Modelling a Speculative Housing Market in a Stock-Flow Consistent Framework

By Paul J. Beckta

7894255

Major Paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M.A. Degree
Supervisor: Professor Marc Lavoie

ECO 6999

Ottawa, Ontario
August 2015

First and foremost I sincerely thank my supervisor Marc Lavoie for the many discussions, advice and knowledge he passed on to me, without which this paper would never have been possible. Second I would like to thank Antoine Godin, Eugenio Caverzasi and Stephen Kinsella who organized the Winter School on stock-flow consistent and agent based modelling at the University of Limerick where I learned to program SFC models. Third I would like to thank my colleague Daniel Detzer who I met at the Winter School and my friends Drew Penner and Dan Clearwater who helped me clarify my early ideas and discussions on programing SFC models. Finally my Father Jim and Aunt Becky for the support and encouragement they gave me throughout the process of writing this paper.

I dedicate this paper to late Mother Annalea, my Father Jim and my Aunt Becky

Introduction

Leading up to the 2007 financial crisis, housing prices in the United States had climbed to unprecedented levels. This was a highly speculative market. Within the post-Keynesian stock-flow consistent (PK-SFC) literature, models focused on speculation in the housing market are quite limited. This paper presents a new stock-flow consistent model, influenced by Gennaro Zezza's paper *U.S. growth, the housing market, and the distribution of income* (2008) and the textbook for stock-flow consistent modelling: *Monetary Economics, An Integrated Approach to Credit, Money, Income, Production and Wealth, 2nd Edition* coauthored by Wynne Godley and Marc Lavoie. The main contribution of this model to existing literature is a new behaviour of the firm in the production and setting of the price of houses. The result is a model that creates business cycles driven by speculation in the housing market, without any exogenous shocks imposed on the model. The first section begins with a brief history and discussion of SFC modelling. The second section is a literature review of SFC models focused on the housing market. The third section is a brief review of U.S. housing data, building the foundation for a theoretical argument about what caused the rise of U.S. house prices. In the fourth section a new SFC model focused on the housing market is presented. The final section is a discussion of the simulations of the baseline model and the three shocks imposed on the model.

SFC 101

Stock-flow consistent models have three components: a balance sheet, a transaction matrix, and a flow of funds matrix. The balance sheet tracks the stock of capital for each of the sectors in the model. Capital can be split into two groups, financial capital and tangible capital. Various forms of financial capital include deposits, loans and equities. Each of these types of capital is part of one economic sector's asset, but also another sector's liability. For example a loan is the asset of the bank that issues the loan and is the liability of the borrower of said loan. Assets and liabilities for financial capital net out to zero. The second form of capital, tangible capital, is physical capital such as machinery used in production, or real estate and does not have the asset/liability party/counter-party framework that govern financial assets. The transaction matrix tracks all transactions between the various sectors in the model. A simple example is the transactions between the firm and household sectors. The household allocates a portion of its income, which they receive from the firm, towards consumption. For the household, consumption is an outflow and the income it receives from the firm is an inflow. The counterparty, a firm, has an inflow of the consumption expenditure by the household and an outflow in the form of wages it pays to the household sector. With each outflow from one sector of the economy having a corresponding inflow from another sector of the economy all transactions net out to zero. The flow of funds matrix tracks the flows of capital between each of the sectors. An example of a flow is the change in the stock of a mortgage. The mortgage is a use of funds (outflow) for the bank and a source of funds (inflow) for the household.

Brief History of Stock-Flow Consistent Modelling

SFC modelling began with Morris Copeland's approach to modelling economic flows in the late 1940s (Copeland, 1949). Based upon the double entry bookkeeping principle, Copeland augmented this approach to record flows from a social perspective creating the quadruple entry bookkeeping principle (Caverzasi & Godin, 2015). An example of the quadruple entry approach is the sale of a house. To purchase houses the buyers must have either sufficient cash on hand or get a mortgage from a bank. For the sake of simplicity let's assume the former, the buyers using the money in their deposit accounts. Upon the sale of the house the buyers transfer the money to the account of the sellers and in exchange they receive the title of the house. In the end four transactions between the buyers and the sellers take place. The buyers receive the houses, thus their tangible capital increases, but their deposits decrease by the same amount to pay for the houses. The sellers lose the tangible capital and their deposits increase by the amount of money received from the sale of the houses.

After Copeland there were two other early major contributors to SFC modelling, James Tobin at Yale and Wynne Godley at Cambridge in the United Kingdom. The first major SFC model, empirical no less, focused on the U.S. economy, was published in a paper by Tobin along with David Backus, William Brainard and Gary Smith (Backus et al., 1980) (Caverzasi & Godin, 2015). Another significant contribution by Tobin was the development of a system of equations for portfolio allocation of wealth into various assets depending on their respective rates of return (Tobin, 1969) (Caverzasi & Godin, 2014). With respect to SFC modelling used by post-Keynesian economists, Godley made the significant early contributions on both theoretical and empirical sides. Perhaps the most important contribution to SFC modelling was the book Godley coauthored with Marc Lavoie: *Monetary Economics: An Integrated Approach to Credit, Money,* *Income, Production and Wealth* in 2007, with a second edition published in 2012. *Monetary Economics* introduces the fundamentals of SFC modelling and explores various facets of macroeconomics, such as open economy models, realistic financial systems and growth. It is the textbook for SFC modelling. A more in-depth discussion of SFC modelling can be found in *Post-Keynesian stock-flow-consistent modelling: a survey* by Eugenio Caverzasi and Antoine Godin (2015).

Literature Review

The PK-SFC literature on modelling a housing market is quite limited. The first two models put forth were by Zezza (2008) and the Eatwell et al. (2008) paper. These two models provided the foundation for modelling the housing market for subsequent papers by other authors.

Gennaro Zezza's *U.S. growth, the housing market, and the distribution of income* (2008) is a PK-SFC model, as the title implies, of the housing market and the conflict over the distribution of income. The conflict over the distribution of income is between workers and capitalists the latter of which he defines as the top 5% percent of income earners, while workers are the bottom 95%. Capitalists derive their income from wages, profits, interest and rental income from houses rented to the worker sector. The workers earn their income from wages and interest.

The relevant features of Zezza's model for this paper is the housing market. Specifically this involves the demand behaviour for houses by the capitalists and worker household sectors, as well as the firm's behaviour with regards to production and price formation of houses. The capitalist demand for houses is determined by the Tobinesque portfolio demand equations, which also determine the capitalists' demand for government bills and equities issued by the firm, based upon their expected rates of return. Worker's demand for houses is determined by population growth, expected real output growth and mortgage repayment from the previous period (Zezza, 2008). On the supply side, the firm produces new houses based upon "expected demand and past capital gains on houses" (Zezza, 2008, p. 390). If the supply of houses exceeds its demand, the firm will end up with a stock of unsold houses at the end of the period. The price of houses is inversely determined by the change in the stock of unsold houses. This creates a problem for the model, since the change of unsold houses is determined at the end of the period, so that the price of houses is only known after they have been sold. Zezza's solution, one he acknowledges is a strong simplification, is that houses are sold at the general price level, which is used for consumption/government services and the price of capital for the firm. This simplification implies that the cost of production of firms is homogenous for all goods and services produced in the model. A possible solution to Zezza's simplification is the price of houses bought at the beginning of a period could be the price of houses determined at the end of the previous period. This is justified because the point in time where one period ends is the same at which the next period begins.

Another problem with the model is the treatment of the stock of houses for both household sectors and the firm. In a model that does not have a housing resale market or does not have housing depreciation, the stock of houses for the household sectors cannot decrease. In Zezza's (2008) model this problem is likely to occur in the capitalist household sector. If the price of houses falls between two periods, the rate of return on houses will be negative, causing the capitalist's demand for houses to decrease. The decline of the capitalist sector's demand for houses will reduce their stock of houses, causing a rise in unsold houses by the firm since the change in capitalist's stock of houses will be negative. The firm's stock of unsold houses is the difference between the new houses produced each period and the change of each of the household sectors stock of houses. Thus the firm's stock of unsold houses is the buffer for houses the capitalist household sector no longer desires to own. It is unrealistic to assume that the firm will buy any amount of houses a household sector does not wish to own. An alternative to this assumption is to impose a floor on the change of the stock of houses held by the household sector. If a household sector's demand for houses is less than their stock of houses at the beginning of the period, then their stock of houses should remain unchanged.

The Eatwell et al. (2008) model was presented in the paper *Liquidity, leverage and the impact of sub-prime mortgages* by John Eatwell, Tarik Mouakil and Lance Taylor (2008). Their paper focused on the housing market and securitization of mortgages. Again the relevant part of Eatwell et al. (2008) for this paper is the demand, supply and price determination of houses. Unlike the Zezza (2008) model, the Eatwell et al. (2008) model only has one household sector. Their demand for houses depends positively upon the rate of return of houses and it has a negative relationship with the price of house, the mortgage rate, the debt-to-net wealth ratio of the household and the banks leverage ratio (Eatwell et al. 2008). It is based upon the works of Miles (1994) and Kenny (1999).

The firm's supply of housing is based upon the paper by Kenny (1999), an econometrics paper analyzing the supply of and demand for houses in Ireland. Eatwell et al. (2008) has the supply of houses positively related to the ratio of the price of the house with respect to the cost of production. They claim it is "Tobin's *q* analysis of investment" (Eatwell et al., 2008, p.14).

The price of houses in the model is determined not by the firm as in Zezza (2008), but by a parameter multiplying the difference in the growth rates of the demand and supply for houses. Thus if the growth in demand is greater than the growth in supply the price of houses will rise. This approach to house prices makes the assumption that prices are immediately determined by market interactions, which is a very strong assumption. This approach is not in accordance with the traditional post-Keynesian approach to pricing final goods, where prices are set by the firm based upon the cost of producing the good and the markup determined by the firm (Lavoie, 2014).

Securitization, Housing Market and Banking Sector Behaviour in a Stock-Flow Consistent Model by Olimpia Fontana and Antoine Godin has a similar objective to the Eatwell et al. (2008) paper of modelling a housing market, financed by mortgages which are then securitized and sold in the financial market. The model, with respect to the housing market, is more closely aligned with Zezza's (2008) model. Like Zezza (2008), the Fontana & Godin (2013) model has two household sectors, capitalists and workers. The capitalist's demand for houses is done through the Tobinesque portfolio decision. The worker's demand for houses follows the Eatwell et al. (2008) model, where the demand for houses is negatively related to the expected rate of return on houses (which is in fact the opposite of the Eatwell model) and negatively related to the expected rate of change of the cost to service their mortgage (Fontana & Godin, 2013). The supply and pricing of houses is modelled exactly as in the Zezza (2008) model.

Saed Khalil's 2011 doctoral thesis *Price Formation, Income Distribution, and the Business Cycle in a Stock-Flow Consistent Monetary Model* examines the impact of the variation of prices in the housing market on the financial sector. An interesting feature in the Khalil (2011) model is housing depreciation, which does not appear in any of the other models. The demand for houses is a function of the change in disposable income and the expected change of the rate of return on houses (Khalil, 2011). A key contribution by Khalil (2011) is the rate of return on houses, which includes the cost of borrowing and income received on renting houses. This is the first model that looks at the rate of return on houses as an opportunity cost, not as the gross return of the houses. The supply of houses is a function of the difference between the expected stock of houses held by the household sector and the stock of the houses of the previous period, the difference between the normal amount of unsold houses and the stock of unsold houses from the previous period, plus the amount related to the housing depreciation (Khalil, 2011). The price of houses is determined by the change in the stock of unsold houses and by the change in the "general price level" (Khalil, 2011, p. 37).

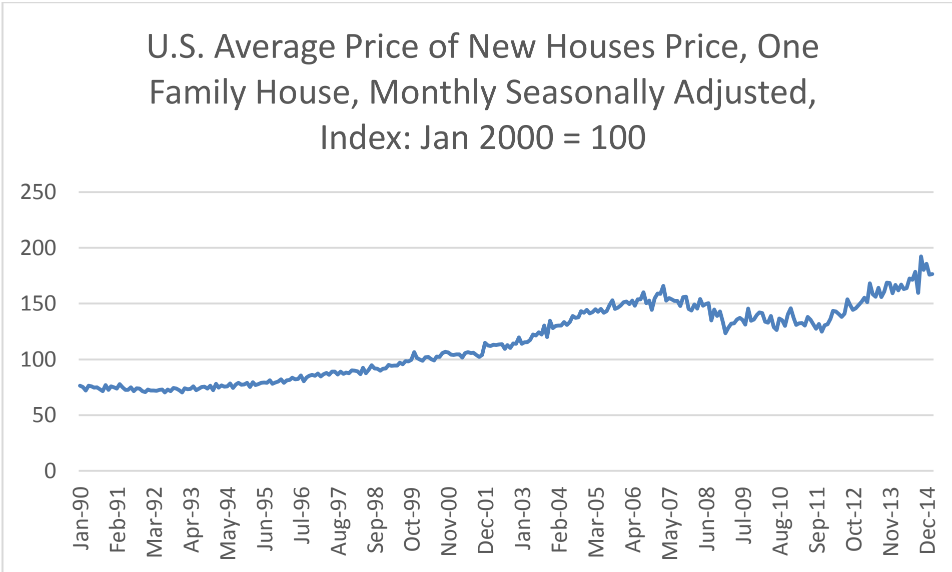
Samuel Effah's 2012 doctoral thesis *Introducing Real Estate Assets and the Risk of Default in a Stock-Flow Consistent Framework* is another SFC model focused on the household sector's role in the real estate market. The main contribution by Effah (2012) was the introduction of nonperforming mortgages, which is a fixed portion of the stock of mortgages from the previous period. Effah (2012) follows Zezza (2008) as the demand for houses is determined through the Tobinesque portfolio decision. The supply of houses by the firm and the price of houses follows Eatwell et al. (2008).

