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THE UNIVERSITY OF OTTAWA

SYNTHESIS:
FROM NUMBERS TO INTENTIONALITY

SUBMITTED TO THE
DEPARTMENT OF PHILOSOPHY
TO COMPLETE M.A. REQUIREMENTS

BY

DEREK FOLLON

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CHAPTER ONE

INTRODUCTION

The two objectives of this introduction will be to state precisely what the aim of this paper is and how it may be reached. As the title indicates, the paper deals with synthesis and number concepts but, and this shall be repeated throughout the paper, this is not a paper in philosophy of mathematics. There is to be no attempt made here to resolve the ongoing debate as to the nature of number. Whether one accepts a formalist, intuitionist or some other position on the nature of number, this is quite incidental to the views contained herein. Even if one does not accept synthesis as an explanation of number, this does not detract from the use made of numbers in this paper. Numbers are probably the best illustration of how synthesis operates.

What then is the purpose of demonstrating the role of synthesis in our development of number concepts? The reason for this is that it serves as a starting point from which to follow this notion of synthesis to where it operates in intentionality. Perhaps this better states the overall aim

of the paper which is to pursue the role of synthesis from number concepts to the part it plays in everyday consciousness.

Before proceeding any further, there are two fundamental notions which should be mentioned as they may serve to anchor the paper during rough waters. For the first point, we must go directly to the nature of synthesis and see it as combining, that is to say, in its most simple form synthesis may be thought of as little more than joining. What makes this issue so important are the contents that are joined and the result that is produced. The second issue that is central to this paper, especially in the later stages where intentionality comes to the fore, is the view that the mind places or directs meaning onto the world we experience.

The question of this thesis is not so much is synthesis a good explanation of number but instead is number a good illustration of synthesis? The foundation for this paper is built upon a most affirmative answer to this question and I would now like to explain why I feel this is so and how such views may be seen to be applied to an interpretation of our conscious activity.

The reasons number was chosen as the means to demonstrate the features of synthesis is that in Kant's work, which serves historically to introduce the idea of synthesis, all that he says about numbers and mathematics provides the most clear and most easily understandable account of what later will be shown to be the most distinguishing feature of synthesis. It is this joining, combining activity which is performed in order to be able to grasp numbers that most perfectly indicates the role of synthesis. By the end of the paper, it will be shown that this combining activity, first demonstrated through numbers, will be reflected in the process that occurs when the intellect places (joins if you will), a particular meaning to some thing, event or phenomenon that we encounter in our conscious experience.

This general theme of the paper, starting with a consideration of synthesis and numbers and moving to the role of synthesis in consciousness, is precisely the motif that sets the tone for each of the three sections of the paper. It is interesting to note that the contributions of these two areas in which synthesis will be examined, numbers and

consciousness, will be in inverse proportions by the end of the paper. By this I mean that in opening with Kant the bulk of the material treats of number and there is but a closing mention of our perception, our experience of the world around us. Frege, on the other hand, presents a more balanced mix and begins to expand the parameters of the paper from the consideration of numbers to the much broader framework of our mode of conscious activity. Husserl begins with the treatment of number, but by then the emphasis shifts to the role of synthesis in consciousness.

I would like to finish this introduction by briefly indicating how these aims are to be met for each of the three men involved. Kant provides ample evidence of his views on the nature of synthesis and its relation to numbers. This is taken as the basis, the building-blocks for the remainder of the paper. The section on geometry is where the door is first opened upon considering synthesis in the expanded content of consciousness in general rather than just with numbers. Our perception of geometrical shapes and their relations is like a skeleton of our full-blown experience of the world.

In the section on Frege, our experience of numbers is put forth on a strictly logical basis. It is in this middle section that the paper takes a most important change in direction from numbers to the consideration of our conscious experience in general. The shift occurs when the paper looks at some of Frege's semantic notions such as concept, object, function; it is then realized that not only numbers provide the focus of the discussion but indeed anything at all may be the subject of our study.

The main body of the paper culminates with Husserl. With the slant of the paper leaning more and more towards an interpretation of our conscious experience, the theory of intentionality provides the frame within which to demonstrate the role of synthesis. Husserl's section begins with a brief look at his explanation of numbers, but much more to the point is the subsequent fitting of the components of his explanation to the theory of intentionality. Husserl has stated that the essence of consciousness is intentionality, in what follows, I hope to convey that the essence of intentionality is synthesis.

If this introduction has been successful then the objective of the paper is clear. In its most simple form it may be stated thus: an attempt is made here to show the essential role synthesis plays in our everyday conscious activity. One may express it in strong terms claiming that without this synthesis one would not have what we commonly refer to as our conscious experience of the world around us and that synthesis alone gives us the required grounds for having such experience. It is to this position that the paper strives and thereby demonstrates the all important role of synthesis.

CHAPTER TWO

KANT

This study of Kant's conception of number will have three main sections to it; first, an explanation of what Kant considers to be the main defining characteristic of number; second, an examination of how this trait may be specifically noted in one particular arithmetical equation and finally the manner in which this principle relates to the study of geometry. The main aim of this section is to establish Kant's use of the concept of synthesis in his formulation of number concepts. Ultimately, the same synthesis that goes into the construction of numbers will be shown to be fundamental to man's everyday conscious experience.

Kant believed mathematics to be a perfect indicator of how reason could be seen to progress successfully without any help from or recourse to experience.

Mathematics gives us a shining example of how far, independently of experience, we can progress in a priori knowledge.

Mathematics presents the most splendid example of the successful extension of pure reason, without the help of experience.¹

Kant thought that this separateness or independence from experience guaranteed mathematical knowledge a certainty that was just not attainable by any empirically based science.

Before considering how Kant explains the manner in which we come to know numbers, let us see what Kant says about how we come to know anything at all. What I refer to is his two by two classification of knowledge based upon the analytic-synthetic and the a priori-a posteriori distinctions.

The first pair to be examined is the a priori-a posteriori division of knowledge. The former, a priori, is that type of knowledge which is not based upon or derives any justification from empirical experience. It is knowledge that does not result from our contact with or awareness of physical, empirical objects or their properties. A posteriori knowledge conversely, is that type of knowledge which is based upon and derived from empirical experience. It is always possible that such knowledge may be either verified or refuted on the basis of such experience.²

The other classification to be looked at is the analytic-synthetic distinction. In an analytic statement such as 'A rainy day is a wet day' it may be noted that the predicate adds nothing to what is already contained in the

subject; it merely explains or elaborates the content that is already present in the subject. Conversely, synthetic statements do add some information that is not necessarily part of the subject, thereby increasing our knowledge. For example, if one says 'A rainy day is a cold day', then in the stipulation of cold, one has added on to the necessary characteristics of what is already contained in the subject 'A rainy day'.³ Even in this preliminary stage the gist of the whole notion of synthesis is apparent; it is essentially a concept by which things are joined, or added on to one another enabling the person to experience the unity that results from such combination.

This brief discussion of Kant's divisions is sufficient for the aims of this paper. The main goal has been to introduce the notion of synthesis, albeit in a rather narrow sense restricted to the examples of sentences. We shall now expand the scope of synthesis and investigate Kant's presentation of it as the main defining characteristic of number.

2.1 Numbers

This section treats two areas of Kant's discussion of number. First of all, we will look at the central theme in

Kant's proposals about how we arrive at specific numbers; then a more general look at how numbers are presented as the schema of the category of Quantity.

Kant states very definitely that all mathematical judgements are without exception synthetic.⁴ This is quite contrary to the more prevalent view that holds the nature of mathematics to be analytic. To make these two views more precise consider the example three and three is six; those who oppose Kant's synthetic claim would say that since the predicate 'six' is already contained in the subject 'three and three' then the proposition is analytic. Kant maintains that the subject 'three and three' consists only of the idea of two groups of three plus the idea of uniting these two groups and as such contains no indication whatsoever of the number six.

Kant presents an explanation of number which is the foundation for the aim of this paper. What he says basically is that number is the result of the successive addition of homogeneous units. I will contend this successive addition to be based upon a synthesis which the units undergo. As pointed out by Kant,⁵ it is even more crucial to our concept of number to be aware of the resulting totality than of the successive addition of unit to unit which goes to make up the totality.

If, in counting, I forget that the units, which now hover before me, have been added to one another in succession, I should never know that a total is being produced through the successive addition of unit to unit, and so would remain ignorant of the number. For the concept of the number is nothing but the consciousness of this unity of synthesis.⁵

Thus our counting, as is easily seen in the case of larger numbers, is a synthesis according to concepts, because it is executed according to a common ground of unity, as for instance, the decade.⁶

Two facts become evident from the above quotes. The first is that the method of counting is employed by Kant in order to arrive at an explanation of number but more importantly, synthesis is presented as the basis upon which such totality or unity may be built.

One also finds many of these central views about numbers restated and emphasized by those who discuss Kant's work. The work of D.P. Dryer points this out as well as introducing some key notions which will be very important in the following development in Frege and Husserl.

Dryer approaches the problem of explaining number concepts through the question of how many people there are in a room. He indicates the necessity of establishing some concept which will serve to provide the units in order for the counting to proceed. In this case, to determine how many people, one adopts the concept of a person.

We count by connecting together what we are successively presented with. This synthesis we repeat with each new application of the unit that we find. Kant points out that several items can only be counted by means of a successive synthesis.⁷

With each step in counting we arrive only at how many times we have found an application of the unit concept. Kant points out that counting is performed by a successive synthesis of homogeneous items.⁸

The counting is complete only when there are no further applications of the unit concept and is correct only if one application of the unit concept has been made to each and every instance that is present. Dryer repeats Kant's claim that the final number arrived at contains a synthesis of all the applications of the concept that have gone before it. This idea is expressed by Dryer when he says that in counting it is necessary at each step that we connect what we find with what we have found.

The attempt to explain number by means of counting was not immediately acceptable to all persons. Russell objected to such a view. He believed mathematics and logic to be analytic in nature and like Frege, Russell felt mathematics could be inferred from logic.

Russell's objection may be summarized along these lines. Numbers may be explained independently of counting.⁹ Against the explanation by counting, Russell employs the

notion of one-to-one correspondence and says that one knows the number of left shoes in a room is equal to the number of right shoes. When someone actually counts the number of left shoes he assigns a number to each, showing Russell contends, that counting is actually a special case of determining one-to-one correspondence between objects and numbers and does not serve to explain numbers.

Russell further charged that an explanation of number in terms of counting could refer only to finite numbers and could not apply to infinite numbers. Such 'infinite numbers' he felt were included in his explanation of number through the notion of sets; as applying to the set of all sets equal to a given set.

Now what is wrong with Russell's explanation of number by means of one-to-one correspondence? If his talk of infinite sets "having the same number" means only that they are equal then this actually is not an objection to Kant for he never held otherwise. But the real area where Russell's account falters is due to the fact that even if we know that the left shoes equal the right shoes, or the members of one set equal another, we still do not know what their number is.

This can only be determined though the one-to-one correspondence between objects and numbers and in this special case, Russell concedes, one is counting. Dryer himself expresses it, "Hence in spite of what he says elsewhere, Russell in effect admits that numbers have to be explained in terms of counting".

In addition to this what I feel is mistaken in Russell's one-to-one correspondence is that we do not learn or explain the number of items simply by corresponding each one to a member in the series of numbers. This appears to be what occurs on the surface. However it is not merely progression down the series that explains number, rather it is the awareness of the result of the synthesis of units that go into making up the number that is the explanation of number. There is more involved than simply linking each of the members to a particular digit in the series of natural numbers.

To conclude, let us see what may be gathered from what both Kant and these secondary sources have stated about the nature of the concept of number. For this paper the most critical part of the various expressions on numbers and counting is the mention of the contribution of synthesis to the attainment of number concepts. Synthesis is the process by which we come to grasp or understand numbers.

2.2 NUMBERS AND THE CATEGORY OF QUANTITY

But pure schema of magnitude (quantitatis), as a concept of the understanding, is number, a representation which comprises successive addition of homogeneous units. Number is simply the unity of the synthesis of the manifold of a homogeneous intuition in general, a unity due to my generating time itself in the apprehension of the intuition.¹⁰

In this quote appears the familiar 'successive addition of units, but how may we understand the other notions such as the schema of the category of Quantity, and the role of time in the unity of the synthesis which is number.

The very difficult Kantian notion of schema is not to be thought of as the mental picture that is matched to the corresponding experience. If one imagines five points, one has what may be said to be the image of the number five. However, the schema is to be thought of as something altogether different from that. The schema is not itself the group of five dots but rather it is the representation of the method whereby a multiplicity, eg. five, may be presented in an image in accord with certain concepts.¹¹ The image of the five dots may be seen as the end or the result of the process of presenting this picture but the schema represents the method whereby such an alignment occurs.

Körner has some helpful remarks on this idea of quantity and he also helps clarify the position 'time' plays in the overall phenomenon.¹² He claims that any perceived object can only be categorized as quantity if it can be compared to other quantities, that is to say, only if it can be measured. In order to determine this measurement numbers would be employed and through these one would come to have an understanding of the quantity involved.

Körner also states that such measurement implies the addition of units (similar to counting) which is 'necessarily a succession in time'. Kant also claims that the successive addition of units is a process that occurs over time and so refers to the numbers arrived at as temporal concepts. This line of thought, as well as a statement of the inclusive, cumulative nature of numbers is well expressed in this quote from Schopenhauer:

In time every moment is conditioned by the preceding...All counting rests upon this nexus of the parts of time; its words merely serve to mark the single steps of this succession. This is true of the whole of arithmetic, which throughout teaches nothing but the methodical abbreviations of counting.

