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Taylor Rule and Structural Change

By

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**Thesis presented to the Department of Economics of
the University of Ottawa in partial fulfillment of the
requirements of the M.A. degree**

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ABSTRACT

We evaluate the Taylor rule and investigate its stability for the period 1963Q2 to 1999Q4. Using a benchmark model, we demonstrate that the equation cannot be evaluated over this period without taking into account parameter instability and structural changes, which reflect changing monetary policy preferences. Neglecting to allow for at least one shift in the equation can lead to artificial results by ignoring the heterogeneity of the long-run relationship, while it is not capturing the changes in monetary policy preferences. To estimate the equation over the 1963Q2-1999Q4 period, we follow Bai and Perron's (1998) recent methodology, with which we find evidence for up to five breaks.

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CHAPTER 1

Introduction

With the end of the 1981-82 recession, in Canada as in most developed countries, began an era of disinflation that brought us to a period where inflation is stable and under control. During the 60's, Canadian inflation rate averaged at about 4% per year, but with the first oil crisis it rose above the 10% mark, hitting 13.25% in the second quarter of 1974, a rate which had not been seen during the preceding twenty years. However high these rates may seem to have been at the time, they still remained well below those observed in many South American countries (*e.g.* Brazil or Argentina) during the late 80's or those witnessed in the post-war Germany of the 20's. Moreover, the 70's were marked with the apparition of a new economic phenomenon known as stagflation. Repeated and simultaneous increases in both inflation and unemployment rates seemed to challenge the orthodox theory of the Phillips curve, where the negative relationship of inflation and unemployment is illustrated. However, after reconsideration, this unusual economic behavior can now be explained by the shifting in the expectations of inflation in an augmented version of the Phillips curve model.

One of the main explanations of those disastrous years, is the accommodating behaviour of the central banks of the most developed economies who conducted activist monetary policies at the time. As Bernanke, Laubach, Mishkin, and Posen (1999, pp. 11) argue, "[...] in the view of most economists, the severe 1981-82

recession was largely the result of restrictive monetary policy, which in turn had been made necessary by surging inflation". And they further add that "[...] the activist monetary policies of the 1960's and 1970's not only failed to deliver their promised benefits, they helped to generate inflationary pressures that could be subdued only at high economic cost" (*ibid*, pp. 11).

In order to ensure that central bankers do not commit themselves to inflationary monetary policies such as those conducted in past decades, a majority of economists have argued that monetary rules should be adopted in order to make the policies more transparent and credible. The adoption of a monetary rule would give the general public and the financial markets an insight on which direction the interest rate is going to change and to what magnitude it will change in order to achieve its policy objective of price stability. As a clear definition of what can be understood as a monetary rule, Bernanke *et al.* (1999, pp. 5), give us the following definition: "Rules are monetary policies that are essentially automatic, requiring little or nothing in the way of macroeconomic analysis or value judgments by the monetary authorities".

The most general class of rule is one where the central bank is setting the money market interest rate as a function of output and inflation in order to implement its policy, which can be written as:

$$i_t = \mu + \beta_1 y_t + \beta_2 \pi_t + v_t \quad (1.1)$$

where $\{i_t\}$ is the overnight interest rate, depicted in Figure 1; $\{y_t\}$ is the logarithm of the gross domestic product (*GDP*) adjusted for the seasonal variations, depicted

in Figure 2¹; $\{\pi_t\}$ is the annual rate of inflation calculated using the Consumer Price Index (*CPI*), depicted in Figure 3 and calculated as $\pi_t = (\ln p_t - \ln p_{t-1}) \times 400$; and $\{v_t\}$ is a noise function.

Of most interest however, is the monetary rule proposed by Taylor (1993). This fairly simple rule, commonly known as the Taylor Rule, advocates that the interest rate is a function of deviations from the potential output level, deviations from an expected inflation rate target, and a smoothing parameter, which is the lagged interest rate. Thus, the Taylor Rule can be written as:

$$i_t = \mu + \beta_1(y_t - y_t^P) + \beta_2(\pi_t - E[\pi_t]) + \beta_3 i_{t-1} + \varepsilon_t \quad (1.2)$$

where $\{\varepsilon_t\}$ can be describe as a monetary policy shock, and is also an *i.i.d.* white noise sequence with mean 0 and variance σ_ε^2 , orthogonal to $\{i_t\}$; $\{y_t^P\}$ is a measure of the potential output level, depicted in Figure 4; and finally, $E[\pi_t] = \{\pi_t^e\}$ is the unconditional expected rate of inflation².

In light of the many arguments in favour of more transparency from central banking authorities, the Bank of Canada has adopted an official target range since 1991; nowadays, the upper and lower bounds are 1 and 3% respectively. In a notorious speech in 1989, governor John Crow announced a plan to reduce considerably the rate of inflation and to keep eventually the price level under a tight control, allowing for small positive price changes and also for the possibility to accommodate for one time price increases due to exogenous factors, such as the increase in

¹ For simplicity, we are implicitly referring to “output” as the real *GDP* or real *GNP* for the entire discussion.

² Appendix II contains the complete data references.

oil prices that we have been witnessing for the past months. By announcing an official target in 1991, Canada, in line with New-Zealand (who first announced the adoption of an inflation target one year before), and Sweden, for example, was the second country to officially act in such a way. Now, Israel, Great Britain and the European Monetary Union countries are following the same kind of policy where public announcements are made³. As for the United States, they do not yet have official targets but the Greenspan administration however remains very concerned with respect to inflation, allowing it to evolve in the neighbourhood of a yearly rate of 4 per cent, a rate slightly higher than that tolerated in Canada⁴.

This thesis therefore proposes to investigate some of the properties of different specifications of Taylor's Rule for Canada when changes in monetary policies are taking place. Using a similar specification to that of Taylor (1993), we mainly build on Clarida, Gali and Gertler (1998a,b), Judd and Rudebusch (1998), Nelson (2000), and Muscatelli, Tirelli, and Trecroci (1996), who all studied reaction functions similar to the one we estimate under policy regime shifting. However, we intend to push further these studies by using more sophisticated econometric techniques in order to evaluate the effect of regime changes that are inherent to the data generating process of the variable used in the model. Essentially, our approach

³ In fact, the European Central Bank (ECB), who considers both the inflation rate and the growth rate of a broad monetary aggregate, has been showing some concerns with this year's fall of the Euro/U.S.\$ exchange rate.

⁴ As Fortin (1996) argues, considering the rates of growth observed in the United-States for the last 10 years, one may wonder whether or not the Bank of Canada has been conducting a policy in an excessively zealous fashion, which led to the first ever recession that was "made in Canada", as it occurred in Canada before it did in the U.S..

consist in treating policy changes as rare and unknown random events.

The rationale for undertaking such an investigation of the structural stability of the model can be best described by Rotemberg and Woodford (1998, pp. 6), who investigated monetary rules for the U.S. for the period 1980Q1 to 1995Q2, arguing that “[...] it is widely recognized that a significant change in the U.S. monetary policy regime occurred around that time [i.e., 1980]; thus at least one equation of [...] the monetary rule cannot be expected to have remain invariant over a longer period of time than the one we use.” While they also add: “Many, of course, would doubt that the monetary policy rule has remained unchanged since then” (*ibid*, pp. 6).

Also, we will estimate an approximate target value for the inflation rate which economic agents anticipate given the underlying economic environment at time t . Although we investigate the relative stability of the rule, we do not enter the debate regarding its optimal properties, which have already been well studied by many authors (see Svensson (1998) and Taylor (1998c) for excellent surveys). Rudebusch and Svensson (1998, pp. 17), studying the case of the U.S., compare the relative performance of monetary targeting versus inflation targeting and finds “that monetary targeting would be quite inefficient for the U.S., in the sense of bringing much higher variability of inflation and output gap than inflation targeting”. Finally, Williams (1999, pp. 27) finds that simple rule are “very effective at minimizing the fluctuations in inflation, output, and interest rates: complicated rules yield trivial stabilization benefits over efficient simple rule”.

Even though we are using contemporary data in the model, we could have followed the suggestion of McCallum (1993), who raised some criticism of models that included contemporary data since policymakers do not actually observe them. Ghysels, Norman, Swansson, and Callan (1998), and Orphanides (1997) discussed the problems related to data availability in real time as well as the frequent updating of data by governmental authorities, which could lead the econometrician to reach for drastically different conclusions. For further details on these issues, the reader may consult the mentioned above references.

Moreover, we do not seek to maximize the fit of the model by examining the effect of additional explanatory variables in the model (*e.g.* the U.S./Canada exchange rate) or different lag/lead structure for the variables introduced in the model. Thus, our sole concern here is that of the structural analysis of the Taylor Rule when the econometrician takes advantage of the longest sample available to him, so that his estimations are not subject to data mining criticism and leaving aside all other issues of concerns mentioned above.

The data considered here consist of quarterly data going from 1961Q1 to 1999Q4. We are investigating the following specifications:

Model I (MI): A benchmark reaction function where some variables are considered to follow an $I(1)$ process. Also, $E[\pi_t] = \{\pi_t^e\}$ is simply described as a constant, as in Clarida *et al.* (1998b).

Model II (MII): Which is an extension of MI, where we take into account the univariate dynamics of $\{i_t\}$, for which we find evidence that it follows a stationary

process when we allow for a shifting mean and a broken trend. Moreover, instead of taking the unconditional expectation, we take the conditional expectations of $\{\pi_t^e\}$, which is given by $E[\pi_t|\psi_{t-1}]$, and is being governed by a first order Markov process, in as proposed by Hamilton (1989) and Garcia and Perron (1996).

Model III (MIII): To illustrate the great instability of the parameters, a reaction function with time-varying parameters (*TVP*) is estimated. This model uses the same variables of those included in **MII**.

Model IV (MIV): This is another extension of **MII**, but now considering the presence of endogenous structural changes in the linear equation in order to capture policy changes. For this, we follow the approach suggested by Bai and Perron (1998a) to detect and select the number of breaks in the model.

The remaining of this thesis is divided as follows. In Chapter 2, we first present a short discussion on the Taylor Rule; secondly, we examine the conceptual framework in which the economy operates; lastly, we investigate the properties of each time series to determine their integration order. In Chapter 3, we present our output gap and inflation expectation measures, and discuss the presence of non-linearities in those two series. In Chapter 4, we estimate and evaluate the Benchmark model, and we apply a battery of tests, which highlight the fact that it is plagued with the violation of many important statistical assumptions on which statistical inference are based. In Chapter 5, we first consider $E[\pi_t|\psi_{t-1}]$ and we introduce other specifications which makes explicit allowance for structural changes in the model. Then, we reestimate the monetary rule in a structural time

series model with time-varying parameters. Finally, in Chapter 6, we search for $m + 1$ monetary regimes in the equation of the rule; after we have detected the m structural breaks, we reestimate the rule in a fully specified equation where we allow for pure structural changes into the population parameter vector. Finally, Chapter 7 briefly concludes.

CHAPTER 2

The Theoretical Framework

In this chapter, we first discuss the modern context in which monetary policy operates. Today's leitmotiv of central banking is to maintain relative price stability while keeping the economy prospering. The Bank of Canada now operates with an explicit inflation target band that is conjointly announced with the Department of Finance in an effort to make monetary policy more transparent. Secondly, we present the basic economic framework in which the central bank operates. Finally, we perform unit root tests on the variables considered in this thesis. We show that two of the three time series used can be described by a trend stationary process with a structural break in their deterministic function.

2.1 Today's Monetary Framework: Inflation Targeting

Back in the 50's and 60's, most economists then agreed that monetary policies should pursue the objective of full employment above all other macroeconomic concerns, including inflation. In the spirit of the Phillips curve, the monetary authorities, strongly encouraged by the Governments of the time, were actively trying to eradicate the business cycles and maintain high employment levels (see Samuelson and Solow (1960)). Then came the critique of Friedman (1968) and Phelps (1970), who both argued that money was neutral - at least in the long-run - and that all activists policies would end up doing, is to create inflation.

Thus, in light of the inflationary events of the 70's, policy-makers in the West-

ern world, who were less enthusiastic about their role in promoting aggregate demand, adopted tighter monetary policies so that inflation could finally be under some degree of control in the single digit figures. Facing uncontrollable narrow money aggregates, central bankers still needed a nominal anchor to conduct monetary policy so that the “policy is most effective” and easy for the public to comprehend (Bernanke *et al.* (1999), pp. 20). Inflation targeting is now seen by a majority of economists as the most appropriate framework for the realization of policies intended to keep prices under control, given that the idea of the nominal anchor still remains relevant in the vast majority of opinions on the matter¹.

Thus, on February 26, 1991, one year after New Zealand went ahead in being the first country to adopt an official inflation target in the Post-War era, Canada became the second country to adopt an inflation targeting monetary policy. For the first months of the new policy (1991M02 to 1992M12), the Bank aimed at bringing down inflation through a target band ranging from 2 % to 4%. Then, until June of 1994, the target range was lowered at 1.5% for the lower limit and 3.5% for the upper limit to finally reach today’s range of 1% to 3%. According to Bernanke *et al.* (1999, pp. 125), “no other inflation-targeting central bank has so explicitly made a virtue of transparency for its benefits to the economy as well as for its role in reducing inflation, although all have made efforts in that direction”.

To implement this policy, the Bank uses the so-called “monetary condition index”, which consists of a weighted average of the exchange rate and the overnight

¹ For an elaborate discussion on the foundations of the nominal anchor, see Wicksell (1899).

rate of interest². Here, we assume that the Bank's operating procedure for conducting its monetary policy can be equivalently characterized with the Taylor Rule, while ignoring exchange rate movements in our model in order to keep it as simple as possible – at least with respect to the number of variables on the right hand side of the equation.

Even though official inflation targets are only a recent phenomenon in most industrialized countries, it would be unreasonable to say that the Bank of Canada was not concerned with the ongoing rate of inflation before the adoption of the official target band in 1991³. For this reason, we treat $\{\pi_t^e\}$ as the implicitly defined and unobservable inflation target considered by the Bank through out the whole sample. However, using the simple unconditional expectations has the major drawback that it does not change through time and according to the underlying economic conditions. Taking expectations conditional upon information at time t , on the other hand, has the net advantage of fluctuating through time, and therefore being more realistic and reflecting more adequately the underlying behaviour of the economy, as it is constantly hit by random shocks, driving it in unanticipated directions. Also, since agents' expectations neither change nor do they adapt through economic conditions, using constant expectations is subject to Lucas' Critique.

However, as for any other public policy, the general public needs to comprehend not only the reason motivating the adoption of a policy, but also the functioning of

² For more details on the Monetary Conditions Index, see Freedman (1994).

³ John Crow's 1988 Hanson Lecture could be seen as the starting point of the public campaign of the Bank in reducing inflation.

the policy itself. With respect to this concern, the *CPI* is an excellent instrument since most news broadcast or newspapers are now reporting monthly rates, which makes the *CPI* an highly visible and comprehensible policy objective for the Bank of Canada, as the general public is kept informed and made aware of the consequences of undesired movements in inflation. However, although more obscure and less visible, core inflation measures are also important since they exclude energy and food prices, which are evidently more volatile than other basic commodity prices.

Another well embraced advantage of inflation targeting over targeting some monetary aggregate is that it is a well defined policy objective that is relatively easier to communicate to the public. Also, it is now well recognized that narrow monetary target can be hard, if not impossible, to exploit as policy instruments because they are subject to various and multiple exogenous shocks - also referred to as "velocity shocks". These shocks made monetary policy at the mercy of randomness that led to relatively high inflation rates during the 70's, when it was used in Canada and the U.S. (*e.g.* Bernanke *et al.* (1999)). This inability to control money and the shocks that affected its demand forced most central banks to abandon money targeting during the late 70's and early 80's⁴. During this period, both Canada and the U.S. were officially using monetary aggregates until Volker took over the Fed chairman's position⁵. Presently, the European Monetary

⁴ A notorious exception is the Bundesbank, which has always claimed to be targeting money. However, in an interesting paper, Bernanke and Mihov (1997) argue that the Bundesbank can be best described as an inflation targeter rather than as a money targeter.

⁵ Volker was Chairman until 1987Q2, and was followed by Greenspan, who still holds the

Union, under the strong influence of the conservative Bundesbank, is considering an hybrid type of targeting, which consist of controlling both inflation and a broad monetary aggregate, as the official policy objective is to maintain relative price stability.

By focusing on the interest rate instead of some money aggregates, the Taylor Rule illustrates the fact that the contemporary framework of monetary policy does not pay as much - if any - attention to money aggregates as it did in the past. In fact, this framework makes us depart from the usual textbook assumption where the money supply is vertical (or money is strictly exogenous), which implies that the central bank has an effective 100% control over money and that a change in the quantity of money (or its rate of growth) automatically translates into a change of opposite directions in the interest rates as well. In addition, contrary to the monetarist approach advocated by Friedman (1960, 1968), many have argued that in reality central banks have basically no power over the quantity of money (or its rate of growth) in the economy, but that they can strongly influence the money market by using the interest rate as an instrument, which operates through the channels of chartered banks' settlement balances⁶. However, "[...] using an interest rate rule does not eliminate the concept of money demand and supply; it simply makes money endogenous" (Taylor (1998), pp. 9). Furthermore, Svensson (1998, pp. 35), describing money as an endogenously driven process, argues that "a

important position as of today.

⁶ For an extended discussion on the matter, see Bernanke and Blinder (1992), and Bernanke and Gertler (1995), where they discuss the channels of credit, which is divided into the balance sheet channel and the bank lending channel, and the cost-of-capital effect.

rationale for monetary aggregates is then to be one set of indicators among many, whose usefulness depend solely on their predictive power for future inflation". Finally, Zha (1997, pp. 29) argues that "[...] central banks can heavily influence broad monetary aggregate such as M2 but cannot control them completely". Zha (1997) illustrates his point by saying that money supply cannot be vertical, which is how macroeconomic textbooks are describing the central bank's behaviour.

Besides, the Canadian banking sector is running under a zero requirement reserve ratio since 1994, which is taking us even further from the textbook world of the multiplier of deposits. In an interesting paper, Clinton (1997) discusses the framework for monetary policy at the Bank of Canada, where the overnight interest rate is targeted in such way that the Bank can monitor the supply of settlement balances in order to implement its policy. In fact, the central bank is targeting the overnight rate of interest in the middle of the spread of the deposit rate and the borrowing rate so that "cost-minimizing banks will target zero balances" at the end of the day. In the same fashion, Henckel, Ize, and Konanen (1999, pp. 24) discuss monetary policies in a world without central bank money, where they argue that in such a moneyless world, "[...] the key is that the central bank remains the uncontested broker/lender of last resort in the case of settlement difficulties, and controls the rate at which it engages in such operations". Nowadays, banks in Great Britain, Australia, Belgium and Sweden operate, as in Canada, under a zero reserve requirement ratio; while the 90's have seen reductions in such requirements in Germany, France, and Japan. As for the United States, where positive reserves

are still required, it is now commonly known that commercial banks do not have frequently recourse to the Discount Window of the Federal Reserve system since commercial banks usually prefer to equilibrate their balance sheet on the Federal Funds Market where the ongoing rate of interest is the well-known Federal Funds Rate⁷. In brief, the modern banking sector is now very flexible, capable of innovations if authorities try to restrict its lending activities; consequently, this makes them less reliant on central bank when they balance their books at the end of the day. For example, in Switzerland, knowing that commercial bank had the capability of creating money despite the central bank wishes, the Government had to put forth a law which constrained the actual quantity of loans – “credit ceiling” – that chartered banks could provide to their customers, so that the Swiss National Bank would have a final say into the quantity of money circulating in the country. However, as Bernanke *et al.* (1999, pp. 64) argue, this idea which advocates that central banks have total control over money does not really hold in fact:

“In theory, the Swiss National Bank has perfect control over the monetary base. In practice, however, the Bank has repeatedly found itself forced to counteract large and sustained exchange-rate movements, usually appreciations of the Swiss franc, that obliged it to accept large, undesired expansions of the monetary base.”

2.2 The Economic Framework

In our model, we suppose that the central bank is fully independent of political pressures and that it wants to maximize a social welfare function which depends

⁷ As Hamilton and Jorda (1999, pp. 15) put it, “Banks’ aversion to the second and third options [i.e., “borrow the reserves from the Fed, or to manage with a lower level of excess reserves”] causes the equilibrium interest rate on loans of Federal funds to be bid up [...]”.

on the utilities of economic agents. However, as Blanchard and Fischer (1989, pp. 568) put it, such a function “rapidly becomes untractable”. As a possible alternative, the central bank considers a simple and commonly used macro function. In their recent work, Clarida, Gali and Gertler (1998a,b, 1999), Rotemberg Woodford (1998), Svensson (1998), Rudebusch and Svensson (1998), and Svensson and Woodford (1999), all consider simple quadratic loss functions which are very similar to the one considered here. We define the intertemporal welfare loss function as:

$$\min_{\{i_t\}} L_t = E \left[\sum_{t=0}^{\infty} (1 + \nu)^{-t} \{ \lambda_p (\pi_t - \bar{\pi}_t)^2 + \lambda_y (y_t - y_t^p)^2 + \lambda_i (i_t - i_{t-1})^2 \} \right] \quad (2.1)$$

In (2.1), $\bar{\pi}_t$, y_t^p and i_{t-1} are the targets considered by the central bank for inflation, output gap and the interest rate, respectively. Moreover, when the bank deviates from the target values, it faces a penalty, which we define as λ_p , λ_y , and λ_i , associated with each variable that enters in the loss function. Finally, ν is the rate at which we discount future deviations. For each period, we can rewrite (2.1) in compact matrix notation as:

$$\min_{\{i_t\}} L_t = \Lambda E (X_t - \bar{X}_t)^2 \quad (2.2)$$

Where Λ is a (3×3) matrix, in which the diagonal elements contain the penalties mentioned above, and the off diagonal elements are zeros. In the X_t (3×1) vector, we include the set of variables π_t , y_t , and i_t ; while we include the target variables in $\bar{X}_t$ vector. An optimal solution to (2.2) is found when $X_t = \bar{X}_t$, i.e. when the bank does not deviate from the targets.

The reason why we consider π_t , y_t , and i_t into (2.1) is that the welfare of society presumably depends upon the level of output, which translates into the level of employment and the wage rate. The potential output level is such that it maximizes production while it does not accelerate the inflation rate through excess demand. Also, by letting π_t enter the function, we explicitly assume that inflation, or disinflation, is costly when unanticipated by agents. It is reasonable to give more weights to deviations from the inflation target than from deviations from potential output or the lagged interest rate. Finally, including i_{t-1} illustrates the fact that we do not wish to see many movements in the interest rate.

According to Cukierman (1994), the central banks among industrialized countries tend to conduct monetary policies in a way that minimizes fluctuations in the financial markets in order to avoid as much as possible the apparition of financial crisis or the bankruptcy of financial institutions. Regarding this point, he argues that “there is little doubt that the Fed is concerned about the stability of the financial system in general and that of the banking sector in particular” (*Ibid*, pp. 119).

Svensson (1998, pp.12) argues that “[...] target variables are variables that appear in the loss function, while variables that appear in the reaction function are indicators, predetermined variables that cause and/or predict the target variables and therefore convey information”. This distinction is important since the central bank only enters the deviations in the reaction function instead of the target itself; “[...] any central bank trying to implement a reaction function [based on target

variables] would have strong incentives to deviate from it; the reaction function is not incentive-compatible" (*Ibid*, pp. 13). Consequently, when dealing with reaction functions, it is preferable to consider that the central bank responds to deviations rather than considering that it is targeting some variables. This is what we consider in equation (1.2).

