Optimal Design and Operation of Two Hydroelectric Reservoirs in Series

by

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CHAPTER I

INTRODUCTION

During the last thirty years, great advances were made in water resources management as far as the development and adoption of optimization techniques for planning, design and management of complex water resources systems are concerned.

Generally speaking, the important objectives for water resource projects are national economic development and environmental quality, regional economic development and other social goals. For water in a reservoir, it is generally accepted that appropriate multiple purposes are hydropower generation, irrigation, industrial and domestic uses, fishing and wildlife enhancement, flood control, water quality improvement and navigation.

The management and use of water resources has long been a problem for decision makers. Over the years, a substantial literature has accumulated that deals with investment in water resources development. For example, in 1972, Haveman presented his book on the appraisal methodology of public investment in water resource development[1]. Economists contend that the problem could be greatly simplified if knowledge of the value of water in alternative uses could be employed as an important, albeit not exclusive (because of external effects), basis for allocating water and investing in water development. So the estimates of value for some major uses are of great importance as a basis for decision about optimal water

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[1] Haveman, R.H., The Economic Performance of Public Investment: An Ex Post Evaluation of Water Resources Investments, The Johns Hopkins University Press, Baltimore, 1972.

resource management.

Interdependencies are particularly prevalent in water resource systems for two reasons. First, water resources occur in a natural resource system that has many components and a high degree of interaction and interdependence among the parts. Using water at one point in the system can set in motion a chain reaction resulting in changes in the natural system far from the point of first use. Second, water uses generally do not "consume" water in the same way as, for example, a heating unit consumes coal. As a result, most uses return a substantial portion of the water that they use to the stream. If the uses are well integrated, the returned water can be put to other valuable uses.

Natural resources can be classified as exhaustible and nonexhaustible according to whether their stock can be depleted. In the exhaustible resource category, we have both renewable and nonrenewable resources. According to the characteristics of water in a reservoir, it can be treated as nonrenewable and renewable resource under the following conditions. On one hand, water in a reservoir during abundance period can be considered renewable because when water inflow exceeds outflow, the demand for service can always be met and the service provided in this case is considered unbounded by the resource stock. On the other hand, water in a reservoir during the scarcity period is considered nonrenewable because it provides a bounded sum of services. We shall have detailed discussion in Chapter II.

The presence of interdependencies in water systems complicates the valuation processes. Value gained from one use may be offset by sizeable decreases that it causes in the value gained from some other use because uses are substitutable. At the other extreme, one use may complement another so that both can expand together. So the resulting value of water can only be estimated when the effect on the other uses is taken into consideration. In other words, we must consider opportunity cost in water usage for interdependent purposes.

For example, river water can be used to generate hydropower, irrigate farm lands and increase fish production. Any overuse for one purpose will directly offset the use for other purposes. If we want to evaluate the value of water used for fish production, we have to take into account the opportunity cost for hydropower generation and for irrigation of farm land. For example, certain amount of water flow has to be maintained for the benefit of fish production and it may cause a loss in the production of hydropower due to diversions of flows through by-pass facilities and over spillways or a loss to agriculture due to reductions in withdrawals of irrigation water. These are considered opportunity cost of water for fish production.

Another kind of opportunity cost we have to consider is the sacrifice of water use in future if we use it now. Not only should we decide for which purpose water should be used, we should also consider when it should be used. In order to make the best use of water, we should consider the trade off between present water use and storing it up for the future use. Both kinds are opportunity cost and constitute a component of the implicit price of water.

In this paper, we take hydroelectric power generation as the major objective for the use of water resources.

The harnessing of water power to generate electricity in Canada dates back to 1882, when a generating facility was installed at Chaudiere Falls on the Ottawa River to provide electric lighting for a local sawmill.

The subsequent development of hydroelectric power was facilitated by significant technical advances that took place around the turn of the century. Developments in protection and control technology improved system reliability and safety and the desire to capitalize on economies of scale began to encourage the development of large generating and distribution networks operated as public utilities.

The Canadian hydro power generating capacity has been growing steadily during the past hundred years. In 1950s, total electricity generating capacity in Canada was about 9800-MW, 91 percent of which was provided from hydraulic sources. By 1970, total capacity had jumped to almost 43,000-MW, although the hydro share had fallen to 66.1 percent. By 1980, capacity had doubled to 81,599-MW and hydro's share had again fallen--to 58.5 percent. This share has been more or less constant ever since.(Statistics Canada No. 57-202; Bernard,Chatel,1984). Nevertheless, with much potential hydro power remaining to be developed, this valuable resource will continue to make a significant contribution to Canada's electricity supply, especially since Canada is the industrialized country which uses hydro-power the most. It is anticipated that a significant amount will be developed over the next twenty to twenty-five years. Thus, the study of making the best use of water resource is of great importance[2].

Since the water resource system is used mainly for the purpose of hydroelectricity generation, we would like to evaluate water resource used for this purpose so that we can optimize the usage of the resource. Methods are sought to meet the demand for hydropower which shall minimize the cost of generating it.

In Chapter II, we shall first recall some basic principles of management of nonrenewable resources. We shall then show some applications to the management of water resources. Next, we shall derive an optimal price path for power generated by the water resource system. In the end, we shall establish a method for the economic evaluation of power generated.

In Chapter III, we shall generally introduce the major techniques of optimization for water resource management. This includes a discussion of different methodologies in water system management and a comparison of the characteristics of each method. We shall mainly focus on deterministic linearization methods. Finally, we shall propose Koopmans and Becker's models and extend them.

In Chapter IV, we first look at the total cost and marginal cost functions of reservoir-capacity expansion. Then we discuss two models of the hydro system at the design and operation stages and apply the methodologies introduced in Chapter III to it. We shall

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[2] Hydroelectric Power Potential in Canada, Energy Mines and Resources Canada, 1988, pp.82-84.

propose two investment models discussed in this paper, Koopmans and Pindyck's models, and compare them. In the end, we suggest optimal operation policies for the optimum usage of the water resource system.

A concluding chapter summarizes the contribution of this paper.

CHAPTER II

REMINDER OF BASIC PRINCIPLES OF ECONOMIC MANAGEMENT OF HYDRO ELECTRIC RESOURCES

Dasguspta and Heal (1979) define a nonrenewable resource in terms of the service it is capable of providing over time. The distinguishing feature of a nonrenewable resource is that it is used up when used as input in production and at the same time its rate of growth is nil. In short, the intertemporal sum of the services provided optimally by a given stock of a nonrenewable resource over a horizon is finite[1].

Water in a reservoir has some similar features, but we do not usually regard water as being nonrenewable in the same way as fossil fuels are. For fossil fuels, we will never be able to recover an entire ton of secondary fossil fuel from a ton of secondary fossil fuel in use. There will be leakage at every round of recycling. Moreover, this leakage rate is bounded away from zero. Consequently, the integral of services provided by a given initial stock of such a resource is bounded. For water, if we carefully manage it, it can, in principle, provide an unbounded amount of service over a certain period of time. But notice that when the amount of water inflow into the reservoir tends to be less than the amount of outflow for a period of time, water in a certain reservoir is considered to provide bounded sum of services, and it is always the condition under which we make our discussion.

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[1] Dasgupta, P.S. and Heal G.M., Economic Theory and Exhaustible Resources, Cambridge University Press, 1979.

To sum up, the water resource of a reservoir in scarcity period is classified as an nonrenewable resource. Therefore, the methods for the management of nonrenewable resources, in a general sense, can be made applicable to the management of water resource.

In this chapter, we will first review Hotelling's model (1931) for the management of nonrenewable resources. Then, we shall proceed with the application of the theory to water resource.

Hotelling (1931) first demonstrated that with constant marginal extraction costs, price minus marginal cost should rise at the discount rate in a competitive market[2]. If the extraction costs rise as the resource is depleted, the competitor will be more 'conservationist', and price minus marginal cost should grow less rapidly relative to the case of constant extraction cost, which means at a rate less than the interest rate[3].

In Hotelling's classic paper, the management of resources is under free competition, and maximum social efficiency is pursued.

Here, we are going to mention some assumptions and results from the literature for maximizing social efficiency because the water resource systems we are going to discuss later on are for public use.

The basic assumptions made in Hotelling's paper are the following:

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[2] Hotelling, Harold, "The Economics of Exhaustible Resources", <u>The Journal of Political Economy</u>, Vol. 39, 1931, pp.137-175.

[3] The case of rising extraction costs has been examined by Levhari and Leviatan (1977) and other scholars. Price trajectories for several empirical examples have been calculated by Pindyck (1978).

(a). The objective for the supply of nonrenewable resource is to maximize the present value of future consumer and producer's surplus.

(b). The extraction cost is assumed to be constant, and $p_{t(n)}$ (n stands for net) is to be interpreted as the net price received after paying the cost of extracting and placing the resource upon the market--a convention to which we shall adhere throughout.

The formula

$p_{t(n)} = p_{0(n)} e^{rt}$

fixes the relative prices at different times and this is the equilibrium result obtained under free competition.

The only way that a given unextracted stock of such resource can yield a return to its owner is by appreciating in value. It follows that under competitive conditions it is the rate of capital gains enjoyed by the resource that must equal the rate of return earned in holding any other asset (i.e. interest rate). It is also the condition of flow equilibrium in the market for nonrenewable resources. When p_t is greater than p_0e^{rt}, no one would wish to extract now. If p_t is less than $p_0\ e^{rt}$, no one would wish to hold on to the stock at all.

We should notice that the computation of present values of consumers' surpluses is a common practice in intertemporal resource-allocation problems. But Blackorby et al. (1984) argued that when this discounted sum is used as an objective function, it is equivalent to assuming that it ranks possible outcomes correctly. In the simple case, society consists of a single individual, and the usefulness of the procedure depends critically on the exactness of the index for the individual's lifetime preferences. In a dynamic model, there do not exist such preferences for which the discounted sum is an exact measure for social efficiency change. However, it does provide a one-way test for social efficiency change. That is, if the present value of the compensating variation is positive, social efficiency has improved; if the present value of the sum of equivalent variation is negative, it has worsened. Ambiguous results occur when the present value of compensating variations is negative and the present value of equivalent variations is positive. In our paper, we assume that the sum of consumer surplus is an approximate, though not perfect measure of social efficiency[4].

The basic Hotelling model has been extended by Pindyck (1978)[5]. Instead of taking fixed reserves as "endowed", he argued that reserves must be developed through the process of exploration. There is no "fixed" reserve base to be exhausted over time. Given economic incentives, reserves can be maintained or increased through further exploration--even though the physical returns to exploration decrease as "depletion" ensues. Thus nonrenewable resources are better thought of as inexhaustible, but decreasing in quality.

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[4] Blackorby, Charles, Donaldson, David, and Moloney, David, "Consumer's Surplus and Welfare Change in a Simple Dynamic Model", The Review of Economic Studies, Vol. LI(1) No.164, 1984, pp.171-176.

[5] Pindyck, R.S., "The Optimal Exploration and Production of Nonrenewable," Journal of Political Economy, pp.841-861, 1978.

He views exploratory activity as the means of accumulating or maintaining a level of reserves and low level of extraction cost, and treats depletion by assuming that reserve additions ("discoveries") resulting from exploratory activity fall as cumulative discoveries increase. The desired level of reserves depends in part on the behaviour of the production costs. Production costs rise as reserves decline. Thus producers must simultaneously determine optimal levels of exploratory activity and production--resulting in an optimal reserve level--that balance revenues with exploration costs, production costs, and the "user cost" of depletion.

L.C. Gray introduced the user cost concept as the present value of the net return for postponing the extraction of a unit of resource[6]. In other words, user cost is the future profit one gives up in order to make one more unit of profit today. J.M. Keynes coined the term "user cost" for the loss in value when a capital asset is reduced by one marginal unit for present use. Modern usage leans toward the term shadow price of capital and it refers to the fact that the asset's value is not its direct sale value, but the value imputed from its future productivity.

Since we always face the dilemma of extracting now or in the future in managing resources, we have to consider user cost in making the decision.

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[6] Crabbe, Philippe J., "The Contribution of L.C. Gray to the Economic Theory of Exhaustible Natural Resources and its Roots in the History of Economic Thought", Journal of Environmental Economics and Management, Vol.10, 1983, pp.195-220.

Pindyck's model under perfectly competitive market condition is as follows.

$$\mathrm{Max}_{q,w} \quad W=\int_0^\infty [q_t*p_t-C_1(R_t)*q_t-C_2(w_t)]*e^{-rt}\,dt \tag{1}$$

Subject to

$$dR_t/dt=(dX_t/dt)-q_t \tag{2}$$

$$dX_t/dt = f(w_t,X_t) \tag{3}$$

$$\text{and } R_t\geq 0;\ q_t\geq 0;\ w_t\geq 0;\ X_t\geq 0; \tag{4}$$

Here, producers take p_t as price given and choose a rate of production q_t from a proven reserve base R_t. The average cost of production $C_1(R_t)$ increases as the proven reserve base is depleted. Additions to the proven reserve base occur in response to the level of exploratory effort w_t. The rate of flow of additions x_t is $dX_t/dt=f(w_t,X_t)$ with $f_w\geq 0$ and $f_x\leq 0$. Thus, as exploration and discovery proceed over time, it becomes more and more difficult to make new discoveries. The cost of exploratory effort $C_2(w_t)$ increases with w. Pindyck assumes that $C_2''(w_t)\geq 0$ and the marginal discovery cost, $C_2'(w_t)/f_w$, increases as w increases. He further assumes that $C_1(R_t)\rightarrow\infty$ as $R_t\rightarrow 0$.

The solution of this optimization is straightforward when we use the maximum principle to solve it.

