THE EPISTEMOLOGY OF HENRI POINCARE

by

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ABBREVIATIONS

The following abbreviations are utilized to denote the titles of Poincaré's works in the original French editions:

S.M.: Science et Méthode

V.S.: Valeur de la Science

S.H.: Science et Hypothèse

D.P.: Dernières Pensées

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INTRODUCTION

The aim of the proposed research is to discover the epistemological foundations of Henri Poincaré's [1] philosophy. To accomplish this goal it is necessary to uncover these foundations by an investigation of the philosophy in which it is embedded.

The task of correctly ascertaining even the philosophy of science of Poincaré, much less concluding from that view his epistemological position, requires a choice of the matter to be explored. The published works of Poincaré number 438, and this includes both articles and books. Poincaré's monographs on mathematics and his treatises on mathematical physics and celestial mechanics lie outside the scope of non-mathematicians. They are therefore outside the scope of the present thesis which is concerned only with the scientific philosophy as it reflects his epistemological foundations.

Henri Poincaré, a scientist and renowned mathematician, [2] became increasingly preoccupied with the philosophy of science and presented his doctrine on the foundations of science in four volumes:

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1 Henri Poincaré was born April 29, 1854 in Nancy, France. He died on July 17, 1912.

2 Henri Poincaré's most important academic honours include: member of l'Académie des Sciences de Paris on Jan. 31, 1887; honorary doctorates from the Universities of Cambridge, Oxford, Christiania, Kolossvar, Glasgow, Brussels, Stockholm, and Berlin. He was awarded the Bolyai prize in mathematics in 1905, and the supreme honour of l'Académie Française in 1908.

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La science et l'hypothèse, Science et méthode, La valeur de la science, and Dernières pensées. These works form the basis for this philosophical investigation of his doctrine.

The epistemology which underlies Poincaré's philosophy of science still remains to be explored nearly sixty years after his death. The importance of such an investigation is indicated in the first instance by the fact that he enunciated a unique conception of science.

> Il se dégage de la philosophie de Poincaré une conception de la Science qui, à peu de chose près, mérite bien le triomphe définitif. 3

A unique conception of science suggests that it might be founded upon an epistemology different in some respects from other positions. An understanding of the epistemology should contribute to a greater appreciation of the other elements in his scientific philosophy. It should help in identifying the proper role and significance of these elements.

Certainly one of the most striking features of the literature on Poincaré is the lack of consensus on the significance of the various elements present in his philosophy of science. Many have seen in his philosophy of science a series of negations and a devaluation

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3 Jules Sageret, Henri Poincaré, p. 40.

of the truth of science rather than a positive position. Poincaré's nephew, Pierre Boutroux, while not subscribing to this interpretation, has noted the tendency.

> A première vue, on pourrait être tenté de croire que Poincaré a surtout été un négateur. 4

In the eyes of the logical analysts, Poincaré is seen as a bitter opponent. Poincaré sought to limit greatly the role of logic in mathematics and denied the ultimate reduction of mathematics to pure logic.

> Peano's earlier attempt to formulate the apparatus of concepts and axioms of mathematics had already evoked a violent reaction on the part of Poincaré. 5

But does not this opposition to expanding the use of logic require interpretation? Must we not seek an affirmative position which acts as a foundation for the negation?

Realistic interpretations of Poincaré's position are quick to note the absence of objective value in scientific constructions and situate this fact in Kantian influences on Poincaré.

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4 Pierre Boutroux, "L'Oeuvre Philosophique", Henri Poincaré: L'Oeuvre Scientifique, L'Oeuvre Philosophique, p. 258.

5 Leon Chwistek, The Limits of Science, p. 20.

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> Poincaré... was led to deny objective value in the conclusions of science. The two fundamental principles of Poincaré ... are essentially Kantian... 6

Yet Kantian philosophers themselves express disappointment in the restrictions which Poincaré places on the synthetic à priori judgment.

> Imagine-t-on, par exemple, la surprise que réserverait à un disciple de Kant... Tout d'abord il reconnaîtrait la pensée du maître dans celle de M. Poincaré: les vérités arithmétiques doivent leur évidence et leur certitude... puis tout à coup, avec la connaissance géométrique, ce n'est pas seulement sa nécessité que l'on rejette, mais la question même de sa vérité que l'on se refuse à poser. 7

An epistemological study is required to indicate in what sense Poincaré can be considered a Kantian. We need to discover the epistemological and psychological foundations which have caused Poincaré to restrict the Kantian foundations. What is there so distinctive in Poincaré's conception of truth that exact thought is limited to the science of pure quantity? What is the epistemological significance of situating all sciences except arithmetic on a foundation of conventions which are postulated and unable to be verified by experience, and yet cannot

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6 Fulton J. Sheen, Philosophy of Science, Milwaukee, Bruce, 1934, p. 12-13.

7 Gaston Milhaud, "La science et l'hypothèse, par H. Poincaré", in Revue de métaphysique et de morale, 11, 1903, p. 783.

be considered true? What is the epistemological importance of the psychological experience which precedes science and scientific discovery?

The solution of these problems requires an analysis of Poincaré's position which goes beyond his negations and seeks a positive synthesis of his philosophical views. The primary manner of exploration of Poincaré's position must be a detailed textual analysis of his scientific philosophy. The analysis will be directed towards the various areas of human knowledge with the view of establishing the reasons Poincaré considers certain types valid and other types invalid. In the discovery of both truth and certainty in the science of pure quantity, we will seek the positive characteristics which cause this science to be the archetype of knowledge. The direct intuition of number and the complete mathematical induction constitute the most basic and positive value the scientist can attain. The pure mathematical ideal will provide the answers to the nature of human knowing and to the question "What is truth for Poincaré?"

CHAPTER I

THE MEANING OF PHILOSOPHY FOR H. POINCARE

Our concern at this stage in determining what is Poincaré's philosophical position - he is after all a professional mathematician - is to inquire regarding his philosophical training, his knowledge of philosophers, his views of the status of philosophy and metaphysics as disciplines, and finally what role he attaches to philosophy. It is only by understanding what Poincaré understood by philosophy that we will be able to situate his notion of science and isolate his own philosophical position.

A. Poincaré's Philosophical Background

Poincaré was not without any formal training in philosophy. His training in philosophy however was minimal, being confined to the years of his study for the Baccalauréat ès Lettres which he completed Aug. 5, 1871 at the lycée de Nancy. A study of his course results reveals that he followed only one course in philosophy with little success; in fact it was among his lowest marks. [1] Upon completing his Baccalauréat ès Lettres, he enrolled immediately for the Baccalauréat ès Sciences where he was not obliged to continue courses in philosophy. His biographer speaks of their walks together and notes:

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1 He received two points in philosophy out of a possible five. cf. Paul Appell, Henri Poincaré, p. 18.

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> Il nous arrivait quelquefois de philosopher; Poincaré souriait doucement de la psychologie et de la théodicée naives qu'on enseignait alors, en vue de baccalauréat. Je me souviens également de longues conversations sur les raisons scientifiques et philosophiques de croire à l'existence de la vie dans d'autres planètes. 2

We can observe at this point that Poincaré was interested early in his career in scientific philosophy but was not enthused by his formal training in philosophy at the French lycée [3] although he does not exhibit the profound disappointment of Descartes.

A second possible source of philosophical influence on Poincaré is the family relationship which resulted when his sister married Emile Boutroux, the eminent French philosopher. [4] His sister was a close collaborator with her husband, M. Boutroux, but Poincaré,

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2 Paul Appell, Henri Poincaré, p. 23.

3 Poincaré received his formal training in philosophy shortly before the changes in teaching philosophy occurred around 1874 as a result of the changes brought about by the Franco-Prussian War. Canivez reveals that the learning of philosophy in the French lycées prior to 1874 consisted of an enormous amount of memory work involving long and tedious dissertations in both French and Latin. Canivez maintains that intelligence was crushed by a surplus of work. The emphasis was mainly upon logic, psychology, and theodicy. The predominant approach was that of French idealism, with little attention being put upon student participation and the sciences. The latter fact gives us an indication of Poincaré's lack of enthusiasm in the philosophy being taught. Speaking of the professors of philosophy in the French lycées, Canivez notes:

> Leur idéalisme moral les empêche de prêter attention au mouvement réel des sciences et de participer à la naissance des sciences humaines...

André Canivez, Jules Lagneau: Professeur de Philosophie: Essai sur la condition de professeur de philosophie jusqu'à la fin du XIXième siècle, Volume I, Paris, Publication de l'Université de Strasbourg, 1965, p. 269. Cf. also pp. 245-291.

4 Paul Appell, Henri Poincaré, p. 13.

while being graceful in his treatment of Boutroux's position, does not seem to be in agreement with his doctrines. Emile Boutroux's doctoral thesis, which was published in 1874, was entitled "De la Contingence des Lois de la Nature". In his treatise on "The Evolution of Laws" in his Mathematics and Science: Last Essays, Poincaré treats of Boutroux:

> Mr. Boutroux, in his writings on the contingency of the laws of Nature, queried, whether natural laws are not susceptible to change and if the world evolves continuously, whether the laws which govern this evolution are alone exempt from all variation. 5

After having correctly noted the position central to E. Boutroux's thesis and the one responsible for his great fame, namely the contingence of the laws of nature, Poincaré continues:

> Such a concept has no chance of ever being accepted by the scientist; in the sense in which he would understand it, the scientist could not accept it without denying the justness and the very possibility of science. 6

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5 Poincaré, Mathematics and Science: Last Essays, p. 1

- M. Boutroux, dans ses travaux sur la contingence des lois de la nature, s'est demandé si les lois naturelles ne sont pas susceptibles de changer, si alors que le monde évolue continuellement, les lois elles-mêmes, c'est-à-dire les règles suivant lesquelles se fait cette évolution, seront seules exemptes de toute variation. (D.P.: p. 5)

6 Ibidem.

- Une pareille conception n'a aucune chance d'être jamais adoptée par les savants; au sens où ils l'entendraient, ils ne sauraient y adhérer sans nier la légitimité et la possibilité même de la science. (D.P.: p. 5)

What then appears to be the status of such a theory? Poincaré notes:

> But the philosopher reserves the right to ask such a question, to consider the various solutions which it entails, to examine their consequences, and to try to reconcile them to the reasonable demands of the scientist. 7

Poincaré was not philosophically impressed by his formal training in philosophy nor in agreement with the philosophical positions of his family. Let us now investigate Poincaré's view concerning various philosophers and natural philosophy.

B. Poincaré's Critique of Philosophers and Natural Philosophy.

In his books on scientific philosophy, Poincaré does not display any great knowledge of, or reliance upon, the works of philosophers. The references he does make to individual philosophers are not made in such a way as to indicate his dependence upon them. An example of this is the mention Poincaré makes of Aristotle in the work, The Value of Science. In his three references to Aristotle,he does not give indication of more than a general secondary awareness of Aristotle's philosophy. In his first reference, Poincaré gives a mention so general that it carries no philosophical import.

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7 Poincaré, Mathematics and Science: Last Essays, p. 1

- Mais le philosophe conserve le droit de se poser la question, d'envisager les diverses solutions qu'elle comporte, d'en examiner les conséquences, et de chercher à les concilier avec les légitimes exigences des savants. (D.P.: p. 5)

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> Yet to work one must stop, as some Greek, Aristotle or another, has said. 8

He demonstrates a greater knowledge of Aristotle's role in the development of logic in his second reference to Aristotle, yet he does not appear aware of the difference between the role of logic in Aristotle's philosophy, as opposed to that of the mathematical analysts.

> ... I have shown how this reasoning... could lift us from the particular to the general by a procedure I have called Mathematical induction. It is by this procedure that the analysts have made science progress, and if we examine the detail itself of their demonstrations, we shall find it there at each instant beside the classic syllogism of Aristotle. We, therefore, see already that the analysts are not simply makers of syllogisms after the fashion of the scholastics. 9

Poincaré does not appear to be aware of the role of induction in the philosophical doctrine of Aristotle. His reference to Aristotle as "simply a maker of syllogisms" indicates that he identifies Aristotle's use of logic with that of the logical analysts. Such an interpretation

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8 Poincaré, The Value of Science, p. 11.

- Et cependant pour agir il faut s'arrêter... comme a dit je ne sais plus quel grec, Aristote ou un autre. (V.S.: pp. 1 & 2)

9 Ibidem, pp. 23-24.

- ...j'ai montré comment ce raisonnement... pouvait nous élever du particulier au général par un procédé que j'ai appelé l'induction mathématique.

C'est par ce procédé que les analystes ont fait progresser la Science et si l'on examine le détail même de leurs démonstrations, on l'y retrouvera à chaque instant à côté du syllogisme classique d'Aristote.

Nous voyons donc déjà que les analystes ne sont pas simplement des faiseurs de syllogismes à la façon des scolastiques. (V.S.: p. 30)

would not have been possible if Poincaré had had a first-hand acquaintance with Aristotelian philosophy. In his third and final reference to Aristotle, Poincaré asks:

> How was the order of the universe understood by the ancients; for instance Pythagoras, Plato, or Aristotle? It was either an immutable type fixed once for all, or an ideal to which the world sought to approach. 10

Poincaré appears, hence, to have no first-hand knowledge of Aristotelian cosmology. Of equal importance is the fact that Poincaré has no references to the works or philosophy of Thomas Aquinas. Hence in his general references to philosophy and metaphysics we must assume that philosophy for him was other than the so-called Aristotelian-Thomistic synthesis, and that it consisted primarily in the then current continental philosophy of his time: post Cartesian. Of Cartesian philosophy Poincaré observes:

> Descartes used to commiserate the Ionians. Descartes in his turn makes us smile, and no doubt some day our children will laugh at us. 11

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10 Poincaré, The Value of Science, p. 87.

- Comment l'ordre de l'univers était-il compris par les anciens; par exemple par Pythagore, Platon ou Aristote? C'était ou un type immuable fixé une fois pour toutes, ou un idéal dont le monde cherchait à se rapprocher. (V.S.: p. 162)

11 Poincaré, Science and Hypothesis, p. 141.

- Descartes avait pitié des Ioniens; Descartes à son tour nous fait sourire; sans aucun doute nos fils rirons de nous quelque jour. (S.H.: p. 168)

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Let us inquire Poincaré's view of natural philosophy in hopes of concluding what according to him is the object of philosophic inquiry and if according to him philosophy has a proper object. Our instrument in making this inquiry will have to be a textual analysis of his references to philosophy.

What is Poincaré's view of philosophy? He has not written any essay in which the role of philosophy is explicitely treated. It is necessary, hence, to attempt such an investigation by focusing on his references to philosophy and, by analysing them, determine what role is open to the philosopher and what contribution in Poincaré's view he can make.

Poincaré is aware that logic is first of all a branch of philosophy. We have seen that he considers the scholastics to have been mere makers of syllogisms but he recognizes that logic itself is properly a branch of philosophy. When investigating the role of intuition and logic in mathematics he notes:

> We (putting himself rhetorically in the place of logical analysts) believe that in our reasoning we no longer appeal to intuition; the philosophers will tell us this is an illusion... In one sense these philosophers are right; to make arithmetic, as to make geometry, or to make any science, something else than pure logic is necessary... we have no word other than intuition. 12

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12 Poincaré, The Value of Science, p. 19.

- Nous croyons dans nos raisonnements ne plus faire appel à l'intuition; les philosophes nous diront que c'est là une illusion. ... Ces philosophes ont raison dans un sens; pour faire l'arithmétique, comme pour faire la géométrie, ou pour faire une science quelconque, il faut autre chose que la logique pure. ... nous n'avons pour le désigner d'autre mot que celui d'intuition. (V.S.: p. 20)

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Although he does not identify any specific philosophers, the fact is clear that philosophy does encompass the sphere of logic and intuition.

In addressing himself to the question of mathematical pedagogy, Poincaré writes:

> Why is it that, in most cases, the definitions which satisfy scientists mean nothing at all to children? Why is it necessary to give them other definitions? This is the question I have set myself in the chapter which follows, and its solution might, I think suggest useful reflections to philosophers interested in the logic of sciences. 13

Again we find confirmed the conclusion that Poincaré attributes the discipline of logic to the domain of philosophers and also the study of the art of defining and perhaps some role in the study of methods of education. There is also a hint that the role of the philosopher is to reflect upon the conclusions of science. A secondary role of philosophy is continued in his essay on the "Search for Postulates".

> Attempts have been made... to enumerate the axioms and postulates more or less concealed which form the foundation of the different mathematical theories... But when everything has been enumerated, there will be many ways of classifying it all. A good librarian

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13 Poincaré, Science and Method, p. 11.

- Pourquoi les enfants ne comprennent-ils rien le plus souvent aux définitions qui satisfont les savants? Pourquoi faut-il leur en donner d'autres? C'est la question que je me pose dans le chapitre suivant et dont la solution pourrait, je crois, suggérer d'utiles réflexions aux philosophes qui s'occupent de la logique des sciences. (S.M.: p. 3)

> always finds work to do, and each new classification will be instructive for the philosopher. 14

It does not appear that Poincaré identifies the role of the philosopher exactly to the role of the librarian. But the role of the philosopher does appear to be subordinated to and dependent upon the results of the scientist. That the philosopher is rightly concerned with classifications seems linked with the philosopher's possession of the domain of logic, intuition, and definition.

Does not the philosopher have a more fundamental role in investigating the foundations of science? Poincaré's position appears to be in the negative; the philosopher will either draw conclusions unacceptable to the scientist or else tackle insoluble problems.

> Some years ago a certain philosopher said that the future was determined by the past, but not the past by the future... It is obvious that no scientist can accept this conclusion... But what can have been the origin of the philosopher's error? 15

The question is rhectorical for Poincaré does not attempt to isolate the "philosopher's error". But must we not ask if the

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14 Poincaré, Science and Method, p. 45.

- On s'est efforcé ... d'énumérer les axiomes et les postulats plus ou moins dissimulés, qui servent de fondement aux diverses théories mathématiques. ... Mais quand on aura tout énuméré, il y aura bien des manières de tout classer; un bon bibliothécaire trouve toujours à s'occuper, et chaque classification nouvelle sera instructive pour le philosophe. (S.M.: p. 41-42)

15 Ibidem, p. 70.

- Un philosophe a dit il y a quelques années que l'avenir était déterminé par le passé, mais que le passé ne l'était pas par l'avenir. ... Il est clair qu'aucun savant ne peut souscrire à cette conclusion. ... Mais quelle a pu être l'origine de l'erreur de ce philosophe? (S.M.: p. 71)

"philosophic error" is not simply the fact that the philosopher has concluded on a matter so fundamental to be bordering on metaphysics? Support of this theory may be drawn from a second example of philosophic speculation concerning the foundations of philosophy.

> But then who gives us the right of attributing to the principle itself more generality and more precision than to the experiments which have served to demonstrate it? This is asking, if it is legitimate to generalise, as we do, every day, empiric data, and I shall not be so foolhardy as to discuss this question, after so many philosophers have vainly tried to solve it. 16

Poincaré proves himself aware of the basic philosophic controversy regarding the possibility of, and foundations of science. He deliberately avoids entering into the controversy - to do so in his opinion would be "foolhardy". The solutions to these questions need not trouble the scientist. Science does not seek or await the permission of philosophy.

C. Poincaré's Critique of Metaphysics

What then is Poincaré's view of metaphysics? Writing of ether he declares:

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16 Poincaré, Science and Hypothesis, p. 129.

- Mais alors qui nous donne le droit d'attribuer au principe lui-même plus de généralité et plus de précision qu'aux expériences qui ont servi à le démontrer? C'est là demander s'il est légitime, comme on le fait tous les jours, de généraliser les données empiriques, et je n'aurai pas l'outrecuidance de discuter cette question, après que tant de philosophes se sont vainement efforcés de la trancher. (S.H.: pp. 155-156)

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> Whether the ether exists or not matters little - let us leave that to the metaphysicians; what is essential for us is, that everything happens as if it existed, and that this hypothesis is found suitable for the explanation of phenomena. 17

Questions relating to existence are understood by Poincaré to be the concern of metaphysics. But such questions do not hinder the scientist - a suitable hypothesis is sufficient. Poincaré understands further that metaphysics is concerned with the nature of causality.

> The day will perhaps come when physicists will no longer concern themselves with questions which are inaccessible to positive methods, and will leave them to the metaphysicians. That day has not yet come; man does not so easily resign himself to remaining for ever ignorant of the causes of things. 18

Regardless of metaphysical interest in causality and existence, such questions are outside the interest of the scientist and furthermore their contemplation would not be productive for him.

> When we say force is the cause of motion, we are talking metaphysics; and this definition if we had to be content with it, would be absolutely fruitless, would lead to absolutely nothing. 19

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17 Poincaré, Science and Hypothesis, p. 211.

- Peu nous importe que l'éther existe réellement, c'est l'affaire des métaphysiciens; l'essentiel pour nous c'est que tout se passe comme s'il existait et que cette hypothèse est commode pour l'explication des phénomènes. (S.H.: pp. 245-246)

18 Ibidem, p. 223.

- Un jour viendra peut-être où les physiciens se désintéresseront de ces questions, inaccessibles aux méthodes positives, et les abandonneront aux métaphysiciens. Ce jour n'est pas venu; l'homme ne se résigne pas si aisément à ignorer éternellement le fond des choses. (S.H.: p. 258)

19 Ibidem, p. 98.

- Quand on dit que la force est la cause d'un mouvement, on fait de la métaphysique, et cette définition, si on devait s'en contenter, serait absolument stérile. (S.H.: p. 120)

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The facts that metaphysicians occupy themselves with questions of existence and causality, and that scientists would be well advised to leave such questions to the metaphysicians, do not in themselves indicate that the metaphysicians can never solve these questions.

> We must address ourselves to reason; and I do not mean to that of the metaphysicist whose constructs are brilliant but ephemeral like soap bubbles which amuse us for an instant, then burst. 20

Not only does Poincaré not find any possibility for metaphysics to bring us in possession of truth, he does not find it to possess any intellectual or practical ethical value.

> When passions - some noble, some ignominious - quarrel over the possession of our conscience, what weight can the metaphysical distinction between the contingent and the necessary carry before such powerful adversaries? 21

and

> Instinct is stronger than all metaphysical systems; and even though this should be proved, even though the secret of its force should become known, its power would not be weakened as a result. Is gravitation less irresistible since Newton? 22

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20 Poincaré, Mathematics and Science: Last Essays, p. 102.

- C'est à la raison qu'il faut nous adresser, et je ne dis pas à celle du métaphysicien dont les constructions sont brillantes, mais éphémères, comme les bulles de savon dont on s'amuse un instant et qui crèvent. (D.P.: p. 223)

21 Ibidem, p. 112.

- Quand des passions - les unes généreuses, les autres basses, se disputent notre conscience, de quel poids, auprès d'adversaires si puissants peut peser la distinction métaphysique du contingent et du nécessaire? (D.P.: p. 244)

22 Ibidem, p. 112-113.

- ... l'instinct est plus fort que toutes les métaphysiques, et quand même on l'aurait démontré, quand même on connaîtrait le secret de sa force, sa puissance n'en serait pas affaiblie. La gravitation est-elle moins irrésistible depuis Newton? (D.P.: p. 246)

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Thus far Poincaré has been considering metaphysics in relation to man's desire to know causes, in comparison to human instinct, but when he considers metaphysics in itself, isolated from the instinct that gave rise to it, he displays outright opposition and hostility to metaphysics.

> Metaphysics obliges us to conform to a general law of being which it claims to have discovered. It is possible to reply: I prefer to obey my own particular law. I do not know what metaphysics will reply, but I can assure you it will not have the last word. 23

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23 Poincaré, Mathematics and Science: Last Essays, p. 102.

- La métaphysique nous engage à nous conformer à la loi générale de l'être qu'elle prétend avoir découverte; j'aime mieux, pourra-t-on lui répondre, obéir à ma loi particulière; je ne sais pas ce qu'elle répliquera, mais je peux vous assurer qu'elle n'aura pas le dernier mot. (D.P.: p. 226)

SUMMARY

In Poincaré's view of philosophy, we must carefully distinguish between metaphysics and natural philosophy. By metaphysics Poincaré understands the discipline concerned with a general law of being, the discussion of contingent versus necessary, causality, and objective existence. He places no intrinsic value in these questions, appears to deny the possibility of ever drawing any valid conclusions to these questions, cautions scientists to avoid speculating on them, and labels them fruitless for science and powerless for ethics. He does explain interest in causality to a natural human desire to know, but looks to the day when man will cease the search for this type of knowledge. To the sphere of natural philosophy he relegates the notions of logic and intuition, classifications, categorizations, and fundamental principles of science. He appears to limit the natural philosopher's field of investigation to one of reflection upon the work and accomplishments of science.

The role of philosophy for Poincaré is greatly limited. Metaphysics and the possibility of metaphysics have been eliminated. Ethics must be based on feeling and instinct rather than on metaphysics. The philosophy of nature assumes a subservient position content to reflect upon the findings of science. The conclusion must be drawn that according to Poincaré philosophy itself has no proper object and possesses no matter for inquiry distinct from that of science. The theory has been advanced that Poincaré was merely naive in

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philosophical matters:

> Non, Henri Poincaré n'a jamais dégradé la science par plaisir de sceptique: hors ses finesses de géomètre, il paraissait plutôt naïf. 24

This theory has little to support it. Poincaré has proved himself sufficiently aware of philosophical and metaphysical questions. His silence need not be interpreted as naiveté. The possibility must be raised if at issue here is not an epistemological position which renders the former philosophical issues meaningless or insoluble.

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24 Gilbert Maire, "Henri Poincaré, La dépréciation de la Science" in Revue Critique des Idées et des Livres, (?), p. 294.

CHAPTER II

NOTIONS OF CONVENTIONAL SCIENCE

Having examined Poincaré's position on the nature and scope of philosophy, we must now direct our attention to his doctrine on the nature and scope of science. This examination of Poincaré's conception of science is crucial, first in order that we may correctly situate the role of intuition in scientific method, and second to enable us to conclude to the epistemological foundations of his thoughts. The fact that Poincaré has enunciated a unique conception of science is universally recognized.

> Il se dégage de la philosophie de Poincaré une conception de la Science qui, à peu de chose près, nous le pensons du moins, mérite bien le triomphe définitif.

But apart from the recognition of a unique conception of science, attempts at situating Poincaré's doctrine of conventionalism have failed to result in universal agreement.

> Of the scientific philosophy of Poincaré his eulogists give but passing notice. Philosophers, however, have not been so circumspect. Nominalism, conventionalism, idealism, realism, even solipsism, and nihilism were among the epithets hurled at him... 2

This lack of consensus regarding the doctrines proper to Poincaré himself has been increased by the fact that frequently

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1 Jules Sageret, Henri Poincaré, p. 40.

2 Tobias Dantzig, Henri Poincaré: Critic of Crisis, p. 10.

studies of Poincaré's position have been directed towards the influence of Poincaré in the development of other positions. The effect of these comparison studies has at times been to emphasize those particular elements of Poincaré's doctrine which may have been an influence on others.

> ... it is not our purpose to defend the doctrines of logical positivism in this dissertation. We wish simply to show that Poincaré must be counted among the principal precursors of the movement. 3

It is not surprising, hence, that discussion regarding Poincaré is frequently debated on the level of either the historical origin of this thought or the historical effect of this thought.

> Poincaré's work in the philosophy of science was in the tradition of Mach and Hertz, and he admitted a debt to Kant. ... Einstein had a profound respect for his work in both mathematics and the philosophy of science. He is often claimed as an ancestor of logical positivism, although the justification is not always easy to see. 4

Valuable as such comparisons undoubtedly are, our present task is but to attempt as complete and accurate a statement of his conception of the nature of science as possible. The primary source of this synthesis must of necessity be the works of Poincaré themselves.

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3 Roger B. Angel, Henri Poincaré's Theory of Conventionalism, p. 146.

4 Peter Alexander, "Jules Henri Poincaré" in Encyclopedia of Philosophy, Vol. 6, p. 361.

NOTIONS OF CONVENTIONAL SCIENCE

Insofar as possible we shall endeavour to state Poincaré's doctrine of the nature of science without citing purely scientific examples. Our procedure in elucidating Poincaré's doctrine will be to classify it under three main headings: "The Objects of Science", "The Constituents of Science", and "The Method of Science".

