilon = 0.0001$ and once this limit is reached the computation stops at that level. In structural problems matrices generally have the following characteristics:

- 1- In general structural matrices have predominating values at the principal diagonal, a property will favour the application of the iterative method.
- 2- The accuracy required is not of high order, the maximum is of an order of 10^{-4} which is suitable for any iterative method.

5.3 Second Iterative Method

This method consists of transforming the original system to another system like in the first iterative method. The difference between the two methods will be evident once the following steps are explained.

Let us consider the transformation of the 3 by 3 matrix similar to the first iteration method.

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{13} \\ \bar{a}_{21} & \bar{a}_{22} & \bar{a}_{23} \\ \bar{a}_{31} & \bar{a}_{32} & \bar{a}_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{bmatrix}$$

As we know $\sum_{j=1}^n \bar{a}_{ij} = 1$.

From this relation, the first iterative vector to satisfy the first equation will be $x_1 = x_2 = x_3 = \bar{b}_1$ and the first row will be

$$\bar{b}_1 \sum_{j=1}^3 \bar{a}_{1j} = \bar{b}_1 \quad (5.7)$$

and for the second equation, for $i=2$,

$$\bar{b}_1 \sum_{j=1}^3 \bar{a}_{2j} \neq \bar{b}_2 \quad (5.8)$$

and the difference is;

$$\Delta \bar{b}_2 = \bar{b}_1 \sum_{j=1}^3 \bar{a}_{2j} - \bar{b}_2 \quad (5.9)$$

Thus, we need to correct the previous x_i^1 vector by the amount of $\frac{\Delta \bar{b}_2}{\bar{a}_{22}}$ to the corresponding x_i (i.e $i = 2$) then,

$$\begin{aligned}
x_1^{12} &= \bar{b}_1 \\
x_2^{12} &= \bar{b}_1 - \frac{\Delta \bar{b}_2}{\bar{a}_{22}} \\
x_3^{12} &= \bar{b}_1
\end{aligned}$$

Now if we apply this modified x_i^{12} to the second equation, this latter one will satisfy it exactly.

We pass to the third equation $i = 3$

$$\bar{a}_{31} + \bar{a}_{32}(\bar{b}_1 - \frac{\Delta \bar{b}_2}{\bar{a}_{22}}) + \bar{a}_{33}\bar{b}_1 \neq \bar{b}_3 \quad (5.10)$$

$$(\bar{a}_{31} + \bar{a}_{32} + \bar{a}_{33})\bar{b}_1 - \bar{a}_{32} \frac{\Delta \bar{b}_2}{\bar{a}_{22}} \neq \bar{b}_3 \quad (5.11)$$

Rewriting equation (5.11) gives:

$$\bar{b}_1 - \bar{a}_{32} \frac{\Delta \bar{b}_2}{\bar{a}_{22}} \neq \bar{b}_3 \quad (5.12)$$

the difference $\Delta \bar{b}_3$ will be :

$$\Delta \bar{b}_3 = (\bar{b}_1 - \bar{a}_{32} \frac{\Delta \bar{b}_2}{\bar{a}_{22}}) - \bar{b}_3 \quad (5.13)$$

To correct the previous vector x_i^{12} in order to satisfy the third equation, we modify it as shown below by adding to the corresponding x_i^{12} the appropriate change;

$$\begin{aligned}
x_1^{13} &= \bar{b}_1 \\
x_2^{13} &= \bar{b}_1 - \frac{\Delta \bar{b}_2}{\bar{a}_{22}} \\
x_3^{13} &= \bar{b}_1 - \frac{\Delta \bar{b}_3}{\bar{a}_{33}}
\end{aligned}$$

As we finish the first iteration, we go back again to the first equation

repeating the same procedure starting with the last vector.

The notation x_i^{km} needs some clarification,

i: defines the position of the unknown.

k: defines the number of iterations.

m: defines the modified vector inside the k^{th} iteration.

The difference between the two iterative methods is clear. In the first method we correct the previous vector, once we have completed all the iterations. In the second method the previous vector is changed internally, when passing from the i^{th} equation to $(i + 1)^{th}$ equation by keeping the $(i + 1)^{th}$ equation satisfied.

5.4 First Direct Method

This direct method transforms the original matrix to a lower triangular matrix and solve the system directly by forward substitution. To transform the original matrix we need to make all the upper triangular elements equal to zero. To explain the new procedure, let us take a system of a 3 by 3 matrix and proceed with direct method of eliminations.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

To apply this technique we have to start from the last column, and begin

the elimination of the elements. For this we need to create a vector V in such way that it will not affect the previous elimination.

To proceed with the elimination we start with the element a_{13} ;

$$V = \{v_1, v_2, v_3\} \quad (5.14)$$

where, $v_1 = -\frac{a_{21}+a_{31}}{a_{13}}$, and $a_{13} \neq 0$, and $v_2 = v_3 = 1$.

Multiplying the system by the vector V .

$$\{V\}[A] = \{V\}[B] \quad (5.15)$$

$$\{\bar{V}\} = \bar{b}_1 \quad (5.16)$$

and the system becomes,

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ b_2 \\ b_3 \end{bmatrix}$$

To eliminate a_{23}

$$V = \{v_1, v_2, v_3\} \quad (5.17)$$

where, $v_2 = -\frac{a_{23}}{a_{23}}$, and $a_{23} \neq 0$, and $v_1 = v_3 = 1$.

Again, multiplying the new system by the vector V gives.

$$\begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & 0 \\ \bar{a}_{21} & \bar{a}_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ b_3 \end{bmatrix}$$

To eliminate \bar{a}_{12}

$$V = \{v_1, v_2, v_3\} \quad (5.18)$$

where, $v_{12} = -\frac{a_{22}}{\bar{a}_{12}}$, and $\bar{a}_{12} \neq 0$, and $v_2 = 1$, $v_3 = 0$.

Multiplying the new system by the vector V gives.

$$\begin{bmatrix} \bar{a}_{11} & 0 & 0 \\ \bar{a}_{21} & \bar{a}_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ b_3 \end{bmatrix}$$

Once we end with this system, its solution is already known starting with the first row, and forward substitution will determine all the unknowns.

It should be noted that during the elimination, the elements forming the V vector should follow certain rules. To eliminate the elements above any diagonal element a_{ii} in the matrix, the elements forming the V vector from the position $i+1$ to n should be all equal to zero, whereas all remaining terms are equal to one except the element corresponding to the position we are searching to make it zero. The element corresponding to the position we are searching to make it zero is equal to the minus sum of all column elements of the diagonal term excluding the corresponding one, divided by the corresponding element.

For example if we have a 5 by 5 system and we are searching to make the element of the fourth column a_{34} equal to zero, the elements of the V vector will be equal to:

$$\{V\} = \left\{ 1 \quad 1 \quad -\frac{(a_{14}+a_{24}+a_{44})}{a_{34}} \quad 1 \quad 0 \right\}$$

5.5 Second Direct Method

This method consists of solving a system of equations using a new technique. The procedure is better described as the technique of change of variables. By changing the variables from x to y results in a diagonal system in terms of y . The solution of this system entails back substitution in the transformed system to solve for x . Back substitution would ultimately yield the inverted matrix of the system. To illustrate the validity of the method and the various steps required, the following 3 by 3 matrix is taken as an example:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Starting with the first equation since we want all the elements equal to zero except the diagonal term, we put:

$$x_1 = y_1 - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 \quad (5.19)$$

and by making use of the first equation we will get:

$$a_{11}y_1 = b_1 \quad (5.20)$$

Let us move to the second equation and substitute the value of x_1 .

$$a_{21}\left(y_1 - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3\right) + a_{22}x_2 + a_{23}x_3 \quad (5.21)$$

After collecting terms it becomes:

$$a_{21}y_1 + \left(-\frac{a_{12}}{a_{11}} + a_{22}\right)x_2 + \left(-\frac{a_{13}}{a_{11}} + a_{23}\right)x_3 \quad (5.22)$$

If we want to change the variable and have all the terms zero except the diagonal term, then:

$$x_2 = -\frac{a_{21}}{\left(-\frac{a_{12}}{a_{11}} + a_{22}\right)}y_1 + y_2 - \frac{\left(-\frac{a_{13}}{a_{11}} + a_{23}\right)}{\left(-\frac{a_{12}}{a_{11}} + a_{22}\right)}x_3 \quad (5.23)$$

If we replace the second equation we will get;

$$\left(-\frac{a_{12}}{a_{11}} + a_{22}\right)y_2 = b_2 \quad (5.24)$$

Proceeding to the third equation, we replace the value of x_1 and then the value of x_2 . Once all the calculations are finished we find:

$$\bar{a}_{31}y_1 + \bar{a}_{32}y_2 + \bar{a}_{33}x_3 \quad (5.25)$$

To have all the terms zero except the diagonal term we put:

$$x_3 = -\frac{\bar{a}_{31}}{\bar{a}_{33}}y_1 - \frac{\bar{a}_{32}}{\bar{a}_{33}}y_2 + y_3 \quad (5.26)$$

If we replace in the third equation we get:

$$\bar{a}_{33}y_3 = b_3 \quad (5.27)$$

where :

$$\bar{a}_{31} = a_{31} + \frac{(a_{31}a_{12} - a_{32})a_{21}}{-a_{12} + a_{22}a_{11}} \quad (5.28)$$

$$\bar{a}_{32} = \frac{a_{32} - a_{31}a_{12}}{a_{11}} \quad (5.29)$$

$$\bar{a}_{33} = \frac{a_{31}a_{12} - a_{32}}{a_{11}} + \frac{a_{23}a_{11} - a_{13}}{a_{11}a_{22} - a_{12}} + \frac{a_{31}a_{13} + a_{33}a_{11}}{a_{11}} \quad (5.30)$$

Now we have the first system;

$$\begin{bmatrix} a_{11}y_1 & 0 & 0 \\ 0 & (-\frac{a_{12}}{a_{11}} + a_{22})y_2 & 0 \\ 0 & 0 & \bar{a}_{33}y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

And the second system is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 & c_{12}x_2 & c_{13}x_3 \\ c_{21}y_1 & y_2 & c_{23}x_3 \\ c_{31}y_1 & c_{32}y_2 & y_3 \end{bmatrix}$$

where the c terms in the second system correspond to the constants of the equations defining the three equation for x_1, x_2, x_3 . From the first system we solve for y and then we back substitute in the second system to determine the x vector. We can remark now that the second system is in terms of x and y. Now we pass to the final operation where we have to express the whole system in terms of y only. The first equation defining x_1 is in term of x_2 and x_3 . First we substitute the expression of x_2 in the the expression of x_1 , then we get:

$$y_1 + c_{12}(c_{21}y_1 + y_2 + c_{23}x_3) + c_{13}x_3 \quad (5.31)$$

$$(c_{12}c_{21} + 1)y_1 + c_{12}y_2 + (c_{12}c_{23} + c_{13})x_3 \quad (5.32)$$

And the general system would yield:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (c_{12}c_{21} + 1)y_1 & c_{12}y_2 & (c_{12}c_{23} + c_{13})x_3 \\ c_{21}y_1 & y_2 & c_{23}x_3 \\ c_{31}y_1 & c_{32}y_2 & y_3 \end{bmatrix}$$

By, substituting x_3 in x_1 and x_2 respectively, row 1 becomes:

$$(c_{12}c_{21} + 1)y_1 + c_{12}y_2 + (c_{12}c_{23} + c_{13})(c_{31}y_1 + c_{32}y_2 + y_3) \quad (5.33)$$

and, after simplification it becomes:

$$[1 + c_{12}c_{21} + c_{31}(c_{12}c_{23} + c_{13})]y_1 + [c_{12} + c_{32}(c_{12}c_{23} + c_{13})]y_2 + (c_{12}c_{23} + c_{13})y_3 \quad (5.34)$$

Following the same procedure, row 2 becomes

$$c_{21}y_1 + y_2 + c_{23}(c_{31}y_1 + c_{32}y_2 + y_3) \quad (5.35)$$

and, after collecting the terms we get:

$$(c_{21} + c_{23}c_{31})y_1 + (1 + c_{23}c_{32})y_2 + c_{23}y_3 \quad (5.36)$$

The system then becomes:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} [(c_{12}c_{21} + 1) + c_{31}(c_{12}c_{23} + c_{13})] & [c_{12} + c_{32}(c_{12}c_{23} + c_{13})] & (c_{12}c_{23} + c_{13}) \\ (c_{21} + c_{23}c_{31}) & (1 + c_{23}c_{32}) & c_{23} \\ c_{31} & c_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

In explicit form this can be written as:

$$\{X\} = [C]\{Y\} \quad (5.37)$$

Once all the terms are in terms of the y variables, we can calculate the y vector from the first system

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} (a_{11})^{-1} & 0 & 0 \\ 0 & (-\frac{a_{12}}{a_{11}} + a_{22})^{-1} & 0 \\ 0 & 0 & (\bar{a}_{33})^{-1} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Writing in condensed form :

$$\{Y\} = [D]\{B\} \quad (5.38)$$

and, the x vector is equal to

$$\{X\} = [C][D]\{B\} \quad (5.39)$$

From this relation we conclude that the product of C by D is nothing but the inverse of the A matrix

$$[A]^{-1} = [C][D] \quad (5.40)$$

5.6 Numerical Application

The numerical application carried in appendix A shows the application of the four techniques using a 3 by 3 matrix. The two iterative techniques show that they converge rapidly compared to the well known Gauss-Seidel iterative technique. Using different values of the relaxation factor to improve the Gauss-seidel technique, the best factor was between 1.60 and 1.80 with 56 iterations, when the Gauss-Seidel technique is applied alone 93 iterations are required. The choice of the relaxation factor to give an optimum solution is very difficult to get if not impossible. In comparison, the two techniques presented herein can give very accurate results with much less computation. An analysis conducted by Bencharif and Ng [61]

shows that the time and storage capacity improved significantly using the direct method of transformation. The comparison of the three techniques is summarized in Table 5.1 . To reduce the time, knowing that 144 numerical applications are involved, the two iterative techniques are not convenient. The second direct method is used to carry the solutions of the linear and nonlinear equations of thick rectangular plates for the 144 applications.

Table 5.1 : Comparison of The Methods

System of Equations	Methods	Computer Time (CPU)	Storage Capacity
60×60	Gauss-Jordan	5	60×60
	LU decomposition	4	2(60×60)
	Present Technique	4	60×60
135×135	Gauss-Jordan	12	135×135
	LU decomposition	8	2(135×135)
	Present Technique	9	135×135
240×240	Gauss-Jordan	52	240×240
	LU decomposition	29	2(240×240)
	Present Technique	32	240×240
375×375	Gauss-Jordan	216	375×375
	LU decomposition	109	2(375×375)
	Present Technique	121	375×375

Chapter 6

Applications and Results

6.1 General

Using the three dimensional formulation as outlined in chapter III, two numerical examples are presented in this chapter. The two examples consist of the use of plate structures with two different boundary conditions. The first one considered is a fully clamped plate and the second one is a simply supported plate with immovable edges.

A computer program has been developed which is applicable to rectangular plates with two boundary conditions as illustrated in Figures (4.11) and (4.12). The objective of this chapter is to demonstrate the applicability of the method to several types of plates with different aspect ratios ranging from thin plates to thick plates. It should be emphasized here that

the finite difference technique developed in this thesis for the linear and nonlinear modelling of thick plates is, in general, applicable to thick plates of various plan forms and boundary conditions provided appropriate adjustments are made in the input data and the boundary conditions. For the variety of applications considered, the Poisson's ratio is kept equal to $\nu = 0.316$. Three plate aspect ratios have been considered $\frac{a}{b} = 1.0, 1.5, 2.0$, as well for the thickness plate ratios $\frac{h}{a} = 0.025, 0.05, 0.10$. At the same time two numerical approaches were used to investigate the convergence of results for both the linear and nonlinear analysis of thick plates since no results for such plates are as yet available in the technical literature.

To investigate convergence, four mesh sizes have been used for both clamped and simply supported rectangular plates. In order to have greater confidence in the results obtained, the finite difference was applied to both thin plates and thick plates. Comparison were made with previous investigators for nonlinear deflection results of thin plates and linear deflection results of thick plates. Data for nonlinear thick plates are as yet unavailable and hence no comparison of results can be made at this time. The number of finite difference applications to the thick plate analysis with various parameters are shown in Table 6.1.

6.2 Convergence of the Results

To improve the convergence and the accuracy of the results, two numerical finite difference schemes were investigated namely forward and central finite

difference techniques, with four mesh sizes in parallel for each case. For different ratio of $\frac{h}{a}$, the results of the dimensionless central deflection W for both linear and nonlinear analysis seem to be very satisfactory for all plates investigated. Tables 6.2-6.13 show the variation of centre plate deflection with the dimensionless load $Q = \frac{q \cdot a^4}{E h^4}$, the plate aspect ratios, and the mesh sizes used in this investigation.

To investigate the convergence of the finite difference techniques, four mesh sizes have been used for both clamped and simply supported rectangular plates. The convergence as a result of increase in mesh size is summarized in Table 6.14. We note that the convergence for the simply supported plates is faster than that of the clamped plates. A maximum mesh size of 125 nodes was adopted yielding 375 simultaneous equations. Convergence results from Table 6.14 indicate that by changing the mesh size from 80 nodes to 125 nodes the deviation in results does not exceed 4% for all plate aspect ratios investigated which is very well accepted.

Table 6.1, Number of Applications used with $\nu = 0.316$

Nodes	Equs	a/b	h/a	Schemes	B. Cond.	No. Applications
20	60	1.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		1.5	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		2.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
45	135	1.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		1.5	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		2.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
80	240	1.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		1.5	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		2.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
125	375	1.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		1.5	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4
		2.0	0.025	2	2	4
			0.050	2	2	4
			0.100	2	2	4

Table 6.2, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 1.0, \mu = 0.316, \text{CC/CC.}$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{q a^4}{E h^4}$								
		17.79	38.3	63.4	95.0	134.0	184.0	245	318	402
20	.025	0.34579	0.74445	1.2323	1.8466	2.6046	3.5765	4.7622	6.1811	7.8138
		0.33458	0.65923	0.96678	1.2642	1.5531	1.8536	2.1598	2.4757	2.7981
	.05	0.34569	0.74424	1.2320	1.8460	2.6039	3.5754	4.7608	6.1793	7.8116
		0.33382	0.65424	0.95343	1.2384	1.5112	1.7900	2.0691	2.3513	2.6334
	.10	0.34407	0.74075	1.2262	1.8374	2.5917	3.5587	4.7385	6.1504	7.7750
		0.33230	0.64456	0.92815	1.1907	1.4347	1.6765	1.9104	2.1384	2.3580
45	.025	0.29484	0.63477	1.0508	1.5745	2.2209	3.0495	4.0605	5.2704	6.6626
		0.28473	0.55821	0.81273	1.0525	1.2771	1.4997	1.7137	1.9201	2.1158
	.05	0.29474	0.63455	1.0504	1.5740	2.2201	3.0485	4.0591	5.2686	6.6603
		0.28384	0.55274	0.79921	1.0284	1.2408	1.4490	1.6469	1.8355	2.0124
	.10	0.29313	0.63107	1.0446	1.5653	2.2079	3.0318	4.0369	5.2397	6.6238
		0.28208	0.54236	0.77428	0.98498	1.1762	1.3598	1.5308	1.6906	1.8373
80	.025	0.27371	0.58927	0.97545	1.4616	2.0617	2.8310	3.7695	4.8926	6.1850
		0.26299	0.51067	0.73775	0.95065	1.1503	1.3486	1.5394	1.7230	1.8963
	.05	0.27361	0.58905	0.97509	1.4611	2.0609	2.8299	3.7681	4.8908	6.1827
		0.26189	0.50434	0.72305	0.92575	1.1141	1.2996	1.4765	1.6453	1.8033
	.10	0.27199	0.58557	0.96933	1.4525	2.0487	2.8132	3.7458	4.8619	6.1462
		0.25972	0.49260	0.69667	0.88197	1.0513	1.2154	1.3693	1.5139	1.6470
125	.025	0.26320	0.56664	0.93799	1.4055	1.9825	2.7222	3.6247	4.7047	5.9475
		0.25152	0.48353	0.69294	0.88806	1.0709	1.2528	1.4283	1.5978	1.7581
	.05	0.26310	0.56642	0.93763	1.4050	1.9817	2.7212	3.6233	4.7029	5.9452
		0.25015	0.47623	0.67682	0.86175	1.0337	1.2035	1.3662	1.5220	1.6684
	.10	0.26148	0.56294	0.93187	1.3963	1.9696	2.7045	3.6011	4.6740	5.9087
		0.24751	0.46296	0.64862	0.81671	0.97080	1.1210	1.2628	1.3968	1.5208

Table 6.3, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{q}{b} = 1.0, \mu = 0.316, CC/CC.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$								
		17.79	38.3	63.4	95.0	134.0	184.0	245	318	402
20	.025	0.34580	0.74447	1.2324	1.8466	2.6047	3.5766	4.7623	6.1812	7.8140
		0.33432	0.65764	0.96295	1.2576	1.5437	1.8410	2.1440	2.4567	2.7762
	.05	0.34580	0.74446	1.2324	1.8466	2.6047	3.5765	4.7622	6.1812	7.8140
		0.33333	0.65117	0.94623	1.2264	1.4940	1.7676	2.0415	2.3188	2.5964
	.10	0.34577	0.74442	1.2323	1.8465	2.6045	3.5763	4.7619	6.1808	7.8134
		0.33426	0.63886	0.91533	1.1701	1.4065	1.6409	1.8681	2.0905	2.3053
45	.025	0.29485	0.63478	1.0508	1.5745	2.2209	3.0496	4.0606	5.2705	6.6628
		0.28437	0.55607	0.80775	1.0442	1.2655	1.4847	1.6954	1.8985	2.0911
	.05	0.29485	0.63478	1.0508	1.5745	2.2209	3.0496	4.0606	5.2705	6.6627
		0.28313	0.54869	0.79004	1.0135	1.2202	1.4228	1.6153	1.7989	1.9710
	.10	0.29483	0.63473	1.0507	1.5744	2.2207	3.0494	4.0603	5.2701	6.6622
		0.28072	0.53507	0.75858	0.96045	1.1434	1.3192	1.4832	1.6367	1.7783
80	.025	0.27372	0.58928	0.97547	1.4617	2.0617	2.8310	3.7696	4.8927	6.1852
		0.26253	0.50807	0.73190	0.94116	1.1371	1.3317	1.5188	1.6988	1.8686
	.05	0.27372	0.58928	0.97547	1.4617	2.0617	2.8310	3.7696	4.8927	6.1851
		0.26098	0.49950	0.71253	0.90906	1.0913	1.2707	1.4417	1.6048	1.7575
	.10	0.27369	0.58923	0.97539	1.4615	2.0615	2.8308	3.7693	4.8923	6.1846
		0.25801	0.48413	0.67931	0.85549	1.0162	1.1720	1.3183	1.4559	1.5829
125	.025	0.26321	0.56665	0.93801	1.4055	1.9825	2.7223	3.6248	4.7048	5.9476
		0.25094	0.48048	0.68634	0.87762	1.0566	1.2347	1.4063	1.5720	1.7287
	.05	0.26320	0.56665	0.93801	1.4055	1.9825	2.7223	3.6248	4.7048	5.9476
		0.24904	0.47064	0.66521	0.84380	1.0096	1.1732	1.3299	1.4799	1.6208
	.10	0.26318	0.56660	0.93793	1.4054	1.9824	2.7221	3.6245	4.7044	5.9471
		0.24543	0.45348	0.63007	0.78915	0.93484	1.0769	1.2112	1.3383	1.4562

Table 6.4, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 1.5, \mu = 0.316, CC/CC.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$										
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0	178.0	225.0	282.0
20	.025	0.19892	0.40348	0.86345	1.4364	2.1465	3.0059	4.1962	5.5277	7.1820	9.0783	11.378
		0.19701	0.38893	0.75954	1.1226	1.4831	1.8478	2.2831	2.7163	3.2097	3.7453	4.3617
	.05	0.19889	0.40342	0.86333	1.4362	2.1462	3.0055	4.1956	5.5269	7.1809	9.0770	11.377
		0.19684	0.38776	0.75231	1.1036	1.4469	1.7888	2.1902	2.5832	3.0233	3.5011	4.0378
	.10	0.19844	0.40251	0.86137	1.4329	2.1414	2.9987	4.1861	5.5144	7.1647	9.0565	11.351
		0.19651	0.38544	0.73844	1.0680	1.3805	1.6825	2.0259	2.3511	2.7035	3.0872	3.4954
45	.025	0.16598	0.33666	0.72446	1.1985	1.7911	2.5082	3.5013	4.6123	5.9926	7.5750	9.4939
		0.16432	0.32395	0.62762	0.91235	1.1766	1.4232	1.6917	1.9345	2.1862	2.4432	2.7051
	.05	0.16595	0.33661	0.72034	1.1983	1.7908	2.5077	3.5007	4.6115	5.9916	7.5737	9.4923
		0.16413	0.32257	0.61946	0.89238	1.1417	1.3713	1.6180	1.8382	2.0633	2.2957	2.5255
	.10	0.16550	0.33570	0.71839	1.1951	1.7859	2.5009	3.4912	4.5990	5.9754	7.5532	9.4666
		0.16374	0.31987	0.60426	0.85652	1.0807	1.2824	1.4945	1.6795	1.8643	2.0546	2.2361
80	.025	0.15383	0.31203	0.66775	1.1108	1.6600	2.3246	3.2451	4.2748	5.5541	7.0207	8.7993
		0.15214	0.29914	0.57500	0.82872	1.0601	1.2727	1.5001	1.7019	1.9066	2.1019	2.3023
	.05	0.15380	0.31197	0.66762	1.1106	1.6597	2.3242	3.2445	4.2740	5.5531	7.0194	8.7977
		0.15190	0.29741	0.56524	0.80584	1.0217	1.2176	1.4251	1.6076	1.7913	1.9651	2.1419
	.10	0.15335	0.31106	0.66567	1.1074	1.6549	2.3174	3.2350	4.2616	5.5369	6.9989	8.7720
		0.15140	0.29407	0.54753	0.76625	0.95730	1.1276	1.3054	1.4598	1.6135	1.7575	1.9025
125	.025	0.14812	0.30044	0.64295	1.0696	1.5983	2.2383	3.1246	4.1161	5.3479	6.7599	8.4725
		0.14634	0.28695	0.54745	0.78307	0.99508	1.1876	1.3915	1.5707	1.7511	1.9221	2.0962
	.05	0.14809	0.30038	0.64282	1.0694	1.5980	2.2379	3.1240	4.1153	5.3468	6.7587	8.4708
		0.14603	0.28482	0.53589	0.75685	0.95213	1.1274	1.3115	1.4723	1.6333	1.7851	1.9391
	.10	0.14764	0.29947	0.64087	1.0661	1.5923	2.2311	3.1145	4.1028	5.3306	6.7382	8.4452
		0.14542	0.28076	0.51544	0.71299	0.88303	1.0334	1.1898	1.3257	1.4611	1.5883	1.7167

Table 6.5, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{a}{b} = 1.5, \mu = 0.316, CC/CC.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$										
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0	178.0	225.0	282.0
20	.025	0.19892	0.40348	0.86346	1.4364	2.1465	3.0060	4.1962	5.5277	7.1820	9.0784	11.3780
		0.19698	0.38873	0.75831	1.1193	1.4769	1.8375	2.2665	2.6921	3.1755	3.6995	4.3017
	.05	0.19892	0.40348	0.86346	1.4364	2.1465	3.0060	4.1962	5.5277	7.1820	9.0784	11.378
		0.19679	0.38736	0.74995	1.0976	1.4357	1.7706	2.1616	2.5422	2.9662	3.4249	3.9388
	.10	0.19891	0.40347	0.86342	1.4363	2.1464	3.0058	4.1961	5.5275	7.1817	9.0780	11.378
		0.19640	0.38466	0.73413	1.0577	1.3621	1.6540	1.9832	2.2922	2.6243	2.9817	3.3617
45	.025	0.16598	0.33667	0.72047	1.1985	1.7911	2.5082	3.5014	4.6124	5.9927	7.5750	9.4941
		0.16429	0.32375	0.62651	0.90981	1.1724	1.4173	1.6836	1.9241	2.1729	2.4259	2.6832
	.05	0.16598	0.33667	0.72047	1.1985	1.7911	2.5082	3.5013	4.6123	5.9927	7.5750	9.4940
		0.16407	0.32218	0.61740	0.88791	1.1347	1.3618	1.6056	1.8228	2.0444	2.2698	2.4934
	.10	0.16597	0.33665	0.72044	1.1985	1.7910	2.5081	3.5012	4.6121	5.9924	7.5747	9.4936
		0.16363	0.31913	0.60071	0.84946	1.0704	1.2696	1.4791	1.6619	1.8445	2.0264	2.2025
80	.025	0.15383	0.31203	0.66775	1.1108	1.6600	2.3247	3.2452	4.2749	5.5542	7.0208	8.7993
		0.15211	0.29895	0.57401	0.82669	1.0572	1.2691	1.4959	1.6972	1.9014	2.0962	2.2959
	.05	0.15383	0.31203	0.66775	1.1108	1.6600	2.3246	3.2451	4.2749	5.5542	7.0208	8.7993
		0.15184	0.29704	0.56347	0.80251	1.0173	1.2125	1.4198	1.6022	1.7859	1.9597	2.1365
	.10	0.15383	0.31202	0.66772	1.1108	1.6599	2.3245	3.2450	4.2747	5.5539	7.0204	8.7989
		0.15129	0.29338	0.54469	0.76161	0.95194	1.1224	1.3009	1.4563	1.6111	1.7562	1.9022
125	.025	0.14812	0.30045	0.64295	1.0696	1.5984	2.2383	3.1246	4.1161	5.3479	6.7600	8.4726
		0.14631	0.28676	0.54661	0.78168	0.99361	1.1865	1.3913	1.5716	1.7534	1.9257	2.1014
	.05	0.14812	0.30044	0.64295	1.0696	1.5984	2.2383	3.1246	4.1161	5.3479	6.7600	8.4725
		0.14598	0.28447	0.53449	0.75488	0.95058	1.1271	1.3132	1.4763	1.6398	1.7942	1.9508
	.10	0.14811	0.30043	0.64292	1.0695	1.5983	2.2382	3.1245	4.1159	5.3476	6.7597	8.4721
		0.14531	0.28013	0.51346	0.71104	0.88277	1.0358	1.1958	1.3349	1.4735	1.6035	1.7345

Table 6.6, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 2.0, \mu = 0.316, CC/CC.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$										
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0	178.0	225.0	282.0
20	.025	0.20738	0.42066	0.90021	1.4975	2.2379	3.1339	4.3749	5.7630	7.4877	9.4648	11.863
		0.20545	0.40599	0.79676	1.1887	1.5912	2.0143	2.5424	3.0932	3.7480	4.4864	5.3615
	.05	0.20736	0.42060	0.90009	1.4973	2.2376	3.1335	4.3743	5.7622	7.4867	9.4635	11.861
		0.20527	0.40473	0.78901	1.1681	1.5510	1.9469	2.4327	2.9310	3.5154	4.1794	4.9516
	.10	0.20691	0.41969	0.89814	1.4941	2.2327	3.1267	4.3648	5.7497	7.4705	9.4430	11.835
		0.20492	0.40223	0.77419	1.1297	1.4775	1.8256	2.2378	2.6457	3.1086	3.6484	4.2430
45	.025	0.16958	0.34398	0.73611	1.2246	1.8300	2.5626	3.5774	4.7125	6.1228	7.7395	9.7002
		0.16799	0.33174	0.64654	0.94738	1.2322	1.5025	1.8014	2.0753	2.3629	2.6589	2.9692
	.05	0.16955	0.34392	0.73599	1.2244	1.8297	2.5622	3.5768	4.7117	6.1218	7.7382	9.6986
		0.16779	0.33033	0.63817	0.92681	1.1959	1.4480	1.7227	1.9706	2.2262	2.4919	2.7606
	.10	0.16910	0.34301	0.73404	1.2211	1.8248	2.5554	3.5673	4.6992	6.1056	7.7177	9.6729
		0.16739	0.32757	0.62269	0.89029	1.1334	1.3562	1.5933	1.8019	2.0112	2.2286	2.4387
80	.025	0.15615	0.31674	0.67781	1.1276	1.6850	2.3597	3.2941	4.3393	5.6379	7.1266	8.9319
		0.15457	0.30465	0.59005	0.85848	1.1092	1.3443	1.6006	1.8313	2.0680	2.2956	2.5306
	.05	0.15612	0.31668	0.67769	1.1274	1.6847	2.3593	3.2935	4.3385	5.6369	7.1253	8.9303
		0.15433	0.30291	0.58022	0.83544	1.0704	1.2885	1.5241	1.7342	1.9477	2.1511	2.3612
	.10	0.15567	0.31577	0.67574	1.1241	1.6799	2.3525	3.2840	4.3260	5.6207	7.1048	8.9047
		0.15384	0.29958	0.56258	0.79620	1.0066	1.1989	1.4035	1.5836	1.7641	1.9338	2.1021
125	.025	0.14991	0.30408	0.65073	1.0825	1.6177	2.2654	3.1624	4.1659	5.4126	6.8418	8.5750
		0.14828	0.29166	0.56139	0.81112	1.0413	1.2553	1.4872	1.6951	1.9078	2.1120	2.3221
	.05	0.14988	0.30402	0.65061	1.0823	1.6174	2.2650	3.1618	4.1651	5.4116	6.8405	8.5734
		0.14798	0.28955	0.54983	0.78493	0.99860	1.1955	1.4073	1.5960	1.7879	1.9709	2.1580
	.10	0.14943	0.30311	0.64866	1.0791	1.6126	2.2582	3.1524	4.1526	5.3954	6.8200	8.5477
		0.14738	0.28555	0.52967	0.74198	0.93116	1.1034	1.2868	1.4486	1.6115	1.7654	1.9214

Table 6.7, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{a}{b} = 2.0, \mu = 0.316, CC/CC.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{q a^4}{E h^4}$										
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0	178.0	225.0	282.0
20	.025	0.20739	0.42066	0.90022	1.4976	2.2379	3.1339	4.3749	5.7631	7.4878	9.4649	11.863
		0.20544	0.40592	0.79631	1.1874	1.5882	2.0086	2.5315	3.0743	3.7166	4.4360	5.2869
	.05	0.20739	0.42066	0.90022	1.4976	2.2379	3.1339	4.3749	5.7631	7.4878	9.4649	11.863
		0.20525	0.40459	0.78815	1.1657	1.5458	1.9372	2.4144	2.9002	3.4649	4.0990	4.8311
	.10	0.20738	0.42065	0.90018	1.4975	2.2378	3.1338	4.3747	5.7628	7.4875	9.4645	11.862
		0.20488	0.40197	0.77264	1.1256	1.4692	1.8110	2.2121	2.6048	3.0444	3.5433	4.0882
45	.025	0.16958	0.34398	0.73612	1.2246	1.8300	2.5627	3.5774	4.7126	6.1229	7.7396	9.7003
		0.16799	0.33173	0.64647	0.94726	1.2320	1.5023	1.8010	2.0747	2.3619	2.6571	2.9662
	.05	0.16958	0.34398	0.73612	1.2246	1.8300	2.5627	3.5774	4.7125	6.1229	7.7396	9.7003
		0.16779	0.33030	0.63806	0.92663	1.1957	1.4477	1.7224	1.9702	2.2256	2.4898	2.7573
	.10	0.16958	0.34397	0.73609	1.2245	1.8299	2.5625	3.5773	4.7123	6.1226	7.7392	9.6999
		0.16738	0.32752	0.62254	0.89012	1.1333	1.3562	1.5936	1.8023	2.0119	2.2278	2.4372
80	.025	0.15615	0.31674	0.67782	1.1276	1.6851	2.3597	3.2941	4.3393	5.6380	7.1266	8.9321
		0.15457	0.30466	0.59014	0.85875	1.1097	1.3451	1.6018	1.8328	2.0699	2.2980	2.5335
	.05	0.15615	0.31674	0.67782	1.1276	1.6850	2.3597	3.2941	4.3393	5.6379	7.1266	8.9320
		0.15433	0.30294	0.58039	0.83592	1.0713	1.2898	1.5259	1.7365	1.9506	2.1546	2.3657
	.10	0.15614	0.31672	0.67779	1.1275	1.6850	2.3596	3.2939	4.3391	5.6377	7.1263	8.9316
		0.15384	0.29963	0.56292	0.79702	1.0079	1.2007	1.4060	1.5865	1.7675	1.9377	2.1069
125	.025	0.14991	0.30408	0.65074	1.0825	1.6177	2.2654	3.1625	4.1659	5.4127	6.8419	8.5751
		0.14829	0.29169	0.56163	0.81176	1.0425	1.2570	1.4896	1.6982	1.9116	2.1166	2.3276
	.05	0.14991	0.30408	0.65073	1.0825	1.6177	2.2654	3.1624	4.1659	5.4127	6.8418	8.5751
		0.14799	0.28962	0.55026	0.78603	1.0005	1.1982	1.4108	1.6009	1.7931	1.9769	2.1650
	.10	0.14991	0.30407	0.65070	1.0825	1.6176	2.2653	3.1623	4.1657	5.4124	6.8415	8.5747
		0.14740	0.28567	0.53042	0.74365	0.93375	1.1068	1.2910	1.4535	1.6171	1.7717	1.9283

Table 6.8, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 1.0, \mu = 0.316, SS/SS.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^3}$							
		12.10	29.40	56.90	99.40	161.0	247.0	358.0	497.0
20	.025	0.45491	1.1053	2.1392	3.7370	6.0529	9.2862	13.459	18.685
		0.40744	0.77809	1.1376	1.5037	1.8874	2.2994	2.7379	3.2240
	.05	0.45484	1.1051	2.1389	3.7364	6.0520	9.2847	13.457	18.682
		0.40518	0.76728	1.1126	1.4577	1.8145	2.1880	2.5765	3.0015
	.10	0.45373	1.1024	2.1337	3.7273	6.0372	9.2621	13.424	18.637
		0.40080	0.74699	1.0667	1.3755	1.6856	1.9957	2.3054	2.6332
45	.025	0.48840	1.1867	2.2967	4.0121	6.4985	9.9698	14.450	20.061
		0.41211	0.73946	1.0376	1.3263	1.6099	1.8914	2.1669	2.4161
	.05	0.48833	1.1865	2.2964	4.0116	6.4976	9.9684	14.448	20.058
		0.40770	0.72372	1.0074	1.2783	1.5417	1.7995	2.0503	2.2643
	.10	0.48722	1.1838	2.2911	4.0024	6.4828	9.9457	14.415	20.012
		0.39945	0.69546	0.95402	1.1950	1.4214	1.6381	1.8445	2.0063
80	.025	0.50474	1.2264	2.3735	4.1464	6.7160	10.303	14.934	20.732
		0.40881	0.71143	0.98292	1.2444	1.4991	1.7497	1.9901	—
	.05	0.50467	1.2262	2.3732	4.1458	6.7151	10.302	14.932	20.729
		0.40266	0.69236	0.94882	1.1931	1.4276	1.6564	1.8736	—
	.10	0.50356	1.2235	2.3680	4.1367	6.7003	10.279	14.899	20.683
		0.39148	0.65930	0.89078	1.1065	1.3042	1.4943	1.6685	—
125	.025	0.51341	1.2475	2.4143	4.2176	6.8313	10.480	15.190	21.088
		0.40287	0.68746	0.94120	1.1855	1.4229	1.6566	1.8784	—
	.05	0.51334	1.2473	2.4140	4.2170	6.8304	10.479	15.188	21.085
		0.39527	0.66596	0.9044	1.1316	1.3484	1.5606	1.7592	—
	.10	0.51223	1.2446	2.4087	4.2079	6.8156	10.456	15.155	21.039
		0.38181	0.62971	0.8435	1.0432	1.2229	1.3966	1.5533	—

Table 6.9, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{a}{b} = 1.0, \mu = 0.316, SS/SS.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$							
		12.10	29.40	56.90	99.4	161.0	247.0	358.0	497.0
20	.025	0.45491	1.1053	2.1392	3.7371	6.0530	9.2862	13.459	18.685
		0.40669	0.77494	1.1314	1.4940	1.8744	2.2829	2.7186	3.2030
	.05	0.45491	1.1053	2.1392	3.7370	6.0530	9.2862	13.459	18.684
		0.40374	0.76137	1.1012	1.4405	1.7915	2.1594	2.5438	2.9663
	.10	0.45489	1.1053	2.1391	3.7368	6.0526	9.2857	13.459	18.684
		0.39807	0.73655	1.0476	1.3479	1.6492	1.9526	2.2576	2.5832
45	.025	0.48840	1.1867	2.2967	4.0122	6.4986	9.9699	14.450	20.061
		0.41055	0.73428	1.0285	1.3131	1.5926	1.8700	2.1420	2.3855
	.05	0.48840	1.1867	2.2967	4.0122	6.4986	9.9699	14.450	20.061
		0.40478	0.71439	0.99129	1.2556	1.5113	1.7624	2.0067	2.2141
	.10	0.48837	1.1866	2.2966	4.0119	6.4982	9.9693	14.449	20.060
		0.39424	0.68003	0.92863	1.1603	1.3746	1.5819	1.7777	1.9405
80	.025	0.50475	1.2264	2.3736	4.1464	6.7161	10.304	14.934	20.732
		0.40653	0.70466	0.97153	1.2284	1.4778	1.7236	1.9590	—
	.05	0.50475	1.2264	2.3736	4.1464	6.7161	10.304	14.934	20.732
		0.39846	0.68054	0.92943	1.1662	1.3913	1.6118	1.8203	—
	.10	0.50472	1.2263	2.3734	4.1462	6.7157	10.303	14.933	20.731
		0.38430	0.64067	0.86157	1.0674	1.2511	1.4297	1.5905	—
125	.025	0.51342	1.2475	2.4143	4.2176	6.8314	10.480	15.190	21.080
		0.39993	0.67944	0.92815	1.1674	1.3989	1.6272	1.8436	—
	.05	0.51341	1.2475	2.4143	4.2176	6.8314	10.480	15.190	21.088
		0.39000	0.65232	0.88277	1.1022	1.3086	1.5113	1.7009	—
	.10	0.51338	1.2474	2.4142	4.2174	6.8310	10.480	15.189	21.087
		0.37314	0.60903	0.81216	1.0021	1.1673	1.3282	1.4734	—

Table 6.10, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 1.5, \mu = 0.316, SS/SS.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^3}$							
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0
20	.025	0.57098	1.1582	2.4785	4.1231	6.1615	8.6284	12.045	15.867
		0.46537	0.74011	1.1179	1.4393	1.7564	2.0791	2.4732	2.8746
	.05	0.57095	1.1581	2.4784	4.1229	6.1612	8.6280	12.044	15.866
		0.46080	0.72735	1.0886	1.3909	1.6876	1.9842	2.3427	2.7050
	.10	0.57049	1.1572	2.4764	4.1196	6.1562	8.6210	12.035	15.853
		0.45214	0.70395	1.0361	1.3062	1.5677	1.8211	2.1211	2.4186
45	.025	0.57562	1.1676	2.4986	4.1566	6.2116	8.6985	12.143	15.996
		0.45111	0.69701	1.0131	1.2633	1.4860	1.6954	1.9315	2.1446
	.05	0.57559	1.1675	2.4985	4.1564	6.2112	8.6981	12.142	15.995
		0.44369	0.67847	0.97578	1.2086	1.4142	1.6062	1.8220	2.0118
	.10	0.57513	1.1666	2.4965	4.1531	6.2063	8.6911	12.133	15.982
		0.42277	0.63350	0.91036	1.1161	1.2941	1.4578	1.6394	1.7926
80	.025	0.57779	1.1720	2.5081	4.1723	6.2350	8.7314	12.189	16.056
		0.44172	0.67304	0.96434	1.1871	1.3861	1.5676	1.7700	1.9409
	.05	0.57777	1.1719	2.5079	4.1721	6.2347	8.7309	12.188	16.056
		0.43195	0.64987	0.91973	1.1236	1.3029	1.4665	1.6488	1.7982
	.10	0.57730	1.1710	2.5059	4.1688	6.2297	8.7240	12.178	16.043
		0.41480	0.61116	0.84800	1.0237	1.1712	1.3077	1.4566	1.5755
125	.025	0.57893	1.1743	2.5130	4.1805	6.2473	8.7485	12.213	16.088
		0.43381	0.65454	0.92935	1.1370	1.3195	1.4849	1.6690	1.8116
	.05	0.57890	1.1742	2.5129	4.1803	6.2470	8.7481	12.212	16.087
		0.42199	0.62751	0.87864	1.0660	1.2259	1.3721	1.5336	1.6569
	.10	0.57844	1.1733	2.5109	4.1770	6.2420	8.7411	12.202	16.074
		0.40184	0.58388	0.80039	0.95930	1.0854	1.2049	1.3295	1.4413

Table 6.11, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{a}{b} = 1.5, \mu = 0.316$, SS/SS.

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{q a^4}{E h^4}$							
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0
20	.025	0.57098	1.1582	2.4785	4.1231	6.1615	8.6284	12.045	15.867
		0.46477	0.73857	1.1146	1.4339	1.7483	2.0673	2.4559	2.8504
	.05	0.57098	1.1582	2.4785	4.1231	6.1615	8.6284	12.045	15.867
		0.45966	0.72451	1.0827	1.3816	1.6736	1.9643	2.3137	2.6646
	.10	0.57096	1.1581	2.4784	4.1230	6.1613	8.6281	12.045	15.866
		0.45008	0.69910	1.0268	1.2922	1.5469	1.7923	2.0801	2.3619
45	.025	0.57562	1.1676	2.4986	4.1566	6.2116	8.6985	12.143	15.996
		0.45010	0.69475	1.0092	1.2582	1.4796	1.6876	1.9220	2.1323
	.05	0.57562	1.1676	2.4986	4.1566	6.2116	8.6985	12.143	15.996
		0.44183	0.67456	0.96947	1.2003	1.4041	1.5940	1.8070	1.9930
	.10	0.57560	1.1675	2.4986	4.1565	6.2114	8.6982	12.142	15.995
		0.41906	0.62648	0.90116	1.1053	1.2816	1.4432	1.6212	1.7714
80	.025	0.57780	1.1720	2.5081	4.1723	6.2350	8.7314	12.189	16.056
		0.44045	0.67051	0.96064	1.1829	1.3815	1.5625	1.7645	1.9345
	.05	0.57779	1.1720	2.5081	4.1723	6.2350	8.7314	12.189	16.056
		0.42972	0.64573	0.91424	1.1179	1.2970	1.4604	1.6417	1.7919
	.10	0.57778	1.1720	2.5080	4.1722	6.2348	8.7311	12.188	16.056
		0.41127	0.60537	0.84159	1.0182	1.1676	1.3048	1.4514	1.5773
125	.025	0.57893	1.1743	2.5130	4.1805	6.2473	8.7486	12.213	16.088
		0.43240	0.65203	0.92641	1.1345	1.3180	1.4844	1.6694	1.8153
	.05	0.57893	1.1743	2.5130	4.1805	6.2473	8.7485	12.213	16.088
		0.41962	0.62370	0.87499	1.0639	1.2264	1.3749	1.5375	1.6723
	.10	0.57891	1.1743	2.5129	4.1804	6.2471	8.7482	12.212	16.087
		0.39832	0.57918	0.79758	0.95998	1.0933	1.2163	1.3416	1.4793

Table 6.12, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Forward Finite Difference $\frac{a}{b} = 2.0, \mu = 0.316, SS/SS.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{q a^4}{E h^4}$							
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0
20	.025	0.69648	1.4127	3.0233	5.0293	7.5158	10.525	14.692	19.355
		0.51930	0.79815	1.1913	1.5410	1.8982	2.2778	2.7605	—
	.05	0.69645	1.4127	3.0231	5.0291	7.5155	10.524	14.692	19.354
		0.51258	0.78214	1.1569	1.4856	1.8132	2.1652	2.6016	—
	.10	0.69599	1.4117	3.0211	5.0258	7.5105	10.517	14.682	19.341
		0.50013	0.75331	1.0968	1.3884	1.6700	1.9558	2.3369	—
45	.025	0.68885	1.3973	2.9901	4.9742	7.4334	10.410	14.531	19.142
		0.49514	0.74217	1.0674	1.3309	1.5722	1.8036	2.0640	—
	.05	0.68882	1.3972	2.9900	4.9740	7.4331	10.409	14.531	19.142
		0.48454	0.71940	1.0255	1.2713	1.4944	1.7070	1.9436	—
	.10	0.68835	1.3963	2.9880	4.9707	7.4281	10.402	14.521	19.129
		0.46595	0.68101	0.95684	1.1736	1.3679	1.5499	1.7492	—
80	.025	0.68613	1.3917	2.9783	4.9546	7.4041	10.369	14.474	19.067
		0.48246	0.71471	1.0152	1.2541	1.4883	1.6728	1.8950	—
	.05	0.68610	1.3917	2.9782	4.9544	7.4038	10.368	14.474	19.066
		0.46867	0.68637	0.96553	1.1859	1.3820	1.5664	1.7665	—
	.10	0.68564	1.3908	2.9762	4.9511	7.3988	10.361	14.464	19.053
		0.44552	0.64107	0.88878	1.0820	1.2486	1.4040	1.5728	—
125	.025	0.68487	1.3892	2.9729	4.9455	7.3905	10.349	14.448	19.032
		0.47329	0.69509	0.97848	1.2015	1.4012	1.5887	1.7913	—
	.05	0.68484	1.3891	2.9727	4.9453	7.3902	10.349	14.447	19.031
		0.45671	0.66202	0.92230	1.1260	1.3048	1.4732	1.6540	—
	.10	0.68438	1.3882	2.9707	4.9420	7.3852	10.342	14.437	19.018
		0.42989	0.61148	0.83965	1.0166	1.1677	1.3075	1.4610	—

Table 6.13, Linear and Nonlinear α Values For The Dimensionless Central Deflection $W = \alpha h$
 Central Finite Difference $\frac{a}{b} = 2.0, \mu = 0.316, SS/SS.$

Nodes	$\frac{h}{a}$	Dimensionless Load $Q = \frac{qa^4}{Eh^4}$							
		4.93	10.0	21.4	35.6	53.2	74.5	104.0	137.0
20	.025	0.69648	1.4127	3.0233	5.0294	7.5158	10.525	14.693	19.355
		0.51907	0.79763	1.1901	1.5388	1.8943	2.2713	2.7490	—
	.05	0.69648	1.4127	3.0233	5.0294	7.5158	10.525	14.692	19.355
		0.51215	0.78121	1.1549	1.4819	1.8068	2.1545	2.5828	—
	.10	0.69646	1.4127	3.0232	5.0292	7.5156	10.525	14.692	19.354
		0.49938	0.75179	1.0938	1.3832	1.6617	1.9426	2.3114	—
45	.025	0.68885	1.3973	2.9901	4.9742	7.4334	10.410	14.531	19.142
		0.49498	0.74200	1.0673	1.3308	1.5720	1.8035	2.0640	—
	.05	0.68885	1.3973	2.9901	4.9742	7.4334	10.410	14.531	19.142
		0.48427	0.71916	1.0255	1.2713	1.4944	1.7070	1.9431	—
	.10	0.68883	1.3972	2.9900	4.9741	7.4332	10.409	14.531	19.142
		0.46557	0.68079	0.95706	1.1739	1.3684	1.5504	1.7496	—
80	.025	0.68613	1.3918	2.9783	4.9540	7.4041	10.369	14.474	19.067
		0.48245	0.71495	1.0159	1.2552	1.4712	1.6745	1.8972	—
	.05	0.68613	1.3917	2.9783	4.9546	7.4041	10.369	14.474	19.067
		0.46869	0.68685	0.96674	1.1876	1.3842	1.5690	1.7703	—
	.10	0.68611	1.3917	2.9783	4.9545	7.4039	10.368	14.474	19.066
		0.44566	0.64195	0.89059	1.0844	1.2511	1.4067	1.5779	—
125	.025	0.68487	1.3892	2.9729	4.9455	7.3905	10.349	14.448	19.032
		0.47349	0.69586	0.98020	1.2040	1.4044	1.5926	1.7967	—
	.05	0.68487	1.3892	2.9729	4.9455	7.3905	10.349	14.448	19.032
		0.45711	0.66336	0.92494	1.1296	1.3092	1.4780	1.6619	—
	.10	0.68485	1.3891	2.9728	4.9454	7.3903	10.349	14.447	19.031
		0.43063	0.61350	0.84301	1.0208	1.1712	1.3107	1.4690	—

Table 6.14, Convergence of The Linear Dimensionless Center Deflection Using Finite Difference Technique

Boundary Conditions	Clamped			Simply Supported			
	$\frac{a}{b}$	1.0	1.5	2.0	1.0	1.5	2.0
20 nodes		14.73%	16.56%	18.22%	6.86%	0.81%	1.1%
45 nodes		7.16%	7.32%	7.92%	3.24%	0.38%	0.4%
80 nodes		3.84%	3.71%	3.99%	1.68%	0.20%	0.18%
125 nodes							

The errors due to finite difference techniques are a combination of discretisation errors and rounding off errors. The rounding off error is the error associated with the accuracy with which the numbers are manipulated in the computations. The discretisation error occurs irrespective of the accuracy of the numerical calculations, and is the result of approximating a continuum which has an infinite number of degrees of freedom with a model having a finite number of degrees of freedom. The discretisation error may tend to zero as the limit when the element size approaches zero. At that stage, the approximate solution is said to converge to the exact solution. To investigate the convergence and accuracy of the method, dimensionless central deflection of various plates were plotted against the number of nodes for the linear analysis in Figures 6.1-6.9 for clamped plates, and Figures

