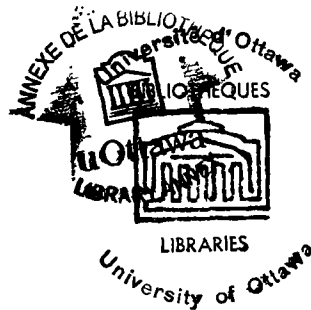


OPTIMIZATION OF A CLASS OF STOCHASTIC
SYSTEMS GOVERNED BY ITO DIFFERENTIAL
EQUATIONS

by

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ABSTRACT

In this thesis, we consider the optimal parameter selection problem of a class of Ito differential equations.

In Chapter 1, we consider the optimal parameter selection of a class of Ito differential systems with Markov terminal time. These problems are reduced to the optimal parameter selection problems of a class of distributed parameter systems. In section 1.3, the necessary condition for determination of optimal parameter of this reduced problem is given. The results are illustrated by an example.

In Chapter 2, certain special cases of hereditary stochastic systems are reduced to ordinary stochastic Ito differential equations. The results are illustrated by an example.

Chapter 3 presents the numerical methods for the determination of optimal parameters as illustrated by solving two examples, one from chapter 1 and the other from chapter 2.

Numerical results are presented in chapter 4.

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CHAPTER 1

NECESSARY CONDITIONS FOR SELECTION OF
OPTIMAL PARAMETERS

1.1. INTRODUCTION

In this chapter, we consider a class of stochastic systems described by Ito differential equations in which the parameters are to be chosen optimally with respect to certain performance criteria. In reference [1] , Teo and Ahmed have shown that these optimal parameter selection problems of stochastic differential systems can be reduced to the optimal parameter selection problems of a class of non-standard distributed parameter systems. The parameter in the reduced problem will appear in the parabolic partial differential operator rather than on the boundary of a fixed domain or distributed in time and space. [2, pp. 1 - 143] , This is where they differ from the standard problems of optimal control of distributed parameter systems [3 , pp. 377] . This result is quoted in section 1.2. Further, to this reduced problem, the necessary condition for determination of optimal parameter is given [section 1.3] . We consider the system described by the following Ito differential equation ,

$$S_1 \left\{ \begin{array}{l} d\xi = f(t, \xi(t), \sigma)dt + g(t, \xi(t), \sigma) d\omega(t) , \\ \qquad \qquad \qquad t \in R_0 \underline{\Delta} [0, \infty) \\ \xi(0) = \xi_0 , (P_0 - \text{initial probability measure supported by } B) \\ \sigma \in \Sigma , (\Sigma - \text{parameters set}), \end{array} \right.$$

where $\xi(t)$ is the dynamic state, and Σ a compact and convex subset of R^r .

$\{\omega(t), t \in I\}$ is the m-dimensional Wiener process independent of ξ_0 .

Further $f: R_0 \otimes R^n \otimes \Sigma \rightarrow R^n$ and $g: R_0 \otimes R^n \otimes \Sigma \rightarrow M$, where M is the space of $(n \times m)$ matrices.

1.2. BASIC ASSUMPTIONS AND PROBLEM STATEMENTS

Let B be an open set in R^n (with compact closure) supporting the initial probability measure P_0 and let ∂B denote the boundary of the set B , which is assumed to be locally representable by functions with Hölder continuous second order partial derivatives. Further, it is assumed that the coefficients f and g satisfy the following hypothesis :

- (i) f is a bounded measurable function in $t \in I \triangleq [0, T]$, $T < \infty$, for every $(x, \sigma) \in B \times \Sigma$; and continuous in $x \in \bar{B}$ for almost all $t \in I$ and for any $\sigma \in \Sigma$. Further $f(t, x, \cdot) \in C^1(\Sigma)$ a.e. in \bar{Q} , where $Q \triangleq (0, T) \times B$ and \bar{Q} denotes its closure ;
- (ii) $g(\cdot, \cdot, \sigma)$ is a continuous function in \bar{Q} for each $\sigma \in \Sigma$ and $g(t, x, \cdot) \in C^1(\Sigma)$ in \bar{Q} .

Define

$$\tau_\sigma \triangleq \inf \{ t \in [0, T] : \xi^\sigma(t) \in \partial B \}$$

to be the first exit time from the set B .

With these preparations the problem P_1 can be stated as : given the system S_1 , find a parameter vector $\sigma^0 \in \Sigma$ that minimizes the following cost functional

$$(1.2.1) \quad J(\sigma) = E \left\{ \int_0^{\tau_\sigma} L(t, \xi(t), \sigma) dt \right\},$$

where L is a bounded real valued function in $t \in I$ for every

$(x, \sigma) \in B \times \Sigma$, and continuous in $x \in B$ for almost all $t \in I$ and for any $\sigma \in \Sigma$. Further $L(t, x, \cdot) \in C^1(\Sigma)$ a.e. in \bar{Q} .

1.3 NECESSARY CONDITIONS FOR OPTIMAL CHOICE OF PARAMETERS OF A CLASS OF STOCHASTIC ITO DIFFERENTIAL EQUATIONS

For convenience, let us denote by A^σ the following differential operator :

$$A^\sigma \psi \triangleq \left\{ \sum_{i=1}^n f_i(t, x, \sigma) \frac{\partial \psi}{\partial x_i} + \sum_{i,j=1}^n a_{ij}(t, x, \sigma) \cdot \frac{\partial^2 \psi}{\partial x_i \cdot \partial x_j} \right\},$$

where $\sigma \in \Sigma$; $a_{ij}(t, x, \sigma) \triangleq \frac{1}{2} (g \cdot g^*)_{ij}(t, x, \sigma)$,

$i, j = 1, \dots, n$; and $*$ denotes matrix transpose.

Suppose that the coefficients f and g satisfy hypotheses (i) and (ii) given in section 1.2 and that there exists a constant $c > 0$ so that $c|\lambda|^2 \leq \sum_{i,j=1}^n a_{ij}(t, x, \sigma) \lambda_i \lambda_j$ for all $\lambda \in \mathbb{R}^n$ uniformly on $\bar{Q} \times \Sigma$. Then, it follows [4, pp.345] that for each $\sigma \in \Sigma$ the system S_1 has a unique solution $\xi(\cdot)$ which is a strong Markov process. Using Ito's lemma [4, pp. 345 - 400], it can be shown that the stochastic optimal parameter selection problem reduces to the optimal parameter selection problem of an equivalent distributed parameter system, as stated in the following lemma :

Lemma (1.3.1) : The problem P_1 reduces to an equivalent problem P_2 which consists of the first boundary value problem S_2

and the cost functional $\min_{\sigma \in \Sigma} J(\sigma) = \min_{\sigma \in \Sigma} \int \phi^\sigma(0, x) P_0(dx)$.

$$S_2 \left\{ \begin{array}{l} - \frac{\partial \phi^\sigma}{\partial t} = A^\sigma \phi^\sigma(t, x) + L(t, x, \sigma), \quad (t, x) \in [0, T] \times B \\ \phi^\sigma(t, x) |_{\partial B} = 0, \quad t \in (0, T] \\ \phi^\sigma(T, x) = 0, \quad x \in B \\ \sigma \in \Sigma \end{array} \right.$$

Proof: The proof is given in [1]

For convenience of further references, let \mathcal{F}_1 denote the class of all real valued functions ϕ on the cylinder \bar{Q} [5, pp. 194 - 214] having properties :

(i) ϕ and ϕ_{x_i} , $i = 1, \dots, n$, are Hölder continuous on \bar{Q} ;

(ii) the partial derivatives $\phi_t, \phi_{x_i x_j}$, $i = 1, \dots, n$,

are square integrable on Q ;

(iii) $\phi(t, x) = 0$ for all $(t, x) \in [0, T] \times \partial B \cup \{T\} \times B$.