A. The Objects of Science

1. The Subject Matter of Science

In approaching the problem of the object of science according to Poincaré, we are seeking an understanding of what is the actual subject matter of science. It is of signal importance to ascertain what is, according to Poincaré, the reality which is the concern of science.

Having already noted the impossibility of identifying a proper object to the field of philosophy, our first concern is to discover if the domain of science is in any sense limited. We discover that the potential subject matter of science is, according to Poincaré, without limits:

> There is still another way to conceive of the relations of science with morality. There is no phenomenon which cannot be the object of science since there is none which cannot be observed. Moral phenomena do not escape this any more than the others. 5

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5 Poincaré, Mathematics and Science: Last Essays, p. 110.

- Il y a encore une autre manière de concevoir les rapports de la science et de la morale; il n'est aucun phénomène qui ne puisse être objet de science, puisqu'il n'en est aucun qui ne puisse être observé. Les phénomènes moraux n'y échappent pas plus que les autres. (D.P.: pp. 241-2)

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Science, hence, is capable of taking unto itself any phenomenon which it chooses. This is so because the scientist observes the phenomenon. Moreover, Poincaré states that all phenomena are capable of being observed. The case might be raised that phenomenon is not the same as reality and that philosophy has as its object the study of reality under the aspect of primary causes. If this distinction could be made, it would be possible to conclude that science has for its object the study of all phenomena while philosophy, the nature of reality. Such an interpretation, however, is not permitted according to Poincaré:

> ... not only can science not teach us the nature of things; but nothing is capable of teaching it to us and if any god knew it, he would not find words to express it. Not only can we not devine the response, but if it were given to us, we could understand nothing of it; I ask myself even whether we really understand the question. 6

We are forced now to two conclusions. Whatever the status of philosophy is, it does not reveal the true nature of things. But not only does philosophy not do this but neither does science. As we have seen, science is capable of having any phenomenon as its object. The

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6 Poincaré, The Value of Science, p. 138.

- ... non seulement la science ne peut faire connaître la nature des choses; mais rien n'est capable de nous la faire connaître et si quelque dieu la connaissait, il ne pourrait trouver de mots pour l'exprimer. Non seulement nous ne pouvons deviner la réponse, mais si on nous la donnait, nous n'y pourrions rien comprendre; je me demande même si nous comprenons même la question. (V.S.: pp. 266-267)

explanation of these phenomena which science proposes is not, hence, an explanation of nature.

> The object of mathematical theories is not to reveal to us the real nature of things; that would be an unreasonable claim. 7

and

> Trying to make science contain nature is like trying to make the part contain the whole. 8

Philosophy does not explain nature. Science does not explain nature. Nature itself is not explained by man and man is incapable of having nature explained to him. We must ask further if any relationship can be established between science and nature at all. Poincaré also rejects this possible alternative:

> For here the mind may affirm because it lays down its own laws. But let us clearly understand that while these laws are imposed on our science, which otherwise could not exist, they are not imposed on nature. 9

We do not know nature; science does not study nature and the scientist does not impose scientific laws on nature. Hence we are unable to discover in Poincaré's doctrine any real relationship between science

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7 Poincaré, Science and Hypothesis, p. 211.

- Les théories mathématiques n'ont pas pour objet de nous révèler la véritable nature des choses; ce serait là une prétention déraisonnable. (S.H.: p. 245)

8 Poincaré, Science and Method, p. 16.

- Vouloir faire tenir la nature dans la science, ce serait vouloir faire entrer le tout dans la partie. (S.M.: p. 8)

9 Poincaré, Science and Hypothesis, p. xxiii.

- Là, notre esprit peut affirmer parce qu'il décrète; mais entendons-nous: ces décrets s'imposent à notre science, qui, sans eux, serait impossible; ils ne s'imposent pas à la nature. (S.H.: p. 3)

and nature. Is it possible that nature may have suggested to us, as in the background, some of the basic areas of scientific concern? Poincaré assures us that all of science is of human construction:

> Do we find it in nature, or have we ourselves introduced it? And if the latter, are we not running the risk of coming to incorrect conclusions all-round? Comparing the rough data of our senses with that extremely complex and subtle conception which mathematicians call magnitude, we are compelled to recognize a divergence. The framework into which we wish to make everything fit is one of our own construction... 10

We are forced now to conclude that no relationship appears to exist between nature in itself and the constructions of science. But Poincaré frequently speaks of "reality". Since science has no relationship with nature itself it must be that the "reality" with which science is concerned is not to be attributed to nature. What then are the characteristics of this "reality" and where is it found?

> ... we can not take such a poor account of reality, and I do not mean merely the reality of the sensible world, which has its value nevertheless, since it is for battling with it that nine-tenths of our pupils are asking for arms. There is a more subtle reality which constitutes the life of mathematical entities... 11

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10 Poincaré, Science and Hypothesis, p. xxv.

- La trouvons-nous dans la nature, ou est-ce nous qui l'y introduisons? Et, dans ce dernier cas, ne risquons-nous pas de tout fausser? Comparant les données brutes de nos sens et ce concept extrêmement complexe et subtile que les mathématiciens appellent grandeur, nous sommes bien forcés de reconnaître une divergence; ce cadre où nous voulons tout faire rentrer, c'est donc nous qui l'avons fait...(S.H.: pp. 4-5)

11 Poincaré, Science and Method, p. 126.

- ... nous ne pouvons faire aussi bon marché de la réalité, et je n'entends pas seulement la réalité du monde sensible, qui a pourtant son prix, puisque c'est pour lutter contre elle que les neuf dixièmes de vos élèves vous demandent des armes. Il y a une réalité plus subtile, qui fait la vie des être mathématiques... (S.M.: p. 133)

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The "reality" with which Poincaré is concerned is more subtle than the reality of the sensible world. What is the relationship between the sensible world of things and the science of this more subtle reality which is of such primary concern? Poincaré has reminded his readers that the existence of science is a fact and that we see the accomplishments of science daily. [12] The problem is to explain how this is possible:

> This could not be unless it taught us something about reality; the aim of science is not things themselves, as the dogmatists in their simplicity imagine, but the relations between things; outside those relations there is no reality knowable. 13

Poincaré has stated that the only reality which is knowable is in the domain of relation and this reality is in the order of the life of mathematical entities. If the relationships between things are all that is knowable, it must be that we do not know the things themselves. But the thing itself is given in sensation. We must inquire if the validity of sensation itself is in question.

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12 Poincaré, Science and Hypothesis, p. xxiv.

- la science... Or, nous la voyons chaque jour agir sous nos yeux. (S.H.: p. 4)

13 Ibidem.

- Cela ne pourrait être si elle ne nous faisait connaître quelque chose de la réalité; mais ce qu'elle peut atteindre, ce ne sont pas les choses elles-mêmes, comme le pensent les dogmatistes naïfs, ce sont seulement les rapports entre les choses; en dehors de ces rapports, il n'y a pas de réalité connaissable. (S.H.: p. 4)

> Sensations are therefore intransmissible, or rather all that is pure quality in them is intransmissible and forever impenetrable. But it is not the same with relations between these sensations. 14

Indeed the validity of sensation is called into question. Rather that which is valid in sensation is limited to relation. All data given by sensation other than relation are either invalid in themselves or at least invalid for any scientific purposes. Poincaré avoids a direct pronouncement on the former but there can be no question of the latter.

> From this point of view, all that is objective is devoid of all quality and is only pure relation. Certes, I shall not go so far as to say that objectivity is only pure quantity (this would be to particularize too far the nature of the relations in question) but we understand how someone could have been carried away into saying that the world is only a differential equation. 15

Poincaré was well aware of the criticisms directed at his doctrine for the alleged demeaning of the importance of the world of sensation.

> Some people will be astonished at such a result. The exterior world, they think must surely count for something. 16

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14 Poincaré, The Value of Science, p. 136.

- Les sensations sont donc intransmissibles, ou plutôt tout ce qui est qualité pure en elles est intransmissible et à jamais impénétrable. Mais il n'en est pas de même des relations entre ces sensations. (V.S.: p. 263)

15 Ibidem.

- A ce point de vue, tout ce qui est objectif est dépourvu de toute qualité et n'est que relation pure. Je n'irai certes pas jusqu'à dire que l'objectivité ne soit que quantité pure (ce serait trop particulariser la nature des relations en question), mais on comprend que je ne sais plus qui se soit laissé entraîner à dire que le monde n'est qu'une équation différentielle. (V.S.: p. 263)

16 Poincaré, Science and Method, p. 113.

- Quelques personnes s'étonneront d'un pareil résultat. Le monde extérieur, penseront-elles, doit bien y être pour quelque chose. (S.M.: p. 118)

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But Poincaré did not consider that he was demeaning the external world. He noted:

> After all, have we any other reason for believing in the existence of material objects? That, too, is only a convenient hypothesis; only, it will never cease to be so... 17

Poincaré remained convinced that relation alone was knowable:

> With due reserve regarding this paradoxical proposition, we must nevertheless admit that nothing is objective which is not transmissible, and consequently that the relations between the sensations can alone have an objective value. 18

We are in a position now to consider the limitations which are imposed on science. These limitations are not caused by the method proper to science but rather by the limits of objectivity proper to human knowledge itself. Within the sphere of human knowledge science has no limits as Poincaré has stated: "There is no phenomenon which cannot be the object of science..." [19] But science is limited to the relations between sensations. As Charles Nordmann concluded in his eulogy on Poincaré's work on the occasion of Poincaré's death

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17 Poincaré, Science and Hypothesis, p. 212.

- Après tout, avons-nous d'autre raison de croire à l'existence des objets matériels? Ce n'est là aussi qu'une hypothèse commode; seulement elle ne cessera jamais de l'être... (S.H.: p. 246)

18 Poincaré, The Value of Science, p. 136.

- Tout en faisant des réserves sur cette proposition paradoxale, nous devons néanmoins admettre que rien n'est objectif qui ne soit transmissible, et par conséquent que les relations entre les sensations peuvent seules avoir une valeur objective. (V.S. pp. 263-4)

19 Poincaré, Mathematics and Science: Last Essays, p. 110.

- Il n'est aucun phénomène qui ne puisse être objet de science... (D.P. pp. 242-3)

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> Science has its limits. It can know only the relative; but in that it is supreme. As to wishing to penetrate into what is called the absolute - the "things in themselves" - these questions are not only insoluble but illusory and devoid of sense. 20

Poincaré has stated that there is no phenomenon which cannot be observed. [21] But that which is observed is observed by means of sensation. In sensation more is revealed than mere relation. Might it not be contended that there is a greater measure of objectivity in the sensation than in the relation which it reveals? Poincaré maintains on the contrary that there is rather a greater measure of objectivity to science than to the phenomenon given in sensation:

> And the scientific syntheses have in a sense even more reality than those of ordinary senses, since they embrace more terms and tend to absorb in them the partial syntheses. 22

The scientific synthesis is, according to Poincaré, every much as real as the phenomenon which is supplied by sense experience. Are we not forced to conclude that the phenomenon which is dependent upon sensation which is in its turn dependent upon the external world, while it is observed, is not really known? If, as we have seen,

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20 Charles Nordmann, "Henri Poincaré: His Scientific Work; His Philosophy", in Annual Report Smithsonian Institution, 1912, p. 763.

21 Poincaré, Mathematics and Science: Last Essays, p. 110.

- ... puisqu'il n'en est aucun qui ne puisse être observé. (D.P.: p. 242)

22 Poincaré, The Value of Science, p. 140.

- Et même les synthèses scientifiques ont en un sens plus de réalité que celles du sens commun, puisqu'elles embrassent plus de termes et tendent à absorber en elles les synthèses partielles. (V.S.: p. 270)

relation alone is knowable, then is it not the case that the phenomenon itself is not known? Poincaré states that sensations give us a "feeling" regarding the external world but that science enables us to see "between phenomena" and the two are of fundamentally different types.

> ... our belief in external objects. These latter are real in this, that the sensations they make us feel appear to us as united to each other by I know not what indestructible cement and not by the hazard of a day. In the same way science reveals to us between phenomena other bonds finer but less solid; these are threads so slender that they long remained unperceived but once noticed there remains no way of not seeing them; they are therefore not less real than those which give their reality to external objects; small matter that they are more recently known since neither can perish before the other. 23

It is necessary to recognize that when Poincaré states that there is no phenomenon which cannot be the object for science, it is not the phenomenon itself which science attempts to know.

> When, therefore, a scientific theory pretends to teach us what heat is, or what electricity, or life is, it is condemned beforehand. All it can give us is only a crude image. It is therefore provisional and crumbling. 24

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23 Poincaré, The Value of Science, p. 139.

- ... notre croyance aux objets extérieurs. Ces derniers sont réels en ce que les sensations qu'ils nous font éprouver nous apparaissent comme unies entre elles par je ne sais quel ciment indestructible et non par un hasard d'un jour. De même la science nous révèle entre les phénomènes d'autres liens plus ténus mais moins solides; ce sont des fils si déliés qu'ils sont restés longtemps inaperçus mais dès qu'on les a remarqués, il n'y a plus moyen de ne pas les voir; ils ne sont donc pas moins réels que ceux qui donnent leur réalité aux objets extérieurs; peu importe qu'ils soient plus récemment connus puisque les uns ne doivent pas périr avant les autres. (V.S.: p. 270)

24 Ibidem, p. 138.

Quand donc une théorie scientifique prétend nous apprendre ce qu'est la chaleur, ou ce qu'est l'électricité ou la vie, elle est condamnée d'avance; tout ce qu'elle peut nous donner, ce n'est qu'une image grossière. Elle est donc provisoire et caduque. (V.S.: p. 267)

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The relation between phenomena which science attempts to explain and predict is neither hence the phenomenon itself as understood by the common man nor even the common man's understanding of the relations between phenomena. The scientist is justified in making any relations between phenomena the object of scientific synthesis if he can but convince others of the relationship. Speaking of the special character of the relations proper to science Poincaré notes:

> It is clear that they are not the same for the scientist and the ignorant person. But that is unimportant, because if the ignorant person does not see them all at once, the scientist may succeed in making him see them by a series of experiments and reasonings. The thing essential is that there are points on which all those acquainted with the experiments made can reach accord. 25

The foregoing discussion enables us to appreciate Poincaré's definition of science.

> Now what is science? I have explained in the preceeding article, it is before all a classification; a manner of bringing together facts which appearances separate, though they were bound together by some natural and hidden kinship. Science,

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25 Poincaré, The Value of Science, p. 138.

- Il est clair qu'ils ne sont pas les mêmes pour le savant et pour l'ignorant. Mais peu importe, car si l'ignorant ne les voit pas tout de suite, le savant peut arriver à les lui faire voir par une série d'expériences et de raisonnements. L'essentiel est qu'il y a des points sur lesquels tous ceux qui sont au courant des expériences faites peuvent se mettre d'accord. (V.S.: pp. 267-268)

> in other words, is a system of relations. Now we have just said, it is in the relations alone that objectivity must be sought; it would be vain to seek it in beings considered as isolated from one another. 26

2. The Goals of Science

Now that we have seen that the definition of science is according to Poincaré, let us inquire further and attempt a penetration of the purpose of science. Science, in order to be of value and to be worth the long, and arduous effort of the scientist, must serve some important function. It was common for the pragmatists to argue from the practical results of science to its value. In arguing in this manner the pragmatists, while being faithful to their criterion of truth, were basing the value of science on a foundation other than reason. It is at this point that there was an essential divergence between Poincaré and the pragmatists.

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26 Poincaré, The Value of Science, p. 137.

- Maintenant qu'est-ce que la science? J'ai expliqué dans l'article précédent, que c'est avant tout une classification, une façon de rapprocher des faits que les apparences séparaient, bien qu'ils fussent liés par quelque parenté naturelle et cachée. La science, en d'autres termes, est un système de relations. Or nous venons de le dire, c'est dans les relations seulement que l'objectivité doit être cherchée; il serait vain de la chercher dans les êtres considérés comme isolés les uns des autres. (V.S.: pp. 265-266)

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> The aim of pragmatism, whence its name, is action, practical service, and if science has a value it is as a means of action and because it furnishes us with practical and useful rules. To Poincaré, on the other hand, it is knowledge which is the end of action. 27

Poincaré in no way shares the pragmatists' distrust and lack of appreciation for theory. He was aware of a general movement against theory in favour of action but he believed that those who held these views had misunderstood the goal of science.

> At that time men valued only the discoveries that can be made without leaving home. Today it is theory rather that is held in low esteem. But this implies a misconception of the aim of science. 28

Tolstoi had given his reasons why the concept of science for its own sake was unacceptable. According to Tolstoi the scientist had to be guided by something in his choice of areas of investigation and this guide according to him should be utility. Poincaré is aware that the term "utility" has a very special meaning for Tolstoi which does not correspond at all with that of

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27 Charles Nordmann, "Henri Poincaré: His Scientific Work; His Philosophy", Op. Cit., p. 761.

28 Poincaré, Science and Method, p. 275.

- Alors, on n'estimait que les découvertes que l'on peut faire sans sortir de chez soi. Aujourd'hui, ce serait plutôt de la théorie qu'on ferait peu de cas. C'est là méconnaître le but de la science. (S.M.: p. 297)

common usage. Poincaré recognized that "utility" for Tolstoi consisted solely in what could make man better. But Poincaré was unable to accept the criterion of utility even in the sense of Tolstoi as the goal of science.

> The search for truth should be the goal of our activities; it is the sole and worthy of them. Doubtless we should first bend our efforts to assuage human suffering, but why? Not to suffer is a negative ideal more surely attained by the annihilation of the world. If we wish more and more to free man from material cares, it is that he may be able to employ the liberty obtained in the study and contemplation of truth. 29

Science exists, according to Poincaré, for its own sake. The scientist is not directed to preoccupations of usefulness but for the delight he receives in scientific theories.

> On the contrary, intellectual beauty is sufficient unto itself, and it is for its sake, more perhaps than for the future good of humanity, that the scientist devotes himself to long and difficult labors. 30

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29 Poincaré, The Value of Science, p. 11.

- La recherche de la vérité doit être le but de notre activité; c'est la seule fin qui soit digne d'elle. Sans doute nous devons nous efforcer de soulager les souffrances humaines, mais pourquoi? Ne pas souffrir, c'est un idéal négatif et qui serait plus sûrement atteint par l'anéantissement du monde. Si nous voulons de plus en plus affranchir l'homme des soucis matériels, c'est pour qu'il puisse employer sa liberté reconquise à l'étude et à la contemplation de la vérité. (V.S.: p. 1)

30 Ibidem, p. 8. (Author's preface to English translation)

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The purpose which science fulfills is then chiefly a mental function. Science provides us with intellectual beauty and answers above all a mental need.

> Briefly stated, the sentiment of mathematical elegance is nothing but the satisfaction due to some conformity between the solution we wish to discover and the necessities of our mind... 31

But while Poincaré disagrees with Tolstoi that the scientist should be motivated by the criterion of utility - even utility in the moral sense - Poincaré does not dismiss the possibility that the possession of intellectual beauty will not have a beneficial moral effect on the scientist. He maintains rather that intellectual beauty does have the effect of making for a more strong and sure intelligence.

> But this disinterested quest of the true for its own beauty is sane also and able to make man better. 32

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31 Poincaré, Science and Method, p. 31.

- En un mot, le sentiment de l'élégance mathématique n'est autre chose que la satisfaction due à je ne sais quelle adaptation entre la solution que l'on vient de découvrir et les besoins de notre esprit. (S.M.: p. 26)

32 Poincaré, The Value of Science, p. 9. (Author's preface to English translation)

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B. The Constituents of Science

In investigating the constituents of science, we are trying to understand the way in which the essential elements of science are so combined as to produce science. Having seen what, according to Poincaré, science is, our present inquiry will attempt to answer the question, "Of what is science composed?" To complete the investigation of Poincaré's general notions of science we will later have to investigate the way in which the scientist makes his constructions. Returning to the investigation of the constituents of science, Poincaré holds that science is essentially composed of theory and experience:

> Isolated, theory is empty and experience blind; and both are useless and of no interest alone. 33

It is important, consequently, for us to study the roles of both theory and experience. However for the purposes of our investigation it is useful to take account of two types of theories: foundation theories which are presupposed, and hypotheses which are consciously created and subject to experimentation. Therefore, the present study will be divided into three distinct areas of

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33 Poincaré, The Value of Science, p. 275.

- Isolées, la théorie serait vide, l'expérience serait myope; toutes deux seraient inutiles et sans intérêt. (S.M.: p. 298)

of concern: "The Role of Presuppositions", "The Role of Experience", and "The Role of Hypothesis".

1. The Role of Presuppositions

Presuppositions are the foundations upon which the various branches of science are built. There are, according to Poincaré, two types of science: the science of pure quantity - i.e. arithmetic - and all other sciences which he terms "conventional". The foundations of all sciences must consist of certain premisses. These premisses, according to Poincaré, are ultimately indemonstrable.

> Every conclusion presumes premisses. These premisses are either self-evident and need no demonstration; or can be established only if based on other propositions; and, as we can not go back in this way to infinity, every deductive science, geometry in particular must rest upon a certain number of indemonstrable axioms. 34

But even more basic is the fact that the very process of thought requires that certain presuppositions be admitted.

> As soon as we wish to think, we are bound to have infinitely precise postulates, since this is the only means of avoiding contradictions.

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34 Poincaré, Science and Hypothesis, p. 35.

- Toute conclusion suppose des prémisses; ces prémisses elle-mêmes ou bien sont évidentes par elles-mêmes et n'ont pas besoin de démonstration, ou bien ne peuvent être établies qu'en s'appuyant sur d'autres propositions, et comme on ne saurait remonter ainsi à l'infini, toute science déductive, et en particulier la géométrie, doit reposer sur un certain nombre d'axiomes indémontrables. (S.H.: p. 49)

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> But among all the possible systems of postulates, there are some that we shall be unwilling to choose, because they do not accord sufficiently with our habits. 35

But the source of indemonstrable axioms can be either their own property of being self-evident or their being pure conventions freely chosen. However, there is only one science which does not rest upon a foundation of postulates of the conventional type. This science is arithmetic, and it is founded upon a direct intuition.

> This rule, as inaccessible to analytic proof and to experiment, is the exact type of the a priori synthetic intuition. On the other hand, we can not see in it a convention as in the case of the postulates of geometry. 36

Because the foundations of arithmetic are not conventional, the characteristics of its presuppositions which are self-evident must be distinguished from the discussion of the role of presuppositions and we will treat the special status of arithmetic

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35 Poincaré, Science and Method, p. 115.

- Dès que nous voulons penser, il nous faut bien des postulats infiniment précis, puisque c'est le seul moyen d'éviter la contradiction; mais parmi tous les systèmes de postulats possibles, il en est que nous répugnerions à choisir, parce qu'ils ne s'accorderaient pas suffisamment avec nos habitudes; ... (S.M.: pp. 120-121)

36 Poincaré, Science and Hypothesis, pp. 12-13.

Cette règle, inaccessible à la démonstration analytique et à l'expérience, est le véritable type du jugement synthétique a priori. On ne saurait d'autre part songer à y voir une convention, comme pour quelques-uns des postulats de la géométrie. (S.H.: p. 23)

in the following chapter. We wish to note, however, this exception:

> In other words, the axioms of geometry (I do not speak of those of arithmetic) are only definitions in disguise. 37

Restricting the types of presuppositions to those which enter into the foundations of the various conventional sciences, that which is common to all of them is that they are (1) not proved; (2) necessary for the construction of science; (3) pure conventions freely chosen; and (4) unable to be contradicted by experience. Presuppositions in the conventional sciences may be referred to also as postulates, as assumptions, as disguised definitions, as principles, as axioms, or as indemonstrable axioms.

The fact of a number of presuppositions is not of great concern to Poincaré. Of concern to him is the fact that there are many presuppositions which are made without the recognition of their purely postulatory character. As his biographer, Robert d'Adhémar notes:

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37 Poincaré, Science and Hypothesis, p. 68.

- En d'autres termes, les axiomes de la géométrie (je ne parle pas de ceux de l'arithmétique) ne sont que des définitions déguisées. (S.H.: p. 67)

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> ... certains postulats sont bien énoncés, l'on en admet certainement d'autres sans que l'on s'en aperçoive à l'instant où on le fait. L'on ne saurait débrouiller bien explicitement tous les postulats et une critique profonde craint toujours l'un ou l'autre de ces écueils. 38

Because Poincaré had stressed that science is built upon systems of postulates, his work has greatly contributed to a destruction of the absolute character of science.

> Poincaré montra une hardiesse que ceux-ci n'avaient jamais eue, et lui, mathématicien, il ruina d'abord, au sein des mathématiques elles-mêmes, le prestige de l'Absolu. 39

The effect of destroying the absolute character of science has led to the recognition that Poincaré's doctrine, outside of arithmetic, does not hold a single necessary truth.

> Comme le remarquait excellemment M. Michaud dans un article récent, "Poincaré, pour avoir vécu au contact des vérités apodictiques de l'analyse abstraite, ne reconnait plus nulle part ailleurs, pas même dans le monde des figures spatiales, une seule vérité nécessaire". 40

Let us now direct our attention to the various postulates which, according to Poincaré, are involved in the foundations of science. We can distinguish levels of postulates: some are very

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38 Robert d'Adhémar, Henri Poincaré, p. 6.

39 Jules Sageret, Henri Poincaré, p. 21.

40 Leon Brunschvicg, "Henri Poincaré: le Philosophe" in Revue de Métaphysique et de Morale, Vol. 21, No. 5, 1913, p. 612.

basic and common to all science; others are disguised definitions and form the foundations of the different sciences. Perhaps the most basic is a belief in order.

> Underlying scientific procedures, Poincaré held, is a belief in a general order in the universe which is independent of us and our knowledge. 41

Belief in general order because it is a belief and hence unable to be demonstrated is a very general postulate and without the belief science would be impossible.

> ... if the belief in continuity were to disappear, experimental science would become impossible. 42

Hence before the scientist can even enunciate the first scientific observations, it is necessary for him to make use of presuppositions.

> Let us first of all observe that every generalization supposes in a certain measure a belief in the unity and simplicity of nature. 43

Belief in order, in continuity, in unity, and in simplicity are all postulates necessarily imposed upon the scientist. While

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41 Peter Alexander, "Jules Henri Poincaré" in Op. Cit., p. 361.

42 Poincaré, Science and Hypothesis, p. 206.

- ... si cette croyance disparaissait à son tour, la science expérimentale deviendra impossible. (S.H.: p. 239)

43 Ibidem, p. 145.

- Observons d'abord que toute généralisation suppose dans une certaine mesure la croyance à l'unité et à la simplicité de la nature. (S.H.: p. 172)

scientists may ignore the fact that they make these presuppositions, Poincaré assures us that they are fundamental postulates which are the basis of the foundation of science.

> Derrière cette croyance à la simplicité que les savants ont répudiée, quoique bien souvent ils soient obligés d'agir comme s'ils l'avaient conservée, demeure, comme le postulat ultime de la foi scientifique, la croyance à la continuité. 44

The very fundamental type of postulates having been noted, let us consider two sets of postulates which ramify in their effects through nearly all of science. We are referring to the fact that Poincaré does not hold in a direct intuition of either space or time.

> A modified form of the Kantian attempt to explain the fruitfulness of mathematical reasoning on an intuitive rather than logical basis is that of Poincaré. He rejects the Kantian a priori intuition of space as Euclidian and in his account of the nature of time ... there certainly is no room for a priori intuitions. 45

Poincaré considers that the notions of space and time which science employs are free creations of the human mind.

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44 Léon Brunschvicg, "Henri Poincaré: le Philosophe", in Op. Cit., p. 610.

45 Morris R. Cohen, Reason and Nature, p. 188.

> Thus the characteristic property of space, that of having three dimensions, is only a property of our distribution board, a property residing, so to speak, in the human intelligence..46

Because the nature of space is postulated and geometry presupposes space, geometry rests necessarily upon a presupposed foundation.