The Hamiltonian is

$$H=q_tp_te^{-\delta t}-C_1(R_t)q_te^{-\delta t}-C_2(w_t)e^{-\delta t}+\lambda_1[f(w_t,X_t)-q_t]+\lambda_2 f(w_t,X_t) \tag{5}$$

Differentiating H with respect to R and x gives the dynamic equations for λ_{1t} and λ_{2t}:

$$d\lambda_1/dt=C_1'(R_t)q_te^{-\delta t} \tag{6}$$

$d\lambda_2/dt=-(\lambda_1+\lambda_2)f_x$ (7)

and from the maximum principle, we have

$\lambda_{1t}=p_te^{-\delta t}-C_1(R_t)e^{-\delta t}$ (8)

Note that λ_{1t} is the change in the present value of future profits resulting from an additional unit of reserves.

Recall that in Hotelling's model, we have the equilibrium result for maximizing social value as

$p_{t(n)}=p_{0(n)}\ e^{\delta t}$ (n stands for net)

where $p_{t(n)}$ denotes the price net of extraction cost. After being transformed, it becomes

$p_{0(n)}=p_{t(n)}\ e^{-\delta t}$ (9)

From the definition, $p_{t(n)}=p_t-C_1(R)$, equation (9) can be written as

$p_0-C_1(R_0)=p_te^{-\delta t}-C_1(R_t)e^{-\delta t}$ (10)

Comparing (8) and (10), we can conclude that equation (10) has the same interpretation as equation (8) and the Hotelling rule.

Differentiating (8) with respect to time, substituting (2) for dR/dt, and equating with (6) gives us the equation describing the dynamics of the price path:

$dp_t/dt=\delta p_t-\delta C_1(R_t)+C_1'(R_t)*f(w_t,X_t)$ (11)

Note that if $C_1'(R_t)$ is zero--that is, if production costs do not depend on reserves--the rate of change of the price path is unaffected by exploration. Also Pindyck pointed out that it is identical with that in the standard constant-cost Hotelling problem.

The level of the price path, however, will be affected by exploration; since "planned" reserves (i.e., the total amount of resource available for production, including what will ultimately be discovered) are greater than initial reserves, our producer can set the initial price at a lower level than the one corresponding to initial reserves.

The optimal rate of exploration is determined by setting $\partial H/\partial w=0$. Equation (12) describes the dynamics of exploratory effort.

$$dw_t/dt=\{C_2'(w_t)[(f_{wx}/f_w)*f-f_x+\delta]+C_1'(R_t)q_tf_w\}/[C_2''(w_t)-C_2'(w_t)*f_{ww}/f_w] \quad (12)$$

The characteristics of the boundary conditions for equations (11) and (13) are also discussed in Pindyck's model.

Given particular functional forms for f, C_1, and C_2, and a demand function relating p and q, equations (11) and (13) can be solved together with the boundary conditions to yield optimal paths for price, production and exploration. The particular pattern of exploration, production and pricing that will result depends critically on the initial value of reserves. The intertemporal tradeoff in exploration involves balancing the gain from postponing exploration (so that its cost can be discounted) with the loss from higher current production costs resulting from a lower reserve base. If initial reserves are large so that $C_1(R_t)$ is small, most exploration can be postponed to the future, whereas if initial reserves are small, exploration must occur early on so as to increase the inventory of proven reserves. In this latter case, production will increase initially (as price falls), and later reserves and production will fall as exploratory effort diminishes[7].

Pindyck summarizes the entire price and reserve profile for a resource from the early period of reserve development, i.e. begins by production increasing and price decreasing, to a later period of rising price and the eventual winding down of both exploration and production. This is his well-known U-shaped price path theory. He pointed out that the exploratory activity must be chosen to build the reserve base up to a level that reduces extraction costs and then is adjusted over time so as to trade off cost savings from postponed exploration with savings from lower extraction costs and revenue gains from greater total production. The pattern of optimal exploratory activity thus depends highly on initial reserve levels and on rates of depletion.

Swierzbinski and Mendelsohn (1985) criticized Pindyck's U-shaped price path as a result of misspecification in the assumptions of cost functions[8]. They argued that the discovery of low cost deposit is unlikely to produce a U-shaped price path. Instead, it would often produce an increasing price path. It is the differences about the assumptions of extraction and exploration cost profile that lead to different shaped price paths. Their paper does not have a direct impact on our paper because we are dealing with water which is of homogeneous quality and hydro power generation which is

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[7] Pindyck, R.S., "The Optimal Exploration and Production of Nonrenewable Resources", Journal of Political Economy, pp.841-861, 1978.

[8] Swierzbinski, J.E. and Mendelsohn, R., "Exploration and Exhaustible Resources: The Microfoundations of Aggregate Models", International Economics Review,30(1), pp.175-186, 1989.

subject to diminishing returns to scale. The assumptions we have in our paper and the features of water coincide with Pindyck's assumptions. So, water management problems can be solved by the application of Pindyck's model rather than using Swierzbinski's model, which is for the general management of mineral resources.

Now, let's discuss some special economic features of water in a reservoir.

We always have to bear in mind that the water resource system discussed in this paper is for hydroelectric power generation. The all-hydroelectric system we studied contains the following convenient assumptions:

(a) The operation of an entire multi-reservoir, multi-plant system is modeled in terms of an 'equivalent' single composite reservoir.

(b) The system is analyzed essentially for one year, for example, we divide one year into two periods, Summer and Winter.

(c) Deterministic water inflows and outflows are used and the effects of uncertainty are ignored.

(d) Investment in turbine, head plant etc. is never constraining, i.e. we assume sufficient power capacity for a given reservoir size.

The system has a single abundance season and a scarcity season where water has to be put into storage in the abundance season to help meet requirements during the scarcity season since service provided by water in the reservoir system as a whole is considered finite and bounded. Especially, during the scarcity period, water is nonrenewable if a sufficient amount of water has not been stored up during the abundance period.

As we discussed before, when we are in the abundance period, water inflow increases until the reservoirs are full. Then there will be spillage as long as energy demands require water outflows below water inflows. So, the marginal cost of energy generated is purely the short run production cost. The long run marginal cost of energy generated in terms of reservoir capacity is zero. We can also state that the opportunity cost of water is zero since the value of water spilled is nil given the choices made.

When we are in the scarcity period, energy demand water requirements exceed water inflow. Additional energy demands then cannot be met without the provision of additional storage capacity, so the marginal cost of energy at any point in time during the scarcity period is the short run marginal cost of production plus the long run marginal cost of providing storage capacity. In the short run, when capacity is given, if energy demand during the scarcity period threatens to exceed potential energy in storage plus potential energy available from scarcity period river inflows, then the price required to keep energy demands down to this amount will reflect short run production costs plus marginal user cost. In the long run, there will be storage capacity expansion and the optimal price of the energy generated will be determined not only by its basic marginal production cost, but also by the marginal cost of storage expansion[9]. In a perfect market, the long run

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[9] Turvey, R. and Anderson, D, "Electricity Economics, Essays and Case Studies", <u>Water Resource Research</u>, 21(12), 1797-1818, 1985.

marginal cost of storage expansion must equal the short run user cost.

Now, let us examine the cost function for hydropower generation.

As we know, electricity surplus to power demands at any moment cannot itself be stored or accumulated for later distribution out of inventories. In hydroelectric systems, however, the potential energy of falling water can be stored in reservoirs for release at times during which the required discharge through the turbines exceeds the streamflows into the reservoirs. The period between the beginning of storage reservoir drafting from full pool elevations to the completion of the refilling operation is referred to as the "critical period". The average generation over the critical period fixes the prime power capability of the system.

When sufficient storage is added to the system to lengthen the critical period, the incremental prime power gain per unit of storage diminishes with each change in the length of the critical period. That is to say, if constant average cost of each additional storage is assumed, the amount of power gain from each unit of construction cost decreases. In other words, there is increasing marginal cost for each additional unit of power gain.

We shall illustrate this with the following example.

Assume the potential energy of a reservoir to be 8,760,000 kilowatt-hours (8,760 kwh= production of one kilowatt over one year). A kilowatt-hour being the output of a kilowatt of capacity operating for one hour, the amount of prime power which such energy in storage can produce will depend on the number of hours during which it is released. That is, if 8,760,000 kwh are drafted during a six-month period each year, the amount of prime power they would provide is 8,760,000 divided by (1/2)*(8,760,000) which is 2mw. If 8,760,000 kwh are drafted during a twenty-four-month period, the amount of prime power they would provide is 0.5mw.

Accordingly, the incremental prime power gain from equal increments of storage from the present to the projected levels will be characterized by markedly diminishing returns.

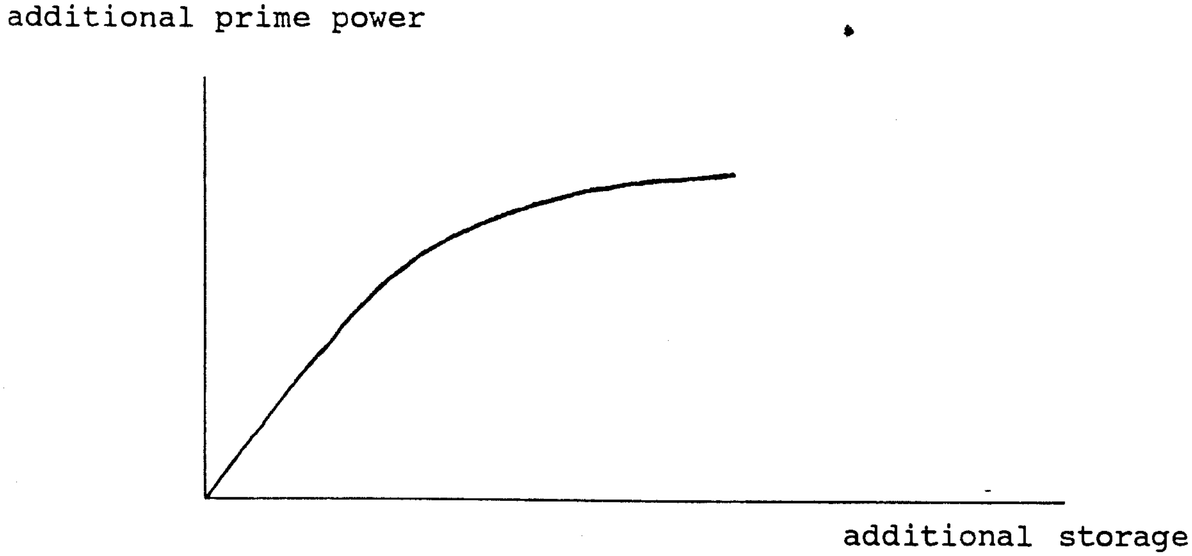

Fig.1. Power gain diminishes with each additional unit of storage capacity

The figures (Fig.1 and Fig.2) depict graphically the diminishing incremental power gain as a function of additional usable storage and correspondingly, the incremental marginal cost as a function of additional power gain.

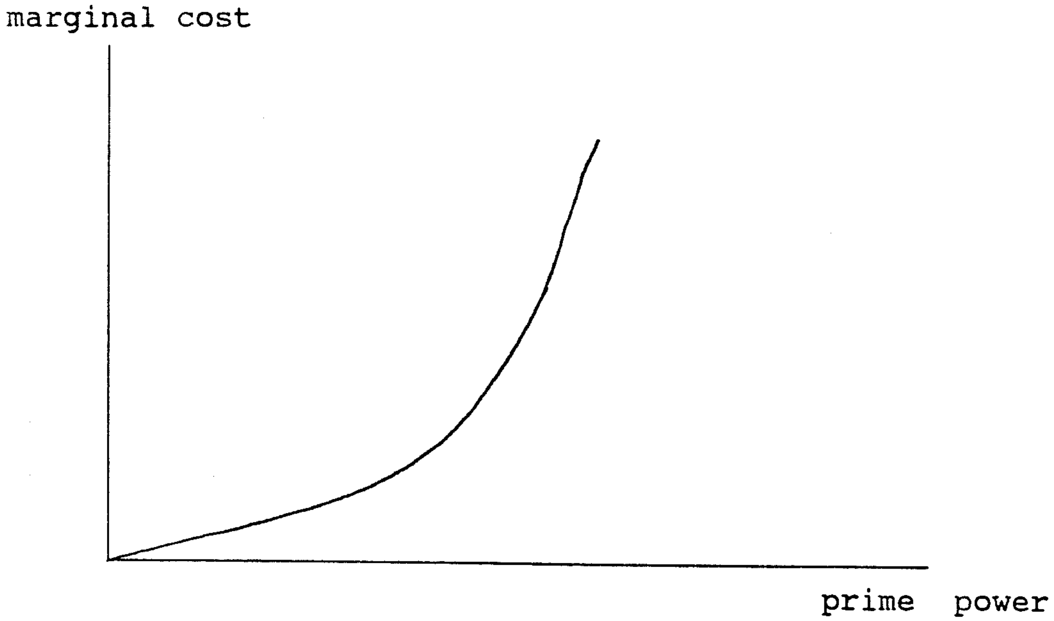

Fig.2. marginal cost increases with each additional unit of prime power

From the above, we know that the marginal benefit of expanding storage capacity decreases and there is increasing marginal cost with additional prime power production. And since the prime power production capacity is limited by the capacity of storage (critical period), we conclude that there is increasing marginal cost for storage expansion and we must pay attention to economies of scale. We shall use this conclusion later on to make the best use of investment in storage[10].

Apart from the decreasing marginal productivity of reservoir theory of Krutilla, Bernard (1989) argued in his paper entitled "A Ricardian theory of hydroelectric power development: some Canadian evidence" that total hydropower production cost increases

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[10] Krutilla, John V., The Columbia River Treaty, The Economics of an International River Basin Development, Johns Hopkins Press, 1967, pp.31-56.

incrementally at each additional power site. In another words, the marginal cost of power generation increases for additional reservoirs. The reasoning is as follows[11].