Denote the operator $(\frac{\partial}{\partial t} + A^\sigma)$ by E^σ and consider the differential equation adjoint to the system S_2 given by

$$S_3 \left\{ \begin{array}{l} - \frac{\partial q^\sigma}{\partial t} + (A^\sigma)^* q^\sigma \stackrel{\Delta}{=} (E^\sigma)^* q^\sigma = 0, \quad (t, x) \in (0, T) \times B \\ q^\sigma(t, x) |_{x \in \partial B} = 0, \quad t \in [0, T] \\ q^\sigma(0, x) = q_0(x), \quad x \in B, \end{array} \right.$$

where $\sigma \in \Sigma$; and

$$(A^\sigma)^* \psi = \sum_{i,j=1}^n \frac{\partial^2}{\partial x_i \partial x_j} (a_{ij} \psi) - \sum_{i=1}^n \frac{\partial}{\partial x_i} (f_i \psi) ;$$

and for every measurable subset D of B , $P_o(D) = \int_D q_o(x) dx$.

For each $\sigma \in \Sigma$, a function q^σ is said to be a weak solution of S_3 if, for every $\psi \in \mathcal{F}_1$ for which $E^\sigma \psi$ is bounded,

$$(1.3.1) \int_Q (E^\sigma \psi) q^\sigma dt dx = - \int_B \psi(0, x) dP_o(x) .$$

The existence and uniqueness of the weak solution was reported in [5, pp. 194 - 214] .

With these preparations, we can now state the necessary condition for determination of optimal parameter vector. This result is derived by Ahmed and Teo [6, to appear] and is quoted in the following theorem.

Theorem (1.3.1) : Consider the problem P_2 . Suppose that $a(\cdot, \cdot, \sigma) \in C^2(\bar{Q})$ for all $\sigma \in \Sigma'$, $a(t, x, \cdot) \in C^1$ for all $(t, x) \in \bar{Q}$ and that there exists a number $c > 0$ such that $\sum_{i,j=1}^n a_{ij}(t, x, \sigma) \beta_i \beta_j \geq c |\beta|^2$ for all $\beta \in R^n$ uniformly on $\bar{Q} \times \Sigma$.

Further, let f and L be bounded measurable in \bar{Q} for all $\sigma \in \Sigma$ and of class C^1 in σ for all $(t, x) \in \bar{Q}$. Then, for $\sigma^o \in \Sigma$

to be an optimal parameter with $\phi^o \in \mathcal{F}_1$ the corresponding solution of the system S_2 , it is necessary that

$$(1.3.2) \quad \int_Q \left\{ \sum_{i,j=1}^n \frac{\partial a_{ij}(t, \mathbf{x}, \sigma^0)}{\partial v} \cdot \frac{\partial^2 \phi^0}{\partial x_i \partial x_j} + \sum_{i=1}^n \frac{\partial f_i(t, \mathbf{x}, \sigma^0)}{\partial v} \cdot \frac{\partial \phi^0}{\partial x_i} + \frac{\partial L(t, \mathbf{x}, \sigma^0)}{\partial v} \right\} q^0(t, \mathbf{x}) dt \, dx \geq 0, \text{ (for } \Sigma \text{ closed);}$$

$$(1.3.3) \quad \int_Q \left\{ \sum_{i,j=1}^n \frac{\partial a_{ij}(t, \mathbf{x}, \sigma^0)}{\partial \sigma_k} \cdot \frac{\partial^2 \phi^0}{\partial x_i \partial x_j} + \sum_{i=1}^n \frac{\partial f_i(t, \mathbf{x}, \sigma^0)}{\partial \sigma_k} \cdot \frac{\partial \phi^0}{\partial x_i} + \frac{\partial L(t, \mathbf{x}, \sigma^0)}{\partial v} \right\} q^0(t, \mathbf{x}) dt \, dx = 0, \text{ } k = 1, \dots, r, \text{ (for } \Sigma \text{ open);}$$

where $\frac{\partial a(t, \mathbf{x}, \sigma^0)}{\partial v} \triangleq \lim_{\varepsilon \rightarrow 0} \frac{a(t, \mathbf{x}, \sigma^0 + \varepsilon v) - a(t, \mathbf{x}, \sigma^0)}{\varepsilon}$,

$\sigma^0 + \varepsilon v \in \Sigma$, $0 \leq \varepsilon \leq 1$; and v any vector directed inward into Σ emanating from σ^0 ; and q^0 is the weak solution of the adjoint system S_3 corresponding to the parameter vector σ^0 .

Corollary 1.3.1 is a consequence of theorem 1.3.1.;

Corollary (1.3.1): Consider the system S_2 . Suppose that all the hypotheses given in Theorem 1.3.1 are satisfied and that the coefficients a_{ij} , f_i , ($i, j = 1, \dots, n$), and L are linear in the parameter σ .

Then, if the parameter restraint set Σ is a closed and convex polyhedron in R^r , the optimal parameter vector σ^0 takes its value at one of the vertices of Σ .

In the linear case as considered in corollary 1.3.1, the optimal parameter in S_1 can be found by solving 2^r systems of type S_2 , one for each of the vertices of the polyhedron Σ , instead of solving a two point boundary value problem for S_2 and S_3 .

1.4 AN EXAMPLE (1.4.1)

For illustration, let us consider the following stochastic time optimal control problem.

Example 1.4.1 : Consider the following Ito differential system S , with its coefficients f and g redefined as $f = \begin{bmatrix} \sigma \cdot x_1 \cdot x_2 \\ x_1 + x_2 \end{bmatrix}$ and $g = \begin{bmatrix} x_1 + 2 & 0 \\ 0 & 5\sqrt{2} \end{bmatrix}$ respectively.

Further, the parameter restraint set Σ and the supporting set Ω are taken as $[-1, 1]$ and $\{(x_1, x_2): -1 \leq x_1 \leq +1 \text{ and } -1 \leq x_2 \leq +1\}$ respectively.

