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**Investigation of Magnetothermal and Critical
Current Hysteresis in Polycrystals of Low and
High T_c Type II Superconductors**

by

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**Thesis submitted to the University of Ottawa
in Partial fulfillment of the requirements for the
degree of Doctor of Philosophy in Physics**

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My deep gratitude to my wife who has been patiently waiting to see this thesis completed and witnessed all the stages of its progress, chapter by chapter. I dedicate her this thesis for her patience, support and love.

Also I dedicate this thesis to my parents, brothers and sisters whose love, support and encouragement have been among the main reasons of my success throughout my life.

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Abstract

The model of Clem and Hao and others is extended to account for the enhancement of the Meissner effect observed in single crystals of hysteretic type II superconductors upon thermal cycling below T_c in static applied magnetic fields. Predictions are made about the features of the final closed thermal hysteresis loop achievable by extensive cycling and their dependence on the temperature limits T_1 and $T_2 < T_c$ chosen for the cycles.

A large variety of observations, by several workers, of a narrow peak of enhanced Meissner effect near T_c in polycrystalline type II superconductors upon slow warming in static applied fields after fast field cooling, are qualitatively and quantitatively accounted for by a model where we introduced the scheme developed above for single crystals into a weak-linked intergranular network (matrix). This "two tier" framework is then extended to describe the enhancement of the Meissner effect observed by Hyun by thermal cycling of weak-linked Nb_3Sn below T_c in a static field.

A simple framework is presented which quantitatively develops the proposal of Evetts and Glowacki that the superposition of the applied field H_a and the return field, H_r , of the magnetized grains, is the cause of the hysteretic behaviour of I_c in weak-linked high T_c superconductors and the occurrence of a peak in I_c versus H_a descending and reascending, after an excursion to various values, denoted H^{cycle} , or after field cooling in different H^{cool} . Observations by several workers on the dependence, of the position of four categories of peaks of I_c , on H^{cycle} and H^{cool} are reproduced by this model and yield estimates of the "compression" factor C in the linear dipole approximation, $H_r = C M_g$. We also show that, ratios of the measured plateau values for the position of these peaks,

lead to an estimate for C which is independent of H_{c2} , the penetration field into the grains, and of the model chosen to calculate the dependence of the magnetization of the grains, M_g , on H_a .

Instead of the artificial pseudo-Josephson - junction expression generally used by other workers in the analysis of I_c hysteresis phenomena, we develop a family of formulae based on the critical state concept applied to idealized planar geometry. Exploiting an especially simple case from this family of formulae we reproduce a panoply of experimental curves of I_c versus H_a displayed in the literature and exhibiting a variety of features. Analysis of the extensive data of List et al reveals that our approach leads to results in accord with observations whereas the Josephson junction format does not.

We calculate the return field H_r along the edge of thin strips and thin disks, for situations where the countercirculating induced critical current densities are independent of field strength. These calculations exploit Ampere's law, the Biot-Savart formula, expressions developed by Brandt-Indenbom, and an empirical framework. Introducing these results for H_r versus the corresponding M_g we reproduce the data curves of Kwasnitza and Widmer who found I_c to be a double-valued function of M_g at chosen final fields H_f and final temperature T_f . Pursuing our model we make detailed predictions for the evolution of these double-valued curves as a function of H_f over the entire range from 0 to a large value.

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Chapter 1

Introduction

The major application of superconductors has been in the construction of large electromagnets. Each of the thousands of Magnetic Resonance Imaging (MRI) installations in the world comprises a large coil which embraces the patient. This coil is wound with NbTi cable which is maintained in the superconducting state at 4.2K by bathing in liquid helium at atmospheric pressure. The gigantic particle accelerators built at the Fermi Laboratory (Chicago), CERN (Geneva) and HERA (Hamburg) each comprises several hundred large electromagnets to guide and focus the beam of high energy particles. Again cables of the conventional low T_c alloy (NbTi) are used in the fabrication of these electromagnets operating at 4.2K in liquid helium.

All of these massive installations exploit the classical low T_c type II superconductor NbTi in the fabrication of the electromagnets because of the ability of this alloy to support large currents without dissipation of energy in intense magnetic fields. Not only is this alloy ductile, hence easily fabricated into kilometer length cables but metallurgists quickly found sequences of heat treatments and cold work which optimized the current carrying properties.

In contrast, high T_c materials, although superconducting at 77K in plentiful and inexpensive liquid nitrogen, are brittle. Further the crystallites of these high T_c ceramic copper oxides are very anisotropic. The ab planes of single crystals can support large current densities in strong magnetic fields even at 77K and orders of magnitude greater at 4.2K. Unfortunately the order parameter is dramatically depressed between the ab planes. As a consequence the critical current density along the c axis is several orders of

magnitude smaller than that along the ab planes. When the crystallites are randomly oriented, the ab planes of one crystal will generally not match the ab planes of the neighbouring crystals at the surfaces of contacts. Consequently the transport current flowing along a polycrystalline ribbon or wire will seldom pass directly and smoothly along their ab planes but will have to detour and percolate at various angles with respect to these planes. Considerable effort has been and continues to be expended in many laboratories to find processes which texture polygranular specimens so as to optimize the pathways. The intricate network comprising the periphery and surfaces of the grains and the contacts between the grains has been referred to as the weak-linked-network, WLN. This network will also be denoted the “matrix” in this thesis. Two basic arrangements of these flat and thin grains are pictorially referred to as the “brick wall” and the “railway switch” models. In this thesis however I will not address any of the details of the complex internal structures of these high T_c materials but will examine some of the wide variety of magnetothermal and hysteretic critical current phenomena which have been reported in the literature and occur as a consequence of the large current densities inside the grains and the weak current densities between the links in these polygranular assemblies.

We will see that the dramatic disparity between the large current carrying capacity of the grains and the small intergranular currents while giving rise to the fascinating behaviours studied in this thesis also allows crucial simplifications to be introduced in the analysis of these phenomena.

Firstly, as a consequence, the polygranular superconducting specimens can, to a first approximation, be regarded as an agglomeration of electrically isolated islands immersed in the homogeneous applied magnetic field H_a . Hence the grains can be viewed

as responding to changes of H_a independently of and unaffected by the concomitant response of the weak-linked-network to these same disturbances. The return field of the grains when they become magnetized will however seriously influence the capacity of the weak-linked-network (matrix) to support persistent currents and affect its distribution in the specimen.

The configurations of the return fields are certainly locally highly inhomogeneous and extremely complicated. In this thesis, and in all published analysis of the observations, this intractable situation is grossly simplified and the effective return field controlling the intergranular currents is regarded as uniform in the weak-linked-network and directed either parallel or antiparallel to $\vec{H}_a$ depending on the sign of the magnetization of the grains. It is certainly amazing that such a crude picture can lead to good qualitative and semi-quantitative descriptions of the observations.

Many of the experimental observations analysed in this thesis received only qualitative verbal interpretation by the authors in the articles reporting on their measurements. These results, although subsequently referenced by the authors or other workers, have remained without quantitative accounts. In these cases a simple mathematical model is proposed in this thesis and shown to reproduce their observations in detail. In some other instances, where although quantitative modeling of their data has been presented by the researchers, a simpler approach is proposed in this thesis which achieves comparable, and sometimes, better results. Also, in these circumstances, our new model, or the published model, is usually extended and broadened to qualitatively describe features of the published data which had not been accounted for by the authors. Finally this thesis also predicts in detail a variety of new behaviour which hopefully will

be explored and verified either in our laboratory by future graduate students or by other groups.

The established framework for describing magnetothermal behaviour in isotropic single crystals of type II superconductors is reviewed in the next chapter. This outline provides background for extensions of the prevailing concepts to weak-linked granular materials in the subsequent chapters. In that chapter however we also extend the existing model and make predictions for new behaviour.

For convenience, the research papers^[1-165] reporting on observations or analysis of the phenomena examined in this thesis and of behaviour related to these phenomena, the review articles^[R1-R7] and monographs^[M1-M4] containing sections developing background concepts and presenting information pertaining to these topics, are listed and numbered in the alphabetic order of the first author in the bibliography hence referenced accordingly in the text.

In this thesis the pronoun we will be used in presenting the work. The reasons for this are the following. Clearly it would be presumptuous and perhaps deceitful to pretend that I have pursued the research by myself and written the thesis without great assistance from my supervisor. Almost every day of the week for four years Dr. LeBlanc met with me to guide and assist in the development of the ideas and application of the models presented in this thesis. The rough drafts of articles jointly prepared for publication and based on the chapters of this thesis were available to me in writing the thesis. Further my supervisor made numerous suggestions for changes and improvements as this material was developed into this thesis.

Chapter 2

Enhancement of the Meissner Effect in Single Crystals of Hysteretic Type II Superconductors by Thermal Cycling

Abstract

In this chapter we extend the model, first developed by other researchers^[25, 34, 66, 67, 96], to qualitatively and quantitatively reproduce the observations of Hyun^[66] of the evolution and gradual enhancement of the magnitude of the diamagnetic magnetization M of field cooled single crystals of Nb_3Sn during subsequent thermal cycling between T_1 and T_2 , where $T_1 < T_2 < T_c$, with the applied field $H_a < H_{c1}(T_2)$ maintained fixed. The model also predicts that thermal cycling between T_1 and T_2 will eventually cause $\langle M \rangle$ versus T to trace a final closed hysteretic loop. The structure of this final hysteresis loop and the sequences of configurations of magnetic flux and persistent currents leading to and accompanying it are prescribed by the model.

I Introduction

The response of type II superconductors to variations in temperature while immersed in static magnetic fields $H_a < H_{c2}$, the upper critical field, has been investigated experimentally and theoretically by several workers^[20, 25, 29, 32, 34, 55, 65-67, 73, 76, 79, 86, 96, 107, 109, 136, 149, 161]. These measurements provide interesting data on the magnetothermal behaviour of these materials in the superconducting state. These studies also yield information on the dependence of the Meissner current I_M , the bulk pinning current

density J_c and any surface barriers^[27, 28, 51, 77, 78, 81] on temperature and magnetic field. These observations also shed light on the evolution of the magnetic flux density configurations and the patterns of circulation of the induced persistent currents in these materials in such circumstances. Consequently such experiments inform us on the interactions of the flux lines with each other, with pinning sites, and with the reversible field shielding Meissner current as a function of field and temperature.

Several simple and standard procedures have been exploited in these measurements. We first outline these procedures and the ensuing flux density distributions. A detailed description of the variety of flux configurations and patterns of persistent current circulation which are visualized to occur during these procedures will be presented in the next section. In these investigations it is the evolution of the magnetization which is observed as the changes in the temperature take place in fixed fields. In our modeling we ignore any decay of the “persistent” critical currents by thermal fluctuations, hence neglect the phenomena of magnetic relaxation^[2, 16, 68, 73, 82, 84, 106, 112, 133, 148, 154, 161, R7].

The procedures are the following:

(i) The specimen, placed in a magnetic field H_a , denoted H_{cool} , at a temperature $T_i > T_c$, is cooled from the normal state to a selected “final” temperature $T_f < T_c$. This procedure is denoted Field Cooling (FC) and the locus of the magnetization M versus T descending has sometimes been referred to as the Meissner curve. If the cooling is made to proceed slowly, so as to ensure that a uniform temperature exists inside the sample which closely corresponds to the descending ambient temperature, the process is labeled Slow Field Cooling (SFC). If however the temperature of the specimen is allowed to drop rapidly from T_i to T_f (for instance, by plunging a sample with no thermal insulation into

the cold bath) the process is labeled Fast Field Cooling (FFC). In this approach the sample is then maintained at the cold temperature for a time sufficiently long so as to ensure that its entire volume has cooled to T_f in the selected $H_a = H_{cool}$.

In the superconducting state just below T_c , the mutual repulsion of the unpinned flux lines, and their confinement in the specimen by the Meissner field shielding surface current, gives rise to the intrinsic reversible Abrikosov diamagnetism, denoted $M_{rev}(T, H)$. At a temperature $T_{irr} < T_c$, the appearance of pinning gives rise to opposition to the expulsion of flux lines from the body of the crystal (see Fig 2.1c). The evolution of the flux line density in the specimen, hence the resultant magnetization M , is now dictated by two competing mechanisms. The expulsion of a fraction of the flux lines threading the specimen is associated with the growth in the magnitude of the intrinsic diamagnetism, M_{rev} , as the temperature is made to descend. Expulsion of flux lines from the specimen can however be partially frustrated by the corresponding rise in the strength of the pinning which opposes their outwards migration to the surfaces. The details of the evolution of the flux density profiles during slow field cooling will be examined in the next section.

(ii) After Slow or Fast Field Cooling, the specimen is warmed to T_c . This generally proceeds slowly and is denoted Field Cooled Warming (FCW or SFCW) after the former case and Fast Field Cooled Warming (FFCW) after the latter. The rise in temperature from T_f causes the effective Meissner field shielding surface current, I_M , to diminish hence allowing flux to enter the specimen since H_a is always present (see Fig 2.1d). The pinning sites adjacent to the surface are then called upon to oppose the entry of this invading flux. Meanwhile, the pinning sites located deeper inside the specimen continue

to oppose any outwards migration of the flux lines which they have trapped during the previous cooling process. Eventually as the rise of T progresses, a temperature $T_c < T_{irr} < T_c$ is reached where all of the pinning sites, although now much less effective, are called

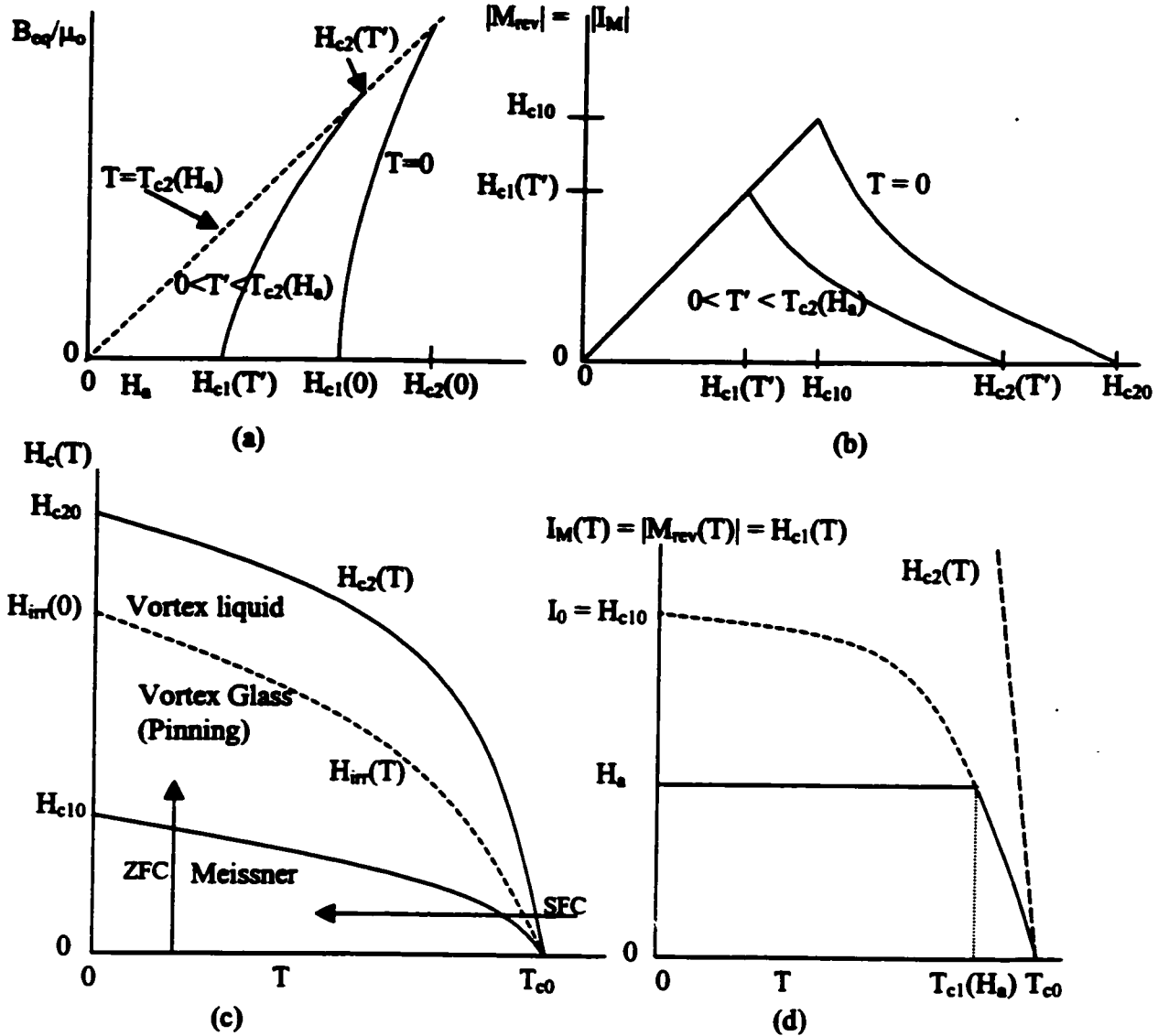


Fig 2.1 Variation of (a) the magnetic flux density and (b) the magnitude of the diamagnetic magnetization of an ideal type II superconductor at various temperatures as a function of the magnetic field H_a . (c) Dependence on T of the boundaries of various "phases" of the flux line lattice. (d) Dependence of the Meissner current on T when $H_a < H_{c10}$ (lower solid curve) and $H_a > H_{c10}$ (upper solid curve).

upon to oppose the entry of flux. This entry is permitted by further diminution of the Meissner field opposing current I_M and of the pinning.

When, during further warming, the temperature $T_{ir} > T_c$ is reached, the pinning is extinguished and subsequently over the range $T_{ir} \leq T \leq T_c$, the specimen exhibits only the reversible Abrikosov diamagnetism M_{rev} .

Hyun^[66, 67], Clem and Hao^[34] and Cave^[25] have separately accounted for the observations that generally, for a given specimen in the same H_a , $|M_{SFCW}(T)|$ is larger than $|M_{SFCC}(T)|$, until the temperature T_{ir} is attained where these two curves meet and then overlap over the range $T_{ir} \leq T \leq T_c$.

(iii) The specimen is first cooled from $T_i > T_c$ to $T_f \ll T_c$ in zero applied magnetic field. Generally the earth's magnetic field is not screened or cancelled and is considered negligible. With T maintained fixed at T_f , a magnetic field H_a is then slowly impressed on the superconducting sample thereby causing the field shielding Meissner surface current I_M to rise. When H_a exceeds $H_{c1}(T_f)$, the lower critical field at T_f , flux lines penetrate across the surface of the specimen. Pinning however opposes the advance of the vortices into the body of the specimen hence gradients in the density of flux lines arise adjacent to the surfaces. As H_a is further increased in strength, the fronts of the flux density gradients advance into the specimen, and eventually meet at the centre at a first full penetration field, denoted $H_p(T)$.

The evolution of the field profiles, as the pinning and $|M_{rev}|$ are made to diminish when the temperature of the magnetized zero field cooled monolithic sample is slowly increased from T_f to T_c in the chosen static magnetic field H_a , is fairly straightforward to model. The procedure just described and the locus of M subsequently measured when T

is increased to T_c are referred to as the shielding curve, and denoted Zero Field Cooled (ZFC). It is easy to see that generally $|M_{ZFC}(T, H_a)| > |M_{FCW}(T, H_a)|$. Clem and Hao^[34] have shown that the curves of these two quantities meet at $T_0(H_a) < T_{irr}(H_a)$, hence the temperature where their overlap begins does not correspond to T_{irr} as was previously assumed by some researchers. These features are illustrated in Fig 2.2.

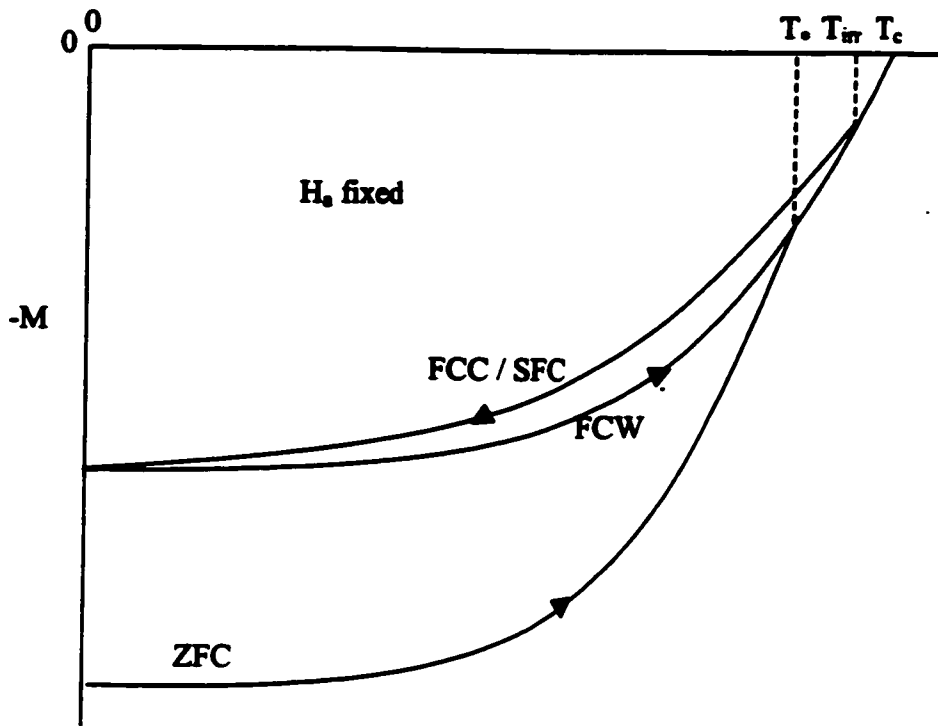


Fig 2.2 Schematic of the dependence of the diamagnetic magnetization of a single crystal on temperature in a static magnetic field $H_a < H_{c10}$ for procedures described in the text.

II Review of the Established Model

A. General Framework

This model describes the evolution of the magnetization M_s of an isotropic single crystal immersed in a static magnetic field H_a during slow cooling from the normal state through the field dependent critical temperature $T_{c2}(H_a)$ to a selected temperature $T_f < T_{c2}(H_a)$ and during subsequent warming to the normal state. The model addresses idealized slab or cylindrical geometry with H_a directed along the surfaces situated at the half thickness $\pm X$ of the infinite slab or at the radius R of the infinitely long cylinder.

When the temperature of the specimen is made to descend below the transition temperature $T_{c2}(H_a)$, the magnetic flux initially permeating the specimen in the normal state acquires the structure of discrete parallel flux lines each containing a quantum of flux, $\Phi_0 = h/2e$.

The mutual repulsion of the flux lines (vortices) causes some of the magnetic flux to be expelled from the sample. In the absence of pinning, the spatial distribution of the flux lines remaining in the specimen will be uniform. The homogeneous flux line density $B(x) = \langle B \rangle$, then adopts a value, $B_{eq}(H_a)$, in equilibrium with H_a which is determined by the temperature $T < T_{c2}(H_a)$ and the Ginzburg-Landau parameter $K = \lambda/\xi$ of the specimen. Here, λ and ξ denote the penetration depth and the coherence length. In this “liquid” vortex state, where $H_{c1}(T) < H_a < H_{c2}(T)$, the mutual repulsion of the flux lines is balanced by their interaction with the field shielding diamagnetic Meissner surface current. The latter can be viewed as confining and imprisoning the flux lines at the equilibrium density $B_{eq}(H_a)$.

For simplicity, the penetration depth $\lambda(T)$, which is occupied by the Meissner field shielding current, is taken to be small compared with X or R . In this context, since the equilibrium (reversible) Abrikosov diamagnetic magnetization reads,

$$\mu_0 M_{rev}(H_a, T) = B_{eq}(H_a, T) - \mu_0 H_a, \quad (2.1)$$

an effective Meissner surface current per unit height of the idealized slab or cylinder can be written,

$$I_M(H_a, T) = M_{rev}(H_a, T) \quad (2.2)$$

As T is made to diminish from $T_{c2}(H_a)$, M_{rev} , hence I_M , both augment in magnitude while B_{eq} decreases from $\mu_0 H_a$ (see Fig 2.1(a) and (b)). For $H_a < H_{c1}(T_f)$, the lower critical field at the preselected final cold temperature, the growth of $|M_{rev}|$ hence $|I_M|$, ceases when a critical temperature, $T_{c1}(H_a)$, is attained where B_{eq} has descended to zero near the surfaces. At this juncture,

$$|I_M(T_{c1})| = |M_{rev}(T_{c1})| = H_{c1}(T_{c1}) = H_a \quad (2.3)$$

All these quantities then remain fixed at these values during all subsequent excursions of T below T_{c1} in the fixed H_a as illustrated in Fig 2.1(d).

I_M is frequently written,

$$I_M = I_0 \left[1 - \left(\frac{T}{T_\infty} \right)^n \right] = H_{c10} \left[1 - \left(\frac{T}{T_{c0}} \right)^n \right] = M_{rev \max} \left[1 - \left(\frac{T}{T_{c0}} \right)^n \right] \quad (2.4)$$

where generally $n \geq 2$.

Consequently, $T_{c1}(H_a)$ reads,

$$T_{c1}(H_a) = T_\infty \left(1 - \frac{H_a}{I_0} \right)^{1/n} = T_{c0} \left(1 - \frac{H_a}{H_{c10}} \right)^{1/n} = T_\infty \left(1 - \frac{H_a}{M_{rev \max}} \right)^{1/n} \quad (2.5)$$

T_{c0} denotes the transition temperature to the superconducting state in zero field, and $I_0 = H_{c10} = M_{rev \max}$ denote these quantities at $T = 0$ in a corresponding H_a .

Clearly eqn 2.5 is not valid when $H_a > H_{c10}$. Under these circumstances, B_{eq} remains finite and can span the range, $0 \leq B_{eq} \leq \mu_0 H_{c2}(T)$ over the temperature range, $0 \leq T \leq T_{c2}(H_a)$, as sketched in Fig 2.1(a). It is then convenient and realistic to retain eqn 2.4 but replace the prefactor by,

$$I_0 f(H_a/H_{c2}(T)) = M_{rev \max} f(H_a/H_{c2}(T)) \quad (2.6)$$

where $f(H_a/H_{c2}(T))$ is a monotonic function of $H_a/H_{c2}(T)$ descending from 1 when $H_a = H_{c1}(T)$ to 0 when $H_a = H_{c2}(T)$.

A simple but realistic expression reads,

$$f(H_a/H_{c2}(T)) = \frac{H_{c2}(T) - H_a}{H_{c2}(T) - H_{c1}(T)} \quad (2.7)$$

where generally,

$$H_{c2}(T) = H_{c20} \left[1 - \left(\frac{T}{T_{c0}} \right)^m \right] \quad (2.8)$$

with $m \approx n \geq 2$. Generally linear near T_c , but quadratic as $T \rightarrow 0$.

At a temperature $T_{irr}(H_a) < T_{c2}(H_a)$, pinning arises in the body of type II superconductors and acts to oppose any tendency of the flux lines to migrate under the influence of their repulsive interactions. However since the difference between $T_{irr}(H_a)$ and $T_{c2}(H_a)$ plays no role in our application and extension of the standard model, we will let $T_{irr} = T_{c2}$ throughout our analysis. Also, for simplicity, the possible existence of any type of surface barriers to the exit or entry of flux lines is ignored^[27, 28, 51, 77, 78, 81]. Now as T descends below $T_{irr} = T_{c2}$, the expulsion of flux lines, associated with $M_{rev}(T, H_a)$ and

arising from their mutual repulsion, will be opposed by pinning forces in the body of the specimen. However, regardless of the strength of the pinning appearing in the body of the specimen, and in the absence of surface barriers, B_s , the density of flux lines just inside the surfaces is expected to exist in equilibrium with the external field H_a , hence, $B_s = B_{eq}(T, H_a)$. Therefore the expressions for $I_M(T, H_a) = M_{rev}(T, H_a)$, listed above are taken to apply unaltered by the growth or decline of the bulk pinning.

As T descends below $T_{ir} < T_{c2}$, the density of flux lines, $B(x)$ at distances $x > \lambda$ from the surfaces, cannot achieve the equilibrium value, $B_{eq}(H_a, T)$, because pinning opposes the outwards migration of flux lines required in order that the initially uniform flux line density profile, $B(x) = \mu_0 H_a$, diminish to $B_{eq}(H_a, T) < \mu_0 H_a$. Consequently $B(x)$ becomes inhomogeneous near the surfaces with flux density gradients $\pm dB/dx$ arising, hence flux trapping persistent currents of density $J_s = \pm dB/\mu_0 dx$, which circulate in an expanding annular volume along the periphery of the specimen.

In our analysis and displays we will focus on the right hand half, $0 \leq x \leq X$, of an infinite slab of thickness $2X$. This framework has the distinct advantage that the expressions for the current density and flux density profiles can be directly applied to cylinder geometry simply by replacing x by r and X by R . As a consequence it is a straightforward exercise to compute the magnetization of the specimen in both geometries by introducing the identical expression, $B(x) = B(r)$ in the corresponding definitions,

$$\mu_0 (M + H_a) = \frac{1}{X} \int_0^X B(x) dx = \frac{2}{R^2} \int_0^R B(r) r dr \quad (2.9)$$

Maxwell's eqn, $\nabla \times \vec{H} = \vec{J}$, for these idealized geometries reads,

$$\frac{dH}{dx} = \frac{dH}{dr} = \pm J \quad (2.10)$$

The mutual repulsion of the flux lines becomes unbalanced and a net force ensues in the volume where the density of flux lines is driven inhomogeneous. Flux line pinning and migration then occur when the inhomogeneity in the field profile causes a Lorentz like driving force density, $\bar{F}_L = \bar{J} \times \bar{B}$ to exceed the local pinning force density $F_p(B)$. Under these circumstances^[M2], the flux density profile establishes a critical configuration and the persistent current density $J_c(B, T)$ exists in a critical state. Because of thermal fluctuations these “persistent” currents are known to decay with time. The simple model which we exploit ignores this feature and regards J_c as truly persistent.

For simplicity, J_c is taken independent of the applied field, and is written,

$$J_g(T) = J_{g0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^k \right\} \quad (2.11)$$

The subscript g is now introduced since in this chapter we address the phenomena in an idealized crystal or grain. In subsequent chapters we will examine magnetothermal behaviour in weak-linked polycrystalline materials where we will need to also consider the intergranular current densities and associated field profiles which we will then identify by the subscript m for “matrix”.

The field profile, $H_g(x, T)$ encountered in the body of the specimen and associated with the bulk pinning current densities reads,

$$H_g(x, T) = H_a + \int_0^x J_g(x', T) dx' \quad (2.12)$$

Frequently in a volume, $x_c \leq x \leq X$, near the surface, $J_g = J_{gc}$. Integration of eqn 2.12 then leads to the linear expression,

$$H_g(x, T) = H_a + J_{cg} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^k \right\} (X - x) \quad (2.13)$$

valid over the range, $x_c \leq x \leq X$.

Quite generally it is convenient to define a local magnetization, denoted $M_H(x)$, which is associated with the field profiles $H_g(x, T)$, and reads,

$$M_H(x, T) = H_g(x, T) - H_a \quad (2.14)$$

The net local magnetization of the grain (crystal) is thus regarded as having two contributions and reads,

$$M_g(x, T) = M_H(x, T) - M_{rev}(H_a, T) = (H_g(x, T) - H_a) - I_M(H_a, T) \quad (2.15)$$

B. A Useful Simple Account of Slow Field Cooling

Hyun^[66] and Clem and Hao^[34] have focused on the special situation where $n = k$ in eqns 2.4 and 2.11. Under these circumstances, the evolution of $I_M(T)$, $H_g(x, T)$ and $M_g(x, T)$ as the temperature descends from T_{c0} is quite simple to visualize and is displayed in Fig 2.3 in two different but equivalent formats. The picture used by Clem and others, which we will refer to as the Clem picture, focuses on the evolution of $M_g(x, T)$ (see Fig 2.3a and b). An alternative approach, especially useful in the description of the behaviour in weak-linked polycrystalline samples, displays $H_g(x, T)$ and $I_M(T) = M_{rev}(T)$ separately (see Fig 2.3c and d).

In Fig 2.3(a) and (c), $J_{cg}(T)X > I_M(T)$, and in (b) and (d) $J_{cg}(T)X < I_M(T)$. When $H_a > H_{c1}(T_f)$, the profiles evolve as shown with descending T until $T = T_f$. If $H_a < H_{c1}(T_f)$, the evolution will also proceed as shown in the displays but $|M_{rev}|$ will cease to increase when

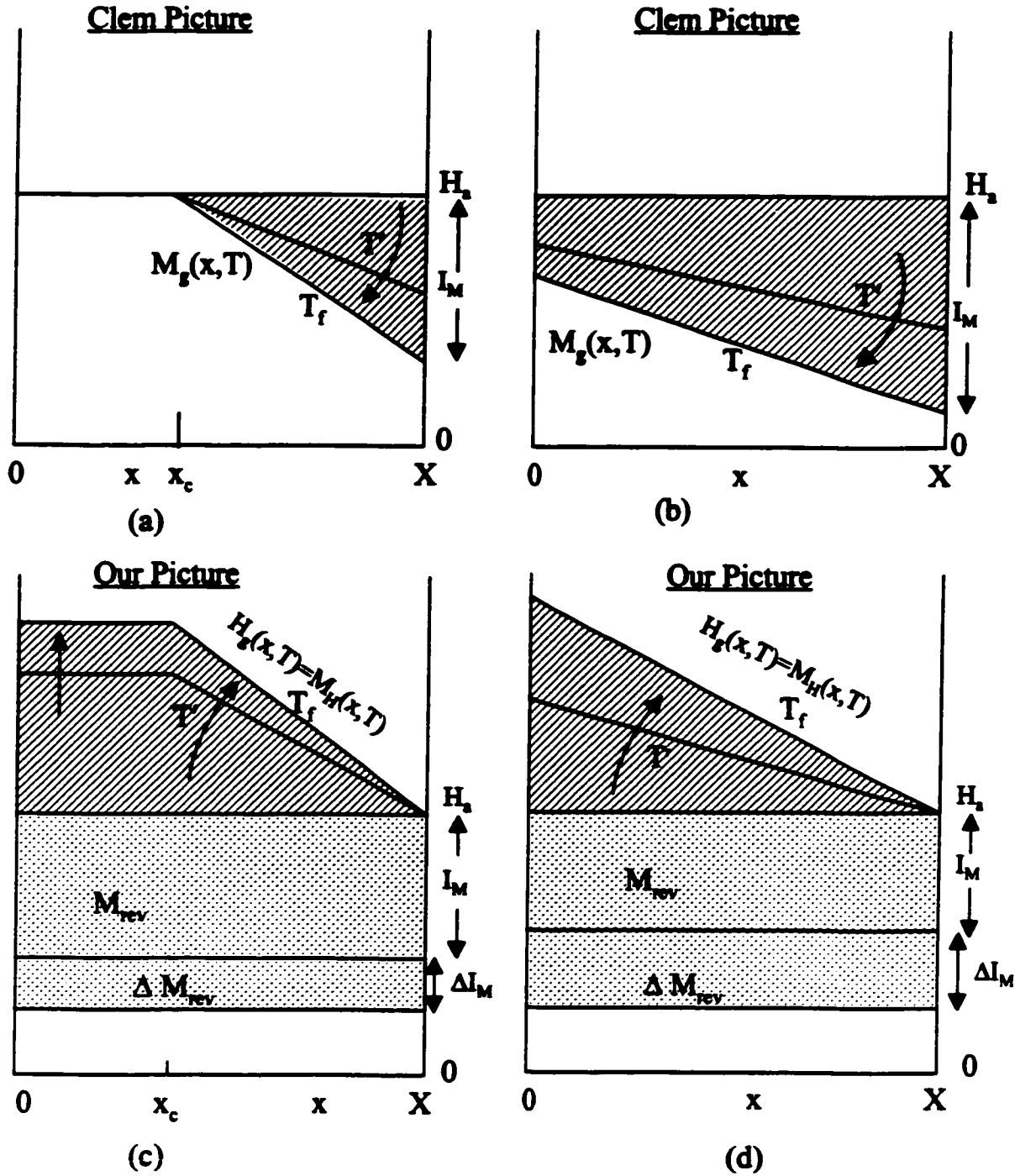


Fig 2.3 Clem picture of the evolution of the magnetic flux density, hence the local magnetization $M_g(x, T)$ as T descends from T_c in a static field H_a when $J_{cg}(T)(X - x_c) = I_M(T)$ in (a) and $J_{cg}(T)X < I_M(T)$ in (b). Our representation of $M_g(x, T)$ in terms of its two components for the situations where (c) corresponds to (a) and (d) to (b).

$|M_{rev}| = |I_M| = H_a$, hence when $T_{c1}(H_a)$ is attained (see Fig 2.1d). During further descent of T below $T_{c1}(H_a)$, the profiles remain frozen at the configuration established at T_{c1} , hence the gradients, dH/dx , and the corresponding J_g remain fixed but become progressively more subcritical.

In the simple situations we first examine here, the flux lines are “frozen” in a fixed inner volume, $0 \leq x \leq x_c$, as shown in Fig 2.3(a) and (c), throughout the descent of T from T_{c2} to T_f . Some of the flux lines in the volume, $x_c \leq x \leq X$, are being expelled from the specimen throughout the descent of T to T_f when $H_a > H_{c1}(T_f)$ but only until $T = T_{c1}(H_a)$ when $H_a < H_{c1}(T_f)$.

At the interior boundary x_c , as T descends, ΔH_g , the growth in the magnetic field H_g generated by the flux retaining currents at the critical density $J_{gc}(T)$, over the entire volume, $x_c \leq x \leq X$, is large enough to successfully oppose the concomitant growth of the intrinsic diamagnetism $|\Delta M_{rev}| = |\Delta I_M|$. Consequently at x_c , from eqns 2.4 and 2.13, we obtain

$$\left| \frac{\Delta H_g}{\Delta T} \right| = \frac{k J_{gc0}}{T_{c0}^k} (X - x_c) T^{k-1} = \left| \frac{\Delta I_M}{\Delta T} \right| = \frac{n I_0 T^{n-1}}{T_{c0}^n} \quad (2.16)$$

$(X - x_c)$ is labeled the flux trapping length L by Clem and Hao^[34]. We will examine later the more complicated situation where x_c migrates outwards as the temperature descends. Here, we pursue the case investigated in detail by Hyun^[66] and Clem and Hao^[34], where x_c is stationary. This behaviour occurs when $k = n$. Eqn 2.16 then leads to a flux trapping length L , which is independent of the temperature and reads,

$$0 < \frac{L}{X} = 1 - \frac{x_c}{X} = \frac{I_0}{J_{gc0} X} = \frac{I_0}{H_{a0}} < 1 \quad (2.17)$$

Eqn 2.17 applies when, $J_{g0}X$, denoted the flux penetration field H_{*0} at $T = 0$, is larger than the maximum reversible diamagnetism $|M_{rev\ max}| = I_0 = H_{c10}$. Clem and Hao^[34] refer to the situations where $H_{*0} > I_0$ as the strong pinning case. Fig 2.3(a) and (c) display the behaviour in the strong pinning case and Fig 2.3(b) and (d) for the “weak” pinning case where $H_{*0} < I_0$.

For both the strong and weak pinning cases, the evolution of M_s versus T , can readily be written in closed form when $n = k$ and reads,

$$\begin{aligned} M_s(x, T) &= J_{g0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} (X - x) - I_0 \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \\ &= \left\{ H_{*0} \left(1 - \frac{x}{X} \right) - I_0 \right\} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \end{aligned} \quad (2.18)$$

valid over the entire volume $0 \leq \frac{x}{X} \leq 1$, when $H_{*0}/I_0 < 1$, and over the outer volume $x_c \leq x \leq X$, when $H_{*0}/I_0 > 1$. In the strong pinning case, $M_s(x, T) = 0$ in the inner volume $0 \leq x \leq x_c$, where the flux lines are “frozen”.

Introducing eqn 2.18 in the definition for the “global” magnetization,

$$M_s(T) = \frac{1}{X} \int_0^X M_s(x, T) dx \quad (2.19)$$

leads to,

$$M_s(T) = -\frac{I_0^2}{2H_{*0}} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \quad (2.20)$$

when $H_{*0}/I_0 \geq 1$, and,

$$M_s(T) = -\left(I_0 - \frac{H_{*0}}{2} \right) \left\{ 1 - \left(\frac{T}{T_0} \right)^n \right\} \quad (2.21)$$

when $H_{c0}/I_0 \leq 1$.

For both strong and weak pinning, when $H_a < I_0 = H_{c10}$, the growth of the diamagnetism ceases when T descends to $T_{c1}(H_a)$. At this juncture, $I_M = H_a$ and introducing eqn 2.4 into eqns 2.20 and 2.21 leads to,

$$M_s(T_{c1}(H_a)) = -\frac{I_0}{2H_{c0}} H_a \quad (2.22)$$

when $H_{c0}/I_0 \geq 1$, and,

$$M_s(T_{c1}(H_a)) = -H_a \left\{ 1 - \frac{H_{c0}}{2I_0} \right\} \quad (2.23)$$

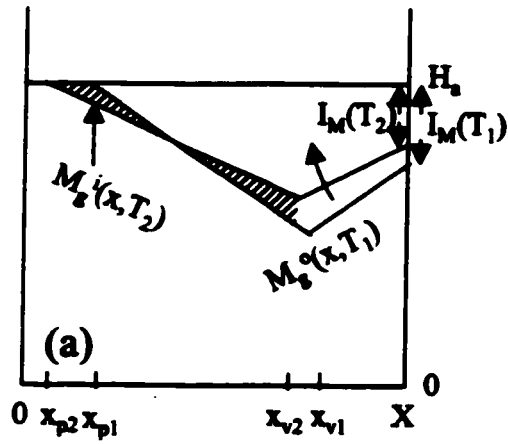
when $H_{c0}/I_0 \leq 1$.

C. Evolution of $M_s(T, H)$ upon Warming

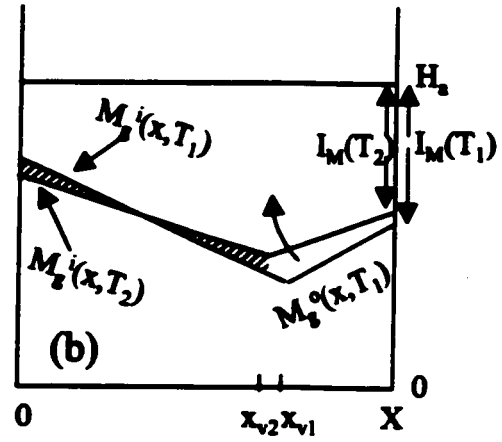
We now examine the evolution of the field profiles and of the magnetization during warming (FCW) to T_{c0} after Slow Field Cooling (FCC) in a static H_a . The evolution of the field profiles is illustrated in Figs 2.4.

The diminution of the Meissner field shielding current I_M due to the rise in temperature now allows flux lines to enter the specimen. The pinning sites adjacent to the surface respond by opposing this invasion. Consequently the sign of the field gradient and the direction of flow of the critical current density J_{gc} in this region are now reversed. This leads to the formation of a valley in the profiles of $M_s(x, T)$ and $H_g(x, T)$. The magnitude of the gradients $\mp dH(T)/dx = \pm J_{gc}$ are however consequently diminished throughout the volume of the specimen by the uniform rise in temperature. Consequently

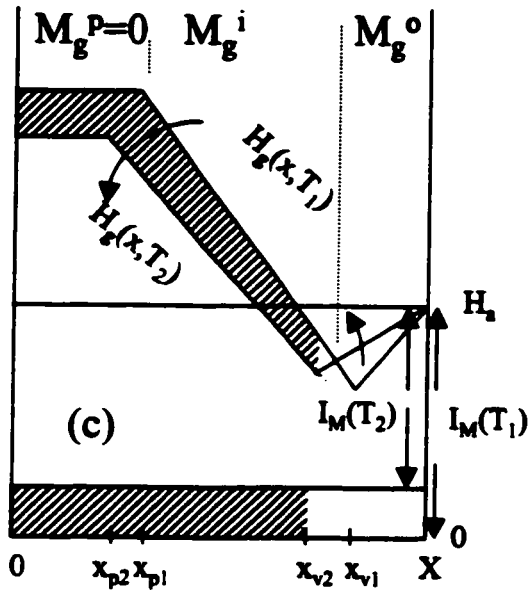
Clem Picture



Clem Picture



Our Picture



Our Picture

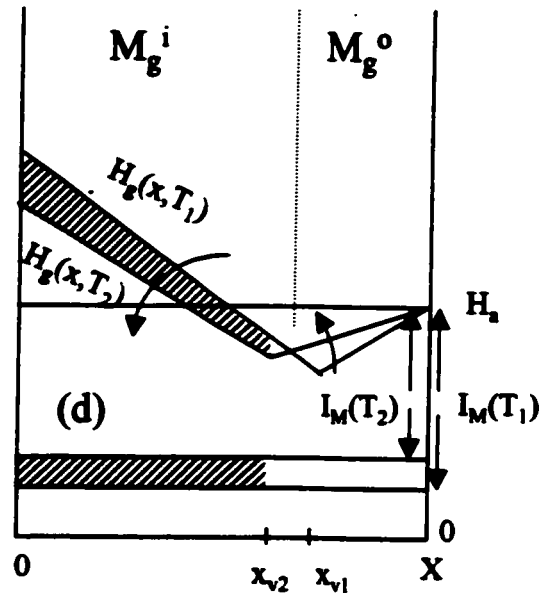


Fig 2.4 Sketches of the evolution of (a) and (b) the local magnetization $M_g(x, T)$, and, (c) and (d) $H_g(x, T)$, and the reversible magnetization $I_M(T) = M_{rev}(T)$ upon slow warming after slow field cooling.

the flux lines inside a shrinking volume, $0 \leq x \leq x_v$, although conserved, are forced to redistribute because $|J_{gc}|$ hence $|dH_g/dx|$ are decreased. Fig 2.4 shows these events using both pictorial approaches side by side. The mathematical description of the events is, of course, identical and is now presented.

We let T_1 and T_2 denote two stages in the rise of the temperature with $T_1 < T_2 < T_{c0}$. In the calculations $T_2 = T_1 + \Delta T$ will be incremented in conveniently small steps. However, for clarity, in the sketches we will exaggerate the magnitude of ΔT . The local magnetization at T_2 reads,

$$M_g^o(x, T_2) = -I_M(T_2) - J_{gc}(T_2)(X - x) \quad (2.24)$$

for the outer region, $x_{v2} \leq x \leq X$, and,

$$\begin{aligned} M_g^i(x, T_2) &= -I_M(T_2) - J_{gc}(T_2)(X - x_{v2}) + J_{gc}(T_2)(x_{v2} - x) \\ &= -I_M(T_2) - J_{gc}(T_2)\{X - 2x_{v2} + x\} \end{aligned} \quad (2.25)$$

for the inner region, $x_{p2} \leq x \leq x_{v2}$. Here

$$M_g^p(T_2) = 0 \quad (2.26)$$

for the plateau region, $0 \leq x \leq x_{p2}$, if such a structure exists.

Identical expressions are written for $T = T_1$ for the previous segments, $M_g^o(x, T_1)$, $M_g^i(x, T_1)$ and $M_g^p(x, T_1)$ where the previously calculated valley and plateau positions are denoted, x_{v1} and x_{p1} . With the new values for $J_{gc}(T_2)$ and $I_M(T_2)$ specified by the choice of $T_2 = T_1 + \Delta T$ we proceed to solve for the variables x_{v2} and x_{p2} as follows.

The expressions for $M_g^i(x, T_2)$ and $M_g^o(x, T_2)$ are introduced into the definition of the global magnetization,

$$M_g(T_2) = \int_{x_{p2}}^{x_{v2}} M_g^i(x, T_2) dx / X + \int_{x_{v2}}^X M_g^o(x, T_2) dx / X \quad (2.27)$$

Conservation of the flux which is redistributing in the inner volume, $0 \leq x \leq x_{v2}$, leads to,

$$M_g^i(T_2) = \int_{x_{p2}}^{x_{v2}} M_g^i(x, T_2) dx / X = M_g^i(T_1) = \int_{x_{p1}}^{x_{v2}} M_g^i(x, T_1) dx / X \quad (2.28)$$

The continuity of the field profile leads to,

$$-I_M(T_2) - J_{eq}(T_2)(X - x_{v2}) + J_{eq}(T_2)(x_{v2} - x_{p2}) = 0 \quad (2.29a)$$

hence,

$$x_{p2} = 2x_{v2} - X - \frac{I_M(T_2)}{J_{eq}(T_2)} \quad (2.29b)$$

valid when $x_{p2} > 0$.

Eqns 2.28 and 2.29 dictate the position of the boundaries x_{v2} and x_{p2} at T_2 , hence the limits of the integration in eqn 2.27. Proceeding in this manner as T is made to increment, we see the plateau region shrink and disappear (if any existed) and the valley x_v migrate inwards. The calculation is of course simpler when $x_p = 0$. In these circumstances the conservation of flux alone (eqn 2.28) determines the inward migration of x_v until the valley front has attained $x = 0$, the centre of the specimen. Let $T_*(H_*)$ denote the corresponding temperature. The evolution, as T is increased from $T_1 = 0$ to $T_2 = T_*$, is illustrated semi-quantitatively in Fig 2.5 for the simple case where $H_* = H_{c10} = I_0 = |M_{rev \max}| = H_{*0} = J_{gc0}X$.

The sequence of profiles occurring as T is augmented from $T_*(H_*)$ to T_{c0} are identical to that encountered at the corresponding temperatures and in the same field H_*

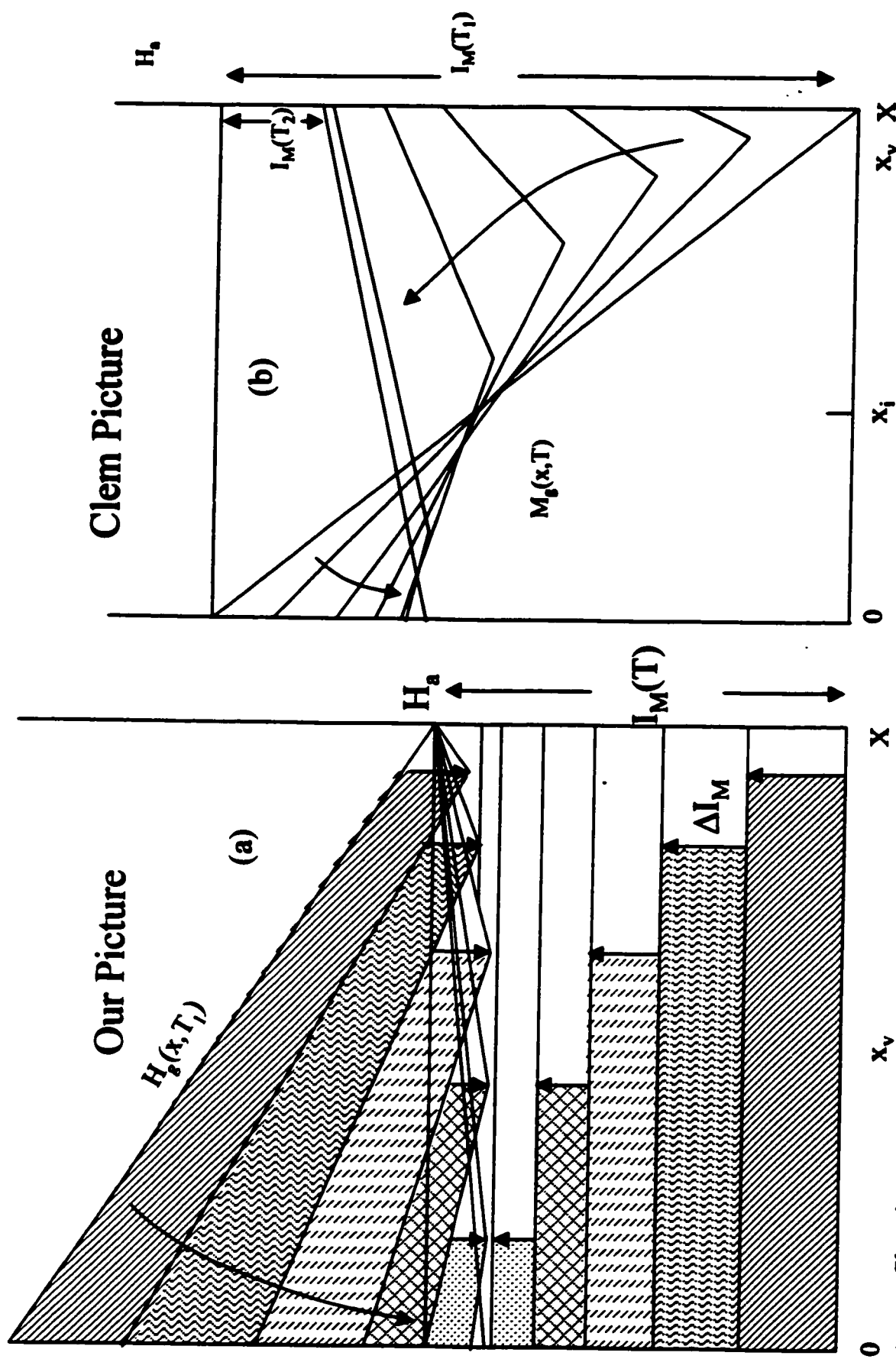


Fig 2.5 Sketches of the evolution of (a) $H_g(x, T)$, $I_M(T) = M_{rev}(T)$, and (b) $M_g(x, T)$ as T is increased from $T_1 < T_*$ to $T_2 = T_*$.

during warming in the zero field cooled case (ZFC). Here, eqn 2.27 simplifies to,

$$M_s(T) = \int_0^x M_s^o(x, T) dx / X = -I_M(T) - \int_0^x J_F(T)(X - x) dx / X = -I_M(T) - J_F(T) \frac{X}{2} \quad (2.30)$$

III Extension of the Model to Thermal Cycling below T_{c0}

A. Introduction

Hyun^[66] investigated the evolution of the magnetization of a Nb_3Sn powder specimen, hence a collection of electrically isolated single crystals, during subsequent thermal cycling between T_1 and T_2 where $T_1 < T_2 < T_{c0}$ with H_a maintained fixed. He observed that this procedure caused the diamagnetic moment to progressively grow in magnitude at both T_1 and T_2 until "saturation" values were attained after some 15 cycles. Hyun however did not extend the model just described so as to account for his observations. Instead he displays a sketch of the field profiles he visualized to occur at the lower temperature limit during the subsequent thermal cycling. However the field profile he proposes is inconsistent with the physical constraints, hence with the established model. We now proceed to develop the standard model to account for these observations. As a first step in this enterprise we focus on the behaviour during the first recooling stage, from various temperatures $T_2 < T_{c0}$.

B. Enhancement of the Meissner Effect by Recooling from $T < T_{c0}$.

The evolution of the flux density configurations where T now descends from some $T_2 > T_1$, but less than T_{c0} , is again governed by the increase in the magnitude of $I_M(T)$ and the consequent generation of critical field gradients adjacent to the surfaces which oppose the exit of flux lines as described above in section IIB. We stress that the first

cooling commenced from $T \geq T_{c0}$, therefore from the normal state where the flux density, $B(x) = \mu_0 H_x$, was uniform in the entire volume of the specimen. Now, depending on the choice of the temperature $T_2 < T_{c0}$ where the warming is made to cease and the recooling begins, any of the nonuniform flux density configurations from the sequence displayed in Fig 2.5b can come into play. We now examine the different types of field profiles which can be encountered when the temperature has been decreased to T_1 and how these are determined by the structure of the field profiles existing when the rise in temperature was reversed at $T_2 < T_{c0}$. Again, for simplicity, in our description of the behaviour we let $T_1 = 0$ and $H_{c0} = J_{c0}X = I_0 = H_{c10} = M_{rev \max}$.

Here the Clem picture illustrates the message more clearly than our format, hence we now focus on that representation of the events.

The diagonal dashed-solid line in Fig 2.6 indicates the local magnetization after the initial cooling from the normal state to $T_1 = 0$ and labeled $M_g(x, T_1)_0$. The curves labeled $M_g(x, T_2)_1$ show the local magnetization at different selected temperatures $T_2 \leq T_*$ where the warming ceased and the recooling begins. The $M_g(x, T_2)$ curves displayed in Fig 2.6a, b, c and d, are chosen from the sequence of curves illustrated in Fig 2.5(b). In Fig 2.6 the curves labeled $M_g(x, T_1)_1$ display the local magnetization after the recooling to T_1 . The shaded area at the top left corners in Fig 2.6 provides a measure of the enhancement in the Meissner effect which is caused by the first thermal cycle from T_1 to T_2 and back to T_1 . From inspection of Fig 2.6 we note that the number of flux lines trapped in the specimen after the recooling from T_2 to T_1 (the area under the lowermost lines), is smaller than the number trapped after the initial cooling from T_{c0} to T_1 (the area under the

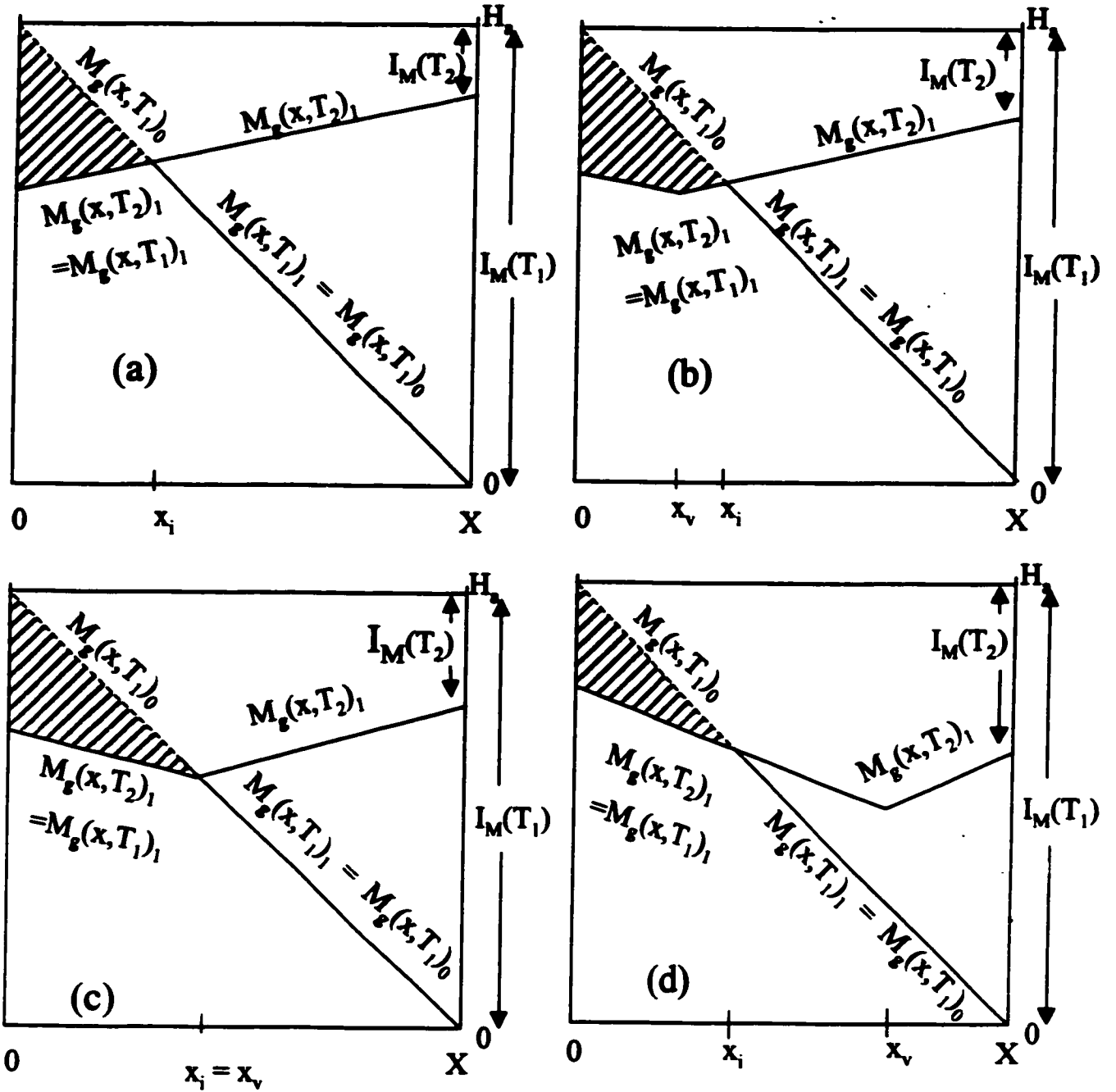


Fig 2. 6 Sketches of the sequences of the local magnetization $M_g(x, T)_n$ when T is made to descend from T_∞ to $T_1 = 0$, then raised to $T_2 < T_\infty$ and then decreased to $T_1 = 0$. The subscripts $n = 0$ and 1 denote $M_g(x, T)_n$ after initial cooling from $T > T_\infty$ to T_1 , first warming to $T_2 < T_\infty$ and first recooling to $T_1 = 0$. Depending on the warm temperature T_2/T_∞ different sets of the intermediate and final configurations of $M_g(x, T)_n$ need to be considered.

diagonal). Hence a net amount of flux has been released as a consequence of the warming to T_2 and the recooling to T_1 . Thus the magnitude of the diamagnetic magnetization (the Meissner effect) after recooling to T_1 , denoted $M_g(T_1)_1$, is larger than that observed after the initial cooling from T_{c0} , denoted $M_g(T_1)_0$. Note that the subscript 0 or 1 outside the parentheses indicates the sequence, with 0 denoting the initial cooling and 1 the first warming and first recooling.

The net release of flux caused by the initial thermal cycle from T_1 to T_2 to T_1 , hence the initial enhancement in the Meissner effect, is controlled and dictated by the following feature. The flux trapped in the inner volume, $0 \leq x \leq x_i$, after the excursion to T_2 and the return to T_1 , possesses a flux density profile governed by the smaller critical current density, $J_{gc}(T_2)$, hence the shallower field gradient, $|dH_g(x, T_2)/dT| = |dM_g(x, T_2)/dT|$, existing at the warmer temperature T_2 . This crucial feature emerges from inspection of all the segments of the lowermost curves in Fig 2.6. By contrast, the flux trapped after the initial cooling to T_1 is controlled by $J_{gc}(T_1) > J_{gc}(T_2)$ over the entire volume of the specimen (i.e. the diagonal dashed–solid line).

Fig 2.7(a) illustrates the dependence of the enhancement in the Meissner effect, ΔM_g at $T_1 = 0$, on two choices of T_2 for the first thermal cycle. Fig 2.7(b) displays these enhancements, denoted $\Delta M_g(T_1, T_2)$, versus T_2/T_{c0} for the cases where $H_{*0}/I_0 = 0.75, 1.0, 1.5, 2.0$.

For weak pinning where $H_{*0} < I_0$, the recooling from $T_2 \geq T_*$, will simply reestablish the same flux configuration at T_1 which existed previously at the end of the initial cooling to T_1 as illustrated in Fig 2.8(a). Consequently the recooling from, $T_* \leq T_2 \leq T_{c0}$ will not generate, any enhancement in the Meissner effect at T_1 , since there are no

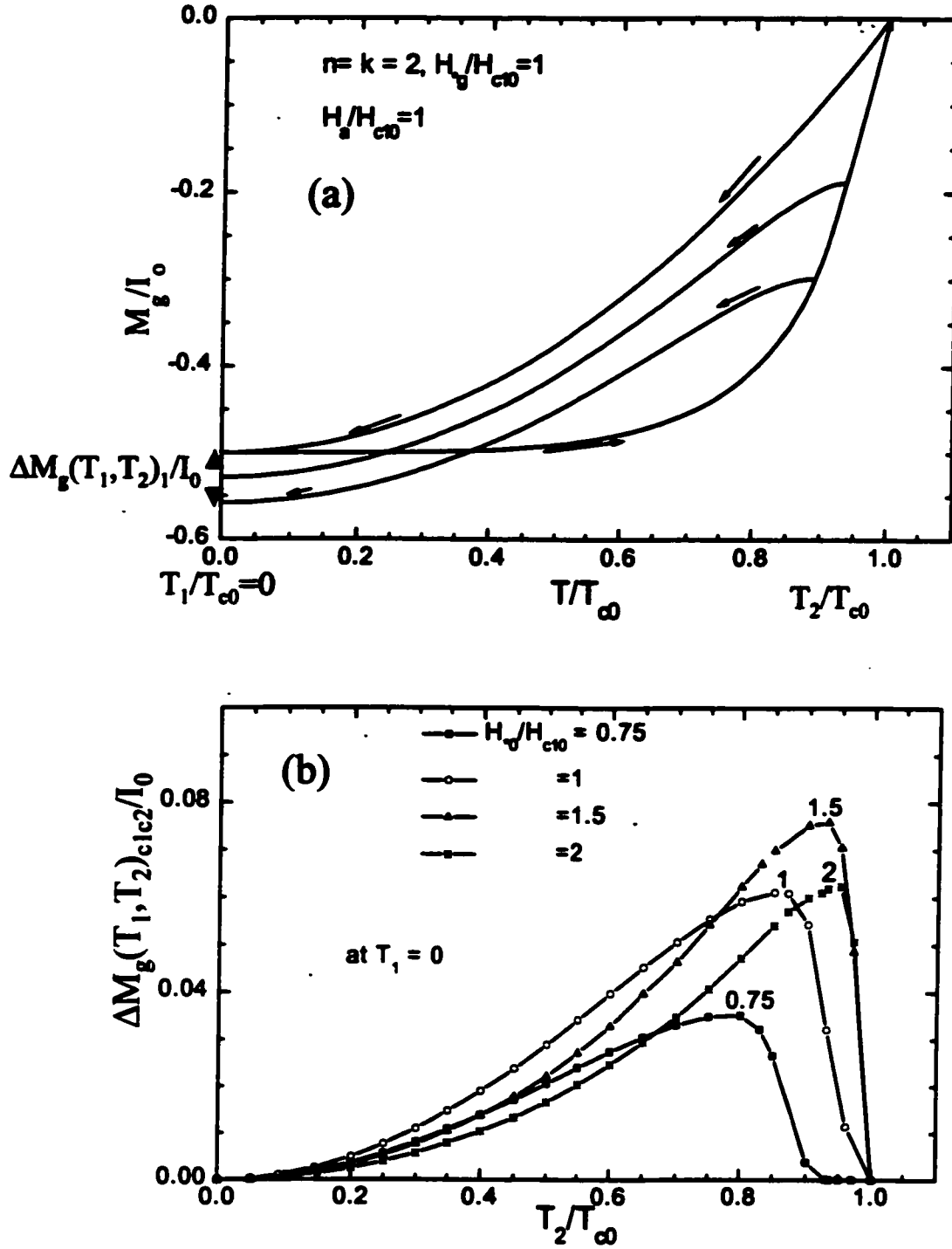


Fig 2. 7 (a) Illustrates the evolution of M_g versus T predicted by our model for thermal cycles where $T_2/T_{c0} = 0.9, 0.95$ and 1 . Here $n = k = 2$ in eqn 2.18, $h_{g0} = H_{g0}/I_0 = 1$ and $H_g/H_{c10} = h_c = 1$ (b) Predictions of our model for the enhancement of the Meissner effect at $T_1 = 0$ after a thermal cycle to T_2/T_{c0} calculated for different values of H_{g0}/H_{c10} with $n = k = 2$ and $H_g/H_{c10} = 1$.