Every number presupposes the preceding numbers as grounds of its existence; I can only reach them through all the preceding and only by means of this insight into the ground of its existence do I know that, where ten are, there are also eight, six, four.¹³

Kant never abandoned the cumulative nature of number, each built upon the preceding units, but he may have compromised his statements as to the necessity of the role of time in numbers. In a letter to Schulze (1788), Kant wrote that the science of number, notwithstanding the succession which every construction of quantity demands, is a purely intellectual synthesis which we represent to ourselves in thought.

Although the act of counting to twelve involves a succession or series of steps transpiring over time, in the number twelve itself, there is not in the least any indication of this need for time.

The process by which we become aware of objects, figures, etc. in our environment is subject to time. But what is not subject to time? Time flows throughout all things, including our counting, but it is hardly appropriate to conclude time as the foundation of number. In similar fashion, simply because a moral act takes a period of time to perform one is hardly justified in concluding time as the basis of morality. The quantitative relations determined by the science of arithmetic are independent of time and are intellectually apprehended.

Husserl expresses quite precisely the view on time which is taken here. While he recognizes the role of time in numbers, he does stop short of declaring time as the absolute foundation of numbers.

But even if the temporal sequence in which objects are colligated were always attended to, it would still remain incapable of grounding by itself alone the unity of the collective whole. (...) Herbart is completely justified in saying that "number has ... no more in common with time than do a hundred other sorts of representations which also can be produced only gradually."¹⁴

In order to allay any concerns that this discussion merely teases the issue of time, may I restate the position taken here. First of all, let it be granted that time be accepted as a psychological pre-condition for our grasp of numbers. However, against such a background, it is felt that there are more specific contributors to our understanding of number. There is no intention here to diminish the importance of time in the issue but rather the purpose is towards highlighting other aspects of the process, such as synthesis. Of course it is agreed that progressing along the steps of a series necessarily takes time but the point here is that although this is what must occur in moving along the series of natural numbers say 1, 2, 3, 4, once the desired number has been reached then in any subsequent use of four it contains no requirement or mention of time for it to be understood.

2.3 SEVEN PLUS FIVE EQUALS TWELVE

This example of simple addition $7 + 5 = 12$ is used by Kant to demonstrate all the salient features of his views on mathematics. Of course, the most important feature Kant wishes to show is the synthetic nature of the proposition. Dryer offers an opposing view and his argument will be examined. I will hope to show where Dryer is mistaken in his consideration of analytic statements.

Let us consider the reasons Kant gives for believing 'seven plus five equals twelve' is a synthetic proposition. His argument hinges particularly in the definition of analytic-synthetic statements which was presented earlier. If one examines the subject of the proposition, one finds that in the number seven or in the number five, or even in the idea of combining them, there is not any indication or thought of the number twelve. Since the predicate twelve is in no wise contained in the subject then the criterion for a synthetic proposition has been met.

The fact that one must reach twelve if the addition is carried out does not disturb Kant. He says this is not to the point, since in an analytic proposition the question is

only whether I actually think the predicate in the representation of the subject.¹⁵ He feels that the process of addition is actually the means for verifying the statement and that it does not affect his claim to the synthetic nature of the proposition at all.

What may be said for the opposing view that this is in fact an analytic proposition? Dryer feels Kant is wrong to claim that mathematics are synthetic. Dryer writes that of course if one adds five to seven correctly, one reaches twelve but he maintains that the statement cannot be verified solely by reflecting on the union of seven and five.

The crux of Dryer's argument is that the verification of the proposition 'seven plus five equals twelve' is achieved through a deduction from a series of analytic propositions. He tries to use Kant's method of verification i.e. carrying out the actual addition of seven and five, against Kant as a means for showing the analytic nature of the propositions.

Dryer says that $12 = 11 + 1$ is analytically true and that $11 = 10 + 1$. It follows from this by substituting $10 + 1$ for 11 that $12 = (10 + 1) + 1$; continuing along these lines we have this.

Since 10 = 9+1,	12 = ((9+1)+1)+1
Since 9 = 8+1,	12 = (((8+1)+1)+1)+1
Since 8 = 7+1,	12 = (((((7+1)+1)+1)+1)+1)
Removing brackets	12 = 7+1+1+1+1+1
Since 2 = 1+1,	12 = 7+2+1+1+1
Since 3 = 2+1,	12 = 7+3+1+1
Since 4 = 3+1,	12 = 7+4+1
Since 5 = 4+1,	12 = 7+5.

Dryer thus concludes that $12 = 7 + 5$ is analytically true.¹⁶

Dryer's counter-argument to Kant's synthetic claim is based upon considering the actual addition one performs. The last step in such an operation is the addition of the one remaining unit to that group which has already been assembled and labelled eleven. Why do we not call the result of adding one to eleven say fifteen, Dryer asks? Dryer now employs definitions and shows the analytic nature of the propositions. By the number fifteen, we do not mean one more than eleven; it is the number twelve that is taken to mean one more than eleven. Therefore the proposition 'seven plus five equals twelve' by definition, is an analytic proposition. Dryer thus feels he has demonstrated that the predicate twelve is already contained in the subject of the proposition.

What Dryer has demonstrated in his treatment of this proposition is that he has completely distorted Kant's idea of the 'successive addition of units.' Although Dryer feels he has shown the analytic nature of the proposition that he finally arrives at by the successive replacement of larger numbers, all that he has accomplished is a complete fragmentation of Kant's requirement of the consciousness of the synthetic unity that makes up a number.

When one thinks of twelve, there is at the same time, an awareness of the twelve individual units that make up the number: It is this synthesis alone that permits the experience of twelve and not, as Dryer claims the addition of one to eleven; this does not constitute twelve any more than thirteen minus one, one-half of twenty-four, square root of hundred forty-four and so on. One must be aware, however dimly, as Paton expresses it, of the twelve distinct units that are put together in order to make twelve.¹⁷

The difficulty in expressing such an idea is that there seems to be a contradiction involved in the experiencing of a unity and at the same time being aware of the units that go into forming the whole. The person must simultaneously be aware of the group and of its members. The solution seems to

be in the fact that the person is directly aware of the synthetic unity only and the individual members only come to mind if the person deliberately reflects on what has been put together to make up twelve.

2.4 Geometry

The main tenets of Kant's position on the formation of number concepts will be seen to prevail in this examination of geometry. The role of synthesis has been traced and developed through its presence in specific numbers up to its role in particular mathematical formulae and will now be shown to be essential in the study of geometry. A discussion of the role of synthesis in our grasping the shapes and figures of geometry, begins to indicate how the same phenomenon which enables us to reach concepts in, such a non-physical, non-sensible realm may also be seen to contribute to our understanding the events and objects of our everyday lives.

The entire issue may be introduced by considering Kant's Axioms of Intuition; in particular, the first axiom which says:

All appearances are, in their intuition, extensive magnitudes. (A)

All intuitions are extensive magnitudes. (B)¹⁸

Kant's main concern is to uncover whether or not anything may be known about an object of which no magnitude may be ascertained. Our aim, however, will be to investigate the nature of the processes involved in the experience of 'extensive magnitudes'. The part played by synthesis in the study of geometrical shapes will be shown to parallel the rôle of synthesis in the development of the concept of number.

How may these processes be discerned in the work of the geometer? Consider what is the object of his attention. While doing geometry, the geometer's focus of attention is directed towards the image or appearance of some particular shape, say, a space enclosed by three straight lines. But before considering the nature of these appearances, let us first look at what Kant means when he says of something that it has or is an 'extensive magnitude'. Kant describes the situation thus:

I entitle a magnitude extensive when the representation of the parts makes possible, and therefore necessarily precedes, the representation of the whole. I cannot represent to myself a line, however small, without drawing it in thought, that is, generating from a point all its parts one after another.¹⁹

The tactic to be employed here is to now apply what Kant said about numbers being formed from the successive addition of homogeneous units combined with an awareness of the unity or totality so formed, to the study of geometry. Our interest is to see how the geometer comes to have an understanding of magnitude.

The question seems to be this: must it be that the magnitude of some figure can only be ascertained after the perception of the individual parts and their subsequent combination? From the above quote, one may note that Kant maintained the view that such a synthesis preceded and was a necessary part of the experiencing of any geometrical shape. Just as the individual units went into making up the number, so too the particular points are put together (synthesized), in order to enable the person to experience a line. This is but the narrow tip of the wedge which eventually will widen to show synthesis pervading all of our experience and perception of figures, objects and events in our day to day living.

The relation between the manner in which synthesis operates in the intuitive, abstract realm of numbers or geometry and the way in which it contributes to our knowing and understanding everyday empirical objects is not immediately evident. The relationship between the intuitive representation of figures and their empirical counterparts is clarified in the following material from Kant:

Take, for instance, the concepts of mathematics, considering them first of all in their pure intuitions. Space has three dimensions; between two points there can only be one straight line, etc.(...)

they [these principles and the representation of the mathematical object] would mean nothing, were we not always able to present their meaning in appearances, that is, in empirical objects... the mathematician meets this demand by the construction of a figure, which, although produced a priori, is an appearance present to the senses. In the same science the concept of magnitude seeks its support and sensible meaning in number, and this in turn in the fingers, in the beads of the abacus, or in strokes and points which can be placed before the eyes.²⁰

In this quote we see two things of importance for the view put forward in this paper. The first thing which Kant emphasizes is the necessary relation between the two seemingly distinct realms. The concepts would, as Kant says 'mean nothing', if they could not be presented in empirical

appearances. Here again, Kant refers to the use of fingers, beads or strokes so as to give meaning to the concept of magnitude expressed through numbers. The similarities between the geometer's understanding of magnitude and the mathematician's approach to number can be seen to closely parallel each other; both require that the totality be divided into separate units and that a synthesis or combining of these individual elements goes to make up the particular entity. Most important, in the resulting experience of a particular magnitude or a particular number there is at the same time an implicit awareness of the parts that make up the whole.

What may be said of the nature of these 'appearances' to which the geometer ascribes such magnitude? Paton, in his summary of Kant, highlights the following features of Kant's views on mathematics. Kant claims that mathematical knowledge depends upon the construction of concepts.²¹ Accordingly, the appearance of, say a triangle, will conform to what the individual has already contained in his concept of a triangle. Paton reiterates Kant's statement that the geometer is exhibiting, a priori, the intuitions

corresponding to particular concepts. That is to say, independent of any empirical, physical evidence, the geometer considers as 'appearance', the corresponding intuition through the representation of one of a manifold of images that satisfy the criteria for the concept.

'Appearance' is enclosed in single quotes to differentiate it from the more common sense of the appearance of a triangle made of wood placed before the geometer. The sense of 'appearance' that is meant here is the 'appearance' of a triangle in the mind of the geometer. It is meant to convey the notion which is generally referred to as presentation.

One may better understand the nature of these 'appearances' by considering their counterparts in empirical intuition.²² In such a comparison the a priori presentation of a triangle is analogous to the drawing of a triangle on a blackboard; the a priori presentation of the number two is analogous to putting one physical thing next to another. The appearance or representation that the geometer reflects on is one and the same with the sensible physical manifestation of the concept. The triangle that the geometer

thinks about or holds before his mind's eye is subject to the same definition, properties and restrictions as the triangle drawn before him on paper.

If a geometer does in fact work with a sensible empirical triangle drawn on a blackboard this in no way diminishes the universal nature of what he finds. The empirical characteristics of the triangle are irrelevant to what the geometer deduces. He considers the triangle before him solely as a representation of the intuition and hence precedes any object. His findings apply to all triangles, not merely the one he faces. The triangle before him is but one of a variety of figures that could have been drawn which would have fulfilled the concept or definition of a triangle. It is in this sense that the particular triangle is also universal, in that it is representative of all triangles.

The occasion of constructing an empirical triangle is exactly the same as the instance in which the intuitions of seven and five were made more clear through their representation by groups of beads, fingers or whatever. Such a device serves merely to facilitate the visualization and operations to be performed on the two numbers and in no way

lessens the generality or universality of the findings. A manoeuvre which presents the figure before the mathematician or geometer does not in the least detract from the pure or a priori nature of the intuitions.

If one accepts the above material, the question immediately arises of how one may ascribe properties to intuitions (appearances) without recourse to experience. This section will conclude with a look at this very question. What Kant expresses along these lines will later be found, in general terms, in Husserl's intentionality. In the preface to the second edition Kant writes:

The true method, so he found, was not to inspect what he discerned either in the figure, or in the bare concept of it, and from this, as it were to read off its properties; but to bring out what was necessarily implied in the concepts that he had himself formed a priori, and had put into the figure in the construction by which he presented it to himself. If he is to know anything with a priori certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with this concept.²³

The aspect of what Kant says here, which may be seen in Husserl's intentionality, is that the person may know of the figure only what he has already put into his concept of it. This Copernican revolution by which objects were to conform to the mind instead of the mind accommodating the objects is similar to Husserl's view in this way. In its most pregnant

sense, the intentional outlook of a person is characterized by the direction of thought towards an object. What this object comes to mean for the individual is thought to be determined, as Husserl puts it, 'by the sedimented layers of experience' built up over one's life. In both cases, the external object receives its meaning, in a sense comes to be, as a result of a subjective action directed out from the individual. In both cases, all that an individual may know of any particular entity is determined by what he ascribes to it; he infers nothing save that which necessarily follows from what he has already placed on the object.

2.5 Synthesis and the Possibility of Knowledge

After having seen the role Kant assigned to synthesis in relation to numbers and synthetic judgements, it is important to realize that Kant also used the notion of synthesis in a much broader context, i.e. as the basis for the possibility of knowledge in general.

It is to synthesis, therefore, that we must first direct our attention, if we would determine the first origin of our knowledge.²⁴

Kant is not discussing here the experience of this or that particular object but the foundation for experience in general (A111). The importance of the terms 'concepts' and 'intuitions' will become more evident in the subsequent explanation of Kant's theory of knowledge but their inseparability for the possibility of knowledge, may be noted even at this introductory stage of the exposition. Knowledge is said to involve two factors: the concept, by which the object is thought and the intuition through which it is given (B146).

