

GAS-LIQUID FLOWS IN VERTICAL ANNULAR CHANNELS

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Abstract

Experimental and computational studies of air bubbles rising in stagnant and flowing water in vertical annular channels for the full range of eccentricity e were performed. The experiments were conducted in a specially designed experimental facility and two video cameras were used to record the flow from two directions perpendicular to each other. Analysis of the videos and use of computer-aided-design (CAD) software provided measurements of the velocity, length, shape, volume and angle of wrap of Taylor bubbles in stagnant water for $0 \leq e \leq 1$. The bubble velocity increased with increasing eccentricity from the concentric value ($e = 0$), peaked for $e \approx 0.2 - 0.3$ and then decreased with further increasing eccentricity, reaching a minimum value for $e = 1$. The maximum angle of wrap decreased monotonically with increasing eccentricity in its full range. The same observations were also made in our numerical simulations. On the other hand, the bubble length and volume were essentially insensitive to eccentricity. Isolated Taylor bubbles in upward flowing water were found to be slimmer and had a smaller maximum angle of wrap than bubbles with comparable lengths rising in stagnant water. Four regimes were identified visually in upwards air-water flows in annular channels, namely, bubbly, cap-bubbly, slug and churn flows. A flow regime map was constructed for the full range of eccentricity. The transition regions of the map were shifted with changes in eccentricity in the range $e = 0.3 - 0.7$. Oscillatory cross-flows, attributed to gap vortex streets, were visually identified in the recordings of bubbly, cap-bubbly and slug flow regimes for $e = 0.5$ and 0.7 . The numerical simulations also provided evidence for the presence of gap vortex streets in gas-liquid flows in highly eccentric annular channels. These vortex streets are deemed to be a main source of the eccentricity effects.

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Nomenclature

Roman symbols

A	Archimedes number
b	bias limit
C	Courant number
C_0	distribution parameter
c_p	specific heat
Ca	capillary number
D	diameter
e	eccentricity
Eo	Eötvös number
f	focal length, friction factor
F_{ST}	surface tension force
Fr	Froude number
g	gravitational acceleration
H	height
j	superficial velocity
k	turbulent kinetic energy
L	length
Mo	Morton number

n	refractive index
N_f	inverse viscosity number
P	pressure
p	precision limit
Q	volumetric flow rate
R	radius
Re	Reynolds number
s	length on image plane
t	time
U	velocity
u	uncertainty
V	volume
x_b	distance of the bubble edge from the inner tube centre
y_b	half width of the liquid column
Z	fully development length
z	axial distance

Greek symbols

α	volume fraction
γ	diameter ratio
Δ	difference

δ	distance between the tube centres
ε	turbulent dissipation rate
θ	angle of wrap
κ	local interface curvature
λ	liquid film thickness
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	surface tension
ω	specific turbulence dissipation rate

Subscripts

atm	atmospheric pressure
ave	average
b	bubble
bf	bubble in flowing liquid
bhyp	hypothetical bubble
bt	bubble in circular tube
CFD	computational simulation
exp	experimental
fs	free surface

g	gas
h	hydraulic
in	inner cylinder
l	liquid
max	maximum
min	minimum
out	outer cylinder
r	release
rot	rotameter
rot,m	measured at the rotameter
t	tube
tbot	bottom of the upper tank
tin	tube inner
tout	tube outer
vt	viewing tank

Acronyms

CFD	computational fluid dynamics
CLHS	camera link high speed
CPU	central processing unit
CSF	continuum surface force

DES	detached eddy simulations
DNS	direct numerical simulation
fps	frames per second
IDDES	improved delayed detached eddy simulations
LDV	laser Doppler velocimetry
LES	large eddy simulations
PIV	particle image velocimeter
PST	pulsed shadow technique
RANS	Reynolds-Averaged Navier-Stokes
RO	reverse osmosis
SCFH	standard cubic feet per hour
sCMOS	complementary metal-oxide semiconductor
SST	shear stress transport
URANS	unsteady Reynolds-Averaged Navier-Stokes
VOF	volume of fluid

Chapter 1

Introduction

1.1 Motivation for this research

Multiphase flows can be classified into five types: gas-liquid, gas-solid, liquid-solid, immiscible liquid-liquid, and gas-liquid-solid ones. The interest of the present work focuses on gas-liquid flows, which, because of the compressibility of gases and the deformability of liquids, are considered to be the most complex two-phase flow type. Gas-liquid flows are commonly encountered in thermal power generation plants, petroleum extraction systems and chemical industries. Among the engineering systems and processes in which the prediction of gas-liquid flow characteristics is essential, one may mention emergency core cooling of nuclear reactors, power station steam boilers, heat exchangers, gas absorption units, oil and gas transport in wells, pipeline transportation of fuels, activated sludge processes and polymer devolatilisation.

Gas-liquid flow characteristics are generally sensitive to the geometry of the vessel that contains the fluid mixture and the flow orientation with respect to the gravitational direction. Gas-liquid flow in vertical circular channels has been the topic of a large number of studies, because of its significance in many engineering applications. Many studies identified the gas-liquid flow regimes in circular and rectangular channels and the transition from one regime to another (Oshinowo & Charles, 1974; Taitel *et al.*, 1980; Sadatomi *et al.*, 1982; Mishima & Ishii, 1984; Barnea, 1987; Hibiki & Mishima, 2001). Although the terminology used by different authors may vary somewhat, it is recognised that there are four main flow regimes in vertical gas-liquid pipe flows as shown in Figure 1.1: bubbly flow, slug flow, churn flow and annular flow. Additional flow regimes found in the literature are cap-bubbly flow, dispersed bubble flow, mist flow, wispy annular flow and dispersed droplet flow.

Gas-liquid flow regimes in vertical annular channels are very similar to the ones in vertical circular channels. The main difference is observed in the slug flow regime. Slug flow in vertical circular channels is characterised by the formation of elongated, large,

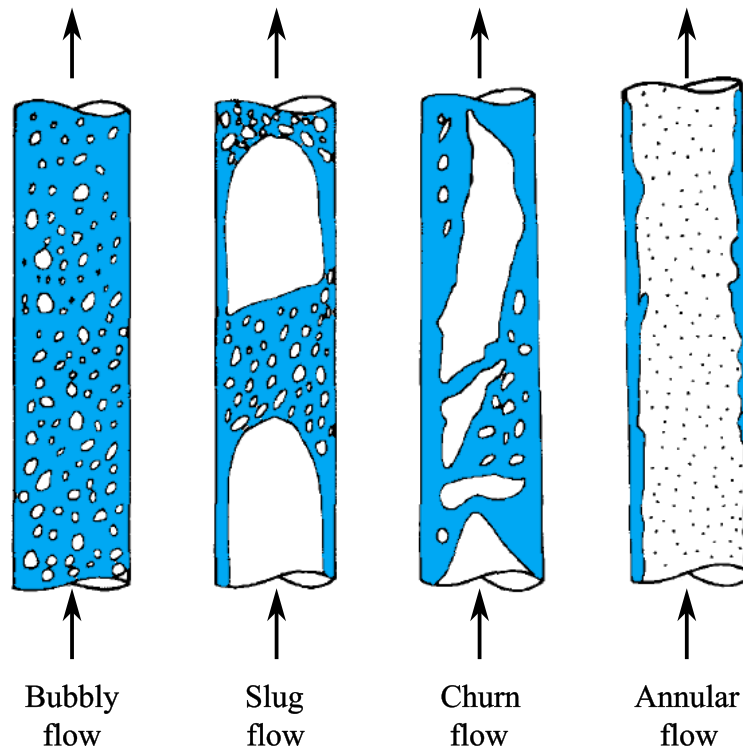


Figure 1.1: Gas-liquid flow regimes in vertical pipes (reproduced from Taitel *et al.* (1980)).

bullet-shaped bubbles, called Taylor bubbles and occupying most of the cross-sectional area of the channel with liquid in narrow films between the bubble and the wall, as shown in Figure 1.2a. Co-flowing gas-liquid streams with multiple Taylor bubbles are said to be in the slug flow regime, in which liquid flows between bubbles in the form of slugs. Buoyancy introduces a relative velocity between the Taylor bubbles and the liquid surrounding them, such that, in the thin liquid film between the bubble and the tube wall, the relative velocity is against the direction of the bubble motion. One may distinguish three regions in the liquid around a Taylor bubble: the nose region, the liquid film region and the wake region. Slug flow in vertical annular channel is characterised by Taylor bubbles, in which the bubbles are wrapped around the inner cylinder, about which they may rotate, as they get convected along the channel. In general, Taylor bubbles in annular channels (shown in Figure 1.2b) are not connected circumferentially, but only occupy part of the annular cross-section, leaving some space between two edges, which is occupied by liquid and is called liquid bridge. Furthermore, the nose of Taylor bubbles is sharper in annular channels than in

tubes, which results in higher rising speed (Hasan & Kabir, 1992). In eccentric channels, Taylor bubbles mainly occupy the wider gap region, but they may oscillate around the inner cylinder (Kelessidis, 1986). The shape and motion of Taylor bubbles in annular channels mainly depends on two geometrical parameters, the ratio γ of the inner and outer diameters of the annulus and the eccentricity, defined as the normalized distance between the axes of the inner and outer cylinders. Due to imperfections in manufacturing, annular channels unavoidably have some eccentricity.

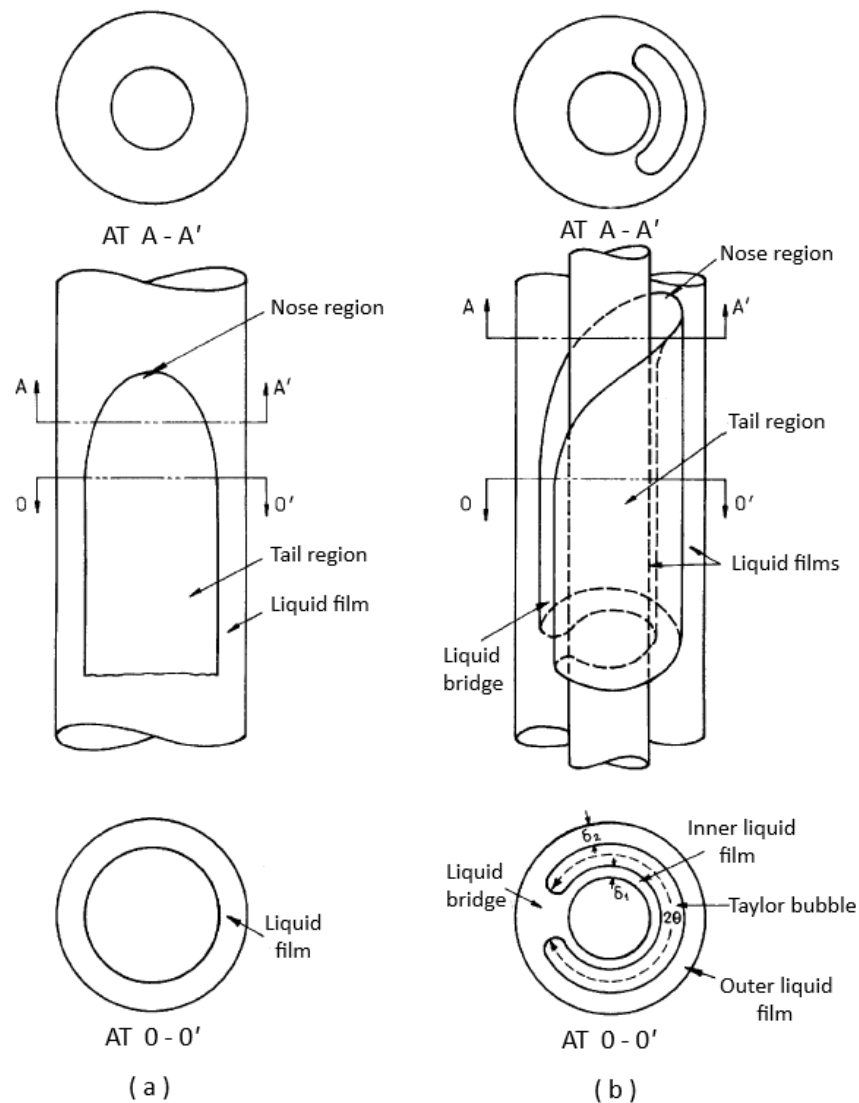


Figure 1.2: Schematic diagram of a Taylor bubble in (a) circular tubes, (b) annular channels (adapted from Das *et al.* (1998)).

The present study is concerned with eccentricity effects on Taylor bubbles and upwards gas-liquid flows in vertical annular channels, for which only limited information is available in the literature (Rader *et al.*, 1975; Kelessidis & Dukler, 1989, 1990; Caetano *et al.*, 1992*a,b*; De Sousa *et al.*, 2014; Rohilla & Das, 2019). In single-phase flows, eccentricity is known to trigger a mechanism that has been referred to as gap instability, which, under certain conditions, may generate a street of quasi-periodic, staggered, counter-rotating vortex pairs (Tavoularis, 2011; Lamarche-Gagnon & Tavoularis, 2021). The possible presence of gap instability in gas-liquid flows in annular channels and its effect on Taylor bubble characteristics have never been investigated in the literature. Such investigations seem worthwhile, first because they would illuminate some interesting and complex flow phenomena and second for their potentially important consequences in different multi-phase flow applications.

1.2 Objectives of the research

The general objective of this research is to conduct experimental and computational investigations of air-water flows in vertical, concentric and eccentric, annular channels. The specific objectives are as follows.

- To investigate the rising of a single Taylor bubble in stagnant water and in upward flowing water in vertical annular channels with different eccentricities.
- To investigate the upward, forced flow of air-water mixtures in vertical annular channels with different eccentricities.

1.3 Outline of the thesis

Following an introduction in Chapter 1, Chapter 2 presents a literature survey of relevant studies in circular and annular channels. Chapter 3 presents the definitions of important properties and some theoretical background on the topic. Chapter 4 presents the experimental facility, instrumentation and measuring procedures and Chapter 5 describes the computational procedures. Chapters 6 and 7 present and discuss the results of the experimental tests and the computational simulations, respectively. Lastly, Chapter 8 summarises the conclusions and recommendations for future work.

Chapter 2

Literature survey

This literature review covers two distinct topics: first, experimental and analytical investigations of Taylor bubbles in vertical circular and annular channels containing a stagnant liquid; and, second, experimental and analytical investigations of flows of gas-liquid mixtures in vertical circular and annular channels. Studies in circular channels are discussed to serve for comparison with the main topic of interest, which is studies in concentric and eccentric annular channels. Recent reviews of gas-liquid flows in vertical pipes and annular channels were presented by Morgado *et al.* (2016), Wu *et al.* (2017) and Holagh & Ahmed (2024). Also, Ghendour *et al.* (2020) discussed void fraction measurement techniques in gas-liquid flows through annular channels.

2.1 Taylor bubbles rising in liquid columns

2.1.1 Studies in circular channels

The earliest investigations of long bubbles were conducted by Dumitrescu (1943) and Davies & Taylor (1950). Dumitrescu (1943) suggested a theoretical model for the rise velocity of long gas slugs rising in a stagnant liquid, based on potential flow analysis. He expressed the bubble rising velocity as $U = 0.351\sqrt{gD}$, where g is the gravitational acceleration and D is the inner diameter of the pipe. Later on, Davies & Taylor (1950) measured the bubble length in an experiment and suggested a correlation for the rise velocity as $U = 0.328\sqrt{gD}$, assuming that the pressure around the nose is the same as in an ideal flow around a sphere.

Laird & Chisholm (1956) studied experimentally Taylor bubbles rising in a stagnant circular liquid column with a diameter of 50.8 mm. They found that the pressure did not change along the bubbles and showed that the shear force acting on the water column was a power function of the bubble length with an exponent 1.5.

Bretherton (1961) carried out experiments to examine the mechanism of bubbles rising at low Reynolds numbers in a vertical tube sealed at one end. He presented a theory that there is no motion when $\Delta\rho g R^2 / \sigma < 0.842$, where $\Delta\rho$ is the density difference between the fluids inside and outside of the bubble and σ is the surface tension. However, his experiments were not in good agreement with the theory.

Goldsmith & Mason (1962) studied the movement of single Taylor bubbles in vertical tubes filled with a stagnant liquid for Reynolds numbers between 10^{-5} and 1 and capillary numbers (Ca, the ratio of viscous forces to the surface tension forces) between 2×10^{-4} and 2×10^2 . They identified the shape of the bubbles from recorded images. They found that the bubble velocity was related to the film thickness and that the bubble deformation, for a given surface tension, was not dependent on the length of the bubble and the liquid viscosity. They also found that the bubbles were spheroidal, with a prolate nose shape and an oblate tail shape, which depended on the surface tension.

White & Beardmore (1962) used non-dimensional parameters to describe the rise velocity of Taylor bubbles in vertical tubes filled with stagnant liquid. They derived an empirical correlation of the Froude number (Fr , the ratio of the inertial force and the gravitational force) in terms of the Morton number (Mo , the ratio of viscous force and the surface tension force) and the Eötvös number (EO , the ratio of the gravitational force and the surface tension force) in the ranges $3 < EO < 400$ and $10^{-12} < Mo < 10^3$. They concluded that interfacial effects were negligible for $EO > 70$, that inertia forces were negligible for $Fr < 0.003$ and that the bubbles had no motion for $EO < 4$.

Brown (1965) derived a correlation for the bubble rise velocity by incorporating viscous effects to the pioneering analyses. He also derived a correlation to predict the equilibrium liquid film thickness under the assumption of a laminar film. These correlations were supported by experimental results performed in a vertical tube with a diameter of 26 mm and a length of 11 m, filled with various stagnant fluids.

Campos & Guedes De Carvalho (1988) observed and recorded photographically the flow in the wake of a single Taylor bubble rising in two different tubes, having internal diameters of 19 mm and 52 mm. They observed laminar, transition and turbulent flow regimes and showed that the only factor determining the flow pattern was the inverse viscosity number $N_f = \sqrt{gD^3}/\nu$ where D is the pipe diameter and ν is the kinematic

viscosity of the liquid.

Bugg *et al.* (1998) developed a two dimensional, unsteady computational model using the finite difference method to investigate Taylor bubbles rising in circular tubes filled with stagnant fluid. They used the volume of fluid (VOF) method to follow fluid particles. They determined the shape and the terminal velocity of the bubbles for various Eötvös and Morton numbers. They found that the shape of nose was a prolate spheroid for all runs, in agreement with previous literature and that the tail was flat or concave, depending on a characteristic number, noting that the tail was flat for all cases for which the Froude number was larger than 0.3. They also found that the falling film thickness was constant for some of the runs, which was an indication of equilibrium.

Viana *et al.* (2003) suggested universal correlations for the Froude number of Taylor bubbles rising in circular pipes filled with various stagnant fluids. They used data from 255 previous experiments and seven experiments of their own. In these correlations, the Froude number is expressed in terms of the Eötvös number and the inverse viscosity number, with different expressions fitted to the ranges $N_f < 10$, $10 < N_f < 200$ and $N_f > 200$. These correlations are used widely for predicting the bubble rise velocity in circular channels.

Van Hout *et al.* (2002) used particle image velocimeter (PIV) to measure liquid velocity around a single Taylor bubble rising in a 4 m long vertical pipe with a 25 mm inner diameter, filled with stagnant water. They presented mean velocity profiles around the nose, in the liquid film and in the wake region by ensemble averaging velocity profiles around 100 different bubbles. They found that the effects of the bubble presence on the liquid velocity became negligible at distances of $0.5D$ from the bubble nose and $12D$ from the bubble tail. On the other hand, measurable velocity fluctuations were observed up until distances of $50D$ from the bubble tail.

Nogueira *et al.* (2006a) performed experiments in a pipe with an inner diameter $D = 32$ mm, filled with two different liquids (water and an aqueous glycerol solution), covering a wide range of viscosities (0.001 to 1.5 Pa s). They measured the velocity field and the shear stress distribution around a Taylor bubble using particle image velocimetry (PIV) and pulsed shadowgraphy (PST). They presented liquid film thickness measurements under various conditions and found that their results in the laminar film region were in good agreement with the correlation of Brown (1965). They further suggested a critical Reynolds number

for the transition of liquid film from laminar to turbulent. Moreover, they showed that the liquid film was stabilised after a certain distance from the bubble nose, and demonstrated that the liquid film had characteristics that were similar to those of a free-falling film.

Kang *et al.* (2010) studied numerically the rising characteristics of a Taylor bubble in stagnant liquids using a front-tracking algorithm in a cylindrical coordinate system and the finite difference method. The simulation results showed that the viscosity ratio has a weak effect on the final shape of the Taylor bubble, whereas Eo and the Archimedes number Ar (defined as the ratio of the gravitational force and the viscous force) have strong effects. Moreover, the bubble shape changed from a longer and slender one to a shorter and wider one with an increase of Ar , which resulted in a smaller liquid film thickness. They fit a correlation to their numerical data, which predicted the dimensionless liquid film thickness as $\lambda' \equiv \lambda/R = 0.64Ar^{-0.1}$.

Llewellyn *et al.* (2012) performed experiments to quantify the equilibrium liquid film thickness around Taylor bubbles rising in stagnant liquids. They used 2 m long tubes with three different inner diameters ($D = 20, 40, 80$ mm) partially filled with various Newtonian liquids (cooking oil, golden syrup, liquid soap and water), leaving an air pocket with a length L_0 . Then the tube was flipped upside down and the air pocket formed a Taylor bubble with a length L_{bt} rising in the liquid, which was recorded by a video camera. The liquid film thickness was determined using the relationship between L_0 and L_{bt} . They suggested a correlation for the equilibrium liquid film thickness by combining their experimental data and the data of Nogueira *et al.* (2006a). They showed that when $Eo > 40$ the film thickness is only a function of N_f . Furthermore, for a thin liquid film ($\lambda \ll D$), they showed that the liquid film Reynolds number Re_f is equal to the bubble Reynolds number Re_b . Using this condition, they suggested that the earlier models for the wavy film thicknesses (Nusselt, 1916; Dukler & Bergelin, 1952; Karapantsios & Karabelas, 1995; Lei *et al.*, 2005), rewritten in non-dimensional forms, also apply to the liquid film thickness around a Taylor bubble.

Araújo *et al.* (2013) studied two consecutive Taylor bubbles rising in a vertical stagnant Newtonian liquid column using the VOF method in the commercial CFD package ANSYS FLUENT. Simulations were run for laminar flow regime over a wide range of Eötvös and Morton numbers. The results accounted for the interaction of two consecutive bubbles and

demonstrated that the separation distance between the bubbles depends on the liquid film and wake region of the trailing bubble.

Santos & Pinheiro (2014) examined how bubble expansion affects the characteristics of a single Taylor bubble rising through a 4 m long vertical tube with an inner diameter of 0.032 m filled with stagnant water. Various pressure conditions were applied at the free liquid surface to obtain several bubble expansion rates. The liquid velocity profile above the nose of the Taylor bubble was measured using a combination of PIV and PST (pulsed shadow technique), as described by Nogueira *et al.* (2003). The upward net volumetric flow rate derived by integrating the velocity profiles was in good agreement with the estimated bubble volume expansion rate, derived under the assumption of ideal gas behaviour. The centreline liquid velocity, relative to the Taylor bubble velocity at the tip of the bubble, dropped rapidly above the bubble nose. Also, an increase in bubble volume caused a significant increase in the rise velocity, because of the liquid movement above the bubble front induced by gas expansion.

De Azevedo *et al.* (2017) investigated experimentally the equilibrium liquid film thickness around Taylor bubbles in 2 m long acrylic tubes with three different inner diameters ($D = 19, 24$, and 34 mm), filled with various mixtures of water and glycerol, using a pulse-echo ultrasonic technique. They used the same procedure as Llewellyn *et al.* (2012) to create the bubbles. The bubble rise velocity, the bubble shape, the equilibrium liquid film thickness, and the development length (Z , the minimum bubble length to achieve the fully developed film thickness) were determined by ultrasonic signal processing. The velocity measurements showed a good agreement with the literature (Dumitrescu, 1943; Nicklin, 1962; Viana *et al.*, 2003). The measurements for the liquid film thickness were in good agreement with correlations from the literature (Nusselt, 1916; Brown, 1965; Karapantsios & Karabelas, 1995; Lel *et al.*, 2005) but some corrections were suggested to their validity range. De Azevedo *et al.* (2017) mentioned that the correlation of Kang *et al.* (2010) was in very poor agreement with their results and that the correlation of Llewellyn *et al.* (2012) overestimated the liquid film thickness for higher N_f values due to the insufficient bubble length (not fully developed bubble). They further concluded that fully developed bubbles were present in their tube with a diameter of 24 when $L_{bt} > 20D$. Lastly, they suggested empirical correlations for the liquid film thickness and the development length, and found

that the former one was in good agreement with Llewellyn *et al.* (2012) and Nogueira *et al.* (2006a), and the latter one was in good agreement with Nogueira *et al.* (2006a) and Esteves & De Carvalho (1993); both were valid in the range $15 < N_f < 12900$. The aforementioned liquid film thickness correlations are listed in Table 2.1.

Table 2.1: The liquid film thickness correlations from the literature.

Author	Correlation	Condition
Nusselt (1916)	$\lambda' = \left(6 \frac{Fr}{N_f}\right)^{1/3}$	Laminar falling film Thin film $N_f < 3000$
Dukler & Bergelin (1952)	$Re_b = \eta(3 + 2.5 \ln \eta) - 256$ where $\eta = N_f \left(\frac{\lambda'}{2}\right)^{1.5}$	Laminar flow and transition $1500 < N_f < 9000$
Karapantsios & Karabelas (1995)	$\lambda' = 0.428 Re_b^{0.538} N_f^{-2/3}$	Thin film $9000 < N_f < 44000$
Lel <i>et al.</i> (2005)	$\lambda' = (2 + 0.641 Re_b^{0.47}) N_f^{-2/3}$	$40 < N_f < 9000$
Kang <i>et al.</i> (2010)	$\lambda' = 0.64 N_f^{-0.2}$	CFD simulations $10 < N_f < 450$
Llewellyn <i>et al.</i> (2012)	$\lambda' = 0.204 + 0.123 \tanh(2.66 - 1.15 \log N_f)$	$0.1 < N_f < 100000$
De Azevedo <i>et al.</i> (2017)	$\lambda' = -0.0419 \ln N_f + 0.425$	$15 < N_f < 12900$

Gutiérrez *et al.* (2017) computationally investigated Taylor bubbles using a three-dimensional Arbitrary Lagrangian-Eulerian formulation using the moving mesh method. They examined the effects of the initial bubble shape and volume, the Eötvös and Morton numbers and the channel inclination. The results were comparable to experimental data from the literature.

Shaban & Tavoularis (2018) conducted numerical simulations of turbulent flow induced by Taylor bubbles moving steadily in a circular pipe using the open source CFD program OpenFOAM 2.3. The volume of fluid method was used to model the two phases and the turbulence was resolved by the Detached Eddy Simulation method. Predictions of turbulence fluctuations, mean velocity and void fraction were in good agreement compared to experimental results in the literature. Also, the wall pressure, the differential wall pressure, and the wall temperature were evaluated. The wall temperature and pressure

underwent large fluctuations during the bubble passage. The same authors performed simulations of the coalescence of two consecutive Taylor bubbles. They found that, when the trailing bubble was in the far wake region of the leading bubble, it was deformed due to its interactions with coherent structures in the wake of the leading bubble. The resulting non-axisymmetry of the trailing bubble reduced its drag and accelerated it, bringing it closer to the leading bubble. When the trailing bubble reached the near wake region of the leading bubble it developed strong disturbances and, under the action of surface tension action, the two bubbles coalesced.

Massoud *et al.* (2018) carried out a mathematical study on a single Taylor bubble rising in a vertical column filled with stagnant Newtonian fluid. They first derived dimensionless parameters and then performed an order of magnitude analysis, which concluded that the Reynolds, Eötvös and Froude numbers were the sole dimensionless groups affecting the flow characteristics. Furthermore, CFD simulations were conducted on ANSYS FLUENT using the VOF method to investigate the hydrodynamic characteristics. Results were presented for the wake and liquid film regions, the Taylor bubble rise velocity and the Taylor bubble shape. They showed that the hydrodynamic characteristics of slug flow are weakly dependent on viscosity ratio, density ratio and the ratio of initial bubble size to the pipe diameter.

Majumdar & Das (2019) examined computationally Taylor bubbles rising in different stagnant non-Newtonian (power-law) fluid columns using the VOF method in OpenFOAM. The main goal of this analysis was to estimate the terminal velocity and the shape of Taylor bubbles. Simulations were run for different consistency coefficients (K) and power law exponents (n). They also derived a semi-analytical solution in a closed form to determine the terminal velocity. The computational results, semi-analytical method and some experimental data from the literature showed a good agreement. For the power fluids, it was shown that the bubble velocity goes up with an increase in its volume, as for Newtonian fluids. Once the change in the bubble shape ended, the rise velocity reached the terminal value. Moreover, the rise velocity was quite sensitive to n and also depended on K . The nose curvature also depended on n . On the other hand, the rise velocity and the shape were insensitive to the surface tension, which, however, had a weak effect on the tail shape.

De Azevedo *et al.* (2020) performed experiments to investigate single Taylor bubbles rising in a stagnant mixture of water-glycerin, inside cylindrical pipes that were inclined with six different angles varying in the range from 0° to 15° , with increments of 2.5° . A pulse-echo ultrasonic technique was used to measure the bubble velocity and the shape of bubbles. Experiments were conducted for various combinations of fluids (pure water, pure glycerin, and water-glycerin mixtures), inner tube diameters of 0.019, 0.024, 0.034 m, and air pocket lengths of 0.1 and 0.2 m, and at least ten bubbles for each case. Results showed that the bubble velocity was proportional to the angle of inclination, and independent of the bubble length for all Morton numbers and initial bubble lengths. Also, the lateral expansion of bubbles increased with the rise of the inclination angle, and the thickness of liquid films decreased as a result. Moreover, for all experiments, correlation performance enhanced with an increase in tube diameter.

2.1.2 Studies in annular channels

Sadatomi *et al.* (1982) studied experimentally the effects of channel shape on air-water flow in vertical channels. Measurements were taken for pressure drop, mean void fraction and velocity of bubbles in seven different vertical channels: four rectangular, an isosceles-triangular, a concentric annular and a circular one. They derived correlations that were in good agreement with Laird & Chisholm (1956). The bubble rise velocity correlation for the concentric annular channel ($D_{in} = 15$ mm and $D_{out} = 30$ mm) filled with stagnant water was given as: $U_b = 0.32\sqrt{g(D_{in} + D_{out})}$, which was in the range of the results of Griffith (1964).

Kelessidis & Dukler (1990) developed a theoretical model to predict the rise velocity of a Taylor bubble and validated the results with experiments. The model proposed elliptical axisymmetric bubbles rather than axisymmetric ones. Experiments were performed in a 6.97 m long annulus with a diameter ratio of 0.67, and eccentricities 0 (concentric) and 0.5. For the velocity measurements and the shape determination, they used two conductance probes that were spaced equally. Experiments were conducted to measure the rise velocity for single Taylor bubbles in stagnant water. The average rise velocities were found to be 0.370 and 0.356 m/s for the concentric and eccentric annuli, respectively.

Caetano *et al.* (1992a) studied experimentally upward two-phase flow in a 16 m long vertical annulus ($D_{out} = 76.2$ mm and $D_{in} = 42.2$ mm). Experiments were carried out for both concentric and fully eccentric configurations using air-water and air-kerosene mixtures. Taylor bubble velocity measurements were taken in a stagnant liquid column, and the results were expressed as a function of the bubble length. The results were validated with the model of Sadatomi *et al.* (1982). It was shown that the bubble rise velocity in annular channels is higher than that in circular channels due to the different nose shapes of bubbles. The rise velocity of the bubble was found to be inversely proportional to the hydraulic diameter $D_h = D_{out} - D_{in}$. Caetano *et al.* (1992a) identified five two-phase flow regimes (bubble, dispersed bubble, slug, churn and annular flow regimes) and derived correlations to predict the transitions between flow regimes by modifying the model of Taitel *et al.* (1980).

Hills & Chety (1998) conducted experiments to determine the rising velocity of Taylor bubbles as functions of gas flow rate in an annulus. A dip tube was placed in a 1.8 m long tube ($D = 140$ mm) concentrically in order to introduce the gas phase into the stagnant liquid. Three different dip tubes with outer diameters of 22 mm, 27 mm and 49 mm were used to conduct the experiments. These tubes were immersed in the stagnant liquid at different depths with various gas flow rates. A video camera was used to record the movement of Taylor bubbles. The rising velocities of Taylor bubbles were found to be proportional to the liquid velocity, and the results were in fair agreement with the results of Griffith (1964) and Caetano *et al.* (1992a).

Das *et al.* (1998) introduced a theoretical method to determine the velocity of Taylor bubbles rising in multiple concentric annulus. The potential flow assumption was applied at the bubble nose for their model, including the viscosity effects in the falling film region. To validate the model, experiments were conducted for three different annuli, and the rise velocity was determined using a pair of conductance probes by measuring the voltage difference. Experiments showed that the rise velocity is independent of the diameter ratio and the annular gap, but it depends strongly on the diameters of inner and outer tubes. Moreover, the theoretical model was in good agreement with the experimental results and the values from the literature. On the other hand, the accuracy of the model was low for very small and very large channels. The model was modified for the unsatisfactory cases.

Later on, Das *et al.* (2002) investigated the shape of Taylor bubbles rising in concentric annular channels and derived a mathematical relationship between the bubble length and its volume. They showed that, once the bubble is shaped, an increase in the bubble volume affects its length but not the nose shape. Moreover, they showed that the angle of wrap was only a function of the channel geometry and independent of bubble volume.

Rohilla & Das (2017) experimentally and numerically investigated the transition mechanism of a fully developed Taylor bubble from a circular tube section to an annular section in stagnant kerosene. Simulations were conducted using the open source flow solver Gerris with the coupled level set volume of fluid (CLSVOF). For the experiments, a dip tube ($D_{in} = 10$ mm) was inserted in a circular tube ($D_{out} = 20$ mm) to create the annular section. Images were taken using a high-speed camera. The bubble velocity was found numerically and experimentally as 0.1513 m/s and 0.1504 m/s, respectively. The transition mechanism was found to consist of six steps, namely, plateau formation, doughnut shape orientation and equipotential nucleation, preferential rise, retraction of lagging lobe, consumption of air thread formed, and manifestation of annular sectorial wrap. Using the same experimental setup, Rohilla & Das (2019) further investigated the transition mechanism in different fluids, including water, kerosene, glycerin and silicone oil. They also introduced eccentricity to the annular section by moving the dip tube. They had two different eccentricities, 0.25 and 0.5, for the water experiments. Results showed that transition mechanism was drastically affected by the presence of eccentricity. Also, the rise velocity of the bubble in the eccentric annular section was found to be lower than that in the concentric one.

Mitchell & Leonardi (2020a) carried out numerical simulations of Taylor bubble dynamics in vertical and inclined annular pipes using the lattice Boltzmann method. They established closure relationships for the rise velocity and the shape of bubbles, expressed in terms of a Froude number correlation that depended on the Eötvös and inverse viscosity numbers. Their model accounts for the pipe inclination and was validated against both simulations and experimental data obtained for different configurations. In a follow-up work, Mitchell *et al.* (2021) conducted a parametric study on the stability of the phase-field lattice Boltzmann model for simulating Taylor bubbles in annular channels. They investigated the effects of pipe eccentricity and diameter ratio on bubble propagation and validated their numerical method with the experimental data of Das *et al.* (1998). The results showed that

the rise velocity of the bubble increases slightly with increasing small eccentricity, peaks for eccentricities of 0.2 and 0.3, and decreases for larger values. The authors mentioned that there are no experimental data to support this result. They also showed that the angle of wrap decreases monotonically with increasing eccentricity.

Liu *et al.* (2023) performed CFD simulations of single Taylor bubbles rising in vertical and inclined, concentric annular channels. 3D unsteady simulations were performed using the commercial CFD code ANSYS Fluent (Release 19.2) with the volume of fluid method. Their interest was to find the effects of non-dimensional parameters, including Eo , N_f and inclination angle, on the bubble velocity and shape. The focus was on the low inverse viscosity number region ($N_f < 320$). The velocity of the Taylor bubble was measured by tracking the bubble tip until it reached the steady state (terminal velocity). The bubble shape was found to be sensitive to the surface tension and the viscosity of the liquid. The simulations were validated against experimental data from the literature (Rader *et al.*, 1975; Sadatomi *et al.*, 1982; Das *et al.*, 1998).

2.2 Gas-liquid flows in vertical channels

2.2.1 Studies in circular channels

Griffith & Wallis (1961) derived mathematical models to predict the average density and the pressure drop in slug flow in an approximately 2 m long vertical pipe. They also performed experiments to compare the theory using pipes with three different inner diameters, 12.7, 19.1 and 25.4 mm. The rise velocity of the bubbles was measured in both still water and water flowing upwards and downwards. It was shown that the effect of the entrance region can be observed for long lengths ($L/D = 300$) and times. Wall shear stresses made a small contribution to the pressure drop in the cases of moderate and low velocities. Furthermore, the flow hydrodynamics around the nose was a determining factor in the rise velocity.

Nicklin (1962) proposed a theory that the superficial liquid velocity made a contribution to the Taylor bubble rise velocity in addition to the buoyancy and supported the theory with a simple experiment which he measured the buoyancy component of velocity by quick closing valve method. The theory showed that the contribution of the moving fluid to the bubble velocity was characterized with a non-dimensional parameter called the distribution

parameter (C_0). Moreover, he showed that the energy loss because of slip was correlated with the buoyancy component of the rise velocity.

Collins *et al.* (1978) described the effects of the liquid flow in tubes on the rise velocity of the bubble and the bubble shape. They described the flow relative to the bubble rising in an upward stream as an inviscid flow with vorticity. They derived models predicting the laminar and turbulent velocity profiles. They carried out experiments to measure the velocity of the bubble and radius of curvature at the nose using a 4 m long tube with an inner diameter of 51.4 mm in the laminar region ($0 < Re < 2100$) and the turbulent region ($2100 < Re < 4000$). Experiments and the laminar model were in agreement only at the lower velocities and the slug velocity was not dependent on the viscosity in laminar flow. On the other hand, the turbulent model was in excellent agreement with the experiments and the data of Nicklin (1962).

Bendiksen (1985) derived analytical expressions using the potential flow theory to find the shape and velocity of a single infinitely long bubble rising through a moving fluid in a vertical pipe. The main objectives were to find out the effects of the surface tension on the rise velocity. The velocity of the bubble was calculated numerically for both laminar and turbulent flow conditions. The results were found to be in agreement with the experimental data of Nicklin (1962) and Collins *et al.* (1978). Moreover, it was found that the radius of curvature was inversely proportional to the surface tension, in agreement with the data of Dumitrescu (1943).

Polonsky *et al.* (1999b) measured the velocity field in the liquid film and around the nose of a single Taylor bubble rising in a stagnant column ($D = 25$ mm) and moving (both upward and downward) liquid. The interlaced image technique was used to take the PIV measurements around the bubble nose; however, it was not applicable to the liquid film because the velocities were higher. Therefore, the particle streaks method was applied in the liquid film region in order to measure the liquid velocity. The bubble shape measurements were performed separately with an image processing procedure, as described by Polonsky *et al.* (1999a), using a 5 W Argon ion laser light sheet with 1 mm thickness. This study found that the effect of bubble length on the rise velocity are relatively weak.

Taha & Cui (2006) investigated computationally different characteristics of single Taylor bubbles rising in vertical tubes in both stagnant and flowing liquids. They used the

VOF method within the ANSYS FLUENT to compute parameters such as shape, velocity and liquid film of Taylor bubbles. Comparison with experimental data from the literature (Griffith & Wallis, 1961; Nicklin, 1962; White & Beardmore, 1962) showed a favourable agreement.

Nogueira *et al.* (2006b) conducted an experimental study of the flow patterns in the wake and near-wake regions of single Taylor bubbles rising in a vertical pipe filled with various Newtonian fluids in co-current flow. Experiments were performed in a pipe ($D = 32$ mm) filled with two different liquids (water and aqueous glycerol solution), covering a wide range of viscosities (1×10^{-3} to 1.5 Pa s). To analyse the flow in the liquid film and around the nose of Taylor bubbles, they employed a combination of two non-intrusive methods, as described by Nogueira *et al.* (2003): PIV and PST. Laminar, transition and turbulent regions were observed in all cases. Results showed that the wake flow pattern (dimensionless wake length and volume) in stagnant liquids depends only on the dimensionless number $N_f = \sqrt{(gD^3)/\nu}$. The flow pattern was found to depend on the Reynolds number, based on the liquid velocity relative to the bubble.

Hua *et al.* (2009) investigated the characteristics of a single Taylor bubble rising in a vertical pipe within a stagnant or co-current flowing liquid, using a CFD based front tracking algorithm. The ratio of bubble equivalent diameter to the diameter of pipe was between 0.1 and 1. The wall effects on the bubble terminal velocity and the bubble shape were investigated in terms of the Archimedes and Eötvös numbers. The results were fitted with a correlation between the bubble terminal velocity and the pipe diameter. The terminal bubble velocity tended to be lower in a small pipe diameter. The characteristics of bubbles were very slightly affected by moving liquid in a large pipe and the bubble terminal velocity was equal to the sum of the flow velocity and the rising velocity in the stagnant fluid. On the other hand, the behaviour was significantly affected by moving liquid in a small pipe.

Lu & Prosperetti (2009) applied a finite volume discretisation method to study axisymmetric Taylor bubbles rising in a stagnant, co- or counter-current flowing liquid. The gas-liquid interface exact position and curvature were tracked using a set of marker points linked by cubic splines. With this method, they were able to track the exact position and curvature of the interface. The calculated data of the rise velocity of Taylor bubbles in stagnant and flowing liquid were validated with the experimental data of Bugg *et al.* (1998)

and White & Beardmore (1962).

Quan (2011) studied numerically a single Taylor bubble in co-current upward and downward flow, using a front tracking method combined with a finite difference based approach. The simulations showed that the bubble shape was significantly affected by the co-current flow. In contrary, the velocity profile in the liquid film was not strongly dependent on the mean velocity of the co-current flow.

Cerqueira (2021) developed a multiscale, 3-D, CFD model to simulate the motion of Taylor bubbles in the presence of dispersed bubbles, using a combination of VOF and Discrete Bubble Model (DBM) techniques. The models were validated using experimental data from the literature. This study showed that the existence of dispersed bubbles has a significant effect on the Taylor bubble terminal velocity, which rises an the increase in the number of dispersed bubbles.

2.2.2 Studies in annular channels

Rader *et al.* (1975) examined experimentally the rise velocity of bubbles in an annulus with various diameter ratios, eccentricities (concentric and fully eccentric), density and viscosity of the liquid, velocity of the liquid, bubble length, surface tension and inclination angle. They had 5 different lab-scale models and a full-scale model training well to perform experiments. They found that the diameter ratio of the annular channel was the most significant parameter affecting the rise velocity and that other important parameters were the viscosity of the liquid, density, velocity of the liquid, and inclination angle. On the other hand, the parameters that had a small contribution to the rise velocity were the length of the bubble, the surface tension and the eccentricity of the annulus. Eccentricity had an effect on the shape of the bubble but not on the rise velocity. Furthermore, they modified the correlation between the rise velocity of the bubble derived by Dumitrescu (1943) for the annular geometry. The proposed correlation performed well for the lab-scale models, but it was inapplicable for the full-scale training well.

Sadatomi *et al.* (1982) studied the flow pattern transitions and discussed the frictional pressure drop for a concentric annular channel ($D_{in} = 15$ mm and $D_{out} = 30$ mm). They concluded that the channel geometry itself has no effect on flow pattern transitions when the hydraulic diameter D_h is larger than 10 mm.

Kelessidis & Dukler (1989) investigated the flow pattern transition for co-current gas-liquid flows in vertical concentric and eccentric ($e = 0.5$) annular channels. They constructed flow regime maps for both concentric and eccentric channels on the basis of the analysis of the probability density function of conductance probe signals and proposed mathematical models to predict the transition between regimes. They also measured the rise velocity for Taylor bubbles in slug flow, found the distribution parameter using Nicklin (1962)'s theory and observed that asymmetric bubbles have higher rise velocity than axisymmetric ones in the concentric annulus.

Caetano *et al.* (1992a) classified the flow regimes and developed flow pattern maps. Observations for the flow regimes showed no significant difference in the transition from churn to annular flow in all configurations. The results demonstrated that the friction factor was dependent on the Reynolds number, diameter ratio and eccentricity. Caetano *et al.* (1992b) developed a mechanistic model for each flow pattern in an annulus to predict the friction factor and the rise velocity. The results compared fairly well with the experimental results by Caetano *et al.* (1992a).

Hasan & Kabir (1992) performed experimental studies and derived a mathematical method to predict the void fraction of gas-liquid flow in vertical and inclined annuli. The regime transition and phase slips were modelled by using the drift-flux approach. Void fraction expressions in the bubbly, slug and churn flow regimes were derived using the phase velocities. The model showed good agreement with their experimental data and data from the literature (Caetano *et al.*, 1992a).

Hibiki *et al.* (2003b) studied vertical upward bubbly flow in an annulus consisting of an inner rod with a diameter of 19.1 mm and an outer tube with an inner diameter of 38.1 mm. Local flow parameters, including void fraction, interfacial area concentration, interfacial velocity, liquid velocity, and turbulence intensity, were measured using a double-sensor conductivity probe and laser Doppler anemometry. Various void fraction profiles were observed, and the interfacial area concentration followed these profiles. Specific trends were noted in the interfacial velocity and Sauter mean diameter. Additionally, liquid velocity and turbulence intensity were analyzed. A modified correlation for the distribution parameter and the existing drift velocity in the drift-flux model were compared with the measurements. Fair agreement was demonstrated between the model predictions

and experimental data.

Sun *et al.* (2004) investigated experimentally the cap bubbly-to-slug-flow regime transition in a vertical annular flow channel with an inner diameter of 19.1 mm and an outer diameter of 38.1 mm. They used impedance void meters and a self-organizing neural network to classify flow regimes based on impedance signal characteristics. An analytical model was also developed to predict the transition boundary. The results of the neural network, analytical model, and existing experimental data (Das *et al.*, 1999b; Kelessidis & Dukler, 1989) were in good agreement, and contributed to a better understanding of flow regime transitions in annular channels.

Agarwal *et al.* (2007) did further experimental research on the effects of the shape and size of the annulus on Taylor bubbles. Various combinations of circular and square-shaped tubes with different diameters were investigated. The bubbles were found to be asymmetric, in agreement with Das *et al.* (1998), except when the inner tube was a thin wire, in which case they were nearly axisymmetric at the bubble nose. Moreover, the liquid bridge was no more observed in that case. To measure rise velocity, a pair of conductance probes 2 m apart from each other were used. The results agreed with the theoretical model of Das *et al.* (1998) for circular tubes but showed huge differences for the square shape outer tube, independently of the inner tube shape or dimension. This was attributed to the strong difference in the shapes of bubbles in the two configurations. The authors suggested a computational study to improve the model.

Ozar *et al.* (2008) investigated the flow structure of air-water two-phase flow in an annular channel with an inner diameter of 19.1 mm and an outer diameter of 38.1 mm. Various flow regimes were identified, including bubbly, cap-slug, and churn-turbulent flows. The authors collected detailed local void fraction and interfacial area concentration (IAC) data, which are essential to understand the flow dynamics in annular channels. They proposed a new correlation for the distribution parameter in two-phase, adiabatic, annular flows and compared their experimental findings with existing theoretical models (Hibiki *et al.*, 2003b,a). This study emphasized the significant influence of bubble dynamics on IAC in bubbly flow and proposed a new correlation for the drift-flux model, suggesting further research in complex geometries relevant to chemical and nuclear systems.

Julia *et al.* (2009) investigated the axial development of flow regimes in a vertical annular channel with an inner diameter of 19.1 mm and an outer diameter of 38.1 mm. The flow regime identification results were validated by comparing them with existing models (Mishima & Ishii, 1984; Das *et al.*, 1999a; Kelessidis & Dukler, 1989) and conducted a sensitivity analysis. The Mishima & Ishii (1984) model, originally developed for circular pipes, provided highly valuable predictions for the observed flow regime transitions. However, it had limitations in predicting the cap-slug-to-churn transition for higher liquid velocities. The Kelessidis & Dukler (1989) model, developed for annular channels, accurately predicted the cap-slug-to-churn transition but underestimated the critical void fraction for the bubbly-to-cap-slug transition. The Das *et al.* (1999b) model, also developed for annular channels, accurately predicted the cap-slug-to-churn transition but underestimated the critical void fraction for the bubbly-to-cap-slug transition. These validation methods established the reliability and accuracy of the results, strengthening the confidence in their conclusions about the axial development of flow regimes in vertical annular channels.

Julia *et al.* (2011) conducted a comprehensive study on the development of the flow regime in a vertical annulus, consisting of an inner rod with a diameter of 19.1 mm and an outer round tube with an inner diameter of 38.1 mm. Local flow regimes were identified at three axial and five radial locations using conductivity probes and neural networks, and a flow regime map was constructed based on void fraction and bubble chord length. The superficial gas velocity range was between 0.1 and 30 m/s, while the superficial liquid velocity range was between 0.2 and 3.5 m/s. It was found that flow regime transitions were typically initiated at the centre of the annulus and propagated radially. Additionally, the axial development of flow regimes was observed, particularly for lower liquid velocities. This study compared experimental results with existing flow regime transition models (Mishima & Ishii, 1984; Das *et al.*, 1999a; Kelessidis & Dukler, 1989) and highlighted their strengths and weaknesses.

Later, Julia & Hibiki (2011) developed a novel flow regime transition model for two-phase flows in a vertical annulus. The model proposed novel criteria for each flow regime transition, incorporating elements from existing models and accounting for the unique characteristics of annular channels. The model demonstrated a great predictive accuracy

compared to existing correlations (Hibiki & Mishima, 2001; Mishima & Ishii, 1984), particularly for the cap-slug-to-churn and churn-to-annular flow transitions. The developed model provides a practical tool for predicting flow regime transitions in various engineering applications involving two-phase flow in annular channels.

De Sousa *et al.* (2014) performed co-current air-water flow in a 40 m long vertical concentric annular channel ($D_{out} = 139.7$ mm and $D_{in} = 60.3$ mm) as well as in the same channel with a fully eccentric setting. The focus was on the slug-churn transition, churn, and churn-annular transition flow regimes. Pressure drop, liquid holdup, Reynolds and Weber numbers were compared between the two configurations. The results showed that the fully eccentric channel experienced a larger pressure drop compared to the concentric channel. The couplings holding the inner tube were identified as the reason for this difference.

Guo *et al.* (2018) conducted an experimental study of air-water co-current flow in two annular channels with the same outer diameter $D_{out} = 102$ mm and two different inner diameters $D_{in} = 89$ and 51 mm and constructed regime maps. They found that transitions from one flow regime to another happened at a lower superficial gas velocity for the narrower annular channel.

Gouidmi *et al.* (2019) performed three-dimensional CFD simulations of two phase flow in an 18 m long vertical annulus ($D_{out} = 38.1$ mm and $D_{in} = 12.7$ mm) using the code ANSYS FLUENT. The VOF method was used to model the two phases and the $k-\varepsilon$ model was used to model turbulence. The results were qualitative consistent with the experiments of Das *et al.* (1998).

Mitchell & Leonardi (2020b) analysed the rise of Taylor bubbles in annular channels using the Lattice Boltzmann method, considering the inclination and flow velocity effects. This study consisted of three parts. For the first part, simulations were run for Taylor bubbles in stagnant fluid for five different annular channel configurations with the aim of recreation of the experiments of Rader *et al.* (1975) and Das *et al.* (1998). Shortcomings were determined for the correlations that were used to predict the velocity of rising bubbles. For the second part, the inclination effect of the channel was investigated for two cases from the first part and a modification for the correlation to the preliminary estimation of the rising velocity of bubbles was proposed. In the final part, they demonstrated the effect

of water flow on the dynamics of bubbles with three different Morton numbers, and the results were compared to the experimental studies (Nicklin, 1962; Martin, 1976).

Kiran *et al.* (2020) examined experimentally and numerically the flow characteristics of air-water two phase flow in a concentric annulus for different superficial gas and liquid velocities. For the first part of the research, experiments were conducted to determine the pressure gradient and void fraction in a 5.5 m long annulus ($D_{out} = 82.5$ mm and $D_{in} = 35$ mm) for two sets of five tests. The superficial air velocity was in the range between 9.2 – 47.2 m/s, while the superficial water velocity was 0.3 m/s for the first set and 0.6 m/s for the second set. The pressure gradient and the void fraction were found to be proportional to the superficial air velocity. In contrast, the pressure gradient was proportional to the superficial liquid velocity, but the void fraction was inversely proportional to it. For the second part of the study, CFD simulations were conducted at the experimental conditions using the VOF technique. The turbulence was modelled by realizable $k-\varepsilon$ and SST $k-\omega$ models. The SST $k-\omega$ model demonstrated a better performance in predicting the void fraction at superficial air velocities greater than 28 m/s. However, the realizable $k-\varepsilon$ model was better for air velocities less than 19 m/s.

Liu *et al.* (2024) extended the work of Liu *et al.* (2023) by conducting computational simulations in vertical and inclined annular channels with counter-current flow in the slug flow regime using identical methodology and test cases. Besides flows in concentric annular channels, they simulated flows in eccentric vertical annuli with eccentricities 0.3, 0.5 and 0.7. This work was applied to drilling operations in oil, gas and geothermal wells and its focus was on the low inverse viscosity number region ($N_f < 320$). Its main goal was to find out the effects of different factors on the bubble shape and velocity as well as the distribution parameter (C_0), which is a non-dimensional parameter that characterizes the contribution of the moving fluid to the bubble velocity. These authors concluded that the liquid velocity has a strong influence on the movement of the bubble. As the liquid velocity increases, some surface waves are formed, the bubble shape is distorted, and the tip is dislocated towards the outer pipe. The distribution parameter increases as the non-dimensional parameters Eu and N_f increase. For the eccentric cases, it was demonstrated that, as the eccentricity increases, the bubble velocity increases and reaches the maximum value at an eccentricity of 0.3. As the eccentricity increases above 0.3, the bubble velocity

decreases while the wrapping angle gets smaller, which means that the bubbles get wider and their drag increases.

In a recent study, Cao *et al.* (2024) proposed a machine-learning-based approach to identify flow regimes in gas-liquid counter-current flow through a vertical annular channel. They classified the flow regimes as bubbly, slug, and churn by analysing differential pressure signals. Their findings revealed that the power spectral density and the probability density function of the signals were key indicators for identifying these regimes. Additionally, this study investigated the impact of eccentricity (0.25, 0.5, 0.75 and 1) on flow regime transitions. It was demonstrated that, while eccentricity does not significantly affect the overall flow regime, it can influence the transition between bubbly and slug flow in certain cases.

Most recently, Yin *et al.* (2025) conducted experimental and numerical investigations of the flow regimes of co-current flow in relatively large vertical concentric annular channels. In their numerical simulations, they applied the VOF method and the SST $k-\omega$ turbulence model to air-water and air-oil flows in an annular channel with $D_{out} = 431.8$ mm and $D_{in} = 149.2$ mm. They also collected results in an experimental facility with $D_{out} = 298$ mm and $D_{in} = 100$ mm to validate the simulations. They constructed flow regime maps, which, oddly enough, did not contain a slug flow regime. They attributed this result to the fact that the annular gap was quite wide, noting that, in wide gaps, the inertia and gravitational forces on the gas phase are much stronger than the surface tension force, which causes the flow regime to transition from cap-slug to churn flow directly, bypassing the slug flow regime. Moreover, they investigated the effect of channel geometry ($D_h = 57, 95, 198$ and 283 mm) on flow regime under the same superficial gas and liquid flow rates. They showed that, for fixed flow rates, the void fraction was reduced significantly as the annular gap was widened and that this affected the flow regime.

Chapter 3

Background

3.1 Annular channel geometry

An annular channel, shown schematically in Figure 3.1, consists of the space between two cylinders that are parallel to each other, an outer cylinder with diameter D_{out} and an inner cylinder with diameter D_{in} . The annular geometry is characterized by the hydraulic diameter

$$D_h = D_{out} - D_{in} , \quad (3.1)$$

the diameter ratio

$$\gamma = \frac{D_{in}}{D_{out}} , \quad (3.2)$$

and the eccentricity

$$e = \frac{2\delta}{D_{out} - D_{in}} , \quad (3.3)$$

where δ is the distance between the two cylinder axes. When $\delta = 0$, the channel is called concentric, otherwise it is called eccentric.

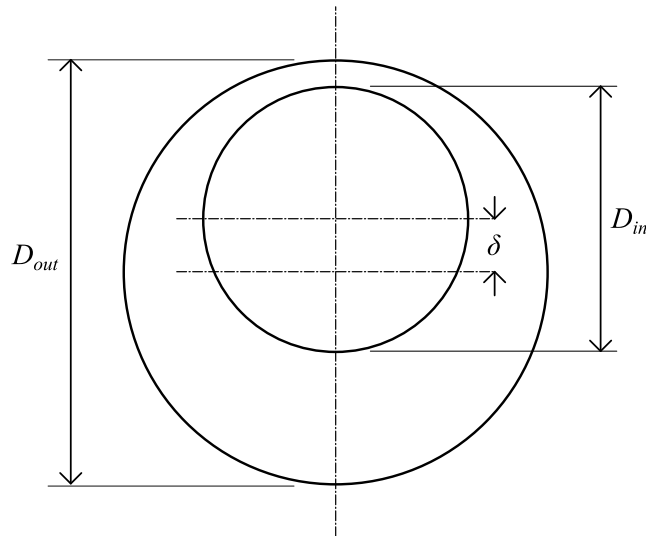


Figure 3.1: Sketch of an annular channel.

3.2 Non-dimensional groups

The hydrodynamic characteristics of gas-liquid flow through annular channels are determined by the interaction of several forces acting on the two phases. These forces include gravitational, inertial, surface tension and viscous forces. The relative magnitudes of these forces are described by the values of several non-dimensional groups, which are listed in Table 3.1. The definitions for these groups apply to circular tubes with a diameter D . In conformity with practices in the literature, D will be replaced by $D_{out} + D_{in}$ for the definition of the Froude number for annular channels. For the other parameters, D_h will be used instead of D , although some authors have used D_{out} .

