

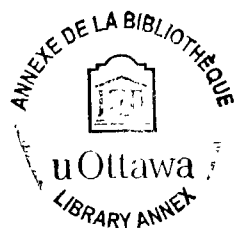
RELIABILITY OF A FRAME SYNCHRONIZATION TECHNIQUE

by

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A thesis submitted to  
the School of Graduate Studies  
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in partial fulfillment of the requirements  
for the degree of Master of Applied Science

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ABSTRACT

The problem of frame synchronization of a time-division multiplexer (TDM) which employs one synchronizing bit per frame, is dealt with. After the introduction of the different levels of synchronization, the upper bound on the number of frames  $N$  which have to be examined, before a decision with a given reliability factor  $M$  is reached, is worked out. The reliability factor  $M$ , is given by the ratio of the probability that the decision reached is correct to the probability that the decision is incorrect.

In practice systems, the reliability factor is never given. This is probably due to the emphasis being put on reframe time, rather than the degree of confidence in the decisions reached with regard to the synchronization strategy.

The analysis carried out in this thesis is good for any system in which information bits are partitioned into equal blocks with one bit per block reserved for frame synchronization. In the case of a burst error environment, a log-normal burst length distribution is assumed.

## TABLE OF CONTENTS

	PAGE
ABSTRACT	i
ACKNOWLEDGEMENTS	ii
<u>CHAPTER 1</u> INTRODUCTION	1
1-1    Synchronous Digital Communication Systems	2
1-2    The Different Levels of Synchronization	5
<u>CHAPTER 2</u> FRAME SYNCHRONIZATION IN AN ERROR-FREE ENVIRONMENT	22
2-1    Introduction	23
2-2    Strategy for Synchronization	28
2-3    Number of Frames to be Examined	30
2-4    Probability of False Lock	34
2-5    Probability of Missing the Synchronizing Bit	35
2-6    Mean Time before Acquisition	37
<u>CHAPTER 3</u> FRAME SYNCHRONIZATION IN A NOISY ENVIRONMENT	39
3-1    Introduction	40
3-2    Number of Frames to be Examined	43
3-3    Random Errors with Bit Error Probability $P_e$	48
3-4    Burst Error Situation	52
3-4.1    Introduction	52
3-4.2    Number of Frames to be Examined	54
3-4.3    Log-Normal Burst Length Distribution	61
3-4.4    Synchronization in a Log-Normal Burst Error Distribution	73

Table of Contents cont'd . . . .		PAGE
3-4.5	Probability of False Lock	85
3-4.6	Probability of Missing the Synchronizing Bit	86
<u>CHAPTER 4</u>	CONCLUDING REMARKS	87
	REFERENCES	89

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CHAPTER 1

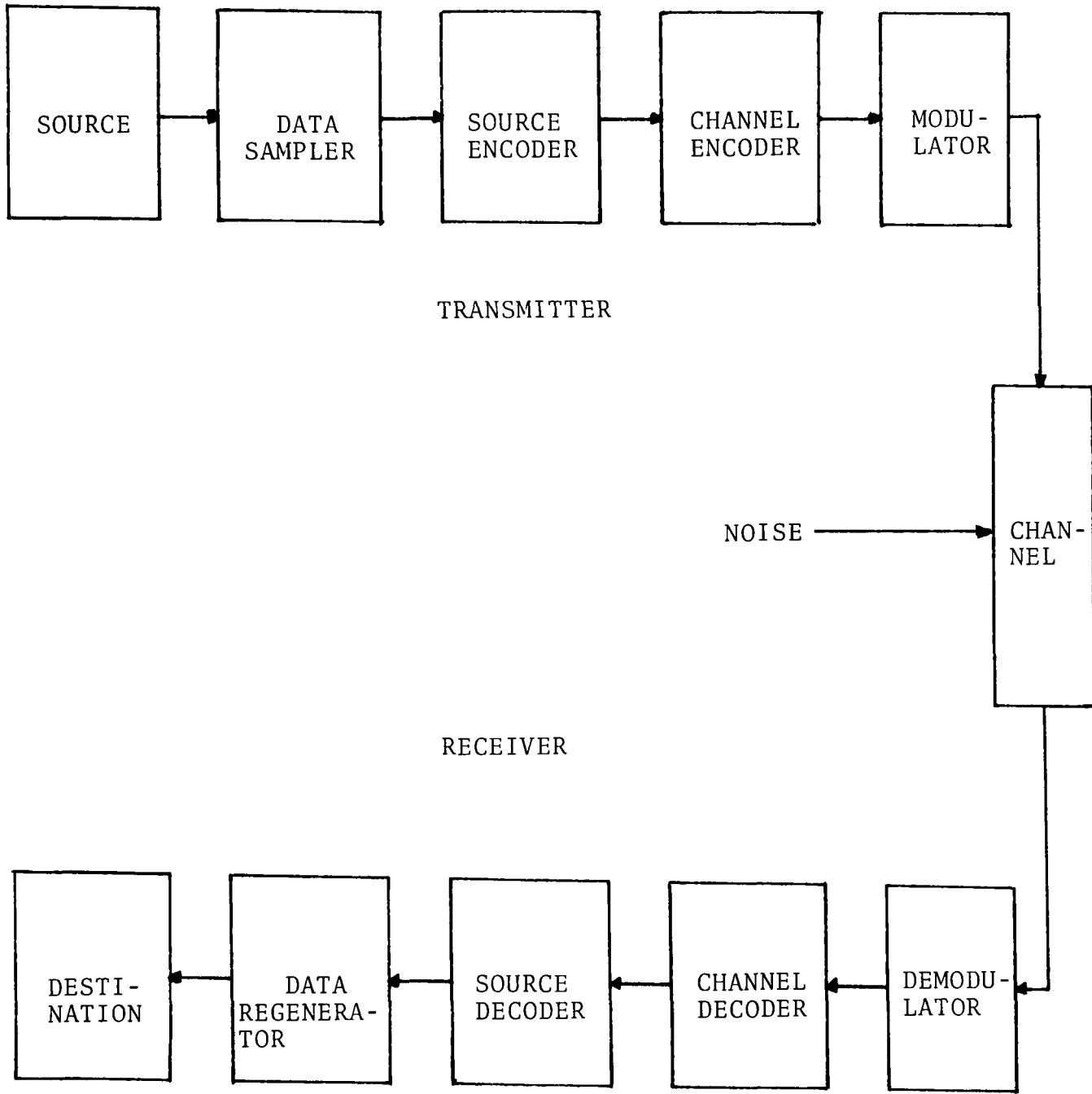
INTRODUCTION

The science and technology by which information is collected from a source, converted into electric signals, transmitted over an electric network or space to another point, where it is reconverted into a form suitable for interpretation by the receiver is known as communication. The need for communication has become more and more important with the rapid advance in science. Not only is there communication between man and machine but there is machine to machine communication too.

There is a trend towards the transmission of information by digital techniques. In part this is due to the large amount of data, that exists in digital form; digital computer data, alphanumeric data such as stock market quotations and control signals associated with automated processes for example. One of the advantages of using digital rather than analog means of transmission is the ability to tolerate lower signal-to-noise ratios and yet come up with comparable performance. This can be achieved by the use of regenerative repeaters and error correcting codes.<sup>1,2</sup> A distinctive characteristic of digital transmission is that a greater bandwidth is required in order to send information at a given rate than it would require by analog means of transmission. If intersymbol interference is to be avoided for example, the bandwidth needed by a digitally encoded signal, has to be considerably higher than the baseband bandwidth of the original analog signal.

Two sequences of events are said to be synchronous if the corresponding events in the two sequences occur simultaneously. A communication system can be classified as being synchronous, if the existence of a time reference common to both the transmitter and the receiver, is a necessary condition for its satisfactory operation.<sup>3</sup> Synchronization is the process of bringing about and retaining a synchronous situation.

The main purpose of this introductory chapter is to give a general view of synchronization and some of the techniques used to achieve and maintain the different levels of synchronization. A generalized digital communication system model given in Figure 1.1 is used to study the different levels of synchronization. It should be noted however, that not all the blocks in Figure 1.1 are always essential for the system to be called digital. If a baseband signal is to be transmitted for example, then there is no need for the modulator or demodulator. On the other hand, if there is no provision for error detection and/or correction, then both the channel encoder and channel decoder are not needed.



## 1.2

THE DIFFERENT LEVELS OF SYNCHRONIZATION

We begin the discussion with what might be considered as the lowest level of synchronization, that is clock synchronization. It involves forcing the clock which regulates the sequence in the receiver to run at the same speed as the clock which regulates the sequence in the transmitter. The receiver clock could be derived from the incoming carrier. Such an arrangement is known as carrier synchronization.<sup>3</sup>

After obtaining clock synchronization, higher levels of synchronization have to be achieved. Some events taking place in each of the blocks of the receiver portion of Figure 1.1 must be synchronized with corresponding events taking place in the analogous block section in the transmitter portion. When dealing with synchronous detection, the phase of the locally generated carrier has got to be synchronized with that of the incoming carrier for proper demodulation to take place.

The importance of phase synchronization is illustrated with the help of Figure 1.2 which is a diagram of synchronous detection procedure of phase-shift-keyed signals, (PSK). In that figure,  $f(t)$  is the binary signal with voltage levels  $\pm V$ .  $\theta_1$  and  $\theta_2$  are phases of the received signal and the locally generated one respectively. As the phase difference between the locally generated carrier and the incoming signal approaches  $\pi/2$ , the demodulated output decreases. When the phase difference between the two is  $\pi/2$ , there is no output and as it goes beyond  $\pi/2$ , the output is inverted.

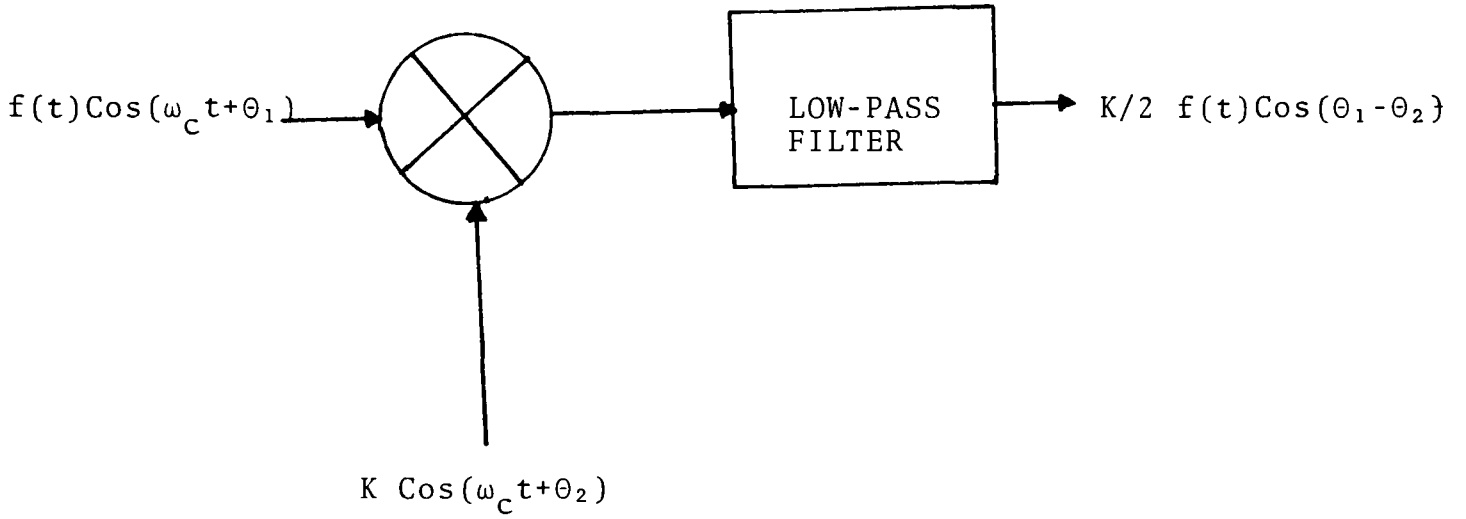


FIGURE 1.2 COHERENT DEMODULATION OF PSK

Phase synchronization can be achieved with the help of a phase locked loop (PLL) and a pilot tone sent by the transmitter. In Figure 1.3 a possible arrangement for the determination of the carrier phase is illustrated. It is assumed that the channel affects the pilot signal  $\sqrt{2}E_0 \sin \omega_c t$  in the same way as it affects the information signal  $\sqrt{2}f(t) \cos \omega_c t$  so as to produce the phase difference  $\theta_1$  by the time the two signals reach the receiver.

Since the pilot tone does not carry information, its transmission represents a waste of power. The use of a self-synchronizing scheme using a squarer and a phase locked loop (PLL) is more economical with respect to the power required for transmission. In this case, the use of a pilot tone is not required as can be seen in Figure 1.4. The incoming signal  $f(t) \cos \omega_c t$  suffers a phase shift of  $\theta_1(t)$  as it goes through the channel.  $\cos(\omega_c t + \theta_2(t))$  is used as the locally generated carrier in the coherent detection.

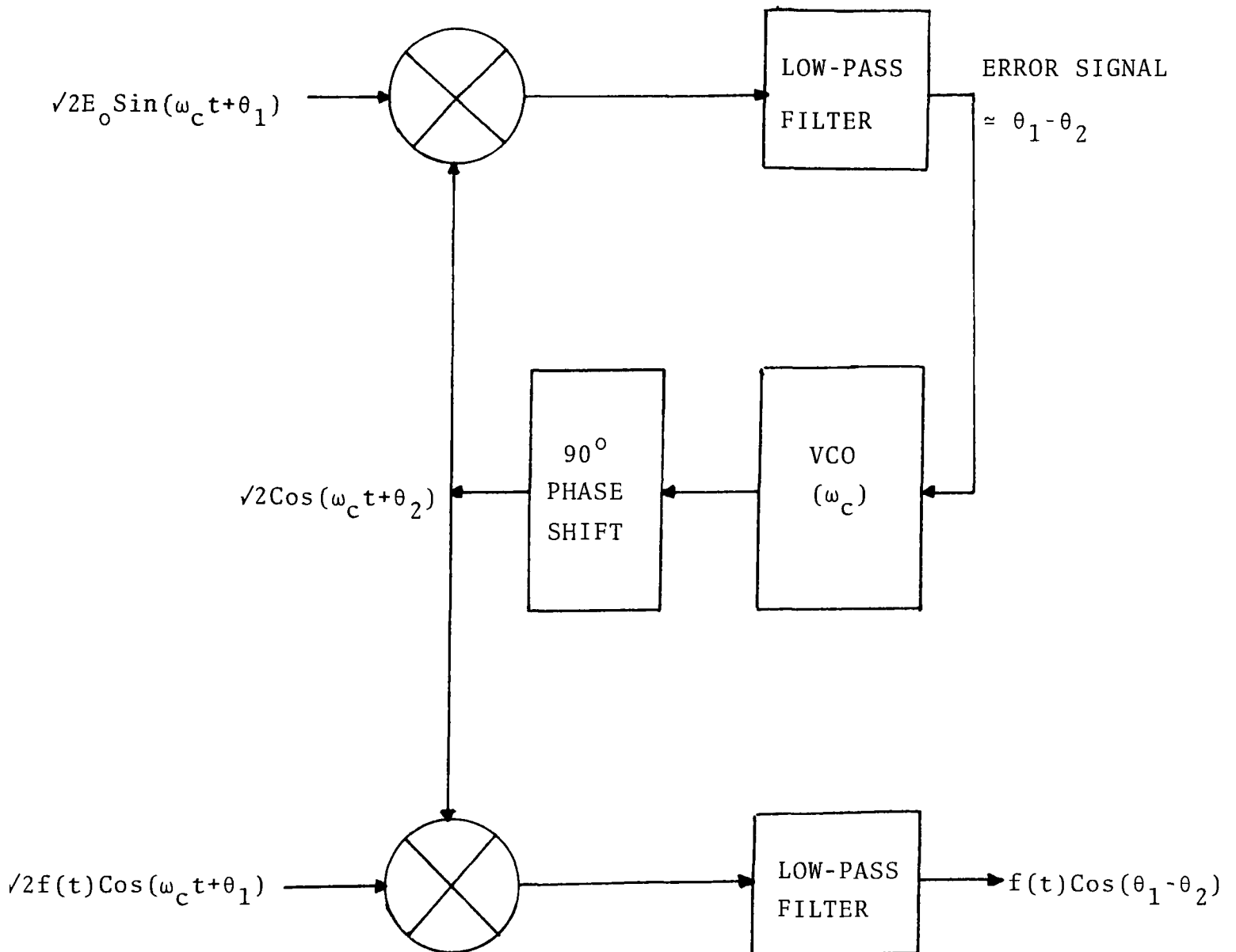


FIGURE 1.3 COHERENT DETECTION OF A PSK SIGNAL USING A PILOT TONE AND A PLL.

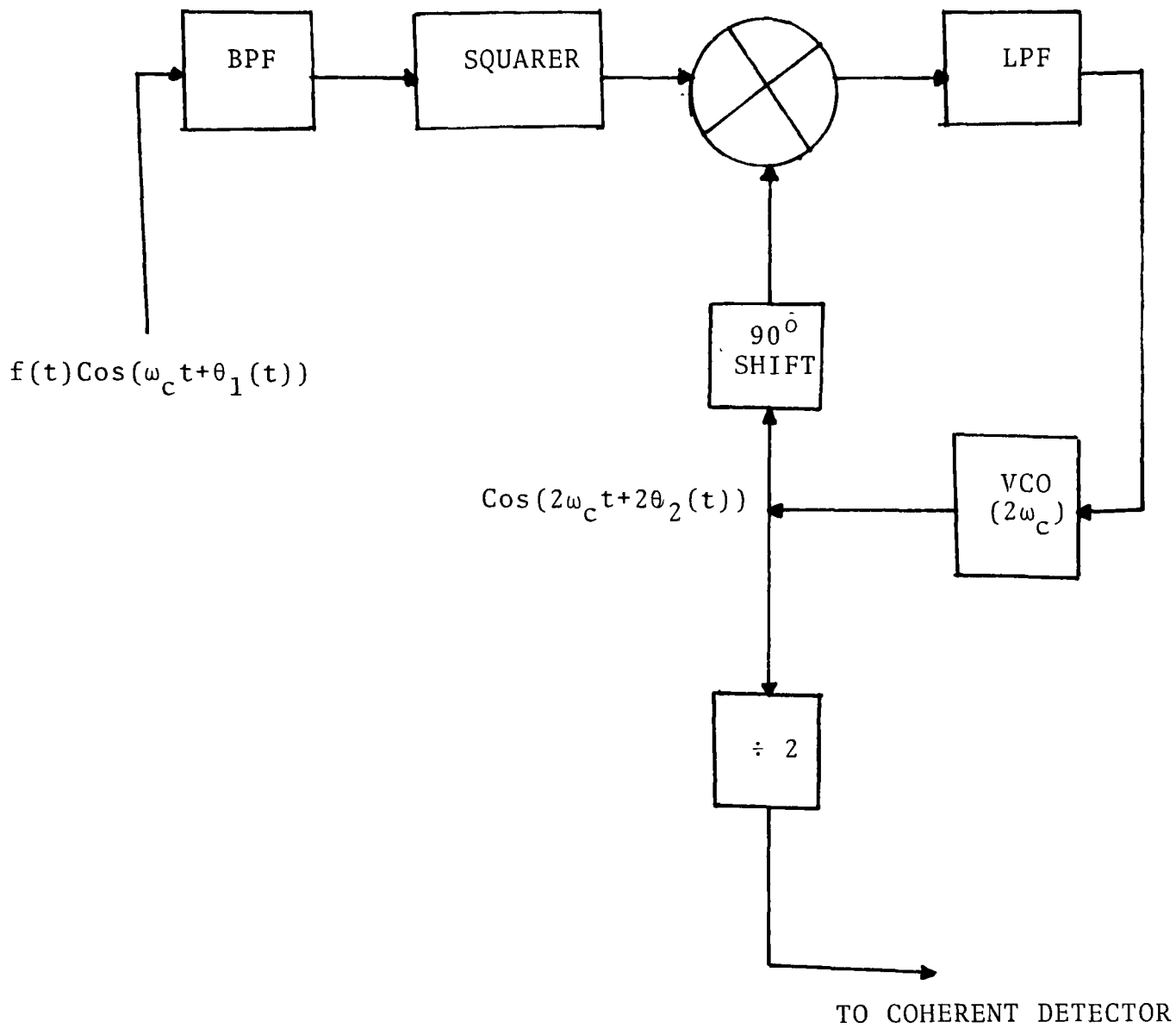


FIGURE 1.4 A SELF-SYNCHRONIZED PSK SYSTEM

In some communication systems, bit synchronization rather than phase synchronization might be required. The demodulator has to be synchronized with the modulator so as to know when the same form representing one bit ceases, and the next bit begins. One method used to keep track of bit synchronization, is through the use of a phase locked loop and a zero crossing detector to derive the bit clock from the incoming signal.