Data

The foundation for the house pricing decision by the firm presented in this paper is based upon two questions that arise from an initial look at U.S. housing prices. First, from a post-Keynesian price formation perspective, what caused the rise of U.S. housing prices? Second, what is the rationale, at least a theoretical one, for the increase?

The average price of new houses in the United States were relatively stagnant over the first half of the 1990s (Figure 1). Beginning in 1995 the average price of new houses began to rise, reaching unprecedented levels and finally reaching a maximum in the summer of 2006. After 2006, the price of new houses reversed and continued on a downward trend until the end of 2011.

Figure 1



U.S. Census Bureau/Haver Analytics

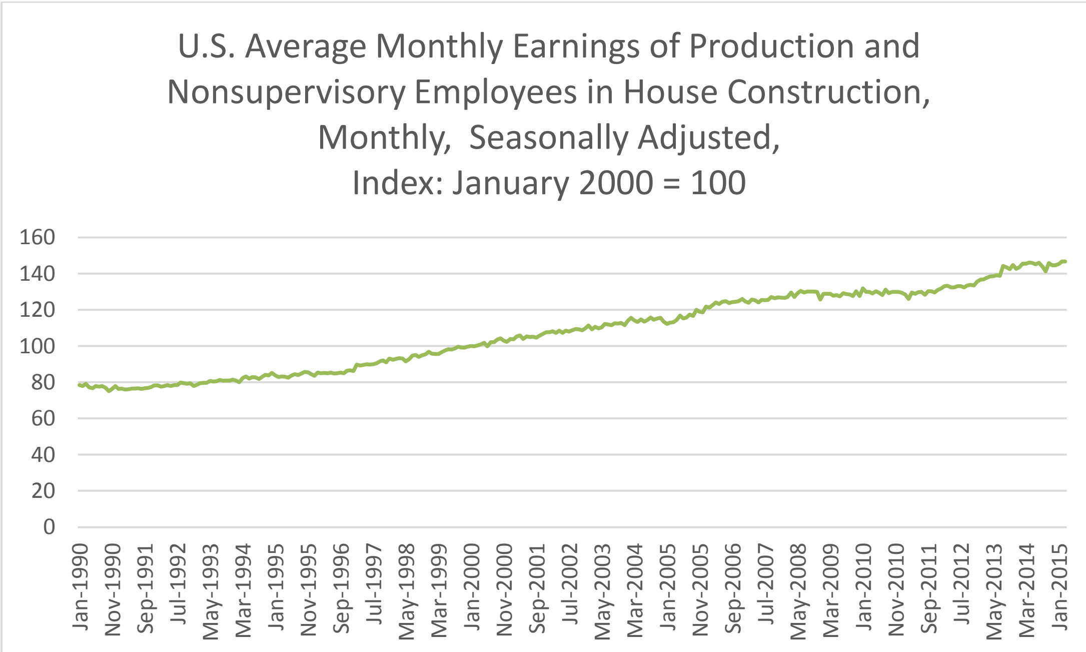
According to post-Keynesian economics the firm sets the price of goods sold in the market. The simplest pricing model of firms is called cost-plus pricing. Equation i. states the price of a good is equal to the unit cost, *UC*, of the good multiplied by a markup, θ, that is determined by the firm, from which they derive their profit. The unit cost of the good (equation ii.) is the average wage paid to workers divided by their average level of productivity (Lavoie, 2014).

i. $P = (1 + \theta) \cdot UC$

ii. $UC = \frac{Wage}{Productivity}$

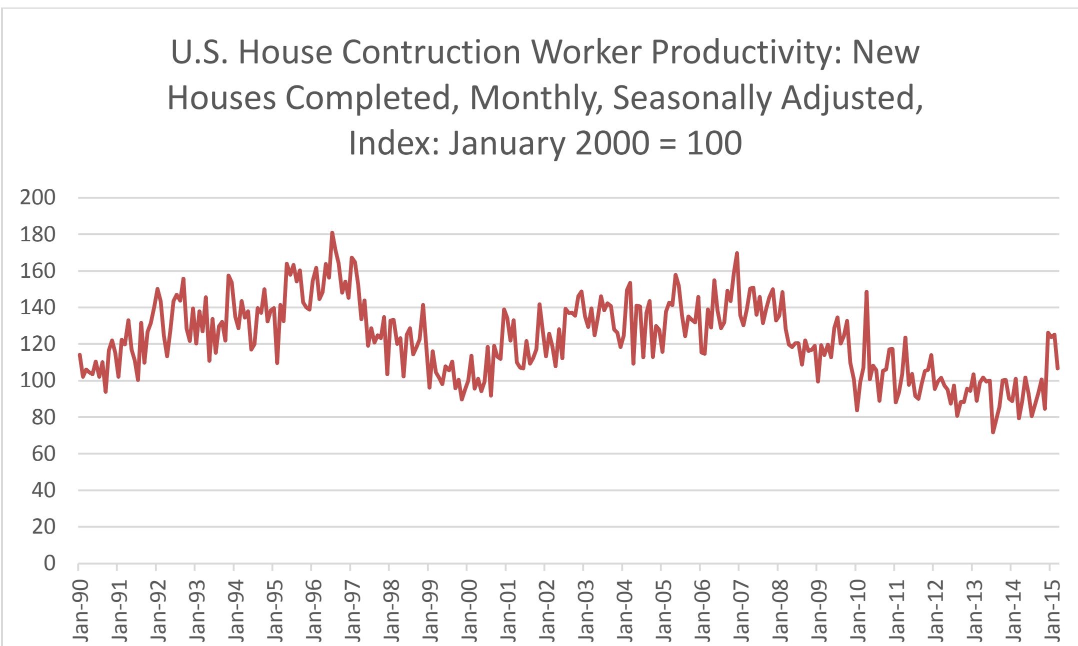
Using these two equations we can gain some insight into what was driving the rise of U.S. house prices. Figures 2 and 3 are monthly earnings and monthly productivity of U.S. residential construction workers from January 1990 to March 2015. The average weekly earnings has a continuous upward trend over time. Residential housing construction worker productivity exhibits two cycles, a strong cyclical movements in short periods, but also longer cycles over 10-year periods. Wages do follow the trend of U.S. house prices, but worker productivity certainly does not.

Figure 2



Source: U.S. Bureau of Labor Statistics

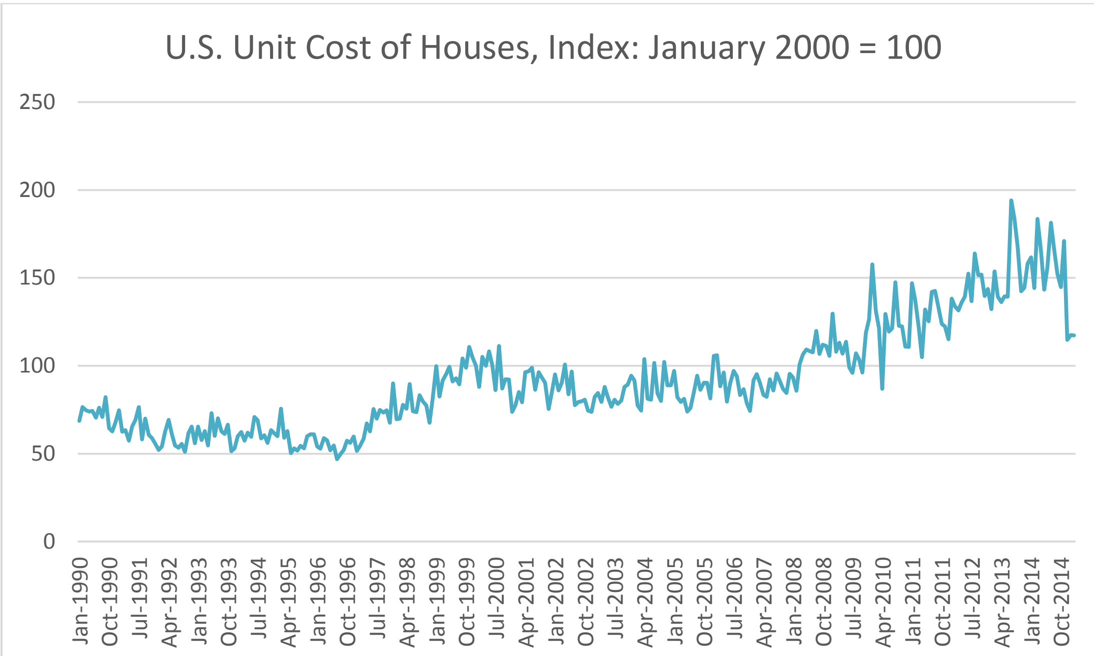
Figure 3



Source: U.S. Bureau of Labor Statistics

Figure 4 is the unit cost of houses. The unit cost is derived from equation ii using the weekly earnings and productivity from Figures 2 and 3. The unit cost is cyclical, but relatively stationary, except for two upward trends, the first being from the mid-1990s to 2000 and the second between the beginning of 2008 and 2015. The unit cost does not follow the trend of U.S. prices. In fact it is moderately countercyclical, with a very small downward trend from 2000 to 2008 and then reversing with a large rise until the last quarter of 2014.

Figure 4



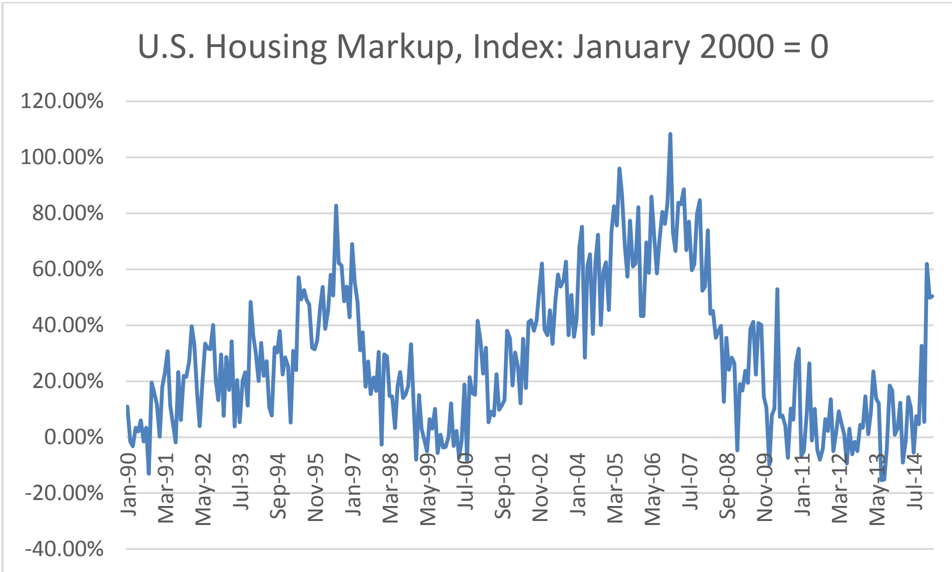
Source: U.S. Bureau of Labor Statistics, Author's Calculation

Rearranging equation i. and solving for the markup, θ, we get equation iii. Using the wage, productivity and housing price indeces we get a markup index (Figure 4). There are two observations to be made from the markup of houses. First, the markup is highly cyclical within a given year. Second, there is distinct upward trend in the markup from the end of 1999 to the end of

2006, followed by a steep downward trend. The markup is procyclical with the price of houses and has a much stronger movement than the unit cost of new houses. There are factors not captured in the pricing equation that, when excluded, could result in a variable markup. Some of these factors are the cost of capital inputs and resources (eg. lumber, concrete and land) necessary for the production of housing. This data is a strong first step in providing evidence that the firm's control over the markup of U.S. houses was a significant driver of the rise of house prices during the boom.

iii. $\theta = P \cdot \frac{Productivity}{Wage} - 1$

Figure 4



Source: U.S. Census Bureau/Haver Analytics, Bureau of Labor Statistics & Author's Calculations

With this strong evidence, indicating the markup was the significant factor for the rise in house prices, we come to the second question: what were the dynamics causing the variation of the markup? A plausible theory is that households assumed the price of houses would rise forever, increasing the demand for houses because houses would be viewed as a credible investment. Knowing the household's assumption that house prices would continuously rise allowed the firm to continuously increase the markup on houses. The rising markup validates the assumption that the price of houses would continue to rise and this process would continue.

The next requirement is a theoretical decision process for when the firm will increase the markup. This begins with the firm's approach to inventory management put forth by Godley & Lavoie (2012), where the firm has an inventory-to-sales ratio target. Connecting the realized inventory-to-sales ratio of the previous period and their target ratio reveals the behaviour of the demand for houses. If the realized ratio is smaller than the target ratio this means the firm's realized inventory of houses is less than the targeted inventory the firm was expecting. This can also be interpreted as the demand for houses being stronger than the firm's expectation. When demand exceeds expected demand, the firm will raise the markup on houses, which will give a capital gain to the existing homeowners and satisfy the assumption that house prices will continue to rise. When the demand for houses falls short of the firm's expectation, the firm lowers the markup. This will result in a capital loss for existing homeowners, invalidating the household's assumption that the price of houses would continue to rise and reduce the demand for houses. This causes the firm's inventory of houses to be above its targeted level, resulting in the firm reducing the markup on houses. With the price of houses falling further, the demand for houses will once again decrease. The result is a downward cycle in the housing market.

