Asset Markets and Index Investment:

On the Impact of a CPP Investment Fund

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Introduction

Index investment is a technical trading rule which invests a pool of capital by buying a portfolio of stocks in proportion to their weight in total market capitalization. The stock index is the usual representation of this capitalization.

The use of this trading rule has its roots in the traditional *Capital Asset Pricing Model* (CAPM) in which market capitalization represents the 'market portfolio' - a composite asset which underlies the pricing and return structure of all other assets in the market. Investing in this composite asset is believed to mimic the trading of all other investors. As a result, index investment is not thought to alter the structure of asset prices or returns.

This paper challenges this conclusion on the grounds that an automatic trading rule does not properly mimic the aggregated trading patterns of individual investors. A modified consumption based CAPM model is used to demonstrate the effect of index investment on asset returns. Given two distinct groups of investors - individual investors who invest according to marginal utility considerations and institutional investors who invest using a stock index - larger price variations are required to equilibrate asset returns to the preferences of marginal utility investors.

This finding is framed in the context of a recent decision to invest the assets of the Canada Pension Plan (CPP) using an equity market investment fund. Given the operational constraints facing institutional investors in Canada, and following the experience of a Canadian fund of comparable size and purpose – the Ontario Teacher's Pension Plan (OTPP) – it is likely that the new CPP fund will use index investment as a key trading strategy.

The last section of the paper examines empirical evidence drawn from Toronto Stock Exchange (TSE) data from the 1980's and the 1990's. To a certain extent, the latter period represents a 'natural experiment' in which a large institutional investor, the Ontario Teacher's Pension Plan – an institution noted for its extensive use of index investments – is introduced into the Canadian equity market. The Canadian market, as represented by the TSE, is small relative to US markets such as the New York Stock Exchange, which suggests that the impact on Canadian stock returns of this increase in index investment may be more dramatic and observable than would be the case elsewhere.

In the 1990's an increase in the return on the stock market index is observed. This is consistent with the conclusions reached using the asset pricing models, which predict that the introduction of a large investment fund, such as the OTTP or the CPP, using an index investment strategy, would result in rising returns for assets that co-vary with the market and falling returns for those assets which are a hedge against the market. In particular the modeling predicts that the return on the stock market index will rise with the proportion of indexed investment, an observation which corresponds to the TSE data. The empirical analysis nevertheless suggests that only a modest impact would result from the introduction of an indexed CPP fund.

Background: Canadian markets and the CPP Investment Fund

In 1997, a dramatic change of course occurred in Canada's social security network. The federal government introduced a number of measures to address a chronically underfunded national pension plan which many Canadians had come to believe would fail to support them in their retirement. While the issue of additional Canada Pension Plan (CPP) payroll deductions continues to be hotly debated[1], many of the changes were technical in nature and have attracted little attention in the media.

In particular, the introduction of an equity market investment plan for the CPP received limited consideration. The traditional structure of the CPP as a current account plan – a plan that pays current claims out of current premiums without drawing from an investment pool – is set to change substantially. The CPP has access to a pool of capital of $35.6 billion dollars, originally set up to hedge against inflation, and currently invested in government bonds. This capital forms the nucleus of the new investment fund, which the government plans to grow with contributions by an estimated $10 billion per year. The goal is to reach a capital base of approximately $80 billion by 2006, which will act as a savings fund, paying pension claims from investment returns rather than from current pension premiums[2].

To add perspective to this number, consider that the country's second largest pension fund, the Ontario Teacher's Pension Plan Board, has assets of $54.5 billion, and the largest Canadian financial institution, the Royal Bank, has assets of $245 billion[3]. The new fund will thus be a substantial player in the Canadian equity market, larger than most existing pension funds and comparable to many banking institutions. Even with investments of $10 billion per year, a small fraction of the available capital, this fund would represent 5% of daily trading volume on the TSE[4]. Further, it is estimated that:

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[1] See for example Dungan (1998) which critically assesses the impact of these payroll deductions.

2 The financial details of the plan given here are drawn from McCarthy (1999)

[3]The figures cited is total assets as of October 31, 1997.

[4] Bertram (1997) p 34.

> *"If all the potential CPP funds were to be invested in the TSE 300 index, the fund would become the largest single Canadian equity investor within three to five years. If the CPP invested its full potential of $100 billion in Canadian equity it would reach an ownership of 12% to 16% of the outstanding float for every company in the index within five years"* [5]

Could this scale of ownership by a public institution influence Canadian asset prices and returns? Since the economy's overall performance is tied closely to investment decisions, it is critical to assess how the CPP, as an institutional investor in the Canadian equity market, will affect the efficiency of this market.

Institutional Investment in Canada

Many of the concerns over institutional investment arise from the question of corporate governance. Pension and mutual funds now own nearly one third of all shares in Canada's publicly traded corporations, compared with only 1% two decades ago[6]. Investors who buy shares in a company are not simply storing their wealth, rather they are influential in determining the allocation of economic resources. With share ownership, investment funds must take on the critical role of overseeing firm management. In 1996, for example, the Ontario Teacher's Pension Plan exercised proxies (i.e. voted on management proposals) in 325 Canadian companies[7]. As a result of being large shareholders, funds carry considerable influence over the governance of firms, which in a large part determines the long run performance of these firms.

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[5] Bertram (1997) p 34.

[6] 1996 Report to Members, Ontario Teachers' Pension Plan Board, p 4.

[7] Ontario Teachers' Pension Plan Board Annual Report 1996, p 3.

Thus, increasing institutional holdings of stocks directly translates into increased control of the economy, a responsibility that cannot be treated lightly.

In the case of the Canadian market, the relatively small amount of issued shares inflates the impact of share ownership on institutional control. In order to reduce concerns over corporate governance an ownership limit of 30% is currently placed on pension funds. This constraint is meant to reduce their market power in the allocation of investment capital. It restricts their ability to purchase large stakes in firms, positions which could potentially allow them to manipulate share prices and dividend payments. Canadian pensions also face an upper limit on their foreign equity investments equal to 20% of total assets[8]. This is meant to ensure that Canadian savings remain in the domestic investment market[9]. Clearly these requirements are conflicting; pensions are required to keep a low profile in Canadian markets, but at the same time they are prevented from investing a significant share of their assets elsewhere. Notwithstanding this contradiction, pension funds continue to be big players in a small market. The introduction of an additional public fund is expected to aggravate already scarce investment opportunities and drive up prices.