Table 3.1: Non-dimensional groups.

Name	Definition	Interpretation
Archimedes number	$Ar = N_f^2$	Gravitational forces / viscous forces
Density ratio	$\rho_{ratio} = \rho_l / \rho_g$	Liquid density / gas density
Eötvös number	$EO = \frac{(\rho_l - \rho_g)gD^2}{\sigma}$	Gravitational forces / surface tension forces
Froude number	$Fr = \frac{U}{\sqrt{gD(\rho_l - \rho_g)/\rho_l}}$	Inertial forces / gravitational forces
Inverse viscosity number	$N_f = \frac{\sqrt{gD^3\rho_l(\rho_l - \rho_g)}}{\mu_l}$	Gravitational forces / viscous forces
Morton number	$Mo = \frac{g\mu_l^4(\rho_l - \rho_g)}{\rho_l^2\sigma^3}$	Viscous forces / surface tension forces
Reynolds number	$Re = \frac{\rho UD}{\mu}$	Inertial forces/viscous forces
Viscosity ratio	$\mu_{ratio} = \mu_l / \mu_g$	Liquid viscosity / gas viscosity

3.3 Void fraction and superficial velocities

The fraction of a volume or area of a fluid mixture that is occupied by gas is defined in three different ways. The volume-averaged void fraction is defined as the ratio of the volume V_g occupied by the gas phase and the total volume $V = V_g + V_l$ of a designated region occupied by a gas-liquid mixture (Figure 3.2a)

$$\alpha_V = \frac{V_g}{V}. \quad (3.4)$$

The instantaneous cross-sectional void fraction is defined as the ratio of the area A_g occupied by the gas and the total cross-sectional area $A = A_g + A_l$ (Figure 3.2b)

$$\alpha_{CS_{ins}} = \frac{A_g}{A}. \quad (3.5)$$

Integrating the instantaneous cross-sectional void fraction in time gives the time-averaged cross-sectional void fraction (Figure 3.2c):

$$\alpha_{CS} = \frac{\overline{A_g}}{A}, \quad (3.6)$$

where $\overline{A_g}$ is the time average of A_g over a sufficiently long time interval.

Phase velocity (in-situ or in-flow velocity) is the volume flow rate of one phase of a multiphase flow per unit area of the section occupied by that phase (Mokhatab *et al.*, 2018). Direct measurement of the phase velocities U_g and U_l is difficult for practical reasons, whereas measurement of the phase volumetric flow rates Q_g and Q_l is easier. The superficial velocity of a phase is a hypothetical velocity that is calculated under the assumption that the flow region is occupied entirely by that phase. The time-averaged gas and liquid superficial velocities j_g and j_l are, respectively, defined as

$$j_g = \frac{Q_g}{A} \text{ and} \quad (3.7)$$

$$j_l = \frac{Q_l}{A}. \quad (3.8)$$

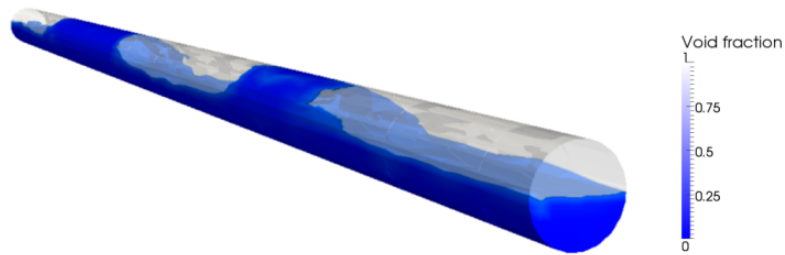
The instantaneous superficial velocities are defined in terms of the instantaneous phase velocities and void fraction as

$$j_{g_{ins}}(t) = \alpha_{CS_{ins}}(t)U_g(t), \quad (3.9)$$

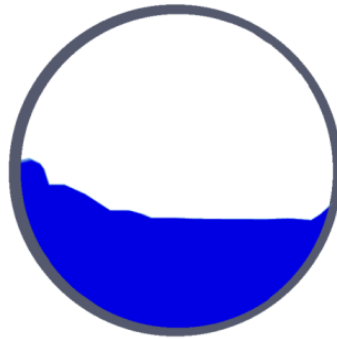
$$j_{l_{ins}}(t) = (1 - \alpha_{CS_{ins}}(t))U_l(t). \quad (3.10)$$

3.4 In-flow velocity of Taylor bubbles

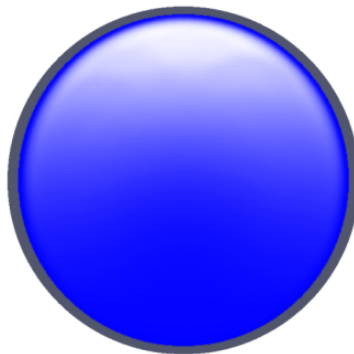
Previous researchers described the in-flow velocity of rising Taylor bubbles in circular and annular channels by superimposing a term that accounts for the interaction between the two phases onto the rise velocity of Taylor bubbles in stagnant liquid. A general expression



(a) Example of a gas-liquid pipe flow pattern
Volume-averaged void fraction = 0.437



(b) Instantaneous void fraction map
Instantaneous cross-section void fraction = 0.682



(c) Time-averaged void fraction map
Time-averaged cross-section void fraction = 0.437

Figure 3.2: Void fraction terminology (Shaban, 2015).

for the velocity of a single Taylor bubble convected in an upward liquid stream was given by Nicklin (1962) as

$$U_{bf} = C_0 j_l + U_b , \quad (3.11)$$

where C_0 is an empirical dimensionless coefficient, called the distribution parameter, which represents the ratio of the maximum and the mean velocities of the liquid, j_l is the superficial liquid velocity, and U_b is the bubble velocity in stagnant liquid. The value of C_0 can be determined from values of the three velocities as

$$C_0 = \frac{U_{bf} - U_b}{j_l} . \quad (3.12)$$

The value of C_0 has been found to be around 1.2 for turbulent flows and around 2 for laminar flows (Nicklin, 1962; Collins *et al.*, 1978; Bendiksen, 1985).

3.5 Flow regimes

A gas-liquid flow regime is defined as a characteristic distribution of individual geometric fluid elements (Brennen, 2005). Before distinguishing the flow regimes, it is helpful to explain what dictates the shape of a bubble and how bubbles get larger. There are four fundamental forces acting on a bubble: the inertia, gravitational, viscous, and surface tension forces. The shape of a small bubble is largely dictated by the surface tension force, which is due to molecular interactions on its surface. Small bubbles tend to be spherical, as this is the shape with the greatest mass-to-surface area ratio (Hendricks, 1990). At low gas flow rates, small bubbles rise following an oscillatory, zig-zag path and mostly do not contact each other. When there is contact between bubbles, they may bounce and move away from each other without coalescing, as evidenced visually in experimental studies. One cause of this bouncing is that the surface tension force is stronger than the inertia force of the bubble. In other words, bubbles do not collide fast enough to break the interfaces that surround them. Another reason of bubble coalescence inhibition might be the presence of surface-active contaminants in the liquid (Hetsroni, 1982). As the gas flow rate increases, the distances between bubbles diminish and bubble collisions occur more frequently. As the bubbles get closer to each other, the liquid interface between them gets drained. If this happens before the bubbles separate from each other, coalescence occurs. Following coalescence, the bubble volume increases and the shape of the bubble is dictated mainly by

the inertia and viscous forces. The balance between the ensuing gas slugs and liquid slugs is maintained by the gravitational and surface tension forces. The falling liquid film around a Taylor bubble gets stabilised and reaches a terminal velocity. Once this happens, the pressure of the liquid film increases in the downward direction, which creates an adverse pressure gradient. Thus, the liquid film becomes unstable and breaks down as it gets exposed to disturbances. This movement cuts off the gas slug and creates unstable gas volumes with an oscillatory movement of liquid around them (Mishima & Ishii, 1984).

3.5.1 Flow regimes in circular tubes

Flow patterns in vertical, upwards, co-current gas-liquid pipe flows are most commonly classified in four main regimes: bubbly flow, slug flow, churn flow and annular flow. These flow regimes were described by Taitel *et al.* (1980) as follows.

- Bubbly flow: The gas phase is dispersed in the liquid phase in the form of small bubbles. This flow regime appears for relatively small gas flow rates and small void fractions.
- Slug flow: As the gas flow rate increases, the small bubbles coalesce and create large bullet-shaped slugs, which are called Taylor bubbles. A Taylor bubble occupies nearly the entire cross section of the pipe. Taylor bubbles are separated by liquid slugs, which contain dispersed small bubbles. A liquid film occurs in the space between the Taylor bubble and the pipe wall, in which the liquid flows against the direction of the bubble motion.
- Churn flow: Further increasing the gas flow rate results in a frothy flow. The shape of Taylor bubbles is distorted. Liquid slugs are still observed but continuously penetrated by the gas phase. The liquid slugs fall as a result of that. Fallen slugs form liquid bridges again and they are lifted by the gas phase. Therefore, the churn flow is characterized by pulsing oscillations.
- Annular flow: This flow regime occurs at very high gas flow rates, in which gas flows in the core and a liquid film flows between the gas and the pipe wall. Some liquid droplets are also observed within the gas. The interface between the phases might be wavy or smooth.

3.5.2 Flow regimes in annular channels

Flow regimes in annular channels are somewhat different from the ones in circular tubes. There are five major flow regimes, namely bubbly flow, dispersed bubble flow, slug flow, churn flow and annular flow (Caetano *et al.*, 1992a), as shown in Figure 3.3. It is worth to mention that there is no universal agreement on the flow regime terminology. In this thesis, we classified each flow in one of the following five regimes.

- Bubbly flow: Bubbly flow in concentric annular channels is very similar to the one in circular tubes. Small dispersed bubbles, having spherical, ellipsoidal or distorted shapes are present in the continuous liquid phase and no large bubbles are present. These bubbles have a zig-zag movement. Smaller bubbles rise more slowly than larger ones. In highly eccentric channels, the dispersed bubbles mostly concentrate in the wider gap region of the channel.
- Cap-bubbly flow: This is a transitional regime, in which the flow is dominated by small dispersed bubbles but also contains cap bubbles, which are larger and resemble hollow hemispheres without a nearly cylindrical main body.
- Slug flow: This regime is dominated by Taylor bubbles. In concentric annular channels Taylor bubbles are wrapped around the inner tube but do not occupy the entire cross-sectional area, which creates a bridge between the two edges of the bubble. Liquid falls in this bridge containing some small dispersed bubbles. A liquid film also exists between a Taylor bubble and the outer tube wall, as seen in circular tubes. The wake regions of Taylor bubbles are highly turbulent due to the backflow in the liquid film and the liquid bridge. In eccentric channels, the liquid bridge is formed in the smaller gap side of the channel.
- Churn flow: Churn flow is very similar to the one in circular channels and independent of eccentricity.
- Annular flow: The liquid film is not only formed between the gas and the outer tube but also between the gas and the inner tube. The former liquid film is always thicker than the latter one. In fully eccentric channels, liquid films merge on the contact line of the tubes.

Flow regime transitions for concentric annular channels have been investigated by several authors (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Das *et al.*, 1999a,b; Hibiki *et al.*, 2003b; Ozar *et al.*, 2008; Julia *et al.*, 2009; Julia & Hibiki, 2011; Wu *et al.*, 2017; Guo *et al.*, 2018; Yin *et al.*, 2025). Flow regime maps based on these studies are shown in Figure 3.4 (Julia & Hibiki, 2011). In comparison, there are very few maps available for eccentric channels (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; De Sousa *et al.*, 2014; Cao *et al.*, 2024).

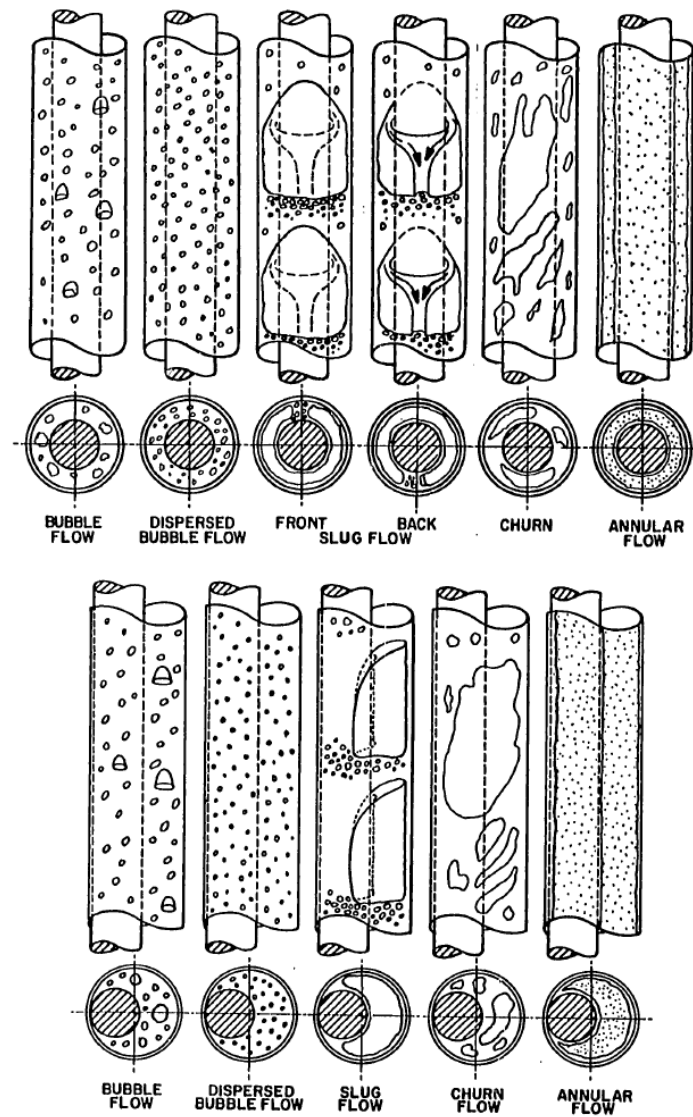


Figure 3.3: Gas-liquid flow regimes in annular channels: concentric (top) and fully eccentric (bottom) (Caetano *et al.*, 1992a).

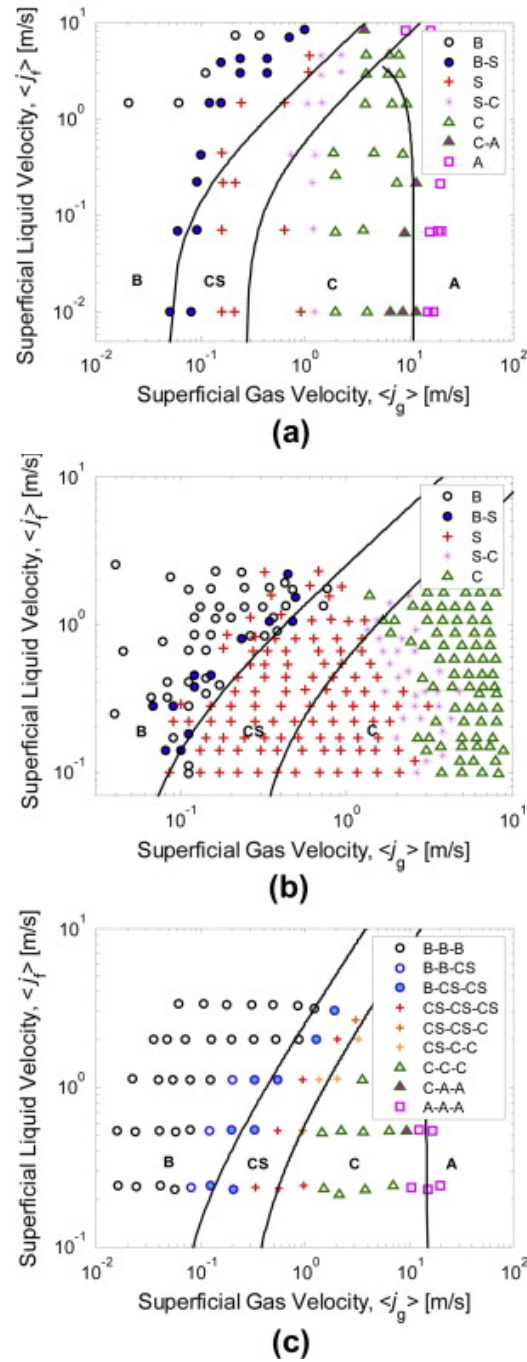


Figure 3.4: Regime maps for gas-liquid flows in concentric vertical annular channels (Julia & Hibiki, 2011), based on measurements by a) Kelessidis & Dukler (1989), b) Das *et al.* (1999a) and c) Julia *et al.* (2009).

Chapter 4

Experimental facility, instrumentation and procedures

4.1 Experimental facility

All experiments were carried out in the Fluid Mechanics Laboratory at the University of Ottawa. The experimental configurations included a vertical circular tube with a column of water and rising Taylor bubbles and vertical annular channels with different eccentricities, both in a water-column/Taylor-bubble setup and with a flowing air-water stream, as described in the following.

4.1.1 Circular channel

Reference tests were conducted in a vertical circular channel, which was an acrylic tube with an outer diameter $D_{out} = 38.1 \text{ mm} \pm 1\%$, an inner diameter $D_{in} = 31.5 \text{ mm} \pm 1\%$ and a length of 3.70 m. The tube was attached to the trumpet in the bottom tank (see next subsection) with a coupling and left open to the atmosphere at its top, as shown in Figure 4.1. The tube diameter was selected so that the flow area of the tube was very close to the flow area of the annular channel, which provided an ideal inner tube diameter $D_{in,ideal} = \sqrt{D_{out}^2 - D_{in}^2} = 32.3 \text{ mm}$. The tube was filled with a column of water that was 3.1 m high and air was released from the bottom tank into the water column from a round nozzle.

4.1.2 Annular channel

Stagnant water column

A sketch of the stagnant water column configuration of the vertical annular channel setup is shown in Figure 4.2 and a photograph of it is shown in Figure 4.3. The test section is fastened on an I-beam, which is mounted on four U-shaped beam segments that are anchored on the wall. The test section has the same annular channel configuration as the horizontal annular

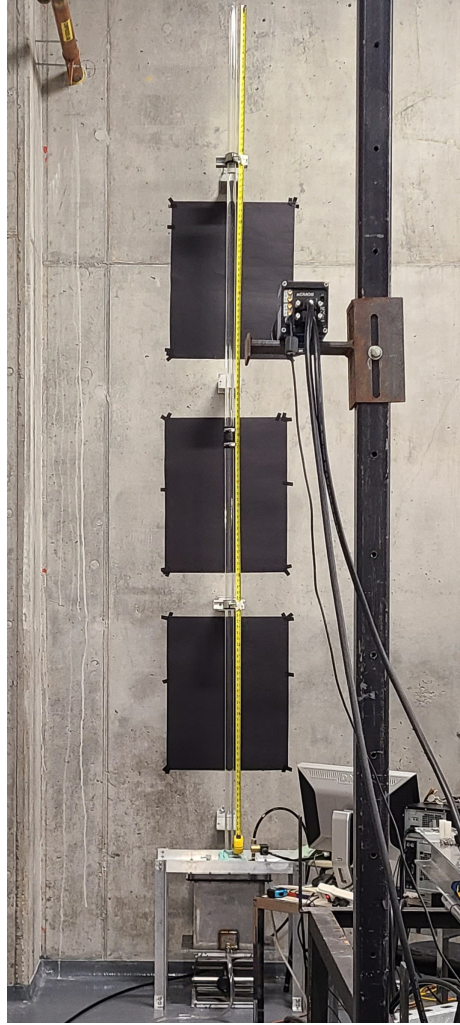


Figure 4.1: A photograph of the vertical circular channel setup.

channel described by Lamarche-Gagnon & Tavoularis (2021). The outer and inner cylinders are glass tubes with diameters $D_{out} = 37 \text{ mm} \pm 1\%$ and $D_{in} = 18 \text{ mm} \pm 1\%$, which give a diameter ratio $\gamma = D_{in}/D_{out} = 0.49$, a hydraulic diameter $D_h = D_{out} - D_{in} = 19 \text{ mm}$, and an ideal area-equivalent circular tube diameter $D_{ideal} = \sqrt{D_{out}^2 - D_{in}^2} = 32.3 \text{ mm}$. The test section consists of two sets of outer and inner tubes, each 1.5 m long and connected in tandem to each other with plastic couplings, which makes the total test section length $L \approx 3 \text{ m} \approx 158D_h \approx 93D_{ideal}$.

The outer tube ends are connected to a bottom tank and an upper tank. The connection to the bottom tank is made with the use of a specially designed flange, referred to as “trumpet” (shown in Figure 4.4), which has an elliptical (bell-shaped) inner wall profile

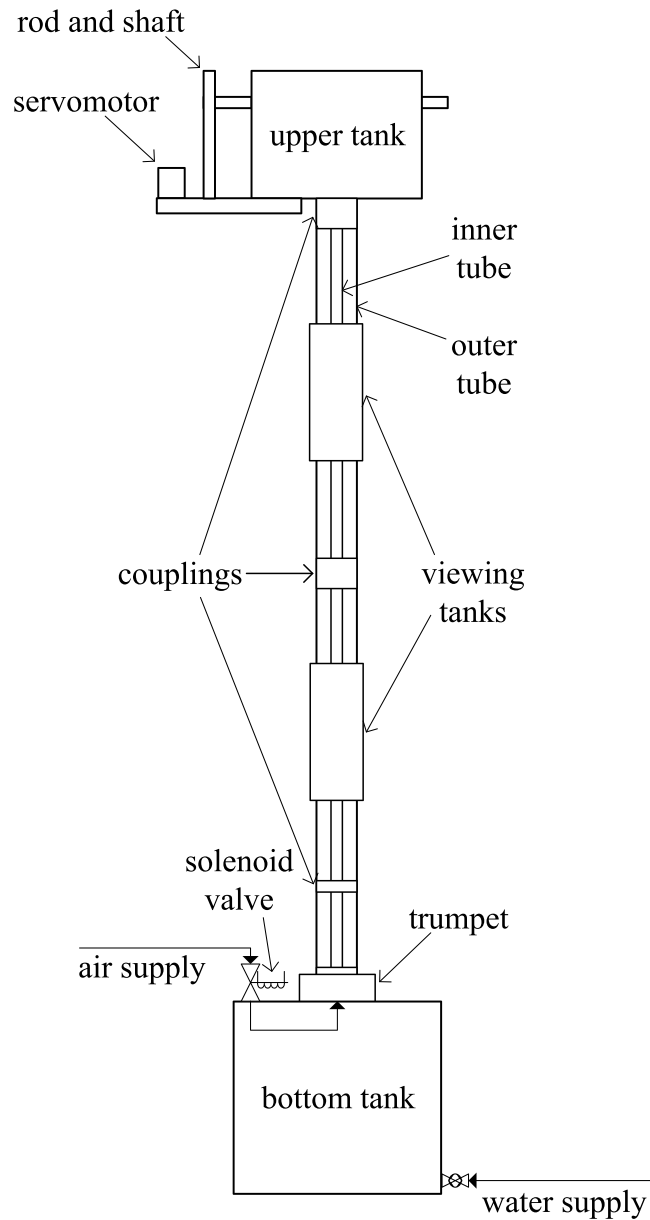


Figure 4.2: Sketch of the stagnant water column configuration.

with an area ratio of 4.2. At its bottom, the inner tube is terminated and sealed by an inlet coupling, which also has an ellipsoidal shape to facilitate a smooth bubble inlet into the annular channel.

A threaded rod with side spacers is placed inside the entire length of the inner tube. The upper end of the threaded rod extends beyond the tube and is fastened on a horizontal traversing mechanism, which is immersed in the upper tank. The bottom end of the threaded

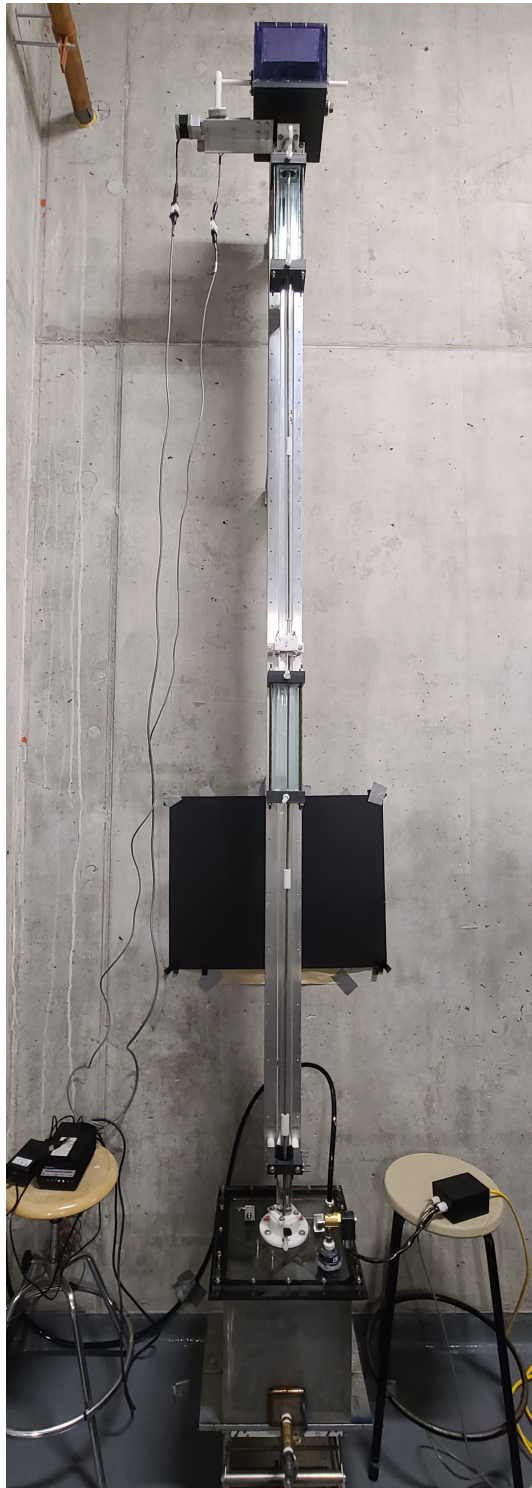


Figure 4.3: A photograph of the stagnant water column configuration.

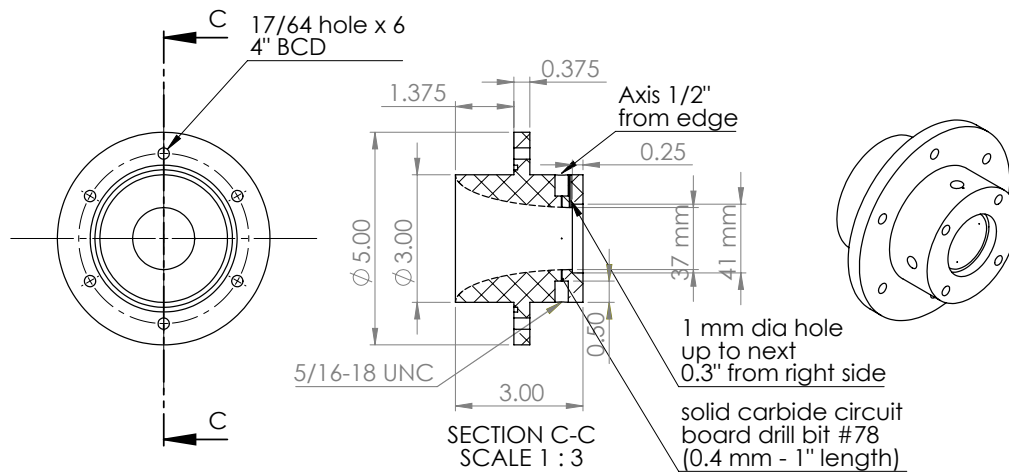


Figure 4.4: Drawings of the trumpet: bottom view (left), C-C cross section (middle) and an isometric view (right); dimensions are in inches unless otherwise specified.

rod is fastened onto the inlet coupling of the inner tube. The threaded rod allows the inner tube to be suspended from the top and free at its bottom. The eccentricity of the inner tube is controlled at three different locations along the tube. The traversing mechanism is powered by a servomotor, which allows adjustment of the eccentricity of the inner tube at the top. In the middle, stainless steel precision pins ($D = 1 \text{ mm}$) are used to adjust the position. The pins go through a hex-head plastic screw and are fixed in the screw with a set screw as shown in Figure 4.5. To ensure the seal, a rubber piece is placed where the screw is facing the hole of the middle coupling. At the bottom, the eccentricity of the inner tube is adjusted by pulling one of two pieces of stainless steel wire with a 0.3 mm diameter, which are mounted on opposite sides of the inner tube, penetrate through the trumpet and are kept taut by guitar tuning pegs (shown in Figure 4.6). The eccentricity is measured with the use of a set of precision pins, which can be inserted through designated holes in the trumpet closely above the wires.

Air is injected into the water at the bottom of the annular channel. Air from the central compressed air system (Figure 4.7) is supplied through a computer-controlled solenoid valve. When the valve is open, air flows into the annular section through a round nozzle ($D = 3.2 \text{ mm}$) located near the inlet of the trumpet (Figure 4.8). The air pressure is adjusted by a regulator. In this configuration, the upper tank is left open to the atmosphere.

Two glass viewing tanks filled with water are positioned around the outer tube to permit

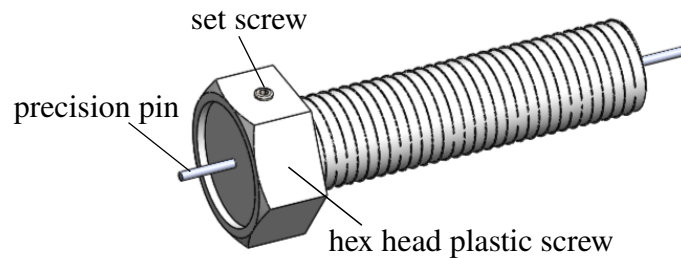


Figure 4.5: The eccentricity adjustment pin.

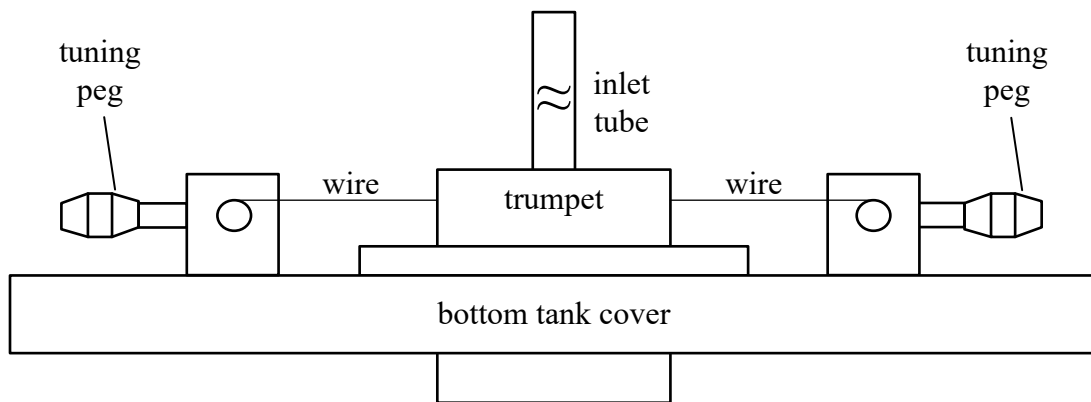


Figure 4.6: The eccentricity adjustment setup at the bottom of the annular channel.

optical access to the flow without excessive distortion due to light refraction.

Co-current flow loop

For tests with a water flow, some parts were added to the stagnant-column configuration. As shown in Figure 4.9, the upper tank was connected to the separation tank by a return branch. The separation tank was used to vent out the trapped air in the return branch and to store the water. A vertical circular tube with a diameter of 19.1 mm (0.75 in) and a length of about 1 m was mounted at the top of the upper tank with its upper end open to the atmosphere. This was done to prevent excessive pressure buildup in the upper tank, if it were sealed, while allowing for some small head increase, as water rose in the tube. The separation tank was also left open to the atmosphere at its top to vent out any air entrained into the return branch. An additional branch was added to connect the separation tank to the bottom tank. This additional branch was composed of the circulation pump, an ultrasonic flow meter, multiple valves, and a bypass branch which redirected part of the

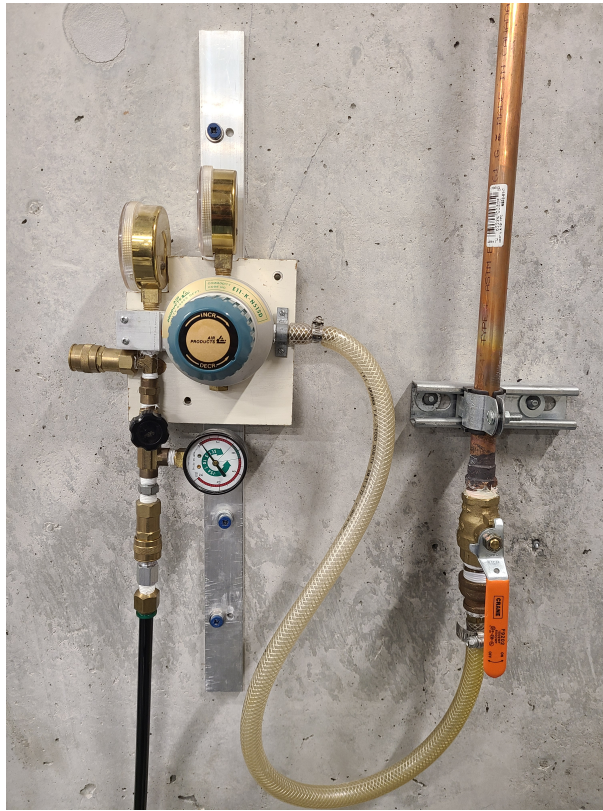


Figure 4.7: Compressed air line and air pressure regulator.

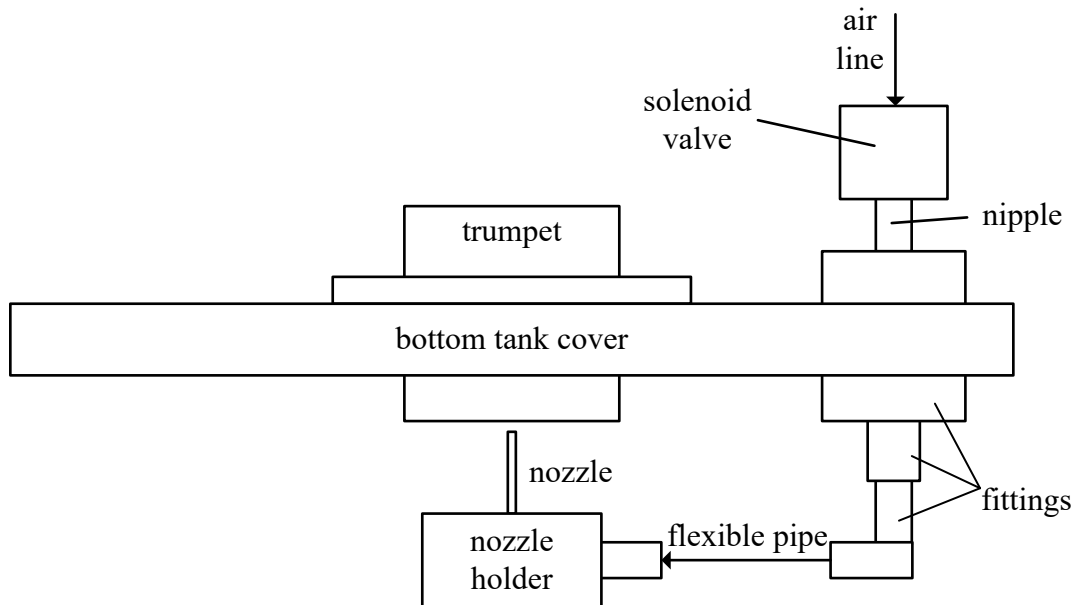


Figure 4.8: Sketch of the air injection setup.

flow through the pump back to the separation tank to adjust the flow rate through the test section. A photograph of the separation tank and other components used in the flow loop are shown in Figure 4.10. Air was released into the stream from a “muffler” (ALWITCO, Model Bantam B28), instead of the round nozzle used in the column experiments, in an effort to reproduce the bubbly water flow regime.

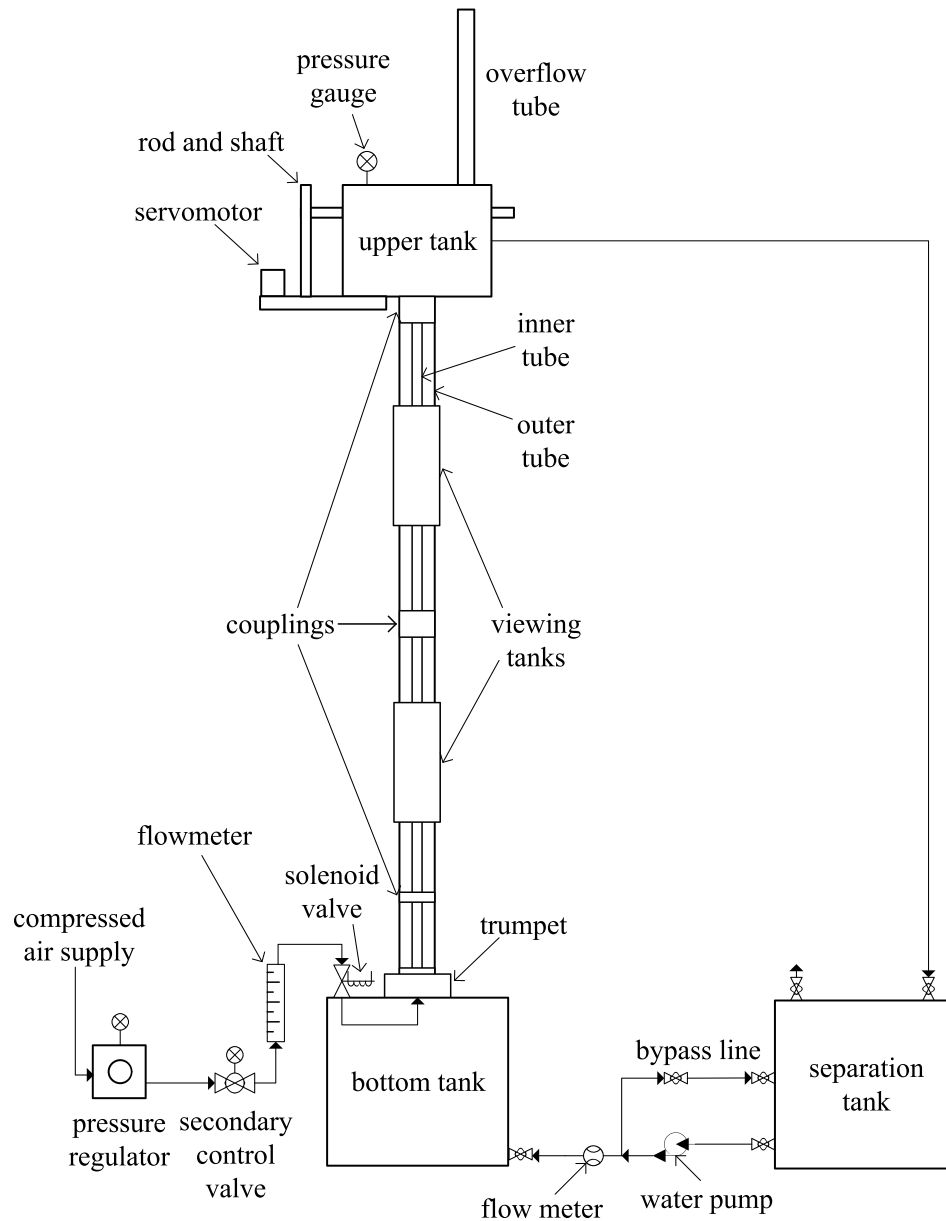


Figure 4.9: Sketch of the co-current flow loop configuration.



Figure 4.10: A photograph of the separation tank, the pump and the flowmeter used in the flow loop.

4.2 Instrumentation

4.2.1 Flow generation and control

The air supply line is instrumented with a pressure regulator, a secondary valve with an attached pressure gauge, a rotameter and a solenoid valve. The pressure regulator (Air Products, Model E11-K-N515D) allows adjustment of the air pressure between 0 and 1400 kPa gauge with a resolution of 50 kPa. Right after this pressure regulator, there is a secondary valve to further adjust the pressure, which is attached to a pressure gauge that shows the air pressure in the narrow range between 0 and 103.4 kPa (0 to 15 psi) with a resolution of 1.7 kPa (0.25 psi). Depending on the flow rate range of interest, one of the two rotameters shown in Figure 4.11 is inserted in the line following the pressure regulation instruments. The first rotameter (Key Instruments, Model MR3A048NVT) is

used for smaller flow rates ($0.01 - 0.14 \text{ m}^3/\text{h}$ ($0.4 - 5 \text{ SCFH}$)), whereas the second rotameter (King Instruments, Model 7510-3-2-2A-13) is used for larger flow rates ($0.28 - 2.83 \text{ m}^3/\text{h}$ ($10 - 100 \text{ SCFH}$)). The air leaving the rotameter flows into the computer-controlled solenoid valve (Parker Hannifin Corp., Model 04F20C1108AAF4C05), which discharges an amount of it through the nozzle into the test section just below the opening of the trumpet. The valve is normally closed and is operated by a digital signal provided by a micro-controller connected to a computer. The user-specified duty cycle of the valve is controlled by software to generate single Taylor bubbles of different volumes and consecutive bubbles after a specified time delay. The solenoid valve was set to open or closed states using a digital switch without a time restriction, providing a continuous air flow into the system. For the flow loop tests, only a digital on/off switch was used to open and close the solenoid valve without a time specification.

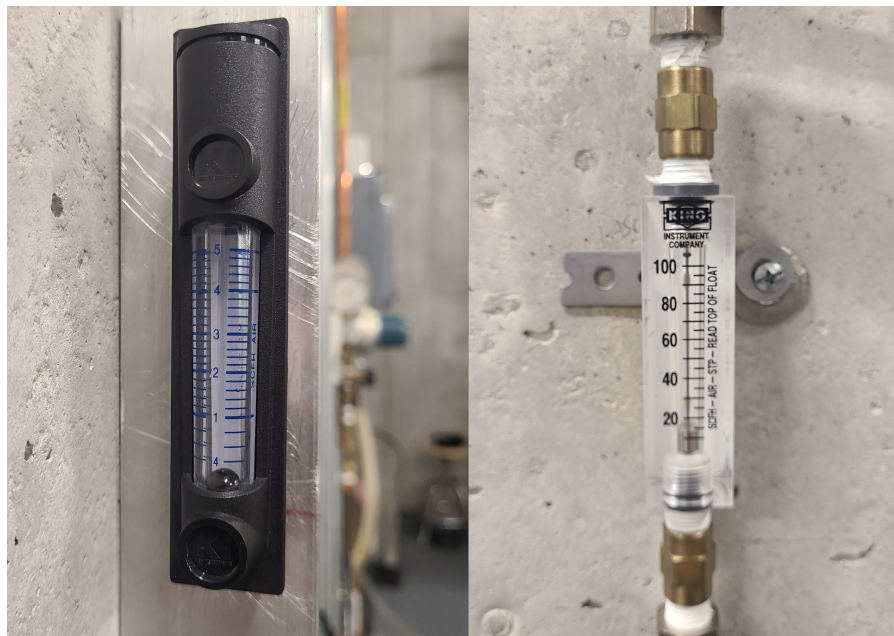


Figure 4.11: Photographs of the smaller flow rate rotameter (left) and the larger flow rate one (right).

For the closed-loop operation of the facility, a magnetic-drive pump (Iwaki, Model MD-70RLT) was connected to the system. This pump has a maximum capacity of 3.79 L/min and a maximum head of 9.7 m . The pump motor speed was kept fixed during the tests and the water flow rate through the test section was controlled by diverting part of the

flow to a bypass branch using the bypass valve. The water flow rate was measured with a calibrated ultrasonic flowmeter (OMEGA, Model FTD-31-T).

4.2.2 Cameras

Two different video cameras (pco.edge sCMOS and pco.edge 5.5 CLHS sCMOS, Excelitas PCO GmbH, Germany), to be referred to as Camera A and Camera B, with main specifications listed in Table 4.1 were used in the present tests. Camera A was used alone for the circular tube tests, whereas both cameras were used for the annular channel tests, mounted as to record simultaneous views of the flow from two directions perpendicular to each other. Camera A recorded the front view of the test section, whereas Camera B recorded a side view, as shown in Figure 4.12. Camera A was mounted on a tall pole, which allowed the traversing of the camera at different elevations, whereas Camera B was mounted on a frame that could be attached at different elevations on the vertical beam that supports the test section.

Table 4.1: Specifications of the cameras.

Camera name	Camera A	Camera B
Test type(s)	Circular tube & annular channel tests	Annular channel tests
Model	pco.edge sCMOS	pco.edge 5.5 CLHS sCMOS
Max. res.	2560 × 2160 pixels	2560 × 2160 pixels
Frame rate at max. res.	100 fps	100 fps
High dynamic range	27000:1	30000:1
Shutter modes	Rolling and global	Rolling and global
Lens	Tamron 90 mm $f/2.5$ (Vivitar for the tube tests)	Vivitar 28 – 90 mm $f/2.8 - 3.5$

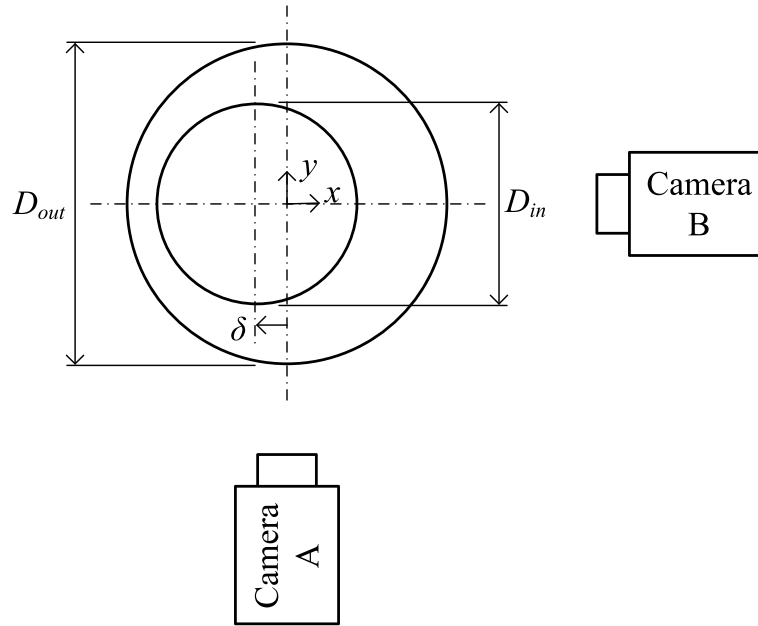


Figure 4.12: Sketch of camera positions for the annular channel tests.

4.3 Experimental procedures

4.3.1 Circular channel tests

One is reminded that the circular tube results were intended to provide a means to estimate the bubble volume in the annular tests. The aim was to collect data for single Taylor bubbles rising in a stagnant water column. Video recordings were collected with Camera A, having the Vivitar lens and placed 1 m away from the tube. A black cardboard sheet was placed behind the tube for a good contrast, and blackout curtains were used to eliminate reflections of the room lights.

The test matrix for a single bubble data collection is given in Table 4.2. Videos were recorded at 50 fps for about 4 s each, with an image frame size of 1440×1440 pixels. The frame height (ΔH) was 292 ± 1 mm. Each recording included 30 single and isolated bubbles for each of the following gauge pressure settings of the secondary pressure valve, measured while the solenoid valve was closed: $P_r = 41, 43, 45, 47, 48, 55, 62, 69, 76$ and 83 kPa (6, 6.25, 6.5, 6.75, 7, 8, 9, 10, 11 and 12 psi). Each bubble was released into the water column by the solenoid valve with an opening time $\Delta t_r = 0.5$ s. To ensure that the previous bubble completely left the system before the next bubble was released, a delay

of 30 s was applied between releases of consecutive bubbles. To avoid spillage, the water level inside the tube was kept at 3.1 m, which is the same as the water level in the annular channel tests.

Table 4.2: Test matrix for the circular channel experiments.

P_r [kPa]	Δt_r [s]	Number of samples
41	0.5	30
43	0.5	30
45	0.5	30
47	0.5	30
48	0.5	30
55	0.5	30
62	0.5	30
69	0.5	30
76	0.5	30
83	0.5	30

The procedure for these experiments is as follows.

- Attach two circular tubes (each $L = 1.85$ m) using a flexible coupling, then attach the tubes on the bottom tank using a coupling. Using tube clamps, secure the tubes to the wall-mounted U-bars.
- Fill the system to the specified water level (3.1 m) with water purified by reverse osmosis (RO) from the separator tank using the pump.
- Inspect the system for possible leaks and seal it further, if necessary.
- Adjust the pressure of the compressed air (P_r) to a specified value, while the valve is closed.
- Release air for a specified time to produce a single Taylor bubble of the desired volume. Start the camera recording right before the bubble reaches the viewing area.
- Repeat the previous step until the number of desired samples is collected.
- Estimate the velocity and the length of each bubble from the recorded images.

4.3.2 Taylor bubble tests in annular channels

In this part of the study, the aim is to collect data for single Taylor bubbles rising in a stagnant water column. The test matrix is given in Table 4.3 for the single bubble data collection. Videos of the Taylor bubbles rising in the annular channel filled with stagnant water were taken synchronously from two perpendicular directions with two cameras, Camera A and Camera B, focused on the upper viewing tank ($H = 2.85$ m). Black cardboard sheets were placed behind the viewing tank on both camera directions for a good contrast, and blackout curtains were used to eliminate reflections of the room lights. Camera A was positioned about 3 m away from the setup and recorded the views with a resolution of 640×2160 pixels. Camera B was positioned approximately 0.6 m away from the upper viewing tank and recorded the views with a resolution of 656×1842 pixels. Each camera recorded 200 frames at a rate of 100 fps for about 2 s. This frame rate was achieved by changing the pixel clock (image sensor readout frequency) from slow scan (95 MHz) to fast scan (286 MHz). Each bubble was released into the channel by the solenoid valve with the specified P_r and Δt_r listed in Table 4.3. To ensure that the previous bubble completely left the system before the next bubble was released, a delay of 25 s was applied between releases of consecutive bubbles.

Table 4.3: Test matrix for stagnant water in annular channel experiments.

P_r [kPa]	Δt_r [s]	e	Number of samples
41	0.25	0.0, 0.1, 0.2, ..., 1.0	220
41	0.50	0.0, 0.1, 0.2, ..., 1.0	220
55	0.25	0.0, 0.1, 0.2, ..., 1.0	220
69	0.25	0.0, 0.1, 0.2, ..., 1.0	220
83	0.25	0.0, 0.1, 0.2, ..., 1.0	220
83	0.50	0.0, 0.1, 0.2, ..., 1.0	220

The procedure below is followed for these experiments.

- Leave the upper tank open to the atmosphere.
- Adjust the eccentricity of the annular channel to a specified value using the guitar tuning pegs, the precision pins and the step-motor driven traverse in the upper tank. Remove the pins at the bottom once the eccentricity is adjusted and insert silicone plugs to block the holes.

- Make sure every other opening in the system is closed.
- Fill the system to the specified water level (3.1 m) with RO water from the separator tank using the pump.
- Inspect the system for leaks and seal if further, if necessary.
- Adjust the pressure of the compressed air (P_r) to a specified value, while the valve is closed.
- Release air for a specified time to produce a single Taylor bubble of the desired volume. Start the camera recording right before the bubble reaches the viewing area.
- Repeat the previous step until the number of desired samples is collected for the specified eccentricity.
- Estimate the velocity and the length of each bubble from the recorded images.
- Repeat the previous steps for a different eccentricity, as in the test matrix.

4.3.3 Air-water flow tests in annular channels

This part of the study aims to collect data to create a flow regime map and to investigate how eccentricity affects the flow regime transition in co-current flow. The setup configuration is changed as explained in Section 4.1.2. The camera setup and settings described in Section 4.3.2 were also used in these tests, except that this time the recording length was 5 s. The test matrix is given in Table 4.4. The procedure below is followed for these experiments.

- Adjust the eccentricity of the annular channel to a specified value using the guitar tuning pegs, the precision pins and the step-motor driven traverse in the upper tank. Remove the pins at the bottom once the eccentricity is adjusted and insert silicone plugs to block the holes.
- Start the pump. Adjust the liquid flow rate using the bypass valve.
- Adjust the pressure of the compressed air to the specified value.

Table 4.4: Test matrix for co-current flow in annular channel experiments.

Gas flow rate [L/s]	Liquid flow rate [L/s]	Eccentricity	Number of samples
0.003	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.006	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.009	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.019	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.039	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.079	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.118	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.157	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.236	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.315	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.393	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.590	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35
0.787	0.03, 0.06, 0.09, 0.13, 0.16, 0.19, 0.22	0.0, 0.3, 0.5, 0.7, 1.0	35

- Open the solenoid valve. Adjust the gas flow rate using the valve before the rotameter.
- Start the camera recording once the system is in steady state.
- Record the flow rates, gas pressure and flow pattern.
- Investigate the stability of the flow and the possible generation of vortex streets triggered by gap instability.
- The flow was analysed qualitatively by visual observation and quantitatively by post-processing of the recorded images.

4.4 Measurement methodology

4.4.1 Measurement limitations

The measurement of local fluid velocity in the present system is subject to severe limitations. First of all, the use of glass walls and the sensitivity of two-phase flow patterns to obstructions do not permit the use of intrusive devices. At the start of the present investigations, we considered the use of non-intrusive optical methods, including laser Doppler velocimetry (LDV) and particle image velocimetry (PIV). Instrumentation for these methods was available in the laboratory and methodologies for measurements in single-phase water flows have been established by previous researchers. It is quite clear, however, that such methods cannot be used for measurements in both the gas and the liquid phases, because of severe light path distortions at the interface. Seeding of the gas with particles would also be impractical and the only option would possibly be to conduct some measurements with the light path entirely in the liquid and away from the gas. Nevertheless, optical access to the annular region was severely hindered by light reflections on the glass walls and refraction between the glass walls and water in the channel and in the viewing tank. Following careful consideration of the prevailing conditions, it was realized that the quality of local velocity measurements that can be collected in this setup would not be acceptable. Consistent with this assessment is the fact that no previous study has been found to apply optical techniques to determine liquid velocity within a vertical annular channel. To compensate for this limitation, we proceeded to collect as much information as possible by analysing video recordings, in some cases also applying corrections based on geometrical models and using measurements in the circular channel and those reported by other authors. Moreover, we relied on the results of parallel computational studies to extend and complement the experimental findings.

4.4.2 Bubble velocity measurement

Individual Taylor bubbles were analyzed frame-by-frame in videos recorded through the viewing tank. The front view was recorded in the circular channel tests and two perpendicular views were recorded simultaneously in the annular channel tests. The location of the bubble upper tip on consecutive images was determined visually. Then the tip displacement

for each time increment and the tip velocity were determined. Because the shape of the bubble is quite complex in annular channels, different for different bubbles and changing as the bubble rises, the tip velocity determination could not always be determined from a single view. In some cases, as for example the case shown in Figure 4.13, the bubble tip was behind the inner tube, as viewed from one of the cameras, which distorted that view and made it difficult to determine the tip location. Nevertheless, having two perpendicular cameras allowed at least one of them to capture fairly accurately the bubble tip. The reported measurements were taken from images of Camera B showing the rear of the bubble, unobstructed by the inner tube, which was on the opposite side of the camera location. In addition to images capturing a portion of the viewing tank, images of a long length of the test section were obtained, from which the velocity of each bubble was determined as a function of distance from the test section inlet. Multiple images were obtained for each set of conditions to allow the statistical characterization of the bubble velocity.

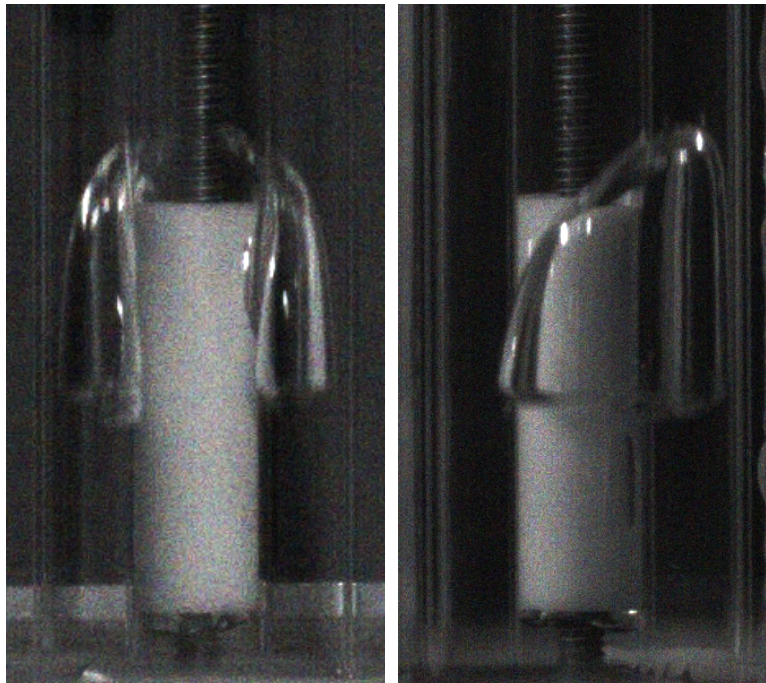


Figure 4.13: A bubble in the concentric annular channel, as seen simultaneously by Camera A (left) and Camera B (right); $P_r = 41$ kPa and $\Delta t_r = 0.25$ s.

4.4.3 Bubble length measurement

The images through the viewing tank were also analysed frame-by-frame to determine the shape and location of the tail of each bubble. The objective was to measure the length of each bubble as the distance between its tip and its tail. However, in many images in both the circular and the annular channel flows, bubble tails had strongly non-axisymmetric shapes, as the air-water interface at the tail departed significantly from a horizontal plane. Tail distortion is well known and is attributed to the interaction of the bubble with its turbulent wake in the liquid. In most cases, the release of a Taylor bubble by the nozzle in the bottom tank was followed by the release of several small bubbles, which trailed the main bubble in its rise, occasionally coalescing with it, but, in general, wandering randomly in the turbulent wake. On the other hand, the Taylor bubble tail interacted with the surrounding turbulent falling liquid film, often getting an irregular shape and occasionally breaking up to create separate small bubbles in the wake (Kren *et al.*, 2023). It was assessed that, in general, the coalescing and breaking up of small bubbles resulted in a practically negligible change of the Taylor bubble volume and length.