The phase locked loop (PLL) can be thought of as a narrowband filter with a centre frequency which tracks the carrier, or any other signal of sufficient energy found within the narrow tracking range of the PLL. Consequently, if the bit-clock frequency is present in the received signal spectrum with sufficient energy to be distinguishable from data or noise, it can be tracked and regenerated by the PLL; otherwise it is first necessary to use a non-linear device to restore the desired Spectral Component. A bit synchronizer is shown in Figure 1.5 where the zero crossing detector contains the fore mentioned non-linear circuit.

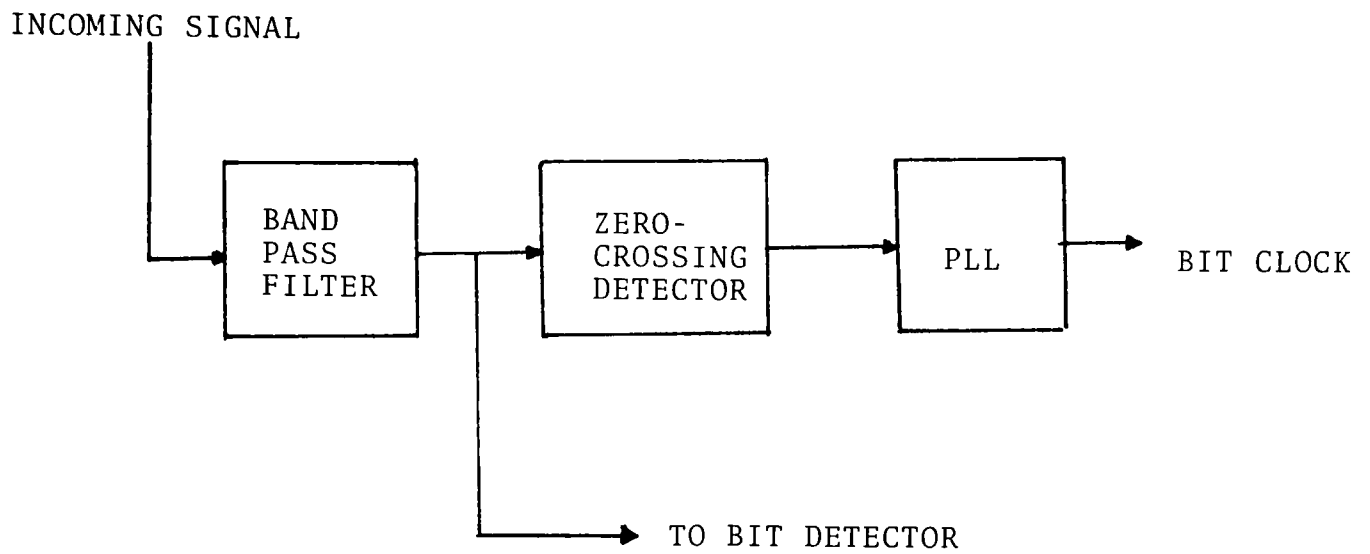


FIGURE 1.5 BIT SYNCHRONIZER

After obtaining phase and/or bit synchronization, the next level of synchronization, involves the channel encoder and decoder with reference to Figure 1.1. The channel encoder adds redundancy to digital sequences presented at its input. The purpose of this is to enable the receiver to correctly identify transmitted data even though some errors may occur during the transmission. The redundancy added to the data by the channel encoder is removed by the channel decoder. In case of block codes for example, the channel encoder adds  $(n-k)$  parity check bits to form an  $(n,k)$  code.<sup>1</sup> In order to decode a block code, the channel decoder must be able to distinguish between information and parity check bits or equivalently to identify the first digit of the received word. This is known as code word synchronization.

Code word synchronization is necessary for correct decoding of the information. Loss of word synchronization typically causes the received word, to be substantially different from any code word. In such situations the decoder can interpret as a synchronization loss a long sequence of received words which contain an unusually large number of errors. Once word synchronization has been lost, there are a few techniques that can be used to recover it, one method of recovering synchronization requires the use of a special synchronization character which is inserted between data words (for example, the letter space in Morse Code). Another approach to the problem is the use of a prefix consisting of a sequence of binary digits which have a sharp auto correlation function. At the receiver, the

incoming bit stream is correlated with a code stored in a register. If the system is in word synchronism, a large output will be obtained if the sequence has not been corrupted by noise.

Another method used in code word synchronization involves self-synchronizing codes.<sup>3</sup> One of the properties of these codes, is the yielding of different results, when some measurements are made on the received sequence of code words when in synchronism and when out of synchronism. A good example of self-synchronizing codes are the comma free codes.<sup>1</sup> With a comma free code, any overlap formed from one code word followed by another does not belong to the code. In absence of errors, the beginning of code words is apparent. A useful self-synchronizing dictionary might thus be obtained by beginning with a comma free dictionary and then selecting from it a sub-dictionary with desirable error correcting properties.

Frame synchronization represents the highest level of synchronization in a communication system, once it has been achieved, the lower levels can easily be maintained by the use of reliable counters. With reference to Figure 1.1, the sequence of events taking place in the Data Regenerator has to be in step with the events in the Data Sampler. Data might be originating from different sources and intended to be time-division multiplexed and each message sent to the proper destination. A conceptual time-division multiplexer (TDM) is given in Figure 1.6 to help in the illustration of the need for frame synchronization.

The rotating 'switches'  $S_T$  and  $S_R$  have to be in step for the system to operate properly. The sampling rate is determined by switch  $S_T$ . At the receiver the data from source  $i$  has to go to destination  $i$  so  $S_R$  has to be in step with  $S_T$ .

A very common arrangement is the allowance of one time slot per frame for the purpose of transmitting synchronizing information. If, say, there are  $B$  baseband channels to be multiplexed,  $B+1$  time slots are provided,  $B$  for the signal samples and one for synchronization. A code inserted in the synchronizing slot at the transmitter is detected at the receiver and helps the receiver to steer the signals to their respective destinies.

In the T-1 carrier system however, only one bit per frame rather than a whole channel is reserved for frame synchronization. In this particular case, the synchronizing bits form a pattern of alternating ONES and ZEROES.<sup>4,5</sup> At the receiver, the synchronizing circuits look for that pattern and lock on to it when found. As will be seen later, there is a possibility of the synchronizing pattern being simulated by an information channel or indeed the pattern could be corrupted by noise thus making acquisition of synchronism less certain.

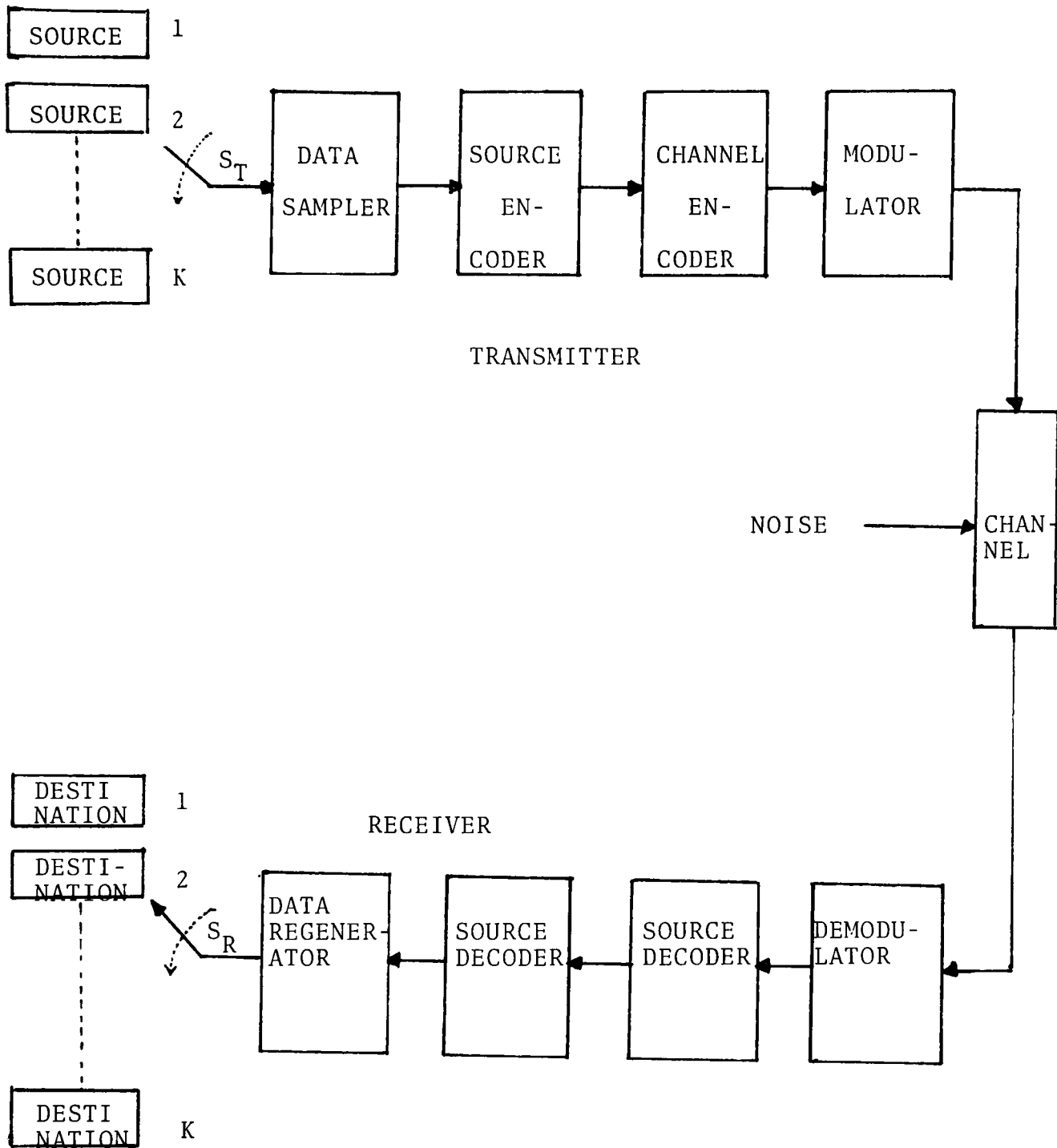


FIGURE 1.6

A CONCEPTUAL TDM SYSTEM

The different levels of synchronization mentioned so far are a must for proper point-to-point communication to take place. The simple scheme of slaving each receiving terminal to its respective transmitting terminal is not adequate for a network. If a signal is to be time-division multiplexed with other signals in order to share a common facility, the bit rates must be exactly synchronous. In a network of geographically separated signal sources, this exact synchronization presents a problem. This leads to the need for network synchronization.<sup>6,7,8,9,10,11</sup> There are a number of schemes used to achieve and maintain network synchronization. Some of those schemes are given here:

i) Master-slave clock system.

In the master-slave clock system one node acts as the master and the other nodes act as slaves. The clock signals used by the slave nodes are derived from the clock generated by the master. In Figure 1.7 a possible master-slave network arrangement is shown.

ii) Mutual Synchronization System.

In case of a mutually synchronized system, each node of the network derives its clock frequency by averaging the phases of all the incoming bit streams. Such arrangement unlike the master-slave one is bound to be more reliable since the network does not depend on only one node for its clocking. A possible configuration of a mutually synchronized system is shown in Figure 1.8.

### iii) Hierarchical Master-slave System

One of the disadvantages of a master-slave system is, should the master clock fail, the other nodes will not function properly since they derive their clock signals from the master. A hierarchical master-slave system is designed to take care of such a situation.

The hierarchical master-slave system has a number of masters arranged in a hierarchy. These masters take over the network in their order of hierarchy should the foremost in the hierarchy fail. In Figure 1.9 for example, should node 1 fail, then node 2 takes over the clocking of the network as shown in Figure 1.9a. If node 2 fails then node 3 takes over and so forth.