I would now like to offer the structure of this section in which the above mentioned terms, synthesis, concepts, and intuitions will be more fully dealt with. The overall aim is to present the role of synthesis and trace it through the various related concepts that eventuate in the individual having an understanding of some object in his world. Of course, as mentioned, these very same principles will be seen to apply to the general notion of understanding. A certain amount of shifting back and forth in these two areas is almost inevitable as examples are supplied in order to render the general concepts more intelligible.

The first objective will be to secure the definition Kant gives of synthesis in general, to distinguish it from the more particular restricted instances of synthesis that were treated earlier in this chapter. Next, the place of synthesis in the three sources of knowledge that Kant presents, namely sense, imagination, apprehension, will be investigated and the relation of these, in the sense of where they finally arrive at, i.e. the unity of apperception, will also be presented with an eye to explaining "knowledge" as well as the part synthesis contributes. The relation of the understanding, which will be seen as the seat of the categories and concepts, to sensibility, will serve to provide a structure for the entire picture of "knowledge" in which both understanding and sensibility will play essential roles. Finally, some criticisms of the overall project re: knowledge and synthesis will finish the section. The material is drawn almost exclusively from "The Deduction Of The Pure Concepts Of Understanding".

2.5 i) Synthesis in General

By synthesis, in its most general sense, I understand the act of putting different representations together, and of grasping what is manifold in them in one (act of) knowledge.²⁵

Kant says that synthesis is pure if the manifold is not empirical but is given a priori, as is the manifold of space and time (B103). This pure synthesis gives us the pure concept of the understanding and is based upon a priori synthetic unity (B104). The reason Kant talks here of pure synthesis and a priori knowledge is due to the following. If our concepts were derived from the objects themselves then they would be empirical and would lack any of the universal necessity Kant posits in his a priori pure concepts of the understanding.

On Kant's view we are not dealing with the object out there as it really is in itself (noumenal realm) but instead, we are in contact with the mere appearance of the object (phenomenal realm). What we are in touch with is the play of

representations in our mind. The unity of experience, to which the entire notion of synthesis is directed, can then in no way come from the object but instead must be a unity in the consciousness of the person. It is on the grounds of these claims that Kant requires a pure, a priori concept of the object to precede our empirical awareness of it.²⁶

To show how the pure synthesis of representations is brought to concepts, Kant suggests we consider the three requisites for the knowledge of an object:

3. concept gives unity to this pure synthesis
(understanding)
2. manifold of intuition is synthesized by imagination
(appearance)
1. manifold of pure intuition (a priori synthesis)
object (x) (given)

One may now see Kant's concept of synthesis, as the putting together of different representations and grasping their manifoldness in one act of knowledge, to be operating in three areas.

2.5 ii) Kant's Threefold Distinction of Synthesis

Kant refers to the understanding, in a not altogether clear manner, as spontaneity (in order to distinguish it from the 'receptivity' of sensibility) and says it is "the ground of a threefold synthesis which must necessarily be found in all knowledge" (A97, my underline). It is these three sources of knowledge which will now be briefly summarized in order to see the crucial role of synthesis in the possibility of knowledge.

1. The Synthesis of Apprehension in Intuition (A99-A100)

Kant begins his discussion of the 'apprehension of representations as modifications of the mind in intuition' by pointing out that modifications of the mind belong to inner sense and hence all our knowledge is subject to time, the formal condition of inner sense. The manifold of intuition results from the mind's distinguishing the sequence of impressions one after another. This manifold must be run through and held together. It is this Kant wishes to call the synthesis of apprehension. The manifold can only be represented as a manifold contained in a single representation due to the synthesis.

2. The Synthesis of Reproduction in Imagination (A100-A102)

If things were different all the time eg. cinnabar was sometimes red, sometimes black, or if certain names applied sometimes to this, sometimes to that, then imagination would never succeed in its reproduction. But, Kant says, if we 'show our purest a priori intuitions yield no knowledge save as a combination of the manifold renders a synthesis of reproduction possible, then this synthesis of imagination is also grounded on a priori principles.' This is illustrated by the drawing of a line in thought which would never be completed if the preceding parts were not reproduced while advancing to the next. 'The synthesis of apprehension is thus inseparably bound up with the synthesis of reproduction.'

3. The Synthesis of Recognition in a Concept (A103-A110)

If we were not aware that what we are thinking now is the same as a moment before, then the manifold of representations would never make a whole since it would lack the unity that only consciousness can ascribe. Kant says that this unitary consciousness...

...must always be present; without it, concepts, and therewith knowledge of objects, are altogether impossible.²⁷

The necessity in the relation of our knowledge to its object, is due in part to the object preventing our knowledge from being haphazard or arbitrary, in that our modes of knowledge 'must possess that unity which constitutes the concept of an object' (A105). Now this unity would be impossible if it were not governed by a rule (concept may serve as the rule) which made the reproduction of manifold a priori necessary.

Kant says all necessity is grounded in the transcendental condition and so there must be a transcendental ground of the unity of consciousness in the synthesis of the manifold of all our intuitions (A106). This transcendental apperception, or unity of apperception as referred to in the second edition, is that to which all the previous material is directed.

2.5 iii) Unity of Apperception

In the unity of apperception, Kant uncovered that centre of experience, that 'I', to which all experiences were said to belong. The unity of the object is realized only in so far as the representations are united in one consciousness. If we follow the representations to the point where they obtain the unity necessary for knowledge, we must begin with pure apperception. Kant refers to the 'I think' which must be able to accompany all of a person's representations or else he would be having the thought of something that he could not think.

In other words, appearances in experience must stand under the conditions of the necessary unity of apperception, just as in mere intuition they must be subject to the formal conditions of space and time.²⁸

Just as we can have no appearance in an intuition without it being founded a priori in the manifoldness of space and time, likewise no object may be thought that corresponds to that intuition unless it stands under the necessary unity of apperception. All the representations

must be seen as belonging to each other and thereby belonging to one single, unitary consciousness. This synthetic unity of consciousness is not just a condition required in the knowledge of an object but is the condition under which every intuition must stand in order to become an object for me. This principle of apperception, by which a person may say of all the representations that they are one and all my representations, joined in a single consciousness, is the highest principle in the whole sphere of human knowledge (B134) . That is to say, no one could know anything without the accompanying awareness that there was a single 'I' to which all knowledge belonged. Without this there would only be a disconnected series of bits of knowledge lacking all wholeness and unity which only a unified consciousness may impart.

We have now looked at Kant's general concept of synthesis, his threefold synthesis in the sources of knowledge as well as his synthetic unity of apperception, in which all of a person's representations were synthesized (combined, if you will) into a unitary consciousness, expressed through 'I think'. To cement all these pieces of Kant's theory together, one may do well to consider the

distinction Kant drew between sensibility and understanding; bearing in mind, of course, that despite the distinction, knowledge depended on the relation (synthesis) of these very two areas sensibility and understanding.

Intuitions, in relation to sensibility, stand subject to the formal conditions of space and time: intuitions, in relation to the understanding stand subject to the necessary conditions of the unity of apperception. So far as the manifold representations of intuition are given to us, they are subject to the former; in so far as they must allow of being combined in one consciousness, they are subject to the latter (B136). Let us begin with the former.

Sensibility is that which yields appearances. Appearances are nothing but sensible presentations (A104). They (appearances) are the only things that we are in immediate contact with. Now in these appearances, that which relates immediately to the object is called intuition (A109). Each intuition is given only as appearance; it is a representation given before thought. As such, and subject to conditions of space and time, it is the mere form of intuition and does not yet provide any knowledge. Only when

this synthesis is brought into relation with the concepts, a function belonging to the understanding, do we first obtain knowledge properly so called (B103).

This brings us then to the latter, the understanding. All combination is an act of the understanding (B130). In the understanding are pure a priori modes of knowledge, the categories, the pure concepts of understanding. It is through these concepts that an object is thought which corresponds to the intuition.

Kant states that the imagination brings the manifold of intuition on one side (sensibility) into connection with the unity of apperception on the other (understanding). Sensibility and understanding must stand in necessary connection with each other or else the former, though yielding appearances would give no objects of empirical knowledge and consequently no experience (A123). It is only by imagination that concepts may be brought into relation with sensible intuition (A124). In the above, we have the intuition but no concept. Similar shortcomings are noted if one has the concept and no intuition. The concept would still be a thought as far as the form is concerned, but would

not have an object and so knowledge would not be possible. The thought could not be applied to anything (B146). Hence Kant's famous dictum; concepts without intuitions are empty, intuitions without concepts are blind.

2.5 iv) Criticisms

It is worthwhile to have looked at this large picture of synthesis in Kant's presentation of the conditions for the possibility of knowledge, in comparison to the specific role of synthesis in arithmetic and in synthetic judgements, presented earlier in this chapter. It also points ahead to later chapters, for example where Kant's mention of time in the apprehension of intuition becomes more fully developed in Husserl's temporal synthesis. There are however certain problems with what Kant presents in his deduction of the categories.

First of all, the fact that pure understanding is accounted for in these and only in these categories can not be explained. Kant's positing of the noumenal realm excludes experience of the objects themselves, thereby restricting us to appearances. Such a situation requires the necessary,

transcendental, pure a priori nature of concepts through which we may secure both the experience of unity and the unity of consciousness to which it must belong.

Further on this point, the fact that we employ synthesis in order to make sense of the world presupposes a chaotic disarray of experiences 'out there' which we must put together. Again this is required by the impossibility of the unity stemming from the object; a unity which must be supplied by the mind. This is in accord with the view maintained in this paper but one still may caution that although we separate the object-in-itself from the appearance, there still could exist the possibility that what we think of as the appearance, does in fact coincide precisely with the object.

Finally, the fact that the knowledge of an object is governed by a rule which is the concept of the object suggests an element of circularity. We would seem to already know the object when supposedly arriving at knowledge of it. The point of the matter would then no longer be cognition but re-cognition.

In defence of Kant, let us remind ourselves of his 'Copernican revolution'. His position that nature is only an

aggregate of appearances, so many representations of the mind, allows him to posit the understanding as the source of the laws of nature and so of its formal unity (A114). If one accepts the entire structure Kant puts forward, then one has a truly seaworthy vessel ready for the roughest waters without fear of leaks or holes. To continue the analogy, what I shall attempt in this paper is to take the hull from Kant and building upon it, construct an entirely new vessel that will sail different waters, but hopefully will prove just as seaworthy.

CHAPTER THREE

FREGE

This section will consider Frege's work in two particular areas. The first part treats his logicist reduction, not with the aim of ultimately explaining his difficult procedures which demonstrate how mathematics may be inferred from logic, but instead, the purpose here will be to get a general feel for the reduction and highlight certain areas which may point to what Frege says about the nature of numbers. The second section considers the shift from mathematical functions to propositional functions. It is hoped that while examining these expressions of ordinary language the same principles may be seen to operate as were found in the case of the mathematical functions. In both sections, the presence of synthesis will be ferretted out from what Frege says.

Frege does not expressly use the term synthesis in the direct manner of Kant or Husserl but it is suggested here that in order for one to make sense of his logical reduction it is necessary for one to perform synthetic acts. The turn

from numbers to linguistic expressions indicates the paper's more general concern with features of all experience instead of restricting the material to the domain of number. It is further contended that the synthetic process may be seen to operate essentially the same in both the areas of numerical formulae and ordinary sentences.

Before dealing with some of the more technical notions and definitions of Frege's reduction it may be profitable to get a general view of the land upon which Frege situates himself. To this end, some views of number which Frege finds unacceptable will be looked at first.

The view that numbers are a physical property is absolutely rejected by Frege.¹ The reasons Frege finds such a physical explanation of number unacceptable are that things other than physical bodies may be counted, such as thoughts feelings, and angels also a physical entity may be described in different terms thereby disassociating number from the physical aspects. Finally, Frege notes there are extreme difficulties in adequately explaining either "one" or "zero" as a physical property, "one" due to its universality

and "zero" due to its intangibility or lack of reference. As Frege puts it, it is unlikely that anyone every held zero pebbles.

Frege was also resolutely against any interpretation that claimed number to be a creation of the mind. He strongly separated logic from psychology and rebuked any attempt to explain number psychologically. Neither could Frege accept an abstractionist account of number; he felt it was incoherent to maintain that from considering a pair of objects and a trio of other objects one ~~could~~ ever reach an understanding of two to the third power.²

In Frege's earlier works he stated that numbers may be understood by examining the sentences in which they occur. This context principle by which the parts of the sentences came to receive meaning, if the whole sentence had meaning, was later rejected by Frege and did not appear in his later works.

Frege did not accept the view that held numbers to be attributes or predicates; such a use altered the meaning of the numbers and for Frege, numbers were not to be used in such a manner.³ In everyday language, numbers are generally used in this manner, as a type of adjective.

Unfortunately, language helps to promote this misuse of number when in fact, attributions of numbers are not distributive in the way that attributions of other properties are. By this I mean that when one says there are green leaves on the tree then the colour green is given as a property of each leaf; on the other hand if one says that there are a thousand leaves on the tree then this number "one thousand" can not be taken as a property of each leaf.

At face value, this seems to be an obvious claim. When one holds a leaf in hand, its property of greenness is much more easily ascertained than whether it evinces a dark or light hue of "thousandness". Personally, I feel each leaf may be said to have the property of belonging to the group of leaves that makes up the thousand leaves on the tree. It must be noted however that it is the mind that bestows such a notion and such a belonging can in no way be found in the leaf.