Power production cost is measured in terms of total output(kwh) produced by each power plant and is represented by an annuitized amount which depends on total electricity generated at each site. Suppose turbines at a hydro power site are driven by hydraulic energy (falling water) which is provided jointly by nature (rainfalls) and by engineering works (dams, river diversion and dredging). Since rainfalls are given over a water basin, except for random fluctuations, increasing hydraulic energy requires higher dams and/or more dredging. Since these construction activities are assumed to exhibit decreasing returns to scale, the annuitized total cost increases incrementally and hence the marginal cost of hydraulic energy is positive and increases at each reservoir.

Thus, the marginal cost for hydroelectric power generation increases for additional reservoirs and this represents decreasing returns to power generated by additional reservoirs.

Since waterflow in the river system is considered as given, we must store water in the abundance season for use in the scarcity season. The policy makers for water resource usage in hydro power generation would face the choice of generating as much power as it can during the year, spilling extra water in the abundance period

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[11] Bernard, J.T., "A Ricardian Theory of Hydroelectric Power Development: Some Canadian Evidence", Canadian Journal of Economics, 1989, pp.328-339.

without reservoir capacity expansion or saving a certain amount of water during the abundance period for use in the scarcity period, keeping a more constant supply of power. To evaluate the tradeoff, policy makers set the goal of making the whole year gain from this activity maximum. Whether or not and how much to expand the reservoir depends on to what extent we can gain from reservoir expansion, that is, the marginal value of power and the marginal cost of expanding a reservoir.

Applying optimal control theory to the river system, the procedure is similar to that of optimizing mining resources with exploration. Here, the extraction of resources can be considered as storing up water which would otherwise be spilled in the abundance season. Accordingly, the extraction cost implies the cost of storing up water--reservoir capacity expansion cost in our model. Water, in our model, is for public usage and the marginal value of power generated is the price of power or the marginal cost of generating power.

One thing which is different from the other models is that we take the time horizon T as one year. Therefore, no discounting is required.

We take price P(t) for electricity power as given, X(t) as reservoir capacity and V(t) as the rate of discharging water for hydro power generation.

We take $C_2(X)$ as the cost function for storage expansion. The marginal cost of reservoir expansion, as we have discussed before, increases. That is, both $C_2'(X)$ and $C_2''(X)$ are greater than zero.

Thus the average and marginal costs of storage expansion increase.

The rate of capacity expansion, dX/dt, depends on the existing capacity level X(t) and water level W(t). This is because the higher the existing expansion level is, the more difficult it is to expand the reservoir. Existing water level is one way of reflecting the size of the existing reservoir if reservoir capacity expansion is necessary. When water is stored in the abundance period for use in the scarcity period, it makes the best use of the existing reservoir. The final storage of the abundance period, which is the existing water storage of the scarcity period, reflects the size of the reservoir. The higher the water level is, the bigger the existing reservoir is, given the same range of reservoir surface. The other explanation could be when water level is higher, it is more difficult to do the expansion work. Thus, the rate of expansion is slower if the water level is higher: $dX/dt=g(W(t),X)$.

Take $C_1(W(t))$ as the average short run cost of hydro power production. This cost increases as the water level W(t) decreases because the conversion factor $\alpha(W(t))$ of turning water into useful energy decreases with the water level, i.e. $C_1'(W(t))\leq 0$. The problem can be written as follows:

$$\text{Maximize}_{x,v} \quad W=\int_0^T[P(t)*\alpha(W(t))*V(t)-C_1(W(t))*\alpha(W(t))*V(t)-C_2(X)]dt \tag{1}$$

subject to

$$dW(t)/dt=dX/dt-V(t) \tag{2}$$

$$dX/dt=g(W,X) \tag{3}$$

and $$C_1'(W(t))\leq 0, g_w\leq 0, g_x\leq 0, W(t)\geq 0, V(t)\geq 0, C_2(X)\geq 0, \alpha'(W(t))\geq 0, X\geq 0 \tag{4}$$

The solution of this optimization problem is straightforward.

The Hamiltonian is written as:

$$H = P*\alpha(W(t))*V(t)-C_1(W(t))*\alpha(W(t))*V(t)-$$
$$C_2(X)+\lambda_1[g(W,X)-V(t)]+\lambda_2*g(W,X) \tag{5}$$

H is a linear function of V(t), but, in general, is a nonlinear function of X, V(t) is the control variable and X is the response variable.

Differentiating H with respect to W(t) and X gives the dynamic equations for λ_1, λ_2, which are adjoint equations.

$$d\lambda_1/dt=-\partial H/\partial W=-P(t)*V(t)*\alpha'(W(t)+C_1'(W(t))*V(t)*\alpha(W(t)+$$
$$C_1(W(t))*\alpha'(W(t))*V(t)-(\lambda_1+\lambda_2)*g_w \tag{6}$$
$$d\lambda_2/dt=-\partial H/\partial X=C_2'(X)-(\lambda_1+\lambda_2)*g_x \tag{7}$$

From (5), we see that

$$H=[P*\alpha(W(t))-C_1(W(t))*\alpha(W(t))-\lambda_1]*V(t)+C_2(X)+(\lambda_1+\lambda_2)*g(W,X)$$

The producer should produce either nothing or at some maximum capacity level, depending on whether the switching function $P*\alpha(W(t))-C_1(W(t))*\alpha(W(t))-\lambda_1$ is negative or positive. The maximum principle implies that $\partial H/\partial V(t)=0$. Singular solution will ensure $P*\alpha(W(t))-C_1(W(t))*\alpha(W(t))-\lambda_1=0$, thus

$$\lambda_1=P*\alpha(W(t))-C_1(W(t))*\alpha(W(t))=(P-C_1(W(t)))*\alpha(W(t)) \tag{8}$$

which defines the marginal rent resulting from an additional unit of storage. We notice that P is the price of power in terms of (\$/e), $\alpha(W(t))$ is the conversion factor denoting the amount of power one unit of water can generate (e/l^3). Therefore, $P*\alpha(W(t))$

in terms of ($\$/l^3$) is the price of water in power generation. A similar interpretation applies to $C_1(W(t))*\alpha(W(t))$ which is the power generation cost per unit of water. The marginal unit in storage should have the same value as the marginal unit used up in the turbine.

Differentiating (8) with respect to time,

$d\lambda_1/dt=[(dP/dt)-C_1'(W)*(dW(t)/dt)]*\alpha(W)+[P-C_1(W)]*\alpha'(W)*(dW(t)/dt)$

substituting (2) for dW/dt, we get

$d\lambda_1/dt =(dP/dt)-C_1'(W)*g(W,X)+C_1'(W)*V(t)]*\alpha(W)+$
$[P*g(W,X)-C_1(W)*g(W,X)-P*V(t)+C_1(W)*V(t)]*\alpha'(W)$

and equating with (6), gives us

$dP/dt=(1/\alpha(W))*[-P*\alpha'(W)*g(W,X)+C_1'(W)*\alpha(W)*g(W,X)+$
$C_1(W)*\alpha'(W)*g(W,X)-(\lambda_1+\lambda_2)g_w]$

(9)

This is the dynamics of the price path of the power generated under the condition of optimal control.

Taking $\partial H/\partial X=0$, we have the optimal rate of storage expansion.

$\lambda_1*g_x'(W,X)+\lambda_2*g_x'(W,X)-C_2'(X)=0$

substituting (8) for λ_1, we have

$\lambda_2 =[C_1(W(t))-P]*\alpha(W)+(1/g_x)*C_2'(X)$ (10)

The value of λ_2 is the marginal rent resulting from an additional unit of capacity expansion. It is equal to the current decrease in profit $[P-C_1(W(t))]*\alpha(W(t))$ plus the marginal degradation cost measured in marginal productivity of capacity units.

Substituting (8) and (10) into (9), we have

$$dP/dt=[1/\alpha(W(t))]*[-P*\alpha'(W)*g(W,X)+C'_1(W(t))*\alpha(W)*g(W,X)+ C_1(W)*\alpha'(W)*g(W,X)-(g_w/g_x)*C_2'(X) \qquad (11)$$

The dynamic price path tells us that the change of price is affected by the existing price, water storage, reservoir capacity, power generation cost function and reservoir expansion cost function.

In Hotelling's constant production (extraction) cost model, hydropower generation would be independent of the reservoir storage and conversion factor is a constant, i.e. $\alpha=1$. Thus, $C_1'(w)=0$ and the whole procedure would be

$$d\lambda_1/dt=-(\lambda_1+\lambda_2)*g_w \qquad (6')$$

Differentiating (8) with respect to time, gives

$d\lambda_1/dt=dp/dt$ which implies that the path of the marginal rent of storage level will follow the same path as the price of electricity.

$$dP/dt=-(\lambda_1+\lambda_2)*g_w \qquad (6')$$

$$dP/dt=(-g_w/g_x)*C_2'(x) \qquad (12)$$

Comparing equations (11) and (12), we conclude that the dynamics of the price path are affected by both hydropower production and storage expansion cost in general, but will only be affected by storage expansion cost under Hotelling's constant production cost assumption.

From what we have discussed above, we know the economic valuation of the decision we made. Being aware of this, we can make an optimal decision of how much electric power we should generate and the reservoir capacity level to which we should expand. The marginal gain from the storage level increase is determined by the price of power and the production cost. The maximum and optimum storage level would enable price to equal production cost. To reach this level, we may need to expand capacity, and the optimum capacity expansion would be decided by the cost of production and reservoir expansion, the price of power. So they are all interrelated and must be determined at the same time.

From (11), we can see that the optimal price is closely related to the marginal gains from either reservoir capacity expansion or storage level increases which are related, as we mentioned above. So we should evaluate the power generated based on the production and capacity expansion cost incurred. The relationship is shown in equation (11).

CHAPTER III

LITERATURE REVIEW ON HYDRO ELECTRIC RESOURCES MANAGEMENT

Reservoirs are generally used for hydropower generation, irrigation, industrial and domestic water uses, fishing and navigation, etc.

Typical reservoir constraints include continuity equations which relate water storage level of the previous period with that in current period, maximum and minimum storage, maximum and minimum releases, turbine and reservoir capacity limitation and contractual, legal and institutional obligations arising from the various purposes of system.

The economic objective for water usage has always been maximizing the net benefit of the usage of water or at least minimizing the cost of providing it.

Thus the complexities of a multipurpose, multiple-reservoir system should require release decisions to be determined by an optimization or simulation model. The same techniques are also required at the system design stage. Most optimization models are based on some type of mathematical programming technique. Optimization models have been applied for planning purposes as well as real time operation.

So, basically there are two areas where optimization models can be used. One is when reservoir capacity is not determined yet--the capacity expansion design stage. The other is when reservoir configuration and capacity are fixed or determined--the real time operation stage.

Bower et al. (1962) pointed out that there should be close[1] correspondence between the performance of the system as optimized at the design stage and that attainable after the system is built. Accordingly, one must use, at the design stage, an operating procedure (releases) that is consistent with feasible management of the existing system. An optimal operating procedure is needed for the purposes of planning a complex water resources system to attain the best economic and social benefits from water usage. Finding the optimal operating rules of a reservoir system for planning purposes has been a major area of study. The optimal operating rules are used jointly with the problem of determining the optimal expansion capacity of a reservoir.

Real time reservoir operation is concerned with the optimal operation of an existing reservoir system, and decisions regarding releases for various purposes have to be made in a very short time period. In addition to the usually explicitly stated categories of system constraints, there is always the formally unstated but very important constraint of continuing operations. That is, whatever the total number of time periods considered for the optimization, the optimal release policy must result in a system state at the end of the last period that is conducive to satisfactory future

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[1] Bower, T.A., Hufschmidit, M.M., and Reedy, W.W., "Operating Procedures: Their Role in Design of Water Resources Systems by Simulation Analysis", Design of Water Resource Systems, Harvard University Press, Cambridge, Mass., 1962.

operations[2].

Many successful applications of optimization techniques have been made in reservoir studies, especially in the above two areas. Due to the subject limitations of this paper, a brief review is given to the literature of water resource operation. In this paper, we choose hydroelectric power generation as our major objective for the water resource system.

Generally speaking, the following methods are reported in the literature.

a. Linear programming (LP), including chance-constrained LP, stochastic LP, and stochastic programming with recourse.

b. Dynamic programming (DP), including incremental DP,discrete differential DP, incremental DP and successive approximations, stochastic DP, reliability-constrained DP, differential DP, and the progressive optimality algorithm.

c. Nonlinear programming.

We are not going to discuss another important method--simulation.

III.1) Theory of Hydroelectric Reservoir Investment

Koopmans (1957) presented us with a model that deals with hydroelectricity generation system that combines one hydroelectric reservoir and one plant with unlimited thermal generating capacity

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[2] Yeh, W.W-G., "Reservoir Management and Operation Models: A State-of-the-art Review", Water Resources Research, 21(12), 1797-1818, 1985.

operating at increasing incremental cost[3]. His article borrows most of its assumptions from Little's (1955) work which we shall introduce later[4]. He constructs a water storage policy that minimizes the operating cost of thermal generation over the planning period while meeting a prescribed demand for power given as a function of time. Associated with this policy are valuations imputed to the power generated and the water used and to the use of reservoir and turbine capacities. This is one of the most useful papers for the model we build later.

Bernard and Chatel (1984) developed a marginal cost approach in determining the optimal usage of gas turbine and capacities of reservoirs in a water resource system while meeting a specified level of hydroelectricity demand[5]. Three characteristics of hydropower generation are taken into consideration: a) the limited availability of hydraulic forces to activate the turbines, b) the addition of turbines at a reservoir site without changing hydraulic forces, c) the possibility of developing more reservoirs or expanding reservoir capacity with rising costs. Their paper highlighted the influence of these characteristics of hydropower

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[3] Koopmans, Tjalling C., "Water Storage Policy in a Simplified Hydroelectric System", Proceedings of the First International Conference on Operational Research, 1957, London, Baltimore, 1957, pp.193-227.