As examples of the tail distortion, Figure 4.14 shows three successive images of the same bubble in the circular channel. Figure 4.14a has a nearly horizontal tail, whereas Figures 4.14b,c have distorted and non-horizontal tails. The uncertainty of bubble length measurement was higher for images where tail distortions and bubble rotation were significant than for images that showed a nearly flat and stable horizontal tail. For this reason, the analysis was mostly based on images of the latter type. The reported bubble length measurements were based mostly on images like the first one in the figure (in this case the length was taken as 104 mm).

In concentric annular channels, the bubble often rotated around the inner tube as it rose and its tail image depended on the observation direction. In eccentric channels, bubbles remained on the side of the wider gap and did not rotate. Figure 4.15 shows three successive images of the same bubble in an eccentric annular channel. The tail of Figure 4.15a is nearly flat and horizontal, whereas the tails in the two other images are quite distorted. It is evident that the first image would provide a more reliable bubble length measurement (in this case 67 mm) and so the reported measurements were based, as much as possible, on images with nearly horizontal tails. As Figure 4.16 shows, tail distortions may be present

in both views. The reported measurements were taken from images of Camera B in which the bubble tail was nearly horizontal.

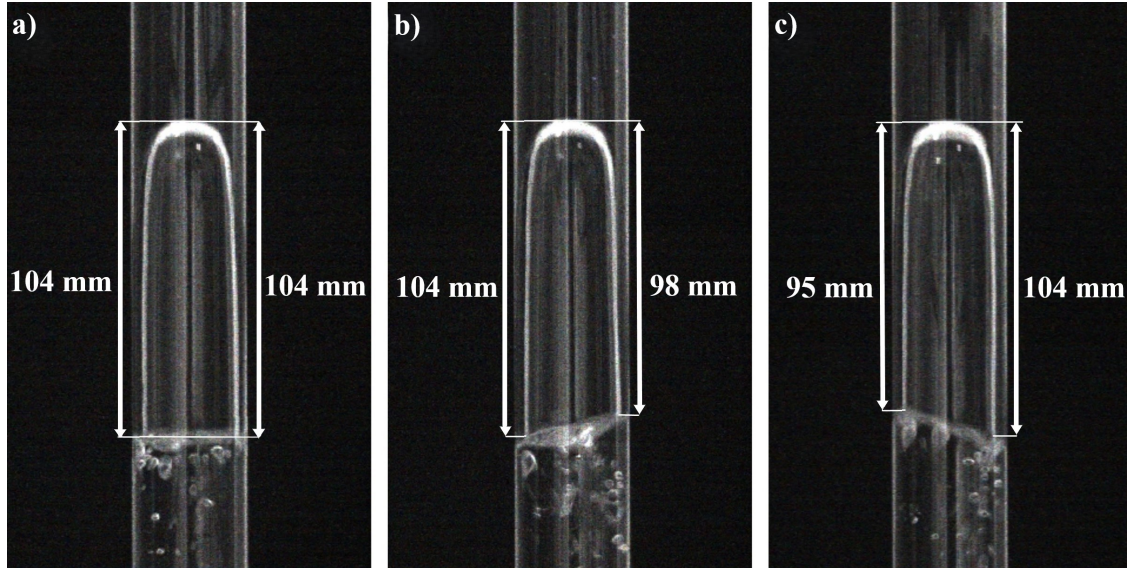


Figure 4.14: Three successive images of the same bubble rising in the circular tube; $P_r = 48$ kPa and $\Delta t_r = 0.5$ s; the time differences between the two consecutive pairs of images were 0.178 and 0.199 s, and the corresponding tip displacements were 34 and 39 mm.

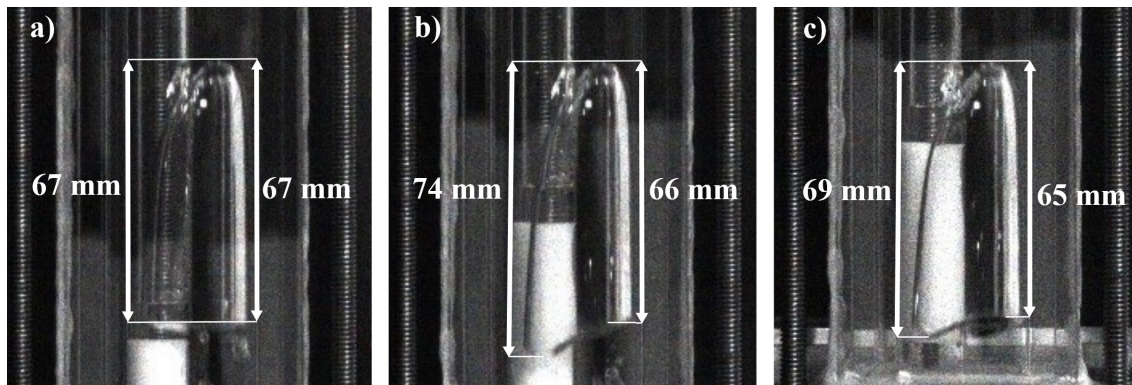


Figure 4.15: Three successive images taken by Camera B of the same bubble rising in the annular channel with an eccentricity of 0.7; $P_r = 55$ kPa and $\Delta t_r = 0.25$ s; the time differences between the two consecutive pairs of images were 0.089 and 0.128 s, and the corresponding tip displacements were 21 and 30 mm; in chronological order, image c was taken first and image a was taken last.

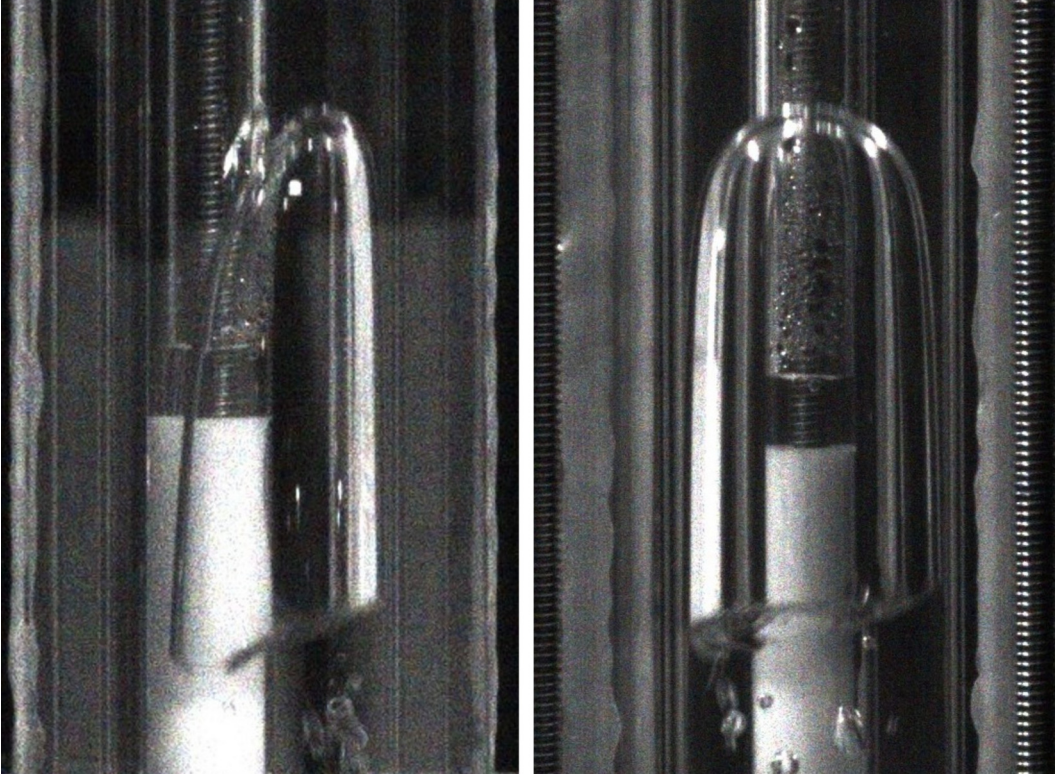


Figure 4.16: A bubble in the annular channel with an eccentricity of 0.7, as seen simultaneously by Camera A (left) and Camera B (right); $P_r = 55$ kPa and $\Delta t_r = 0.25$ s.

4.4.4 Estimation of liquid film thicknesses

To construct a geometrical model of the Taylor bubble and to determine the bubble shape and volume, we need the outer liquid film thickness λ_{out} and the inner liquid film thickness λ_{in} . The literature contains measurements and correlations of the film thicknesses in terms of the inverse viscosity number N_f , which, however, is defined differently by different authors. First, we examined λ_{out} in concentric channels. From images for the concentric case, we get a very rough estimate of the outer film thickness as 2 – 3 pixels on the camera, which gives approximately $\lambda_{out} = 0.4 - 0.6$ mm. Our own CFD studies for concentric channels give $\lambda_{out} = 0.50 - 0.55$ mm. Rohilla & Das (2019) measured $\lambda_{out} = 0.5$ mm at $N_f = \sqrt{\rho_l(\rho_l - \rho_g)gD_{out}^3/\mu_l} = 8,500$, whereas, in our case, $N_f = 22,800$. Liu *et al.* (2023) found numerically $\lambda_{out} = 0.5$ mm, validated with an experimental case of Das *et al.* (1998) for $N_f = \sqrt{\rho_l(\rho_l - \rho_g)gD_h^3/\mu_l} = 4,500$, whereas, in our case, $N_f = 8,300$. In both of these studies, $D_{in}/D_{out} = 0.5$ but the diameters were smaller than the ones in our setup. Lou

et al. (2022) fitted the correlation $\lambda_{out} = (-0.009 \ln N_f + 0.1064) \sqrt{D_{out}^2 - D_{in}^2}$, where $N_f = \rho_l \sqrt{g D_{out}^3} / \mu_l$, to data by Rohilla & Das (2019) and Das *et al.* (2002). For the present value of $N_f = 22,800$, this correlation gives $\lambda_{out} = 0.52$ mm. Next, we examined λ_{out} in eccentric channels. We do not have reliable estimates of this parameter based on our images. Rohilla & Das (2019) fitted the empirical correlation $\lambda_{out} = (0.0991e + 0.0816)(D_{out} - D_{in})/2$ to their data in air-kerosene and air-water mixtures within the range $4,375 < N_f < 8,437$ and $0 \leq e \leq 0.5$. For the geometry of our channel, this correlation gives $\lambda_{out} = 0.78$ mm for concentric channels ($e = 0$) and $\lambda_{out} = 1.25$ mm for middle eccentricity channels ($e = 0.5$). Nevertheless, these values are likely to be significantly higher than the actual ones because our value of $N_f = 22,800$ is about three times the highest value in the experiments of Rohilla & Das (2019) and these authors, as well as previous literature, clearly state that λ_{out} decreases with increasing N_f . Indeed, we have already shown that our own experimental and CFD estimates of λ_{out} for the concentric channel were about 0.5 mm, which is 35% lower than the Rohilla & Das (2019) correlation estimate. Moreover, our own CFD studies for $e = 0.5$ give $\lambda_{out} = 0.80 - 0.85$ mm, which is also much lower than the correlation estimate. Considering all previous estimates from the literature, our images and our simulations, we estimated the bubble shape and volume using the value $\lambda_{out} = 0.5$ mm for concentric channels and our CFD results for eccentric channels, to be presented in Chapter 7.

Concerning λ_{in} , we could not extract any estimate from our images and there is no relevant experimental information in the literature. The only usable value we found was $\lambda_{in} = 0.28$ mm from the concentric channel CFD simulations of Liu *et al.* (2023), which was validated by the experimental work of Das *et al.* (1998) for a diameter ratio of 0.5. It is also noted that this value is comparable to our own numerical results. Although the liquid film thicknesses would depend on the axial distance from the bubble nose, we had no means of estimating this dependence and so we approximated λ_{in} by the constant value 0.28.

4.4.5 Bubble maximum angle of wrap measurement

As mentioned previously, Taylor bubbles rising in vertical annular channels do not occupy the entire annular cross section at any axial location. Instead, they are partially wrapped around the inner cylinder, with a liquid column separating the two sides of the bubble so that each horizontal cross section of the bubble has a meniscus-like shape. For each cross-section, one may define the angle of wrap θ as the angle between two radial lines of the inner cylinder that are tangent to either side of the bubble edge and such that it contains the bubble and not the liquid column, as illustrated in Figure 4.17. The angle of wrap varies with axial distance from the nose along the bubble and has a minimum value of zero at the nose tip and a maximum value θ_{max} near the tail. The same θ_{max} is also the angle of wrap of the projection of the entire bubble on a transverse plane. This angle increases with increasing bubble length until the bubble reaches a length, beyond which the angle of wrap remains essentially constant.

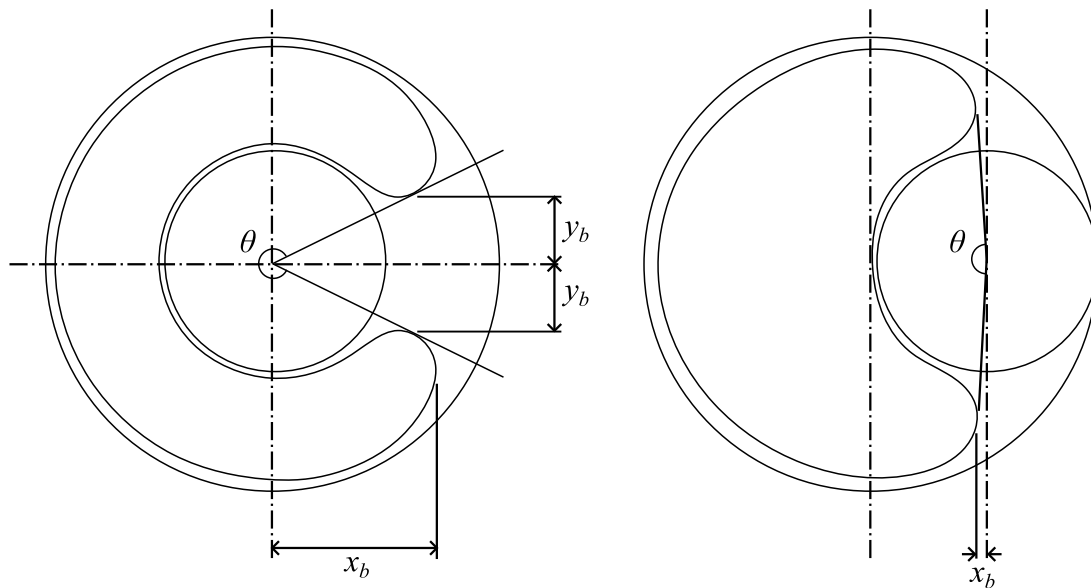


Figure 4.17: Sketches of hypothetical horizontal cross-sections of a bubble in a concentric (left) and a fully eccentric (right) annular channel defining the angle of wrap θ and the bubble edge coordinates x_b, y_b .

The bubble maximum angle of wrap was determined roughly from two images obtained simultaneously with the two cameras so that the recorded views were perpendicular to each other. In eccentric channels, the bubbles were always located approximately symmetrically

with respect to the geometrical plane of symmetry of the channel and such that the liquid column between the bubble edges was located in the narrow gap. Therefore, Camera A always viewed one side of the bubble, whereas Camera B viewed the liquid column between the edges of the bubble. In a perfectly concentric channel, the orientation of the liquid column should be arbitrary and not necessarily facing any camera. Indeed, the bubbles rotated azimuthally as they rose up to an elevation of roughly 2 m in the test section; however, towards the end of the test section, they settled at an orientation such that the liquid column between the edges faced Camera A, presumably due to a very slight eccentricity of the upper support of the inner tube. A representative pair of images by the two cameras for the concentric channel is shown in Figure 4.18. These images define the observed bubble edge coordinates x_b, y_b . The side coordinate x_b , which is the distance of the bubble edge from the inner cylinder axis, could always be determined from the images. However, the frontal coordinate y_b , which is half the width of the liquid column, could only be determined along the bubble beyond some distance from the nose that depended on the eccentricity. Moreover, for relatively large eccentricities ($e \geq 0.6$), the bubble edges were never close enough to each other for y_b to be defined. Figure 4.17 shows how the bubble edge coordinates were defined for the two types of bubble shapes. Therefore, the determination of bubble geometrical properties from the images was done somewhat differently for cases with measurable x_b and y_b and for cases with only x_b available.

To determine the maximum angle of wrap, we created a 2D model of a cross-section of the bubble close to its tail using a computer aided design software package (SolidWorks Education Edition 2019, Dassault Systèmes) according to the following procedure. a) The procedure starts with the construction of a sketch showing two circles representing the inner and outer tube contours for a particular eccentricity. b) Two additional circles, partially approximating the bubble inner and outer interfaces in the channel were drawn; each circle perimeter was separated from the inner or outer tube surface by the estimated corresponding liquid film thickness. c) The bubble edge coordinates x_b, y_b , when available, or just x_b , otherwise, at the bubble tail were determined from the corresponding video images. d) A construction line parallel to the y -axis at a distance x_b and two construction lines parallel to the x -axis at distances $\pm y_b$ were drawn on the sketch. e) Two rounding circles were drawn, each tangent to a pair of construction lines, when both x_b and y_b were available, or

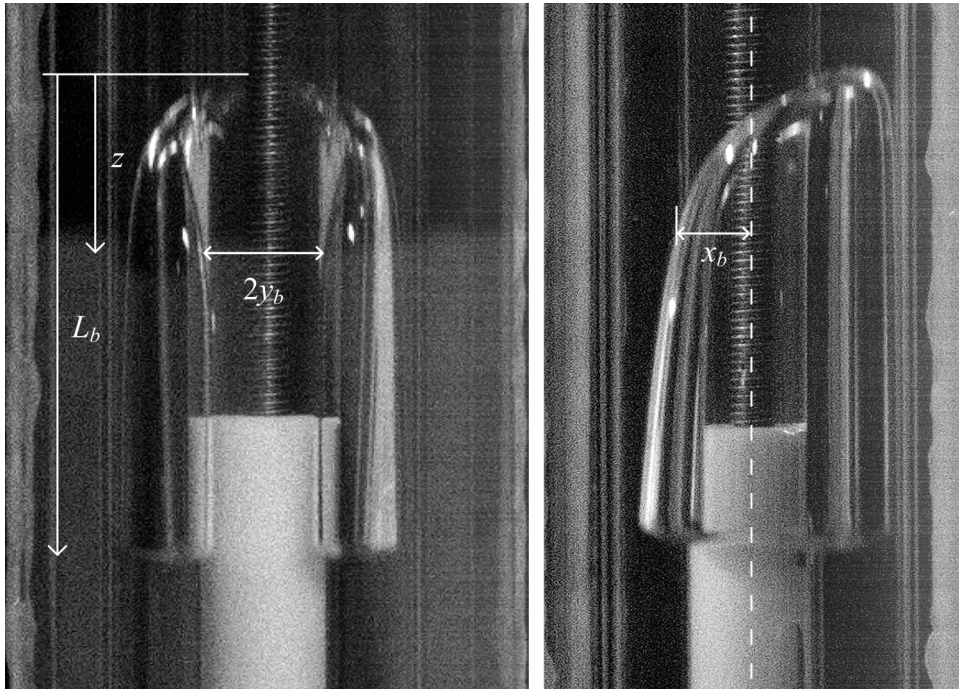


Figure 4.18: Two perpendicular images of the same bubble in a concentric channel, defining the bubble length L_b and edge coordinates x_b, y_b .

one construction line for cases when only x_b was available (in the latter case, the rounding circles were drawn tangent to the corresponding construction line at the midpoint between the inner and outer bubble circles). f) Each rounding circle was joined to the inner and outer bubble circles using splines and a portion of each rounding circle was trimmed to form a continuous smooth bubble contour model. g) The trimmed drawing, shown in Figure 4.17, was used as a 2D model of the cross section of the bubble at its tail. h) Radial lines centred on the inner tube axis and tangent to the rounding circles were drawn; the angle between these radial lines in the direction facing the bubble was taken to be (approximately) the maximum angle of wrap.

A shortcoming of this approach was that all bubble surfaces, and especially the edges, were approximated by simple geometrical models. The uncertainty of this approximation was estimated by changing the position and diameter of the rounding circles within realistic ranges and finding the minimum and maximum θ_{max} . The half difference between these values, which may serve as a measure of uncertainty, was 3° for cases constructed using x_b and y_b , and 5° for cases constructed using x_b only.

The previous procedure, which was specific for the bubble tail and the maximum angle of wrap, could be applied to each cross section along the bubble so that one can estimate the local angle of wrap. As an example, Figure 4.19 shows the axial variation of the angle of wrap in the concentric channel for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s and $L_b = 49.5$ mm, based on Figure 4.18. This figure shows that $\theta = 0$ at the nose tip and then increases with distance from the nose tip to reach θ_{max} at the tail.

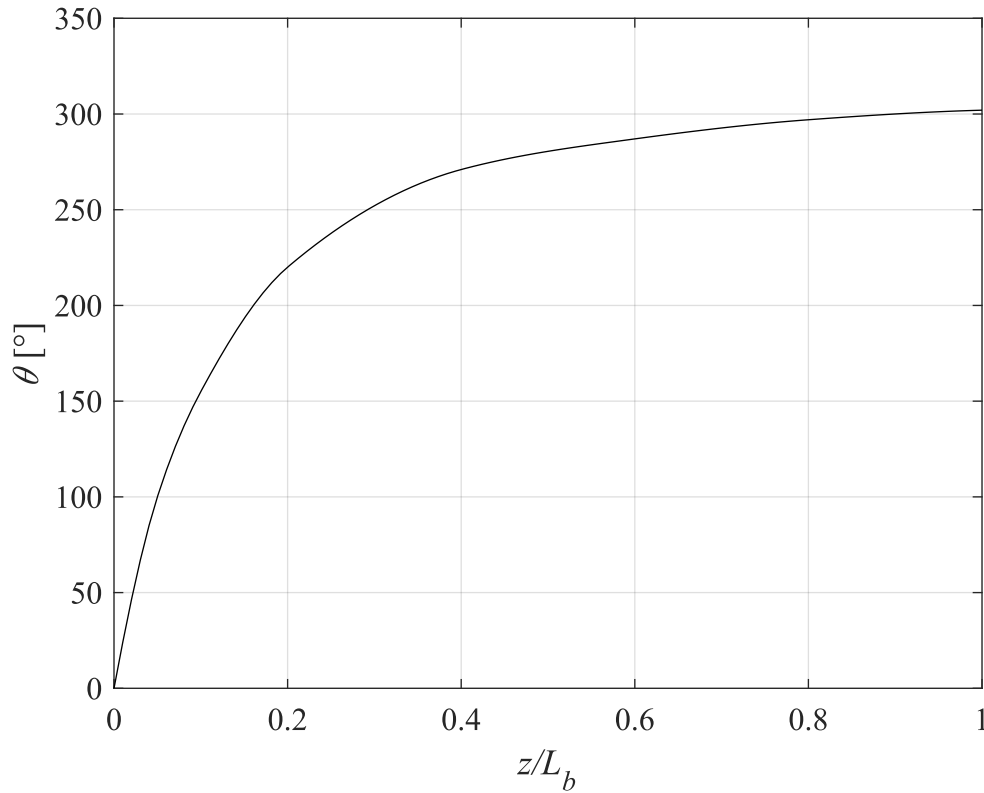


Figure 4.19: Axial variation of the angle of wrap of a representative bubble in the concentric annular channel.

Besides the actual angle of wrap, we also determined an ideal angle of wrap θ_{ideal} , which is independent of axial location. This angle is based on the estimated bubble volume and so its determination procedure will be presented at the end of the next section.

4.4.6 Estimation of bubble volume

The volume of the bubble in the annular channel could not be determined accurately from a direct analysis of the recorded images because of the complexity of the geometry, especially the small liquid film thickness. Although refraction in the water was essentially eliminated by enclosing the viewed section of the channel in a viewing tank filled with water, light refraction through the relatively thin glass wall of the outer tube somewhat distorted the images. As shown in Figure 4.20, the image of a bubble in the annular channel was significantly wider when viewed outside the viewing tank than the image through the viewing tank. Among several methods that we considered for estimating the bubble volume, we retained the following two, one that used images from the annular channel and another that used images from the circular channel.

The first method was to approximate the volume of a bubble in the annular channel by the volume of a reference Taylor bubble released in the circular tube with the same water column height under the same pressure and valve opening time. This approach is based on the assumption that the presence of the inner tube of the annular channel at different eccentricities has a negligible effect on the pressure field near the air release nozzle exit. This analysis was performed only for bubbles with 41 and 83 kPa air release pressures, which have equivalent test conditions in both the tube and the annular channel tests. The same method was applied to both concentric and eccentric cases.

An image of a reference bubble in the circular channel is shown in Figure 4.21. The image resolution is sufficient to allow the fitting of points on the bubble contour and the outer edge of the tube and we further marked the inner edge of the tube by using the known wall thickness. Unfortunately, the bubble shape and dimensions in this image have been distorted by light refraction, which was significant in the case of the cylindrical channel that, unlike the annular one, was not equipped with a viewing tank and also had a thicker wall. We put considerable effort to correct the bubble dimensions obtained from images for refraction effects using geometrical optics. The results of this analysis were not deemed to be reliable because they produced a liquid film thickness that was several times larger than values estimated from the literature. Instead, we calculated the volume of the bubble as

$$V_b = \pi \int_{z=0}^{L_{bt}} R_{bt}^2 dz, \quad (4.1)$$

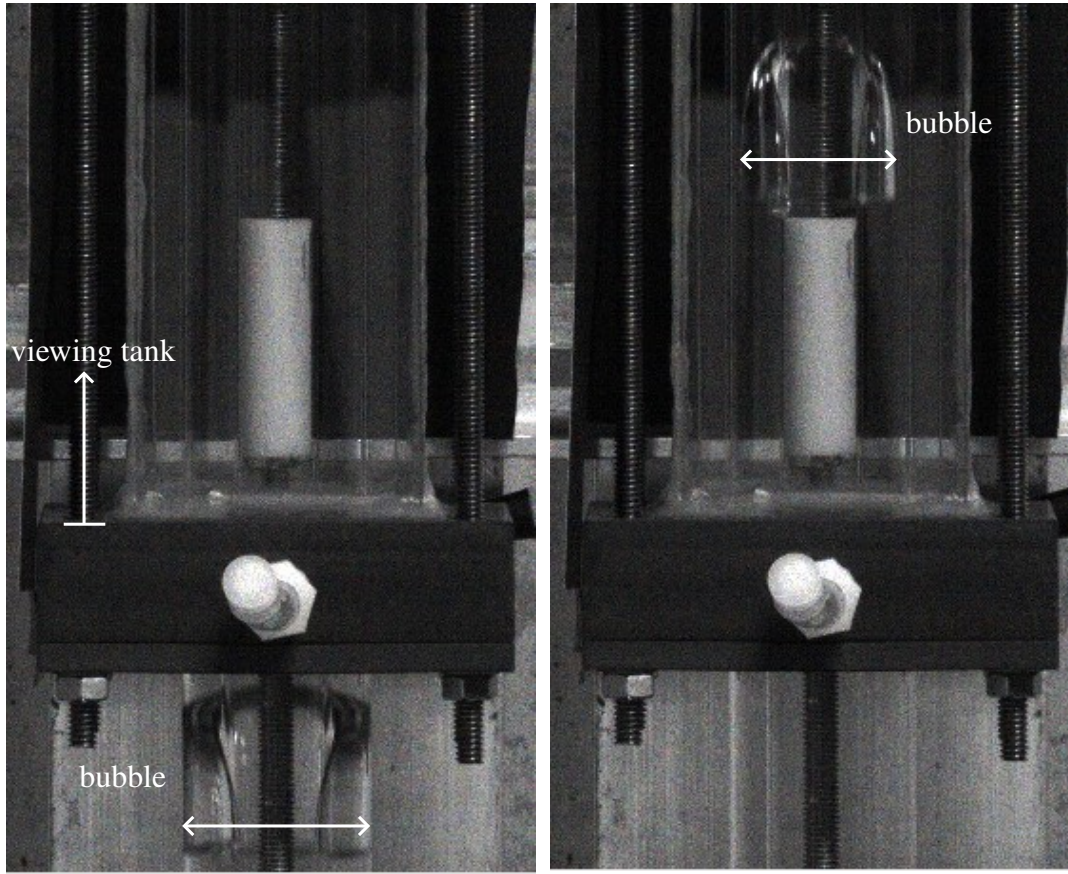


Figure 4.20: Two images of the annular test section with the image on the right taken 0.583 s after the image on the left; the upper parts of both images were viewed through the viewing tank, whereas the lower parts viewed the bare test section; $P_r = 41$ kPa and $\Delta t_r = 0.25$ s.

where R_{bt} is the bubble radius, calculated from the following analytical expressions, suggested by Dumitrescu (1943) and confirmed by Nogueira *et al.* (2006b)

$$R_{bt}/D_{tin} = \begin{cases} 0.375 [1 - (1 - 2.67z/D_{tin})^2]^{1/2}, & z/D_{tin} \leq 0.25, \\ 0.5 \left\{ 1 - [0.0615/(z/D_{tin})]^{1/2} \right\}^{1/2}, & z/D_{tin} \geq 0.25. \end{cases} \quad (4.2)$$

Figure 4.22 shows the radius of two reference bubble images without any correction for refraction effects. It is interesting to see that the contours of the two bubbles, both relatively short ($L_{bt}/D_{tin} = 1.69$ and 6.17) but one being 3.65 times longer than the other, essentially collapse in the overlapping lengths. The same figure shows that Eq. (4.2) produces significantly slimmer bubbles than the ones seen in the present images. In fact, the longer of the two reference bubble images appears to extend beyond the tube inner

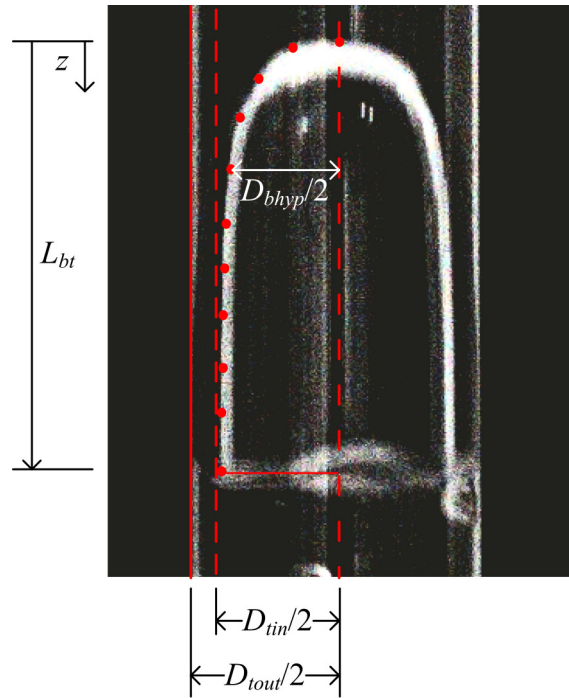


Figure 4.21: Image of a reference Taylor bubble in the cylindrical channel; solid lines are fitted along the bottom of the bubble and an outer edge of the channel, a dot-dash line is fitted to the axis of the channel and dashed line is fitted to the inner edge of the channel and solid circles mark points on half the bubble contour image.

wall. The liquid film thicknesses produced by Eq. (4.2) at the tails of the two reference bubbles were, respectively, 1.6 and 0.8 mm, which are indeed too small to be resolved from the images. The volumes of the models of the two reference bubbles were, respectively, $V_{bt} = 27.4 \times 10^{-6} \text{ m}^3$ and $122.9 \times 10^{-6} \text{ m}^3$. The average bubble volumes for the same release pressures were $V_{bt} = 28.1 \times 10^{-6} \text{ m}^3$ and $109.0 \times 10^{-6} \text{ m}^3$.

Besides the previous bubble volume estimation method, we approximated the bubble volume in concentric and eccentric channels by the volume of a 3D model that was constructed by the CAD software from the recorded images viewed through the viewing tank, such as the ones shown in Figure 4.18. As discussed in Section 4.4.5, these images were used to construct a 2D model of a transverse cross section of the bubble near its tail. 3D models of selected bubbles, an example of which is shown in Figure 4.23b, were created using the following procedure. a) A bubble was selected for each set of release conditions such that its length was close to the average. b) A preliminary 3D model was constructed by extruding the 2D model of the bubble tail; this preliminary 3D model had a uniform

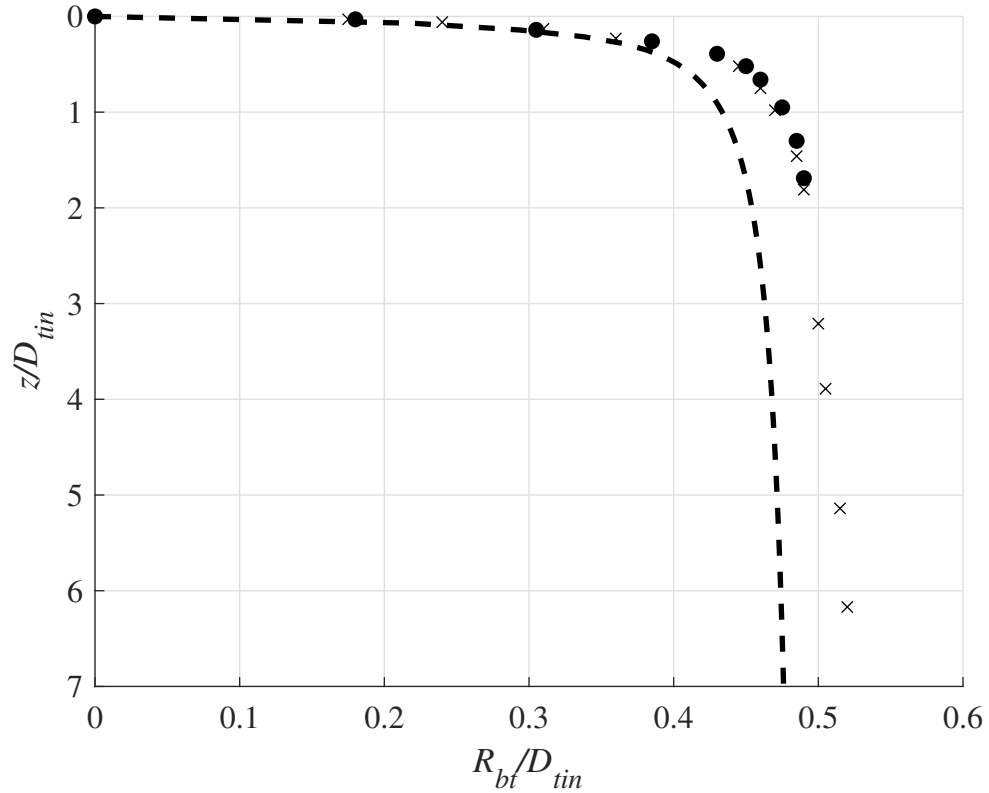


Figure 4.22: Axial variation of the radius of the uncorrected image of the reference bubble at $P_r = 41$ kPa, $\Delta t_r = 0.5$ (●) and the one at $P_r = 83$ kPa, $\Delta t_r = 0.5$ s (×); the dashed line is Eq. (4.2) (Dumitrescu, 1943).

cross-section along the entire length of the bubble. c) An image of a bubble with length approximately equal to the average length, taken from Camera A, was superimposed on the corresponding view of the preliminary 3D model. d) The experimental outer bubble profile was drawn on the image using splines; this profile was then used as a template to trim the preliminary 3D body. e) A similar procedure was applied to the image of the same bubble taken from Camera B. f) When possible, a more realistic model of the bubble, in which the bubble edges were curved, was created by using the fillet feature of the software. Once the CAD model was finalised, its volume was computed by the software. It is noted that the present procedure resulted in bubbles having nearly flat edges, rather than curved ones. This difference resulted in a model volume that was likely slightly larger than the bubble volume. This method determined the volumes of bubbles with the average length in the concentric channel, released at the same pressure and valve opening time as

the two reference bubbles in the cylindrical channel, as, respectively, $V_b = 23.8 \times 10^{-6}$ and $86.7 \times 10^{-6} \text{ m}^3$. These two values are, respectively, 18% and 26% smaller than the corresponding average volumes determined by the first volume estimation method.

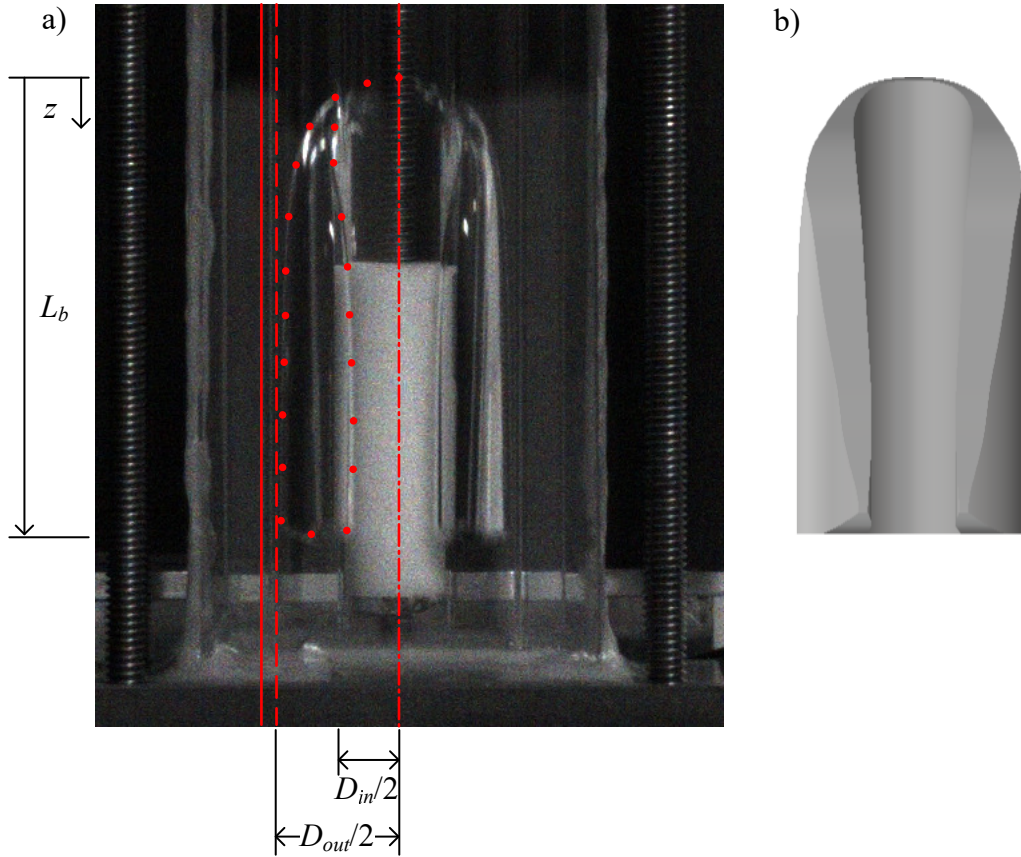


Figure 4.23: a) Representative frontal image of a Taylor bubble in the concentric annular channel with a solid line fitted along an outer edge of the channel, a dot-dash line fitted to the axis of the channel, a dashed line fitted to the inner edge of the channel and points marked on half the bubble contour image; b) a 3D model of this bubble created by the CAD software.

Having estimated the bubble volume, we may now determine an ideal angle of wrap θ_{ideal} , which is defined by assuming that the cross section of the bubble is uniform and its end surfaces are planar, as shown in Figure 4.24a. This angle was estimated from a model constructed using the CAD software. Inputs to this model were the bubble length L_b that was measured in the annular test, the bubble volume V_b from the CAD model and the previously specified film thicknesses $\lambda_{out}, \lambda_{in}$. The cross-sectional area of a bubble that

would occupy the entire annulus would be

$$A_{bfull} = \frac{\pi}{4}(D_{out} - 2\lambda_{out})^2 - \frac{\pi}{4}(D_{in} + 2\lambda_{in})^2 . \quad (4.3)$$

Such a bubble would have a volume

$$V_{bfull} = A_{bfull}L_b , \quad (4.4)$$

which is obviously larger than the actual bubble volume V_b . The ideal angle of wrap in radians was found as

$$\theta_{ideal} = 2\pi \frac{V_b}{V_{bfull}} . \quad (4.5)$$

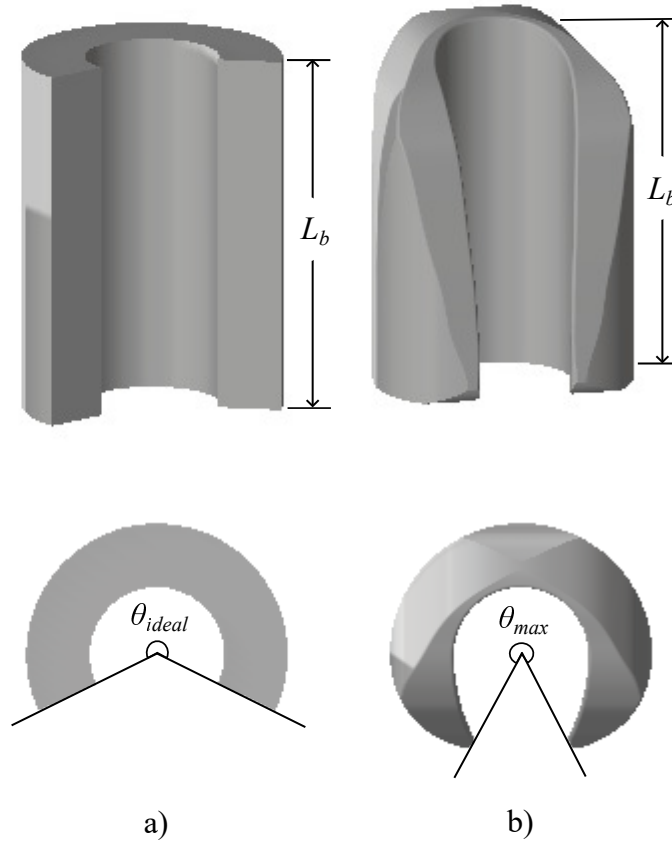


Figure 4.24: a) An idealised 3D model of a bubble in the concentric channel and its projection on a transverse plane, which defines the idealised angle of wrap; b) a model of the same bubble based on its recorded image and its projection, which defines the maximum angle of wrap.

4.4.7 Identification of co-current flow regimes

In order to measure the gas flow rate, one of the two available rotameters was added to the air entrainment system, as described in Section 4.2.1. Because the rotameters were calibrated at standard atmospheric pressure P_{atm} and temperature as a factory setting, the readings of the rotameters were corrected to account for the higher air pressure in the unit, while neglecting any errors for temperature effects. Using the ideal gas law, we may approximate the air volume flow rate through the rotameter as

$$Q_{rot} = Q_{rot,m} \sqrt{\frac{P_r}{P_{atm}}} , \quad (4.6)$$

where $Q_{rot,m}$ is the measured flow rate and the air pressure at the rotameter has been approximated by the pressure at the compressed air regulator outlet. Then, the air mass flow rate through the loop can be determined as

$$\dot{m} = \rho_{rot} Q_{rot} , \quad (4.7)$$

where ρ_{rot} is the air density at a pressure P_{rot} and room temperature, which was taken to be 21°C. Finally, the air superficial volume flow rate through the test section at the middle elevation of the viewing tank, where the images are taken, can be found as

$$Q_{vt} = \frac{\dot{m}}{\rho_{vt}} = Q_{rot,m} \sqrt{\frac{P_r}{P_{atm}}} \frac{\rho_{rot}}{\rho_{vt}} \approx Q_{rot,m} \sqrt{\frac{P_r}{P_{atm}}} \frac{P_r}{P_{vt}} . \quad (4.8)$$

In this expression, the air pressure inside the bubble was approximated by the pressure P_{vt} of water at mid-tank, which was calculated by applying the energy equation between the mid-tank elevation and the water free surface in the upper tank, which was open to the atmosphere. Considering that the kinetic energy of water in the test section is lost entirely upon discharge to the upper tank, the energy equation is simplified into

$$P_{vt} \approx P_{atm} + \rho_l g (z_{fs} - z_{vt}) + f \frac{z_{tbot} - z_{vt}}{D_h} \frac{\rho_l j_l^2}{2} . \quad (4.9)$$

In this expression, z_{vt} , z_{tbot} and z_{fs} are, respectively, the elevations of the viewing tank mid-point, the bottom of the upper tank and the water free surface, equal to, respectively, 2.8, 3 and 3.1 m with respect to the test section inlet; ρ_l is the density of water, f is the friction factor for smooth channels, obtained from the Moody diagram; and j_l is the

superficial velocity of water through the test section. For the relatively high water speed of $j_l = 0.292$ m/s, the frictional loss in Eq. 4.9 was found to be negligible by comparison to P_{atm} . For the typical elevation difference $z_{fs} - z_{vt} \approx 0.3$ m, the elevation pressure difference in Eq. 4.9 was about 3 kPa and so a correction for this effect was applied to P_{vt} .

Flow regime identification of the co-current flow in the annular channel was performed by visual observation. Four flow regimes were identified, namely, the bubbly, the cap-slug, the slug, and the churn regime. The bubbly flow is characterized by small bubbles, some of which have nearly spherical shapes, while others have distorted shapes, but is free of cap bubbles and slugs. The cap-bubbly flow represents a transitional regime between bubbly and slug flows, distinguished by the appearance of small hollow nearly hemispherical bubbles within the bubbly flow regime. These cap bubbles are identified as noses of slugs without a nearly cylindrical body. The slug flow regime is characterized by Taylor bubbles, which are long gas slugs partially wrapped around the inner cylinder. Liquid slugs (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Das *et al.*, 1998) occupy the space between Taylor bubbles. Each Taylor bubble is wrapped around the inner tube and has two longitudinal edges, the space between which is occupied by liquid. It may be thought as consisting of a partial, nearly hemispherical nose, from which the core has been removed, and a partial, quasi-annular body. The churn flow regime is a highly disturbed extension of the slug flow regime. The liquid flow is frothy and highly turbulent, which disrupts the Taylor bubble shape. Moreover, the liquid slugs are incessantly penetrated by the gas slugs and fall in the form of liquid bridges in a pulsatile motion. The length of the liquid slugs in churn flow is much smaller than that in the slug flow regime, as the gas flow rate is higher in the churn flow regime.

Representative images of the four observed flow regimes in an eccentric annular channel are shown in Figure 4.25. Although the visual regime identification approach is subjective, it is considered to be adequate for the present purposes. It is noted that Cao *et al.* (2024) compared results of visual regime identification with those of a machine learning regime identification method using pressure signals and found differences up to 4% occurring only in the transition zones.

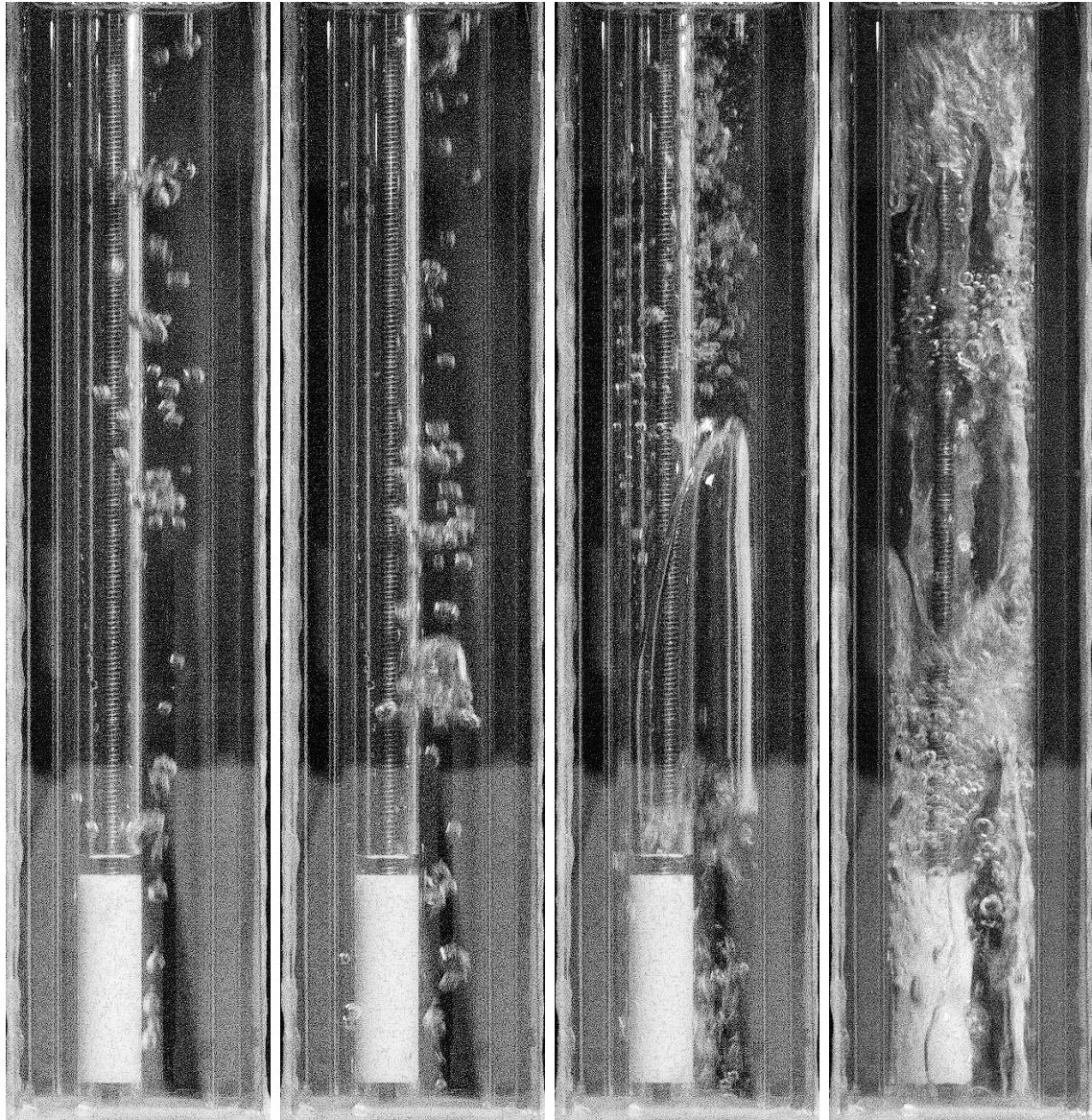


Figure 4.25: Representative images of the four identified flow regimes in the annular channel with an eccentricity of 0.7: bubbly, cap-bubbly, slug, and churn regime (left to right).

4.4.8 Measurement uncertainty

An analysis was conducted to estimate the uncertainty of all measured properties. The procedures follow current international standards, as described by Tavoularis & Nedić (2024). The sources of measurement uncertainty of the reported properties are the uncertainties of the facility dimensions and alignment, the measurement devices, the measurement procedures, and the post-processing of the data. Some of these uncertainties are quantifiable and some are not. Geometrical imperfections of the apparatus include deviations from circularity and straightness of the inner and outer cylinders and misalignment of the two cylinder axes. The estimated tolerance of the tube diameters was roughly 1%. The maximum deviation of eccentricity from the nominal value along the test section was about 0.05. The resolution of the pressure gauge used for the measurements, approximated as half of the scale increments on the gauge, was 1.7 kPa (0.25 psi). The manufacturer of the ultrasonic flowmeter specified that it measured the liquid flow rate within 1%. The resolution for the lower-range rotameter was 0.003 m³/h (0.1 SCFH) and the resolution of the wider-range rotameter was 0.071 m³/h (2.5 SCFH). The response time of the solenoid valve was given by the manufacturer as 5 – 10 ms. The cameras were calibrated using a measurement tape on the viewing tank and the pixel values were converted to mm. The approximate sizes of the physical space that corresponds to one pixel are 0.20 mm and 0.14 mm for Camera A and Camera B, respectively. The bias limit of bubble tip location within the same time frame was estimated as 2 pixels, which is 0.4 mm for Camera A and 0.3 mm for Camera B, and the uncertainty of time difference between frames was estimated as 0.01 s. The uncertainty of bubble length and velocity measurements was quantified using the standard propagation of uncertainty relationship (Tavoularis & Nedić, 2024)

$$u_y = \sqrt{\sum_{m=1}^M \left(\frac{\partial y}{\partial x_m} u_{x_m} \right)^2}, \quad (4.10)$$

where y is a property derived from direct measurements of M measurable properties. The bubble velocity is a function of the pixel distance on the camera and the time difference between frames. The uncertainty of bubble velocity, which is also affected by the eccentricity of the channel, was estimated as 2%. The bubble length solely depends on the pixel distance between bubble nose tip and tail. Based on this calculation, the uncertainty of bubble length was found as at most 1%. However, the bubble tail swung from each frame

to the next, as discussed in Section 4.4.3, which introduced an unquantifiable random error in determining the location of the bubble tail. In the extreme example of this swing shown in Figure 4.15b, the tail location swung by ± 20 pixels; in this example, the swing of the tail location dominates the uncertainty, which is estimated as roughly 7%.

Because the maximum angle of wrap and bubble volume were found from CAD drawings, the propagation of uncertainty method is not suitable to estimate the uncertainty of these properties. For the angle of wrap measurements, the limits of the bubble edges in the corresponding views could be located in the images but the edge shape and dimensions could not be identified. To proceed, a 2D model representing the bubble tail cross-section was created and the maximum angle of wrap was estimated, as explained in Section 4.4.5. The uncertainty of the angle of wrap was estimated very roughly as half the difference between two values obtained from two extreme rounding circles. Realistic values of this uncertainty were 3° for $e < 0.6$ and 5° for $e \geq 0.6$. A 3D bubble model was created, based on the 2D model and using the average bubble length and the bubble outer profile, which was obtained from two perpendicular images of bubbles. These parameters contribute to the uncertainty but it is difficult to quantify their effects. Based on comparisons of volumes of bubbles reconstructed by CAD software with the use of extreme values of the maximum angle of wrap and bubble length, we estimated the uncertainty of the bubble volume as roughly 10%.

The measurement uncertainty u_x of a measured property x is given by Tavoularis & Nedić (2024) as follows

$$u_x = \sqrt{b_x^2 + p_x^2}, \quad (4.11)$$

where b_x is the bias limit and p_x is the precision limit of x . The precision limit for the mean of a set of N repeat measurements is

$$p_x = \frac{t_v \sigma_{xN}}{\sqrt{N}}, \quad (4.12)$$

where σ_{xN} is the standard deviation of the set and t_v is the student t -score, which specifies the 95% confidence limits and depends on N . When N is sufficiently large, the t -score value approaches the value of 2 (Tavoularis & Nedić, 2024). In this thesis, the precision uncertainty of repeat measurements is estimated as $t_v \sigma_x$ for $N < 30$ and $2\sigma_x$ for $N \geq 30$.

Chapter 5

Computational procedures

5.1 The volume of fluid method

The present computations of air-water flows were performed with the use of the Volume of Fluid (VOF) method. This method, pioneered by Hirt & Nichols (1981), is an Eulerian interface tracking technique, which solves a single set of momentum equations for a gas-liquid mixture and an additional equation for the volume fraction of one phase. The volume fraction of phase i is defined as (Siemens, 2018)

$$\alpha_i = \frac{V_i}{V}, \quad (5.1)$$

where V_i is the volume of phase i and V is the total volume of the cell. The sum of the volume fractions must be equal to one. The VOF method is suitable for flow of immiscible fluids, which are separated by a deformable interface. Thus, it is well suited for capturing slug flows and Taylor bubbles. The commercial CFD code STAR-CCM+ was used to solve the governing equations.

As part of the solution, each cell in the computational mesh is assigned a value of the volume fraction between 0 and 1. This modelling approach assumes that the mesh resolution is sufficient to resolve the position and the shape of the interface between the phases. The distribution of phases and the position of the interface are resolved in terms of the phase volume fraction α_i . The material properties of the fluid mixture in the cells containing the interface are expressed in terms of the material properties of the constituent fluids, as follows:

$$\rho = \sum_i \rho_i \alpha_i, \quad (5.2)$$

$$\mu = \sum_i \mu_i \alpha_i, \quad (5.3)$$

$$c_p = \sum_i \frac{c_{pi}\rho_i}{\rho} \alpha_i, \quad (5.4)$$

where ρ is the density, μ is the dynamic viscosity and c_p is the specific heat under constant pressure.

The governing continuity and momentum equations are, respectively,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{U}) = 0 \quad (5.5)$$

and

$$\frac{\partial}{\partial t}(\rho \vec{U}) + \nabla \cdot (\rho \vec{U} \otimes \vec{U}) = -\nabla p + \vec{\nabla} \cdot (\mu(\vec{\nabla} \vec{U} + \vec{\nabla} \vec{U}^T)) + \rho \vec{g} + \vec{F}_{ST}, \quad (5.6)$$

where p is pressure and T denotes the transpose. $\vec{F}_{ST}$ is the surface tension force, computed with the use of the continuum surface force (CFS) model of Brackbill *et al.* (1992) as

$$\vec{F}_{ST} = \sigma \kappa \vec{\nabla} \alpha, \quad (5.7)$$

where κ is the local interface curvature. The tracking of the interface between the phases is accomplished by solving a model continuity equation for the volume fraction in the form

$$\frac{\partial \alpha}{\partial t} + \vec{\nabla} \cdot (\alpha \vec{U}) = 0. \quad (5.8)$$

5.2 Turbulence modelling

For the present work, we expected that the flow would be turbulent in at least some part of the computational domain and that the available computer resources would be insufficient for resolving accurately all turbulent motions, as would be the case, if it were possible to perform Direct Numerical Simulations (DNS). Therefore, we needed to implement a turbulence model, by which some of the turbulence motions are not resolved, but they are accounted for by the model. Many turbulence models have been developed, each with different capabilities, accuracy and computational cost. The turbulence models that are available in STAR-CCM+ can be classified in the following two categories.

- Models that provide closure of the Reynolds-Averaged Navier-Stokes (RANS) equations.

- Scale-resolving simulations that solve for large scales of turbulence and model small-scale motions. These include Detached Eddy Simulations (DES) and Large Eddy Simulations (LES).