> It will be seen that though geometry is not an experimental science, it is a science born in connexion with experience; that we have created the space it studies, but adapting it to the world in which we live. 47

The instance of time is similar to the case of space. We have created certain rules which define the nature of time. These rules could be varied, but to do so would radically change those sciences which make use of time in their constructions. Time thus is another instance of a presupposition.

> ... we have not a direct intuition of simultaneity nor of the equality of two durations. If we think we have this intuition, this is an illusion: we replace it by the aid of certain rules which we

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46 Poincaré, Science and Method, p. 112.

- Ainsi la propriété caractéristique de l'espace, celle d'avoir trois dimensions, n'est qu'une propriété de notre tableau de distribution, une propriété interne de l'intelligence humaine pour ainsi dire. (S.M.: pp. 117-118)

47 Ibidem, p. 115.

- On voit que si la géométrie n'est pas une science expérimentale, c'est une science née à propos de l'expérience; que nous avons créé l'espace qu'elle étudie, mais en adaptant au monde où nous vivons. (S.M.: p. 121)

> apply almost always without taking count of them.
>
> These rules are not imposed upon us and we might amuse ourselves in inventing others; but they could not be cast aside without greatly complicating the enunciation of the laws of physics, mechanics, and astronomy. 48

Since the notions of space and time are presupposed or postulated, all sciences which depend upon these notions are founded upon postulates which are freely chosen by the scientist. Because the scientist was not forced to any particular set of postulates, he could have chosen other sets of postulates. Had the scientist chosen another set, he would have constructed a science quite different from the one he has created.

> La géométrie soulève bien d'autres questions: l'espace, sa relativité, ses trois dimensions... Retenons seulement que notre géométrie, celle dont on se sert dans toutes les applications, n'est point la seule possible, in abstracto. 49

Poincaré assures us that the choice of postulates which functions as the foundation of a science in no way makes one science more true

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48 Poincaré, The Value of Science, p. 36.

- Nous n'avons pas l'intuition directe de la simultanéité, pas plus que celle de l'égalité de deux durées.
Si nous croyons avoir cette intuition, c'est une illusion.
Nous y suppléons à l'aide de certaines règles que nous appliquons presque toujours sans nous en rendre compte...
Ces règles ne s'imposent pas à nous et on pourrait s'amuser à en inventer d'autres; cependant on ne saurait s'en écarter sans compliquer beaucoup l'énoncé des lois de la physique, de la mécanique, de l'astronomie. (V.S.: p. 57)

49 Robert d'Adhémar, Henri Poincaré, p. 35.

than another founded upon other choices of postulates.

> What are we to think of the question: is Euclidean geometry true? It has no meaning... One can not be more true than another; it can only be more convenient. 50

Not only are the foundations of the sciences presupposed, but also the first axioms of the sciences are conventions which are neither imposed upon the scientist nor capable of being confirmed or contradicted by experience.

> The geometrical axioms are therefore neither synthetic a priori intuitions nor experimental facts. They are conventions. 51

The immunity to empirical verification is not a characteristic of geometric axioms alone. The same immunity applies equally to mechanics.

> Pas plus que ceux de la géométrie, les axiomes de la mécanique ne sont ni ne seront jamais vérifiables par expérience. Cela est évident du principe d'inertie. 52

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50 Poincaré, Science and Hypothesis, p. 50.

- Dès lors, que doit-on penser de cette question: La géométrie Euclidienne est-elle vraie? Elle n'a aucun sens. (...) Une géométrie ne peut pas être plus vraie qu'une autre; elle peut seulement être plus commode. (S.H.: p. 67)

51 Ibidem.

- Les axiomes géométriques ne sont donc ni des jugements synthétiques à priori ni des faits expérimentaux. Ce sont des conventions. (S.H.: p. 66)

52 Jules Sageret, Henri Poincaré, p. 28.

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Speaking of the principle of inertia, Poincaré assures us that other principles could have served as a foundation for another construction of that science.

> The law of inertia, as I have said above, is not imposed on us a priori, other laws would be just as compatible with the principle of sufficient reason. 53

Let us use the law of inertia as but an instance in which the physicist by means of a definition in disguise constructs his science upon a postulate which is indifferent to empirical verification. There seems to be no point in multiplying examples of this type of postulate. Poincaré held that such was the case with all mechanical models.

> Mechanical models certainly have not as much relevance to physical inquiry as diagrams in geometry, since it can be shown, as Poincaré has done, that whenever a mechanical model is invented to explain physical phenomena, an infinity of other models is possible. 54

We have seen the various roles played by postulates in the formation of science. The first type consisted of very general principles held on belief which enabled science to exist. These include belief in continuity, simplicity, unity, and order. A second type consisted of

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53 Poincaré, Science and Hypothesis, p. 92.

- La loi d'inertie, je l'ai dit plus haut, ne s'impose pas à nous à priori; d'autres lois seraient, tout aussi bien qu'elle, compatibles avec le principe de raison suffisante. (S.H.: p. 114)

54 Morris R. Cohen, Reason and Nature, p. 215.

free creations of the mind itself into which the scientist fits his scientific constructions. These are notions of space and time. The third type consisted of postulates proper to specific sciences which the scientist enunciates as definitions. They are according to Poincaré conventions which cannot be verified by experience. They serve as foundations for the scientists' experiments and future hypotheses. There are many instances of these; the examples we cited were the axioms of geometry and the law of inertia.

2. The Role of Experience

Experimentation is but a particular instance of experience. If the scientist had an indefinite amount of time, he would not have to conduct experiments. He would have only to observe events at sufficient length and these observations would be stimulus enough for the mind's creative action to form scientific laws and hypotheses. Poincaré holds that science is engendered by the union of the mind's creation and experience.

> Isolated, theory is empty and experience blind:
> and both are useless and of no interest alone. 55

The principal value of experience is that it supplies the mind with data upon which the mind can operate. It is the stimulus for mental operation.

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55 Poincaré, Science and Method, p. 275.

- Isolées, la théorie serait vide, l'expérience serait myope; toutes deux seraient inutiles et sans intérêt. (S.M.: p. 298)

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> Experiment is the sole source of truth. It alone can teach us something new; it alone can give us certainty. These are two points that cannot be questioned. 56

The necessity of both theory and experience having been seen, we need to discern the nature of the relationship between the two. George Bruce Halsted summarized this relationship in the translator's introduction to Poincaré's work The Value of Science as follows:

> Sense may act as stimulus, as suggestive, yet not to awaken a dormant depiction, or to educe the conception of an archetypal form, but rather to strike the hour for creation, to summon to work a sculptor capable of smoothing a Venus of Milo out of the formless clay. Knowledge is not a gift of bare experience, nor even made solely out of experience. The creative activity of mind is in mathematics particularly clear. 57

If knowledge is not derived or abstracted solely out of experience, then there must be in knowledge elements which derive from the mind itself. We ask if such an interpretation is consistent with Poincaré's examples. The origin of the mathematical continuum is such an instance. Poincaré notes:

> We are therefore forced to conclude that this notion has been created entirely by the mind, but it is experiment that has provided the opportunity. 58

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56 Poincaré, Science and Hypothesis, p. 140.

- L'expérience est la source unique de la vérité: elle seule peut nous apprendre quelque chose de nouveau; elle seule peut nous donner la certitude. (S.H. p. 167)

57 George Bruce Halsted, "Translator's Introduction", in The Value of Science, p. 1.

58 Poincaré, Science and Hypothesis, p. 22.

- On est donc forcé de conclure que cette notion a été créée de toutes pièces par l'esprit, mais que c'est l'expérience qui lui fourni l'occasion. (S.H.: p. 35)

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Where Poincaré's position regarding the role of experience begins to take on unique characteristics is in the fact that the experimental facts (1) place no limit on the scientist's freedom and (2) do not require any given hypothesis for a correct interpretation. Poincaré maintains that the scientist retains his freedom throughout the experimental process.

> Experience leaves us our freedom of choice, but it guides us by helping us to discern the most convenient path to follow. 59

Poincaré further maintains that there is no one true explanation for any experimental facts.

> In a word, to get the law from experiment, it is necessary to generalise; ... But how generalise? Every particular truth may evidentally be extended in an infinity of ways. 60

If the experimental data do not limit the various explanations which the scientist may wish to propose, we may wonder if there are any intrinsic limitations placed upon the scientific explanation at all. Poincaré assures us that there is only one such limitation - the duty of the scientist to have no contradictions in his theory.

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59 Poincaré, Science and Hypothesis, p. xxiii. (Poincaré's preface to the English edition)

60 Poincaré, The Value of Science, p. 77.

- En un mot, pour tirer la loi de l'expérience, il faut généraliser; ... Mais comment généraliser? Toute vérité particulière peut évidemment être étendue d'une infinité de manières. (V.S.: p. 142)

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> Our choice among all possible conventions is guided by experimental facts; but it remained free, and it is only limited by the necessity of avoiding every contradiction. 61

Because any given experimental fact can be explained by an infinite number of generalizations, then clearly the generalization that the scientist chooses is based on criteria other than experimental data. The consequent is that experimental sciences are open to a process of frequent exchanging and transforming of hypotheses.

Regarding experimental sciences Poincaré notes:

> If geometry were an experimental science, it would not be an exact science. It would be subjected to continual revision. 62

The scientist conducts an experiment in order to obtain data which will enable him to generalize. Since his goal is a generalization, the scientist may have some idea of the type of generalization he is seeking. Poincaré considers that these goals are required in order that the scientist may conduct a good experiment.

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61 Poincaré, Science and Hypothesis, p. 50.

- ... notre choix, parmi toutes les conventions possibles, est guidé par des faits expérimentaux; mais il reste libre et n'est limité que par la nécessité d'éviter toute contradiction. (S.H.: p. 66)

62 Ibidem, pp. 49-50.

- Si la géométrie était une science expérimentale, elle ne serait pas une science exacte, elle serait soumise à une continuelle revision. (S.H.: p. 66)

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> It is often said that experiments should be made without preconceived ideas. That is impossible. Not only would it make every experiment fruitless, but ... it could not be done. 63

The scientist hence begins his experiment with preconceived ideas, conducts his experiment and considers it a success if it is the occasion whereby he is enabled to create a further hypothesis.

> What, then, is a good experiment? It is that which teaches us something more than an isolated fact. It is that which enables us to predict, and generalise. 64

3. The Role of Hypothesis

The aim of experimentation is to enable the scientist to form a hypothesis. It is in the hypothesis that the creative power of the scientist is most clearly seen. It is in the hypothesis that science most properly can be said to exist. Poincaré considers hypothesis of paramount importance:

> Every generalization is a hypothesis. Hypothesis therefore plays a necessary rôle, which no one has ever contested. 65

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63 Poincaré, Science and Hypothesis, p. 143.

- On dit souvent qu'il faut expérimenter sans idée préconçue. Cela n'est pas possible; non seulement ce serait rendre toute expérience stérile, mais on le voudrait qu'on ne le pourrait pas. (S.H.: p. 170)

64 Ibidem, p. 142.

- Qu'est-ce donc qu'une bonne expérience? C'est celle qui nous fait connaître autre chose qu'un fait isolé; c'est celle qui nous permet de prévoir, c'est-à-dire celle qui nous permet de généraliser. (S.H.: p. 169)

65 Ibidem, p. 150.

- Toute généralisation est une hypothèse; l'hypothèse a donc un rôle nécessaire que personne n'a jamais contesté. (S.H.: p. 178)

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Poincaré refuses to investigate the foundations upon which the power of generalization rests on the grounds that the attempts of philosophers have proved fruitless.

> But then who gives us the right of attributing to the principle itself more generality and more precision than to the experiments which have served to demonstrate it? This is asking, if it is legitimate to generalise, as we do every day empiric data, and I shall not be so foolhardy as to discuss this question, after so many have vainly tried to solve it. 66

While not offering any justification for the process of generalization, Poincaré recognizes that without it science would be impossible.

> Throughout we have seen the part played by intuition and the spirit of generalization, without which these three grades of mathematicians, if I may venture so to express myself, would be reduced to equal impotence. 67

Our present concern is to indicate the role played by hypothesis. Poincaré distinguished three types of hypotheses. We must also distinguish these three types, since each type has a distinct

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66 Poincaré, Science and Hypothesis, p. 129.

- Mais alors qui nous donne le droit d'attribuer au principe lui-même plus de généralité et plus de précision qu'aux expériences qui ont servi à le démontrer? C'est là demander s'il est légitime, comme on le fait tous les jours, de généraliser les données empiriques, et je n'aurai pas l'outrecuidance de discuter cette question, après que tant de philosophes se sont vainement efforcés de la trancher. (S.H.:pp.155-156)

67 Poincaré, Science and Method, p. 286.

- Partout nous avons vu le rôle de l'intuition et de l'esprit de généralisation sans lequel ces trois étages de mathématiciens, si j'ose m'exprimer ainsi, seraient réduits à une égale impuissance. (S.M.: p. 309)

role. Poincaré presents his doctrine on the three types of hypotheses twice in his work Science and Hypothesis. While the doctrine is the same in both instances, the order of presentation is reversed; hence, it is not to our advantage to enumerate the types by number but rather by their properties. Poincaré prepares the reader by summarizing the three types in his introduction.

> ... there are several kinds of hypotheses; that some are verifiable, and once confirmed by experiment become truths of great fertility; that others may be useful in fixing our ideas; and others that hypotheses only in appearance, and reduce to definitions or to conventions in disguise. 68

The hypotheses which can be reduced to definitions or conventions in disguise are very frequently presupposed and their role has been seen in our treatment on postulates. Their role basically is to form the foundation of our scientific theories. When this type of hypothesis is applied to mathematics the mind is laying down its own laws and they form the bases for the exact characteristics of mathematics:

> From then, indeed, the sciences derive their rigour; such conventions are the result of the unrestricted activity of the mind, which in this domain recognises no obstacle. 69

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68 Poincaré, Science and Hypothesis, p. xxii. (Poincaré's preface to the English Edition).

69 Ibidem, p. xxiii.

Poincaré considers this type of hypothesis to be both natural and necessary. They also have importance for the experimental sciences. In the physical sciences they are very general and it is because of them that we are capable of making judgments of relevance. According to Poincaré, examples of hypotheses of this type in the physical sciences include the belief that the effect of very distant bodies is negligible and the belief that an effect is the continuous function of its cause. Because this type of hypothesis plays a fundamental role, a hypothesis of this type should be rejected with reluctance and only if the explanation of phenomena cannot be satisfactorily affected by changes in the other two types of hypotheses.

> All these hypotheses affirm, so to speak, the common basis of all the theories of mathematical physics. They are the last that should be abandoned. 70

The role of the second type of hypothesis is to fix our ideas. They are useful artifices for calculation in mathematics or pictorial aids to understanding in the physical sciences. The scientist at times finds this type of hypothesis useful because (1) the scientist must choose between various explanations and he finds it convenient when

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70 Poincaré, Science and Hypothesis, p. 152.

- Toutes ces hypothèses forment pour ainsi dire le fonds commun de toutes les théories de la physique mathématique. Ce sont les dernières que l'on doit abandonner. (S.H.: p. 180)

experimentation cannot be a guide to fix his ideas around a model, and (2) a model is also useful because, according to Poincaré, the real nature of the thing is forever hidden. Examples of this type of hypothesis given by Poincaré include electricity and ether.

> ... only, the something which we then called motion, we now call electric current. But these are merely names of the images we substitute for the real objects which nature will hide forever from our eyes. 71

and

> Whether the ether exists or not matters little - let us leave that to the metaphysicians; what is essential for us is, that everything happens as if it existed, and that this hypothesis is found to be suitable for the explanation of phenomena. 72

Now this type of hypothesis is sometimes referred to by Poincaré as an "indifferent hypothesis". That is to say that they are unable to be confirmed by experimentation. While the use of this type of hypothesis is not rejected by Poincaré, he does warn against the danger of misunderstanding its character. They are merely useful, they may be changed.

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71 Poincaré, Science and Hypothesis, p. 161.

- ... seulement, ce quelque chose nous l'appelions autrefois mouvement, nous l'appelons maintenant courant électrique. Mais ces appellations n'étaient que des images substituées aux objets réels que la nature nous cachera éternellement. (S.H.: p. 190)

72 Ibidem, p. 211.

- Peu nous importe que l'éther existe réellement, c'est l'affaire des métaphysiciens; l'essentiel pour nous c'est que tout se passe comme s'il existait et que cette hypothèse est commode pour l'explication des phénomènes. (S.H.: pp. 245-246)

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> The true relations between the real objects are the only reality we can attain; and the sole condition is that the same relations shall exist between these objects as between the images we are forced to put in their place. If the relations are known to us, what does it matter if we think it convenient to replace one image by another. 73

It is the third type of hypothesis in which the scientist offers an explanation, i.e. prediction of phenomena. This type of hypothesis is subject to verification. Although the scientist still retains a choice among the various hypotheses, this type must be confirmed by experimentation in order to be accepted. Not only must they be confirmed but they are indefinitely open to further testing. According to Poincaré this type of hypothesis, a "real generalization" is always valuable. If it is condemned by experiment, it will lead the scientist to either further refinements or creation of a new hypothesis. If confirmed by experiment, they will offer an explanation of phenomena hitherto seen as complex and unrelated. The only warning Poincaré gives us against these hypotheses is the danger of multiplying them more than necessary for explaining the phenomena.

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73 Poincaré, Science and Hypothesis, p. 161.

- Les rapports véritables entre ces objets réels sont la seule réalité que nous puissions atteindre, et la seule condition, c'est qu'il y ait les mêmes rapports entre ces objets qu'entre les images que nous sommes forcés de mettre à leur place. Si ces rapports nous sont connus, qu'importe si nous jugeons commode de remplacer une image par une autre. (S.H.: p. 190)

> Let us also notice that it is important not to multiply hypotheses indefinitely. If we construct a theory based upon multiple hypotheses, and if experiment condemns it, which of the premisses must be changed? It is impossible to tell. Conversely, if the experiment succeeds, must we suppose that it has verified all these hypotheses at once? 74

C. The Method of Science

Poincaré's conception of science renders certain distinctions which are commonly made between the various sciences and particularly between the various types of sciences no longer justified. Little justification can be found in Poincaré's position for a real distinction between pure and applied sciences, between mathematical and experimental sciences, or even between the natural and social sciences. This fact can be observed when Poincaré notes the foundation common to physics, mathematics, and history.

> The first question, then, is to know how to make this selection. This question confronts the physicist as well as the historian; it also confronts the mathematician, and the principles which should guide them all are not very dissimilar. 75

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74 Poincaré, Science and Hypothesis, pp. 151-152.

- Remarquons, d'autre part, qu'il importe de ne pas multiplier les hypothèses outre mesure et de ne les faire que l'une après l'autre. Si nous construisons une théorie fondée sur des hypothèses multiples, et, si l'expérience la condamne, quelle est parmi nos prémisses celle qu'il est nécessaire de changer? Il sera impossible de le savoir. Et inversement, si l'expérience réussit, croira-t-on avoir vérifié toutes ces hypothèses à la fois? (SH.: pp. 179-180)

75 Poincaré, Science and Method, p. 9.

- La première question est donc de savoir comment il doit faire ce choix. Cette question se pose au physicien comme à l'historien; elle se pose également au mathématicien, et les principes qui doivent les guider les uns les autres ne sont pas sans analogie. (S.M.: p. 1)

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All the sciences aim at the formulation of laws. But the discovery of laws is dependent upon the successful refinement of scientific theories.

The difference between law, theory, and hypothesis does not appear to be due to any difference in their relationship to facts. Law, theory, and hypothesis are all equally within the conventional framework. To designate a scientific explanation as a law does invest it with a greater value within the context of the accepted conventions. That is to say that within a given set of conventions the law is the most perfect type of scientific creation.

> The best expression of this harmony is law. Law is one of the most recent conquests of the human mind... 76

But by recalling the conventional foundations of science, the designation of a scientific construction as a law does not imply any objective and absolute perfection. Hence any scientific law is always replaceable by another law resting on other conventions.

> We know the laws only imperfectly. Experience merely limits our choice; and from among all the laws which experience permits us to choose, it is always possible to find some which will not lead us to a contradiction. 77

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76 Poincaré, The Value of Science, p. 13.

- La meilleure expression de cette harmonie, c'est la loi; la loi est une des conquêtes les plus récentes de l'esprit humain; ... (V.S.: p. 4)

77 Poincaré, Mathematics and Science: Last Essays, p. 5.

- Nous ne connaissons les lois qu'imparfaitement; l'expérience ne fait que limiter notre choix, et parmi toutes les lois qu'elle nous permet de choisir, on en trouvera toujours qui ne nous exposent pas à une contradiction... (D.P.: p. 12)

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A scientific theory is not based on experience alone. It is formed rather by the combination of intellectual activity and imposed facts. But the imposed facts are numberless and the scientist must choose from among them. The value of a scientific theory rests first in a relevant choice of facts coupled with an intellectual activity which reveals new relations. Now two characteristics follow from the above considerations: the first that the methods of all the sciences are basically the same since the principles underlying a selection of facts are the same and second that choice of facts is synonymous with scientific method. Scientific method is determined by the underlying scientific principles which make possible fruitful intellectual activity.

In our first conclusion we see the reason why traditional distinctions between the various sciences are not made by Poincaré.

> These are generalities which apply in the main to all sciences. For instance, there is no appreciable difference between the mechanism of mathematical discovery and the mechanism of discovery in general. 78

The mechanism of discovery, while composed of several elements as we shall see, is an instance of a basically univocal procedure.

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78 Poincaré, Mathematics and Science: Last Essays, p. 10.

- Ce sont là des généralités qui s'appliquent en somme à toutes les sciences; et par exemple le mécanisme de l'invention mathématique ne diffère pas sensiblement du mécanisme de l'invention en général. (S.M.: p. 2)

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Some commentators have contended that Poincaré reduced scientific method to mathematical method:

> In "Science and Method" the thesis is, that the discovery of laws is by methods substantially the same as those of mathematical investigation, deducing from a significant particular a wide-reaching generalization, selecting our facts because of their significance. 79

This interpretation can undoubtedly be defended as correct in itself. The question can be raised if even the mechanism of mathematical discovery is not capable of being further reduced. Charles Nordmann maintains that Poincaré regarded psychology as destined to be the archetype of discovery.

> Since what we know of the universe comes only from its image as reflected to us through our brain, that knowledge is affected by all the properties and deformations of that inner mirror, and psychology will without doubt one day be the master science. 80

This interpretation does have much in its favour. While it is true that Poincaré prefers mathematical discoveries as examples, it may not be just because he was himself a mathematician, but also because mathematics is more obviously a product of mental activity. Poincaré has stated several times that observation of the mathematician's method would be very fruitful for the psychologist.

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79 James Byrnie Shaw, "Henri Poincaré as an Investigator" in Popular Science Monthly, issue of March, 1913, Vol. 82, p. 217.

80 Charles Nordmann, "Henri Poincaré: His Scientific Works, His Philosophy", in Op. Cit., p. 749.

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> The genesis of mathematical discovery is a problem which must inspire the psychologist with the keenest interest. For this is the process in which the human mind seems to borrow least from the exterior world, in which it acts, or seems to act, only by itself and on itself. 81

In presenting Poincaré's method of science we must investigate both the choice of facts and the requirements which lead the scientist to choose in a given way.

1. The Choice of Facts

A scientific theory consists of intellectual activity performed upon imposed facts. While the facts are imposed upon the scientist, it needs to be noted that an unlimited number of facts are imposed upon the scientist. Thus it is necessary for the scientist to select from among these facts.

> Method is precisely the selection of facts, and accordingly our first care must be to devise a method. 82

The facts which the scientist selects must be common to many minds. But this is not enough; it is necessary for the scientist to know ahead of the selection which facts will be profitable in the construction of his theories. Poincaré maintains that often the

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81 Poincaré, Science and Method, p. 46.

- La genèse de l'invention mathématique est un problème qui doit inspirer le plus vif intérêt au psychologue. C'est l'acte dans lequel l'esprit humain semble emprunter au monde extérieur, où il n'agit ou ne parait agir que par lui-même et sur lui-même... (S.M.: p. 43)

82 Ibidem, p. 19.

- ... la méthode, c'est précisément le choix des faits, il faut donc se préoccuper d'abord d'imaginer une méthode,... (S.M.: p. 12)

scientist knows instinctively the principles to follow. The problem of fact selection is identical with the search for method.

a. Nature of Scientific Fact

Henri Poincaré's exposition on the nature of the scientific fact was the result of a controversy in which M. LeRoy, a nominalist scientist, made use of Poincaré's doctrine in support of his own contention that science is only a rule of action teaching nothing of the truth. At issue in M. LeRoy's position is the doctrine that the scientist creates the scientific fact. Poincaré makes clear that he is unable to view the scientific fact as a pure creation of the scientist although he does not regard the distinction between "scientific fact" and the "fact in the rough" to be unfounded. Poincaré says:

> Finally, I can not admit that the scientist creates without restraint the scientific fact since it is the crude fact which imposes it upon him. 83

Poincaré does not however consider that it is possible to distinguish clearly where the crude fact is changed into a scientific fact. Basically the distinction is that the crude fact is individual, completely distinct from all other facts, and susceptible

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83 Poincaré, The Value of Science, p. 116.

- Enfin je ne puis admettre que le savant crée librement le fait scientifique puisque c'est le fait brut qui le lui impose. (V.S.: pp. 221-222)

to verification as in the case of an eclipse where the crude fact could be enunciated simply by the statement "It grows dark". But the transformation of a crude fact according to Poincaré goes through a scale of stages which can apparently be subdivided almost indefinitely. In order for the crude fact to be rendered into a scientific one, it needs to be interpreted by means of conventions which are of the mind's free creation. Among the conventions invoked in transforming the crude fact into a scientific fact, Poincaré includes all methods of measurements, ranging from clocks and meters to galvanometers and geometry. Poincaré compares the transforming of the crude fact into a scientific fact with the translation of one language into another:

> The scientific fact is only the crude fact translated into a convenient language. 84

There is consequently a certain limit on the level of the crude fact where the scientist's creative activity is unable to operate. There is a certain basic fact of experience which is imposed upon the scientist, and he is unable to contradict this. The crude fact does limit the scientist.

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84 Poincaré, The Value of Science, p. 120.

- Le fait scientifique n'est que le fait brut traduit dans un langage commode. (V.S.: p. 231)

> First of all, he does not create it from nothing, since he makes it (the scientific fact) with the fact in the rough. Consequently he does not make it freely and as he chooses. However able the worker may be, his freedom is always limited by the properties of the raw material on which he works. 85

Although the crude fact does impose itself on the scientist, this does not result in a complete loss of freedom to the scientist. He still remains free to select from among crude facts.

> What then remains of M. LeRoy's thesis? This remains: the scientist intervenes actively in choosing the facts worth observing. 86

2. Search for Harmony, Simplicity, and Unity

The areas in which the scientist has the freedom of choice in Poincaré's conception of science is indeed vast. The scientist chooses from among the crude facts. He chooses the presuppositions, although he often does so unconsciously.

He also chooses among hypotheses and finally from among theories. While Poincaré defends this vast area as belonging to the scientist's free choice, he does not contend that these

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85 Poincaré, The Value of Science, p. 121.

- Tout d'abord il ne le crée pas ex nihilo puisqu'il le fait avec le fait brut. Par conséquent il ne le fait pas librement et comme il veut. Quelque habile que soit l'ouvrier, sa liberté est toujours limitée par les propriétés de la matière première sur laquelle il opère. (V.S.: p. 232)

86 Ibidem.

- Que reste alors de la thèse de M. LeRoy? Il reste ceci: le savant intervient activement en choisissant les faits qui méritent d'être observés. (V.S.: p. 233)

choices can be made on an arbitrary or an unmethodical basis. Method consists in those principles which must guide the scientist in making his choice. The nature of these guiding principles is the search by the scientist for harmony, simplicity, and unity. Harmony, simplicity, and unity could be compared with what scholastic philosophers term the final object since they are for Poincaré both first in the scientist's intention and last in the order of the scientist's execution. Regarding the necessity of harmony, simplicity, and unity, Poincaré appears to leave no freedom of choice.