Given the existence of the uncertainty problem relative to the potential output level or relative to inflation forecasts, Tetlow (1999a) argues in favour of giving less weight to deviations from the potential output level than from the expectations in inflation, in order to achieve the main goal of keeping prices under control. Moreover, we could also have used a forward-looking approach by using a forecast of $t + s$ period ahead in such a way that maximizes the goodness of the adjustment of the model. In effect, this is what have done Muscatelli *et al.* (1998), who find that a four quarters ahead forecast gives the best adjustment for their reaction function.

The specification used in (2.1) implicitly allows for nominal rigidities in the economy. For example, wages and prices are slow to adjust since labour agreements are contracted for long periods of time, based upon expectations regarding future economic growth and costs of living⁸. Thus, given the existence of nominal rigidities in the economy, monetary policy can affect the real economy in the short run while it has no tangible effects in the long run. In other words, we al-

⁸ Taylor (1979) elaborates a macro model where wages do not adjust instantaneously although expectations are rational. He shows that wages' persistent behaviour "[...] must be traded off against real output and employment stability" (Taylor (1979), pp. 110).

low for a short-run trade off between inflation and output, as is described by the Phillips curve. In the short-run, the Phillips curve has a negative slope while in the long-run its slope is strictly vertical, which characterizes the Friedman-Phelps accelerationist hypothesis. Therefore, reducing the rate of inflation may necessitate either a reduction in the level of actual output, a rise in the potential level of output, which may be caused by a positive technological shock, or a reduction in the agents' inflation expectations, which could be the result of an announcement by the central bank that it is changing its policy objectives towards a tighter monetary policy.

Furthermore, if such factors can reduce rates of inflation, adverse shock evidently can have opposite consequences on prices and output. In effect, the oil price shocks of the 70's can be seen as negative technological shocks, which lead to a reduction of the potential output level, while at the same time leading to an increase in the expected rate of inflation, as agents anticipated that oil price increases would eventually feed into the aggregate price level. According to Friedman (1968) and Phelps (1970), this phenomenon is explained by the fact that agents can be surprised by unanticipated shocks. Consequently, a period of time is needed so that the economy and the agents can gradually adapt to their new environment until a new equilibrium point is reached. This type of framework involving short-run and long-run relationship is similar to those found in most textbooks where Keynesian and Neoclassical macroeconomic policies are compared (*e.g.* Dornbusch, Fischer, and Sparks (1993)).

Next, we examine the unit root hypothesis of the time series used to estimate the coefficients in (1.2).

2.3 Stationarity Analysis

2.3.1 Methodology

Since Nelson and Plosser (1982), where they demonstrated that many of the macroeconomic time series are characterized by a stochastic process instead of a deterministic one, macroeconometricians have been engaged in a debate about whether or not the random shocks that hit the economy have temporary or permanent effects on the level of macroeconomic time series. The question of persistence is an important one since it determines how the econometrician must deal with non-stationary time series; that is, can she/he simply extract the trend function from the series, or must he difference the series in order to have a time series with finite second moment.

However, with the seminal work of Perron (1989), the debate goes a step further by now considering the fact that most shocks in the economy may only be of a conjunctural nature and thus be limited to temporary effects; while certain rare shocks of a structural nature may have, on the contrary, permanent effects. According to Perron (1989), structural changes can lead the econometrician to conclude that a time series follows an integrated process, while it is in fact an artefact creation, caused by a one-time shift in the intercept of the deterministic function and/or in the slope of the trend function. Perron (1989) illustrates this by considering the Nelson-Plosser macroeconomic time series and by allowing for

a single exogenous structural changes at the time of the great crash and of the oil crisis of 1973. For the model with the break occurring in 1929, Perron (1989) calls it the “crash model” by letting the intercept of the trend shift; while for the model with the break occurring in 1973, he allows for a growth slowdown by introducing a shift in the slope of the trend function.

Although most appealing, the approach of Perron (1989) has a major drawback in imposing the date of the break on the data-generating process of the series⁹. As an extension, Zivot et Andrews (1992) and Banerjee, Lumsdaine and Stock (1992), proposed a unit root test based on a data-dependent method to detect the date of the break in a time series. With less evidence, they nevertheless reach the conclusion that some of the Nelson-Plosser times series are in fact $I(0)$ after accounting for a break, which is treated as an unknown event¹⁰. Perron (1997), on the other hand, with a slightly different approach, reaffirms the previous findings of Perron (1989). In effect, after accounting for a structural change, Perron (1997) finds that eight of the fourteen Nelson-Plosser series are in fact stationary. In other words, the observations where the shift occurred are causing the series to be contaminated, which then makes it difficult to distinguish between an $I(0)$ versus an $I(1)$ process.

In order to investigate the univariate process of each of the time series that we are considering, we apply five tests. They are the augmented test proposed

⁹ For a criticism of the method, see Christiano (1992).

¹⁰ Using the 10% finite sample critical values they tabulated, Zivot and Andrews (1992) reject the null hypothesis of a unit root for seven of the fourteen series; while using the 5% critical values, they reject the hypothesis only for five of the series.

by Dickey and Fuller (1979) and by Said and Dickey (1984) (hereafter *ADF*), the *ADF* based on the Generalized Least Squares detrending procedure proposed by Elliot, Rothenberg and Stock (1996) (hereafter *ERS*), the nonparametric test proposed by Phillips and Perron (1988) (hereafter *PP*), Zivot and Andrews (1992) (hereafter *ZA*), and finally, the test proposed by Perron (1997) (hereafter *P97*). The last two tests allow for a more complex dynamics by introducing the possibility that the series are best represented by a shifting mean, a broken trend, or both simultaneously.

However, unit root tests like the *ADF* or the *PP*, are well known for having major weaknesses when the generating process is relatively more complex than “pure” linear autoregressive processes. For example, Schwert (1989), demonstrates, via Monte Carlo simulations, that the *PP* test may suffer major size distortions by rejecting the null hypothesis too often when the true data-generating processes are exhibiting strong moving average component. Moreover, if the true generating process of a series is one containing a structural change in either the intercept and/or the trend function, then both the *ADF* and the *PP* tests have little power in recognizing an $I(0)$ series form an $I(1)$. Perron (1989) performed Monte Carlo experiments to show that if a shift is important enough, unit root tests will not be able to reject the null of a root despite the condition that the process is truly stationary.

2.3.2 Testing for Unit Roots

In order to implement the unit root tests on the series, let $\{x_t\}$ be any one of

the three series. For the *ADF* test, we estimate the following test equation using ordinary least squares (*OLS*):

$$\Delta x_t = d_t + (\alpha - 1)x_{t-1} + \sum_{j=1}^k \phi_j \Delta x_{t-j} + \varepsilon_t \quad (2.3)$$

with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $E[\varepsilon'_t \varepsilon_\tau] = 0, \forall t \neq \tau$, and where Δ represents the difference operator. The deterministic component, d_t , can either be equal to $\{\emptyset\}$, the empty set; $\{1\}$, an intercept, μ ; or $\{1, t\}'$, an intercept and a slope, $\mu + \delta t$.

Under the null hypothesis, the *ADF* test considers a unit root in the *AR* component of the series ($\alpha = 1$). In this case, the series follows a random walk with or without a drift. Under the alternative hypothesis, the series is said to be stationary around its mean value and/or around a deterministic trend; which is frequently denoted by an $I(0)$ process.

To determine the lag length of the autoregressive structure in the expression (2.3), we use the sequential procedure proposed by Campbell and Perron (1991), with $k^{Max} \simeq (12 \times \frac{T}{100})^{1/4}$. Starting with k^{Max} , we estimate (2.3) sequentially until we have the k 'th significant lag at the 10% significance level, denoted here as k^* . Then, we can perform the unit root test on a series.

Similarly, the *ERS* approach consists of first locally removing the deterministic component of $\{x_t\}$ via *GLS*. However, the specification of d_t is now: $\psi' z_t$, where z_t is equal to $\{1\}$ or $\{1, t\}'$. Using the *GLS* method instead of *OLS* “permits a more accurate estimation of the deterministic component than what is possible using an *OLS* approach” (Perron and Rodríguez (1998), pp. 2). Denoting $\{\tilde{x}_t^{\hat{\alpha}}\}$ and $\{z_t^{\hat{\alpha}}\}$

as:

$$\tilde{x}_t^{\bar{\alpha}} = [x_1, (1 - \bar{\alpha}L)x_2, \dots, (1 - \bar{\alpha}L)x_T] \quad (2.4)$$

$$z_t^{\bar{\alpha}} = [z_1, (1 - \bar{\alpha}L)z_2, \dots, (1 - \bar{\alpha}L)z_T] \quad (2.5)$$

We define $\hat{\psi}$ as the estimator which minimizes the squared sum of residuals for $t = 1, 2, \dots, T$:

$$S(\psi, \bar{\alpha}) = \sum_{t=0}^T (\tilde{x}_t^{\bar{\alpha}} - \psi' z_t^{\bar{\alpha}})^2 \quad (2.6)$$

where $\bar{\alpha} = 1 + \bar{c}T^{-1}$, with $\bar{c} = -7.0$ for the case where $z_t = \{1\}$, and $\bar{c} = -13.5$ when $z_t = \{1, t\}'$. We Use $\hat{\psi}$ to construct $\tilde{x}_t = x_t - \psi' z_t^{\bar{\alpha}}$, to apply the standard *ADF* test on $\tilde{x}_t$.

For the *PP* test, based on a non-parametric approach, we must first remove the deterministic function of the series before estimating the test equation:

$$\Delta \tilde{x}_t = \alpha \tilde{x}_{t-1} + \varepsilon_t \quad (2.7)$$

where $\{\tilde{x}_t\}$ is a variable from which the deterministic function, d_t , has been removed. To compute the *PP* test, we need σ_ε^2 and $\sigma^2 = \lim_{T \rightarrow \infty} T^{-1} E(S_T^2)$, where $S_T = \sum_{j=1}^T \hat{\varepsilon}_j$. We correct for serial correlation using the Newey-West heteroscedasticity autocorrelation consistent estimator, denoted as $\hat{\omega}^2$ (a spectral density estimator at frequency zero):

$$\hat{\omega}^2 = \hat{\gamma}_0 + 2 \sum_{j=1}^q \left(1 - \frac{j}{q+1}\right) \hat{\gamma}_j \quad (2.8)$$

where $\hat{\gamma}_0 = E[\hat{\varepsilon}_t' \hat{\varepsilon}_t]$, and $\hat{\gamma}_j$ is the autocovariance function between $\hat{\varepsilon}_t$ and $\hat{\varepsilon}_{t-j}$, for $t, j = 1, 2, \dots, q$, where q is the truncation lag.

The *PP* test statistic, $Z_{t_{\hat{\alpha}}}$, is calculated as follows:

$$Z_{t_{\hat{\alpha}}} = \frac{\sqrt{\gamma_0} t_{\hat{\alpha}}}{\hat{\omega}} - \frac{T(\omega^2 - \gamma_0) \sigma_{\hat{\alpha}}}{2\hat{\omega} \hat{\sigma}_{\varepsilon}} \quad (2.9)$$

where $\sigma_{\hat{\alpha}}$ is the standard error of $\hat{\alpha}$.

As a more refined alternative, the *ZA* test considers instead the three following test equations:

$$\Delta x_t = \mu + \theta DU_t(\lambda) + (\alpha - 1)x_{t-1} + \sum_{j=1}^k \phi_j \Delta x_{t-j} + \varepsilon_t \quad (2.10)$$

$$\Delta x_t = \mu + \delta t + \gamma DT_t(\lambda) + (\alpha - 1)x_{t-1} + \sum_{j=1}^k \phi_j \Delta x_{t-j} + \varepsilon_t \quad (2.11)$$

$$\Delta x_t = \mu + \theta DU_t(\lambda) + \delta t + \gamma DT_t(\lambda) + (\alpha - 1)x_{t-1} + \sum_{j=1}^k \phi_j \Delta x_{t-j} + \varepsilon_t \quad (2.12)$$

The first model, called the “crash model”, given by expression (2.10), allows only for a change in the intercept. The second model, the “growth slowdown model”, given by expression (2.11), allows for a change in the slope of the trend function. Finally, for the third model, given by expression (2.12), a break is allowed in both components simultaneously. The dichotomous variables, $DU_t(\lambda)$ and $DT_t(\lambda)$, are defined by: $DU_t(\lambda) = 1(t > T_B)$ if $t > T\lambda$, and is equal to 0 otherwise, $1(\cdot)$ being the indicator function. Also, $\lambda = T_B/T$, which is the relative exogenous break point, T_B being the date at which the break point occurred, and T is the sample size. $DT_t(\lambda) = 1(t = T_B + 1)$, if $t > T\lambda$; and 0 otherwise. Under these specifications of the *ZA* test, the structural change occurs only gradually: this method is named the “innovative outlier”.

In order to choose T_B , we estimate one of the test equation described above through $t = k + 1, k + 2, \dots, T$, and we select the date, T_B , where we are the most

likely to reject the null hypothesis that the series contains a unit root. That is, the value which minimizes the t -statistics for testing $\alpha = 1$. This method is called the “infimum” procedure.

As for the $P97$ test, which is slightly different, we again remove the trending component of the series to estimate the following equation:

$$\Delta \tilde{x}_t = (\alpha - 1)\tilde{x}_{t-1} + \sum_{j=1}^k \phi_j \Delta \tilde{x}_{t-j} + \varepsilon_t \quad (2.13)$$

The major difference between the $P97$ test and the ZA test is that under the null hypothesis of $P97$, $\{x_t\}$ follows an integrated process in addition to a break; while under the alternative, the two tests are equivalent. Moreover, the $P97$ test considers that the break occurs immediately; this model is called the “additive outlier”. Perron (1997) derived the critical values for three different ways to select the break point; amongst them, we have only chosen the T_B which minimizes $t_{\hat{\alpha}}$ since our intention is not to compare their capacity to yield similar results.

2.3.3 Unit Root Tests Results

Before applying the unit root tests, we analyzed the univariate representation of each series, following the methodology proposed by Box and Jenkins (1976) to select the most appropriate $ARMA(p, q)$ model. Tests and estimations results are presented in Table 1.

Based on the Bayesian Information Criterion (BIC), it appears that the interest rate, $\{i_t\}$, can be best described as an $AR(1)$ process, with $\hat{\alpha} = 0.94$, which is close enough to unity to make us believe that it follows an integrated process and that it is exhibiting an highly persistent behaviour despite that the $AR(1)$ process

is stationary in the sense that $\hat{\alpha} < 1$. We estimated the *ADF* test equation using only an intercept and, for the lag structure, we have selected $k^* = 7$. Given this parameterization, we were not able to reject the null hypothesis, since $\hat{\alpha} = 0.94$ and $t_{\hat{\alpha}} = -2.01$. When applying the *PP* test, we arrived at the same conclusion that the interest rate is not stationary with $\hat{\alpha} = 0.94$ and $Z_{t_{\hat{\alpha}}} = -2.33$. Following the *ERS* detrending procedure, selecting $k^* = 7$, we have $t_{\hat{\alpha}}$ of -1.58 and an $\hat{\alpha}$ of 0.96 , which is not statistically different than one when we compare the calculated statistic to the critical values tabulated in Elliot *et al.* (1996)¹¹. Thus, according to *ERS*, we can conclude there is a unit root in the autoregressive component of $\{i_t\}$.

However, using the *ZA* approach with a $k^* = 9$, we found that the series suffers from a structural break in both the intercept and the trend function, with $\hat{\alpha} = 0.51$, and $t_{\hat{\alpha}} = -6.10$, which is well below the 1% asymptotic critical value of -5.57 . The consequences of the structural break are such that the interest rate can be said to be stationary around a broken trend and a shifting mean. Therefore, taking the first difference of the series would lead the model to be misspecified while also introducing a root in the *MA* representation. According to this test, we find that the date of the break is 1981Q1. This date is close enough to the major change in the Canadian monetary policy to say that the break is in fact caused by this important historical event.

Similarly, the *P97* approach test led us to the same conclusion that the series

¹¹ For a T of 200 and $d_t = \{1, t\}'$, the critical values for the 10%, 5% and 1% significance levels, are -2.64 , -2.93 and -3.46 , respectively.

is $I(0)$. In effect, with a $k^* = 7$, $\hat{\alpha} = 0.61$, and $t_{\hat{\alpha}} = -5.12$, once again above the 1% asymptotic critical value which is, in this case, equal to -4.91 ; using the finite sample critical values, we find the same conclusion, but this time at the 2.5% level. The break point was found to be at the observation 1979Q3. Although this time $\hat{T}_B$ is not as near the 1982 monetary policy change, the proximity of the break is sufficient to allow us to conclude that the $P97$ test captures the 1982 policy shift. In effect, the turning point of the inflation process, by looking at the graph, is 1982Q2, where a peak of 12.96% was reached.

For $\{\pi_t\}$, we select an $AR(4)$ process, which gives us¹² $\sum_{j=1}^4 \hat{\alpha}_j = 0.89$. With the ADF test, including only a drift and selecting a $k^* = 8$, we find that $\hat{\alpha} = 0.89$, and $t_{\hat{\alpha}} = -1.96$, which is well below the 10% critical value. Interestingly, the PP test first led us to believe that the inflation rate was following a stationary process since the test statistic was equal to -4.01 , and thus we reject at the 1% critical level. However, since we doubt this result to be robust, we investigated further by estimating a $MA(1)$ for $\{\Delta\pi_t\}$. We found that the moving average coefficient, $\hat{\theta}$, is equal to -0.59 , which we consider to be a highly negative moving average component. As we mentioned earlier, we suspect this negative MA process to be the reason why the PP test is leading to the rejection of the null hypothesis of a unit root. Using the method proposed by *ERS*, we reach to the same conclusion of a unit root in $\{\pi_t\}$. In effect, selecting a $k^* = 15$ (which is equal to k^{Max}), we have $t_{\hat{\alpha}} = -0.875$ and $\hat{\alpha} = 0.957$.

¹² According to *BIC*, dropping lag 2 and 3, which are not statistically significant, is more adequate. This, however, has little effect on the sum of $\hat{\alpha}$'s since it is lowered by only 0.02.

However, with the *ZA* test, we find that $\{\pi_t\}$ is $I(0)$ at the 2.5% significance level, with a broken trend and a shifting mean. In effect, selecting a $k^* = 12$, we have $\hat{\alpha} = 0.151$, and $t_{\hat{\alpha}} = -5.305$. The date of the break is found to be 1982Q4. In the case of the *P97* test, we arrive to the same conclusion at the 1% level. Selecting a $k^* = 12$, we found that $\hat{\alpha} = 0.051$, and $t_{\hat{\alpha}} = -5.12$. Interestingly, the break occurs in 1982Q4 as well.

Lastly, for the output series, we selected an *AR*(2), where we have $\hat{\alpha}_1 + \hat{\alpha}_2 = 0.99$, which is at the limit of describing a random walk. Applying the *ADF* test, we find, as expected, that $\{y_t\}$ is an integrated process, $\hat{\alpha}$ being equal to 0.97, and $t_{\hat{\alpha}} = -2.31$. With the *PP* test, we cannot reject the null hypothesis since $Z_{t_{\hat{\alpha}}} = -2.43$ and $\hat{\alpha} = 0.98$. Similarly, with the *ERS* procedure, selecting a $k^* = 14$, we have $t_{\hat{\alpha}} = -0.72$ and $\hat{\alpha} = 0.92$.

In fact, using either *ZA* or *P97*, it is not possible to reject the null that the series is integrated of order one. However, it is interesting to note that the *P97* test finds a break in 1979Q2, and that at the 10% asymptotic critical level, $\{y_t\}$ could be said to be an $I(0)$ process when allowance for a break in both the intercept and the slope of the trend function is made.

2.4 Conclusion

In conclusion to this section, we have showed that $\{i_t\}$ and $\{\pi_t\}$ can be described as stationary time series after allowing for a structural break in the deterministic function, a result in contradiction with standard testing procedures like the *ADF*, *ERS* or *PP* tests. But for the output series, the hypothesis that the

series follows a random walk appears to be the most appropriate to work with.

In the next chapter, we estimate the potential output level and the ex ante rate of inflation using Markov-Switching models.

CHAPTER 3

A Markov-Switching Approach in Modeling Output and Inflation

In this chapter, we estimate the output gap and the ex ante rate of inflation through a Markov-Switching approach. Firstly, to extract the business cycles from the output series, we use the approach proposed by Kim (1994), which is an extension of the $AR(4)$ model with two Markov-Switching regimes. This model was originally proposed by Hamilton (1989), where a shift in mean is allowed for the U.S. output growth¹. Secondly, we present our model for $\{\pi_t^e\}$, which is the same as that used by Garcia and Perron (1996) to model the U.S. real rate of interest. Hence, we use a $MS(3) - AR(2)$ to model the inflation rate, where both the mean and the variance are allowed to shift according to the state variable, s_t . Under this specification, we show that $\{\pi_t^e\}$ follows a stationary process since the sum of the autoregressive coefficients is near zero.

3.1 Estimating the Potential Output

3.1.1 Introduction

When comes the time to evaluate the potential output level, econometricians are facing the problem of estimating an unobservable variable which, evidently, has led to the rise of many approaches. If $\{y_t\}$ could be said to be a trend stationary process, then removing the trend function would yield a consistent measure of

¹ Hereafter, we write $MS(m) - AR(p)$ for an autoregressive model of order p with Markov-Switching with m regimes.

deviations from the deterministic trend components of $\{y_t\}$. However, rejection of the unit root hypothesis for the output series is very unlikely, as the application of unit root tests used in the first chapter suggests and as many other evidence on the matter tends to argue that output follows a random walk despite some notorious exceptions of Perron (1989, 1997), and Zivot and Andrews (1992), among others.

Clarida, Gali and Gertler (1998a) use the method proposed by Hodrick and Prescott (1997)²; while Tetlow (1999), Svensson and Woodford (2000), and Muscatelli *et al.* (1996) use a state-space representation as that proposed by Harvey (1981, 1989). Clark (1987) uses a method of unobserved components to model the output for the United States into a stochastic trend and cycle components, where output results in being an $I(2)$ process³. Another possibility is that considered by Rotemberg *et al.* (1998), who simply removed the linearly trending component of the U.S. output. As an alternative, we could also consider a polynomial trend function of an order of two or three, so that the model can capture the growth slowdown observed in output since the 70's.

More interestingly, Lam (1990) proposed a generalization of Hamilton's (1989) model. The main distinction with Lam's (1990) model is that the innovations in the economy have both permanent and transitory effects, whereas Hamilton's (1989) assumes that all shocks affect the level of output permanently (*i.e.* supply

² However, this approach is not free of drawbacks. See, for example, St-Amant and van Norden (1996) for a detailed discussion.

³ Nelson (1988) demonstrates via Monte Carlo simulations that when the true generating process of the data is a random walk, the state-space representation will lead to the extraction of spurious cycles.

shocks). Building on both Hamilton (1989) and Lam (1990), Kim (1994) proposed a state-space representation that yields results very similar to those obtained by Lam (1990) but which facilitated computations. Hence, to estimate the potential output level for the Canadian economy, we use the algorithm proposed by Kim (1994).

