

Dynamic modelling and fault feature analysis of gear tooth pitting and spalling

by

Yang Luo

A Thesis Submitted to the University of Ottawa in Partial Fulfillment of
the Requirements for the Degree of

Doctorate of Philosophy

in

Mechanical Engineering

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering
University of Ottawa

© Yang Luo, Ottawa, Canada, 2019

Abstract

Fault feature analysis of gear tooth spall plays a vital role in gear fault diagnosis. Knowing the characteristic of fault features and their evolution as a gear tooth fault progresses is key to fault severity assessment. This thesis provides a comprehensive (both theoretical and experimental) analysis of the fault vibration features of a gear transmission with progressive localized gear tooth pitting and spalling. A dynamic model of a one-stage spur gear transmission is proposed to analyze the vibration behavior of a gear transmission with tooth fault. The proposed dynamic model considers the effects of Time Varying Mesh Stiffness (TVMS), tooth surface roughness changes and geometric deviations due to pitting and spalling, and also incorporates a time-varying load sharing ratio, as well as dynamic tooth contact friction forces, friction moments and dynamic mesh damping ratios. The gear dynamical model is validated by comparison with responses obtained from an experimental test rig under different load and fault conditions. In addition, several methods are proposed for the evaluation of the TVMS of a gear pair with tooth spall(s) with curved bottom and irregular shapes, which fills the current research gap on modelling tooth spalls with irregular shapes and randomly distribution conditions. Experiments are conducted and the fault vibration features and their evolution as the tooth fault progresses are analyzed. Based on feature analysis, a new health indicator is proposed to detect progressive localized tooth spall.

Table of Contents

Abstract.....	ii
List of Figures	vii
List of Tables	xiii
Acronyms	xiv
Acknowledgements	xvi
Chapter 1 Introduction	1
1.1 Overview	1
1.2 Motivation	3
1.3 Deficiencies and improvements of existing methods.....	6
1.4 Proposed Study.....	9
1.5 Organization of the thesis.....	10
Chapter 2 Literature Review	12
2.1 Review of gear TVMS evaluation.....	12
2.1.1 Review of general TVMS methods.....	12
2.1.2 Review of gear TVMS evaluation for tooth pitting and spalling.....	14
2.2 Review of gear dynamic models	16
2.2.1 Early gear dynamic models	16
2.2.2 Gear dynamic model with faults	19
2.2.3 Dynamic modelling for gear tooth pitting and spalling	24
2.3 Review of fault feature analysis by HCIs.....	26
Chapter 3 Gear mesh kinematic model.....	28
3.1 Abstract	29
3.2 Introduction	29
3.3 The proposed gear mesh kinematic model	30
3.3.1 Limits of Contact.....	30

3.3.2	Single and double tooth pair mesh durations	34
3.3.3	Geometric model of the spur gear profile	35
3.4	Potential energy method in TVMS calculation	38
3.5	Evaluation of gear TVMS with the proposed kinematic model.....	40
3.5.1	TVMS evaluation with constant gear center distance error	40
3.5.2	TVMS with eccentric error	42
3.6	Conclusion.....	45
Chapter 4	Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features.....	47
4.1	Abstract	48
4.2	Introduction	48
4.3	The proposed ellipsoid shape-based method on modelling gear tooth spalls	49
4.3.1	The geometric model of ellipsoid tooth spall.....	49
4.3.2	Evaluation of the TVMS of a spur gear pair with ellipsoid tooth spall	54
4.4	Validation and analysis	57
4.4.1	Validation of the proposed method	57
4.5	Conclusions	63
Chapter 5	A shape-independent approach to modelling gear tooth spalls for time varying mesh stiffness evaluation of a spur gear pair.....	64
5.1	Abstract	65
5.2	Introduction	65
5.3	Introduction of the proposed shape-independent method	66
5.3.1	The shape-independent method on modelling gear tooth spalls	66
5.3.2	The shape-independent method for gear TVMS with spall	72
5.4	The shape-independent method for modelling localized tooth spalls	76
5.4.1	Modelling a localized rectangular tooth spall	76

5.4.2	Modelling localized tooth spalls in round/elliptical shape.....	80
5.4.3	Modelling localized tooth spalls with irregular shapes	82
5.5	The shape-independent method for modelling distributed tooth spalls	87
5.5.1	Modelling multi-tooth spalls with a Gaussian distribution	88
5.5.2	Modelling randomly distributed tooth spalls	92
5.6	Conclusions	95
5.6.1	Summary of the proposed method	95
5.6.2	Advantages of the proposed method	96
Chapter 6 Dynamical modeling and experimental validation for tooth pitting and spalling in spur gears..... 97		
6.1	Abstract	98
6.2	Introduction	98
6.3	Gear contact model with tooth pitting and spalling	99
6.4	Modelling and evaluation of the TVMS of gear pair with tooth spalling	112
6.5	Gear mesh dynamic model.....	117
6.6	Evaluation of the system physical and mechanical parameters	121
6.7	Evaluation of the bearing stiffness and damping coefficient	122
6.8	Evaluation of the stiffness and damping coefficient of bolts	126
6.9	Measuring the gear tooth surface roughness	129
6.10	Dynamic simulation and model validation.....	130
6.10.1	Description of the experimental setup	130
6.10.2	Dynamic simulation and model validation.....	130
6.10.3	Experiment 1- Healthy conditions low speed.....	131
6.10.4	Experiment 2- Healthy conditions high speed.....	132
6.10.5	Experiment 3- Comparisons under initial faulty conditions.....	133
6.10.6	Experiment 4 - Comparison under severe tooth spalling conditions.....	137

6.11	The effects of friction forces in gear dynamic models.....	140
6.12	Conclusion.....	142
Chapter 7 Fault feature analysis and detection of progressive localized gear tooth pitting and spalling		
		144
7.1	Abstract	145
7.2	Introduction	145
7.3	Experimental description.....	147
7.4	Vibration feature analysis of gear transmission in healthy conditions.....	149
7.5	Fault feature analysis of gear system with progressive spalls.....	152
7.5.1	Fault feature analysis of spalls in lower speed condition.....	152
7.5.2	Fault feature analysis of spalls in higher speed condition.....	156
7.6	Fault detection of progressive tooth spall	160
7.7	Conclusion.....	162
Chapter 8 Contributions, conclusions and future work		
		164
8.1	Thesis Contributions	164
8.2	Conclusions	165
8.3	Future work	167
References.....		168
Appendix 1.....		187
Appendix 2.....		188

List of Figures

Figure 2-1 TVMS of spur gear pair with spalling defect modeled in rectangular shape..	15
Figure 2-2 The early gear contact model	17
Figure 2-3 The early gear vibration model	17
Figure 2-4 The 1-DOF dynamic model of a gear pair	18
Figure 2-5 The 3-DOF gear dynamic model.....	19
Figure 2-6 The 6-DOF gear dynamic model.....	20
Figure 2-7 The 8-DOF gear dynamic model.....	21
Figure 2-8 The one stage 16-DOF gear dynamic model.....	22
Figure 2-9 The 6-DOF model consider friction effect.....	23
Figure 2-10 9-DOF gear dynamic model.....	23
Figure 2-11 A planetary gear dynamic model	24
Fig. 3-1 Meshing process of a spur gear pair (initial position).....	31
Fig. 3-2 Mating process of a spur gear pair (separation position)	33
Fig. 3-3 Geometric model of the spur gear profile	36
Fig. 3-4 Mating process of a spur gear pair (arbitrary contact point)	37
Fig. 3-5 The effect of constant center distance deviation on gear mesh process.....	40
Fig. 3-6 Simulated comparison of TVMS of perfectly mounted gear pair and gear pairs subjected to constant center distance variations	41
Fig. 3-7 Gear position changes due to run out	42
Fig. 3-8 Gear mating process with run-out error	43
Fig. 3-9 TVMS of spur gear pair for each tooth pair	44
Fig. 3-10 Comparison of TVMS of the perfectly mounted spur gear pair and a spur gear pair with gear a run-out error	44
Fig. 4-1 Gear tooth spall modeled by rectangular shape	48

Fig. 4-2 Gear tooth spalling in practice	49
Fig. 4-3 Ellipsoid tooth spalling and a cross-section view passing through the center of the ellipsoid.....	50
Fig. 4-4 Cross-section model of the proposed tooth spall	50
Fig. 4-5 Geometric model of the spur gear profile	54
Fig. 4-6 Gear tooth spall modeled by the proposed method under different conditions ..	58
Fig. 4-7 The variation of cross-section area and area moment of inertia under different spalling conditions	59
Fig. 4-8 Zoomed cross-section area and area moment of inertia under different spalling conditions.....	59
Fig. 4-9 Finite element model of a mating gear pair for case (a) as an example.....	60
Fig. 4-10 TVMS of spur gear pair with multiple spalls.....	62
Fig. 4-11 Zoomed TVMS of case (a) and case (b)	62
Fig. 5-1 Simplified model of gear tooth spalling.....	67
Fig. 5-2 Illustration of the shape-independent method for modelling gear tooth spalling	69
Fig. 5-3 Gear tooth spalling modeled by a rectangular shape.....	69
Fig. 5-4 Cross-section model of the tooth spall	70
Fig. 5-5 Gear tooth modeled as a cantilever beam.....	72
Fig. 5-6 Finite element model of gear tooth spall in rectangular shape.....	78
Fig. 5-7 Comparison of the TVMS with rectangular spall evaluated by different methods	79
Fig.5-8 Gear tooth spall in elliptical/round shape.....	80
Fig.5-9 Gear tooth spalling modeled in round/elliptical shape by geometric based method	81
Fig.5-10 Comparison of the TVMS evaluated by different methods	82
Fig.5-11 Gear tooth spalls of irregular shapes	83

Fig.5-12 Tooth spall modelled in irregular shape	84
Fig.5-13 Gear tooth spall in complicated shape.....	84
Fig.5-14 Area and area moment of inertia of a gear tooth with irregular tooth spall	85
Fig.5-15 The TVMS of a gear tooth pair with irregular tooth spall	85
Fig.5-16 Tooth spall with irregular shape and varying depth.....	86
Fig.5-17 Gear tooth spall in complicated shape and varying depth.....	86
Fig.5-18 TVMS of the gear pair with irregular spall	87
Fig.5-19 Gear tooth spalls with a Gaussian distribution.....	88
Fig.5-20 Geometric model of distributed gear tooth spalls	88
Fig.5-21 Model of tooth spall in Gaussian distribution	91
Fig.5-22 Area and area moment of inertia of a gear tooth with Gaussian distributed tooth spalls	91
Fig.5-23 TVMS of gear tooth pair with Gaussian distributed tooth spalls.....	92
Fig.5-24 Tooth spalls in randomly distributed conditions	93
Fig.5-25 The features of tooth spalls and corresponding TVMS.....	94
Fig. 6-1 Gear tooth pitting and spalling.....	100
Fig. 6-2 The contact model of a spur gear pair with tooth pitting and spalling.....	101
Fig. 6-3 Mating process of a spur gear pair with friction forces.....	102
Fig. 6-4 The effects of gear tooth pitting and spalling on EHL film and pressure (under the conditions of $w=7\times 10^4$ N/m, $R=0.0071$ m, $T=25^\circ\text{C}$, $\omega_1=1500$ rpm, $\rho_0=864.80$ kg/m ³ , $\eta_0=0.0479$ Pa·s; and $L_e=0.016$ m & $Rc=0.001$ for (b), $L_e=0.016$ m & $Rc=0.05$ for (c), $L_e=0.014$ m & $Rc=0.1$ for (d), where Rc is surface roughness coefficient)	105
Fig. 6-5 Comparison of the gear tooth mating properties and EHL features under healthy and faulty conditions along the line of action for one mating gear tooth pair (roughness is assumed as isotropic $\gamma=I$; for healthy condition $L_e=0.016$ m; for pitting $L_e=0.016$ m; for spalling $L_e=0.014$ m).....	109

Fig. 6-6 Damping coefficient of the EHL film under different fault conditions	109
Fig. 6-7 EHL film thickness variation and film stiffness ($F_H=1000\text{ N}$, $\Delta F=50\text{ N}$).....	113
Fig. 6-8 Geometric model of the spur gear profile	115
Fig. 6-9 Dynamic model of a one stage spur gear transmission	120
Fig.6-10 Structure and internal sub-model data exchange of the proposed gear dynamic model.....	120
Fig. 6-11 Experimental devices and dynamic model of a simple rotor bearing system .	123
Fig. 6-12 The impulse response of the bearing test rig.....	126
Fig. 6-13 Evaluation of the bolts' stiffness and damping coefficient based on the Logarithmic decrement method	128
Fig. 6-14 The impulse response of the bolt joint	129
Fig. 6-15 Measuring tooth surface roughness.....	129
Fig. 6-16 Gear fault simulator.....	130
Fig. 6-17 Comparison between the experimental and simulated signals in vertical direction	131
Fig. 6-18 Comparison between the experimental and simulated signals in horizontal direction	131
Fig. 6-19 Comparison between the experimental and simulated signals in vertical direction	132
Fig. 6-20 Comparison between the experimental and simulated signals in horizontal direction	132
Fig. 6-21 Tools utilized for making progressive tooth pitting and spalling.....	134
Fig. 6-22 Gear tooth pitting and spalling and corresponding TVMS	134
Fig. 6-23 Surface roughness and effective tooth contact length of gear pair with and without faults.....	135

Fig. 6-24 Comparison between the experimental and simulated signals with gear tooth spalling (8.1 Hz).....	135
Fig. 6-25 Comparison between the experimental and simulated signals with gear tooth spalling (zoomed, 8.1 Hz).....	136
Fig. 6-26 Comparison between the experimental and simulated signals with gear tooth spalling (25.2 Hz).....	136
Fig. 6-27 Gear tooth pitting and spalling and corresponding TVMS	138
Fig. 6-28 Modeling of tooth surface changes under severe pitting and spalling conditions	138
Fig. 6-29 Comparison between the experimental and simulated signals with severe tooth spalling (8.1 Hz).....	139
Fig. 6-30 Frequency comparison between the experimental and simulated signals with severe tooth spalling (8.1 Hz)	139
Fig. 6-31 Frequency comparison between the experimental and simulated signals with severe tooth spalling (25.2 Hz)	140
Fig. 6-32 Simulation of the dynamic model considered only the modification of TVMS due to tooth pitting.....	140
Fig. 6-33 The dynamic forces of the gear system with consideration of surface roughness changes under tooth pitting and spalling conditions (test rig 1, 8.1 Hz)	142
Fig.7-1 Examples of localized tooth spall.....	145
Fig. 7-2 Progressive tooth spalls on the same pinion tooth	148
Fig. 7-3 Comparison of the vibration behavior of the gear system under different speed conditions.....	151
Fig. 7-4 Comparison of the vibration features of progressive tooth spall under lower speed conditions.....	153
Fig. 7-5 The time domain impulsive vibration features of 8 Hz speed cases	154

Fig. 7-6 Frequency comparison of progressive tooth spall with constant speed 8 Hz...	154
Fig. 7-7 Sideband property of progressive tooth spall under lower speed conditions ...	156
Fig. 7-8 Comparison of the vibration features of progressive tooth spall under higher speed conditions.....	157
Fig. 7-9 The time domain impulsive vibration features of 25 Hz speed cases	158
Fig. 7-10 Frequency comparison of progressive tooth spall with constant speed 25 Hz	158
Fig. 7-11 Sideband property around the 2 nd GMF of progressive tooth spall under higher speed conditions.....	159
Fig. 7-12 The 3 rd , 4 th and 6 th order peaks of the vibration signals of sever at 25 Hz speed conditions.....	161
Fig. 7-13 MPn of signals with progressive localized tooth spall-8 Hz speed conditions	162
Fig. 7-14 MPn of signals with progressive localized tooth spall-25 Hz speed conditions	162

List of Tables

Table 3-1 Reference equations for calculating the angles in Eq. (3.3)	32
Table 3-2 Reference equations for calculating the angles in Eq.(3.4)	33
Table 4-1 Parameters of the gear-pinion set of chapter 4	58
Table 5-1 Parameters of the gear-pinion set of chapter 5	78

Acronyms

AM	Analytical Method
CBM	Condition-Based Maintenance system
CF	Crest Factor
DOF	Degree-of-Freedom
DSFM	Discrete Sampling and Fit Method
EHL	Elastohydrodynamic lubrication
EOP	Energy Operator
ER	Energy Ratio
FE	Finite Element method
FGP	Fault-growth parameter
FGP1	The improved gear fault-growth parameter
FM0	Zero Order Figure of Merit
FM4	Fourth Normalized Statistical Moment
GDS	Gearbox Dynamics Simulator
GMF	Gear Mesh Frequency
HCI	Healthy Condition Indicators
IF	Instantaneous frequency
IVHMS	Integrated Vehicle Health Management System
M6A	Sixth normalized statistical moment
M8A	Eighth normalized statistical moment
MLS	Multiscale Local Statistics

MSPU	Modern Signal Processing Unit
NA4	Quasi-normalized statistical moments
NB4	Quasi-normalized statistical moments of band passed signal
NLLS	Non-Linear Least Squares
OLOA	Off Line of Action
PE	Potential Energy method
PHM	Prognostics and Health Management system
PP	Peak to Peak
RMS	Root Mean Square
RUL	Remaining Useful Life
SI	Sideband Index
SK	Spectral Kurtosis
SLF	Sideband level factor
STFT	Short Time Fourier Transform
TVMS	Time Varying Mesh Stiffness
WT	Wavelet Transform
WVD	Wigner-Ville Distribution

Acknowledgements

I would like to express my deepest gratitude to my tremendous mentors Professor Natalie Baddour and Professor Ming Liang. I would like to thank you for your continuous support, patience, motivation, and guidance on my Ph.D. research. Your immense knowledge and professional experience have enlightened me all the time of research.

I would like to thank the rest of my thesis committee: Professor Chris Mechefske (Queen's University, Canada), Professor Jie Liu, Professor Dan-Sorin Neculescu and Professor Balbir Dhillon for serving as my committee and their valuable time spent on this thesis.

My sincere thanks to my parents and my elder sister for supporting me spiritually throughout my life in general. Special thanks to my beloved Mengyuan Huan for years of unimpeded companionship, encouragement and support.

I thank my research fellows Shuai Yang, Yunpeng Guan, Zhuwen Sun, Xueyang Yao, Juanjuan Shi, and Huan Huang for their help and suggestions.

Chapter 1 Introduction

1.1 Overview

A gear transmission is one of the most important devices found in machines such as automobiles, wind turbines, airplanes, spacecraft, robots, mining equipment, etc. [1]. Due to complex structures, severe working conditions or inappropriate operation, gear transmissions are susceptible to damage and failure [2, 3]. The degradation of the transmission system will directly affect equipment safety and reliability. Severe faults may result in catastrophic failures, leading to productivity losses, maintenance costs or danger to human life. Condition monitoring and online diagnosis of this critical component are thus crucial to prevent system failures and reduce financial costs [1].

A strong desire and need for an intelligent condition monitoring system have existed for a long time [4]. Over the past few decades, numerous investigations have been carried out to construct reliable Condition-Based Maintenance (CBM) systems, as well as more advanced Prognostics and Health Management (PHM) systems. The goal of a CBM system is to replace the traditional prescheduled and labor-intensive planned maintenance methods so that maintenance work is performed only when a fault is detected [5]. The CBM system can largely reduce maintenance costs, minimize system downtime and increase machine life. Meanwhile, the PHM system is more ambitious; the primary emphasis is on providing an early indication of incipient faults, identifying the corresponding fault types and predicting the remaining useful life of the part or system. It not only provides valuable information for maintenance decisions but also increases system reliability and insures operational safety [4, 6]. Despite many investigations devoted to this area, the current CBM and PHM systems are still far from mature, with many reporting a significant number of false alarms or failure to detect the occurrence of faults. The incorrect responses of a CBM

or PHM system, such as the reporting of false alarms, may actually increase the maintenance work and costs due to unnecessary shut downs and subsequent required inspections [5].

Fault diagnosis is part of constructing a reliable PHM system for machinery, yet it is also the most challenging and difficult part. There is a tremendous body of work on fault detection and diagnosis based on vibration signals. Methods such as time-domain analysis, frequency domain analysis and time-frequency analysis, etc. have been extensively studied. Health Condition Indicators (HCIs) such as Root Mean Square (RMS), Kurtosis, Fourth Normalized Statistical Moment (FM4) [7], Quasi-normalized statistical moments (NA4) [8], Quasi-normalized statistical moments of band passed signal (NB4) [9], Energy Operator (EOP) [10], etc. are typical time-domain techniques. The values obtained via these statistical means can often quickly indicate the presence of a fault [11], but cannot provide a precise fault diagnosis [12]. Frequency and time-frequency techniques can provide more information about the system vibrations, and thus have the potential to precisely localize the failure through visually observing some 2D or 3D graphics [13].

To date, many detection/diagnosis methods based on the frequency components of vibration signals have been proposed, such as the well-known Short Time Fourier Transform (STFT), Wavelet Transform (WT), Wigner-Ville Distribution (WVD), Q-factors [14, 15], Envelope analysis [16], Cepstrum analysis [17], and numerous advanced methods derived from these methods. The diagnosis of gear damage mainly depends on the detection or extraction of some distinct vibration characteristics excited by the fault, regardless of the kind of method utilized. The key issue is “what kinds of fault symptoms are sought?” A clear answer to this question is the most essential and important issue for the success of fault diagnosis.

In practice, it is expected that a gear tooth fault (e.g. tooth spall and crack) can become more severe due to long duration continuous loading of the gear, which leads to another important research topic on the evaluation of the severity of a fault. It is often assumed that it is unnecessary to assess severity because the faulty component, once detected, should be replaced. However, this assumption is often incorrect because a faulty component (e.g. a gear with initial tooth spall or wear) may often be kept in service for a long time without jeopardizing the operation, and it is expensive and inconvenient to frequently change parts.

In fact, fault severity assessment has been attracting considerably increasing attention, due to its importance in making maintenance decisions as well as the evaluation of the fault severity of the failing component. The assessment of the fault severity based on vibration signal requires thorough understanding of the fault evolution characteristics as the fault becomes more severe. However, little work has provided a systematic analysis of the evolution of fault features of progressive gear tooth faults, especially for gear tooth pitting and spalling [18, 19].

1.2 Motivation

The diagnosis of gear tooth damage mainly depends on the detection of some vibration characteristics excited by the fault. For example, the diagnosis of a gear tooth crack is generally based on detecting a specific periodic impulse in the vibration signal [20, 21], and the identification of gear runout or unbalance relies on the examination of the sidebands around the gear mesh frequency and its harmonics [21, 22]. However, unlike the above defects, the fault symptoms of tooth pitting and spalling and the corresponding fault mechanisms remain unclear due to the complicated interactions of the mating gear tooth pairs. The diagnosis of a spalling defect is widely based on the assumption that the fault characteristics of tooth spalling are the same as those of a tooth crack, such as found in [23-

28]. This assumption needs to be further validated or corrected since there are fundamental differences between the failure mechanisms of tooth crack and tooth spalling. For example, the occurrences of a gear tooth crack mainly results in tooth bending and affects the contact stiffness of mating teeth [29]. However, tooth pitting and spalling mainly modifies the surface contact properties, which not only reduces contact stiffness but also increases surface roughness and changes the contact lubrication conditions and friction forces [18, 30, 31]. Therefore, an improved understanding of the distinct vibration characteristics excited by tooth pitting and spalling is needed for gear fault detection and diagnosis.

The development of an advanced CBM or PHM system not only requires the successful detection of the existence of fault(s), but also requires the corresponding fault severity of the faulty components. Knowing the evolution characteristics of fault vibration features as the fault becomes more severe plays a vital role in fault severity assessment. However, little work has provided analysis of the evolution of fault features of progressive gear tooth faults, especially for tooth spalling. The assessment of fault severity levels is widely based on the hypothesis that ‘the more severe the fault, the stronger the fault symptom’, which appears to be a reasonable assumption but still requires experimental validation. For example, the existence of periodic impulses and sidebands around a meshing frequency is often used to detect and diagnose localized gear tooth faults. However, can we say that a gear fault with higher-sideband-amplitude is more severe than one with lower-sideband-amplitude? Does this imply that the more sidebands, the more severe the fault? What does it mean if the sidebands appear in a certain meshing frequency harmonic? Would the fault severity be different if the sidebands appeared around a certain harmonic, e.g., the 3rd harmonic of the meshing frequency instead of other harmonics? Does the distribution of the sidebands change when fault becomes more severe? Is it true that the more severe a localized gear tooth fault, the higher the amplitude of the fault vibration impulses? A clear

understanding of the evolution of fault features of a progressive tooth fault via experiments remains an important research topic for gear fault detection and severity assessment. Knowing the fault features of a gear system under different fault severity conditions is critical for gear fault severity assessment and maintenance decision making.

The assessment of fault severity is generally based on the change of amplitude of some condition indicator(s) as the fault progresses and the severity of the fault is judged by comparing the measured values with pre-set thresholds [1]. Most of the proposed condition indicators were designed for detecting a specific type of gear fault. For example, the Multiscale Local Statistics (MLS), NB4, Sideband Level Factor (SLF), Sideband Index (SI), Crest Factor (CF) etc. targeted the detection of localized gear tooth faults, such as tooth root crack or initial pitting (pitting only on a few teeth). The EOP, the Sixth Normalized Statistical Moment (M6A), the Eighth Normalized Statistical Moment (M8A) and the Energy Ratio (ER) have primarily been used for detecting distributed faults such as heavy tooth wear, severe pitting or scuffing. Kurtosis, the Zero Order Figure of Merit (FM0), the Fourth Normalized Statistical Moment (FM4), RMS as well as delta-RMS have generally been used in detecting both gear tooth crack and wear. Based on Zakrajsek's [8] description, only the NA4 has the ability to detect progressive damage where the fault transformed from localized to distributed. Therefore, new condition indicators that can robustly detect a fault and evaluate the corresponding fault severity are still required.

As such, the motivations of this thesis work are to:

1. Analyze the fault features of a gear transmission with tooth pitting and spalling comprehensively (theoretically and experimentally). Determine the fault vibration characteristics of a gear transmission with tooth pitting and spalling. Determine if these fault characteristics can be found in the time domain or frequency domain.

2. Investigate the evolution of characteristics of fault vibration features as the fault progresses from healthy state to severe condition. Determine if and how the vibration signal changes when a fault become more severe. Determine if a gear fault with higher-sideband-amplitudes is more severe than one with lower-sideband-amplitudes. Determine if number, location and distribution of sidebands imply an increase in the severity of the fault.
3. Propose a novel condition indicator to detect progressive tooth spalling.

1.3 Deficiencies and improvements of existing methods

Dynamic models simulating gear tooth pitting and spalling are generally the same models used to analyze the effects of a gear tooth crack, such as in [32, 33]. In modelling a gear tooth crack, the changes on the magnitude of the Time Varying Mesh Stiffness (TVMS) due to the crack is significant and can be assumed as the dominant factor. The tooth surface roughness as well as the mesh friction conditions, can be assumed to remain unchanged if no pitting exists. However, unlike modelling a tooth crack, the changes to the TVMS due to tooth pitting and spalling are small, especially in the case of initial tooth pitting and pitting with limited depths, such as for the results in [34-37]. The small changes in TVMS result in limited effects on the dynamical vibration behavior of the system and thus modelling small levels of pitting and spalling as changes in TVMS will fail to explain any observed vibration features. The modelling of gear tooth pitting and spalling is much more complicated compared to modelling a tooth crack, as other factors, such as surface roughness changes and geometric deviations due to tooth pitting and spalling, can have significant effects on the vibration response of the system and thus need to be taken into consideration.

The validation of the dynamic model requires comparison of the predictions of the model with signals that can be experimentally obtained. Due to experimental limitations, most vibration signals are obtained from the outside of the gearbox casing, instead of from the rotating gear pairs. Therefore, dynamic modelling of a gear system should also consider the effect of the gearbox casing in order to generate signals comparable to those obtained in practice. It is also worth mentioning that the gearbox casing itself is one of the most important components of the gear system, and so plays an important role in vibration and acoustic signal propagation, as studied in [38-42]. Therefore, its effects cannot be neglected and should be taken into consideration when modelling a gear transmission. Moreover, the validation of the dynamic vibration behavior of a real gear transmission also requires the actual values of the physical properties of some important components, such as stiffness and damping ratios of bearings, shaft couplings and gearbox-fixing bolts. However, these physical properties are mostly unknown and are affected by working and mounting conditions. Therefore, theoretical or experimental evaluation of these critical parameters is often necessary.

The analysis of the vibration behavior of a gear transmission by dynamic simulation requires accurate values of the gear Time Varying Mesh Stiffness (TVMS) and its corresponding changes when a tooth fault occurs. The occurrence of gear tooth spalls reduces the amount of effective tooth material supporting the dynamic loads of the transmission, which results in a decreased gear TVMS. The decreased gear tooth stiffness is one of the main sources of impulse vibrations and increases the noise in a gear system. The purpose of gear TVMS evaluation is to accurately calculate the stiffness and capture its changes when a fault occurs. Common approaches assume that the tooth spall has a flat bottom with a constant tooth spall depth. This assumption implies an abrupt change in depth and cliff-like material loss. However, in practice, the flake of the surface material

usually results in a gradually changing depth with a curve-shaped bottom. Thus, the modelling of tooth spalls with a flat bottom may result in mesh stiffness differences.

Existing work on modelling gear tooth spalls has also focused on using a specific geometry (e.g. rectangular, round, elliptical, pixel, etc.) to approximate the features of the tooth spall. The accuracy of the geometry-based methods on modeling gear tooth spall depends on how close the utilized geometry is to the actual shape of the spall. However, in practice, tooth spall(s) are usually irregular in shape and hence it is often difficult to find a suitable geometry to match the shape of the defect. The issue of how to analytically evaluate the TVMS of the gear pair with irregular tooth spalls as observed in practice remains unsolved. Moreover, many existing methods are suitable for application to only a specific type of tooth spall with a specific geometry or distribution. Changing the type or shape of the tooth spall requires a corresponding change in the evaluation equations and algorithms. However, the equations used by the different methods for modelling different types of spalls can vary significantly, such as the ones presented in [35, 36, 43, 44], which significantly hinders the practical applications of these methods. Hence, a general method that covers various situations and is unconstrained by the geometry and distribution of the tooth spalls is highly desirable in order to simplify the evaluation process when modelling spalls of different types or shapes.

Analyzing the evolution of the vibration features of a gear system with a progressive tooth fault requires that the obtained signals are comparable after each test. Data consistency not only requires the signals to be obtained under the same working conditions (e.g. the same speed and load conditions) but also requires the status of the other healthy components to remain the same, e.g. changes that might be introduced by disassembling the gears, shafts, bearings and couplings, or loosening the bolts of the gearbox and motors

must be avoided. Since any change in these components may change the vibration features of the system, this may result in inaccurate fault feature comparison conclusions. Moreover, the disassembly and reassembly of a gear system also has the potential to introduce additional assembly errors or to change existing mounting errors. The introduction or modification in assembly errors can directly affect the system vibration behavior, which in turn results in difficulties in determining which source has caused the change in the fault feature symptom. To avoid modifying the mounting conditions of the transmission and to ensure that the obtained signals are accurate and comparable under different faulty severity conditions, tooth pitting and spalling should be introduced directly on the gear tooth without disassembly of the gearbox during experiments.

1.4 Proposed Study

Understanding of fault vibration features and the evolution of these features as the fault progresses requires both improved theoretical methods and advanced experimental techniques. In light of the summary presented in section 1.3, the objectives of this thesis are to:

1. Propose a gear mesh kinematic model which can be used to evaluate the actual contact positions of tooth engagement with respect to gear rotational angle.
2. Propose a curved-bottom shaped tooth spall model to best fit the shape of the spalling geometric features as observed in practice.
3. Propose an advanced method to model tooth spall(s) with irregular shapes and randomly distributed features.
4. Propose a novel spur gear dynamical model that can be used to understand the dynamic behavior of a gear system under faulty conditions. Validate the proposed

- gear dynamical model by comparison with responses obtained from an experimental test rig under different conditions.
5. Analyze the fault features of a gear transmission with tooth pitting and spalling through the proposed dynamic model and compare with features obtained by experiments.
 6. Investigate the evolution characteristics of fault vibration features as the fault progresses from healthy to severe faulty condition.
 7. Propose a novel condition indicator to detect progressive tooth spalling.

1.5 Organization of the thesis

The thesis is organized as follows. A literature review that encompasses gear TVMS evaluation with and without tooth fault, dynamic modelling of gear transmission and HCIs on extracting fault features is given in Chapter 2. Chapter 3 addresses objective 1, which is to propose a gear mesh kinematic model to evaluate the actual contact position of tooth engagement. The proposed model can be used to evaluate the actual tooth mesh contact positions with either constant or time varying gear center distance. Additional analysis on the effect of center distance variation on the TVMS of a spur gear pair is also provided. Chapter 4 proposes a curved-bottom shaped tooth spall to best fit the shape of the spalling geometric features as observed in practice. This addresses objective 2. Chapter 5 addresses objective 3, which is to propose an advanced method to model tooth spall(s) with irregular shapes and randomly distributed features. The performance of the proposed method on modeling localized tooth spall with different regular shapes (e.g. rectangular, round, elliptical, etc.), irregular shapes, and spalls under normal or randomly distributed conditions are then evaluated. Chapter 6 proposed an advanced spur gear dynamical model, validated by various experimental tests, to analytically investigate the effects of tooth

pitting and spalling on the vibration responses of a gear transmission to address objectives 4 and 5. Chapter 7 addresses objectives 6 and 7, the evolution characteristics of fault vibration features as the gear tooth spall progresses from initial to severe. A novel condition indicator is designed to detect progressive gear tooth spalling. Conclusions and future work are presented in Chapter 8.

Chapter 2 Literature Review

This chapter presents a literature review of prior work that on gear time varying mesh stiffness evaluation, gear transmission dynamic modelling, and gear healthy condition indicators.

2.1 Review of gear TVMS evaluation

2.1.1 Review of general TVMS methods

Gear TVMS plays a vital role in the dynamic simulation of a gear system. It has been shown that many gear faults, such as gear tooth crack, tooth pitting and spalling, can significant change the value of the TVMS. Therefore, many types of gear faults have been modeled as a change of the TVMS, as was explained in the previous sections.

Much research has been carried out to investigate TVMS with or without gear faults. Methods such as the Finite Element method (FE), Analytical Method (AM), AM-FE hybrid method, and experimental methods have been developed and/or validated [45]. The FE method has been widely used to calculate the TVMS of both healthy and cracked gear tooth cases, because of its high accuracy and the ability to simulate complex gear tooth geometry [45, 46]. Sirichai, et al. evaluated the torsional mesh stiffness of a spur gear pair by a two-dimensional FE method [47]. Ma et al. applied the FE method to simulate the TVMS of spur gears with tooth spalling defects [34]. Zouari et al. predicted the effects of crack size, position, and direction on the behavior of the gear meshing using a three-dimensional FE model [48]. The analysis of successive discrete TVMS values needs to redefine loads and boundary conditions for each contact point. This is not an onerous work if the gears are perfectly mounted, where the gear center distance always equals its theoretical value. However, if the actual center distance differs from the theoretical value because of gear

assembly errors, run-out errors, shaft bending, or simply bearing deformation caused by heavy loads, then the actual tooth contact points will be modified. Under this condition, the corresponding evaluation process will need a different finite element model for every modified contact point, which could be difficult and very time consuming to perform [46, 49].

Analytical methods (AMs) have been considered to be the most promising approach to evaluating TVMS, and some studies, e.g., those reported in [34, 50, 51], show that very accurate TVMS values can be obtained using AMs with reduced computation time compared with FE methods. Yang and Lin modeled the gear tooth as a cantilever of variable cross-section starting from the base circle, and then utilized the Potential Energy method (PE), with consideration of Hertzian contact stiffness, bending stiffness and axial compressive stiffness, to calculate the TVMS of spur gear pairs [52]. This work was further improved by Tian et al. [53] and Wu et al. [50] through considering shear stiffness. Later, Zhou et al. [54] and Wan et al. [55] pointed out that the deformation of the gear body should also be taken into consideration. Soon after, Ma et al. [56] and Liang et al. [57] indicated that ignoring the misalignment between the base circle and root circle led to significant deviation from the actual TVMS. This observation prompted them to propose a more advanced PE method based on the gear root circle instead of the base circle. Recently, Ma et al. [45] revised the calculation error of the fillet foundation stiffness during double-tooth engagement, and investigated the effect of extended tooth contact on TVMS [58]. Saxena et al. [59] modified the PE model and studied the effect of angular shaft misalignment and friction on TVMS of spur gear pair. The continuous improvement of the PE method has made it a more accurate analytical tool for evaluating TVMS of spur gear pairs compared with the FE method.

In addition to these methods, Rincon et al. considered the advantages of both AM and FE methods and proposed a hybrid AM-FE method to evaluate TVMS of spur gear pairs [46, 49, 60]. In this method, the deformations of the mating teeth were resolved into global and local terms and evaluated using the FE method and Hertzian contact theory, respectively. Eritenel and Parker proposed a lumped-parameter model to evaluate the TVMS, which captures partial contact-loss and nonlinear load distribution caused by tooth surface modifications, misalignments, and elastic deflections [61]. Yu and Mechefske [62] proposed an analytical method to study the effects of corner contact (contact outside the normal path of contact) on gear mesh stiffness. Moreover, some experimental methods such as the photo-elasticity technique [63, 64] and strain gauge technique [65] were used to measure the actual TVMS in real applications.

2.1.2 Review of gear TVMS evaluation for tooth pitting and spalling

The occurrence of tooth pitting and spalling not only has effects on the tooth surface roughness and effective length of the contact line, but also has a significant influence on the TVMS of mating tooth pairs. Some work has been carried out in modeling and evaluating the TVMS of a tooth pair with tooth spalling by either finite element analysis or the analytic method. The modelling of the spalling defect was generally assumed to be a rectangular shape, e.g. papers [34, 37, 43, 66], and the severity of the fault was controlled by the length l_s , width w_s , and depth h_s of the rectangular, Figure 2-1(a). The use of a rectangular shape leads to a sharp tooth stiffness reduction, as is shown in Figure 2-1(b), which does not reflect the actual effect of tooth spalling on the TVMS in practice, where the actual TVMS should have a gradually decreasing property.

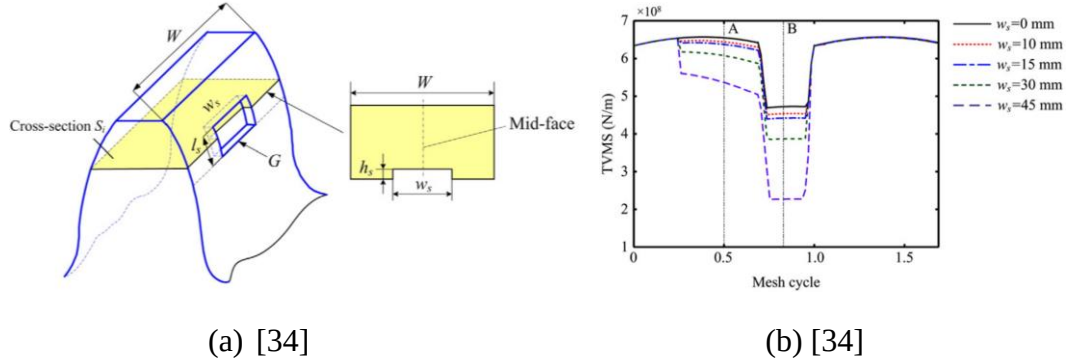


Figure 2-1 TVMS of spur gear pair with spalling defect modeled in rectangular shape

Recently, Rincon [36] considered the shape of spalling as elliptic, Liang et al. [35] modeled the tooth spalling in a round shape, Ankur Saxena [37] et al. proposed a V-shaped spall and compared the effect of shape difference on the TVMS of spur gear pair. These newly proposed spall shapes avoided the sharp decrease of the TVMS of the faulty gear pairs and got a closer tooth stiffness evaluation to the real spalling features, but largely increased the complexity of the calculation formulas and still didn't cover all of the actual spalling shapes. Moreover, the effect of tooth spalling on the mesh stiffness after the defect area has been always ignored. Due to the micro surface cracks and the decreased tooth surface thickness within the spalling area, the reduction of the bending stiffness of the following rest of the tooth surface could be a significant value, especially in severe spalling conditions where large and deep dents occur.

Existing methods utilized to estimate the TVMS of a gear pair with tooth spalling are widely targeted on one tooth spall. In practice, it is normal that multiple spalls appear on the same tooth. Liang et al. [35] evaluated the TVMS with multiple tooth spalls. In that work, all of the tooth spalls were arranged in a straight line, assumed to be in round shapes with no overlap. In practice, the occurrence of multiple spalls on one tooth may display various sizes and shapes, and the shapes are usually very irregular. Moreover, the multiple

spalls on one tooth are also usually randomly distributed in an area, and do not usually stay in lines. The occurrence of multiple spalls with irregular shapes results in difficulties if attempting to model with only one specific geometric shape. Also, the random distribution of features over a tooth spall area will increase the complexity of the evaluation process. Therefore, a new method that can consider the randomness of the features of tooth spalls and does not depend on a specific tooth spall geometric shape is highly desired.

2.2 Review of gear dynamic models

2.2.1 Early gear dynamic models

The study of gear dynamic models dates back to the early 1920s [67]. The targets of the early models were mainly focused on noise control, contact stress evaluation, natural frequency analysis, noise radiation, dynamic stabilities, system unbalance responses etc. [68]. Buckingham [69] first presented a dynamic load equation and studied the dynamic loads of a gear pair by experiments. Later, Tuplin [70], Reswick [71], Harris [72], Tobe [73], etc. developed several gear vibratory models, in which the contact of the mating teeth was simplified to massless springs, see Figure 2-2. These early vibratory models were too simple - limited to analyzing the dynamic loads of the gear system. Soon after, Gregory, et al. [74], Aida [75], Nakamura [76] extended the early models and considered the gear mesh stiffness to be time-varying. In their studies, the non-linear effects, stability regions, steady state responses and effects of transmission errors of the gear system were analyzed. A typical vibration model for a mating gear pair utilized at that time (around 1967) is shown in Figure 2-3, in which M_p , M_g and K_p , K_g are the effective mass and tooth stiffness of the pinion and gear, respectively, and $e(t)$ is the gear transmission error between the contact of the pinion and gear. However, these models were too simple and could not fully reflect the vibration behavior of the gear system.

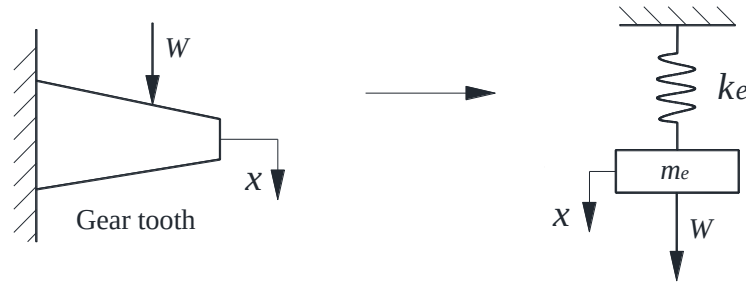


Figure 2-2 The early gear contact model [73]

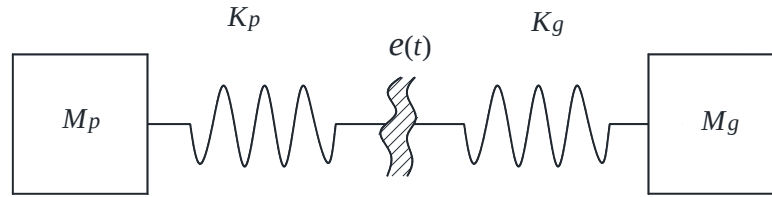


Figure 2-3 The early gear vibration model [76]

In 1974, Ichimaru and Hirano [77] improved the early gear vibration model and studied the interaction between the tooth deformation and dynamic load behavior of heavy-loaded spur gears with the consideration of the effect of teeth manufacturing errors. Later, Kubo [78] proposed a calculation method for evaluating the load sharing and load distribution of helical gear tooth pairs with manufacturing and alignment error. Remmers [79] described an analytical method to analyze the influence of spacing errors, load, design contact ratio and profile modifications on gear vibration, noise and tooth dynamic loads. The same year, Benton and Seireg [80] introduced a computer simulation procedure for predicting the gear system resonances and instabilities under sinusoidal excitation conditions. At that time, the investigation of the gear dynamics was extended to include the effects of tooth surface roughness, tooth contact lubrication problems and dynamic model solving methods (numerical or analytical). Ishida and Matsuda [81] studied the effect of tooth surface roughness on gear noise and the gear noise transmission path. Wang and Cheng [82] developed a numerical method to predict the lubrication film thickness and contact surface

temperature based on a single degree of freedom lumped model. Yang and Lin [52] studied the effect of Hertzian damping, tooth friction and bending elasticity in gear dynamics.

In 1988, Ozguven and Houser [67] discussed two calculation methods for evaluating the dynamic mesh forces and dynamic factors. The dynamic model utilized in their study is a one degree of freedom (DOF) torsional model, in which the gears are represented as rigid disks and the contact of the gear pair is simplified as a spring and a damper with a constant damping ratio, as shown in Figure 2-4, where ω_1 , ω_2 are the rotational speed, r_{b1} , r_{b2} and I_1, I_2 are base radii and moment of inertias of the pinion and gear, respectively. k_m , c_m and $e(t)$ are respectively the mesh stiffness, damping ratio, and transmission error.

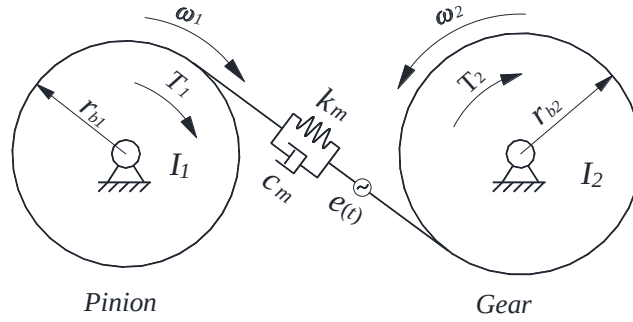


Figure 2-4 The 1-DOF dynamic model of a gear pair [67]

Later, Kahraman and Singh [83] extended the 1-DOF model and developed a 3-DOF non-linear dynamic model which was utilized to examine the nonlinear frequency response characteristics of the gear system. This model considered the effect of the supporting bearings and shafts, which were modeled as with an equivalent bearing stiffness k_b and damping coefficient c_b , see Figure 2-5.

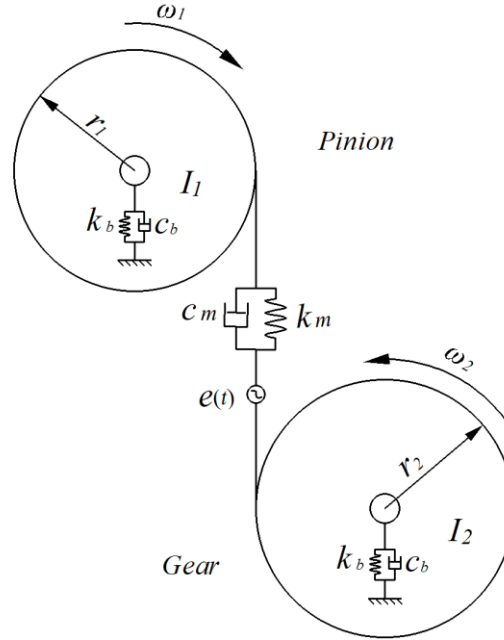


Figure 2-5 The 3-DOF gear dynamic model

The dynamic models summarized above are the typical early models which were mainly targeted on noise control, contact stress evaluation, natural frequency analysis, noise radiation, dynamic stabilities, and system unbalance responses.

2.2.2 Gear dynamic model with faults

Dynamic modelling of gear tooth faults started at the end of 20th century. In 1992, Kahraman [84] et al. analyzed the effects of gear eccentricities and unbalances on the system natural frequencies and mode shapes. The dynamic model with 6-DOF utilized in their study is shown in Figure 2-6, the gear eccentricities were modeled as periodic excitations (sinusoidal functions) of the gear mesh forces which can be assumed as periodic transmission errors. Their conclusion indicated that there was no feature difference between the gear geometric eccentricities and mass unbalances. Later, Velex and Maatar [85] introduced a mathematical model for analyzing the effects of gear shape deviations and mounting errors (misalignments and eccentricities) on the dynamic tooth loads and

transmission errors of the gear system. The eccentricity error was expressed as sinusoidal functions with the magnitude determined by the distance between the gear rotation centre and the centre of inertia.

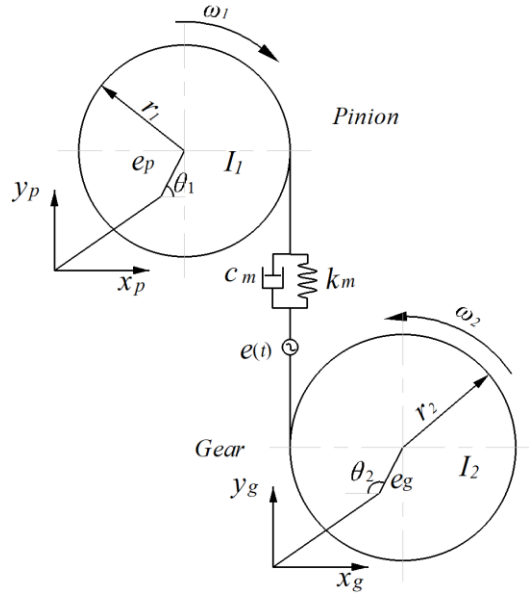


Figure 2-6 The 6-DOF gear dynamic model

In 2001, Bartelmus [86] extended the 6-DOF model (see Figure 2-6) to an 8-DOF model, as shown in Figure 2-7. These models incorporated the inertia of the input motor and output load, considered the lateral and rotational vibrations of the system, took the mesh stiffness and damping ratio as time varying variables, and ignored the effect of friction. The tooth errors (e.g. manufacture errors, tooth shape deviations) were modeled as periodic inter-tooth force excitations. At the same time, Howard [87] proposed a one stage 16 DOF gear dynamic model (Figure 2-8) to analyze the effect of friction and tooth crack on the vibration response of the gear system. FEM modelling was utilized to evaluate the TVMS with and without tooth crack. The occurrence of a tooth crack directly results in a decreased TVMS, therefore excites impulse vibrations of the dynamic system. Choi and Mau [88] analyzed the steady-state responses of a gear pair due to geometric eccentricity, unbalance, and

transmission errors through dynamic simulation. The eccentricity fault was modeled as one part of the transmission errors expressed by a sinusoidal function and coupled in the dynamic force function.

Later, Jia et al. [33] extended this model to a two stage gear system with 26-DOF for comparison of the vibration behavior of localized tooth crack and spall. In their study, both of the tooth crack and tooth spalling defects were modeled as the reduction of TVMS. Driot and Liaudet [89] utilized a 1-DOF dynamic model to simulate the variations of natural frequencies and gear mesh dynamic forces due to manufacturing errors and shaft misalignments. Wu et al. [50] utilized the 8-DOF model and simulated the vibration features of the gear system with tooth crack. In that work, the tooth crack was modeled as a reduction of TVMS, which is the main excitation of the gear system. Endo et al. [20] utilized a one stage gear dynamic model to examine the difference between the fault features of a tooth crack and tooth pitting with simulations. In that work, the effect of the tooth crack was modeled as a change of TVMS and localized tooth spall was represented by transmission errors.

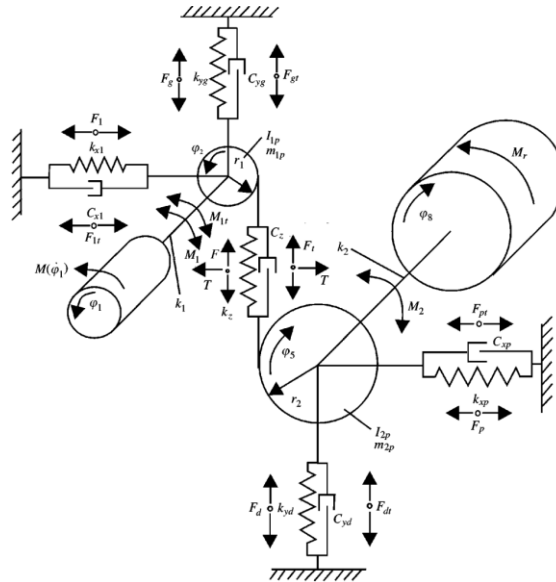


Figure 2-7 The 8-DOF gear dynamic model

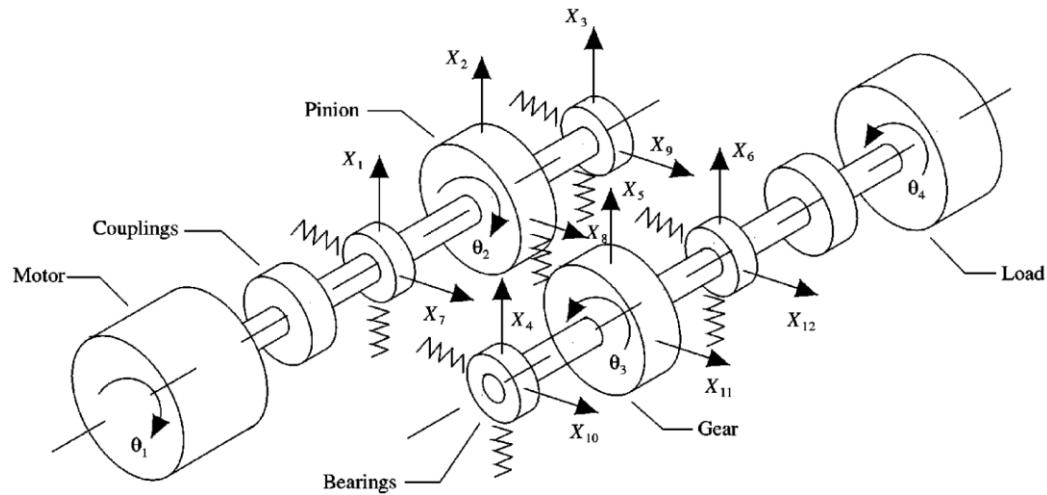


Figure 2-8 The one stage 16-DOF gear dynamic model

Zhang et al. [90] introduced a dynamic model for helical gear pairs to study the effects of geometric eccentricity on the dynamic mesh forces and vibration modes of the gear transmission. In that work, the gear mesh stiffness is modeled as a linear spring with constant stiffness and the eccentric error was directly incorporated into the dynamic function. Chen and Shao [91], Mohammed et al. [92] modified the 6-DOF model with the consideration of the effects of friction (the friction coefficient was assumed constant, see Figure 2-9) and simulated the vibration responses of the gear system under different tooth crack progression situations.

Considering the effect of the transmission path between the gear shaft and the accelerometer, Omar et al. [93] developed a 9-DOF gear dynamic model, as shown in Figure 2-10. The gear transmission error was modeled as a displacement excitation, and the effect of tooth crack was modeled as a reduction of TVMS.