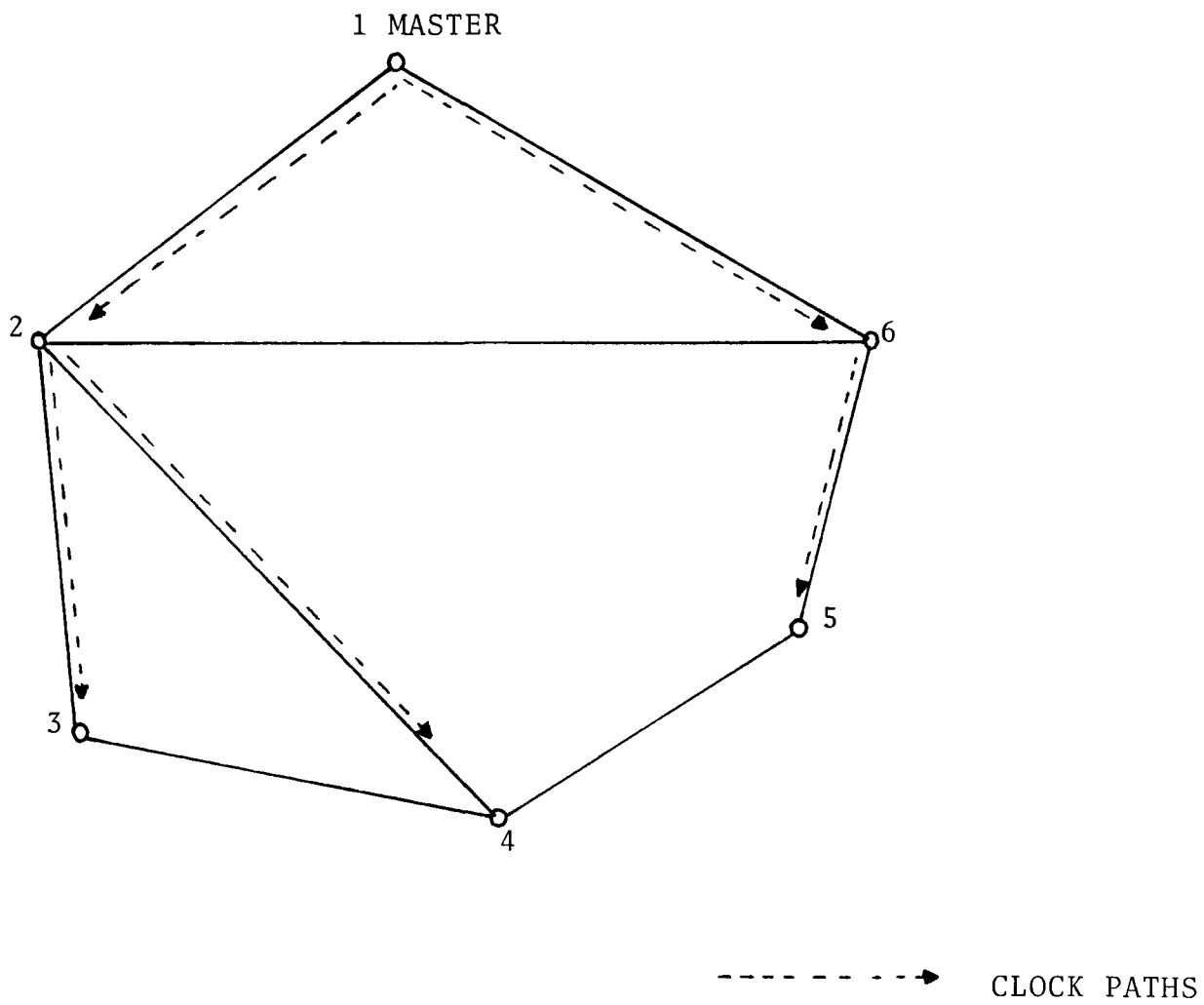


FIGURE 1.7 A MASTER-SLAVE NETWORK

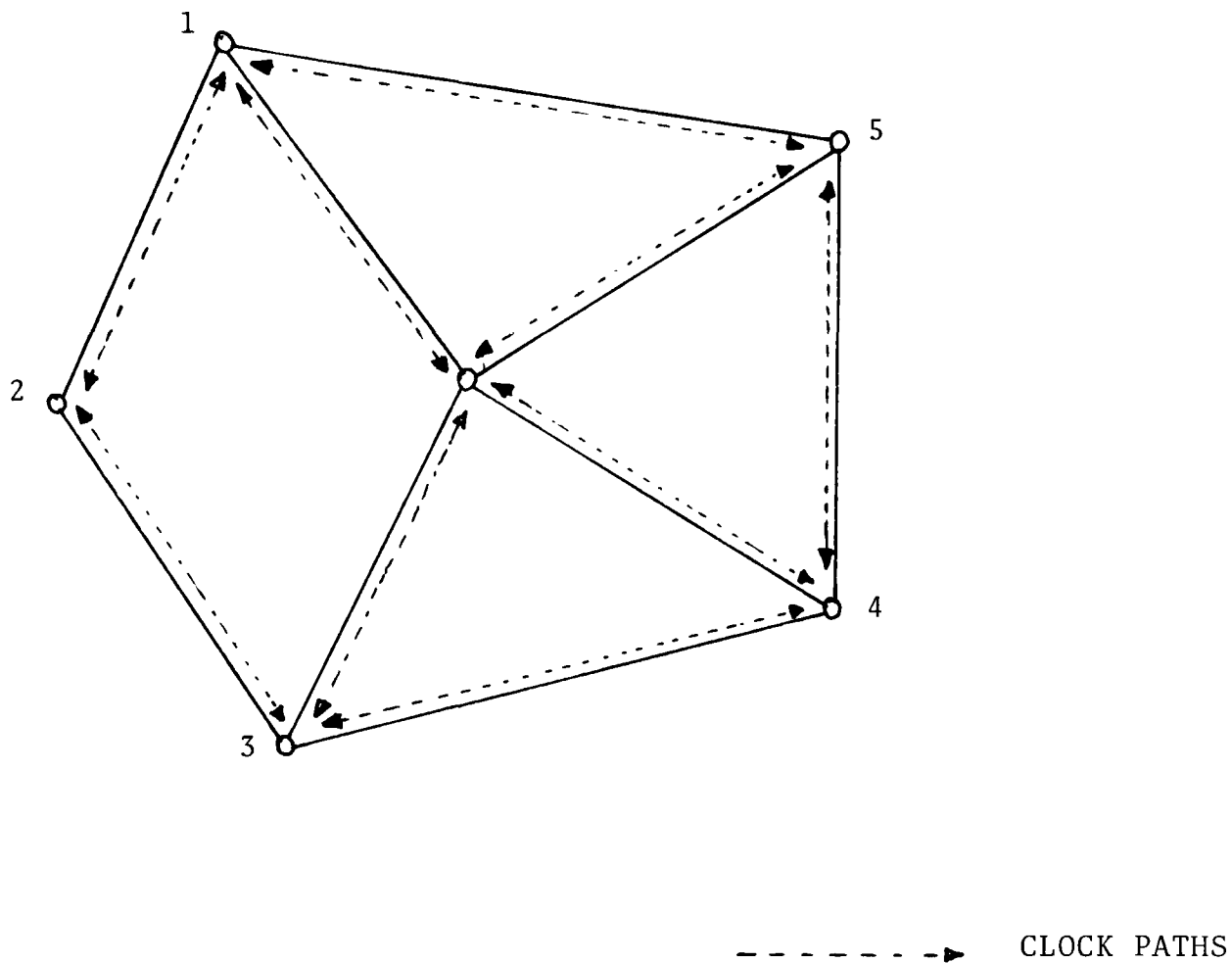


FIGURE 1.8 A MUTUALLY SYNCHRONIZED NETWORK

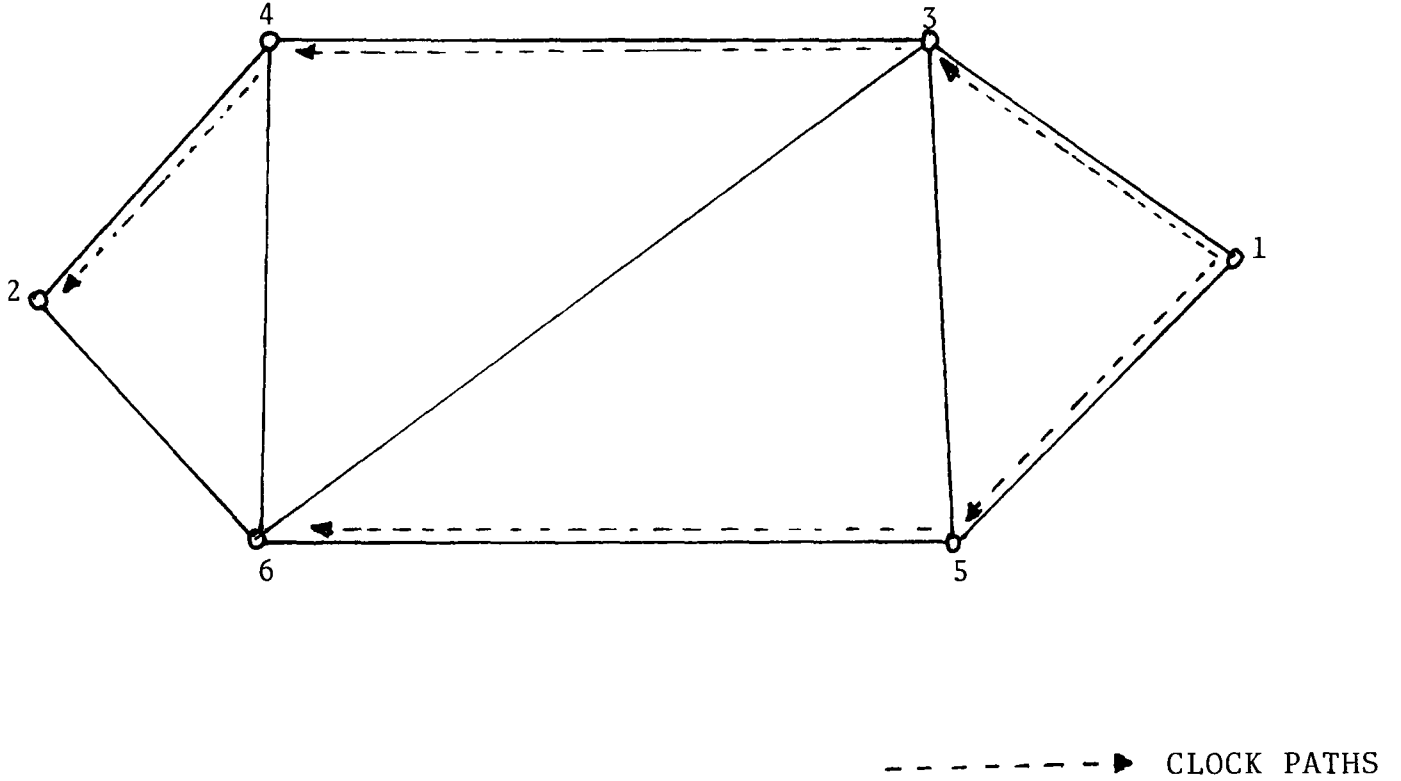


FIGURE 1.9 A HIERARCHICAL MASTER-SLAVE SYSTEM WITH ALL NODES OPERATIONAL.

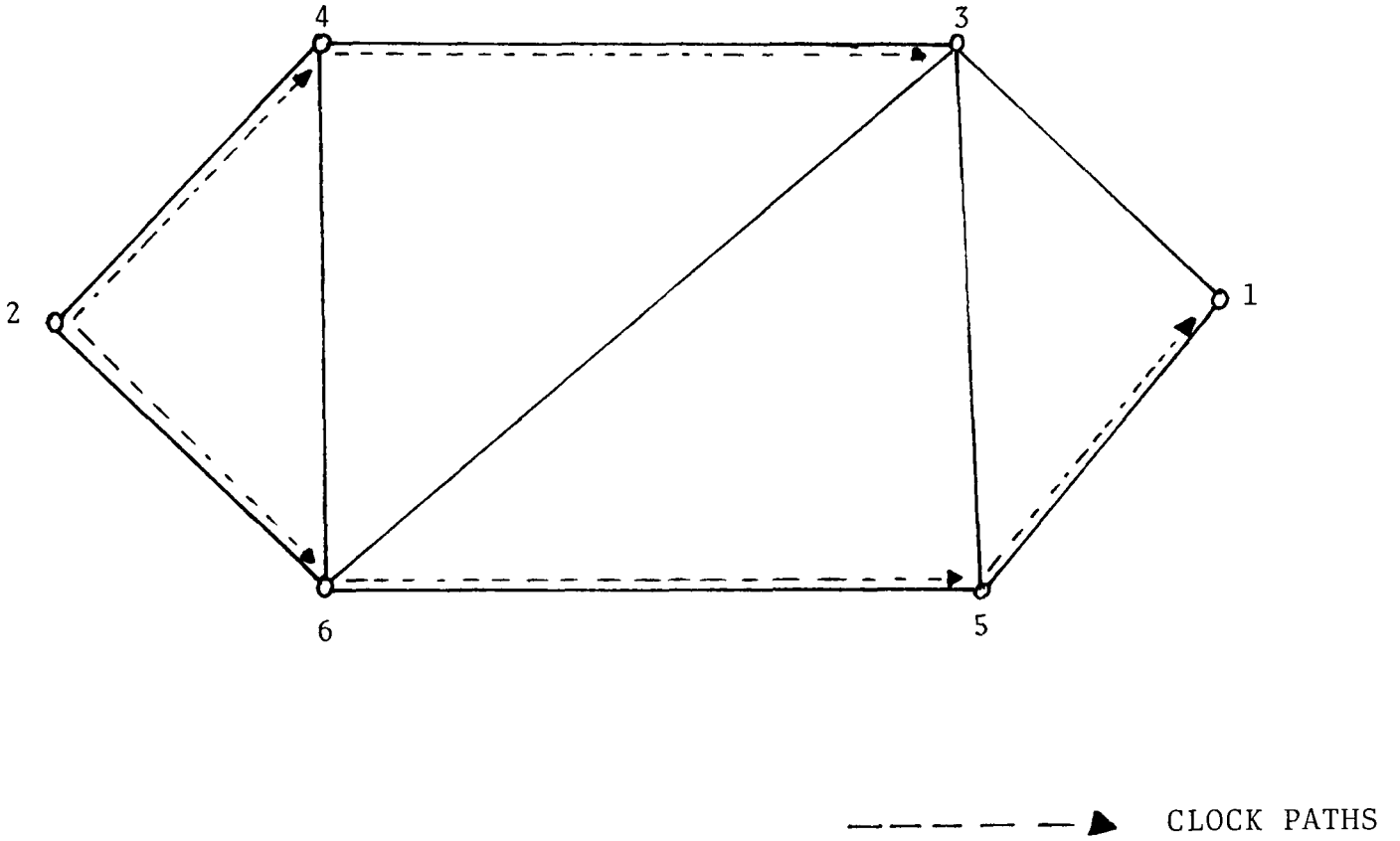


FIGURE 1.9a A HIERARCHICAL MASTER-SLAVE SYSTEM AFTER THE FAILURE OF NODE 1.

CHAPTER 2

FRAME SYNCHRONIZATION IN AN ERROR-FREE ENVIRONMENT

## 2.1 INTRODUCTION

A frame could be defined as a time interval, from the beginning of the time slot allocated to a particular channel until the next time of recurrence of that time slot.

In telecommunication systems where time division multiplexing is used, a provision for frame synchronization is usually made. In case of a time division multiple access (TDMA) system using geosynchronous satellites for example, each frame starts with a synchronizing burst.<sup>12,13,14,15,16,17,18</sup> The ground stations transmit data in preassigned time slots as shown in Figure 2.1.

In Figure 2.2 a typical frame format of the TDMA system is shown. The preamble is responsible for achieving and maintaining the lower levels of synchronization such as bit synchronization. The guard time which is a non-transmit period, is inserted to prevent the bursts from accidentally overlapping each other because of tolerance variations in the system and uncertainties in the burst synchronization technique.<sup>14</sup> In real time systems, the guard time usually lies between 10 and 100 nanoseconds and the frame is 125 micro-seconds long.

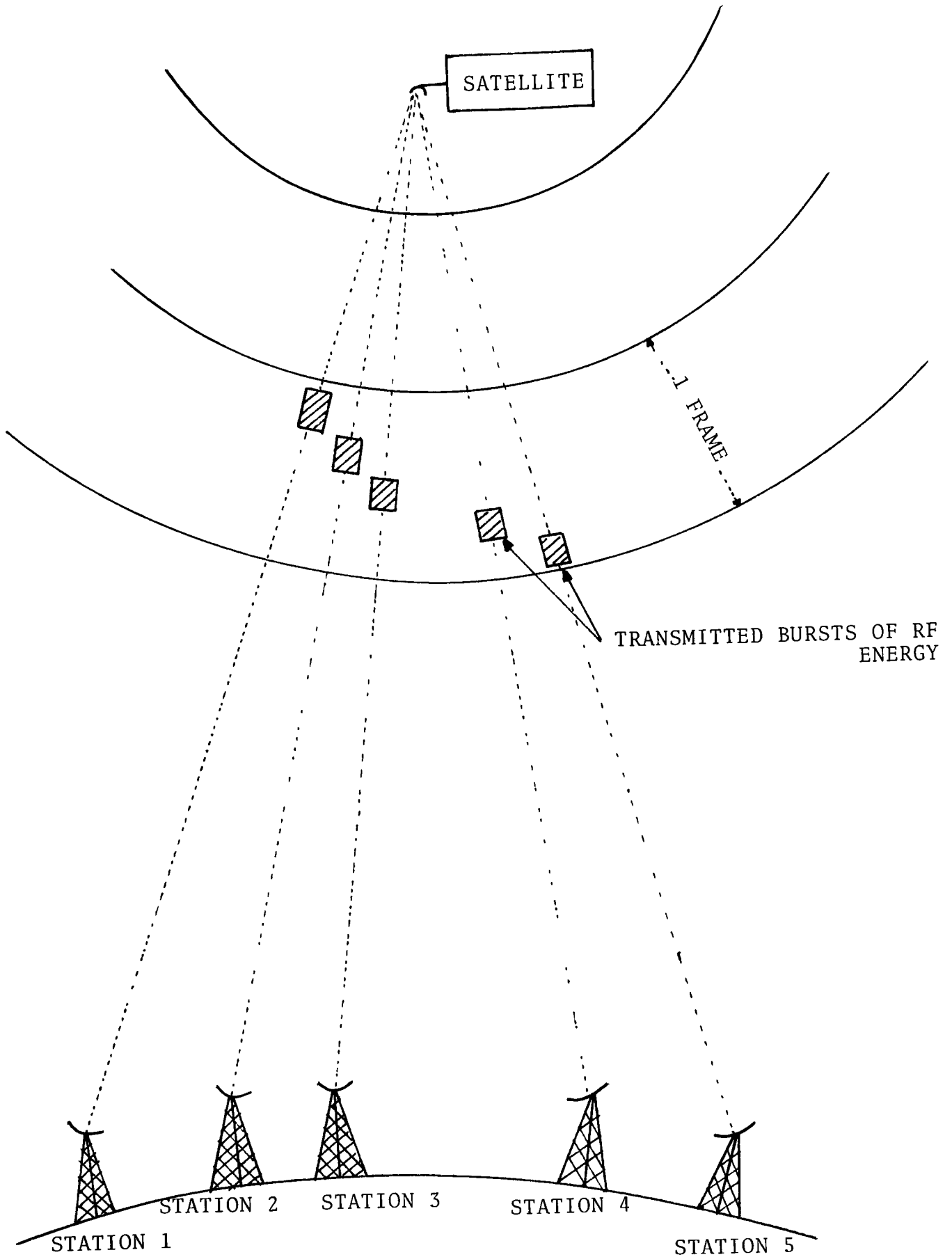


FIGURE 2.1 POSSIBLE ARRANGEMENT OF A TDMA SYSTEM.

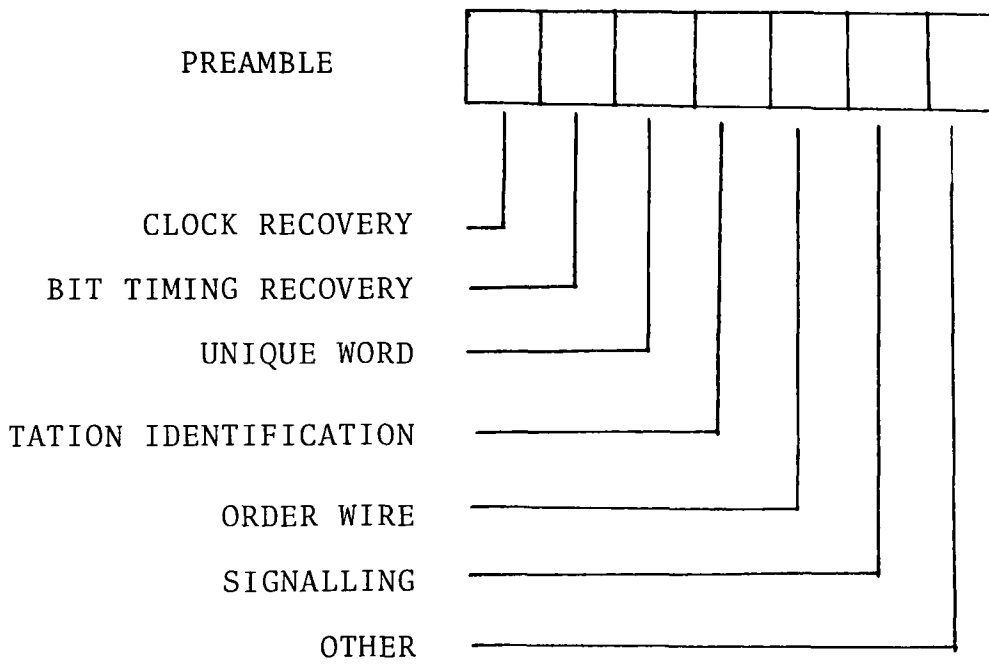
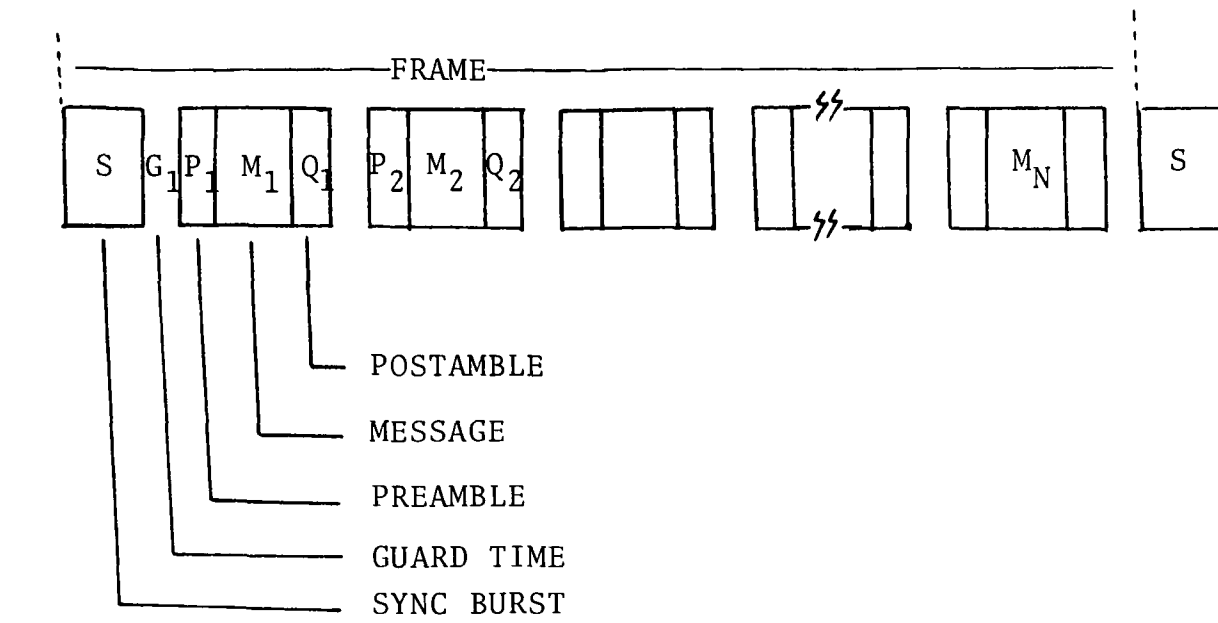


FIGURE 2.2 TYPICAL TDMA FRAME FORMAT

In the case of the T-1 carrier system, a PCM time-division multiplexed system, only one bit per frame is reserved for frame synchronization.<sup>4,5</sup> The other 192 bits are information or data bits. Each frame takes 125 $\mu$ s which leads to a line bit rate of 1.544 M b/s. A frame format for the multiplexer is given in Figure 2.3. The synchronizing bits form an alternating pattern of ZEROES and ONES.

The subsequent discussion on frame synchronization will be essentially based on the T-1 carrier system. Although the noiseless environment is considered first, it is a well-known fact that in practice errors do occur in communication systems. The noiseless case however, could be considered as a special case of the noisy environment. The noiseless case provides a good starting point for the discussion regarding frame synchronization in communication systems. The terms frame synchronization and synchronization will hereafter be used synonymously.