All of these different approaches are valid under the strong assumption that the resulting output gap is following a stationary process. This might be an assumption which is too strong, especially in the case of the linearly detrended output of Rotemberg *et al.* (1998). Thus, removing the trend component of output – or that of another time series – is a delicate task since spurious cycles can be extracted from the series.

Using Kim's (1994) approach, we find, among other results, that the mean growth rate for the Canadian output when the economy is running at full steam is 2.63%, with an expected duration of 1.54 quarters only; while the mean growth rate when the economy is running at a moderate pace is 0.78% with an expected duration of 6.47 quarters; and finally, for periods of slowdown, the mean growth rate is -0.75% with an average duration of 3.1 quarters.

Before presenting Hamilton's (1989) and Kim's (1994) models, the next section briefly presents some of the most important work that have been done in this recent econometric field of Markov-Switching.

3.1.2 MS(m)-AR(p) Models

Hamilton (1989) modeled the quarterly growth rate of the U.S. output for the

period 1952-84. He considers an $AR(4)$ process with 2 regimes for the mean growth rate of output, which is denoted as a $MS(2) - AR(4)$ model. Let $\tilde{y}_t = y_t - y_{t-1}$, and μ_{s_t} as the mean growth rate of output, which depends on the unobservable state of the economy, s_t , characterized by the business cycles. So we can write the $MS(2) - AR(4)$ representation as:

$$(\tilde{y}_t - \mu_{s_t}) = \phi_1(\tilde{y}_{t-1} - \mu_{s_t}) + \dots + \phi_4(\tilde{y}_{t-4} - \mu_{s_t}) + w_t \quad (3.1)$$

$$\mu_{s_t} = \mu_{0s_t} + \mu_{1s_t} \quad (3.2)$$

where s_t is a discrete variable such that $s_t = 1$ if $s_{t-1} = j$, for $j = 0, 1$; otherwise $s_t = 0$; $\{w_t\}$ is a white noise error term with mean 0 and variance σ_w^2 . Thus, the representation of $\{y_t\}$ in (3.1) is simply an $ARIMA(4, 1, 0)$ process with a shifting mean.

Among other things, Hamilton (1989) finds that the average period of expansions to last approximately 10 quarters, while the periods of recessions to last only 4 quarters. Also, the mean growth rate during the expansion period is 1.2%, while the growth rate during recessions is -0.4%.

One of the very interesting aspect of this model is that it treats the breaks as endogenous, inherent to the data-generating process of the variable, from which the econometrician must make inference on the probabilities of being in one of the m possible regimes. In other words, in the model $\{\tilde{y}_t\}$ is drawn from a mixture of normal distributions, each following a well defined stochastic process so that it is possible to find the log likelihood that the observations are drawn from one of the regimes that generated the data process. If the econometrician were actually

able to observe with certainty from which regime and distribution the data are generated, the model would then collapse to a simple regression with dichotomous variables that would capture the regime shifts, with the probabilities being known and taking the extreme values of either 1 or 0.

Following Hamilton (1989), there has been an extensive amount of work done on Markov-Switching models, which were a major breakthrough in modelling non-stationary time series by introducing non-linearities and asymmetries into autoregressive processes. Engle and Hamilton (1990), for example, examine the exchange rate behaviour. Lam (1990, 1997) extends the model of Hamilton (1989) model by using the method proposed by Durland and McCurdy (1994), which is equivalent to incorporating “mean growth rates that are dependent upon the duration of the current phase and transition probabilities that are dependent upon the durations of both the current and the previous phases” (Lam (1997), pp. 3). His main conclusion is that “[...] as an expansion ages, the growth rate of output declines” (Lam (1997), pp. 2) and that the probabilities of the economy to fall into a recession phase are declining as the expansion period ages. On the other hand, Simon (1996) applied a Markov-Switching model for the Australian inflation rate, where he allows for two different autoregressive coefficients depending on whether inflation is high or low. Kim (1993, pp. 341), demonstrates, using a state-space representation with four different regimes for the mean and the variance, that inflation is costly to the economy when it is in a higher phase, as the latter suggests “long term uncertainty”. Investigating the stock market, van Norden and Schaller

(1993) find that *MS-AR* models are better at representing the market than *ARCH* models.

Goodwin (1993) applies Hamilton's (1989) model to the growth rate of output for the G-7 economies plus Switzerland. In the case of estimates for Canada, the data between 1957Q2 and 1989Q3 period, Goodwin (1993) estimates the mean growth rates of $\{y_t\}$ to be -1.06% and 2.19% for the regimes 1 and 2 respectively. However, he finds that the expected duration of expansion periods to last 100 quarters, while the periods of contractions to last approximately 3.84 quarters. This expected duration for the period of expansions raises some questions regarding whether or not the model with only 2 regimes is the most appropriate when modelling the Canadian output.

We next discuss and develop our approach to obtain the output gap measure, which we later introduce in the reaction functions.

3.2 Modeling Output

Our specification for the measure of potential output draws essentially from Kim's (1994) algorithm with approximations for Lam's (1990) generalization of Hamilton's (1989) model. We therefore have the following representation:

$$y_t = x_t + n_t \tag{3.3}$$

$$n_t = n_{t-1} + \mu_{s_t} \tag{3.4}$$

$$\mu_{s_t} = \mu_{0s_t} + \mu_{1s_t} + \mu_{2s_t} \tag{3.5}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t \tag{3.6}$$

Here (3.3) decomposes $\{y_t\}$ into two components: n_t , a stochastic trend function, and x_t , the cyclical deviation from the stochastic trend; this last term is specified as being an $AR(2)$ process, μ_{s_t} is the growth rate of output following a first order Markovian process, subject to discrete shift, with $\mu_0 < 0$, $\mu_1 > 0$, and $\mu_0 < \mu_1 < \mu_2$. The unobservable state of the economy, $s_t = 1$ if $s_{t-1} = j$, otherwise, $s_t = 0$, and the error term, $\{e_t\}$, is an *i.i.d.* $N(0, \sigma_e^2)$ sequence. In general, we have $i, j = 1, 2, \dots, m$; with m being the number of regimes. Here, we have selected m to be equal to three.

For the state-space representation, we rewrite equations (3.3) to (3.6) in order to have both the measurement and transition equations and defining $\tilde{y}_t = y_t - y_{t-1}$:

$$\tilde{y}_t = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} + \mu_{s_t} \quad (3.7)$$

and

$$\begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \quad (3.8)$$

with

$$E(e'_t e_t) = Q = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.9)$$

Furthermore, we assume that e_t and e_τ are not correlated through time when $t \neq \tau$. The roots of the polynomial in the lag operator, $\phi(L)$, must lie outside the unit circle. Also, we define the probabilities of transitions of going from one

regime to the other as:

$$\Pr[s_t = j | s_{t-1} = i_{t-1}, s_{t-2} = i_{t-2}, \dots, s_1 = i_{t-T+1}, s_0 = i_1] = \Pr[s_t = j | s_{t-1} = i] = p_{ij} \quad (3.10)$$

for $i, j = 1, 2, \dots, m$. Expression (3.10) collapses to being conditional on only time $t - 1$ since we have defined s_t as being governed by a first order Markovian process. If s_t was instead evolving independently of its past values, the calculations would then be straightforward: $\Pr[s_t = j]$. However, this is not the case and inference is needed, which is what we do below.

The transition probabilities, denoted as p_{ij} , for $i, j = 0, 1, 2$, can be grouped in a (3×3) matrix named the matrix of transition probabilities:

$$P_y = \begin{bmatrix} p_{00} & p_{10} & p_{20} \\ p_{01} & p_{11} & p_{21} \\ p_{02} & p_{12} & p_{22} \end{bmatrix} \quad (3.11)$$

Obviously, we specify the constraint that the sum of the probabilities for each regime is equal to unity, which applies on the elements of each column of the P_y matrix:

$$\sum_{j=0}^2 p_{ij} = 1 \quad (3.12)$$

Using the elements of the main diagonal of (3.11) and letting D_j be the duration of the j -th regime, we can find the unconditional expected duration of a regime, which is given by:

$$E(D_j) = \sum_{j=1}^{\infty} j \Pr[D_j = j] = \frac{1}{1 - p_{jj}} \quad (3.13)$$

Let ξ_t be an identity matrix of dimension $(m \times m)$, where the j -th element of each column is equal to 1 if $s_t = j$. The steady state probabilities are then given by⁴:

$$E(\xi_t) = \Pr[s_t = j] = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad (3.14)$$

that we can use to initialize the filter. The conditional expectation of $\hat{\xi}_{t+1|t}$ is given by $P\xi_t$. Using the Chapman-Kolmogorov equations, we can make a prediction for τ -periods ahead regarding the probabilities that $\{\tilde{y}_t\}$ will be in state j given that we were in state i initially. This can be obtained by the expression: $\hat{\xi}_{t+\tau|t} = P^\tau \xi_t$. Finally, if $p_{jj} < 1$, when $\tau \rightarrow \infty$, then $P^\tau \xi_t \rightarrow 0^5$. In other words, the further we iterate into the future, the less likely we are to still be in regime j after τ transitions.

We also suppose that disturbances, $\{e_t\}$, are not correlated with precedent regimes:

$$E(e_t | s_{t-1} = 0) = E(e_t | s_{t-1} = 1) = E(e_t | s_{t-1} = 2) = 0 \quad (3.15)$$

However, their conditional variance is different from 0:

$$E(e_t' e_t | s_{t-1} = j) = p_{jj}(1 - p_{jj}) \quad (3.16)$$

The non-linearities arise since $\{\tilde{y}_t\}$ is drawn from a mixture of normal dis-

⁴ Note that in general ω is denoted by π , but in order to avoid confusion with the symbol of inflation, we use ω instead.

⁵ For further details on expectations of probabilities, see Ross (1989).

tributions, which in our case are three⁶. Therefore, we are maximizing the log likelihood function for all $t = 1, 2, \dots, T$. We denote the joint density function as $f(\tilde{y}_t | s_t, s_{t-1}, s_{t-2}, \psi_{t-1}; \theta)$, where $\psi_{t-1} = \{\tilde{y}_{t-1}, \tilde{y}_{t-2}, \dots, \tilde{y}_1\}'$ is the set of information available at time $t - 1$ and where $\theta = \{\phi_1, \phi_2, \mu_0, \mu_1, \mu_2, \sigma, p_{ij}\}'$ is the population parameter vector estimated to maximize the log likelihood function. First, denote Π as:

$$\text{vec}(\Pr[s_t = i, s_{t-1} = j, s_{t-2} = l | \psi_{t-1}]) = \text{vec}(\Pr[s_t = i | s_{t-1} = j] \times \Pr[s_{t-1} = j | \psi_{t-1}]) \quad (3.17)$$

Thus, Π is a vectorization of the transition probabilities, which are multiplied by a weighting factor, given by the expression: $\Pr[s_{t-1} = j | \psi_{t-1}]$. Then, we can write the log likelihood function as:

$$\ln L(\theta | \tilde{y}_t) = \sum_{t=1}^T \ln \left[\sum_{s_t=0}^2 \sum_{s_{t-1}=0}^2 \sum_{s_{t-2}=0}^2 f(\tilde{y}_t | s_t, s_{t-1}, s_{t-2}, \psi_{t-1}; \theta) \times \Pi \right] \quad (3.18)$$

The joint density function, $f(\cdot | \cdot)$, may be written as:

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{\{(\tilde{y}_t - \mu_{s_t}) - \phi_1(\tilde{y}_{t-1} - \mu_{s_{t-1}}) - \phi_2(\tilde{y}_{t-2} - \mu_{s_{t-2}})\}^2}{2\sigma^2} \right) \quad (3.19)$$

which is the usual form for the density function of a variable normally distributed.

We also see in (3.18) that Π acts as a weight on the m^m conditional density functions.

In order to be maximized, (3.18) must be twice differentiable, and we further assume that $\{\tilde{y}_t\} \sim N(\mu, \sigma_y^2)$. To find the maximum, let $\mathcal{L}(\theta | \tilde{y}_t) = \ln L(\theta | \tilde{y}_t)$, for some $\theta \in \tilde{\theta}$. We then set the first derivatives of $\mathcal{L}(\theta | \tilde{y}_t)$ with respect to θ equal to

⁶ For further details on mixtures of normal distributions, see Hamilton (1990, 1994).

0. For $\mathcal{L}(\theta|\tilde{y}_t)$ to be a local maximum, we must have:

$$\frac{\partial^2 \mathcal{L}(\theta|\tilde{y}_t)}{\partial \theta \partial \theta'} = H < 0 \quad (3.20)$$

Differentiating once (3.18) yields a $(l \times 1)$ *score* vector, while differentiating it twice yields the Hessian matrix, H , of dimension $(l \times l)$. Then, using the negative of the inverse of the matrix second derivatives of $\mathcal{L}(\theta|\tilde{y}_t)$, we have H^{-1} , which yields the variance-covariance matrix for the population parameters contained in the vector θ^7 .

To calculate (3.11) based upon $\psi_t = \{\psi_{t-1}, \tilde{y}_t\}'$, we iterate through $t = 1, 2, \dots, T$, to get the filtered probabilities, $\Pr[s_t = i, s_{t-1} = j, s_{t-2} = l | \psi_t]$, given by:

$$\frac{f(\tilde{y}_t | s_t = i, s_{t-1} = j, s_{t-2} = l, \psi_{t-1}; \theta) \times \Pi}{\sum_{s_t=0}^2 \sum_{s_{t-1}=0}^2 \sum_{s_{t-2}=0}^2 f(\tilde{y}_t | s_t = i, s_{t-1} = j, s_{t-2} = l, \psi_{t-1}; \theta) \times \Pi} \quad (3.21)$$

However, Kim's (1994) algorithm also involves an approximation of Hamilton (1989) via a state-space representation for which the Kalman filter is needed to estimate the coefficients contained in θ . According to Kim and Nelson (1999, pp. 70), Kim's (1994) algorithm is "[...] vastly more efficient than those in Hamilton (1989) and Lam (1990) in terms of simplicity and computation time". In the Appendix I (section 6.1), we briefly present the state-space model in the case of time-varying parameters (*TVP*). The state-space model we use to estimate the output gap is essentially the same with one important distinction: the coefficients for the mean are now a function of $\Pr[s_t = j | s_{t-1} = i, s_{t-2} = l]$.

⁷ For excellent discussions on likelihood functions, see Cramer (1986) and Hamilton (1994).

Next, we present the maximization results of (3.18), which provided us with the estimation of θ .

3.2.1 Maximum Likelihood Results

To initialize the algorithm, we did a systematic grid search in the space of $\tilde{\theta}$, confining the range in a closed interval for each parameters with a small increment at each step of the grid search for which we stored the estimated maxima of $\mathcal{L}(\theta|\tilde{y}_t)$ into a vector. Then, we simply estimated θ , given the starting value that yielded the global maximum for $\mathcal{L}(\theta|\tilde{y}_t)$, which we can denote as:

$$\arg \max_{\{\theta\}} \mathcal{L}(\theta|\tilde{y}_t) \quad (3.22)$$

Convergence was achieved when the gradients reached 0.000001 or until the maximization algorithm reached one thousand iterations, whichever came first. Estimations were done on Gauss386, and Kim and Nelson's (1999) source codes were used as a basis to construct the codes for the $MS(m) - AR(p)$ models. Also, due to notorious initial instability in the state-space algorithm, we started the iterations at the 9-th observation⁸. Maximum likelihood estimation results are presented in Table 2.

In Figure 4, we show the estimated potential output and output measures. Since the 60's, we notice that there are three major periods where the output has been well below its potential level. First during the period of the late 70's; second, the severe contractionary times of the early 80's, when the monetary policy became

⁸ Kim and Nelson (1999) start at the 23rd observation; but after consideration, we find no big difference between the two in our case, and starting at the 9th instead of the 23rd give us the advantage of losing less observations.

tight. Finally, it appears as if the early 90's recession and the major cutbacks in government spending during the 90's caused the Canadian output to remain well below its potential level during most of the last decade. Given our results for the output gap, we have only recently entered a phase where the production level would be above a sustainable potential level. These results would also partly explain why inflation has remained relatively low during the last decade, while it would also explain its recent resurgence as the strong current expansion brought y_t above y_t^P , thus creating excess demand. Another interesting feature of y_t^P , is that the resulting y_t^G encompasses the idea of Okun's law since y_t , during quarters where growth is positive but small, can actually be below y_t^P .

The maximum likelihood estimation results for the three mean growth rates of output are $\hat{\mu}_0 = 2.63$, $\hat{\mu}_1 = 0.78$, and $\hat{\mu}_2 = -0.75$. The autoregressive coefficients are $\hat{\phi}_1 = 1.35$ and $\hat{\phi}_2 = -0.46$. The matrix of transition is:

$$\hat{P}_y = \begin{bmatrix} 0.35 & 0.12 & 0.28 \\ 0.65 & 0.85 & 0.05 \\ 0 & 0.03 & 0.68 \end{bmatrix} \quad (3.23)$$

Figure 5 depicts the filtered probabilities of being in one of the three regimes. An important feature is that the filtered probabilities are in general near the boundary of the $[0,1]$ interval, which indicates a good fit of the model to the data. As shown in Figure 6, our model appears to be capable of capturing the main dynamic and cyclical movements of output as the filtered probabilities of being in one of the three regimes allowed are in connivance with times of recessions or booms. Moreover, we notice that the probabilities of being in the state of high growth remain

near zero after the boom of 1987. This is consistent with the Golden Age period, which lasted through the sixties until the oil price shock of 1973, where output grew at a much faster pace than what we have been observing since (see Figure 6). Most interestingly however, are the filtered probabilities of being in a state of contraction. They illustrate quite well the periods which are significantly marked with contractionary quarters, especially from 1981Q3 to 1983Q4, where they rose to nearly 100%. A similar behaviour is observed from 1990Q2 to 1991Q2, as they again rose from 26% to peak at nearly 100% during 1990Q4 until 1991Q1.

From (3.23), we notice some interesting characteristics. First of all, the probabilities of going into the regime of high growth, given that we were previously in the regime moderate growth, are 0.65; while it is impossible to go into the regime of high growth when we were previously in a recession. Moreover, the probabilities of going into a recession after having been in the regime of high growth are 0.28. Finally, the regime of moderate growth, given that we were in this regime previously, is the most dominating regime with a probability of 0.85.

Given the transition probabilities and using expression (3.13), we find that the expected duration of being in high growth, moderate growth, and negative growth, is 1.54, 6.47 and 3.1 quarters, respectively.

Thus, our representation of output illustrates the different features of output and the business cycles. As Hamilton (1989) demonstrated analyzing the case of the U.S., there are non-linearities in output, which translates into asymmetric dynamics of fluctuations. For the state-space representation of $\{\tilde{y}_t\}$ and testing for

one versus two regimes, we easily reached a conclusion in favour of the alternative of two regimes. Furthermore, when testing for two against three regimes, we reject the null hypothesis that $\{\tilde{y}_t\}$ is best characterized by two regimes, which consists of a regime of high, medium, and negative growth. Thus, the most appealing model is also the one selected according to Davies (1987) likelihood ratio test, for which we present the procedure in section 3.4. Moreover, we tested for the stationarity of $\{y_t^G\}$ using the *ADF* and *ERS* tests. Using a linear trend in the deterministic function, we concluded that $\{y_t^G\}$ was an $I(0)$ process⁹.

In the next section, we present the model for the ex-ante rate of inflation.

3.3 Modeling Inflation

For a number of years during the twentieth century, inflation rates across the industrialized countries were generally stable during the years when they except for war episodes. However, with the Korean and Vietnam wars the gold, prices rose despite the gold standard and the Bretton Woods monetary system. Nixon's decision to put an end to the gold window in 1971 and the first oil price shock of 1973 led inflation to levels that had not been seen in the U.S. or Canada for over twenty five years. Thus, after a period of stable and low inflation, the 70's were severely marked with both high and unstable rates. During the 80's, central banks in the western world tightened their monetary policies in order to bring down inflation, while during the 90's inflation has remained stable and low.

⁹ With *ADF*, using the method proposed by Campbell and Perron (1991) to select a $k^* = 3$, we can reject the null of a unit root at the 1% significance level. With *ERS*, selecting a $k^* = 4$, we reject the null of a unit root at the 5% significance level. All other results are unreported.

In light of this apparent random and uncertain behaviour of inflation, rational economic agents do their best in forming the optimal inflation forecast, so that they take the most efficient decisions with respect to their future wealth. Moreover, in modelling such forecasts over long periods of time that are characterized by randomness, the econometrician is facing the challenge of extracting agents' expectations under the assumption that agents are rational. The implications of the rational behaviour are that expectations cannot take a constant value throughout the years of ever changing underlying economic conditions. Therefore, to arrive with a model capable of capturing such time-varying expectations, we use a Markov-Switching model to infer that $\{\pi_t\}$ is generated by the dynamic of a particular regime, thus making possible discrimination amongst the regimes and the estimation of discrete shifts in agents' expectations.

If prices or inflation could be characterized by a linear stationary processes, we would not be facing any problems in capturing expectations. But, Perron (1993) showed the behaviour of the Canadian rate of inflation appears to be characterized by both non-stationarity and non-linearities, although these conclusions are sometimes subject to the sample taken under consideration¹⁰.

In the literature related to modeling the Canadian rate of inflation with Markov-Switching methodology, some interesting work has been done by Laxton, Ricketts and Rose (1994), and Ricketts and Rose (1995). In both cases, they use the representation of Hamilton (1989), but they also let the autoregressive coefficient be

¹⁰ Perron (1994) also considered different sub-samples for the period 1918 to 1992, at monthly and quarterly frequencies.

dependent on the state variable, s_t . After having experimented many different specifications, they conclude that inflation can be best characterized as following three different regimes. Also, they find that $\{\pi_t\}$ follows a random walk when it is generated by the higher regime. Seeing the world “as stochastic, subject to change in the underlying objectives of monetary policy”, they interpret the transition probabilities as reflecting the agents’ measure of the degree of confidence towards the monetary policy (Laxton *et al.* (1994), pp. 177). In other words, “agents’ forward-looking inflation expectations are formed using a weighted average of the forecast from the models, for each state, where the weights are the ex-ante probabilities that the various states will occur next period [...]” (Ricketts *et al.* (1995), pp. 1).

On the other hand, Garcia *et al.* (1996) modelled the ex-ante real interest rate, $\{r_t^e\}$, as a Markov-Switching model where the mean level and the variance are allowed to take three different values, depending on the state variable, s_t . They arrive to the conclusion that $\{r_t^e\}$ is stationary but is subject to discrete shifts, which, if not incorporated into the model, would then give the econometrician the false impression that he is facing an integrated process and that shocks to $\{r_t^e\}$ are permanent. In effect, using a linear representation, they find that $\sum_{j=1}^2 \hat{\phi}_j = 0.84$, which is quite different from the one they find when $\{r_t^e\}$ is modelled non-linearly: $\sum_{j=1}^2 \hat{\phi}_j = 0.02$. This results implies that in general the mean is reverting to its level after a shock although for some rare shocks the effects are long lasting. After modelling $\{r_t^e\}$, they manipulate r_t^e and i_t in order to get the ex-ante rate of

inflation, simply defined as $i_t - r_t^e = \pi_t^e$.