In this study, the DES model was applied, because, although LES is considered to be more accurate, its implementation near walls would require an extremely fine computational mesh, which is untenable with the available resources and time. DES is a hybrid model that combines the features of RANS simulation in wall regions and LES in regions away from flow walls. The unsteady RANS equations are applicable to transient situations where the unsteadiness is either imposed, such as by a time-varying boundary condition, or is inherent, such as vortex shedding in a massively separated flow. In the latter case, transient simulations often yield better results than attempting to use a steady-state approach. However, successful unsteady RANS (URANS) simulations require that the time scales of the turbulence be disparate from the mean-flow unsteadiness. Furthermore, the limitations of the turbulence model may preclude good unsteady results. DES turbulence models are set up so that boundary layers and irrotational flow regions are solved using a RANS closure model. However, the turbulence model is intrinsically modified so that, if the grid is fine enough, it will emulate a basic LES subgrid-scale model in detached flow regions. In this way, one gets the best of both worlds: a RANS simulation in the boundary layers and an LES in the unsteady separated regions (Siemens, 2018).

STAR-CCM+ supports DES with the following three RANS models.

- The SST (Menter) $k-\omega$ model.
- The Spalart-Allmaras model.
- The elliptic blending $k-\varepsilon$ model.

5.3 Numerical procedures

All simulations were performed using the commercial finite volume CFD package Star-CCM+. The computational method consists of solving a system of three-dimensional, time dependent equations. In order to resolve the interface between air and water in the annular channel, the Volume of Fluid (VOF) method was applied. For turbulence modelling, the

Improved Delayed Detached Eddy Simulation (IDDES) was used. In comparison with the traditional DES, IDDES has an improved performance during transition from LES to RANS near walls. The SST (Menter) $k-\omega$ model was selected as the RANS model.

Simulations were performed to find implicit unsteady solutions. The segregated flow solver, which solved the equations separately (one equation for pressure and one for each velocity component), was implemented, as it required less memory and is suitable for 3D incompressible flows.

The specification of appropriate mesh size and time step are crucial for getting accurate solutions in CFD analyses. Details of the mesh generation is given in the next chapter. The time step was determined by consideration of the Courant number, which is defined as

$$C = U \frac{\Delta t}{\Delta x}, \quad (5.9)$$

where U is the maximum velocity in the region, Δt is the time step, and Δx is the minimum cell size in the region. All simulations were run with the adaptive time step control that targeted the mean Courant number of 0.5, which allowed the solution to advance at a Δt that was appropriate to maintain numerical stability and accuracy.

Multiple cases were run simultaneously, using four dedicated servers (44 processors per server) at the University of Ottawa, along with the servers available through the Digital Research Alliance of Canada in the Beluga (32 processors per node), Cedar (32 processors per node), Graham (44 processors per node), and Narval (64 processors per node) systems.

5.4 Mesh generation

Mesh generation is the process of dividing the computational domain into smaller finite volumes (cells) in order to solve numerically the governing equations. Mesh generation is a vital part of any computational simulation. The mesh shape and size are determined by a trade-off between accuracy and computational time and cost. The mesh was generated in STAR-CCM+ (v11 or v13) using directed mesh setting, which allows the flexibility of creating patches and generates a uniform or non-uniform structured mesh. This type of mesh is easy to generate for circular and annular channels. Hernandez-Perez *et al.* (2011) performed numerical simulations to compare different mesh generation techniques in STAR-CCM+ by simulating Taylor bubbles rising in vertical circular channel and highly

suggested the butterfly mesh for this type of geometry. Numerical studies on annular channels also showed that structural O-type mesh works well for this geometry in both concentric (Gouidmi *et al.*, 2019; Kiran *et al.*, 2020; Liu *et al.*, 2023) and eccentric (Liu *et al.*, 2024) channels. Therefore, in this work we used types of mesh like the ones shown in Figure 5.1. For meshing the circular channel, we used larger uniform-size hexahedral cells in the middle of the channel, and cells with diminishing size with one-sided geometric mesh distribution towards the wall. For meshing the annular channel, we used a two-sided hyperbolic mesh distribution with a minimum radial cell size of 0.1 mm, which ensured that the wall regions had smaller cells. Circumferentially, the cross-section was divided into 120 cells of equal width. Because there was no mechanism to control the radial distribution of the mesh in the narrow and wide gaps separately, the number of radial mesh divisions were identical for both gaps. In the cases with $e = 0.8$ and 0.9 , where the narrow gap was very small, using the two-sided hyperbolic mesh distribution caused the mesh cells in the narrow gap to become wider towards the walls for the assigned number of radial divisions, for which the minimum radial cell size was set to 0.1 mm. This could have been prevented by setting a smaller minimum cell size; however, if this approach were followed, the cells in the wide gap would have become too large, unless the total number of cells were increased. To keep the computational cost within acceptable levels, and to sustain the same number of cells for all simulations, no action was taken to adjust the mesh generation method for different eccentricities. For $e = 1$, the geometry was not suitable for the type of meshing used for the non-zero eccentricity cases, because of the vanishing distance between the inner and outer boundaries. To resolve this issue, a small section near the contact point was excluded from the computational domain and the usual mesh type was used for the rest of the cross section, shown in Figure 5.2. For both the circular and the annular channels, all cells had a uniform axial mesh distribution. For the co-current flow simulations, the nozzle for the injection of air was meshed using the automated mesh generation, which created approximately the same number of cells as the stagnant-column cases using a polyhedral mesher, a prism layer mesher and a surface remesher. This mesh type is shown in Figure 5.3. Mesh dependence tests are presented in Chapter 7.

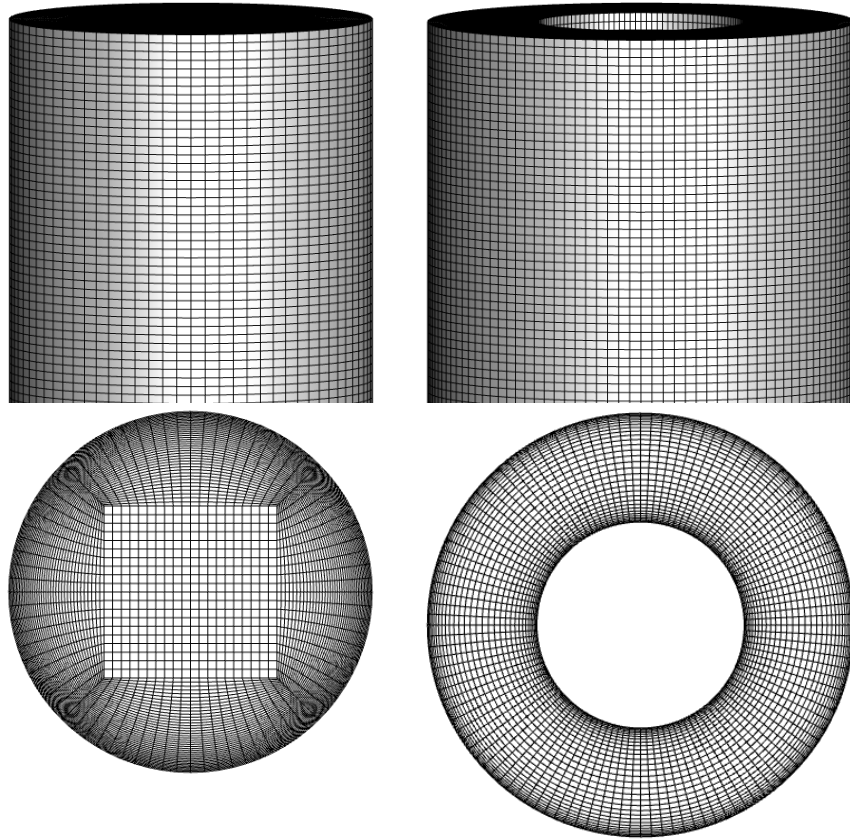


Figure 5.1: Typical cross-sectional and axial mesh images for the circular channel (left) and the concentric annular channel (right).

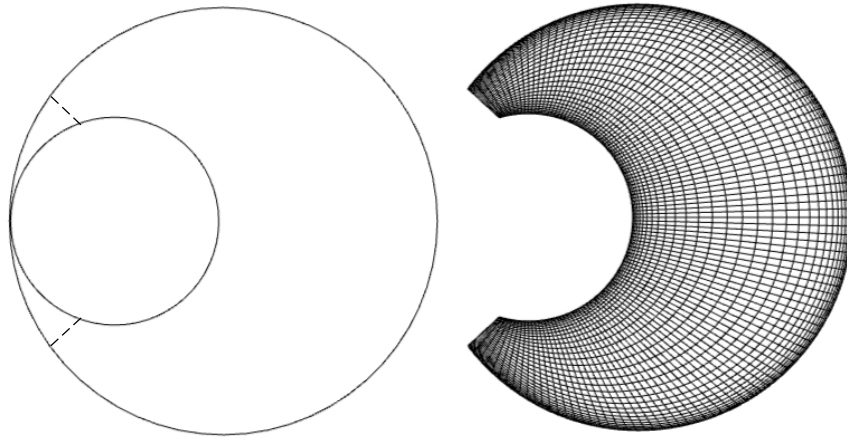


Figure 5.2: Fully eccentric ($e = 1$) channel cross section (left) and mesh fitted to the section outside the narrow gap (right).

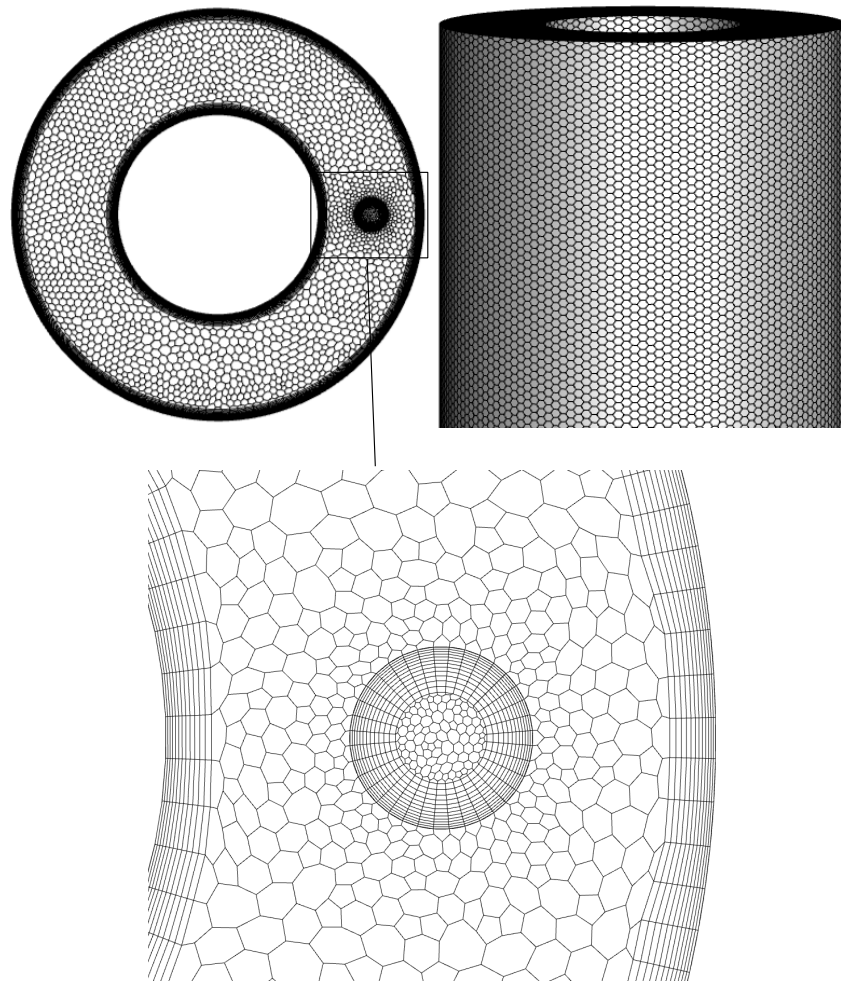


Figure 5.3: Typical cross-sectional and axial mesh images for co-current flow simulations in the concentric annular channel; the detail shows the cross-sectional mesh around and in the air-injection nozzle.

5.5 Boundary and initial conditions

The boundary conditions in the computational studies were set to correspond as much as possible to the experimental conditions. In all studies, the side boundaries were set as solid walls and the top boundary was set as an outlet at atmospheric pressure. For the stagnant column cases, the bottom boundary was also set as a solid wall. For the co-current flow simulations, we used two uniform-velocity inlets, one for air through a tube with a cross-sectional area of 8 mm^2 and a length of 50 mm , and another for water through the rest of the bottom area of the channel, having an area of 813 mm^2 . The use of this short tube as air inlet

ensured control of the superficial velocity of air. The height of the circular channel was set at 3 m, whereas simulations for the stagnant liquid column in the annular channel were run for two heights, 3 m and $20D_{out} = 0.74$ m. Previous studies (Kiran *et al.*, 2020; Liu *et al.*, 2024) demonstrated that an essentially fully-developed state was achieved after an axial distance from the inlet that was $30 - 40D_h$. Other authors (Mitchell & Leonardi, 2020b; Liu *et al.*, 2023) found that their simulations of Taylor bubbles rising with an experimentally specified terminal velocity in stagnant liquid columns with a length of $10 - 11D_{out}$ were in good agreement with experimental results. Co-current flow simulations were only run for the shorter tube because the required solution times for the longer tube were excessive. The simulated geometries and boundary conditions are shown in Figure 5.4. For the co-current flow cases, the initial water velocity in the entire domain was set to a uniform value, which matched the water inlet velocity.

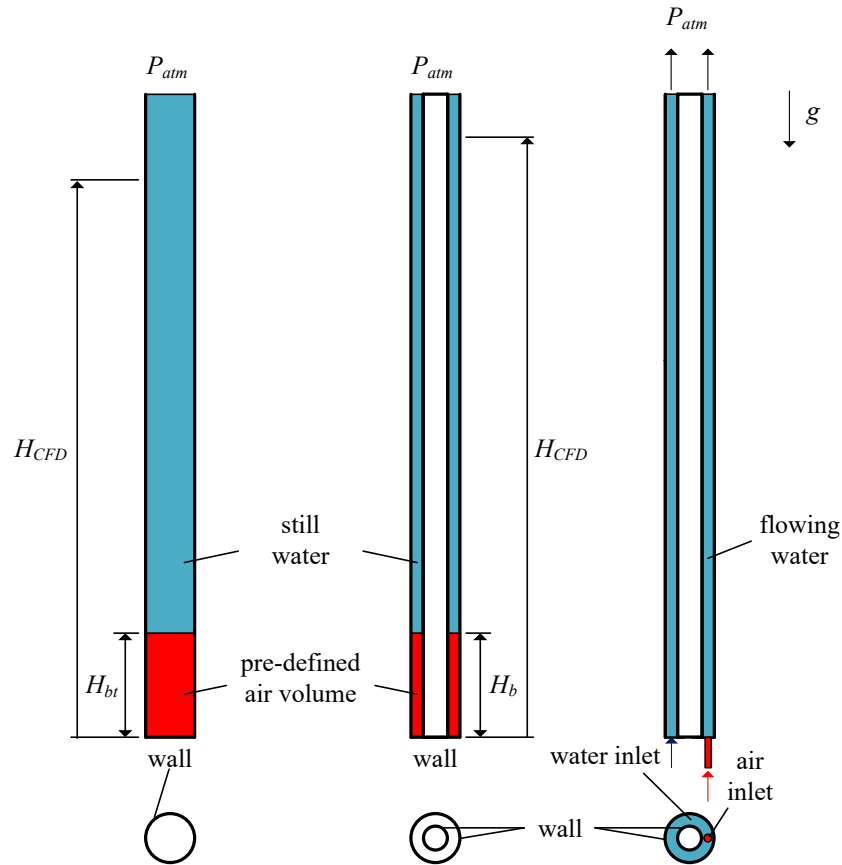


Figure 5.4: Computational domains and boundary conditions for the numerical simulations; stagnant water case in a circular (left) and an annular channel (centre); co-current flow case in an annular channel (right); the drawings are not to scale.

Chapter 6

Experimental results

6.1 Taylor bubbles rising in the circular channel

Each bubble was tracked frame by frame vertically and the entry and exit times of its nose tip in the field of view of the camera were noted. The bubble velocity was found by dividing the vertical height of the field of view, which was $\Delta H = 292$ mm (see Figure 6.1), by the difference between the tip exit and entry times. Measurements were taken at different elevations H of the mid-point of the field of view, relative to the test section inlet. As explained in Section 4.4.3, the bubble length was measured as the distance between the tip and the tail using the line profile tool in the camera software (PCO Camware 4.14, Excelitas PCO GmbH, Germany). Note that the bubble length appeared to change somewhat from one frame to another, as the bubble tail swung in different directions. Moreover, the pressure in the bubble decreased as it rose due to the change in hydrostatic pressure, which resulted in the gradual enlargement of each bubble along the tube. This increase in length was also accompanied by an increase in the bubble volume; the volume increase was in good agreement with predictions of the ideal gas law $PV_b = \text{const.}$, which were roughly 12 – 13% as the bubble rose from $H = 1.5$ m to 2.9 m, for the samples in Table 6.1. These data were obtained from additional recordings that were taken with the Camera A positioned at a longer distance (5 – 6 m) from the test section in order to allow the observation of the axial development of individual bubbles over a long section of the tube. Each bubble was tracked in the images and the axial development of its velocity and length were determined. The selected examples in Table 6.1 show that the bubble velocity did not change significantly for $H \geq 2$ m. Present results also demonstrated that smaller bubbles reached their terminal velocity faster than larger ones.

The average bubble velocity and length for different pressures at an elevation $H = 2.6$ m are listed in Table 6.2. Figure 6.2 shows that the dependence of the bubble rise velocity on the bubble length was extremely weak, in agreement with many studies in

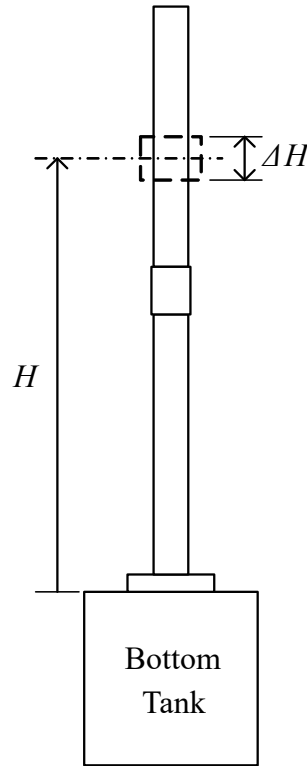


Figure 6.1: A sketch of the circular channel setup; the dashed rectangle shows the camera field of view (not to scale).

Table 6.1: Axial development of the velocity and length of representative bubbles at two different release pressures; $\Delta t_r = 0.5$ s.

P_r [kPa]	H [m]	L_{bt} [mm]	U_{bt} [mm/s]
48	1.5	69.4	199
48	2.0	73.3	196
48	2.5	76.2	196
48	2.9	78.1	195
69	1.5	109.9	187
69	2.0	113.8	201
69	2.5	119.6	199
69	2.9	123.4	196

the literature (Griffith & Wallis, 1961; White & Beardmore, 1962; Viana *et al.*, 2003; Llewellyn *et al.*, 2012; Lizarraga-Garcia *et al.*, 2017; De Azevedo *et al.*, 2020; Liu *et al.*, 2023). Consequently, the Froude number was also very weakly dependent on the bubble length, as shown in Figure 6.3. The presently found Froude number values are in good agreement with predictions of the correlation by Dumitrescu (1943), in which $Fr = 0.351$. Figure 6.4 shows that the bubble length was approximately related to the release pressure by the parabolic expression

$$L_{bt} = -0.039P_r^2 + 7.48P_r - 183 . \quad (6.1)$$

Consecutive images of a representative bubble as it travels through the frame are shown in Figure 6.5, whereas images of bubbles at four different pressures are shown in Figure 6.6.

Table 6.2: Average bubble length and velocity for different release pressures and $\Delta t_r = 0.5$ s at an elevation $H = 2.6$ m; the uncertainty values represent two standard deviations of each data set.

P_r [kPa]	L_{bt} [mm]	U_{bt} [mm/s]
41	54 ± 4	193.2 ± 1.2
43	63 ± 8	193.4 ± 0.6
45	77 ± 16	194.2 ± 2.2
47	85 ± 18	194.9 ± 2.0
48	96 ± 28	195.5 ± 2.4
55	120 ± 44	196.5 ± 2.0
62	127 ± 46	196.0 ± 1.6
69	140 ± 38	196.3 ± 2.0
76	167 ± 36	199.1 ± 4.2
83	174 ± 30	199.7 ± 2.4

The main goal of the circular tube tests was to provide estimates of the gas volume inside each bubble. As mentioned in Section 4.4.6, the circular tube test section was not equipped with a viewing tank, and so the obtained bubble images were distorted excessively by light refraction. Despite considerable effort in optical correction of the bubble shape, the liquid film thickness values that were extracted from the images were significantly larger than values calculated from correlations reported in the literature and listed in Table 2.1. Such

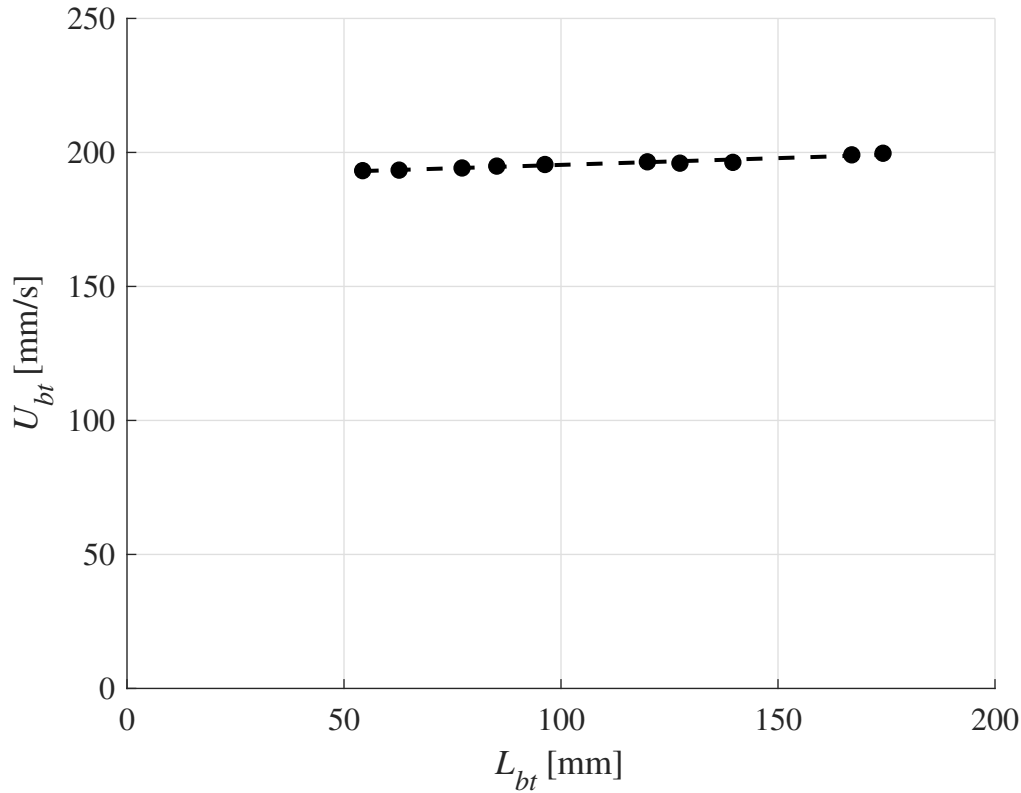


Figure 6.2: Bubble velocity vs. bubble length; the dashed line is a linear least squares fit.

estimates, shown in Table 6.3, were calculated from the corresponding correlations using the fluid properties at 21°C and the non-dimensional groups shown in Table 6.4. Note that Re_b depends on the bubble velocity and the value in Table 6.4 was calculated by using the average velocity at $P_r = 48$ kPa.

As can be seen in Table 6.3, the predictions of different correlations for high inverse viscosity numbers ($N_f > 10000$) vary significantly. There are multiple explanations for this disagreement. Firstly, some correlations were derived for wavy annular films, whereas some others were derived for the liquid film thickness around a Taylor bubble. Although the wavy film thickness models have been shown to be relevant to the film thickness around a Taylor bubble (Llewellyn *et al.*, 2012), their validity is unknown outside of their specified N_f range. Secondly, these correlations were based on different theoretical assumptions. Lastly, the empirical correlation by Llewellyn *et al.* (2012) and De Azevedo *et al.* (2017), which describes the liquid film thickness around a Taylor bubble, was fitted to results

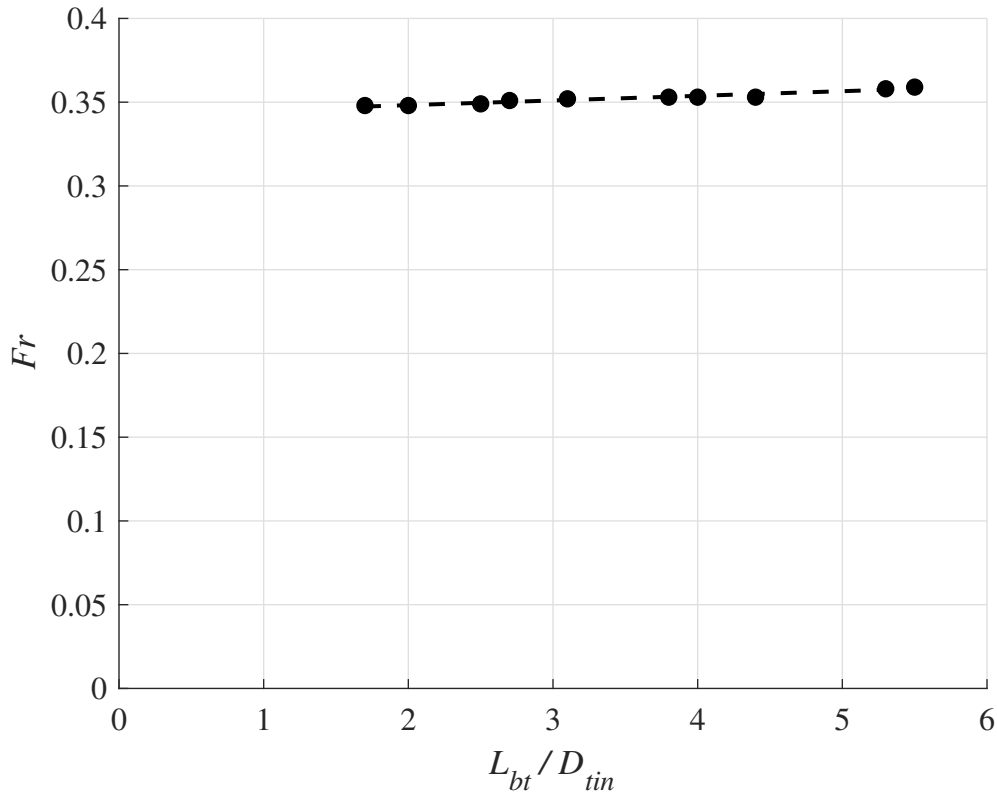


Figure 6.3: Froude number *vs.* non-dimensional bubble length; the dashed line is a linear least squares fit.

obtained for different tube diameters, fluid mixtures and bubble lengths. The tube diameter and the liquid viscosity are parameters that affect strongly the values of Re_b and N_f , and the film thickness is a function of the axial distance from the bubble nose until the bubble reaches a fully developed length, after which the liquid film thickness is taken to be constant. In summary, differences in the predicted values of the film thickness may be to a large extent attributed to differences in the geometrical, material and dynamic conditions of the different studies.

Among all available expressions, we considered as more reliable the one that was based on the pioneering work of Dumitrescu (1943), which was validated by Nogueira *et al.* (2006a). This expression was used to calculate the bubble volumes in the circular channel using average bubble length data obtained in the experiments. As seen in Figure 6.7, there is an approximately linear relationship between the bubble length and the bubble volume.

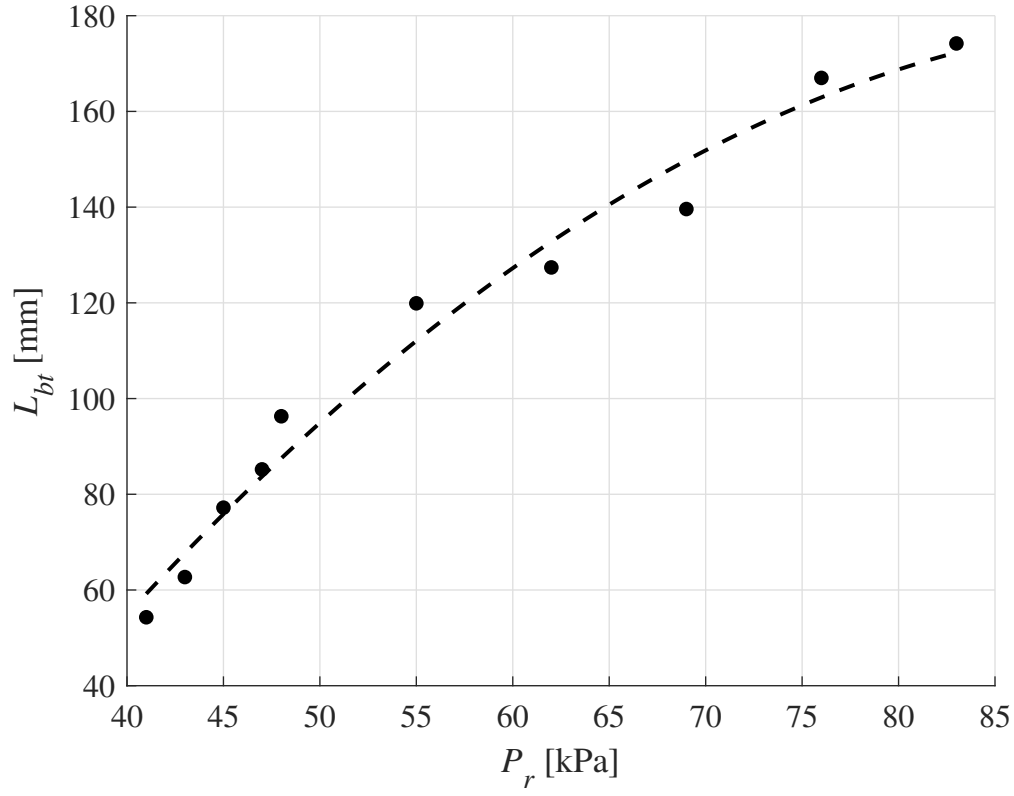


Figure 6.4: Average bubble length vs. bubble release pressure; the dashed line is Equation (6.1).

Table 6.3: Estimates of the liquid film thickness from correlations in the literature for circular tubes.

Authors	λ [mm]
Nusselt (1916)	0.77
Dukler & Bergelin (1952)	1.06
Karapantsios & Karabelas (1995)	1.09
Lel <i>et al.</i> (2005)	0.95
Kang <i>et al.</i> (2010)	1.42
Llewellyn <i>et al.</i> (2012)	1.32
De Azevedo <i>et al.</i> (2017)	0.23

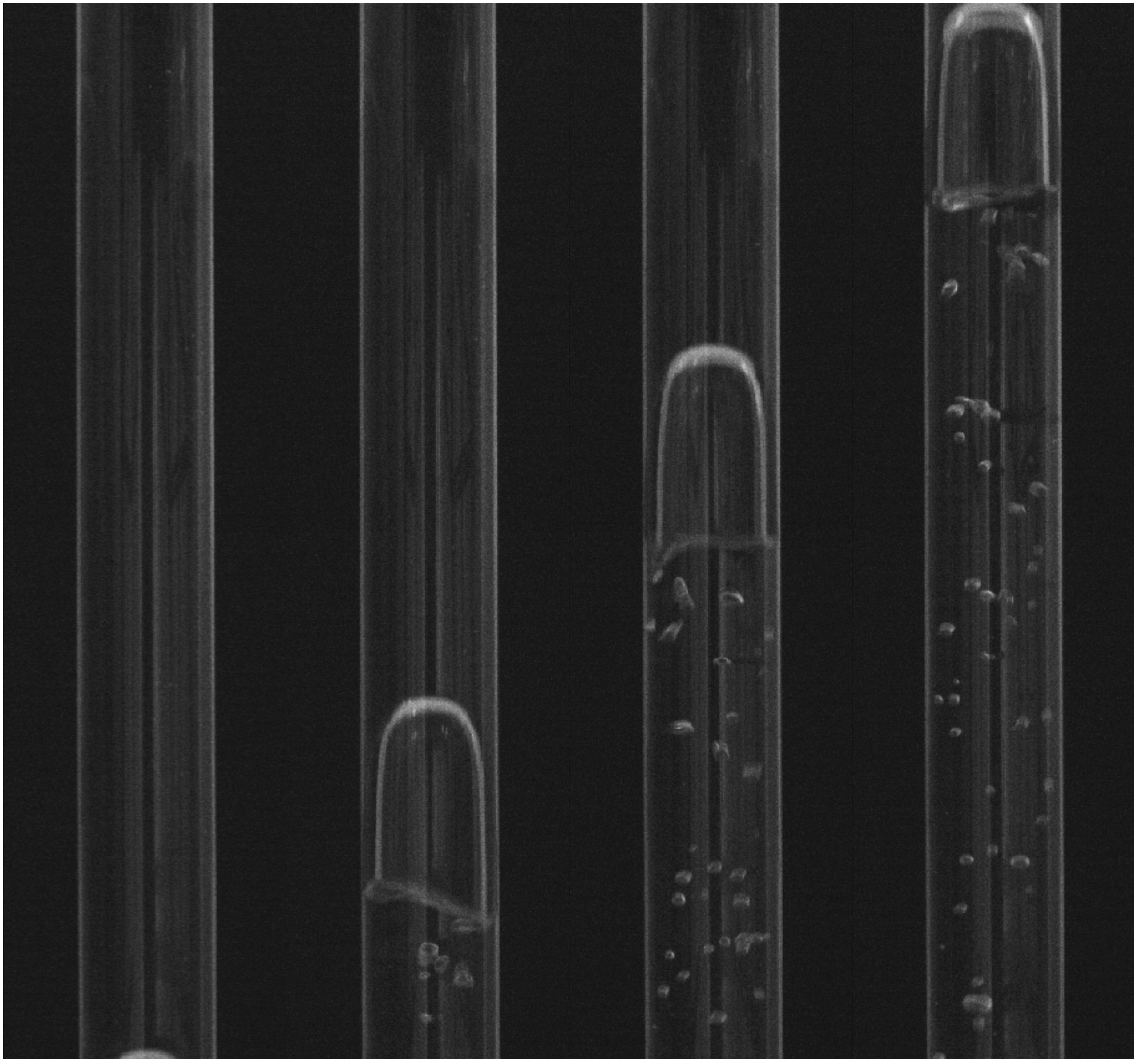


Figure 6.5: Consecutive images of a bubble taken with Camera A at four different times ($t = 0, 0.5, 1$ and 1.5 s, from left to right).

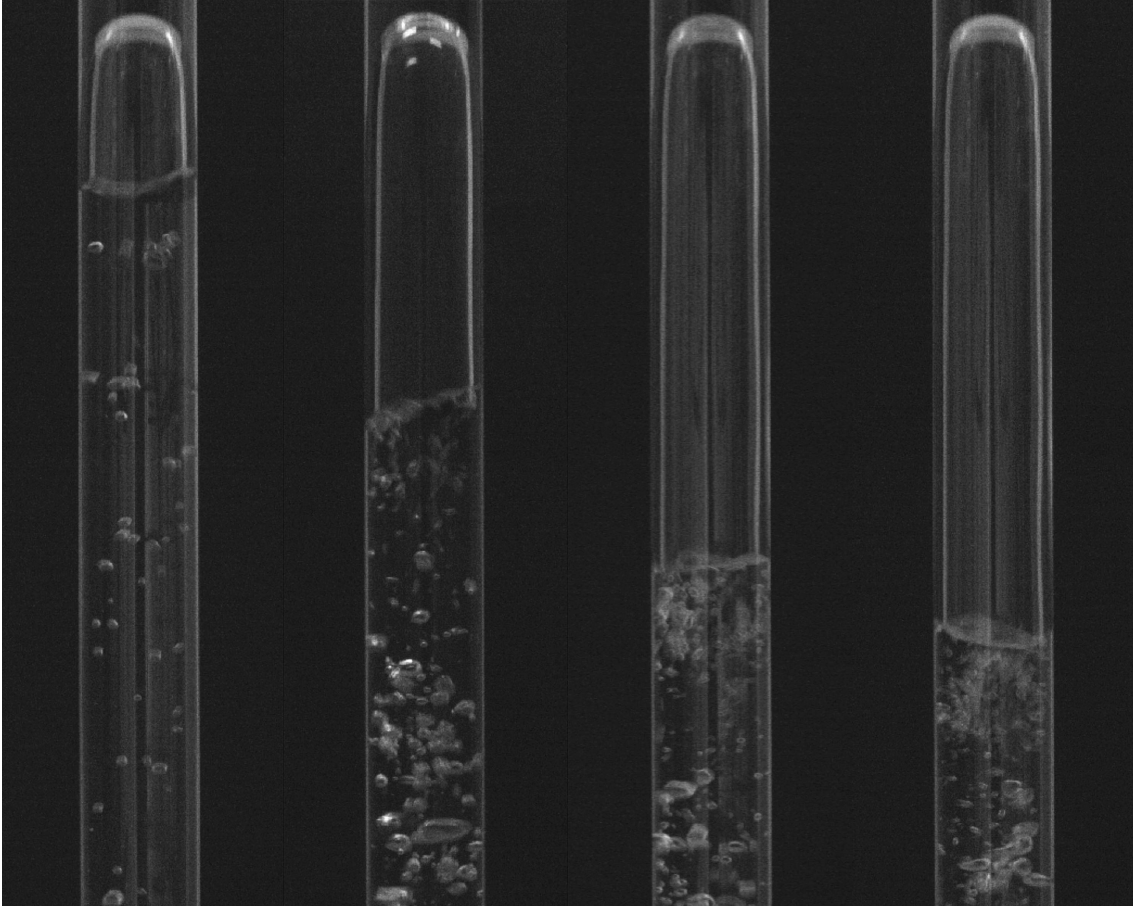


Figure 6.6: Images of bubbles taken with Camera A at four different pressures (41, 55, 69 and 83 kPa from left to right).

Table 6.4: Values of fluid properties at 21°C and values of non-dimensional groups in the present circular channel tests.

ρ_g [kg/m ³]	1.2
ρ_l [kg/m ³]	998
μ_l [kgm/s]	9.8×10^{-4}
σ [N/m]	0.0728
N_f	17,900
EO	133
Mo	2.3×10^{-11}
Re_b	6,300
Fr	0.351

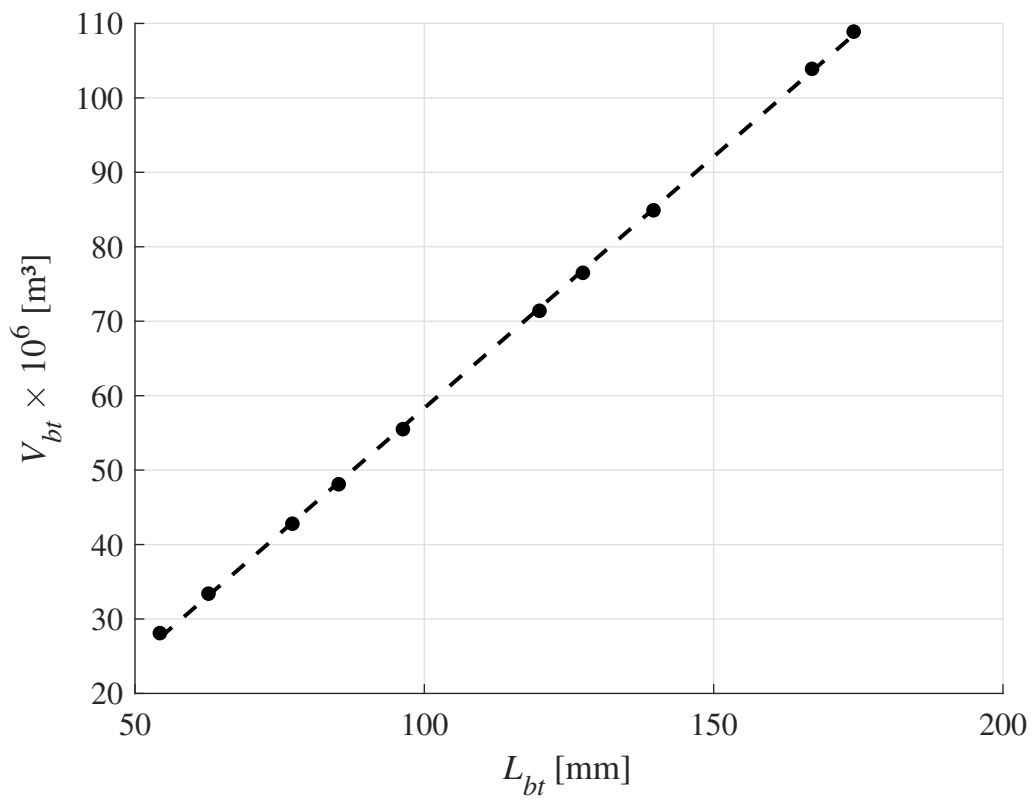


Figure 6.7: Average bubble length vs. average bubble volume calculated from the analytical expression of Dumitrescu (1943).

6.2 Taylor bubbles rising in the annular channel

6.2.1 Experimental conditions

The images for the bubble velocity and length measurements were taken with Camera B. For most tests, the eccentricity of the channel was adjusted by moving the inner tube in the positive δ direction ($e > 0$), see Figure 4.12), which showed the back of the bubble on Camera B and allowed us to view the shape of the entire bubble nose tip. Some additional tests were performed with the eccentricity adjusted in the negative δ direction ($e < 0$), which showed the liquid bridge length ($2y_b$ in Figure 4.18). The bubbles were tracked frame by frame vertically in the upper viewing tank located at $H = 2.8$ m. The axial locations of the bubble nose tip and the corresponding times were recorded at two different vertical locations approximately 0.10 – 0.15 m away from each other. The bubble velocity was found by dividing the vertical distance ΔH between the two locations of the bubble tip by the time difference Δt_b between the two frames. The bubble length was estimated with the line profile tool in the camera software (PCO Camware 4.16, Excelitas PCO GmbH, Germany), which measures the distance between the tip and the tail in pixels, and then those values were converted to physical lengths. Note that the bubble length changed from one frame to another, as its tail swung in different directions, which is explained in Section 4.4.3. An example of two measurement frames is shown in Figure 6.8.

Fluid properties were calculated at 21°C and were assumed to be constant throughout the experiments, as the laboratory temperature was maintained in the range $21^\circ \pm 1^\circ\text{C}$. The non-dimensional groups for identifying the experimental conditions and comparisons with the literature were determined by using the hydraulic diameter $D_h = D_{out} - D_{in}$ instead of D , except for the Froude number, for which we used $D_{out} + D_{in}$ instead of D . The corresponding dimensionless groups were thus defined as follows

$$N_f = \frac{\sqrt{\rho_l(\rho_l - \rho_g)gD_h^3}}{\mu_l}, \quad (6.2)$$

$$Eo = \frac{(\rho_l - \rho_g)gD_h^2}{\sigma}, \quad (6.3)$$

$$Mo = \frac{(\rho_l - \rho_g)g\mu_l^4}{\rho_l^2\sigma^3} \approx \frac{g\mu_l^4}{\rho_l\sigma^3}, \quad (6.4)$$

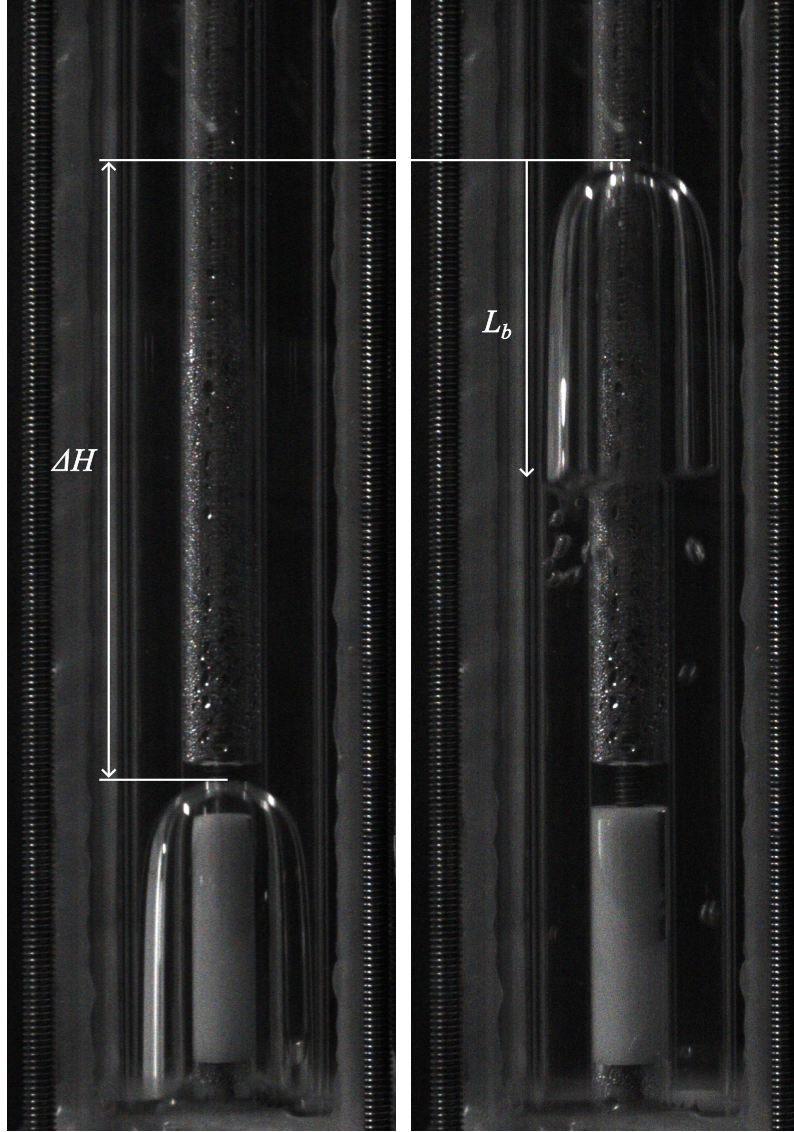


Figure 6.8: Two images of a rising bubble taken with Camera B; $e = 0.5$, $\Delta H = 0.12$ m, $P_r = 55$ kPa, $\Delta t_r = 0.25$ s, $\Delta t_b = 0.52$ s and $L_b = 60.9$ mm.

$$Fr = \frac{U_b}{\sqrt{g(D_{out} + D_{in})(\rho_l - \rho_g)/\rho_l}} \approx \frac{U_b}{\sqrt{g(D_{out} + D_{in})}}. \quad (6.5)$$

The fluid properties and values of the relevant non-dimensional groups for these experiments are presented in Table 6.5.

An objective of the present work is to extend the ranges of parametric experimental studies for eccentric vertical annular channels. First, we examined the effect of eccentricity and bubble release conditions on bubble length, velocity and Froude number. This is the

Table 6.5: Values of fluid properties and non-dimensional groups at 21°C for the annular channel tests.

ρ_g [kg/m ³]	1.2
ρ_l [kg/m ³]	998
μ_l [kgm/s]	9.8×10^{-4}
σ [N/m]	0.0728
N_f	8,300
Eu	49
Mo	2.3×10^{-11}

first experimental study investigating these effects for many values of eccentricity within its entire possible range. Moreover, the reported estimates of the maximum angle of wrap for multiple eccentricities are the first ones in the experimental literature; these values are compared to available numerical results. Another addition to the literature is a discussion of the estimated bubble volumes in concentric and eccentric channels.

6.2.2 Bubble velocity

Before discussing the bubble velocity behaviour far away from the test section inlet, it seems useful to describe the bubble motion near the inlet. Following the opening of the solenoid valve, the released air displaces water and forms a Taylor bubble, which grows upwards. As soon as the valve is closed, the bubble stops moving upwards and even moves downwards very briefly. After this momentary stall, the Taylor bubble rises again and smaller bubbles coalesce with it. Then, the bubble stalls again momentarily and afterwards rises continuously as buoyancy dominates its motion. These observations indicate that the bubble generation process is fairly complex, which implies that the velocity and pressure fields near the test section inlet are strongly time dependent and subjected to oscillations.

Some preliminary recordings were taken with the camera 5 – 6 m away from the test section in order to allow the observation of the axial development of individual bubbles over a long section of the channel. Selected bubbles released at multiple P_r were tracked in the images and the axial development of their velocity is reported in Table 6.6. The reported data show that the bubble velocity was nearly constant for $H \geq 2$ m as well as

insensitive to the release pressure within the experimental range of conditions.

Table 6.6: Axial development of the velocity of representative bubbles in the concentric channel for three release pressures ($\Delta t_r = 0.5$ s).

P_r [kPa]	H [m]	U_b [mm/s]
48	1.5	242
48	2.0	245
48	2.5	244
48	2.9	244
69	1.5	248
69	2.0	244
69	2.5	245
69	2.9	243
103	1.5	241
103	2.0	243
103	2.5	247
103	2.9	249

The average bubble velocities measured at approximately $H = 2.8$ m for all test conditions are listed in Table 6.7. The dependence of the bubble velocity on the eccentricity for different release pressures and a fixed release time $\Delta t_r = 0.25$ s is shown in Figure 6.9, whereas the dependence of the bubble velocity on the eccentricity for different air release times and two release pressures is shown in Figure 6.10. For a fixed eccentricity and a bubble release time of $\Delta t_r = 0.25$ s, the velocity varied by at most 2% within the investigated range of release pressures P_r . For a fixed eccentricity and release pressure, the velocity was essentially insensitive to a change in Δt_r for $P_r = 41$ kPa, and changed up to 3% for $P_r = 83$ kPa. In summary, the velocity was very weakly sensitive to the bubble size, an observation consistent with the literature (Caetano *et al.*, 1992a; Das *et al.*, 1998; Liu *et al.*, 2022).

Table 6.7: Average bubble velocity for different eccentricities and different combinations of release pressure and release time; the uncertainty values denote the standard deviation of the data set multiplied by the student t -score for 20 samples (2.093).

e	U_b [mm/s]					
	$P_r = 41$ kPa	$P_r = 41$ kPa	$P_r = 55$ kPa	$P_r = 69$ kPa	$P_r = 83$ kPa	$P_r = 83$ kPa
	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.5$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.5$ s
0	230.0 $\pm$ 0.8	230.2 $\pm$ 1.0	230.2 $\pm$ 0.6	229.7 $\pm$ 0.8	229.9 $\pm$ 1.7	235.2 $\pm$ 1.7
0.1	234.9 $\pm$ 1.6	234.9 $\pm$ 1.3	235.0 $\pm$ 1.2	235.1 $\pm$ 2.1	237.0 $\pm$ 3.4	240.6 $\pm$ 3.4
0.2	236.2 $\pm$ 1.0	236.0 $\pm$ 0.7	236.0 $\pm$ 1.0	236.7 $\pm$ 1.9	237.1 $\pm$ 2.2	242.4 $\pm$ 2.3
0.3	234.7 $\pm$ 1.2	234.7 $\pm$ 0.8	234.9 $\pm$ 3.3	235.6 $\pm$ 2.2	237.3 $\pm$ 2.9	239.8 $\pm$ 5.4
0.4	232.2 $\pm$ 0.8	233.0 $\pm$ 1.3	233.3 $\pm$ 1.2	232.9 $\pm$ 1.2	233.1 $\pm$ 2.0	241.0 $\pm$ 2.4
0.5	228.8 $\pm$ 0.9	229.8 $\pm$ 1.0	230.8 $\pm$ 1.5	231.3 $\pm$ 2.4	231.0 $\pm$ 1.8	237.6 $\pm$ 1.5
0.6	224.5 $\pm$ 0.9	226.0 $\pm$ 1.1	227.2 $\pm$ 1.4	228.0 $\pm$ 1.3	227.8 $\pm$ 1.0	233.1 $\pm$ 5.6
0.7	220.4 $\pm$ 1.1	221.1 $\pm$ 1.5	222.0 $\pm$ 1.5	224.0 $\pm$ 1.4	224.7 $\pm$ 1.0	226.9 $\pm$ 3.3
0.8	216.1 $\pm$ 0.8	216.5 $\pm$ 0.9	217.4 $\pm$ 1.2	219.3 $\pm$ 2.2	220.2 $\pm$ 2.4	221.3 $\pm$ 3.6
0.9	213.3 $\pm$ 0.8	214.0 $\pm$ 0.9	214.0 $\pm$ 1.3	214.7 $\pm$ 1.7	215.7 $\pm$ 2.1	219.9 $\pm$ 1.9
1	209.8 $\pm$ 2.0	210.5 $\pm$ 1.3	210.2 $\pm$ 1.6	211.7 $\pm$ 1.6	212.0 $\pm$ 2.3	216.8 $\pm$ 1.8

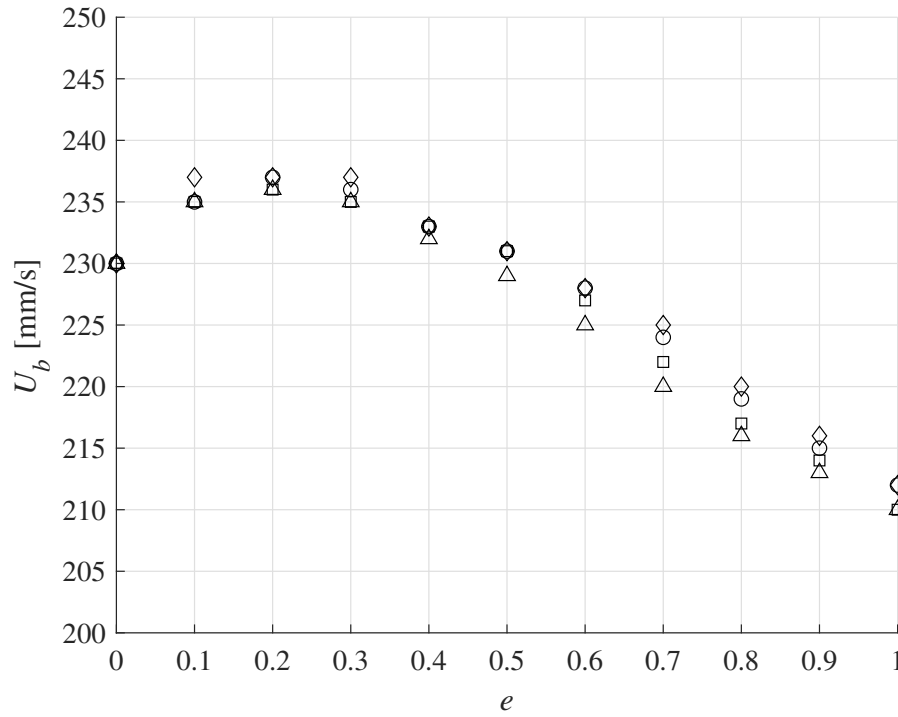


Figure 6.9: Average bubble velocity vs. eccentricity for $\Delta t_r = 0.25$ s and different release pressures; $P_r = 41$ kPa (Δ), 55 kPa ($\square$), 69 kPa ($\circ$) and 83 kPa ($\diamond$).

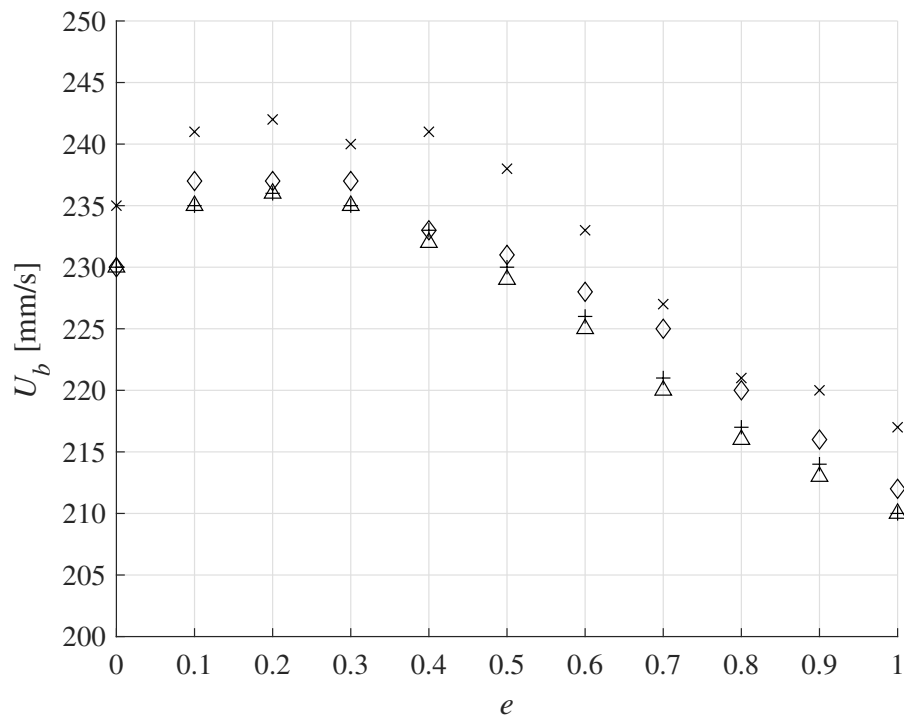


Figure 6.10: Average bubble velocity vs. eccentricity for two different release times and two different release pressures; $\Delta t_r = 0.25$ s, $P_r = 41$ kPa (Δ); $\Delta t_r = 0.5$ s, $P_r = 41$ kPa (+); $\Delta t_r = 0.25$ s, $P_r = 83$ kPa ($\diamond$); and $\Delta t_r = 0.5$ s, $P_r = 83$ kPa ($\times$).