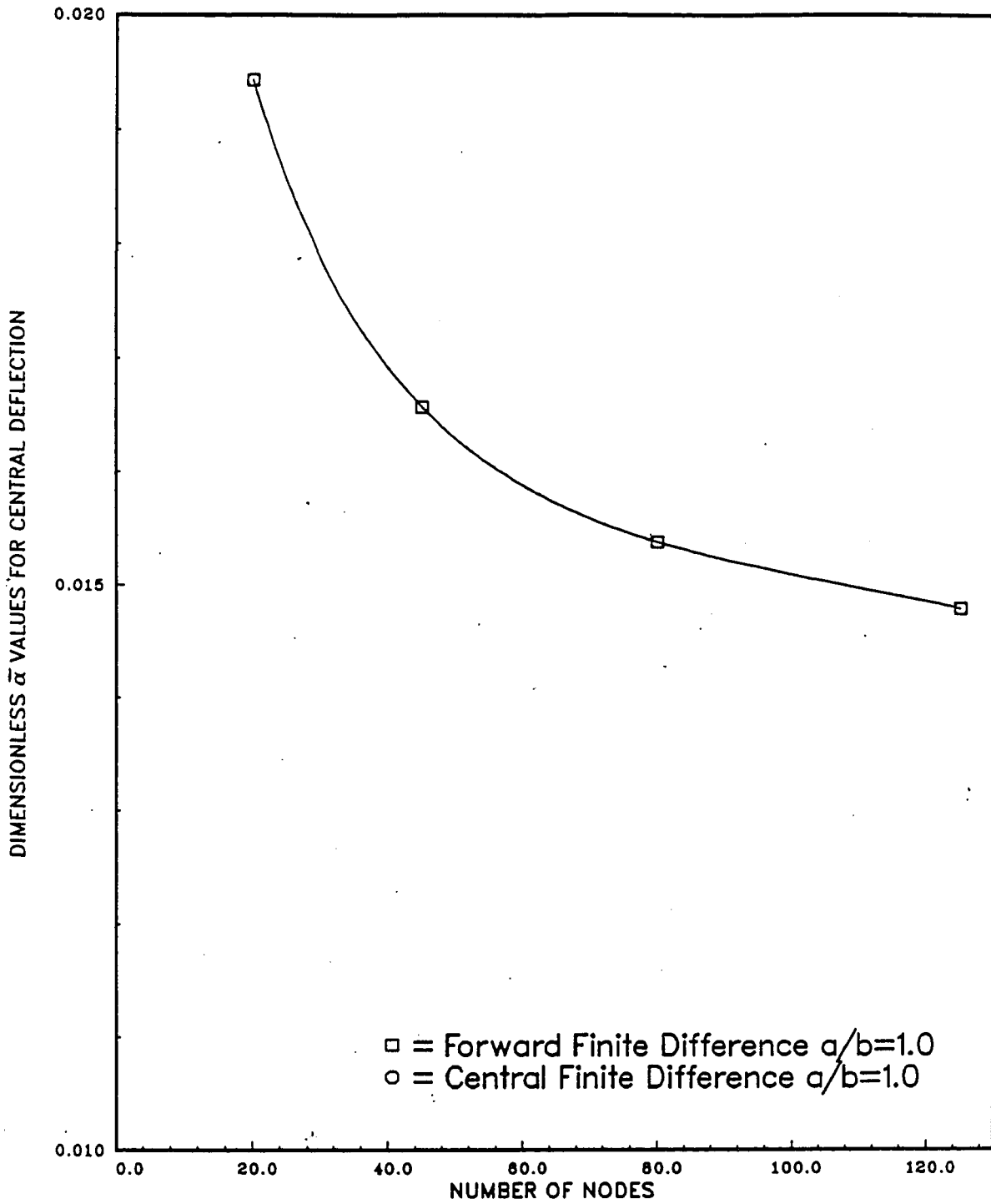


FIGURE 6.1, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.025$

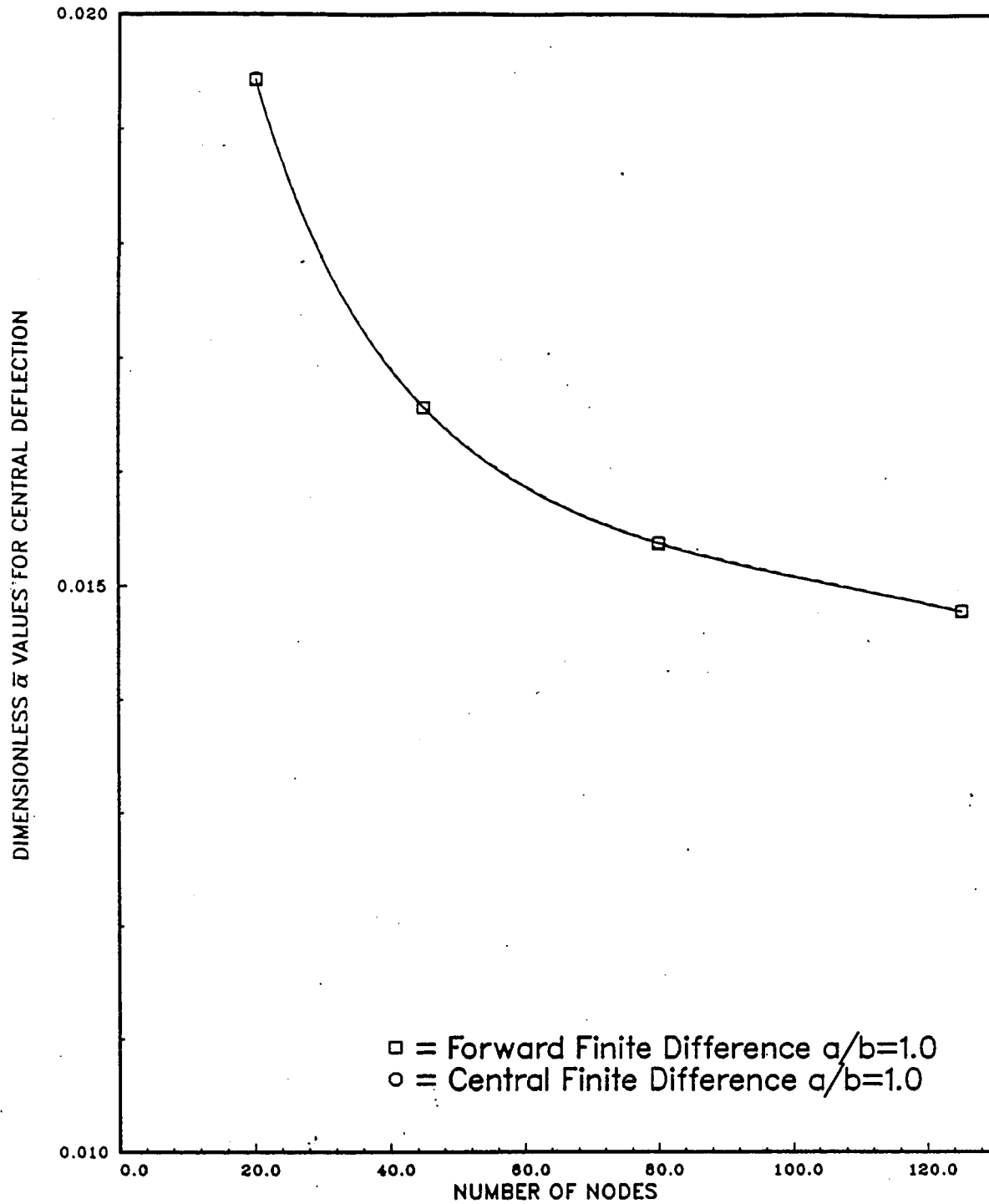


FIGURE 6.2, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.050$

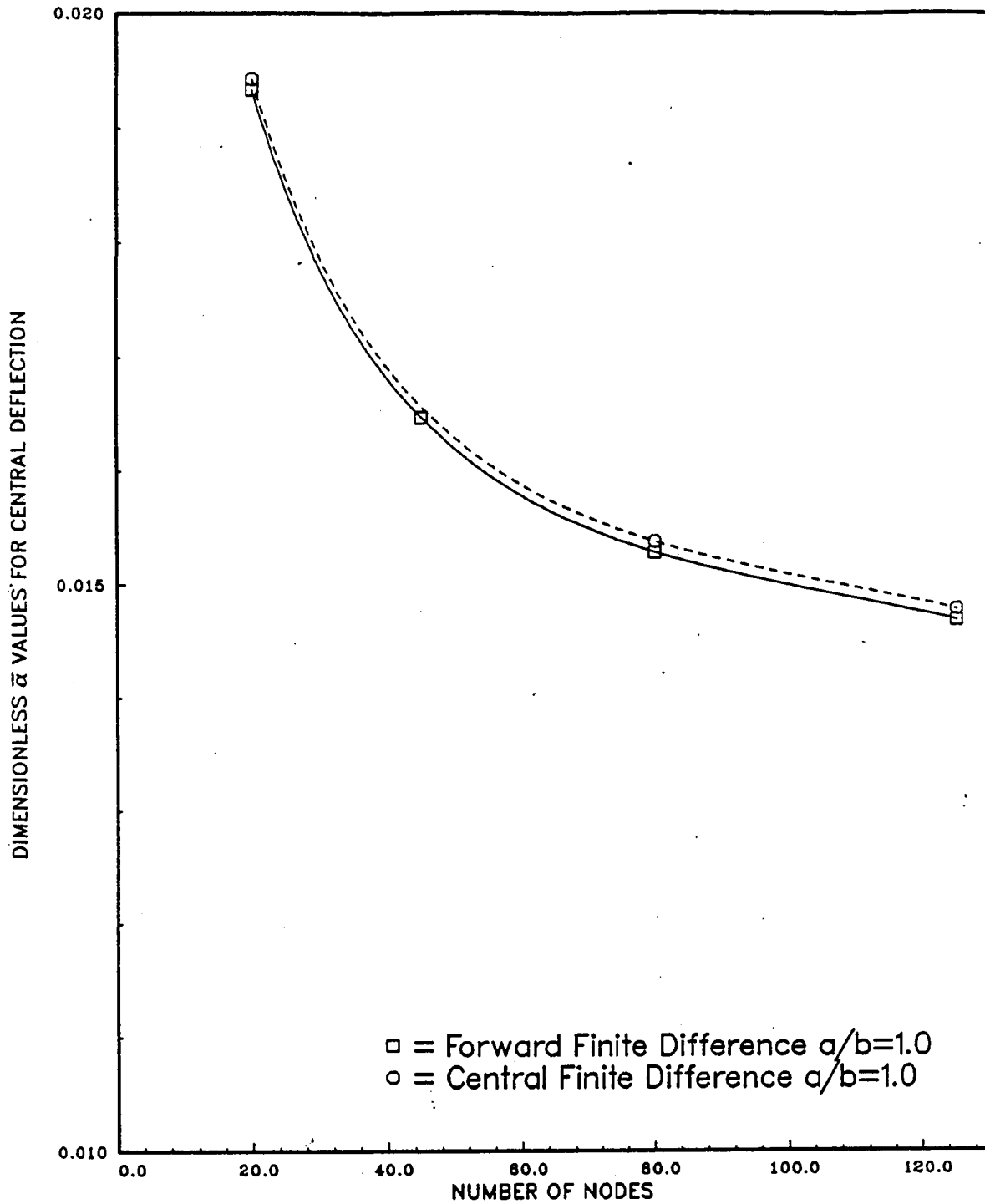


FIGURE 6.3, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.100$

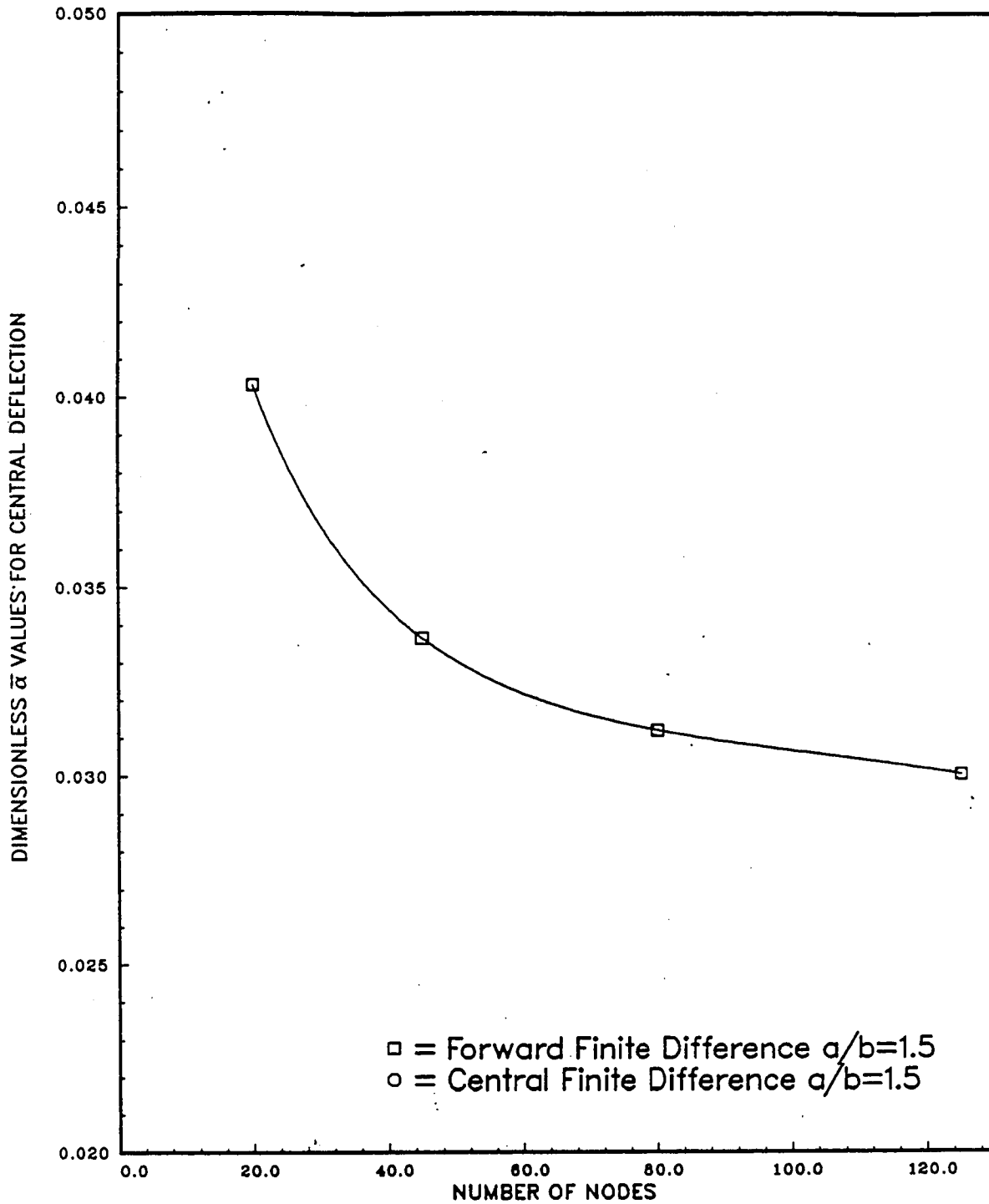


FIGURE 6.4, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.025$

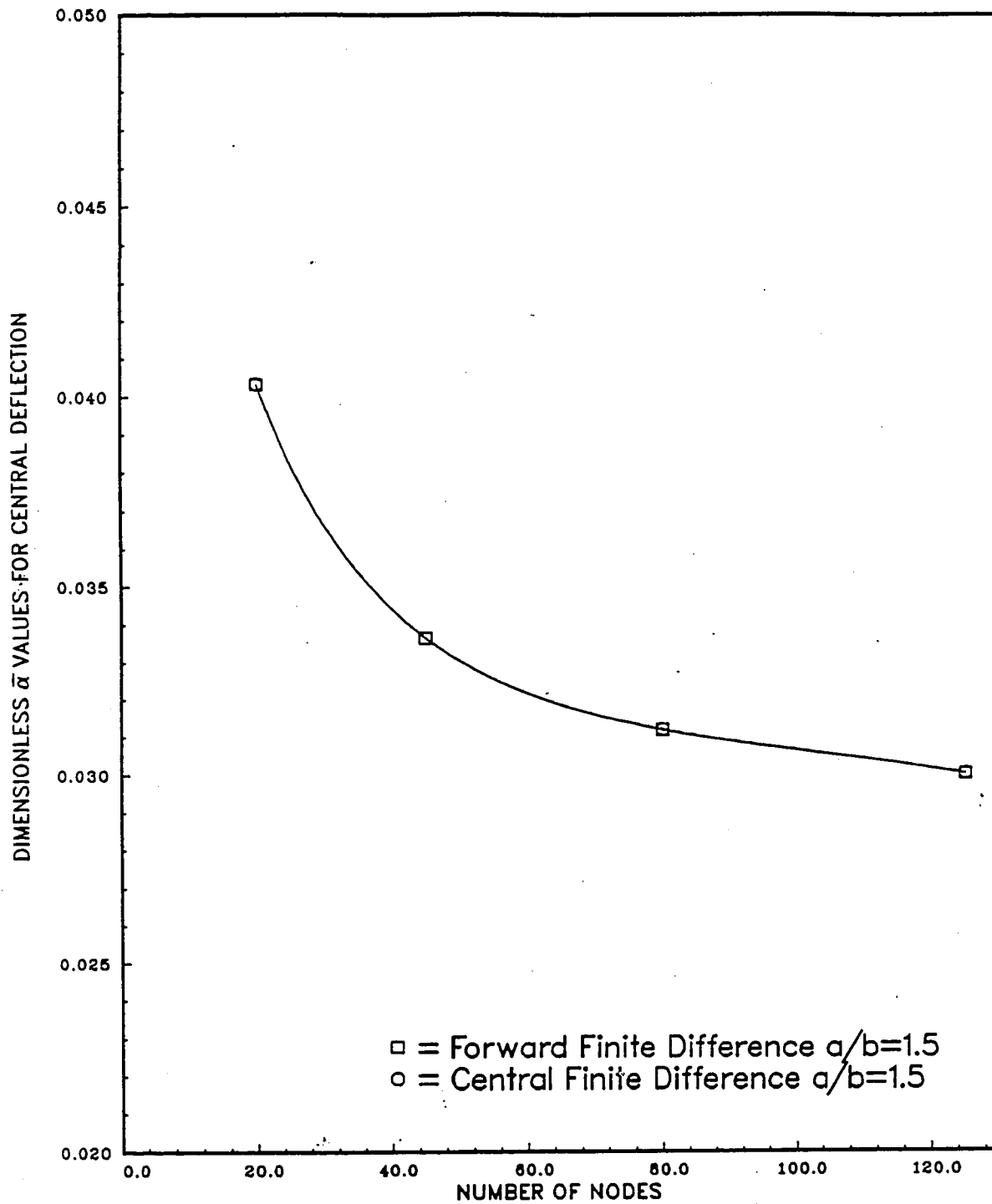


FIGURE 6.5, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.050$

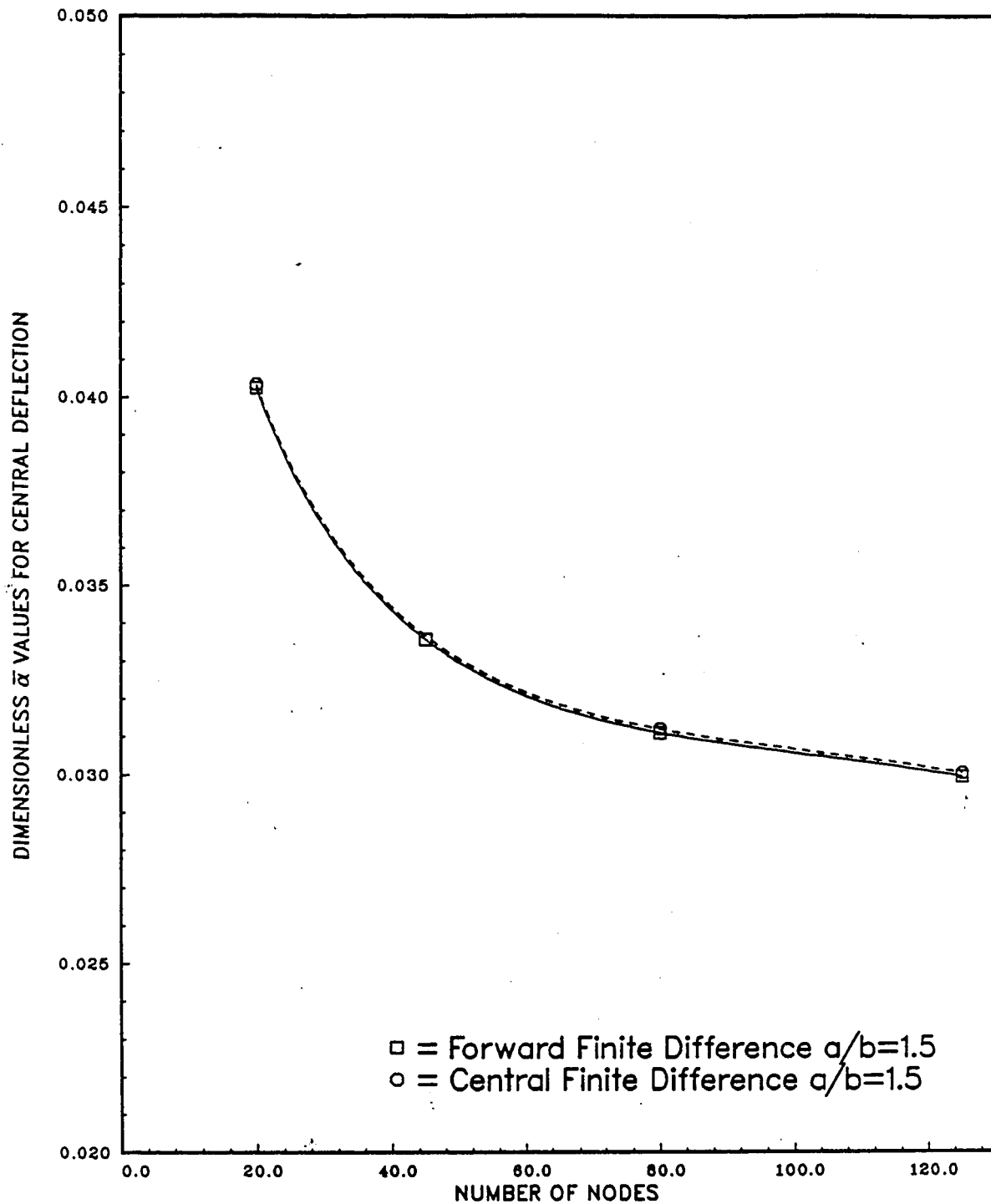


FIGURE 6.6, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.100$

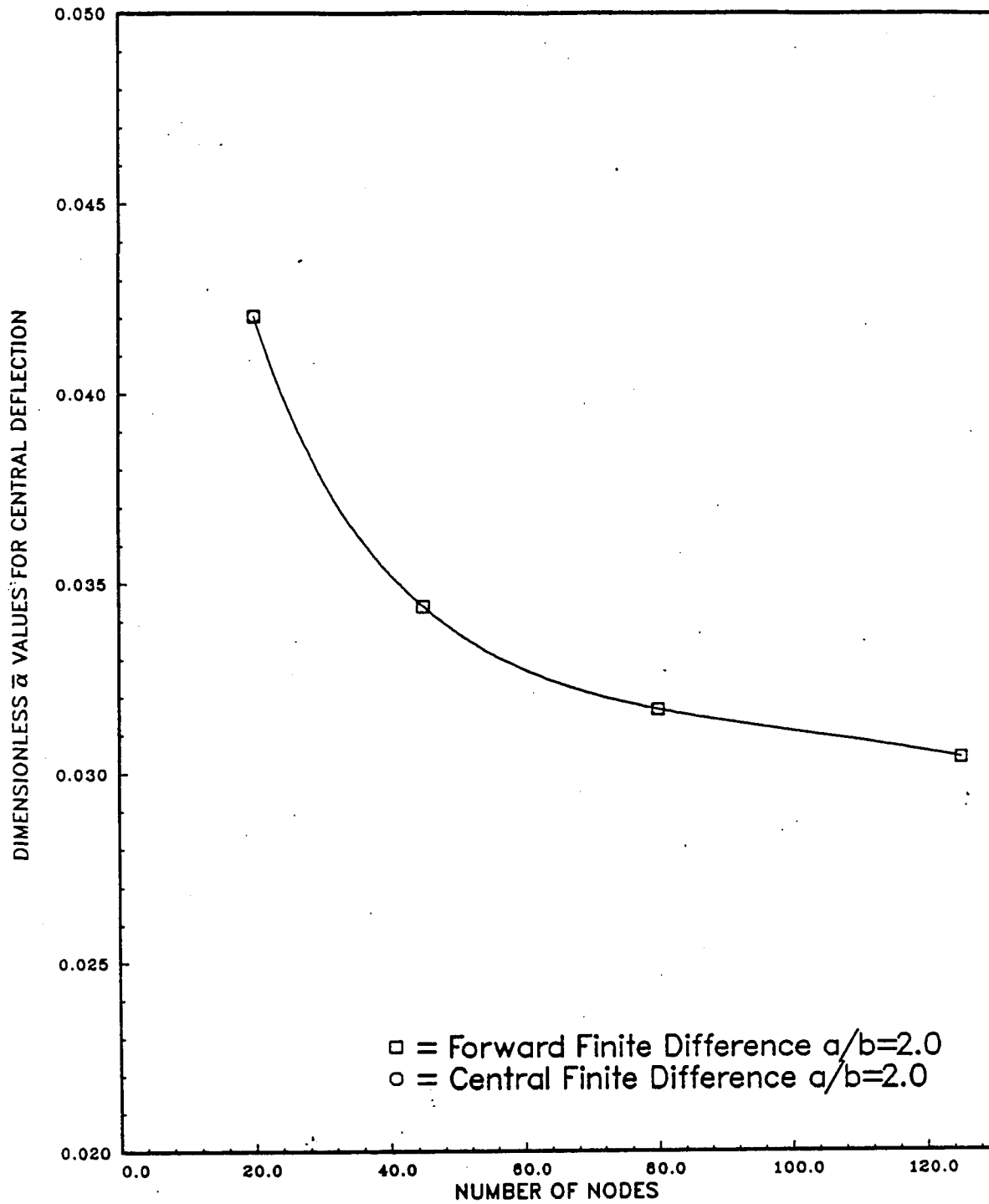


FIGURE 6.7, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.025$

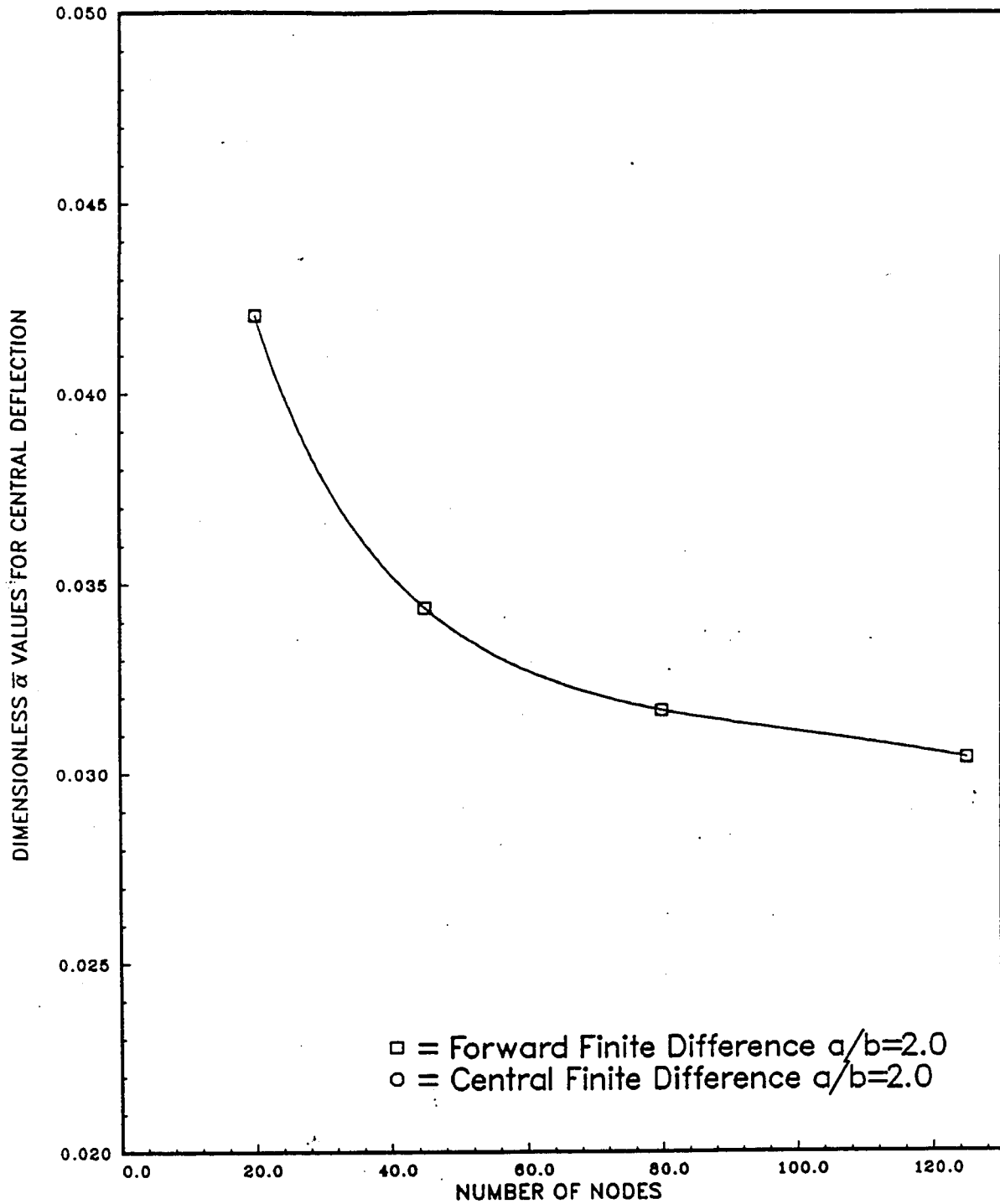


FIGURE 6.8, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.050$

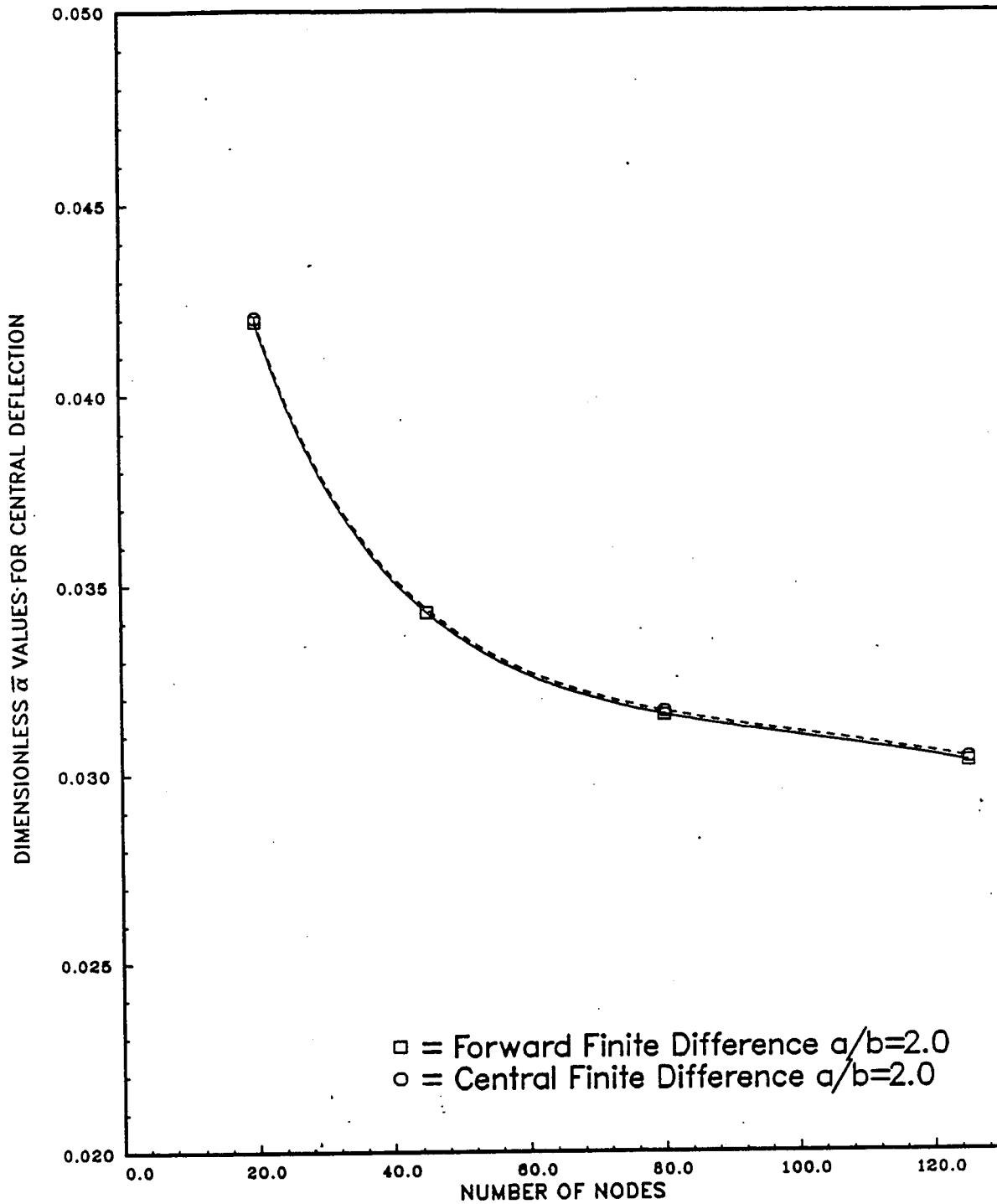


FIGURE 6.9, LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.100$

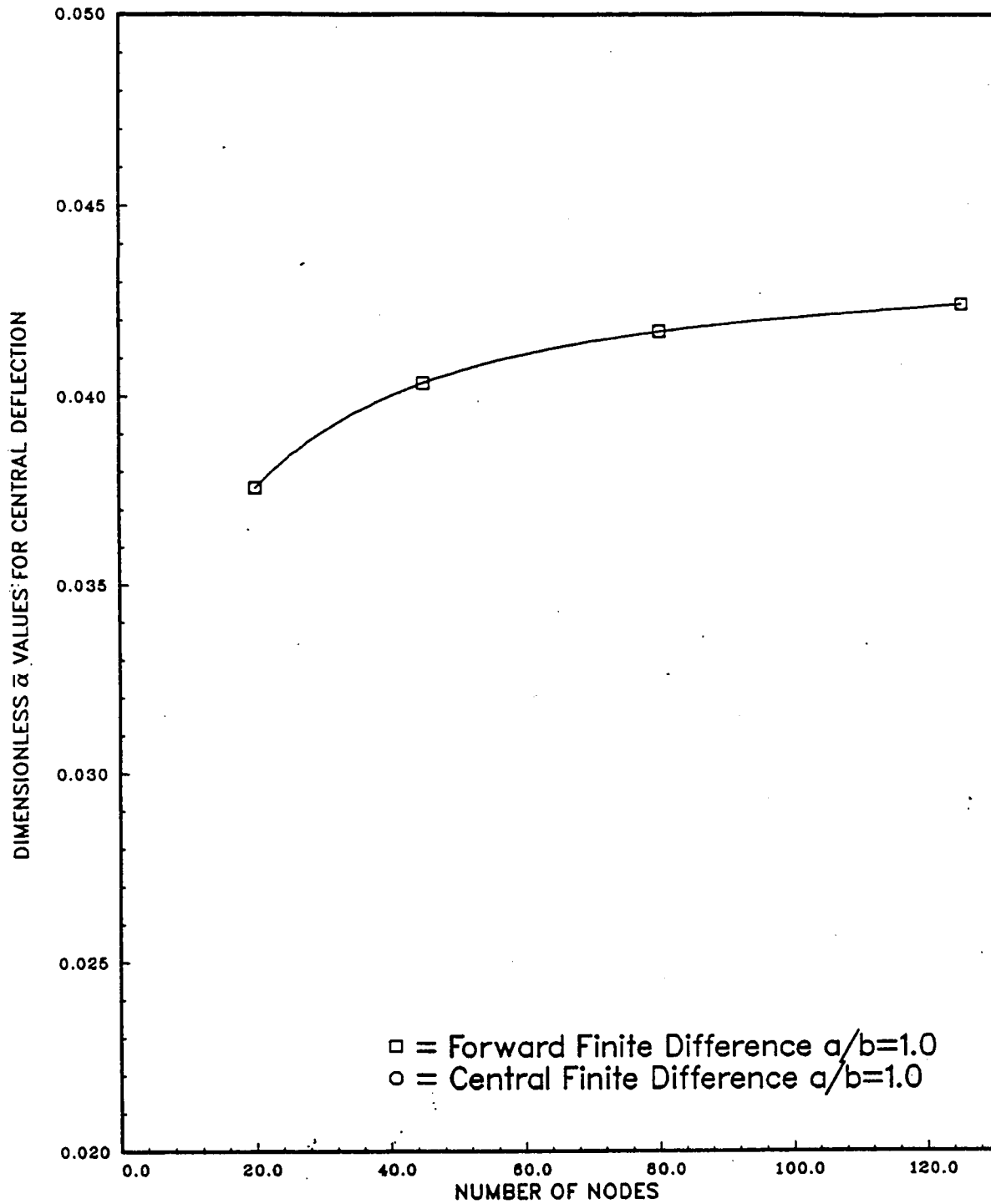


FIGURE 6.10, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

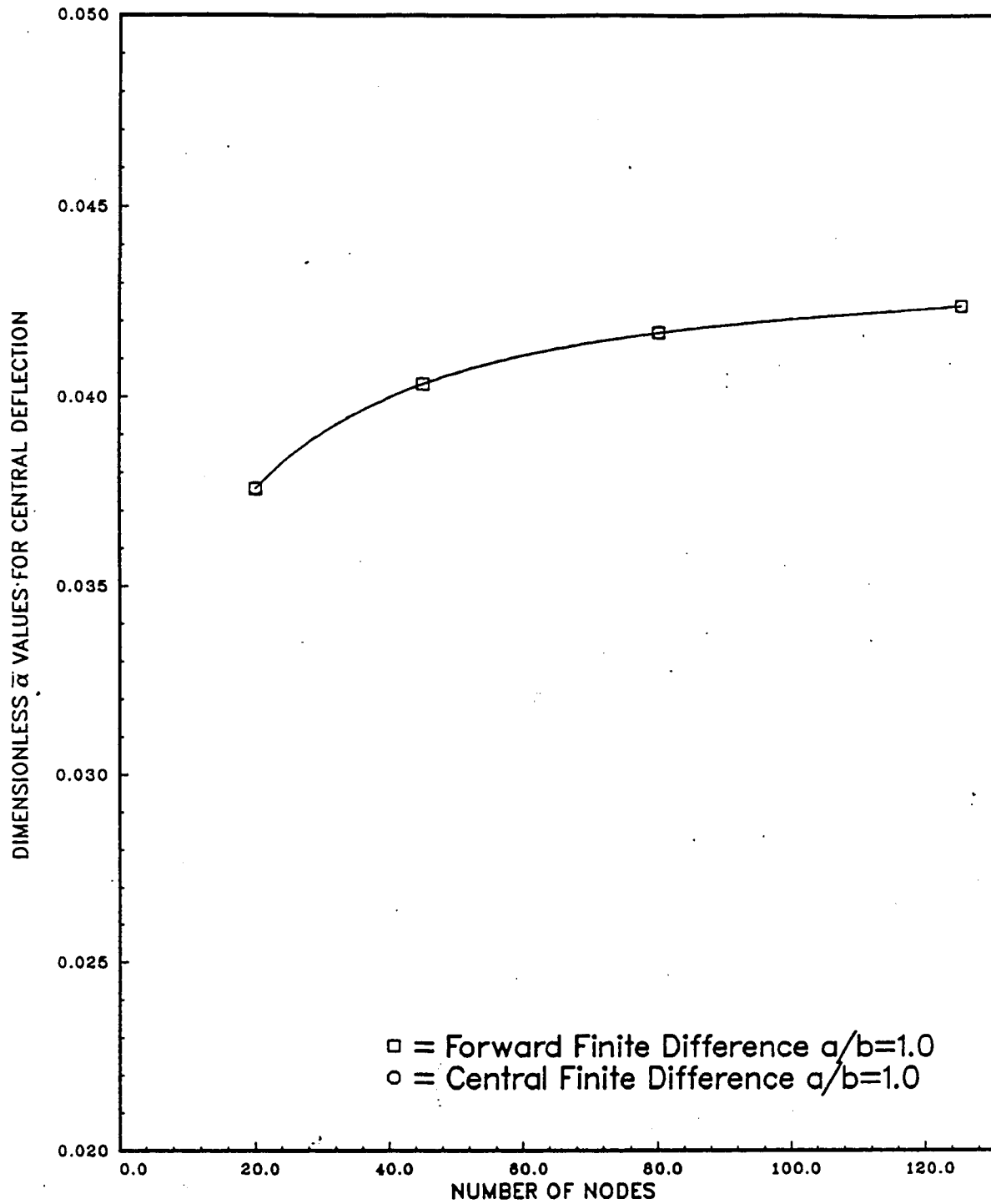


FIGURE 6.11, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

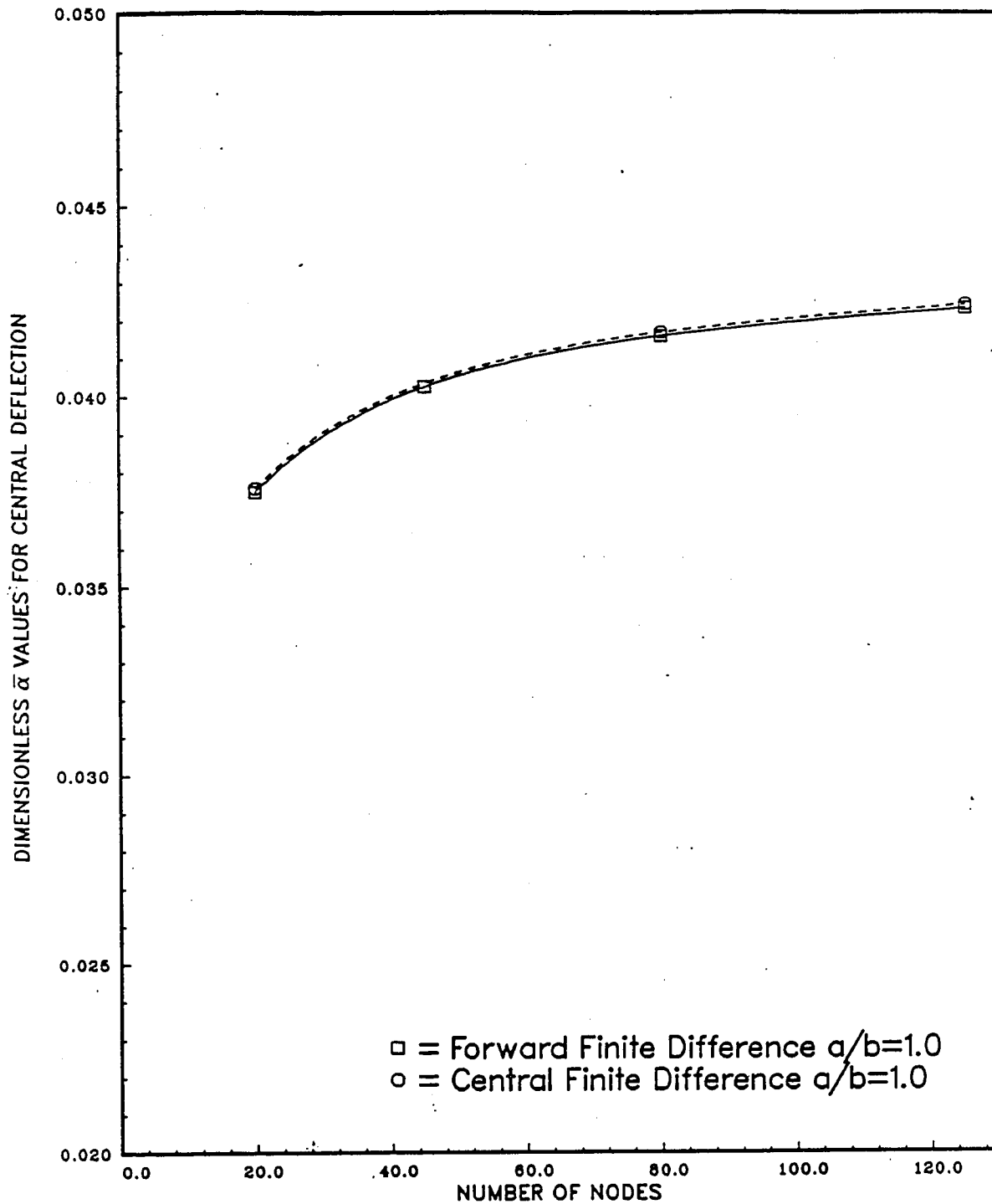


FIGURE 6.12, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

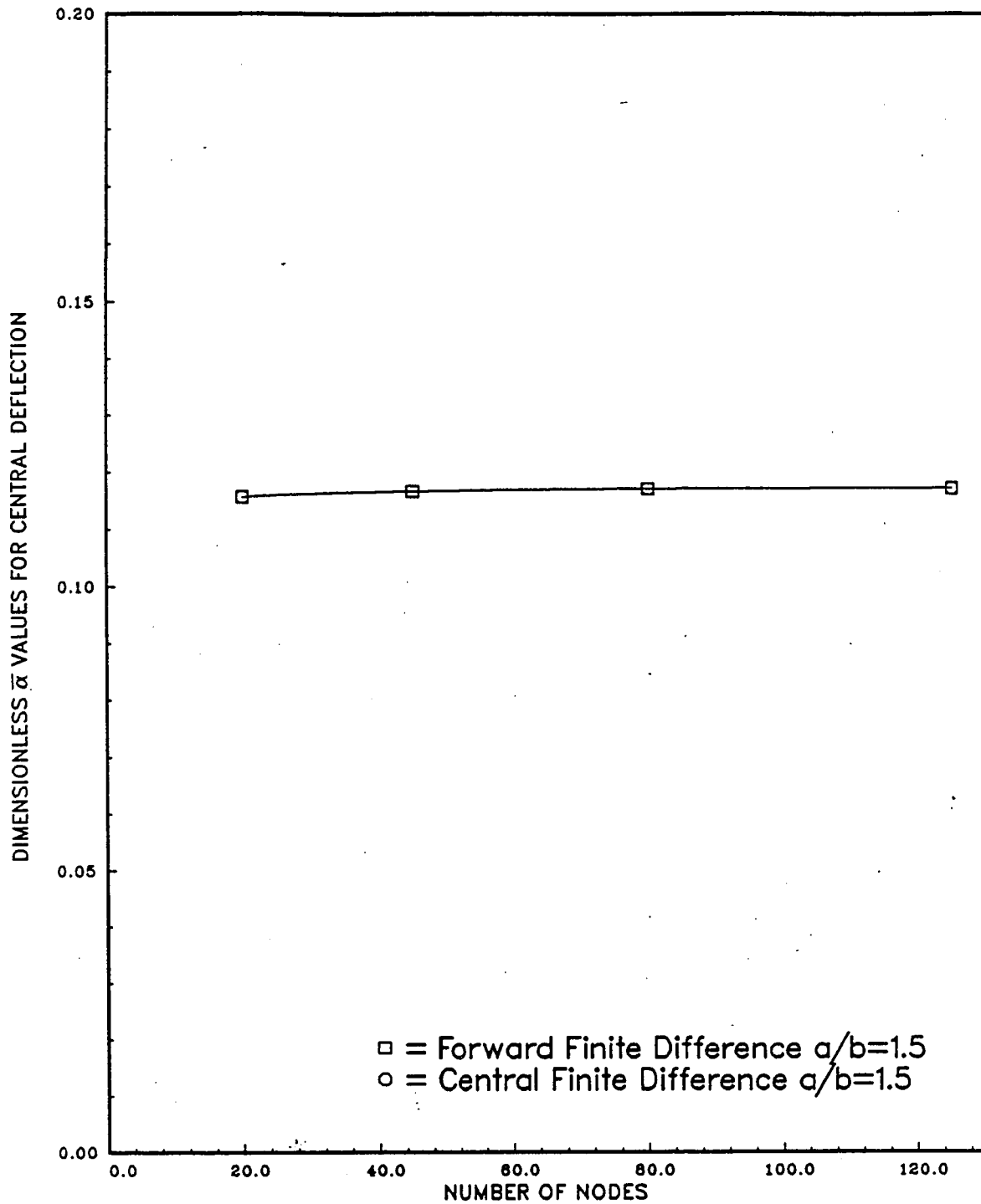


FIGURE 6.13, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

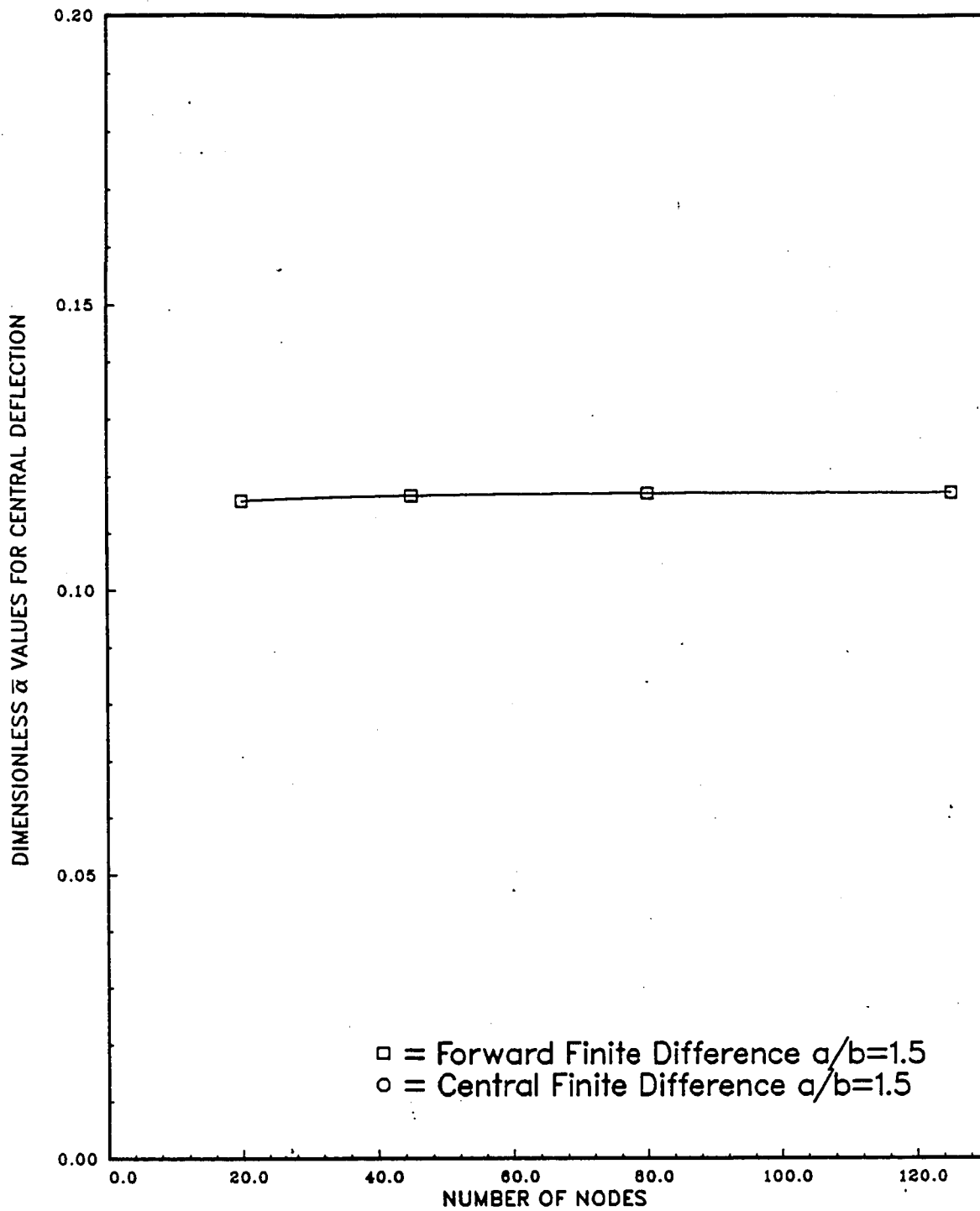


FIGURE 6.14, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

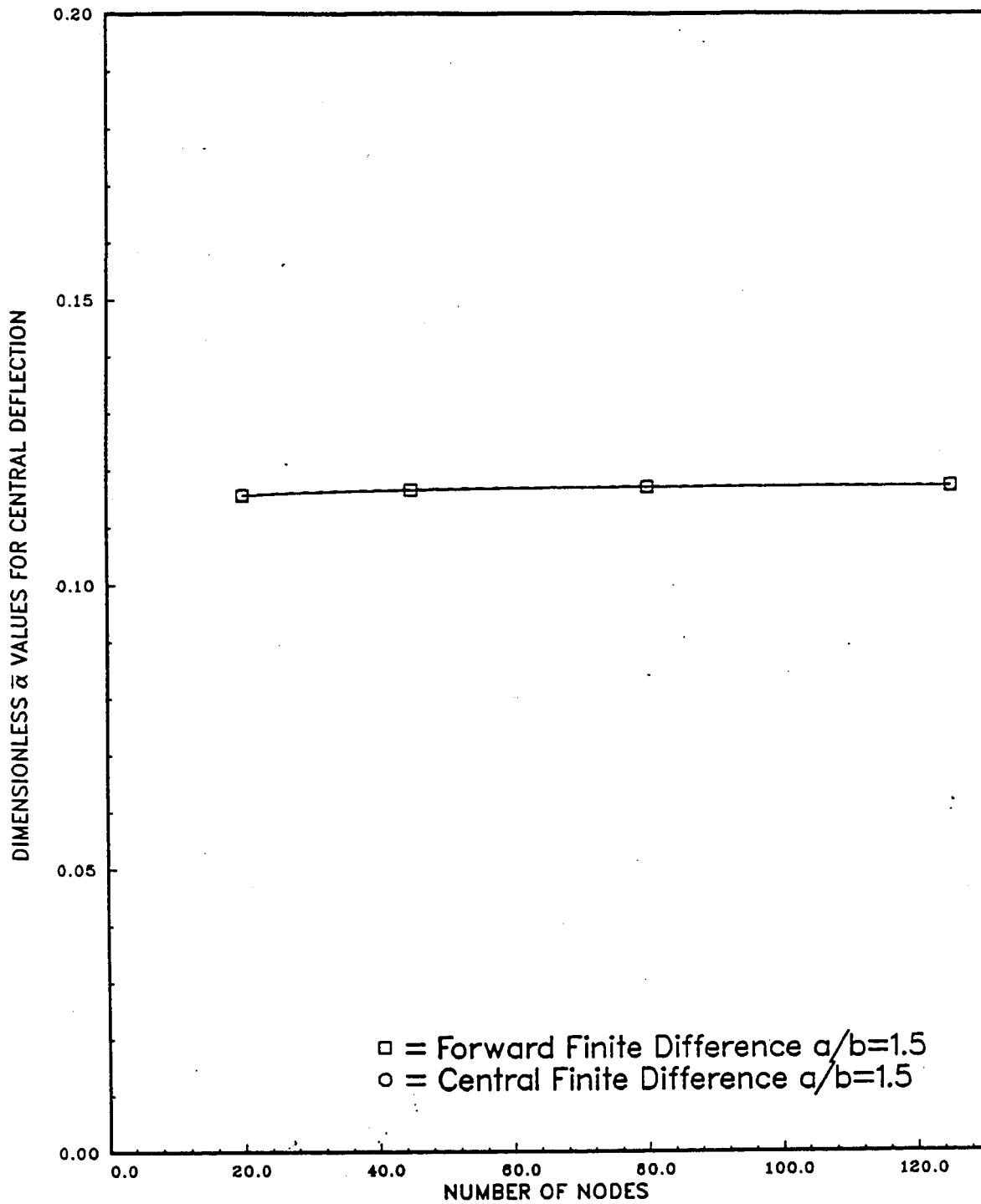


FIGURE 6.15, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

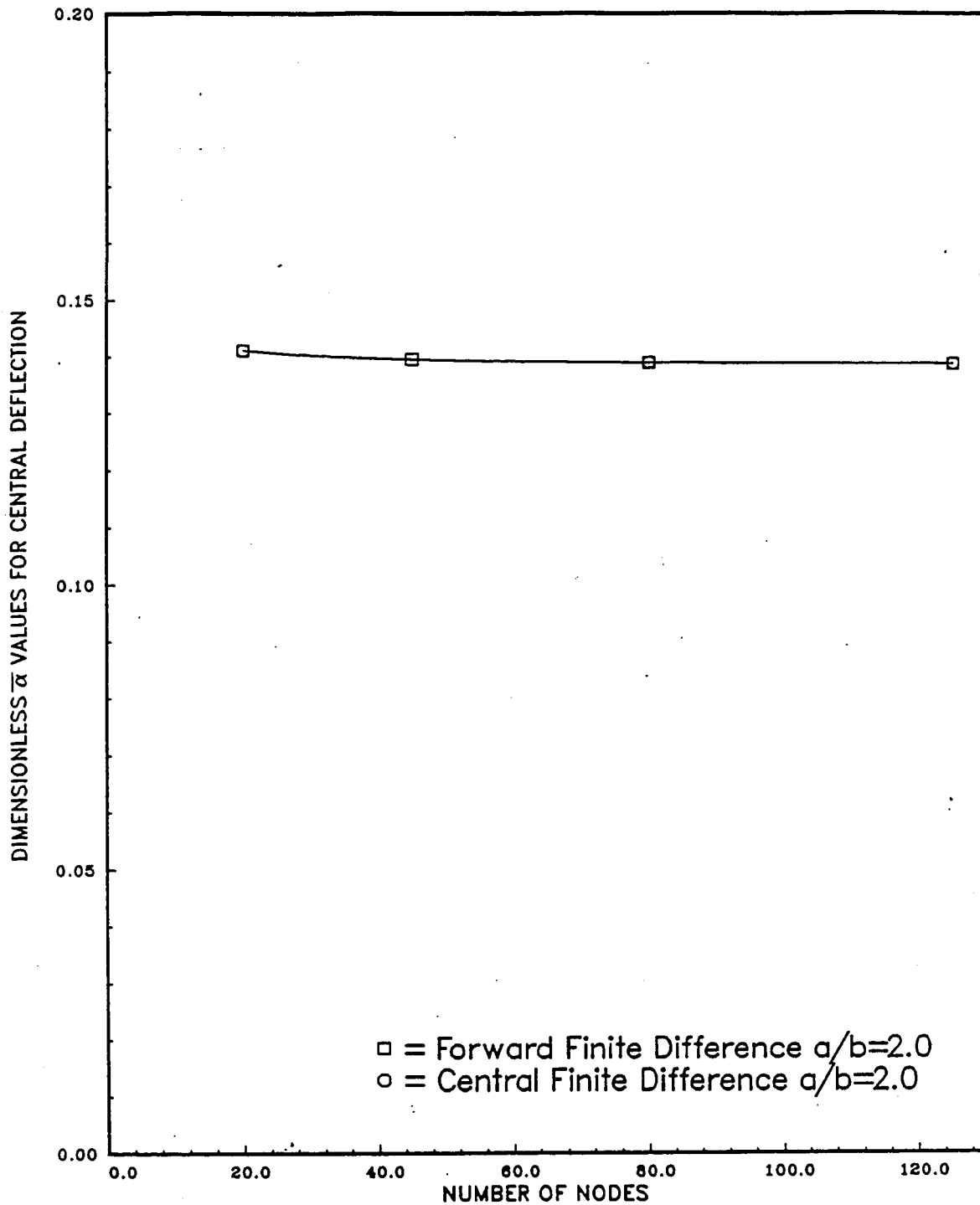


FIGURE 6.16, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

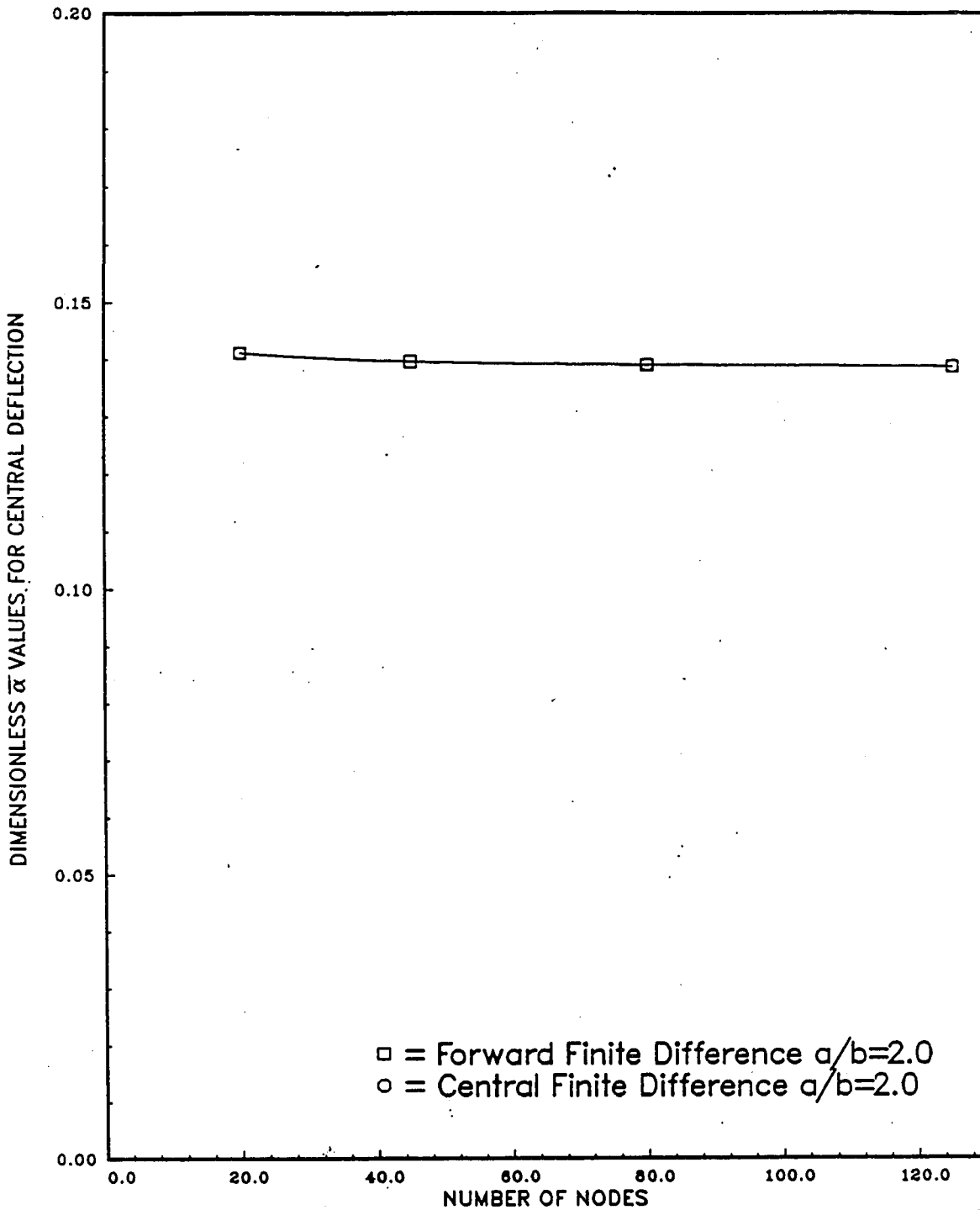


FIGURE 6.17, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

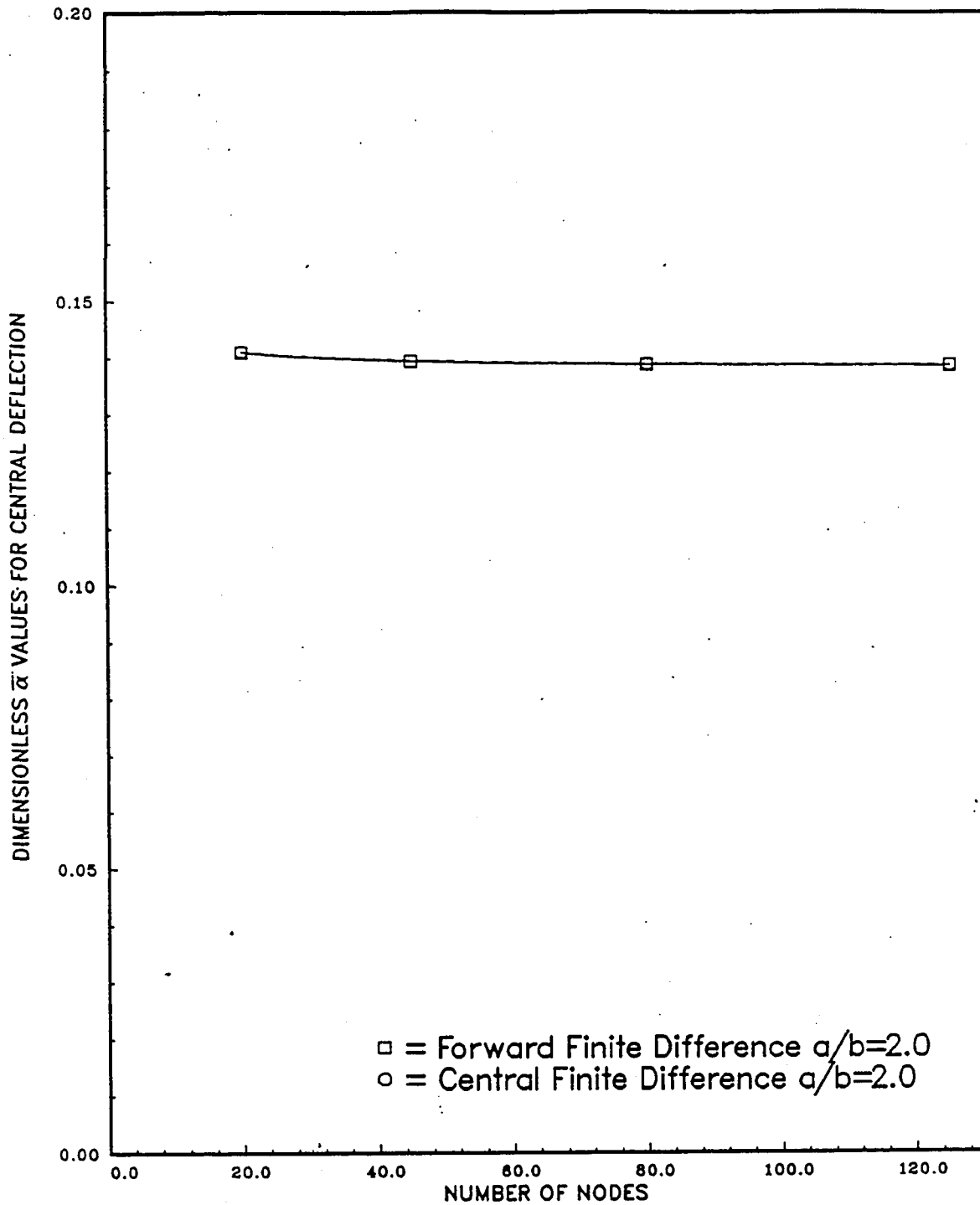


FIGURE 6.18, LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

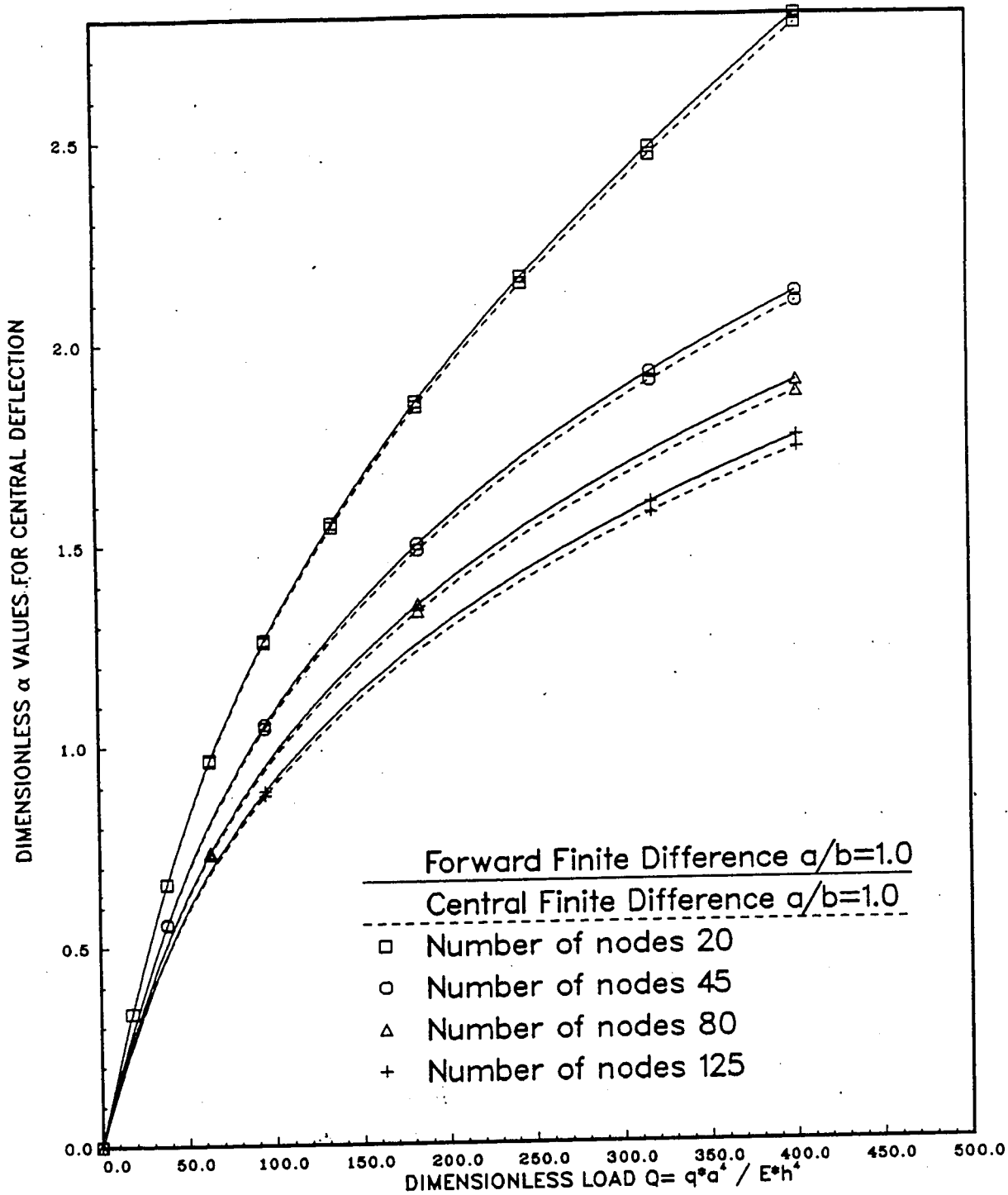


FIGURE 6.19, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.025$

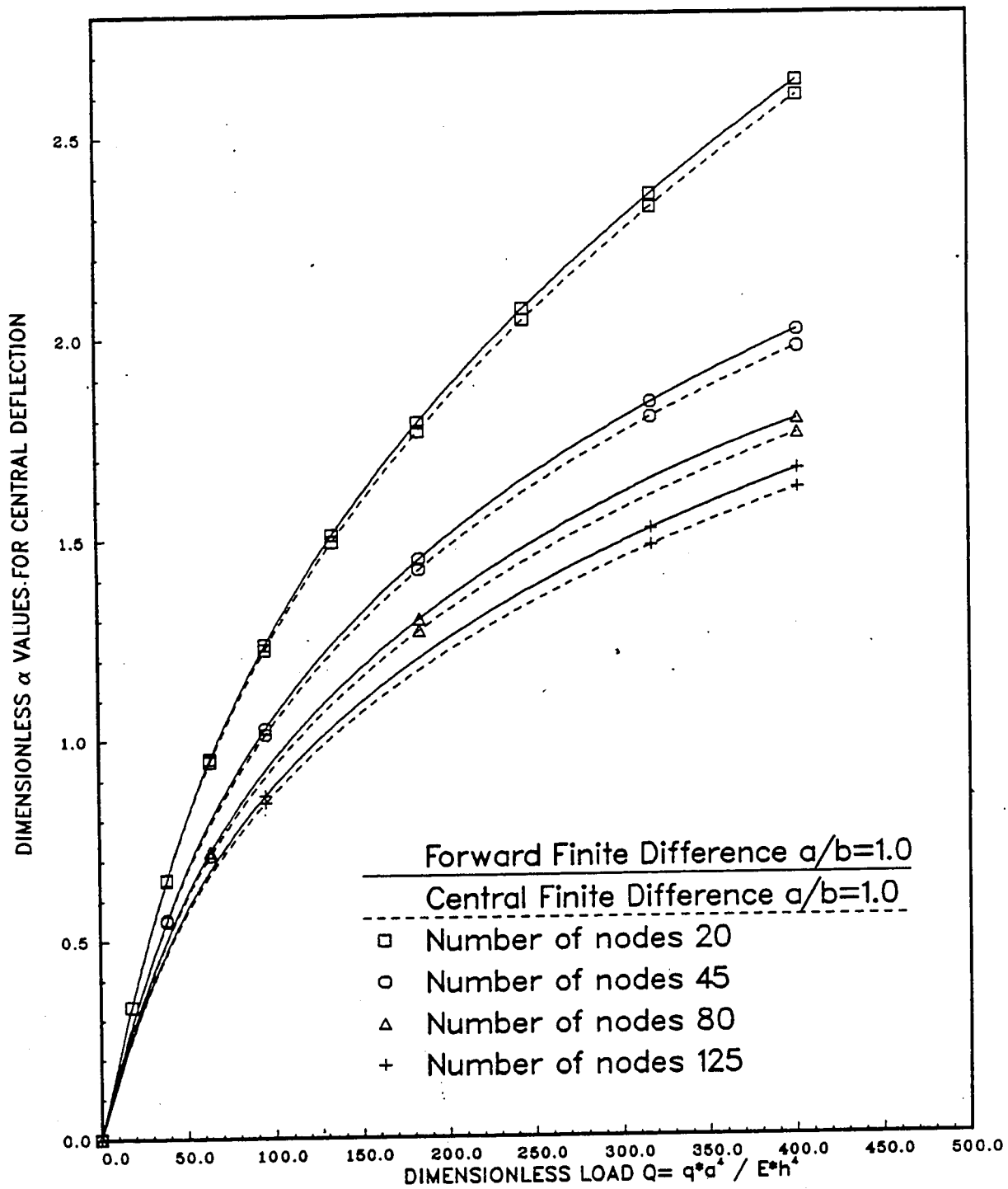


FIGURE 6.20, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.050$

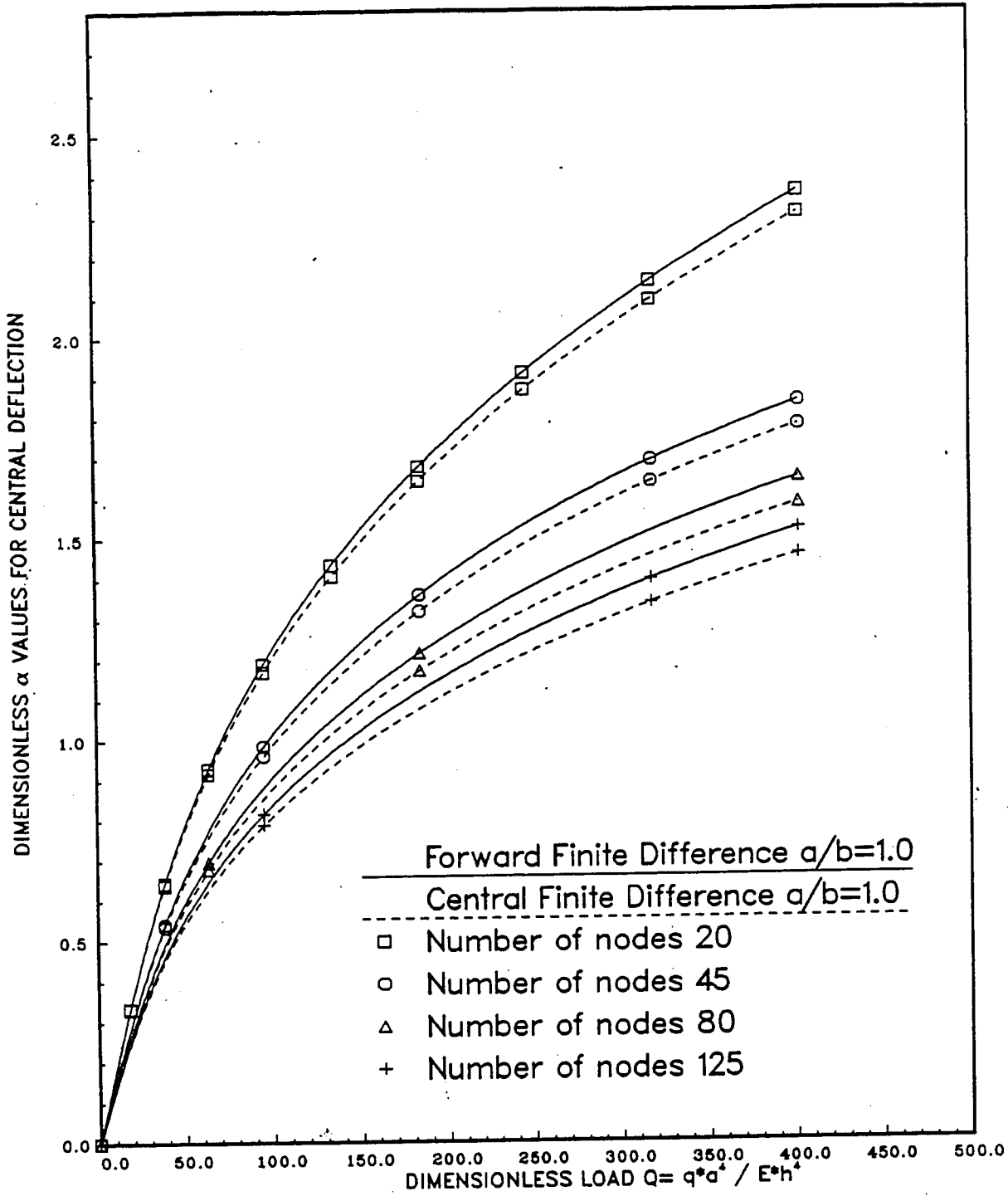


FIGURE 6.21, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.100$

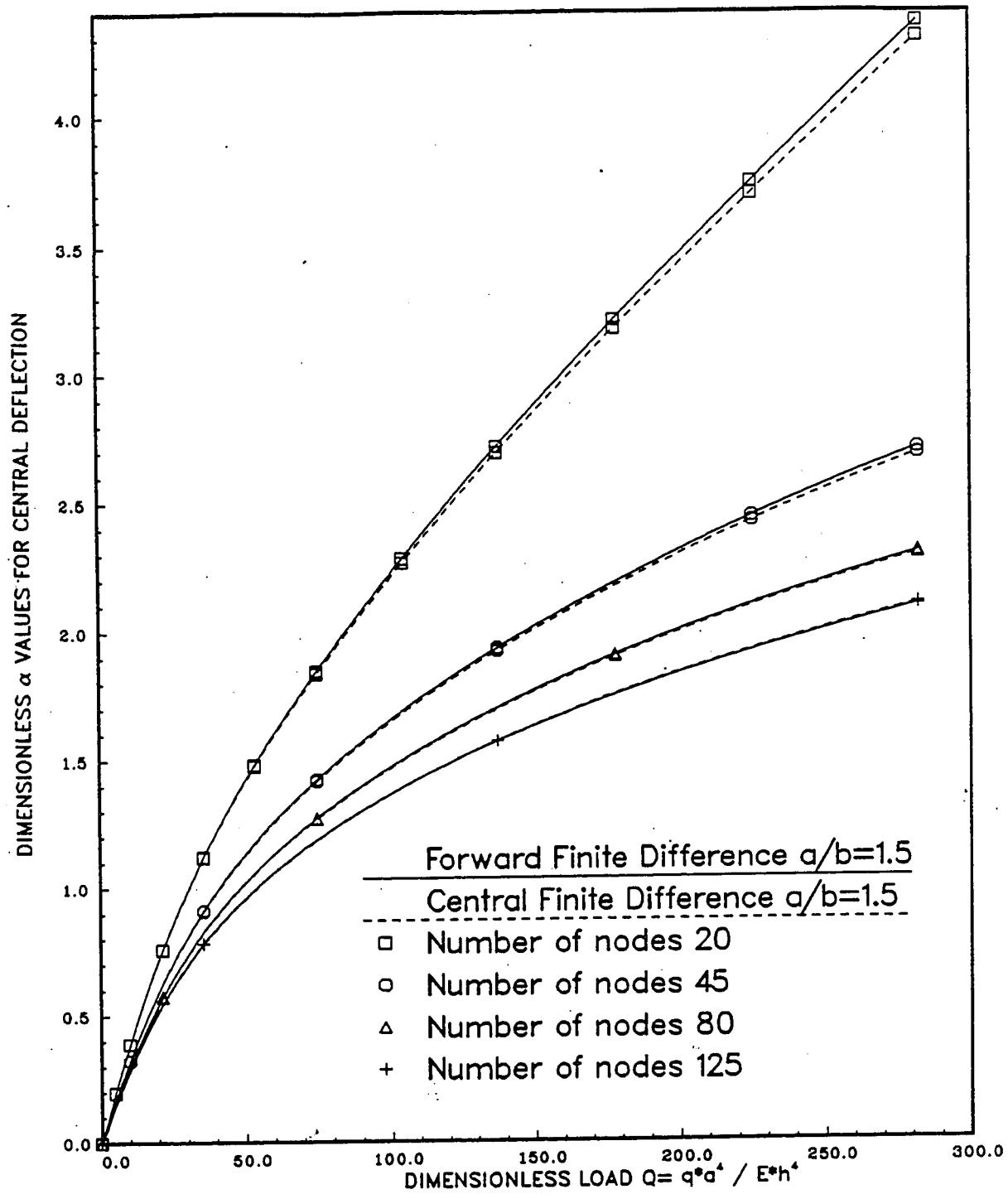


FIGURE 6.22, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.025$

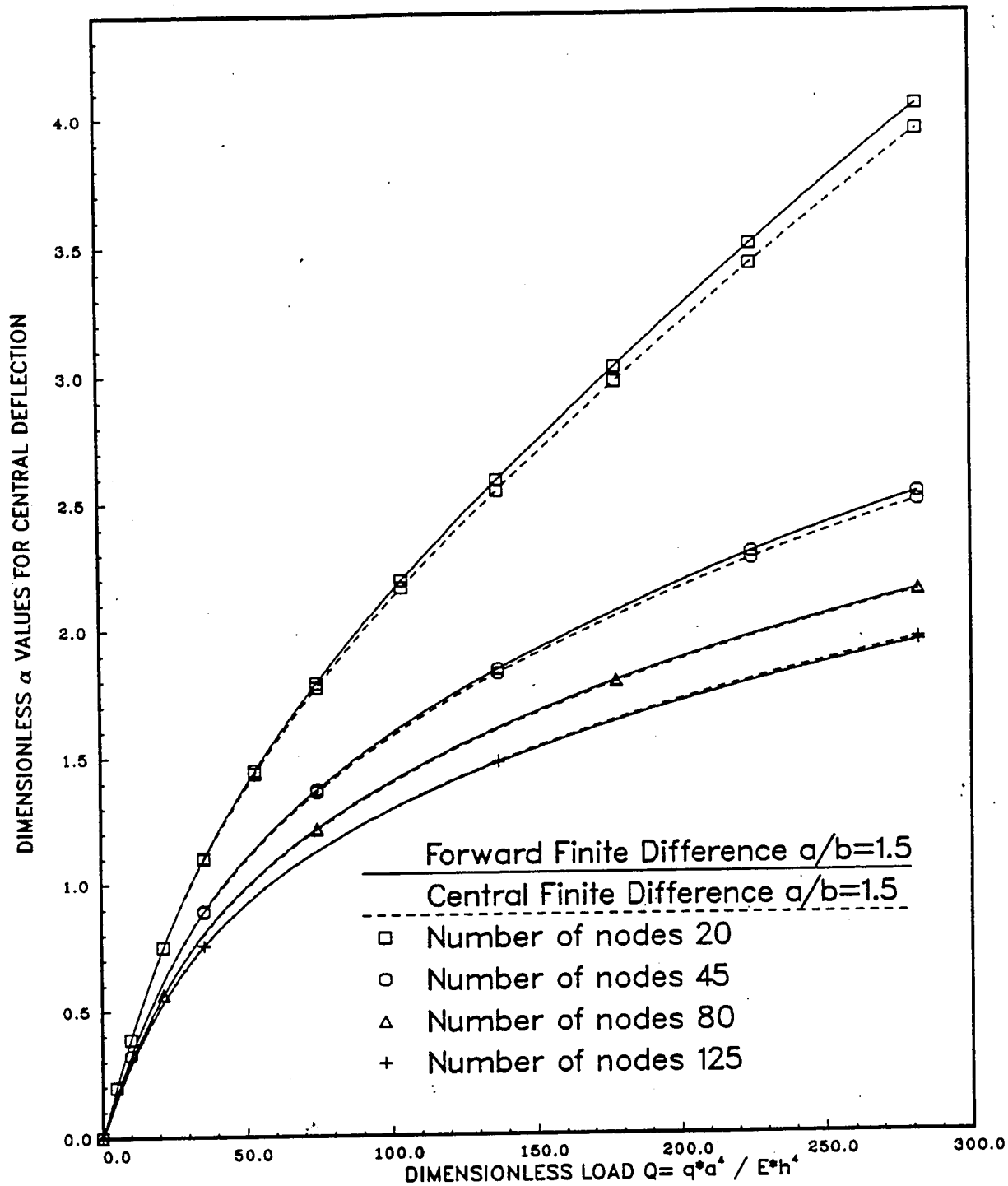


FIGURE 6.23, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.050$

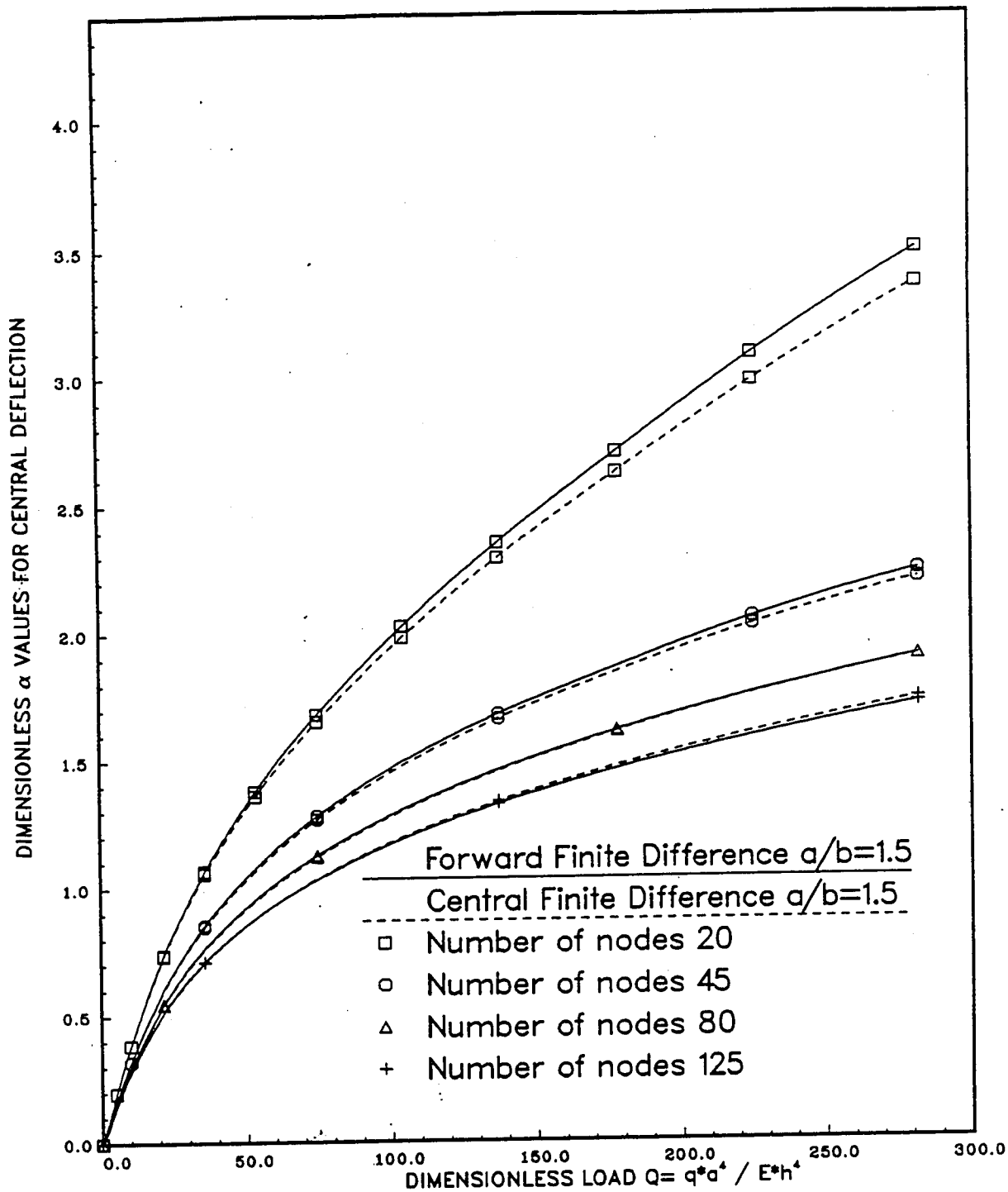


FIGURE 6.24, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.100$

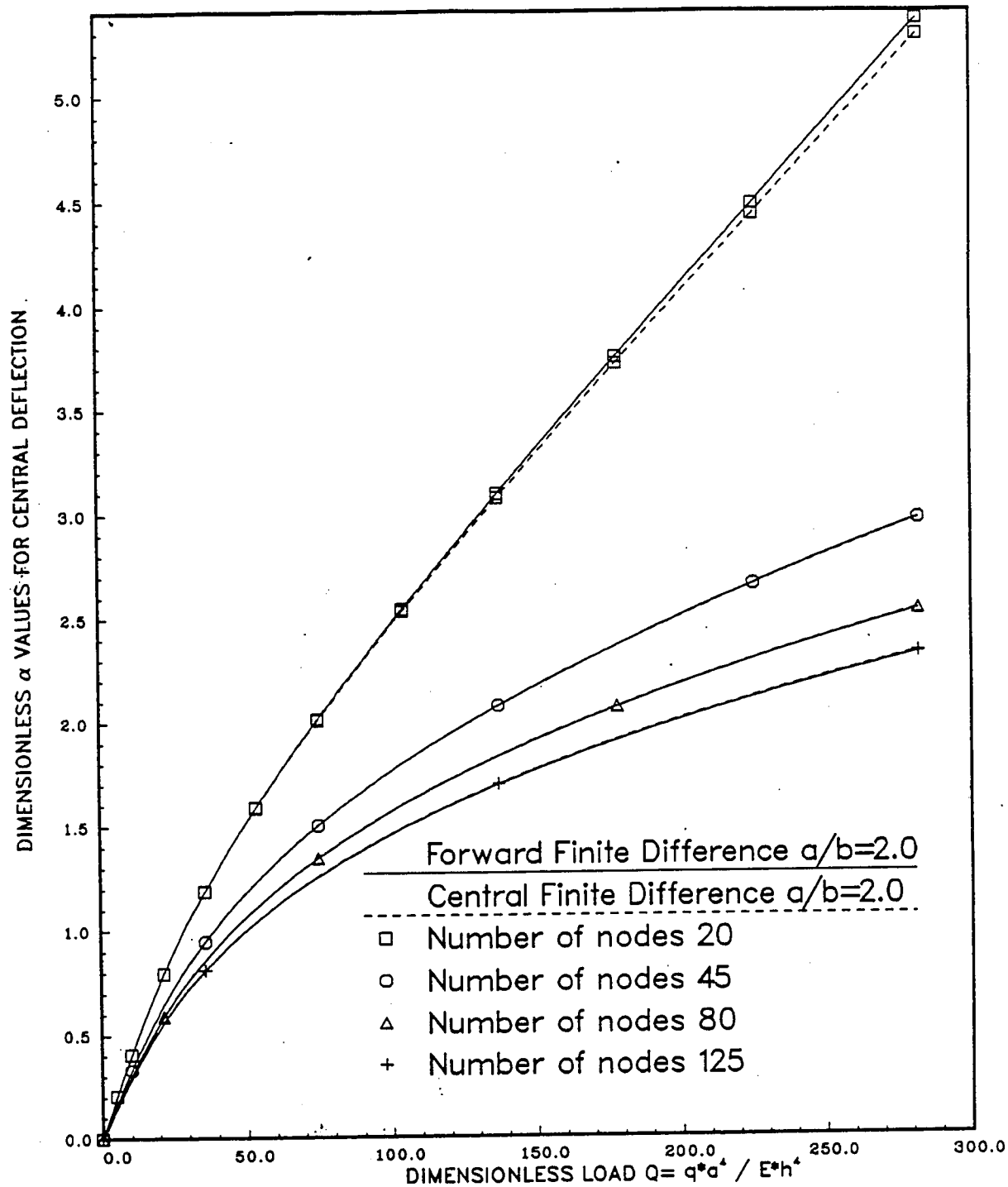


FIGURE 6.25, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.025$

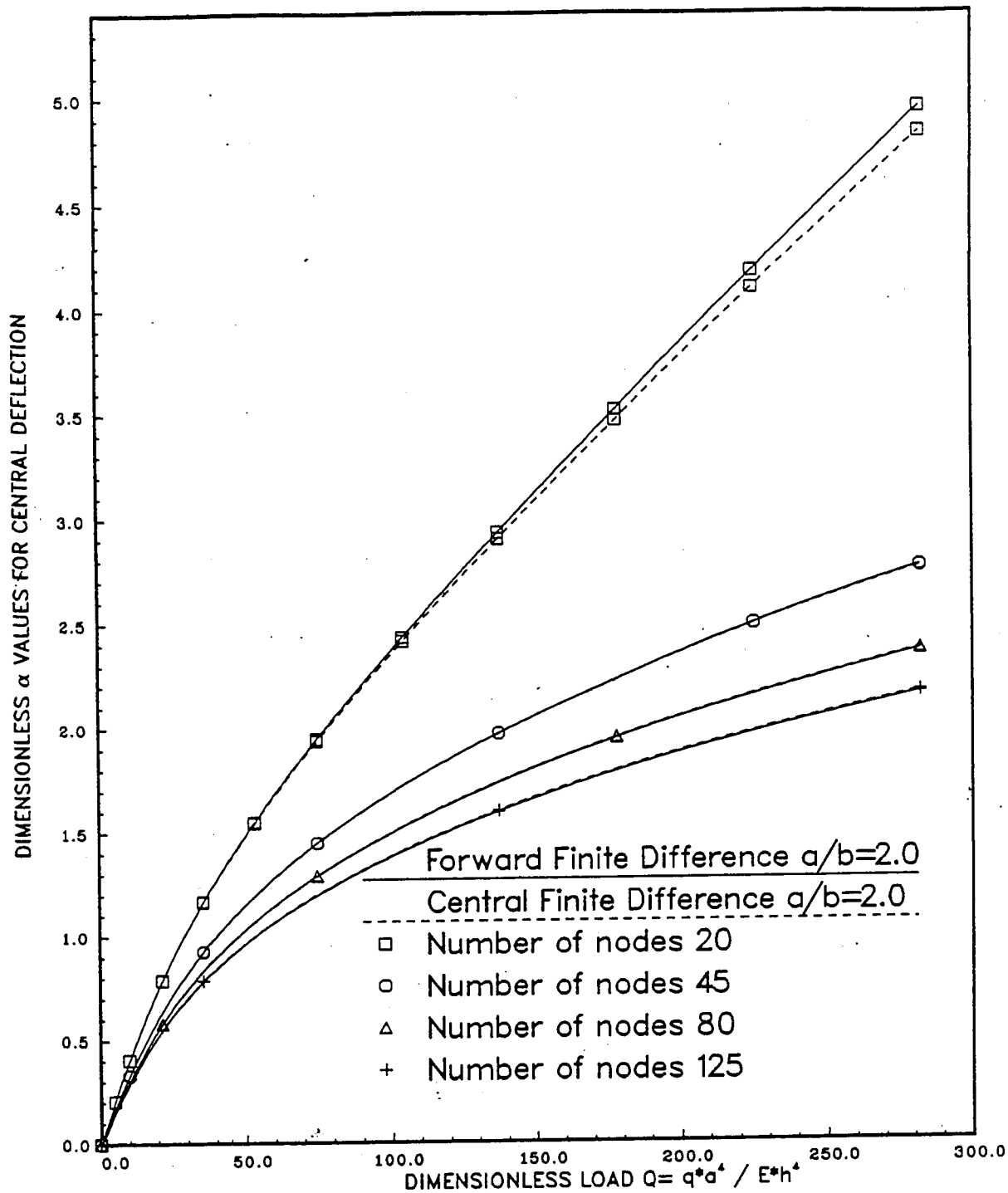


FIGURE 6.26, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CALMPED PLATE. $h/a=0.050$

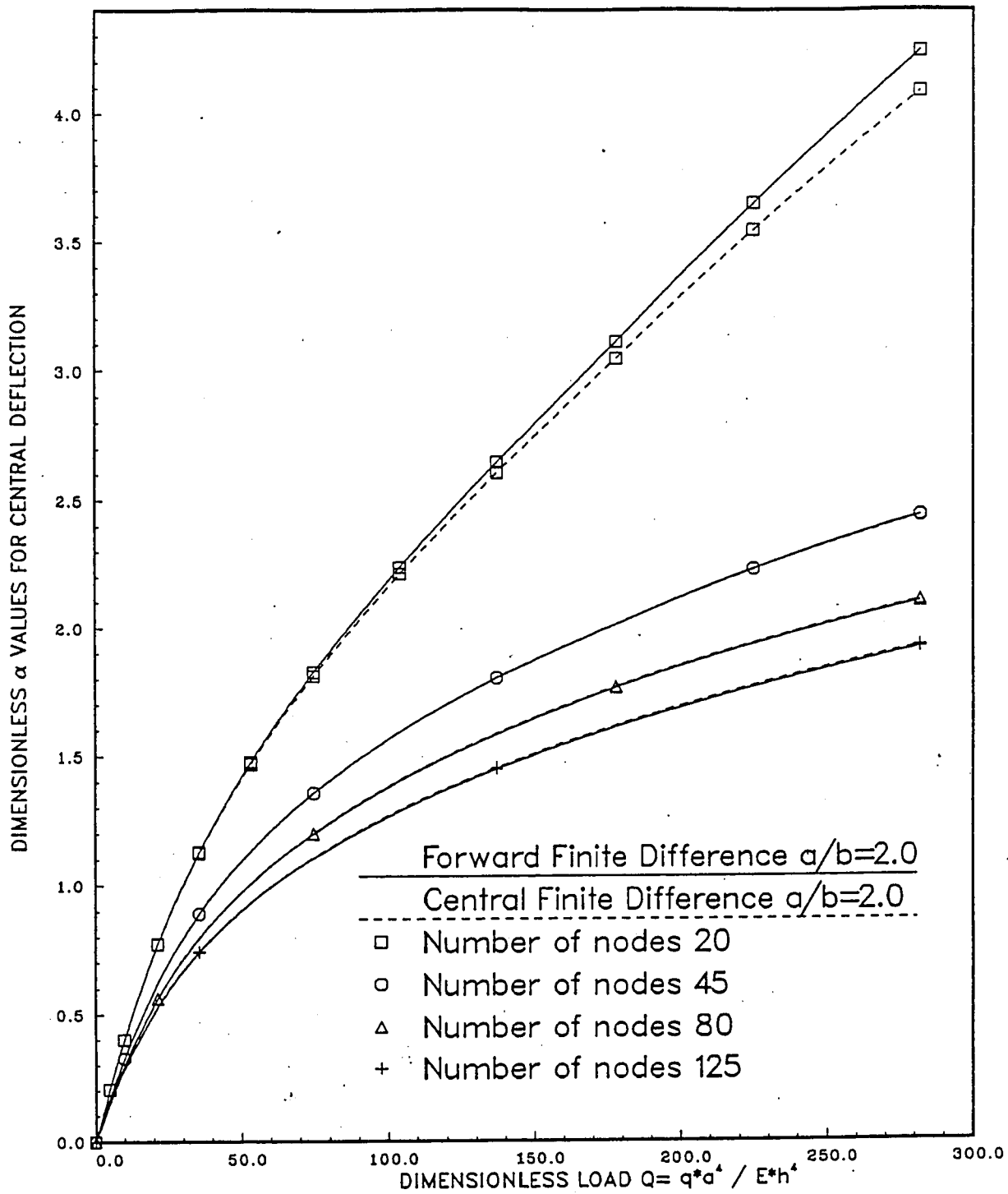


FIGURE 6.27, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE CLAMPED PLATE. $h/a=0.100$

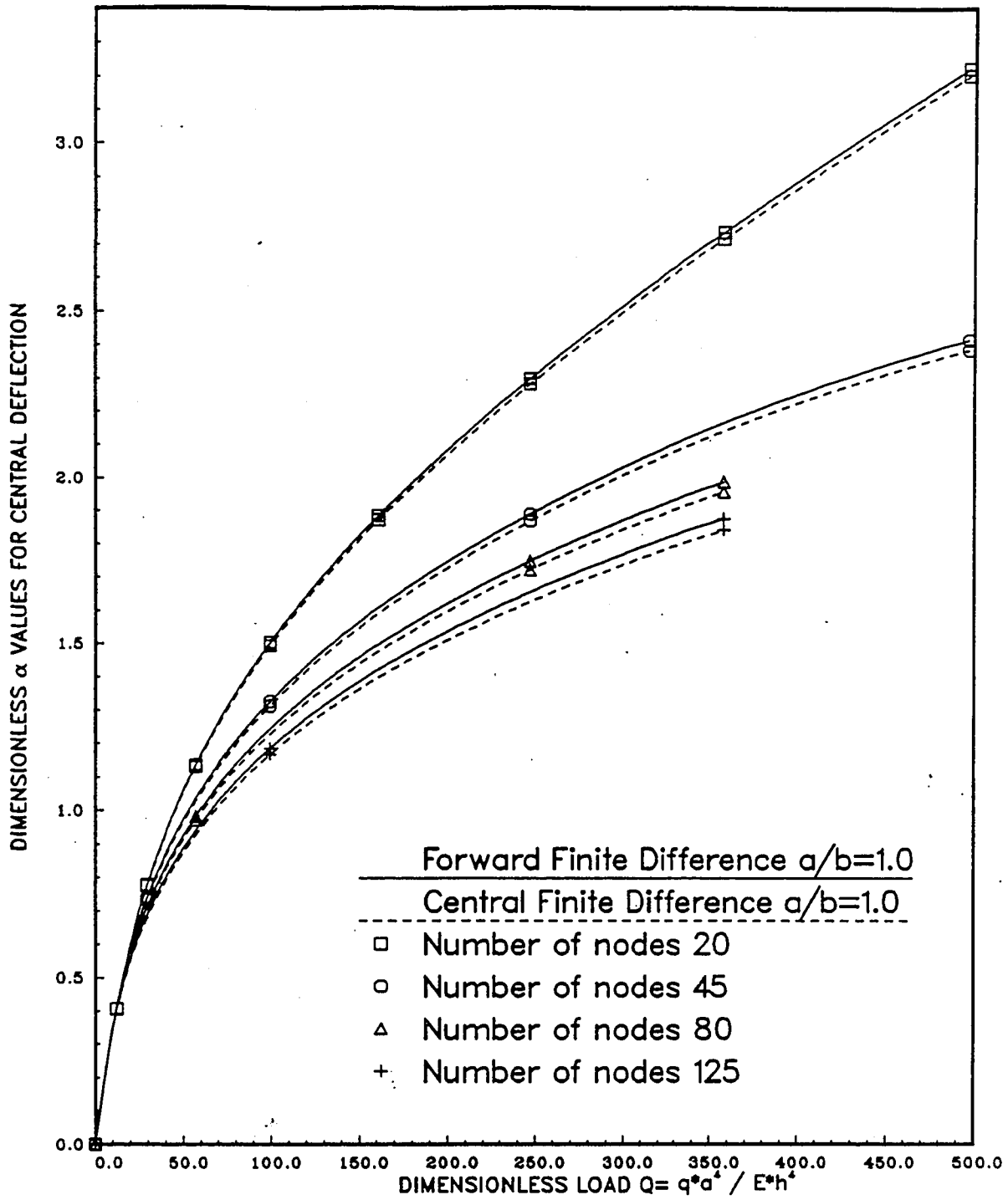


FIGURE 6.28, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

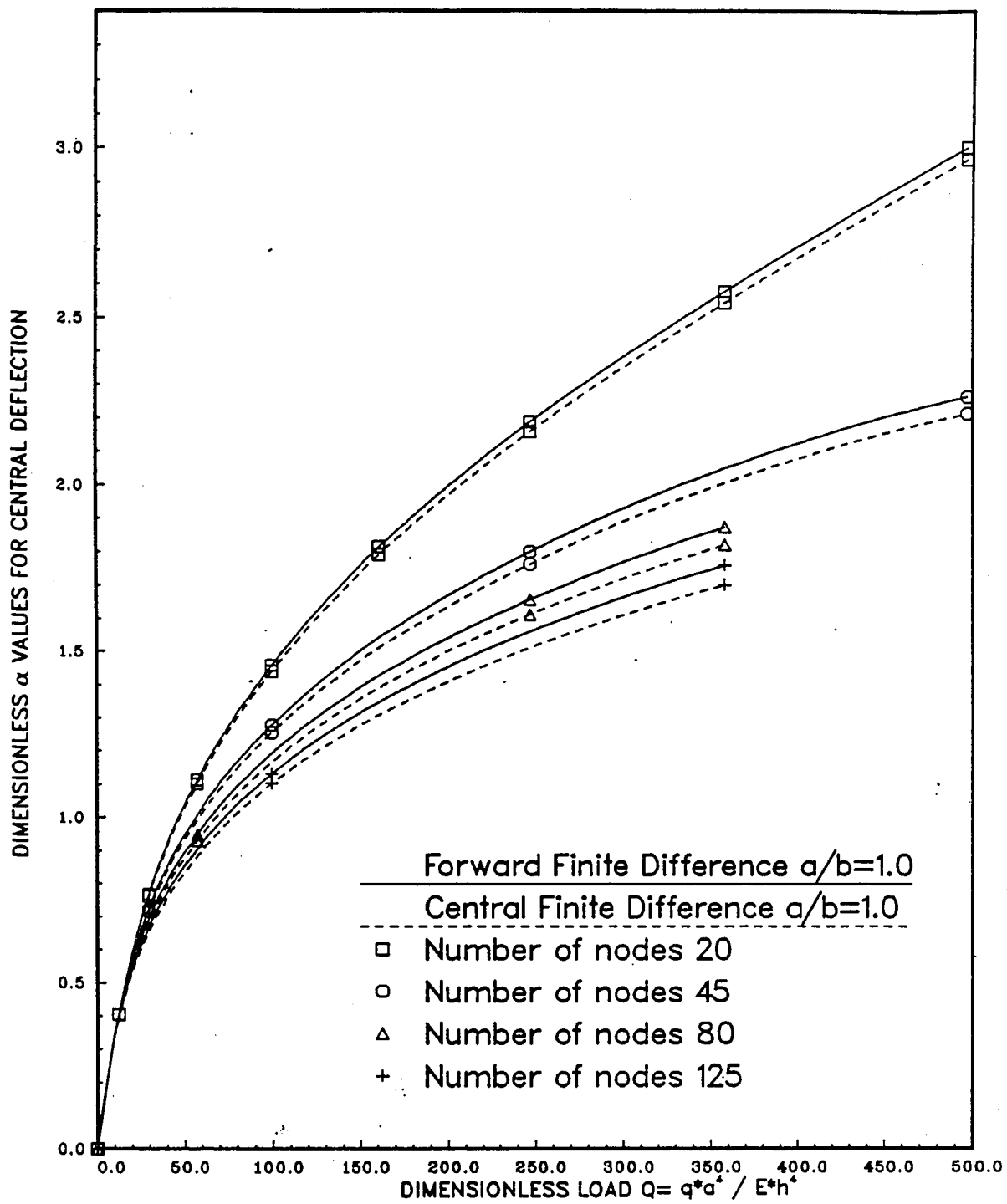


FIGURE 6.29, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

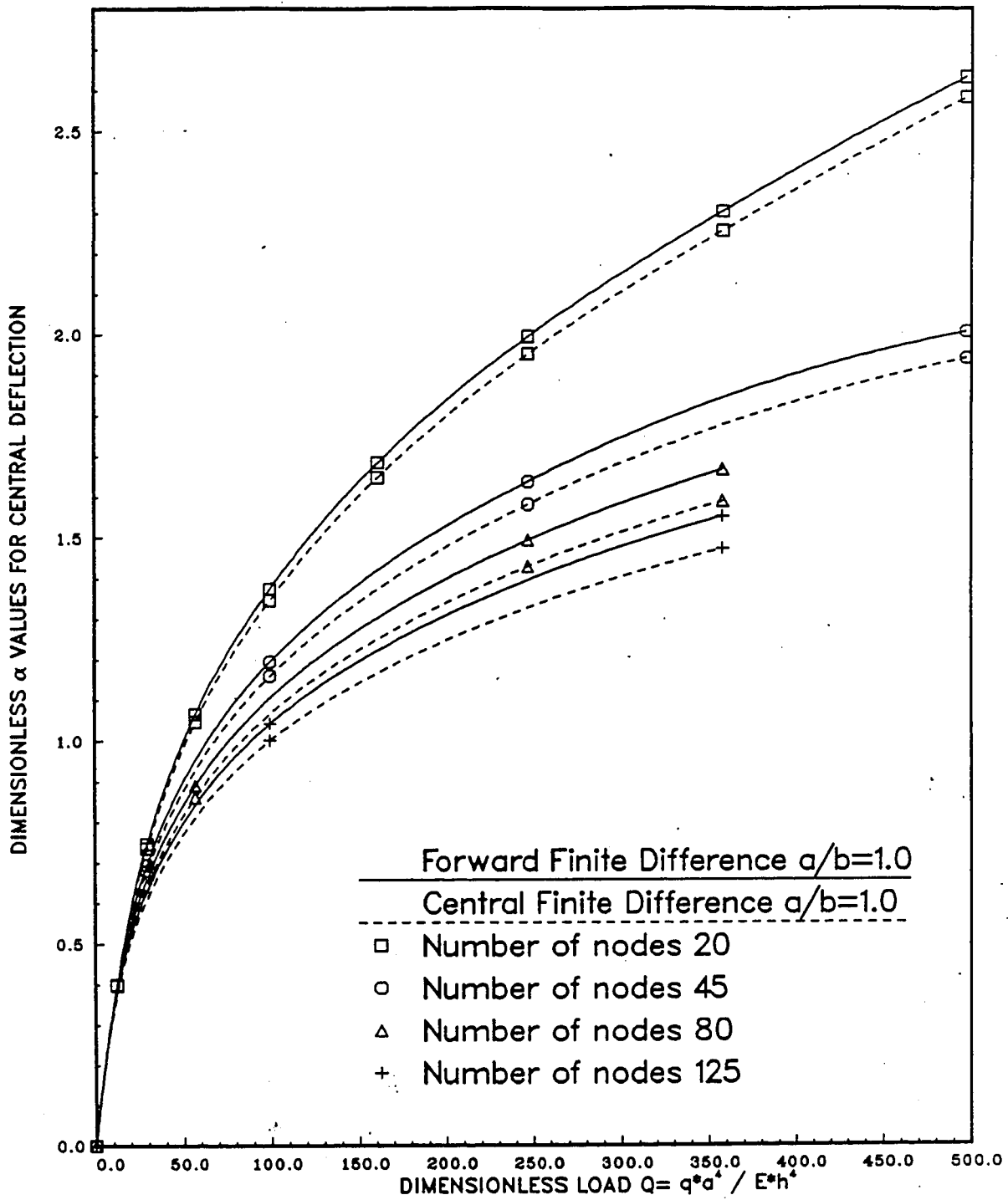


FIGURE 6.30, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

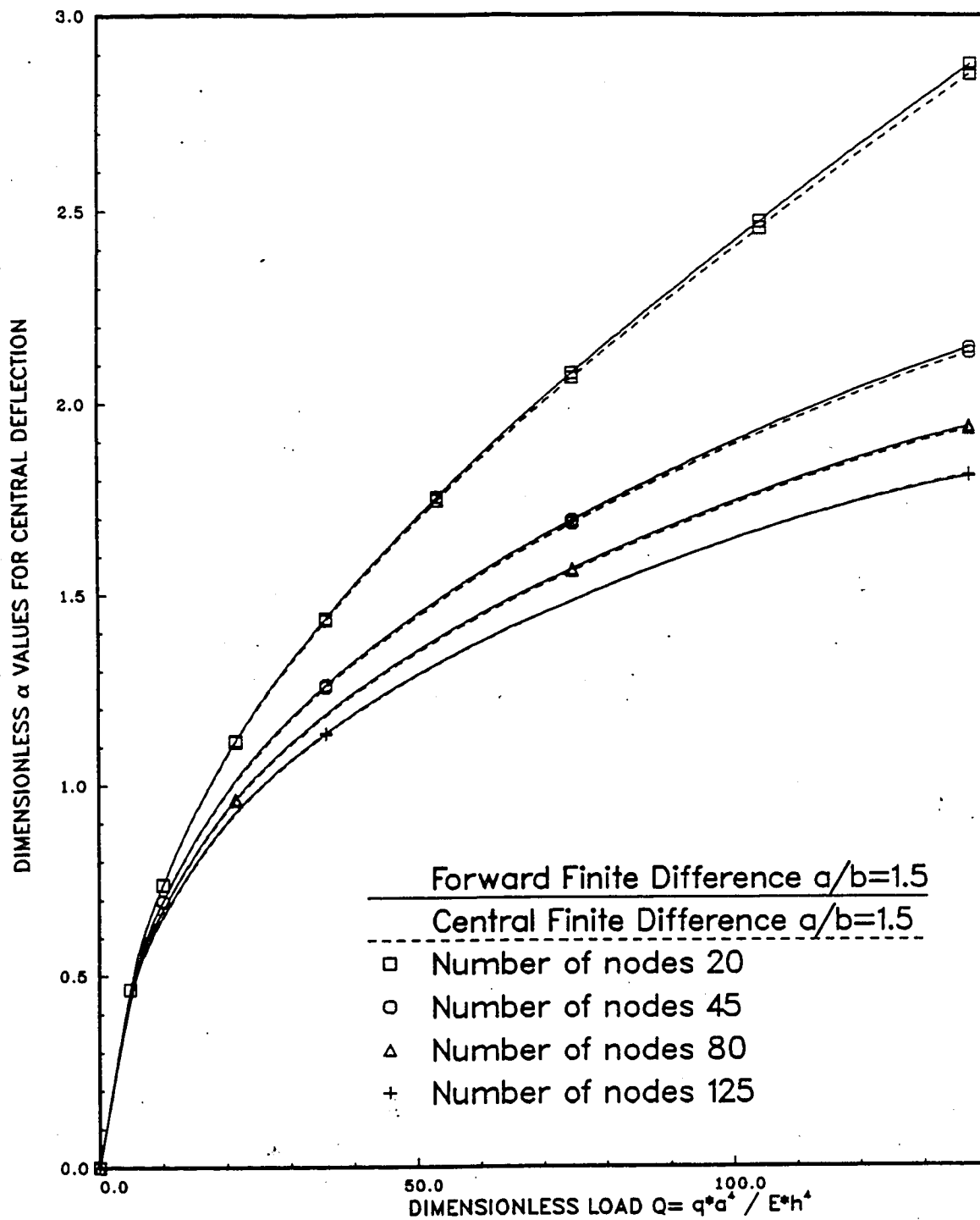


FIGURE 6.31, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

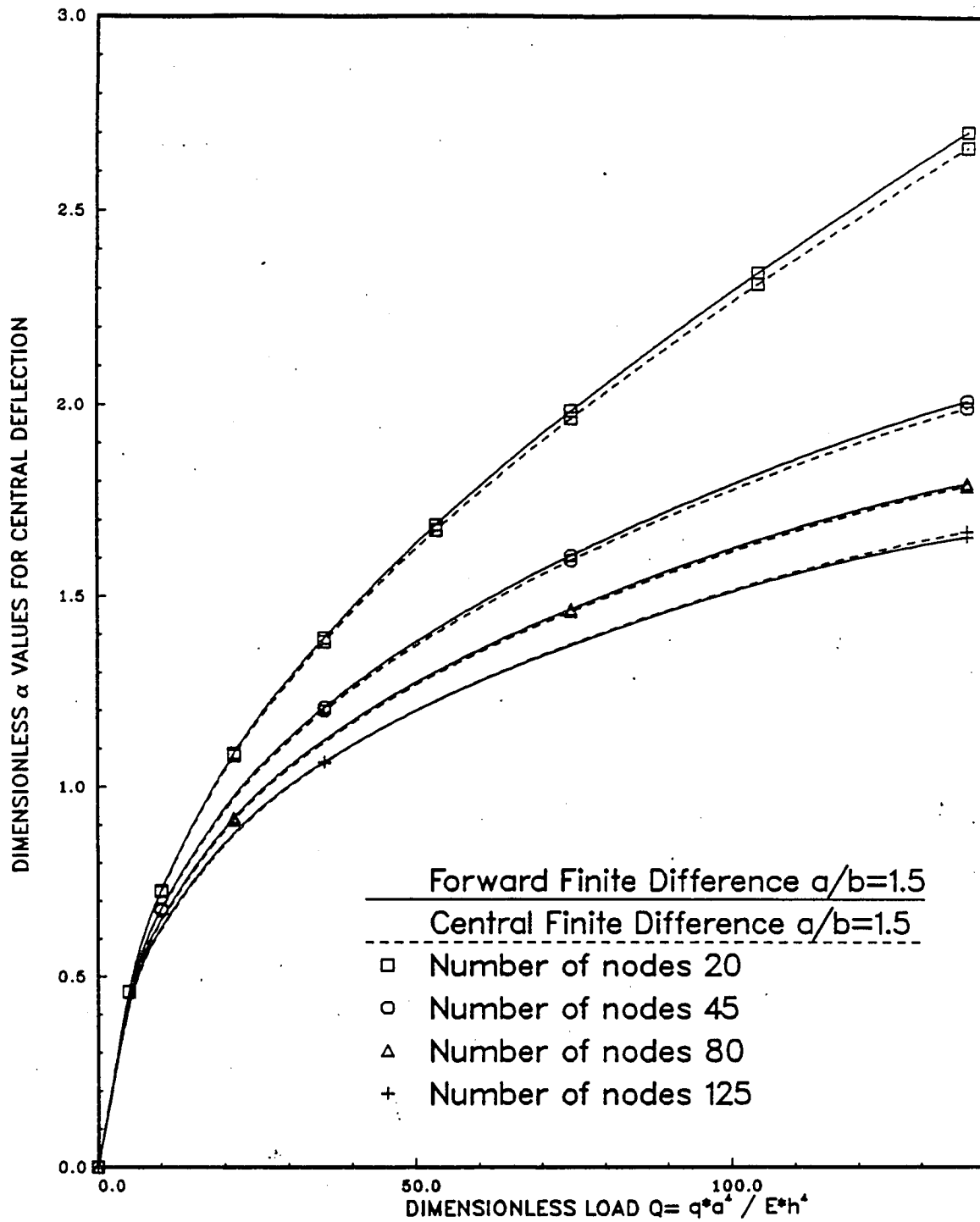


FIGURE 6.32, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

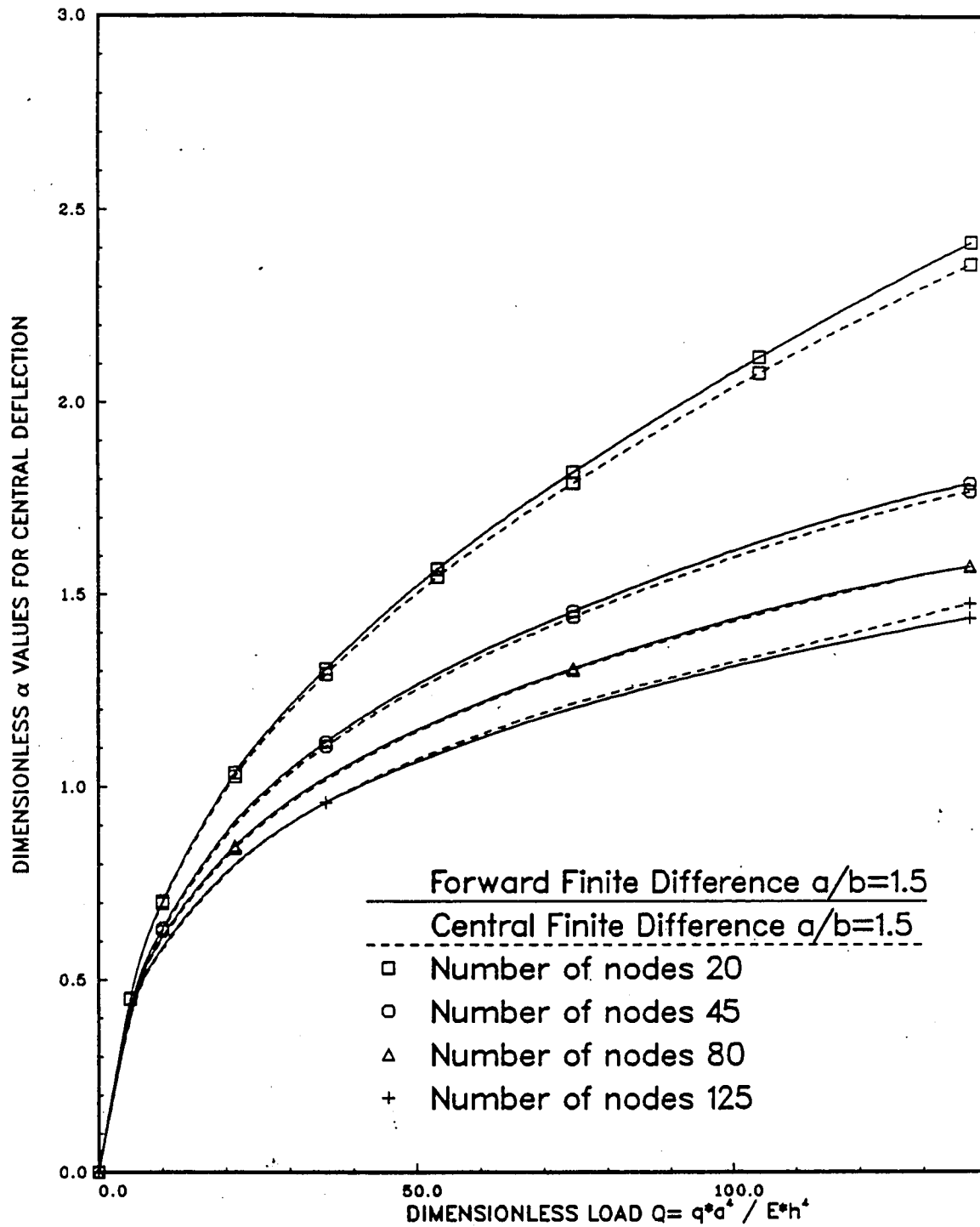


FIGURE 6.33, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

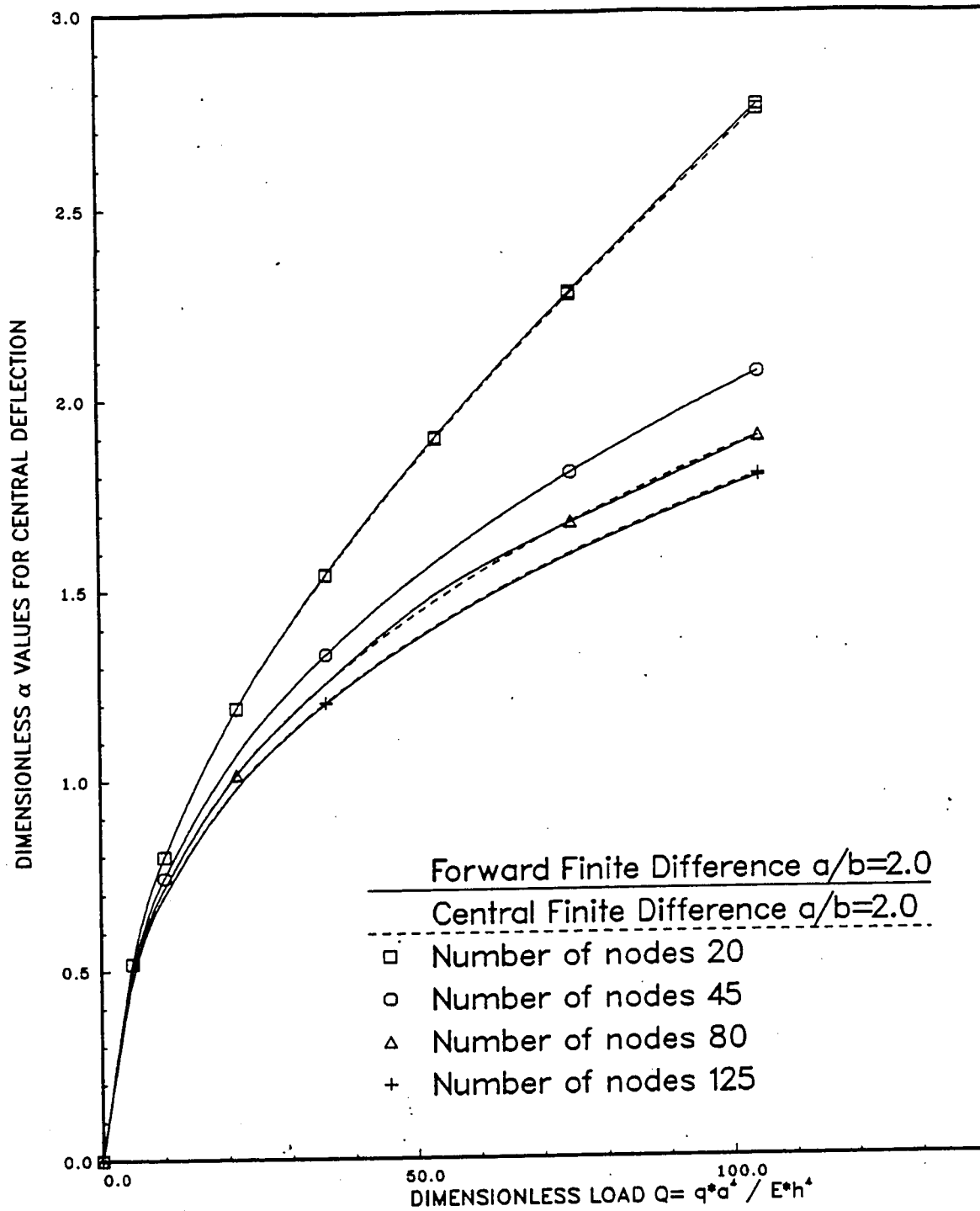