The Model

The SFC model presented follows the six sector closed economy model put forth by Zezza (2008). The six sectors are: capitalist household; worker household; firm; commercial bank; central bank and the government. The purpose of this model is to test the plausibility of the evolution of a speculative housing market put forth in the previous section.

Starting with the balance sheet (Table 1) we have the assets (denoted with a (+) in front of the variable) and liabilities (denoted with a (-) in front of the variable) of the six sectors. The capitalist household sector's (column one) net wealth is distributed among the following financial assets: high powered money (HPM_c), deposits (M_c), savings account (SA_c), government bills (B_c) and commercial bank capital (OF). Following the *Growth Model* in Godley & Lavoie (2012) the commercial bank is privately owned by the capitalist household sector. Therefore the commercial bank's capital is an asset of the capitalist household sector and a liability of the commercial bank (Godley & Lavoie, 2012). The tangible capital is the number of houses owned by the capitalist household sector (h_c), multiplied by the current price of houses (P_h). The sum of the capitalist households' assets is its net worth (V_c). The worker household sector (column two) distributes its wealth among high powered money (HPM_w), deposits (M_w), savings account (SA_w) and houses ($P_H h_w$). The worker's liability is the mortgages (MO_w) it borrows from the commercial bank to finance its acquisition of houses. The net worth of the worker household sector (V_w) is the difference between its assets and liabilities.

The firm's asset is the nominal value of the stock of its housing inventory, which is the number houses in its inventory (h_{INV}) multiplied by the cost of production (UC_h). The firm finances its stock of housing inventory through loans (L) it borrows from the commercial bank. The commercial bank's assets are the loans (L) issued to the firm, mortgages issued to the worker household sector (MO) and the government bills (B_B) it owns. The commercial bank's liabilities are its capital (OF) and the deposits (M) and savings accounts (SA) of the household sectors.

The central bank's asset is government bills (B_{CB}) and its liability is the high powered money held by the household sectors (HPM). The government's liability is the bills (B) it issues. All financial assets and liabilities net out to zero and the sum of the net worth of all the sectors is equal to the nominal stock of houses in the economy (H).

The transaction flow matrix is given in table two. The capitalist household sector derives its income (F) from the profits of the firm (F_F) and dividends (D_B) from the commercial bank. Worker's income is the wages earned (WB) for the labour provided to the firm. Both household sectors earn interest from their respective financial assets. The household sectors consume services (C_c & C_w) and buys houses ($P_h \Delta h_c$ & ($P_h \Delta h_w$) from the firm sector and pays taxes (T_c & T_w) to the government. Both household sectors save (SAV_c & SAV_w) a portion of their income each period. Their saving will change their net worth and will be allocated amongst their various assets.

The firm's revenue comes from consumption (C) and government (G) services and the sale of houses ($P_H \Delta h_S$). The firm pays wages to its workers (WB), interest on its loans ($r_{L-1}L_{-1}$) to the commercial bank and its profits to the capitalist household sector. The remaining unsold houses ($UC_H \Delta h_{INV}$) are transferred from the current account to the capital account changing the stock of housing inventory each period. The commercial bank earns interest on its assets and pays interest on its liabilities. A portion of its earnings is retained each period (RE_B) to meet the capital requirement and pay the remaining amount out as dividends (D_B).

The central bank receives interest on the bills they hold ($r_{B-1}B_{CB-1}$) which is paid back to the government (F_{CB}), here referred to as public profits. The government purchases services each period (G) and pays interest on bills ($r_{B-1}B_{-1}$). Its income comes from taxes (T) and the central bank's public profits. The government's deficit is the difference between its expenditures and revenues, which changes the amount of bills supplied each period (ΔB).

The equations such as consumption and saving functions are based on the model in Zezza (2008) and chapters 9 and 11 from Godley & Lavoie (2012). There are also many new equations presented in this model, in particular the determination of the price of houses, an endogenously determined central bank interest rate and the heavy use of logic operator functions.

Following Zezza (2008) and Godley & Lavoie (2012), unless otherwise indicated, the expectation variables are determined by the adaptive expectation equation below. All of the generic adaptive expectation variables have: $\mu = 0.5$, except for expected housing sales where $\mu = 0.25$.

$$X^e = \mu X^e_{-1} + (1 - \mu) X_{-1}$$

Table 1 Balance Sheet

	Capitalist Household	*Worker Household*	*Firm*	*Commercial Bank*	*Central Bank*	*Government*	*Σ*
Cash	$+HPM_c$	$+HPM_w$			$-HPM$		0
Deposits	$+M_C$	$+M_W$		$-M$			0
							0
Savings Account	$+SA_c$	$+SA_w$		$-SA$			0
Mortgages		$-MO_w$		$+MO$			0
Loans			$-L$	$+L$			0
Bills	$+B_c$			$+B_B$	$+B_{CB}$	$-B$	0
Houses	$+P_h h_c$	$+P_h h_w$	$+UC_h h_{INV}$				$+H$
Bank Capital	$+OF$			$-OF$			
Net Worth	$-V_C$	$-V_W$	0	0	0	$-V_G$	$-H$

Table 2 Transaction Flow Matrix

	Capitalist Household		*Worker Household*		*Firm*		*Commercial Bank*		*Central Bank*	*Government*	*Σ*
	Current	Capital	Current	Capital	Current	Capital	Current	Capital			
Consumption	$-C_c$		$-C_w$		$+C$						0
Wages			$+WB$		$-WB$						0
Taxes	$-T_c$		$-T_w$							$+T$	0
Private Profits	$+F$				$-F_F$		$-D_B$				0
Public Profits									$-F_{CB}$	$+F_{CB}$	0
Government Expenditure					$+G$					$-G$	0
Houses Sold		$-P_h\Delta h_c$		$-P_h\Delta h_w$	$+P_h\Delta h_s$						0
Housing Inventory					$+UC_h\Delta h_{INV}$	$-UC_h\Delta h_{INV}$					0
Interest Payments											0
Deposits	$+r_{M-1}\ M_{c-1}$		$+r_{M-1}\ M_{w-1}$				$-r_{M-1}M_{-1}$				0
Savings Account	$+ r_{-1}SA_{c-1}$		$+ r_{-1}SA_{w-1}$				$- r_{-1}SA_{-1}$				0
Mortgages			$- r_{-1}MO_{w-1}$				$- r_{-1}MO_{-1}$				0
Loans					$-r_{L-1}L_{-1}$		$+r_{L-1}L_{-1}$				0
Bills	$+r_{B-1}B_{c-1}$						$+r_B B_{B-1}$		$+r_{B-1}B_{CB-1}$	$-r_{B-1}B_{-1}$	0
Savings	$-SAV_c$	$+SAV_c$	$-SAV_w$	$+SAV_w$	0	0	$-RE_B$	$+RE_B$	0	-Deficit	0
Cash		$-\Delta\ HPM_c$		$-\Delta\ HPM_w$					$+\Delta\ HPM$		0
Deposits		$-\Delta M_c$		$-\Delta M_w$			$+\Delta M$				0
Savings Account		$-\Delta SA_c$		$-\Delta SA_w$			$+\Delta SA$				0
Mortgages Issued				$+\Delta MO_w$			$-\Delta MO$				0
Loans						$+\Delta L$	$-\Delta L$				0
Bills		$-\Delta B_c$					$-\Delta B_B$		$-\Delta B_{CB}$	$+\Delta B$	0
Bank Capital		$-\Delta OF$					$+\Delta OF$				0

Capitalist Equations

The capitalist sector derives its income from the profits of the firm, dividends paid out by the commercial bank and interest received on its government bills, savings and deposit accounts (Equation 1). Equation 2 is disposable income, which is income minus the taxes it owes. Capitalists pay the percentage τ_c of their income in taxes to the government (Equation 3). Taxes have a floor of zero in case capitalist income is negative. Capitalist consumption is determined by the marginal propensities to consume out of expected disposable income and the previous period's consumable wealth (Equation 4). Consumable wealth is net wealth less the market value of houses owned by the capitalist sector and the bank capital (Equation 5). Capitalist demand for high-powered money (herein after referred to as cash) is a fraction ω_c of its consumption (Equation 6).

1. $Y_c = F_F + D_B + r_{B-1} \cdot B_{c-1} + r_{SA-1} \cdot SA_{c-1} + r_{M-1} \cdot M_{c-1}$
2. $Yd_c = Y_c - T_c$
3. $T_c = \max\{\tau_c \cdot Y_c , 0\}$
4. $C_c = \alpha_1 \cdot Yd_c^e + \alpha_2 \cdot V_{c,c-1}$
5. $V_{c,c} = V_c - H_c - OF_b$
6. $HPM_c = \omega_c \cdot C_c$

The capitalist demand for houses (in unit terms) is the maximum of the fraction ∂ of its expected total wealth deflated by the price of houses for that period and the number of houses held last period (Equation 7). This is to ensure that the capitalist stock of houses cannot decrease since there does not exist a resale market for houses between the two household sectors. From an intra-sectorial view, some capitalist households may be selling their houses to other capitalist households, however these transactions do not change the total stock of houses held by that sector. The market value of its houses is the number of houses owned in the current period multiplied by the current house price (Equation 8). Capitalist saving is disposable income less consumption (Equation 9). The capital gain on houses is the change in house prices multiplied by the number of houses held at the beginning of the period (Equation 10). Equation 11 is the capital gains from the commercial bank capital. The total wealth of the capitalist household at the end of the period is the wealth held at the beginning of the period plus its savings and capital gains (Equation 12).

7. $h_c = \max\{\partial \cdot \frac{V_c^e}{P_h}, h_{c-1}\}$
8. $H_c = P_h \cdot h_c$
9. $SAV_c = Yd_c - C_c$
10. $CG_{h,c} = \Delta P_h \cdot h_{c-1}$
11. $CG_{B,c} = \Delta OF_B$
12. $V_c = V_{c-1} + SAV_c + CG_{B,c} + CG_{h,c}$

Equation 13 is the expected investible wealth, which is the expected total wealth less the bank capital, the market value of houses currently owned and the wealth held in cash. Equation 13 takes into account the reality that a person cannot consume its house. In the real world when a homeowner earns a capital gain on his/her house he/she can borrow against the new equity from the bank (of course this depends on the current liabilities as well) and put this liability towards consumption, financial investments or pay off current debt obligations. For the sake of simplicity neither of the household sectors are able to borrow against the equity in their house. Equations 14 and 15 are the Tobinesque demand equations for government bills and portfolio demand for the savings account respectively. The capitalist sector allocates its wealth between government bills and its savings account based upon its expected rates of return. Following Zezza (2008) deposits act as the buffer stock for allocation of total wealth into the various assets held by the capitalist household sector. Equation 16 is the amount of deposits held based upon the portfolio decision for the savings account. There is a possibility that the allocation of wealth to the savings account may be too large, thus returning a negative value for the deposit account. Equations 17 through 20 solve this problem. Equation 17 is the realized amount of wealth in the savings account, which is equal to the amount determined by the portfolio decision plus the portfolio decision for the deposit account, multiplied by the logic operator C_1. Equation 18 states C_1 is equal to one if and only if the amount of deposits determined by equation 16 is negative. If that is the case then the realized amount of wealth held in the savings account will decrease by the amount returned in equation 16. If equation 16 turns out to be positive the realized savings account will be equal to the amount determined by the portfolio decision. Equations 19 and 20 state that the amount of deposits will be equal to equation 16 if it is positive, which is determined through the logic operator C_2 and otherwise deposits will be zero.

13. $V_{fa,c}^e = V_c^e - OF_b - P_h h_{c-1} - HPM_c$

14. $\frac{B_c}{V^e_{fa,c}} = \lambda_{20} + \lambda_{21} \cdot r^e_b - \lambda_{22} \cdot r^e_{SA} - \lambda_{23} \cdot \frac{Y^e_c}{V^e_{fa,c}}$
15. $\frac{SA^{Port}_c}{V^e_{fa,c}} = \lambda_{10} - \lambda_{11} \cdot r^e_b + \lambda_{12} \cdot r^e_{SA} - \lambda_{13} \cdot \frac{Y^e_c}{V^e_{fa,c}}$
16. $M^{Port}_c = V_c - HPM_c - SA^{Port}_c - B_c - H_c - OF_B$
17. $SA_c = SA^{Port}_c + C_1 \cdot M^{Port}_c$
18. $C_1 = 1 \; iff \; M^{Port}_c < 0$
19. $M_c = C_2 \cdot M^{Port}_c$
20. $C_2 = 1 \; iff \; M^{Port}_c > 0$

Worker Equations

The worker household derives its income from wages and interest received from its deposit and savings accounts (Equation 21). The total amount of wages received is the sum of wages the firm pays to workers in consumption/government services and workers building houses (Equation 22). Disposable income is the gross income, minus the tax they paid to the government and the interest owed to the commercial bank on the net amount of mortgages held at the beginning of the period (Equation 23). Net mortgages is the total amount of mortgages held at the beginning of the period minus the amount of nonperforming mortgages from the previous period. The worker household taxes and consumption (Equations 24 & 25) are determined in the same manner as the capitalist sector, except for the maximum function being dropped. Consumable wealth is the net wealth less the market value of houses owned (Equation 26). The demand for cash is determined by some fraction ω_w of their consumption (Equation 27). Worker's saving, capital gain and net wealth (Equations 28, 29 & 30) are the same as in the capitalist household sector.