Frege did propose that such number statements could be made more clear if they were written in the form of identity

statements. By this one was less likely to consider the number as an adjective. The remedy hinges on the two senses of the copula "is", which is no longer to be thought of as the "is" of existence, but instead with the sense of "is identical to". A statement like "Five men crossed the street", would be re-arranged to "The number of men who crossed the street is five". In other words, one could say that the concept "men who crossed the street" may have five asserted about it.⁴

The view that numbers may be thought of as sets of units was also declined by Frege.⁵ Since this was the Euclidean definition of number which had prevailed for over two thousand years a detailed presentation of why Frege rejected it will be given here. In the large frame of the question, the idea of set as a spatial juxtaposition of things was rejected because, as mentioned before, numbers could in no way be thought of as physical. Also the idea of a set implied something more general than numbers. This points to the idea which will be developed in Husserl, that number is but a species of the genus plurality.

More specifically the question of units in numbers gives rise to three possible confusions. The use of unit may be taken to mean number one or several number ones: the unit could also refer to having unity or to be indivisible; or finally the unit may be thought to mean the unit or object as any object may be regarded as a unit to be counted. To the first position Frege proposes that several ones or for example one and one and one do not make the number three; what one needs is one plus one plus one to equal three. It is suggested that "and" and "+" are not the same thing. The second contention, that unit refers to an indivisible whole, is rejected since there can be no such property as being one. Since every object may be said to be one object this property vanishes due to its universality. The last case where the unit may be any object to be counted encounters the problem of whether the units are to be thought of as identical or different. The problem is expressed thus: if the units are identical, they in effect collapse into a unique unit and if they are different, then the number reached is not an individual entity. The paradox was presented by Aristotle in

similar fashion; if two is a collection of units, then two will not be one entity, and if two is one entity, it will not be a collection of units.

In order to see how Frege regards the development of number concepts we shall turn our attention to some points contained in his Grundlagen der Arithmetik, particularly sections 55-91.⁶ Frege begins with some attempts at defining numbers which he himself says are unsatisfactory. The initial definitions he considers do not sufficiently explain numbers as independent objects but rather provide explanations for the expression "the number 1 applies to ..." and likewise for zero. Frege then considers an attempt to define numbers through abstraction. He first defines the direction of a straight line and then hopes to present the number of a concept in a manner analogous to his explanation of the direction of a line. This also fails for reasons very similar to the first attempt; it fails to give sufficient criteria for recognizing number as an independent separate entity.

Frege continued his efforts to explain number by considering the case of the direction of parallel lines. He established that all that could be said about this was that they were equal in direction. He then needed a characteristic of groups that would be equivalent to direction in the case of lines that were parallel; this he found in equinumerosity.⁷

The attempt to reformulate Hume's idea of one-to-one correspondence i.e. groups that were equinumerous, and the claim that numbers concerned concepts and not the objects falling within these concepts represent two of the cornerstones of Frege's approach to explaining numbers. Consideration of these two features will provide a good frame of reference before entering the specific steps of the reduction that are pertinent to the aims of this paper.

Although Frege claimed numbers were objects and then proceeded to explain them by means of concepts, which may seem a bit contradictory to some, it was nevertheless an invaluable Fregean insight that tied numbers to concepts. The concept tells us which particular things we are concerned

with and the number tells us how many of the particular things there are. The necessity of the relation of numbers to concepts is made more clear in the case where one is handed a deck of cards and asked "How many?" Before answering one must know how many what is to be counted. Is it how many pairs, Jacks or red cards? One must have a concept, explicit or otherwise, in order to make sense of number statements. Further on this point one may add that the same physical situation is subject to different interpretations, indicating that numbers are dependent on the concept that is adopted; one may say there are five trees or that there is one group of trees, four boots may be described as two pairs.⁸

The other device employed by Frege in his explanation of number, equinumerosity, will begin to take us into his logical definitions. Frege wished to be able to express this basic idea without making use of the expressions "the number of f's". He stated that the number of a concept, would be the property of the class of all concepts equinumerous with that concept.⁹

To achieve a purely logical formulation of this notion, Frege produced the following:

If \underline{d} stands in relation \emptyset to \underline{a} , and if \underline{d} stands in relation \emptyset to \underline{e} , then, no matter what \underline{d} , \underline{a} , \underline{e} , may be, \underline{a} is always the same as \underline{e} .

If \underline{d} stands in relation \emptyset to \underline{a} , and if \underline{b} stands in relation \emptyset to \underline{a} , then no matter what \underline{d} , \underline{b} , and \underline{a} may be, \underline{d} is always the same as \underline{b} .¹⁰

It is important to realize that Frege had not yet managed to define number but rather had achieved a means of identifying a number as the same number in different instances. It is interesting that equinumerosity is more fully legitimized upon acceptance of Frege's view that numbers are objects. As soon as one assumes this position, it follows that there is at least one meaningful statement that can be made about the objects .i.e. the identity statement.

After Frege had this mechanism for identifying numbers in general, he directed his efforts towards explaining particular numbers. He took as his starting points the definitions of 1 and 0. After rejecting some initial attempts Frege decides upon the following definition of zero:

Because nothing falls under the concept "unequal to itself", I define: 0 is the number which applies to the concept "unequal to itself".¹¹

Frege now feels that he has a satisfactory definition of zero, since there can not possibly be any object which is not equal to itself. He therefore has a definition of the null class and is now on his way to a secure foundation for the series of natural numbers. The next step for Frege is to present in a logical manner, how he arrives at the first number in the series.

In what may seem to be a paradox of terms, Frege proposes to define one as the number which applies to the concept "equal to zero".¹² The reason a person can not simply produce an object of any sort and suggest that it indicates the number one is that since all singular objects possess this trait it becomes meaningless due to its universality.

Admittedly, Frege's approach does require a bit of effort, but is well worth it. One may approach it in this manner. Imagine your outstretched hand with all the objects that are not equal to themselves in it, that is, there is nothing in it. Now anything, any object, any matter of the

universe that you put in your other hand, in the attempt to demonstrate one, will fail for the above reason of universality. One may note that everything else that exists will in fact be not equal to 0. There is only one thing that equals nothing and that is nothing. Therefore Frege presents us with the definition that one is the number which applies to the concept "equal to 0".

Frege then faced the problem of demonstrating, in logical fashion, how it could be shown that one immediately followed zero in the series of natural numbers. As a prelude to this, Frege presented the logical description of the relation between any member of the series and its successor. This he defined as follows:

'There is a concept F and an object x falling under it such that that number which applies to the concept F is n, and that the number which applies to the concept "Falling under F but not identical with x" is m',

is to be synonymous with

'n immediately follows m in the series of natural numbers'.¹³

In order that one may now see where Frege justified his definitions of 0 and 1, we will now look at how he demonstrated that there was something (one) which immediately followed 0. This he set out in the following manner:

Let us consider the concept - or, if you prefer - the predicate 'equal to 0'. 0 falls under this. On the other hand, no object falls under the concept "equal to 0 but not equal to 0," so that 0 is the number which applies to this concept. We have therefore, a concept "equal to 0" and an object 0 falling under it, for which it holds that:

The number which applies to the concept "equal to 0", is equal to the number which applies to the concept "equal to 0";

The number which applies to the concept "equal to 0 but not equal to 0" is 0.

Therefore, according to our definition, the number which applies to the concept "equal to 0" follows immediately after 0 in the series of natural numbers.

If we define, then,

1 is the number which applies to the concept "equal to 0",

then we can express the last statement so:

1 immediately follows 0 in the series of natural numbers.¹⁴

So far we have ~~seen~~ Frege's reduction in its preliminary steps, first was his equinumerosity, his notion of belonging to a series, his definition of 1 and 0 and now there remained for him to show that the series was infinite. To reduce such a notion as infinity to a logical explanation seems almost a paradox of terms and his attempt shall not be dealt with here. However, the final technical definition, some may think thankfully so, to be dealt with here is the one Frege uses to begin his effort to show that every number is followed by another number. Towards this end Frege drew upon a basic defining trait of the number series which he had put forward in his Begriffsschrift, i.e. the idea of belonging to or following in a series. Such a notion was logically defined in this way:

The statement

'if every object to which x stands in the relation \emptyset falls under the concept F , and if, from the fact that d falls under the concept F , it always follows, no matter what d may be, that every object to which d stands in the relation \emptyset falls under the concept F , then y falls under the concept F , no matter what concept F might be",

is to be synonymous with

' y follows x in the \emptyset series'

and with

' x precedes y in the \emptyset series".¹⁵

Thus in the above one sees that any object either receives the property of belonging to the series, denoted by the fact that it has a successor, directly from being related to d , or indirectly from being related to an object which is in turn related to d .

3.1 SYNTHESIS IN FREGE

It is precisely at this point that I would like to step back from the technicalities of Frege's logicist reduction and investigate where synthesis may be seen to be present in what he says. The two notions I wish to focus upon are his ideas of "belonging to a series" and the "successor relation". Either of these will be seen to amply demonstrate the presence of synthesis as a necessary component in order to come to understand what Frege offers in his definitions.

Let us first of all examine the definition whereby Frege arrives at the explanation of "x preceeds y in the 0 series", the relation of any two members belonging to a series. To

arrive at this he begins in the following manner- "There is a concept F and an object x falling under it" - and this is as far as we have to go. Already in this fragment of the definition we see that a synthetic act is required of the person.

First of all, Frege states "There is a concept F"; this immediately requires the person to be aware of all that is necessary to have a concept - there must be an identifiable trait and it must be evidenced in the members, mind you such a trait does not have to be physical and in the most dissimilar group of objects the trait may be just that they are so joined by the combining (synthesizing) act of the intellect. When no definite concept is specified, as in Frege's concept F, then it is just this abstract notion of a concept that the person must assume.

The second point concerns the phrase "an object x falling under it", in which "an object x" presupposes understanding of one individual character; an independent object which implicitly draws upon what is meant by, the number one. Even more problematic is the idea of "falling under" - this is never made absolutely clear but in general

conveys the notion of belonging to a concept, being a member of the group of instances of the concept. The act by which an object is realized to belong to a concept is, in the view expressed in this paper, a synthetic act. That is to say there is no necessity to the fact that some particular object due to its particular characteristics falls under some concept. It is entirely dependent upon an act of the intellect, in each case a synthetic act occurs by which the object is said to belong to the concept.

The other area where I feel synthesis may be seen to be present in the reduction of Frege is in his idea of the "successor relation". Frege arrives at the position where he feels he has demonstrated the inference from n to $n + 1$ which was generally thought to be mathematical, to be logical.¹⁶ Besides the fact that Frege makes use of "+" an arithmetical operation, in his deduction of mathematics from logic, there are more critical nuances contained in this expression $n + 1$.

The crux of the matter, in which synthesis may be seen to be at the center, is to be absolutely clear on what is required of a person in order that they have an understanding of what is meant by the very next number in a series or group

or that every member has a successor. The person must have the idea of the original group represented by n and then have the idea of the next number, represented by $n + 1$. Although this may be presented in logical symbolism what really is at point here in $(n + 1)$ is that by this the person understand that one has the original group plus one more member which thus forms a new group. This is precisely the synthetic act by which Kant explained how a new number was reached by the synthesis of one unit to the original collection. The same synthesis is required in adding one to eleven to arrive at twelve as is required to add one to n in order to arrive at $n + 1$.

3.2 EXPANSION OF FUNCTIONS

How then may we move from arithmetic to ordinary language and how can we carry along the notion of synthesis with us? The transition from numbers to expressions comprised of ordinary words is legitimate due to the extended use Frege made of the term function. Frege's philosophical writings grew from his work in mathematics; he was first of all a mathematician and it was his attempt to remove some of the inconsistencies in the field of mathematics that eventually developed into his logico-philosophical works. One such confused notion that Frege found in mathematics was that of function.¹⁷

Frege states that we may consider any expression as a function if we regard it as unsaturated. For example, the function X^2 contains an empty space, expressed by X , that must be completed in order for the function to have a value. One sees that in the case of functions, each and every one of them is characterized by this empty place, this argument place, which needs supplementation in order for the

expression to have sense. Thus, in Fregean terms, substitution of the argument 3, in the above function x^2 , will yield the value 9.

One sees then a result of Frege's efforts to remedy such ills in mathematics that beset both teachers and pupils over such functions as $x^2 = y$. Frege began to treat arithmetical expressions in the manner that was formerly the domain of algebraic functions, $x^2 = 9$. Now, obviously in such a function if one substitutes for the numerical variable X the number 3, one obtains a true equation; but if one substitutes the number 4, the function yields $4^2 = 9$, which is False. According to Frege, 'True' and 'False' make out the range of values of the function expression.¹⁸ That the True and the False, as values of such expressions, are to be thought of as objectual, is not the least difficult segment of Frege's work to grasp.

The shift to sentences of ordinary language is the second aspect of Frege's extended use of the notion of functions.¹⁹ Again, it is founded on the basis that one part of the expression be considered unsaturated (the function), while the other part may be thought of as complete (the argument).

In fact, such a manoeuvre is not really too startling if one considers that the arithmetical expressions themselves could have been written out in words, such as, there is a number which multiplied by itself is equal to nine.

There are however certain difficulties encountered when one begins to consider linguistic expressions. Frege himself puts it this way:

We cannot avoid a certain inappropriateness of linguistic expression; there is nothing for it but to realize this and always take it into account.²⁰

The inadequacies of everyday language were the prime motivators for Frege's attempts to construct a more reliable logical language. His Begriffsschrift was such an effort which tried to provide a more foolproof method of inference. A very grave consequence of such a careless manner of speaking or writing was that very often labels were accepted as proper names when there was in fact no corresponding object for which the name was appropriate, eg. the extension of the concept.²¹ Also, one could easily get into difficulty by using the definite article "the" which seems

to confer some sort of independent objectivity on whatever it precedes.