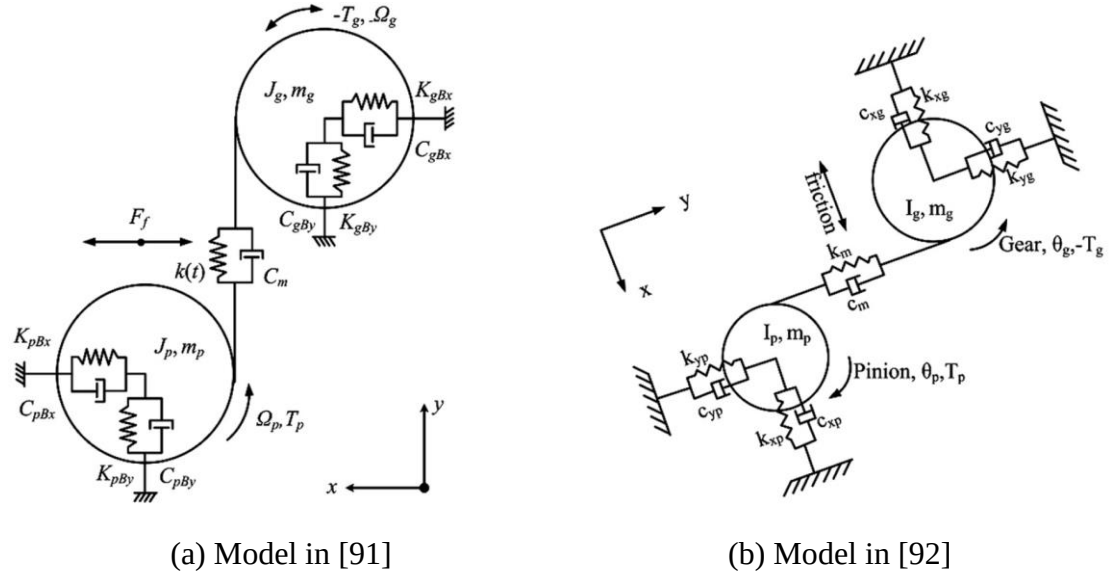


Figure 2-9 The 6-DOF model consider friction effect

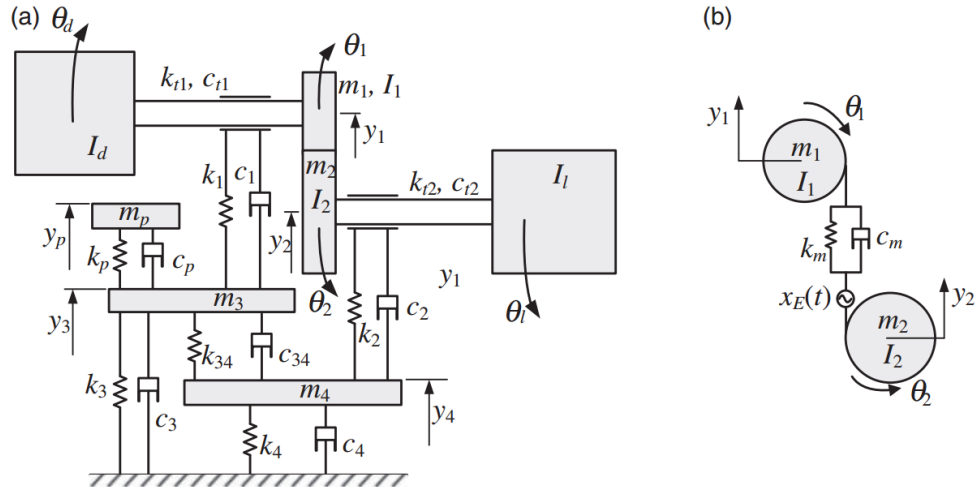


Figure 2-10 9-DOF gear dynamic model [94]

Chaari et al. [95, 96] considered the elasticity of the input and output shafts and proposed a two stage 12-DOF gear dynamic model to study the influence of time varying load on the dynamic behavior of a gearbox. Liang et al. [42, 97], Chen et al. [98] and Shao and Chen [99] dynamically simulated the vibration responses of a planetary gear system

To date, the research on modeling gear tooth pitting and spalling and the corresponding fault features is very limited [35]. Choy et al. [103] proposed an analytical model to simulate the effect of tooth surface pitting and wear on the vibration response of a gear transmission. In their analysis, tooth pitting and wear were modeled as phase and magnitude changes of the gear mesh stiffness. Ma et al. [104] used a one Degree-of-Freedom (DOF) gear dynamic model of torsional vibrations to simulate the vibration response excited by local tooth spall. In their analysis, tooth spall was modeled as a reduction of TVMS and was the only defect-excitation of the dynamic system. Abouel-seoud et al. [32] simulated the vibration response of tooth crack, pitting and wear through a dynamic model which only considered motions in the torsional and lateral y direction; vibrations in the x direction and friction forces were ignored. In practice, the appearance of pitting and spalling will directly result in a change in surface contact conditions [31, 105]. The friction coefficient and forces vary significantly when gear teeth contact in a pitting or spalling area, thus friction effects cannot be neglected if an understanding of the effects of tooth pitting or spalling is desired. Jiang et al. [66] simulated the vibration response of a gear system with a tooth spalling defect. The dynamic model used in their analysis assumed a constant friction coefficient between the mating teeth and ignored the gear tooth sharing ratio. The assumption of a constant friction coefficient failed to reflect the changes in tooth surface contact conditions within the fault area. More specifically, the contact of the tooth surfaces will transform the lubrication condition from elastohydrodynamic lubrication (where two surfaces in contact are fully separated by a lubrication film as in the healthy case) to mixed-elastohydrodynamic lubrication (where contact load is shared by both the lubrication film and the asperities under fault conditions) [106]. This observation implies that friction coefficients would not be expected to remain constant as tooth contact surfaces change and should be estimated dynamically [106]. Jia and Howard [33] compared the

differences between the vibration responses of tooth pitting and cracking through dynamic simulation. However, their dynamic model does not show how to model tooth contact changes within the pitting and spalling area.

2.3 Review of fault feature analysis by HCIs

Gear health condition indicators is a statistical method to measure the vibration features of a dynamic system. The investigation of HCIs for gear transmissions dates back to the early 1970s. Stewart [107] first proposed a zero-order figure of merit (FM0) used to detect peak-to-peak variations of a vibration signal, and introduced the fourth normalized statistical moment (FM4) to measure the vibration feature changes in the difference signal. Subsequently, Swansson [108] introduced the Crest Factor (CF) defined as the ratio of the maximum positive peak value and the RMS of the signal, and proposed the energy ratio (ER) expressed as the RMS of the difference signal divided by the RMS of the signal containing only the gear mesh frequency and its harmonics. Later, Martin [109] developed the sixth and eighth normalized statistical moments (M6A and M8A) of a vibration signal to detect gear surface damage, which is more sensitive than the Kurtosis (the fourth order normalized moment of the vibration signal) in measuring the peakedness of the signal. Around the same time, Szczepanik [110] introduced the Sideband Level Factor (SLF) to measure the amplitude and frequency modulations caused by gear mesh misalignment.

Soon after, Lyon [111] created the Instantaneous Frequency (IF) metric to measure the instantaneous phase change of a vibration signal caused by gear tooth surface defects or cracks. Zakrajsek et al. [8, 9] proposed the use of quasi-normalized statistical moments (NA4) of the residual signal and quasi-normalized statistical moments (NB4) of the envelope signal to detect the onset of damage. Soon after, NA4* was developed to improve the performance of NA4 by [112]. Then, Ma [10] proposed the Energy Operator (EOP) to

estimate the impulsive energy caused by a damaged gear tooth. Loughlin et al. [113] applied low-order spectral moments which considered the mean and standard deviation of the time series and the median and mode frequency to detect the changes of the vibration signal. Capdessus et al. [114] proposed a spectral correlation (coherence) density function to study the evolution of the frequency energy change due to a local tooth spall. Polyshchuk et al. [115] introduced the fourth order normalized power (NP4) to measure the change of instantaneous power distribution due to gear faults. Zhang et al. [116] and Loutridis [117] utilized a local energy density indicator to identify the impulse component excited by gear tooth crack. Decker and Lewicki [118] reported that ΔRMS , which is defined as the difference between two successive measured RMS, can be used to measure the energy change of the vibration signal under different faulty conditions.

Later, Lin et al. [119] proposed the fault-growth parameter (FGP) and the improved gear fault-growth parameter (FGP1). Zhan et al. [13] proposed an autoregressive model-based gear status indicator to estimate the fault-induced non-stationary features (the percentage of outliers) of a vibration signal under varying load conditions. Loutridis [120] proposed a Multiscale Local Statistics (MLS) approach used to estimate the severity of a gear tooth crack. McDonald et al. [121] introduced a maximum correlated kurtosis to monitor the periodic impulse-like vibration behaviour of a local gear tooth fault. Recently, Sharma and Parey [122] proposed the PS-I and PS-II based on the residual signal to evaluate the impulses and sidebands excited by a gear tooth crack.

Chapter 3 Gear mesh kinematic model

This chapter addresses objective 1, which is to propose a gear mesh kinematic model to evaluate the actual contact position of tooth engagement. Further discussions are included on the effect of center distance variation on the time varying mesh stiffness of a spur gear pair based on the proposed gear mesh kinematic model.

The contents of this chapter have been published in the journal *Engineering Failure Analysis*:

Y. Luo, N. Baddour, and M. Liang, "Effects of gear center distance variation on time varying mesh stiffness of a spur gear pair," *Engineering Failure Analysis*, vol. 75, pp. 37-53, 2017.

3.1 Abstract

Gear mesh kinematic models are generally utilized to evaluate the contact positions of tooth engagement with respect to gear rotational angle. The existing gear mesh kinematic models used to evaluate TVMS are generally based on the assumption that the gear pair is perfectly mounted and that all mesh points are at their theoretical positions. This assumption prevents these methods from modeling deviations in gear center distance. Therefore, this chapter proposes a new gear mesh kinematic model that can evaluate the actual contact positions and the time varying equivalent contact radii of tooth engagement. The proposed model enables the evaluation of the actual TVMS under conditions of perfect mounting, constant gear center distance deviation, and also time-varying gear center distance.

3.2 Introduction

Gear mesh kinematic models play an important role in analyzing the gear mesh contact positions of the mating process of a spur gear pair. With the gear mesh kinematic model, the actual gear tooth contact limits (the starting and separation point of contact), the double and single tooth pair duration and the equivalent contact radii, as well as relative sliding contact speed of the contacting surfaces can be obtained. These are important parameters for the dynamic simulation of the gear system. Existing gear mesh kinematic models mainly focus on evaluating the involute curves of the gear tooth and ignore the actual achievable contact regions due to tooth chamfer. The tooth chamfer directly affects the starting and separation point of tooth contact, which results in different effective contact durations (double or single tooth contact regions). Moreover, the common methods used to evaluate TVMS are generally based on the assumption that the gear pair is perfectly mounted and that all mesh points are at their theoretical positions. This assumption prevents these methods from modeling deviations in gear center distance.

This chapter proposes a new gear mesh kinematic model that can accurately calculate the contact positions of tooth engagement with respect to gear rotational angle under the condition of time varying gear center distance. The proposed kinematic model is validated by comparing with the results presented in [123]. With the proposed kinematic model, the actual TVMS of healthy gear teeth are computed under conditions of perfect mounting, constant center distance deviation, and time-varying center distance. The calculated results demonstrate that the proposed gear mesh kinematic model can be used to examine the effects of assembly errors, gear run-out errors, and shaft bending on the TVMS of spur gear pairs. These effects are important for the analysis of the vibration behavior of a gear transmission system with such flaws.

3.3 The proposed gear mesh kinematic model

3.3.1 Limits of Contact

The mating process of a spur gear pair, as shown in Fig. 3-1, keeps continuous power transmission from one gear to another. The tooth of the driving gear (pinion) contacts the mating tooth of the driven gear along the action line $\overline{B_1B_2}$, which is tangent to both pinion and gear base circles at points B_1 and B_2 , respectively. Based on the involute gear mesh properties, the limit contact positions of two mating teeth are governed by the radius of the outermost point (or the break radius) of teeth involute profiles (R_{op} and R_{og}) [124], and create the actual effective action line $\overline{P_1P_2}$, point P_1 corresponds to the starting point of contact, and P_2 corresponds to the separation point of the pair of mating teeth.

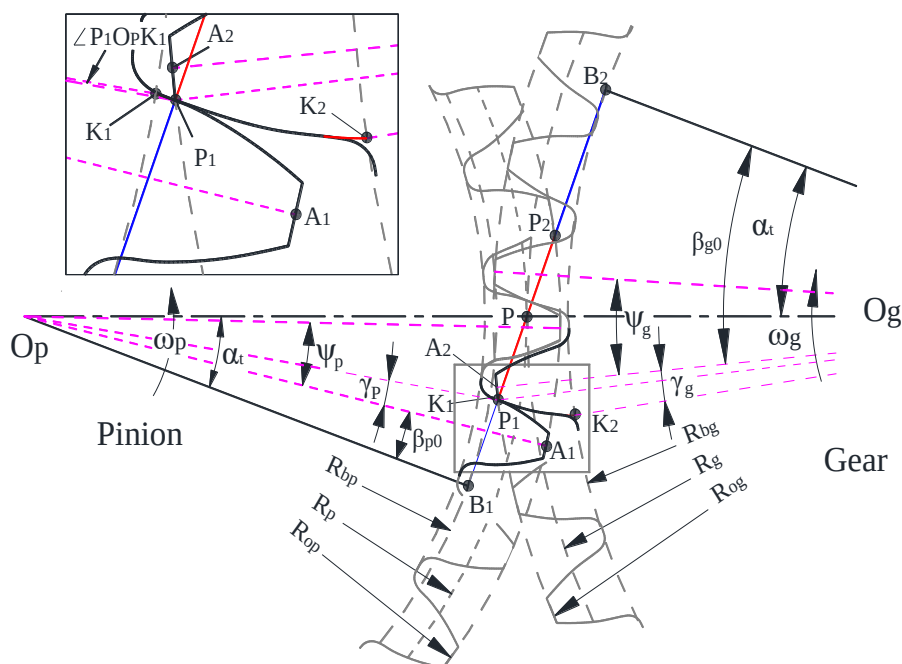


Fig. 3-1 Meshing process of a spur gear pair (initial position)

In Fig. 3-1, the target teeth (with A_1 and A_2 at the middle of each tooth tip) are just starting to engage at point P_1 . R_{bp} and R_p , R_{bg} and R_g are the radii of the base circle and pitch circle of the pinion and gear, respectively. Point P is the pitch point, O_p and O_g represent the geometric centers, K_1 and K_2 are the starting points of the tooth involute curves on the base circles of the pinion and gear, respectively. ω_p and ω_g are the rotational speeds of the pinion and gear. α_t is the actual pressure angle. When the gear pair is perfectly mounted, α_t equals to the theoretical pressure angle α_0 .

The actual center distance of a gear pair can be expressed as:

$$\overline{O_p O_g} = \frac{1}{2}(N_1 + N_2)m + \Delta\delta \quad (3.1)$$

in which m is tooth module, N_1, N_2 are the numbers of teeth for the pinion and gear, respectively. $(N_1 + N_2)m/2$ is the theoretical center distance of a perfectly mounted gear pair, and $\Delta\delta$ is the variation of the center distance. Hence when $\Delta\delta = 0$, the gear pair is perfectly mounted.

The actual pressure angle α_t can be expressed as:

$$\alpha_t = \arccos\left(\frac{1}{2} \frac{(N_1 + N_2)m}{O_p O_g} \cos(\alpha_0)\right) \quad (3.2)$$

Setting $\overline{O_p B_1}$ and $\overline{O_g B_2}$ as the reference lines, the angle of the center line of the target teeth (β_{p0} and β_{g0}) at the starting contact point P_1 can be expressed as:

$$\begin{cases} \beta_{p0} = \angle B_1 O_p P_1 - \gamma_p + \angle P_1 O_p K_1 \\ \beta_{g0} = \angle P_1 O_g B_2 - \angle A_2 O_g P_1 \end{cases} \quad (3.3)$$

where the angles are determined by the equations in Table 3-1.

Table 3-1 Reference equations for calculating the angles in Eq. (3.3)

$\overline{B_1 P} = R_{bp} \tan(\alpha_t)$	$\overline{B_2 P} = R_{bg} \tan(\alpha_t)$
$\overline{B_1 P_2} = \sqrt{R_{op}^2 - R_{bp}^2}$	$\overline{B_2 P_1} = \sqrt{R_{og}^2 - R_{bg}^2}$
$\angle O_p O_g P_1 = \arccos(R_{bg}/R_{og}) - \alpha_t$	$\overline{O_p P_1} = \sqrt{R_{og}^2 + \overline{O_p O_g}^2 - 2R_{og} \cdot \overline{O_p O_g} \cos(\angle O_p O_g P_1)}$
$\gamma_p = \angle K_1 O_p A_1 = \pi/2N_1 + \text{inv}(\alpha_0)$	$\text{inv}(\alpha_0) = \tan(\alpha_0) - \alpha_0$
$\angle B_1 O_p P_1 = \arccos(R_{bp}/\overline{O_p P_1})$	$\angle P_1 O_p K_1 = \tan(\angle B_1 O_p P_1) - \angle B_1 O_p P_1$
$\angle P_1 O_g B_2 = \arctan(\overline{B_2 P_1}/R_{bg})$	$\gamma_g = \angle K_2 O_g A_2 = \pi/2N_2 + \text{inv}(\alpha_0)$
$\angle K_2 O_g P_1 = \tan(\angle P_1 O_g B_2) - \angle P_1 O_g B_2$	$\angle A_2 O_g P_1 = \gamma_g - \angle K_2 O_g P_1$

The mating process at the separation position of the target teeth is explained in Fig. 3-2, where A_1', A_2', K_1', K_2' are A_1, A_2, K_1, K_2 when they rotate to the separation position.

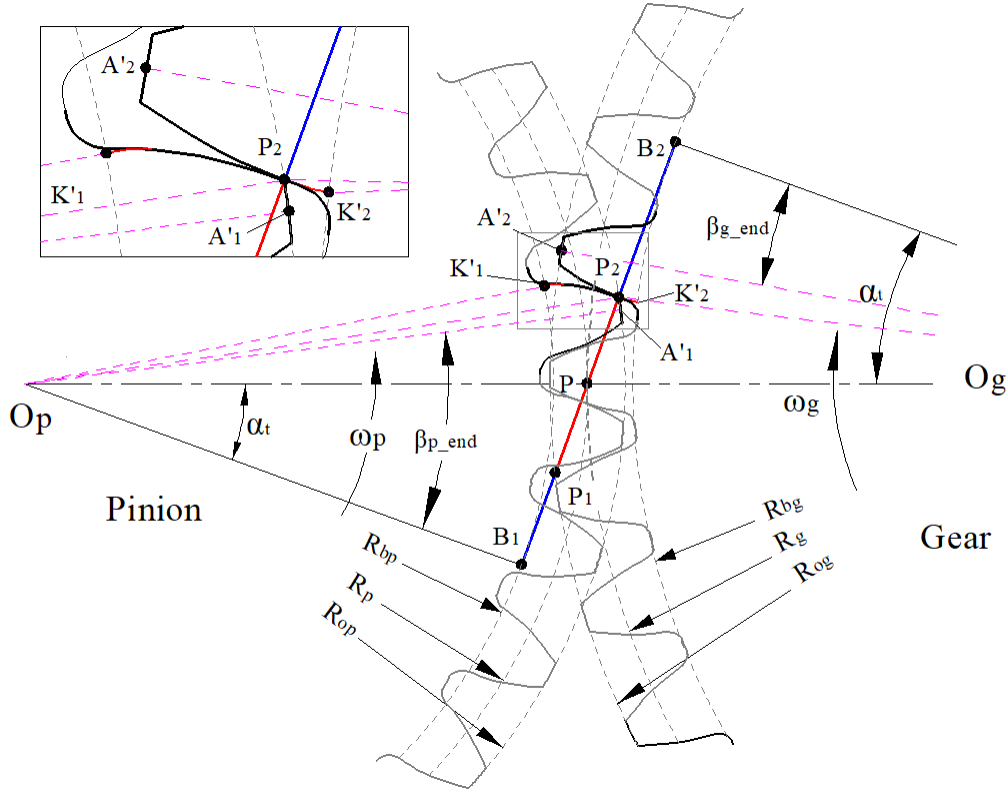


Fig. 3-2 Mating process of a spur gear pair (separation position)

Referring to $\overline{O_p B_1}$ and $\overline{O_g B_2}$ (in Fig. 3-2), the angle of the center line of the target teeth (β_{p_end} and β_{g_end}) at the separate point P_2 can be expressed as:

$$\begin{cases} \beta_{p_end} = \angle P_2 O_p B_1 - \angle P_2 O_p A'_1 \\ \beta_{g_end} = \angle P_2 O_g B_2 - \angle A'_2 O_g P_2 \end{cases} \quad (3.4)$$

The equations used to calculate the angles in Eq.(3.4) are listed in Table 3-2.

Table 3-2 Reference equations for calculating the angles in Eq.(3.4)

$\angle P_2 O_p B_1 = \arccos(R_{bp}/R_{op})$	$\angle P_2 O_p A'_1 = \gamma_p - \text{inv}(\angle P_2 O_p B_1)$
$\angle P_2 O_p O_g = \arccos(R_{bp}/R_{op}) - \alpha_t$	$\overline{P_2 O_g} = \sqrt{R_{op}^2 + \overline{O_p O_g}^2 - 2R_{op} \cdot \overline{O_p O_g} \cos(\angle P_2 O_p O_g)}$
$\angle P_2 O_g B_2 = \arccos(R_{bg}/\overline{P_2 O_g})$	$\angle A'_2 O_g P_2 = \gamma_g - \tan(\angle P_2 O_g B_2) + \angle P_2 O_g B_2$

3.3.2 Single and double tooth pair mesh durations

Based on the involute gear mating properties, the number of meshing tooth pairs is a function of gear/pinion rotation angle. As can be seen in Fig. 3-1 and Fig. 3-2, for a gear pair with contact ratio $1 < \xi < 2$, each mating tooth pair experiences the process of load pickup at its starting contact point P_1 from its preceding tooth pair, and load transfer to its succeeding tooth pair when it rotates an angle of $\psi_p = 2\pi / N_1$ (taking the driving pinion as an example). Setting the pinion rotation angle $\theta_p = 0$ when the target tooth pair starts contacting at point P_1 , the total angular displacement of the pinion target tooth in mesh can be calculated as:

$$\theta_{p_total} = \beta_{p_end} - \beta_{p0}, \quad (3.5)$$

The double and single tooth pair mesh durations within the total mesh angle of one tooth pair can be expressed as:

$$\begin{cases} \theta_{p_double} \in [0, \theta_{p_total} - \psi_p] \cup [\psi_p, \theta_{p_total}] \\ \theta_{p_single} \in [\theta_{p_total} - \psi_p, \psi_p] \end{cases} \quad (3.6)$$

where θ_{p_double} is the double tooth pair mesh duration, and θ_{p_single} is the single tooth pair mesh duration.

Based on Eqs.(3.1) to (3.5), the values of θ_{p_double} and θ_{p_single} are functions of the actual center distance $\overline{O_p O_g}$ and the radii of the outermost points of the involute profiles R_{op} and R_{og} . The center distance can be modified by assembly errors (installation problems), manufacturing errors (the gears are perfect but there often exist manufacturing errors of the distance of bearing housings), gear run-out errors (eccentricity error), shaft bending or simply inadequate bearing stiffness. Different types of faults exhibit different fault characteristics. For example, the assembly and

manufacturing errors normally lead to a constant center distance error, while eccentricity error causes time-varying center distance deviations. Moreover, the center distance variations caused by shaft bending or inadequate bearing stiffness are load-related faults, which mainly occur when the system is heavily loaded or a flaw appears on shaft or bearings. Obviously, the values of R_{op} , R_{og} directly affect the teeth mesh starting and ending positions. In some previous studies such as those reported in [125] and [56], the radius of the addendum circle (outer most tooth tip) was used to model the healthy and/or cracked tooth. However, for most of the gears, especially those designed for heavy load transmission, their tooth tips incorporate chamfers or rounded edge breaks [124] as shown in Fig. 3-3, which reduces the length of the actual contact line, and therefore changes the values of θ_{p_double} and θ_{p_single} .

3.3.3 Geometric model of the spur gear profile

The tooth profile of the spur gear can be divided into transition curve AB, involute curve BC (non-working area), involute curve CD (working area), and tip chamfer DE. As shown in Fig. 3-3, xoy is the coordinate system of the spur gear tooth, O is the geometric center of the gear, x is the axis along the horizontal direction, y is the axis perpendicular to x . Positions x_A , x_B , x_C , x_{Pm} correspond to the abscissas of points A, B, C, and current tooth meshing point P_m , respectively. Variables x and y are used to define the position of an arbitrary point on tooth profile. τ is the pressure angle of an arbitrary point, α_{pm} is the operating pressure angle, and θ is the current angular displacement of the tooth with respect to its initial position β_0 .

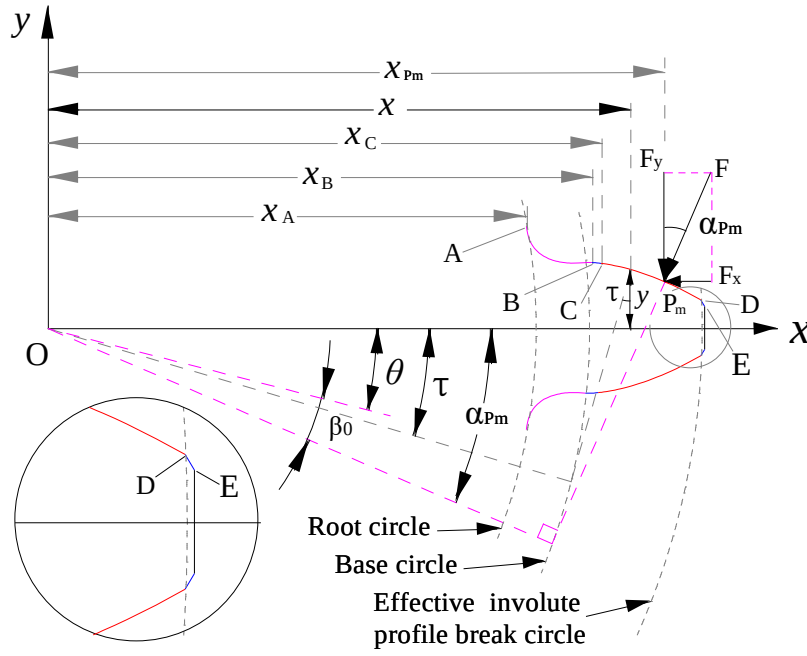


Fig. 3-3 Geometric model of the spur gear profile [56]

The transition curve AB is formed by the trajectory of the cutter tip during manufacturing, and depends on the shape of the cutting tool [57, 126]. Ma et al. [56] used the following equations to depict the transition curve when the cutter tip is ordinary fillet:

$$\begin{cases} x_{tr} = R_p \cos(\phi) - \left(\frac{A}{\sin(\gamma)} + R_{tr} \right) \sin(\gamma - \phi) \\ y_{tr} = R_p \sin(\phi) - \left(\frac{A}{\sin(\gamma)} + R_{tr} \right) \cos(\gamma - \phi) \end{cases}, (\alpha_0 \leq \gamma \leq \pi/2), \quad (3.7)$$

where x_{tr} and y_{tr} , which are functions of γ , define the position of an arbitrary point on the transition curve (see Fig. 3-3), R_p is the radius of pitch circle, $R_{tr} = c^* m / (1 - \sin(\alpha_0))$,

$A = (h^* + c^*) \times m - R_{tr}$, $\phi = (A / \tan(\gamma) + B) / R_p$, in which $B = \pi m / 4 + h_a^* m \tan(\alpha_0) + R_{tr} \cos(\alpha_0)$.

For standard spur gears $h^* = 1$, $c^* = 0.25$, and $\alpha_0 = 20^\circ$.

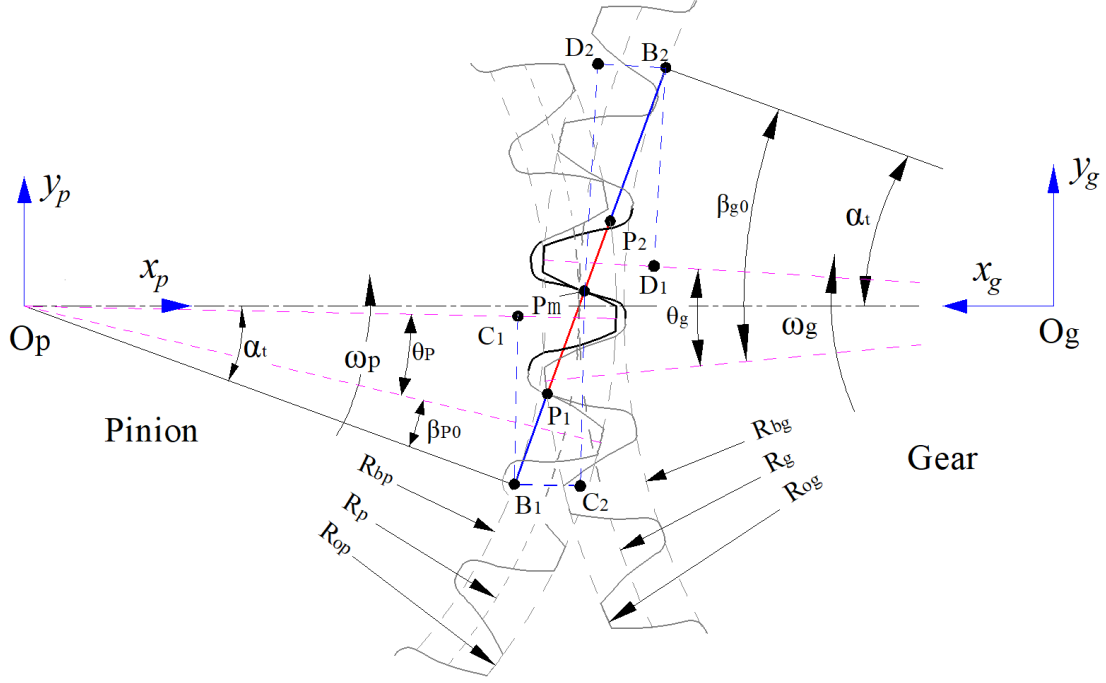


Fig. 3-4 Mating process of a spur gear pair (arbitrary contact point)

Fig. 3-4 describes the mating process of a spur gear pair in an arbitrary contact point. The position of the current mesh point P_m described respectively in the pinion and gear coordinate systems can be expressed as:

$$\begin{cases} x_p = \overline{O_p C_1} + \overline{B_1 C_2} \\ \quad = R_{bp} \cos(\beta_{p0} + \theta_p) + R_{bp} (\beta_{p0} + \theta_p + \gamma_p) \sin(\beta_{p0} + \theta_p) \\ y_p = \overline{P_m C_2} - \overline{B_1 C_1} \\ \quad = R_{bp} (\beta_{p0} + \theta_p + \gamma_p) \cos(\beta_{p0} + \theta_p) - R_{bp} \sin(\beta_{p0} + \theta_p) \end{cases} \quad (3.8)$$

and

$$\begin{cases} x_g = \overline{O_g D_1} + \overline{B_2 D_2} \\ \quad = R_{bg} \cos(\beta_{g0} - \theta_g) + R_{bg} (\beta_{g0} - \theta_g + \gamma_g) \sin(\beta_{g0} - \theta_g) \\ y_g = \overline{P_m D_2} - \overline{B_2 D_1} \\ \quad = R_{bg} (\beta_{g0} - \theta_g + \gamma_g) \cos(\beta_{g0} - \theta_g) - R_{bg} \sin(\beta_{g0} - \theta_g) \end{cases}, \quad (3.9)$$

where x_p and y_p , x_g and y_g are the coordinates of contact point P_m relative to pinion and gear, respectively. θ_p and θ_g are the angular displacement of pinion and gear relative to their initial positions, respectively.

3.4 Potential energy method in TVMS calculation

Based on beam theory, the total energy, U_{total} , stored in a meshing gear pair is the sum of axial compressive energy U_a , bending energy U_b , shear energy U_s , fillet foundation energy U_f , and Hertzian energy U_h , which can be expressed as[56]:

$$\begin{aligned} U_{total} &= U_b + U_s + U_a + U_f + U_h \\ &= \frac{F^2}{2} \left(\frac{1}{k_b} + \frac{1}{k_s} + \frac{1}{k_a} + \frac{1}{k_f} + \frac{1}{k_h} \right) \end{aligned} \quad (3.10)$$

where

$$\begin{cases} U_a = \frac{F^2}{2k_a} = \int_{x_A}^{x_B} \frac{F_x^2}{2EA_{x1}} dx_1 + \int_{x_B}^{x_{pm}} \frac{F_x^2}{2EA_{x2}} dx_2 \\ U_b = \frac{F^2}{2k_b} = \int_{x_A}^{x_B} \frac{M_1^2}{2EI_{y1}} dx_1 + \int_{x_B}^{x_{pm}} \frac{M_2^2}{2EI_{y2}} dx_2 \\ U_s = \frac{F^2}{2k_s} = \int_{x_A}^{x_B} \frac{1.2F_y^2}{2GA_{x1}} dx_1 + \int_{x_B}^{x_{pm}} \frac{1.2F_y^2}{2GA_{x2}} dx_2 \\ U_f = \frac{F^2}{2k_f}, \quad U_h = \frac{F^2}{2k_h} \end{cases} \quad (3.11)$$

Here, F is the teeth contact force, $F_x = F \sin(\alpha_{pm})$, $F_y = F \cos(\alpha_{pm})$, $x_1 = x_{tr}$, $x_2 = x_p$ for the pinion and $x_2 = x_g$ for the gear (see Eqs. (3.7), (3.8) and (3.9)). $M_1 = F_y(x_{pm} - x_1) - F_x y_{pm}$, $M_2 = F_y(x_{pm} - x_2) - F_x y_{pm}$. A_{x1} and A_{x2} , I_{y1} and I_{y2} are the cross-sectional areas and area moments of inertia, respectively. Stiffnesses k_a , k_s , k_b , k_f , and k_h are respectively tooth axial compressive

stiffness, shear stiffness, bending stiffness, fillet foundation stiffness, and Hertzian contact stiffness, and can be determined using the following equations [56]:

$$\frac{1}{k_a} = \int_{\frac{\pi}{2}}^{\alpha_0} \frac{\sin^2 \alpha_{pm}}{EA_{x1}} \frac{dx_1}{d\gamma} d\gamma + \int_{\alpha_B}^{\alpha_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_{x2}} \frac{dx_2}{d\tau} d\tau \quad (3.12)$$

$$\begin{aligned} \frac{1}{k_b} = & \int_{\frac{\pi}{2}}^{\alpha_0} \frac{\left(\cos \alpha_{pm} (x_{pm} - x_1) - y_{pm} \sin^2 \alpha_{pm} \right)^2}{EI_{x1}} \frac{dx_1}{d\gamma} d\gamma \\ & + \int_{\alpha_B}^{\alpha_{pm}} \frac{\left(\cos \alpha_{pm} (x_{pm} - x_2) - y_{pm} \sin^2 \alpha_{pm} \right)^2}{EI_{x2}} \frac{dx_2}{d\tau} d\tau \end{aligned} \quad (3.13)$$

$$\frac{1}{k_s} = \int_{\frac{\pi}{2}}^{\alpha_0} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{x1}} \frac{dx_1}{d\gamma} d\gamma + \int_{\alpha_B}^{\alpha_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{x2}} \frac{dx_2}{d\tau} d\tau \quad (3.14)$$

$$\frac{1}{k_f} = \frac{\cos^2 \alpha_{pm}}{EW} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_{pm}) \right\} \quad (3.15)$$

$$k_h = \frac{\pi EL}{4(1-\nu^2)} \quad (3.16)$$

where $\tau = \theta_p$ in the pinion tooth, $\tau = \theta_g$ in the gear tooth. G , E , W and ν represent shear modulus, Young's modulus, tooth width and Poisson's ratio, respectively. u_f , S_f , L^* , M^* , P^* and Q^* are parameters utilized to calculate the fillet foundation stiffness, details are given in [127]. α_B is the pressure angle at the start point of involute curve which can be expressed as:

$$\alpha_B = \arccos \left(\frac{R_{bp}}{\sqrt{\left(R_{bp} \tan(\alpha_0) - h_a^* m / \sin(\alpha_0) \right)^2 + R_{bp}^2}} \right) \quad (3.17)$$

The total energy stored in a gear pair can therefore be expressed as[56]:

$$\begin{aligned}
 U_{total} &= \frac{F^2}{2k_{total}} = U_h + U_{b1} + U_{s1} + U_{a1} + U_{f1} + U_{b2} + U_{s2} + U_{a2} + U_{f2} \\
 &= \frac{F^2}{2} \left(\frac{1}{k_h} + \frac{1}{k_{b1}} + \frac{1}{k_{s1}} + \frac{1}{k_{a1}} + \frac{1}{k_{f1}} + \frac{1}{k_{b2}} + \frac{1}{k_{s2}} + \frac{1}{k_{a2}} + \frac{1}{k_{f2}} \right)
 \end{aligned} \tag{3.18}$$

Accordingly, the total mesh stiffness of a gear pair can be expressed as:

$$k = \frac{1}{\frac{1}{k_h} + \frac{1}{k_{b1}} + \frac{1}{k_{s1}} + \frac{1}{k_{a1}} + \frac{1}{k_{f1}} + \frac{1}{k_{b2}} + \frac{1}{k_{s2}} + \frac{1}{k_{a2}} + \frac{1}{k_{f2}}} \tag{3.19}$$

where the subscript 1, 2 correspond to the pinion and gear respectively.

3.5 Evaluation of gear TVMS with the proposed kinematic model

3.5.1 TVMS evaluation with constant gear center distance error

Constant center distance errors of a gear system can be caused by manufacturing errors of bearing housings, assembly errors, shaft bending, and bearing deformation etc., which lead to center deviations from the theoretical value.

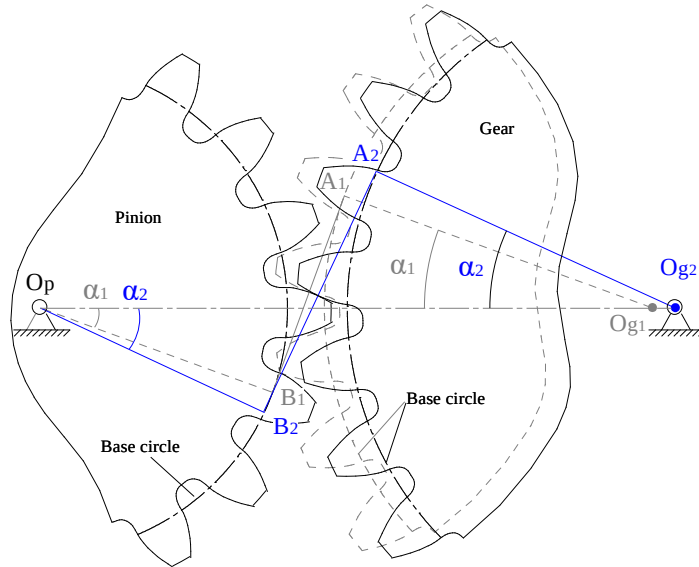


Fig. 3-5 The effect of constant center distance deviation on gear mesh process

The constant center distance variation has an effect on the gear contact ratio and pressure angle. However, the gear ratio will remain unchanged. The extended center distance reduces the actual line of action during gear meshing, which causes earlier separation and delayed engagement of the gear pair compared with their theoretical timings. Another effect of center distance deviation is the change of the actual tooth working area. For example, if the center distance is increased, the meshing teeth will lose some working areas near the tooth root. Accordingly, the contact points and hence the force will move up towards the tooth tip, and the bending stiffness will be reduced. The following simulation examines four constant center distance deviation cases, $\delta = -0.2, 0, 0.2, 0.4$ (mm), where a negative value, $\delta = -0.2$ (mm), indicates a reduction in center distance.

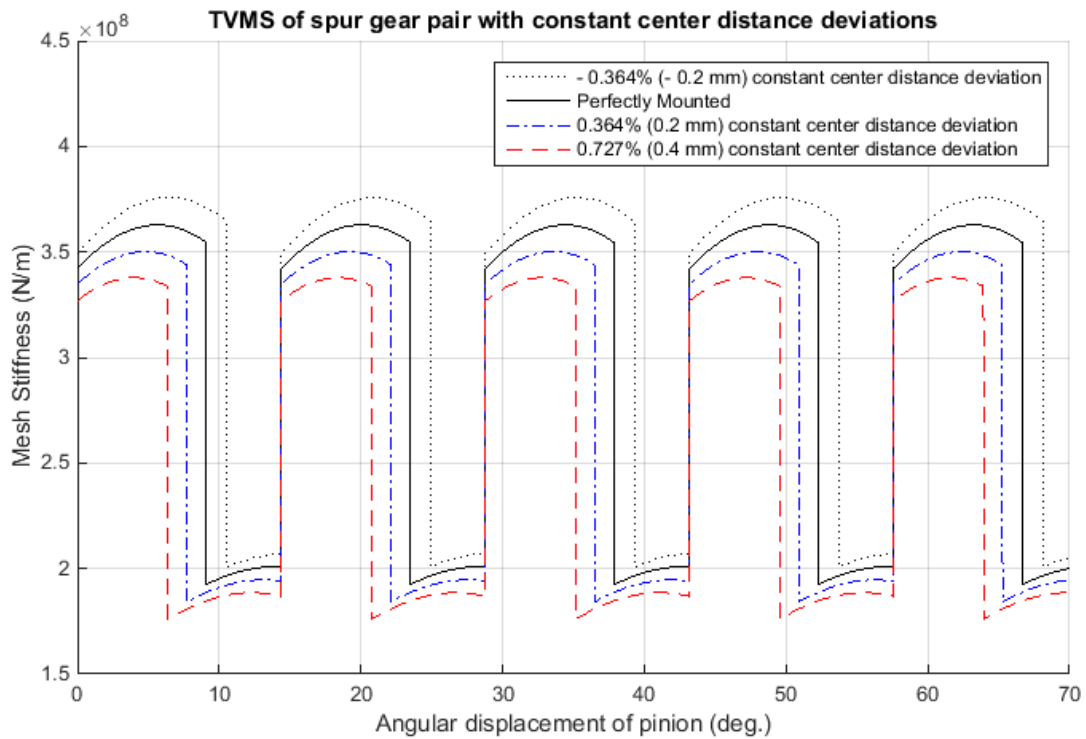


Fig. 3-6 Simulated comparison of TVMS of perfectly mounted gear pair and gear pairs subjected to constant center distance variations

In Fig. 3-6, the center distance deviations are transformed to a percentage through $(\Delta\delta / \text{theoretical center distance}) \times 100\%$. It can be observed that with the increase in center distance, the TVMS performs a mesh stiffness and contact ratio (or effective working area) decrease compared to shorter center distances. It should be noted that the constant center distance variation leads to both phase modulation and amplitude modulation effects on the mesh stiffness. The phase modulation is reflected by the contact position deviations from its corresponding healthy case, and the amplitude modulation is revealed by the mesh stiffness value changes from its corresponding healthy case.

3.5.2 TVMS with eccentric error

Gear eccentric errors are mainly due to inaccurate gear manufacturing processes that cause inconsistency between the geometric center and gear rotation center. As shown in Fig. 3-7, the gear run-out has a constant value of $\Delta\delta$, and the pinion is assumed to be perfect. Points O_p and O_g are, respectively, the rotational centers of the pinion and gear, O_{g1} is the actual gear geometrical center, O_{g1}' is the projection of O_{g1} onto x-axis. $\Delta\delta_x = \Delta\delta \cos(\theta)$ and $\Delta\delta_y = \Delta\delta \sin(\theta)$, in which θ is the angular displacement of the gear.

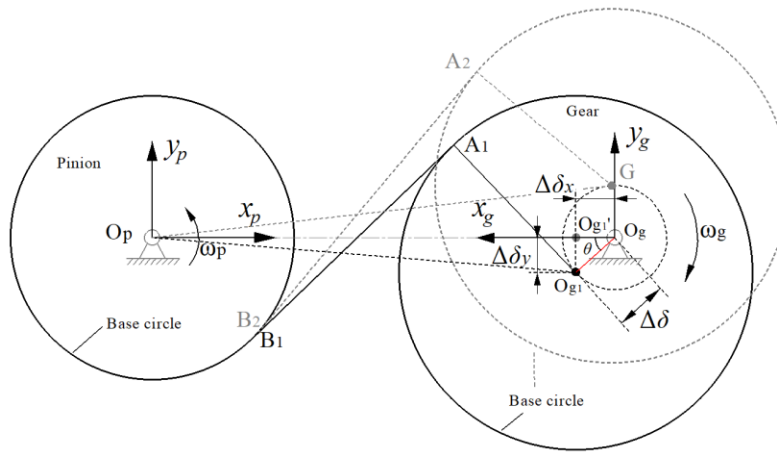


Fig. 3-7 Gear position changes due to run out

The run out error $\Delta\delta$ leads to a time varying gear mesh center distance $\overline{O_p O_{g1}}$ while operating, which can be expressed as:

$$\overline{O_p O_{g1}} = \sqrt{\left(\overline{O_p O_g} - \Delta\delta \cos(\theta)\right)^2 + \left(\Delta\delta \sin(\theta)\right)^2} \quad (3.20)$$

The mating process of a spur gear pair with run-out error on the driven gear is shown in Fig. 3-8. Here, α_1 and α_2 , β_1 and β_2 are the pressure angles and initial angles of starting contact points (see Eq.(3.3)) respectively correspond to the gear geometric center positions O_{g1} and O_{g2} .

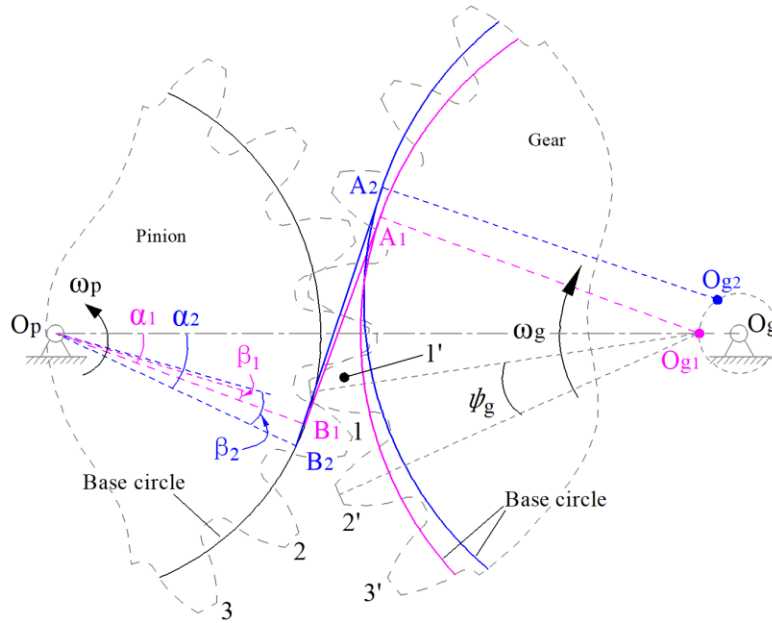


Fig. 3-8 Gear mating process with run-out error

As can be seen in Fig. 3-8, assuming that the gear pair 1-1' is just starting to engage corresponds to initial angle β_1 . If there were no gear run-out errors, the following gear pair 2-2' would start engaging when gear pair 1-1' rotates an angle of $\psi_p = 2\pi / N_1$ corresponding to the same initial angle β_1 . However, because of gear run-out error, the initial contact angle of gear pair

2-2' is actually β_2 , therefore, the actual phase difference between two successive gear pair meshing will be:

$$\theta_{diff} = 2\pi / N_1 + (\beta_2 - \beta_1) \quad (3.21)$$

where β_1 and β_2 are two status of β_{p0} given in Eq. (3.3).

An example TVMS of a spur gear with $\Delta\delta = 0.2\text{mm}$ ($=0.71\%$ calculated by $a\% = \Delta\delta / R_{bg} \times 100\%$) run-out error on the driven gear for one cycle is shown in Fig. 3-9 and Fig. 3-10.

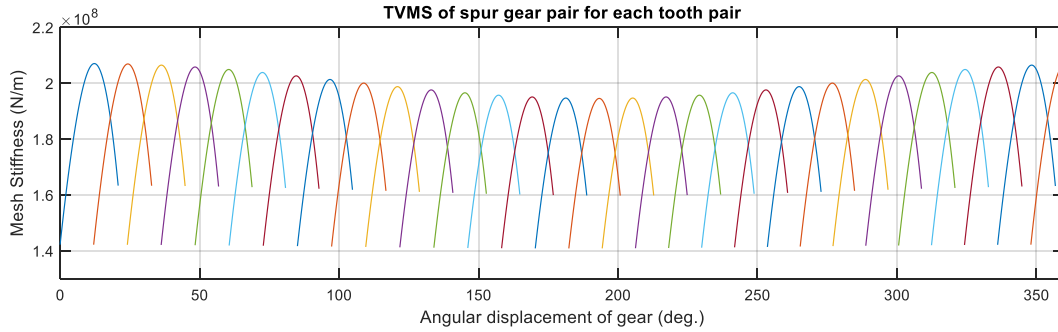


Fig. 3-9 TVMS of spur gear pair for each tooth pair

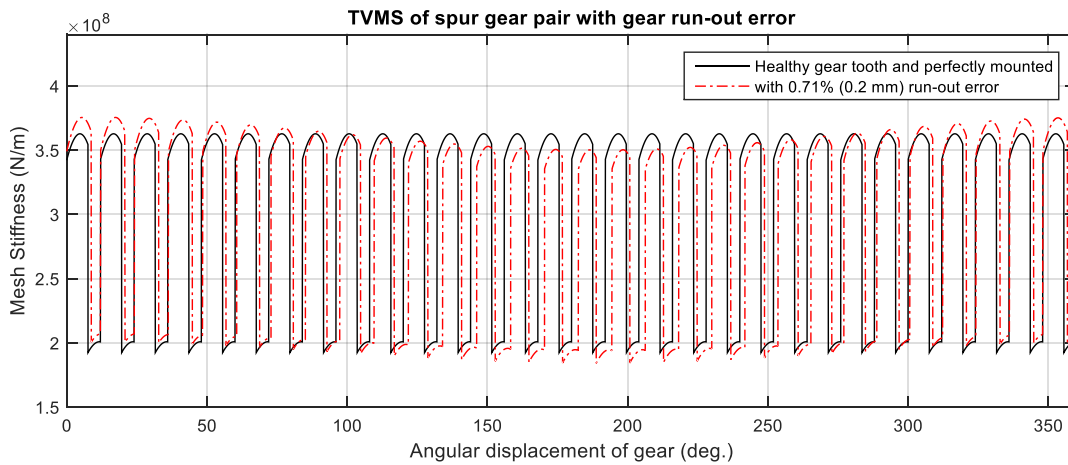


Fig. 3-10 Comparison of TVMS of the perfectly mounted spur gear pair and a spur gear pair with gear a run-out error

In this simulation, it is assumed that the gear meshing process starts at the smallest gear center distance point (point O_{g1} in Fig. 3-8). The 0.2mm run-out error causes a time-varying change in gear center distance that ranges between -0.2 and 0.2 mm (-0.36% and 0.36%) for each cycle. The mesh stiffness then varies from the higher values to the lower values and finally back to the higher values during one cycle, as observed in Fig. 3-10.

From Fig. 3-10, it can be seen that the run-out error leads to both time-varying frequency modulation and amplitude modulation effects on the mesh stiffness. The time varying frequency modulation is caused by the time varying single and double tooth mesh duration changes over one cycle, e.g. shorter center distance corresponds to longer double tooth pair mesh duration and shorter single tooth pair mesh duration, and vice versa. For the time-varying amplitude modulation, the peak values of TVMS due to the runout error vary over one gear revolution.

3.6 Conclusion

In this chapter, a new gear mesh kinematic model was proposed to evaluate the contact positions of tooth engagement with time-varying gear mesh center distance. This model considered the actual gear center distance, pressure angle and the actual working areas of teeth involute profiles of a mating spur gear pair. The advantages of the proposed model are: a) it can evaluate the actual teeth mesh contact positions with either constant or time varying gear center distance; b) with the precise teeth meshing position obtained, the actual TVMS of a mating spur gear pairs can be calculated; c) this permits the modelling and evaluation of the effects of assembly errors, gear run-out errors, shaft bending, or bearing deformation (alone or combined) on TVMS.

The TVMS simulation results for constant center distance variation indicate a stable and significant mesh stiffness and contact ratio changes, while the time-varying center distance variation shows both time varying phase and amplitude modulation effects on the mesh stiffness.

The evaluated actual TVMS in this chapter can be further used in gear system dynamic simulations, through which the dynamic behavior and vibration signal of a gear system (with single or multiple faults) can be obtained. This ability to model these effects, as enabled by the proposed model in this chapter, is important in gear fault detection and diagnosis.

Chapter 4 Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features

This chapter addresses the objective 2, which is to propose a curved-bottom shaped tooth spall to best fit the shape of the spalling geometric features as observed in practice.

The contents of this chapter have been published in the journal *Engineering Failure Analysis*.

Y. Luo, N. Baddour, G. Han, F. Jiang, and M. Liang, "Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features," *Engineering Failure Analysis*, vol. 92, pp. 430-442, 2018. Special thanks to the co-authors (Guosheng Han and Fei Jiang) for providing suggestions and assistance on the finite element analysis of this paper.

Another proposed method (not included in this thesis) which models gear tooth spalls in spherical shapes to address the objective 2 has also been published in the *2018 Prognostics and System Health Management International Conference, Chongqing, China*.

Y. Luo, N. Baddour and M. Liang, "Evaluation of Time-Varying Mesh Stiffness of Gears with Tooth Spalls Modeled as Spherical Shapes," *2018 Prognostics and System Health Management Conference (PHM-Chongqing)*, Chongqing, pp. 94-99, 2018.

4.1 Abstract

This chapter proposes a curved-bottom shaped tooth spall to model the tooth spalling geometric features as observed in practice. The proposed method is constructed based on an ellipsoid geometry which is capable of varying radii in three dimensions to best fit the shape of the tooth spall. The foundation stiffness within the double tooth contact area in the proposed method is corrected and the non-linearity of Hertzian contact stiffness is considered. The effectiveness of the proposed method on modelling single and multiple tooth spalls with different shapes and severity conditions is then validated.

4.2 Introduction

The existing geometric-shape based methods assume that the tooth spall has a flat bottom, and the severity of the fault is generally controlled by the length l_s , width w_s , depth h_s and/or radius of the indentation, see Fig. 4-1.

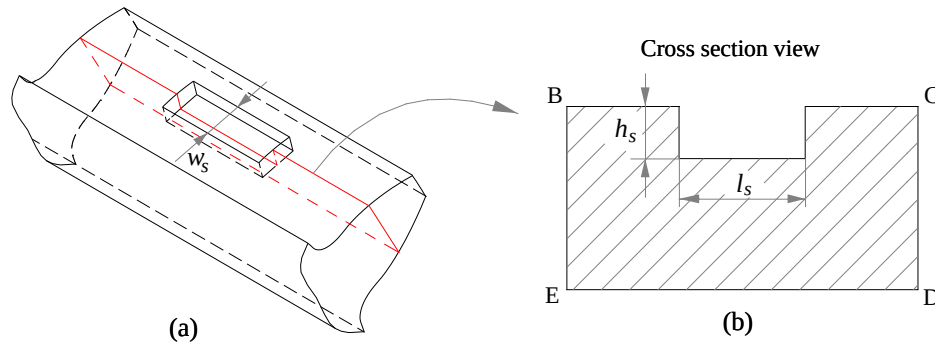


Fig. 4-1 Gear tooth spall modeled by rectangular shape

As shown in Fig. 4-2, in practice, the flake of the surface material usually possesses a curve-shaped bottom with a gradually changed depth instead of a suddenly decreased tooth thickness. Thus, the modelling of tooth spalls with a flat bottom rather than a curved bottom may result in mesh stiffness differences, especially for some sub-stiffness components (e.g. bending stiffness,

shear stiffness, etc.) which are sensitive to the variation of the gear tooth cross-sectional area and area moment of inertia.