21. $Y_w = WB + r_{M-1} \cdot M_{w-1} + r_{SA-1} \cdot SA_{w-1}$
22. $WB = WB_h + WB_{c,g}$
23. $Yd_w = Y_w - T_w - r_{MO-1} \cdot (MO_{w-1} - NPMO)$
24. $T_w = \tau_w \cdot Y_w$
25. $C_w = \gamma_1 \cdot Yd^e_w + \gamma_2 \cdot V_{c,w-1}$
26. $V_{c,w} = V_w - H_w$
27. $HPM_w = \omega_w \cdot C_w$
28. $SAV_w = Yd_w - C_w$
29. $CG_{h,w} = \Delta P_h h_{w-1}$
30. $V_w = V_{w-1} + SAV_w + CG_{h,w}$

The worker sector's demand for mortgages is the amount of mortgages held at the beginning of the period, plus the new amount of mortgages which is a fraction ϵ of their income less the amount of nonperforming mortgages (Equation 31). The worker sector will increase the amount of mortgages demanded when it buys new houses (Equation 32). q_1 will equal to 1 if and only if the stock of worker houses increases and zero otherwise. The amount of nonperforming mortgages is the stock of mortgages from the previous period multiplied by some fraction $nmpo$ (default rate) and the logic operator θ_d (Equation 33). The fraction $npmo$, given by equation 34, is composed of a fixed default rate , $npmo_{fix}$, which acts as a floor rate, and a variable default rate, $npmo_{var}$. $npmo_{var}$ is a function of the $npmo_{var}$ from the previous period, plus $nmpo_{adj}$, which is a fixed adjustment value, all multiplied by the logic operator k_1 (Equation 35). k_1 is equal to one if and only if the mortgage-to-income ratio (since mortgages is the only liability for workers this is their debt-to-income ratio and will be referred to as such) is above the $Debt_{Threshold}$, and will otherwise be zero (Equation 36). Equation 37 gives the logical operator θ_d, which will equal one if q_1 and/or k_1 are equal to one, otherwise it will equal zero. Returning to equation 33, there will be nonperforming mortgages if the stock of housing increases and/or the debt-to-income ratio is above the $Debt_{Threshold}$ value. $npmo_{var}$ will increase when the debt-to-income ratio of the previous period is above $Debt_{Threshold}$. Once the debt-to-income ratio is equal to or less than $Debt_{Threshold}$ then $npmo_{var}$ will return to zero and $npmo$ will equal the base rate $npmo_{fix}$.

31. $MO_w = MO_{w-1} + q_1 \cdot \epsilon \cdot Y_w - NPMO$
32. $q_1 = 1 \; iff \; h_w > h_{w-1}$
33. $NPMO_w = nmpo \cdot \theta_d \cdot MO_{w-1}$
34. $npmo = \; npmo_{fix} + npmo_{var}$
35. $npmo_{var} = (npmo_{var-1} + nmpo_{adj}) \cdot k_1$
36. $k_1 = 1 \; iff \; \frac{MO_{w-1}}{Y_{w-1}} > Debt_{Threshold}$
37. $\theta_d = \min\{q_1 + k_1, 1\}$

One simplification in the model is not linking the stock of the worker sector's houses to its nonperforming mortgages. The reason for the simplification is the complexity created in dealing with defaulted houses. A more realistic approach would be for the worker household sector's stock of houses to decrease by the number of nonperforming mortgages (in terms of defaulted house units) which would pass through the current account of the bank and then be sold to the firm, being added to its inventory. This simplification does not alter the key dynamics of the model. Commercial bank profits will be higher since it would not be taking a loss of the difference between the unit cost of the house and the price of house multiplied by the number of houses defaulted upon. This is assuming the firm would purchase the defaulted houses from the commercial bank at the unit cost using a loan from the bank. Since the volume of loans for the firm is lower, the firm's profits are higher and the commercial bank's profits are lower by the amount of interest that would have been paid on the loans.

The worker household's expected investible wealth is the expected net wealth less the market value of houses held at the beginning of the period and the cash it holds (Equation 38). The portfolio decisions for the worker sector's savings account and demand for houses is given by equations 39 and 40. The savings account and demand for houses are deflated by the prices of houses because the model needs to ensure the stock of houses (in terms of units) does not decrease. The portfolio decisions are determined by the expected savings rate and by the real rate of return on houses, following the method presented in Khalil (2011), the difference between the rate of return on houses (equation 41) and the expected interest rates for mortgages. Since the capital gain is assumed to be known, the rate of return on housing is also known, therefore the worker household does not make the portfolio decisions based upon an expected rate of return for houses. In the firm section the price determination for housing is discussed, which is entirely backward looking. Therefore the price in the current period is one of the first pieces of information known since it is strictly determined by the events of the previous period. The number of houses owned by the worker sector at the end of the period is the maximum between the house demand from the portfolio decision and the amount of houses owned at the end of the last period (Equation 42). This is to prevent the stock of houses from decreasing. Equation 43 is the market value of houses held by the worker sector.

38. $V_{fa,w}^{e} = V_{w}^{e} - P_h h_{w-1} - HPM_w$

39. $\frac{sa_w^{Port}}{V_{fa,w}^{e}} = (\rho_{10} - \rho_{11} \cdot (r_h - r_{MO}^{e}) + \rho_{12} \cdot r_{SA}^{e} + \rho_{13} \cdot \frac{Y_w^e}{V_{fa,w}^{e}} - \frac{MO_w}{V_{fa,w}^{e}})/P_h$

40. $\frac{h_w^{Port}}{V_{fa,w}^{e}} = (\rho_{20} + \rho_{21} \cdot (r_h - r_{MO}^{e}) - \rho_{22} \cdot r_{SA}^{e} - \rho_{23} \cdot \frac{Y_w^e}{V_{fa,w}^{e}} + \frac{MO_w}{V_{fa,w}^{e}})/P_h$

41. $r_h = \frac{\Delta P_h}{P_{h-1}}$

42. $h_w = \max\{h_w^{Port}, h_{w-1}\}$

43. $H_w = P_h \cdot h_w$

Equation 44 gives the nominal value of wealth allocated to the savings account by the portfolio choice. Equations 45 through 49 are the same as the capitalist household sector, ensuring the deposit and savings accounts never take on negative values.

44. $SA_w^{Port} = P_h \cdot sa_w^{Port}$
45. $M_w^{Port} = V_w - SA_w^{Port} - HPM_w - H_w$
46. $SA_w = SA_w^{Port} + L_1 \cdot M_w^{Port}$
47. $L_1 = 1\ iff\ \ M_w^{Port} < 0$
48. $M_w = L_2 \cdot M_w^{Port}$
49. $L_2 -= 1\ iff\ \ M_w^{Port} > 0$

The Firm

The firm in this model provides consumption and government services as well as producing houses for both household sectors. The firm derives its profits from consumption and government services, houses sold, the change in housing inventory, minus the interest it pays on loans to the commercial bank and wages it pays to its workers (Equation 50). Total consumption is the sum of consumption from the worker and capitalist households (Equation 51). Total output of services is the sum of consumption and government services (Equation 52). Labour demanded to provide consumption and government services is the total output of consumption and government services, $Y_{c,g}$, divided by the productivity of the workers in the firm, $pr_{c,g}$ (Equation 53). $pr_{c,g}$ is a fixed value in the model. The wage bill for workers providing consumption and government services is equal to the wage rate, $W_{c,g}$, exogenously determined, multiplied by the number of workers demanded (Equation 54).

50. $F_F = C + G + H_s + \Delta H_{INV} - r_{l-1}L_{-1} - WB$
51. $C = C_w + C_c$
52. $Y_{c,g} = C + G$
53. $N_{c,g} = \frac{Y_{c,g}}{pr_{c,g}}$
54. $WB_{c,g} = W_{c,g} \cdot N_{c,g}$

The number of houses sold each period is the maximum between the change in worker and capitalist houses between each period and 1×10^{-20} (Equation 55). The minimum value is not exactly zero because the inventory-to-sales ratio used to determine the change in price (Equations 65 – 68) would be infinity if there were no sales. Therefore 1×10^{-20} is used as an approximation for zero. The number of houses produced each period is the maximum of the sum of expected houses to be sold, the difference between the expected housing inventory of this period and the realized housing inventory from the previous period and zero (Equation 56). The realized stock of housing inventory is housing inventory from the previous period plus the difference between the expected house sales and realized house sales for the current period (Equation 57). The firm's targeted inventory of houses it seeks to achieve each period is the product of σ^T, the firm's inventory to sales ratio, and the expected sale of houses (Equation 58). The expected inventory of houses is a unique expectation equation to the model (Equation 59). The firm determines their expected house inventory based upon a weighted average parameter γ_F of the housing inventory from the previous period and their targeted inventory. Equations 60 and 61 are the demand for labour to produce houses and the wage bill paid to those workers, which are the same as equations 53 and 54 for consumption and government services. The unit cost, the cost to produce a house, is the wage rate divided by productivity of workers building houses (Equation 62). The inventory of houses in nominal terms is the product of the unit cost of houses and the housing inventory in terms of units (Equation 63). The nominal value of houses sold is equal to the price of houses multiplied by the number of units of houses sold (Equation 64).

55. $h_s = \max\{\Delta h_w + \Delta h_c\,, 1 \times 10^{-20}\}$
56. $y_h = \max\{h_s^e + h_{inv}^e - h_{inv-1}, 0\}$
57. $h_{inv} = \max\{h_{inv-1} + h_s^e - h_s, 0\}$
58. $h_{inv}^T = \sigma^T h_s^e$
59. $h_{inv}^e = h_{inv-1} + \gamma_F\left(h_{inv}^T - h_{inv-1}\right)$
60. $N_h = \frac{y_h}{pr_h}$
61. $WB_h = W_h \cdot N_h$
62. $UC_h = \frac{W_h}{pr_h}$
63. $H_{INV} = UC_h \cdot h_{inv}$
64. $H_s = P_h \cdot h_s$

As previously discussed in the literature review, this model presents a new method that the firm uses to determine the price of houses. The process is built on the simple classic post-Keynesian cost-plus pricing approach (Lavoie, 2014). Equation 65 states that the price of the house is equal to the unit cost of the house multiplied by the markup, θ_h, set by the firm. The firm will adjust the markup depending on the realized inventory-to-sales ratio of houses from the previous period. If the realized ratio is less than the targeted ratio, the firm will increase the markup, thus increasing the price. If the realized ratio is greater than the targeted ratio the firm will lower the markup. This behaviour is based on the idea that if the firm increases the price and the demand for houses in the next period is such that the realized ratio is less than the targeted ratio, then home buyers are indicating that they welcome the price increase. Equations 66 through 68 change the markup. To make sense of equation 66, equations 67 and 68 will be discussed first. Equation 67 states x_1 is equal to one if and only if the targeted inventory-to-sales ratio of houses is greater than the realized inventory-to-sales ratio and zero otherwise. If $x_1$is equal to one then equation 66 will be $\theta_h = \theta_{h-1}(1 + \theta_x)$ and the markup will increase. Equation 68 states x_2 is equal to one if and only if the targeted inventory-to-sales ratio of houses is less than the realized inventory-to-sales ratio and zero otherwise. If $x_2$is equal to one then equation 66 will be $\theta_h = \theta_{h-1}(1 - \theta_x)$ and the markup will decrease. Because the realized inventory-to-sales ratio of houses is known at the beginning of the period and the unit cost of houses in the model is constant, the price of houses is one of the first pieces of information determined in the model for a given period. The markup of houses, given by equation 66, is the maximum of one and the markup adjustment equation just discussed. The maximum function ensures that the firm does not sell houses below their cost of production.

65. $P_h = \theta_h \cdot UC_h$
66. $\theta_h = \max\{\theta_{h-1}(1 + \theta_x(x_1 - x_2)), 1\}$
67. $x_1 = 1\ iff\ \sigma^T > \frac{h_{inv-1}}{h_{s-1}}$
68. $x_2 = 1\ iff\ \sigma^T < \frac{h_{inv-1}}{h_{s-1}}$

Commercial Bank

The commercial bank sector is drawn from chapter 11 of Godley & Lavoie (2012); however the sector presented in this paper has been simplified. The commercial bank derives its profit from interest earned on bills, loans and the net volume of mortgages minus the interest paid on deposits and savings accounts (Equation 69). The commercial bank's demand for government bills, which acts as the banks buffer stock, is such that it will bring its assets (bills, loans and mortgages) and liabilities (deposits, savings accounts and bank capital) to equality (Equation 70). Total deposits and savings accounts are equal to the sum of capitalist and worker deposits and savings accounts respectively (Equations 71 & 72). The change in loans demanded by firms is equal to the change of nominal value of the firm's housing inventory (Equation 73).