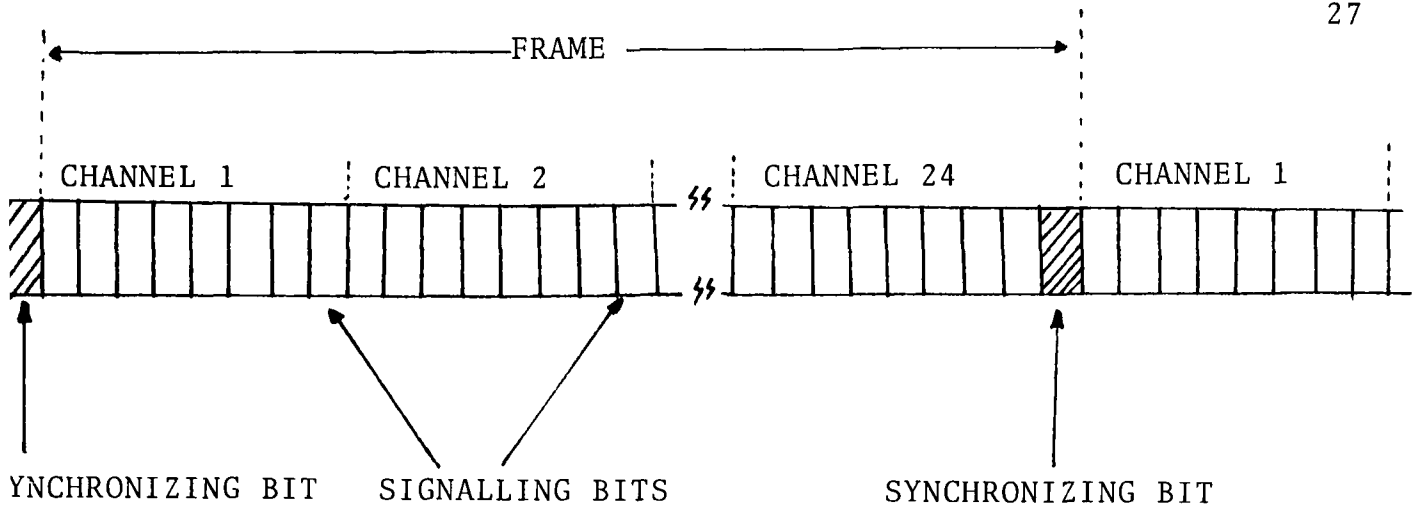


FIGURE 2.3 FRAME FORMAT FOR THE T 1 SYSTEM

## 2.2 STRATEGY FOR SYNCHRONIZATION

Let the following sequence be considered.

$$S = \dots a_{-5} a_{-4} a_{-3} a_{-2} a_{-1} a_0 a_1 a_2 a_3 \dots a_n a_{n+1} \dots a_{2n} a_{2n+1} \dots$$

Where  $a_{-n}, a_0, a_n, a_{2n}, \dots, a_{kn}, \dots$  are synchronizing bits and the others are information or data bits. The information bits are assumed to be random. The problem of frame synchronization could be considered as being equivalent to partitioning the given sequence  $S$  into blocks of  $n$  bits such that each block starts with an  $a_{i_{n+1}}$  and ends with an  $a_{(i+1)_n}$  where  $i$  is  $\dots -5, -4, -3, -2, 1, 0, 1, 2, 3, \dots$ . Usually in practice (the T-1 system) the  $N$ -bit synchronizing sequence  $\sigma_N = a_{i_n} a_{(i+1)_n}$  is 101010....or 010101....depending on  $i$ . For the purpose of our discussion,  $\bar{\sigma}_N = a_{i_n} a_{(i+1)_n} \dots a_{(i+N-1)_n}$  will be 010101.... $N$ -bits and  $\sigma_N = a_{i_n} a_{(i+1)_n} \dots a_{(i+N-1)_n}$  will be 101010.... $N$ -bits.

The problem of synchronization reduces to one of picking in a given sequence  $S$  a synchronizing bit  $a_{i_n}$ . If an  $a_j$  is picked and an  $N$ -tuple ( $N$ -bit sequence)  $\psi_j = a_j a_{j+n} a_{j+2n} \dots a_{j+(N-1)n}$  is formed and it is found that  $\psi_j$  is identical to  $\sigma_N$  or  $\bar{\sigma}_N$ , there is no guarantee that  $a_j$  is a synchronizing bit since information bits could very well be such that  $\psi_j$  looks like  $\sigma_N$  or  $\bar{\sigma}_N$ . The situation becomes even more uncertain in the noisy case as will be seen later.

Initial frame synchronization is achieved by observing the synchronizing pattern (010101....or 101010....) for a given number of frames. In an error-free environment, data bits having a pattern similar to the synchronizing one are the only cause of

false declaration of 'Lock', on the other hand in a noisy situation, the synchronizing sequence at the receiving end might turn out to be different from  $\sigma_N = 101010\dots$  or  $\bar{\sigma}_N = 010101\dots$  after being corrupted by noise and mistaken for information bits.

In this context, the following strategy which is essentially the one used in practice is considered. Given the sequence  $S$  an  $a_j$  is picked and the  $N$ -tuple  $\psi_j$  formed where  $\psi_j = a_j a_{j+n} a_{j+2n} \dots a_{j+(N-1)n}$ . If  $\psi_j$  belongs to an acceptable set  $Z$ , of  $N$ -tuples which will be defined later, it is concluded that a synchronizing bit was picked. If not  $a_{j+1}$  is picked and  $\psi_{j+1}$  formed. If it belongs to  $Z$ , it is concluded that  $a_{j+1} a_{j+1+n} a_{j+1+2n} \dots$  are synchronizing bits, if not the process is repeated until the synchronizing bits are found. The reliability factor  $M$ , is hereby defined as the ratio of the probability that the decision reached is correct to the probability that the decision is incorrect. The value of  $M$  depends on the number of frames examined as will be seen later.

### 2.3 THE NUMBER OF FRAMES TO BE EXAMINED

In the noiseless case, the acceptable set Z of N-tuples consists of 101010.... or 010101.... The following probabilities are investigated with respect to the strategy referred to in the previous section.

$$\begin{aligned}\Phi_1 &= \text{Prob \{A synchronizing bit has been picked} \\ &\quad \psi_j \text{ belongs to Z\}} \\ &= \frac{1}{n} \dots\dots\dots(2-1)\end{aligned}$$

$$\begin{aligned}\Phi_2 &= \text{Prob \{A synchronizing bit has been picked} \\ &\quad \text{and } \psi_j \text{ does not belong to Z\}} \\ &= 0 \dots\dots\dots(2-2)\end{aligned}$$

$$\begin{aligned}\Phi_3 &= \text{Prob \{A non-synchronizing bit has been picked} \\ &\quad \text{and } \psi_j \text{ belongs to Z\}} \\ &= \frac{n-1}{n} \left(\frac{2}{2^N}\right) \dots\dots\dots(2-3)\end{aligned}$$

$$\begin{aligned}\Phi_4 &= \text{Prob \{A non-synchronizing bit has been picked} \\ &\quad \text{and } \psi_j \text{ does not belong to Z\}} \\ &= \frac{n-1}{n} \left(\frac{2^N-2}{2^N}\right) \dots\dots\dots(2-4)\end{aligned}$$

By way of verification, it can be realized that  $\Phi_1, \Phi_2, \Phi_3,$  and  $\Phi_4$  add up to unity.

For the synchronization strategy to be reliable it is required that  $\Phi_1 + \Phi_4 \gg \Phi_3 + \Phi_2$

$$\text{Let } \Phi_1 + \Phi_4 = M(\Phi_3 + \Phi_2) \dots\dots\dots(2-5)$$

where M is the reliability factor introduced in the previous section. Using equations (2-1), (2-2), (2-3), (2-4) and (2-5), the following equation is obtained.

$$\frac{1}{n} + \frac{n-1}{n} \left( \frac{2^N-2}{2^N} \right) = M \left( \frac{n-1}{n} \right) \frac{2}{2^N}$$

$$\text{or } 2^N + (n-1)(2^N-2) = 2(n-1)M$$

$$\text{or } n2^N = (n-1)(2M + 2)$$

$$2^N = \frac{2(n-1)(M+1)}{n}$$

$$N = \log_2 \left( \frac{n-1}{n} \right) + \log_2(M+1) + 1 \dots \dots \dots (2-6)$$

Where N = The number of frames to be examined

n = Number of bits in one frame

M = Reliability factor

Noting that in many cases of practical interests, n, the number of bits per frame is usually such that  $\frac{n-1}{n} \approx 1$  and noting that it is required to make  $M \gg 1$ , then 2-6 can be approximated to (2-6a)

$$N \approx 1 + \log_2 M \dots \dots \dots (2-6a)$$

From (2-6) or (2-6a) the value of N can be worked out. It is interesting to note that N varies approximately linearly with  $\log_2 M$ .

The values of N for certain selected values of M are computed from (2-6) and shown in Table 2-1. The contents of Table 2-1 are shown graphically in Figure 2-4.

It is known that in practice, the pattern 1010.... or 0101.... cannot occur for long among the information bits.<sup>5</sup> If it did, it would imply a 4KHZ component in the signal and the input filters do not pass the 4KHZ. So  $\phi_3$  would generally be less than  $\left( \frac{n-1}{n} \right) \frac{2}{2^N}$  and  $\phi_4$  would be greater than  $\left( \frac{n-1}{n} \right) \left( \frac{2^N-2}{2^N} \right)$ .

TABLE 2.1 THE RELATIONSHIP BETWEEN THE RELIABILITY FACTOR  
M AND NUMBER OF FRAMES N FOR THE NOISELESS CASE.

M	N
$2^0$	2
$2^3$	4
$2^4$	5
$2^6$	7
$2^8$	9
$2^{10}$	11
$2^{12}$	13
$2^{14}$	15
$2^{16}$	17
$2^{18}$	19
$2^{20}$	21
$2^{22}$	23
$2^{24}$	25
$2^{26}$	27

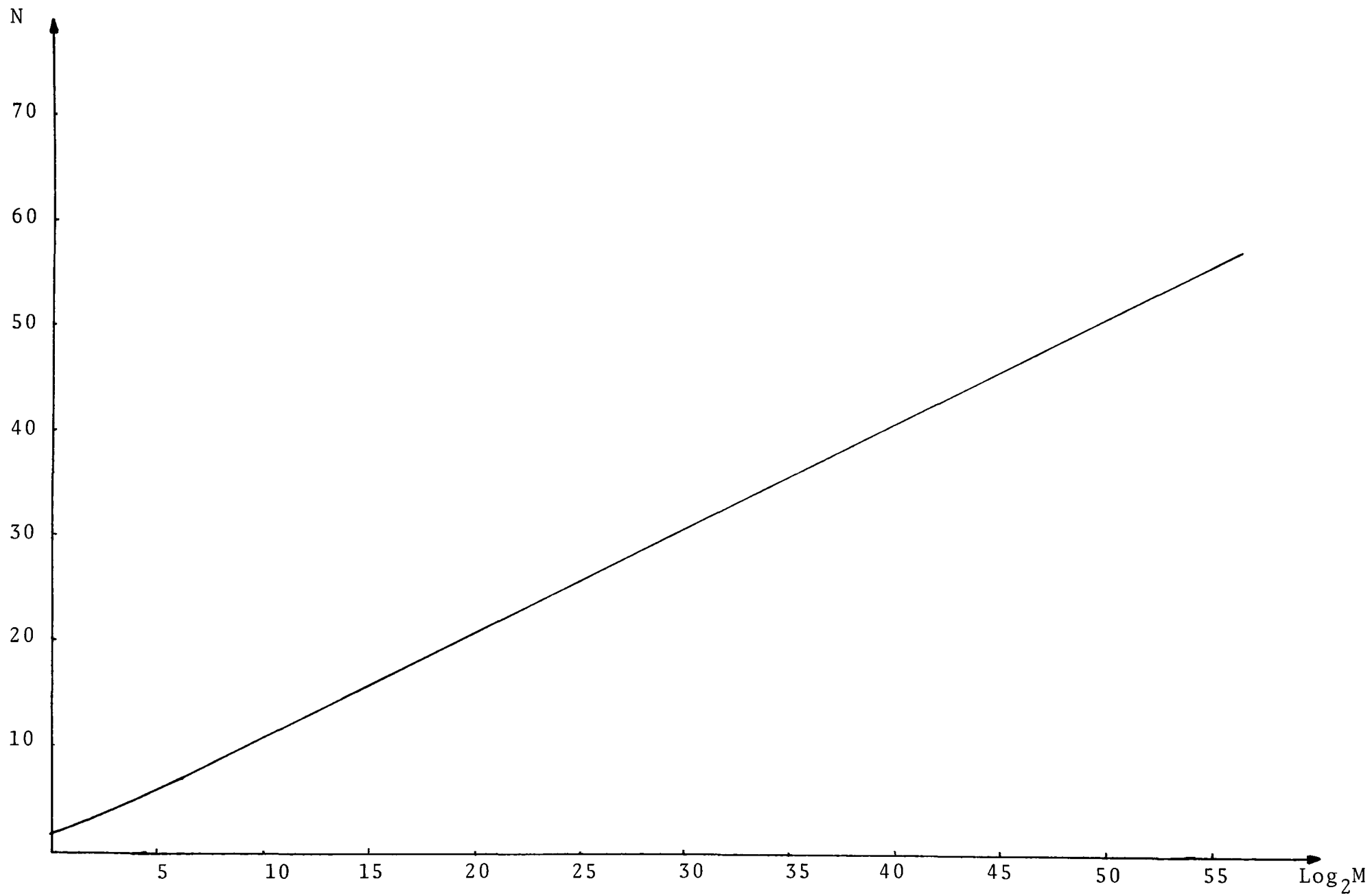


FIGURE 2.1 THE NUMBER OF FRAMES IN THE NOISELESS CASE

## 2.4 PROBABILITY OF FALSE LOCK

After observing the synchronizing pattern 101010.... or 010101.... for N frames, the system goes into Frame Lock Mode. From the strategy adopted, False 'Lock' will take place if a pattern similar to the synchronizing one is observed for N frames or more among the information bits.

Let the probability of False Lock be PFL.

PFL = Prob {of picking a non-synchronizing bit and yet observe the pattern 1010.... or 0101.... for N frames}

$$PFL = \phi_3$$

$$= \left(\frac{n-1}{n}\right) \frac{2}{2^N} \quad (\text{from equation 2-3})$$

$$\text{But } 2^N = 2 \left(\frac{n-1}{n}\right) (M+1) \quad \text{from equation 2-6}$$

$$\begin{aligned} \text{So PFL} &= \left(\frac{n-1}{n}\right) \frac{2}{2 \left(\frac{n-1}{n}\right) (M+1)} \\ &= \frac{1}{(M+1)} \dots\dots\dots (2-7) \end{aligned}$$

In the cases where M is chosen to be  $\gg 1$ ,  $PFL \approx \frac{1}{M}$ ... (2-7a)

It is interesting to note that equation 2-7 gives an inverse relationship between the probability of a false lock and the reliability factor. This means that the probability of false lock can be reduced as much as desired by simply increasing the reliability factor required. The number of frames to be examined however, would have to increase as well.

## 2.5 PROBABILITY OF MISSING THE SYNCHRONIZING BIT

If a synchronizing bit is picked when forming  $\psi_j$  and yet a decision is reached that we are out of synchronism, then we say that a misframe has occurred. The probability of a misframe PMF can be worked out as follows.

$$\text{PMF} = \text{Prob} \{ \text{A synchronizing bit is picked and yet } \psi_j \text{ is outside } Z \}$$

$$\text{PMF} = \phi_2 \text{ (Equation 2-2)}$$

$$\text{PMF} = 0 \dots\dots\dots(2-8)$$

From the strategy adopted, the probability of a misframe in the noiseless environment is zero.

The values obtained for the probability of false lock and the probability of misframe can be verified as follows.

Recalling the definition of the reliability factor M,

$$M = \frac{\text{Prob} \{ \text{decision reached is correct} \}}{\text{Prob} \{ \text{decision reached is incorrect} \}}.$$

or

$$M = \frac{1 - \text{Prob} \{ \text{decision reached is incorrect} \}}{\text{Prob} \{ \text{decision reached is incorrect} \}}$$

But the decision reached being incorrect leads to either a misframe or a false lock.

$$\text{So } M = \frac{1 - (\text{PMF} + \text{PFL})}{(\text{PMF} + \text{PFL})} \dots\dots\dots(2-9)$$

or

$$M = \frac{1}{(\text{PMF} + \text{PFL})} - 1$$

$$(M+1) = \frac{1}{(\text{PMF} + \text{PFL})}$$

or

$$\text{PMF} + \text{PFL} = \frac{1}{(M+1)} \dots\dots\dots(2-10)$$

Equation (2-10) is in agreement with equations (2-7) and (2-8).

I would like to point out however, that the probability of a misframe (PMF) being zero is a result of the strategy adopted and the channel being noiseless. In some situations as will be seen later, noise could drive the sequence  $\psi_n$  out of the acceptable set Z. In that case, PMF is no longer zero.

## 2.6 MEAN TIME BEFORE AQUISITION OF SYNCHRONISM

From the strategy adopted, the search for synchronizing bits is both unidirectional and sequential in that if position  $j$  is not the synchronization position then we look at position  $j+1$ . If position  $j+1$  is not the one, then we look at  $j+2$  and so forth. The time taken before synchronism depends on the starting position  $j$  as well as the number of frames  $N$  examined while in each position.

The incoming bit stream  $S$  is  $S = \dots a_{-5} a_{-4} a_{-3} a_{-2} a_{-1} a_0 a_1 a_2 \dots \dots a_n a_{n+1} \dots a_{2n} a_{2n+1} \dots$  with  $a_0, a_n, a_{2n}, \dots$  as the synchronizing bits.

The formed  $N$ -tuple is:

$$\psi_j = a_j a_{j+n} a_{j+2n} \dots a_{j+(N-1)n} \quad 1 \leq j \leq n$$

If  $a_j$  is not the synchronizing bit, sequence  $\psi_{j+1}$  is examined after the  $N$  frames of  $\psi_j$ .

$$\psi_{j+1} = a_{j+1} a_{j+1+n} a_{j+1+2n} \dots a_{j+1+(N-1)n}$$

The number of sequences ( $\psi'_s$ ) which have to be looked at before acquiring synchronism is  $(n - j + 1)$ . The total number of frames which have to be looked at before acquisition is  $N\{n - j + 1\}$ .

Since  $a_j$  is picked at random, the probability of picking a particular value of  $j$  is  $P(j) = 1/n$ .

Let the frame period be  $t_f$  seconds.

$$\text{Mean time before acquisition (MTBA)} = t_f \sum_{j=1}^n N(n-i+1) P(j)$$

$$\begin{aligned}
\text{MTBA} &= t_f \sum_{j=1}^n N(n-j+1) \\
&= t_f N \{n + (n-1) + (n-2) + \dots + 1\} \\
&= \frac{N}{2} \frac{(n+1)}{2} t_f \\
&\quad \text{using (2-6)} \\
\text{MTBA} &= \frac{(n+1)}{2} t_f \{1 + \log_2(M+1) + \log_2 \frac{(n-1)}{n}\} \dots \dots \dots (2-10)
\end{aligned}$$

In the T-1 carrier system where the frame period is 125 $\mu$ s. and the number of bits in a frame is 193, the mean time before acquisition using equation (2-10) would be

$$\begin{aligned}
\text{MTBA} &= \frac{194}{2} \times 125 \{1 + \log_2(M+1) + \log_2 \frac{192}{193}\} \text{ microseconds} \\
&\approx 12.125 \{1 + \log_2(M+1)\} \text{ milliseconds}
\end{aligned}$$

If a reliability factor of  $2^{10}$  is required for example, then the mean time before acquisition would be approximately 133.4 milliseconds.

