Permanent and Transitory Components in Latin American Real Exchange Rates: A Gibbs-Sampling Approach

by

Indira Romero

Major Paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M.A. Degree

Supervisor

Professor Gabriel Rodríguez

ECO 7997

Ottawa, Canada
August 2002

Acknowledgments

I would like to thank my parents and siblings for their constant encouragement. Special thanks to my supervisor Gabriel Rodríguez for suggesting me this interesting topic and for his helpful comments. I greatly appreciate to the financial support given by the Economics Department of the University of Ottawa.

I also thank Professors Arturo Huerta and Ruben Tellez who encouraged me to pursue graduate studies, I want them to know that I really appreciate all their support, you have a very special place in my heart.

Abstract

In this paper, it is estimated an unobserved component model (UCM) for four Latin American real exchange rates. The Kalman filter is used to identify a permanent and a transitory component. The estimation of the model is made using a Gibbs-sampling approach as used in Engel and Kim (1999). Unlike them, we only consider two regimes in the behavior of the variance of the transitory component given the non significance of a third regime. The four Latin American countries are Argentina, Brazil, Chile and Mexico. The information is monthly and it spans the period 1957-2002. A detailed presentation of the economic history related to the exchange rates in these countries is presented. Overall, the results show the significant importance of the transitory component to explain the behavior of the real exchange rates. The Brazilian case shows to be the most persistent. The average duration of the state of low-variance goes from 3 years (Brazil) until a little more than 7 years (Chile). In the case of the high-variance regime, the duration goes from 2 months (Brazil) until 6 months (Argentina).

Keywords: Real Exchange Rates, Gibbs-Sampling, Permanent and Transitory Components, Regimes, Persistence.

JEL Classification: C2, C5.

Contents

1 Introduction

Nominal and real exchange rates always play a crucial role in any country, since their movements affect key economic variables. This role is particularly important for developing countries, given the external restriction they face, specially after the second half of 1970 with the breakdown of the Bretton Woods system. Exchange rates have been used extensively as an economic policy tool by many Latin-American countries. For example, exchange rates were kept fixed under inflationary episodes[1], while under restrictions of foreign capital, a policy of devaluation was often followed. Generally, an "intermediate" exchange rate policy is adopted given a context of abundant foreign capital and low inflation. Currently, the major Latin-American countries have a floating exchange rate as their exchange rate policy. Unlike the industrialized countries, the Latin-American economies have experienced large devaluations and inflationary episodes . Not surprisingly, the evolution of the real exchange rate in these countries has showed periods of large appreciations, severe depreciations, and periods of stability. Given the importance of the real exchange rate, a vast amount of literature has been devoted to the analysis of its behavior, in particular to test the Purchasing Power Parity (PPP) theory. For this purpose, a variety of econometric methods have

[1] Argentina experienced hyperinflation in the eighties and early nineties

been used. The most common methods are: the unit-root test, and the co integration test. The results from these methods have been so different[2] that researchers have appropriately named it the "PPP puzzle" (Rogoff, 1996). However, there is little disagreement about the high persistence of the real exchange rate. It seems that "shocks" to the real exchange rates are of a long duration, causing its deviation -for long periods of time- from what is considered its PPP value.

In this paper, the real exchange rate is seen as consisting of two elements, a non- stationary component and a stationary component. Based on the methodology of Engel and Kim (1999), an unobserved component model of the real exchange rate is then used. In this model, the real exchange rate is decomposed into two components: a transitory and a permanent component, while the Kalman filter is used to identify them. The model is estimated using the technique of Gibbs sampling[3]. It is applied to the real exchange rate of four Latin-American countries, namely, Argentina, Brazil, Chile, and Mexico. The period of analysis is observed from January 1957 to April 2002.

[2] As surveyed in Froot and Rogoff (1995), while early research suggests the presence of a unit root in the real exchange rate, the recent evidence supports stationarity.

[3] Gibbs-sampling, a Markov chain Monte Carlo method, is a technique for generating random variables from a distribution indirectly, without having to calculate the density. See Kim and Nelson (1999) for details.

The rest of this paper is organized as follows. Section 2 presents the characteristics of the model to be estimated. Included is a discussion of the stationarity -or non- stationarity- of the real exchange rate. In section 3, we describe exchange rate policies experienced by the four Latin-American countries. Section 4 presents the data and the estimation results. Conclusions are drawn in the section 5. An appendix includes some technical explanations about Bayesian estimation and the Gibbs sampler.

2 The Model

The estimation of the real exchange rate is based on the unobserved component model used by Engel and Kim (1999), who applied it to the United States /United Kingdom real exchange rate. The real exchange rate is by definition equal to:

$$Q_t = E_t * \left(\frac{P_t^*}{P_t}\right) \tag{1}$$

or in logarithm form:

$$q_t = e_t + p_t^* - p_t \tag{2}$$

where e_t is the logarithm of the nominal exchange rate (E_t), p_t^* is the logarithm of the foreign price level (P_t^*) and p_t represents the logarithm of the home country price level (P_t).

Now, assume that the domestic and foreign prices are weighted averages of the prices of traded (T) and non-traded goods (N):

$$p_t = (1-\pi)p_t^T + \pi p_t^N \quad (3)$$

$$p_t^* = (1-\lambda)p_t^{T*} + \lambda p_t^{N*} \quad (4)$$

Then, we can write the logarithm of the real exchange rate, q_t, as the sum of two components:

$$q_t = y_t + x_t \quad (5)$$

where y_t and x_t are defined by:

$$y_t = (p_t^{N*} - p_t^{T*}) - (p_t^N - p_t^T) \quad (6)$$

$$x_t = e_t + p_t^{T*} - p_t^T \quad (7)$$

Thus, the real exchange rate, q_t, is assumed to consist of a permanent component, y_t, and a transitory component, x_t. On the one hand, the relative price of traded goods, x_t, is likely to be a stationary random variable. If all goods in the traded goods price indices have the same weight at home and abroad, then changes in x_t occurs only because of deviations from the law of one price (Engel, 1999). Even if the deviations from the law of one price are large and persistent, they are almost certainly stationary[4]. The temporary component mainly captures the slow movement of nominal prices to economic shocks. We adopt the approach of Engel and Kim (1999) who consider the shocks to the transitory component as coming primarily from a nominal demand sources, such as monetary shocks. On the other hand, permanent shocks to productivity could impart a non-stationary component to the relative price of non-traded to traded goods, thus y_t could have a unit root[5]. It is believed that the permanent component represents the shocks to tastes and technologies that cause permanent movements in relative price levels.

Given the above explanation, the permanent component is modeled as a homoskedastic random walk

$$y_t = y_{t-1} + v_t \tag{8}$$

where $v_t \sim N(0, \sigma_v^2)$. The transitory component is assumed to follow an $AR(2)$ process with a heteroskedastic error term

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \epsilon_t \tag{9}$$

[4]Almost any theory of international price determination implies that deviations from the law of one price for traded goods are stationary.

[5]See, for example, Balassa (1964) and Samuelson (1964).

where $\epsilon_t \sim N(0, \sigma^2_{\epsilon,t})$. In particular, it depends on a discrete-valued first-order Markov switching variable, S_t, $(t = 1, 2, 3)$ which evolves independently of v_t and ϵ_t, according to the following transition probabilities:

$$Pr[S_t = j | S_{t-1} = i] = p_{ij} \qquad i, j = 1, 2, 3; \sum_{j=1}^{3} p_{ij} = 1$$

Thus, we can write the variance of the transitory component as:

$$\sigma^2_{\epsilon,t} = \sigma^2_1 S_{1t} + \sigma^2_2 S_{2t} + \sigma^2_3 S_{3t}, \qquad \sigma^2_1 \prec \sigma^2_2 \prec \sigma^2_3,$$

where $S_{kt} = 1$ if $S_t = k$ $(k = 1, 2, 3)$, and $S_{kt} = 0$ otherwise. One last assumption is that shocks v_t and ϵ_t are independent. This assumption is important in helping to distinguish the temporary from the permanent component. The aggregate nominal shocks, which correspond to the transitory component, and the sectoral real shocks, which correspond to the permanent component, can be thought of as independent.

2.1 Real Exchange Rate: Stationary or Nonstationary?

When standard unit root tests, such as the ADF statistic (see Dickey and Fuller, 1979; Said and Dickey, 1984), were applied to the series of real exchange rates, they did not allow us to reject the null hypothesis of a unit root. Although this finding seems to support the decomposition of the real exchange rate into a transitory and permanent component, we cannot rely on it given the nature of the series. Specifically, they exhibit several structural breaks and combined periods of stability and high volatility[6]. In fact, the series of real exchange rates we are dealing with cover periods where different exchange rate regimes were in place.

It is widely recognized by econometricians that the ADF test presents important problems in several aspects. First, it is sensitive to the choice of the number of lags included in the test. Second, the ADF test has low power when the true data-generating process (DGP) is stationary but close to a unit root. A different type of critique that also shows the problems of power of the ADF test arises when the true DGP is that of a series having occasional changes in level or trend (Perron, 1989). Accordingly, statistical procedures for testing the presence of unit roots by considering its univariate time series properties present a constant struggle between size and power. Even though the same could be said when testing any null hypothesis, the statistical and economic implications of a unit root make this null hypothesis

[6] One issue that arises frequently in the analysis of the exchange rate (real or nominal) is whether combining the low volatility data before the Bretton Woods agreement with high volatility data after the agreement will affect the size of unit root tests. Moreover, there is some concern when including very long time series since it combines fixed and floating rate data (see Mussa, 1986).

particularly important.

Several authors[7] have tried to gain power using unit-root tests by pooling bilateral real exchange rates.They have found that the real exchange rate is mean stationary. However, O'Connell (1998) pointed out that the unit-root tests are prone to over-reject the null hypothesis in the presence of a contemporaneous correlation among bilateral real exchange rates. Adjusting the panel unit root tests to account for this issue implies that the null hypothesis of a unit root could no longer be rejected. Another problem of the panel unit root tests, as pointed out by Rogoff (1996), is that when the panel tests generate a rejection, they cannot indicate which exchange rates are responsible for that result. Recently, Taylor and Sarno (1998) have overcome this difficulty by applying tests to a group of real exchange rates. In their setting, rejection of the null occurs only when all the series are stationary. However, one flaw of this technique is that it can only be applied to a limited number of exchange rates one at a time.

Costa and Crato (2000) discuss the unit-root issue in a more general and flexible setting by using fractional integrated models. They consider that the "bias" of the unit-root tests towards the non- rejection of the null hypothesis of unit root, is caused either by a flaw in the economic and

[7] See for example, Frankel and Rose (1996), and Papell (1997).

statistical methodology or a problem with the available data. To implement their methodology they used as an input the real exchange rate of the British pound and the US. dollar against the Portuguese escudo. Using annual and monthly data[8], they found that the monthly U.K. and U.S. real exchange rate can be considered as a unit root process. However, this picture is different for the annual data. For both annual series, since the unit root hypothesis is rejected, the hypothesis of parity reversion is reinforced.

Recently, Cushman (2001) applied the Bayesian unit root test procedure of Phillips and Ploberger (1994) to 51 bilateral OECD real exchange rates for the floating period. He also applied the DF-GLS test of Elliot, Rothenberg, and Stock (1996). He found very modest support for the stationarity of the real exchange rate. Finally, evidence from Ng and Perron (2001), analyzing the validity of unit root tests for series like the real exchange rate suggests that it can have a unit root. The authors applied the new (and old) tests of unit-roots to the real exchange rate of eighteen countries[9] using quarterly data for the period 1973:1-1997:2.They could only reject, barely, a unit root in the real exchange rate for Canada (vis à vis US dollar).

[8]Monthly data covers the period from January 1973 to December 1994 whereas annual data covers the period from 1891 to 1994 for U.K. and from 1900 to 1994 for U.S.

[9]They include Australia, Austria, Canada, Denmark, France, Germany, Greece, Ireland, Italy, Japan, Korea, Luxembourg, Netherlands, Norway, Portugal, Sweden, Switzerland, and United Kingdom.

Unfortunately, all these pieces of evidence in favor of stationarity are for developed countries and only for the post-Bretton Woods period. As can be see from the above discussion, there are mixed results about the stationarity of the real exchange rate, but most of them support the unit-root hypothesis. Regardless of these, economic and historic analysis of the four countries considered here suggests that the way the real exchange rate is modeled in this paper is probably correct.

3 Exchange Rate Policies: 1957-2002

This section describes the exchange rate policies followed by Argentina, Brazil, Chile, and Mexico[10]. This section is included because we are convinced that an analysis of results, combining figures with economic history, is more appealing than the mere description of econometric results. Moreover, the economic history of the exchange rate regimes in these countries is so interesting that it deserves to be mentioned.

It is undeniable that the evolution of the nominal exchange rate (and its real counterpart) is closely related to the business cycle. Since the mid-70s, these economies have experienced severe crises that have been characterized,

[10]Information about exchange rate policies in Argentina, Brazil and Chile is based on papers cited in the references.

among other things, by sharp devaluations of their currencies and hyperinflation (Argentina being an outstanding case). During the second half of the '90s, Brazil, Chile and Mexico dropped a fixed or quasi-fixed exchange rate regime. In January 2002, Argentina was forced to give up its currency board regime that had lasted almost ten years.

3.1 Argentina (1954-2002)

In the period considered, Argentina has utilized a variety of exchange rate regimes, from a floating to a fixed exchange rate in a dollar-standard currency board. Three main periods are distinguished in line with the behavior of the nominal exchange rate: periodic adjustments in the peg until the mid-70s, accelerated devaluations through the 1980s, and stability since the beginning of the 1990s.

The period 1950-1975 corresponds to the import-substitution industrialization (ISI). This phase shows periodic, step-wise adjustments in the nominal exchange rate, and it ends with the military intervention of March 1976. The step-wise adjustments in the nominal exchange rate were associated with the internal economic limits of the inward-oriented industrialization strategy. The exchange rate regime was usually fixed (but adjustable) rates to try to maintain lower levels of inflation and support the process of industrialization. The deterioration of the external accounts forced sharp adjustments of the exchange rate in the last quarter of 1974, and, particularly during the second quarter of 1975. This last episode, known as *"El Rodrigazo"* (after the then Minister of Finance, Celestino Rodrigo), was the first of a series of maxi devaluations.

The exchange rate regimes adopted during this period can be summarized in the following way: important devaluation in 1959; dirty float system in the period 1960-1961; fixed exchange rate regime in 1961; important devaluation in the first quarter of 1962; adoption of a flexible exchange rate from the second quarter of 1962 until the first quarter of 1967; a strong initial devaluation and then a fixed exchange rate from the second quarter of 1967 until the second quarter of 1970; a multiple exchange rate with a changing mix of commercial and financial rates for different transactions until the first quarter of 1973; a fixed exchange rate system from the second quarter of 1973 until the last quarter of 1974; a strong devaluation in the last quarter of 1974; another strong devaluation in the second quarter of 1975 (transition to the inflationary period).

During the period of March of 1976 until June of 1985, the Argentinean economy experienced an acceleration of inflation and growing dollarization. In December 1978, the government implemented a new economic program.