Often pensions represent a diverse membership, which may tie the hands of fund managers faced with defending controversial investment decisions. This concern would be particularly acute for a fund such as the CPP, which is owned entirely by the Canadian public. Investments would come under close scrutiny for political interference or favoritism. An

8 It is the combination of these two constraints which sets the stage for investment scarcity in the Canadian market. As profitable investments become more difficult to obtain in the domestic market, a move towards foreign markets would normally occur. Holt (1996) suggests that relaxing the foreign content limit on Canadian institutional investors could entirely offset the introduction of the CPP investment fund.

9 To a certain extent, funds have been able to avoid the restriction on foreign content by investing in derivative products which are not covered under the rule, but are considerably more risky.

appointed board at arm's length from government is essential to prevent such interference, but the potential for public criticism remains.

In order to minimize the impact on corporate governance and on market prices, larger funds are currently required to invest some portion of their assets on the index. Index funds select a leading group of stocks by their capitalization in the market. For example, the TSE operates with TSE 300, 200, 100 and 35 index funds[10]. Each stock in the group is purchased relative to its capital weight in the total capital represented by the group of stocks. Thus the amount of stock purchased is independent of the stock's identity. As Bertram (1997) notes:

> *"An index process independently determines the investability and the amounts invested in each company in the index. The selection of stocks to be included in the index is largely a rules driven transparent process that operates independently from investment management. In terms of performance the index will equal the performance of the average return in the stock market over any reasonable time period."* [11]

Most large institutional investors find this practice to be desirable, as it reduces some of the transaction costs associated with trading. The OTPP, for instance, invested $13.8 billion of its $50 billion in assets in 1996 using such funds[12]. As one might expect, the benefits of index investment increase with the size of the plan. The larger the fund, the more difficult it is for the fund to achieve greater than market returns, since their asset holdings move closer and closer to

[10] The TSE has recently combined the TSE 200, 100 and 35 indexes into a TSE 60, which will be managed by Standard and Poor.

[11] Bertram (1997) p 34.

[12] Ontario Teachers' Pension Plan Board Annual Report 1996, p 22.

the market holdings. Rather than trying to outperform the market and incurring the costs of actively picking stocks, index investment is often a better alternative. Index investment can also help to spread investment capital thin enough to prevent pension funds from reaching the 30% ownership limit while still maintaining healthy returns.

Further, index investment is also a clear trend among individual investors who have seen most mutual funds routinely underperform the market. At the end of April 1998 there were 27 index funds available to Canadians with assets of $3.5 Billion, up 142% from the previous year[13]. Since 1991, when index funds accounted for only $157 million, they have grown at an average annual rate of 74%[14]. In the US, $140 of every $1000 invested in 1998 were invested using index funds, up from $50 in 1995 [15]. Index funds in the U.S. represent about 10% of total mutual fund assets. While the current proportion for Canada is only about 1%-2% of total mutual fund assets (not including pension fund investments), it is expected that index funds will grow to occupy 10% of mutual fund assets over the next 10 years[16]. Over the course of the next decade the growth of index investment will be a key factor in equity markets.

It seems likely then, that in order to mitigate the impact of the CPP fund on corporate control and on price movements, while at the same time achieving a reasonable rate of return, some amount of index investment for the CPP is inevitable. Given this background on the CPP investment fund and on the importance of index investment in asset markets, an exploration of asset pricing theories can now be attempted, with the assurance that situating index investment among these theories will provide insight into current concerns over the impact of CPP investments.

13 Greenwood (1998) pp. 1-2.
14 Ibid
15 ibid
16 ibid

The Capital Asset Pricing Model (CAPM)

As most discussions of asset pricing do, it is a good idea to begin with the dominant model in pricing theory, the capital asset pricing model or CAPM. The behavioral assumption of the CAPM is that investors treat expected (or mean) consumption as a good and variance of consumption as a bad. Their investment choices minimize variance given some expected return.

Is it possible to apply the CAPM to a model of institutional investment? Institutions generally target a benchmark return against which their investment managers are judged. For this reason, variance minimization for given expected return is also a plausible behavioral assumption for institutional investors. The following is a simple exposition of the traditional CAPM model drawn from Varian (1992):

Let x_a be the proportion of a portfolio invested in asset 'a'. Of course in the total portfolio, the sum of all these proportions must be equal to one:

$$\sum_{a=0}^{A} x_a = 1$$

Among these assets *0* is a riskless asset with a guaranteed return. The investor seeks to minimize the variance of the portfolio return subject to this constraint, and the constraint that a specified expected return *E* [R*] is achieved. The equivalent problem can be written as:

$$\min_{x_0,\ldots,x_A} \quad \sum_{a=0}^{A} \sum_{b=0}^{A} x_a x_b \sigma_{ab}$$

$$\text{such that} \quad \sum_{a=0}^{A} x_a E\,[R_a] = E\,[R^*]$$

$$\text{and} \quad \sum_{a=0}^{A} x_a = 1$$

Where σ_{ab} is the covariance between asset a and b. Using λ as the Lagrange multiplier for the first constraint and μ for the second constraint, the first order conditions take the form:

$$2 \sum_{b=0}^{A} x_b \sigma_{ab} - \lambda E R_a - \mu = 0 \qquad \text{for} \quad a = 0, \ldots, A \qquad \text{i)}$$

Given a convex objective function and linear constraints, the second order conditions are satisfied.

These first order conditions can be simplified in a clever way. Let there be a portfolio $(x_1^e, \ldots, x_A^e)$ consisting only of risky assets that satisfies the mean variance efficient criteria given above. Suppose that one of the available assets is a 'mutual fund' that holds this efficient portfolio (x_a^e). A portfolio that invests 0 in every asset except for asset e and 1 in asset e would then also be mean variance efficient. This means that such a portfolio satisfies the first order condition (i) for each asset a = 0 , ... , A. Noting that for this portfolio $x_b = 0$ for $b \neq e$, then the a^{th} first order condition becomes:

$$2\sigma_{ae} - \lambda E R_a - \mu = 0 \qquad \text{ii)}$$

Two special cases occur when $a = 0$ and when $a = e$:

$$- \lambda R_0 - \mu \quad = 0$$

$$2\,\sigma_{ae} - \lambda\, E\, R_e - \mu = 0$$

When $a = 0$, σ_{ae} is zero since asset 0 is not risky. When $a = e$, $\sigma_{ae} = \sigma_{ee}$ since the covariance of a variable with itself is simply the variance of the random variable. Solving these two equations for λ and μ yields:

$$\lambda = \frac{2\,\sigma_{ae}}{E\, R_e - R_0} \qquad \text{and} \qquad \mu = -\lambda R_0 = \frac{-2\,\sigma_{ae}\, R_0}{E\, R_e - R_0}$$