In practice, the time needed to acquire frame synchronism depends on how far from the synchronizing bit the hunting starts. In the worst case, when all the 193 positions have to be examined, on the average, the total time taken before acquisition is approximately 50 milliseconds.<sup>5</sup>

## CHAPTER 3

### FRAME SYNCHRONIZATION IN A NOISY ENVIRONMENT

### 3.1 INTRODUCTION

In practice, the process of frame synchronization involves three major modes of operation which will be described briefly as follows:<sup>19</sup>

#### 1) LOCK

This is the normal mode of operation in which the synchronizing pattern is checked every frame and found to differ from  $\sigma_N$  or  $\bar{\sigma}_N$  in no more than a certain number of positions (threshold).

#### 2) SEARCH

When the number of violations exceeds the threshold, the system moves from LOCK to SEARCH mode in which the synchronizing bit is searched in accordance with the procedure mentioned in the strategy.

#### 3) VERIFICATION

When the system observes a pattern similar to the synchronizing one, it goes into VERIFICATION mode during which an attempt is made to try and confirm that the synchronizing bits have actually been found. After a number of verifications, the multiplexer moves to LOCK otherwise it goes back to SEARCH, if the required number of verifications is not met.

The modes of operation are shown in Figure 3-1.

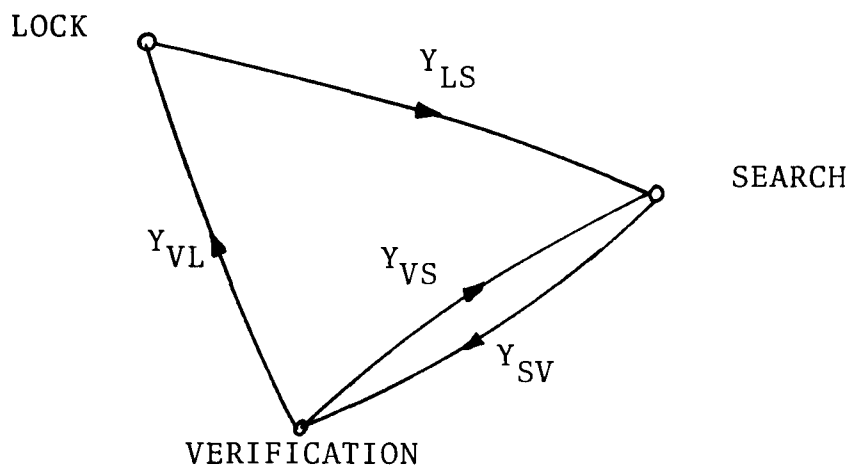


FIGURE 3-1 THE THREE MODES OF A FRAME SYNCHRONIZER

$Y_{LS}$  = Move from LOCK to SEARCH after the number of violations is greater than the threshold.

$Y_{SV}$  = Move from SEARCH to VERIFICATION after observing the synchronizing pattern for a given number of frames.

$Y_{VS}$  = Move from VERIFICATION to SEARCH if the number of verifications is less than the required number  $V_x$ .

$Y_{VL}$  = Move from VERIFICATION to LOCK after  $V_x$  or more verifications.

In the T-1, it takes 0.4 to 6 milliseconds to detect loss of frame synchronization. Before moving on from VERIFICATION to LOCK, the alternating pattern has to be observed for more than 8 frames.<sup>5</sup>

The basic objectives of frame synchronization in practice are first of all to make possible a quick restoration from the state of out-of-frame and secondly to make it difficult for the signals to lose synchronism once they are in LOCK mode.<sup>19,20,21,22,23</sup> The detection of an out-of-frame condition and reconfirmation of the framing bit following reacquisition of frame are situations involving the line error environment. System inertia is essential to ensure that there is no false declaration of out-of-frame (MISFRAME) and to minimize the probability of 'Locking" on the wrong bit after the synchronizing bit search.

In this chapter, the problem of frame acquisition in both random and burst error channels is studied. No attempt is made to try and achieve frame synchronism as soon as possible however, the emphasis is put on the degree of confidence in the decision reached with regard to the adopted synchronization strategy.

### 3.2 NUMBER OF FRAMES TO BE EXAMINED

We begin by assuming that in  $N$  consecutive blocks each of  $n$  bits in the sequence  $S$ , the maximum number of blocks in error is  $e > 0$ .

This assumption means that in  $\sigma_N$  or  $\bar{\sigma}_N$  the maximum number of bits corrupted is  $e$ . Let us take for the set  $Z$  all of the  $N$ -tuples which differ from  $\sigma_N$  or  $\bar{\sigma}_N$  in  $e$  or fewer positions. The number  $|Z|$  of  $N$ -tuples in  $Z$  is given by:

$$|Z| = 2\left\{ \binom{N}{0} + \binom{N}{1} + \binom{N}{2} + \cdots + \binom{N}{e} \right\} \dots\dots\dots(3-1)$$

here we assume that  $2e < N$  an assumption which is automatically satisfied for a reasonably large value of the reliability factor  $M$ , as will be seen later. We note that  $Z$  consists of all the  $N$ -tuples that result when  $e$  or fewer errors occur in  $\sigma_N$  or  $\bar{\sigma}_N$ .

At this point we consider with reference to the synchronization strategy under consideration the following probabilities.

$$\begin{aligned} P_1 &= \text{Prob} \{ \text{A synchronizing bit is picked and the } N\text{-tuple formed } \psi_j \\ &\quad \text{belongs to } Z \} \\ &= \frac{1}{n} \dots\dots\dots(3-2) \end{aligned}$$

$$\begin{aligned} P_2 &= \text{Prob} \{ \text{A synchronizing bit is picked and } \psi_j \text{ does not} \\ &\quad \text{belong to } Z \} \\ &= 0 \dots\dots\dots(3-3) \end{aligned}$$

$$P_3 = \text{Prob} \{ \text{A non-synchronizing bit is picked and } \psi_j \text{ belongs to } Z \}$$

$$P_4 = \text{Prob} \{ \text{A non-synchronizing bit is picked and } \psi_j \text{ does not belong to } Z \}$$

The number of N-Tuples which could go to set Z on the occurrence of e or fewer errors is  $|Z| + 2\{ \binom{N}{e+1} + \binom{N}{e+2} \dots \binom{N}{2e} \}$  therefore

$$\chi_3 < \frac{\binom{n-1}{n} [2\{ \binom{N}{0} + \binom{N}{1} + \binom{N}{2} + \dots + \binom{N}{2e} \}]}{2^N} \dots \dots \dots (3-4)$$

so that

$$\chi_4 > \frac{\binom{n-1}{n} [2^N - 2\{ \binom{N}{0} + \binom{N}{1} + \dots + \binom{N}{2e} \}]}{2^N} \dots \dots \dots (3-5)$$

It is required to make  $\chi_1 + \chi_4 \gg \chi_3 + \chi_2$

$$\text{Let } \chi_1 + \chi_4 = M(\chi_3 + \chi_2)$$

Where M as in the noiseless case is the reliability factor.

Using (3-2), (3-4), (3-5), and (3-6)

we set

$$\frac{1}{n} + \frac{\binom{n-1}{n} [2^{N'} - 2\{ \binom{N'}{0} + \binom{N'}{1} + \dots + \binom{N'}{2e} \}]}{2^{N'}} = M \frac{\binom{n-1}{n} 2^{\sum_{j=0}^{2e} \binom{N'}{j}}}{2^{N'}}$$

Where  $N'$  is an approximation to  $N$ .

$$2^{N'} + (n-1) \{ 2^{N'} - 2\{ \binom{N'}{0} + \binom{N'}{1} + \binom{N'}{2} + \dots + \binom{N'}{2e} \} \} = 2M(n-1) \sum_{j=0}^{2e} \binom{N'}{j}$$

$$2^{N'} n = (n-1) (M+1) \sum_{j=0}^{2e} \binom{N'}{j}$$

$$2^{N'} = \frac{n-1}{n} (M+1) \sum_{j=0}^{2e} \binom{N'}{j}$$

$$N' = 1 + \log_2 \frac{n-1}{n} + \log_2 (M+1) + \log_2 \{ \binom{N'}{0} + \binom{N'}{1} + \dots + \binom{N'}{2e} \} \dots \dots \dots (3-7)$$

In practice  $n$  is usually  $\gg 1$  and it is desired to make  $M \gg 1$  so that

(3-7) can be approximated by (3-7a)

$$N' \approx 1 + \log_2 M + \log_2 \{ \binom{N'}{0} + \binom{N'}{1} + \dots + \binom{N'}{2e} \} \dots \dots \dots (3-7a)$$

TABLE 3-1 THE NUMBER OF FRAMES IN THE NOISY CASE

	e	1	2	3	4	5	6	7	8	9	10
$M=2^0$	N'	7	12	16	20	24	28	33	36	41	45
$M=2^5$	N'	13	18	23	28	33	38	43	48	52	56
$M=2^{10}$	N'	19	25	31	36	42	47	53	58	63	68
$M=2^{15}$	N'	24	31	38	44	50	55	61	66	72	77
$M=2^{20}$	N'	30	37	44	50	57	63	68	74	80	85
$M=2^{25}$	N'	35	43	50	57	63	70	75	81	87	93
$M=2^{30}$	N'	41	49	56	63	70	76	82	88	94	100
$M=2^{35}$	N'	46	55	62	69	76	83	89	95	102	108
$M=2^{40}$	N'	51	60	68	75	82	89	96	102	108	115
$M=2^{45}$	N'	57	66	74	81	88	95	102	109	115	121
$M=2^{50}$	N'	62	71	79	87	94	101	108	115	122	128
$M=2^{55}$	N'	67	76	85	93	100	108	115	121	128	135

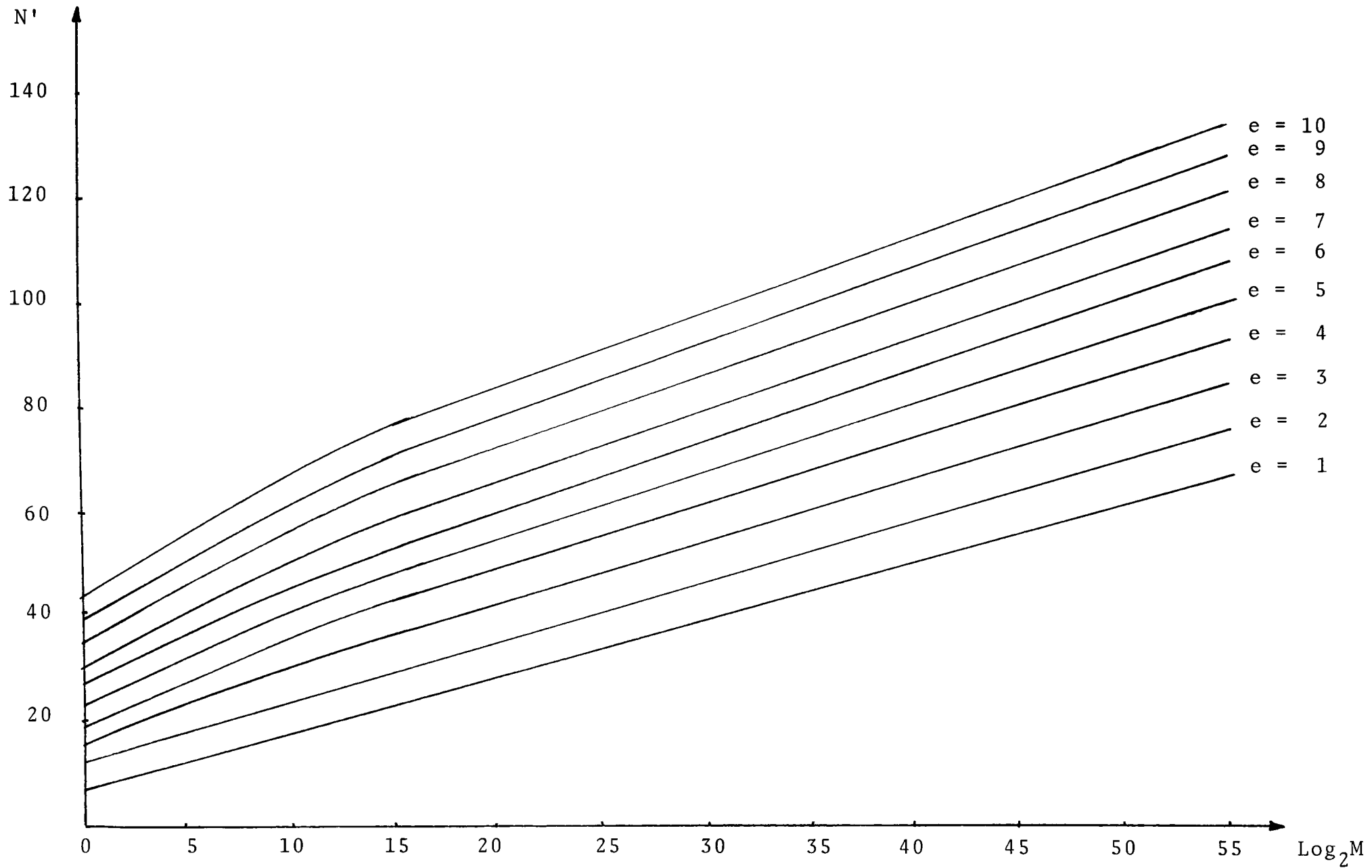


FIGURE 3.2 THE NUMBER OF FRAMES IN THE NOISY CASE.

Table 3-1 shows the values of  $N'$  as computed from equation 3-7 for certain selected values of  $e$  and  $M$ . The results of table 3-1 are shown in Figure 3-2. It is interesting to note that for  $M > 2^{15}$ ,  $N'$  varies approximately lineally with  $\log_2 M$ . This leads us to believe that for large values of the reliability factor  $M$ ,  $N'$  could be approximated to  $N' \approx C_1 + C_2 \log_2 M$  where  $C_1$  is a constant which depends on  $e$ ,  $C_2$  is another constant and  $M$  is larger than  $2^{15}$ .

The values of  $N'$  shown in Table 3-1 might appear to be too high in some cases however, as mentioned earlier, the values of  $N'$  are upper bounds on the number of frames which have to be examined. With the error probabilities known, a better estimate on the number of frames which have to be examined can be obtained.

### 3.3 RANDOM ERRORS WITH BIT ERROR PROBABILITY $P_e$

From equation 3-7, the number of frames which have to be examined is approximately  $N'$  where

$$N' \approx 1 + \log_2(M+1) + \log_2 \sum_{j=0}^{2e} \binom{N'}{j}$$

Where  $e$  is the number of frames which have at least one error each.

As mentioned earlier, when looking for the synchronizing bits, the sequence  $\psi_j$  is formed where

$$\psi_j = a_{j+n} a_{j+3n} \dots a_{j+(N'-1)n}$$

Supposing the errors in the channel under consideration are randomly distributed with bit error probability  $P_e$ . The expected number of errors in  $\psi_j$  will be  $N'P_e$ .

In using our decision on the expected number of errors and the strategy adopted for synchronization, the number of  $N$ -tuples in the acceptable set  $Z$  is given by  $\sum_{j=0}^{2\bar{e}} \binom{N'}{j}$ .

The number of frames which have to be examined in this case is

$$N' \approx 1 + \log_2(M+1) + \log_2 \sum_{j=0}^{2\bar{e}} \binom{N'}{j} \dots \dots \dots (3-8)$$

Where  $\bar{e}$  is the expected number of errors in  $N'$  bits

$$N' \approx 1 + \log_2(M+1) + \log_2 \sum_{j=0}^{2N'P_e} \binom{N'}{j} \dots \dots \dots (3-9)$$

In general  $N'P_e$  is not an integer so (3-9) has to be approximated by (3-9a).

$$N' \approx 1 + \log_2(M+1) + \log_2 \sum_{j=0}^{2\acute{e}} \binom{N'}{j} \dots \dots \dots (3-9a)$$

Where  $\acute{e}$  is the closest integer to  $N'Pe$ .

Equation (3-9) gives the approximate number of frames  $N'$  which have to be examined in a noisy channel with bit error probability  $Pe$ . Table 3-2 gives some values of  $N'$  for different values of  $M$  and  $Pe$  as computed from equation (3-9a). The results of Table 3-2 are shown in Figure 3-3.

It is interesting to note that for bit error probabilities less than  $10^{-3}$ , the values of  $N'$  which are obtained from equation (3-9a) are the same as those obtained from equation (2-6) for the same values of  $M$ . In other words for  $Pe < 10^{-3}$  and  $M \leq 2^{55}$ , the random error case could be considered as being practically noiseless.

For the telephone channel, the average error rate for medium speed transmission is about  $10^{-5}$ . Even at higher speeds like the one for the T-1 carrier system for example, a bit error rate of  $10^{-4}$  is easily achieved. From Figure 3-3, it can be deduced that random errors do not pose a big problem to frame synchronization in the T-1 or any other practical system of comparable error performance.

In the next section, we attempt to analyse synchronization in a burst error environment.