> The only facts worthy of our attention are those which introduce order into this complexity and so make it accessible to us. 87

Clearly it is only in the introduction of order that harmony and simplicity can be found. But since as we have seen the scientist along with all men does not know the extra-mental world, is the scientist certain that the world is in fact simple and harmonious? Poincaré maintains that the scientist does not know this.

> Is nature governed by caprice or is harmony the reigning influence? That is the question. It is when science reveals this harmony that it becomes

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87 Poincaré, Science and Method, p. 30.

- Les seuls faits dignes de notre attention sont ceux qui introduisent de l'ordre dans cette complexité et la rendent ainsi accessible. (S.M.: p. 25)

> beautiful, and for that reason worthy of being cultivated. 88

and

> It is not certain that nature is simple. Can we without danger act as if she was? 89

A further justification certainly is indicated. The scientist must choose that which introduces order, harmony, and simplicity yet he is not certain that the world is really ordered, harmonious, and simple. What is the source of this necessity? Let us note that Poincaré does not deny that the world is really ordered and harmonious but rather that we know it to be so.

> ... it (science) is tending more and more to show us the mutual dependence of the different parts of the universe, to unveil its harmony to us. Is it because this harmony is real or because it is a need of our mind and consequently a postulate of science? This is a question which I shall not attempt to resolve. 90

While Poincaré assumes a skeptical position regarding the harmonious and simple character of the universe, it is certainly a

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88 Poincaré, Science and Method, p. 275.

- La nature est-elle gouvernée par le caprice, ou l'harmonie y règne-t-elle? voilà la question; c'est quand elle nous révèle cette harmonie que la science est belle et par là digne d'être cultivée. (S.M.: pp. 297-298)

89 Poincaré, Science and Hypothesis, p. 145.

- Il n'est pas sûr que la nature soit simple. Pouvons-nous sans danger faire comme si elle l'était? (S.H.: p. 173)

90 Poincaré, Mathematics and Science: Last Essays, p. 109.

- ... elle tend de plus en plus à nous montrer la solidarité des diverses parties de l'univers, à nous en dévoiler l'harmonie; est-ce parce que cette harmonie est réelle, ou parce qu'elle est un besoin de notre intelligence, et par conséquent un postulat de la science? C'est une question que je n'entreprendrai pas de décider. (D.P.: pp. 237-238)

mental need and sufficiently strong to necessitate the scientists' selecting only those facts which introduce them. Speaking of facts which would not introduce order, Poincaré notes:

> It (science) would not longer be to us of any value, since it could not satisfy our needs of order and harmony, and because it would be at the same time incapable of prediction. 91

The need for simplicity and harmony is accordingly primarily if not exclusively a mental need. Regarding this mental need there is no freedom.

> In formulating a general, simple, and formal law, based on a comparatively small number of not altogether consistent experiments, we have only obeyed a necessity from which the human mind can not free itself. 92

The achievement of simplicity and harmony ordinarily results in bringing about unity.

> The fact remains that science tends toward unity and leads us toward unity. 93

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91 Poincaré, Science and Hypothesis, p. 130.

- ... elle n'aurait pour nous aucun prix, puisqu'elle ne pourrait donner satisfaction à notre besoin d'ordre et d'harmonie et qu'elle serait en même temps incapable de prévoir. (S.H.: p. 156)

92 Ibidem.

- En formulant une loi générale, simple et précise après des expériences relativement peu nombreuses et qui présentent certaines divergences, nous n'avons donc fait qu'obéir à une nécessité à laquelle l'esprit humain ne peut se soustraire. (S.H.: p. 157)

93 Poincaré, Mathematics and Science: Last Essays, p. 109.

- Toujours est-il que la science va vers l'unité et nous fait aller vers l'unité. (D.P.: p. 238)

NOTIONS OF CONVENTIONAL SCIENCE

In a sense unity is the supreme principle which guides the scientist in all his endeavours and choices. The possession of harmony, simplicity, and unity has two salutary effects on the scientist: it results in a saving of thought and mental satisfaction.

a) Economy of Thought

The "economy of thought" and "mental or aesthetic satisfaction" which are caused by the harmonious, simple, and unified constructions are not really distinct. They are the same effect viewed from two different points of view. The economy of thought tends to be practical or pragmatic in the sense that the simple and unified construction, whether scientific fact or generalization, results in a saving of time and thought for the scientist.

> Thus the importance of a fact is measured by the return it gives - that is, by the amount of thought it enables us to economize. 94

Likewise the economy of thought is not distinct from the search for unity. It is rather the result of unity viewed subjectively from the standpoint of the scientist as a person.

> It is the economy of thought that we should aim at and therefore it is not sufficient to give models to be copied. 95

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94 Poincaré, Science and Method, p. 29.

- L'importance d'un fait se mesure donc à son rendement, c'est-à-dire à la quantité de pensée qu'elle nous permet d'économiser. (S.M.: p. 23)

95 Ibidem, p. 33.

- C'est à l'économie de pensée que l'on doit viser, ce n'est donc pas assez de donner des modèles à imiter. (S.M.: p. 28)

b) Mental Satisfaction

Viewing the possession of an harmonious, simple, and unified construction in its primary effect upon the scientist, Poincaré maintains that it results in a true satisfaction.

> Briefly stated, the sentiment of mathematical elegance is nothing but the satisfaction due to some conformity between the solution we wish to discover and the necessities of our mind, ... this aesthetic satisfaction is consequently connected with the economy of thought. 96

This is not surprising since it was the initial mental need for harmony and simplicity that caused the scientist to follow the method of selection which leads to harmony, simplicity, and unity. The final construction of the harmonious, simple, and unified, by answering a mental need naturally results in mental satisfaction.

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96 Poincaré, Science and Method, p. 31.

- En un mot, le sentiment de l'élégance mathématique n'est autre chose que la satisfaction due à je ne sais quelle adaptation entre la solution que l'on vient de découvrir et les besoins de notre esprit, ... Cette satisfaction esthétique est par suite liée à l'économie de pensée. (S.M.: p. 26)

SUMMARY

The notion of conventional science applies to a vast area of human knowledge. Philosophy, having no proper object, does not have a place in Poincaré's hierarchy of valid knowledge. Valid knowledge is the exclusive domain of science. There are, however, two types of science. Arithmetic, the science of pure quantity, is founded upon a direct intuition of number and it alone among the sciences cannot be classed as a conventional science. But all other sciences from geometry to sociology are in Poincaré's division to be classed as conventional. The nearly universal domain of conventional sciences at times results in diminished attention being focused on the non-conventional science. But the two types are distinct and essential for a comprehensive interpretation of Poincaré's position.

The reality which the conventional sciences seek to understand is confined to the area of relation. It is a mistake to consider that conventional sciences attempt to explain nature. Any phenomenon is capable of serving as an object of scientific hypothesis but it is only the relation between phenomena which science attempts to explain. Science exists for its own sake, for its own intellectual beauty, and not for utilitarian aims.

All conventional sciences are founded upon postulates which may or may not always be recognized. The existence of science requires these presuppositions, but there is no necessity for any given postulate in itself. The postulates are freely chosen by

SUMMARY

the scientist, and while they are unproved they cannot be contradicted by existence. They provide the framework so to speak upon which science is constructed. Because they are unproved, some may be considered beliefs, such as the belief in continuity, simplicity, and unity. Because they are freely chosen they may be considered artificial, such as definitions of time, space, and the laws of inertia. However they are all conventions.

Experience is unable to confirm or condemn the conventions upon which science is erected. Experience may provide the scientist with data which will be the occasion for the creation of a generalization, i.e. a hypothesis. But experience does not determine any given hypothesis; any empiric fact can be generalized in an unlimited number of ways. Empiric verification can be useful if applied to an explanatory hypothesis. Verification is of primary value if it fails to confirm a hypothesis; the scientist must seek a further refinement of it or the creation of a new hypothesis.

All sciences seek laws. A scientific theory is the result of intellectual activity performed upon imposed facts. The scientist must however interject himself actively into the process of fact selection since there is an unlimited number of brute facts. The brute fact must be rendered suitable for scientific purposes. In selecting the brute facts and in converting them into scientific facts the scientist has no method other than mathematical analogy.

SUMMARY

In seeking mathematical analogies within the relations of phenomena the scientist follows a method which seeks the establishment of order. The scientist chooses that which introduces simplicity, harmony, and unity. It is not certain that nature is simple, harmonious, and unified; the mind itself is ordained toward these properties and consequently scientific method conforms to a mental need.

CHAPTER III

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

In the designation of Poincaré's doctrine of science by the term which he himself so frequently used, i.e. "conventionalism", one must note carefully that arithmetic is an exception and does not rest upon this foundation.

> On the other hand, we cannot see in it a convention as in the case of the postulates of geometry. 1

The various branches of mathematics do possess an exact character but their exactness is a consequent of the initial postulates made by the scientist. The postulates prevent them from being experimental sciences and thereby give them a permanence which saves them from continuous revision. But the existence of these postulates also causes these branches of mathematics to be conventional sciences in which the objectivity of postulates is eliminated because there is no necessity for one set instead of another. The science of arithmetic holds a unique status and he who would understand the nature of mathematical reasoning is obliged to find it in arithmetic.

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1 Poincaré, Science and Hypothesis, p. 13.

- On ne saurait d'autre part songer à y voir une convention, comme pour quelques-uns des postulats de la géométrie. (S.H.: p. 23)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

> First of all, geometry must be excluded, or the question becomes complicated by the difficult problems relating to the rôle of the postulates, the nature and origin of the idea of space. For analogous reasons we cannot avail ourselves of the infinitesimal calculus. We must seek mathematical thought where it has remained pure - i.e., in Arithmetic. 2

A special investigation of the characteristics of the science of pure quantity is required since the fundamental epistemology must apply to all valid branches of knowledge. Further it must apply most directly to the science of arithmetic since it is the purest mathematical type and it occupies a special status in the hierarchy of knowledge by being true, certain, and necessary. The explanation of these characteristics is our present aim.

A. The Nature of Discovery in the Science of Pure Quantity

1. Psychology of Discovery

All science is the result of the creative activity of the scientist. Mathematics holds a special status among the sciences because it has less dependence upon the exterior world.

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2 Poincaré, Science and Hypothesis, p. 5.

- Nous devons exclure d'abord la géométrie où la question se complique des problèmes ardus relatifs au rôle des postulats, à la nature et à l'origine de la notion d'espace. Pour des raisons analogues nous ne pouvons nous adresser à l'analyse infinitésimale. Il nous faut chercher la pensée mathématique là où elle est restée pure, c'est-à-dire en arithmétique. (S.H.: p. 14)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

> ... mathematical science must reflect upon itself, and this is useful because reflecting upon itself is reflecting upon the human mind which created it; the more so because, of all its creations, mathematics is the one for which it has borrowed least from outside. 3

The sciences can be distinguished according to their dependence upon sense experience. The greater the dependence is, the less the results of the science will be. [4] Conversely, the less the dependence the greater are the results:

> It is by withdrawing from reality that it (science of mathematics) has acquired this perfect purity. 5

The science of mathematics is more pure than the other sciences because it is more withdrawn from sense experience. Within the mathematical sciences arithmetic is the most pure since it is not complicated by any postulates. But mathematics like all sciences is dependent upon facts. [6] The problem then is to explain the origin of the facts which are essential to the

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3 Poincaré, Science and Method, p. 36.

- ... la science mathématique doit réfléchir sur elle-même et cela est utile, parce que réfléchir sur elle-même, c'est réfléchir sur l'esprit humain qui l'a créée, d'autant plus que c'est celle de ses créations pour laquelle il a fait le moins d'emprunts au dehors. (S.M.: p. 31)

4 Poincaré cites sociology as an example of the science with the least results. Science and Method, pp. 20-21.
Cf. Science et Méthode, pp. 12-13.

5 Poincaré, Science and Method, p. 124.

- C'est en s'éloignant de la réalité qu'elles ont acquis cette pureté parfaite. (S.M.: p. 131)

6 The first chapter of Science and Method is entitled "The Selection of Facts" and the problem is applied to all sciences. The first chapter of Science and Hypothesis is a demonstration of the factual foundation of arithmetic.

development of mathematical thought and which are not given by sense experience. Harold Chapman Brown has noted that Poincaré makes use of two types of experience:

> Mathematical propositions are, then, transcriptions of experience. It may be of psychological experience ... or again they may express complex relations among physical objects, and both psychological and physical experiences are invoked. 7

The genesis of mathematics is situated in the psyche of man. The nature of this psychological experience is difficult to define. Poincaré describes it as a special aesthetic sensibility, [8] an intuition of mathematical order, [9] an aesthetic feeling, [10] an aesthetic emotion, [11] and a special intuition. [12] This

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7 Harold Chapman Brown, "The Work of Henri Poincaré", in The Journal of Philosophy, Psychology, and Scientific Methods, 1914, p. 229.

8 Poincaré, Science and Method, p. 60.
- "sensibilité esthétique spéciale" (S.M.: p. 59)

9 Ibidem, p. 50.
- "intuition de l'ordre mathématique" (S.M.: p. 47)

10 Ibidem, p. 59.
"sensibilité esthétique" (S.M.: p. 58)

11 Poincaré, The Value of Science, p. 136.
"émotion esthétique" (V.S.: p. 57)

12 Poincaré, Science and Method, p. 50.
"intuition spéciale" (S.M.: p. 57)

psychological experience corresponds to the artistic emotion. [13] This psychological experience is the cause not only for the historical birth of mathematics but also necessary for a true understanding of the mathematical creations of others and for the ability to make further mathematical discoveries. Furthermore, this psychological experience is not to be identified with memory.

> Lastly, others possess the special intuition I have spoken of more or less highly developed, and they can not only understand mathematics, even though their memory is in no way extraordinary, but they can become creators, and seek to make discovery with more or less chance of success, according as their intuition is more or less developed. 14

The moot problem concerning the psychological experience is the question of whether it depends in any way on sensation. The chief characteristic of it is its high degree of independence from sense experience. The question of its total independence from sense experience is treated by Poincaré with great delicacy and while the

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13 Poincaré, The Value of Science, p. 137.

- "émotion artistique" (V.S.: p. 264)

14 Poincaré, Science and Method, p. 50.

- Les autres enfin posséderont à un plus ou moins haut degré l'intuition dont je viens de parler et alors non seulement ils pourront comprendre les mathématiques, quand même leur mémoire n'aurait rien d'extraordinaire, mais ils pourront devenir créateurs et chercher à inventer avec plus ou moins de succès, suivant que cette intuition est chez eux plus ou moins développée. (S.M.: p. 48)

question remains doubtful, Poincaré sees sufficient evidence for making an essential distinction in the types of intuition.

> I have said how much the intuition of pure number, whence comes rigorous mathematical induction, differs from sensible intuition to which the imagination, properly so called, is the principal contributor. ... Could we recognize with a little attention that this pure intuition itself could not do without the aid of the senses?... But the things being doubtful is enough to justify me in recognizing and affirming an essential difference between the two kinds of intuition... 15

The source of the psychological experience [16] being doubtful, Poincaré has taken great care in describing the sequence of the process itself. [17] The interesting characteristic of the process of discovery is that conscious work does not perform the most important function although it plays a necessary rôle. The process

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15 Poincaré, The Value of Science, pp. 24-25.

- J'ai dit combien l'intuition du nombre pur, celle d'où peut sortir l'induction mathématique rigoureuse, diffère de l'intuition sensible dont l'imagination proprement dite fait tous les frais.

... Reconnaîtrait-on avec un peu d'attention que cette intuition pure elle-même ne saurait se passer du secours des sens? ... Mais il suffit que la chose soit douteuse pour que je sois en droit de reconnaître et d'affirmer une divergence essentielle entre les deux sortes d'intuition. (V.S.: pp. 32-33)

16 In response to the statement of Moreau of Tours that "genius is a nervous disease", Poincaré submitted to a complete psychological study by Dr. Toulouse. In his medico-psychological inquiry into intellectual superiority Dr. Toulouse found Poincaré perfectly normal psycho-physiologically although he was struck by his instability of attention. The results are discussed in Charles Nordmann's "Henri Poincaré: His Scientific Work: His Philosophy", Op. Cit., pp. 743-744.

17 Poincaré describes his personal experiences in Chapter III, Science and Method.

consists of three stages: the first stage is a period of conscious activity in which one attempts a solution without success; the second stage is a period of sub-conscious activity in which various solutions are combined and the fruitless ones rejected; the third stage is the instantaneous projection into consciousness of one of the solutions of sub-conscious activity which the mathematician then verifies. Let us examine the three stages more in detail.

The act of mathematical creation consists in forming a useful combination of mathematical entities. The number of new combinations is without limit but of the possible new combinations only a very few are of interest. Attempts at consciously forming a useful combination are usually unsuccessful.

> Often when a man is working at a difficult question, he accomplishes nothing the first time he sets to work. Then he takes more or less of a rest, and sits down again at his table. During the first half hour he still finds nothing, and then all at once the decisive idea presents itself to his mind. 18

The subliminal ego or the unconscious ego works only upon material which is prepared for it by the conscious self. The subliminal

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18 Poincaré, Science and Method, p. 55.

- Souvent, quand on travaille une question difficile, on ne fait rien de bon la première fois qu'on se met à la besogne; ensuite on prend un repos plus ou moins long, et on s'asseoit de nouveau devant sa table. Pendant la première demi-heure, on continue à ne rien trouver et puis tout à coup l'idée décisive se présente à l'esprit. (S.M.: pp. 53-54)

ego is active, sifting various combinations while the conscious self is directed elsewhere.

> Poincaré est d'avis que la phase transitoire qui sépare les deux phases de travail conscient n'est une période de repos qu'en apparence. En réalité il intervient selon lui une phase de travail inconscient qui produit finalement la solution recherchée. 19

The second period of conscious activity is necessary to verify the results of the combinations which are formed by the subliminal ego and work out their consequences. The nature of the subliminal ego is of great interest since it is able to create what the conscious self is unable to accomplish.

> The subliminal ego is in no way inferior to the conscious ego; it is not purely automatic; it is capable of discernment; it has tact and lightness of touch; it can select, and it can devine. 20

How can we explain the fact that the creation of the subliminal ego which does illuminate the conscious self is a useful combination while those which never break forth into consciousness are not useful? The solution is crucial to our deliberation. The solution is found in the nature of the useful combination.

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19 Evert W. Beth and Jean Piaget, Epistemologie, Mathématique et Psychologie, p. 97.

20 Poincaré, Science and Method, p. 57.

- Le moi subliminal n'est nullement inférieur au moi conscient; il n'est pas purement automatique, il est capable de discernement, il a du tact, de la délicatesse; il sait choisir, il sait deviner. (S.M.: p. 56)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

> The useful combinations are precisely the most beautiful, I mean those that can most charm that special sensibility that all mathematicians know, but of which laymen are so ignorant that they are often tempted to smile at it. 21

Poincaré describes further the nature of the beautiful combination.

> It may appear surprising that sensibility should be introduced in connexion with mathematical demonstrations, which, it would seem, can only interest the intellect. But not if we bear in mind the feeling of mathematical beauty, of the harmony of numbers and forms of geometric elegance. It is a real aesthetic feeling that all true mathematicians recognize, and this is truly sensibility. 22

In short, the subliminal self forms a great number of worthless combinations. When the subliminal ego forms a beautiful, harmonious, and elegant combination, this very construction affects the special aesthetic sensibility of the subliminal ego which in turn attracts the attention of the conscious self.

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21 Poincaré, Science and Method, p. 59.

- Les combinaisons utiles, ce sont précisément les plus belles, je veux dire celles qui peuvent le mieux charmer cette sensibilité spéciale que tous les mathématiciens connaissent, mais que les profanes ignorent au point qu'ils sont souvent tentés d'en sourire. (S.M.: p. 58)

22 Ibidem.

- On peut s'étonner de voir invoquer la sensibilité à propos de démonstrations mathématiques qui, semble-t-il, ne peuvent intéresser que l'intelligence. Ce serait oublier le sentiment de la beauté mathématique, de l'harmonie des nombres, et des formes, de l'élégance géométrique. C'est un vrai sentiment esthétique que tous les vrais mathématiciens connaissent. Et c'est bien là de la sensibilité. (S.M.: p. 57)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

> More commonly the privileged unconscious phenomena, those that are capable of becoming conscious, are those which, directly or indirectly, most deeply affect our sensibility. 23

The essence of mathematical discovery lies in the fact that by its nature the subliminal ego is affected by and attracted to a beautiful, harmonious, and elegant mental phenomenon. The most harmonious construction turns out to be the most useful.

2. Nature and Role of the Direct Intuition

The problem is to understand how the science of arithmetic is possible and upon what foundation it rests. The science of arithmetic is the one that borrows least from the external world and is perfectly rigorous and unchallenged. Because there is only science of the general, Poincaré maintains that arithmetic must be founded upon proof rather than verification:

> Verification differs from proof precisely because it is analytical, and because it leads to nothing because the conclusion is nothing but the premisses translated into another language.

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23 Poincaré, Science and Method, p. 58.

- Plus généralement, les phénomènes inconscients privilégiés, ceux qui sont susceptibles de devenir conscients, ce sont ceux qui, directement ou indirectement, affectent le plus profondément notre sensibilité. (S.M.: p. 57)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

> A real proof, on the other hand, is fruitful, because the conclusion is in a sense more general than the premisses. 24

The problem is then to explain the general character of arithmetic. The deductive and analytic explanations are not sufficient since they cannot explain the origin of the general.

> Pour obtenir une démonstration générale, portant sur la suite illimitée des nombres naturels, il faut pouvoir passer du fini à l'infini; et ce passage rend le raisonnement mathématique irréductible aux formes purement analytiques de la déduction. 25

The solution of the problem of the existence of mathematics is found in the role of intuition. There can be no question that there are several types of intuition in Poincaré's doctrine. There is also solid recognition of the fact that Poincaré has not been successful in identifying the roles of the different types.

> Une complication est causée par le fait que Poincaré n'emploie pas toujours le terme intuition au même sens... En premier lieu, nous voyons qu'il distingue trois cas, qui se laissent décrire comme suit: 1) l'intuition sensorielle, 2) l'intuition généralisante, 3) l'intuition pure. 26

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24 Poincaré, Science and Hypothesis, p. 4.

- La vérification diffère précisément de la véritable démonstration, parce qu'elle est purement analytique et parce qu'elle est stérile. Elle est stérile parce que la conclusion n'est que la traduction des prémisses dans un autre langage. La démonstration véritable est féconde au contraire parce que la conclusion y est en un sens plus générale que les prémisses. (S.H.: p. 13)

25 Léon Brunschvicg, "Henri Poincaré: Le Philosophe", Op. Cit., p. 602.

26 J.J.A. Mooij, La Philosophie des Mathématiques de Henri Poincaré, pp. 113-114.

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

Pierre Boutroux, the nephew of Poincaré, notes the difficulty of isolating the types of intuition:

> Cependant le mot "intuition" est ambigu. Henri Poincaré lui-même ne l'a pas toujours employé dans le même sens. L'intuition dont il parlait naguère était le plus souvent une intuition de sens, ou tout au moins de l'imagination... Poincaré nous avertit, il est vrai, qu'il existe une autre intuition que celles des sens ou de l'imagination: l'intuition de nombre pure. 27

There are two sources of the difficulty in identifying the different kinds of intuition. The nature of "l'intuition généralisante" is difficult to reconcile if it is truly distinct from a sense foundation and yet not a pure intuition. The second source of difficulty in identifying the different kinds of intuition is the question of whether there may not be more than three types. Support of this theory is found in the fact that Poincaré says there are <u>many</u> kinds of intuition even though he only enumerates the three types cited by J.J.A. Mooij.

> We have then many kinds of intuition; first, the appeal to the senses and the imagination; ... finally, we have the intuition of pure number. 28

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27 Pierre Boutroux, "L'Oeuvre philosophique" in <u>Henri Poincaré: L'Oeuvre scientifique, L'Oeuvre philosophique</u>, p. 257.

28 Poincaré, <u>The Value of Science</u>, p. 20.

- Nous avons donc plusieurs sortes d'intuitions; d'abord, l'appel aux sens et à l'imagination; ... nous avons enfin l'intuition du nombre pur, ... (V.S.: p. 22)

THE CHARACTERISTICS OF THE SCIENCE OF PURE QUANTITY

Fortunately there can be no doubt that the foundation of arithmetic is in the pure and direct intuition. This must be the case since arithmetic is the purest of the sciences.

> I have shown ... that the first two can not give us certainty; but who will seriously doubt the third, who will doubt arithmetic? ... there can be nothing but syllogisms or appeals to this intuition of pure number, the only intuition that can not deceive us. 29

Let us now direct our research to the way in which the intuition of pure number is capable of founding the science of arithmetic by a process which begins with the particular and rises to the general. The reasoning process itself is variously titled by Poincaré: synthetic à priori intuition, [30] reasoning by recurrence, [31] principle of complete induction, [32] true mathematical induction, [33] and sometimes simply mathematical reasoning.[34] By the phrase synthetic à priori Poincaré understands:

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29 Poincaré, The Value of Science, p. 20.

- Les deux premières ne peuvent nous donner la certitude, je l'ai montré ... mais qui doutera sérieusement de la troisième, qui doutera de l'arithmétique? ... il n'y a plus que des syllogismes ou des appels à cette intuition du nombre pur, la seule qui ne puisse nous tromper. (V.S.: pp. 22-23)

30 Poincaré, Science and Hypothesis, pp. 12-13.
"jugement synthétique à priori" (S.H.: p. 23)

31 Ibidem, p. 9.
"raisonnement par récurrence" (S.H.: p. 20)

32 Poincaré, Science and Method, pp.149-150.
"le principe d'induction complète" (S.M.: pp. 160-161)

33 Ibidem, p. 150.
"véritable induction mathématique" (S.M.: p. 160)

34 Poincaré, The Value of Science, p. 23.
"raisonnement mathématique" (V.S.: p. 156)

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> ... the term employed by Kant to designate the judgments that can neither be demonstrated analytically, nor reduced to identity, nor established experimentally... 35

In more positive terms the synthetic a priori intuition is a direct intuition of the mind's own power.

> It is because it is only the affirmation of the power of the mind which knows it can conceive of the indefinite repetition of the same act, when the act is once possible. The mind has a direct intuition of this power, and experiment can only be for it an opportunity of using it, and thereby of becoming conscious of it. 36

Poincaré shows that the foundations of arithmetic rest upon the processes of addition and multiplication. Now the associative and commutative laws of addition and the distributive and commutative laws of multiplication can only be established by recurrence. But to verify their recurrence analytically would require

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35 Poincaré, Science and Method, p. 146.

- ... comme disait Kant pour désigner les jugements qui ne peuvent être ni démontrés analytiquement, ni réduits à des identités, ni établis expérimentalement, ... (S.M.: p. 156)

36 Poincaré, Science and Hypothesis, p. 13.

- C'est qu'il n'est que l'affirmation de la puissance de l'esprit qui se sait capable de concevoir la répétition indéfinie d'un même acte dès que cet acte est une fois possible. L'esprit a de cette puissance une intuition directe et l'expérience ne peut être pour lui qu'une occasion de s'en servir et par là d'en prendre conscience. (S.H.: pp. 23-24)

an infinite number of syllogisms since these properties would have to be verified for each number.

These laws can be proved by means of a construction which is a complete induction. In one intuition it is able to include an infinity of syllogisms.

> The essential characteristic of reasoning by recurrence is that it contains, condensed, so to speak, in a single formula, an infinite number of syllogisms. 37

The statement of this principle is simply:

> If a property is true of the number 1, and if it is established that it is true of $n + 1$ provided it is true of n, it will be true of all whole numbers. 38

This intuition which is the foundation of arithmetic is active in the sense that the mind experiences itself capable of performing the operation indefinitely without obstacle.

> Une telle intuition, qui est d'ordre dynamique et idéaliste, ne peut pas se transformer en l'intuition directe d'un donné au sens réaliste du mot. 39

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37 Poincaré, Science and Hypothesis, p. 9.