Subject to this dynamic constraint, the problem is to find a scalar parameter $\sigma^0 \in \Sigma \Delta [-1, 1]$ so that $J(\sigma^0) = \varepsilon \{ \tau^{\sigma^0} \}$ is minimum, where the Markov time τ is as defined in section 1.2. Call this problem P_3 . As a direct consequence of Lemma 1.3.1, we note that the problem P_3 reduces to an equivalent problem of optimal control of the following first boundary value problem,

$$\begin{aligned}
 S_4 \left\{ \begin{aligned}
 - \frac{\partial \phi(\sigma)}{\partial t} &= \frac{1}{2} (x_1 + 2)^2 \frac{\partial^2 \phi(\sigma)}{\partial x_1^2} + 25 \frac{\partial^2 \phi(\sigma)}{\partial x_2^2} \\
 &+ \sigma \cdot x_1 x_2 \frac{\partial \phi(\sigma)}{\partial x_1} + (x_1 + x_2) \frac{\partial \phi(\sigma)}{\partial x_2} \\
 &+ 1, \quad (t, x) \in [0, T] \times \Omega, \\
 \\
 \phi(\sigma)(T, x) &= 0 \quad \text{for } x \in \Omega, \\
 \\
 \phi(\sigma)(t, x) &= 0 \quad \text{for } (t, x) \in [0, T] \times \partial\Omega, \\
 \\
 \sigma &\in \Sigma \underline{\Delta} [-1, 1], \\
 \\
 \Omega &\underline{\Delta} \{(x_1, x_2) : -1 \leq x_1 \leq +1 \text{ and } -1 \leq x_2 \leq +1\}, \\
 \\
 \min_{\sigma \in \Sigma} J(\sigma) &= \min_{\sigma \in \Sigma} \int \phi(\sigma)(o, x) dx
 \end{aligned} \right.
 \end{aligned}$$

Denote this problem by P_4 .

The procedures for solving this problem will be given in chapter 3 and numerical results will be presented in chapter 4.

CHAPTER 2

OPTIMIZATION OF PROPORTIONAL AND
INTEGRAL CONTROLS FOR A STOCHASTIC
SYSTEM DESCRIBED BY ITO DIFFERENTIAL
EQUATIONS.

2.1 INTRODUCTION

The theory of Markov processes has been widely used [5, pp. 194 - 214 , 7] in the solution of stochastic control problems. However, if the system involved is described by a stochastic hereditary differential equation [8, pp 13 - 30], then in general it is not possible to take advantage of the well known results like the forward and backward Kolmogorov partial differential equations arising in the study of Markovian control problems. There are certain special cases of hereditary stochastic systems that, by augmenting the dimension of the state space, can be reduced to Ito stochastic differential equations. The reduced problem can then be solved using the theory of Markov processes.

2.2 PROBLEM STATEMENT AND ITS SOLUTION

The system to be optimized is described by the following stochastic hereditary Ito differential equation :

$$S_a \left\{ \begin{array}{l} d\xi(t) = f(t, \xi(t), \gamma \xi(t) + \int_0^t K(t, \tau)v(\tau, \xi(\tau))d\tau)dt + g(t, \xi(t))d\omega \\ \xi(0) = \xi_0 \quad \text{a random variable, } t \geq 0 . \end{array} \right.$$

The functions f , v and g are assumed to be given, ω is a 1-dimensional Wiener process for $t \geq 0$, and the state ξ is a real-valued stochastic process. The system S_a arises, from the proportional and (not necessarily linear) integral control of the plant :

$$P_a \quad \begin{cases} d\xi(t) &= f(t, \xi(t), u(t)) dt + g(t, \xi(t)) d\omega(t) \\ u(t) &= \gamma \xi(t) + \int_0^t K(t, \tau) v(\tau, \xi(\tau)) d\tau \end{cases}$$

where u is the control based on the present and past history of the process ξ .

Let $\Omega = \{y \in \mathbb{R}: |y| < a\}$ be an open interval on the real line and $I = [0, T]$, $T < \infty$. Let $\tau \stackrel{\Delta}{=} \inf \{t \in I : \xi(t) \in \partial\Omega\}$ be the first exit time from the set Ω and define the performance index as the expected value $E(\tau)$ of the random time τ . $\partial\Omega =$ boundary of Ω . Let Γ be a closed bounded interval on the real line from which the proportionality factor γ (amplifier gain) can take its values, and let H be a class of functions where K can take its values.

The optimization problem is $\max_{(\gamma, K) \in \Gamma \times H} J(\gamma, K)$; that is,

choose the proportionality factor γ and the kernel K that maximizes the expected value of the period of stay of the trajectory $\{\xi(t), t \in I\}$ in Ω . Obviously this problem is meaningless unless further specification is provided for the class H . For this, let Σ be a compact (closed and bounded) subset of the Euclidean space $E^{2(n-1)}$ and define H to be the class of functions $K(\cdot, \cdot)$ such that

$$K(t, \tau) = \begin{cases} \sum_{i=1}^{n-1} a_i e^{-\beta_i(t-\tau)} & \text{for } t \geq \tau \\ 0 & \text{for } t < \tau \end{cases}$$

and $\sigma \triangleq (a_1, a_2, \dots, a_{n-1}, \beta_1, \beta_2, \dots, \beta_{n-1}) \in \Sigma$.

For the simplicity of notations $\{a_i, \beta_i\}$ are assumed to be real. Note that the impulse response of a large class of linear systems can be approximated arbitrarily closely by appropriate choice of the number n and the parameters $\{a_i, \beta_i\}$. The optimization problem reduces to

$$\max_{(\gamma, \sigma) \in \Gamma \times \Sigma} J(\gamma, \sigma) .$$

Under this situation the hereditary system S_a can be reduced to a differential system.

Define $x_1 = \xi$

$$\text{and } x_m = \int_0^t a_{m-1} e^{-\beta_{m-1}(t-\tau)} v(\tau, \xi(\tau)) d\tau ,$$

for $m = 2, 3, \dots, n$. The hereditary system S_a can then be reduced to an equivalent stochastic differential system in n - space,

$$S'_a \left\{ \begin{array}{l} dx_1 = f(t, x_1, \gamma x + \sum_{k=2}^n x_k) dt + g(t, x_1) d\omega \\ dx_2 = (a_1 v(t, x_1) - \beta_1 x_2) dt + 0 \\ \vdots \\ dx_n = (a_{n-1} v(t, x_1) - \beta_{n-1} x_n) dt + 0 \end{array} \right. .$$

In vector form we have

$$dx(t) = F(t, x(t)) dt + G(t, x(t)) d\omega .$$

Define $D \subset E^n$ to be

$$D = \{ x \in E^n : x_1 \in \Omega, -\infty < x_i < \infty, i = 2, \dots, n \}$$

The performance index $J(\gamma, \sigma)$ is then given by

$$J(\gamma, \sigma) = \varepsilon(\tau_D) \text{ where } \tau_D = \inf \{ t \in I : x(t) \in \partial D \}$$

The system S'_a is a degenerate stochastic system, since the diffusion matrix is not positive (i.e. $G G' \not\geq 0$). However if $g^2(t, x_1) > 0$ for all $x_1 \in \Omega$, then by adding small perturbation terms the system S'_a can be converted into a non-degenerate problem.