TABLE 3-2 THE NUMBER OF FRAMES FOR EXAMINATION IN THE RANDOM ERROR SITUATION

	Pe	$\leq 10^{-3}$	$10^{-2}$	$2.5 \times 10^{-2}$	$5 \times 10^{-2}$	$7.5 \times 10^{-2}$	$10^{-1}$	$1.25 \times 10^{-1}$	$1.5 \times 10^{-1}$	$2 \times 10^{-1}$
$M = 2^0$	N'	2	2	2	2	2	3	4	7	20
$M = 2^5$	N'	6	6	6	9	13	15	23	33	80
$M = 2^{10}$	N'	11	11	15	19	25	30	47	68	
$M = 2^{15}$	N'	16	16	20	28	35	50	72	106	
$M = 2^{20}$	N'	21	21	26	37	46	65	96		
$M = 2^{25}$	N'	26	30	35	46	60	80			
$M = 2^{30}$	N'	31	36	40	54	75	100			
$M = 2^{35}$	N'	36	41	46	62	86				
$M = 2^{40}$	N'	41	47	55	70	95				
$M = 2^{45}$	N'	46	52	61	81					
$M = 2^{50}$	N'	51	57	67	90					
$M = 2^{55}$	N'	56	62	76	100					

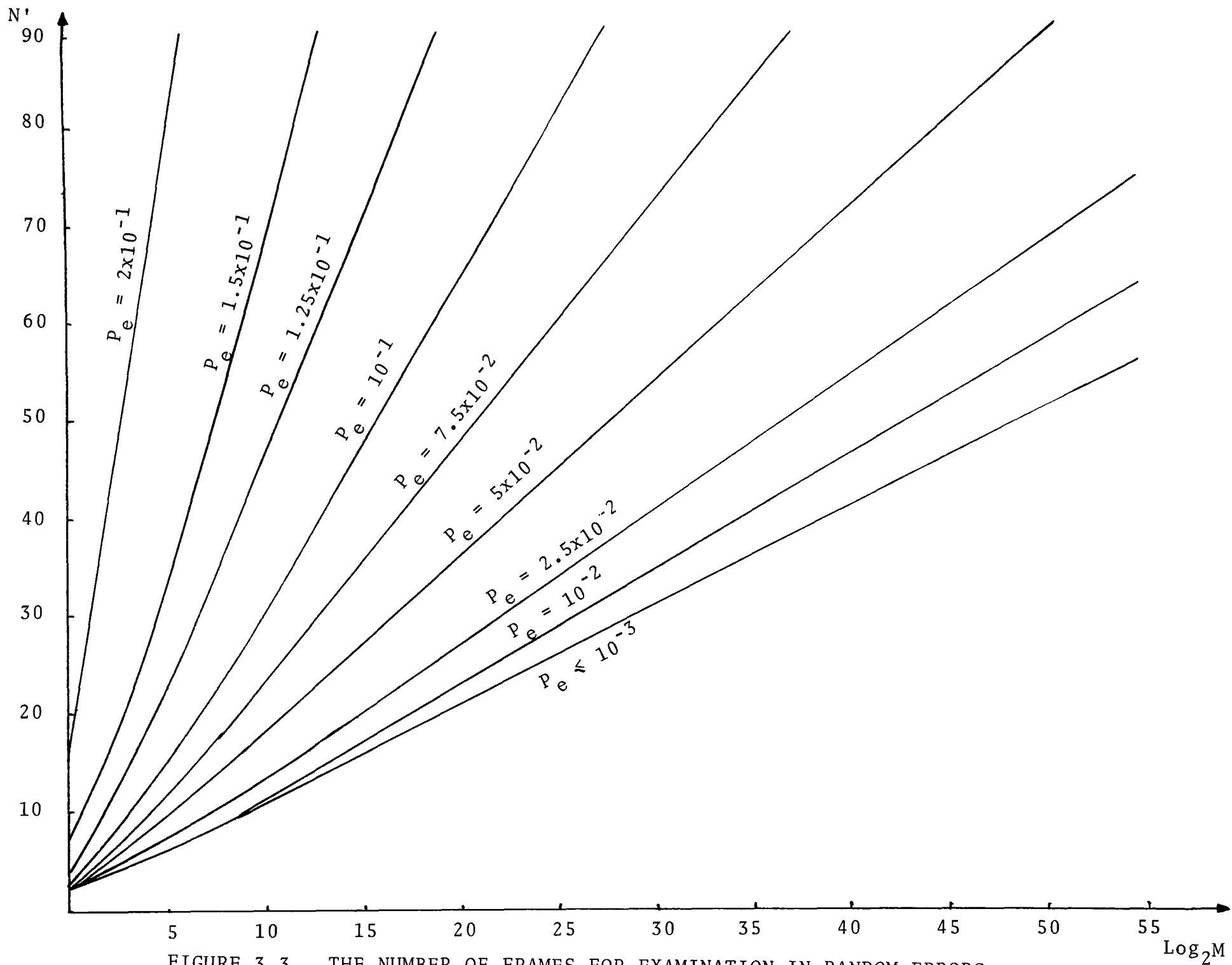


FIGURE 3.3 THE NUMBER OF FRAMES FOR EXAMINATION IN RANDOM ERRORS.

### 3.4 BURST ERROR SITUATION

#### 3.4.1 INTRODUCTION

It has been discovered that in general errors do not occur completely at random in voice or data transmission systems. They tend to be bunched together in bursts.<sup>24,25,26,27,28,29,30,31</sup> This led to the proposals of other channel models<sup>32,33,34,35,36,37</sup> other than the binary symmetric channel to characterize real communication channels. In view of this, we continue our discussion with some definitions of terms often encountered when dealing with bursty channels.

Definition of a burst.

A burst is defined as a region of serial data or information stream where the following properties hold.

- i) It begins with a bit in error and ends with a bit in error.
- ii) The ratio  $\Delta$  of the number of bits in error to the total number of bits is equal to or greater than a predetermined quantity  $\Delta_0$ .
- iii) The total number of bits in the region (burst length) is greater than one.

Definition of an interval.

An interval is defined as a region of serial data or information stream where the following hold.

- i) It begins on a correct bit and ends on a correct bit.
- ii) The ratio  $\Delta$  of the number of bits in error to the total number of bits in the region is less than a predetermined quantity  $\Delta_0$ .

- iii) The total number of bits in the region  
(Interval Length) is greater than one.

Definition of a Gap.

A gap is a region in serial data stream where all the bits are correct. In other words an interval where  $\Delta$  is zero.

Definition of a cluster.

A cluster is a region of serial data stream where all bits are in error. In other words a cluster is a burst where  $\Delta$  is unity.

It should be pointed out that, the definitions given in this section are not unique. In one of the papers<sup>31</sup> for example, an error burst is defined to be a collection of one or more bits beginning and ending with an error and separated from neighboring bursts by 50 or more error free bits.

The definitions introduced here, are identical to the ones used in the Markov characterization of the H.F. channel.<sup>32</sup>

In the next section we go on to work out the number frames which have to be examined for the purpose of synchronization, in burst errors.

### 3.4.2 THE NUMBER OF FRAMES TO BE EXAMINED

With regard to frame synchronization of a TDM in a burst error environment, the number of N-tuples in the acceptable set Z can be worked out as follows.

Let a burst of length  $\ell$  bits occur where

$$(K-1)n < \ell < Kn$$

and K is an integer greater than Zero.

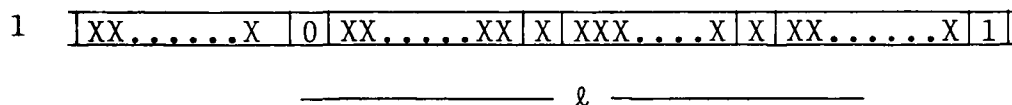
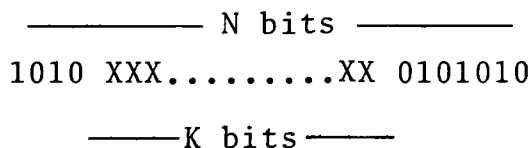


FIGURE 3-4 EFFECT OF BURST ERRORS ON INCOMING BITS

The maximum number of synchronizing bits that would be affected is K. In Figure 3-4, the number of frames that would be affected by the burst of length  $\ell$  is K. In that particular case, K is 4.

Let set  $Z\sigma$  which consists of N-tuples similar to the one given below be considered.



The K-bit block can be located anywhere among the N-K bits. The total number of N-tuples in  $Z\sigma$  is given by

$$|Z\sigma| = 2^{K-1}(N-K) + 2^K$$

or

$$|Z\sigma| = 2^{K-1}(N+2-K) \dots \dots \dots (3-10)$$

Equation (3-10) was developed from N-tuples generated by a computer program.

Let set  $Z\bar{\sigma}$  consisting of N-tuples similar to one given below be introduced.

0101 XXXX.....XX 010101

— K-bits —

The K-bit block can be located anywhere among the N-K bits. Similar to the case of set  $Z\sigma$ ,

$$|Z\bar{\sigma}| = 2^{K-1} \{N+2-K\} \dots \dots \dots (3-11)$$

With regard to the synchronization strategy, the acceptable set of N-tuples is given by

$$Z = Z\sigma \cup Z\bar{\sigma}$$

or

$$|Z| = |Z\sigma| + |Z\bar{\sigma}| - |Z\sigma \cap Z\bar{\sigma}| \dots \dots \dots (3-12)$$

For set  $Z\sigma$  and  $Z\bar{\sigma}$  to be disjoint,

$$N > 2K, \text{ and } |Z\sigma| + |Z\bar{\sigma}| < 2^N \dots \dots \dots (3-13)$$

From equations (3-10) and (3-11),

$$|Z\sigma| + |Z\bar{\sigma}| = 2^{(k-1)} [N+2-K] + 2^{K-1} [N+2-K] \dots \dots \dots (3-14)$$

Combining (3-13) and (3-14)

$$\begin{aligned} 2^K [N+2-K] &< 2^N \\ 2^{N-K} - (N-K+2) &> 0 \\ N-K &> 2 \end{aligned}$$

$$\text{or } N \geq 3+K \dots \dots \dots (3-15)$$

Combining (3-12) and (3-14)

$$|Z| = 2^K \{N+2-K\} - |Z\sigma \cap Z\bar{\sigma}| \dots \dots \dots (3-16)$$

$$|Z| \leq 2^K \{N+2-K\} \dots \dots \dots (3-16a)$$

It can be shown that  $|Z \cap Z^c|$  is zero when  $N > 2K$ ,  $N \geq 3+K$  then the equality in (3-16a) would hold.

Let the following probabilities be assigned:

$$\begin{aligned} \delta_1 &= \text{The probability of picking a synchronizing bit} \\ &\quad \text{and observe } \psi_j \text{ belonging to } Z. \\ &= 1/n \dots \dots \dots (3-17) \end{aligned}$$

$$\begin{aligned} \delta_2 &= \text{The probability of picking a synchronizing bit} \\ &\quad \text{and observe } N\text{-tuple } \psi_j \text{ outside } Z. \\ &= 0 \dots \dots \dots (3-18) \end{aligned}$$

$$\delta_3 = \text{The probability of picking a non-synchronizing bit and yet observe an } N\text{-tuple belonging to } Z.$$

$$\delta_4 = \text{The probability of picking a non-synchronizing bit and observe an } N\text{-tuple outside of } Z.$$

$$\delta_3 = \text{Probability of picking a non-synchronizing bit.}$$

X

Probability of getting an N-tuple belonging to Z.

$$= \left(\frac{n-1}{n}\right) \times \frac{|Z|}{2^N} \dots \dots \dots (3-19)$$

Combining (3-16a) and (3-19)

$$\delta_3 = \left(\frac{n-1}{n}\right) \frac{2^K(N+2-K)}{2^N} \dots \dots \dots (3-20)$$

$$\delta_4 = \text{Probability of picking a non-synchronizing bit.}$$

X

Probability of getting an N-tuple outside Z.

$$= \left(\frac{n-1}{n}\right) \frac{2^N - |Z|}{2^N} \dots \dots \dots (3-21)$$

Combining (3-16a) and (3-21) gives

$$\delta_4 > \left(\frac{n-1}{n}\right) \frac{2^N - 2^K [N+2-K]}{2^N} \dots\dots\dots (3-22)$$

It is desirable to make  $\delta_2 + \delta_3 \ll \delta_1 + \delta_4$

$$\text{Let } \delta_1 + \delta_4 = M(\delta_3 + \delta_2) \dots\dots\dots (3-23)$$

Where M represents the reliability factor and Let  $N' \approx N$  for those cases where the equality in (3-16a) holds. If we combine (3-20), (3-21), (3-22), (3-23) we get

$$\frac{1}{n} + \frac{(n-1)}{n} \left[ \frac{2^{N'} - 2^K \{N'+2-K\}}{2^{N'}} \right] = \frac{(n-1)}{n} \left\{ \frac{2^K [N'+2-K]}{2^{N'}} \right\} M$$

or  $2^{N'} + (n-1) \{2^{N'} - 2^K [N'+2-K]\} = (n-1) \{2^K [N'+2-K]\} M$

or

$$2^{N'} n = (n-1) \{2^K (N'+2-K)\} (M+1)$$

$$N' = K + \log_2 \frac{(n-1)}{n} + \log_2 (M+1) + \log_2 (N'+2-K) \dots\dots\dots (3-24)$$

Where K is the number of frames affected by the burst and  $N' \approx N$ .

TABLE 3-3 THE NUMBER OF FRAMES FOR EXAMINATION IN BURST ERRORS

	K	1	2	3	4	5	6	7	8	9	10
$M= 2^0$	N'	4	5	6							
$M= 2^5$	N'	9	10	11	12	13	14	15			
$M= 2^{10}$	N'	15	16	17	18	19	20	21	22	23	24
$M= 2^{15}$	N'	20	21	22	23	24	25	26	27	28	29
$M= 2^{20}$	N'	25	27	28	29	30	31	32	33	34	35
$M= 2^{25}$	N'	31	32	33	34	35	36	37	38	39	40
$M= 2^{30}$	N'	36	37	38	39	40	41	42	43	44	45
$M= 2^{35}$	N'	41	42	43	44	45	46	47	48	49	50
$M= 2^{40}$	N'	47	48	49	50	51	52	53	54	55	56
$M= 2^{45}$	N'	52	53	54	55	56	57	58	59	60	61
$M= 2^{50}$	N'	57	58	59	60	61	62	63	64	65	66
$M= 2^{55}$	N'	62	63	64	65	66	67	68	69	70	71

Some of the values of  $N'$  for different values of  $K$  and  $M$  as computed from equation (3-24) are shown in Table 3-3. In those cases where the condition that  $N' > 2K$  is not met, no value for  $N'$  for the corresponding values of  $K$  and  $M$  are given. It is obvious that  $N'$  varies linearly with  $\log_2 M$  for any given value of  $K$ . The curve for the noiseless case is shown in dotted line for comparison purposes in Figure 3.5 which shows the contents of Table 3-3.

If we compare Table 3-1 with Table 3-3, we realize that for the same value of  $e$  and  $M$ , the value of  $N'$  given in Table 3-3 is smaller than that in Table 3-1. Since burst errors are bunched together, the number of  $N$ -tuples in  $Z$  is smaller for the burst error case than it is for the case where the errors are not bunched together. This leads to the upper bound on  $N$  in the burst errors being tighter than that in random errors, for a given number of frames affected by errors, and a given reliability factor.

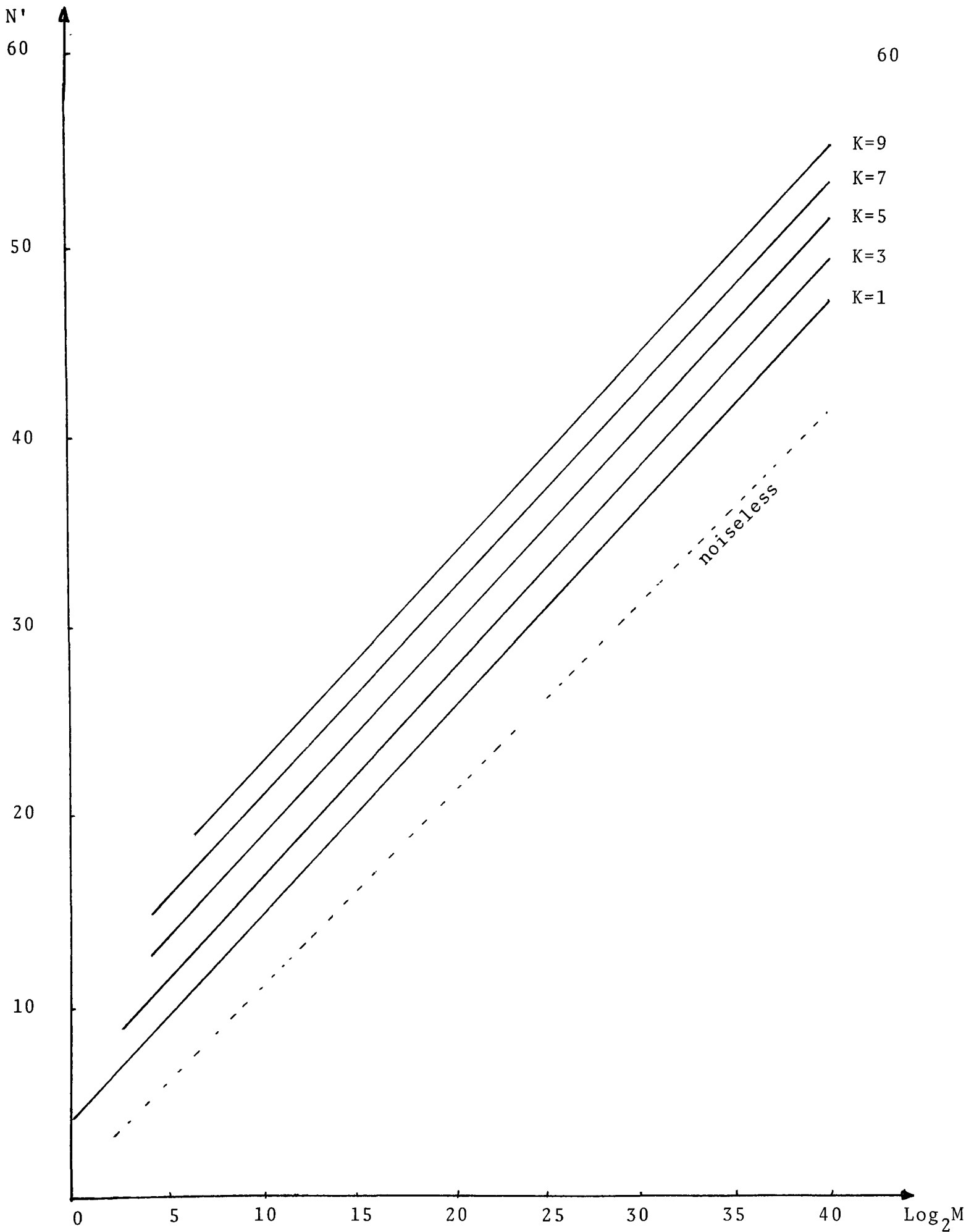


FIGURE 3.5 NUMBER OF FRAMES FOR EXAMINATION IN BURST ERRORS  $K_n$  BITS LONG.

### 3.4.3 LOG-NORMAL BURST LENGTH DISTRIBUTION

If we assume that the burst have a log-normal length distribution,

$$P(\ell) = \frac{1}{\sigma \ell \sqrt{2\pi}} \exp[-(\ln \ell - a)^2 / 2\sigma^2] \quad \sigma > 0$$

Where  $a$  and  $\sigma$  constants and  $P(\ell)$  is the probability density function of the burst lengths.

The mean burst length  $\bar{\ell}$  is

$$\bar{\ell} = \sum_{\ell=2}^{\infty} \ell P(\ell)$$

$$\bar{\ell} \approx \frac{1}{\sigma \sqrt{2\pi}} \int_2^{\infty} \exp[-(\ln \ell - a)^2 / 2\sigma^2] d\ell$$

$$\bar{\ell} \approx \exp[a + \sigma^2 / 2]$$

Mean burst length (MBL)  $\approx \exp[a + \sigma^2 / 2]$ .....(3-25)

Let the value of  $\sigma$  be chosen as 1 (for convenience).

From equation (3-25), the mean burst length (MBL) is given by

$$\text{MBL} = \exp[a + \sigma^2 / 2]$$

$$\text{MBL} = \exp[a + 1/2]$$
.....(3-26)

The cumulative distribution function

$$F(\ell) \approx \int_2^{\ell} P(\ell) d\ell$$

$$= \frac{1}{\sigma \sqrt{2\pi}} \int_2^{\ell} \frac{1}{t} \exp[(\ln t - a)^2 / 2\sigma^2] dt$$

$$\text{for } \sigma = 1,$$

$$F(\ell) = \frac{1}{\sqrt{2\pi}} \int_2^{\ell} \frac{1}{t} \exp[-(\ln t - a)^2/2] dt$$

which gives

$$F(\ell) = \text{erf}(\ln \ell - a) - \text{erf}(-\infty)$$

But  $\text{erf}(-\infty) = -0.5$

so

$$F(\ell) = 0.5 + \text{erf}(\ln \ell - a) \dots \dots \dots (3-27)$$

where

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp[-1/2 t^2] dt$$

Tables 3-4 to 3-11 give some values of burst length and the corresponding values of the burst length distribution  $F(\ell)$  for different values of mean burst length (MBL). The contents of Tables 3-4 to 3-11 are shown graphically in Figure 3-6.