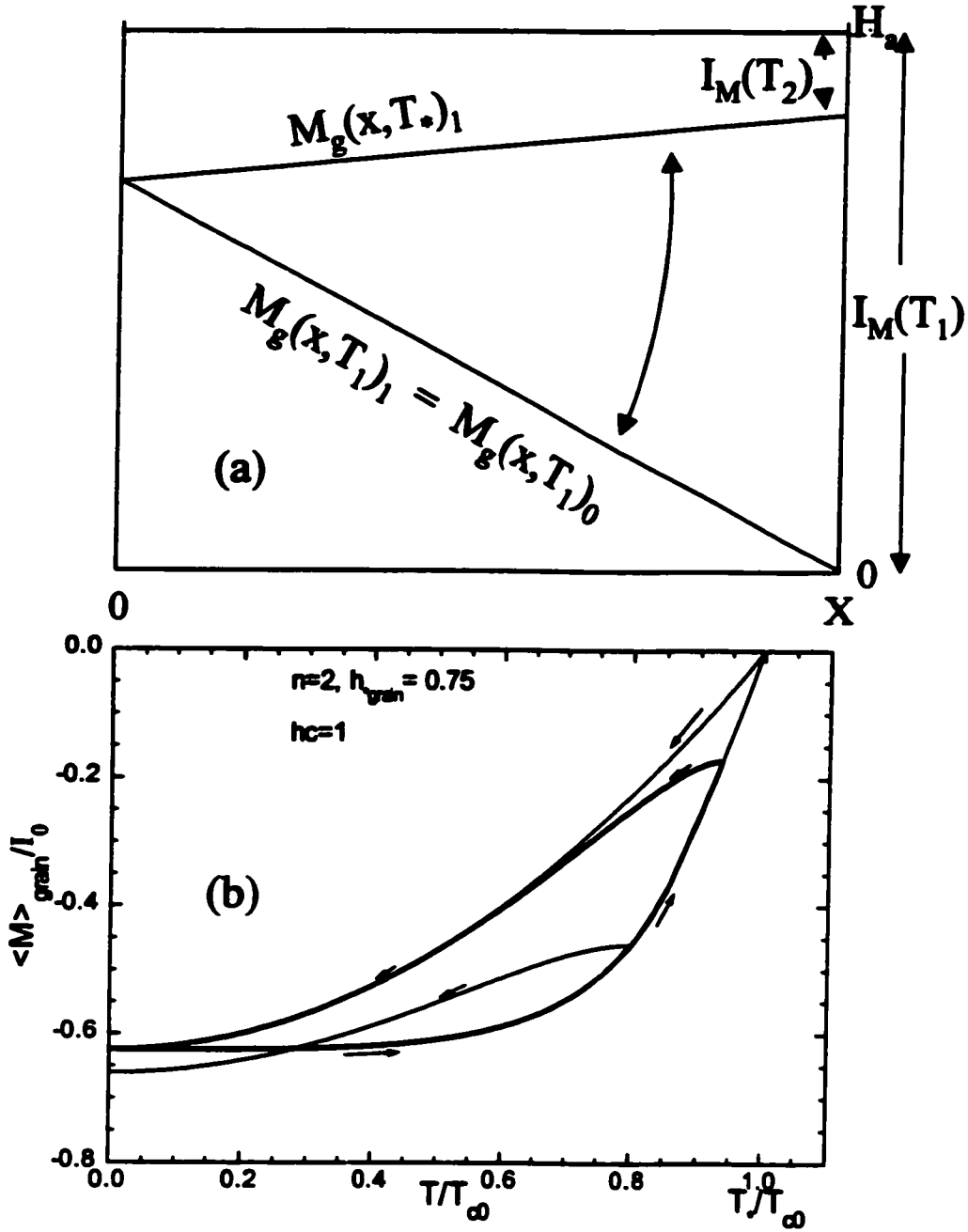


Fig 2. 8 (a) Illustrates the configurations of $M_g(x, T)$ when T is made to swing between $T_* \leq T_2 < T_\infty$ and $T_1 = 0$ when $H_{*g} < I_0$. (b) The thick curves show the corresponding thermal hysteresis loops for the situations where $T_* \leq T_2 \leq T_\infty$, hence no enhancement of the Meissener effect is generated on recooling.

differences in the profiles of the trapped flux at T_1 after the initial cooling from T_{c0} , and after the recooling from T_2 . We note also that subsequent "hysteresis" loops traced by M_z versus T cycling between T_1 and $T_c \leq T_2 \leq T_{c0}$ will overlap and repeat the curve traced during the first warming as illustrated by the heavy hysteresis curve in Fig 2.8b.

The condition for no enhancement of the Meissner effect to occur at T_1 from thermal cycles extending between T_1 and $T_c \leq T_2 \leq T_{c0}$, when $H_{c0} < I_0$, can formally be written from inspection of Fig 2.8(a),

$$[J_E(T_1) + J_E(T_2)]X + I_M(T_2) \leq I_M(T_1) \quad (2.31)$$

where the expressions denote absolute values. Consequently the calculated curve where $H_{c0}/I_0 = 0.75$ in Fig 2.7b displays no enhancement of the Meissner effect at $T_1 = 0$, when $T_2/T_{c0} > 0.935 = T_c/T_{c0}$.

C. Progressive Enhancement of the Meissner Effect by Further Thermal Cycling

We have seen in the previous section how the first recooling from $T_2 < T_{c0}$ to T_1 leads to flux configurations at T_1 which are appreciably different from and more diamagnetic than that which were established by the first cooling from $T > T_{c0}$ to T_1 . Careful inspection of the flux density profiles during subsequent thermal cycling between the same chosen values for T_1 and $T_2 < T_c$ reveals that, in many cases, the flux density configurations encountered upon the first warming to T_2 and the first recooling to T_1 cannot be reestablished during subsequent cycles. Indeed we find that the $B(x)$ profiles must, in many cases, evolve towards predictable limit configurations dependent on the choices of T_1 and T_2 for a given specimen and H_a .

We now endeavour to show why further thermal cycling can cause such an evolution in the $B(x)$ profiles, hence a corresponding progressive enhancement in the

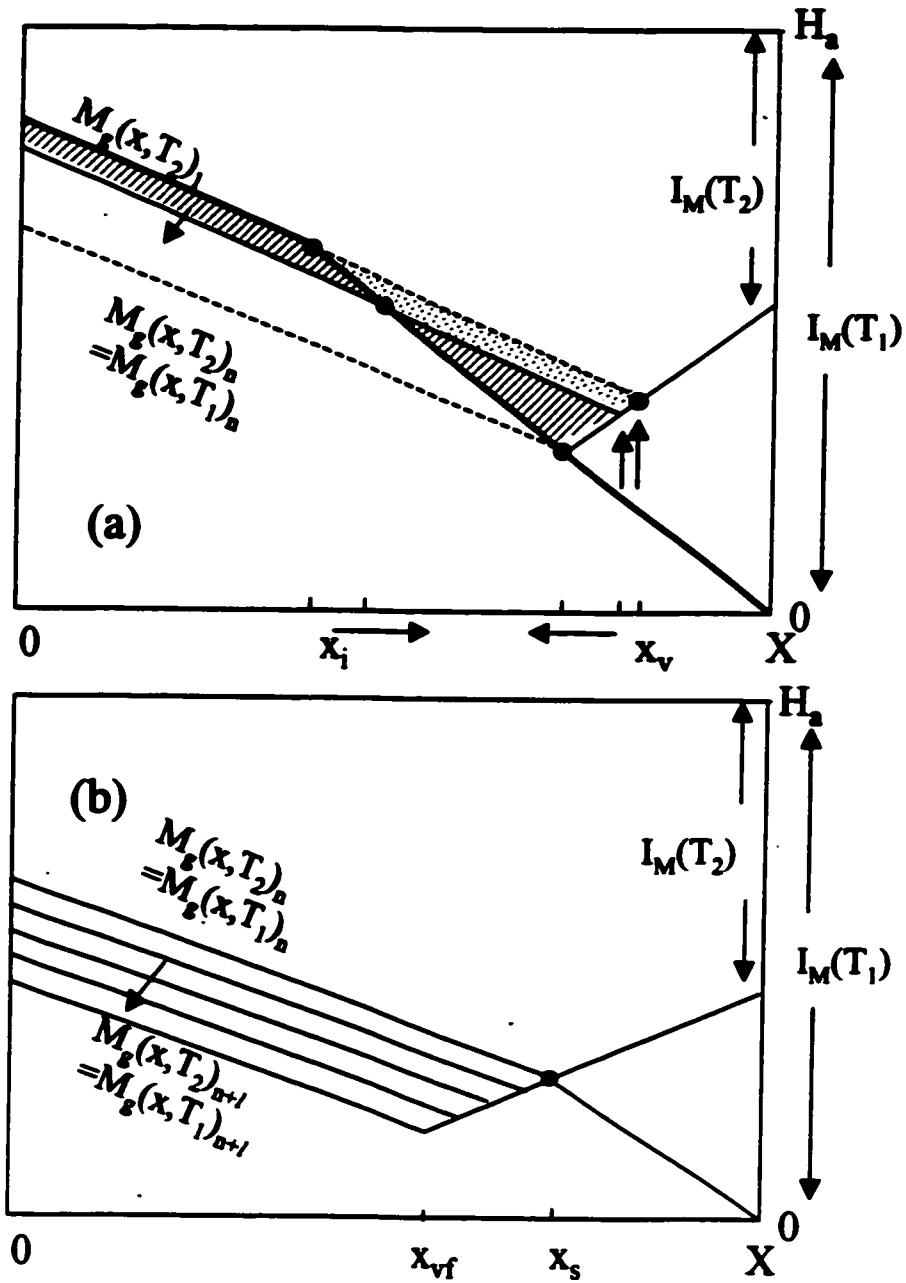


Fig 2. 9 (a) Configurations of the local magnetization after the 1st warming to T_2 (uppermost heavy-dashed-thin lines), after the 1st recooling to T_1 (inner and outer heavy lines), and after 2nd warming to T_2 (inner and outer thin lines). Eventually the configurations represented by the lower dashed-outside thin line occurs at T_2 , and that shown by the lower dashed-outside solid line occurs at T_1

(b) Displays the subsequent evolution of the profiles at T_2 and T_1 . Eventually the descent of the inner profile ceases and now a final closed thermal hysteresis loop is traced whose upper and lower limits differ by the area enclosed in the right hand triangle.

Meissner effect at T_1 and at $T_2 < T_c(H_0)$. To illustrate this behaviour we select the case illustrated in Fig 2.6(d). In Fig 2.9(a) we reproduce the flux density configuration, hence the local magnetization, of Fig 2.6(d) encountered after the 1st warming (uppermost thick-dashed-thin lines) and after the 1st recooling (thick lines). Here the valley bottom x_v after the 1st warming is situated to the right of x_i , the intersection of the inner shallow field gradient with the outer steep field gradient which existed after 1st recooling. Now as the temperature is again raised from T_1 , the sequence of events illustrated in Fig 2.5(b) take place and some flux lines depin and migrate from the inner volume $0 \leq x \leq x_i$ towards the surface. The two areas diagonally shaded in Fig 2.9(a) are sketched equal to illustrate the operation of the constraint of flux conservation in the interior of the specimen. As a consequence of this constraint, the $B(x)$ profile after the 2nd warming from T_1 to T_2 must have dropped from the previous level (uppermost solid-dashed curve of Fig 2.9(a)) to a lower configuration (uppermost solid-solid curve of Fig 2.9(a)). Consequently the Meissner effect after this second excursion to T_2 is enhanced with respect to that which was established after the 1st excursion to T_2 . This enhancement is schematically proportional to the uppermost rectangle comprising the diagonally shaded and stipled areas. We note that the position of x_v has now migrated inwards.

Next when the specimen undergoes a 2nd recooling from T_2 , flux is again expelled and the $B(x)$ profile displayed by the lowermost inner and outer solid lines of Fig 2.9(a) is established. The flux trapped in the specimen at T_1 is now seen to have diminished compared with that trapped after the previous descent to T_1 . The inner diagonally shaded area in Fig 2.9(a) provides a measure of this decrease in the total flux threading the specimen. Because of the drop in the flux density in the inner volume of $0 \leq x \leq x_i$, the

intersection x_i of the inner shallow and the outer steep field gradients of the $B(x)$ profile is seen to have migrated outwards from its previous location after the previous excursion to T_1 .

Thus the thermal cycle has caused x_v to move inwards and x_i to move outwards, hence to approach each other. Subsequent cycles will eventually cause x_i and x_v to meet. This juncture is displayed in the lowermost curve of Fig 2.9(a). The approach and meeting of x_i and x_v can be seen to take place via the following processes.

(i) The decrease of the flux trapping current density during the warming part of the thermal cycle releases flux from the centre of the sample towards the surface. Consequently less flux is allowed to enter through the surface as the rise in temperature causes the Meissner field shielding current to diminish. Because of this diminution in the entry of flux, the diamagnetism at the warm limit $T_2 < T_c$ is stronger than that encountered after the previous warming to T_2 .

(ii) During each cooling phase of the thermal cycles, the flux, which had just previously migrated towards the surface, is now expelled when the Meissner current now grows in magnitude.

The processes we have just described will continue to operate after x_i and x_v have coincided. Because of flux line conservation inside the specimen, the transfer of flux from the inner volume to the outer volume takes place during the first part of each warming phase. Concomitantly flux is penetrating into the sample through the surface (see Fig 2.5(b)). However the outward transfer of flux from the inner to the outer volume leaves the inner region slightly depleted in trapped flux, hence more diamagnetic, as illustrated in Fig 2.9(b). Consequently, x_v continues to migrate inward at the end of each

warming phase of the thermal cycle. Upon recooling, the intersection of the outer steep (cold) field gradient with the inner shallow field gradient of opposite sign now constructs a summit x_s which remains stationary at the position reached earlier when x_i and x_v met as illustrated in Fig 2.9(b). Consequently the increment in the magnitude of the diamagnetism caused by the warmings to T_2 is “preserved” as T descends to T_1 and the Meissner effect expels the flux enclosed in the “triangle” along the surface.

Typical behaviour generated by our model is displayed in Fig 2.10. The locus of M_g versus the swings in temperature starting from $T/T_{c0} = 1$ can be traced by following the arrows. We next discuss the origin of the closed thermal hysteresis loop at the bottom of Fig 2.10.

The evolution of $M_g(T)$ as the thermal cycling is continued leads to progressively smaller growth in the magnitude of the diamagnetism attained at T_1 and T_2 . In our model this enhancement ceases when the thermal cycling has caused x_v to migrate to a final position, denoted x_{vf} , which is determined by the choices for T_1/T_{c0} , T_2/T_{c0} and H_g/H_{c0} , and the functions characterizing the specimen, namely, $I_M = I_0 F_M(T/T_{c0})$ and $J_{cg} = J_{cg0} F_J(T/T_{c0})$, where F_M and F_J denote the dependences on temperature. Now although flux from the volume between the valley x_{vf} and the summit x_s , is transferred to the outer volume, $x_s \leq x \leq X$, during the initial part of the warming stage, an equal amount of flux is returned to that volume during the last part of the warming stage. Hence, with x_v remaining fixed at x_{vf} , the diamagnetism at T_2 will no longer increase in magnitude. Also because x_{vf} is fixed, the diamagnetism at T_1 will accordingly grow no further in magnitude. Consequently now that x_v has migrated to a “stable” final location, the exact amount of flux that enters through the surfaces during the warming phase is subsequently

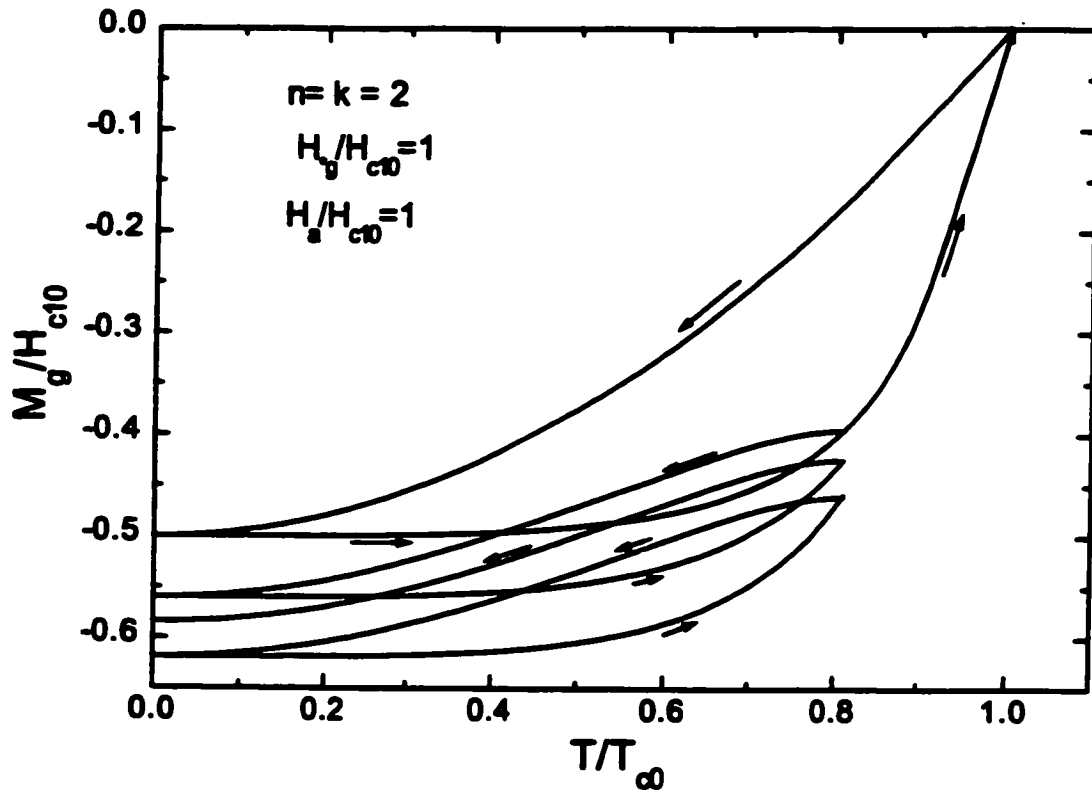


Fig 2.10 Evolution of the magnetization of a single crystal calculated with our model upon cooling from $T = T_{\infty}$ to $T_1 = 0$ and then subjected to two complete thermal cycles between T_1 and $T_2 = 0.8T_{\infty}$. The closed thermal hysteresis loop at the bottom is generated after numerous thermal cycles between the chosen T_1 and T_2 . Here $n = k = 2$ in eqns 2.4 and 2.11. $H_g/H_{c10} = 1$ and $H_s/H_{c10} = 1$.

expelled during the cooling stage. This flux is schematically represented by the area enclosed by the triangle on the right-hand side of Fig 2.9(b). Now, the “hysteresis” curves of M_g versus T swinging between T_1 and T_2 will terminate at fixed values, denoted $M_g(T_1)_f$ and $M_g(T_2)_f$, hence the locus of M_g versus T will trace closed loops. An example of such a closed loop calculated from our model is displayed at the bottom of Fig 2.10.

The measurements of Huyn^[66] are reproduced in Fig 2.11(a) and curves generated by our model are displayed in (b) for comparison. It is not clear from Huyn’s paper whether the data between E and D was taken during warming or cooling after 15 thermal cycles were traversed.

Huyn^[66] presented a model, identical to ours, to reproduce the behaviour during the 1st cooling and 1st warming (i.e the curve from T_c to A and then from A to B in Fig 2.11a). The profile of $B(x)$ labeled C, in the inset of Fig 2.11a, displays his suggestion, which he does not develop, to account for the enhancement of the Meissner effect at C. This proposal however is inconsistent with the critical state hypothesis since it introduces a field gradient which is sub-critical to oppose the exit of flux from the sample as T decreases from B to C and causes the field expelling Meissner current to increase. Huyn^[66] also presents no explanation for his observation that 15 thermal cycles between 5 and 16 K led to the enhanced diamagnetic curve labeled E D in fig 2.11a.

From inspection of the right-hand side triangle in Fig 2.9(b) we can write,

$$I_M(M_1) - I_M(T_2) = J_{\#}(T_1)(X - x_s) + J_{\#}(T_2)(X - x_s) \quad (2.32)$$

Hence,

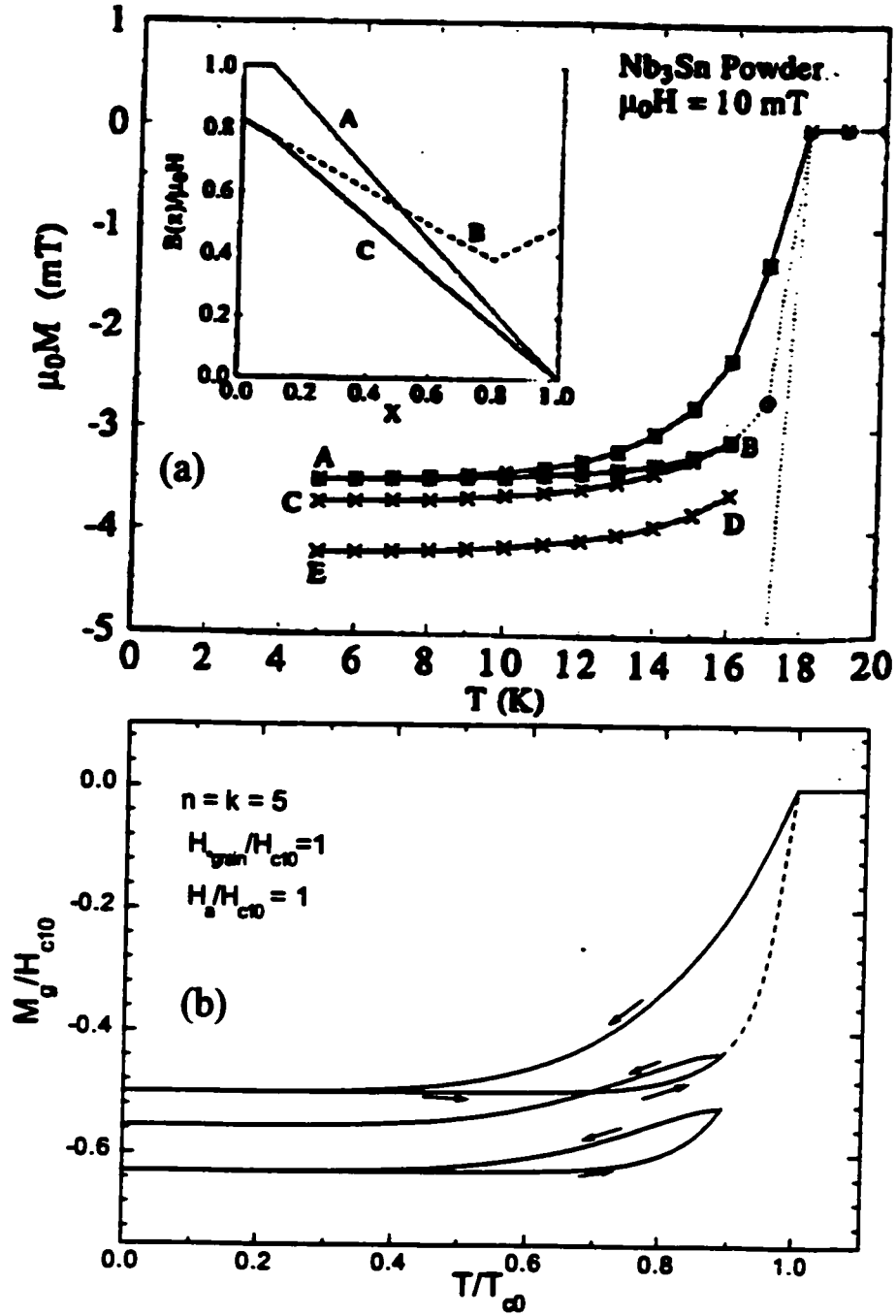


Fig 2.11 (a) Reproduces the observations of Huyn^[66] on Nb_3Sn powder, hence electrically isolated single crystals, upon cooling to A, warming to B and recooling to C. It is not clear from his paper whether the data between E and D was taken during warming or cooling after 15 thermal cycles. (b) Displays curves generated by our model for comparison.

$$1 - \frac{x_s}{X} = \frac{I_M(T_1) - I_M(T_2)}{\{J_g(T_1) + J_g(T_2)\}X} = \frac{I_0 \left\{ \left(\frac{T_2}{T_{c0}} \right)^n - \left(\frac{T_1}{T_{c0}} \right)^n \right\}}{H_{c0} \left\{ 2 - \left(\frac{T_1}{T_{c0}} \right)^n - \left(\frac{T_2}{T_{c0}} \right)^n \right\}} \quad (2.33)$$

valid when, $0 \leq x_s / X \leq 1$.

The area embraced by the right-hand side triangle in Fig 2.9(b) therefore corresponds to the difference between the “final” diamagnetic magnetization, hence the “ultimate” enhanced Meissner effect at T_1 and T_2 which can be achieved by thermal cycling between these chosen limits in the chosen static field H_s . This difference then reads,

$$\begin{aligned} \Delta M_g(T_1, T_2)_{\text{cyc}} &= M_g(T_1)_f - M_g(T_2)_f \\ &= \frac{\{J_g(T_2) + J_g(T_1)\}(X - x_s)^2}{2X} \\ &= \frac{H_{c0}}{2} \left\{ 2 - \left(\frac{T_2}{T_{c0}} \right)^n - \left(\frac{T_1}{T_{c0}} \right)^n \right\} \left(1 - \frac{x_s}{X} \right)^2 \\ &= \frac{I_0}{2} \left(\frac{I_0}{H_{c0}} \right) \left\{ \frac{\left[\left(T_2 / T_{c0} \right)^n - \left(T_1 / T_{c0} \right)^n \right]^2}{2 - \left(T_1 / T_{c0} \right)^n - \left(T_2 / T_{c0} \right)^n} \right\} \end{aligned} \quad (2.34)$$

Various features of the enhancement of the diamagnetism of single crystals by thermal cycling predicted by our model are displayed in Fig 2.12. The four quantities selected for exploration are illustrated and labeled in Fig 2.12a. Again for simplicity we let $T_1 = 0$ in these calculations. This choice does not significantly restrict our message since the initial field cooled Meissner effect curve is quite flat over the low temperature range of T/T_{c0} even when $H_s > H_{c10}$. The evolution calculated for the four selected quantities versus T_2/T_{c0} is displayed in Fig 2.12b for the case where $H_{c0}/H_{c10} = 1$, $H_s/H_{c10} = 1$ and $n = k = 2$. Such curves could provide guidance for the choice of the “optimum”

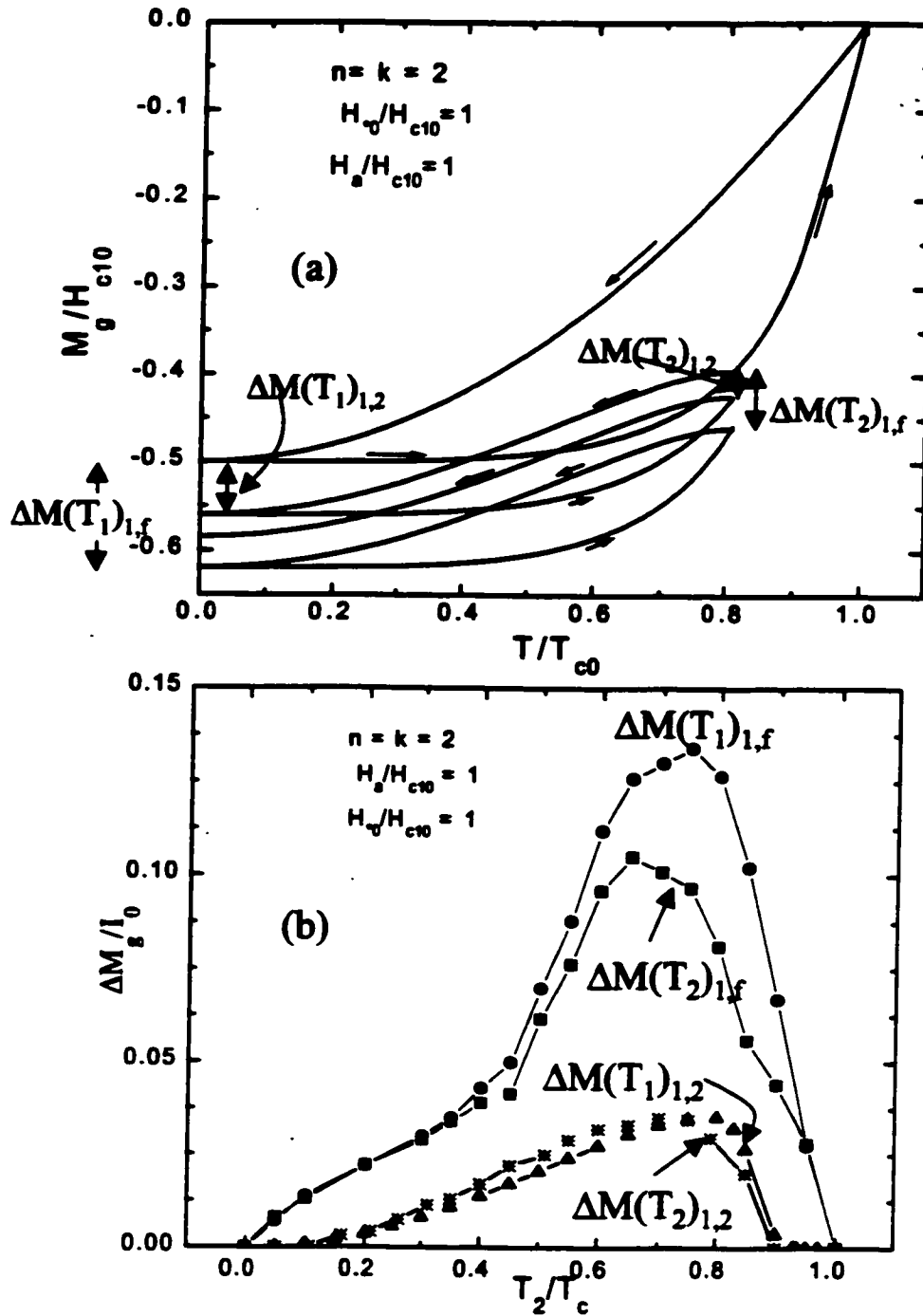


Fig 2.12 (a) Displays and labels 2 quantities at the cold limit and 2 at the warm limit of the thermal cycles whose dependence on T_2 we have investigated computationally. Again for simplicity we let $T_1 = 0$. The subscripts C and W denote the “cold” and “warm” limit and the subscripts 1, 2, and f indicate 1st, 2nd and final visit to the corresponding limit. (b) Dependence of $\Delta M_g(T_1)_{1,f}$, $\Delta M_g(T_2)_{1,f}$, $\Delta M_g(T_1)_{1,2}$ and $\Delta M_g(T_2)_{1,2}$ on T_2/T_{c0} calculated with $n = k = 2$, for $H_s/H_{c10} = 1$, $H_w/H_{c10} = 1$.

value for T_2/T_{c0} in experimental work seeking to determine the largest Meissner effects for a specimen hence a good estimate of H_{c10} .

The number of thermal cycles required to achieve the “final” enhanced levels of diamagnetism was found to be quite large by Huyn in his experiment and also very large computationally. In our calculations to find the final hysteresis loops we circumvent this obstacle by making an “educated” guess for x_{vf} , the final position of the valley for the selected T_1 and T_2 limits and the chosen parameters, H_{c0}/I_0 , n and k . Then we examine how well a calculated hysteresis loop comes to closing under these circumstances and whether the deviation from closing at T_2 was upwards or downwards. Guided by the initial result we then proceed stepwise to find a satisfactory improved value for x_{vf} which leads to a thermal hysteresis loop which closes “satisfactorily”.

IV Summary and Conclusion

The established model^[25, 34, 66, 67, 96], which describes the locus of the diamagnetic magnetization of isotropic hysteretic type II superconductors of idealized geometry during slow cooling from T_{c0} and subsequent slow warming, has been extended to account for the enhancement of the Meissner effect observed by Hyun^[66] during thermal cycling between a very low temperature T_1 and a warmer temperature T_2 near but below T_{c0} . Exploiting our extension of the existing model we have developed detailed quantitative and qualitative predictions for the magnitude of the maximum enhancement which can be achieved by extensive thermal cycling between various pairs of limiting temperatures T_1 and $T_2 < T_{c0}$ for specimens exhibiting low, intermediate and strong pinning.

Chapter 3

Enhancement of the Meissner Effect upon Warming of Weak-linked Granular High T_c Superconductors and Thermal Cycling of a Low T_c Polycrystal

Abstract

Several workers^[66, 69, 73, 149] have observed that, after fast field cooling of weak-linked granular high T_c type II superconductors, the diamagnetic magnetization will trace a narrow valley at a temperature T_v near T_c during slow warming to the normal state with H_a kept fixed throughout the temperature excursion. Huyn^[66] also observed this phenomenon after slow field cooling of a polycrystal of Nb_3Sn (a classical low T_c type II superconductor) and found that thermal cycling between $T \ll T_v$ and T_v generated further enhancement in the Meissner effect. In this chapter we extend the simple model described in the preceding chapter to account for these observations.

I. Introduction

A. General Framework

In chapter II we examined the evolution of M_g , the magnetization of an isolated isotropic single crystal (grain) of idealized planar geometry during slow cooling in a static field H_a and during subsequent warming and thermal cycling below T_c . Now for simplicity, weak-linked granular type II superconductors, are first regarded as an assembly of identical uncoupled idealized isotropic single crystals with their surfaces parallel to each other and to the applied field H_a . Consequently the magnetization of the agglomeration of uncoupled grains, now denoted $\langle M_g(T, H_a) \rangle$, corresponds exactly to that developed earlier and denoted $M_g(T, H_a)$ for the isolated idealized grain.

Next we visualize that the grains in this assembly are electrically connected by numerous bridges along their adjacent parallel surfaces and that these links can carry persistent currents with a critical current density, denoted $J_{cm}(T, H)$, which is very small compared with the intragranular critical current density $J_{gc}(T, H)$. Again, for simplicity, we take $J_{cm}(T, H)$ independent of H and write,

$$J_{cm} = J_{cm0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \quad (3.1)$$

where for simplicity we let the intergranular medium possess the same critical temperature as the grains.

The volume of the entire network through which the intergranular currents are flowing (i.e. the bridges and the surfaces of the grains), together with the voids between the grains and the normal state inclusions, will be referred to as the "matrix". The fraction

of a unit volume of the specimen which is occupied by the “matrix” and by the grains will be taken into account in the modeling by a normalization procedure.

Again for simplicity, we focus on a specimen of idealized slab geometry of thickness $2X_m$ with surfaces parallel to H_a . The analysis, however, can readily be extended to idealized cylindrical geometry.

B. Field Cooling of a Weak-Linked Polycrystal

(i) General Framework

The flux in the voids and normal inclusions is never quantized. Below T_c , the flux permeating the weak links is thought to exist in the form of Josephson vortices whose mutual repulsion is small. Since there are no strong mechanisms for the occurrence of a Meissner effect in the matrix, the lower critical field, H_{c1} , in this medium is found to be negligible and will be ignored in our modeling.

In the previous chapter we have examined the growth of the diamagnetic magnetization $M_g(T, H_a)$ of an isolated grain as it expelled flux during cooling below T_{c0} . Now in weak-linked granular materials, the grains, as they cool from T_{c0} , expel magnetic flux to the outside world through the surface of the specimen and into the “matrix”. By Faraday’s law of induction, the expulsion of flux from the grains will generate electric fields hence induce persistent currents to circulate through the intergranular network to oppose the exit of this flux from the specimen. Let $J_m(x, T)$ denote a “coarse” local spatial average of the density of the persistent currents induced to circulate around the specimen through the intergranular network, and $H_m(x, T)$ denote the “coarse” local average

magnetic field associated with the “coarse” local persistent current density $J_m(x,T)$. Maxwell’s equation for idealized slab and cylinder geometry then reads,

$$\frac{dH_m(x,T)}{dx} = \frac{dH_m(r,T)}{dr} = \pm J_m(x,T) \quad (3.2)$$

where for simplicity we consider $J_m(x,T)$ and $H_m(x,T)$ to describe the current density and the associated magnetic field as “coarse” local averages over the dimensions of several grains and including the volume of the grains. Consequently we ignore the detailed structure of the intricate intergranular current distribution and the complicated magnetic field configuration on a granular scale in the matrix and describe the macroscopic magnetic behaviour of the specimen via quasi-microscopic averages for $J_m(x,T)$ and $H_m(x,T)$. Integration of eqn 3.2 then leads to,

$$H_m(x,T) = H_s + \int_x^{x_m} J_m(x',T) dx' \quad (3.3)$$

Frequently in a volume, $x_{cm} \leq x \leq X_m$, near the surface, J_m exists in a critical state denoted J_{cm} . Under these circumstances, eqns 3.1 and 3.3 lead to,

$$\begin{aligned} H_m(x,T) &= H_s + J_{cm}(X_m - x) \\ &= H_s + J_{cm0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} (X_m - x) \end{aligned} \quad (3.4)$$

valid over the outer volume, $x_{cm} \leq x \leq X_m$.

Quite generally it is convenient to define a local magnetization associated with the field profile $H_m(x,T)$ which reads,

$$M_m(x,T) = H_m(x,T) - H_s \quad (3.5)$$

$M_{WL}(x,T)$, the net “coarse” local magnetization of the specimen can be viewed as comprising two contributions. (i) The spatially varying “coarse” local magnetization of

the matrix, and (ii) the spatially uniform diamagnetic magnetization of the grains in the static H_s . Consequently we write,

$$\begin{aligned} M_{wl}(x, T) &= M_m(x, T) - \langle M_g(T, H_s) \rangle \\ &= \{H_m(x, T) - H_s\} - \langle M_g(T, H_s) \rangle \end{aligned} \quad (3.6a)$$

Here $\langle M_g(T, H_s) \rangle$ denotes the magnetization of the grains averaged over the entire volume of the specimen. Alternatively the superposition of the two components (i) and (ii) can read,

$$M_{wl}(x, T) = (1-f)M_m(x, T) - f \langle M_g(T, H_s) \rangle \quad (3.6b)$$

where f and $(1-f)$ indicate the relative weights of the two contributions to $M_{wl}(x, T)$. For simplicity we use eqn 3.6a, where the relative weights are taken into account by the choice of the relative magnitudes of the pertinent adjustable parameters, namely, $H_{cm0} = J_{cm0}X_m$ for the matrix, and $H_{g0} = J_{cg0}X_g$, and H_{c10} for the grains. We thereby avoid writing $(1-f)$ and f throughout our formulae and diagrams.

(ii) A Useful Simple Account of Slow Field Cooling

We have seen in the previous chapter that the evolution of M_g , the magnetization of an isolated grain during slow cooling from T_{c0} can be readily described if the Meissner current I_M and the intragranular critical current density J_{cg} are assumed to have the same temperature dependence. Again, the assumption that J_{cm} has the same temperature dependence as J_{cg} and I_M , hence the same as $\langle M_g(T) \rangle$, leads to simple expressions for the evolution of M_{wl} , the magnetization of the weak-linked granular specimen during slow field cooling.

Under these circumstances, inside an inner volume, $0 \leq x \leq x_{cm}$, the flux expelled by the grains is completely trapped inside the matrix by the intergranular persistent

currents induced in the surrounding volume $x_{cm} \leq x \leq X_m$, as illustrated in Fig 3.1. Since the flux is conserved in the inner volume, the local magnetization, $M_{WL}(x,T)$ does not change there and remains zero. Consequently, for this region, we can write,

$$\langle M_s(T) \rangle = H_m(x,T) - H_s = M_m(x,T) \quad (3.7)$$

Hence at the boundary $x = x_{cm}$,

$$H_m(x_{cm},T) - H_s = J_{cm}(T)(X_m - x_{cm}) = J_{cm} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} (X_m - x_{cm}) = \langle M_s(T) \rangle \quad (3.8)$$

$$\text{where, } \langle M_s(T) \rangle = \left(I_0 - \frac{H_{s^*0}}{2} \right) \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \quad (3.9a)$$

$$\text{or, } \langle M_s(T) \rangle = \frac{I_0^2}{2H_{s^*0}} \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \quad (3.9b)$$

depending on whether $H_{s^*0} < I_0$, or $H_{s^*0} > I_0$ as we have seen in chapter 2 (eqns 2.20 and 2.21).

From inspection of Fig 3.1(a) and 3.1(c) we see that the upper right hand triangle is proportional to the diamagnetic magnetization of the weak-linked idealized slab specimen and, depending on whether $H_{s^*0} < I_0$, or $H_{s^*0} > I_0$ we write,

$$M_{WL}(T) = -\frac{J_{cm}(T)(X_m - x_{cm})^2}{2X_m} = \frac{-\langle M_s(T) \rangle^2}{2J_{cm}(T)X_m} = \frac{-\left(I_0 - \frac{H_{s^*0}}{2} \right)^2 \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\}}{2H_{m^*0}} \quad 3.10a$$

$$\text{or, } M_{WL}(T) = -\left(\frac{I_0^2}{2H_{s^*0}} \right)^2 \frac{\left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\}}{2H_{m^*0}} \quad 3.10b$$

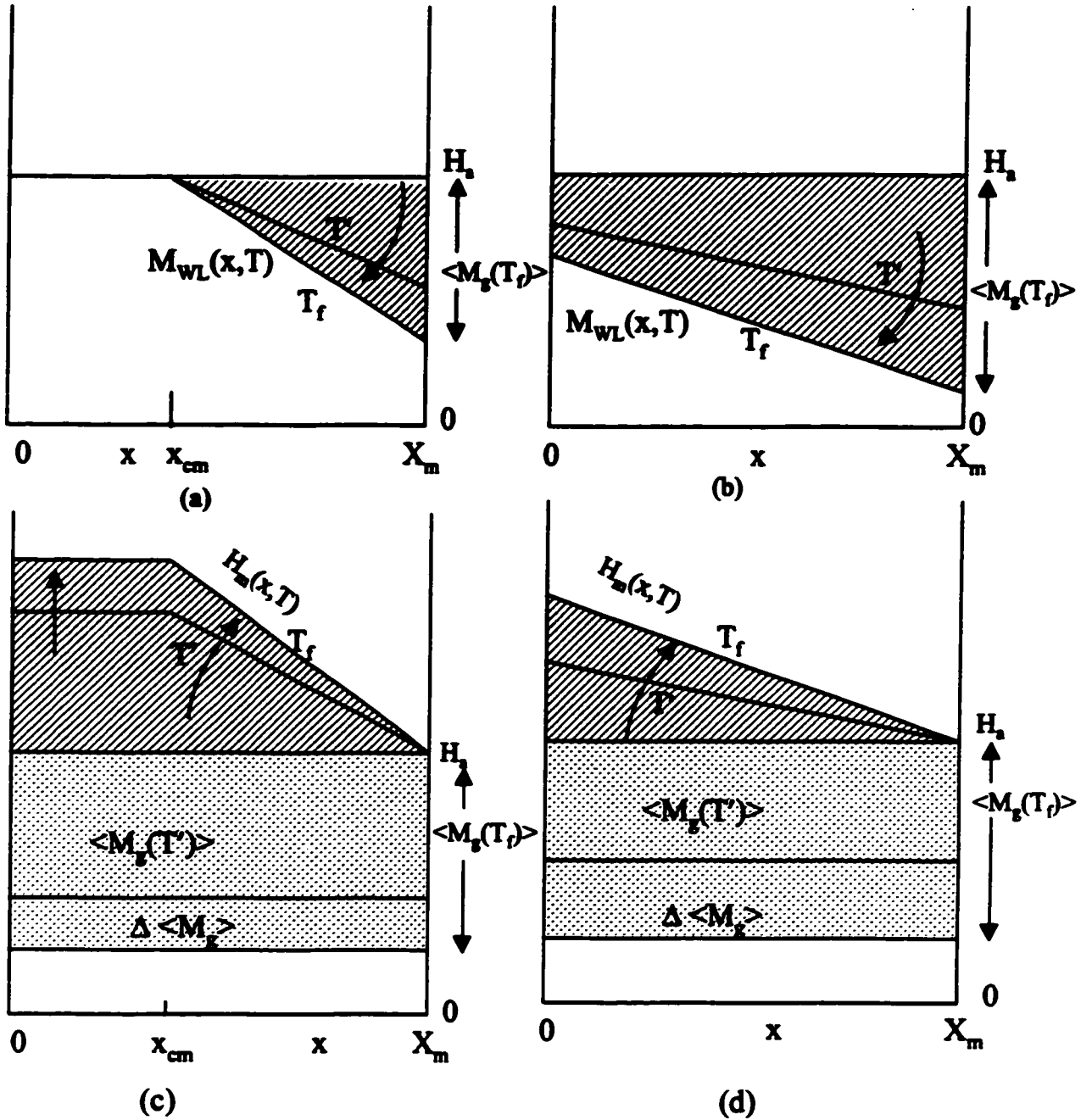


Fig 3.1. Schematics of the evolution of, (a) and (b) $M_{WL}(x, T)$, the “coarse” local magnetization (shaded), $H_m(x, T)$, the “coarse” local magnetic field, and (c) and (d) $\langle M_g(T, H_g) \rangle$, the magnetization of the grains as the specimen cools from T_c through T to T_f in a static H_a . The temperature dependences of J_{cm} , J_{cg} and H_{c1} are taken to be the same. $J_{cm}(T) X_m$ is less than $\langle M_g(T) \rangle$, for (a) and (b), whereas $J_{cm}(T) X_m$ is greater than $\langle M_g(T) \rangle$ for (b) and (d). Note that $\langle M_g(T) \rangle = J_{cm}(T) (X - x_{cm})$ in (a) and (c).

where $H_{m0} = J_{cm0}X_m$. Eqns 3.10(a) and (b) are valid when $0 \leq x_{cm} < X_m$.

Also from inspection of Fig 3.1(b) and 3.1(d) we can write,

$$M_{wL}(T) = -\langle M_s(T) \rangle + \frac{J_{cm}(T)X_m}{2} \quad (3.11)$$

where, $\langle M_s(T) \rangle > J_{cm0}(T)X_m$

(iii) A more General Description of Slow Field Cooling

We now examine the more complicated field profiles encountered in field cooling when the intergranular (matrix) critical current density, $J_{cm}(T)$, and $\langle M_s(T) \rangle$, the magnetization of the grains, do not exhibit the same temperature dependence. For clarity in our discussion we focus on specific simple dependences on temperature which nevertheless illustrate the different features in the evolution of the field profiles which now occur. For simplicity we let $J_{cm}(T)$ depend linearly on temperature, hence write,

$$J_{cm}(T) = J_{cm0} \left(1 - \frac{T}{T_{c0}} \right) \quad (3.12)$$

We take the diamagnetic magnetization of the grains upon cooling to grow in magnitude according to the simple prescription,

$$\langle M_s(T) \rangle = \langle M_{s0} \rangle \left\{ 1 - \left(\frac{T}{T_{c0}} \right)^n \right\} \quad (3.13)$$

where, generally, $n \geq 2$. Again, for simplicity, J_{cm0} and $\langle M_{s0} \rangle$ are taken independent of H_a . We have seen in chapter 2 that when $H_a \leq H_{c10}$, as T descends from T_{c0} , $\langle M_s(T) \rangle$ grows in magnitude until the Meissner screening current $I_M(T)$ in the grains equals the static applied field H_a . For simplicity in our description we will let $I_M(T)$, hence $\langle M_s(T) \rangle$,

evolve until $T = 0$ and take $H_a = H_{c10}$. Application of the model to the situations where $H_a < H_{c10}$ will be straightforward since in these circumstances, $I_M(T)$, $J_{sc}(T)$, $\langle M_g(T) \rangle$, and $J_{cm}(T)$ all cease to evolve at a temperature $T_{c1}(H_a)$ where $I_M(T) = H_{c1}(T) = H_a$.

First we address the situations where the magnetic flux permeating the weak-linked specimen is seen to diminish throughout its entire volume during cooling until a temperature $T_{cr} < T_{c0}$ is attained. In these cases, as illustrated in Figs 3.2 and 3.3, at T_1 and $T_2 > T_{cr}$, the rate of flux expulsion from the grains, $|d\langle M_g \rangle/dT|$, is greater than $X dJ_{cm}/dT$ the rate of the growth of the flux trapping current induced to flow through the entire intergranular network (matrix), hence greater than $dH_m(x, T)/dT$, the increment of the magnetic field at the centre of the matrix. Consequently we write,

$$\left| \frac{d\langle M_g(T) \rangle}{dT} \right| = \frac{\langle M_{g0} \rangle n T^{n-1}}{T_{c0}^n} \geq X \frac{dJ_{cm}(T)}{dT} = \frac{X J_{cm0}}{T_{c0}} = \frac{H_{m=0}}{T_{c0}} \quad (3.14)$$

where we have introduced eqns 3.12 and 3.13. The equality sign applies when $T = T_{cr}$, hence, introducing this in eqn 3.14 leads to,

$$\frac{T_{cr}}{T_{c0}} = \left(\frac{H_{m=0}}{n \langle M_{g0} \rangle} \right)^{1/(n-1)} < 1 \quad (3.15)$$

Next when $T < T_{cr}$, the flux, being expelled by the grains located in an inner volume $0 < x \leq x_{cm}(T)$, is completely trapped in that volume by the growth of the currents induced to flow in the matrix in the surrounding volume $x_{cm}(T) \leq x \leq X_m$. Hence now as illustrated in Fig 3.4, $\Delta H_m(x_{cm}(T))$, the rise in the magnetic field at the interface, $x_{cm}(T)$, is equal to, $\Delta \langle M_g(T) \rangle$, the corresponding growth in the magnitude of the diamagnetism of the grains. Consequently we write,

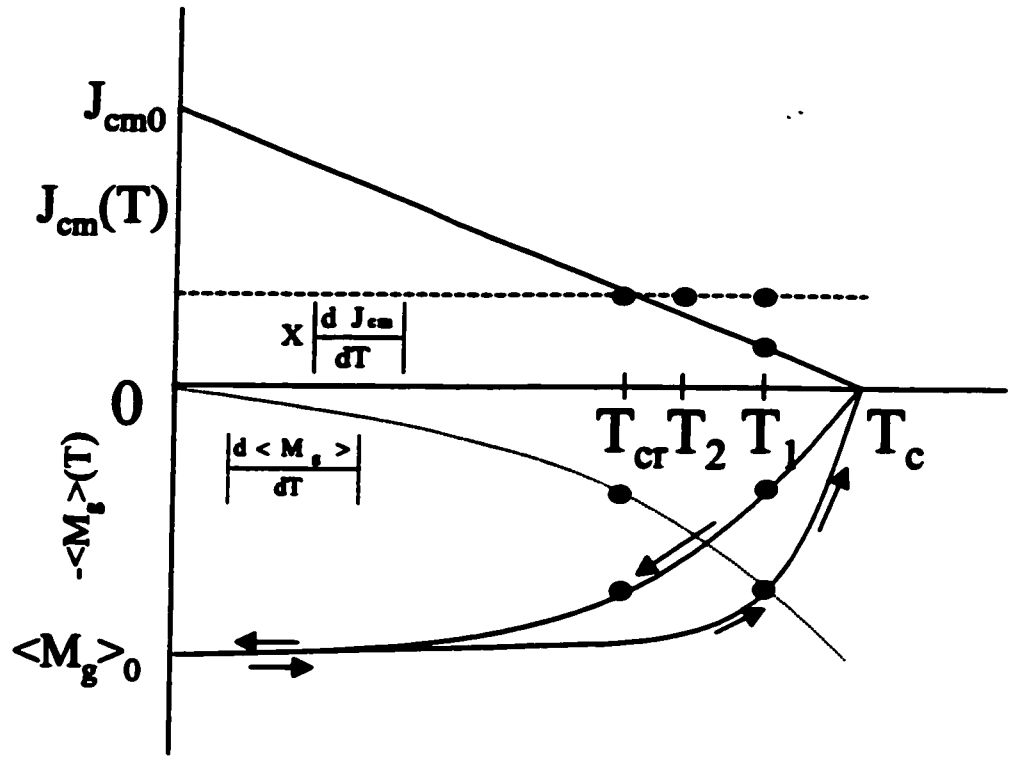


Fig 3.2. Illustrates the dependence on T of $J_{cm}(T)$, the intergranular (matrix) critical current density and $\langle M_g \rangle$, the diamagnetism of the grains and their temperature derivatives. Over the range $T_{cr} < T < T_c$, the rate of flux expulsion $|d\langle M_g \rangle/dT|$ is greater than the ability of the matrix to trap flux.

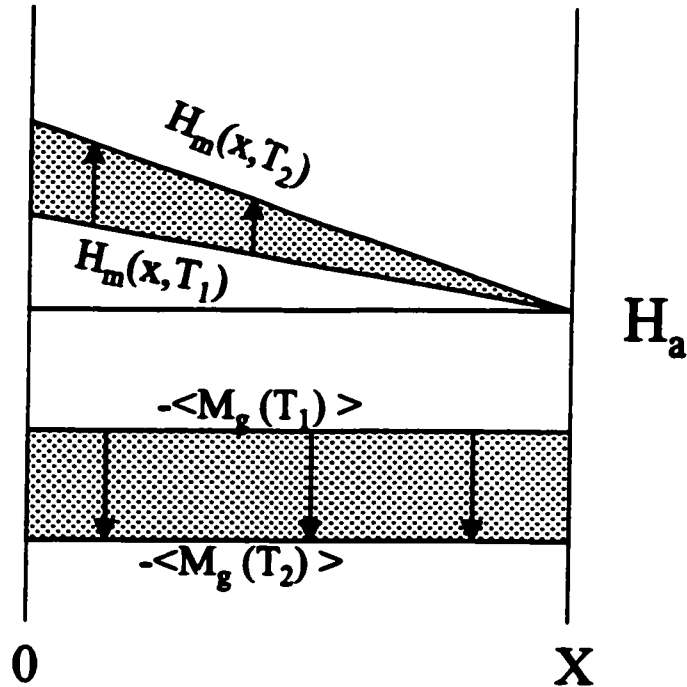


Fig 3.3 Displays the evolution of $H_m(x, T)$, the field profile in the matrix and of $\langle M_g(T) \rangle$, the magnetization of the grains as the temperature descends from T_1 to T_2 where T_1 and T_2 lie in the range $T_{cr} < T < T_c$. Here $|\Delta \langle M_g(T) \rangle| > \Delta H_m$ at $x = 0$, consequently $J_{cm}(x)$ is in a critical state throughout the specimen.

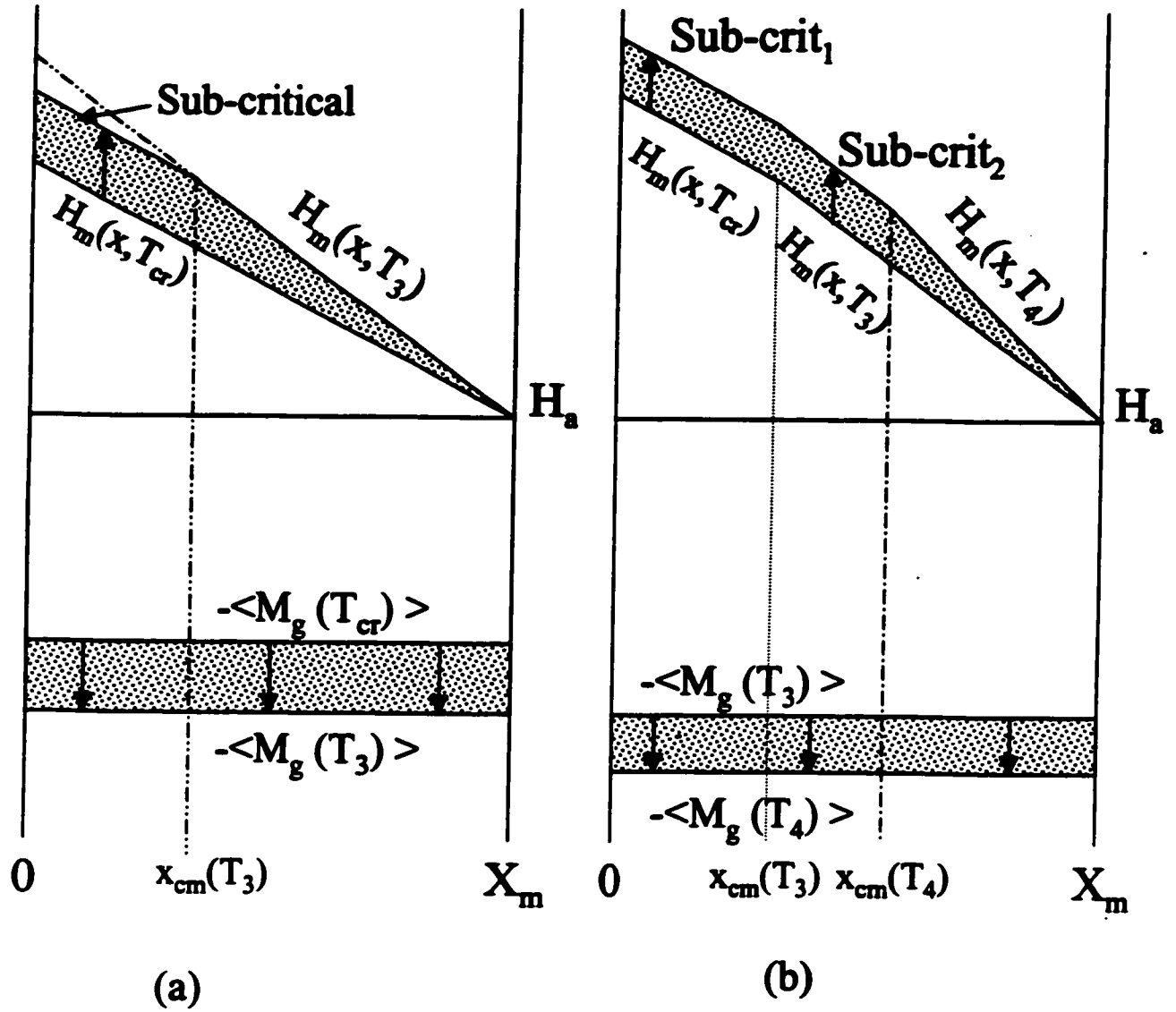


Fig 3.4. Schematics of the evolution of $H_m(x, T)$, the field profiles, and $J_{cm}(T) = |dH_m(x, T)/dT|$, the current density in the matrix when T is descending from T_{cr} to T_3 in (a) and from T_3 to T_4 in (b). Inside the expanding inner volume, $0 \leq x \leq x_{cm}(T)$, all the flux expelled by the grains is trapped in the local matrix since, $(X - x_{cm}(T))|dJ_{cm}(T)/dT| = |d<M_g(T)>/dT|$ at the outward advancing boundary $x_{cm}(T)$. Each element of the volume to the left of $x_{cm}(T)$ retains the corresponding critical value of $J_{cm}(T)$ established there earlier at the corresponding temperature (e.g. $J_{cm} = J_{cm}(T_{cr})$ at $x = x_{cm}(T_{cr}) = 0$, and $J_{cm} = J_{cm}(T_3)$ at $x = x_{cm}(T_3)$, and $J_{cm} = J_{cm}(T_4)$ at $x = x_{cm}(T_4)$ etc).

$$\begin{aligned}
\left| \frac{d \langle M_s(T) \rangle}{dT} \right| &= \frac{\langle M_{s0} \rangle n T^{n-1}}{T_{c0}^n} = \frac{dH_m(x_{cm}(T))}{dT} = \frac{dJ_{cm}(T)}{dT} \{X_m - x_{cm}(T)\} \\
&= \frac{J_{cm0} X_m}{T_{c0}} \left\{ 1 - \frac{x_{cm}(T)}{X_m} \right\} = \frac{H_{m0}}{T_{c0}} \left\{ 1 - \frac{x_{cm}(T)}{X_m} \right\}
\end{aligned} \tag{3.16a}$$

hence,

$$\frac{x_{cm}(T)}{X_m} = 1 - \frac{n \langle M_{s0} \rangle}{H_{m0}} \left(\frac{T}{T_{c0}} \right)^{n-1} \tag{3.16b}$$

describes the advance of the boundary $x_{cm}(T)$ separating the inner zone, $0 < x \leq x_{cm}(T)$, from which flux is not escaping, and the outer region, $x_{cm}(T) \leq x \leq X_m$ which releases flux to the outside world as the cooling progresses from T_{cr} . In the inner zone the flux expelled by the grains is now all stored in the “overlapping” volume of the matrix.

Fig 3.4 illustrates the advance of the boundary $x_{cm}(T)$ towards the surface as T descends below T_{cr} . Fig 3.4 also shows schematically the important feature that in the inner volume $0 < x \leq x_{cm}(T)$, the current density $J_m(x, T)$ spans a “spectrum” of previously critical but presently subcritical states at the temperature T_w “now” existing in the specimen. The values for $J_m(x, T)$, hence $dH_m(x, T)/dT$, range from $J_{cm}(0, T_{cr})$ at $x = 0$, the critical value established at the centre of the specimen when T traversed T_{cr} to the larger value, $J_{cm}(x_m, T_w)$ at the “present” position of the boundary $x = x_{cm}(T_w)$.

The field profile in the inner volume, $0 < x \leq x_{cm}(T)$ then reads,

$$\begin{aligned}
H_m(x, T) &= H_s + J_{cm}(T) \{X_m - x_{cm}(T)\} - \int_x^{x_{cm}(T)} \left(\frac{dH_m(x, T)}{dx} \right) dx \\
&= H_s + H_{m0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right) \right\} \left\{ 1 - \frac{x_{cm}(T)}{X_m} \right\} + J_{cm0} X_m \int_x^{x_{cm}(T)} \left(1 - \frac{T}{T_{c0}} \right) d \left(\frac{x}{X_m} \right)
\end{aligned} \tag{3.17}$$

Introducing,

$$\frac{T}{T_{c0}} = \left\{ \frac{H_{m=0}}{n \langle M_{s0} \rangle} \left(1 - \frac{x_{cm}(T)}{X_m} \right) \right\}^{1/(n-1)} \quad (3.18)$$

from eqn 3.16(b) in the last term of eqn 3.17 and integrating leads to,

$$\begin{aligned} H_m(x, T) = H_s + H_{m=0} \left(1 - \frac{x}{X_m} \right) - \langle M_{s0} \rangle \left(\frac{T}{T_{c0}} \right)^n \\ - (n-1) \langle M_{s0} \rangle \left\{ \frac{H_{m=0}}{n \langle M_{s0} \rangle} \left(1 - \frac{x}{X_m} \right) \right\}^{n/(n-1)} \end{aligned} \quad (3.19)$$

valid over the temperature range, $0 < \frac{T}{T_{c0}} < \frac{T_\alpha}{T_{c0}}$,

hence in the inner volume, $0 \leq x \leq x_{cm} = X_m \left\{ 1 - \frac{n \langle M_{s0} \rangle}{H_{m=0}} \left(\frac{T}{T_{c0}} \right)^{n-1} \right\}$

The critical field profile in the outer region, $x_{cm} \leq x \leq X_m$, reads,

$$H_m(x, T) = H_s + H_{m=0} \left\{ 1 - \left(\frac{T}{T_{c0}} \right) \right\} \left\{ 1 - \frac{x}{X_m} \right\} \quad (3.20)$$

The magnetization of the weak-linked specimen as it cools from T_{c0} is calculated in closed form or numerically from the definition,

$$M_{wl}(T, H_s) = -\langle M_s(T, H_s) \rangle + \frac{1}{X_m} \int_0^{x_m} H_m(x, T) dx \quad (3.21)$$

where we introduce the appropriate expressions for $\langle M_s(T) \rangle$ and $H_m(x, T)$. In pursuing more general accounts of the slow field cooling of weak-linked specimens, dependences of J_{cm} and $\langle M_s \rangle$ on T other than the simple expressions given in eqns 3.12 and 3.13, can readily be exploited and developed in closed form or numerically in the framework we have outlined in this section.

II First Warming of a Weak-Linked Polycrystal

A. General Framework

We now examine the evolution of the magnetization $M_{WL}(T, H_a)$ of weak-linked granular samples during first warming after slow field cooling in a field H_a which is maintained fixed throughout the process.

The rise in the temperature weakens the Meissner field shielding current $I_M(T)$ and the critical current density $J_{c2}(T)$ in the grains thereby allowing some of the flux expelled into the matrix during cooling to now reenter into the grains. A universal and crucial feature of the thermal hysteresis loops, $\langle M_g(T) \rangle$ versus T of the grains which is illustrated in Fig 3.5, now comes into play. We note that in Fig 3.5 the rate of flux entry into the grains during warming, $|d\langle M_g(T) \rangle / dT|_{\text{warming}}$, is much smaller than the rate of flux expulsion during cooling, $|d\langle M_g(T) \rangle / dT|_{\text{cooling}}$, at corresponding temperatures above and near $T_{c1}(H_a)$. It is this difference in the thermomagnetic behaviour of the grains during first cooling and first warming which is responsible for the generation of peaks^[66, 73, 149] in the diamagnetic magnetization $M_{WL}(T)$ of weak-linked specimens upon 1st warming after 1st cooling, whether the latter process occurred rapidly or slowly.

In section I B (ii) of this chapter we have explored various cases where the expulsion of flux by the grains, during cooling from T_c , caused the intergranular current density, $J_{cm}(x, T)$ induced in the matrix, to exist in critical states in the volume $x_{cm} \leq x \leq X_m$ adjacent to the surface and, in some instances, throughout the specimen. This state of affairs came about because the rate of flux expulsion by the grains, $|d\langle M_g(T) \rangle / dT|$, during cooling exceeded the rate of growth of the trapping field in the matrix, $dH_m(x, T)/dT = |dJ_{mc}/dT|(X_m - x_{cm})$, over the range, $x_{cm} \leq x \leq X_m$ which may have

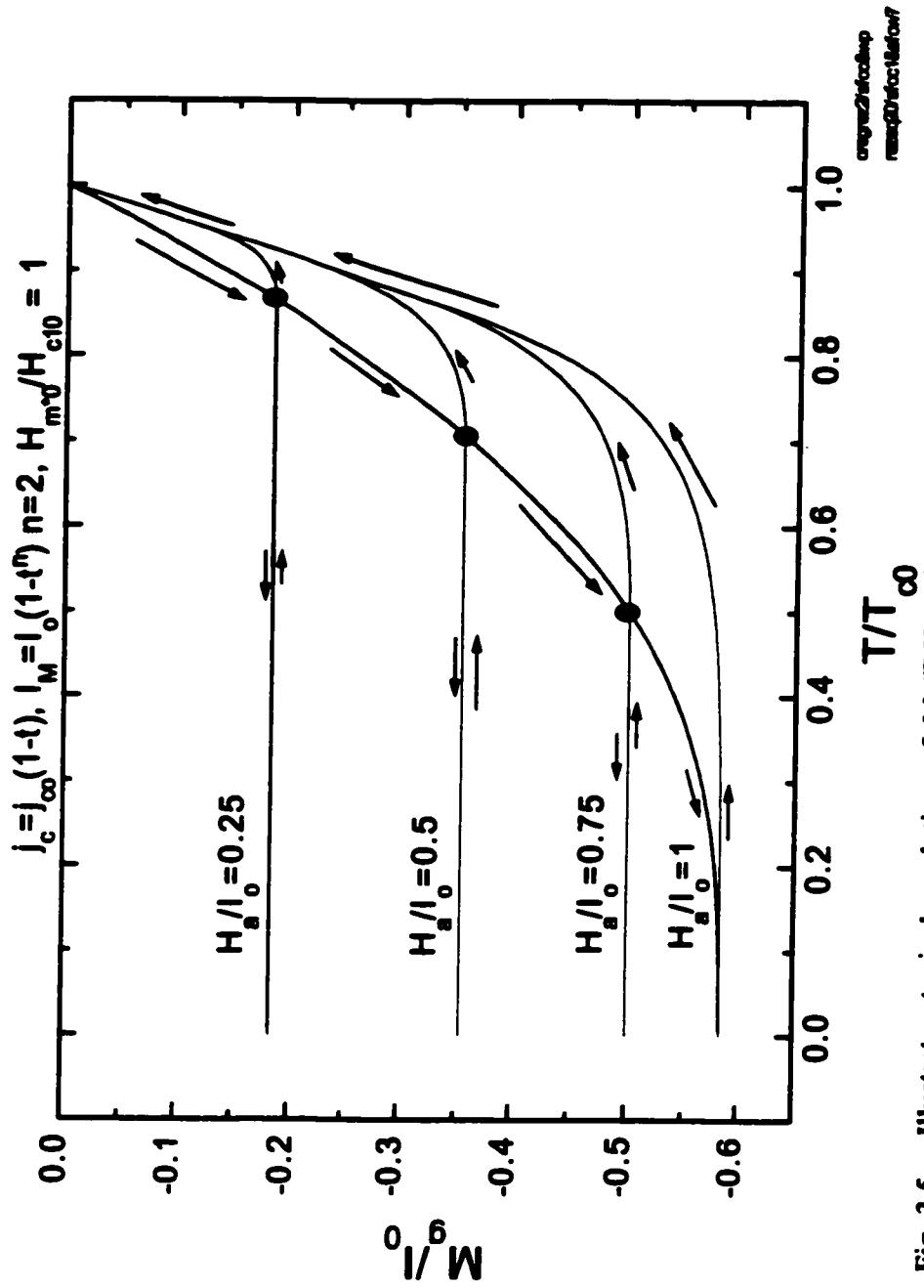


Fig 3.5. Illustrates typical evolution of $M_g(T, H_g)$, the magnetization of a single crystal of a type II superconductor during cooling from T_{c0} and subsequent slow warming in various static fields $0 < H_g \leq H_{c10}$. The dots indicate $T_{ci}(H_g)$.

extended over the entire specimen. Note that $|J_{cm}(T)|$, the critical current density in the matrix, and, $|dJ_{cm}(T)/dT|$, its rate of the change with T , are single-valued functions of T . Consequently the capacity for the matrix to trap or to release flux remains the same, at the same temperatures, on cooling or warming. In contrast however the response of the grains is very different during warming than during cooling as illustrated in Fig 3.5. Now over a wide range of the rise in temperature, $|<M_g(T)>|$ is diminishing very slowly, indicating that the grains can accept only a small fraction of the flux released by the matrix. Consequently, the specimen is compelled to release much of this flux to the outside world. This is illustrated for different types of matrix profiles in Fig 3.6. The escape of flux, previously trapped in the matrix during cooling but now made to exit from the specimen by warming, is responsible for the diamagnetic valley observed by Huyn^[66], Jung et al^[73], Ishida and Goldfarb^[99] and Wang and Joiner^[149] upon slow warming in their weak-linked low and high T_c granular samples.

When T approaches T_{c0} , the rate at which the grains can absorb flux is increasing appreciably as can be seen from inspection of the curves displayed in Fig 3.5. Consequently, the matrix can no longer release trapped flux at a rate sufficient to meet the growing demand of the grains for flux. Now flux must enter from outside to help fill this growing need of the grains for flux. By Faraday's law of induction, the entry of flux through the surface induces persistent critical current densities $J_{cm}(T)$ in the matrix which oppose this penetration. Consequently the scenarios we have described in detail in chapter 2 during warming of the isolated grains are now duplicated in the matrix of the weak-linked granular specimen. The sequences of events are shown schematically in Fig 3.7.