6.2.3 Froude number

The geometrical and dynamic conditions in the present and previous air-water experiments and computations of flows in concentric annular channels and the corresponding Froude numbers, calculated from Eq. (6.5) using the corresponding measured average bubble velocity, are listed in Table 6.8. When comparing present and previous measurements, it is important to note that, in general, the present bubbles were significantly shorter than those in the literature. For example, the average length for the longest bubbles in the present tests was 0.15 m ($8D_h$), whereas Kelessidis (1986) reported an average length of 0.42 m ($22D_h$) and Caetano (1986) reported an average length of 0.30 m ($16D_h$). The present and previous results have demonstrated that the bubble rise velocity increases, although very mildly with increasing bubble length. As the Froude number is proportional to the bubble velocity, one may attribute the small differences between the values of Fr , at least to some extent, to differences in bubble lengths. Table 6.8 shows that the reported average Froude number for our longest bubbles in concentric channels lies within the range between 5% from the upper bound of the range of values in the experimental literature and 1% from the lower bound. The Froude number in the numerical simulations of Mitchell *et al.* (2021) is 8% lower than the present value, whereas the value in the numerical simulations of Liu *et al.* (2024) is 44% lower, which is well outside the tolerable range. Considering that N_f in the latter simulations is three orders of magnitude lower than the present one, we may assess that a quantitative comparison with these simulations would be inappropriate and so any comparison should be at most qualitative.

Some previous authors have suggested correlations for the Froude number. Das *et al.* (1998) suggested a general theoretical correlation in terms of the inner and outer channel diameters and validated it against their experimental results in concentric annular channels and other experimental results in the literature. They reported values of the Froude number that were based on a velocity model instead of the experimental velocity values, which introduced differences up to 6%. Mitchell & Leonardi (2020a) suggested a correlation for the Froude number in concentric channels that was based on a correlation by Viana *et al.* (2003) for flows of various fluids in circular tubes. They reported that this correlation predicted values from the literature with an average error of 5%. Later on, Liu *et al.* (2023) modified the model of Mitchell & Leonardi (2020a) and suggested a new correlation (Eq.

Table 6.8: Conditions and Froude number values in air-water flows in concentric annular channels: present experiments, previous air-water experiments (E), numerical simulations (N) and predictions of correlations (C) for the present conditions; percentages denote differences from the present experimental value of 0.320 for the case with the longest bubbles.

Source	D_{out} [mm]	$\frac{D_{out}}{D_{in}}$	EO	N_f	Fr
Present experiments	37.0	2.06	49	8,300	
$P_r = 41$ kPa, $\Delta t_r = 0.25$ s					0.313
$P_r = 55$ kPa, $\Delta t_r = 0.25$ s					0.313
$P_r = 69$ kPa, $\Delta t_r = 0.25$ s					0.313
$P_r = 83$ kPa, $\Delta t_r = 0.25$ s					0.313
$P_r = 41$ kPa, $\Delta t_r = 0.5$ s					0.313
$P_r = 83$ kPa, $\Delta t_r = 0.5$ s					0.320
Rader <i>et al.</i> (1975) (E)	50.8	1.60	49	8,300	0.325 (+2%)
Sadatom <i>et al.</i> (1982) (E)	30.0	2.00	31	5,700	0.318 (−1%)
Kelessidis & Dukler (1989) (E)	76.2	1.50	87	5,100	0.331 (+3%)
Caetano <i>et al.</i> (1992a) (E)	76.2	1.20	155	19,600	0.328 (+3%)
Das <i>et al.</i> (1998) (E)	50.8	2.00	87	12,600	0.334 (+4%)
	38.1	3.00	87	12,600	0.337 (+5%)
Agarwal <i>et al.</i> (2007) (E)	50.8	2.00	87	12,600	0.335 (+5%)
Rohilla & Das (2019) (E)	20.0	2.00	13	3,100	0.316 (−1%)
Mitchell <i>et al.</i> (2021) (N)	19.2	2.00	12	3,000	0.294 (−8%)
Liu <i>et al.</i> (2024) (N)	—	2.00	140	40	0.180 (−44%)
Das <i>et al.</i> (1998) (C)	—	—	—	—	0.323 (+1%)
Mitchell & Leonardi (2020a) (C)	—	—	—	—	0.330 (+3%)
Liu <i>et al.</i> (2023) (C)	—	—	—	—	0.327 (+2%)

(24) in their article), which had an average error of 4% in predicting the cases in the literature. The differences of the predictions of these three correlations from our own value of 0.320 for the concentric case are, respectively, 1%, 3% and 2% (see Table 6.8).

The dependence of the Froude number on the release conditions and the eccentricity in the present experiments is shown in Figure 6.11. The Froude number in the present results for the longest bubbles was 0.320 for $e = 0$, reached a maximum of 0.329 for $e \approx 0.2$, beyond which it decreased monotonically with increasing e to 0.295 for $e = 1$. Similar behaviours can be seen in Figure 6.11 for all the other release conditions. The same figure also shows that there are slight differences between the Froude numbers of the shortest and longest bubbles for all eccentricities. For example, the Froude number for the

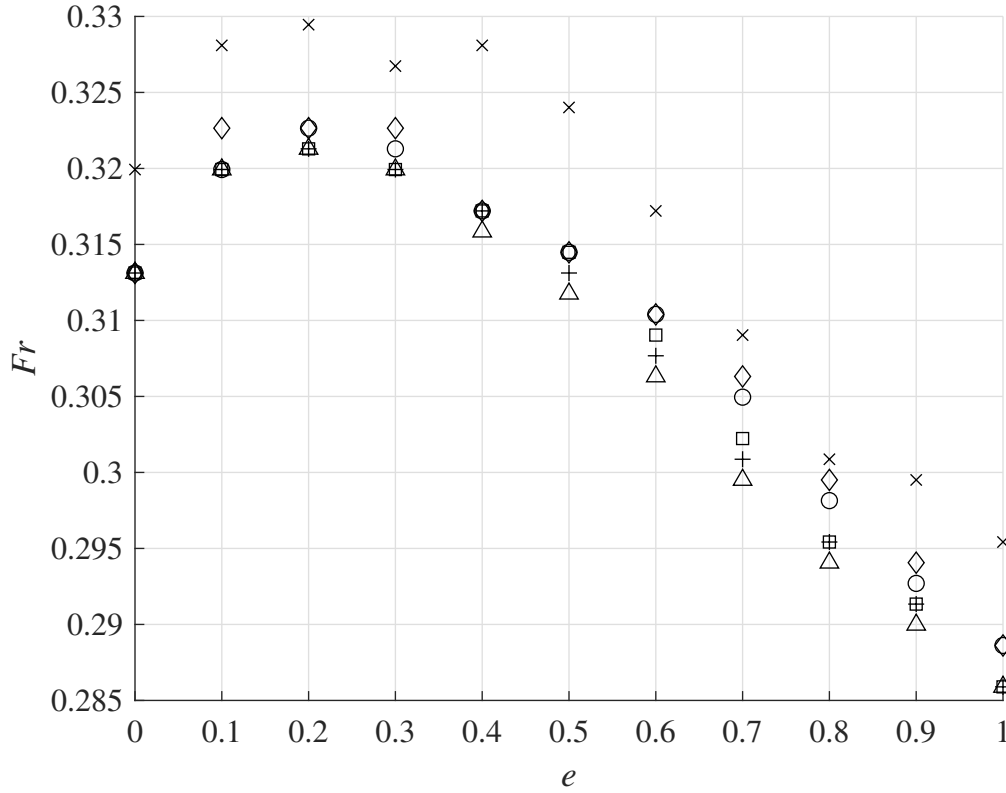


Figure 6.11: Froude number vs. eccentricity in the present experiments: $\Delta t_r = 0.25$ s, $P_r = 41$ kPa (Δ); $\Delta t_r = 0.25$ s, $P_r = 55$ kPa ($\square$); $\Delta t_r = 0.25$ s, $P_r = 69$ kPa ($\circ$); $\Delta t_r = 0.25$ s, $P_r = 83$ kPa ($\diamond$); $\Delta t_r = 0.5$ s, $P_r = 41$ kPa ($+$) and $\Delta t_r = 0.5$ s, $P_r = 83$ kPa ($\times$).

shortest bubbles ($\Delta t_r = 0.25$ s, $P_r = 41$ kPa) is approximately 2 – 4% smaller than that for the longest bubbles ($\Delta t_r = 0.5$ s, $P_r = 83$ kPa). As Figure 6.12 shows, the present Froude numbers for two sets of release conditions were not very different from the sparse available experimental results in the literature (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Rohilla & Das, 2019). In all previous experiments, Fr decreased monotonically with increasing eccentricity at very mild rates, but none of these experiments had sufficient resolution of the eccentricity effect to demonstrate the presence or absence of a maximum at some intermediate e . Therefore, the present experiments are the first ones to show conclusively that Fr has a maximum. On the other hand, the presence of a maximum at some small e is evident in numerical results (Mitchell *et al.*, 2021; Liu *et al.*, 2024), also shown in Figure 6.12. To explain this observation, we note that eccentricity creates

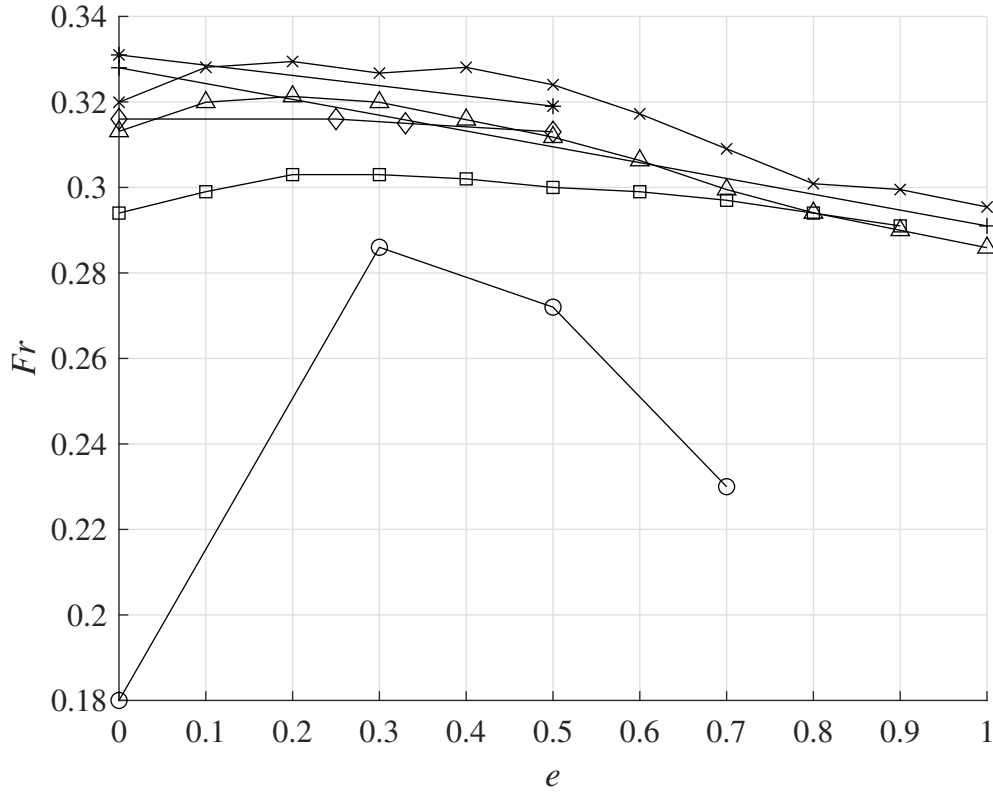


Figure 6.12: Froude number vs. eccentricity; present results for $\Delta t_r = 0.25$ s, $P_r = 41$ kPa (Δ) and $\Delta t_r = 0.5$ s, $P_r = 83$ kPa ($\times$); experiments by Kelessidis & Dukler (1989) (*), Caetano *et al.* (1992a) (+) and Rohilla & Das (2019) ($\diamond$); numerical simulations by Mitchell *et al.* (2021) ($\square$) and Liu *et al.* (2024) ($\circ$); data points are connected by lines for clarity.

geometric asymmetry in the annular channel, which affects the shape of the rising bubbles. When $e = 0.2 - 0.3$, the bubble nose is more streamlined than for lower and higher e , which is accompanied by somewhat lower drag and a somewhat higher bubble rising speed.

6.2.4 Bubble length

As shown in Table 6.9, the bubble length increased measurably with increasing elevation. This is attributed mainly to the hydrostatic pressure decrease as the bubble moved towards the free surface of the water, which reduced the pressure inside the bubble, thus reducing the air density and increasing the bubble volume and length. Using the ideal gas law, and neglecting secondary pressure change effects, we estimated that the bubble volume would

increase by roughly 13% as the bubble rose from $H = 1.5$ m to 2.9 m. Further assuming that the bubble was stretched uniformly as it rose, one may infer that the relative change of bubble length is approximately equal to the relative change of bubble volume. Due to the low resolution of the recordings of long test section segments, estimates of the bubble shape and volume from these images were not deemed to be sufficiently accurate, but the measurements of bubble length were fairly reliable. The rate of change of the measured length for different release conditions as the bubble rose from $H = 1.5$ m to 2.9 m was 9 – 12% or 6 – 9%/m. Such values are deemed to be compatible with the hydrostatic pressure change effect.

The measured bubble lengths, averaged over 20 bubbles for each set of test conditions at $H = 2.8$ m, are listed in Table 6.10. Figure 6.13 shows that the average bubble length in the concentric channel for $\Delta t_r = 0.25$ s could be fitted well by the following parabolic function of P_r , which had a monotonically decreasing slope as P_r was increased

$$L_b = -0.024P_r^2 + 4.40P_r - 109 . \quad (6.6)$$

The rate of increase of L_b was 2.4 mm/kPa for $P_r = 41$ kPa, 1.8 mm/kPa for $P_r = 55$ kPa, 1.1 mm/kPa for $P_r = 69$ kPa, and 0.4 mm/kPa for $P_r = 83$ kPa.

The bubble lengths for a fixed $\Delta t_r = 0.25$ s and different release pressures are plotted vs. eccentricity in Figure 6.14, whereas the bubble lengths for different air release times at fixed release pressures are plotted in Figure 6.15. It is noted that the value $P_r = 41$ kPa was slightly larger than the minimum pressure at the regulator that permitted the release of air from the nozzle against the local hydrostatic pressure, which was approximately 31 kPa, and considering the air pressure drop along the 4 m tube from the regulator to the nozzle exit. The table and the plots show that the bubble length is fairly insensitive to the eccentricity value but very sensitive to the release pressure and release time and that the effects of release conditions are non-linear. For example, a doubling of Δt_r resulted in a mere 45% increase of L_b for $P_r = 41$ kPa and a 71% increase of L_b for $P_r = 83$ kPa. Example images of bubbles at different P_r and the same Δt_r are shown in Figure 6.16, while images for different combinations of Δt_r and P_r are shown in Figure 6.17. These images show that an increase in bubble length makes the bubbles wrap around the inner tube more.

Table 6.9: Axial development of the length of representative bubbles in the concentric channel for three release pressures and $\Delta t_r = 0.5$ s.

P_r [kPa]	H [m]	L_b [mm]
48	1.5	92.3
48	2.0	95.1
48	2.5	99.3
48	2.9	103.5
69	1.5	111.9
69	2.0	113.3
69	2.5	116.0
69	2.9	121.6
103	1.5	190.2
103	2.0	198.5
103	2.5	202.7
103	2.9	206.9

Table 6.10: Average bubble length for different eccentricities and different combinations of release pressure and release time; the uncertainty values denote the standard deviation of the data set multiplied by the student t -score for 20 samples (2.093).

e	L_b [mm]					
	$P_r = 41$ kPa	$P_r = 41$ kPa	$P_r = 55$ kPa	$P_r = 69$ kPa	$P_r = 83$ kPa	$P_r = 83$ kPa
	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.5$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.25$ s	$\Delta t_r = 0.5$ s
0	31.2 $\pm$ 4.4	48.6 $\pm$ 3.1	60.9 $\pm$ 9.2	80.8 $\pm$ 22.2	91.8 $\pm$ 17.2	150.9 $\pm$ 21.3
0.1	34.3 $\pm$ 3.6	45.8 $\pm$ 3.2	61.6 $\pm$ 10.0	80.2 $\pm$ 20.6	94.7 $\pm$ 30.1	160.8 $\pm$ 24.5
0.2	35.8 $\pm$ 3.7	48.7 $\pm$ 2.4	63.5 $\pm$ 10.3	80.1 $\pm$ 19.1	94.8 $\pm$ 24.6	161.7 $\pm$ 23.4
0.3	35.2 $\pm$ 4.7	48.7 $\pm$ 8.4	64.1 $\pm$ 8.4	79.2 $\pm$ 19.5	92.2 $\pm$ 18.0	157.1 $\pm$ 42.7
0.4	33.6 $\pm$ 4.1	48.6 $\pm$ 3.7	63.5 $\pm$ 8.5	81.3 $\pm$ 22.1	96.7 $\pm$ 25.2	160.8 $\pm$ 13.5
0.5	33.6 $\pm$ 4.9	46.6 $\pm$ 2.6	63.3 $\pm$ 9.1	80.5 $\pm$ 18.1	89.1 $\pm$ 20.6	157.3 $\pm$ 18.5
0.6	31.0 $\pm$ 2.7	48.2 $\pm$ 7.9	60.5 $\pm$ 10.0	82.3 $\pm$ 17.0	91.4 $\pm$ 21.2	162.5 $\pm$ 59.3
0.7	33.2 $\pm$ 5.6	47.8 $\pm$ 2.0	60.3 $\pm$ 8.3	78.1 $\pm$ 17.5	95.7 $\pm$ 30.2	161.6 $\pm$ 17.7
0.8	33.0 $\pm$ 4.3	48.6 $\pm$ 3.6	60.1 $\pm$ 11.8	82.1 $\pm$ 23.2	93.7 $\pm$ 28.4	164.1 $\pm$ 32.0
0.9	33.5 $\pm$ 5.4	47.7 $\pm$ 3.0	61.6 $\pm$ 13.3	76.8 $\pm$ 24.2	93.2 $\pm$ 26.4	163.4 $\pm$ 29.7
1	30.9 $\pm$ 4.6	48.4 $\pm$ 4.2	61.1 $\pm$ 9.0	82.2 $\pm$ 22.1	95.9 $\pm$ 27.5	159.2 $\pm$ 21.9

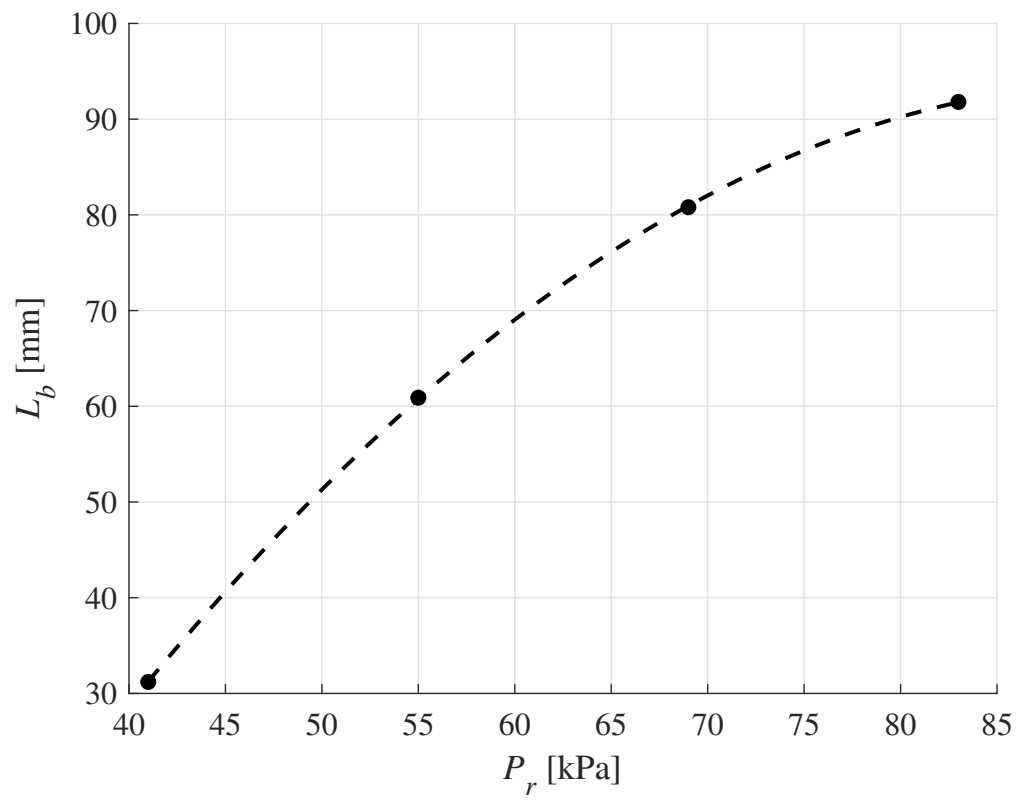


Figure 6.13: Average bubble length vs. bubble release pressure for the concentric channel for $\Delta t_r = 0.25$ s; the dashed line is Equation (6.6).

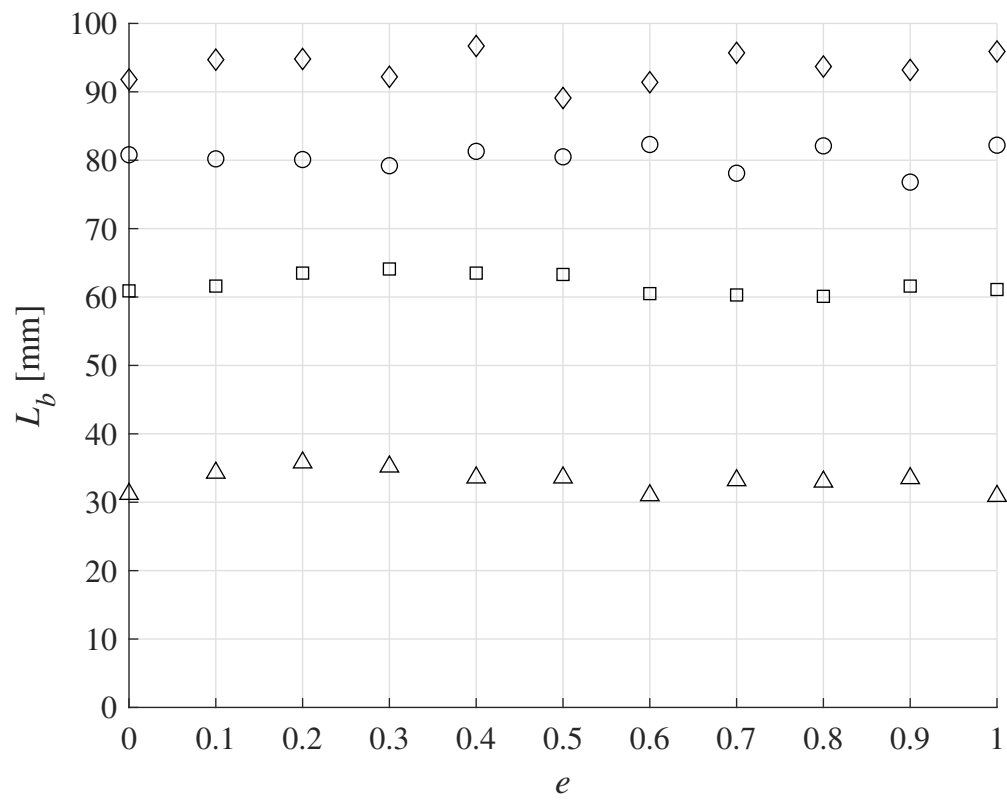


Figure 6.14: Average bubble length vs. eccentricity for $\Delta t_r = 0.25$ s and different release pressures; $P_r = 41$ kPa ($\triangle$), 55 kPa ($\square$), 69 kPa ($\circ$) and 83 kPa ($\diamond$).

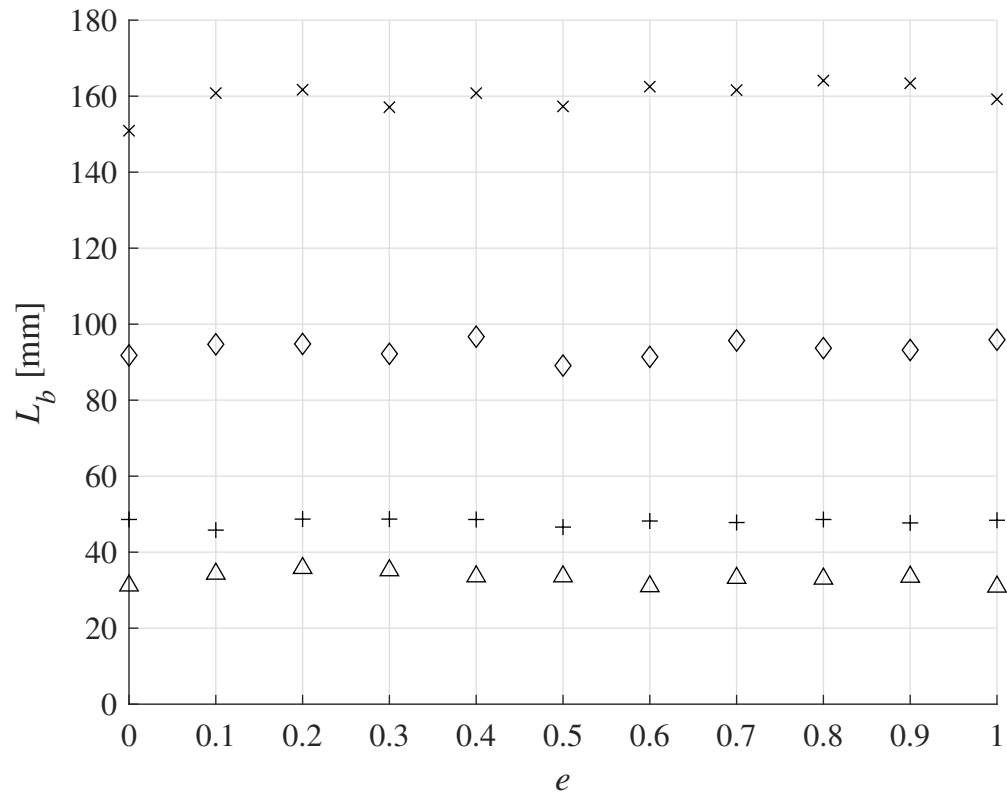


Figure 6.15: Average bubble length vs. eccentricity for two different release times and two different release pressures; $\Delta t_r = 0.25$ s, $P_r = 41$ kPa (Δ); $\Delta t_r = 0.5$ s, $P_r = 41$ kPa (+); $\Delta t_r = 0.25$ s, $P_r = 83$ kPa ($\Diamond$); and $\Delta t_r = 0.5$ s, $P_r = 83$ kPa ($\times$).

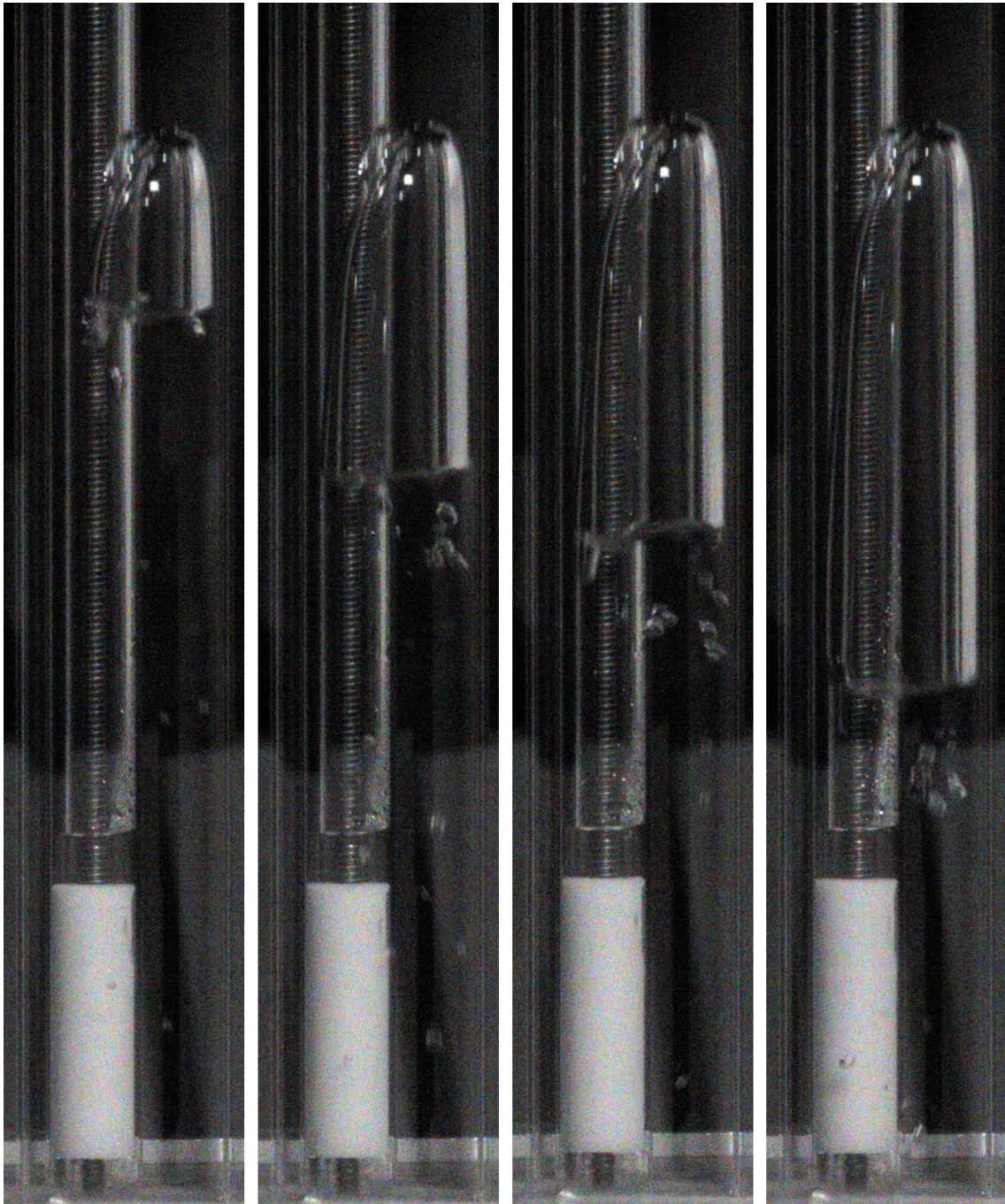


Figure 6.16: Bubble images recorded with Camera A for $e = 0.7$, $\Delta t_r = 0.25$ s and four different P_r values; $P_r = 41$ kPa, 55 kPa, 69 kPa and 83 kPa (left to right).

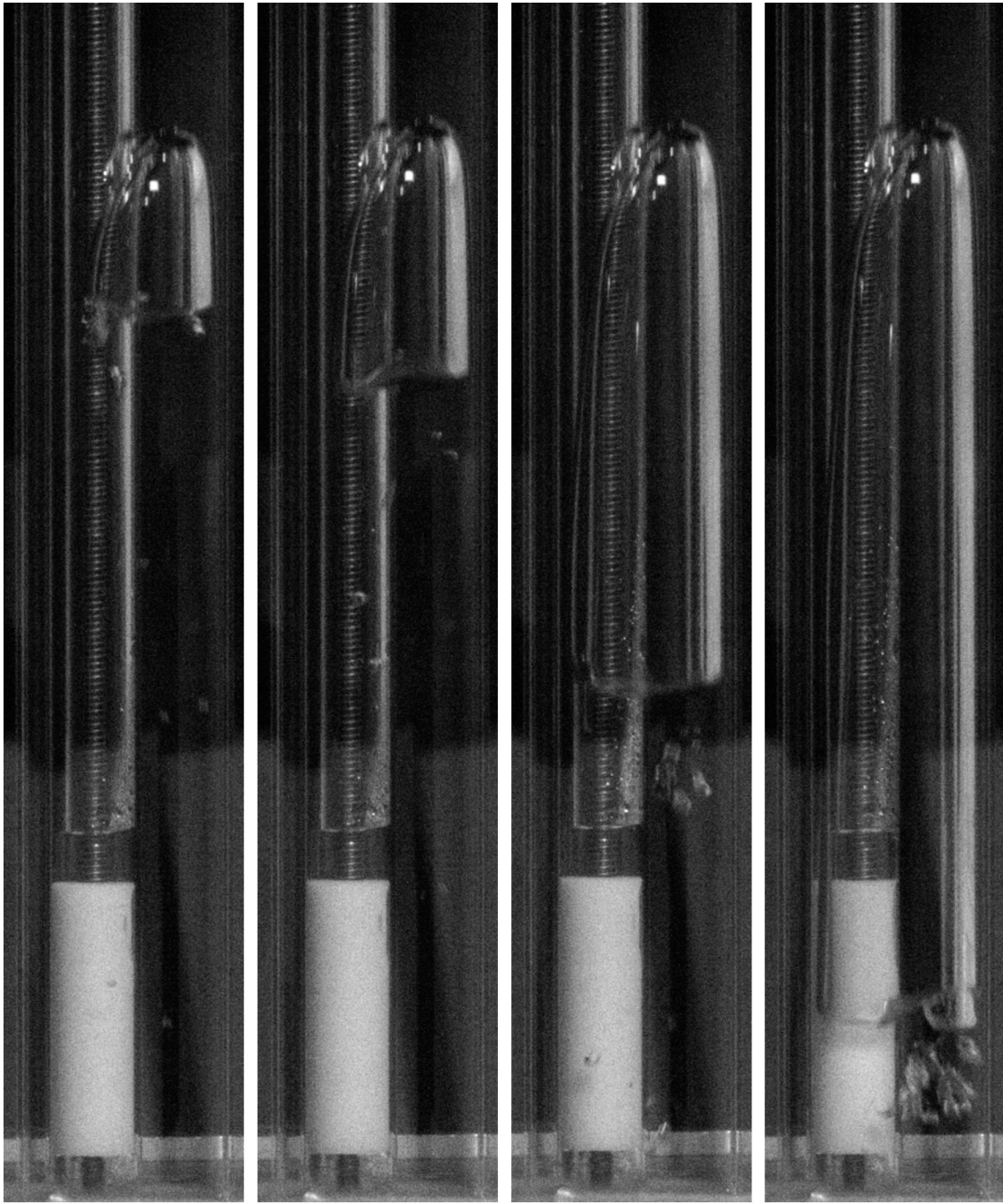


Figure 6.17: Bubble images recorded with Camera A for $e = 0.7$ and four different combinations of release pressure and release time; from left to right: $\Delta t_r = 0.25$ s, $P_r = 41$ kPa; $\Delta t_r = 0.5$ s, $P_r = 41$ kPa; $\Delta t_r = 0.25$ s, $P_r = 83$ kPa; and $\Delta t_r = 0.5$ s, $P_r = 83$ kPa.

6.2.5 Bubble wrapping

The bubble shape changed significantly as the eccentricity was increased from 0 to 1, as shown in the images in Figure 6.18, which were recorded with Camera A for $P_r = 55$ kPa and $\Delta t_r = 0.25$ s. With an increase in eccentricity, the bubble got wrapped around a decreasing sector of the inner tube.

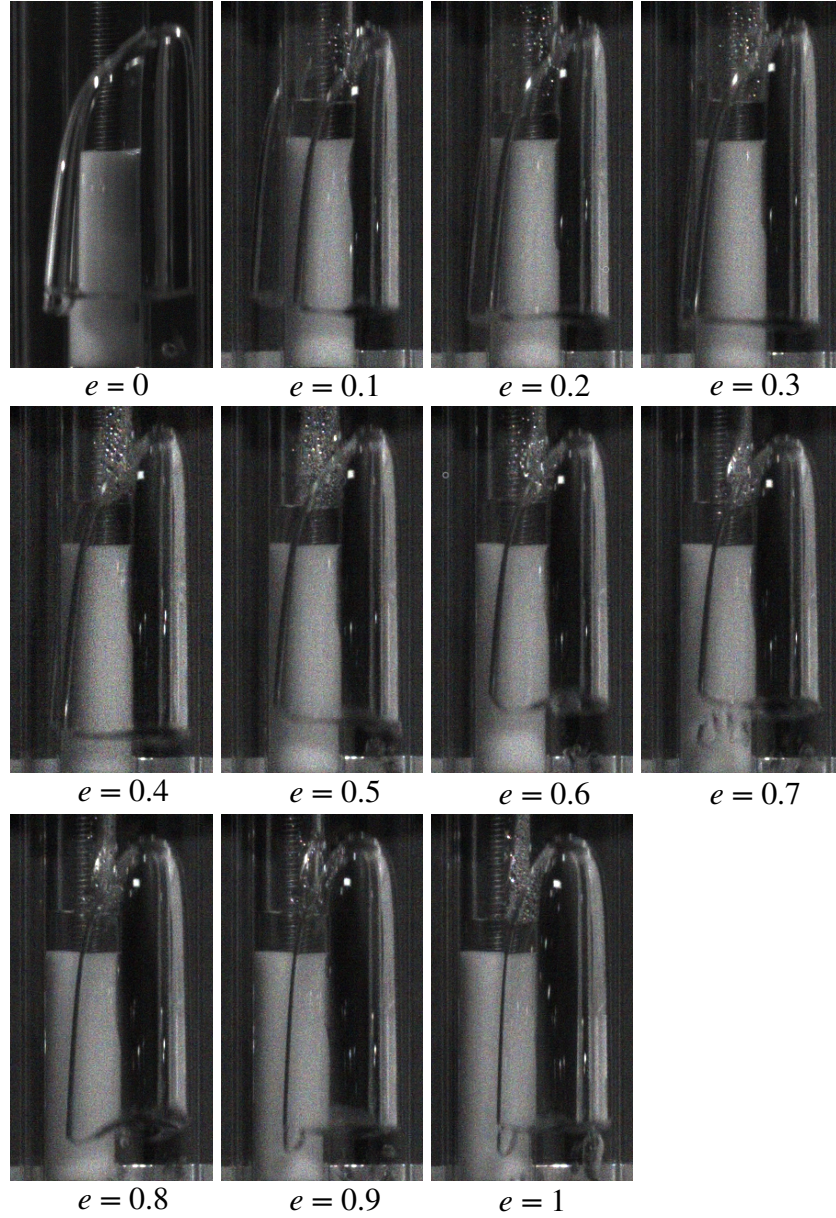


Figure 6.18: Bubble images recorded with Camera A for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s and different eccentricities.

The coordinates x_b and y_b (when possible) of the bubble edge at the tail were determined from images collected by both cameras with the eccentricity adjusted in the negative δ direction ($e = -0.2, -0.4, -0.6, -0.8, -1$). Two bubble images in which the eccentricity of the inner tube is adjusted in the positive and negative directions are shown in Figure 6.19. Using these coordinates and values of liquid film thicknesses obtained from our CFD simulations, as discussed in Section 4.4.4, we constructed bubble contours on the transverse ($x-y$) plane, from which we found the maximum angle of wrap θ_{max} . Representative cases are shown in Figure 6.20. Each shown contour corresponds to a θ_{max} that is the average of values for five bubbles at $P_r = 55$ kPa. The precision limit (random uncertainty) of θ_{max} , assuming that it has a Student t distribution, was roughly 10° . Table 6.11 lists the present experimental data, where each value is the average of five representative bubbles except for the bubbles at $P_r = 69$ kPa and eccentric conditions, which only had a single sample. The bubble length measurements are also included in this table as θ_{max} depends on this length. The present values at $P_r = 55$ kPa and $P_r = 69$ kPa are plotted in Figure 6.21, along with values from the experiments of Kelessidis & Dukler (1990) for a bubble length $L_b \approx 6D_h$ and the numerical simulations of Mitchell *et al.* (2021) and Liu *et al.* (2024). All data show a monotonic decreasing trend for the wrapping angle with an increase in eccentricity. The rates of decrease of θ_{max} in the data of Kelessidis & Dukler (1990) and Mitchell *et al.* (2021) are comparable to ours, however, their values are smaller, with this difference attributed to differences in conditions ($Eu = 49$ and $N_f = 8,300$ vs. $Eu = 12$ and $N_f = 3,000$). On the other hand, the trend of the data of Liu *et al.* (2024) is visibly different; it is noted that not only the geometry of these simulations is different from the present one, but they also used a high viscosity fluid ($Eu = 140$ and $N_f = 40$). To the best of the author's knowledge, this is the first experimental study that documents the maximum angle of wrap upon the full range of eccentricity of vertical annular channels.

For the bubbles in the concentric channel, we estimated θ_{ideal} using Eq. (4.5). For eccentric channels, for which this equation is not valid, θ_{ideal} was estimated using CAD software and iterating the value of this angle until it produced the volume that was calculated from the CAD model. Although it is not a precisely defined average angle of wrap, θ_{ideal} may be considered as a rough measure of this average. The ratio $\theta_{max}/\theta_{ideal}$ indicates the level of non-uniformity in the wrapping of the bubble around the inner tube, with the

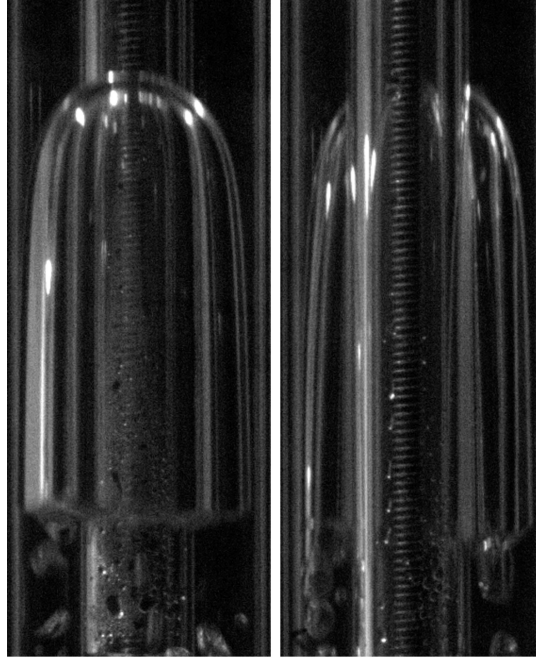


Figure 6.19: Bubble images recorded with Camera B for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s; $e = 0.8$ (left) and $e = -0.8$ (right).

Table 6.11: Estimated θ_{max} , θ_{ideal} and their ratio.

P_r [kPa]	Δt_r [s]	e [–]	L_b [mm]	θ_{max} [°]	θ_{ideal} [°]	$\frac{\theta_{max}}{\theta_{ideal}}$ [–]
41	0.5	0	48.0	292	225	1.30
83	0.5	0	152.2	321	276	1.16
41	0.25	0	31.1	282	197	1.43
55	0.25	0	59.6	300	238	1.26
55	0.25	–0.2	60.7	288	216	1.33
55	0.25	–0.4	63.9	266	187	1.42
55	0.25	–0.6	61.8	246	161	1.53
55	0.25	–0.8	61.0	209	136	1.54
55	0.25	–1	67.3	163	116	1.41
69	0.25	0	83.9	309	252	1.23
69	0.25	–0.2	80.0	297	232	1.28
69	0.25	–0.4	84.0	276	199	1.38
69	0.25	–0.6	96.6	259	179	1.45
69	0.25	–0.8	88.5	224	149	1.50
69	0.25	–1	84.5	166	123	1.35
83	0.25	0	88.8	313	258	1.22

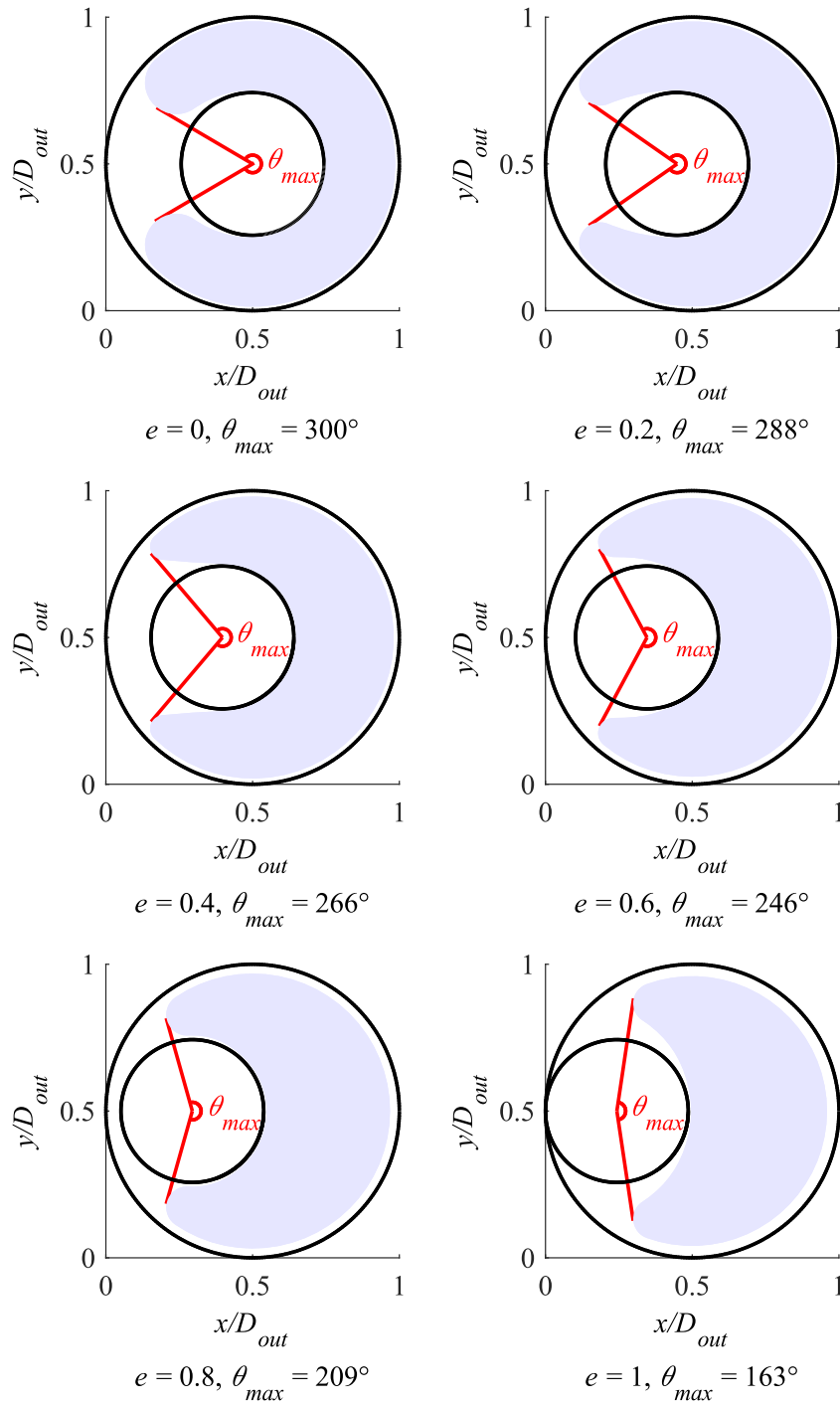


Figure 6.20: Maximum angle of wrap estimated from the images for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s and different eccentricities; shading marks the cross-section of the bubble, black lines mark the inner and outer tube surfaces in contact with the flow, and red lines connect the inner tube centre and the bubble edge.

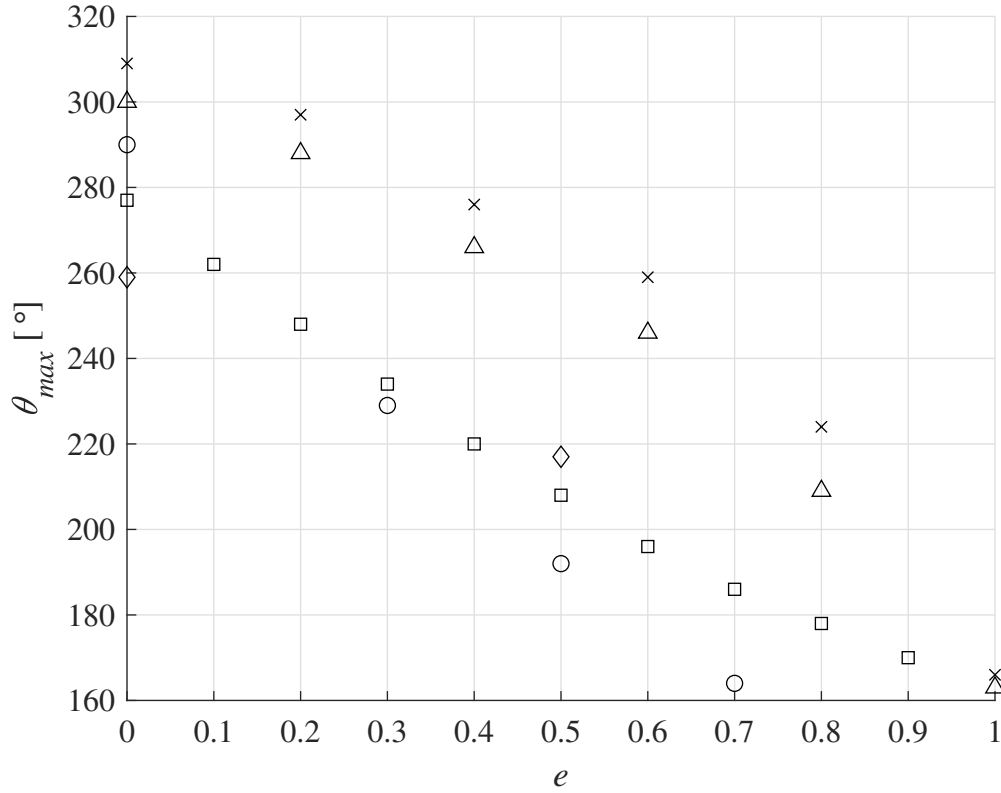


Figure 6.21: The maximum angle of wrap *vs.* eccentricity; present experiments for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s (Δ) and $P_r = 69$ kPa, $\Delta t_r = 0.25$ s ($\times$); experiments of Kelessidis & Dukler (1990) ($\diamond$); numerical simulations of Mitchell *et al.* (2021) ($\square$) and Liu *et al.* (2024) ($\circ$).

value of 1 corresponding to uniform wrapping. Figure 6.22 shows that both θ_{max} and θ_{ideal} decrease monotonically with increasing eccentricity. The ratio of these two values decreases towards unity with an increase in bubble length. On the other hand, Figure 6.23 shows that this ratio increases measurably from the concentric value as eccentricity is increased in the range from 0 to 0.8, it peaks around the latter value and drops again as e approaches 1.

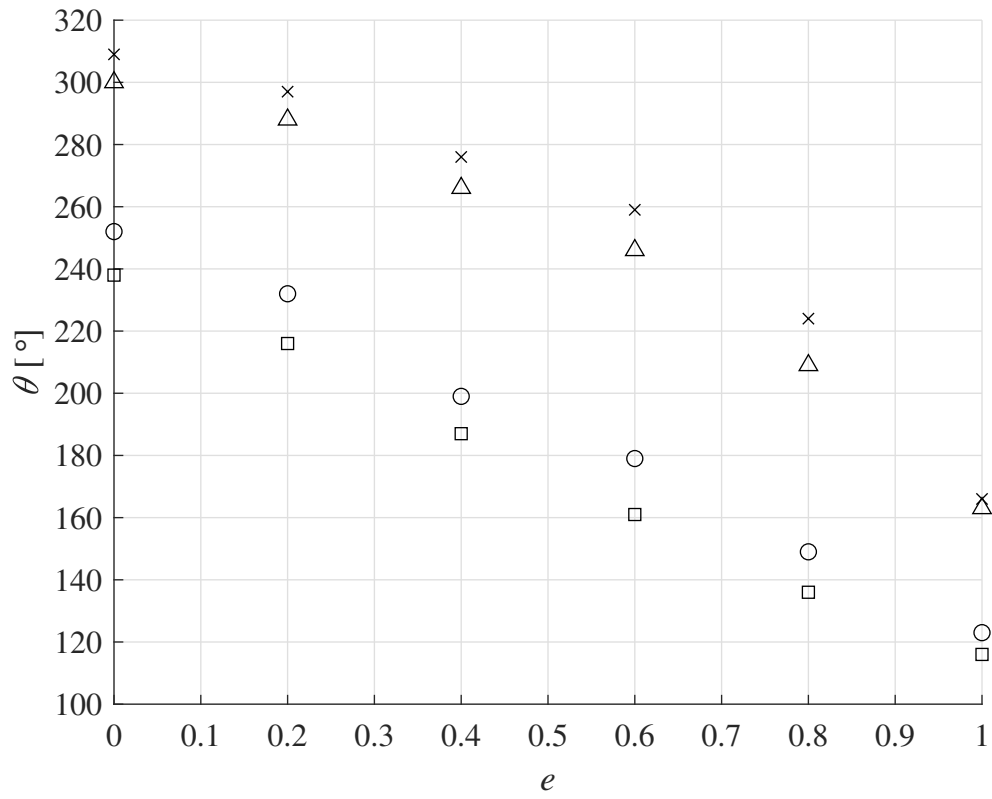


Figure 6.22: The maximum angle of wrap for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s (Δ) and $P_r = 69$ kPa, $\Delta t_r = 0.25$ s ($\times$) and the ideal angle of wrap for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s ($\square$) and $P_r = 69$ kPa, $\Delta t_r = 0.25$ s ($\circ$).

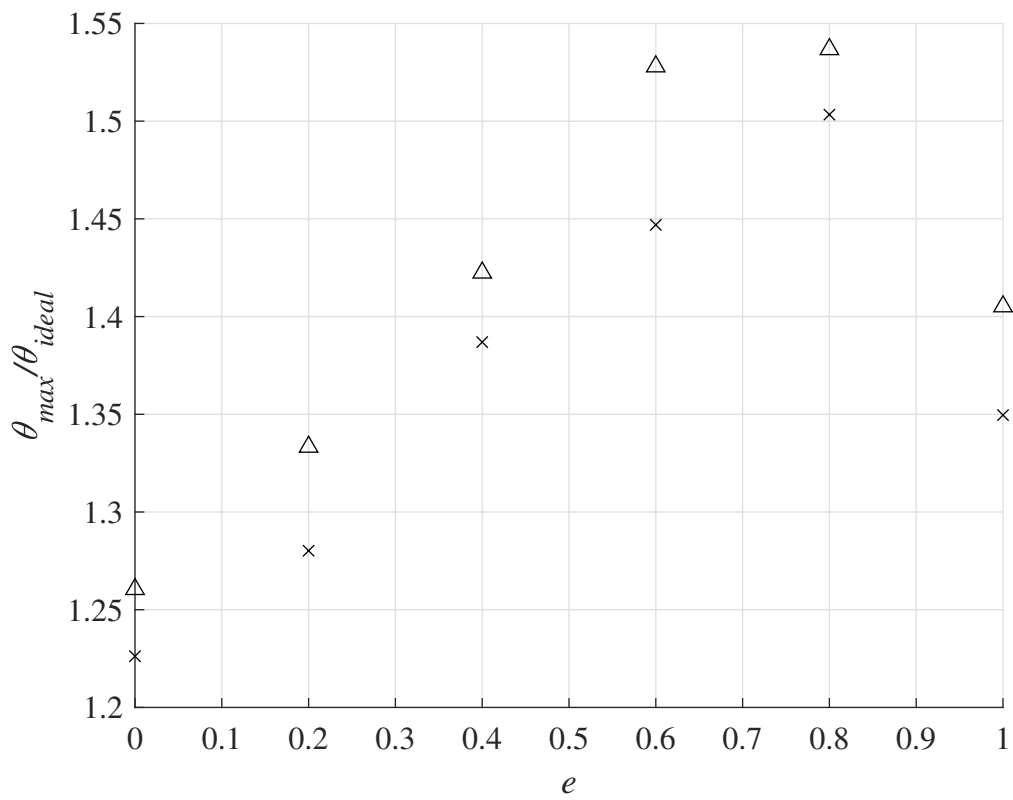


Figure 6.23: The ratio of maximum and ideal angles of wrap vs. eccentricity; $P_r = 55$ kPa, $\Delta t_r = 0.25$ s (Δ) and $P_r = 69$ kPa, $\Delta t_r = 0.25$ s ($\times$).

6.2.6 Bubble volume

As explained in Section 4.4.6, two estimates of the bubble volume in the annular channel were obtained. The first one was by the estimated volume of a bubble in the circular channel for the same release pressure and release time. The second one was by the volume of a 3D CAD model that was based on bubble images in the annular channel and was created by the Solidworks software. Either of these methods may have significant bias (systematic) and precision (random) uncertainties. The main premise of the first method was that the volumes of bubbles released under the same conditions in the tube and in the annular channel were equal. However, the air flow rate released from the nozzle for a specified regulator pressure depends on the local water pressure in the vicinity of the nozzle. The presence of the inner tube in the annular channel poses additional obstruction to the air flow, thus tending to increase the water pressure towards the total pressure and so to decrease the flow rate of air while the nozzle is open. This effect would tend to make the bubble volume in the annular channel smaller than that in the tube. The effects of differences in the liquid films and the bubble shapes and wakes on the bubble volume in the circular and annular channels may also introduce biases but are difficult to estimate. Refraction would affect both methods, although by different amounts, which also cannot be estimated accurately. A comparison of the bubble volumes that were estimated using the two methods for various bubble release conditions and eccentricities is presented in Table 6.12. As tests under common bubble release conditions in the circular and annular channels were performed for $P_r = 41$ kPa, $\Delta t_r = 0.5$ s (shorter bubbles) and $P_r = 83$ kPa, $\Delta t_r = 0.5$ s (longer bubbles) comparisons are only shown for these cases. The circular channel bubble volume estimate was 18–21% larger for the shorter bubbles and 26% larger for the longer bubbles. This is qualitatively consistent with the expectation that entrance losses in the annular channel would be larger than those in the circular channel and that this effect would become stronger with increasing bubble length.

Figure 6.24 shows the volumes estimated from the CAD model for the concentric channel for all release pressures and times and the volumes that were calculated using the linear expression from Das *et al.* (2002) (Eq. (12) in their article). Inputs to this equation were the measured average bubble length and maximum angle of wrap and the inner and outer liquid film thicknesses from our CFD analysis. The linear expression fitted to our

Table 6.12: Bubble volumes estimated using two different methods; L_{bt} and V_{bt} denote, respectively, the average length and volume from the circular channel tests.