So a great part of the work of a philosopher consists - or at least ought to consist - in a struggle against language. But perhaps only a few people are aware of the need for this.²²

Frege at this point warns against use of the definite article but earlier, he did advocate this article as indicating the objectivity of numbers. Frege maintained that numbers were objects and for one who should have been aware of the struggle against language he certainly created enough confusion through his claim that numbers were objects. Of course he did not mean object in the same sense as the objects that surround us in the world. Here one may point out Frege's weak sense of objectivity in which numbers were thought of as accessible to all minds and were the same for all who used them.²³ The fact that it made no sense to talk of the location of numbers did not mean that they were not objects, only that they were not spatial objects. Likewise since objects could be pictured in the mind, the objectivity of numbers was questioned since they were not

picturable. In the face of these problems, Frege was asked if he saw any difficulty with his view that numbers were objects, to which he replied "Sometimes I seem to see a difficulty - but then again I don't see it".²⁴

Bearing in mind the problems one may get into with language and the inappropriateness of certain expressions we shall now conclude this section by dealing with the shift from mathematical functions to propositional functions which was indicated earlier as the second important extension that Frege made in the concept of function.

In these instances, proper names will be substituted in the argument places of the linguistic expressions whereas before it was numbers that were substituted in the algebraic or numerical formulae. Now the value of the linguistic expression will be a complete thought instead of the quantitative result expressed in number which was produced in the mathematical functions.

The following two examples make very clear what Frege meant by treating expressions in ordinary language as functional. Recall the mathematical function X^2 . When the

number 3 was substituted in the argument-place, indicated by x, the value was found to be equal to 9. In similar fashion, Frege offers as example the propositional function - 'the capital of x' - and if one fills the argument place with say, France, then the value of the function for that particular instances is Paris. The unsaturatedness of the function, a characteristic of all functions, must be remedied by filling the argument place in order for the expression to have any sense.²⁵

The second example illustrates how the same procedure may be applied to the phrase e.g. 'the cat is on the mat.'²⁶ Either the word 'cat' or 'mat' may be removed and replaced by the argument place signified by x, which would result in a function dealing either with the cat or the mat. The former will be chosen here, thus yielding the function - 'The cat is on the x' - which has its unsaturated component as well as the variable part or argument place. When the argument place is filled by the word 'mat', then for this particular instance the value of the function is the True. If one substituted for the argument place say the 'square root of four' or the 'roof' or the 'Milky Way' then the resulting expression would have the value the False. This is

the same as when only 3 or -3 in the function $x^2 = 9$ yielded the True and everything else, '4'² or 'the Moon' squared, would give the value the False. Interestingly, it was Frege's demand that since the reduction was to be purely logical, then there could be no restriction on the range of possible objects that may be used to fill the argument places.²⁷ Also every expression formed in accordance with the rules of his system must be a meaningful expression with a reference. Hence, expressions like "the Moon -3" were said to refer to the object which Frege called the False.²⁸ It is strange that Frege's requirement that the reduction be logical and therefore be concerned with the form of expressions regardless of their content was what precisely opened up the door for the emergence of nonsensical, meaningless expressions. This lack of restriction paved the way for the entry of linguistic expressions into his domain since there was logically no ground for excluding such sentences from the study of functions.

CHAPTER FOUR

HUSSERL

In this final section, the investigation of synthesis in numbers will be directed towards an interpretation of the contribution synthesis makes to consciousness as it organizes and gives meaning to our world. In maintaining the general structure of the paper we shall first of all be concerned with investigating where synthesis may be found in Husserl's approach to numbers. After this, the presence of synthesis in our everyday experience will be looked at, with particular attention being paid to how synthesis figures in Husserl's theory of intentionality. The section will conclude with a demonstration of how the essential notion of synthesis, that of joining, may be seen to appear in precisely the same manner in both our experiencing of the world in general and in our experience of numbers in particular.

Husserl's views of number are taken from accounts of his Philosophie der Arithmetik given by Farber, Willard and Sokolowski. Of particular interest here will be the unravelling of what exactly is contained in Husserl's idea of collective combination. This very important notion goes by a variety of labels such as collective connection, collective unification and collective liaison, but in all cases the idea of a collection is maintained. Certain aspects of Frege's review of this book will also be treated here.

The treatment of intentionality is derived from several Husserlian sources, as is appropriate for such a central theme of his writings. The connection of synthesis with intentionality and especially the linking of synthesis with the phenomenon of meaning-bestowing performed by the individual, is to be drawn primarily from Cartesian Meditations, with some of the supplementary material being drawn from Husserl's Ideas and Experience and Judgement.

4.1 Nature of Number

What does Husserl present as his view on the nature of number? Let us turn to his Philosophie der Arithmetik,

keeping in mind that Husserl essentially feels that concepts like multiplicity, equality, plurality and number are of such a basic, fundamental nature that they are neither capable of nor stand in need of any definition.¹ In such an instance the process is not to analyze or define the concept but rather to indicate how it arises or comes into operation.

One gets an excellent overview of Husserl's approach to numbers from Sokolowski's account of Philosophie der Arithmetik. There are five stages in Husserl's investigation of the concept of number.² The first step considers number and multiplicity: Husserl claims the latter is presupposed by numbers which are thought to be the distinct species of the concept of multiplicity. The difficulty then lies in explaining multiplicity and the next step proposes that multiplicity is an abstraction from groups, i.e., actual concrete phenomena, determined objects like a group of trees.

The third step then turns to investigate these groups. Husserl suggests that the group may be explained in terms of relations. He further states that the status of a member of

a group is due to its relationship to the group as a whole. In the fourth step, the relation which is characteristic of and necessary for there to be a group or multiplicity, is presented as collective combination. The fifth stage deals with the origin of a group from the reflection upon this all important act through which the various contents of the group are related. It is to this notion of collective combination as the source of multiplicity and therefore of number that our attention will now be directed.

4.2 Collective Combination

And it arises in a way analogous to that of the concepts of other sorts of wholes, all of which are come by through reflection upon the modes of combination peculiar to those wholes. From here on, we shall use the name "collective combination" to designate that sort of combination which is characteristic of the totality.³

Since Husserl deemed this notion of collective combination to be the essential characteristic of any group or multiplicity, it is necessary that we be absolutely clear on what he means by it. It may be noted that it describes the

same operation that Kant said was performed by synthesis. For Husserl, this collective combination described the relation of the members of a group and solely in virtue of this relationship did the group come into being. Likewise for Kant, the number came into being solely due to the synthesis of the homogeneous units.

It is in the third chapter of der Arithmetik that Husserl investigates the nature of the collective combination. The idea that this particular relation is the same as any other theory of relation is rejected. Five theories of relations, which generally could account for all cases, were found to be unacceptable; these included co-presence to one mind, temporal co-existence, temporal succession, spatial relation and relation of difference.⁴ Husserl thought this relation to be a very special one indeed, unlike any other type of relation. To understand how Husserl thought of this relation, one first of all has to be aware of the distinction he drew between primary and psychological relations. As a result of this particular terminology Husserl opened himself to several attacks; the charge of psychologism being perhaps the most discussed.

Returning to the two types of relations, we may note that the first, the primary relations, are those in which the relation is given with the element and is inseparable from it such as colour to surface. Other examples of these primary relations, one may refer to them as physical relations or content relations, would be identity, gradation of degree, relations in a continuum and combination of properties in physical objects. The most important thing in all of these cases is that the relation is there, contained in the elements and can not be separated from them. It is exactly the opposite case when one considers psychical relations.

The psychical relation has its origin solely in the mental act which relates the elements. It is possible that this relation brings the most disparate collection of entities into a group. In fact, the group may seem to be characterized primarily by unrelatedness. There is nothing in the properties of the objects that necessitates the grouping. It is supplied entirely by the mind, a relation which as it were, is placed upon the contents to be joined. The contents may be varied without altering the relation.

Therefore the collective combination, since it is entirely separate from the nature of its content must be a psychical relation.⁵

Husserl's explanation of number depends upon having the group, that is to be counted, abstracted so as to be represented by units and then collectively combined by use of the term "and" which Husserl claims adequately expresses the notion of collective combination. How he achieves this abstraction and forms the number group by the use of "and" certainly raised a few eyebrows and two of them belonged to Frege. We shall now proceed to a closer look at how Husserl developed his number concepts as well as considering some of the criticisms Frege directed at this area in his review of Philosophie der Arithmetik.

4.3 Husserl's Explanation of Number

Here we shall examine more closely the structure of the groups which Husserl says are characterized by collective combination. These groups serve to explain Husserl's view of how the particular numbers come into being.

Husserl claims that we form a group by a special type of noticing which is generally associated with but not restricted to enumeration.⁶ In this manner one may survey the articles on the desk in front of him and thus serially notice and bring into being the group which has as its members the book, the pen and the piece of paper. This is still far removed from the number three and it is a very interesting process by which Husserl leads us from this noticing to the specific number three. A process in which, as we shall see, Frege noted there were several difficult, if not impossible, steps which had to be performed.

The first step that is required for us to arrive at number is that we abstract from the particular nature of the objects or contents that are to make up the group. It is through this that one reaches some sort of basic element or

indeterminate unit which may be used to build the number. What Husserl means by abstraction may be thought of as inattention, a procedure which disregards certain properties of the particular contents of the group.⁷ One may, in the case of the articles on the desk, cease to notice their colour, their location, their size and so on until all distinguishing features have been removed and they can be regarded as little more than "somethings". The only requirement that an object must meet for it to become a member of a group that forms a number, is that it be possible for it to become an object of consciousness.⁸ That one be able to think of the object as "something" is another way of expressing this requirement.

Husserl considered this "something" (Etwas) to be the most general and empty of all concepts.⁹ It was capable of taking any type of object whatsoever as its contents. Instead of "something", one could also use "one" as an equivalent means of indicating that which remained after the abstraction of all the particular defining characteristics of the object. One then would unite the series of "ones" by use

of the word "and". This syncategorematic term "and" was believed by Husserl to convey in everyday language all that was contained in the idea of collective combination.¹⁰

Frege did not agree at all with such an explanation of number. He charged Husserl with attempting to give some sort of scientific justification for what was a naive view of number. The whole notion of abstraction was particularly objectionable to Frege. He described it as the process by which things were altered and yet remained the same. To Frege, such a transformation could only be achieved by immersing the contents, as he put it, in the "psychological washtub". Continuing the washday analogy Frege said that the suds were provided by the lye of inattention. Here Frege proposed the trick was to find just the right degree of dilution not too weak that it affected nothing, and not too powerful that all distinction was obliterated.

Frege had equally seething criticisms for Husserl's use of the conjunction "and" to arrive at number, saying that it completely missed the point of number statements.¹¹ No doubt the point it missed was Frege's point of numbers

concerning concepts and not the objects that fall under them. It seemed, to Frege, that Husserl had seriously distorted the sense of number statements since in ordinary usage one does not inquire "How many are England and France?" Furthermore Frege questioned what could possibly be gained from such a formulation.

By Frege's approach when one asked "How many moons does Mars have?", there is a concept involved and some worthwhile information may be gained through approaching numbers in such a manner. I do not believe Husserl intended his approach to be expressed in statements like "How many are England and France?" The series of items joined by and, is used as an explanation of number after the individual members have been abstracted to be represented by "ones" and not while they retain their individual defining characteristics as witnessed by proper names 'England', 'France'.

The difficulties that Frege has with Husserl's theory of abstraction do not seem to have much significance for Husserl's overall explanation. I question why Husserl even thought it was necessary to explain how one could get from a

set of different objects book, pen, paper to three basic units such as one, one and one. Every single object, can also be termed a thing, even thoughts, angels, wishes and such may be considered as entities for the purposes of counting. This is self-evidently clear and needs no justification. Hence any critique, of what appears to be an unnecessary step, can have little effect on the outcome of Husserl's argument.

I do however, agree with Frege's objection to the employment of the term "and". Frege complained that this expressed number as mere conjunction and missed the relation to concepts.¹² I feel the problem with the term "and" is that it falls short of adequately expressing all that is contained in the notion of collective combination. Granted, it does convey the idea of conjunction but this can only result in the formation of a group of connected individual elements which does not convey the idea of a unitary whole in which the members combine to form one entity which may be expressed by a number. The term, "and" does indicate the association of an object to another but does not go beyond

this to the combining or synthesizing of one object with another which is required for the experience of number.

Frege complained that "and" did not properly express what was required to explain the number three in "one and one and one".¹³ He suggested that "plus" be substituted, thereby yielding "one plus one plus one". I would agree that "and" is not the same as "plus". The latter does indicate, in mathematical fashion, that the units are to be added together. This still falls short of explaining how it is that the fusion of the elements comes about to produce a new combined totality. The problem may lie in the fact that we are trying to pick a word that describes the action performed by the mind in synthesis. Perhaps mixing "and" with synthesis to produce a new term such as, syn-and, would more closely approximate what is required to explain numbers as in "one syn-and one syn-and one" equals three.

4.4 SYNTHESIS IN INTENTIONALITY: AN OVERVIEW

It is at this point that we would like to turn from what Husserl says about numbers and consider one of his most dominant themes, intentionality. The presence of synthesis in the explanation of number has been shown and we shall now see the role of synthesis within the much broader frame of intentionality.

Unlike Frege, Husserl does make explicit use of the concept of synthesis. One major distinction he makes in the concept, which we may mention in this prelude to the more intense study of the matter to come, is the distinction between active and passive synthesis. In general terms, the difference may be thought of as this. Husserl states that active synthesis is employed in the construction of abstract notions such as groups or parts; passive synthesis results in the experiencing of ordinary objects that confront us in everyday life.¹⁴

I feel that this active-passive distinction may be applied to the previous discussion of number in the following manner. When we are determining a number or arriving at a specific number, it is an active synthesis that is involved. It is active in the sense of combining each of the members and arriving at a total. However, when one later refers to that group, say seven, all that is required is a passive synthesis for the mind to grasp the number. By this I mean that in the mere utterance of the word seven, one presents an already formed, previously determined entity which is readily understandable to others. There is immediate awareness of that which may be denoted by seven.