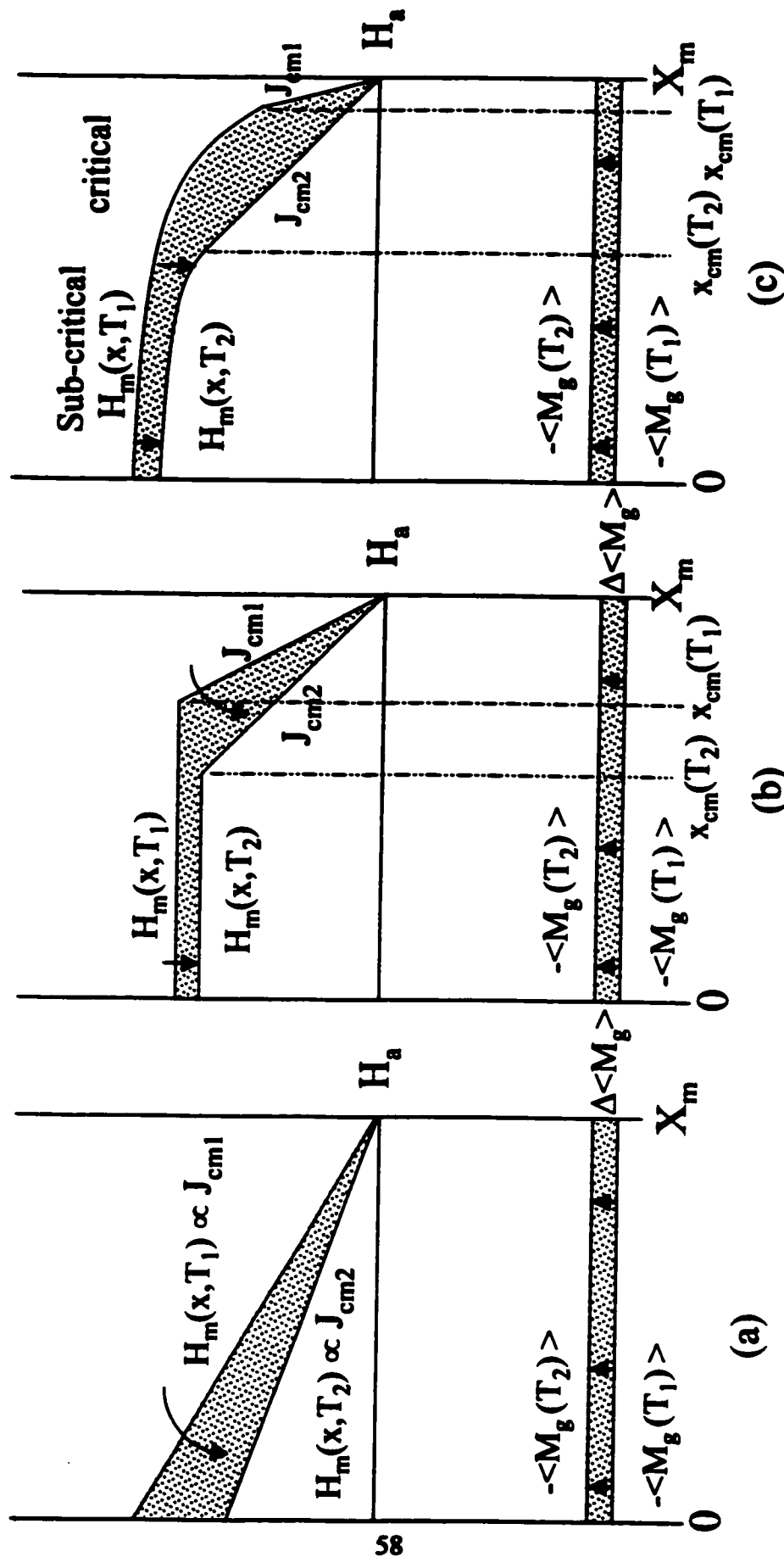


Fig 3.6. Evolution of the field profiles in the matrix during warming from T_1 to T_2 . The intergranular current density diminishes from $J_{cm}(T_1)$ to $J_{cm}(T_2)$. This causes $\Delta\Phi$, the flux released by the matrix, to exceed the amount which can be accommodated by the grains whose diamagnetism is concomitantly diminishing in magnitude by an amount $|\Delta\langle M_g \rangle|$. In (a), $|\Delta\Phi| > |\Delta\langle M_g \rangle|$. In (b) and (c) $|\Delta H_m(x, T)| = |\Delta\langle M_g \rangle|$ in the volume, $0 < x < x_{cm}(T_2)$ but some flux is released to the outside from the outer volume, $x_{cm}(T) \leq x \leq X$, while an amount $\Delta\Phi_i = |\Delta\langle M_g \rangle|(X_m - x_{cm})$ is dumped into the grains. In (c), J_{cm} was initially sub-critical at T_1 in the volume $0 \leq x \leq x_{cm}(T_1)$ but became critical at the smaller value J_{cm2} in the volume $x_{cm}(T_2) \leq x \leq x_{cm}(T_1)$ as T increased from T_1 to T_2 . In (a), (b) and (c), since flux is being released to the outside world by the specimen, warming causes an increase in the magnitude of the total diamagnetism. This accounts for the diamagnetic valley observed by many workers on warming weak-linked type II superconductors.

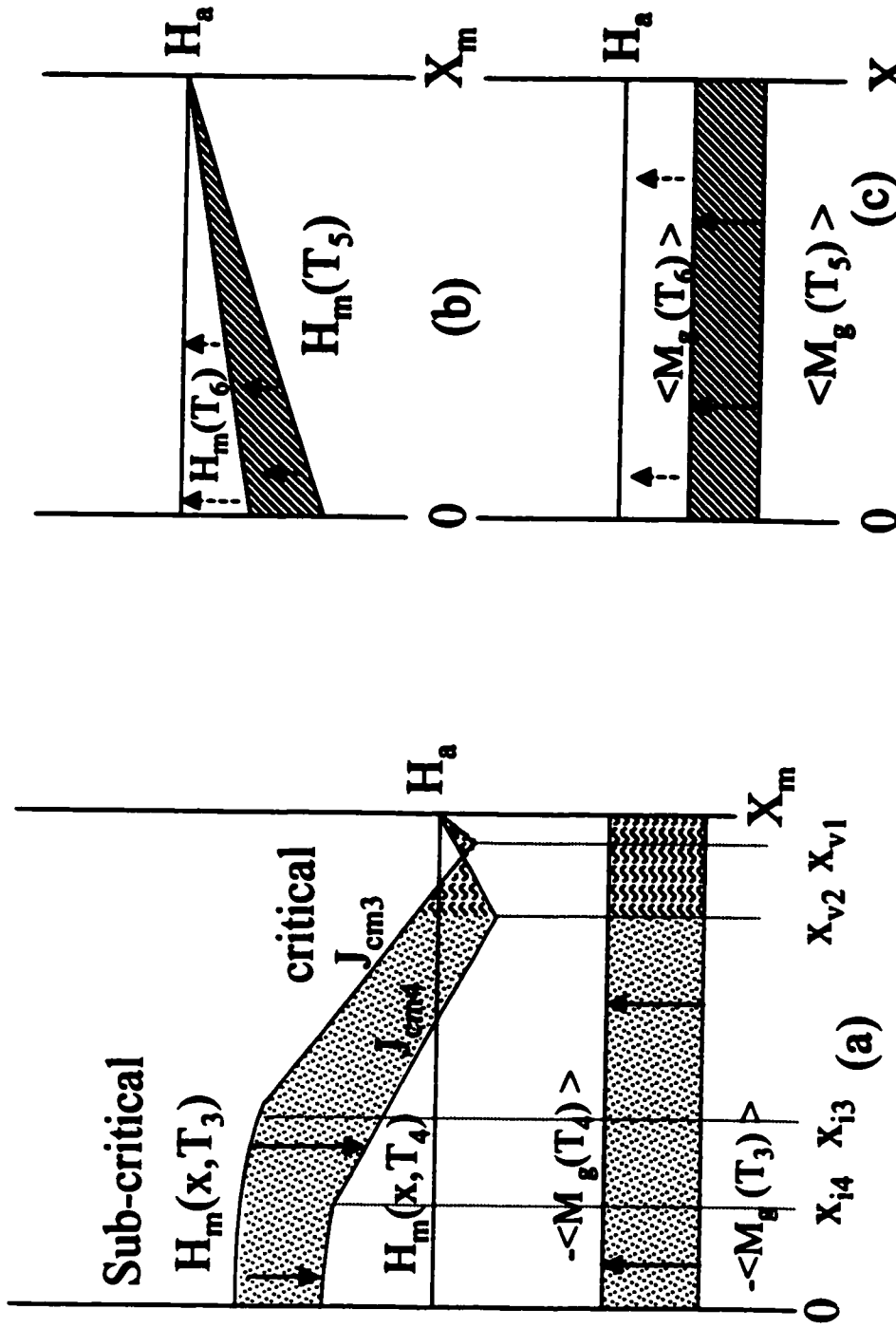


Fig 3.7. Continuation of the evolution of the field profiles from the situation illustrated in Fig 3.6(c). (a) As the specimen is warmed from T_3 to T_4 , the flux released by the matrix in the inner volume, $0 \leq x \leq x_v(T)$, is entirely absorbed by the grains in that space. However, in the outer volume, $x_v(T) \leq x \leq X$, the demand for flux by the grains, whose diamagnetism is diminishing in magnitude, cannot be satisfied by the contribution from the matrix, hence the deficit must be supplied by flux entering from outside. Pinning in the matrix opposes this entry, hence the field gradient and J_{cm} reverse sign in the outer volume. (b) Eventually as T increases from T_4 to T_5 , the bottom of the valley in the field profiles shown in (a) attains the centre of the specimen. Subsequently the gradients become shallower as T increases from T_5 to T_6 and disappear at T_c . (c) Concomitantly with the events shown in (b), the diamagnetism of the grains, $|\langle M_g \rangle|$ diminishes to zero.

In the next section we compare experimental curves obtained by Wang and Joiner^[149] and Jung et al^[73] with curves generated by our model. Inspection shows that our simple model reproduces their observations quite successfully both qualitatively and semi-quantitatively.

B. Comparison of Observations with Theoretical Curves

(i) Modeling of the Data of Dr. W. C. H. Joiner

Dr. W. C. H. Joiner of the Physics Dept at the University of Cincinnati provided us with an extensive compilation of measurements of the evolution of the magnetization of a weak-linked YBCO polycrystal during slow warming after fast field cooling which include and complement the results reported in the article with J. P. Wang^[149]. This family of data curves spans a broad range of static applied magnetic fields. The set of observations are displayed in Figs 3.8, 3.9, and 3.10 and are grouped to show the behaviour in weak, intermediate and strong magnetic fields. In each figure we also display for direct qualitative and quantitative comparison, a corresponding family of curves calculated with our model.

In these calculations, for simplicity, and for computational convenience we let $J_{gc}(T)$, $I_m(T)$ and $J_{cm}(T)$ all have the same dependence on temperature, namely, $f(T) = \{1 - (T/T_{c0})^n\}$ and chose $n = 5$ for these calculations. The theoretical curves are not very sensitive to the choice of n over a modest range (3-7). The effect of a change in the choice of n on the structure of the calculated curves is also illustrated in this section.

To account for the observations of Dr. Joiner over the large range of fields used in his measurements we have found it necessary to introduce dependences of the critical

current densities J_{cg} and J_{cm} on the applied magnetic field H_a . However, in the calculations of the complicated evolution of the field profiles in the grains and in the matrix during cooling and warming in a chosen fixed field H_a , we have, for computational simplicity, ignored the functional dependence on H_a and taken J_{cg} and J_{cm} to be determined by the selected H_a . This approach has been frequently exploited by workers in similar situations and is valid provided that the variations of the field profiles in a chosen H_a are significantly smaller than the increments in the values of H_a . This criterion is fully satisfied in all our calculations. Fig 3.11 displays the dependences of H_{cg}/H_{c10} and H_{cm}/H_{c10} on H_a which we have introduced in these calculations. We achieve a good fit to the data using,

$$h_{cg} = \frac{J_{cg0}X_g}{H_{c10}} = \frac{H_{cg}}{H_{c10}} \approx 2.5 \left(\frac{H_{c10}}{H_a} \right)^{3/4} \quad (3.22)$$

when $H_a > H_{c10}$, and letting, $H_{cg} = 2.5H_{c10}$ over the range $0 \leq H_a \leq H_{c10}$ with $H_{c10} = 40$ Gauss.

In the low field range of the measurements, namely, $0.025 \leq H_a / H_{c10} \leq 1.25$, we obtain a good fit using,

$$h_{cm} = \frac{J_{cm0}X_m}{H_{c10}} = \frac{H_{cm}}{H_{c10}} \approx 0.3 \left(\frac{H_{c10}}{H_a} \right)^{1/2} \quad (3.23)$$

No simple expressions describe the dependence of H_{cm} on H_a in the high field range above $1.25H_{c10}$ (see Fig 3.11(b)).

We recall that when $H_a < H_{c10}$, the field profiles in the grains and the matrix cease to change when the temperature descends below,

$$\frac{T}{T_{c0}} = \frac{T_{cl}(H_a)}{T_{c0}} = \left\{ 1 - \left(\frac{H_a}{H_{c10}} \right) \right\}^{1/n} \quad (3.24)$$

The profiles then remain “frozen” in the configurations established at $T_{cl}(H_a)$ during subsequent excursions of T below $T_{cl}(H_a)$ in the chosen static H_a . Since in our modeling for Figs 3.8, 3.9 and 3.10 we exploited identical temperatures dependences for J_{cs} , J_{cm} and I_M , it follows that,

$$\frac{J_{cs}(T_{cl})}{J_{cs0}} = \frac{J_{cm}(T_{cl})}{J_{cm0}} = \frac{I_M(T_{cl})}{I_0} = \frac{H_a}{H_{c10}} = \left\{ 1 - \left(\frac{T_{cl}}{T_{c0}} \right)^n \right\} \quad (3.25)$$

when $H_a/H_{c10} \leq 1$. Consequently although the values for H_{cm}/H_{c10} displayed in Fig 3.11(b) are appreciable fractions when $H_a < H_{c10}$, the profiles in the matrix are governed by the much smaller critical current density established when $T = T_{cl}(H_a)$, hence,

$$J_{cm}(T_{cl}) = J_{cm0} \left(\frac{H_a}{H_{c10}} \right) \quad (3.26)$$

Therefore, the maximum variation which the field profile of the matrix can exhibit is then,

$$J_{cm}(T_{cl})X_m = (J_{cm0}X) \left(\frac{H_a}{H_{c10}} \right) = H_{cm0} \left(\frac{H_a}{H_{c10}} \right) \quad (3.27a)$$

or

$$\frac{J_{cm}(T_{cl})X_m}{H_{c10}} = \frac{H_{cm0}}{H_{c10}} \left(\frac{H_a}{H_{c10}} \right) = h_{cm0} \left(\frac{H_a}{H_{c10}} \right) \quad (3.27b)$$

when the quantities are normalized to H_{c10} .

Inspection of Fig 3.8, 3.9 and 3.10 shows that our crude model quite successfully generates the prominent features and trends of the measured curves. The main feature, which intrigued the researchers who first encountered this phenomenon, is the occurrence

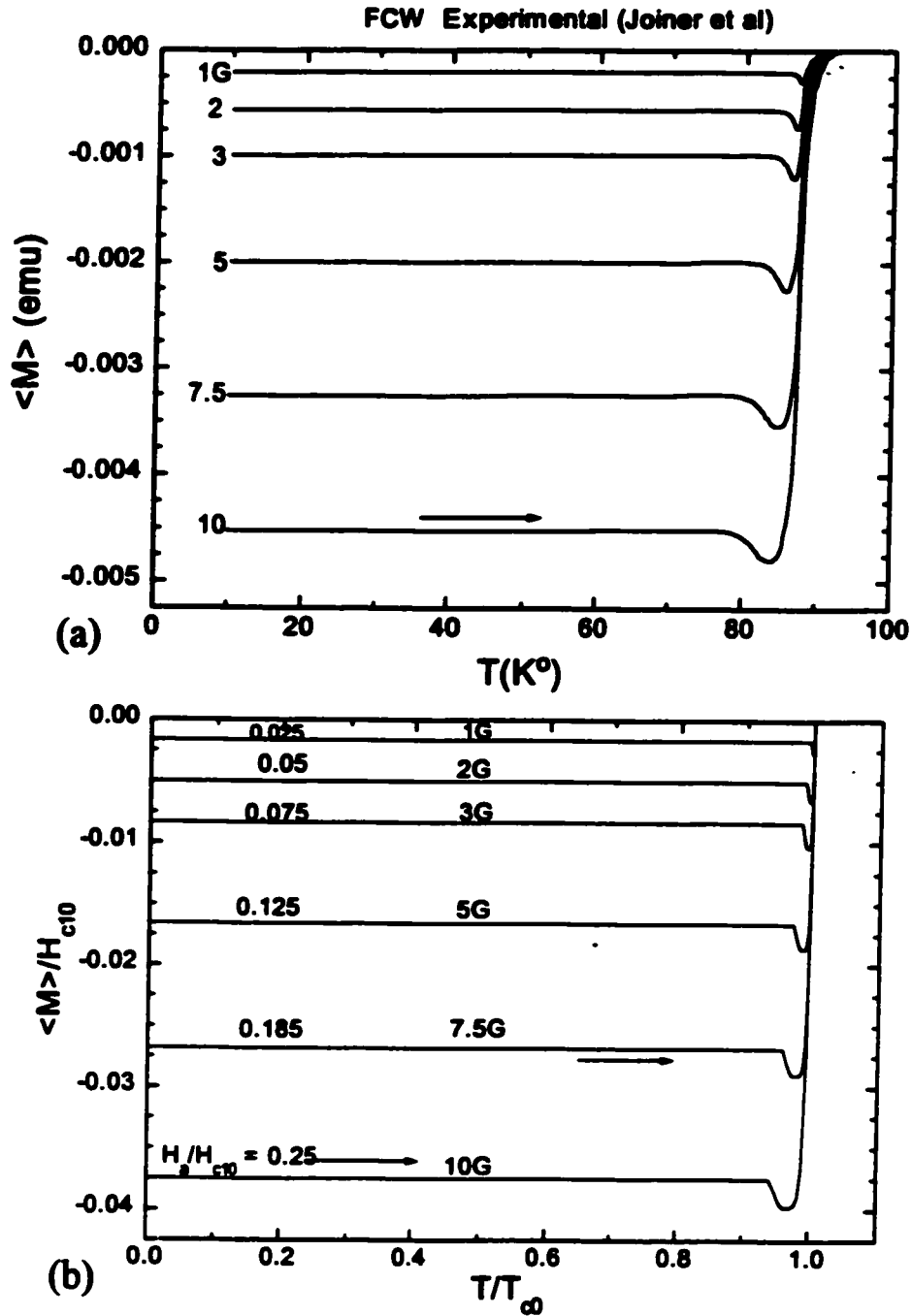


Fig 3.8 (a) Unpublished observations by Dr. W. C. H. Joiner on a polycrystalline YBCO specimen during slow warming after fast field cooling in weak static applied fields, $H_a \ll H_{c10}$. The data was provided in arbitrary (emu) units. (b) Curves calculated with our model taking $n = 5$ in eqn 3.25 with H_m independent of H_a and using $H_m = 2.5H_{c10}$. The dependence of H_m on H_a is shown in the insert to Fig 3.11(b). To convert the theoretical applied fields H_a to that in the measurements we let $H_{c10} = 40$ G throughout the calculations in this and in the next figures.

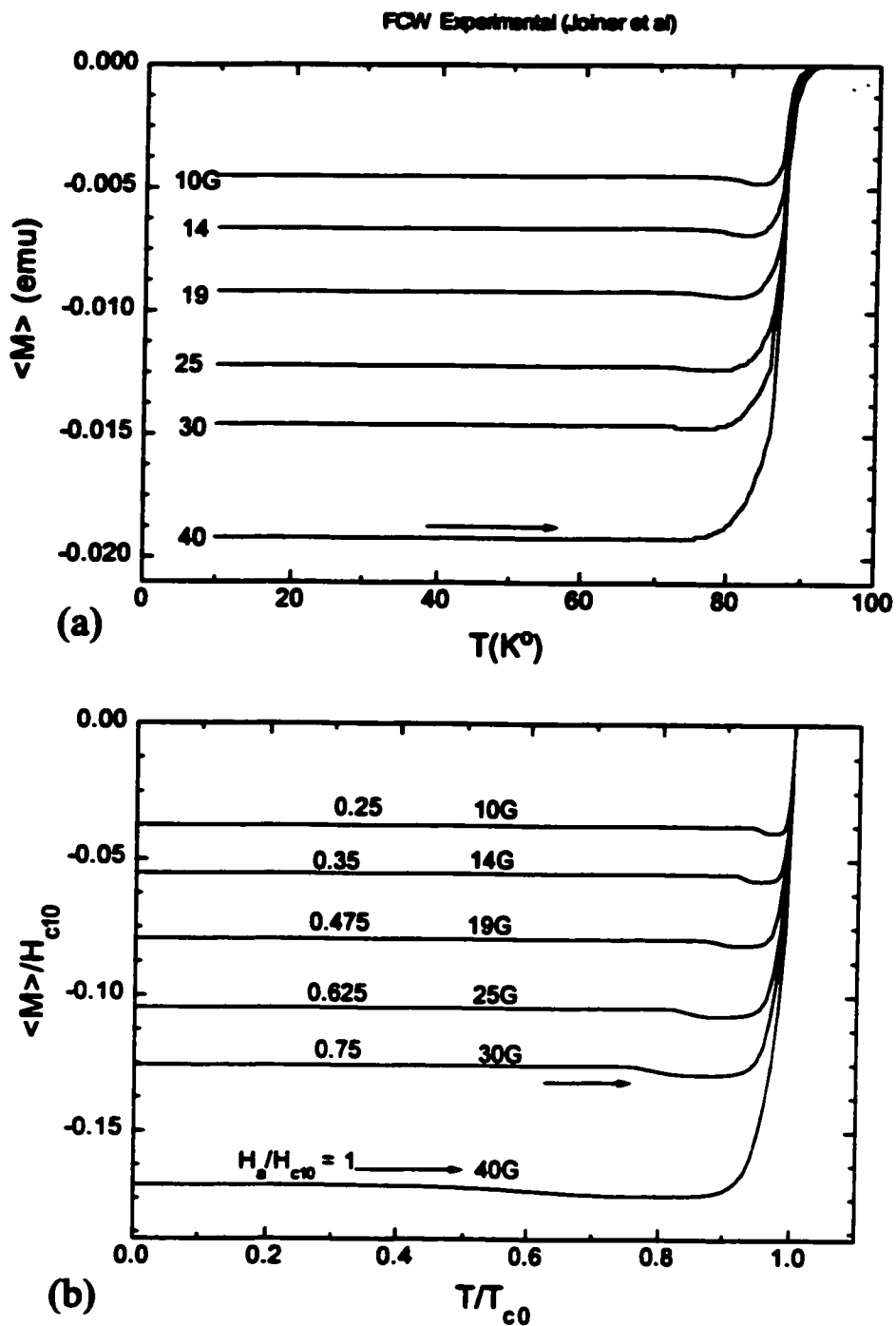


Fig 3.9. Continuation of the previous figure to the range of intermediate applied fields, $H_a \leq H_{c10}$. Here in the calculated curves H_{a2} is again taken independent of H_a with $H_{a2} = 2.5H_{c10}$. The dependence of H_{a2} on H_a is displayed in the insert to Fig 3.11b.

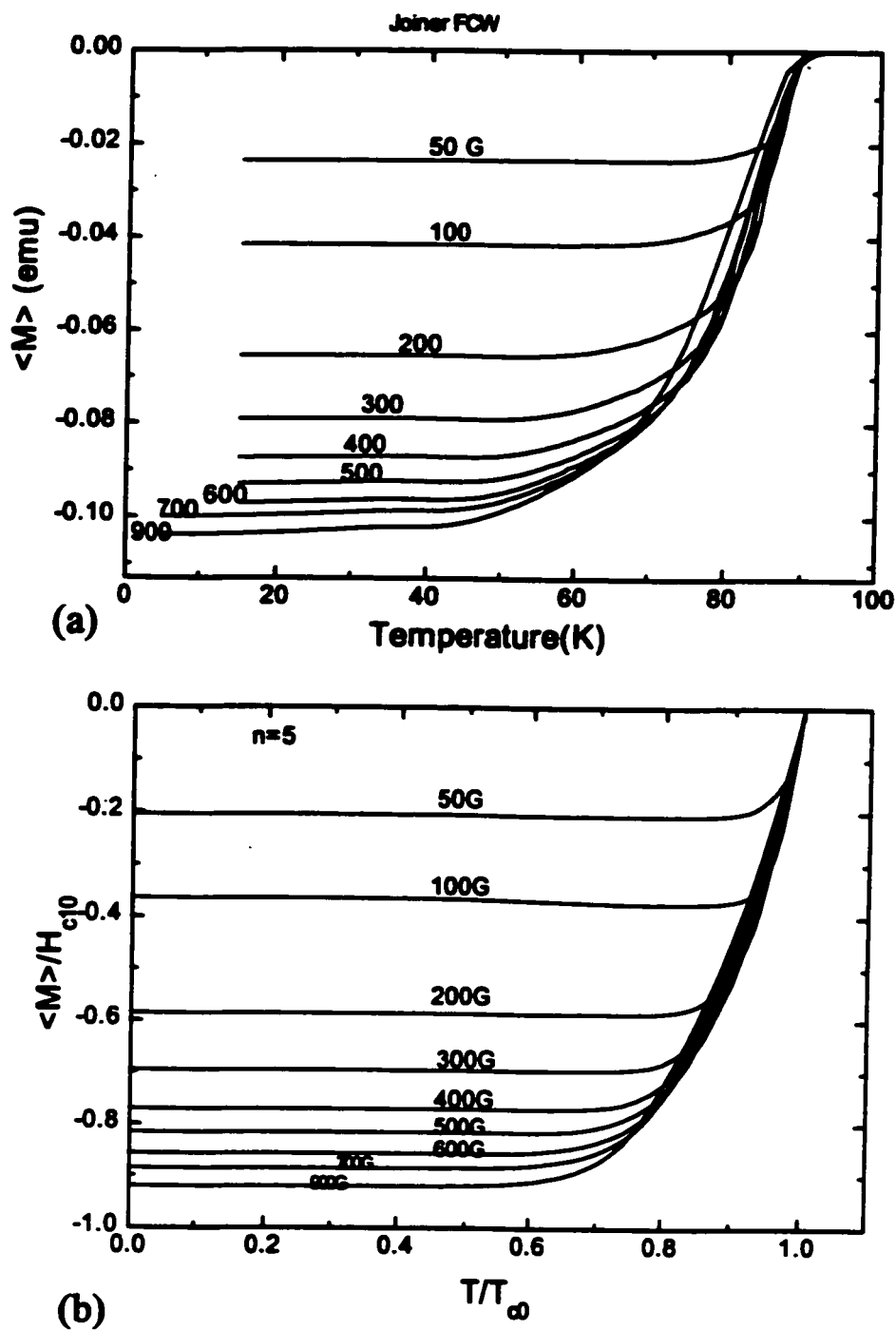


Fig 3.10. (a) Observations of Dr. W. C. H. Joiner in the high field range $H_a > H_{c10}$. (b) The dependences of the parameters H_{a_0} and H_{a_m} on H_a introduced in the calculations are displayed in Fig 3.11.

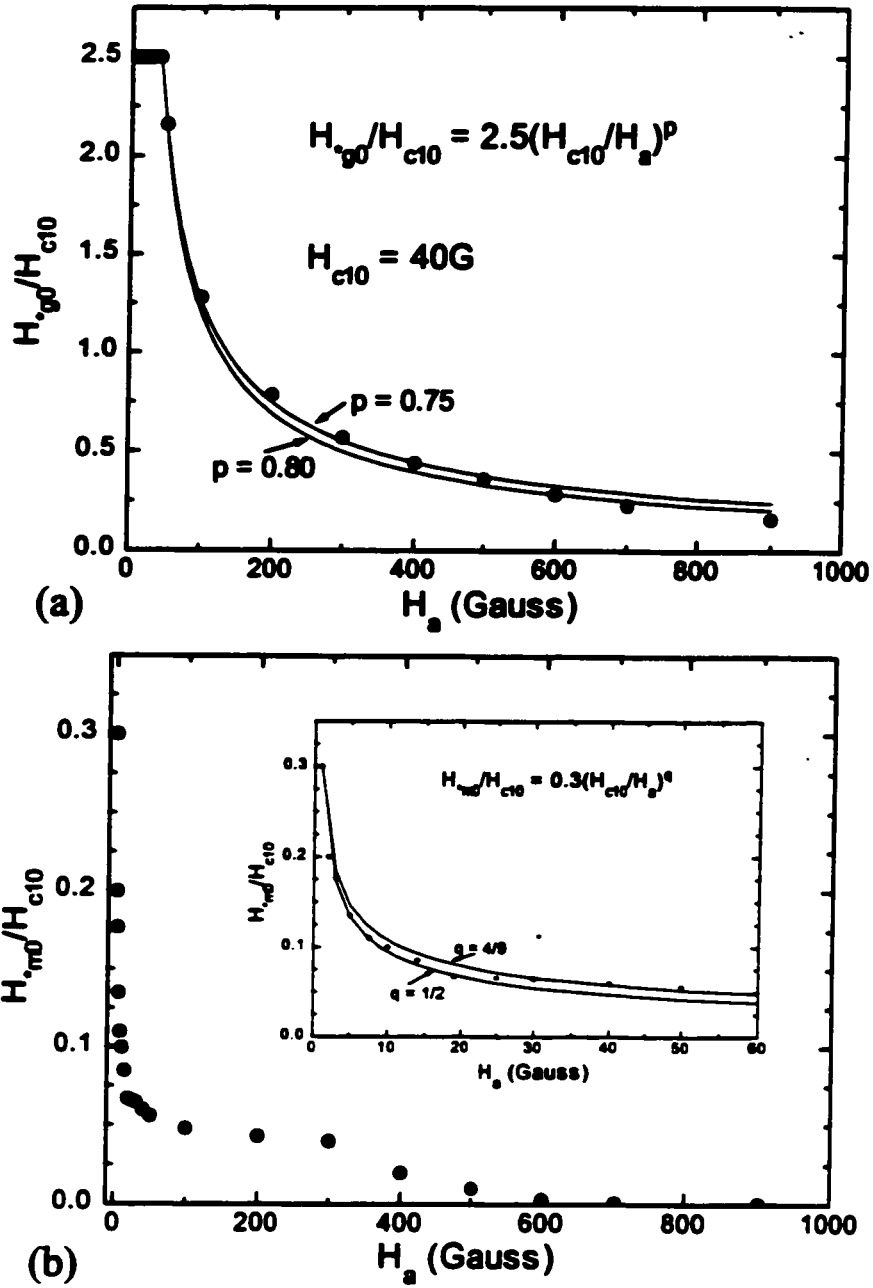


Fig 3.11. Displays the dependence of $H_{g0} = J_{cg0}X_g$ and $H_{m0} = J_{cm0}X_m$ on H_g introduced in the model calculations in order to reproduce the evolution of the experimental curves of Dr. Joiner as the behaviour is explored over a large range of static applied fields. The dots indicate the actual numerical values which were used for these parameters to optimize the fit of the theoretical curves to the corresponding measured curves. The continuous curves in (a) and (b) and in the inset display the simple functions $H_{g0}/H_{c10} = 2.5 (H_{c10}/H_g)^p$, and, $H_{m0}/H_{c10} = 0.3 (H_{c10}/H_g)^q$ which fit these chosen values. Thus fitting of the data with our model provides information of the dependence of J_{cg0} and J_{cm0} on H_g .

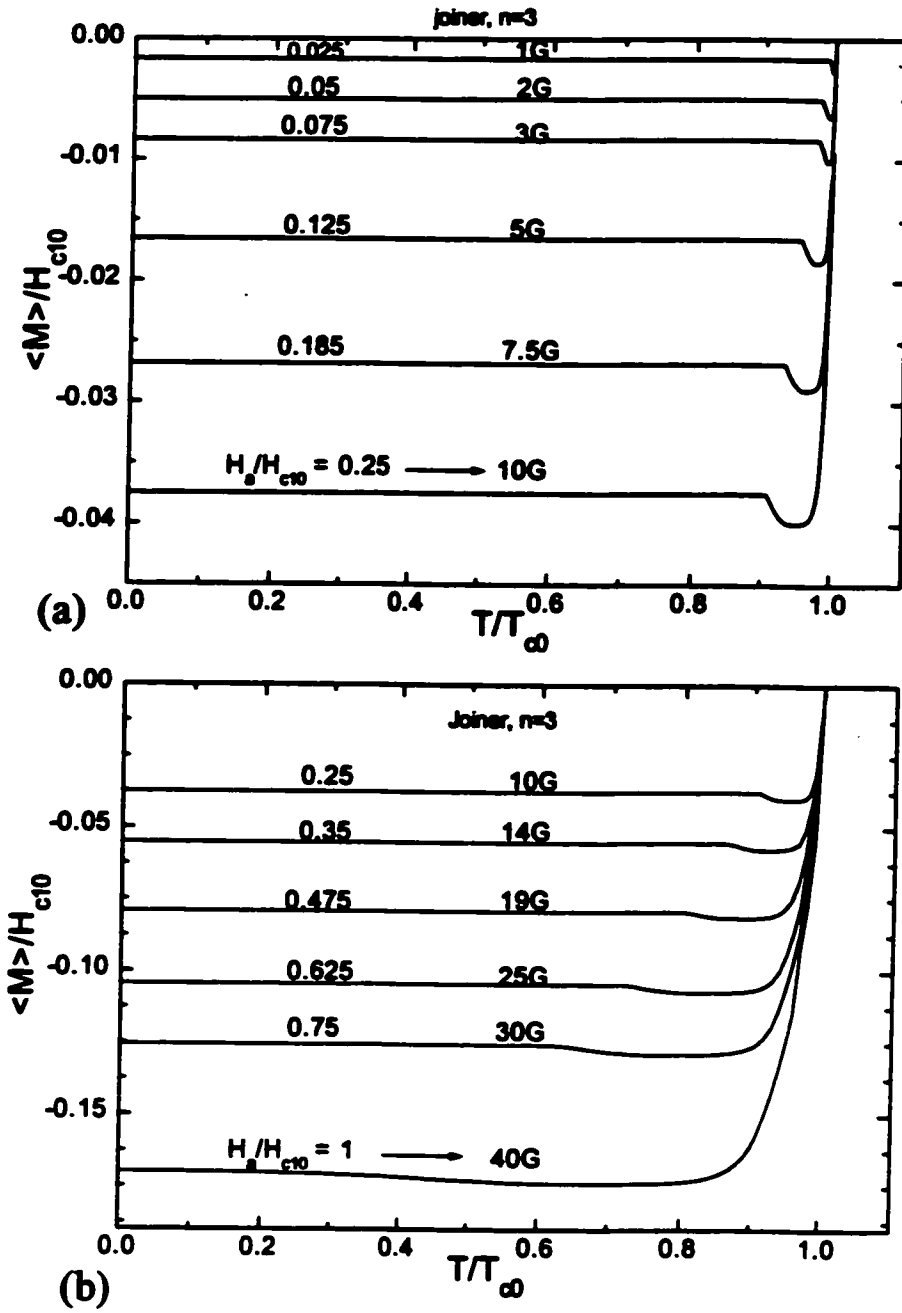


Fig 3.12. Illustrates the effect on the theoretical curves of a different choice for the power n in $f(T) = \{1 - (T/T_{c0})^n\}$ for the dependence of $J_{cm}(T)$, $J_{cg}(T)$ and $I_M(T)$ on temperature. (a) Should be compared with Fig 3.8(b) and (b) with Fig 3.9(b). Clearly, the model is not very sensitive to the choice of this exponent.

of a valley or dip, hence a sharp enhancement of the Meissner effect, in the diamagnetic magnetization of polycrystals just below T_c during warming. The depth and breadth of this valley are seen to evolve as a function of the static field H_s and our model also reproduces this behaviour as we will demonstrate.

In our modeling, when $H_s < H_{c10}$ the valley commences when T increasing attains $T_{c1}(H_s)$. At this juncture, the onset in the diminution of $J_{cm}(T)$, $J_{cs}(T)$ and $I_M(T)$, causes the field profiles, ("frozen" when T descending traversed $T_{c1}(H_s)$), to start changing. However, the change in the grains, is initially more gradual than that in the matrix, since in the grains the rise in temperature is mainly causing an internal distribution of flux (see Fig 2.5 in the preceding chapter) and consequently generates only a small diminution of the diamagnetism of the grains (see Fig 3.5). As a consequence only a small fraction of the flux being released by the decrease of the field trapping current density $J_{cm}(T)$ in the matrix can be accommodated by the grains. The rest of this flux therefore escapes from the sample, thereby causing the diamagnetism to increase.

The bottom of the valley is traced when the rate of release of the flux by the matrix becomes equal to the growing "need" for flux by the grains. This need for flux occurs because the Meissner field shielding current $I_M(T)$ is decreasing in the grains. Next, on the right hand side of the valley, the matrix, while continuing to release flux into the grains, is also compelled to oppose entry of flux across the surface of the specimen as illustrated in Fig 3.7a. Eventually the entire matrix is participating in the opposition to entry of flux into the specimen (see Fig 3.7b).

Thus in our model the temperature for the onset of the valley reads,

$$T_{\text{onset}} = T_{c1}(H_s) \quad (3.28)$$

when $H_a < H_{c10}$, and $T_{onset} = 0$ when $H_a > H_{c10}$. The temperature T_{VB} , where the valley bottom occurs is controlled by,

$$\frac{d\langle M_s \rangle_{\text{valley}}}{dT} = \frac{d}{dT} \int_0^x H_m(x, T) \frac{dx}{X} \quad (3.29)$$

which applies until the intergranular current density J_{cm0} is destroyed by the strong magnetic field H_a , hence when $H_m = 0$. The sample then consists of an assembly of decoupled grains and our model generates no valley in these circumstances.

Although it may not be evident from inspection of Fig 3.10a, all of these measured curves trace a shallow valley which is masked however by the background magnitude of the magnetization and the thickness of the lines. Here T_{VB} ranges from 69K at $H_a = 50G$ to 40K at $H_a = 700G$. It is of interest to note that our corresponding theoretical curves also trace a comparable broad and shallow valley since H_{m0} although very small at these fields is still finite in our calculations (see Fig 3.11b).

The depth of the valley is not sensitive to the choice of the exponent n for the temperature dependence of $J_{cm}(T)$, $J_{cg}(T)$ and $I_M(T)$. This feature is illustrated by comparing the calculated curves displayed in Fig 3.12 with the corresponding theoretical curves of Fig 3.8b and 3.9b. However, as expected, the breadth of the valleys is seen to increase as n is chosen smaller since $T_{onset} = T_{c1}(H_a)$ is dependent on the exponent n (see eqn 3.25). For the families of curves in Fig 3.12, the exponent n is chosen equal to 3 whereas $n = 5$ in Figs 3.8b and 3.9b. All other quantities are as shown in Fig 3.11a and b.

(ii) Modeling of Data of Jung et al after Fast Field Cooling

Fig 3.13 compares the observations of Jung et al^[73], during slow warming of a 4.5mm diameter, 3.5mm length cylinder of YBCO after fast cooling, with a corresponding family of curves generated by our model. Again in the calculations we chose identical temperature dependences of the form $f(T) = \{1 - (T/T_{c0})^n\}$ with $n = 4$ for the three quantities, $J_{cg}(T)$, $I_M(T)$ and $J_{cm}(T)$. We take $H_{g0}/H_{c10} = 3.2$ constant over the range of the modeling and allowed H_{m0} to vary slowly and nearly linearly as a function of H_a as tabulated in the caption to Fig 3.13b. We took $\mu_0 H_{c10} = 10\text{mT}$.

In Fig 3.14 we compare the measured and calculated values for the temperatures where the diamagnetic valley begins, denoted T_{onset} , and where the bottom of this valley is attained, denoted T_{VB} . There is no ambiguity in identifying T_{onset} in our model (see eqn 3.24) where the theoretical $T_{\text{onset}}/T_{c0} = T_{c1}(H_a)/T_{c0} = \{1 - (H_a/H_{c10})\}^{1/n}$. The theoretical T_{VB} does not emerge clearly from the shallow valleys displayed in Fig 3.13 but the accurate numerical values indicate the minimum unambiguously. There is appreciable uncertainty however in identifying T_{onset} and T_{VB} in the experimental curves.

Jung et al^[73] visualize that the intergrain flux depinning commences when T_{onset} is attained. This picture is in harmony with our model where the field profiles in the matrix and in the grains are regarded immobile (frozen) when $T < T_{c1}(H_a) = T_{\text{onset}}$ and migration of flux from the matrix into the grains and out of the specimen begins when T exceeds $T_{c1}(H_a) = T_{\text{onset}}$. Jung et al^[73] refer to T_{VB} as the temperature where the intergrain flux depinning reaches completion. What is meant by “completion” is not defined. In our scenario, flux depinning in the grains and in the matrix are both occurring at T_{VB} and the

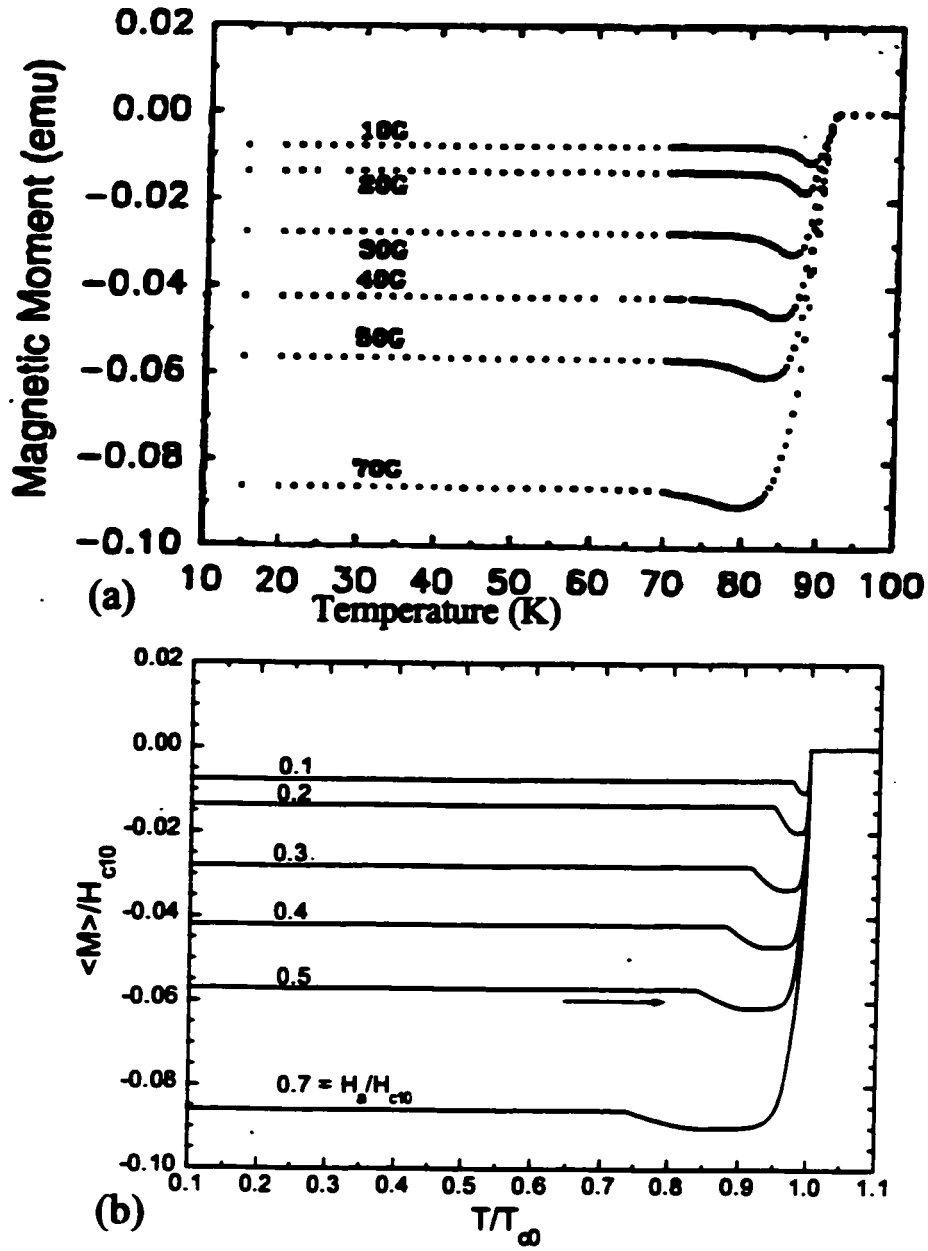


Fig 3.13. (a) Evolution of the diamagnetism observed by Jung et al^[73] for a polycrystalline YBCO specimen upon slow warming to T_c after fast field cooling in the static applied magnetic fields indicated. (b) Corresponding theoretical curves generated by our model with $n = 4$ in $f(t) = \{1 - (T/T_{c0})^n\}$ for the temperature dependence of $J_{cm}(T)$, $J_{cs}(T)$ and $I_M(T)$. Here $H_{s0}/H_{c10} = 3.2$, and $H_{sm}/H_{c10} = 0.162, 0.18, 0.126, 0.103, 0.0845$ and 0.057 for the sequence of $h_c = H_c/H_{c10} = 0.1, 0.2, \dots, 0.7$. Consequently $H_{sm} = J_{cm}X_{cm}$ is seen to diminish approximately linearly versus H_s .

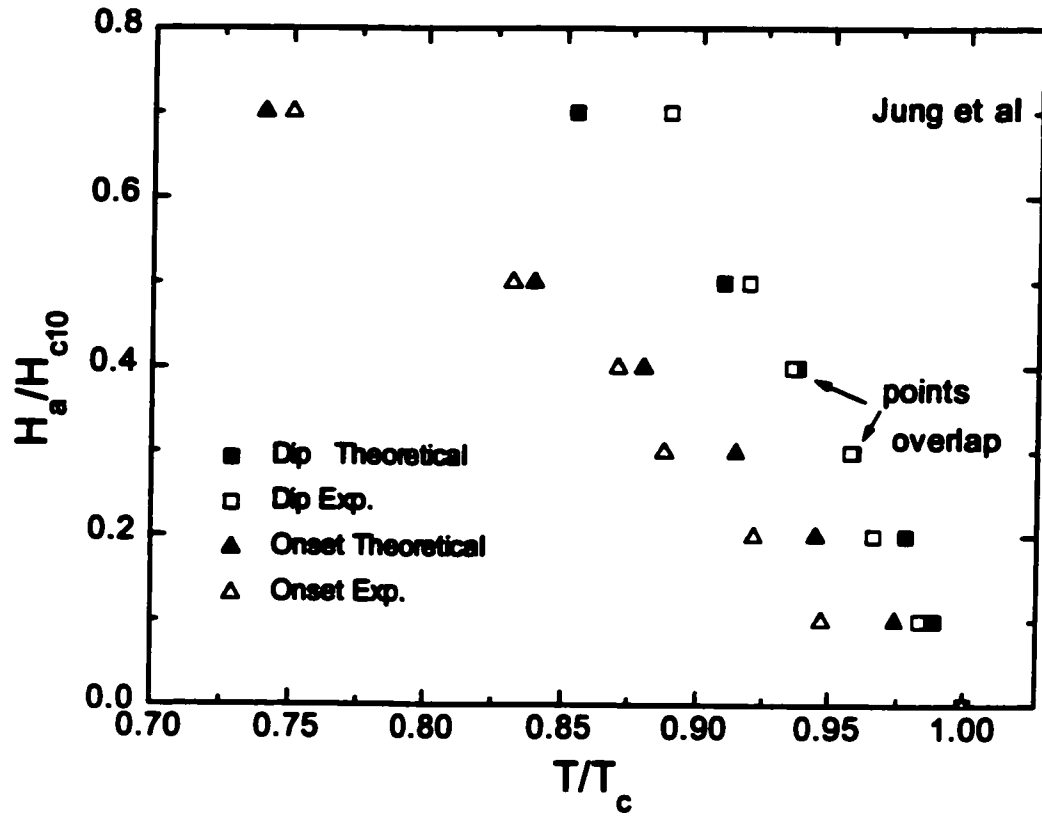


Fig 3.14. Compares the dependences on T/T_c of T_{onset} (△) and T_{dip} (□) experimental with T_{onset} (▲) and T_{dip} (■) theoretical from the observed and calculated curves of the previous figure.

intricate interplay of these two processes concurrently with the decrease of the Meissner current in the grains leads to $dM_{WL}/dT = 0$. These events have been described in some detail in section IIA of this chapter.

(iii) Effect of Slow and Fast Field cooling on the Magnetization of Polycrystals

Jung et al^[73], Wang and Joiner^[149] and others^[66, 69] have observed that the amount of flux expelled from polycrystals during slow field cooling (SFC) is appreciably larger than that released by fast field cooling (FFC) to the same final temperature T_f in the same static applied field H_a . This behaviour is illustrated in fig 3.15 which is reproduced from Jung et al^[73]. Generally, no valley or only a shallow valley appears during warming after slow field cooling. The measurements of Jung et al^[73] and Wang and Joiner^[49] also indicate that the flux expelled when the valley in the magnetization of weak-linked polycrystals is traced during warming after fast field cooling corresponds closely to $\Delta M_{WL}(T_f)$. Here,

$$\Delta M_{WL}(T_f) = |M_{WL}(SFC)| - |M_{WL}(FFC)| \quad (3.30)$$

is the difference in the diamagnetic magnetizations observed at T_f after slow and fast field cooling (see Fig 3.15). In this section we present an explanation for these features and extend our simple model to account for the data of Jung et al^[73] displayed in Fig 3.15 which compares the effect of the two modes of cooling over an appreciable range of H_a .

It is well established that intergranular currents (J_{cm}) decay quite rapidly especially at temperatures close to T_c ^[37, 68, 82, R7, M1]. This phenomenon has been referred to as giant flux creep^[R7]. Intragrain currents (J_{cg}) are also observed to decay but at rates appreciably slower than the former at corresponding temperatures and applied magnetic fields^[R7]. The

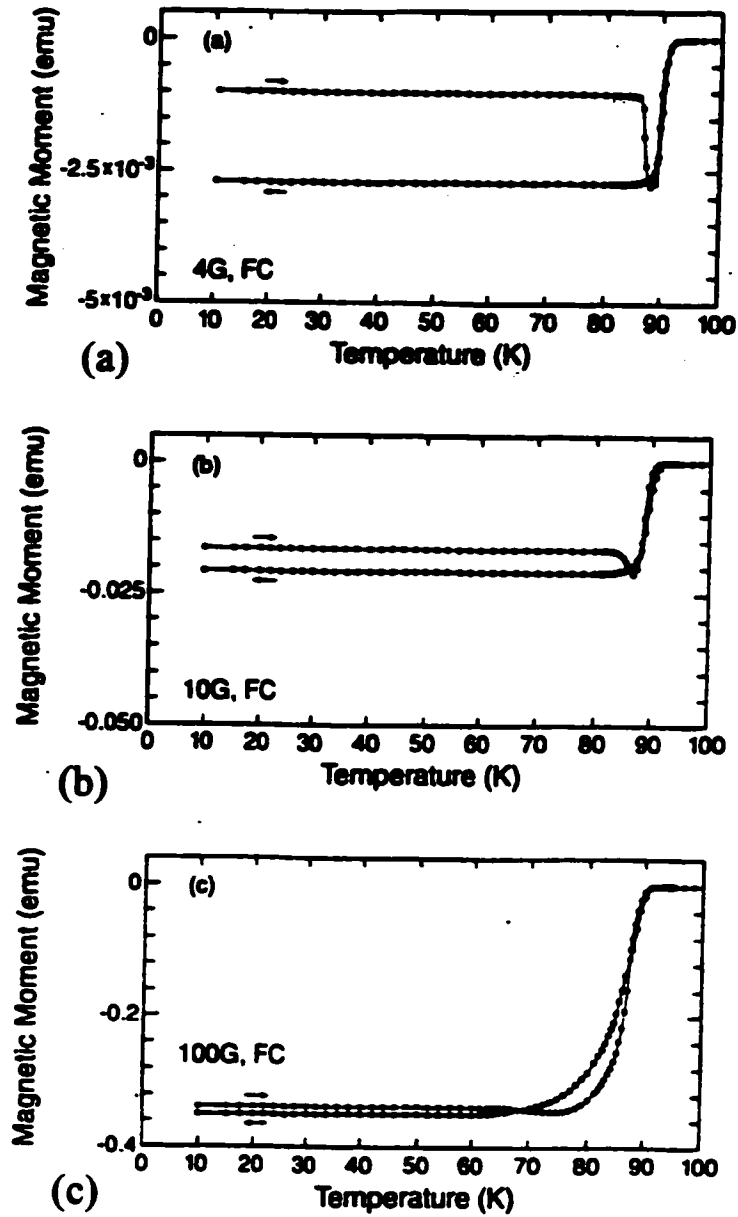


Fig 3.15. Reproduced from the article by Jung et al^[73] reporting on the same specimen examined in the 2 previous figures. The data curves labeled by $\rightarrow$ were measured after fast cooling from T_c in the static applied fields indicated. The diamagnetism observed at 10K after fast field cooling is clearly smaller than that measured after slow field cooling shown by the data curves labeled $\leftarrow$. Warming after fast field cooling is seen to generate a valley in the data curves whose depth corresponds to the difference between each pair of data curves.

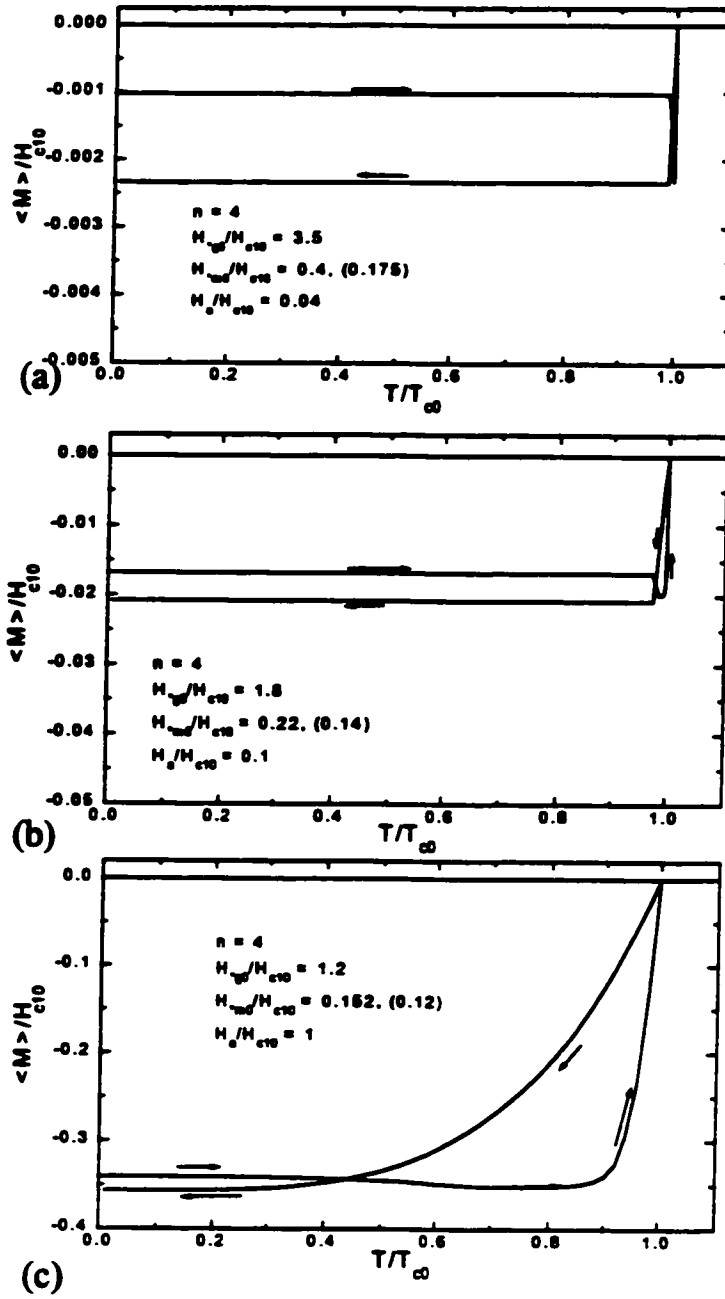


Fig 3.16 (a), (b) and (c) display theoretical curves generated by our model for comparison with the corresponding observations displayed in (a), (b) and (c) of Fig 3.15. In view of the large range of H_a we again let $H_{m0} = J_{cm0} X_m$ and $H_{g0} = J_{cg0} X_g$ depend on H_a , thus $H_{m0}/H_{c10} = 3.5, 1.8$ and 1.2 for $H_a/H_{c10} = 0.04, 0.1$, and 1 . The intergranular current densities are chosen larger after fast field cooling than that for slow cooling cases since negligible decay took place during the fast cooling. Hence $H_{m0}/H_{c10} = 0.4$ (0.175), 0.22 (0.14), 0.152 (0.12) for $H_a/H_{c10} = 0.04, 0.1$ and 1 where the values in the parentheses indicate the parameter for the slow cooling curves.

reversible (equilibrium) Meissner currents appear to be truly persistent. Since the decline of J_{cm} can be appreciable over the duration of the slow field cooling process we will focus on this property to account for the observed ΔM_{WL} .

First however we note that $M_{WL}(T)$ traces horizontal curves during slow cooling and also during slow warming over the range $0 < T < T_{c1}(H_a)$ (see Fig 3.15). These plateaus exhibited by $M_{WL}(T)$ versus T ascending or descending show that the decay of the persistent currents is negligible when $T < T_{c1}(H_a)$. Consequently the decay of the intergranular currents during slow field cooling seems to occur during the time elapsed while the sample cools from T_{c0} to $T_{c1}(H_a)$. The time required for the sample to descend through the temperature range $\Delta T_1 = T_{c0} - T_{c1}(H_a)$ can be made shorter by fast cooling the specimen to temperatures which are far below $T_{c1}(H_a)$. Jung et al^[73] have shown that the samples expel less flux, hence $\Delta M_{WL}(T_f)$ increases as $\Delta T_f = T_c - T_f$ is made larger thereby effectively augmenting the rate of the cooling through the critical range $\Delta T_1 = T_{c0} - T_{c1}(H_a)$.

To model the observations of Jung et al^[73] displayed in Fig 3.15 we first address the curves measured during warming after fast cooling. Here we proceed as in the two preceding sections where we assumed, for simplicity, that no decay occurred for the intergranular and intragranular currents during the fast field cooling. Again, for computational convenience, we take $J_{cm}(T)$, $J_{cg}(T)$ and $I_m(T)$ to possess the same exponent n in the temperature dependence, $f(T) = \{1 - (T/T_{c0})^n\}$. Since the data spans a wide range of static applied fields (4, 10, 100 G), it is necessary to take into account that J_{cm} and J_{cg} diminish with H_a . Consequently we select an appropriate value for the adjustable parameters $H_{m \rightarrow 0} = J_{cm0} X_m$ and $H_{g \rightarrow 0} = J_{cg0} X_g$ for each value of H_a . Also we

consider J_{cm0} and J_{c0} as constants in the process of fast cooling, hence ignore any decay of these currents in the short time elapsed when cooling occurred rapidly. We take $H_{c10} = 100$ G for the specimen. The results of this exercise for M_{WL} versus T during the subsequent warming after fast field cooling are displayed in Fig 3.16 where the arrows indicate the direction of the sweep in temperature.

Now to model the curves measured during slow cooling we take into account a feature we neglected until now since it did not need to be addressed in our analysis of observations. We now take into consideration that the intergranular currents can experience significant decay when the time elapsed during the slow cooling is appreciable. To “mimic” in a simple empirical but physically plausible way any decay of J_{cm} which occurs because of the duration of the slow cooling process we introduce values for $H_{m0} = J_{cm0}X_m$ in the calculation which are smaller. The curves labeled $\leftarrow$ in Fig 3.16 display the results of these calculations.

III Enhancement of the Meissner Effect of Polycrystals Nb_3Sn by Thermal Cycling.

We have seen that weak-linked granular high T_c superconductors exhibit a diamagnetic valley during slow warming after fast field cooling. This is thought to occur because the fast field cooling minimizes the opportunity for the intergranular currents to decay. The thermomagnetic hysteresis in the reentry of flux into the grains upon warming causes the flux trapped in the matrix during the cooling to be initially mainly released to the outside world thereby augmenting the Meissner effect. Hyun^[66] investigated the magnetothermal behaviour of polycrystals of Nb_3Sn and found that these also exhibit a

deep valley in the diamagnetic magnetization during slow warming after slow field cooling. His measurements are reproduced in Fig 3.17 (a). Specimens of powdered Nb_3Sn , hence electrically decoupled grains, did not manifest such a feature.

It is well established that the electrical contacts (links) between the grains of conventional polycrystalline type II superconductors can support current densities much larger than that in bulk high T_c materials. More crucial for the present context, these intergranular currents exhibit negligible decay rate. Consequently Hyun^[66] emphasizes that the behaviour of the Nb_3Sn polycrystal during warming after slow field cooling is very similar to that of fast field cooled high T_c weak-linked granular samples since in both cases the field profiles in the specimens have suffered negligible relaxation during the field cooling.

It is consequently a straightforward exercise to apply our model, which neglects any time decay of the induced currents, to describe the observations by Hyun^[66] of a deep diamagnetic valley in the magnetization of a strong-linked polycrystal of Nb_3Sn during warming after slow field cooling. Let $M_{\text{SL}}(T)$ now denote the magnetization of such a strong-linked specimen. The calculated curves generated by our model during 1st cooling and 1st warming are displayed in Fig 3.17b.

Hyun^[66] also measured the magnetization of the Nb_3Sn polycrystal during recooling from T just below T_{VB} , the temperature where the bottom of the diamagnetic valley occurs, and found that this caused further enhancement of the diamagnetism (see Fig 3.17a). Since we have seen in chapter 2, that the Meissner effect in grains of type II superconductors is enhanced by recooling from any temperature below $T_c < T_{c0}$, we attribute this behaviour to the grains of the polycrystal. The introduction of the evolution

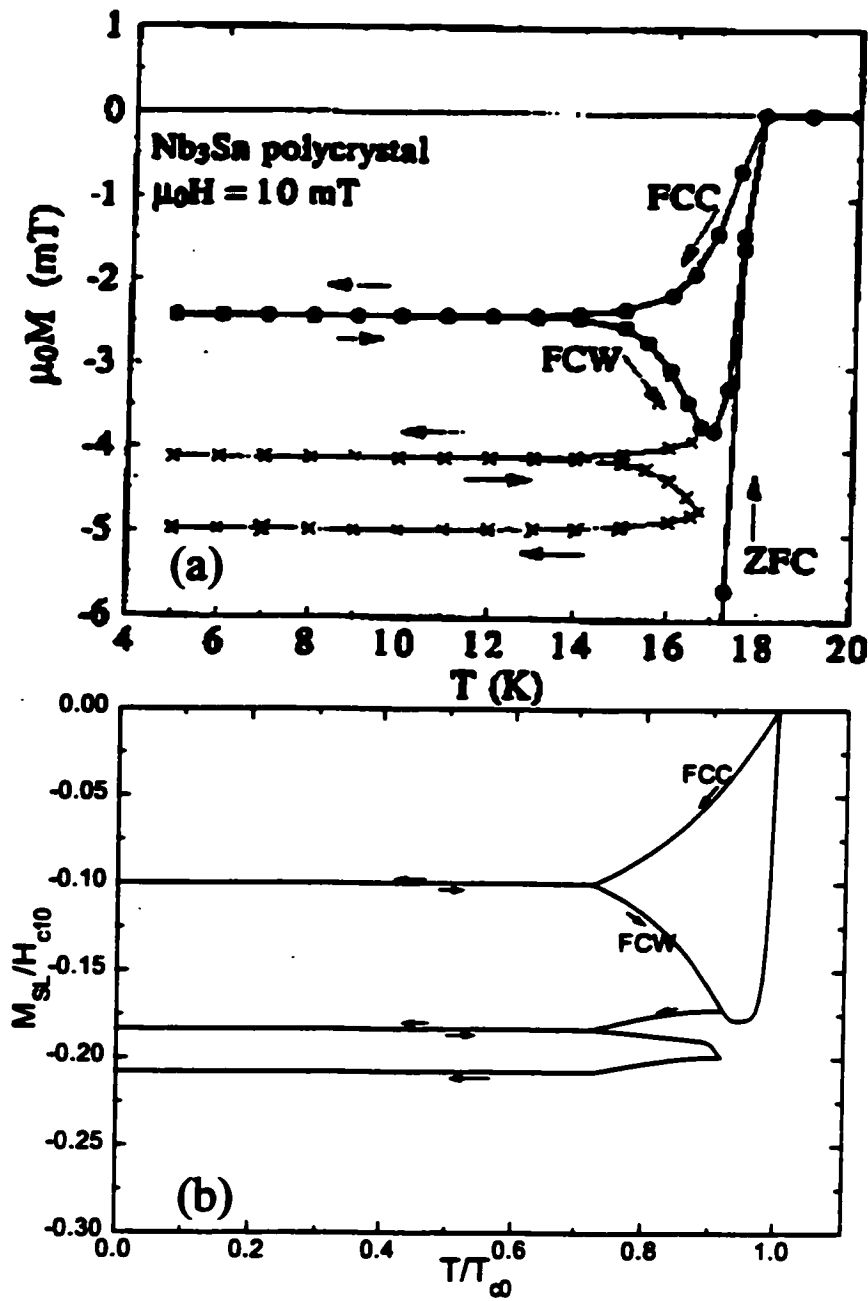


Fig 3.17. (a) Reproduced from the paper by Hyun^[66]. Displays the evolution of the magnetization of a compacted polycrystalline sample of Nb₃Sn during slow-field cooling from T_c in a static field $\mu_0 H_s = 10$ mT and during subsequent warming and cooling cycles. Also shown is the steep disappearance of $\mu_0 M$ during warming after applying $\mu_0 H_s = 10$ mT to the sample at 5K (denoted Zero Field Cooled curve). (b) Curves generated by our model for comparison with the experimental observations of Hyun. Here $H_s/H_{c10} = 1$ and $n = 5$ in $f(T) = \{1 - (T/T_c)^n\}$, the dependence of J_{cm} , J_{cg} and I_M on temperature.

of $\langle M_g(T) \rangle$, the magnetization of the grains during 1st recooling which is generated by our model, into a matrix where J_{cm} does not decay with time is seen to reproduce this feature. This is illustrated by the 1st recooling curve of M_{SL} displayed in Fig 3.17b.

Hyun also examined the evolution of $M_{SL}(T)$ in the polycrystal of Nb_3Sn during another thermal cycle between T_f and $T \leq T_{VB}$. His data and our corresponding theoretical curves can be compared from inspection of Fig 3.17(a) and (b). Both sets of curves show that this thermal processing leads to further enhancement of the Meissner effect. In our development of these phenomena for a polycrystal we simply introduced our model for the evolution of the magnetization of a grain, $\langle M_g(T) \rangle$, during thermal cycles. Our model then simply pursues the effect of the cyclical changes of $\langle M_g(T) \rangle$ on $H_m(x, T)$, the field profiles in the matrix, which are then dictated by, (i) Faraday's law of induction, (ii) conservation of flux inside type II superconductors and (iii) the temperature dependence of the intergranular critical current density $J_{cm}(T)$.

IV Summary and Conclusion

The model developed in chapter 2 enabled us to calculate the evolution of $M_g(T)$, the magnetization of isotropic single crystals of idealized geometry during slow sweeps of the temperature below T_c in fixed fields H_a . $M_g(T)$, now denoted $\langle M_g(T) \rangle$ to represent an assembly of identical grains, is then introduced into a medium (matrix) wherein temperature dependent critical currents, identified as intergranular currents of density denoted $J_{cm}(T)$, are induced by changes of $\langle M_g(T) \rangle$. Pursuing Faraday's law of induction, the critical state concept and conservation of flux inside the grain-intergrain medium, we map out the evolution of $H_m(x, T)$, the field profiles associated with the

patterns of circulation of $J_{cm}(T)$ which are induced, as changes of temperature cause $\langle M_g(T) \rangle$ to evolve.

Exploiting this simple framework, which ignores any decay or relaxation with time of $\langle M_g(T) \rangle$ and $J_{cm}(T)$, we account for many observations reported by Wang and Joiner^[149] and Jung et al^[73] of diamagnetic valleys in M_{WL} , the magnetization of the weak-linked polycrystals of YBCO during warming after fast field cooling. Our model is successful in reproducing these data since fast field cooling ensures that a minimum decay of the hysteretic persistent currents takes place. Also since negligible decay of the intergranular currents occurs in polycrystals of conventional low T_c type II superconductors, our model is successful in accounting for the observations of Hyun on a compacted sample of Nb_3Sn crystals during warming after slow field cooling.