Substituting this into (ii) and rearranging yields the standard CAPM result:

$$E\, R_a = R_o + (\sigma_{ae} / \sigma_{ee})(E\, R_e - R_o) \qquad \text{iii)}$$

$$\text{Or:} \quad E\, R_a - R_o = \beta\,(E\, R_e - R_o) \qquad \text{iv)}$$

How are an asset's returns generated? Equation (iii) states that the expected return on any asset is equal to the risk free rate of return plus a 'risk premium' that depends on its covariance with some efficient portfolio of risky assets. It is important to note that it is not solely the asset's variance itself that determines its risk premium, but its variance in relation to the variance on a variance efficient portfolio, given as the ratio (σ_{ae}/σ_{ee}). By holding one asset over another a specific risk is incurred. This risk is the opportunity cost of not holding a variance minimizing portfolio asset (the asset $E\,R_e$), expressed in terms of variance on the excess return ($E\,R_e - R_o$). An adjustment to the asset's return must be made to reward the investor for incurring this specific risk – this reward is the 'risk premium'.

The equation may be rewritten, as in equation (iv), in terms of the spread of returns on a given asset over the return on the risk-less asset. Here a 'Beta' is substituted for the fraction (σ_{ae}/σ_{ee}). This beta is commonly understood to be the regression coefficient obtained by regressing the excess return on a specific asset ($E\,R_a - R_o$) against the excess return on a variance efficient portfolio ($E\,R_e - R_o$).

The empirical challenge facing the CAPM is to concretely establish what this variance efficient portfolio might actually be. The logic commonly used to connect relative market capitalization to the weights used in the variance efficient portfolio is somewhat circular. The argument runs as follows: since each investor chooses a variance minimizing portfolio, with individual asset weights that minimize variance, the aggregation of these weighted portfolios, represented by the relative market capitalization of stocks, must also represent variance minimization[17]. It is argued that any broad based portfolio, such as an indexed portfolio, approaches variance minimization since it diverges less and less from the average portfolio of investors, in itself is based on variance minimization.

Under index investment, however, not every investor adopts variance minimization. One group of investors chooses simply to mimic the investment choices of the market using an automatic trading rule. Suppose that trading were to take place in two discrete stages. First, non-index investors buy their portfolios according to variance minimization, with their purchases setting the structure of prices and by extension asset returns. In response to this trading, index investors then move to purchase their portfolios in proportion to these aggregate investments. This causes prices to vary and with them asset returns. Suddenly, the portfolios held by non-index investors are no longer variance efficient since asset returns have been altered. (Recall that variance efficiency is expressed in terms of the total variance of excess returns, which depends on the magnitude of these returns). Non-index investors can of course redistribute their portfolios, eventually arriving at a variance efficient equilibrium given synchronous trading, but index investment has clearly altered the process of generating returns.

Prices, (and by extension asset returns), depend on aggregate asset investments. For convenience we will label these investment decisions 'consumption' in the aggregate. This behaviour, however, is no longer entirely the product of simple and identical variance minimization, rather it includes the price feedback effects produced by automatic trading. Clearly, the assumption of identical investors fails to hold. It is then necessary to go beyond the traditional CAPM, whose derivation relies heavily on this premise, to adopt a more robust model that explicitly describes individual investment decisions in relation to overall asset consumption. The next section presents a more elaborate consumption based asset pricing model.

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17 While this logical sequence can be demonstrated mathematically, the exposition is not included here as it is argued that the underlying logic is flawed. These flaws, however, are discussed.

Arrow Debreu Securities - The Aggregate Consumption Beta

From a theoretical standpoint, consumption based CAPM models are much more flexible to work with. Behavioral assumptions can be added or modified to better reflect the economic question at hand. In this section a consumption based CAPM model is presented which better represents the decisions of the individual investor and explicitly includes prices and investment scarcity in the determination of asset returns. Since the model satisfies these criteria, it will be possible in the following section to modify it to include index investment.

The model presented in this section can be found in Varian (1992). It presents a simplified version of Rubinstein (1976) in a way that is both intuitive and rigourous. To begin the argument, suppose that there are s different states of nature and for each state s there is an asset that pays \$1 if state s occurs and \$0 otherwise. An asset of this form is known as an *Arrow Debreu security*. Let p_s be the equilibrium price of Arrow Debreu security s.

Consider an arbitrary asset a with value V_{as} in state s. How much is this asset worth in period 0? The following argument will be used: construct a portfolio holding V_{as} units of Arrow Debreu security s. Since Arrow Debreu security s is worth \$1 in state s, this portfolio will be worth V_{as} in state s. This portfolio will then have exactly the same distribution of payoffs as asset a. It follows from arbitrage considerations that the value of asset a must be the same as the value of this portfolio. Hence:

$$p_a = \sum_{s=1}^{S} p_s V_{as}$$

In this way, the value of any asset can be determined from the values of the Arrow Debreu assets. Letting π_s be the probability of state s, we can write:

$$p_a = \sum_{s=1}^{S} (p_s / \pi_s)\, V_{as}\, \pi_s = E[\,(p/\pi)\, V_a\,]$$

where E is the expectation operator. This expression states that the value of asset a is the expectation of the product of the value of asset a and the random variable (p/π). Using the covariance identity:

$$cov(A, B) = E[AB] - E[A]\,E[B]$$

The expression: $p_a = E[\,(p/\pi)\, V_a\,]$

can be rewritten as: $p_a = \text{cov}[\,(p/\pi), V_a\,] + E[(p/\pi)]E[V_a]$ 1)

By definition: $E[(p/\pi)] = \sum_{s=1}^{S} (p_s/\pi_s)\,\pi_s = \sum_{s=1}^{S} (p_s)$

Hence $E[(p/\pi)]$ is the value of a portfolio that pays $1 for certain next period. Letting R_o be the risk free return on this portfolio, we have:

$$E[(p/\pi)] = 1/R_o$$

Substituting this into equation 1) above and rearranging:

$$p_a = E\,V_a / R_o + \text{cov}\,[\,(p/\pi), V_a\,] \qquad 2)$$

Hence the price of asset *a* must be its discounted expected value plus a risk premium.