FIGURE 6.34, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.025$

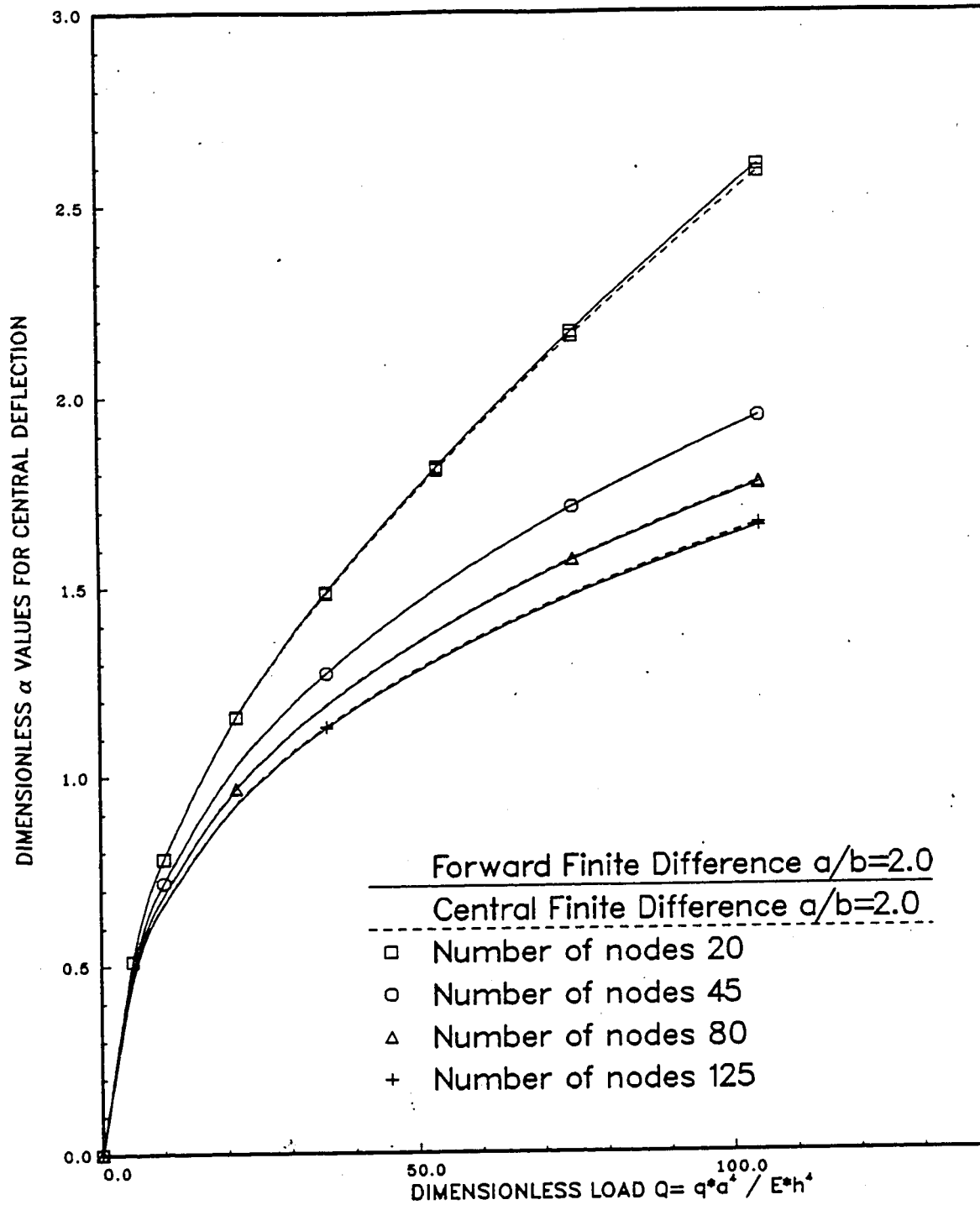


FIGURE 6.35, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.050$

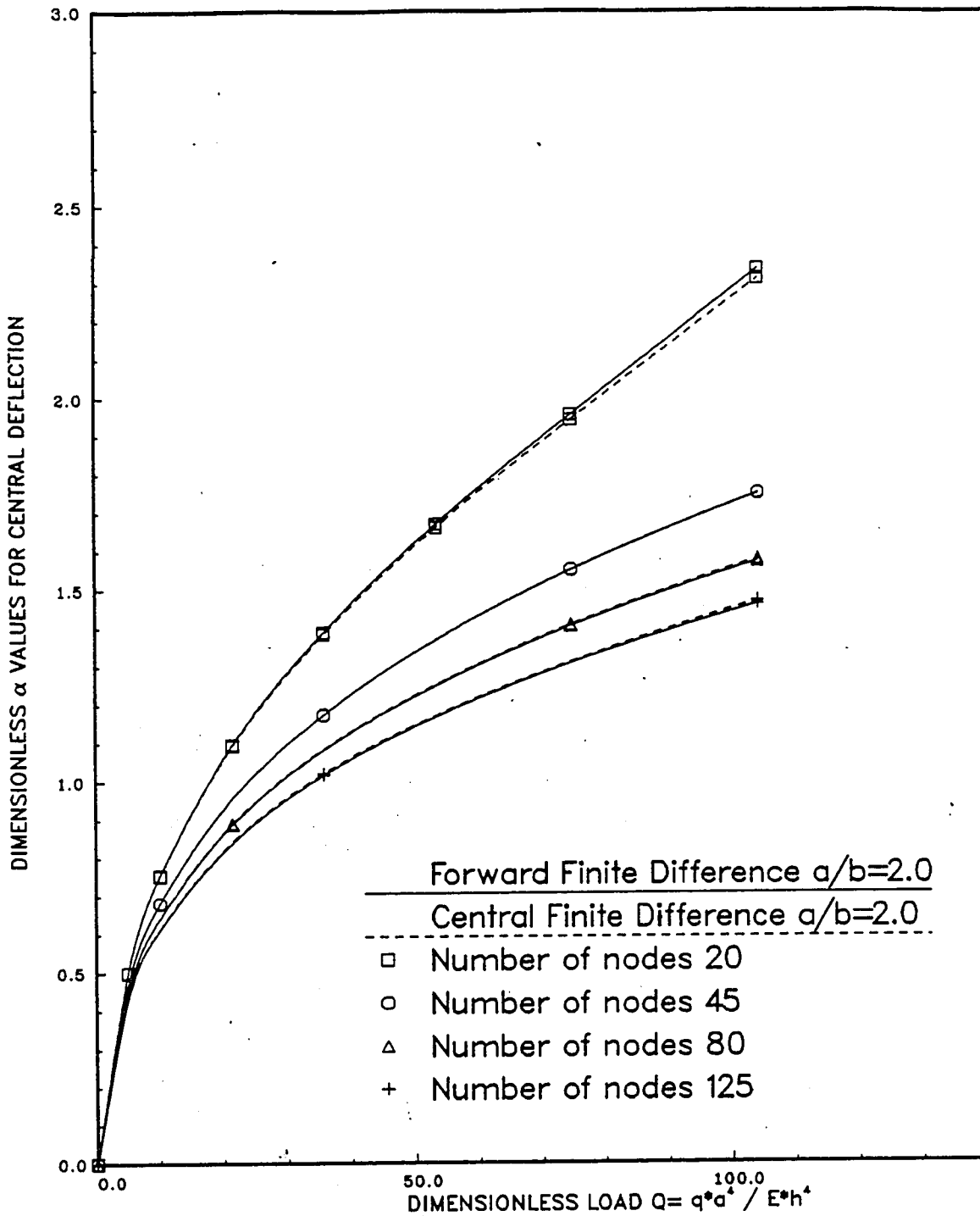


FIGURE 6.36, NON-LINEAR CONVERGENCE CHARACTERISTICS OF THE SIMPLY SUPPORTED PLATE. $h/a=0.100$

6.10-6.18 for simply supported plates.

Figures 6.19-6.36 show the convergence of the dimensionless nonlinear central deflection versus the loading. For all the figures cited the solid line and the dash line represent the forward and central finite difference techniques respectively. The two techniques show a difference ranging from 0 to 4%.

6.3 Comparison of Results

In order to show that the finite difference technique adopted is a good numerical scheme for the deflection analysis of thick nonlinear plates where no data is available, nonlinear thick plates results were reduced to thin plates to the limit $\frac{h}{a} = 0.025$ and results are compared with nonlinear analysis results of thin plates where the Von-Karman equations are used. In addition linear thick plates results were also compared with other investigators whenever possible.

6.3.1 Linear Analysis

For the linear analysis of thick plates, comparison has been made for the case of the square plate. The results presented by two investigators Srinivas [31] and Rajabhau [38] show good agreement with the present analysis for the case of clamped thick plates as shown in Figure 6.37. In the case of simply supported square plates the present analysis is compared with four

investigators Srinivas [30], [31], Rajabhau [38], Reissner [20], MIF [43] and good agreements are also found as shown in Figure 6.38.

6.3.2 Nonlinear Analysis

In Figure 6.39, the finite difference results presented here for clamped thin square plates are compared with those by Way [1] using polynomials and by Levy [3] using series solution. As can be observed from the diagram, the agreement is good. The difference between the present analysis and those of Way [1] and Levy [3] is more evident once the dimensionless central deflection approaches the thickness of the plate. At this point, it is well reported [54] that for thin plate theory using the Von-Karman equations, membrane forces resist virtually all the loading when the deflection is greater than the thickness, and since the participation of the inplane forces in the three dimensional analysis is higher than in the two dimensional analysis, the three dimensional thick plate analysis would give the plate more resistance to the external loading. Rectangular thin plate analysis compared with Way [1], and Levy [4] for the plate aspect ratios 1.5 and 2.0 Figures 6.40-6.41 show that the two dimensional analysis is more rigid. For square simply supported thin plates, the results are compared with those of Chia [46]. Again, good agreements are obtained as shown in Figure 6.42. Large deflection results for thick rectangular plates for both the simply supported and clamped boundary conditions based on the present finite difference technique are presented in Figures 6.43-6.48. Even though

no nonlinear thick plate results are as yet available, based on convergence analysis and good comparison of results of the same techniques applied to large deflection of thin plate as well as the small deflection of thick plates, the author is confident that the large deflection results for thick rectangular plates as presented in Figures 6.43-6.48 are of acceptable engineering accuracy.

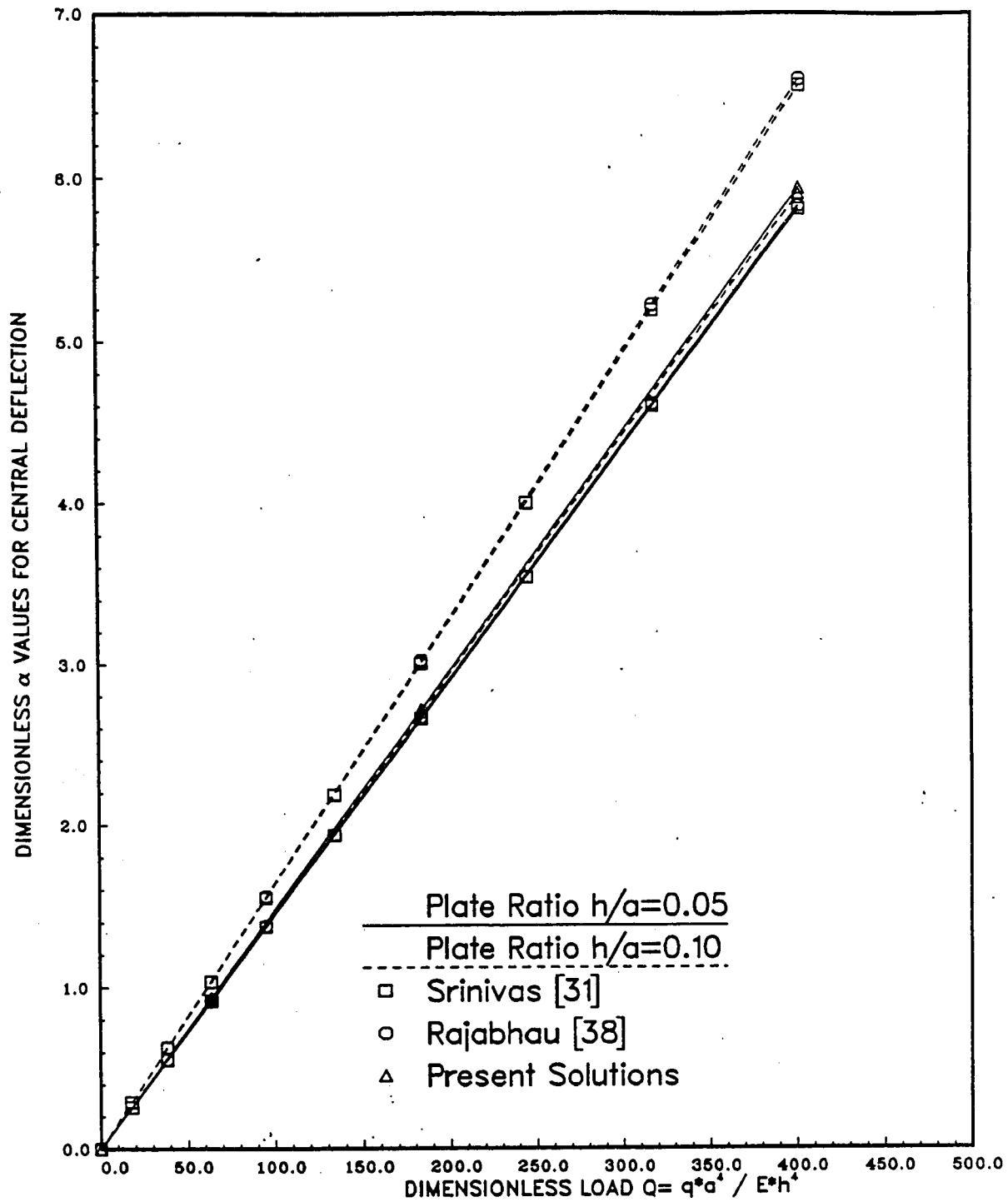


FIGURE 6.37, COMPARISON OF THE LINEAR DEFLECTIONS OF SQUARE CLAMPED THICK PLATES.

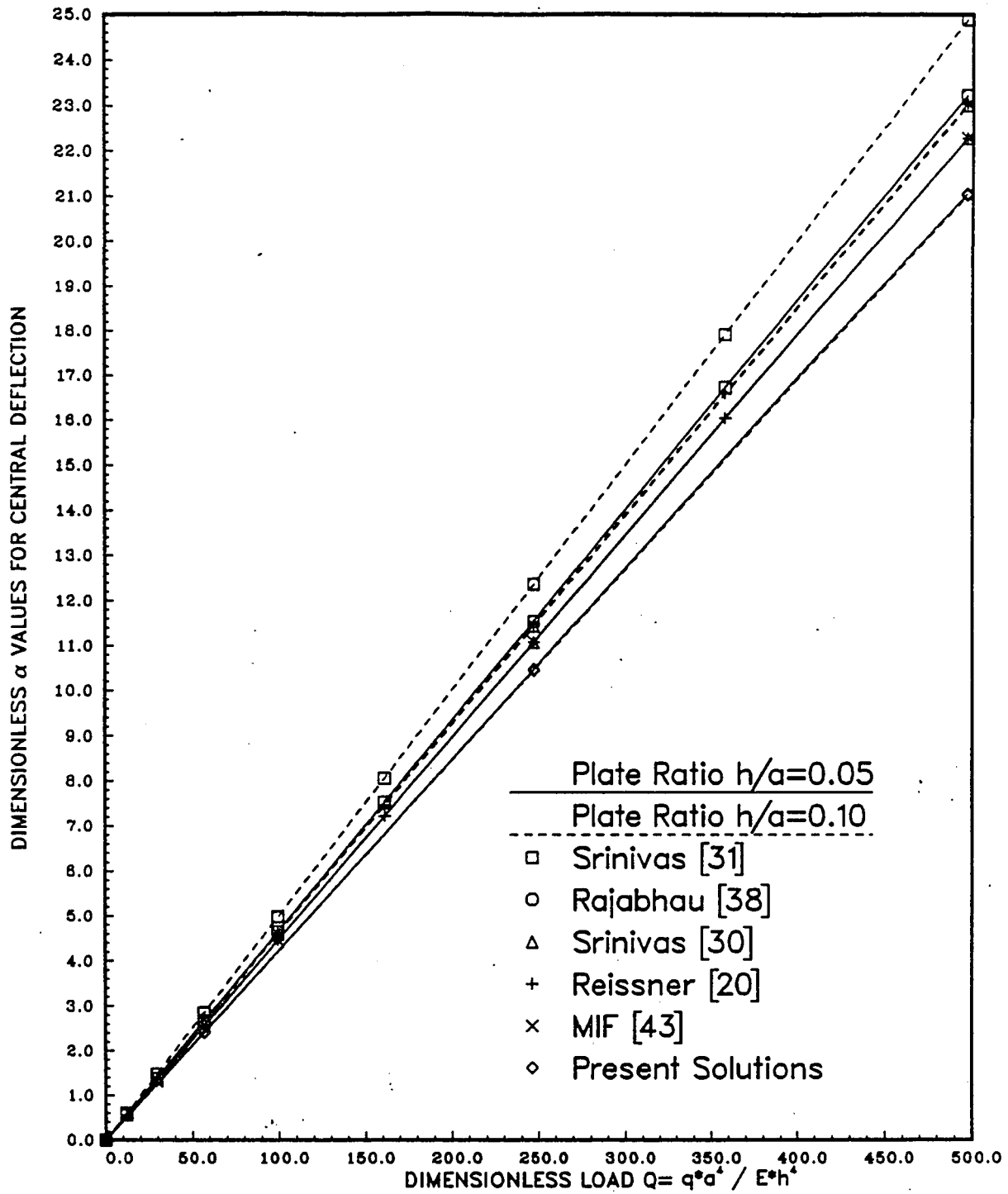


FIGURE 6.38, COMPARISON OF THE LINEAR DEFLECTIONS OF SQUARE SIMPLY SUPPORTED THICK PLATES.