Since Hyun also investigated the behaviour of the Nb_3Sn polycrystal when subjected to thermal cycles extending between 4.2K and 17K (just below T_{VB}) we introduced our model for the behaviour of $\langle M_g(T) \rangle$ under these circumstances to account for his measurements.

In closing we recall that our model for $M_g(T)$ in chapter 2 predicts that extensive thermal cycling between various appropriate limits T_1 and $T_2 < T_c$, will cause $M_g(T)$ to eventually trace closed thermal hysteresis loops exhibiting strongly enhanced diamagnetism. Consequently our model can predict similar behaviour for strong-linked polycrystals. However we do not pursue this prediction computationally in this thesis.

Chapter 4

I_c Hysteresis in High T_c Superconductors: Dependence of the Position of Peaks in I_c versus H_a on Previous H – T History

Abstract

The critical conduction current I_c which weak-linked granular high T_c superconductors can support is not uniquely determined by the temperature T and the applied magnetic field H_a but depends dramatically on the prior magnetothermal (H-T) history of the specimen in the superconducting state. When the applied field H_a is descending from large values, I_c becomes appreciably larger than that observed at the same temperature $T_f < T_c$, and at the corresponding H_a which existed during the prior application of H_a to the virgin specimen. I_c is now seen to trace a peak versus H_a descending whose location, denoted $H_{p\downarrow}^{cycle}$, depends on H_{max}^{cycle} , the magnitude of the excursion of H_a . Another peak of I_c is observed when H_a is then impressed again from zero in the same direction as previously. The position of this peak, denoted $H_{p\uparrow}^{cycle}$, also depends on H_{max}^{cycle} , but now $H_{p\uparrow}^{cycle} < H_{p\downarrow}^{cycle}$ for the same H_{max}^{cycle} . The curve of I_c versus H_a descending after cooling of the specimen from T_c to T_f in a field, denoted H^{cool} , also traces a peak whose location, denoted $H_{p\downarrow}^{cool}$, depends on H^{cool} . Again another peak of I_c is traced, when H_a is then reimpressed from zero, whose position, denoted $H_{p\uparrow}^{cool}$, depends on H^{cool} . Evetts and Glowacki^[48] proposed that these behaviours, labeled I_c hysteresis,

are due to the superposition of the return field H_r of the magnetized grains and the applied field H_a in the weak links of these granular ceramics.

In this chapter we pursue the superposition proposal of Evetts and Glowacki and develop a simple framework which qualitatively and quantitatively describes the dependence of $H_{p\downarrow}^{cycle}$ and $H_{p\uparrow}^{cycle}$ on H_{max}^{cycle} , and of $H_{p\downarrow}^{cool}$ and $H_{p\uparrow}^{cool}$ on H^{cool} .

I Introduction

Soon after the discovery of high T_c superconductors, researchers found that the ability of a sample of these ceramic materials to carry a lossless conduction current I_c , was not uniquely determined by the temperature T and the applied magnetic field H_a [3-5, 7-14, 24, 30, 31, 35, 36, 40, 46, 49, 52-54, 57, 59, 70, 72, 82-85, 91, 93, 97-102, 108, 116-121, 134, 135, 143, 145, 151, 155-157, 160].

Indeed I_c for a given specimen was observed to be a multiple-valued function controlled by the path travelled by the variables T and H_a once the superconducting state was established, hence with $H_a < H_{c2}(T)$ and $T < T_c(H_a)$. Fig 4.1 schematically illustrates the typical behaviour encountered in measurements of I_c versus H_a with $T < T_c(H_a)$ kept constant.

In Fig 4.1 the curve labeled ZFCV (Zero Field Cooled Virgin) is obtained when the sample is first cooled from T_{c0} to T_f in zero field (the earth's field), then H_a is impressed and kept fixed at a selected value denoted H_f while the conduction current I is introduced and gradually increased until I_c is attained. The appearance of a detectable voltage along the tape, ribbon or wire provides the criterion for establishing I_c . In this series of measurements, I_{c0} denotes I_c observed when no magnetic field has yet been applied, hence $H_a = 0$. Subsequent measurements of I_c are performed after an increment ΔH_a is

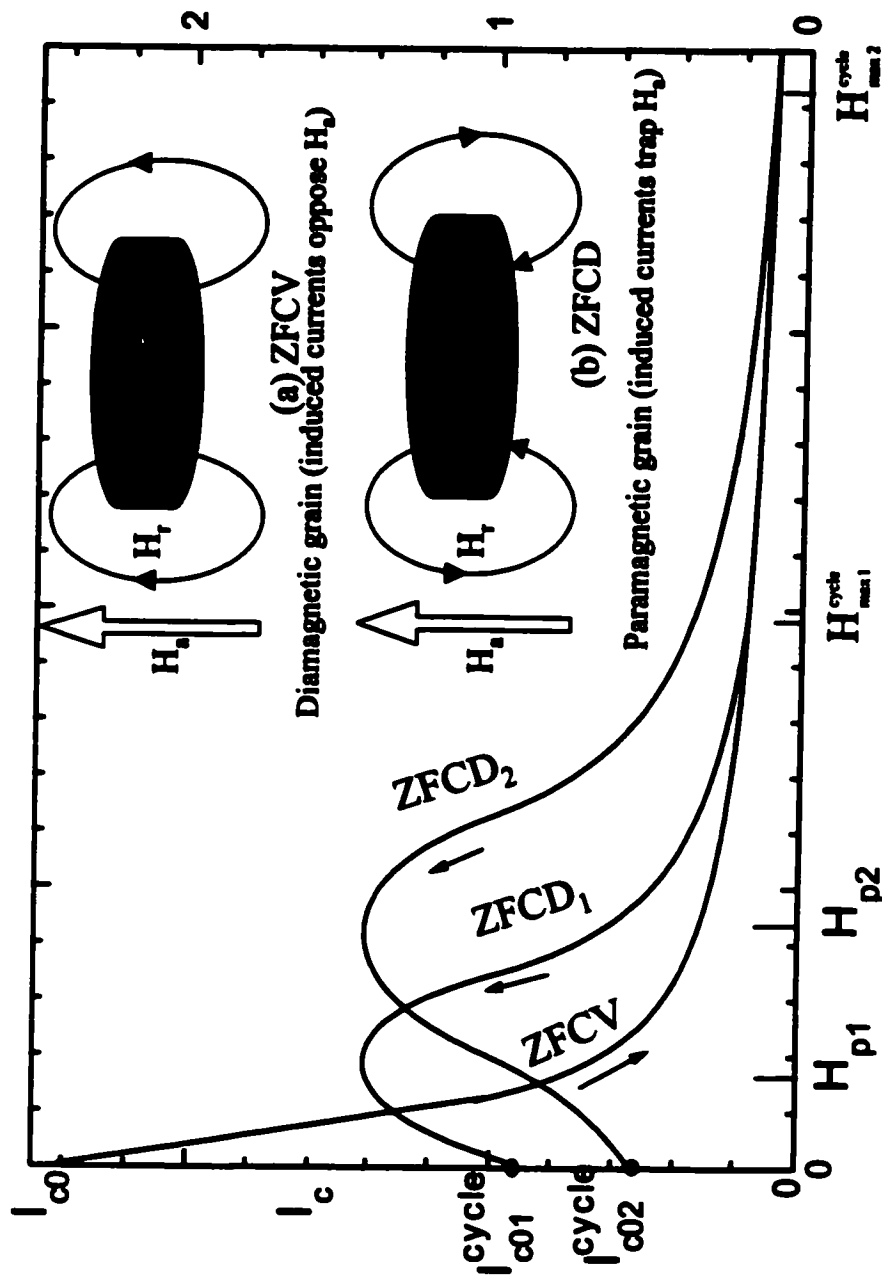


Fig 4.1. Illustrates the behaviour universally observed for the critical current I_c when, (a) H_a is applied to the "virgin" sample (i.e. cooled from T_c in zero field) (curve denoted ZFCV), and (b) H_a is made to descend after an excursion to a value denoted H_{max}^{cycle} . The insets (a) and (b) illustrate the basic concept that the field opposing (trapping) induced currents in the grains generate a diamagnetic (paramagnetic) moment whose return field H_i aids (opposes) H_a . The peaks of I_c occur when H_a , denoted H_p , is cancelled by H_i .

made to the applied magnetic field. Consequently, $H_a = \sum \Delta H_a$, follows a continuous stepwise progression in magnitude where the size of the increments is determined by the patience of the researchers to obtain a smooth and accurate data curve.

In Fig 4.1, the curve labeled ZFCD (Zero Field Cooled Descending) is determined by measuring I_c for a sequence of fixed values of H_a where H_a is now decreased from H_{max} to zero in steps of arbitrary convenient magnitude. As we shall see, many factors will play a role in the choice of $H_{max} \ll H_{c2}(T)$ where the augmentations of H_a ceased.

It is evident from a glance at Fig 4.1 that I_c is a multiple-valued function of H_a for a fixed temperature. In particular we note that each curve of I_c (ZFCD) traces a peak lying above I_c (ZFCV) and that H_p , the value of H_a where the peak appears, depends on H_{max} .

II The Model of Evetts and Glowacki

(i) Introduction to the model

Evetts and Glowacki^[48] showed that, the superposition of H_r , the return field of the magnetized grains, and H_a , the applied magnetic field, qualitatively accounts for the variety of behaviour displayed in Fig 4.1. We now give a brief outline of their model in a framework which we will exploit in this and subsequent chapters of this thesis.

In weak-linked granular high T_c superconductors, the intergranular critical current density J_{cm} declines rapidly as the strength of the magnetic field is augmented. Further J_{cm} is orders of magnitude weaker than the intragranular current density J_{cg} . As a consequence the applied magnetic field H_a enters or leaves the intergranular medium (the “matrix”) with little opposition by the weak J_{cm} . Entry or exit of magnetic flux into or out

of the grains, is however, strongly resisted since, because of the strong intragranular pinning, these processes induce large current densities J_{cg} and steep flux density gradients in the grains, hence large grain magnetizations. The insets in Fig 4.1 schematically display the superposition of the return field H_r of a magnetized grain and the applied field H_a . In (a) the application of H_a has induced currents to circulate clockwise in the grains to oppose the entry of H_a hence generated a diamagnetic magnetization and a return field which aids H_a . In (b) the diminution of H_a from a large value, has induced currents to circulate counterclockwise in the grains in order to trap the magnetic field thereby generating a paramagnetic magnetization whose return field H_r then opposes H_a . Consequently the superpositions of the two fields, $H_{a\uparrow} = H_a + H_r$ and $H_{a\downarrow} = H_a - H_r$, are very different when H_a is ascending, then denoted $H_{a\uparrow}$ than for H_a descending in magnitude, then denoted $H_{a\downarrow}$.

Since the intergranular current density J_{cm} is controlled by the intergranular field strength H_a , and J_{cm} diminishes with H_a increasing, and the critical conduction current I_c is the sum of the currents across the weak-links, we can readily see why the curve of I_c (ZFCV) descends precipitously as H_a is impressed since here $H_{a\uparrow} = H_a + H_r$. Also we see why I_c (ZFCD) can lie above the curve for I_c (ZFCV) since when H_a descends from H_{max} , denoted H_{max}^{cycle} , the return field of the magnetized grains reverses polarity and now $H_{a\downarrow} = H_a - H_r$. Also we then expect a peak to occur in the curve of I_c (ZFCD) versus $H_{a\downarrow}$ when $H_{a\downarrow} = 0$, hence $H_{a\downarrow} = H_r$. Fig 4.1 also illustrates the feature that the position of the peak of I_c versus $H_{a\downarrow}$, denoted $H_{p\downarrow}$, depends on H_{max}^{cycle} . This is also expected since M_g , the magnetization of the grains, hence H_r , both depend on $H_{max}^{cycle} - H_{a\downarrow}$ as H_a

descends from $H_{\max}^{\text{cyclic}}$. Clearly then to describe the dependence of H_p on $H_{\max}$ we need to address the evolution of M_p , hence H_p , as a function of H_a .

(ii) First Application of H_a to a Granular Type II Superconductor

For simplicity we regard the grains as identical circular disks with their axis // to H_a and focus on the response of one isolated grain to the applied field.

The depth of the penetration of the applied magnetic flux into the grain increases as H_a is impressed, hence an annular region occupied by field shielding currents of density J_{cs} concomitantly expands inwards. Eventually some value of $H_{a\uparrow}$, denoted H_{a*} , is reached where the entire volume of the grain is filled with induced currents and its magnetization attains a saturation level denoted M_{*g} .

At this juncture it is useful to recall that an element of current dI circulating along the periphery of a circle of radius r generates a magnetic moment, $d\mu = \pi r^2 dI$. Consequently a grain in the form of a disk of radius R and thickness Z occupied by persistent currents of uniform density J_{cs} in an annular region $r_i \leq r \leq R$ produces a magnetic moment of magnitude,

$$\mu_g = \int d\mu_g = \int_{r_i}^R \pi r^2 dI = \int_{r_i}^R \int_0^Z \pi r^2 (J_{cs} dr dz) = \pi J_{cs} Z \frac{(R^3 - r_i^3)}{3} \quad (4.1)$$

Hence the magnetization of this grain reads

$$M_g(r_i) = \mu_g / \pi R^2 Z = \frac{J_{cs}}{3R^2} (R^3 - r_i^3) = \frac{H_{a*}}{3R} \left\{ 1 - \left(\frac{r_i}{R} \right)^3 \right\} \quad (4.2)$$

The boundary r_i migrates inwards as H_a is impressed and the saturation value,

$$M_g(0) = M_{cg} = \frac{J_{cg} R}{3} = \frac{H_{cg}}{3} \quad (4.3)$$

is attained when $r_i = 0$. For simplicity we take $H_{a1} = H_{cg}$ when $r_i = 0$, and regard M_{cg} to remain constant as H_{a1} is increased beyond H_{cg} . This is consistent with idealized cylinder geometry and the assumption that J_{cg} is insensitive to the strength of the magnetic field.

The sign of μ_g , hence of M_g , depends on the sense of circulation of J_{cg} . Regarding $\vec{H}_a$ to be impressed along the $+\hat{z}$ axis, the induced persistent current density $\vec{J}_{cg}$ opposing its entry into the disk reads $\vec{J}_{cg} = -\hat{\phi}J_{cg}$, hence here M_g is diamagnetic and reads $\vec{M}_g = -\hat{z}M_g$.

(iii) I_g, M_g and H_{g1}^{cycle} versus H_{a1} (ZFCD)

Subsequently when H_a is decreased from H_{max}^{cycle} , the currents circulating in an annular zone, $r_i < r_r \leq r \leq R$, reverse their sense of circulation in order to oppose the exit of magnetic flux from the grain. Consequently the grain is now occupied by two concentric annular zones filled with counter circulating current densities $\pm J_{cg}$. This is illustrated schematically by the inserts in Fig 4.2. The magnetization now reads,

$$M_g(r_i, r_r) = \frac{J_{cg}}{3R^2} \{2r_r^3 - r_i^3 - R^3\} = \frac{H_{cg}}{3} \left\{ 2\left(\frac{r_r}{R}\right)^3 - \left(\frac{r_i}{R}\right)^3 - 1 \right\} \quad (4.4)$$

and the magnetization vanishes when,

$$r_r^3 = \frac{R^3 + r_i^3}{2} \quad (4.5)$$

Hence $M_g = 0$ when $r_r/R = 0.794$ as H_a descends from a value of H_{max}^{cycle} which was sufficiently large to have caused the penetration of r_i to the centre of the disk. M_g , and

hence its return field H_r , then both reverse sign upon further diminution of $H_{a\downarrow}$. Consequently when $H_{a\downarrow}$ is still greater than zero, the return field $-H_r$ will cancel $H_{a\downarrow}$ hence the resultant field $H_{a\downarrow} = H_{a\downarrow} - H_r = 0$.

For simplicity we assume that, the advance of the flux front r_i into the disk when H_a is first applied, and the advance of the boundary r_r of the zone of reversed current circulation when H_a is subsequently decreased from $H_{\max}^{\text{cycle}}$, can be described by the evolution of these boundaries which is encountered in the case of idealized cylindrical geometry. In the latter situation, Maxwell's equation, $\nabla \times \vec{H} = \vec{J}$ and the critical state concept that $J = \pm J_{c0}$, lead to $dH/dr = \pm J_{c0}$, hence for H_a first ascending, integration of $dH/dr = \pm J_{c0}$ leads to,

$$H_r(r) = H_{a\uparrow} - J_{c0}(R - r) = H_{a\uparrow} - H_{c0} \left(1 - \frac{r}{R}\right) \quad (4.6)$$

where $H_{a\uparrow} \geq 0$ and $H_{c0} = J_{c0} R$.

At the inner boundary r_i of the zone of the field shielding currents, hence at the flux front, $H_r(r_i) = 0$ when $H_{a\uparrow} < H_{c0}$, eqn 4.6 leads to,

$$\frac{r_i}{R} = 1 - \frac{H_{a\uparrow}}{H_{c0}} \quad (4.7)$$

with $r_i = 0$ when $H_{a\uparrow} / H_{c0} \geq 1$.

When H_a is descending from $H_{a\uparrow} = H_{\max}^{\text{cycle}}$, integration of $dH/dr = -J_{c0}$ leads to,

$$H_{\downarrow}(r) = H_{a\downarrow} + J_{c0}(R - r) = H_{a\downarrow} + H_{c0} \left(1 - \frac{r}{R}\right) \quad (4.8)$$

where $H_{a\downarrow} \leq H_{\downarrow}(r) \leq H_{\max}^{\text{cycle}}$.

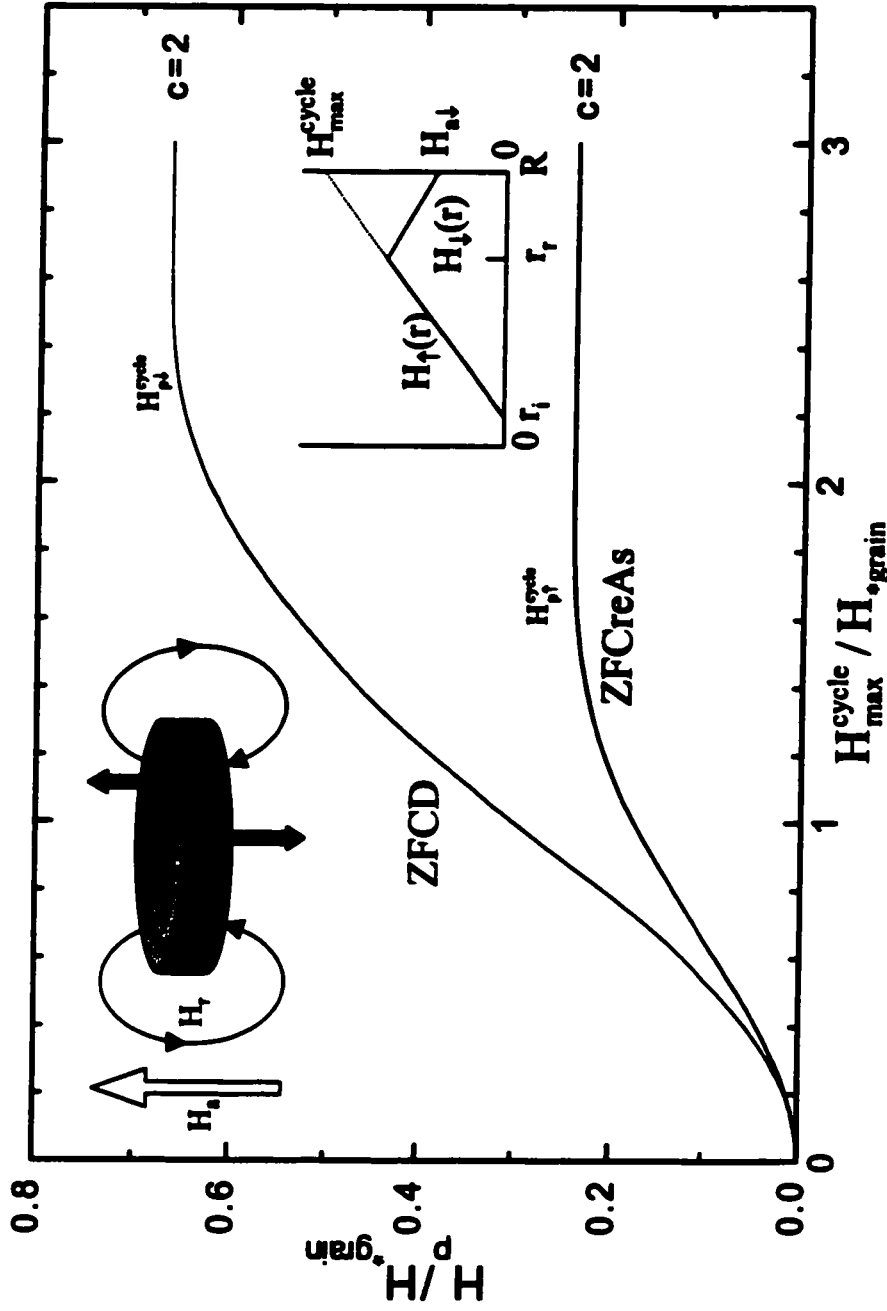


Fig 4.2. The left insert displays the patterns of induced currents in a grain when H_s is descending from $H_{\max}^{\text{cycle}}$. The induced currents in the outer annulus have reversed direction and generate a return field H_r which opposes H_s . The shaded arrows indicate that the magnetic moment comprises two coexisting but opposing components. The right insert sketches the magnetic field profile $H_t(r)$ inside the idealized grain when H_s is descending from $H_{\max}^{\text{cycle}} < H_{c_g}$. The curves display calculated values for H_s , denoted H_p , where a peak of I_c is predicted to occur when, (i) H_s is descending from $H_{\max}^{\text{cycle}}$ applied to a zero field cooled specimen, (denoted ZFCd), and (ii) H_s is reascending from 0 after (i), hence, procedure is denoted ZFCreAs. H_p and $H_{\max}^{\text{cycle}}$ are normalized to $H_{c_g} = J_{c_g} R$. Here $C = 2$ in the linear relation $H_r = -C M_g(H_g)$ for the return field.

As illustrated by the inserts in Fig 4.2, at the interface r_r of the two annular zones filled with counter circulating currents, continuity of the magnetic field requires that $H_L(r_r) = H_T(r_r)$. Hence the two curves described by eqns 4.6 and 4.8 meet at r_r , therefore we write,

$$H_{\max}^{\text{cycle}} - H_{\text{ex}} \left(1 - \frac{r_r}{R}\right) = H_{s\downarrow} + H_{\text{ex}} \left(1 - \frac{r_r}{R}\right) \quad (4.9a)$$

hence,

$$\frac{r_r}{R} = 1 - \frac{1}{2} \left(\frac{H_{\max}^{\text{cycle}} - H_{s\downarrow}}{H_{\text{ex}}} \right) \quad (4.9b)$$

with $r_r = 0$ when $(H_{\max}^{\text{cycle}} - H_{s\downarrow}) \geq 2 H_{\text{ex}}$.

Evetts and Glowacki and other researchers have implicitly or explicitly regarded the effective return field H_r to be proportional to the magnetization of the grains. In this framework they write,

$$H_r = -C M_g(r_i, r_r) \quad (4.10)$$

where the coefficient C is regarded as representing the compression of the return field of adjacent grains in the small volume of the weak link between them. Consequently C is a parameter determined by the geometry, size, orientation and packing of the grains.

Since, in the Evetts and Glowacki model, the position of the peak in each of the ZFCD curves of Fig 4.1 is thought to correspond to the situations where the descending field $H_{s\downarrow}$ and the evolving return field $H_r(H_{s\downarrow})$ cancel each other, we can then write,

$$\begin{aligned} H_{p\downarrow}^{\text{cycle}} = H_{s\downarrow} = H_r &= -C M_g(r_i, r_r) \\ &= -C \frac{H_{\text{ex}}}{3} \left[2 \left\{ 1 - \left(\frac{H_{\max}^{\text{cycle}} - H_{s\downarrow}}{2H_{\text{ex}}} \right) \right\}^3 - \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{H_{\text{ex}}} \right\}^3 - 1 \right] \end{aligned} \quad (4.11)$$

where we have introduced eqns 4.7 and 4.9b into eqn 4.4. Note that, on the right hand side, the second term vanishes when $H_{\text{max}}^{\text{cycle}} / H_{\text{ag}} \geq 1$ and the 1st term vanishes when $(H_{\text{max}}^{\text{cycle}} - H_{\text{a}\downarrow}) \geq 2 H_{\text{ag}}$. Consequently when $(H_{\text{max}}^{\text{cycle}} - H_{\text{a}\downarrow}) \geq 2 H_{\text{ag}}$,

$$H_{\text{pl}}^{\text{cycle}} = H_{\text{a}\downarrow} = H_{\text{r}} = C \frac{H_{\text{ag}}}{3} \quad (4.12)$$

Hence the location of the peak becomes independent of $H_{\text{max}}^{\text{cycle}}$, and the maximum value is denoted $H_{\text{pl max}}^{\text{cycle}}$. The value of $H_{\text{max}}^{\text{cycle}}$ when this onset of the independence begins is given by

$$H_{\text{max}}^{\text{cycle}} = H_{\text{a}\downarrow} + 2H_{\text{ag}} = H_{\text{pl}}^{\text{cycle}} + 2H_{\text{ag}} = \left(\frac{C}{3} + 2\right)H_{\text{ag}} \quad (4.13)$$

The uppermost curve in Fig 4.2 displays $H_{\text{pl}}^{\text{cycle}}$ versus $H_{\text{max}}^{\text{cycle}}$ predicted by eqn 4.11.

Note that $H_{\text{pl}}^{\text{cycle}}$ asymptotically rises to a plateau whose height is given by $H_{\text{pl max}}^{\text{cycle}} / H_{\text{ag}} = C/3$ and which begins when $H_{\text{max}}^{\text{cycle}} / H_{\text{ag}} = \{(C/3) + 2\}$.

The critical current at the peaks, denoted $I_{\text{c peak}}$, is observed to be much smaller than I_{c0} measured with the specimen in the virgin (ZFC) state at $H_{\text{a}} = 0$. This behaviour is not surprising since the numerous different weak links in the “real” inhomogeneous granular sample are expected to bathe in a spectrum of values of H_{r} when $H_{\text{a}\downarrow}$ traces the top of the peak of the measured curve of I_{c} versus $H_{\text{a}\downarrow}$. By contrast, only the weak self-field $H_{\text{m}}(x)$ generated by the small conduction current I_{c0} , and the feeble attendant return field of the grains magnetized by $H_{\text{m}}(x)$, will play a role when I_{c0} is measured in $H_{\text{a}} = 0$ after zero field cooling. We will examine these features in a later chapter.

The critical current I_c , denoted I_{c0}^{cycle} , measured when H_a returns to zero after an excursion to $H_{\text{max}}^{\text{cycle}}$, is expected and observed to diminish monotonically and then trace a plateau as a function of $H_{\text{max}}^{\text{cycle}}$. This occurs because the magnitude of the return field at $H_{a\downarrow} = 0$ is determined by the amount of flux trapped in the grains, hence by their remanent magnetization which grows and saturates as a function of $H_{\text{max}}^{\text{cycle}}$. The dependence of I_{c0}^{cycle} on $H_{\text{max}}^{\text{cycle}}$ will be discussed in the next chapter.

(iv) I_c , M_g and $H_{p\uparrow}^{\text{cycle}}$ versus H_a Reascending (ZFCrAs) after ZFCD.

I_c is also observed to trace a peak when H_a is made to reascend from zero after the excursion to $H_{\text{max}}^{\text{cycle}}$ has been completed. This feature is displayed schematically in Fig 4.3 by the curve labeled ZFCrAs. Let $H_{a\uparrow\uparrow}$ denote H_a under these circumstances and, $H_{p\uparrow}^{\text{cycle}}$, the position of the summit of the peak. The reascent of H_a will induce field shielding persistent currents of density $J_{c\theta}$ in an annular zone, $r_0 \leq r \leq R$, and leave the pattern of circulating currents undisturbed in the volume, $0 \leq r \leq r_0$ in our idealized model. This is illustrated schematically by the inserts in Fig 4.3. Again applying expressions valid for idealized cylindrical geometry, eqn 4.6 is seen to describe $H(r)$, now denoted $H_{\uparrow\uparrow}(r)$, which then reads,

$$H_{\uparrow\uparrow}(r) = H_{a\uparrow\uparrow} - H_{c0}(1 - \frac{r}{R}) \quad (4.14a)$$

in the annular zone, $r_0 \leq r \leq R$,

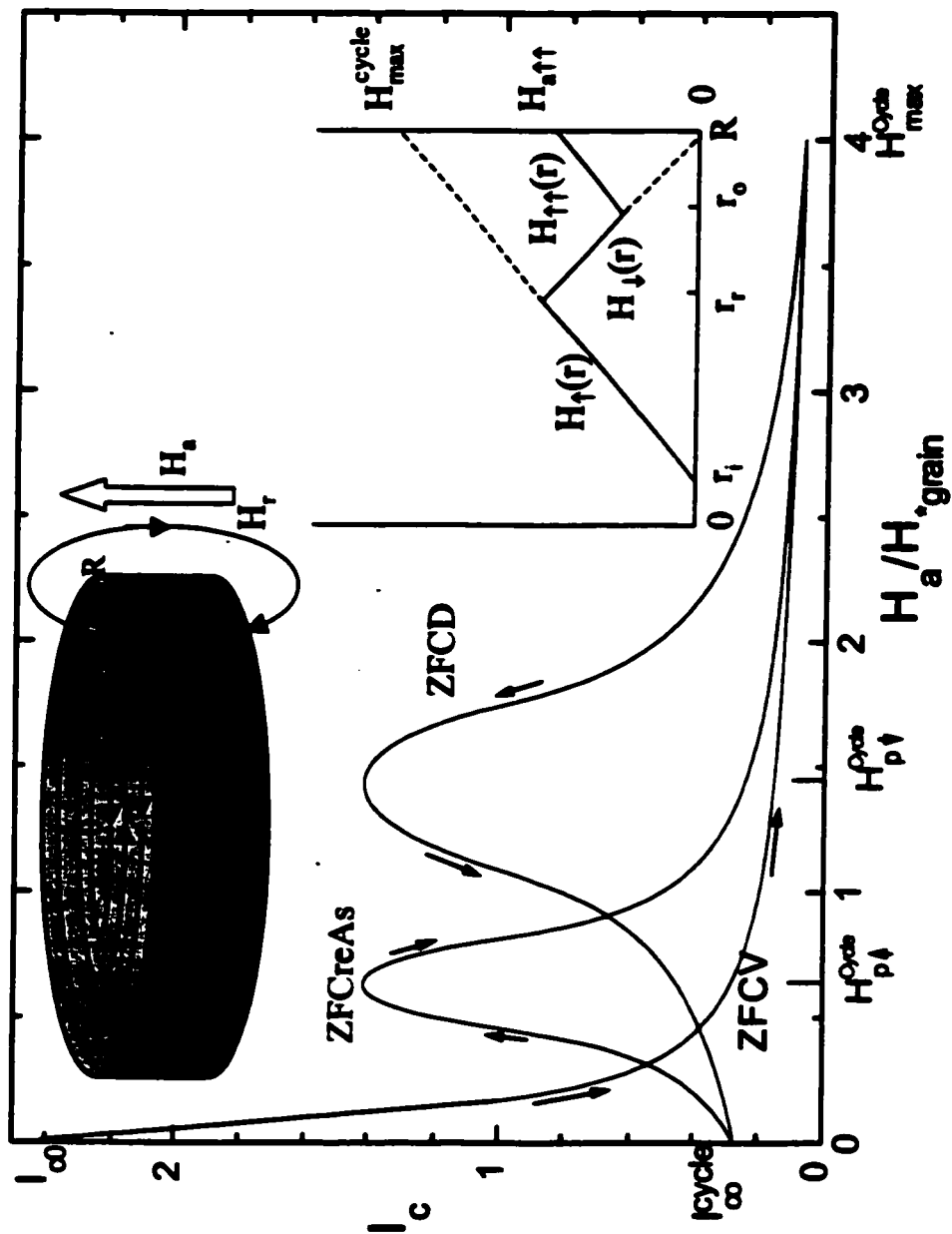


Fig 4.3. Left insert displays the patterns of circulation of the induced currents when H_a is reascending. The return field generated by the outermost induced currents opposes H_a and eventually cancellation occurs. The right insert sketches the sequence of the configurations of the magnetic field $H(r)$ inside the grains as H_a swings from 0 to $H_{\text{max}}^{\text{cycle}}$ to 0 to $H_{a\uparrow\uparrow}$. The curve labeled ZFCrAs illustrates the behaviour of I_c when H_a is reapplied after zero field cooling and an excursion of H_a to $H_{\text{max}}^{\text{cycle}}$ (curves labeled ZFCV and ZFCD). The peak traced by I_c for ZFCrAs occurs at a field denoted H_p^{cycle} which is always smaller than the corresponding H_p^{cycle} traced by the ZFCD curve.

$$H_{\downarrow}(r) = H_{\bullet g} \left(1 - \frac{r}{R}\right) \quad (4.14b)$$

in the annular zone, $r_i \leq r \leq r_o$, and as before,

$$H_{\uparrow}(r) = H_{\max} - H_{\bullet g} \left(1 - \frac{r}{R}\right) \quad (4.14c)$$

in the annular zone, $r_i \leq r \leq r_o$. Here, as before, since $H_{\uparrow}(r_i) = 0$ when $H_{\max} < H_{\bullet g}$,

$$\frac{r_i}{R} = 1 - \frac{H_{\max}}{H_{\bullet g}} \quad (4.15a)$$

valid when $H_{\max} \leq H_{\bullet g}$. Afterwards $r_i = 0$.

Since $H_{\downarrow}(r_o) = H_{\uparrow}(r_o)$, eqns 4.14b and c lead to,

$$\frac{r_o}{R} = 1 - \frac{H_{\max}}{2H_{\bullet g}} \quad (4.15b)$$

and since $H_{\uparrow\uparrow}(r_o) = H_{\downarrow}(r_o)$, eqns 4.14a and b lead to,

$$\frac{r_o}{R} = 1 - \frac{H_{\uparrow\uparrow}}{2H_{\bullet g}} \quad (4.15c)$$

The definition of the magnetic moment of the disk (grain) and its magnetization (eqns 4.1 and 4.2) now leads to,

$$\begin{aligned} M_g(r_i, r_r, r_o) &\equiv M_g(H_{\uparrow\uparrow}, H_{\max}^{\text{cycle}}) \\ &= \frac{J_g}{3R^2} \{R^3 - 2r_o^3 + 2r_r^3 - r_i^3\} \\ &= \frac{H_{\bullet g}}{3} \left[1 - 2 \left\{ 1 - \frac{H_{\uparrow\uparrow}}{2H_{\bullet g}} \right\}^3 + 2 \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{2H_{\bullet g}} \right\}^3 - \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{H_{\bullet g}} \right\}^3 \right] \end{aligned} \quad (4.16)$$

where the 2nd term vanishes when $H_{\uparrow\uparrow} \geq 2H_{\bullet g}$, the 3rd term vanishes when $H_{\max}^{\text{cycle}} \geq 2H_{\bullet g}$,

and the 4th term vanishes when $H_{\max}^{\text{cycle}} \geq H_{\bullet g}$.

Fig 4.4a displays the evolution of $H_r = -CM_g$ generated by the model of a magnetized disk as described above when H_a is first applied to the virgin sample, then is reduced to zero from various values H_{max}^{cycle} , then is reapplied from zero after these excursions. It is convenient to display a diagonal line showing H_a in the lower quadrant since then the intersections of this line with the curves of H_r identify the values of H_a where the peaks in the measurements of I_c are encountered when H_a descends from various values of H_{max} and subsequently reascends from zero. Therefore in Fig 4.4a the intersections labeled $\square$ and $\bullet$ indicate $H_{p\uparrow}^{cycle}$ and $H_{p\downarrow}^{cycle}$ respectively. It is clear from inspection that $H_{p\uparrow}^{cycle} < H_{p\downarrow}^{cycle}$ for the corresponding H_{max}^{cycle} in qualitative agreement with the observations of all researchers.

The behaviour obtained from introducing eqn 4.16 in the Evetts and Glowacki proposal that peaks of I_c occur when $H_r = -CM_g = H_{a\uparrow\uparrow}$ is displayed by the lowermost curve in Fig 4.2. This model predicts that $H_{p\downarrow}^{cycle}$ and $H_{p\uparrow}^{cycle}$ increase monotonically with H_{max}^{cycle} until each traces a plateau. Also the values for the relative height of the two plateaus and for their onsets are predicted by the model. Consequently, in the framework of the model of Evetts and Glowacki, the measurements of $H_{p\downarrow}^{cycle}$ and $H_{p\uparrow}^{cycle}$ provide quantitative information on the effective return field of the magnetized grains in weak-linked granular low and high T_c type II superconductors.

Fig 4.4b displays the evolution of $H_r = -CM_g$ calculated with the disk model when the specimen is first cooled from T_c to T_f in a static field denoted H^{cool} . The field is then reduced to zero and then made to reascend. Again the intersections of the H_a line with the $H_r(H_a)$ curves, labeled $\bullet$ and $\square$ indicate the location of the peaks observed in the curves of

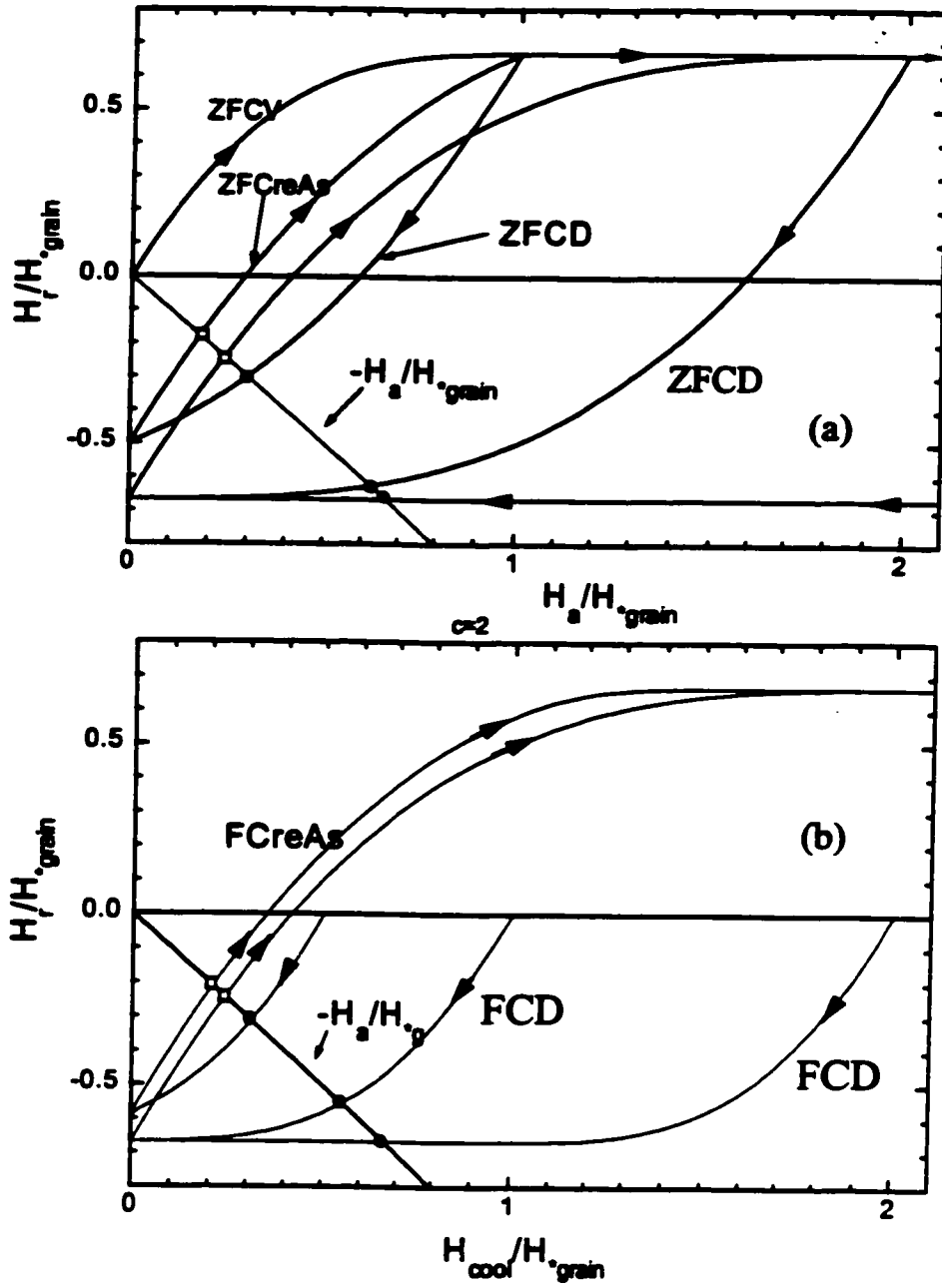


Fig 4.4. Displays the evolution of $H_f(H_a) = -C M_g(H_a)$ calculated for an idealized disk for the various $H - T$ histories exploited in laboratories in the measurements of I_c hysteresis behaviour. The intersections of the curves of H_f and the diagonal line for H_a identify their values when the peaks in the curves of I_c versus H_a are encountered. Here $C = 2$, and H_f and H_s are normalized with respect to $H_{sg} = J_{cg}R$.

I_c versus H_a when H_a descends and reascends after field cooling. We will develop the corresponding expressions for $H_c(H_a)$ and for the location of the peaks in the next section.

(v) I_c , M_z , $H_{p,d}^{\text{cool}}$ and $H_{p,r}^{\text{cool}}$ versus H_a Descending (FCD) and Reascending (FCReAs) after Field Cooling

For completeness in our development of the model of Evetts and Glowacki we now address the behaviour of I_c after the specimen has cooled from T_c to T_f in a static applied field H_a , then denoted H^{cool} . Again, for simplicity, we ignore the Meissner current hence take $H_{c1}(T) = 0$. The field profile in the disk after field cooling in H^{cool} is consequently taken to be uniform with $H(r) = H^{\text{cool}}$. Flux trapping persistent currents are induced to circulate in an annular zone adjacent to the surface when H_a , denoted H_d , descends from H^{cool} , hence as illustrated by sketches (a) and (c) in Fig 4.5,

$$H_d(r) = H_d + J_{cs}(R - r) = H_d + H_{cs}\left(1 - \frac{r}{R}\right) \quad (4.17)$$

in the volume, $r_p \leq r \leq R$. Since, $H_d(r_p) = H^{\text{cool}}$, then

$$\frac{r_p}{R} = 1 - \frac{(H^{\text{cool}} - H_d)}{H_{cs}} \quad (4.18)$$

valid until $r_p = 0$. Consequently,

$$M_z(r_p) = \frac{J_{cs}}{3R}(R^3 - r_p^3) = \frac{H_{cs}}{3} \left[1 - \left\{ 1 - \frac{(H^{\text{cool}} - H_d)}{H_{cs}} \right\}^3 \right] \quad (4.19)$$

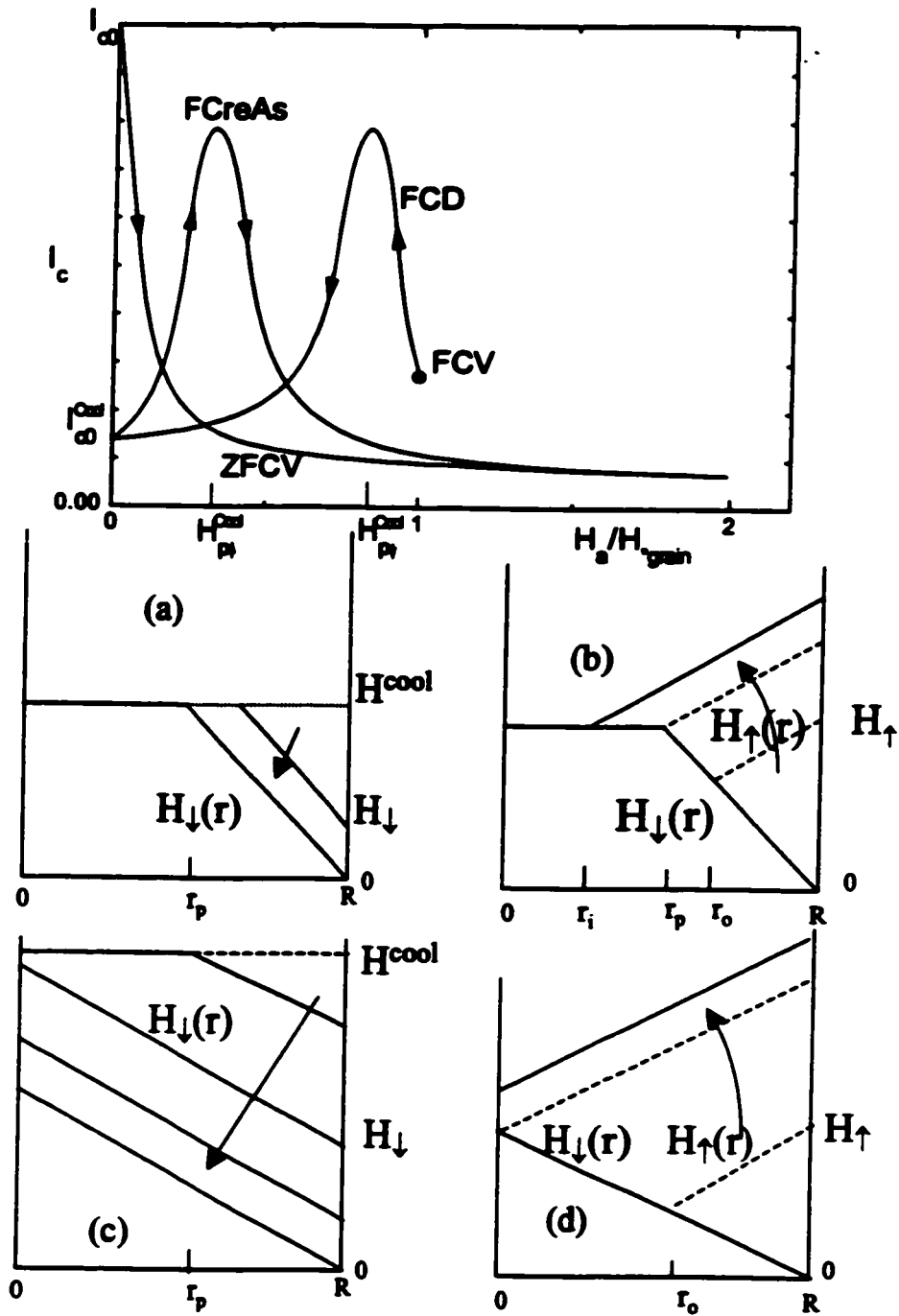


Fig 4.5. Upper figure illustrates the behaviour of I_c versus H_a descending from a static value denoted, H_{cool} where the sample was made to cool from T_c to T_f . I_c then measured at H_{cool} and T_f is denoted I_c^{FCV} . The curve of I_c where H_a is subsequently diminished is denoted FCD. When H_a returns to 0, I_c is denoted I_{c0}^{cool} . The curve of I_c measured when H_a is then reapplied is denoted FCrAs. Sketches of the magnetic field configurations in the idealized grain when H_a descends after field cooling in H_{cool} ((a) and (b)), and then reascends from zero ((b) and (c)). $H_{\text{cool}} < H_g$ in (a) and (b), and $H_{\text{cool}} > H_g$ in (c) and (d).

which applies until $r_p = 0$ hence until, $(H^{\text{cool}} - H_i) = H_{\text{cs}}$. If as illustrated in Fig 4.5c, $H_i > 0$ when r_p has migrated to the centre, M_g remains equal to $M_g(r_p = 0) = H_{\text{cs}}/3$ as the descent of H_i continues to zero. A peak is traced by the curve of I_c versus H_i whose summit occurs when, $H_r = C M_g(r_p) = H_i$ and is denoted $H_{r,i}^{\text{cool}}$. The curve labeled FCD in Fig 4.5 schematically illustrates this behaviour.

When H_o , now denoted H_r , is made to reascend from zero, field shielding currents are induced to circulate in an outer zone adjacent to the surface where the sense of circulation is reversed from that existing there when the descent to zero terminated. Let r_o denote the inner boundary of this zone, hence,

$$H_r(r) = H_r - J_{\text{cs}}(R - r) = H_r - H_{\text{cs}} \left(1 - \frac{r}{R}\right) \quad (4.20)$$

in the volume, $r_o \leq r \leq R$. Since, as illustrated by sketch (b) in Fig 4.5, $H_r(r_o) = H_i(r_o)$, then eqn 4.17, where $H_i = 0$, and eqn 4.20 lead to,

$$\frac{r_o}{R} = 1 - \frac{H_r}{2H_{\text{cs}}} \quad (4.21)$$

valid until $r_o = r_p$. From eqn 4.18, $r_p/R = 1 - (H^{\text{cool}}/H_{\text{cs}})$ when $H_i = 0$ and $(H^{\text{cool}}/H_{\text{cs}}) < 1$. The definition of M_g then leads to,

$$M_g(r_p, r_o) = \frac{J_{\text{cs}}}{3R} \{-R^3 + 2r_o^3 - r_p^3\} = \frac{H_{\text{cs}}}{3} \left[-1 + 2 \left\{ 1 - \frac{H_r}{2H_{\text{cs}}} \right\}^3 - \left\{ 1 - \frac{H^{\text{cool}}}{H_{\text{cs}}} \right\}^3 \right] \quad (4.22)$$

valid over the range, $0 < H^{\text{cool}}/H_{\text{cs}} \leq 1$, and, $0 < H_r \leq 2H^{\text{cool}} \leq 2H_{\text{cs}}$.

As illustrated in Fig 4.5b if $H^{\text{cool}}/H_{\text{cs}} < 1$, then $r_0 = r_p$, when $H_{\uparrow} = 2H^{\text{cool}}$ and the zone occupied by the field trapping currents has then vanished. Subsequently as $H_{\uparrow}^{\text{cool}}$ continues to increase, the grains are occupied only by field shielding currents in an annular zone $r_i \leq r \leq R$ as illustrated by the uppermost curve in Fig 4.5b. In these situations, at r_i , eqn 4.20 reads,

$$H_{\uparrow}(r_i) = H^{\text{cool}} = H_{\uparrow} - H_{\text{cs}} \left(1 - \frac{r_i}{R}\right) \quad (4.23)$$

hence
$$\frac{r_i}{R} = 1 - \left(\frac{H_{\uparrow} - H^{\text{cool}}}{H_{\text{cs}}} \right) \quad (4.24)$$

The definition of M_g then leads to

$$M_g(r_0) = \frac{J_{\text{cs}}}{3R} \{-R^3 + r_i^3\} = \frac{H_{\text{cs}}}{3} \left[-1 + \left\{ 1 - \frac{(H_{\uparrow} - H^{\text{cool}})}{H_{\text{cs}}} \right\}^3 \right] \quad (4.25)$$

valid until $r_i = 0$, hence $H_{\uparrow} - H^{\text{cool}} = H_{\text{cs}}$. Subsequently, $M_g = -H_{\text{cs}}/3$.

No plateau region exists in the field profiles when $H_{\uparrow}^{\text{cool}}$ has descended to zero from $H^{\text{cool}} > H_{\text{cs}}$, as illustrated in Fig 4.5c. In these situations, when H_{cs} reascends from zero, as illustrated in Fig 4.5d, eqn 4.21 applies and the definition of M_g leads to

$$M_g(r_0) = \frac{J_{\text{cs}}}{3R} \{-R^3 + 2r_0^3\} = \frac{H_{\text{cs}}}{3} \left[-1 + 2 \left\{ 1 - \frac{H_{\uparrow}}{2H_{\text{cs}}} \right\}^3 \right] \quad (4.26)$$

valid until $r_0 = 0$, hence $H_{\uparrow} = 2H_{\text{cs}}$. Subsequently, $M_g = -H_{\text{cs}}/3$.

A peak is traced by the curve of I_c versus $H_{\uparrow}$ whose summit occurs when,

$H_r = -CM_g(H_r, H^{\text{cool}}) = H_r$ and is denoted, $H_{p\uparrow}^{\text{cool}}$.

Figs 4.6 a, b and c display the predicted dependence of the location of the peaks, $H_{p\downarrow}^{\text{cool}}$ and $H_{p\uparrow}^{\text{cool}}$, on H^{cool} , and of the peaks $H_{p\downarrow}^{\text{cycle}}$ and $H_{p\uparrow}^{\text{cycle}}$ on $H_{\text{max}}^{\text{cycle}}$. The effect of the “compression” parameter C on the position of the peaks as well as on the onset and height of the plateaus can be assessed by comparing the three sets of displays where C is made to vary from 0.2 to 1 to 5. Here we exploited idealized disk geometry for the magnetization of the grains. We will examine later in this chapter the influence of different grain geometries on such predictions.

III Measurements of H_p versus $H_{\text{max}}^{\text{cycle}}$ and H^{cool} and Model Curves

Figs 4.7 and 4.8 display measured values for the positions of the peaks observed in curves of $I_c(H_a)$ as a function of $H_{\text{max}}^{\text{cycle}}$ and H^{cool} . The sets of data displayed in these figures present the most complete systematic investigations we have found in the literature on the dependence of the position of the peaks of I_c on the previous magnetic field-temperature history of specimens in the superconducting state. These sets of measurements were performed in three different labs at different temperatures and on two different high T_c superconductors. It is gratifying therefore to note that the data display features and trends which our simple model can reproduce satisfactorily.

In order to achieve a good qualitative and quantitative fit to a data set we need to select the two parameters which appear in the model, namely, (i) H_{*p} , the full penetration field into the grains, and (ii) the coefficient C in the Evetts-Glowacki relationship

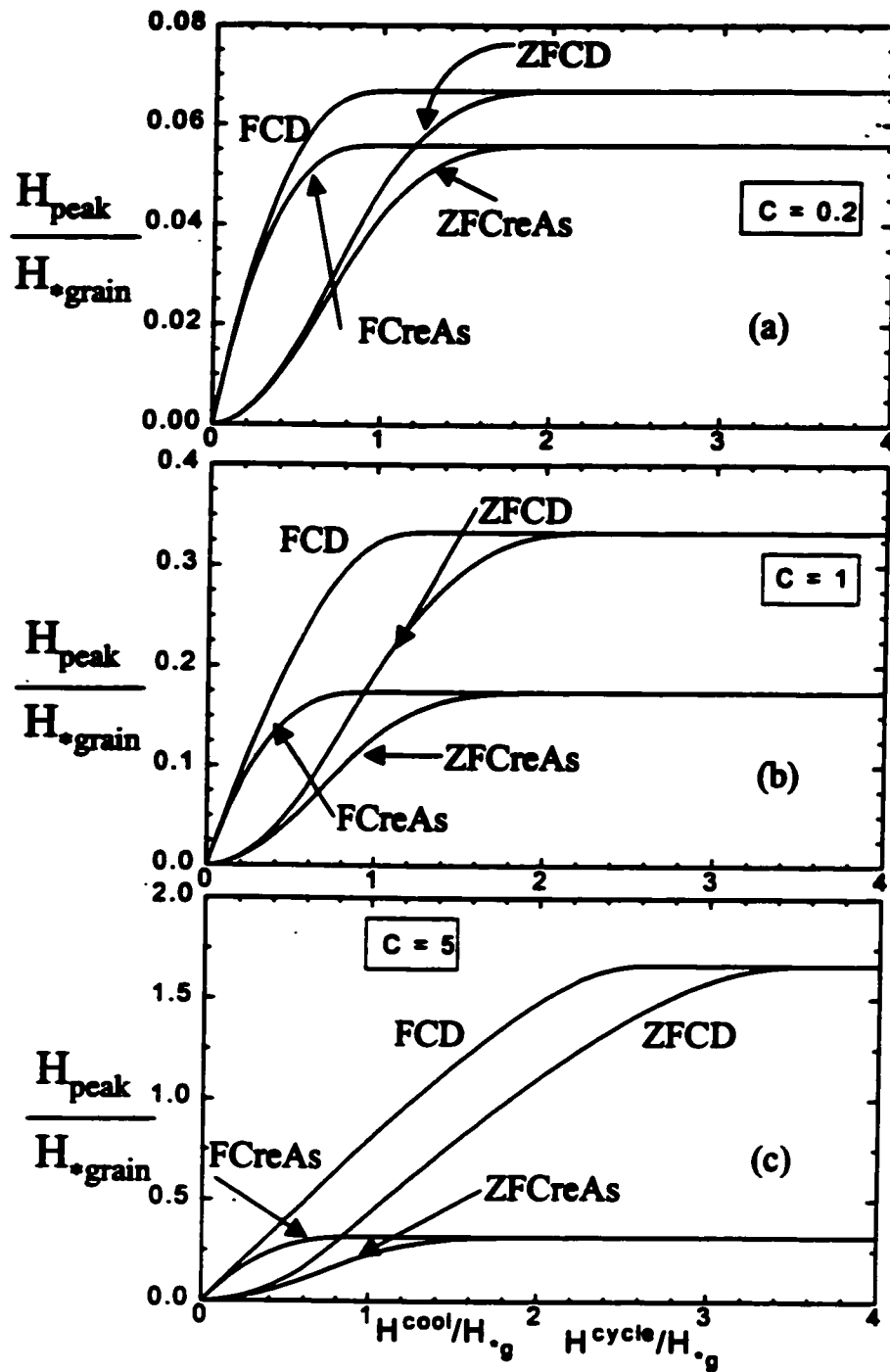


Fig 4.6. (a), (b) and (c) display the theoretical predictions for the dependence on H^{cool} and H^{cycle} of the position of the peaks for the four “categories” of curves of I_c versus H_a which have been explored by researchers. The effect of the compression coefficient C on the evolution of these four types of peaks emerges from comparison of the three families of curves. The calculations used idealized disk grain where, $M_{\text{g}}^{\text{cyl}} = H_{\text{g}}/3 = J_{\text{cg}}R/3$. These normalized curves remain unchanged by letting, $C_{\text{slab}} = 2/3 C_{\text{cyl}}$, where C_{cyl} denotes the values listed in these figures, since for idealized slab geometry, $M_{\text{g}}^{\text{slab}} = M_{\text{g}}/2 = J_{\text{cg}}X/2$.

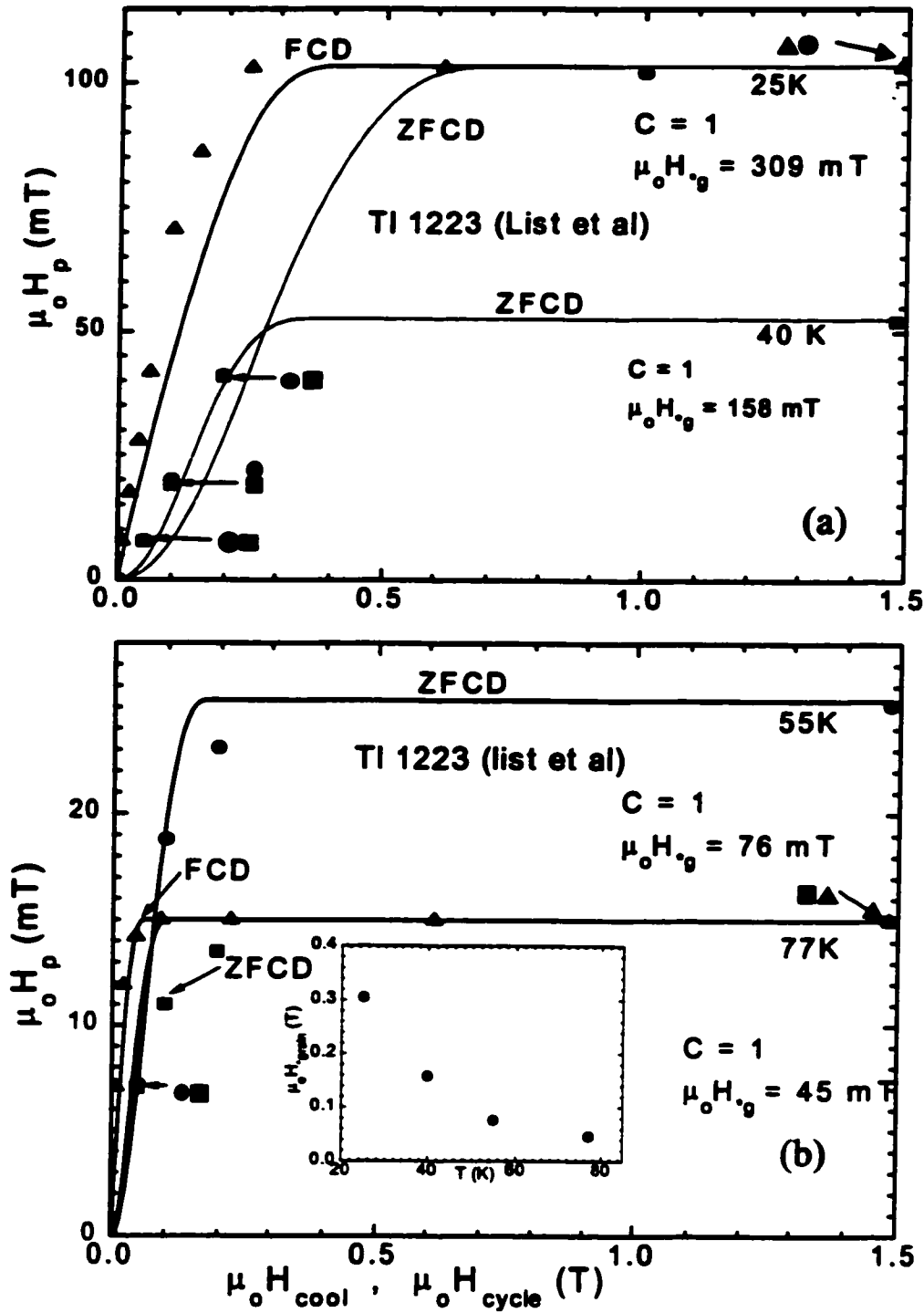


Fig 4.7. Compares all of the data obtained by List et al^[91] for the position of the peaks in I_c versus H_a for the FCD and ZFCD processes at different temperatures, with theoretical curves where we take $C = 1$, hence independent of temperature. The calculations used idealized disk geometry and the values chosen for $\mu_0 H_{a0}(T)$ are indicated along the curves and displayed in the insert. ZFCD data was taken at the four chosen temperatures and are indicated by $\bullet$ at 25 and 55K, and $\blacksquare$ at 40 and 77K. FCD measurements only at 25K and 77K and are indicated by $\blacktriangle$. Where data points overlap, they are also displayed again separately along the tail of an arrow.

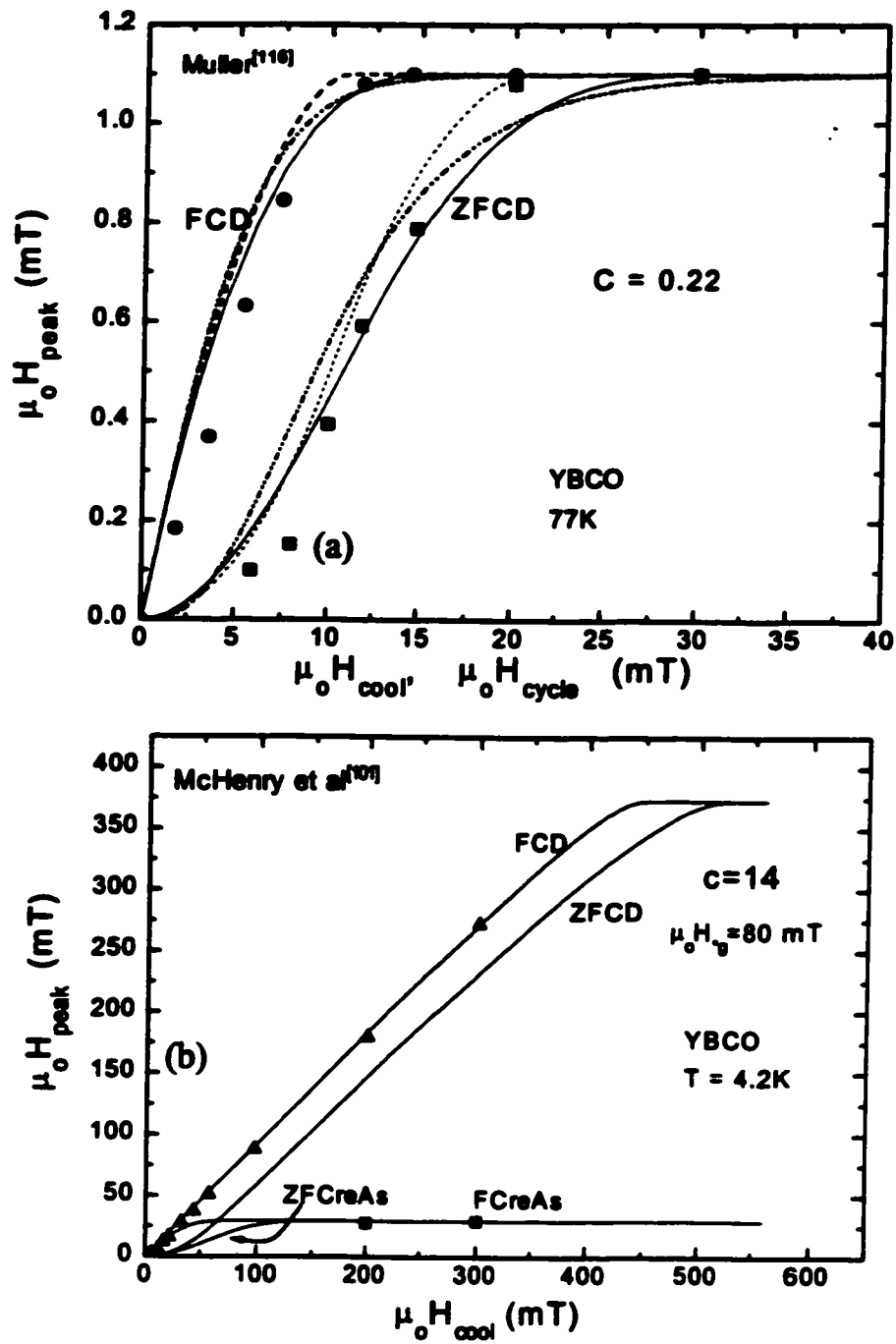


Fig 4.8. (a) Compares the data of Müller and Matthews^[116] at 77K on an YBCO sample, with calculated curves for H_p for the FCD (●) and ZFCD (■) processes. Here we display the differences in the behaviour of these theoretical curves for the evolution of the peaks when idealized disk (—), slab (---) and the Brandt function (····) are exploited to calculate the evolution of the magnetization $M_p(H_p)$ of the grains. We let $C = 0.22$ and $\mu_0 H_g = 15, 10$ and 5 mT for the three descriptions of $M_p(H_p)$. (b) Compares the data of McHenry et al^[101] at 4.2K on an YBCO sample for H_p for FCD and FCreAs processes with model curves for idealized disks. For completeness we also display calculated curves for the ZFCD and ZFCrAs histories. The large difference in the values for C for these two YBCO samples measured at 77K and 4.2K is discussed in detail in the text.

$H_r(H_a, T) = -CM_g(H_a, T)$, between H_r , the effective return field of the magnetized grains which threads the intergranular links and depends on the magnetic field-temperature history. The coefficient C is often referred to as the “enhancement” or “compression” parameter since it can be viewed as resulting from the funneling of the applied magnetic flux through the contact spaces across which the intergrain current flows from a cluster of grains to adjacent grains.

$H_{eg}(T)$ can be determined from separate measurements of the magnetization of the specimen, denoted $\langle M_s \rangle$, and generally associated with the grains^[83, 101, 130, 141]. The contribution of the induced intergranular (matrix) currents to $\langle M_s \rangle$ is generally negligible since $J_{cm} \ll J_{cg}$, and if necessary the small contribution of J_{cm} can be identified via a variety of simple schemes in the performance of the measurements of $\langle M_s \rangle$. Unfortunately, among the sets of data we display, only McHenry et al^[101] report on magnetization measurements of their specimen. List et al^[91] and Müller and Mathews^[116] do not report any magnetization measurements hence provide no direct information on H_{eg} .

The parameter C is the crucial and novel physical quantity which emerges from measurements of H_p versus H_{max}^{cycle} and H^{cool} in the framework of the Evetts-Glowacki concept. We expect this parameter to be determined by, (i) the granular structure of the specimen, i.e. the size distribution, orientation, geometry and packing of the grains, and (ii) the geometry of the specimen, the orientation of its surfaces, and of the ab planes of the grains with respect to $\vec{H}_a$ (eg. $H_a \perp$ or $\parallel$ to the surfaces of a rectangular ribbon or tape specimen). Consequently we may anticipate C to be insensitive to the temperature. This expectation is confirmed by the good fit we obtain introducing a common value $C = 1$ in

the modeling of the measurements of List et al^[91] at four different temperatures on a specimen of $(\text{Ti}_{0.5}\text{Pb}_{0.5})(\text{Sr}_{0.8}\text{Ba}_{0.2})_2\text{Ca}_2\text{Cu}_3\text{O}_x$ (abbreviated to Tl 1223). However in this exercise we had to select values for $\mu_0 H_{c2}(T)$ for the 4 different temperatures, since none were reported. The variation of these chosen values for $\mu_0 H_{c2}$ (309, 158, 76 and 45mT) versus the temperatures of the measurements (25, 40, 55 and 77K) is displayed in the insert to Fig 4.7b and is seen to be typical for high T_c grains. We will show in the next chapter that measurements of List et al^[91] of I_c at different temperatures with $H_a = 0$, after the sample has been subjected to H^{cycle} excursions sufficiently large to ensure that a saturation amount of flux has been trapped in the grains, also indicate that C is nearly independent of T .