These operations have simply manipulated definitions, now a behavioral assumption is added. If agent *i* purchases $c_{i\,s}$ units of Arrow Debreu security (*s*), he must satisfy the first order condition:

$$\pi_s\, u_i{}'(c_{i\,s}) = \lambda\, p_s \qquad \text{or} \qquad u_i{}'(c_{i\,s}) / \lambda = p_s / \pi_s$$

It is apparent then that p_s / π_s must be proportional to the marginal utility of consumption of investor *i*. Under risk aversion, the left side of this expression is a strictly decreasing function of consumption. Let f_i be the inverse of $u_i{}'(c_{i\,s})/\lambda$; this is also a decreasing function. We can then write:

$$c_{i\,s} = f_i\,(p_s / \pi_s)$$

Summing over *i* and using C_s to denote aggregate consumption in state *s* we have:

$$C_s = \sum_{i=1}^{I} f_i\,(p_s / \pi_s)$$

Since each f_i is a decreasing function, the right hand side of this expression is also decreasing function. As a result, there exists an inverse *F* , and this expression can be written as:

$$(p_s / \pi_s) = F(C_s)$$

Where $F(C_s)$ is a decreasing function of aggregate consumption. Substituting this into 2) we have:

$$p_a = E V_a / R_o + \text{cov}[F(C_s), V_a] \qquad 3)$$

Interpreting this equation, the value of the asset *a* is given by its discounted expected value adjusted by a risk premium that depends on the covariance of the value of the asset with a decreasing function of aggregate consumption. Assets that are positively correlated with aggregate consumption will have a negative adjustment; assets that are negatively correlated will have a positive adjustment. The covariance term is a representation of a 'consumption beta'.

It is relatively straightforward to translate this result into an asset return formulation. The value returned on investment can be divided by purchase price to give a rate of return for that asset. Rearranging the above equation yields:

$$E V_a / R_o = p_a - \text{cov}[F(C_s), V_a]$$

$$E V_a = R_o p_a - R_o \text{cov}[F(C_s), V_a]$$

Dividing through by p_a:

$$E R_a = R_o - R_o \text{cov}[F(C_s), R_a] \qquad 4)$$

If an asset's value is negatively correlated with consumption in that asset, the return on that asset can afford to be lower since it provides a hedge against aggregate consumption, paying off more when aggregate consumption is low. The benefits of this negative correlation are reflected by an upward adjustment in price and a corresponding downward adjustment in return.

If an asset's value is positively correlated with aggregate consumption, then it represents an undiversified risk subject not only to fluctuations in its own value but also to the variance of consumption. Its return must be higher to attract investors. Thus there is a negative adjustment in price and a positive adjustment in return resulting from its positive covariance with consumption.

Extending the Model to Include Institutional Investors and Index Investment

Suppose that an institutional investor is introduced. The impact of such an investor will depend on the initial marginal utility condition given by:

$$u_i{}'(c_{is}) / \lambda = p_s / \pi_s$$

In large part arguments against institutional investors in the marketplace rely on their influence in determining aggregate consumption. Because their c_{is} is large, when all c_{is} are summed to provide the aggregate consumption term C_{is} , it will be skewed towards the preferences of larger investors. The function of aggregate consumption against which an asset's price is determined through covariance $F(C_s)$, will thus be increased for assets which pensions prefer to hold, and decreased for those they do not, which decreases/increases the covariance adjustment to price respectively.

It is interesting to consider the alternative, however. Suppose that institutional investors are restricted in their trading practices such that a certain portion of their investments must be made on the index. In this case each stock is bought according to its share in total market capitalization. For institutional investors this condition can be expressed as follows:

$$(p_s c_{is})/w_i = (p_s C_s)/W \qquad 5)$$

That is to say, the proportion of an institution's wealth that is invested in some asset $(p_s c_{is})/w_i$, where their total wealth is (w_i), exactly matches the relative market capitalization of that asset, given by the total value invested in that asset ($p_s C_s$) divided by the total level of investment in the marketplace (W). This rule is an acceptable proxy for index investment. Dividing out (p_s) this equation can be rewritten as:

$$c_{is} = C_s w_i / W$$

In this way, it is clear that index investors react only to movements in aggregate consumption, and not to movements in prices or asset returns, except via their effects on aggregate consumption. The aggregation proceeds as before, but now there are two pools of investors: one which follows the marginal condition rule (which may include institutional funds not indexed) and a second group of investors with investments tied to an index. Here (N) denotes the number of non-index investors and (I) the number of index investors:

$$C_s = \sum_{i=1}^{N} f_i (p_s/\pi_s) + \sum_{i=1}^{I} C_s w_i / W$$

The sum of (w_i)'s is just the total amount invested on the index, call this I*. (C_s) and (W) are common to each term in the summation and thus can be factored out. The summation then reduces to:

$$C_s = \sum_{i=1}^{N} f_i \, (p_s / \pi_s) + C_s \, I^*/W$$

Let ε denote (I*/W) – the proportion of funds invested on the index:

$$C_s = \sum_{i=1}^{N} f_i \, (p_s / \pi_s) + C_s \, \varepsilon$$

Which can then be simplified to:

$$(1 - \varepsilon) \, C_s = \sum_{i=1}^{N} f_i \, (p_s / \pi_s)$$

Each f_i is a decreasing function as before and the right hand side of this expression is a decreasing function. Hence, there is an inverse F, so we can write:

$$(p_s / \pi_s) = F \, [\, (1 - \varepsilon) \, C_s \,]$$

Where $F \, [\, (1 - \varepsilon) \, C_s \,]$ is a decreasing function of aggregate consumption.

Substituting this into 2) we have:

$$p_a = E\,V_a / R_o + \text{cov}\,[\,F\,[\,(1-\varepsilon)\,C_s\,]\,,V_a\,] \qquad 6)$$

The value of the asset *a* is still the discounted expected value adjusted by a risk premium that depends on the covariance of the value of the asset with a decreasing function of aggregate consumption. Assets that are positively correlated with aggregate consumption will continue to have a negative adjustment; assets that are negatively correlated will continue to have a positive adjustment. It will be convenient to denote assets in terms of their return. Rearranging the above equation yields:

$$E\,V_a / R_o = p_a - \text{cov}\,[\,F\,[\,(1-\varepsilon)\,C_s\,]\,,V_a\,]$$

$$E\,V_a = R_o\,p_a - R_o\,\text{cov}\,[\,F\,[\,(1-\varepsilon)\,C_s\,]\,,V_a\,]$$

Dividing through by p_a: $$E\,R_a = R_o - R_o\,\text{cov}\,[\,F\,[\,(1-\varepsilon)\,C_s\,]\,,R_a\,] \qquad 7)$$