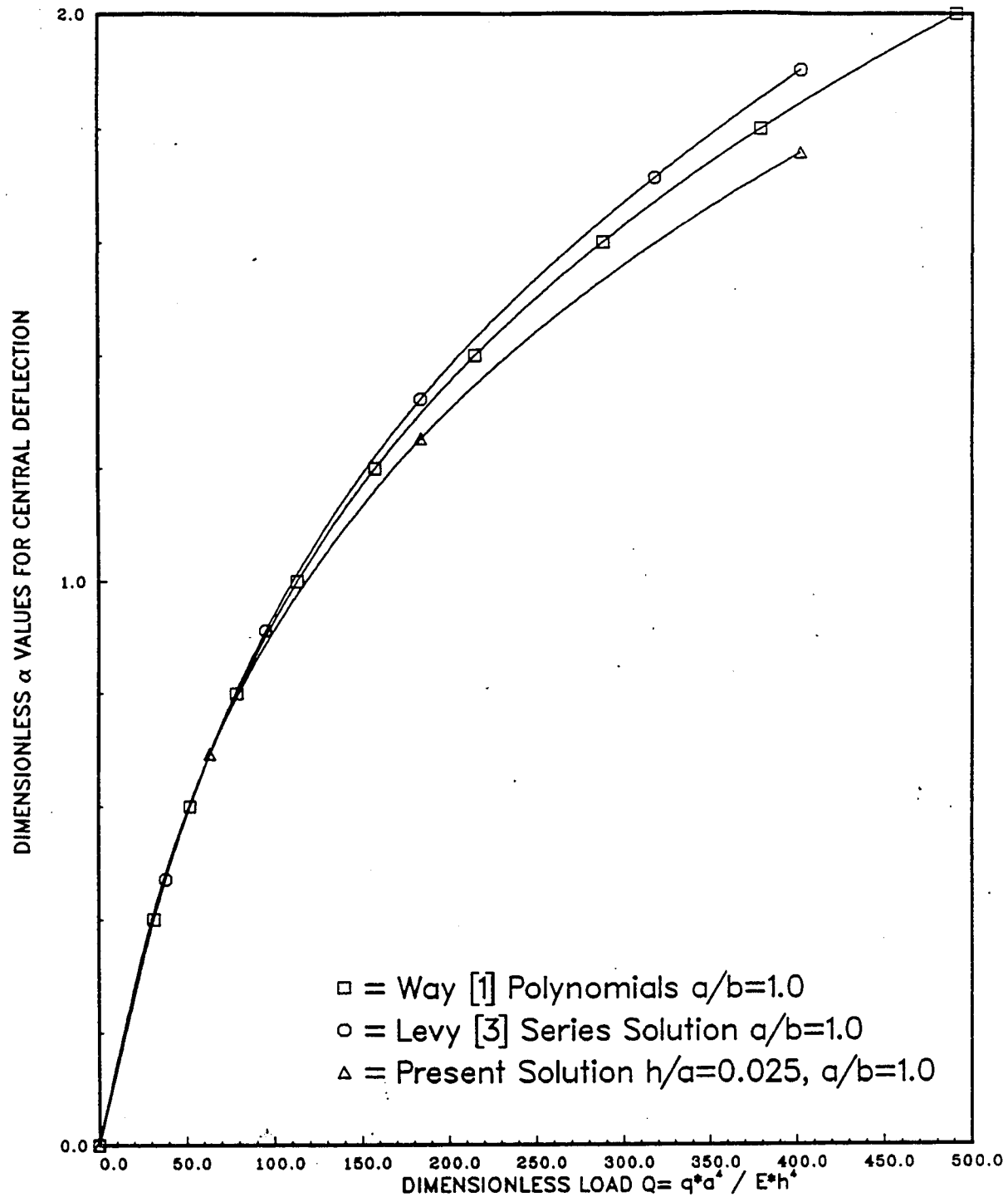


FIGURE 6.39, COMPARISON OF THE NONLINEAR DEFLECTIONS OF SQUARE CLAMPED THIN PLATES. $a/b=1.0$

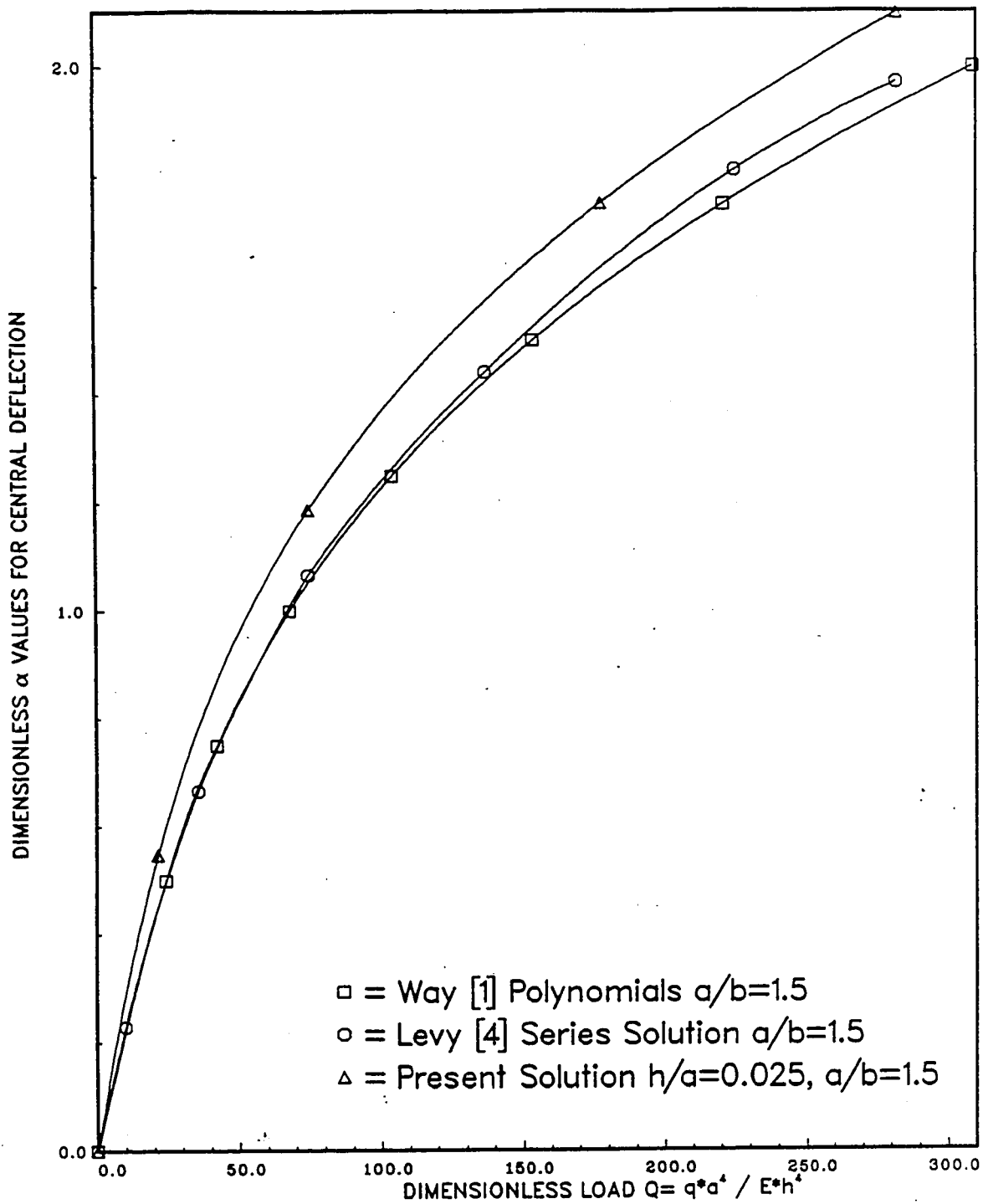


FIGURE 6.40, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULAR CLAMPED THIN PLATES. $a/b=1.5$

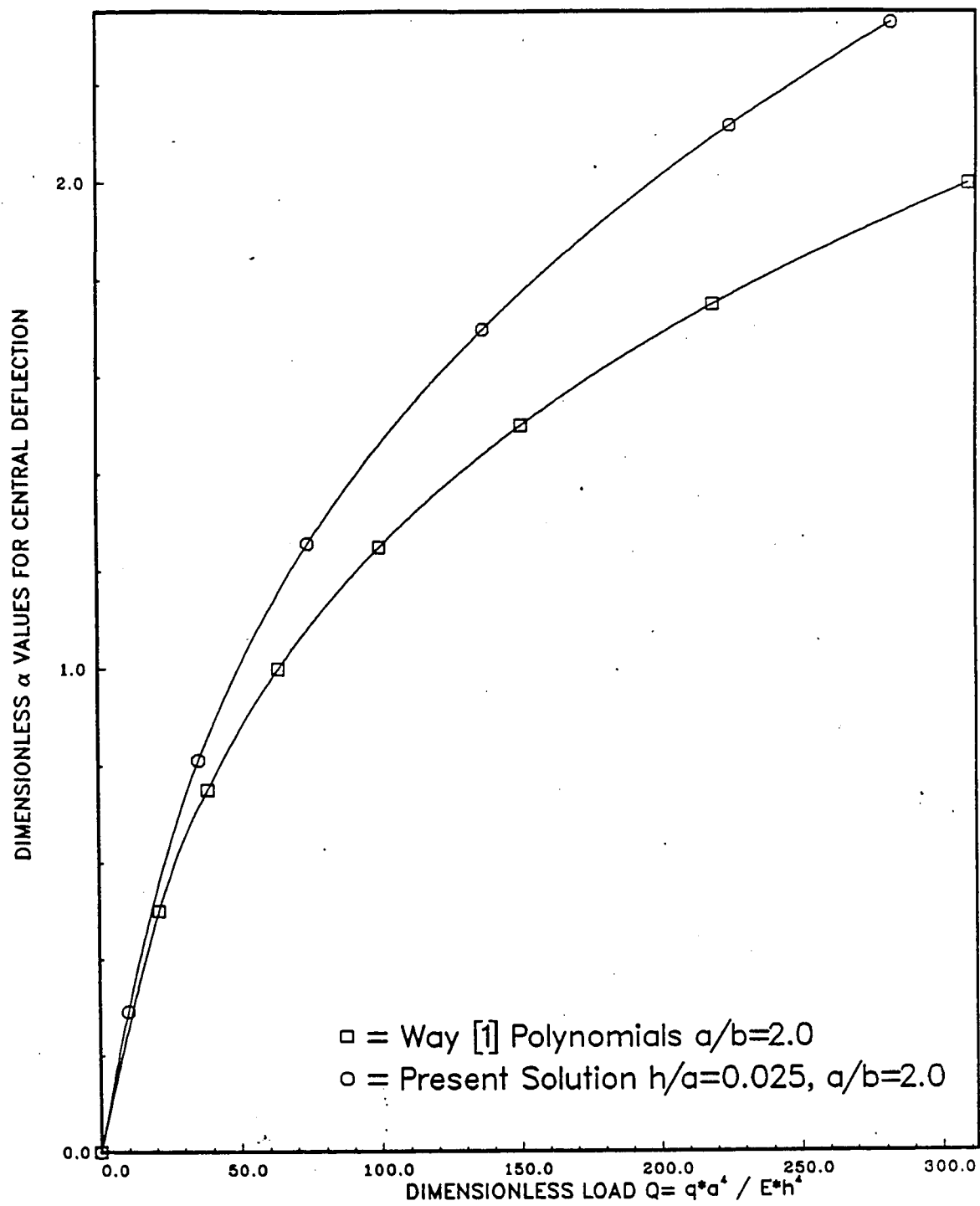


FIGURE 6.41, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULAR CLAMPED THIN PLATES. $a/b=2.0$

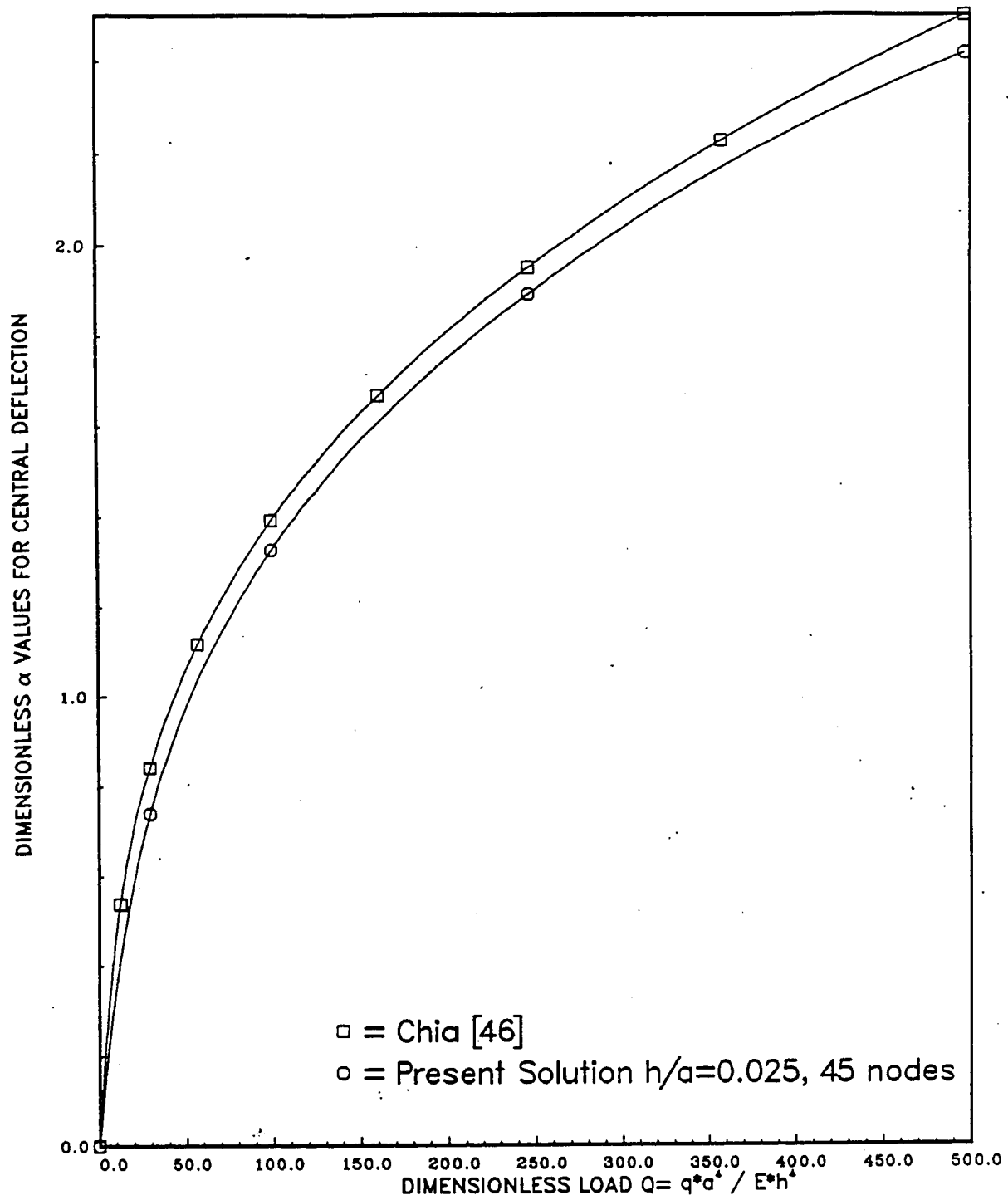


FIGURE 6.42, COMPARISON OF THE NONLINEAR DEFLECTIONS OF SQUARE SIMPLY SUP. THIN PLATES. $a/b=1.0$

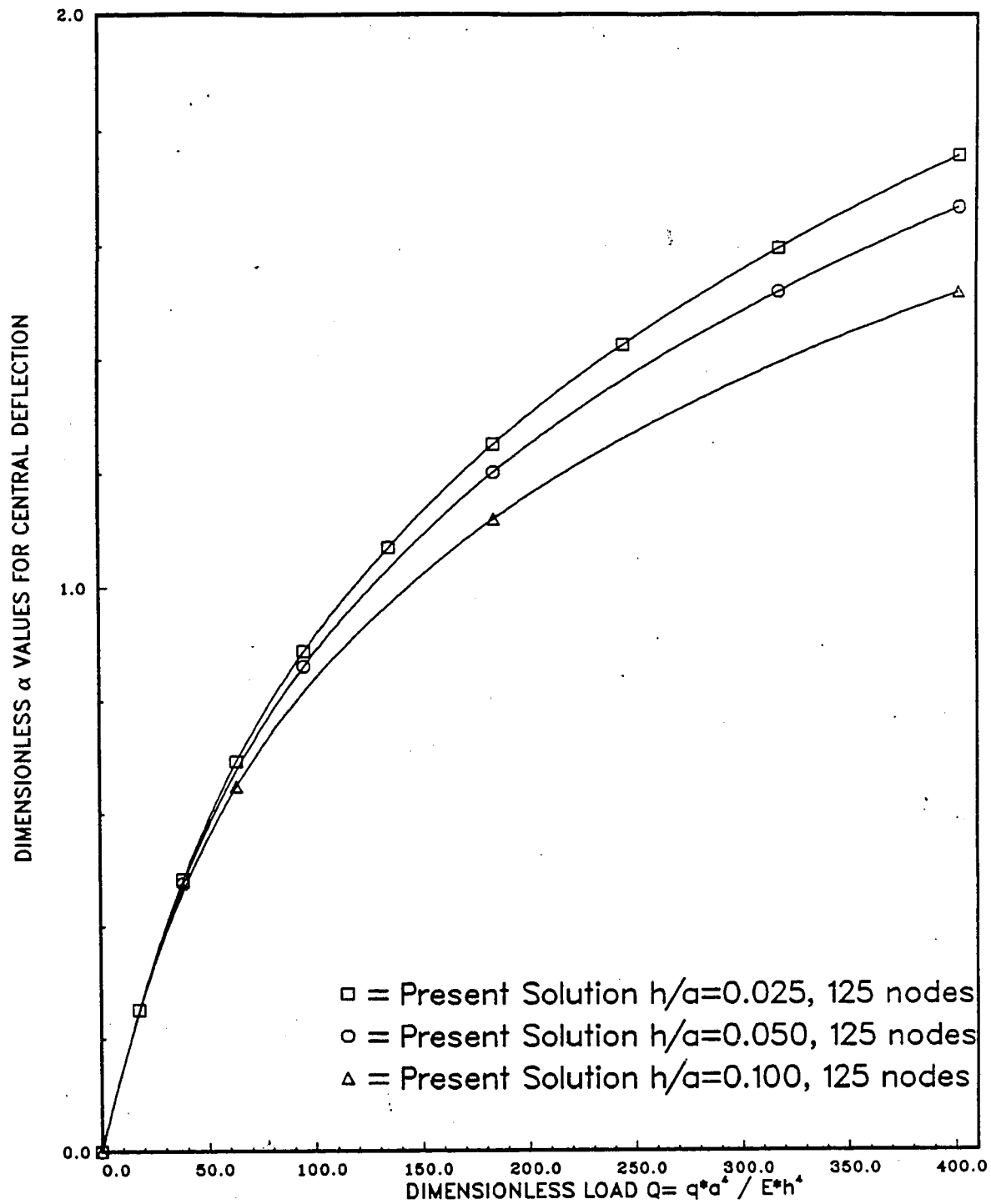


FIGURE 6.43, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE CLAMPED PLATES. $a/b=1.0$

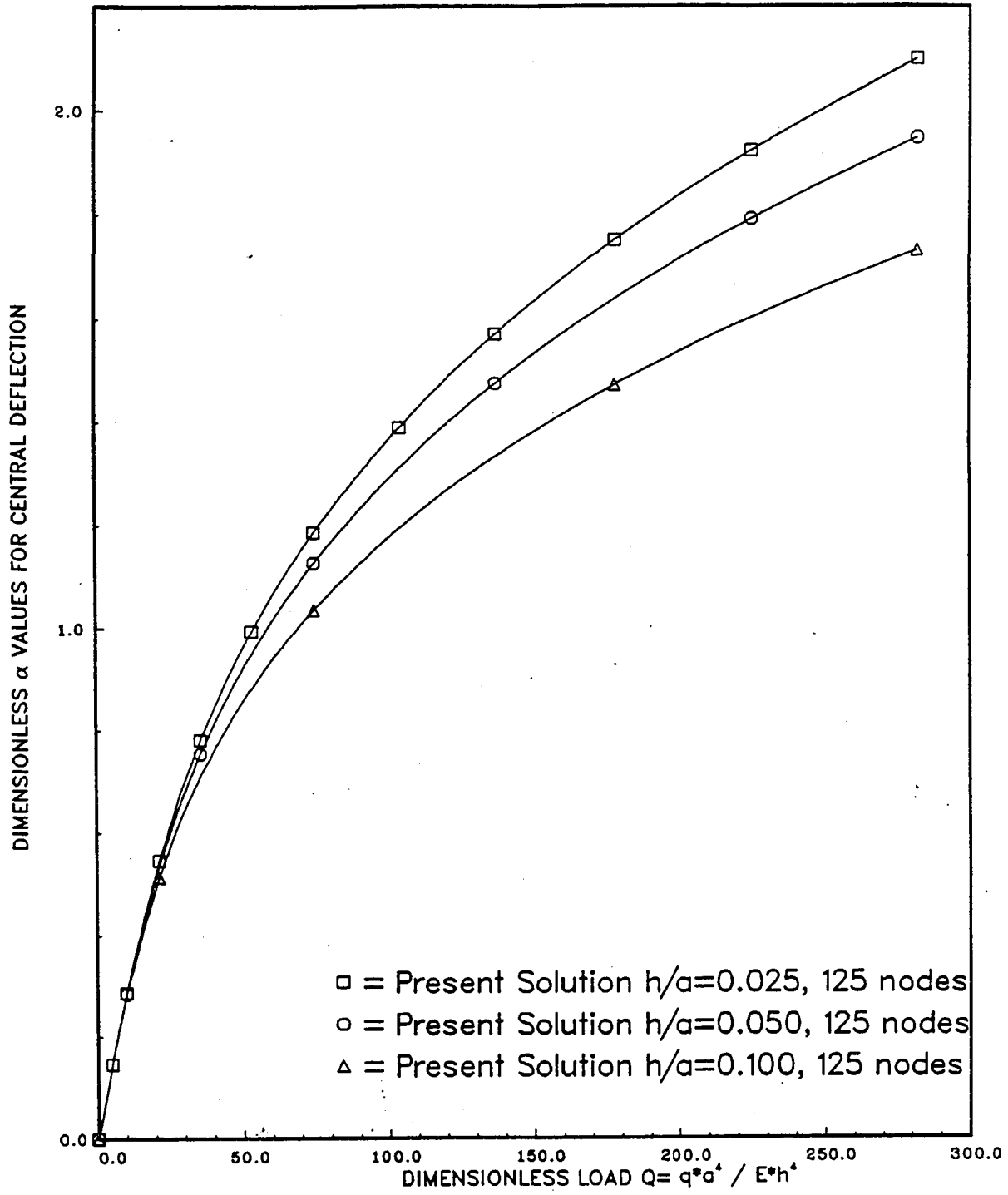


FIGURE 6.44, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE CLAMPED PLATES. $a/b=1.5$

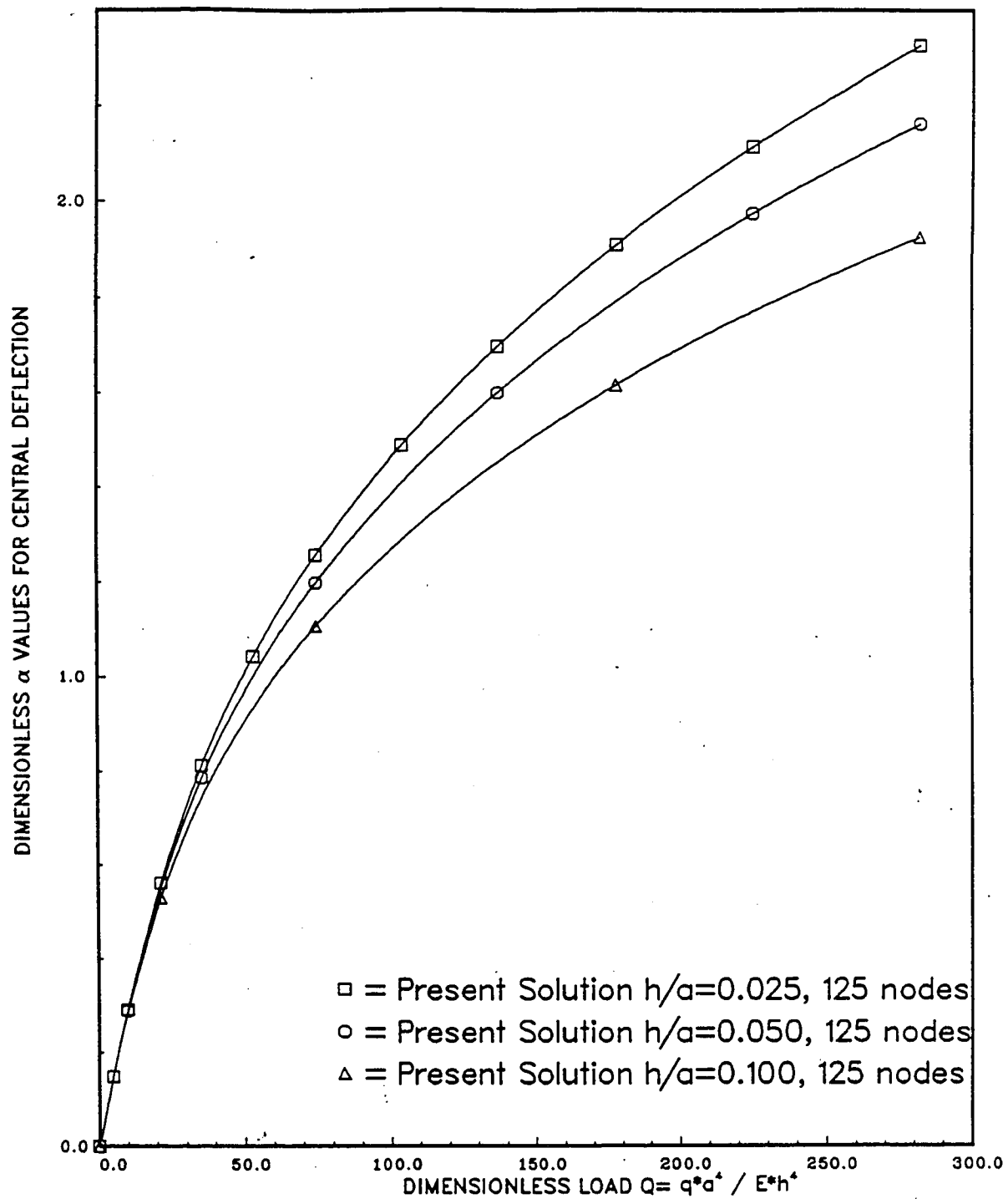


FIGURE 6.45, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE CLAMPED PLATES. $a/b=2.0$.

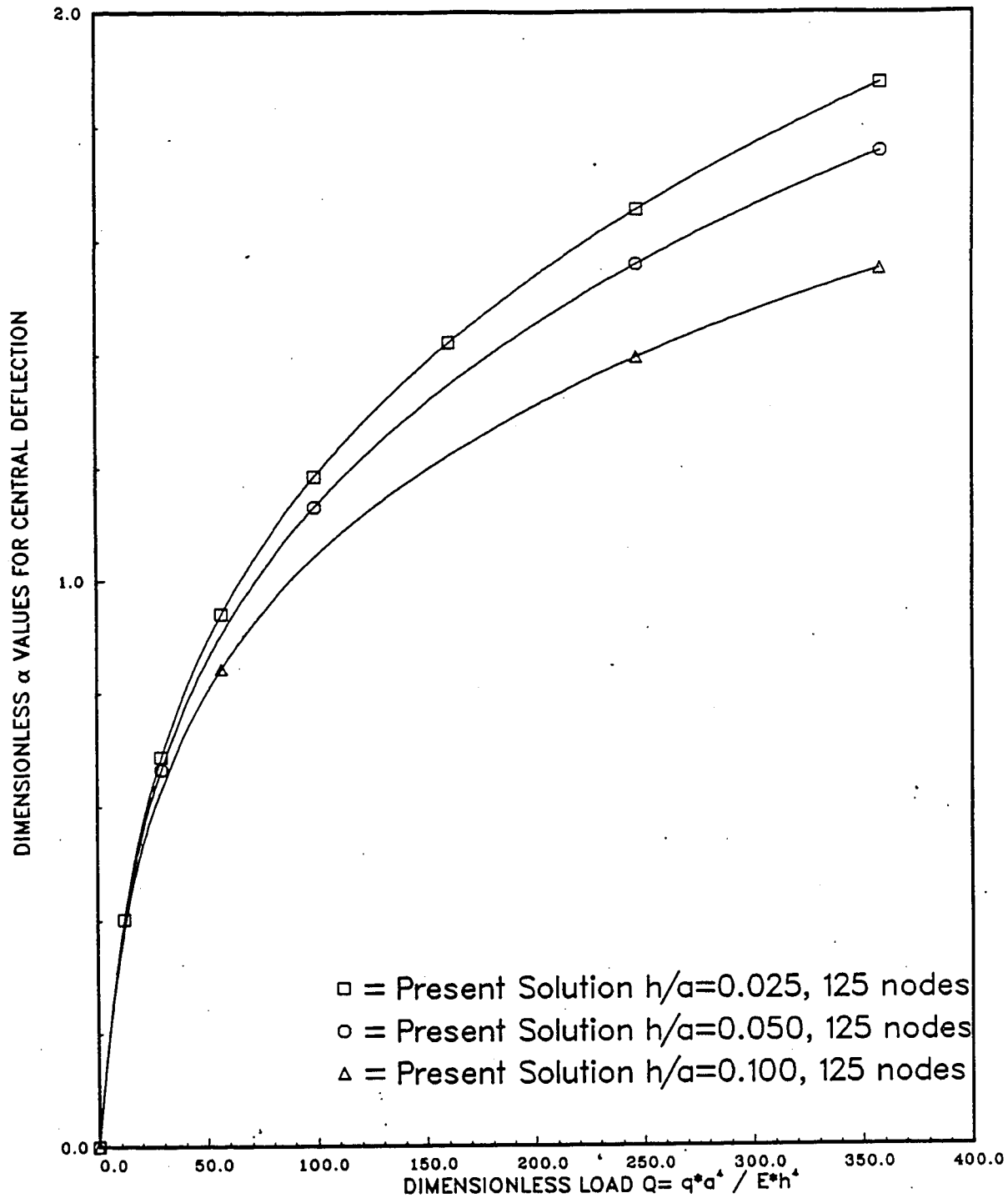


FIGURE 6.46, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE SIMPLY SUPPORTED PLATES. $a/b=1.0$

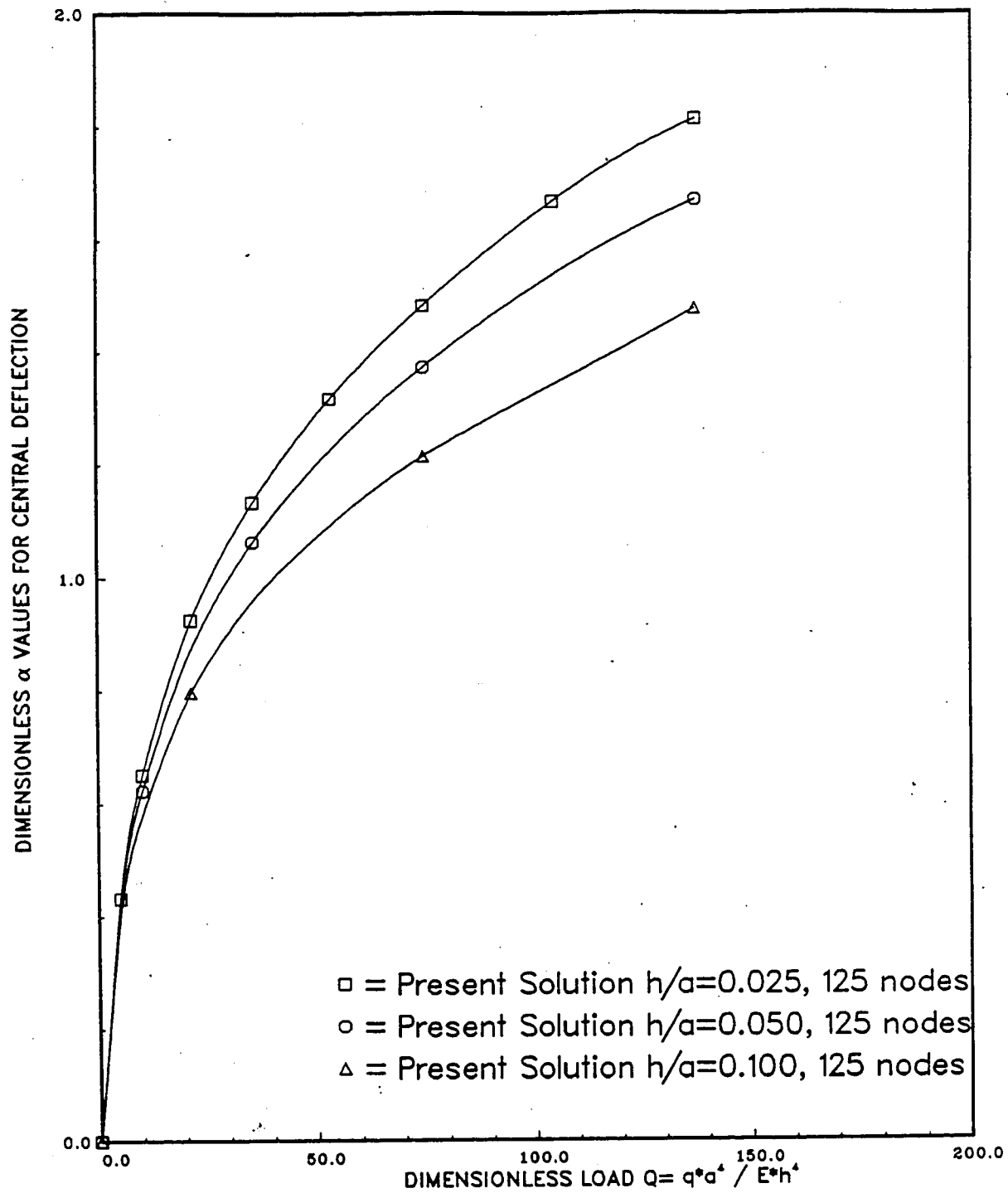


FIGURE 6.47, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE SIMPLY SUPPORTED PLATES. $a/b=1.5$

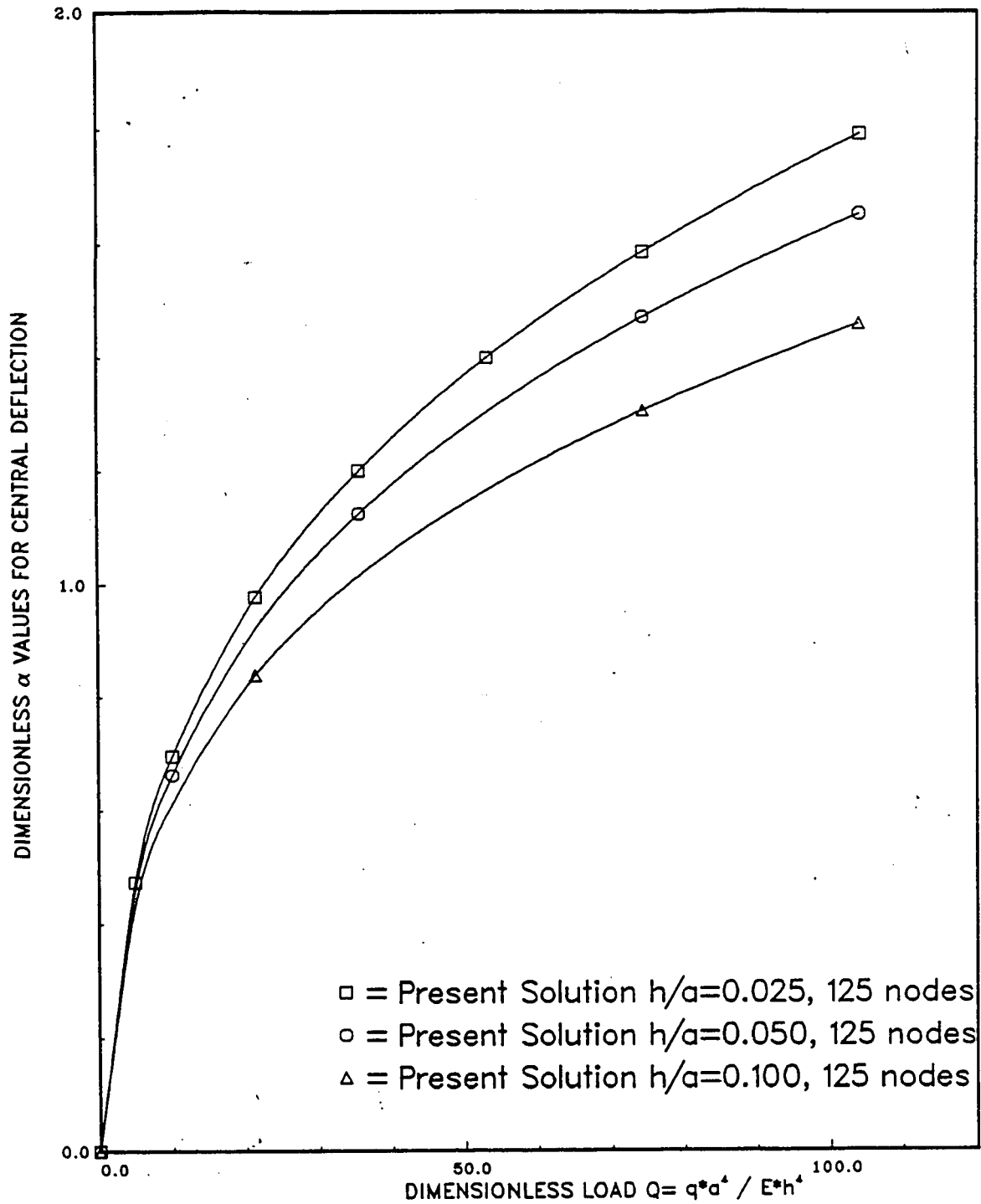


FIGURE 6.48, COMPARISON OF THE NONLINEAR DEFLECTIONS OF RECTANGULARE SIMPLY SUPPORTED PLATES. $a/b=2.0$

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

Three dimensional nonlinear analysis of thick plates was formulated using finite difference techniques. To use the finite difference modelling, the plate was divided in 5 layers, with the top and bottom layers satisfying the thick plate equilibrium equations at the boundaries.

The thick plate analysis considered here is quiet different from the Reissner's and Mindlin's theories. In this analysis the general three dimensional equations of equilibrium are involved, whereas in Reissner and Mindlin theories the vertical shear is considered, but the analysis conducted is a two

dimensional analysis. Two finite difference techniques were used, namely, forward finite difference and central finite difference to support the analysis. Both those techniques give excellent convergence pattern and acceptable results. It can be concluded that the finite difference technique as adopted here represents quite well the thick plate equilibrium equations. No results for similar nonlinear analysis of thick plates are available in the technical literature to confirm the validity of the solutions. However, for comparison purposes, the large deflections differential equations for thick plates were simplified to solve small deflection problem of thick plates and to analyse the large deflection problem of thin plates. Limited results are available by other researchers in the area of small deflection of thick plates and large analysis of thin plates. All comparisons of results indicate that the present finite difference modelling yields good agreement with other investigators. As a result of the present research, the following main conclusions can be drawn:

- (1)- A three dimensional analysis of thick plates was formulated as three very complex coupled partial differential equations. These equations are solved by finite difference modelling for both small and large deflections.
- (2)- By dividing the thickness of the plate into layers and by adopting a new finite difference modelling technique, the problem of small and large deflection of thick plates was successfully completed.
- (3)- A computer program using forward and central finite difference techniques included in the appendix creates the general matrix, the linear and nonlinear force vectors from relatively simple input. Each node is identified with a function label. The submatrices are formed automatically in

the general matrix to save time and space.

(4)- Analytical nonlinear results compared with large deflection of thin plate theory show excellent agreement with earlier investigators.

(5)- Linear analysis of thick plates show excellent convergence and for square thick plates, results are in excellent agreement with previous results available in the technical literature.

(6)- For nonlinear analysis results of thick plates, no data is yet available in the literature and hence no comparison can be made. However, convergence and comparison of limited data seem to indicate the results are of acceptable accuracy for engineering purposes.

7.2 Recommendations

Further works can be recommended for research based on this study.

(1)- Application of improved finite difference methods involving more accurate approximations to the derivatives.

(2)- The function labels can be modified to make them applicable to orthotropic and nonhomogeneous materials.

(3)- Due to the absence of experimental sources to guarantee the accuracy and practicability of the results, an experimental analysis of such plates from the convergence and stability point of view should be useful.

(4)- The analysis could be extended for the analysis of pre- and post-buckling of thick plates, vibration, and thick plates with variable stiffness.

(5)- The computer programme developed here was mainly aimed to solve

particular problems. It is more general than needed for the problems solved here. Programme although used only for rectangular thick plate with simply supported and clamped boundary conditions, input can be adjusted to solve thick plates of other plan forms and other boundary conditions.

(6)- An analysis using finite element method in three dimensional analysis applied to a simple cantilever [64] can be extended to thick plates for the analysis of linear and nonlinear deflection.

The boundary integral method might also be considered. Considering the same case and comparing numerical results on convergence, accuracy, would be of interest.

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Appendix A

Numerical Examples

A.1 Examples

A.1.1 First iterative method

Let us consider the 3 by 3 system of equations:

$$[A]\{X\} = \{B\} \quad (\text{A.1})$$

$$\begin{bmatrix} 20 & -16 & 2 \\ -16 & 24 & -8 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For the application of the iterative methods, this system need to be transformed.

First row:

$$20 - 16 + 2 = 6 \quad (\text{A.2})$$

Second row:

$$-16 + 24 - 8 = 0 \quad (\text{A.3})$$

The sum is zero, and by adding the first row to the second row will get:

$$4 + 8 - 6 = 6 \quad (\text{A.4})$$

Third row:

$$-8 - 32 + 20 = -4 \quad (\text{A.5})$$

The transformed system will be:

$$\begin{bmatrix} \frac{20}{6} & \frac{-16}{6} & \frac{2}{6} \\ \frac{4}{6} & \frac{8}{6} & \frac{-6}{6} \\ \frac{8}{-4} & \frac{-32}{-4} & \frac{20}{-4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} \\ \frac{2}{6} \\ \frac{1}{-4} \end{bmatrix}$$
$$\begin{bmatrix} 3.33333 & -2.66667 & 0.33333 \\ 0.66667 & 1.33333 & -1.00000 \\ -2.00000 & 8.00000 & -5.00000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.16667 \\ 0.33333 \\ -0.25000 \end{bmatrix}$$
$$[\bar{A}]\{X\} = \{\bar{B}\} \quad (\text{A.6})$$

The initial guess is: $X^0 = \bar{B}$

$$\begin{bmatrix} 3.33333 & -2.66667 & 0.33333 \\ 0.66667 & 1.33333 & -1.00000 \\ -2.00000 & 8.00000 & -5.00000 \end{bmatrix} \begin{bmatrix} 0.16667 \\ 0.33333 \\ -0.25000 \end{bmatrix} = \begin{bmatrix} -0.41667 \\ 0.80556 \\ 3.58333 \end{bmatrix}$$

The first iterative vector is:

$$\begin{bmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{bmatrix} = \begin{bmatrix} (0.16667 - (-0.41667))/3.33333 \\ (0.33333 - (0.80556))/1.33333 \\ (-0.25000 - (3.58333))/-5.00000 \end{bmatrix} + \begin{bmatrix} 0.16667 \\ 0.33333 \\ -0.25000 \end{bmatrix} = \begin{bmatrix} 0.34167 \\ -0.02083 \\ 0.51667 \end{bmatrix}$$

The second iterative vector is:

$$\begin{bmatrix} 3.33333 & -2.66667 & 0.33333 \\ 0.66667 & 1.33333 & -1.00000 \\ -2.00000 & 8.00000 & -5.00000 \end{bmatrix} \begin{bmatrix} 0.34167 \\ -0.02083 \\ 0.51667 \end{bmatrix} = \begin{bmatrix} 1.36667 \\ -0.31667 \\ -3.43333 \end{bmatrix}$$

$$\begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix} = \begin{bmatrix} (0.16667 - (1.36667))/3.33333 \\ (0.33333 - (-0.31667))/1.33333 \\ (-0.25000 - (-3.43333))/-5.00000 \end{bmatrix} + \begin{bmatrix} 0.34167 \\ -0.02083 \\ 0.51667 \end{bmatrix} = \begin{bmatrix} -0.01833 \\ 0.46667 \\ -0.12000 \end{bmatrix}$$

From this stage we see that $\frac{B^1+B^2}{2} \neq \bar{B}$ is not satisfied then we continue our iteration till this relation is verified. For this particular problem, the above relation is satisfied at the 58th iteration. And the X and B vectors are:

$$\{X^{57}\} = \begin{bmatrix} 0.74170 \\ 0.55517 \\ 1.42091 \end{bmatrix}, \{B^{57}\} = \begin{bmatrix} -1.13222 \\ 0.85287 \\ 3.64666 \end{bmatrix}$$

$$\{X^{58}\} = \begin{bmatrix} 0.35204 \\ 0.94483 \\ 0.64159 \end{bmatrix}, \{B^{58}\} = \begin{bmatrix} 1.46555 \\ -0.18621 \\ -4.14666 \end{bmatrix}$$

Then, if

$$\left\{ \frac{B^{57} + B^{58}}{2} \right\} = \begin{bmatrix} 0.16667 \\ 0.33333 \\ -0.25000 \end{bmatrix}$$

So the solution vector is:

$$\left\{ \frac{X^{57} + X^{58}}{2} \right\} = \begin{bmatrix} 0.54687 \\ 0.75000 \\ 1.03125 \end{bmatrix}$$

A.1.2 Second iterative method

As mentioned before, this method transforms the original system as shown in the first method. Then the transformed system is:

$$[\bar{A}]\{X\} = \{\bar{B}\} \quad (\text{A.7})$$

The first initial guess is:

$$\{X^{11}\} = \begin{bmatrix} 0.16667 \\ 0.16667 \\ 0.16667 \end{bmatrix}$$

And this vector will satisfy the first equation.

$$0.16667(3.33333 - 2.66667 + 0.33333) = 0.16667 \quad (\text{A.8})$$

and for the second equation,

$$0.16667(0.66667 + 1.33333 - 1.00000) \neq 0.33333 \quad (\text{A.9})$$

The vector does not satisfy it, then we correct this first vector,

$$\{X^{12}\} = \begin{bmatrix} 0.16667 \\ 0.16667 - \frac{0.16667 - 0.33333}{1.33333} \\ 0.16667 \end{bmatrix} = \begin{bmatrix} 0.16667 \\ 0.29167 \\ 0.16667 \end{bmatrix}$$

If we apply this vector, the second equation is satisfied.

Proceeding to the third equation,

$$0.16667(-2.00000) + 0.29167(8.00000) - 5.00000(0.16667) \neq -0.25000 \quad (\text{A.10})$$

as a result, we need to correct this vector to satisfy the third equation.

$$\{X^{13}\} = \begin{bmatrix} 0.16667 \\ 0.29167 \\ 0.16667 - \frac{0.16667 + 0.33333}{-5.00000} \end{bmatrix} = \begin{bmatrix} 0.16667 \\ 0.29167 \\ 0.45000 \end{bmatrix}$$

At this stage the first iteration is finished and we have to go back to the first equation and repeat the same procedures using this last vector.

By using this new approach we need only 19 iteration to solve this particular example.

$$\{X^{19}\} = \begin{bmatrix} 0.54687 \\ 0.75000 \\ 1.03125 \end{bmatrix}$$

A.1.3 First direct method

Let us consider the same system,

$$\begin{bmatrix} 20 & -16 & 2 \\ -16 & 24 & -8 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

we want to transform this system to a lower triangular system and solve it by forward substitution.

First we eliminate the element above the diagonal element a_{33} . We start with $a_{13} = 2$. The corresponding vector to eliminate this element is:

$$\{V\} = [-6.0 \quad 1.0 \quad 1.0]$$

Premultiplying the system by the vector V will give a new equation.

$$-128x_1 + 88x_2 + 0x_3 = -4 \quad (\text{A.11})$$

And the new system will look like:

$$\begin{bmatrix} -128 & 88 & 0 \\ -16 & 24 & -8 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$$

Proceeding to the element $a_{23} = -8$, the corresponding vector is:

$$\{V\} = [1.0 \quad 2.5 \quad 1.0]$$

Premultiplying the system by the vector V will give the new equation.

$$-160x_1 + 116x_2 + 0x_3 = -0.5 \quad (\text{A.12})$$

resulting in the new system:

$$\begin{bmatrix} -128 & 88 & 0 \\ -160 & 116 & 0 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -0.5 \\ 1 \end{bmatrix}$$

Proceeding to the last term $a_{12} = 88$, and the corresponding vector is:

$$\{V\} = \left[-\frac{116}{88} \quad 1.0 \quad 0.0 \right]$$

Premultiplying the system by the vector V will give the new equation.

$$8.72727x_1 + 0x_2 + 0x_3 = 4.77273 \quad (\text{A.13})$$

And the final system is:

$$\begin{bmatrix} 8.72727 & 0 & 0 \\ -160 & 116 & 0 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4.77273 \\ -0.5 \\ 1 \end{bmatrix}$$

By forward substitution we determine the X vector.

$$x_1 = \frac{4.77273}{8.72727} = 0.54688 \quad (\text{A.14})$$

$$x_2 = \frac{-0.5 - (-160 \times 0.54688)}{116} = 0.75001 \quad (\text{A.15})$$

$$x_3 = \frac{1.0 - (8 \times 0.54688) - (-32.0 \times 0.75001)}{20} = 1.03126 \quad (\text{A.16})$$

which is the solution of the system.

A.1.4 Second direct method .

Let us consider the same 3 by 3 system of equation:

$$\begin{bmatrix} 20 & -16 & 2 \\ -16 & 24 & -8 \\ 8 & -32 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

We start with the first unknown, we put x_1 equal to:

$$x_1 = y_1 - \frac{-16}{20}x_2 - \frac{2}{20}x_3 \quad (\text{A.17})$$

substituting it into the three equations,

the first equation becomes:

$$20(y_1 - \frac{-16}{20}x_2 - \frac{2}{20}x_3) - 16x_2 + 2x_3 = 1 \quad (\text{A.18})$$

$$20y_1 = 1 \quad (\text{A.19})$$

the second equation becomes:

$$-16(y_1 - \frac{-16}{20}x_2 - \frac{2}{20}x_3) + 24x_2 - 8x_3 = 1 \quad (\text{A.20})$$

$$-16y_1 + 11.2x_2 - 6.4x_3 = 1 \quad (\text{A.21})$$

From this expression we put the value of x_2 equal to:

$$x_2 = -\frac{-16}{11.2}y_1 + y_2 - \frac{-6.4}{11.2}x_3 \quad (\text{A.22})$$

substituting it in the previous equation will give:

$$-16y_1 + 11.2(-\frac{-16}{11.2}y_1 + y_2 - \frac{-6.4}{11.2}x_3 - 6.4x_3) = 1 \quad (\text{A.23})$$

$$11.2y_2 = 1 \quad (\text{A.24})$$

and after substituting the value of x_1 and then x_2 The third equation becomes: third equation respectively.

$$8(y_1 - \frac{-16}{20}x_2 - \frac{2}{20}x_3) - 32x_2 + 20x_3 = 1 \quad (\text{A.25})$$

$$8y_1 - 25.6x_2 + 19.2x_3 = 1 \quad (\text{A.26})$$

then,

$$8y_1 - 25.6(-\frac{-16}{11.2}y_1 + y_2 - \frac{-6.4}{11.2}x_3) + 19.2x_3 = 1 \quad (\text{A.27})$$

$$-28.57143y_1 - 25.6y_2 + 4.57143x_3 = 1 \quad (\text{A.28})$$

From this we put the value of x_3 equal to:

$$x_3 = -\frac{-28.57143}{4.57143}y_1 - \frac{-25.6}{4.57143}y_2 + y_3 \quad (\text{A.29})$$

After we substitute it in the previous equation we get:

$$4.57143y_3 = 1 \quad (\text{A.30})$$

we have now formed two systems of equations the first system from (1.59), (1.64), and (1.70),

$$\begin{bmatrix} 20 & 0 & 0 \\ 0 & 11.2 & 0 \\ 0 & 0 & 5.37143 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and the second system from (1.57), (1.62), and (1.69)

$$x_1 = y_1 - \frac{-16}{20}x_2 - \frac{2}{20}x_3 \quad (\text{A.31})$$

$$x_2 = -\frac{-16}{11.2}y_1 + y_2 - \frac{-6.4}{11.2}x_3 \quad (\text{A.32})$$

$$x_3 = -\frac{-28.57143}{4.57143}y_1 - \frac{-25.6}{4.57143}y_2 + y_3 \quad (\text{A.33})$$

By replacing the x_2 expression in the x_1 expression,

$$x_1 = y_1 - \frac{-16}{20} \left(-\frac{-16}{11.2} y_1 + y_2 - \frac{-6.4}{11.2} x_3 \right) - \frac{2}{20} x_3 \quad (\text{A.34})$$

$$x_1 = 2.14286y_1 + 0.8y_2 + 0.35714x_3 \quad (\text{A.35})$$

And, by replacing the x_3 expression in the x_2 expression and, with the new x_1 expression.

$$x_2 = 1.42857y_1 + y_2 + 0.57143(6.25y_1 + 5.6y_2 + y_3) \quad (\text{A.36})$$

$$x_2 = 5.00001y_1 + 4.20001y_2 + 0.57143y_3 \quad (\text{A.37})$$

Then

$$x_1 = 2.14286y_1 + 0.8y_2 + 0.35714(6.25y_1 + 5.6y_2 + y_3) \quad (\text{A.38})$$

$$= 4.37499y_1 + 2.79998y_2 + 0.35714y_3 \quad (\text{A.39})$$

The system is now reduced to:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4.37499 & 2.79998 & 0.35714 \\ 5.00001 & 4.20001 & 0.57143 \\ 6.25 & 5.6 & 1.0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

By replacing the first system in the second system we get:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4.37499 & 2.79998 & 0.35714 \\ 5.00001 & 4.20001 & 0.57143 \\ 6.25 & 5.6 & 1.0 \end{bmatrix} \begin{bmatrix} 20 & 0 & 0 \\ 0 & 11.2 & 0 \\ 0 & 0 & 5.37143 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

or we can write;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4.37499 & 2.79998 & 0.35714 \\ 5.00001 & 4.20001 & 0.57143 \\ 6.25 & 5.6 & 1.0 \end{bmatrix} \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ 0 & \frac{1}{11.2} & 0 \\ 0 & 0 & \frac{1}{5.37143} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Thus the final system is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.21875 & 0.25000 & 0.07812 \\ 0.25000 & 0.37500 & 0.12500 \\ 0.31250 & 0.50000 & 0.21875 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

This system gives the exact solution of the problem the 3 by 3 matrix. The solution from this system is therefore:

$$x_1 = 0.54687$$

$$x_2 = 0.75000$$

$$x_3 = 1.03125$$

Appendix B

Program Elements

Two programs are written for the three dimensional finite difference techniques described in this thesis. The two programs are used to compute the linear and nonlinear deflection of clamped and simply supported thick plates.

The program DIFF9 FORTRAN considers forward finite difference.

The program DIFF91 FORTRAN considers central finite difference.

For the two programs, the submatrices described in chapter IV are formulated automatically in the general matrix AA to save the storage.

Nine subroutines are involved in each program, namely:

SUBROUTINE FORDIF OR SUBROUTINE CENDIF

SUBROUTINE GENR

SUBROUTINE SCHEM

SUBROUTINE BOUNDA

SUBROUTINE SOLINV

SUBROUTINE MATMUL

SUBROUTINE FORCE

SUBROUTINE BOUNDF

SUBROUTINE NOLINE

In these programs, double precision arithmetic is used for all the calculations.

B.1 Description of Input Data

1-Free format card to define, the number of points to generate the scheme (N), the increment number (INC), the relaxation factor (GAMA), the number of iteration maximum (IMAX), the type of boundary condition used (KD).

2-Master Control Card (I5,F10.4,F5.3,F10.5,2F5.3,2I5,F10.5,I5)

One Card to define the number of nodes, the modulus of elasticity, Poisson's ratio, the principal length, the ratio a/b, the ratio h/a, number of mesh sizes in x,y directions, number of mesh sizes in z direction, the convergence ratio, number of loads used.

Columns 1-5 : number of nodes in the plate [NOD].

6-15 : Modulus of elasticity (E).

16-20 : Poisson's ratio (P).

21-30 : The principal length (XA).

31-35 : The ratio a/b (AB).

36-40 : The ratio h/a (AH).

41-45 : Number of mesh sizes in the x and y direction (NTX).

46-50 : Number of mesh sizes in z direction (NTZ).

51-60 : Convergence ratio (DELTA).

61-65 : Number of loads (NIL).

3-Element Data (F5.2,4I5,3I5).

One Card for each node in order.

Columns 1-5 : Load factor [FF(NOD,1)].

FF(NOD,1)=1.0 loaded.

FF(NOD,1)=0.0 not loaded.

5-10 : Incrementation in the Y direction (IJKL(I,1)).

11-15 : Incrementation in the plan XZ (IJKL(I,2)).

16-20 : Incrementation in the plan YZ (IJKL(I,3)).

21-25 : Label of the functions related to each node (IJKL(I,4)).

Nodes relationship for the boundary layers [NP(13)].

26-30 : Row position of the node to generate the scheme (IM).

31-35 : Column position to generate the scheme (JM).

36-40 : To generate the appropriate boundary conditions,

ITEST=0 Clamped plate

ITEST=1 Simply Supported plate

For clamped plate the points beyond the edges are affected with a sign (+).

For simply supported plate the points beyond the edges are affected with a sign (-).