The observation of the location of the peaks of I_c for various previous H-T histories on samples of YBCO at 77K by Müller and Mathews^[116], and at 4.2K by McHenry et al^[101] are displayed by the data points in Fig 4.8. The curves displayed in the figure are calculated using our modeling of the Evetts-Glowacki concept. For brevity in our discussion we will refer to features in Fig 4.8a (4.8b) with the labels 77K (4.2K) and also omit the symbol $\approx$ in front of estimates of quantities in the data or in the calculations.

In our view it is clear from inspection that the calculated curves lead to a good fit to the data points. As noted above, to fit data we need to assume an idealized geometry for the grains, then select values for both $H_{c2}(T)$ and the crucial new parameter C . Our analysis of the data of List et al^[91] supports the assumption that C is insensitive to temperature. In Fig 4.8a and b we are confronted with the dramatic feature that for one YBCO specimen we take $C = 0.22$ to fit the 77K data and for the other YBCO sample we take $C = 14$ to fit the 4.2K results. Evidently we must first address this discrepancy.

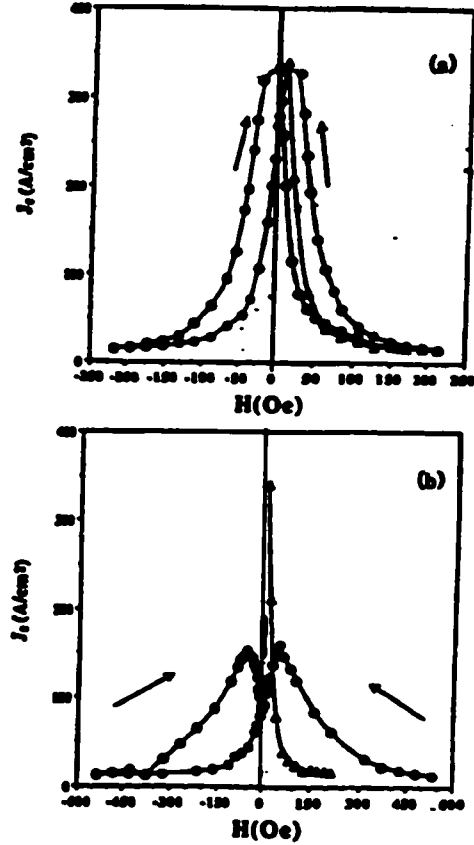
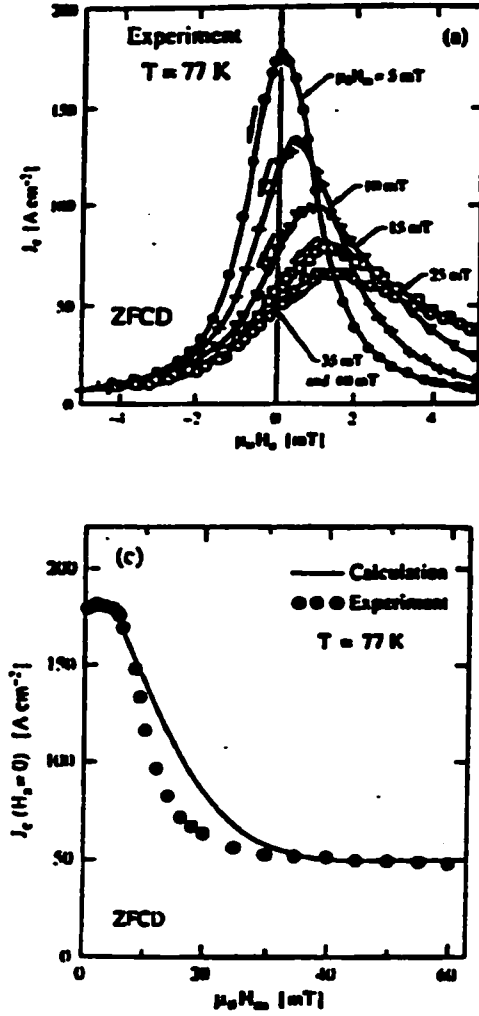


FIG. 1. Hysteresis in the critical current density J_c of polycrystalline Y 1:2:3 cooled in zero field to 4 K. (a) Virgin curve showing $J_c(H)$ in an increasing field (triangles), and decreasing- (filled circles) and increasing-field (open circles) branches after cycling to $H_{max} = 225$ Oe. (b) Virgin curve (triangles), decreasing- (filled squares) and increasing-field (open squares) branches after cycling to $H_{max} = 550$ Oe.

Fig 4.9. Figures (a) and (c) on the left are reproduced from Müller and Matthews^[116], and those on the right are Fig 1 from McHenry et al^[101]. Comparison shows that the values for the current densities J_c of the two YBCO specimens and their dependence on applied fields H_a are quite similar although the measurements were made at 77 and 4.2K respectively. We speculate that much larger return fields arise in the McHenry et al specimen at 4.2K which dramatically weaken the intergranular current carrying capacity. In the model these large H_r values are generated by a large M_r , hence H_{gr} , and a large compression factor C .

Unfortunately neither paper provides information on the internal structure of their specimen. Also both papers only outline the general standard procedure they followed in the sample preparation. We note that they indicate slightly different sequences in the temperature annealing treatment of the samples. No magnetization measurements accompany the 77K data hence H_{c2} at that temperature is not known. The 4.2K paper estimates $\mu_0 H_{c2} \approx 150\text{mT}$ at 7K from measurements of the applied field for the onset of the maximum diamagnetism upon first application of H_a . This procedure however tends to overestimate H_{c2} since it reflects the response of the largest grains and includes the shielding against the applied field by the Meissner currents (i.e. H_{c1}) and by surface barriers of various kinds (eg. image force, surface pinning, geometric^[27, 28, 51, 77, 78, 81]). However this possible overestimate of H_{c2} does not resolve the problem of our large value for C for the 77K sample. To shed light on this feature we now compare the J_c versus H_a data presented in the two papers and reproduce this pertinent information in Fig 4.9.

Inspection shows that the critical current density J_c descends from 175 A/cm^2 (350 A/cm^2) at $H_a = 5 \text{ mT}$ (15 mT) for the 77K (4.2K) results. Therefore the properties of the 4.2K sample appear enhanced by a factor of 2 relative to that exhibited by the 77K specimen. Such data curves for the same YBCO specimen typically scale by 2 orders of magnitude upon lowering T from 77 to 4.2K. Clearly then the current carrying performance of the 4.2K sample is pathetic relative to that of the 77K sample. The difference in the geometry and orientation of the two samples with respect to H_a can account for some and perhaps all of the difference in the relative performance of the two samples. The 4.2K sample is a “cube” of 0.03cm^2 cross section with H_a directed along

one of the edges. The 77K is a slab of thickness $d = 0.5\text{mm}$, width $w = 2.5\text{mm}$ and length $L = 13\text{mm}$, with H_a directed along the width. It is well established that the small demagnetization factor of the slab with H_a directed along its width can enhance J_c by a factor of ≈ 2 with respect to the “cube” geometry^[24]. More importantly however may be the feature that pressing of the powders will cause the ab planes of the grains to align along the plane // to the width-length of the slab. Hence, in the 77K specimen, H_a will be preferentially directed along the ab planes of the grains. Since the grains are platelets flat and wide along the ab plane and thin along the c axis, their magnetization along the planes and their return field will consequently be minimized. The orientation of the crystals in the “cube” with respect to $\vec{H}_a$ is unknown since no mention is made regarding this aspect in the cutting of the cube from the original piece cold-pressed to 50000psi (3.4 kbar). It emerges from the foregoing that no definite conclusion can be drawn regarding any clear link between the values of C assigned to these two YBCO samples and the temperature of the measurements. It is regrettable also that we can only present conjectures about the physical reason for the large difference in the parameter C for these two YBCO samples. The observations and our analysis in the Evetts-Glowacki framework indicate unambiguously however that the parameter C can span a very wide range.

It is important to note that the large values for C which emerge in quantitatively modeling the 4.2K data were anticipated by McHenry et al^[101] from inspection of their observations. These considerations led these workers to make the following statements in the final paragraph of their paper. “The field-cooled results presented here reveal new features not previously observed in ZFC studies”. “These new systematics are difficult to

reconcile with the Evetts-Glowacki model, which is based upon a vectorial addition of the applied field and dipolar fields proportional to the magnetic moment of the grains”
 We will examine the dipolar fields picture in a different perspective later in this thesis.

IV Various Observations on our Model of the Position of I_c Peaks.

(i) General Remarks

Pursuing the proposal of Evetts and Glowacki, that peaks in curves of I_c versus H_a are encountered in weak-linked type II superconductors when the return field H_r of the magnetized grains cancels the applied field H_a , we have presented a simple formalism for quantitatively relating the position of four categories of such peaks to specific prior values of the applied fields in the previous field-temperature history of the specimen. The reader will note that this formalism does not invoke any knowledge whatsoever on the actual or postulated dependence of the intergranular current density J_{cm} on the magnetic field and temperature. We have not found in the literature any such systematic development of the Evetts-Glowacki concept regarding the field-temperature dependence of the position of the various types of peaks. Several qualitative descriptions of the relative positions of the I_c peaks for ZFCD and ZFCrAs procedures have been published. Müller and Mathews^[116] appear to be the only researchers who have explicitly and quantitatively addressed the evolution of the location of these two peaks. Their model however is based upon and incorporates the hypothesis, that the links between the grains are Josephson junctions, in the development of expressions for the dependence of position of the I_c peaks on H_{max}^{cyc} and H^{cool} .

In analyzing various features of the phenomenon of I_c hysteresis workers have exploited idealized slab geometry to describe the magnetic behaviour of the grains in the context of the critical state concept. For variety and for comparison with previous work we have used idealized cylinder geometry to describe the magnetization of the grains visualized as circular disks. For completeness appendix 4.A presents the expressions for M_g versus H_g for idealized slab geometry for the various field-temperature histories addressed in section II above. Again J_{cg} is assumed independent of H . The dash-dash curves in Fig 4.8a display the fit achieved by our model when these expressions are introduced in the calculations for the location of the I_c peaks. The difference between the theoretical curves using these two different idealized geometries appears unimportant compared with the deviations (although small) of the calculated curves with the data points. An important feature however which enters into the fitting is that for slab geometry we need to choose $\mu_0 H_{g2}$ smaller than for a cylinder (disk geometry) by a “geometrical” factor of 2/3, hence now $\mu_0 H_{g2} = 10$ mT instead of 15 mT. This appreciable difference indicates the importance of separately measuring H_{g2} for the specimen in order to evaluate the parameter C more reliably with our detailed development of the Evetts-Glowacki proposal.

(ii) Plateaus in the Positions of the I_c Peaks.

We now focus on the relative heights of the various plateaus in Figs 4.6 and 4.8b. Here we are looking at the position of the peaks of I_c encountered when the magnetic field, H^{cycle} or H^{cool} in the prehistory, was sufficiently large so that the position of the I_c peaks remained fixed regardless of any increase in H^{cycle} or H^{cool} . This behaviour is also

illustrated by the extremal $\square \bullet$ intersections displayed in Fig 4.4. Let the location of the I_c peaks given by the height of the plateaus be denoted $H_{\text{plateau}}^{\text{ZPCD}}$, $H_{\text{plateau}}^{\text{PCD}}$, $H_{\text{plateau}}^{\text{ZFCrAs}}$ and $H_{\text{plateau}}^{\text{FCrAs}}$, and the threshold values of H^{cycle} or H^{cool} where the plateaus begin be denoted by $H_{\text{onset}}^{\text{ZPCD}}$, $H_{\text{onset}}^{\text{PCD}}$, $H_{\text{onset}}^{\text{ZFCrAs}}$ and $H_{\text{onset}}^{\text{FCrAs}}$. We will develop expressions for the latter four quantities in appendix 4.C.

Firstly we note, in the theoretical results shown in Fig 4.6, that $H_{\text{plateau}}^{\text{PCD}} = H_{\text{plateau}}^{\text{ZPCD}}$ and, $H_{\text{plateau}}^{\text{FCrAs}} = H_{\text{plateau}}^{\text{ZFCrAs}}$. The first of these simple predictions of our model is confirmed by the 25K and 77K data presented in Fig 4.7a and b and by the data displayed in fig 4.8a. Unfortunately these are the only pertinent data sets we have found in the literature. The model developed by Müller and Mathews^[116] also generates this behaviour. No measurements were found however to enable us to ascertain whether the second of these straightforward predictions is correct, namely $H_{\text{plateau}}^{\text{FCrAs}} = H_{\text{plateau}}^{\text{ZFCrAs}}$.

Next and more importantly we note from inspection of Fig 4.6 that the model predicts a relationship between the compression factor C and the ratio $H_{\text{plateau}}^{\text{ZPCD}} / H_{\text{plateau}}^{\text{ZFCrAs}}$, denoted R_p , and that the model predicts this ratio to be the same as the ratio $H_{\text{plateau}}^{\text{PCD}} / H_{\text{plateau}}^{\text{FCrAs}}$ for a given sample at the same temperature. In our model it is a straightforward exercise to develop the expressions for these ratios.

Firstly we note from eqn 4.12 and as illustrated in Fig 4.4, that,

$$H_{\text{plateau}}^{\text{PCD}} = H_{\text{plateau}}^{\text{ZPCD}} = \frac{C}{3} H_{\text{e}_g} \quad 4.27$$

From eqn 4.16, where the 3rd and 4th terms have vanished (since to generate I_c peaks along the plateau, $H_{\text{max}}^{\text{cycle}} \geq 2H_{\text{e}_g}$) we write,

$$H_{\text{plateau}}^{\text{ZFCrAs}} = -\frac{CH_{\text{g}}}{3} \left[1 - 2 \left\{ 1 - \frac{H_{\text{plateau}}^{\text{ZFCrAs}}}{2H_{\text{g}}} \right\}^3 \right] \quad 4.28$$

We see that eqn 4.26 (page 101) leads, as expected from inspection of Fig 4.4, to the identical expression for $H_{\text{plateau}}^{\text{FCrAs}}$, namely,

$$\frac{H_{\text{plateau}}^{\text{FCrAs}}}{H_{\text{g}}} = -\frac{C}{3} \left[1 - 2 \left\{ 1 - \frac{H_{\text{plateau}}^{\text{FCrAs}}}{2H_{\text{g}}} \right\}^3 \right] = \frac{H_{\text{plateau}}^{\text{ZFCrAs}}}{H_{\text{g}}} \quad 4.29a$$

Let $2x \equiv H_{\text{plateau}}^{\text{ZFCrAs}} / H_{\text{g}} = H_{\text{plateau}}^{\text{FCrAs}} / H_{\text{g}}$. Then we obtain,

$$2x^3 - 6x^2 + 6x \left[1 + \frac{1}{C} \right] - 1 = 0 \quad 4.29b$$

Let R_p^{cyl} denote the ratio of the positions of the two I_c peaks obtained for cylindrical (disk) geometry from eqn 4.27 to the positions of the two peaks obtained from solving eqn 4.29, namely,

$$R_p^{\text{cyl}} \equiv \frac{H_{\text{plateau}}^{\text{ZPCD}}}{H_{\text{plateau}}^{\text{ZFCrAs}}} = \frac{H_{\text{plateau}}^{\text{PCD}}}{H_{\text{plateau}}^{\text{FCrAs}}} \quad 4.30$$

We see that R_p^{cyl} does not depend on H_{g} but only the compression parameter C . This dependence is displayed by the thin curve in Fig 4.10. Consequently we note that a determination of C can be obtained from two judiciously chosen sets of measurements, namely, “tracing” the position of the I_c peak ensuing from the zero field cooled procedure when H_{g} is descending from a large $H_{\text{max}}^{\text{cycle}}$ established to be greater than $2H_{\text{g}}$, and then cause H_{g} to reascend from $H = 0$ until the second peak in I_c is observed. The estimate of C from the ratio of the position of these two peaks can then be confirmed and a prediction of our model verified by also measuring the position of the two I_c peaks

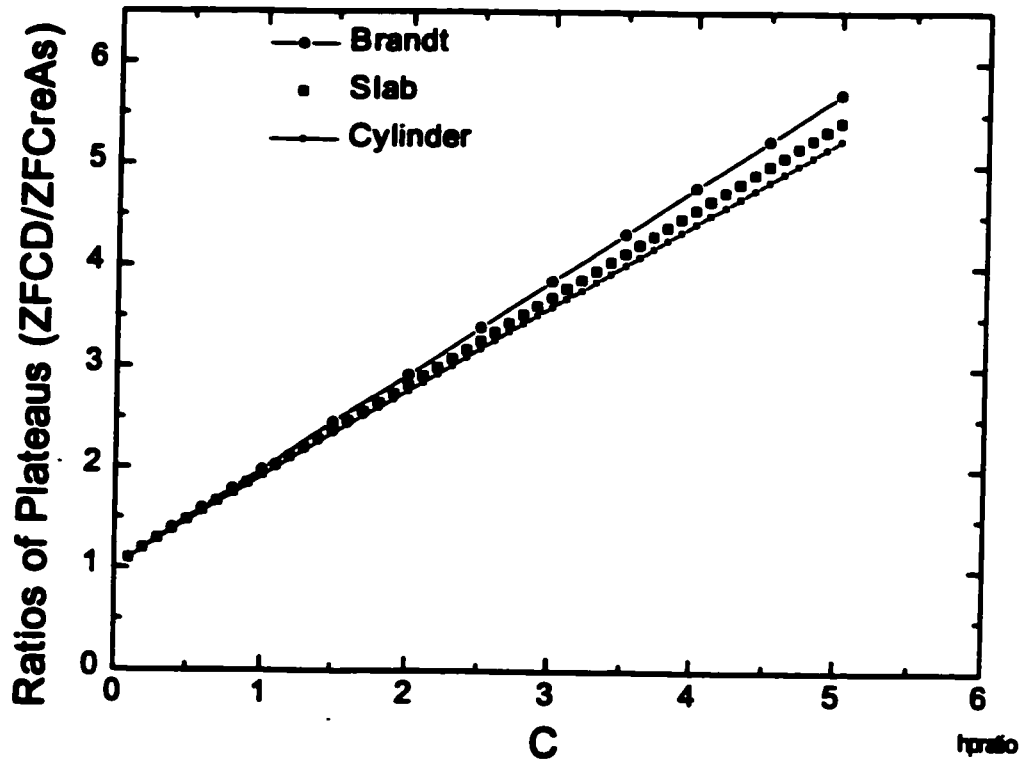


Fig 4.10. Our model predicts that the ratio $H_{\text{plateau}}^{\text{ZFCD}} / H_{\text{plateau}}^{\text{ZFCrAs}} = H_{\text{plateau}}^{\text{FCD}} / H_{\text{plateau}}^{\text{FCrAs}}$, and that this provides an unambiguous measure of the compression factor C , and that this value is independent of the calculated value of H_{g} for the specimen, and the choice made between the very different idealized geometries for the grains, eg. cylinder, slab or ribbon (Brandt-Indenbom equations). The four quantities appearing in these two ratios denote the values of H_{g} when the corresponding peaks in I_{c} curves are observed when $H^{\text{cycle}} > 2H_{\text{g}}$ and $H^{\text{cool}} > H_{\text{g}}$. The researcher however need not measure H_{g} but simply ensure that it is exceeded by two simple magnetization measurements.

ensuing from the field cooled procedure where H^{cool} is chosen greater than $H_{\bullet g}$ and H_a reascends from $H = 0$ until the corresponding peak is established.

From appendix 4.A we note that for grains of idealized slab geometry,

$$H_{\text{plateau}}^{\text{ZPCD}} = H_{\text{plateau}}^{\text{PCD}} = \frac{CH_{\bullet g}}{2} \quad 4.31$$

and that,

$$\frac{H_{\text{plateau}}^{\text{ZPCrAs}}}{H_{\bullet g}} = -\frac{C}{2} \left[1 - 2 \left\{ 1 - \frac{H_{\text{plateau}}^{\text{ZPCrAs}}}{2H_{\bullet g}} \right\}^2 \right] = \frac{H_{\text{plateau}}^{\text{PCrAs}}}{H_{\bullet g}} = -\frac{C}{2} \left[1 - 2 \left\{ 1 - \frac{H_{\text{plateau}}^{\text{PCrAs}}}{2H_{\bullet g}} \right\}^2 \right] \quad 4.32$$

These identical quadratic eqns can readily be solved and lead to,

$$\frac{H_{\text{plateau}}^{\text{ZPCrAs}}}{H_{\bullet g}} = \frac{H_{\text{plateau}}^{\text{PCrAs}}}{H_{\bullet g}} = 2 \left[\left(1 + \frac{1}{C} \right) - \sqrt{\left(1 + \frac{1}{C} \right)^2 - 0.5} \right] \quad 4.33$$

We then note that the ratios,

$$R_p^{\text{slab}} = \frac{H_{\text{plateau}}^{\text{ZPCD}}}{H_{\text{plateau}}^{\text{ZPCrAs}}} = \frac{H_{\text{plateau}}^{\text{PCD}}}{H_{\text{plateau}}^{\text{PCrAs}}} = \frac{C}{4 \left[\left(1 + \frac{1}{C} \right) - \sqrt{\left(1 + \frac{1}{C} \right)^2 - 0.5} \right]} \quad 4.34$$

are independent of $H_{\bullet g}$ and depend only on the parameter C . This dependence or function of C is displayed by the ■■■ curve in Fig 4.10. We stress that it nearly overlaps with the curve for R_p^{cyl} versus C . This result reveals that the ratios are not only independent of $H_{\bullet g}$ but are also independent of which of the two idealized geometries we have exploited to describe the magnetization of the grains.

In Appendix 4.B we present expressions developed by Brandt and Indenbom^[19] to describe the magnetization of a slab of width $W = 2a$, thickness $d \ll W$, and infinite length L , taking $j_{\bullet g}$ independent of the flux density, and with $\vec{H}_a$ directed // to the small

thickness d . In that appendix the basic formulae of Brandt and Indenbom are adapted and extended to describe the evolution of the magnetization for the ZFCD, ZFCrAs, FCD and FCrAs situations we have addressed in this chapter.

From these expressions we note that, the magnitude of the magnetizations approach H_{*g} asymptotically when the functions inside the [] brackets approach unity, consequently, $|H_r|_{\text{max}} = C_1 H_{*g}$, hence,

$$H_{\text{plasma}}^{\text{ZFCD}} = H_{\text{plasma}}^{\text{FCD}} = C_1 H_{*g} \quad 4.35$$

and,

$$\frac{H_{\text{plasma}}^{\text{ZFCrAs}}}{H_{*g}} = -C_1 \left[1 - \tanh \left(C_2 \frac{H_{\text{plasma}}^{\text{ZFCrAs}}}{H_{*g}} \right) \right] \quad 4.36a$$

$$\frac{H_{\text{plasma}}^{\text{FCrAs}}}{H_{*g}} = -C_1 \left[1 - \tanh \left(C_2 \frac{H_{\text{plasma}}^{\text{FCrAs}}}{H_{*g}} \right) \right] \quad 4.36b$$

Solving eqn 4.36 numerically for arbitrary values of C_1 and C_2 and introducing these results and eqn 4.35 into the definition,

$$R_p^{\text{Brandt}} = \frac{H_{\text{plasma}}^{\text{ZFCD}}}{H_{\text{plasma}}^{\text{ZFCrAs}}} = \frac{H_{\text{plasma}}^{\text{FCD}}}{H_{\text{plasma}}^{\text{FCrAs}}} \quad 4.37$$

we find that R_p^{Brandt} is also independent of H_{*g} and depends only on the parameter C_1 when the coefficient $C_2 = 1$. The variation of R_p^{Brandt} as a function of C_1 is displayed by the topmost line in Fig 4.10 and is seen to coincide closely with the curves for R_p^{cyl} and R_p^{slab} , indicating that these measured ratios are independent of the magnitude of the full penetration field H_{*g} and of the idealized geometries assumed for the grains.

In the physical and mathematical limit where $C = 0$, there is no return field of the magnetized grains inside the specimen, hence the I_c hysteresis vanishes and the unique maximum for I_c for the various H-T histories is encountered at $H_a = 0$.

V Summary and Conclusion

Evetts and Glowacki proposed that a simple superposition of H_r , the return field of the magnetized grains and the applied field H_a accounted for the large hysteresis of the critical transport current I_c in polycrystalline high T_c superconductors. The return fields are determined by previous magnetic field-temperature history of the specimen in the superconducting state. In this framework, the dramatic peaks, observed in the behaviour of I_c versus H_a descending to zero and then reascending after previous histories labeled zero field cooled (ZFC) or field cooled (FC) are attributed to the situations where H_r is caused to cancel H_a . In the modeling of the phenomenon of I_c hysteresis, workers have regarded the return field to be proportional to M_g , the magnetization of the grains, hence write $H_r = -C M_g$. We have pursued this framework to quantitatively examine the migration of the position of four “categories” of peaks generated in the curves of I_c versus H_a for ZFC and FC prior histories. Experimental studies show that the migration of the peaks ceases when the magnitude of the initial magnetic fields, denoted H_{max}^{cycle} and H^{cool} , exceed threshold values, denoted H_{onset}^{ZFC} , $H_{onset}^{ZFCreAs}$, H_{onset}^{FC} and H_{onset}^{FCreAs} .

We stress that $J_{cm}(H)$ the detailed dependence of the intergranular transport critical current density J_{cm} on the total magnetic field H_{total} plays no role in the modeling of the location of the peaks of I_c . Consequently, in this chapter we ignored the detailed structure and the quantitative behaviour of the hysteretic curves of I_c versus H_{total} .

In order to develop quantitative and qualitative expressions to describe the dependence of H_c on H_0 , we have assumed the intragranular current density J_{cg} to be independent of the magnetic flux density and visualized an assembly of identical grains. Further, for simplicity we addressed idealized geometries in our calculations of the magnetization of the grains, namely disks (cylinders) and slabs with the wide surfaces // and $\perp$ to H_0 . We find that these various cases lead to only small differences in the shape of the curves for the evolution of the position of the I_c peaks as a function of $H_{\max}^{\text{cycle}}$ or H^{cool} . Consequently these differences may not warrant the long and tedious experimental effort demanded for verification.

Experimental studies of the migration of the peaks have shown that their displacements cease when the fields $H_{\max}^{\text{cycle}}$ and H^{cool} exceed threshold values. Again we have developed expressions for these threshold fields which depend on the “compression” parameter C , the field for full penetration of the grains, H_0 , and the geometry introduced in describing the magnetization of the grains. Since the approach to the threshold values is asymptotic, the experimental determination of these thresholds is also problematic.

The determination of the position of the four peaks, encountered when $H_{\max}^{\text{cycle}}$ and H^{cool} exceed the onset values, denoted $H_{\text{plateau}}^{\text{ZFC D}}$, $H_{\text{plateau}}^{\text{ZFC ReAs}}$, $H_{\text{plateau}}^{\text{FCD}}$ and $H_{\text{plateau}}^{\text{FC ReAs}}$, demand a minimum of only four measurements of I_c versus H_0 for a chosen specimen and temperature. Our model predicts that $H_{\text{plateau}}^{\text{ZFC D}} = H_{\text{plateau}}^{\text{FCD}}$ and $H_{\text{plateau}}^{\text{ZFC ReAs}} = H_{\text{plateau}}^{\text{FC ReAs}}$ under these circumstances. No measurements were found however to enable us to ascertain whether the second of these straightforward predictions is correct, namely $H_{\text{plateau}}^{\text{FC ReAs}} = H_{\text{plateau}}^{\text{ZFC ReAs}}$.

Hence our model predicts that the two ratios of these pairs of plateaus will be the same. More importantly, our model reveals that these two ratios, namely $H_{\text{plateau}}^{\text{ZFCD}} / H_{\text{plateau}}^{\text{ZFCrAs}}$, $H_{\text{plateau}}^{\text{FCD}} / H_{\text{plateau}}^{\text{FCrAs}}$, are not only equal but are uniquely dictated by the compression parameter C . Hence these ratios are independent of the three idealized grain geometries we have exploited in the development of these ratios. These ratios are also independent of H_p , the penetration field into the grains. Fig 4.10, which displays the variation of these ratios versus C , presents the central message emerging from our systematic and formal pursuit of the Evetts-Glowacki concept. Since H_p is appreciable relative to $H_{c1}(T)$ for the grains when these peaks of I_c are traced, we expect that the neglect of H_{c1} in our analysis will not significantly affect this message.

Chapter 5

I_c Hysteresis: A Critical State Model

Abstract

We develop a simple critical state model to describe the critical transport current I_c supported by weak-linked granular type II superconductors of idealized slab geometry where the intergranular (matrix) critical current density is dictated by $J_{cm} = J_{\infty} H_1 / \{H_0 + H_{total}\}^n$, $J_{cm} = J_{\infty} e^{-H_{total}/H_1}$, or, $J_{cm} = J_{\infty} H_1^n / \{H_0^n + H_{total}^n\}$, where $H_{total} = H_a \pm H_f$. The model is exploited to examine the dependence of I_c at a final field $H_f = 0$ after zero field cooling excursions to various H^{cycle} (ZFC), and after field cooling in various H^{cool} (FCD). The model is also exploited to reproduce various hysteretic I_c versus H_a curves reported in the literature. In particular the model can generate the valley observed by several researchers in the curve of I_c versus H_a ascending after zero field cooling (ZFCV).

I Introduction

Soon after the discovery of high T_c superconductors Ekin and co-workers^[46] suggested that the very low critical transport current densities J_{cm} of bulk sintered samples and the rapid decrease of the total transport current I_c at low magnetic fields result from the feature that contacts at the grain boundaries are Josephson junctions. Consequently to describe the hysteretic behaviour of I_c versus the magnetic field -

temperature history of their specimens, almost all researchers have introduced the Josephson junction Fraunhofer diffraction pattern^[2, 7, 13, 49, 52, 91, 116, 118-120],

$$I_c = I_\infty \left| \frac{\sin(\pi H_{\text{total}} / H_J)}{\pi H_{\text{total}} / H_J} \right| \quad 5.1$$

Here the characteristic magnetic field, $H_J = \Phi_0 / \mu_0 (t + 2\lambda)L$, where μ_0 is the permeability of free space, $\Phi_0 = h/2e = 2(10^{-15})$ Tesla-m², is the quantum of flux, λ is the London penetration depth, t is the thickness of the junction barrier, L is the width of the junction whose height h , // to H_{total} , is taken large relative to L , and $H_{\text{total}} = H_a \pm H_r$, the superposition of the applied field H_a , and the return field H_r of the magnetized grains.

Workers however retain only the initial segment of the sophisticated Fraunhofer pattern over the range,

$$0 \leq H_{\text{total}} / H_J \leq \frac{1}{2} \quad 5.2$$

and assume simple Kim-like behaviour, namely,

$$I_c = \frac{I_\infty}{(\pi H_{\text{total}} / H_J)} \quad 5.3a$$

when $H_{\text{total}} / H_J \geq \frac{1}{2}$, hence when

$$\frac{I_c}{I_\infty} \leq 0.637 \quad 5.3b$$

It is instructive to set aside the arbitrary constraints imposed by the above equations. In this chapter we develop expressions for I_c in the critical state framework and display their usefulness in describing many features of I_c hysteretic behaviour.

II Expressions for I_c versus H_{total}

We assume that the intergranular (matrix) transport critical current density $J_{cm}(H,T)$ can be described by the well known empirical critical state equations of the form,

$$J_{cm} = J_{cm0} \left\{ \frac{H_1}{H_{total} + H_0} \right\}^n \quad 5.4a$$

Müller et al^[116] have developed equations for $I_c(H,T)$ based on the following expressions for $J_{cm}(H,T)$, but have not applied their formulae to account for hysteretic I_c phenomena,

$$J_{cm} = J_{cm0} e^{-H_{total}/H_1}, \quad J_{cm} = J_{cm0} \frac{H_1^2}{H_0^2 + H_{total}^2} \quad 5.4b, 5.4c$$

In these expressions H_1 is an adjustable scaling magnetic field, H_0 serves as a convenient temperature dependent parameter and avoids infinite current densities when $H_{total} = 0$. H_{total} , which we will abbreviate to H_t , reads,

$$H_{total} \equiv H_t = H_a + H_r + H_m(x) \quad 5.5$$

where H_a is the applied magnetic field, H_r is the “effective” return field of the magnetized grains which are assumed uniform in size, shape and orientation, hence H_r is regarded as uniform in the contact volume between grains. Simple expressions for H_r versus previous field-temperature history have been presented in the preceding chapter. $H_m(x)$ is the self-field generated by the intergranular current and represents a local average over several contacts, hence several grains.

To develop simple expressions for I_c versus H_t we address idealized planar geometry for the specimen. Consequently we consider a slab of infinite height along the $\hat{z}$ axis, infinite length along the $\hat{y}$ axis, and of width W along $\hat{x}$ with surfaces located at $x = 0$

and $x = W$. The conduction current flows along the $-\hat{y}$ direction and I_c denotes the current traversing a cross-section of unit height $Z = 1$ meter, and width W . The applied magnetic field H_a , the effective return field H_r of the grains magnetized by the application and changes of H_a , and the self-field $H_m(x)$ are all directed along the $\pm \hat{z}$ axis. The application and changes of H_a will induce field shielding or flux trapping intergranular currents. These induced currents circulating in the intergranular medium (matrix) will all be replaced by conduction currents flowing in the direction of the impressed I_c when I_c is attained. Since J_{cm} is orders of magnitude weaker than the critical current density J_{cs} circulating inside the magnetized grains, the effect of the self-field generated by I_c on the magnetization of the grains, hence on their return field is regarded as negligible.

For a specimen of idealized slab geometry, Maxwell's eqn, $\nabla \times \vec{H} = \vec{J}$, and the critical state concept that, $J = \pm J_{cm}(x, H_i)$, leads to,

$$\frac{dH_m(x)}{dx} = \pm J_{cm}(H_i) = \frac{\pm J_{cm} H_i^n}{\{H_o + H_i + H_m(x)\}^n} \quad 5.6$$

where,

$$H_i = H_a \pm H_r \quad 5.7$$

denotes the superposition of the applied and return fields. To fix ideas we have focused on J_{cm} of the form given by eqn 5.4a. In appendices 5A and 5B we develop formulae based on eqns 5.4b and c. The integral formulation of eqn 5.6 reads,

$$\int_{H_i - (I_c/2)}^{H_i + (I_c/2)} \{H_o + H_i + H_m(x)\}^n dH_m(x) = \pm J_{cm} H_i^n \int_0^W dx \quad 5.8$$

where the limits of integration follow from the feature that the "homogeneous" field H_e is enhanced or opposed by the self-field, $\pm I_c / 2$, of the intergranular conduction current I_c at the boundaries $x = 0$, $x = W$, of the slab.

We note that since H_o , H_r , hence H_e , and H_o , are independent of x , we can write,

$$dH_m(x) = d\{H_o + H_r + H_m(x)\} \quad 5.9$$

inside the left hand side integral.

First we consider the case where $H_e > I_c/2$, illustrated in Fig 5.1a. Here eqn 5.8, where we have introduced eqn 5.9, leads to,

$$\left(H_o + H_r + \frac{I_c}{2}\right)^{n+1} - \left(H_o + H_r - \frac{I_c}{2}\right)^{n+1} = (n+1)J_{cm}WH_r^n = K^{n+1} \quad 5.10$$

When, $H_e < I_c/2$, it is useful to first identify a plane, denoted x_o where $H_e + H_m(x) = 0$ as illustrated in Fig 5.1b. Then integrating eqn 5.8 from x_o to W leads to,

$$\left(H_o + H_r + \frac{I_c}{2}\right)^{n+1} - H_o^{n+1} = (n+1)J_{cm}(W - x_o)H_r^n \quad 5.11a$$

Integrating eqn 5.8 from $x = 0$ to x_o and noting that J_{cm} is dictated by the absolute value of the magnetic field, leads to,

$$\left(H_o - H_r + \frac{I_c}{2}\right)^{n+1} - H_o^{n+1} = (n+1)J_{cm}H_r^n x_o \quad 5.11b$$

Combining eqn 5.11a and b to eliminate x_o leads to,

$$\left(H_o + H_r + \frac{I_c}{2}\right)^{n+1} + \left(H_o - H_r + \frac{I_c}{2}\right)^{n+1} - 2H_o^{n+1} = K^{n+1} \quad 5.12$$

It is convenient and useful to pursue the computations with the equations in normalized form. We have elected to normalize the physical quantities H_o , H_e , H_r (hence

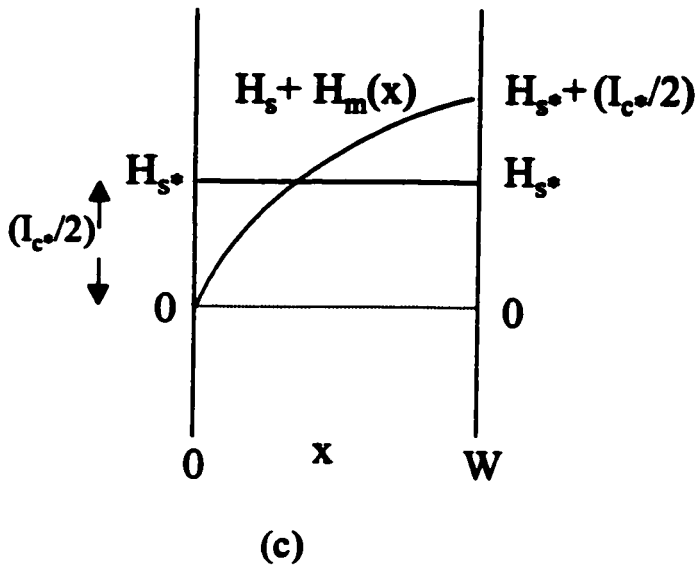
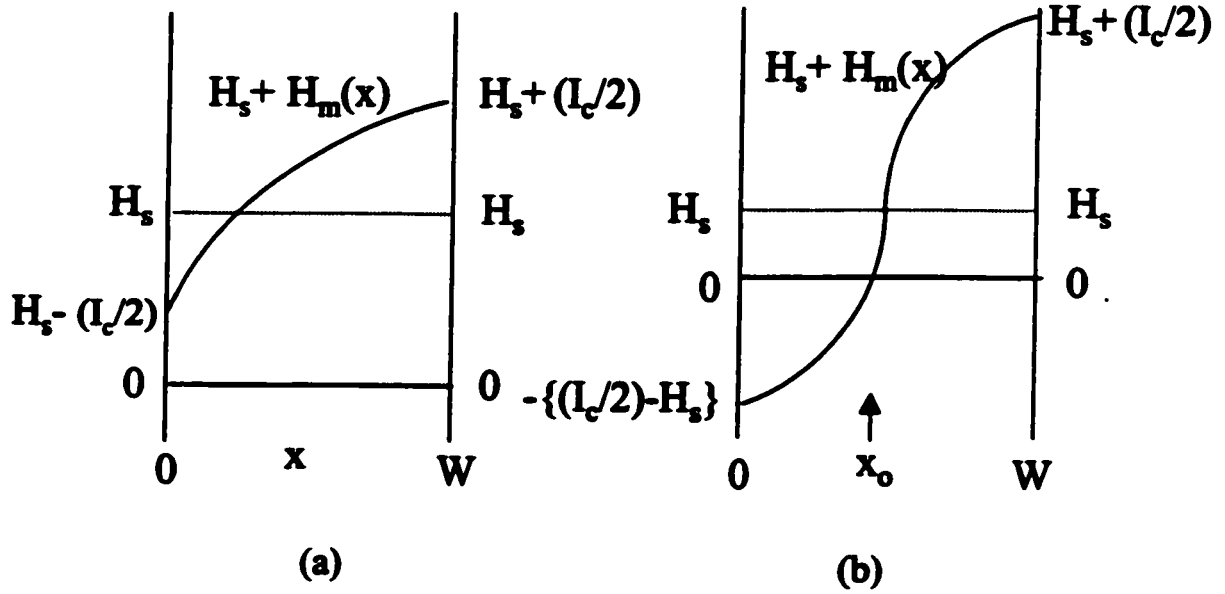


Fig 5.1. Illustrates the configuration of the total magnetic field $H_t = H_s + H_m(x)$ permeating an infinite slab of thickness W along the x -axis with the critical conduction current I_c directed out of the page, and H_s , directed upwards, is the superposition of the applied field H_s , and the return field H_r . $H_m(x)$ is the self-field generated by I_c inside the slab. In (a), $H_s > I_c/2$, and in (b) $0 < H_s < I_c/2$. In (c) $H_s = I_c/2$ and the quantities are then denoted H_{s*} and I_{c*} .

$H_s = H_a \pm H_r$) and K with respect to a specific value for I_c denoted I_{c0} . The configuration of the self-field $H_m(x)$ and the boundary conditions where $I_c = I_{c0}$, and H_s denoted H_{s0} , are displayed in Fig 5.1c. Here, $H_{s0} + (I_{c0}/2) = I_{c0}$ at $x = W$, and $H_{s0} - (I_{c0}/2) = 0$ at $x = 0$. The normalized formulae for this scenario are given in appendix 5.C. A special advantage of this procedure is that all the curves of I_c versus H_s exhibit $I_{c0}/I_{c0} = i_{c0} = 1$ when $H_{s0}/I_{c0} = h_{s0} = 0.5$ for all choices of $n > 0$ in eqn 5.4a. Consequently the relative quantitative and qualitative differences in the structure of the variety of the theoretical curves can readily be displayed and compared.

Several workers^[17, 49, 70, 102, 113, 115] have described the dependence of I_c on $\langle B \rangle$, the spatial average of the magnetic flux density permeating a specimen of idealized slab geometry and directed // to the broad surfaces. These researchers exploit critical current expressions for I_c such as,

$$I_c = I_\infty \left(\frac{B_1}{B_0 + \langle B \rangle} \right)^n, \quad I_c = I_\infty e^{-\langle B \rangle / B_1}, \quad I_c = \frac{I_\infty B_1^n}{B_0^n + \langle B \rangle^n} \quad 5.13a, b, c$$

These are similar to eqns 5.4(a) (b), and (c) which explicitly address the dependence of the critical current density J_{cm} on the "local" magnetic field inside the specimen. However, eqns 5.13a, b and c as well as eqns 5.1 and 5.3, ignore any effect of the configuration and magnitude of the self-field of I_c on itself. Clearly the self-field, $H_m(x)$, plays a significant role on the dependence of I_c on $H_s = H_a + H_r$ in the range where $H_s \approx 0$ hence when the peaks of I_c are encountered.

III Applications of our Model for I_c versus H_a

In the preceding chapter we accounted for many aspects of the relative position and migration of peaks in the curves of I_c versus H_a for a variety of magnetic field-temperature histories (eg. ZFCV, ZFCD, ZFCrAs, FCD, FCrAs) without introducing any specific model and formulae to describe the actual detailed dependence of I_c on $H_a = H_a \pm H_r(H_a)$. In this chapter we exploit our basic prescriptions, namely eqns, 5.10 and 5.12, to reproduce some of the variety of features in I_c hysteresis curves which have been reported in the literature.

(i) Butterfly Wing Curves of I_c versus H_a .

Fig 5.2 displays the typical hysteretic locus of I_c when the applied field H_a is made to swing slowly back and forth between $+$ and $-H_{max}$. The experimental curve is reproduced from the 1997 paper by Fukami^[52] and shows the behaviour of a BSCCO thin film where H_a is directed $\perp$ to the wide surfaces of the thin film and $\parallel$ to the c -axis of the grains. Since the dimensions of the specimen are not given we cannot estimate the magnitude of the self field, hence we cannot compare the magnitudes of I_c observed (Fig 5.2a) with that calculated (Fig 5.2b). The similarity of the two sets of butterfly wing curves is quite evident.

In Fig 5.2b we have taken $n=1$, $H_0/I_{c0} = 6.2$, and introduced $H_r(H_a) = C_1 M_g = C_1 I_{c0} F_B(C_2 H_a / H_p)$, with $C_1 = 1$, $C_2 = 1$. The ratio $H_{max}/H_p = 20$ for the calculation was chosen to correspond to the experimental ratio of $|\mu_0 H_{max}| = 2$ Tesla with respect to the location of the peak $|\mu_0 H_p| = 0.1$ Tesla. $F_B(C_2 H_a / H_p)$ denotes the

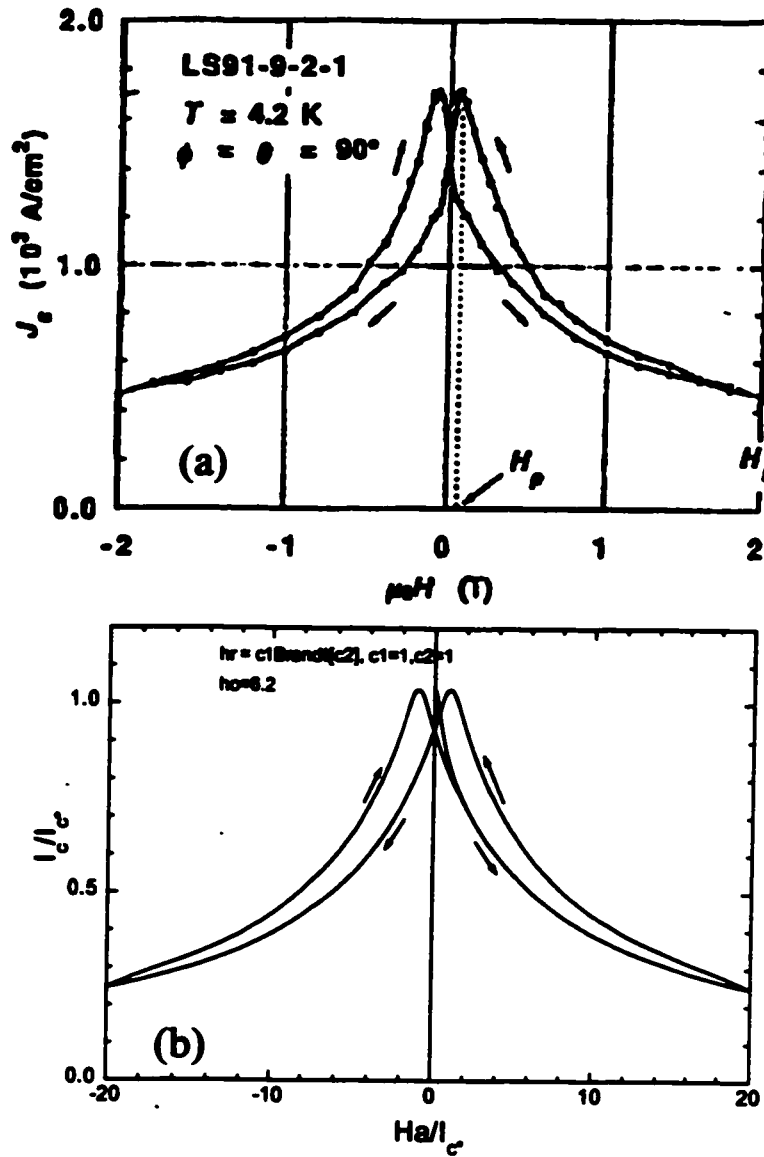


Fig 5.2. Displays (a) a typical measured, and (b) a theoretical, symmetric major I_c hysteresis loop where H_p , after the initial increase to a large maximum value, denoted $H_{\max}$, is then made to vary through a full cycle between $+H_{\max}$ and $-H_{\max}$. The experimental results are reproduced from Fukami^[52] and were observed on a BSCCO thin film with H_a directed $\perp$ to the wide surfaces and $\parallel$ to c -axis of the grains. The theoretical curves were calculated using eqns 5.10 and 5.12 with $n = 1$ and $H_0/I_{c0} = 6.2$. Here $H_r = -C M_r = C_1 I_{c0} F_B(C_2 H_a / I_{c0})$ with $C_1 = C_2 = 1$ and F_B denotes the Brandt-Indenbom formulae (Appendix 4.B). I_{c0} , H_p , H_r are normalized to $I_{c0} = 2H_{p0}$ illustrated in Fig 5.1(c).

Brandt-Indenborn formulae for the magnetization of the grains presented in appendix 4.B. Here the evolution of F_B is determined by H_a with respect to the appropriate parameter for the grains, hence can be regarded to be independent of $H_{c0} = I_{c0}/2$.

For completeness in Fig 5.2b we have also displayed the calculated zero field cooled virgin curve (ZFCV) although the corresponding data curve is not presented in the article by Fukami. We note that the ZFCV curve joins the bottom envelope of the butterfly wing when $H_a/H_{c0} \approx 2$ as expected since here the grains have effectively reached saturation magnetization when $H_a = 2 H_p = 2 I_{c0} = 4 H_{c0}$.

(ii) Minor Hysteresis Loops of I_c versus H_a .

In section (i) above we have examined the behaviour of a symmetric major hysteresis loop of I_c versus H_a where H_a swings between values for $\pm H_{\max}$ which are sufficiently large to ensure that, over most of the sweep of H_a , the return field $\pm H_r$ exists at the “saturation” level where the grains are completely magnetized, hence filled with induced currents circulating in the same sense.

Fig 5.3 displays traversals of I_c versus H_a between two segments of the upper and lower envelope of an asymmetric major hysteresis curve of I_c versus H_a . Any pair of the oppositely directed traversals between the envelopes trace a minor hysteresis loop of I_c versus H_a . The experimental curves are reproduced from an article by Grasso et al^[59]. Since the authors do not indicate the thickness and width of their ribbon specimen we cannot calculate I_c per unit meter for comparison with calculations. The theoretical curves are displayed with I_c normalized to I_{c0} . However from the current I_{c0} and current density

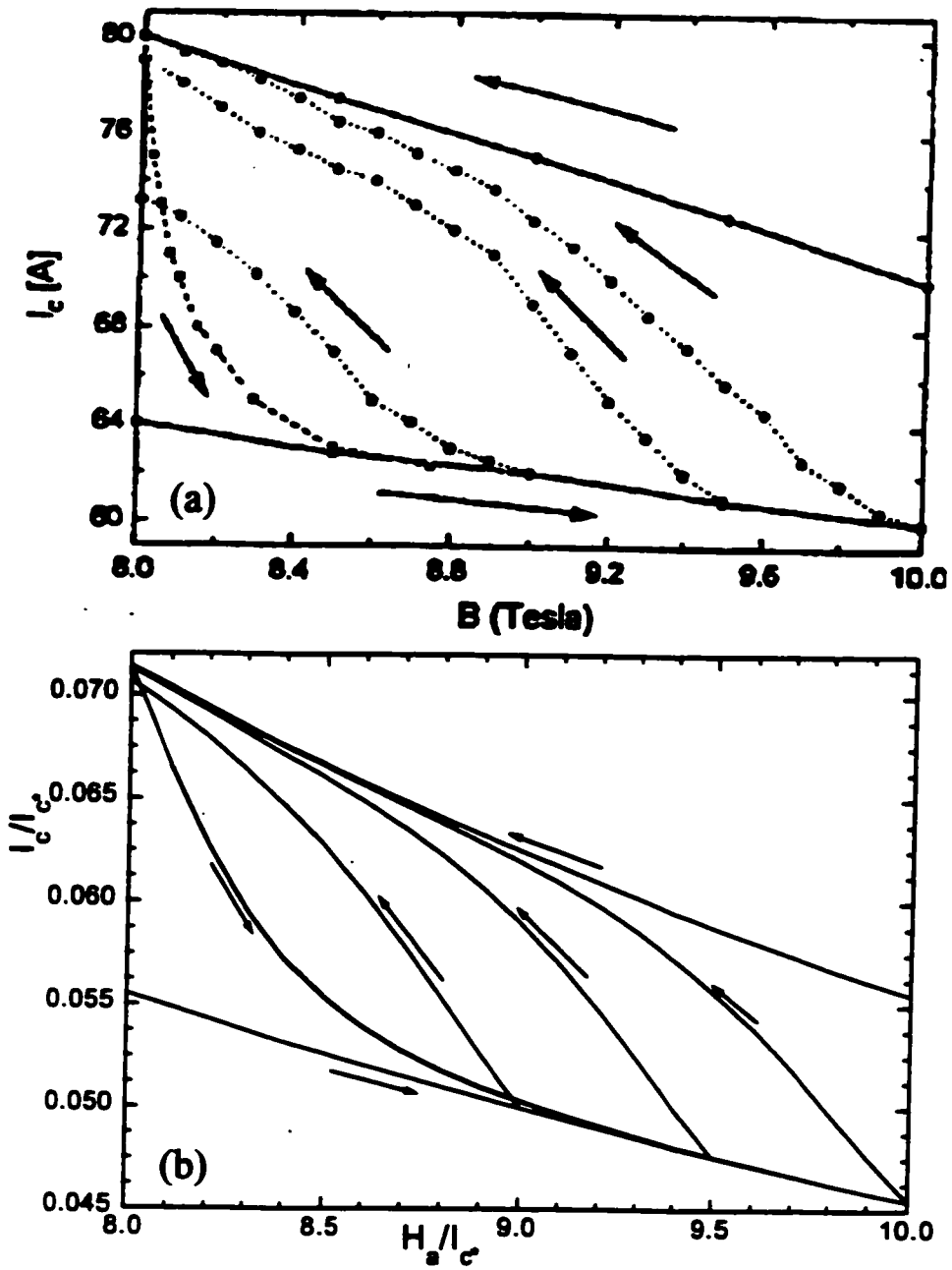


Fig 5.3. (a) Experimental curves reproduced from Grasso et al^[59] of traversals of I_c versus B , over the range $8 \leq B \leq 10$ Tesla, between two segments of the upper and lower envelopes of an asymmetric major hysteresis curve of I_c versus B . (b) Curves calculated using eqn 5.10 with $n = 1$, $H_0 = 0$ to compare with the experimental curves in (a). The return field of the grains has been calculated using the Brandt-Indenbom formulae for $M_g(H_g)$, hence $H_r(H_g) = -C_1 M_g(H_g)$ was calculated using $M_g = I_c F_B(C_2 H_g / I_c)$ with $C_1 = 1$, $C_2 = 4$.

J_{c0} which are reported we can establish that the area $A = I_{c0} / J_{c0} = 150 \text{ A} / 1.80(10^9) \text{ A/m}^2 = 8.33 (10^{-6}) \text{ m}^2$. Consequently Ampere's law, $B_\phi = \mu_0 I_\infty / 2\pi R$, leads to an estimate of a maximum self-field $B_\phi \approx 0.184 \text{ T}$ when we regard the specimen as a circular wire, hence write $A = \pi R^2$. We note that the range, $8 \leq H_\phi / I_\infty \leq 10$, selected for the excursions in the theoretical curves, is chosen proportional to the magnitude of the swings in the measurements. Also, in the calculations, the position of the minor hysteresis loop along the horizontal axis was chosen to correspond ratiowise with the experimental case.

(iii) Effect of Trapped Flux on I_c at $H_a = 0$.

A. Introduction

Several workers^[2, 4, 7, 14, 70, 83, 91, 116, 156] have investigated the effect of flux trapped in the specimen, hence in the grains, on the critical current in zero applied field (the earth's field). Generally the flux is trapped and the measurement of I_c made by following two simple but different procedures.

- a) The sample is cooled from T_c to a chosen temperature $T_f < T_c$ in zero field. H_a is then applied and brought to a chosen value denoted H_{cycle} , and removed. I_c , denoted $I_{c \text{ rem}}$, is then measured. We stress that in this procedure it is not necessary to cool the specimen from T_c to T_f in zero field before each measurement, provided that each value chosen for H_{cycle} in the sequence of the measurements is greater than the previous one. Of course the sequence of values of H_{cycle} is immaterial if the specimen is zero field cooled before each measurement.

b) The sample is cooled from T_c to T_f in a static applied field H_a , now denoted H_{cool} . The field H_{cool} is then removed and I_c , also denoted $I_{c \text{ rem}}$, is then measured. Here the sequence of the values chosen for H_{cool} is arbitrary.

$I_{c \text{ rem}}$ is then plotted versus H_{cycle} or H_{cool} . It is always found that $I_{c \text{ rem}}$ descends much more steeply versus H_{cool} than versus H_{cycle} . Also all workers observe that, (i) for large H_{cycle} and H_{cool} , $I_{c \text{ rem}}$ traces plateau values which overlap, and, (ii) H_{cool} for the onset of the plateau of $I_{c \text{ rem}}$ is always much smaller than the threshold value found for H_{cycle} . In both cases the approach of $I_{c \text{ rem}}$ to the plateau level depends asymptotically on H_{cycle} and H_{cool} , hence these threshold values, which can provide a measure of the full penetration field H_{p0} into the grains, cannot readily be determined accurately from these data.

The curves of $I_{c \text{ rem}}$ versus H_{cycle} and H_{cool} have been successfully described qualitatively and semi-quantitatively exploiting the framework of eqns 5.1, 5.2 and 5.3. Here the weak links between the grains are regarded as acting like Josephson junctions when the trapped flux, $\Phi_{rem} = B_{rem} A$, threading the effective area, $A = (t+2\lambda)L$, of the junction is less than $\Phi_0/2 = 10^{-15} \text{ T-m}^2$. i.e. less than half a quantum of flux. When $\Phi_{rem} > \Phi_0/2$, hence $I_{c \text{ rem}}/I_{c0} < 0.637$, the Josephson junction is now arbitrarily taken to obey a simple Kim critical state formula,

$$I_{c \text{ rem}} = \frac{I_{c0} \Phi_0}{\pi \Phi_{rem}} = \frac{I_{c0} \Phi_0}{\pi B_{rem} (t + 2\lambda)L} \quad 5.14$$

Taking the length of the contact between two grains $L \approx 1 \text{ micron}$, and $\lambda \approx 0.2 \text{ micron}$ yields $B_{rem} = \mu_0 H_{rem} \approx 5 \text{ mT}$ when $\Phi_{rem} \approx \Phi_0/2$, hence leads to satisfactory agreement with the observations of List et al^[91]. Eqn 5.14 however is equivalent to our critical state

model (eqn 5.10) where $n = 1$. It is therefore instructive to pursue our model to examine the behaviour of $I_{c \text{ rem}}$ for the entire range of H_{cycle} and H_{cool} , hence including the situations where $0.637 I_{c0} < I_{c \text{ rem}} < I_{c0}$, and $0 \leq B_{\text{rem}} \leq 5\text{mT}$.

In the 2 following subsections we apply our critical state model to sets of data of $I_{c \text{ rem}}$ versus H_{cycle} and H_{cool} published by two different groups.

B. Application to data of Müller and Matthews.

Fig 5.4a reproduces measurements of $I_{c \text{ rem}}$ versus H_{cool} (FCD) and H_{cycle} (ZFCD) of Müller and Matthews^[116] and the theoretical curves they obtained. The specimen is a thin slab of polycrystalline YBCO of thickness $W = 0.5\text{mm}$, width $Z = 2.5\text{mm}$ and length $Y = 13\text{mm}$, immersed in liquid N_2 at 77K. H_a is directed along the width Z . The preferred orientation of the grains with respect to the plane of the slab is not indicated.

The solid curves in Fig 5.4a display their theoretical calculations of $I_{c \text{ rem}}$ exploiting eqns 5.1, 5.2 and 5.3, hence a Josephson junction framework. The magnetization M_g of the grains is calculated using idealized slab geometry and the Bean approximation where J_{cg} is independent of the field strength. They introduce a lower critical field $\mu_0 H_{c1g} = 5\text{mT}$ in the calculation of M_g and take the full penetration field $\mu_0 H_{c2g} = 11\text{mT}$. Instead of writing $H_r = C M_g$, they introduce a demagnetization factor n_g in their development of M_g . More importantly they visualize that $I_{c \text{ rem}} = \sum \Delta I_{c \text{ rem}}$, hence arises from a summation of parallel elements of currents (“percolation paths”) each of which is

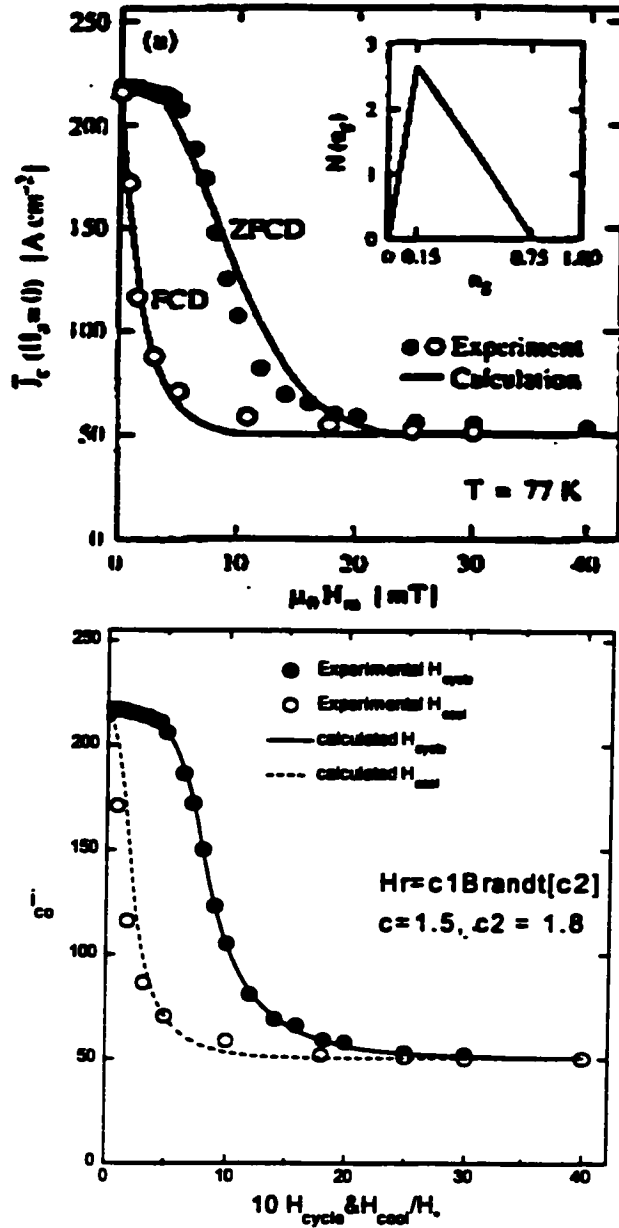


Fig 5.4. (a) Open and closed circles display measurements of $I_{c \text{ rem}}$ versus H_{cool} (FCD) and H_{cycle} (ZFCD) reproduced from Müller and Matthews^[116]. The specimen is a slab of YBCO of thickness $W = 0.5\text{mm}$, width $Z = 2.5\text{mm}$ and length $Y = 13\text{mm}$. The solid curves display their theoretical calculations of $I_{c \text{ rem}}$ exploiting eqns 5.1, 5.2 and 5.3. The insert displays $N(n_g)$, the distribution of demagnetization factor (n_g), for the grains. (b) Open and closed circles again display the above data of Müller and Matthews, the solid and dashed curves display the calculated $I_{c \text{ rem}}$ versus H_{cool} and H_{cycle} , exploiting eqns 5.10 and 5.12 with $n = 1$ and $H_0 = 0$. The return field $H_r = -CM_g$ is calculated using Brandt-Indenbom expressions, $M_g = I_{c0} \cdot F_B(C_2 H_g / I_{c0})$, where $C = 1.5$, $C_2 = 1.8$.

determined by a different demagnetization factor n_g for the grains. $N(n_g)$, the spectrum of distribution of demagnetization factors n_g is displayed in the insert to Fig 5.4a. The current density for the specimen is chosen to correspond to the measured value, $J_{co} = 218 \text{ A/cm}^2$.

Fig 5.4b again displays the data points of Müller and Matthews. Here the theoretical curves are calculated exploiting our critical state model for I_c (eqns 5.10 and 5.12) where we use only one component (percolation path) where $H_o = 0$ and $n = 1$. The return fields H_r were calculated using $H_r/I_{c*} = C_1 F_B(C_2 H_o/H_{g*})$ where F_B is given in appendix 4C with $C_1 = 1.5$, $C_2 = 1.8$. The adjustable parameters, I_{c*} , C_1 and C_2 are chosen to fit the data. I_{c*} however is a factor of ≈ 5 smaller than that observed.

Clearly both theoretical schemes lead to excellent qualitative and semi-quantitative descriptions of the observations. In both cases the quantitative agreement is achieved by appropriate choices for the adjustable parameters. We stress that when $J_c/J_{co} < 2/3$, both approaches are exploiting the same functional dependence for J_c versus the trapped flux density B_{rem} , namely $J_c \propto 1/B_{rem}$. Although we neglect H_{c1g} in our modeling we nevertheless achieve a good fit at low fields. The role played by the intricate distribution of demagnetization factors (see insert of Fig 5.4a) in the summation calculations of Müller and Matthews is unclear. The good fit we obtain using only one percolation path supports our view that a summation procedure (i.e. superposition of parallel paths) is not needed to describe the behaviour of $I_{c, rem}$ versus previous field-temperature histories (H_{cool} , H_{cycle} , etc).

C. Application to the Data of List et al.

List et al^[91] measured $I_{c \text{ rem}}$ versus H_{cycle} and H_{cool} at several temperatures for a Tl 1223 (i.e. $(\text{Tl}_{0.5}\text{Pb}_{0.5})(\text{Sr}_{0.8}\text{Ba}_{0.2})_2\text{Ca}_2\text{Cu}_3\text{O}_x$) polycrystalline slab of thickness $W = 40 \text{ um}$, width $Y = 0.6 \text{ cm}$, length $l = 5 \text{ cm}$ embedded in 40 um thick silver cladding. Typical grain sizes ranged from 1 to 10 um . The authors give no indication on the preferred orientation of the grains. H_a was directed // to the thickness W , hence pierced the flat faces.

Their data curves are reproduced in Fig 5.5a and b. The solid curves are not theoretical curves but guides to the eye. The salient features of these measurements is that $I_{c \text{ rem}}$ is clearly independent of the temperature when H_{cool} and H_{cycle} cause the trapped flux in the grains to reach saturation levels for each chosen temperature. The authors address this special behaviour in the framework that I_c is determined by Josephson junctions hence by eqns 5.1, 5.2 and 5.3. They then conclude that $J_{cm}(T)$ and $J_{cg}(T)$, hence $H_{cm}(T)$ and H_{cg} , must possess the same temperature dependence.

We now present their approach to this conclusion. In all cases in their experiment $I_{c \text{ rem sat}} < 0.637 I_{co}(T)$, when $I_{c \text{ rem}}(T)$ has descended to the plateau values $I_{c \text{ rem sat}}(T) \approx 8.6 \text{ A}$. Along the plateaus, $B_{\text{rem}}(T)$, the trapped flux density in the grains, has attained saturation values $B_{\text{rem sat}}(T)$.

Consequently in their Josephson junction model they are applying eqn 5.3a in the form,

$$I_{c \text{ rem sat}}(T) = \frac{I_{co}(T)\Phi_0}{\pi B_{\text{rem sat}}(T)(\mu_0 2\lambda(T)L)} \quad 5.14$$

in their analysis. Since they expect $\lambda(T)$ to be nearly constant over the temperature range 25 to 77K $< T_c \approx 116$ K of their specimen, they conclude that $I_{co}(T)$ and $B_{rem sat}(T)$ must have the same temperature dependence. However,

$$B_{rem sat}(T) \propto M_s(T) \propto H_{sg}(T) \propto J_{cgo}(T) \quad 5.15$$

Consequently, they conclude that the intergranular and intragranular current densities of their specimen have the same temperature dependence.

We now apply our critical state model in its simplest form to the description of their observations. The results of these calculations are displayed in Fig 5.5c and 5.5d. Here we have used eqns 5.10 and 5.12 in the following simple form with $n = 1$ and $H_0 = 0$.

$$\left(H_{rem} + \frac{I_c}{2}\right)^2 - \left(H_{rem} - \frac{I_c}{2}\right)^2 = 2H_{rem}I_c = K^2 = 2J_{cno}WH_1 \quad 5.16$$

valid when, $H_{rem} > I_c/2$, and,

$$\left(\frac{I_c}{2} + H_{rem}\right)^2 + \left(\frac{I_c}{2} - H_{rem}\right)^2 = 2\left(\frac{I_c}{2}\right)^2 + 2H_{rem}^2 = K^2 \quad 5.17$$

valid when, $0 \leq H_{rem} < \frac{I_c}{2}$.