The amount of mortgages issued by the bank is equal to the amount of mortgages demanded by the worker household (Equation 74). The commercial bank is required to have a certain amount of capital which is some fraction CAR (capital adequacy ratio) of the liabilities it issues, to act as a buffer for a decrease of its total assets (Equation 75). The retained earnings of the commercial bank is the difference between the required amount of bank capital and the capital it has at the beginning of the period (Equation 76). The bank capital at the end of the period is equal to the bank capital at the beginning of the period plus its retained earnings (Equation 77).The amount of dividends paid out by the commercial bank is equal to its profits less its retained earnings (Equation 78).

69. $F_B = r_{B-1} \cdot B_{B-1} + r_{L-1} \cdot L_{-1} + r_{MO-1} \cdot (MO_{-1} - NPMO) - r_{M-1} \cdot M_{-1} - r_{SA-1} \cdot SA_{-1}$
70. $B_B = M + SA + OF_B - L - MO$
71. $M = M_w + M_c$
72. $SA = SA_w + SA_c$
73. $\Delta L = \Delta H_{INV}$
74. $MO = MO_w$
75. $OF_B^{REQ} = CAR(L + MO - NPMO)$
76. $RE_B = OF_B^{REQ} - OF_{B-1}$
77. $OF_B = OF_{B-1} + RE_B$
78. $D_B = F_B - RE_B$

Equations 79 through 82 are the various interest rates set by the commercial bank. In this model the interest rates move with the central bank interest rate r by their respective premiums. All interest rate premiums in the model are fixed.

79. $r_M = r + r_{Mprem}$
80. $r_{SA} = r + r_{SAprem}$
81. $r_L = r + r_{Lprem}$
82. $r_{MO} = r + r_{MOprem}$

Central Bank and Government

In this model government expenditure is exogenous and grows at a constant rate g_r (Equation 83). The change in bills supplied by the government is the total amount of government services provided, G, and the interests paid on the amount of government bills from the previous period minus the public profits received from the central bank and taxes collected from the worker and capitalist household sectors (Equation 84). The interest rate for bills adjusts to the interest rate set by the central bank with an exogenous premium r_{Bprem} (Equation 85).

83. $G = G_{-1} \cdot (1 + g_r)$
84. $\Delta B_s = (G + r_{B-1} \cdot B_{-1}) - (F_{cb} + T_w + T_c)$
85. $r_B = r + r_{Bprem}$

The central bank sets the interest rate according to a simplified Taylor rule (Taylor, 1993) (Equation 86). In this model the general price is assumed to be constant, therefore inflation can be omitted from the Taylor rule. This Taylor rule is determined by an exogenous natural rate of interest, r^*, and the spread between real growth between the current and previous period (Equation 87) and the growth trend (Equation 88). For this model a floor interest rate of 0.25 percent and a ceiling interest rate of 10 percent are imposed. The growth trend is the rate of change of the weighted average growth rate of the economy. Equation 89 is the weighted average level of output of the economy. The total output of the economy is the sum of consumption, government expenditure, houses sold and the change in housing inventory (Equation 90).

86. $r = \min\{\max\{r^* + a_y \cdot (Y^g - \bar{Y}^g), 0.0025\}, 0.1\}$
87. $Y^g = \frac{Y - Y_{-1}}{Y_{-1}}$
88. $\bar{Y}^g = \frac{Y_{AVG} - Y_{AVG-1}}{Y_{AVG-1}}$
89. $Y_{AVG} = \emptyset_1 \cdot Y + \emptyset_2 \cdot Y_{-1}$
90. $Y = C + G + H_s + \Delta H_{INV}$

The central bank acts as the buffer for government bills, acquiring the remaining bills the commercial bank and the capitalist household do not wish to own (Equation 91). The change in cash supplied each period is equal to the change in bills held by the central bank (Equation 92). The central bank's profit is the interest earned on the bills it holds at the end of the previous period (Equation 93). The redundant equation in this model is that cash supplied by the central bank is equal to the cash demanded by the two household sectors (Equation 94).

91. $B_{CB} = B_s - B_B - B_c$
92. $\Delta HPM_s = \Delta B_{CB}$
93. $F_{cb} = r_{B-1} B_{cb-1}$

94. $HPM_S = HPM_w + HPM_c$

Simulations

The simulations are separated into two sections. The first section will review the results from the baseline simulation. The second section will explore the impact of various shocks to the model. The simulations were run using the open source statistical software R and the PK-SFC package for R developed by Antoine Godin.

Baseline Simulation

This is a large model, therefore the analysis of the baseline simulation will focus on the key elements of the model: the firm's decision for adjusting the markup, the worker household sector's realized demand for houses and mortgages, and the commercial bank's profits.

The firm sector produces houses, allocating their production to house sales and to inventories. Over time the firm's expected house sales becomes more accurate, which causes its inventories to decrease, converging to the targeted inventories. This will cause the realized inventory to sales ratio to decline each period and once it falls below the targeted ratio, the firm will increase the price of houses. Figure 5 shows the evolution of the price of houses and the logic operator when the realized inventory to sales ratio is less that the targeted ratio. If the logic operator is 1 in period *t* this means the realized inventory to sales ratio is less than the targeted ratio in period *t-1*. The logic operator is graphed instead of the realized inventories to sales ratio since sales could be near zero resulting in the ratio being quite large.

The firm's profits (Figure 6) drastically increase the same period of the initial increase of the markup. The firm's profits fall the next period due to the decline in the number of houses sold. The reason the firm continues to increase the markup after its profits has already fallen is because the firm is basing this decision upon the previous period sales, where the realized inventory to sales ratio was below the targeted ratio.

Figure 5

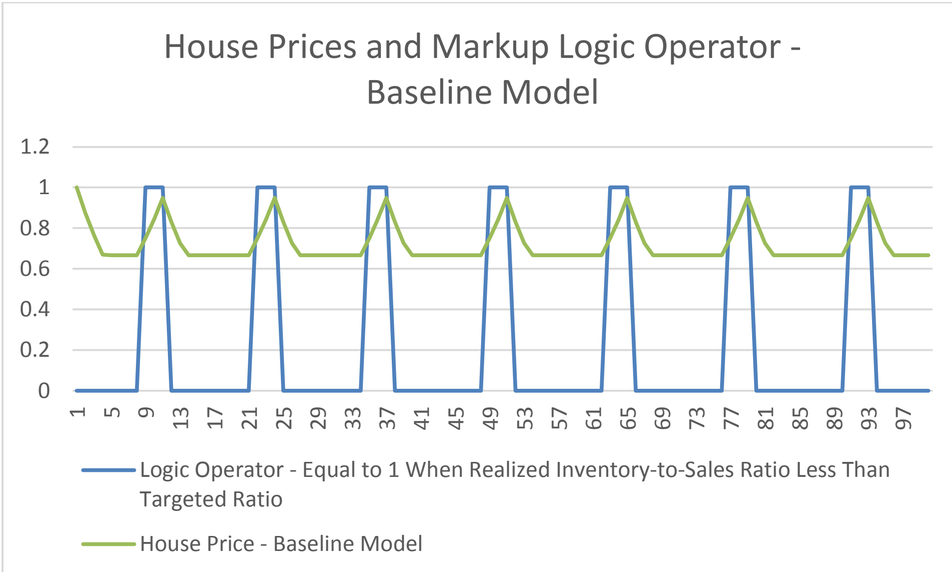
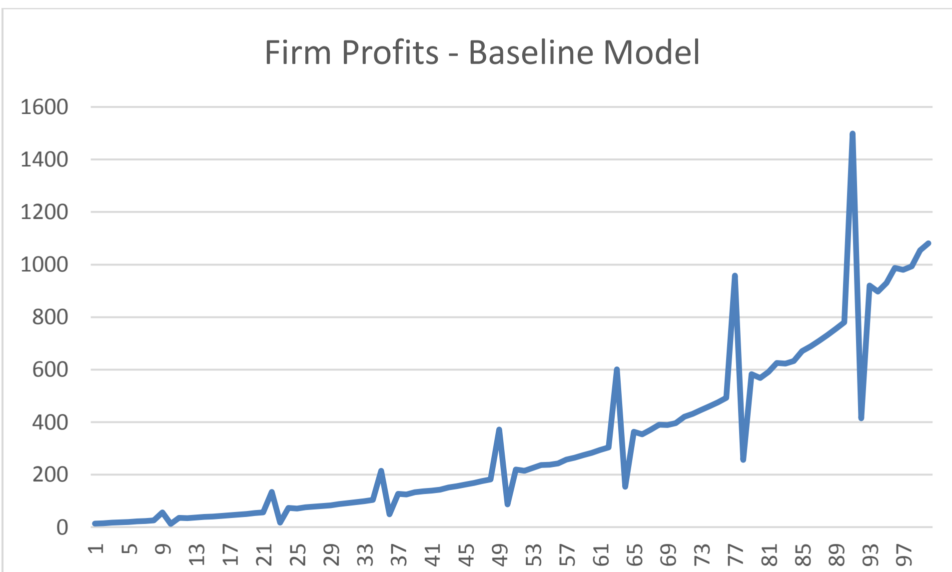
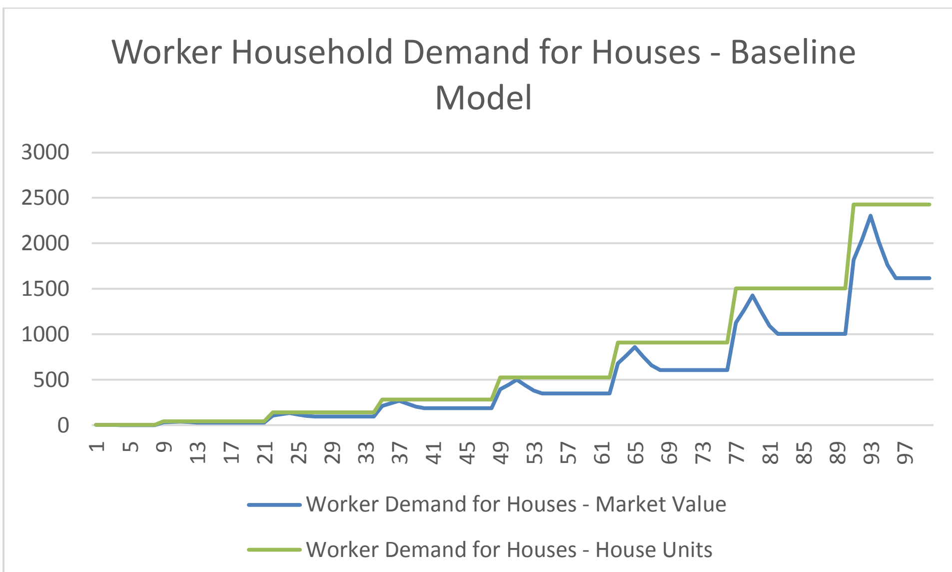


Figure 6



The rise in the price of houses increases its rate of return increasing the demand for houses by the worker household sector. The worker sector's realized demand for houses is shown in Figure 7. This shock in demand keeps the realized inventory-to-sales ratio below the firm's, thus the firm will increase the markup, increasing the price of houses. As this process continues the firm's inventories accelerate to keep up with the rise in demand. However, the firm produces too many houses and its inventory to sales ratio rises above its target. This causes the firm to drop the markup on houses the following period, creating a negative rate of return on houses. The worker household sector reacts to the negative return by no longer demanding houses. This keeps the realized inventory to sales ratio above the firm's target causing the firm to further reduce the markup. The decline continues until the price of the house is equal to its unit cost.

Figure 7



When there is a positive rate of return on houses, the workers demand for mortgages rises (Figure 8) and so does its debt-to-income ratio (Figure 9). The rise of the debt-to-income ratio is due to the worker sector buying more houses. The fall of the workers debt-to-income ratio is due to three reasons. First, because this is a growth model the income of the workers sector will be growing over time. Second, the rise of nonperforming loans (discussed later, Figure 11) increases when the debt-to-income ratio is above 40 percent. Third, when the worker sector buys new houses. The commercial bank's profits (Figure 10) rise when the worker household's demand for mortgages increases.