Instead, we directly modelled $\{\pi_t^e\}$ as also being governed by a three-state Markov-Switching model. In addition, as Ricketts and Rose (1995) did, we interpret the filtered probabilities as reflecting the relative beliefs of agents that the central bank will actually follow one of the three policy regime with respect to the rate of inflation it will tolerate. Moreover, given this interpretation of the filtered probabilities, we can also interpret $\{\pi_t^e\}$ as being the rate anticipated by agents, or, in other words, as being an implicit inflation target. This point is very interesting since Canada has been using an explicit target only since 1991, but the bank's authorities evidently paid a lot of attention to inflation in the past despite the fact there was no public announcement made relative to the inflation rate it would tolerate.

Thus, our objective is to estimate $\pi_t^e = E(\pi_t | \psi_{t-1})$, where $\psi_t = \{\pi_0, \pi_1, \dots, \pi_{t-1}\}'$ is the information vector available at time t . We can then specify $\{\pi_t\}$ by:

$$(\pi_t - \mu_{s_t}) = \phi_1(\pi_{t-1} - \mu_{s_{t-1}}) + \phi_2(\pi_{t-2} - \mu_{s_{t-2}}) + v_t \quad (3.24)$$

where $v_t \sim N(0, \sigma_{s_t}^2)$. We consider equivalent conditions as in the specification in (3.15) and (3.16). The mean rate and the variance of inflation can be written as follows:

$$\mu_{s_t} = \mu_{0s_t} + \mu_{1s_t} + \mu_{2s_t} \quad (3.25)$$

$$\sigma_{s_t}^2 = \sigma_{0s_t}^2 + \sigma_{1s_t}^2 + \sigma_{2s_t}^2 \quad (3.26)$$

After passing μ_{s_t} to the right hand side of (3.24) and taking the conditional expectations on both sides, we can write the expression for $\{\pi_t^e\}$ as:

$$E(\pi_t|\psi_t) = E(\mu_{s_t}|\psi_t) + \phi_1 E(\pi_{t-1} - \mu_{s_{t-1}}|\psi_t) + \phi_2 E(\pi_{t-2} - \mu_{s_{t-2}}|\psi_t) \quad (3.27)$$

The estimation procedure of (3.27) is very similar to that of $\{\tilde{y}_t\}$, although this time variance is also allowed to shift. Thus, the joint density function is now given by:

$$\frac{1}{\sqrt{2\pi\sigma_{s_t}^2}} \exp \left(-\frac{\{(\pi_t - \mu_{s_t}) - \phi_1(\pi_{t-1} - \mu_{s_{t-1}}) - \phi_2(\pi_{t-2} - \mu_{s_{t-2}})\}^2}{2\sigma_{s_t}^2} \right) \quad (3.28)$$

Since the variance is shifting as well, two additional parameters are now added to the vector of the population parameters, θ , that is: $\{\phi_1, \phi_2, \mu_0, \mu_1, \mu_2, \sigma_0^2, \sigma_1^2, \sigma_2^2, p_{ij}\}'$.

3.3.1 Maximum Likelihood Results

Given the three regimes specification, we get the following maximum likelihood results, which are presented in Table 3. For the autoregressive coefficients we have $\hat{\phi}_1 = 0.24$, and $\hat{\phi}_2 = -0.19$. These results contrast very much with those previously estimated under a linear specification where the sum of the ϕ 's was near unity. For the mean of $\{\pi_t\}$, we have $\hat{\mu}_0 = 9.24$; $\hat{\mu}_1 = 4.01$; and $\hat{\mu}_2 = 1.56$. The estimated variances are $\hat{\sigma}_0^2 = 4.64$, $\hat{\sigma}_1^2 = 2.39$, and $\hat{\sigma}_2^2 = 1.17$. All these parameters are significant at the 1% significance level. An interesting feature of the estimated coefficients is that $\hat{\mu}_2$, the mean rate of inflation corresponding to a monetary policy concerned with keeping prices under control, is about half a point below the official mid-range target announced by the Bank of Canada since 1994. Moreover, these results are in line with those of Ruge-Murcia (1998), who

shows that since the beginning of the official policy of inflation targeting by the Bank, inflation rates in Canada have been more likely to be below the mid-range of the target than above. Moreover, Ruge-Murcia (1998) estimates the mid-value of the target to be 2%, which is the same as the official target. The results of Ruge-Murcia (1998) suggest that the Bank of Canada has asymmetric preferences with respect to inflation, which helps explaining why we have been observing rates of inflation below the mid-point of the target during the 90's. Furthermore, our results are also highlighting quite well previous allegations (*e.g.* Ball and Cecchetti (1990)) which stated that inflation was more volatile when it was evolving at higher rates than at lower rates since the variance increases significantly as we go from a regime of low inflation to a regime of medium inflation, while it increases even more when we are under the regime of high inflation, being nearly twice as much as that of the medium regime.

We have estimated the following matrix of transition probabilities:

$$\hat{P}_{\pi} = \begin{bmatrix} 0.95 & 0.04 & 0.00 \\ 0.02 & 0.96 & 0.01 \\ 0.03 & 0.00 & 0.99 \end{bmatrix} \quad (3.29)$$

where we can see that once we enter in one of the regimes, the persistence is very high. Using expression (3.13), we calculated the expected durations for the regime of high, moderate and low inflation to be of 18.48, 25.38 and 81.96 quarters, respectively. It is most interesting to note that the expected duration for the regime of low inflation is the one who's $E(D_j)$ is the highest, and therefore the most persistent also.

Under the specification given by equation (3.27), we can see that the probabilities of changing regime are very small, as the element of the main diagonal are all near unity. In other words, each one of the three regimes are exhibiting an high degree of persistence, which is well characterized in Figure 6.

As for the case of the potential output estimations, the filtered probabilities (showed in Figure 7) are well distributed, being always near the boundary of the $[0,1]$ interval. Interestingly, in 1973Q2 the filtered probabilities of going into the regime of high inflation suddenly increased, to decrease only in 1983Q1. Furthermore, with a lag of one quarter, our model captured well the sudden and brief rise in inflation that occurred in 1991Q1. For the regime of moderate inflation, the filtered probabilities are high from 1966Q2 to 1972Q3, and from 1983Q3 to 1991Q1. Finally, the filtered probabilities for the regime of low inflation are close to zero from the beginning of the sample until 1966Q1, while remaining close to zero until 1991Q3. Afterwards, the filtered probabilities remain high until 1999Q4. All of this period corresponds to the period of official inflation targeting.

3.4 A Non-linear Testing Procedure

Even before testing for a $MS(3)$ model for $\{\tilde{y}_t\}$ against linear or non-linear specifications, the estimated model that yielded the most attractive results was the $MS(3) - AR(2)$. The model used in Kim (1994), an $MS(2) - AR(2)$, did not prove to be able to provide us with parameters that made any economic sense. For example, the filtered probabilities of being in one of the two regime were always equal to unity for the first half of the sample while it was equal to zero for the

second half. The consequences of such results are that once we enter the second regime we are to remain there for ever as $1/(1 - p_{jj})$ diverges to infinity. These results were robust to the choice of initial values used. Finally, another problem with the $MS(2) - AR(2)$ model is that we were confronted with having either a model where the filtered probabilities captured well the main features of the known business cycles but where the estimated potential output was not well behaved at all, or a model with opposite characteristics.

When testing Markov-Switching specifications, the commonly used tests such as the likelihood ratio or the Wald test cannot be applied as usual since a number of variables (denote them by κ) are not identified under the null hypothesis. This is a problem of having a pair of non-nested models, which is equivalent to have different vectors of population parameters to estimate. For the likelihood ratio test to be asymptotically distributed as a χ^2 , all the parameters must be present under the null hypothesis. But in the case of a $MS(m) - AR(p)$ model when comparing for a model with m regimes (alternative hypothesis) against a model with $m - 1$ regimes (null hypothesis), the parameters present under the alternative hypothesis with m regimes are not identified under the null hypothesis of $m - 1$ regimes. When some parameters are not identified under the null hypothesis, the distribution will then depend on nuisance parameters, leading to non-standard testing procedures. Therefore, to be able to verify if our model is truly justified despite its attractive results, we used the testing procedure proposed by Davies (1987), where the null hypothesis consists of m regimes with the alternative of $m - 1$ regimes.

The test of Davies' (1987) test consists of an upper bound for the significance level of the model stated under the null hypothesis. Define the likelihood ratio, which is a function of θ , by: $LR(\theta) = 2(\ln L_1(\theta_g) - \ln L_0)$, where $L_1(\theta_g)$ is the likelihood value calculated for the model with $m - 1$ regimes present under the alternative hypothesis, L_0 denotes the likelihood calculated under the null hypothesis, θ is a vector of the population parameters previously defined, given an observation on θ that is located in the space $\tilde{\theta}$, which contains all possible θ 's. Let θ^* be the argmax of $L_1(\theta_g)$ and denote the likelihood function under the alternative hypothesis at θ^* by L_1^* . Then:

$$\sup_{\theta \in \tilde{\theta}} LR(\theta) \equiv 2(\ln L_1^*(\theta_g^*) - \ln L_0^*) \quad (3.30)$$

Denote by Q the empirically observed value of $2(\ln L_1^*(\theta_g^*) - \ln L_0^*)$. Davies (1987) derived the following upper bound of Q :

$$\Pr \left[\sup_{\theta \in \tilde{\theta}} LR(\theta) > Q \right] \leq \Pr [\chi_\kappa^2 > Q] + V Q^{(\kappa-1)/2} \exp^{-(Q/2)} \frac{2^{-\kappa/2}}{\Gamma(\kappa/2)} \quad (3.31)$$

where V is simply equal to¹¹ $2Q^{1/2}$, and $\Gamma(\cdot)$ is the gamma function. Thus, even though the χ_κ^2 distribution enters expression (3.31), an additional weight, which is a function of the values of κ and Q , is added so that the non-standard test is not only compared with the χ_κ^2 distribution.

Now, we consider the results for the test of Davies (1987) for $\{\pi_t\}$. Testing for 1 regime against 2 ($\kappa = 3$) gives a p -value of 0.09. Thus, we are able to reject the

¹¹ Here, we follow Garcia and Perron (1996) in setting $V = 2Q^{1/2}$, which simplifies the calculations tremendously. In effect, this simplification is valid under the assumption that the likelihood ratio has a single peak.

null that $\{\pi_t\}$ is generated by a linear autoregressive process. Testing for the case of 2 regimes against 3 regimes ($\kappa = 5$), we are once again able to reject the null hypothesis with a p -value of only 0.01.

Thus, our 3 regimes Markov-Switching specification for $\{\pi_t\}$ appears to be strongly justified according to the test of Davies (1987).

For the $\{\tilde{y}_t\}$ specifications, we have the following results. Testing for a linear specification (*i.e.* a single regime) against a non-linear specification with two regimes ($\kappa = 3$), we can reject the null hypothesis of a linear state-space representation of a single growth rate of output in favour of a non-linear representation with two regimes since the p -value is 0.0008. Finally, testing for the null of two regimes against the alternative of three ($\kappa = 5$), we can still reject the null hypothesis in favour of the alternative of three regimes since the p -value is 0.0003. Therefore, our model for $\{\tilde{y}_t\}$, which yields the most attractive results, is also very well supported by Davies' (1987) test.

3.5 Conclusion

In this chapter, we have estimated the potential output level for Canada, and extracted the resulting business cycles. Among other interesting results, we have shown that the Canadian output has recently been unlikely to experience periods of high growth, such as those observed in the 60's. Moreover, our results suggest that business cycles are asymmetric since we estimated that a recession is likely to last approximately four times less than a period of moderate growth, and half the time of a period of strong growth.

The non-linear specification for the inflation rate process also demonstrates interesting characteristics. In addition to being the regime with the highest expectation of duration, the estimated ex ante rate of inflation for the 90's is nearly half a point lower than the centre of the target band set by the Bank of Canada. This gives strong evidence in favour of the "credibility" hypothesis advocated by the Bank since economic agents actually expect inflation to remain low as output grows. However, as we have showed, this is possible only when output remains below its potential level, which has been the case during the most of the 90's.

CHAPTER 4

The Benchmark Model

In this chapter, we present the Benchmark model regression results, which we demonstrate to be an unstable relationship during the period considered. In particular, the period around 1980 exhibits severe monetary shocks which are causing the Benchmark to contain a structural change. In addition to structural instability, after running a battery of diagnostic tests, we find evidence that the residuals of the model are not properly distributed. However, notice that our sole concern is to raise the existence of those problems without solving them directly since we consider them as emanating from structural instability as a result of monetary policy changes.

4.1 The Benchmark, Model I

We estimated equation (1.2), which we rewrite for convenience:

$$i_t = \mu + \beta_1(y_t - y_t^P) + \beta_2(\pi_t - E[\pi_t]) + \beta_3 i_{t-1} + \varepsilon_t \quad (4.1)$$

where we use the variables defined previously. Denote $y_t^G = y_t - y_t^P$, and for the this model, we have $\pi_t^G = \pi_t - E[\pi_t]$. To estimate $E[\pi_t]$, we use the intercept estimated for the linear $AR(4)$ univariate representation of π_t , which is equivalent to 4.519.

The lagged interest rate, i_{t-1} , reflects the degree to which the central bank is smoothing the variations in i_t . In addition, this assures us of having white noise

perturbations when there is first order autocorrelation.

4.1.1 Regression Results

The regression results for the Benchmark model are presented in Table 4. The estimated coefficients for Model I are indicating to us that the Bank of Canada has been reacting to inflation with very little aggressiveness, with $\hat{\beta}_2 = 0.13$, which is significant at the 0.01% significance level. Interestingly, it appears that the Bank of Canada has not been concerned with deviations from the output gap since the p -value of $\hat{\beta}_1$ is only 0.36. The value of the coefficient of the smoothing parameter, $\hat{\beta}_3$, demonstrates a lot of persistence in the behaviour of i_t , such as is showed by a $\hat{\beta}_3 = 0.89$. We have tested for a higher order of autocorrelation in the residuals using the test proposed by Ljung and Box (1978), but there are no evidence that the remaining residuals are correlated through time. Overall, the fit of the model is good with an adjusted $R^2 = 0.89$. But these results must be taken with great caution since the regressors are either trending or integrated, which, if not specified into the model, may lead the variance to be a function of time and thus to spurious results due to an inefficient estimate of the variance.

4.1.2 Diagnostic Tests

Our first step in analyzing this model, is by looking at the estimated residuals, $\{\hat{\varepsilon}_t\}$, whose behaviour is depicted in Figure 8. The most striking feature of this graph is that located around the period 1980, when major changes in monetary policy were implemented. In effect, Paul Volker then took over the Chairman's seat of the Board of Governors of the Federal Reserve System of the United States,

putting an end to money aggregates targeting and bringing about new ways to conduct monetary policy in the U.S., while shortly after, Canada followed up¹.

Figure 8 also depicts the variance of the forecast error, given by:

$$\hat{\eta}_{t|t} = E \left[\left((i_t(\cdot) - \hat{i}_{t|t}(\cdot)) \right)^2 \right] \quad (4.2)$$

During the third quarter of the year 1980 there as has been a major monetary policy shock, while shortly after we can see a decrease in the following quarters, it peaked once again in 1981Q1. It is also interesting to note that only a few quarters following the first oil price shock, in 1975Q1, there was another period of great uncertainty as $\hat{\eta}_{t|t}$ substantially rose. Finally, for the rest of the sample, $\hat{\eta}_{t|t}$ was generally low and stable with only minor peaks in 1984Q3, 1986Q1, 1992Q4, and lastly, in 1995Q1.

Since the distribution of the *OLS* estimator and its finite sample properties are based on having Gaussian residuals, an examination of their distribution is essential to determine whether or not the model is well estimated. Figure 9 depicts the kernel density of $\{\hat{\varepsilon}_t\}$. By looking at the graph, we see that $\{\hat{\varepsilon}_t\}$ does not seem to be following a normal distribution – non-Gaussian –, having a lot of mass centered around the mean, 0, and long and fat tails, indicating possible heteroscedasticity or outliers. By considering the third and fourth moments, we applied the test proposed by Jarque and Bera (1987), with which we strongly rejected the null hypothesis of normality, with a *JB* statistics of 198.37, yielding a *p*-value of 0.0.

¹ It is a reasonable assumption to say that Canada has been following closely the American policy since the end of the Bretton Woods monetary system – which Canada never officially adhered to.

Finally, we applied Engle's (1982) Lagrange Multiplier test (*LM*) test for the presence of autoregressive conditional heteroscedasticity (*ARCH*), which is described by the following autoregressive representation:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_m \varepsilon_{t-m}^2 + w_t \quad (4.3)$$

where $w_t \sim N(0, \sigma_w^2)$. Thus, with this representation, not only the conditional variance of $i_{t|t}$ is changing through t , but the conditional variance of ε_t^2 as well. We must also note that σ_w^2 is nothing else than the fourth moment of ε_t . To select the *AR*(m) structure, we followed Bollerslev (1988), who suggested to follow the Box-Jenkins method. Thus, after comparing among 4 specifications, based on *AIC*, we selected $m = 2$. Given this parameterization, we rejected the null that the conditional variance is homogenous at the 0.001% critical level. Thus, we say that $\varepsilon_t \sim ARCH(2)$ follows a stationary process since $\sum_{j=1}^m \hat{\alpha}_j < 1$. The innovation vector, $\{\hat{\varepsilon}_t\}$, is then distributed as $N(0, \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2)$. This presence of *ARCH* component in the model simply reflects the fact that there were periods where volatility was increasing due to monetary policy innovations; in particular around the years 1980-82.

4.1.3 Searching for a Structural Break

In order to evaluate the stability of the model, we first calculated a recursive ordinary least squares to (1.2). In Figure 10, we can see the recursive estimate of each coefficients. It clearly appears that $\hat{\beta}_2$ has gone through some fluctuations

² For a deeper discussion on the existence of the fourth moment of ε_t , the reader may consult Hamilton (1994).

around the first oil price shock, and once again during the year 1980 as it increased dramatically without returning to its previous level, thus indicating that the Bank has been allowing increasing attention to inflation over the years. Secondly, the coefficient for the output gap has gone through a similar dynamics, but in the opposite direction, therefore showing the fact that the Bank of Canada is now less concerned with output relative to inflation. In fact, for this model, we can say, without considering the standard error of the estimate, that the Bank does not respond to $\{y_t^G\}$ since 1980. As for the smoothing parameter, $\hat{\beta}_3$, it has been relatively stable, although with some movements during the known period of policy uncertainty that have hit the economy during the years 1973-75 and 1980-82.

Moreover, Figure 11 depicts the Cumulative Sums of Squares (*CUSUM*), as suggested by Brown, Durbin, and Evans (1975), which indicates that the model is unstable. However, the *CUSUM* square testing procedure does not have clear hypotheses identification³.

In a more rigorous way to test for parameter instability and structural changes, we first followed the method proposed by Andrews (1993). This test comes as a more practical extension to the tests originally proposed by Chow (1960), where the date of the break point is considered to be known a priori, and that proposed by Quandt (1960) where the alternative consist of a single break at some unknown t . However, in our case, we have no such certainty regarding the break point and it is considered as an unknown event, endogenous to the model and to the underlying

³ For a discussion on this issue, see Andrews (1993).

generating process of the data. Therefore, Andrews' (1993) testing procedure is non-standard since the "change point parameter only appears under the alternative hypothesis and not under the null" (Andrews (1993), pp. 821)⁴.

Under the null hypothesis the model is strictly linear. Therefore, the null of parameter stability is:

$$H_0 : \beta_t = \beta, \forall t \geq 1, \text{ for some } \beta \in \tilde{\beta} \quad (4.4)$$

where $\tilde{\beta}$ is the population parameter space.

Under the alternative, we are considering the case of "pure" structural change; that is, the whole population parameter vector is subject to change. Thus, we specify that:

$$H_{1T}(\lambda) : \beta_t = \begin{cases} \beta_1(\lambda), & \text{for } t = 1, 2, \dots, \lambda T \\ \beta_2(\lambda), & \text{for } t = \lambda T + 1, \dots, T \end{cases} \quad (4.5)$$

Denoting Y_t as the dependent variable vector and X_t as the vector of explanatory variables, including an intercept, and the β 's as the vector of the population parameters, we can rewrite the model under the alternative hypothesis in the following form in compact matrix notation:

$$Y_t = X_t \beta_1 + \varepsilon_{1t}; \text{ for } t = 1, 2, \dots, \lambda T \quad (4.6)$$

$$Y_t = X_t \beta_2 + \varepsilon_{2t}; \text{ for } t = \lambda T + 1, \dots, T \quad (4.7)$$

⁴ Before Andrews (1993), applying the Chow test randomly or systematically to every possible t may have lead the analyst to false or weak conclusions in spite of the strong appearance of results in favour of instability since the critical values of the F -distribution are too liberal and may lead to over-rejection of the null hypothesis that the model is stable, therefore accepting the alternative hypothesis that the model is heterogenous.

for some $\lambda \in [0, 1]^5$. In this case the variables are drawn from two distinct normal distributions, with different population parameters characterizing the alternative hypothesis ($\beta_1 \neq \beta_2$). This also implies that $E(\varepsilon'_{1t}\varepsilon_{1t}) \neq E(\varepsilon'_{2t}\varepsilon_{2t})$, or $\sigma_1^2 \neq \sigma_2^2$.

Andrews (1993) proposed three different ways to compute a test statistics: a supremum of Wald (SupW), a supremum of the Lagrange Multiplier (SupLM), and supremum of the likelihood ratio (SupLR). Let the squared sum of residuals (SSR) of the model under the null hypothesis be equal to $\varepsilon'_t\varepsilon_t$. While for the model given by expression (4.6) and (4.7), we have $SSR_1 = \varepsilon'_{1t}\varepsilon_{1t}$ and $SSR_2 = \varepsilon'_{2t}\varepsilon_{2t}$, respectively. Then, the statistics are calculated as⁶:

$$\text{SupW} = \max_{\{\lambda\}} T \left[\frac{SSR - SSR_1 - SSR_2}{SSR_1 + SSR_2} \right] \quad (4.8)$$

$$\text{SupLM} = \max_{\{\lambda\}} T \left[\frac{SSR - SSR_1 - SSR_2}{SSR} \right] \quad (4.9)$$

$$\text{SupLR} = \max_{\{\lambda\}} T \left[\frac{SSR}{SSR_1 + SSR_2} \right] \quad (4.10)$$

We calculated each statistics and found that there is strong evidence of a break using the asymptotic critical values tabulated by Andrews (1993). However, we must take with some precaution these results since the limiting distribution is derived only for $I(0)$ times series, and in our case we are facing non-stationary processes. Figure 12 depicts the behaviour of the SupW, SupLM and the SupLR with their respective critical values. The date of the break selected by the three

⁵ Andrews (1993) demonstrated that if λ is not bounded away from 0 or 1, then the statistics will diverge to infinity. Therefore, we use an arbitrary $\lambda \in [0.15, 0.85]$ as recommended by Andrews (1993).

⁶ Denote the SupW as the supremum of Wald; the SupLM as the supremum of the Lagrange multiplier; and finally, denote the SupLR as the supremum of the likelihood ratio.

tests is 1978Q2. For the $\text{Sup}W$ and $\text{Sup}LM$, which behave almost identically, we also note a smaller peak in the early 70's, but the tremendous peak that begins in 1977 and lasts until 1985 is by far the period marked with the highest supremum statistics and the most likely to exhibit structural changes.