Alternately this may be expressed as:

$$E\,R_a / R_o = 1 - \text{cov}\,[\,F\,[\,(1-\varepsilon)\,C_s\,]\,,R_a\,] \qquad 8)$$

Assume that the function F (*) is separable in the following way:

$$F\,[\,(1-\varepsilon)\,C_s\,] = F\,[\,(1-\varepsilon)\,]\,F\,[\,C_s\,]$$

In testing consumption CAPM's, a common practice is to assume a utility function with constant relative risk aversion (CRRA)[18]. If this assumption is used then the function $F\,(*)$ relates to the marginal utility of a CRRA function and will be separable in this way. For example:

CRRA utility

$$U = C^{(1-a)} / (1-a)$$

$$U' = C^{(-a)}$$

$$U'[\,(1-\varepsilon)\,C_s\,] = [\,(1-\varepsilon)\,C_s\,]^{(-a)}$$

$$= (1-\varepsilon)^{(-a)}\,C_s^{\,(-a)}$$

$$= U'[(1-\varepsilon)]\,U'[\,C_s\,]$$

The ratio $E\,R_a / R_o$ represents the ratio of the return on a given risky asset to the return on the riskless asset. What will be the effect on this ratio of an increase in the proportion of investments which are indexed? To see this we take the derivative of equation 8) with respect to the proportion of index investment ε:

$$E\,R_a / R_o = 1 - \operatorname{cov}[\,F[\,(1-\varepsilon)\,C_s\,], R_a\,] \qquad 8)$$

$$E\,R_a / R_o = 1 - \operatorname{cov}[\,F[\,(1-\varepsilon)]\,F[\,C_s\,], R_a\,]$$

$$E\,R_a / R_o = 1 - F[\,(1-\varepsilon)]\operatorname{cov}(\,F[\,C_s\,], R_a)$$

$$\frac{\delta\,(E\,R_a / R_o)}{\delta\,\varepsilon} = F'\,[\,(1-\varepsilon)]\operatorname{cov}(\,F[\,C_s\,], R_a)$$

18 Mankiw and Shapiro (1986) is a typical example of the use of the CRRA function in testing the consumption CAPM.

given $F(*)$ decreasing therefore $F'[*] < 0$

given a positive correlation between C_s and R_a then $\text{cov}(F[C_s], R_a) < 0$

given a negative correlation between C_s and R_a then $\text{cov}(F[C_s], R_a) > 0$

$$\frac{\delta(E R_a/R_o)}{\delta \varepsilon} = F'[(1-\varepsilon)] \text{cov}(F[C_s], R_a) > 0 \quad \text{for positively correlated assets}$$

$$\frac{\delta(E R_a/R_o)}{\delta \varepsilon} = F'[(1-\varepsilon)] \text{cov}(F[C_s], R_a) < 0 \quad \text{for negatively correlated assets}$$

As a result of an increase in the proportion of indexed investments, assets whose returns are positively correlated with aggregate consumption will find their returns increasing relative to the return on the riskless asset. A larger return is now required in order to attract capital from marginal utility investors. This result can be attributed to the automatic trading rule used by institutional index investors. Rather than reacting via price movements, with prices incorporating changes in the marginal utility of consumption and in risk, these investors react only to aggregate consumption. When the marginal utility investor makes a decision to increase their consumption, it marginally increases the consumption of index investors as well, since the index simply 'follows the crowd'. As a result the specific risks generated by individual consumption are higher than under a non-indexed market. These higher risks require higher returns. (A similar argument can be made for assets that are negatively correlated with the market).

Connecting the traditional CAPM to the consumption CAPM

It is possible to demonstrate the equivalence of the traditional and consumption based formulations of the CAPM given certain preconditions. In making this connection, the conclusions of the previous section can be shown to hold even if we adhere to the original CAPM model. Taking the return formulation of equation 4) and rearranging it:

$$E R_a = R_o - R_o \operatorname{cov}[F(C_s), R_a] \qquad 4)$$

$$E R_a - R_o = - R_o \operatorname{cov}[F(C_s), R_a] \qquad 9)$$

Suppose that: $F(C_s) = R_e$

That is to say, the function of aggregate consumption, where consumption C_s represents a random state dependant variable, exactly matches the returns on some composite asset – this asset can be thought of as the security market line of the traditional CAPM. Using this assumption:

$$E R_a - R_o = - R_o \operatorname{cov}[F(C_s), R_a]$$

$$= - R_o \frac{\operatorname{cov}[F(C_s), R_a]}{\operatorname{cov}[F(C_s), R_e]} \operatorname{cov}[F(C_s), R_e]$$

$$= - R_o \frac{\operatorname{cov}[R_e, R_a]}{\operatorname{var}[R_e]} \operatorname{cov}[F(C_s), R_e]$$

$$= \beta(- R_o \operatorname{cov}[F(C_s), R_e])$$

$$E R_a - R_o = \beta(E R_e - R_o)$$

Of course, this is simply the traditional CAPM formulation[19]. Given this result, in order for the traditional CAPM to continue to hold the condition:

The condition: $F\,(C_s) = R_e$

must adjust to the condition: $F\,[\,(1-\varepsilon)\,C_s\,] = R_e^{/}$

Given that $F\,(\,*\,)$ is a decreasing function:

$$F\,(C_s) \quad < \quad F\,[\,(1-\varepsilon)\,C_s\,]$$

And therefore: $R_e \quad < \quad R_e^{/}$

As the proportion of index investment rises, the return on this 'security market line' must be higher to re-establish a stable market structure of returns. The conclusion of the consumption based model with respect to the derivative of $E\,R_a\,/\,R_o$ is consistent with this conclusion. This result goes further by suggesting an increase independent of any changes in the risk free rate of return.

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19 This approach is a variation on the strategy presented in Blanchard and Fisher (1993) pp 506-510, save that here a different formulation of the consumption CAPM is being adapted.

Empirical Evidence

Several conclusions have been reached regarding the effects of index investment on asset pricing and returns. It is now possible to weigh these findings against market evidence. This section discusses empirical observations of Toronto Stock Exchange (TSE) data from the 1980's and the 1990's. To a certain extent, the latter period represents a 'natural experiment' in which a large institutional investor, the Ontario Teacher's Pension Plan – an institution noted for its extensive use of index investments – began investing in Canadian equities[20]. The Canadian market, as represented by the TSE, is small relative to US markets such as the New York Stock Exchange, which suggests that the impact on Canadian stock returns of an increase in index investment may be more dramatic and observable than would be the case elsewhere.