[4] Little, John D.C., "The Use of Storage Water in a Hydroelectric System", Journal of the Operations Research Society of America, 3, pp.187-197, 1955.

[5] Bernard, J.T., and Chatel, J., "The Role of Energy Limited Hydro Equipment in an Optimal Plant Mix", Energy Economics, vol.6, pp.139-144, 1974.

on the choice of an optimal generating plant mix, i.e. optimal usage of gas turbine and optimal capacity of reservoir. Marginal conditions for a cost minimizing approach are developed while incorporating the effects of the rising cost of hydraulic forces, the cost of added hydro-turbines and the costs of building more reservoirs. An example involving Quebec Hydro is reviewed.

III.2) Theory of Reservoir Operation

III.2.i) Linear Programming Models

LP has been a valuable tool in the optimization of reservoir operations, both for real time and planning purposes.

The essential advantages of LP include a) its ability to accommodate relatively high dimensionality with comparative ease, b) global optima are obtained, c) no initial policy is needed, d) standard computer codes are readily available. Decomposition techniques are necessary for large, complex reservoir systems where the numbers of operating variables and constraints are high. Combination of LP and DP have also been used to alleviate the problem of dimensionality[6].

The application of LP to water resource management varies from relatively simple models of straightforward allocation of resources to complex situations of operation and management. Under certain assumptions, nonlinear problems can be linearized and solved by iteration or approximation procedures.

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[6] Yeh, W.W-G., "Reservoir Management and Operations Model: A State-of-the-art Review", <u>Water Resource Research</u>, 21(12), 1797-1818, 1985.

The typical LP model requires that all relations among the variables are linear, both in constraints and in the objective function to be optimized. It is unrealistic for a practical problem to meet this requirement strictly. Thus, we can either linearize the problem according to certain assumptions or combine linear and dynamic programming together to solve the problem. So we are especially interested in the development of the literature of applying LP-DP in water resource management.

Hall and Shephard (1967) developed a DP-LP technique for river reservoir optimization in which the multiple-reservoir system is decomposed into a master problem and subproblems[7]. The master problem could be seen as a system coordination agency and the subproblem as simple reservoir managers. In that work, the subproblems were solved by DP. The schedule of releases and energy were reported to the system coordination agency (master problem) which is LP.

As we know, if the master problem is solved by LP and the subproblem is solved by DP, it is called a DP-LP problem. If the master problem is solved by DP while the subproblem is solved by LP, it is called a LP-DP problem.

Becker and Yeh (1974) suggested a combined solution methodology of LP-DP for the determination of optimal real time reservoir

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[7] Hall, W.A. and Shephard, R.W., Optimum Operations for Planning of a Complex Water Resources System, Tech. Rep. 122(UCLA-ENG 67-54), Water Resource Center, School of Engineering and Applied Science, University of California, Los Angeles, 1967.

operations associated with the California Central Valley project[8]. Incorporating some of the other objectives into the constraint set, LP minimizes the loss in potential energy of the stored waters in the reservoirs resulting from any release policy in each period. LP solutions are embedded in a deterministic forward DP for multi-period optimization. This method is of considerable value for the determination of release policies in real time operation of an existing system and it results in relatively easy computer implementation and high flexibility in that changes in constraints are simply made and additional reservoirs are easily added. Also, this method can be extended to the planning and design stages for such a facility. It is used in our model developed in Chapter IV.

Marino and Mohammadi (1983) extended this method for the monthly operation of a system of two parallel multipurpose reservoirs using a personal computer[9]. Their model provides the reservoir operator with an inexpensive, yet efficient tool for performing frequent updating of optimal releases due to changing input data (i.e. inflow). Additional reservoirs will cause LP size to increase linearly while DP is unaffected. Therefore, the model can easily be used in a system with more reservoirs without any computation difficulties. An application to the California Central Valley

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[8] Becker, L. and Yeh, W.W-G, "Optimization of Real Time Operation of Multiple-reservoir System", Water Resource Research, 10(6), 1107-1112, 1974.

[9] Marino, M.A. and Mohammadi, B., "Reservoir Operation by Linear and Dynamic Programming", Journal of Water Resource, Planning Management Division American Society of Civil Engineers, 109(4), pp.303-319, 1983.

project is demonstrated.

Now we are going to introduce a series of articles concerning the application of a linear decision rule using LP-DP method in the management of reservoirs.

A linear decision rule is defined by a simple rule shown as A=B-C where A is the release during a period of reservoir operation; B is the storage at the end of the previous period; and C is a commensurable variable chosen to optimize some criterion function. This rule is to be interpreted as an aid to the reservoir operator's judgement in selecting a release commitment to be honoured under normal conditions[10].

In 1969 and 1970, ReVelle et al. wrote two articles entitled 'the linear decision rule in reservoir management'. The first one is devoted to the development of the stochastic model and the second one focuses on performance optimization.

The first paper concluded that a linear decision rule derived from a chance-constrained linear environment is far better in terms of the objective function than that from a deterministic environment.

The objective of the problem is to minimize the size, and hence the cost of the reservoir required while providing a regulated outflow to meet various demands and keep a satisfactory water

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[10] ReVelle, C.S., Joeres, E. and Kirby, W., "The Linear Decision Rule in Reservoir Management and Design, 1. Development of the Stochastic Model". Water Resource Research, 5(4), p.767, 1969.

ReVelle, C.S. and Kirby, W., "The Linear Decision Rule in Reservoir Management and Design, 2. Performance Optimization", Water Resource Research, 6(4) p.1033, 1970.

storage level in the reservoir.

The paper uses first a deterministic formulation of a reservoir design problem with input specified in advance. There are, however, shortcomings in the deterministic formulation concerning the reliability of the decision made. While it is easier to have direct economic interpretations of the constraints, there is no explicit prescription for reliable release commitments and for explicit control over the reliabilities with which the various performance objectives will be met. When there is a change in one of the constraints, we have to obtain exogenous physical or economic data in order to calculate its effect. Sensitivity study is carried out through a complicated procedure and there is no direct interpretation of the economic meaning for the constraints. Chance-constrained programming brought forth the notion of the linear decision rule which can be used to eliminate these deficiencies in the deterministic formulation of the reservoir management problem.

The problem is then restructured in a stochastic environment in the second stage. Flows in particular period are not specified and are known only with some probability. The objective function and constraints are reformulated accordingly. The latter formulation improves the reliability of each performance and is under the direct control of the designer. Moreover, the probabilistic formation clarifies the operational significance of the decision rule but doesn't permit more direct economic interpretation of the constraints than the deterministic formulation.

In this case, we have probabilistic equations to explain when the magnitude of any input, when any of the requirement constraints changes, what economic consequences these will bring about and what the possible benefits and costs will be. The economic consequences of the change in constraint are associated with the changes in reliability. With the change in any of the constraint variables, reliability changes and this leads to either benefits or costs which are shown clearly.

The linear decision rule in either of the two frameworks presented in this paper have both advantages and limitations. First, although the linear rule may not be optimal under all circumstances, it is intuitively appealing and simple to apply in practice. Another significant advantage of the linear decision rule used jointly with chance constraints is its clarification of the role of operating policies in optimal reservoir design.

Finally, the most advantageous feature of such formulations is that risk is made explicit in stochastic problems. Reservoirs operating under linear rules based on chance-constrained formulations should perform at the levels of certainty that the designer or decision maker specifies[11].

The second paper about performance optimization of reservoirs is a modification and extension of the first paper. In the first paper, ReVelle et al. have used linear decision rules to formulate a reservoir management and design problem as a chance-constrained linear programming problem. That formulation sought the smallest possible reservoir that would meet certain operational requirements

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[11] Ibid.

with high reliability.

In the second one, the application of linear decision rules to reservoir management and design is extended in two directions. First, a technique is introduced that modifies the original linear decision rule for a situation in which evaporation losses occur. A more refined treatment of the chance constraints is presented. Second, several reservoir performance measures are formulated as objective functions that can be optimized by linear programming.

The release policy generated by this rule is based on the storage at the beginning of the period, the predicted evaporation depth in the period and the linearized storage area curve of the reservoir rather than using only the actual storage level at the beginning period as in the previous simple case. Then, a clear and more rigorous derivation of the chance constraints, including the effects of both evaporation losses and possible failures to meet previous release commitments is presented. This is done because the previous assumption that it is sufficiently accurate to interpret the reliability requirements on the conditional probabilities of end-of-period storage levels, given that the linear rule yields a realizable release commitment for the period, is legitimate.

The second part of the paper is concerned with the potential improvement in the performance of the reservoir if the reservoir could be built to a size larger than the minimum. Or, in the situation in which a reservoir is already built, one might wish to know how its performance could be upgraded. In both situations, measures of performance in addition to those implied by original chance constraints are suggested. These performance measures include expected and reliable values of storages and releases, deviations from targets and reliabilities of achieving stated goals.

As stated above, methods used in this paper also provide direct economic interpretation of the constraints.

As an extension of the two papers written by ReVelle et al., Nayak and Arora (1971) published a paper entitled "Optimal capacities for multireservoir systems using the linear decision rule"[12]. In the paper by ReVelle et al. (1969), a linear decision rule for making release decisions for a single reservoir system with stochastic inflows is presented. The management of releases becomes quite complex in a multireservoir system with stochastic inflows. This paper extends the linear decision rule proposed by ReVelle et al. for a single reservoir system to that for a multireservoir system. The paper chooses the Minnesota Basin as the example, and the method can be generalized.

The objective function minimizes the annual amortized cost of reservoirs given demand and price. A chance constrained formulation of a multireservoir system is considered. The problem is to define a linear decision rule for each reservoir such that the given water demands of the basin during each period are satisfied with a specified probability at a minimum annual amortized cost. Relative cost rather than absolute cost is used. Minimum and maximum release

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[12] Nayak, S.C. and Arora, S.R., "Optimal Capacities for a Multireservoir System Using the Linear decision Rule", <u>Water Resource Research</u>, 7(3), p.485, 1971.

requirements, minimum pool level,and minimum freeboard capacity are specified by the management for each reservoir. The management also specifies the maximum probabilities beyond which these requirements are not violated.

By proposing linear decision rules for the releases, the authors reduce the chance constrained formulation to an ordinary linear programming formulation because otherwise some constraints are just not easily solved by mathematical programming techniques. The linear decision rule states that the release from a reservoir during a given time period is defined by the difference between the "net initial storage" and a "decision parameter" for that reservoir during that period.

Sensitivity analysis is carried out to study the effect of varying initial relative cost, minimum storage levels, capacity requirements for reservoirs, and minimum and maximum flow requirements. By doing this, one can obtain a relationship between the performance level and the total control volumes which will be useful to management in selecting to the optimal performance level.

A combination of LP with stochastic DP was also used by Takeuchi and Morreau (1974)[13]. The paper presents the development of a realistic methodology for determining the optimal operating policies for a multi-unit interbasin water resource system under uncertain inflows. In this model, the objective function consists of two parts: immediate economic losses within the month and the

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[13] Takenchi, K. and Morreau, D.H., "Optimal Control of Multi-unit Interbasin Water Resource System", <u>Water Resources Research</u>, 10(3) pp.407-414, 1974.

expected present value of future losses as a function of end-of-storage levels in the reservoirs. The latter function is estimated by imbedding the linear programming problem in a stochastic dynamic programming problem. The problem of determining optimal values for control variables within a monthly time interval, after initial values of state variables have been observed, is formulated as a convex piecewise LP problem in which the objective function contains, in addition to immediate losses, the expected value of economic efficiency losses over all future months, an unknown function of end-of-month state variables. That function can be estimated, however, by solving a stochastic DP problem within which the LP problem is nested. Special techniques are adapted to the task of obtaining a large number of solutions to similar LP problems which are required to obtain approximate solutions to the stochastic DP problem. The latter task involves the use of simulation in a recursive algorithm. Simulation is also used to test the derived policy on the 40 years of actual streamflow data existing in the case study area.

Successive linear programming (SLP) has been applied to water resources and similar problems by a number of researchers (Yeh et al.,1979; Bechard et al.,1981; Palicios-Gomez et al.,1982; Martin,1983; Pereira and Pinto,1983.). The basic SLP method consists of solving a large linear program using a first-order approximation to the nonlinear objective function and minimum energy, capacity constraints. After finding the optimal solution to the linearized problem, a new approximation to the objective function is developed at the best recent solution, and the LP is solved[14].

Dagli and Miles (1980) proposed a simple solution approach called adaptive planning for a system of four dams in series[15]. In the objective function, the annual total head of water retained in the reservoir is maximized. Since the hydraulic head is a function of the volume of water stored in each reservoir, the problem was linearized, and an LP was solved based on a forecast of future events, with updates at regular intervals to take into account the latest available information.

The stochastic nature of water resource systems suggests that deterministic linear programming model cannot effectively handle the problem. The research in the application of stochastic programming methods is needed.

III.2.ii) <u>Dynamic Programming Models</u>

Dynamic programming is a procedure for optimizing a multistage decision process. DP is used extensively in the optimization of water resources systems. The popularity and success of this technique can be attributed to the fact that the non-linear and stochastic features which characterize a large number of water resources systems can be translated into a DP formulation. In

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[14] Grygier, J.C. and Stedinger, J.R., "Algorithms for Optimization Hydropower System Operation", <u>Water Resources Research</u>, 21(1), pp.1-10, 1985.

[15] Dagli, C.H. and Miles, J.F.,"Determining Operation Policies for a Water Resource Systems", <u>Journal of Hydrology</u>, 47(34), pp.297-306, 1980.

addition, it has the advantage of effectively decomposing highly complex problems with a large number of variables into a series of subproblems which are solved recursively.