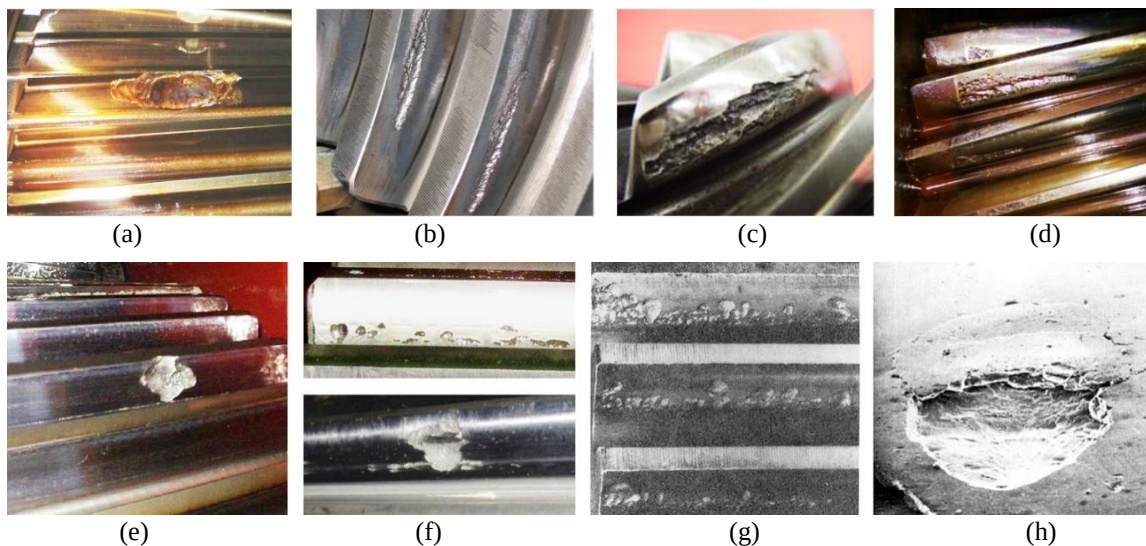


Fig. 4-2 Gear tooth spalling in practice [128-134]

In view of the above, this chapter proposes a curved bottom shaped method to model gear tooth spalling that better represents the spall geometries observed in practice. The curve bottom shaped method is constructed by removing the intersection between the ellipsoid and gear tooth. The new proposed spalling geometry generates a gradually changing depth and is capable of stretching and shrinking in size in different dimensions, thus better approximating the geometric features of tooth spall observed in practice. The effectiveness of the proposed method on modelling practical gear tooth spalls under different shapes and severities is then validated.

4.3 The proposed ellipsoid shape-based method on modelling gear tooth spalls

4.3.1 The geometric model of ellipsoid tooth spall

Practical gear tooth spalls in many cases have gradually changing depths and curved bottom surfaces. This characteristic can be approximated by part of an ellipsoid. A simplified model of a

gear tooth spall with an ellipsoid shape is shown in Fig. 4-3 and Fig. 4-4. The construction of the ellipsoid tooth spall was formed by removing the intersection between the gear and ellipsoid. Fig. 4-3 (b) is the x - y cross-section view passing through the geometrical center of the ellipsoid. The geometry information of the k -th ellipsoid spalling can be defined by its maximum width w_{sk_max} , maximum length l_{sk_max} , maximum center depth h_{sk_max} , and its starting position x_{sk_start} and the incline angle θ_{sk} which is the angle of the tangent line of the tooth spall to the involute curve with respect to the x axis, see Fig. 4-3.

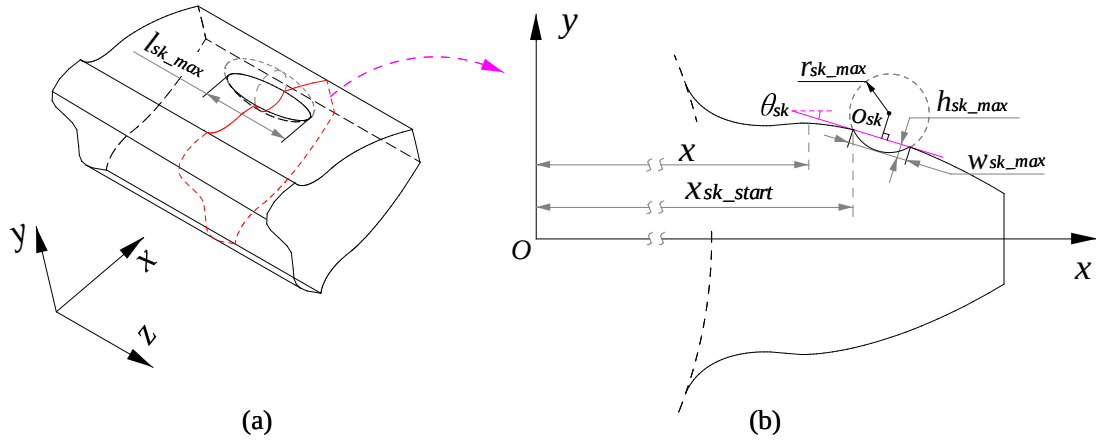


Fig. 4-3 Ellipsoid tooth spalling and a cross-section view passing through the center of the ellipsoid

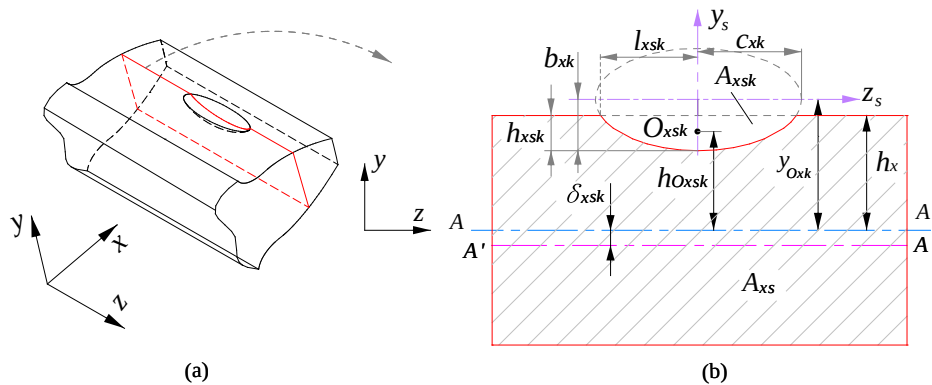


Fig. 4-4 Cross-section model of the proposed tooth spall

The parametric function of an ellipsoid is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (4.1)$$

where a, b, c , are the radii of the ellipsoid in the x, y, z directions. If any two of the radii are equal, the cross-section of the ellipsoid (in the plane of those two radii) forms a circle. Fig. 4-3 shows the proposed ellipsoid spalling model with radii $a=b$. For an ellipsoid tooth spall in practice, the position (x_{sk_start} and θ_{sk}) and the severity (w_{sk_max} , l_{sk_max} and h_{sk_max}) can be directly measured or estimated. With knowledge of these position and geometrical parameters, the radii of the ellipsoid in the x and y axis can be defined as

$$a = b = r_{sk_max} = \frac{\left(0.5w_{sk_max}\right)^2 + h_{sk_max}^2}{2h_{sk_max}}. \quad (4.2)$$

The coordinates of the center of the ellipsoid in the x - y plane can be evaluated from

$$\begin{cases} x_{Osk} = x_{sk_start} + r_{sk_max} \cos \left[\frac{\pi}{2} - \theta_{sk} - \arcsin \left(\frac{0.5w_{sk_max}}{r_{sk}} \right) \right] \\ y_{Osk} = -\tan(\theta_{sk}) (x_{Osk} - x_{sk_start}) + y(x_{sk_start}) + \frac{r_{sk_max} - h_{sk_max}}{\cos \theta_{sk}}, \quad (h_{sk_max} < r_{sk_max}) \end{cases}, \quad (4.3)$$

where $y(x_{sk_start})$ is the height of the point on the tooth involute curve corresponding to the abscissa x_{sk_start} . The radius of the ellipsoid in the z axis can be expressed as

$$c = \frac{1}{2} \sqrt{\frac{r_{sk_max}^2 l_{sk_max}^2}{2r_{sk_max} h_{sk_max} - h_{sk_max}^2}}. \quad (4.4)$$

The cross-section area of the k^{th} tooth spall at displacement x (when x is within the spalling area) is constructed by a rectangular cut by an ellipse, as is shown in Fig. 4-4 (b), the function of the ellipse can be written as

$$\frac{y^2}{b_{xk}^2} + \frac{z^2}{c_{xk}^2} = 1, \quad (4.5)$$

with $b_{xk} = \sqrt{b^2 \left(1 - \frac{(x_{Osk} - x_t)^2}{a^2} \right)}$ and $c_{xk} = \sqrt{c^2 \left(1 - \frac{(x_{Osk} - x_t)^2}{a^2} \right)}$, where a , b and c are radii of the

ellipsoid evaluated by Eq. (4.2) and Eq.(4.4). The length l_{xsk} can be evaluated by

$$l_{xsk} = \sqrt{c_{xk}^2 \left(1 - (y_{Osk} - h_x)^2 / b_{xk}^2 \right)}, \quad (4.6)$$

where h_x is the half height of the gear tooth cross-section at displacement x , see Fig. 4-5. The effective gear tooth contact length can be expressed as

$$L_e = L - \sum_{k=1}^K 2l_{xsk}, \quad (4.7)$$

where L is the width of the gear tooth. The maximum depth of the tooth spall at displacement x of the k^{th} tooth spall can be estimated by

$$h_{xsk} = b_{xk} - (y_{Osk} - h_x). \quad (4.8)$$

The corresponding area of the ellipse segment can be written as

$$A_{xsk} = 2 \int_{-b_{xk}}^{-(y_{Osk} - h_x)} \sqrt{\left(1 - \frac{y_s^2}{b_{xk}^2} \right)} c_{xk}^2 dy_s, \quad (4.9)$$

or approximated by

$$A_{xsk} = b_{xk} c_{xk} \left(\arccos \left(\frac{b_{xk} - h_{xsk}}{b_{xk}} \right) - \left(\frac{b_{xk} - h_{xsk}}{b_{xk}} \right) \sqrt{\frac{2h_{xsk}}{b_{xk}} - \frac{h_{xsk}^2}{b_{xk}^2}} \right). \quad (4.10)$$

The actual value of the cross-section area A_{xs} with multiple tooth spalls at displacement x can be expressed as

$$A_{xs} = 2h_x L - \sum_{k=1}^K A_{xsk}. \quad (4.11)$$

The modulation of A_{xs} will result in a change of the center line (neutral axis) of the cross-section, e.g. moving from AA to $A'A'$. The corresponding displacement δ_{xs} between these two center lines can be evaluated by

$$\delta_{xs} = \frac{\sum_{k=1}^K A_{xsk} h_{Oxsk}}{A_{xs}}, \quad (4.12)$$

with

$$h_{Oxsk} = y_{Osk} - abs \left(\frac{2}{A_{xsk}} \int_{-b_{xk}}^{-(y_{Osk} - h_x)} y_s \sqrt{\left(1 - \frac{y_s^2}{b_{xk}^2} \right)} c_{xk}^2 dy_s \right), \quad (4.13)$$

where y_s is the distance along the vertical axis. The $abs()$ is the absolute operation. The area moment of inertia of the ellipse segment (the removed area) of the k^{th} spall, see Fig. 4-4 (b), can be expressed as

$$I_{zsk} = \int_{-b_{xk}}^{-(y_{Osk} - h_x)} 2y_s^2 \sqrt{\left(1 - \frac{y_s^2}{b_{xk}^2} \right)} c_{xk}^2 dy_s - A_{xsk} (y_{Osk} - h_{Oxsk})^2, \quad (h_x \leq y_{Osk}). \quad (4.14)$$

The area moment of inertia of the gear tooth cross-section with respect to the neutral axis $A'A'$ under tooth spalling condition can be evaluated from

$$I_{zs} = \frac{2Lh_x^3}{3} + 2Lh_x\delta_{xsk}^2 - \sum_{k=1}^K \left(I_{zsk} + A_{xsk} (h_{Oxsk} + \delta_{xsk})^2 \right). \quad (4.15)$$

4.3.2 Evaluation of the TVMS of a spur gear pair with ellipsoid tooth spall

Fig. 4-5 shows the geometric model of the spur gear tooth profile, where O is the gear geometrical center, x is the axis consistent with the middle line of the gear tooth, and the y axis is perpendicular to the x axis. AB and BC denote the gear transition curve and involute curve, respectively. P_m is the current tooth meshing point corresponding to a contact pressure angle α_{pm} . The variables x_A , x_B , and x_{pm} are respectively the abscissas of points A , B , and P_m . The gear mesh force F is resolved into F_x and F_y along the x and y directions, respectively. Variables x and h_x represent the coordinates of an arbitrary point and τ is the corresponding pressure angle. θ is the angular displacement of the tooth.

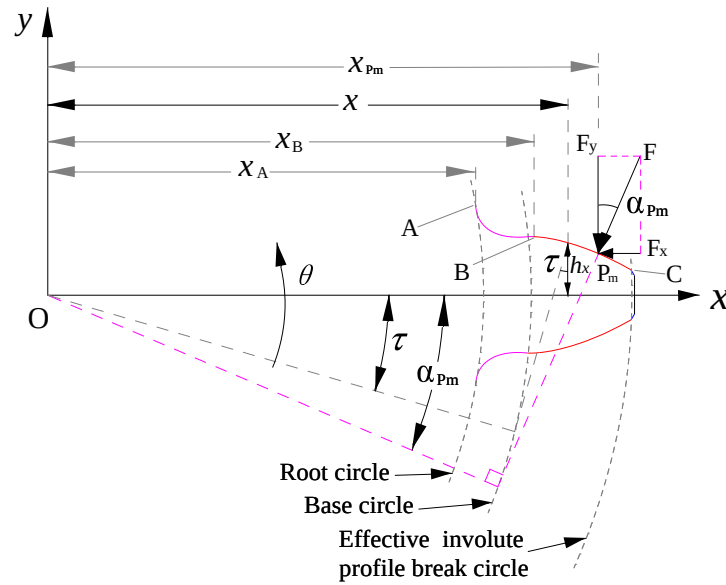


Fig. 4-5 Geometric model of the spur gear profile[56]

Based on the potential energy method, the total energy in a meshing gear pair includes both of the energy stored in the pinion and gear, which can be expressed as

$$\begin{aligned}
 U_{total} &= \frac{F^2}{2k_{total}} = \sum (U_{ai} + U_{bi} + U_{si} + U_{fi}) + U_h \\
 &= \frac{F^2}{2} \left(\sum \left(\frac{1}{k_{ai}} + \frac{1}{k_{bi}} + \frac{1}{k_{si}} + \frac{1}{k_{fi}} \right) + \frac{1}{k_h} \right), \quad (4.16)
 \end{aligned}$$

where the subscript $i=p, g$ represents pinion and gear, respectively. U_{ai} , U_{bi} , U_{si} , U_{fi} and U_h are respectively axial compressive energy, bending energy, shear energy, fillet foundation energy, and Hertzian energy. Variables k_{ai} , k_{si} , k_{bi} , k_{fi} , and k_h are the tooth axial compressive stiffness, shear stiffness, bending stiffness, fillet foundation stiffness, and Hertzian contact stiffness, respectively. Based on the gear tooth contact model (Fig. 3-1) and the gear tooth geometric model (Fig. 4-5), each of the stiffnesses can be evaluated by [29, 135]

$$\frac{1}{k_{ai}} = \begin{cases} \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx, & (x_B < x \leq x_{s_start}) \\ \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx + \int_{x_{s_start}}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_{xs}} dx, & (x_{s_start} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx + \int_{x_{s_start}}^{x_{s_end}} \frac{\sin^2 \alpha_{pm}}{EA_{xs}} dx + \int_{x_{s_end}}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx, & (x_{s_end} < x) \end{cases}, \quad (4.17)$$

$$\frac{1}{k_{bi}} = \begin{cases} \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{pm}} \frac{f_x}{EI_z} dx, & (x_B < x \leq x_{s_start}) \\ \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{f_x}{EI_z} dx + \int_{x_{s_start}}^{x_{pm}} \frac{f_x}{EI_{zs}} dx, & (x_{s_start} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{f_x}{EI_z} dx + \int_{x_{s_start}}^{x_{s_end}} \frac{f_x}{EI_{zs}} dx + \int_{x_{s_end}}^{x_{pm}} \frac{f_x}{EI_z} dx, & (x_{s_end} < x) \end{cases}, \quad (4.18)$$

$$\frac{1}{k_{si}} = \begin{cases} \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx, & (x_B < x \leq x_{s_start}) \\ \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx + \int_{x_{s_start}}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xs}} dx, & (x_{s_start} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{s_start}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx + \int_{x_{s_start}}^{x_{s_end}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xs}} dx + \int_{x_{s_end}}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx, & (x_{s_end} < x) \end{cases}, \quad (4.19)$$

$$\frac{1}{k_f} = \frac{\cos^2 \alpha_{pm}}{EWC_f^*} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_{pm}) \right\}, \quad (4.20)$$

$$k_h = \frac{\pi E L_e C_h^*}{4(1-\nu^2)} \left(\ln \left(\frac{2\sqrt{h_{1pm} h_{2pm}}}{b_{pm}} \right) - \frac{\nu}{2(1-\nu)} \right)^{-1} \quad \text{with } b_{pm} = \sqrt{\frac{8F(1-\nu^2)r_{1pm}r_{2pm}}{\pi L_e E (r_{1pm} + r_{2pm})}}, \quad (4.21)$$

where $f_{xt} = (\cos \alpha_{pm} (x_{pm} - x_t) - h_{pm} \sin \alpha_{pm})^2$, $f_x = (\cos \alpha_{pm} (x_{pm} - x) - h_{pm} \sin \alpha_{pm})^2$, in which h_{pm} is the half tooth thickness at the mesh point P_m , α_{pm} is the corresponding pressure angle. x_t represents the coordinate during the transition curve AB evaluated by Eq.(3.7). x is the coordinate of the point on the involute curve BC. A_{xt} , I_{xt} represent the cross-sectional areas and corresponding area moments of inertia within the transition curve. A_x , I_z and A_{xs} , I_{zs} are cross-sectional areas and corresponding area moments of inertia within the involute curve for healthy and spalling conditions, respectively. For a healthy involute gear tooth part, $A_x = 2h_x L$, $I_z = (1/12)(2h_x)^3 L$. For the spalling portion, A_{xs} , I_{zs} are evaluated by Eq.(4.11) and (4.15), respectively. E , G , L_e and ν are Young's modulus, shear modulus, effective tooth contact width and Poisson's ratio. u_f , S_f , L^* , M^* , P^* and Q^* are constant parameters given in [127]. F is gear mesh force, C_f^* is a coefficient which corrects the foundation stiffness within the double-tooth contact zone, similar to the correction approach proposed in [34]. In the double tooth pair contact area $C_f^* = 0.8$, and within the single tooth pair mesh zone $C_f^* = 1$, r_{1pm} and r_{2pm} are the radii of the tooth curvatures, determined by $r_{1pm} = P_m B_1 = R_{bp} (\beta_{p0} + \theta_p + \angle K_1 O_p A_1)$ and $r_{2pm} = P_m B_2 = R_{bg} (\beta_{g0} - \theta_g + \angle K_2 O_g A_2)$ [29]. Furthermore, b_{pm} is the half Hertzian contact width, h_{1pm} and h_{2pm} are the sear lengths of teeth 1 and 2 at the mesh point [135], where sear length is defined as the length of a line normal to the gear profile, from the mesh point to the centerline of the gear. $C_h^* = 0.5C_d L_e / L$ is a coefficient

to correct the nonlinear Hertzian stiffness, in which $C_d = \max\left(\ln\left(2\sqrt{h_{1Pm}h_{2Pm}}/b_{Pm}\right) - \nu/2(1-\nu)\right) \cdot L_e$ is evaluated by Eq.(4.7), where L is the width of the gear.

4.4 Validation and analysis

4.4.1 Validation of the proposed method

The effectiveness of the proposed method is validated by modelling four types of gear tooth spalls with different shapes that might be observed in practice. The single ellipsoid tooth spall shown in Fig. 4-6 (a) is a simplified model of the practical gear tooth spall in Fig. 4-2 (a), the corresponding parameters for the tooth spall are $x_{s1_start} = 37$ (mm), $w_{s1_max} = 2.08$ (mm), $l_{s1_max} = 5.5$ (mm), $h_{s1_max} = 0.8$ (mm) and $\theta_{s1} = 16.29^\circ$. The ellipsoid tooth spall with longer spalling length shown in Fig. 4-6 (b) models the common local tooth spalls shown in Fig. 4-2(b), (c), and (d). The corresponding parameters for this tooth spall are $x_{s1_start} = 37$ (mm), $w_{s1_max} = 2.08$ (mm), $l_{s1_max} = 15.5$ (mm), $h_{s1_max} = 0.8$ (mm) and $\theta_{s1} = 16.29^\circ$. The tooth spall shown in Fig. 4-6 (c) is in a spherical shape which is a special case of an ellipsoid utilized to model the practical tooth spalls shown in Fig. 4-2 (e) and (h). The spherical shape can be easily described with an ellipsoid by setting $a=b=c$ in Eq.(4.1). The corresponding parameters for the spherical shaped tooth spall are $x_{s1_start} = 37.3$ (mm), $l_{s1_max} = w_{s1_max} = 3.4$ (mm), $h_{s1_max} = 0.7$ (mm) and $\theta_{s1} = 19.84^\circ$. The last case shown in Fig. 4-6 (d) is a simplified model of the multiple tooth spalls shown in Fig. 4-2 (f) and (g), where the five in-line spalls are identical and their parameters are the same as those of the single spall in Fig. 4-6 (a). The parameters for the four identical smaller tooth spalls are $x_{s2_start} = 39.35$ (mm), $w_{s2_max} = 1.48$ (mm), $l_{s2_max} = 4.0$ (mm), $h_{s2_max} = 0.38$ (mm) and $\theta_{s2} = 24.16^\circ$. The parameters for the spur gear pair are shown in Table 4-1.

Table 4-1 Parameters of the gear-pinion set of chapter 4

Parameters	Value	Parameters	Value
Number of pinon teeth	24	Diametral Pitch (inch ⁻¹)	8
Number of gear teeth	24	Module (mm)	3.175
Teeth width (mm)	38.1	Theoretical contact ratio	1.6019
Pressure angle (deg.)	20	Young's modulus (N/mm ²)	2×10^5
Bore diameter (mm)	25.4	Poisson's ratio	0.3

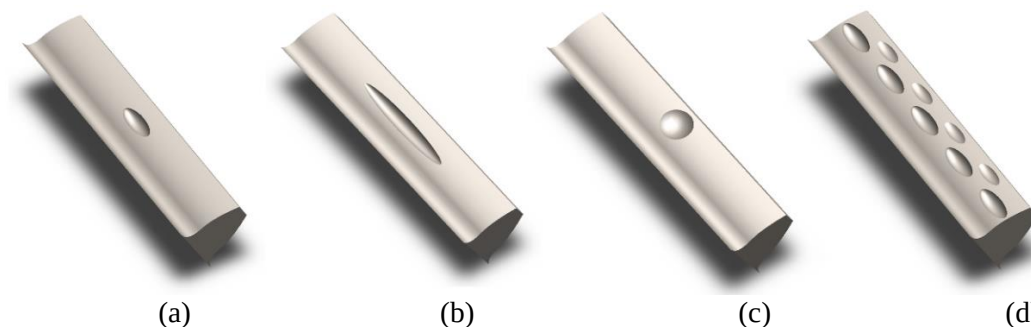


Fig. 4-6 Gear tooth spall modeled by the proposed method under different conditions

The removal of surface material due to tooth spalling changes the magnitudes of the cross-sectional area, area moment of inertia, and the effective surface contact length. The variation of the cross-section area and the corresponding area moment of inertia of the four spalling conditions are shown in Fig. 4-7 and Fig. 4-8, where case (a), case (b), case (c) and case (d) correspond to the spalling conditions shown in Fig. 4-6. It can be observed that the occurrence of the tooth spall decreases the magnitude of the crossing-area and its area moment of inertia. The reduction of these parameters (A_{xs} , I_{zs}) and the effective tooth surface contact length (not shown) reduce the strength of the gear tooth.

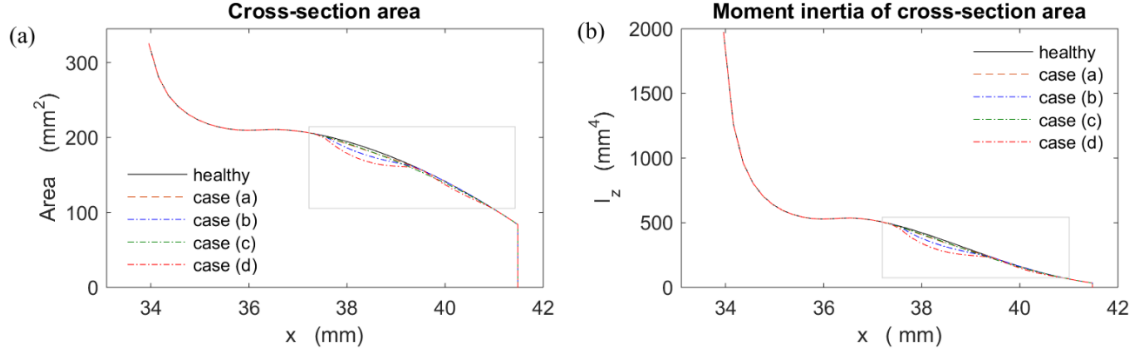


Fig. 4-7 The variation of cross-section area and area moment of inertia under different spalling conditions

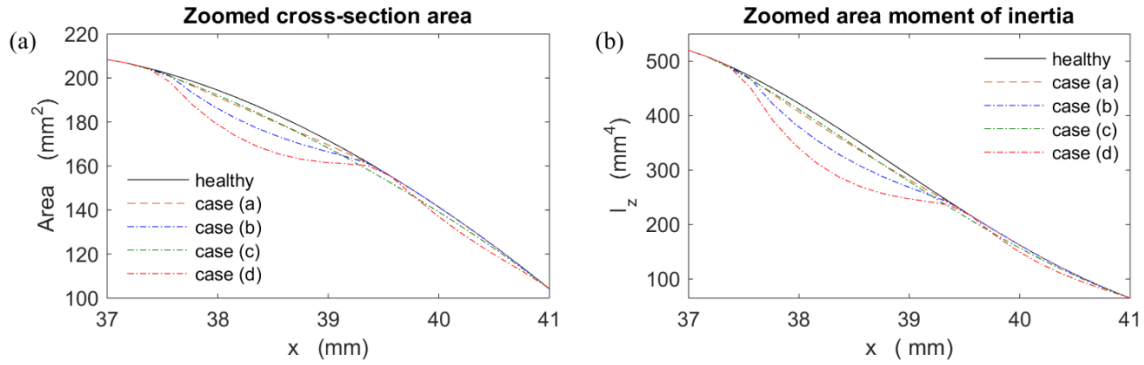


Fig. 4-8 Zoomed cross-section area and area moment of inertia under different spalling conditions

The TVMS of the healthy and spalling conditions evaluated with the proposed method were validated by comparing with the results obtained using commercial software ANSYS. The finite element model of case (a) is shown in Fig. 4-10 as an example. The FEM methods proposed in [136] were adopted to evaluate the TVMS of the spur gear pair. Specifically, the experimental procedure of the ‘proposed model 2’ in [136] is applied. An ANSYS Parametric Design Language (APDL) script code was written to control the contact points as well as to measure the rotational angle of the nodes on the gear bore. A torque with $T=80\text{Nm}$ was applied to the gear bore surface (the gear in green color shown in Fig. 4-9). The contact force (approximated by $F=T/R_{bp}$) is

$F=2235\text{N}$. The contact point at the handover regions (from two to one tooth pair contact or one to two tooth pair contact) is neglected in order to avoid convergence issues of the FEM method. To obtain the FEM results at the handover regions, see [137].

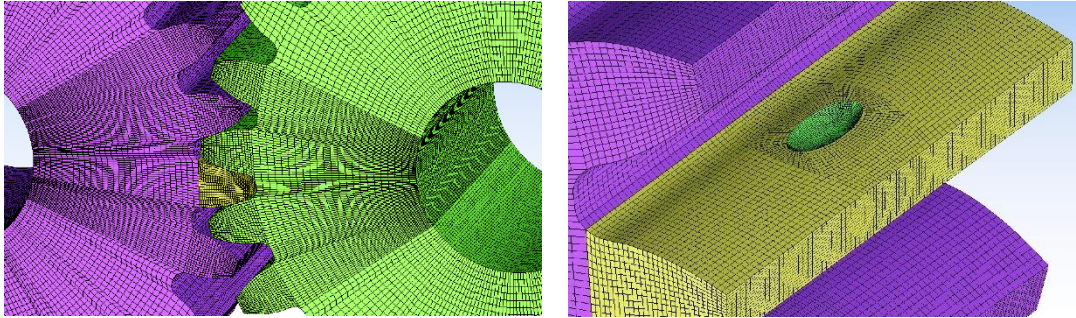


Fig. 4-9 Finite element model of a mating gear pair for case (a) as an example.

The corresponding TVMS of the gear tooth under different spalling conditions evaluated by the proposed analytical method and the FEM method are shown in Fig. 4-10. In this work, the rotation of the pinion is set to be counter-clockwise. In this way, the contact of the pinion tooth starts at point P_1 near the tooth base circle and processes to the separation point P_2 , see Fig. 3-1. Thus, the mesh points are in a healthy status until the defect zone comes into the mesh. It can be observed that the gear mesh stiffness obtained by ANSYS software predicts results consistent to those evaluated by the proposed analytical method. The perfect match between the proposed analytical method and the FEM method is mainly due to the correction of the foundation stiffness within the double tooth contact zone, as well as consideration of the non-linearity of the Hertzian contact stiffness, Eq.(4.21). The reduction of the TVMS of the gear pair depends on the shape and severity of the tooth spall. As the fault becomes more severe, e.g. changing from case (a) to (b), the greater the stiffness reduction can be observed within the spalling area. Moreover, noticeable stiffness reductions can also be observed after the defect area. This is due to the fact that the reduction of structural materials affects the stiffness of the rest of the gear tooth.

For case (a), the spall starts at 7.036° and ends at 16.63° of pinion rotation angle. The spalling area of case (b) is the same as case (a) due to the same starting position and width of tooth spall. However, in case (b) the length of the elliptical tooth spall is much longer than in case (a), therefore larger TVMS reductions can be observed in the same area (Fig. 4-10), as well as larger TVMS reductions after the defect area, Fig. 4-11. In Fig. 4-11, there are subtle differences between the analytical and FEM results after the defect area, where the TVMS evaluated by FEM methods is slightly smaller than that of the analytical results. This is normal because the FEM method itself has limited resolution and the algorithm of the FEM is more complicated than that of the analytical method, which may explain the difference. The difference between the FEM and the analytical results is less than 1% (evaluated by $(R_{analytical}-R_{FEM})/R_{FEM}$), which is acceptable for most engineering applications.

For case (c), the spalling area is from 8.46° to 21.56° of pinion rotation angle. Both the length and depth of the spherical spall are smaller than the spalls in cases (a) and (b), therefore smaller TVMS reductions are observed compared to the previous cases. For case (d), there are two stiffness reduction areas due to the two sets of gear tooth spalls, namely, $[7.036^\circ, 16.63^\circ]$ for the first set and $[18^\circ \text{ to } 22.74^\circ]$ for the second set, where the first set is greater in spall length and depth. Therefore, the reduction of the TVMS within the first spalling area is much larger than the reduction in the second area.

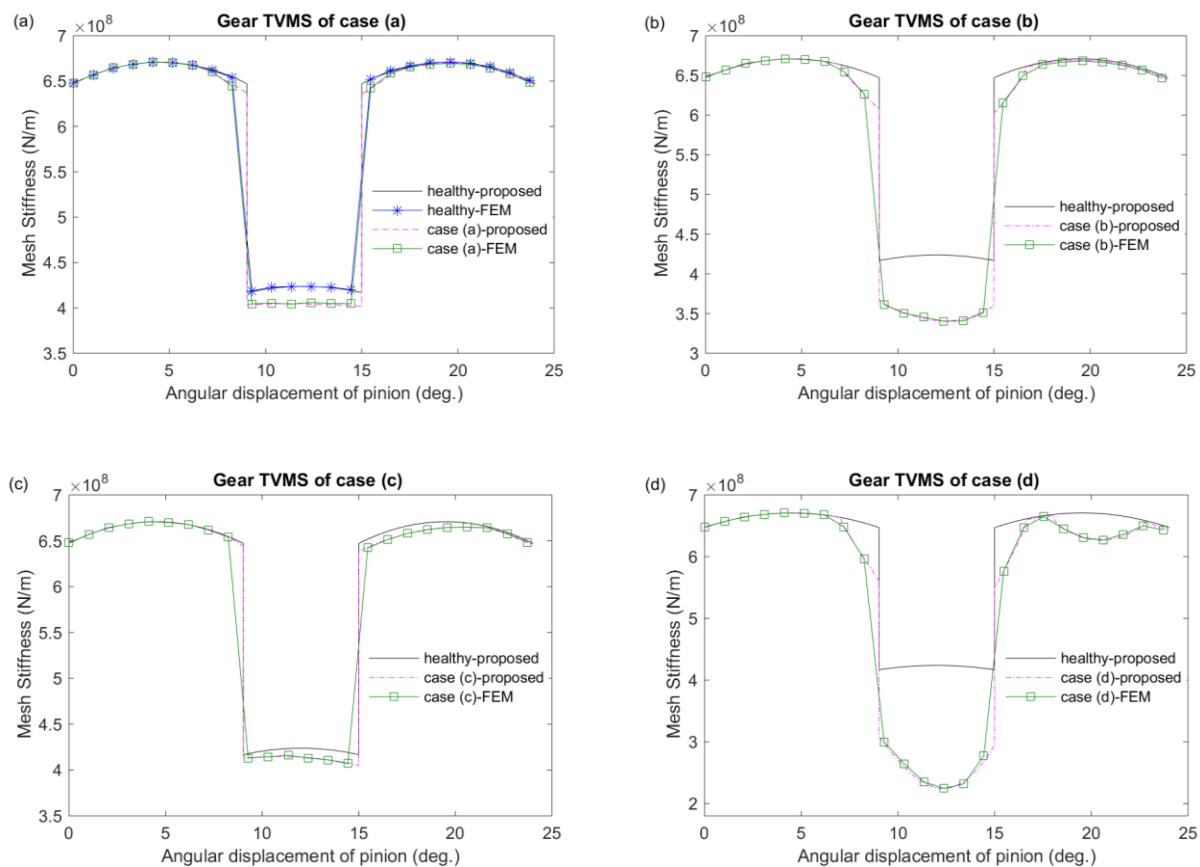


Fig. 4-10 TVMS of spur gear pair with multiple spalls

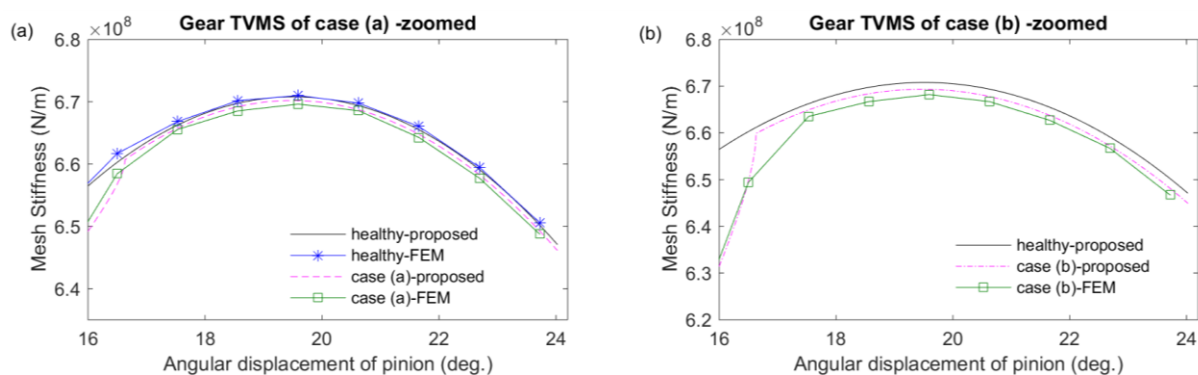


Fig. 4-11 Zoomed TVMS of case (a) and case (b)

4.5 Conclusions

This chapter proposed a curved bottom shape method to model the geometric features of gear tooth spalls that typically occur in practice. The proposed tooth spall is constructed by removing the intersection between an ellipsoid and the gear tooth surface. The radii of the ellipsoid can be varied in three dimensions, which makes it possible to closely model the geometric features of tooth spall. The proposed method corrects the foundation stiffness within the double tooth contact area and considers the non-linearity of Hertzian contact stiffness. The effectiveness of the proposed method for modelling single and multiple tooth spalls with different shapes and severity conditions was validated by finite element analysis. Results indicate that the proposed method can predict gear TVMS with the results consistent with those of FEM under healthy and various tooth spalling conditions.

Chapter 5 A shape-independent approach to modelling gear tooth spalls for time varying mesh stiffness evaluation of a spur gear pair

This chapter addresses objective 3, which is to propose an advanced method to model tooth spall(s) with irregular shapes and randomly distributed features.

This chapter has been published as a journal paper in the journal of *Mechanical Systems and Signal Processing*.

Y. Luo, N. Baddour, and M. Liang, "A shape-independent approach to modelling gear tooth spalls for time varying mesh stiffness evaluation of a spur gear pair," *Mechanical Systems and Signal Processing*, vol. 120, pp. 836-852, 2019.

5.1 Abstract

This chapter proposes a shape-independent method to model tooth spall(s) based on defect ratios instead of a specific geometry. The proposed method is applicable to tooth spalls of various sizes, shapes, numbers and distribution conditions, and thus is capable of modeling any kind of tooth spall. A powerful advantage of the proposed method is that to change the type of spall being modelled, only the functions of the defect ratios need to be modified rather than the entire algorithm. Therefore, the proposed method is general in nature and much easier to implement in practice. The effectiveness of the proposed shape-independent approach to modelling different kinds of tooth spalls is validated by finite element results.

5.2 Introduction

Gear tooth spall is one of the most common gear surface defects. The causes of tooth spall range from surface fatigue, excessive load, mounting errors, lubrication problems, outside foreign particles or inside debris, etc.[138] The occurrence of tooth spall increases system noise and vibration, affects transmission accuracy and reduces gear service life. Fault diagnosis of a gear transmission based on vibration signals requires the knowledge of the fault symptoms and the difference between damaged and undamaged vibration responses [1]. Dynamic simulation of gear transmissions with faults has played an important role in analyzing system vibration behavior with and without gear fault(s) [29, 139, 140]. The analysis of the vibration behavior of a gear transmission by dynamic simulation requires accurate values of the gear Time Varying Mesh Stiffness (TVMS) and its corresponding changes when tooth fault occurs.

Existing work on modelling gear tooth spalls has focused on using a specific geometry (e.g. rectangular, round, elliptical, pixel, etc.) to approximate the features of the tooth spall. In practice, tooth spall(s) are usually irregular in shape and hence it is often difficult to find a suitable geometry

to match the shape of the defect. The issue of how to analytically evaluate the TVMS of the gear pair with irregular tooth spalls as observed in practice remains unsolved. In light of this, a shape-independent approach to modeling gear tooth spalls as observed in practice is proposed in this chapter. The proposed method models tooth spalls from a different perspective, in which two defect ratios, namely the defect length ratio C_{Lsx} and the defect depth ratio C_{hsx} , are utilized to directly model the features and severity of the gear tooth spall(s) instead of using a specific geometry to represent the characteristics of the spall(s).

As the shape-independent method is not restricted by the pre-specified geometry and distribution of tooth spalls, it can be applied practically to modeling any kind of spall, such as a localized spall with simple/complicated shape and/or multiple spalls with Gaussian or randomly distributions. The effectiveness of the proposed shape-independent approach to modelling different kinds of tooth spall is then examined and validated by the Finite Element Method (FEM). Based on the proposed shape-independent method, an advanced energy method for evaluating the TVMS of distributed spall is also proposed. The advanced energy method simplifies the existing energy method on modelling tooth spalls.

5.3 Introduction of the proposed shape-independent method

5.3.1 The shape-independent method on modelling gear tooth spalls

Fig. 5-1 shows a simplified model of a localized gear tooth spall of irregular shape. The removal of surface material due to tooth spalling reduces the amount of material available to support the contact loads, which results in a reduction of gear TVMS. The effect of the tooth stiffness reduction due to spalling is generally modeled by: a) reducing the effective surface contact length L_e ; and b) changing the gear tooth cross-sectional properties, namely, the cross-section area A_{xs} and the area moment of inertia I_{zs} .

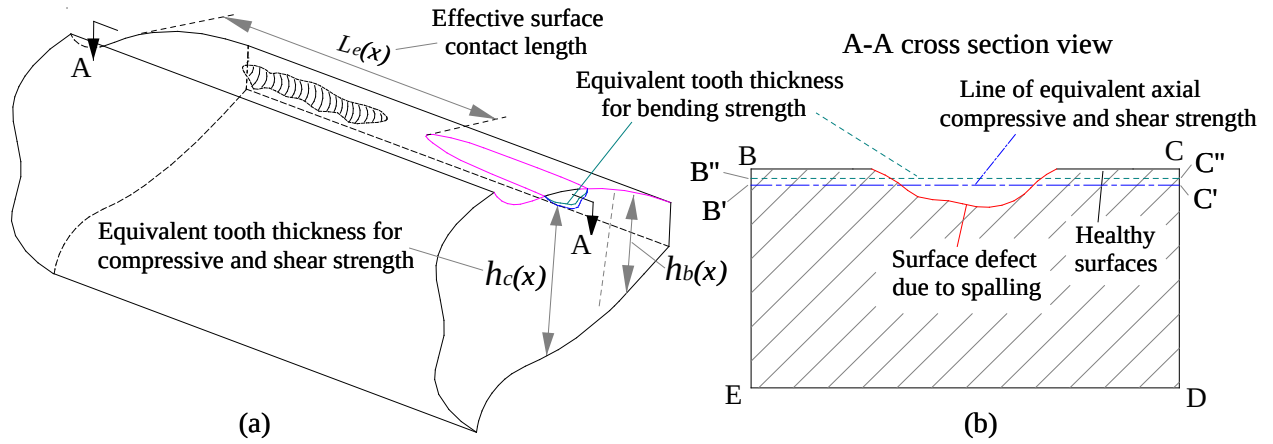


Fig. 5-1 Simplified model of gear tooth spalling

The goal of modelling the effects of tooth spall on gear TVMS is to capture the change in the cross-sectional properties and the change in the effective surface contact length within the spalling area. Geometry-based methods focus on using simple geometries (rectangular, round, elliptical, etc.) to approximate the shape of the tooth spall. With the approximated geometry of the defect, the time varying contact length and cross-sectional properties can be evaluated, and thus the gear mesh stiffness with tooth spall can be calculated. The accuracy of the geometric based methods of modelling tooth spalling depends on how close the utilized geometry is to the actual shape of the spall. However, in practice, tooth spalls are usually irregular and complex in shape, which may result in large calculation errors when being approximated using simple geometries.

The variations of the effective contact length and tooth cross-sectional properties due to spalling can be directly modelled with some simple functions instead of using a specific geometric shape, e.g. by $L_e(x) = f_L(x)$, $A_{xs} = f_A(x)$ and $I_{zs} = f_I(x)$, where $f_L(x)$ is the function of effective contact length, $f_A(x)$ and $f_I(x)$ are respectively the functions of cross-sectional area and area moment of inertia of a gear tooth, x is the displacement as shown in Fig. 5-2. The magnitudes of $f_L(x)$, $f_A(x)$ and $f_I(x)$ depend on the severity of the tooth spall and reflect the health status of the

gear tooth. However, directly expressing $f_L(x)$, $f_A(x)$ and $f_I(x)$ to model the features of gear tooth spall(s) could be difficult. The shape-independent method provides an alternative way to more easily and accurately evaluate the values of these functions with tooth spalls.

Due to the physical properties of the gear tooth, the magnitudes of $f_L(x)$, $f_A(x)$ and $f_I(x)$ are correlated with each other. For example, an increase in the diameter of a tooth spall will lead to reduction of magnitude of $L_e(x)$ as well as a decrease in the values of $f_A(x)$ and $f_I(x)$. Therefore, it is unnecessary to evaluate the values of $L_e(x)$, $f_A(x)$ and $f_I(x)$ independently. Knowing the relationships between these functions can simplify the evaluation process and reduce the number of functions needed to model the tooth spall.

In order to clearly explain the effects of tooth spall on the cross-sectional properties of the gear tooth, two rectangles $B'C'DE$ and $B''C''DE$ satisfied by the conditions of $f_A(x) = S_{B'C'DE} = Lh_c(x)$ and $f_I(x) = I_{B''C''DE} = (1/12)h_b^3(x)L$ are introduced, where $S_{B'C'DE}$ is the area of $B'C'DE$ which is equal to the actual area of the irregular shape under spalling conditions and $h_c(x)$ is the corresponding height of the rectangle and is defined as the equivalent tooth thickness for the axial compressive and shear strength of the gear tooth. $I_{B''C''DE}$ is the area moment of inertia of $B''C''DE$ which is equal to the actual area moment of inertia of the tooth cross-section with spall, $h_b(x)$ is the height of rectangle $B''C''DE$ and is defined as the equivalent tooth thickness for the bending stiffness of the gear tooth. Fig. 5-2 shows an example of equivalent tooth thickness curves under spalling conditions.

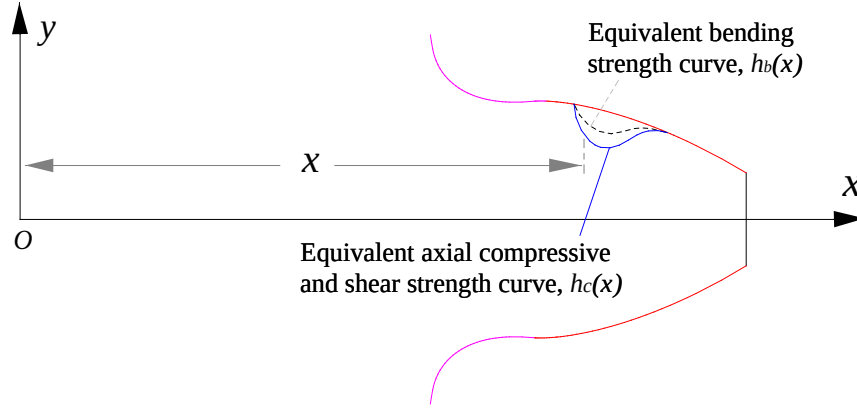


Fig. 5-2 Illustration of the shape-independent method for modelling gear tooth spalling

For a simple gear tooth spall with rectangular shape, as shown in Fig. 5-3 and Fig. 5-4, the cross-sectional area at displacement x within the spalling area can be written as

$$A_{xs} = 2Lh_x - L_s h_s, \quad x \in [x_{s_st}, x_{s_end}] \quad (5.1)$$

where L_s is the length of the tooth spall, h_s is the depth. The reduction of the cross-sectional area results in a change of the neutral axis of the area moment of inertia which is given by

$$\delta_{xs} = \frac{(h_x - 1/2 h_s) L_s h_s}{A_{xs}}. \quad (5.2)$$

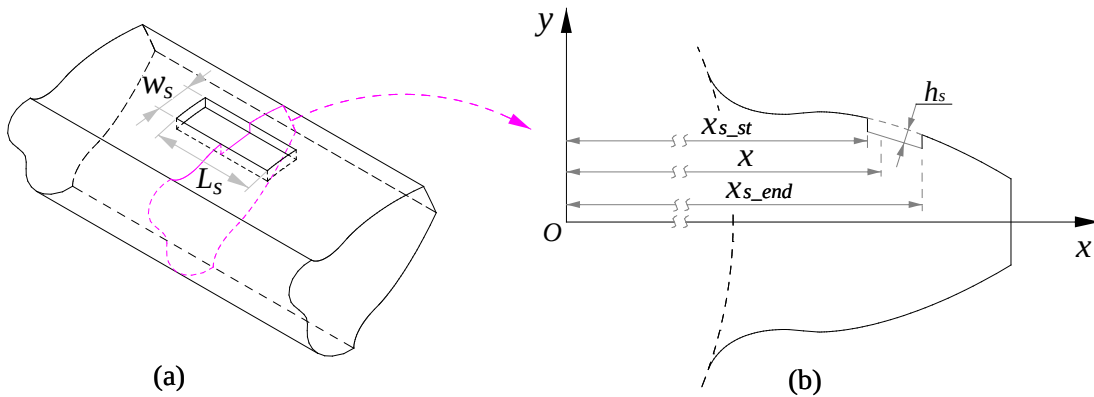


Fig. 5-3 Gear tooth spalling modeled by a rectangular shape

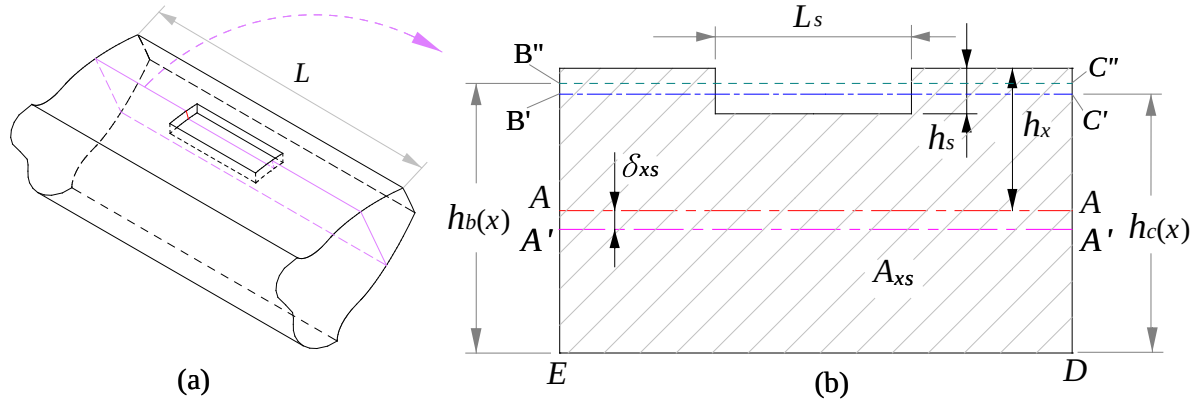


Fig. 5-4 Cross-section model of the tooth spall

The area moment of inertia with respect to the neutral axis $A'A'$ can be evaluated by

$$I_{zs} = \frac{2Lh_x^3}{3} + 2Lh_x\delta_{xs}^2 - \left(\frac{L_s h_s^3}{12} + L_s h_s (h_x - 0.5h_s + \delta_{xs})^2 \right), \quad x \in [x_{s_st}, x_{s_end}] \quad (5.3)$$

The values of $h_b(x)$ and $h_c(x)$ can be determined from

$$\begin{cases} h_c(x) = A_{xs}/L \\ h_b(x) = (12I_{zs}/L)^{\frac{1}{3}} \end{cases} \quad (5.4)$$

To obtain the relationships between $L_e(x)$ and $h_b(x)$, $h_c(x)$, two defect ratios are defined

$$\begin{cases} C_{Lsx} = L_s/L \\ C_{hsx} = h_s/(2h_x) \end{cases} \quad (5.5)$$

where C_{Lsx} and C_{hsx} are the defect length ratio and defect depth ratio of the tooth spall at displacement x , respectively. Substituting Eq. (5.5) into Eq.(5.1), the effective cross-sectional area can be expressed as $A_{xs} = 2Lh_x(1 - C_{Lsx}C_{hsx})$. Based on Eq.(5.4), the magnitude of the equivalent tooth thickness $h_c(x)$ can be written as

$$h_c(x) = 2(1 - C_{Lsx}C_{hsx})h_x, \quad (5.6)$$

A similar process can be performed to obtain $h_b(x)$. After substituting Eq. (5.5) into Eq.(5.3) and Eq.(5.4) , one can obtain

$$h_b(x) = 2h_x (K_{co})^{\frac{1}{3}} \quad (5.7)$$

with

$$K_{co} = \frac{1 - C_{hsx}^5 C_{Lsx}^3 + (4C_{hsx}^2 - 6C_{hsx}^3 + 5C_{hsx}^4) C_{Lsx}^2 - (5C_{hsx} + 4C_{hsx}^3 - 6C_{hsx}^2) C_{Lsx}}{(1 - C_{hsx} C_{Lsx})^2}. \quad (5.8)$$

The relationship between $h_c(x)$ and $h_b(x)$ can be expressed as

$$h_b(x) = K_r h_c(x), \text{ with } K_r = (K_{co})^{\frac{1}{3}} / (1 - C_{Lsx} C_{hsx}). \quad (5.9)$$

It can be observed that with the knowledge of the tooth fault severity (C_{Lsx} and C_{hsx}), the equivalent tooth thickness curves $h_c(x)$ and $h_b(x)$ can be simply evaluated by Eqs. (5.6) and (5.9). With these two parameters, the effective contact length, cross-sectional area and area moment of inertia can then be written as

$$\begin{cases} L_e(x) = L(1 - C_{Lsx}) \\ A_{xs} = Lh_c(x) = 2L(1 - C_{Lsx} C_{hsx}) h_x \\ I_{zs} = (1/12) h_b^3(x) L = (1/12) (2h_x)^3 K_{co} L \end{cases}, \quad (5.10)$$

Eq.(5.10) can be applied under both tooth healthy and damaged conditions. In practice, the value of C_{Lsx} is determined by the shape of the tooth spall. For a healthy condition, $C_{Lsx} = 0$, and therefore, $K_{co} = 1$, and Eq.(5.10) is the same as Eq.(5.18). For a rectangular tooth spall, C_{Lsx} is a constant due to a constant L_s . For tooth spall with a complicated shape, C_{Lsx} is a function of displacement x . The value of C_{hsx} is determined from the depth of tooth spall h_s , Fig. 5-4.

5.3.2 The shape-independent method for gear TVMS with spall

A gear tooth can be simplified as a cantilever beam with variable cross-sectional area, as shown in Fig. 5-5. In the figure, the coordinate system xoy is located at the geometrical center of the gear and the x axis is consistent with the center line of the tooth. AB and BC are the tooth transition curve and involute curve, respectively. x_A and x_B denote the positions of the transition curve starting point and ending point. x_t and h_{xt} are the position and half tooth thickness of an arbitrary point on curve AB . P_m denotes the mesh point and x_{Pm} denotes the position of P_m , F is the corresponding contact force which is tangent to the gear base circle and is decomposed into F_x and F_y , α_{Pm} is the corresponding operation angle. x and h_x correspond to the position and half tooth thickness of a point on the involute curve. x_{s_st} and x_{s_end} are the starting and ending positions of the tooth spall, h_{xs} is the equivalent positive half tooth thickness under tooth spalling conditions.