P_r [kPa]	Δt_r [s]	e [–]	L_{bt} [mm]	V_{bt} [$\times 10^6 \text{ m}^3$]	L_b [mm]	V_b [$\times 10^6 \text{ m}^3$]	$(V_b - V_{bt})/V_b$ [%]
41	0.5	0	54.3	28.1	48.6	23.8	–18
41	0.5	0.2	54.3	28.1	48.7	23.6	–19
41	0.5	0.4	54.3	28.1	48.6	23.2	–21
41	0.5	0.6	54.3	28.1	48.2	23.7	–19
41	0.5	0.8	54.3	28.1	48.6	23.6	–19
41	0.5	1	54.3	28.1	48.4	23.6	–19
83	0.5	0	174.3	109.0	150.9	86.7	–26
41	0.25	0	–	–	31.2	12.8	–
41	0.25	0.2	–	–	35.8	15.9	–
41	0.25	0.4	–	–	33.6	14.5	–
41	0.25	0.6	–	–	31.0	13.6	–
41	0.25	0.8	–	–	33.0	14.4	–
41	0.25	1	–	–	30.9	13.5	–
55	0.25	0	–	–	60.9	30.8	–
55	0.25	0.2	–	–	63.5	32.4	–
55	0.25	0.4	–	–	63.5	32.2	–
55	0.25	0.6	–	–	60.5	30.9	–
55	0.25	0.8	–	–	60.1	31.3	–
55	0.25	1	–	–	61.1	31.0	–
69	0.25	0	–	–	80.8	42.3	–
69	0.25	0.2	–	–	80.1	43.2	–
69	0.25	0.4	–	–	81.3	43.2	–
69	0.25	0.6	–	–	82.3	44.7	–
69	0.25	0.8	–	–	82.1	44.3	–
69	0.25	1	–	–	82.2	43.3	–
83	0.25	0	–	–	91.8	49.2	–

data was

$$V_b = 0.619L_b - 7.26, \quad (6.7)$$

whereas the linear expression to the values based on the discussion by Das *et al.* (2002) was

$$V_b = 0.625L_b - 5.31. \quad (6.8)$$

These two equations have very close rates of change of volume with respect to length, and a small difference in their origins.

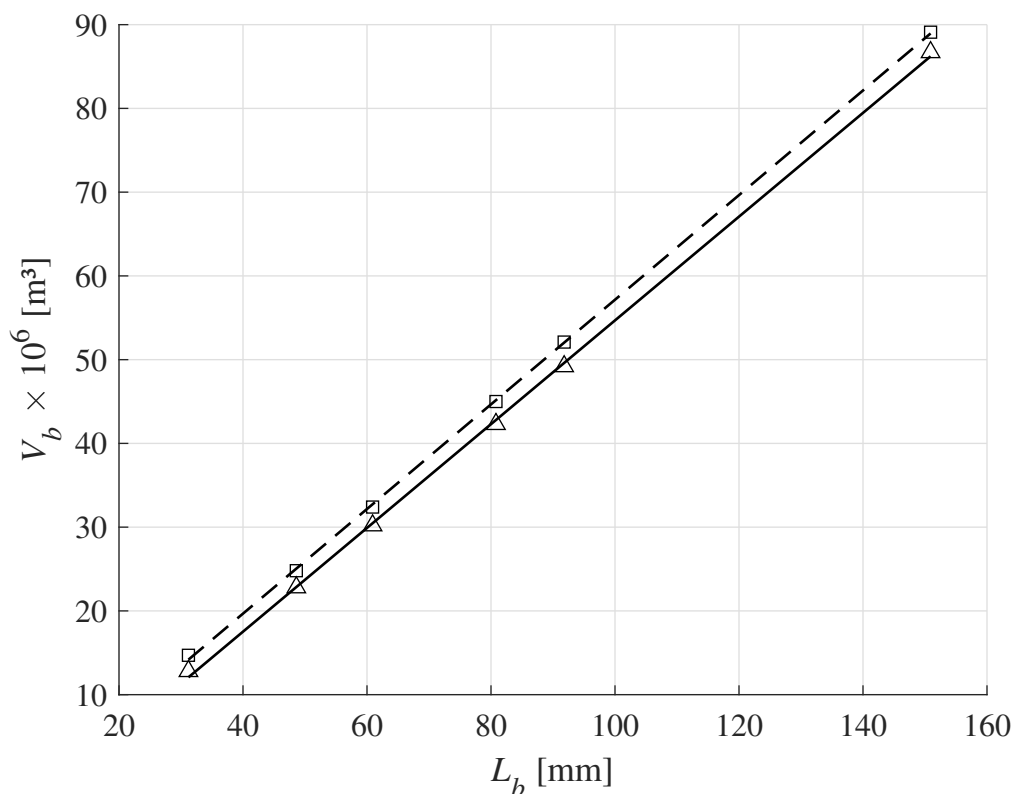


Figure 6.24: Average bubble volume vs. average bubble length for the concentric annular channel; present experiments (Δ) and their linear fit (solid line); values based on the expression by Das *et al.* (2002) ($\square$) and their linear fit (dashed line).

Table 6.12 also contains bubble volume estimates for cases for which only tests in annular channels were performed and no corresponding results in the circular channel are available. Figure 6.25 shows the change of bubble volume with respect to eccentricity.

This graph shows that, although the bubble shape changed with changing eccentricity, the effect of eccentricity on the bubble volume was not significant.

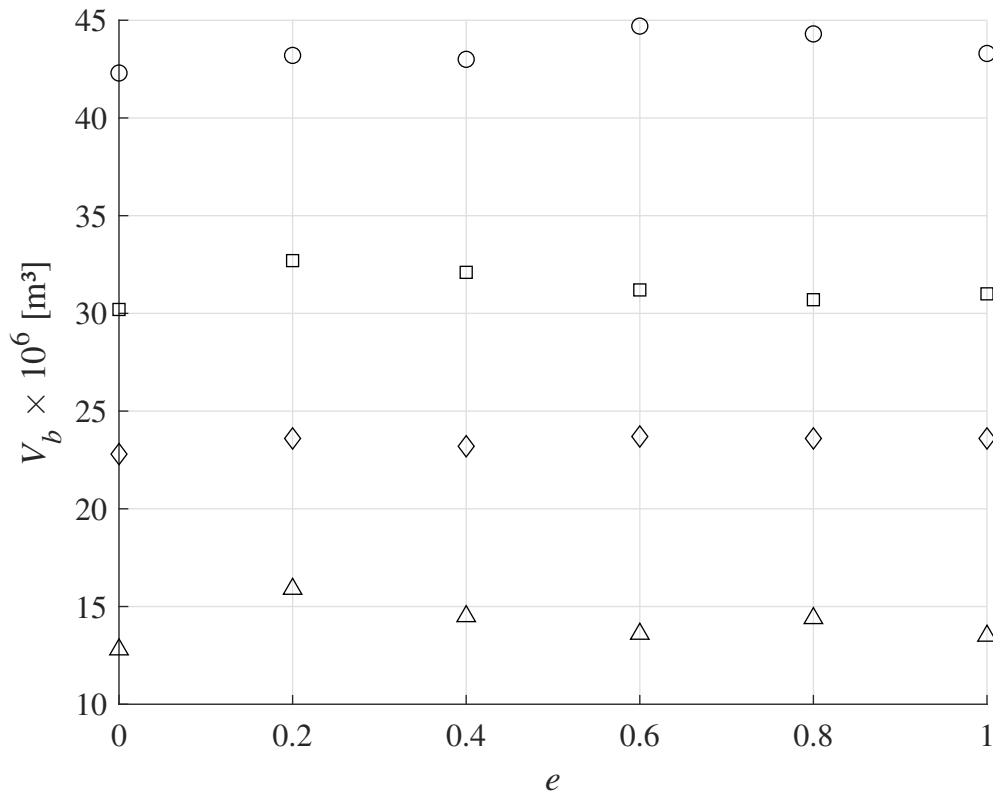


Figure 6.25: Average bubble volume vs. eccentricity; $P_r = 41 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ (Δ); $P_r = 41 \text{ kPa}$, $\Delta t_r = 0.5 \text{ s}$ ($\diamond$); $P_r = 55 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ ($\square$); $P_r = 69 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ ($\circ$).

6.3 Co-current air-water flow in the annular channel

Experiments were performed for two bubble release conditions: first, by releasing isolated Taylor bubbles in the water stream and, second, by releasing air continuously in the liquid stream. In the first set of experiments, the bubble velocity was measured and compared to the bubble velocity in the liquid column under corresponding conditions. In the second set of experiments, the velocity of individual bubbles could not be measured accurately because their shape and the location of the nose tip were hard to determine. The main objective of the second set was to investigate the effect of eccentricity on the flow regime, particularly on the transition from one regime to another.

6.3.1 Velocity of isolated bubbles in a liquid stream

In-flow velocity measurement of single, isolated Taylor bubbles was conducted for two different flow rates, selected eccentricities ($e = 0, +0.3, +0.5, +0.7, +1$), different release pressures and $\Delta t_r = 0.25$ s. Two isolated bubbles were tracked on the camera recordings for each set of conditions and their velocities were determined using the same methodology as for the measurements in the liquid column. All results are reported in Table 6.13. The velocity values for the two bubbles under the same conditions were at most 5% different from each other. Using the velocities in Table 6.13, the Froude numbers were calculated. The Froude number, averaged for all release pressures, presented in Figure 6.26, was weakly dependent on eccentricity and appeared to have a maximum around $e \approx 0.3$, as observed for the liquid column tests. For the lower flow rate, the measurements had significant scatter, which is attributed to fluctuations in the instantaneous liquid flow rate during the release of the bubble into the annular channel. This fluctuation was less than 2% for the higher flow rate, whereas, for the lower flow rate, fluctuations reached up to 17% of the average. The bubble velocity and Froude number increase drastically with increasing liquid flow rate.

Investigations of the in-flow velocity of isolated Taylor bubbles rising in co-current flow have been conducted theoretically and experimentally in vertical circular tubes (Nicklin, 1962; Collins *et al.*, 1978; Bendiksen, 1985; Polonsky *et al.*, 1999b), concentric annular channels (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Hasan & Kabir, 1992; Hibiki *et al.*, 2003b; Ozar *et al.*, 2008) and, to a lesser amount, in eccentric vertical annular

Table 6.13: Measurements of isolated Taylor bubble velocities in flows with two liquid superficial velocities for five eccentricities, four release pressures and $\Delta t_r = 0.25$ s; the uncertainty values are equal to the standard deviation of the data set multiplied by the student t -score (2.365) for eight samples.

e	j_l [mm/s]	Re_l	U_{bf} [mm/s]							
			$P_r = 41$ kPa		$P_r = 55$ kPa		$P_r = 69$ kPa		$P_r = 83$ kPa	
0	47.2 ± 7.8	900	296	297	302	302	308	308	311	319
	150.6 ± 2.5	2,900	420	426	420	425	424	419	435	436
0.3	36.3 ± 5.1	700	298	300	301	299	302	304	306	301
	151.5 ± 1.3	2,900	404	421	421	428	429	440	445	431
0.5	36.0 ± 3.9	700	301	304	308	313	310	306	320	306
	151.2 ± 2.5	2,900	420	418	408	415	430	416	434	419
0.7	41.6 ± 5.5	800	304	295	306	302	305	299	304	292
	150.6 ± 2.0	2,900	410	398	419	407	425	419	419	399
1	40.2 ± 5.2	800	282	282	287	286	293	291	295	290
	151.1 ± 0.2	2,900	396	391	392	405	400	399	394	404

channels (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Mitchell & Leonardi, 2020b). The state of the liquid flow is characterised by the Reynolds number, which was defined in Table 3.1 as

$$Re_l = \frac{\rho_l j_l D_h}{\mu_l}. \quad (6.9)$$

For single-phase water flow in a horizontal annular channel with the same inner and outer diameters as the present one, Lamarche-Gagnon & Tavoularis (2021) reported that the flow was laminar for $Re_l \lesssim 3000$. The present tests were performed at $Re_l \approx 700$ and 2900. Observation showed that, for both cases, the flow was turbulent within the bubble wake, where small bubbles were seen to move randomly as the liquid films separated from the main bubble. Away from the wake, the flow was laminar for the lower flow rate but appeared to be transitional or turbulent for the higher flow rate.

Values of the distribution parameter C_0 , calculated using Eq. (3.12) and the corresponding U_b values from Table 6.7 are listed in Table 6.14. The C_0 numbers for each eccentricity, averaged for all release pressures, are presented in Figure 6.27. For the higher flow rate, C_0 was in the range between 1.2 and 1.3, which, according to the literature, indicates that the flow was turbulent or possibly transitional. The values appear to be weakly dependent on eccentricity. It is noted that, for the lower flow rate, the C_0 values were scattered between 1.6 and 2.2, which may be the result of fluctuations in the instantaneous liquid flow rate

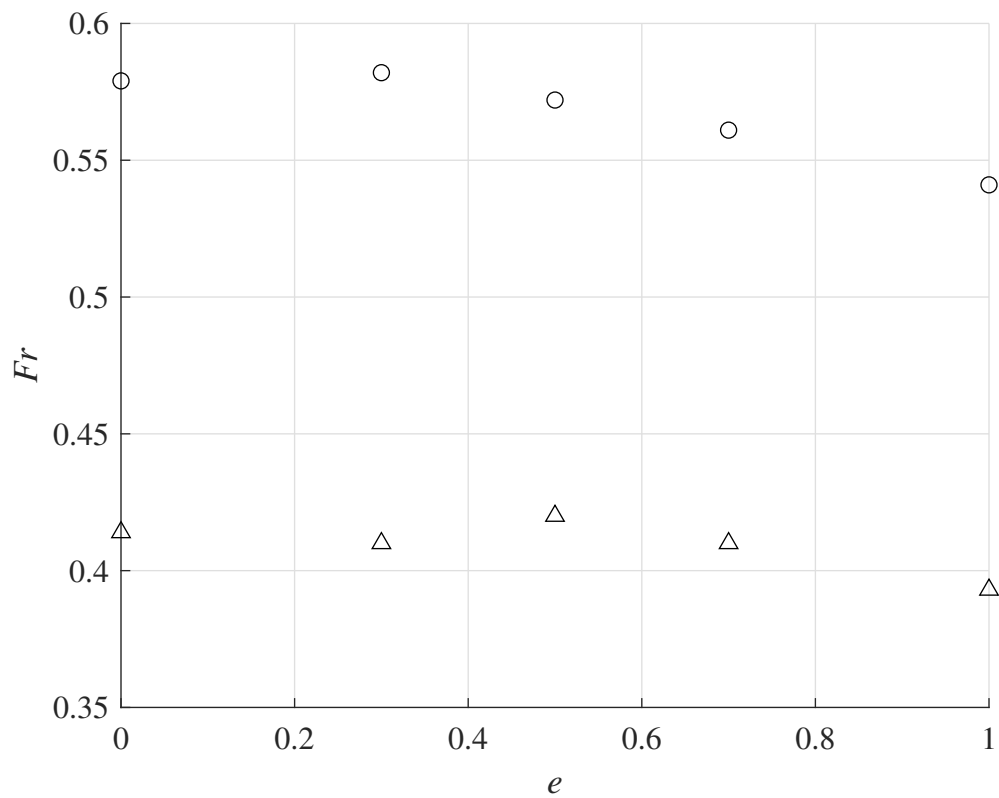


Figure 6.26: Average Froude number vs. eccentricity; lower flow rate (Δ); higher flow rate ($\circ$).

rather than a change in eccentricity.

Table 6.14: Representative values of the distribution parameter for different conditions.

e	j_l [mm/s]	C_0							
		$P_r = 41$ kPa		$P_r = 55$ kPa		$P_r = 69$ kPa		$P_r = 83$ kPa	
0	47.2 ± 7.8	1.4	1.4	1.5	1.5	1.7	1.6	1.7	1.9
	150.6 ± 2.5	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.4
0.3	36.3 ± 5.1	1.7	1.8	1.8	1.8	1.8	1.9	1.9	1.8
	151.5 ± 1.3	1.1	1.2	1.2	1.3	1.3	1.3	1.4	1.3
0.5	36.0 ± 3.9	2.0	2.1	2.1	2.3	2.2	2.1	2.5	2.1
	151.2 ± 2.5	1.3	1.3	1.2	1.2	1.3	1.2	1.3	1.2
0.7	41.6 ± 5.5	2.0	1.8	2.0	1.9	1.9	1.8	1.9	1.6
	150.6 ± 2.0	1.3	1.2	1.3	1.2	1.3	1.3	1.3	1.2
1	40.2 ± 5.2	1.8	1.8	1.9	1.9	2.0	2.0	2.1	1.9
	151.1 ± 0.2	1.2	1.2	1.2	1.3	1.2	1.2	1.2	1.3

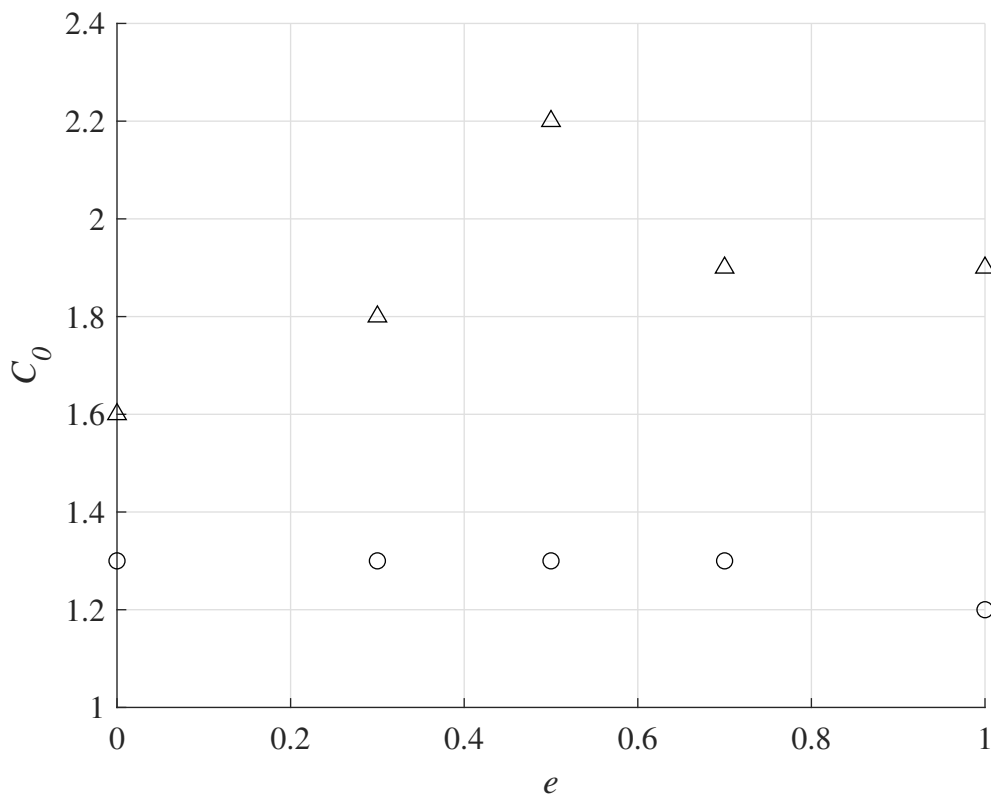


Figure 6.27: Average C_0 vs. eccentricity; lower flow rate (Δ) and higher flow rate ($\circ$).

It was observed that the Taylor bubbles in the flowing stream for given P_r and Δt_r were larger than those released under the same conditions in stagnant water. This is attributed to the enhancement of bubble growth by the surrounding liquid stream. The shape of isolated Taylor bubbles in the flowing stream was also visibly different from that in stagnant water: in the stream, bubbles were slimmer and had smaller corresponding angles of wrap, as shown in Figure 6.28 for two different eccentricities. In this figure, the corresponding bubbles in the two configurations were selected to have roughly comparable volumes and were created with different release conditions. These observations are in good agreement with the literature (Kelessidis, 1986; Quan, 2011; Mitchell & Leonardi, 2020b).

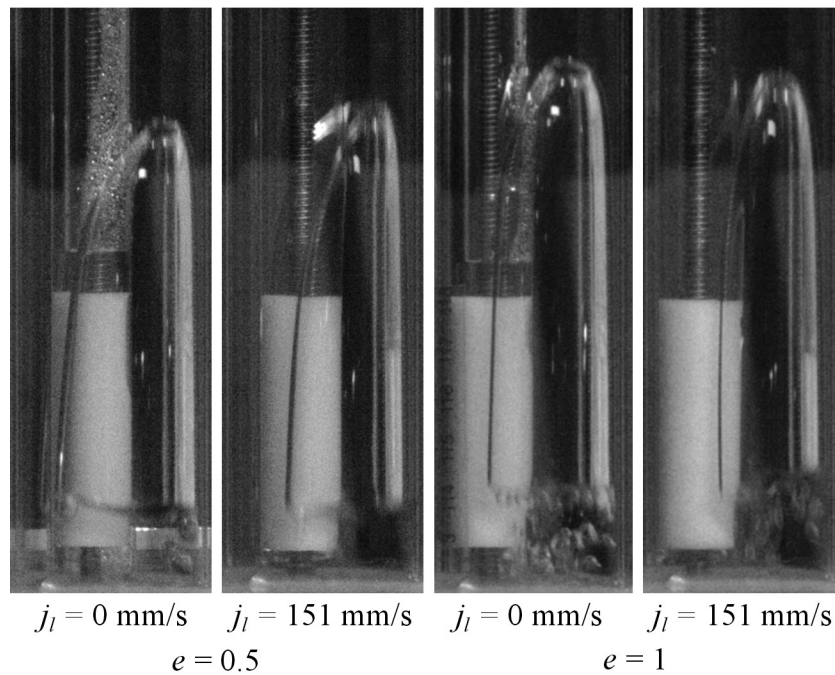


Figure 6.28: Representative images of bubbles in stagnant water and in co-current flow for two different eccentricities; the release conditions for the in-flow bubbles were adjusted so that the bubble volumes in each pair were comparable.

6.3.2 Flow regime map

Co-current gas-liquid flow regime maps for concentric vertical annular channels have been reported by many investigators (Kelessidis & Dukler, 1989; Caetano *et al.*, 1992a; Das *et al.*, 1999b; Hibiki *et al.*, 2003b; Sun *et al.*, 2004; Ozar *et al.*, 2008; Julia *et al.*, 2009; Julia & Hibiki, 2011; Julia *et al.*, 2011; Guo *et al.*, 2018; Yin *et al.*, 2025) but such maps for eccentric vertical annular channels are only available for $e = 0.5$ Kelessidis & Dukler (1989) and $e = 1$ Caetano *et al.* (1992a); De Sousa *et al.* (2014). The present investigation includes cases with $e = 0, +0.3, +0.5, +0.7, +1$, within the following ranges of the superficial liquid and gas velocities: $0.032 < j_l < 0.292 \text{ m/s}$ and $0.006 < j_g < 2.028 \text{ m/s}$. The flow regimes were identified by visual observation of the flow in real time and by inspecting videos of the flow through the upper viewing tank ($H = 2.8 \text{ m}$), replayed at a slower speed when necessary. When comparing the present results with results from the literature, one should consider the fact that flow regime transitions are in general sensitive to the purity of water. In the present experiments, the apparatus was filled with water processed by reverse

osmosis (RO), but it is possible that this water was contaminated by an unknown small amount of impurities, which may have had a non-negligible effect on surface tension and regime transition.

Depending on the gas and liquid flow rates and the eccentricity, it was possible to identify four regimes: bubbly, cap-bubbly, slug, and churn flow. Representative images of these flows are shown in Figure 6.29. The bubbly flow contained only small, nearly spherical bubbles, which did not coalesce to create larger ones. The cap-bubbly flow contained short cap bubbles, which had a length that was less than $D_h = 19$ mm and a shape resembling the nose region of the Taylor bubbles. Cap-bubbly flow may be viewed as a transitional regime from bubbly to slug flow. As the gas flow rate was increased, cap bubbles grew into Taylor bubbles, the length of which increased with increasing flow rate. Further increase in gas flow rate made the Taylor bubbles coalesce with each other and create very long bubbles with length exceeding 600 mm, while the liquid slugs between these bubbles got shorter and developed an increasingly frothy appearance. Once these long bubbles started to coalesce with each other, they became highly distorted, with liquid occasionally dropping through the gas. Under such conditions, the regime was identified as churn flow. It is noted that the transition from slug to churn flow was not as obvious as transition from bubbly to slug flow. This is in agreement with the observation of Cao *et al.* (2024).

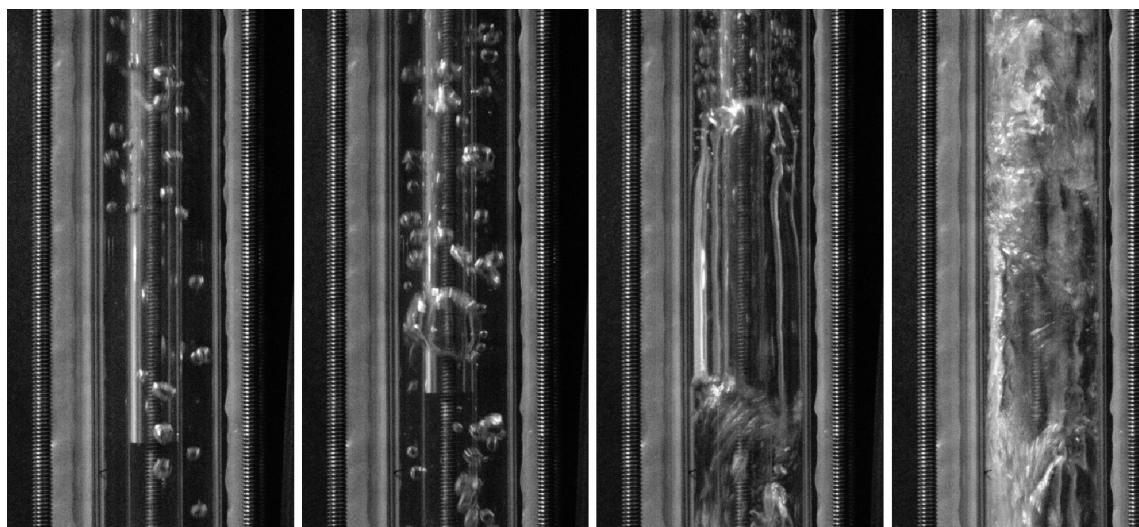


Figure 6.29: Representative images of flow patterns: bubbly flow, cap-bubbly flow, slug flow and churn flow (left to right).

The flow regime map for all eccentricities is shown in Figure 6.30. This map covers all four regimes that were discussed previously. For a fixed liquid flow rate within the entire range of values tested, the flow regime progressed from bubbly to churn flow as the gas flow rate was increased. Because the symbols for different eccentricities overlap in parts of the map, it is not possible to discern the effect of eccentricity near transition regions. It is clear, however, that, away from transition regions, eccentricity did not affect the type of regime. On the other hand, near transition regions, a change in eccentricity could trigger transition from one regime to another. These map regions have been marked by shaded areas in Figure 6.30. A general observation can be made for all transition trends. When considering all cases with the same liquid flow rate, all transitions, regardless of eccentricity, were shifted towards higher gas flow rates as the liquid flow rate was increased. Thus, the transition areas of the map were tilted clockwise; this tilt is obvious for the two transitions at lower j_g and may be inferred for the higher- j_g transition, where the data density is relatively low.

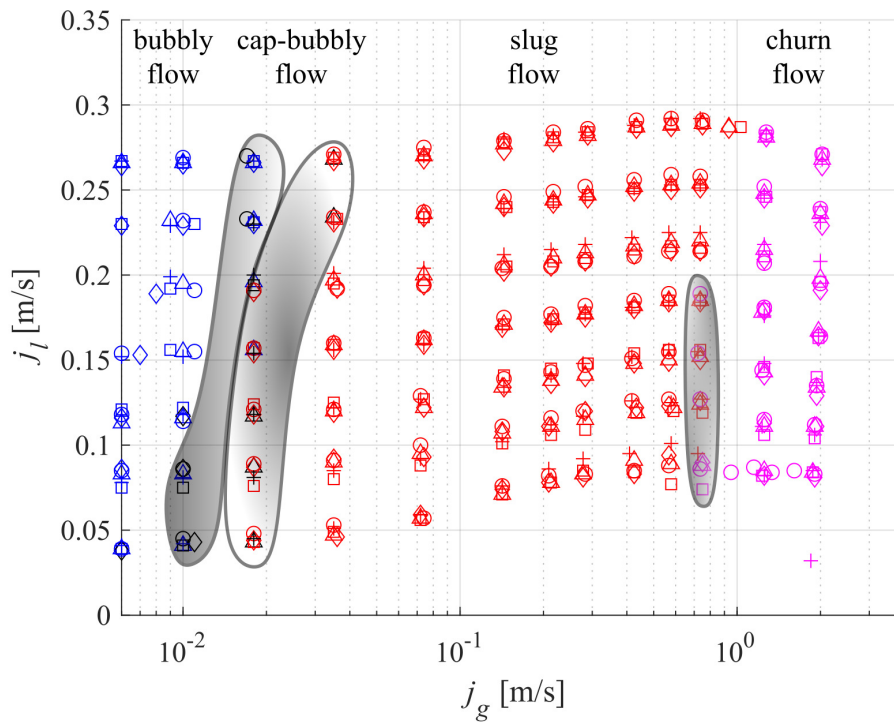


Figure 6.30: Regime map of air-water flows in annular channels with $e = 0$ (Δ), 0.3 ($\square$), 0.5 ($\diamond$), 0.7 ($\circ$), and 1 ($+$); symbols are blue for bubbly flow, black for cap-bubbly flow, red for slug flow, and magenta for churn flow; shaded areas show the transition regions.

The literature contains observations showing that gas-liquid flow regime in vertical channels is sensitive to the inlet conditions. Although we did not attempt to quantify the inlet effect, we noticed that, in some cases, the flow regime changed along the channel between the entrance region and the upper viewing tank. For relatively low gas flow rates, dispersed bubbles were seen to coalesce and so a bubbly flow at the channel bottom transitioned into cap-bubbly flow at the channel middle and slug flow at the top of the channel. For relatively high gas flow rates, the flow near the entrance had a churn-like appearance, which transitioned into a slug flow further downstream. These observations are in agreement with the study of Julia *et al.* (2009).

Experiments were performed for 13 specific values of j_g and wide ranges of values of j_l . For nine values of j_g among these sets, the flow regime was the same for all eccentricities, but for four sets, those with $j_g = 0.010, 0.018, 0.035$ and 0.70 m/s, the flow regimes were different for different eccentricities. To illustrate clearly this effect of eccentricity, we have marked the flow regimes for these four j_g values and the full available ranges of j_l and e in Figures 6.31, 6.32, 6.34 and 6.35. These plots demonstrate that eccentricity affects measurably the conditions for transition from one regime to another and that this effect does not depend monotonically on e , but is strongest for intermediate values of e . Specific observations that can be made for each of these figures are as follows. For $j_g = 0.010$ m/s (Figure 6.31), bubbly flow was observed for all j_l examined, when $e = 0$ and 1 , but, for intermediate eccentricities, bubbly flow occurred when j_l exceeded a specific threshold, which depended on e , whereas cup-bubbly flow occurred when j_l dropped below this threshold. For $j_g = 0.018$ m/s (Figure 6.32), two flow regimes, bubbly and cap-bubbly flows, were observed, when $e = 0$ and 1 , but, for intermediate eccentricities, three regimes, bubbly, cap-bubbly and slug flows occurred. Figure 6.33 shows flow regime images for $j_g = 0.018$ m/s and $j_l = 0.124, 0.194$ and 0.267 m/s, which correspond to conditions for $e = 0.3$ shown in Figure 6.32. For $j_g = 0.035$ (Figure 6.34), only a small part of the map ($e = 0, j_l \gtrsim 0.2$ m/s) was in cap-bubbly flow, whereas the rest of the map was in slug flow. Although this figure, viewed in isolation, cannot describe clearly the general effect of eccentricity on regime transition, it does illustrate that such effect exists and shows a transition trend that is consistent with the trends that may be extracted from the general regime map (Figure 6.30). For $j_g = 0.70$ (Figure 6.35), the void fraction is relatively high

for all examined j_l and only two flow regimes, slug and churn flows, are observed. In general, slug flow occurs when j_j exceeds a threshold that depends on e and churn flow occurs when j_l drops below this threshold. Although for $e = 1$ the map does not extend to sufficiently low values of j_l for slug-to-churn-flow transition to be observable, one may infer that such transition would have occurred if j_l were dropped below the lowest value in the present tests. This figure shows that, for an eccentricity of 0.7, transition occurred at value of j_l that was more than twice the transition threshold for the concentric channel and for channels with lower eccentricities.

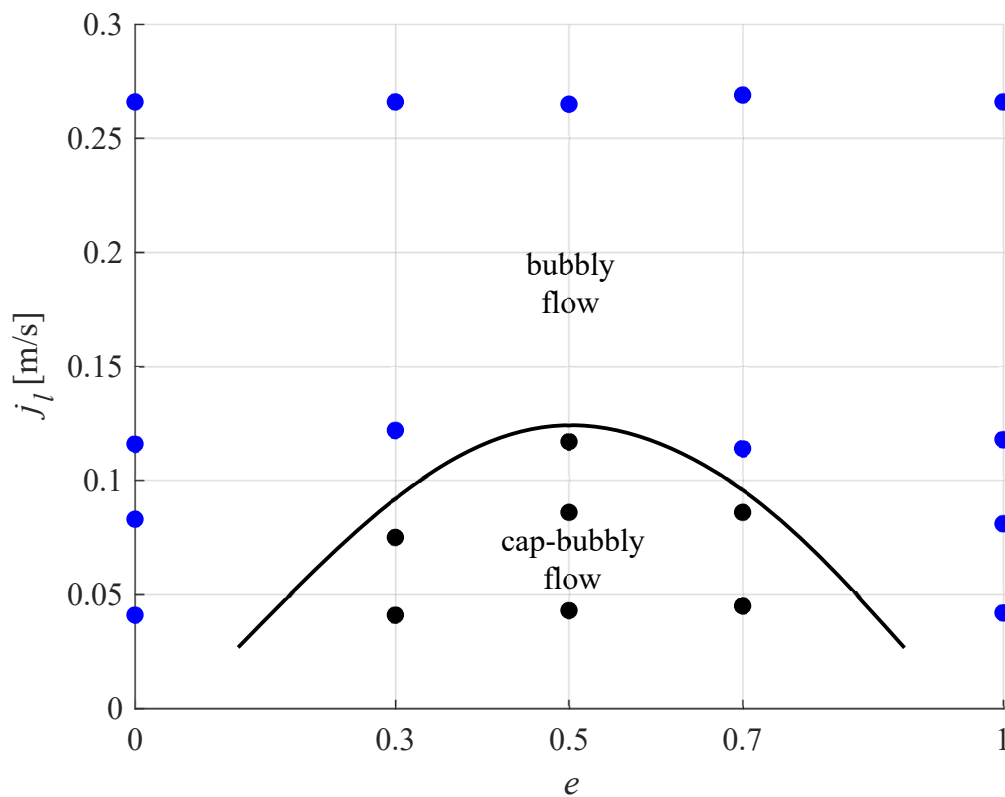


Figure 6.31: Flow regime map for $j_g = 0.010$ m/s and different liquid flow rates and eccentricities; symbols are blue for bubbly flow and black for cap-bubbly flow.

6.3.3 Gap vortices and their effect on flow regime transition

Previous literature (Tavoularis, 2011; Choueiri & Tavoularis, 2014, 2015; Lamarche-Gagnon & Tavoularis, 2021) has documented the phenomena of gap instability and gap

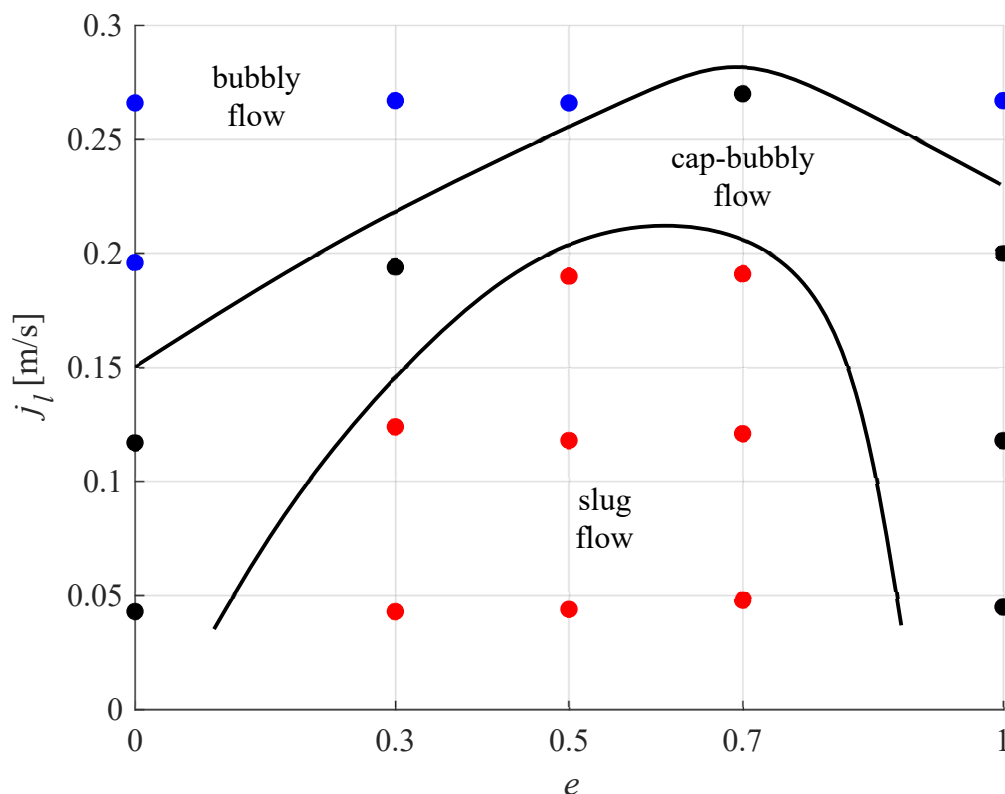


Figure 6.32: Flow regime map for $j_g = 0.018$ m/s and different liquid flow rates and eccentricities; symbols are blue for bubbly flow, black for cap-bubbly flow and red for slug flow.

vortex streets in single-phase flows in annular channels, rod bundles and other channels with axial gaps. Gap vortices transport fluid across the gap and generate strong cross flows. The presence of gas-filled bubbles disrupts the continuity of the liquid phase, thus complicating the structure of the gap vortices. Nevertheless, because the vortex street has a wavelength that is several inner diameters long, it seems unlikely that it would be annihilated by relatively small bubbles, at least in the bubbly and cap-bubbly regimes. This expectation was actually confirmed by observation of video recordings of air-water flows in these regimes taken with the camera focused across the narrow gap at eccentricities in the range $0.5 \lesssim e \lesssim 0.75$ and replayed at low speed. These observations demonstrated that both dispersed and cap bubbles had roughly quasi-periodic cross motions of a type that is consistent with the actions of gap vortex streets. Because air has much lower inertia than water, gas bubbles would tend to oscillate with larger amplitudes, thus larger



Figure 6.33: Representative images of flow patterns in the bubbly to slug flow transition region for $e = 0.3$ and $j_g = 0.018$ m/s: bubbly flow (left, $j_l = 0.267$ m/s), cap-bubbly flow (centre, $j_l = 0.194$ m/s) and slug flow (right, $j_l = 0.124$ m/s).

velocities, than the adjacent liquid (Tavoularis & Nedić, 2024). Therefore, the magnitude of the velocity of bubbles would tend to increase significantly above their axial velocity component, which is accounted for by the superficial velocity of air. Bubbles embedded in different parts of a liquid vortex would have different velocity magnitudes and directions and so the probability of bubble collisions and coalescence would be higher than in the absence of gap vortices. One may thus infer that, for a fixed liquid superficial velocity, in a strongly eccentric channel the flow would cross the regime transition boundary at a gas superficial velocity that is lower than that in a concentric channel. Alternatively, for a fixed gas superficial velocity, in a strongly eccentric channel the flow would cross the regime transition boundary at a higher liquid superficial velocity than that in a concentric channel.

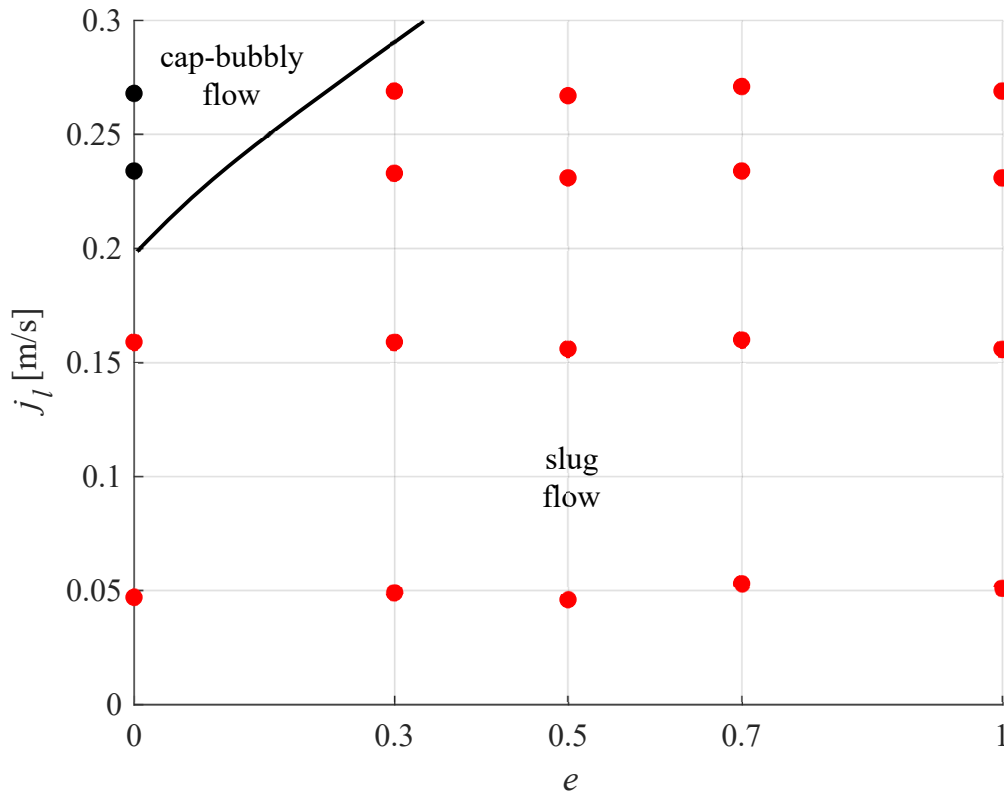


Figure 6.34: Flow regime map for $j_g = 0.035$ m/s and different liquid flow rates and eccentricities; symbols are black for cap-bubbly flow and red for slug flow.

A related but separate effect is that bubbles, which otherwise would tend to concentrate in the wider part of the channel where friction effects are less significant, would cross the narrow gap, where they are more likely to collide with other bubbles and coalesce. Although hydrodynamic stability analysis (Moradi & Tavoularis, 2019) has shown that gap instability may be triggered for eccentricities as small as 0.05, in practice, strong gap vortex streets in turbulent flows would likely occur for $e \gtrsim 0.5$ (Lamarche-Gagnon & Tavoularis, 2021). In view of these considerations, we may plausibly attribute the apparent shift of the regime boundaries in Figure 6.30 towards higher j_g to the action of gap vortex streets. This explanation seems to be well justified for the bubbly-to-cap-bubbly and cap-bubbly-to-slug-flow transition. The effect of strong eccentricity on the slug-flow-to-churn-flow transition seems to be also a shift towards higher j_g (or lower j_l), although the available information is insufficient for us to describe the precise mechanism that causes this phenomenon.

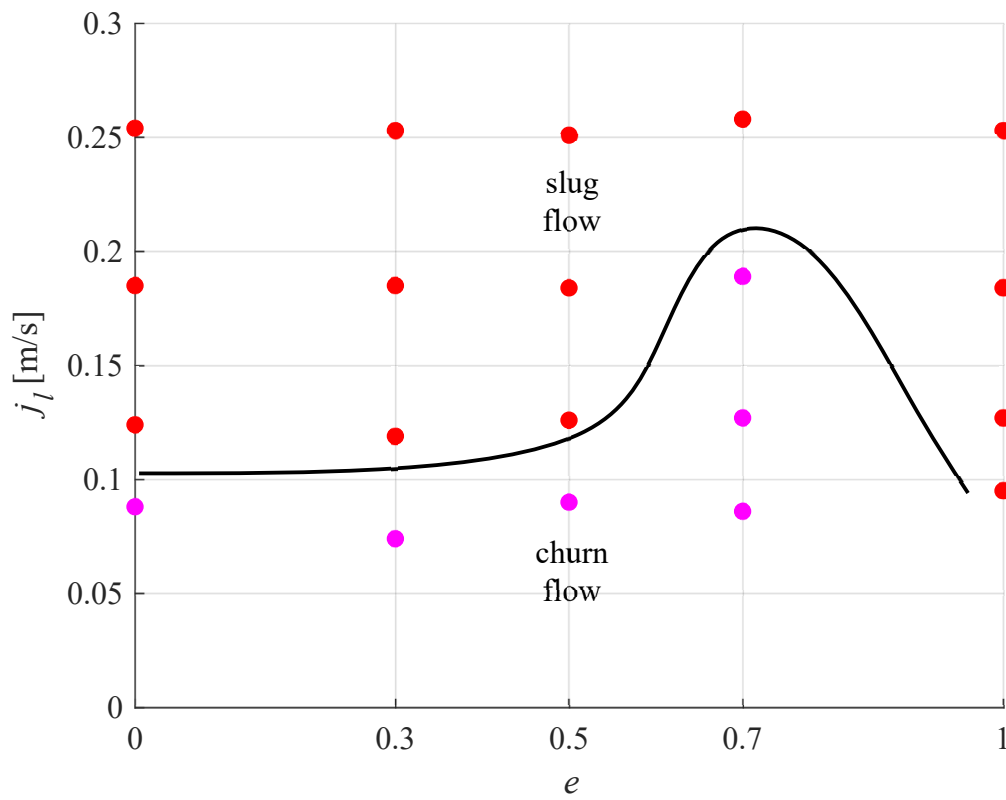


Figure 6.35: Flow regime map for $j_g = 0.70$ m/s and different liquid flow rates and eccentricities; symbols are red for slug flow and magenta for churn flow.

Chapter 7

Computational results

7.1 Simulated cases

The computational analysis included the following cases:

1. Simulations of Taylor bubbles rising in a cylindrical liquid column; these results were compared with the present experimental results and results from the literature.
2. Simulations of Taylor bubbles rising in a liquid column contained in concentric and eccentric annular channels at conditions matching the experimental ones; cases with different combinations of eccentricity and bubble size were examined.
3. Simulations for gas-liquid flows in concentric and eccentric annular channels at conditions matching the experimental ones; of particular interest was the possible occurrence of gap vortices and its effect on the flow regime transitions.

7.2 Taylor bubbles in a cylindrical tube

A mesh sensitivity analysis was performed using three different cross-sectional meshes, having respectively 2000 (T1), 3265 (T2) and 3600 (T3) planar cells, as shown in Figure 7.1. The mesh near the wall was refined using a one-sided geometric distribution and the minimum radial size of the cells was set to 0.1 mm for T1 and T3 and 0.5 mm for T2. Each cross-sectional mesh was then extruded to form prismatic volume cells with a uniform axial length. The entire domain was meshed with 1000, 3000, 4000 or 5000 axial cells, with the corresponding mesh name having the letter a, b, c or d, respectively. Six different 3D meshes, listed in Table 7.1, were tested for their accuracy in predicting the bubble velocity following the release of a cylindrical bubble with a height $H_{bt} = 1.6D_{tin} = 50$ mm. The temporal evolutions of the Froude number $Fr_{i,CFD}$ for these six cases, shown in Figure 7.2, clearly demonstrate the quick attainment of a steady state. Table 7.2 lists

the Froude number for each simulation at an elevation $H_{CFD} \approx 2.6$ m from the bottom of the channel (see Figure 5.4) and its normalised difference from the experimental value of $Fr_{t,exp} = 0.348$ for a bubble with a comparable length (Table 6.2) and the value of $Fr_{t,Dum} = 0.351$ predicted by the expression given by Dumitrescu (1943). It is clear that the finest of the tested meshes, which had 18 million cells, had the highest accuracy so this is the mesh that we selected for the main simulations. Simulations with the T3d mesh using 256 computer cores took about three weeks to reach 15 s in simulation time, whereas simulations with the T1a mesh using 192 cores took 13 hours.

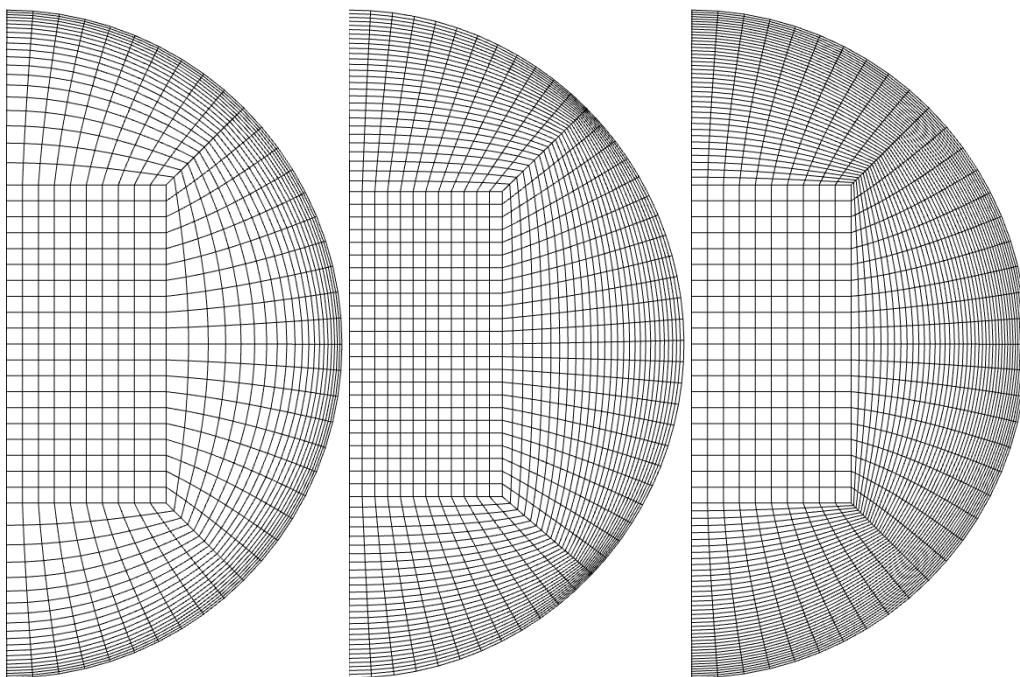


Figure 7.1: The three tested cross-sectional meshes for the cylindrical tube: T1 (left), T2 (centre) and T3 (right).

Table 7.1: Meshes used for the cylindrical tube mesh sensitivity analysis.

Symbol	Mesh name	Cross-sectional cells	Axial cells	Total cells
+	T1a	2000	1000	2 million
$\triangle$	T1c	2000	4000	8 million
$\diamond$	T2d	3265	5000	16.3 million
$\square$	T3b	3600	3000	10.8 million
$\bigcirc$	T3c	3600	4000	14.4 million
$\times$	T3d	3600	5000	18 million

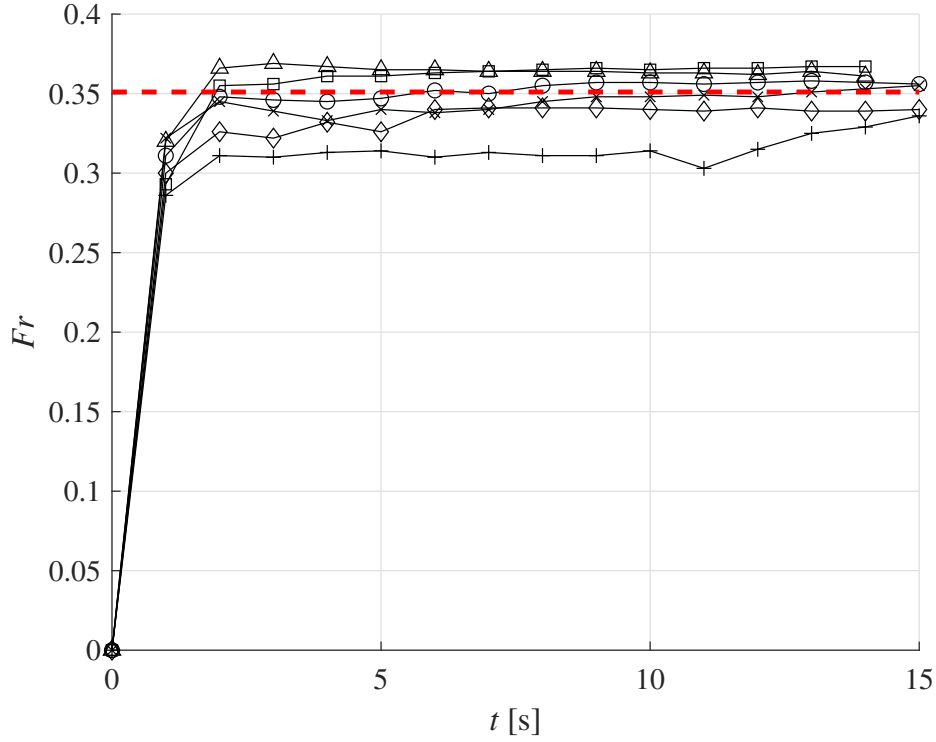


Figure 7.2: Temporal evolution of the Froude number in the cylindrical tube, obtained with the following meshes: T1a (+); T1c (Δ); T2d ($\diamond$); T3b ($\square$); T3c ($\circ$); T3d ($\times$); experimental value (red dashed line).

Table 7.2: Froude numbers in the annular channel, obtained using different meshes, and their differences from the experimental value and the value obtained from the expression of Dumitrescu (1943).

Symbol	$Fr_{t,CFD}$	$\frac{Fr_{t,CFD} - Fr_{t,exp}}{Fr_{t,exp}}$ [%]	$\frac{Fr_{t,CFD} - Fr_{t,Dum}}{Fr_{t,Dum}}$ [%]
+	0.333	-4.3	-5.1
Δ	0.362	4.0	3.1
$\diamond$	0.339	-2.6	-3.4
$\square$	0.367	5.5	4.6
$\circ$	0.358	2.9	2.0
$\times$	0.352	1.1	0.3

As explained in Section 5.1, each computational cell in the VOF method is assigned to a volume fraction value α_g between 0 and 1, with 0 being entirely liquid and 1 being entirely gas. For all simulations, a cylindrical air pocket was initially defined at the bottom of the channel by assigning $\alpha_g = 1$ to all cells in the pocket and $\alpha_g = 0$ to all cells

outside it. As the simulation run, the gas phase at the bottom was redistributed throughout the computational domain and rose towards the surface. In VOF analyses, the interface separating the gas and liquid phases is approximated by a surface with a constant gas volume fraction α_g . Within this approximation, a gas bubble is considered to be a region where α_g exceeds a certain value. It is obvious that the shape and size of bubbles would depend on the mesh and on the α_g threshold. Figure 7.3 shows isosurfaces obtained with different α_g and different meshes at $t = 14$ s. All meshes, except the coarsest one (T1a), produced a realistic main Taylor bubble, albeit with a different shape and size. The main bubble was followed by smaller ones. A comparison of the three plots obtained with the T3 mesh shows that, as the axial cell size was refined, the main bubble size increased. This can be explained as follows: the volume fraction of the gas phase in each cell near the interface tends to diminish as the cell size is increased and so more cells tend to have an α_g that is below the set threshold for the detection of the interface; consequently, mesh refinement identifies bubbles with increasing volume and length and decreasing liquid film thickness. Another observation from Figure 7.3 is that the bubble volume decreased and the liquid film thickness increased as the threshold of α_g was increased from 0.05 to 0.9. The dependence of the bubble volume and liquid film thickness on the mesh and α_g is not very significant in the upstream part of the bubble, as shown in Table 7.3), which shows these properties at an axial location with $z/D_{tin} = 0.7$, which is the furthest distance at which the air bubble extends in all cases.

Besides the simulations with a “short” ($H_{bt} = 50$ mm) initial air pocket, a simulation with the T3d mesh was performed following the release of a larger air pocket with a height $H_{bt} = 12D_{tin} = 380$ mm. It is noted that at H_{CFD} , the longer bubble is approximately 10 times longer than the shorter bubble and twice as long as the longest bubble in the experiments. The temporal evolution of the Froude numbers of the short and long bubbles using the T3d mesh is shown in Figure 7.4. The Froude number of the longer bubble was measurably larger than that of the shorter bubble, in agreement with the observed experimental trend. Isocontours of the gas volume fraction on a plane of symmetry of the cylindrical tube for different ranges of α_g are shown in Figure 7.5. For all ranges, the nearly pure gas regions of the main Taylor bubble and a few smaller trailing bubbles can be distinguished quite clearly, but the contours for the ranges $\alpha_g \geq 0.5$ and $\alpha_g \geq 0.9$ separate

Table 7.3: Bubble volume up to $z/D_{tin} = 0.7$ and liquid film thickness at that location.

Mesh	α_g	$V_{bt,CFD} \times 10^6 [\text{m}^3]$	$\lambda_{CFD} [\text{mm}]$
T1c	0.05	8.5	2.6
T1c	0.5	8.4	2.6
T1c	0.9	8.1	3.0
T2d	0.05	9.3	2.3
T2d	0.5	9.2	2.3
T2d	0.9	9.1	2.6
T3b	0.05	8.5	2.8
T3b	0.5	8.4	3.0
T3b	0.9	8.3	3.0
T3c	0.05	8.6	2.4
T3c	0.5	8.5	2.6
T3c	0.90	8.4	2.6
T3d	0.05	8.4	2.3
T3d	0.5	8.3	2.4
T3d	0.9	8.2	2.4

Table 7.4: Comparison of calculated bubble volumes in the cylindrical tube and the values from the expression of Dumitrescu (1943).

$L_{bt} [\text{mm}]$	$V_{bt,CFD} \times 10^6 [\text{m}^3]$	$V_{bt} \times 10^6 [\text{m}^3]$	$(V_{bt,CFD} - V_{bt})/V_{bt} [\%]$
54	26.3	28.1	-6.4
174	104.6	108.9	-3.9

the gas and liquid regions most effectively.