Figure 8

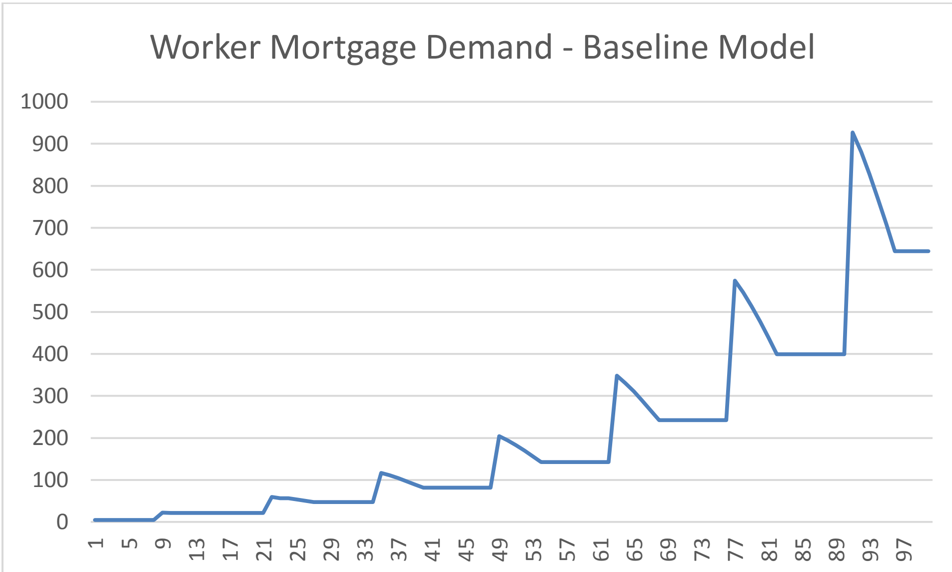


Figure 9

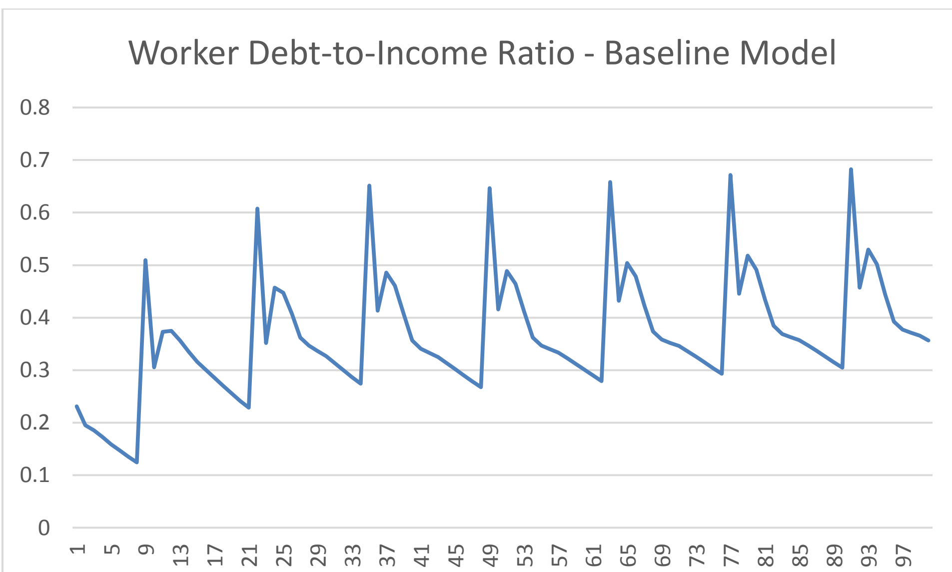
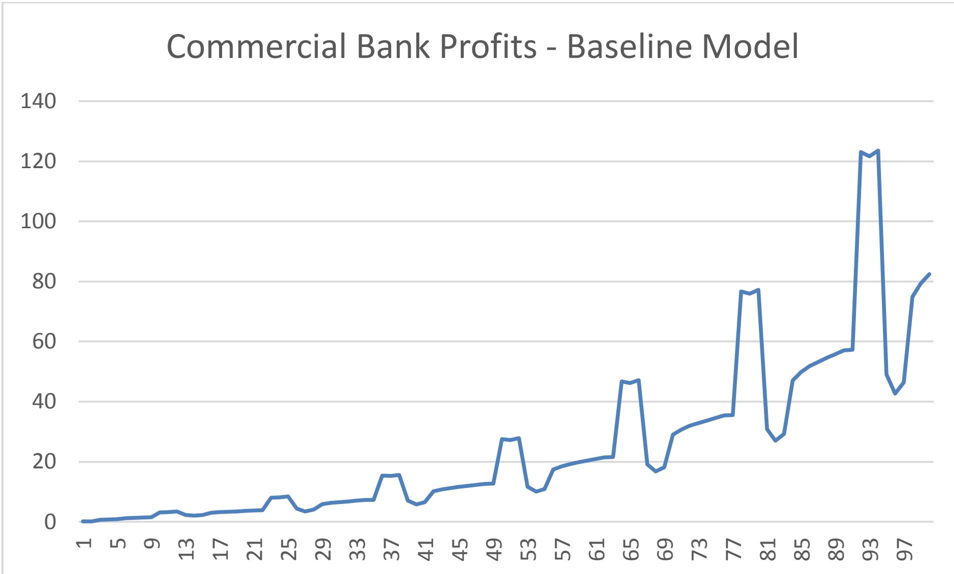


Figure 10



The rise in the debt-to-income ratio triggers an increase in the nonperforming mortgages in the worker household, which is the cause of the significant decrease of the commercial bank's profits after the housing booms. Nonperforming mortgages will increase when the stock of houses increased in the previous period. Typically the default rate will only be the base rate $npmo_{fix}$. However the default rate will rise whenever the workers debt-to-income ratio in the previous period is above 40 percent, which is the source of the large increase of nonperforming loans in Figure 11. The economic cycles of the model are very clear in the economic growth data (Figure 12). One observation from the growth rate is the incredible expansion then contraction. This is attributed to the worker household purchasing a large volume of houses when the firm is raising the prices of houses. The magnitude is certainly not realistic. However, in a model that is focused on only a few markets this is to be expected, especially when modelling a speculative market.

Figure 11

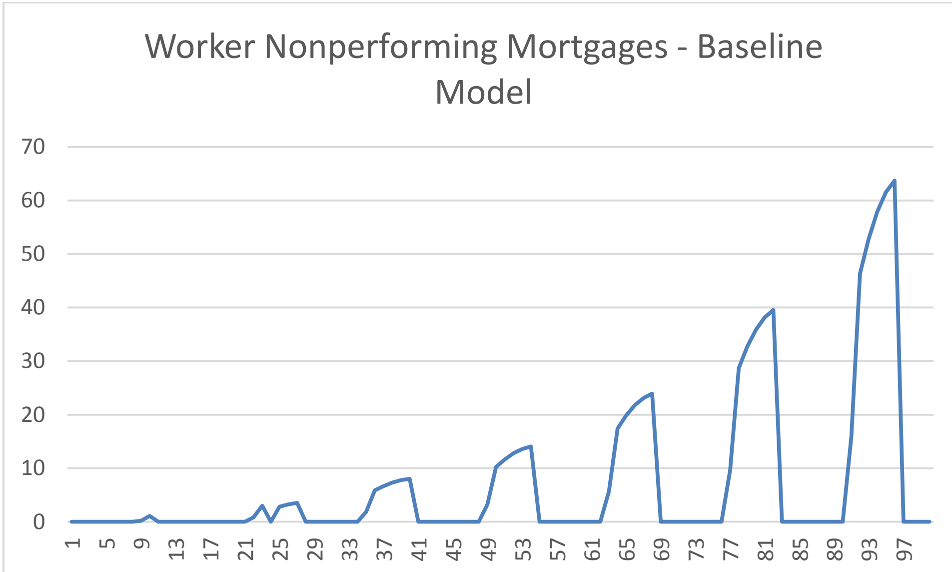
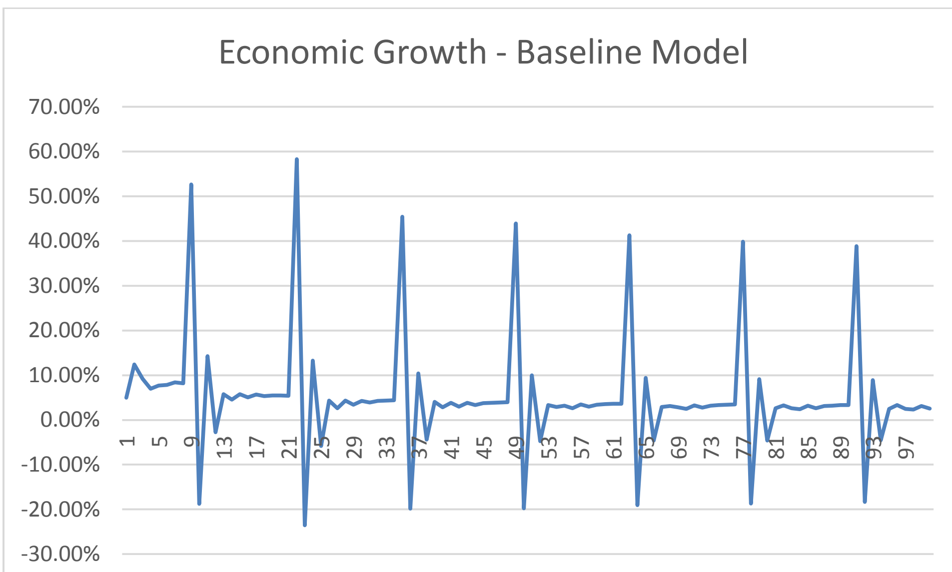


Figure 12



Shocks

This model allows for us to examine the impacts of various shocks. The three shocks tested were:

a) A comparison between an austerity versus fiscal expansion policy responding to an economic contraction.
b) A change of the weights in the expected inventory equation, shifting more weight to the targeted inventory away from the previous period's realized inventory.
c) A decrease of what the central bank perceives to be the natural rate interest.

The reason for choosing these three shocks is to evaluate the impacts coming from three major, but different macroeconomics angles, being fiscal policy, the firm's production process and monetary policy. The shocks for scenarios a) and c) happen in the 55th period of the simulation.

Austerity versus Government Intervention

The first shock to the model is to test the effect of three types of government expenditure policies: laissez-faire, fiscal expansion and austerity policies. The three policies are reacting to an economic contraction. This requires modifications to equation 83 for the fiscal expansion and austerity policies. The laissez-faire scenario has the government not reacting to variations in economic growth. In this scenario the government will implement a fiscal expansion when total output contracts by more than one percent. When this contraction happens, the government will increase its expenditures by $\vartheta \cdot Y_{-1}^{g}$. Otherwise the government will maintain its standard exogenous fiscal growth rate g_r. The fiscal expansion reaction function is equation 83.i) using the logic operator equations 83.ii) and 83.iii).

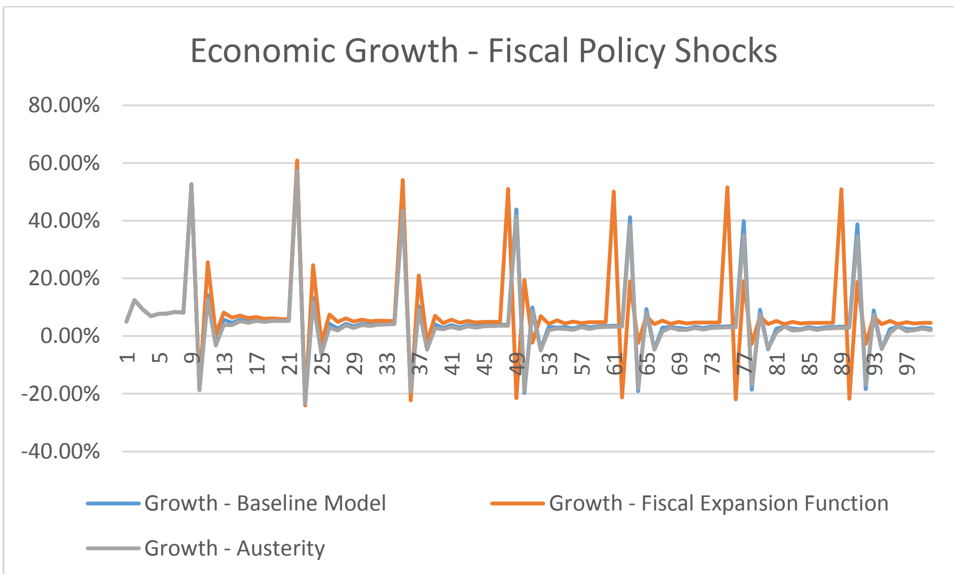
83. i) $G = G_{-1} \cdot \left(1 + M_1 \cdot g_r - \vartheta \cdot M_2 \cdot Y_{-1}^{g}\right)\ ,\ \ \vartheta = 2$
83. ii) $M_1 = 1\ iff\ Y_{-1}^{g} > -1\%$
83. iii) $M_2 = 1\ iff\ Y_{-1}^{g} < -1\%$

The austerity reaction function (Equation 83. iv)) has the government reduce their expenditure by π when economic growth contracts by more than one percent.

83. iv) $G = G_{-1} \cdot (1 + M_1 \cdot g_r - M_2 \cdot \pi)$, $\pi = 0.02$

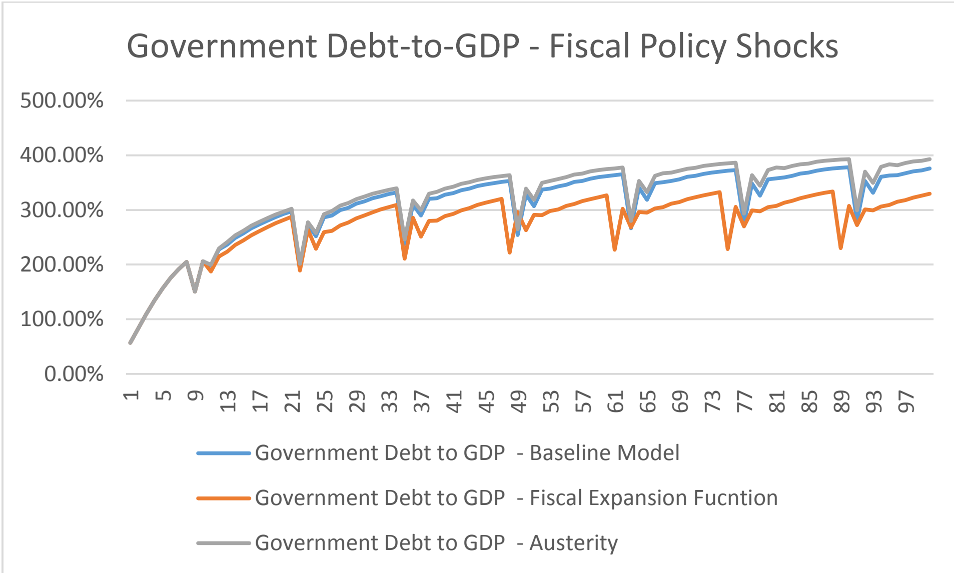
The best approach for comparing the three policies is by independently simulating each policy (Figure 13). Fiscal expansion contributes to higher growth rates and faster recovery, which creates shorter cycles. The austerity function leads to slightly smaller economic growth and has no impact on the frequency of the cycles.