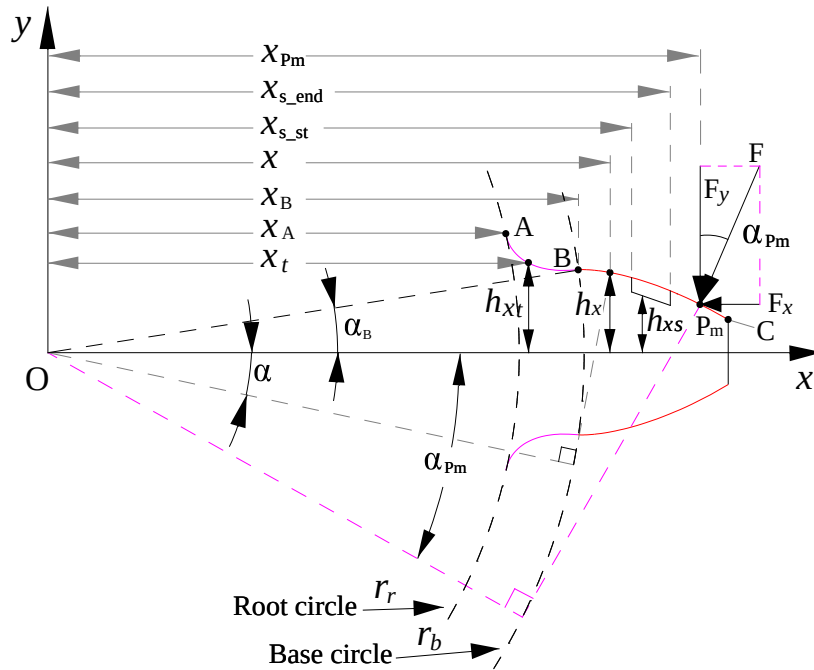


Fig. 5-5 Gear tooth modeled as a cantilever beam

The involute function can be expressed as the function of the dynamic gear mesh pressure angle [56],

$$\begin{cases} x = \frac{r_b}{\cos \alpha_i} \cos \left(\frac{\pi}{2N} - (\tan \alpha_i - \alpha_i - \tan \alpha_0 + \alpha_0) \right) \\ y = \frac{r_b}{\cos \alpha_i} \sin \left(\frac{\pi}{2N} - (\tan \alpha_i - \alpha_i - \tan \alpha_0 + \alpha_0) \right) \end{cases}, \quad (\alpha_c \leq \alpha_i \leq \alpha_a) \quad (5.11)$$

where r_b is the radius of gear base circle, α_0 is the pressure angle at the pitch circle, α_i is the pressure angle of an arbitrate point on the involute curve.

$\alpha_c = \arccos \left(r_b / \sqrt{(r_b \tan \alpha_0 - h_a^* m / \sin \alpha_0)^2 + r_b^2} \right)$ is the pressure angle of the involute starting point,

and $\alpha_a = \arccos(r_b / r_a)$ is the pressure angle of the point on the addendum circle [56]. The function of the transition curve depends on the trajectory of the cutter tip during manufacturing process [57, 126]. Based on the work of Ma et al. [56], the transition curve is expressed as a function of an angle parameter γ as explained in Eq. (3.7) in chapter 3.

Based on the potential energy method, the total contact energy of a mating gear tooth pair can be expressed as [29, 56]

$$\begin{aligned} U_{total} &= \frac{F^2}{2k_{total}} = \sum_{i=1}^n (U_{ai} + U_{bi} + U_{si} + U_{fi}) + U_h \\ &= \frac{F^2}{2} \left(\sum_{i=1}^n \left(\frac{1}{k_{ai}} + \frac{1}{k_{bi}} + \frac{1}{k_{si}} + \frac{1}{k_{fi}} \right) + \frac{1}{k_h} \right), \end{aligned} \quad (5.12)$$

where n denotes the number of tooth pairs in meshing at the same time, the subscript $i = p, g$ represents pinion and gear, respectively. U_{ai} , U_{bi} , U_{si} , U_{fi} and U_h are respectively axial compressive energy, bending energy, shear energy, fillet foundation energy, and Hertzian energy. k_{ai} , k_{bi} , k_{si} , k_{fi} , and k_h are the corresponding axial compressive stiffness, bending stiffness, shear stiffness, fillet foundation stiffness and Hertzian contact stiffness. The fundamental idea of

the shape-independent method is to use the defect ratios, namely C_{Lsx} and C_{hsx} , to model the features of gear tooth spall instead of using a specific geometric shape. For a gear tooth with spalling defect, see Fig. 5-5, each of the sub-stiffness can be expressed as [141]

$$\frac{1}{k_{ai}} = \begin{cases} \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx, & (x_B < x \leq x_{s_st}) \\ \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx + \int_{x_{s_st}}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{2EL(1-C_{Lsx}C_{hsx})h_x} dx, & (x_{s_st} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx + \int_{x_{s_st}}^{x_{s_end}} \frac{\sin^2 \alpha_{pm}}{2EL(1-C_{Lsx}C_{hsx})h_x} dx + \int_{x_{s_end}}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx, & (x_{s_end} < x) \end{cases} \quad (5.13)$$

$$\frac{1}{k_{bi}} = \begin{cases} \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{pm}} \frac{f_x}{EI_z} dx, & (x_B < x \leq x_{s_st}) \\ \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{f_x}{EI_z} dx + \int_{x_{s_st}}^{x_{pm}} \frac{f_x}{(1/12)(2h_x)^3 K_{co} LE} dx, & (x_{s_st} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{f_x}{EI_z} dx + \int_{x_{s_st}}^{x_{s_end}} \frac{f_x}{(1/12)(2h_x)^3 K_{co} LE} dx + \int_{x_{s_end}}^{x_{pm}} \frac{f_x}{EI_z} dx, & (x_{s_end} < x) \end{cases} \quad (5.14)$$

$$\frac{1}{k_{si}} = \begin{cases} \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx, & (x_B < x \leq x_{s_st}) \\ \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx + \int_{x_{s_st}}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{2GL(1-C_{Lsx}C_{hsx})h_x} dx, & (x_{s_st} < x \leq x_{s_end}) \\ \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{s_st}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx + \int_{x_{s_st}}^{x_{s_end}} \frac{1.2 \cos^2 \alpha_{pm}}{2GL(1-C_{Lsx}C_{hsx})h_x} dx + \int_{x_{s_end}}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx, & (x_{s_end} < x) \end{cases} \quad (5.15)$$

$$\frac{1}{k_f} = \frac{\cos^2 \alpha_{pm}}{EWC_f^*} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_{pm}) \right\} \quad (5.16)$$

$$k_h = \frac{\pi EL_e C_h^*}{4(1-\nu^2)} \left(\ln \left(\frac{2\sqrt{h_{1pm} h_{2pm}}}{b_{pm}} \right) - \frac{\nu}{2(1-\nu)} \right)^{-1} \text{ with } b_{pm} = \sqrt{\frac{8F(1-\nu^2)r_{1pm}r_{2pm}}{\pi L_e E(r_{1pm} + r_{2pm})}} \quad (5.17)$$

where $f_{xt} = (\cos \alpha_{pm}(x_{pm} - x_t) - y_{pm} \sin \alpha_{pm})^2$, $f_x = (\cos \alpha_{pm}(x_{pm} - x) - y_{pm} \sin \alpha_{pm})^2$, in which x_{pm} is the position of the contact point on the x axis and y_{pm} is the half tooth thickness at the mesh point P_m . x and x_t are defined in Eqs.(5.11) and (3.7), respectively. E , G , L_e and ν are Young's modulus, shear modulus, effective tooth contact width and Poisson's ratio. u_f , S_f , L^* , M^* , P^* and Q^* are constant parameters given in [127]. C_f^* is a coefficient utilized to correct the foundation stiffness within the double-tooth pair mesh region[141]. The theoretical value of C_f^* in the double tooth pair contact zone can be determined based on the model proposed in [34, 142] from $1/C_f^* = k_{fA}/k_{fB}$, where k_{fA} and k_{fB} are the foundation stiffness of double and single tooth pairs at the transaction zone, see [142]. In this work, $C_f^* \approx 0.8$ in the double tooth pair contact area and $C_f^* = 1$ in the single tooth pair mesh zone. F is the contact force, r_{1pm} and r_{2pm} are the radius of tooth curvature, b_{pm} is the half Hertzian contact width, h_{1pm} and h_{2pm} are the sear length of tooth 1 and 2 at the mesh point [135]. C_h^* is a coefficient to correct the nonlinear Hertzian stiffness, which is constructed by comparing the analytical Hertzian stiffness with the stiffness evaluated with FEM results. In this work, $C_h^* = 0.5C_d(1 - C_{Lsx})$ in which $C_d = \max\left(\ln\left(2\sqrt{h_{1pm}h_{2pm}}/b_{pm}\right) - \nu/2(1 - \nu)\right)$. A_{xt} , A_x and I_{zt} , I_z are, respectively, the cross-sectional area and area moment of inertia within the transition curve and involute curve under healthy conditions, and are given by

$$\begin{cases} A_{xt} = 2h_{xt}L, & I_{zt} = (1/12)(2h_{xt})^3L \\ A_x = 2h_xL, & I_z = (1/12)(2h_x)^3L \end{cases}, \quad (5.18)$$

where h_{xt} is the half tooth thickness of the transaction part evaluated by Eq.(3.7). h_x is the half tooth thickness of an arbitrary point on the involute curve given by Eq.(5.11). The detailed evaluation process of the TVMS equations is similar to the procedure provided in [34]. The following sections

introduce the application of the proposed shape-independent method on modelling tooth spalls under various defect conditions.

5.4 The shape-independent method for modelling localized tooth spalls

The factors that lead to gear tooth surface damage vary from lubrication problems to surface fatigue, manufacturing errors, mounting errors, etc. Different causes result in different shapes and types of tooth spalls. A localized tooth spall usually results from surface overload where excessive stress is concentrated [143]. The corresponding shape of the tooth spall is usually irregular and varies from one to another. The following subsections validate the effectiveness of the proposed shape-independent approach to modelling localized tooth spalls with different shapes.

5.4.1 Modelling a localized rectangular tooth spall

In practice, a tooth spall with a rectangular shape is rare, however it is a good simplifying assumption for the purpose of analyzing the effects of tooth spall on the gear TVMS. The effectiveness of the shape-independent method for modelling a simple rectangular tooth spall is validated by comparing it with both existing geometry-based method and FEM results. For the geometric based method and the finite element method, the severity of the spall is controlled by the dimensions of the defect geometry. Fig. 5-6 shows the finite element model of a mating gear pair with rectangular-shaped tooth spall. The parameters for the rectangular tooth spall, are $L_s = 6$ mm, $W_s = 2$ mm, $h_s = 0.9$ mm, $x_{s_st} = 37.2$ mm. The parameters for the gear pair are shown in Table 4-1. For the geometry-based method, the required cross-sectional properties (A_{xs} and I_{zs}) for evaluating the TVMS under spalling condition are evaluated by Eqs.(5.1) and (5.3), the detailed TVMS evaluation process for the geometric based method can be found in [34].

For a spur gear pair, the mesh stiffness is defined as the stiffness along the action line (namely the rectilinear mesh stiffness). In finite element analysis, the rectilinear mesh stiffness of a pair of

gears can be directly evaluated from $k = F/\delta$, where F is the contact force and δ is the displacement produced by the force along the action line [136]. However, directly measuring the displacement δ is complicated, since all the sub-deformations of each gear tooth for every mesh point must be calculated [136]. Howard et al.[87] and Liang et al.[136] proposed to transform the rectilinear deformation into a torsional displacement, and evaluated the gear mesh stiffness from $k = T/\theta r_b^2$, where r_b is gear base circle radius, and θ is the angular displacement of the gear body. This method works for both healthy and defective gear teeth. In this work, the evaluation process of mesh stiffness by FEM is based on method 2 proposed in [136]. The element is a hexahedral shape and elements around the defect area are refined, multiple refined element layers created to obtain better stress penetration on the tooth surface. Within the spalling area, the load is assumed to be carried by the remaining healthy surfaces and the bottom of the tooth spall is not defined as a potential contacting surface. In the analysis, the gear bore is set to be fixed and the load acting on the pinion bore surface is 80 Nm. The pinion rotates counter-clockwise with a rotational angle step of 1 degree. In this way, the contact point starts near the pinion base circle and proceeds to the addendum circle. The contact points are healthy until the defect zone is included in the mesh. To avoid convergence issues with the FEM approach, the contact point at the handover regions (from two to one tooth pair contact or one to two tooth pair contact) is neglected. An ANSYS Parametric Design Language (APDL) script code was written to measure the angular displacement of the nodes on the pinion flange for each mesh point. The surface contact friction force in the finite element analysis is not defined, consistent with the proposed analytical method where friction effects are neglected.

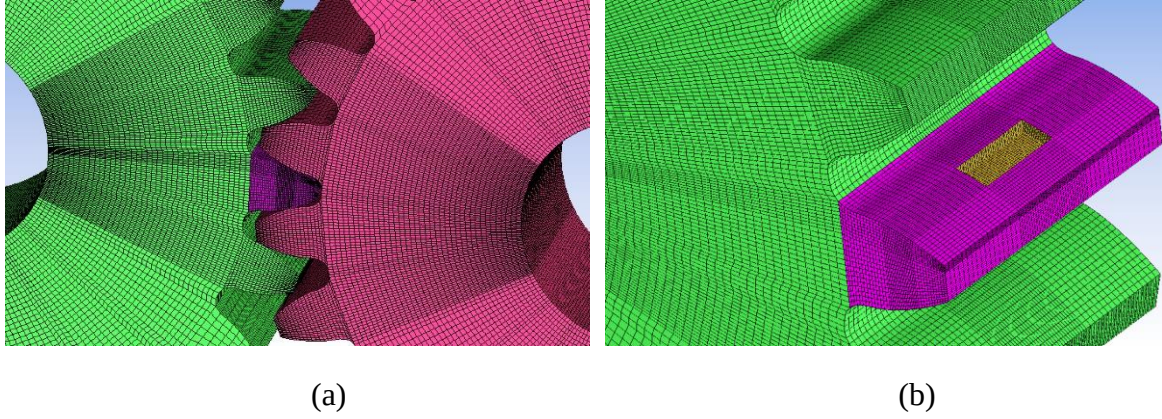


Fig. 5-6 Finite element model of gear tooth spall in rectangular shape

Table 5-1 Parameters of the gear-pinion set of chapter 5

Parameters	Value	Parameters	Value
Number of pinon teeth	24	Diametral Pitch (inch ⁻¹)	8
Number of gear teeth	24	Module (mm)	3.175
Teeth width L (mm)	19.5	Theoretical contact ratio	1.6019
Pressure angle (deg.)	20	Young's modulus (N/mm ²)	2×10^5
Bore diameter (mm)	25.4	Poisson's ratio	0.3

For the shape-independent method, the severity of the tooth spall is controlled by the defect ratios. For modelling the rectangular spall shown in Fig. 5-6, $C_{hsx} = h_s / (2h_x) = 0.9 / (2h_x)$, $C_{Lsx} = L_s(x) / L = 6 / 19.5$. The required cross-sectional properties (A_{xs} and I_{zs}) for calculating the TVMS under spalling condition in the shape-independent method are evaluated using Eq.(5.10). The corresponding TVMS evaluated by the proposed shape-independent method, the widely-used geometry-based method and the finite element method are shown in Fig. 5-7.

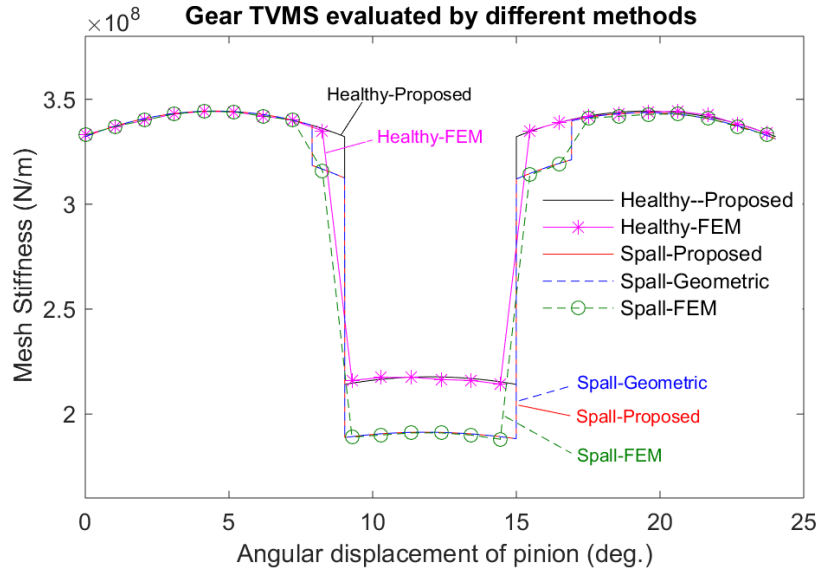


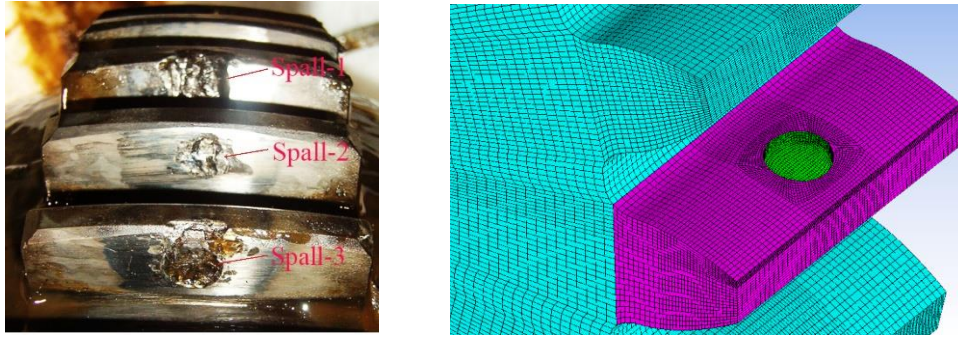
Fig. 5-7 Comparison of the TVMS with rectangular spall evaluated by different methods

As can be observed in Fig. 5-7, the results evaluated by the shape-independent and FEM methods under both healthy and spalling conditions are close. This is mainly because the proposed shape-independent method corrects the foundation stiffness within the double tooth contact zone in a similar way to that proposed in [34], and also considers the non-linearity of the Hertzian contact stiffness. The maximum difference between the FEM and the analytical results is less than 1% (evaluated by $|R_{analytical}-R_{FEM}|/R_{FEM}$), which satisfies most engineering applications. The difference between the FEM and analytical method are probably due to: a) the calculations for the FEM are more complicated and different from the analytical method proposed. The FEM is greatly affected by many parameters such as element type, element size, etc. b) the analytical method assumes that the load is uniformly allocated on the contact line which is different from the finite element method [44]. In order to show the performance of the general geometry-based method for modelling gear tooth spall, the evaluation of the gear TVMS with the geometry-based method is based on the same stiffness equations provided in section 5.3.2, the only difference is that the geometry-based method utilizes different equations, Eqs.(5.1) and (5.3), to evaluate the cross-

sectional area and the corresponding area moment of inertia. It can be seen that the result of the geometry-based method is identical to the result of the shape-independent method.

5.4.2 Modelling localized tooth spalls in round/elliptical shape

Elliptical and round shapes are good geometries to approximate some of the gear tooth spalls that occur in practice, e.g. the spall-2 and spall-3 shown in Fig.5-8(a). The corresponding finite element model and geometric model are shown in Fig.5-8(b) and Fig.5-9, respectively. The severity of the tooth spall in the finite element method and geometric based method is controlled by the length $l_{\max}$, the width $w_{\max}$, and the depth h_s , as shown in Fig.5-9. In this case, $l_{\max} = 4$ (mm), $w_{\max} = 3$ (mm), $h_s = 1$ (mm). The position of the spall is controlled by the center position x_{center} of the spall and the incline angle θ_s , see Fig.5-9 (b). In this case, $x_{center} = 38.55$ (mm) and $\theta_s = 18.40^\circ$.



(a) Gear tooth spalling in practice[144] (b) Finite element model of gear with elliptical spall

Fig.5-8 Gear tooth spall in elliptical/round shape

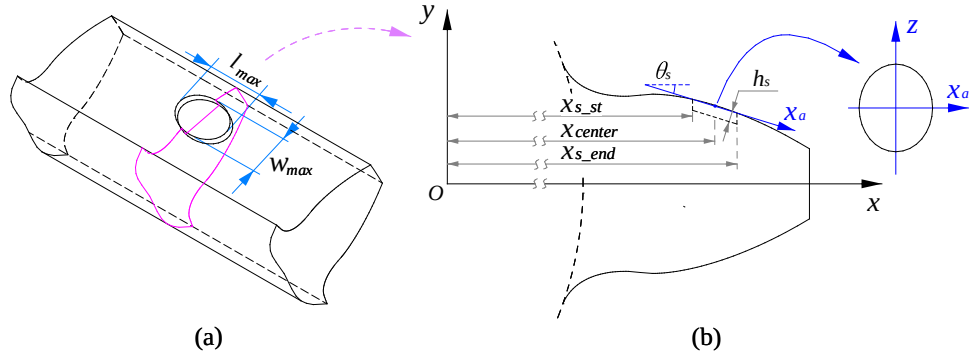


Fig.5-9 Gear tooth spalling modeled in round/elliptical shape by geometric based method

For the geometry-based method, the length of the spall can be expressed as a function of displacement x by

$$L_s(x) = \sqrt{l_{\max}^2 \left(1 - \frac{4(x_{\text{center}} - x)^2}{w_{\max}^2 \cos^2 \theta_s} \right)}, \quad x \in [x_{s_st}, x_{s_end}], \quad (5.19)$$

where $x_{s_st} = x_{\text{center}} - w_{\max} \cos \theta_s / 2$ is the starting position of the tooth spall, $x_{s_end} = x_{\text{center}} + w_{\max} \cos \theta_s / 2$ is the end position of the spall. When $l_{\max} = w_{\max}$, the tooth spall has a round shape. The cross-sectional properties (A_{xs} and I_{zs}) required for evaluating the TVMS with elliptical spall by the existing geometry-based method can be expressed as [35]

$$\begin{cases} A_{xs} = 2h_x L - \Delta A_x, \quad L_e(x) = L - L_s(x), \quad \Delta A_x = h_s L_s(x) \\ I_{xs} = \frac{2}{3} h_x^3 L - \frac{1}{12} L_s(x) h_s^3 - \frac{2h_x L \Delta A_x (h_x - h_s/2)^2}{2h_x L - \Delta A_x} \end{cases}, \quad x \in [x_{s_st}, x_{s_end}]. \quad (5.20)$$

For the shape-independent approach to modelling the elliptical tooth spall shown in Fig.5-8 (b), the defect ratios can be expressed as $C_{hsx} = h_s / (2h_x) = 1 / (2h_x)$, $C_{Lsx} = L_s(x) / L$. Eq.(5.10) was utilized to evaluate the effective contact length and cross-sectional properties in the TVMS calculation. The TVMS results evaluated by the proposed method, the geometry-based method and the finite element method are shown in Fig.5-10.

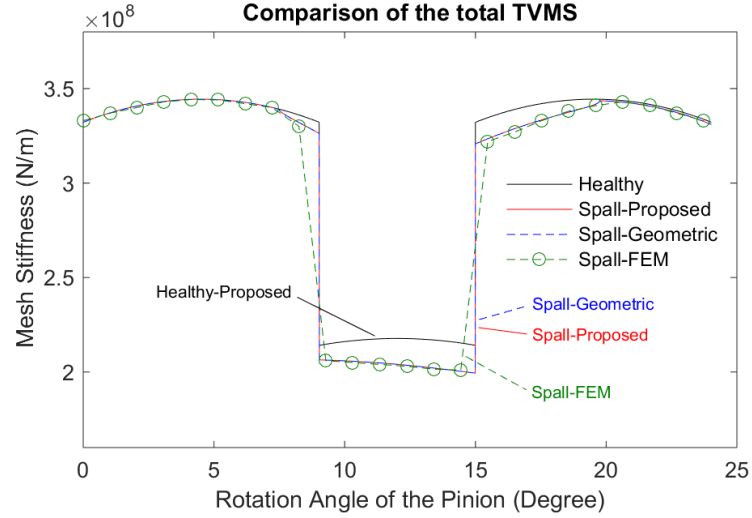


Fig.5-10 Comparison of the TVMS evaluated by different methods

As it can be observed in Fig.5-10, the TVMS evaluated by the shape-independent method and geometry-based method are consistent. Due to the computational cost of the finite element method, the distances between two successive discrete points in FEM results are much larger than the distances in the analytical methods. However, one can still observe that the FEM results are similar to those of the analytical methods.

5.4.3 Modelling localized tooth spalls with irregular shapes

As mentioned above, the actual shape of a localized tooth spall can be irregular, as shown in Fig.5-11. Due to shape irregularities, it is difficult to apply existing geometry-based methods to accurately evaluate the corresponding gear TVMS. Knowing the accurate TVMS of gear tooth with such spalls helps to present the actual dynamic responses of a gear system, as well as to provide better explanations of vibration behavior of a gear transmission with faults. The following two examples demonstrate the application of the proposed shape-independent method in modelling localized gear tooth spalls with irregular shapes.

5.4.3.1 Irregular tooth spall with constant depth.

Fig.5-11 shows some localized tooth spalls with irregular shapes that occur in practice. A similar-shaped irregular tooth spall with a constant depth feature is created, as shown in Fig.5-12. The parameters for the spall are $x_{s_st}=36.1$ (mm), $x_{s_end}=39.5$ (mm), $h_s=1.2$ (mm).

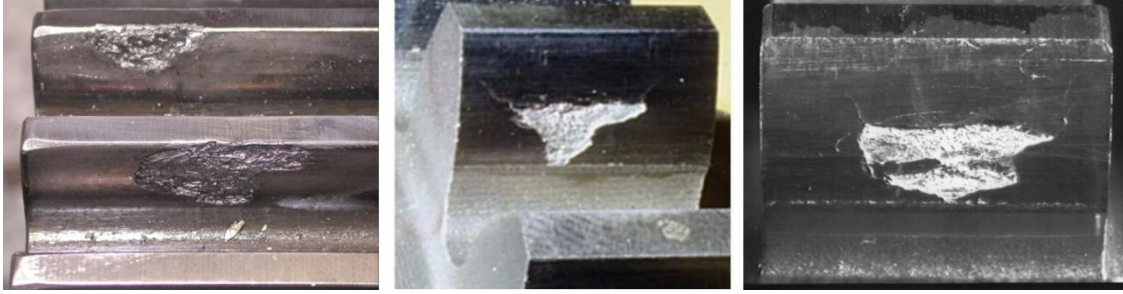


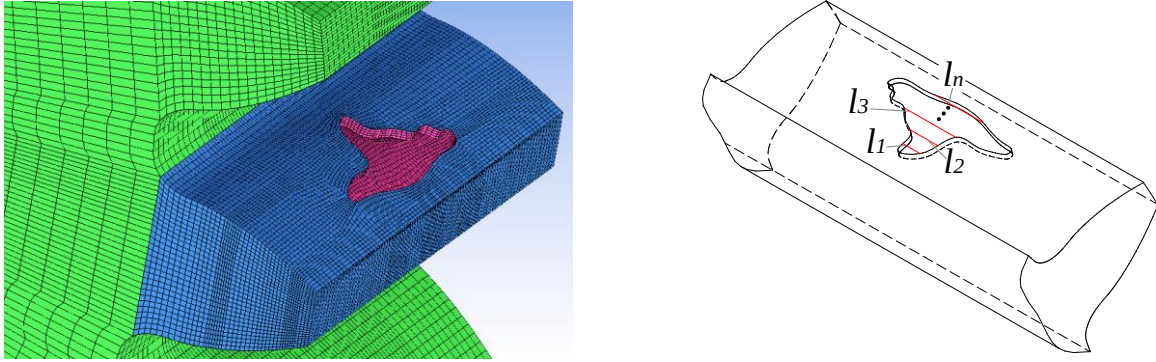
Fig.5-11 Gear tooth spalls of irregular shapes[145-147]

The key point of applying the shape-independent method to modelling tooth spall with irregular shapes is to define the two defect ratios C_{Lsx} and C_{hsx} , as mentioned earlier. In this case, the depth of the tooth spall h_s can be assumed to be constant. Therefore, the depth ratio can be simply expressed as $C_{hsx} = h_s / (2h_x)$. Due to the shape irregularity, the length of the spall cannot be expressed by a simple function. A Discrete Sampling and Fit Method (DSFM) is utilized to get the information of the length of the tooth spall, see Fig.5-12(b), where $l_1, l_2, l_3, \dots, l_n$ are the discrete lengths of the tooth spall at different positions. With the obtained discrete lengths of the spall, the continuous length function $L_s(x)$ can be approximated by a polynomial function expressed as

$$L_s(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0, \quad x \in [x_{s_st}, x_{s_end}], \quad (5.21)$$

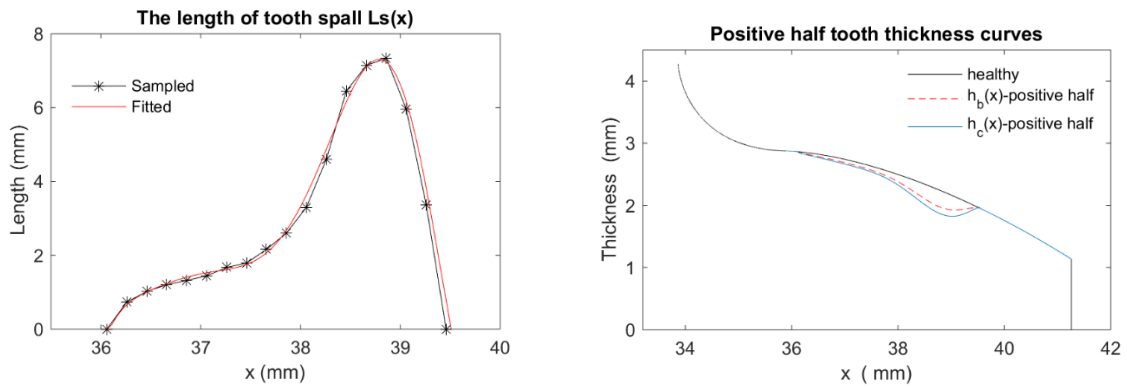
where $a_0 \sim a_n$ are constants of the fitted function, which can be obtained in MATLAB using the ‘polyfit’ function. An example of the fitted curve is shown in Fig.5-13 (a). With knowledge of $L_s(x)$, the defect length ratio can be written as $C_{Lsx} = L_s(x)/L$. Fig.5-13 (b) shows the corresponding equivalent tooth thickness curves $h_c(x)$ and $h_b(x)$ evaluated from Eqs.(5.6) and (5.7)

with the defined defect ratios (C_{Lsx} and C_{hsx}). Substituting $h_c(x)$ and $h_b(x)$ into Eq.(5.10), the corresponding cross-sectional properties of the gear tooth can be obtained, see Fig.5-14, and thus, the final TVMS can be obtained by the equations in section 5.3.2.



(a) Finite element model of an irregular spall (b) Geometric model of an irregular spall

Fig.5-12 Tooth spall modelled in irregular shape



(a) Length of tooth spall

(b) Equivalent tooth thickness with spall

Fig.5-13 Gear tooth spall in complicated shape

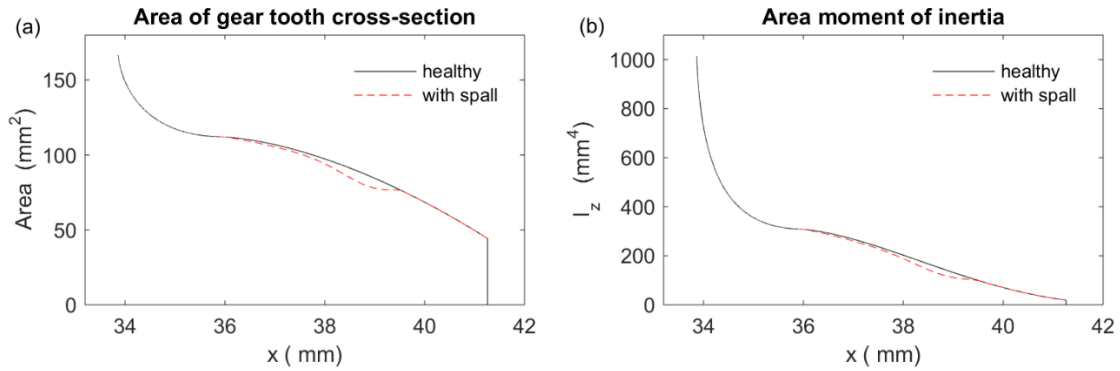


Fig.5-14 Area and area moment of inertia of a gear tooth with irregular tooth spall

Fig.5-14 presents the evaluated area and area moment of inertia of a gear tooth with irregular spall. It can be observed that with the increase of the length of the tooth spall, the effective area and area moment of inertia decrease. The reductions in the cross-sectional properties and effective contact length result in a decrease in the effective stiffness of the gear tooth. The TVMS results of the shape-independent method and the FEM results are plotted in Fig.5-15.

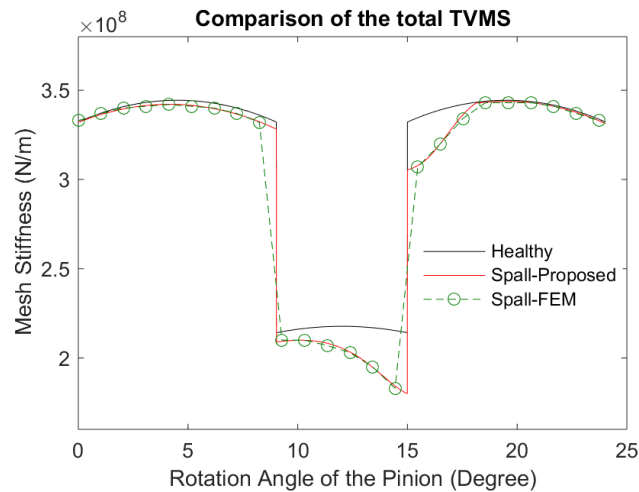


Fig.5-15 The TVMS of a gear tooth pair with irregular tooth spall

It can be observed that the results evaluated by the proposed method and the FEM are close. Due to the shape irregularity, the general geometry-based method cannot be applied in this case, therefore, the result of geometry-based method is not provided.

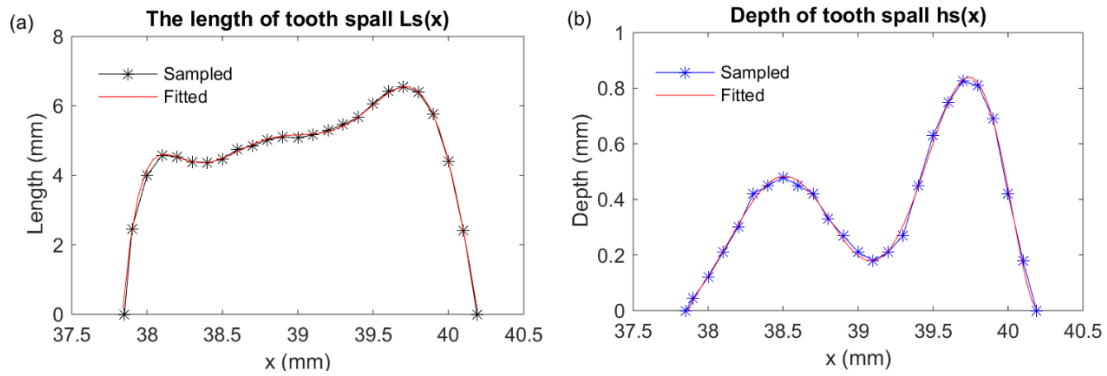
5.4.3.2 Tooth spall with complicated shape and varying depth

In practice, a localized tooth spall can have both irregular shape and varying depth, as shown in Fig.5-16 (a) and (b). A similar-shaped tooth spall is created and is shown in Fig.5-16 (c). The effective contact length of the tooth spall, evaluated by the DSFM, is shown in Fig.5-17 (a).



(a) Practical tooth spall (b) Practical tooth spall [148] (c) Finite element model of spall

Fig.5-16 Tooth spall with irregular shape and varying depth



(a) Length of tooth spall

(b) Depth of tooth spall

Fig.5-17 Gear tooth spall in complicated shape and varying depth

In this case, the depth of the tooth spall h_s varies at different mesh positions within the defect area. The DSFM can be utilized to obtain the varying depth and express the h_s as a function of the distance x . The corresponding sampling points and fitted curve are shown in Fig.5-17 (b). The TVMS of the gear pair with this irregular-shaped tooth spall is shown in Fig.5-18.

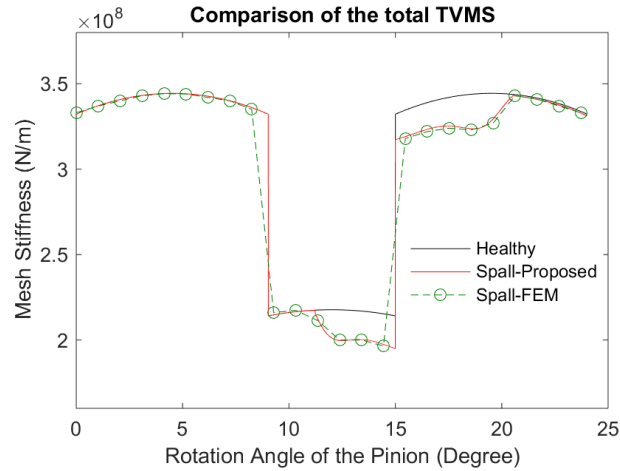


Fig.5-18 TVMS of the gear pair with irregular spall

5.5 The shape-independent method for modelling distributed tooth spalls

Distributed tooth spalls are one of the most common tooth surface defects observed in practice. The occurrence of distributed tooth spalls is mainly due to surface wear and fatigue. Lei et al. [44] proposed a probability method to model the distribution and number of such tooth spalls. The probability method is based on the hypothesis that the tooth spalls have a round shape, are uniformly distributed in the direction of tooth width and are normally distributed in the direction of tooth height. The fundamental idea of the probability method is to provide a way to approximate how many tooth spalls that occur on one tooth under a specific severity level. The shape-independent method proposed in this work models tooth spalls from a different perspective; it does not depend on the number, shape, size and distribution of the tooth spalls as required by the probability method. It directly models the reduction of tooth contact length (by C_{Lsx}) and tooth spall depth (by C_{hsx}). Therefore, it is applicable to practically model all kinds of tooth spalls. The following subsections demonstrate the effectiveness of the shape-independent method in modelling multi-tooth spalls under different distribution conditions.

5.5.1 Modelling multi-tooth spalls with a Gaussian distribution

Fig.5-19 shows some typical distributed tooth spalls that occur in practice. One of the common features of these spalls is that they are distributed around a center line and then decrease, the further from the axis, the sparser the spalls. This distribution feature is a typical Gaussian distribution, see Fig.5-20 (a). In some cases, the bottom of the tooth spalls may not be flat. It is possible that the tooth spalls are deeper around the center line and then become shallow further away from the center line, as is the case shown in Fig.5-19 (a). This feature can also be approximated by a Gaussian function, see Fig.5-20 (b).

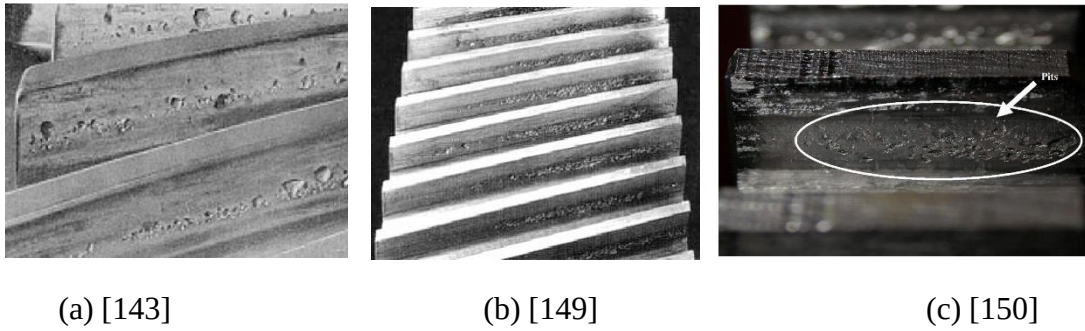


Fig.5-19 Gear tooth spalls with a Gaussian distribution

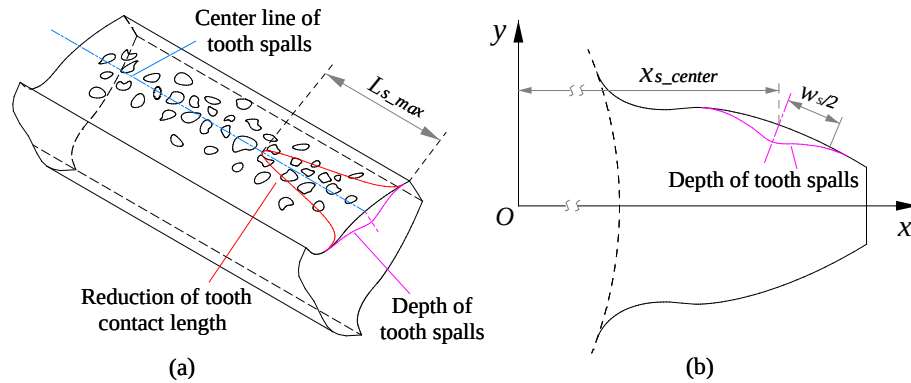


Fig.5-20 Geometric model of distributed gear tooth spalls

For the Gaussian distributed tooth spalls, the defect length ratio can be expressed as

$$C_{Lsx} = \frac{L_s(x)}{L} = \frac{L_{s_max}}{L} \exp\left(-\frac{(x - x_{s_center})^2}{2w_s^2}\right) \quad (5.22)$$

where $L_s(x)$ is the reduction of the total contact length, w_s is the Gaussian width which controls the width of the tooth spall, L_{s_max} and x_{s_center} represent the maximum total length reduction and the position of the center line of the spalls, respectively.

When the bottoms of the spalls are flat and equal in depth, h_s can be assumed to be a constant, as is the case for Fig.5-19 (c). Under a constant depth condition, the defect depth ratio can be simply defined as $C_{hsx} = h_s/(2h_x)$. When the spalls vary in depth, the defect depth ratio approximated by a Gaussian function can be written as

$$C_{hsx} = \frac{h_s}{h_x} = \frac{h_{s_max}}{h_x} \exp\left(-\frac{(x - x_{s_center})^2}{2w_s^2}\right), \quad (5.23)$$

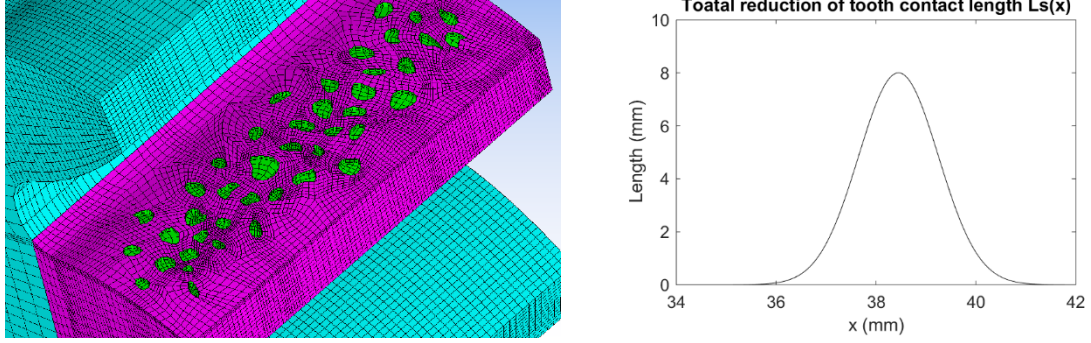
where h_{s_max} is the averaged maximum depth of the tooth spalls at the center line.

The equations of the energy method for gear TVMS evaluation provided section 5.3.2, as well as the equations proposed in other literatures, e.g. [34], are separated into several sections by $x_B < x \leq x_{s_st}$, $x_{s_st} < x \leq x_{s_end}$ and $x_{s_end} < x$. This is due to the fact that the occurrence of tooth spall(s) leads to discontinuities of the functions of cross-sectional area and area moment of inertia. However, for the spalls modelled by Gaussian functions, it is unnecessary to segment the TVMS equations. This is due to the special property of the Gaussian function where the magnitude gradually decreases to zero at both sides of the function. Therefore, the equivalent tooth thickness functions and the cross-sectional properties still maintain their continuities. Thus, the equations for evaluating the TVMS with distributed tooth spalls modeled by Gaussian functions can be simply expressed as

$$\left\{ \begin{array}{l} \frac{1}{k_{ai}} = \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{2LE(1-C_{Lsx}C_{hsx})h_x} dx, \\ \frac{1}{k_{bi}} = \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx_t + \int_{x_B}^{x_{pm}} \frac{f_x}{(1/12)(2h_x)^3 K_{co} LE} dx, \\ \frac{1}{k_{si}} = \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx_t + \int_{x_B}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{2LG(1-C_{Lsx}C_{hsx})h_x} dx, \\ \frac{1}{k_f} = \frac{\cos^2 \alpha_{pm}}{EWC_f^*} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_{pm}) \right\}, \\ k_h = \frac{\pi EL_e C^*}{4(1-\nu^2)} \left(\ln \left(\frac{2\sqrt{h_{1pm}h_{2pm}}}{b_{pm}} \right) - \frac{\nu}{2(1-\nu)} \right)^{-1}, b_{pm} = \sqrt{\frac{8F(1-\nu^2)r_{1pm}r_{2pm}}{\pi L_e E (r_{1pm} + r_{2pm})}}. \end{array} \right. \quad (5.24)$$

In the above equations, C_{Lsx} is expressed by Gaussian function(s) while the C_{hsx} can be any function. It is obvious that Eq.(5.24) is much simpler than the TVMS equations provided in section 5.3.2 and the equations of the probability method provided in [44]. It is worth mentioning that Eq.(5.24) is a simplified version of the equations in section 5.3.2, and the equations in section 5.3.2 are still valid for evaluating the TVMS with tooth spalls with Gaussian distribution conditions.

A finite element model of a distributed spall case is shown Fig.5-21(a). It should be noted that it is very difficult and time consuming to create a refined FEM model of distributed spalls with varying depth. Therefore, the depth of the spalls in this case are assumed equal to a constant value $h_s = 0.7$ mm. The parameters of the spalls modeled by the Gaussian functions are $L_{s_max} = 8$ mm, $w_s = 0.8$ mm, $x_{s_center} = 38.5$ mm. Fig.5-21(b) shows the corresponding curve of the reduction of contact length due to spalls.



(a) Finite element model of distributed tooth spalls (b) The reduction of contact length

Fig.5-21 Model of tooth spall in Gaussian distribution

The corresponding area A_x and area moment of inertia I_z of the gear tooth with distributed tooth spalls are shown in Fig.5-22. It can be observed that, within the spalling area, the magnitudes of A_x and I_z are reduced. The decreases of the effective contact length and cross-sectional properties result in gear TVMS reductions.

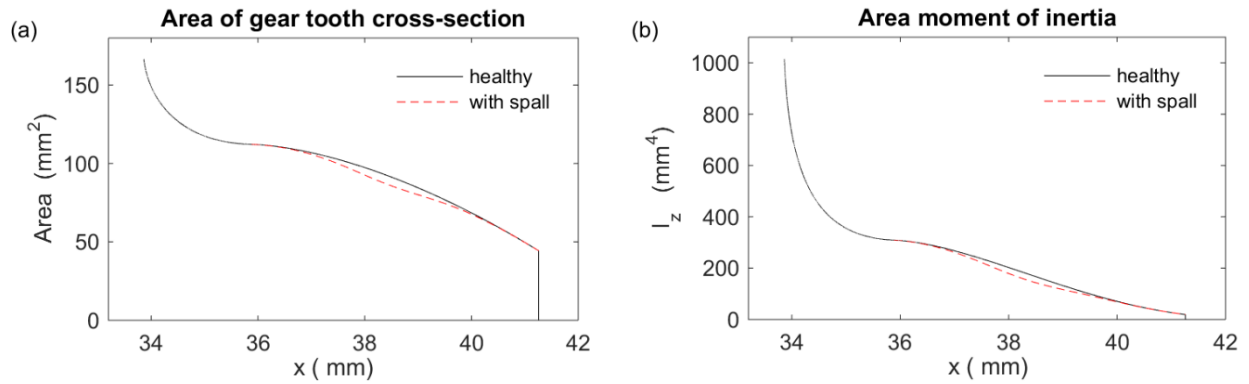


Fig.5-22 Area and area moment of inertia of a gear tooth with Gaussian distributed tooth spalls

The total TVMS with and without defects are shown in Fig.5-23. In this case, the results of the shape-independent method are smoother than the TVMS results evaluated by the FEM method. However, both results of the two methods provide similar trends, which implies that utilizing the Gaussian function to approximate the features of the distributed spall is effective.

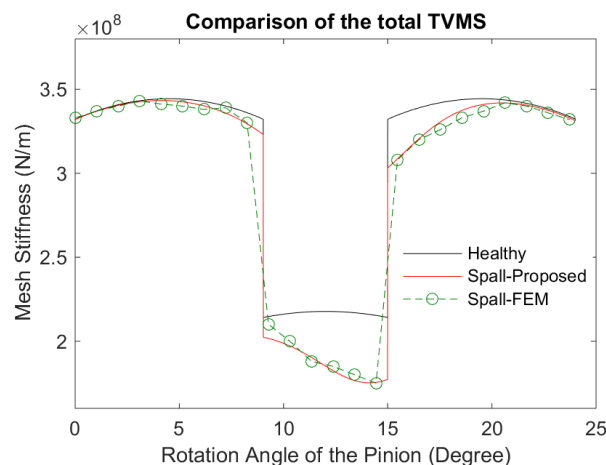
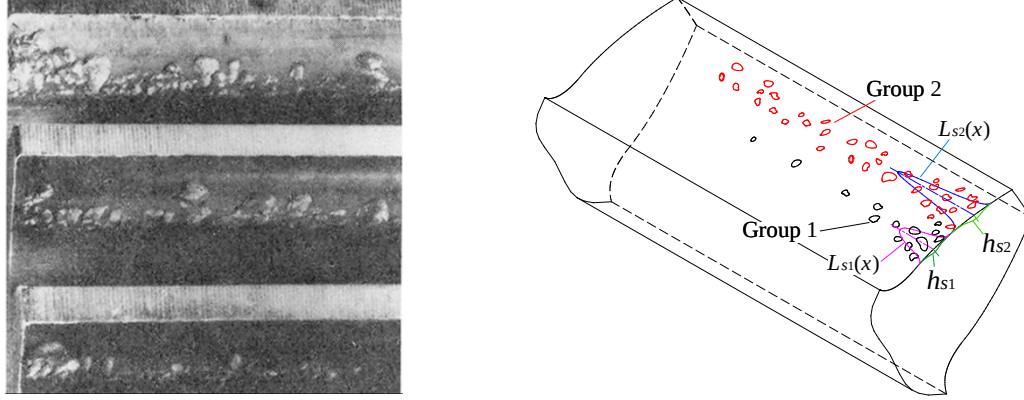


Fig.5-23 TVMS of gear tooth pair with Gaussian distributed tooth spalls

5.5.2 Modelling randomly distributed tooth spalls

In practice, gear tooth spalls may also be randomly distributed, such as the one shown in Fig.5-24 (a). A geometric model of tooth spalls with a similar random distribution feature is shown in Fig.5-24 (b). The shape-independent method directly considers the reduction of the effective contact length and models the length changes by a function. Theoretically, any continuous function can be utilized to model the features of the spall. However, simple functions are always the first choice in solving any engineering problem. For the spalls shown in Fig.5-24 (b), the spalls are divided into two groups, and each group can be modeled by an independent Gaussian function.



(a) Tooth spalls in practice [134, 151] (b) Geometric model of randomly distributed spalls

Fig.5-24 Tooth spalls in randomly distributed conditions

For the randomly distributed multi-tooth spalls, the spalls can be separated into multi-groups, the features of each group can be described by Gaussian functions. The final defect length ratio can be expressed as the sum of the independent sub-functions by

$$\begin{aligned}
 C_{L_{sx}} &= \frac{L_{s1}(x)}{L} + \frac{L_{s2}(x)}{L} + \dots + \frac{L_{sn}(x)}{L} \\
 &= \frac{L_{s1_max}}{L} \exp\left(-\frac{(x - x_{s1_center})^2}{2w_{s1}^2}\right) + \frac{L_{s2_max}}{L} \exp\left(-\frac{(x - x_{s2_center})^2}{2w_{s2}^2}\right) + \dots + \frac{L_{sn_max}}{L} \exp\left(-\frac{(x - x_{sn_center})^2}{2w_{sn}^2}\right)
 \end{aligned}
 \tag{5.25}$$

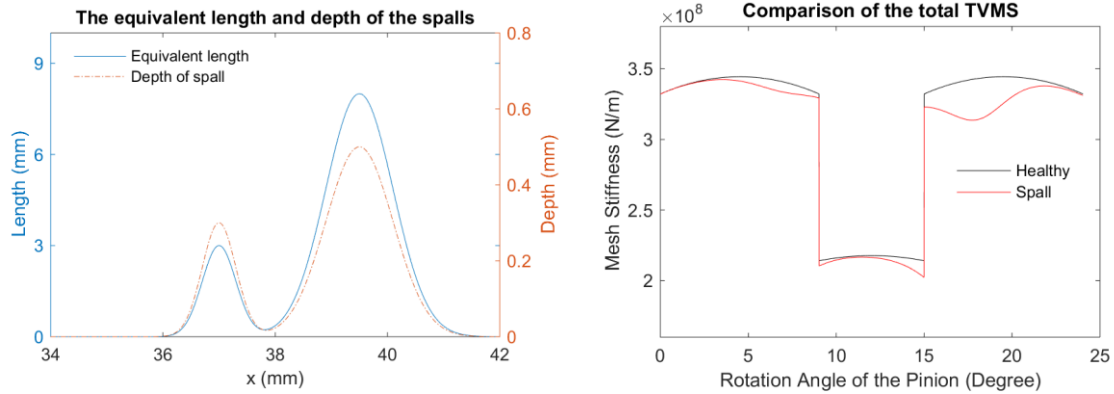
where $L_{sn}(x)$ is the reduction of the total contact length of the n^{th} group Gaussian distributed spalls. w_{sn} , L_{sn_max} and x_{sn_center} define the width, maximum total length reduction and the position of the center line of the n^{th} group spalls.

Under varying depth condition, the defect depth ratio of randomly distributed spalls expressed by Gaussian functions can be written as

$$\begin{aligned}
 C_{hsx} &= \frac{h_{s1}}{h_x} + \frac{h_{s2}}{h_x} + \dots + \frac{h_{sn}}{h_x} \\
 &= \frac{h_{s1_max}}{h_x} \exp\left(-\frac{(x - x_{s1_center})^2}{2w_{s1}^2}\right) + \frac{h_{s2_max}}{h_x} \exp\left(-\frac{(x - x_{s2_center})^2}{2w_{s2}^2}\right) + \dots + \frac{h_{sn_max}}{h_x} \exp\left(-\frac{(x - x_{sn_center})^2}{2w_{sn}^2}\right)
 \end{aligned}
 \quad (5.26)$$

where h_{n_max} is the averaged maximum depth of the n^{th} group of Gaussian distributed tooth spalls at the center line.

For the case shown in Fig.5-24 (b), the tooth spalls are divided into two groups. The corresponding parameters for group 1 are $L_{s1_max} = 3$ mm, $h_{s1_max} = 0.3$ mm, $w_{s1} = 0.3$ mm, $x_{s1_center} = 37$ mm, and the parameters for group 2 are $L_{s2_max} = 8$ mm, $h_{s2_max} = 0.5$ mm, $w_{s2} = 0.6$ mm, $x_{s2_center} = 39.5$ mm. The equivalent length and depth of the spalls and the corresponding TVMS of the gear pair are shown in Fig.5-25.



(a) Properties of the tooth spalls

(b) TVMS evaluated by the proposed method

Fig.5-25 The features of tooth spalls and corresponding TVMS

In this case, both of the equivalent length and depth of the spalls are modeled by Gaussian functions. The TVMS of the gear pair is evaluated by Eq.(5.24). As can be observed in Fig.5-25(b), the TVMS of the gear tooth decreases within the spalling area. The tooth spalls of group 1 result

in much less TVMS reductions compared to group 2. This is due to the fact that both of the equivalent length and depth of the spalls of group 1 are much less than the values of the group 2.