The new program aimed at breaking inflationary expectations and reducing uncertainties. This occurred through the pre-announcements of three main variables which were controlled by the government: the devaluation rate[11], the price of goods and services provided by public enterprises and minimum wages. The exchange rate came to receive the greatest attention and had the strongest impact on the economy in years that followed, until the scheme was abandoned in 1981.

The exchange rate was pre-established in advance for the next eight months. When that period was about to expire, the rate for the subsequent period was established according to the desired goal of inflation during the next period. Domestic inflation was supposed to converge to international inflations adjusted by the present rate of devaluation. In February 1981, the government devalued the peso by 10%, followed by devaluations in April and June over 20% each, the last two under a new military regime.

The exchange rate policy during this period can be summarized in the following way: a passive crawling peg from March 1976 until December 1978; a pre-announced sliding peg (the *"tablita"*) from December 1978 until the first quarter of 1981; a floating exchange rate /adjusted passively to inflation from first quarter of 1981 until June of 1985. In this month, the government

[11] The schedule indicating the rate of devaluation was known as the *"tablita"*.

was launched the *Austral Plan.*

The *Austral Plan* was applied from June 1985 until February 1991. This program included a fixed exchange rate, along with several fiscal and monetary adjustments to control inflation. Eventually, the exchange rate had to be readjusted after three quarters from the initial peg, and then it was changed periodically, until the *Primavera Plan* in August 1988, when there was a brief attempt to fix the exchange rate again. The *Primavera Plan* collapsed, leading to the hyperinflation in the second half of 1989[12]. Thus, from June 1985 to February 1991, the government tried to fix or control the exchange rate four times. However, all attempts ended in episodes of sharp devaluations and very high inflation.

Although the initial economic programs implemented by the Menem Administration ended up in economic turbulence, the pattern of short-lived program, which attempted to fix the exchange rate only to end in an inflationary explosion continued during the first 18 months of the Menem Administration. In addition to the shared hyperinflation of 1989, the Argentinean economy had two other similar episodes. One of full hyperinflation in 1990 -quarterly inflation of 334% and devaluation of 267%- and another of very high inflation in the first quarter of 1991 -quarterly inflation of 39%

[12]The quarterly rate of inflation was 816% in the second quarter, and almost 370% in the third quarter.

and devaluation of 60%.

This economic programs established the foundations for the period of stability that began in 1991. The cornerstone of the anti-inflationary program was the exchange rate policy, as defined in the "Convertibility Law" passed by Congress in March 1991. This plan pegged the peso to the U.S. dollar one-to-one, and transformed the monetary and exchange rate functions of the Central Bank into a currency board. The fixed exchange rate had survived the so-called *"tequila effect"* in 1994-1995, a Presidential election in 1995, and the financial crises in Asia, Russia and Brazil, but it had to be abandoned in January 2002. In fact, on January 6 of 2002, the government was forced to adopt a new exchange rate policy: a floating exchange rate. In January 2002, the currency had experienced a devaluation of 40%.

3.2 Brazil (1953-2001)

During most of the time period studied, the exchange rate regime in Brazil has been a crawling peg. The two exceptions are from 1964 to 1967, when the exchange rate policy was characterized by infrequent and large devaluations, and the period from July 1994 until February 1995, when the government adopted a floating exchange rate regime.

In 1953, a system of multiple exchange rates was introduced in the context of the Bretton Woods agreement. In the period 1964-1967, the exchange rate was unified. Until 1967, the exchange rate policy was characterized by infrequent and large devaluations, causing substantial real exchange rate variability.

A long period of nominal exchange rate mini-devaluations started in 1968 in order to keep the real exchange rate stable. The real exchange rate was in fact invariant over the period. There were almost monthly devaluations. During the period 1974-1979, the real exchange rate was kept constant, but by the end of 1979, the exchange rate was devalued in 30%, causing a real exchange rate depreciation. For the year of 1980, the exchange rate devaluation was predetermined at 40%. By conclusion, it can be said that the main policy was the application of currency maxi-devaluations.

In 1980, the exchange rate policy was reverted to the old mini-devaluations policy based on inflation differential, which stabilized the real exchange rate. In February 1983, the government devalued the currency by 30%, the second time in a little more than three years. After the maxi devaluation, the government pursued a continuous devaluation scheme. Thus, it returned to the previous policy of maxi-devaluations.

The period from 1985 until 1992 corresponds to real exchange rate appreciations, indicating that the government prioritized price stabilization in detriment of the balance of payments. During 1985, the inherited exchange rate policy of daily mini-devaluations was maintained. In 1986, *The Cruzado Plan* was launched. The nominal exchange rate was fixed at the level prevalent the day before the plan was introduced. When the black market premium increased to 90%, the government broke up the fixed exchange rate rule and devalued it by 1.8% in October. At the same time, a return to the mini-devaluations policy was announced, without specifying the exact timing of devaluations.However, the exchange rate was still considered appreciated.

In 1989, the exchange rate was devalued by 18%, to reach the parity of one new Cruzado for one US. dollar. As in the *Cruzado Plan*, the nominal exchange rate rigidity would be used to affect future inflation. In contrast, the exchange rate was devalued beforehand to a very comfortable level. Furthermore, the parity of one to one was chosen to influence psychological expectations of further devaluations. On April 18, 1989, the exchange rate was devalued by 3.2%, but it was not enough to prevent further devaluation speculation. Devaluations were resumed, but at irregular time intervals. On June 15, the government decided to devaluate the currency by 4.5%, and to announce the return of the mini-devaluations and a rule of adjustments.The total devaluation rate would be equal to the current inflation rate. In order to reach that target, it would be necessary to implement six mini-devaluations.

In 1990, the government announced an important exchange rate regime change. The real exchange rate was devalued from March to July. Moreover, the government maintained the nominal exchange rate stable for the next two months. As trade surplus narrowed, and inflation persisted, the Central Bank intervened, causing a continuous real currency devaluation from September to January. On September 30, 1991, two months of fast erosion of reserves prompted a devaluation of 14%. The Central Bank kept slowly devaluing the currency until February 1992, when the attained real level was kept stable until mid 1993. On average, the real exchange rate was kept more appreciated than in the preceding periods.

In March 1994, the government created an alternative unit of account (URV), indexed by inflation. Since the degree of dollarization was low in the Brazilian economy, it was necessary to create a new unit of account with a stable purchase power. Its value in domestic currency was initially fixed at the same amount as the value of one US. dollar. During this preparation phase, the government would intervene in the dollar market whenever the dollar value would become greater than the UVR value. Although the exchange rate policy was important to fight inflation, this was not an exchange rate based stabilization, since some flexibility in the exchange rate policy was preserved.

In July 1994, a new stabilization plan was launched which was named *The Real Plan.* When the new currency, the Real, was introduced, the initial parity to the US. dollar was one to one. The government decided not to intervene, by letting the currency to appreciate up to R$ 0.83 by the end of October, because of capital inflows. For the next months, the government would intervene to maintain the currency in an informal mini-band with limits 0.83-0.86. This policy lasted until February 1995.

In March 1995, an exchange rate band regime was announced. The exchange rate was devalued by 6%. The currency was allowed to float in a band of roughly five percent which changed from time to time. Exchange rates became very stable since the government established periodic spread exchange rate auctions in July 1995. The Central Bank had been able to fix the exchange rate within very narrow limits, and the wider band, although it still existed, lost its importance. Thus, in practice the regime was a crawling peg. The nominal exchange rate has been devalued at a rate of roughly 0.6% since the end of 1996.

Because of both the 1995 Mexican crisis and the 1997 Asian crisis, it was impossible to continue the policy of a managed exchange rate regime.

In January 1999, Brazil adopted a floating exchange rate regime. This accounted for the third main change in less than five years. From January 2000 to September 2000, the nominal exchange rate was fluctuating within the interval 1.74-1.84. Between the end of 2000 and the beginning of 2001, the Argentinean crisis changed the path that the nominal exchange rate had been previously following. Thus, between January 2001 and September 2001, the monthly depreciation of the nominal exchange rate raised to 36%.

3.3 Chile (1955-2001)

Although Chile has experienced almost all the options of exchange rate regimes in the last forty-eight years, it has not adopted a foreign currency. From the hard pegging in the early 60s and 80s, to the current clean floating, this country has even developed some innovative intermediate regimes that were subsequently adopted by a number of other countries.The Chilean economy has changed from a crawling peg, adjusted to the past inflation scheme of the second half of the 60s, to the "active" crawling peg arrangement of 1978 (later popularized in Argentina as the *"tablita"*), and then to a crawling band of the late 80s and most of the 90s. A floating exchange rate policy has been in place since September 1999.The following is a description of the evolution of the exchange rate policies in Chile.

The "Mission Klein-Sacks",an economic program designed by an American consulting firm, was implemented from 1955 to 1958, they proposed a broad reform program that showed partial success. After reducing inflation levels from around 70-80% in 1954-55 to 17% in 1957, inflation jumped again to around 30% in 1958. The Chilean currency was devalued by 18% in late 1958, and by 25% in January 1959.

In January 1959, the exchange rate was fixed at 1.05 Escudos per US. dollar. The foreign exchange market was unified, and free access to it was established. Between 1960 and 1962, there was a decline in inflation and a sharp increase in the deficit of the current account. With the external situation deteriorating and the liberalization of the foreign exchange, the year 1962 started with a reversal of liberalization, and the introduction of foreign trade restrictions. The exchange rate system was abandoned in October 1962.

From November 1962 until March 1965, the exchange rate policy and capital account management were oriented to close the external gap. Restrictions in the foreign exchange market were reintroduced and intensified during 1963-64. A crawling peg system was initiated in April 1965, with a couple of small devaluations a month. This policy remained essentially the same until 1970, with varying degree of restrictions on foreign exchange and foreign trade, depending to a large extent on the short-term balance of payments conditions. By the late 1960s, there were three official exchange rates. In late 1970, there was a new attempt to fix the exchange rate, but the rapid rise in inflation caused a substantial appreciation of the real exchange rate. The policy response was to devalue the currency and to increase segmentation in the foreign exchange market. After the militaries took power in 1973, the first measure was a 300% devaluation to restore competitiveness. A crawling peg with four devaluations a month was implemented, with different adjustments for the different rates, for the purpose of achieving exchange rate unification. In August 1974, there were only two exchange rates. During 1976-1977 the crawling peg system was based on pre- announced daily devaluations , and indexed to the domestic inflation. However, in June 1976 and March 1977 two revaluations of 10% were implemented.

Between February 1978 and May 1979, the system of daily indexed devaluations was abandoned by a *"tablita"* that set the rate of crawl according to inflationary targets. The *"tablita"* was announced to start with a 2.5% monthly devaluation (in February) to end in 0.75% (in the last quarter of 1978). In late 1978, it was announced a *"tablita"* for the whole year of 1979, with a devaluation of 15%. The year of 1978 closed with 30% inflation, the same rate prevailing in June 1979.

The exchange rate was fixed in June 1979. The value of the dollar was set at \$39, which was 6% higher than the prevailing *"tablita"*. After deteriorating internal and external conditions, the fixed exchange rate collapsed in June 1982, and the currency was devalued. It was followed by a period of instability, with discrete devaluations, a short period of a floating exchange rate, and segmentation of the exchange rate market.

In August 1982, the exchange rate was again set according to a crawling peg based on a PPP-rule. In late September 1982, the exchange rate was allowed to move within a narrow band (±2 %).

Several devaluations were implemented during the 1980s, in addition to the daily devaluations. The first one was 23.7% in September 1984, 9% in February 1985, and 5% in June 1985. The crawling rate had been set following a PPP rule. In late 1985, a downward drift of 2% was added to the PPP rule. The central parity was set according to a basket of currencies, while the weights were changed several times. For the first time, a narrow band (2%) was applied. In mid 1989, the band was increased to 5%, further to 10 % in January of 1992 and to 12.5 % in January 1997.

As in the 1980s, the central parity was changed on several occasions during the 1990s, but through revaluations instead of devaluations. There had been four revaluations: in June 1991 (2%); in January 1992 (5%); in December 1994 (10%); and finally in January 1997 (5%). The Central Bank decided by the end of June 1998 to narrow the band width, from the prevailing 2.5% to 5.5%. Thus, the Chilean economy adopted a crawling band from August 1984 to September 1999. Since September 1999, a period of floating exchange rate is the prevailing exchange rate policy.

3.4 Mexico (1954-2001)

Mexico has had to change its exchange rate regime many times during the forty-five years of exchange rate policy. Most often, this change was originated by inflation pressures. Since 1976, it had been almost a routine to end a presidential period[13] with a devaluation of the currency, causing a subsequent inflationary episodes it brings about, and a slowdown in production. Several measures were implemented to face this sort of crisis, the exchange rate policy being the foremost. Fortunately, the last presidential election of 2000 ended this cycle. Currently, a floating exchange rate regime prevails.

Between 1954 and the first eight months of 1976, the nominal peso-US. dollar exchange rate was fixed at 0.0125. It had been associated with price stability and a high rate of growth. The beginning of the world-wide

[13]Six- year Presidential term.

economic crisis in 1971 -characterized by floating exchange rates, permanent supply side shocks and structural adjustment policies, even in developed countries- combined with the fledging trade opening in 1978-1981, intensified the serious structural problems of the Mexican economy.

With the outbreak of the structural crisis in 1976, and particularly since February 1982 until December 1987, the exchange rate became an important instrument for the correction of the current account deficit. In addition, since Mexico was outside of the international financial markets until the late 1980s due to its so-called debt crisis, it was necessary to maintain a trade surplus in order to fulfill financial commitments abroad. The resulting permanent devaluations and inflation-devaluation spiral brought the country to the brink of hyperinflation. An adjustment policy was implemented in 1987 in which changes in the exchange rate and the design of an income policy would be crucial. The currency experienced five major devaluations between 1976 and 1987: in September 1976 (59%); in February 1982 (68%); in December 1982 (93%); in July 1985 (23%); and in December 1987 (25%).

During the period from December 1987 until November 1994, the prevailing exchange rate policy can be characterized as semi-fixed, with the intention of reducing inflation by using it as a price anchor -that is, as an instrument which would contribute to the elimination of inertial inflation and, at the same time, guarantee that the fiscal policy would maintain the necessary discipline. Formally, the exchange rate policy in this period was called crawling peg, and it was in use from January 1989 to November 10, 1991, while the exchange rate bands were in use from November 11, 1991 to December 21, 1994. It was impossible to maintain the stability of the exchange rate, due to its high level reached in the previous phase and to the enormous capital inflows that Mexico enjoyed until 1993. In December 1987, The Pact of Economic Solidarity, better known as *"The Pacto"*, was signed simultaneously by the government and representatives of workers, agricultural producers and the business sector. Its basic components included a commitment to further reductions of the fiscal deficit, tighten monetary policy, liberalize trade and, for the first time since the crisis of 1982, it implemented an income policy, which would cover wages, prices and the exchange rate.