Figure 3-7 shows the burst length distribution obtained from measurements made on telephone lines between Pretoria and Riverhead.<sup>24</sup> Comparing Figure 3-6 and Figure 3-7, one notices that the practical case (Figure 3-7) is very similar to the curve of 50 bits MBL in Figure 3-6. This could be viewed as backing the assumption made with regard to the burst lengths having a log-normal distribution.

TABLE 3-4 COMMULATIVE BURST DISTRIBUTION FUNCTION FOR MBL OF 10 bits

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
2	0.1335	30	0.9452
3	0.2420	40	0.9706
4	0.3409	50	0.9826
5	0.4246	60	0.9890
6	0.4960	70	0.9927
7	0.5596	80	0.9951
8	0.6103	90	0.9965
9	0.6553	100	0.9975
10	0.6915	150	0.9993
20	0.8849	200	0.9998

TABLE 3-5 COMMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 20 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
2	0.0351	30	0.8159
3	0.0808	40	0.8830
4	0.1335	50	0.9207
5	0.1867	60	0.9441
6	0.2388	70	0.9599
7	0.2912	80	0.9699
8	0.3372	90	0.9772
9	0.3821	100	0.9826
10	0.4207	200	0.9974
20	0.6915	300	0.9993

TABLE 3-6 COMMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 50 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
2	0.0033	50	0.6915
3	0.0107	60	0.7518
4	0.0217	70	0.7996
5	0.0359	80	0.8340
6	0.0526	90	0.8621
7	0.0721	100	0.8849
8	0.0918	200	0.9713
9	0.1151	300	0.9890
10	0.1335	400	0.9951
20	0.3409	500	0.9974
30	0.4960	600	0.9986
40	0.6103	700	0.9992

TABLE 3-7 CUMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 100 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
2	0.0003	70	0.5557
3	0.0013	80	0.6064
4	0.0030	90	0.6517
5	0.0062	100	0.6915
6	0.0102	200	0.8830
7	0.0154	300	0.9441
8	0.0228	400	0.9699
9	0.0280	500	0.9821
10	0.0351	600	0.9890
20	0.1335	700	0.9927
30	0.2388	800	0.9949
40	0.3372	900	0.9964
50	0.4207	1000	0.9974
60	0.4920	2000	0.9998

TABLE 3-8      CUMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 200 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
2	0.0000	80	0.3372
3	0.0001	90	0.3821
4	0.0003	100	0.4246
5	0.0007	200	0.6915
6	0.0013	300	0.8159
7	0.0022	400	0.8830
8	0.0033	500	0.9207
9	0.0047	600	0.9452
10	0.0062	700	0.9599
20	0.0359	800	0.9699
30	0.0808	900	0.9772
40	0.1357	1000	0.9826
50	0.1867	2000	0.9974
60	0.2388	3000	0.9993
70	0.2912	4000	0.9998

TABLE 3-9 CUMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 400 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
5	0.0001	200	0.4246
6	0.0001	300	0.5832
7	0.0002	400	0.6915
8	0.0003	500	0.7642
9	0.0005	600	0.8186
10	0.0007	700	0.8554
20	0.0064	800	0.8830
30	0.0183	900	0.9049
40	0.0359	1000	0.9222
50	0.0571	1500	0.9656
60	0.0808	2000	0.9821
70	0.1075	3000	0.9941
80	0.1335	4000	0.9974
90	0.1611	5000	0.9988
100	0.1894	6000	0.9993

TABLE 3-10 CUMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 800 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
10	0.0001	700	0.6443
20	0.0007	800	0.6915
30	0.0027	900	0.7324
40	0.0064	1000	0.7673
50	0.0116	1500	0.8708
60	0.0183	2000	0.9222
70	0.0268	3000	0.9664
80	0.0359	4000	0.9826
90	0.0465	5000	0.9904
100	0.0582	6000	0.9951
200	0.1894	7000	0.9962
300	0.3156	8000	0.9974
400	0.4246	9000	0.9982
500	0.5120	10000	0.9988
600	0.5871	20000	0.9999

TABLE 3-11 COMMULATIVE DISTRIBUTION FUNCTION FOR MBL OF 1000 bits.

$\ell$	$F(\ell)$	$\ell$	$F(\ell)$
20	0.0003	700	0.5557
30	0.0013	800	0.6064
40	0.0033	900	0.6517
50	0.0062	1000	0.6915
60	0.0102	2000	0.8830
70	0.0154	3000	0.9452
80	0.0212	4000	0.9699
90	0.0287	5000	0.9826
100	0.0359	6000	0.9890
200	0.1335	7000	0.9927
300	0.2388	8000	0.9951
400	0.3372	9000	0.9964
500	0.4207	10000	0.9974
600	0.4960	20000	0.9998

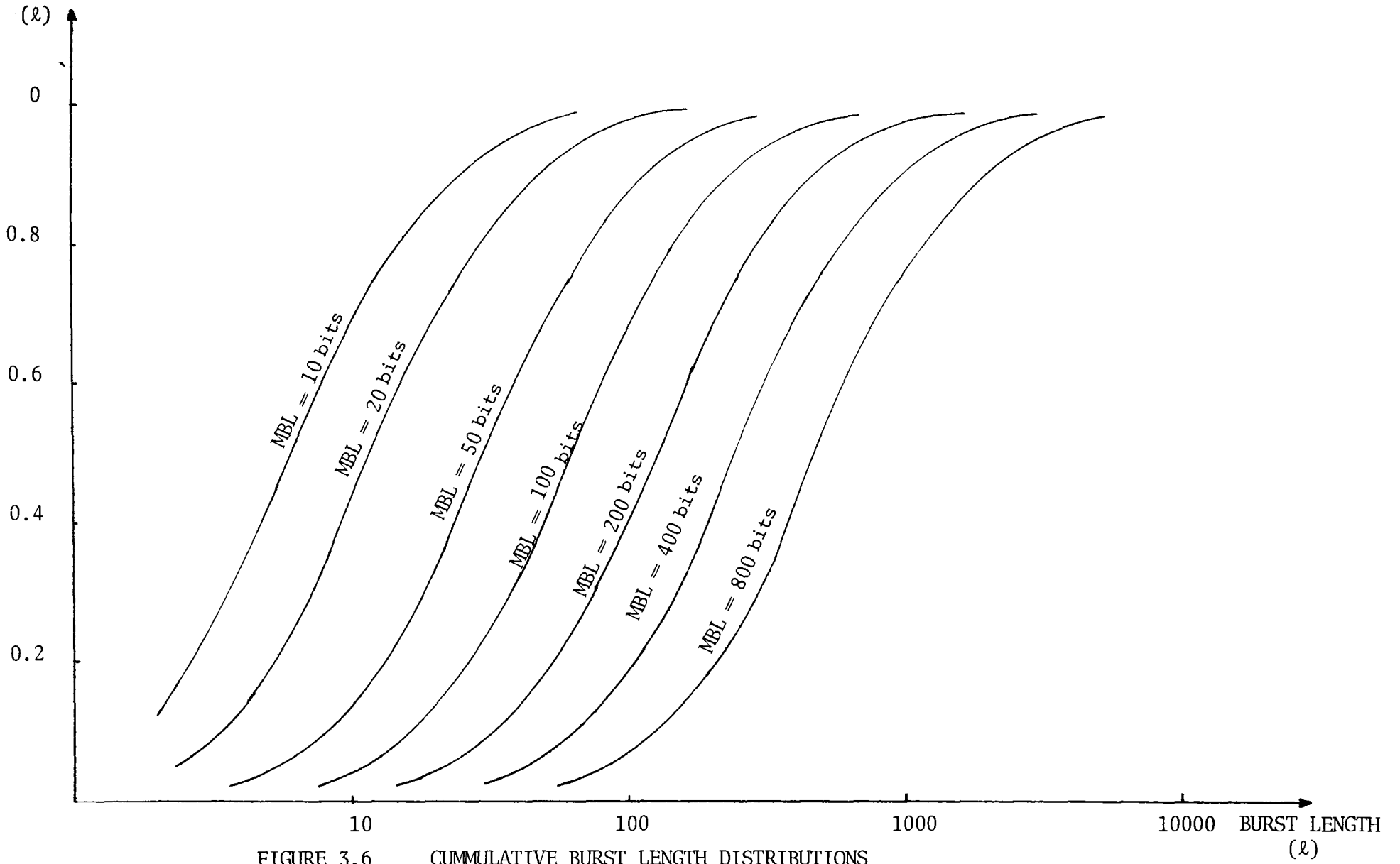


FIGURE 3.6 CUMULATIVE BURST LENGTH DISTRIBUTIONS

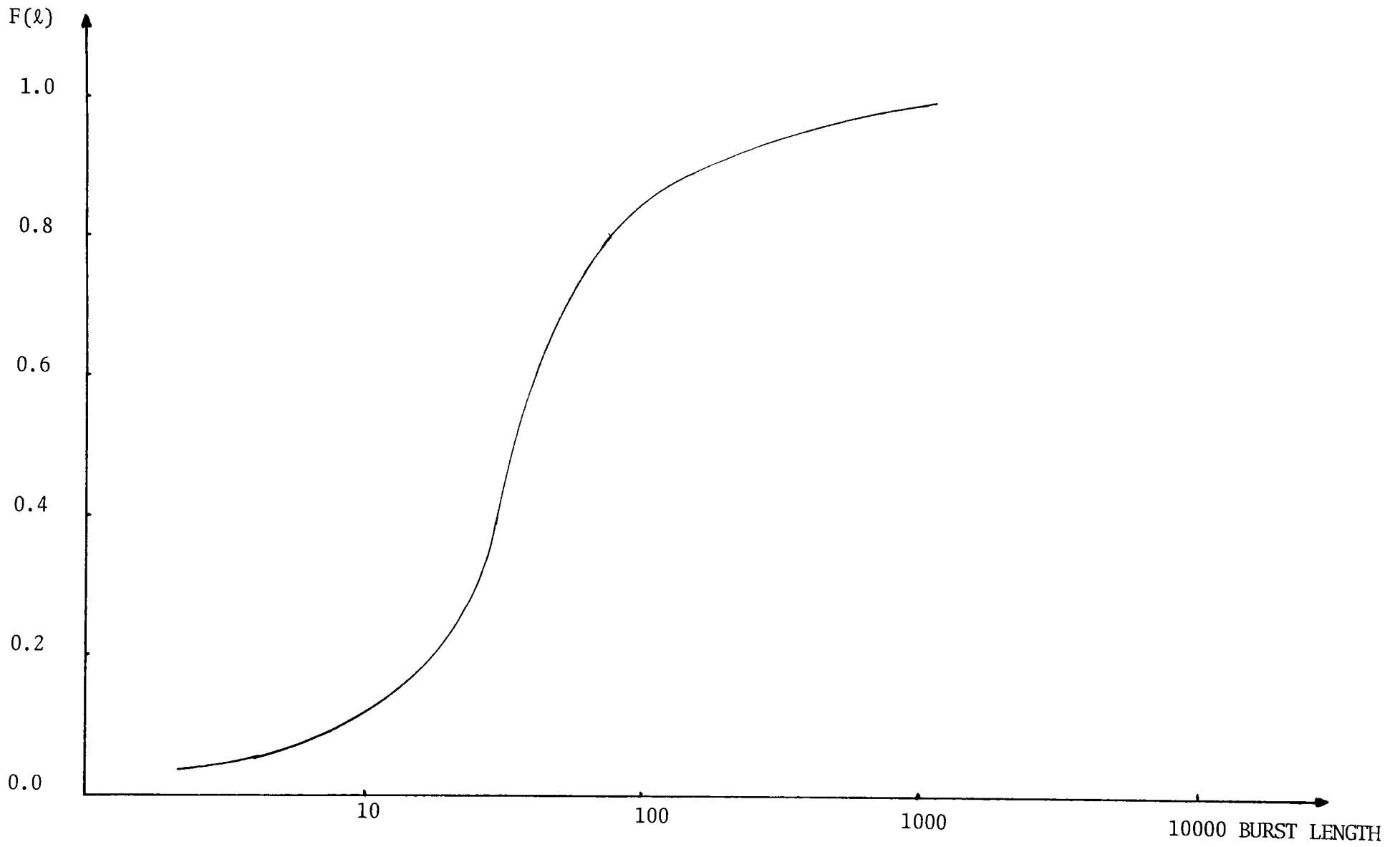


FIGURE 3.7 BURST LENGTH DISTRIBUTION FOR BURSTS MEASURED ON PRETORIA-RIVERHEAD ROUTE.

### 3.4.3 SYNCHRONIZATION IN A LOG-NORMAL BURST ERROR DISTRIBUTION

In practice, the actual burst length might not be known. However, the burst error characteristics of the channel might be known.

In this section, an attempt is made to determine the number of frames which have to be examined for a given reliability factor, in the event of burst errors. It is assumed that the bursts have a log-normal length distribution.

The incoming bit stream is

$$\dots a_{-5} a_{-4} a_{-3} a_{-2} a_{-1} a_0 a_1 a_2 a_3 \dots a_n a_{2n} a_{2n+1} \dots$$

where  $a_n a_{2n} a_{3n} \dots$  are synchronizing bits.

The strategy adopted here is as follows:

When a burst occurs, it is assumed that  $K'$  frames are affected by the burst. In other words, the burst length of  $K'n$  bits is assumed. Then the sequence  $\psi_j$  is formed where

$$\psi_j = a_j a_{j+n} a_{j+2n} \dots a_{j+(N-1)n}$$

If the  $N$ -tuple formed by  $\psi_j$  belongs to the acceptable set  $Z$ , it is concluded that  $a_j a_{j+n} a_{j+2n} \dots$  are synchronizing bits. If not, then  $\psi_{j+1}$  is formed where

$$\psi_{j+1} = a_{j+1} a_{j+1+n} a_{j+1+2n} \dots a_{j+1+(N-1)n}$$

And the process continues.

From equation (3-16), the number of N-tuples in the set Z is

$$|Z| = 2^{K'} \{N+2-K'\} \text{ for } N > 2K'.$$

For the case where K' frames are affected by the burst.

Let the following probabilities be examined:

$\beta_1$  = Probability of picking a synchronizing bit and observe an N-tuple  $\psi_j$  belonging to Z.

$\beta_2$  = Probability of picking a synchronizing bit and observe an N-tuple outside Z.

$\beta_3$  = Probability of picking a non-synchronizing bit and observe an N-tuple belonging to Z.

$\beta_4$  = Probability of picking a non-synchronizing bit and observe an N-tuple outside Z.

$F(K'n)$  = The probability of the burst length being less than or equal to K'n bits.

$\beta_1$  = (Probability of picking a synchronizing bit)

X

(Probability of the burst length  $\leq$  K'n bits)

$$= \frac{1}{n} \times F(K'n) \dots \dots \dots (3-28)$$

$\beta_2$  = (Probability of picking a synchronizing bit) \

× (Probability of burst being greater than K'n bits)

$$= \frac{1}{n} \{1-F(K'n)\} \dots \dots \dots (3-29)$$

$\beta_3$  = (Probability of picking a non-synchronizing bit)

X

(Probability of getting an N-tuple belonging to Z)

$$= \left(\frac{n-1}{n}\right) \frac{|Z|}{2^N}$$

$$= \left(\frac{n-1}{n}\right) \frac{2^{K'} [N+2-K']}{2^N} \dots \dots \dots (3-30)$$

$$\beta_4 = (\text{Probability of picking a non-synchronizing bit}) \times$$

$$(\text{Probability of getting N-tuple outside } |Z|)$$

$$= \left(\frac{n-1}{n}\right) \frac{2^{N-2^{K'}} [N+2-K']}{2^N} \dots \dots \dots (3-31)$$

$$\text{Let } \beta_1 + \beta_4 = M(\beta_2 + \beta_3) \dots \dots \dots (3-32)$$

From equations (3-28), (3-29), (3-30) and (3-31)

$$\beta_2 = \frac{1}{n} - \beta_1, \quad \beta_3 = \frac{n-1}{n} - \beta_4$$

Putting that in (3-32),

$$\beta_1 + \beta_4 = M \left[ \frac{1}{n} - \beta_1 + \frac{n-1}{n} - \beta_4 \right]$$

$$= M [1 - (\beta_1 + \beta_4)]$$

$$(M+1) \{\beta_1 + \beta_4\} = M \dots \dots \dots (3-33)$$

Substituting for  $\beta_1$  and  $\beta_4$  in equation (3-33)

$$(M+1) \left[ \frac{1}{n} F(K'n) + \frac{(n-1)}{n} \left\{ \frac{2^{N-2^{K'}} [N+2-K']}{2^N} \right\} \right] = M$$

or

$$(M+1) [2^N F(K'n) + (n-1) \{2^{N-2^{K'}} [N+2-K']\}] = 2^N M n$$

or

$$2^N \{F(K'n) [M+1] + (n-1) (M+1) - M n\} = 2^{K'} (n-1) (M+1) [N+2 - K']$$

or

$$2^N \{(M+1) F(K'n) - M + n - 1\} = 2^{K'} (n-1) (M+1) [N+2 - K']$$

or

$$2^N \{(M+1) [F(K'n) - 1] + n\} = 2^{K'} (n-1) (M+1) [N+2 - K']$$

or

$$\begin{aligned}
 N &= K' + \log_2(n-1)[N+2-K'] \\
 &\quad - \log_2\{(M+1)[F(K'n)-1]+n\} \\
 N &= K' + \log_2(M+1) + \log_2(n-1) + \log_2(N+2-K') \\
 &\quad - \log_2\{(M+1)[F(K'n)-1]+n\} \dots \dots \dots (3-34)
 \end{aligned}$$

Where  $K'$  is the assumed number of frames affected by the burst.

Examination of equation 3-34 reveals that in the limit when  $F(K'n)$  is equal to 1, the equation reduces to

$$N = K' + \log_2 \frac{(n-1)}{n} + \log_2(M+1) + \log_2[N+2-K'] \dots \dots \dots (3-34a)$$

Which is similar to equation (3-24), which gives the number of frames to be examined when a burst of length  $Kn$  bits occurs. In the above equation (3-34a) however, the number of bits affected by the burst is  $K'n$ .

In case of the assumed burst length distribution

$$F(K'n) \approx \frac{1}{\sqrt{2\pi}} \int_0^{K'n} \frac{1}{t} e^{- (Int-a)^2/2} dt.$$

where  $a$  is a constant  $> 0$ .

From equation (3-34), it can easily be deduced that for a given number of bits in a frame  $N$ , an assumed number of frames affected by the burst  $K'$ , and a given burst length distribution  $F(K'n)$ , the maximum value of  $M$  is limited. This is due to the term  $-\log_2[n - \{1 - F(K'n)\}(M+1)]$ .

For that term to be meaningful,

$$n - \{1 - F(K'n)\}(M+1) > 0$$

or

$$n > \{1 - F(K'n)\}(M+1)$$

or

$$(M+1) < \frac{n}{[1-F(K'n)]}$$

or

$$M < \frac{n}{[1-F(K'n)]} - 1$$

But  $n$  is usually  $\gg 1$

so

$$M_{\max} \approx \frac{n}{[1-F(K'n)]} \dots\dots\dots (3-35)$$

Where  $M_{\max}$  is the maximum possible attainable reliability factor. When we assume that the burst length is  $K'n$  bits, there is no guarantee that the burst is actually that length. If a burst of length greater than  $K'n$  occurs then a wrong conclusion will probably be reached with regard to which bits are the synchronizing ones. That is why there is a limit to the maximum attainable reliability factor when burst lengths are probabilistic rather than deterministic. It is interesting to note that for  $F(K'n) = 1$ ,  $M_{\max} = n/0 = \infty$  the situation becomes deterministic in that case.