Besides this active-passive distinction, which among others will be returned to in the next section, by far the most noteworthy topic will be the question of meaning. A preliminary sketch of the material may serve to situate the topic and indicate the parameters within which the role of synthesis in meaning will be pursued.

In what may be thought of as the opening statement in a legal case, I would now like to lay out what the objectives are in the investigations of synthesis in intentionality. The motivation for each aspect of synthesis studied here rests ultimately on the question of meaning. We shall see how Husserl explains, through intentionality, the fashion in

which the world and its objects receive meaning. Pursuant to this, the role of synthesis in intentionality will be presented, including the major types and their contribution to intentionality.

The most crucial debate, implicit throughout these early discussions, finally comes to the fore in the section on the relation between synthesis and meaning. It will be shown, through various forms of syntheses, how it is that the composite nature of what things mean is arrived at and joined to the particular person, object or whatever that we experience.

I believe that the role of synthesis in our development of number concepts, as it was demonstrated in the initial stages of this paper, is the most adequate, straightforward, most basic paradigm upon which to construct and demonstrate the processes that give meaning to our lives. This transition from numbers to meaning is not immediately evident and so I would again like to offer a preliminary sketch of the issue as a guideline for the discussion to follow.

We do not simply perceive the world around us as appearances, presented to us through child-like innocence or blissful ignorance. Instead we encounter, in the case of healthy, rational beings, a world that is replete with significance for us, objects that are almost bubbling over

with meaning. The point here is that we do however, experience a single unit. The perception is put together and the meaning is included in the unitary experience that we have. For example, I do not perceive various shades of gray with certain bright spots, a flat surface, sides which I know are at right angles but do not appear so, legs which appear flat even although I know they are round, and so on; but instead, I have the experience of this table. Besides the obvious physical attributes that are assembled, there are also the intangible dimensions involved. I am aware of who built the table, the circumstances under which it came into my possession and so on. Now all these aspects are not present to me immediately when I am aware of the table, but they are in a sense, contained in the meaning of the object and do come forward if reflected upon.

Just as the various units went into the building up of a number, in similar fashion, the variety of perceptions, memories, judgements go into building up the meaning of something for us. In both cases, the person meets a single entity but only as the result of synthesis which has joined the members so as to produce the single object we experience.

4.5 INTENTIONALITY, SYNTHESIS, MEANING

In this section we shall attempt to present a full and complete picture of the prominent part played by synthesis in the concept of intentionality. When dealing with as pervasive a concept as intentionality, there are many related topics or concepts such as constitution, relation, meaning, activity-passivity and so on that arise in the work of Husserl. Each of these areas could not be treated exhaustively and the amount of explanation accorded to each is based upon the extent to which they contribute toward the understanding of synthesis and how they help in demonstrating the role of synthesis in intentionality.

This look at synthesis will have three main sections to it. The first will consider, in its most pregnant sense, the concept of intentionality. Here, we will look briefly at the primary historical roots from which grew Husserl's idea of intentionality. Developing this idea, the main fundamental characteristic of intentionality will be presented and finally, what is felt here to be the most important aspect of intentionality, its meaning-bestowing component will be given an introductory presentation as it serves as a bridge to the next section, synthesis.

In the second section, the focus of attention will be directed specifically upon synthesis. The first important

issue to be dealt with will be the question of temporality.¹⁵ After that, several different types of synthesis will be investigated along with some basic divisions within the concept, such as the active-passive issue will also be presented. Perhaps the most important feature of synthesis, for the purposes of this paper, is that of unity. This issue will be pursued along several different avenues including the unity of the perceived object, its horizons and to a lesser extent, the perceiving ego.

In the final section, the various strands will be pulled together to complete the picture with an examination of the question of synthesis and meaning. Here it will be necessary to consider such notions as fulfillment, constitution as well as the term "and", almost an anachronism from the earlier treatment of arithmetic.

4.5.1 INTENTIONALITY

4.5.1.1 i) SOURCES

It is a generally accepted view that Husserl's concept of intentionality had its beginnings in the work of his teacher Franz Brentano. Although Brentano's aim was to clearly demarcate the mental from the physical, he did give Husserl the raw material from which to draw his own concept,

fashioned towards his own ends. The original seed may yet be noted in Brentano's presentation of the issue:

Every mental phenomenon is characterized by what the Scholastics of the Middle Ages called the intentional (and also mental) inexistence of an object, and what we call, although in not entirely unambiguous terms, the reference to a content, a direction upon an object (by which we are not to understand a reality in this case), or an immanent objectivity.¹⁶

In this brief quote one finds a rich and complex diversity of issues pertaining to the concept of intentionality. Despite this, one does find contained in this quote the cornerstone of what was later to become Husserl's full-blown use of intentionality. Leaving aside the very problematic nature of the intentional object, what I am referring to here is of course the idea of "the reference to a content, a direction upon an object". It is this directedness of consciousness towards an object that was to be characteristic of Husserl's later approach to intentionality.

4.5.1 ii) HUSSERL'S DEFINITION

The importance of intentionality to the overall program of Husserl's phenomenological approach to understanding man cannot be overstated. It is at the core of his explanation

of how man perceives and gives meaning to his world and its objects. Accordingly, the notion of synthesis may be a less studied but, as we shall soon see, a no less important or necessary tool to attain a full understanding of Husserl's work. In the Paris Lectures, which, in his introductory essay Kostenbaum says, "serve as the foundation of Husserl's definitive work Cartesian Meditations", Husserl affirms the central position of intentionality and presents us with a fundamental defining feature of consciousness.

The essence of consciousness, in which I live as my own self, is the so-called intentionality. Consciousness is always consciousness of something.¹⁷

Here we have Husserl's most straightforward statement of the importance of this key concept, the essence of consciousness is intentionality. I would hope to carry this a step further and by the close of this section shall have demonstrated that the essence of intentionality is synthesis. The other point of note in the above quotation, i.e. consciousness is always consciousness of something, indicates the relation in this key area to the view of Brentano presented earlier. Husserl's teacher made mention of the fact that in

all mental acts, there was always a reference to a content, attention was directed towards an object.. Husserl maintained this theme that consciousness was always directed towards something; it was always consciousness of or consciousness about such and such. Continuing on a historical note, this particular notion could be traced even further back and in fact, was given excellent description by Hume:

When I enter most intimately into what I call myself, I always stumble on some particular perception or other, of heat or cold, light or shade, love or hatred, pain or pleasure. I never catch myself at any time without a perception, and can never observe anything, but the perception.¹⁸

Although each of these three men had his own particular purpose in presenting such a position, it is nevertheless undeniably clear that each gave adequate presentation to what Husserl considered the most fundamental feature of intentionality: that it, in every instance, was directed toward an object and this object became the single focus of attention of the individual.

4.5.1 iii) INTENTIONALITY AND MEANING

The ultimate aim, it must be remembered, of this section on Husserl is to show the operation of synthesis in his formation of number concepts but more importantly to demonstrate the role of synthesis in our ascribing meaning to things, people and other phenomena in our daily lives.

So it is with the risk of getting ahead of ourselves that this sub-section on meaning is included here. Here the basic role of intentionality, as it provides meaning and significance to our experiences in life, will be investigated. One may think of this look at meaning as a sort of preliminary establishing of the fact that the mind does attribute meanings, values, and so on to our world. Whereas at the conclusion of this section, the return to the examination of synthesis and intentionality will provide the "how", rather than merely expressing the fact that meaning is given to the world.

We are not talking about meanings in the narrow dictionary sense, although these meanings are not excluded; there may be some particular significance or memory that is associated with the meaning of a word for some person. In general, a much broader sense of meaning is meant or intended here. It includes, but goes far beyond the intellectual,

rationalistic side of man, and attempts to unravel why it is that people feel the way they do as a result of the meanings they have placed upon all that they encounter in their lives and more specifically how it is that these meanings become attached to that which bears them. What we hope to investigate is the process or processes by which one builds up the meaning and the manner in which this meaning is given to or associated with the particular object or event. Later, we shall see that the foundation for these processes is synthesis, but first let us return to Husserl's expression of how the world and its objects attain meaning for us.

The objective world, the world that exists for me, that always has and always will exist for me, the only world that ever can exist for me - this world, with all its Objects, I said, derives its whole sense and its existential status which it has for me, from myself, from me as the transcendental Ego, the Ego who comes to the fore only with the transcendental-phenomenological *epoché*.¹⁹

In this selected quote we are given one formulation of the view that the meaning of the world and its objects derives from the transcendental Ego. It is a view that finds repeated expression in Husserl's works. The key phrase in the above is "this world... derives its whole sense... from myself". This captures the most important point of this

section on meaning; it is the person who gives meaning to the world and its contents. The other important issues mentioned in this quote, such as the transcendental Ego or the transcendental-phenomenological epoché (reduction, or perhaps more accurately bracketing) must, regrettably be omitted from any further consideration here. These are important issues but it is felt that any detailed exploration of them would take us too far from, and would begin to blur the main point of the person giving or bestowing meaning upon the world.

4.5.2 SYNTHESIS

4.5.2 i) SYNTHESIS AND TIME

It is in the second of the Cartesian Meditations that Husserl directs his full efforts towards presenting the extremely difficult idea of synthesis. We will begin our considerations with the simple idea of synthesis, as combining, reminiscent of Kant's original presentation of synthesis, and then the topic will be developed along some of the more difficult, involved areas of application to which

synthesis is put. The most essential synthesis, according to Husserl, is the synthesis of the individual temporal moments and we see mention of this as Husserl introduces synthesis as the primal form of consciousness.

The sort of combination uniting consciousness with consciousness can be characterized as synthesis, a mode of combination exclusively peculiar to consciousness.²⁰

This quote shows Husserl agrees that the most definitive trait of synthesis is combination. Despite the possible equivocations in the three instances of consciousness, the first two may be thought of as acts of consciousness or specific moments of consciousness, one may still note that synthesis is employed, in a Kantian manner to indicate the combining activity that occurs in consciousness. More importantly, Husserl points out that this type of combination is restricted to the mental realm and is excluded from the physical realm. Husserl goes on, in the second meditation, with his presentation of synthesis, bringing the perception of objects and of course, the necessary temporal synthesis involved, into the picture.

For example, if I take the perceiving of this die as the theme of my description, I see in pure reflection that "this" die is given continuously as an objective unity in a multiform and changeable multiplicity of manners of appearing, which belong determinately to it. These, in their temporal flow, are not an incoherent sequence of subjective processes. Rather they flow away in the unity of a synthesis, such that in them, "one and the same" is intended as appearing.²¹

Two major points are introduced to the discussion in this quote; the first relates to the unity of the perceived object in its multiplicity of manners of appearing, which will be returned to later and the second point involves the temporal flow of these appearances plus their synthetic unity. It is this particular notion that we wish to pursue to a deeper, clearer level of understanding.

It is not conducive to our aims here, to attempt an explanation of time as such. Whether one considers time as a sort of duration, or a sequential series of moments or tantamount to change is not to the point, it will be necessary only that all agree on the most fundamental division of this medium. By this I mean that certain events may be situated in time as past, present or future. From such a basis, Husserl's temporal synthesis may be thought of in the following manner.

An individual experiences an object in his world. He has the perception of it at one particular moment, say, t . This same object remains as the focus of his attention in his field of experience throughout an extended length of time, which we shall denote as t^2 to t^n . Husserl states that the reason this person does not experience a static, unconnected series of stages of perceiving this object x at t^1 , then x at t^2 or does not come under the impression of a variety of different objects, is due to the fact that as a result of the synthesis of time, a unity is arrived at. The object is seen to be "one and the same", enduring throughout the different temporal phases and not only this, but the phases themselves are combined to make the one "time", so to speak, of the object.

Husserl does distinguish objective time (external) from internal time, that is to say, the time in which an object persists outside us in its environment is separated from the subjective internal time in which that object appears to us. He further distinguishes internal time from the consciousness of internal time but such considerations take us too far away from the essential point concerning time to be made here.

The crucial point to be remembered is that through the synthesis of temporal phases we are able to identify objects in our world as enduring persisting entities and that we perceive "one and the same" object over an extended period of time. It does not take Husserl long to expand this concept of synthesis from perception of mere objects to that of the entire world around us. In quite straightforward terms Husserl says that the whole of conscious life is unified synthetically, and furthermore..

The fundamental form of this universal synthesis, the form that makes all other synthesis of consciousness possible, is the all-embracing consciousness of internal time.²²

4.5.2 ii) ACTIVITY-PASSIVITY

After considering the question of time and seeing the position of prominence accorded to it by Husserl, as the precondition for the possibility of all other types of synthesis, then perhaps the next most fundamental question that arises around the issue of synthesis is the discussion

of the active and the passive nature of the synthetic acts. Some have slighted the issue claiming passive acts to be somewhat of a contradiction in terms, thus setting the question aside. It is felt here that a more strenuous pursuit of the question will yield profitable insights into the nature of synthetic acts and the processes involved. Husserl does distinguish between active and passive synthesis but the question of which comes first or whether one is the foundation of the other remains a philosophically unanswerable question. Rather than one being built upon the other, they seem to operate in a more symbiotic manner, an interweaving of the two, in which yet distinct areas may be perceived.

This relationship between active and passive synthesis will come to the fore in the following discussion but if a decision had to be made one way or the other, then it is felt that in the question of which is primary between the two, there is a slight leaning towards passive synthesis as presupposed in the other type. It is for this reason that the discussion here will commence with the examination of passive synthesis.

The "ready made" object that confronts us in life as an existent mere physical thing (when we disregard all the "spiritual" or "cultural" characteristics that make it knowable as, for example, a hammer, a table, an aesthetic creation) is given, with the originality of the "it itself", in the synthesis of a passive experience.²³

Husserl states that the empirical objects that confront us in daily life are knowable through passive experience. That which we meet as a hammer, a table may be subject to an active synthesis, to be discussed later, by which refinements or more detail or a richer sense of appearing or meaning may be reached for the object but as Husserl expresses it "the passive synthesis that supplies all their material still goes on" (Cart. Med., p. 78). It is as if the passive synthesis supplies the raw material and active synthesis contributes to the finished product..