Since $I_c = I_{co}$ when $H_{rem} = 0$, then eqn 5.17 leads to,

$$K^2(T) = \frac{I_{co}^2(T)}{2} = I_{co}^2(T) \quad 5.18$$

Introducing eqn 5.18 into eqn 5.16 and focusing again on the situations where $H_a = 0$ but the magnetization of the grains is saturated hence the return field exists at the saturation level, $B_{rem sat}(T)/\mu_0 = H_{rem sat}(T)$ and $I_{c rem}(T) = I_{c rem sat}(T)$ leads to,

$$I_{c rem sat}(T) = \frac{K^2(T)}{2H_{rem sat}(T)} = \frac{I_{co}^2(T)}{4H_{rem sat}(T)} = \frac{J_{cno}(T)WH_1(T)}{2H_{rem sat}(T)} = \frac{I_{co}^2(T)}{4CM_{gs}(T)} \quad 5.19a$$

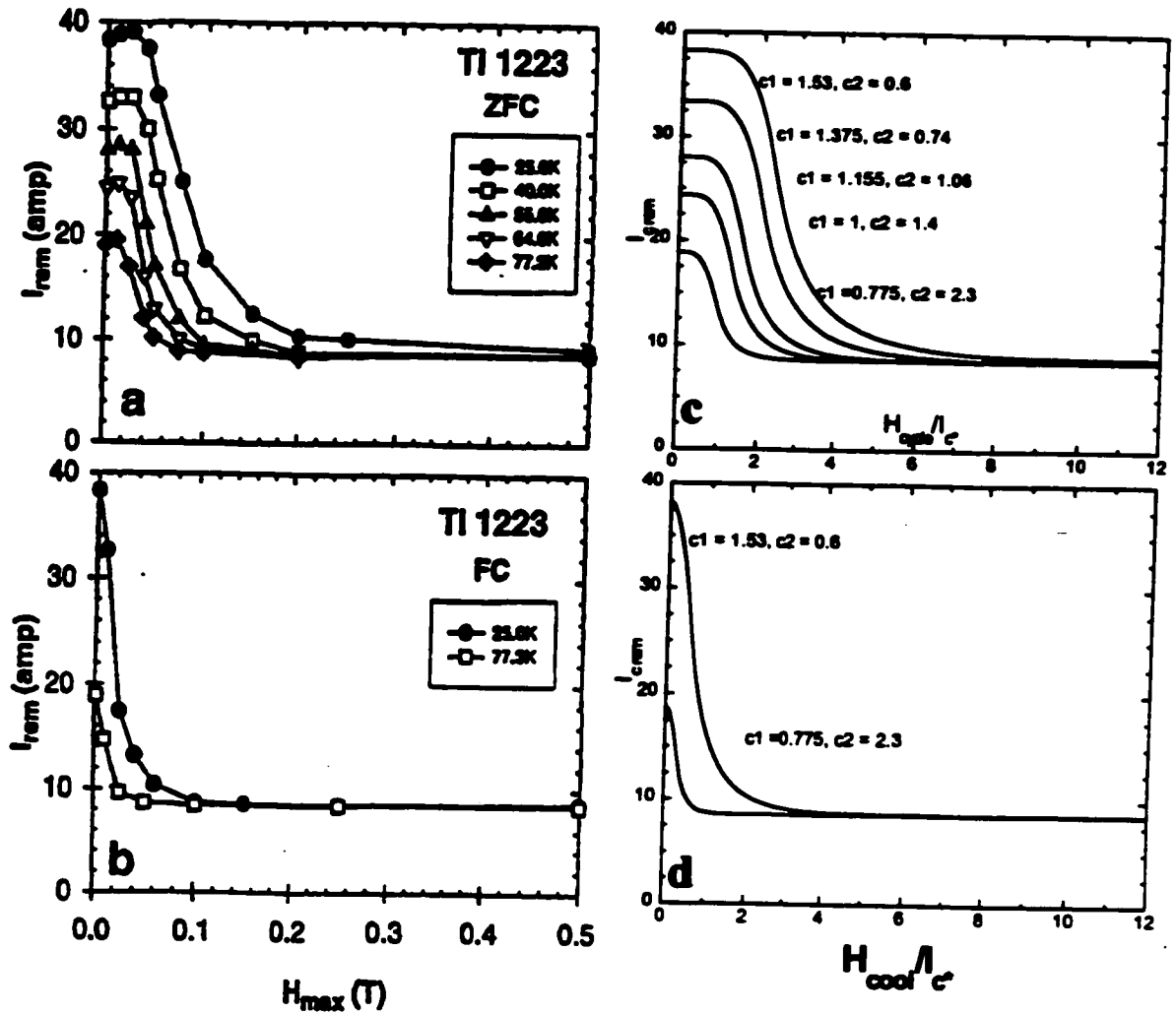


Fig 5.5. (a) Displays measurements of List et al^[91] for $I_{c \text{ rem}}$ versus H_{cycle} on Tl 1223 slab of thickness $W = 40 \mu\text{m}$, width $Y = 0.6 \mu\text{m}$, length $l = 5 \text{ cm}$ embedded in $40 \mu\text{m}$ thick silver cladding at 25, 40, 55, 64 and 77.3K. H_a was directed // to the thickness W . (b) Displays $I_{c \text{ rem}}$ versus H_{cool} for this sample at 25 and 77.3K. (c). Displays $I_{c \text{ rem}}$ versus H_{cycle} using eqns 5.16 and 5.17 and the Brandt-Indenbom formulae for $H_T = -C_1 M_g (C_2 H_T / I_c)$, where $C_1 = 1.53, 1.375, 1.155, 1, 0.775$, and $C_2 = 0.6, 0.74, 1.06, 1.4, 2.3$ from top to bottom curves. (d) Displays $I_{c \text{ rem}}$ versus H_{cool} with corresponding parameters as in (c).

Therefore in our critical state model which is similar in form to the popular Josephson junction framework (eqn 5.14) when we take $n = 1$ and $H_0 = 0$, we see that $I_{c \text{ remanent}}(T)$ will be independent of temperature if, $I_{co}^2(T)/2 = J_{co}(T)H_1(T)W_1$ has the same temperature dependence as $H_{\text{remnant}}(T) \propto C M_g(T) \propto C' H_{*g}(T)$. Consequently, the intergranular and intragranular critical current densities will display the same temperature dependence only if the reference field $H_1(T)$ in eqn 5.16 is taken independent of the temperature or if the product of the compression factor C and the grain magnetization has the same temperature dependence as the square of the conduction current of the virgin sample in zero field. The crucial difference between the Josephson junction and the critical state model arises from the important feature that the former introduces the ratio of the temperature independent flux quantum Φ_0 with respect to $\lambda(T)$, the penetration depth of the grains. However $\lambda(T)$ is nearly constant when T/T_c is fractionally smaller than unity.

Table 5.1 lists the values for $\mu_0 H_{\text{remnant}}(T)$ introduced in the calculation of the model curves displayed in Fig 5.5c and d. The values for $I_{co}(T)$ are experimental. To describe the variation of $H_{\text{rem}}(T,H)$ versus T , and the variations of H_{cool} and H_{cycle} , we have exploited the Brandt-Indenbom function $F_B(H_g/H_{*g})$, (see appendix 4C) hence write, $H_{\text{rem}} = C_1 M_g(T) = C_1 H_{*g}(T) F_B(C_2 H_g/H_{*g}(T))$ with $C_1 = 1$, $C_2 = 1$.

Table 5.1

T K°	$I_{co}(T)$ A	$\mu_0 H_{\text{remnant}}(T)$ mT
25	38.25	8.29
40	33.4	6.8
55	28	4.78
64	24.33	3.61
77.3	18.84	2.17

Since the specimen has a width = 0.6 cm, hence a “geometric” factor $G = 100/0.6$ cm is introduced, thus the critical currents per meter read,

$$I_{c, \text{theoretical}} = I_{c, \text{measured}} G = I_{c, \text{measured}} / 0.6(10^{-2}) \quad 5.20$$

In table 5.1, $\mu_0 H_{\text{return}}$ is calculated using the experimental I_{c0} and $I_{c, \text{return}} = 8.6A$, hence eqn 5.19a now reads,

$$\mu_0 H_{\text{return}}(T) = \frac{\mu_0 I_{c0}^2(T) G}{4 \{I_{c, \text{return}}(T)\}} \quad 5.19b$$

The dependences on temperature of the measured $I_{c0}(T)$ and of H_{return} (calculated from eqn 5.19b and listed in table 5.1) are displayed in Fig 5.6. The solid and dashed curves show $I_{c0}(T) = I_{c00} \left\{1 - \frac{T}{T_{cm}}\right\}^{3/4}$ and $H_{\text{return}}(T) = H_{\text{return}0} \left\{1 - \frac{T}{T_{cg}}\right\}^{3/2}$ where we let the critical transition temperature for the matrix, $T_{cm} = 110$ K, and for the grains, $T_{cg} = 110$ K. Here I_{c00} denotes I_{c0} at $T = 0$, and $H_{\text{return}0}$ denotes H_{return} at $T = 0$. It is interesting and gratifying to note that the temperature dependence of H_{return} for the grains which emerges from this analysis is similar to that calculated in chapter 4 from the position of the peaks of I_c for this specimen at the corresponding temperatures (see insert to Fig 4.7).

However, the magnitudes of $\mu_0 H_{\text{return}}$, the return field of the saturated trapped flux calculated using eqn 5.19 are smaller by an order of magnitude when compared with the saturation return fields obtained from the position of the peaks of I_c in chapter 4. This discrepancy ensues because we introduce the self-field of an idealized slab in our critical state model for I_c . The self-fields, based on the measured intergranular critical current densities $J_{cm}(H, T)$, are generally too small relative to the large applied fields in the

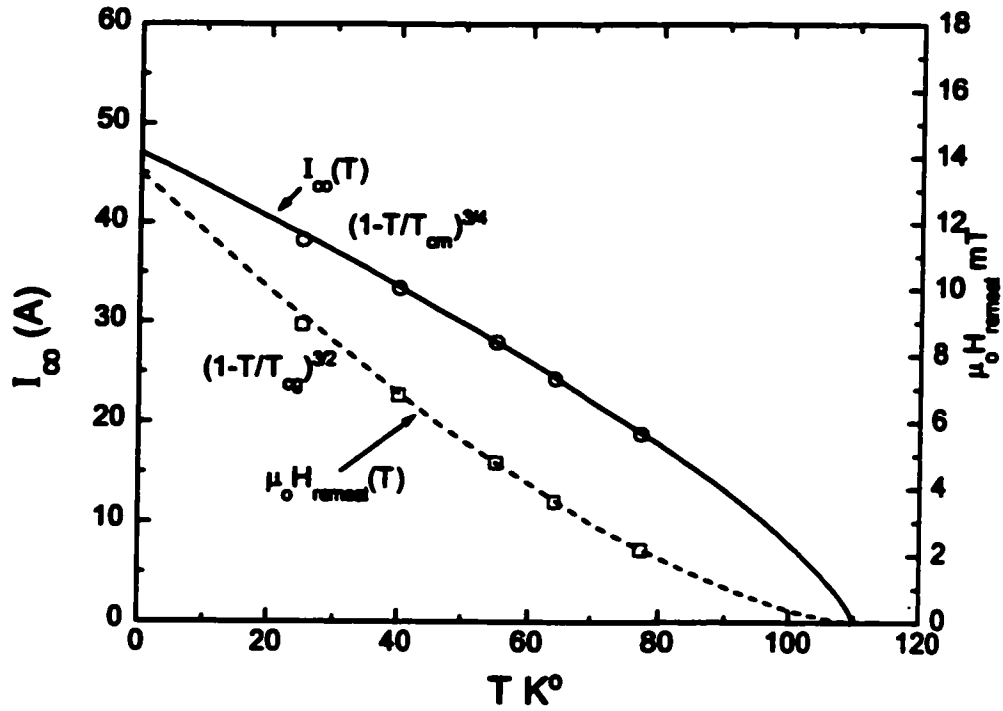


Fig 5.6. Open circles, o, display the experimental values for $I_{c0}(T)$, and the solid curve shows $I_{c0}(T) = I_{c00} \{1 - (T/T_{cm})\}^{3/4}$, where the transition temperature for the matrix reads $T_{cm} = 110\text{K}$. Open squares, $\square$, display the values for $\mu_0 H_{0 \text{ remnant}}$ (right hand scale) obtained by applying eqn 5.19b from our critical state model. The dashed curve displays $H_{0 \text{ remnant}}(T) = H_{0g0} \{1 - (T/T_{cg})\}^{3/2} = H_{0g}(T)$ where the transition temperature for the grains reads $T_{cg} = 110\text{K} = T_{cm}$. This curve displays the same convex downward structure as our results for $H_{0g}(T)$, shown in the insert of Fig 4.7, obtained from the position of the peaks in the I_c versus H_a curves. From their observations that $I_{c \text{ remnant}} = 8.6 \text{ A}$ at the five different temperatures of their measurements, and application of their Josephson junction based model, List et al conclude that $I_{c0}(T)$ and $H_{0g}(T) \propto H_{0 \text{ remnant}}(T)$ display identical temperature dependences.

experiments. The quantitative situation however is not improved, but indeed is worse, for the Josephson junction model since here List et al have to estimate $\lambda(T)$ to be two orders of magnitude smaller than generally observed to achieve agreement with their data of Fig 5.5.

It is of interest to point out that the initial flat segment in the calculated curves of $J_{c \text{ rem}}$ versus H_{cycle} in Fig 5.5c, which correspond to the nearly horizontal initial part of the measured curves in Fig 5.5a, occur even though, we have not introduced $H_{c1}(T)$ in the description of the magnetic response of the grains to H_{cycle} in our modeling. Workers generally attribute the initial horizontal segments to the exclusion of the applied field H_a from the grains by the Meissner shielding current $I_M(T) = H_a$ when $0 \leq H_a \leq H_{c1}(T)$. In our model the flat segments arise because the critical current density due to the pinning in the grains is very large when H_a is $< H_{c1}$, hence very little flux can penetrate into the grains, and thus only a very small amount of flux remains trapped in the grains when $H_{\text{cycle}} < H_{c1}$ is removed.

Yang, Beduz and Ashworth ^[156] have measured $J_{c \text{ rem}}$ and J_{co} at two different temperatures on two samples of YBCO fabricated differently. The values they obtain for $J_{c \text{ remast}}(T)$, after field cooling in H_{cool} sufficiently large to ensure that its removal causes the grains to contain a saturation amount of trapped flux, are listed in Table 5.2 together with the measured $J_{co}(T)$.

Table 5.2 Yang, Beduz and Ashworth ^[156]

	T (K)	$J_{c \text{ remast}}$ (A/m ²)	J_{co} (A/m ²)
Sample # 1	80	6	12.5
	65	15	80
Sample # 2	65	70	240
	4.2	136	1500

We note that Yang, Beduz and Ashworth find $J_{c \text{ remax}}$ to increase appreciably in both specimens when their temperature is decreased, in sharp contrast with the behaviour encountered by List et al in their T1 1223 sample where $I_{c \text{ remax}}$ is found independent of the temperature.

The prevailing Josephson junction model to describe the behaviour of the conduction current in granular superconductors arbitrarily takes $I_c \propto 1/H$ when $I_c < 0.637I_{co}$. In the above applications of our critical state model to the $I_{c \text{ rem}}$ data of Müller and Matthews and List et al we have focused on the case where $n = 1$ and $H_0 = 0$ in eqn 5.10, hence $I \propto 1/H$, in order to compare apples with apples so to speak. For completeness we now indicate that different choices of n in our eqn 5.10, hence different dependences of the intergranular critical current density J_{cm} on H will lead to correspondingly different relationships between $I_{c \text{ remax}}(T)$, $I_{co}(T)$ and $H_{\text{remax}}(T)$. To illustrate this aspect we now focus on the frequently encountered case in the analysis of experiments where $n = 1/2$ and $H_0 = 0$. Eqn 5.10 now leads to,

$$\left(H_{\text{rem}} + \frac{I_{c \text{ rem}}}{2} \right)^{3/2} - \left(H_{\text{rem}} - \frac{I_{c \text{ rem}}}{2} \right)^{3/2} = 2 \left(\frac{I_{co}}{2} \right)^{3/2} \quad 5.21$$

valid when, $(I_{c \text{ rem}}/2) < H_{\text{rem}}$. Since in these experiments, $I_{c \text{ rem}}/2 H_{\text{rem}} < 1$ we can exploit the expansion of $(1+x)^{3/2}$ and $(1-x)^{3/2}$ where $x = I_{c \text{ rem}}/2 H_{\text{rem}}$. This leads to,

$$I_{c \text{ rem}}(T) \approx \frac{\sqrt{2}}{3} \frac{I_{co}^{3/2}(T)}{H_{\text{rem}}^{1/2}(T)} \quad 5.22$$

It is instructive to compare this simple result with eqns 5.14 and 5.19(a).

D. Application to the Data of Kwasnitza and Widmer

Kwasnitza and Widmer^[83] measured the evolution of the critical current of slab specimens of YBCO at 4.2K in zero field as the amount of net trapped flux in the grains is made to diminish from a saturation value of a chosen polarity, go through zero, and then augment to the saturation magnitude in the opposite direction. The locus of the measured magnetization curves of the specimen corresponding to the magnetic field-temperature histories which precede their measurements of $I_{c,rem}$ at $H_a = 0$, are reproduced in Fig 5.7a. Fig 5.7b illustrates the variety of flux density profiles $B(r)$ trapped in a grain, of idealized cylinder or slab geometry, at $B_a = 0$ after the applied field sweeps displayed in Fig 5.7a to different values of B_r . These are denoted B_{s1} , B_{s2} , B_{s3} and B_{s4} . For completeness and comparison, in Fig 5.7b, c and d, we have also sketched the trapped flux density profiles which are visualized to occur after the H_{cycle} and H_{cool} procedures examined in the previous section.

$I_{c,rem}$ versus the magnitude of the reversal field, denoted B_s , observed by Kwasnitza and Widmer, is displayed in Fig 5.8a. Kwasnitza and Widmer have not published any modeling of their experimental curves (Fig 5.7a or 5.8a). The theoretical curve we calculated for $I_{c,rem}$ versus $H_s = B_s/\mu_0$ is displayed in Fig 5.8b. Here we used eqn 5.10 and 5.12 with $H_0 = 0$ and $n = 1$, hence the simple form given by eqn 5.16 and 5.17. The evolution of the magnetization of the grains and the associated return fields are again assumed to be described by the Brandt-Indenbom expressions listed in appendix 4C hence we write, $H_r = C M_{0g} = C_1 H_{0g} F_B(C_2 H_{0g}/H_{0g})$, and take $C_1 = 1$, $C_2 = 2$, and let $H_{0g} = I_{c0}$.

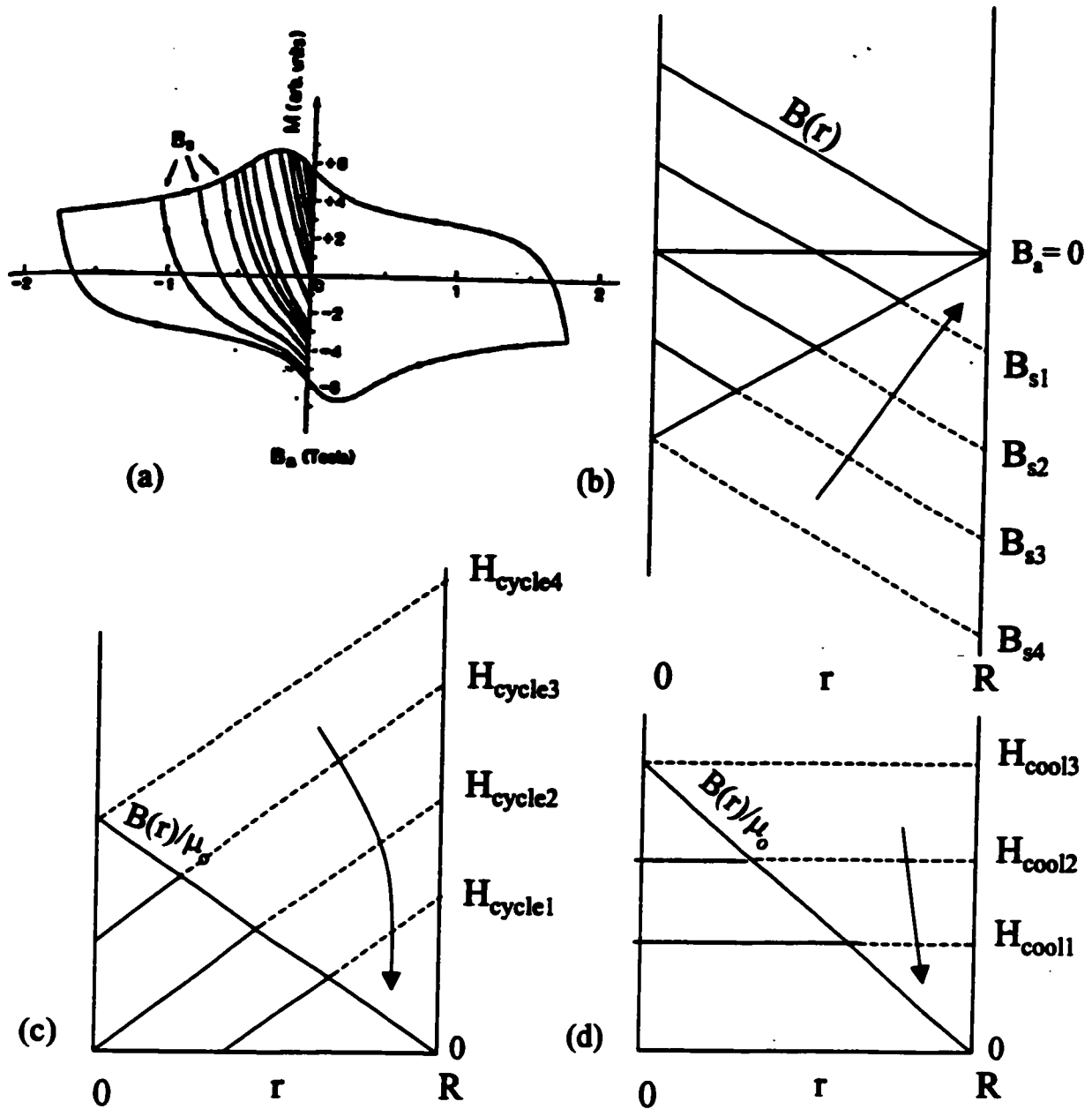


Fig 5.7. (a) Reproduced from Kwasnitza and Widmer^[83], and displays their measurements of the locus of the magnetization M , of their YBCO slab specimen at 4.2K, when, with M initially tracing the envelopes of the major hysteresis loop, the direction of the sweep of the applied magnetic field is reversed at values, denoted B_s , and “descends” in magnitude to zero. (b) Illustrates the flux density profiles in an idealized grain at different values of B_s along the major hysteresis envelope (solid-dashed lines), and after B_s is removed (solid-solid lines). Note that when $|B_s|$ is small (eg. B_{s1}), the trapped flux at $B_s = 0$ comprises two regions of opposite polarity. (c) and (d) Display again for comparison and completeness the variety of field profiles encountered at H_{cycle} , and H_{cool} and after their removal.

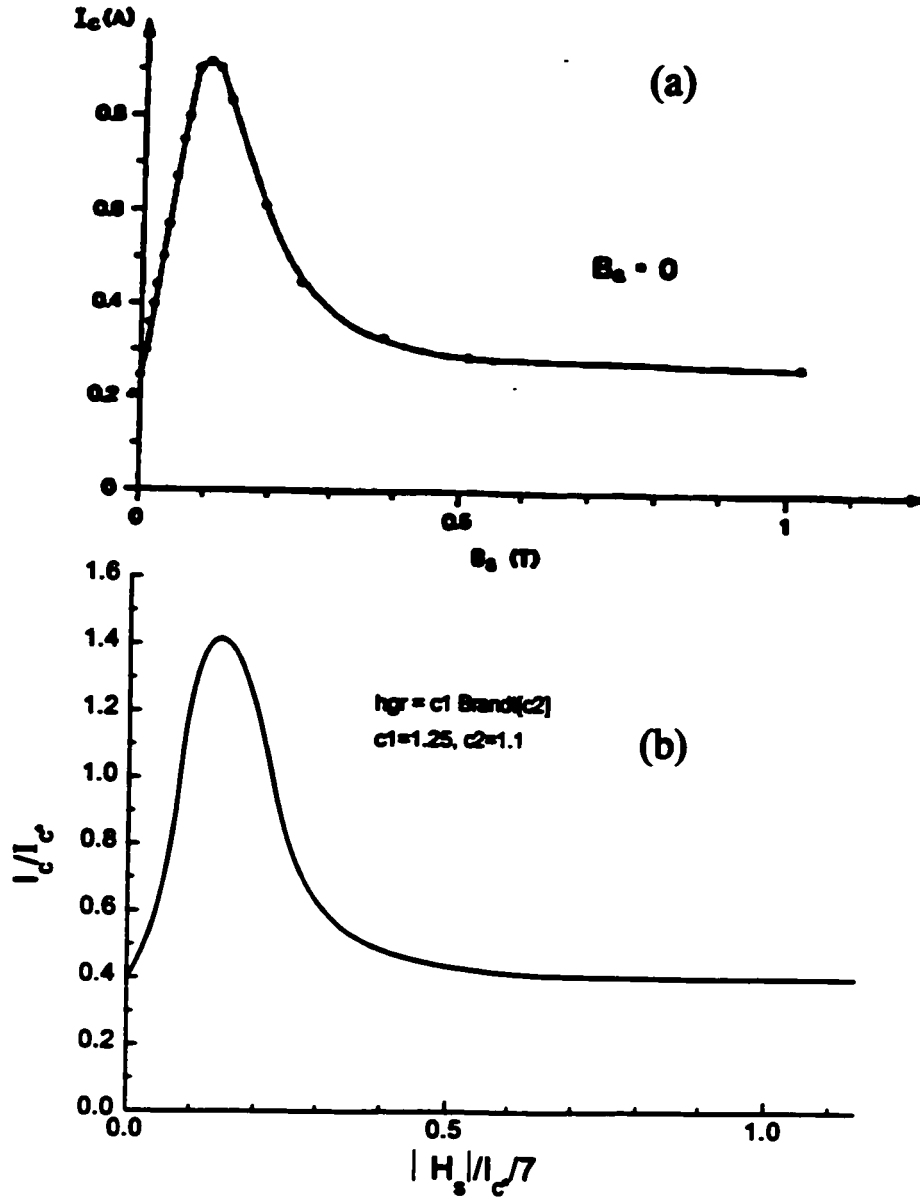


Fig 5.8. (a) Reproduced from Kwasnitza and Widmer^[83], and displays their measurements of I_c at $B_s = 0$ after a reversal of the applied magnetic field at B_s and its removal as shown in Fig 5.7(a). (b) Displays I_c/I_{c0} calculated using eqns 5.10 and 5.12 with $n = 1$, $H_0 = 0$ and the Brandt-Indenbom expressions for the magnetization with $C_1 = 1.25$ and $C_2 = 1.1$. Here $B_s = \mu_0 H_s$ is normalized to $\mu_0 I_{c0}$ and divided by 7 to "match" the numbers from the applied field in the experiments.

Clearly, the model generates a good quantitative description of the observed I_c versus B_z data curves. The quantitative fit however is far from satisfactory since the measured critical current and corresponding critical current density, $J_{cno} \approx 10^6 \text{ A/m}^2$, leads to a self-field which is some three orders of magnitude smaller than that envisaged for the idealized infinite slab specimen.

(iv) Valley in ZFCV Curves of I_c versus H_a Ascending

A. Introduction

Many workers^[36, 40, 50, 93, 102, 156] have observed that the curves of I_c versus the first application of H_a after zero field cooling, hence I_c denoted ZFCV, trace a valley. Some examples of this type of behaviour are reproduced in Fig 5.9A. In many other measurements I_c versus H_a (ZFCV) displays an inflection "point" instead of a valley. This feature is displayed in Fig 5.9B (e,f).

Fisher et al^[50] developed a mathematically complicated model which, in effect, attributes the occurrence of the valley, and the peak which follows it, to the return field associated with the reversible diamagnetic Meissner response of the grains to H_a . Consequently in their model the bottom of the valley appears when $H_a = H_{c1}$ hence $\log(H_a/H_{c1}) = 0$. The curves of Fig 5.9B(g) are reproduced from their paper and display the predictions of their model for different values of H_{c1} . These curves are intended to reproduce their data curves for I_c (ZFCV) Fig 5.9A(a) and (b). The valley bottom however appears in their measurements when $\mu_0 H_a \approx 10 \text{ mT}$, and $T = 77\text{K}$ hence at values somewhat larger than H_{c1} at that temperature. Further we note that, along ZFCV curves of I_c , the magnetic field experienced by the grains, $H_s = H_a + H_r$, will be

appreciably stronger than H_a , in the range $0 < H_a < H_{c1}$, since here the Meissner surface shielding currents will be most effective in generating return fields appreciably enhancing H_a .

B. Generation of the Valley in I_c (ZFCV) Curves.

We now apply our critical state model to the description of this valley phenomenon. The results of this exercise are displayed in Fig 5.10A and B for direct sequential comparison with the corresponding family of experimental curves presented in Fig 5.9A and B. We now describe the modification and extension of our critical state model which leads to these results. Our approach is similar to and broadens the model developed by Mishra et al^[102]

Until now in this thesis we have, for simplicity, described the magnetization of the grains, $M_g(H_a)$, using the Bean approximation for the intragranular current density, hence taken J_{cg} to be independent of H_a .

We believe that the valley in the intergranular critical current I_c (ZFCV) curves arises because in some specimens the intragranular current density J_{cg} diminishes as a function of the flux density inside the grains. We now address these situations.

Expressions for the sequences of critical state flux density profiles $B_g(r) = \mu_0 H_g(r)$, for idealized slab and cylinder geometry, as H_a is impressed to virgin (zero field cooled) grains and subsequently removed, are developed in appendix 5.D for Kim-like dependences of J_{cg} on H where $J_{cg} = J_{c0} H_1^p / (H_0 + H)^p$.

The behaviour of the magnetization $M_g(H_a)$ which ensues from these expressions for various values of p , where for simplicity $H_0 = 0$, are displayed in Fig 5.11a. The superpositions of $H_t = H_a + H_r(H_a) = H_a + CM_g(H_a)$ displayed in Fig 5.11b clearly can

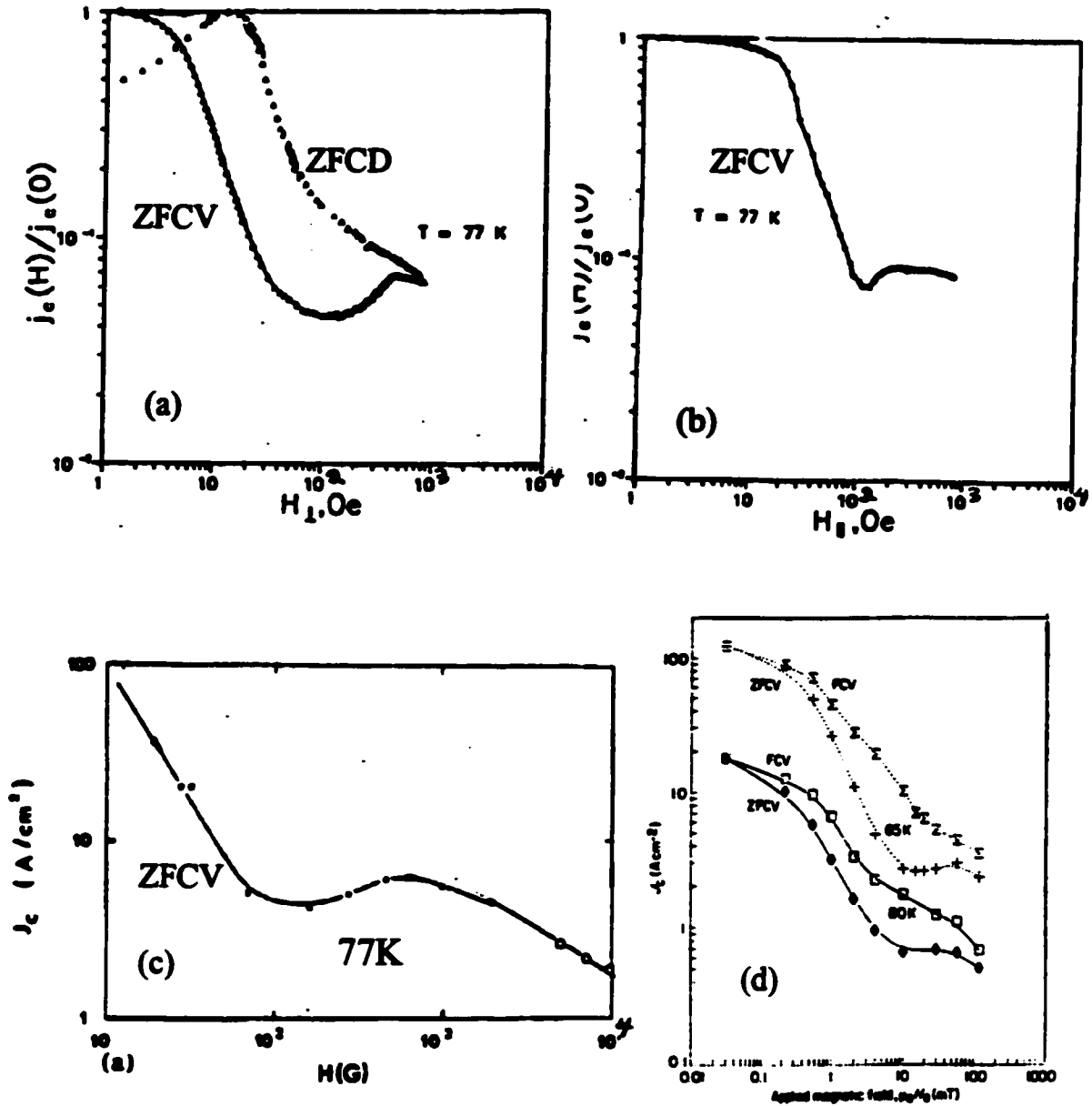


Fig 5.9A. Displays measurements of j_c versus H_a , both on logarithmic scales, where in (a), (b) and (c) a deep valley and in (d) a shallow valley or an inflection point are evident in the ZFCV curves. (a) and (b) are reproduced from Fisher et al^[50] where $H_a \perp$ and $\parallel$ to j_c in (a) and (b). (c) Is reproduced from Uri Dai et al^[36] and (d) from Yang et al^[156].

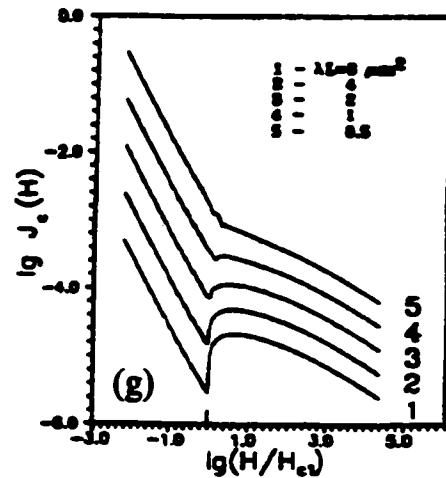
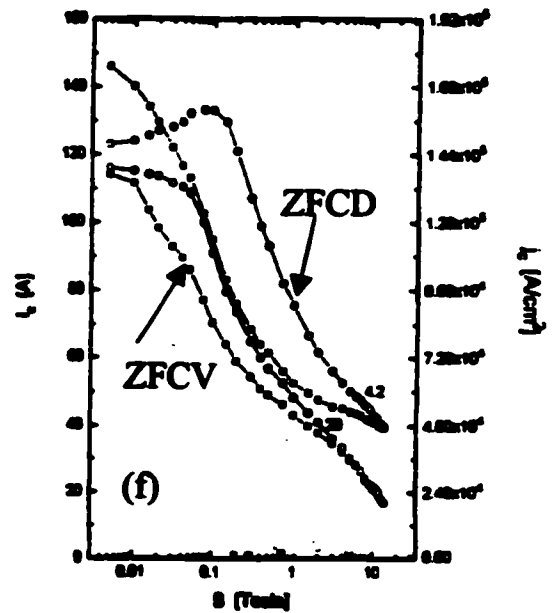
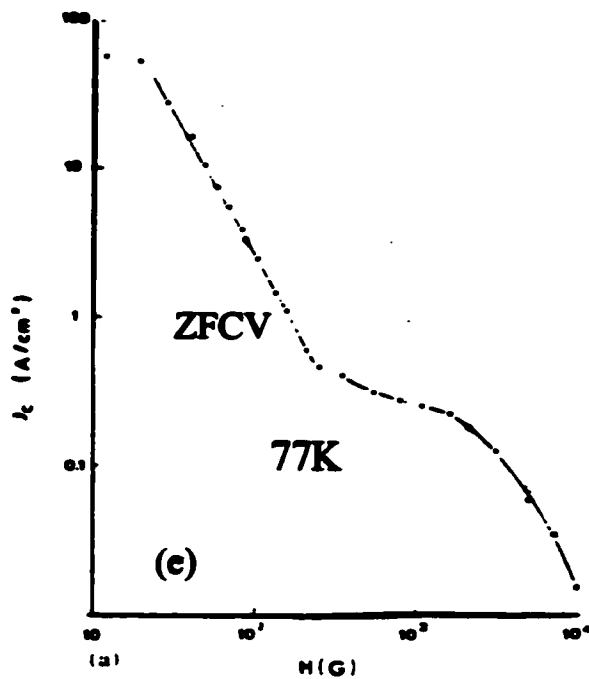


Fig 5.9B. (e) Reproduced from Uri Dai et al^[35]. Displays an inflection point in a ZFCV curve at 77K in YBCO. (f) Reproduced from Grasso et al^[59]. Upper (lower) • data points display ZFCV, and o display FCV curves at 4.2 (20)K in YBCO. (g) Theoretical curves reproduced from and generated by the $H_{c1}(T)$ based model of Fisher et al^[50] to account for their ZFCV curves displayed in our Fig 5.9A (a) and (b).

Fig-6 The calculated dependences of critical current on magnetic field at different values of product λL .

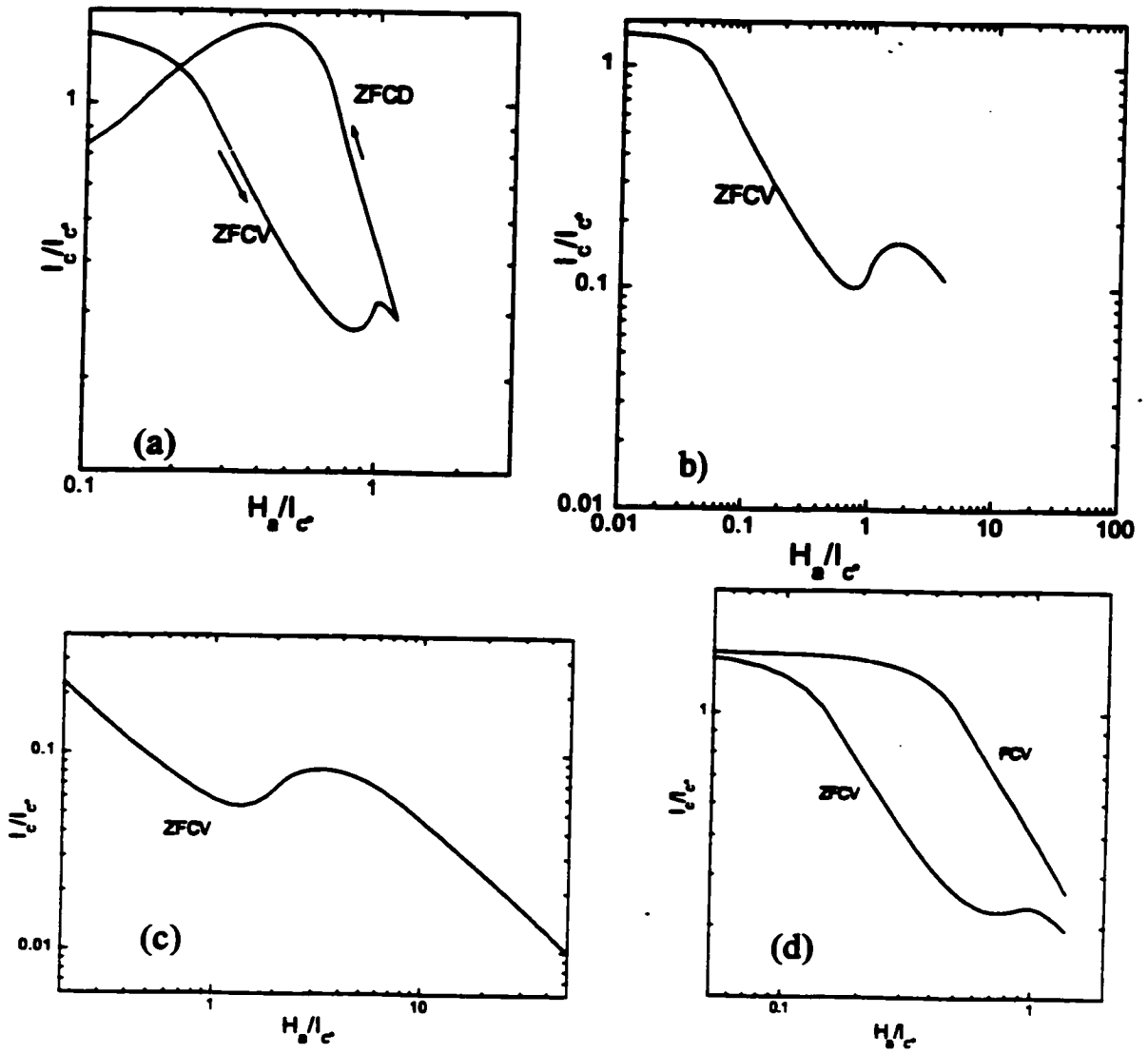


Fig 5.10A

Figs 5.10A and B. display theoretical curves generated by our model for comparison with the corresponding (a), (b), (c), (d), (e), and (f) of Fig 5.9A and B. Eqns 5.10 and 5.12 are used for I_c with $H_0 = 0$ in all cases. The value for n is indicated below. The intergrain critical current densities are also described by Kim-like expressions, namely $J_{cg} = J_{c0} H_1^p / (H_0 + H)^p$ where we write p to avoid confusion with the exponent n for the matrix current density.

(a) $p = 3, n = 1, C = 0.2$

(b) $p = 1, n = 1, C = 3$

(c) $p = 1, n = 1, C = 2, C_2 = J_{c0} X_g / I_{c0} = 5$

(d) $p = 1, n = 1, C = 0.5$

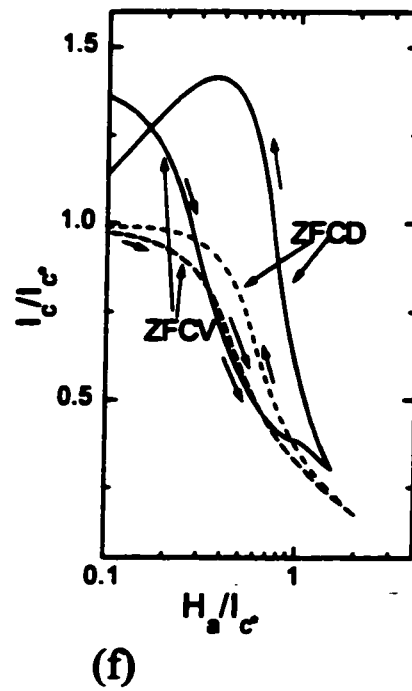
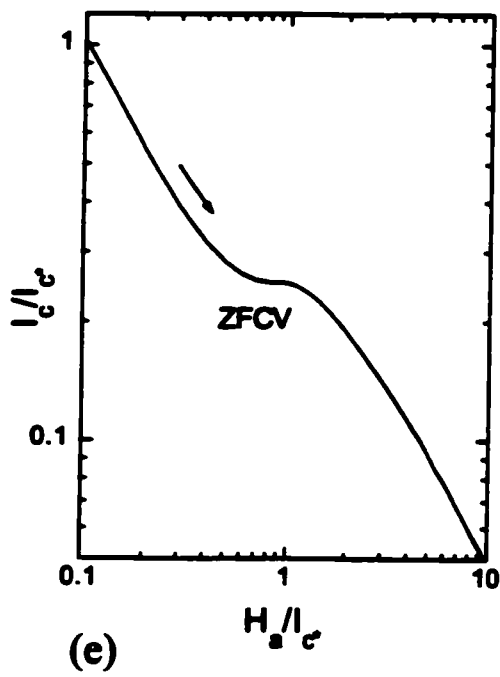


Fig 5.10B

- (e) $p = 0.5, n = 1, C = 1$
 (f) $p = 1, n = 1, C = 0.3, 0.07$

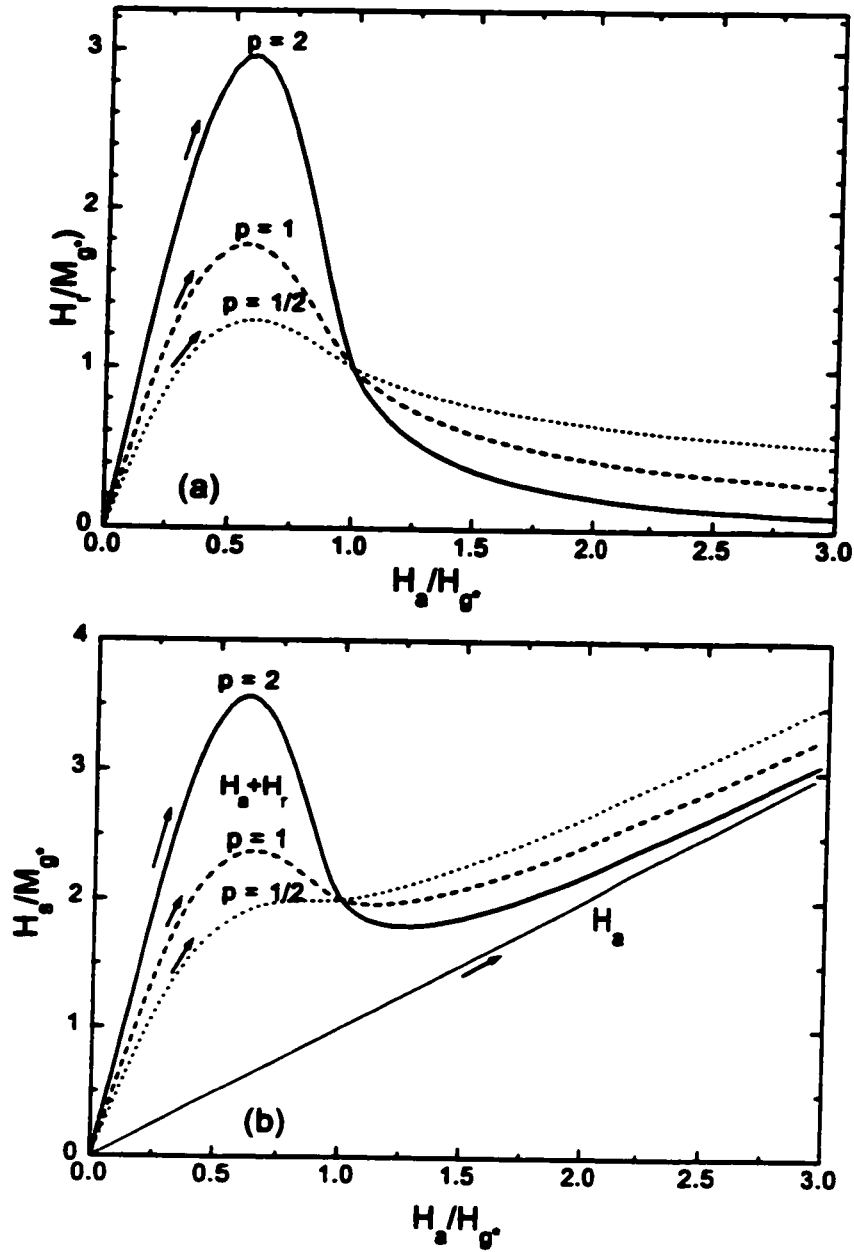


Fig 5.11. (a) Displays $H_t = C M_g(H_s/H_g)$ versus H_s/H_g for various values of p , with $C = 1$, where $M_g(H_s/H_g)$ is calculated using $J_{cg} = J_{cg} H_1^p / (H_0 + H)^p$ (see appendix 5.D). H_t and M_g are normalized with respect to M_g , the magnetization of an idealized slab or cylinder when $H_s = H_g$. Consequently all the curves for slab and cylinder geometry intersect at $H_s/H_g = 1$ when $H_t/H_g = 1$. (b) Displays $H_s = H_s + H_t(H_s)$ with $C = 1$ where H_t from (a) is introduced. Here all the curves for slab and cylinder geometry intersect at $H_t/M_g = 2$ when $H_s = H_g$.

generate a variety of curves of I_c versus H_a when the intergranular (matrix) critical current density is taken as previously also of the form $J_{cm} \propto 1/(H_o + H)^n$, where $n > 1$.

For completeness and comparison in Fig 5.10A(d) we have also shown an I_c versus H_a curve predicted to be encountered when the specimen is field cooled in the chosen static H_a before each measurement of I_c . These curves are denoted field cooled virgin curves (FCV). In our calculation of the FCV curve we have regarded the grains as not magnetized, hence taken $H_r = 0$, and write $H_a = H_o + H_r = H_o$. This approximation is justified since, during field cooling, the strong flux pinning in the grains opposes the Meissner expulsion of the flux. Hence the resulting diamagnetic magnetization of the grains is much smaller than the idealized Abrikosov reversible value M_{rev} . Our approach is self consistent since we have also neglected any contribution of M_{rev} in our calculations of M_g for the ZFCV and ZFCD curves.

(v) Width of the peaks in I_c versus H_a curves

All published observations of the low field peaks in the I_c versus H_a curves show common features and trends which we now enumerate.

- a) The peaks of I_c for ZFCD curves are wider than the peaks for the corresponding ZFCreAs curves.
- b) The peaks of I_c for FCD curves are wider than the peaks for the corresponding FCreAs curves.
- c) The width of the all four types of peaks increases to a maximum as H_{cycle} or H_{cool} are chosen larger. The maximum widths are attained when H_{cycle} and H_{cool} have reached threshold "plateau" values where the migration of the location of the peaks ceases.

- d) The ratio of the width of the I_c peak of the ZFCD (FCD) curves to the width of the corresponding peak in the ZFCrAs (FCrAs) curves increases to a maximum as H_{cycle} (H_{cool}) increases to the threshold value.

Some of these features are illustrated schematically, via theoretical I_c versus H_a curves in Figs 5.12 and 5.13.

The curves of H_r versus H_a in Fig 5.12 and 5.13 are displayed below the corresponding I_c versus H_a curves to help in showing why the behaviours listed above are a direct consequence of the Evetts - Glowacki concept that H_s , the superposition of the return field H_r of the magnetized grains with the applied field H_a , controls the intergranular critical current I_c . In these figures, I_c , H_r and H_a are normalized to the same quantity I_{c0} , and $H_r = C M_g$ was calculated for idealized cylindrical grains with $C = 1$. We let $n = 1$ and $H_0 = 0$ in the expressions for I_c .

We note that the difference between H_a and H_r for any pair of intersecting curves displays $H_s = H_r - H_a$ versus H_a . We stress that it is the magnitude of H_s which determines $I_c(H_s)$, and that I_c traces a peak when $\pm H_s$ straddles $H_s = 0$ hence $H_a = H_r$. Careful examination of the range of variation of $|H_s|$, spanning the intersections of the H_r and H_a curves, hence the locations where $H_s = 0$, over corresponding ranges of H_a as H_{cycle} or H_{cool} are chosen larger together with inspection of the width of the corresponding peaks of I_c will show that the trends and features enumerated above are indeed qualitatively predicted by the Evetts - Glowacki model.

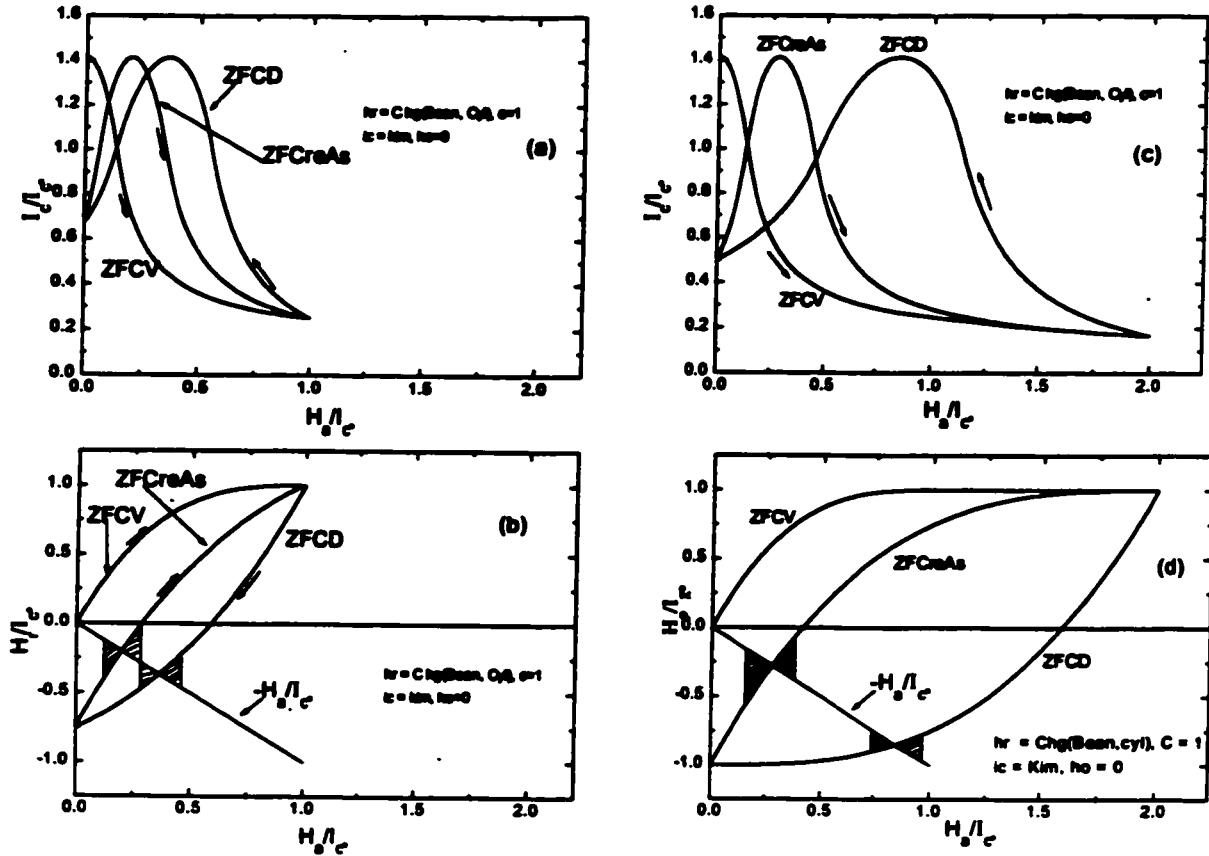


Fig 5.12. (a) and (b) illustrate the observed feature that the width of the peak of the ZFCD curve is greater than that of the corresponding ZFCrAs curve and also that as the reversal field H^{cycle} is made larger, both these peaks get fatter. The widening ceases when H^{cycle} exceeds a threshold value (see appendix 4C). (a) and (d) illustrate why the above behaviour occurs. Since $H_y = 0$ at the intersections of the curves, the changes here of H_x , indicated by the height of the shaded triangles, is clearly greater for the ZFCrAs case than for the ZFCD case hence the former peak in I_c is sharper than the latter. The narrowness of the peaks is then quantitatively determined by the sensitivity of I_c on H_x .

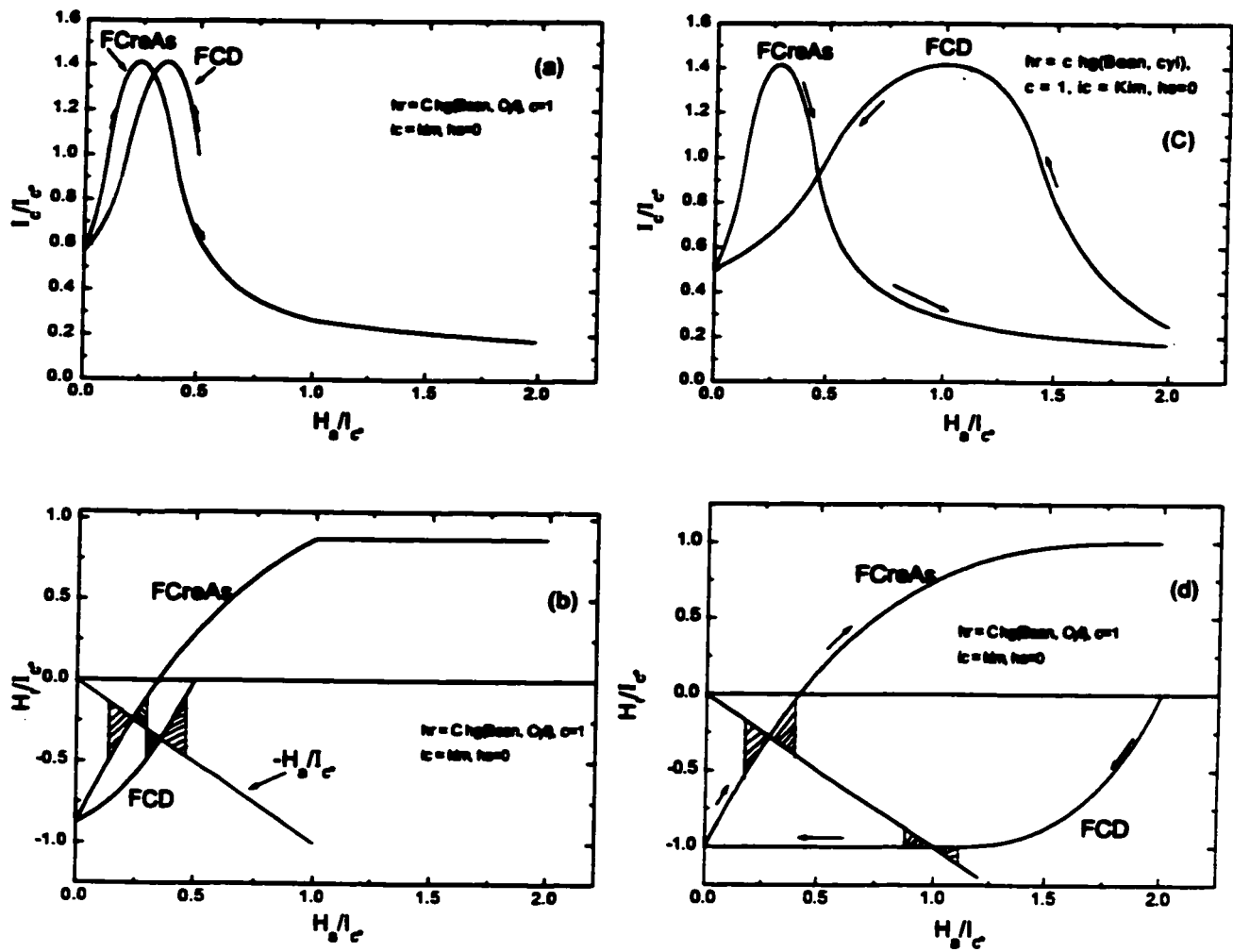


Fig 5.13. Same as caption for Fig 5.12 except that here we focus on the behaviour of the peaks of I_c versus H_s for the FCD and FCDs situations

(vi) Determination of H_r from Hysteretic I_c Curves

$H_r(H_a, T)$, the effective return field of the magnetized grains is the important new quantity which can be extracted from measurements of I_c hysteresis curves by exploiting the Evetts - Glowacki concept that the superposition of H_a and $H_r(H_a, T)$ dictate $I_c(H_a, T)$. First we review the schemes which have been exploited by various workers then we present a framework embracing and extending these methods.

(a) Review of Methods Used.

In section III(iii) of this chapter we examined the relationship between $I_{c \text{ rem}}$ the critical current in $H_a = 0$ and H_{rem} the return field of the residual magnetization of the grains. Although H_{rem} depends on the magnitudes of H_{cycle} or H_{cool} , experiments show that H_{rem} then denoted $H_{\text{rem sat}}(T)$, is independent of H_{cycle} or H_{cool} when the latter quantities exceed the temperature dependent threshold values. Consequently we can obtain an estimate of $H_{\text{rem sat}}(T)$ from the measurements of the threshold value for H_{cycle} or H_{cool} . However, the asymptotic dependence of $I_{c \text{ rem}}$ on H_{cycle} and H_{cool} introduces large uncertainty in the determination of these threshold values. Further the link between $H_{\text{rem sat}}$ and the threshold values for H_{cycle} and H_{cool} is dependent on the choice for the function $J_{cg}(H)$ and the geometry of the grains.

We have seen in chapter 4 that the location of the peaks of I_c versus H_a descending (ZFCD or FCD) provides a direct and unambiguous measurement of H_r since the centre of a peak indicates that here $H_a = H_r$. Therefore from the measurements of the sequences of peaks as a function of H_{cycle} or H_{cool} at a chosen temperature we can map out the evolution of $H_r(H_a, T)$ from $H_r = 0$ to $H_r = H_{\text{r sat para}}$, the return field for the saturation

paramagnetic magnetization of the grains. However this approach leads to only one value for $H_{r \text{ sat para}}$, namely, the plateau value we have discussed in chapter 4 where,

$$H_{r \text{ sat para}}(T, H_a) = H_a = H_{\text{plateau}}^{\text{ZFC}} = H_{\text{plateau}}^{\text{FC}} \quad 5.23$$

Other methods must then be pursued to determine $H_{r \text{ sat para}}(T, H_a)$ over the entire range of H_a both above and below the plateau value. Also it is of interest to determine $H_r(T, H_a)$ versus H_a for the full range of H_a for the situations where the grains are diamagnetically magnetized to saturation, which we denote as $H_{r \text{ sat dia}}(T, H_a)$.

Yang, Beduz and Ashworth^[156] devised and exploited a direct and unambiguous method for measuring $H_{r \text{ dia}}(H_a, T)$, the return field for diamagnetically magnetized grains, as a function of H_a ascending (ZFC). This approach can also determine $H_{r \text{ sat dia}}(T, H_a)$, the saturation value versus H_a and T . This scheme can readily be understood from consideration of Fig 5.14a. Here, the central dot on the cross indicates a critical current, denoted I_{c2} , measured after field cooling in H_a , denoted H_2 . Since the magnetization of the grains is negligible after field cooling, the return field H_{r2} is also negligible, then $H_{a2} = H_2 + H_{r2} \approx H_2$. The dot on the left of the cross indicates a critical current $I_c = I_{c2}$ on the ZFCV curve in an applied field denoted H_1 . Since these two data points have the same I_c , the superposition field, $H_{a1} = H_1 + H_{r1}(H_1)$ should be the same as $H_{a2} = H_2$ regardless of the functional dependence of I_c on H_a . Consequently we can write,

$$H_{a1} = H_1 + H_{r1}(H_1) = H_{a2} = H_2 \quad 5.24a$$

hence,
$$H_{r1}(H_1, T) = H_2 - H_1 \quad 5.24b$$

Pursuing this scheme, Yang Beduz and Ashworth^[156] obtained the results reproduced in our Fig 5.15a from their data curves displayed in our Fig 5.9d. Presumably the grains are

diamagnetically magnetized to saturation over the range of large applied magnetic fields hence here,

$$H_e = H_a + H_r \approx \mu_0 M_s(H_a, T) \quad 5.25$$

It is evident from inspection of their data (Fig 5.15a) that they measure very large values for the ratio of the local field, $H_e = H_a + H_r$, with respect to H_a , hence large compression factors, $C = H_r/M_s(H_a) \approx H_r/H_a$.

Exploiting an AC susceptibility technique to measure the penetration of an alternating magnetic field into the matrix, and into the grains, after field cooling and zero field cooling, Yang, Beduz and Scurlock^[157] determined $B_j = \mu_0 H_e$, the effective field in the intergranular medium versus H_a at various temperatures just below T_c in an YBCO specimen. Their results are reproduced in Fig 5.15b. Note that the ratios $B_j/\mu_0 H_a$ are ≈ 10 , hence the compression ratios are correspondingly large, and saturate at the same value in the temperature range examined.

Jones et al^[72] extended the approach of Yang, Beduz and Ashworth^[156] and measured I_c versus H_a (ZFCV, FCV and ZFCD) curves for an YBCO bar of dimensions 1X4X12 mm³, supporting $J_{co} = 1.5(10^5)$ A/m² at 77K. They studied three arrangements for the orientation of H_a with respect to the surfaces of the ribbon and the current.

- (i) $H \parallel$ to I_c and along the length $Y = 12$ mm,
- (ii) $H \perp$ to I_c and along the width $Z = 4$ mm, and
- (iii) $H \perp$ to I_c and along the thickness $X = 1$ mm, hence piercing the flat YZ surfaces of the slab. Further they measured the magnetization of the ribbon with H_a directed along its length Y as H_a is applied to the virgin specimen (ZFCV) and then removed (ZFCD), and

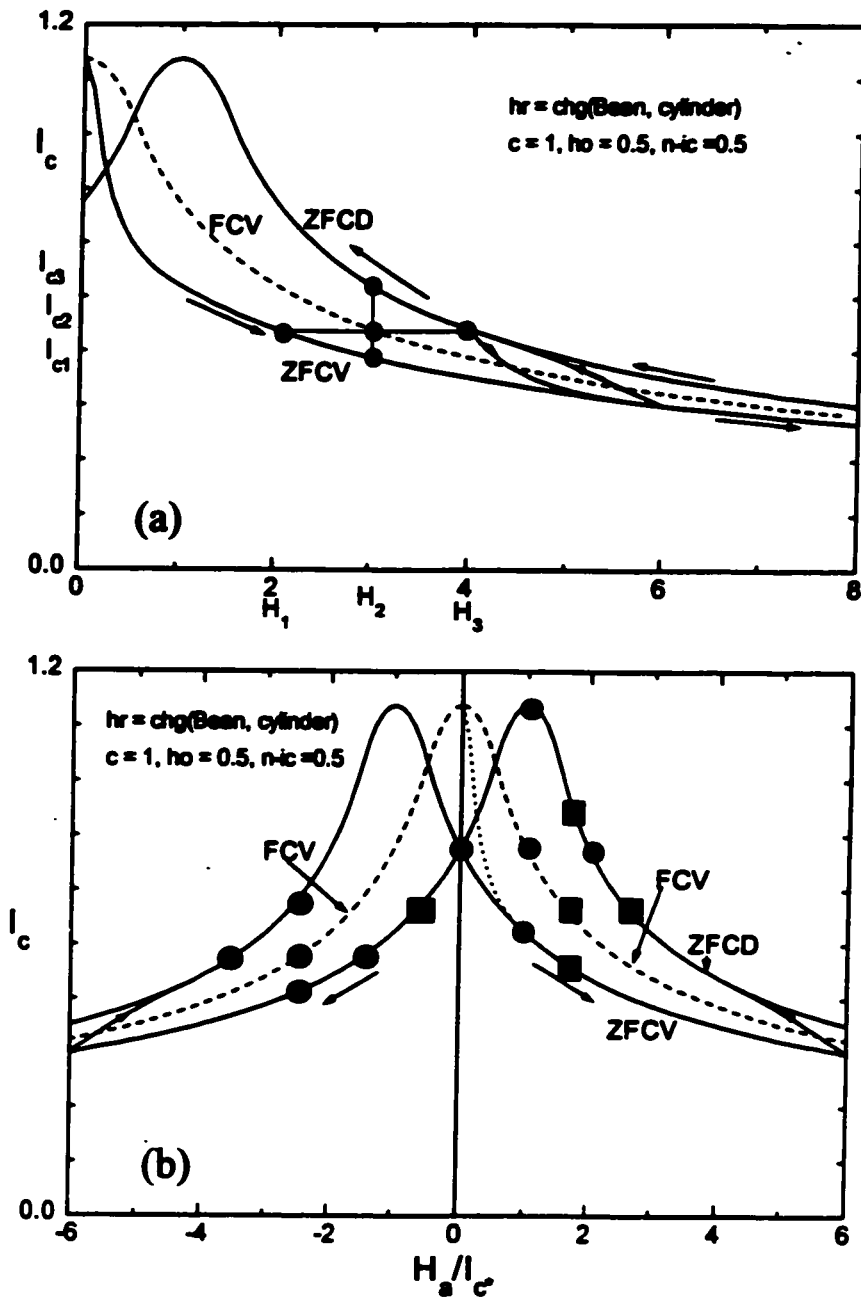


Fig 5.14. Displays two different approaches to extracting values for H_T from the family of curves of I_c versus H_a . (a) Any chosen I_c (eg. I_{c2} indicated by three horizontal dots) is encountered at three different fields, H_1 along ZFCV, H_2 along FCV and H_3 along ZFCD. Assuming a negligible Meissner effect for the grains upon field cooling, $H_{a2} = H_2$ along FCV, $H_{a1} = H_1 + H_{r1}$ along ZFCV, and $H_{a3} = H_3 - H_{r3}$ along ZFCD. (b) Illustrates that the sets of three "horizontal" and "vertical" data points used for determinations of $H_T(H_a)$ can be selected at any place along the major symmetric I_c hysteretic butterfly wings. Hence the saturation diamagnetic and paramagnetic values for H_T can be determined since along these envelopes, $M_p(H_a)$ exists with the induced currents all circulating in the same + or - sense in the grains.