Figure 13



The second observation from these simulations is the government debt-to-GDP ratio seen in Figure 14. Like the growth rates, the value of the government debt-to GDP is its relative comparison. Under austerity the debt-to-GDP ratio is slightly above the laissez-faire scenario. The debt-to-GDP under a fiscal expansion reaction function is much less than both the laissez-faire and austerity policies, which begins immediately after the first contraction. What is most striking is the separation over-time between the government debt-to-GDP under a fiscal expansion policy and the two other policies.

Figure 14



Firm's Expected Inventory Weight Change

The second shock is a change of the firm's behaviour of their expected inventories (Equation 59). In the baseline model γ_F is set to 0.5, so the firm's expected inventories is equally weighted between their targeted inventory-to-sales ratio and the stock of housing inventory from the previous period. The shock is an increase of γ_F to 0.75, shifting more weight to the targeted inventories away from the stock of inventory from the previous period. The weight shift causes the firm's realized inventory-to-sales ratio to drop more frequently below the targeted ratio (Figure 15). The increase in the frequency of the markup cycle is due to the decline of the expected housing inventory, which, after the shock is driven more by the firm's expectations than by realized outcomes. The expected housing inventory lags behind the realized outcome, which, after the shock will reduce the firm's output. This causes the stock of housing inventory to be lower, thus the realized housing inventory to sales ratio remains closer to the targeted ratio. The outcome is the realized ratio moves more frequently back and forth around the targeted ratio. The change of the frequency of the markup appears in the change of the price of houses (Figure 16).

Figure 15

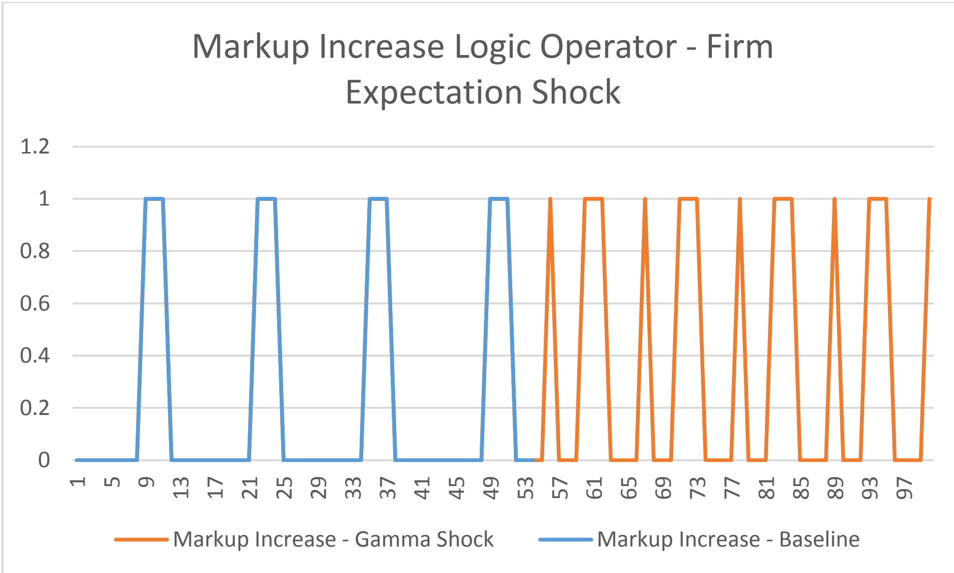
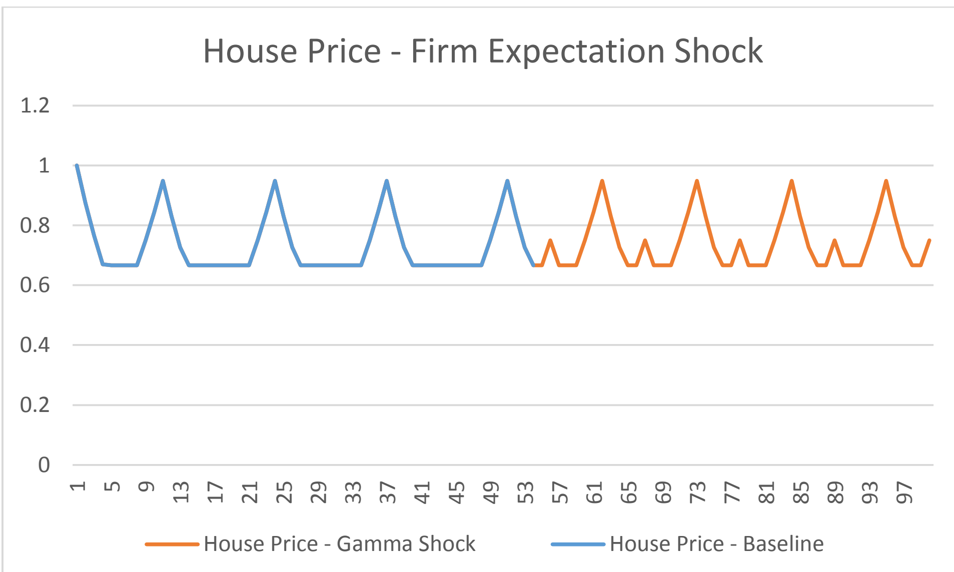


Figure 16



Decrease of the Natural Rate of Interest

The third shock is a decrease of what the central bank perceives to be the natural rate of interest from 5 to 2.5 percent. The decrease will immediately reduce the cost of borrowing for all sectors in the model (Figure 17) which reduces the commercial bank's profits (Figure 18).

Figure 17

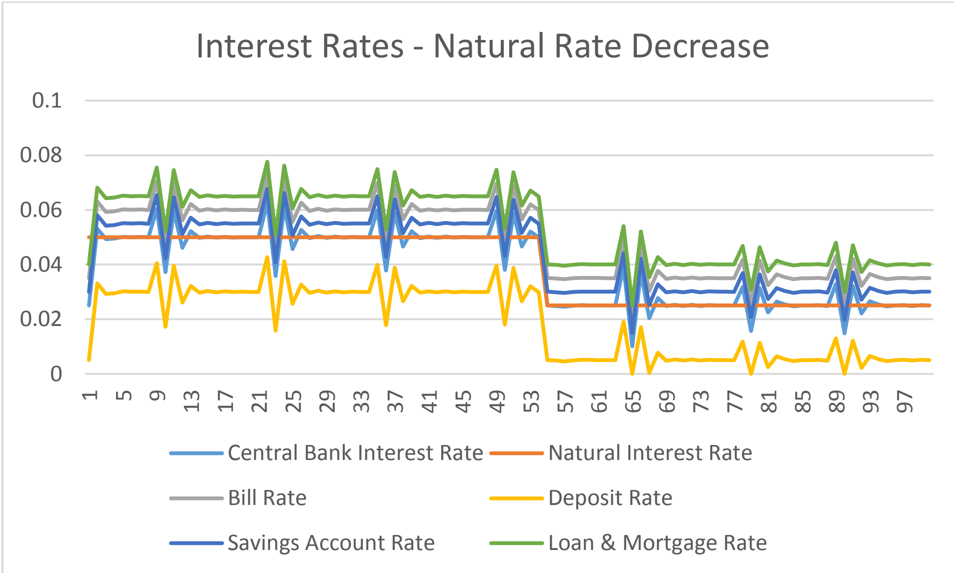
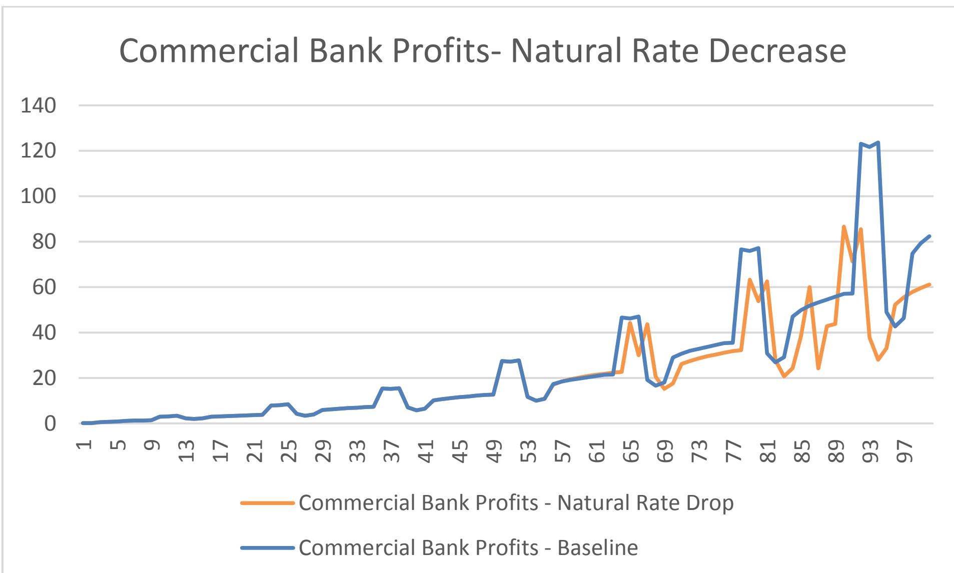


Figure 18



A drop in the interest rate has a short-run positive impact on growth. However, in the long-run growth is smaller than in the baseline model (Figure 19). The period of the growth cycles decreases after the rate decrease. The short-run impact is the real rate of return on houses rises due to the decline of the mortgage interest rate (Figure 20).

Figure 19

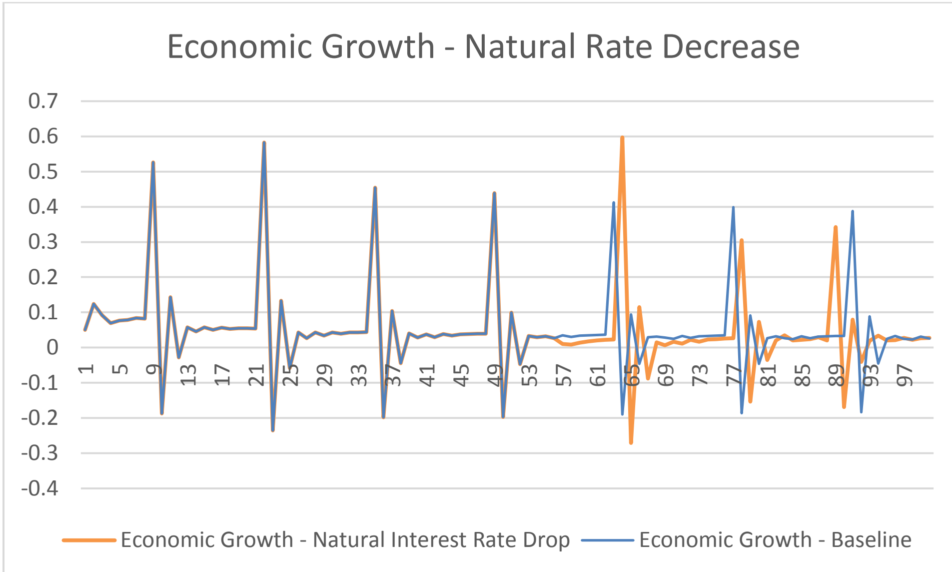
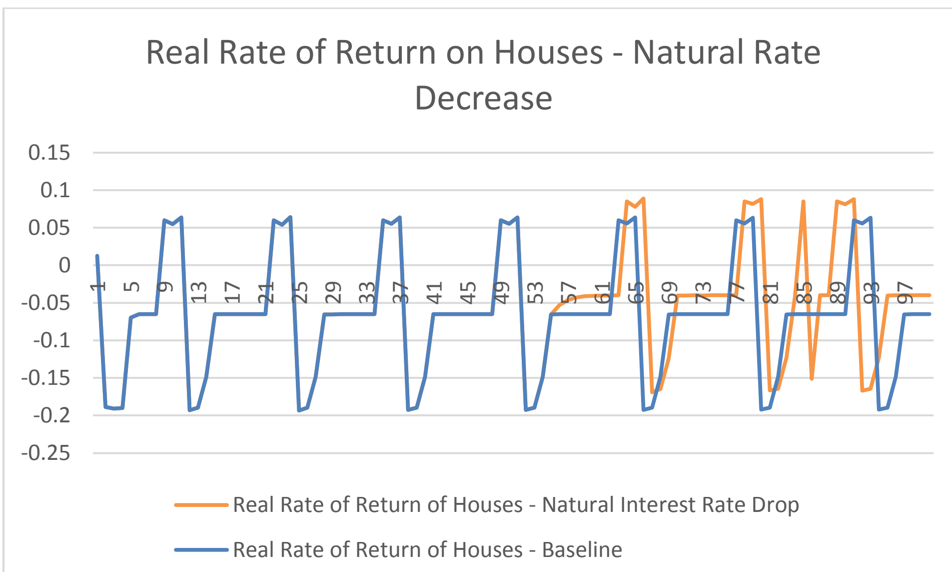


Figure 20



The long-run decline of economic growth is due to the decrease of houses sold (Figure 21). The capitalist sector's stock of houses falls behind the baseline model after the natural interest rate decrease (Figure 22). The worker sector's stock of houses first increases, due to the increase of the real rate of return on houses, but then falls behind the baseline model in terms of the volume of each increase, omitting the difference in timing of the respective increases (Figure 23).