There are two areas which are said to be fashioned through passive synthesis which would tip the scales in favour of it being the more basic, fundamental notion, the two areas are time and the world. If we consider the original presentation of an object to consciousness eg. there is a book in front of me, then this awareness is due to a synthesis of moments of time and it precedes the further acts by which the book becomes identifiable to me. This is passive, accomplished without any particular intervention or activity on the part of the ego. It is the first level of encounter that one experiences of the world around him. Husserl describes the ego as a nexus of synthetically congruous performances - "at levels, all of which fit the universal persisting form, temporality, because the latter itself is built up in a continual, passive and completely

universal genesis." (Cart. Med., p. 81). Thus temporality is described as built upon a passive genesis i.e. a passive synthesis by which the objective time of the object existing out there and the internal subjective time in which the object appears, are each formed from phases into a continuous time allowing the existence of the object and the perception of the object each in their own time.

Kostenbaum, in his introduction to Paris Lectures, says that Husserl refers to the passive constitution of the world by the ego. Without going into the issue of constitution at this point, suffice it to say that by constitution here, is not meant any sort of physical creation but rather a sort of bringing-into-being-for-a-subject. By this, one means that the world is already there and physically existing, this is not ever intended otherwise by Husserl.²⁴ The constitution of the world, a world that is already there, however does seem to be a bit paradoxical. Especially, if it is supposed to be constructed, synthesized, given meaning by an ego which is characterized as free and spontaneous. Kostenbaum considers the voluntaristic connotations of such terms as intentionality and constitution to be unfortunate and misleading.

A hint of the interdependency between activity and passivity as well as a reminding statement of the main role of passive synthesis in its contribution to the experience of objects in the world is contained in the following quote:

Thanks to the aforesaid passive synthesis (into which the performance of active synthesis also enter), the Ego always has an environment of "objects".²⁵

Prior to this Husserl presented a more complete account of active genesis in which the Ego was said to function as productively constitutive by means of subjective processes. He continues:

Here belong all the works of practical reason, in a maximally broad sense... The characteristic feature is that Ego-acts - become combined in a manifold, specifically active synthesis and, on the basis of objects already given, constitute new objects originally. These then present themselves for consciousness as products. Thus, in collecting, the collection is constituted; in counting, the number, in dividing, the part; in predicating, the predicate ... and so forth.²⁶

One sees that opposed to passive synthesis of objects, the active synthesis realizes, as products of the Ego, more abstract, conceptual notions such as collections, numbers. The members of such products may already have been given in a passive sense to the Ego and as such, any clear demarcation of the two areas is at least questionable if not impossible.

How much of the experience depends on the active synthesis and how much is already constituted by the passive synthesis is extremely difficult to gauge. It is not a deliberate, conscious effort by which attributes or significance is given to some particular object. The case in which the Ego spontaneously identifies a single object as the focus of its attention can only occur when a receptive apprehension and explication have gone before. That is to say, there must be some material already there from which this particular object is specifically selected from. The active turning of the ego presupposes something already given to which the ego turns.²⁷

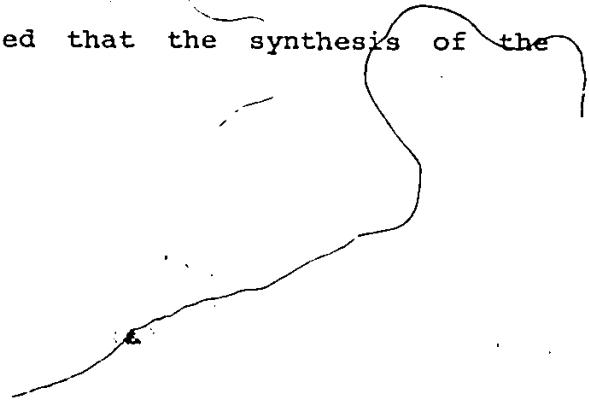
This haziness over whether the active synthesis of the ego brings the object to be or whether there is first of all a passive synthesis, which allows the object to spring forth, indicates the confusion of which underlies the other. Husserl further contends that this field of passive givenness is already a field which is constituted by unifying synthesis.²⁸ Any unifying synthesis would suggest an activity on the part of the ego. It is however more to the point that we recognize that there are two dimensions active and passive involved with synthesis and that there is some form of interrelation between the two rather than attempting to determine which is the more basic.

4.5.2 iii) UNITY

The topic of unity is here presented as subject, although in much of what has gone before, unity was seen to be the result of whatever was the particular subject of investigation of that section. In what may seem as a sort of description of the other side of the same coin, unity is introduced as a subject of investigation to highlight the importance of two unities gained through synthesis; the unity of the object and the unity of the ego perceiving the object. Not only are the manifold, variety of appearances combined to make one object but we must equally emphasize that it is one act that thus combines the appearances and an act which belongs to a single unitary ego to which all such perceptions of the individual may be said to belong. One hears distinct echoes of Kant's unity of apperception here but before the unity of the ego let us examine the unity of the object.

A. UNITY IN TIME

The temporal unity of the object has already been discussed so a brief summation will be all that is required here to emphasize this unitary aspect of the perception of an object. It should be noted that the synthesis of the



temporal phases of perception is described as passive synthesis. It is the uniting of the successive apprehensions that allow the person to realize the perception of a single object throughout the successive steps of perception. The ability to recognize one and the same object on different occasions is accordingly called the synthesis of identification. The foundational level at which Husserl places this notion is evident in the following.

If we consider the fundamental form of synthesis, namely identification, we encounter it first of all as an all-ruling, passively flowing synthesis, in the form of the continuous consciousness of internal time.²⁹

B. UNITY IN SPACE

We shall now direct our attention to the spatial unity of the object, its features and characteristics which will, in effect, reflect, the most important aspect of unity dealt with here, that is the unity of the perceiving ego. Bearing this in mind, let us consider two examples of perception in which the object is realized as a single, unitary entity.

In a quotation given earlier, which served to introduce the notion of synthesis in general as well as the importance of its temporal aspects, Husserl did suggest that he would take the perception of a particular die as the theme of his description. At that point, it was noted that the die was given as an objective unity despite the "changeable multiplicity of manners of appearing".³⁰ Whether the die appeared near or far, from one side or the other or whether attention was directed to one of its particular features, such as its shape or colour, all of these appearances were seen to belong to a passing flow of features of the object in question. All of these appearances belonged to a synthetic unity by which the object was recognized as the same enduring object throughout these various appearances at different times.

The difference between the perception of the die as a multiplicity of changing shapes and impressions and the person's conception of it as one, unchanging, solid, cubic object is dealt with by Kostenbaum.³¹ The discrepancy between the two is indicated by reference to the angles of the die which are all at right angles to each other. Most of the time, physical location does not permit the angles to be sensorily perceived as right angles.

Husserl also describes the maintenance of a single unity of the identity of an object throughout various apprehensions. He considers the appearance of an object, indistinctly in the field of vision, and concludes that although the position of the eyes may change and various prominences stand out from the object it is still the case that the ego does not yield to these tendencies of apprehension but "remains solely oriented on the unity of identity of this continuous synthesis".³²

Our final illustration of the unity of the object involves a commonplace item, a table. Husserl says that as one walks around the table while keeping it in view, the perception of the table changes continuously. Husserl expresses the situation thus.

Under no circumstance does it (perception of the table) return to me individually the same. Only the table is the same, known as identical through the synthetic consciousness which connects the new perception with the recollection.³³

Just as in the case of perceiving the die, the table, which actually was perceived as a variety of changing shapes and shades depending upon perspective position, degree of illumination and so on, was yet grasped by the mind, due to the synthesis performed, as a single, objective unity. It is very difficult to single out one particular area for discussion in this material. The preceding material, aiming

at a treatment of unity touched upon such related areas as passive temporal synthesis, continuous synthesis and synthesis of identification. The closeness of all these topics makes their exclusion from each other next to impossible. There is an area, even more strongly related to the unity of the object and that is what we shall turn to now, the unity of the ego.

C. UNITY OF EGO

Perhaps more important for a phenomenological approach to understanding man, or the unitary consciousness of man, is what Husserl presents as the unified ego, a single entity to which all the previously described experiences belong. It is not only necessary to present the combining of various features of an object into the experience of a single object but one must also present the unity of this act. Furthermore, it must be made clear that this act is experienced by a single ego; a consciousness to which these experiences belong and which is related, through these experiences, to the objects in its world.

Hopefully the example of temporal synthesis has not yet been used to the point of overkill, but its place of importance in the work of Husserl merits its use once again to demonstrate its key role in arriving at the unity of the

ego. Husserl states that the connectedness of appearances is such that it makes the unity of one consciousness.³⁴ The case is also made for various modes of consciousness i.e. perception, recollection, expectation, valuation and so on again being united by a synthesis, "that as a unitary consciousness" makes possible any knowledge of identity.

The case of the perception of a table as a single object is also seen to reflect the unity of the ego or consciousness that experiences it.

...an empirical consciousness of a self-same thing that looks "all-round" its object, and in so doing is continually confirming the unity of its own nature.³⁵

The perception of an object, as a single experience is then seen to belong to a single, unitary ego which makes up the core or centre to which all these experiences are directed. The memory of objects or experiences from the past brings one to realize that this past is the past of this very same subject, that it is his own past that is remembered.³⁶

I would like, at the close of this section, to calm any objections to the interchanging of the terms ego and consciousness. Most people are familiar with the more common

usage of the term consciousness as the all-emcompassing, ubiquitous label used to designate the totality of man's mental activities. The term ego, on the other hand, is more likely to bring to mind the psychological "I", the active agent directing experience, bearing personal identity and so on.

It is maintained here, that for this discussion, the two notions be thought of as essentially inseparable. Any mention of an ego necessarily suggests that it stands for some consciousness, whereas mention of a consciousness implies that there is an ego, albeit unspecified, which may be thought to represent the person of such consciousness. Although distinguishing one from the other may be beneficial for other purposes it is felt that in this case it would be a superficial, artificial device which would contribute nothing to the theme of unity being pursued here.

4.5.2 iv) HORIZONS

The last feature, that figures prominently in any questions of intentionality and synthesis, to be treated here is the issue of horizons. Husserl contends that each experience, in its intentionality, goes beyond the mere givenness, to anticipatory, as yet unfulfilled expectations.

Every experience can be extended in a continuous chain of explicative individual experiences, united synthetically as a single experience, open without limit, of the same.³⁷

In the experience of some object there are its internal horizons i.e. the features that are not at that moment being perceived such as the underneath side of it, the back of it and such. Husserl expresses it as "every actuality involves its potentialities" (Cart. Med., p. 44). Whatever is perceived, the mind does not rest content with such a state but, as it were moves on to consider what if, or begins to imagine, as if such and such. Each conscious process has the ability to envision the object of perception in relation to either its past or future. Each process thus has retention or protention.³⁸ Every perception has its horizon of before and after.

Thus, as consciousness of something, every consciousness has the essential property, not just of being somehow able to change into continually new modes of consciousness of the same object (which throughout the unity of synthesis, is inherent in them as an identical objective sense), but of being able to do so according to - indeed, only according to those horizon intentionalities.³⁹

Besides the horizons belonging to the specific object which have not been realized by the person as modes of consciousness of experiencing the particular object, indeed there are an infinite variety of possible experiences of the

object, there is a further dimension of the topic of horizons that we have not yet mentioned. As well as the internal horizons of the object, there is also an external horizon that is given with every experience.⁴⁰ By this is meant the surrounding objects that are present in the field of perception along with the particular object that is the focus of attention. A person's attention may turn to any of these objects in the background, for a variety of reasons, and thus shift the object from the external horizon to the centre of attention in which the object's internal horizons may be uncovered in further detail. The attention, may however be fleeting, and quickly move on to further objects in what is an open infinite possibility of experiences for the individual.

More pertinent to the direction of this paper are the internal horizons by which an object is disclosed in a richer, fuller more explicated level for the person. This does involve the question of how objects come to be for the person; the question of constitution which Husserl admits is the most difficult problem facing consciousness. As mentioned earlier, this idea must be distinguished from creation. In relation to uncovering new horizons of some object there are two aspects of constitution that should be noted. First of all, the synthesis of features of an object that result in the possibility of it being known by a consciousness is not a mere random or accidental process. It is a rule governed process in which an "essentially necessary conformity to type

prevails and can be apprehended in strict concepts". The second point concerns the possibility that if consciousness moves continually from one horizon of an object to another of its features; or even outwards from the object to other ones in the external horizon; or if in the temporal flow of every "now" flowing into the past, while a new moment is synthesized to permit the enduring experience of the same object, then there must surely be a general ongoing state of chaos and scatteredness characteristic of conscious processes. The answer to this is definitely not. The processes of synthesis and constitution result in a world of objects and experiences that have relevance and meaning for the persons that undergo them. The next section deals extensively with this question of meaning and the role of synthesis in it.

4.5.3 SYNTHESIS AND MEANING

In the most general, broadest sense of meaning, it is the individual who posits meaning upon the world. In the earlier discussions of intentionality, as well as in many instances where Husserl considers any type of transcendental, phenomenological beginnings for his study of man there is

generally ample reference to this placing or determining of meaning upon the world by the individual. In his efforts to find an indubitable starting point for his philosophy (in Cartesian manner), Husserl determined that the justification for what he accepted as "being", had to be found within his own intentionality and more to the point here, any other meaning of being was also to be so determined.⁴¹ It was through such positions, in which the world received its whole sense, meaning and existential status, that Husserl concluded the priority of the ego with its cogitationes (cognitive mental act/processes) over the natural being of the world.