The policy, based on a fixed exchange rate, designed to fight inflationary pressures, was implemented at the end of February 1988. Beginning in 1989, the fixed exchange rate regime was replaced by a crawling peg. Originally, the crawling peg was fixed at one peso per day (equivalent to the annual depreciation rate of 16%). In 1990, this was reduced to 80 cents (11% annual depreciation) and, in 1991, the crawl was fixed at 40 cents daily (5% annual depreciation). In November 1991, the crawling peg was replaced by a band within which the exchange rate was allowed to fluctuate. The ceiling of the band was adjusted daily by 0.0002. This adjustment was increased in October 1992 to 0.0004 daily while the floor was maintained at 3.0512 pesos per US. dollar. After April 1994, the dollar often was at the ceiling of the band (the ceiling of the band in this period was equal to a little more than 3.4 pesos per dollar). On December 20, 1994, the ceiling of the band was increased by 15% while its daily increase of 0.0004 was maintained.

However, the adjustments made to the band, within which the exchange rate was allowed to fluctuate, did not work and this exchange rate regime was abandoned on December 22, 1994, when the Mexican authorities were forced to adopt a floating exchange rate regime since they had run out of international reserves. In the last month of 1994, the currency had to be devalued in 54%. The floating exchange rate regime is combined with an automatic intervention mechanism in the exchange market by the Bank of Mexico in response to unusual events.

Summary

Argentina

1950-1975: Periodic, step-wise adjustments in the nominal exchange rate. This policy ended with the military intervention of March 1976.

March 1976-December 1978: Passive crawling peg.

December 1978-First quarter of 1981: Pre-announced sliding peg (the "tablita").

First quarter of 1981-June 1985: Floating exchange rate adjusted passively to inflation.

June 1985-February 1991: Attempts to fix the exchange rate

March 1991-January 2002: Currency board.

January 2002- ? : Floating exchange rate

It is important to point out that this country exhibited both the sharpest devaluations and the highest inflation rate of the four economies considered in the analysis. Those episodes of devaluation and inflation are particularly severe during the 70's and 80's.

Brazil

1955-1960: Multiple nominal exchange rates, quasi-fixed exchange rate policy

!960-1968: Fixed exchange rate subject to occasional maxi-devaluations.

January 1969-November 1979: Consolidation of a regime of mini-devaluations that began in August 68 consisting of frequent devaluations of small magnitude.

December 1979-January 1983: Policy of mini-devaluation of the real exchange rate.

February 1983-February 1990: This period began with a devaluation of the nominal exchange rate of 30%. The main exchange rate policy was that of mini-devaluations mixed with intervals of "freezing" exchange rate[14].

March 1990- June 1994: In 1990, under the Collor Plan, a floating exchange rate regime was adopted.

July 1994-December 1998: A new stabilization plan was adopted (Real Plan). The floating exchange rate policy prevailed until September 1994. On March 6, 1995, a exchange rate band regime was adopted; the limits of the band would change from time to time.

January 1999- ?: Floating exchange rate regime.

Chile

January 1959-October 1962: The exchange rate was fixed.

November 1962-March 1965: The goal of the exchange rate policy was to close the external gap, active exchange rate policy.

April 1965- 1975 : Crawling peg system. The Chilean economy experienced sharp devaluations of its currency during the first half of de 70's.

1976-77: Crawling peg, pre-announced daily devaluations.

February 1978-May 1979: During this period, it was adopted a exchange

[14]These episodes of "freezing" exchange rate occurred during the Cruzado Plan (1986), Bresser Plan (1987), and Verao Plan (1989).

rate regime named "tablita". This policy set the rate of crawl according to inflationary targets.

June 1979-June 1982: Fixed exchange rate. In June, 1982 there was a devaluation of the nominal exchange rate.

August 1982: The exchange rate was again set according to a crawling peg based on a Purchasing Power Parity- rule.

September 1982: The exchange rate was allowed to move within a narrow band (±2%).

There were several devaluation during the 80's: In September 1984, February 1985, June 1985.

August 1984- September 1999: Crawling band.

September 1999- ?: Floating exchange rate.

Mexico

1954-August 1976: Fixed exchange rate.

September 1976-January 1982: Policy of mini-devaluations, depreciation of the real exchange rate.

February 1982-December 1987: The exchange rate became an important instrument for the correction of the current account deficit. This period was characterized by a policy of maxi-devaluation of the nominal exchange rate.

There were several sharp devaluation of the currency in the following dates: August 1976, February 1982, December 1982, July 1985, and December 1987.

December 1987-November 1991: During this period, the exchange rate policy can be characterized as semi-fixed.

January 1989-November 1991: Crawling peg.

November 1991-December 1994: Exchange rate bands.

December 1994- ?: Floating exchange rate policy with sporadic interventions by the Central Bank.

4 Data and Estimation Results

In a previous section, we set up the characteristics of the model to be estimated. The real exchange rate was thought to consist of two parts: a transitory component and a permanent component. In this setting, the transitory component, x_t, represents the relative price of traded goods; the permanent component, y_t, represents the relative price of non-traded to traded goods. Although the decomposition is theoretically correct, there are not available figures for the estimation to be empirically possible, that is, we only observe the real exchange rate as a whole. In this context, the Kalman filter is used to identify the two components.

Real exchange rates are calculated in the usual way, that is, using expression (2). For our purpose, P_t^* represents the consumer (or producer) price index of United States; P_t is the consumer (or producer) price indices of Argentina, Brazil, Chile or Mexico and the nominal exchange rate is represented by e_t. In the estimation of the model, we use the logarithm of the real exchange rate multiplied by 100. Because the time span matters in this econometric estimation, a consumer or producer price index was chosen according to availability of information. Thus, for Argentina, Chile and Mexico, consumer price indices were used, and for Brazil, a producer price index was chosen[15].Monthly data is used and it corresponds to the period from January 1957 until April 2002[16]. Information on price indices and nominal exchange rates are taken from the "International Financial Statistics", the CD-ROM of the International Monetary Fund (IMF).

In the estimation of this model, the Gibbs sampler is run for twelve thousand iterations, and we discard the first two thousand. The distributions of $\{X^T\}$, $\{Y^T\}$ and $\{S^T\}$[17]are based on the last ten thousand iterations. For the distribution of $\Theta = \{\phi_1, \phi_2, \sigma_v^2, \sigma_{\epsilon1}^2, \sigma_{\epsilon2}^2, p_{11}, p_{22}\}$, it is taken every fifth observation from the final ten thousand observations (because of potential

[15]It is important to point out that the behaviour of the real exchange rate under both indices is very similar; comparisons were made under equal periods.

[16]Given the nature of the model estimated a longer time span would be desirable, but information before 1957 is not available.

[17]X^T refers to the vector of T values of the transitory component, whereas Y^T refers to the vector of T values of the permanent component. S^T is the vector of T values of the state S_t.

serial correlation across iterations).

Observe that the parameters of the model are treated as random variables with prior distributions in the Bayesian setting. At first, the model was specified following Engel and Kim's (1999) mode, but this specification was inappropriate.. Thus, a slightly different model was implemented. The new model allows the variance of the transitory component to evolve according to a Markov switching process between a low and high variance states (not three states like in the original model). The variance of the permanent component is kept homoskedastic.

4.1 Argentina

It is not a surprise that the volatility observed in the real exchange rate of these Latin-American countries is closely related to the devaluations of their currencies. Argentina is a special case since it has experienced periods of hyperinflation and sharp devaluations that are reflected in the behavior of its real exchange rate. From Figures 1 and 2, it is easily seen how the pattern of devaluation is reflected on the movements of the real exchange rate.

Results from the model applied to this country are presented in Table 1. It shows the statistical properties of all the unknown parameters of the econometric model. Figure 3 illustrates the similarity between the transitory component and the real exchange rate, it shows that most of the volatility of the real exchange rate is due to the transitory component. Interestingly enough, a glance at Figure 4 demonstrates that the probability of not being in the low variance state corresponds, precisely, to periods of high inflation and sharp devaluation. From 1991 to 2001, the Figure 5 shows that the probability of being in the low-variance state is almost one. It is known that the Argentinean economy adopted a currency board in March 1991 and that it lasted until December 2001. During that period, this country exhibited relative stability and it is well reflected in the estimation. In January 2002, the currency board policy was over; figure 6 reflects this fact very well, since the probability of being in the high- variance state is close to one.

Table 1. Results for Argentina

Parameter	Mean	SD	Median	95% Posterior Band
p_{11}	0.986	0.005	0.987	0.975,0.998
p_{22}	0.834	0.008	0.845	0.681,0.986
ϕ_1	0.774	0.052	0.774	0.671,0.877
ϕ_2	0.203	0.059	0.203	0.102,0.303
$\phi_1 + \phi_2$	0.977	0.012	0.981	0.954,1.000
σ_v^2	10.634	5.411	9.610	0.028,21.239
$\sigma_{\epsilon 1}^2$	37.117	7.124	36.604	23.154,51.080
$\sigma_{\epsilon 2}^2$	1504.778	568.245	1394.378	391.018,2618.538
$eigen$	0.981	0.009	0.984	0.962,1.000

In all tables of results, p_{11} is the probability of staying in the state 1 (low-variance state); p_{22} is the probability of staying in the state 2 (high-variance state); ϕ_1 is the coefficient associated to the first-order autoregressive term; ϕ_2 is the coefficient associated to the second-order autoregressive term; σ_v^2 is the error term variance of the permanent component; $\sigma_{\epsilon 1}^2$ is the error term variance of the transitory component (state 1); $\sigma_{\epsilon 2}^2$ is the error term variance of the transitory component (state 2). Finally, *eigen* refers to the largest eigenvalue of the transitory component. From figures of Table 1 we can compute the expected duration of both the low-variance and high variance states. The expected duration of the low-variance state can be calculated from $1/(1 - p_{11})$, which corresponds to 75.93 (approximately 6 years). Likewise, the expected duration of the high-variance state is calculated from $1/(1 - p_{22})$, and it is approximately 6 months.

Values of the coefficients associated with the autoregressive terms of the transitory component, tell us that this component of the real exchange rate for Argentina is relatively less persistent than those of Brazil, Chile or Mexico as we will see. The variance of the permanent component, not surprisingly, is the smallest of the three variances. It might be of some concern that the high-state variance of the transitory component is too high, but if we think about the extremely high devaluations experienced by this country[18], that figure makes sense.

[18] Especially during the 80s, there is a year where the monthly devaluation of the currency was of almost 400%. Moreover, the model for the three countries was estimated

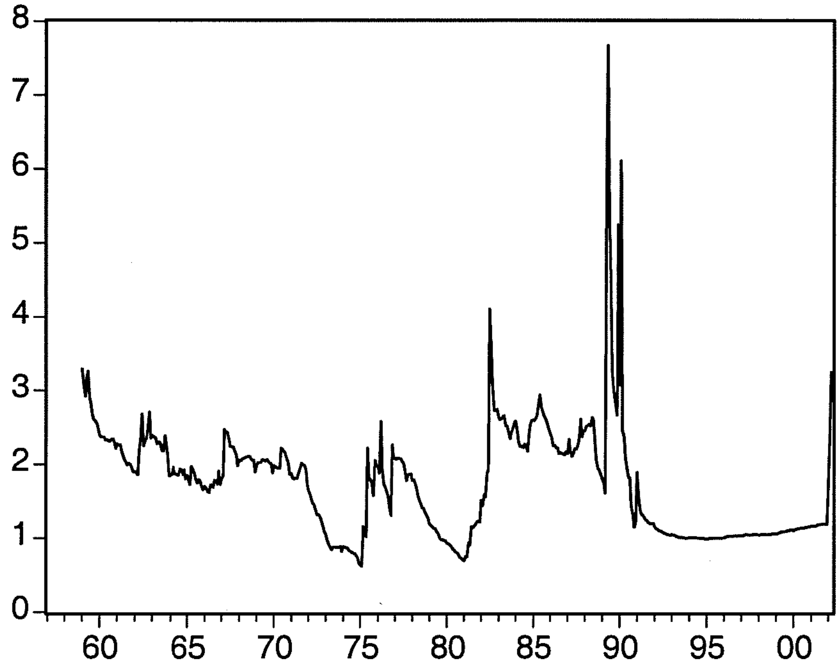

Figure 1: Argentina: Monthly Real Exchange Rate [$Q_t = E_t(P_t^*/P_t)$]

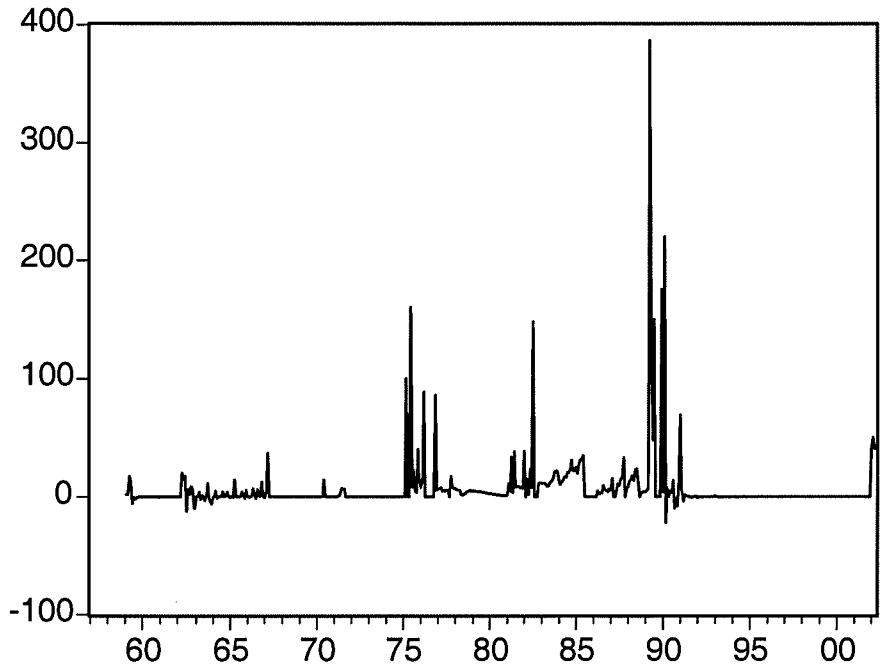

Figure 2: Argentina: Monthly Rate of Devaluation (%)

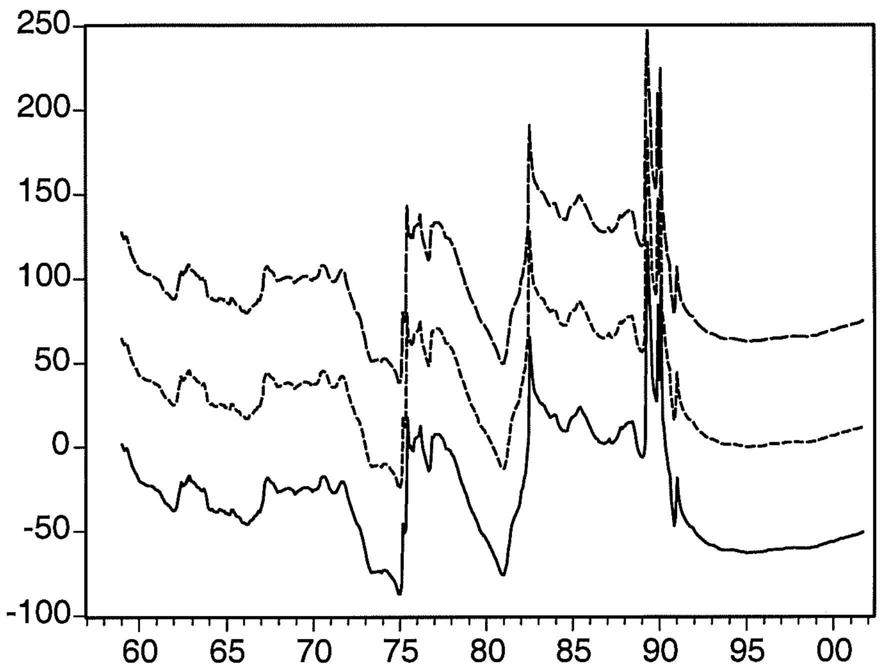

Figure 3: Argentina: Transitory Component with Lower and Upper Bands [Units: Logarithm times 100]