Appendix C

Input-Output

**C.1 Input-Output: Clamped Plate 20 nodes,
Using DIFF9**

**C.2 Input-Output: Simply supp. Plate 45
nodes, Using DIFF91**

NUMBER OF ITERATIONS = 105

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.38300E+02 0.41371E+03 0.74075E+00 0.64456E+00

NUMBER OF ITERATIONS = 126

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.63400E+02 0.68483E+03 0.12262E+01 0.92815E+00

NUMBER OF ITERATIONS = 112

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.95000E+02 0.10262E+04 0.18374E+01 0.11907E+01

NUMBER OF ITERATIONS = 104

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.13400E+03 0.14474E+04 0.25917E+01 0.14347E+01

NUMBER OF ITERATIONS = 100

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.18400E+03 0.19875E+04 0.35587E+01 0.16765E+01

NUMBER OF ITERATIONS = 97

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.24500E+03 0.26464E+04 0.47385E+01 0.19103E+01

NUMBER OF ITERATIONS = 101

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.31800E+03 0.34349E+04 0.61504E+01 0.21384E+01

NUMBER OF ITERATIONS = 110

(Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.40200E+03 0.43423E+04 0.77750E+01 0.23580E+01

FILE: DIFF9 RESULT *

VM/SP CONVERSATIONAL MONITOR SYSTEM

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NUMBER OF ITERATIONS = 108

FILE: DIFF2 DATA *

VM/SP CONVERSATIONAL MONITOR SYSTEM

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1.00	3	-9	-9	107	5	4	1
1.00	3	-9	-9	108	5	5	1

SIMPLY SUPPORTED ISOTROPIC THICK RECTANGULAR PLATE

TOP LAYER SCHEME GENERATED BY NA MATRIX

0	0	-1	-2	-3	-2	-1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
-1	0	1	2	3	2	1	0	0	0	0
-4	0	4	5	6	5	4	0	0	0	0
-7	0	7	8	9	8	7	0	0	0	0
-4	0	4	5	6	5	4	0	0	0	0
-1	0	1	2	3	2	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

NUMBER OF NODES = 45
 YOUNG MODULOUS = 1.000000
 POISSON S RATIO = 0.316
 LONGITUDINAL LENGTH = 1.000000
 RATIO A/B = 1.000
 RATIO H/A = 0.025
 MESH NUMBER ON X, Y = 6
 MESH NUMBER ON Z = 4
 MESH SIZE ON X AXIS = 0.16667
 MESH SIZE ON Y AXIS = 0.16667
 MESH SIZE ON Z AXIS = 0.00625
 DELTA = 0.99999
 GAMMA = 0.05000
 (0*A**4)/(E*H**4) (0*A**4)/(D*H) LINEAR W

0.12100E+02 0.13070E+03 0.48840E+00
 4.0000E+00

NUMBER OF ITERATIONS = 218
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.29400E+02 0.31757E+03 0.11867E+01 0.73428E+00

NUMBER OF ITERATIONS = 218
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.56900E+02 0.61462E+03 0.22967E+01 0.10285E+01

NUMBER OF ITERATIONS = 215
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.99400E+02 0.10737E+04 0.40122E+01 0.13131E+01

NUMBER OF ITERATIONS = 225
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.16106E+03 0.17391E+04 0.64986E+01 0.15926E+01

NUMBER OF ITERATIONS = 301
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.24700E+03 0.26680E+04 0.99699E+01 0.18700E+01

NUMBER OF ITERATIONS = 325
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.35800E+03 0.38670E+04 0.14450E+02 0.21420E+01

NUMBER OF ITERATIONS = 367
 (Q*A**4)/(E*H**4) (Q*A**4)/(D*H) LINEAR W NONLINEAR W
 0.49700E+03 0.53685E+04 0.20061E+02 0.23855E+01

NUMBER OF ITERATIONS = 399

Appendix D

Computer Programmes

D.1 DIFF9 FORTRAN

D.2 DIFF91 FORTRAN


```

CCC READ(S,*) (VAL(IL), IL=1,NIL)
CCC AB=B/A B>A AB>1.0 A=XA=1.0
CCC HA=H/A H<A HA<1.0
CCC TX=XA/NTX TV=TX/AB TZ=(XA*HA)/NTZ
CCC TX=XA/NTX
    TV=(XA*AB)/NTX
    TZ=(XA*HA)/NTZ
WRITE(6,1016)NOD,E,P,XA,AB,HA,NTX,NTZ, TX,TV,TZ,DELTA,GAMA
B=(E*P)/((1.+P)*(1.-2.*P))
G=E/(2.*(1.+P))
G1=B+2.*G
G2=B+G
G3=B
G11=G1
G22=G2
G11=E/(1.-P**2)
G22=E/(2.0*(1.-P))
D=(E*(XA*HA)**3)/(12.*(1.-P**2))
TV=TV*AB
TZ=TZ*HA
HX=HA

```

```

C R(1)=(1.0/TV**4)*(1./HX)
R(2)=(2.0/((TX**2)*(TV**2)))*(1./HX)
R(3)=-((-4.0/TV**2)*((1.0/TV**2)+(1.0/TV**2)))*(1./HX)
R(4)=R(2)
R(5)=(1.0/TX**4)*(1./HX)
R(6)=-((-4.0/TX**2)*((1.0/TV**2)+(1.0/TV**2)))*(1./HX)
R(7)=-((6.0/TX**4)+(6.0/TV**4)+(8.0/((TX**2)*(TV**2)))*(1./HX)
R(8)=R(6)
R(9)=R(5)
R(10)=R(4)
R(11)=R(3)
R(12)=R(2)
R(13)=R(1)
DO 4 I=1,NOD/5
DO 4 K=1,3
UWV(I,K)=0
4 CONTINUE
DO 1 I=1,3*NOD
DO 1 J=1,3*NOD
AA(I,J)=0.0
1 CONTINUE

```

```

CCC UWV: MATRIX TO SATISFY THE SYMMETRY
CCC DO 3 I=1,NOD/5
CCC 3 READ(5,700)(UWV(I,K),K=1,3)
CCC 700 FORMAT(3I5)
CCC CALL FORDIF(AA,NA,NP,R,N,HA, TX,TV,TZ,NOD,G,G1,G2,IJKL)
CCC CALL BOUNDA(UWV,NOD,AA)

```

```

D1F00560
D1F00570
D1F00580
D1F00590
D1F00600
D1F00610
D1F00620
D1F00630
D1F00640
D1F00650
D1F00660
D1F00670
D1F00680
D1F00690
D1F00700
D1F00710
D1F00720
D1F00730
D1F00740
D1F00750
D1F00760
D1F00770
D1F00780
D1F00790
D1F00800
D1F00810
D1F00820
D1F00830
D1F00840
D1F00850
D1F00860
D1F00870
D1F00880
D1F00890
D1F00900
D1F00910
D1F00920
D1F00930
D1F00940
D1F00950
D1F00960
D1F00970
D1F00980
D1F00990
D1F01000
D1F01010
D1F01020
D1F01030
D1F01040
D1F01050
D1F01060
D1F01070
D1F01080
D1F01090
D1F01100

```

```

CCC      CALL SOLINV(AA,60)
C        IUNIT=1
        NIL=IUNIT
        DO 250 IL =IUNIT,NIL
        DO 13 I=1,3*NOD
        DIS(I,1)=0.0
        DISN(I,1)=0.0
        13 CONTINUE
        O=VAL(IL)
CCC      CALL FORCE(O,P,HA,NOD,E,FF,DFOR)
CCC      WRITE(6,1014) (I,DFOR(I,1),DFOR(I+NOD,1),DFOR(I+2*NOD,1),I=1,NOD)
CCC      CALL BOUNDF(UVW,NOD,DFOR)
CCC      CALL MATMUL(AA,60,60,DFOR,1,DIS)
        NL=NOD/5
        NN=2*NOD+3*NL
        DLINEA=DIS(NN,1)
C        PRINT 1013
C        WRITE(6,1014) (I,DIS(I,1),DIS(I+NOD,1),DIS(I+2*NOD,1),I=1,NOD)
CCCCCCC
CCCCCCC
CCCCCCC
        ICONT =0
        50 CALL NOLINE(DIS,IJKL,NOD,AB,XA,HA,E,O,P,B,G,NTX,NTZ,FOR)
        CALL BOUNDF(UVW,NOD,FOR)
C        WRITE(6,1014) (I,FOR(I,1),FOR(I+NOD,1),FOR(I+2*NOD,1),I=1,NOD)
CCCCCCC
CCCCCCC
        CALL MATMUL(AA,60,60,FOR,1,DISN)
CCC
C        PRINT 1013
C        WRITE(6,1014) (I,DISN(I,1),DISN(I+NOD,1),DISN(I+2*NOD,1),I=1,NOD)
C        WRITE(6,301) 0.12,0*(1,-P**2)+0.0,DIS(NN,1),DISN(NN,1)
C        IF(ICONT.EQ.IMAX) GO TO 251
        ICONT=ICONT+1
        DO 51 I=1,3*NOD
        IF(DABS(DISN(I,1))-DABS(DIS(I,1))) 52,51,53
        52 IF(DELTA-DABS(DISN(I,1)/DIS(I,1))) 51,54,54
        53 IF(DELTA-DABS(DIS(I,1)/DISN(I,1))) 51,54,54
        51 CONTINUE
        251 PRINT 804
        WRITE(6,301) 0.12,0*(1,-P**2)+0.0,DLINEA,DISN(NN,1)
        301 FORMAT(AE15.5,/)
        WRITE(6,803) ICONT
        803 FORMAT(/,5X,'NUMBER OF ITERATIONS =',15,/)
        GO TO 250
        54 DO 56 I=1,3*NOD
        TEMP=DIS(I,1)
        DIS(I,1)=GAMA*DISN(I,1)+(1.0-GAMA)*TEMP
        56 CONTINUE

```

DIF01110
DIF01120
DIF01130
DIF01140
DIF01150
DIF01160
DIF01170
DIF01180
DIF01190
DIF01200
DIF01210
DIF01220
DIF01230
DIF01240
DIF01250
DIF01260
DIF01270
DIF01280
DIF01290
DIF01300
DIF01310
DIF01320
DIF01330
DIF01340
DIF01350
DIF01360
DIF01370
DIF01380
DIF01390
DIF01400
DIF01410
DIF01420
DIF01430
DIF01440
DIF01450
DIF01460
DIF01470
DIF01480
DIF01490
DIF01500
DIF01510
DIF01520
DIF01530
DIF01540
DIF01550
DIF01560
DIF01570
DIF01580
DIF01590
DIF01600
DIF01610
DIF01620
DIF01630
DIF01640
DIF01650

```

C      GO TO 50
C      250 CONTINUE
C      PRINT 1003
C      WRITE(6,*)((AA(M,N),N=1,3*NOD),M=1,3*NOD)
C      PRINT 1003
C      PRINT 1013
C      WRITE(6,1014) (I,DISN(I,1),DISN(I+NOD,1),DISN(I+2*NOD,1),
C      &I=1,NOD)
C      PRINT 1013
C      WRITE(6,1014) (I,DIS(I,1),DIS(I+NOD,1),DIS(I+2*NOD,1),
C      &I=1,NOD)
C      801 FORMAT('SIMPLY SUPPORTED ISOTROPIC THICK RECTANGULAR PLATE',/)
C      802 FORMAT('CALMPED ISOTROPIC THICK RECTANGULAR PLATE',/)
C      804 FORMAT(' (Q*A**4)/(E*H**4) ',.2X, '(Q*A**4)/(D*H) ',.2X, 'LINEAR W',.6X,
C      &, 'NONLINEAR W',.7)
C      805 FORMAT(10X, ' TOP LAYER SCHEME GENERATED BY NA MATRIX',/)
C      900 FORMAT(15, F10.4, F5.3, F10.5, 2F5.3, 2I5, F10.5, I5)
C      1001 FORMAT(/, /, 2(IX,10E13,7, /))
C      1002 FORMAT(6(IX,10E13,4, /), /)
C      1003 FORMAT(/, 5X, 'MATRIX AA',)
C      1014 FORMAT(/, 5X, I3, 2X, 3(E12.4, 2X), /)
C      1013 FORMAT(/, 5X, 'NODE X-DIRECTION', ' Y-DIRECTION', ' Z-DIRECTION', /)
C      1016 FORMAT(9X, 'NUMBRE OF NODES', ' =', I5, /, 9X, 'YOUNG MODULOUS', ' =',
C      &F10.5, /, 9X, 'POISSON S RATIO', ' =', F5.3, /, 9X, 'LONGITUDINAL LENGTH', ' =',
C      &F10.5, /, 9X, 'RATIO A/B', ' =', F5.3, /, 9X, 'RATIO H/A', ' =',
C      &, 'F5.3, /, 9X, 'MESH NUMBER ON X, Y, Z', ' =', I5, /, 9X, 'MESH NUMBER ON X',
C      &, ' =', I5, /, 9X, 'MESH SIZE ON X AXIS', ' =', F10.5, /, 9X, 'MESH SIZE ON Y',
C      &AXIS', ' =', F10.5, /, 9X, 'MESH SIZE ON Z AXIS', ' =', F10.5, /, 9X, 'DELTA', ' =',
C      &F10.5, /, 9X, 'GAMA', ' =', F10.5)
C      STOP
C      END
C
C      *****
C      *SUBROUTINE PROGRAM FOR CALCULATING MATRIX *
C      *MULTIPLICATION *
C      *****
C
C      SUBROUTINE MATMUL(AA, NN, MM, BB, LL, CC)
C      IMPLICIT REAL*8 (A-H, O-Z)
C      DIMENSION AA(NN, MM), BB(MM, LL), CC(NN, LL)
C      DO 100 I=1, NN
C      DO 200 J=1, LL
C      DO 300 K1=1, MM
C      CC(I, J)=CC(I, J)+AA(I, K1)*BB(K1, J)
C      END
C      300 CONTINUE
C      200 CONTINUE
C      100 CONTINUE
C      RETURN
C      END
C
C      *****
C      *SUBROUTINE PROGRAM DETERMINING THE FINITE *
C      *****

```

DIF01660
DIF01670
DIF01680
DIF01690
DIF01700
DIF01710
DIF01720
DIF01730
DIF01740
DIF01750
DIF01760
DIF01770
DIF01780
DIF01790
DIF01800
DIF01810
DIF01820
DIF01830
DIF01840
DIF01850
DIF01860
DIF01870
DIF01880
DIF01890
DIF01900
DIF01910
DIF01920
DIF01930
DIF01940
DIF01950
DIF01960
DIF01970
DIF01980
DIF01990
DIF02000
DIF02010
DIF02020
DIF02030
DIF02040
DIF02050
DIF02060
DIF02070
DIF02080
DIF02090
DIF02100
DIF02110
DIF02120
DIF02130
DIF02140
DIF02150
DIF02160
DIF02170
DIF02180
DIF02190
DIF02200

```

C * DIFFERENCE SCHEME FOR BIHARMONIC OPERATOR *
C *****
C

```

```

SUBROUTINE SCHE(N,N,IM,JM,NOD,ITEST,NP)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION NA(11,11),NP(13)
NP(1)=NA(IM-2,JM)
NP(2)=NA(IM-1,JM-1)
NP(3)=NA(IM-1,JM)
NP(4)=NA(IM-1,JM+1)
NP(5)=NA(IM,JM-2)
NP(6)=NA(IM,JM-1)
NP(7)=NA(IM,JM)
NP(8)=NA(IM,JM+1)
NP(9)=NA(IM,JM+2)
NP(10)=NA(IM+1,JM-1)
NP(11)=NA(IM+1,JM)
NP(12)=NA(IM+1,JM+1)
NP(13)=NA(IM+2,JM)
IF(ITEST.EQ.0) GO TO 10
IF(ITEST.EQ.1) GO TO 11
11 DO 12 IK=1,13
13 NP(IK)=NP(IK)+(NP(IK)/ABS(NP(IK)))*(4*NOD)/5
12 CONTINUE
10 RETURN
END

```

```

*****
* SUBROUTINE PROGRAM GENERATING THE FINITE *
* DIFFERENCE MESH POINTS ON T/B LAYER *
*****

```

```

C
C SUBROUTINE GENR(N,INC,NA,KD)
C IMPLICIT REAL*8 (A-H,O-Z)
C DIMENSION NA(11,11)
C DO 9 I=1,N
C DO 9 J=1,N
C NA(I,J)=0
C DO 10 I=3,N-2
C DO 11 J=3,N-2
C IF(I.GT.3) GO TO 14
C NA(I,J)=NA(I,J-1)+1
C NA(I,1)=NA(I,3)
C NA(I,J)=((-1)**KD)*NA(3,J)
C NA(1,N)=NA(1,N-4)
C NA(1,N-1)=NA(1,N-3)
C GO TO 11
C 14 NA(I,J)=NA(I-1,J)+INC
C 11 CONTINUE
C NA(I,1)=((-1)**KD)*NA(I,3)
C NA(I,N)=NA(I,N-4)
C NA(I,N-1)=NA(I,N-3)

```

```

DIF02210
DIF02220
DIF02230
DIF02240
DIF02250
DIF02260
DIF02270
DIF02280
DIF02290
DIF02300
DIF02310
DIF02320
DIF02330
DIF02340
DIF02350
DIF02360
DIF02370
DIF02380
DIF02390
DIF02400
DIF02410
DIF02420
DIF02430
DIF02440
DIF02450
DIF02460
DIF02470
DIF02480
DIF02490
DIF02500
DIF02510
DIF02520
DIF02530
DIF02540
DIF02550
DIF02560
DIF02570
DIF02580
DIF02590
DIF02600
DIF02610
DIF02620
DIF02630
DIF02640
DIF02650
DIF02660
DIF02670
DIF02680
DIF02690
DIF02700
DIF02710
DIF02720
DIF02730
DIF02740
DIF02750

```



```

D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
DT=DIS(I2+I+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
D4=0.0
D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+I+IK,1)-DIS(I2+1-IK,1)-DIS(I2-1+IK,1)+DIS(I2-1-IK,1)
D7=0.0
D8=(-2.*DIS(I-1,1))/(2.*TX)
D9=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)
D10=0.0
D11=0.0
D12=0.0
DT=-2.*DIS(I-1+IK,1)+2.*DIS(I-1-IK,1)
D13=DT/(4.*TX*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1)
D14=0.0
D15=0.0
D16=(DIS(I1+JK,1)-DIS(I1-JK,1))/(2.*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1)
D17=DT/(4.*TV*TZ)
DT=DIS(I1+I+IK,1)-DIS(I1+1-IK,1)-DIS(I1-1+IK,1)+DIS(I1-1-IK,1)
D18=0.0
D19=(DIS(I1-JK,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2)
D22=(DIS(I2+IK,1)-DIS(I2-IK,1))/(2.*TZ)
D23=0.0
D24=(DIS(I1+IJ,1)-2.*DIS(I1,1)+DIS(I1-IJ,1))/(TV**2)
D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-IK,1))/(TZ**2)
DT=-2.*DIS(I-1+IJ,1)+2.*DIS(I-1-IJ,1)
D26=DT/(4.*TX*TV)
DT=DIS(I1+I+IJ,1)-DIS(I1+1-IJ,1)-DIS(I1-1+IJ,1)+DIS(I1-1-IJ,1)
D27=0.0
D28=0.0
D29=(-2.*DIS(I1,1))/(TX**2)

GO TO Z0

C
CCC POINTS ON THE X AXIS
C
2103 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
      D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
      D3=0.0
      DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
      D4=0.0
      D5=(-2.*DIS(I2,1))/(TV**2)
      DT=DIS(I2+I+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
      D6=0.0
      DT=DIS(I2+I+IK,1)-DIS(I2+1-IK,1)-DIS(I2-1+IK,1)+DIS(I2-1-IK,1)
      D7=DT/(4.*TX*TZ)
      D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
      D9=(-2.*DIS(I1-IJ,1))/(2.*TV)
      D10=0.0
      D11=0.0
      D12=(DIS(I+IK,1)-DIS(I-IK,1))/(2.*TZ)
      DT=DIS(I+I+IK,1)-DIS(I+1-IK,1)-DIS(I-1+IK,1)+DIS(I-1-IK,1)
      D1F03860
      D1F03870
      D1F03880
      D1F03890
      D1F03900
      D1F03910
      D1F03920
      D1F03930
      D1F03940
      D1F03950
      D1F03960
      D1F03970
      D1F03980
      D1F03990
      D1F04000
      D1F04010
      D1F04020
      D1F04030
      D1F04040
      D1F04050
      D1F04060
      D1F04070
      D1F04080
      D1F04090
      D1F04100
      D1F04110
      D1F04120
      D1F04130
      D1F04140
      D1F04150
      D1F04160
      D1F04170
      D1F04180
      D1F04190
      D1F04200
      D1F04210
      D1F04220
      D1F04230
      D1F04240
      D1F04250
      D1F04260
      D1F04270
      D1F04280
      D1F04290
      D1F04300
      D1F04310
      D1F04320
      D1F04330
      D1F04340
      D1F04350
      D1F04360
      D1F04370
      D1F04380
      D1F04390
      D1F04400

```

```

D13=DT/(4.*TX+TZ)
DT=DIS(I+I+J+JK,1)-DIS(I+I+J-JK,1)-DIS(I-I+J+JK,1)+DIS(I-I+J-JK,1)
D14=0.0
D15=(DIS(I-I-K,1)-2.*DIS(I,1)+DIS(I+I+K,1))/(TZ**2)
D16=0.0
DT=-2.*DIS(I+I+J+JK,1)+2.*DIS(I+I+J-JK,1)
D17=DT/(4.*TV+TZ)
DT=DIS(I+I+I+K,1)-DIS(I+I+I-K,1)-DIS(I+I+I+I+K,1)+DIS(I+I+I-K,1)
D18=0.0
D19=0.0
D22=(DIS(I+I+K,1)-DIS(I+I-K,1))/(2.*TZ)
D23=(DIS(I+I,1)-2.*DIS(I,1)+DIS(I+I,1))/(TX**2)
D24=0.0
D25=(DIS(I+I+K,1)-2.*DIS(I,1)+DIS(I+I+K,1))/(TZ**2)
DT=DIS(I+I+I+J,1)-DIS(I+I+I+J,1)-DIS(I+I+I+I+J,1)+DIS(I+I+I+I+J,1)
D26=0.0
DT=-2.*DIS(I+I+I+J,1)+2.*DIS(I+I+I+I+J,1)
D27=DT/(4.*TX+TV)
D28=(-2.*DIS(I,1))/(TV**2)
D29=0.0

```

GO TO 20

C
CCC CENTRAL POINT

```

2104 D1=0.0
D2=(-2.*DIS(I,1))/(TX**2)
D3=0.0
DT=DIS(I+I+I+J,1)-DIS(I+I+I+J,1)-DIS(I+I+I+J,1)+DIS(I+I+I+J,1)
D4=0.0
D5=(-2.*DIS(I,1))/(TV**2)
DT=DIS(I+I+I+JK,1)-DIS(I+I+I+JK,1)-DIS(I+I+I+J+JK,1)+DIS(I+I+I+J+JK,1)
D6=0.0
DT=DIS(I+I+I+K,1)-DIS(I+I+I-K,1)-DIS(I+I+I+I+K,1)+DIS(I+I+I+I-K,1)
D7=0.0
D8=(-2.*DIS(I,1))/(2.*TX)
D9=(-2.*DIS(I+I+I,1))/(2.*TV)
D10=0.0
D11=0.0
D12=0.0
DT=-2.*DIS(I+I+I+K,1)+2.*DIS(I+I+I-K,1)
D13=DT/(4.*TX+TZ)
DT=DIS(I+I+I+JK,1)-DIS(I+I+I+JK,1)-DIS(I+I+I+J+JK,1)+DIS(I+I+I+J+JK,1)
D14=0.0
D15=0.0
D16=0.0
DT=-2.*DIS(I+I+I+JK,1)+2.*DIS(I+I+I+J+JK,1)
D17=DT/(4.*TV+TZ)
DT=DIS(I+I+I+K,1)-DIS(I+I+I-K,1)-DIS(I+I+I+I+K,1)+DIS(I+I+I+I-K,1)
D18=0.0
D19=0.0
D22=(DIS(I+I+K,1)-DIS(I+I-K,1))/(2.*TZ)
D23=0.0
D25=(DIS(I+I+K,1)-2.*DIS(I,1)+DIS(I+I+K,1))/(TZ**2)

```

D1F04410
D1F04420
D1F04430
D1F04440
D1F04450
D1F04460
D1F04470
D1F04480
D1F04490
D1F04500
D1F04510
D1F04520
D1F04530
D1F04540
D1F04550
D1F04560
D1F04570
D1F04580
D1F04590
D1F04600
D1F04610
D1F04620
D1F04630
D1F04640
D1F04650
D1F04660
D1F04670
D1F04680
D1F04690
D1F04700
D1F04710
D1F04720
D1F04730
D1F04740
D1F04750
D1F04760
D1F04770
D1F04780
D1F04790
D1F04800
D1F04810
D1F04820
D1F04830
D1F04840
D1F04850
D1F04860
D1F04870
D1F04880
D1F04890
D1F04900
D1F04910
D1F04920
D1F04930
D1F04940
D1F04950

D1=DIS(I+1+IJ,1)-DIS(I+1-IJ,1)-DIS(I-1+IJ,1)+DIS(I-1-IJ,1)
 D26=0.0
 D1=DIS(I+1+IJ,1)-DIS(I+1-IJ,1)-DIS(I-1+IJ,1)+DIS(I-1-IJ,1)
 D27=0.0
 D28=0.0
 D29=0.0

GO TO 20

C
 CCC CORNER POINT

2100 D1=(DIS(I2+1,1))/(2.*TX)
 D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2)
 D3=(DIS(I2+IJ,1))/(2.*TV)
 D1=DIS(I2+1+IJ,1)
 D4=DT/(4.*TX*TV)
 D5=(DIS(I2+IJ,1)-2.*DIS(I2,1))/(TV**2)
 D1=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 D1=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
 D7=DT/(4.*TX*TZ)
 D8=(DIS(I+1,1))/(2.*TX)
 D9=(DIS(I+1+IJ,1))/(2.*TV)
 D10=(DIS(I+IJ,1))/(2.*TX)
 D11=(DIS(I+1,1))/(2.*TX)
 D12=(DIS(I+IK,1)-DIS(I-1K,1))/(2.*TZ)
 D1=DIS(I+1+IK,1)-DIS(I+1-1K,1)
 D13=DT/(4.*TX*TZ)
 D1=DIS(I+IJ+JK,1)-DIS(I+IJ-JK,1)
 D14=DT/(4.*TV*TZ)
 D15=(DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)
 D16=(DIS(I+1+JK,1)-DIS(I-1-JK,1))/(2.*TZ)
 D1=DIS(I+1+IJ+JK,1)-DIS(I+1-IJ-JK,1)
 D17=DT/(4.*TV*TZ)
 D1=DIS(I+1+IK,1)-DIS(I+1-1K,1)
 D18=DT/(4.*TX*TZ)
 D19=(DIS(I-1JK,1)-2.*DIS(I,1)+DIS(I+1+JK,1))/(TZ**2)
 D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ)
 D23=(DIS(I+1,1)-2.*DIS(I,1))/(TX**2)
 D24=(DIS(I+1+IJ,1)-2.*DIS(I,1))/(TV**2)
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
 D1=DIS(I+1+IJ,1)
 D26=DT/(4.*TX*TV)
 D1=DIS(I+1+IJ,1)
 D27=DT/(4.*TX*TV)
 D28=(DIS(I+IJ,1)-2.*DIS(I,1))/(TV**2)
 D29=(DIS(I+1,1)-2.*DIS(I,1))/(TX**2)

GO TO 20

C
 CCC POINTS PARALLEL TO X AXIS

2099 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
 D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
 D3=(DIS(I2+IJ,1))/(2.*TV)
 D1F04960
 D1F04970
 D1F04980
 D1F04990
 D1F05000
 D1F05010
 D1F05020
 D1F05030
 D1F05040
 D1F05050
 D1F05060
 D1F05070
 D1F05080
 D1F05090
 D1F05100
 D1F05110
 D1F05120
 D1F05130
 D1F05140
 D1F05150
 D1F05160
 D1F05170
 D1F05180
 D1F05190
 D1F05200
 D1F05210
 D1F05220
 D1F05230
 D1F05240
 D1F05250
 D1F05260
 D1F05270
 D1F05280
 D1F05290
 D1F05300
 D1F05310
 D1F05320
 D1F05330
 D1F05340
 D1F05350
 D1F05360
 D1F05370
 D1F05380
 D1F05390
 D1F05400
 D1F05410
 D1F05420
 D1F05430
 D1F05440
 D1F05450
 D1F05460
 D1F05470
 D1F05480
 D1F05490
 D1F05500

```

D1=DIS(I2+1+I,J,1)-DIS(I2-1+I,J,1)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+I,J,1)-2.*DIS(I2,1))/(TV**2)
D7=DIS(I2+I+J+JK,1)-DIS(I2+I+J,1)
D6=DT/(4.*TV*TZ)
D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
D8=DIS(I+1,1)-DIS(I-1,1)/(2.*TX)
D9=(DIS(I+I,J,1))/(2.*TV)
D10=(DIS(I+I,J,1))/(2.*TV)
D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
D12=(DIS(I+1+IK,1)-DIS(I-1K,1))/(2.*TZ)
D13=DT/(4.*TX*TZ)
D14=DIS(I+I+J+JK,1)-DIS(I+I+J,1)
D15=DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)
D16=(DIS(I+I+J,1)-DIS(I+I+J+JK,1))/(2.*TZ)
D17=DT/(4.*TV*TZ)
D18=DIS(I+1+IK,1)-DIS(I+1-1K,1)+DIS(I-1-1K,1)
D19=(DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+J+K,1))/(TZ**2)
D22=(DIS(I+1,1)-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)
D23=(DIS(I+1,1)-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)
D24=(DIS(I+1+J,1)-2.*DIS(I,1))/(TV**2)
D25=(DIS(I+1+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
D26=DT/(4.*TX*TV)
D27=DIS(I+1+I,J,1)-DIS(I-1+I,J,1)
D28=(DIS(I+I,J,1)-2.*DIS(I,1))/(TV**2)
D29=(DIS(I+1,1)-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)

```

GO TO 20

C
CCC POINT PARALLEL TO X AXIS ON THE Y AXIS

```

2097 D1=0.0
D2=(-2.*DIS(I2,1))/(TX**2)
D3=(DIS(I2+I,J,1))/(2.*TV)
D4=DIS(I2+1+I,J,1)-DIS(I2-1+I,J,1)
D5=(DIS(I2+I,J,1)-2.*DIS(I2,1))/(TV**2)
D6=DT/(4.*TV*TZ)
D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
D8=(-2.*DIS(I-1,1))/(2.*TX)
D9=(DIS(I+I,J,1))/(2.*TV)
D10=0.0
D11=0.0
D12=0.0
D13=DT/(4.*TX*TZ)

```

D1F05510
D1F05520
D1F05530
D1F05540
D1F05550
D1F05560
D1F05570
D1F05580
D1F05590
D1F05600
D1F05610
D1F05620
D1F05630
D1F05640
D1F05650
D1F05660
D1F05670
D1F05680
D1F05690
D1F05700
D1F05710
D1F05720
D1F05730
D1F05740
D1F05750
D1F05760
D1F05770
D1F05780
D1F05790
D1F05800
D1F05810
D1F05820
D1F05830
D1F05840
D1F05850
D1F05860
D1F05870
D1F05880
D1F05890
D1F05900
D1F05910
D1F05920
D1F05930
D1F05940
D1F05950
D1F05960
D1F05970
D1F05980
D1F05990
D1F06000
D1F06010
D1F06020
D1F06030
D1F06040
D1F06050

DT=DIS(I+I+J+JK,1)-DIS(I+I+J-JK,1) DIF06060
 D14=0.0 DIF06070
 D15=0.0 DIF06080
 D16=(DIS(I+I+J+JK,1)-DIS(I+I-JK,1))/(2.*TZ) DIF06090
 DT=DIS(I+I+J+JK,1)-DIS(I+I+J-JK,1) DIF06100
 D17=DT/(4.*TV*TZ) DIF06110
 DT=DIS(I+I+I+IK,1)-DIS(I+I+I-IK,1)-DIS(I+I+I+IK,1)+DIS(I+I-I-IK,1) DIF06120
 D18=0.0 DIF06130
 D19=(DIS(I+I-JK,1)-2.*DIS(I+I,1)+DIS(I+I+JK,1))/(TZ**2) DIF06140
 D22=(DIS(I+I+IK,1)-DIS(I+I-IK,1))/(2.*TZ) DIF06150
 D23=0.0 DIF06160
 D24=(DIS(I+I+I+J,1)-2.*DIS(I+I,1))/(TV**2) DIF06170
 D25=(DIS(I+I+IK,1)-2.*DIS(I+I,1)+DIS(I+I+JK,1))/(TZ**2) DIF06180
 D1=(DIS(I+I+I+J,1)-DIS(I+I+I+J,1)) DIF06190
 D26=DT/(4.*TX*TV) DIF06200
 DT=DIS(I+I+I+J,1)-DIS(I+I+I+J,1) DIF06210
 D27=0.0 DIF06220
 D28=0.0 DIF06230
 D29=(-2.*DIS(I+I,1))/(TX**2) DIF06240
 DIF06250
 DIF06260
 DIF06270
 DIF06280
 DIF06290
 DIF06300
 DIF06310
 DIF06320
 DIF06330
 DIF06340
 DIF06350
 DIF06360
 DIF06370
 DIF06380
 DIF06390
 DIF06400
 DIF06410
 DIF06420
 DIF06430
 DIF06440
 DIF06450
 DIF06460
 DIF06470
 DIF06480
 DIF06490
 DIF06500
 DIF06510
 DIF06520
 DIF06530
 DIF06540
 DIF06550
 DIF06560
 DIF06570
 DIF06580
 DIF06590
 DIF06600

GO TO 20

C
 CCC POINTS PARALLEL TO V AXIS

2098 D1=(DIS(I+I+I,1))/(2.*TX) DIF06300
 D2=(DIS(I+I+I,1)-2.*DIS(I+I,1))/(TX**2) DIF06310
 D3=(DIS(I+I+I,1)-DIS(I+I,1))/(2.*TV) DIF06320
 DT=DIS(I+I+I+J,1)-DIS(I+I+I+J,1) DIF06330
 D4=DT/(4.*TX*TV) DIF06340
 D5=(DIS(I+I+I+J,1)-2.*DIS(I+I,1)+DIS(I+I+J,1))/(TV**2) DIF06350
 DT=DIS(I+I+I+J+JK,1)-DIS(I+I+I+J-JK,1)-DIS(I+I+I+J+JK,1)+DIS(I+I+I+J-JK,1) DIF06360
 D6=DT/(4.*TV*TZ) DIF06370
 DT=DIS(I+I+I+I+IK,1)-DIS(I+I+I-IK,1) DIF06380
 D7=DT/(4.*TX*TZ) DIF06390
 D8=(DIS(I+I+I,1))/(2.*TX) DIF06400
 D9=(DIS(I+I+I+J,1)-DIS(I+I+I,1))/(2.*TV) DIF06410
 D10=(DIS(I+I+I,1)-DIS(I+I+I,1))/(2.*TV) DIF06420
 D11=(DIS(I+I+I,1))/(2.*TX) DIF06430
 D12=(DIS(I+I+I,1)-DIS(I+I-IK,1))/(2.*TZ) DIF06440
 DT=DIS(I+I+I+I+IK,1)-DIS(I+I+I-IK,1) DIF06450
 D13=DT/(4.*TX*TZ) DIF06460
 DT=DIS(I+I+I+J+JK,1)-DIS(I+I+I+J-JK,1)-DIS(I+I+I+J+JK,1)+DIS(I+I+I+J-JK,1) DIF06470
 D14=DT/(4.*TV*TZ) DIF06480
 D15=(DIS(I+I-IK,1)-2.*DIS(I+I,1)+DIS(I+I+IK,1))/(TZ**2) DIF06490
 D16=(DIS(I+I+JK,1)-DIS(I+I-JK,1))/(2.*TZ) DIF06500
 DT=DIS(I+I+I+J+JK,1)-DIS(I+I+I+J-JK,1)-DIS(I+I+I+J+JK,1)+DIS(I+I+I+J-JK,1) DIF06510
 D17=DT/(4.*TV*TZ) DIF06520
 DT=DIS(I+I+I+I+IK,1)-DIS(I+I+I-IK,1) DIF06530
 D18=DT/(4.*TX*TZ) DIF06540
 D19=(DIS(I+I-JK,1)-2.*DIS(I+I,1)+DIS(I+I+JK,1))/(TZ**2) DIF06550
 D22=(DIS(I+I+IK,1)-DIS(I+I-IK,1))/(2.*TZ) DIF06560
 D23=(DIS(I+I,1)-2.*DIS(I+I,1))/(TX**2) DIF06570
 D24=(DIS(I+I+I+J,1)-2.*DIS(I+I,1)+DIS(I+I+J,1))/(TV**2) DIF06580
 D25=(DIS(I+I+IK,1)-2.*DIS(I+I,1)+DIS(I+I-IK,1))/(TZ**2) DIF06590
 DT=DIS(I+I+I+I,1)-DIS(I+I+I-I,1) DIF06600

```

D26=DT/(4.*TX*TV)
DT=DIS(I1+1+IJ,1)-DIS(I1+1-IJ,1)
D27=DT/(4.*TX*TV)
D28=(DIS(I1+IJ,1))-2.*DIS(I,1)+DIS(I-IJ,1))/(TV**2)
D29=(DIS(I1+1,1))-2.*DIS(I1,1))/(TX**2)
    
```

GO TO 20

C
CCC POINT PARALLEL TO V AXIS ON THE X AXIS

2096 D1=(DIS(I2+1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1))/(TX**2)

D3=0.0
DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)

D4=0.0
D5=(-2.*DIS(I2,1))/(TV**2)

DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)

D6=0.0
DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)

D7=DT/(4.*TX*TZ)
D8=(DIS(I1+1,1))/(2.*TX)

D9=(-2.*DIS(I1-IJ,1))/(2.*TV)
D10=0.0

D11=0.0
D12=(DIS(I1+IK,1)-DIS(I1-1K,1))/(2.*TZ)

DT=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)
D13=DT/(4.*TX*TZ)

DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1)

D14=0.0
D15=(DIS(I1-1K,1))-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)

D16=0.0
DT=-2.*DIS(I1-IJ+JK,1)+2.*DIS(I1-IJ-JK,1)

D17=DT/(4.*TV*TZ)
DT=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+1K,1)+DIS(I1-1-1K,1)

D18=0.0
D19=0.0

D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ)
D23=(DIS(I1+1,1))-2.*DIS(I,1))/(TX**2)

D24=0.0
D25=(DIS(I2+IK,1))-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)

DT=DIS(I1+1+IJ,1)-DIS(I1+1-IJ,1)
D26=0.0

DT=-DIS(I1+1-IJ,1)-DIS(I1+1-IJ,1)
D27=DT/(4.*TX*TV)

D28=(-2.*DIS(I,1))/(TV**2)
D29=0.0

GO TO 20
C
CCC GENERAL POINT (B,L)

2105 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)

D1F06610
D1F06620
D1F06630
D1F06640
D1F06650
D1F06660
D1F06670
D1F06680
D1F06690
D1F06700
D1F06710
D1F06720
D1F06730
D1F06740
D1F06750
D1F06760
D1F06770
D1F06780
D1F06790
D1F06800
D1F06810
D1F06820
D1F06830
D1F06840
D1F06850
D1F06860
D1F06870
D1F06880
D1F06890
D1F06900
D1F06910
D1F06920
D1F06930
D1F06940
D1F06950
D1F06960
D1F06970
D1F06980
D1F06990
D1F07000
D1F07010
D1F07020
D1F07030
D1F07040
D1F07050
D1F07060
D1F07070
D1F07080
D1F07090
D1F07100
D1F07110
D1F07120
D1F07130
D1F07140
D1F07150

D4=DT/(4.*TX*TV) D1F07160
 D5=(DIS(I2+IJ,1))-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2) D1F07170
 D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1) D1F07180
 D6=DT/(4.*TV*TZ) D1F07190
 D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1) D1F07200
 D7=DT/(4.*TX*TZ) D1F07210
 D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX) D1F07220
 D9=(DIS(I+IJ,1)-DIS(I-1-IJ,1))/(2.*TV) D1F07230
 D10=(DIS(I+IJ,1)-DIS(I-1-IJ,1))/(2.*TV) D1F07240
 D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX) D1F07250
 D1F07260
 D1F07270
 D1F07280
 D1F07290
 D1F07300
 D1F07310
 D1F07320
 D1F07330
 D1F07340
 D1F07350
 D1F07360
 D1F07370
 D1F07380
 D1F07390
 D1F07400
 D1F07410
 D1F07420
 D1F07430
 D1F07440
 D1F07450
 D1F07460
 D1F07470
 D1F07480
 D1F07490
 D1F07500
 D1F07510
 D1F07520
 D1F07530
 D1F07540
 D1F07550
 D1F07560
 D1F07570
 D1F07580
 D1F07590
 D1F07600
 D1F07610
 D1F07620
 D1F07630
 D1F07640
 D1F07650
 D1F07660
 D1F07670
 D1F07680
 D1F07690
 D1F07700

GO TO 30

C
 CCC POINTS ON THE V AXIS (B.L)

2106 D1=0.0
 D2=(-2.*DIS(I2,1))/(TX**2)
 D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
 D7=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
 D4=0.0
 D5=(DIS(I2+IJ,1))-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
 D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
 D7=0.0
 D8=(-2.*DIS(I-1,1))/(2.*TX)
 D9=(DIS(I+IJ,1)-DIS(I-1-IJ,1))/(2.*TV)
 D10=0.0
 D11=0.0

GO TO 30

C
 CCC POINTS ON THE X AXIS (B.L)

2107 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
 D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
 D3=0.0
 D7=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
 D4=0.0
 D5=(-2.*DIS(I2,1))/(TV**2)
 D7=DIS(I2+1+JK,1)-DIS(I2+1-JK,1)-DIS(I2-1+JK,1)+DIS(I2-1-JK,1)
 D6=0.0
 D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
 D7=DT/(4.*TX*TZ)
 D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
 D9=(-2.*DIS(I-1-IJ,1))/(2.*TV)
 D10=0.0
 D11=0.0

GO TO 30

C
 CCC CENTRAL POINT (B.L)

2108 D1=0.0
 D2=(-2.*DIS(I2,1))/(TX**2)

D3=0.0
 DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
 D4=0.0
 D5=(-2.*DIS(I2,1))/(TV**2)
 DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
 D6=0.0
 DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
 D7=0.0
 D8=(-2.*DIS(I-1,1))/(2.*TX)
 D9=(-2.*DIS(I-1J,1))/(2.*TV)
 D10=0.0
 D11=0.0

GO TO 30

C
 CCC CORNER POINT (B.L)

2109 D1=(DIS(I2+1,1))/(2.*TX)
 D2=(DIS(I2+1,1))-2.*DIS(I2,1)/(TX**2)
 D3=(DIS(I2+IJ,1))/(2.*TV)
 DT=DIS(I2+1+IJ,1)
 D4=DT/(4.*TX*TV)
 D5=(DIS(I2+IJ,1))-2.*DIS(I2,1)/(TV**2)
 DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
 D7=DT/(4.*TX*TZ)
 D8=(DIS(I+1,1))/(2.*TX)
 D9=(DIS(I+IJ,1))/(2.*TV)
 D10=(DIS(I+IJ,1))/(2.*TV)
 D11=(DIS(I+1,1))/(2.*TX)

GO TO 30

C
 CCC POINTS PARALLEL TO X AXIS (B.L)

2110 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
 D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1)/(TX**2)
 D3=(DIS(I2+IJ,1))/(2.*TV)
 DT=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1)
 D4=DT/(4.*TX*TV)
 D5=(DIS(I2+IJ,1))-2.*DIS(I2,1)/(TV**2)
 DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
 D7=DT/(4.*TX*TZ)
 D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
 D9=(DIS(I+IJ,1))/(2.*TV)
 D10=(DIS(I+IJ,1))/(2.*TV)
 D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)

GO TO 30

C
 CCC POINTS PARALLEL TO X AXIS ON THE Y AXIS

D1F07710
 D1F07720
 D1F07730
 D1F07740
 D1F07750
 D1F07760
 D1F07770
 D1F07780
 D1F07790
 D1F07800
 D1F07810
 D1F07820
 D1F07830
 D1F07840
 D1F07850
 D1F07860
 D1F07870
 D1F07880
 D1F07890
 D1F07900
 D1F07910
 D1F07920
 D1F07930
 D1F07940
 D1F07950
 D1F07960
 D1F07970
 D1F07980
 D1F07990
 D1F08000
 D1F08010
 D1F08020
 D1F08030
 D1F08040
 D1F08050
 D1F08060
 D1F08070
 D1F08080
 D1F08090
 D1F08100
 D1F08110
 D1F08120
 D1F08130
 D1F08140
 D1F08150
 D1F08160
 D1F08170
 D1F08180
 D1F08190
 D1F08200
 D1F08210
 D1F08220
 D1F08230
 D1F08240
 D1F08250

```

2112 D1=0.0
      D2=(-2.*DIS(I2,1))/(TX**2)
      D3=(DIS(I2+IJ,1))/(2.*TV)
      DT=DIS(I2+IJ,1)-DIS(I2-1+IJ,1)
      D4=0.0
      D5=(DIS(I2+IJ,1)-2.*DIS(I2,1))/(TV**2)
      DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)
      D6=DT/(4.*TV*TZ)
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D7=0.0
      D8=(-2.*DIS(I-1,1))/(2.*TX)
      D9=(DIS(I+IJ,1))/(2.*TV)
      D10=0.0
      D11=0.0
  
```

GO TO 30

C
CCC POINTS PARALLEL TO Y AXIS

```

2111 D1=(DIS(I2+1,1))/(2.*TX)
      D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2)
      D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
      DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)
      D4=DT/(4.*TX*TV)
      D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
      DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
      D6=DT/(4.*TV*TZ)
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
      D7=DT/(4.*TX*TZ)
      D8=(DIS(I+1,1))/(2.*TX)
      D9=(DIS(I+IJ,1)-DIS(I-IJ,1))/(2.*TV)
      D10=(DIS(I+IJ,1)-DIS(I-IJ,1))/(2.*TV)
      D11=(DIS(I+1,1))/(2.*TX)
  
```

GO TO 30

C
CCC POINTS PARALLEL TO Y AXIS ON THE X AXIS

```

2113 D1=(DIS(I2+1,1))/(2.*TX)
      D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2)
      D3=0.0
      DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)
      D4=0.0
      D5=(-2.*DIS(I2,1))/(TV**2)
      DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
      D6=0.0
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
      D7=DT/(4.*TX*TZ)
      D8=(DIS(I+1,1))/(2.*TX)
      D9=(-2.*DIS(I-IJ,1))/(2.*TV)
      D10=0.0
      D11=0.0
  
```

GO TO 30

```

D1F08260
D1F08270
D1F08280
D1F08290
D1F08300
D1F08310
D1F08320
D1F08330
D1F08340
D1F08350
D1F08360
D1F08370
D1F08380
D1F08390
D1F08400
D1F08410
D1F08420
D1F08430
D1F08440
D1F08450
D1F08460
D1F08470
D1F08480
D1F08490
D1F08500
D1F08510
D1F08520
D1F08530
D1F08540
D1F08550
D1F08560
D1F08570
D1F08580
D1F08590
D1F08600
D1F08610
D1F08620
D1F08630
D1F08640
D1F08650
D1F08660
D1F08670
D1F08680
D1F08690
D1F08700
D1F08710
D1F08720
D1F08730
D1F08740
D1F08750
D1F08760
D1F08770
D1F08780
D1F08790
D1F08800
  
```

```

20 OX=0.5*(D3-D16)
   OY=0.5*(D12-D1)
   XOX=0.5*(D4-D17)
   YOY=0.5*(D5-D18)
   ZOX=0.5*(D6-D19)
   XOY=0.5*(D13-D2)
   YOY=0.5*(D14-D4)
   ZOV=0.5*(D15-D7)
   SX=G1*D8+G3*(D9+D22/(HA**2))
   SY=G1*D9+G3*(D8+D22/(HA**2))
   SZ=G1*D22/(HA**2)+G3*(D8+D9)
   TXV=G*(D10+D11)
   TZ=G*(D1+D12)
   TVZ=G*(D3+D16)
   XSXX=G1*D23+G3*(D27+D7/(HA**2))
   VSVY=G1*D24+G3*(D26+D6/(HA**2))
   ZSZZ=G1*D25/(HA**2)+G3*(D13+D18)
   XTXY=G*(D26+D29)
   VTXV=G*(D28+D27)
   XTZX=G*(D2+D13)
   ZTXZ=G*(D7+D15)
   VTYZ=G*(D18+D5)
   ZTYZ=G*(D6+D19)
   H1=2.0*OX*XOX
   H2=2.0*OY*YOY
   H3=OX*XOV+OY*XOX
   H4=2.0*OX*VOX
   H5=2.0*OY*VOY
   H6=OX*VOY+OY*VOX
   H7=2.0*OX*ZOX
   H8=2.0*OY*ZOV
   FX=-(XTXZ*OY+TXZ*XOV)-(VTYZ*OY+TVZ*VOY)-(ZSZZ*OY+SZZ*ZOV)
   FV=+(XTXZ*OX+TXZ*XOX)+(VTYZ*OX+TVZ*VOX)+(ZSZZ*OX+SZZ*ZOX)
   FZ1=(XSXX*OY+SXX*XOV-XTXY*OX-TXV*XOX)*HA
   FZ2=(VTXY*OY+TXV*YOY-VSVY*OX-SVV*YOX)*HA
   FZ3=(ZTXZ*OY+TXZ*ZOV-ZTYZ*OX-TVZ*ZOX)/HA
   FOR(1,1)=FOR(1,1)-G2*H2+G*H6-G3*H1+FX
   FOR(1,1)=FOR(1,1)+G*H3-G2*H4-G3*H5+FY
   FOR(1,1)=FOR(1,1)-G2*H2+G*H6-G3*H1+FX
   FOR(1,1)=FOR(1,1)+G*H3-G2*H4-G3*H5+FY
   FOR(12,1)=-G2*(H7+H8)/HA+(FZ1+FZ2+FZ3)
   FOR(1,1)=+FX*HA
   FOR(1,1)=+FY*HA
   FOR(12,1)=(FZ1+FZ2+FZ3)*HA
GO TO 18
30 FOR(1,1)=-G1*1*D1*D2-G22*D3*D4-G*D1*D5)*HA
   FOR(1,1)=-G1*1*D3*D5-G22*01*D4-G*D3*D2)*HA
   DT1=(D8+0.5*D1**2+p*D9+0.5*p*D3**2)*D2
   DT2=(D9+0.5*D3**2+p*D8+0.5*p*D1**2)*D5
   DT3=(1.-P)*(D10+D11+D1*D3)*D4
   C=12.0
   C=12.0/(XA*HA)
D1F08810
D1F08820
D1F08830
D1F08840
D1F08850
D1F08860
D1F08870
D1F08880
D1F08890
D1F08900
D1F08910
D1F08920
D1F08930
D1F08940
D1F08950
D1F08960
D1F08970
D1F08980
D1F08990
D1F09000
D1F09010
D1F09020
D1F09030
D1F09040
D1F09050
D1F09060
D1F09070
D1F09080
D1F09090
D1F09100
D1F09110
D1F09120
D1F09130
D1F09140
D1F09150
D1F09160
D1F09170
D1F09180
D1F09190
D1F09200
D1F09210
D1F09220
D1F09230
D1F09240
D1F09250
D1F09260
D1F09270
D1F09280
D1F09290
D1F09300
D1F09310
D1F09320
D1F09330
D1F09340
D1F09350

```

```

C FOR(12,1)=(12.0*Q*(1.-P**2))/(FF(1,1)*E*(HA**4))+C*(DT1+DT2+DT3) D1F09360
C FOR(12,1)=(12.0*Q*(1.-P**2))*FF(1,1)+C*(DT1+DT2+DT3) D1F09370
C FOR(12,1)=(12.0*Q*(1.-P**2))/HA+C*(DT1+DT2+DT3) D1F09380
C FOR(12,1)=FOR(12,1)+C*(DT1+DT2+DT3) D1F09390
18 CONTINUE D1F09400
RETURN D1F09410
END D1F09420

```

```

C *****
C * SUBROUTINE PROGRAM GENERATING THE FORCE *
C * VECTORS BASED ON FF INPUT VECTOR *
C *****

```

```

C SUBROUTINE FORCE(Q,P,HA,NOD,E,FF,VAR)
C IMPLICIT REAL*8 (A-H,O-Z)
C DIMENSION FF(20,1),VAR(60,1)

```

```

C DO 8 K=1,3*NOD
C VAR(K,1)=0.0
C CONTINUE
C DO 9 K=1,3*NOD
C IF(K.GT.2*NOD) GO TO 14
C IF(K.LE.2*NOD) GO TO 9
C 14 VAR(K,1)=(12.0*Q*(1.-P**2))/(FF(K-2*NOD,1))*E*(HA**4)
C 14 VAR(K,1)= 12.0*Q*(1.-P**2)/HA
C 9 CONTINUE
C RETURN
C END

```

```

C *****
C * SUBROUTINE PROGRAM TO SATISFY THE BOUNDARY *
C * CONDITIONS IN THE GLOBAL MATRIX 'AA' *
C *****

```

```

C SUBROUTINE BOUNDA(UVW,NOD,AA)
C IMPLICIT REAL*8 (A-H,O-Z)
C DIMENSION AA(60,60),UVW(4,3)

```

```

C NL=5
C DO 501 I=1,NOD/NL
C DO 502 K=1,3
C IF(UVW(I,K).EQ.0) GO TO 502
C DO 504 M=1,NL
C L=I+(M-1)*(NOD/NL)+(K-1)*NOD
C TEMP=AA(L,L)
C DO 503 J=1,3*NOD
C AA(L,J)=0.0
C CONTINUE
C AA(L,L)=1.0

```

```

504 CONTINUE
502 CONTINUE
501 CONTINUE

```

RETURN
END

* SUBROUTINE PROGRAM TO SATISFY THE BOUNDARY *
* CONDITIONS IN THE FORCE VECTORS 'FOR' *

SUBROUTINE BOUNDF(UVW,NOD,WAR)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION WAR(60,1),UVW(4,3)

NL=5
DO 501 I=1,NOD/NL
DO 502 K=1,3
IF(UVW(I,K).EQ.0) GO TO 502
DO 504 KI=1,NL
L=I+(KI-1)*(NOD/NL)+(K-1)*NOD
WAR(L,1)=0.0

504 CONTINUE
502 CONTINUE
501 CONTINUE
END

* SUBROUTINE PROGRAM TO INVERT AND SOLVE THE *
* SIMULTANEOUS EQUATIONS USING CHANGE OF *
* VARIABLE TECHNIQUE *

SUBROUTINE SOLINV(A,N)
IMPLICIT REAL*8 (A-H,O-Z)
PARAMETER (NMAX=375)
DIMENSION A(N,N),V(NMAX)

DO 10 I=1,N
V(I)=A(I,1)
DO 20 J=1,N
A(I,J)=-A(I,J)/V(I)

20 CONTINUE
A(I,1)=-A(I,1)
IF(I.EQ.N) GOTO 10
DO 30 II=I+1,N
IF(A(II,1).EQ.0.0) GO TO 30
TEMP=A(II,1)
A(II,1)=0.0
DO 40 J=1,N
A(II,J)=A(II,J)+A(I,J)*TEMP
40 CONTINUE
30 CONTINUE
10 CONTINUE

DIF09910
DIF09920
DIF09930
DIF09940
DIF09950
DIF09960
DIF09970
DIF09980
DIF09990
DIF10000
DIF10010
DIF10020
DIF10030
DIF10040
DIF10050
DIF10060
DIF10070
DIF10080
DIF10090
DIF10100
DIF10110
DIF10120
DIF10130
DIF10140
DIF10150
DIF10160
DIF10170
DIF10180
DIF10190
DIF10200
DIF10210
DIF10220
DIF10230
DIF10240
DIF10250
DIF10260
DIF10270
DIF10280
DIF10290
DIF10300
DIF10310
DIF10320
DIF10330
DIF10340
DIF10350
DIF10360
DIF10370
DIF10380
DIF10390
DIF10400
DIF10410
DIF10420
DIF10430
DIF10440
DIF10450


```

IF(IJKL(I,4).EQ.100) GO TO 100
IF(IJKL(I,4).EQ.101) GO TO 101
IF(IJKL(I,4).EQ.102) GO TO 102
IF(IJKL(I,4).EQ.103) GO TO 103
IF(IJKL(I,4).EQ.104) GO TO 104
IF(IJKL(I,4).EQ.105) GO TO 105
IF(IJKL(I,4).EQ.106) GO TO 106
IF(IJKL(I,4).EQ.107) GO TO 107
IF(IJKL(I,4).EQ.108) GO TO 108
IF(IJKL(I,4).EQ.109) GO TO 109
IF(IJKL(I,4).EQ.110) GO TO 110
IF(IJKL(I,4).EQ.111) GO TO 111
IF(IJKL(I,4).EQ.112) GO TO 112
IF(IJKL(I,4).EQ.113) GO TO 113
IF(IJKL(I,4).EQ.114) GO TO 114
IF(IJKL(I,4).EQ.115) GO TO 115
IF(IJKL(I,4).EQ.116) GO TO 116
IF(IJKL(I,4).EQ.117) GO TO 117
IF(IJKL(I,4).EQ.118) GO TO 118
IF(IJKL(I,4).EQ.119) GO TO 119
IF(IJKL(I,4).EQ.120) GO TO 120
IF(IJKL(I,4).EQ.121) GO TO 121
IF(IJKL(I,4).EQ.122) GO TO 122
IF(IJKL(I,4).EQ.123) GO TO 123
IF(IJKL(I,4).EQ.124) GO TO 124
IF(IJKL(I,4).EQ.125) GO TO 125

```

```

C
CCC POINT PARALLEL TO Y AXIS ON THE X AXIS

```

```

96 AA(I,1)=AA(I,1)-(2.0*G1/TX**2)
AA(I,1+1)=AA(I,1+1)+(G1/TX**2)
AA(I,1-IJ)=AA(I,1-IJ)+(2.0*G/TV**2)
AA(I,1)=AA(I,1)-(2.0*G/TV**2)
AA(I,1-IK)=AA(I,1-IK)+G/TZ**2
AA(I,1)=AA(I,1)-2.0*G/TZ**2
AA(I,1+IK)=AA(I,1+IK)+G/TZ**2

```

```

CCCCC

```

```

AA(I,11)=AA(I,11)+(G2/(TX*TV))
AA(I,11+1)=AA(I,11+1)-(G2/(TX*TV))
AA(I,11-IJ)=AA(I,11-IJ)+(G2/(TX*TV))
AA(I,11-IJ+1)=AA(I,11-IJ+1)-(G2/(TX*TV))
AA(I,12)=AA(I,12)+G2/(TX*TZ*HA)
AA(I,12+1)=AA(I,12+1)-G2/(TX*TZ*HA)
AA(I,12+IK)=AA(I,12+IK)-G2/(TX*TZ*HA)
AA(I,12+IK+1)=AA(I,12+IK+1)+G2/(TX*TZ*HA)

```

```

CCCCC
AA(I,1,1)=AA(I,1,1)+(G2/(TX*TV))
AA(I,1,1+1)=AA(I,1,1+1)-G2/(TX*TV)
AA(I,1,1-IJ)=AA(I,1,1-IJ)-G2/(TX*TV)
AA(I,1,1-IJ+1)=AA(I,1,1-IJ+1)+G2/(TX*TV)
AA(I,1,11)=AA(I,1,11)-(2.0*G/TX**2)
AA(I,1,11+1)=AA(I,1,11+1)+(G/TX**2)

```

```

DIF11010
DIF11020
DIF11030
DIF11040
DIF11050
DIF11060
DIF11070
DIF11080
DIF11090
DIF11100
DIF11110
DIF11120
DIF11130
DIF11140
DIF11150
DIF11160
DIF11170
DIF11180
DIF11190
DIF11200
DIF11210
DIF11220
DIF11230
DIF11240
DIF11250
DIF11260
DIF11270
DIF11280
DIF11290
DIF11300
DIF11310
DIF11320
DIF11330
DIF11340
DIF11350
DIF11360
DIF11370
DIF11380
DIF11390
DIF11400
DIF11410
DIF11420
DIF11430
DIF11440
DIF11450
DIF11460
DIF11470
DIF11480
DIF11490
DIF11500
DIF11510
DIF11520
DIF11530
DIF11540
DIF11550

```

```

CC      AA(I1,I1-IJ)=AA(I1,I1-IJ)+(2.0*G1/TV**2)          DIF11560
      AA(I1,I1)=AA(I1,I1)-(2.0*G1/TV**2)                DIF11570
      AA(I1,I1-JK)=AA(I1,I1-JK)+G/TZ**2                DIF11580
      AA(I1,I1)=AA(I1,I1)-2.0*G/TZ**2                    DIF11590
      AA(I1,I1+JK)=AA(I1,I1+JK)+G/TZ**2                  DIF11600
      AA(I1,I1)=AA(I1,I1)+G/TZ**2                          DIF11610
      AA(I1,I2)=AA(I1,I2)+(G2/(TV*TZ*HA))                DIF11620
      AA(I1,I2-IJ)=AA(I1,I2-IJ)-(G2/(TV*TZ*HA))         DIF11630
      AA(I1,I2+JK)=AA(I1,I2+JK)-(G2/(TV*TZ*HA))         DIF11640
      AA(I1,I2+JK-IJ)=AA(I1,I2+JK-IJ)+(G2/(TV*TZ*HA))  DIF11650
      AA(I2,I1)=AA(I2,I1)+(G2/(TV*TZ))                    DIF11660
      AA(I2,I1+1)=AA(I2,I1+1)-(G2/(TV*TZ))               DIF11670
      AA(I2,I1+1)=AA(I2,I1+1)-(G2/(TV*TZ))               DIF11680
      AA(I2,I1+1K)=AA(I2,I1+1K)-(G2/(TV*TZ))            DIF11690
      AA(I2,I1+1K+1)=AA(I2,I1+1K+1)+(G2/(TV*TZ))        DIF11700
      AA(I2,I1)=AA(I2,I1)+(G2/(TV*TZ))                    DIF11710
      AA(I2,I1-IJ)=AA(I2,I1-IJ)+(G2/(TV*TZ))             DIF11720
      AA(I2,I1+JK)=AA(I2,I1+JK)-(G2/(TV*TZ))             DIF11730
      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-(G2/(TV*TZ))      DIF11740
      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-(G2/(TV*TZ))      DIF11750
      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-(G2/(TV*TZ))      DIF11760
      AA(I2,I2)=AA(I2,I2)-2.0*G/(HA*TX**2)               DIF11770
      AA(I2,I2+1)=AA(I2,I2+1)+G/(HA*TX**2)               DIF11780
      AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(HA*TV**2)         DIF11790
      AA(I2,I2)=AA(I2,I2)-2.0*G/(HA*TV**2)                DIF11800
      AA(I2,I2-IK)=AA(I2,I2-IK)+G1/(HA*TX**2)           DIF11810
      AA(I2,I2-IK)=AA(I2,I2-IK)+G1/(HA*TX**2)           DIF11820
      AA(I2,I2)=AA(I2,I2)-2.0*G1/(HA*TX**2)              DIF11830
      AA(I2,I2+IK)=AA(I2,I2+IK)+G1/(HA*TX**2)            DIF11840
      AA(I2,I2)=AA(I2,I2)+G/TZ**2                          DIF11850
      AA(I2,I1)=AA(I2,I1)+(G2/(TV*TZ))                    DIF11860
      AA(I2,I1-IJ)=AA(I2,I1-IJ)+(G2/(TV*TZ))             DIF11870
      AA(I2,I1+JK)=AA(I2,I1+JK)-(G2/(TV*TZ))             DIF11880
      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-(G2/(TV*TZ))      DIF11890
      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-(G2/(TV*TZ))      DIF11900
      AA(I2,I1)=AA(I2,I1)-2.0*G/TV**2                    DIF11910
      AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2                      DIF11920
      AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2                  DIF11930
      AA(I1,I1-IK)=AA(I1,I1-IK)+G/TZ**2                  DIF11940
      AA(I1,I1)=AA(I1,I1)-2.0*G/TZ**2                      DIF11950
      AA(I1,I1+IK)=AA(I1,I1+IK)+G/TZ**2                    DIF11960
      AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)                       DIF11970
      AA(I1,I1-IJ)=AA(I1,I1-IJ)-G2/(TX*TV)                DIF11980
      AA(I1,I1+IJ)=AA(I1,I1+IJ)-G2/(TX*TV)                DIF11990
      AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)+G2/(TX*TV)           DIF12000
      AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)+G2/(TX*TV)           DIF12010
      AA(I1,I2)=AA(I1,I2)+G2/(TX*TV*HA)                   DIF12020
      AA(I1,I2-1)=AA(I1,I2-1)-G2/(TX*TV*HA)              DIF12030
      AA(I1,I2+1K)=AA(I1,I2+1K)-G2/(TX*TV*HA)            DIF12040
      AA(I1,I2+1K)=AA(I1,I2+1K)-G2/(TX*TV*HA)            DIF12050
      AA(I1,I2+1K-1)=AA(I1,I2+1K-1)+G2/(TX*TV*HA)        DIF12060
      AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)                       DIF12070
      AA(I1,I1-1)=AA(I1,I1-1)+G2/(TX*TV)                  DIF12080
      AA(I1,I1+IJ)=AA(I1,I1+IJ)-G2/(TX*TV)                DIF12090
      AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(TX*TV)           DIF12100

```

GO TO 2

CCC POINT PARALLEL TO X AXIS ON THE Y AXIS

CC 97 AA(I1,I1-I)=AA(I1,I1-I)+2.0*G1/TV**2

CC 97 AA(I1,I1)=AA(I1,I1)-2.0*G1/TV**2

CC 97 AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2

CC 97 AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2

CC 97 AA(I1,I1-IK)=AA(I1,I1-IK)+G/TZ**2

CC 97 AA(I1,I1)=AA(I1,I1)-2.0*G/TZ**2

CC 97 AA(I1,I1+IK)=AA(I1,I1+IK)+G/TZ**2

CC 97 AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)

CC 97 AA(I1,I1-1)=AA(I1,I1-1)-G2/(TX*TV)

CC 97 AA(I1,I1+IJ)=AA(I1,I1+IJ)-G2/(TX*TV)

CC 97 AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)+G2/(TX*TV)

CC 97 AA(I1,I2)=AA(I1,I2)+G2/(TX*TV*HA)

CC 97 AA(I1,I2-1)=AA(I1,I2-1)-G2/(TX*TV*HA)

CC 97 AA(I1,I2+1K)=AA(I1,I2+1K)-G2/(TX*TV*HA)

CC 97 AA(I1,I2+1K)=AA(I1,I2+1K)-G2/(TX*TV*HA)

CC 97 AA(I1,I2+1K-1)=AA(I1,I2+1K-1)+G2/(TX*TV*HA)

CC 97 AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)

CC 97 AA(I1,I1-1)=AA(I1,I1-1)+G2/(TX*TV)

CC 97 AA(I1,I1+IJ)=AA(I1,I1+IJ)-G2/(TX*TV)