DP is capable of handling adaptive, nonlinear and stochastic problems in reservoir systems. DP is specifically applicable to those reservoir planning and operation problems which can be represented as either a progressive or serial directed network problem. Among the assumptions of monotonicity and separability in the DP model, the separability condition of the objective function limits some model applications and thus choice of stage and state is required. In addition to the multipurpose and multifacility nature of a large reservoir system, considerations of reliability and real time operation have aggravated the so-called curse of dimensionality, and thus numerous efforts have been attempted to alleviate this problem during the past decade.

Applications of DP to reservoir optimization have been reported by Little (1955), Hall and Buras (1961), Young (1967), Meier and Beightler (1967), Hall et al. (1968), Schweig and Cole (1968), Fitch et al. (1970), Liu and Tedrow (1973), Opricobic and Djordjivic (1976), and Collins (1977)[16].

John D.C. Little (1955) presented an article entitled 'the use of storage water in a hydroelectric system'[17]. It developed a

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[16] Yeh, W.W-G., "Reservoir Management and Operations Models: A State-of-the-art Review", Water Resource Research, 21(12), 1791-1818, 1979.

[17] Little, John D.C., "The Use of Storage Water in a Hydroelectric System", Journal of the Operation Research Society of America, 3, pp.187-197, 1955.

computational procedure that recognized uncertainty about future water inflow into the reservoir, and assessed the average saving obtainable by a policy of minimizing the mathematical expectation of cost in one particular instance--an approximate model of the Grand Coulee plant on the Columbia River. It constructed a mathematical model of a hydroelectric system which served as a means of determining optimum water use--an unconventional inventory problem. Using the method of DP, he determined the optimal rule curve of the reservoir. Koopmans (1957) simplified and developed Little's model.

In the class of deterministic DP procedures with separable objective functions for the general problem in discrete time, the subproblem at each stage was solved by linear programming (as in our model discussed later) or other techniques. The selection of a method for a particular problem depends upon both the nonlinearity associated with the objective function and complexity associated with constraints.

Decomposition is essential in the application of DP to a multiple reservoir system. Becker and Yeh (1974) developed an optimization algorithm for the real time monthly operation of a large-scale water resources system. The procedure makes use of the best features of LP and DP in that LP serves for month to month optimization, and DP is used for the selection of an optimal release policy through the specified number of months. The use of a combination of LP and DP was mentioned before and we shall return to it in our model later[18].

Other DP methods are not directly related to the technique we use later. Thus, they are not discussed here.

III.2.iii) <u>**Nonlinear Programming (NLP) Models**</u>

Nonlinear programming has not enjoyed the popularity that LP and DP have in water resource systems analysis. This is due to the fact that the optimization process is usually slow and takes up large amounts of computer storage and time when compared with other methods. The mathematics involved in the nonlinear models are much more complicated than in the linear case, and NLP unlike LP cannot easily accommodate the stochastic nature of inflows to the system[19].

NLP does offer, however, a more general mathematical formulation and may provide a foundation for analysis by other methods. NLP can effectively handle a nonseparable objective function and nonlinear constraints which many programming techniques cannot. NLP includes quadratic programming, geometric programming, and separable programming as special cases which can be used iteratively as a master program or as a subprogram in large-scale system problems. Search techniques have also been used in conjunction with simulation in order to evaluate the performance functions of

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[18] Bechard, D., et al., "The Ottawa River Regulation Modelling System (ORRMS)", <u>Proceedings of International Symposium on Real-Time Operation of Hydrosystems</u>, vol.1, pp.179-198, University of Waterloo, Waterloo, Ont., 1981.

[19] Yeh, W.W-G.,"Reservoir Management and Operation Models: A State-of-the-art Review", <u>Water Resource Research</u>, 21(12), pp.1797-1818, 1985.

alternative systems (Maass et al., 1962). For a system of reservoirs, the number of constraints is large because they deal with similar subsystems, repeated in time or location. Therefore, NLP will gain its practical importance in water resources systems with the development of computer technology and effective algorithms for large-scale, multiobjective optimization[20].

III.3) Methods Adopted in Our Model

Our model is a deterministic model with future power demand and water inflow to the reservoir known and the effect of variation in the head of water ignored.

In our first model, we deal with a kind of inventory or production smoothing and we solve it by direct construction.

In our second model, we deal with a problem which can be divided into a master problem and subproblem. We combine linear and dynamic programming to solve the problem.

So, generally speaking, we use a deterministic, comprehensive method in our model.

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[20] Cohon, J.L. and Marks, D.H., "A Review and Evaluation of Multiobjective Programming Techniques", Water Resource Research, 11(2), pp.208-220, 1975.

CHAPTER IV

INVESTMENT AND OPERATING HYDROELECTRIC RESERVOIR OPTIMIZATION MODELS

In this chapter, we are going to discuss two optimization models for investment and operation on hydroelectric reservoir system at two different stages.

The first model deals with the planning stage of a second reservoir when there is only one reservoir available. What optimal operation policy should we adopt in managing the existing reservoir and how should we design a second reservoir when there is excess demand for hydropower. The rule obtained in designing two reservoirs can be generalized to the design of one system with K reservoirs. The second model deals with the operation stage when there are K reservoirs available in the system. What optimal operation policy should we choose to make the best use of water resource in the system? After presenting the models, the economic valuation imputed to the power generated is discussed.

In the first model, we minimize the cost of building a second reservoir while making water release meet the constraints of the reservoir, of the turbine and of the continuation function. The existing reservoir has a fixed prime power capacity, therefore, the second reservoir is built to provide supplementary electric power when there is excess demand. So, the amount of power required to be provided by the second reservoir is equal to the amount of power demanded, which is constant in our model, less the amount that could be generated by the first (existing) reservoir. Therefore, the rate of water released from the first reservoir for power generation determines the rate of supplementary power generation from the second reservoir. Since the storage of water in the reservoir directly reflects the rate of water release, it is much easier if we intuitively monitor the storage of water in order to control the power generation rate. Indirectly, the rate of supplementary power generation from the second reservoir is determined by the rule curve for water storage of the first reservoir.

Corresponding reservoir capacity is required to meet the rate of power generation and the reservoir construction cost increases incrementally with each additional unit of power. With economies of scale in reservoir expansion, we should make the best use of the existing reservoir and minimize the amount of supplementary power required from the second reservoir, i.e. the cost of reservoir expansion.

By deriving the optimal rule curve of the water storage in the first reservoir, we can obtain from it: (a) the optimal rate of water release for power generation from the first reservoir; (b) the optimal rate of supplementary power required to be generated by the second reservoir; (c) the optimal reservoir capacity needed for the second reservoir.

Once an optimal storage policy has been specified, it is possible to associate with it imputed valuations for reservoir expansion that occur in the system, also known as "shadow price". The shadow price of power is the increase in reservoir expansion cost of one extra unit of electric power to be generated.

In the second model, we maximize the energy generated by a hydro system over one year plus the expected future returns from water storage at the end of the year, given constraints of reservoir, turbine and continuity equation. Dynamic programming is employed to determine the power targets for each month, while linear programming is used to allocate power generation among the reservoirs at each stage. We use LP to optimize reservoir water release for each month, with the objective of maximizing potential value of water in storage at the end of each month, subject to the constraints of power demand, turbine and reservoir capacity, and given states of power target. LP provides us with feasible solutions and we use DP to select the optimal one. The optimal path is selected corresponding to the most desirable storage vector or the greatest cumulative energy generation. The shadow price of water in this model would be the sacrifice of future value for power generation if we use one more unit now because using one more unit of water would increase the value of power generated now, but would decrease the value of future return from water storage. In other words, the shadow price of water is its marginal user cost.

IV.1) Model 1[1]

A similar kind of problem was discussed by both Little (1955) and Koopmans (1957). The difference between their models is that the former is a stochastic problem while the latter is a much

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[1] For the definitions of terms, please refer to the list of notations.

simplified deterministic one. Both deal with a system consisting of a reservoir and a thermal plant enabling it to have unlimited generation capacity. While our model borrows most of its assumptions from them, we tackle a different system. We assume that capacity can be expanded through additional reservoir rather than through additional thermal plants.

To make things simple, we are going to start with a situation in which there is only one reservoir available and we are going to build a second one due to increasing demand in hydropower.

Our objective is to minimize the new construction costs while making the best use of the available reservoir. In order to minimize the cost, we have to know the cost function of constructing an additional reservoir, in other words, the cost of expanding reservoir capacity. As we have discussed in Chapter 2, we know that there are increasing marginal costs for storage expansion. Corresponding to the power generation rate $S(t)$, there is a reservoir capacity requirement, therefore, a cost function. If we denote $f(S(t))$ as the cost function of storage expansion, $f'(S(t))\geq 0$ when S increases. The cost function looks like the one in figure 3.

The objective function to minimize is the cost of building a second reservoir. That is

Minimize $C=\int_0^T f(S(t))\,dt$

where $S(t)$ is the rate of supplementary power that has to be generated by the second reservoir with the corresponding reservoir capacity. If demand for power is known as $D(t)$, and the amount that can be generated by the available first reservoir is known as α_1 * $V_1(t)$, then the amount required to be generated by the second reservoir is

$S(t)=D(t)-\alpha_1 * V_1(t)$.

where $\alpha_i(W(t))$=conversion factor which is affected by W(t) for ith reservoir;

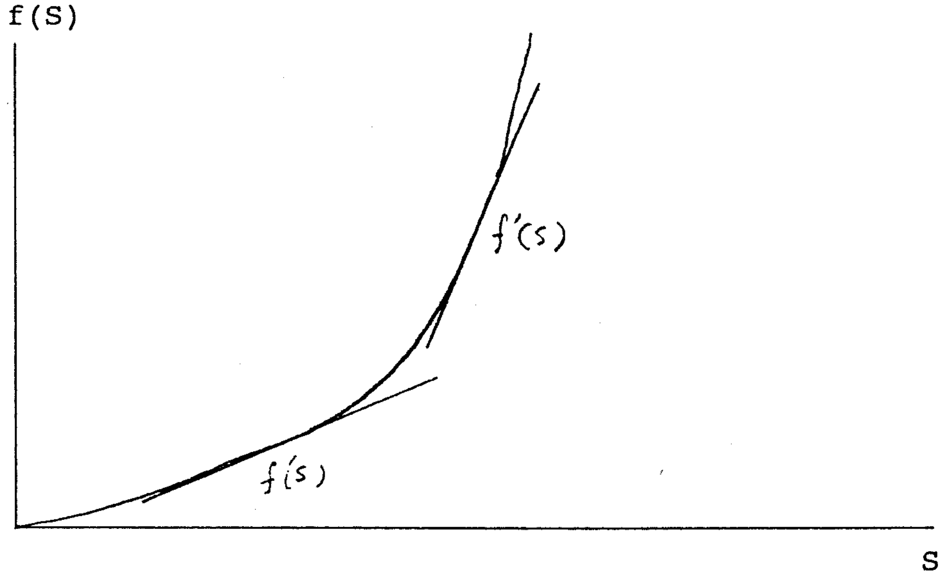

Fig.3. Incremental total cost with storage expansion

$V_i(t)$=rate of discharge of water through turbine for ith reservoir;

The problem can be written as follows.

Consider the other reservoir as a newly designed and we tried to minimize the cost of building it.

Minimize $C=\int_0^T f(S(t))\,dt$

where $S(t)=D(t)-\alpha_1 * V_1(t)$ $\quad 0\leq t\leq T$

U(t)=rate of spillage of water

and the choice of policy functions U(t), V(t) meets the three restraints stated as reservoir, turbine, demand, feasible functions and the continuation function.

The constraints can be expressed as follows.

The reservoir constraints are

$0 \le W_i(t) \le \Omega$, for $0 \le t \le T$, $W_i(0)=\Omega_0$, $W(T) \ge \Omega_T$

where $W_i(t)=\Omega_t$=water storage;

Ω_0, Ω_T=initial and minimum final store of water of the reservoir currently under consideration;

We assume that the area of reservoir floor is a constant, the height of reservoir reflects the capacity of the reservoir, the height of water storage reflects the volume of water stored.

The turbine constraints are

$0 \le U_i(t)$, $0 \le V_i(t) \le \phi$ for $0 \le t \le T$

where ϕ=turbine capacity; and water spilled doesn't go through the turbine, so there is no upper bound for $U_i(t)$.

The demand constraint requires that the hydropower generated by the two reservoirs must at least meet the demand.

The continuation functions are

$W_i(t)=\Omega_0+\int I_i(t)*dt-\int V_i(t)dt-\int U_i(t)dt$

$I_2(t)=V_1(t)+U_1(t)$ for $0 \le t \le T$

where $I_i(t)$=inflow of water into the ith reservoir.

What we want to mention is $\alpha_1(W(t))$. α is the conversion factor which depends on the head of water and turbine efficiency. If we assume constant turbine efficiency, α then depends on the water level W. In the model, we assume that the range in reservoir surface levels is very small compared with the drop in elevation from the intake gate to the tailwater pool, so we can ignore variations in the conversion factor. Here we take $\alpha_1(W_1(t))=1$ because reservoir 1 has the highest head in the system. It will considerably simplify the analysis.

Here, spillage is not considered as an important factor in the model.

In order to minimize the cost of building the new reservoir, i.e., to make full use of the second reservoir, keeping as constant usage as possible, we try to make $S(t)$ as stable as possible, so that the capacity of the reservoir can be fully used most of the time.

To achieve this goal, we try to find the optimal discharge policy for the first reservoir.

The "rule curve" is closely related to the discharge policy. A rule curve is a plot specifying reservoir elevation as a function of time. The rule is that a reservoir must be operated within constraints. The curve is constructed by finding the reservoir operation which meets the requirements and achieves the objective. The resulting reservoir elevation as a function of time is the rule curve.