In the case where the regressors are following $I(1)$ processes, Hansen (1992) proposed three testing procedures which can be denoted by a $\text{Sup}F$, $\text{Mean}F$, and L_c . All have the same null hypothesis of structural stability, and finite critical values were tabulated by Hansen (1992). According to Hansen (1992), the $\text{Sup}F$ and $\text{Mean}F$ tests have both local power against an alternative of cointegration, while the L_c is more appropriate for the case when we are interested in seeing if the model is only shifting slowly (the innovative model) rather than suffering from an abrupt structural break (the additive model) considered with the $\text{Sup}F$ and $\text{Mean}F$. Another interpretation of the L_c , is also to determine if the parameters are stable through time.

Since his testing procedure is fairly complex and requires a long development that is beyond the scope of this thesis, we do not enter into the details of constructing and estimating the cointegrated regression model⁷. In brief, his procedure consists of a fully modified (FM) estimation method based on a $VAR(1)$, where the covariance matrix of the residuals is pre-whitened using a kernel⁸. Afterwards, we get a transformed $\{y_t\}$ so that we can finally estimate the regression parameters

⁷ To apply the test, we used Hansen's (1992) accompanying Gauss source codes.

⁸ We have chosen Andrews (1991) quadratic spectral kernel.

and get the residuals series to construct the test. The approach of Hansen (1992) consist of a set of multiple equations in a cointegration framework based on the fully modified *OLS* method proposed by Phillips and Hansen (1990)⁹.

We computed the $\text{Sup}F$ (shown in Table 5), which is constructed as the $\text{Sup}W$ of Andrews (1993) and depicted in Figure 13¹⁰. Once again, we are able to reject strongly the null of structural stability with a $\text{Sup}F$ of 241.6, which we compared with the asymptotic 1% critical value of 21.4 tabulated in Hansen (1992). Furthermore, interpreting our results one step further, we can say that there is no cointegration relationship amongst the variables of the model. For the $\text{Mean}F$, which is simply defined as the mean of all the F -statistics computed previously, for $t \simeq \lambda T, \lambda T + 1, \dots, (1 - \lambda)T$, with $\lambda = 0.15$, the calculated value is 91.03, again well above the 1% critical value of 12.0, yielding a p -value of 0.01. The estimated coefficients with the FM estimator are: $\hat{\mu} = 0.95$, with a p -value of 0.004; $\hat{\beta}_1 = 0.11$, with a p -value of 0.004; $\hat{\beta}_2 = 0.03$, with a p -value of 0.33; $\hat{\beta}_3 = 0.88$, with a p -value of 0.0.

4.1.4 Testing for a Unit Root in the Presence of Outliers

In effect, if there exist a cointegration relationship in the model, we could say that the problems with the equation are somewhat less severe since in the long-run the variables are moving together in such a way that the residuals are stationary. To examine this matter under a different angle, we follow the two-step method

⁹ For all details regarding the development of the testing procedure, see Hansen (1992).

¹⁰ The sample is adjusted once again following the recommendations of Andrews (1993).

proposed by Engle and Granger (1987) and we apply an *ADF* test to the residuals series of equation (1.2). At first, we are led to believe that $\{\hat{\varepsilon}_t\}$ is in fact following a stationary process. Fitting an *AR*(1) process to $\{\hat{\varepsilon}_t\}$, we find that $\hat{\alpha}$ is close to zero at 0.02. But when we reestimate the model by adding a *MA*(1) term, $\hat{\alpha}$ goes up to 0.96 while the estimated *MA* coefficient is strongly negative at -0.99 . Thus, it is possible that this strongly negative *MA* component, which is near unity, causes the *ADF* test to suffer dramatic size distortions, leading to over rejection of the null hypothesis of a unit root.

However, a further examination of the residuals series of (1.2) suggests that there may be outliers present in $\{\hat{\varepsilon}_t\}$. Franses and Haldrup (1994) have demonstrated that the presence of outliers in time series are equivalent to observing a negative *MA* component, which could explain the controversial nature of our results in favour of the alternative hypothesis that the series does not contain a unit root¹¹. Therefore, we must remove the influence of additive outliers in a time series when we are testing for the presence of a unit root. To do so, one could simply remove the observations that are considered additive outliers, or one could also use dummy variables in the *ADF* test equation to remove their influence, which is what Vogelsang (1994) proposed. Vogelsang (1994) argues that this latter method is more appropriate than removing the outlier observations before applying an *ADF* test since the latter can lead to misleading inference. To investigate this matter thoroughly, we use the two robust unit root testing procedures in the

¹¹ For theoretical and simulation evidence on the matter, see Franses and Haldrup (1994) and Vogelsang (1999).

presence of additive outliers proposed by Perron and Rodríguez (2000), which are an extension of Vogelsang (1999).

In opposition to Vogelsang (1999), where a single critical value is used to detect outliers, the first method proposed by Perron and Rodríguez (2000) (called τ_c) uses different asymptotic critical values at each step of the searching process. In effect, Perron and Rodríguez (2000) argue that different critical values for finite samples must be considered since the limiting distribution is changing at each step of the search. The second method that Perron and Rodríguez (2000) proposed (called τ_d), is a procedure based on first-differenced data, which is more powerful than τ_c at detecting outliers when they are not of a very large size, a case that is often encountered in macroeconomic time series. Moreover, the finite critical distribution of the τ_d statistics does not change at each step of the search, and thus only a single finite critical value is necessary to apply the test.

Let $\{x_t\}$ be any time series, for $t = 1, 2, \dots, T$. Iterating through $t = 1, 2, \dots, T$, we generate a dummy variable, call it $T_{ao,j}$, such that $T_{ao,j} = 1$ if $t = T_{ao,j}$, otherwise $T_{ao,j} = 0$, for $j = 1, 2, \dots, m$. Then, we can estimate the following equation in level to detect outliers:

$$x_t = d_t + \gamma_j D(T_{ao})_t + u_t \quad (4.11)$$

where $D(T_{ao})_t = 1$ if $t = T_{ao,j}$ and 0 otherwise, with $T_{ao,j}$ ($j = 1, 2, \dots, m$) defined as the dates where additive outliers have been detected, and where $d_t = \{\beta' z_t\}$, with $z_t = \{1\}$, and $\{u_t\}$ is a *i.i.d.* $N(0, \sigma_u^2)$ sequence. At each step, we store the calculated $t_{\hat{\gamma}}$ value into a vector of dimension $(T \times 1)$. We denote $t_{\hat{\gamma}}$ as the t -statistic

for testing $\hat{\gamma} = 0$ in (4.11), and $\tau_c = \sup_{T_{ao}} |t_{\hat{\gamma}}(T_{ao})|$. Thus, if τ_c is greater than the appropriate critical value¹², an outlier is detected at date $\hat{T}_{ao} = \arg \max_{T_{ao}} |t_{\hat{\gamma}}(T_{ao})|$. Having detected an outlier, we remove the corresponding observation from $\{x_t\}$ and we reestimate (4.11) with the adjusted sample of $\{x_t\}$. This iterative detection process stops when τ_c is less than the critical value.

For the τ_d method, we use instead $\{\Delta x_t\}$ to detect outliers, estimating the following equation:

$$\Delta x_t = \gamma[D(T_{ao})_t - D(T_{ao})_{t-1}] + v_t \quad (4.12)$$

where $v_t = u_t - u_{t-1}$, and is a *i.i.d.* $N(0, \sigma_v^2)$ sequence. Similarly, we calculate the statistics $\tau_d = \sup_{T_{ao}} |t_{\hat{\gamma}}(T_{ao})|$. As for τ_c , the searching process stops when τ_d is less than the critical value. After we have generated the vector $D(T_{ao})_t$, we use it to apply the *ADF* test in the same fashion as for τ_c .

We can apply the *ADF* test equation on $\{x_t\}$ accounting for the presence of additive outliers found in the previous step (according to τ_c or τ_d):

$$x_t = \mu + \alpha x_{t-1} + \sum_{i=1}^{k+1} \phi \Delta x_{t-i} + \sum_{i=0}^{k+1} \delta_i D(T_{ao})_{t-i} + \varepsilon_t \quad (4.13)$$

In all, we have added $k + 2$ dummy variables to remove the influence of outliers in (4.13).

With the τ_c method, using 5% critical values¹³, we found evidence for the presence of two outliers, corresponding to observations 1980Q3 and 1981Q1. After

¹² The five asymptotic critical values at the 10% significance level for τ_c are 2.81, 3.38, 3.88, 4.33, 4.78 (Table 3, Perron and Rodríguez (2000))

¹³ The results were identical using 10% critical values.

accounting for those observations, we applied the *ADF* test and concluded that $\{\hat{\varepsilon}_t\}$ is stationary at the 1% significance level¹⁴. Using the τ_d method, with the 10% critical value¹⁵, we find evidence of three outliers, corresponding to observations 1980Q3, 1984Q4 and 1992Q2, while using the 1 and 5% critical values, the test is only able to detect a single outlier: 1980Q3. In all cases however, we are able to reject the null hypothesis that $\{\hat{\varepsilon}_t\}$ contains a unit root¹⁶.

Therefore, $\{\hat{\varepsilon}_t\}$ can be describe by a stationary process, which implies that there is a cointegration relationship –in the sense of Engle and Granger (1987)– amongst the variables. This result comes in contradiction with the results of Hansen’s (1992) test that we previously applied.

4.2 Conclusion

In conclusion, the Benchmark model does not exhibit an homogenous relationship and inference based upon this model may very well lead the analyst to spurious conclusions for a variety of reasons, as we have shown above. Especially, trying to identify and estimate the central bank’s reaction to inflation and output would not be appropriate since the estimated coefficients may only reflect a long-run average of the central bank’s behaviour.

In effect, we have found very strong evidence in favour of structural instability

¹⁴ Selecting a $k^* = 6$, we find that $t_{\hat{\alpha}} = -4.27$, with $\hat{\alpha} = 0.11$.

¹⁵ We simulated the critical values for $T = 150$.

¹⁶ Given the 10% significance level, and selecting a $k^* = 6$, we find that $t_{\hat{\alpha}} = -4.13$, with $\hat{\alpha} = 0.22$.

of parameters using different testing procedures. In all cases, the period of the early 80's appears to exhibit a great deal of uncertainty as the new monetary policy was implemented by the Bank of Canada.

However, an interesting feature of the behaviour of the monetary policy shock, $\{\hat{\varepsilon}_t\}$, is that it evolves according to a stationary process. Nevertheless, the existence of a cointegration relationship in the case of equation (1.2) is something for which more efforts should be devoted to in future studies, as we have found mixed evidence using two tests. Perhaps investigation in the Vector Autoregression (*VAR*) context with structural changes, as proposed by Ng and Vogelsang (1997), would be an alternative to consider.

In the next chapter, after taking the conditional expectations of $\{\pi_t\}$, we explicitly introduce a structural break into equation (1.2) in order to determine whether or not this is what is causing the instability in the parameters. Moreover, we estimate a time-varying parameters model to illustrate the evolution of the coefficients of the Taylor Rule.

CHAPTER 5

Non-Stationary Processes and Instability of Parameters, Model II

In this chapter, we extend the Benchmark model in two ways. First, we now take into account the dynamic process of $\{i_t\}$, for which we previously found evidence that it is following a stationary process when we allow for a shifting intercept and a broken trend. Furthermore, we now take the conditional expectation of $\{\pi_t\}$, as we have defined it in section (3.3).

Since a singularity arise when using a dichotomous variable for the structural break, no test based on a recursive approach can be applied in order to investigate the structural stability of the model. Therefore, we use a structural approach to illustrate the parameter instability of the monetary rule using a state-space representation. Among other results, we show that the period of the early 80's is once again exhibiting great parameter instability.

5.1 Model II

The unit root test results showed us that the overnight rate of interest was stationary when we introduced a structural change in its representation. Thus $\{i_t\}$ can then be written as:

$$i_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \delta_1 t + (\delta_2 - \delta_1)DT_t + \varepsilon_t \quad (5.1)$$

In section (2.3), we found $\hat{T}_B$ to be equal to 1981Q1 with the test proposed by Zivot and Andrews (1992), and 1979Q3 with the test proposed by Perron (1997).

We now estimate the model according to both selections of $\hat{T}_B$. Thus, introducing (5.1) and using $E[\pi_t|\psi_t]$ instead of $E[\pi_t]$ to have $\{\pi_t^G\}$, we can now rewrite the monetary Rule as:

$$i_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \delta_1 t + (\delta_2 - \delta_1)DT_t + \beta_1 y_t^G + \beta_2 \pi_t^G + \beta_3 i_{t-1} + \varepsilon_t \quad (5.2)$$

5.1.1 Regression Results

The regression results can be found in Table 6. Firstly, we estimated the model using the break found with the test of Zivot and Andrews (1992). The estimated intercept, $\hat{\mu}_1$, before 1981Q1 is equal to 0.03 and is not statistically significant since the p -value is 0.93. While the slope parameter of the trend function, $\hat{\delta}_1$, is equal to 0.03 and is significant at the 1% significance level. After 1981Q1, the intercept shifts up and $\hat{\mu}_2 - \hat{\mu}_1$ is equal to 3.64 with a p -value of 0.01. Subsequent to the structural break, $\hat{\delta}_2 - \hat{\delta}_1$ goes from being positive to being negative and is equal to -0.03 and is also highly significant with a p -value of 0.002, indicating a decrease in the process of $\{i_t\}$. As with Model I, $\hat{\beta}_2$ being equal to 0.136 with a p -value of 0.01, indicates little aggressiveness from the Bank of Canada in responding to $\{\pi_t^G\}$. For the coefficient of $\{y_t^G\}$, we estimated $\hat{\beta}_1 = 0.18$, significant at the 10% critical level. Finally, for the coefficient of $\{i_{t-1}\}$, β_3 is almost identical even after the introduction of a structural change into the model. In effect, using a Wald test with the null hypothesis that $\hat{\beta}_3 = 0.87$, which is the value of $\hat{\beta}_3$ estimated in the Benchmark model. Comparing with a χ^2 statistic, we could not reject the null hypothesis of homogenous slope coefficients with a p -value of 0.38. According to the normality test of Jarque and Bera (1987), the residuals are not

distributed normally since $JB = 186.72$, with a p -value of 0. We also tested for remaining *ARCH* effects, and as for the Benchmark estimations, Model II follows an autoregressive conditionally heteroscedastic process of order two. Finally, the calculated $\bar{R}^2$ of 0.89 is comparable to that of the Benchmark estimation.

We applied the statistics τ_c and τ_d to search for additive outliers in $\{\hat{\varepsilon}_t\}$. According to these procedures and the subsequent *ADF* test, we still find that ε_t contains outliers and that $\hat{\varepsilon}_t \sim I(0)$. In effect, with the τ_c method, at the 5 and 10% critical levels, we detect two outliers: 1980Q3 and 1981Q1. After accounting for these observations, the *ADF* results are, selecting a $k^* = 6$, $t_{\hat{\alpha}} = -4.37$ and $\hat{\alpha} = 0.06$; thus can we reject the null of a unit root at the 1% critical level. At the 1 and 5% critical levels, using the τ_d method, we detect a single outlier at observation 1980Q3. Selecting a $k^* = 6$, we have: $t_{\hat{\alpha}} = -4.69$ and $\hat{\alpha} = 0.06$, and thus we reject the null of a unit root at the 1% critical level. Instead, using the 10% critical value, we detect three outliers, at observation 1980Q3, 1982Q4 and 1992Q4. After accounting for these two observations, the *ADF* results are, selecting a $k^* = 9$, $t_{\hat{\alpha}} = -3.06$ and $\hat{\alpha} = 0.19$; thus we reject the null of a unit root at the 5% critical level.

Reestimating equation (5.2) using the break found with the unit root test of Perron (1997), we have almost identical results to those estimated previously. However, using a Wald test, we note that this time the autoregressive coefficient is statistically different from the one estimated in the Benchmark model. Testing for $\hat{\beta}_3 = 0.89$, the p -value is equal to 0. Thus, the addition of a structural break

for both the intercept and the slope of the trend function at date 1979Q3, makes the rate of interest exhibit a less persistent behaviour than in the previous two models. Finally, the $\bar{R}^2 = 0.89$, $JB = 198.97$ (p -value of 0), and $\hat{\varepsilon}_t^2 \sim ARCH(2)$.

Similarly, when testing for the presence of a unit root in $\{\hat{\varepsilon}_t\}$, we find that in $\hat{\varepsilon}_t \sim I(0)$. In effect, with the τ_c method, at the 5 and 10% critical levels, we detect two outliers: 1980Q3 and 1981Q1. After accounting for these observations, the ADF results are, selecting a $k^* = 6$, $t_{\hat{\alpha}} = -4.2$ and $\hat{\alpha} = 0.17$; thus we reject the null of a unit root at the 1% critical level. At the 1 and 5% critical levels, using the τ_d method, we detect a single outlier at observation 1980Q3. Selecting a $k^* = 6$, we have: $t_{\hat{\alpha}} = -4.59$ and $\hat{\alpha} = 0.15$, and thus we reject the null of a unit root at the 1% critical level. Instead, using the 10% critical value, we detect three outliers, at observation 1980Q3, 1982Q4 and 1992Q4. After accounting for these observations, the ADF results are, selecting a $k^* = 9$, $t_{\hat{\alpha}} = -3.11$ and $\hat{\alpha} = 0.17$; thus we reject the null of a unit root at the 5% critical level.

In conclusion, introducing an endogenous structural break does not render a satisfactory model in the sense that the residuals still exhibit major anomalies (*e.g.* non-normality), which we suspect to be caused by unstable parameters. Moreover, the innovation vector is still characterized by a complex dynamic where outliers are present, although $\hat{\varepsilon}_t \sim I(0)$. In the next section, we present the time-varying parameters model which depicts the unstable behaviour of the parameters through time.

5.2 The Time-Varying Parameter Model, Model III

To be able to capture the dynamics of a structural model, the state-space representation can be used to compute the estimates of a state vector for $t = 1, 2, \dots, T$ using the Kalman filter. To illustrate the evolution of the coefficients of the Taylor Rule, we estimated a time-varying parameters (*TVP*) model of the following form in compact matrix notation:

$$i_t = X_t \beta_t + e_t \quad (5.3)$$

where

$$\beta_t = F \beta_{t-1} + v_t \quad (5.4)$$

The X_t matrix, of dimension $(T \times k)$, contains our usual explanatory variables. However, we now have the state vector β_t , a $(k \times 1)$ vector that contains all the slope coefficients, which are now varying through time. The F matrix, of dimension $(k \times k)$, contains the autoregressive coefficients of β_t . We can either let β_t follow a random walk or have β_t follows a stationary $AR(1)$ process, which is what we consider here.

Although the estimation of (5.3) and (5.4) is also possible through a Generalized Least Squares (*GLS*) estimator, the Kalman filter is the most appropriate tool since the *GLS* requires the inversion a $(t \times t)$ matrix through out each updating step of the estimation, going from $t = k + 1, k + 2, \dots, T$.

The method of time-varying parameters was first introduced by Rosenberg (1972), and Cooley and Prescott (1973) and has been widely applied since. In a

Bayesian fashion, the Kalman filter allows us to estimate $\beta_{t|t}$ for $t = 1, 2, \dots, T$, so that we minimize the variance of the forecast error at each point in time given all the information available at time t . Therefore, when we face a model where the coefficients are evolving stochastically over time, or where the parameters are instable through time, the *TVP* model is most appropriate technique to illustrate such a changing behaviour.

The Kalman filter has been widely applied to econometrics for the last twenty years. The main interest of the Kalman filter is its capability to estimate the unobservable component of a state vector. Additional to some of the previous work mentioned above, Burmeister, Wall, and Hamilton (1986), applied the Kalman filter to extract the expected rate of inflation in a multivariate approach. Kim and Nelson (1989) applied it to estimate a *TVP* model for a U.S. monetary growth function. Finally, Stock and Watson (1991) used it to develop their coincident economic indicator for the macroeconomic conditions in the U.S.. In the appendix section, we present the main development of the state space representation.

5.2.1 Maximum Likelihood Results

We started the estimation at $t = 11$ so that some of the noisy behaviour which appears at the beginning of the estimation would be ignored. The maximum likelihood results for the population parameters are presented in Table 7. Given a time-varying parameter specification, we can rewrite the monetary rule as:

$$i_t = \mu_t + \delta_t t + \beta_{1t} y_t^G + \beta_{2t} \pi_t^G + \beta_{3t} i_{t-1} + \varepsilon_t \quad (5.5)$$

In Figure 14, we can see the evolution of each coefficients. Interestingly, they

all appear to be undergoing major changes through out the sample and especially around the period of 1980. First, for the intercept, $\hat{\mu}_t$, we notice that it starts a about 2 and remains there until 1971, to quickly increase and reach a peak of 3.6; it remains in the neighbour of 3 until 1980, where it falls to 1.15 in 1981. For the remainder of the sample, it slowly rises to 2.5, with a small abrupt upward shift around the period of 1991. Secondly, for the coefficient, $\hat{\beta}_{1t}$, although the picture is not as clear, we nevertheless see that it starts above one in 1965 but falls to 0.3 in 1970, to reach a steady-state low of 0.2 in 1980. For the coefficient, $\hat{\beta}_{2t}$, it starts low but positive. However, from 1972 until 1980, it is negative at -0.05 . The economic implications of having a negative coefficient for $\{\pi_t^G\}$ are very important since it implies that the Bank of Canada was in fact conducting an inflationary (accommodating) monetary policy during this period. Then, from 1980 until today, $\hat{\beta}_{2t}$ kept on rising, hitting a peak of 0.13 in 1994, where the inflation rate was actually at it lowest in years. Nowadays, it is approximately 0.1. For the coefficient of the smoothing parameter, $\hat{\beta}_{3t}$, we see an interesting behaviour. In effect, until 1975 $\{i_t\}$ seems to be following a stationary process since the value of the coefficient, $\hat{\beta}_{3t}$, is close to zero. However, starting in 1975, it jumps above the 0.75 mark and remains above 0.5 until the early 80's, reflecting a persistent behaviour and possibly an integrated process. Afterwards, it decreases to become negative, reaching a low of -0.5 in 1993. Finally, for the trending component coefficient, $\hat{\delta}_{2t}$, it also exhibits a very volatile behaviour, with the years 1971 and 1980 being periods of important monetary policy changes.

When looking at the variance of the forecast error, depicted in Figure 15, 1981Q4 comes as the period where the variance is the highest, illustrating agents great uncertainty regarding their revisions of the coefficients. In a *TVP* model, uncertainty arises from two sources. The first is simply caused by random monetary shock hitting the economy, while the second emanates from the forecast of $\beta_{t|t}$.