Indexed mutual fund holdings, at $3.5 billion in 1998, now account for about 1.5% of Canadian market holdings[21]. The Ontario Teacher's Pension Plan, one of the larger pension funds in Canada, currently indexes an additional $20 billion. These figures suggest that, as a conservative estimate, 5-10% of Canadian holdings are indexed. Given that a decade ago index investment was virtually non existent, the phenomenon can be isolated to the 1990's.

The theoretical models presented in this paper predict a rise in the market return as a result of increased institutional investment. By comparing TSE returns from a period unaffected by index investment (1981-1987), to TSE returns in a period with substantial index investment (1991-1997), it should be possible to assess these predictions.

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20 The OTPP became an equity investor as of the end of the fiscal year 1990. Prior to this date the plan consisted only of investments in government debentures.

21 Greenwood (1998)

The consumption based model of index investment and asset pricing provided the following result:

$$\frac{\delta (E R_a / R_o)}{\delta \varepsilon} = F' \, [(1-\varepsilon)] \operatorname{cov} (F [C_s] , R_a) > 0 \quad \text{for positively correlated assets}$$

In general, the return on the stock market index features a small positive correlation with consumption. As a result, the ratio $E\,R_e/ R_o$, where $E\,R_e$ is the mean return on the stock market index and R_o is the return on a risk free asset, would be expected to rise with an increase in index investment.

A test of this conclusion can be made by examining the variable $E\,R_e/ R_o$, over the periods 1981-1987 and 1991-1997, to determine if an increase has occurred. The return on the stock market index is obtained using data drawn from monthly summary tables published in the TSE Review from 1981-1997. This summary gives the percentage capital gain on the index and its percentage dividend yield. These percentages are combined to calculate the return an investor would have received in the previous month by investing on the index, the variable R_e. Using a methodology presented in Eckbo (1986), the returns, R_e are averaged over three months to provide an unbiased estimate of the monthly rate of return on the index.[22] The risk free rate of return, R_o , is obtained from the monthly yield on 6 month Canadian Treasury Bills at auction,[23] with the ratio R_e/ R_o calculated using the two figures.

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22 Eckbo (1986) points out that individual securities are faced with non synchronous trading since a significant portion of TSE firms trade only sporadically. Such non synchronous trading means that the return on securities should not be measured over the same fixed one month interval. Averaging the return over three months provides a more consistent estimate provided the market index is serially uncorrelated. Although, the stock market index represents an average which is more resistant to these effects, the averaging is nonetheless adopted as a precaution.

23 This data, and other interesting monetary aggregates, can be obtained from the Bank of Canada site: http://www.bank-banque-canada.ca/english/fmd.htm

Using a simple comparison of means for the observations of R_e / R_o in the two periods, the return on the stock market index is seen to rise by a statistically significant amount after 1991, between – 0.1% and 2.0% relative to the risk free rate of return. This statistical comparison, given in more detail in the appendix, is consistent with the predictions of the models. Nevertheless, there are a number of possible explanations for the increase between the two periods. An opening of the Canadian economy to international trade, changes in the composition of the TSE index, or efficiencies from electronic and computer trading, are all possible factors in the rising market return. Based on the theoretical results given in this paper, however, the impact of index investment should also be considered as a factor contributing to this increase.

In any event, the statistical results suggest an upper bound on the impact of a 5-10% increase in the proportion of the market that is invested on the index. As a result of this increase in index investment, the market return has risen by no more than 2% relative to the risk free rate of return. Using a fairly generous estimate, the introduction of the CPP investment fund is likely to produce a similar 5-10% increase in the proportion of the market that is indexed. Based on this look at TSE market data, however, there is little to evidence to conclude that an increase of this magnitude would have more than a moderate impact on TSE returns.

Conclusions

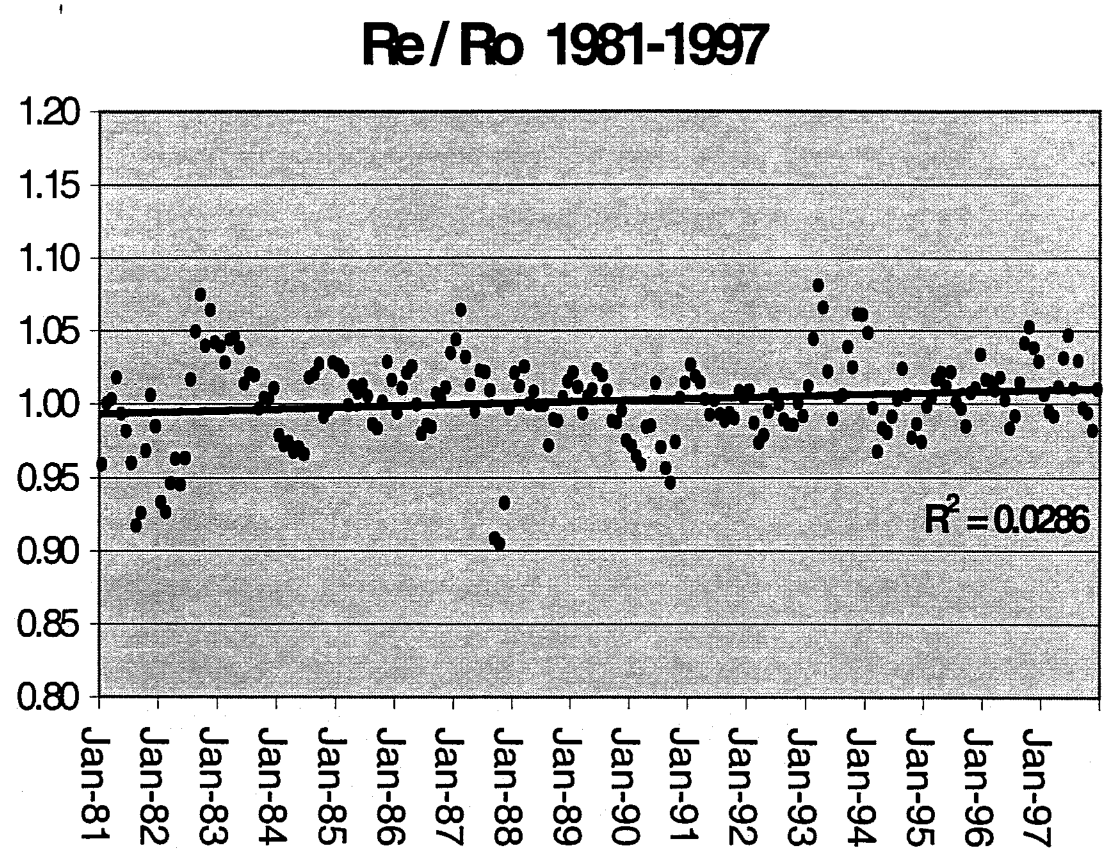
It is often difficult to assess novel economic policies for which there are few precedents. Nevertheless, using equal parts intuition, empirical evidence and economic theory, it is usually possible to develop a confident understanding of any new market developments. In order to support a number of conclusions regarding the introduction of a large indexed CPP investment plan, this paper has developed a model of consumption based asset pricing to include indexed investment. In doing so, an attempt has been made to reconcile the consumption CAPM model with the traditional CAPM formulation.