Two assumptions are made in this model. One is that spillage takes place only when reservoir capacity is fully used. That is, spillage always takes the form of automatic overflow. The other assumption is that the maximal discharge policy will be taken as the one which allows discharge equal to its turbine-feasible upper bound (turbine capacity or power demand whichever is smaller).

Here we consider one year as a period in which we have Summer and Winter. In the Summer, there is abundant water flow and we may even have water spillage. We call this the abundance period. In the winter, there is not enough water flow, all the outflow is used to generate hydropower. No spillage exists. We define this as the scarcity period.

Consider the first reservoir as our main operation reservoir.

If $b_{i-1} \le t \le a_i$ is the period of abundance, we have

$\Omega(b_{i-1})+\Omega \le \Omega(a_i)$ which means the reservoir rises by at least Ω from the beginning of that period to a maximum at its end;

$\Omega(b_{i-1}) \le \Omega(t) \le \Omega(a_i)$ for $b_{i-1} \le t \le a_i$ which means the reservoir fluctuates within the range during the period;

$\Omega(t)-\Omega(t') \le \Omega$ for $b_{i-1} \le t \le t' \le a_i$ which means any downward swing within the period cannot exceed Ω in range.

If $a_i \le t \le b_i$ is the period of scarcity, we have

$\Omega(a_i)-\Omega \ge \Omega(b_i)$ which means the reservoir level falls by more than Ω from a unique maximum at the beginning to a unique minimum at the end;

$\Omega(a_i) \ge \Omega(t) \ge \Omega(b_i)$ for $a_i \le t \le b_i$ which means the reservoir level fluctuates within the range during the time period;

$\Omega(t')-\Omega(t) \le \Omega$ for $a_i \le t \le t' \le b_i$

which means any upward swing within the period cannot exceed Ω in range.

We can use diagrams(Fig.4 and Fig.5) to illustrate the above statement.

So, we can divide the minimization of the entire period into two separate periods: cost in abundance period and cost in scarcity period.

For the abundance period,

$C=\Sigma\int_{b(i-1)}^{ai} f(D(t)-V_1(t))\,dt$

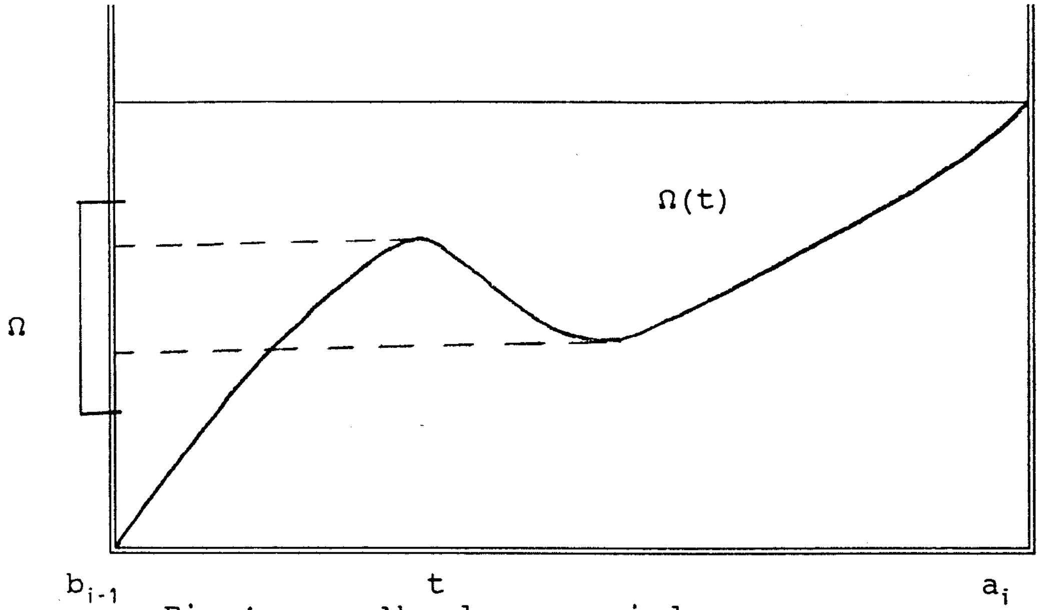

Fig.4. Abundance period

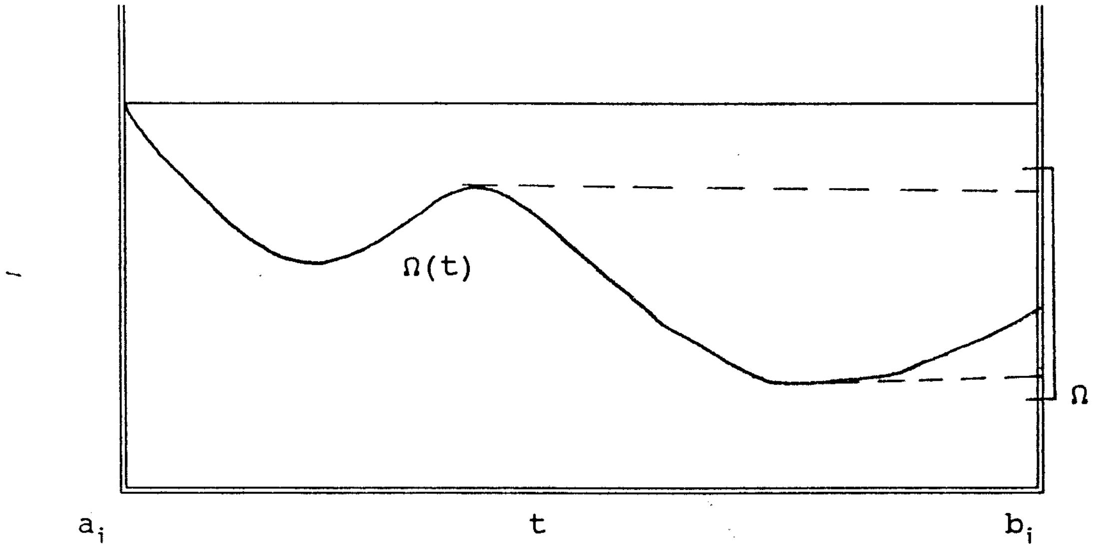

Fig.5. Scarcity period

To minimize this, we should make maximum use of the first reservoir for hydro-electricity generation. That is, the discharge throughout the planning period equal to its turbine-feasible upper bound (turbine capacity or power demand whichever is smaller), and this policy would be optimal.

That is, the rule curve of the first reservoir should be

$W_1(t)=\Omega_0+\int I_1(t)dt-\int H(t)dt-\int U_1(t)dt$

and the release policy shall be

$V_1(t)=H(t)=\min(D(t), \phi)$ for $b_{i-1}\leq t \leq a_i$

where H(t)=maximum rate of turbine discharge and it is determined by either the capacity of turbine or the rate of water release corresponding to energy demand, whichever is less;

Thus, the initial store $W(b_{i-1})$ of a period of abundance doesn't limit the choice of a discharging policy and the final store $W(a_i)$ is independent of the discharge policy because spillage may occur and $W(a_i)$ is usually the maximum level. In all these cases, the maximal discharge policy is feasible and because of increasing marginal cost of additional reservoir storage capacity which leads to higher cost of supplementary hydro-power generation, it follows that the maximal discharge policy is uniquely optimal in each period of abundance.

The discharge policy in period of scarcity of water is:

Minimize $C=\int_{ai}^{bi} f(D(t)-V_1(t))\ dt$

subject to

$0\leq V_1(t) \leq H(t)=\min(\phi, D(t))$

all three feasible constraints and the reservoir water level are as follows:

$$W(a)=\begin{cases}\Omega_0 \\ \\ \Omega\end{cases} \qquad \text{if } a \begin{cases} =0 \\ \\ \geq 0\end{cases} \qquad \text{and}$$

$$\left.\begin{matrix}0 \\ \\ \Omega_T\end{matrix}\right\} \leq W(b) \leq \Omega \quad \text{if } b \begin{cases}\leq T \\ \\ = T\end{cases}$$

In order to avoid higher cost supplementary hydro-generation, we should end up with no more than required minimum storage of water in the scarcity period in the first reservoir, which means

$$W(b_i) = \begin{cases} 0 \\ \\ \Omega_T \end{cases} \qquad \text{for } \; b \begin{cases}\leq T \\ \\ = T\end{cases}$$

The total amount of hydro generation of the second reservoir for the period is set equal to the water deficit,

$S(b)-S(a)=(Z(b)-I(b)-W(b))-(Z(a)-I(a)-W(a))$

where $Z(b)$ is the amount of water required for power generation to meet the demand at time b. $S(b)$ is the amount of supplementary power needed corresponding to water deficit at time b. Thus the average supplementary power generation rate by the second reservoir is $S(t)=(S(b)-S(a)) \;/\; (b-a)$ for $a \leq t \leq b$.

Given $S(t)$, we then can work out the release policy for the first reservoir. That is

$S(t)=D(t)-\alpha_1 {*} V_1(t)$ where $\alpha_1=1$

$V_1=D(t)-S(t)$ and $0 \le V_1(t) \le \phi$

What we have to be aware about is our optimization rule. Our release policy must end up the period of scarcity with only the required store of water and make the supplementary power generation $S(t)$ as constant as possible.

Apart from that, our solution must also satisfy all the constraints of reservoir and turbine, etc.

We shall use a graphical procedure to solve the problem which makes the answer easier to understand. But before that, a relationship should be sorted out. $W_1(t)$ is the rule curve we want to get which is affected by the release policy $V_1(t)$. When $V_1(t)$ increases, $W_1(t)$ will decrease and vice versa. Now, we know that for a given power demand, $V_1(t)$ and $S(t)$ change in the opposite direction. That is, when $S(t)$ increases, $V_1(t)$ decreases, and $W_1(t)$ increases. This implies when $S(t)$ increases, $W_1(t)$ will increase and vice versa. We now find out when $S(t)$ increases, the whole rule curve $W_1(t)$ will shift upward.

Illustrated in Fig.6.A is a family of rule curves $W(t,s)$, all starting at the full reservoir level. The curve for $W(t,s)$ represents what would be the store of water at any time t if a constant rate s of second reservoir hydro-electricity generation were maintained within turbine restraints and if no reservoir restraints existed.

Starting with the curve labelled $S=0$, we find it violating a reservoir bound for the first time for $t \ge t_0$, and the violated restraint is the lower bound. This suggests increasing S until such a value S_1 is reached such that a further increase in S would violate the upper bound at time a_1, even before the lower bound is violated. The segment of the curve W(t,s) for which $a \leq t \leq a_1$ is now moved down to Fig.6.B,C. Since the continuation of that curve would still leave the lower bound violated at t_1 , we search the diagram for curves with still higher S until we meet one, labelled $S=S_2$, for which the decline from its highest peak after a_1 does not exceed its lower bound. If this decline runs from a_2 to b_2 the segment $a_2 \leq t \leq b_2$ of the curve $W(t,S_2)$ is now reduced by a suitable constant to form another piece of the solution,

$$W(t)=W(t,S_2)-W(a_2,S_2)+\Omega \qquad \text{for} \quad a_2 \leq t \leq b_2$$

which just fits between the reservoir bounds.

In the intervening interval $a_1 \leq t \leq b_1$, the supplementary power generation rate is equal to an increasing function S(t) of time determined in such a way that the reservoir remains full ($W_1(t)=\Omega$) throughout this interval. This function is traced out by the locus of maxima of $W_1(t,s)$ (dotted line) if S increases from S_1 to S_2. Beyond b_2, the curve $W_1(t, S_2)-W_1(b_2,S_2)$ rises again to a value at b exceeding the value 0 in this case. Therefore, S(t) is gradually decreased so as to keep the reservoir empty ($W_1(t)=0$) from b_2 t0 b_3, where a curve labelled $S=S_3$ with a horizontal double tangent (double supporting line) is reached. This curve is adjusted by a constant to serve as the solution $W_1(t)$ between the two points of contact, b_3 and b_3', thereafter, the reservoir is again kept empty by further decreases of S(t) until the value S_4 is reached at b.