5.6 Conclusions

5.6.1 Summary of the proposed method

In this chapter, a shape-independent method was proposed to model tooth spalls from a different perspective. The fundamental idea of this method is to use two defect ratios, namely the defect length ratio C_{Lsx} and the defect depth ratio C_{hsx} , to directly model the features and severity of the gear tooth spalls instead of using a pre-specific geometry to represent the properties of the spalls. In the shape-independent method, the relationship between the effective contact length $L_e(x)$ and the tooth cross-sectional properties (A_x and I_z) under spalling conditions is identified, which makes it possible to directly express these parameters ($L_e(x)$, A_x and I_z) as functions of the defect ratios (C_{Lsx} and C_{hsx}). Therefore, with the knowledge of these two defect ratios, the values of $L_e(x)$, A_x and I_z can be evaluated, and thus the corresponding TVMS of the tooth pair with spall can be calculated. The defect ratios can be expressed by a function which is determined by the type and feature of the tooth spall. The shape-independent method is capable of modeling most kinds of spalls, such as a localized tooth spall with simple shapes (rectangular, round, elliptical, etc.) or irregular shapes, and spalls under normal or randomly distributed conditions. The effectiveness of the proposed shape-independent method in modelling different kinds of tooth spall(s) has been examined and validated by FEM analysis.

The proposed shape independent method provides an alternative way to model tooth spalls of a spur gear pair operating under normal tooth contact conditions. In practice, mating gear tooth pairs may also be subject to tooth profile modification [152], eccentric errors [29], gear manufacturing errors, and misalignment. These effects are not considered in this chapter. The

proposed method can be modified to consider these effects by including the corresponding fault effects into the calculation of gear TVMS with multiple types of faults. In addition, further extensions and applications are possible with the proposed method, for example modelling the effect of a localized spall on a bearing race e.g. [153-156] and the effect of extended tooth contact [58, 62, 142, 157] under heavy load conditions.

5.6.2 Advantages of the proposed method

Existing methods use either geometry-based methods or probability methods and mainly focus on modeling a specific type of spall with a specific shape. Changing the type or shape of the spall implies changing the TVMS evaluation method. Since the methods for evaluating different types of spalls vary significantly, coding different methods is time consuming and inconvenient. The use of the defect ratio functions to model the features of tooth spalls in the proposed shape-independent method makes the method “adaptive” to the spalls of various sizes and shapes at different locations and with different distribution conditions. The only change to the algorithm of the shape-independent method when modelling a different type of spall is in the functions of the defect ratios (C_{Lsx} and C_{hsx}). Therefore, the proposed method is general in nature and is much easier to implement compared with existing methods.

Chapter 6 Dynamical modeling and experimental validation for tooth pitting and spalling in spur gears

This chapter addresses objectives 4 and 5 which are to: a) propose a novel spur gear dynamical model to better understand the dynamic behavior of a gear system with tooth pitting and spalling. b) analyze the fault features of a gear transmission with tooth pitting and spalling through the proposed dynamic model and compare with features obtained by experiments.

The contents of this chapter have been published in the Journal of Mechanical Systems and Signal Processing.

Y. Luo, N. Baddour, and M. Liang, "Dynamical modeling and experimental validation for tooth pitting and spalling in spur gears," *Mechanical Systems and Signal Processing*, vol. 119, pp. 155-181, 2019.

Part of the contents have also been published in the *Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2018.

Y. Luo, N. Baddour, and M. Liang, Dynamical Modeling of Gear Transmission Considering Gearbox Casing. ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 8: 30th Conference on Mechanical Vibration and Noise: V008T10A024, 2018. doi:10.1115/DETC2018-85656.

6.1 Abstract

This chapter proposes a novel spur gear dynamical model, validated by various experimental tests, to analytically investigate the effects of tooth pitting and spalling on the vibration responses of a gear transmission. The proposed dynamical model considers the effects of tooth surface roughness changes and geometric deviations due to pitting and spalling, and also incorporates Time Varying Mesh Stiffness (TVMS), a time-varying load sharing ratio, as well as dynamic tooth contact friction forces, friction moments and dynamic mesh damping ratios. The proposed gear dynamical model is validated by comparison with responses obtained from an experimental test rig under different conditions. Comparisons indicate that the responses of the proposed dynamical model are consistent with experimental results, in both time and frequency domains under different rotation speeds and fault severity conditions.

6.2 Introduction

Gear tooth pitting and spalling are typical tooth surface fatigue damages in a gear transmission. The appearance of tooth pitting is mainly due to prolonged, repeated heavy contact loads, whereby excessive local Hertzian contact fatigue stress flakes the asperity particles out of the contact surface [100-102]. The occurrence of pitting and spalling directly changes the tooth contact conditions by increasing the roughness of the tooth surfaces, modifying the tooth geometric profiles and reducing the effective tooth contact length. Due to the complicated structures and intricate interactions between the components of the gear system, fault vibration features and corresponding vibration mechanisms due to tooth spalling remain mostly unknown.

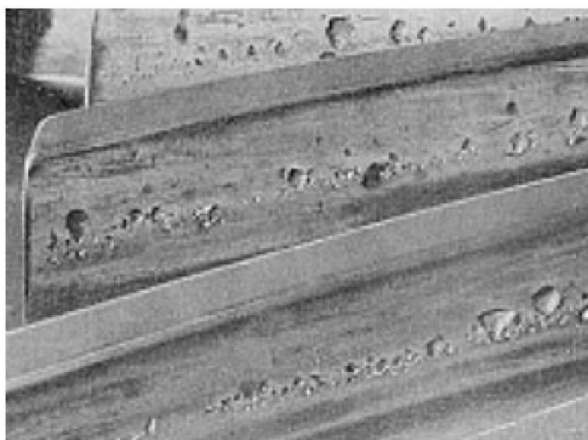
Dynamical modeling of gear transmissions with faults has been an important research topic for understanding gear fault vibration characteristics [139]. Dynamic models simulating gear tooth pitting and spalling are generally the same models used to analyze the effect of a gear tooth crack,

which ignored the effects of tooth geometric profile errors and surface contact condition changes (e.g. increase of surface roughness, change of dynamic friction forces and damping of lubrication film etc.), and thus inadequate to fully understand the complicated internal interactions between the mating gear pairs, as well as to explain the gear fault vibration mechanisms.

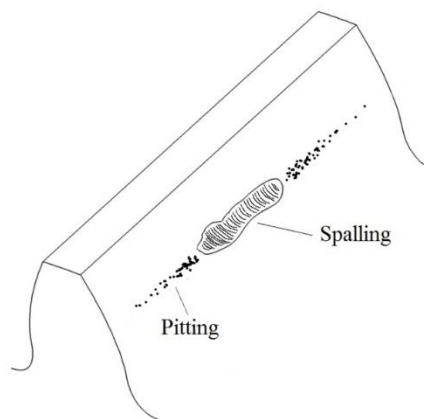
In light of this, the goal of this chapter is to propose a new dynamic model to simulate the vibration behavior of a spur gear transmission with pitting and spalling defects. A gear contact model is proposed to account for the effects of tooth surface roughness changes and geometric deviations due to pitting and spalling with lubrication effects. The proposed model considers the effect of gearbox casing, incorporates TVMS, time-varying load sharing ratio, dynamical tooth contact damping ratios, friction forces and friction moments. Experimental tests are performed to validate the proposed dynamic model under healthy cases as well as different fault conditions. The system physical properties (e.g. mass, mass moment of inertia) and mechanical parameters (e.g. stiffness and damping coefficients of bearings and bolts) used in the simulations were obtained by direct measurement or were calculated based on experimental tests combined with dynamic theories. The results were examined in both time and frequency domains, under different rotation speeds and fault severity conditions. Spectrum comparisons are provided to identify the difference between damaged and healthy cases.

6.3 Gear contact model with tooth pitting and spalling

In practice, tooth pitting and spalling are normally first initiated on the smaller gear (which undergoes more revolutions and therefore more stress cycles) and close to the tooth pitch line [100], see Fig. 6-1. The size of what delineates pitting and spalling has not been uniformly defined and both terms are often used interchangeably. Based on [158, 159], the size of pitting can range from 0.01 mm (micro-pitting) to 0.8 mm (macro-pitting) in diameter, and the size of tooth spall is considerably larger than tooth pitting.



(a) Pitting and spalling in practice [160]



(b) Simplified pitting and spalling

Fig. 6-1 Gear tooth pitting and spalling

The contact model of a spur gear pair with lubrication under tooth pitting and spalling is shown in Fig. 6-2, where the contact line just crosses the defect area near the dedendum of tooth 1. In practice, a gear transmission is always lubricated in order to reduce contact friction and wear [31, 105]. As defined in [105, 158], micro-pitting is usually in small size, therefore, its effect can be modeled as the increase of the tooth surface roughness. The tooth spalling (or macro-pitting) is often in large size which is capable of blocking the formation of a lubricant film and reducing the effective tooth contact length, see Fig. 6-2. The reduced tooth contact length directly decreases the strength of the contacting teeth on supporting the dynamic loads, which is generally modeled by decreased gear TVMS within the spalling area [141].

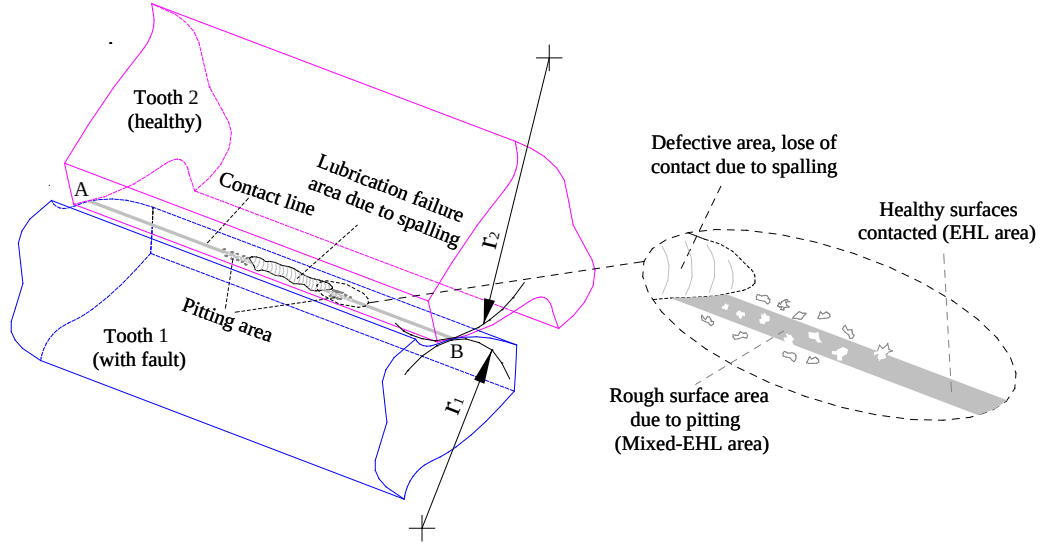


Fig. 6-2 The contact model of a spur gear pair with tooth pitting and spalling

The mating process of a spur gear pair with tooth contact friction forces is shown in Fig. 6-3. The line of action B_1B_2 is tangent to the base circles of the pinion and gear at points B_1 and B_2 . Line segment P_1P_2 is the actual effective contact locus, where point P_1 is the starting contact point (approach point), and P_2 corresponds to the separation point. α_0 is the pressure angle, ω_1 and ω_2 are rotational speeds. F_{pg} is the total dynamic mesh force of the mating process (along the action line B_1B_2). F_{g1} and F_{p1} , F_{g2} and F_{p2} , represent the tooth contact friction forces of each mating tooth pair. The mating tooth can be simplified as equivalent cylinders in contact with time-varying radii of the tooth curvatures r_1 and r_2 (Fig. 6-2) or r_{p1} , r_{p2} , r_{g1} , r_{g2} (Fig. 6-3) and forming a lubrication film with width of $2b$. $u_{p1} = \omega_1 \cdot r_{p1}$, $u_{g1} = \omega_2 \cdot r_{g1}$ are the moving speeds of the contact point of each surface along the off-line of action [30]. For ordinary spur gear pairs with contact ratio less than 2, the length of the time-varying radii (r_{pi} , r_{gi}) can be determined from

$$\begin{cases} r_{pi} = r_{bp} (\theta_{pi} + \tan(\angle B_1 O_p P_1)) \\ r_{gi} = (r_{pp} + r_{pg}) \sin \alpha_0 - r_{pi} \end{cases}, \quad i = 1, 2, \quad (6.1)$$

in which i denotes the i th gear tooth, θ_{pi} is the contact angle of the i th tooth pair. For a spur gear pair, the contact angle of the followed second gear tooth lags with respect to the first gear tooth pair by $\theta_{p1} - 2\pi/N$ where N is the number of gear teeth. If $\theta_{p2} = \theta_{p1} - 2\pi/N < 0$, this implies that the second gear pair has not started to mesh and there is only one gear tooth pair in contact. The

$$\angle B_1 O_p P_1 = \arccos(R_{bp}/\overline{O_p P_1}) \quad \text{with} \quad \overline{O_p P_1} = \sqrt{R_{ag}^2 + \overline{O_p O_g}^2 - 2R_{ag} \cdot \overline{O_p O_g} \cos(\angle O_p O_g P_1)} \quad \text{and}$$

$$\angle O_p O_g P_1 = \arccos(R_{bg}/R_{ag}) - \alpha_0 \quad [29],$$

r_{bp} and r_{bg} , r_{pp} and r_{pg} , r_{ap} and r_{ag} are the radii of the base circle, pitch circle, and addendum circle of pinion and gear, respectively (see Fig. 6-3).

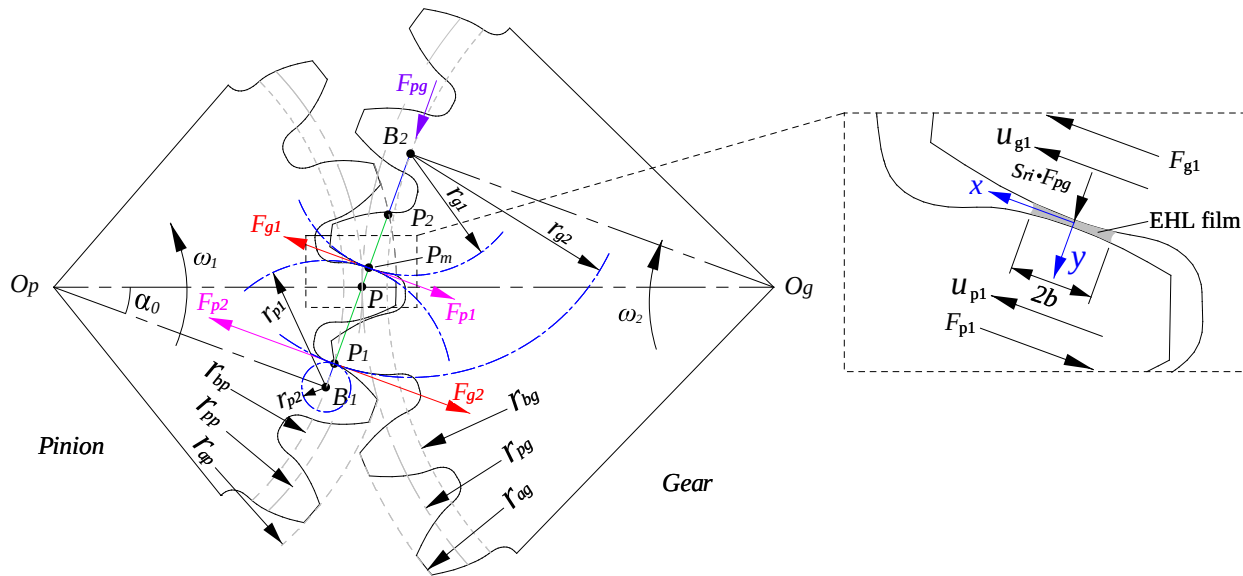


Fig. 6-3 Mating process of a spur gear pair with friction forces

The evaluation of the dynamical friction forces and damping ratios of the contacting tooth surfaces with lubrication effects involves the analysis of the lubrication film thickness, contact pressure, oil viscosity, surface roughness, moving velocity, effective contact area, and film shear stress. The mixed elastohydrodynamic lubrication (mixed-EHL) theories are widely applied to analyze the contact properties of a mating gear tooth pair with gear tooth pitting and spalling. The gear tooth in the healthy surface area is assumed smooth, in which the asperity has limited effects

on film thickness and pressure distribution. Therefore, the two surfaces in contact are assumed supported by a lubrication film [161]. For an arbitrary gear mesh point, the line contact mixed-EHL equations under isothermal conditions can be expressed as [162]

$$\begin{cases} \frac{d}{dx} \left(\frac{\rho h^3}{\eta} \frac{dp}{dx} \right) = 12u_s \frac{d(\rho h)}{dx} \\ h = h_c + \frac{x^2}{2R} + v(x) + r(x) \\ v(x) = -\frac{2}{\pi E} \int_{x_0}^{x_e} p(s) \ln(s-x)^2 ds + c_k \\ w - \int_{x_0}^{x_e} p(x) dx = 0 \end{cases} \quad (6.2)$$

where x corresponds to the x -axis of the coordinate system, as shown in the zoomed area of Fig. 6-3. x_0 and x_e are the coordinates of inlet and outlet positions, respectively. R is the equivalent radius, u_s is the equivalent speed, p is the contact pressure, h_c is the central film thickness, h is the film thickness, $v(x)$ is the surface elastic deformation, s is the integration variable on the x axis, $r(x)$ is surface roughness function, c_k is a constant, w is the load per unit contact length. ρ is the oil density which is a function of contact pressure and can be approximated as $\rho = \rho_0 (1 + 0.6p / (1 + 1.7p))$ in which ρ_0 is the lubricant density at ambient pressure [162]. η is the oil viscosity as a function of the contact pressure, based on [161, 163], it can be expressed as

$$\eta = \begin{cases} \eta_0 \exp(\alpha_1 p), & p < p_a \\ \eta_0 \exp(c_0 + c_1 p + c_2 p^2 + c_3 p^3), & p_a \leq p \leq p_b \\ \eta_0 \exp(\alpha_1 p_t + \alpha_2 (p - p_t)), & p > p_b \end{cases}, \quad (6.3)$$

where $\alpha_{1,2}$ are pressure exponential parameters, c_0 to c_3 are third-order fitting coefficients in order to smoothly connect the first and third section, specifically, the values of c_0 to c_3 are dominated by the end and start values as well as the trends of the viscosity curve of the first and third section. Based on [161], one set of basic parameters under 25°C working condition is given as:

$$\eta_0=0.0479[Pa\cdot s], \quad \alpha_1=18.7[GP a^{-1}], \quad \alpha_2=2.9[GP a^{-1}], \quad p_a=0.406[GP a], \quad p_b=0.812[GP a], \\ p_t=0.580[GP a].$$

The contact energy of a mating gear pair can be dissipated by both lubricant film and solid structures [164]. Therefore, the total damping coefficient of a contacting gear tooth pair generally includes both of the damping of the lubricant film c_{film} and the mating gear teeth c_{str} . The total damping of a meshing gear tooth pair can be evaluated by $c_m = c_{film} + c_{str}$. The dynamic viscous damping coefficient of the lubrication film along the Off Line of Action (OLOA) direction is the function of oil viscosity, film thickness and contact area which can be expressed as [161]

$$c_{film} = A \left[\sum_{j=1}^J \left(\frac{\eta}{h} \right)_j \right], \quad (6.4)$$

where A is the uniform area of the tooth surface grid element, j is the j -th grid element, η is the dynamic oil viscosity evaluated by Eq.(6.3), and h is the dynamic lubrication film thickness solved from Eq.(6.2). The existence of gear tooth pitting and spalling increases the gear tooth surface roughness and may decrease the effective tooth contact length, which results in lubricant condition variation. Fig. 6-4 shows the change in the distribution of instantaneous EHL film and pressure of a mesh point as a result of gear tooth pitting and spalling.

Fig. 6-4 (a) is the dimensionless tooth surface roughness (with amplitudes between 0 to 1) due to tooth pitting and spalling. The actual surface roughness during the simulation is controlled by the roughness coefficient (R_c) times the amplitudes of the dimensionless roughness. For healthy condition, the gear tooth surface is assumed as smooth and, therefore, R_c is low. With the appearance of tooth pitting, surface roughness increases. Comparing with Fig. 6-4 (b) and (c), there are some local pressure fluctuations due to the occurrence of the tooth dent, and the lubricant film thickness of the pitting case is slightly higher than that of the healthy conditions ($h_c=0.62 \mu m$ in

the healthy case, $h_c=0.66 \mu\text{m}$ and $h_c=0.64\mu\text{m}$ for case *b* and *c*, respectively). As tooth pitting progresses to a diameter larger than the width of the EHL film (about 0.4 mm, see the *x*-axis of Fig. 6-4 (b)), tooth spall forms and the effective tooth contact line L_e decreases. Under this condition the tooth pitting and spalling co-exist, therefore one can observe strong lubricant pressure fluctuation.

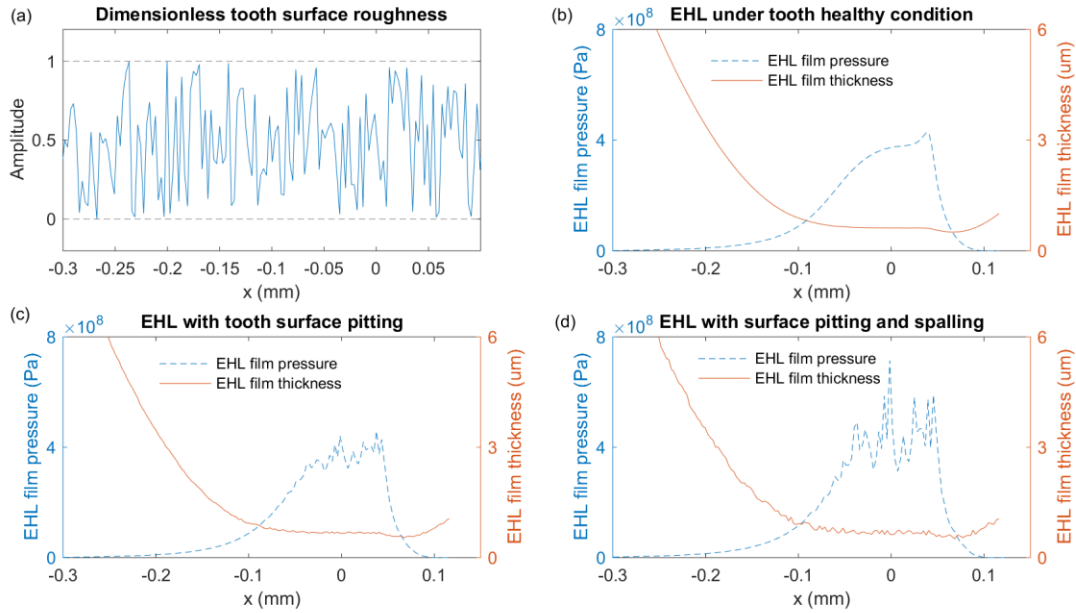


Fig. 6-4 The effects of gear tooth pitting and spalling on EHL film and pressure (under the conditions of $w=7 \times 10^4 \text{ N/m}$, $R=0.0071 \text{ m}$, $T=25^\circ\text{C}$, $\omega_1=1500 \text{ rpm}$, $\rho_0=864.80 \text{ kg/m}^3$, $\eta_0=0.0479 \text{ Pa}\cdot\text{s}$; and $L_e=0.016 \text{ m}$ & $R_c=0.001$ for (b), $L_e=0.016 \text{ m}$ & $R_c=0.05$ for (c), $L_e=0.014 \text{ m}$ & $R_c=0.1$ for (d), where R_c is surface roughness coefficient)

Given the film thickness and pressure distribution, as presented in Fig. 6-4, the damping coefficient of the lubricant film c_{film} along the OLOA direction can be obtained when combined with Eq.(6.3). However, the evaluation process can be computationally expensive, especially when coupling with a gear dynamic model. Recently, Masjedi and Khonsari [165] proposed curve-fit formulas to estimate the dynamic lubrication film thickness and asperity load sharing ratio under

different surface asperity orientation conditions (e.g. isotropic, longitudinal and transverse). The formulas proposed in their work were fitted based on 2000 simulations, and the average evaluation errors were less than 3% compared with the results obtained directly from the mixed-EHL model. The dynamical central film thickness of a mating gear tooth pair are given by [165, 166]

$$h_c = 2.691K_{Hc}W^{-0.135}U^{0.705}G^{0.556}R\left(1+0.2\bar{\sigma}^{1.222}V^{0.223}W^{-0.229}U^{-0.748}G^{-0.842}\right), \quad (6.5)$$

where W is the dimensionless load given by $W = w/E'R$, in which w is load per unit contact length, R is the equivalent contact radius given by $R = (1/r_1 + 1/r_2)^{-1}$, $E' = 2\left[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2\right]^{-1}$ is the effective Young's modulus, where ν_1, ν_2 and E_1, E_2 are Poisson's ratio and Young's modulus of tooth one and two, respectively. U is dimensionless speed expressed as $U = \mu_0 u/E'R$ where μ_0 is lubricant viscosity at zero pressure, $u = (u_{p1} + u_{g1})/2$ is the equivalent rolling speed, G is a dimensionless material coefficient given as $G = E'\alpha_v$ where α_v is pressure-viscosity coefficient. V is dimensionless hardness, $\bar{\sigma} = \sigma/R$ is dimensionless surface roughness, where σ is the standard deviation of the surface roughness [165]. K_{Hc} is coefficient utilized to model the orientation effects of the surface asperities. When the surface asperities are isotropic, $K_{Hc} = 1$. When surface asperities have longitudinal or transverse properties, K_{Hc} is given by [165]

$$K_{Hc} = \begin{cases} 1 + 0.354\Lambda^{-1.351}(1-\gamma)^{2.261}; & \gamma < 1 \\ 1 - 0.135\Lambda^{-1.430}(\gamma-1)^{0.329}; & \gamma > 1 \end{cases}, \quad (6.6)$$

where Λ is the film parameter defined in [165], γ is a surface pattern parameter, $\gamma < 1$ for surface with transverse asperity properties, $\gamma = 1$ for isotropic, and $\gamma > 1$ for longitudinal [165]. With the ledge of h_c , the evaluation of the damping coefficient of the lubricant film can be approximated by

$$c_{film} = \frac{\eta^* A^*}{h_c}, \quad (6.7)$$

where η^* is the oil viscosity of the film center given by Eq.(6.3) with the dynamic lubricant pressure approximated by the Hertz pressure $p = 2w/\pi b$. A^* is the total area of the lubrication film approximated as $A^* = 2bL_e$ in which b is the half width of the lubricant film given by $b = \sqrt{8wR/(\pi E')}$ [162]. It should be noted that, Eq.(6.5) works best for dimensionless load $w \in [2, 50] \times 10^{-5}$, for a load higher or lower than this range, Eq.(6.2) and Eq.(6.4) should be utilized to validate the results.

Fig. 6-5 shows the effects of tooth pitting and spalling on the tooth contact load sharing ratio (LSR), EHL central pressure and film thickness evaluated by Eq.(6.5) under constant load ($F_{pg}=1000\text{ N}$) and speed conditions ($\omega=1500\text{ rpm}$). The number of teeth of pinion and gear are $N_1=16$ and $N_2=48$ respectively. The horizontal-axis in Fig. 6-5 is the distance of the mesh point relative to the pitch point (point P in Fig. 6-3) along the line of action, where a negative distance value indicates the mesh point is on the left side of point P (within line segment P_1P), and vice versa for positive distance values. Due to the considerable high gear ratio (3:1) and small base radius of the driving pinion, the contact equivalent radius and speed are in small values at the approach point (P_1 in Fig. 6-3). As the mating process progresses from P_1 to P_2 , the equivalent contact radius and speed increase, see Fig. 6-5 (a), and the lubricant condition improves as a result of increased lubricant film thickness and decreased central pressure, Fig. 6-5 (b). The high central pressure at the starting contact point is mainly due to the low equivalent radius at the beginning. Lower equivalent radius R indicates higher dimensionless load ($W = w/(E'R)$) and smaller Hertzian contact width ($b = \sqrt{8wR/(\pi E')}$), both increasing the contact pressure, as explained in [167]. It also can be observed that within the area where $LSR=1$, the central pressure is

significantly increased due to the total load shared by only one tooth pair. These results are consistent with the conclusions made in [165, 167, 168].

Gear tooth pitting is modelled as an increase of surface roughness, therefore, it has no effect on the magnitudes of central pressure. However, within the pitting area ($x \in [-5, -3]$), the lubricant film thickness ripples when the surface roughness standard deviation gradually increases from healthy ($\sigma=0.35 \mu\text{m}$) to the maximum value of $\sigma=0.7 \mu\text{m}$ and then back to the normal governed by a Gaussian function. The result of increasing film thickness due to increasing surface roughness is consistent with the conclusions made in [165, 166], and this phenomenon is explained by [106] that “small asperities play a useful role as a reservoir for the lubricant by entrapment between asperities”.

Under gear tooth spalling conditions (within $x \in [-5, -3]$), the effective tooth contact length L_e gradually decreases from 0.016 m to 0.014 m and then increases back to 0.016 m, which increases the load per unit length $w = F_{pg} / L_e$, and hence, results in an increased central pressure within the spalling area, Fig. 6-5 (b). The change of the increased contact pressure should result in decreased film thickness. However, it may not be the case when the surface roughness also increases. As can be observed in Fig. 6-5 (b), the film thickness increases as the roughness gradually changes from $\sigma=0.35 \mu\text{m}$ to a maximum value of $\sigma=1.2 \mu\text{m}$ and then returns to $0.35 \mu\text{m}$ within the defect area.

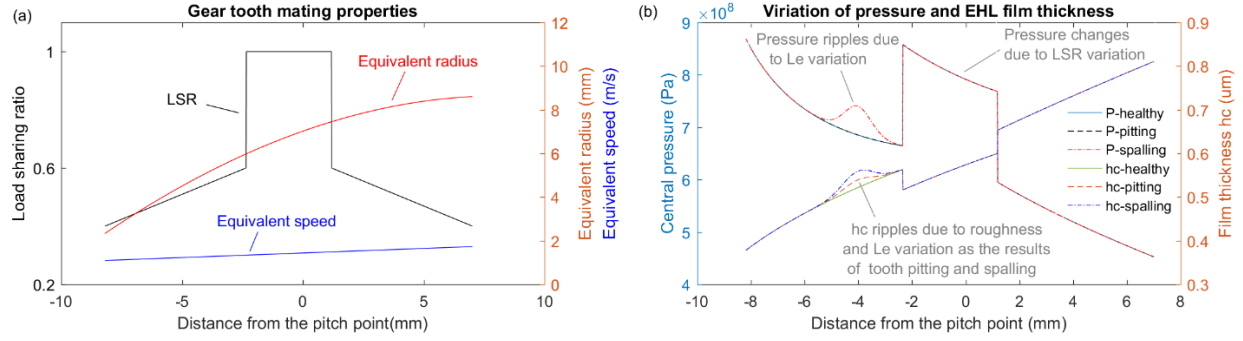


Fig. 6-5 Comparison of the gear tooth mating properties and EHL features under healthy and faulty conditions along the line of action for one mating gear tooth pair (roughness is assumed as isotropic; for healthy condition $L_e=0.016$ m; for pitting $L_e=0.016$ m; for spalling $L_e=0.014$ m)

The corresponding damping coefficient of the EHL film c_{film} along the OLOA direction for one tooth pair over one mesh cycle evaluated by Eq.(6.7) is shown in Fig. 6-6. Within the single tooth contact area ($x \in [-2.35, 1.2]$), c_{film} is much higher than the magnitude in the double tooth contact zone ($x \in [-8.19, -2.35) \cup (1.2, 7.01]$) which is similar to the conclusion made in [161]. The increased damping ratio within the single tooth pair contact area is mainly due to the increased lubricant viscosity, which has an exponential dependence on the hydrodynamic pressure as presented in Eq.(6.3). It also can be observed that both gear tooth pitting and spalling result in a decreased damping coefficient within the faulty area, see Fig. 6-6 (b). The decrease of c_{film} within the faulty area is mainly due to the increase of lubricant film thickness.

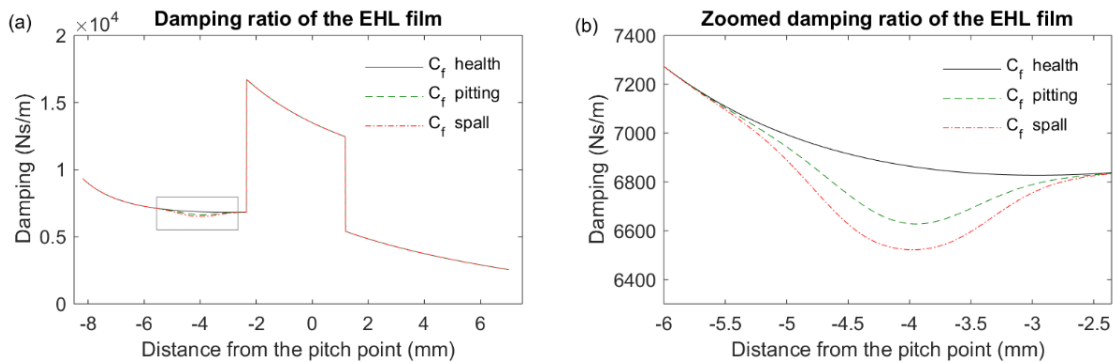


Fig. 6-6 Damping coefficient of the EHL film under different fault conditions

The structural damping coefficient c_{str} is different from the lubricant film damping c_{film} . c_{str} is relative to the material properties (e.g. stiffness) of the gear tooth and it exists even if there is no lubrication. Based on [169-171], c_{str} is proportional to the time varying mesh stiffness of the gear pairs. A typical damping equation given by [171] is

$$c_{str} = \frac{6(1-e)}{(2e-1)^2 + 3} \frac{K_{str}}{V_i} \quad (6.8)$$

where V_i is the initial relative impact velocity between the teeth, $e = 1 - 0.022V_i^{0.36}$, K_{str} is the stiffness of the mating gear pair without lubricant. It can be seen that c_{str} is the damping coefficient of the gear tooth structure determined by the mechanical material properties of the mating gear teeth which is a function of the gear mesh stiffness, and can be expressed as $c_{str} = c_0 K_{str}$, where c_0 is a coefficient.

The dynamic contact friction coefficient under mixed-EHL conditions can be calculated using a friction model proposed by Xu et al. [172], as given in Eq.(6.9) and (6.10). This model is based on regression of experimental tests under various working conditions and has been validated by both simulated and experimental data in [172]. The dynamic friction coefficient as per Xu et al. [172] is given by

$$\mu(t) = \pm e^f P_h^{b_2} |S_r|^{b_3} V_e^{b_6} \nu_0^{b_7} R^{b_8}, \quad (6.9)$$

where

$$\begin{cases} f = b_1 + b_4 |S_r| P_h \log_{10}(\nu_0) + b_5 e^{-|S_r| P_h \log_{10}(\nu_0)} + b_9 e^S \\ S_r = 2(u_{pi} - u_{gi}) / (u_{pi} + u_{gi}) \\ V_e = 0.5(u_{pi} + u_{gi}) \end{cases}, \quad (6.10)$$

and where u_{pi} and u_{gi} are the moving speed of the contact surfaces of the i th gear pair. When $u_{pi} > u_{gi}$, $\mu(t)$ is negative, when $u_{pi} = u_{gi}$, $\mu(t) = 0$, and when $u_{pi} < u_{gi}$, $\mu(t)$ is positive. ν_0 is the inlet oil viscosity (a function of the inlet oil temperature), S is the RMS composite surface roughness, S_r is slide to roll ratio, V_e is entrainment velocity, P_h is Hertzian pressure, and b_1 to b_9 are regression coefficients, with $b_{1-9} = -8.92, 1.03, 1.04, -0.35, 2.81, -0.10, 0.75, -0.39, 0.62$ [172]. The total friction force of the gear pair can be written as

$$F_{pf} = -F_{gf} = \sum_{i=1}^n \mu_i s_{ri} F_{pg}, \quad i = 1, 2, \quad (6.11)$$

where μ_i is the dynamic friction coefficient of the i th gear tooth pair evaluated by Eq.(6.9), F_{pg} is the total gear mesh contact force, and s_{ri} is the load sharing ratio between the mating tooth pairs. The equation of the load sharing ratio can be expressed as

$$s_{ri} = \frac{F_{pgi}}{F_{pg}} = \frac{k_i(t)\xi}{k_1(t) \cdot \xi + k_2(t) \cdot \xi} = \frac{k_i(t)}{k_1(t) + k_2(t)}, \quad i = 1, 2, \quad (6.12)$$

where F_{pgi} is the contact load shared by the i th gear tooth pair, ξ is the contact strain, and $k_1(t)$, $k_2(t)$ are the mesh stiffness of each gear pair at time t . The value of $k_1(t)$ and $k_2(t)$ can be evaluated based on the potential energy method explained in Section 3.

The total moment of friction forces (M_p, M_g) acting on the pinion and gear can be written as

$$\begin{cases} M_p = \sum_{i=1}^n r_{pi} \mu_i s_{ri} F_{pg} \\ M_g = \sum_{i=1}^n r_{gi} \mu_i s_{ri} F_{pg} \end{cases}, \quad (6.13)$$

where r_{pi} and r_{gi} are the length of the time-varying radii of the i th tooth curvatures and can be evaluated by Eq.(6.1).

In this study, the effect of gear tooth pitting is modeled as an increased roughness of the contact surfaces, which is captured by the change in the value of S in Eq. (6.10). The occurrence of tooth spall is modeled as a result of decreasing the effective tooth contact length L_e . However, under severe tooth spalling conditions, the geometric shape of the defect tooth may also change significantly; therefore, another parameter ς is utilized to represent the periodic tooth deviations under severe fault conditions. The occurrence of tooth geometric deviations is modeled as an effect of increasing the gear transmission errors, which is directly incorporated in the tooth contact force equation, Eq. (6.30) in section 4.

The decrease in the effective tooth contact length due to tooth spall leads to an increased load per unit length (w) and results in an increased hydrodynamic pressure in Eq. (6.3), as well as increased Hertzian pressure P_h in Eq.(6.10). The change of the tooth surface roughness S and Hertzian pressure P_h due to tooth pitting and spalling directly modifies the value of the dynamic friction coefficient in Eq.(6.9), which finally results in different contact friction forces and friction moments.

6.4 Modelling and evaluation of the TVMS of gear pair with tooth spalling

The occurrence of tooth spalling affects the amplitude of the time-varying mesh stiffness (TVMS) of a mating tooth pair. Under lubrication conditions, the deformation of a mating gear tooth pair includes both the gear tooth structure and the lubricant film. Based on [173], the total tooth contact stiffness can be expressed as

$$\frac{1}{K_m} = \frac{1}{K_{str}} + \frac{1}{K_{film}} \quad (6.14)$$

where K_m is the total gear mesh stiffness, K_{str} is the tooth structure stiffness (stiffness without lubrication). K_{film} is the stiffness of the lubrication film defined as $K_{film} = \partial F_H / \partial h_c$ in which F_H is the hydrodynamic force and h_c is the corresponding central film thickness evaluated by Eq.(6.5). The exact value of K_{film} needs to be found from Reynolds equations, e.g. Eq.(6.2). A simpler way to get a reasonable value of K_{film} is to use Eq.(6.5) by redefining the stiffness equation as

$$K_{film} = \frac{\partial F_H}{\partial h_c} = \frac{\Delta F}{\Delta h_c} = \frac{\Delta F}{h_c(F_H) - h_c(F_H + \Delta F)} \quad (6.15)$$

where F_H is the contact load of the gear tooth pair, ΔF is the increment of the contact load, and $h_c(F_H)$ is the center film thickness under the load condition of F_H . The stiffness of the lubricant film of one tooth pair along the line of action, under the same load conditions of Fig. 6-5 and Fig. 6-6, is shown in Fig. 6-7.

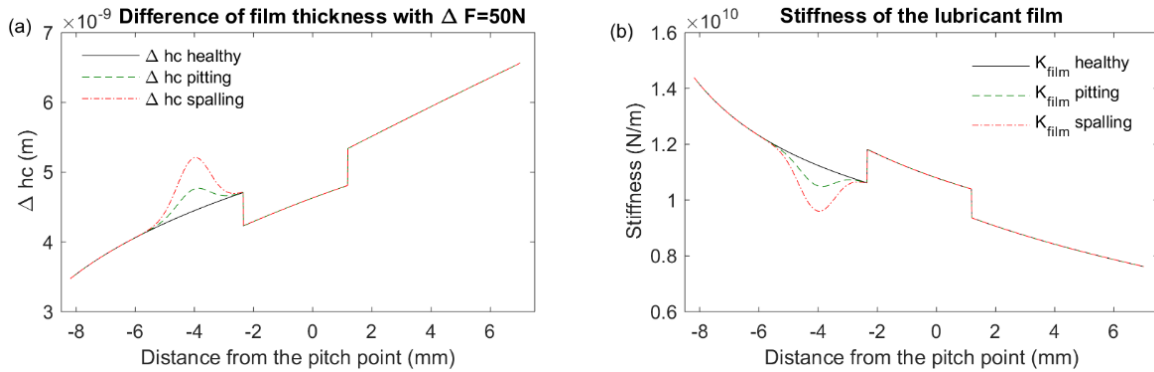


Fig. 6-7 EHL film thickness variation and film stiffness ($F_H=1000 N$, $\Delta F=50 N$)

As can be observed in Fig. 6-7, the effect of tooth pitting and spalling result in decreased lubricant film stiffness due to increased Δh_c within that area. This is due to the fact that as the load increases, the film thickness decreases within the faulty area, and the thickness of the oil film decreases much more than that in the healthy condition. This results in a larger Δh_c in the defect area. The film thickness within the defect area is larger than that under healthy conditions due to

the increase in surface roughness (see Fig. 6-5 (b)) which results in lower film stiffness as shown in Fig. 6-7 (b). Based on [174], under heavy operating conditions, the lubricant film stiffness K_{film} can be several orders larger than K_{str} , therefore, the effect of K_{film} on the total mesh stiffness is limited.

The gear tooth structure stiffness K_{str} can be either obtained by the Potential Energy (PE) method or finite element techniques, where the PE method is the most widely used analytical method in modeling the structural TVMS of a gear-tooth pair [34-37]. The loss of tooth surface material due to gear tooth spalling reduces the strength (or stiffness) of the gear tooth. The modelling of the spalling defect is generally represented by a specific geometric shape, such as rectangular, round, V-shaped or ellipsoid spalls. The severity of the fault is generally controlled by shape parameters such as length, width, radius and depth.

The geometric model of a typical gear tooth profile with spalling defect, as reported in [35], is shown in Fig. 4-5, where the tooth spalls are assumed as identical in-line round shapes. Here, r and δ are the radius and depth of the tooth spalling, ΔL is the magnitude of the reduced contact length at the mesh point. AB and BC are the tooth transition curve and involute curve, P_m is an arbitrary contact point, α_{pm} is the corresponding pressure angle. x_A , x_B , and x_{pm} are distances of points A, B, and P_m from the gear geometrical center O. F is the tooth contact force and is resolved into F_x and F_y . Here, h_x represents the half-tooth thickness at an arbitrary point and τ is the corresponding pressure angle.

axial compressive strength, shear strength, and Hertzian contact strength of the gear pair. Based on the geometric model (see Fig. 4-5), these stiffnesses can be calculated as [135]

$$\left\{ \begin{array}{l} \frac{1}{k_{ai}} = \int_{x_A}^{x_B} \frac{\sin^2 \alpha_{pm}}{EA_{xt}} dx + \int_{x_B}^{x_{pm}} \frac{\sin^2 \alpha_{pm}}{EA_x} dx, \\ \frac{1}{k_{bi}} = \int_{x_A}^{x_B} \frac{f_{xt}}{EI_{zt}} dx + \int_{x_B}^{x_{pm}} \frac{f_x}{EI_z} dx, \\ \frac{1}{k_{si}} = \int_{x_A}^{x_B} \frac{1.2 \cos^2 \alpha_{pm}}{GA_{xt}} dx + \int_{x_B}^{x_{pm}} \frac{1.2 \cos^2 \alpha_{pm}}{GA_x} dx, \\ \frac{1}{k_f} = \frac{\cos^2 \alpha_{pm}}{EW} \left(L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha_{pm}) \right), \\ k_h = \frac{\pi EL_e}{4(1-v^2)}, \end{array} \right. \quad (6.17)$$

where $f_{xt} = (\cos \alpha_{pm}(x_{pm} - x_t) - h_{pm} \sin \alpha_{pm})^2$, $f_x = (\cos \alpha_{pm}(x_{pm} - x) - h_{pm} \sin \alpha_{pm})^2$, in which α_{pm} is the pressure angle of the contact point P_m , h_{pm} is the corresponding half tooth thickness, see [29] for further details. E , G , and v are respectively Young's modulus, shear modulus and Poisson's ratio. u_f , S_f , L^* , M^* , P^* and Q^* are parameters for calculating the fillet foundation stiffness, further details can be found in [127]. A_{xt} , A_x and I_{zt} , I_z are cross-sectional areas and area moments of inertia within the transition curve and involute curve, respectively. L_e is the effective tooth contact length. The detailed evaluation process of Eq.(4.17) can be found in [34]. For a healthy involute gear tooth pair, $A_x = 2h_x L$, $I_z = (1/12)(2h_x)^3 L$ and $L_e = L$. For the spalling portion [35],

$$\left\{ \begin{array}{l} A_x = 2h_x L - \Delta A_x, \quad L_e(x) = L - \Delta L_x, \\ I_x = \frac{2}{3} h_x^3 L - \frac{1}{12} \Delta L_x h_s^3 - \frac{2h_x L \Delta A_x (h_x - h_s/2)^2}{2h_x L - \Delta A_x} \end{array} \right. \quad (6.18)$$

where $\Delta L_x = \sum_{k=1}^K 2\sqrt{r_k^2 - (x - x_{center_k})^2}$, $\Delta A_x = h_s \Delta L_x$, in which the subscript k represents the k -th tooth spall. The total structural stiffness of a tooth pair with tooth spalling can then be expressed as [29]

$$K_{str} = \frac{1}{\frac{1}{k_h} + \frac{1}{k_{bp}} + \frac{1}{k_{sp}} + \frac{1}{k_{ap}} + \frac{1}{k_{fp}} + \frac{1}{k_{bg}} + \frac{1}{k_{sg}} + \frac{1}{k_{ag}} + \frac{1}{k_{fg}}}. \quad (6.19)$$

6.5 Gear mesh dynamic model

Fig. 6-9 shows the proposed one-stage gear mesh dynamic model with the consideration of the effects of the gearbox casing. The proposed dynamic model is based on the geometric structure of the gear transmission, as shown in Fig. 6-16 where the shaft of the pinion and gear are parallel on the horizontal plane. The proposed model considers the tooth surface roughness changes and tooth geometric deviations due to pitting and spalling based on mixed Elastohydrodynamic Lubrication (EHL) models, and incorporates TVMS, a time-varying load sharing ratio, and dynamic damping coefficients, tooth contact friction forces and friction moment. The equations of motion of the dynamic model are given by

$$I_M \ddot{\theta}_M + c_c (\dot{\theta}_M - \dot{\theta}_p) + k_c (\theta_M - \theta_p) = T_M, \quad (6.20)$$

$$I_p \ddot{\theta}_p + c_c (\dot{\theta}_p - \dot{\theta}_M) + k_c (\theta_p - \theta_M) + r_{bp} F_{pg} - M_p = 0, \quad (6.21)$$

$$I_g \ddot{\theta}_g + c_c (\dot{\theta}_g - \dot{\theta}_L) + k_c (\theta_g - \theta_L) - r_{bg} F_{pg} + M_g = 0, \quad (6.22)$$

$$I_L \ddot{\theta}_L + c_c (\dot{\theta}_L - \dot{\theta}_g) + k_c (\theta_L - \theta_g) = T_L, \quad (6.23)$$

$$m_p \ddot{x}_p + c_b (\dot{x}_p - \dot{x}_f) + k_b (x_p - x_f) + F_{pg} \cos \varphi_{pg} + F_{fp} \sin \varphi_{pg} = 0, \quad (6.24)$$

$$m_g \ddot{x}_g + c_b (\dot{x}_g - \dot{x}_f) + k_b (x_g - x_f) - F_{pg} \cos \varphi_{pg} - F_{fp} \sin \varphi_{pg} = 0, \quad (6.25)$$

$$m_p \ddot{y}_p + c_b (\dot{y}_p - \dot{y}_f) + k_b (y_p - y_f) + F_{pg} \sin \varphi_{pg} + F_{fp} \cos \varphi_{pg} = 0, \quad (6.26)$$

$$m_g \ddot{y}_g + c_b (\dot{y}_g - \dot{y}_f) + k_b (y_g - y_f) - F_{pg} \sin \varphi_{pg} - F_{fp} \cos \varphi_{pg} = 0, \quad (6.27)$$

$$m_f \ddot{x}_f + c_{xf} \dot{x}_f + c_b (\dot{x}_f - \dot{x}_p) + c_b (\dot{x}_f - \dot{x}_g) + k_{xf} x_f + k_b (x_f - x_p) + k_b (x_f - x_g) = 0, \quad (6.28)$$

$$m_f \ddot{y}_f + c_{yf} \dot{y}_f + c_b (\dot{y}_f - \dot{y}_p) + c_b (\dot{y}_f - \dot{y}_g) + k_{yf} y_f + k_b (y_f - y_p) + k_b (y_f - y_g) = 0, \quad (6.29)$$

Where $\theta_p, \theta_g, \theta_M, \theta_L$ are rotational displacements of pinion, gear, motor and load, respectively. The sign of the friction force F_{fp} is controlled by Eq.(9) and is initialized to be negative at the beginning. x_p, y_p, x_g, y_g, x_f and y_f are the vibration displacements of pinion, gear and gearbox casing, respectively. r_{bp} and r_{bg} are the radii of the base circle pinion and gear. φ_{pg} is the inclination angle of the line-of-action with respect to the x axis. I_p, I_g, I_M, I_L are mass moments of inertia of pinion, gear, motor and load, respectively. T_M is the motor input torque, T_L is the torque of the load. m_f is the mass of the gearbox casing, m_p, m_g are the equivalent lumped masses of the pinion and gear together with the corresponding mass of the shaft. k_b and k_c, c_b and c_c are the stiffness and damping coefficients of the supporting bearings and shaft couplings, respectively. F_{fp}, M_p and M_g are explained in Eq.(6.11) and (6.13). F_{pg} is the total mesh force which can be expressed as

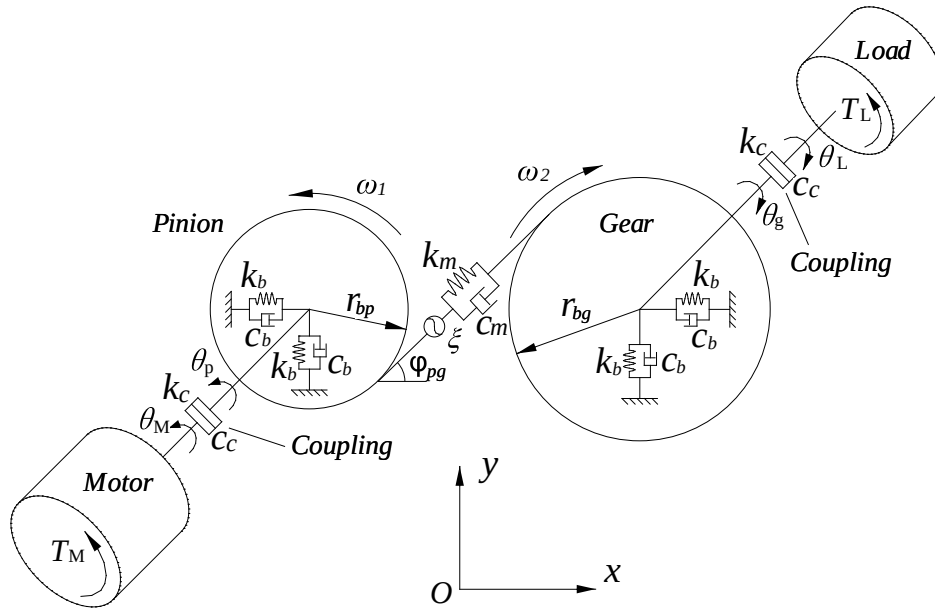
$$F_{pg} = c_m \left[r_{bp} \dot{\theta}_p - r_{bg} \dot{\theta}_g - (\dot{x}_p - \dot{x}_g) \cos \varphi_{pg} - (\dot{y}_p - \dot{y}_g) \sin \varphi_{pg} - \dot{\zeta} - \dot{\xi} \right] + k_m \left[r_{bp} \theta_p - r_{bg} \theta_g - (x_p - x_g) \cos \varphi_{pg} - (y_p - y_g) \sin \varphi_{pg} - \zeta - \xi \right], \quad (6.30)$$

where ξ is the transmission errors. ζ is the tooth geometric deviation due to tooth spalling which has non-zero values when the gear tooth is under severe fault condition, when the tooth involute curve is modified considerably. Under healthy and initial tooth pitting conditions $\zeta = 0$. c_m and k_m are the damping coefficient and time varying mesh stiffness, respectively.

The $k_{xf}, k_{yf}, c_{xf}, c_{yf}$ in Eq.(6.28) and (6.29) are the total stiffness and damping coefficients of the bolts in the x and y directions which can be expressed as

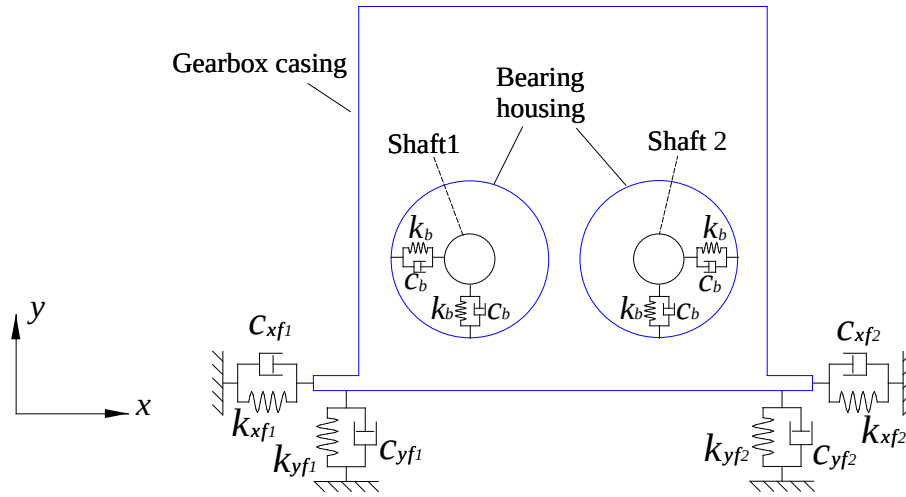
$$k_{xf} = \sum_{k=1}^n k_{xfk}, k_{yf} = \sum_{k=1}^n k_{yfk}, c_{xf} = \sum_{k=1}^n c_{xfk}, c_{yf} = \sum_{k=1}^n c_{yfk}, \quad (6.31)$$

where k is the k th bolt, n is the total number of bolts fixing the gearbox casing, $k_{xfk}, k_{yfk}, c_{xfk}, c_{yfk}$ are the stiffness and damping coefficient of the k th bolt in the x and y directions, respectively.



(a) Dynamic model of gear mesh

The dynamic vibrations of the gear transmission with time-varying mesh stiffness, friction coefficient, load sharing ratio, and damping ratios essentially represent a parametric vibration problem [175]. The relationships between the main dynamic model and the sub-models are shown in Fig.6-10.