CC 97 AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(TX*TV)


```

AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)
AA(I1,I1+1)=AA(I1,I1+1)-G2/(TX*TV)
AA(I1,I1+J)=AA(I1,I1+J)-G2/(TX*TV)
AA(I1,I1+J+1)=AA(I1,I1+J+1)+G2/(TX*TV)
DIF12660
DIF12670
DIF12680
DIF12690
DIF12700
DIF12710
DIF12720
DIF12730
DIF12740
DIF12750
DIF12760
DIF12770
DIF12780
DIF12790
DIF12800
DIF12810
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DIF12870
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DIF12940
DIF12950
DIF12960
DIF12970
DIF12980
DIF12990
DIF13000
DIF13010
DIF13020
DIF13030
DIF13040
DIF13050
DIF13060
DIF13070
DIF13080
DIF13090
DIF13100
DIF13110
DIF13120
DIF13130
DIF13140
DIF13150
DIF13160
DIF13170
DIF13180
DIF13190
DIF13200

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CCCCC
AA(I1,I1)=AA(I1,I1)-2.0*G/TX**2
AA(I1,I1+1)=AA(I1,I1+1)+G/TX**2
AA(I1,I1-IJ)=AA(I1,I1-IJ)+G1/TX**2
AA(I1,I1)=AA(I1,I1)-2.0*G1/TV**2
AA(I1,I1+IJ)=AA(I1,I1+IJ)+G1/TV**2
AA(I1,I1-JK)=AA(I1,I1-JK)+G/TZ**2
AA(I1,I1)=AA(I1,I1)-2.0*G/TZ**2
AA(I1,I1+JK)=AA(I1,I1+JK)+G/TZ**2
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CCCCC
AA(I1,I2)=AA(I1,I2)+G2/(TV*TZ*HA)
AA(I1,I2+IJ)=AA(I1,I2+IJ)-G2/(TV*TZ*HA)
AA(I1,I2+JK)=AA(I1,I2+JK)-G2/(TV*TZ*HA)
AA(I1,I2+JK+IJ)=AA(I1,I2+JK+IJ)+G2/(TV*TZ*HA)
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CCCCC
AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2+1)=AA(I2,I2+1)+G/(TX*TX*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+G/(TV*TV*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TV*TV*HA)
AA(I2,I2+IJ)=AA(I2,I2+IJ)+G/(TV*TV*HA)
AA(I2,I2-IK)=AA(I2,I2-IK)+G1/(TZ*TZ*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G1/(TZ*TZ*HA)
AA(I2,I2+IK)=AA(I2,I2+IK)+G1/(TZ*TZ*HA)
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GO TO 2

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C
CCC POINTS PARALLEL TO X AXIS
C

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99 AA(I,I-1)=AA(I,I-1)+G1/TX**2
AA(I,I) =AA(I,I)-2.0*G1/TX**2
AA(I,I+1)=AA(I,I+1)+G1/TX**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+IJ)=AA(I,I+IJ)+G/TV**2
AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
AA(I,I)=AA(I,I)-2.0*G/TZ**2
AA(I,I+IK)=AA(I,I+IK)+G/TZ**2
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CCCCC
AA(I,I1)=AA(I,I1)+G2/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G2/(TX*TV)
AA(I,I1+IJ)=AA(I,I1+IJ)-G2/(TX*TV)
AA(I,I1+IJ+1)=AA(I,I1+IJ+1)+G2/(TX*TV)
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AA(I, I2)=AA(I, I2)+G2/(TX*TZ*HA) DIF13210
 AA(I, I2+1)=AA(I, I2+1)-G2/(TX*TZ*HA) DIF13220
 AA(I, I2+IK)=AA(I, I2+IK)-G2/(TX*TZ*HA) DIF13230
 AA(I, I2+IK+1)=AA(I, I2+IK+1)+G2/(TX*TZ*HA) DIF13240
 AA(I, I2+IK+1)=AA(I, I2+IK+1)+G2/(TX*TZ*HA) DIF13250
 AA(I, I2+IK+1)=AA(I, I2+IK+1)+G2/(TX*TZ*HA) DIF13260

CCCCC

AA(I, I1)=AA(I, I1)+G2/(TX*TV) DIF13270
 AA(I, I1+1)=AA(I, I1+1)-G2/(TX*TV) DIF13280
 AA(I, I1+IJ)=AA(I, I1+IJ)-G2/(TX*TV) DIF13290
 AA(I, I1+IJ+1)=AA(I, I1+IJ+1)+G2/(TX*TV) DIF13300
 AA(I, I1-1)=AA(I, I1-1)+G/TV**2 DIF13310
 AA(I, I1)=AA(I, I1)-2.0*G/TV**2 DIF13320
 AA(I, I1+1)=AA(I, I1+1)+G/TV**2 DIF13330
 AA(I, I1)=AA(I, I1)-2.0*G/TV**2 DIF13340
 AA(I, I1+1)=AA(I, I1+1)+G/TV**2 DIF13350
 AA(I, I1+IJ)=AA(I, I1+IJ)+G1/TV**2 DIF13360
 AA(I, I1+IJ+1)=AA(I, I1+IJ+1)+G1/TV**2 DIF13370
 AA(I, I1-JK)=AA(I, I1-JK)+G/TZ**2 DIF13380
 AA(I, I1)=AA(I, I1)-2.0*G/TZ**2 DIF13390
 AA(I, I1+JK)=AA(I, I1+JK)+G/TZ**2 DIF13400

CCCCC

AA(I, I1, I2)=AA(I, I1, I2)+G2/(TV*TZ*HA) DIF13410
 AA(I, I1, I2+IJ)=AA(I, I1, I2+IJ)-G2/(TV*TZ*HA) DIF13420
 AA(I, I1, I2+JK)=AA(I, I1, I2+JK)-G2/(TV*TZ*HA) DIF13430
 AA(I, I1, I2+JK+IJ)=AA(I, I1, I2+JK+IJ)+G2/(TV*TZ*HA) DIF13440
 AA(I, I1, I2+JK+IJ)=AA(I, I1, I2+JK+IJ)+G2/(TV*TZ*HA) DIF13450
 AA(I, I2, I1)=AA(I, I2, I1)+G2/(TX*TZ) DIF13460
 AA(I, I2, I1+1)=AA(I, I2, I1+1)-G2/(TX*TZ) DIF13470
 AA(I, I2, I1+IK)=AA(I, I2, I1+IK)-G2/(TX*TZ) DIF13480
 AA(I, I2, I1+JK+IJ)=AA(I, I2, I1+JK+IJ)+G2/(TX*TZ) DIF13490
 AA(I, I2, I1+JK+IJ)=AA(I, I2, I1+JK+IJ)+G2/(TX*TZ) DIF13500

CCCCC

AA(I, I2, I1)=AA(I, I2, I1)+G2/(TV*TZ) DIF13510
 AA(I, I2, I1+IJ)=AA(I, I2, I1+IJ)-G2/(TV*TZ) DIF13520
 AA(I, I2, I1+JK)=AA(I, I2, I1+JK)-G2/(TV*TZ) DIF13530
 AA(I, I2, I1+JK+IJ)=AA(I, I2, I1+JK+IJ)+G2/(TV*TZ) DIF13540
 AA(I, I2, I1+JK+IJ)=AA(I, I2, I1+JK+IJ)+G2/(TV*TZ) DIF13550
 AA(I, I2, I2-1)=AA(I, I2, I2-1)+G/(TX*TX*HA) DIF13560
 AA(I, I2, I2)=AA(I, I2, I2)-2.0*G/(TX*TX*HA) DIF13570
 AA(I, I2, I2+1)=AA(I, I2, I2+1)+G/(TX*TX*HA) DIF13580
 AA(I, I2, I2+1)=AA(I, I2, I2+1)+G/(TX*TX*HA) DIF13590
 AA(I, I2, I2)=AA(I, I2, I2)-2.0*G/(TV*TV*HA) DIF13600
 AA(I, I2, I2+IJ)=AA(I, I2, I2+IJ)+G/(TV*TV*HA) DIF13610
 AA(I, I2, I2-1K)=AA(I, I2, I2-1K)+G1/(TZ*TZ*HA) DIF13620
 AA(I, I2, I2)=AA(I, I2, I2)-2.0*G1/(TZ*TZ*HA) DIF13630
 AA(I, I2, I2+1K)=AA(I, I2, I2+1K)+G1/(TZ*TZ*HA) DIF13640

GO TO 2

C
 CCC CORNER POINT

100 AA(I, I1) =AA(I, I1)-2.0*G1/TX**2 DIF13650
 AA(I, I1+1)=AA(I, I1+1)+G1/TX**2 DIF13660
 AA(I, I1)=AA(I, I1)-2.0*G/TV**2 DIF13670
 AA(I, I1+IJ)=AA(I, I1+IJ)+G/TV**2 DIF13680
 AA(I, I1+IJ)=AA(I, I1+IJ)+G/TV**2 DIF13690
 AA(I, I1-1K)=AA(I, I1-1K)+G/TZ**2 DIF13700
 AA(I, I1)=AA(I, I1)-2.0*G/TZ**2 DIF13710
 AA(I, I1+IK)=AA(I, I1+IK)+G/TZ**2 DIF13720
 AA(I, I1+IK)=AA(I, I1+IK)+G/TZ**2 DIF13730
 AA(I, I1+IK)=AA(I, I1+IK)+G/TZ**2 DIF13740
 AA(I, I1+IK)=AA(I, I1+IK)+G/TZ**2 DIF13750

CCCCC

AA(I, I)=AA(I, I)+G2/(TX*TV)
 AA(I, I+1)=AA(I, I+1)-G2/(TX*TV)
 AA(I, I+I)=AA(I, I+I)-G2/(TX*TV)
 AA(I, I+I+I)=AA(I, I+I+I)+G2/(TX*TV)

DIFF13760
 DIFF13770
 DIFF13780
 DIFF13790
 DIFF13800
 DIFF13810

AA(I, I2)=AA(I, I2)+G2/(TX*TZ*HA)
 AA(I, I2+1)=AA(I, I2+1)-G2/(TX*TZ*HA)
 AA(I, I2+IK)=AA(I, I2+IK)-G2/(TX*TZ*HA)
 AA(I, I2+IK+1)=AA(I, I2+IK+1)+G2/(TX*TZ*HA)

DIFF13820
 DIFF13830
 DIFF13840
 DIFF13850
 DIFF13860

CCCCC

AA(I, I)=AA(I, I)+G2/(TX*TV)
 AA(I, I+1)=AA(I, I+1)-G2/(TX*TV)
 AA(I, I+I)=AA(I, I+I)-G2/(TX*TV)
 AA(I, I+I+I)=AA(I, I+I+I)+G2/(TX*TV)

DIFF13870
 DIFF13880
 DIFF13890
 DIFF13900
 DIFF13910
 DIFF13920

CCCCC

AA(I, I, I)=AA(I, I, I)-2.0*G/TX**2
 AA(I, I, I+1)=AA(I, I, I+1)+G/TX**2
 AA(I, I, I+I)=AA(I, I, I+I)-2.0*G/TX**2
 AA(I, I, I+I+I)=AA(I, I, I+I+I)+G/TX**2
 AA(I, I, I+I+I+I)=AA(I, I, I+I+I+I)+G/TX**2
 AA(I, I, I+I+I+I+I)=AA(I, I, I+I+I+I+I)+G/TX**2
 AA(I, I, I+I+I+I+I+I)=AA(I, I, I+I+I+I+I+I)+G/TX**2

DIFF13930
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 DIFF13980
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 DIFF14000

AA(I, I, I2)=AA(I, I, I2)+G2/(TV*TZ*HA)
 AA(I, I, I2+1)=AA(I, I, I2+1)-G2/(TV*TZ*HA)
 AA(I, I, I2+JK)=AA(I, I, I2+JK)-G2/(TV*TZ*HA)
 AA(I, I, I2+JK+I)=AA(I, I, I2+JK+I)+G2/(TV*TZ*HA)

DIFF14010
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 DIFF14090

CCCCC

AA(I2, I)=AA(I2, I)+G2/(TX*TZ)
 AA(I2, I+1)=AA(I2, I+1)-G2/(TX*TZ)
 AA(I2, I+IK)=AA(I2, I+IK)-G2/(TX*TZ)
 AA(I2, I+IK+1)=AA(I2, I+IK+1)+G2/(TX*TZ)

DIFF14100
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 DIFF14170
 DIFF14180

AA(I2, I2)=AA(I2, I2)-2.0*G/(TX*TV*HA)
 AA(I2, I2+1)=AA(I2, I2+1)+G/(TX*TV*HA)
 AA(I2, I2+I)=AA(I2, I2+I)-2.0*G/(TX*TV*HA)
 AA(I2, I2+I+I)=AA(I2, I2+I+I)+G/(TX*TV*HA)
 AA(I2, I2+I+I+I)=AA(I2, I2+I+I+I)+G/(TX*TV*HA)
 AA(I2, I2+I+I+I+I)=AA(I2, I2+I+I+I+I)+G/(TX*TV*HA)
 AA(I2, I2+I+I+I+I+I)=AA(I2, I2+I+I+I+I+I)+G/(TX*TV*HA)

DIFF14190
 DIFF14200
 DIFF14210
 DIFF14220
 DIFF14230
 DIFF14240
 DIFF14250
 DIFF14260
 DIFF14270

GO TO 2

C

C

C

GENERAL POINTS
 101 AA(I, I-1)=AA(I, I-1)+G1/TX**2
 AA(I, I)=AA(I, I)-2.0*G1/TX**2
 AA(I, I+1)=AA(I, I+1)+G1/TX**2
 AA(I, I-I)=AA(I, I-I)+G1/TX**2

DIFF14280
 DIFF14290
 DIFF14300

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AA(I, I)=AA(I, I)-2.0*G/TV**2
AA(I, I+I)=AA(I, I+I)*G/TV**2
AA(I, I-IK)=AA(I, I-IK)+G/TZ**2
AA(I, I)=AA(I, I)-2.0*G/TZ**2
AA(I, I+IK)=AA(I, I+IK)+G/TZ**2

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CCCCC
AA(I, I)=AA(I, I)+GZ/(TX*TV)
AA(I, I+I)=AA(I, I+I)-GZ/(TX*TV)
AA(I, I+I+I)=AA(I, I+I+I)-GZ/(TX*TV)
AA(I, I+I+I+I)=AA(I, I+I+I+I)+GZ/(TX*TV)

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AA(I, I2)=AA(I, I2)+GZ/(TX*TZ*HA)
AA(I, I2+I)=AA(I, I2+I)-GZ/(TX*TZ*HA)
AA(I, I2+IK)=AA(I, I2+IK)-GZ/(TX*TZ*HA)
AA(I, I2+IK+I)=AA(I, I2+IK+I)+GZ/(TX*TZ*HA)

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CCCCC
AA(I1, I1)=AA(I1, I1)+GZ/(TX*TV)
AA(I1, I1+I)=AA(I1, I1+I)-GZ/(TX*TV)
AA(I1, I1+I+I)=AA(I1, I1+I+I)-GZ/(TX*TV)
AA(I1, I1+I+I+I)=AA(I1, I1+I+I+I)+GZ/(TX*TV)

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CCCCC
AA(I1, I1-1)=AA(I1, I1-1)+G/TV**2
AA(I1, I1)=AA(I1, I1)-2.0*G/TV**2
AA(I1, I1+1)=AA(I1, I1+1)+G/TV**2
AA(I1, I1-1-1)=AA(I1, I1-1-1)+G1/TV**2
AA(I1, I1)=AA(I1, I1)-2.0*G1/TV**2
AA(I1, I1+1+1)=AA(I1, I1+1+1)+G1/TV**2
AA(I1, I1-1-1-1)=AA(I1, I1-1-1-1)+G1/TV**2
AA(I1, I1)=AA(I1, I1)-2.0*G1/TV**2
AA(I1, I1+1+1+1)=AA(I1, I1+1+1+1)+G1/TV**2
AA(I1, I1-1-1-1-1)=AA(I1, I1-1-1-1-1)+G1/TV**2

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CCCCC
AA(I1, I2)=AA(I1, I2)+GZ/(TV*TZ*HA)
AA(I1, I2+I)=AA(I1, I2+I)-GZ/(TV*TZ*HA)
AA(I1, I2+IK)=AA(I1, I2+IK)-GZ/(TV*TZ*HA)
AA(I1, I2+IK+I)=AA(I1, I2+IK+I)+GZ/(TV*TZ*HA)

```

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DIF14850

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```

AA(I2, I1)=AA(I2, I1)+GZ/(TX*TZ)
AA(I2, I1+I)=AA(I2, I1+I)-GZ/(TX*TZ)
AA(I2, I1+IK)=AA(I2, I1+IK)-GZ/(TX*TZ)
AA(I2, I1+IK+I)=AA(I2, I1+IK+I)+GZ/(TX*TZ)

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DIF14840
DIF14850

```

```

CCCCC
AA(I2, I2-1)=AA(I2, I2-1)+G/(TX*TX*HA)
AA(I2, I2)=AA(I2, I2)-2.0*G/(TX*TX*HA)
AA(I2, I2+1)=AA(I2, I2+1)+G/(TX*TX*HA)
AA(I2, I2-1-1)=AA(I2, I2-1-1)+G/(TV*TV*HA)
AA(I2, I2)=AA(I2, I2)-2.0*G/(TV*TV*HA)
AA(I2, I2+1+1)=AA(I2, I2+1+1)+G/(TV*TV*HA)
AA(I2, I2-1-1-1)=AA(I2, I2-1-1-1)+G/(TV*TV*HA)
AA(I2, I2)=AA(I2, I2)-2.0*G1/(TZ*TZ*HA)
AA(I2, I2+1+1+1)=AA(I2, I2+1+1+1)+G1/(TZ*TZ*HA)
AA(I2, I2-1-1-1-1)=AA(I2, I2-1-1-1-1)+G1/(TZ*TZ*HA)

```

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DIF14690
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DIF14780
DIF14790
DIF14800
DIF14810
DIF14820
DIF14830
DIF14840
DIF14850

```

AA(I2,I2+IK)=AA(I2,I2+IK)+G1/(TZ*TZ*HA)
 GO TO 2

C
 CCC POINTS ON THE V AXIS

CC102 AA(I,I-1)=AA(I,I-1)+2.0*G1/TX**2
 102 AA(I,I)=AA(I,I)-2.0*G1/TX**2
 AA(I,I-1J)=AA(I,I-1J)+G/TV**2
 AA(I,I)=AA(I,I)-2.0*G/TV**2
 AA(I,I+1J)=AA(I,I+1J)+G/TV**2
 AA(I,I-1K)=AA(I,I-1K)+G/TZ**2
 AA(I,I)=AA(I,I)-2.0*G/TZ**2
 AA(I,I+1K)=AA(I,I+1K)+G/TZ**2

CCCCC

AA(I,I1)=AA(I,I1)+G2/(TX*TV)
 AA(I,I1-1)=AA(I,I1-1)-G2/(TX*TV)
 AA(I,I1+1J)=AA(I,I1+1J)-G2/(TX*TV)
 AA(I,I1+1J-1)=AA(I,I1+1J-1)+G2/(TX*TV)
 AA(I,I2)=AA(I,I2)+G2/(TX*TZ*HA)
 AA(I,I2-1)=AA(I,I2-1)-G2/(TX*TZ*HA)
 AA(I,I2+1K)=AA(I,I2+1K)-G2/(TX*TZ*HA)
 AA(I,I2+1K-1)=AA(I,I2+1K-1)+G2/(TX*TZ*HA)

CCCCC

AA(I,I,I)=AA(I,I,I)+G2/(TX*TV)
 AA(I,I,I-1)=AA(I,I,I-1)+G2/(TX*TV)
 AA(I,I,I+1J)=AA(I,I,I+1J)-G2/(TX*TV)
 AA(I,I,I+1J-1)=AA(I,I,I+1J-1)-G2/(TX*TV)
 AA(I,I,I-1)=AA(I,I,I-1)+2.0*G/TX**2
 AA(I,I,I)=AA(I,I,I)-2.0*G/TX**2
 AA(I,I,I-1J)=AA(I,I,I-1J)+G1/TV**2
 AA(I,I,I)=AA(I,I,I)-2.0*G1/TV**2
 AA(I,I,I+1J)=AA(I,I,I+1J)+G1/TV**2
 AA(I,I,I-1K)=AA(I,I,I-1K)+G/TZ**2
 AA(I,I,I)=AA(I,I,I)-2.0*G/TZ**2
 AA(I,I,I+1K)=AA(I,I,I+1K)+G/TZ**2

CCCCC

AA(I,I,I2)=AA(I,I,I2)+G2/(TV*TZ*HA)
 AA(I,I,I2+1J)=AA(I,I,I2+1J)-G2/(TV*TZ*HA)
 AA(I,I,I2+1J-1)=AA(I,I,I2+1J-1)+G2/(TV*TZ*HA)
 AA(I,I,I2+1JK)=AA(I,I,I2+1JK)+G2/(TV*TZ*HA)
 AA(I,I,I2+1JK-1)=AA(I,I,I2+1JK-1)-G2/(TV*TZ*HA)

CCCCC

AA(I2,I,I1)=AA(I2,I,I1)+G2/(TV*TZ)
 AA(I2,I,I1-1)=AA(I2,I,I1-1)+G2/(TV*TZ)
 AA(I2,I,I1+1K)=AA(I2,I,I1+1K)-G2/(TV*TZ)
 AA(I2,I,I1+1K-1)=AA(I2,I,I1+1K-1)-G2/(TV*TZ)
 AA(I2,I,I1)=AA(I2,I,I1)+G2/(TV*TZ)
 AA(I2,I,I1+1J)=AA(I2,I,I1+1J)-G2/(TV*TZ)
 AA(I2,I,I1+1JK)=AA(I2,I,I1+1JK)-G2/(TV*TZ)
 AA(I2,I,I1+1JK)=AA(I2,I,I1+1JK)+G2/(TV*TZ)

CCCCC

AA(I2,I2-1)=AA(I2,I2-1)+2.0*G/(TX*TX*HA)
 DIF14860
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 DIF15380
 DIF15390
 DIF15400

```

AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+G/(TV*TV*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TV*TV*HA)
AA(I2,I2+IJ)=AA(I2,I2+IJ)+G/(TV*TV*HA)
AA(I2,I2-IK)=AA(I2,I2-IK)+G/(TZ*TZ*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TZ*TZ*HA)
AA(I2,I2+IK)=AA(I2,I2+IK)+G/(TZ*TZ*HA)

```

GO TO 2

C
CCC POINTS ON THE X AXIS

```

103 AA(I,I-1)=AA(I,I-1)+G1/TX**2
AA(I,I)=AA(I,I)-2.0*G1/TX**2
AA(I,I+1)=AA(I,I+1)+G1/TX**2
AA(I,I-IJ)=AA(I,I-IJ)+2.0*G/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+IJ)=AA(I,I+IJ)+2.0*G/TV**2
AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
AA(I,I)=AA(I,I)-2.0*G/TZ**2
AA(I,I+IK)=AA(I,I+IK)+G/TZ**2

```

CCCCC
AA(I,I1)=AA(I,I1)+G2/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G2/(TX*TV)
AA(I,I1-IJ)=AA(I,I1-IJ)+G2/(TX*TV)
AA(I,I1-IJ+1)=AA(I,I1-IJ+1)-G2/(TX*TV)

```

AA(I,I2)=AA(I,I2)+G2/(TX*TV)
AA(I,I2+1)=AA(I,I2+1)-G2/(TX*TV*HA)
AA(I,I2+IK)=AA(I,I2+IK)-G2/(TX*TV*HA)
AA(I,I2+IK+1)=AA(I,I2+IK+1)+G2/(TX*TV*HA)

```

CCCCC
AA(I,I1-I1)=AA(I,I1-I1)+G/TX**2
AA(I,I1)=AA(I,I1)-2.0*G/TX**2
AA(I,I1+1)=AA(I,I1+1)+G/TX**2
AA(I,I1-IJ)=AA(I,I1-IJ)+2.0*G/TV**2
AA(I,I1)=AA(I,I1)-2.0*G/TV**2
AA(I,I1+JK)=AA(I,I1+JK)+G/TZ**2
AA(I,I1)=AA(I,I1)-2.0*G/TZ**2

CC
AA(I,I1-IJ)=AA(I,I1-IJ)+2.0*G/TV**2
AA(I,I1)=AA(I,I1)-2.0*G/TV**2
AA(I,I1+JK)=AA(I,I1+JK)+G/TZ**2
AA(I,I1)=AA(I,I1)-2.0*G/TZ**2

CCCCC
AA(I,I1,I2)=AA(I,I1,I2)+G2/(TV*TV*HA)
AA(I,I1,I2-IJ)=AA(I,I1,I2-IJ)-G2/(TV*TV*HA)
AA(I,I1,I2+JK)=AA(I,I1,I2+JK)-G2/(TV*TV*HA)
AA(I,I1,I2+JK-IJ)=AA(I,I1,I2+JK-IJ)+G2/(TV*TV*HA)

```

AA(I2,I1)=AA(I2,I1)+G2/(TX*TZ)
AA(I2,I1+1)=AA(I2,I1+1)-G2/(TX*TZ)
AA(I2,I1+IK)=AA(I2,I1+IK)-G2/(TX*TZ)
AA(I2,I1+IK+1)=AA(I2,I1+IK+1)+G2/(TX*TZ)

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- DIF15410
- DIF15420
- DIF15430
- DIF15440
- DIF15450
- DIF15460
- DIF15470
- DIF15480
- DIF15490
- DIF15500
- DIF15510
- DIF15520
- DIF15530
- DIF15540
- DIF15550
- DIF15560
- DIF15570
- DIF15580
- DIF15590
- DIF15600
- DIF15610
- DIF15620
- DIF15630
- DIF15640
- DIF15650
- DIF15660
- DIF15670
- DIF15680
- DIF15690
- DIF15700
- DIF15710
- DIF15720
- DIF15730
- DIF15740
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- DIF15870
- DIF15880
- DIF15890
- DIF15900
- DIF15910
- DIF15920
- DIF15930
- DIF15940
- DIF15950

```

AA(I2,I1)=AA(I2,I1)+G2/(TV*TZ)
AA(I2,I1-IJ)=AA(I2,I1-IJ)+G2/(TV*TZ)
AA(I2,I1+JK)=AA(I2,I1+JK)-G2/(TV*TZ)
AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)-G2/(TV*TZ)

```

```

CCCCC
AA(I2,I2-1)=AA(I2,I2-1)+G/(TX*TX*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2+1)=AA(I2,I2+1)+G/(TX*TX*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(TV*TV*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TV*TV*HA)
AA(I2,I2-IK)=AA(I2,I2-IK)+G1/(TZ*TZ*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G1/(TZ*TZ*HA)
AA(I2,I2+IK)=AA(I2,I2+IK)+G1/(TZ*TZ*HA)

```

GO TO 2

C CCC CENTRAL POINT

```

CC104 AA(I,I-1)=AA(I,I-1)+2.0*G1/TX**2
104 AA(I,I)=AA(I,I)-2.0*G1/TX**2
AA(I,I-IJ)=AA(I,I-IJ)+2.0*G1/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
AA(I,I)=AA(I,I)-2.0*G1/TZ**2
AA(I,I+IK)=AA(I,I+IK)+G/TZ**2

```

```

CCCCC
AA(I,I1)=AA(I,I1)+G2/(TX*TV)
AA(I,I1-1)=AA(I,I1-1)-G2/(TX*TV)
AA(I,I1-IJ)=AA(I,I1-IJ)+G2/(TX*TV)
AA(I,I1-IJ-1)=AA(I,I1-IJ-1)-G2/(TX*TV)

```

```

AA(I,I2)=AA(I,I2)+G2/(TX*TZ*HA)
AA(I,I2-1)=AA(I,I2-1)-G2/(TX*TZ*HA)
AA(I,I2+IK)=AA(I,I2+IK)-G2/(TX*TZ*HA)
AA(I,I2+IK-1)=AA(I,I2+IK-1)+G2/(TX*TZ*HA)

```

```

AA(I,I)=AA(I,I)+G2/(TX*TV)
AA(I,I-1)=AA(I,I-1)+G2/(TX*TV)
AA(I,I-IJ)=AA(I,I-IJ)-G2/(TX*TV)
AA(I,I-IJ-1)=AA(I,I-IJ-1)-G2/(TX*TV)

```

```

CCCCC
AA(I,I1-1)=AA(I,I1-1)+2.0*G/TX**2
AA(I,I1)=AA(I,I1)-2.0*G/TX**2
AA(I,I1-IJ)=AA(I,I1-IJ)+2.0*G1/TV**2
AA(I,I1)=AA(I,I1)-2.0*G1/TV**2
AA(I,I1-IJK)=AA(I,I1-IJK)+G/TZ**2
AA(I,I1)=AA(I,I1)-2.0*G/TZ**2
AA(I,I1+JK)=AA(I,I1+JK)+G/TZ**2

```

```

CCCCC
AA(I,I,I2)=AA(I,I,I2)+G2/(TV*TZ*HA)
AA(I,I,I2-IJ)=AA(I,I,I2-IJ)-G2/(TV*TZ*HA)
AA(I,I,I2+JK)=AA(I,I,I2+JK)-G2/(TV*TZ*HA)
AA(I,I,I2+JK-IJ)=AA(I,I,I2+JK-IJ)+G2/(TV*TZ*HA)
AA(I2,I)=AA(I2,I)+G2/(TX*TZ)

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- DIF15960
- DIF15970
- DIF15980
- DIF15990
- DIF16000
- DIF16010
- DIF16020
- DIF16030
- DIF16040
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- DIF16330
- DIF16340
- DIF16350
- DIF16360
- DIF16370
- DIF16380
- DIF16390
- DIF16400
- DIF16410
- DIF16420
- DIF16430
- DIF16440
- DIF16450
- DIF16460
- DIF16470
- DIF16480
- DIF16490
- DIF16500

```

AA(I2,I-1)=AA(I2,I-1)+G2/(TX*TZ)
AA(I2,I+IK)=AA(I2,I+IK)-G2/(TX*TZ)
AA(I2,I+IK-1)=AA(I2,I+IK-1)-G2/(TX*TZ)
AA(I2,I1)=AA(I2,I1)+G2/(TV*TZ)
AA(I2,I1-IJ)=AA(I2,I1-IJ)+G2/(TV*TZ)
AA(I2,I1+JK)=AA(I2,I1+JK)-G2/(TV*TZ)
AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-G2/(TV*TZ)
CCCCC
AA(I2,I2-1)=AA(I2,I2-1)+2.0*G/(TX*TX*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(TV*TV*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(TV*TV*HA)
AA(I2,I2-IK)=AA(I2,I2-IK)+G1/((TZ*TZ*HA))
AA(I2,I2)=AA(I2,I2)-2.0*G1/((TZ*TZ*HA))
AA(I2,I2+IK)=AA(I2,I2+IK)+G1/((TZ*TZ*HA))
GO TO 2
C
CCC GENERAL POINT (B.L)
C
105 AA(I,I-1)=AA(I,I-1)+G11/TX**2
AA(I,I)=AA(I,I)-2.0*G11/TX**2
AA(I,I+1)=AA(I,I+1)+G11/TX**2
AA(I,I-IJ)=AA(I,I-IJ)+G/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+IJ)=AA(I,I+IJ)+G/TV**2
IRI=ISIGN(1,IK)
AA(I,I-1-IRI)*ABS(IK)=AA(I,I-1-IRI)*ABS(IK)+(-G3)/TZ**2
AA(I,I+IRI)*ABS(IK)=AA(I,I+IRI)*ABS(IK)-2.0*(-G3)/TZ**2
AA(I,I+(1+IRI)*ABS(IK))=AA(I,I+(1+IRI)*ABS(IK))+(-G3)/TZ**2
C
AA(I,I1)=AA(I,I1)+G22/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G22/(TX*TV)
AA(I,I1+IJ)=AA(I,I1+IJ)-G22/(TX*TV)
AA(I,I1+IJ+1)=AA(I,I1+IJ+1)+G22/(TX*TV)
AA(I,I1-IJ)=AA(I,I1-IJ)+G22/(TX*TV)
AA(I,I1+1-IJ)=AA(I,I1+1-IJ)+G/TX**2
AA(I,I1-IJ)=AA(I,I1-IJ)+G11/TV**2
AA(I,I1)=AA(I,I1)-2.0*G11/TV**2
AA(I,I1+IJ)=AA(I,I1+IJ)+G11/TV**2
IR2=ISIGN(1,JK)
AA(I,I1-1-IR2)*ABS(JK)=AA(I,I1-1-IR2)*ABS(JK)+(-G3)/TZ**2
AA(I,I1+1+IR2)*ABS(JK)=AA(I,I1+1+IR2)*ABS(JK)-2.0*(-G3)/TZ**2
AA(I,I1+1+IR2)*ABS(JK)=AA(I,I1+1+IR2)*ABS(JK)+(-G3)/TZ**2
CALL SCHE(NA,IM,JM,NOD,ITEST,NP)
DO 61 I1=1,13

```

```

DIF16510
DIF16520
DIF16530
DIF16540
DIF16550
DIF16560
DIF16570
DIF16580
DIF16590
DIF16600
DIF16610
DIF16620
DIF16630
DIF16640
DIF16650
DIF16660
DIF16670
DIF16680
DIF16690
DIF16700
DIF16710
DIF16720
DIF16730
DIF16740
DIF16750
DIF16760
DIF16770
DIF16780
DIF16790
DIF16800
DIF16810
DIF16820
DIF16830
DIF16840
DIF16850
DIF16860
DIF16870
DIF16880
DIF16890
DIF16900
DIF16910
DIF16920
DIF16930
DIF16940
DIF16950
DIF16960
DIF16970
DIF16980
DIF16990
DIF17000
DIF17010
DIF17020
DIF17030
DIF17040
DIF17050

```

```

NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 61
IF(NP(II).NE.0) GO TO 62
62 AA(I2,NPA(II)+2*NOD)=AA(I2,NP(II)+2*NOD)+(NP(II)/NPA(II))*R(II)
61 CONTINUE
GO TO 2

C
CCC POINTS ON THE V AXIS (B.L)
C
CC106 AA(I,1-1)=AA(I,1-1)+2.0*G11/TX**2
106 AA(I,1)=AA(I,1)-2.0*G11/TX**2
AA(I,1-IJ)=AA(I,1-IJ)+G/TV**2
AA(I,1)=AA(I,1)-2.0*G/TV**2
AA(I,1+IJ)=AA(I,1+IJ)+G/TV**2
C
IR1=ISIGN(1,IK)
AA(I,1-(1-IR1)*ABS(IK))=AA(I,1-(1-IR1)*ABS(IK))+(-G3)/TZ**2
C
AA(I,1+IR1*ABS(IK))=AA(I,1+IR1*ABS(IK))-2.0*(-G3)/TZ**2
C
AA(I,1+(1+IR1)*ABS(IK))=AA(I,1+(1+IR1)*ABS(IK))+(-G3)/TZ**2
C
AA(I,11)=AA(I,11)+G22/(TX*TV)
AA(I,11-1)=AA(I,11-1)+G22/(TX*TV)
AA(I,1+IJ)=AA(I,1+IJ)-G22/(TX*TV)
AA(I,1+IJ-1)=AA(I,1+IJ-1)-G22/(TX*TV)
AA(I,11+IJ)=AA(I,11+IJ)+G11/TV**2
AA(I,11-1)=AA(I,11-1)+2.0*G/TX**2
AA(I,11)=AA(I,11)-2.0*G/TX**2
AA(I,11-IJ)=AA(I,11-IJ)+G11/TV**2
AA(I,11)=AA(I,11)-2.0*G11/TV**2
AA(I,11+IJ)=AA(I,11+IJ)+G11/TV**2
IR2=ISIGN(1,JK)
AA(I,11-(1-IR2)*ABS(JK))=AA(I,11-(1-IR2)*ABS(JK))+(-G3)/TZ**2
C
AA(I,11+IR2*ABS(JK))=AA(I,11+IR2*ABS(JK))-2.0*(-G3)/TZ**2
C
AA(I,11+(1+IR2)*ABS(JK))=AA(I,11+(1+IR2)*ABS(JK))+(-G3)/TZ**2
C
CALL SCHE(NA,IM,JM,NOD,ITEST,NP)
DO 63 II=1,13
NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 63
IF(NP(II).NE.0) GO TO 64
64 AA(I2,NPA(II)+2*NOD)=AA(I2,NP(II)+2*NOD)+(NP(II)/NPA(II))*R(II)
63 CONTINUE
GO TO 2

C
CCC POINTS ON THE X AXIS (B.L)
C
107 AA(I,1-1)=AA(I,1-1)+G11/TX**2
AA(I,1)=AA(I,1)-2.0*G11/TX**2
AA(I,1+1)=AA(I,1+1)+G11/TV**2

```

DIF17060
DIF17070
DIF17080
DIF17090
DIF17100
DIF17110
DIF17120
DIF17130
DIF17140
DIF17150
DIF17160
DIF17170
DIF17180
DIF17190
DIF17200
DIF17210
DIF17220
DIF17230
DIF17240
DIF17250
DIF17260
DIF17270
DIF17280
DIF17290
DIF17300
DIF17310
DIF17320
DIF17330
DIF17340
DIF17350
DIF17360
DIF17370
DIF17380
DIF17390
DIF17400
DIF17410
DIF17420
DIF17430
DIF17440
DIF17450
DIF17460
DIF17470
DIF17480
DIF17490
DIF17500
DIF17510
DIF17520
DIF17530
DIF17540
DIF17550
DIF17560
DIF17570
DIF17580
DIF17590
DIF17600


```

IF(NP(II).EQ.0) GO TO 69
IF(NP(II).NE.0) GO TO 70
70 AA(I2,NPA(II)+2*NOD)=AA(I2,NP(II)+2*NOD)+(NP(II)/NPA(II))*R(II)
69 CONTINUE

```

GO TO 2

```

C
CCC POINTS PARALLEL TO X AXIS (B.L.)

```

110

```

AA(I,I-1)=AA(I,I-1)+G11/TV**2
AA(I,I)=AA(I,I)-2.0*G11/TV**2
AA(I,I+1)=AA(I,I+1)+G11/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+J)=AA(I,I+J)+G/TV**2
IR1=ISIGN(1,IK)
AA(I,I-(-1-IR1)*ABS(IK))=AA(I,I-(-1-IR1)*ABS(IK))+(-G3)/TZ**2
AA(I,I+IR1*ABS(IK))=AA(I,I+IR1*ABS(IK))-2.0*(-G3)/TZ**2
AA(I,I+(1+IR1)*ABS(IK))=AA(I,I+(1+IR1)*ABS(IK))+(-G3)/TZ**2

```

```

AA(I,I1)=AA(I,I1)+G22/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G22/(TX*TV)
AA(I,I1+J)=AA(I,I1+J)-G22/(TX*TV)
AA(I,I1+J+1)=AA(I,I1+J+1)+G22/(TX*TV)

```

```

AA(I1,I1)=AA(I1,I1)+G22/(TX*TV)
AA(I1,I1+1)=AA(I1,I1+1)-G22/(TX*TV)
AA(I1,I1+J)=AA(I1,I1+J)-G22/(TX*TV)
AA(I1,I1+J+1)=AA(I1,I1+J+1)+G22/(TX*TV)

```

```

AA(I1,I1-1)=AA(I1,I1-1)+G/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1+1)=AA(I1,I1+1)+G/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G11/TV**2
AA(I1,I1+J)=AA(I1,I1+J)+G11/TV**2
IR2=ISIGN(1,JK)
AA(I1,I1-(-1-IR2)*ABS(JK))=AA(I1,I1-(-1-IR2)*ABS(JK))+(-G3)/TZ**2
AA(I1,I1+IR2*ABS(JK))=AA(I1,I1+IR2*ABS(JK))-2.0*(-G3)/TZ**2
AA(I1,I1+(1+IR2)*ABS(JK))=AA(I1,I1+(1+IR2)*ABS(JK))+(-G3)/TZ**2

```

```

C
CALL SCHE(NA,IM,JM,NOD,ITEST,NP)

```

```

DO 71 I1=1,13
NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 71
IF(NP(II).NE.0) GO TO 72
72 AA(I12,NPA(II)+2*NOD)=AA(I2,NP(II)+2*NOD)+(NP(II)/NPA(II))*R(II)
71 CONTINUE

```

GO TO 2

```

C
CCC POINTS PARALLEL TO V AXIS

```

```

111 AA(I,I)=AA(I,I)-2.0*G11/TV**2
AA(I,I+1)=AA(I,I+1)+G11/TV**2
AA(I,I-IJ)=AA(I,I-IJ)+G/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2

```

DIFF18710
DIFF18720
DIFF18730
DIFF18740
DIFF18750
DIFF18760
DIFF18770
DIFF18780
DIFF18790
DIFF18800
DIFF18810
DIFF18820
DIFF18830
DIFF18840
DIFF18850
DIFF18860
DIFF18870
DIFF18880
DIFF18890
DIFF18900
DIFF18910
DIFF18920
DIFF18930
DIFF18940
DIFF18950
DIFF18960
DIFF18970
DIFF18980
DIFF18990
DIFF19000
DIFF19010
DIFF19020
DIFF19030
DIFF19040
DIFF19050
DIFF19060
DIFF19070
DIFF19080
DIFF19090
DIFF19100
DIFF19110
DIFF19120
DIFF19130
DIFF19140
DIFF19150
DIFF19160
DIFF19170
DIFF19180
DIFF19190
DIFF19200
DIFF19210
DIFF19220
DIFF19230
DIFF19240
DIFF19250

IF(NP(II).NE.0) GO TO 78
 78 AA(I2,NPA(II))+2*NOD)=AA(I2,NP(II))+2*NOD)+(NP(II)/NPA(II))*R(II)
 77 CONTINUE

DIF20360
 DIF20370
 DIF20380
 DIF20390

GO TO 2
 114 AA(I,I-1K)=AA(I,I-1K)+G/TZ**2
 AA(I,I)=AA(I,I)-2.*G/TZ**2
 AA(I,I+1K)=AA(I,I+1K)+G/TZ**2

DIF20400
 DIF20410
 DIF20420
 DIF20430
 DIF20440

AA(I,I)=AA(I,I)+G3/(TX*TV)
 AA(I,I+1)=AA(I,I+1)-G3/(TX*TV)
 AA(I,I+1J)=AA(I,I+1J)-G3/(TX*TV)
 AA(I,I+1J+1)=AA(I,I+1J+1)+G3/(TX*TV)

DIF20450
 DIF20460
 DIF20470
 DIF20480
 DIF20490

AA(I,I+1J)=AA(I,I+1J)-2.*G1/TV**2
 AA(I,I+1J+1)=AA(I,I+1J+1)+G1/TV**2
 AA(I,I,I-1K)=AA(I,I,I-1K)+G/TZ**2
 AA(I,I,I)=AA(I,I,I)-2.*G/TZ**2
 AA(I,I,I+1K)=AA(I,I,I+1K)+G/TZ**2

DIF20500
 DIF20510
 DIF20520
 DIF20530
 DIF20540
 DIF20550

GO TO 2
 115 AA(I,I-1K)=AA(I,I-1K)+G/TZ**2
 AA(I,I)=AA(I,I)-2.*G/TZ**2
 AA(I,I+1K)=AA(I,I+1K)+G/TZ**2

DIF20560
 DIF20570
 DIF20580
 DIF20590

AA(I,I,I)=AA(I,I,I)+G3/(TX*TV)
 AA(I,I,I+1)=AA(I,I,I+1)-G3/(TX*TV)
 AA(I,I,I+1J)=AA(I,I,I+1J)-G3/(TX*TV)
 AA(I,I,I+1J+1)=AA(I,I,I+1J+1)+G3/(TX*TV)

DIF20600
 DIF20610
 DIF20620
 DIF20630
 DIF20640
 DIF20650

AA(I,I,I-1J)=AA(I,I,I-1J)+G1/TV**2
 AA(I,I,I)=AA(I,I,I)-2.*G1/TV**2
 AA(I,I,I+1J)=AA(I,I,I+1J)+G1/TV**2
 AA(I,I,I-1K)=AA(I,I,I-1K)+G/TZ**2
 AA(I,I,I)=AA(I,I,I)-2.*G/TZ**2
 AA(I,I,I+1K)=AA(I,I,I+1K)+G/TZ**2

DIF20660
 DIF20670
 DIF20680
 DIF20690
 DIF20700
 DIF20710
 DIF20720

GO TO 2
 116 AA(I,I-1K)=AA(I,I-1K)+G/TZ**2
 AA(I,I)=AA(I,I)-2.*G/TZ**2
 AA(I,I+1K)=AA(I,I+1K)+G/TZ**2

DIF20730
 DIF20740
 DIF20750
 DIF20760
 DIF20770

AA(I,I,I)=AA(I,I,I)+G3/(TX*TV)
 AA(I,I,I+1)=AA(I,I,I+1)-G3/(TX*TV)
 AA(I,I,I+1J)=AA(I,I,I+1J)-G3/(TX*TV)
 AA(I,I,I+1J+1)=AA(I,I,I+1J+1)+G3/(TX*TV)
 AA(I,I,I-1J)=AA(I,I,I-1J)+2.*G1/TV**2
 AA(I,I,I)=AA(I,I,I)-2.*G1/TV**2
 AA(I,I,I+1K)=AA(I,I,I+1K)+G/TZ**2
 AA(I,I,I)=AA(I,I,I)-2.*G/TZ**2
 AA(I,I,I+1K)=AA(I,I,I+1K)+G/TZ**2

DIF20780
 DIF20790
 DIF20800
 DIF20810
 DIF20820
 DIF20830
 DIF20840
 DIF20850
 DIF20860
 DIF20870
 DIF20880
 DIF20890

GO TO 2
 117 AA(I,I-1)=AA(I,I-1)+G1/TX**2

DIF20900


```
AA(I, I1)=AA(I, I1)+G3/(TX*TV)
AA(I, I1+1)=AA(I, I1+1)-G3/(TX*TV)
AA(I, I1+I1)=AA(I, I1+I1)-G3/(TX*TV)
AA(I, I1+I1+1)=AA(I, I1+I1+1)+G3/(TX*TV)
GO TO 2
122 AA(I, I1)=AA(I, I1)-2.*G1/TX**2
AA(I, I-1)=AA(I, I-1)+2.*G1/TX**2
AA(I, I1)=AA(I, I1)+G3/(TX*TV)
AA(I, I1-1)=AA(I, I1-1)-G3/(TX*TV)
AA(I, I1+I1)=AA(I, I1+I1)-G3/(TX*TV)
AA(I, I1+I1+1)=AA(I, I1+I1+1)+G3/(TX*TV)
GO TO 2
123 AA(I, I1)=AA(I, I1)-2.*G1/TX**2
AA(I, I+1)=AA(I, I+1)+G1/TX**2
AA(I, I1)=AA(I, I1)+G3/(TX*TV)
AA(I, I1+1)=AA(I, I1+1)-G3/(TX*TV)
AA(I, I1+I1)=AA(I, I1+I1)-G3/(TX*TV)
AA(I, I1+I1+1)=AA(I, I1+I1+1)+G3/(TX*TV)
GO TO 2
124 AA(I, I1)=AA(I, I1)+G3/(TX*TV)
AA(I, I1+1)=AA(I, I1+1)-G3/(TX*TV)
AA(I, I1+I1)=AA(I, I1+I1)-G3/(TX*TV)
AA(I, I1+I1+1)=AA(I, I1+I1+1)+G3/(TX*TV)
AA(I, I1)=AA(I, I1)-2.*G1/TV**2
AA(I, I1+I1)=AA(I, I1+I1)+G1/TV**2
GO TO 2
125 AA(I, I1)=AA(I, I1)-2.*G1/TX**2
AA(I, I+1)=AA(I, I+1)+G1/TX**2
AA(I, I-IK)=AA(I, I-IK)+G/TZ**2
AA(I, I)=AA(I, I)-2.*G/TZ**2
AA(I, I+IK)=AA(I, I+IK)+G/TZ**2
AA(I, I1)=AA(I, I1)+G3/(TX*TV)
AA(I, I1+1)=AA(I, I1+1)-G3/(TX*TV)
AA(I, I1+I1)=AA(I, I1+I1)-G3/(TX*TV)
AA(I, I1+I1+1)=AA(I, I1+I1+1)+G3/(TX*TV)
AA(I, I1-JK)=AA(I, I1-JK)+b/TZ**2
AA(I, I1)=AA(I, I1)-2.*G/TZ**2
AA(I, I1+JK)=AA(I, I1+JK)+G/TZ**2
GO TO 2
2 CONTINUE
RETURN
END
DIF21460
DIF21470
DIF21480
DIF21490
DIF21500
DIF21510
DIF21520
DIF21530
DIF21540
DIF21550
DIF21560
DIF21570
DIF21580
DIF21590
DIF21600
DIF21610
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DIF21770
DIF21780
DIF21790
DIF21800
DIF21810
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DIF21830
DIF21840
DIF21850
DIF21860
DIF21870
DIF21880
DIF21890
DIF21900
DIF21910
DIF21920
DIF21930
DIF21940
DIF21950
DIF21960
DIF21970
DIF21980
```



```

CCC READ(S,*) (VAL(IL),IL=1,NIL)
CCC AB=B/A B>A AB>1.0 A=XA=1.0
CCC HA=H/A H<A HA<1.0
CCC TX=XA/NTX TV=TX/AB TZ=(XA*HA)/NTZ

```

```

TX=XA/NTX
TV=(XA*AB)/NTX
TZ=(XA*HA)/NTZ
WRITE(6,1016)NOD,E,P,XA,AB,HA,NTX,NTZ,TX,TV,TZ,DELTA,GAMA
B=(E*P)/((1.+P)*(1.-2.*P))
G=E/(2.*(1.+P))
G1=B*2.*G
G2=B*G
G3=B
G4=-G3
G11=G1
G22=G2
G11=E/(1.-P**2)
G22=E/(2.0*(1.-P))
D=(E*(XA*HA)**3)/(12.*(1.-P**2))
TV=TV*AB
TZ=TZ*HA
HX=HA

```

```

R(1)=(1.0/TV**4)*(1./HX)
R(2)=(2.0/((TX**2)*(TV**2)))*(1./HX)
R(3)=((-4.0/TV**2)*((1.0/TX**2)+(1.0/TV**2)))*(1./HX)
R(4)=R(2)
R(5)=(1.0/TX**4)*(1./HX)
R(6)=((-4.0/TX**2)*((1.0/TX**2)+(1.0/TV**2)))*(1./HX)
R(7)=(6.0/TX**4)+(6.0/TV**4)+(8.0/((TX**2)*(TV**2)))*(1./HX)
R(8)=R(6)
R(9)=R(5)
R(10)=R(4)
R(11)=R(3)
R(12)=R(2)
R(13)=R(1)
DO 4 I=1,NOD/5
DO 4 K=1,3
UVM(I,K)=0
4 CONTINUE
DO 1 I=1,3*NOD
DO 1 J=1,3*NOD
AA(I,J)=0.0
1 CONTINUE

```

```

CCC UVM: MATRIX TO SATISFY THE SYMMETRY
CCC
CCC DO 3 I=1,NOD/5
CCC 3 READ(5,700)(UVM(I,K),K=1,3)
CCC 700 FORMAT(3I5)
CCC CALL CENDIF,AA,NA,NP,R,N,HA,IX,TX,TV,TZ,NOD,G,DELTA,GAMA
CCC CALL BOUNDA(UVM,NOD,AA)

```

- DIF00560
- DIF00570
- DIF00580
- DIF00590
- DIF00600
- DIF00610
- DIF00620
- DIF00630
- DIF00640
- DIF00650
- DIF00660
- DIF00670
- DIF00680
- DIF00690
- DIF00700
- DIF00710
- DIF00720
- DIF00730
- DIF00740
- DIF00750
- DIF00760
- DIF00770
- DIF00780
- DIF00790
- DIF00800
- DIF00810
- DIF00820
- DIF00830
- DIF00840
- DIF00850
- DIF00860
- DIF00870
- DIF00880
- DIF00890
- DIF00900
- DIF00910
- DIF00920
- DIF00930
- DIF00940
- DIF00950
- DIF00960
- DIF00970
- DIF00980
- DIF00990
- DIF01000
- DIF01010
- DIF01020
- DIF01030
- DIF01040
- DIF01050
- DIF01060
- DIF01070
- DIF01080
- DIF01090
- DIF01100

```

CCC      CALL SOLINV(AA,135)
CCC
C        IUNIT=1
          NIL=IUNIT
          DO 250 IL =IUNIT,NIL
          DO 13 I=1,3*NOD
            DIS(I,1)=0.0
            DISN(I,1)=0.0
          13 CONTINUE
            Q=VAL(IL)
CCC
          CALL FORCE(Q,P,HA,NOD,E,FF,DFOR)
          WRITE(6,1014) (I,DFOR(I,1),DFOR(I+NOD,1),DFOR(I+2*NOD,1),I=1,NOD)
CCC
          CALL BOUNDF(UVW,NOD,DFOR)
CCC
          CALL MATMUL(AA,135,135,DFOR,1,DIS)
          NL=NOD/5
          NN=2*NOD+3*NL
          DLINEA=DIS(NN,1)
          PRINT 1013
C        WRITE(6,1014) (I,DIS(I,1),DIS(I+NOD,1),DIS(I+2*NOD,1),I=1,NOD)
          CCCCCCCC
          CCCCCCCC
          ICONT =0
          50 CALL NOLINE(DIS,IJKL,NOD,AB,XA,HA,E,Q,P,B,G,NTX,NTZ,FOR)
          CALL BOUNDF(UVW,NOD,FOR)
          WRITE(6,1014) (I,FOR(I,1),FOR(I+NOD,1),FOR(I+2*NOD,1),I=1,NOD)
          CCCCCCCC
          CCCCCCCC
          CALL MATMUL(AA,135,135,FOR,1,DISN)
          CCCCCCCC
          PRINT 1013
C        WRITE(6,1014) (I,DISN(I,1),DISN(I+NOD,1),DISN(I+2*NOD,1),I=1,NOD)
          C        WRITE(6,301) 0,12.0*(1,-P**2)*Q,DIS(NN,1),DISN(NN,1)
          IF(ICONT.EQ.IMAX) GO TO 251
          ICONT=ICONT+1
          DO 51 I=1,3*NOD
            IF(DABS(DISN(I,1))-DABS(DIS(I,1))) 52,51,53
            52 IF(DELTA-DABS(DISN(I,1)/DIS(I,1))) 51,54,54
            53 IF(DELTA-DABS(DIS(I,1)/DISN(I,1))) 51,54,54
          51 CONTINUE
          251 PRINT 804
          WRITE(6,301) 0,12.0*(1,-P**2)*Q,DLINEA,DISN(NN,1)
          301 FORMAT(AE15.5,/)
          WRITE(6,803) ICONT
          803 FORMAT(/,5X,NUMBER OF ITERATIONS = ,15,/)
          GO TO 250
          54 DO 56 II=1,3*HGT
            TEMP=DIS(II,1)
            DIS(II,1)=GAMA*DISN(II,1)+(1.0-GAMA)*TEMP
          56 CONTINUE
          D1F01110
          D1F01120
          D1F01130
          D1F01140
          D1F01150
          D1F01160
          D1F01170
          D1F01180
          D1F01190
          D1F01200
          D1F01210
          D1F01220
          D1F01230
          D1F01240
          D1F01250
          D1F01260
          D1F01270
          D1F01280
          D1F01290
          D1F01300
          D1F01310
          D1F01320
          D1F01330
          D1F01340
          D1F01350
          D1F01360
          D1F01370
          D1F01380
          D1F01390
          D1F01400
          D1F01410
          D1F01420
          D1F01430
          D1F01440
          D1F01450
          D1F01460
          D1F01470
          D1F01480
          D1F01490
          D1F01500
          D1F01510
          D1F01520
          D1F01530
          D1F01540
          D1F01550
          D1F01560
          D1F01570
          D1F01580
          D1F01590
          D1F01600
          D1F01610
          D1F01620
          D1F01630
          D1F01640
          D1F01650

```

```

GO TO 50
250 CONTINUE
C PRINT 1003
C WRITE(6,*)((AA(M,N),N=1,3*NOD),M=1,3*NOD)
C PRINT 1003
C 55 PRINT 1013
C WRITE(6,1014) (I,DISN(I,1),DISN(I+NOD,1),DISN(I+2*NOD,1),
C &I=1,NOD)
C PRINT 1013
C WRITE(6,1014) (I,DIS(I,1),DIS(I+NOD,1),DIS(I+2*NOD,1),
C &I=1,NOD)
801 FORMAT('SIMPLY SUPPORTED ISOTROPIC THICK RECTANGULAR PLATE',/)
802 FORMAT('CALMPED ISOTROPIC THICK RECTANGULAR PLATE',/)
804 FORMAT('(Q*A**4)/(E*H**4)'.2X,'(Q*A**4)/(D*H**2X'.LINEAR W'.6X,
&NONLINEAR W'./)
805 FORMAT(10X,'TOP LAYER SCHEME GENERATED BY NA MATRIX',/)
900 FORMAT(15,F10.4,F5.3,F10.5,2F5.3,2I5,F10.5,15)
1001 FORMAT(/,2(1X,10E13.7,/) )
1002 FORMAT(6(1X,10E13.4,/) )
1003 FORMAT(/,5X,'MATRIX AA')
1014 FORMAT(/,5X,13,2X,3(E12.4,2X),/)
1013 FORMAT(/,5X,'NODE X-DIRECTION',Y-DIRECTION',Z-DIRECTION',/)
1016 FORMAT(9X,'NUMBRE OF NODES',15,/,9X,'YOUNG MODULOUS',
&F10.5,/,9X,'POISSON S RATIO',F5.3,/,9X,'LONGITUDINAL LENGTH=',
&F10.5,/,9X,'RATIO A/B',F5.3,/,9X,'MESH NUMBER ON X',Y',15,/,9X,'MESH NUMBER ON Z',
&',15,/,9X,'MESH SIZE ON X AXIS',F10.5,/,9X,'MESH SIZE ON Y',
&F10.5,/,9X,'MESH SIZE ON Z AXIS',F10.5,/,9X,'DELTA',
&F10.5,/,9X,'GAMA',F10.5)
STOP
END
C
C *****
C *SUBROUTINE PROGRAM FOR CALCULATING MATRIX *
C * MULTIPLICATION *
C *****
C
SUBROUTINE MATMUL(AA,NN,MM,BB,LL,CC)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AA(NN,MM),BB(MM,LL),CC(NN,LL)
DO 100 I=1,NN
DO 200 J=1,LL
CC(I,J)=0.0
DO 300 K1=1,MM
DO 300 K2=1,MM
CC(I,J)=CC(I,J)+AA(I,K1)*BB(K1,K2)
END
C
C *****
C *SUBROUTINE PROGRAM DETERMINING THE FINITE *
C *****
300 CONTINUE
200 CONTINUE
100 CONTINUE
RETURN
END
DIF01660
DIF01670
DIF01680
DIF01690
DIF01700
DIF01710
DIF01720
DIF01730
DIF01740
DIF01750
DIF01760
DIF01770
DIF01780
DIF01790
DIF01800
DIF01810
DIF01820
DIF01830
DIF01840
DIF01850
DIF01860
DIF01870
DIF01880
DIF01890
DIF01900
DIF01910
DIF01920
DIF01930
DIF01940
DIF01950
DIF01960
DIF01970
DIF01980
DIF01990
DIF02000
DIF02010
DIF02020
DIF02030
DIF02040
DIF02050
DIF02060
DIF02070
DIF02080
DIF02090
DIF02100
DIF02110
DIF02120
DIF02130
DIF02140
DIF02150
DIF02160
DIF02170
DIF02180
DIF02190
DIF02200

```

C * DIFFERENCE SCHEME FOR BIHARMONIC OPERATOR *
C *****
C

SUBROUTINE SCHE(NA,N,IM,JM,NOD,ITEST,NP)

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION NA(11,11),NP(13)

NP(1)=NA(IM-2,JM)
NP(2)=NA(IM-1,JM-1)
NP(3)=NA(IM-1,JM)
NP(4)=NA(IM-1,JM+1)
NP(5)=NA(IM,JM-2)
NP(6)=NA(IM,JM-1)
NP(7)=NA(IM,JM)
NP(8)=NA(IM,JM+1)
NP(9)=NA(IM+1,JM-1)
NP(10)=NA(IM+1,JM)
NP(11)=NA(IM+1,JM+1)
NP(12)=NA(IM+2,JM)
NP(13)=NA(IM+2,JM)
IF(ITEST.EQ.0) GO TO 10
IF(ITEST.EQ.1) GO TO 11
11 DO 12 IK=1,13
12 IF(NP(IK)) 13,12,13
13 NP(IK)=NP(IK)+(NP(IK)/ABS(NP(IK)))*(4*NOD)/5
12 CONTINUE
10 RETURN
END

C *****
C * SUBROUTINE PROGRAM GENERATING THE FINITE *
C * DIFFERENCE MESH POINTS ON T/B LAYER *
C *****
C

SUBROUTINE GENR(N,INC,NA,KD)

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION NA(11,11)
DO 9 I=1,N
DO 9 J=1,N
9 NA(I,J)=0
DO 10 I=3,N-2
DO 11 J=3,N-2
IF(I.GT.3) GO TO 14
NA(I,J)=NA(I,J-1)+1
NA(I,1)=NA(I,3)
NA(I,J)=((-1)**KD)*NA(3,J)
NA(1,N)=NA(1,N-4)
NA(1,N-1)=NA(1,N-3)
GO TO 11
14 NA(I,J)=NA(I-1,J)+INC
CONTINUE
NA(I,1)=((-1)**KD)*NA(1,3)
NA(I,N)=NA(I,N-4)
NA(I,N-1)=NA(I,N-3)

DIFF02210
DIFF02220
DIFF02230
DIFF02240
DIFF02250
DIFF02260
DIFF02270
DIFF02280
DIFF02290
DIFF02300
DIFF02310
DIFF02320
DIFF02330
DIFF02340
DIFF02350
DIFF02360
DIFF02370
DIFF02380
DIFF02390
DIFF02400
DIFF02410
DIFF02420
DIFF02430
DIFF02440
DIFF02450
DIFF02460
DIFF02470
DIFF02480
DIFF02490
DIFF02500
DIFF02510
DIFF02520
DIFF02530
DIFF02540
DIFF02550
DIFF02560
DIFF02570
DIFF02580
DIFF02590
DIFF02600
DIFF02610
DIFF02620
DIFF02630
DIFF02640
DIFF02650
DIFF02660
DIFF02670
DIFF02680
DIFF02690
DIFF02700
DIFF02710
DIFF02720
DIFF02730
DIFF02740
DIFF02750

```

10 CONTINUE
DO 12 I=N-1,N
DO 12 J=1,N
12 NA(I,J)=NA(I-2*(2+I-N),J)
RETURN
END

```

```

*****
* SUBROUTINE PROGRAM GENERATING THE FINITE *
* DIFFERENCE NONLINEAR OPERANDS *
*****

```

```

SUBROUTINE NOLINE(DIS,IJKL,NOD,AB,XA,HA,E,Q,P,B,G,NTX,NTZ,FOR)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION IJKL(45,4),DIS(135,1),FOR(135,1),FF(45,1)

```

```

TX=XA/NTX
TY=TX*(AB**2)
TZ=(XA*HA)/(NTZ)
G1=B+2.*G
G2=B+G
G3=B
G11=G1
G22=G2
G11=E/(1.-P**2)
G22=E/(2.0*(1.-P))
DO 18 I=1,NOD
IJ=IJKL(I,1)
IK=IJKL(I,2)
JK=IJKL(I,3)
I1=I+NOD
I2=I+2*NOD

```

```

IF(IJKL(I,4).EQ.89) GO TO 2089
IF(IJKL(I,4).EQ.90) GO TO 2090
IF(IJKL(I,4).EQ.91) GO TO 2091
IF(IJKL(I,4).EQ.92) GO TO 2092
IF(IJKL(I,4).EQ.93) GO TO 2093
IF(IJKL(I,4).EQ.94) GO TO 2094
IF(IJKL(I,4).EQ.95) GO TO 2095
IF(IJKL(I,4).EQ.96) GO TO 2096
IF(IJKL(I,4).EQ.97) GO TO 2097
IF(IJKL(I,4).EQ.98) GO TO 2098
IF(IJKL(I,4).EQ.99) GO TO 2099
IF(IJKL(I,4).EQ.100) GO TO 2100
IF(IJKL(I,4).EQ.101) GO TO 2101
IF(IJKL(I,4).EQ.102) GO TO 2102
IF(IJKL(I,4).EQ.103) GO TO 2103
IF(IJKL(I,4).EQ.104) GO TO 2104
IF(IJKL(I,4).EQ.105) GO TO 2105
IF(IJKL(I,4).EQ.106) GO TO 2106
IF(IJKL(I,4).EQ.107) GO TO 2107
IF(IJKL(I,4).EQ.108) GO TO 2108
IF(IJKL(I,4).EQ.109) GO TO 2109

```

- DIF02760
- DIF02770
- DIF02780
- DIF02790
- DIF02800
- DIF02810
- DIF02820
- DIF02830
- DIF02840
- DIF02850
- DIF02860
- DIF02870
- DIF02880
- DIF02890
- DIF02900
- DIF02910
- DIF02920
- DIF02930
- DIF02940
- DIF02950
- DIF02960
- DIF02970
- DIF02980
- DIF02990
- DIF03000
- DIF03010
- DIF03020
- DIF03030
- DIF03040
- DIF03050
- DIF03060
- DIF03070
- DIF03080
- DIF03090
- DIF03100
- DIF03110
- DIF03120
- DIF03130
- DIF03140
- DIF03150
- DIF03160
- DIF03170
- DIF03180
- DIF03190
- DIF03200
- DIF03210
- DIF03220
- DIF03230
- DIF03240
- DIF03250
- DIF03260
- DIF03270
- DIF03280
- DIF03290
- DIF03300

GO TO 20

CCC POINTS PARALLEL TO V AXIS NEAR X AXIS

C

2089

```

D1=(DIS(I2+1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1))/(TX**2)
D3=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)
DT=DIS(I2+1,1)-DIS(I2+1-1,1)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TV**2)
DT=DIS(I2+1,1)+JK,1)-DIS(I2+1,1)-JK,1)-DIS(I2-1,1)-JK,1)+DIS(I2-1,1)-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+1,1)+IK,1)-DIS(I2+1-1,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(I+1,1))/(2.*TX)
D9=(-DIS(I-1,1))/(2.*TV)
D10=(DIS(I+1,1))-DIS(I-1,1))/(2.*TV)
D11=(DIS(I+1,1))/(2.*TX)
D12=(DIS(I+1,1))-DIS(I-1,1))/(2.*TZ)
DT=DIS(I+1,1)+JK,1)-DIS(I+1-1,1)
D13=DT/(4.*TX*TZ)
DT=DIS(I+1,1)+JK,1)-DIS(I+1,1)-JK,1)+DIS(I-1,1)-JK,1)
D14=DT/(4.*TV*TZ)
D15=(DIS(I-1,1))-2.*DIS(I,1)+DIS(I+1,1))/(TZ**2)
D16=(DIS(I+1,1))-DIS(I-1,1))/(2.*TZ)
DT=DIS(I-1,1)+JK,1)+DIS(I-1,1)-JK,1)
D17=DT/(4.*TV*TZ)
DT=DIS(I+1,1)+IK,1)-DIS(I+1-1,1)
D18=DT/(4.*TX*TZ)
D19=(DIS(I-1,1))-2.*DIS(I,1)+DIS(I+1,1))/(TZ**2)
D22=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TZ)
D23=(DIS(I+1,1))-2.*DIS(I,1))/(TX**2)
D24=(-2.*DIS(I,1)+DIS(I-1,1))/(TV**2)
D25=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TZ**2)
DT=DIS(I+1,1)+JK,1)-DIS(I+1,1)-JK,1)
D26=DT/(4.*TX*TV)
DT=DIS(I+1,1)-1,1)
D27=DT/(4.*TX*TV)
D28=(DIS(I+1,1))-2.*DIS(I,1)+DIS(I-1,1))/(TV**2)
D29=(DIS(I+1,1))-2.*DIS(I,1))/(TX**2)

GO TO 20

```

CCC GENERAL POINTS NEAR X & Y AXIS

C

2092

```

D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
D3=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TV)
DT=DIS(I2+1,1)+JK,1)-DIS(I2+1,1)-JK,1)-DIS(I2-1,1)-JK,1)+DIS(I2-1,1)-JK,1)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TV**2)
DT=DIS(I2+1,1)+JK,1)-DIS(I2+1,1)-JK,1)-DIS(I2-1,1)-JK,1)+DIS(I2-1,1)-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+1,1)+IK,1)-DIS(I2+1,1)-IK,1)+DIS(I2-1,1)-IK,1)

```

- DIF03860
- DIF03870
- DIF03880
- DIF03890
- DIF03900
- DIF03910
- DIF03920
- DIF03930
- DIF03940
- DIF03950
- DIF03960
- DIF03970
- DIF03980
- DIF03990
- DIF04000
- DIF04010
- DIF04020
- DIF04030
- DIF04040
- DIF04050
- DIF04060
- DIF04070
- DIF04080
- DIF04090
- DIF04100
- DIF04110
- DIF04120
- DIF04130
- DIF04140
- DIF04150
- DIF04160
- DIF04170
- DIF04180
- DIF04190
- DIF04200
- DIF04210
- DIF04220
- DIF04230
- DIF04240
- DIF04250
- DIF04260
- DIF04270
- DIF04280
- DIF04290
- DIF04300
- DIF04310
- DIF04320
- DIF04330
- DIF04340
- DIF04350
- DIF04360
- DIF04370
- DIF04380
- DIF04390
- DIF04400

```

D7=DT/(4.*TX*TZ)
D8=(-DIS(I-1,1))/(2.*TX)
D9=(-DIS(I-1-J,1))/(2.*TV)
D10=(DIS(I+J,1)-DIS(I-J,1))/(2.*TV)
D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
D12=(DIS(I+IK,1)-DIS(I-1K,1))/(2.*TZ)
D13=DT/(4.*TX*TZ)
D14=DT/(4.*TV*TZ)
D15=(DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+1K,1))/(TZ**2)
D16=(DIS(I+JK,1)-DIS(I-JK,1))/(2.*TZ)
D17=DT/(4.*TV*TZ)
D18=DT/(4.*TX*TZ)
D19=(DIS(I-1-JK,1)-2.*DIS(I,1)+DIS(I+1+JK,1))/(TZ**2)
D22=(DIS(I2+1K,1)-DIS(I2-1K,1))/(2.*TZ)
D23=(-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)
D24=(-2.*DIS(I,1)+DIS(I-1,1))/(TV**2)
D25=(DIS(I2+1K,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
D26=DT/(4.*TX*TV)
D27=DT/(4.*TX*TV)
D28=(DIS(I+J,1)-2.*DIS(I,1)+DIS(I-1+J,1))/(TV**2)
D29=(DIS(I+1,1)-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)

```

GO TO 20

C
CCC . GENERAL POINTS NEAR V AXIS

```

2091 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
D3=(DIS(I2+J,1)-DIS(I2-J,1))/(2.*TV)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+J,1)-2.*DIS(I2,1)+DIS(I2-1+J,1))/(TV**2)
D6=DT/(4.*TV*TZ)
D7=DT/(4.*TX*TZ)
D8=(-DIS(I-1,1))/(2.*TX)
D9=(DIS(I+1+J,1)-DIS(I-1-J,1))/(2.*TV)
D10=(DIS(I+J,1)-DIS(I-J,1))/(2.*TV)
D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
D12=(DIS(I+1K,1)-DIS(I-1K,1))/(2.*TZ)
D13=DT/(4.*TX*TZ)
D14=DT/(4.*TV*TZ)
D15=(DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+1K,1))/(TZ**2)
D16=(DIS(I+JK,1)-DIS(I-JK,1))/(2.*TZ)
D17=DT/(4.*TV*TZ)
D18=DT/(4.*TX*TZ)
D19=(DIS(I-1-JK,1)-2.*DIS(I,1)+DIS(I+1+JK,1))/(TZ**2)
D22=(DIS(I2+1K,1)-DIS(I2-1K,1))/(2.*TZ)
D23=(-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)
D24=(-2.*DIS(I,1)+DIS(I-1,1))/(TV**2)
D25=(DIS(I2+1K,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
D26=DT/(4.*TX*TV)
D27=DT/(4.*TX*TV)
D28=(DIS(I+J,1)-2.*DIS(I,1)+DIS(I-1+J,1))/(TV**2)
D29=(DIS(I+1,1)-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)