The radii of the simulated short and long bubbles, calculated from volume fraction isocontours with $\alpha_g = 0.05$ and $\alpha_g = 0.9$, are plotted in Figure 7.6 along with the bubble radius based on Eq. (4.2) by Dumitrescu (1943). A comparison shows that the short-bubble profile was very close to the other ones, whereas the long-bubble profile was close to the others only for $z/D_{tin} \lesssim 3$, but, for higher z/D_{tin} values, the simulated long bubble had a larger liquid film thickness. The effect of different α_g threshold values is also shown in this graph, which shows a slight difference in liquid film thicknesses. These differences are also noticed in the calculated bubble volumes, which, as shown in Table 7.4, were measurably smaller than the experimental values for $P_r = 41$ and 82 kPa. The CFD bubble volumes were found by integrating the volume fraction of air between the bubble nose tip and tail locations for the experimental lengths L_{bt1} and L_{bt2} shown in Figure 7.4.

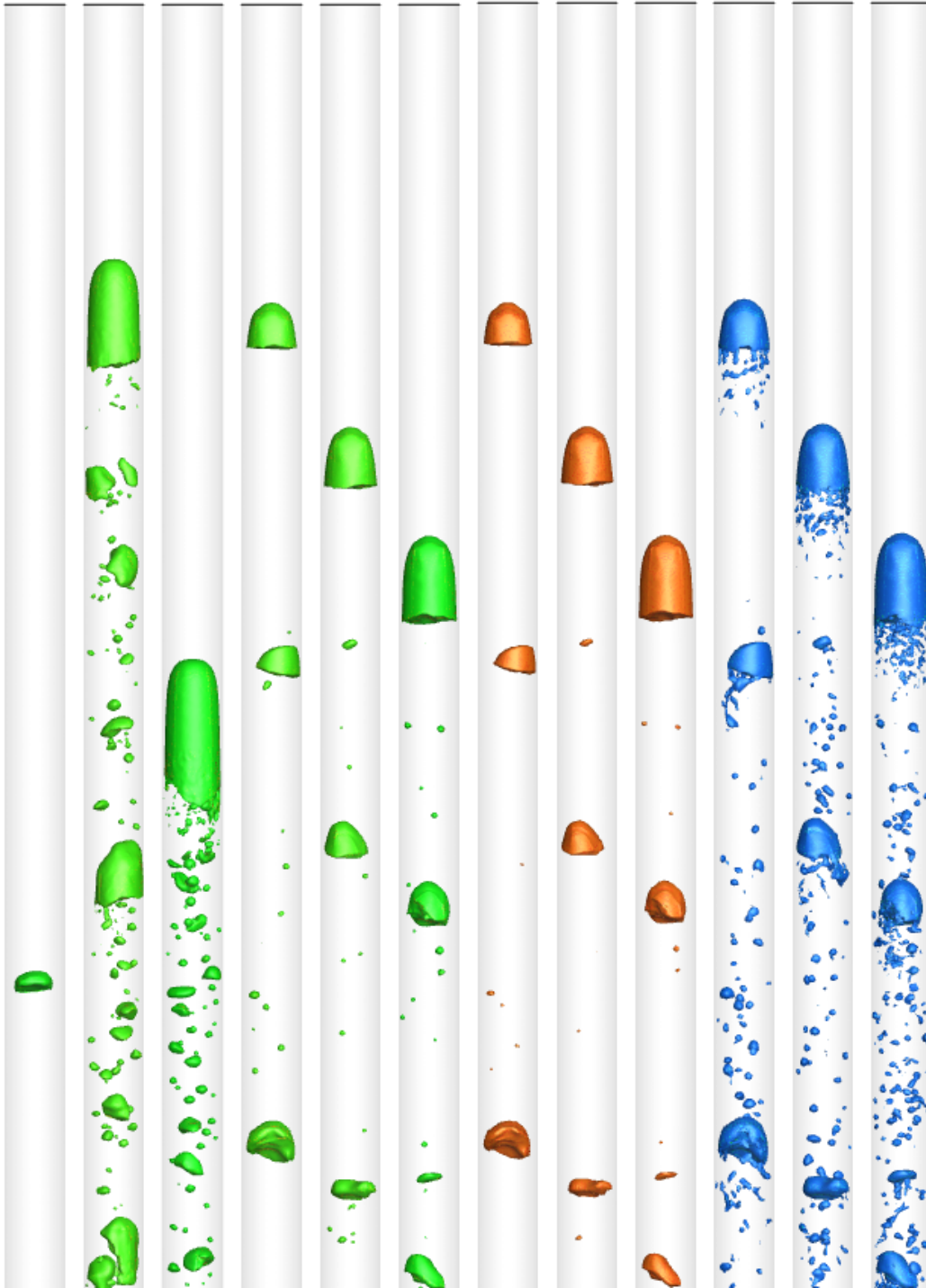


Figure 7.3: Gas volume fraction isosurfaces in the cylindrical tube simulations at $t = 14$ s for the section of the tube in the range $2.3 \text{ m} \leq H_{CFD} \leq 3 \text{ m}$; from left to right, images show isosurfaces with $\alpha_g = 0.5$ (green) obtained with meshes T1a, T1b, T2d, T3b, T3c and T3d, isosurfaces with $\alpha_g = 0.9$ (orange) obtained with meshes T3b, T3c and T3d, and isosurfaces with $\alpha_g = 0.05$ (blue) obtained with meshes T3b, T3c and T3d.

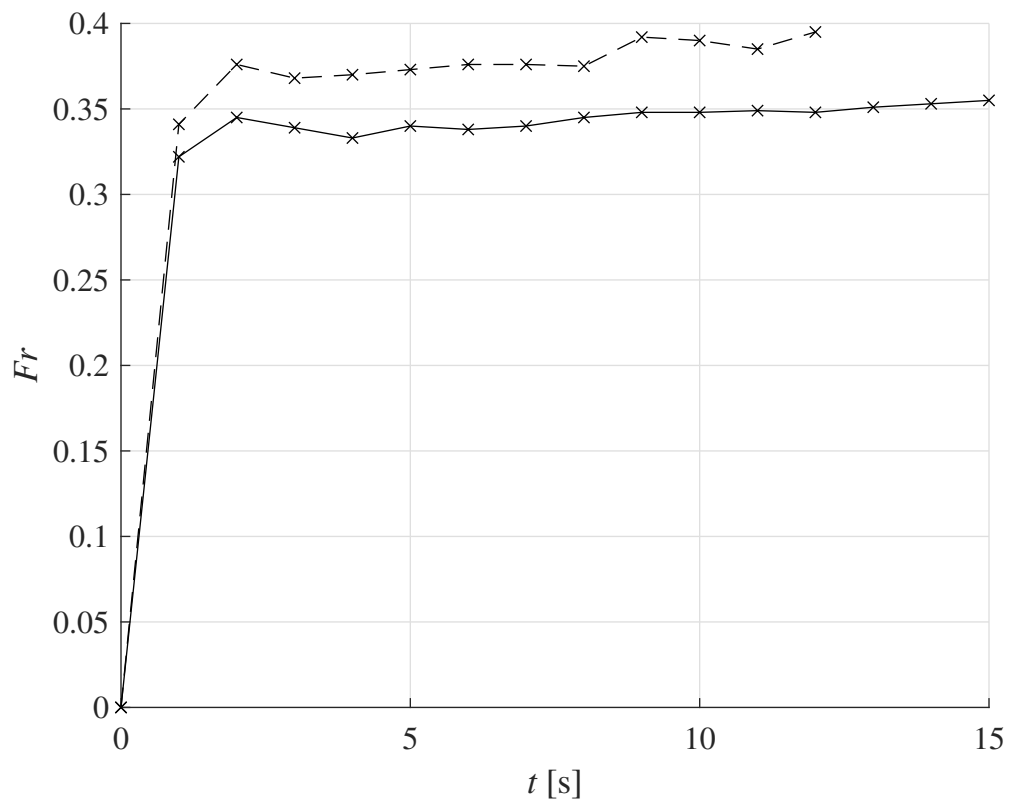


Figure 7.4: Temporal evolution of the Froude number in the cylindrical tube obtained with the T3d mesh: short bubble (solid line) and long bubble (dashed line).

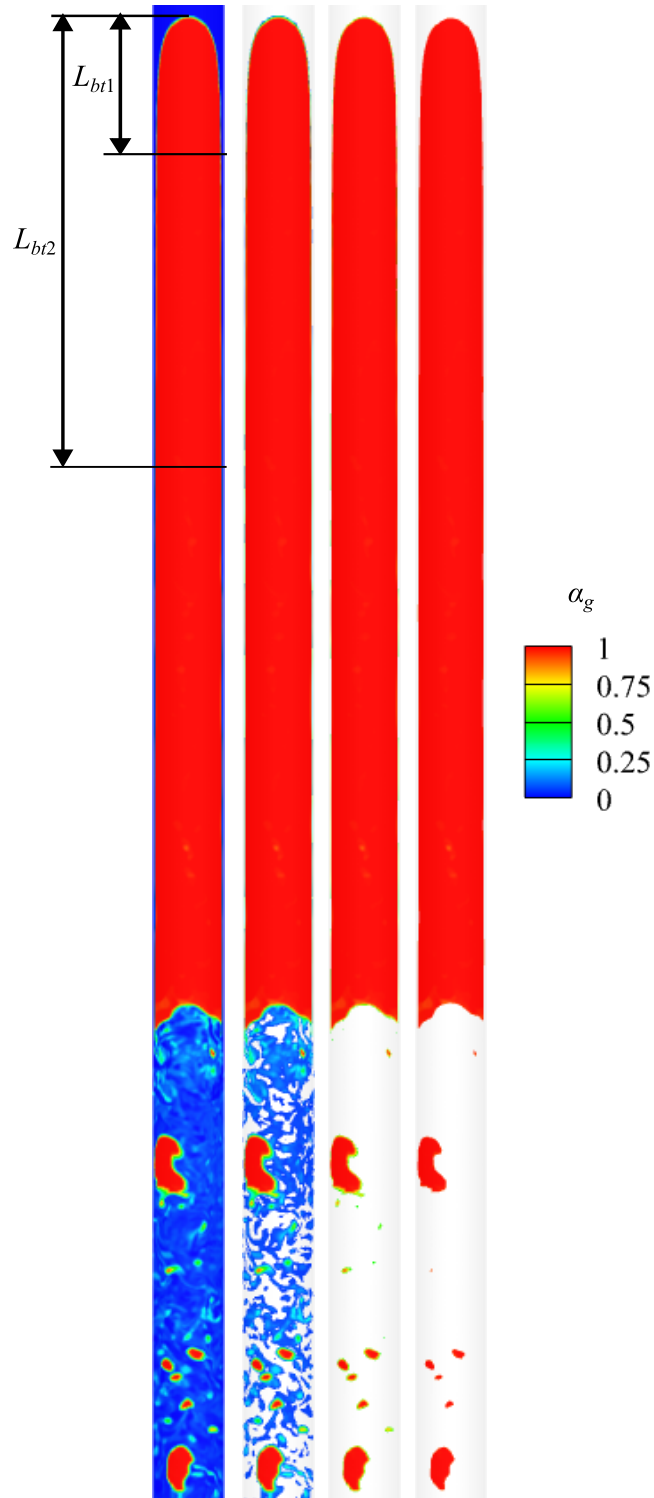


Figure 7.5: Gas volume fraction isocontours on a plane of symmetry of the cylindrical tube for the long bubble simulation ($H_{bt} = 12D_{out}$); in order from left to right, contours with $\alpha_g \geq 0$, $\alpha_g \geq 0.05$, $\alpha_g \geq 0.5$ and $\alpha_g \geq 0.9$.

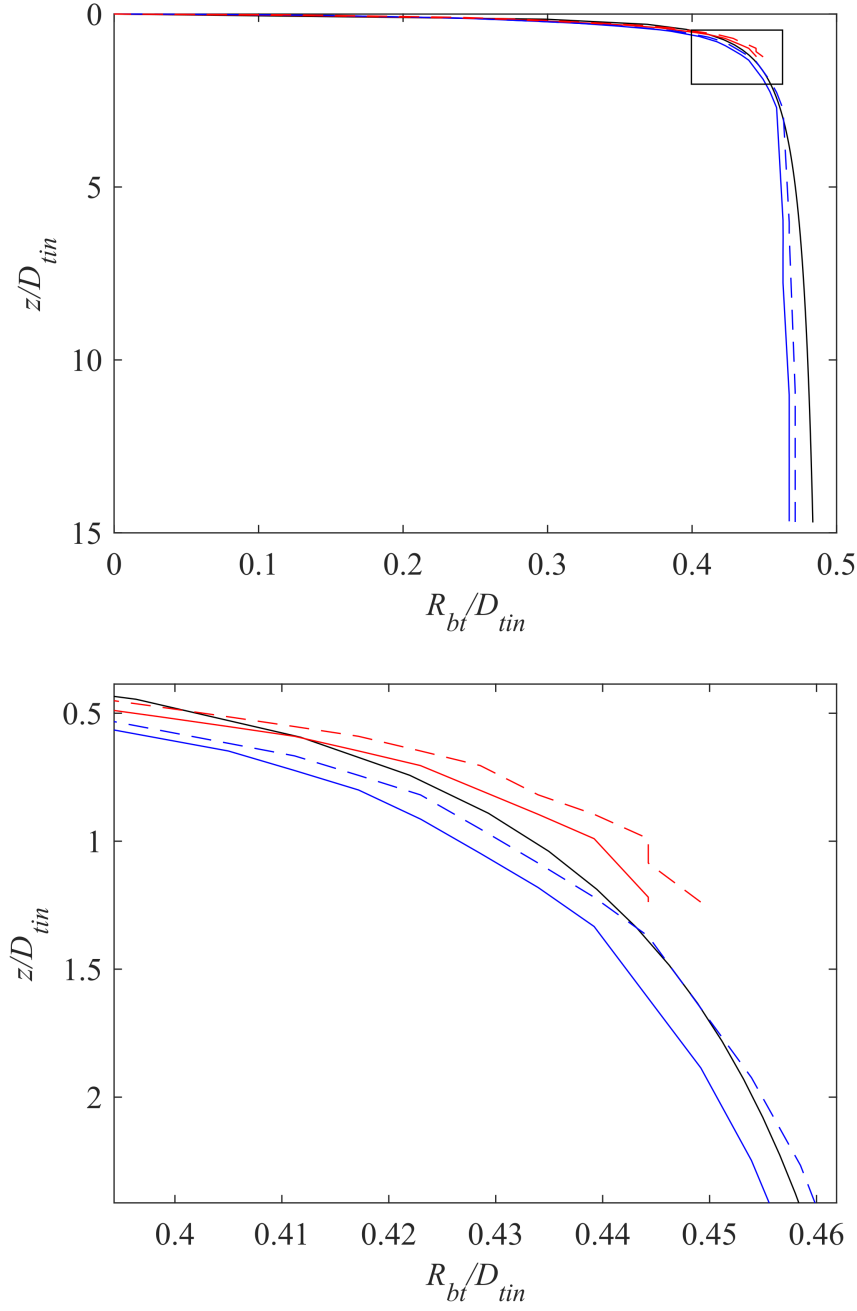


Figure 7.6: Axial variation of the radius of the longer bubble as identified by simulations using $\alpha_g \geq 0.05$ (blue dashed line) and $\alpha_g \geq 0.9$ (blue solid line); also axial variation of the shorter bubble using $\alpha_g \geq 0.05$ (red dashed line) and $\alpha_g \geq 0.9$ (red solid line); Eq. (4.2) (Dumitrescu, 1943) (black solid line); the bottom graph shows the rectangle in the top graph in zoom.

7.3 Taylor bubbles in annular channels

Simulations of a Taylor bubble rising in the annular channel filled with stagnant water were performed for all the eccentricities tested experimentally. A mesh sensitivity analysis was performed for an example case with $e = 0.7$, using four different cross-sectional meshes with 2000 (A1), 3000 (A2), 3600 (A3) and 4200 (A4) cells, as shown in Figure 7.7, and three different axial cell divisions, namely, 1500, 2000 and 4000. The mesh near the walls was refined using a two-sided hyperbolic distribution and the minimum cell size was set to 0.1 mm. The characteristics of five different meshes used are listed in Table 7.5. Table 7.6 lists the Froude numbers of a bubble released from the bottom of the channel and occupying a 50 mm long hollow cylinder, as predicted by these meshes, and its difference from the average experimental value 0.302 for a bubble with $P_r = 55$ kPa, $\Delta t_r = 0.25$ s and $L_b = 60.3$ mm. The temporal evolutions of the Froude number $Fr_{AC,CFD}$ for these five cases, shown in Figure 7.8, clearly demonstrate the quick attainment of a steady state. Because the difference between the Froude number predictions of A3c and A4c was very small, the A3c mesh was used for all subsequent simulations to keep the computational cost as low as possible. The corresponding cross-sectional mesh is shown in more detail in Figure 7.9.

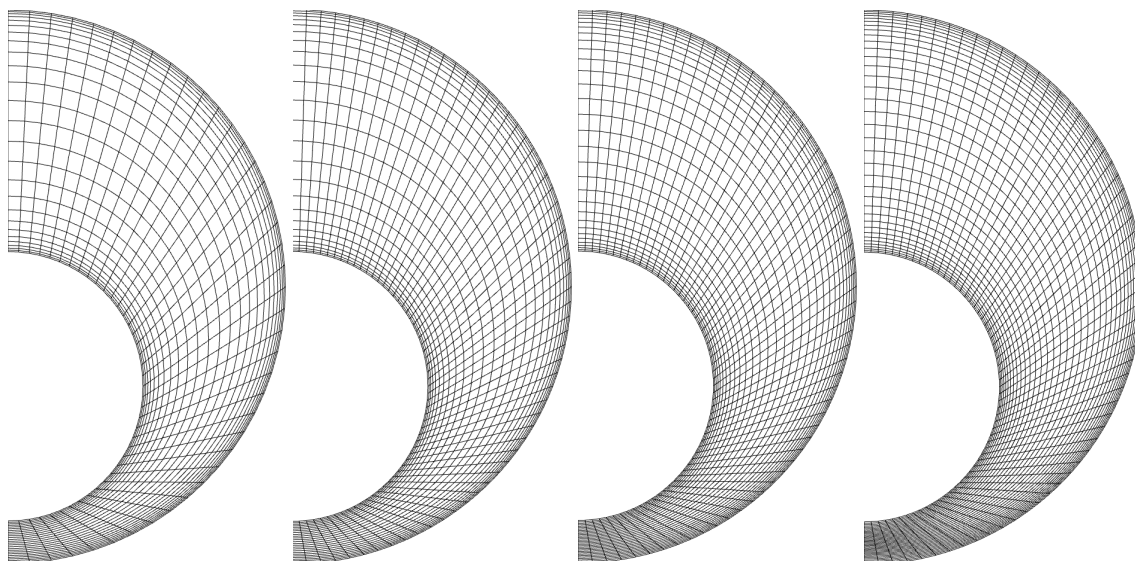


Figure 7.7: Four different cross-sectional meshes used for the annular channel studies with $e = 0.7$: A1, A2, A3 and A4 (from left to right).

Table 7.5: Characteristics of the meshes tested for mesh sensitivity analysis in simulations of flow in the eccentric annular channel with $e = 0.7$.

Symbol	Mesh name	Radial divisions	Circumferential divisions	Axial divisions	Total cells
+	A1a	25	80	1500	3 million
$\triangle$	A2c	25	120	4000	12 million
$\bigcirc$	A3c	30	120	4000	14.4 million
$\square$	A4b	35	120	2000	8.4 million
$\times$	A4c	35	120	4000	16.8 million

Table 7.6: The Froude numbers in the annular channel with $e = 0.7$ using five different meshes and their differences from the experimental value.

Symbol	$Fr_{AC,CFD}$	$\frac{Fr_{AC,CFD} - Fr_{AC,exp}}{Fr_{AC,exp}} [\%]$
+	0.295	-2.3
$\triangle$	0.294	-2.6
$\bigcirc$	0.308	2.0
$\square$	0.294	-2.6
$\times$	0.307	1.7

Simulations were performed for annular channels with the full experimental length of $L = 3$ m as well as for channels with the shorter length of $L = 20D_{out} = 0.74$ m in order to reduce the computational time. For both cases, the same cross-sectional mesh (A3) was used, but the shorter domain had 1000 axial cells, whereas the longer one had 4000 axial cells. The initial air pocket at the bottom was given a height of $H_b = 2D_{out} = 74$ mm. The temporal evolutions of the Froude numbers for the full-length and short-length simulations are plotted in Figure 7.10. After about 2 s from the start, both simulations reached steady state and their bubble velocities approached essentially the same asymptote. This result is in agreement with the literature cited in Section 5.5. The full-length simulations (14.4 million cells) were run at Alliance Canada computer clusters using 128 to 256 cores and took about two to four weeks to reach 13 s in simulation time, whereas the short-length simulations (3.6 million cells) were run at the local clusters using 44 cores and took one week to reach 3 s in simulation time. Although it was faster to run simulations on Alliance Canada clusters, these are shared resources and occasionally required long wait times between submitted jobs. Therefore, Alliance Canada resources were used mainly for

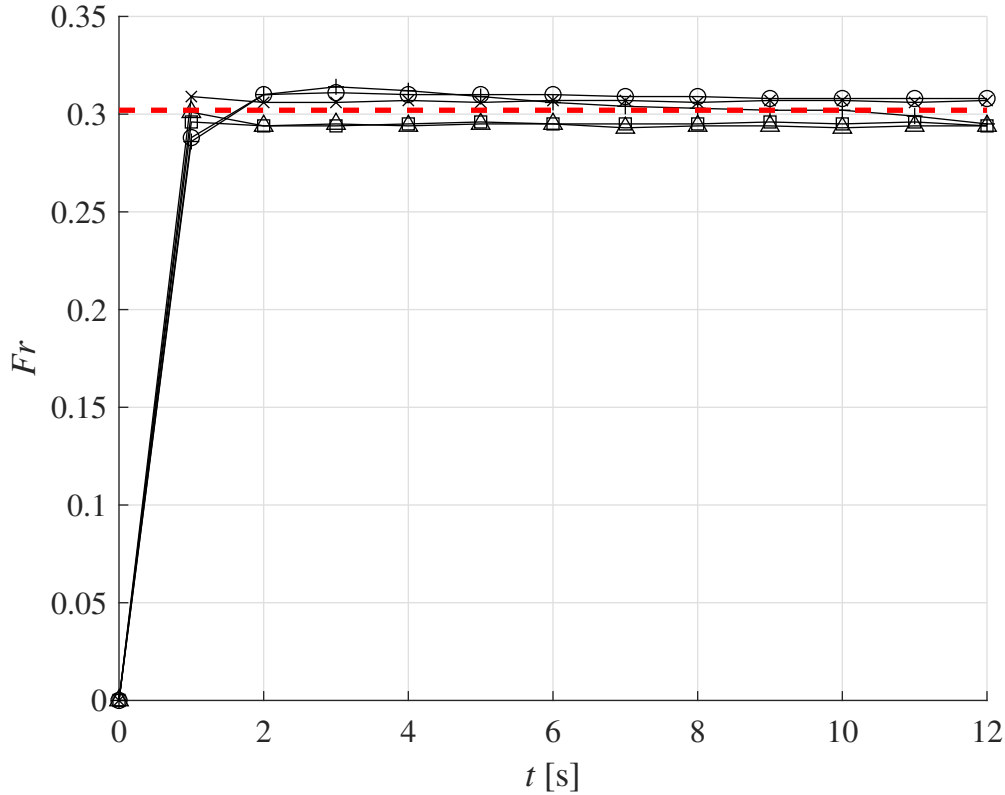


Figure 7.8: Temporal evolution of the Froude number in the annular channel with $e = 0.7$ for the following meshes: A1a (+); A2c (Δ); A3c ($\circ$); A4b ($\square$); A4c ($\times$); experimental value (red dashed line).

simulations requiring a higher resolution. Moreover, short-length simulations allowed us to run relatively fast many cases with different meshes and bubble lengths.

The cross-sectional meshes for different eccentricities are illustrated in Figure 7.11. The details of the mesh generation were explained in Section 5.4. Simulations were run to predict the bubble velocity, angle of wrap, bubble lengths equal to the reported average experimental values. All the results are presented for the short-length simulations except the Froude number, which is presented for all simulations. The dependence of the Froude number on the eccentricity is shown in Figure 7.12. The experimental results shown in this graph are for $P_r = 69$ kPa and $\Delta t_r = 0.25$ s, which had average lengths that were intermediate between the average lengths produced in the two CFD simulations. The difference between the Froude numbers in the CFD and the experimental results was 5% at most. The largest difference was observed for the short-length concentric channel

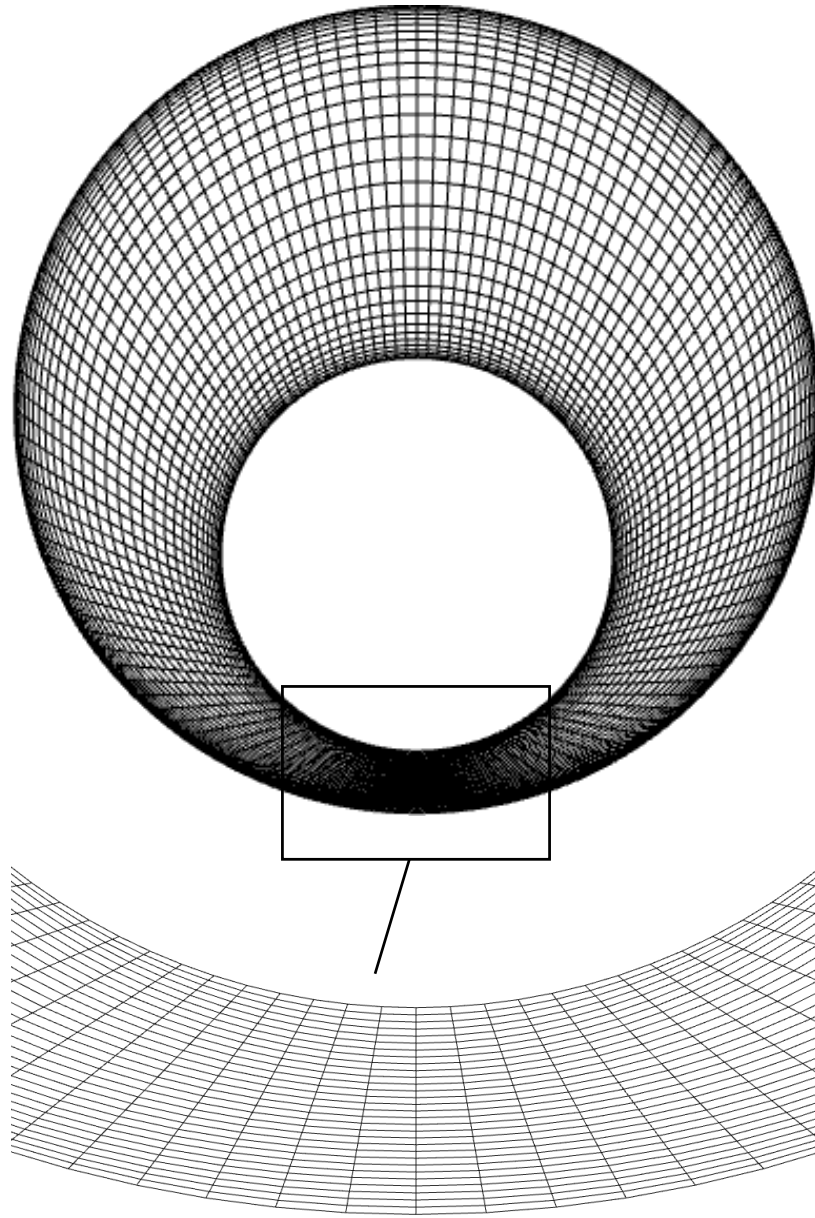


Figure 7.9: The A3 cross-sectional mesh for $e = 0.7$, showing details in the narrow gap.

simulation, where the Froude number was 0.330. The concentric geometry is exceptional as it does not have a preferential circumferential position of the bubble, which may thus move sideways as it rises. Such motion may require a higher axial resolution than axially moving bubbles to be resolved accurately. In fact, a simulation using the same cross-sectional mesh (A3) with twice as many axial divisions in the mesh for the short-length concentric channel (7.2 million cells) gave $Fr_{AC,CFD} = 0.312$, a value that is very close to

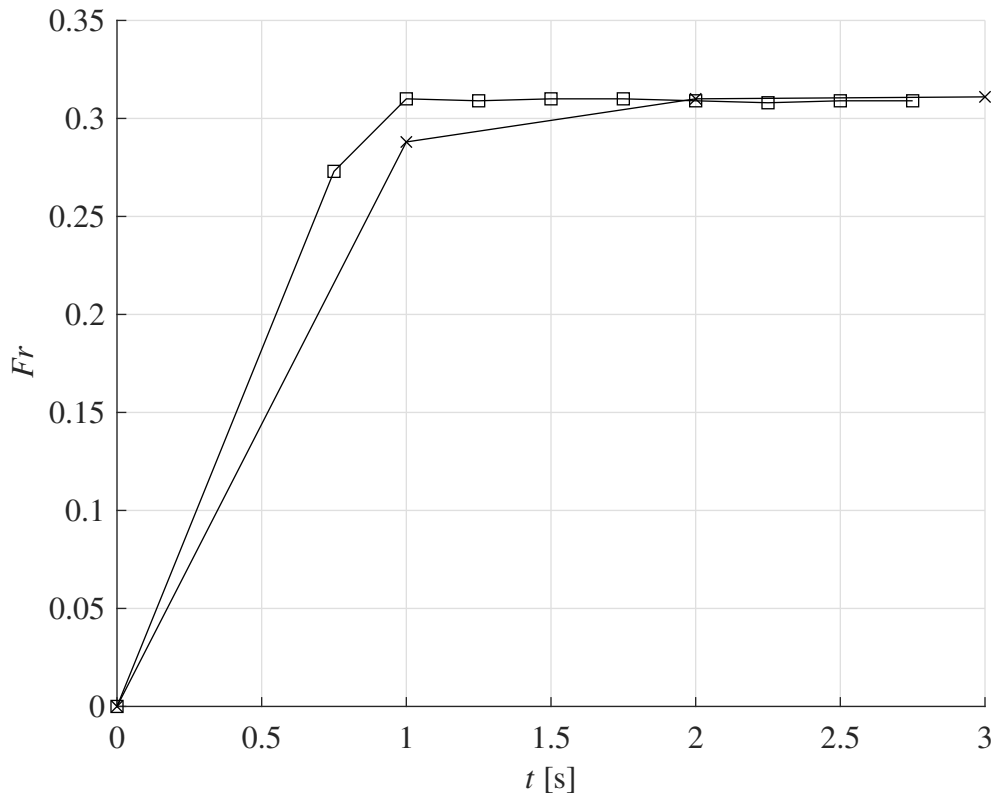


Figure 7.10: Temporal evolution of the Froude number in the shorter (□) and longer (×) annular channels with $e = 0.7$.

the corresponding experimental value $Fr = 0.313$. Therefore, as an exception, the results presented in the following figures and tables for the concentric annulus were obtained with the higher mesh resolution.

Analysis of the bubble shape was done using Tecplot 360 EX 2020 R2 post-processing visualization and analysis tool. Instead of running separate simulations for each desired bubble length, a single simulation was performed for a bubble that was longer than most bubbles observed experimentally and the simulated bubble was truncated to each experimental length for comparisons. An example of this procedure is shown in Figure 7.13 for the short-length simulation with $e = 0.3$. Once the bubble was truncated, the minimum liquid film thickness and the maximum angle of wrap were determined visually from the top view. The bubble interface was determined as the isosurface with $\alpha_g \geq 0.9$. Side views of the simulated bubbles at different eccentricities are shown in Figure 7.14. The bubble

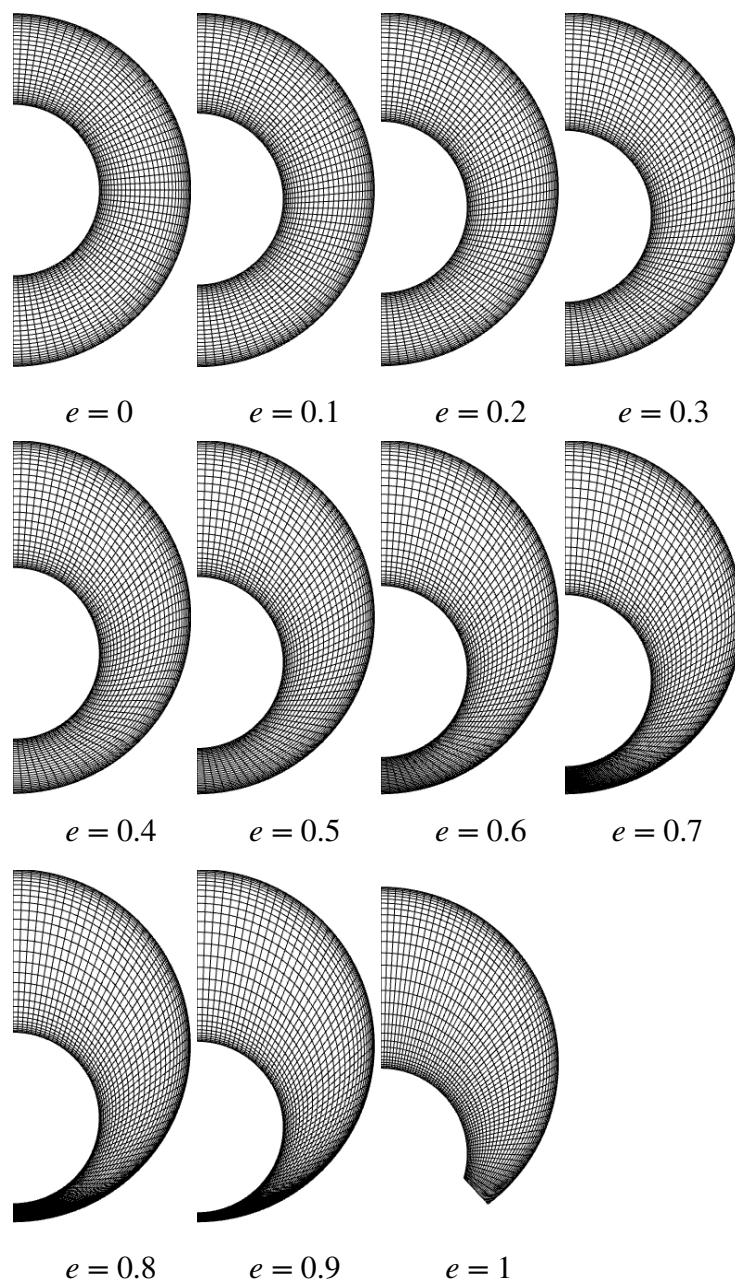


Figure 7.11: The A3 cross-sectional meshes for all eccentricities simulated.

shape changed significantly, which also was evident in the experimental images shown in Figure 6.18. As the eccentricity was increased, the bubble sides became less wrapped around the inner tube and moved towards the wide gap. The rate of change of the angle of wrap in the downward direction showed a similar trend as the velocity; it increased from a lower value for $e = 0$, reached a maximum for $e = 0.2 - 0.3$ and then decreased towards

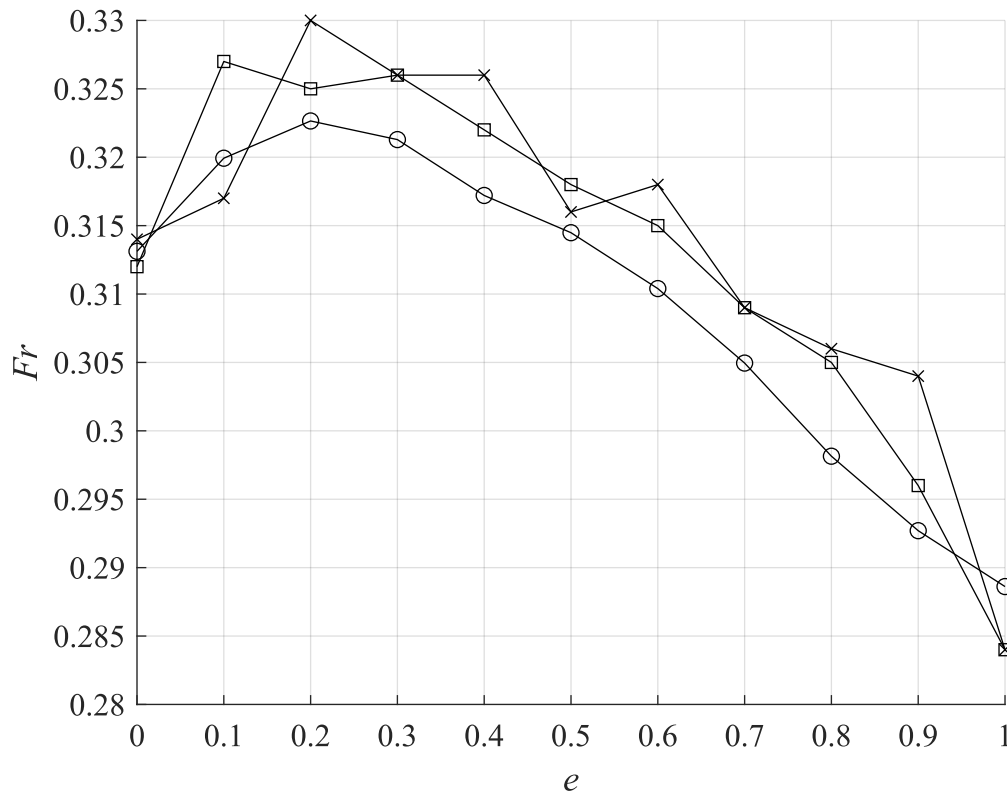


Figure 7.12: Froude number dependence on the eccentricity for the full-length (×) and short-length (□) simulations along with the experimental results (○) for $P_r = 69$ kPa and $\Delta t_r = 0.25$ s.

its lowest value for $e = 1$. The change of angle of wrap illustrates the ensuing change in bubble shape: the bubble nose was more streamlined for $e = 0.2 - 0.3$ and increasingly blunter at higher eccentricities.

The maximum angle of wrap values for calculated bubbles rising in channels with different eccentricities, each truncated to a length equal to the corresponding average experimental length, are listed in Table 7.7. The same bubble release conditions in the simulations ended up creating bubbles with sizes that changed with changing eccentricity, as also observed in the experiments. It is noted that, in the simulations for $e = 0.7, 0.8$ and 0.9 , the bubbles were found to be slightly shorter (up to 6%) than the corresponding average experimental lengths at $P_r = 83$ kPa and $\Delta t_r = 0.25$ s. To distinguish these cases, values corresponding to these three eccentricities at the specified release pressure and time are marked with an asterisk in the tables, which indicates that these values likely under predict

slightly the bubble volume and the maximum angle of wrap while slightly over predicting the liquid film thicknesses. As observed in the experimental results, and in agreement with the previous simulation results of Mitchell *et al.* (2021); Liu *et al.* (2024), the maximum angle of wrap decreased as the eccentricity was increased. Figure 7.15 shows the results of the present simulations compared to the corresponding experimental results and numerical simulations from the literature. Although the conditions of these simulations are different, both showed the same trend that the maximum angle of wrap decreased with increasing eccentricity. Figures 7.16 and 7.17 show planar views of the experimental and simulated bubbles for the average experimental lengths for $P_r = 55$ kPa and $\Delta t_r = 0.25$ s. The difference between corresponding results was at most 4%.

Table 7.7: Maximum angle of wrap for different eccentricities and air release condition; values marked with an asterisk are likely underestimated by a small percentage.

e	$\theta_{max} [^\circ]$				
	$P_r = 41$ kPa $\Delta t_r = 0.25$ s	$P_r = 41$ kPa $\Delta t_r = 0.5$ s	$P_r = 55$ kPa $\Delta t_r = 0.25$ s	$P_r = 69$ kPa $\Delta t_r = 0.25$ s	$P_r = 83$ kPa $\Delta t_r = 0.25$ s
0	285	297	302	305	309
0.1	281	287	294	300	301
0.2	269	278	285	292	294
0.3	257	268	274	281	283
0.4	243	255	263	271	274
0.5	231	242	252	259	260
0.6	216	229	237	244	245
0.7	205	215	220	227	228*
0.8	187	196	202	205	207*
0.9	177	184	187	190	191*
1	155	161	163	165	165

The minimum inner and outer liquid film thicknesses were determined from the top views of the truncated bubbles, as shown in Figure 7.13. Table 7.8 lists the calculated outer liquid film thicknesses corresponding to different experimental conditions. It is evident that the outer liquid film thickness increased significantly with increasing eccentricity. This result is in agreement with the experimental results of Rohilla & Das (2019) and a qualitative observation by Liu *et al.* (2024), although the conditions of these experiments

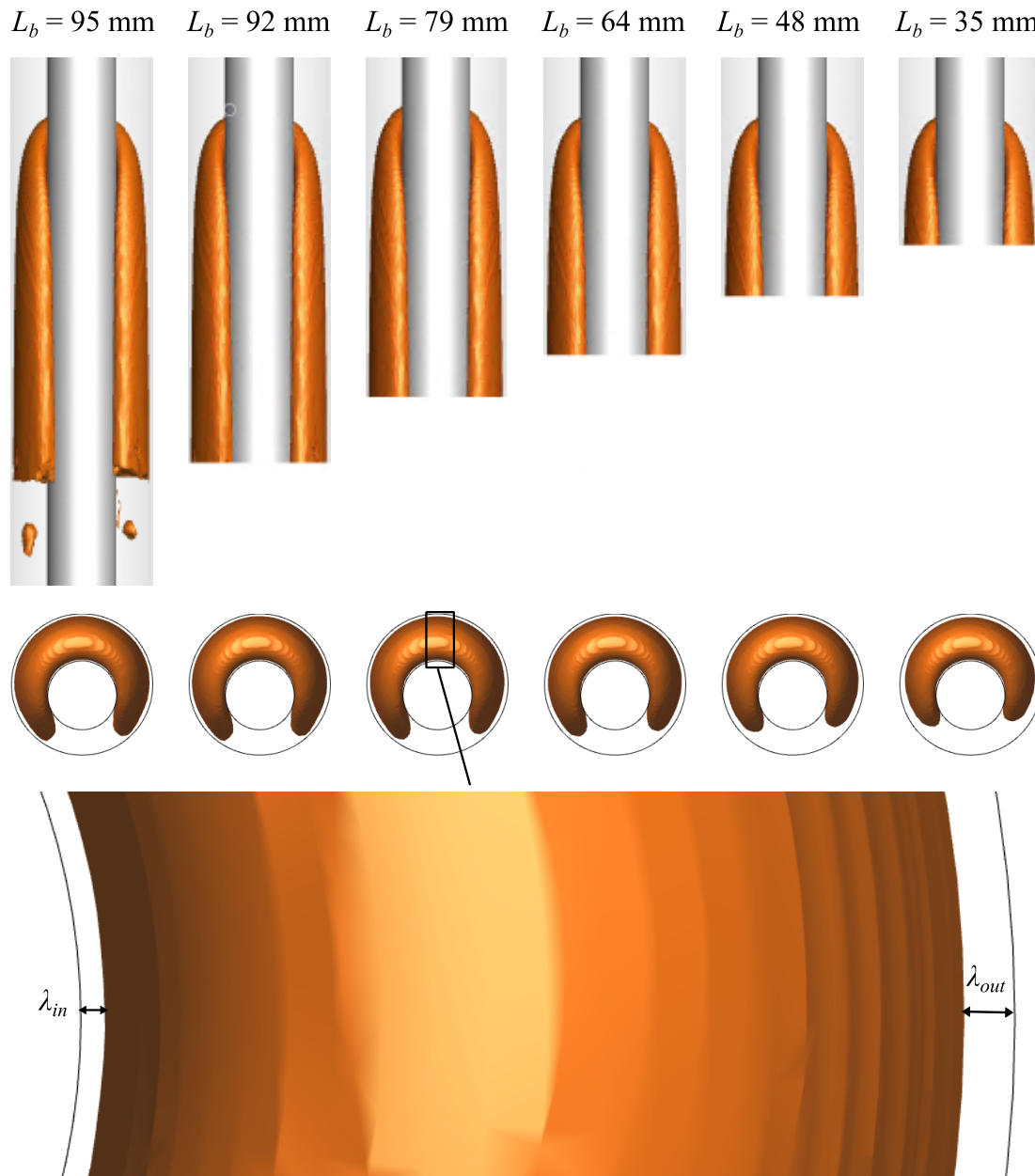


Figure 7.13: Top: front views of a simulated bubble (orange) truncated to different average experimental lengths; bottom: top views of the truncated bubbles and measurement location for the liquid film thicknesses.

and simulations are different. The calculated inner liquid film thicknesses corresponding to different experimental conditions are listed in Table 7.9. It is evident that the outer film thickness is thicker than the inner liquid film thickness for all the cases. The inner liquid film thickness showed a slight increase as the eccentricity was changed from 0 to 1,

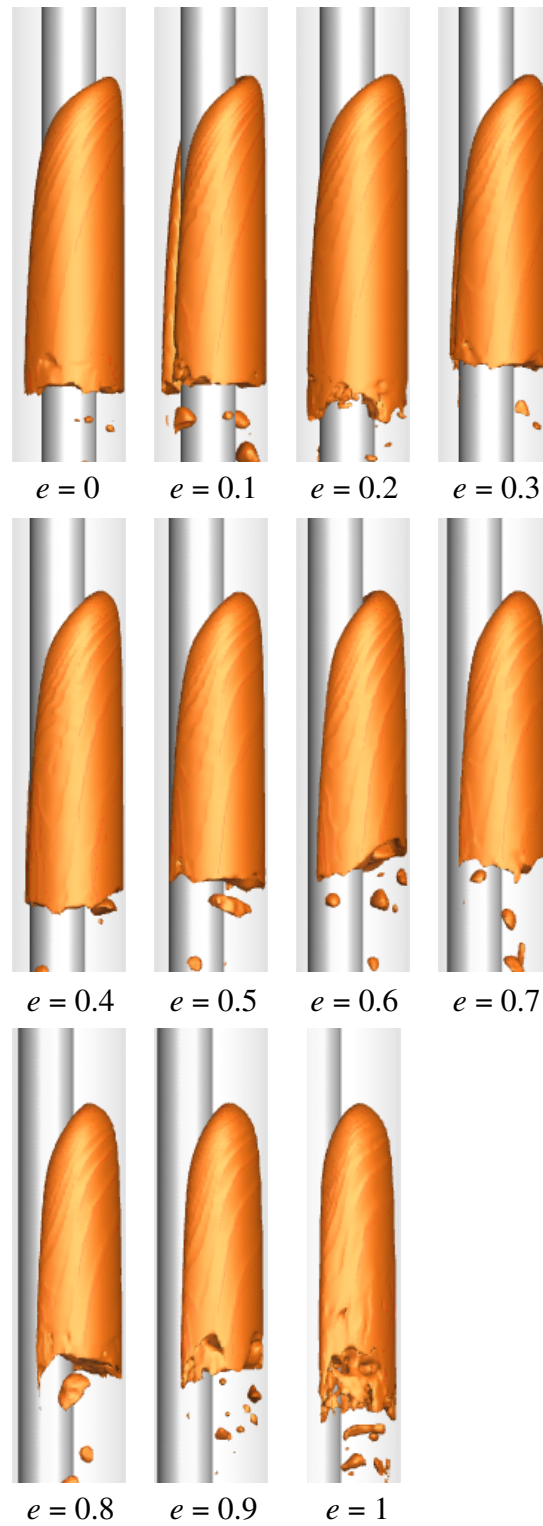


Figure 7.14: Side views of simulated bubbles at different eccentricities.

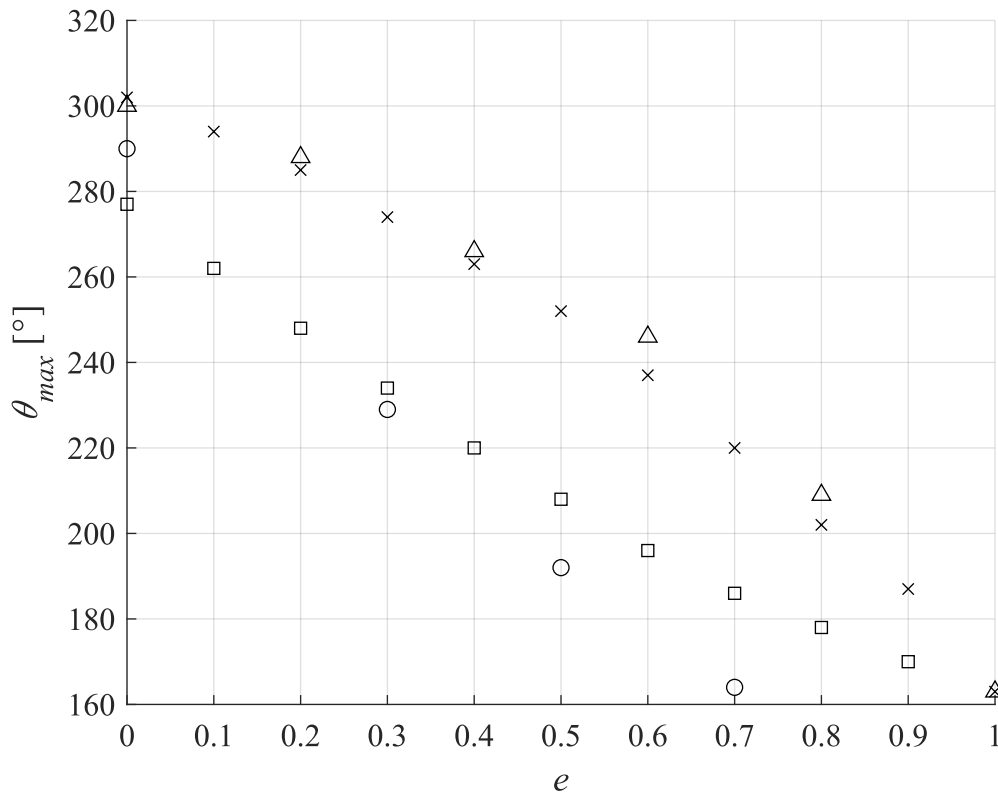


Figure 7.15: The maximum angle of wrap vs. eccentricity; CFD results for average experimental lengths for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s ($\times$); present experiments for $P_r = 55$ kPa, $\Delta t_r = 0.25$ s (Δ); numerical simulations of Mitchell *et al.* (2021) ($\square$) and Liu *et al.* (2024) ($\circ$).

however, there is not sufficient evidence to support the presence of a consistent trend. A much finer mesh near the walls would be required for this analysis to describe accurately the liquid film characteristics. As there are no experimental studies of the inner film thickness around Taylor bubbles in eccentric channels, the present CFD results are at present the only relevant source of information.

The bubble volumes from the CFD analysis were calculated by integrating the volume fraction of gas and they are listed in Table 7.10. The effect of eccentricity on bubble volume, if any, cannot be discerned from these data. A comparison of bubble volumes determined from the experiments with those calculated from the CFD simulations, all of which have been listed in Figure 7.18, shows that the CFD volumes were lower than the experimental ones. The maximum difference, which was observed for the shorter bubbles,

Table 7.8: Outer liquid film thickness for different eccentricities and corresponding air release conditions; values marked with an asterisk are likely overestimated by a small percentage.

e	$\lambda_{out} [mm]$				
	$P_r = 41 \text{ kPa}$	$P_r = 41 \text{ kPa}$	$P_r = 55 \text{ kPa}$	$P_r = 69 \text{ kPa}$	$P_r = 83 \text{ kPa}$
	$\Delta t_r = 0.25 \text{ s}$	$\Delta t_r = 0.5 \text{ s}$	$\Delta t_r = 0.25 \text{ s}$	$\Delta t_r = 0.25 \text{ s}$	$\Delta t_r = 0.25 \text{ s}$
0	0.64	0.52	0.51	0.50	0.50
0.1	0.70	0.66	0.54	0.53	0.52
0.2	0.75	0.70	0.66	0.59	0.55
0.3	0.87	0.75	0.71	0.67	0.65
0.4	1.03	0.91	0.77	0.74	0.73
0.5	1.24	1.05	0.91	0.80	0.78
0.6	1.40	1.11	1.05	0.90	0.85
0.7	1.52	1.34	1.13	1.08	1.06*
0.8	1.75	1.47	1.34	1.15	1.13*
0.9	2.00	1.60	1.51	1.40	1.25*
1	2.19	1.84	1.60	1.57	1.50

was 9%.

Table 7.9: Inner liquid film thickness for different eccentricities and corresponding air release conditions; values marked with an asterisk are likely overestimated by a small percentage.

e	$\lambda_{in} [mm]$				
	$P_r = 41 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 41 \text{ kPa}$ $\Delta t_r = 0.5 \text{ s}$	$P_r = 55 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 69 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 83 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$
0	0.31	0.30	0.30	0.30	0.26
0.1	0.33	0.33	0.32	0.31	0.31
0.2	0.33	0.33	0.32	0.30	0.30
0.3	0.34	0.34	0.32	0.32	0.32
0.4	0.35	0.35	0.31	0.27	0.27
0.5	0.34	0.33	0.31	0.31	0.31
0.6	0.35	0.33	0.30	0.30	0.30
0.7	0.35	0.34	0.34	0.34	0.34*
0.8	0.35	0.33	0.33	0.33	0.33*
0.9	0.36	0.34	0.34	0.34	0.34*
1	0.37	0.37	0.37	0.36	0.35

Table 7.10: Bubble volumes for different eccentricities and corresponding air release conditions; values marked with an asterisk are likely underestimated by a small percentage.

e	$V_b \times 10^6 [m^3]$				
	$P_r = 41 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 41 \text{ kPa}$ $\Delta t_r = 0.5 \text{ s}$	$P_r = 55 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 69 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$	$P_r = 83 \text{ kPa}$ $\Delta t_r = 0.25 \text{ s}$
0	11.9	22.1	29.3	41.8	49.0
0.1	14.0	20.4	29.8	41.9	51.0
0.2	14.5	22.3	31.3	41.6	51.1
0.3	14.5	22.3	31.7	41.1	49.6
0.4	13.7	22.3	31.3	42.5	52.5
0.5	14.1	21.4	31.8	42.6	48.3
0.6	12.6	22.6	30.3	43.9	48.5
0.7	13.9	22.7	30.4	41.7	49.1*
0.8	13.8	23.0	30.3	43.9	47.2*
0.9	14.7	23.1	31.4	41.3	47.8*
1	12.7	23.0	30.6	43.1	51.1

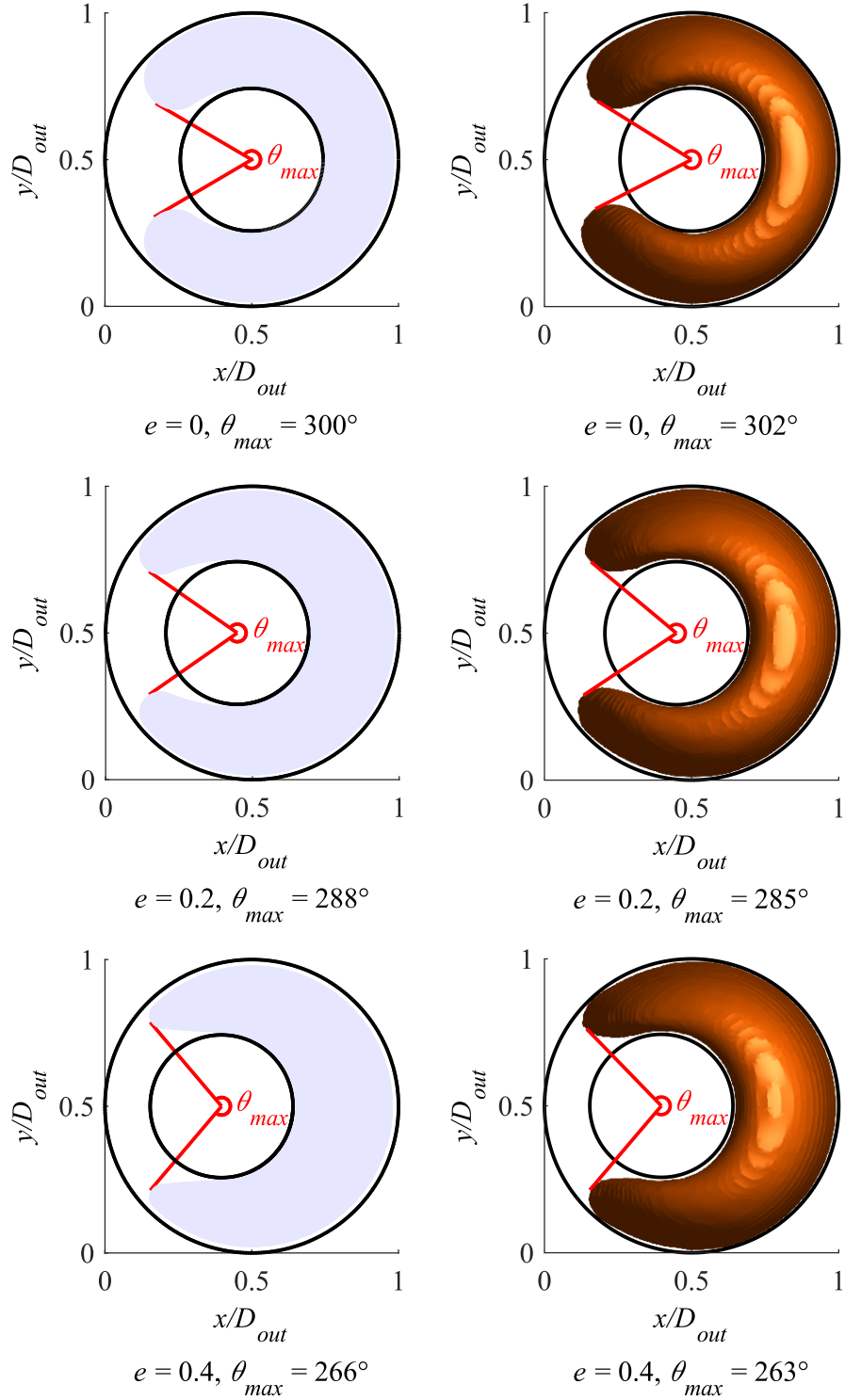


Figure 7.16: Left column: experimental values of the maximum angle of wrap for $e = 0, 0.2$ and 0.4 at $P_r = 55$ kPa and $\Delta t_r = 0.25$; right column: maximum angle of wrap for simulated bubbles at $H_b = 2D_{out} = 74$ mm).