The analysis rejects the notion, often implicitly tied to the traditional CAPM, that index investment is neutral to asset pricing and returns. The modeling predicts that those assets that co-vary with market returns with find their returns rising in response to increased index investment. Empirical evidence suggests, however, that this increase in returns is modest relative to the return on the riskless asset. Despite increased index investment in this decade, the market return on the TSE index, which generally co-varies with consumption, has risen by no more than 2% relative to the return on a 6 month treasury bill. This suggests that similar increases in index investment generated by the new CPP fund will not significantly alter asset returns.

Appendix

This appendix provides a more detailed presentation of the statistical comparison of stock returns cited in the text.

The graph below shows monthly observations of the variable R_e / R_o made from 1981-1997. Given that the linear time trend appears to be upward sloping (despite a low R^2 value), a test to determine if the returns are independent of time is required. If this were not the case then any comparison would be biased towards finding an increase in later periods.



A simple test for time independence is to regress the returns in each period, 1981-1987 and 1991-1997, and in the pooled sample of both periods, using time as the sole explanatory variable, using the residual sums of squares to perform a Chow test. The F statistic for this test is:

$$F = \frac{(RSS_P - RSS_1 - RSS_2) / (K + 1)}{(RSS_1 + RSS_2) / (N_1 + N_2 - 2K - 2)}$$

Where: $K = 1$ the number of independent Variables

$N_1 = 83$ the number of observations in sample 1

$N_2 = 83$ the number of observations in sample 2

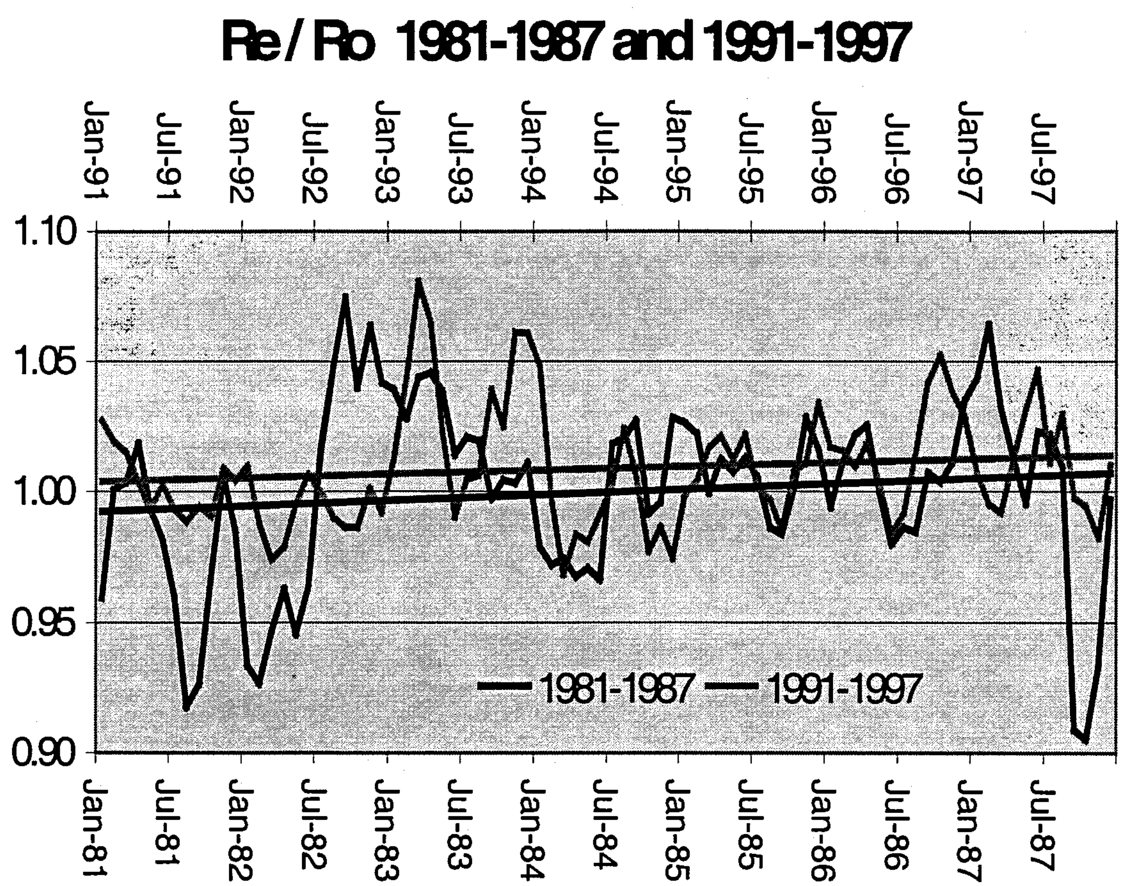
RSS_P , RSS_1 , RSS_2 are the residual sums of squares for the regressions on the pooled sample, sample 1 and sample 2 respectively

The linear regressions provide the following results:

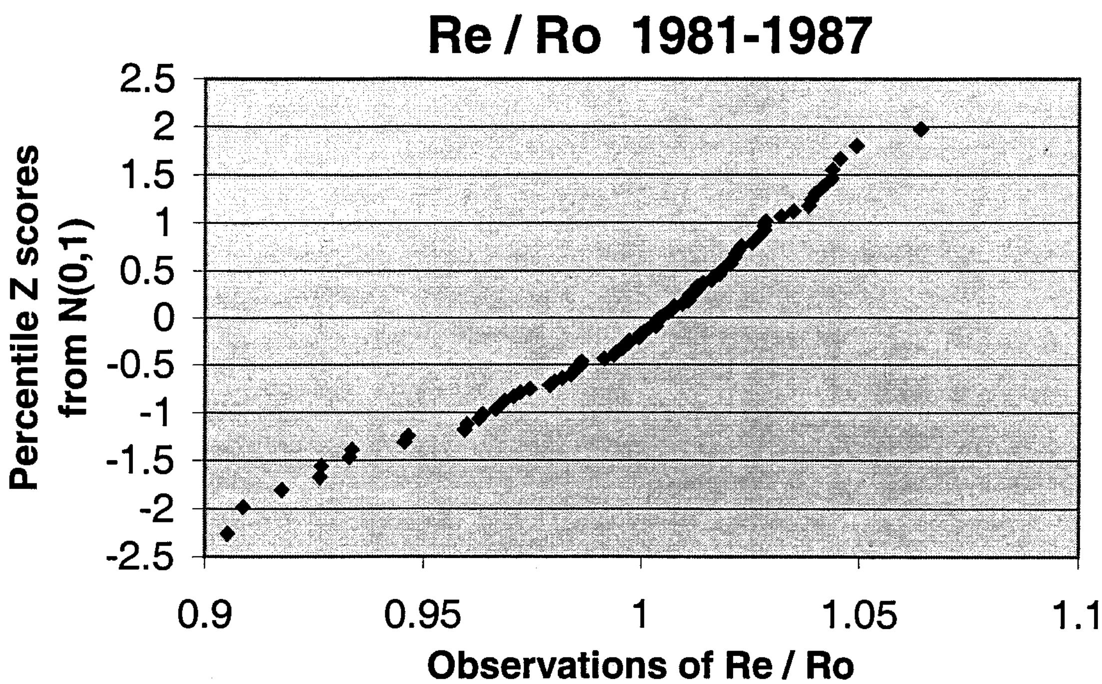
RSS_P 41.225
RSS_1 20.409
RSS_2 20.815

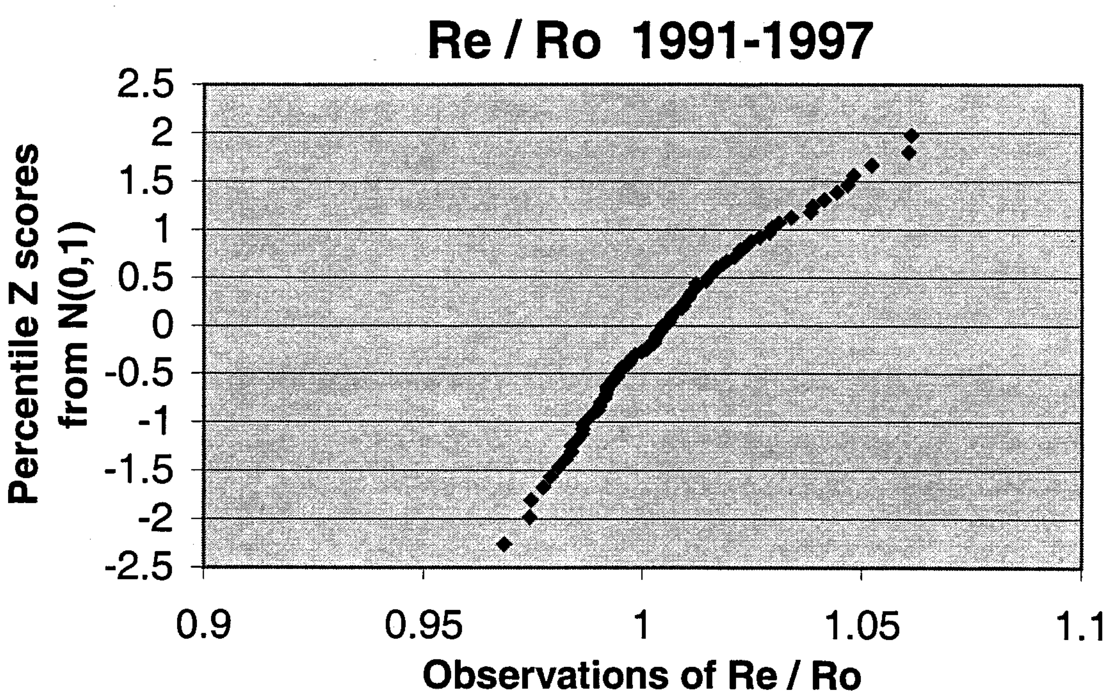
The relevant Chow test statistic is 0.0043. The critical F value, at a 95% confidence level, for a distribution with 2 degrees of freedom in the numerator and 162 in the denominator is 0.0513. Given that the Chow test statistic is less than the F critical value, we cannot reject the null hypothesis that the coefficients with respect to time, in the regression of returns against time, are the same in both periods. In other words, there is no strong case that time affects returns differently in the two chosen periods. The regressions also demonstrate very low interaction between time and returns, generating R^2 values of 0.001 in both cases, suggesting that returns are not by nature increasing over time.

Another potential source of bias arises from the business cycle. If either of the data sets were to include an extra boom or bust period, their returns would be biased. When graphed together, however, the two observation periods demonstrate a strikingly similar pattern of returns over time. This suggests that the two periods are drawn from the same portion of the business cycle and are free from this bias.



The simplest techniques for the comparison of means between sample populations require approximate normality in each population. The following graphs are normal quantile plots of the variable R_e / R_o over the two periods:





The general linearity of the normal quantile plots suggests that the use of analysis techniques for normal means is acceptable[24]. Because the aim is to examine whether the mean has increased in the second period the test hypotheses are:

$H_o: \quad \mu_1 = \mu_2$

$H_a: \quad \mu_1 < \mu_2$

The means and variances calculated using the 83 observations in each period are:

	Mean	Variance	Sample Size
1981-1987	1.000	0.0013	84
1991-1997	1.009	0.0005	84

The T - test statistic for comparing the means of two normally distributed populations is:

$$t^* = \frac{x_2 - x_1}{\sqrt{(s_1^2 / n_1 + s_2^2 / n_2)}}$$

Where x_1, x_2 are the sample means with sample variances s_1, s_2
drawn from populations of size n_1, n_2

Given the sample data this yields a two sample test statistic of: 1.981

24 It should be noted that the two distributions of the test statistic are slightly skewed. From 1981-1987 this skew is towards returns higher than the mean, while from 1991-1997 the distribution is skewed towards lower returns than the mean. These minor deviations would only bias the test towards a rejection of an increase in the mean, and not a false acceptance of this increase. As a result, the assumption of approximate normality is a still a viable one.

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