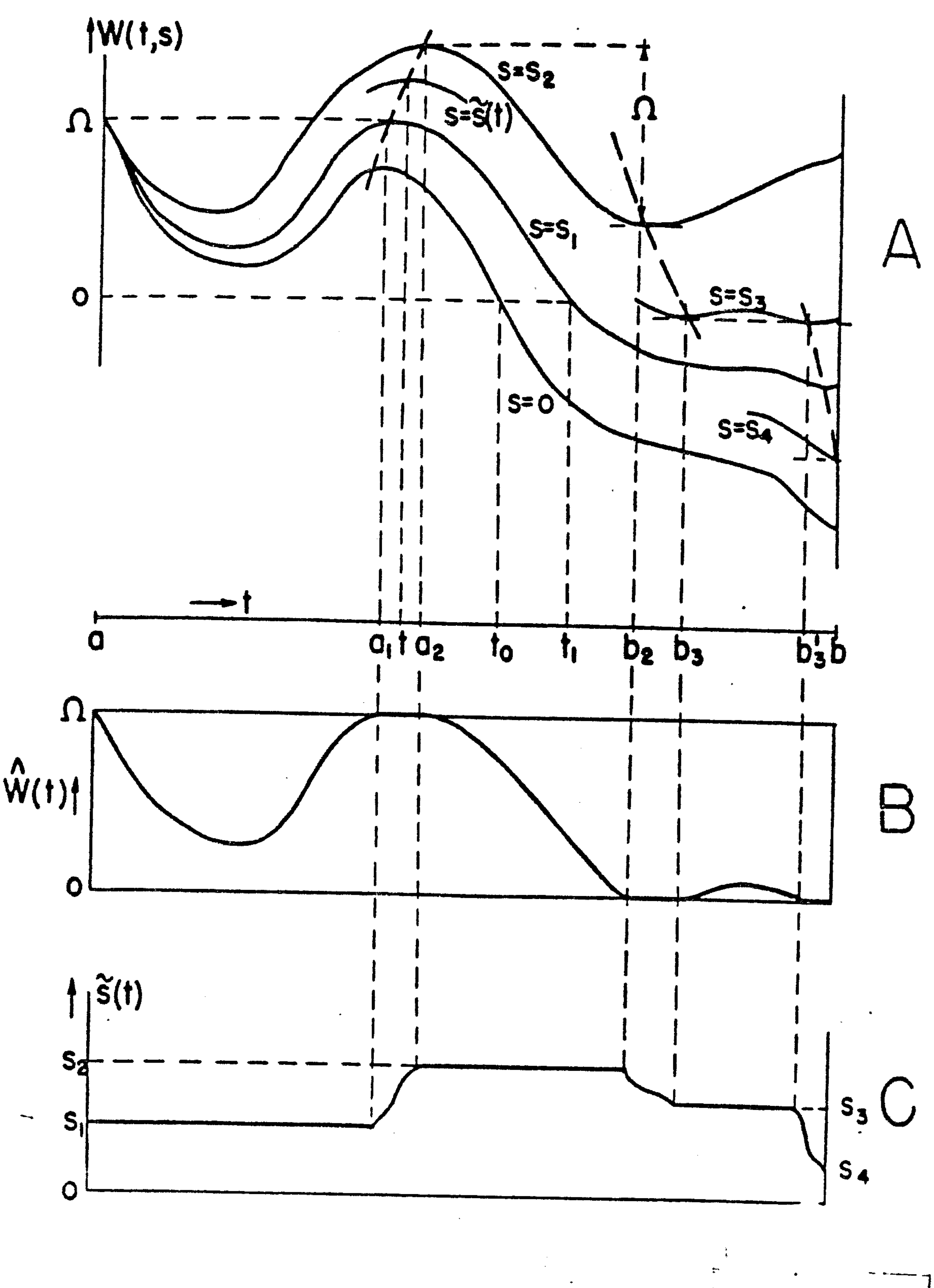

Fig.6. Construction of the optimal storage policy W(t) in a non-terminal period of scarcity of reservoir 1.

Assume two reservoirs are serially located with the first reservoir in the upper reaches and the second reservoir in the lower reaches. For the second reservoir, the water inflow from the first reservoir is as follows:

$I_2(t)=V_1(t)+U_1(t)$

So, we know the amount of hydro-electric power S(t) required during the period, and since we have assumed that the conversion factor is 1 in the first reservoir, we can assume that the conversion factor is less than 1 ($\alpha_1 \leq 1$) in the second reservoir because the second reservoir has a lower head of water than the first one. Under the assumption that the efficiency of turbines are the same, the reservoir located at the upper reaches will have an upper head of water and energy conversion from water is easier. Water will pass the upper reservoir first, and then be released to the lower one. Consequently, the second reservoir will not have as large a conversion factor as the first one. So, the amount of water outflow through turbine of second reservoir would necessarily be

$V_2(t)=S(t) / \alpha_2$

And this is the releasing policy of the second reservoir.

Thus, assuming that the starting level of reservoir is Ω_0, we can determine the rule curve of the second reservoir

$W_2(t)=\Omega_0+\int I_2(t)dt-\int V_2(t)dt$

In the period of abundance, $U_1(t)$ may not be zero

$W_2(t)=\Omega_0+\int V_1(t)dt+\int U_1(t)dt-\int (S(t)/\alpha_2)dt-\int U_2(t)dt$

where S(t) is given corresponding to the abundance period in first reservoir.

In the period of scarcity, $U_1(t)=0$

$W_2(t)=\Omega_0+\int V_1(t)dt-\int(S(t)/\alpha_2)dt$

where $S(t)$ is given, corresponding to the scarcity period in first reservoir.

To sum up, we have discussed a model in which there is excess demand for hydropower and a new reservoir should be built. The capacity of the second reservoir should be corresponding to power generation S_2 which takes both abundance and scarcity periods into consideration. Our objective was to minimize the cost of reservoir expansion which has increasing incremental costs and our solution should satisfy various constraints. Now we have found the optimal operation policies as follows:

In the abundance period, the release policies should be

$V_1(t)=H(t)=\min(\phi, D(t))$ for reservoir 1

$V_2(t)=S(t)/\alpha_2$ for reservoir 2

and the rule curves should be

$W_1(t)=\Omega_0+\int I_1(t)dt-\int H(t)dt-\int U_1(t)dt$ for reservoir 1

$W_2(t)=\Omega_0+\int V_1(t)dt+\int U_1(t)dt-\int(S(t)/\alpha_2)dt-\int U_2(t)dt$ for reservoir 2

In scarcity period, the release policies should be

$V_1(t)=D(t)-S(t)$ for reservoir 1

$V_2(t)=S(t)/\alpha_2$ for reservoir 2

and the rule curves should be

$W_1(t)=\Omega_0+\int I_1(t)dt-\int V_1(t)dt$ for reservoir 1

$W_2(t)=\Omega_0+\int V_1(t)dt-\int(S(t)/\alpha_2)dt$ for reservoir 2

Apart from that, the storage capacity should correspond to the power generation of S_2.

This conclusion can be extended to K-reservoirs system operation at the design stage. We can do it step by step. We know, when there is only one reservoir available, how we should design a second one. Now we can consider these two available reservoirs as a whole, and design another new reservoir according to the same procedure as above. This can go on and on until the K-1th reservoir is built. Once there is excess demand for power, we can consider all K-1 reservoirs as a whole, follow the same rule, and build up the Kth one. Thus, the whole system of K reservoirs is built.

In Chapter II, we applied Pindyck's model in water resource management. Pindyck's model is also considered as an investment problem because optimal capacity of reservoir is determined in the model.

Comparing Pindyck and Koopmans' models, we conclude that the latter is more or less a simplified case of the first one.

In Pindyck's model maximizes the net return of production. Koopmans' model allow us to derive the cost function given demand.

In Pindyck's model, expanding reservoir capacity can be done by means of enlarging the present one or building a second one. When the present reservoir is enlarged, the range of water surface levels varies a lot which brings variation in conversion factor. While in Koopmans' model, present reservoir capacity is fixed and the capacity of water storage is expanded by building a second one.

Production cost varies corresponding to the variation of conversion factor. In Pindyck's model, enlarging a reservoir brings a big variation in water surface level, which leads to the variation of conversion factor. Thus, in this more generalized situation, conversion factor changes when water storage level changes and production cost changes correspondingly. Both production cost and expansion cost are considered in Pindyck's model. However, in Koopmans' model, we assumed for a specific reservoir, the range of reservoir surface is very small compared with the drop in elevation from the intake gate to the tailwater pool, so we can ignore variation in conversion factor for this reservoir. But there is a big difference in water storage level among different reservoirs, so they have different conversion factors. Production cost is constant for one specific reservoir and it is not going to affect the optimum size of the second reservoir. In Koopmans' model, production cost is not considered.

We notice that when production cost is considered constant in Pindyck's model, we get Hotelling's solution for optimum price path, reservoir size.

When we refer to Chapter II, it is interesting to discover that the assumption of constant production cost (pp.25-28) is made by Hotelling and hence the conclusion is that the optimal price setting is not affected by production cost, but is affected by storage expansion cost. If we assume, like Koopmans, that both production cost and market price level for hydro power generated are constant in Pindyck's model, we can derive the following conclusion.

Since net price path is only affected by expansion cost, given price level, total cost for power generation is also only affected by expansion cost. Thus when our objective is to minimize the total cost for power generation, we choose an optimal expansion size of the reservoir. Therefore, the optimal size of the reservoir is affected only by the expansion cost which is the same conclusion we had in Koopmans' model. In this sense, we could say that Koopmans' model is a special case of Pindyck's investment model.

IV.2) Model 2[1]

The dual problem of this original problem of minimising cost of supplementary power generation is to maximize the value of energy generated by a hydropower system over the planning period plus the expected future returns from water left in storage at the end of that period. The constraints include minimum and maximum bounds on storage volumes and flow rates as well as continuity equations to maintain conservation of mass. What is more, we have minimum a contract energy production level.

It can be summarized as below:

Maximize $J=g(\Omega_T)+\Sigma B_t(\Omega_{t-1}, V_t)$

Subject to

$\Omega_t=\Omega_{t-1}+I(t)-V(t)-U(t)$

$\Omega_T=\Omega_t^{min}\leq\Omega_t\leq\Omega_t^{max}=\Omega$

$\alpha * V(t)\geq D(t)$

$0 \leq V(t) \leq \phi$

where t=1,2,....T

This design and operation of integrated multiple water supply and hydroelectric facilities to utilize best a regional water resource constitutes a complex and difficult optimization problem. A number of methods have been proposed for such optimization. A combination of linear and dynamic programming is presented here to solve the problem.

The algorithm is hierarchical: the upper level employs an

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[1] For the definitions of the terms, see the list of notations on pp.78.

algorithm resembling dynamic programming to determine power targets for each month, while a linear programming solution allocates energy generation among reservoirs at each stage.

The state of the system at the upper level is represented by a single variable G_t which equals the accumulated value of generated energy. The ending storage volumes are carried along implicitly, since they are not uniquely determined by the single-state variable.

The LP-DP algorithm optimizes over one period at a time. In each month t, the algorithm solves the optimization problem

Maximize $K_t=\Delta E_{\Omega t} * \Omega_t$

for a given value of energy generated by

$G_T=\Sigma B_l(\Omega_{l-1}, V_l)$

where $\Delta E_{\Omega t}$ is an estimate of the marginal value of water stored in each reservoir at the end of period t.

Overall, we want to find the optimum release policy, over the specified release periods, which results in storage in the final period of the largest total value of potential energy generation, while keeping the energy supply over the contract level. That is, we use LP to optimize reservoir releases V(t) and storage states Ω_{t-1}, Ω_t for each period of the total time interval of interest, with the objective of maximizing K_t subject to the demand, turbine and reservoir constraints, as we mentioned before, and given states of G_{t-1}, G_t. Then, we use DP to decide the power target G_t which gives us the optimum amount of energy we would like to generate above contract level. So, the methodology of the multireservoir operation model employs linear programming (LP) nested in dynamic programming (DP).

Let us look at the methods we used to solve the problem in detail.

IV.2.i.) Linear programming

At every stage of DP (i.e. months), a series of LP's are solved. An LP formulation is used to determine the optimal reservoir releases and storage states for each period of the total time interval of interest. The LP algorithm consists of the following steps.

a) Objective function

There are several ways to express the same objective function. We can maximize the value of stored energy at the end of this specific period while meeting the energy demand. That is, we minimize the stored potential energy losses due to unnecessary spillage, or we can express it as minimizing the sum of the reservoir releases in meeting a given energy demand. The objective of LP we choose in this article is to maximize the value of stored energy while meeting the energy demand in each period so that the whole problem of LP-DP can constitute an approximate dual problem to the one we discussed in the first part.

So the objective is to maximize $K_t=\Delta E_{\Omega t} * \Omega_t$

According to Hall (1972), the stored energy $E_{\Omega t}$ in K reservoirs in series at any given time period can be expressed in terms of the storage of reservoirs Ω^i and the corresponding energy conversion factor function α_i which is a function of W(t).

For a system with two reservoirs, K=2:

$$E_{\Omega t}=\Omega^1*\alpha_2+\int_0^{\Omega 2}\alpha_2 d\Omega^2+\alpha_1+\int_0^{\Omega 1} \alpha_1 d\Omega^1$$

For a system with K reservoirs

$$\begin{aligned}E_{\Omega_t} &= [\Omega^1+\Omega^2+\cdots+\Omega^{K-1}]\alpha_K+\int_0^{\Omega^K} \alpha_K d\Omega^K \\ &+[\Omega^1+\Omega^2+\cdots+\Omega^{K-2}]\alpha_{K-1}+\int_0^{\Omega^{K-1}} \alpha_{K-1} d\Omega^{K-1} \\ &+\cdots+\int_0^{\Omega^1} \alpha_1 d\Omega^1\end{aligned}$$

where the most upstream reservoir is numbered 1 and the last downstream reservoir is numbered K.

Becker and Yeh (1974) developed it by using this particular potential energy function to estimate the value of the water. They derived the change in stored energy produced by a set of changes in storage level as:

For K=k, $\Delta E_{\Omega t} = [\Omega^1(\partial\alpha_1/\partial^1)\Delta\Omega^1+\alpha_1*\Delta\Omega^1+\alpha_2*\Delta\Omega^2]+\alpha_1*\Delta\Omega^1$

For system with K reservoirs,

$$\begin{aligned}\Delta E_{\Omega_t} &= [\sum_{k=1}^{K-1}\Omega^k\frac{\partial\alpha_k}{\partial\Omega^k}\Delta\Omega^k+\alpha_k\sum_{k=1}^{K-1}\Delta\Omega^k+\alpha_K\Delta\Omega^K] \\ &+[\sum_{k=1}^{K-2}\Omega^k\frac{\partial\alpha_{k-1}}{\partial\Omega^{k-1}}\Delta\Omega^{k-1}+\alpha_{k-1}\sum_{k=1}^{K-2}\Delta\Omega^k+\alpha_{K-1}\Delta\Omega^{K-1}] \\ &+\cdots+\alpha_1\Delta\Omega^1\end{aligned}$$

After simplifying, it becomes

$$\Delta E_{\Omega_t} = \Delta\Omega^1[\sum_{k=1}^{K}\alpha_k] + \Omega^2[\Omega^1\frac{\partial\alpha_2}{\partial\Omega^2} + \sum_{k=2}^{K}\alpha_k] + \cdots + \Delta\Omega^K[\frac{\partial\alpha_k}{\partial\Omega^k}\sum_{k=1}^{K-1}\Omega^k + \alpha_K]$$

So, the objective function can be written as a function of Ω_t--the water storage level which is determined by release policy V(t) implicitly.