(b) Dynamic model of gearbox casing

Fig. 6-9 Dynamic model of a one stage spur gear transmission

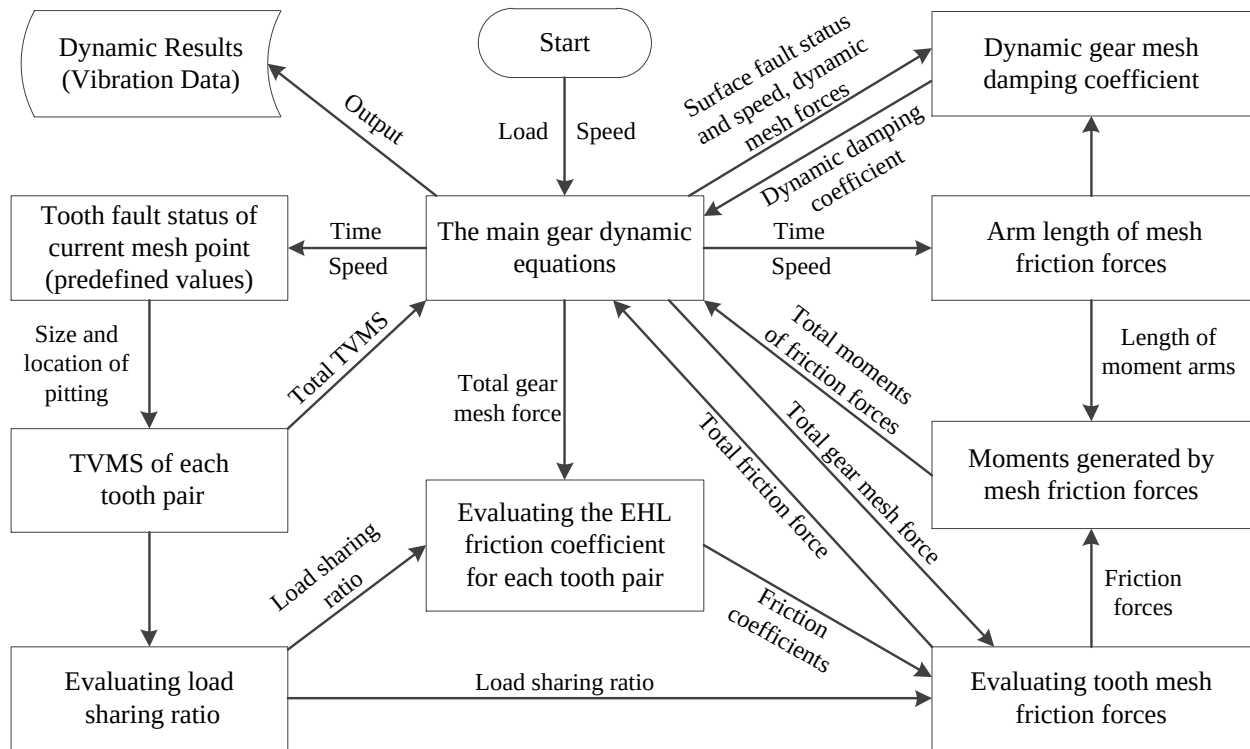


Fig.6-10 Structure and internal sub-model data exchange of the proposed gear dynamic model

A gear transmission is a highly dynamic system. The values of the mesh stiffness, load sharing ratio, friction coefficient, damping ratios, mesh forces, and friction forces vary at each mesh point. As is shown in Fig.6-10, the dynamic model of the gear system consists of the main functions and some sub-functions. The arrows in Fig.6-10 indicate the internal data exchange directions between the functions for each solution step. The solving process of the dynamic model is similar to the process provided in [30]. In practice, the values of gear tooth structure stiffness, time-varying load sharing ratio and time-varying length of moment arms are dominated by the gear geometric parameters and material properties, which can be assumed to be independent of the meshing process. Therefore, with knowledge of the gear geometric information and state of the tooth fault (or tooth surface healthy status), the values of these time-varying parameters can be pre-evaluated as functions of gear rotational angle, and then time-synchronized in the dynamic model in order to accelerate the solution process. The other parameters such as the tooth contact friction coefficient, mesh damping ratio, mesh forces, friction forces, and friction moments are highly dependent on tooth contact conditions, such as load, speed, pressure and surface roughness which must be evaluated dynamically.

6.6 Evaluation of the system physical and mechanical parameters

The vibration behavior of a gear system is governed by many physical and mechanical parameters. Besides the dynamic values (e.g. gear mesh stiffness, friction forces and friction moments, etc.) explained in the previous sections, some other parameters such as bearing stiffness, bearing damping coefficients, bolts' stiffness and damping ratios of the gearbox as well as the system physical properties (mass, mass moment of inertia, inclination angle of the line of action, etc.) are also important in the determination of the system vibration characteristics [176, 177]. For example, the stiffness and damping coefficients of the supporting bearing or fixing bolts have dominant effects on the system dynamic response: lower values of stiffness generally decrease

system natural frequencies, while high damping typically suppresses both nonlinear behaviour and parametric sub-harmonic resonance [161]. Therefore, accurately evaluating these parameters is crucial to obtaining correct gear vibration characteristics. However, most of the prior gear dynamic models adopt some user-defined constant values for the dynamic system based on experience without validation, which may lead to significant inaccuracies.

The physical properties (such as mass and mass moment of inertia) of each component can be directly measured or calculated based on the geometric shape of the corresponding part. However, the evaluation of the stiffness and damping coefficients of the bearings and bolts is much more complicated. There are some mathematical models and finite element methods that can be used to evaluate the stiffness of these components, however the damping coefficients are usually difficult to calculate theoretically, and therefore are usually determined by experimental tests. In this chapter, both stiffness and damping coefficients were determined through experimental tests combined with some necessary mathematical models.

6.7 Evaluation of the bearing stiffness and damping coefficient

The measuring method proposed in [177, 178] was adopted and a similar test rig was built, as shown in Fig. 6-11 (a), where the rotor is supported by two bearings and the corresponding bearing housings. The shaft is short and thick, and the stiffness of the rotor itself is significantly higher than that of the bearings, therefore, the bending effect of the shaft and rotor can be ignored in this experiment. A standard impact hammer (equipped with a built-in force sensor) was used to provide an impulse excitation, and the vibration responses were then obtained by accelerometers (PCB Piezotronics, Inc. Model-623C01, one axis, frequency range: 0.8 to 15000 Hz) mounted on the rotor to measure the vibration accelerations in the vertical and horizontal directions. The accelerometers were connected to a Sensor Signal Conditioner (PCB Piezotronics, Inc. Model 482C). The obtained analog signals are then transferred to a Digital-signal Acquisition Board

(National Instruments, Inc. Model-NI USB-6212). The final digital signals were collected by MATLAB installed in a computer with sampling rate 40 kHz. In this study, three types of ball bearings, namely SER205-16, ER-16K and ER 10K, were tested, and the green impulse hammer tip was utilized.

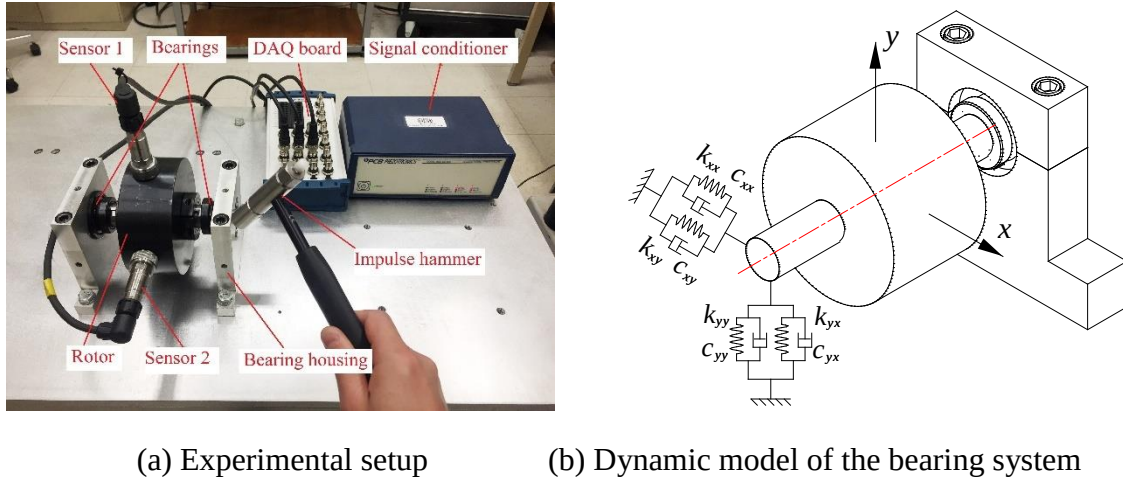


Fig. 6-11 Experimental devices and dynamic model of a simple rotor bearing system

For a rotor system, the applied force in one direction excites a strong vibration corresponding to that direction and a weaker response perpendicular to the applied force direction. Therefore, the stiffness and damping coefficients of a bearing are generally defined as k_{ij} and c_{ij} (with $i=x, y, j=x, y$), which indicate the stiffness and damping coefficient in the i direction when the force is applied in the j direction. The dynamic model of the simple rotor system can be expressed as [177]

$$\begin{cases} m_l \ddot{x} + 2(c_{xx}\dot{x} + c_{xy}\dot{y} + k_{xx}x + k_{xy}y) = f_x(t) \\ m_l \ddot{y} + 2(c_{yx}\dot{x} + c_{yy}\dot{y} + k_{yx}x + k_{yy}y) = f_y(t) \end{cases}, \quad (6.32)$$

where m_l is the lumped mass of the load and shaft, $k_{xx}, k_{xy}, k_{yx}, k_{yy}, c_{xx}, c_{xy}, c_{yx}, c_{yy}$ are the bearing stiffness and damping coefficients, $f_x(t), f_y(t)$ are excitation forces applied in the x and y direction, respectively.

The input force and output responses of the bearing system can be considered as sinusoidal with different frequencies

$$\begin{cases} f_x = F_x e^{j\omega t}, f_y = F_y e^{j\omega t} \\ \ddot{x} = X e^{j\omega t}, \ddot{y} = Y e^{j\omega t} \end{cases}, \quad (6.33)$$

with $\dot{x} = -jX e^{j\omega t} / \omega$, $x = -X e^{j\omega t} / \omega^2$, $\dot{y} = -jY e^{j\omega t} / \omega$, $y = -Y e^{j\omega t} / \omega^2$, where ω is the frequency of the input force, F_x and F_y are the corresponding force amplitudes, and X, Y are the amplitudes of the acceleration responses. For an arbitrary force with frequency ω , the acceleration response of the rotor system can be expressed as

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{H} \begin{bmatrix} m_l - \frac{2jc_{yy}}{\omega} - \frac{2k_{yy}}{\omega^2} & \frac{2k_{xy}}{\omega^2} + \frac{2jc_{xy}}{\omega} \\ \frac{2k_{yx}}{\omega^2} + \frac{2jc_{yx}}{\omega} & m_l - \frac{2jc_{xx}}{\omega} - \frac{2k_{xx}}{\omega^2} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad (6.34)$$

where

$$H = \left(m_l - \frac{2jc_{xx}}{\omega} - \frac{2k_{xx}}{\omega^2} \right) \left(m_l - \frac{2jc_{yy}}{\omega} - \frac{2k_{yy}}{\omega^2} \right) - \left(\frac{2k_{xy}}{\omega^2} + \frac{2jc_{xy}}{\omega} \right) \left(\frac{2k_{yx}}{\omega^2} + \frac{2jc_{yx}}{\omega} \right). \quad (6.35)$$

During the measuring process, the excitation force is applied in the x and y directions separately. For an input force with frequency ω in the x direction ($F_y = 0$), the corresponding input and output response relationship can be defined as

$$\begin{cases} R_{xx} = \frac{m_l \omega^2 - 2j\omega c_{yy} - 2k_{yy}}{\omega^2 H} = \frac{X}{F_x} \\ R_{yx} = \frac{2k_{yx} + 2j\omega c_{yx}}{\omega^2 H} = \frac{Y}{F_x} \end{cases}. \quad (6.36)$$

For the excitation force applied in the y direction ($F_x = 0$), one can obtain

$$\begin{cases} R_{xy} = \frac{2k_{xy} + 2j\omega c_{xy}}{\omega^2 H} = \frac{X}{F_y} \\ R_{yy} = \frac{m_l \omega^2 - 2j\omega c_{xx} - 2k_{xx}}{\omega^2 H} = \frac{Y}{F_y} \end{cases}. \quad (6.37)$$

Eqs.(6.36) and (6.37) contain all of the unknown bearing stiffness and damping coefficients. The actual acceleration responses and input impulse forces at the right-hand side of the equations can be directly measured by the accelerometers and impulse hammer. In practice, directly evaluating the ratio of the amplitude of the acceleration and input force can be difficult because time domain signals may contain many other frequency components. Exploiting the linearity of the Fourier transform on the amplitude of a signal, the input and output relationship can be easily obtained in the frequency domain [176]. Consider R_{xx} as an example,

$$R_{xx}(\omega) = \frac{X}{F_x} = \frac{FT(\ddot{x})}{FT(f_x)} = \frac{\int \ddot{x}(t)e^{-j\omega t} dt}{\int f_x(t)e^{-j\omega t} dt}, \quad (6.38)$$

where $FT()$ is the Fourier transform.

The main concept of the measurement method is to fit the theoretical frequency response of the rotor dynamic model to be consistent with the measured response of the real system by altering the model coefficients. The consistent vibration behavior between the dynamical and real systems indicates that the coefficients (stiffness and damping) of the two systems are also consistent. In practice, the actual excitation force consists of a wide frequency band which excites multiple frequency responses of the rotor system, as shown in Fig. 6-12. An accurate estimate of the bearing coefficients should obtain a minimal deviation error for all considered frequencies, that is

$$\varepsilon = \min \sum_{i=x,y} \sum_{j=x,y} \sum_{\omega} \left(\left| R_{ij}(\omega)_{\text{theoretical}} - R_{ij}(\omega)_{\text{experimental}} \right| \right). \quad (6.39)$$

The minimum value of Eq.(6.39) can be achieved via the Non-Linear Least Squares (NLLS) method or some intelligent methods such as the one presented in [179]. During the optimization process, the optimization method changes the value of the bearing stiffness and damping

coefficients iteratively. The best combination of the bearing coefficients, which results in the lowest deviation error ε , is the final solution.

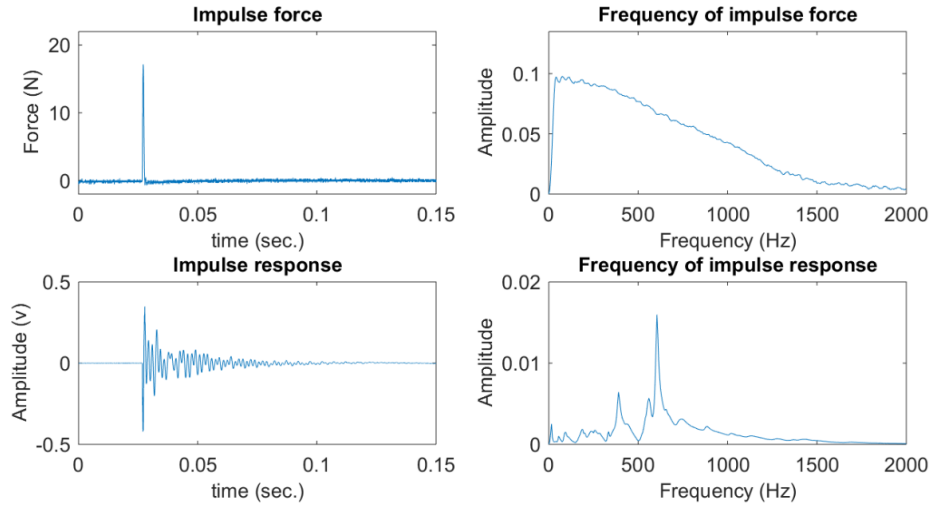


Fig. 6-12 The impulse response of the bearing test rig

6.8 Evaluation of the stiffness and damping coefficient of bolts

In this study, a test rig was set to evaluate the stiffness and damping properties of the bolt. As is shown in Fig. 6-13 (a), where a cylindrical steel mass block (with diameter 76.2 mm and height 38.2 mm, weight 1.36kg) was mounted on the base by a hexagon socket bolt with a rubber mat (thickness of 1 mm) between the two contact surfaces. The base is a large solid aluminum block (1200mm × 600mm × 50mm) with mass properties of about 100kg. A torque wrench was used to measure the preload of the bolt and a torque angle gauge was used to lock the bolt precisely. Sensors 1 and 2 mounted on the mass block were used to measure the impulse vibration of the mass along the vertical and horizontal directions, respectively. Sensors 3 and 4 mounted on the base were utilized to measure the natural frequency of the base under the impulse excitations which were further utilized to check whether the whole system is in resonance or interacting with each other. The impact hammer was used to generate impulse forces to excite the system. In the test,

the applied load of torque wrench was set to 15 ft-lb and the rotation angle of the torque angle gauge utilized to lock the bolt was set to 60 degrees, which are the same mounting conditions as that of the gearbox casing fixed on the work bench.

Due the large mass of the base compared to the mass of the cylindrical block including the mass of the two sensors mounted on it, the base can be assumed as a sturdy ground, and the vibration of the cylindrical mass can be simplified as one degree-of-freedom (DOF) spring-mass-damper systems with stiffness and damping coefficients k_y, c_y and k_x, c_x along the vertical and horizontal directions, respectively, see Fig. 6-13 (b). The dynamic model of a one DOF system can be expressed as:

$$m\ddot{x} + c\dot{x} + kx = F \cos(\Omega t) \quad (6.40)$$

Dividing by the mass m on both side of Eq.(6.40), one obtains

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = F \cos(\Omega t) / m \quad (6.41)$$

where $\omega_n = \sqrt{k/m}$ is the natural frequency of the system, $\xi = c/2m\omega_n = c/2\sqrt{mk}$ is the damping ratio, and Ω is the frequency of the impulse force.

Based on the Logarithmic decrement method, the damping ratio of the vibration can be evaluated by

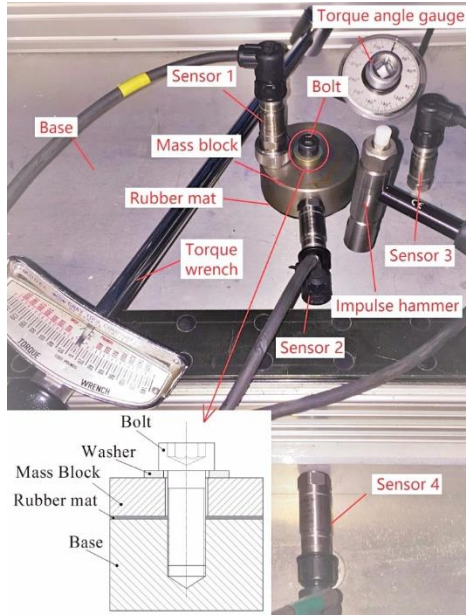
$$\begin{cases} \zeta = \frac{1}{\sqrt{1 + (2\pi/\delta)^2}} \\ \delta = \frac{1}{n} \ln \frac{A_0}{A_n} \end{cases}, \quad (6.42)$$

where ζ is the damping ratio, A_0 is the amplitude of the reference point, A_n is the amplitude of the positive peak n periods away, and n is an integer, see Fig. 6-14(a). With the calculated ζ and the measured damped natural frequency (e.g. $\omega_d = 8800 \text{ Hz}$ for the vibration in the vertical

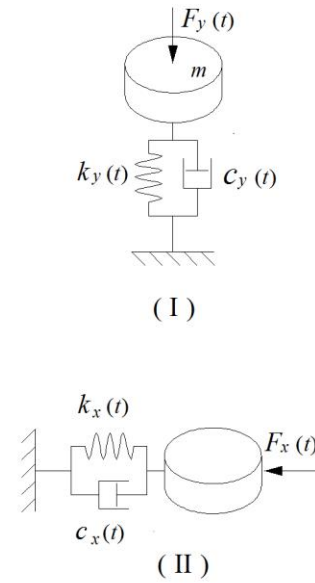
direction), the actual natural frequency ω_n of the mass block can be evaluated by $\omega_n = \omega_d / \sqrt{1 - \zeta^2}$. The evaluated natural frequencies can be further used to determine the total stiffness of the bolts via $k = m\omega_n^2$, in which m is the total mass of the block and the sensors mounted on it. With the known mass of the block and the evaluated ζ and k , the damping coefficient of the bolts can be evaluated by

$$c = 2\zeta\sqrt{mk} \quad (6.43)$$

The impact hammer with a steel tip, see Fig. 6-13 (a), can excite a wide range in the frequency band. In this test, the frequency with the highest amplitude was considered as the main damped natural frequency and utilized to calculate the bolt stiffness and damping coefficient.



(a) The test rig for bolt stiffness



(b) The dynamic model of the mass-bolt system

Fig. 6-13 Evaluation of the bolts' stiffness and damping coefficient based on the Logarithmic decrement method

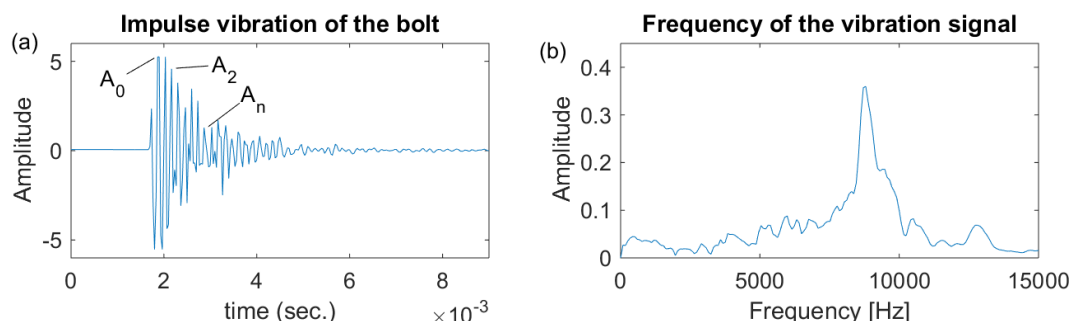


Fig. 6-14 The impulse response of the bolt joint

6.9 Measuring the gear tooth surface roughness

In this study, the roughness of the gear tooth surface was directly measured by the Surface Roughness Tester (SRT, phase II SRG-4000), as shown in Fig. 6-15. The SRT has a high level of accuracy and offers many different surface roughness profilometer parameters, such as the Roughness Average (Ra), Root Mean Square (RMS) Roughness, and Maximum Height of the Profile. The Ra and RMS values of the gear tooth surfaces were measured and utilized in the dynamical analysis. The changes of the tooth surface during the pitting process were directly measured inside of the gearbox without removing the target gear see Fig. 6-15 (b).

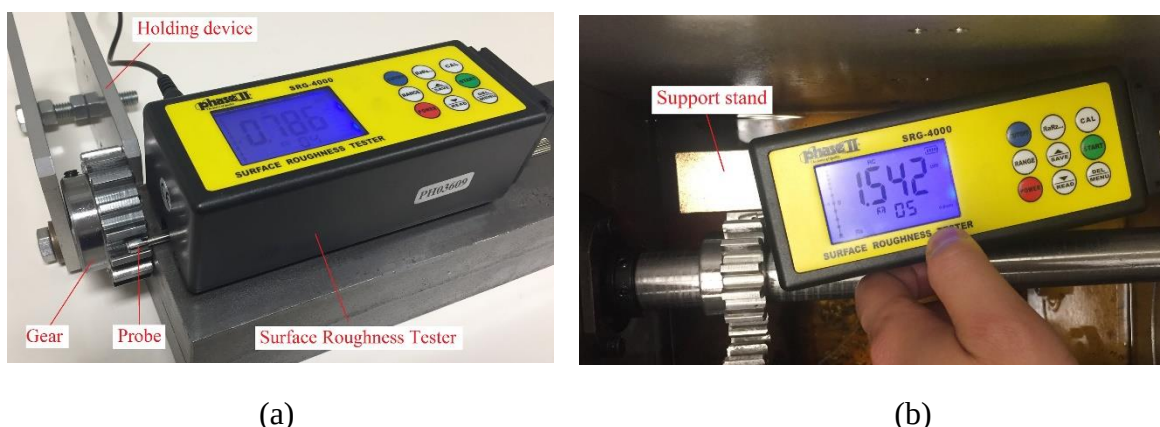


Fig. 6-15 Measuring tooth surface roughness

6.10 Dynamic simulation and model validation

The validation of the proposed dynamic model is based on the comparison of the simulated vibration signals with the experimentally measured signals both in time and frequency domain under different rotation speed and faulty conditions.

6.10.1 Description of the experimental setup

The test rig shown in Fig. 6-16 is a standard Gearbox Dynamics Simulator (GDS) by Spectra Quest, Inc. This GDS consists of a controller, a three-phase motor, a 2-stage parallel shaft gearbox (can be changed to a 1 stage transmission, and the material of the gearbox casing is aluminum) with shafts of 1 inch in diameter, and a load applied by a brake. Sensors 1 and 2 were mounted on the gearbox casing to measure the vibrations in the vertical and horizontal directions, respectively. The same data acquisition equipment shown in Fig. 6-11 was utilized.

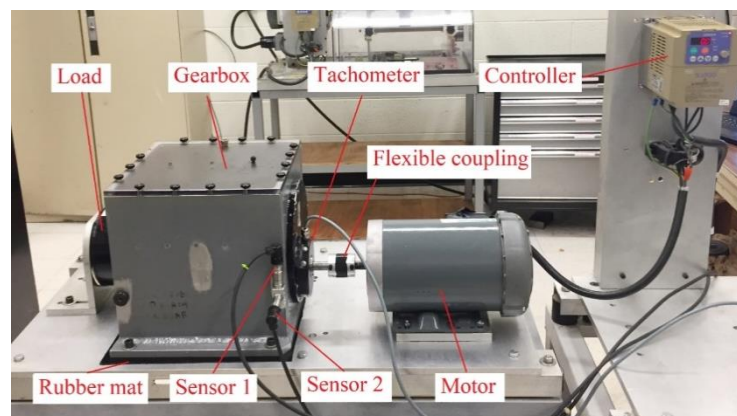


Fig. 6-16 Gear fault simulator

6.10.2 Dynamic simulation and model validation

The validation of the proposed dynamic model was based on a comparison between the simulated signal and experimental data.

6.10.3 Experiment 1- Healthy conditions low speed

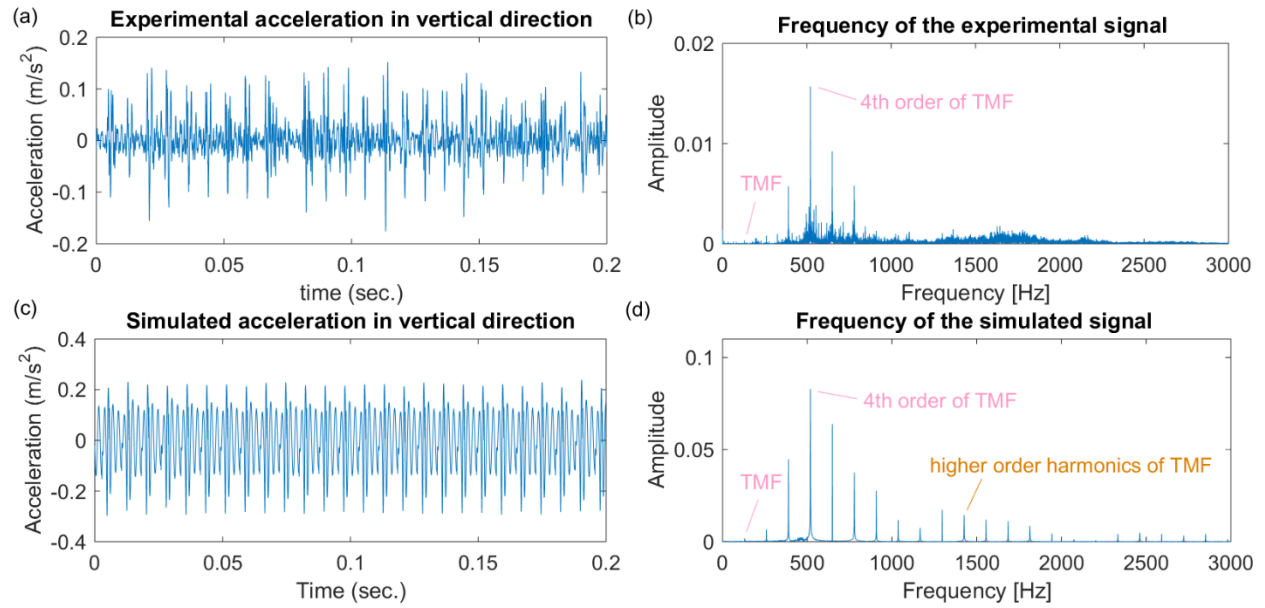


Fig. 6-17 Comparison between the experimental and simulated signals in vertical direction

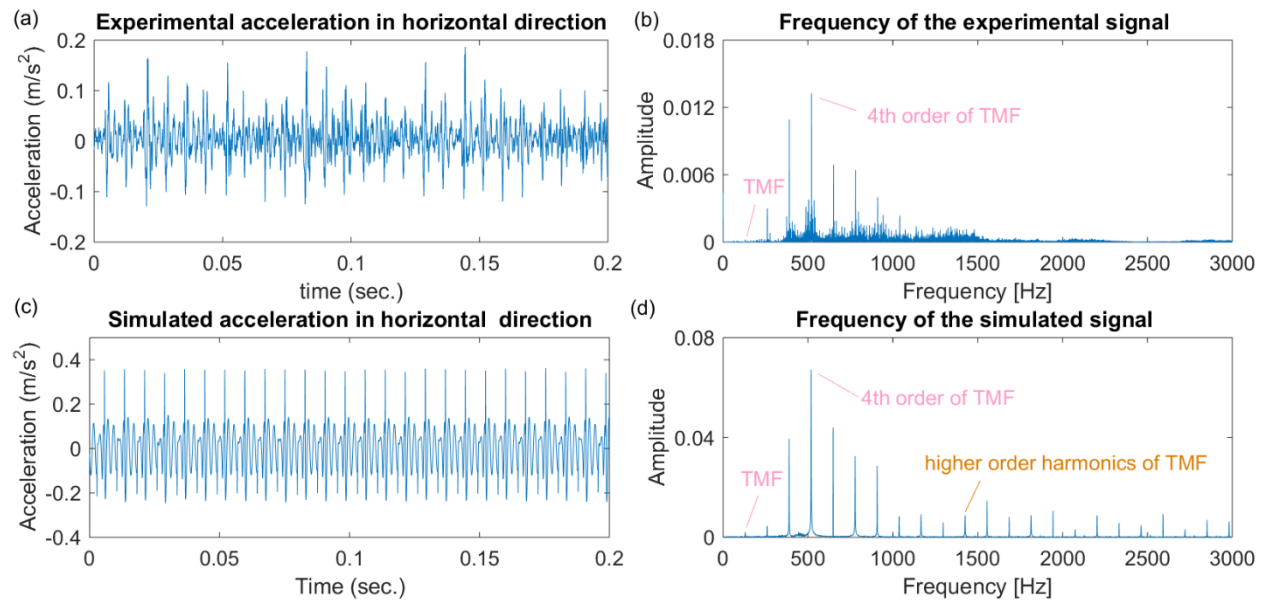


Fig. 6-18 Comparison between the experimental and simulated signals in horizontal direction

As can be observed from Fig. 6-17 to Fig. 6-18, the simulated vibration signals in both vertical and horizontal directions with 8.1 Hz pinion shaft rotational speed are close to the experimental

signals in both the time and frequency domains.

6.10.4 Experiment 2- Healthy conditions high speed

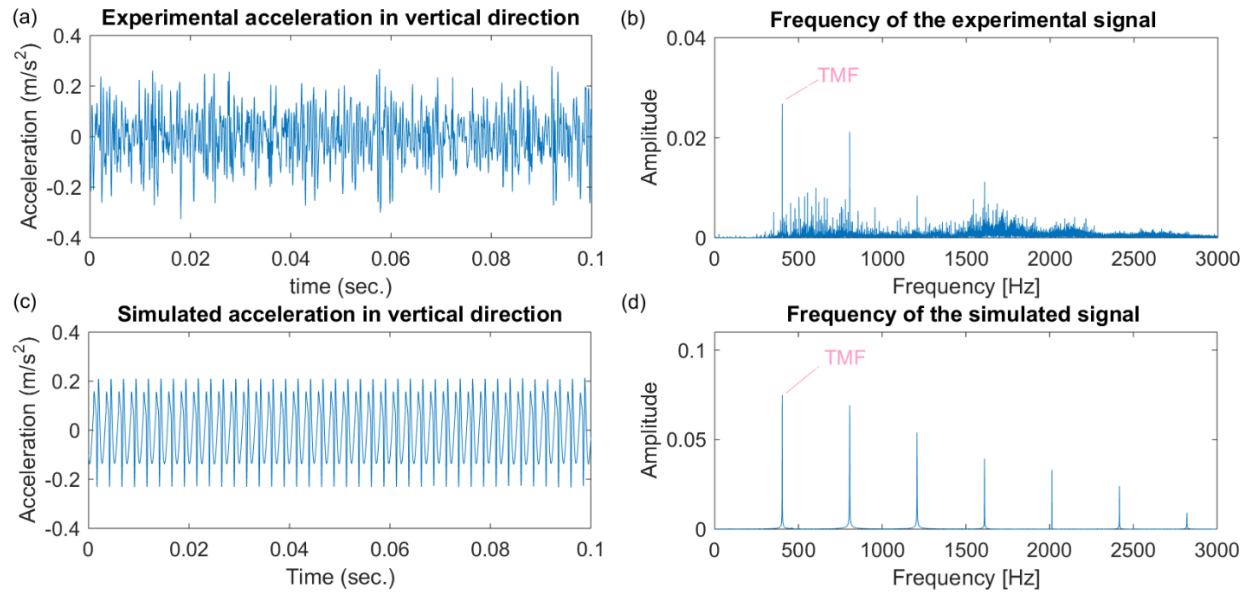


Fig. 6-19 Comparison between the experimental and simulated signals in vertical direction

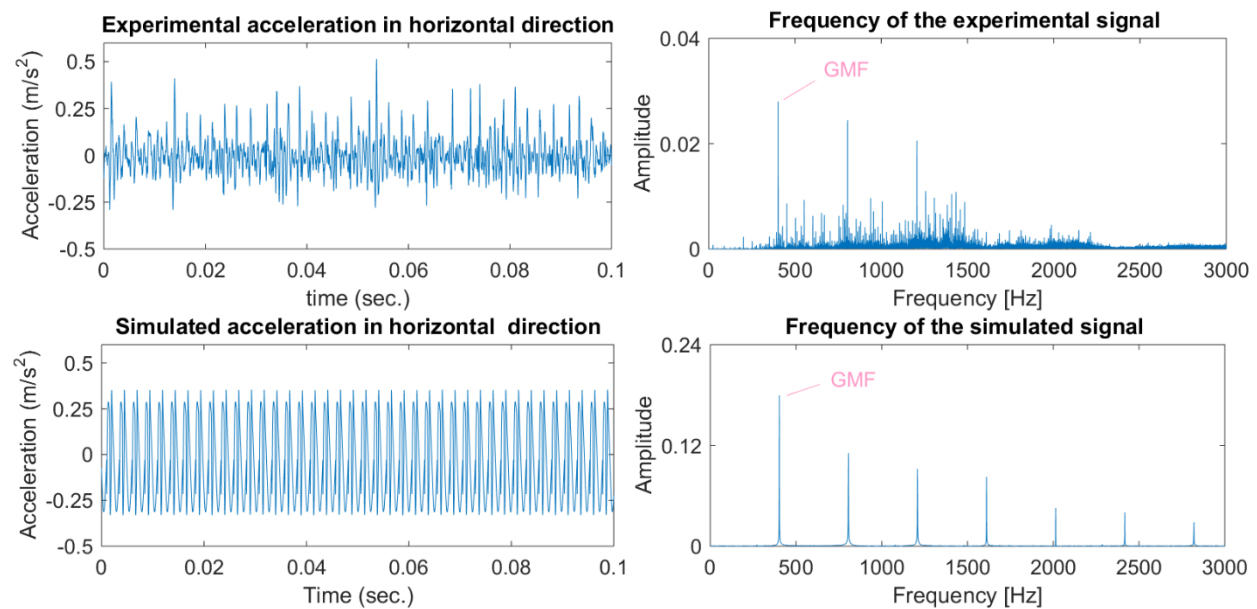


Fig. 6-20 Comparison between the experimental and simulated signals in horizontal direction

As can be observed from Fig. 6-19, the vibration behavior of a gear transmission in time domain varies with the increase of the shaft rotation speed. There are no obvious free vibration features (impulse vibration behavior with exponentially decreased amplitudes) compared with the signal at lower speed conditions (8.1 Hz) as displayed in Fig. 6-17. This is mainly due to the fact that the time duration between two consecutive gear meshing is reduced and there is not enough time for the impulse vibration to exhibit the free vibration feature. Both of the simulated and experimental results (Fig. 6-20) show that the trends of the frequency components also change significantly with the increase of rotational speed. The gear mesh frequency (GMF) which has negligible amplitude under the low speed conditions (see Fig. 6-18) turns out to be one of the dominant components in the higher speed conditions.

6.10.5 Experiment 3- Comparisons under initial faulty conditions

In this experiment, a gear tooth with local tooth pitting and spalling defects was tested. In order to avoid system assembly errors and ensure that the obtained signal is comparable under different gear tooth faulty conditions, the gear tooth pitting and spalling defects were engraved using an electric engraver progressively on the pinion. Tungsten carbide metric drill bits with diameters ranging from 0.2 mm ~ 0.6 mm were utilized to create tooth pitting, and the carburizing strengthened drill bits with diameters ranging from 0.8 mm ~ 3 mm were used to create tooth spall, see Fig. 6-21. The main advantage of using the electric engraver is that it can be used to create the tooth fault directly on the gear tooth without disassembling the system. Because disassembly and reassembly are avoided, this eliminates the potential for introducing assembly error into the system.

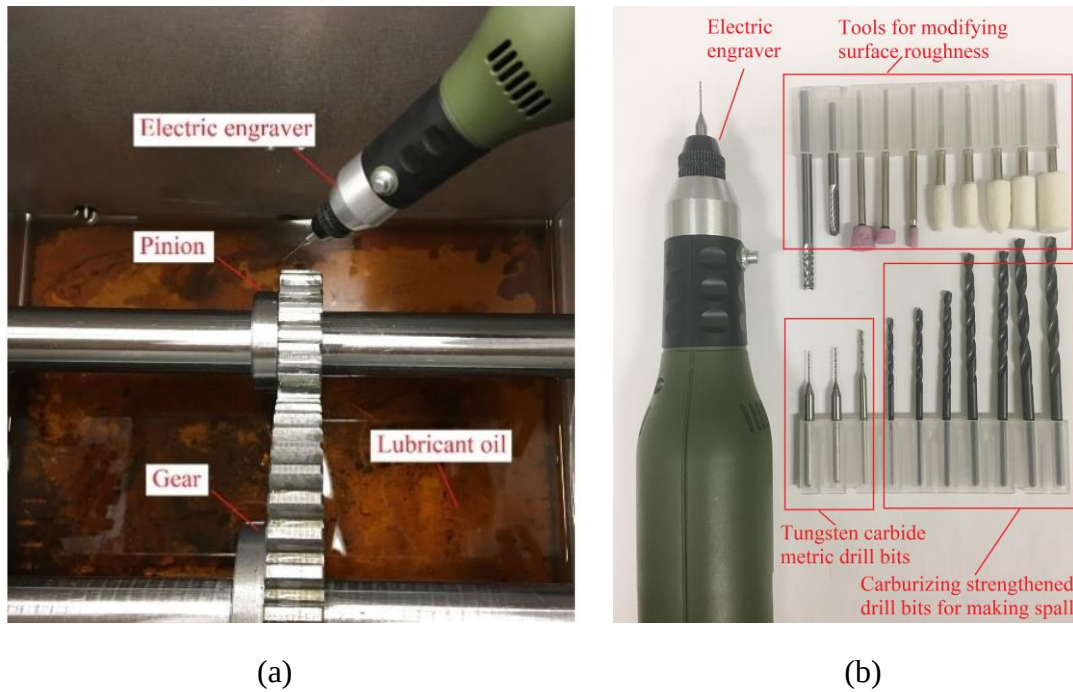


Fig. 6-21 Tools utilized for making progressive tooth pitting and spalling

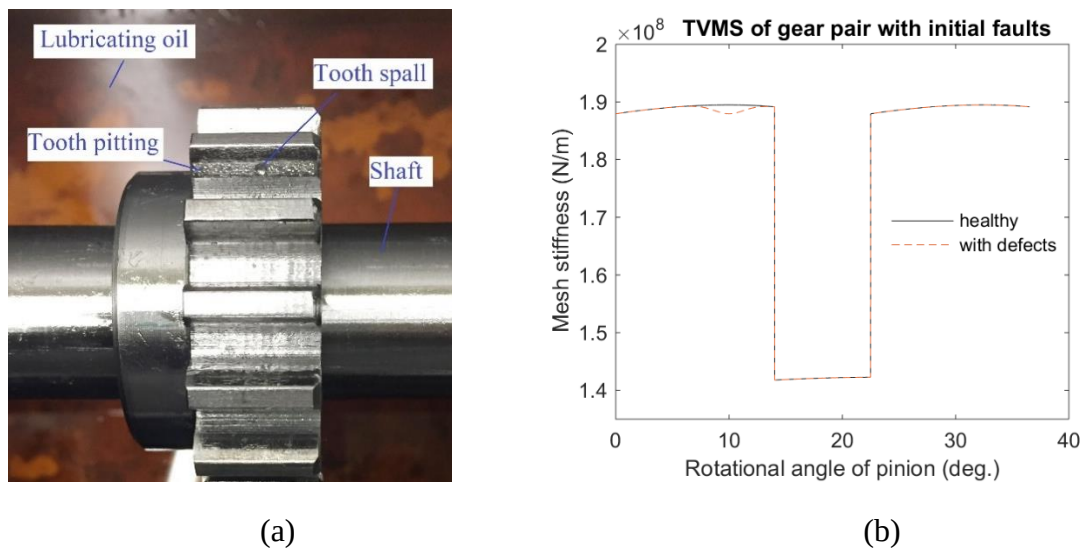


Fig. 6-22 Gear tooth pitting and spalling and corresponding TVMS

As shown in Fig. 6-22 (a), the tooth pitting and spalling close to the pitch line is significant. There is one larger tooth spalling on pinion tooth, as labeled in the figure, which is 1.2 mm in diameter and 0.5 mm in depth. The corresponding TVMS, surface roughness, and effective tooth

contact length are shown in Fig. 6-22(b) and Fig. 6-23, respectively. In this case, the tooth pitting and spalling defects are small in both diameter and depth, which results in only roughness, contact length and TVMS changes. Therefore, tooth geometric deviation due to tooth spalling was set to zero and was not plotted.

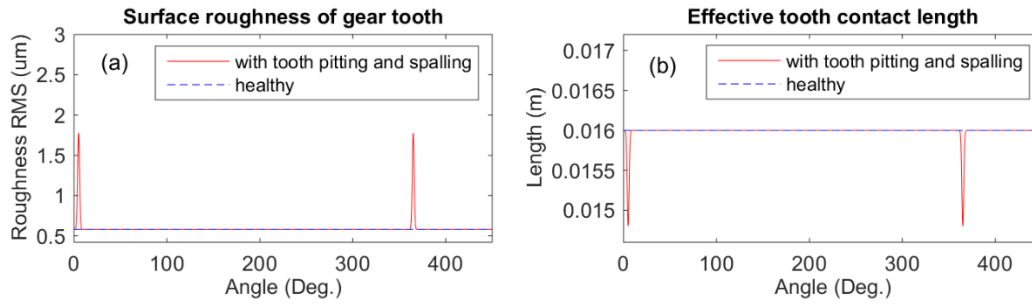


Fig. 6-23 Surface roughness and effective tooth contact length of gear pair with and without faults

The comparisons between the experimental and simulated signals with initial gear tooth spalling are shown in Fig. 6-24 to Fig. 6-26.

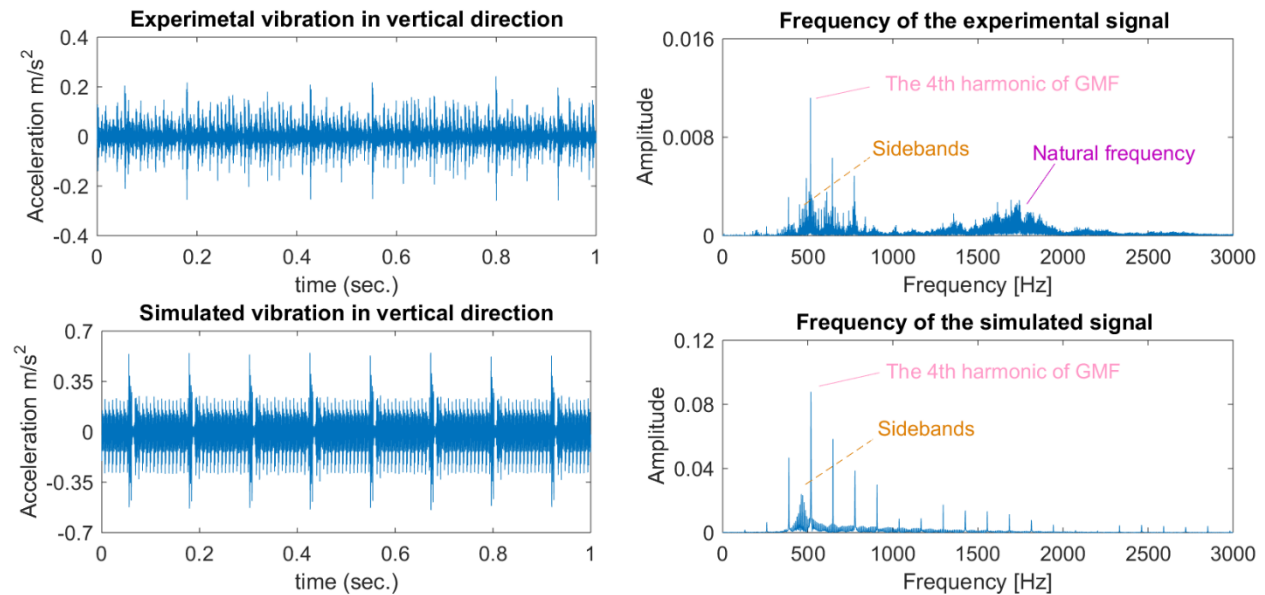


Fig. 6-24 Comparison between the experimental and simulated signals with gear tooth spalling (8.1 Hz)

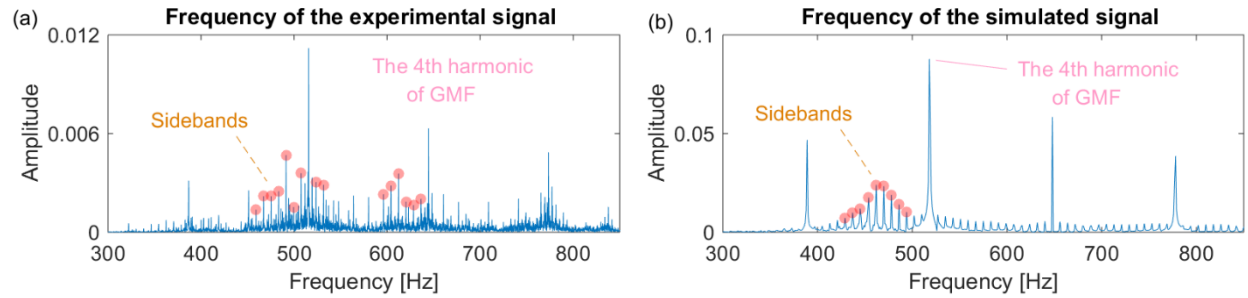


Fig. 6-25 Comparison between the experimental and simulated signals with gear tooth spalling (zoomed, 8.1 Hz)

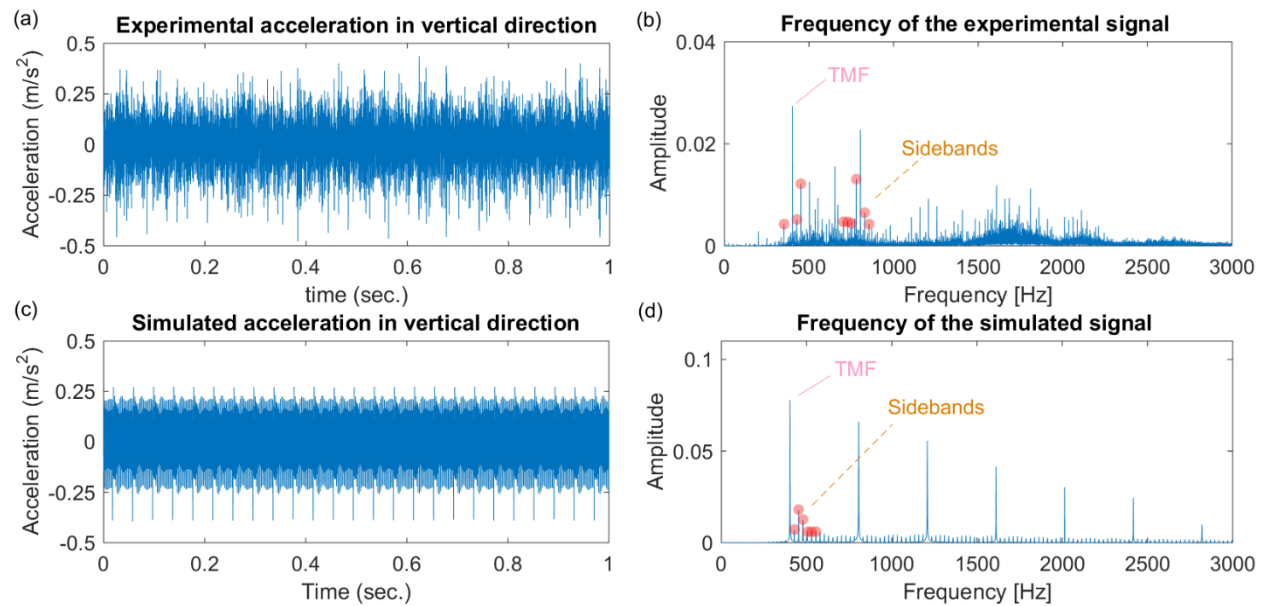


Fig. 6-26 Comparison between the experimental and simulated signals with gear tooth spalling (25.2 Hz)

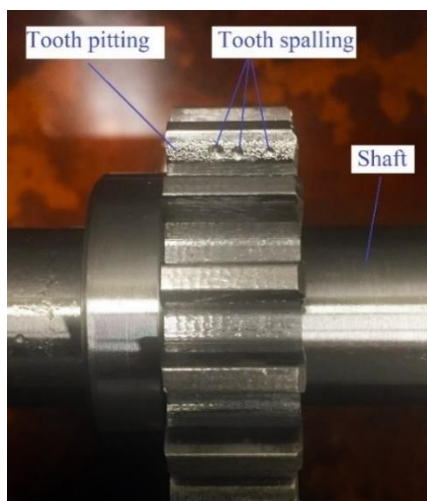
The comparison between the experimental and simulated signals with gear tooth pitting and spalling from Fig. 6-24 to Fig. 6-26 shows very good agreement. As concluded in [180-182], localized faults (e.g. tooth crack and localized spall) can excite periodic impulse vibrations, and sidebands can be observed spaced at the rotational frequency of the faulty gear in the frequency domain. As stated in [181], “because of the short-period nature of the impact, the corresponding modulation sidebands will spread over a wide frequency range, which can produce high-order

sidebands with low amplitudes.” These characteristics were successfully captured by the proposed dynamic model. Both Fig. 6-24, Fig. 6-25 and Fig. 6-26 show periodical short-period impulse excitations at the shaft rotational frequency. In the frequency domain, see Fig. 6-25 and Fig. 6-26 (b, d), one can clearly observe sidebands (marked with red dots in the figures) with significant amplitudes starting in the resonance region (450 Hz) and expanded to higher frequencies. The sidebands are spaced with the distance of shaft rotation speed of 8.1 Hz and 25.2 Hz for lower and higher speed cases, respectively. The amplitudes of the high-order sidebands decrease gradually to zero. These impulse and sideband features are consistent with the conclusion made in [180-182].

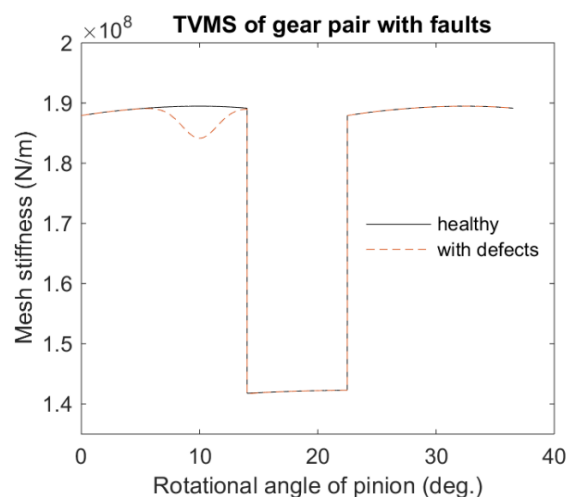
The results shown in Fig. 6-24 to Fig. 6-26 are vibration results in the vertical direction. The vibration features of both experimental and simulated signals in the horizontal direction are very similar to the vibration behavior shown in the vertical direction, therefore the results in the horizontal direction are not shown.

6.10.6 Experiment 4 - Comparison under severe tooth spalling conditions

As shown in Fig. 6-27(a), there are three large tooth spalls and a significant number of tooth pits on the pinion tooth. In this case, both of the tooth pitting and spalling defects are in large size and depth, and more importantly, the defect area has been worn significantly. Therefore, the tooth shape geometric deviation (ζ) cannot be ignored. The corresponding TVMS, surface roughness, effective tooth contact length, tooth geometric errors and corresponding velocity of the geometric errors ($\dot{\zeta}$) under the pinion rotation speed of 8.1 Hz, are shown in Fig. 6-27 (b) and Fig. 6-28.



(a)



(b)

Fig. 6-27 Gear tooth pitting and spalling and corresponding TVMS

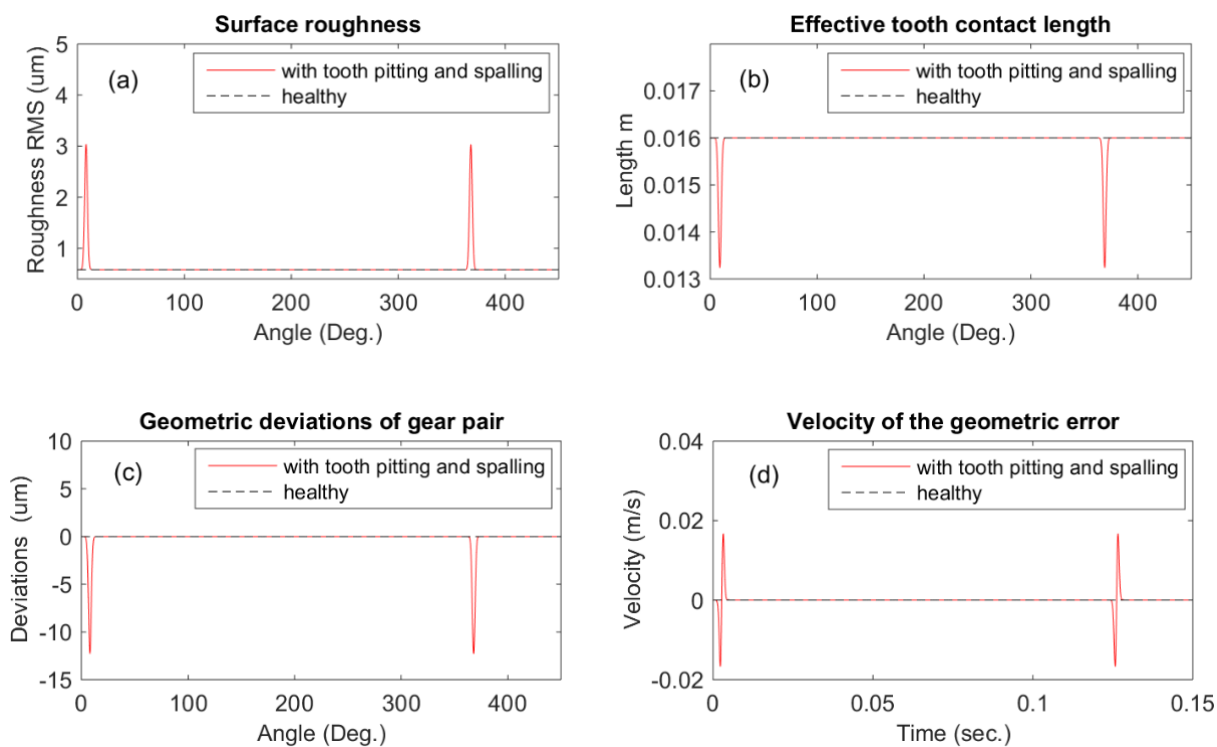


Fig. 6-28 Modeling of tooth surface changes under severe pitting and spalling conditions

The comparison between the experimental and simulated signals are shown in Fig. 6-29 to Fig. 6-31. Both time and frequency comparisons show very good agreement. It can be observed that

with the increase in severity of the tooth pitting and spalling, the magnitudes of the periodic impulse vibrations in the time domain and the amplitudes of the sidebands in the frequency domain increase significantly.