Figure 4: Argentina: Permanent Component with Lower and Upper Bands [Units: Logarithm times 100]

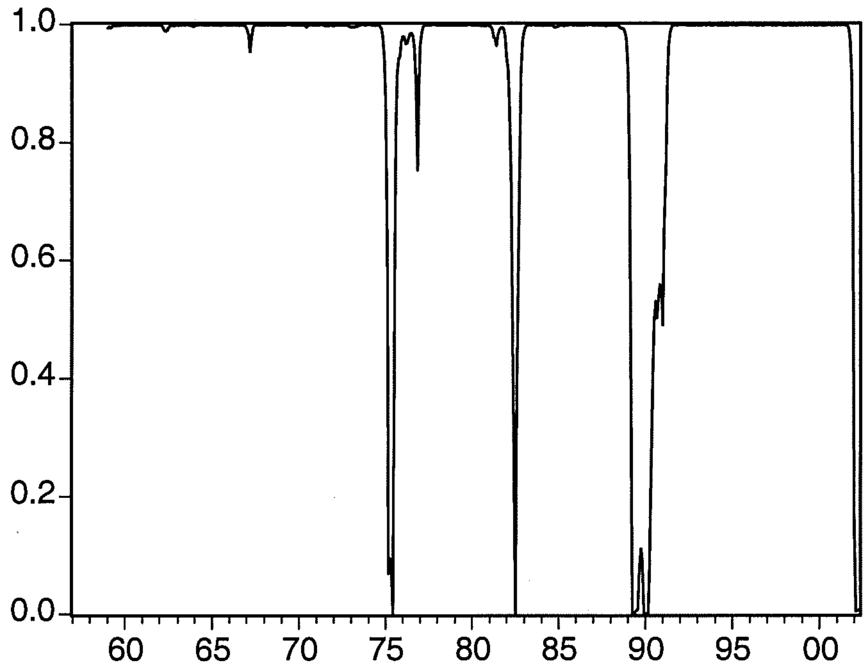

Figure 5: Argentina: Smoothed Probabilities of State 1 [Low-variance state]

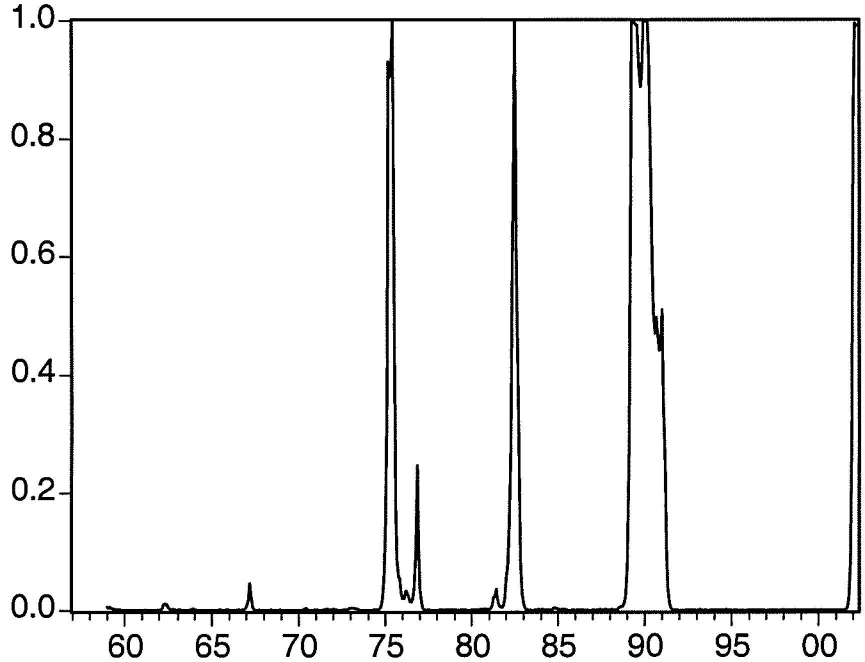

Figure 6: Argentina: Smoothed Probabilities of State 2 [High-variance state]

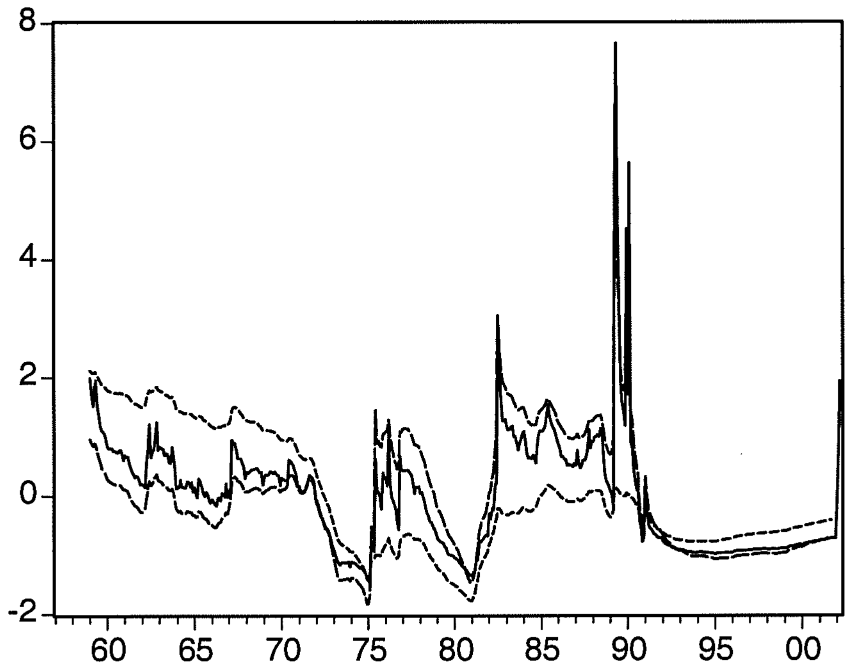

Figure 7: Argentina: Real Exchange Rate (—), Permanent Component (....) and Transitory Component (- - -). Normalized data.

4.2 Brazil

The history of the real exchange rate in Brazil is a little different from the other countries. From Figure 9, we can see that mini and maxi devaluations have been a permanent characteristic of the Brazilian currency, but maxi devaluations have not been as severe as the Argentinean case. Most of the sharp movements of the Brazilian currency happened during the 1980s and 1990s. This period, already described, was characterized by frequent changes in the exchange rate policy and the launching of several stabilization plans. The real exchange rate of this economy does not exhibit large periods of appreciation and depreciation.

Once again, the movements of the real exchange rate resemble the movements of its nominal value. Unlike the case of the Argentinean economy, these series do not exhibit many "peaks" or volatility.

From the figures of the transitory and permanent component (Figures 10 and 11), it is easily seen that they are similar. The values of the coefficients associated with the autoregressive terms tell us that the transitory component of the real exchange rate is highly persistent (see Table 2). This can make the distinction between the permanent and transitory component

using the logarithm of the real exchange rate multiplied by 100 as input, not just the real exchange rate.

very difficult.

Table 2. Results for Brazil

Parameter	Mean	SD	Median	95% Posterior Band
p_{11}	0.973	0.025	0.979	0.923,1.022
p_{22}	0.514	0.184	0.522	0.152,0.876
ϕ_1	1.006	0.065	1.008	0.878,1.135
ϕ_2	-0.032	0.061	-0.032	-0.151,0.086
$\phi_1 + \phi_2$	0.974	0.024	0.981	0.926,1.022
σ_v^2	18.227	3.742	18.087	10.892,25.563
$\sigma_{\epsilon 1}^2$	8.753	1.698	8.544	5.425,12.081
$\sigma_{\epsilon 2}^2$	168.148	182.409	130.495	-189.373,525.670
eigen	0.973	0.026	0.981	0.922,1.024

For this country, the expected duration of the low-variance state is of approximately three years, while the expected duration of the high-variance state is two months. These results reflect the characteristics that are present in the Brazilian economy. Figures 12 and 13 show the probabilities of being in the low or high-variance state. They reflect the exchange rate policies followed by this country very well. After the crisis of 1999, Figure 12 shows that the probability of being in the low-variance state is almost one.

With respect to the values of the variance, we can see that the variance of the permanent component is twice the value of the transitory component's variance. This is not surprising once we look at figure 8 showing the real exchange rate.

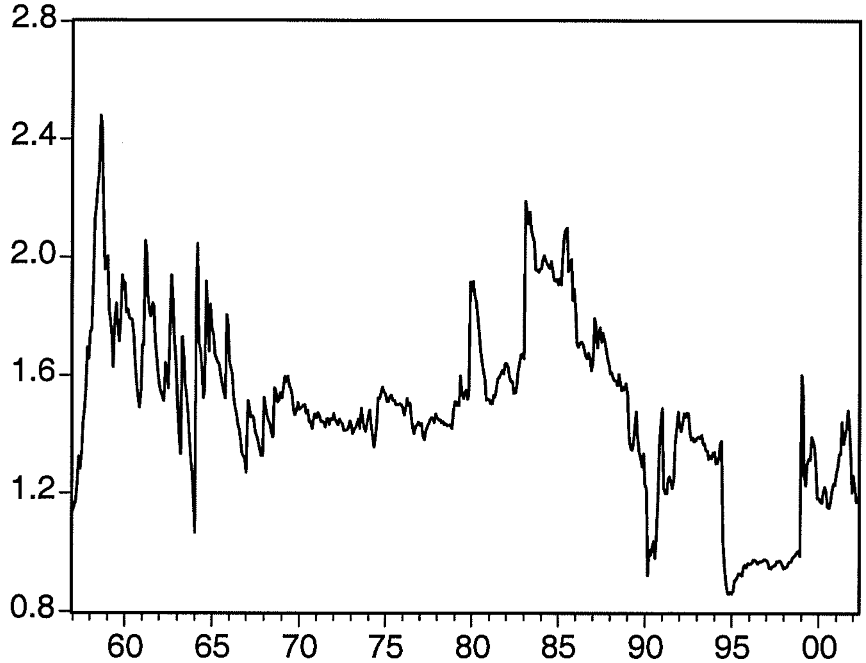

Figure 8: Brazil: Real Exchange Rate $[Q_t = E_t(P_t^*/P_t)]$

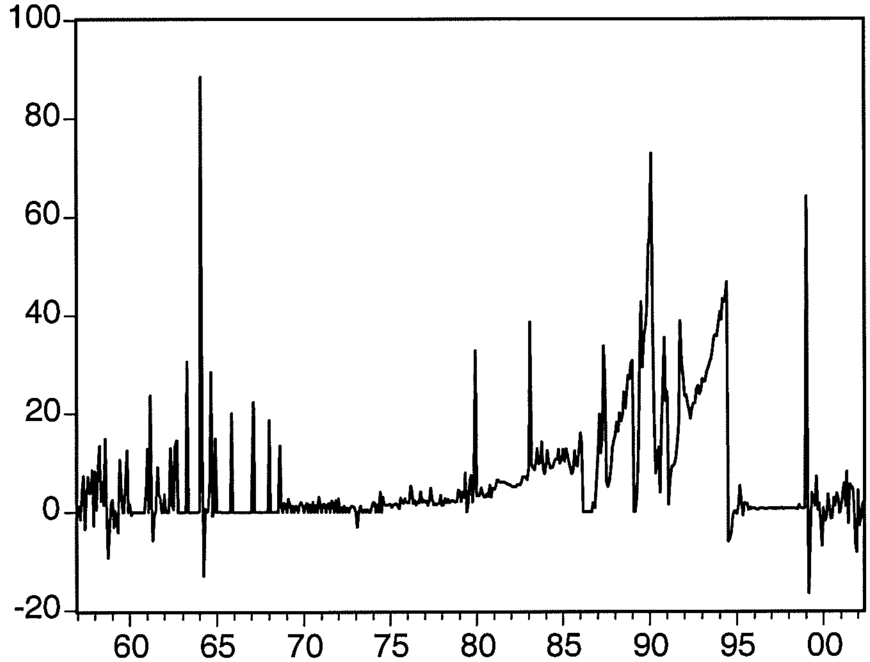

Figure 9: Brazil: Monthly Rate of Devaluation (%)

Figure 10: Brazil: Transitory Component with Lower and Upper Bands [Units: Logarithm times 100]

Figure 11: Brazil: Permanent Component with Lower and Upper Bands [Units: Logarithm times 100]

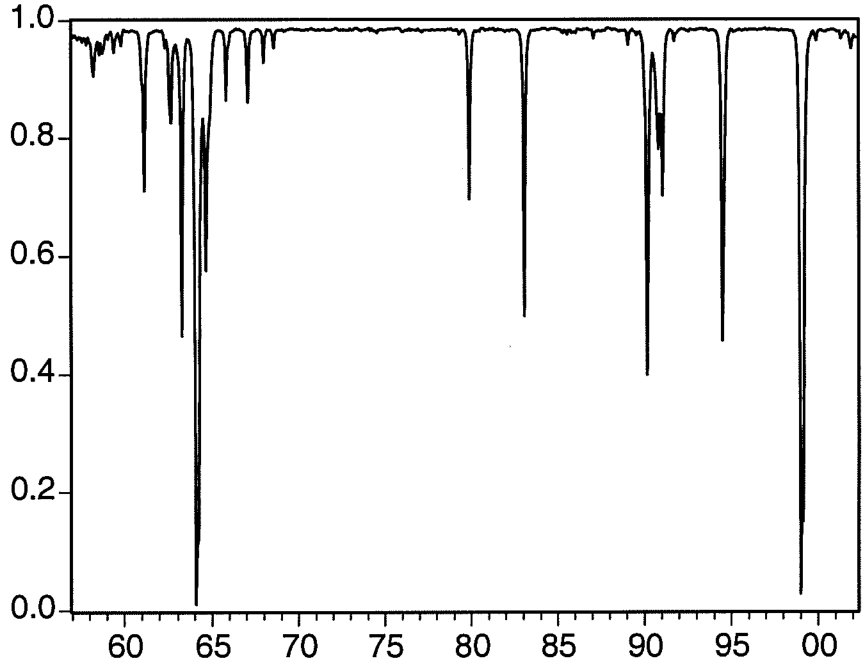

Figure 12: Brazil: Smoothed Probabilities of State 1 [Low-variance state]