Figure 21

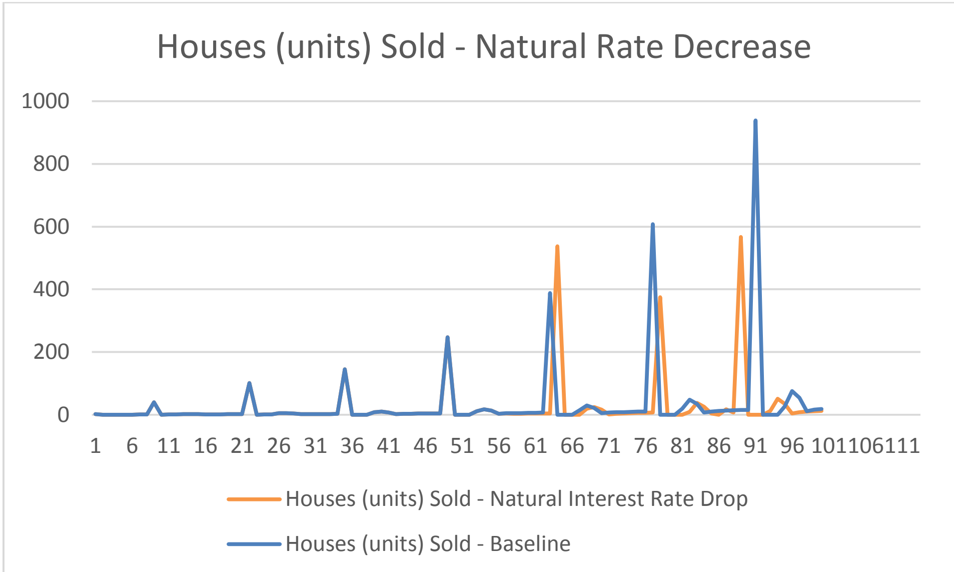


Figure 22

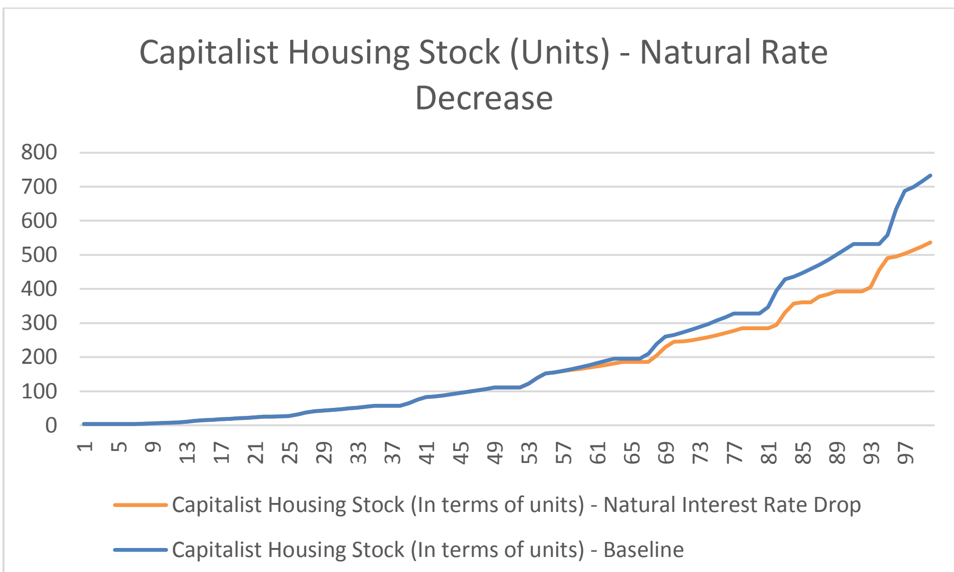
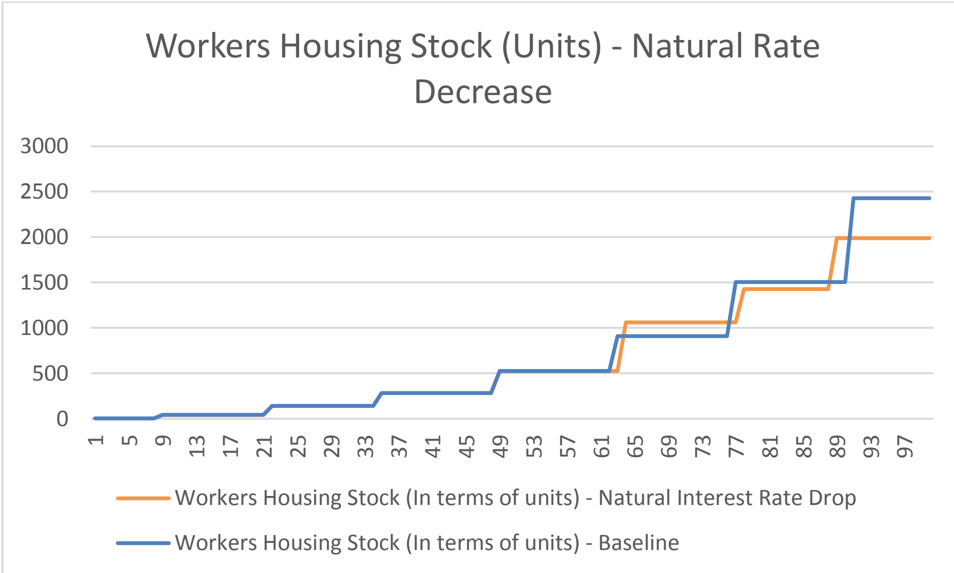
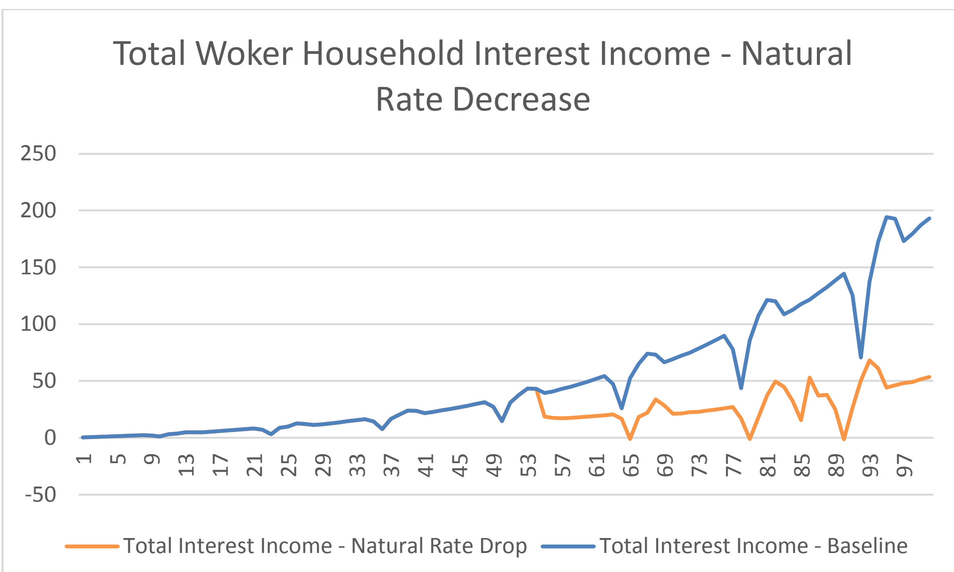


Figure 23



The reason for the relative fall of the worker household sector's stock of houses is the decline of interest income they receive from their deposits and savings accounts (Figure 24). A drop in the central bank's interest rate will lower the interest the worker household's sector earns on its assets thus reducing the amount of wealth accumulated by this sector. The decline in wealth reduces the amount of wealth workers can allocate towards houses.

Figure 24



Conclusion

In conclusion, the SFC model presented in this paper provides a strong theoretical explanation of the dynamics of speculation in the housing market. The simulation of the baseline model produces speculation in the housing market causing periods of high growth followed by large economic contractions. The three shocks to the model provide useful information about economic policies and changes in the dynamics of the housing market. Government intervention is shown to have a positive effect for recoveries from economic contractions and on the government's debt-to-GDP ratio. The firm's greater weighting of targeted inventory of houses in the expected inventory equation increases the frequency of the firm increasing the markup. Finally, a decrease of the central bank's perceived natural rate of interest in the short-run increases the demand for houses during the next period of speculation in the housing market. However, in the long-run, interest based income declines, thus reducing the demand for houses during speculative periods in the housing market. The next step in this research would be to conduct an empirical analysis of the U.S. housing market to verify the firm's price setting behaviour proposed in the model. Now we have a post-Keynesian SFC model with the firm playing an active role in setting the price of houses in a speculative housing market.

Bibliography

Backus, D., Brainard, W. C., Smith, G. and Tobin, J. 1980. 'A model of US financial and nonfinancial economic behaviour', Journal of Money, Credit and Banking, vol. 12, no. 2, 259–93

Caverzasi, E. and Godin, A. 2015. 'Post-Keynesian stock-flow-consistent modelling: a survey', *Cambridge Journal of Economics*, vol. 39, 157 - 187

Copeland, M. A. 1949. 'Social accounting for moneyflows', *Accounting Review*, vol. 24, no. 3, 254–64

Godley, W. and Lavoie, M. 2012 *Monetary Economics: An Integrate Approach to Credit, Money, Income, Production and Wealth, Second Edition*, New York, Palgrave Macmillan

Godley, W. and Zezza, G. 1989. 'A Simple Real Stock-flow Model Illustrated with the Danish Economy', Working Paper no. 8901, Department of Applied Economics, University of Cambridge

Eatwell, J.,Mouakill, T., and Taylor, L. (2008). 'Liquidity, leverage and the impact of sub-prime turbulence' Found in stage.newschool.edu.

Effah, S. 2012. 'Introducing Real Estate Assets and the Risk of Default in a Stock-Flow Consistent Framework', PhD dissertation, University of Ottawa

Fontana, O. and Godin, A. 2013. 'Securitization, Housing Market and Banking Sector Behavior in a Stock-Flow Consistent Model', Kiel Institute for the World Economy, Economics Discussion Papers, No 2013-13

Kenny, G., 1999. 'Modelling the demand and supply sides of the housing market: evidence from Ireland', *Economic Modelling*, vol. 16, 389-409.

Khalil, S. 2011. 'Price Formation, Income Distribution, and the Business Cycle in a Stock-Flow Consistent Monetary Model', PhD dissertation, University of Trento

Lavoie, M. 2014. *Post-Keynesian Economics: New Foundations*, Cheltenham, Edward Elgar Publishing

Miles, D. 1994, *Housing, Financial Markets and the Wider Economy*, New York, John Wiley and Sons.

Taylor, J.B., 1993. ‘Discretion versus policy rules in practice’, Carnegie-Rochester Conference Series on Public Policy, vol. 39, 195-214

Tobin, J. 1969. ‘A General Equilibrium Approach to Monetary Economics’, *Journal of Money, Credit and Banking*, vol. 1, no. 1, 15-29

Zezza, G. 2008. US growth, the housing market, and the distribution of income, Journal of Post Keynesian Economics, vol. 30, no. 3, 375–401

Appendix - Baseline Model Parameter Values & Initial Interest Rates

Capitalist Household Sector

PARAMETER	VALUE
τ_c	0.1
α_1	0.4
α_2	0.1
ω_c	0.1
∂	0.1
$\leftthreetimes_{10}$	0.5
$\leftthreetimes_{11}$	6
$\leftthreetimes_{12}$	6
$\leftthreetimes_{13}$	-0.25
$\leftthreetimes_{20}$	0.5
$\leftthreetimes_{21}$	6
$\leftthreetimes_{22}$	6
$\leftthreetimes_{23}$	0.5
r_b^e	0.05
r_{SA}^e	0.04

Worker Household Sector

PARAMETER	VALUE
τ_w	0.25
γ_1	0.8
γ_2	0.05
ω_w	0.1
ϵ	0.4
$npmo_{fix}$	0.04
$npmo_{var}$	0
$nmpo_{adj}$	0.01
$Debt_{Threshold}$	0.4
ρ_{10}	0.5
ρ_{11}	4
ρ_{12}	4
ρ_{13}	-0.25
ρ_{20}	0.5
ρ_{21}	4
ρ_{22}	4
ρ_{23}	0.25
r_h	0.1
r_{MO}^e	0.0875

The Firm

PARAMETER	VALUE
$pr_{c,g}$	1.67
$W_{c,g}$	1
σ^{T}	0.01
γ_F	0.5
pr_h	1.5
W_h	1
P_h	1
θ_h	1.5
θ_x	0.125

Commercial Bank

PARAMETER	VALUE
CAR	0.1
r_M	0.005
r_{Mprem}	-0.02
r_{SA}	0.03
r_{SAprem}	0.005
r_L	0.04
r_{Lprem}	0.015
r_{MO}	0.04
r_{MOprem}	0.015

Government & Central Bank

PARAMETER	VALUE
g_r	0.03
r_B	0.035
r_{Bprem}	0.01
r	0.025
r^*	0.05
a_y	0.1
$\emptyset_1$	0.75
$\emptyset_2$	0.25