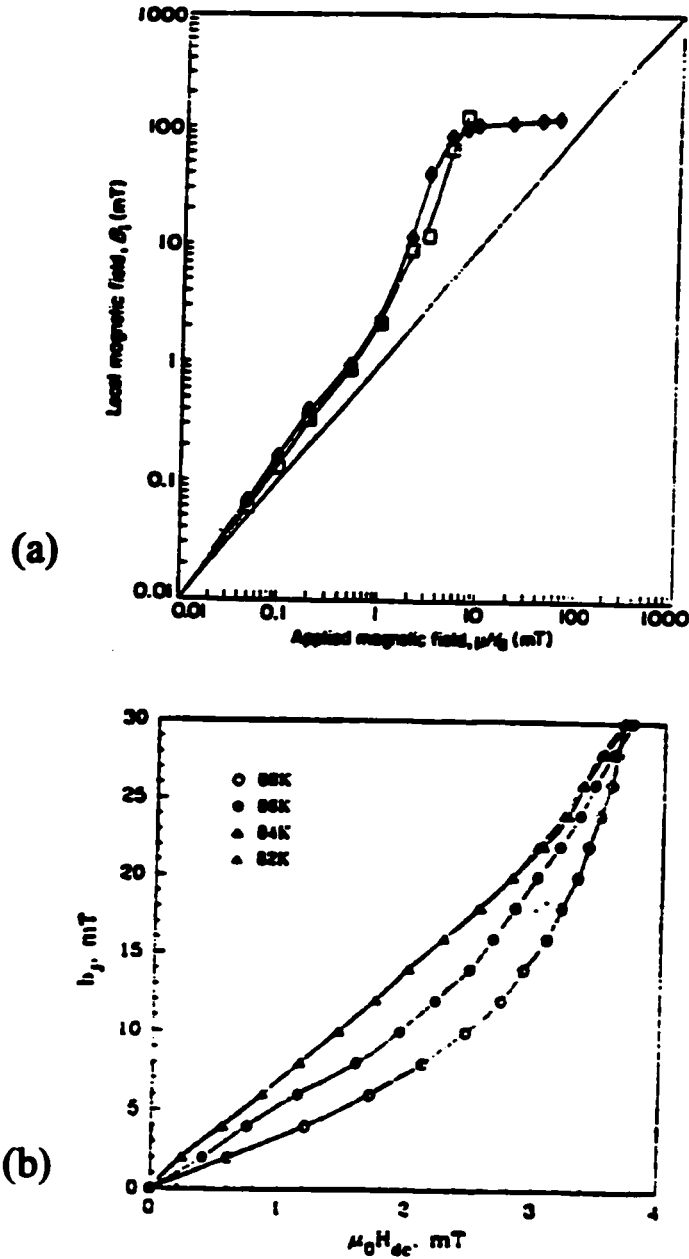


Fig 5.15. (a) Reproduced from Yang, Beduz and Ashworth^[156]. Displays the values they obtained for the intergranular magnetic field, denoted $B_j \equiv \mu_0 H_j$, from their data presented in Fig 5.9A(d), which shows I_c versus $\mu_0 H_a$ measured along the ZFCV and FCV curves, and from their application of eqn 5.24. ($\square$) data points at 65K, and ($\blacklozenge$) at 80K. (b) Reproduced from Yang, Beduz and Scurlock^[157]. Displays the intergranular magnetic field, $B_j \equiv \mu_0 H_j$, versus $\mu_0 H_a$, obtained by applying eqn 5.24 to the measurements of the real part of the AC Susceptibility versus H_a for the ZFCV and FCV procedures at various temperatures near T_c . Note the large ratios $B_j/\mu_0 H_a$ hence large compression factors C in both figures.

also after field cooling (FCV) over the corresponding ranges of H_a . Since the field cooled magnetization was observed to be appreciable relative to the hysteretic contribution to the magnetization measured during the cycle of H_a , the simplification exploited by Yang et al^[156, 157] (eqn 5.24) could not be applied to their data. Jones et al ascribe the relatively large flux expulsion they observed on field cooling to the small size of the grains ($1.8 \times 1.3 \mu\text{m}$), insufficient annealing, and the weak intergrain J_{co} .

We now describe their method to determine $H_i(H_a) = -C M_g(H_a)$ from their data. For any choice of I_c , labeled I_{c2} in Fig 5.15a, the superposition field H_s can be written,

$$H_{s1} = H_1 + H_{r1} = H_1 + CM_{g1} \quad 5.26a$$

$$H_{s2} = H_2 + H_{r2} = H_2 + CM_{g2} \quad 5.26b$$

$$H_{s3} = H_3 + H_{r3} = H_3 + CM_{g3} \quad 5.26c$$

for the ZFCV, FCV and ZFCD curves respectively. Here all the symbols denote absolute values since the magnetizations they observed for the ZFCV, FCV and ZFCD curves were diamagnetic except when H_a descended to very small values along the ZFCD curve where M_g switches to the paramagnetic quadrant. Applying the Evetts-Glowacki concept that H_s controls I_c , we write $H_{s1} = H_{s2} = H_{s3}$ for any set of data points exhibiting the same I_c . Consequently eqns 5.26a, b and c lead to,

$$C = \frac{H_2 - H_1}{M_{g1} - M_{g2}} = \frac{H_3 - H_2}{M_{g2} - M_{g3}} = \frac{H_3 - H_1}{M_{g1} - M_{g3}} \quad 5.27$$

Jones et al then display the values they obtained for C versus I_c from their measurements listed under (i), (ii) and (iii) above. However, for the denominators in eqn 5.27 they always introduce the values of M_g they measured when H_a was applied along the length of the ribbon. Since M_g is very sensitive to the orientation of the magnetic field

with respect to the surfaces of the ribbon, it is not surprising that they obtain large variations and changes of sign for C in case (iii) where H_r pierces the wide surfaces of the specimen. However, for the case (i), where their magnetization data applies, they find “well-behaved” compression factors ranging from ≈ 0.5 to ≈ 1 .

The measurements of Kwasnitza and Widmer of I_c hysteresis and the corresponding magnetization curves^[83] show unambiguously that H_r is not a linear function of M_g . Consequently, the expressions on the far right of eqns 5.26a, b and c, hence eqns 5.27 are not generally valid. We will address this important matter in the next chapter.

Mishra et al^[102] measured I_c versus H_a (ZFCV) and (FCV) curves for an YBCO bar of dimensions $0.5 \times 0.5 \times 9 \text{ mm}^3$, where $J_{c0} \approx 5(10^5) \text{ A/m}^2$ at 77K. They compare the ratios for $I_c^{\text{FCV}}(H_a)/I_c^{\text{ZFCV}}(H_a)$ obtained from their data with the corresponding ratios obtained from their model. This procedure is illustrated by the central and lowest dots in Fig 5.14a. In their model they write,

$$I_c = I_{c0} e^{-H_r/H_1} \quad 5.28$$

with, $H_r = H_a + C M_g(H_a)$, and calculate $M_g(H_a)$ for slab geometry using $J_{cg} = J_{c0} e^{-H/H_1}$, with, $\mu_0 J_{c0} W = 40 \text{ mT}$, $\mu_0 H_0 = 20 \text{ mT}$, and $\mu_0 H_1 = 42 \text{ mT}$.

Fig 5.16 reproduces the three pertinent figures from their paper. Note that Fig 5.16c is very similar to our Fig 5.11b. This is expected since they illustrate the same concept using different but equivalent formulae.

This method of extracting return fields from the measured I_c curves is clearly more complicated than the previous ones since it involves, first a model for I_c versus H_a , then a model for M_g , hence H_r versus H_a .

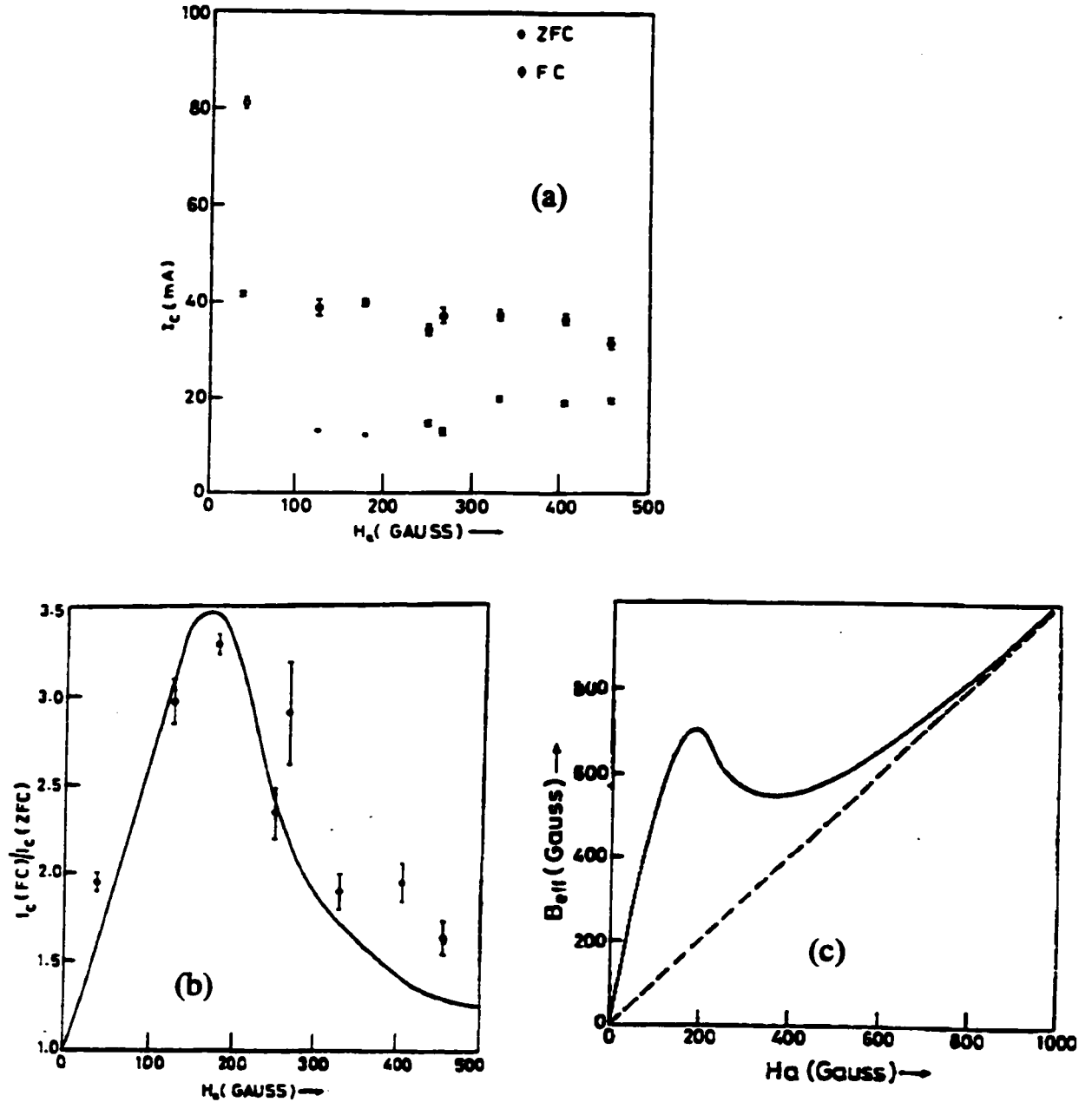


Fig 5.16. Reproduced from Mishra et al^[102]. (a) Displays their measurements of J_c versus H_a at 77K on an YBCO bar. Open circles (o) display data for field cooled virgin and filled circles (•) data for zero field cooling virgin procedure. (b) Data points display the ratios for $J_c^{FCV}(H_a)/J_c^{ZFCV}(H_a)$ versus H_a obtained from the data in (a). The solid curve is calculated with their model. (c) The solid curve displays the theoretical intergranular magnetic field, denoted $B_{gII}(H_a)$, which they introduced in their calculation of the solid curve displayed in (b). Dashed curve shows $B_{gII}(H_a) = \mu_0 H_a$ after field cooling.

(b) Proposed Program for Identifying $H_r(H_a, T)$ from Curves of I_c versus H_a .

Figure 5.14b displays the complete family of curves of I_c versus H_a which experimental researchers should measure in order to obtain reliable and detailed information on the saturated return fields, $H_{r \text{ sat dia}}(H_a, T)$, and $H_{r \text{ sat para}}(H_a, T)$, of their specimens for the entire range of H_a from 0 to H_{max} , the highest value accessible with their apparatus.

The reader will readily recognize the butterfly wing curves already addressed in section III(i) of this chapter, and denoted the symmetric I_c hysteresis loop. The upper envelope of the “wings” is obtained when, after zero field cooling, H_a is descending from a very large value. The same curve will be traced when $|H_a|$ is descending after field cooling in a very large value. Here, however an initial decrement $|\Delta H_a|$ will be required in order to cause I_c to transit from the F.C.V. curve to the top envelope of the wings.

In principle it should not be necessary to measure both, the right and left wings of the butterfly, nor the FCV curves for both the + and $-H_a$, since, in the absence of strong bias fields in the laboratory and apparatus, + and $-H_a$ should be equivalent. However, in order to trace the low field segment of the lower envelope of the wings, it is necessary that H_a be made to change sign after descending to zero from $|H_{\text{max}}|$. This lower segment is seen to intersect and then merge with the ZFCV curve as illustrated by the curve in the +quadrant in Fig 5.14b. This juncture presumably occurs when $H_{r \text{ dia}}$ along the ZFCV curves has attained the first saturation value $H_{r \text{ sat dia}}(H_a, T)$.

Next we strongly recommend that ZFCV, FCV and ZFCD magnetization curves be measured with H_a applied in the same direction with respect to the flat surfaces of the specimen that is used for the I_c hysteresis family of curves. Here, one of the purposes of

these measurements is to determine the relative magnitude of the hysteretic (pinning) and reversible (Meissner) contributions to the magnetization of the grain. This information is needed in order to ascertain the validity of taking $H_r(H_a, T) \approx 0$ when I_c lies along the F.C.V. curve. If $|<M>_{FCV}|$ is small compared to $|<M>_{ZFCV}|$ and $|<M>_{FCD}|$, then the following equations,

$$H_2 - H_1 = H_{r1}(H_1, T) = H_{r, sat dia}(H_1, T) \quad 5.28a$$

$$H_3 - H_2 = H_{r2}(H_2, T) = H_{r, sat para}(H_2, T) \quad 5.28b$$

may be applied with some confidence to any set of three data points with the same I_c along the butterfly wing curves and should yield reliable values for $H_r(H_a, T)$.

We shall show in the next chapter that H_r is a multiple valued function of M_g , when M_g lies along traversals between the envelopes of the major hysteresis loops. Consequently the experimental magnetization data along such traversals should not be used in the determination of compression factors. However, eqn 5.27 is valid when saturation (i.e envelope) value, $M_{g, sat dia}(H_a)$ and $M_{g, sat para}(H_a)$ are exploited.

We stress that, if in the light of the magnetization data, H_r can be considered negligible along the $I_c(FCV)$ curve, it is then not an arduous task to establish a simple expression which approximately reproduces the observed behaviour of $I_c(FCV)$ versus H_a .

Now for any field, denoted H_2 , we can calculate the experimental ratios $I_c(ZFCD)/I_c(FCV)$ and $I_c(FCV)/I_c(ZFCV)$ along the butterfly wings (illustrated by points on vertical parts of crosses in Fig 5.14). Exploiting these ratios and a mathematical formula (or table) for $I_c(FCV)$ versus H_a one can readily calculate $H_{r, sat dia}(H_2, T)$ and $H_{r, sat para}(H_2, T)$. Since $M_{rev}(H_a)$ is a single-valued function of H_a at a chosen T , its

contribution to $H_r(H_a)$, is the same for any three "vertical" data points (i.e at the same H_a). Consequently in this procedure it is not required that M_{rev} , hence its contribution to $H_r(H_a)$ be negligible.

The values of H_r extracted by this somewhat tedious exercise can then be compared for consistency with that obtained from the application of eqn 5.28. Alternatively, the values of H_r from the application of eqn 5.28 can be introduced in the formula or table fabricated to reproduce the $I_c(FCV)$ versus H_a curve. The ensuing curves can then be compared with the corresponding measured curves as a test of the merits of the results.

IV Summary and Conclusion

In chapter 4 we showed that simple considerations of the evolution of H_r , the return field of magnetized grains, versus H_a descending and reascending after Zero Field Cooling and Field Cooling, accurately described the evolution of the position of the peaks of I_c , if as proposed by Evetts and Glowacki, the centre of these peaks coincided with situations where $H_r = -H_a$. This analysis required no knowledge of the actual dependence of I_c on H_a , hence needed no information on the detailed structure of the hysteretic I_c curves.

In this chapter we focused on the immense variety of critical current curves encountered in the study of the I_c hysteresis phenomenon. We developed simple formulae for I_c versus H_a for idealized slab geometry exploiting simple critical state expressions for the dependence of the intergranular critical current density J_{cm} on the magnetic field. Then, again pursuing the Evetts-Glowacki idea that $H_r = H_a + H_r(H_a)$, and using various

convenient and simple expressions for $H_r(H_a)$ we reproduced a large variety of observations on the behaviour of I_c hysteresis curves reported by several workers.

In the analysis of the effect of the trapped flux on I_c in $H_a = 0$ we show that the popular Josephson junction formalism exploited by many workers to describe the behaviour of the intergranular critical current is not only artificial and restrictive but leads to incorrect predictions. Finally we indicate that the simple magnetic dipole picture for the return field of the magnetized grains, hence the linear relationship, $H_r = CM_g(H_a)$, need to be modified. In the next chapter we pursue this important feature.

Chapter 6

I_c Hysteresis: Double-Valued Dependence of I_c on M_g:

A Simple Model

Abstract

In the analysis of I_c hysteresis phenomena, all researchers have assumed that the return field $H_r(H_a, T)$ obeys a linear relationship, $H_r = C M_g(H_a, T)$ where C is a parameter of the specimen which may depend on T , and M_g is the magnetization of the grains. Kwasnitza and Widmer^[83] measured both I_c and M_g at a chosen temperature after various magnetic field cycles terminating at the same chosen final field H_f . They found I_c to be an intricate double-valued function of M_g and that the two different curves traced by I_c versus M_g changed their structure as a function of H_f . These results show unambiguously that the outer configurations of the induced currents circulating in the grains exert a strong influence on H_r . We apply the textbook definition $\bar{M} = (1/2) \int (\bar{r} \times \bar{J}) dV / V$ to calculate the magnetization M_g , and Ampere's law, or the Biot-Savart equation to estimate the return field along the edges of thin ribbons and thin disks containing two concentric zones of countercirculating induced persistent currents of density J_{c2} independent of H . Then exploiting our critical state expressions for I_c and the Evetts-Glowacki superposition concept we reproduce all the observations of Kwasnitza and Widmer, and make predictions on the evolution of the double-valued curves of $I_c(H_f)$ versus $M_g(H_f)$ as a function of H_f over a wide range extending from 0 to high fields.

I Introduction

We noted in chapter 5 that Jones et al^[72] measured both M_g and I_c versus H_a for their YBCO specimen. Focusing on pairs of data points with the same I_c along the family of measured ZFCV, FCV and ZFCD curves, hence where, according to the Evetts-Glowacki proposal, H_r the superposition of the applied field H_a and the return field $H_r = C M_g$ should be equal, they wrote,

$$H_{a1} = H_{a1} + C M_{g1} = H_{a2} = H_{a2} + C M_{g2} \quad 6.1$$

Introducing the measured H_{a1} , H_{a2} , M_{g1} and M_{g2} into eqn 6.1 they calculated C versus I_c . They found that C was far from constant but varied dramatically. They speculated that the different configurations of induced currents circulating in the grains near their contacts with their grains might account for these anomalies but do not seem to have pursued this idea.

McHenry et al^[101] observed that, after zero field cooling, the magnetization of their YBCO specimen reaches a saturation value, $M_{g \text{ sat.}}$ for a range of applied ΔH (M_g) $\approx$ 150 mT. In contrast, the traversal of I_c , from the FCV curve in large H_{cool} to a peak value, required a change in applied field $\Delta H(I_c) = H_{\text{cool}} - H_{\text{peak}} \approx 25$ mT. This large discrepancy between these two ranges led them to conjecture that the induced currents near the edges of the grains played a major role in governing the intergranular currents. These workers however did not publish any follow up on this proposal.

Kwasnitza and Widmer^[83] systematically measured both $\langle M_g \rangle$ and I_c versus H_a for corresponding field-temperature histories for their YBCO sample. They focused on the behaviour generated by a specific but basic family of magnetic field cycles. Their results show that along traversals between the envelopes of the hysteresis curves of M_g and I_c ,

the linear magnetic dipole assumption that $H_r = C M_g$ is grossly invalid. They find that $I_c(H_f)$ in a chosen final field H_f is a double-valued function of $M_g(H_f)$. Consequently, in the framework of the Evetts-Glowacki concept, $H_s(H_f) = H_f + H_r(H_f) = H_f + CM_g(H_f)$, and $H_r(H_f)$ are double-valued functions of $M_g(H_f)$.

In the next section we describe the procedures followed in their measurements of I_c and M_g , then present their fascinating results and outline our explanation. Then in the following section we develop simple equivalent models which account for their observations.

II Observations of Kwasnitza & Widmer

A. Observations of I_c versus $M_g(H_f)$ in a Final Field $\mu_0 H_f = 0.8T$.

It is well known that in hysteretic type II superconductors, a magnetization $M_g(H_f, T_f)$ of a chosen magnitude and sign, in a chosen final magnetic field H_f and chosen final temperature T_f can be generated by numerous different field-temperature histories. It is less well known however that the configurations of induced persistent currents associated with these identical $M_g(H_f, T_f)$ for the chosen specimen can be dramatically different. Kwasnitza and Widmer exploited simple and convenient paths in the H - T plane to generate identical pairs of values for $M_g(H_f, T_f)$. Three pairs of such paths are illustrated in fig 6.1a. For the three paths labeled (1) in that figure, the locus of M_g versus H_s traveling along the upper envelope of the major hysteresis curve is made to reverse direction at some chosen value $H_{rev} < H_f$. Then the sweep of H_s ceases when H_s attains

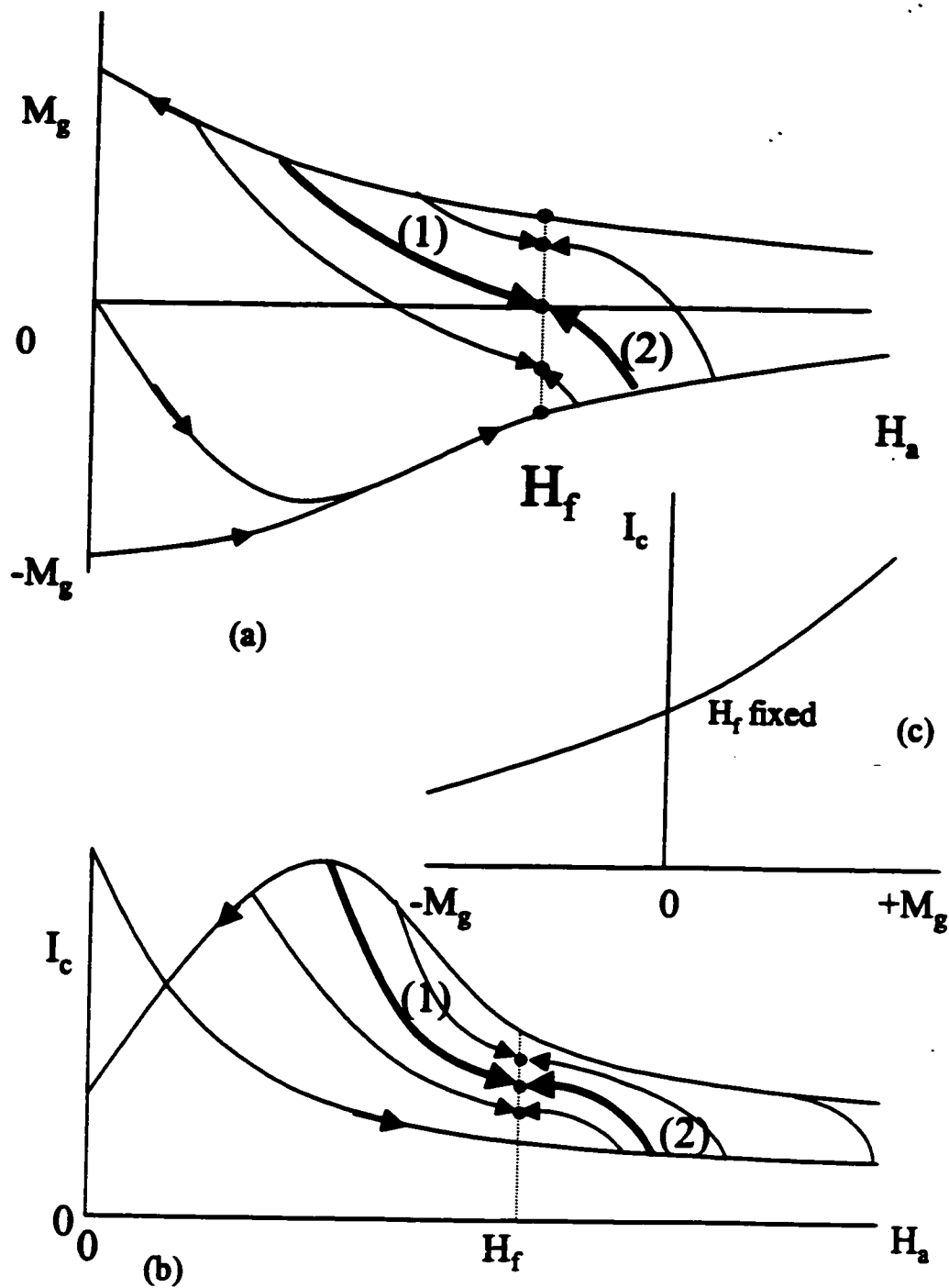


Fig 6.1 (a) Paths (1) and (2) illustrate that a magnetization $M_g(H_f)$ of a chosen magnitude and sign can be generated by two opposite approaches to H_f from the envelopes of the major hysteresis curves. (b) Paths (1) and (2) illustrate that a critical current $I_c(H_f)$ of a chosen magnitude can be observed after tracing two opposite approaches to H_f from the envelopes of the I_c versus H_a curves. (c) Illustrates the single-valued prediction for the behaviour of $I_c(H_f)$ versus $M_g(H_f)$ from the idealized dipole assumption that $H_f(H_f) = C M_g(H_f)$.

H_f . At this juncture I_c is measured. Fig 6.1b illustrates the family of paths which I_c versus H_a is expected to trace if I_c were measured at several points corresponding to the family in Fig 6.1a of M_g versus H_a curves terminating at H_f .

The linear magnetic dipole relationship, $H_f = C M_g$, and the Evetts-Glowacki superposition proposal predicts that $I_c(H_f)$ versus $M_g(H_f)$ display a single-valued curve as sketched in Fig 6.1c. Here the extreme $\pm M_g$ values correspond to the saturation paramagnetic and diamagnetic magnetizations at H_f along the upper and lower envelopes of the major magnetic hysteresis loop. The extreme values for I_c correspond to that at H_f along the upper and lower envelopes of the I_c hysteresis curves. Fig 6.2a reproduces the observations of Kwasnitza and Widmer where $\mu_0 H_f = 0.8$ Tesla ($T_f = 4.2K$). The labels (1) and (2), along the data points indicate the corresponding families of paths followed in tracing the M_g versus H_a and I_c versus H_a families of curves sketched in Fig 6.1a. Especially noteworthy is the large difference in the two values measured for I_c when $M_g(H_f) = 0$ is generated following the opposing M_g versus H_a paths labeled 1 and 2. The solid curves in Fig 6.2b display the theoretical curves, corresponding to that shown in Fig 6.2a, generated by the model we now describe. The dashed curve shows the behaviour predicted by taking $H_f = C M_g$.

B. Qualitative Explanation of the Observations in Large H_f .

Figure 6.3 illustrates the configuration of the magnetic flux density $B(r)$ and persistent current density $J_{cg}(r)$ that are visualized to exist in our idealized disk at H_f after paths 1 and 2 have been traced in the $M_g - H_a$ history. Here the sketches on the left hand (right hand) side correspond to the paths labeled 1 (2).

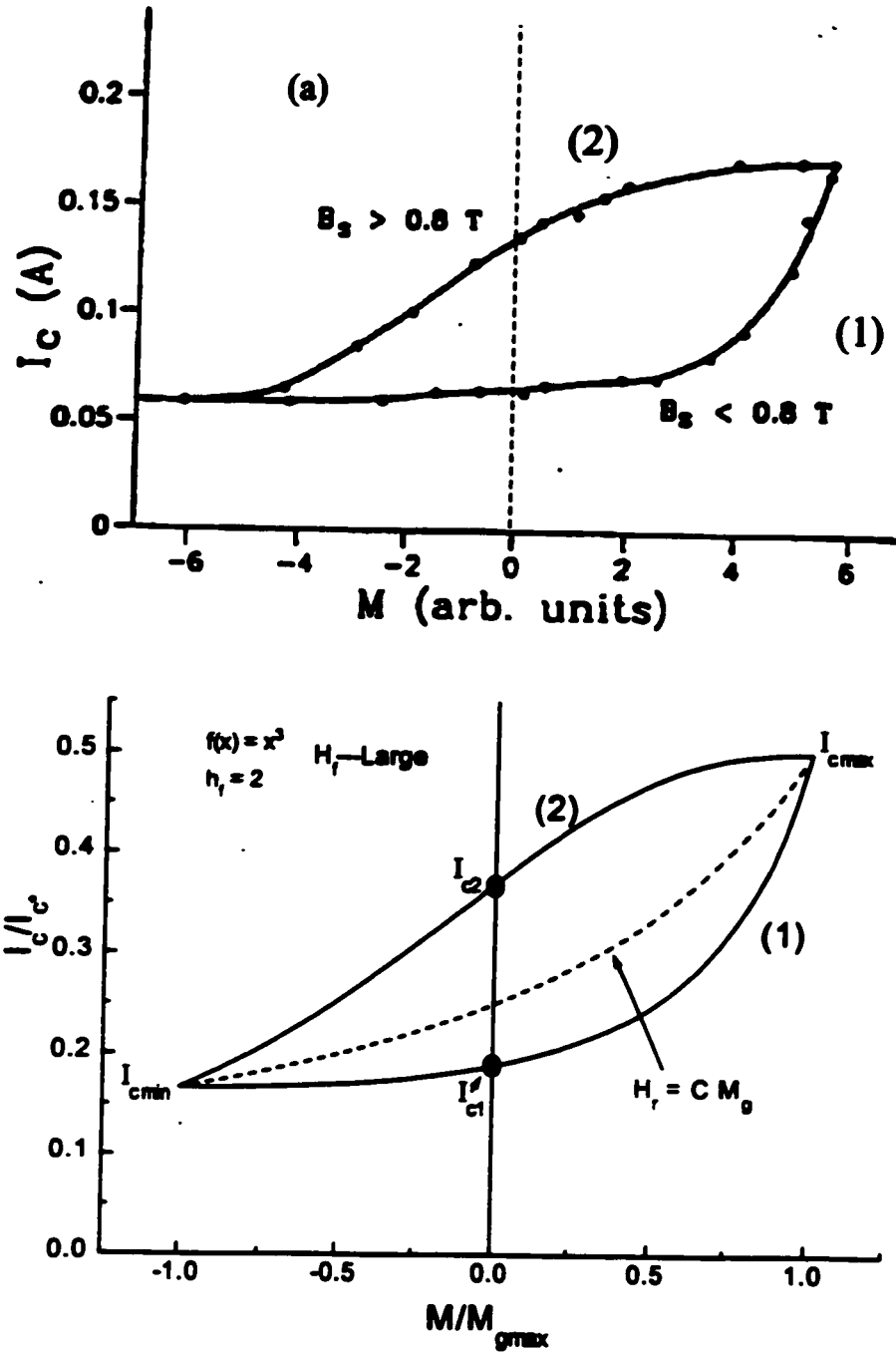


Fig 6.2. (a) Reproduced from Kwasnitza and Widmer^[83] and shows their data for an YBCO strip at 4.2K for $I_c(H_f)$ versus $M_g(H_f)$ at a final field $\mu_0 H_f = 0.8\text{T}$ after following paths (1) and (2) of Fig 6.1 where the reversal field, denoted B_s , is $< 0.8\text{T}$ for data (1) and $> 0.8\text{T}$ for data (2). The solid lines are guides to the eye. (b) The solid lines display the corresponding theoretical curves generated by our models. Here we exploited an empirical expression for H_r versus M_g which is described later in this chapter. The dashed line is calculated using $H_r = C M_g$.

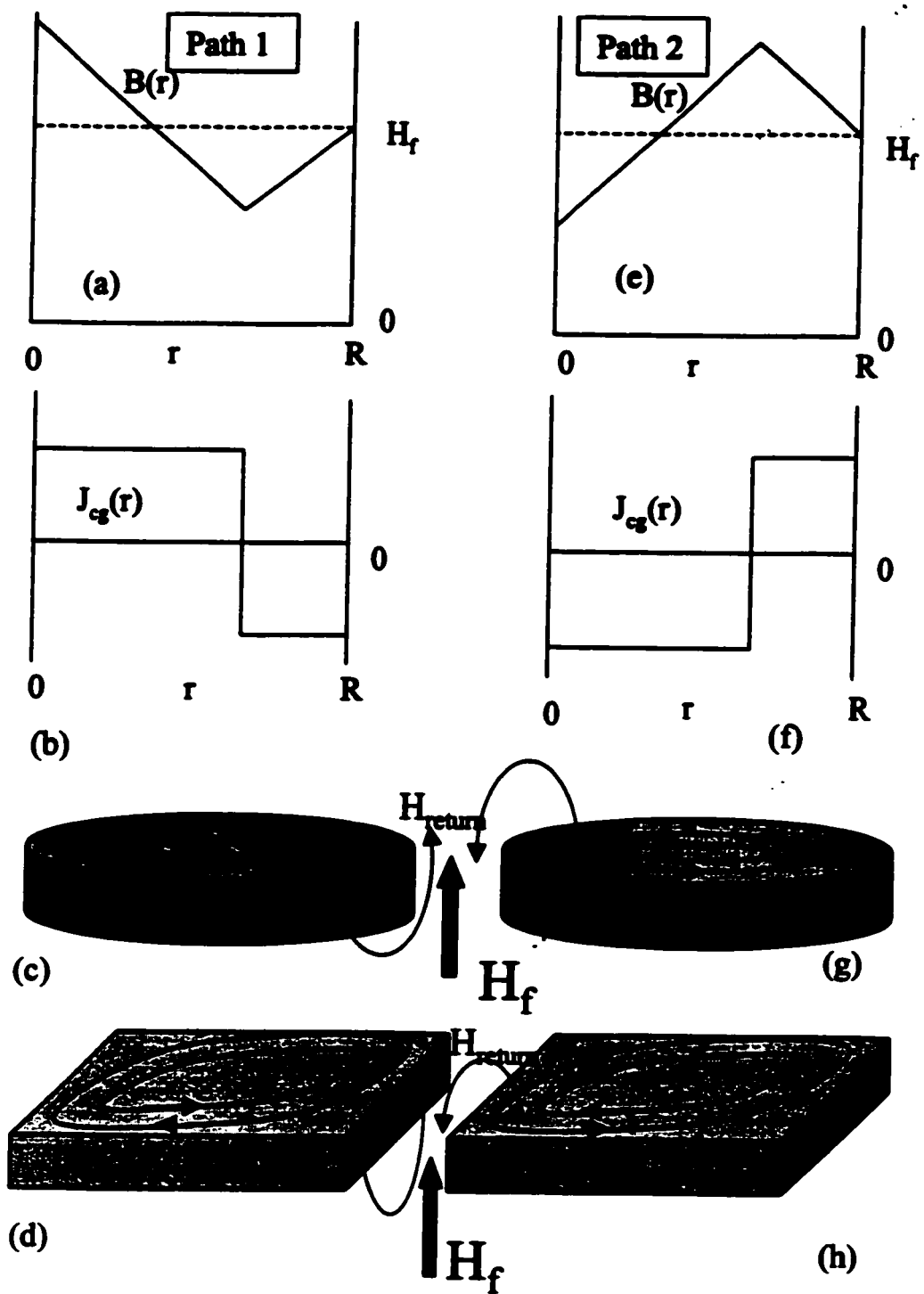


Fig 6.3. Sketches on the left (right) hand sides of the figure illustrate the configurations of the flux density and critical current density for situations where the magnetization $M_s \approx 0$ after following paths (1) and (2) displayed in Fig 6.1. Here (c), (d), (g) and (h) show that reversal of the direction of the circulation of currents, although letting M_s remain small and constant in magnitude, will cause the large return field H_r to change sign since it is generated mainly by the currents near the edges of the disks or plates.

In Figs 6.3 we try to illustrate the feature that thin disks and plates which have net magnetic moments $\mu \approx 0$, hence a resultant magnetization $M_g = \mu/V \approx 0$, can nevertheless generate return fields $H_r(r')$ along their edges which are not only very large but also change direction when the patterns of circulation of the persistent induced currents are reversed. The physical reason for this is that the dominant contributions to the return field $H_r(r')$ along the edges arises from the elements of current $\Delta I = J_{\phi} \Delta r Z$ flowing near the edges. Here Z is the height of the disk or plate. This result follows from consideration and application of Ampere's law in the form,

$$\Delta H_r(r') = \frac{\Delta I}{2\pi \Delta r'} = \frac{\Delta I}{2\pi(r' - r)} \quad 6.2a$$

where r' denotes the location of the return field outside the disk or plate but close to the edge, and r denotes the location of the element of current. Clearly ΔH_r can be very large along the edges due to the elements of current adjacent to the edges. In section III of this chapter we develop simple expressions for H_r for disk and slab geometry exploiting the Ampere and Biot-Savart laws.

However we can readily see without calculations and as illustrated in Fig 6.3 that the return field $H_r(r')$ will aid H_f when path 1 has been followed to generate $M_g = 0$ at H_f , whereas $H_r(r')$ will oppose H_f when path 2 has been followed. Consequently I_c after following path 1 to H_f will be smaller than I_c after the corresponding path 2 to H_f also leading to $M_g = 0$.

Pursuing these considerations and sketches we can see that, in Fig 6.2b, I_c will rise from I_{c1} and I_{c2} , the two different values at $M_g = 0$, to a common maximum $I_{c_{max}}$ when

$M_g = M_{g \text{ max para}}$, the saturation paramagnetic value at H_f . Conversely we can see that I_c will diminish from I_{c1} and I_{c2} to the same minimum $I_{c \text{ min}}$ when $M_g = M_{g \text{ max dia}}$, the saturation diamagnetic value for the chosen H_f .

C. Observations of I_c versus H_{rev} and $M_g(H_f)$ in $H_f = 0$

Kwasnitza and Widmer also measured $I_c(H_f)$ versus $M_g(H_f)$ where the final field $H_f = 0$, hence as a function of the remanent trapped flux. The paths traced in the $M_g - H_a$ plane before each measurement of I_c are illustrated in Fig 6.4a and b. The configurations of the flux density profiles visualized to exist for the trapped flux at $H_f = 0$ are sketched in Fig 6.4c and d for different fields H_{rev} where the reversal of the field sweep occurs. Fig 6.4a is reproduced from the paper by Kwasnitza and Widmer. Fig 6.4b is a representation of the corresponding behaviour which would be encountered when H_{rev} occurs in the fourth quarter.

We stress that, when the earth's weak ambient magnetic field can be ignored, which is the case in the data of Kwasnitza and Widmer, the measured upper envelope of the symmetric major magnetization hysteresis curve displayed in the quadrants 1 and 2, should be reproduced, within experimental accuracy, in quadrants 3 and 4, unless bias background fields existing in the laboratory and in the apparatus make a significant contribution to the observations. For these reasons Kwasnitza and Widmer and other workers do not carry out again the redundant duplicate measurements which, for "pedagogical" purposes, we display in Fig 6.4b. The reader should also note that Fig 6.4d is a rotation of fig 6.4c by 180° about an axis vertical to the page.

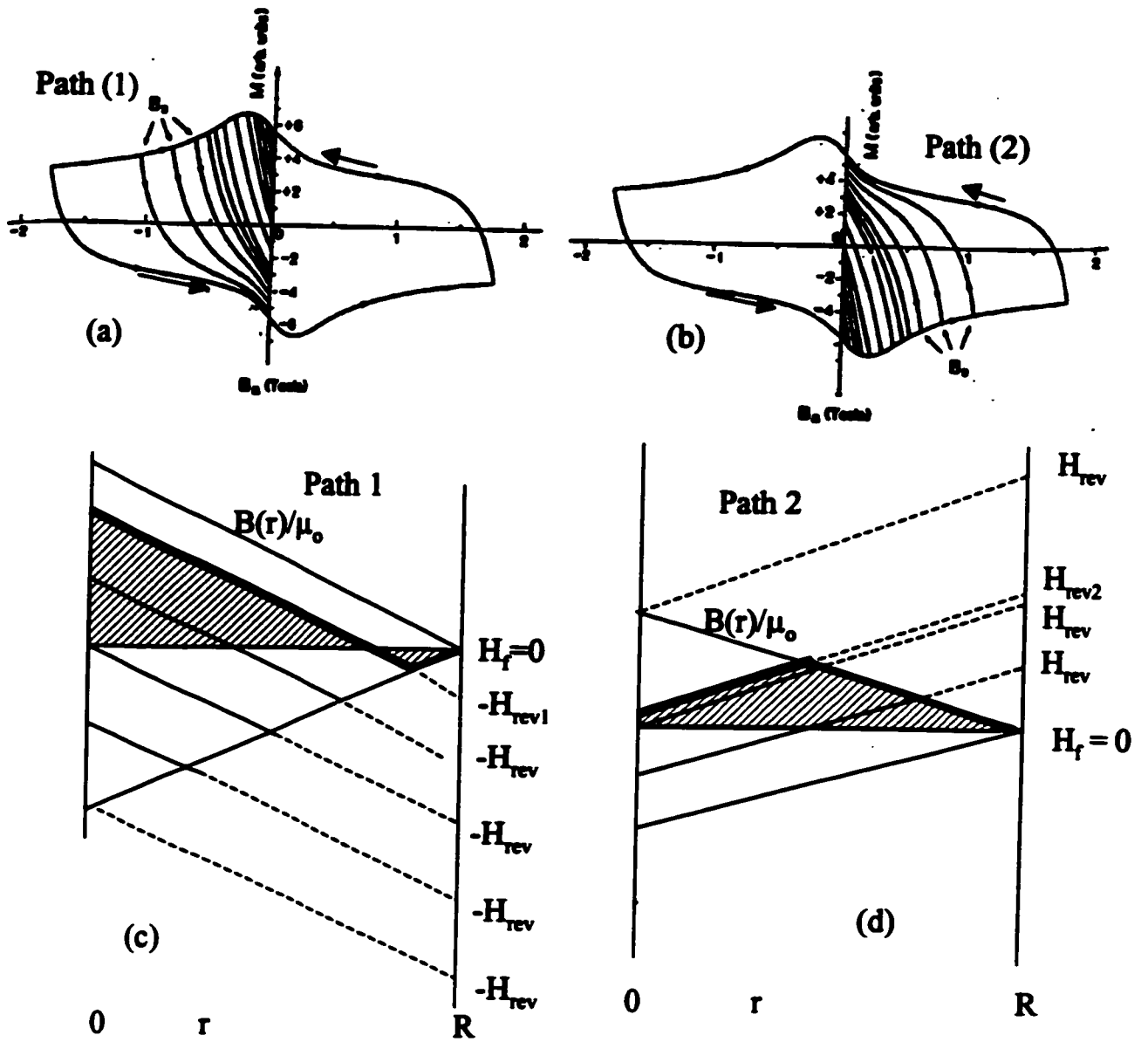


Fig 6.4. (a) Reproduced from Kwasnitza and Widmer^[83]. Displays their measurements of the locus of M for an YBCO strip at 4.2K when H_a reverses direction at values denoted B_s and descends to $H_f = 0$ thereby causing M to migrate from the envelope of the major hysteresis curve to final values $M_s(H_f = 0)$ where I_c is then measured. As in Fig 6.1a we let path (1) denote the situations where M and H_a migrate from left to right. (b) Same as (a) but here H_a reverses direction in the 4th quadrant, hence M and H_a migrate from right to left. This situation is denoted path (2) as in Fig 6.1a. (c) and (d) Sketches of the flux density profiles at various values of H_{rev} in quadrant 2 (4), displayed by solid-dashed lines, and after the removal of H_a (displayed by solid-solid lines). The heavy lines in (c) and (d) display flux configurations at $H_f = 0$ where the values of M , corresponding to the "net" dashed areas, are nearly the same in magnitude and sign. The return field at the edge of the grains after path (2) is, however, considerably stronger in magnitude and perhaps, opposite in sign than that after path (1) since the effect of the currents near the surface in (c) cancel each other.

Fig 6.5a illustrates the behaviour which our model predicts that Kwasnitza and Widmer would have observed for I_c versus H_a if these tedious measurements had been carried out in conjunction with their measurements of M_g versus H_a shown in Fig 6.4a.

The mirror image of these curves across the vertical line at $H_f = 0$ would be generated if the measurements were repeated but with the reversals at the various values of H_{rev} taking place in the first quadrant. Fig 6.5a is presented to illustrate the feature that the curves of I_c versus H_a terminating at $H_f = 0$ will intersect as $|H_{rev}|$ is chosen larger. Consequently, the values of I_c at $H_f = 0$, are seen to, (i) start from I_c minimum when $H_{rev} = H_f = 0$ and the remanent magnetization $|M_{g\text{ rem}}|$ is a maximum (see Fig 6.4a), (ii) increase to I_c maximum at a value of H_{rev} which would lead to $|M_{g\text{ rem}}| = 0$ if $H_f = C M_g$, and then, (iii) descend to I_c minimum again when the excursion to a large $|H_{rev}|$ has caused $|M_{g\text{ rem}}|$ to grow to a maximum of opposite polarity (see Fig 6.4a). Fig 6.5b displays the measurements of Kwasnitza and Widmer^[83] of I_c at $H_f = 0$ versus the reversal fields H_{rev} (denoted B_r). Fig 6.5c shows a theoretical curve where we introduced the simple magnetic dipole linear approximation $H_f = C M_g$.

The peak of I_c in the theoretical curve (Fig 6.5c) occurs when the descent to $H_f = 0$ starts from a value for $|H_{rev}|$, denoted H_{rev}^{peak} which leads to $M_g = 0$ at $H_f = 0$. However, the peak of I_c in the data curve does not correspond to a descent from a value of $|H_{rev}|$ which leaves the specimen with $M_g = 0$ but from a value H_{rev}^{peak} which leaves the specimen with an appreciable net magnetization. This feature is evident in Fig 6.6a where the data curve labeled(1) is reproduced from Kwasnitza and Widmer^[83] and displays their measurements

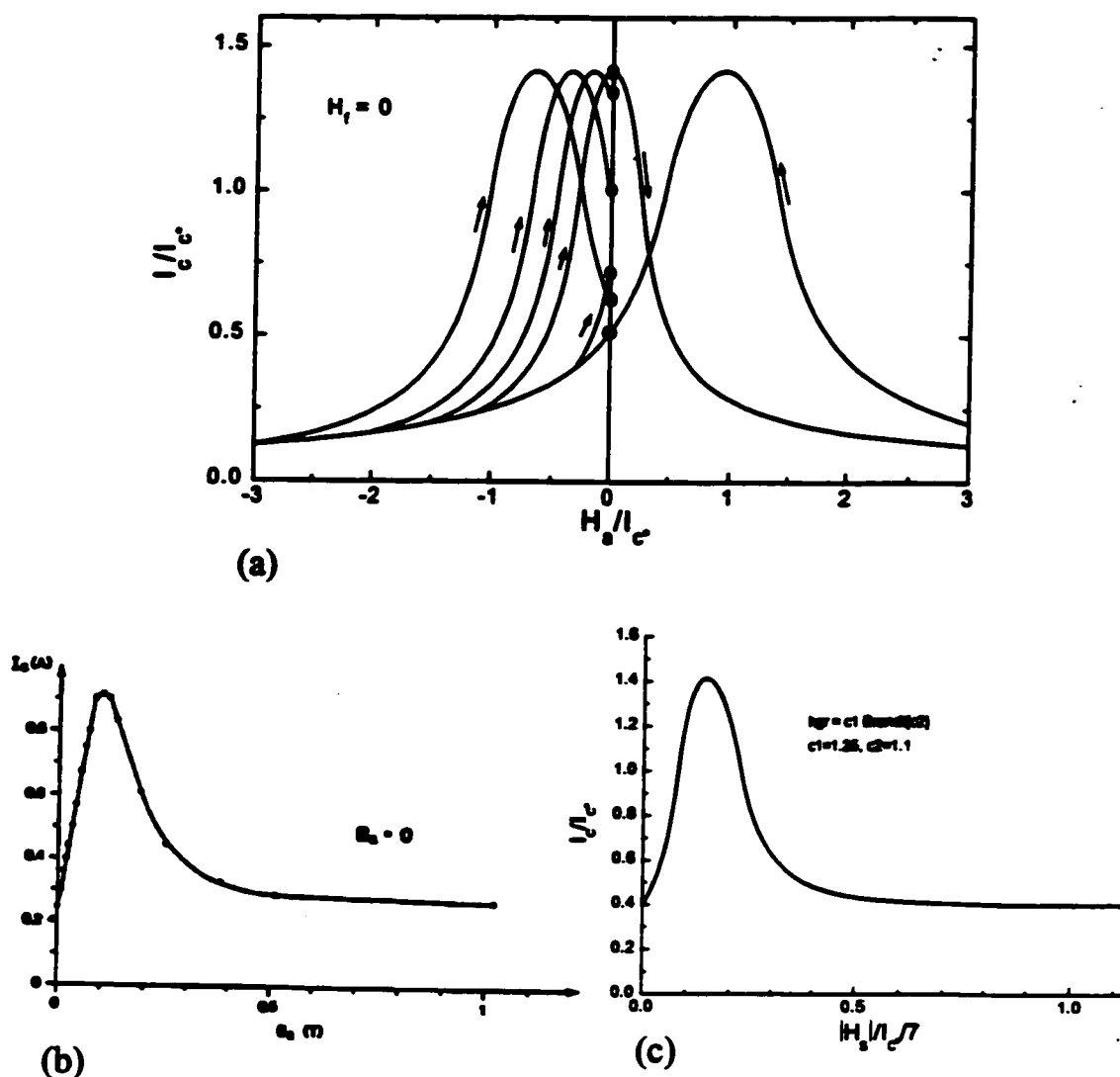


Fig 6.5. (a) Sketches of curves of I_c versus H_s which are expected to be measured when the magnetization M of the specimen is concomitantly made to evolve along the paths displayed in Fig 6.4 (a). Note that as $|H_{rev}|$ is chosen larger, I_c at $H_f = 0$ initially increases, reaches a peak, and then diminishes to the initial value where H_{rev} was 0 hence, where $H_{rev} = H_f = 0$. (b) Displays the data of Kwasnitza and Widmer^[83] for I_c at $H_f = 0$ versus the reversal fields, denoted B_s . (c) Shows a theoretical curve generated by the model described in chapter 5 where we let $H_f = C M_g$. Therefore, when I_c is displayed versus the reversal fields, the double-valuedness of the data curve does not reveal that H_f is a double-valued function of M_g .

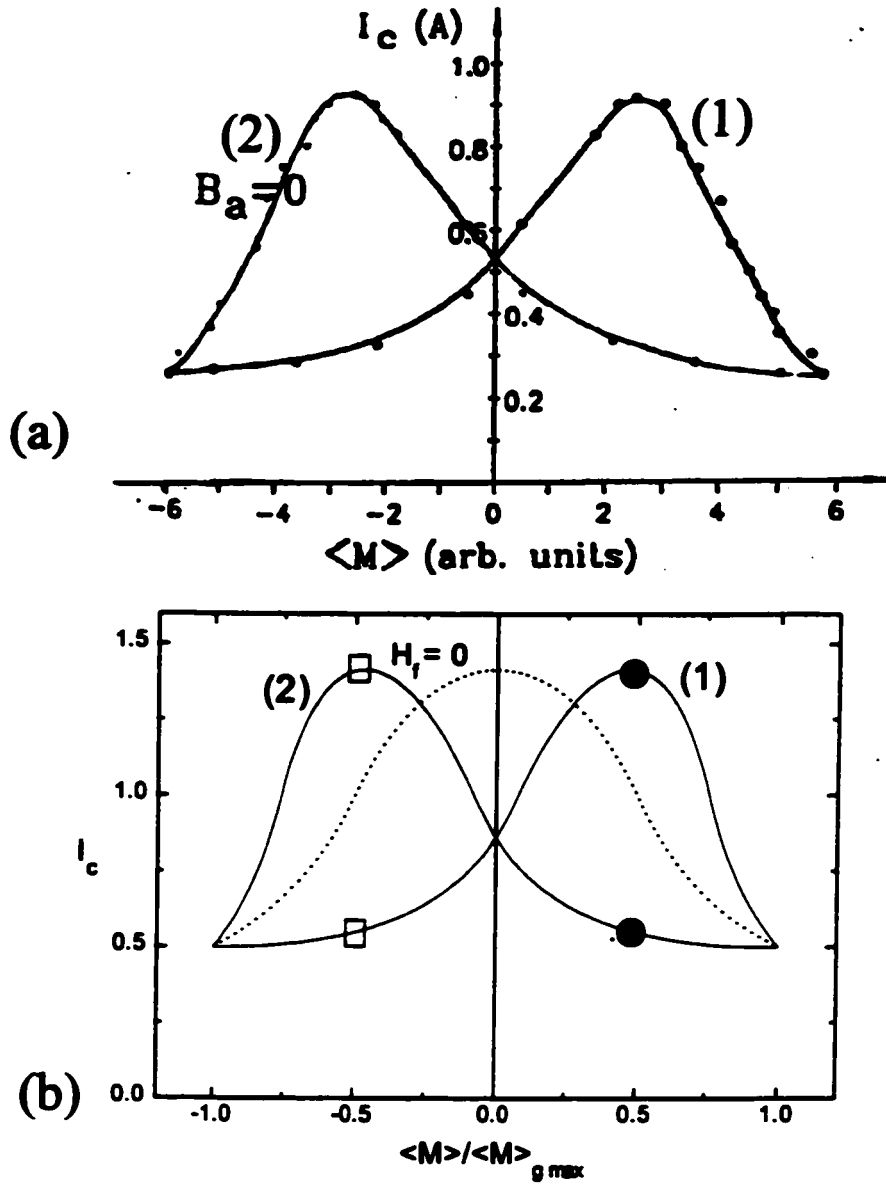


Fig 6.6 (a) Data curve labeled (1) is reproduced from Kwasnitza and Widmer^[83]. Curve labeled (2) is the mirror image of the data curve (1) constructed by us to emphasize the feature that values of $\langle M \rangle$ of the same magnitude and sign lead to two different values for I_c . These data curves display I_c at $H_r = 0$ versus $\langle M \rangle$, the magnetization of the specimen at $H_r = 0$. See Fig 6.4(a) and (b) for a description of paths (1) and (2) in the $\langle M \rangle$ versus H_a plane. The solid lines are guides for the eye. (b) Displays theoretical curves for I_c at $H_r = 0$ versus M_g at $H_r = 0$. The dashed curve is symmetric since here $H_r = C M_g$ is assumed, hence the peak occurs when $M_g = 0$. The solid curves are generated using one of the four approaches described in the text. In each of these schemes, H_r is calculated along the edge of the specimen, and lead to very similar results. Note that when $H_r = 0$, I_c for path (1) is the mirror image of I_c for path (2) and that two very different values for I_c are obtained for two values $+\langle M \rangle$ of the same magnitude (●) and of $-\langle M \rangle$ of the same magnitude (□).

of I_c at $H_r = 0$ versus $M_g(H_r = 0)$. Figs 6.5b and 6.6a complement each other, the former showing I_c at $H_r = 0$ versus the reversal fields B_r , and the latter also showing I_c at $H_r = 0$ but versus M_g at $H_r = 0$. In their paper, Kwasnitza and Widmer present only the data curve labeled 1 in our Fig 6.6a. They considered the repetition of these measurements with $H_r = 0$ following the paths labeled 2 in Fig 6.4b as redundant since here the family 2 of paths is physically identical to family 1. However for symmetry, completeness, and pedagogical reasons, we have constructed the “mirror image” data curve labeled 2 in Fig 6.6a from the data labeled (1).

The important message of the data displayed in Fig 6.6a is that the peaks of I_c do not correspond to $\langle M \rangle = 0$. Here, since $H_r = 0$, only H_r the return field of the magnetized grains will dictate I_c . Consequently, at the peaks of I_c , $|H_r|$ must possess a minimum, although $\langle M \rangle$ is seen to be appreciable. Further, H_r must be significant when $M = 0$ since here I_c is much smaller than its peak values.

The dashed curve in Fig 6.6b displays the predictions of our critical state model for I_c versus M_g when the linear approximation $H_r = C M_g$ is exploited in the calculations. As expected, this theoretical curve is symmetric with respect to $M_g = 0$ since there $H_r = 0$ when $M_g = 0$. The solid curves in Fig 6.6b are obtained exploiting a simple model, which we describe in the next section, where the return field is calculated along the edges of magnetized thin plates and disks using Ampere and Biot-Savart laws. However we can see intuitively or pictorially without any calculations that two very different configurations of currents circulating in a thin plate or disk which, although generating

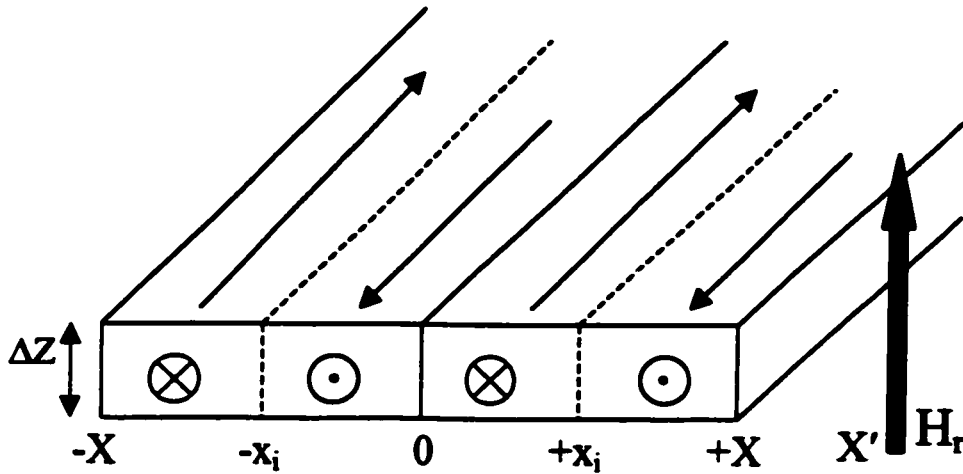
identical magnetic moments, can produce dramatically different return fields along the edges of the specimens. The heavy lines in Fig 6.4c and d schematically display two internal fields where, at $H_r = 0$, the spatial average of the flux densities $\langle B \rangle_{\text{path 1}} \approx \langle B \rangle_{\text{path 2}}$, hence $M_g(\text{path 1}) \approx M_g(\text{path 2})$ in magnitude and sign. Noting again that the creation of the return field just outside the edge of the specimen is dominated by the currents circulating near the edge we can visualize that, for the left hand case, H_r will be small since here the effect of the adjacent countercirculating currents will tend to cancel each other. By contrast, for the right hand case, all currents near the surface combine to create a large return field. As a consequence, for any chosen value for M_g , very different values are encountered for I_c as shown by the pairs of dots and squares in Fig 6.6b.

III Models for H_r when $H_r \neq C M_g$

We have calculated the return field H_r outside but near the narrow edge of thin ribbons and thin disks, magnetized $\perp$ to their flat surfaces, exploiting various approaches which we now describe. In all cases H_r is a double-valued function of the corresponding M_g , which we also calculate, since two configurations of induced persistent currents in the specimen comprising two concentric zones of countercirculating currents can generate the same magnitude and sign for M_g . Such patterns of current circulation are encountered when the direction of the sweep of the applied magnetic field is reversed thereby causing the locus of the magnetization to migrate from one envelope of the major hysteresis curve towards the opposite envelope. We now describe the four models we have exploited in these calculations.

A. Thin Ribbon. Application of Ampere's Law

We consider a thin ribbon of thickness ΔZ , width $2X$, and infinite length along Y , carrying induced currents of uniform density $\pm J_{cy}$ flowing in the $\pm Y$ directions, (see accompanying sketch). The edges of the ribbon are situated at $\pm X$. We calculate the return field at X' outside the right edge and along the mid plane of the ribbon. To fix ideas we let an outer zone of the induced currents circulating clockwise embrace an inner zone filled with induced currents circulating counterclockwise as viewed from a $+Z$ location. Let $\pm x_i$ denote the boundaries between the two zones. Applying Ampere's law, $\oint \vec{H} \cdot d\vec{l} = \Delta I$, to describe the total return field H_r at X' , which is generated by an element



of current $\Delta I = J_{cy} \Delta Z \Delta x$ located at x , leads to,

$$\Delta H_r = \frac{J_{cy} \Delta Z \Delta x}{2\pi(X' - x)} \quad 6.2b$$

hence, the integral,

$$\frac{2\pi H_r}{J_{cy} \Delta Z} = -\int_0^{x_i} \frac{dx}{X' - x} + \int_{x_i}^X \frac{dx}{X' - x} + \int_0^{x_i} \frac{dx}{X' + x} - \int_{x_i}^X \frac{dx}{X' + x} \quad 6.3$$

leads to,

$$\frac{2\pi}{J_{cg}\Delta Z} H_r = \ln \left\{ \frac{(X' - x_i)^2 (X' + x_i)^2}{(X')^2 (X' - X)(X' + X)} \right\} = 2 \ln \left\{ \frac{(d+1-x'_i)(d+1+x'_i)}{(d+1)\sqrt{d(d+2)}} \right\} \quad 6.4a$$

hence, $|H_{r,max}| = H_c \ln \left\{ \frac{d+1}{\sqrt{d(d+2)}} \right\}$ 6.4b

where, $H_c = J_{cg}\Delta Z/\pi$, $d = (X' - X)/X$, and $x'_i = x_i/X$. Note that $H_r = 0$ when

$$x'_i = (d+1) \left\{ 1 - \frac{\sqrt{d(d+2)}}{(d+1)} \right\}^{1/2} \quad 6.5$$

The corresponding magnetization $M_g(x'_i)$ can readily be calculated from the concept that a magnetic moment $\Delta\mu$ is constructed by a current, $\Delta I = J_{cg} \Delta Z \Delta x$, embracing an area $A = 2 xY$, hence $\Delta\mu = (\Delta I) A$, consequently,

$$M_g = \frac{\mu}{V} = \frac{\int d\mu}{2XY\Delta Z} = \frac{J_{cg}}{X} \left[\int_0^{x_i} x dx - \int_{x_i}^X x dx \right] = \frac{J_{cg}}{X} \left\{ x_i^2 - \frac{X^2}{2} \right\} = H_{cg} \left\{ \left(\frac{x_i}{X} \right)^2 - \frac{1}{2} \right\} \quad 6.6$$

where $H_{cg} = J_{cg} X$.

Fig 6.7A(a) displays $H_r(x_i)/H_{r,max}$ versus $M_g(x_i)/M_{g,max}$ for various values of $d = (X' - X)/X$. Generally the contact between grains are regarded to be ≈ 1 nanometers hence for a grain of half width $X \approx 5$ microns, $d = 2(10^{-4})$. It is of interest to compare the maximum magnitudes of the return field and the magnetization. This occurs when $x_i/X = 0$ or 1 hence eqn 6.4 and 6.6 lead to,

$$\frac{|H_{r,max}|}{|M_{g,max}|} = \frac{\Delta Z}{\pi X} \ln \left\{ \frac{d+1}{\sqrt{d(d+2)}} \right\} \approx 0.25 \quad 6.7$$

where we take $\Delta Z \approx 1\mu m$

Grains of YBCO and BSCCO exhibit current densities in the ab plane which are in the range of 10^{11} to 10^{12} A/m² at 4.2K, hence $\mu_0 M_g$ in the range of Teslas have been observed for grains. Indeed the estimate of return fields in the range of a Tesla from eqn 6.7 is consistent with the observations of Kwasnitza and Widmer of $\mu_0 M_g$ of a Tesla or more for their specimen.

B. Thin Disk: Application of Biot-Savart Law

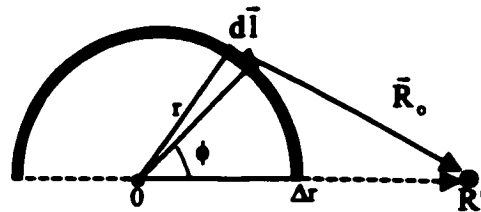
We calculate the return field along the edge of a thin disk of thickness ΔZ and radius R filled with induced persistent currents of uniform density J_{cg} circulating clockwise in an outer zone $r_i \leq r \leq R$, and counterclockwise in the inner zone $0 \leq r \leq r_i$. To fix ideas we write the Biot-Savart law

$$\Delta \vec{H}_r = \frac{\Delta I}{4\pi} \oint \frac{d\vec{l} \times \vec{R}_o}{R_o^3} \quad 6.8$$

for an elemental ring, of width Δr and height ΔZ , situated in the outer zone. The element of current reads $\Delta I = J_{cg} \Delta r \Delta Z$ and the element of length reads,

$$d\vec{l} = \hat{\phi} r d\phi = r \{\hat{x} \sin \phi - \hat{y} \cos \phi\} d\phi \quad 6.9$$

We calculate ΔH_r at a position R' along the +x axis with $R' > R$. The vector $\vec{R}_o$ directed from $d\vec{l}$ to the position R' then reads,



$$\vec{R}_o = \hat{x}(R' - r \cos \phi) - \hat{y} r \sin \phi \quad 6.10a$$

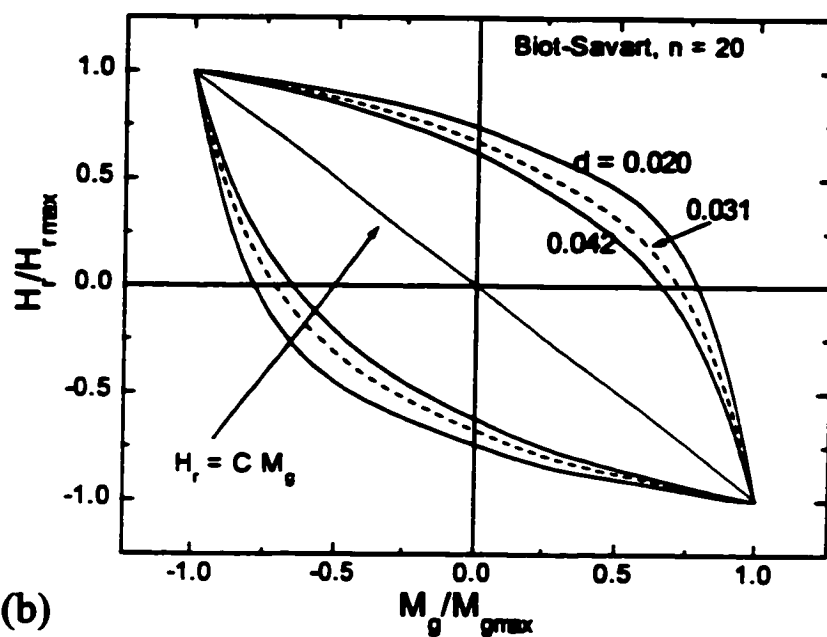
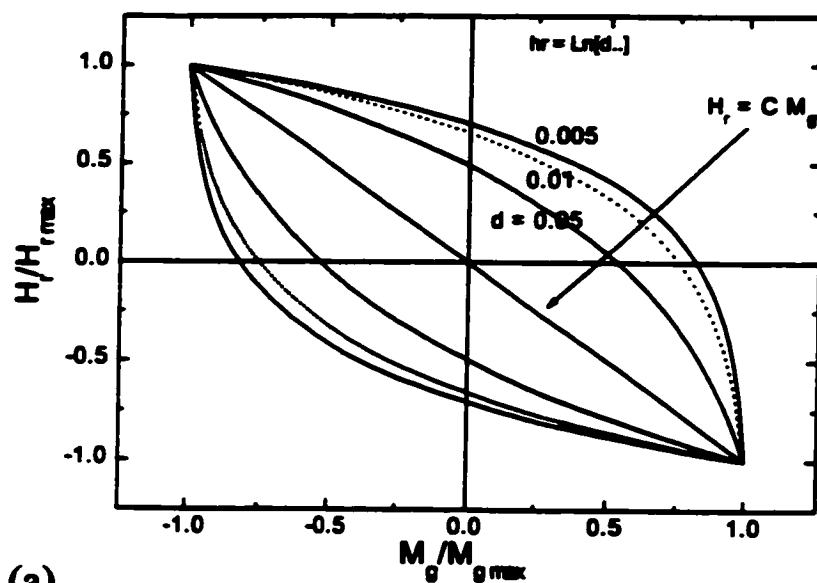


Fig 6.7A. Displays calculated dependences of the return field $H_r/H_{r,max}$ on the magnetization $M_g/M_{g,max}$ at different distances from the edge of the specimen. (a) Ampere's law is applied to an infinitely long thin strip of half width X . Here $d = (X' - X)/X$. (b) The law of Biot-Savart is applied to a disk of radius R . Here $d = (R' - R)/R$. The disk was divided into 20 concentric rings of equal width. J_g is taken independent of field in both cases. Quantities are normalized to their respective maxima.

hence its magnitude reads,

$$R_o = \{r^2 + R'^2 - 2rR' \cos \phi\}^{3/2} \quad 6.10b$$

The ring then generates a contribution $\Delta H_r(R', r)$ to the total return fields $H_r(R')$ of the disk, which reads,

$$\hat{z} \Delta H_r(R', r) = \hat{z} J_\alpha \frac{\Delta Z \Delta r}{2} \frac{r}{(R')^2} \int_0^\pi \frac{\left(\frac{r}{R'} - \cos \phi\right) d\phi}{\left\{1 + \left(\frac{r}{R'}\right)^2 - 2\left(\frac{r}{R'}\right) \cos \phi\right\}^{3/2}} \quad 6.11$$

The disk is divided into $N = R/\Delta r$ rings and the calculations are performed numerically as the boundary r_i between the concentric zones of countercirculating currents is made to migrate from $r = R$ to $r = 0$.