As expected after a visual inspection of $\{\hat{\varepsilon}_t\}$, which is also depicted in Figure 15, the residuals series contains outliers, reflecting periods of monetary policy uncertainty. In effect, using the τ_c method, we detected two outliers: 1980Q3 and 1981Q1 using the 1% critical value¹. In effect, after accounting for those observations, selecting a $k^* = 2$, the *ADF* statistics is $t_{\hat{\alpha}} = -5.22$, with a $\hat{\alpha} = 0.23$. Moreover, using the τ_d method with the 10% critical values, we detected a total of six outliers²: 1975Q1, 1980Q1, 1980Q3, 1981Q2, 1986Q1, and 1992Q4. After accounting for those dates and estimating the *ADF* test on $\{\hat{\varepsilon}_t\}$, we can reject the null hypothesis that $\{\hat{\varepsilon}_t\} \sim I(0)$ at the 1% significance level. Interestingly, if we separate these six outliers into three different periods: the mid 70's, the early 80's, and the early 90's, we can say that these three periods roughly correspond to important dates. In effect, the first oil price shock, the monetary policy change of 1981, and the other monetary policy change of 1991.

Thus, the *TVP* model, with its capacity to capture parameter instability, gives additional proof in demonstrating that a monetary reaction function cannot

¹ Results are equivalent whether we use the 5 or 10% critical values.

² At the 5% significance level, we actually found five outliers (1992Q4 was not detected).

be characterized by an homogenous model over long periods of time.

5.3 Conclusion

In this chapter, we have showed that even after allowing explicitly for breaks in the equation, we still find evidence that the noise function is not well behaved as significant outliers still remain and the normality assumption of $\{\hat{\varepsilon}_t\}$ is also violated. Moreover, the hypothesis that $\beta_t = \beta$, for all $t \geq 1$, is also challenged by our estimations of the *TVP* model. The maximum likelihood results are illustrating well the common belief that Bank of Canada changed its behaviour throughout the years.

Therefore, if the analyst is concerned with using all data available to her/him, she/he must investigate and directly model this changing behaviour in an appropriate manner, treating these changes as inherent to the data and the model itself. This is what we intend to do in the remaining parts of this thesis.

CHAPTER 6

Detecting and Modeling Structural Changes, Model IV

In this chapter, we present the results for Model IV, which is based on the method proposed by Bai and Perron (1998a, 1998b) to detect and model structural changes in the multivariate context. Based upon various tests, we show that the Taylor Rule for Canada can contain up to five structural breaks. However, the most interesting results are for the case where three breaks are allowed. According to this specification, we show that the coefficient β_2 has become significant after the monetary policy change of the early 80's, which illustrates the behaviour of a central bank determined to fight inflation. However, when using the method of Bai and Perron (1998a, 1998b), the detection of a structural change around the period of 1990 is impossible, as most of the instability appears to be located in the 70's and the early 80's.

6.1 Introduction

In Chapter 2, we searched for structural changes in the univariate context, where we allowed a single structural change in the deterministic component of the series, while in Chapter 3 we investigated the behaviour of the Benchmark model and search for an unknown single break using the tests proposed by Andrews (1993) and Hansen (1992). In Model II, we also introduced directly into the equation a structural change in the deterministic function, d_t , as an additional regressor.

However, this did not turn out to be a solution to the instability problem of the Benchmark model.

Recent developments in the context of the multivariate case with multiple structural changes have been made by Liu, Wu and Zidek (1997), who proposed an information criterion to select the number of changes for linear models estimated with *OLS*. More interestingly, Bai and Perron (1998a, 1998b) investigated the *SupW* to test for the null hypothesis of no break versus an alternative hypothesis where “an arbitrary number of changes” are allowed. Moreover, they also proposed a testing procedure where the null consist of l changes versus an alternative of $l + 1$ changes. This procedure, as Bai and Perron (1998b) argue, is attractive since it allows the analyst to specify and estimate the model according to the number of changes. Thus, the work of Bai and Perron (1998a, 1998b) can be seen as an extension of the important body of work that has been done in recent years in the context of unit root tests with structural breaks¹.

In our case, where we investigate the behaviour of the reaction function for the Bank of Canada over the period 1963Q2 to 1999Q4, the methodology proposed by Bai and Perron (1998b) allows us to capture statistically the monetary policy changes for which the information are contained in the underlying dynamic of the monetary reaction function.

¹ See, among others, Perron (1989, 1997), Zivot and Andrews (1992), and Banerjee, Lumsdaine, and Stock (1992).

6.2 The Methodology of Bai and Perron

Following Bai and Perron (1998b) and using a similar notation, we consider the following general specification:

$$\begin{aligned}
 i_t &= z_t' \gamma_1 + x_t' \beta + \varepsilon_t, \quad t = 1, 2, \dots, T_{B_1} \\
 i_t &= z_t' \gamma_2 + x_t' \beta + \varepsilon_t, \quad t = T_{B_1} + 1, \dots, T_{B_2} \\
 &\vdots \\
 i_t &= z_t' \gamma_{m+1} + x_t' \beta + \varepsilon_t, \quad t = T_{B_m} + 1, \dots, T
 \end{aligned} \tag{6.1}$$

According to this specification, we have the $(q \times 1)$ vector z_t , and the $(p \times 1)$ vector x_t , with their respective vector of coefficients, γ_j and β ; we also denote the unknown break points as $T_{B_j} = \{T_{B_j}\}$, for $j = 1, 2, \dots, m+1$, with $T_{B_0} = 1$ and $T_{B_{m+1}} = T$, such that $1 < T_{B_1} < \dots < T_{B_m} < T$, and $\{\varepsilon_t\}$ is an *i.i.d.* noise sequence with mean zero and variance σ_ε^2 . The vectors x_t and z_t may contain either one of the explanatory variables: $\pi_t^G, y_t^G, i_{t-1}, d_t$. If the analyst has reasons to believe that one or more coefficients are stable through time, than we have a model where both $q > 0$ and $p > 0$, which is known as a “partial structural change model” since the coefficients contained in β are not allowed to shift. This is also nesting Model II as a special case when only the deterministic component, d_t , is allowed to shift. Whereas if $p = 0$, we then have a “pure structural change model” and all the coefficients are shifting at dates T_{B_j} .

To estimate (6.1), we first define $Z = (z_1, z_2, \dots, z_T)'$, that we diagonally partition so that we have the block diagonal matrix $\bar{Z} = \text{diag}(Z_1, \dots, Z_{m+1})$ at date

T_{B_j} , yielding an $m + 1$ partitioned matrix. Then, for each partition, we calculate β and γ_j . In compact matrix notation, we can rewrite (6.1) as:

$$I = X\beta + \bar{Z}\gamma + \varepsilon \quad (6.2)$$

where $I = \{i_1, \dots, i_T\}'$, $X = \{x_1, \dots, x_T\}$, $\beta = \beta_i$, for $i = 1, \dots, p$, $\gamma = (\gamma'_1, \dots, \gamma'_{m+1})'$, and $\varepsilon = \{\varepsilon_1, \dots, \varepsilon_T\}'$. So that the *OLS* estimates and the squared sum of residuals are:

$$(I - X\beta - \bar{Z}\gamma)'(I - X\beta - \bar{Z}\gamma) = \sum_{i=1}^{m+1} \sum_{t=T_{B_{i-1}}+1}^{T_{B_i}} [I - X\beta - \bar{Z}\gamma]^2 \quad (6.3)$$

$$SSR_T(\hat{T}_{B_1}, \dots, \hat{T}_{B_m}) = \sum_{i=1}^{m+1} \sum_{t=\hat{T}_{B_{i-1}}}^{\hat{T}_{B_i}} [I - X\hat{\beta} - \bar{Z}\hat{\gamma}]^2 \quad (6.4)$$

Where $\hat{\beta}(\{\hat{T}_{B_j}\})$ and $\hat{\gamma}_j(\{\hat{T}_{B_j}\})$ are the estimated coefficients of (6.1), with the estimated break points $(\hat{T}_{B_1}, \dots, \hat{T}_{B_m})$ such that they are minimizing (6.4) over all the m -partitions:

$$(\hat{T}_{B_1}, \dots, \hat{T}_{B_m}) = \arg \min_{\{T_{B_j}, \dots, T_{B_m}\}} SSR_T(T_{B_1}, \dots, T_{B_m}) \quad (6.5)$$

Depending on the relative distance we allow in between each break point (call it ϵ), we can have at most $T(T+1)/2$ segments to estimate, which creates an enormous computing burden as $T \rightarrow \infty$. Since we impose a minimum of observations in between each break point, $h = \epsilon \cdot T$, we are reducing the maximum number of cases to $(h-1)T - h(h-2)(h-1)/2$. After we have estimated all the segments², the algorithm proposed by Bai and Perron (1998b) can be used to find the optimal

² Bai and Perron (1998b) argue that by setting $h = 1$, $q = 1$, and $p = 0$, with $z_t = \{1\}$, the algorithm can be used to detect outliers in the univariate context.

partition³.

Thus, we wish to find a global minimum for the squared sum of residuals using the algorithm, going from a single regime model to a model with $m + 1$ regimes. Denoting θ as the vector of population parameters $\{\gamma_j, \hat{T}_{B_1}, \dots, \hat{T}_{B_m}\}'$, we iterate on $SSR(\beta, \theta)$ to find the estimated minimization of the residuals:

$$\arg \min_{\{T_{B_1}, \dots, T_{B_m}\}} SSR_T(\beta, \theta) \quad (6.6)$$

6.2.1 Testing for Multiple Structural Changes

To test for the presence of structural breaks, Bai and Perron (1998b) are considering six procedures: $UDmax$, $WDmax$, $\sup F_T(k; q)$, $\sup F_T(l + 1|l)$, $BIC(m)$, $LWZ(m)$, and the sequential procedure.

The $UDmax$ test has the null hypothesis of no structural break and an alternative hypothesis of an unknown number of breaks, m , given some upper bound denoted here as M . This test can be defined as:

$$UDmax F_T(M, q) = \max_{1 \leq m \leq M} F_T(\hat{\lambda}_1, \dots, \hat{\lambda}_m; q) \quad (6.7)$$

for $\hat{\lambda}_j = \hat{\lambda}_1, \dots, \hat{\lambda}_m$, $\hat{\lambda}_j = \hat{T}_{B_j}/T$, and $\hat{\lambda}_j \in \Lambda$, with $\hat{T}_{B_j}$ based on (6.5). Under (6.7), the $UDmax$ test statistics allows the same weight for each segment.

On the other hand, the $WDmax$ test considers different weights, which are depending on the size of q and the significance level, denoted here as α . This test

³ The algorithm of Bai and Perron (1998b) is based on a dynamic programming approach; however, we do not enter into the details of the algorithm, since it is beyond the scope of this thesis (the reader may consult Bai and Perron (1998b) for more details).

can be defined as:

$$WD\max F_T(M, q) = \max_{1 \leq m \leq M} \frac{c(q, \alpha, 1)}{c(q, \alpha, m)} F_T(\hat{\lambda}_1, \dots, \hat{\lambda}_m; q) \quad (6.8)$$

where $c(q, \alpha, m)$ is the asymptotic critical value of the $\sup_{(\lambda_1, \dots, \lambda_m)} F$ test.

The $\sup F_T(k; q)$ test, proposed by Bai and Perron (1998a), is based on a sequential procedure to test for presence of zero break versus an alternative of $m = k$ breaks. This test is defined as:

$$\sup F_T(k; q) = F_T(\hat{\lambda}_1, \dots, \hat{\lambda}_m; q) \quad (6.9)$$

Similarly, the $\sup F_T(l+1|l)$ test considers the null hypothesis of l breaks versus an alternative of $l+1$ breaks. Based upon $\hat{\lambda}_j$, we can reject the null in favour of the alternative hypothesis if the model under the alternative sufficiently reduces the SSR .

For the sequential procedure, which was proposed by Bai (1997), we also apply a F -test, where the method first consists of testing for the case of no regime against one. If we reject the null hypothesis, we then test for the presence of one regime against two, and so forth until we cannot reject the null⁴.

To select the number of possible breaks, we can use the $BIC(m)$, which is calculated as:

$$BIC(m) = \ln \hat{\sigma}(m) + g \ln(T)/T \quad (6.10)$$

where $g = (m+1)q + m + p$, and $\hat{\sigma}(m) = T^{-1} S_T(\hat{T}_{B_1}, \dots, \hat{T}_{B_m})$. Moreover, we can also use a modified version of the Schwarz Information Criterion (SIC) suggested

⁴ For the construction of the F -test, see Bai and Perron (1998b), pp. 18-19.

by Liu, Wu and Zidek (1994), which is calculated as follows:

$$LWZ(m) = \ln \left(\frac{S_T(\hat{T}_{B_1}, \dots, \hat{T}_{B_m})}{T - g} \right) + \left(\frac{g}{T} \times c_0(\ln(T))^{2+\delta_0} \right) \quad (6.11)$$

Liu *et al.* (1997) suggest using $c_0 = 0.299$ and $\delta_0 = 0.1$.

Using different data generating processes, Bai and Perron (1998b) performed Monte Carlo experiments to investigate the behaviour of each method. Their main conclusions are that “[...] the *BIC* works well when breaks are present but less so under the null hypothesis” (Bai and Perron (1998b), pp. 33). According to Bai and Perron (1998b), if we suspect that no break is present in the model, then the *LWZ* method is more appropriate. Finally, they argue that the sequential procedure works best when compared with the other methods they investigated. In brief, testing for the presence of multiple structural changes in a model is a delicate thing to do since every case has different characteristics which must be taken into consideration before reaching to a conclusion.

6.2.2 Modeling the Taylor Rule Under the Presence of Multiple Structural Changes

Since our goal is to capture all possible monetary policy changes, with respect to all the population parameters, we only consider the case of a pure structural change model (*i.e.*, $p = 0$). Therefore, we may rewrite (6.2) as:

$$I = \bar{Z}\gamma + \varepsilon \quad (6.12)$$

We estimated (6.12) using different size for the trimming (ϵ) of the sample: 0.05, 0.1 and 0.15, for which the asymptotic critical values are tabulated in Bai and Perron

(1998b). Moreover, we also imposed a maximum number of breaks ($M = 5$). The estimations were done using the accompanying Gauss codes of Bai and Perron (1998a). All results are presented in Table 8(a,b,c), 9(a,b,c) and 10(a,b).

First, we estimated the model with $\epsilon = 0.05$, yielding an $h = 7$. The global optimization finds the breaks that minimize the SSR , which are corresponding to the following observations: 1978Q3, 1980Q3, 1982Q3, 1986Q1 and 1990Q3. For example, the SSR when no breaks are allowed is 176.56, while the SSR with five breaks is only 71.86.

According to the $\sup F_T(k; q)$ test, we can reject the null hypothesis of no break in favour of an alternative with a single break at the 5% critical level since the calculated $\sup F(1; 6)$ statistics is 21.26, which is compared with a critical value of 19.5. At this significance level, we rejected the null hypothesis for all cases (*i.e.*, $k = 1, \dots, 5$). Thus, the $\sup F_T(k; q)$ test provides evidence in favour of the presence of six monetary regimes.

The $UD_{\max}$ calculated statistics is 56.61, which is above the 1% asymptotic critical value of 24.0. While the $WD_{\max}$ calculated statistics, given an $\alpha = 99\%$ and a $q = 5$, is 74.66, which is above the 1% asymptotic critical value of 25.46. Thus, both “double maximum” tests strongly suggest that the model for the reaction function contains at least one break.

With the $\sup F_T(l+1|l)$ test, where the null consists of the models based upon the global optimization results, we can reject the null sequentially for $l+1$ versus l breaks at the 1% asymptotic critical level, for $\forall l = 1, \dots, 4$. In their order of

selection, the five breaks are: 1978Q3, 1976Q4, 1974Q4, 1973Q1 and 1970Q3.

Using the $BIC(m)$ and the $LWZ(m)$, we find evidence of two breaks (1978Q4 and 1981Q2) with the $BIC(m)$, while the $LWZ(m)$ selects the model without any break. These results are in line with the simulation evidence found in Bai and Perron (1998b), where they argue that if the true data generating process contains some breaks, the $BIC(m)$ is better at finding them; while the $LWZ(m)$ appears to be working well only when the true data generating process does not contain a break (*i.e.*, under the null hypothesis).

According to the sequential procedure, there are five breaks in the model at the 5% significance level, while the method does not find any breaks at the 2.5% level. However, the sequential procedure is known to find too many breaks when $\epsilon = 0.05$; but as ϵ reaches 0.1, the bias then disappears⁵.

Thus, we have evidence supporting two different break structures: a model with two breaks, and a model with five. Based upon those results, we estimated the coefficients according to the two selections.

In the first case, where three monetary regimes are present, we observe the following characteristics. The intercept term appears to suffer dramatic changes as it goes from zero to 216.01 when the first break occurs, to go down to 3.05 with the second break. The coefficient of the output gap is equal to 0.27 in the first regime, while it is strongly negative in the second regime (-9.16). In the third regime, however, it returns to the previous value of 0.28. In all three regimes, $\hat{\beta}_1$

⁵ For further details, see Bai and Perron (1998b).

is significant at least at the 10% critical level. The most interesting feature of this sudden change in the coefficient is that during the second monetary regime, the Bank of Canada reacted in a unusual fashion towards the business cycles. In effect, whenever $\beta_1 < 0$, we can say that the central bank is trying push output either beyond or above its full capacity level procyclically, which goes against general wisdom. Since this negative coefficient appears in the period where there are both high inflation rates and severe contractions in output, we can also interpret this as an illustration of the efforts made by the Bank of Canada to fight inflation which led to a “necessary” deep recession in the early 80’s.

The inflation coefficient, $\hat{\beta}_2$, is not statistically significant in the first and second regimes. Interestingly, however, it becomes very significant in the last regime and is equal to 0.15. For the smoothing parameter (or the persistence parameter), $\hat{\beta}_3$ goes from 0.81 to 0.43 after the first break, and returns to 0.84 in the last regime. Thus, for most of the sample, the interest rate exhibits an highly persistent behaviour, indicating that the Bank was “smoothing” interest movements during that period.

When modelling the monetary rule according to the model with five breaks, as selected by the sequential procedure, we can see great movements in the coefficients. The only notorious result to mention here, is the coefficient of inflation. In effect, we see that $\hat{\beta}_2$ is significant only in the last regime, which starts in 1978Q3. This is most interesting since the central bank authorities in North America started fighting inflation around that period – *i.e.*, when Paul Volker became Chairman

of the Fed.

When we use $\epsilon = 0.1$, $h = 14$, the global optimization finds the following five dates: 1970Q3, 1974Q4, 1979Q4, 1983Q2 and 1988Q2. According to the $\sup F_T(k; q)$ test at the 5% significance level, we can still conclude in favour of five breaks. We can reach the same conclusion with the $\sup F_T(l + 1|l)$ at the 1% significance level. Both $D_{\max}$ tests are strongly rejecting the null hypothesis of no break, as the calculated $UD_{\max}$ statistic is 30.56, which is greater than the 1% critical value of 23.16; and the calculated $WD_{\max}$ statistic, given an $\alpha = 99\%$, is 44.73, which is also greater than the 1% critical value of 24.81.

As before, the $BIC(m)$, is selecting the model with two breaks, while the $LWZ(m)$ selects the model with no break. At the 2.5% significance level, the sequential procedure now finds only three breaks (1970Q3, 1974Q4 and 1980Q4), while it finds none at the 1% significance level. In Table 8c, we report estimation results based upon the $BIC(m)$ and the sequential procedure. However, we do not discuss those results further as the estimated coefficients are exhibiting a similar behaviour as those in the case where $\epsilon = 0.05$.

Now, using $\epsilon = 0.15$, $h = 22$, we have the following dates for the global minimum of the SSR : 1969Q2, 1974Q4, 1980Q4, 1986Q3 and 1992Q3. In Table 9b, we present the estimation results based on the sequential procedure. At the 10% significance level, we can still find evidence of five breaks with the $\sup F_T(k; q)$ test. However, with the $\sup F_T(l + 1|l)$ test, we can now only find evidence in favour of two breaks at 10% significance level. With both $D_{\max}$ tests, we still

have strong results in favour of rejecting the null hypothesis of no break since the calculated $UD_{\max}$ statistics is 21.26, which is greater than the 2.5% critical value of 20.1; while the calculated $WD_{\max}$ statistic, given an α of 99%, is 28.19, which is also greater than the 1% critical value of 24.19. At the 2.5% critical level, the sequential procedure can only find a single break at date 1978Q3. Finally, this time both the $BIC(m)$ and the $LWZ(m)$ are selecting the model without any break.

6.3 Conclusion

Thus, using the method proposed by Bai and Perron (1998b), we have found much strong evidence suggesting that the monetary reaction function for Canada, as described by the Taylor Rule, is exhibiting structural instability. The most important of those changes appears to be located once again around the period of 1980, while the 70's also seem to be best characterized by shifting coefficients. These are essentially in line with the previous results obtained in Section 5.2.

Although notorious, the monetary policy shift of 1991 appears to be hard to capture with the several testing procedures we have used here. As a possible explanation, one can argue that –at least statistically– the behaviour of the Bank of Canada was no different during the period before 1991 than the period after. It is also possible that as economic agents were warned that a new monetary policy would be adopted in 1991, they were able to adjust perfectly to the new inflation targeting policy, and thus, the Bank did not need to react in a different manner with respect to inflation or output. This possible explanation fits well the “credibility”

hypothesis which is so important to the monetary authorities in Canada, as we mentioned earlier in Chapter 2.

Nevertheless, in both an economical and historical point of view, the model where the structural breaks are occurring in 1970Q3, 1974Q4, and 1980Q4 is the most interesting since these dates are capturing some important monetary policy changes or events. In effect, the first break is marking the end of the Bretton Woods monetary system, the second is capturing the first oil price shock, while the last break is capturing the end of monetary targeting and the beginning of disinflation in Canada.

CHAPTER 7

Concluding Remarks

In the second chapter, we have showed that both the inflation rate and the interest rate can be described as stationary autoregressive processes when allowance for a structural change is made in the deterministic function, while the output level appears to behave as a random walk. In the third chapter, we showed that output could also be decomposed into a stochastic trend component, with a growth rate which was driven by a three state Markov process, and a cyclical component. For the rate of inflation, we showed that when allowance for three different generating processes in the *AR* representation is made, the ex-ante rate of inflation is following a stationary process and is exhibiting almost no persistence, which is an important distinction to the high persistence that can be found with linear *AR* processes.

After investigating the processes of the data and modeling output and inflation, we have provided strong statistical evidence in favour of structural instability for the Taylor Rule when estimated over the period 1963Q2 to 1999Q4. Estimating the Taylor Rule using the Kalman filter, we have also showed that the central bank has changed its reaction to macroeconomic variables through time. The *TVP* model performed quite well at capturing the changes in the coefficients, especially the changes located near the first oil price shock, the policy change of the late 70's and early 80's, and the policy change of the early 90's.

Although we could find evidence for up to six monetary regimes using the

method of Bai and Perron (1998a, 1998b), we were not able to capture a structural break for the last monetary policy change of 1991. However, the first oil price shock and the policy change of the early 80's are still found to be important periods of policy changes, which gives strong support to a reaction function for Canada where three monetary policy regimes are considered, with policy changes located at dates 1970Q3, 1974Q4, and 1980Q4.

Evidently, further work would be needed under a broader approach. In effect, now that we have provided the statistical evidences highlighting structural instability of the reaction function, additional work could be done by considering a more sophisticated model. Perhaps the inclusion of an exchange rate variable, the Federal Funds Rate, or lag/lead variables would be an interesting path for further research.