- Le caractère essentiel du raisonnement par récurrence c'est qu'il contient, condensés pour ainsi dire en une formule unique, une infinité de syllogismes. (S.H.: p. 20)

38 Poincaré, Science and Method, p. 149.

- Si une propriété est vraie du nombre 1, et si l'on établit qu'elle est vraie de $n + 1$ pourvu qu'elle le soit de n, elle sera vraie de tous les nombres entiers. (S.M.: p. 159)

39 Léon Brunschvicg, "L'Oeuvre d'Henri Poincaré: Le Philosophe", in Op. Cit., p. 603.

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In short, the existence of arithmetic is founded upon the fact that the mind possesses a direct intuition of number. This direct intuition which is independent of sensory experience does not appear capable of a realistic interpretation.[40] The validity of performing operations upon the numbers given in the direct and pure intuition is guaranteed by a second intuition which is synthetic a priori and a complete induction. For the purpose of brevity we can designate the combination of the direct intuition of pure number and the resulting complete induction by the term "pure mathematical ideal". The aim of the next section is to consider the characteristics of the "pure mathematical ideal".

40 If the direct intuition of number could be given a realistic interpretation, Poincaré would not be justified in making an essential distinction between it and those intuitions in which the imagination contributes. Cf. footnote 15.

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B. Characteristics of the Pure Mathematical Ideal.

1. The Truth and Certitude of the Pure Mathematical Ideal

The pure mathematical ideal is both true and certain. The fact that truth and certainty coincide here is of special significance since it occurs at no other place in knowledge. As a general rule Poincaré has cautioned against their coincidence:

> Il ne faut pas croire que l'amour de la vérité se confonde avec celui de la certitude; loin de là, dans notre monde relatif toute certitude est un mensonge. 41

Since truth and certainty are not ordinarily found combined, the special instance found in the mathematical ideal is incapable of establishing a form of reasoning which can be applied identically elsewhere.

> Mais cette grande simplicité ne marque qu'un degré dans l'ensemble des objets de connaissance. M. Poincaré distingue radicalement et d'une façon absolue la nécessité apodictique de l'arithmétique, en refusant d'y voir le mode ordinaire de raisonnement de l'esprit, en faisant dépendre les vérités énoncées d'un postulat spécial, synthétique et a priori. 42

Our concern is in establishing the conditions which have made possible the truth and certitude of the pure mathematical ideal.

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41 Poincaré, Savants et Ecrivains, p. vii.

42 G. Milhaud, "La Science et l'Hypothèse par M. H. Poincaré" in Revue de Métaphysique et de Morale, Vol. 11, 1903, p. 782.

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We must investigate the ordinary origins of both truth and certitude. The source of certitude is the application of the principles of definition and logic:

> It was not long before it was recognized that exactness cannot be established in the arguments unless it is first introduced into the definitions. 43

This cannot be the source of the certitude of the pure mathematical ideal since the pure mathematical ideal is not capable of being expressed in analytic terms. The pure mathematical ideal appears to possess a priority of nature over logic.

> The fact is that it is impossible to give a definition without enunciating a phrase, and difficult to enunciate a phrase without putting in a name of number or at least the word several, or at least a word in the plural. 44

Definition is impossible without the concept of number and this concept is given in the pure intuition. The priority of this intuition over logic enables the pure mathematical ideal to be both true and certain. The truth and certainty of the pure mathematical

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43 Poincaré, Science and Method, pp. 123-124.

_ On n'a pas tardé à s'apercevoir que la rigueur ne pourrait pas s'établir dans les raisonnements, si on ne la faisait entrer d'abord dans les définitions. (S.M.: p. 130)

44 Ibidem, p. 155.

- C'est qu'il est impossible de donner une définition sans énoncer une phrase, et difficile d'énoncer une phrase sans y mettre un nom de nombre, ou au moins le mot plusieurs, ou au moins un mot au pluriel. (S.M.: p. 166)

ideal are found in the distinguishing characteristics of the mathematical intuition. This differs from the other types of intuition which are only capable of supplying truth but not certainty.

> ... it is by logic that we prove, but by intuition that we discover. 45

The complete mathematical induction is not the only type of intuition which is employed by the mathematician:

> I did not mean to say, as had been supposed, that all mathematical arguments can be reduced to an application of this principle. 46

But it is the origin of arithmetic in the direct intuition and complete mathematical induction which confers on it certainty as well as truth.

> Mais Poincaré ne voudrait pas appliquer le raisonnement par récurrence, avec la nécessité intuitive et synthétique qui lui est propre, à toutes les sciences... A la rigueur, le raisonnement synthétique et intuitif n'est applicable que dans l'arithmétique et dans l'analyse pure. 47

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45 Poincaré, Science and Method, p. 129.

- ... c'est par la logique qu'on démontre, c'est par l'intuition qu'on invente... (S.M.: p. 137)

46 Ibidem, p. 149.

- Je ne voulais pas dire, comme on l'a cru, que tous les raisonnements mathématiques peuvent se réduire à une application de ce principe. (S.M.: pp. 159-160)

47 J. Benrubi, Les Sources et les Courants de la Philosophie Contemporaine en France, p. 369.

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Dr. Leon Chwistek is critical of the fact that Poincaré is unwilling to extend the area of certitude:

> He (Poincaré) was of the generation of mathematicians who wished to see exact thought confined within the limits of mathematics and he did not realize that if rigorous criticism is not undertaken in connection with all the problems of life a detrimental confusion of concepts will occur. 48

We need to note that Poincaré limited the area of exact thought even further than Dr. Chwistek indicates. It is only in arithmetic that one is able to speak of absolute exactness, and this is a result of a total withdrawal from intuitions dependent on sense experience.

> Induction applied to the physical sciences is always uncertain, because it is based on the belief in a general order of the universe, an order which is external to us. Mathematical induction - i.e. proof by recurrence - is, on the contrary, necessarily imposed on us, because it is only the affirmation of a property of the mind itself. 49

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48 Leon Chwistek, The Limits of Science, p. 108.

49 Poincaré, Science and Hypothesis, p. 13.

- L'induction, appliquée aux sciences physiques, est toujours incertaine, parce qu'elle repose sur la croyance à un ordre général de l'Univers, ordre qui est en dehors de nous. L'induction mathématique, c'est-à-dire la démonstration par récurrence, s'impose au contraire nécessairement, parce qu'elle n'est que l'affirmation d'une propriété de l'esprit lui-même. (S.H.: p. 24)

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Thus far our considerations have led us to an appreciation of the exact character of pure mathematics. The mathematical induction is the result of a necessary imposition of the mind's power upon itself. This situation is certainly sufficient to explain the certitude experienced by the mathematician who finds himself unable to conceive the opposite. But is this necessary imposition by the mind sufficient to explain the truth value of the pure mathematical ideal? According to Poincaré, intuition is the source of truth, but does it follow that a necessary intuition is thereby necessarily true? Poincaré has never maintained that all intuitions are true:

> They trusted to intuition, but intuition cannot give us exactness, nor even certainty, and this has been recognized more and more. It teaches us, for instance, that every curve has a tangent - that is to say, that every continuous function has a derivative - and that is untrue. 50

The issue will be further clarified if we recall the characteristics that invest the theories of the experimental science with value. In the preceding chapter we saw that those theories of value to the scientist were those that introduced simplicity,

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50 Poincaré, Science and Method, p. 123.

- On se fiait à l'intuition; mais l'intuition ne peut nous donner la rigueur, ni même la certitude, on s'en est aperçu de plus en plus. Elle nous apprend par exemple que toute courbe a une tangente, c'est-à-dire que toute fonction continue a une dérivée, et cela est faux. (S.M.: p. 130)

harmony, and unity within the scientist's mental order. The effect of these characteristics was mental satisfaction and economy of thought. Now in the present case of the direct intuition which is necessarily imposed in consciousness, can we not recognize that these characteristics are likewise present - actually present in a most perfect manner? Because the intuition is direct is it not the most simple of all intuitions?

> In this category of principles, that of the complete induction is only the simplest of all, and it is for that reason that I selected it as a type. 51

This tends to confirm the observation of G. Milhaud already cited that the simplicity of the mathematical induction is not a difference in kind from other intuitions but merely a difference in degree among the various objects of knowledge.

Let us consider also the unitive function of the direct intuition. Is not the synthesis produced by the mathematical induction, since it is complete, an instance of a reasoning which possesses perfect unity?

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51 Poincaré, Science and Method, p. 150.

- Dans cette catégorie de principes, celui de l'induction complète est seulement le plus simple de tous et c'est pour cela que je l'ai choisi pour type. (S.M.: p. 160)

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> It is therefore mathematical reasoning par excellence, and we must examine it closer. The essential characteristic of reasoning by recurrence is that it contains, condensed, so to speak, in a single formula, an infinite number of syllogisms. 52

What could convey to the mind greater unity than an intuition capable of containing in a single formula an infinite number of syllogisms? The economy of thought must be achieved in the highest form possible. What then remains of Dr. Leon Chwistek's objection that Poincaré should have extended the fields of exactness? The notions of truth and certitude (exactness) having been separated, Poincaré was unable to do so. Only in the mathematical ideal is perfect simplicity and unity united with necessity. What remained for Poincaré was rather to defend the pure mathematical ideal in arithmetic and seek its approximation outside arithmetic.

Finally we must not fail to take note of the purely idealistic character of the pure mathematical ideal. In itself it is not relative while in no way dependent upon any extra-mental phenomena. As René poirier notes:

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52 Poincaré, Science and Hypothesis, p. 9.

- C'est donc bien là le raisonnement mathématique par excellence et il nous faut l'examiner de plus près. Le caractère essentiel du raisonnement par récurrence c'est qu'il contient, condensés pour ainsi dire en une formule unique, une infinité de syllogismes. (S.H.: pp. 19-20)

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> Son relativisme se satisfait d'une connaissance purement formelle, pourvu que cette forme s'exprime mathématiquement, car elle est encore foncièrement rationelle et résume même toute la rationalité possible du monde. 53

2. The Mathematical Ideal: The Archetype of Science

The pure mathematical ideal with its special characteristics of truth and certainty is unable to be duplicated anywhere else in human knowledge. Its purity remains intact only in arithmetic. Its foundation in the direct intuition of number and its accomplishment in the principle of complete induction are the most simple and unifying operations anywhere performed. We can see in this mental structure, incapable of being perfectly reproduced elsewhere, the original pattern of which all knowledge is a copy. The role of the pure mathematical form in the formation of the other branches of sciences is not limited to a mere exemplary cause. We find on the contrary that it is a most functional principle not at all dissimilar to the functional role of God in the philosophy of Descartes. We need now direct our attention to the delineation of the functions of the pure mathematical intuition in the birth of science. The pure mathematical ideal functions in three ways: (1) it makes thought of extra-mental phenomena possible; (2) it directly produces

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53 René Poirier, "Henri Poincaré et le Problème de la Valeur de la Science" in Revue Philosophique de la France et de l'Etranger, Vol. 144, issue of 1954, p. 503.

the science of arithmetic; and (3) it creates the conditions upon which all other sources depend. Let us now examine these functions in greater detail.

The first function of the pure mathematical intuition and induction is that it constitutes a sine qua non of thinking itself. As J.J.A. Mooij observes,

> Sa croyance (est) que nous posséderions l'intuition du nombre naturel et que nous ne serions pas capable de penser sans nous servir de cette intuition... 54

We have already observed that the very possibility of definition depends upon the notion of the natural number which is given in the direct intuition. But the function is even more basic. The very intelligibility of the sensations themselves depend upon the direct intuition of number. The notion of continuity is derived from the mathematical continuum and this is the power which enables the knower to unite sensations.

> I shall conclude that there is in all of us an intuitive notion of the continuum of any number of dimensions whatever because we possess the capacity to construct a physical and mathematical continuum; and that this capacity exists in us before any experience because, without it, experience

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54 J.J.A. Mooij, La Philosophie des Mathématiques de Henri Poincaré, p. 84.

> properly speaking would be impossible and would be reduced to brute sensations, unsuitable for any organization... 55

The direct intuition of number provides the mental framework which makes the organization of sensations possible. The fact that sensations owe their intelligibility to the pure mathematical forms is responsible for the justification of the mind's power of correcting sensations - interpreting them.

> The day Copernicus proved that what was most stable was in motion, that what was thought moving was fixed, he showed how deceptive could be infantile reasonings which spring directly from the immediate data of our senses. 56

The second function of the pure mathematical intuition and induction is in the formation of the science of arithmetic. The notion of the whole numbers is given in the direct intuition but it is the complete mathematical induction which elevates the isolated, and particular numbers into the unified and general science.

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55 Poincaré, Mathematics and Science: Last Essays, p. 44.

- Je conclurai que nous avons tous en nous l'intuition du continu d'un nombre quelconque de dimensions, parce que nous avons la faculté de construire un continu physique et mathématique; que cette faculté préexiste en nous à toute expérience parce que sans elle, l'expérience proprement dite serait impossible et se réduirait à des sensations brutes, impropres à toute organisation... (D.P.: p. 96)

56 Poincaré, The Value of Science, p. 87.

- Le jour où Copernic a prouvé que ce qu'on croyait le plus stable était en mouvement, que ce qu'on croyait mobile était fixe, il nous a montré combien pouvaient être trompeurs les raisonnements enfantins qui sortent directement des données immédiates de nos sens... (V.S.: p. 163)

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> Throughout we have seen the part played by intuition and the spirit of generalization, without which these three grades of mathematicians, if I may venture so to express myself, would be reduced to equal impotence. 57

We need not dwell upon the second function as it has already been the subject for investigation and it is the most obvious function since the science of arithmetic is directly constituted by the complete mathematical induction.

The third function of the pure mathematical intuition and induction is that it provides the conditions which make all other sciences possible. It is by the harmony and symmetry of mathematical entities that the pure mathematical ideal functions as an archetype for future knowledge.

> What has taught us to know the true profound analogies, those the eyes do not see but reason divines? It is the mathematical spirit, which disdains matter to cling only to pure form. 58

How is the pure mathematical ideal able to perform this function? It does so in two ways: it establishes an instance of

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57 Poincaré, Science and Method, p. 286.

- Partout nous avons vu le rôle de l'intuition et de l'esprit de généralisation sans lequel ces trois étages de mathématiciens, si j'ose m'exprimer ainsi, seraient réduits à une égale impuissance. (S.M.: p. 309)

58 Poincaré, The Value of Science, p. 77.

- Qui nous a appris à connaître les analogies véritables, profondes, celles que les yeux ne voient pas et que la raison devine? C'est l'esprit mathématique, qui dédaigne la matière pour ne s'attacher qu'à la forme pure. (V.S.: pp. 142-143)

universality in knowledge thus producing science, and it prepares the other sciences to seek relations of the mathematical type outside the field of pure mathematics itself.

Because the mathematical induction is a true construction, Poincaré considers that without it there could be no scientific invention. Mathematical induction hence serves as an archetype for subsequent physical inductions.

> We can only ascend by mathematical induction, for from it alone can we learn something new. Without the aid of this induction, which in certain respects differs from, but is as fruitful as, physical induction, construction would be powerless to create science. 59

The pure mathematical ideal provides the mind with the power to discern the harmony of various mathematical relations and series. The knowledge of these mathematical relations provides the mind with the instrument for seeking similar series of relations in the physical world.

> Thus mathematical analogies not only may make us foresee physical analogies... It is besides, it is above all, to reveal to him the hidden harmony of things in making him see them in a new way. 60

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59 Poincaré, Science and Hypothesis, p. 16.

- Nous ne pouvons nous élever que par l'induction mathématique, qui seule peut nous apprendre quelque chose de nouveau. Sans l'aide de cette induction différente à certains égards de l'induction physique, mais féconde comme elle, la construction serait impuissante à créer la science. (S.H.: p. 28)

60 Poincaré, The Value of Science, p. 79.

- Ainsi les analogies mathématiques, non seulement peuvent nous faire pressentir les analogies physiques ... Il est encore, il est surtout de lui faire connaître l'harmonie cachée des choses en les lui faisant voir d'un nouveau biais. (V.S.: pp. 146-147)

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This search for pure mathematical relations in the various branches of science rightly lead René Poirier to observe:

> ... il semble que la réalité à laquelle il (Poincaré) songe soit d'ordre arithmétique d'une part, d'ordre esthétique de l'autre. 61

The fact has been noted that Poincaré's doctrine of conventionalism reduces all sciences except arithmetic to a relative status, thus creating an unbridgeable gap between pure mathematics which is objective and all the other sciences which are relative. This fact is often presented as a regrettable situation:

> ... on dirait que M. Poincaré, pour avoir connu dans l'analyse mathématique une necessité inéluctable, apodictique, s'exprimant en jugement synthétiques a priori, en garde désormais au fond de son esprit un souvenir ineffaçable... Et en fait, malgré ses efforts pour justifier par leur opportunité les conventions et les définitions, il donnera toujours le sentiment qu'à ses yeux elles impliquent quelque chose de contraire à l'objectivité vraie... La nécessité une fois perdue, M. Poincaré semble avoir peine à retrouver l'objectivité et la vérité. 62

While not in any way seeking to dispute the facts of this situation, we wonder whether a more positive interpretation might not be appropriate. Instead of regretting the fact that the sciences founded upon conventions are only relative in terms of objectivity and truth, is it not also the case that they would not even have a

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61 René Poirier, "Henri Poincaré et le problème de la valeur de la science", Op. Cit., p. 508.

62 G. Milhaud, "La science et l'hypothèse, par H. Poincaré", Op. Cit., p. 784.

relative value were it not for the insertion of the pure mathematical ideal within their structure. Central to the position of Poincaré is the doctrine that measurement alone is capable of making an object suitable for science.

> In fact it had to become measurable; that which can not be measured cannot be an object of science. Thus, time which can be measured is also relative. 63

Thus interpreted, it is only by means of the objective value of the pure mathematical ideal that the other sciences are able to participate in an even relative value.

The mathematician has three aims: (1) he provides an instrument which makes possible the study of nature; (2) he provides the philosopher with subject matter for reflection (number, space, and time); and (3) an esthetic aim. 64

Having already noted the secondary role of the philosopher as that of reflecting upon the findings of science and the mental satisfaction resultant upon the pure mathematical ideal, it is the

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63 Poincaré, Mathematics and Science: Last Essays, p. 18.

- Il a fallu en effet qu'elle devint mesurable; ce qui ne se mesure pas ne peut être objet de la science. Or, le temps mesurable est aussi essentiellement relatif. (D.P.: p. 41)

64 Poincaré, The Value of Science, pp. 75-76.

Cf. Valeur de la science, p. 130.

first aim of the mathematician which deserves our attention. It is in being the only truly objective and true knowledge that the pure mathematical ideal is the only instrument capable of giving us even relative knowledge of the extra-mental world.

> ... a special language is needed; ordinary language is too poor, it is besides too vague, to explain relations so delicate, so rich, and so precise. This therefore is one reason why the physicist can not do without mathematics; it furnishes him the only language he can speak. 65

We can recognize now that any organization of extra-mental phenomenon, if it is to be a science, must be organized into a mathematical synthesis.

> ... les rapports demeurent qui sont purement intellectuels, et les rapports constituent la science. Cette conception domine la philosophie scientifique de Poincaré: par elle s'expliquent les merveilleux services dont la science de la nature est redevable à la méthode moderne de l'interprétation mathématique. 66

The pure mathematical ideal alone is objective. Other sciences are possible only if they employ the instruments provided by this ideal. Some have questioned the capacity to render the world intelligible by mathematics alone:

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65 Poincaré, The Value of Science, p. 76

- ... il faut une langue spéciale; le langage ordinaire est trop pauvre, il est d'ailleurs trop vague, pour exprimer des rapports si délicats, si riches et si précis. Voilà donc une première raison pour laquelle le physicien ne peut se passer des mathématiques; elles lui fournissent la seule langue qu'il puisse parler. (V.S.: p. 141)

66 Léon Brunschvicg, "Henri Poincaré: Le Philosophe", Op. Cit., p. 593.

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> Il (Poincaré) n'a pas expliqué en quel sens les mathématiques, parce qu'elles expriment une certaine réalité, peuvent rendre foncièrement intelligible une Nature, alors même qu'elles n'en figurent rationellement que les rapports. 67

This question can have no meaning within the context of the epistemological position of Poincaré:

> To say that science can not have objective value since it teaches us only relations, this is to reason backwards, since, precisely, it is relations alone which can be regarded as objective. 68

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67 René Poirier, "Henri Poincaré et le problème de la valeur de la science", Op. Cit., p. 508.

68 Poincaré, The Value of Science, p. 137.

- Dire que la science ne peut avoir de valeur objective parce qu'elle ne nous fait connaître que des rapports, c'est raisonner à rebours, puisque précisément ce sont les rapports seuls qui peuvent être regardés comme objectifs. (V.S.: p. 266)

SUMMARY

By our analysis of the characteristics of the science of pure quantity, we have discovered that the pure mathematical ideal is more than just a unique instance of true and certain knowledge. The pure mathematical ideal is highly functional. It provides the criterion of truth - the simple, harmonious, and unified mental intuition. It provides the mental framework which is able to unite brute sensations and thus render them intelligible. Finally, it provides the mathematical analogies which the sciences of physical phenomena must seek to discover duplicated within extra-mental phenomena.

If our analysis of the role of the pure mathematical ideal is correct, we can see that it is a very valuable possession of the scientist, and because of its irreplaceable functions must never be degraded. Poincaré's repeated attacks on logical analysis at times have appeared excessive to some observers. Within the present context, we believe we can offer a new interpretation of these criticisms. If the pure mathematical ideal is as valuable as we have shown it to be, the defense of this ideal is absolutely necessary for the justification of all sciences. We submit that Poincaré's criticism of logical analysis is such a defense and thus tends towards confirmation of the importance of the pure mathematical ideal. Our next chapter is an analysis of Poincaré's criticism of logical analysis.

CHAPTER IV

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Logic has been chosen as the vehicle for investigating Poincaré's position on the importance of intuition in general and the pure mathematical ideal in particular. Nowhere does Poincaré appear more amusing and more vehement than in his treatment of the role of logic. [1]

In this chapter we will analyse both the valid and invalid uses of logic. Concerning the valid uses of logic, Poincaré is prepared to admit the utility of logic provided that (1) the application of logic remains totally dependent upon the direct intuition of number and the pure mathematical ideal; (2) logic is used in verification of particular mathematical combinations, not in their discovery; (3) the use of logic even in verification must be

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1 Each of Poincaré's works on scientific philosophy contains sections or chapters devoted to the use and misuse of logic. One can note chronologically an increased preoccupation with the theme. His first work Science and Hypothesis contains sections on logic and the syllogism in chapters I and II entitled "On the Nature of Mathematical Reasonings" and "Mathematical Magnitude and Experiment". His second work, The Value of Science, opens with the very problem of logic in Chapter I entitled "Intuition and Logic in Mathematics". However his third work, Science and Method, contains three chapters explicitely devoted to the subject of the role of logic in science. Chapter III is entitled "Mathematics and Logic"; Chapter IV is "The New Logics"; Chapter V is "The Last Efforts of the Logisticians". In his posthumous work, Mathematics and Science: Last Essays, Poincaré replies to his critics by writing a new chapter entitled "Mathematics and Logic" as well as a separate chapter devoted to "The Logic of Infinity".

subordinated to intuitions other than the pure mathematical ideal; and (4) logic never be considered the ideal and its use restricted to instances in which the aim is rigour.

Two types of analysis will be made of Poincaré's denial of the ultimate reduction of mathematics to pure logic. In the first analysis, which is the external critique of the logical ideal, Poincaré criticises the logical ideal from the point of view of his own position. Here we will note that the foundations of Poincaré's objection is that the logical ideal violates his own criterion of truth: harmony, simplicity, unity, and the economy of thought. In the second analysis, which is the internal critique of the ultimate reduction of mathematics to pure logic, Poincaré considers the theoretical reasons why the logisticians will never be able to accomplish the logical ideal. Here we will note that the basic reasons are that Poincaré considers the syllogism only capable of leading to tautologies and that the logisticians are making use of the principle of complete induction without admitting it.

A. The Valid Role of Logic in Scientific Method

Poincaré never considers the analytic approach an ideal for the scientific mind. Nevertheless in viewing the history of

mathematics and the causes of its development, Poincaré is forced to admit that logic and analysis have made a contribution:

> The two sorts of minds are equally necessary for the progress of science; both the logicians and the intuitionalists have achieved great things that others could not have done. ... analysis and synthesis have then both their legitimate roles. But it is interesting to study more closely in the history of science the part that belongs to each. 2

The history of mathematics reveals two types of approach to the problems of mathematics: analytic and synthetic. But the approach chosen is not a result of the subject matter itself. Poincaré situates the duality of approaches rather as a property of the mind of the individual scientist.

> It is the very nature of their minds which makes them logicians or intuitionalists, and they can not lay it aside when they approach a new subject. 3

But even in admitting the legitimate contribution of the two approaches in the development of mathematics and the usefulness of logic to mathematics, we cannot fail to note Poincaré's preference

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2 Poincaré, The Value of Science, p. 17.

- Les deux sortes d'esprits sont également nécessaires aux progrès de la Science; les logiciens, comme les intuitifs, ont fait de grandes choses que les autres n'auraient pas pu faire. ... L'analyse et la synthèse ont donc toutes deux leur rôle légitime. Mais il est intéressant d'étudier de plus près quelle est dans l'histoire de la science la part qui revient à l'une et à l'autre. (V.S.: pp. 15-16)

3 Poincaré, The Value of Science, p. 15.

- C'est la nature même de leur esprit qui les fait logiciens ou intuitifs, et ils ne peuvent la dépouiller quand ils abordent un sujet nouveau. (V.S.: p. 12)

for the intuitional approach.

> ... at the first stroke (they) make quick but sometimes precarious conquests, like bold cavalrymen of the advance guard. 4

While the works of mathematicians possessing minds preoccupied with logic leads Poincaré to note:

> ... to read their works one is tempted to believe they have advanced only step by step, after the manner of a Vauban who pushes on his trenches against the place besieged, leaving nothing to chance. 5

Let us now investigate the nature of the contribution which the application of logic has made and the conditions which have enabled it to render a contribution.

1. Necessity of Analysis and Synthesis in the Progress of Science

Poincaré is quite prepared to admit the fact that both analysis and synthesis are necessary.

> Thus logic and intuition have each their necessary role. Each is indispensible. Logic, which alone can give certainty, is the instrument of demonstration; intuition is the instrument of invention. 6

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4 Poincaré, The Value of Science.

- ... les autres ... font du premier coup des conquêtes rapides, mais quelquefois précaires, ainsi que de hardis cavaliers d'avant-garde. (V.S.: p. 12)

5 Ibidem.

- ... à lire leurs ouvrages, on est tenté de croire qu'ils n'ont avancé que pas à pas, avec la méthode d'un Vauban qui pousse ses travaux d'approche contre une place forte, sans rien abandonner au hasard. (V.S.: p. 11)

6 Ibidem, p. 23.

- Ainsi, la logique et l'intuition ont chacune leur rôle nécessaire. Toutes deux sont indispensables. La logique qui peut seule donner la certitude de la démonstration: l'intuition est l'instrument
V.S.: p. 29)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Why is Poincaré led to admit the necessity of analysis? The solution lies in the fact that if mathematics is to progress it must extend beyond the confines of the pure mathematical ideal which is limited to the whole number. But in the act of progressing beyond this ideal the intuitions which the mathematicians must employ contain elements furnished from sense experience. The admixture of sense data to intuition opens the intuition to the possibility of error.