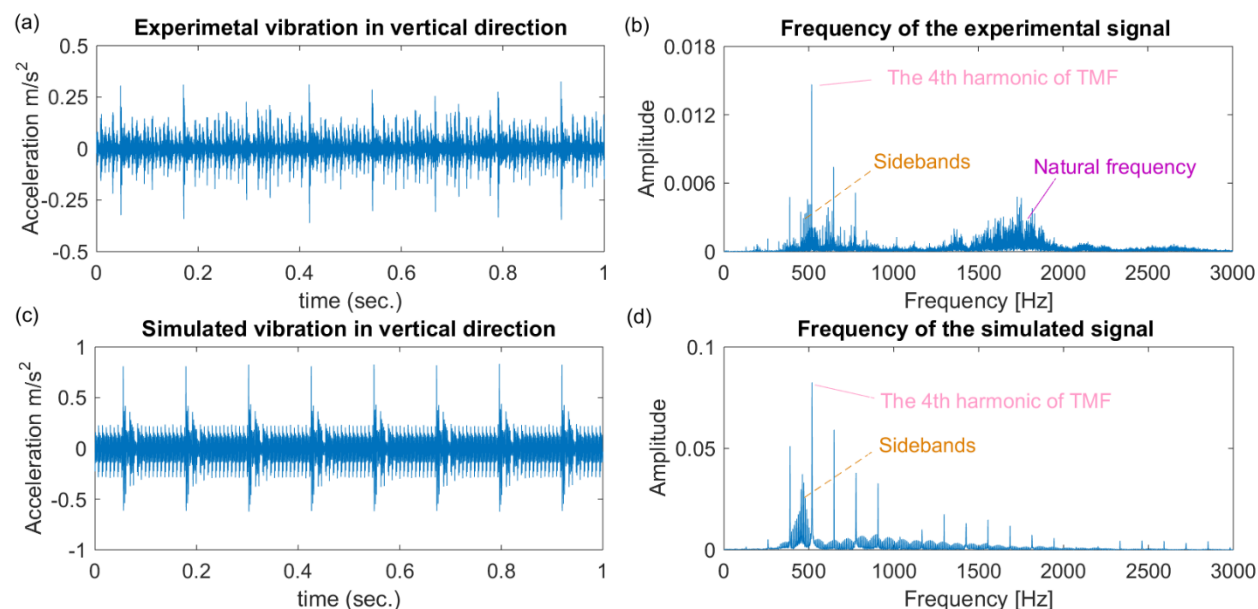


Fig. 6-29 Comparison between the experimental and simulated signals with severe tooth spalling (8.1 Hz)

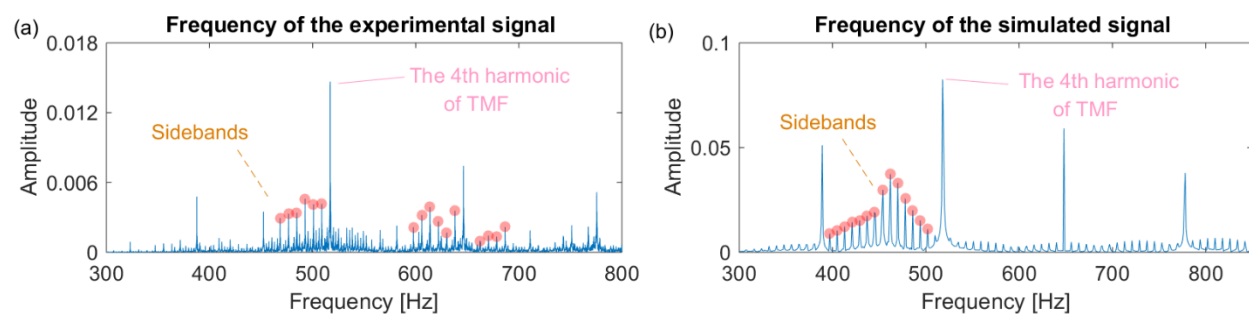


Fig. 6-30 Frequency comparison between the experimental and simulated signals with severe tooth spalling (8.1 Hz)

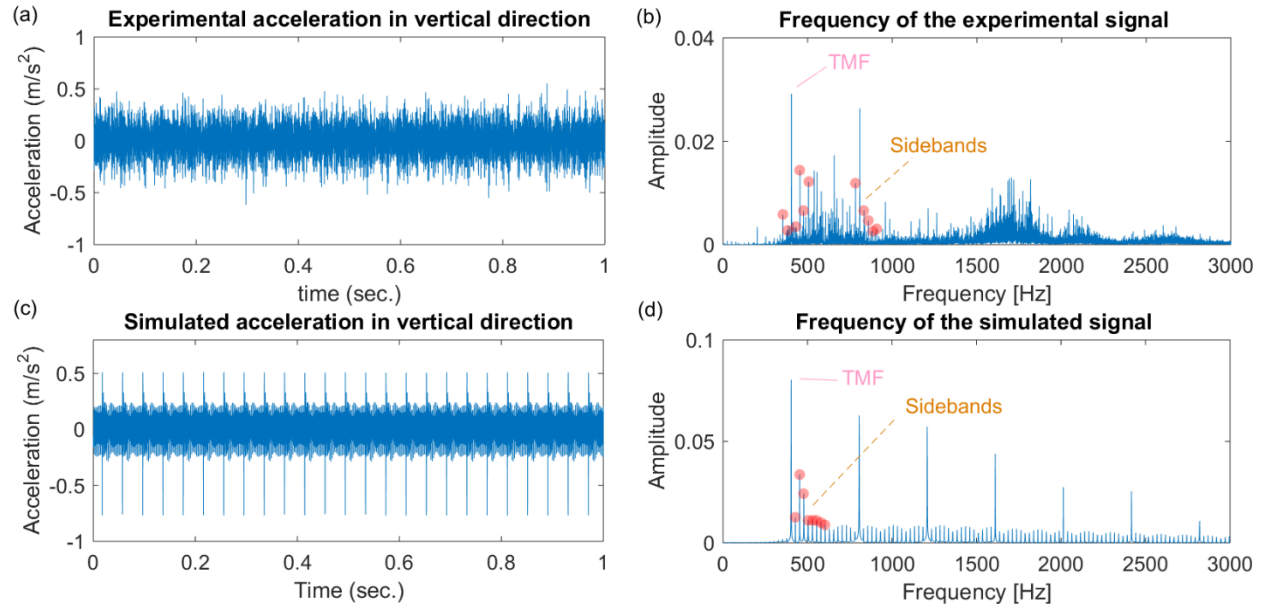


Fig. 6-31 Frequency comparison between the experimental and simulated signals with severe tooth spalling (25.2 Hz)

6.11 The effects of friction forces in gear dynamic models

The effects of friction forces and friction moments have widely been ignored in most existing gear dynamic models. The effect of friction on the gear dynamical system is difficult to quantify without a validated gear dynamic model. The following analysis compares the differences between the dynamic responses with and without friction effects under the same working conditions of the simulation shown in Fig. 6-24 and Fig. 6-25.

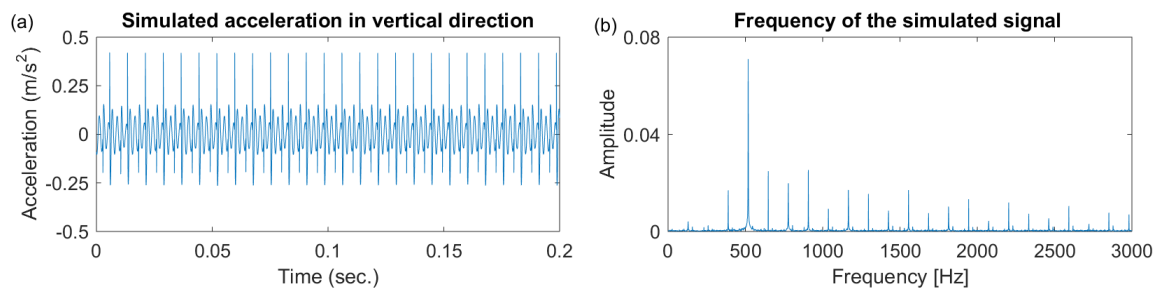


Fig. 6-32 Simulation of the dynamic model considered only the modification of TVMS due to tooth pitting

As shown in Fig. 6-32, the vibration signal under frictionless conditions changes significantly in both time and frequency domains. The vibration features are not consistent with the signal with dynamic friction and the actual experimental signal (see Fig. 6-24 a, c). The trend of the frequency components in the simulated signal under frictionless conditions is significantly different from the phenomenon shown in Fig. 6-24 (b). Therefore, a dynamic model which neglects friction effects may not be capable of capturing the vibration behavior of an actual gear transmission. More importantly, the model that considers only the reduction of TVMS due to tooth pitting fails to capture the fault vibration responses of the system in this simulation, as there is hardly any change in the time domain compared to the simulation shown in Fig. 6-24. Also, there are no obvious sidebands or other fault symptoms shown in the frequency domain. The comparisons between the amplitudes of the signals in the time and frequency domains between Fig. 6-32 and Fig. 6-24 indicate that the accelerations under frictionless conditions are much lower than that with friction effects. This phenomenon indicates that the friction forces not only modify the vibration features (shapes) of the signal, but also affect vibration amplitude.

The dynamic forces of the gear system with tooth pitting and spalling associated with the simulation shown in Fig. 6-24 are shown in Fig. 6-33. Based on [183], the increase of surface roughness results in an increase of the EHL traction coefficient (friction coefficient), thus increases the traction forces within the rough surface area. This phenomenon is captured by the proposed dynamic model, as one can observe the significant periodical dynamical friction force impulses (with a peak at about -24N) and friction moment impulses (-0.38 Nm for pinion, -0.45 Nm for gear). The variation friction forces and moments in turn affect the dynamic contact forces (see the changes in the ellipse of Fig. 6-33 (a)).

The gear mesh dynamic forces presented in Fig. 6-33 shows the ability of the proposed dynamic model in analyzing the internal interactions between the components of the gear system

under faulty conditions. It has also demonstrated that gear tooth pitting can be modeled as surface roughness change, which has been widely neglected previously.

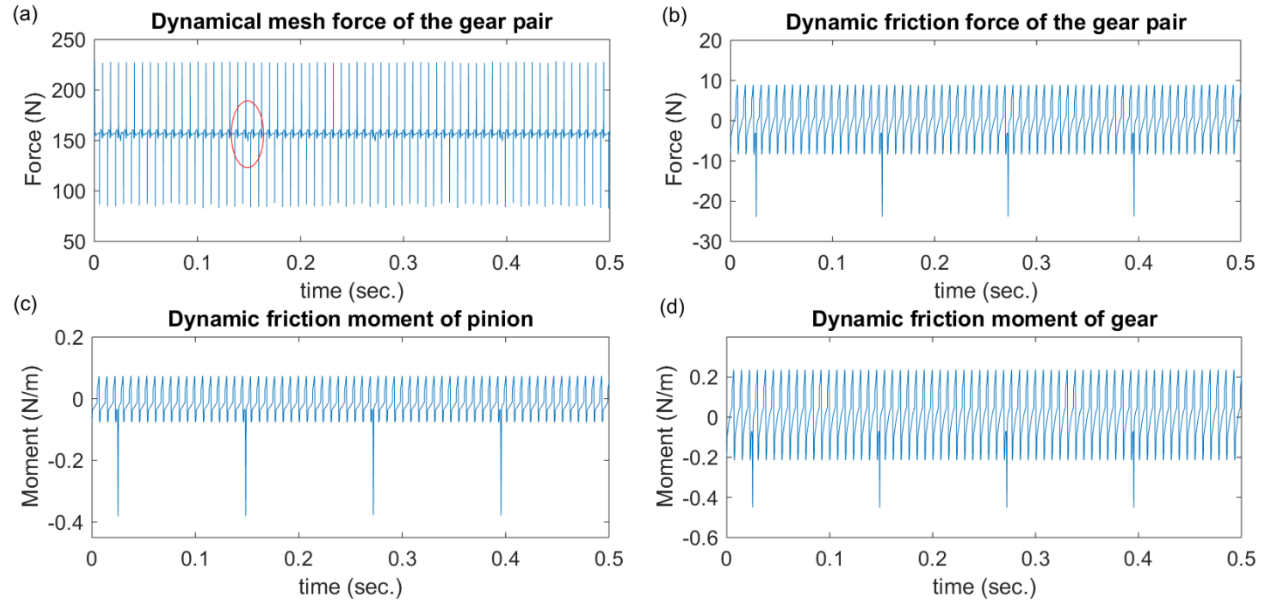


Fig. 6-33 The dynamic forces of the gear system with consideration of surface roughness changes under tooth pitting and spalling conditions (test rig 1, 8.1 Hz)

6.12 Conclusion

This chapter proposes a new gear dynamical model which considers the effects of the tooth surface roughness changes and geometric deviations due to pitting and spalling, in order to analytically investigate the fault features of tooth pitting and spalling. The proposed dynamic model considers the effect of gearbox casing, and incorporates TVMS, time-varying load sharing ratio, and dynamic tooth contact friction forces, friction moments and mesh damping ratios.

The effectiveness of the proposed gear dynamic model in simulating gear tooth pitting and spalling was validated by comparison with responses obtained from experimental test rigs under different rotation speeds and faulty conditions. Comparisons between the experimental and simulated signals have shown very good agreements in both time and frequency domains. The

vibration features, e.g. the periodic impulse response in the time domain and the amplitude trends in the frequency, of a gear system vary with rotation speed. It has been demonstrated that the proposed dynamic model effectively keeps track of the changes of these features and captures the time-varying features for reliable fault analysis.

Both of the experimental tests and dynamic simulation results have shown that the tooth surface roughness changes and contact length variations will generate impulse vibrations and friction force changes with considerable amplitudes. In the frequency domain, one can clearly observe the corresponding sidebands with significant amplitudes starting in the resonance region and expanded to higher frequencies. Thus, the approach proposed in this study to model gear tooth pitting and spalling as tooth surface contact variations is effective.

Chapter 7 Fault feature analysis and detection of progressive localized gear tooth pitting and spalling

This chapter addresses objectives 6 and 7, which are to:

- a) Investigate the evolution characteristics of fault vibration features as the fault progresses from healthy to severe condition.
- b) Propose a novel condition indicator to detect progressive tooth spalling.

A conference paper (not shown in this thesis) focusing on the fault feature evolution and performance of condition indicators with a progressing gear tooth crack has been published in the *Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2017.

Y. Luo, N. Baddour, and M. Liang, Performance Assessment of Gear Condition Indicators in Detecting Progressive Gear Tooth Crack. ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 8: 29th Conference on Mechanical Vibration and Noise ();V008T12A010, 2017. doi:10.1115/DETC2017-67460.

7.1 Abstract

This chapter provides a comprehensive experimental analysis of the evolution of fault vibration features of a gear transmission with progressive localized gear tooth spalling. The effects of rotational speed on the vibration features of the gear transmission are analyzed. The change in fault features (e.g. periodic impulses and sidebands phenomenon) under different fault severity levels and speed conditions are compared. Results indicate that the number, amplitude and distribution of sidebands increase non-linearly as the fault progresses. Based on feature analysis, a new health indicator of the mean of the n^{th} order peaks is proposed to detect progressive localized tooth spall. Results indicate that the proposed indicator shows very good performance for tracking the severity of the progressive tooth spall under different speed conditions.

7.2 Introduction

The localized gear tooth spall (where spall exists mainly located on a few teeth) is tooth surface damage that usually occurs when contacting surfaces are over stressed [159]. Fig.7-1 shows some practical, localized gear tooth spalls, in which the spall shown in Fig.7-1(c) occurred during an endurance test in [184].



(a) [185]

(b) [186]

(c)[184, 187]

Fig.7-1 Examples of localized tooth spall

The construction of a reliable healthy monitoring system, e.g. a Prognostics and Health Management (PHM) system, requires the reliable detection and diagnosis of gear faults as well as the evaluation of the corresponding fault severity. The assessment of fault severity directly relates to the evaluation of the Remaining Useful Life (RUL) of the failing component, which is important for maintenance decisions. Much work has been carried out to investigate fault detection and diagnosis of a gear system. Countless detection/diagnosis methods have been proposed, such as healthy indicators (Kurtosis, NA4, FM4, etc.) analyzed in [1], the Short Time Fourier Transform (STFT), Wavelet Transform (WT), Wigner-Ville Distribution (WVD), Q-factors [14, 15], Envelope analysis [16], Cepstrum analysis, and numerous advanced methods derived from these methods. However, work on severity assessment of a faulty component is limited.

Irrespective of the method utilized for detecting a gear fault, knowing the evolution characteristics of fault vibration features as the fault progresses is vital for fault severity assessment. However, limited work has provided analysis of fault feature evolution of progressive localized gear tooth spall. The assessment of fault severity levels is widely based on the hypothesis that ‘the more severe the fault, the stronger the fault symptom’, which appears to be a reasonable assumption but requires experimental validation. Knowing the fault features of a gear system under different faulty severity conditions is critical for gear fault severity assessment, RUL evaluation and maintenance decision making.

In the light of this, the goals of this chapter are to provide a comprehensive analysis of the evolution of fault features of localized progressive gear tooth spall by experiment, and to propose a new method to detect progressive tooth spall. In this chapter, the effects of the rotational speed on the vibration behavior of the gear transmission under healthy conditions are analyzed first in order to identify the possible fault feature differences due to speed. The tooth spalls are directly seeded on the pinion tooth progressively without disassembly of the gearbox in order to avoid

introducing assembly errors. Vibration signals with different fault severity are obtained under the same load and speed conditions, to ensure that the signals are comparable under different severity conditions. The evolution of the fault vibration behavior (e.g. periodic impulses, and the number, amplitude, strength, and distribution of sidebands) of the progressive localized gear tooth spall under different speed conditions are analyzed. A new fault severity assessment method which is constructed by the mean of the n^{th} order peak values is proposed to assess the fault severity of the progressive localized tooth spall. Results indicate that the proposed indicator shows very good performance for tracking the severity of progressive tooth spall under different speed conditions with constant load.

7.3 Experimental description

The vibration features with localized tooth spall under different fault severity conditions were obtained from a standard Gearbox Dynamics Simulator (GDS) by Spectra Quest, Inc. as shown in Fig. 6-16 in chapter 6. Analyzing the evolution of the vibration features of a gear system with a progressive tooth fault requires that the obtained signals are comparable after each test. Data consistency not only requires the signals to be obtained under the same working conditions (e.g. the same speed and load conditions) but also requires the status of the other healthy components to remain the same, e.g. changes that might be introduced by disassembling the gears, shafts, bearings and couplings, or loosening the bolts of the gearbox and motors must be avoided. To avoid modifying the mounting conditions of the transmission and to ensure that the obtained signals are accurate and comparable under different faulty severity conditions, the tooth pitting and spalling were introduced directly on the pinion tooth without disassembly by a handheld electric engraver with different cutting tools, as shown in Fig. 6-21 in chapter 6.

The test started with a tooth healthy condition running under constant load and speed conditions. After collecting data under healthy conditions, several tooth pits were seeded on one

of the pinion teeth by a 0.3 mm drill bit (see the initial fault in Fig. 7-2), and the vibration signals were collected under the same pinion shaft rotational speeds (8 Hz and 25 Hz) and load conditions as the healthy cases. Subsequently, additional tooth pits were added to the same tooth and the process was repeated. In some cases, the polishing bits were utilized to change the surface roughness as well as to create surface wear. The large tooth spalls shown in the moderate spall condition were created by the 2 mm drill bit. In each test, tooth spalling severity was increased by either introducing new tooth pits/spalls or by increasing the size of the existing spalls. The experiment ended when the tooth surface was severely damaged, see Fig. 7-2. It is proposed to describe the severity of tooth pitting and spalling by $S_e = (C_p * A_{pit} + A_{spall}) / A_s$, in which S_e is the fault severity in percent, A_s is the total area of the tooth surface, A_{pit} and A_{spall} are the area of tooth pitting and spalling on the gear tooth surface, respectively. In this formula, the pitting area is corrected by a coefficient C_p due to some small isolated healthy areas distributed within the pitting area which share the load during the meshing process. The value of C_p is determined by the intensity of the tooth pitting within the pitting area. Based on this definition, the healthy case (see Fig. 7-2) corresponds to 0% damage, the initial fault indicates about 5% severity, the moderate fault counts as about 50% fault severity and the last severe fault implies 100% tooth surface damage.

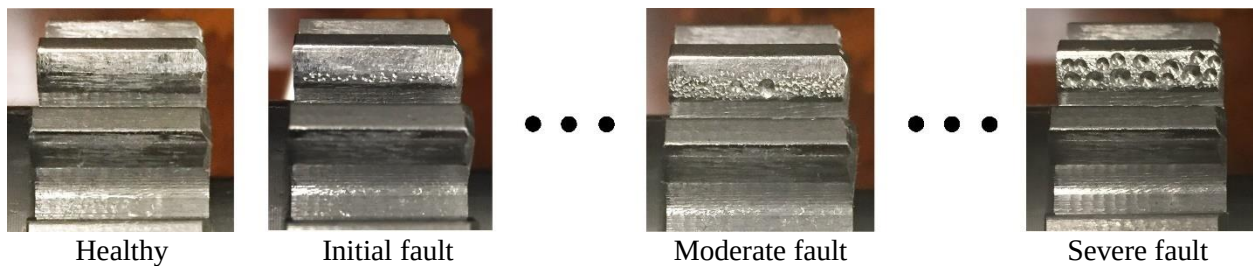


Fig. 7-2 Progressive tooth spalls on the same pinion tooth

7.4 Vibration feature analysis of gear transmission in healthy conditions

The rotation of a spur gear system maintains continuous power transmission from one gear to another [29]. Due to the abrupt change in the number of mating teeth, the mesh stiffness of a spur gear pair fluctuates periodically, which excites periodical vibration of the system. Hohn [188], Agemi [189] and Ognjanovic [190] classified the dynamic vibration behavior of a gear transmission into three resonance ranges, namely, the subcritical resonance range (when $f_m < 0.85f_n$, where f_m is gear tooth mesh frequency, f_n is the system dominant natural frequency), the resonance range (when $0.85f_n \leq f_m \leq 1.15f_n$) and the supercritical resonance range (when $1.15f_n < f_m$). Most industrial gear systems operate in the subcritical resonance range, some specific high-speed gear transmissions can operate in the supercritical resonance range. As concluded in [190], the vibration features of a gear transmission may vary significantly when the system operates under different resonance ranges or speed conditions. The change in the vibration behavior of a system due to speed changes can result in a corresponding change in fault vibration features. Analyzing the fault features under different vibration response conditions is critical for gear fault diagnosis.

In this experimental test, the test rig was first operated under healthy conditions with different rotational speeds to examine whether there are significant vibration differences due to speed changes. Fig. 7-3 shows the vibration features of the test rig with different speeds (8 Hz, 25Hz, and 35Hz) under health tooth conditions. The first and second natural frequencies of the test rig are approximately 550 Hz and 1700 Hz, respectively. As shown in Fig. 7-3, the vibration behavior of the gear transmission in the time and frequency domains changes significantly as the speed increases from 8 Hz to 35Hz. In the low speed condition (8 Hz), the system works within the subcritical resonance range and the impulse vibration behavior with exponentially decreased amplitudes of each mesh tooth pair are obvious, see Fig. 7-3 (a). The periodic impulse vibrations appearing in the low frequency condition are mainly caused by the mesh stiffness fluctuation and

teeth impacting. With an increase in the pinion rotational speed, the time between two consecutive tooth meshing is reduced, the impulse vibrations are compressed and thus no obvious impulse vibrations can be observed. The waveform of the vibration under higher speeds conditions becomes more like sinusoid waves combined with noise, see Fig. 7-3 (c) and (e). By comparison, the 35Hz speed case (with GMF=560 Hz) indicates that the system operates within the 1st resonance range ($0.85f_n \leq f_m \leq 1.15f_n$). It was found that by operating in this frequency range, the vibration of the system is much stronger than that of the lower speed conditions, e.g. the vibration amplitudes of the 35 Hz speed case are about 4 times those of the 8 Hz case, see Fig. 7-3 (e).

The frequency components also change significantly with an increase in the shaft rotational speed. The Gear Mesh Frequency (GMF) (which is also known as gear tooth mesh frequency) has negligible amplitude in the low speed condition turns out to be the dominant component with the highest amplitude in the higher speed conditions, see Fig. 7-3 (b), (d) and (f). It can be seen that the amplitudes of the GMF and its harmonics do not decrease with order for all cases. The feature that the frequency amplitudes do not decrease successively is one of the most natural phenomena observed in practice. Li [191] explained that the amplitude irregularity of the frequency components is mainly due to the convolution of nonlinear periodic exciting force and nonlinear feedback of the system. This explanation gave a possible reason for the frequency irregularity phenomenon; however, it is insufficient to explain why the frequency amplitudes are always higher (or lower) in a specific area.

As can be observed in Fig. 7-3 (b), (d) and (f), a common feature of the dominant frequencies with higher amplitudes (e.g. the 4th GMF harmonic in the 8 Hz case, the GMF and its 2nd and 4th harmonics in the 25 Hz case, and the GMF and its 3rd harmonic in the 35 Hz case) is that they are located either close to the 1st natural frequency area or close to the 2nd natural frequency area. The frequency components which are far from the natural frequency area, such as the GMF in the 8 Hz

case, the 3rd order GMF in the 25 Hz case, and the 2nd order GMF in the 35 Hz case, have lower amplitudes compared to the frequency components located within the natural frequency area. This phenomenon allows us to conclude that the amplitudes of the frequency components near the natural frequency area tend to be higher than the amplitudes of the frequency components that are far away from the natural frequency area. This discovery confirms the explanation of why the GMF and its harmonics do not decrease successively and why the frequency amplitudes tend to be higher (or lower) in a specific area. This conclusion can be helpful for gear fault diagnosis since many detection/diagnosis methods focus on filtering the vibration signal in a specific frequency range to examining the fault symptoms (e.g. sidebands). Based on this discovery, the filter design can be narrowed to the 1st and/or 2nd natural frequency areas where the gear tooth mesh frequencies and corresponding fault symptoms tend to be more obvious.

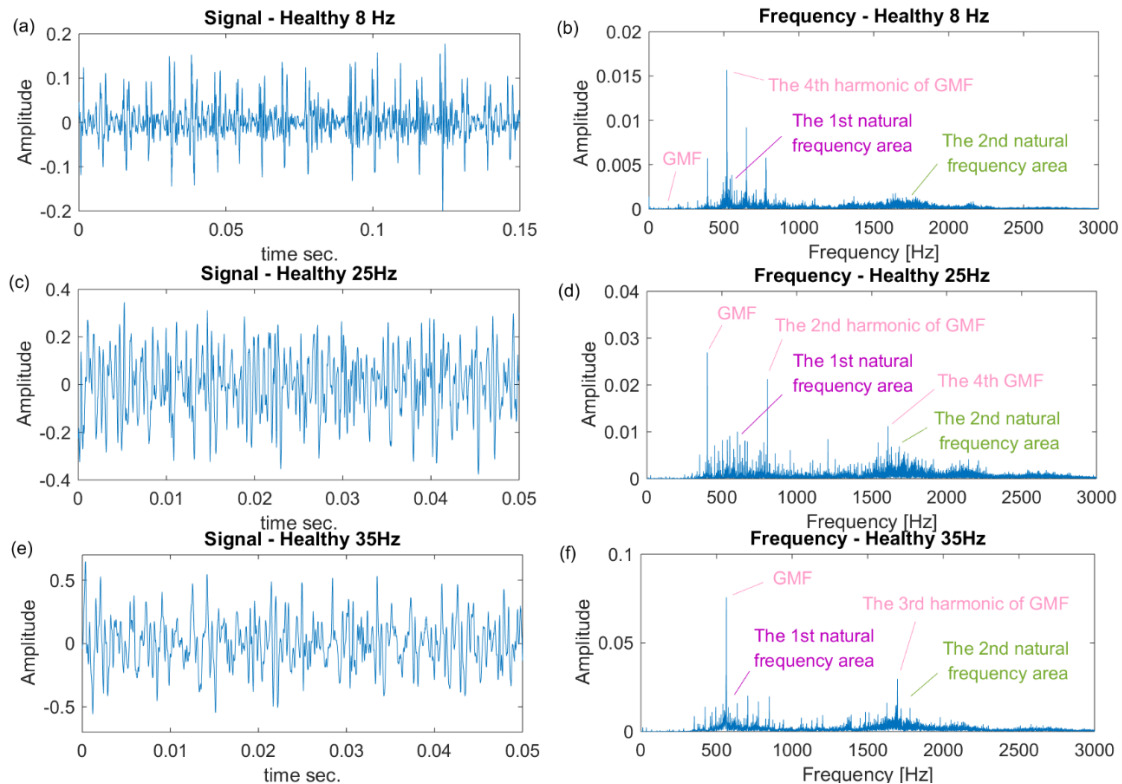


Fig. 7-3 Comparison of the vibration behavior of the gear system under different speed conditions

7.5 Fault feature analysis of gear system with progressive spalls

The significant differences of the vibration responses and frequency features of the gear system due to speed differences under healthy conditions may correspondingly result in different fault vibration features under different speed conditions. Analyzing fault features under different vibration response conditions is important for gear fault diagnosis and it helps to answer the question whether the same gear fault is more easily diagnosed under some conditions.

The transmission running at 35Hz speed condition is operating within the system natural frequency region, which is not typically permitted in real applications. Operating the system with tooth faults under this resonant condition is dangerous and may lead to failures of other components. In addition, the 8 Hz and 25 Hz speed operating conditions already covered the two typical vibration behavior (with and without gear mesh impulse vibrations) of the system, thus, only these two speed conditions are considered in this test to analyze the fault features of progressive tooth spall.

7.5.1 Fault feature analysis of spalls in lower speed condition

The evolution of fault vibration features of the gear transmission with progressive tooth spall at constant speed 8 Hz condition of pinion shaft is shown in Fig. 7-4, in which all of the plots are set in the same ranges in order to easily compare the amplitude changes between each test. Due to the large number of tests, only a selection of the signals, namely the initial fault, moderate fault, and server fault cases (see Fig. 7-2) are selected to show their vibration features in this comparison.

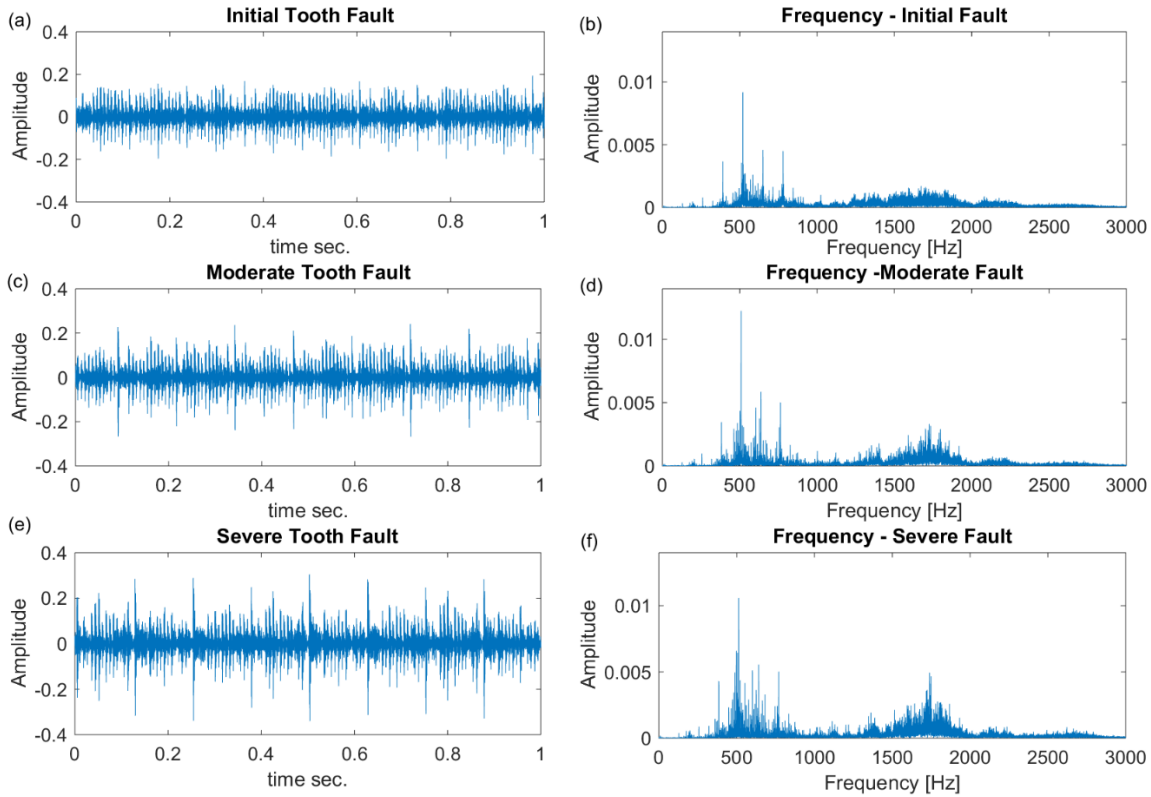


Fig. 7-4 Comparison of the vibration features of progressive tooth spall under lower speed conditions

As can be seen in Fig. 7-4 (c and e), there are obvious periodic impulse vibrations (once per revolution) in the time domain. The occurrence of the vibration impulses is mainly due to the sudden change of gear tooth stiffness, surface roughness, load distribution and increased transmission error due to the localized tooth spall. Fig. 7-5 shows detailed statistical properties of the impulse features under different severity conditions (from the healthy condition of the 1st point to the case of 100% surface damage condition). The Peak to Peak (PP) value shown in Fig. 7-5 (a) is evaluated by $P = \max(x) - \min(x)$ where x is the vibration signal. Though the amplitude of the PP fluctuates from case to case, there is a general trend that the amplitude of the fault vibration impulse increases with an increase in the fault severity. The Impulse Factor (IF), defined as $IF = \max(|x|) / \text{mean}(|x|)$ which is a healthy indicator utilized to evaluate the impulse feature of a

vibration signal [192], shows similar feature as the PP indicator, see Fig. 7-5 (b). The “peakedness” of vibration signals is measured by the kurtosis indicator [50] and is shown in Fig. 7-5 (c). It can be observed that the value of the kurtosis has a general increasing trend with respect to fault severity.

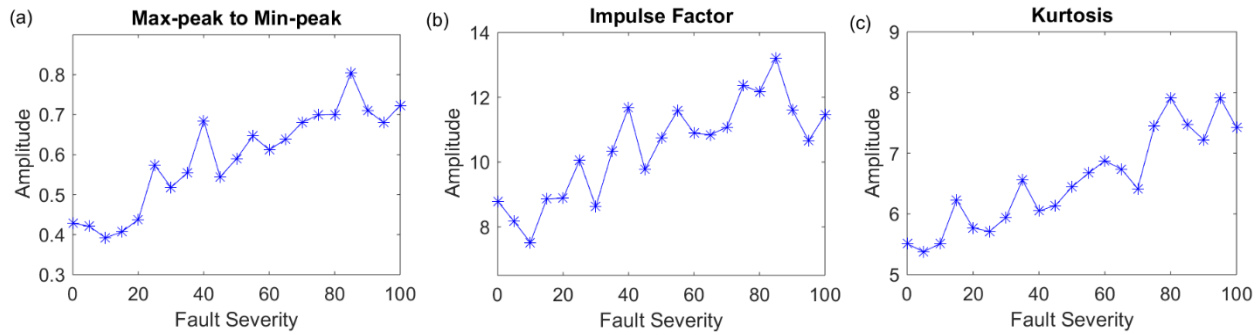


Fig. 7-5 The time domain impulsive vibration features of 8 Hz speed cases

The frequency components and amplitudes also change significantly with progressing tooth spall, as shown in Fig. 7-4 (b, d, f). The most obvious change of the frequency components is near the resonant frequencies area. It can be observed that the sidebands mainly appear in the 1st natural frequency area and became more and more prominent as the fault progresses. Fig. 7-6 shows detailed frequency comparison within the frequency area 350 Hz ~ 820 Hz of the corresponding cases.

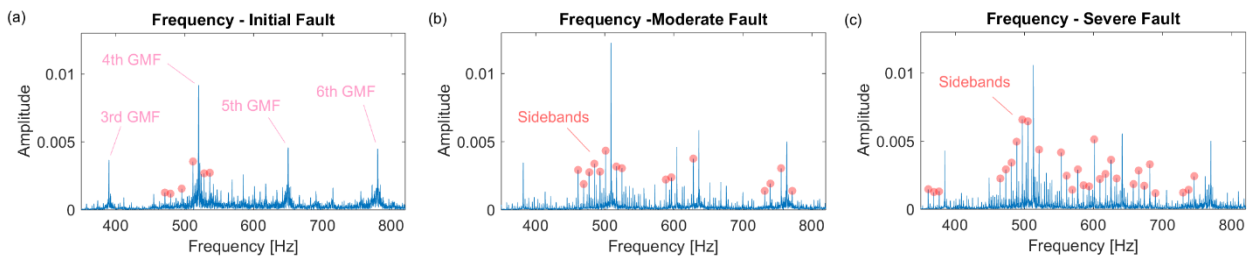


Fig. 7-6 Frequency comparison of progressive tooth spall with constant speed 8 Hz

As shown in Fig. 7-6, under initial tooth spall conditions, the frequency components mainly consist of GMF and its higher order harmonics. For the moderate and sever spalling cases,

sidebands (with frequencies marked with red dots) spaced at the rotational frequency of the faulty gear around the harmonics of the GMF can be clearly observed. The appearance of the sideband phenomenon is mainly due to the periodic short-period impulse vibration due to localized gear tooth fault, as explained in [180-182, 191]. It can be seen that the sidebands are distributed unsymmetrically to each main frequency. The asymmetric sidebands phenomenon is mainly due to the natural frequency modulation and system damping effect, as explained in [191].

Comparing the sideband features under different severity conditions shown in Fig. 7-6, there exists a general trend that the number and amplitudes of the sidebands increase with increasing severity of the tooth spall. Moreover, with the increase in fault severity, the distribution of the sidebands also changes. For example, the sidebands which were mainly distributed on the left side of the 4th and 5th GMF in the case of moderate fault case gradually extend to the right side of the 4th and 5th GMF as the fault severity increases. In addition, the amplitudes of the early sidebands (e.g. the sidebands shown in the moderate fault case) increase significantly as the tooth spall progresses.

The typical indicators utilized to describe the strength of the sidebands are the sideband index (SI) [107] and sideband level factor (SLF) [110], expressed as

$$SI = \frac{1}{K} \sum_{k=1}^K S_k \quad (7.1)$$

and

$$SLF = \sum_{k=1}^K S_k / Std \quad (7.2)$$

where S_k is the amplitude of the k^{th} sideband around the 1st GMF, Std is the standard deviation of the vibration signal. These two indicators were designed to mainly consider the sidebands around the 1st GMF area. However, for the 8 Hz speed cases, the 1st GMF has negligible amplitude and few sidebands can be found around this area. Therefore, these two indicators were modified to

consider the sidebands (with amplitude $\geq 2 \times 10^{-3}$ in this work) around the 4th and 5th GMF frequency area. Fig. 7-7 shows the sideband features measured by the SI, SLF, and sideband number.

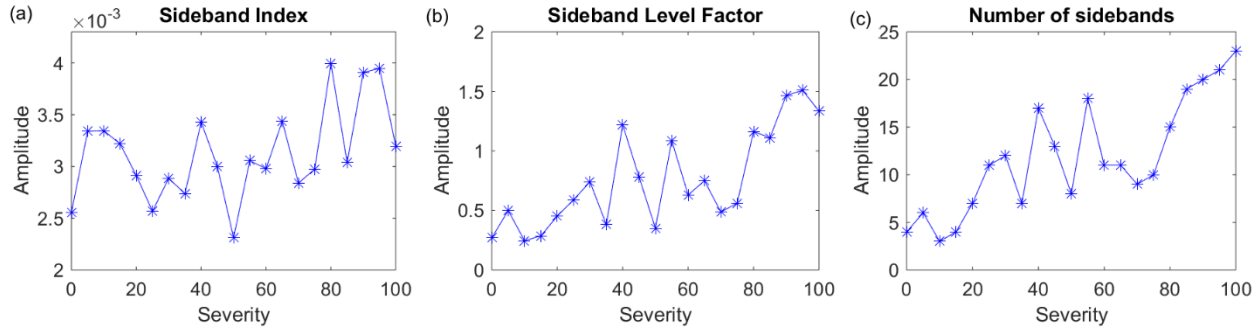


Fig. 7-7 Sideband property of progressive tooth spall under lower speed conditions

As can be observed in Fig. 7-7(a), the SI varies irregularly and loses sensitivity to track the severity of the tooth spall. The reason for the ineffectiveness of SI to track the severity is because both of the amplitudes and numbers of sidebands increase as the severity of the tooth spall increases, as shown in Fig. 7-6 and Fig. 7-7(c). Therefore, the mean value of the sidebands may not increase with the increase of the fault severity. The SLF shows better performance than the SI indicator; it has a general increasing trend with increasing fault severity, see Fig. 7-7(b). The evolution of the sideband feature of progressive tooth spall has demonstrated that the change of number, amplitude and distribution of sidebands indicate a change of fault severity.

7.5.2 Fault feature analysis of spalls in higher speed condition

As discussed in section 7.4, the significant difference between the vibration responses of the lower and higher speed conditions may result in corresponding fault feature differences. This section examines the fault features of progressive tooth spall under higher speed conditions and compares the fault feature differences with the lower speed cases.

The vibration signals and the corresponding frequency features of the initial fault, moderate fault, and severe fault cases at the 25 Hz speed case are shown in Fig. 7-8. It can be seen that the impulse vibration due to tooth spall is less obvious compared to the impulse features shown in the lower speed conditions (see Fig. 7-4), which indicates that the gear fault is harder to identify under higher speed conditions in this test. The weakened impulse feature is mainly due to the increase of the amplitude of the whole vibration signal as a result of the speed increase. For example, the average peak amplitude of the vibration signal was about 0.15v for the 8 Hz speed case under initial fault condition, while the average peak amplitude is increased to 0.3v for the 25 Hz initial fault case, which is large enough to submerge most of the impulse vibrations caused by the tooth spalls.

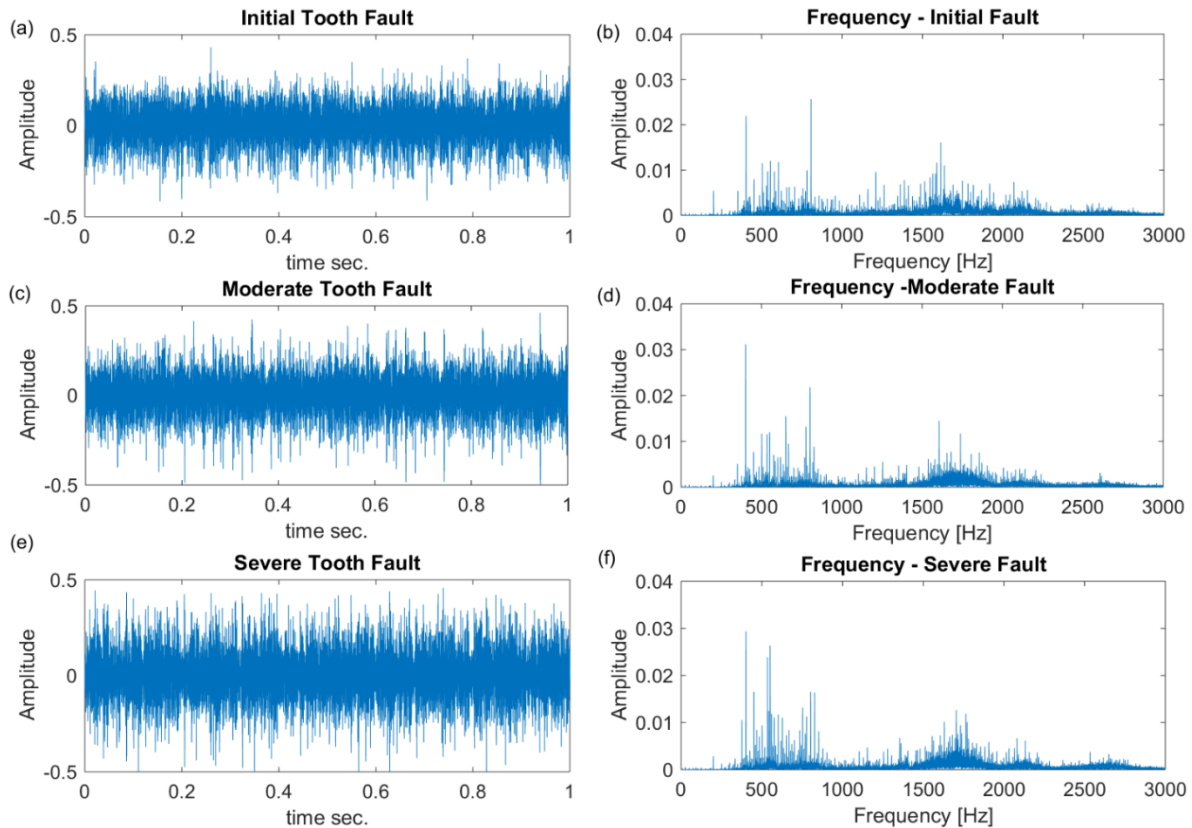


Fig. 7-8 Comparison of the vibration features of progressive tooth spall under higher speed conditions

The impulsive vibration features of the 25 Hz speed case are shown in Fig. 7-9. The PP value varies more irregularly compared to the 8 Hz case shown in Fig. 7-5(a), but still indicates that the maximum vibration peak of the signal has an increasing trend for increasing fault severity. The impulse feature and the peakedness of the vibration signals, measured by the impulse factor and kurtosis, are very unstable in this case, see Fig. 7-9 (b) and (c), which indicate that the impulsive features of the signals due to tooth spall under higher speed conditions are more difficult to be captured or described.

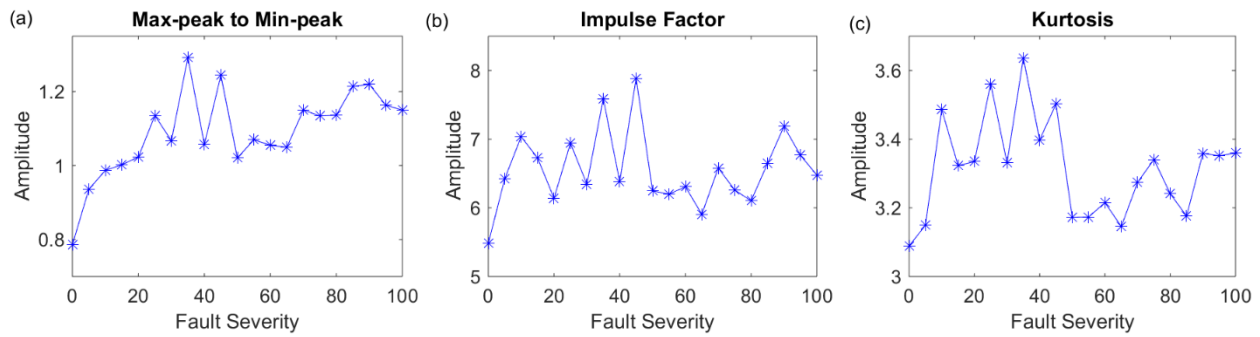


Fig. 7-9 The time domain impulsive vibration features of 25 Hz speed cases

As can be observed in Fig. 7-8 (b, d, f), the evolution of the sidebands is obvious in the 25 Hz speed conditions. Fig. 7-10 shows the zoomed frequency features within 300Hz~1000Hz. It can be seen that there is a general trend that the number and amplitudes of the sidebands increases with increasing severity of the tooth spall, similar to the trend of the lower speed (8Hz) cases.

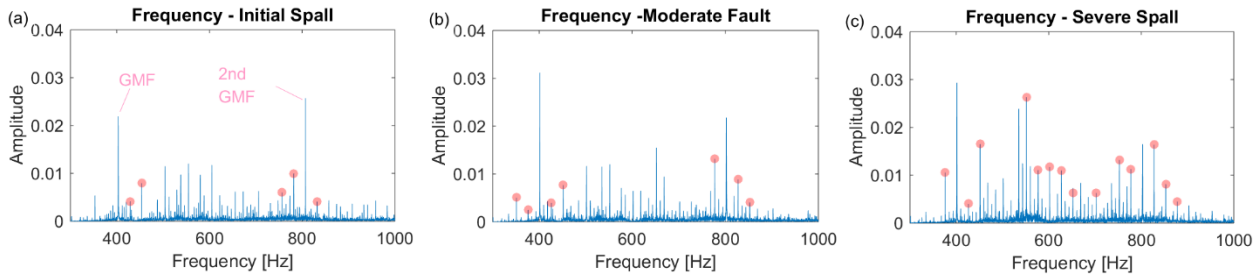


Fig. 7-10 Frequency comparison of progressive tooth spall with constant speed 25 Hz

The sideband property around the 1st natural frequency area of the progressive tooth spall at the 25Hz speed condition is shown in Fig. 7-11. It can be seen that the mean amplitude of the sidebands measured by the SI indicator varies irregularly. The SLF, Fig. 7-11 (b), fluctuates from case to case but still has a general increasing trend with increasing fault severity. The number of the sidebands slightly increases with increasing tooth spall. It can be seen that the amplitudes of the sidebands close to the 1st natural frequency area are slightly higher than those further away from the natural frequency area. This phenomenon is consistent with the conclusion made in [191] which it states “the amplitudes of those modulation sidebands around the mesh frequency harmonics are asymmetric, and the side close to the natural frequency would be higher than the other side.”

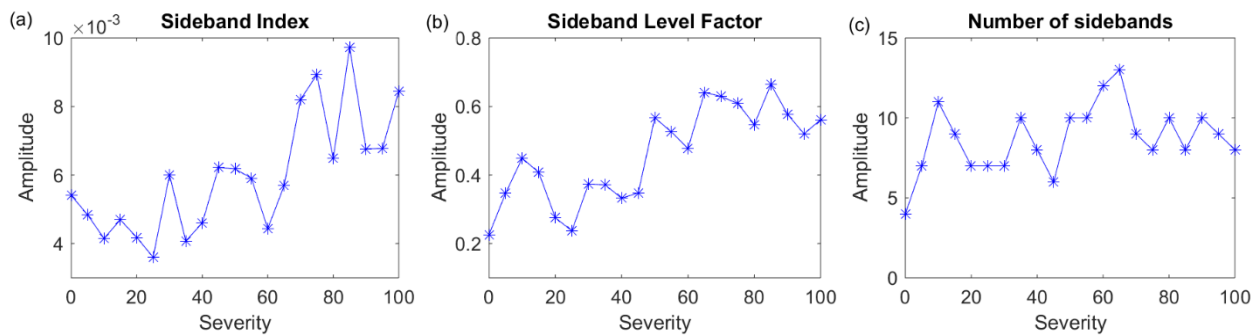


Fig. 7-11 Sideband property around the 2nd GMF of progressive tooth spall under higher speed conditions

The fault feature analysis of the progressive tooth spall at 8 Hz and 25 Hz speed conditions indicates that the rotational speed can result in significant difference in fault features, as well as affect the diagnosability of the gear tooth fault. First, the time domain impulse features of the signals under the lower speed condition (8 Hz) is more obvious than that in the higher speed condition under the same fault severity level. The condition indicators (peak to peak, impulse factor, and kurtosis) have general increasing trends with increasing fault severity at the 8 Hz speed conditions, while the values of these same indicators become unstable and fluctuate irregularly

when the rotational speed of the input shaft is increased to 25 Hz. Second, the sideband features in the frequency domain of the lower speed cases are also more obvious than the features of the higher speed cases. The sidebands phenomenon is much clearer in the case of moderate fault for the 8 Hz speed condition than the 25 Hz speed condition. The fault feature difference between the lower and higher speed conditions in this comparison indicates that the localized tooth spall may easier to be diagnosed in the lower speed conditions.

7.6 Fault detection of progressive tooth spall

Based on the fault feature analysis of the progressive tooth spall in section 7.5, there is evidence that the general vibration level and strength of the periodic impulses increase with increasing fault severity. In this section, a new health indicator which is based on the mean of the n^{th} order peaks (MPn) is proposed to track and assess fault severity of localized gear tooth spall.

The main reason that the existing indicators (PP and IF) failed to follow the severity of the progressive tooth spall is due to the impulse outliers (with extreme high or low amplitudes) in the vibration signal. Therefore, taking the maximum value of a vibration signal may not reflect the overall vibration features of the periodic impulses. A better way to capture the impulse energy due to spall is to consider all of the periodic impulses of the signal instead of considering only the maximum and/or minimum value. A new indicator which takes the mean of the n^{th} order peaks (MPn) is proposed as

$$MPn = \frac{1}{K} \sum_{k=1}^K y_n, \quad (7.3)$$

where $y_n = \text{peak}^n(x)$, denotes the n^{th} ($n \geq 0$) order peaks of the time domain vibration signal x . The 0^{th} order peak is the vibration signal itself, $y_1 = \text{peak}^1(x)$ denotes the peak values of the original vibration signal, $y_2 = \text{peak}^2(x)$, are the 2^{nd} order peaks of the original vibration signal, which are

the peak values of the 1st order peaks. Using Fig. 7-12 as an example, the 4th order peaks (red line) are the peaks of the 3rd order peaks (blue line) of the vibration signal. The peak value is defined as a local maximum point of a signal, which is a point with amplitude larger than its two neighboring points. If the amplitude of a point is equal to both of its neighboring points, the first point is recognized as a local peak. The n^{th} order peaks of a signal are defined as the peaks of the $(n-1)^{th}$ order peaks of the signal.

The property of the n^{th} order peaks is that with the increase of peak order, fewer peaks with higher amplitudes are selected. As can be seen in Fig. 7-12, the 3rd order peaks contain details of the original signal, and the 4th order peak values, which are the peak values of the 3rd order peak curve, mainly select the higher peaks of the 3rd order peaks, and the 6th order peaks mainly contain the periodic impulses.

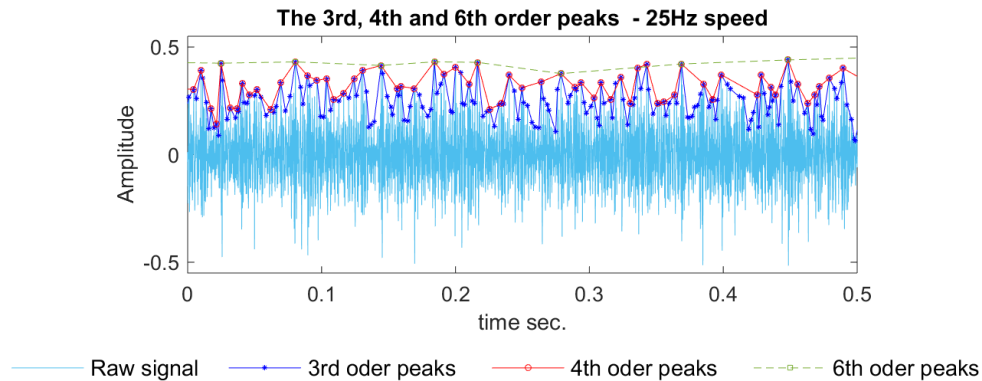


Fig. 7-12 The 3rd, 4th and 6th order peaks of the vibration signals of sever at 25 Hz speed conditions

The mean of the 3rd, 4th and 6th order peaks of the vibration signals with progressive tooth spall under lower and higher speeds conditions are shown in Fig. 7-13 and Fig. 7-14, respectively.