Consider

$$W = (\omega_1 = \omega, \omega_2, \dots, \omega_n)$$

to be an n-dimensional normalized and coordinatewise independent Wiener process. For every $\varepsilon > 0$, let us construct the system

$$S'_\varepsilon \left\{ \begin{array}{l} dx_1 = f(t, x_1, \gamma x_1 + \sum_{k=2}^n x_k) dt + g d\omega_1 \\ dx_2 = (a_1 v(t, x_1) - \beta_1 x_2) dt + \varepsilon d\omega_2 \\ \vdots \\ dx_n = (a_{n-1} v(t, x_1) - \beta_{n-1} x_n) dt + \varepsilon d\omega_n \end{array} \right.$$

Clearly the Ito-differential equation

$$dx = F dt + G^\varepsilon dW$$

is non-degenerate since $(G^\varepsilon G^{\varepsilon'}) > 0$.

It is clear that S'_0 is equivalent to S'_a . The performance index corresponding to the system S'_ϵ can be written as

$$J_\epsilon(\gamma, \sigma) = \mathcal{E}(\tau_D^\epsilon)$$

where $\tau_D^\epsilon = \inf \{t \in I : x_\epsilon(t) \in \partial D\}$

and x_ϵ is the solution of the Ito differential equation S'_ϵ , and consequently a Markov process on E^n . This problem P'_ϵ , can be reduced to a first boundary value problem of parabolic partial differential equations [1, to appear]. The proof of the following lemma follows from minor modifications of the above result.

Lemma 2.2.1: The problem P'_ϵ is equivalent to the problem

$$P'_\epsilon \left\{ \begin{array}{l} - \frac{\partial \varphi}{\partial t} = f \frac{\partial \varphi}{\partial x_1} + \sum_{k=2}^n (\alpha_{k-1} v(t, x_1) - \beta_{k-1} x_k) \frac{\partial \varphi}{\partial x_k} \\ \quad + \frac{1}{2} g^2(t, x_1) \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{1}{2} \epsilon \sum_{k=2}^{2n} \frac{\partial^2 \varphi}{\partial x_k^2} + 1 \\ \text{for } (t, x) \in [0, T) \times B. \\ \varphi(T, x) = 0 \quad \text{for } x \in B \\ \varphi(t, x) = 0 \quad \text{for } x \in \{x \in B : x_1 = \pm a\} \\ \text{and } t \in [0, T), \\ \max_{(\gamma, \sigma) \in \Gamma \times \Sigma} J_\epsilon(\gamma, \sigma) = \max_{(\gamma, \sigma) \in \Gamma \times \Sigma} \int_{\Omega} \varphi(0; x_1, 0, \dots, 0) d\pi_0(x_1); \end{array} \right.$$

where π_0 is the probability distribution of the random variable $x_1(0)$ at time $t = 0$ with Ω being the supporting set.

In view of the above lemma our original (optimal) control problem reduces to the problem of optimal selection of parameters (γ, σ) from the set $\Gamma \times \Sigma$ for the parabolic partial differential system. The necessary condition for optimal selection of parameters of a system described by a general parabolic partial differential equation was given in chapter 1. (Thm 1.3.1).

In view of corollary 1.3.1, the problem of finding the optimal parameter (γ^0, σ^0) reduces to the problem of solving the first boundary value problem P'_ϵ for φ for each of the vertices of the polyhedra and then choosing the one that yields the maximum for the integral J_ϵ .

The question that now arises is what happens to the optimality as $\epsilon \downarrow 0$. The following result deals with this question.

Theorem 2.2.1: For every fixed $(\gamma, \sigma) \in \Gamma \times \Sigma$, let φ^ϵ be the solution of the differential equation of the problem P'_ϵ and φ the solution of the same problem with $\epsilon = 0$. Then $\varphi^\epsilon(t, x)$ converges to a limit $\varphi^0(t, x)$ as $\epsilon \downarrow 0$ uniformly on compact sets in D and that $\varphi^0(t, x) = \varphi(t, x)$ for almost all $(t, x) \in D$.

Proof :

Since for fixed $(\gamma, \sigma) \in \Gamma \times \Sigma$ the condition (b) of theorem 3 (Fleming, p. 74) is obviously satisfied. The proof is an immediate

consequence of Fleming's result.

In view of the above theorem, we solve the optimality problem P'_ϵ for a fixed ϵ using theorem 1.3.1. In other words, for a fixed $\epsilon > 0$ the differential equation of the problem P'_ϵ is solved for all the vertices (in case $\Gamma \times \Sigma$ is a polyhedron) and the optimal parameter (γ^0, σ^0) is found by comparing the integrals $J_\epsilon(\gamma, \sigma)$. Then the parameter (γ^0, σ^0) is fixed and the differential equation of the problem P'_ϵ is solved for decreasing values of $\epsilon > 0$, till $J_\epsilon(\gamma^0, \sigma^0)$ as a function of ϵ converges to a limit. This limit is the value of the performance integral J of the original problem.

2.3 AN EXAMPLE (2.3.1)

The results of the preceding sections are illustrated by solving the following example,

$$\left\{ \begin{array}{l} d\xi(t) = f(t, \xi(t), \gamma \xi(t) + \int_0^t K(t, \tau) v(\tau, \xi(\tau)) dt \\ \quad + g(t, \xi(t)) d\omega(t), t \in [0, 1] \\ \xi(0) = \xi \end{array} \right.$$

with $f(t, \xi, y) = \xi \cdot y$, $\gamma = 0$, $K(t, \tau) = \begin{cases} a e^{\beta(t-\tau)}, & \tau \leq t, \\ 0, & \text{otherwise,} \end{cases}$

$\sigma = (a, \beta) \in \Sigma \triangleq \{(a, \beta): |a| \leq 2, |\beta| \leq 2\}$, $v(t, \xi) = \xi$,

$g(t, \xi) = \xi + 2$, the set of confinement of the process

$\{ \xi(t), t \in I \}$ is $\Omega \triangleq (-1, +1)$, i.e. $a = 1$, and the initial probability density of the random variable ξ_0 is

$$q_0(x_1) = \begin{cases} \frac{1}{2}, & \text{for } x_1 \in \Omega, \\ 0, & \text{elsewhere.} \end{cases}$$

The corresponding ϵ -problem is

$$\left\{ \begin{array}{l} -\frac{\partial \varphi}{\partial t} = x_1 x_2 \frac{\partial \varphi}{\partial x_1} + (a x_1 - \beta x_2) \frac{\partial \varphi}{\partial x_2} + \\ \quad \frac{1}{2} (x_1 + 2)^2 \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{1}{2} \epsilon^2 \frac{\partial^2 \varphi}{\partial x_2^2} + 1, \quad \text{for} \\ \quad (t, x) \in I \times B \triangleq (0, 1] \times \{(x_1, x_2) : |x_1| < 1, x_2 \in \mathbb{R}\} \\ \\ \varphi(1, x) = 0 \quad \text{for } x \in B \\ \\ \varphi(t, x) = 0 \quad \text{for } x \in \partial B \text{ and } t \in [0, 1]. \\ \\ \max_{(\alpha, \beta) \in \Sigma} J_{\epsilon}(\alpha, \beta) = \max_{(\alpha, \beta) \in \Sigma} \int_{-1}^1 \varphi(0, x_1, 0) q_0(x_1) dx_1. \end{array} \right.$$

The procedures for solving this problem are similar to those given in chapter 3. The numerical results will be presented in chapter 4.