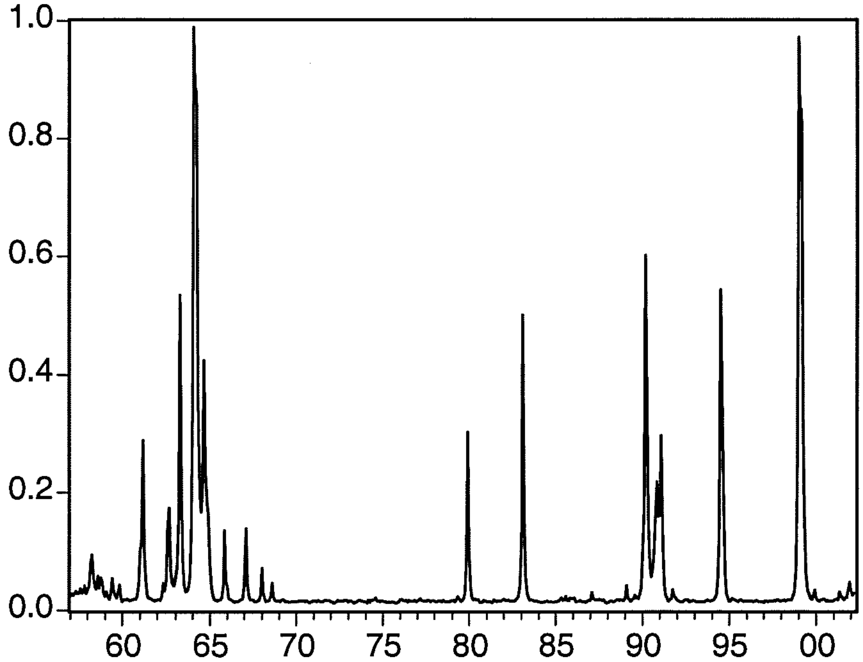

Figure 13: Brazil: Smoothed Probabilities of State 2 [High-variance state]

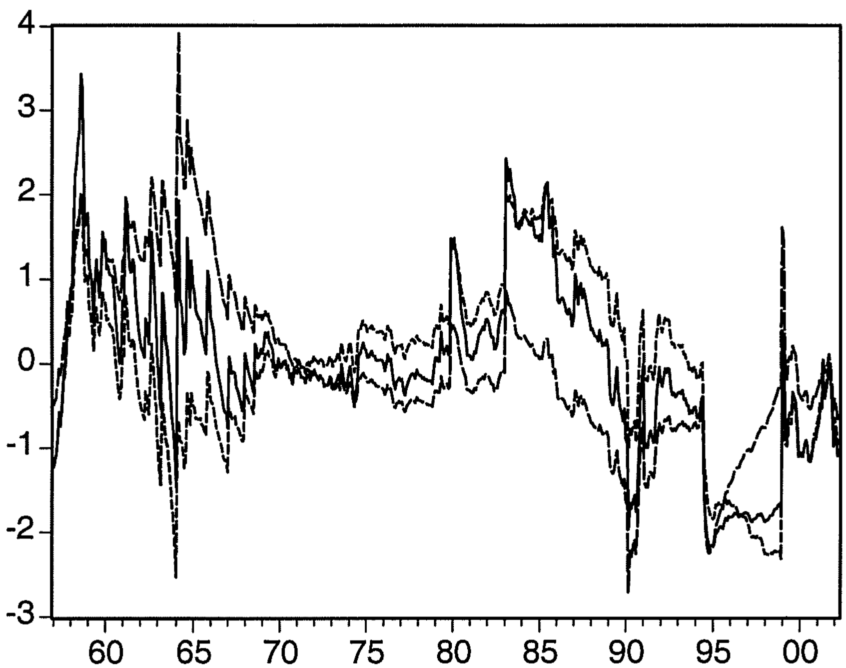

Figure 14: Brazil: Real Exchange Rate (—), Permanent Component (....) and Transitory Component (- - -). Normalized data.

4.3 Chile

This economy adopted an exchange rate band that lasted eighteen years (1982-1999). This corresponds to the exchange rate regime with the longest duration within our period of analysis for this country[19]. An interesting characteristic of the exchange rate policy in Chile is that it only had two episodes of a fixed exchange rate, both of them of short duration (January 1959-October 1962 and June 1979-June 1982). This contrasts sharply with other countries in the analysis. Mexico, for example, had several episodes of fixed exchange rates, with the longest one lasting almost twenty-three years. Despite this fact, we can see from Figure 16 that most of the stability of the nominal exchange rate coincides with both the period of the exchange rate band and the floating exchange rate period.

The results of applying the model to the real exchange rate of this country are shown in Table 3. They show that the expected duration of the high-state variance is almost four months (3.94) and the expected duration of the low-state variance is seven years and six months, the highest of the four countries. Since most of the volatility of the real exchange rate is due to the transitory component and since the movements of the nominal exchange rate drive the movements of this component, this result is not surprising.

[19] Mexico adopted an exchange rate band, too, but it lasted less than six years.

Figures 17 and 18 show the permanent and transitory components of the real exchange rate together with their 95% posterior bands. We can see that the transitory component has been very stable since 1982. Meanwhile, Figures 19 and 20 show the smoothed probabilities of being in the low or high-variance states. Between 1973 and 1975, the probability of being in the low-variance state is close to zero, especially around 1973. The Chilean economic history shows this period as the deepest economic and political crisis of the century. From 1987 onwards, the probability of being in the low-variance state is almost one.

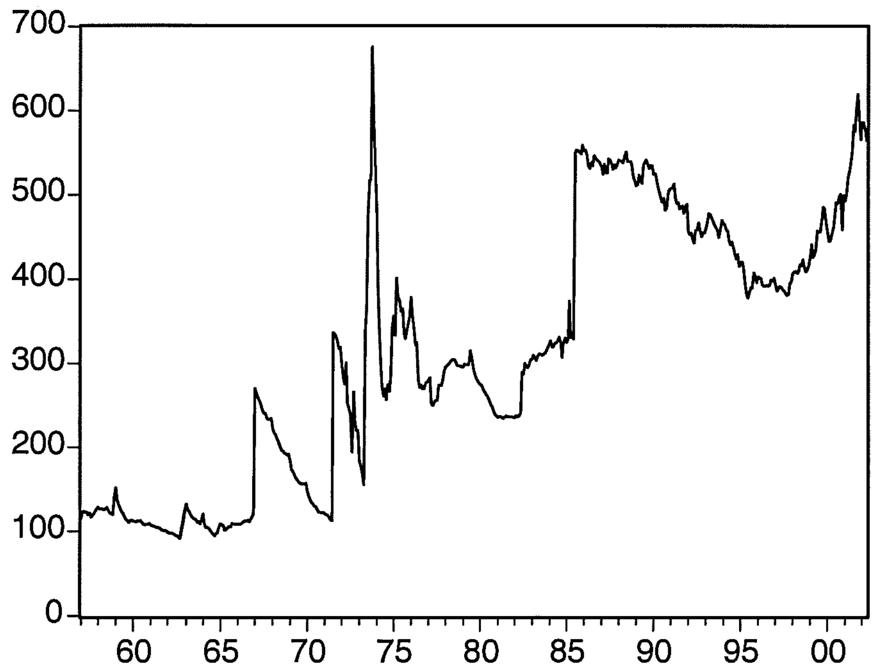

Figure 15: Chile: Real Exchange Rate $[Q_t = E_t(P_t^*/P_t)]$

Table 3. Results for Chile

Parameter	Mean	SD	Median	95% Posterior Band
p_{11}	0.989	0.005	0.989	0.978,0.999
p_{22}	0.747	0.128	0.768	0.496,0.998
ϕ_1	0.972	0.070	0.971	0.835,1.109
ϕ_2	0.007	0.068	0.008	-0.127,0.141
$\phi_1 + \phi_2$	0.979	0.010	0.982	0.959,0.999
σ_v^2	9.026	3.015	8.632	3.117,14.936
$\sigma_{\epsilon 1}^2$	20.022	4.399	19.613	11.399,28.644
$\sigma_{\epsilon 2}^2$	846.494	636.418	714.039	-400.886,2093.875
$eigen$	0.979	0.010	0.983	0.959,0.999

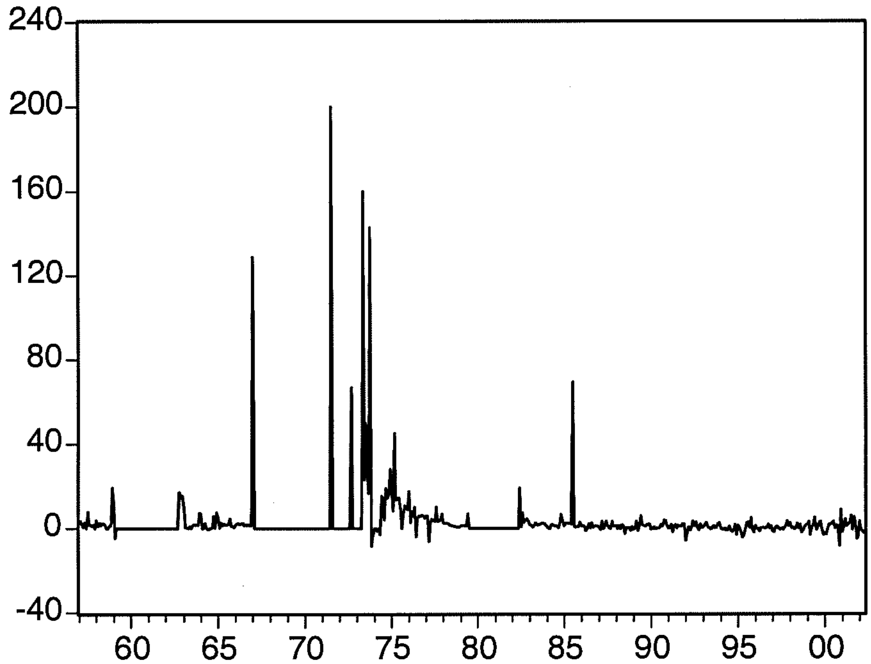

Figure 16: Chile: Monthly Rate of Devaluation (%)

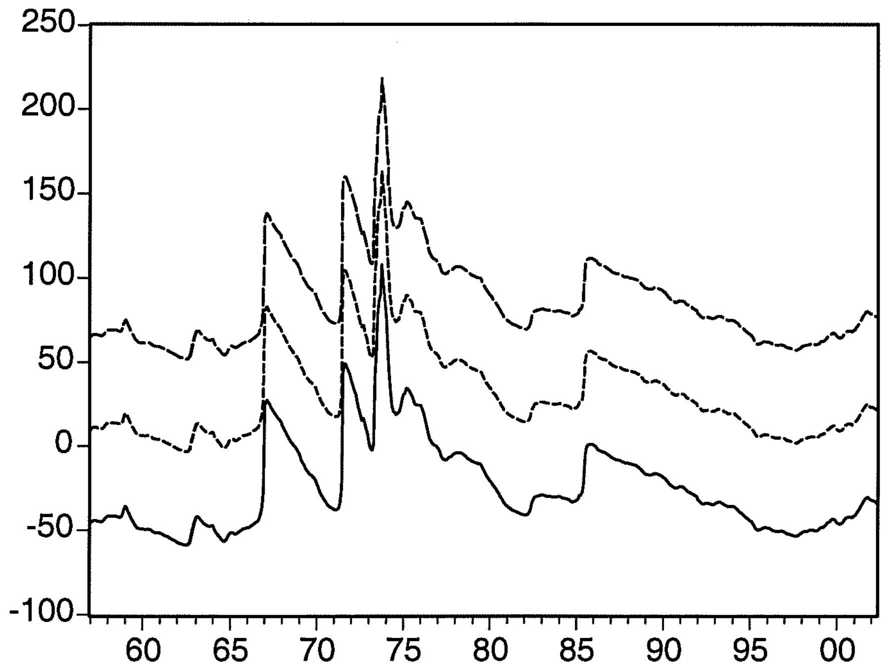

Figure 17: Chile: Transitory Component with Lower and Upper Bands [Units: Logarithm times 100]

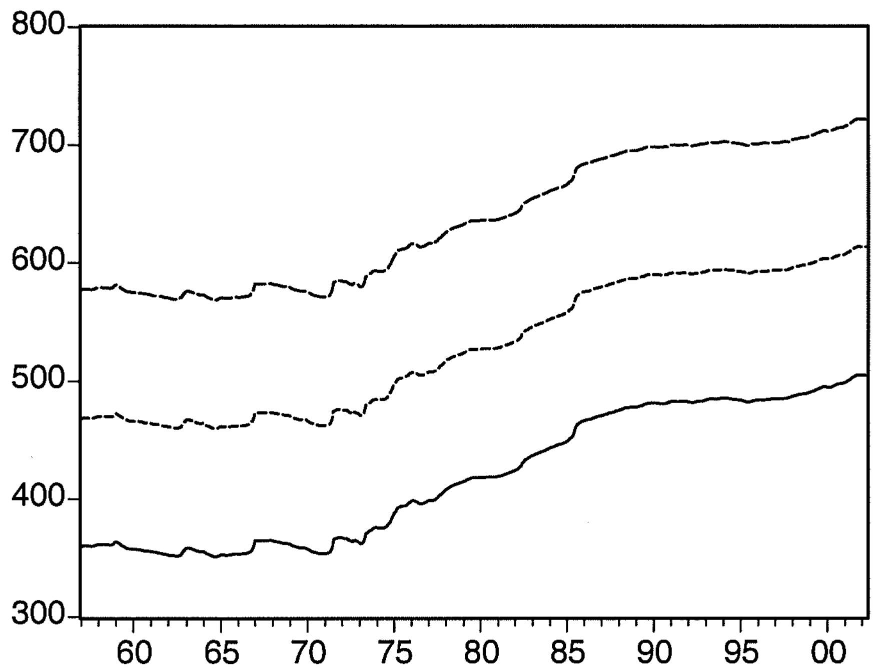

Figure 18: Chile: Permanent Component with Lower and Upper Bands [Units: logarithm times 100]

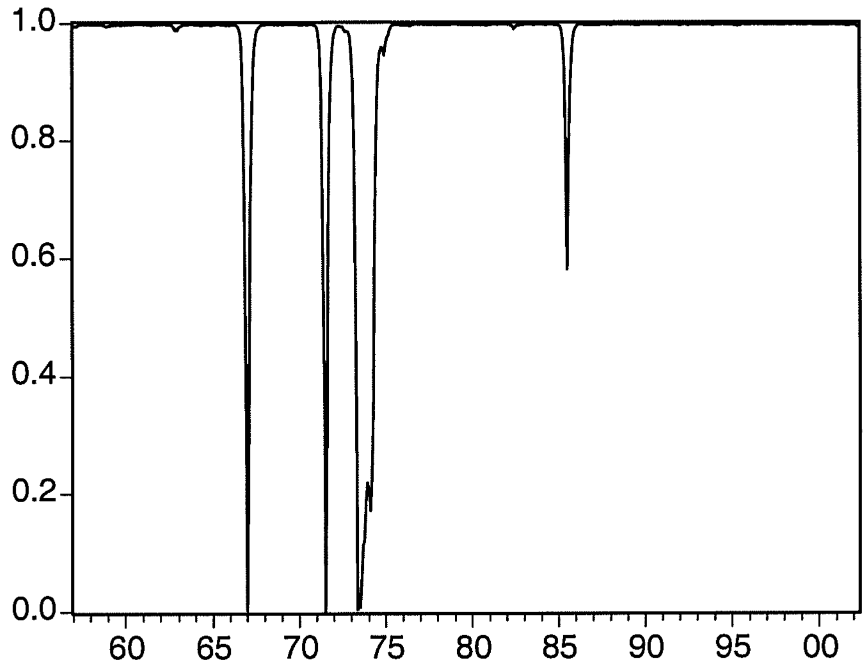

Figure 19: Chile: Smoothed Probabilities of State 1 [Low-variance state]