> How can intuition deceive us on this point? It is because when we seek to imagine a curve, we can not represent it to ourselves without width... We well know these lines have no width; we try to imagine them narrower and narrower ... but we shall never attain the limit... We shall thus be led, unless warned by a rigorous analysis, to conclude that a curve always has a tangent. 7

Thus viewed, the necessity of analysis is to correct the errors which the scientist may be tempted to make as a result of an intuition influenced by sense data. We have had reason to note before that sense phenomena are observed but not known. 8 We find confirmation of the fact that what the scientist really knows directly is his own concept in the above quote by the statement

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7 Poincaré, The Value of Science, p. 18. (Emphasis added)

- Comment l'intuition peut-elle nous tromper à ce point? C'est que quand nous cherchons à imaginer une courbe, nous ne pouvons pas nous la représenter sans épaisseur; ... Nous savons bien que ces lignes n'ont pas d'épaisseur; nous nous efforçons de les imaginer de plus en plus minces... mais nous n'atteindrons jamais cette limite... Nous serons ainsi amenés, à moins d'être avertis par une analyse rigoureuse, à conclure qu'une courbe a toujours une tangente. (V.S.: p. 17-18)

8 Cf. "The Subject Matter of Science".

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"We well know these lines have no width". The possibility of error is introduced by sense intuition. We notice that the power of sense intuition is limited but the mathematician needs to free himself from the limitations imposed by sense intuition.

> Intuition is not necessarily founded on the evidence of the senses; the senses soon would become powerless; for example, we can not represent to ourselves a chiliagon, and yet we reason on polygons in general, which include the chiliagon as a particular case. 9

The necessity of analysis and hence the justification of the use of logic in science consists in the fact that the sense intuition may need to be corrected. It is a characteristic of the sense intuition that it may not provide the mind with a pure enough concept upon which to reason.

> For the most part the objects treated by mathematicians were long ill defined; they were supposed to be known because represented by means of the senses or the imagination; but one had only a crude image of them and not a precise idea on which reasonings could take hold. It was there first that the logicians had to direct their efforts. 10

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9 Poincaré, The Value of Science, p. 19.

- L'intuition n'est pas forcément fondée sur le témoignage des sens; les sens deviendraient bientôt impuissants; nous ne pouvons, par exemple, nous représenter le chilogone, et cependant nous raisonnons par intuition sur les polygones en général, qui comprennent le chilogone comme cas particulier. (V.S.: p. 21)

10 Ibidem, p. 18.

- Longtemps les objets dont s'occupent les mathématiques étaient pour la plupart mal définis; on croyait les connaître, parce qu'on se les représentait avec les sens ou l'imagination; mais on n'en avait qu'une image grossière et non une idée précise sur laquelle le raisonnement pût avoir prise. C'est là d'abord que les logiciens ont dû porter leurs efforts. (V.S.: p. 19)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Now two characteristics of this justified use of logic must be borne in mind: (1) the use of logic improves only the sense intuition, not the pure intuition, and (2) the use of logic in correcting sense intuition does not supplant the importance and necessity of the sense intuition.

The use of logic is always dependent upon the use of intuition. The use of logic may purify sense intuition but pure logic alone is never adequate for creating science.

> Pure logic could never lead us to anything but tautologies; it could create nothing new; not from it alone can any science issue... to make arithmetic, as to make geometry, or to make any science, something else than pure logic is necessary. To designate this something else we have no word other than intuition. 11

Concerning the first characteristic noted above, that the use of logic does not improve the pure intuition, Poincaré maintains that this intuition is absolutely essential. While analysists have certainly rendered the development of mathematics a service, they have never accomplished it by denigrating the pure intuition. Rather their only capacity to create is dependent upon their use of the pure mathematical ideal.

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11 Poincaré, The Value of Science, p. 19. (Emphasis is Poincaré's)

- La logique toute pure ne nous mènerait jamais qu'à des tautologies; elle ne pourrait créer du nouveau; ce n'est pas d'elle toute seule qu'aucune science peut sortir. ... pour faire l'arithmétique, comme pour faire la géométrie, ou pour faire une science quelconque, il faut autre chose que la logique pure. Cette autre chose, nous n'avons pour la désigner d'autre mot que celui d'intuition. (V.S.: p. 20)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> It is the intuition of pure number, that of pure logical forms, which illumines and directs those we have called analysts. This it is which enables them not alone to demonstrate, but also to invent. By it they perceive at a glance the general plan of a logical edifice, and that too without the senses appearing to intervene. In rejecting the aid of the imagination, which, as we have seen, is not always infallible, they can advance without fear of deceiving themselves. 12

Logic and intuition have complemented each other and have together rendered possible the progress of science and rigorous demonstrations in science. However, at this point Poincaré distinguishes between the analysts of the past and the current efforts of the new logisticians, or mathematical analysts among whom he numbers Cantor, Signor Peano, Couturat, Burali-Forti, Russell, and Hilbert. This distinction is justified on the fact that the former, according to Poincaré, never made a reduction of mathematical method to pure logic.

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12 Poincaré, The Value of Science, p. 25.

- C'est l'intuition de nombre pur, celle des formes logiques pures qui éclaire et dirige ceux que nous avons appelés analystes. C'est elle qui leur permet non seulement de démontrer, mais encore d'inventer. C'est par elle qu'ils aperçoivent d'un coup d'oeil le plan général d'un édifice logique, et cela sans que les sens paraissent intervenir. En rejetant le secours de l'imagination, qui, nous l'avons vu, n'est pas toujours infaillible, ils peuvent avancer sans crainte de se tromper. (V.S.: p. 33)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> ... do you think these logicians have always proceeded from the general to the particular, as the rules of formal logic would seem to require of them? Not thus could they have extended the boundaries of science; scientific conquest is to be made only by generalization. 13

Poincaré denies that the analysts of the past can be considered pure analysts. The analysts of the past did make use of intuition to some degree and particularly the pure mathematical ideal.

The analysts of the past discovered not because of their analytical method alone but because they integrated it with the fruits of induction.

> ... I have shown how this reasoning (i.e. mathematical reasoning) without ceasing to be absolutely rigorous, could lift us from the particular to the general by a procedure I have called mathematical induction. It is by this procedure that the analysts have made science progress, and if we examine the detail itself of their demonstrations, we shall find it there at each instant beside the classic syllogisms of Aristotle... the analysts are not simply makers of syllogisms after the fashion of the scholastics. 14

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13 Poincaré, The Value of Science, p. 23.

- Croit-on d'abord que ces logiciens ont toujours procédé du général au particulier, comme les règles de la logique formelle semblaient les y obliger? Ce n'est pas ainsi qu'ils auraient pu étendre les frontières de la science; on ne peut faire de conquête scientifique que par la généralisation. (V.S.: p. 30)

14 Ibidem, pp. 23-24.

- ... j'ai montré comment ce raisonnement (i.e. raisonnement mathématique), sans cesser d'être absolument rigoureux, pouvait nous élever du particulier au général par un procédé que j'ai appelé l'induction mathématique. C'est par ce procédé que les analystes ont fait progresser la science et si l'on examine le détail même de leurs démonstrations, on l'y retrouvera à chaque instant à côté du syllogisme classique d'Aristote ... Nous voyons donc déjà que les analystes ne sont pas simplement des faiseurs de syllogismes à la façon des scolastiques. (V.S.: p. 30)

and

> Is there room for a new distinction, for distinguishing among the analysts those who above all use this pure intuition and those who are first of all preoccupied with formal logic? 15

The absolute dependence of logic upon the direct intuition and the continued use of other types of intuition having been seen, the characteristics of the valid use of logic need to be shown in greater detail.

2. Logic: Source of Rigour

It is by means of logic that the possibility of exactness has entered in the mathematical sciences. It is able to accomplish this by giving the mathematician a number of procedures of analysis which he may apply to his creations and thereby verify if his creations involve a contradiction. The proper role of logic hence appears to be criticism of the mathematical creations which are the fruit of intuition. If analysis shows that a creation leads to a contradiction, the mathematician will recognize the error to be a result of an intuition represented by a rough image. Logic makes possible exactness and certitude - i.e. rigour - but it is able to do so only in confirming

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15 Poincaré, The Value of Science, pp. 25

- Y a-t-il lieu de faire une nouvelle coupure et de distinguer parmi les analystes ceux qui se servent surtout de cette intuition pure ou ceux qui se préoccupent d'abord de la logique formelle? (V.S.: p. 34)

particular instances of mathematical creations. As J.J.A. Mooij notes:

> La certitude que la logique est en mesure de fournir, concerne selon Poincaré, seulement la vérification de cas particuliers. La certitude des vérités générales reposerait sur l'induction complète ou sur d'autres méthodes intuitives. 16

It is unquestionably true that Poincaré considered the acquisition of rigour by mathematics as a contribution.

> ... it is by logic that we prove but by intuition that we discover... You know how to recognize whether a combination is correct, but much use this will be if you do not possess the act of selecting among all the possible combinations. 17

Logic by rendering rigor to mathematical combinations performs a negative function by giving either confirmation or condemnation to the creations of intuition. But let us recall that Poincaré was not committed to the search for certitude:

> Il ne faut pas croire que l'amour de la vérité se confond avec celui de la certitude; loin de là, dans notre monde relatif toute certitude est un mensonge. 18

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16 J.J.A. Mooij, La Philosophie des Mathématiques de Henri Poincaré, p. 132.

17 Poincaré, Science and Method, p. 129.

- ... c'est par la logique qu'on démontre, c'est par l'intuition qu'on invente, ... Vous savez reconnaître si une combinaison est correcte; la belle affaire si vous ne possédez pas l'art de choisir entre toutes les combinaisons possibles. (S.M.: p. 137)

18 Poincaré, Savants et Ecrivains, p. vii.

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Logic imparts rigour; intuition is the source of truth. Should logic show that mathematical combination leads to a contradiction, it renders it uncertain and it is to another intuition that the mathematician must look for a true combination. Should logic show that a mathematical combination does not lead to any contradiction, it renders it certain but does not add in any way to the truth value of the intuition. Consequently it is not surprising that Poincaré does not wish to see mathematics perfectly rigorous.

> Each individual enunciation in mathematics may be always verified... But if mathematics could be reduced to a series of such verifications it would not be a science ... There is no science but the science of the general. It may even be said that the object of the exact sciences is to dispense with these direct verifications. 19

Thus the making of mathematics rigorous is only of limited value. Logic is of value only when it clarifies either the intuition of the imagination or the mathematician's method of expressing the mathematical entity.

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19 Poincaré, Science and Hypothesis, p. 4.

- Tout énoncé particulier en mathématiques pourra toujours être vérifié... Mais si la mathématique devait se réduire à une suite de pareilles vérifications, elle ne serait pas une science ... Il n'y a de science que du général. On peut même dire que les sciences exactes ont précisément pour objet de nous dispenser de ces vérifications directes. (S.H.: p. 13)

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Dr. Leon Chwistek, a mathematical analyst, criticises Poincaré on forgetting that very point.

> Poincaré forgot that getting rid of ambiguity has very great merit and importance and that a symbolic language does just this. He forgot that there is a great difference between the construction of expressions in terms of certain symbols and the use of everyday language with all its confusions and traps. 20

The "getting rid of ambiguity" was the one merit that Poincaré accepted that logic had contributed to mathematics.

Poincaré did welcome the fact that exactness had entered into mathematical demonstration. The search for exactness was in Poincaré's view inevitable and its achievement of great value for mathematics.

> Since the middle of the last century, mathematicians have been more and more anxious to attain to absolute exactness. They are quite right, and this tendency will become more and more marked. In mathematics, exactness is not everything, but without it there is nothing: a demonstration which lacks exactness is nothing at all. 21

What Henri Poincaré did object to was the implication that "getting rid of ambiguity" was the only objective of science of great merit and importance, and that a symbolic language was able to improve on the direct intuition of number.

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20 Leon Chwistek, The Limits of Science, p. 107.

21 Poincaré, Science and Method, p. 32.

- Depuis le milieu du siècle dernier, les mathématiciens sont de plus en plus soucieux d'atteindre à l'absolue rigueur; ils ont bien raison et cette tendance s'accentuera de plus en plus. En mathématiques la rigueur n'est pas tout, mais sans elle il n'y a rien; une démonstration qui n'est pas rigoureuse, c'est le néant. (S.M.: p. 26)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> ... in becoming rigorous, mathematical science takes a character so artificial as to strike every one; it forgets its historical origins; we see how the questions can be answered, we no longer see how and why they were put.
>
> This shows us that logic is not enough; that the science of demonstration is not all science and that intuition must retain its rôle as complement, I was about to say, as counterpoise or as antidote of logic. 22

Since rigour is not the primary ideal of science, and since logic alone gives rigour, the analytical approach or logic alone can not suffice as a model for the scientific mind.

3. Logic: Tool Rather than Ideal

The extent of Poincaré's acceptance of logic is quite limited. It is not a question of rejecting the truth of logic but rather a rejection of the self-sufficiency of logic for mathematical progress.

> In mathematics logic is called analysis and analysis means division, dissection... It can have, therefore, no tool other than the scalpel and the microscope. 23

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22 Poincaré, The Value of Science, p. 21.

- ... en devenant rigoureuse, la science mathématique prend un caractère artificiel qui frappera tout le monde; elle oublie ses origines historiques; on voit comment les questions peuvent se résoudre, on ne voit plus comment et pourquoi elles se posent.

Cela nous montre que la logique ne suffit pas; que la science de la démonstration n'est pas la science tout entière et que l'intuition doit conserver son rôle comme contrepoids ou comme contrepoison de la logique. (V.S.: pp. 24-25)

23 Ibidem, p. 23.

En mathématiques, la logique s'appelle analyse et analyse veut dire division, dissection. Elle ne peut donc avoir d'autre outil que le scalpel et le microscope. (V.S.: p. 29)

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As a tool it can be of value in verification, and in imparting rigor.

> It is not wings you (logistics) have given us, but leading strings. But we have the right to demand that these leading-strings should keep us from falling; this is their only excuse. 24

The use of logic in science, for Poincaré, is not indicated as an ideal but rather as a necessity (at times) required for the benefit of rendering demonstrations possible and exact. The work of analysis once having made others should proceed over these quickly, reap their conclusions and proceed to new problems.

> Thus the hair-splitters can render us a double service, first by teaching us to do as they do if necessary, but more especially by enabling us as often as possible not to do as they do, and yet make no sacrifice of exactness. 25

We have said that the principal use of logic consists in verification of particular mathematical combinations. Poincaré substantiates this view by an analogy drawn from a game of chess. In this analogy the man of pure logic is compared with a man who

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24 Poincaré, Science and Method, p. 178.

- Ce ne sont pas des ailes que vous nous donnez, ce sont des lisières. Et alors nous avons le droit d'exiger que ces lisières nous empêchent de tomber. Ce sera leur seule excuse (S.M.: p. 193)

25 Ibidem, p. 33

- Les coupeurs de difficultés en quatre peuvent donc nous rendre un double service; c'est d'abord de nous apprendre à faire comme eux au besoin, mais c'est surtout de nous permettre le plus souvent possible de ne pas faire comme eux, sans pourtant rien sacrifier de la rigueur. (S.M.: p. 29)

understands the rules for moving each individual chess piece but who lacks knowledge of the goal of the game. The knowledge such a man possesses is of very little value because even though he can verify each piece's move he cannot grasp why one piece was moved instead of another. True understanding of chess requires more than verification of individual moves; it requires a grasp of the whole.

> To understand the game is wholly another matter; it is to know why the player moves this piece rather than that other which he could have moved without breaking the rules of the game. It is to perceive the inward reason which makes of this series of successive moves a sort of organized whole. This faculty is still more necessary for the player himself, that is for the inventor. 26

As the true understanding of chess requires a grasp of the unity of moves, so also does true understanding of mathematics require the faculty of intuition which gives the mind the synthetic unity of mathematical combinations.

The analysis of the valid role of logic in mathematics having been completed, we must now examine Poincaré's criticism of the logisticians. These views which are quite negative will give greater contrast to the valid roles which we have already investigated.

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26 Poincaré, The Value of Science, p. 22.

- Comprendre la partie, c'est tout autre chose; c'est savoir pourquoi le joueur avance telle pièce plutôt que telle autre qu'il aurait pu faire mouvoir sans violer les règles du jeu. C'est apercevoir la raison intime qui fait de cette série de coups successifs une sorte de tout organisé. A plus forte raison, cette faculté est-elle nécessaire au joueur lui-même, c'est-à-dire à l'inventeur. (V.S.: p. 27)

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B. Denial of the Ultimate Reduction of Mathematics to Pure Logic

Poincaré's doctrine on the value of logic within scientific method having been seen, the inquiry must now focus on the reasons why he held the impossibility of an ultimate reduction of mathematics to pure logic. By the ultimate reduction of mathematics to pure logic, Poincaré means the attempts

> ... in proving all the propositions relating to this small class (that is to say, our whole arithmetic and algebra) without making use of a single principle foreign to logic. 27

Poincaré advances a number of arguments in attempting to refute this position both as a theoretical possibility and also as a desirable goal. A classification of these arguments is needed and the distinction between his internal critique and external critique is particularly appropriate. The internal critique centers on the theoretical reasons why the ultimate reduction is impossible. The external critique focuses on the reasons why the ultimate reduction should not even be considered desirable. The external critique is of particular significance within our context because it is here that Poincaré advances all the disadvantages he sees in

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27 Poincaré, Science and Method, p. 144.

- ... à démontrer toutes les propositions relatives à cette petite classe (c'est-à-dire toute notre arithmétique et notre algèbre) sans se servir d'aucun principe étranger à la logique. (S.M.: p. 154)

the logical goal. These disadvantages are made by Poincaré from his own epistemological viewpoint; consequently we find here not only a strong defence of the pure mathematical ideal, but even more, criticism of the logical ideal for its violation of his own criterion of truth - i.e. the simple, harmonious and unifying intuition.

1. External Critique of the Logical Ideal

a) Artificial Nature of the Logical Ideal

Poincaré notes that the mathematical analysts in their attempts to prove that mathematics can be reduced to logic have been compelled to develop their own language in which signs replace words and hence also intuition. This evolution must be regretted since it thereby restricts mathematics to but a few initiated individuals. The effect of this procedure on the uninitiated, according to Poincaré, is the instilling in them of the inclination "to bow before the decisive affirmations of the adepts". [28] Poincaré, however, attempts to reassure the uninitiated that their affirmations do not justify their claims.

Poincaré, holding as his doctrine the direct intuition of number, and that this intuition is prior in nature to logic, criticises the

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28 Poincaré, Science and Method, p. 143.

- ... à s'incliner devant les affirmations tranchantes des adeptes. (S.M.: p.152)

attempts of the logisticians to define numbers. He concludes from their attempts that their definitions will ultimately contain a petitio principii. Poincaré suspects that their exaggerated estimation for symbolic language is an attempt to conceal these difficulties. Poincaré maintains:

> The fact that it is impossible to give a definition without enunciating a phrase, and difficult to enunciate a phrase without putting in the name of number, or at least the word several, or at least a word in the plural. 29

Poincaré notes Signor Burali-Forti's definition of the number 1:

$$I = LT' \left\{ KO \frown (U,h) \in \left(u \quad One \right\} \right. .$$ 30

Using this as an example of the artificiality of the analytical approach and of the exaggerated importance of logic, Poincaré notes that 1) Signor Burati-Forti's definition could never convey the idea of one to a person not already familiar with it and 2) it must contain a petitio principii by containing the figure "1" in the first half and the word "one" in the second half.

If mathematics has its origin in the direct intuition of number which Poincaré maintains, then any attempts to establish these numbers

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29 Poincaré, Science and Method, p. 155.

- C'est qu'il est impossible de donner une définition sans énoncer une phrase, et difficile d'énoncer une phrase sans y mettre un nom de nombre, ou au moins le mot plusieurs, ou au moins un mot au pluriel. (S.M.: p. 166)

30 Ibidem, p. 157.

Cf. Science et méthode, p. 168.

by definition must become (were it possible) at least artificial. But Poincaré maintains that it is the very intuition of number which makes definition itself possible. (Clearly it would be a vicious circle to hold that the direct intuition of number makes definition possible and that definition is capable of establishing the nature of number.) Thus Poincaré holds that attempts at defining whole numbers will always involve petitio principii and thus always be artificial.

Concerning M. Couturat's definition of zero as the number of the objects that satisfy a condition that is never fulfilled, Poincaré considers this attempt as "an abuse of the wealth of language". [31] Defining zero by the term "never" which means "in no case" which means "none" which means "nil" confirms Poincaré's suspicions of artificiality and petitio principii in the analytic ideal.

Continuing his charge of artificiality, Poincaré notes that it is not sufficient for the mathematical analysts to define the number one. If they endeavour to be faithful to their own method, they must prove that one is a number. This Signor Burali-Forti claims to have established after twenty seven equations.

In calling attention to this circuitous and artificial process Poincaré notes:

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31 Poincaré, Science and Method, p. 158.

- ... c'est vraiment abuser de la richesse de la langue française... (S.M.: p. 169)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> ... if it requires twenty seven equations to prove that "1" is a number, how many will it require to demonstrate a real theorem? 32

The result of this artificial character is a positive harm to mathematics. It causes the mathematician to become disengaged from the initial problems and insights of mathematics which are provided by the direct intuition of number and found in the pure mathematical ideal.

> ... in becoming rigorous, mathematical science becomes so artificial as to strike everyone; it forgets its historical origins; we see how the questions can be answered, we no longer see how and why they are put. 33

But despite the fact that the logical ideal adds both length and symbolic language, Poincaré considers that they only succeed in displacing the problem of the origin of mathematics.

> Thus, in spite of all this pasigraphical apparatus, the question is not solved. 34

The logical ideal, hence, renders mathematics artificial by 1) imposing an artificial language of symbols and signs which force mathematics to be greatly lengthened (which lengthening does not

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32 Poincaré, Science and Method, p. 178.

- ... et s'il faut 27 équations pour établir que 1 est un nombre, combien en faudra-t-il pour démontrer un vrai théorème? (S.M.: p. 193)

33 Poincaré, The Value of Science, p. 21.

- ... en devenant rigoureuse, la science prend un caractère artificiel qui frappera tout le monde; elle oublie ses origines historiques; on voit comment les questions peuvent se résoudre, on ne voit plus comment et pourquoi elles se posent. (V.S.: pp. 24-25)

34 Poincaré, Science and Method, p. 159.

- Ainsi, malgré tout cet appareil pasigraphique, la question n'était pas résolue. (S.M.: p. 171)

answer the questions proposed) and 2) concealing in their symbolic language petitio principii.

b) Error and the Logical Ideal

Let us assume temporarily that mathematics rests on nothing other than logic, and further that logic is common to all men. Now the problem is to explain the lack of mathematical comprehension in some, and secondly, to explain the occurrence of mathematical errors. By doing this Poincaré is able to place the burden of explanation on the mathematical analysts.

> How does it happen that there are people who do not understand mathematics? If the science invokes only the rules of logic, those accepted by all well-formed minds, if its evidence is founded on principles that are common to all men, and that none but a mad man would attempt to deny, how does it happen that there are so many people who are entirely impervious to it? 35

The second part of the problem continues:

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35 Poincaré, Science and Method, p. 47.

- Comment se fait-il qu'il y ait des gens qui ne comprennent pas les mathématiques? Si les mathématiques n'invoquent que les règles de la logique, celles qui sont acceptées par tous les esprits bien faits; si leur évidence est fondée sur des principes qui sont communs à tous les hommes et que nul ne saurait nier sans être fou, comment se fait-il qu'il y ait tant de personnes qui soient totalement réfractaires? (S.M.: p. 44)

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> And still further, how is error possible in mathematics? A healthy intellect should not be guilty of any error in logic, and yet there are very keen minds ... which yet are incapable of following or repeating without error the demonstration of mathematics... 36

Poincaré's argument thus is that if mathematics is only logic, then all failures (either in comprehension or in construction) must be failures in logic. Poincaré leaves the question open to the mathematicians of the analyst school to answer.

Poincaré remains firm in maintaining that even a mathematical demonstration is never totally reducible to logic. It is the nature of mathematical demonstration that

> ... it consists of syllogisms placed in a certain order, and the order in which these elements are placed is much more important than the elements themselves. If I have the feeling, so to speak, the intuition, of the order, so that I can perceive the whole of the argument at a glance, I need no longer be afraid... 37

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36 Poincaré, Science and Method, p. 47.

- Et il y a plus: comment l'erreur est-elle possible en mathématiques? Une intelligence saine ne doit pas commettre de faute de logique, et cependant il y a des esprits très fins... qui sont incapables de suivre ou de répéter sans erreur les démonstrations des mathématiques... (S.M.: pp. 44-45)

37 Ibidem, p. 49.

- Une démonstration mathématique n'est pas une simple juxtaposition de syllogismes, ce sont des syllogismes placés dans un certain ordre, et l'ordre dans lequel ces éléments sont placés est beaucoup plus important que ne le sont ces éléments eux-mêmes. Si j'ai le sentiment, l'intuition pour ainsi dire de cet ordre, de façon à apercevoir d'un coup d'oeil l'ensemble du raisonnement, je ne dois plus craindre... (S.M.: p. 47)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Presumably Poincaré might admit that some errors in mathematics are due to failures in logic but this would have to be to the degree that he had admitted its validity. The source of error, according to Poincaré, is frequently found in failures in the "intuition of the whole", [38] and lack of "special aesthetic sensibility". [39]

A corollary to the problem of error in mathematics is also developed. This argument focuses around the problem of mathematical discovery. Again the assumption is made that if mathematics is nothing other than logic then all mathematical discoveries are nothing but new logical combinations. But if this is so how can we distinguish between new logical combinations that are valuable and others that are devoid of significance. Poincaré insists that this evaluation requires something other than logic:

> What in fact is mathematical discovery? It does not consist in making new combinations with mathematical entities already known. That can be done by anyone, and the combinations that could be formed would be infinite in number, and the greater part of them would be absolutely devoid of interest.

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38 Poincaré, Science and Method, p. 49.

- Si j 'ai le sentiment, l'intuition pour ainsi dire de cet ordre, de façon à apercevoir d'un coup d'oeil l'ensemble du raisonnement... (S.M.: p. 47)

39 Ibidem, p. 60.

- Ainsi c'est cette sensibilité esthétique spéciale... (S.M.: p. 59)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> Discovery consists precisely in not constructing useless combinations but in constructing those that are useful, which are an infinitely small minority. Discovery is discernment, selection. 40

The faculty or power which has the capacity for truth must also be the same faculty with a capacity for error. By arguing that the logical ideal is incapable of explaining the nature of mathematical errors, Poincaré is arguing indirectly to its inadequacy in explaining the nature of mathematical truth. His entire criticism can be recognized as a defense of the necessity of intuition.

c) Psychological and Historical Invalidity of the Logical Ideal

We have already had occasion to note the importance with which Poincaré invests the psychological experience which produces science. [41] The explanation and necessity of this importance is found in the criterion of truth which applies to the intuition. For an intuition to be considered true it must fulfill certain psychological needs -

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40 Poincaré, Science and Method, p. 51.

Qu'est-ce, en effet, que l'invention mathématique? Elle ne consiste pas à faire de nouvelles combinaisons avec des être mathématiques déjà connus. Cela, n'importe qui pourrait le faire, mais les combinaisons que l'on pourrait former ainsi seraient en nombre infini, et le plus grand nombre serait absolument dépourvu d'intérêt. Inventer, cela consiste précisément à ne pas construire les combinaisons inutiles et à construire celles qui sont utiles et qui ne sont qu'une infime minorité. Inventer, c'est discerner, c'est choisir. (S.M.: p. 48)

41 Cf. "The Method of Science" in Chapter II and "The Psychology of Discovery" in Chapter III.

simplicity, harmony, unity - in short, it must be mentally satisfying. Thus when Poincaré claims that the logical ideal is psychologically invalid, he is accusing it of violating his criterion of truth.

One of Poincaré's chief objections to the logical method is the fact that it shows down mental processes.

> Logistic forces us to say all that we assume, it forces us to advance step by step; it is perhaps surer, but it is not more expeditious. 42

Poincaré considers that the progress of science and scientific discovery would be jeopardized by modelling science on the criteria of logic.

> It is the economy of thought that we should aim at, and therefore it is not sufficient to give models to be copied. We must enable those that come after us to do without the models, and not to repeat a previous reasoning, but summarize it in a few lines. 43

Poincaré feared that the acceptance of logic as an ideal rather than as a tool to be utilized when required would have the effect of lengthening scientific efforts and thereby endanger the economy of

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42 Poincaré, Science and Method, p. 178.