CHAPTER 3

NUMERICAL METHOD

3.1 INTRODUCTION

In order to compute the optimal parameter for the problem considered in Example 1.3.1, we note from the statement following Corollary 1.3.1 that it would be required to solve 2 partial differential equations with first boundary conditions. Using Crank-Nicolson Method [10, pp. 25-27], we will show [section 3.3] that each of the two first boundary value problems can be reduced to a matrix equation. The solution of this matrix equation is discussed below.

3.2 SOLUTION OF MATRIX EQUATION $\sum_{i=1}^N A_i X B_i = C$

Consider a class of matrix equations given by

$$(3.2.1) \quad \sum_{\ell=1}^N A_{\ell} X B_{\ell} = C ,$$

where A_{ℓ} , B_{ℓ} and C are $m \times m$, $n \times n$ and $m \times n$ constant matrices respectively; and X is an $m \times n$ unknown matrix.

For the solution of the matrix equation (3.2.1), we need the following lemma.

Lemma 3.2.1 : Consider the matrix equation of the form

$$(3.2.2) \quad A X B = C ,$$

where $A \triangleq (a_{ij})$, $B \triangleq (b_{ij})$ and $C \triangleq (c_{ij})$ are $m \times m$, $n \times n$ and $m \times n$ constant matrices respectively ; and $X \triangleq (x_{ij})$ is an $m \times n$ unknown matrix. Then,

$$(3.2.3) \quad \hat{X} = [B^* \otimes A]^{-1} \hat{C}$$

if and only if

$$(3.2.4) \quad | B^* \otimes A | \neq 0$$

where \hat{X} and \hat{C} denote the transposes of the row vectors

$[x_{11}, \dots, x_{1m}, \dots, x_{n1}, \dots, x_{mn}]$ and $[c_{11}, \dots, c_{1m}, \dots, c_{n1}, \dots, c_{mn}]$ respectively; $B^* \otimes A \triangleq \{ a_{ij} B^* \}$; \otimes is known as Kronecker product; $'^*$, $'^{-1}$ and $|\cdot|$ denote matrix transpose, matrix inversion and matrix determinant respectively.

Proof : Taking the p th column and q th row of both sides of the matrix equation (3.2.2), we have

$$(3.2.4) \quad \sum_{i=1}^n \sum_{j=1}^m a_{pi} \cdot x_{ij} \cdot b_{jq} = c_{pq}$$

Letting c_{ij} , ($i=1, \dots, m; j=1, \dots, n$), be the ij th element of the matrix C and denoting by \hat{C} the transpose of the mn row vector $(c_{11}, \dots, c_{n1}, \dots, c_{1m}, \dots, c_{mn})$, the above expression can be rearranged as

$$(3.2.5) \quad \sum_{i=1}^n \sum_{j=1}^m a_{pi} \cdot b_{jq} \cdot x_{ij} = \hat{C}_{(p-1)n+q}$$

Thus, the matrix equation (3.2.2) reduced to the following equivalent relation

$$(3.2.6) \quad [B^T \otimes A] \hat{X} = \hat{C},$$

where \otimes denotes the Kronecker product as defined in lemma 3.2.1; \hat{X} is the transpose of the row vector $(x_{11}, \dots, x_{n1}, \dots, x_{1n}, \dots, x_{mn})$; and x_{ij} are the ij th element of the unknown matrix X .

From the above relation, it is clear that the condition (3.2.4) is necessary and sufficient for the components of the solution of the matrix equation (3.2.2) to be expressed by the equation (3.2.3). This completes the proof.

Using the similar procedure as given in the proof of the above lemma, we have our main result of this chapter presented in the following theorem.

Theorem 3.2.1 Consider the matrix equation (3.2.1). Then

$$(3.2.7) \quad \hat{X} = \left[\sum_{\ell=1}^N B_{\ell}^* \otimes A_{\ell} \right]^{-1} \hat{C}$$

if and only if

$$(3.2.8) \quad \left| \sum_{\ell=1}^N B_{\ell}^* \otimes A_{\ell} \right| \neq 0,$$

where \hat{X} , \hat{C} , $|\cdot|$, $'*'$, $'-1'$ and \otimes are as defined in Lemma 3.2.1.

Proof : The proof is similar to that given for Lemma 3.2.1.

3.3 DERIVATION OF THE ITERATION SCHEME

Consider the first boundary value problem given in example 1.4.1. It follows from the statement after Corollary 1.3.1 that if σ^0 is the optimal parameter, then it is necessary that σ^0 take a value of 1 or -1. In this section, Crank-Nicolson method will be used to convert the first boundary value problem of this kind into a matrix equation of the form (3.2.1).

Partition the domain Ω , where

$$\Omega = \{ (x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1 \},$$

into ten equals rows, each row having ten equal squares. The grid points will consist of all the corners of the squares, excluding the corners on the boundary.

Let Δ denote the time step and let h denote the spatial step in either of the directions x_1 or x_2 . Using the Crank-Nicolson method, we have to express the first and second partial derivatives of ϕ at $t + \frac{\Delta}{2}$ in terms of ϕ at t and $t + \Delta$. Applying Taylor series expansion, we obtain the following relations,

$$(3.3.1) \quad \phi_t(t + \frac{\Delta}{2}, x_1, x_2) = \frac{\phi(t + \Delta, x_1, x_2) - \phi(t, x_1, x_2)}{\Delta} + O(\Delta^2);$$

$$(3.3.2) \quad \begin{aligned} & \phi_{x_1 x_1}(t + \frac{\Delta}{2}, x_1, x_2) = \\ & = \frac{1}{2} \left\{ \frac{\phi(t, x_1 + h, x_2) - 2\phi(t, x_1, x_2) + \phi(t, x_1 - h, x_2)}{h^2} \right. \\ & + \left. \frac{\phi(t + \Delta, x_1 + h, x_2) - 2\phi(t + \Delta, x_1, x_2) + \phi(t + \Delta, x_1 - h, x_2)}{h^2} \right\} \\ & + O(h^2) + O(\Delta^2); \end{aligned}$$

$$(3.3.3) \quad \begin{aligned} & \phi_{x_2 x_2}(t + \frac{\Delta}{2}, x_1, x_2) = \\ & = \frac{1}{2} \left\{ \frac{\phi(t, x_1, x_2 + h) - 2\phi(t, x_1, x_2) + \phi(t, x_1, x_2 - h)}{h^2} \right. \\ & + \left. \frac{\phi(t + \Delta, x_1, x_2 + h) - 2\phi(t + \Delta, x_1, x_2) + \phi(t + \Delta, x_1, x_2 - h)}{h^2} \right\} \\ & + O(h^2) + O(\Delta^2); \end{aligned}$$