```

D1=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+IK,1)+DIS(I1-1-1K,1) D1F04960
 D18=DT/(4.*TX*TZ) D1F04970
 D19=(DIS(I1-1K,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2) D1F04980
 D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ) D1F04990
 D23=(-2.*DIS(I,1)+DIS(I-1,1))/(TX**2) D1F05000
 D24=(DIS(I1+IJ,1)-2.*DIS(I1,1)+DIS(I1-IJ,1))/(TV**2) D1F05010
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2) D1F05020
 D1=-DIS(I-1+IJ,1)+DIS(I-1-IJ,1) D1F05030
 D26=DT/(4.*TX*TV) D1F05040
 D7=DIS(I1+1+IJ,1)-DIS(I1+1-IJ,1)-DIS(I1-1+IJ,1)+DIS(I1-1-IJ,1) D1F05050
 D27=DT/(4.*TX*TV) D1F05060
 D28=(DIS(I1+IJ,1)-2.*DIS(I,1)+DIS(I-IJ,1))/(TV**2) D1F05070
 D29=(DIS(I1+1,1)-2.*DIS(I1,1)+DIS(I1-1,1))/(TX**2) D1F05080
 D1F05090
 D1F05100
 D1F05110
 D1F05120
 D1F05130
 D1F05140
 D1F05150
 D1F05160
 D1F05170
 D1F05180
 D1F05190
 D1F05200
 D1F05210
 D1F05220
 D1F05230
 D1F05240
 D1F05250
 D1F05260
 D1F05270
 D1F05280
 D1F05290
 D1F05300
 D1F05310
 D1F05320
 D1F05330
 D1F05340
 D1F05350
 D1F05360
 D1F05370
 D1F05380
 D1F05390
 D1F05400
 D1F05410
 D1F05420
 D1F05430
 D1F05440
 D1F05450
 D1F05460
 D1F05470
 D1F05480
 D1F05490
 D1F05500

GO TO 20

C
 CCC GENERAL POINTS NEAR X AXIS

2090 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX) D1F05110
 D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2) D1F05120
 D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV) D1F05130
 D7=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1) D1F05140
 D4=DT/(4.*TX*TV) D1F05150
 D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2) D1F05160
 D6=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1) D1F05170
 D6=DT/(4.*TV*TZ) D1F05180
 D7=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1) D1F05190
 D7=DT/(4.*TX*TZ) D1F05200
 D8=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX) D1F05210
 D9=(DIS(I1-IJ,1))/(2.*TV) D1F05220
 D10=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV) D1F05230
 D11=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX) D1F05240
 D12=(DIS(I1+IK,1)-DIS(I1-1K,1))/(2.*TZ) D1F05250
 D13=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+IK,1)+DIS(I1-1-1K,1) D1F05260
 D13=DT/(4.*TX*TZ) D1F05270
 D14=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1) D1F05280
 D14=DT/(4.*TV*TZ) D1F05290
 D15=(DIS(I1-1K,1)-2.*DIS(I,1)+DIS(I+1K,1))/(TZ**2) D1F05300
 D16=(DIS(I1+JK,1)-DIS(I1-JK,1))/(2.*TZ) D1F05310
 D17=DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1) D1F05320
 D17=DT/(4.*TV*TZ) D1F05330
 D18=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+IK,1)+DIS(I1-1-1K,1) D1F05340
 D18=DT/(4.*TX*TZ) D1F05350
 D19=(DIS(I1-1K,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2) D1F05360
 D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ) D1F05370
 D23=(DIS(I1+1,1)-2.*DIS(I1,1)+DIS(I1-IJ,1))/(TX**2) D1F05380
 D24=(-2.*DIS(I1,1)+DIS(I1-IJ,1))/(TV**2) D1F05390
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2) D1F05400
 D1=DIS(I1+1+IJ,1)-DIS(I1+1-IJ,1)-DIS(I1-1+IJ,1)+DIS(I1-1-IJ,1) D1F05410
 D26=DT/(4.*TX*TV) D1F05420
 D1=-DIS(I1+1-IJ,1)+DIS(I1-1-IJ,1) D1F05430
 D27=DT/(4.*TX*TV) D1F05440
 D28=(DIS(I1+IJ,1)-2.*DIS(I,1)+DIS(I-IJ,1))/(TV**2) D1F05450
 D29=(DIS(I1+1,1)-2.*DIS(I1,1)+DIS(I1-1,1))/(TX**2) D1F05460
 D1F05470
 D1F05480
 D1F05490
 D1F05500

GO TO 20

C
CCC POINTS ON THE Y AXIS NEAR THE CENTRE

C
2093

```

D1=0.0
D2=(-2.*DIS(12,1))/(TX**2)
D3=(DIS(12+IJ,1)-DIS(12-IJ,1))/(2.*TV)
D4=DIS(12+1+IJ,1)-DIS(12+1-IJ,1)-DIS(12-1+IJ,1)+DIS(12-1-IJ,1)
D5=(DIS(12+IJ,1)-2.*DIS(12,1)+DIS(12-IJ,1))/(TV**2)
D6=DIS(12+IJ+JK,1)-DIS(12+IJ-JK,1)-DIS(12-IJ+JK,1)+DIS(12-IJ-JK,1)
D7=0.0
D8=(-2.*DIS(1-1,1))/(2.*TX)
D9=(-DIS(1-IJ,1))/(2.*TV)
D10=0.0
D11=0.0
D12=0.0
D13=DIS(1-1+IK,1)+2.*DIS(1-1-IK,1)
D14=DIS(1+IJ+JK,1)-DIS(1+IJ-JK,1)-DIS(1-IJ+JK,1)+DIS(1-IJ-JK,1)
D15=0.0
D16=(DIS(1+1+JK,1)-DIS(1+1-JK,1))/(2.*TZ)
D17=DIS(1+1+JK,1)+DIS(1+1-JK,1)
D18=DIS(1+1+IK,1)-DIS(1+1-IK,1)-DIS(1-1+IK,1)+DIS(1-1-IK,1)
D19=(DIS(1-1-JK,1)-2.*DIS(1,1)+DIS(1+1+JK,1))/(TZ**2)
D20=(DIS(12+IK,1)-DIS(12-IK,1))/(2.*TZ)
D21=0.0
D22=(-2.*DIS(1,1)+DIS(1+1+IK,1))/(TV**2)
D23=DIS(12+IK,1)-2.*DIS(12,1)+DIS(12-IK,1))/(TZ**2)
D24=DIS(1-1+IJ,1)+2.*DIS(1-1-IJ,1)
D25=DIS(1-1+IJ,1)+2.*DIS(1-1-IJ,1)
D26=DIS(1+1+IK,1)+2.*DIS(1-1-IJ,1)
D27=0.0
D28=0.0
D29=(-2.*DIS(1,1))/(TX**2)

```

GO TO 20

C
CCC POINTS ON THE X AXIS NEAR THE CENTRE

C
2094

```

D1=(DIS(12+1,1)-DIS(12-1,1))/(2.*TX)
D2=(DIS(12+1,1)-2.*DIS(12,1)+DIS(12-1,1))/(TX**2)
D3=0.0
D4=DIS(12+1+IJ,1)-DIS(12+1-IJ,1)-DIS(12-1+IJ,1)+DIS(12-1-IJ,1)
D5=(-2.*DIS(12,1))/(2.*TV)
D6=DIS(12+IJ+JK,1)-DIS(12+IJ-JK,1)-DIS(12-IJ+JK,1)+DIS(12-IJ-JK,1)
D7=DIS(12+1+IK,1)-DIS(12+1-IK,1)-DIS(12-1+IK,1)+DIS(12-1-IK,1)
D8=DIS(12+1+IK,1)+DIS(12-1-IK,1)

```

```

08=(-DIS(I-1,1))/(2.*TX)
09=(-2.*DIS(I1-IJ,1))/(2.*TV)
D10=0.0
D11=0.0
D12=(DIS(I+IK,1)-DIS(I-1K,1))/(2.*TZ)
DT=-DIS(I-1+K,1)+DIS(I-1-K,1)
D13=DT/(4.*TX*TZ)
DT=DIS(I+IJ+JK,1)-DIS(I+IJ-JK,1)-DIS(I-IJ+JK,1)+DIS(I-IJ-JK,1)
D14=0.0
D15=(DIS(I-1K,1)-2.*DIS(I,1)+DIS(I+1K,1))/(TZ**2)
D16=0.0
DT=-2.*DIS(I1-IJ+JK,1)+2.*DIS(I1-IJ-JK,1)
D17=DT/(4.*TV*TZ)
DT=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+1K,1)+DIS(I1-1-1K,1)
D18=0.0
D19=0.0
D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ)
D23=(-2.*DIS(I,1)+DIS(I-1,1))/(TX**2)
D24=0.0
D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
DT=DIS(I1+1+IJ,1)-DIS(I1+1-IJ,1)-DIS(I-1+IJ,1)+DIS(I-1-IJ,1)
D26=0.0
DT=-2.*DIS(I1+1-IJ,1)+2.*DIS(I1-1-IJ,1)
D27=DT/(4.*TX*TV)
D28=(-2.*DIS(I,1))/(TV**2)
D29=0.0

GO TO 20

C
CCC GENERAL POINTS
2101 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+1K,1)+DIS(I2-1-1K,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(I1+1,1)-DIS(I-1,1))/(2.*TX)
D9=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)
D10=(DIS(I1+IJ,1)-DIS(I-1J,1))/(2.*TV)
D11=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX)
D12=(DIS(I1+IK,1)-DIS(I-1K,1))/(2.*TZ)
D7=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)+DIS(I-1-1K,1)
D13=DT/(4.*TX*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1)
D14=DT/(4.*TV*TZ)
D15=(DIS(I1+1,1)-2.*DIS(I,1)+DIS(I+1K,1))/(TZ**2)
D16=(DIS(I1+1+K,1)-DIS(I1-1+K,1))/(2.*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)-DIS(I1-IJ+JK,1)+DIS(I1-IJ-JK,1)
D17=DT/(4.*TV*TZ)
DT=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+1K,1)+DIS(I1-1-1K,1)
D1F06060
D1F06070
D1F06080
D1F06090
D1F06100
D1F06110
D1F06120
D1F06130
D1F06140
D1F06150
D1F06160
D1F06170
D1F06180
D1F06190
D1F06200
D1F06210
D1F06220
D1F06230
D1F06240
D1F06250
D1F06260
D1F06270
D1F06280
D1F06290
D1F06300
D1F06310
D1F06320
D1F06330
D1F06340
D1F06350
D1F06360
D1F06370
D1F06380
D1F06390
D1F06400
D1F06410
D1F06420
D1F06430
D1F06440
D1F06450
D1F06460
D1F06470
D1F06480
D1F06490
D1F06500
D1F06510
D1F06520
D1F06530
D1F06540
D1F06550
D1F06560
D1F06570
D1F06580
D1F06590
D1F06600

```

D18=DT/(4.*TX*TZ)
 D19=(DIS(I1-JK,1))-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2)
 D22=(DIS(I2+IK,1))-DIS(I2-1K,1))/(2.*TZ)
 D23=(DIS(I1,1))-2.*DIS(I1,1)+DIS(I1,1))/(TX**2)
 D24=(DIS(I1+I,1))-2.*DIS(I1,1)+DIS(I1-I,1))/(TV**2)
 D25=(DIS(I2+IK,1))-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
 DT=DIS(I1+I,1)-DIS(I1-I,1)+DIS(I1+I,1)+DIS(I1-I,1)
 D26=DT/(4.*TX*TV)
 DT=DIS(I1+I,1)-DIS(I1-I,1)-DIS(I1-I,1)+DIS(I1-I,1)
 D27=DT/(4.*TX*TV)
 D28=(DIS(I1+I,1))-2.*DIS(I1,1)+DIS(I1-I,1))/(TV**2)
 D29=(DIS(I1+I,1))-2.*DIS(I1,1)+DIS(I1-I,1))/(TX**2)

GO TO 20

C
 CCC POINTS ON THE Y AXIS

C
 2102 D1=0.0
 D2=(-2.*DIS(I2,1))/(TX**2)
 D3=(DIS(I2+I,1))-DIS(I2-I,1))/(2.*TV)
 DT=DIS(I2+I,1)-DIS(I2-I,1)+DIS(I2-I,1)+DIS(I2-I,1)
 D4=0.0
 D5=(DIS(I2+I,1))-2.*DIS(I2,1)+DIS(I2-I,1))/(TV**2)
 DT=DIS(I2+I,1)-DIS(I2-I,1)-DIS(I2-I,1)+DIS(I2-I,1)
 D6=DT/(4.*TV*TZ)
 DT=DIS(I2+I,1)-DIS(I2-I,1)-DIS(I2-I,1)+DIS(I2-I,1)
 D7=0.0
 D8=(-2.*DIS(I1,1))/(2.*TX)
 D9=(DIS(I1+I,1))-DIS(I1-I,1))/(2.*TV)
 D10=0.0
 D11=0.0
 D12=0.0
 DT=-2.*DIS(I1+I,1)+2.*DIS(I1-I,1)
 D13=DT/(4.*TX*TZ)
 DT=DIS(I1+I,1)-DIS(I1-I,1)-DIS(I1-I,1)+DIS(I1-I,1)
 D14=0.0
 D15=0.0
 D16=(DIS(I1+JK,1))-DIS(I1-JK,1))/(2.*TZ)
 DT=DIS(I1+I,1)+DIS(I1-I,1)+DIS(I1-I,1)+DIS(I1-I,1)
 D17=DT/(4.*TV*TZ)
 DT=DIS(I1+I,1)-DIS(I1-I,1)-DIS(I1-I,1)+DIS(I1-I,1)
 D18=0.0
 D19=(DIS(I1-JK,1))-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2)
 D22=(DIS(I2+IK,1))-DIS(I2-1K,1))/(2.*TZ)
 D23=0.0
 D24=(DIS(I1+I,1))-2.*DIS(I1,1)+DIS(I1-I,1))/(TV**2)
 D25=(DIS(I2+IK,1))-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
 D1=-2.*DIS(I1+I,1)+2.*DIS(I1-I,1)
 D26=DT/(4.*TX*TV)
 DT=DIS(I1+I,1)-DIS(I1-I,1)-DIS(I1-I,1)+DIS(I1-I,1)
 D27=0.0
 D28=0.0
 D29=(-2.*DIS(I1,1))/(TZ**2)

D1F06610
 D1F06620
 D1F06630
 D1F06640
 D1F06650
 D1F06660
 D1F06670
 D1F06680
 D1F06690
 D1F06700
 D1F06710
 D1F06720
 D1F06730
 D1F06740
 D1F06750
 D1F06760
 D1F06770
 D1F06780
 D1F06790
 D1F06800
 D1F06810
 D1F06820
 D1F06830
 D1F06840
 D1F06850
 D1F06860
 D1F06870
 D1F06880
 D1F06890
 D1F06900
 D1F06910
 D1F06920
 D1F06930
 D1F06940
 D1F06950
 D1F06960
 D1F06970
 D1F06980
 D1F06990
 D1F07000
 D1F07010
 D1F07020
 D1F07030
 D1F07040
 D1F07050
 D1F07060
 D1F07070
 D1F07080
 D1F07090
 D1F07100
 D1F07110
 D1F07120
 D1F07130
 D1F07140
 D1F07150

GO TO 20

D9=(-2.*DIS(I1-IJ,1))/(2.*TV) D1F07710
 D10=0.0 D1F07720
 D11=0.0 D1F07730
 D12=0.0 D1F07740
 DT=-2.*DIS(I-1+IK,1)+2.*DIS(I-1-IK,1) D1F07750
 D13=DT/(4.*TX*TZ) D1F07760
 DT=DIS(I+IJ+JK,1)-DIS(I+IJ-JK,1)-DIS(I-IJ+JK,1)+DIS(I-IJ-JK,1) D1F07770
 D14=0.0 D1F07780
 D15=0.0 D1F07790
 D16=0.0 D1F07800
 DT=-2.*DIS(I1-IJ+JK,1)+2.*DIS(I1-IJ-JK,1) D1F07810
 D17=DT/(4.*TV*TZ) D1F07820
 DT=DIS(I1+I1+IK,1)-DIS(I1+I1-IK,1)-DIS(I1+I1+IK,1)+DIS(I1+I1-IK,1) D1F07830
 D18=0.0 D1F07840
 D19=0.0 D1F07850
 D20=0.0 D1F07860
 D21=0.0 D1F07870
 D22=(DIS(I2+IK,1)-DIS(I2-IK,1))/(2.*TZ) D1F07880
 D23=0.0 D1F07890
 D24=0.0 D1F07900
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-IK,1))/(TZ**2) D1F07910
 DT=DIS(I1+I1+IJ,1)-DIS(I1+I1-IJ,1)-DIS(I1+I1+IJ,1)+DIS(I1+I1-IJ,1) D1F07920
 D26=0.0 D1F07930
 DT=DIS(I1+I1+IJ,1)-DIS(I1+I1-IJ,1)-DIS(I1+I1+IJ,1)+DIS(I1+I1-IJ,1) D1F07940
 D27=0.0 D1F07950
 D28=0.0 D1F07960
 D29=0.0 D1F07970
 D30=0.0 D1F07980
 D31=0.0 D1F07990
 D32=0.0 D1F08000
 D33=0.0 D1F08010
 D34=0.0 D1F08020
 D35=0.0 D1F08030
 D36=0.0 D1F08040
 D37=0.0 D1F08050
 D38=0.0 D1F08060
 D39=0.0 D1F08070
 D40=0.0 D1F08080
 D41=0.0 D1F08090
 D42=0.0 D1F08100
 D43=0.0 D1F08110
 D44=0.0 D1F08120
 D45=0.0 D1F08130
 D46=0.0 D1F08140
 D47=0.0 D1F08150
 D48=0.0 D1F08160
 D49=0.0 D1F08170
 D50=0.0 D1F08180
 D51=0.0 D1F08190
 D52=0.0 D1F08200
 D53=0.0 D1F08210
 D54=0.0 D1F08220
 D55=0.0 D1F08230
 D56=0.0 D1F08240
 D57=0.0 D1F08250

GO TO 20

C CCC CORNER POINT

2100 D1=(DIS(I2+1,1))/(2.*TX) D1F08000
 D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2) D1F08010
 D3=(DIS(I2+IJ,1))/(2.*TV) D1F08020
 DT=DIS(I2+1+IJ,1) D1F08030
 D4=DT/(4.*TX*TV) D1F08040
 D5=(DIS(I2+IJ,1)-2.*DIS(I2,1))/(TV**2) D1F08050
 DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1) D1F08060
 D6=DT/(4.*TV*TZ) D1F08070
 DT=DIS(I2+1+IK,1)-DIS(I2+1-IK,1) D1F08080
 D7=DT/(4.*TX*TZ) D1F08090
 D8=(DIS(I1+1,1))/(2.*TX) D1F08100
 D9=(DIS(I1+IJ,1))/(2.*TV) D1F08110
 D10=(DIS(I1+IJ,1))/(2.*TV) D1F08120
 D11=(DIS(I1+1,1))/(2.*TX) D1F08130
 D12=(DIS(I1+IK,1)-DIS(I1-IK,1))/(2.*TZ) D1F08140
 DT=DIS(I1+1+IK,1)-DIS(I1+1-IK,1) D1F08150
 D13=DT/(4.*TX*TZ) D1F08160
 DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1) D1F08170
 D14=DT/(4.*TV*TZ) D1F08180
 D15=(DIS(I1-IJ+JK,1)+2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2) D1F08190
 D16=(DIS(I1+JK,1)-DIS(I1-JK,1))/(2.*TZ) D1F08200
 DT=DIS(I1-IJ+JK,1)-DIS(I1+IJ-JK,1) D1F08210
 D17=DT/(4.*TV*TZ) D1F08220
 DT=DIS(I1+1+IK,1)-DIS(I1+1-IK,1) D1F08230
 D18=DT/(4.*TX*TZ) D1F08240
 D19=DT/(4.*TX*TZ) D1F08250

D19=(DIS(I1-JK,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2) DIF08260
 D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ) DIF08270
 D23=(DIS(I1+1,1)-2.*DIS(I,1))/(TX**2) DIF08280
 D24=(DIS(I1+IJ,1)-2.*DIS(I1,1))/(TV**2) DIF08290
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2) DIF08300
 D1=DIS(I1+IJ,1) DIF08310
 D26=DT/(4.*TX*TV) DIF08320
 D1=DIS(I1+IJ,1) DIF08330
 D27=DT/(4.*TX*TV) DIF08340
 D28=(DIS(I1+IJ,1)-2.*DIS(I,1))/(TV**2) DIF08350
 D29=(DIS(I1+1,1)-2.*DIS(I1,1))/(TX**2) DIF08360
 DIF08370
 DIF08380
 DIF08390
 DIF08400
 DIF08410
 DIF08420
 DIF08430
 DIF08440
 DIF08450
 DIF08460
 DIF08470
 DIF08480
 DIF08490
 DIF08500
 DIF08510
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 DIF08580
 DIF08590
 DIF08600
 DIF08610
 DIF08620
 DIF08630
 DIF08640
 DIF08650
 DIF08660
 DIF08670
 DIF08680
 DIF08690
 DIF08700
 DIF08710
 DIF08720
 DIF08730
 DIF08740
 DIF08750
 DIF08760
 DIF08770
 DIF08780
 DIF08790
 DIF08800

GO TO 20

C
 CCC POINTS PARALLEL TO X AXIS

2099 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX) D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2) D3=(DIS(I2+IJ,1))/(2.*TV) DT=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1) D4=DT/(4.*TX*TV) D5=(DIS(I2+IJ,1)-2.*DIS(I2,1))/(TV**2) DT=DIS(I2+1+JK,1)-DIS(I2+1J-JK,1) D6=DT/(4.*TV*TZ) D1=DIS(I2+1+I,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1) D7=DT/(4.*TX*IZ) D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX) D9=(DIS(I1+IJ,1))/(2.*TV) D10=(DIS(I1+IJ,1))/(2.*TV) D11=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX) D12=(DIS(I1+IK,1)-DIS(I1-1K,1))/(2.*TZ) DT=DIS(I1+1+IK,1)-DIS(I1-1K,1)-DIS(I1+1+IK,1)+DIS(I1-1-1K,1) D13=DT/(4.*TX*TZ) DT=DIS(I1+1J+JK,1)-DIS(I1+1J-JK,1) D14=DT/(4.*TV*TZ) D15=(DIS(I1-1K,1)-2.*DIS(I,1)+DIS(I1+IK,1))/(TZ**2) D16=(DIS(I1+JK,1)-DIS(I1-1K,1))/(2.*TZ) DT=DIS(I1+1J+JK,1)-DIS(I1+1J-JK,1) D17=DT/(4.*TV*TZ) DT=DIS(I1+1+1K,1)-DIS(I1+1-1K,1)-DIS(I1-1+1K,1)+DIS(I1-1-1K,1) D18=DT/(4.*TX*TZ) D19=(DIS(I1-1K,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2) D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ) D23=(DIS(I1+1,1)-2.*DIS(I,1)+DIS(I1-1,1))/(TX**2) D24=(DIS(I1+IJ,1)-2.*DIS(I1,1))/(TV**2) D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2) DT=DIS(I1+IJ,1)-DIS(I1-1+IJ,1) D26=DT/(4.*TX*TV) DT=DIS(I1+1+IJ,1) D27=DT/(4.*TX*TV) D28=(DIS(I1+IJ,1)-2.*DIS(I,1))/(TV**2) D29=(DIS(I1+1,1)-2.*DIS(I1,1)+DIS(I1-1,1))/(TX**2)

GO TO 20

C

CCC POINT PARALLEL TO X AXIS ON THE Y AXIS

C

2097

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D1=0.0
D2=(-2.*DIS(I2,1))/(TX**2)
D3=(DIS(I2+IJ,1))/(2.*TV)
DT=DIS(I2+IJ,1)-DIS(I2-1+IJ,1)
D4=0.0
D5=(DIS(I2+IJ,1)-2.*DIS(I2,1))/(TV**2)
DT=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)
D7=0.0
D8=(-2.*DIS(I-1,1))/(2.*TX)
D9=(DIS(I1+IJ,1))/(2.*TV)
D10=0.0
D11=0.0
D12=0.0
DT=-2.*DIS(I-1+IK,1)+2.*DIS(I-1-1K,1)
D13=DT/(4.*TX*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)
D14=0.0
D15=0.0
D16=(DIS(I1+J,1)-DIS(I1-JK,1))/(2.*TZ)
DT=DIS(I1+IJ+JK,1)-DIS(I1+IJ-JK,1)
D17=DT/(4.*TV*TZ)
DT=DIS(I1+1+IK,1)-DIS(I1+1-1K,1)-DIS(I1-1+IK,1)
D18=0.0
D19=(DIS(I1-JK,1)-2.*DIS(I1,1)+DIS(I1+JK,1))/(TZ**2)
D22=(DIS(I2+IK,1)-DIS(I2-1K,1))/(2.*TZ)
D23=0.0
D24=(DIS(I1+IJ,1)-2.*DIS(I1,1))/(TV**2)
D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-1K,1))/(TZ**2)
DT=-DIS(I1+IJ,1)-DIS(I1+IJ,1)
D26=DT/(4.*TX*TV)
DT=DIS(I1+1+IJ,1)-DIS(I1-1+IJ,1)
D27=0.0
D28=0.0
D29=(-2.*DIS(I1,1))/(TX**2)

```

GO TO 20

CCC POINTS PARALLEL TO V AXIS

C

2098

```

D1=(DIS(I2+1,1))/(2.*TX)
D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2)
D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)
D4=DT/(4.*TX*TV)
D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
DT=DIS(I2+1+JK,1)-DIS(I2+1-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(I1+1,1))/(2.*TX)
D9=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)

```

- D1F08810
- D1F08820
- D1F08830
- D1F08840
- D1F08850
- D1F08860
- D1F08870
- D1F08880
- D1F08890
- D1F08900
- D1F08910
- D1F08920
- D1F08930
- D1F08940
- D1F08950
- D1F08960
- D1F08970
- D1F08980
- D1F08990
- D1F09000
- D1F09010
- D1F09020
- D1F09030
- D1F09040
- D1F09050
- D1F09060
- D1F09070
- D1F09080
- D1F09090
- D1F09100
- D1F09110
- D1F09120
- D1F09130
- D1F09140
- D1F09150
- D1F09160
- D1F09170
- D1F09180
- D1F09190
- D1F09200
- D1F09210
- D1F09220
- D1F09230
- D1F09240
- D1F09250
- D1F09260
- D1F09270
- D1F09280
- D1F09290
- D1F09300
- D1F09310
- D1F09320
- D1F09330
- D1F09340
- D1F09350

```

D10=(DIS(I+I,J,1))-DIS(I-I,J,1))/(2.*TV)
D11=(DIS(I+I+1,1))/(2.*TX)
D12=(DIS(I+IK,1))-DIS(I-IK,1))/(2.*TZ)
D13=DIS(I+I+IK,1)-DIS(I-IK,1)
D13=DT/(4.*TX*TZ)
D14=DIS(I+I+J+JK,1)-DIS(I+I+J-JK,1)+DIS(I-I+J-JK,1)
D14=DT/(4.*TV*TZ)
D15=(DIS(I-IK,1))-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)
D16=(DIS(I+I+JK,1))-DIS(I-I+JK,1))/(2.*TZ)
D17=DIS(I+I+J+JK,1)-DIS(I+I+J-JK,1)+DIS(I-I+J+JK,1)
D17=DT/(4.*TV*TZ)
D18=DIS(I+I+IK,1)-DIS(I+I-IK,1)
D18=DT/(4.*TX*TZ)
D19=(DIS(I+I-IK,1))-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)
D20=(DIS(I+I+JK,1))-DIS(I-I+JK,1))/(2.*TZ)
D21=(DIS(I+I,1))-2.*DIS(I,1)/(TX**2)
D22=(DIS(I+I+1,1))-2.*DIS(I,1)/(TX**2)
D23=(DIS(I+I,1))-2.*DIS(I,1)/(TX**2)
D24=(DIS(I+I+J,1))-2.*DIS(I,1)+DIS(I-I+J,1))/(TV**2)
D25=(DIS(I+I+JK,1))-2.*DIS(I,1)+DIS(I-I+JK,1))/(TZ**2)
D26=DIS(I+I+J,1)-DIS(I+I-IJ,1)
D26=DT/(4.*TX*TV)
D27=DIS(I+I+J,1)-DIS(I+I-IJ,1)
D27=DT/(4.*TX*TV)
D28=(DIS(I+I+J,1))-2.*DIS(I,1)+DIS(I-IJ,1))/(TV**2)
D29=(DIS(I+I+1,1))-2.*DIS(I,1)/(TX**2)

```

GO TO 20

C
CCC POINT PARALLEL TO V AXIS ON THE X AXIS

```

2096 D1=(DIS(I2+1,1))/(2.*TX)
D2=(DIS(I2+1,1))-2.*DIS(I2,1))/(TX**2)
D3=0.0
D4=DIS(I2+1+I,J,1)-DIS(I2+1-IJ,1)
D4=0.0
D5=(-2.*DIS(I2,1))/(TV**2)
D6=DIS(I2+I+J+JK,1)-DIS(I2+I+J-JK,1)+DIS(I2-I+J+JK,1)
D6=0.0
D7=DIS(I2+I+JK,1)-DIS(I2+I-IK,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(I+1,1))/(2.*TX)
D9=(-2.*DIS(I-I+J,1))/(2.*TV)
D10=0.0
D11=0.0
D12=(DIS(I+IK,1))-DIS(I-IK,1))/(2.*TZ)
D13=DIS(I+I+IK,1)-DIS(I-IK,1)
D13=DT/(4.*TX*TZ)
D14=0.0
D15=(DIS(I+I+J+JK,1))-DIS(I+I+J-JK,1)+DIS(I-I+J+JK,1)
D15=0.0
D16=(DIS(I-IK,1))-2.*DIS(I,1)+DIS(I+IK,1))/(TZ**2)
D17=DT/(4.*TV*TZ)
D18=DIS(I+I+IK,1)-DIS(I+I-IK,1)+DIS(I-I+I-IK,1)
D18=0.0
D19=0.0

```

D22=(DIS(I2+IK,1)-DIS(I2-IK,1))/(2.*TZ)
 D23=(DIS(I1+1,1)-2.*DIS(I,1))/(TX**2)
 D24=0.0
 D25=(DIS(I2+IK,1)-2.*DIS(I2,1)+DIS(I2-IK,1))/(TZ**2)
 D7=DIS(I1+IJ,1)-DIS(I1-IJ,1)
 D26=0.0
 D7=-DIS(I1+1-IJ,1)-DIS(I1+1-IJ,1)
 D27=DT/(4.*TX*TV)
 D28=(-2.*DIS(I,1))/(TV**2)
 D29=0.0

GO TO 20

C
 CCC GENERAL POINT (B.L)

2105 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
 D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
 D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
 D7=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
 D4=DT/(4.*TX*TV)
 D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
 D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 D7=DIS(I2+1+IK,1)-DIS(I2+1-IK,1)-DIS(I2-1+IK,1)+DIS(I2-1-IK,1)
 D7=DT/(4.*TX*TZ)
 D8=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX)
 D9=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)
 D10=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)
 D11=(DIS(I1+1,1)-DIS(I1-1,1))/(2.*TX)

GO TO 30

C
 CCC POINTS ON THE Y AXIS (B.L)

2106 D1=0.0
 D2=(-2.*DIS(I2,1))/(TX**2)
 D3=(DIS(I2+IJ,1)-DIS(I2-IJ,1))/(2.*TV)
 D7=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
 D4=0.0
 D5=(DIS(I2+IJ,1)-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)
 D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)
 D6=DT/(4.*TV*TZ)
 D7=DIS(I2+1+IK,1)-DIS(I2+1-IK,1)-DIS(I2-1+IK,1)+DIS(I2-1-IK,1)
 D7=0.0
 D8=(-2.*DIS(I1,1))/(2.*TX)
 D9=(DIS(I1+IJ,1)-DIS(I1-IJ,1))/(2.*TV)
 D10=0.0
 D11=0.0

GO TO 30

C
 CCC POINTS ON THE X AXIS (B.L)

2107 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
 D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)

DIFF09910
 DIFF09920
 DIFF09930
 DIFF09940
 DIFF09950
 DIFF09960
 DIFF09970
 DIFF09980
 DIFF09990
 DIFF10000
 DIFF10010
 DIFF10020
 DIFF10030
 DIFF10040
 DIFF10050
 DIFF10060
 DIFF10070
 DIFF10080
 DIFF10090
 DIFF10100
 DIFF10110
 DIFF10120
 DIFF10130
 DIFF10140
 DIFF10150
 DIFF10160
 DIFF10170
 DIFF10180
 DIFF10190
 DIFF10200
 DIFF10210
 DIFF10220
 DIFF10230
 DIFF10240
 DIFF10250
 DIFF10260
 DIFF10270
 DIFF10280
 DIFF10290
 DIFF10300
 DIFF10310
 DIFF10320
 DIFF10330
 DIFF10340
 DIFF10350
 DIFF10360
 DIFF10370
 DIFF10380
 DIFF10390
 DIFF10400
 DIFF10410
 DIFF10420
 DIFF10430
 DIFF10440
 DIFF10450

```

D3=0.0
DT=DIS(12+1+IJ,1)-DIS(12+1-IJ,1)-DIS(12-1+IJ,1)+DIS(12-1-IJ,1)
D4=0.0
D5=(-2.*DIS(12,1))/(TV**2)
DT=DIS(12+IJ+JK,1)-DIS(12+IJ-JK,1)-DIS(12-IJ+JK,1)+DIS(12-IJ-JK,1)
D6=0.0
DT=DIS(12+1+IK,1)-DIS(12+1-1K,1)-DIS(12-1+IK,1)+DIS(12-1-1K,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(1+1,1)-DIS(1-1,1))/(2.*TX)
D9=(-2.*DIS(1-1J,1))/(2.*TV)
D10=0.0
D11=0.0

```

GO TO 30

C
CCC CENTRAL POINT (B.L)

```

2108 D1=0.0
D2=(-2.*DIS(12,1))/(TX**2)
D3=0.0
DT=DIS(12+1+IJ,1)-DIS(12+1-IJ,1)-DIS(12-1+IJ,1)+DIS(12-1-IJ,1)
D4=0.0
D5=(-2.*DIS(12,1))/(TV**2)
DT=DIS(12+IJ+JK,1)-DIS(12+IJ-JK,1)-DIS(12-IJ+JK,1)+DIS(12-IJ-JK,1)
D6=0.0
DT=DIS(12+1+IK,1)-DIS(12+1-1K,1)-DIS(12-1+IK,1)+DIS(12-1-1K,1)
D7=0.0
D8=(-2.*DIS(1-1,1))/(2.*TX)
D9=(-2.*DIS(1-1J,1))/(2.*TV)
D10=0.0
D11=0.0

```

GO TO 30

C
CCC CORNER POINT (B.L)

```

2109 D1=(DIS(12+1,1))/(2.*TX)
D2=(DIS(12+1,1)-2.*DIS(12,1))/(TX**2)
D3=(DIS(12+1J,1))/(2.*TV)
DT=DIS(12+1+IJ,1)
D4=DT/(4.*TX*TZ)
D5=(DIS(12+1J,1)-2.*DIS(12,1))/(TV**2)
DT=DIS(12+1J+JK,1)-DIS(12+1J-JK,1)
D6=DT/(4.*TV*TZ)
DT=DIS(12+1+IK,1)-DIS(12+1-1K,1)
D7=DT/(4.*TX*TZ)
D8=(DIS(1+1,1))/(2.*TX)
D9=(DIS(1+1J,1))/(2.*TV)
D10=(DIS(1+1J,1))/(2.*TV)
D11=(DIS(1+1,1))/(2.*TX)

```

GO TO 30

C
CCC POINTS PARALLEL TO X AXIS (B.L)

- D1F10460
- D1F10470
- D1F10480
- D1F10490
- D1F10500
- D1F10510
- D1F10520
- D1F10530
- D1F10540
- D1F10550
- D1F10560
- D1F10570
- D1F10580
- D1F10590
- D1F10600
- D1F10610
- D1F10620
- D1F10630
- D1F10640
- D1F10650
- D1F10660
- D1F10670
- D1F10680
- D1F10690
- D1F10700
- D1F10710
- D1F10720
- D1F10730
- D1F10740
- D1F10750
- D1F10760
- D1F10770
- D1F10780
- D1F10790
- D1F10800
- D1F10810
- D1F10820
- D1F10830
- D1F10840
- D1F10850
- D1F10860
- D1F10870
- D1F10880
- D1F10890
- D1F10900
- D1F10910
- D1F10920
- D1F10930
- D1F10940
- D1F10950
- D1F10960
- D1F10970
- D1F10980
- D1F10990
- D1F11000

```

2110 D1=(DIS(I2+1,1)-DIS(I2-1,1))/(2.*TX)
      D2=(DIS(I2+1,1)-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
      D3=(DIS(I2+1J,1))/(2.*TV)
      DT=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1)
      D4=DT/(4.*TX*TV)
      D5=(DIS(I2+1J,1)-2.*DIS(I2,1))/(TV**2)
      DT=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)
      D6=DT/(4.*TV*TZ)
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D7=DT/(4.*TX*TZ)
      D8=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
      D9=(DIS(I+1J,1))/(2.*TV)
      D10=(DIS(I+1J,1))/(2.*TV)
      D11=(DIS(I+1,1)-DIS(I-1,1))/(2.*TX)
  
```

GO TO 30

C POINTS PARALLEL TO X AXIS ON THE Y AXIS

```

2112 D1=0.0
      D2=(-2.*DIS(I2,1))/(TX**2)
      D3=(DIS(I2+1J,1))/(2.*TV)
      DT=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1)
      D4=0.0
      D5=(DIS(I2+1J,1)-2.*DIS(I2,1))/(TV**2)
      DT=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)
      D6=DT/(4.*TV*TZ)
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D7=0.0
      D8=(-2.*DIS(I-1,1))/(2.*TX)
      D9=(DIS(I+1J,1))/(2.*TV)
      D10=0.0
      D11=0.0
  
```

GO TO 30

C POINTS PARALLEL TO Y AXIS

```

2111 D1=(DIS(I2+1,1))/(2.*TX)
      D2=(DIS(I2+1,1)-2.*DIS(I2,1))/(TX**2)
      D3=(DIS(I2+1J,1)-DIS(I2-1J,1))/(2.*TV)
      DT=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)
      D4=DT/(4.*TX*TV)
      D5=(DIS(I2+1J,1)-2.*DIS(I2,1)+DIS(I2-1J,1))/(TV**2)
      DT=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)-DIS(I2-1J+JK,1)+DIS(I2-1J-JK,1)
      D6=DT/(4.*TV*TZ)
      DT=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)
      D7=DT/(4.*TX*TZ)
      D8=(DIS(I+1,1))/(2.*TX)
      D9=(DIS(I+1J,1)-DIS(I-1J,1))/(2.*TV)
      D10=(DIS(I+1J,1)-DIS(I-1J,1))/(2.*TV)
      D11=(DIS(I+1,1))/(2.*TX)
  
```

GO TO 30

- DIFF11010
- DIFF11020
- DIFF11030
- DIFF11040
- DIFF11050
- DIFF11060
- DIFF11070
- DIFF11080
- DIFF11090
- DIFF11100
- DIFF11110
- DIFF11120
- DIFF11130
- DIFF11140
- DIFF11150
- DIFF11160
- DIFF11170
- DIFF11180
- DIFF11190
- DIFF11200
- DIFF11210
- DIFF11220
- DIFF11230
- DIFF11240
- DIFF11250
- DIFF11260
- DIFF11270
- DIFF11280
- DIFF11290
- DIFF11300
- DIFF11310
- DIFF11320
- DIFF11330
- DIFF11340
- DIFF11350
- DIFF11360
- DIFF11370
- DIFF11380
- DIFF11390
- DIFF11400
- DIFF11410
- DIFF11420
- DIFF11430
- DIFF11440
- DIFF11450
- DIFF11460
- DIFF11470
- DIFF11480
- DIFF11490
- DIFF11500
- DIFF11510
- DIFF11520
- DIFF11530
- DIFF11540
- DIFF11550

CCC POINTS PARALLEL TO Y AXIS ON THE X AXIS

2113 D1=(DIS(I2+1,1))/(2.*TX)

D2=(DIS(I2+1,1))-2.*DIS(I2,1)/(TX**2)

D3=0.0

D4=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)

D5=0.0

D6=(-2.*DIS(I2,1))/(TV**2)

D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)

D8=0.0

D9=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)

D10=DT/(4.*TX*TV)

D11=(DIS(I+1,1))/(2.*TX)

D12=(-2.*DIS(I1-IJ,1))/(2.*TV)

D13=0.0

D14=0.0

D15=0.0

D16=0.0

D17=0.0

D18=0.0

D19=0.0

D20=0.0

D21=0.0

D22=0.0

D23=0.0

D24=0.0

D25=0.0

D26=0.0

D27=0.0

D28=0.0

D29=0.0

D30=0.0

D31=0.0

D32=0.0

D33=0.0

D34=0.0

D35=0.0

D36=0.0

D37=0.0

D38=0.0

D39=0.0

D40=0.0

D41=0.0

D42=0.0

DIFF1560

DIFF1570

DIFF1580

DIFF1590

DIFF1600

DIFF1610

DIFF1620

DIFF1630

DIFF1640

DIFF1650

DIFF1660

DIFF1670

DIFF1680

DIFF1690

DIFF1700

DIFF1710

DIFF1720

DIFF1730

DIFF1740

DIFF1750

DIFF1760

DIFF1770

DIFF1780

DIFF1790

DIFF1800

DIFF1810

DIFF1820

DIFF1830

DIFF1840

DIFF1850

DIFF1860

DIFF1870

DIFF1880

DIFF1890

DIFF1900

DIFF1910

DIFF1920

DIFF1930

DIFF1940

DIFF1950

DIFF1960

DIFF1970

DIFF1980

DIFF1990

DIFF2000

DIFF2010

DIFF2020

DIFF2030

DIFF2040

DIFF2050

DIFF2060

DIFF2070

DIFF2080

DIFF2090

DIFF2100

CCC POINTS PARALLEL TO X AXIS NEAR Y AXIS (B.L.)

2126 D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)

D3=(DIS(I2+1,1))/(2.*TV)

D4=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1)

D5=DT/(4.*TX*TV)

D6=(-2.*DIS(I2,1))/(TV**2)

D7=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)

D8=0.0

D9=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)

D10=DT/(4.*TX*TV)

D11=(DIS(I+1,1))/(2.*TX)

D12=(-2.*DIS(I1-1,1))/(2.*TV)

D13=0.0

D14=0.0

D15=0.0

D16=0.0

D17=0.0

D18=0.0

D19=0.0

D20=0.0

D21=0.0

D22=0.0

D23=0.0

D24=0.0

D25=0.0

D26=0.0

D27=0.0

D28=0.0

D29=0.0

D30=0.0

DIFF1990

DIFF2000

DIFF2010

DIFF2020

DIFF2030

DIFF2040

DIFF2050

DIFF2060

DIFF2070

DIFF2080

DIFF2090

DIFF2100

GO TO 30

CCC GENERAL POINTS NEAR Y AXIS (B.L.)

2127 D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)

D3=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TV)

D4=DIS(I2+1+IJ,1)-DIS(I2-1+IJ,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)

D5=DT/(4.*TX*TV)

D6=DIS(I2+IJ+JK,1)-DIS(I2+IJ-JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ-JK,1)

D7=DT/(4.*TV*TV)

D8=(-DIS(I-1,1))/(2.*TX)

D9=(DIS(I+1,1))-DIS(I-1,1))/(2.*TV)

D10=(DIS(I+1,1))-DIS(I-1,1))/(2.*TV)

D11=(DIS(I+1,1))-DIS(I-1,1))/(2.*TV)

GO TO 30

GENERAL POINTS NEAR X & Y AXIS (B.L.)

```

C
CCC
2128 D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)
      D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
      D3=(DIS(I2+1J,1))-DIS(I2-1J,1))/(2.*TV)
      D4=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
      D5=(DIS(I2+1J,1))-2.*DIS(I2,1)+DIS(I2-1J,1))/(TV**2)
      D6=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)-DIS(I2-1J+JK,1)+DIS(I2-1J-JK,1)
      D7=DT/(4.*TV*TZ)
      D8=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D9=DT/(4.*TX*TZ)
      D10=(-DIS(I1-1J,1))/(2.*TV)
      D11=(DIS(I1+1,1))-DIS(I1-1,1))/(2.*TX)

```

GO TO 30

POINTS ON THE Y AXIS NEAR THE CENTRE (B.L.)

```

C
CCC
2129 D1=0.0
      D2=(-2.*DIS(I2,1))/(TX**2)
      D3=(DIS(I2+1J,1))-DIS(I2-1J,1))/(2.*TV)
      D4=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
      D5=0.0
      D6=(DIS(I2+1J,1))-2.*DIS(I2,1)+DIS(I2-1J,1))/(TV**2)
      D7=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)-DIS(I2-1J+JK,1)+DIS(I2-1J-JK,1)
      D8=DT/(4.*TV*TZ)
      D9=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D10=0.0
      D11=(-2.*DIS(I1,1))/(2.*TX)
      D12=(-DIS(I1-1J,1))/(2.*TV)
      D13=0.0
      D14=0.0

```

GO TO 30

POINTS ON THE X AXIS NEAR THE CENTRE (B.L.)

```

C
CCC
2130 D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)
      D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)
      D3=0.0
      D4=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)-DIS(I2-1+IJ,1)+DIS(I2-1-IJ,1)
      D5=(-2.*DIS(I2,1))/(TV**2)
      D6=DIS(I2+1J+JK,1)-DIS(I2+1J-JK,1)-DIS(I2-1J+JK,1)+DIS(I2-1J-JK,1)
      D7=DT/(4.*TX*TZ)
      D8=DIS(I2+1+IK,1)-DIS(I2+1-1K,1)-DIS(I2-1+IK,1)+DIS(I2-1-1K,1)
      D9=(-DIS(I1-1,1))/(2.*TX)
      D10=0.0

```

D11=0.0

GO TO 30

C
CCC GENERAL POINTS NEAR X AXIS (B.L)

C

2131 D1=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

D2=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TX**2)

D3=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TV)

D4=DIS(I2+1,1)-DIS(I2+1-IJ,1)+DIS(I2-1-IJ,1)

D5=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-1,1))/(TV**2)

D6=DIS(I2+1,1)-DIS(I2-IJ+JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ+JK,1)

D7=DIS(I2+1+IK,1)-DIS(I2+1+IK,1)-DIS(I2-1+IK,1)+DIS(I2-1+IK,1)

D8=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

D9=(-DIS(I2-1,1))-DIS(I2-1,1))/(2.*TV)

D10=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

GO TO 30

C
CCC POINTS PARALLEL TO Y AXIS NEAR X AXIS (B.L)

C

2132 D1=(DIS(I2+1,1))/(2.*TX)

D2=(DIS(I2+1,1))-2.*DIS(I2,1))/(TX**2)

D3=(DIS(I2+1,1))-DIS(I2-IJ,1))/(2.*TV)

D4=DIS(I2+1+IJ,1)-DIS(I2+1-IJ,1)

D5=(DIS(I2+1,1))-2.*DIS(I2,1)+DIS(I2-IJ,1))/(TV**2)

D6=DIS(I2+1+IJ+JK,1)-DIS(I2+1+IJ+JK,1)-DIS(I2-IJ+JK,1)+DIS(I2-IJ+JK,1)

D7=DIS(I2+1+IK,1)-DIS(I2+1+IK,1)

D8=(DIS(I2+1,1))/(2.*TX)

D9=(-DIS(I2-1,1))-DIS(I2-1,1))/(2.*TV)

D10=(DIS(I2+1,1))-DIS(I2-1,1))/(2.*TX)

GO TO 30

20 OX=0.5*(D3-D16)

OY=0.5*(D12-D1)

XOX=0.5*(D4-D17)

VOX=0.5*(D5-D18)

ZOX=0.5*(D6-D19)

XOV=0.5*(D13-D2)

VOV=0.5*(D14-D4)

ZOV=0.5*(D15-D7)

SXX=G1*D8+G3*(D9+D22/(HA**2))

SYY=G1*D9+G3*(D8+D22/(HA**2))

SZZ=G1*D22/(HA**2)+G3*(D8+D9)

TXV=G*(D10+D11)

TXZ=G*(D1+D12)

DIFF12660

DIFF12670

DIFF12680

DIFF12690

DIFF12700

DIFF12710

DIFF12720

DIFF12730

DIFF12740

DIFF12750

DIFF12760

DIFF12770

DIFF12780

DIFF12790

DIFF12800

DIFF12810

DIFF12820

DIFF12830

DIFF12840

DIFF12850

DIFF12860

DIFF12870

DIFF12880

DIFF12890

DIFF12900

DIFF12910

DIFF12920

DIFF12930

DIFF12940

DIFF12950

DIFF12960

DIFF12970

DIFF12980

DIFF12990

DIFF13000

DIFF13010

DIFF13020

DIFF13030

DIFF13040

DIFF13050

DIFF13060

DIFF13070

DIFF13080

DIFF13090

DIFF13100

DIFF13110

DIFF13120

DIFF13130

DIFF13140

DIFF13150

DIFF13160

DIFF13170

DIFF13180

DIFF13190

DIFF13200

```

TVZ=G*(D3+D16)
XSXX=G1*D23+G3*(D27+D7/(HA**2))
YSVY=G1*D24+G3*(D26+D6/(HA**2))
ZSZZ=G1*D25/(HA**2)+G3*(D13+D18)
XTXV=G*(D26+D29)
VTXV=G*(D28+D27)
XTXZ=G*(D2+D13)
ZTXZ=G*(D7+D15)
VTXZ=G*(D18+D5)
ZTVZ=G*(D6+D19)
H1=2.0*OX*XOX
H2=2.0*OY*XOX
H3=OX*XOY+OY*XOX
H4=2.0*OX*VOX
H5=2.0*OY*VOY
H6=OX*VOY+OY*VOX
H7=2.0*OX*ZOX
H8=2.0*OY*ZOX
FX=- (XTXZ*OY+TXZ*XOV)-(VTXZ*OY+TVZ*VOV)-(ZSZZ*OY+SZZ*ZOV)
FY+ (XTXZ*OX+TXZ*XOX)+(VTXZ*OX+TVZ*VOX)+(ZSZZ*OX+SZZ*ZOX)
FZ1=(XSXX*OY+SXX*XOV-XTXV*OX-TXV*XOX)*HA
FZ2=(VTXV*OY+TVV*VOV-VSVV*OX-SVV*VOX)*HA
FZ3=(ZTXZ*OY+TXZ*ZOV-ZTVZ*OX-TVZ*ZOX)/HA
FOR(I,1)= -G2*H2+G*H6-G3*H1+FX
FOR(I,1)= +G*H3-G2*H4-G3*H5+FY
FOR(I,1)= -G2*(H7+H8)/HA+(FZ1+FZ2+FZ3)
FOR(I,1)= FX*HA
FOR(I,1)= FV*HA
FOR(I2,1)= (FZ1+FZ2+FZ3)*HA

GO TO 18

30 FOR(I,1)= (-G11*D1+D2-G22*D3+D4-G*D1*D5)*HA
FOR(I,1)= (-G11*D3+D5-G22*D1+D4-G*D3*D2)*HA
DT1=(D8+0.5*D1**2+P*D9+0.5*P*D3**2)*D2
DT2=(D9+0.5*D3**2+P*D8+0.5*P*D1**2)*D5
DT3=(1.-P)*(D10+D11+D1*D3)*D4
C=12.0
C=12.0/(XA*HA)
FOR(I2,1)=(12.0*Q*(1.-P**2))/(F(I,1)*E*(HA**4))+C*(DT1+DT2+DT3)
FOR(I2,1)=(12.0*Q*(1.-P**2))/HA+*(DT1+DT2+DT3)
FOR(I2,1)=(12.0*Q*(1.-P**2))*(0.025/HA)**4+C*(DT1+DT2+DT3)
FOR(I2,1)=-C*(DT1+DT2+DT3)
18 CONTINUE
RETURN
END
END

```

 SUBROUTINE PROGRAM GENERATING THE FORCE
 VECTORS BASED ON FF INPUT VECTOR

- D1F13210
- D1F13220
- D1F13230
- D1F13240
- D1F13250
- D1F13260
- D1F13270
- D1F13280
- D1F13290
- D1F13300
- D1F13310
- D1F13320
- D1F13330
- D1F13340
- D1F13350
- D1F13360
- D1F13370
- D1F13380
- D1F13390
- D1F13400
- D1F13410
- D1F13420
- D1F13430
- D1F13440
- D1F13450
- D1F13460
- D1F13470
- D1F13480
- D1F13490
- D1F13500
- D1F13510
- D1F13520
- D1F13530
- D1F13540
- D1F13550
- D1F13560
- D1F13570
- D1F13580
- D1F13590
- D1F13600
- D1F13610
- D1F13620
- D1F13630
- D1F13640
- D1F13650
- D1F13660
- D1F13670
- D1F13680
- D1F13690
- D1F13700
- D1F13710
- D1F13720
- D1F13730
- D1F13740
- D1F13750

```

SUBROUTINE FORCE(Q,P,HA,NOD,E,FF,VAR)
  IMPLICIT REAL*8 (A-H,O-Z)
  DIMENSION FF(45,1),VAR(135,1)
  DO 8 K=1,3*NOD
    VAR(K,1)=0.0
  CONTINUE
  DO 9 K=1,3*NOD
    IF(K.GT.2*NOD) GO TO 14
    IF(K.LE.2*NOD) GO TO 9
  14 VAR(K,1)=(12.0*Q*(1.-P**2)/(FF(K-2*NOD,1))*E*(HA**4))
  14 VAR(K,1)=(12.0*Q*(1.-P**2)*FF(K-2*NOD,1))/HA
  9 CONTINUE
  RETURN
  END
  14 VAR(K,1)=(12.0*Q*(1.-P**2)*FF(K-2*NOD,1))
  14 VAR(K,1)= 12.0*Q*(1.-P**2)/HA
  9 CONTINUE
  RETURN
  END
*****
* SUBROUTINE PROGRAM TO SATISFY THE BOUNDARY *
* CONDITIONS IN THE GLOBAL MATRIX 'AA' *
*****
SUBROUTINE BOUNDA(UVW,NOD,AA)
  IMPLICIT REAL*8 (A-H,O-Z)
  DIMENSION AA(135,135),UVW(9,3)
  NL=5
  DO 501 I=1,NOD/NL
    DO 502 K=1,3
      IF(UVW(I,K).EQ.0) GO TO 502
      DO 504 M=1,NL
        L=I+(M-1)*(NOD/NL)+(K-1)*NOD
        TEMP=AA(L,L)
        DO 503 J=1,3*NOD
          AA(L,J)=0.0
        CONTINUE
        AA(L,L)=1.0
      504 CONTINUE
    502 CONTINUE
  501 CONTINUE
  RETURN
  END
*****
* SUBROUTINE PROGRAM TO SATISFY THE BOUNDARY *
* CONDITIONS IN THE FORCE VECTOR 'FF' *
*****

```

DIFF13760
DIFF13770
DIFF13780
DIFF13790
DIFF13800
DIFF13810
DIFF13820
DIFF13830
DIFF13840
DIFF13850
DIFF13860
DIFF13870
DIFF13880
DIFF13890
DIFF13900
DIFF13910
DIFF13920
DIFF13930
DIFF13940
DIFF13950
DIFF13960
DIFF13970
DIFF13980
DIFF13990
DIFF14000
DIFF14010
DIFF14020
DIFF14030
DIFF14040
DIFF14050
DIFF14060
DIFF14070
DIFF14080
DIFF14090
DIFF14100
DIFF14110
DIFF14120
DIFF14130
DIFF14140
DIFF14150
DIFF14160
DIFF14170
DIFF14180
DIFF14190
DIFF14200
DIFF14210
DIFF14220
DIFF14230
DIFF14240
DIFF14250
DIFF14260
DIFF14270
DIFF14280
DIFF14290
DIFF14300


```

IF(IJKL(I,4),EQ,109) GO TO 109
IF(IJKL(I,4),EQ,110) GO TO 110
IF(IJKL(I,4),EQ,111) GO TO 111
IF(IJKL(I,4),EQ,112) GO TO 112
IF(IJKL(I,4),EQ,113) GO TO 113
IF(IJKL(I,4),EQ,114) GO TO 114
IF(IJKL(I,4),EQ,115) GO TO 115
IF(IJKL(I,4),EQ,116) GO TO 116
IF(IJKL(I,4),EQ,117) GO TO 117
IF(IJKL(I,4),EQ,118) GO TO 118
IF(IJKL(I,4),EQ,119) GO TO 119
IF(IJKL(I,4),EQ,120) GO TO 120
IF(IJKL(I,4),EQ,121) GO TO 121
IF(IJKL(I,4),EQ,122) GO TO 122
IF(IJKL(I,4),EQ,123) GO TO 123
IF(IJKL(I,4),EQ,124) GO TO 124
IF(IJKL(I,4),EQ,125) GO TO 125
DIF15410
DIF15420
DIF15430
DIF15440
DIF15450
DIF15460
DIF15470
DIF15480
DIF15490
DIF15500
DIF15510
DIF15520
DIF15530
DIF15540
DIF15550
DIF15560
DIF15570
DIF15580
DIF15590
DIF15600
DIF15610
DIF15620
DIF15630
DIF15640
DIF15650
DIF15660
DIF15670
DIF15680
DIF15690
DIF15700
DIF15710
DIF15720
DIF15730
DIF15740
DIF15750
DIF15760
DIF15770
DIF15780
DIF15790
DIF15800
DIF15810
DIF15820
DIF15830
DIF15840
DIF15850
DIF15860
DIF15870
DIF15880
DIF15890
DIF15900
DIF15910
DIF15920
DIF15930
DIF15940
DIF15950

```

CCC POINT PARALLEL TO Y AXIS ON THE X AXIS

```

96 AA(I,1)=AA(I,1)-(2.0*G1/TX**2)
AA(I,1+1)=AA(I,1+1)+(G1/TX**2)
AA(I,1-J)=AA(I,1-J)+(2.0*G/TV**2)
AA(I,1)=AA(I,1)-(2.0*G/TV**2)
AA(I,1-IK)=AA(I,1-IK)+G/TZ**2
AA(I,1)=AA(I,1)-2.0*G/TZ**2
AA(I,1+IK)=AA(I,1+IK)+G/TZ**2
DIF15610
DIF15620
DIF15630
DIF15640
DIF15650
DIF15660
DIF15670
DIF15680
DIF15690
DIF15700
DIF15710
DIF15720
DIF15730
DIF15740
DIF15750
DIF15760
DIF15770
DIF15780
DIF15790
DIF15800
DIF15810
DIF15820
DIF15830
DIF15840
DIF15850
DIF15860
DIF15870
DIF15880
DIF15890
DIF15900
DIF15910
DIF15920
DIF15930
DIF15940
DIF15950

```

```

CCCCC
AA(I,1,1)=AA(I,1,1)+(G2/(TX*TV))
AA(I,1,1+1)=AA(I,1,1+1)-(G2/(TX*TV))
AA(I,1,1-J)=AA(I,1,1-J)-(G2/(TX*TV))
AA(I,1,1-IJ)=AA(I,1,1-IJ)+(G2/(TX*TV))
AA(I,1,1-IK)=AA(I,1,1-IK)+G/TZ**2
AA(I,1,1)=AA(I,1,1)-2.0*G/TZ**2
AA(I,1,1+IK)=AA(I,1,1+IK)+G/TZ**2
DIF15810
DIF15820
DIF15830
DIF15840
DIF15850
DIF15860
DIF15870
DIF15880
DIF15890
DIF15900
DIF15910
DIF15920
DIF15930
DIF15940
DIF15950

```

```

CCCCC
AA(I,1,1)=AA(I,1,1)-(2.0*G/TX**2)
AA(I,1,1+1)=AA(I,1,1+1)+(G/TX**2)
AA(I,1,1-J)=AA(I,1,1-J)-(2.0*G/TV**2)
AA(I,1,1)=AA(I,1,1)-(2.0*G/TV**2)
AA(I,1,1-IJ)=AA(I,1,1-IJ)+(2.0*G/TV**2)
AA(I,1,1-IK)=AA(I,1,1-IK)+G/TZ**2
AA(I,1,1)=AA(I,1,1)-2.0*G/TZ**2
AA(I,1,1+JK)=AA(I,1,1+JK)+G/TZ**2
DIF15910
DIF15920
DIF15930
DIF15940
DIF15950

```

```

CCCCC
AA(I,1,1,1)=AA(I,1,1,1)+(G2/(TV*TZ*HA))
AA(I,1,1,2)=AA(I,1,1,2)-(G2/(TV*TZ*HA))
AA(I,1,1,1-J)=AA(I,1,1,1-J)-(G2/(TV*TZ*HA))
AA(I,1,1,1+JK)=AA(I,1,1,1+JK)-(G2/(TV*TZ*HA))
DIF15910
DIF15920
DIF15930
DIF15940
DIF15950

```

```

C      AA(I1,I2+JK-IJ)=AA(I1,I2+JK-IJ)+(G2/(TV*TZ*HA))
DIF15960
C      AA(I2,I1)=AA(I2,I1)+(G2/(TX*TZ))
DIF15970
C      AA(I2,I+1)=AA(I2,I+1)-(G2/(TX*TZ))
DIF15980
C      AA(I2,I+1K)=AA(I2,I+1K)-(G2/(TX*TZ))
DIF15990
C      AA(I2,I+1K+1)=AA(I2,I+1K+1)+(G2/(TX*TZ))
DIF16000
C      AA(I2,I1)=AA(I2,I1)-(G2/(TV*TZ))
DIF16010
C      AA(I2,I1+JK)=AA(I2,I1+JK)-(G2/(TV*TZ))
DIF16020
C      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)+(G2/(TV*TZ))
DIF16030
C      AA(I2,I1)=AA(I2,I1)+(G2/(TV*TZ))
DIF16040
C      AA(I2,I1+JK)=AA(I2,I1+JK)-(G2/(TV*TZ))
DIF16050
C      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)+(G2/(TV*TZ))
DIF16060
CCCCC
AA(I2,I2)=AA(I2,I2)-2.0*G/(HA*TX**2)
DIF16070
AA(I2,I2+1)=AA(I2,I2+1)+G/(HA*TX**2)
DIF16080
AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(HA*TV**2)
DIF16090
AA(I2,I2)=AA(I2,I2)-2.0*G/(HA*TV**2)
DIF16100
AA(I2,I2-1K)=AA(I2,I2-1K)+G1/(HA*TZ**2)
DIF16110
AA(I2,I2-1K+1)=AA(I2,I2-1K+1)+G1/(HA*TZ**2)
DIF16120
AA(I2,I2)=AA(I2,I2)-2.0*G1/(HA*TX**2)
DIF16130
AA(I2,I2+1K)=AA(I2,I2+1K)+G1/(HA*TX**2)
DIF16140
GO TO 2
DIF16150
C XD
AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)
DIF16160
AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)-G2/(4.0*TX*TV)
DIF16170
AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(4.0*TX*TV)
DIF16180
AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)+G2/(4.0*TX*TV)
DIF16190
C VD
AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)
DIF16200
AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)-G2/(4.0*TX*TV)
DIF16210
AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)+G2/(4.0*TX*TV)
DIF16220
AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)-G2/(4.0*TX*TV)
DIF16230
AA(I1,I2-1K-1)=AA(I1,I2-1K-1)+G2/(4.0*TX*TZ*HA)
DIF16240
AA(I1,I2-1K+1)=AA(I1,I2-1K+1)-G2/(4.0*TX*TZ*HA)
DIF16250
AA(I1,I2+1K-1)=AA(I1,I2+1K-1)+G2/(4.0*TX*TZ*HA)
DIF16260
AA(I1,I2+1K+1)=AA(I1,I2+1K+1)-G2/(4.0*TX*TZ*HA)
DIF16270
C ZD
AA(I1,I2-1K-1)=AA(I1,I2-1K-1)+G2/(4.0*TV*TZ*HA)
DIF16280
AA(I1,I2-1K+1)=AA(I1,I2-1K+1)-G2/(4.0*TV*TZ*HA)
DIF16290
AA(I1,I2+1K-1)=AA(I1,I2+1K-1)+G2/(4.0*TV*TZ*HA)
DIF16300
AA(I1,I2+1K+1)=AA(I1,I2+1K+1)-G2/(4.0*TV*TZ*HA)
DIF16310
AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)+G2/(4.0*TX*TV)
DIF16320
AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)
DIF16330
AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)-G2/(4.0*TX*TV)
DIF16340
AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(4.0*TX*TV)
DIF16350
AA(I1,I2+JK-IJ)=AA(I1,I2+JK-IJ)+G2/(4.0*TV*TZ*HA)
DIF16360
AA(I1,I2+JK+1-IJ)=AA(I1,I2+JK+1-IJ)+G2/(4.0*TV*TZ*HA)
DIF16370
AA(I2,I1-1K-1)=AA(I2,I1-1K-1)+G2/(4.0*TX*TZ)
DIF16380
AA(I2,I1-1K+1)=AA(I2,I1-1K+1)-G2/(4.0*TX*TZ)
DIF16390
AA(I2,I1+1K-1)=AA(I2,I1+1K-1)+G2/(4.0*TX*TZ)
DIF16400
AA(I2,I1+1K+1)=AA(I2,I1+1K+1)-G2/(4.0*TX*TZ)
DIF16410
AA(I2,I1-1K-1)=AA(I2,I1-1K-1)+G2/(4.0*TV*TZ)
DIF16420
AA(I2,I1-1K+1)=AA(I2,I1-1K+1)+G2/(4.0*TV*TZ)
DIF16430
AA(I2,I1-1K-1)=AA(I2,I1-1K-1)+G2/(4.0*TV*TZ)
DIF16440
AA(I2,I1-1K+1)=AA(I2,I1-1K+1)+G2/(4.0*TV*TZ)
DIF16450
AA(I2,I1+1K-1)=AA(I2,I1+1K-1)+G2/(4.0*TV*TZ)
DIF16460
AA(I2,I1+1K+1)=AA(I2,I1+1K+1)+G2/(4.0*TV*TZ)
DIF16470
GO TO 2
DIF16480
DIF16490
DIF16500