- La logistique nous force à dire tout ce qu'on sous-entend d'ordinaire; elle nous force à avancer pas à pas; c'est peut-être plus sure, mais ce n'est pas plus rapide. (S.M.: p. 193)

43 Ibidem, p. 33.

- C'est à l'économie de pensée que l'on doit viser, ce n'est donc pas assez de donner des modèles à imiter. Il faut qu'on puisse après nous se passer de ces modèles et, au lieu de répéter un raisonnement déjà fait, le résumer en quelques lignes. (S.M.: p. 28)

thought, the role of intuition, and consequently scientific discovery itself.

> Those who were the first to pay special attention to exactness have given us reasonings that we may attempt to imitate; but if the demonstrations of the future are to be constructed on this model, mathematical works will become exceedingly long and ... I fear as they grow in length our demonstrations will lose that appearance of harmony which plays such a useful part. 44

The logical ideal is more lengthy, therefore it violates the mental need for simplicity. The construction of logical models violates the principle of the economy of thought. Lengthy models destroy the mind's grasp of harmony. Thus we see that the logical ideal tends toward the destruction of all of Poincaré's criteria of truth. It destroys the feeling of mathematical elegance which is so mentally satisfying to him. In this, no doubt we find the reason that Poincaré, ordinarily such a mild writer, attacks the logical analysts with such vengence when he writes:

> Logic sometimes breeds monsters. For half a century there have been springing up weird functions, which seem to strive to have as little

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44 Poincaré, Science and Method, p. 33.

- ... ceux qui les premiers se sont préoccupés avant tout de la rigueur nous ont donné des raisonnements que nous pouvons essayer d'imiter; mais si les démonstrations de l'avenir doivent être bâties sur ce modèle, les traités de mathématiques vont devenir très longs;... je crains qu'en s'allongeant, nos démonstrations perdent cette apparence d'harmonie dont j'ai expliqué tout à l'heure le rôle utile. (S.M.: p. 28)

> resemblance as possible to honest functions that are of some use. No more continuity... 45

And again he writes:

> It is time that these exaggerations were treated as they deserve. I have no hope of convincing these logicians, for they have lived too long in this atmosphere. Besides, when we have refuted one of their demonstrations, we are quite sure to find it cropping up again with insignificant changes... Such in old terms was the Lernaean hydra, with its famous heads that always grew again ... And so I appeal only to unprejudiced people of common sense. 46

The logical analysts in attacking the pure mathematical ideal are eliminating what in the epistemology of Poincaré is the source of all knowledge and truth. Mathematics without intuition is not only an impossibility but also an epistemological and psychological disaster. As Leon Brunschvicg notes:

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45 Poincaré, Science and Method, p. 125.

- La logique parfois engendre des monstres. Depuis un demi-siècle on a vu surgir une foule de fonctions bizarres qui semblent s'efforcer de ressembler aussi peu que possible aux honnêtes fonctions qui servent à quelque chose. Plus de continuité... (S.M.: p. 132)

46 Ibidem, pp. 145-146.

- Il est temps de faire justice de ces exagérations. Je n'espère pas les convaincre; car ils ont trop longtemps vécu dans cette atmosphère. D'ailleurs, quand on a réfuté une de leurs démonstrations, on est sûr de la voir naître avec des changements insignifiants, ... Telle autrefois l'hydre de Lerne avec ses fameuses têtes qui repoussent toujours... Je ne fais donc appel qu'aux hommes de bon sens sans parti pris. (S.M.: p. 155)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

> Par le sentiment de la beauté, il rend compte de ce que l'esprit doit ajouter à la logique proprement dite, pour avoir pleine et familière possession de la science, de cette sorte d'intuition, dans l'acceptation large que l'on peut donner à ce mot, qui fait rentrer les articulations successives d'une démonstration dans l'unité d'un tout organisé. 47

The history of the evolution of ideas of the various sciences is also a factor which Poincaré does not discount.

> The educator must make the child pass through all that his fathers have passed through, more rapidly, but without missing a stage. On this account, the history of any science must be our first guide. 48

But in the logical ideal Poincaré sees a radical break with the history of mathematics. Speaking of the logical ideal, he notes:

> This method is evidently contrary to all healthy psychology. It is certainly not in this manner that the human mind proceeded to construct mathematics... 49

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47 Leon Brunschvicg, "L'Oeuvre d'Henri Poincaré: Le Philosophe" in Op. Cit., p. 615.

48 Poincaré, Science and Method, p. 127.

- L'éducateur doit faire repasser l'enfant par où ont passé ses pères; plus rapidement mais sans brûler d'étape. A ce compte, l'histoire de la science doit être notre premier guide. (S.M.: p. 135)

49 Ibidem, pp. 144-145.

- Cette méthode est évidemment contraire à toute saine psychologie; ce n'est certainement pas comme cela que l'esprit humain a procédé pour construire les mathématiques... (S.M.: p. 154)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

d) The Sterile Character of the Logical Ideal

Granting that the logical ideal is artificial, psychologically and historically invalid, that the logical ideal should not be productive is but a corollary. Yet the only defense with which the mathematical analysts could and did counter Poincaré's arguments was to claim that their method could lead to mathematical progress. But Poincaré maintains that not only do their works not satisfy his mathematical demands but that they themselves have not accomplished what they attempted.

> As regards fruitfulness, it seems that M. Couturat has most childish illusions. Logistic, according to him, lends "stilts and wings" to discovery, and... he says, "It is ten years since Sinor Peano published the first edition of Formulaires".
>
> What! You have had wings for ten years and you haven't flown yet!
>
> ... He (Sinor Peano) has not gone any further, or higher, or faster than the majority of wingless mathematicians, and he could have done everything just as well on his feet. 50

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50 Poincaré, Science and Method, pp. 177-178.

- En ce qui concerne la fécondité, il semble que M. Couturat se fasse de naïves illusions. La Logistique, d'après lui, prête à l'invention "des échasses et des ailes" et à la page suivante: "Il y a dix ans que M. Peano a publié la première édition de son Formulaire".

Comment, voilà dix ans que vous avez des ailes, et vous n'avez pas encore volé!

... il n'est allé ni plus loin, ni plus haut, ni plus vite que la plupart des mathématiciens aptères, et il aurait pu faire tout aussi bien avec ses jambes. (S.M.: pp. 192-193)

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Poincaré, not content with claiming the logical ideal fruitless, advances a further distinction. Should it occur that a mathematician who claims to be an analyst would make a genuine mathematical creation, one would have to guard against considering it a vindication of the logical ideal. This occurrence, admittedly rare, would rather be the result of a very special type of intuition - an intuition of a logical edifice. [51] This intuition of a logical edifice would be a second instance of a pure intuition. Thus Poincaré postulates the possibility that some analysts possess a special high type of intuition - an intuition of logical structures.

> Is there room for a new distinction for distinguishing among the analysts those who above all use this pure intuition and those who are first of all preoccupied with formal logic? 52

The possibility of mathematical contributions by the application of pure logic without the concomitant assistance of intuition is precluded.

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51 Poincaré, The Value of Science, p. 25.

- Cf. Valeur de la Science, p. 33.

52 Ibidem.

Y a-t-il lieu de faire une nouvelle coupure et de distinguer parmi les analystes ceux qui se servent surtout de cette intuition pure ou ceux qui se préoccupent d'abord de la logique formelle? (V.S.: p. 34)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

2. Internal Critique of the Logical Ideal

a) The Principle of Complete Induction and Axiomatic Systems

In denying that mathematics can be reduced to pure logic, Poincaré is actually asserting that the validity of logical operations must ultimately be reduced to the principle of complete induction.

> ... the principle of complete induction appears to me at once necessary to the mathematician, and irreducible to logic. 53

In attempting to prove all of mathematics by pure logic, the axiomatic systems consider the principle of complete induction as the disguised definition of the whole number. Poincaré's thesis is that in denying that the principle of complete induction is a synthetic judgment and in considering it a mere definition, axiomatic systems are forced by their very nature to fall into contradiction by committing a petitio principii. The reason for this is that (1) deductive systems must rest upon a system of postulates and (2) that the meaning of existence in mathematics is freedom from contradiction. Now the system of postulates in an axiomatic system must all be

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53 Poincaré, Science and Method, p. 149.

- ... "le principe d'induction complète" ne paraissait à la fois nécessaire au mathématicien et irreductible à la logique. (S.M.: p. 159)

affirmed by definition since otherwise they would have to be furnished by intuition. But before a definition can be accepted it is necessary to establish its existence - i.e. that it does not involve any contradictions. But the consequences of some of these definitions are infinite; establishing thus their freedom from contradictions would require an infinite series of deductions. But the construction of an infinite series of deductions is impossible.

In short, if the principle of complete induction is reduced to the level of an axiom, it is necessary to show that it does not involve contradiction. But the only way to show that it will never involve contradiction is by a complete induction which requires the very principle we are seeking to establish.

Now for Poincaré the science of pure quantity is not purely deductive, since the notion of number is directly intuited and since the continuum is produced by a complete induction. Every deductive system must ultimately rest upon a small number of indefinable notions and a small number of indemonstrable propositions. [54]

The truth value of science depends upon the truth of these indefinable notions and indemonstrable propositions. In geometry Poincaré admits that they are conventions, but the result is that geometry cannot be considered objectively true. But in arithmetic Poincaré considers these notions intuitively true.

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54 Cf. "The Role of Presuppositions" in Chapter II.

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Now Poincaré's criticism of the logical analysts is (1) that they cannot deduce the notion of number and (2) that they cannot establish their axioms without employing the principle of complete induction.

Let us examine these two criticisms. Logical analysis, according to Poincaré, claims to be prior to the notion of number.

> M. Couturat is fond of repeating that this new logic is quite independent of the idea of number. I will not amuse myself by counting how many instances his statement contains of adjectives of number, cardinal as well as ordinal, or of indefinite adjectives such as several. 55

However, Poincaré considers this a theoretical impossibility.

> A relation is incomprehensible without two terms. It is impossible to have the intuition of a relation without having at the same time the intuition of its two terms. 56

Poincaré's second criticism is that an axiomatic system must demonstrate that its axioms do not lead to contradiction but that this can only be demonstrated by the principle of complete induction.

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55 Poincaré, Science and Method, p. 164.

- M. Couturat répète souvent que cette logique nouvelle est tout à fait indépendante de l'idée de nombre. Je ne m'amuserai pas à compter combien son exposé contient d'adjectifs numéraux, tant cardinaux qu'ordinaux, ou d'adjectifs indéfinis, tels que plusieurs. (S.M.: pp. 176-177)

56 Ibidem.

- Une relation est incompréhensible sans deux termes; il est impossible d'avoir l'intuition de la relation, sans avoir en même temps celle de ses deux termes,... (S.M.: p. 177)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

Concerning the meaning of existence for mathematical entities, Poincaré notes:

> Mathematics is independent of the existence of material objects. In mathematics the word exist can only have one meaning; it signifies exemption from contradiction. 57

Thus the first duty of an axiomatic system is to establish that its axioms do not lead to contradiction.

> But we have seen that in order to allow of a definition by postulates being accepted, we must be able to establish that it implies no contradiction. 58

But, according to Poincaré, the only way of establishing the absence of contradiction in an infinite sequence is by the principle of complete induction. This he maintains is actually done in geometry.

> What is, in fact, the fundamental theorem of geometry? It is that the axioms of geometry do not involve contradiction, and this can not be demonstrated without the principle of induction. 59

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57 Poincaré, Science and Method, pp. 151-152.

- Les mathématiques sont indépendantes de l'existence des objets matériels; en mathématiques le mot exister ne peut avoir qu'un sens, il signifie exempt de contradiction. (S.M.: p. 162)

58 Ibidem, p. 165. (Emphasis added)

- Mais nous avons vu que, pour qu'une définition par postulats puisse être acceptée, il faut que l'on puisse établir qu'elle n'implique pas contradiction. (S.M.: p. 178)

59 Ibidem, p.171. (Emphasis added)

-Quel est en somme le théorème fondamental de la géométrie? C'est que les axiomes de la géométrie n'impliquent pas contradiction et, cela, on ne peut pas le démontrer sans le principe d'induction. (S.M.: p. 185)

But it is impossible to demonstrate by the methods of mere logical analysis that the principle of complete induction does not lead to contradiction.

> What does this mean? It means that when he comes to demonstrate that the definition of the whole number by the axiom of complete induction does not involve contradiction, Mr. Hilbert breaks down, just as Mr. Russell and M. Couturat broke down, because the difficulty is too great. 60

We are led to agree with the observation of Robert d'Adhémar that Poincaré considers that they have misunderstood the dynamic character of mathematical reasoning. 61

b) The Nature of the Syllogistic Reasoning

Basic to Poincaré's criticism of logical analysis is his objection to syllogistic reasoning. If mathematics were capable of being reduced to the principles of formal logic, Poincaré considers that arithmetic would be one vast tautology.

> The syllogism can lead us to nothing essentially new, and if everything must spring from the principle of identity, then everything should be capable of being reduced to that principle. 62

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60 Poincaré, Science and Method, p. 170. (Emphasis Poincaré's)

- Qu'est-ce à dire? Au moment de démontrer que la définition du nombre entier par l'axiome d'induction complète n'implique pas contradiction, M. Hilbert se dérobe comme se sont dérobés MM. Russell et Couturat, parce que la difficulté est trop grande. (S.M.: p. 184)

61 Ses raisons paraissent écrasantes pour ses adversaires auxquels d'ailleurs il reconnait un grand talent, mais qui méconnaissent le caractère essentiellement dynamique du raisonnement scientifique. Robert d'Adhémar, Henri Poincaré, p. 280.

62 Poincaré, Science and Hypothesis, p. 1

- Le syllogisme ne peut rien nous apprendre d'essentiellement nouveau et, si tout devait sortir du principe d'identité, tout devrait aussi pouvoir s'y ramener. (S.H.: p. 10)

IN DEFENSE OF THE PURE MATHEMATICAL IDEAL: INTUITION VS. LOGIC

But since mathematics is not one vast tautology, it is not capable of being reduced to the principles of formal logic.

> Syllogistic reasoning remains incapable of adding anything to the data that are given it; the data are reduced to axioms, and that is all we should find in the conclusions. 63

and

> ... it must be granted that mathematical reasoning has of itself a kind of creative virtue, and is therefore to be distinguished from the syllogism. 64

Here we have the key to all of Poincaré's criticisms of the logical method. Mathematical reasoning is creative; the mind is active and hence intuition must play a part. Poincaré has defended the pure mathematical ideal:

> Je crois qu'en insistant sur la méthode de récurrence, Poincaré a voulu donner l'exemple, le plus facile à saisir du raisonnement dynamique, de l'induction sûre. 65

Poincaré's critique of the role of logic is at the same time a defense of the pure mathematical ideal; the defense of the pure mathematical ideal was made primarily from the viewpoint of his criterion of truth and always consistently with his epistemological position.

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63 Poincaré, Science and Hypothesis, p. 2

- ... le raisonnement syllogistique reste incapable de rien ajouter aux données qu'on lui fournit; ces données se réduisent à quelques axiomes et on ne devrait pas retrouver autre chose dans les conclusions. (S.H.: p. 10)

64 Ibidem, p. 3.

- ... il faut bien concéder que le raisonnement mathématique a par lui-même une sorte de vertu créatrice et par conséquent qu'il se distingue du syllogisme. (S.H.: p. 11)

65 Robert d'Adhémar, Henri Poincaré, p. 259.

SUMMARY

In Chapter III we stressed the importance of the pure mathematical ideal, not only for arithmetic, but for all knowledge. We submitted that the essential functions performed by the pure mathematical ideal in the hierarchy of knowledge would make its defense a matter of extreme concern. In the foregoing analysis we have seen that logic can have a valid or an invalid use. The question of whether logic is being used validly or invalidly is directly situated in its relationship to intuition. There can be no valid use of logic if it is separated from the direct intuition of number, and the complete mathematical induction. There can be no valid use of logic if it attempts to renounce the use of secondary intuitions. In short the valid use of logic is always subordinated to the direct intuition and secondary intuitions.

The value of logic consists not in discovery, but in the verification of particular mathematical combinations. Logic is capable of rendering rigorous a mathematical concept already known. But its analytic character precludes its functioning in the construction of science; it goes from the particular to the general.

Mathematics cannot be reduced to pure logic. Such an attempt would require that all errors in mathematics be reduced to faults in logic and all discoveries in mathematics be attributed to advances in logic. Poincaré denies this possibility. The logical ideal is psychologically invalid and there is for Poincaré no valid

SUMMARY

logic and epistemology independent of psychology. The logical ideal denies the mental need for simplicity, harmony, and unity. By so doing, the logical ideal results not only in fruitlessness but also in antimonies.

Poincaré maintains rather that logic is dependent by its own nature on the pure mathematical ideal, without which there can be no definition and no universality. The pure mathematical ideal is consequently both irreducible and irreplaceable.

CHAPTER V

PHILOSOPHICAL CONCLUSIONS: EPISTEMOLOGICAL PROBLEMS

In this chapter we seek a synthesis of Poincaré's fundamental epistemology. Before proceeding to formulate this position in which the foregoing analyses are the chief contributors, we need to take account of the opinions rendered by others. Thus we will first examine several critiques of Poincaré's philosophy which we cannot accept at least without reservations. Here we recognize the inadequacy of considering Poincaré's position a positivism since, according to Poincaré himself, the data of sense experience are unable to establish any scientific hypotheses. Likewise we must note the inadequacy of considering Poincaré's position a pragmatism - unless considered in a restricted sense. We notice that, in order to preserve the intellectualistic character of science and thereby reject the notion of science as a rule of action, Poincaré places the foundation of science upon a philosophy of thought.

In rendering explicit the epistomology common to the two types of science, the following characteristics are presented: that both logic and experience perform negative functions and that the primacy of intuition is linked to Poincaré's psychological position in which the mind is creative and synthetic. The necessity and nature of Poincaré's criterion of truth is examined. The necessity is a result of some theoretical basis for distinguishing

between the various intuitions. The nature of the criterion consists in the satisfaction and beauty found in simplicity, harmony, and unity. Finally the relative nature of truth is considered and we note that Poincaré is optimistic even in his relativism since this relativism is supreme within the mental order.

A. Examination of Selected Critiques of Poincaré's Philosophy

Now that the essential elements of Poincaré's doctrine of the two types of science have been examined, we can compare the epistemology common to these elements against the various interpretations which have been made of it. The thesis we wish to establish in this section is that Poincaré's doctrine is consistent only by means of an intellectualistic interpretation. Let us reserve the analysis of the characteristics of this intellectualism for the final section. In saying that Poincaré's doctrine requires an intellectualistic interpretation, we are saying that his doctrine stands in contrast to the positions of empiricism, extreme pragmatism, logical positivism, and positivistic utilitarianism. Two trends can be recognized in the interpretation of Poincaré's thought: the one rightly preserves the intellectualist character of his position, the second attempts to interpret the doctrine in terms of pragmatism and utilitarianism. Let us consider the case against the latter interpretation and inquire the reasons such an interpretation might have been suggested.

PHILOSOPHICAL CONCLUSIONS: EPISTEMOLOGICAL PROBLEMS

1. Inadequacy of Positivistic Interpretations

Let us examine the evidence against an interpretation which would accommodate Poincaré's position with empiricism. The treatment of "The Role of Presuppositions" in Chapter II provides the doctrine which precludes an empiricist interpretation. All sciences (except arithmetic) rest necessarily upon presuppositions and common to all presuppositions are the characteristics of being unproved, freely chosen, and unable to be contradicted by experience. Generalization is incapable of being explained within Poincaré's doctrine by experience alone.

> Pour Poincaré comme pour Claude Bernard l'expérience n'est qu'une occasion pour la vérification d'une idée qui préexiste dans notre esprit. Mais Poincaré va plus loin que Claude Bernard, en ce sens qu'il limite plus le rôle de l'expérience. 1

The case in favour of interpreting Poincaré's position as even a modified form of positivism faces insurmountable problems. One is forced to explain away a great number of rationalistic statements which stress the fact that science is primarily a result of mental operations and consequently only incidentally related to sense data while at the same time attempting to establish that

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1 J. Benrubi, Les sources et les courants de la philosophie contemporaine en France, p. 352.

scientific hypothesis is really applicable in a direct way to nature. In short these attempts must seek to make Poincaré's position appear as some form of realism. Harold Chapman Brown, in attempting this interpretation, states the problem quite adequately:

> M. Poincaré's idealism accords verbally with traditional statements, ... Either this scientist whose thinking is most exact in the field of science, experiences an astounding laspe of logical intuition when he enters the realm of philosophy, or else his idealism must be read without those Berkeleian connotations the above quoted idealistic phrases usually carry with them. 2

Harold Chapman Brown suggests that a possible way of avoiding the idealistic interpretation is to interpret Poincaré's theory of consciousness in terms of the "new behaviorism" instead of a "representative theory". But such a behavioristic interpretation of Poincaré's theory of consciousness appears difficult to justify. According to Poincaré, the qualities within sensations are impenetrable and further the beauty of a sensation is not the result of the qualities present within it but rather its relationship to ideas.

> Sensations are therefore intransmissible, or rather all that is pure quality in them is intransmissible and forever impenetrable. 3

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2 Harold Chapman Brown, "The Work of Henri Poincaré" in Op. Cit., p. 231.

3 Poincaré, The Value of Science, p. 136. (Emphasis added)

- Les sensations sont donc intransmissibles, ou plutôt tout ce qui est qualité pure en elles est intransmissible et à jamais impénétrable. (V.S.: p. 263)

and

> Such a sensation is beautiful, not because it possesses such a quality, but because it occupies such a place in the woof of our associations of ideas... 4

Poincaré is explicit in assuring his readers that the qualities and appearances which strike the senses have nothing to do with science.

> I am not speaking, of course, of that beauty which strikes the senses, of the beauty of qualities and appearances. I am far from dispising this, but it has nothing to do with science. 5

In "The Subject Matter of Science" of Chapter II we have seen that Poincaré has called the validity of sense knowledge into question. Poincaré restricts the use of "know" to relation and employs "observe" to phenomenon. Moreover, in attributing the possibility of error to intuitions of sense and the immunity of error to the pure intuition, we find added difficulties for the behavioristic thesis.

The second difficulty which a posivistic interpretation of Poincaré's thought must establish is that scientific theories

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4 Poincaré, The Value of Science, p. 137.

- Telle sensation est belle, non parce qu'elle possède telle qualité, mais parce qu'elle occupe telle place dans la trame de nos associations d'idées, ... (V.S.: p. 264)

5 Poincaré, Science and Method, p. 22.

- Je ne parle pas ici, bien entendu, de cette beauté qui frappe les sens, de la beauté des qualités et des apparences; non que j'en fasse fi, loin de là, mais elle n'a rien à faire avec la science; ... (S.M.: p. 15)

really do apply in a direct way to nature. But in "The Subject Matter of Science" of Chapter II we have shown that Poincaré holds such an application to be impossible.

2. Inadequacy of a Pragmatic Interpretation

It is in the designation of Poincaré's philosophy as pragmatism that the analysis becomes more involved and more cognent. For the present let us direct our investigation of the question of pragmatism and Poincaré to these critiques which have maintained that his philosophy ultimately reduces to pragmatism and defer until later those critiques which attribute the characteristic of pragmatism to him in a very restricted sense such as J. Benrubi and René Berthelot have done. It is with the charge of extreme pragmatism that we are concerned. There appears to be two different types of analysis which may be followed in the attempt to establish that there are pragmatic foundations of Poincaré's doctrine. Let us term them the direct and indirect reduction; as an instance of the direct reduction let us take the analysis of A.C. Benjamin, and as an example of indirect reduction let us take the example of M. LeRoy.

By the direct reduction we mean that analysis of Poincaré's doctrine which interprets the notion of convenience directly in terms

of utility and practical application thereby really avoiding the intellectualistic character of Poincaré's doctrine. By the indirect reduction we mean that analysis which recognizes the idealistic character of Poincaré's position but then attempts to reduce it to nominalism which has the effect of destroying all objectivity to science, leaving it with no function except serving as a rule of action.

Let us consider first the attempts at a direct reduction of Poincaré's philosophy to pragmatism. The question centers on the meaning of "convention".

> Unfortunately Poincaré has not told us precisely what is meant by convention. Presumably a convenient symbol would be one which is useful... 6

If this interpretation were correct it would mean that the scientist would choose among various hypotheses on the basis of an utilitarian criterion rather than on an epistemological criterion. Science would thereby have as its goal application rather than knowledge. But Poincaré denies this evaluation of science.

> It should not even be said that action is the goal of science;... To my eyes on the contrary, it is the knowledge which is the end, and the action which is the means. 7

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6 A.C. Benjamin, An Introduction to the Philosophy of Science, p. 162.

7 Poincaré, The Value of Science, p. 115.

- On ne peut même pas dire que l'action soit le but de la science; ... A mes yeux au contraire, c'est la connaissance qui est le but, et l'action qui est le moyen. (V.S.: p. 220)

and

> Besides, a science made solely in view of applications is impossible; truths are fecund only if bound together. If we devote ourselves solely to those truths whence we expect an immediate result, the intermediary links are wanting and there will no longer be a chain. 8

The direct reduction of Poincaré's doctrine of conventions to a pragmatic foundation appears an oversimplification. Poincaré could not have held the utilitarian concept of convention while at the same time denying the utilitarian concept of science. While rejecting the pragmatic interpretation of the notion of convention, we can recognize four reasons why such an interpretation might have suggested itself.

First, in employing the term "convenient" (commode), Poincaré has chosen a common word but he applies it in a technical sense. Our thesis however is that we must seek the meaning of the word "convenient" within the context of Poincaré's doctrine of psychology rather than within grammatical usage. Poincaré does, as we have seen, [9] place great emphasis on the psychological foundations of science.

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8 Poincaré, The Value of Science, p. 75.

- D'ailleurs, une science uniquement faite en vue des applications est impossible; les vérités ne sont fécondes que si elles sont enchaînées les unes aux autres. Si l'on s'attache seulement à celles dont on attend un résultat immédiat, les anneaux intermédiaires manqueront, et il n'y aura plus de chaîne. (V.S.: p. 138)

9 Cf. "The Method of Science" in Chapter II, "The Psychology of Discovery" in Chapter III, and "Psychological and Historical Invalidity of the Logical Ideal" in Chapter IV.

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A second reason why a direct pragmatic interpretation of Poincaré's position might have suggested itself is the fact that Poincaré has not given any indications regarding the origins of his conventionalism. Moreover he has left few clues on his relationship to other philosophical positions and he does not acknowledge possible sources of philosophical influence.

> Poincaré left no written material explicitely answering the questions. Nor do any of his letters in the Bibliothèque Nationale, l'Académie des Sciences, or l'Institut de France reveal his relationship to the philosophy of his age. In his correspondence he talks of his scientific contemporaries, and occasionally of men interested in the philosophy of science, but never of his intellectual debt to these or other men. 10

A third factor which may have contributed to the tendency of interpreting Poincaré's position in terms of a pragmatism is the fact that he so frequently philosophizes from very esoteric experiences and in isolation from others rather than from common experiences. The description of the psychological experience which serves as the foundation of science by such terms as "special aesthetic sensibility"[11] and "special intuition" [12] shows the highly individualistic character of Poincaré's point of philosophical departure.

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10 James A. Gould, "The Origin of Poincaré's Conventionalism" in Revue Internationale de Philosophie, issue of 1961, p. 116.

11 Poincaré, Science and Method, p. 60.

Cf. Science et Méthode, p. 59.

12 Ibidem, p. 50.

Cf. Science et Méthode, p. 48.