In Tables 3-12, 3-13 and 3-14, some values of  $N$  are worked out for different values of  $K'$  and  $M$  based on equation (3-34). Three values of mean burst length (MBL) are considered namely 50 bits, 200 bits and 800 bits. The contents of Tables 3-12, 3-13 and 3-14 are shown in Figures 3-8, 3-9 and 3-10 respectively. It is assumed that the number of bits in each frame is 200.

From the graphs in Figures 3-8, 3-9 and 3-10, it seems as though for reliability factor  $M < M_{\max}/2^3$ ,  $N$  varies

approximately linearly with  $\log_2 M$  for a given value of  $K'$ . As  $M$  approaches  $M_{\max}$  however,  $N$  increases very rapidly with little increase in  $\log_2 M$ . In Figure 3-8 with  $MBL = 50$  bits for example. If it is required to achieve a reliability factor of  $2^{10}$ , then a burst length of 1 frame (200 bits) can be assumed in which case 15 frames have to be examined. On the other hand if a reliability factor of  $2^{15}$  is required then the assumed burst length has to be 3 frames (600 bits) in which case 22 frames have to be examined before making the decision on whether we are in synchronism or not. In a way, the required reliability factor dictates the assumed number of frames affected by the burst.

TABLE 3.12 NUMBER OF FRAMES TO BE EXAMINED IN BURST  
 ERRORS WITH MBL OF 50 BITS

	K'	1	2	3	4	5
$M=2^0$	N	5	6	7		
$M=2^2$	N	7	8	8	9	
$M=2^4$	N	9	10	10	11	12
$M=2^6$	N	11	12	12	13	14
$M=2^8$	N	13	14	15	15	17
$M=2^{10}$	N	15	16	17	18	19
$M=2^{12}$	N	19	18	19	20	21
$M=2^{12.5}$	N	20	18	19	20	21
$M=2^{12.76}$	N	26	19	20	21	22
$M=2^{13}$	N		19	20	21	22
$M=2^{14}$	N		21	21	22	23
$M=2^{15}$	N		24	22	23	24
$M=2^{15.3}$	N		30	23	23	24
$M=2^{16}$	N			25	25	25
$M=2^{17}$	N			28	26	26
$M=2^{18}$	N				28	28
$M=2^{18.6}$	N				35	29
$M=2^{19}$	N					30

TABLE 3-13 NUMBER OF FRAMES TO BE EXAMINED IN BURST ERRORS WITH MBL OF 200 bits.

	K'	1	3	5	7	10
$M = 2^0$	N	4				
$M = 2^2$	N	6	8			
$M = 2^4$	N	8	10	12		
$M = 2^6$	N	11	13	15	17	
$M = 2^8$	N	14	15	17	19	22
$M = 2^9$	N	15	16	18	20	23
$M = 2^{9.34}$	N	26	16	18	20	23
$M = 2^{10}$	N		17	19	21	24
$M = 2^{11}$	N		20	20	22	25
$M = 2^{11.5}$	N		21	20	22	25
$M = 2^{11.83}$	N		28	21	23	26
$M = 2^{12}$	N			22	23	26
$M = 2^{13}$	N			24	25	27
$M = 2^{13.48}$	N			31	25	28
$M = 2^{14}$	N				27	29
$M = 2^{14.57}$	N				35	30
$M = 2^{15}$	N					30
$M = 2^{16}$	N					33
$M = 2^{16.23}$	N					42

TABLE 3-14 NUMBER OF FRAMES TO BE EXAMINED IN BURST ERRORS WITH  
MBL OF 800 bits.

	K'	1	3	5	7	10
$M = 2^0$	N	4				
$M = 2$	N	5	7			
$M = 2^2$	N	6	8			
$M = 2^3$	N	7	9	11		
$M = 2^4$	N	8	10	12		
$M = 2^5$	N	9	11	13	16	
$M = 2^6$	N	11	13	15	17	
$M = 2^7$	N	13	14	16	18	21
$M = 2^{7.5}$	N	14	15	17	18	21
$M = 2^{7.9}$	N	18	16	17	19	22
$M = 2^8$	N		16	18	19	22
$M = 2^{8.5}$	N		18	18	20	22
$M = 2^{8.9}$	N		23	19	21	23
$M = 2^9$	N			19	21	23
$M = 2^{9.7}$	N			24	22	24
$M = 2^{10}$	N				23	25
$M = 2^{10.38}$	N				33	36
$M = 2^{11}$	N					28
$M = 2^{11.5}$	N					32

MBL = 50 bits

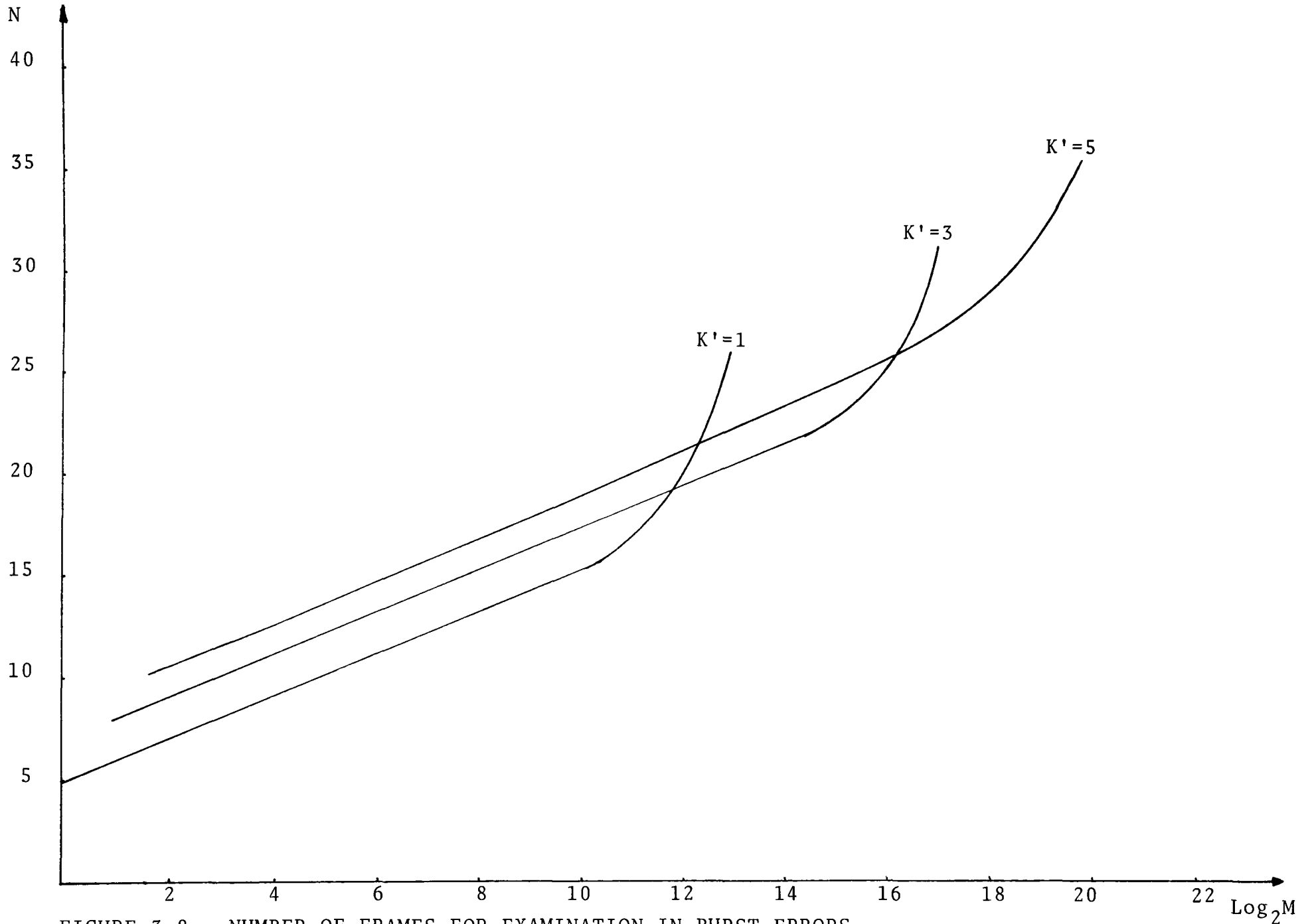


FIGURE 3.8 NUMBER OF FRAMES FOR EXAMINATION IN BURST ERRORS.

MBL = 200

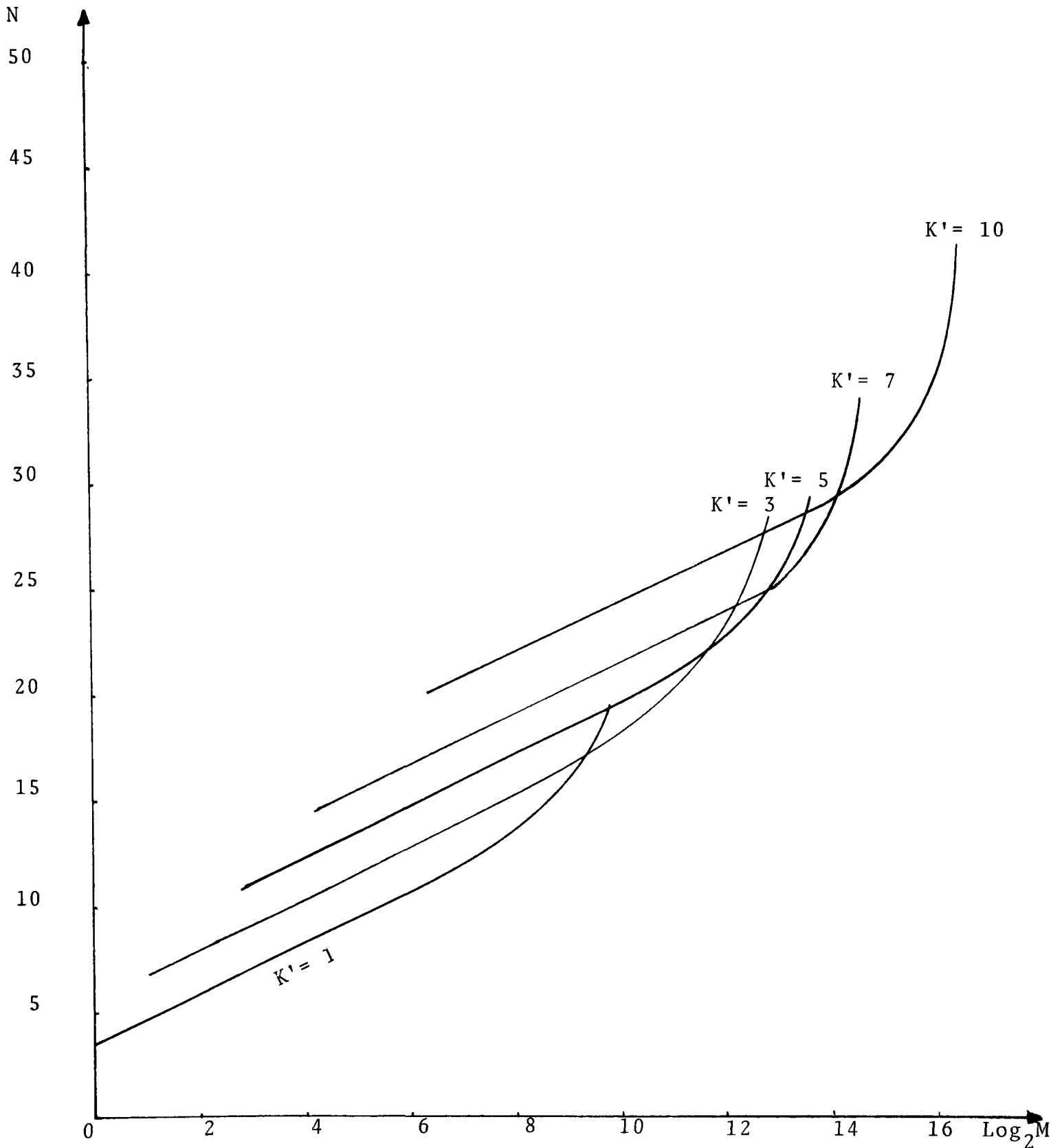


FIGURE 3.9 NUMBER OF FRAMES FOR EXAMINATION IN BURST ERRORS.

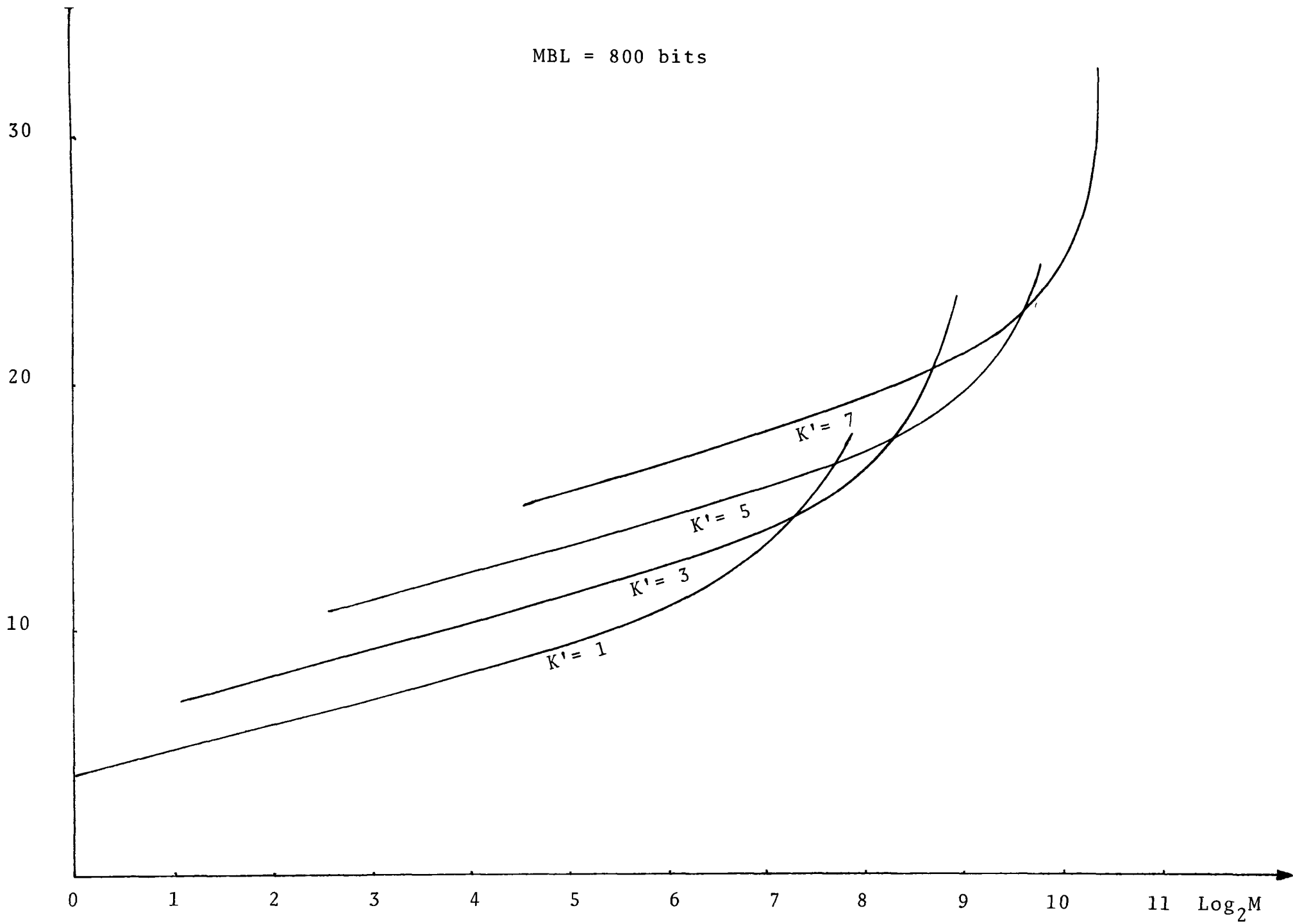


FIGURE 3.10 NUMBER OF FRAMES FOR EXAMINATION IN BURST ERRORS.

## 3-4.5 PROBABILITY OF FALSE LOCK

The system will 'Lock' on the wrong bit, if the sequence  $\psi_j$  belongs to the acceptable set Z, even though the bits

$$a_j \ a_{i+n} \ a_{j+2n} \ \dots \ a_{j+(N-1)n}$$

are not synchronizing bits.

From equation (3-30), the probability of picking a non-synchronizing bit and yet observing an N-tuple belonging to Z is given by

$$\beta_3 = \frac{n-1}{n} \frac{2^{K'} [N+2-K']}{2^N}$$

so the probability of false lock  $PFL = \beta_3 = \frac{(n-1)}{n} \frac{2^{K'} [N+2-K']}{2^N} \dots (3-36)$

using (3-34) and (3-36) PFL works out to

$$PFL = \frac{2^N \{ (M+1) [F(K'n) - 1] + n \}}{n \times 2^N (M+1)}$$

or

$$PFL = \frac{[F(K'n) - 1]}{n} + \frac{1}{(M+1)} \dots (3-37)$$

### 3-4.6 PROBABILITY OF MISSING THE SYNCHRONIZING BITS (PMF)

The synchronizing bits will be interpreted as information or data bits if the formed sequence  $\psi_n$  does not belong to the acceptable set Z.

That will happen when a burst larger than the assumed burst length  $K'n$  takes place and drives  $\psi_n$  out of Z.

From equation (3-29), the probability of picking a synchronizing bit and yet observe an N-tuple  $(\psi_n)$  outside Z is  $\beta_2$ . The probability of missing the synchronizing bits is PMF.

$$\text{PMF} = \beta_2 = \frac{1}{n} [1 - F(K'n)] \dots\dots\dots(3-38)$$

As expected,  $\text{PMF} + \text{PFL} = 1/(M+1)$

In other words the ratio of the probability of making the wrong decision to the probability of making the right one is  $1/(M+1)$ .

Supposing the system lost frame synchronism once a day and the reliability factor of  $2^{10}$  were chosen. The probability of making the wrong decision would be approximately  $10^{-3}$  which would mean that on the average, a wrong decision would be made once every three years.

CHAPTER 4  
SOME CONCLUDING REMARKS

The importance of synchronization in communication systems cannot possibly be over-emphasized. The results of loss of synchronism can range from a simple click in the telephone receiver to something as serious as a breakdown of communication leading to a request for re-transmission.

The values of  $N$  worked out in this paper are in some cases too high due to the large values of reliability factor associated with them. In practice, it is desirable to acquire synchronism as soon as possible after it has been lost. This puts a practical limit to the number of frames which have to be examined before reaching a decision. This in turn limits the value of reliability factor which can be achieved.

The results of this thesis however, give us some idea of what is required in the acquisition of frame synchronism in a time division multiplexer employing one bit per frame for synchronization. As cited in Section 3.3, random errors are not the major source of loss of synchronism in the T-1 carrier system. That means that burst errors are responsible for the the largest number of loss of synchronism incidences in that system.

In the cases where rapid acquisition of frame synchronism is required, it is possible to use more than one bit<sup>19,38,39</sup> per frame or indeed a whole channel for synchronization.<sup>2</sup>

As opposed to the synchronization scheme considered in here, which requires synchronizing bits, it is possible to use self-synchronizing schemes which require some sort of coding of the data bits.<sup>2,40</sup>

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