$$(3.3.4) \quad \phi_{x_1} \left(t + \frac{\Delta}{2}, x_1, x_2 \right) = \frac{1}{2} \left\{ \frac{\phi(t, x_1+h, x_2) - \phi(t, x_1, x_2)}{h} \right. \\ \left. + \frac{\phi(t + \Delta, x_1+h, x_2) - \phi(t + \Delta, x_1, x_2)}{h} \right\}$$

$$+ O(h) + O(\Delta^2) ; \quad \text{and}$$

$$(3.3.5) \quad \phi_{x_2} \left(t + \frac{\Delta}{2}, x_1, x_2 \right) = \frac{1}{2} \left\{ \frac{\phi(t, x_1, x_2+h) - \phi(t, x_1, x_2)}{h} \right. \\ \left. + \frac{\phi(t + \Delta, x_1, x_2+h) - \phi(t + \Delta, x_1, x_2)}{h} \right\}$$

$$+ O(h) + O(\Delta^2) .$$

Using these approximations and denoting by ϕ_{ij} the solution of P_4 at the intersection of the i th row and j th column of grid points the first boundary value problem P_4 reduces to ,

$$(3.3.6) \quad - \frac{\phi_{ij}(t + \Delta) - \phi_{ij}(t)}{\Delta} \\ = \frac{1}{4h^2} (x_1 + 2)^2 \{ \phi_{i,j+1}(t) - 2\phi_{i,j}(t) + \phi_{i,j-1}(t) + \phi_{i,j+1}(t + \Delta) \\ - 2\phi_{i,j}(t + \Delta) + \phi_{i,j-1}(t + \Delta) \} \\ + \frac{\epsilon^2}{4h} \{ \phi_{i+1,j}(t) - 2\phi_{i,j}(t) + \phi_{i-1,j}(t) + \phi_{i+1,j}(t + \Delta) - 2\phi_{i,j}(t + \Delta) \\ + \phi_{i-1,j}(t + \Delta) \}$$

$$\begin{aligned}
 & + \frac{x_1 x_2}{2h} \{ \phi_{i,j+1}(t) - \phi_{i,j}(t) + \phi_{i,j+1}(t + \Delta) - \phi_{i,j}(t + \Delta) \} \\
 & + \frac{1}{2h} (\alpha x_1 - \beta x_2) \{ \phi_{i+1,j}(t) - \phi_{i,j}(t) + \phi_{i+1,j}(t + \Delta) - \phi_{i,j}(t + \Delta) \} \\
 & + 1 + O(\Delta^2) + O(h).
 \end{aligned}$$

In order to convert the expression (3.3.6) into a matrix equation of the form (3.2.1) so that $\phi_{ij}(t)$, $i, j = 1, \dots, 9$, can be solved recursively, we define the following constant 9×9 matrices,

$$(3.3.7) \quad X_1 = \{ x_{ij} \};$$

$$\text{with } x_{ij} = \begin{cases} (h \cdot i)^{-1} & \text{if } i = j, \\ 0 & \text{otherwise,} \end{cases}$$

$$(3.3.8) \quad X_2 = -X_1$$

$$(3.3.9) \quad P_1 = \{ (p_1)_{ij} \};$$

$$\text{with } (p_1)_{ij} = \begin{cases} 1 & \text{if } i=j, \\ -1 & \text{if } i-j=1, \\ 0 & \text{otherwise,} \end{cases}$$

$$(3.3.10) \quad P_2 = -P_1,$$

$$(3.3.11) \quad Q = \{ q_{ij} \};$$

$$\text{with } q_{ij} = \begin{cases} 2 & \text{if } i=j, \\ -1 & \text{if } |i-j| = 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$(3.3.12) \quad Z = \{ z_{ij} \},$$

where $z_{ij} = 1$. Further, let I be the identify matrix. With these preparations, the equation (3.3.6) can be rearranged into the following matrix equation,

$$(3.3.13) \quad -\frac{1}{\Delta} [U_{\ell+1} \quad - U_{\ell}] = \frac{1}{4h^2} [-U_{\ell+1}Q - U_{\ell}Q] [X_1^2 + 4X_1 + 4I]$$

$$+ \frac{\epsilon^2}{4h^2} [-QU_{\ell+1} - QU_{\ell}] + \frac{1}{2h} X_2 [-U_{\ell+1}P_1 - U_{\ell}P_1] X_1$$

$$+ \frac{a}{2h} [P_2 U_{\ell+1} + P_2 U_{\ell}] X_1 - \frac{\beta}{2h} X_2 [P_2 U_{\ell+1} + P_2 U_{\ell}]$$

$$+ Z,$$

where $U = \{ \phi_{ij} \}$, $i = 1, \dots, 9$ and $j = 1, \dots, 9$.

(3.3.13) can be reduced to

$$(3.3.14) \quad U_{\ell+1} - U_{\ell} = \frac{\Delta}{4h^2} [U_{\ell+1}Q + U_{\ell}Q] [X_1^2 + 4X_1 + 4I]$$

$$+ \frac{\Delta \cdot \epsilon^2}{4h^2} [QU_{\ell+1} + QU_{\ell}] + \frac{\Delta}{2h} X_2 [U_{\ell+1}P_1 + U_{\ell}P_1] X_1$$

$$- \frac{\Delta \cdot a}{2h} [P_2 U_{\ell+1} + P_2 U_{\ell}] X_1 + \frac{\Delta \cdot \beta}{2h} X_2 [P_2 U_{\ell+1} + P_2 U_{\ell}]$$

$$+ \Delta \cdot Z.$$

After some manipulations, (3. 3. 14) becomes (3. 3. 15),

$$\begin{aligned}
 (3. 3. 15) \quad & U_{l+1} - U_{l+1} Q A - \frac{\Delta \cdot \epsilon^2}{4h^2} Q U_{l+1} - \frac{\Delta}{2h} X_2 U_{l+1} P_1 X_1 \\
 & + \frac{\Delta \cdot a}{2h} P_2 U_{l+1} X_1 - \frac{\Delta \cdot \beta}{2h} X_2 P_2 U_{l+1} + \Delta \cdot Z \\
 & = U_l + U_l Q A + \frac{\Delta \cdot \epsilon^2}{4h^2} Q U_l - \frac{\Delta a}{2h} P_2 U_l X_1 \\
 & + \frac{\Delta}{2h} X_2 U_l P_1 X_1 + \frac{\Delta \cdot \beta}{2h} X_2 P_2 U_l,
 \end{aligned}$$

where

$$A = \frac{\Delta}{4h^2} X_1^2 + \frac{\Delta}{h^2} X_1 + \frac{\Delta}{2h} I.$$

In view of the relation (3. 3. 15), if U_{l+1} is known, then the sum on the left hand side is known. Denote this sum by the matrix C. Then (3. 3. 15) can be put in the form $\sum_{i=1}^5 A_i X B_i = C$ and solved by the method outlined in section 3. 2.