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A. Appendix I: The Kalman Filter and the TVP model

For a general state-space model with time-varying parameters, consider the two sequence: $\{y_t\}$ of dimension $(T \times 1)$ and $\{x_t\}$ of dimension $(T \times k)$, for $t = 1, 2, \dots, T$. We can write the following specification:

$$y_t = x_t \beta_t + e_t \quad (\text{A.1})$$

$$\beta_t = F_t \beta_{t-1} + v_t \quad (\text{A.2})$$

with $e_t \sim N(0, R_t)$ and $v_t \sim N(0, Q_t)$, β_t is a $(k \times 1)$ unobservable state vector, F_t is a $(k \times k)$ matrix containing autoregressive coefficients. Thus, we have (A.1), the measurement equation, and (A.2), the transition equation characterizing the dynamics of the state vector. The objective of the Kalman filter is to get an optimal recursive estimate of $\{\beta_t\}$, given the information vector available at time t , $\{\psi_t\}$.

First, we define:

$$\beta_{t|t-1} = F_t \beta_{t-1|t-1} \quad (\text{A.3})$$

$$\zeta_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t-1|t-1})'] = F_t \zeta_{t-1|t-1} F_t' + Q_t \quad (\text{A.4})$$

$$y_{t|t-1} = x_t \beta_{t|t-1} \quad (\text{A.5})$$

$$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - x_t \beta_{t|t-1} \quad (\text{A.6})$$

$$\xi_{t|t-1} = E[(y_t - x_t \beta_{t|t-1})(y_t - x_t \beta_{t|t-1})'] = x_t \zeta_{t|t-1} x_t' + R_t \quad (\text{A.7})$$

Thus, at the beginning of time t , we make a prediction for $\{\beta_t\}$ based on $\{\psi_{t-1}\}$. Once we have calculated $\{\beta_t\}$, we also calculate the forecast error, given by (A.6).

Then having $\{\beta_{t|t-1}\}$, at the end of period t , we update the forecast using $\{\eta_{t|t-1}\}$ and the new information to estimate $\{\beta_t\}$, given by:

$$\beta_{t|t} = \beta_{t|t-1} + K_t \eta_{t|t-1} \quad (\text{A.8})$$

and

$$\zeta_{t|t} = \zeta_{t|t-1} - K_t x_t \zeta_{t|t-1} \quad (\text{A.9})$$

with

$$K_t = \zeta_{t|t-1} x_t' \xi_{t|t-1}^{-1} \quad (\text{A.10})$$

K_t being the well known Kalman gain matrix, which is a weight given to the new information after we have updated $\{\beta_t\}$. To initialize the filter, we have set the unconditional expectation, $\beta_{0|0}$ and $\zeta_{0|0}$, equal to 0.

The essential difference between the *TVP* model and a fixed parameter model, is that in the fixed parameter model the matrix R and Q are no longer a function of time, while for the most part the development of the model is similar.

B. Appendix II: Data References

The CANSIM data label for each series are:

- Overnight interest rate: B114011.
- Real Gross Domestic Product: D14816.
- Consumer Price Index: P100000.

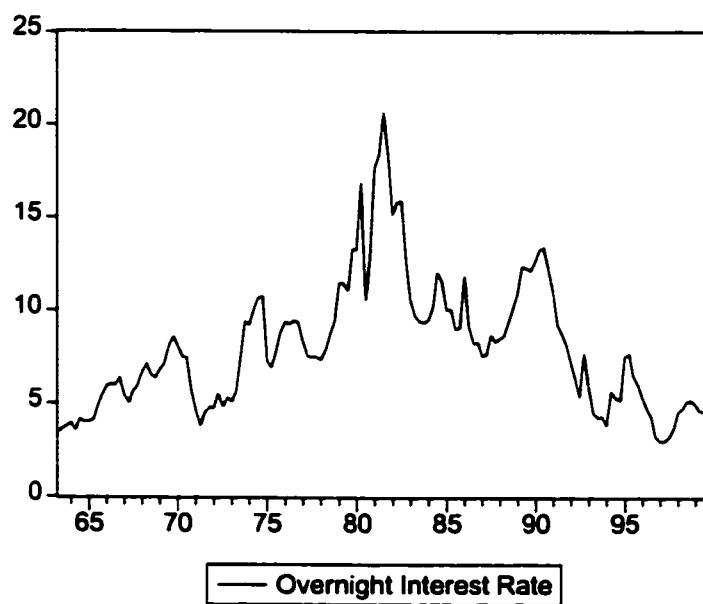


Figure 7.1:

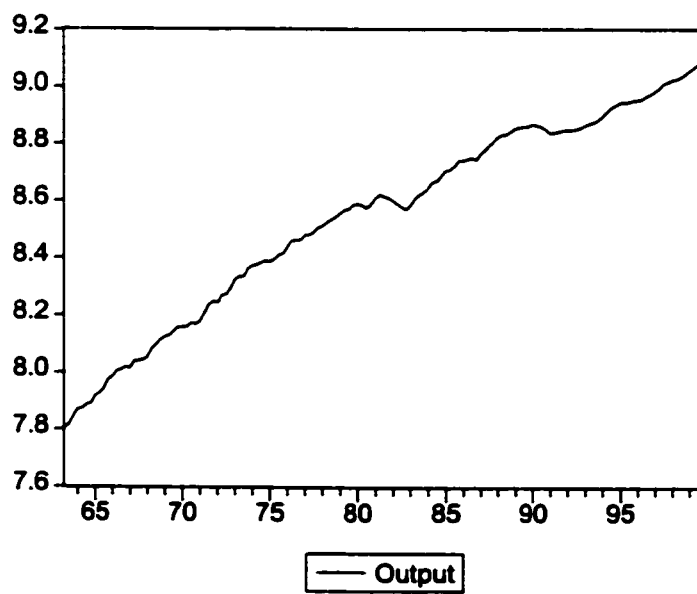


Figure 7.2:

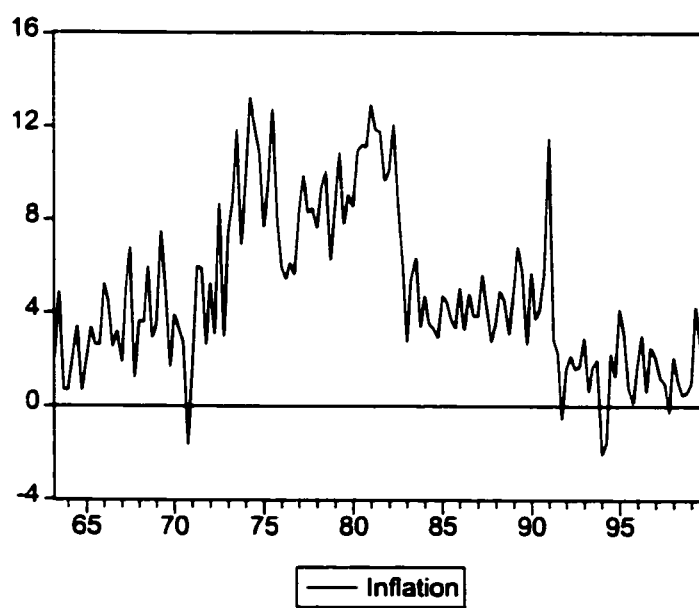


Figure 7.3:

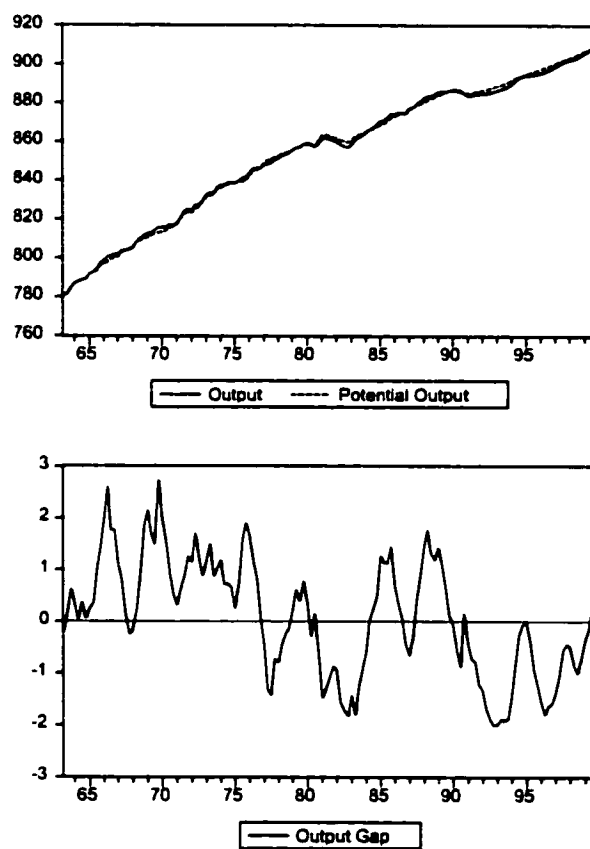


Figure 7.4:

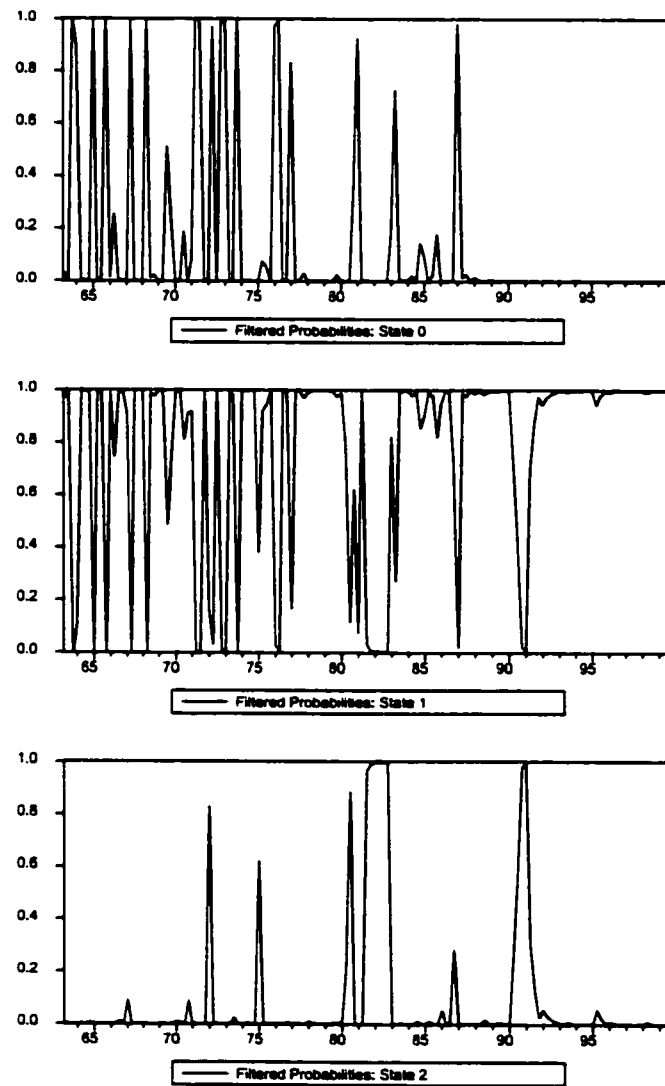


Figure 7.5:

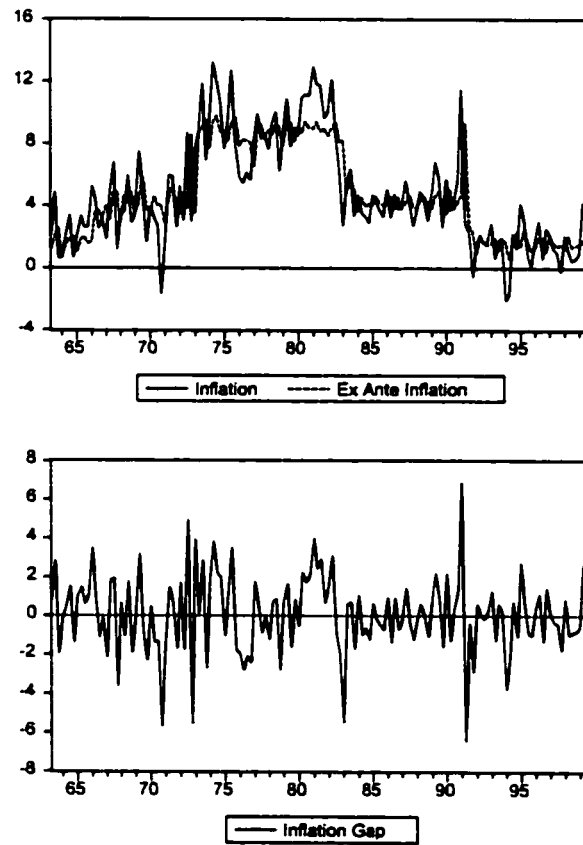


Figure 7.6:

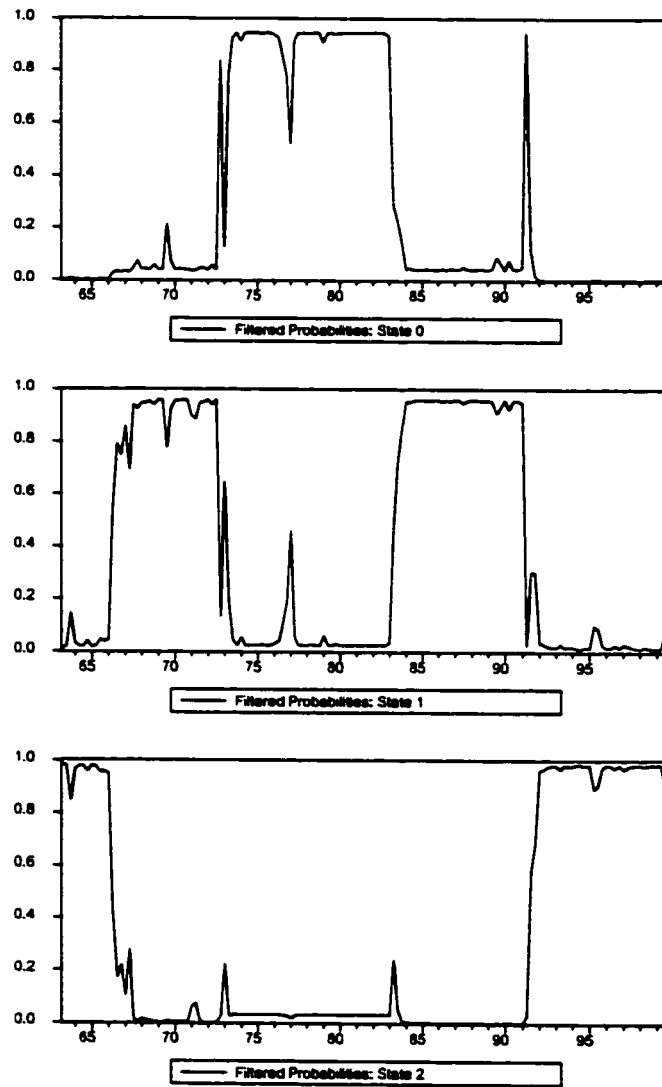


Figure 7.7:

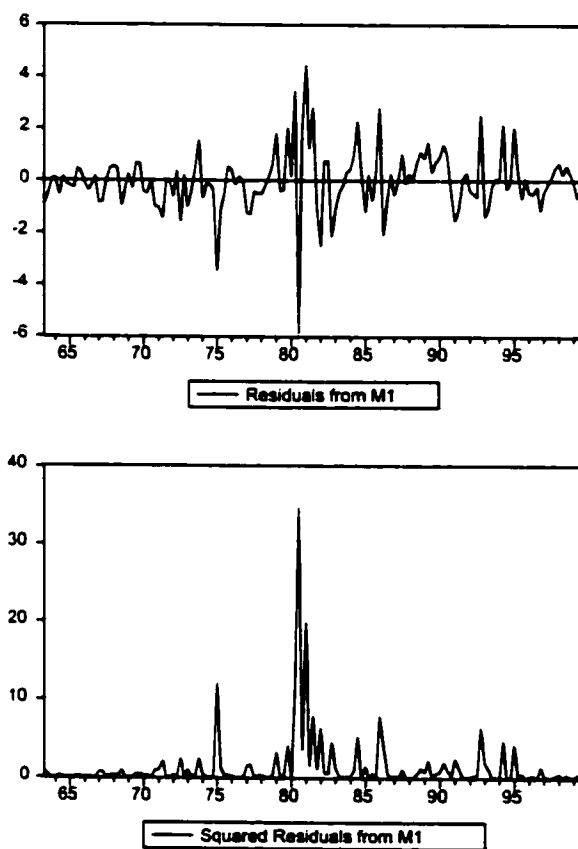


Figure 7.8:

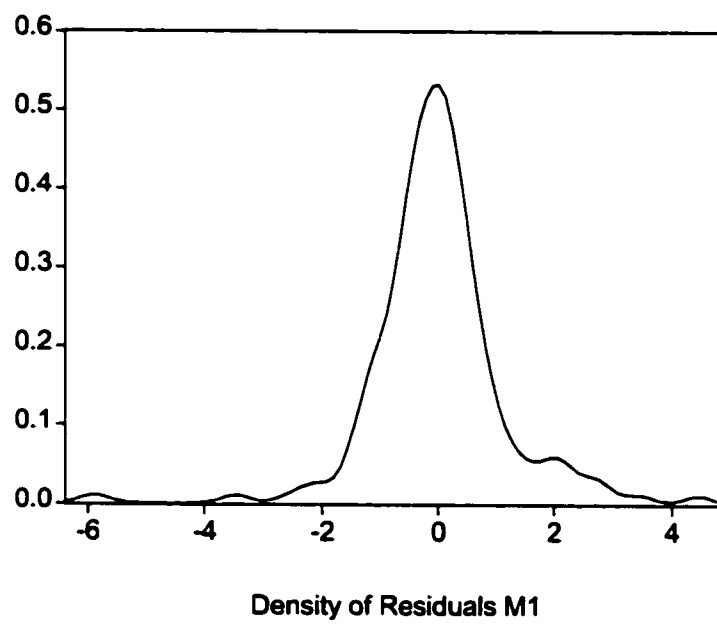


Figure 7.9:

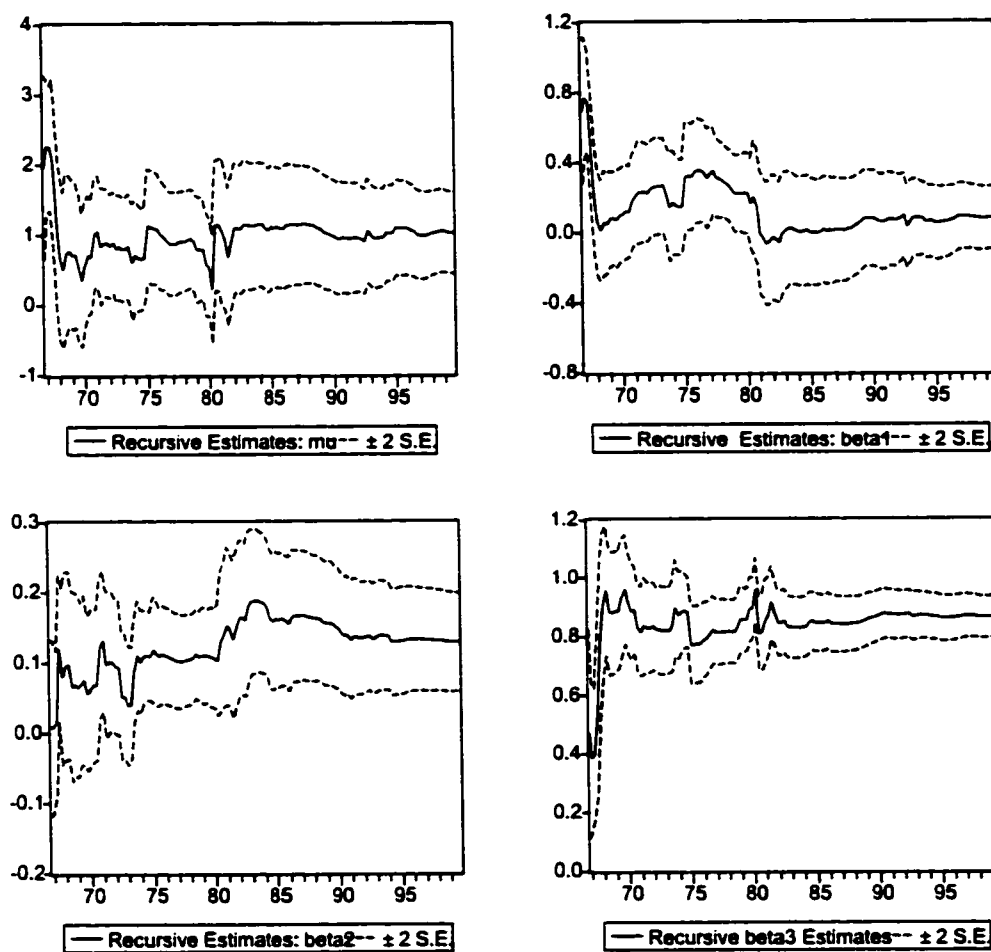


Figure 7.10:

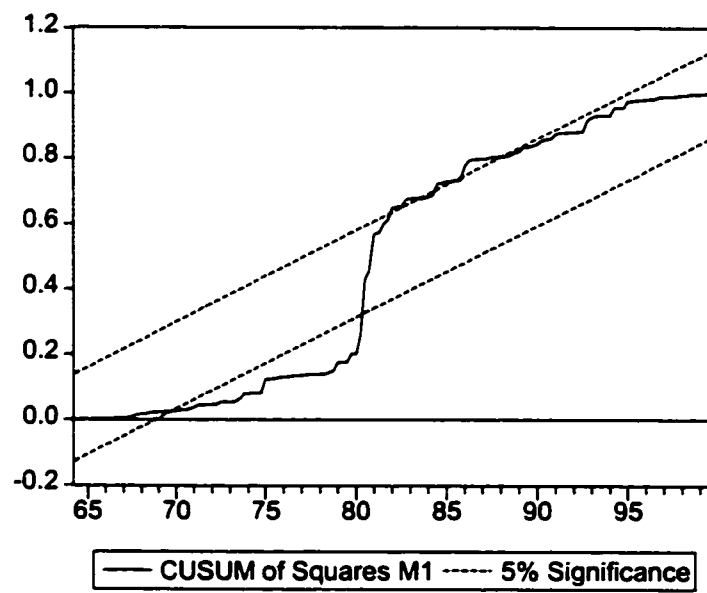


Figure 7.11:

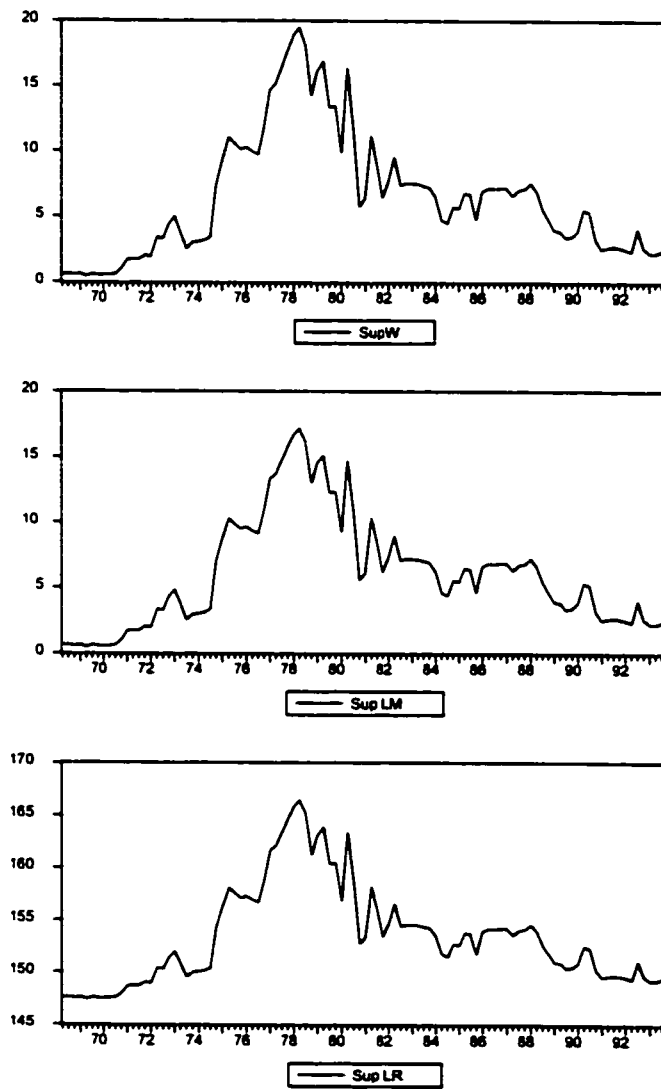


Figure 7.12:

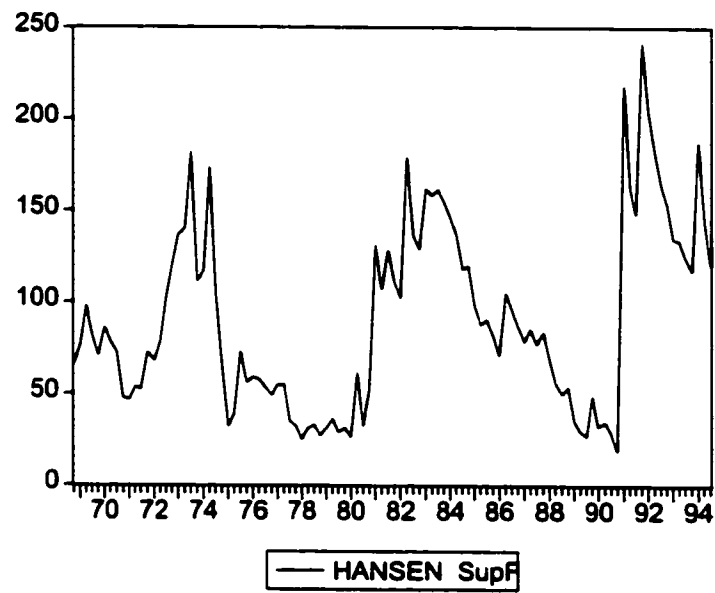


Figure 7.13:

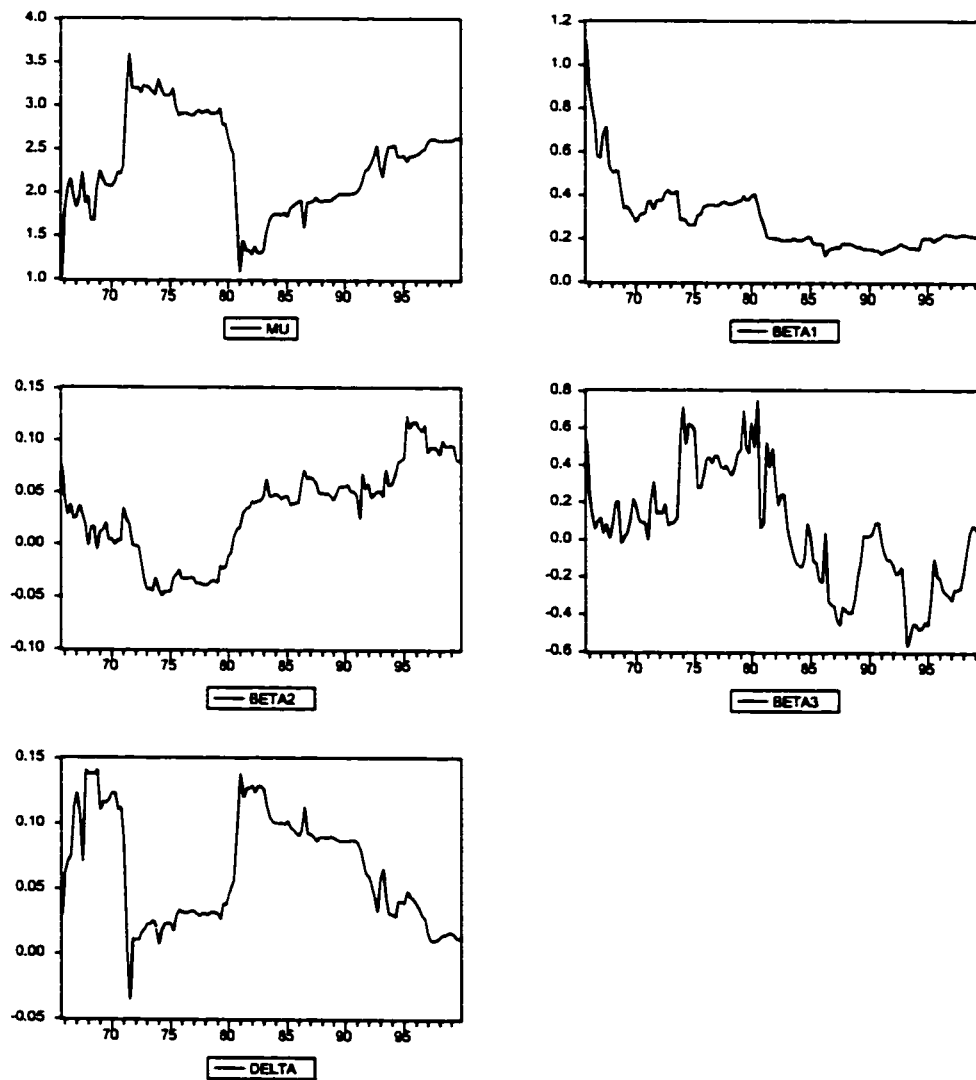


Figure 7.14:

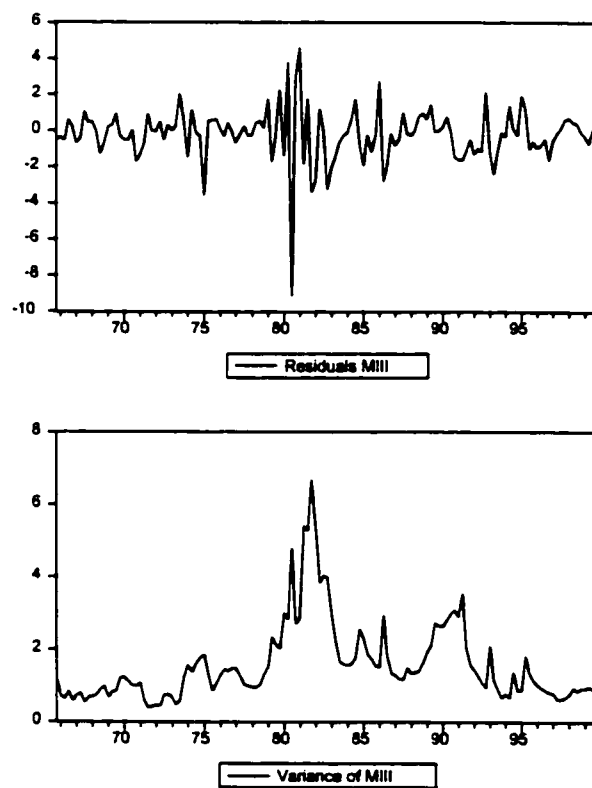


Figure 7.15:

**Table 1. Unit Root Tests for the Output, the Inflation and the
Interest Rate: 1961Q1-1999Q4**

Output

Parameter	$\hat{\alpha}$	$t_{\hat{\alpha}}$	k^*	$\bar{T}_B$
<i>ADF</i>	0.97	-2.31	9	—
<i>ADF^{GLS}</i>	0.99	-0.72	14	—
<i>PP</i>	0.92	-2.43	9	—
<i>ZA</i>	0.90	-3.97	9	1972Q2
<i>P97</i>	0.83	-4.25	10	1979Q2

Inflation

Parameter	$\hat{\alpha}$	$t_{\hat{\alpha}}$	k^*	$\bar{T}_B$
<i>ADF</i>	0.89	-1.96	8	—
<i>ADF^{GLS}</i>	0.96	-0.88	15	—
<i>PP</i>	0.76	-4.01 ^a	8	—
<i>ZA</i>	0.15	-5.31 ^b	12	1982Q4
<i>P97</i>	0.05	-5.01 ^a	12	1982Q4

Interest Rate

Parameter	$\hat{\alpha}$	$t_{\hat{\alpha}}$	k^*	$\bar{T}_B$
<i>ADF</i>	0.94	-2.01	7	—
<i>ADF^{GLS}</i>	0.96	-1.58	7	—
<i>PP</i>	0.94	-2.33	7	—
<i>ZA</i>	0.51	-6.10 ^a	9	1981Q1
<i>P97</i>	0.61	-5.12 ^b	7	1979Q3

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%

**Table 2. Maximum Likelihood Estimates of State-Space Models
for Output: 1961Q1-1999Q4**

Parameter	AR(2)		MS(2)-AR(2)		MS(3)-AR(2)	
p_{00}	—	—	0.997 ^a	(0.005)	0.352 ^a	(0.089)
p_{01}	—	—	—	—	0.645 ^a	(0.089)
p_{10}	—	—	0.985 ^a	(0.017)	0.121 ^a	(0.028)
p_{11}	—	—	—	—	0.846 ^a	(0.031)
p_{20}	—	—	—	—	0.276 ^c	(0.135)
p_{21}	—	—	—	—	0.046	(0.054)
ϕ_1	0.28 ^a	(0.079)	1.212 ^a	(0.079)	1.352 ^a	(0.025)
ϕ_2	0.059	(0.081)	-0.277 ^a	(0.079)	-0.457 ^a	(0.017)
μ_0	0.898 ^a	(0.109)	1.322 ^a	(0.11)	2.623 ^a	(0.089)
μ_1	—	—	-0.663 ^a	(0.106)	-1.854 ^a	(0.089)
μ_2	—	—	—	—	-3.384 ^a	(0.028)
σ^2	0.798	—	0.832 ^a	(0.049)	0.446 ^a	(0.024)
<i>Log Lik</i>	-197.814		-186.176		-170.962	

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, standard error in parenthesis

Table 3. Maximum Likelihood Estimates of $AR(p)$ Models for the Inflation: 1961Q1-1999Q4

Parameter	AR(4)	MS(2)-AR(3)	MS(2)-AR(3)
p_{00}	—	0.889 ^a (0.09)	0.95 ^a (0.04)
p_{01}	—	—	0.02 (0.03)
p_{10}	—	0.96 ^a (0.04)	0.09 ^a (0.03)
p_{11}	—	—	0.96 ^a (0.03)
p_{20}	—	—	0.0 (0.0)
p_{21}	—	—	0.011 ^a (0.01)
ϕ_1	0.52 ^a (0.06)	0.545 ^a (0.08)	0.24 ^a (0.09)
ϕ_2	—	0.0 (0.09)	-0.19 ^b (0.09)
ϕ_3	—	0.35 ^a (0.09)	—
ϕ_4	0.37 ^a (0.06)	—	—
μ_0	4.54 ^b (1.47)	9.93 ^b (4.0)	1.56 ^a (0.20)
μ_1	—	-0.66 ^a (0.11)	4.01 ^a (0.22)
μ_2	—	—	9.24 ^a (0.37)
σ_0^2	4.04	9.29 ^b (4.0)	4.64 ^a (1.11)
σ_1^2	—	9.29 ^a (0.67)	2.39 ^a (0.49)
σ_2^2	—	—	1.17 ^a (0.35)
<i>Log lik</i>	-329.53	-321.33	-170.96

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, standard error in parenthesis

**Table 4. Regression Estimates of Benchmark Model, MI:
1962Q3-1999Q4**

Parameter	Estimate	Diagnostic Tests			
μ	1.028 ^a (0.289)	$\sup W = 12.49$ $\sup LM = 11.52$ $\sup LR = 159.49$ $\hat{T}_B = 1978Q2$			
β_1	0.081 (0.089)				
β_2	0.128 ^a (0.034)				
β_3	0.878 ^a (0.034)				
		LM Tests			
		AR(1)	0.77 (0.68)*	AR(2)	0.09 (0.76)*
R^2	0.889	ARCH(1)	9.65 (0.0)*	ARCH(2)	25.61 (0.0)*
F	387.85				
σ	1.17	JB	198.37 (0.00)*		
$Log\ lik$	-230.23				

a,b,c,d denotes significance levels at 1%, 2.(0.023)5%, 5.0% and 10%, standard error in parenthesis

*denotes p-value

Table 5. Fully-Modified Regression Estimates of Benchmark Model, MI: 1962Q3-1999Q4

Parameter		Hansen's (1992) tests	
$\hat{\mu}$	0.95 ^a	Tests for parameter instability	
	(0.195)	<i>SupF</i>	241.6
$\hat{\beta}_1$	0.114 ^a		(0.0)
	(0.023)	<i>MeanF</i>	91.03
$\hat{\beta}_2$	0.029		(0.0)
	(0.06)	<i>LC</i>	7.924
$\hat{\beta}_3$	0.878 ^a		(0.0)
	(0.023)	$\hat{T}_B = 1978Q2$	
kernel lag window* : 17.708			

a,b,c,d denotes significance levels at 1%, 2.(0.023)5%, 5.0% and 10%, standard error in parenthesis

*Andrews' (1991) quadratic spectral kernel was used

Table 6. Regression Results for Partial Structural Change Models, MII: 1963Q2-1999Q4

Parameter	$\hat{T}_B = 1981Q1$	$\hat{T}_B = 1979Q3$
μ_1	0.03 (0.378)	0.413 (0.4)
μ_2	3.642 ^a (1.401)	4.943 ^a (1.293)
β_1	0.178 ^d (0.104)	0.198 ^c (0.104)
β_2	0.136 ^a (0.05)	0.132 ^a (0.048)
β_3	0.825 ^a (0.048)	0.782 ^a (0.049)
δ_1	0.03 ^a (0.008)	0.024 ^a (0.009)
δ_2	-0.048 ^a (0.015)	-0.053 ^a (0.014)
σ	1.114	1.116
$\bar{R}^2$	0.889	0.891
LM Tests		
AR(1)	0.05 (0.82)*	0.37 (0.55)*
AR(2)	0.95 (0.76)*	0.95 (0.70)*
ARCH(1)	9.97 (0.002)*	6.62 (0.01)*
ARCH(2)	30.32 (0.0)*	28.11 (0.0)*
JB	186.72 ^a (0.0)*	7.68 (0.021)*

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, standard error in parenthesis
 * denotes p-value

Table 7. Maximum Likelihood Estimates of a Time-Varying Parameter Model, MIII: 1963Q2-1999Q4

Parameter	Estimate
μ_t	0.0 (0.101)
β_{1t}	0.0 (0.076)
β_{2t}	0.012 (0.01)
β_{3t}	-0.125 ^a (0.01)
δ_t	-0.005 ^a (0.001)
σ	0.0 (0.162)
<i>Log lik</i>	-210.05
LM Tests	
AR(1) = 2.72 (0.25)*	ARCH(1) = 105.8 (0.0)*
AR(2) = 4.08 (0.13)*	ARCH(2) = 106.2 (0.0)*
JB	617.4 (0.0)*

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, standard error in parenthesis
 * denotes p-value

Table 8a: Estimation Results of Multiple Strutural Change
Model: 1963Q2-1999Q4, $\epsilon = 0.05$

Specification		$M = 5$	$q = 5$	$h = 7$	
Tests					
$UD \max$	$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$	$\sup F_T(5)$
56.61 ^a	21.26 ^a	41.43 ^a	56.61 ^a	37.41 ^a	41.05 ^a
$WD \max(1\%)$	$\sup F_T(2 1)$	$\sup F_T(3 2)$	$\sup F_T(4 3)$	$\sup F_T(5 4)$	
7.66 ^a	43.26 ^a	79.5 ^a	42.42 ^a	43.26 ^a	
Number of Breaks Selected					
Sequential Procedure			5		
LWZ			0		
BIC			2		

Table 8b: Estimates with 5 Breaks

$\hat{T}_{B_1} = 1970Q3$	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
$\hat{T}_{B_2} = 1973Q1$	1.24	-0.21	38.6	-14.63	-14.73	5.75 ^a
$\hat{T}_{B_3} = 1974Q4$	(1.14)	(7.4)	(57.95)	(18.66)	(58.37)	(1.32)
$\hat{T}_{B_4} = 1976Q4$	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$	$\beta_{1,4}$	$\beta_{1,5}$	$\beta_{1,6}$
$\hat{T}_{B_5} = 1978Q3$	0.23	0.7	-4.16	0.73	-0.14	0.13
$\sigma = 1.02$	(0.33)	(1.3)	(3.3)	(0.92)	(2.25)	(0.13)
$F = 124.4$	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$	$\beta_{2,4}$	$\beta_{2,5}$	$\beta_{2,6}$
$\bar{R}^2 = 0.9$	0.01	-0.08	-0.22	-0.07	0.13	0.25 ^a
	(0.13)	(0.12)	(0.25)	(0.41)	(0.81)	(0.07)
	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{3,3}$	$\beta_{3,4}$	$\beta_{3,5}$	$\beta_{3,6}$
	0.39	0.38	1.11	0.24	0.73	0.76 ^a
	(0.48)	(0.44)	(0.98)	(0.69)	(1.92)	(0.05)
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
	0.09	0.06	-0.66	0.34	0.24	-0.03
	(0.07)	(0.17)	(1.22)	(0.25)	(0.65)	(0.01)

a, b, c, d denotes significance levels at the 1%, 2.5%, 5% and 10%

Table 8c: Estimates with 2 Breaks

$\hat{T}_{B1} = 1978Q4$	μ_1	μ_2	μ_3
$\hat{T}_{B2} = 1981Q2$	0.29	216.01 ^a	3.05 ^b
$\sigma = 1.04$	(0.41)	(48.18)	(1.42)
$F = 113.4$	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$
$\bar{R}^2 = 0.93$	0.27	-9.16 ^a	0.28 ^a
	(0.14)	(1.7)	(0.11)
	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$
	0.03	-0.4	0.15
	(0.05)	(0.51)	(0.06)
	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{3,3}$
	0.81 ^a	0.43 ^a	0.84 ^a
	(0.09)	(0.19)	(0.05)
	δ_1	δ_2	δ_3
	0.02 ^b	-2.69 ^a	0.01
	(0.01)	(0.64)	(0.01)

a, b, c, d denotes significance levels at the 1%, 2.5%, 5% and 10%

Table 9a: Estimation Results of Multiple Structural Change
Model: 1963Q2-1999Q4, $\epsilon = 0.1$

Specifications	$M = 5$		$q = 5$	$h = 14$	
Tests					
$UD_{\max}$	$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$	$\sup F_T(5)$
30.56 ^a	21.26 ^a	19.77 ^a	30.56 ^a	26.13 ^a	27.9 ^a
$WD_{\max}$	$\sup F_T(2 1)$	$\sup F_T(3 2)$	$\sup F_T(4 3)$	$\sup F_T(5 4)$	
44.73 ^a	43.26 ^a	45.5 ^a	35.38 ^a	35.38 ^a	
Number of Breaks Selected					
Sequential procedure			3		
LWZ			0		
BIC			2		

Table 9b: Estimates with 3 Breaks

$\hat{T}_{B_1} = 1978Q3$	μ_1	μ_2	μ_3	μ_4
$\hat{T}_{B_2} = 1974Q4$	1.241	-8.974 ^a	-6.345	5.754 ^a
$\hat{T}_{B_3} = 1970Q3$	(1.12)	(3.414)	(8.78)	(1.288)
$\sigma = 1.109$	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$	$\beta_{1,4}$
$F = 66.88^a$	0.231	-0.331	0.836 ^c	0.13
$\bar{R}^2 = 0.9$	(0.319)	(1.017)	0.425	(0.125)
	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$	$\beta_{2,4}$
	0.009	-0.086	-0.09	0.255 ^a
	(0.127)	(0.1)	(0.204)	(0.067)
	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{3,3}$	$\beta_{3,4}$
	0.387	0.596 ^b	0.415	0.756 ^a
	(0.466)	(0.242)	(0.358)	(0.05)
	δ_1	δ_2	δ_3	δ_4
	0.089	0.258 ^b	0.173	-0.031 ^a
	(0.072)	(0.105)	(0.109)	(0.008)

a, b, c, d denotes significance levels at 1%, 2.5%, 5% and 10%

Table 9c: Estimates with 2 Breaks

$\hat{T}_{B_1} = 1979Q4$	μ_1	μ_2	μ_3
$\hat{T}_{B_2} = 1983Q2$	0.102	77.069 ^a	2.984 ^d
$\sigma = 0.976$	(0.418)	(13.337)	(1.573)
$F = 117.4^a$	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$
$\bar{R}^2 = 0.923$	0.275 ^d	-5.185 ^a	0.331 ^b
	(0.147)	(0.891)	(0.136)
	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$
	0.038	0.08	0.077
	(0.056)	(0.158)	(0.073)
	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{3,3}$
	0.835 ^a	0.661 ^a	0.826 ^a
	(0.088)	(0.092)	(0.064)
	δ_1	δ_2	δ_3
	0.023 ^b	-0.941 ^a	-0.013
	(0.01)	(0.173)	(0.009)

a, b, c, d denotes significance levels at 1%, 2.5%, 5% and 10%

Table 10a: Estimation Results of Multiple Strutural Change
Models: 1963Q2-1999Q4, $\epsilon = 0.15$

Specifications		$M = 5$	$q = 5$	$h = 22$	
Tests					
$UD_{\max}$	$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$	$\sup F_T(5)$
20.49 ^b	17.3 ^d	12.07	20.49	17.2	16.38
$WD_{\max}$	$\sup F_T(2 1)$	$\sup F_T(3 2)$	$\sup F_T(4 3)$	$\sup F_T(5 4)$	
30.23 ^a	17.27 ^d	17.27	14.95	14.95	
Number of Breaks Selected					
Sequential Procedure			1		
LWZ			0		
BIC			0		

Table 10b: Estimates with 1 Break

$\hat{T}_{B_1} = 1978Q3$	μ_1	μ_2
$\sigma = 1.116$	0.936 ^d	4.368 ^a
$F = 131^a$	(0.564)	(1.228)
$\bar{R}^2 = 0.899$	$\beta_{1,1}$	$\beta_{1,2}$
	0.006	-0.014 ^d
	(0.014)	(0.008)
	$\beta_{2,1}$	$\beta_{2,2}$
	0.285 ^d	0.155
	(0.169)	(0.126)
	$\beta_{3,1}$	$\beta_{3,2}$
	0.792 ^a	0.695 ^a
	(0.106)	(0.054)
	δ_1	δ_2
	0.098 ^d	0.246 ^a
	(0.06)	(0.061)