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A fourth factor contributing to the interpretation of Poincaré's position as a pragmatism is the philosophical trend of logical positivism which developed shortly after his death. By starting from his position on the equivalence of hypotheses they see in his position a precursor to their own position, i.e. that the choice from among hypotheses must be made on the basis of verification. Such an interpretation in a sense uses Poincaré's intellectualism against himself. Because of his intellectual arguments to prove the equivalence of hypotheses, one does find a real need for a criterion which can explain the preference of one hypothesis over another, and by inserting the logical positivist criterion, Poincaré could be considered a precursor to the Vienna Circle Movement. René Poirier notes the highly rationalistic means used by Poincaré in establishing the equivalence of hypotheses:

> On a plutôt le sentiment qu'il y a toujours en lui deux hommes: le philosophe qui formule des interprétations logiquement possibles, le savant qui juge certaines d'entre elles pratiquement infécondes et s'en désintéresse. 13

Let us now proceed to an examination of the indirect reduction of Poincaré's position to a pragmatism. This approach preserves

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13 René Poirier, "Henri Poincaré et le Problème de la Valeur de la Science", Op. Cit., p. 491.

much better than the former, the idealistic character of Poincaré's position. However by laying stress on the idealistic character of his position in the creative activity of the scientist and contrasting it with the practical needs of science, it concludes that Poincaré's position must result in a nominalism. In such a view science becomes totally unable to teach us anything of objective reality. Accordingly science can have no other function than serving as a rule of action. Such an interpretation of Poincaré's position is most frequently attributed to M. Edouard LeRoy; [14] Poincaré certainly interpreted it as such. Fortunately, M. LeRoy, who uses Poincaré's position in support of his own doctrine, advanced his views during Poincaré's lifetime and thus we can investigate the nature of Poincaré's reply. Now Poincaré does not appear to be as concerned by the charge of nominalism as with the resulting anti-intellectualism which its application to science entails.

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14 We say "attributed" since M. LeRoy denies that Poincaré interpreted his position correctly. "Je n'accepte pas une seule des objections de M. Poincaré. Elles reposent toutes sur une fausse interprétation de ma doctrine." Edouard LeRoy, Dogme et Critique, Paris, Librairie Bloud, 1907, pp. 333-334.

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> Here we recognize the philosophic theory known under the name of nominalism; all is not false in this theory; its legitimate domain must be left out, but out of this it should not be allowed to go.
>
> This is not all; M. LeRoy's doctrine is not only nominalistic; it has besides another characteristic which it doubtless owes to M. Bergson, it is anti-intellectualistic. According to M. LeRoy, the intellect deforms all it touches... 15

Poincaré appears to face great difficulty in denying the philosophical reduction of his position to nominalism. Although he denies that the scientist creates the brute fact, he has difficulty in seeking to establish that the scientist does not create the scientific fact.

> ... to say ... that scientific fact itself was created by the scientist. This is going too far in the direction of nominalism. No, scientific laws are not artificial creations; we have no reason to regard them as accidental, though it be impossible to prove they are not. 16

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15 Poincaré, The Value of Science, p. 112.

- On reconnait là la théorie philosophique connue sous le nom de nominalisme; tout n'est pas faux dans cette théorie; il faut lui réserver son légitime domaine, mais il ne faudrait pas non plus l'en laisser sortir. Ce n'est pas tout, la doctrine de M. LeRoy n'est pas seulement nominaliste; elle a encore un autre caractère qu'elle doit sans doute à l'influence de M. Bergson, elle est anti-intellectualiste. Pour M. LeRoy, l'intelligence déforme tout ce qu'elle touche... (V.S.: p. 214)

16 Ibidem, p. 14. (Emphasis added)

- ... que le fait scientifique lui-même étaient créés par le savant. C'est là aller beaucoup trop loin dans la voie du nominalisme. Non, les lois scientifiques ne sont pas des créations artificielles; nous n'avons aucune raison de les regarder comme contingentes, bien qu'il nous soit impossible de démontrer qu'elles ne le sont pas. (V.S.: p. 9)

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Unable thus to refute the charge of nominalism which is imputed to his doctrine, Poincaré directs his refutation of the pragmatic and anti-intellectual consequences of this nominalism by situating the very limits of objectivity to thought itself.

> Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt, a reality completely independent of the mind which conceives it, sees or feels it, is an impossibility. A world as exterior as that, even if it existed, would for us be forever inaccessible. 17

In order to reject the conclusion that science is a rule of action, Poincaré must refute the anti-intellectualism of M. LeRoy's argument.

> I wish to speak of science, and about it there is no doubt; ... it will be intellectualist or it will not be at all. 18

But the intellectualism of science can be maintained only if science teaches us something of objective reality. As we have seen, [19] Poincaré maintains that science does not teach us about nature and

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17 Poincaré, The Value of Science,

- Cette harmonie que l'intelligence humaine croit découvrir dans la nature, existe-t-elle en dehors de cette intelligence? Non, sans doute, une réalité complètement indépendante de l'esprit qui la conçoit, la voit ou la sent, c'est une impossibilité. Un monde si extérieur que cela, si même il existait, nous serait jamais inaccessible. (V.S.: p. 9)

18 Ibidem, p. 114.

- ... je veux parler de la science et pour elle, il n'y a pas de doute; ... elle sera intellectualiste ou elle ne sera pas. (V.S.: p. 217)

19 Cf. "The Subject Matter of Science" in Chapter II.

hence the reason he cannot refute the charge of nominalism which has reference to the extra-mental world. Poincaré maintains the intellectualism of science by an identification of objective reality itself with mathematical harmony.

> But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all; this common part, we shall see, can only be the harmony expressed by mathematical laws. It is this harmony then which is the sole objective reality, the only truth we can attain;... 20

Thus Poincaré is able to maintain that science does teach us something objective by equating objective reality with mathematic harmony. Mathematical analogy applied to the extra-mental is only capable of revealing the relations between things; but the relations between things are thus the only objective reality which can be attributed to them. Thus in defending the intellectualistic character of science, Poincaré has grounded it in a metaphysical position.

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20 Poincaré, The Value of Science, p. 14. (Emphasis added)

- Mais ce que nous appelons la réalité objective, c'est, en dernière analyse, ce qui est commun à plusieurs êtres pensants, et pourrait être commun à tous; cette partie commune, nous le verrons, ce ne peut être que l'harmonie exprimée par des lois mathématiques. C'est donc cette harmonie qui est la seule réalité objective, la seule vérité que nous puissions atteindre... (V.S.: pp. 9-10)

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> ... the metaphysical bases of this doctrine are essentially obscure. He (Poincaré) seems to maintain an agnostic realism - insisting that we cannot know the nature of things themselves - but are limited to a knowledge of their relations. 21

Poincaré is forced into an agnostic realism:

> But better; in the same language it may very well be said: these two propositions, the external world exists, or, it is more convenient to suppose that it exists, have one and the same meaning. 22

Poincaré refutes the position imputed by M. LeRoy - that anti-intellectualism which would reduce science to a rule for action - and defends his own position, of science for its own sake, not by repelling the charge of nominalism but by positing a philosophy of thought.

> All that is not thought is pure nothingness; since we can think only thoughts and all the words we use to speak of things can express only thoughts, to say that there is something other than thought, is therefore an affirmation which can have no meaning. 23

From our examination of selected critiques of Poincaré's philosophy let us note the inadequacy of any designation which does

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21 Morris Cohen, Reason and Nature, p. 202.

22 Poincaré, The Value of Science, p. 140.

- Mais il y a mieux; dans le même langage on dira très bien: ces deux propositions, le monde extérieur existe, ou, il est plus commode de supposer qu'il existe, ont un seul et même sens. (V.S.: p. 272)

23 Ibidem, p. 142.

not take note of his fundamental position of rationalism. The classification of Poincaré's position as an extreme pragmatism, or logical positivism, or positivistic utilitarianism insofar as they fail to take account of his idealism must be avoided. That which remains is to attempt to make a positive synthesis of Poincaré's epistemological foundations.

B. The Epistemological Foundations of Poincaré's Philosophy

Our present task is to synthesize the various elements which the analysis of Poincaré's position has revealed. It is a further precision of the nature of Poincaré's idealism which must be considered. Poincaré's epistemology and psychology are intimately linked, and common to both is the belief in the synthetic characteristic of the mind as it reveals itself in intuition. There can be no epistemology according to Poincaré independent of the psychological experience.

> Mr. Russell will tell me no doubt that it is not a question of psychology, but of logic and epistemology; and I shall be led to answer that <u>there is no logic and epistemology independent of psychology</u>... 24

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24 Poincaré, <u>Mathematics and Science: Last Essays</u>, p. 64. (Emphasis added)

- M. Russell me dira sans doute qu'il ne s'agit pas de psychologie, mais de logique et d'épistemologie; et moi, je serai conduit à répondre qu'il n'y a pas de logique et d'épistemologie indépendantes de la psychologie. (D.P.: p. 139)

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Poincaré's rationalism is defined by the nature and characteristics of the psychological experience. It is the essentially creative function of the human psyche joined with a representation theory of consciousness which results in the primacy of intuition. It is the primacy of intuition which locates all the types of discovery in the inventive and creative mind of the scientist.

The primacy of intuition in Poincaré is noted by his nephew, Pierre Boutroux:

> Quelle idée doit-on faire de ces principes scientifiques qui ne sont pas empiriques, mais sont néanmoins tirés de l'expérience, qui ne sont pas synthétiques a priori, mais sont néanmoins créés par l'esprit, qui ne sont pas des définitions arbitraires,mais sont néanmoins des conventions?... C'est ainsi qu'il a été amené à formuler plus explicitement qu'il ne l'avait fait tout d'abord - ses idées sur l'intuition. 25

Thus we can see that the two fundamental principles of Poincaré, namely the creative freedoms of the mind and the essentially synthetic (conventional) character of science, are basically Kantian. By limiting the synthetic a priori to the science of pure quantity, he provides the conventional sciences with greater fluidity than Kant.

> In saying that they (conventions) are not synthetic a priori judgments he (Poincaré) is breaking free from the Kantian rigidity. 26

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25 Pierre Boutroux, Henri Poincaré: L'Oeuvre Scientifique, l'Oeuvre Philosophique, pp. 258-259.

26 Gavin Ardley, Aquinas and Kant, p. 105.

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However by reducing all knowledge and even objective reality itself to the pure mathematical ideal, or imitations of it by means of analogy, Poincaré can be considered a hyper-Kantian.

Our aim is Poincaré's notion of truth. The recognition of Kantian influences and similarities is not sufficient for such an undertaking. The epistemology common to the two types of science - conventional and synthetic a priori - provides the basis for this doctrine.

1) Theory of Conventions Implies an Epistemology

That is the epistemological significance of the doctrine of conventions? The answer to this question lies in the very great importance such a theory places within the knowing operations. Scientific theories are not constructed from the crude fact which is given through sensation; they are not derived from sense experience by means of abstraction; thus scientific theories have no real applicability to extra-mental events. The true applicability of laws according to Poincaré must be limited to the intellect which conceives theory.

> Up to this point we did not seem to worry whether the laws really vary but only whether man can consider them variable. Are laws, considered as existing outside the mind that

> creates or observes them, unalterable in themselves? Not only is the question impossible of solution but it is meaningless. 27

The method of discovery in the conventional sciences - even the experimental ones - rests upon the primacy of intuition.

> The method of demonstration is not the same for the physicist as for the mathematician. But their methods of discovery are very similar. 28

What characteristics does the primacy of intuition establish within the conventional sciences? The primacy of intuition reduces the theories of the conventional sciences to creative constructions of the mind of the scientist, produced by the activity of the unconscious human psyche. The result of this is a purely negative function for the process of empirical verification. Since there is no one true explanation for any experimental fact and since

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27 Poincaré, Mathematics and Science: Last Essays, p. 13.

- Jusqu'ici nous n'avons pas semblé nous inquiéter de savoir si les lois varient réellement, mais seulement si les hommes peuvent les croire variables. Les lois considérées comme existant en dehors de l'esprit qui les crée ou qui les observe sont-elles immuables en soi? Non seulement la question est insoluble, mais elle n'a aucun sens. (D.P.: p. 29)

28 Poincaré, Science and Method, pp. 285-286.

- La méthode de démonstration n'est pas la même pour le physicien et pour le mathématicien. Mais les méthodes d'invention se ressemblent beaucoup. (S.M.: p. 309)

all experiments begin from preconceived ideas, [29] the value of experiments is negative and quite analogous to the valid role of logic in the science of pure quantity.

If an experiment produces data which do not harmonize with a hypothesis, that is if the empiric facts contradict the hypothesis, it is accordingly condemned.

> Only, it should always be as soon as possible submitted to verification. It goes without saying that, if it can not stand this test, it must be abandoned without any hesitation. 30

But if the empiric facts do not contradict the hypothesis, this lack of contradiction is not sufficient to enable us to consider this hypothesis true, since other hypotheses also lacking contradictions could replace the hypothesis in question. Hence experience can condemn some hypotheses, but it is incapable of establishing the truth of a hypothesis. In the final analysis we can conclude that experience (of the extra-mental) is powerless in the establishing of truth.

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29 Cf. "The Role of Experience" in Chapter II.

30 Poincaré, Science and Hypothesis, p. 150.

- Seulement elle doit toujours être, le plus tôt possible et le plus souvent possible, soumise à la vérification. Il va sans dire que, si elle ne supporte pas cette épreuve, on doit l'abandonner sans arrière-pensée.
(S.H.: p. 178.

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2) Logical Position Implies an Epistemology

What is the epistemological significance of Poincaré's logical position? The importance of this logical position is that it establishes the primacy of intuition and the synthetic character of the mind in the science of pure quantity. It does so by limiting the valid use of logic to a purely negative function. The valid use of logic is in the verification of particular mathematical combinations previously discovered. If the verification process shows that a particular combination leads to a contradiction, this process does condemn the mathematical combination. But if the verification shows no contradiction, this process is insufficient in establishing the truth of a given combination. This is because the number of contradiction-free combinations is infinite and the majority of them are worthless. [31]

The capacity of mathematical discovery is located in the power of intuition. It is here that Poincaré has emphasized the importance of the unconscious in mathematical creation. The nature of the unconscious is difficult for Poincaré to define [32] and the way in which it functions is obscure.

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31 Cf. "Error and the Logical Ideal" in Chapter IV.

32 Cf. "Psychology of Discovery" in Chapter III.

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> The question why the correct combination, and no other is presented to the conscious mind, does not permit of any very satisfactory answer. But it is significant that the right combination is the one which most appeals to the aesthetic sense of the mathematician. 33

But the obscurity of the nature and processes of the unconscious does not prevent us from grasping the characteristics of a correct solution. It is the characteristics of truth that we are seeking.

3) Notions of Truth

Poincaré's epistemological foundations require a criterion of truth. This is both a theoretical and a practical necessity. It is first a theoretical necessity for both types of science. This is because the mind is capable of conceiving an infinite number of intuitions. Now there must be some way of distinguishing true intuitions. But the logical method in the science of pure quantity is capable of establishing the falsehood of some intuitions but cannot establish the truth of the others. A criterion of truth is also required for the conventional sciences. The process of empirical verification is capable of establishing the falsehood of some hypotheses but cannot establish the truth of the remaining possible hypotheses.

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33 J.W.N. Sullivan, "Henri Poincaré - The Passing of a Great Mathematician and Philosopher" in Scientific American, Vol. 102, issue of July 27, 1912, p. 78.

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From the fact that scientific laws cannot be considered as applying outside the mind two consequences result: Poincaré's notion of truth cannot be reduced to a realistic position and his criterion of truth must be a mental criterion. As an instance of Poincaré's esoteric conception of truth let us note:

> The truth for which Galileo suffered remains, therefore, the truth, although it has not altogether the same meaning as for the vulgar, and its true meaning is much more subtle, more profound, and more rich. 34

As an instance of Poincaré's idealistic notion of truth let us observe his proof for the existence of the world by means of the thoughts of others.

> What guarantees the objectivity of the world in which we live is that this world is common to us with other thinking beings. Through the communications that we have with other men, we receive from them ready-made reasonings; we know these reasonings do not come from us and at the same time we recognize in them the work of reasonable beings like ourselves. 35

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34 Poincaré, The Value of Science, p. 141.

- La vérité, pour laquelle Galilee a souffert, reste donc la vérité, encore qu'elle n'ait pas tout à fait le même sens que pour le vulgaire, et que son vrai sens soit bien plus subtil, plus profond et plus riche. (V.S.: p. 274)

35 Ibidem, p. 135.

- Ce qui nous garantit l'objectivité du monde dans lequel nous vivons, c'est que ce monde nous est commun avec d'autres êtres pensants. Par les communications que nous avons avec les autres hommes, nous recevons d'eux des raisonnements tout faits; nous savons que ces raisonnements ne viennent pas de nous et en même temps nous y reconnaissons l'oeuvre d'êtres raisonnables comme nous. (V.S.: p. 262)

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Let us now attempt a synthesis of Poincaré's doctrine concerning truth. From Poincaré's writings let us consider the following characteristics:

(1) The search for truth should be the goal of our activities... 36

(2) ... truth ... it alone is beautiful. 37

(3) ... profounder beauty which comes from the harmonious order of parts... 38

(4) ... the harmonious order of parts and which a pure intelligence can grasp. 39

By combining the first two quotes we can conclude that the search for truth is a search for the beautiful. By combining this conclusion with the third quote we can conclude again that the search for truth is the search for the harmonious order of parts. Finally by combining this conclusion with the fourth quote we can conclude that truth is grasped by a pure intelligence. But Poincaré also assures us that simplicity is beautiful.

(5) And it is because simplicity ... is beautiful that we preferably seek simple facts. 40

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36 Poincaré, The Value of Science, p. 11.

- La recherche de la vérité doit être le but de notre activité... (V.S.: p.1)

37 Ibidem.

- Et pourtant il ne faut pas avoir peur de la vérité parce qu'elle seule est belle. (B.S.: p. 2)

38 Ibidem, p. 8.

39 Ibidem.

40 Ibidem.

(38, 39, 40 - These three quotes are not translations. They are from Poincaré's preface for the English edition of Value of Science)

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By combining the first two quotes with number five we can conclude that the search for truth is a search for simplicity. Now we can see that the search for truth is a search for simplicity and harmony which is grasped by a pure intelligence. In saying that simplicity and harmony are grasped by a pure intelligence, it is clear that we are dealing with the realm of intuition.

Thus we are led to recognize that truth is a property of certain intuitions. An intuition can be attributed as true if it possesses certain properties. These properties are simplicity and harmony. Simplicity and harmony lead to unity, so we may speak of a simple, harmonious, and unified intuition. Although we have many intuitions we can distinguish them on the basis of simplicity, harmony and unity. The greater an intuition possesses these properties, the greater our justification of attributing truth to it. But let us recall that simplicity, harmony and unity are requirements of the human psyche. [41] Thus we see that the essence of truth for Poincaré consists in the correspondence between an intuition and the requirements of the human psyche.

The degree of correspondence determines the degree of truth. The correspondence is perfect but once in the direct intuition of

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41 Cf. Poincaré, Science and Hypothesis, p. 130, and "The Search for Simplicity, Harmony, and Unity" in Chapter II.

Cf. Science et Hypothèse, pp. 156-157.

number and the complete mathematical induction since this intuition is the product of the psyche alone, working only upon itself. It is the mind experiencing its own power.

a) Conventionalism as a Criterion of Truth

It is in the light of the foregoing analysis that the meaning of conventionalism needs to be interpreted. Poincaré's criterion of truth is the simple, harmonious, and unified intuition. However since intuitions other than the pure mathematical ideal do not possess these properties perfectly, the necessary character of all other intuitions is eliminated. Consequently the choice among other types of intuitions does entail practical necessity. But this practical necessity is still within the mental order. Thus the description of conventionalism as a pragmatism applies only in a limited sense. Conventionalism is in essence merely the choice of intuitions where all necessity has been eliminated. The basis of the choice remains the criteria of simplicity, harmony, and unity. Conventionalism is actually a synonym for the criterion of truth when it is utilized by the scientist with full freedom. Conventionalism thusly understood may be considered a type of idealistic utilitarianism.

> De même on peut parler chez Poincaré d'un utilitarisme idéaliste... 42

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42 J. Benrubi, Les Sources et les Courants de la Philosophie Contemporaine en France, p. 361.

The designation is appropriate when we consider that Poincaré considers utility a result of beauty.

> L'astronomie ... est utile parce qu'elle est grande; elle est utile parce qu'elle est belle. 43

While there are many types of intuition, there is only the one criterion of truth. The criterion of truth is simplicity, harmony, and unity in an intuition. The criterion of truth corresponds to the demands of the psyche. An intuition which is attributed with truth because of these properties but without necessity is termed a convention.

b) Evolutionary Character of Truth

The mind in affirming that a given intuition is true does so on the basis of its correspondence with the criteria of truth. But there is nothing which prevents this concept from later being recognized as more complex and unharmonious than was first thought or again there is nothing preventing the formation of a new intuition which may correspond better to the criteria of truth. Thus apart from the pure mathematical ideal, we cannot speak of the attainment of truth.

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43 Poincaré, La Valeur de la Science, p. 157. (French text is used owing to omission of last phrase in the English translation.)

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> But sometimes truth frightens us. And in fact we know that it is sometimes deceptive, that it is a phantom never showing itself for a moment except to ceaselessly flee, that it must be pursued further and ever further without ever being attained. 44

The truth which we seek is not permanent owing to the unlimited power of the mind to form new intuitions, some of which better conform to the criteria of truth than do the formerly held ones.

> No doubt, if our means of investigation became more and more penetrating, we should discover the simple beneath the complex, and then the complex from the simple, and again the simple beneath the complex, and so on, without ever being able to predict what the last term will be. 45

But despite the transitory character of the truth which the human mind is permitted to glimpse, Poincaré must be considered an optimistic relativist.

> De même que l'humanité est immortelle, bien que les hommes subissent la mort, de même la vérité est éternelle, bien que les idées soient périssables, parce que les idées engendrent les idées comme les hommes engendrent les hommes. 46

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44 Poincaré, The Value of Science, p. 11

- Cependant quelquefois la vérité nous effraye. Et en effet, nous savons qu'elle est quelquefois décevante, que c'est un fantôme qui ne se montre à nous un instant que pour fuir sans cesse, qu'il faut la poursuivre plus loin et toujours plus loin, sans jamais pouvoir l'atteindre. (V.S.: p.1)

45 Poincaré, Science and Hypothesis, pp. 148-149.

- Sans doute, si nos moyens d'investigation devenaient de plus en plus pénétrants, nous découvririons le simple sous le complexe, puis le complexe sous le simple, puis de nouveau le simple sous le complexe, et ainsi de suite, sans que nous puissions prévoir quel sera le dernier terme. (S.H.: p. 176)

46 Poincaré, Savants et Ecrivains, p. 175.

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We have seen the idealistic characteristics of Poincaré's notion of truth. The nature of truth consists in the correspondence between the intuition and mental (psychological) criteria - the needs for simplicity, harmony, and unity. The mind is essentially creative but its creations must correspond with the psychological needs to be considered true. Excepting the pure mathematical ideal, all other "truths" are relative but with this mental order they are paramount.

SUMMARY

Poincaré holds that science must be intellectualistic or else cease to exist. This view precludes any interpretation of Poincaré's philosophy which would consider science primarily a rule of action. The qualities given by sensation are impenetrable and empiric data in no way determines any particular scientific hypothesis. These considerations force us to reject positivistic interpretations of Poincaré's philosophy.

Poincaré's conventionalism can be considered pragmatic only in a very limited sense. The choice of scientific hypotheses is not made in view of a pragmatic desire for function or application to extra-mental phenomena. Science exists for its own sake; its end is knowledge, not action. Nevertheless there are reasons why a pragmatic interpretation has appeared to be an appropriate interpretation. These factors include: the ambiguity of the word "convenient", a silence on Poincaré's part to trace the origins of his conventionalism and to indicate the sources which have influenced him philosophically, the esoteric psychological experiences which serve as his point of departure in philosophizing, and the popularity of positivism which developed in the decades following his death. But the interpretation of his conventionalism requires that it be situated within his psychological position. The psychological factors which produce the science of pure quantity

SUMMARY

are paramount in Poincaré's position. It is in this psychological experience that the mind is clearly seen in its true nature, i.e. active, creative, synthetic, and completely self-sufficient. Poincaré defended these intellectualistic properties against M. LeRoy to the point of situating them in a philosophy of thought.

Nevertheless, owing to the equivalence of hypothesis, there must be some way of judging the truth of mental constructions. Empiric verification in the experimental sciences and logical verification in the science of pure quantity and the deductive sciences by performing a negative function are unable to serve as a guide for the required choice. Hence the real necessity for a criterion of truth. The criterion of truth is situated within the psychological needs of the human psyche. These needs express themselves in the intellectual beauty and aesthetic feeling resultant from harmony, simplicity, and unity. Truth is attributed to those mental constructs in the degree that they conform to the psychological need for harmony, simplicity, and unity. The mind's creative activity results in an unending intuition of new ideas, some of which may conform to the criterion of truth better than previous "truths". Consequently the attributing of truth becomes a process of pursuit rather than a state of attainment. Truth may be glimpsed but certitude is never attained.

CONCLUSION

In this research, the first type of knowledge to be evaluated has been philosophy. The task here was to establish Poincaré's view of the status of philosophy. By understanding what he means by philosophy and his judgments on it, we were able to situate more accurately his notion of science and ultimately isolate his own philosophical position. It would seem that of all the philosophical disciplines the only one accepted by Poincaré is logic. Metaphysics is categorically denied, and no intrinsic value is even admitted to questions of this order.

The analysis of the nature and scope of science has been the most crucial undertaking for the solution of the epistemological problem. This is the case since the conclusions regarding the role of metaphysics are solely negative and because Poincaré holds a negative conception of philosophy, while science alone is true knowledge. The scope of science, that is the areas of investigation in which a science may be established, is unlimited. But despite the fact that science has an unlimited field for development, it is likewise the case that science does not reveal the true nature of things. We sought to understand how science can be true knowledge while being incapable of revealing nature in any way. The solution of this apparent dilemma led us to the distinction of two types of science: sciences which

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are founded upon conventions, and the science of pure quantity which is for Poincaré the archetype science since it is not founded upon conventions but rather upon a direct intuition.

The first area of research was to examine the philosophical elements of those sciences which rest upon conventions. Before the scientist can even enunciate the first scientific observations, it is necessary for him to make use of conventions, frequently called presuppositions. These conventions include a belief in order, continuity, unity, and simplicity. The nature of space and time are also postulated. The most important features of these notions are that they are the result of the creative activity of the scientist and cannot be verified by experience. They are mental needs, not physical facts. By an analysis of the constitutive elements of sciences founded upon conventions, it was noted that there is not a single necessary truth which they are capable of enunciating.

The criterion of truth for these sciences is found in the mental satisfaction and economy of thought which is the product of a harmonious, simple, and unified theory.

The only science which does not rest upon conventions is arithmetic, that is, the science of pure quantity. The truths of arithmetic are immutable and necessary because they rest upon

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a direct intuition. This direct intuition of number as well as operations performed upon numbers is absolutely rigorous and it lifts us from the particular to the general by a complete mathematical induction. This is the sole instance of the coincidence of truth, certainty, and immutability. This is due to the fact that the pure intuition requires no aid from the senses.

The pure intuition of number and the complete induction which results in the creation of the mathematical continuum are incapable of a realistic interpretation. By nature they precede all sense experiences; moreover they provide the very mental structure which makes human sense experience possible by the uniting of brute sensations into a synthetic whole. The pure intuition of number and the complete induction are founded on none other than the mind's intuition of itself and its own powers. The pure intuition of number and the resulting complete induction constitute the pure mathematical ideal.

The importance attached to the pure mathematical ideal is best understood by studying the relationship between intuition and logic in mathematics. Poincaré denies that mathematics can be reduced to pure logic. He asserts rather that logic is impossible without the direct intuition since without the notion of number definition is impossible and without the complete induction

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it is impossible to show that a logical definition does not lead to contradiction. Accordingly the use of logic is subordinated to that of intuition. The use of logic must never be allowed to be substituted for the goal of simplicity, harmony, and unity in mental creations. Logic gives rigour but no advances in truth. Intuition alone is the element of discovery and truth.

The criterion of truth for Poincaré is the harmonious, simple, and unified concept.

The basic epistemological position is highly rationalistic. Only in the science of pure quantity, however, is truth ever completely and permanently attained. The character of the pure intuition is supremely harmonious, simple, and unified. In all other branches of knowledge there is an evolutionary form of rationalism since concepts can be continually refined and rendered more simple or more unified. Truth is never permanently attained in the sciences based on conventions; it is pursued endlessly.

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