CHAPTER 4
COMPUTATIONAL RESULTS

In this Chapter we consider two examples [Example 1.4.1 and example 2.3.1]. In example 1.4.1, the diffusion matrix is uniformly positive definite. In example 2.3.1, there is no such restriction on the diffusion matrix.

4.1. Example 1.4.1 With Diffusion Matrix Uniformly Positive Definite

The problem is duplicated here for convenience. Consider the stochastic optimal control problem given in example 1.4.1. As indicated in section 1.4, the stochastic optimal control problem is reduced to the following problem of optimal parameter selection of distributed parameter systems.

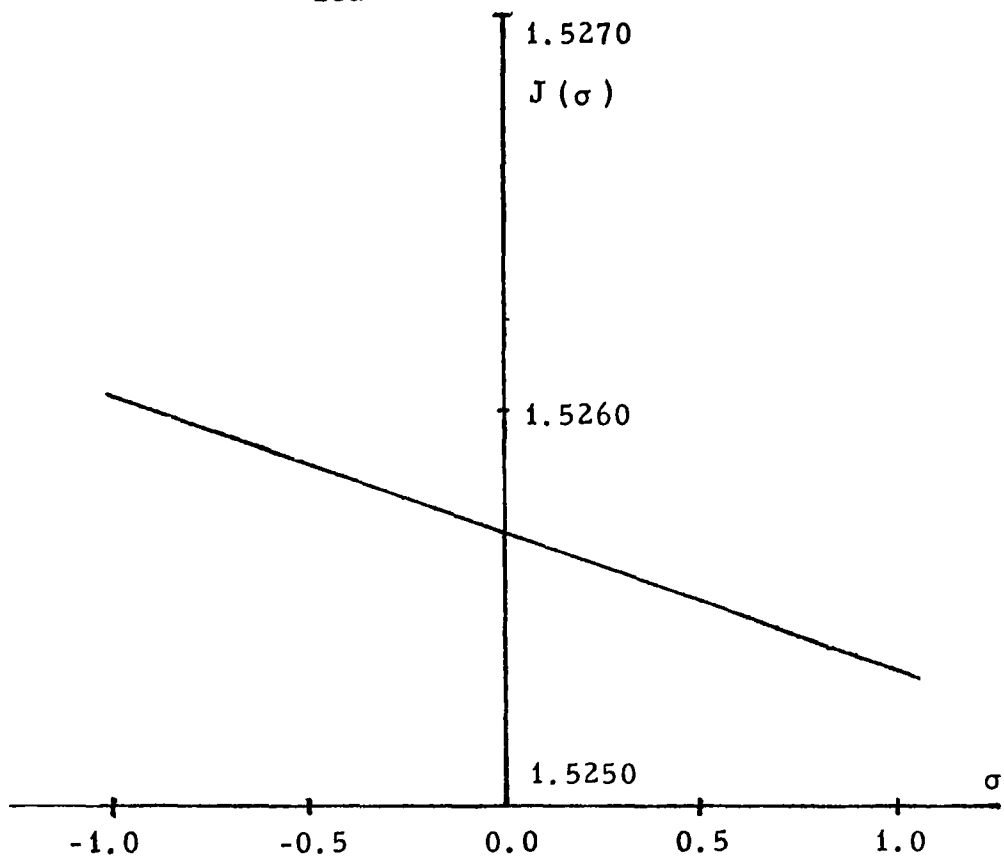
$$\left. \begin{aligned}
 & \frac{\partial \varphi(\sigma)}{\partial t} = \frac{1}{2} (x_1 + 2)^2 \frac{\partial^2 \varphi(\sigma)}{\partial x_1^2} + 25 \cdot \frac{\partial^2 \varphi(\sigma)}{\partial x_2^2} \\
 & \quad + \sigma x_1 x_2 \frac{\partial \varphi(\sigma)}{\partial x_1} + (x_1 + x_2) \frac{\partial \varphi(\sigma)}{\partial x_2} \\
 & \quad + 1 \qquad \qquad \qquad (x, t) \in \Omega \times (0, T] \\
 & \varphi(\sigma)(x, 0) = 0 \qquad \qquad \text{for } x \in \Omega \\
 & \varphi(\sigma)(x, t) = 0 \qquad \qquad \text{for } (x, t) \in \partial \Omega \times [0, T] \\
 & \sigma \in \Sigma \triangleq [-1, 1],
 \end{aligned} \right\}$$

where $\Omega \triangleq \{ (x_1, x_2) : -1 \leq x_1 \leq +1 \text{ and } -1 \leq x_2 \leq +1 \}$.

The problem is to find the scalar parameter $\sigma \in \Sigma$ so that

$$J(\sigma) = \int_0^T \int_{\Omega} \varphi(\sigma)(x, T) dx$$

is minimum.



σ	$J(\sigma)^*$
-1.0	1.52602
-0.5	1.52585
0.0	1.52568
0.5	1.52550
1.0	1.52531

* $J(\sigma)$ the integration of $z(\sigma)$ over the area Ω at T .

Figure 4.1

The system equation T_σ is solved for each $\sigma \in \Sigma_\Delta [-1, 1]$ using the Crank Nicolson Method [Chapter 3] on IBM 360/65. The cost functional $J(\sigma)$ is plotted as a function of σ as shown in figure 4.1. It is clear from the figure that the parameter corresponding to the extremal lies on the boundary of Σ as expected from Corollary 1.3.1.

4.2 Example 2.3.1 with Diffusion Matrix Not Necessarily Positive.

For convenience, example 2.3.1 is duplicated as follows: -

$$\left\{ \begin{array}{l} d\xi(t) = f(t, \xi(t), \gamma \xi(t) + \int_0^t K(t, \tau) v(\tau, \xi(\tau)) d\tau) dt \\ \quad + g(t, \xi(t)) d\omega(t) . \\ \xi(0) = \xi_0 \quad , \quad t \in I = [0, 1] \end{array} \right.$$

with $f(t, \xi, \gamma) = \xi \cdot \gamma$, $\gamma = 0$, $K(t, \tau) = \begin{cases} \alpha e^{\beta(t-\tau)}, & \tau \leq t \\ 0 & \text{elsewhere} \end{cases}$

$\sigma = (\alpha, \beta) \in \Sigma_\Delta \{(\alpha, \beta) : |\alpha| \leq 2, |\beta| \leq 2\}$, $v(t, \xi) = \xi$, $g(t, \xi) = (\xi + 2)$, the set of confinement of the process $\{\xi(t), t \in I\}$ is $\Omega_\Delta (-1, 1)$, (i.e. $a = 1$) and the initial probability density of the random variable ξ_0 is

$$q_0(x_1) = \begin{cases} \frac{1}{2} & \text{for } x_1 \in \Omega , \\ 0 & \text{elsewhere} \end{cases}$$

The problem is to find the vector $(\alpha^0, \beta^0) \in \Sigma$ that will maximize the confinement of x in Ω .