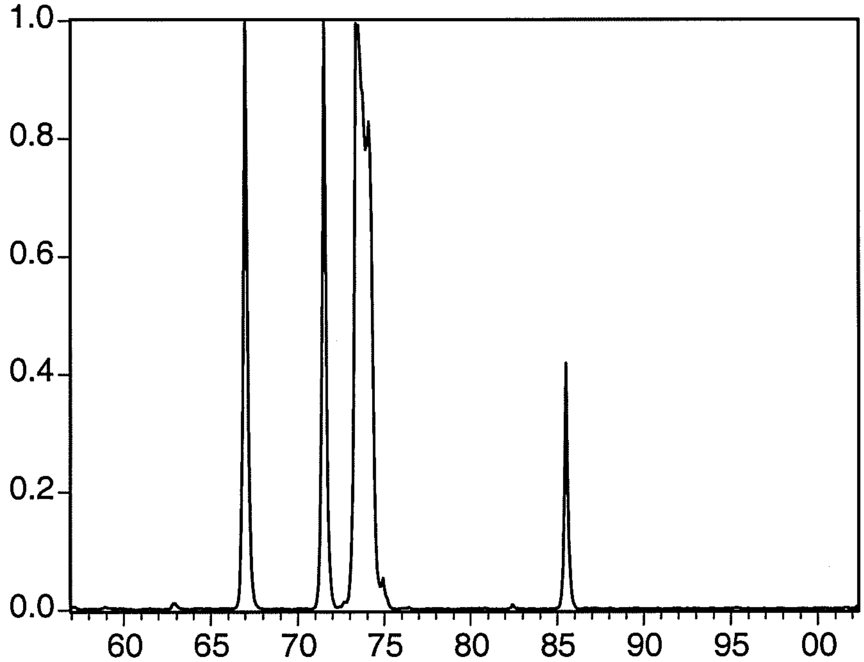

Figure 20: Chile: Smoothed Probabilities of State 2 [High- variance state]

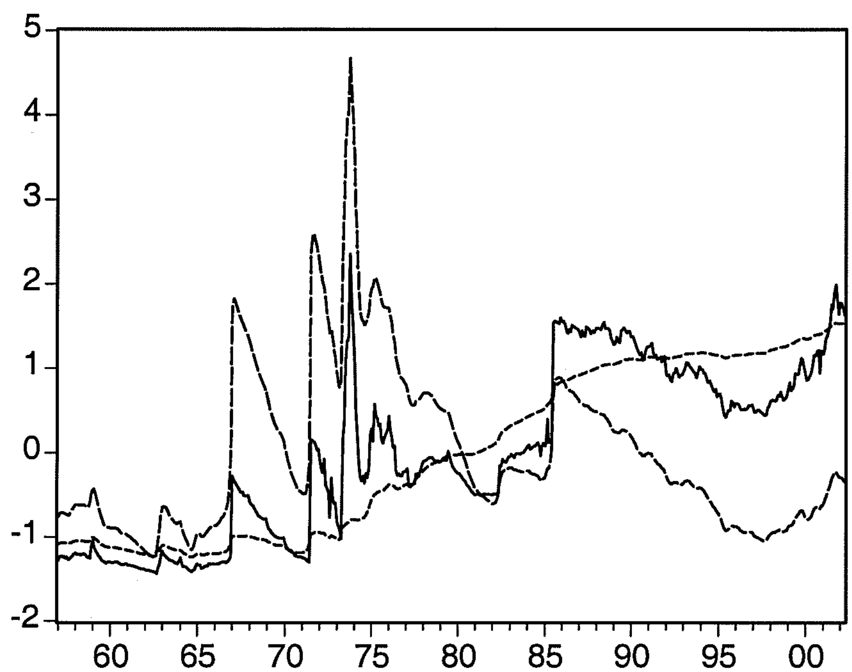

Figure 21: Chile: Real Exchange Rate (—), Permanent Component (....) and Transitory Component (- - -). Normalized data

4.4 Mexico

The real exchange rate of this country exhibits periods of long depreciation, long appreciation and periods of stability (see Figure 22). It shows many "peaks" which are associated with the devaluation of the currency (see Figure 23). The behavior of the series from 1957 to 1976, which corresponds to the Bretton Woods period, contrasts sharply with that for the rest of the period. Periods of continuous appreciation of the real exchange rate correspond to periods of abundance of external capital, while periods of depreciation correspond to a shortage of foreign capital, which was especially sharp during the eighties with the so called debt-crisis. During this time, the Mexican currency had to be devalued many times in order to boost exports and fill the savings gap.

Many exchange rate regimes were adopted by this economy. Most of them were imposed rather than a result of a careful analysis about what exchange rate regime was optimal in a specific period. Since 1995, a floating exchange rate regime has been the prevailing policy, and no devaluation has since occurred.

When applying the unobserved component model to the real exchange rate of Mexico (see Table 4), we found that most of the volatility of this economic variable is due to the transitory component (see Figure 24). The similarity between the figures of the real exchange rate and its transitory component is undeniable.

According to the coefficients of the autoregressive terms, the persistence of the transitory component is high (see Table 4), but not as high as in the case of Brazil. Thus, even if this series seems to deviate from its equilibrium value, it will return to it eventually. For this country, the expected duration of the low-variance state was found to be 83.47 months, or almost seven years. The expected duration of the high-variance state is a little more than three months.

The variance of the permanent component is low in comparison to the variances of the transitory component, but this is not odd, given the remarkably different behavior of the real exchange rate under both components.

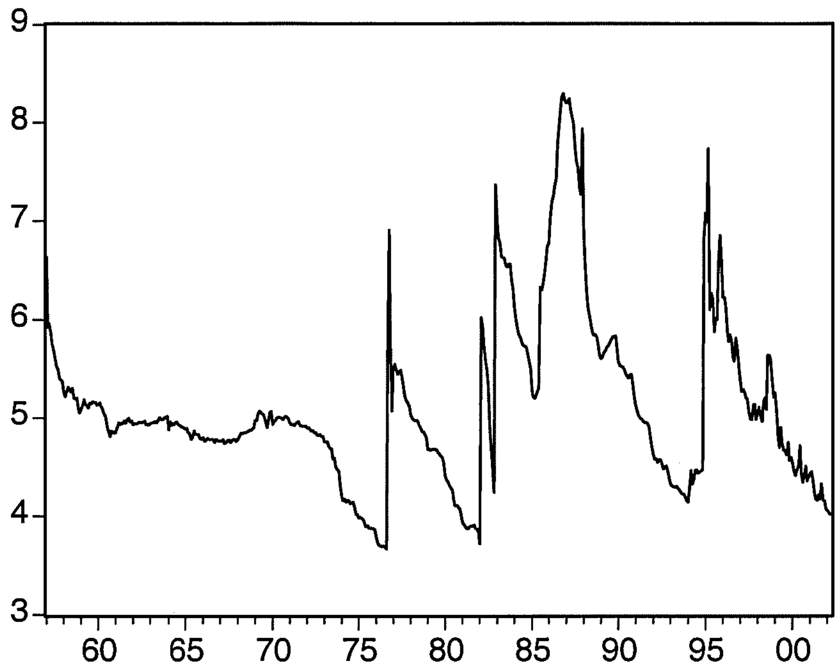

Figure 22: Mexico: Real Exchange Rate $[Q_t = E_t(P_t^*/P_t)]$

Table 4. Results for Mexico

Parameter	Mean	SD	Median	95% Posterior Band
p_{11}	0.988	0.005	0.988	0.976,0.999
p_{22}	0.704	0.137	0.720	0.436,0.973
ϕ_1	0.929	0.069	0.927	0.792,1.065
ϕ_2	0.053	0.068	0.054	-0.081,0.187
$\phi_1 + \phi_2$	0.982	0.006	0.984	0.970,0.994
σ_v^2	0.494	0.739	0.047	-0.954,1.953
$\sigma_{\epsilon 1}^2$	7.994	1.581	7.817	4.893,11.094
$\sigma_{\epsilon 2}^2$	373.727	243.58	311.734	-103.705,851.159
eigen	0.983	0.005	0.984	0.971,0.994

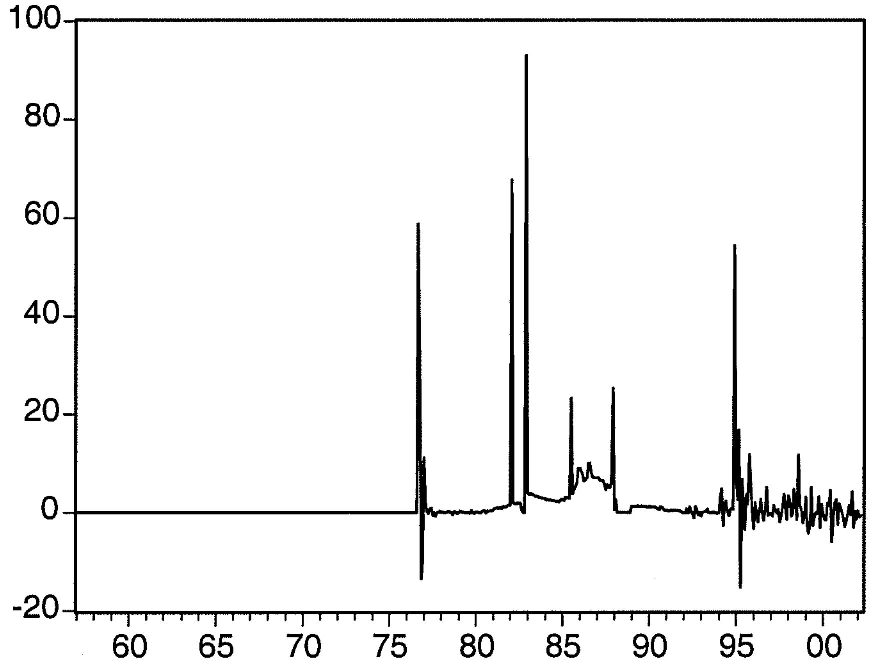

Figure 23: Mexico: Monthly Rate of Devaluation (%)

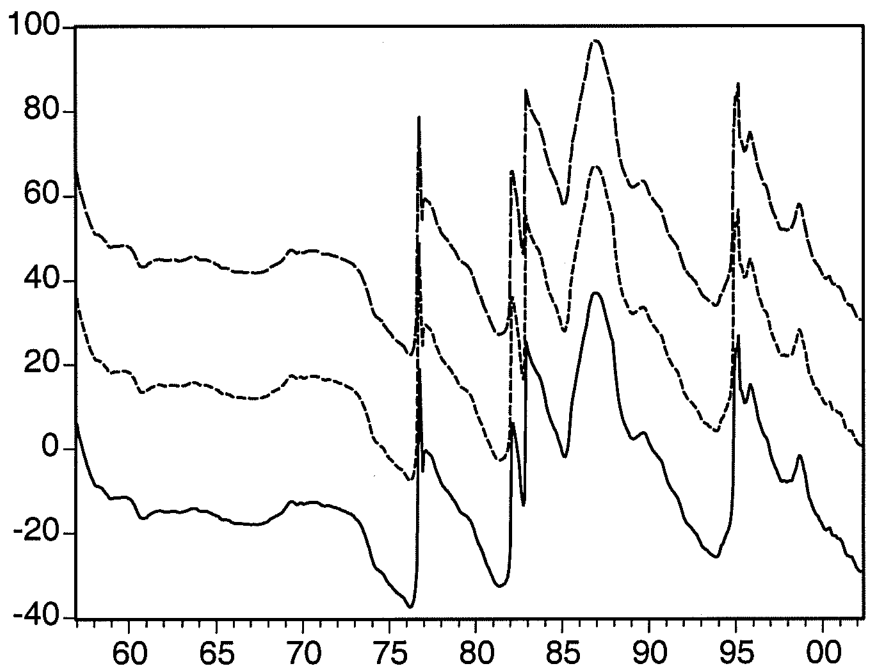

Figure 24: Mexico: Transitory Component with Lower and Upper Bands [Units: Logarithm times 100]

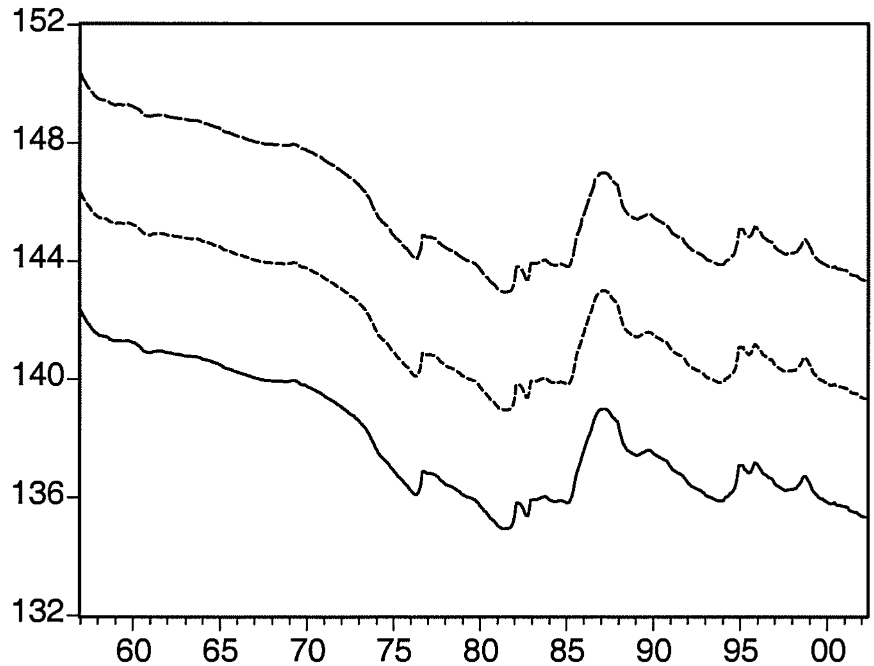

Figure 25: Mexico: Permanent Component with Lower and Upper Bands [Units: Logarithm times 100]