To begin to concentrate our study from the meaning of the world in general to that of particular objects the point at which we hope to arrive, we shall, as intermediary, consider the tripartite formula Husserl gave to describe the conscious processes. The expansion of Descartes' "cogito ergo sum" (I think therefore I am) into Husserl's description of experience "ego cogito cogitatum" (I think an object) was thought to more accurately describe experience and more closely reflect language as well as noting the intentional facet of experience.⁴² In every mental act there was the object to which the meaning was directed:

Each cogito, each conscious process, we may also say, "means" something or other and bears in itself, in this manner peculiar to the meant, its particular cogitatum.⁴³

We thus see this meaning component of mental acts was to be included in all acts. It is contained in "each conscious process" and is not restricted to just one particular act of meaning involving a single particular object. Yet it is of interest to us here to investigate how such specific objects come to bear the meaning that they do acquire. One prominent notion in such occurrences is that of fulfillment. Its specific nature and how synthesis contributes to it will be dealt with in the next section.

4.5.3 i) Fulfillment

A much deeper level of clarity, about how the meanings of things are arrived at, is gained through discovering what takes place during the fulfillment of experiences of objects. The evidence we have for the existence of objects in our world, how they appear to us, may be graded according to a greater or lesser degree of perfection. Husserl says that as a rule, imperfection signifies incompleteness and that perfecting them requires a synthetic course of harmonious experiences in which the meanings become fulfilled in actual experience.⁴⁴ This completion results in more perfect evidence which yields a fuller, more complete appearance or perception of the object. Referring to the evidence of particular objects with their unfulfilled horizons, Husserl

writes that although through synthetic transitions from evidence to evidence, the imperfect may become more perfect, they will never become completely adequate.⁴⁵ The appearing of an object may at first be unclear and indistinct, as not yet understood, and through successive synthesis it may be brought to a more distinct and understandable position. However, no object will ever be presented to anyone as totally and completely knowable for what it is; there necessarily are always unfulfilled, expectant meanings in any such synthesis.⁴⁶

Husserl elaborates further on this idea of how the sense or meaning of an object of cognition is achieved through a step by step progression in a developing cognitive context if, he cautions, it has the proper forms belonging to the object of experience.⁴⁷ It is in this sense that the object becomes progressively constituted. The expression constitution, thought of as the piece by piece putting together of an object, matches quite well here with the image of fulfillment presented as the idea of an object becoming more knowable, definable through stages of fulfillment in which what was at first a vague and almost indiscernible perception is then realized in a much more clear and distinct manner.

4.5.3 ii) Overlapping

This particular expression "overlapping" is, to my mind, a most suitable description of the synthetic process that I wish to describe in relation to, the building up of meaning. It represents quite adequately, as if borrowed from a Venn diagram, the image of one or more attributes, properties of a thing flowing over into each other and merging as unified and combined in the eventual experience of the object. Such activities are presented by Husserl in his discussion of explicative synthesis in which the perception of an object and its determination are dealt with.

Explicative synthesis comes into play due to the interest of the ego refusing to be content with simply looking at and apprehending an object. It is, in a sense, the pursuit of what was previously described as the internal horizons of the object. Imagine one has an object S, and its internal determinations a, b; the process of explication maintains or holds-in-grasp the interest in S as we come to know it more fully through the determinations a, ~~248~~

Husserl notes that such overlapping is common to all transitions from a substrate S, to its determination a.

For such an overlapping of all apprehensions is common to all cases of explication and all cases in which the ego advances from apprehension to apprehension in a synthetic activity unified by the bond of a single interest.⁴⁹

There can also be the case in which the substrate S is further enriched by p, q, r. We thus move from the original first form of predication S is p, on to further explications q, r. Each of these is apprehended not only for itself but attached to what precedes it and "as belonging to S, they enter passively with one another into a synthetic overlapping". Husserl states that the coincidence of the members, based on their coincidence with S, may be expressed by "and", yielding S is p and q and r.⁵⁰ This use of "and" is labelled collective linkage, and brings to mind the collective combination through the use of "and" in the earlier sections treating arithmetic.

It is possible for these determinations to become substrate themselves rather than being collectively linked as above. If the transition from S to a is further enriched by b, it may be expressed as (Sa) b, in which a is retained not for itself but as something of S, as belonging to it.⁵¹ Husserl states that the synthesis of transition is no longer accomplished when the predicative adjective of the subordinate clause (S is p, read as S which is p), becomes an

attributive adjective Sp. Then in such cases where there is transition to q, Sp is q (eg. The cold air is refreshing) there is only synthetic transition to q whereas p is taken along with S, as the character of the result.⁵²

I do not feel that shifting from "The air is cold" to "The cold air" in the expression "The cold air is refreshing" in any way alters the synthesis which is performed when the property cold is combined or joined to "air", the ordering or re-arrangement does not alter the underlying process of synthesis.

In the explicative synthesis, dealt with at the beginning of this section, in which an object S, say a house, was progressively enriched by attention being directed to its various determinations a, b, such as its roof, its colour, a particular spot or area of its surface, it may be easy to confuse this with the experience of a plurality.

Husserl does require a "rigorous differentiation" between explication and the apprehension of a plurality such as a cluster of stars or a cluster of dots. He concedes that a plurality may be conceived unitarily but insists that in general the individual members are joined thematically and linked together as interest follows the likeness of each of the members.⁵³ The difference is that in explication the substrate or general theme S, is grasped in advance and is

held onto as that to which the explicates belong. The plurality on the other hand, has its source not in activity itself, but in connection arising from passivity.⁵⁴ If the members of a plurality are actively taken up, then this is quite another matter and results in the plurality being constituted as a set.⁵⁵

To conclude this section it will be sufficient to outline the steps that have been taken to show the importance of synthesis in intentionality and how synthesis predominates in the ascription of meaning to our world. First of all, an overview of Husserl's intentionality was given in which the mind's contribution to the experiencing of the world was indicated, along with the role of synthesis. Next, a more detailed examination of synthesis was given with emphasis on its fundamental types and its contribution to our experiencing of one world by one individual ego. Finally, the relation of synthesis to meaning concluded the section. Here the operation of synthesis, in the combining of different features or properties of things, was presented.

It has been demonstrated in this paper that the synthesis of qualities in objects and the synthesis of units in numbers is, at the core of the matter, essentially the same. Just as in the case of the number four, there is the infinite variety of members that could be represented by four, in

similar fashion there exists the infinite variety of constituting members that could go into making up the experience of a die. It is essential to realize that the process remains the same, irrespective of the particular nature of the contents. In each case the individual members are joined and a unity is produced. The perception of the single entity has within it, the implicit awareness of all that has gone into making up the resulting experience of one single object and not a disconnected chaotic series of perceptions. One may add that a synthetic activity is required in order to perceive the unity of this essay itself so that the various sections which may at first seem disparate, in effect coalesce into a unified whole. It is by doing synthesis that we may hope to ever fully understand synthesis.

CHAPTER FIVE

Conclusion

In this conclusion we shall briefly summarize the main points arrived at in each of the sections dealing with Kant, Frege, and Husserl. Such a manner shall provide a means of seeing how synthesis operates in an almost identical manner whether it is involved with combining units to form numbers or joining meanings to objects so that our world becomes relevant and meaningful for us. Finally, some recurring themes will be noted: themes which serve to justify the choice of the three philosophers selected for the purposes of this paper.

5.1 Recapitulation

5.1.1 Kant presented us with the basis for our investigation into synthesis. He gave us the notion of synthesis in its most elementary guise, that of combining. Throughout the paper this essential core of the concept was held onto with the elaboration and complexity arising from

the different elements that went into the combinations and the nature of the results produced through the synthesis.

Kant held that a number, say that which described the length of a room, depended upon our choosing a particular unit and determining how many applications of it were necessary to cover the length of the room. It was upon each successive placing of the unit, that a synthesis was required on the part of the person so that by the final application one was also aware of the accumulation of units represented by the final number.

5.1.2 The section on Frege contains the pivotal and perhaps most important, transition for the more general aim of the paper, i.e. to demonstrate the parallel nature of the operation of synthesis as it applies to our understanding of numbers and to our comprehension of the everyday world around us.

Admittedly, the symbolic, logical definitions of Frege are not easily grasped, especially on first reading, and do appear to be in sharp contrast to the rest of the paper. They had to be dealt with, so that the reader could meet the

very same notion of synthesis dressed in logical garb. The success or failure of Frege's account of number (cf. Russell's paradox) is not at the centre of the question here. Rather the aim was to tease out the strands of synthesis in such notions as "belonging to a series", which were part of the logical framework of Frege.

As mentioned, the crucial turn in the paper was contained in the shift from logical functions of a mathematical nature to propositional functions. The belief that this adequately expressed the aim of this paper was based upon the following argument: synthesis was seen to be operative in the logicist definition of number given by Frege; plus the structure of algebraic-mathematical functions could be directly transferred to sentences of ordinary words and therefore the features of the functions such as concepts, arguments as well as the presence of synthesis, would likewise be transferred and accordingly would play a part in everyday language.

5.1.3 Husserl's view of number which was given here was to a large extent revised in later writings after Philosophie der Arithmetik. He did maintain a key element of this early

exposition, collective combination, as the characteristic defining element of groups. It was this idea of collective combination which was believed to best express the presence of synthesis. Husserl indicated that this relation could not be found in the objects but had to be contributed by the individual and only then could the group be said to come into being. Husserl considered numbers to be determinable portions of an undefined multiplicity and defined them by one and one and one, say for number three. He further believed we could not envisage or have any kind of clear representation to ourselves of any number larger than twelve. Again, this is not a paper in philosophy of mathematics and to attack any material contained herein with questions of how it could explain ten thousand, or infinite numbers is off the mark.

The culmination of Husserl's section where attention is shifted to his theory of intentionality is really the heart of the matter. It is through intentionality that our world comes to have meaning for us, and it is the thesis here that it is through synthesis that meanings are attached to objects

persons, phenomena in our world. It is the same combining of many into one that occurs whether it is units that go into making a particular number or whether it is perspectives, experiences and whatever else that eventuate in particular meanings for us. The other dimension, awareness of the many in the perception of one item, is also contained in each synthetic act. Although we are aware of a single unitary experience, whether the number twelve or the cup on the table, we are at the same time cognizant of all that which is contained in the original awareness. Of course, it is extremely difficult to express in what manner one has this awareness; as a sort of catch-all one may say the subconscious but this is scarcely an acceptable explanation. Unfortunately, I am not able to better express the notion at the present time.

5.2 **Themes** There are several themes which run through this paper providing unity to the overall project. The most obvious being that there is *only* one series of natural numbers. Regardless of how each of these men approached the issue and despite the divergence in methods employed to explain numbers, they were all dealing with the same series of natural numbers. When either Kant, Frege or Husserl, on a

warm sunny day, had a glass of water and still thirsty proceeded to have another, they all would have said they had two glasses of water.

Another important theme which resurfaces in various contexts is the idea that since anything at all may be counted then there can be no restrictions placed upon that which makes up the members of any particular number. It receives its weakest formulation in Kant where he merely refers to the homogenous units, which in the examples of measurement may be specified but if not, then the person is at liberty to choose a suitable unit. With Frege, the lack of restriction is demanded by the nature of his logical approach. As soon as he employs the symbol x to express a variable term in a theorem or definition then it is the nature of the beast, so to speak, that anything may be substituted for it. Husserl likewise felt that anything at all could be utilized as the ingredients before an abstraction was performed to reach the basic indeterminable unit "Etwas", one or something. Husserl opposed any view that limited the domain of numbers such as Mills's empiricist view that restricted numbers to physical entities.

Perhaps the most important thread which holds the material together is the idea of synthesis expressed by means of the image of combining. Whether it is Kant's suggestion that number is a representation which comprises the successive addition of homogenous units (CPR 183) or simply Frege's $n+1$ or expressions like "n immediately follows m" in a series (GRL sec. 76) or more obviously in Husserl's explanation of three as "one and one and one," (PA 85) these all point inescapably to the general theme of joining or combining which is so prevalent and so important throughout the entire paper.

I would like to finish this conclusion with a brief look at two themes which, rather than remaining constant for the entire paper, oscillate in pendulum fashion returning to a similar position at the end of the paper as that from which they began albeit, an improved or more inclusive position at the end. The first such example I would like to offer is the use of sentence as a vehicle for presenting synthesis. Indeed, Kant's first presentation of the notion was in his contrast between analytic and synthetic statements or

sentences. Frege's section seemed a reversal from the treatment of everyday sentences as he attempted to show how arithmetic (therefore numbers) could be inferred solely from logic. Finally, in Husserl's section the importance of treating sentences as expressions of our experience of the world around us reappears full strength to the centre of the explanation of synthesis.

The last issue involves people, appropriately enough, as the paper is an attempt to better understand how people give the meanings that they do to that which makes up their lives. The paper essentially attempted to unravel the role of synthesis and the part it plays in our understanding of all that we experience from numbers to everyday articles around us.

Over the entire paper I would suggest that the role of the individual may be thought of in this perhaps over simplified manner. Kant revitalizes the part played by the person in ordering and structuring his experience, he in effect, takes us into the person. Contrary to this, Frege takes us out of the person into the realm of logic, independent of any subjectivity. Husserl then takes us back into the person reaffirming the importance of the individual

and the contribution made by consciousness towards organizing and giving meanings to our lives.

This shifting of emphasis in and out of the person, may be thought of as analogous to breathing, if you would permit. It is in this manner that such issues as those contained in this paper may be said to breath life into the controversies surrounding not only synthesis but the very life of man himself.

CHAPTER TWO

FOOTNOTES

- 1) I. Kant, Critique of Pure Reason, trans. by N.K. Smith (New York: St Martins's Press, 1965), B 8, B 740.
- 2) I. Kant, CPR, B 8, B 34.
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CHAPTER THREE

FOOTNOTES

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