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C CCC POINT PARALLEL TO X AXIS ON THE Y AXIS

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C 97 AA(I,1,1-1)=AA(I,1,1-1)+2.0*G1/TX**2          DIF16510
C 97 AA(I,1,1)=AA(I,1,1)-2.0*G1/TX**2              DIF16520
AA(I,1,1)=AA(I,1,1)-2.0*G/TV**2                   DIF16530
AA(I,1+1J)=AA(I,1+1J)-2.0*G1/TV**2                DIF16540
AA(I,1+1K)=AA(I,1+1K)+G/TZ**2                     DIF16550
AA(I,1)=AA(I,1)-2.0*G/TZ**2                        DIF16560
AA(I,1+1K)=AA(I,1+1K)+G/TZ**2                      DIF16570
C CCCCC
AA(I,1,1)=AA(I,1,1)+G2/(TX*TV)                     DIF16580
AA(I,1,1-1)=AA(I,1,1-1)-G2/(TX*TV)                 DIF16590
AA(I,1+1J)=AA(I,1+1J)-G2/(TX*TV)                   DIF16600
AA(I,1+1J)=AA(I,1+1J)-G2/(TX*TV)                   DIF16610
AA(I,1+1J)=AA(I,1+1J)-G2/(TX*TV)                   DIF16620
C AA(I,1,12)=AA(I,1,12)+G2/(TX*TV*HA)                DIF16630
C AA(I,1,12-1)=AA(I,1,12-1)-G2/(TX*TV*HA)           DIF16640
C AA(I,1,12+1K)=AA(I,1,12+1K)-G2/(TX*TV*HA)        DIF16650
C AA(I,1,12+1K-1)=AA(I,1,12+1K-1)+G2/(TX*TV*HA)    DIF16660
C AA(I,1,1)=AA(I,1,1)+G2/(TX*TV)                     DIF16670
C AA(I,1,1-1)=AA(I,1,1-1)-G2/(TX*TV)                 DIF16680
C AA(I,1,1+1J)=AA(I,1,1+1J)-G2/(TX*TV)               DIF16690
C AA(I,1,1+1J)=AA(I,1,1+1J)-G2/(TX*TV)               DIF16700
C AA(I,1,1+1J)=AA(I,1,1+1J)-G2/(TX*TV)               DIF16710
C AA(I,1,1+1J)=AA(I,1,1+1J)-G2/(TX*TV)               DIF16720
C AA(I,1,1+1J-1)=AA(I,1,1+1J-1)+G2/(TX*TV)          DIF16730
C CCCCC
AA(I,1,1,1-1)=AA(I,1,1,1-1)+2.0*G/TX**2            DIF16740
AA(I,1,1,1)=AA(I,1,1,1)-2.0*G/TX**2                DIF16750
AA(I,1,1,1)=AA(I,1,1,1)-2.0*G/TX**2                DIF16760
AA(I,1,1,1)=AA(I,1,1,1)-2.0*G1/TV**2               DIF16770
AA(I,1,1+1J)=AA(I,1,1+1J)+G1/TV**2                 DIF16780
AA(I,1,1,1-JK)=AA(I,1,1,1-JK)+G/TZ**2              DIF16790
AA(I,1,1,1)=AA(I,1,1,1)-2.0*G/TZ**2                DIF16800
AA(I,1,1+1JK)=AA(I,1,1+1JK)+G/TZ**2                 DIF16810
C CCCCC
AA(I,1,1,12)=AA(I,1,1,12)+G2/(TV*TZ*HA)             DIF16820
AA(I,1,1,12+1J)=AA(I,1,1,12+1J)-G2/(TV*TZ*HA)     DIF16830
AA(I,1,1,12+JK)=AA(I,1,1,12+JK)-G2/(TV*TZ*HA)     DIF16840
AA(I,1,1,12+JK)=AA(I,1,1,12+JK)-G2/(TV*TZ*HA)     DIF16850
AA(I,1,1,12+1J+JK)=AA(I,1,1,12+1J+JK)+G2/(TV*TZ*HA) DIF16860
C AA(I,1,1)=AA(I,1,1)+G2/(TX*TV)                     DIF16870
C AA(I,1,1-1)=AA(I,1,1-1)-G2/(TX*TV)                 DIF16880
C AA(I,1,1+1K)=AA(I,1,1+1K)-G2/(TX*TV)               DIF16890
C AA(I,1,1+1K-1)=AA(I,1,1+1K-1)+G2/(TX*TV)          DIF16900
C AA(I,1,1,12)=AA(I,1,1,12)+2.0*G/(TX*TV*HA)        DIF16910
C AA(I,1,1,12-1)=AA(I,1,1,12-1)-2.0*G/(TX*TV*HA)   DIF16920
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TX*TV*HA)        DIF16930
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16940
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16950
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16960
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16970
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16980
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF16990
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF17000
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF17010
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G/(TV*TV*HA)        DIF17020
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G1/(TV*TV*HA)       DIF17030
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G1/(TV*TV*HA)       DIF17040
C AA(I,1,1,12)=AA(I,1,1,12)-2.0*G1/(TV*TV*HA)       DIF17050
GO TO 2

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C      AA(I1,I1)=AA(I1,I1)+G2/(TX*TV)
C      AA(I1,I1+1)=AA(I1,I1+1)-G2/(TX*TV)
C      AA(I1,I1+J)=AA(I1,I1+J)-G2/(TX*TV)
C      AA(I1,I1+J+1)=AA(I1,I1+J+1)+G2/(TX*TV)
CCCCC
AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1+1)=AA(I1,I1+1)+G/TV**2
AA(I1,I1-J)=AA(I1,I1-J)+G1/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G1/TV**2
AA(I1,I1+J)=AA(I1,I1+J)+G1/TV**2
AA(I1,I1-JK)=AA(I1,I1-JK)+G1/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1+JK)=AA(I1,I1+JK)+G1/TV**2
CCCCC
AA(I1,I2)=AA(I1,I2)+G2/(TV*TZ)
AA(I1,I2+J)=AA(I1,I2+J)-G2/(TV*TZ)
AA(I1,I2+JK)=AA(I1,I2+JK)-G2/(TV*TZ)
AA(I1,I2+JK+J)=AA(I1,I2+JK+J)+G2/(TV*TZ)
C      AA(I2,I1)=AA(I2,I1)+G2/(TV*TZ)
C      AA(I2,I1+J)=AA(I2,I1+J)-G2/(TV*TZ)
C      AA(I2,I1+JK)=AA(I2,I1+JK)-G2/(TV*TZ)
C      AA(I2,I1+JK+J)=AA(I2,I1+JK+J)+G2/(TV*TZ)
CCCCC
AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2+1)=AA(I2,I2+1)+G/(TX*TX*HA)
AA(I2,I2-J)=AA(I2,I2-J)+G/(TV*TV*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TV*TV*HA)
AA(I2,I2+J)=AA(I2,I2+J)+G/(TV*TV*HA)
AA(I2,I2-K)=AA(I2,I2-K)+G1/(TZ*TZ*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G1/(TZ*TZ*HA)
AA(I2,I2+K)=AA(I2,I2+K)+G1/(TZ*TZ*HA)
C9003 GO TO 2
C XD
CCC   AA(I1,I1-J)=AA(I1,I1-J)+G2/(4.0*TX*TV)
CCC   AA(I1,I1+J)=AA(I1,I1+J)-G2/(4.0*TX*TV)
CCC   AA(I1,I1+J-1)=AA(I1,I1+J-1)-G2/(4.0*TX*TV)
CCC   AA(I1,I1+J+1)=AA(I1,I1+J+1)+G2/(4.0*TX*TV)
CCC   AA(I1,I2-K-1)=AA(I1,I2-K-1)+G2/(4.0*TX*TZ*HA)
CCC   AA(I1,I2-K+1)=AA(I1,I2-K+1)-G2/(4.0*TX*TZ*HA)
CCC   AA(I1,I2+K-1)=AA(I1,I2+K-1)-G2/(4.0*TX*TZ*HA)
CCC   AA(I1,I2+K+1)=AA(I1,I2+K+1)+G2/(4.0*TX*TZ*HA)
C YD
CCC   AA(I1,I1-J-1)=AA(I1,I1-J-1)+G2/(4.0*TX*TV)
CCC   AA(I1,I1-J+1)=AA(I1,I1-J+1)-G2/(4.0*TX*TV)
CCC   AA(I1,I1+J-1)=AA(I1,I1+J-1)-G2/(4.0*TX*TV)
CCC   AA(I1,I1+J+1)=AA(I1,I1+J+1)+G2/(4.0*TX*TV)
DIF17610
DIF17620
DIF17630
DIF17640
DIF17650
DIF17660
DIF17670
DIF17680
DIF17690
DIF17700
DIF17710
DIF17720
DIF17730
DIF17740
DIF17750
DIF17760
DIF17770
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DIF17790
DIF17800
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DIF17980
DIF17990
DIF18000
DIF18010
DIF18020
DIF18030
DIF18040
DIF18050
DIF18060
DIF18070
DIF18080
DIF18090
DIF18100
DIF18110
DIF18120
DIF18130
DIF18140
DIF18150

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AA(I1,I2-JK-IJ)=AA(I1,I2-JK-IJ)+G2/(4.0*TV*TZ*HA)
AA(I1,I2-JK+IJ)=AA(I1,I2-JK+IJ)-G2/(4.0*TV*TZ*HA)
AA(I1,I2+JK-IJ)=AA(I1,I2+JK-IJ)-G2/(4.0*TV*TZ*HA)
AA(I1,I2+JK+IJ)=AA(I1,I2+JK+IJ)+G2/(4.0*TV*TZ*HA)
C ZD
CCC
AA(I2,I-IK-1)=AA(I2,I-IK-1)+G2/(4.0*TV*TZ)
AA(I2,I-IK+1)=AA(I2,I-IK+1)-G2/(4.0*TV*TZ)
AA(I2,I+IK-1)=AA(I2,I+IK-1)-G2/(4.0*TV*TZ)
AA(I2,I+IK+1)=AA(I2,I+IK+1)+G2/(4.0*TV*TZ)
AA(I2,I1-JK-IJ)=AA(I2,I1-JK-IJ)+G2/(4.0*TV*TZ)
AA(I2,I1-JK+IJ)=AA(I2,I1-JK+IJ)-G2/(4.0*TV*TZ)
AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-G2/(4.0*TV*TZ)
AA(I2,I1+JK+IJ)=AA(I2,I1+JK+IJ)+G2/(4.0*TV*TZ)
GO TO 2
C
CCC POINTS PARALLEL TO X AXIS
C
99 AA(I,I-1)=AA(I,I-1)+G1/TX**2
AA(I,I) =AA(I,I)-2.0*G1/TX**2
AA(I,I+1)=AA(I,I+1)+G1/TX**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+IJ)=AA(I,I+IJ)+G/TV**2
AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
AA(I,I)=AA(I,I)-2.0*G/TZ**2
AA(I,I+IK)=AA(I,I+IK)+G/TZ**2
CCCCC
AA(I,I1)=AA(I,I1)+G2/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G2/(TX*TV)
AA(I,I1+IJ)=AA(I,I1+IJ)-G2/(TX*TV)
AA(I,I1+IJ+1)=AA(I,I1+IJ+1)+G2/(TX*TV)
C
AA(I,I2)=AA(I,I2)+G2/(TX*TZ*HA)
AA(I,I2+1)=AA(I,I2+1)-G2/(TX*TZ*HA)
AA(I,I2+IK)=AA(I,I2+IK)-G2/(TX*TZ*HA)
AA(I,I2+IK+1)=AA(I,I2+IK+1)+G2/(TX*TZ*HA)
C
AA(I,I1,I)=AA(I,I1,I)+G2/(TX*TV)
AA(I,I1,I+1)=AA(I,I1,I+1)-G2/(TX*TV)
AA(I,I1,I+IJ)=AA(I,I1,I+IJ)-G2/(TX*TV)
AA(I,I1,I+IJ+1)=AA(I,I1,I+IJ+1)+G2/(TX*TV)
CCCCC
AA(I,I1,I1-1)=AA(I,I1,I1-1)+G/TX**2
AA(I,I1,I1)=AA(I,I1,I1)-2.0*G/TX**2
AA(I,I1,I1+1)=AA(I,I1,I1+1)+G/TX**2
AA(I,I1,I1)=AA(I,I1,I1)-2.0*G1/TV**2
AA(I,I1,I1+IJ)=AA(I,I1,I1+IJ)+G1/TV**2
AA(I,I1,I1-JK)=AA(I,I1,I1-JK)+G/TZ**2
AA(I,I1,I1)=AA(I,I1,I1)-2.0*G/TZ**2
AA(I,I1,I1+JK)=AA(I,I1,I1+JK)+G/TZ**2
CCCCC
AA(I,I1,I2)=AA(I,I1,I2)+G2/(TV*TZ*HA)
AA(I,I1,I2+IJ)=AA(I,I1,I2+IJ)-G2/(TV*TZ*HA)
AA(I,I1,I2+JK)=AA(I,I1,I2+JK)-G2/(TV*TZ*HA)

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- DIFF18160
- DIFF18170
- DIFF18180
- DIFF18190
- DIFF18200
- DIFF18210
- DIFF18220
- DIFF18230
- DIFF18240
- DIFF18250
- DIFF18260
- DIFF18270
- DIFF18280
- DIFF18290
- DIFF18300
- DIFF18310
- DIFF18320
- DIFF18330
- DIFF18340
- DIFF18350
- DIFF18360
- DIFF18370
- DIFF18380
- DIFF18390
- DIFF18400
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- DIFF18590
- DIFF18600
- DIFF18610
- DIFF18620
- DIFF18630
- DIFF18640
- DIFF18650
- DIFF18660
- DIFF18670
- DIFF18680
- DIFF18690
- DIFF18700

C GO TO 2 DIF19810

C XD AA(1,11-IJ-1)=AA(1,11-IJ-1)+G2/(4.0*TX*TV) DIF19820

C AA(1,11-IJ+1)=AA(1,11-IJ+1)-G2/(4.0*TX*TV) DIF19830

C AA(1,11+IJ-1)=AA(1,11+IJ-1)-G2/(4.0*TX*TV) DIF19840

C AA(1,11+IJ+1)=AA(1,11+IJ+1)+G2/(4.0*TX*TV) DIF19850

C AA(1,12-IK-1)=AA(1,12-IK-1)+G2/(4.0*TX*TZ*HA) DIF19860

C AA(1,12-IK+1)=AA(1,12-IK+1)-G2/(4.0*TX*TZ*HA) DIF19870

C AA(1,12+IK-1)=AA(1,12+IK-1)-G2/(4.0*TX*TZ*HA) DIF19880

C AA(1,12+IK+1)=AA(1,12+IK+1)+G2/(4.0*TX*TZ*HA) DIF19890

C VD AA(1,1-IJ-1)=AA(1,1-IJ-1)+G2/(4.0*TX*TV) DIF19900

C AA(1,1-IJ+1)=AA(1,1-IJ+1)-G2/(4.0*TX*TV) DIF19910

C AA(1,1+IJ-1)=AA(1,1+IJ-1)-G2/(4.0*TX*TV) DIF19920

C AA(1,1+IJ+1)=AA(1,1+IJ+1)+G2/(4.0*TX*TV) DIF19930

C AA(1,12-IK-1)=AA(1,12-IK-1)+G2/(4.0*TX*TZ) DIF19940

C AA(1,12-IK+1)=AA(1,12-IK+1)-G2/(4.0*TX*TZ) DIF19950

C AA(1,12+IK-1)=AA(1,12+IK-1)-G2/(4.0*TX*TZ) DIF19960

C AA(1,12+IK+1)=AA(1,12+IK+1)+G2/(4.0*TX*TZ) DIF19970

C AA(1,12-IK-1)=AA(1,12-IK-1)+G2/(4.0*TX*TZ) DIF19980

C AA(1,12-IK+1)=AA(1,12-IK+1)-G2/(4.0*TX*TZ) DIF19990

C AA(1,12+IK-1)=AA(1,12+IK-1)-G2/(4.0*TX*TZ) DIF20000

C AA(1,12+IK+1)=AA(1,12+IK+1)+G2/(4.0*TX*TZ) DIF20010

C AA(1,12-IK-1)=AA(1,12-IK-1)+G2/(4.0*TX*TZ) DIF20020

C AA(1,12-IK+1)=AA(1,12-IK+1)-G2/(4.0*TX*TZ) DIF20030

C AA(1,12+IK-1)=AA(1,12+IK-1)-G2/(4.0*TX*TZ) DIF20040

C AA(1,12+IK+1)=AA(1,12+IK+1)+G2/(4.0*TX*TZ) DIF20050

C AA(1,12-IK-1)=AA(1,12-IK-1)+G2/(4.0*TX*TZ) DIF20060

C AA(1,12-IK+1)=AA(1,12-IK+1)-G2/(4.0*TX*TZ) DIF20070

C AA(1,12+IK-1)=AA(1,12+IK-1)-G2/(4.0*TX*TZ) DIF20080

C AA(1,12+IK+1)=AA(1,12+IK+1)+G2/(4.0*TX*TZ) DIF20090

GO TO 2 DIF20100

GO TO 2 DIF20110

GO TO 2 DIF20120

GO TO 2 DIF20130

GO TO 2 DIF20140

GO TO 2 DIF20150

GO TO 2 DIF20160

GO TO 2 DIF20170

GO TO 2 DIF20180

GO TO 2 DIF20190

GO TO 2 DIF20200

GO TO 2 DIF20210

GO TO 2 DIF20220

GO TO 2 DIF20230

GO TO 2 DIF20240

GO TO 2 DIF20250

GO TO 2 DIF20260

GO TO 2 DIF20270

GO TO 2 DIF20280

GO TO 2 DIF20290

GO TO 2 DIF20300

GO TO 2 DIF20310

GO TO 2 DIF20320

GO TO 2 DIF20330

GO TO 2 DIF20340

GO TO 2 DIF20350

GENERAL POINTS

AA(1,1)=AA(1,1)-2.0*G1/TX**2

AA(1,1+1)=AA(1,1+1)+G1/TX**2

AA(1,1-IJ)=AA(1,1-IJ)+G/TV**2

AA(1,1+IJ)=AA(1,1+IJ)+G/TV**2

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C      AA(I, I2+IK+1)=AA(I, I2+IK+1)+G2/(TX*TZ*HA)
DIF20360
C      AA(I, I, I)=AA(I, I, I)+G2/(TX*TV)
DIF20370
C      AA(I, I, I+1)=AA(I, I, I+1)-G2/(TX*TV)
DIF20380
C      AA(I, I, I+J)=AA(I, I, I+J)-G2/(TX*TV)
DIF20390
C      AA(I, I, I+J+1)=AA(I, I, I+J+1)+G2/(TX*TV)
DIF20400
C      AA(I, I, I+J+1)=AA(I, I, I+J+1)+G2/(TX*TV)
DIF20410
C      AA(I, I, I-1)=AA(I, I, I-1)+G/TX**2
DIF20420
C      AA(I, I, I)=AA(I, I, I)+G/TX**2
DIF20430
C      AA(I, I, I)=AA(I, I, I)-2.0*G/TX**2
DIF20440
C      AA(I, I, I+1)=AA(I, I, I+1)+G/TX**2
DIF20450
C      AA(I, I, I-1)=AA(I, I, I-1)+G/TX**2
DIF20460
C      AA(I, I, I)=AA(I, I, I)-2.0*G1/TV**2
DIF20470
C      AA(I, I, I+J)=AA(I, I, I+J)+G1/TV**2
DIF20480
C      AA(I, I, I-JK)=AA(I, I, I-JK)+G/TZ**2
DIF20490
C      AA(I, I, I)=AA(I, I, I)-2.0*G/TZ**2
DIF20500
C      AA(I, I, I+JK)=AA(I, I, I+JK)+G/TZ**2
DIF20510
C      AA(I, I, I)=AA(I, I, I)-2.0*G/TX**2
DIF20520
C      AA(I, I, I)=AA(I, I, I)+G/TX**2
DIF20530
C      AA(I, I, I2+J)=AA(I, I, I2+J)-G2/(TV*TZ*HA)
DIF20540
C      AA(I, I, I2+JK)=AA(I, I, I2+JK)-G2/(TV*TZ*HA)
DIF20550
C      AA(I, I, I2+JK+1J)=AA(I, I, I2+JK+1J)+G2/(TV*TZ*HA)
DIF20560
C      AA(I, I, I2+JK+1J)=AA(I, I, I2+JK+1J)+G2/(TV*TZ*HA)
DIF20570
C      AA(I, I, I)=AA(I, I, I)+G2/(TV*TZ)
DIF20580
C      AA(I, I, I+1)=AA(I, I, I+1)+G2/(TV*TZ)
DIF20590
C      AA(I, I, I+JK)=AA(I, I, I+JK)-G2/(TV*TZ)
DIF20600
C      AA(I, I, I+JK)=AA(I, I, I+JK)-G2/(TV*TZ)
DIF20610
C      AA(I, I, I+JK+1)=AA(I, I, I+JK+1)+G2/(TV*TZ)
DIF20620
C      AA(I, I, I)=AA(I, I, I)+G2/(TV*TZ)
DIF20630
C      AA(I, I, I+1)=AA(I, I, I+1)+G2/(TV*TZ)
DIF20640
C      AA(I, I, I+JK)=AA(I, I, I+JK)-G2/(TV*TZ)
DIF20650
C      AA(I, I, I+JK)=AA(I, I, I+JK)-G2/(TV*TZ)
DIF20660
C      AA(I, I, I+JK+1J)=AA(I, I, I+JK+1J)+G2/(TV*TZ)
DIF20670
C      AA(I, I, I2-1)=AA(I, I, I2-1)+G/(TX*TX*HA)
DIF20680
C      AA(I, I, I2)=AA(I, I, I2)-2.0*G/(TX*TX*HA)
DIF20690
C      AA(I, I, I2+1)=AA(I, I, I2+1)+G/(TX*TX*HA)
DIF20700
C      AA(I, I, I2-1J)=AA(I, I, I2-1J)+G/(TV*TV*HA)
DIF20710
C      AA(I, I, I2-1J)=AA(I, I, I2-1J)+G/(TV*TV*HA)
DIF20720
C      AA(I, I, I2)=AA(I, I, I2)-2.0*G/(TV*TV*HA)
DIF20730
C      AA(I, I, I2+1J)=AA(I, I, I2+1J)+G/(TV*TV*HA)
DIF20740
C      AA(I, I, I2-1K)=AA(I, I, I2-1K)+G1/(TZ*TZ*HA)
DIF20750
C      AA(I, I, I2)=AA(I, I, I2)-2.0*G1/(TZ*TZ*HA)
DIF20760
C      AA(I, I, I2+1K)=AA(I, I, I2+1K)+G1/(TZ*TZ*HA)
DIF20770
C      GO TO 2
DIF20780
DIF20790
DIF20800
DIF20810
DIF20820
DIF20830
DIF20840
DIF20850
DIF20860
DIF20870
DIF20880
DIF20890
DIF20900

```

C YD

AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+GZ/(4.0*TX*TV)
 AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)-GZ/(4.0*TX*TV)
 AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)-GZ/(4.0*TX*TV)
 AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)+GZ/(4.0*TX*TV)

DIFF20910
 DIFF20920
 DIFF20930
 DIFF20940
 DIFF20950
 DIFF20960
 DIFF20970
 DIFF20980
 DIFF20990
 DIFF21000

AA(I1,I2-JK-IJ)=AA(I1,I2-JK-IJ)+GZ/(4.0*TV*TZ*HA)
 AA(I1,I2-JK+IJ)=AA(I1,I2-JK+IJ)-GZ/(4.0*TV*TZ*HA)
 AA(I1,I2+JK-IJ)=AA(I1,I2+JK-IJ)-GZ/(4.0*TV*TZ*HA)
 AA(I1,I2+JK+IJ)=AA(I1,I2+JK+IJ)+GZ/(4.0*TV*TZ*HA)

DIFF21010
 DIFF21020
 DIFF21030
 DIFF21040
 DIFF21050
 DIFF21060
 DIFF21070
 DIFF21080
 DIFF21090
 DIFF21100

AA(I2,I1-IK-1)=AA(I2,I1-IK-1)+GZ/(4.0*TX*TZ)
 AA(I2,I1-IK+1)=AA(I2,I1-IK+1)-GZ/(4.0*TX*TZ)
 AA(I2,I1+IK-1)=AA(I2,I1+IK-1)-GZ/(4.0*TX*TZ)
 AA(I2,I1+IK+1)=AA(I2,I1+IK+1)+GZ/(4.0*TX*TZ)

DIFF21110
 DIFF21120
 DIFF21130
 DIFF21140
 DIFF21150
 DIFF21160
 DIFF21170
 DIFF21180
 DIFF21190
 DIFF21200

AA(I2,I1-JK-IJ)=AA(I2,I1-JK-IJ)+GZ/(4.0*TV*TZ)
 AA(I2,I1-JK+IJ)=AA(I2,I1-JK+IJ)-GZ/(4.0*TV*TZ)
 AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-GZ/(4.0*TV*TZ)
 AA(I2,I1+JK+IJ)=AA(I2,I1+JK+IJ)+GZ/(4.0*TV*TZ)

DIFF21210
 DIFF21220
 DIFF21230
 DIFF21240
 DIFF21250
 DIFF21260
 DIFF21270
 DIFF21280
 DIFF21290
 DIFF21300

GO TO 2

C CC POINTS ON THE V AXIS

C 102 AA(I,I-1)=AA(I,I-1)+2.0*G1/TV**2
 C AA(I,I)=AA(I,I)-2.0*G1/TV**2
 C 102 AA(I,I)=AA(I,I)-2.0*G1/TV**2
 C AA(I,I-IJ)=AA(I,I-IJ)+G/TV**2
 C AA(I,I)=AA(I,I)-2.0*G/TV**2
 C AA(I,I+IJ)=AA(I,I+IJ)+G/TV**2
 C AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
 C AA(I,I)=AA(I,I)-2.0*G/TZ**2
 C AA(I,I+IK)=AA(I,I+IK)+G/TZ**2

DIFF21310
 DIFF21320
 DIFF21330
 DIFF21340
 DIFF21350
 DIFF21360
 DIFF21370
 DIFF21380
 DIFF21390
 DIFF21400

CCCCC
 C AA(I,I1)=AA(I,I1)+GZ/(TX*TV)
 C AA(I,I1-1)=AA(I,I1-1)-GZ/(TX*TV)
 C AA(I,I1+1)=AA(I,I1+1)+GZ/(TX*TV)
 C AA(I,I1+IJ-1)=AA(I,I1+IJ-1)+GZ/(TX*TV)
 C AA(I,I2)=AA(I,I2)+GZ/(TX*TZ*HA)
 C AA(I,I2-1)=AA(I,I2-1)-GZ/(TX*TZ*HA)
 C AA(I,I2+1K)=AA(I,I2+1K)-GZ/(TX*TZ*HA)
 C AA(I,I2+1K-1)=AA(I,I2+1K-1)+GZ/(TX*TZ*HA)

DIFF21410
 DIFF21420
 DIFF21430
 DIFF21440
 DIFF21450

CCCCC
 C AA(I1,I1-1)=AA(I1,I1-1)+2.0*G/TV**2
 C AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
 C AA(I1,I1+1)=AA(I1,I1+1)+G/TV**2
 C AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
 C AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2
 C AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
 C AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2
 C AA(I1,I1-JK)=AA(I1,I1-JK)+G/TZ**2
 C AA(I1,I1)=AA(I1,I1)-2.0*G/TZ**2

DIFF21390
 DIFF21400
 DIFF21410
 DIFF21420
 DIFF21430
 DIFF21440
 DIFF21450


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C      AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-G2/(4.0*TV*TZ)
C      AA(I2,I1+JK+IJ)=AA(I2,I1+JK+IJ)+G2/(4.0*TV*TZ)
C      WRITE(6,890)((AA(I,J),J=1,3*NOD)
C      WRITE(6,890)((AA(I,J),J=1,3*NOD)
C      WRITE(6,890)((AA(I2,J),J=1,3*NOD)
C 890  FORMAT(9E12.4,/)
      GO TO 2
CCC   POINTS ON THE X AXIS
C
C 103 AA(I,I-1)=AA(I,I-1)+G1/TX**2
C      AA(I,I)=AA(I,I)-2.0*G1/TX**2
C      AA(I,I+1)=AA(I,I+1)+G1/TX**2
C      AA(I,I-IJ)=AA(I,I-IJ)+2.0*G/TV**2
C      AA(I,I-IJ)=AA(I,I-IJ)+2.0*G/TV**2
C      AA(I,I-IK)=AA(I,I-IK)+G/TZ**2
C      AA(I,I)=AA(I,I)-2.0*G/TZ**2
C      AA(I,I+IK)=AA(I,I+IK)+G/TZ**2
CCCCC
C      AA(I,I1)=AA(I,I1)+G2/(TX*TV)
C      AA(I,I1+1)=AA(I,I1+1)-G2/(TX*TV)
C      AA(I,I1-IJ)=AA(I,I1-IJ)-G2/(TX*TV)
C      AA(I,I1-IJ+1)=AA(I,I1-IJ+1)+G2/(TX*TV)
C
C      AA(I,I2)=AA(I,I2)+G2/(TX*TV*HA)
C      AA(I,I2+1)=AA(I,I2+1)-G2/(TX*TV*HA)
C      AA(I,I2+IK)=AA(I,I2+IK)-G2/(TX*TV*HA)
C      AA(I,I2+IK+1)=AA(I,I2+IK+1)+G2/(TX*TV*HA)
CCCCC
C      AA(I,I1,I1)=AA(I,I1,I1)+G/TX**2
C      AA(I,I1,I1)=AA(I,I1,I1)-2.0*G/TX**2
C      AA(I,I1,I1+1)=AA(I,I1,I1+1)+G/TX**2
C      AA(I,I1,I1-IJ)=AA(I,I1,I1-IJ)+2.0*G1/TV**2
C      AA(I,I1,I1)=AA(I,I1,I1)-2.0*G1/TV**2
C      AA(I,I1,I1-JK)=AA(I,I1,I1-JK)+G/TZ**2
C      AA(I,I1,I1)=AA(I,I1,I1)-2.0*G/TZ**2
C      AA(I,I1,I1+JK)=AA(I,I1,I1+JK)+G/TZ**2
CCCCC
C      AA(I,I1,I2)=AA(I,I1,I2)+G2/(TV*TZ*HA)
C      AA(I,I1,I2-IJ)=AA(I,I1,I2-IJ)-G2/(TV*TZ*HA)
C      AA(I,I1,I2+JK)=AA(I,I1,I2+JK)-G2/(TV*TZ*HA)
C      AA(I,I1,I2+JK-IJ)=AA(I,I1,I2+JK-IJ)+G2/(TV*TZ*HA)
C
C      AA(I2,I1)=AA(I2,I1)+G2/(TX*TZ)
C      AA(I2,I1+1)=AA(I2,I1+1)-G2/(TX*TZ)
C      AA(I2,I1+IK)=AA(I2,I1+IK)-G2/(TX*TZ)
C      AA(I2,I1+IK+1)=AA(I2,I1+IK+1)+G2/(TX*TZ)
DIF22010
DIF22020
DIF22030
DIF22040
DIF22050
DIF22060
DIF22070
DIF22080
DIF22090
DIF22100
DIF22110
DIF22120
DIF22130
DIF22140
DIF22150
DIF22160
DIF22170
DIF22180
DIF22190
DIF22200
DIF22210
DIF22220
DIF22230
DIF22240
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DIF22470
DIF22480
DIF22490
DIF22500
DIF22510
DIF22520
DIF22530
DIF22540
DIF22550

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C      AA(I2,I1)=AA(I2,I1)+G2/(TV*TZ)
C      AA(I2,I1-IJ)=AA(I2,I1-IJ)-G2/(TV*TZ)
C      AA(I2,I1+JK)=AA(I2,I1+JK)-G2/(TV*TZ)
C      AA(I2,I1-IJ+JK)=AA(I2,I1-IJ+JK)+G2/(TV*TZ)
CCCCC
AA(I2,I2-1)=AA(I2,I2-1)+G/(TX*TX*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TX*TX*HA)
AA(I2,I2+1)=AA(I2,I2+1)+G/(TX*TX*HA)
AA(I2,I2-IJ)=AA(I2,I2-IJ)+2.0*G/(TV*TV*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G/(TV*TV*HA)
AA(I2,I2-1K)=AA(I2,I2-1K)+G/(TZ*TZ*HA)
AA(I2,I2)=AA(I2,I2)-2.0*G1/(TZ*TZ*HA)
AA(I2,I2+1K)=AA(I2,I2+1K)+G1/(TZ*TZ*HA)
C9008 GO TO 2

C XD
AA(I,I-IJ-1)=AA(I,I-IJ-1)+G2/(4.0*TX*TV)
AA(I,I-IJ+1)=AA(I,I-IJ+1)-G2/(4.0*TX*TV)
AA(I,I-IJ)=AA(I,I-IJ)+G2/(4.0*TX*TV)
AA(I,I-IJ+1)=AA(I,I-IJ+1)-G2/(4.0*TX*TV)
AA(I,I-IJ-1)=AA(I,I-IJ-1)+G2/(4.0*TX*TV)
AA(I,I2-1K-1)=AA(I,I2-1K-1)+G2/(4.0*TX*TZ*HA)
AA(I,I2-1K+1)=AA(I,I2-1K+1)-G2/(4.0*TX*TZ*HA)
AA(I,I2+1K-1)=AA(I,I2+1K-1)-G2/(4.0*TX*TZ*HA)
AA(I,I2+1K+1)=AA(I,I2+1K+1)+G2/(4.0*TX*TZ*HA)
C YD
AA(I,I-IJ-1)=AA(I,I-IJ-1)+G2/(4.0*TX*TV)
AA(I,I-IJ+1)=AA(I,I-IJ+1)-G2/(4.0*TX*TV)
AA(I,I-IJ)=AA(I,I-IJ)+G2/(4.0*TX*TV)
AA(I,I-IJ+1)=AA(I,I-IJ+1)-G2/(4.0*TX*TV)
AA(I,I-IJ-1)=AA(I,I-IJ-1)+G2/(4.0*TX*TV)
AA(I,I2-1K-1)=AA(I,I2-1K-1)+G2/(4.0*TV*TZ*HA)
AA(I,I2-1K+1)=AA(I,I2-1K+1)-G2/(4.0*TV*TZ*HA)
AA(I,I2+1K-1)=AA(I,I2+1K-1)+G2/(4.0*TV*TZ*HA)
AA(I,I2+1K+1)=AA(I,I2+1K+1)-G2/(4.0*TV*TZ*HA)
C ZD
AA(I2,I-IK-1)=AA(I2,I-IK-1)+G2/(4.0*TX*TZ)
AA(I2,I-IK+1)=AA(I2,I-IK+1)-G2/(4.0*TX*TZ)
AA(I2,I+1K-1)=AA(I2,I+1K-1)-G2/(4.0*TX*TZ)
AA(I2,I+1K+1)=AA(I2,I+1K+1)+G2/(4.0*TX*TZ)
AA(I2,I1-JK-IJ)=AA(I2,I1-JK-IJ)+G2/(4.0*TV*TZ)
AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-G2/(4.0*TV*TZ)
AA(I2,I1-JK-IJ)=AA(I2,I1-JK-IJ)+G2/(4.0*TV*TZ)
AA(I2,I1+JK-IJ)=AA(I2,I1+JK-IJ)-G2/(4.0*TV*TZ)
WRITE(6,891)(AA(I,J),J=1,3*NOD)
WRITE(6,891)(AA(I,J),J=1,3*NOD)
WRITE(6,891)(AA(I2,J),J=1,3*NOD)
C 89: FORMAT(9E12.4,' ')
GO TO 2

CCC  CENTRAL POINT
C
C 104 AA(I,I-1)=AA(I,I-1)+2.0*G1/TX**2

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- DIF22560
- DIF22570
- DIF22580
- DIF22590
- DIF22600
- DIF22610
- DIF22620
- DIF22630
- DIF22640
- DIF22650
- DIF22660
- DIF22670
- DIF22680
- DIF22690
- DIF22700
- DIF22710
- DIF22720
- DIF22730
- DIF22740
- DIF22750
- DIF22760
- DIF22770
- DIF22780
- DIF22790
- DIF22800
- DIF22810
- DIF22820
- DIF22830
- DIF22840
- DIF22850
- DIF22860
- DIF22870
- DIF22880
- DIF22890
- DIF22900
- DIF22910
- DIF22920
- DIF22930
- DIF22940
- DIF22950
- DIF22960
- DIF22970
- DIF22980
- DIF22990
- DIF23000
- DIF23010
- DIF23020
- DIF23030
- DIF23040
- DIF23050
- DIF23060
- DIF23070
- DIF23080
- DIF23090
- DIF23100


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AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ-1)=AA(I,11-IJ-1)-G2/(4.0*TX*TV)
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ-1)=AA(I,11-IJ-1)-G2/(4.0*TX*TV)
AA(I,12-IK-1)=AA(I,12-IK-1)+G2/(4.0*TX*TZ*HA)
AA(I,12-IK-1)=AA(I,12-IK-1)-G2/(4.0*TX*TZ*HA)
AA(I,12-IK-1)=AA(I,12-IK-1)+G2/(4.0*TX*TZ*HA)
AA(I,12-IK-1)=AA(I,12-IK-1)-G2/(4.0*TX*TZ*HA)
AA(I,12+JK-1)=AA(I,12+JK-1)+G2/(4.0*TX*TV)
AA(I,1,1-IJ-1)=AA(I,1,1-IJ-1)+G2/(4.0*TX*TV)
AA(I,1,1-IJ-1)=AA(I,1,1-IJ-1)-G2/(4.0*TX*TV)
AA(I,1,1-IJ-1)=AA(I,1,1-IJ-1)+G2/(4.0*TX*TV)
AA(I,1,1-IJ-1)=AA(I,1,1-IJ-1)-G2/(4.0*TX*TV)
AA(I,1,12+JK-IJ)=AA(I,1,12+JK-IJ)+G2/(4.0*TV*TZ*HA)
AA(I,1,12+JK-IJ)=AA(I,1,12+JK-IJ)-G2/(4.0*TV*TZ*HA)
AA(I,1,12+JK-IJ)=AA(I,1,12+JK-IJ)+G2/(4.0*TV*TZ*HA)
AA(I,1,12+JK-IJ)=AA(I,1,12+JK-IJ)-G2/(4.0*TV*TZ*HA)
AA(I,12,1-IK-1)=AA(I,12,1-IK-1)+G2/(4.0*TX*TZ)
AA(I,12,1-IK-1)=AA(I,12,1-IK-1)-G2/(4.0*TX*TZ)
AA(I,12,1-IK-1)=AA(I,12,1-IK-1)+G2/(4.0*TX*TZ)
AA(I,12,1-IK-1)=AA(I,12,1-IK-1)-G2/(4.0*TX*TZ)
AA(I,12,1+IK-1)=AA(I,12,1+IK-1)+G2/(4.0*TX*TZ)
AA(I,12,1+IK-1)=AA(I,12,1+IK-1)-G2/(4.0*TX*TZ)
AA(I,12,1+IK-1)=AA(I,12,1+IK-1)+G2/(4.0*TX*TZ)
AA(I,12,1+IK-1)=AA(I,12,1+IK-1)-G2/(4.0*TX*TZ)
AA(I,12,11-JK-IJ)=AA(I,12,11-JK-IJ)+G2/(4.0*TV*TZ)
AA(I,12,11-JK-IJ)=AA(I,12,11-JK-IJ)-G2/(4.0*TV*TZ)
AA(I,12,11-JK-IJ)=AA(I,12,11-JK-IJ)+G2/(4.0*TV*TZ)
AA(I,12,11-JK-IJ)=AA(I,12,11-JK-IJ)-G2/(4.0*TV*TZ)
AA(I,12,11+JK-IJ)=AA(I,12,11+JK-IJ)+G2/(4.0*TV*TZ)
AA(I,12,11+JK-IJ)=AA(I,12,11+JK-IJ)-G2/(4.0*TV*TZ)
AA(I,12,11+JK-IJ)=AA(I,12,11+JK-IJ)+G2/(4.0*TV*TZ)
AA(I,12,11+JK-IJ)=AA(I,12,11+JK-IJ)-G2/(4.0*TV*TZ)
GO TO 2
C
C
C
CCC GENERAL POINT (B.L)
C
C
C
105 AA(I,1,1-1)=AA(I,1,1-1)+G11/TV**2
AA(I,1,1)=AA(I,1,1)-2.0*G11/TV**2
AA(I,1,1+1)=AA(I,1,1+1)+G11/TV**2
AA(I,1,1-1)=AA(I,1,1-1)+G/TV**2
AA(I,1,1-IJ)=AA(I,1,1-IJ)+G/TV**2
AA(I,1,1)=AA(I,1,1)-2.0*G/TV**2
AA(I,1,1+IJ)=AA(I,1,1+IJ)+G/TV**2
IR1=ISIGN(1,IK)
AA(I,1,1-IR1)*ABS(IK)=AA(I,1,1-IR1)*ABS(IK)+G4/(TZ**2)
AA(I,1,1+IR1)*ABS(IK)=AA(I,1,1+IR1)*ABS(IK)-2.0*G4/(TZ**2)
AA(I,1,1+(1+IR1)*ABS(IK))=AA(I,1,1+(1+IR1)*ABS(IK))+G4/(TZ**2)
AA(I,1,1+(1+IR1)*ABS(IK))=AA(I,1,1+(1+IR1)*ABS(IK))-2.0*G4/(TZ**2)
AA(I,1,1)=AA(I,1,1)+G22/(TX*TV)
AA(I,1,1+1)=AA(I,1,1+1)-G22/(TX*TV)
AA(I,1,1+IJ)=AA(I,1,1+IJ)-G22/(TX*TV)
AA(I,1,1+IJ)=AA(I,1,1+IJ)+G22/(TX*TV)
AA(I,1,1)=AA(I,1,1)+G22/(TX*TV)
AA(I,1,1+1)=AA(I,1,1+1)-G22/(TX*TV)
AA(I,1,1+IJ)=AA(I,1,1+IJ)-G22/(TX*TV)
AA(I,1,1+IJ)=AA(I,1,1+IJ)+G22/(TX*TV)
AA(I,1,1+IJ+1)=AA(I,1,1+IJ+1)+G22/(TX*TV)
AA(I,1,1+IJ+1)=AA(I,1,1+IJ+1)-G22/(TX*TV)
AA(I,1,1+IJ+1)=AA(I,1,1+IJ+1)+G22/(TX*TV)
AA(I,1,1+IJ+1)=AA(I,1,1+IJ+1)-G22/(TX*TV)

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```

C XD
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ+1)=AA(I,11-IJ+1)-G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)-G2/(4.0*TX*TV)
AA(I,11+IJ+1)=AA(I,11+IJ+1)+G2/(4.0*TX*TV)
C VD
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ+1)=AA(I,11-IJ+1)-G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)-G2/(4.0*TX*TV)
AA(I,11+IJ+1)=AA(I,11+IJ+1)+G2/(4.0*TX*TV)
AA(I,11-1)=AA(I,11-1)+G/TV**2
AA(I,1,11)=AA(I,1,11)-2.0*G/TV**2
AA(I,11+1)=AA(I,11+1)+G/TV**2
AA(I,11-IJ)=AA(I,11-IJ)+G11/TV**2
AA(I,1,11)=AA(I,1,11)-2.0*G11/TV**2
AA(I,11+IJ)=AA(I,11+IJ)+G11/TV**2
IR2=ISIGN(1,JK)
AA(I,1,11-(1-IR2)*ABS(JK))=AA(I,1,11-(1-IR2)*ABS(JK))+G4/(TZ**2)
AA(I,1,11+IR2*ABS(JK))=AA(I,1,11+IR2*ABS(JK))-2.0*G4/(TZ**2)
AA(I,1,11+(1+IR2)*ABS(JK))=AA(I,1,11+(1+IR2)*ABS(JK))+G4/(TZ**2)
C
CALL SCHE(NA,IM,JM,NOD,ITEST,NP)
DO 61 II=1,13
NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 61
IF(NP(II).NE.0) GO TO 62
61 CONTINUE
62 AA(I2,NPA(II)+2*NOD)=AA(I2,NP(II)+2*NOD)*(NP(II)/NPA(II))*R(II)
C
GO TO 2
C
CCC POINTS ON THE V AXIS (B.L)
C
C 106 AA(I,1-1)=AA(I,1-1)+2.0*G11/TV**2
C
C 106 AA(I,1)=AA(I,1)-2.0*G11/TV**2
C
C 106 AA(I,1-IJ)=AA(I,1-IJ)+G/TV**2
C
C AA(I,1)=AA(I,1)-2.0*G/TV**2
C
C AA(I,1+IJ)=AA(I,1+IJ)+G/TV**2
C
C IR1=ISIGN(1,IK)
C
C AA(I,1-(1-IR1)*ABS(IK))=AA(I,1-(1-IR1)*ABS(IK))+G4/(TZ**2)
C
C AA(I,1+IR1*ABS(IK))=AA(I,1+IR1*ABS(IK))-2.0*G4/(TZ**2)
C
C AA(I,1+(1+IR1)*ABS(IK))=AA(I,1+(1+IR1)*ABS(IK))+G4/(TZ**2)
C
C AA(I,11)=AA(I,11)+G22/(TX*TV)
C
C AA(I,11-1)=AA(I,11-1)-G22/(TX*TV)
C
C AA(I,11+1J)=AA(I,11+1J)-G22/(TX*TV)
C
C AA(I,11+1J-1)=AA(I,11+1J-1)+G22/(TX*TV)
C
C AA(I,1,1)=AA(I,1,1)+G22/(TX*TV)
C
C AA(I,1,1-1)=AA(I,1,1-1)-G22/(TX*TV)
C
C AA(I,1,1+1J)=AA(I,1,1+1J)-G22/(TX*TV)
C
C AA(I,1,1+1J-1)=AA(I,1,1+1J-1)+G22/(TX*TV)

```

DIF24210

DIF24220

DIF24230

DIF24240

DIF24250

DIF24260

DIF24270

DIF24280

DIF24290

DIF24300

DIF24310

DIF24320

DIF24330

DIF24340

DIF24350

DIF24360

DIF24370

DIF24380

DIF24390

DIF24400

DIF24410

DIF24420

DIF24430

DIF24440

DIF24450

DIF24460

DIF24470

DIF24480

DIF24490

DIF24500

DIF24510

DIF24520

DIF24530

DIF24540

DIF24550

DIF24560

DIF24570

DIF24580

DIF24590

DIF24600

DIF24610

DIF24620

DIF24630

DIF24640

DIF24650

DIF24660

DIF24670

DIF24680

DIF24690

DIF24700

DIF24710

DIF24720

DIF24730

DIF24740

DIF24750

```

C XD
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ-1)=AA(I,11-IJ-1)-G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)-G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)+G2/(4.0*TX*TV)

```

```

C YD
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11-IJ-1)=AA(I,11-IJ-1)+G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)-G2/(4.0*TX*TV)
AA(I,11+IJ-1)=AA(I,11+IJ-1)-G2/(4.0*TX*TV)

```

```

C
AA(I,11-1)=AA(I,11-1)+2.0*G/TV**2
AA(I,11)=AA(I,11)-2.0*G/TV**2
AA(I,11-IJ)=AA(I,11-IJ)+G11/TV**2
AA(I,11)=AA(I,11)-2.0*G11/TV**2
AA(I,11+IJ)=AA(I,11+IJ)+G11/TV**2
AA(I,11+IJ)=AA(I,11+IJ)+G11/TV**2
IR2=ISIGN(1,JK)
AA(I,11-1-IR2)*ABS(JK)=AA(I,11-1-IR2)*ABS(JK)+G4/(TZ**2)
AA(I,11+1+IR2)*ABS(JK)=AA(I,11+1+IR2)*ABS(JK)-2.0*G4/(TZ**2)
AA(I,11+(1+IR2)*ABS(JK))=AA(I,11+(1+IR2)*ABS(JK))+G4/(TZ**2)

```

```

C CALL SCHE(NA,IM,JM,NOD,ITEST,NP)
DO 63 I1=1,13
NPA(I1)=ABS(NP(I1))
IF(NP(I1).EQ.0) GO TO 63
IF(NP(I1).NE.0) GO TO 64
64 AA(I2,NPA(I1)+2*NOD)=AA(I2,NP(I1)+2*NOD)+(NP(I1)/NPA(I1))*R(I1)
63 CONTINUE
GO TO 2

```

```

C
CCC POINTS ON THE X AXIS (B.L.)
C
107 AA(I,1-1)=AA(I,1-1)+G11/TV**2
AA(I,1)=AA(I,1)-2.0*G11/TV**2
AA(I,1+1)=AA(I,1+1)+G11/TV**2
AA(I,1-IJ)=AA(I,1-IJ)+2.0*G/TV**2
AA(I,1)=AA(I,1)-2.0*G/TV**2

```

```

C
IR1=ISIGN(1,IK)
AA(I,1-1-IR1)*ABS(IK)=AA(I,1-1-IR1)*ABS(IK)+G4/(TZ**2)
AA(I,1+1+IR1)*ABS(IK)=AA(I,1+1+IR1)*ABS(IK)-2.0*G4/(TZ**2)
AA(I,1+(1+IR1)*ABS(IK))=AA(I,1+(1+IR1)*ABS(IK))+G4/(TZ**2)

```

```

C
AA(I,11)=AA(I,11)+G22/(TX*TV)
AA(I,11+1)=AA(I,11+1)-G22/(TX*TV)
AA(I,11-IJ)=AA(I,11-IJ)-G22/(TX*TV)
AA(I,11+IJ+1)=AA(I,11+IJ+1)+G22/(1*TV)
C
AA(I,11)=AA(I,11)+G22/(TX*TV)
AA(I,11+1)=AA(I,11+1)-G22/(TX*TV)
AA(I,11-IJ)=AA(I,11-IJ)-G22/(1*TV)
AA(I,11-IJ+1)=AA(I,11-IJ+1)+G22/(1*TV)
C XD
DIF24760
DIF24770
DIF24780
DIF24790
DIF24800
DIF24810
DIF24820
DIF24830
DIF24840
DIF24850
DIF24860
DIF24870
DIF24880
DIF24890
DIF24900
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DIF24980
DIF24990
DIF25000
DIF25010
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DIF25200
DIF25210
DIF25220
DIF25230
DIF25240
DIF25250
DIF25260
DIF25270
DIF25280
DIF25290
DIF25300

```



```

C VD
C AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)
C AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)-G2/(4.0*TX*TV)
C AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(4.0*TX*TV)
C AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)+G2/(4.0*TX*TV)
C AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
C AA(I1,I1+1)=AA(I1,I1+1)+G/TV**2
C AA(I1,I1-1)=AA(I1,I1-1)-2.0*G/TV**2
C AA(I1,I1+IJ)=AA(I1,I1+IJ)+G11/TV**2
C IR2=ISIGN(1,JK)
C AA(I1,I1-(-IR2)*ABS(JK))=AA(I1,I1-(-IR2)*ABS(JK))+G4/(TZ**2)
C AA(I1,I1+IR2*ABS(JK))=AA(I1,I1+IR2*ABS(JK))-2.0*G4/(TZ**2)
C AA(I1,I1+(1+IR2)*ABS(JK))=AA(I1,I1+(1+IR2)*ABS(JK))+G4/(TZ**2)
C CALL SCHE(NA,IM,JM,NOD,ITEST,NP)
C DO 69 I1=1,13
C NPA(I1)=ABS(NP(I1))
C IF(NP(I1).EQ.0) GO TO 69
C IF(NP(I1).NE.0) GO TO 70
C 70 AA(I2,NPA(I1)+2*NOD)=AA(I2,NP(I1)+2*NOD)+(NP(I1)/NPA(I1))*R(I1)
C 69 CONTINUE
C GO TO 2
C CC POINTS PARALLEL TO X AXIS (B.L)
C 110 AA(I1,I1-1)=AA(I1,I1-1)+G11/TV**2
C AA(I1,I1)=AA(I1,I1)-2.0*G11/TV**2
C AA(I1,I1+1)=AA(I1,I1+1)+G11/TV**2
C AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
C AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2
C IR1=ISIGN(1,IK)
C AA(I1,I1-(-IR1)*ABS(IK))=AA(I1,I1-(-IR1)*ABS(IK))+G4/(TZ**2)
C AA(I1,I1+IR1*ABS(IK))=AA(I1,I1+IR1*ABS(IK))-2.0*G4/(TZ**2)
C AA(I1,I1+(1+IR1)*ABS(IK))=AA(I1,I1+(1+IR1)*ABS(IK))+G4/(TZ**2)
C AA(I1,I1)=AA(I1,I1)+G22/(TX*TV)
C AA(I1,I1+1)=AA(I1,I1+1)-G22/(TX*TV)
C AA(I1,I1+IJ)=AA(I1,I1+IJ)-G22/(TX*TV)
C AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)+G22/(TX*TV)
C AA(I1,I1)=AA(I1,I1)+G22/(TX*TV)
C AA(I1,I1+1)=AA(I1,I1+1)-G22/(TX*TV)
C AA(I1,I1+IJ)=AA(I1,I1+IJ)-G22/(TX*TV)
C AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)+G22/(TX*TV)
C XD
C AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)
C AA(I1,I1-IJ+1)=AA(I1,I1-IJ+1)-G2/(4.0*TX*TV)
C AA(I1,I1+IJ-1)=AA(I1,I1+IJ-1)-G2/(4.0*TX*TV)
C AA(I1,I1+IJ+1)=AA(I1,I1+IJ+1)+G2/(4.0*TX*TV)
C VD
C AA(I1,I1-IJ-1)=AA(I1,I1-IJ-1)+G2/(4.0*TX*TV)

```

```

C AA(I1,I-IJ+1)=AA(I1,I-IJ+1)-G2/(4.0*TX*TV)
AA(I1,I-IJ-1)=AA(I1,I-IJ-1)-G2/(4.0*TX*TV)
AA(I1,I+IJ+1)=AA(I1,I+IJ+1)+G2/(4.0*TX*TV)

```

```

C AA(I1,I1-1)=AA(I1,I1-1)+G/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1+1)=AA(I1,I1+1)+G/TV**2
AA(I1,I1+1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2
AA(I1,I1+IJ)=AA(I1,I1+IJ)+G/TV**2
AA(I1,I1-IR2)*ABS(JK)=AA(I1,I1-IR2)*ABS(JK)+G4/(TZ**2)
AA(I1,I1+IR2)*ABS(JK)=AA(I1,I1+IR2)*ABS(JK)-2.0*G4/(TZ**2)
AA(I1,I1+(1+IR2)*ABS(JK))=AA(I1,I1+(1+IR2)*ABS(JK))+G4/(TZ**2)

```

```

C CALL SCHE(NM,IM,JM,NOD,I,TEST,NP)
DO 71 I1=1,13
NPA(I1)=ABS(NP(I1))
IF(NP(I1).EQ.0) GO TO 71
IF(NP(I1).NE.0) GO TO 72
72 AA(I2,NPA(I1)+2*NOD)=AA(I2,NP(I1)+2*NOD)+(NP(I1)/NPA(I1))*R(I1)
71 CONTINUE
GO TO 2

```

```

C CCC POINTS PARALLEL TO Y AXIS
111 AA(I,I)=AA(I,I)-2.0*G11/TV**2
AA(I,I+1)=AA(I,I+1)+G11/TV**2
AA(I,I-IJ)=AA(I,I-IJ)+G/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
AA(I,I+IJ)=AA(I,I+IJ)+G/TV**2

```

```

C IRI=ISIGN(1,IK)
AA(I,I-IR1)*ABS(IK)=AA(I,I-IR1)*ABS(IK)+G4/(TZ**2)
AA(I,I+IR1)*ABS(IK)=AA(I,I+IR1)*ABS(IK)-2.0*G4/(TZ**2)
AA(I,I+(1+IR1)*ABS(IK))=AA(I,I+(1+IR1)*ABS(IK))+G4/(TZ**2)
AA(I,I)=AA(I,I)+G22/(TX*TV)
AA(I,I+1)=AA(I,I+1)-G22/(TX*TV)
AA(I,I-IJ)=AA(I,I-IJ)-G22/(TX*TV)
AA(I,I+IJ)=AA(I,I+IJ)+G22/(TX*TV)

```

```

C AA(I1,I)=AA(I1,I)+G22/(TX*TV)
AA(I1,I+1)=AA(I1,I+1)-G22/(TX*TV)
AA(I1,I-IJ)=AA(I1,I-IJ)-G22/(TX*TV)
AA(I1,I+IJ)=AA(I1,I+IJ)+G22/(TX*TV)
C XD AA(I,I-IJ-1)=AA(I,I-IJ-1)+G2/(4.0*TX*TV)
AA(I,I-IJ+1)=AA(I,I-IJ+1)-G2/(4.0*TX*TV)
AA(I,I+IJ-1)=AA(I,I+IJ-1)-G2/(4.0*TX*TV)
AA(I,I+IJ+1)=AA(I,I+IJ+1)+G2/(4.0*TX*TV)

```

```

C YD AA(I1,I-IJ-1)=AA(I1,I-IJ-1)+G2/(4.0*TX*TV)
AA(I1,I-IJ+1)=AA(I1,I-IJ+1)-G2/(4.0*TX*TV)

```



```

AA(I1,I1-1)=AA(I1,I1-1)+2.0*G/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G/TV**2
AA(I1,I1)=AA(I1,I1)-2.0*G11/TV**2
AA(I1,I1+1J)=AA(I1,I1+1J)+G11/TV**2
IR2=ISIGN(1,JK)
C
AA(I1,I1-1-IR2)*ABS(JK)=AA(I1,I1-1-IR2)*ABS(JK))+G4/(TZ**2)
AA(I1,I1+1R2)*ABS(JK)=AA(I1,I1+1R2)*ABS(JK))-2.0*G4/(TZ**2)
AA(I1,I1+(1+IR2)*ABS(JK)=AA(I1,I1+(1+IR2)*ABS(JK))+G4/(TZ**2)
C
CALL SCHE(NM, JM, NOD, ITEST, NPA)
DO 75 I1=1,15
NPA(I1)=ABS(NP(I1))
IF(NP(I1).EQ.0) GO TO 75
IF(NP(I1).NE.0) GO TO 76
76 AA(I2,NPA(I1)+2*NOD)=AA(I2,NP(I1)+2*NOD)+(NP(I1)/NPA(I1))*R(I1)
75 CONTINUE
GO TO 2
C
CCC POINTS PARALLEL TO Y AXIS ON THE X AXIS
C
113 AA(I,I)=AA(I,I)-2.0*G11/TV**2
AA(I,I+1)=AA(I,I+1)+G11/TV**2
AA(I,I-1J)=AA(I,I-1J)+2.0*G/TV**2
AA(I,I)=AA(I,I)-2.0*G/TV**2
IR1=ISIGN(1,IK)
C
AA(I,I-1-IR1)*ABS(IK)=AA(I,I-1-IR1)*ABS(IK))+G4/(TZ**2)
AA(I,I+1R1)*ABS(IK)=AA(I,I+1R1)*ABS(IK))-2.0*G4/(TZ**2)
AA(I,I+(1+IR1)*ABS(IK)=AA(I,I+(1+IR1)*ABS(IK))+G4/(TZ**2)
C
AA(I,I1)=AA(I,I1)+G22/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G22/(TX*TV)
AA(I,I1-1J)=AA(I,I1-1J)-G22/(TX*TV)
AA(I,I1-1J+1)=AA(I,I1-1J+1)+G22/(TX*TV)
C
AA(I,I1)=AA(I,I1)+G22/(TX*TV)
AA(I,I1+1)=AA(I,I1+1)-G22/(TX*TV)
AA(I,I1-1J)=AA(I,I1-1J)-G22/(TX*TV)
AA(I,I1-1J+1)=AA(I,I1-1J+1)+G22/(TX*TV)
C
AA(I,I1-1J-1)=AA(I,I1-1J-1)+G2/4.0*TX*TV)
AA(I,I1-1J+1)=AA(I,I1-1J+1)-G2/4.0*TX*TV)
AA(I,I1+1J-1)=AA(I,I1+1J-1)-G2/4.0*TX*TV)
AA(I,I1+1J+1)=AA(I,I1+1J+1)+G2/4.0*TX*TV)
C
AA(I,I1)=AA(I,I1)-2.0*G/TV**2
AA(I,I1+1)=AA(I,I1+1)+G/TV**2

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- D1F28060
- D1F28070
- D1F28080
- D1F28090
- D1F28100
- D1F28110
- D1F28120
- D1F28130
- D1F28140
- D1F28150
- D1F28160
- D1F28170
- D1F28180
- D1F28190
- D1F28200
- D1F28210
- D1F28220
- D1F28230
- D1F28240
- D1F28250
- D1F28260
- D1F28270
- D1F28280
- D1F28290
- D1F28300
- D1F28310
- D1F28320
- D1F28330
- D1F28340
- D1F28350
- D1F28360
- D1F28370
- D1F28380
- D1F28390
- D1F28400
- D1F28410
- D1F28420
- D1F28430
- D1F28440
- D1F28450
- D1F28460
- D1F28470
- D1F28480
- D1F28490
- D1F28500
- D1F28510
- D1F28520
- D1F28530
- D1F28540
- D1F28550
- D1F28560
- D1F28570
- D1F28580
- D1F28590
- D1F28600

AA(I,1,I-IJ)=AA(I,1-IJ)-G3/(TX*TV)
 AA(I,1,I-IJ+1)=AA(I,1-IJ+1)+G3/(TX*TV)
 AA(I,1,I-IJ)=AA(I,1,I-IJ)+2.*G1/TV**2
 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2

GO TO 2

121 AA(I,1,I-1)=AA(I,1-I)+G1/TV**2
 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2
 AA(I,1,I+1)=AA(I,1,I+1)+G1/TV**2

AA(I,1,I)=AA(I,1,I)+G3/(TX*TV)
 AA(I,1,I+1)=AA(I,1,I+1)-G3/(TX*TV)
 AA(I,1,I+I+I)=AA(I,1,I+I+I)-G3/(TX*TV)
 AA(I,1,I+I+I+1)=AA(I,1,I+I+I+1)+G3/(TX*TV)

GO TO 2

122 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2
 AA(I,1,I-1)=AA(I,1,I-1)+2.*G1/TV**2

AA(I,1,I)=AA(I,1,I)+G3/(TX*TV)
 AA(I,1,I-1)=AA(I,1,I-1)-G3/(TX*TV)
 AA(I,1,I+I)=AA(I,1,I+I)-G3/(TX*TV)
 AA(I,1,I+I+1)=AA(I,1,I+I+1)+G3/(TX*TV)

GO TO 2

123 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2
 AA(I,1,I+1)=AA(I,1,I+1)+G1/TV**2

AA(I,1,I)=AA(I,1,I)+3/(TX*TV)
 AA(I,1,I+1)=AA(I,1,I+1)-G3/(TX*TV)
 AA(I,1,I+I)=AA(I,1,I+I)-G3/(TX*TV)
 AA(I,1,I+I+1)=AA(I,1,I+I+1)+G3/(TX*TV)

GO TO 2

124 AA(I,1,I)=AA(I,1,I)+G3/(TX*TV)
 AA(I,1,I+1)=AA(I,1,I+1)-G3/(TX*TV)
 AA(I,1,I+I)=AA(I,1,I+I)-G3/(TX*TV)
 AA(I,1,I+I+1)=AA(I,1,I+I+1)+G3/(TX*TV)
 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2
 AA(I,1,I+I)=AA(I,1,I+I)+G1/TV**2

GO TO 2

125 AA(I,1,I)=AA(I,1,I)-2.*G1/TV**2
 AA(I,1,I+1)=AA(I,1,I+1)+G1/TV**2
 AA(I,1,I-IK)=AA(I,1-IK)+G/TZ**2
 AA(I,1,I)=AA(I,1,I)-2.*G/TZ**2
 AA(I,1,I+IK)=AA(I,1+IK)+G/TZ**2

AA(I,1,I)=AA(I,1,I)+G3/(TX*TV)
 AA(I,1,I+1)=AA(I,1,I+1)-G3/(TX*TV)
 AA(I,1,I+I)=AA(I,1,I+I)-G3/(TX*TV)
 AA(I,1,I+I+1)=AA(I,1,I+I+1)+G3/(TX*TV)

D1F29710

D1F29720

D1F29730

D1F29740

D1F29750

D1F29760

D1F29770

D1F29780

D1F29790

D1F29800

D1F29810

D1F29820

D1F29830

D1F29840

D1F29850

D1F29860

D1F29870

D1F29880

D1F29890

D1F29900

D1F29910

D1F29920

D1F29930

D1F29940

D1F29950

D1F29960

D1F29970

D1F29980

D1F29990

D1F30000

D1F30010

D1F30020

D1F30030

D1F30040

D1F30050

D1F30060

D1F30070

D1F30080

D1F30090

D1F30100

D1F30110

D1F30120

D1F30130

D1F30140

D1F30150

D1F30160

D1F30170

D1F30180

D1F30190

D1F30200

D1F30210

D1F30220

D1F30230

D1F30240

D1F30250

```
AA(I1,I1-JK)=AA(I1,I1-JK)+G/TZ**2  
AA(I1,I1)=AA(I1,I1)-2.*G/TZ**2  
AA(I1,I1+JK)=AA(I1,I1+JK)+G/TZ**2
```

```
GO TO 2  
2 CONTINUE  
RETURN  
END
```

```
DIF30260  
DIF30270  
DIF30280  
DIF30290  
DIF30300  
DIF30310  
DIF30320  
DIF30330
```