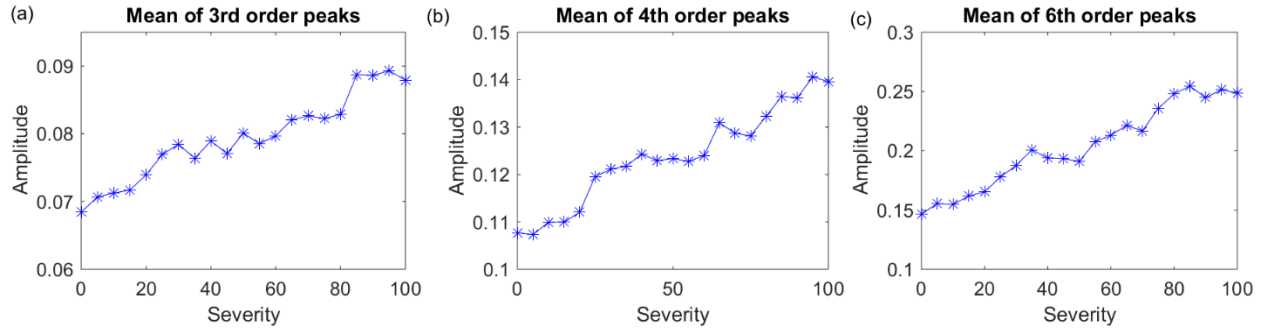


Fig. 7-13 MPn of signals with progressive localized tooth spall-8 Hz speed conditions

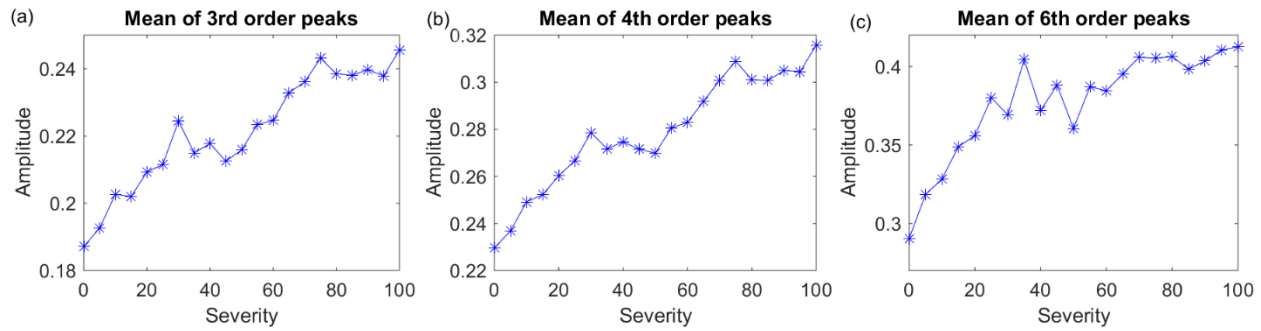


Fig. 7-14 MPn of signals with progressive localized tooth spall-25 Hz speed conditions

As can be observed in Fig. 7-13 and Fig. 7-14, the MPn reacts robustly to progressing localized tooth spall under both lower and higher speed conditions. It has a good monotonic feature and continuously reacts to the progressive tooth spall from the healthy state to the severely damaged level. The proposed MPV4 out-performs all of the other three indicators shown in Fig. 7-5 and Fig. 7-9. The reason behind the success of the proposed method is that gear tooth faults can excite extra impulses, which increase the system vibration amplitudes.

7.7 Conclusion

Fault feature analysis plays a vital role in gear fault diagnosis. Understanding the fault characteristics and corresponding fault feature evolution is the key to success of fault diagnosis and severity assessment. The experimental analysis of fault features of progressive tooth spall in this work indicates that:

- 1) The change of the rotational speed can result in a corresponding change of the vibration behavior of a system and corresponding fault features. It is found that the amplitudes of the frequency components near the natural frequency area tend to be higher than the amplitudes of the frequency components that are far away from the natural frequency area. The phenomenon that the amplitudes of the GMF and its harmonics do not decrease with order is mainly due to the modulation of the system natural frequencies.
- 2) The existence of localized tooth spall can result in periodic impulse vibrations once per revolution in the time domain. The occurrence of the vibration impulses is mainly due to the sudden change of gear tooth stiffness, surface roughness, load distribution and the increased transmission error due to the seeded tooth spall.
- 3) Vibration impulse features (Peak to Peak, Impulse Factor and Kurtosis) fluctuate from case to case and have non-linear increasing trends with progress of the tooth spall.
- 4) In the frequency domain, the sidebands appear due to periodic vibration impulses of the tooth spall. The sidebands are unsymmetrically located around the dominant gear mesh frequencies due to natural frequency modulation and system damping effects.
- 5) The changes of the number, amplitude and distribution of sidebands indicate a possible change of in fault severity and these features have general increasing trends as the fault becomes more severe.
- 6) The fault feature comparison between the lower and higher speed conditions indicates that rotational speed can result in a significant difference on the fault features, as well as affect the gear tooth fault diagnosability. Both the vibration impulse features and the sidebands phenomenon of the lower speed cases (8 Hz) are more obvious than that at the higher (25 Hz) speed conditions.
- 7) The proposed mean of the n^{th} order peaks (MP_n , especially MP_3 and MP_4) has a very good monotonic feature and is able to continuously react to progressive tooth spall, which outperforms other traditional indicators analyzed in this work.

Chapter 8 Contributions, conclusions and future work

8.1 Thesis Contributions

The contributions of the thesis include the following:

- 1) A novel gear mesh kinematic model was proposed. This model is capable of valuating the actual contact positions of tooth engagement even under the condition of changes in gear center distance. This not only enables the TVMS of a mating spur gear pair to be obtained under normal assembly conditions but also permits the modelling and evaluation of the effects of assembly errors, gear run-out errors, shaft bending, bearing deformation etc. (alone or combination) on the TVMS.
- 2) A curved-bottom shaped tooth spall model was developed based on an ellipsoid geometry. This model allows for changes in radii in three dimensions to best fit the shape of a tooth spall. Hence, this model enables the simulation of precise gear TVMS with tooth spalls with curved bottom features.
- 3) A shape-independent method was proposed that is capable of modeling almost any kind of spall, such as a localized tooth spall with simple shapes (rectangular, round, elliptical, etc.) or irregular shapes, and additionally spalls under normal or randomly distributed conditions. In other words, more realistic spall models can be developed, which is of great practical significance.
- 4) A gear dynamical model was proposed that considers the effects of tooth surface roughness changes and geometric deviations due to pitting and spalling. This model also incorporates Time Varying Mesh Stiffness (TVMS), a time-varying load-sharing ratio, as well as dynamic tooth-contact friction forces, friction moments and dynamic mesh damping ratios. This model that allows for a wider range of the gear mating properties to be simulated

compared with existing gear dynamic models. Comparison between simulated vibrations and experimental results indicate that the proposed gear dynamical model is effective for modeling gear transmission vibrations under both healthy and faulty conditions.

- 5) The thesis contributed a comprehensive experimental analysis of the evolution of fault vibration features of a gear transmission with progressive localized gear tooth spalling. The effects of rotational speed on the vibration features of the gear transmission were analyzed. The change in fault features (e.g. periodic impulses and sidebands phenomenon) under different fault severity levels and speed conditions were also compared.
- 6) A new health indicator that uses the mean of the n th order peaks was proposed for detecting progressive localized tooth spall. Experiments showed that it had a very good monotonic feature and was able of continuously reacting to progressive tooth spall under constant load and speed conditions.

8.2 Conclusions

The gear mesh kinematic model proposed in chapter 3 considers the actual gear center distance, pressure angle and the working areas of teeth involute profiles of a mating spur gear pair. The inclusion of these factors enables the evaluation of the teeth mesh contact positions with either constant or time varying gear center distances. The proposed kinematic model was applied to evaluate the TVMS of a spur gear pair with eccentric error; results indicate that the time-varying center distance variation shows both time varying frequency and amplitude modulation effects on the mesh stiffness.

The ellipsoid shape-based method proposed in chapter 4 utilized to model the gear tooth spalls with curved bottom geometric features corrects the foundation stiffness within the double tooth contact area and considers the non-linearity of Hertzian contact stiffness. The effectiveness of the method for modelling tooth spalls with different shapes and severity conditions was validated by

finite element analysis. Results indicate that the proposed method can provide precise TVMS of practical tooth spalls.

The shape-independent method proposed in chapter 5 is capable of modeling most kinds of spalls, such as a localized tooth spall with simple shapes (rectangular, round, elliptical, etc.) or irregular shapes, and spalls under normal or randomly distributed conditions. The method uses the defect length ratio and the defect depth ratio to model the features and severity of the gear tooth spalls. The effectiveness of the proposed shape-independent method in modelling different kinds of tooth spalls was analyzed and validated by FEM analysis.

The dynamical model proposed in chapter 6 is an advanced model which considers the effects of the gearbox casing, tooth surface roughness changes and geometric deviations due to pitting and spalling, incorporates TVMS, time-varying load sharing ratio, and dynamic tooth contact friction forces, friction moments and mesh damping ratios. The proposed gear dynamic model was validated by comparison with experimental results under different rotation speeds and fault conditions. Results indicate that the proposed model can effectively reflect the actual vibration behavior of the real transmission and successfully capture the fault features when the transmission is operating under faulty conditions.

The experimental study on the features of gear tooth pitting and spalling in chapter 7 shows that rotational speed can result in a corresponding change on the vibration behavior of a system and corresponding fault features. Localized tooth spall can result in periodic impulse vibrations once per revolution in the time domain. In the frequency domain, sidebands appear which are unsymmetrically located around the natural frequency area. It is found that the number, amplitude and distribution of sidebands have a general tendency to increase as the fault becomes more severe. Experimental results shown that the proposed indicator MP_n (mean of the n th order peaks) has a

very good monotonic feature and is capable of continuously reacting to progressive tooth spall under constant load and speed conditions.

8.3 Future work

The main work of this thesis focused on localized gear tooth pitting and spalling. One possible avenue for future work the dynamical modeling of distributed gear tooth pitting and spalling as well the analysis their corresponding fault features.

The obtained fault evolution features of progressive localized gear tooth spall in this thesis are important for designing advanced methods to evaluate the remaining useful life (RUL) of a failing component. The design of RUL methods based on obtained results is an area of potential future research.

The dynamic model proposed in this thesis develops the model of gear tooth pitting and spalling in a one-stage spur gear transmission. This model can be can be further upgraded to a multiple stage gear dynamic model to analyze the corresponding vibration behavior of a multi-stage system. Moreover, future work may also apply the proposed gear dynamic model to analyze the fault vibration behavior of bearings in a gear transmission.

The effectiveness of the proposed MPn on detecting fault severity under varying load and speed conditions needs to be further examined and explored.

References

- [1] Y. Luo, N. Baddour, and M. Liang, "Performance Assessment of Gear Condition Indicators in Detecting Progressive Gear Tooth Crack," *ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, vol. 8: 29th Conference on Mechanical Vibration and Noise () :V008T12A010, 2017.
- [2] P. D. Samuel and D. J. Pines, "A review of vibration-based techniques for helicopter transmission diagnostics," *Journal of sound and vibration*, vol. 282, pp. 475-508, 2005.
- [3] I. Yesilyurt, F. Gu, and A. D. Ball, "Gear tooth stiffness reduction measurement using modal analysis and its use in wear fault severity assessment of spur gears," *NDT & E International*, vol. 36, pp. 357-372, 2003.
- [4] G. J. Vachtsevanos, F. Lewis, A. Hess, and B. Wu, *Intelligent fault diagnosis and prognosis for engineering systems*. Hoboken, NJ: Wiley Online Library, 2006.
- [5] C. Scheffer and P. Girdhar, "Practical Machinery Vibration Analysis & Predictive Maintenance," *Burlington, MA: Newnes*, vol. 252, 2004.
- [6] L. H. Chiang, E. L. Russell, and R. Braatz, "Fault detection and diagnosis in industrial systems," ed: IOP Publishing, 2001.
- [7] R. Stewart, "Some Useful Data Analysis Techniques for Gearbox Diagnostics. Machine Health Monitoring Group Report MHM," R/10/77, University of Southampton1977.
- [8] J. J. Zakrajsek, D. P. Townsend, and H. J. Decker, "An analysis of gear fault detection methods as applied to pitting fatigue failure data," DTIC Document1993.
- [9] J. J. Zakrajsek, R. F. Handschuh, and H. J. Decker, "Application of fault detection techniques to spiral bevel gear fatigue data," *NASA STI/Recon Technical Report N*, vol. 94, 1994.

References

- [10] J. Ma, "Energy operator and other demodulation approaches to gear defect detection," in *Proceedings of the 49th Meeting of the Society for Machinery Failure Prevention Technology*, 1995, pp. 127-140.
- [11] P. Rzeszucinski, J. Sinha, R. Edwards, A. Starr, and B. Allen, "Normalised root mean square and amplitude of sidebands of vibration response as tools for gearbox diagnosis," *Strain*, vol. 48, pp. 445-452, 2012.
- [12] V. Sharma and A. Parey, "A review of gear fault diagnosis using various condition indicators," *Procedia Engineering*, vol. 144, pp. 253-263, 2016.
- [13] Y. Zhan, V. Makis, and A. K. Jardine, "Adaptive state detection of gearboxes under varying load conditions based on parametric modelling," *Mechanical Systems and Signal Processing*, vol. 20, pp. 188-221, 2006.
- [14] H. Huang, N. Baddour, and M. Liang, "Auto-OBSD: Automatic parameter selection for reliable Oscillatory Behavior-based Signal Decomposition with an application to bearing fault signature extraction," *Mechanical Systems and Signal Processing*, vol. 86, pp. 237-259, 2017.
- [15] D. M. Linli Cui, La Wu, Qian Wang, Cunguang Wang, "Bearing fault diagnosis method based on composite Q-factor base algorithm," *Chinese invention patent*, vol. CN 10998119 A, 2013.
- [16] I. S. Bozchalooi and M. Liang, "Parameter-free bearing fault detection based on maximum likelihood estimation and differentiation," *Measurement Science and Technology*, vol. 20, p. 065102, 2009.
- [17] I. M. Howard, "A review of rolling element bearing vibration detection, diagnosis and prognosis," *NASA STI/Recon Technical Report N*, vol. 95, 1994.
- [18] Y. Luo, N. Baddour, and M. Liang, "Dynamical modeling and experimental validation for tooth pitting and spalling in spur gears," *Mechanical Systems and Signal Processing*, vol. 119, pp. 155-181, 2019.

References

- [19] Y. Luo, N. Baddour, and M. Liang, "A shape-independent approach to modelling gear tooth spalls for time varying mesh stiffness evaluation of a spur gear pair," *Mechanical Systems and Signal Processing*, vol. 120, pp. 836-852, 2019.
- [20] H. Endo, R. Randall, and C. Gosselin, "Differential diagnosis of spall vs. cracks in the gear tooth fillet region: Experimental validation," *Mechanical Systems and Signal Processing*, vol. 23, pp. 636-651, 2009.
- [21] E. Ebrahimi, "Fault diagnosis of Spur gear using vibration analysis," *Journal of American Science*, vol. 8, pp. 133-138, 2012.
- [22] D. Dewell and L. Mitchell, "Detection of a misaligned disk coupling using spectrum analysis," *Journal of vibration, acoustics, stress, and reliability in design*, vol. 106, pp. 9-16, 1984.
- [23] F. Combet and L. Gelman, "Optimal filtering of gear signals for early damage detection based on the spectral kurtosis," *Mechanical Systems and Signal Processing*, vol. 23, pp. 652-668, 2009.
- [24] F. Combet, L. Gelman, and G. LaPayne, "Novel detection of local tooth damage in gears by the wavelet bicoherence," *Mechanical Systems and Signal Processing*, vol. 26, pp. 218-228, 2012.
- [25] M. El Badaoui, J. Antoni, F. Guillet, J. Daniere, and P. Velez, "Use of the moving cepstrum integral to detect and localise tooth spalls in gears," *Mechanical Systems and Signal Processing*, vol. 15, pp. 873-885, 2001.
- [26] C. Rahmoune and D. Benazzouz, "Early detection of pitting failure in gears using a spectral kurtosis analysis," *Mechanics & Industry*, vol. 13, pp. 245-254, 2012.
- [27] H. Öztürk, M. Sabuncu, and I. Yesilyurt, "Early detection of pitting damage in gears using mean frequency of scalogram," *Journal of Vibration and Control*, vol. 14, pp. 469-484, 2008.
- [28] W. Teng, F. Wang, K. Zhang, Y. Liu, and X. Ding, "Pitting fault detection of a wind turbine gearbox using empirical mode decomposition," *Strojniški vestnik-Journal of Mechanical Engineering*, vol. 60, pp. 12-20, 2014.

References

- [29] Y. Luo, N. Baddour, and M. Liang, "Effects of gear center distance variation on time varying mesh stiffness of a spur gear pair," *Engineering Failure Analysis*, vol. 75, pp. 37-53, 2017.
- [30] S. Li and A. Kahraman, "A tribo-dynamic model of a spur gear pair," *Journal of Sound and Vibration*, vol. 332, pp. 4963-4978, 2013.
- [31] S. Li and A. Kahraman, "A micro-pitting model for spur gear contacts," *International Journal of Fatigue*, vol. 59, pp. 224-233, 2014.
- [32] S. A. Abouel-seoud, E. S. Dyab, and M. S. Elmorsy, "Influence of tooth pitting and cracking on gear meshing stiffness and dynamic response of wind turbine gearbox," *Int. J. Sci. Adv. Technol*, vol. 2, pp. 151-165, 2012.
- [33] S. Jia and I. Howard, "Comparison of localised spalling and crack damage from dynamic modelling of spur gear vibrations," *Mechanical Systems and Signal Processing*, vol. 20, pp. 332-349, 2006.
- [34] H. Ma, Z. Li, M. Feng, R. Feng, and B. Wen, "Time-varying mesh stiffness calculation of spur gears with spalling defect," *Engineering Failure Analysis*, vol. 66, pp. 166-176, 2016.
- [35] X. Liang, H. Zhang, L. Liu, and M. J. Zuo, "The influence of tooth pitting on the mesh stiffness of a pair of external spur gears," *Mechanism and Machine Theory*, vol. 106, pp. 1-15, 2016.
- [36] A. F. d. Rincon, F. Viadero, M. Iglesias, A. de-Juan, P. Garcia, and R. Sancibrian, "Effect of cracks and pitting defects on gear meshing," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 226, pp. 2805-2815, 2012.
- [37] A. Saxena, A. Parey, and M. Chouksey, "Time varying mesh stiffness calculation of spur gear pair considering sliding friction and spalling defects," *Engineering Failure Analysis*, vol. 70, pp. 200-211, 2016.
- [38] R. G. Parker, Y. Guo, T. Eritenel, and T. M. Ericson, "Vibration propagation of gear dynamics in a gear-bearing-housing system using mathematical modeling and finite element analysis," Technical Report, NASA Contract NNC08CB03C, 2011.

References

- [39] Y. Guo, T. Eritenel, T. M. Ericson, and R. G. Parker, "Vibro-acoustic propagation of gear dynamics in a gear-bearing-housing system," *Journal of Sound and Vibration*, vol. 333, pp. 5762-5785, 2014.
- [40] G. Dalpiaz, A. Rivola, and R. Rubini, "Effectiveness and sensitivity of vibration processing techniques for local fault detection in gears," *Mechanical systems and signal processing*, vol. 14, pp. 387-412, 2000.
- [41] P. McFadden and J. Smith, "Effect of transmission path on measured gear vibration," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 108, pp. 377-378, 1986.
- [42] X. Liang, M. J. Zuo, and M. R. Hoseini, "Vibration signal modeling of a planetary gear set for tooth crack detection," *Engineering Failure Analysis*, vol. 48, pp. 185-200, 2015.
- [43] F. Chaari, W. Baccar, M. S. Abbes, and M. Haddar, "Effect of spalling or tooth breakage on gearmesh stiffness and dynamic response of a one-stage spur gear transmission," *European Journal of Mechanics-A/Solids*, vol. 27, pp. 691-705, 2008.
- [44] Y. Lei, Z. Liu, D. Wang, X. Yang, H. Liu, and J. Lin, "A probability distribution model of tooth pits for evaluating time-varying mesh stiffness of pitting gears," *Mechanical Systems and Signal Processing*, vol. 106, pp. 355-366, 2018.
- [45] H. Ma, X. Pang, R. Feng, J. Zeng, and B. Wen, "Improved time-varying mesh stiffness model of cracked spur gears," *Engineering Failure Analysis*, vol. 55, pp. 271-287, 2015.
- [46] A. F. del Rincon, F. Viadero, M. Iglesias, A. de-Juan, P. Garcia, and R. Sancibrian, "Effect of cracks and pitting defects on gear meshing," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 226, pp. 2805-2815, 2012.
- [47] S. Sirichai, I. Howard, L. Morgan, and K. Teh, "Finite element analysis of gears in mesh," *Fifth International Congress on Sound and Vibration*, vol. 1, pp. 869-876, 1997.
- [48] S. Zouari, M. Maatar, T. Fakhfakh, and M. Haddar, "Three-dimensional analyses by finite element method of a spur gear: effect of cracks in the teeth foot on the mesh stiffness," *Journal of failure analysis and prevention*, vol. 7, pp. 475-481, 2007.

References

- [49] A. F. Del Rincon, F. Viadero, M. Iglesias, P. García, A. De-Juan, and R. Sancibrian, "A model for the study of meshing stiffness in spur gear transmissions," *Mechanism and Machine Theory*, vol. 61, pp. 30-58, 2013.
- [50] S. Wu, M. J. Zuo, and A. Parey, "Simulation of spur gear dynamics and estimation of fault growth," *Journal of Sound and Vibration*, vol. 317, pp. 608-624, 2008.
- [51] H. Ma, J. Yang, R. Song, S. Zhang, and B. Wen, "Effects of tip relief on vibration responses of a geared rotor system," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 228, pp. 1132-1154, 2014.
- [52] D. Yang and J. Lin, "Hertzian damping, tooth friction and bending elasticity in gear impact dynamics," *Journal of mechanisms, transmissions, and automation in design*, vol. 109, pp. 189-196, 1987.
- [53] X. Tian, M. J. Zuo, and K. R. Fyfe, "Analysis of the Vibration Response of a Gearbox With Gear Tooth Faults," vol. 1, pp. 785-793, 2004.
- [54] X. Zhou, Y. Shao, Y. Lei, and M. Zuo, "Time-varying meshing stiffness calculation and vibration analysis for a 16DOF dynamic model with linear crack growth in a pinion," *Journal of Vibration and Acoustics*, vol. 134, p. 011011, 2012.
- [55] Z. Wan, H. Cao, Y. Zi, W. He, and Z. He, "An improved time-varying mesh stiffness algorithm and dynamic modeling of gear-rotor system with tooth root crack," *Engineering Failure Analysis*, vol. 42, pp. 157-177, 2014.
- [56] H. Ma, R. Song, X. Pang, and B. Wen, "Time-varying mesh stiffness calculation of cracked spur gears," *Engineering Failure Analysis*, vol. 44, pp. 179-194, 2014.
- [57] X. Liang, M. J. Zuo, and M. Pandey, "Analytically evaluating the influence of crack on the mesh stiffness of a planetary gear set," *Mechanism and Machine Theory*, vol. 76, pp. 20-38, 2014.
- [58] H. Ma, X. Pang, R. Feng, R. Song, and B. Wen, "Fault features analysis of cracked gear considering the effects of the extended tooth contact," *Engineering Failure Analysis*, vol. 48, pp. 105-120, 2015.

References

- [59] A. Saxena, A. Parey, and M. Chouksey, "Effect of shaft misalignment and friction force on time varying mesh stiffness of spur gear pair," *Engineering Failure Analysis*, vol. 49, pp. 79-91, 2015.
- [60] A. Fernández-del-Rincón, M. Iglesias, A. de-Juan, A. Diez-Ibarbia, P. García, and F. Viadero, "Gear transmission dynamics: Effects of index and run out errors," *Applied Acoustics*, vol. 108, pp. 63-83, 2016.
- [61] T. Eritenel and R. G. Parker, "An investigation of tooth mesh nonlinearity and partial contact loss in gear pairs using a lumped-parameter model," *Mechanism and Machine Theory*, vol. 56, pp. 28-51, 2012.
- [62] W. Yu and C. K. Mechefske, "Analytical modeling of spur gear corner contact effects," *Mechanism and Machine Theory*, vol. 96, pp. 146-164, 2016.
- [63] Y. Pandya and A. Parey, "Experimental investigation of spur gear tooth mesh stiffness in the presence of crack using photoelasticity technique," *Engineering Failure Analysis*, vol. 34, pp. 488-500, 2013.
- [64] N. K. Raghuwanshi and A. Parey, "Mesh stiffness measurement of cracked spur gear by photoelasticity technique," *Measurement*, vol. 73, pp. 439-452, 2015.
- [65] N. K. Raghuwanshi and A. Parey, "Experimental measurement of gear mesh stiffness of cracked spur gear by strain gauge technique," *Measurement*, vol. 86, pp. 266-275, 2016.
- [66] H. Jiang, Y. Shao, and C. K. Mechefske, "Dynamic characteristics of helical gears under sliding friction with spalling defect," *Engineering Failure Analysis*, vol. 39, pp. 92-107, 2014.
- [67] H. N. Özgüven and D. Houser, "Dynamic analysis of high speed gears by using loaded static transmission error," *Journal of Sound and Vibration*, vol. 125, pp. 71-83, 1988.
- [68] H. N. Özgüven and D. R. Houser, "Mathematical models used in gear dynamics—a review," *Journal of sound and vibration*, vol. 121, pp. 383-411, 1988.
- [69] E. Buckingham, "Dynamic loads on gear teeth," *ASME Research Publication, New York*, vol. 10, 1931.

References

- [70] W. Tuplin, "Dynamic loads on gear teeth," *Machine Design*, vol. 25, p. 203, 1953.
- [71] J. Reswick, "Dynamic loads on spur and helical gear teeth," *Trans. ASME*, vol. 77, pp. 635-644, 1955.
- [72] S. L. Harris, "Dynamic loads on the teeth of spur gears," *Proceedings of the Institution of Mechanical Engineers*, vol. 172, pp. 87-112, 1958.
- [73] T. TOBE, "Dynamic Loads on Spur Gear Teeth," *Bulletin of JSME*, vol. 4, pp. 417-422, 1961.
- [74] R. Gregory, S. Harris, and R. Munro, "Dynamic behaviour of spur gears," *Proceedings of the institution of mechanical engineers*, vol. 178, pp. 207-218, 1963.
- [75] T. Aida, S. Sato, and H. Fukuma, "Properties of gear noise and its generating mechanism," in *Proceedings of the Japanese Society of Mechanical Engineers Semi-International Gearing Symposium*, 1967, pp. 151-160.
- [76] K. Nakamura, "Tooth separations and abnormal noise on power-transmission gears," *Bulletin of JSME*, vol. 10, pp. 846-854, 1967.
- [77] K. Ichimaru and F. Hirano, "Dynamic behavior of heavy-loaded spur gears," *Journal of Engineering for Industry*, vol. 96, pp. 373-381, 1974.
- [78] A. Kubo, "Stress condition, vibrational exciting force, and contact pattern of helical gears with manufacturing and alignment error," *ASME J. Mech. Des.*, vol. 100, pp. 77-84, 1978.
- [79] E. Remmers, "Gear mesh excitation spectra for arbitrary tooth spacing errors, load and design contact ratio," *Journal of Mechanical Design*, vol. 100, pp. 715-722, 1978.
- [80] M. Benton and A. Seireg, "Simulation of resonances and instability conditions in pinion-gear systems," *Journal of Mechanical Design*, vol. 100, pp. 26-32, 1978.
- [81] K. Ishida and T. Matsuda, "Effect of tooth surface roughness on gear noise and gear noise transmitting path," *American Society of Mechanical Engineers*, p. 80C2/DET69, 1980.

References

- [82] K. Wang and H. Cheng, "A numerical solution to the dynamic load, film thickness, and surface temperatures in spur gears, part I: analysis," *Journal of Mechanical Design*, vol. 103, pp. 177-187, 1981.
- [83] A. Kahraman and R. Singh, "Non-linear dynamics of a geared rotor-bearing system with multiple clearances," *Journal of sound and vibration*, vol. 144, pp. 469-506, 1991.
- [84] A. Kahraman, H. N. Ozguven, D. R. Houser, and J. J. Zakrajsek, "Dynamic analysis of geared rotors by finite elements," *Journal of Mechanical Design*, vol. 114, pp. 507-514, 1992.
- [85] P. Velex and M. Maatar, "A mathematical model for analyzing the influence of shape deviations and mounting errors on gear dynamic behaviour," *Journal of Sound and Vibration*, vol. 191, pp. 629-660, 1996.
- [86] W. Bartelmus, "Mathematical modelling and computer simulations as an aid to gearbox diagnostics," *Mechanical Systems and Signal Processing*, vol. 15, pp. 855-871, 2001.
- [87] I. Howard, S. Jia, and J. Wang, "The dynamic modelling of a spur gear in mesh including friction and a crack," *Mechanical systems and signal processing*, vol. 15, pp. 831-853, 2001.
- [88] S.-T. Choi and S.-Y. Mau, "Dynamic analysis of geared rotor-bearing systems by the transfer matrix method," *Journal of mechanical design*, vol. 123, pp. 562-568, 2001.
- [89] N. Driot and J. Perret-Liaudet, "Variability of modal behavior in terms of critical speeds of a gear pair due to manufacturing errors and shaft misalignments," *Journal of Sound and Vibration*, vol. 292, pp. 824-843, 2006.
- [90] Y. Zhang, Q. Wang, H. Ma, J. Huang, and C. Zhao, "Dynamic analysis of three-dimensional helical geared rotor system with geometric eccentricity," *Journal of Mechanical Science and Technology*, vol. 27, pp. 3231-3242, 2013.
- [91] Z. Chen and Y. Shao, "Dynamic simulation of spur gear with tooth root crack propagating along tooth width and crack depth," *Engineering Failure Analysis*, vol. 18, pp. 2149-2164, 2011.

References

- [92] O. D. Mohammed, M. Rantatalo, J.-O. Aidanpää, and U. Kumar, "Vibration signal analysis for gear fault diagnosis with various crack progression scenarios," *Mechanical systems and signal processing*, vol. 41, pp. 176-195, 2013.
- [93] F. K. Omar, K. A. Moustafa, and S. Emam, "Mathematical modeling of gearbox including defects with experimental verification," *Journal of Vibration and Control*, vol. 18, pp. 1310-1321, 2012.
- [94] F. K. Omar, K. A. Moustafa, and S. Emam, "Mathematical modeling of gearbox including defects with experimental verification," *Journal of Vibration and Control*, p. 1077546311403791, 2011.
- [95] F. Chaari, R. Zimroz, W. Bartelmus, T. Fakhfakh, and M. Haddar, "Modeling of local damages in spur gears and effects on dynamics response in presence of varying load conditions," *Proceedings of Surveillance*, vol. 6, pp. 1-19, 2011.
- [96] F. Chaari, W. Bartelmus, R. Zimroz, T. Fakhfakh, and M. Haddar, "Gearbox vibration signal amplitude and frequency modulation," *Shock and Vibration*, vol. 19, pp. 635-652, 2012.
- [97] X. Liang, M. J. Zuo, and M. R. Hoseini, "Understanding vibration properties of a planetary gear set for fault detection," *Prognostics and Health Management (PHM), 2014 IEEE Conference on*, vol. 1, pp. 1-6, 2014.
- [98] Z. Chen, Z. Zhu, and Y. Shao, "Fault feature analysis of planetary gear system with tooth root crack and flexible ring gear rim," *Engineering Failure Analysis*, vol. 49, pp. 92-103, 2015.
- [99] Y. Shao and Z. Chen, "Dynamic features of planetary gear set with tooth plastic inclination deformation due to tooth root crack," *Nonlinear dynamics*, vol. 74, pp. 1253-1266, 2013.
- [100] P. Fernandes and C. McDuling, "Surface contact fatigue failures in gears," *Engineering Failure Analysis*, vol. 4, pp. 99-107, 1997.
- [101] T. Davidson and P. Ku, "The effect of lubricants on gear tooth surface fatigue," *ASLE Transactions*, vol. 1, pp. 40-50, 1958.

References

- [102] L. S. Rossino, D. B. V. d. Castro, J. A. Moreto, C. O. F. T. Ruchert, D. Spinelli, and J. R. Tarpani, "Surface contact fatigue failure of a case hardened pinion shaft," *Materials Research*, vol. 17, pp. 535-541, 2014.
- [103] F. Choy, V. Polyshchuk, J. Zakrajsek, R. Handschuh, and D. Townsend, "Analysis of the effects of surface pitting and wear on the vibration of a gear transmission system," *Tribology international*, vol. 29, pp. 77-83, 1996.
- [104] R. Ma, Y. Chen, and Q. Cao, "Research on dynamics and fault mechanism of spur gear pair with spalling defect," *Journal of Sound and Vibration*, vol. 331, pp. 2097-2109, 2012.
- [105] J. Wei, A. Zhang, and P. Gao, "A study of spur gear pitting under EHL conditions: Theoretical analysis and experiments," *Tribology International*, vol. 94, pp. 146-154, 2016.
- [106] G. Stachowiak and A. W. Batchelor, *Engineering tribology*: Butterworth-Heinemann, 2013.
- [107] R. Stewart, *Some useful data analysis techniques for gearbox diagnostics*: University of Southampton, 1977.
- [108] N. Swansson, "Application of Vibration Signal Analysis Techniques to Condition Monitoring," in *Conference on Lubrication, Friction and Wear in Engineering 1980, Melbourne: Preprints of Papers*, 1980, p. 262.
- [109] H. Martin, "Statistical moment analysis as a means of surface damage detection," *Proceedings of the 7th International Modal Analysis Conference*, vol. 1, pp. 1016-1021, 1989.
- [110] A. Szczepanik, "Time synchronous averaging of ball mill vibrations," *Mechanical Systems and Signal Processing*, vol. 3, pp. 99-107, 1989.
- [111] R. Lyon, "Using the Hilbert Transform in Machine Diagnostics," in *Proceedings of the 3rd International Machine Monitoring and Diagnostics Conference, Society for Experimental Mechanics, Bethel, CT*, 1991, pp. 204-206.

References

- [112] H. J. Decker, R. F. Handschuh, and J. J. Zakrajsek, "An enhancement to the NA4 gear vibration diagnostic parameter," *NASA, 18th Annual Meeting, Hershey, Pennsylvania*, vol. ARL-TR-389, pp. 68-88, 1994.
- [113] P. Loughlin, F. Cakrak, and L. Cohen, "Conditional moments analysis of transients with application to helicopter fault data," *Mechanical Systems and Signal Processing*, vol. 14, pp. 511-522, 2000.
- [114] C. Capdessus, M. Sidahmed, and J. Lacoume, "Cyclostationary processes: application in gear faults early diagnosis," *Mechanical systems and signal processing*, vol. 14, pp. 371-385, 2000.
- [115] V. Polyshchuk, F. Choy, and M. Braun, "New gear-fault-detection parameter by use of joint time-frequency distribution," *Journal of Propulsion and Power*, vol. 16, pp. 340-346, 2000.
- [116] H. Zheng, Z. Li, and X. Chen, "Gear fault diagnosis based on continuous wavelet transform," *Mechanical systems and signal processing*, vol. 16, pp. 447-457, 2002.
- [117] S. Loutridis, "A local energy density methodology for monitoring the evolution of gear faults," *NDT & E International*, vol. 37, pp. 447-453, 2004.
- [118] H. J. Decker and D. G. Lewicki, "Spiral bevel pinion crack detection in a helicopter gearbox," *59th Annual Forum and Technology Display; Phoenix, AZ; United States*, vol. NASA/TM-27, pp. 290-306, 2003.
- [119] D. Lin, M. Wiseman, D. Banjevic, and A. K. Jardine, "An approach to signal processing and condition-based maintenance for gearboxes subject to tooth failure," *Mechanical Systems and Signal Processing*, vol. 18, pp. 993-1007, 2004.
- [120] S. Loutridis, "Gear failure prediction using multiscale local statistics," *Engineering structures*, vol. 30, pp. 1214-1223, 2008.
- [121] G. L. McDonald, Q. Zhao, and M. J. Zuo, "Maximum correlated Kurtosis deconvolution and application on gear tooth chip fault detection," *Mechanical Systems and Signal Processing*, vol. 33, pp. 237-255, 2012.

References

- [122] V. Sharma and A. Parey, "Gear crack detection using modified TSA and proposed fault indicators for fluctuating speed conditions," *Measurement*, vol. 90, pp. 560-575, 2016.
- [123] O. D. Mohammed, M. Rantatalo, and J.-O. Aidanpää, "Improving mesh stiffness calculation of cracked gears for the purpose of vibration-based fault analysis," *Engineering Failure Analysis*, vol. 34, pp. 235-251, 2013.
- [124] R. J. Drago, *Fundamentals of gear design*: Butterworth-Heinemann, 1988.
- [125] X. Tian, "Dynamic Simulation for System Response of Gearbox Including Localized Gear Faults," Department of Mechanical Engineering, University of Alberta, Edmonton, MS thesis, 2004.
- [126] A. Kapelevich and Y. Shekhtman, "Tooth fillet profile optimization for gears with symmetric and asymmetric teeth," *Gear technology*, vol. 26, pp. 73-79, 2009.
- [127] P. Sainsot, P. Velez, and O. Duverger, "Contribution of gear body to tooth deflections—a new bidimensional analytical formula," *Journal of mechanical design*, vol. 126, pp. 748-752, 2004.
- [128] NTAC, "Expert Forensic Engineering Testing, Analysis, and Testimony," Retrieved from <http://ntaclab.com/metallurgical.html>, Retrieval date Feb, 2018
- [129] A. Parey, N. K. Jain, and S. Koria, "Failure analysis of air cooled condenser gearbox," *Case Studies in Engineering Failure Analysis*, vol. 2, pp. 150-156, 2014.
- [130] R. Latino, "The 4 Basic Physical Root Causes of Component Failure: Fatigue & Overload," Retrieved from <https://www.linkedin.com/pulse/part-ii-4-basic-physical-root-causes-component-failure-bob-latino>, Retrieval date, Feb, 2018 2017.
- [131] ADDINOL, "Severe gear tooth fatigue and pitting," Retrieved from <http://www.addinol.com.cn/pro.asp?classid=59>, Retrieval date, Feb, 2018.
- [132] A. G. M. A. AGMA Technical Committee, "Appearance of Gear Teeth: Terminology of Wear and Failure," *American Gear Manufacturers Association*, vol. 40, 1995.

References

- [133] Marilyn, "Gear And Reducer Inspection And Analysis," Retrieved from <https://www.efficientplantmag.com/2008/03/shedding-light-on-gear-and-reducer-inspection-and-analysis/>, Retrieval date, Feb, 2018 2008.
- [134] K.Gopinath and M.M.Mayuram, "Gear Failure," Retrieved from https://nptel.ac.in/courses/112106137/pdf/2_6.pdf, Indian Institute of Technology Madras Retrieval date, Feb, 2018.
- [135] P. Langlois, B. Al, and O. Harris, "Hybrid Hertzian and FE-based helical gear-loaded tooth contact analysis and comparison with FE," *Gear Technology*, pp. 54-63, 2016.
- [136] X. Liang, H. Zhang, M. J. Zuo, and Y. Qin, "Three new models for evaluation of standard involute spur gear mesh stiffness," *Mechanical Systems and Signal Processing*, vol. 101, pp. 424-434, 2018.
- [137] S. Xue, R. Entwistle, I. Mazhar, and I. Howard, "The spur planetary gear torsional stiffness and its crack sensitivity under quasi-static conditions," *Engineering Failure Analysis*, vol. 63, pp. 106-120, 2016.
- [138] G. DeLange, "Analyzing gear failures," *Hydrocarbon Processing*, vol. 79, pp. 97-97, 2000.
- [139] X. Liang, M. J. Zuo, and Z. Feng, "Dynamic modeling of gearbox faults: A review," *Mechanical Systems and Signal Processing*, vol. 98, pp. 852-876, 2018.
- [140] H. Ma, J. Zeng, R. Feng, X. Pang, Q. Wang, and B. Wen, "Review on dynamics of cracked gear systems," *Engineering Failure Analysis*, vol. 55, pp. 224-245, 2015.
- [141] Y. Luo, N. Baddour, G. Han, F. Jiang, and M. Liang, "Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features," *Engineering Failure Analysis*, vol. 92, pp. 430-442, 2018.
- [142] H. Ma, J. Zeng, R. Feng, X. Pang, and B. Wen, "An improved analytical method for mesh stiffness calculation of spur gears with tip relief," *Mechanism and Machine Theory*, vol. 98, pp. 64-80, 2016.
- [143] H. Ozturk, I. Yesilyurt, and M. Sabuncu, "Detection and advancement monitoring of distributed pitting failure in gears," *Journal of Nondestructive Evaluation*, vol. 29, pp. 63-73, 2010.

References

- [144] SEA, "Surface Engineering Associates," *Retrieved from* <http://www.tribolite.com/energymining.html>, Retrieval date (Feb, 2018).
- [145] R. T. R. T. P. LtdLaboratory, "Gear tooth failure," *Retrieved from* http://www.rttech.com.au/?page_id=56, Retrieval date, Feb, 2018 2015.
- [146] I. Boiadjev, J. Witzig, T. Tobie, and K. Stahl, "Tooth Flank Fracture–Basic Principles and Calculation Model for a Sub-Surface-Initiated Fatigue Failure Mode of Case-Hardened Gears," in *International Gear Conference, Lyon Villeurbanne, France. Woodhead Publishing*, 2014.
- [147] U. W. Dipl.-Ing. Max Fromberger, M. Sc., Dipl.-Ing. Bernhard, D.-I. T. U. Kohn, Dr.-Ing. Michael Otto and Prof. Dr.-Ing., and K. Stahl, "Evaluating signal processing methods for use in gearbox condition monitoring," *24th International Congress on Sound and Vibration (ICSV 24)*, vol. 23, pp. 23-27, July, 2017.
- [148] T. Production, "Advanced Sensor to Monitor Gearbox Condition Could Save Thousands of Dollars in Downtime," *Retrieved from* <http://www.toolingandproduction.com>, 2018.
- [149] B. T. Kuhnell, "Wear in rolling element bearings and gears- How age and contamination affect them," *Machinery Lubrication*, vol. 10, pp. 62-64, 2004.
- [150] C. K. Tan, P. Irving, and D. Mba, "A comparative experimental study on the diagnostic and prognostic capabilities of acoustics emission, vibration and spectrometric oil analysis for spur gears," *Mechanical Systems and Signal Processing*, vol. 21, pp. 208-233, 2007.
- [151] G. Niemann, "Maschinenelemente," *Springer-Verlag Berlin Heidelberg*, 1950.
- [152] S.-Y. Ye and S.-J. Tsai, "A computerized method for loaded tooth contact analysis of high-contact-ratio spur gears with or without flank modification considering tip corner contact and shaft misalignment," *Mechanism and Machine Theory*, vol. 97, pp. 190-214, 2016.
- [153] J. Liu and Y. Shao, "Dynamic modeling for rigid rotor bearing systems with a localized defect considering additional deformations at the sharp edges," *Journal of Sound and Vibration*, vol. 398, pp. 84-102, 2017.

References

- [154] J. Liu and Y. Shao, "An improved analytical model for a lubricated roller bearing including a localized defect with different edge shapes," *Journal of Vibration and Control*, vol. 24, pp. 3894-3907, 2018.
- [155] J. Liu, H. Wu, and Y. Shao, "A theoretical study on vibrations of a ball bearing caused by a dent on the races," *Engineering Failure Analysis*, vol. 83, pp. 220-229, 2018.
- [156] J. Liu, Z. Shi, and Y. Shao, "An analytical model to predict vibrations of a cylindrical roller bearing with a localized surface defect," *Nonlinear Dynamics*, vol. 89, pp. 2085-2102, 2017.
- [157] Q. Han, J. Wang, and Q. Li, "Analysis of parametric stability for a spur gear pair system considering the effect of extended tooth contact," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 223, pp. 1787-1797, 2009.
- [158] G. Morales-Espejel, P. Rycerz, and A. Kadiric, "Prediction of micropitting damage in gear teeth contacts considering the concurrent effects of surface fatigue and mild wear," *Wear*, vol. 398, pp. 99-115, 2018.
- [159] E. E. Shipley, "Gear failures," *Machine Design*, vol. 39, pp. 152-162, 1967.
- [160] G. Niemann and H. Winter, *Maschinenelemente: Band 2: Getriebe allgemein, Zahnradgetriebe-Grundlagen, Stirnradgetriebe*: Springer-Verlag, 2013.
- [161] S. Li and A. Kahraman, "A spur gear mesh interface damping model based on elastohydrodynamic contact behaviour," *International Journal of Powertrains*, vol. 1, pp. 4-21, 2011.
- [162] H. Ping, "Numerical calculation methods of elastohydrodynamic lubrication," ed: Tsinghua University Press, Tsinghua, 2013.
- [163] P. Goglia, T. Conry, and C. Cusano, "The effects of surface irregularities on the elastohydrodynamic lubrication of sliding line contacts. Part I—single irregularities," *Journal of Tribology*, vol. 106, pp. 104-112, 1984.

References

- [164] M. Ankouni, A. Lubrecht, and P. Velex, "Modelling of damping in lubricated line contacts—Applications to spur gear dynamic simulations," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 230, pp. 1222-1232, 2016.
- [165] M. Masjedi and M. Khonsari, "Mixed elastohydrodynamic lubrication line-contact formulas with different surface patterns," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 228, pp. 849-859, 2014.
- [166] M. Masjedi and M. Khonsari, "Film thickness and asperity load formulas for line-contact elastohydrodynamic lubrication with provision for surface roughness," *Journal of Tribology*, vol. 134, pp. 23-33, 2012.
- [167] D. Hua and M. Khonsari, "Application of transient elastohydrodynamic lubrication analysis for gear transmissions," *Tribology transactions*, vol. 38, pp. 905-913, 1995.
- [168] S. Akbarzadeh and M. Khonsari, "Performance of spur gears considering surface roughness and shear thinning lubricant," *Journal of Tribology*, vol. 130, pp. 503-511, 2008.
- [169] H. H. Lin and C.-H. Liou, "A parametric study of spur gear dynamics," Memphis University TN1998.
- [170] R. Kasuba and J. W. Evans, "An extended model for determining dynamic loads in spur gearing," *Journal of mechanical design*, vol. 103, pp. 398-409, 1981.
- [171] D. Yang and Z. Sun, "A rotary model for spur gear dynamics," *Journal of mechanisms, transmissions, and automation in design*, vol. 107, pp. 529-535, 1985.
- [172] H. Xu, A. Kahraman, N. Anderson, and D. Maddock, "Prediction of mechanical efficiency of parallel-axis gear pairs," *Journal of Mechanical Design*, vol. 129, pp. 58-68, 2007.
- [173] S. H. Yuan, H. L. Dong, and X. Y. Li, "Analysis of lubricating performance for involute gear based on dynamic loading theory," *Journal of Mechanical Design*, vol. 134, pp. 1004-1013, 2012.

References

- [174] S. Li and A. Kahraman, "Influence of dynamic behaviour on elastohydrodynamic lubrication of spur gears," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 225, pp. 740-753, 2011.
- [175] J. Wang, R. Li, and X. Peng, "Survey of nonlinear vibration of gear transmission systems," *Applied Mechanics Reviews*, vol. 56, pp. 309-329, 2003.
- [176] R. Tiwari, "Analysis and Identification in Rotor-Bearing Systems," *Department of Mechanical Engineering, IIT Guwahati, Guwahati*, vol. 1, 2010.
- [177] R. Nordmann, "Identification of stiffness and damping coefficients of journal bearings by means of the impact method," in *Dynamics of Rotors*, ed: Springer, 1984, pp. 395-409.
- [178] R. Zeillinger, H. Springer, and H. Kötttritsch, "Experimental determination of damping in rolling bearing joints," in *ASME 1994 International Gas Turbine and Aeroengine Congress and Exposition*, 1994, pp. V005T14A016-V005T14A016.
- [179] Y. Luo, Q. Li, and Z. Liu, "Design and optimization of wheel-legged robot: Rolling-Wolf," *Chinese Journal of Mechanical Engineering*, vol. 27, pp. 1133-1142, 2014.
- [180] C. Scheffer and P. Girdhar, *Practical machinery vibration analysis and predictive maintenance*: Elsevier, 2004.
- [181] W. Wang, "Early detection of gear tooth cracking using the resonance demodulation technique," *Mechanical Systems and Signal Processing*, vol. 15, pp. 887-903, 2001.
- [182] R. Randall, "A new method of modeling gear faults," *Journal of Mechanical Design*, vol. 104, pp. 259-267, 1982.
- [183] M. Masjedi and M. Khonsari, "Theoretical and experimental investigation of traction coefficient in line-contact EHL of rough surfaces," *Tribology International*, vol. 70, pp. 179-189, 2014.
- [184] D. G. Lewicki, H. J. Decker, and J. T. Shimski, "Development of a full-scale transmission testing procedure to evaluate advanced lubricants," 1992.

References

- [185] W. D. Mark, "Analytical approximations to damaged gear tooth transmission-error contributions for gear-health monitoring," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 230, pp. 1157-1182, 2016.
- [186] B. Gaylord, "Rebuilding the MG gearbox- GT-203D," Retrieved from <http://mgaguru.com/mgtech/gearbox/gt203d.htm>, Retrieval date (June, 2018) 2018.
- [187] D. G. Lewicki, K. E. LaBerge, R. T. Ehinger, and J. Fetty, "Planetary gearbox fault detection using vibration separation techniques," *67th Annual Forum and Technology Display (Forum 67); Virginia Beach, VA; United States*, vol. E-17816, pp. 42-63, 2011.
- [188] B.-R. Hohn, "Modern gear calculation," *VDI-Berichte*, vol. 1665, pp. 23-44, 2002.
- [189] F. M. Agemi and M. Ognjanović, "Gear vibration in supercritical mesh-frequency range," *FME Transactions*, vol. 32, pp. 87-94, 2004.
- [190] M. Ognjanovic and F. Agemi, "Gear Vibrations in Supercritical Mesh-Frequency Range Caused by Teeth Impacts," *Strojniski Vestnik/Journal of Mechanical Engineering*, vol. 56, pp. p653-662, 2010.
- [191] Y. Li, K. Ding, G. He, and H. Lin, "Vibration mechanisms of spur gear pair in healthy and fault states," *Mechanical Systems and Signal Processing*, vol. 81, pp. 183-201, 2016.
- [192] C. Shen, D. Wang, F. Kong, and W. T. Peter, "Fault diagnosis of rotating machinery based on the statistical parameters of wavelet packet paving and a generic support vector regressive classifier," *Measurement*, vol. 46, pp. 1551-1564, 2013.

Appendix 1

Publications during Ph.D. study

1. Y. Luo, N. Baddour, and M. Liang, "Dynamical modeling and experimental validation for tooth pitting and spalling in spur gears," *Mechanical Systems and Signal Processing*, vol. 119, pp. 155-181, 2019.
2. Y. Luo, N. Baddour, and M. Liang, "A shape-independent approach to modelling gear tooth spalls for time varying mesh stiffness evaluation of a spur gear pair," *Mechanical Systems and Signal Processing*, vol. 120, pp. 836-852, 2019.
3. Y. Luo, N. Baddour, and M. Liang, "Effects of gear center distance variation on time varying mesh stiffness of a spur gear pair," *Engineering Failure Analysis*, vol. 75, pp. 37-53, 2017.
4. Y. Luo, N. Baddour, G. Han, F. Jiang, and M. Liang, "Evaluation of the time-varying mesh stiffness for gears with tooth spalls with curved-bottom features," *Engineering Failure Analysis*, vol. 92, pp. 430-442, 2018.
5. Y. Luo, N. Baddour, and M. Liang, Performance Assessment of Gear Condition Indicators in Detecting Progressive Gear Tooth Crack. ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 8: 29th Conference on Mechanical Vibration and Noise (): V008T12A010. doi:10.1115 /DETC2017-67460, 2017.
6. Y. Luo, N. Baddour, and M. Liang, Dynamical Modeling of Gear Transmission Considering Gearbox Casing. ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 8: 30th Conference on Mechanical Vibration and Noise ():V008T10A024. doi:10.1115/DETC2018-85656, 2018.
7. Y. Luo, N. Baddour, and M. Liang, "Evaluation of Time-Varying Mesh Stiffness of Gears with Tooth Spalls Modeled as Spherical Shapes," 2018 Prognostics and System Health Management Conference (PHM-Chongqing), Chongqing, pp. 94-99, 2018.

Appendix 2

Parameters for the dynamic simulation of the spur gear transmission utilized in chapter 6

Number of pinion teeth	$N_1 = 16$
Number of gear teeth	$N_2 = 48$
Teeth module	$\text{mod} = 3.175 \text{ mm}$
Teeth width	$L = 16 \text{ mm}$
Pressure angle	$\alpha_0 = 20^\circ$
Young's modulus	$E = 2 \times 10^5 \text{ N / mm}^2$
Poisson's ratio	$\nu = 0.3$
Radius of pinion base circle	$r_{bp} = 0.023868 \text{ m}$
Radius of gear base circle	$r_{bg} = 0.0716045 \text{ m}$
Mass of pinion	$m_{g1} = 1.272 \text{ kg}$
Mass of gear	$m_{g2} = 3.5367 \text{ kg}$
Mass of gearbox casing	$m_f = 18.509 \text{ kg}$
Mass moment inertia of pinion	$I_{g1} = 0.0001751 \text{ Kg} \cdot \text{m}^2$
Mass moment inertia of gear	$I_{g2} = 0.006828 \text{ Kg} \cdot \text{m}^2$
Mass moment inertia of motor	$I_m = 0.016107 \text{ Kg} \cdot \text{m}^2$
Mass moment inertia of load	$I_L = 0.005153 \text{ Kg} \cdot \text{m}^2$
Inclination angle of the line-of-action (clockwise)	$\varphi = 70^\circ$
Torque of motor	$T_M = 21 \text{ N} \cdot \text{m}$
Torque of load	$T_L = 7 \text{ N} \cdot \text{m}$
Damping coefficient of bearing	$c_b = 2.134 \times 10^4 \text{ N} \cdot \text{s/m}$
Stiffness of bearing	$k_b = 8.5364 \times 10^7 \text{ N/m}$
Damping coefficient of gear box casing screw in x axis	$c_{xf} = 1995.56 \text{ N} \cdot \text{s/m}$
Stiffness of gear box casing screw in x axis	$k_{xf} = 1.9912 \times 10^8 \text{ N/m}$
Damping coefficient of gear box casing screw in y axis	$c_{yf} = 2005.80 \text{ N} \cdot \text{s/m}$
Stiffness of gear box casing screw in y axis	$k_{yf} = 2.036 \times 10^8 \text{ N/m}$
Damping of coupling	$c_c = 23.1 \text{ N} \cdot \text{m} \cdot \text{rad/s}$
Coupling stiffness	$k_c = 330 \text{ N} \cdot \text{m/rad}$