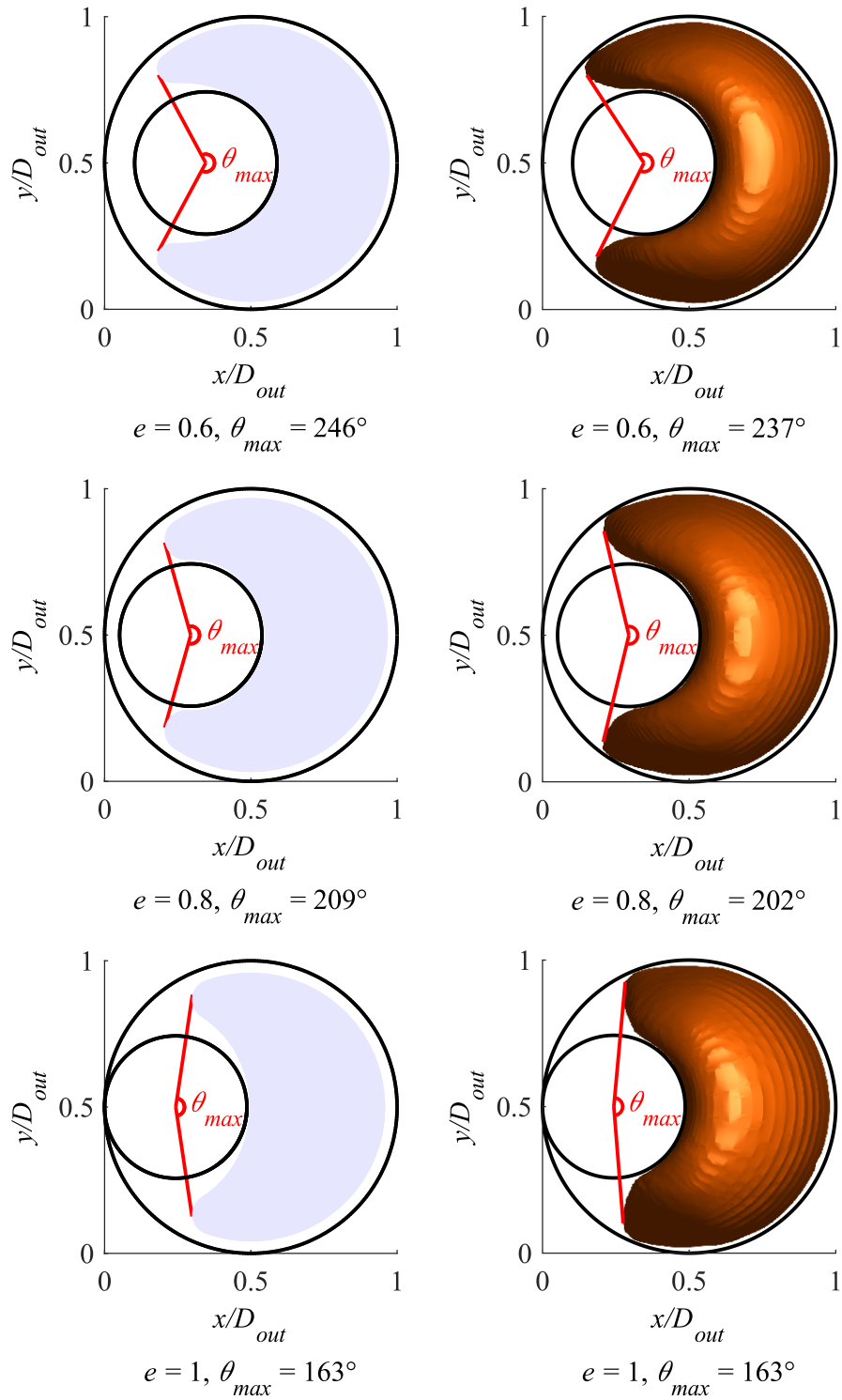


Figure 7.17: Left column: experimental values of the maximum angle of wrap for $e = 0.6, 0.8$ and 1 at $P_r = 55$ kPa and $\Delta t_r = 0.25$; right column: maximum angle of wrap for simulated bubbles at $H_b = 2D_{out} = 74$ mm).

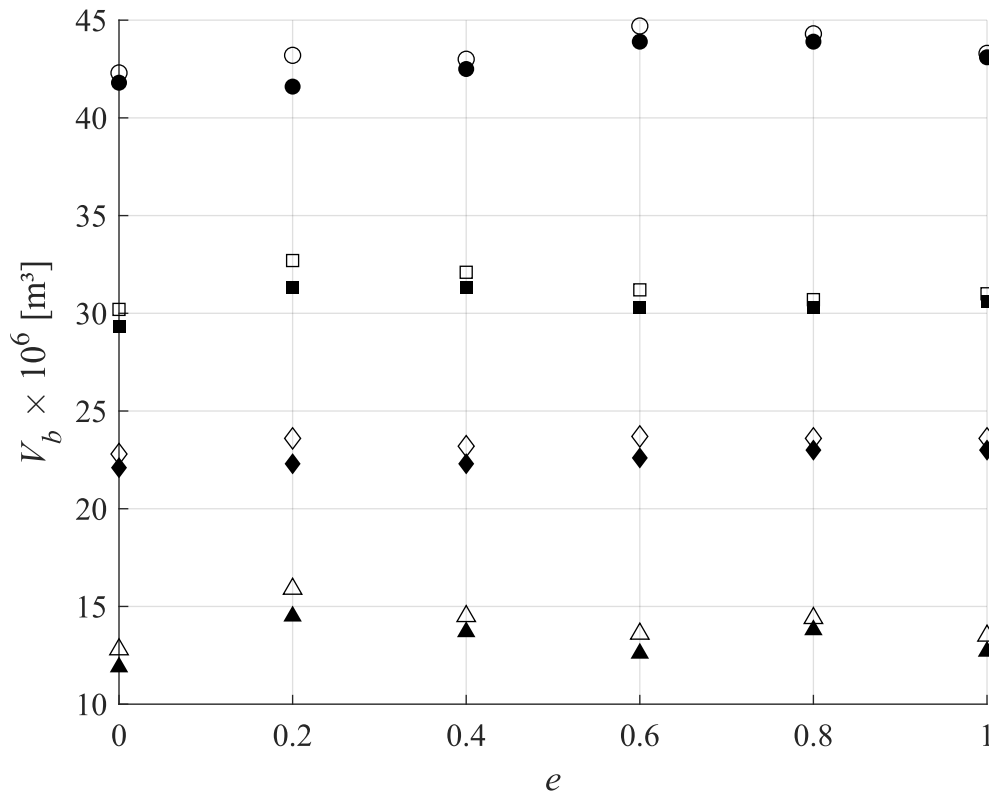


Figure 7.18: Average bubble volume vs. eccentricity; experimental results for $P_r = 41 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ ($\triangle$), $P_r = 41 \text{ kPa}$, $\Delta t_r = 0.5 \text{ s}$ ($\diamond$), $P_r = 55 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ ($\square$), $P_r = 69 \text{ kPa}$, $\Delta t_r = 0.25 \text{ s}$ ($\circ$); corresponding CFD results presented with the filled symbols.

7.4 Air-water flows in annular channels

Simulations of air-water flows in eccentric annular channels were performed for three representative sets of flow rates, each of which corresponded to those in our experiments with flows in the bubbly, slug and churn regime. For each of these three cases, simulations were performed for three eccentricities, namely, $e = 0.3, 0.5$ and 0.7 , deemed to be subject to relatively weak, medium and strong eccentricity effects, if any. Simulations of the concentric channel flow in the slug regime were also performed. The addition of an inlet nozzle to the stagnant column geometry made it unsuitable for the directed mesh used in the previous simulations. Therefore, an automated mesh generator was used to generate the mesh using polyhedral mesher, a surface remesher and a prism layer mesher. The target surface size was set to 0.75 mm with a surface growth rate of 1.2 . 12 prism layers were used with a total thickness of 1 mm to capture the flow close to the wall regions. A mesh with 3.7 million cells was used, which was considered to be sufficient in view of our experience in the stagnant column mesh dependence studies. The cross-sectional meshes used in all simulations are shown in Figure 7.19; a mesh close to walls was shown previously in Section 5.4. Although an adaptive time step was used for these simulations, the presence of an inlet nozzle with relatively high air velocity required a small time step, which was around 10^{-5} s. This extended the time required to get the solution. Analysis of a single full-length channel would take over a year (16 months) of CPU time in the local computers and 4 – 6 months in Alliance Canada computers. Such requirement was not deemed to be feasible and, for this reason, all analyses were performed for short-length channels, having a length of $20D_{out} = 0.74$ m. Each of these cases took approximately 4 months of CPU time in local computers and 1 – 2 months in Alliance Canada computers. It is noted that the simulations were run uninterrupted on local computers, whereas they were subjected to time limitations and queues on Alliance Canada computers. The short-length simulations produced realistic flow patterns but it is possible that, had the channel been of full length, the observed flow patterns may have evolved further. In particular, it is possible that, in a much longer channel, bubbles may have coalesced to larger ones, as observed in the experiments, and may have undergone additional transitions from bubbly to cap-bubbly flows and from cap-bubbly to slug flows. Results of the simulations and comparisons to the experimental results are discussed below.

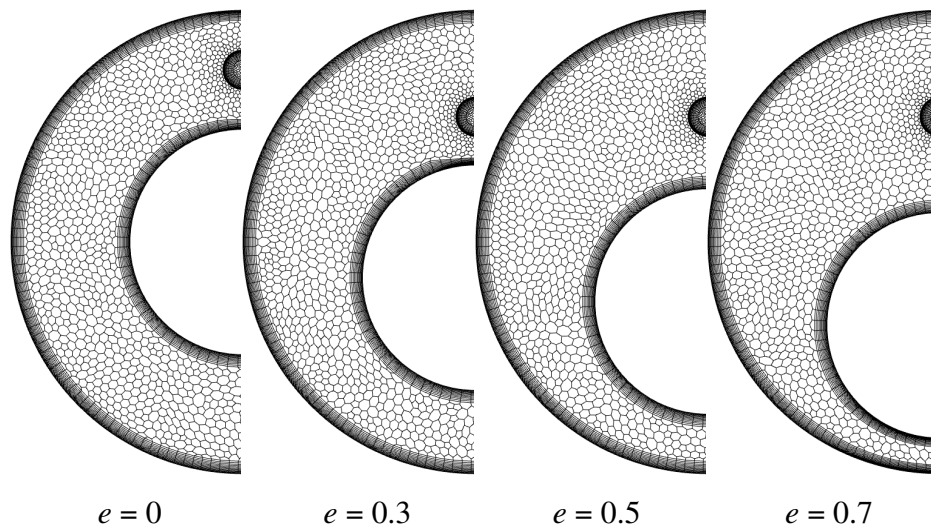


Figure 7.19: Cross-sectional meshes for all eccentricities used in the air-water flow simulations.

a) **Bubbly flow:** Simulations were performed for $j_l = 0.270$ m/s, $j_g = 0.004$ m/s and $e = 0.3, 0.5$ and 0.7 , for all of which we have also collected experimental results. Isocontours of the computed air volume fraction on the $x-z$ and $y-z$ planes (Figure 4.12) are presented in Figures 7.20 and 7.21. These simulations were able to capture small bubbles and their oscillatory zig-zag movement, similar to those observed in the experiments. The circumferential amplitude of this movement is visibly larger for the case with $e = 0.7$, especially towards the downstream end of the channel. None of the calculated bubbles seem to move across the narrow gap in these simulations, which may be a result of the air nozzle placement in the wide gap. In the experiments, the bubbles travelled mostly in the wide gap, but a few occasionally moved into the narrow gap. Moreover, the concentration of bubbles observed in these simulations was smaller than that in the experimental visualizations. One of the reasons for this may be the difference in inlet geometry between the simulations and the experiments. A muffler-type nozzle with multiple small holes was used in the experiments, whereas a single-hole nozzle was used for the simulations. Another difference between the simulations and the experiments was that the channel length in the simulations was much shorter. However, the effect of this difference could not be assessed for the bubbly flow simulations because a change in the flow regime, in this case transition from bubbly to cap-bubbly flow, was observed only for j_g values that were higher than the value in these simulations.

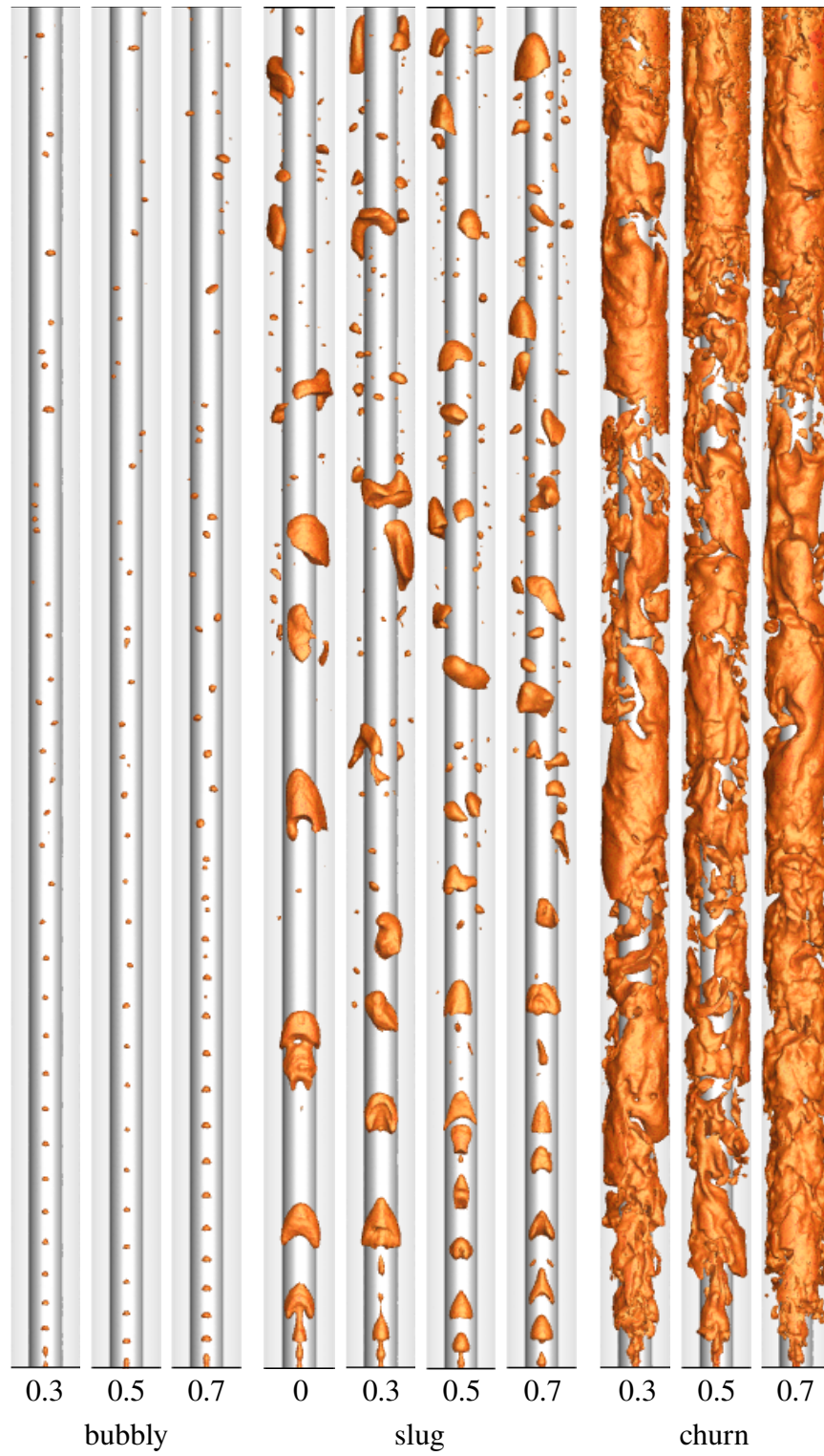


Figure 7.20: Isosurfaces of the volume fraction of air on the $x - z$ plane having $\alpha_g \geq 0.9$; numbers denote the eccentricity values.

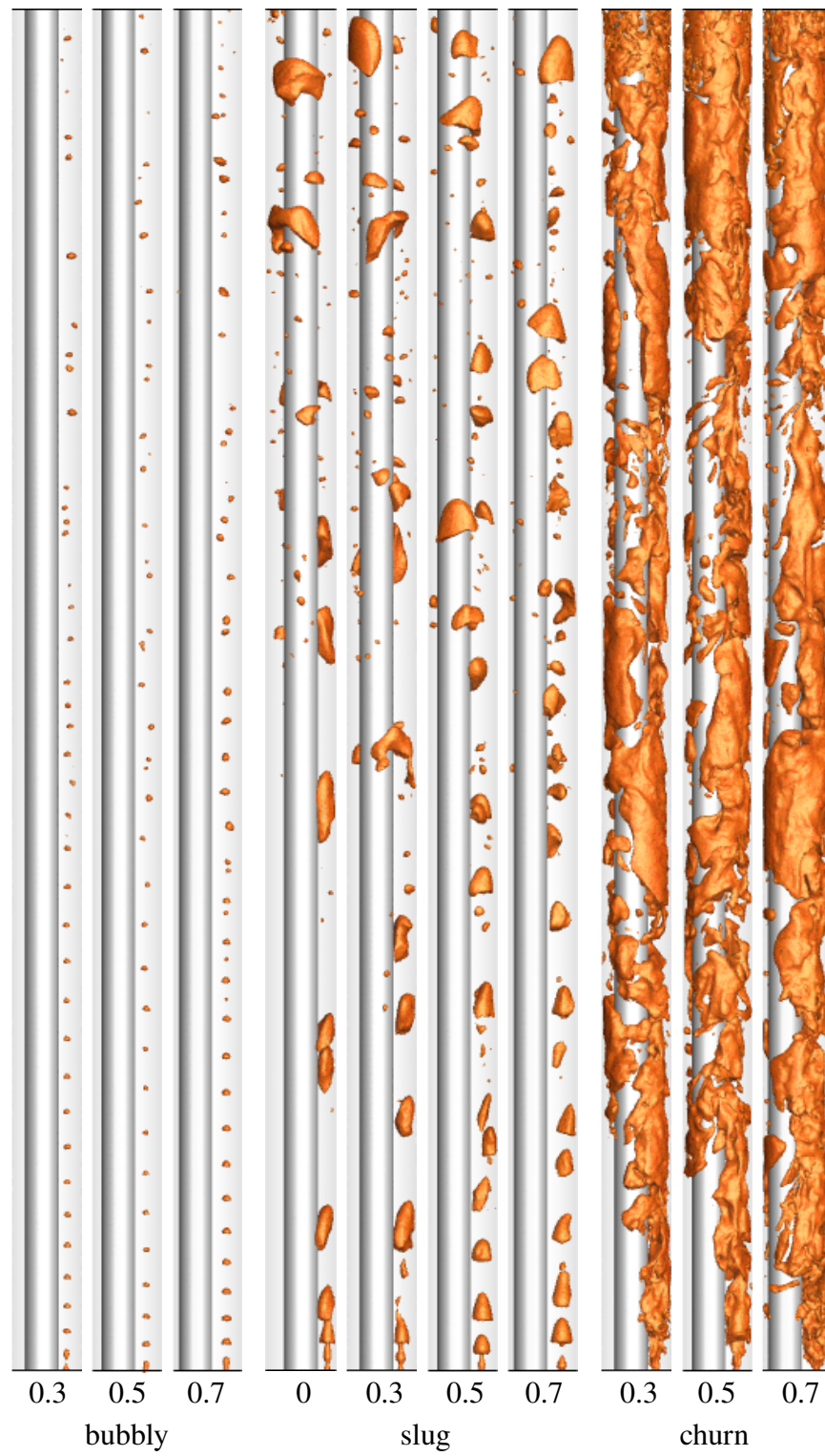


Figure 7.21: Isosurfaces of the volume fraction of air on the $y-z$ plane having $\alpha_g \geq 0.9$; numbers denote the eccentricity values.

Figure 7.22 shows instantaneous cross-sectional maps of the calculated axial velocity U_z , normalized by j_l , at multiple axial locations of the eccentric annular channels. The isocontours were created after 4 s in simulation time to allow sufficient flow development. Near the inlet ($z/D_h \leq 10.5$), the maps did not differ from each other significantly. Around mid-channel and beyond ($z/D_h \geq 20$), however, the flow difference between the narrow and wide gap became obvious and cross-flow oscillations were observed, as the high axial velocity region swung strongly circumferentially. The amplitude of cross-flows became larger with an increase in eccentricity from 0.3 to 0.7. These cross-flows are evident more clearly in Figure 7.23, in which axial velocity maps are presented on closely spaced $x - y$ planes for the strongly eccentric channel ($e = 0.7$). From these images, one may estimate very roughly the axial wavelength of the oscillations as $10D_h \approx 10D_{in}$. These observations are in fair qualitative agreement with results by Choueiri & Tavoularis (2014) and many other authors and clearly prove the presence of gap vortex streets in the eccentric channel.

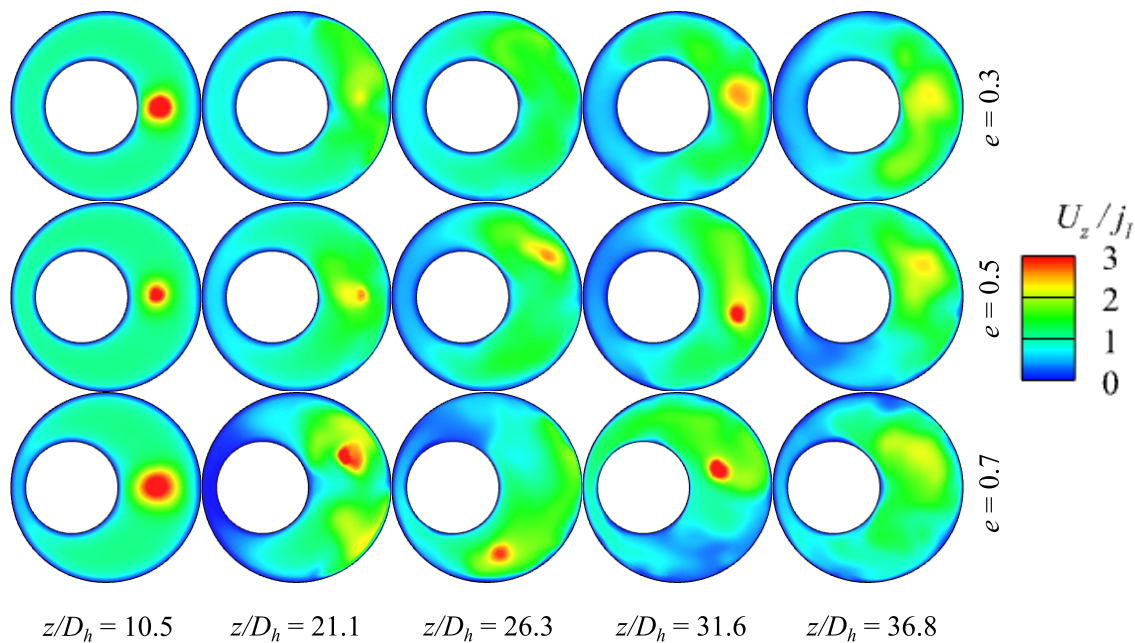


Figure 7.22: Isocontours of the normalised instantaneous axial velocity on $x - y$ planes at different axial locations and selected eccentricities (bubbly flow).

b) **Slug flow:** Simulations were performed for $j_l = 0.270$ m/s and $j_g = 0.056$ m/s, for which the experimental flow was in the slug flow regime. Besides eccentric geometries with $e = 0.3, 0.5$ and 0.7 , the concentric geometry was simulated as well. Unlike the

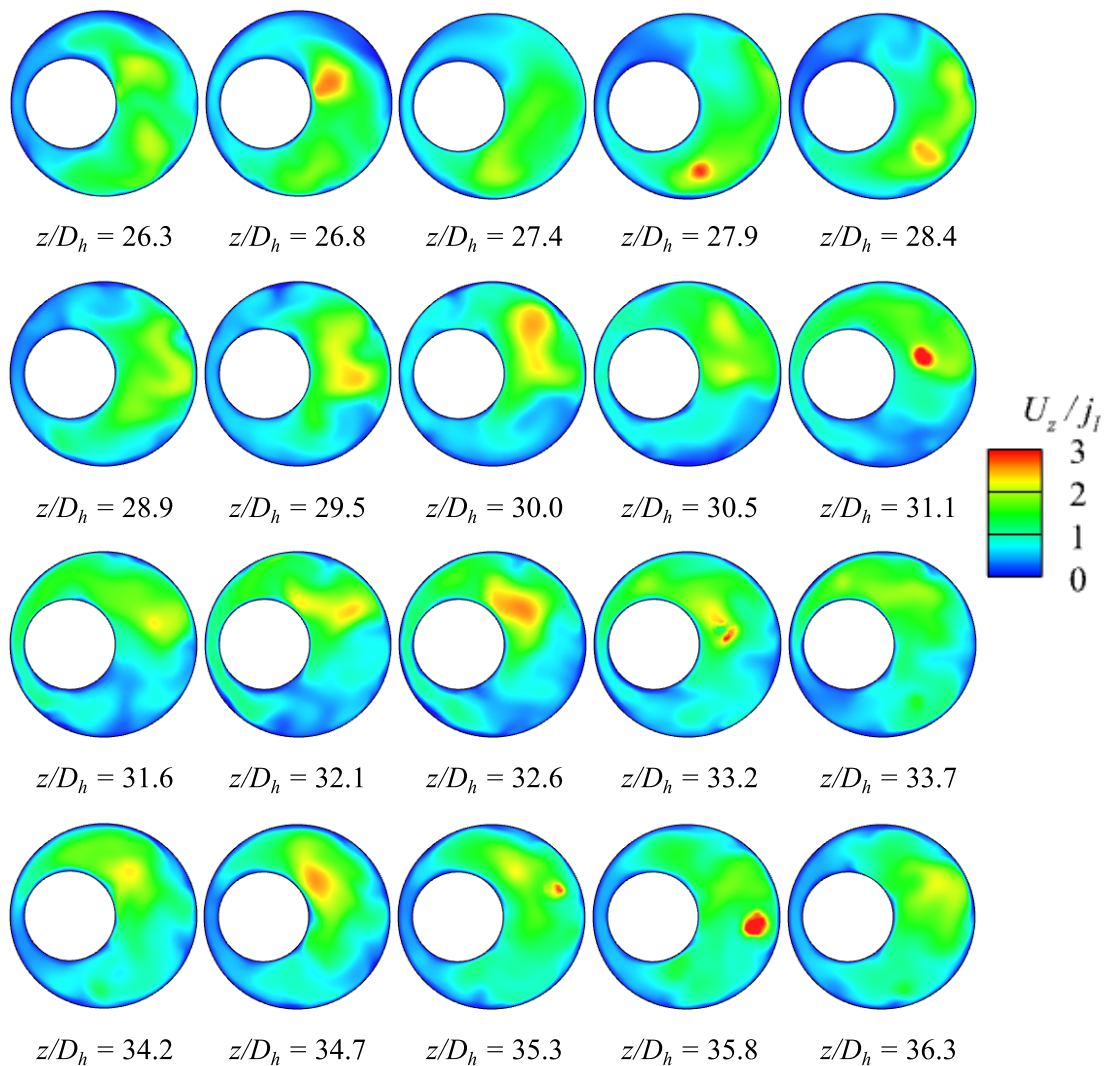


Figure 7.23: Isocontours of the normalised instantaneous axial velocity on closely spaced $x - y$ planes for $e = 0.7$ (bubbly flow).

experimental flow, however, the simulated flow appeared to be in the cap-bubbly flow regime, as gas volume fraction isocontours show in Figures 7.20 and 7.21. It is noted that, at the gas flow rate selected for these simulations, the experiments showed axial development of the flow regime from cap-bubbly to slug flow at the viewing tank location of the annular channel. Because these simulations were run for a short channel length, it is possible that this length was insufficient to capture this regime transition. This possibility is supported by the observation that the cap bubbles increased in size from the inlet to the top of the channel.

Figure 7.24 shows computed normalised axial velocity contours on transverse planes at multiple axial locations for cap-bubbly flow. In the concentric channel, the peak axial velocity region rotated around the inner cylinder in a counterclockwise sense. However, in the eccentric channels, the peak axial velocity region oscillated across the plane of symmetry in the wide gap in a manner that was similar to that in bubbly flow. The oscillation amplitude was larger for the $e = 0.7$ case. Although the presence of larger bubbles seems to complicate the shape of the axial velocity contours, it is clear that vortex streets had a strong effect on the bubble motion in highly eccentric channels both in the bubbly and the cap-bubbly regimes. One may speculate that vortex street effects, with possible occasional interruptions, would be also identifiable in the slug flow regime.

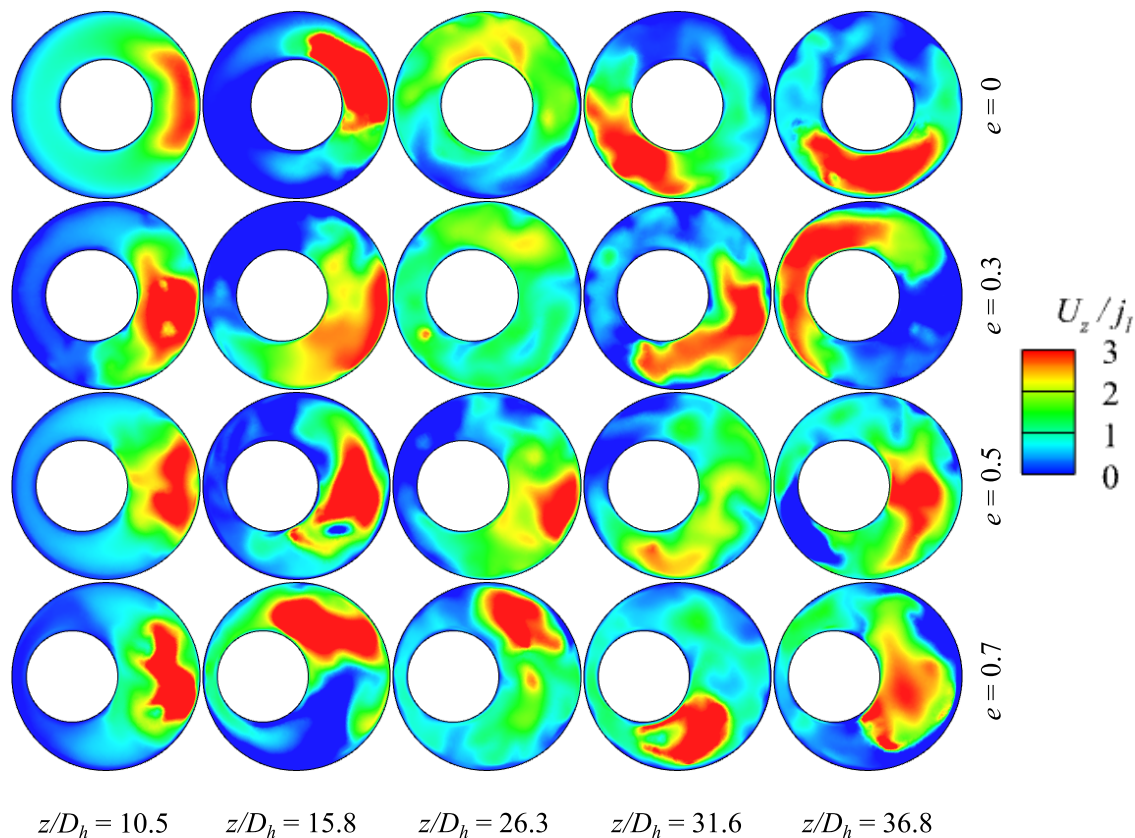


Figure 7.24: Isocontours of the normalised instantaneous axial velocity on $x - y$ planes at different axial locations and selected eccentricities (cap-bubbly/slug flow).

c) **Churn flow:** Simulations were performed for $j_l = 0.270$ m/s and $j_g = 1.217$ m/s, with the latter being the largest gas flow rate in the experiments. The appearance of the

simulated flow was very similar to that in the experiments: in both cases, most of the cross-section of the annular channel, including the narrow gap, was occupied by highly distorted gas columns. The effect of eccentricity, if any, was not apparent in either the simulations or the experiments. It is noted that the present work is essentially of qualitative nature and the available results are deemed to be insufficient for a conclusive determination of the effect of eccentricity on the transition from slug to churn flow and the axial development of churn flow (see also the discussion by Cao *et al.* (2024)).

Chapter 8

Conclusions and recommendations for future research

8.1 Conclusions

The present dissertation reports the results of experimental and numerical investigations of upwards air-water flows in annular channels. The effects of channel eccentricity e on Taylor bubbles rising in stagnant water and in co-current water flow and on air-water flow regimes were examined. The main conclusions of these investigations are listed in the following.

Taylor bubbles rising in stagnant water:

- As the eccentricity of the annular channel increased from $e = 0$, the Taylor bubble velocity first increased, peaked around $e \approx 0.2 - 0.3$, and then decreased to a value at $e = 1$ that was lower than the value in the concentric case. This is the first experimental study to document this effect. This finding was also observed in our numerical simulations as well as in previous numerical studies in the literature.
- The Taylor bubble length and volume were found to be essentially insensitive to the eccentricity.
- The maximum angle of wrap monotonically decreased with increasing eccentricity. This is the first study to document this finding experimentally for the entire eccentricity domain. This finding was also observed in our numerical simulations as well as in the previous numerical studies in the literature.

Taylor bubbles rising in co-flowing water:

- The distribution parameter C_0 , which is a normalised difference between the velocities of isolated Taylor bubbles in upward liquid flow and in stagnant liquid columns, was found to have a weak, if any, sensitivity to the eccentricity.

- Isolated Taylor bubbles rising in flowing water were found to be longer compared to Taylor bubbles rising in stagnant water and released under the same conditions. For bubbles with comparable lengths released in the two setups under different conditions, the ones in flowing water were slimmer and had smaller maximum angle of wrap than those in stagnant water.

Air-water flow regimes:

- A flow regime map was created by using experimental observations for co-current air-water flow and the full range of eccentricity. This is the first study of its kind. The map extended over four flow regimes, namely, bubbly, cap-bubbly, slug and churn flow and had sufficient resolution to identify the regime transition regions.
- It was found that eccentricity in the range from 0.3 to 0.7 affected the transition from one regime to another. In particular, transition from bubbly to cap-bubbly flow under a constant water superficial velocity was shifted towards higher air superficial velocities. A similar eccentricity effect was also observed for the transition from cap-bubbly to slug flow.
- Experimental flow visualisation showed the presence of gap vortex streets for flows in the bubbly, cap-bubbly and slug flow regimes for $e = 0.5$ and 0.7 . These findings were supported by our numerical analysis. The presence of a vortex street was identified as the cause for the previously mentioned effect of eccentricity on regime transition.

The present study has complemented and expanded the literature on gas-liquid flows in annular channels with a full range of eccentricity and has shown that eccentricity has significant effects on several aspects of such flows. It has further provided evidence for the presence of gap vortex streets in gas-liquid flows, thus connecting to the substantial literature for single-phase flows in annular channels and other channels with narrow gaps. The present findings may find application in the development of improved procedures for the prediction of flow regime transitions in shell-and-tube type heat exchangers, nuclear reactors and other thermal power plants, and geothermal and natural gas extraction systems.

8.2 Recommendations for future research

The present study made several original contributions to gas-liquid flows in eccentric channels. It collected extensive experimental and computational results, which led to essentially identical conclusions. In its preliminary phase, it considered using quantitative measurement methods, including particle image velocimetry, but, in view of the challenges of using such methods in two-phase flows and the time limitations, the experimental results were based on analysis of video recordings. Innovative processing of the information provided by this approach permitted us to quantify as much as possible the available observations, but cannot be extended much further. Future extensions of the experimental work may consider developing quantitative non-intrusive measurement methods that can be used in the present setup and enlarging the scope of the study.

Measurement methods may be used to map the velocity field and its time dependence and to identify the flow regimes. Particle image velocimetry and laser Doppler velocimetry may prove feasible for the wake of the Taylor bubble and the liquid in the bubbly flow regime, particularly for documenting the characteristics of the vortex street. Quantitative investigations of the eccentricity effect on flow regime transitions using non-intrusive methods, such as flush-mounted ring probe or electrical resistance tomography, may be also applied. Development of a method for visualising the flow on cross sectional planes would be a very challenging task, but, if successful, would provide important additional information about the bubble and flow characteristics.

Future work with an enlarged scope may examine eccentricity effects in flows of various fluid mixtures by replacing the water with liquids that have substantially different physical properties. The results of such work could be of interest in chemical processing and in petroleum extraction. Another extension could be an examination of the eccentricity effect on bubble coalescence and breakup mechanisms. Finally, future work may consider replacing the annular test section with a more complex one, such as a simple rod bundle. Such work could be of interest in nuclear power applications.

The scope of the computational work may also be extended. One important issue that can be examined is the effect of eccentricity on heat and mass transfer, which has multiple applications. Although quite challenging to model accurately, phase change (e.g., boiling or condensation) may be also introduced in the analysis.

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Appendix A

Estimation of bubble volume in the circular channel

Besides the previously described method of estimating the Taylor bubble volume from images in the annular channel, consideration was given to approximate this volume by the average volume of Taylor bubbles released in the circular tube under the same pressure and valve opening time, as well as the same water column height. This analysis was performed only for bubbles with 41 and 83 kPa air release pressures, which have equivalent test conditions in both the tube and the annular channel tests.

If there were no light refraction, the volume of the bubble would be estimated from an image, such as the one shown in Figure A.1. The image would be analysed in a computer aided design software package (SolidWorks Education Edition 2019, Dassault Systèmes) as follows: a number of points would manually be marked on one half of the contour of the bubble image, an analytical curve would be fitted to these points and then this curve would be rotated about the tube axis to construct an axisymmetric model of the bubble. Finally, the volume of this model would be computed by the software.

Unfortunately, this method is inaccurate because of light refraction, which was significant in the case of the circular channel that, unlike the annular one, was not equipped with a viewing tank. As shown in Figure A.2, the width of a bubble in the annular channel was significantly larger in the image outside the viewing tank than in the image through the viewing tank. For this reason, dimensions obtained from images of the circular channel were corrected for refraction effects using geometrical optics, as described in the following.

Sketches of light rays illustrating refraction effects are shown in Figure A.3. Let f be the focal distance of the camera and x the distance between the focus and the tube axis, under the assumption that $x \gg f$. The height of the tube image s_t corresponds to the viewed tube height, which is approximately equal to the tube outer diameter $D_t = 2r_t$. The dimensions f, x, s_t, D_t are known and so one can determine the magnification factor in the absence of refraction. Then, one can determine a hypothetical bubble diameter that would correspond

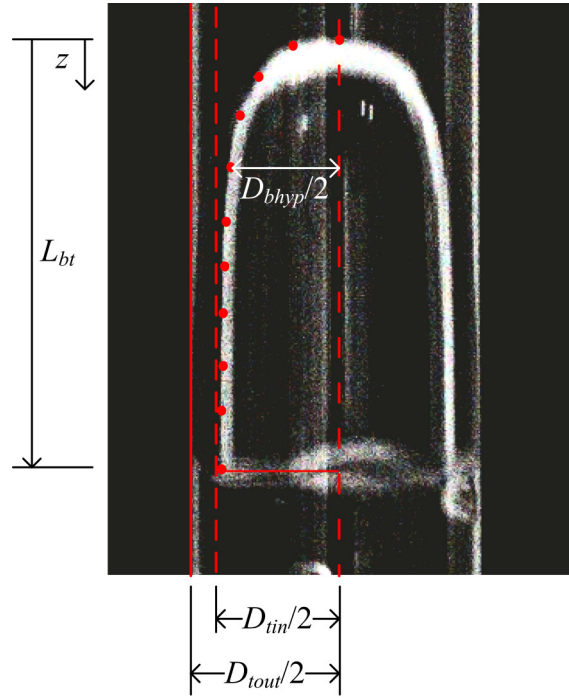


Figure A.1: Representative image of a Taylor bubble in the circular channel with solid lines fitted along the bottom of the bubble and an outer edge of the channel, a dot-dash line fitted to the axis of the channel, a dashed line fitted to the inner edge of the channel and points marked on half the bubble contour image.

to the image of the bubble if no refraction were present as $D_{bhyp} = s_b D_t / s_t$. Refraction, however, makes the magnification factor of the actual bubble diameter $D_b = 2r_b$ at each horizontal cross section of the bubble a function of this diameter. As light originating at the edges of the bubble propagates through the water (having a refractive index n_{water}), the acrylic wall of the tube (having a refractive index $n_{acrylic}$) and air (having a refractive index n_{air}), it gets refracted according to Snell's law. From accurate CAD drawings of the setup (SolidWorks), following light rays from the image plane to the edges of bubbles with different diameters as they got refracted according to Snell's law, we calculated the ratio D_b / D_{bhyp} . The ratios D_b / D_{tin} and D_{bhyp} / D_{tin} , where D_{tin} is the inner diameter of the tube, have been plotted vs. the normalised distance from the nose z / D_{tin} in Figure A.4. It can be seen that the bubble diameter correction for refraction effects was significant and increased with distance from the nose. It is also interesting to note that, although the two

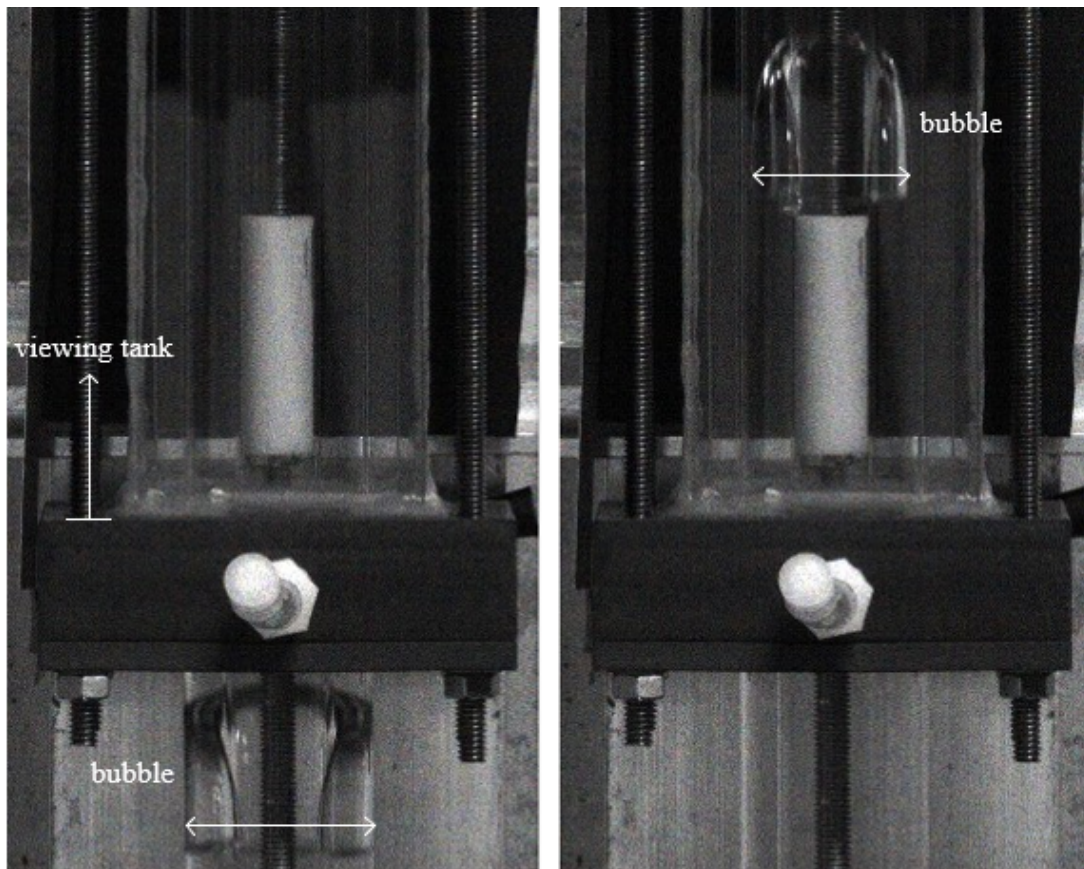


Figure A.2: Two images of the annular test section with the image on the right taken 0.583 s after the image on the left; the upper parts of both images were viewed through the viewing tank, whereas the lower parts viewed the bare test section; $P_r = 41$ kPa and $\Delta t_r = 0.25$ s.

reference bubble had very different lengths, their contours essentially coincided over the length of the shorter bubble. Then, we entered the corrected bubble diameter into the CAD software, from which we calculated the bubble volume V_{bt} of the reference bubble.

To apply this method to all bubbles (typically 30) for each set of conditions would be quite cumbersome, as it requires the fitting of multiple points to each bubble contour. The reported bubble volume in the circular tube tests was calculated with the use of a relatively simple approximate method, which only required the measurement of each bubble length L_{bt} . Under the assumption that the film thickness near the tail of all bubbles was approximately constant along the bubble and equal for all bubbles, we corrected the volume of each individual bubble by adding algebraically the volume of a cylinder with a diameter D_{bmax} , determined as the maximum diameter of the reference bubble, and a

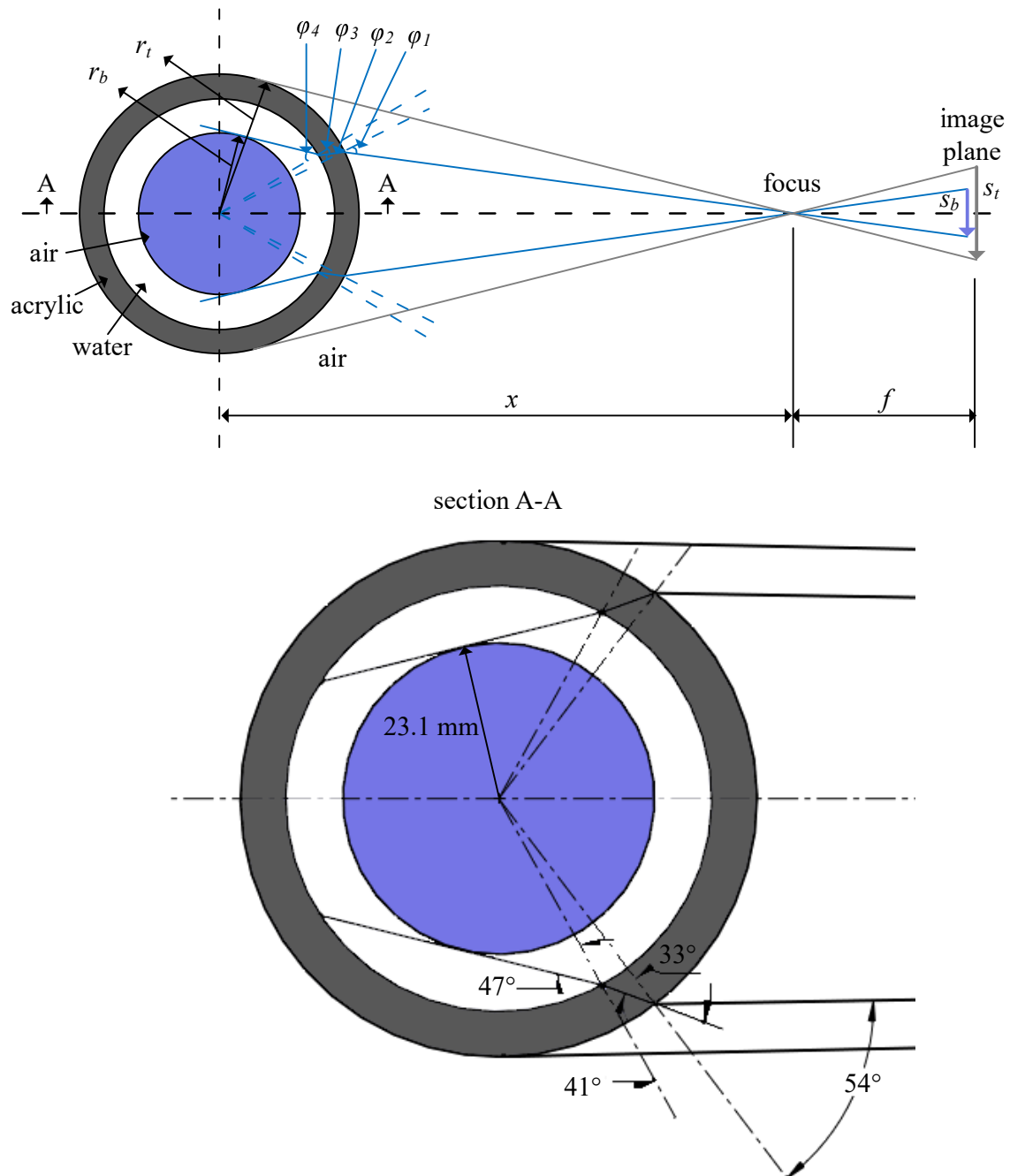


Figure A.3: Sketches of an idealised bubble in the circular tube and its image in the camera (not to scale), showing refraction effects; the sketch at the bottom is zoomed at the tube and shows more details.

height $L_{bt} - L_{bref}$, where L_{bref} is the height of the reference bubble.

In the remainder of this section, we will present examples of measured and calculated

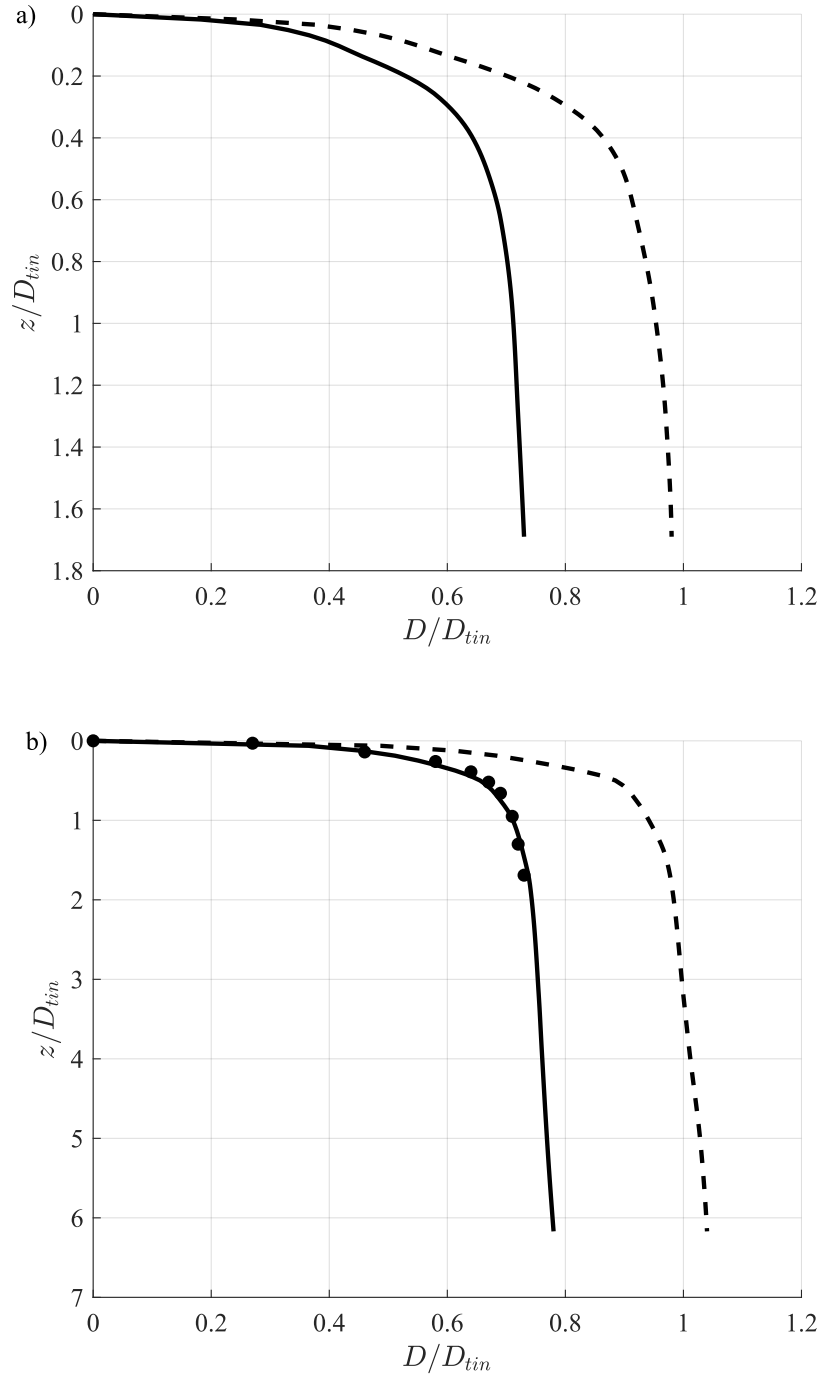


Figure A.4: Axial variation of the diameter of the reference bubbles, normalised by the tube inner diameter (solid line) and the corresponding bubble diameter uncorrected for refraction effects (dashed line); a) $P_r = 41$ kPa, $\Delta t_r = 0.5$ s; b) $P_r = 83$ kPa, $\Delta t_r = 0.5$ s; the solid circles in (b) mark values from (a).

values of relevant properties of two reference bubbles. These values are listed in Table A.1. The minimum liquid film thickness was estimated as $\lambda_{min} = (D_{tin} - D_{bmax})/2$. These values were compared to values obtained from available correlations in the literature for fully developed Taylor bubbles rising in stagnant water columns (Nogueira *et al.*, 2006a; Llewellyn *et al.*, 2012; De Azevedo *et al.*, 2017). Such bubbles were long enough to have a rear section with an approximately constant film thickness, which typically happened when $L_{bt}/D_{tin} > 20$, whereas the present bubbles had average values $L_{bt}/D_{tin} = 1.7$ for 41 kPa and 5.5 for 83 kPa. It is also noted that each of the cited correlations was fitted to data in different ranges of the inverse viscosity number N_f , which in general had lower upper bounds than the present value $N_f = 17,000$. The minimum film thickness values for the present reference bubbles were 4.2 mm for $P_r = 41$ kPa and 3.5 mm for $P_r = 83$ kPa. Values from the literature were as follows. Nogueira *et al.* (2006a) report $\lambda = 1.17$ mm for a single test in a tube with a diameter $D = 32$ mm and $N_f = 18,000$; they did not specify the bubble length and volume, although they gave the range of bubble volumes in all their tests as between 40 and 265 cm³). Llewellyn *et al.* (2012) report $\lambda = 1.32$ mm for fully developed bubbles in the range $0.1 < N_f < 100000$, although De Azevedo *et al.* (2017) suggested that these results do not apply for $N_f > 5703$. De Azevedo *et al.* (2017) report $\lambda = 0.23$ mm for fully developed bubbles in the range $15 < N_f < 12900$. The fully developed film thickness values in the literature are significantly smaller than the presently measured values, which, however, were based on relatively short bubbles.

Table A.1: Properties of reference bubbles for two sets of release conditions.

Property	Value	Value
P_r	41 kPa	83 kPa
Δt_r	0.5 s	0.5 s
N_f	17,900	17,900
n_{air}	1.00	1.00
$n_{acrylic}$	1.49	1.49
n_{water}	1.33	1.33
f	28 mm	28 mm
x	915 mm	915 mm
pixel size	6.48×10^{-3} mm	6.48×10^{-3} mm
s_t	1.21 mm	1.21 mm
s_b	0.97 mm	1.03 mm
D_{tout}	38.1 mm	38.1 mm
D_{tin}	31.5 mm	31.5 mm
L_{bt}	53.1 mm	194.5 mm
L_{bt}/D_{tin}	1.69	6.17
D_{bmax}	23.1 mm	24.5 mm
λ_{min}	4.2 mm	3.5 mm
V_{bt}	18.2×10^{-6} m ³	84.8×10^{-6} m ³

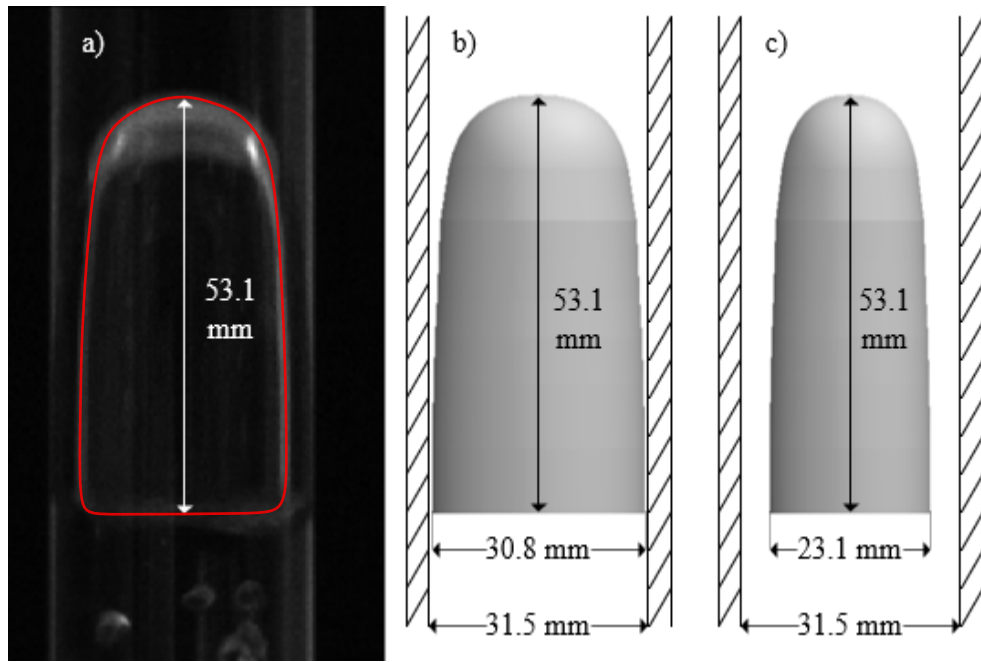


Figure A.5: Examples of two types of axisymmetric models of a bubble produced with $P_r = 41$ kPa and $\Delta t_r = 0.5$ s; a) an image from the camera and a fitted contour in red; b) a model created from the fitted contour, which gave a volume $V_{bt} = 33.1$ cm³; c) a model corrected for refraction, which gave a volume $V_{bt} = 18.2$ cm³.