The corresponding ϵ - problem is

$$\left\{ \begin{array}{l} - \frac{\partial \varphi}{\partial t} = x_1 x_2 \frac{\partial \varphi}{\partial x_1} + (\alpha x_1 - \beta x_2) \frac{\partial \varphi}{\partial x_2} \\ \quad + \frac{1}{2} (x_1 + 2)^2 \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{1}{2} \epsilon^2 \frac{\partial^2 \varphi}{\partial x_2^2} + 1 \\ \text{for } (t, x) \in I \times B \triangleq (0, 1] \times \{(x_1, x_2) : \\ \qquad \qquad \qquad |x_1| < 1, x_2 \in \mathbb{R}\} \\ \\ \varphi(1, x) = 0 \quad \text{for } x \in B \\ \varphi(t, x) = 0 \quad \text{for } x \in \partial B \text{ and } t \in [0, 1] \\ \\ \max_{(\alpha, \beta) \in \Sigma} J_{\epsilon}(\alpha, \beta) = \max_{(\alpha, \beta) \in \Sigma} \int_{-1}^{+1} \varphi(0, x_1, 0) q_0(x_1) dx_1 \end{array} \right.$$

The above boundary value problem is solved using the method outlined in Chapter 3 on IBM 360/65 for all the four vertices $(2, 2)$, $(2, -2)$, $(-2, 2)$ and $(-2, -2)$.

The results are shown in table 4.2 and figure 4.2. It is clear from the table that the vertex $(\alpha^0, \beta^0) = (+2, -2)$ is optimal and that it maximizes the confinement time of the process ξ in Ω . The value of $J_{\epsilon}(+2, -2)$ is plotted as a function of ϵ in figure 4.2 which shows that as $\epsilon \rightarrow 0$, $J_{\epsilon}(+2; -2)$ converges to a definite limit and consequently the performance integral corresponding to the original problem $J_0(2, -2) \cong 3.50613$.

α	β	$J_{.1}(2, -2)$
+2	+2	3.45371
+2	-2	3.53073*
-2	-2	2.53278
-2	+2	2.52668

TABLE 4.2

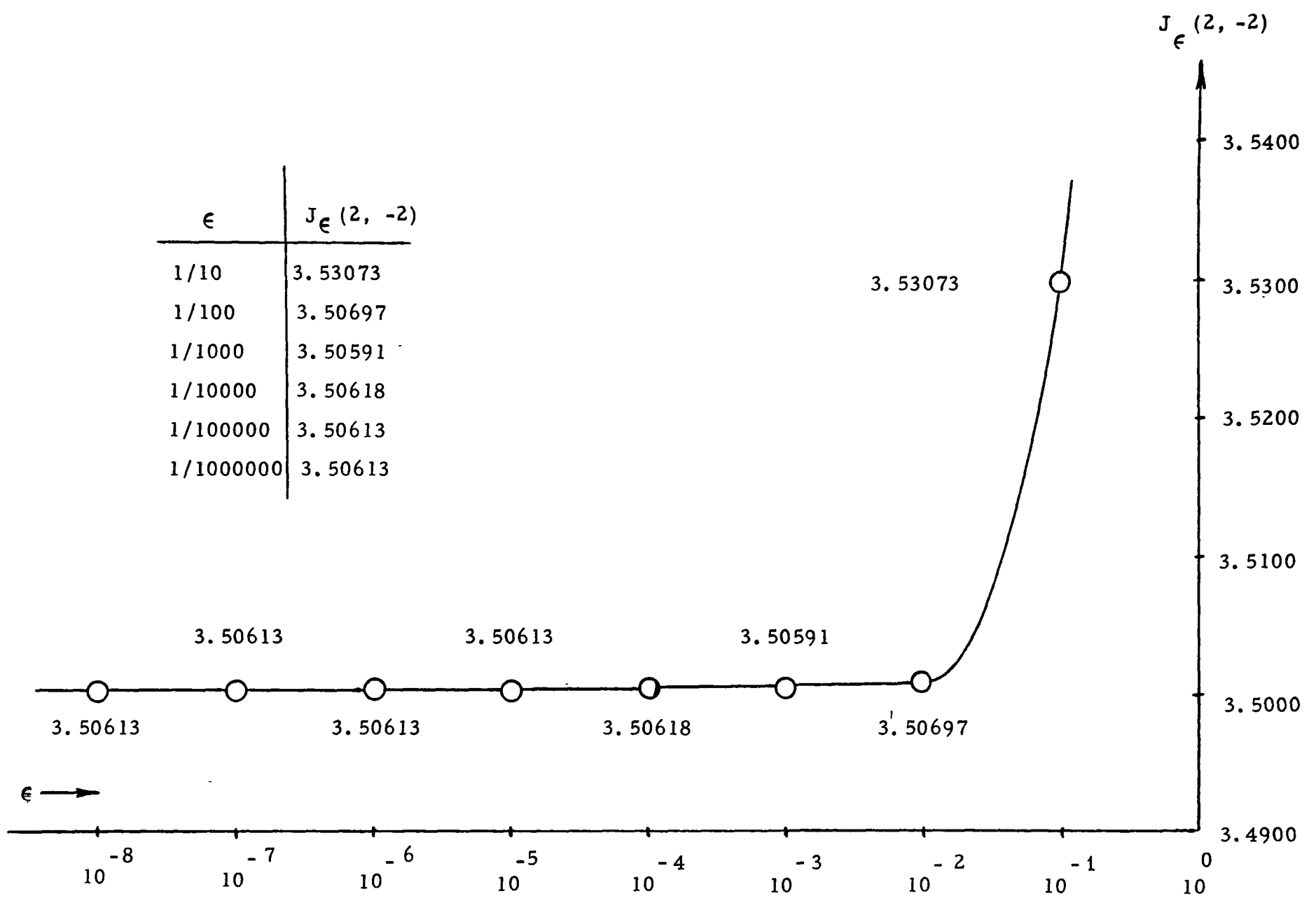


Fig. 4.2

CONCLUSIONS

In chapter 1, the stochastic optimal parameter selection problem was reduced to an equivalent problem of optimal parameter selection of a class of first boundary value problems, in which the parameters appear in the coefficients of the differential operator. Necessary conditions for optimal parameter of the reduced problem were given.

In chapter 2, the optimal parameter selection of stochastic hereditary systems was first reduced to an equivalent problem of degenerate stochastic Ito differential systems. By adding a perturbation term to the diffusion term, the system for the latter problem was converted into a non-degenerate one. The corresponding problem of optimal parameter selection is the one which approximates the problem of optimal parameter selection of stochastic hereditary systems. This approximated problem was further reduced to an equivalent optimal parameter selection problem of a class of first boundary value problems.

The numerical methods for the examples, as given in chapter 3, required the solution of the matrix equation $\sum_{i=1}^N A_i X B_i = C$, which in turn required the inversion of an $N^2 \times N^2$ matrix. The amount of memory needed for this matrix increased rapidly with N . Therefore it appears to the author that a method which could avoid to involve such an $N^2 \times N^2$ matrix would be very useful.

Chapter 4 included the computational results of the examples.

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