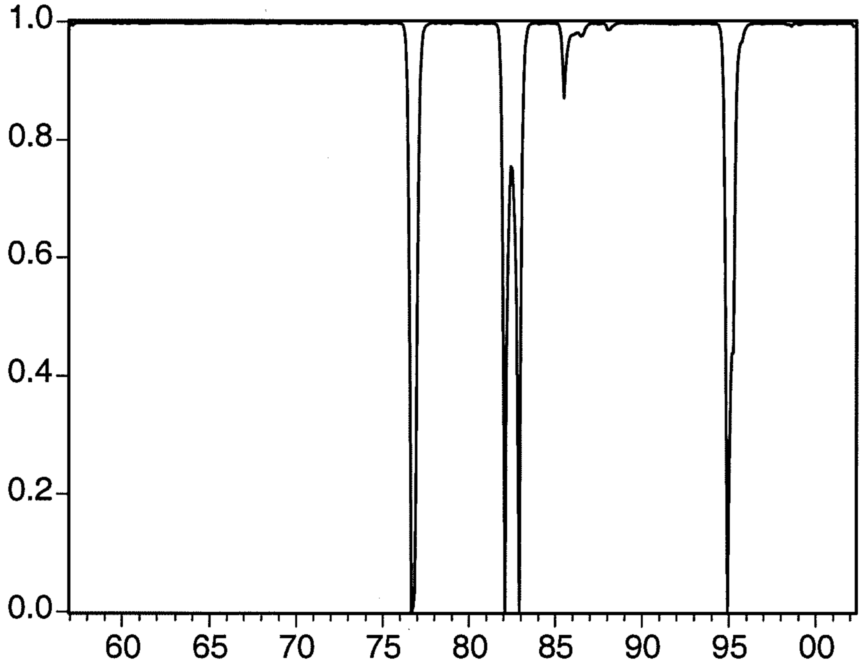

Figure 26: Mexico: Smoothed Probabilities of State 1 [Low-variance state]

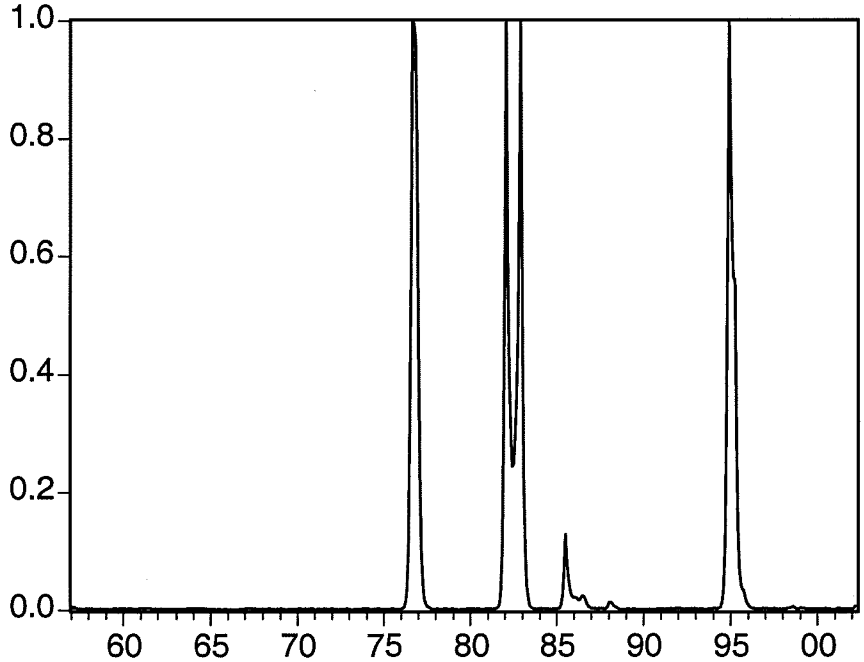

Figure 27: Mexico: Smoothed Probabilities of State 2 [High-variance state]

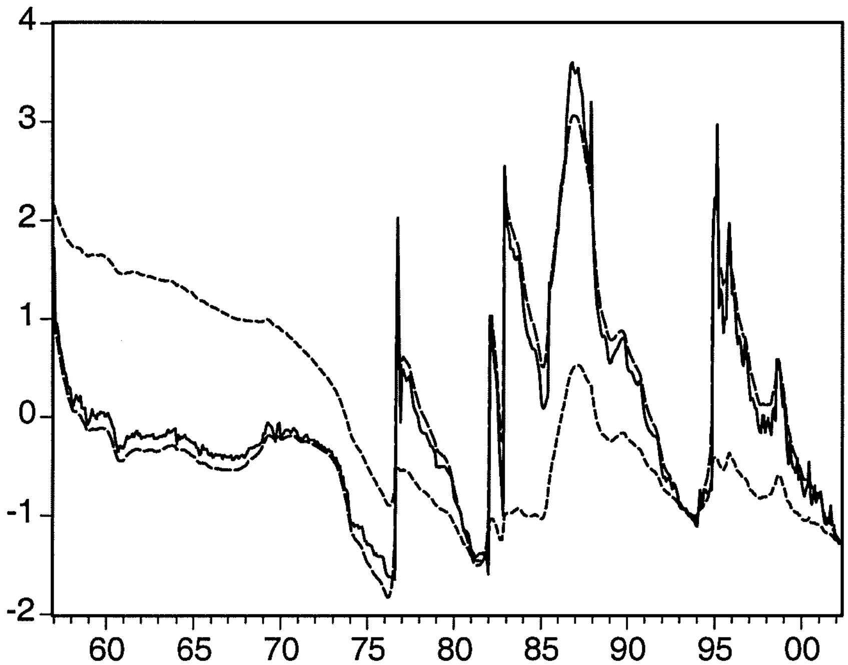

Figure 28: Mexico: Real Exchange Rate (—), Permanent Component (....) and Transitory Component (- - -). Normalized data

5 Conclusions

Using a Gibbs-Sampling approach, we estimated all the parameters of the model, while the Kalman filter is used to identify the permanent and the transitory components of the real exchange rates in Argentina, Brazil, Chile and Mexico. We used a similar approach as in Engel and Kim (1999) but we restricted the variance of the transitory component to be generated for two regimes, namely, a low-variance and a high-variance state. We found that a third regime, medium-variance state (as in Engel and Kim, 1999) is not significant for our four time series. The data used was monthly and it spans the period 1957-2002. Coefficients associated to the permanent and transitory components are obtained. Using these estimations, we calculated smoothed probabilities of both states of the variance in the transitory component. Overall, the results show a significant importance of the transitory component to explain the behavior of the real exchange rates. Brazil presented the most persistent case.

References

[1] Balassa, Bela. (1964), "The Purchasing Power Parity Doctrine: A Reappraisal," *Journal of Political Economy* **72**, 584-96.

[2] Bonomo, M. and Terra, C. (1999), "The Political Economy of Exchange Rate Policy in Brazil," Graduate School of Economics, Getulio Vargas Foundation, Rio.

[3] Campbell, J. and Perron P. (1991), "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," *NBER Macroeconomic Annual*, **Vol.6**, M.I.T. Press, 141-201.

[4] Costa, A. A., and Crato N. (2000), "Long-Run Versus Short-Run Behaviour of the Real Exchange Rates", mimeo, Institute of Economics and Management- ISEG, Portugal.

[5] Cochrane, J.H. (1991), "A Critique of the Application of Unit Root Tests," *Journal of Economic Dynamics and Control* **15**, 275-84.

[6] Cushman, D. O. (2001), "Bayesian and DF- GLS Unit Root Tests of Real Exchange Rates over the Current Floating Period," *Economics Bulletin* **6**, 1-7.

[7] De Gregorio, J. (1998), "Exchange Rate Policy in Chile since 1960: Political Economy and the Choice of Regime," Centre of Applied Economics, University of Chile.

[8] Diaz E. and Schamis H. (1999), "The Political Economy of Exchange Rate Policies in Argentina, 1950-1998," Working Paper R-379, Inter-American Development Bank.

[9] Elliot, G., Rothenberg, T. J., and Stock J. H. (1996), "Efficient Tests for an Autoregressive Unit Root," *Econometrica* **64**, 813-836.

[10] Dickey, D. and Fuller W. (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* **74**, 427-431.

[11] Engel C., and Kim C. J. (1999), "The Long-Run U.S./U.K. Real Exchange Rate," *Journal of Money, Credit and Banking* **31**, 335-356.

[12] Engel, C. (1999), "Long-Run PPP May not Hold after All", *Journal of International Economics.*

[13] International Monetary Fund, *International Financial Statistics*, CD-ROM.

[14] Frankel, J. A., and Rose A. K. (1996), "A Panel Project on Purchasing Power Parity: Mean Reversion within and between Countries," *Journal of International Economics* **40**, 209-224.

[15] Froot, K. and Rogoff, K. (1995), "Perspectives on PPP and Long-Run Real Exchange Rates," in G. Grossman and K. Rogoff (eds), *Handbook of International Economics*, North Holland.

[16] Geman, S., and D. Geman (1984), "Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images", *IEEE Transactions on Pattern Analysis and Machine Intelligence* **6**, 721-41.

[17] Hamilton, C. and Getulio, B. (2002), "Mudanças de Regime no Câmbio Brasileiro," Trabalhos para Discussão 41, Banco Central do Brasil.

[18] Hamilton, J. (1993), "Estimation, Inference and Forecasting of Time Series Subject to Change in Regime," in Maddala G. S., Rao C. R. and Vinoud H. D. (eds), *Handbook of Statistics*, **Vol. 11**. Elsevier Science Publishers B.V.

[19] Hamilton, J. (1994), *Time Series Analysis*, Princeton University Press.

[20] Kim C. J. and Kim M. J. (1996), "Transient Fads and the Crash of '87," *Journal of Applied Econometrics* **11**, 41-58.

[21] Judge, G.G., R.C. Hill, W.E. Griffiths, H. Lutkepohl, and T.-C. Lee (1988), *Introduction to the Theory and Practice of Econometrics*, (2nd ed.), New York: John Wiley and Sons.

[22] Kim, C. J. and Nelson, Ch. (1999), *State-Space Models with Regime Switching*, The MIT Press.

[23] Kleijn ,R., and Van Dijk H. K. (2001), "A Bayesian Analysis of the PPP Puzzle using an Unobserved Component Model," Discussion Paper Tinbergen Institute 105/4, 1-20.

[24] Morandé G. F. and Matias T. (2002), "Exchange Rate Policy in Chile: From the band to floating and beyond," Working Paper 152, April 2002, Central Bank of Chile.

[25] Mussa, M. (1986), "Nominal Exchange Rate Regimes and the Behavior of the Real Exchange Rate," in K. Brunner and A. H. Meltzer (eds), *Real Business Cycles, Real Exchange Rates and the Actual Policies*, New York: North Holland, 117-213.

[26] Ng, S. and Perron, P. (2001), "PPP May not Hold After all: A Further Investigation," Working Paper, Boston College.

[27] Norris, J.R. (1997), *Markov Chains*, Cambridge, Cambridge University Press.

[28] O'Connell, P. (1998), "The Overvaluation of Purchasing Power Parity," *Journal of International Economics* **44**, 1-19.

[29] Papell, D. H. (1997), "Searching for Stationarity: Purchasing Power Parity Under the Current Float," *Journal of International Economics* **43**, 313-332.

[30] Perron, P. (1989), "The Great Crash, The Oil Price Shock, and the Unit Root Hypothesis," *Econometrica* **57**, 361-401.

[31] Phillips, P. C. B., and Ploberger W. (1994), "Posterior odds testing for a unit root with data based model selection," *Econometric Theory* **10**, 774-808.

[32] Phillips, P.C.B., and Perron, P. (1988), "Testing for a Unit Root in Time Series Regression," *Biometrika* **75**, 335-346.

[33] Rivera-B, F., and Rivera-B, L. (1994), *International Finance and Open Economy Macroeconomics*, Mac Millan.

[34] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle," *Journal of Economic Literature* **34**, 647-68.

[35] Romero, I. (2000), "The Real Exchange Rate in Mexico: A Long-Term Analysis," mimeo.

[36] Said, S.E., and Dickey D.A. (1984), "Testing for Unit Root in Autoregressive-Moving Average Model of Unknown Order," *Biometrika* **71**, 599-607.

[37] Samuelson, Paul A. (1964), "Theoretical Notes on Trade Problems," *Review of Economics and Statistics* **46**, 145-54.

[38] Sarno, L., and Taylor, M. P. (1998), "Real Exchange Rates under the Recent Float: Unequivocal Evidence of Mean Reversion," *Economic Letters* **60**, 131-137.

[39] Schwert, G.W. (1989), "Test for Unit Roots: A Monte Carlo Investigation," *Journal of Business and Economic Statistics* **7**, 147-159.

[40] Taylor M. P., and Sarno L. (1998), "The Behavior of Real Exchange Rates during the Post-Bretton Woods Period," *Journal of International Economics* **46**, 281-312.

6 Appendix

Following lines are adapted from Judge, et. al (1988) and from Norris (1997).

6.1 Bayesian Reasoning

In order to have a clearer understanding of the Gibbs-sampling algorithm, we begin with a brief discussion of the Bayesian setting. Let $\{Y_1, Y_2, ., Y_m\}$ represents a set of equally likely outcomes or propositions for an experiment. If the event A represents some subset of the possibility space that identifies an event of interest for that experiment, the probability of the event is defined, via the classical probability concept, as $P(A) = n(A)/m$, where $n(A)$ is the number of elements in A. Following Bernoulli, if we acquire n independent outcomes from the experiment, and if we find that A occurs t times, it is of interest to compare observed relative frequencies of occurrence t/n with the conceptual, true fixed probability of the occurrence of event A, $P(A)$. The random relative frequency T/n is a minimum variance unbiased estimator of $P(A)$.

In contrast to the classical approach, Bayes inverted the classical reasoning and focused his attention on the problem of inferring the probabilities that $P(A)$ takes on various values, given what has been observed for the sample outcome T/n. Thus, in the problem posed by Bayes, we observe data, and thereby know the values of the data outcomes, and wish to know what probabilities are consistent with these outcomes. Given propositions A and B and the rules of conditional probability, Bayes's principle of inverse probability (Bayes theorem)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{A.1}$$

reflects what we know about A after making use of the (new) information in B. The analog to Bayes theorem when we are dealing with probability densities, and not necessarily probabilities, is then given by the natural representation

$$f(x|y) = \frac{f(y|x)f_x(x)}{f_y(y)} \tag{A.2}$$

where $f_x(x)$ and $f_y(y)$ are the marginal probability density functions of X and Y.

Consider applying Bayes's reasoning to a coin-tossing experiment. Let the outcome of the random variable T represents the number of heads in n independent tosses of the coin, so that $Y = T/n$ represents the relative frequency of heads. We can represent the probability distribution of potential outcomes Y for the relative frequency of heads, as

$$L(\theta|y) = f(y|\theta) = \binom{n}{ny}\theta^{ny}(1-\theta)^{n-ny}\mathbf{1}_{\{0,1,\ldots,n\}}(ny), \qquad \text{(A.3)}$$

where $\theta \in [0,1]$, θdenotes a possible value for $P(H)$, the probability of a head on any coin toss. Now, let the PDF p(θ), for $\theta \in$ [0,1], be our prior (previous to observing the outcome $y = t/n$) information representing the distribution of probabilities (or degrees of belief) that the various values of θ represent the true value of $P(H)$. Then, setting $x \equiv \theta$ and $fx(x) = p(\theta)$ in the statement (A.2) of Bayes theorem leads to the following representation of the probability distribution of θ, given that the value $y = t/n$ has been observed, as

$$\begin{aligned} p(\theta|y) &= \frac{L(\theta|y)p(\theta)}{f_y(y)} = \frac{f(y|\theta)p(\theta)}{f_y(y)} \\ &\propto f(y|\theta)p(\theta) \end{aligned} \qquad \text{(A.4)}$$

Note that the marginal probability distribution of Y can be defined by

$$f_y(y) = \int_0^1 f(y|\theta)p(\theta)d\theta \qquad \text{(A.5)}$$

because by the probability multiplication rule, $f(y,\theta) = f(y|\theta)p(\theta)$ is the joint probability distribution of the pair (Y, Θ). The distribution (A.5) is referred to as the posterior distribution of Θ because it reflects that our information about θ is posterior to, or after, the observed data information has been incorporated into the information base. Thus, Bayes has inferred a probability distribution on potential values of $\theta \in [0,1]$ of $P(H)$ from prior and data information on the sampling process underlying the data.