For K=k, $$K_t = \{\Delta\Omega^1[\sum_{k=1}^{K}\alpha_k] + \Omega^2[\Omega^1\frac{\partial\alpha_2}{\partial\Omega^2} + \sum_{k=2}^{K}\alpha_k] + \cdots + \Delta\Omega^K[\frac{\partial\alpha_k}{\partial\Omega^k}\sum_{k=1}^{K-1}\Omega^k + \alpha_K]\}\Omega_t$$

That is, the objective function is implicitly determined by the release policy.

b) Constraints

The LP algorithm has the following constraints:

continuity equations

$\Omega_t=\Omega_{t-1}+I(t)-V(t)-U(t)$;

maximum and minimum reservoir storage

$\Omega_t=\Omega_t^{min}\leq\Omega_t \leq\Omega_t^{max}=\Omega$;

turbine capacity

$0\leq V(t)\leq\phi$.

Energy generation constraint parametrically varied from the contract level to the maximum amount can be produced by the system

$D(t) \leq \alpha * V(t) \leq E^{max}$.

All the constraints should be nearly linear in reservoir releases during the policy period in order to satisfy the requirements of linear programming.

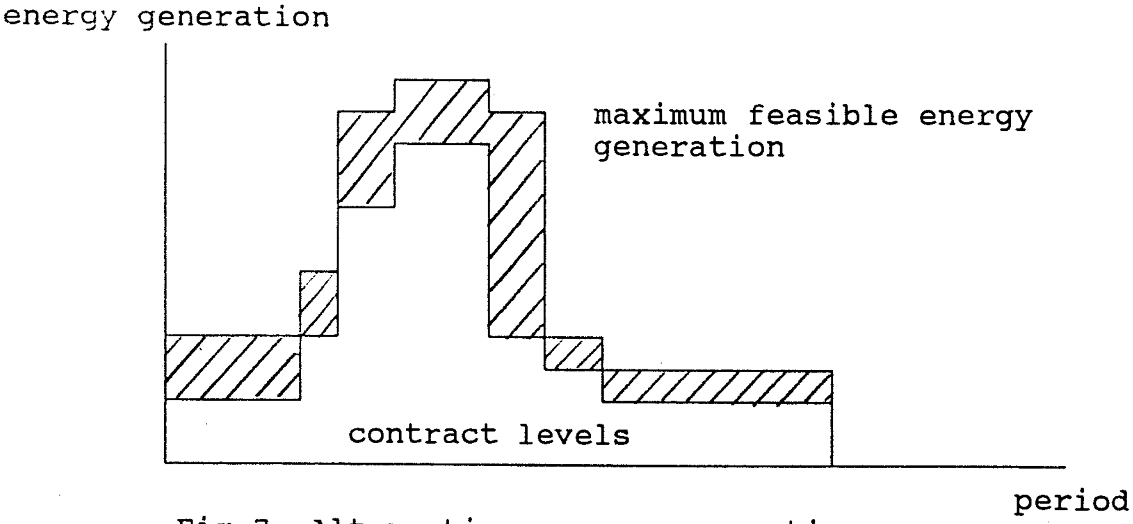

Fig.7. Alternative energy generation

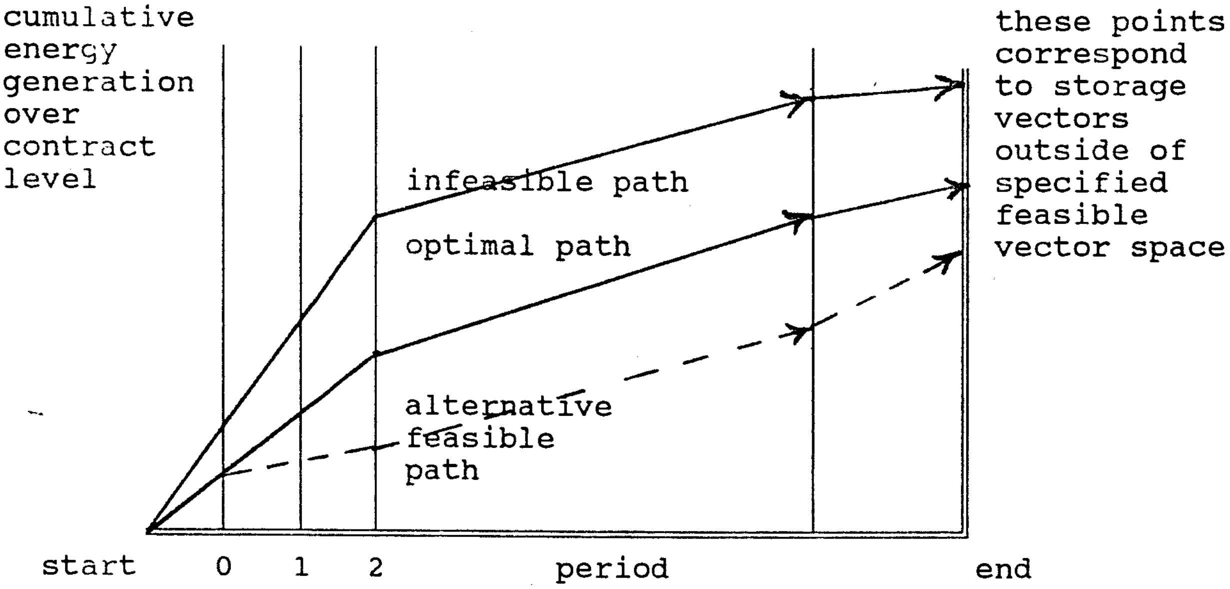

Fig.8. Policy path alternatives

An important thing that we need to mention about is the fourth constraint. The contract (demand) energy generation is usually based partly on conservative estimates of natural inflows so that the probability of not meeting these contract levels would be low. Therefore, the system is likely to be capable of providing more than the contracted levels. To allow for this extra water and power production, the values of the right hand side of the contract constraints are parametrically increased from their contract levels to the maximum possible value in each month, thus requiring the solution of a set of LP's rather than a single LP in each month. The parametric solution of LP in each month results more than one V_t and thus Ω_{t+1}, so that we have a feasible set for water releases and thus ending storage of reservoirs. Therefore, to select the "best" ending storage level of each month, we need to use DP procedure.(Fig.7)

IV.2.ii) Dynamic programming

As illustrated in Figure 8, there are alternative paths from the starting storage vector of two or K reservoirs to the ending vector space. A DP routine is used to make a selection from these alternatives representing an optimal path from period i to each of the incremental energy levels of period i+1.

The state of the DP is designated by the cumulative number of increments of energy generation over the contract level up to the current month, the cumulative energy E_c, and the decision made at every stage about the number of increments of energy generation added to the contract levels of the current month, that is, the feasible energy constraint E for the period. In going from one period to the next, there are as many optimal paths as there are feasible increments of energy in the cumulative total. The criterion for making the selection is the maximization of the sum of monthly energy generation.

This process continues until, on reaching the final period, the number of alternatives is truncated by the ending period requirement provided by the predetermined storage vector space. The path that is then taken as being best is that which corresponds to the most advantageous ending storage vector or to the maximum cumulative excess energy generated above the firm contract level.

The recursive equations for the DP are

if K=2

$$f_{i+1}(E_{c(i+1)})=\max_{E(i+1)}\{(\Omega_{i+1}{}^{1}(E_{c(i+1)},E_{i+1})$$
$$+\Omega_{i-1}{}^{2}(E_{c(i+1)},E_{i+1})+f_i(E_{ci})\}$$

For system with K reservoir,

$$f_{i+1}(E_{c(i+1)})=\max_{E(i+1)}\{\Sigma_K\Omega_{i+1}{}^{K}(E_{c(i+1)},E_{i+1})+f_i(E_{ci})\}$$

where $f_{i+1}(E_{c(i+1)})$ represents the storage vector having the maximum sum of each value of $E_{c(i+1)}$ in the period i+1 and $f_0(E_{c0})$ is the given beginning storage vector. In actual practice, the state variable E_c is discretized.

When the final period is reached, the surviving optimal path is selected corresponding to the most desirable storage vector or the greatest cumulative energy generation.

So, the net result is the selection of a policy path that will maximize total energy generation over and above the contract level and that will, under our assumptions, reach a satisfactory ending storage state (or starting state of the next period) to be considered. Further, the technique tends to reduce unnecessary spillage of water and maximum utilization is the principle carried through to the final period under consideration[2].

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[2] Becker, L. and Yeh, W.W-G, "Optimization of Real Time Operation of Multiple-reservoir System", Water Resource Research, 10(6), 1107-1112, 1974.

CHAPTER V

CONCLUSION

Canada largely relies on water resource to generate its hydro electricity. For example, the Columbia River across British Columbia and the American Northwest is the largest producer of hydroelectricity in the world. So, much attention has been drawn to the optimization of hydroelectric systems in Canada. Especially with the development of thermal plants and hydro reservoirs, various mathematical models have been applied for the best use of the resource and system.

As we have discussed in Chapter II, water can be treated as a nonrenewable or a renewable resource. It is a nonrenewable resource during scarcity period and a renewable resource during abundance period. These features are incorporated in the models we proposed.

In Chapter II, we introduced Hotelling's extraction model for the economic management of nonrenewable natural resources. Thereafter, we discussed Pindyck's exploration model for these resources. The latter developed the classical Hotelling model by treating nonrenewable resources as inexhaustible, but of variable quality; both the rate of exploration and usage of natural resources are to be optimized at the same time.

Water has its own specific characteristics which differ from other nonrenewable and renewable resources, i.e. in abundance period, it is renewable, but in scarcity periods, it is nonrenewable. Apart from this, hydro electricity generation has an incremental cost function which meets our assumption as we have discussed in Chapter II. The paper took these into consideration when the general theory of managing nonrenewable resources was applied to water resource management. In the same chapter, economic features of water in hydro system were discussed and the model of optimizing the usage of water for power generation was presented.

In the water resource management model which is the extension of Pindyck's exploration model, we used optimal theory to maximize the net benefit of water usage for power generation. We evaluated from the economic point of view the decision made and the power generated. The marginal gain from capacity expansion or storage level increase is determined and the price path of power generated is derived. We conclude that optimal of power generation, reservoir capacity expansion and price setting must be determined simultaneously because they are closely related, given exogeneous demand in the model.

A review of the literature on hydro-electric resource management was given in the following chapter. The paper introduced methods for reservoir management at both investment and operating stages. Different programming methods were summarized and the conditions under which these methods should apply were also discussed. The paper mainly focused on a series of models dealing with linear decision rules. Nevertheless, Little, Koopmans and Becker's dynamic programming models were also introduced.

In Chapter IV, the paper discussed a deterministic, multi-reservoir hydroelectricity generation system which is an extension of the Koopmans' model. The optimal operation policy differed at the design stage from that at the operation stage of the system. At the design stage, we treated the available reservoirs as a whole. While making the best use of the available reservoirs, we designed a new additional reservoir at the minimum cost. So, optimal operation rule curves for both available and designed ones were derived at the same time. By deriving the optimal rule curves for the reservoirs, we could determine the optimal rate of power generation by each reservoir and the optimal capacity needed for the newly designed reservoir.

As concluded in the paper, this investment model is a simplified case of Pindyck's model discussed in Chapter II. A comparison was performed between these two models.

At the operation stage, when reservoirs were built as one system, we divided the problem into a master problem and a subproblem and applied the LP-DP method. The optimal operation policy was derived from the maximum usage of available water in the system, i.e. maximize the value of power generation during the planning period plus the future return from water storage at the end of the period.

Simple examples of hydro system with two reservoirs were discussed in detail with optimal solutions derived. We also generalized the solutions to the situation when there were K reservoirs in a hydro system. Theoretically speaking, these models can be applied to any river with hydro reservoirs located in a series.

LIST OF NOTATIONS

t-------------time, (t)

T-------------planning or operation period, (t)

Ω-------------reservoir capacity, (l^3)

Ω_0, Ω_T---------initial and minimum final, store of water, (l^3)

$\Omega_t, W(t)$-------storage of water, (l^3)

ϕ-------------turbine capacity, (l^3/t)

$f(s)$----------cost of the second reservoir power generation function, $(\$/t)$

$\alpha(W(t))$-------conversion factor which is affected by $W(t)$, (e/l^3)

$I(t)$----------rate of inflow of water into reservoir, (l^3/t)

$D(t)$----------demand for power, (e/t)

$V(t)$----------rate of discharge of water through turbine, (l^3/t)

$U(t)$----------spillage of water, (l^3/t)

H-------------maximum rate of turbine discharge, (l^3/t)

$S(t)$----------rate of supplementary power generation of the second reservoir, (e/t)

$B_t(\Omega_{t-1}, V_t)$---value of energy generated during period t, $(\$)$

K_t------------value of stored water, $(\$)$

G_t------------sum of energy generated by K reservoirs during time t, (e)

$E_{\Omega t}$-----------stored energy at the storage level Ωt, (e)

E^{max}, E^{min}----maximum, minimum possible energy generated, (e)

Ω^i------------water storage of the ith reservoir, (l^3)

P-------------price set for hydro electricity power, $(\$/e)$

X-------------reservoir capacity expanded, (l^3)

$C_1(W(t))$------average cost of power generation excluding expansion cost, (\$/e)

$C_2(X)$---------cost of storage expansion, (\$/t)

$g(W(t)),X)$----rate of capacity expansion function, (l^3/t)

*Note: In brackets, t=time, l=length, l^3=volume, \$=money, and e=electric energy.

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