Given the above explication, we can say that the fundamental components of Bayesian inference consists of the sample data, prior density of the parameters, the posterior density of the parameters, a loss function and the corresponding optimal posterior risk-minimizing postdata Bayes estimate, and the predictive distribution of observations outside the sample. These components, now stated in the context of data and parameter vectors, translate into

1. sample information $\mathbf{y} = (y1, y2, ., yn)$ having a joint probability density function $f(y/\theta)$ and associated likelihood function $L(\boldsymbol{\theta}/\mathbf{y})$,$\theta \in \Omega$,

2. prior information in the form of a prior probability density $p(\boldsymbol{\theta})$,$\boldsymbol{\theta} \in \Omega$ for the parameter $\boldsymbol{\theta}$ in the sampling-probability model $f(\boldsymbol{\theta}/\mathbf{y})$, $\boldsymbol{\theta} \in \Omega$, and

3. the likelihood function $L(\boldsymbol{\theta}/\mathbf{y})$ and prior density $p(\boldsymbol{\theta})$ combines by Bayes theorem to yield the posterior density of $\boldsymbol{\theta}$.

It is worthwhile to emphasize that in the Bayesian approach, unknown and unobservable elements of the statistical models underlying an inverse problem, including parameters, are treated (operationally) as if they are random variables. This is not to say that a parameter is actually a random variable. Rather, this is a conceptual device that allows the analyst to identify the possible values considered as candidates for the generally fixed unknown parameter value as well as corresponding probabilities, or degrees of belief, relating to the validity of these values.

Prior Distributions The Bayesian approach, like maximum likelihood-based methods, uses the likelihood function to provide a linkage between the information contained in a sample of data and the value of the unknown parameters of a statistical model. In addition, the Bayesian method factors additional information into inverse problem solutions via the specification of a prior PDF, $p(\boldsymbol{\theta})$, on the parameter vector of the statistical model. The information represented by $p(\boldsymbol{\theta})$ can be objective, subjective, or a combination of the two.

Posterior Probability Distributions Because, within Bayesian paradigm, both $\mathbf{Y}$ and $\boldsymbol{\Theta}$ are considered to be random variables, one can then conceive of a joint probability distribution, say $f(\mathbf{y}, \boldsymbol{\theta})$, existing for the pair ($\mathbf{Y}$, $\boldsymbol{\Theta}$). Letting $fy(\mathbf{y})$ represents the marginal probability distribution of the random sample, if follows directly form the definition of conditional PDFs that we can represent the joint PDF of $(\mathbf{Y}, \Theta)$ as

$$f(\mathbf{y}, \boldsymbol{\theta}) = f(\mathbf{y}|\boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\boldsymbol{\theta}|\mathbf{y})f_y(\mathbf{y}) \tag{A.6}$$

The probability distribution of the parameter vector, conditional on the observed data, is given by

$$p(\boldsymbol{\theta}|\mathbf{y}) = \frac{f(\mathbf{y}|\boldsymbol{\theta})}{f_y(\mathbf{y})} \tag{A.7}$$

Given (A.7), the conditional PDF of the parameter vector can be represented in terms of only a kernel of the PDF. Noting that$fy(\mathbf{y})$ is a constant when we are conditioning on y, we can represent the PDF up to a positive scale factor as

$$\begin{aligned} p(\boldsymbol{\theta}|\mathbf{y}) &\propto f(\mathbf{y}|\boldsymbol{\theta})p(\boldsymbol{\theta}) \\ &\equiv L(\boldsymbol{\theta}|\mathbf{y})p(\boldsymbol{\theta}) \end{aligned} \tag{A.8}$$

where this follows from both (A.6) and (A.7) and the definition that $L(\boldsymbol{\theta}|\mathbf{y}) \equiv f(\mathbf{y}|\boldsymbol{\theta})$.

The conditional PDF $p(\boldsymbol{\theta}/\mathbf{y})$ is called the posterior distribution of the parameters because it is defined after observing the data and incorporates the data information into the distribution of Θ. The result A.8 is Bayes theorem, which can be expressed in words as the posterior PDF is proportional to the likelihood function multiplied by the prior PDF.

6.2 Gibbs-Sampling Methodology

Gibbs- sampling is a Markov chain Monte Carlo (MCMC) simulation method for approximating joint and marginal distributions by sampling from conditional distributions. It was introduced to the statistic literature by Geman and Geman(1984).

In order to gain a better understanding of this algorithm we begin with a brief review of the theory of Markov Chains, the random processes that form the mathematical backbone of MCMC method.

6.2.1 Markov Chains

A Markov Chain is a random process $(X_n)_{0 \leq n \leq N}$, (i.e. a sequence of random variables[20] X indexed by n) satisfying the Markov property:

$$\begin{aligned} \mathbf{P}(X_m &= i_m, ..., X_{m+n} = i_{m+n} | X_0 = i_0, ..., X_m = i) \\ &= \mathbf{P}(X_m = i_m, ..., X_{m+n} = i_{m+n} | X_m = i) \end{aligned} \quad \text{(A.9)}$$

In other words, the distinguishing feature of a Markov Chain is that the evolution of the process after period m depends solely on the state of the process at period m and is independent of the states prior to period m. The index is normally associated with time, and Markov Chains are often understood in this context.

Markov Chains can be described in terms of a transition matrix P and an initial distribution λ. A transition matrix has elements p_{ij}, the probability that the process is in state j given that it was in state i in the previous period, which can be represented by

$$p_{ij} = \mathbf{P}(X_{n+1} = j \mid X_n = i) \quad \text{(A.10)}$$

In another hand, the initial distribution λ is a vector describing the distribution for X_0:

$$\lambda_i = \mathbf{P}(X_0 = i) \quad \text{(A.11)}$$

From this initial distribution and the transition matrix, we can determine the distribution of X_n for $n = 1, 2, 3$ recursively as follows:

$$\begin{aligned} \mathbf{P}(X_1 &= i_1) = \sum_{i_0} \mathbf{P}(x_0 = i_0) p_{i_0 i_1} = (\lambda P)_i \quad \text{(A.12)} \\ \mathbf{P}(X_2 &= i_2) = \sum_{i_0} \mathbf{P}(x_1 = i_1) p_{i_1 i_2} = (\lambda P^2)_i \\ &\vdots \end{aligned}$$

[20] In the case of a Markov chain, the random variable is assumed to take integer values and frequently this is limited to finite subsets of the integers.

$$\mathbf{P}(X_n = i_n) = \sum_{i_{n-1}} \mathbf{P}(x_{n-1} = i_{n-1}) p_{i_{n-1} i_n} = (\lambda P^n)_i$$

then, Markov Chain Monte Carlo methods rely on the limiting distribution of a Markov Chain as $n \longrightarrow \infty$.

6.2.2 Markov Chain Monte Carlo Methods

Norris (1997) defines Monte Carlo simulation as computer based simulation; in fact, methods that rely on Monte Carlo algorithms are extremely intensive computationally. Markov Chain Monte Carlo refers to the specific use of Markov Chains to carry out simulation.

6.2.3 The Gibbs Sampler in Bayesian Inference

Suppose that we have a random sample $X = (X_1, , X_T)$ from a distribution $f(X_i|\theta)$where the parameter $\theta = (\theta_1, ..., \theta_k)$ is unknown but prior beliefs about θ follow a distribution $\pi(\theta)$. Then the posterior distribution is represented by

$$\pi(\theta \mid X = x) \propto \pi(\theta) f(X \mid \theta) \tag{A.13}$$

where

$$f(X \mid \theta) = \Pi_i f(X_i \mid \theta) \tag{A.14}$$

which can be estimated by sampling from θ, given X. In fact, the Gibbs sampler creates a sample as follows:

1. Take an initial vector $\theta^{(0)} = (\theta_1^{(0)}, ..., \theta_k^{(0)})$.

2. Generate a sample $\theta_1 = \theta_1^{(1)}$ from the conditional distribution $\pi(\theta_1^{(1)} | \theta_2^{(0)}, ..., \theta_k^{(0)}, X)$. This can be done by generating a random number, p, uniformly between 0 and 1 and then taking the value t that satisfies

$$\mathbf{P}(\pi(\theta_1^{(\mathbf{1})} \mid \theta_2^{(0)}, ..., \theta_k^{(0)} X) \leq t) = p \tag{A.15}$$

3. Generate samples for $\theta_2^{(1)}$,..., $\theta_k^{(1)}$ using the conditional distribution:

$$\pi(\theta_i^{(1)} \mid \theta_1^{(1)}, ..., \theta_{i-1}^{(1)}, \theta_{i+1}^{(0)}, ..., \theta_k^{(0)}, X)$$

4. Repeat steps 2 and 3 for $\theta^{(n)} = (\theta_1^{(n)}, ..., \theta_k^{(n)})$ using $\theta^{(n-1)}$

5. For large s and t, record the n vectors $\theta^{(s)}, \theta^{(s+t)}, ..., \theta^{(s+(n-1)t)}$.

These n vectors should be independent random samples from the distribution $\pi(\theta|X)$.

We now have a set of n vectors[21] , $\theta^{[1]}, ..., \theta^{[n]}$assumed to be samples from $\pi(\theta|X)$. We can now use this sample to approximate the mean of θ given the data X

$$\theta = n^{-1} \sum_i \theta^{(i)}$$

Alternatively, we can form confidence intervals for the true value of the vector θ by assuming that for large n, the $\theta^{[i]}$are approximately normally distributed with mean $\theta$and sample variances matrix

$$\widehat{\sigma}^2 = (n-k)^{-1} \sum_i (\theta^{(i)} - \widehat{\theta})(\theta^{(i)} - \widehat{\theta})' \tag{A.16}$$

then the confidences intervals are given by

$$\mathbf{P}[(\theta - \widehat{\theta})' \widehat{\sigma}^2 (\theta - \widehat{\theta}) \leqslant F_{n,m}^{\alpha}] = 1 - \alpha \tag{A.17}$$

Calculation of $\pi(\theta|X)$ is a specific application of a more general calculation of $f(\theta|\phi)$ where the θ are unknown parameters and the ϕ are parameters specified in full. Notice that the Gibbs sampler, when used in Bayesian inference, requires an explicit calculation of the conditional distribution of $\theta_i$given the other elements of θ and X. Often, the prior distribution $\pi(\theta)$ is chosen such that the conditional posterior distribution takes on a standard form like a normal or beta distribution.

6.2.4 A simple example of Gibbs-sampling in Econometrics

This example is adapted from Kim and Nelson (1999). Consider an $AR(2)$ model for an economic time series:

$$\begin{aligned} yt &= \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t \\ \epsilon_t &\sim i.i.d. \quad N(0, \sigma^2) \end{aligned} \tag{A.18}$$

[21]Here we define $\theta^{[i]} \equiv \theta^{(s+it)}$.

where the roots of $(1-\phi_1 L - \phi_2 L^2) = 0$ lie outside the complex unit circle. In matrix notation we have

$$\begin{aligned} Y &= X\beta + \epsilon \\ \epsilon &\sim N(0, \sigma^2 I_T) \end{aligned} \tag{A.19}$$

To apply the Gibbs sampling algorithm, we need conditional posterior distributions of β and σ^2 (the unknown "parameters"), given appropriate conditional prior distributions. We need the following two conditional distributions for Gibbs-sampling which are the consitional distribution of β given σ^2 and the conditional distribution of σ^2 given β.

6.2.5 Conditional Distributions of β, given σ^2

The prior distribution can be represented by

$$\beta|\sigma^2 \sim N(\beta_0, \Sigma_0)_{1[S(\phi)]}, \tag{A.20}$$

where β_0 and Σ_0 are known and $1[s(\phi)]$ is an indicator function to indicate that the roots of $\phi(L) = 0$, are outside the unit circle. In another hand, the posterior distribution will be

$$\beta|\sigma^2, Y \sim N(\beta_1, \Sigma_1)_{1[S(\phi)]}, \tag{A.21}$$

where

$$\beta_1 = (\Sigma_0^{-1} + \sigma^{-2} X'X)^{-1} (\Sigma_0^{-1} \beta_0 + \sigma^{-2} X'Y) \tag{A.22}$$

$$\Sigma_1 = (\Sigma_0^{-1} + \sigma^{-2} X'X)^{-1} \tag{A.23}$$

6.2.6 Conditional Distribution of σ^2, Given β

In this case, the prior distribution can be represented by

$$\sigma^2|\beta \sim IG(v_0/2, \delta_0/2), \tag{A.24}$$

where IG is the inverted gamma distribution and v_0 and δ_0 are known. The posterior distribution will be

$$\begin{aligned} \sigma^2|\beta, Y &\sim IG(v_1/2, \delta_1/2), \\ v_1 &= v_0 + T, \\ \delta_1 &= \delta_0 + (Y - X\beta)'(Y - X\beta) \end{aligned} \tag{A.25}$$

Given the conditional posterior distributions in (A.21) and (A.25). we can proceed to implement the Gibbs-sampling algorithm. It is possible to start the iteration of Gibbs sampling with an arbitrary starting value for $\sigma^2 = \{\sigma^2\}^0$.Then the following is iterated for $j = 1, 2, , L + M$:

1. Conditional on $\sigma^2 = \{\sigma^2\}^{j-1}$, a generated value of σ^2 at the previous iteration generate β^j from the conditional posterior distribution in [25],

2. Conditional on $\beta = \beta^j$, a generated value of β from step 1, generate $\{\sigma^2\}^{j-1}$ from the conditional posterior distribution in (A.25).

3. Set $j = j - 1$, and go to step (1).

In generating $\beta' = (\mu, \phi_1, \phi_2)'$ from (A.21), it is employed rejection sampling in order to ensure that roots of $(1 - \phi_1 L - \phi_2 L^2) = 0$ lie outside the complex unit circle. As a result, we have the following sets of generated values for β and σ^2:

$$\beta^1, \beta^2, \beta^3, ..., \beta^{L+M} \tag{A.26}$$

$$\{\sigma^2\}^1, \{\sigma^2\}^2, \{\sigma^2\}^3, ..., \{\sigma^2\}^{L+M} \tag{A.27}$$

where the first L generated values are discarded to ensure the convergence of the Gibbs sampler, then the inferences on β and σ^2 (mean, median, standard deviation, posterior probability bands) are made based on the remaining M generated values. These remaining values of β and σ^2 provide us with the marginal and joint distributions.