The corresponding magnetization of the disk is calculated from,

$$M_s(r_i) = \frac{J_\alpha R}{3} \left\{ 2 \left(\frac{r_i}{R} \right)^3 - 1 \right\} \quad 6.12$$

obtained by applying the definition

$$\bar{M}_s = \frac{1}{2} \int (\bar{r} \times \bar{J}_\alpha) dV \quad 6.13$$

where $V = \pi R^2 \Delta Z$.

Fig 6.7A(b) displays $H_r(r_i)/H_{r \max}$ versus $M_s(r_i)/M_{s \max}$ for comparison with the curves displayed in Fig 6.7A(a).

C. Thin Ribbon: Application of Formulae of Brandt-Indenbom

Brandt and Indenbom^[19] developed expressions describing the magnetization and the return field at the edge of thin infinitely long strips magnetized by a magnetic field

applied $\perp$ to the flat surfaces. The current density J_{c2} is taken independent of H . The crucial feature in their result is that concentric zones occur where the induced (and transport) current densities span the range $0 \leq J \leq J_{c2}$, hence where J is subcritical.

The expressions which we develop from their central formula for the return field are the following where $H_c = J_{c2} \Delta Z / \pi$.

$$\frac{H_r(X')}{H_c} = \tanh^{-1} \left[\frac{\tanh(H_s / H_c)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh(H_s / H_c)} \right)^2 \right\}^{1/2}} \right] - \frac{H_s}{H_c} \quad 6.14a$$

which applies when H_s is first impressed after zero field cooling. Then when H_s reverses direction we write,

$$\begin{aligned} \frac{H_r(X')}{H_c} = & \tanh^{-1} \left[\frac{\tanh(H_{\max} / H_c)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh(H_{\max} / H_c)} \right)^2 \right\}^{1/2}} \right] - 2 \tanh^{-1} \left[\frac{\tanh\left(\frac{H_{\max} - H_s}{2H_c}\right)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh\left(\frac{H_{\max} - H_s}{2H_c}\right)} \right)^2 \right\}^{1/2}} \right] \\ & - \frac{H_s}{H_c} \end{aligned} \quad 6.14b$$

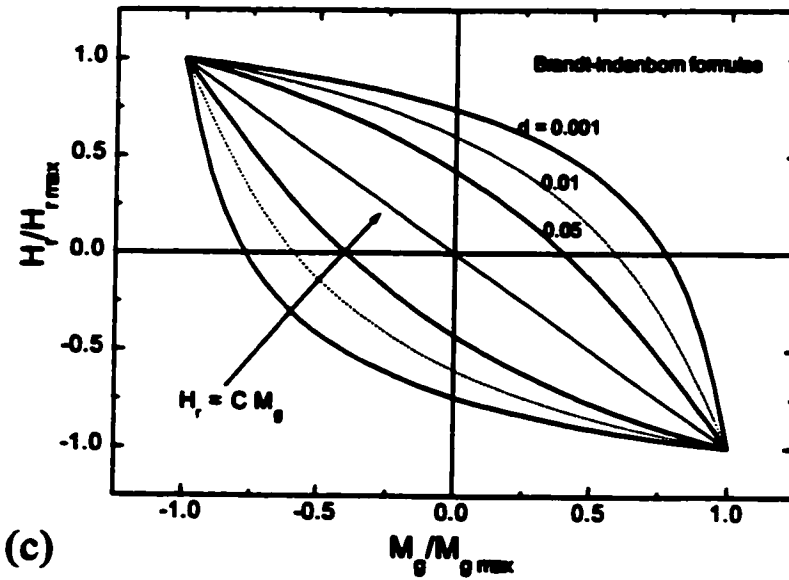
which applies when H_s is descending from $H_{\max}$. Then when H_s attains a value $H_{\min}$ where the direction of the swing is again reversed we write,

$$\begin{aligned}
\frac{H_r(X')}{H_c} = & \tanh^{-1} \left[\frac{\tanh(H_{\text{max}}/H_c)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh(H_{\text{max}}/H_c)} \right)^2 \right\}^{1/2}} \right] - 2 \tanh^{-1} \left[\frac{\tanh\left(\frac{H_{\text{max}} - H_{\text{min}}}{2H_c}\right)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh\left(\frac{H_{\text{max}} - H_{\text{min}}}{2H_c}\right)} \right)^2 \right\}^{1/2}} \right] \\
& + 2 \tanh^{-1} \left[\frac{\tanh\left(\frac{H_s - H_{\text{min}}}{2H_c}\right)}{\left\{ 1 - \left(\frac{1}{(X'/X) \cosh\left(\frac{H_s - H_{\text{min}}}{2H_c}\right)} \right)^2 \right\}^{1/2}} \right] - \frac{H_s}{H_c}
\end{aligned}$$

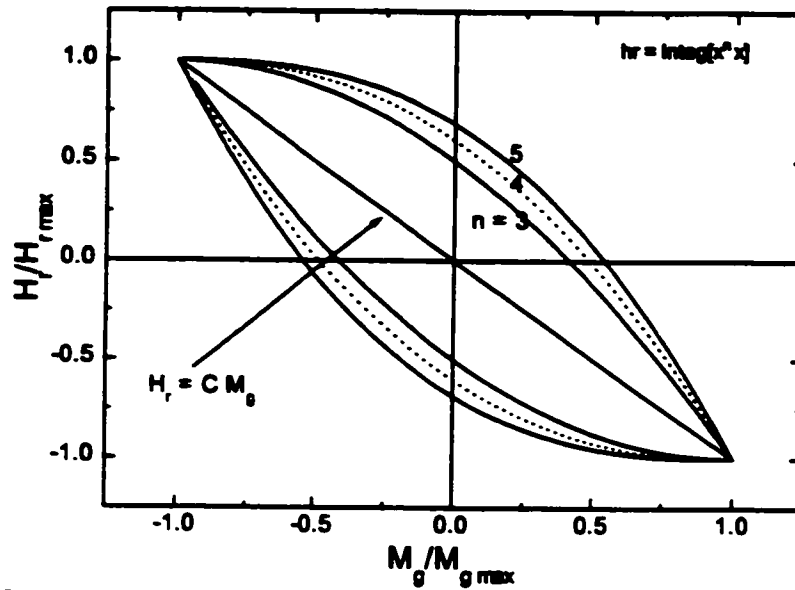
6.14c

The corresponding expressions for the evolution of the magnetization are listed in appendix 4.B.

Fig 6.7B(c) displays $H_r(X', H_s)/H_{r \text{ max}}$ versus $M_g(H_s)/M_{g \text{ max}}$ for various values of X'/X for comparison with the curves shown in the preceding figure. In all these figures the diagonal shows H_r versus M_g at a point along X very distant from the specimen hence corresponds to the linear approximation $H_r = C M_g$.



(c)



(d)

Fig 6.7B. Displays calculated dependences of the return field $H_r/H_{r,max}$ on the magnetization $M_g/M_{g,max}$ at different distances from the edge of the specimen. (c) The formulae of Brandt-Indenbom are applied to an infinitely long thin strip of half width X . Here $d = (X' - X)/X$ and although J_{cg} is independent of H , zones exist where J_{cg} is subcritical. (d) Simple empirical expressions described in the text for the return field of either a thin disk or an infinitely long thin strip are applied in these calculations. Here the exponent n is linked to the distance from the specimen where H_r is calculated but there is no explicit relationship.

D. Thin Ribbon and Disks: Empirical Expressions for H_r .

Since the return field and the magnetization are dictated mainly by the induced currents near the edges of the grains we have examined the behaviour generated by simple expressions for H_r which are analogous to that which describe the corresponding magnetization of thin ribbons and disks. Consequently in analogy with the formulae already encountered for M_g which read, for strip and disk geometries,

$$\frac{M_g(x)}{J_{cs}X} = \frac{1}{X^2} \left[\int_0^{x_i} x dx - \int_{x_i}^x x dx \right] \quad 6.15a$$

$$\frac{M_g(r_i)}{J_{cs}R} = \frac{1}{R^3} \left[\int_0^{r_i} r^2 dr - \int_{r_i}^R r^2 dr \right] \quad 6.15b$$

we write, for the return field for strip and disk geometries,

$$\frac{H_r(x_i)}{J_{cs}X} = \frac{C}{X^{2+n}} \left[\int_0^{x_i} x^{1+n} dx - \int_{x_i}^x x^{1+n} dx \right] \quad 6.16a$$

$$\frac{H_r(r_i)}{J_{cs}R} = \frac{C}{R^{3+n}} \left[\int_0^{r_i} r^{2+n} dr - \int_{r_i}^R r^{2+n} dr \right] \quad 6.16b$$

where $n \geq 0$.

These expressions lead to,

$$\frac{H_r(x_i)}{J_{cs}X} = \frac{C}{n+2} \left\{ 2 \left(\frac{x_i}{X} \right)^{n+2} - 1 \right\}, \quad \frac{H_r(r_i)}{J_{cs}R} = \frac{C}{n+3} \left\{ 2 \left(\frac{r_i}{R} \right)^{n+3} - 1 \right\} \quad 6.17a,b$$

which reduce to the linear magnetic dipole relation, $H_r = C M_g$, when $n = 0$.

Fig 6.7B(d) displays $H_r(x_i)/H_{r \max}$ versus $M_g(x_i)/M_{g \max}$ calculated with various values for n using eqns 6.15 and 6.16. Comparison of these curves with that calculated using the three previous approaches and displayed in Fig 6.7(a), (b) and (c) reveals that

this simple empirical model yields results very similar to those obtained exploiting well known basic laws. Indeed all four schemes provide equally good descriptions of the observations of Kwasnitza and Widmer reproduced in Fig 6.2a and 6.6a. Hence all four schemes provide information on the magnitudes of the return fields threading the contacts between the grains and the dependence of H_r on the configuration of the induced magnetization currents near the contact edges of the grains.

IV Predictions of the Model

A. Evolution of the curves of I_c versus M_g as a Function of H_r .

Kwasnitza and Widmer^[33] measured $I_c(H_r)$ versus H_a at $H_r = 0$ and at a large final field, namely $\mu_0 H_r = 0.8$ Tesla. The four models we have developed to calculate H_r for the circumstances employed in their experiment all quite satisfactorily reproduce the two sets of observations these researchers reported. The experimental and theoretical curves were compared in Fig 6.2 and 6.6. The theoretical models also make predictions regarding the behaviour of $I_c(H_r)$ versus $M_g(H_r)$ at intermediate fields between $H_r = 0$ and H_r "large" as well as beyond H_r "large". Fig 6.8 displays such a sequence of predicted behaviour for the evolution as a function of H_r of the curves of $I_c(H_r)$ versus $M_g(H_r)$ for paths 1 and 2.

We stress that the four models described in the preceding section predict essentially the same evolution in the curves of $I_c(H_r)$ versus $M_g(H_r)$ as a function of H_r , as displayed in Fig 6.8. This is expected since the four models generate very similar curves for H_r versus M_g as illustrated in Fig 6.7A and B. The dashed curves in Fig 6.8 display the behaviour predicted by the linear dipole (single valued) approximation $H_r = C M_g$.

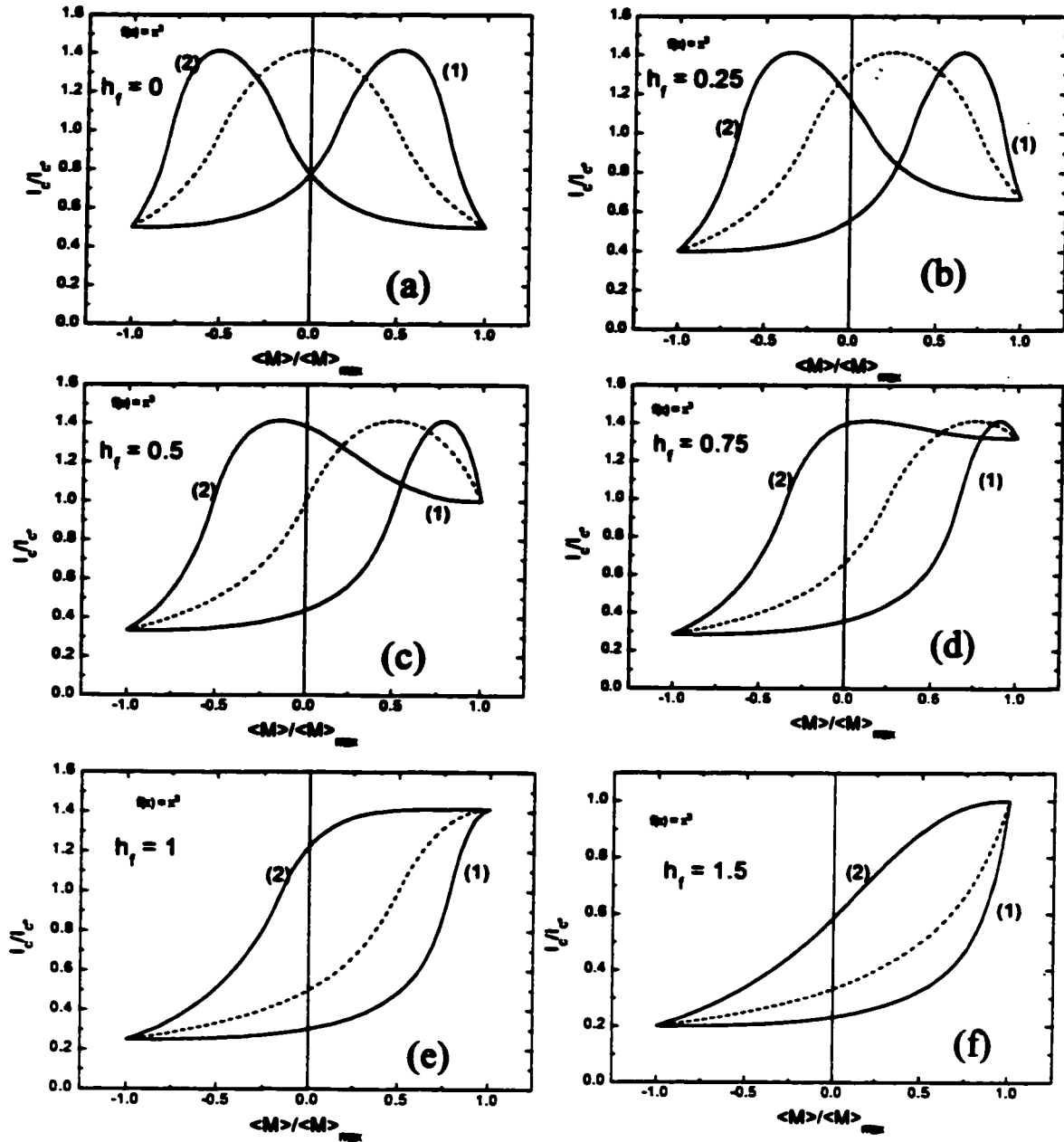


Fig 6.8. Kwasnitza and Widmer^[83] measured the dependence of I_c on the magnetization M_g at $\mu_0 H_f = 0.8$ Tesla, hence at a “large” field, and at $H_f = 0$. Figs 6.2 and 6.6 display their data at these two “extremes” and corresponding theoretical curves. The latter are reproduced in (a) and (f) above. The sequences of (a), (b), (c), (d), (e) and (f) illustrate the evolution of the double-valued I_c versus M_g curves as a function of H_f predicted by our models for H_f along the edge of the specimen. The dashed curves display the behaviour for $H_f = C M_g$.

The theoretical curves of Fig 6.8 were calculated using the simple Kim approximation, $J_{cm} = J_{co}(H_1/H)$, for the intergranular critical current density. Qualitatively similar behaviours are generated using $J_{cm} = J_{co}(H_1/H)^{1/2}$. Here, as expected from the weaker dependence of J_{cm} on H , the spreads in the variations of I_c are narrower than that encountered for the Kim case.

B. Evolution of the Curves of I_c versus H_{rev} as a Function of H_f .

In Fig 6.5b we displayed the data of Kwasnitza and Widmer on the behaviour of I_c at $H_f = 0$ as a function of the remanent trapped flux produced by reversal of the sweep of H_a at various values, denoted $B_r = \mu_0 H_{rev}$, along the envelope of the major symmetric hysteresis curve (see Fig 6.4a). In the discussion in section II C we noted that, barring strong bias magnetic fields, the image of their I_c versus H_{rev} curve would be observed if path 2 had been traced prior to the measurements of I_c at $H_f = 0$. In Fig 6.5c we displayed our modeling of their experimental curve shown in Fig 6.5b. Now in Fig 6.9a we reproduce again this calculated curve and also display its theoretical mirror image. The labeling of the families of paths corresponds to that of Fig 6.4a and b.

The evolution of the curves of I_c versus H_{rev} , as a function of the final H_f ranging from 0 to large values, which our model predicts is displayed in fig 6.9. Again, these curves are insensitive to the choice among our four models for H_f versus M_g . Indeed a similar family of curves is obtained using the linear approximation $H_f = C M_g(H_a)$. The curves in Fig 6.9 were calculated assuming the Kim approximation $J_{cm} = J_{co}(H_1/H)$. Again, qualitatively similar behaviours are obtained using, $J_{cm} = J_{co}(H_1/H)^{1/2}$. Here, as

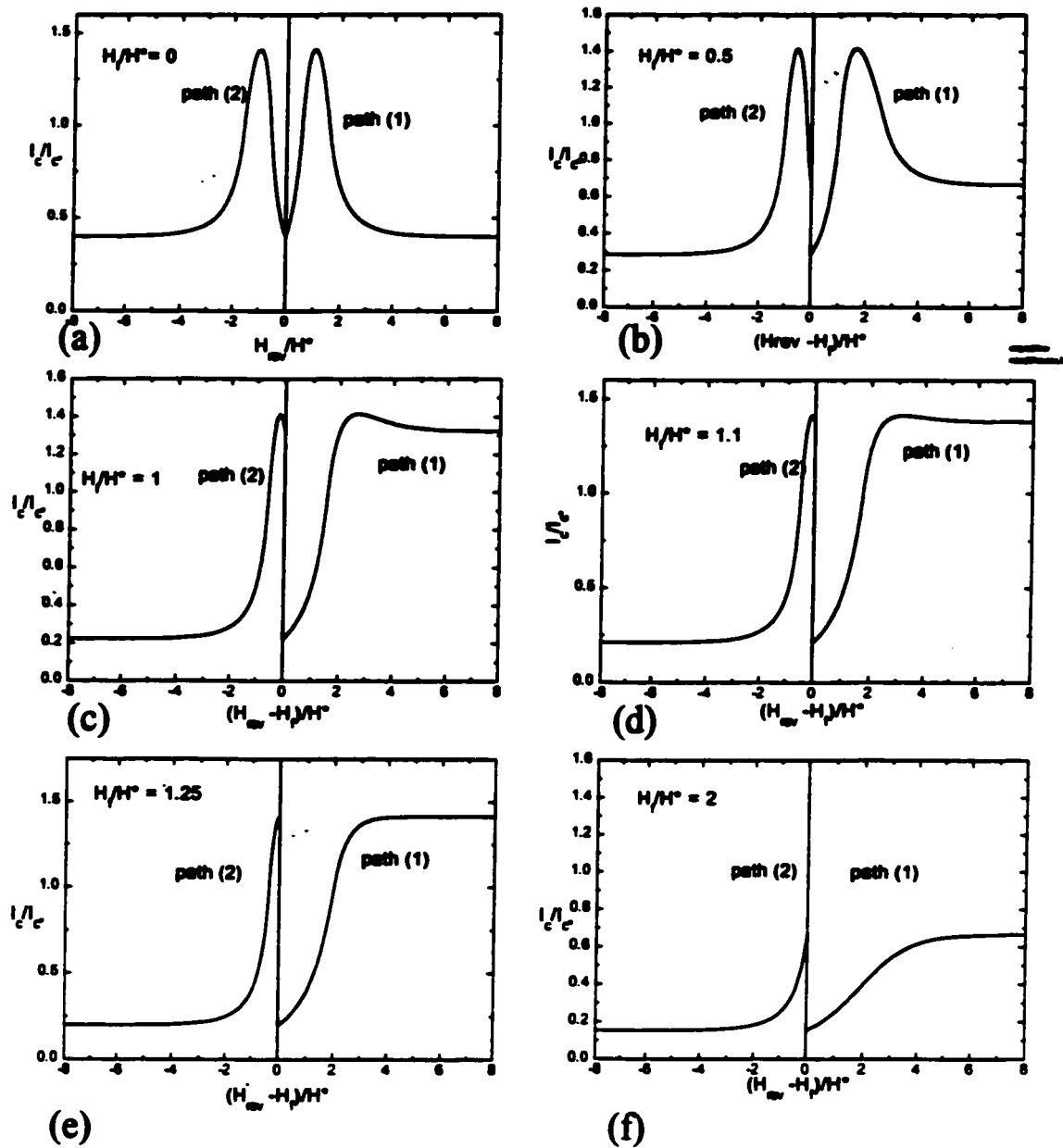


Fig 6.9. Displays the evolution of the dependence on H_r of the curves of I_c versus the reversal fields H_{rev} which is predicted by our four models for H_r near the edge of the specimens and also taking $H_r = C M_p$. These curves are insensitive to the approach exploited in calculating H_r . Unfortunately the only measurements available in the literature to compare with these predictions are those of Kwasnitza and Widmer^[83] at $H_r = 0$ reproduced in Fig 6.5(b).

expected from the weaker dependence of J_{cm} on H , the spreads in the variations of I_c are narrower than that encountered for the Kim case.

It is unfortunate that Kwasnitza and Widmer did not tabulate the reversal fields associated with their measurements of I_c versus M_g at $\mu_0 H_f = 0.8$ Tesla by paths 1 and 2. As a consequence we have no corresponding data to compare with the high field theoretical curves of Fig 6.9.

V Comparison of H_r dipole and H_r edge versus H_a

As we conclude this chapter which studies the behaviour of I_c in circumstances where H_r is clearly a double-valued function of M_g , the reader can quite correctly question the validity of the messages developed in the two preceding chapters where we focused on and applied the simple linear approximation $H_r(H_a) = C M_g(H_a)$ to describe several features of I_c hysteresis phenomena. It is therefore important that we comment on this dichotomy.

In chapters 4 and 5 we focused on experimental observations of I_c hysteresis behaviour where the measurements were carried out and reported as a function of H_a for a variety of “standard” $H_a - T$ histories (eg. ZFCV, ZFCD, FCV, FCD, ZFCrAs, FCrAs). To pursue this analysis we exploited simple well known schemes to describe the evolution of $M_g(H_a, T)$ for this variety of $H_a - T$ histories. Then to describe the associated return fields as a function of these $H_a - T$ histories we introduced the universally accepted simple magnetic dipole picture where $H_r(H_a) = C M_g(H_a)$.

In this chapter however, we have focussed on the return field $H_r(H_a, T)$ along the edges of the grains. We estimated these return fields and their relationship with the

corresponding evolution of $M_g(H_a, T)$ and found H_r , hence I_c , to be double-valued functions of M_g for opposite pairs of paths in the $M_g - H_a$ plane terminating at the same H_r and the same M_g in magnitude and sign. Note that the double-valued feature of H_r comes into play in the situations where M_g is made to migrate from one envelope of a major or minor hysteresis curve towards the opposite.

To place these two different ways of estimating H_r in perspective it is useful to compare the curves for the behaviour of $M_g(H_a)$ {denoted $H_{r, \text{dipole}}(H_a) = C M_g(H_a)$ } with the corresponding curves for $H_r(H_a)$ calculated along the edge of the grains, denoted $H_{r, \text{edge}}(H_a)$. Such comparisons are displayed in Fig 6.10. Here we show $M_g(H_a)/M_{g, \text{max}}$ and $H_{r, \text{edge}}/H_{r, \text{edge max}}$ for the basic $H_a - T$ histories, hence the basic I_c hysteresis curves pursued in experiments (ZFC, ZFCD, ZFC reAS, FCD, FCreAS).

The reader will note that corresponding curves are qualitatively similar but that the $H_{r, \text{edge}}$ curves rise and descend more steeply than the related M_g curves. Consequently the curves for the evolution towards plateaus and other behaviour as a function of H_a which we have examined in chapters 4 and 5 are only slightly different if $H_{r, \text{edge}}$ is introduced in the modeling. We have seen already however that the different models for $M_g(H_a)$ lead to curves which differ from each other semi-quantitatively. Consequently, the detailed descriptions of the evolutions of various features can be viewed as qualitative only. Indeed, because of the inevitable inaccuracies in the measurements of I_c , the data curves generally only provide a qualitative picture of the evolution towards “plateaus”. Consequently we finally focused on “plateau” values, hence saturation states for the magnetization where $H_{r, \text{edge}}$ is single valued, to obtain reliable quantitative information on the return fields. At these “plateaus”, M_g lies along the envelopes of the hysteresis curves.

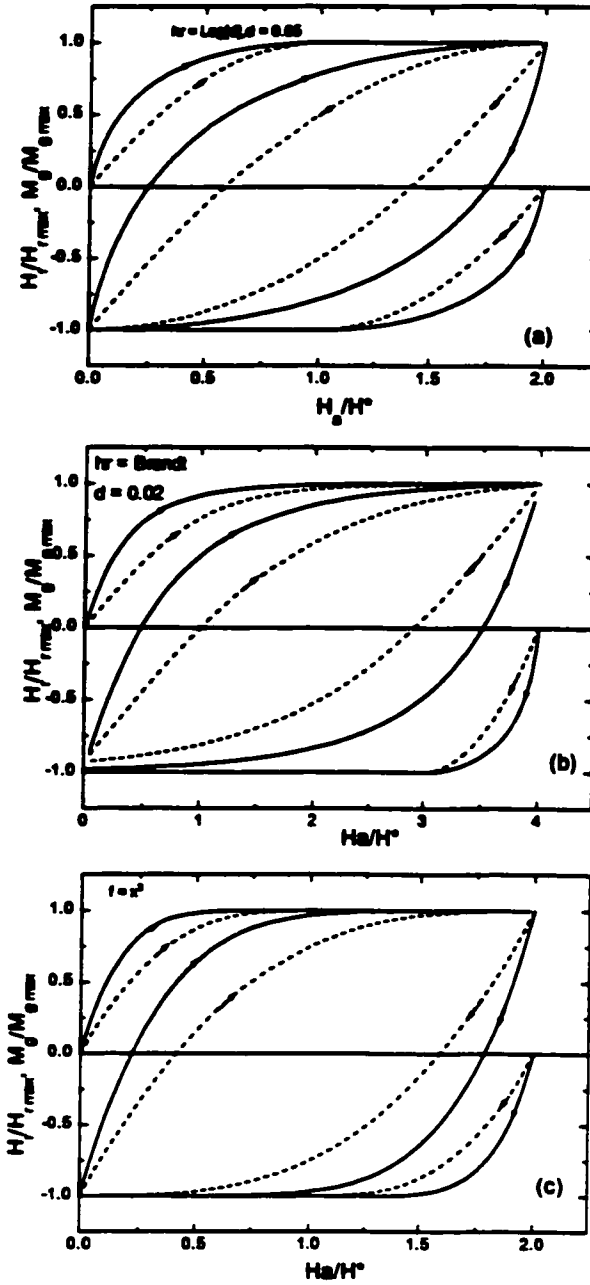


Fig 6.10. Illustrates and compares the evolution versus H_a for $M_g/M_{g,max}$, hence $H_{r,dipole} = C M_g$ (dashed curves), and, $H_{r,edge}/H_{r,max}$ (solid curves) for the ZFCV, ZFCD, ZFCrAs, FCD and FCrAs procedures. The curves for H_r move towards (away from) the corresponding curve for M_g as d is made larger (smaller) in (a) and (b) and n chosen larger (smaller) in (c). (a) Calculation for thin infinite slab at $d = 0.05$ using Ampere's law. (b) Calculations at $d = 0.02$ using the Brandt-Indenbom formulae. (c) Calculation for a slab using empirical formulae with $n = 3$. Calculations for a disk using Biot-Savart are very time consuming and were not pursued.

Here the induced currents in the grains all circulate in the same sense, hence $H_{r, \text{edge}}$ is single-valued, and thus $H_{r, \text{edge}} = H_{r, \text{dipole}}$. Therefore the central messages of chapters 4 and 5 are not modified if we now introduce the models for $H_{r, \text{edge}}$ in the calculations. Since the observations discussed and analyzed in chapter 4 and 5 are not sensitive to, and do not discriminate between, the choice of a single-valued $H_{r, \text{dipole}}$, or a double-valued function $H_{r, \text{edge}}$ we elected there, for simplicity, to apply the "traditional" linear dipole approximation $H_r = C M_g$.

VI Summary and Conclusion

Kwasnitza and Widmer investigated the dependence of the critical current I_c in granular strips of YBCO on M_g , the magnetization of the grains in a final field, $H_f = 0$, and a large final field $\mu_0 H_f = 0.8$ Tesla. In both cases they observed that, magnetizations of the same magnitude and polarity, but generated by opposite paths in the $M_g - H_a$ plane, led to very different values for I_c in the same chosen H_f . These results indicate unambiguously that H_r , the return field of the magnetized grains is a double-valued function of the magnetization when M_g is migrating from an envelope of a hysteresis curve towards the opposite. Under these circumstances two concentric zones of countercirculating currents are generated in the grains. In these situations the ideal dipole linear approximation $H_r = C M_g$ is not correct.

Applying Ampere's law to a thin infinitely long strip and the Biot-Savart eqn to a thin disk we develop expressions for H_r corresponding to the magnetizations which comprise two concentric zones of oppositely circulating persistent currents. Formulae developed by Brandt and Indenbom for thin infinitely long strips are extended to describe

H_f and M_g for these configurations of current flow. We also propose empirical expressions for H_f analogous to the well known formulae for the magnetization of thin strips and thin disks sustaining countercirculating patterns of induced currents. All of these simple prescriptions successfully describe the dramatic observations of Kwasnitza and Widmer for the double-valued behaviour of curves of I_c versus M_g at $H_f = 0$ and at a large H_f . Consequently we pursue our model to make predictions for the behaviour of curves of I_c versus M_g at intermediate values for H_f . We also make predictions on the behaviour as a function of H_f of the curves of I_c versus the reversal fields H_{rev} in the field cycles leading to the final magnetizations. Here however the only data available for comparison with our family of theoretical curves are the unique measurements of Kwasnitza and Widmer at $H_f = 0$.

Chapter 7

General Concluding Remarks

A. Research Reported in this Thesis

In this closing chapter, as we summarize the main message of this thesis, we endeavour to delineate clearly the ideas which already exist in the literature and our new original contributions to the understanding of magnetothermal and critical current behaviour of weakly-linked granular type II superconductors.

The monotonic expulsion and reentry of magnetic flux during slow cooling from T_c and rewarming to T_c of single crystals of type II superconductors in static applied magnetic fields has been examined theoretically by several workers^[25, 34, 66, 67, 96]. The models these workers present to account for the observations are conceptually identical and the mathematical formulations very similar. For completeness and clarity we have presented this standard background picture and a simple mathematical framework in some detail. The enhancement of the flux expulsion, hence of the Meissner effect, upon recooling from T below T_c has not been previously accounted for in the literature. Explanation of this behaviour and the observed subsequent growth of the diamagnetism to a final “stable” thermal hysteresis curve are novel developments by us of existing ideas. Consequently the various detailed qualitative and quantitative predictions we generate with our model regarding specific features of the “ultimate” diamagnetic hysteretic magnetothermal curves are entirely new and could stimulate experimental study.

The observation by several workers^[66, 73, 149] of a strong and sharp enhancement of the diamagnetism of weak-linked granular type II superconductors near T_c , upon slow rewarming after fast cooling in static applied fields, has not previously been formally modeled either qualitatively or quantitatively. The researchers who have reported on this phenomenon have discussed possible explanations for this behaviour but no detailed physical and mathematical model were put forward. The simple framework we develop mathematically, which incorporates, our extension of the established model for the magneto-thermal behaviour of single crystals in static applied magnetic fields, into a weak-linked medium (matrix), is new and original. We show that this simple model successfully reproduces a panoply of experimental curves and a variety of features and trends exhibited by these measured curves.

This model also accounts for the progressive growth of the diamagnetism of weak-linked granular specimens subjected to thermal cycling below T_c reported but not explained by Hyun^[66] or by other workers. Further we indicate that the model predicts the generation of “ultimate” stable thermal hysteresis loops after extensive cycling. Publication of details of such predictions may encourage experimental verification.

All workers examining the phenomenon of I_c hysteresis in weak-linked high T_c superconductors have pursued the Evetts-Glowacki concept^[48] in the framework that the return field H_r of the grains is linked to the evolution of their magnetization via the linear dipole approximation where $H_r(H_a) = C M_g(H_a)$. In chapter 4 we apply this simple picture and show how a pictorial display of the curves of H_r , hence M_g , versus H_a , and the intersection of these curves with the $-H_a$, line readily accounts qualitatively and quantitatively for a large variety of features in the evolution of the position of the peaks

in the curves of I_c versus H_a . Application of such a pictorial approach is new, fruitful, and pedagogically useful. Most importantly however, we find that measurements of the location of the peaks of I_c , when threshold values for H^{cycle} and H^{cool} are exceeded, unambiguously lead to determination of the compression coefficient C . This is obtained quite simply by taking the ratio of $H_{\text{plateau}}^{\text{ZPCD}} / H_{\text{plateau}}^{\text{ZPCoAS}}$ which is predicted to be the same as the ratio $H_{\text{plateau}}^{\text{PCD}} / H_{\text{plateau}}^{\text{PCoAS}}$. Unfortunately we find that only one measurement of the latter ratio has been published in the literature. We hope that publication of our theoretical analysis will serve as a guide to future experimental work.

All theoretical examinations of the structure and evolution of the peaks of I_c versus H_a , except the analysis of Mishra et al^[102], exploited the same crude formulae distantly and artificially related to the Josephson junction Fraunhofer diffraction pattern. In chapter 5 we apply the critical state current density concept to develop simple formulae to describe the dependence of I_c on H_a for an idealized slab specimen.

Families of formulae for I_c versus H_a , are then developed using different generic empirical expressions for $J_c(H_a)$. Focussing on one especially simple member of this panoply of critical state equations we show that this framework readily reproduces a vast assortment of experimental observations on the structure of and behaviour of hysteretic I_c curves^[36, 40, 50, 52, 59, 83, 91, 102, 116, 156, 157]. In particular we show that the “Josephson” approach applied by List et al^[91] in their analysis of their measurements of the effect of trapped flux on I_c leads to conclusions regarding the temperature dependence of the grains and junctions which are in disagreement with other data^[156] and with the results of our critical state model.

Kwasnitza and Widmer^[83] measured the dependence of I_c versus both H_a and versus M_g . Their plots of I_c versus M_g revealed that two appreciably different values for $I_c(H_f)$ were encountered for values of $M_g(H_f)$ of the same magnitude and sign lying between the limits of hysteresis loops. This shows most dramatically and unambiguously that the linear dipole assumption, $H_r = C M_g$, is not exact when M_g is made to migrate away from the hysteresis envelopes. This failure of the dipole approximation remains hidden however when data are plotted versus H_a , eg. H^{cycle} , H^{cool} , H_{reversal} , etc.

In chapter 6 we introduce four simple methods for calculating the return field at the edge of thin strips and thin disks as a function of their magnetization. We show that the curves of $H_{r, \text{edge}}$ versus H_a , and of the corresponding $H_{r, \text{dipole}} = C M_g$ versus H_a , are very similar (although the ascents and descents are always initially much steeper in the former). As a consequence of the similarity of these curves, analysis of I_c hysteresis behaviours as a function of H_a does not discriminate between these two approaches.

We find that, all of the four approaches we have explored to estimate $H_{r, \text{edge}}$ as a function of M_g , are equally successful in reproducing the double-valued I_c versus M_g curves measured by Kwasnitza and Widmer at $H_f = 0$ and H_f large. Consequently we make detailed predictions on the evolution of these double-valued curves at intermediate values of H_f . We also make predictions for the corresponding behaviour of the curves of I_c versus the reversal fields H_{rev} over the range of H_f from 0 to large values. Again, hopefully such predictions may invite experimental investigation.

Kwasnitza and Widmer did not present an analysis or model for their observations and none has subsequently appeared in the literature. Consequently the account we present in chapter 6 is completely new work.

B. Published Research in which I Participated.

Research I have pursued in collaboration with other graduate students under the supervision of Dr. Marcel LeBlanc is not included nor referred to in this thesis, hence I mention this work here and the role I played.

- (i) "Generation of quasi-reversibility in a commercial Bi:2223/ A_g tape by vortex shaking with varying orthogonal magnetic fields" M. A. R. LeBlanc, S. Celebi and M. Rezeq, *Physica C* 361, 251-259 (2001). This paper reports on novel experimental results and discusses their meaning. I assisted in all of the measurements.
- (ii) Comment on "Angle dependence of magnetization in a single-domain $YBa_2Cu_3O_x$ sphere" M. A. R. LeBlanc, Moh'd Rezeq, Daniel S. M. Cameron and David LeBlanc, *Phys. Rev. B* 61, 3745 – 47 (2000).

A group from the University of Cincinnati and Brookhaven National Laboratory, B. A. Tent, D. Qu, Donglu Shi, W. J. Bresser, P. Boolchand and Zhi - Xiong Cai, *Phys. Rev. B* 58, 11761-67 (1998) reported on their extensive measurements and presented a detailed model for their results. The formulae which the authors developed from their model contained errors which led to strange extra features in their calculated curves. The correct expressions were developed by Dr. LeBlanc and the families of curves calculated with these reproduced the data. Dr. LeBlanc, placed my name second on this paper in recognition of the major role I played in the analysis and computations.

C. Research Completed but not Reported in this Thesis

The concepts and framework we have presented in this thesis have been extended and adapted to model observations of the behaviour of weak-linked granular type II superconductors in a variety of other situations. This analysis and the computations are essentially completed. Dr. LeBlanc however decided that I didn't need to include this work in this already long thesis, and recommended however that I should briefly describe their nature.

(i) Structure of $\langle M \rangle_{\text{rem}}$ versus H^{cycle} and H^{cool} procedures.

Many workers^[74, 75, 102, 104, 105, 107] have observed that $\langle M \rangle_{\text{rem}}$, the remanent magnetization of weak-linked granular type II superconductors, whether low or high T_c , displays sometimes quite dramatic peaks just before tracing common plateaus as a function of H^{cycle} and H^{cool} . By contrast single crystals exhibit monotonic rises to common plateaus. The peak structure is attributed to the effect of the return field of the magnetized grains on the currents induced to circulate through the intergranular network. Our simple model based on this idea reproduces the extensive variety of observations.

(ii) Peaks in dI_c/dt versus $\langle M \rangle_{\text{rem}}$

Altshuler et al^[2] have found that the critical current in high T_c polycrystalline ribbons permeated with trapped flux increases with time. dI_c/dt , the time rate of increase, exhibits a sharp peak as a function of $\langle M \rangle_{\text{rem}}$, the remanent magnetization. These workers present a mathematically complicated model with several ingredients to account for this behaviour. As a consequence the physics of the phenomenon remains unexplained.

Exploiting a simple critical state expression for I_c for an idealized slab and introducing the standard description of $\langle M \rangle_{\text{rem}}$ versus H^{cool} and H^{cycle} we reproduce their

observations and present predictions for a variety of related behaviour. In our model it is quite clear how the magnitude of $\langle M \rangle_{\text{rem}}$ affects the configuration of the intergranular current density in the specimen and thereby dictates its rate of change with time.

(iii) Expansion of Flux by Hollow Cylinders Cooling from T_c in Static $H//$

Workers have measured the fraction of the magnetic flux, initially threading the wall of a hollow cylinder of a polycrystalline type II superconductors immersed in a static field $H//$ directed along the axis, which is expelled into the cavity and to the outside world as the specimen cools from T_c . Exploiting a simple extension of our model presented in chapter 3 we can successfully reproduce the published observations and make predictions of a variety of behaviour expected as the relative magnitudes of the parameters $I_M(T)$, $J_{cg}(T)$ and $J_{cm}(T)$ are modified.

(iv) Summation of Parallel ΔI_c

Pursuing the simplified percolation picture exploited by several workers^[7, 13, 16, 118, 119, 120], where the total critical current I_c is a non-linear summation of contributions, ΔI_c , flowing along parallel paths each affected by a different return field, we have accounted for the intricate spectrum of FCD and FCrAs peaks in I_c versus H^{cool} observed by McHenry et al^[101]. Here the observed spectrum of FCD peaks descend dramatically in height and increases in width whereas the FCrAs peaks exhibit the same height and width as a function of H^{cool} over a broad range extending from 50 to 3000 Gauss.

D. Abstracts of posters presented at CAP Conferences

(i) CAP Congress, June 2-5, 2002, Laval University, Quebec, QC

1- Effect of Trapped Flux on the Critical Current of Weak-Linked Granular High T_c Superconductors. Moh'd Rezeq and M. A. R. LeBlanc, Physics Dept, U. of Ottawa, ON K1N 6N5- Several researchers[1-6] have studied the dependence of $I_{c, rem}$, the critical current in zero field of weak-linked high T_c superconductors, on the remanent flux density $\langle B_{rem} \rangle$ trapped in the grains of the specimen by $H_a = H_{cycle}$ in the zero field cooled (ZFC) and $H_a = H_{cool}$ in the field cooled (FC) procedures. $\langle B_{rem} \rangle$ trapped in the grains by these procedures has the polarity of H_{cycle} and H_{cool} . Kwasnitza and Widmer[5], after ZFC, followed a field cycling procedure which leaves the grains filled with two concentric zones of countercirculating persistent currents, hence generating configurations where a zone with $+B_{rem}(r)$ embraces one with $-B_{rem}(r)$. A simple critical state model for $I_{c, rem}$ versus $\langle B_{rem} \rangle$ based on the Evetts-Glowacki concept[6] reproduces the published results.

1. A. J. Batista-Leyva, R. Cobas, E. Estévez-Rams, M. T. D. Orlando, C. Noda and E. Altshuler, *Physica C* **331**, 57 (2000).
2. F. A. List, D. M. Kroeger, V. Selvamanickam, *Physica C* **275**, 220 (1997)
3. K. -H. Müller and D. N. Matthews, *Physica C* **206**, 275 (1993)
4. E. Altshuler, S. Garcia, J. Barroso, *Physica C* **177**, 61 (1991)
5. K. Kwasnitza and Ch. Widmer, *IEEE Trans. Magn.* **27**, 1202 (1991)
6. J. E. Evetts, B. A. Glowacki, *Cryogenics* **28**, 641-649 (1988)

2- Observations of a Valley in I_c vs H_a in Zero Field Cooled Weak-Linked High T_c Superconductors. M. A. R. LeBlanc and Moh'd Rezeq. Physics Dept, U. of Ottawa, ON, K1N 6N5- Several workers [1-6] have reported that the critical conduction current I_c sustained by granular high T_c specimens during the first application of the magnetic field H_a after zero field cooling (denoted ZFCV curves) traces a valley then a broad summit. An elaborate model by Fisher et al[1] attributes the valley to the effect of the reversible magnetization of the grains on the intergranular current density J_{cm} . Valley bottoms however occur at values of H_a larger than $H_{c1}(T)$. We present a simple critical state model for I_c where $J_{cm} \propto 1/(H_a + H_r)^n$, $n > 0$, where H_r is the return field of the grains magnetized by H_a . The magnetization of the grains, $M_g(H_a)$ is calculated for idealized slab and cylinder geometry with the intragranular current density $J_{cg} \propto 1/H_r^p$ with $p > 0$. This model exploiting the Evetts-Glowacki concept[6] reproduces the published results.

1. L. M. Fisher, N. V. Il'in, N. A. Podlevskikh and S. I. Zakharchenko, *Sov. St. Comm.* **73**, 687-690 (1990)
2. Uri Dai, Guy Deutsher, C. Lacour, F. Laher-Lacour, P. Mocaër and M. Laguës, *Appl. Phys. Lett* **56**, 1284-1286 (1990).
3. Y. Yang, C. Beduz, and S. P. Ashworth, *Cryogenics* **30**, 618-625 (1990)
4. M. Majoros, M. Polak, V. Strbik, S. Benaka, S. Chromik, F. Hanic and V. Plechacek, *Supercond. Sci. Technol.*, **3**, 227 (1990)

5. P. K. Mishra, G. Ravikumar, P. Chaddah, S. Kumar and B. A. Dasannacharya, *Jap. J. Appl. Phys.* **29**, L1612-1614 (1990)
6. J. E. Evetts, B. A. Glowacki, *Cryogenics* **28**, 641-649 (1988)

(ii) CAP Congress, June 17-21, 2001, University of Victoria, Victoria, BC.

1- Evolution of the Magnetization of Polycrystalline Type II Superconductors with Temperature in Static Magnetic Fields. Moh'd Rezeq and M. A. R. LeBlanc, *U. of Ottawa*, – The model of Clem and Hao^[1] and of Hyun^[2], which describes the evolution of the diamagnetic magnetization $\langle M \rangle$ of hysteretic isotropic single crystals of type II superconductors during slow cooling from T_c and the subsequent behaviour during slow warming in various static magnetic fields $H < H_{c1}$, $H > H_{c1}$ and $H < H_*$, $H > H_*$, the full penetration field, has been extended to account for the observation of $\langle M \rangle$ vs T for weak-linked granular high and low T_c materials in these circumstances (Jung et al^[3] and Wang and Joiner^[4]). The model also explains the appearance of a deep diamagnetic valley in the locus of $\langle M \rangle$ vs T during warming after slow and fast cooling which they report.

[1] J. R. Clem and Z. Hao, *Phys. Rev.* **B48**, 13774 (1993)

[2] O.B. Hyun, *Phys. Rev.* **B48**, 1244 (1993)

[3] J. Jung, M. A – K. Mohamed, I. Issac and L. Friedrich, *Phys. Rev.* **B49**, 12188 (1994)

[4] J. P. Wang and W. C. H. Joiner, *Phys. Rev.* **B50**, 1253 (1994)

2- Role of the Return Field of the Magnetized Grains in I_c Hysteresis of Polycrystalline High T_c Samples. M. A. R. LeBlanc and Moh'd Rezeq, *U. of Ottawa* – The effect of the magnetization $\langle M \rangle$ on the critical transport current (I_c hysteresis) of weak-linked granular YBCO plate-like samples at 4.2K was studied by Kwasnitza and Widmer^[1]. They observed two very different values for I_c in the same final applied field B_a after two different field-temperature histories where both led to zero net magnetization before I_c was introduced. A simple model which implements the proposal of Evetts and Glowacki^[2] that the superposition of the return field of the magnetized grains and the applied field B_a is responsible for the hysteresis in I_c , is shown, to account for the above apparently paradoxical observations.

[1] K. Kwasnitza and Ch. Widmer, *IEEE Trans. MAGN.* **27**, 1202 (1991)

[2] J. E. Evetts and B. Glowacki, *Cryogenics* **28**, 641 (1988)

Appendix 4.A

Magnetization Curves for an Idealized Slab with Surfaces // to $\vec{H}_a$

For completeness we present the expressions for the magnetization M_s for a grain with idealized slab (planar) geometry with H_a directed parallel to the surfaces situated at $x = \pm X$ and taking J_{cs} independent of the flux density. The boundaries of segments of the field profiles $H(x)$, are now labeled x_i , x_r , x_o instead of r_i , r_r and r_o . The magnetization can be calculated by focusing on the magnetic moment μ of a volume of height Z along H_a , length Y along $J_{cs} \perp$ to H_a and thickness X , and writing,

$$M_s = \frac{\mu}{XYZ} = \frac{J_{cs}}{X} \int_0^x x dx = H_{cs} \int_0^1 \left(\frac{x}{X} \right) d\left(\frac{x}{X} \right) \quad \text{A-1}$$

For all the temperature-field histories the profiles of the magnetic flux density are identical to that encountered for idealized cylinder geometry.

$$M_s^{ZFC} = -\frac{H_{cs}}{2} \left\{ 1 - \left(\frac{x_i}{X} \right)^2 \right\} = -\frac{H_{cs}}{2} \left[1 - \left\{ 1 - \frac{H_{s\uparrow}}{H_{cs}} \right\}^2 \right] \quad \text{A-2}$$

valid for, $0 \leq \frac{H_{s\uparrow}}{H_{cs}} \leq 1$, then $M_s = -\frac{H_{cs}}{2}$ when $H_{s\uparrow} \geq H_{cs}$.

For ZFCD situations where $H_{\max}^{\text{cycle}} \geq H_{cs}$,

$$M_s^{ZFCD} = \frac{H_{cs}}{2} \left\{ 1 - 2 \left(\frac{x_r}{X} \right)^2 \right\} = \frac{H_{cs}}{2} \left\{ 1 - 2 \left[1 - \left(\frac{H_{\max}^{\text{cycle}} - H_{s\downarrow}}{2H_{cs}} \right)^2 \right] \right\} \quad \text{A-3}$$

valid over the range, $(H_{\max}^{\text{cycle}} - H_{s\downarrow}) \leq 2H_{cs}$. If $H_{\max}^{\text{cycle}} \geq 2H_{cs}$, the 2nd term vanishes when

$$(H_{\max}^{\text{cycle}} - H_{s\downarrow}) = 2H_{cs}.$$

For cases where $H_{\max}^{\text{cycle}} < H_{\text{cg}}$,

$$\begin{aligned} M_s^{\text{ZPCD}} &= \frac{H_{\text{cg}}}{2} \left[1 - 2 \left(\frac{x_r}{X} \right)^2 + \left(\frac{x_i}{X} \right)^2 \right] \\ &= \frac{H_{\text{cg}}}{2} \left[1 - 2 \left\{ 1 - \frac{(H_{\max}^{\text{cycle}} - H_{s\downarrow})}{2H_{\text{cg}}} \right\}^2 + \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{H_{\text{cg}}} \right\}^2 \right] \end{aligned} \quad \text{A-4}$$

For H_s reascending from 0 after the ZFC descent from $H_{\max}^{\text{cycle}} < H_{\text{cg}}$,

$$\begin{aligned} M_s^{\text{ZFCnAs}} &= -\frac{H_{\text{cg}}}{2} \left\{ 1 - 2 \left(\frac{x_o}{X} \right)^2 + 2 \left(\frac{x_r}{X} \right)^2 - \left(\frac{x_i}{X} \right)^2 \right\} \\ &= -\frac{H_{\text{cg}}}{2} \left[1 - 2 \left\{ 1 - \frac{H_{s\uparrow}}{2H_{\text{cg}}} \right\}^2 + 2 \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{2H_{\text{cg}}} \right\}^2 - \left\{ 1 - \frac{H_{\max}^{\text{cycle}}}{H_{\text{cg}}} \right\}^2 \right] \end{aligned} \quad \text{A-5}$$

The 4th term vanishes if $H_{\max}^{\text{cycle}} \geq H_{\text{cg}}$, and the 3rd term vanishes if $H_{\max}^{\text{cycle}} \geq 2H_{\text{cg}}$.

For H_s descending after field cooling in H^{cool} ,

$$M_s^{\text{FCD}} = \frac{H_{\text{cg}}}{2} \left\{ 1 - \left(\frac{x_p}{X} \right)^2 \right\} = \frac{H_{\text{cg}}}{2} \left[1 - \left(\frac{H^{\text{cool}} - H_{\downarrow}}{H_{\text{cg}}} \right)^2 \right] \quad \text{A-6}$$

valid until $(H^{\text{cool}} - H_{\downarrow}) = H_{\text{cg}}$, then $M_s = \frac{H_{\text{cg}}}{2}$.

When H_s , denoted $H_{\uparrow}$, is reascending after a descent to zero from $H^{\text{cool}} \geq H_{\text{cg}}$,

$$M_s^{\text{FCnAs}} = -\frac{H_{\text{cg}}}{2} \left[1 - 2 \left(\frac{x_o}{X} \right)^2 \right] = -\frac{H_{\text{cg}}}{2} \left[1 - 2 \left\{ 1 - \frac{H_{\uparrow}}{2H_{\text{cg}}} \right\}^2 \right] \quad \text{A-7}$$

valid until $H_{\uparrow} = 2H_{\text{cg}}$, then $M_s = -\frac{H_{\text{cg}}}{2}$.

When H_s is reascending after a descent to zero from $H^{\text{cool}} < H_{\text{cg}}$,

$$\begin{aligned}
M_s^{\text{PCrAs}} &= -\frac{H_{cs}}{2} \left[1 - 2 \left(\frac{x_o}{X} \right)^2 + \left(\frac{x_p}{X} \right)^2 \right] \\
&= -\frac{H_{cs}}{2} \left[1 - 2 \left\{ 1 - \frac{H_f}{2H_{cs}} \right\}^2 + \left\{ 1 - \frac{H^{\text{cool}}}{H_{cs}} \right\}^2 \right]
\end{aligned}
\tag{A-8}$$

valid until $H_f = 2H^{\text{cool}}$. As H_f continues to reascend beyond $H_f = 2H^{\text{cool}} < 2H_{cs}$,

$$M_s^{\text{PCrAs}} = -\frac{H_{cs}}{2} \left[1 - \left(\frac{x_i}{X} \right)^2 \right] = -\frac{H_{cs}}{2} \left[1 - \left\{ 1 - \left(\frac{H_f - H^{\text{cool}}}{H_{cs}} \right) \right\}^2 \right]
\tag{A-9}$$

valid until $x_i = 0$, hence $(H_f - H^{\text{cool}}) = H_{cs}$. As H_f continues to reascend $M_s = -H_{cs}/2$.

Appendix 4.B

Magnetization Curves for Ribbon with Surfaces $\perp$ to $\vec{H}_a$

Brandt and Indenbom^[19] have developed expressions for the magnetization of a slab of thickness d , width $2X$ and infinite length L where $d \ll 2X$. The critical current density J_{cs} is independent of the flux density and the magnetic field is directed // to the thickness d , hence $\perp$ to the XL plane. We adapt these expressions to describe the magnetization of an idealized grain for the various field-temperature histories we have addressed in chapter 4.

For H_a increasing after zero field cooling

$$M_s^{\text{ZFC}} = -[H_{cs} \tanh(C_2 H_{sf} / H_{cs})]
\tag{B-1}$$

When H_a is decreasing after an excursion to H_{max} ,

$$M_s^{ZPCD} = -H_{\sigma} \left[\tanh \left(C_2 \frac{H_{\max}^{\text{cycle}}}{H_{\sigma}} \right) - 2 \tanh \left(\frac{C_2}{2} \frac{(H_{\max}^{\text{cycle}} - H_{\sigma\uparrow})}{H_{\sigma}} \right) \right] \quad \text{B-2}$$

$$M_s^{ZPCrAs} = -H_{\sigma} \left[\tanh \left(C_2 \frac{H_{\max}^{\text{cycle}}}{H_{\sigma}} \right) - 2 \tanh \left\{ \frac{C_2}{2} \left(\frac{H_{\max}^{\text{cycle}} - H_{\min}}{H_{\sigma}} \right) \right\} \right. \\ \left. + 2 \tanh \left\{ \frac{C_2}{2} \left(\frac{H_{\sigma\uparrow} - H_{\min}}{H_{\sigma}} \right) \right\} \right] \quad \text{B-3}$$

where H_{σ} , denoted $H_{\sigma\uparrow}$ is reascending after descending to $H_{\min}$, and $-H_{\max}^{\text{cycle}} \leq H_{\min} < H_{\max}$. In our application of eqn B-3 we have focused on the special case where $H_{\min} = 0$.

For H_{σ} , denoted $H_{\downarrow}$, descending from any arbitrary value of H^{cool} ,

$$M_s^{\text{PCD}} = -H_{\sigma} \tanh \left\{ C_2 \left(\frac{H^{\text{cool}} - H_{\min}}{H_{\sigma}} \right) \right\} \quad \text{B-4}$$

For H_{σ} , denoted $H_{\downarrow}$, reascending after a descent to $H_{\min}$ from H^{cool} ,

$$M_s^{\text{PCrAs}} = -H_{\sigma} \left[\tanh \left\{ C_2 \left(\frac{H^{\text{cool}} - H_{\min}}{H_{\sigma}} \right) \right\} - 2 \tanh \left\{ \frac{C_2}{2} \left(\frac{H_{\downarrow} - H_{\min}}{H_{\sigma}} \right) \right\} \right] \quad \text{B-5}$$

where $-H^{\text{cool}} \leq H_{\min} < H^{\text{cool}}$. In our application of eqn B-5 we have focussed on the special case where $H_{\min} = 0$.

Again we introduce the magnetic dipole framework to describe the return field and write,

$$H_r = -C_1 M_s(H_s) \quad \text{B-6}$$

Appendix 4.C

Onset of Plateaus in Curves of Peaks of I_c versus $H_{\text{max}}^{\text{cycle}}$ or H^{cool}

For completeness we present expressions for the minimum values of $H_{\text{max}}^{\text{cycle}}$ and H^{cool} required to generate the peaks of I_c versus H_a for ZFCD, ZFCrAs, FCD and FCrAs procedures. The edge of the plateaus of the various curves illustrated in Fig 4.6 display the applied magnetic field present when the summit of these peaks of I_c are encountered.

Since for idealized cylinder geometry the maximum magnitude of the return field $H_r^{\text{max}} = CH_{c0}/3$ and for the slab $H_r^{\text{max}} = CH_{c0}/2$ we see from inspection that the lowest intersection, labeled • in Fig 4.4a, is generated by a prior excursion to a field $H_{\text{max}}^{\text{cycle}}$ denoted $H_{\text{onset}}^{\text{ZFCD}}$, which reads,

$$H_{\text{max}}^{\text{cycle}} = H_{\text{onset}}^{\text{ZFCD}} = H_r^{\text{max}} + 2H_{c0} = \left(\frac{C}{3} + 2\right)H_{c0} \quad \text{C-1}$$

for cylinder geometry, and

$$H_{\text{max}}^{\text{cycle}} = H_{\text{onset}}^{\text{ZFCD}} = H_r^{\text{max}} + 2H_{c0} = \left(\frac{C}{2} + 2\right)H_{c0} \quad \text{C-2}$$

for slab geometry.

In these expressions, the first term states that the peak of I_c occurs when $H_r^{\text{max}} = H_a = C H_{c0}/3$, or $C H_{c0}/2$. The second term arises because in the critical state model for idealized cylinder and slab geometry, when J_{c0} is taken independent of H , a descent of H_a over a range $2H_{c0}$ is required to cause M_g to swing from the maximum diamagnetism at $H_{\text{max}}^{\text{cycle}} = H_{\text{onset}}^{\text{ZFCD}}$ to the saturation paramagnetism.

The lowest intersection labeled by a data point $\square$ in Fig 4.4a indicates the onset of $H_{\text{plateau}}^{\text{ZFCrAs}}$. To cause the locus of H_r to travel to this intersection requires that the remanent magnetization at $H_a = 0$ be generated by an excursion of $H_{\text{max}}^{\text{cycle}}$ to a minimum strength $= 2H_{\text{a}}$ for both idealized cylinder and slab geometry. Consequently, we write,

$$H_{\text{onset}}^{\text{cycle}} = H_{\text{max}}^{\text{cycle}} = 2H_{\text{a}} \quad \text{C-3}$$

The lowest intersection, indicated by a $\bullet$ in Fig 4.4b shows that the descent of H_a in the FCD procedure begins from a minimum value of H^{cool} which reads,

$$H_{\text{onset}}^{\text{FCD}} = H^{\text{cool}} = H_{\text{a}} + H_r^{\text{max}} = H_{\text{a}} + \frac{C}{3}H_{\text{a}} \quad \text{C-4}$$

for cylinder geometry and,

$$H_{\text{onset}}^{\text{FCD}} = H_{\text{a}} + \frac{C}{2}H_{\text{a}} \quad \text{C-5}$$

for slab geometry.

Note that a decrease of H_a by an amount not less than H_{a} is required in both of these geometries to generate the maximum trapped flux in the specimen.

The lowest intersections labeled by a $\square$ in Fig 4.4b indicates the threshold for $H_{\text{plateau}}^{\text{FCrAs}}$. To cause the locus of H_r to travel to this intersection requires that the remanent magnetization at $H_a = 0$ correspond to the saturation amount, and a descent of H_a after field cooling in a minimum field,

$$H^{\text{cool}} = H_{\text{onset}}^{\text{FCrAs}} = H_{\text{a}} \quad \text{C-6}$$

is required to achieve this result in both idealized geometries.

We note that the Brandt function for $M_g(H_a)$ increases monotonically to a plateau over the entire range $0 < H_a / H_{a_g} < \infty$, hence no precise onset values can be identified in this case.

Since for idealized cylinder, slab and Brandt-Indenbom geometries the values for the position of the peaks in the I_c curves approach the plateaus asymptotically, we do not expect that experimental researchers will devote much effort to studying these onset values in the behaviour of I_c hysteresis. Consequently this appendix is perhaps of "academic" interest only.

Appendix 5.A

I_c versus H_s : Modified Kim $J_{cm}(H)$

Müller et al^[117] addressed infinite slab geometry with H_s // to the large surfaces at $x = 0$, and $x = W$, and $\perp$ to the critical current I_c , taking $H_{total} = H_s + H_m(x)$, they write,

$$J_{cm} = \frac{J_{cm0} H_1^n}{H_s^n + (H_s + H_m(x))^n} \quad A-1$$

The critical state concept and Maxwell's eqn, $dH_m(x)/dx = \pm J_{cm}$ leads to the integral formula,

$$\int_{H_s - I_c/2}^{H_s + I_c/2} [H_s^n + (H_s + H_m(x))^n]^{1/n} dH_m(x) = J_{cm0} H_1^n \int_0^W dx \quad A-2$$

When $H_s > I_c/2$, the integral leads to,

$$(n+1)H_s^n I_c + \left(H_s + \frac{I_c}{2}\right)^{n+1} - \left(H_s - \frac{I_c}{2}\right)^{n+1} = J_{cm0} H_1^n W(n+1) \quad A-3$$

When $H_s < I_c/2$, it is convenient to introduce a boundary x_0 where $(H_s + H_m(x)) = 0$ as illustrated in Fig 5.1. Integrating eqn A-2 from x_0 to W leads to,

$$(n+1)H_s^n \left(H_s + \frac{I_c}{2}\right) + \left(H_s + \frac{I_c}{2}\right)^{n+1} = J_{cm0} H_1^n (n+1)(W - x_0) \quad A-4$$

Integrating from $x = 0$ to x_0 and taking into account that J_{cm} depends on the absolute value of H_{total} , leads to,

$$(n+1)H_s^n \left(\frac{I_c}{2} - H_s\right) + \left(\frac{I_c}{2} - H_s\right)^{n+1} = J_{cm0} H_1^n (n+1)x_0 \quad A-5$$

where $H_s < I_c/2$.

Combining eqns A-4 and A-5 leads to,

$$(n+1)H_0^n I_c + \left(H_s + \frac{I_c}{2}\right)^{n+1} + \left(\frac{I_c}{2} - H_s\right)^{n+1} = J_{\text{ano}} H_1^n (n+1)W \quad \text{A-6}$$

When $H_0 = 0$, eqn A-1 is identical to eqn 5.6 for all n , hence eqn A-3 and A-6 are identical to eqn 5.10 and 5.12.

When $H_0 \neq 0$ and $n = 1$, eqn A-3 is identical to eqns 5.10 and eqn A-6 to eqn 5.12.

Workers have found that eqns A-3 and A-6 describe the behaviour of I_c for Josephson junctions in weak fields when $n \approx 3/2$ ^[49].

It is useful and convenient to normalize eqns A-3 and A-6 with respect to the situation where $I_c/2 = H_s$, then denoted I_{c^*} and H_{s^*} .

Under these circumstances eqns A-3 and A-6 lead to,

$$(n+1)H_0^n I_{c^*} + I_{c^*}^{n+1} = K^{n+1} = J_{\text{ano}} W H_1^n (n+1) \quad \text{A-7}$$

hence,

$$k^{n+1} = \left(\frac{K}{I_{c^*}}\right)^{n+1} = (n+1)h_0^n + 1 \quad \text{A-8}$$

where we write $h_0 = H_0/I_{c^*}$.

Consequently eqn A-3 reads,

$$(n+1)h_0^n (i_c - 1) + \left(h_s + \frac{i_c}{2}\right)^{n+1} - \left(h_s - \frac{i_c}{2}\right)^{n+1} = 1 \quad \text{A-9}$$

where $i_c = I_c/I_{c^*}$, $h_s = H_s/I_{c^*}$. Eqn A-9 applies when $h_s > i_c/2$.

Eqn A-6 now reads,

$$(n+1)h_0^n (i_c - 1) + \left(h_s + \frac{i_c}{2}\right)^{n+1} + \left(\frac{i_c}{2} - h_s\right)^{n+1} = 1 \quad \text{A-10}$$

which applies when $h_s < i_c/2$.

Appendix 5.B

I_c versus H_s : Exponential $J_{\infty}(H)$

Müller et al^[117] exploit eqn 5.4b and write

$$\frac{dH_m(x)}{dx} = \pm J_{\infty}(H_s) = \pm J_{\infty} e^{-(H_s + H_m(x))/H_1} \quad \text{B-1}$$

The integral formulation reads,

$$\int_{H_s - (I_c/2)}^{H_s + (I_c/2)} e^{-(H_s + H_m(x))/H_1} dH_m(x) = \pm J_{\infty} \int_0^W dx \quad \text{B-2}$$

The case where $H_s > I_c/2$ leads to,

$$e^{(H_s + (I_c/2))/H_1} - e^{(H_s - (I_c/2))/H_1} = \frac{J_{\infty} W}{H_1} \quad \text{B-3a}$$

which can be written,

$$\sinh\left(\frac{I_c}{2H_1}\right) = \left(\frac{J_{\infty} W}{2H_1}\right) e^{-H_s/H_1} \quad \text{B-3b}$$

Let I_{c*} denote I_c when $H_s = I_c/2$, then eqn B-3a leads to,

$$I_{c*} = H_1 \ln\left\{1 + \frac{J_{\infty} W}{H_1}\right\} \quad \text{B-4}$$

When $0 \leq H_s \leq I_c/2$, it is again useful to identify a plane, denoted x_0 , where

$H_s + H_m(x) = 0$, and perform the integration in two steps. The integration from x_0 to W leads to,

$$e^{(H_s + \frac{I_c}{2})/H_1} - 1 = \frac{J_{\infty} (W - x_0)}{H_1} \quad \text{B-5a}$$

Since J_{cm} depends on the absolute value of $H_s + H_m(x)$, the integration over the volume $0 \leq x \leq x_0$, leads to,

$$e^{\left(\frac{1}{2} - H_s\right)/H_1} - 1 = \frac{J_{cm} x_0}{H_1} \quad B-5b$$

Combining eqns B-5a and b leads to,

$$e^{\left(H_s + \frac{1}{2}\right)/H_1} + e^{\left(\frac{1}{2} - H_s\right)/H_1} = 2 + \frac{J_{cm} W}{H_1} \quad B-6a$$

which can be written,

$$e^{\frac{1}{2H_1}} = \frac{1 + \frac{J_{cm} W}{2H_1}}{\cosh(H_s / H_1)} \quad B-6b$$

Let $I_{c\bullet}$ denote I_c when $H_s = 0$, then eqn B-6a reads,

$$I_{c\bullet} = 2H_1 \ln \left\{ 1 + \frac{J_{cm} W}{2H_1} \right\} \quad B-7$$

I_c , $J_{cm} W$, H_s (hence H_a and H_r) in eqn B-3a through B-7 are normalized with respect to H_1 which controls the exponential rate of descent of J_{cm} versus H . It is again useful to renormalize the calculated curves with respect to $I_{c\bullet}/2 = H_{s\bullet}$ since in this format the structure of the various curves of $i_c = I_c/I_{c\bullet}$ with different $J_{cm} W$ and H_1 intersect at $h_s = 0.5$ when $i_c = 1$.

Appendix 5.C

I_c versus H_s : Kim J_{cm} in Normalized Form

In the computations it is useful to exploit eqns 5.10 and 5.12 in normalized form. It is convenient to normalize the quantities H_a , H_r (hence $H_s = H_a + H_r$), I_c and K to the value of I_c denoted $I_{c\bullet}$ in the situation where, for idealized planar geometry, the self field $H_m(x)$

= 0 at $x = 0$ and $I_c = I_{c0}$ at $x = W$. The boundary conditions under these circumstances then dictate (see Fig 5.1c) that, H_e , denoted H_{e0} , is equal to $I_{c0}/2$. We let $h_0 = H_0/I_{c0}$, $h_s = H_s/I_{c0}$, $h_r = H_r/I_{c0}$, $h_e = H_e/I_{c0}$, $k = K/I_{c0}$, $i_c = I_c/I_{c0}$ hence, $i_{c0} = I_{c0}/I_{c0} = 1$. Introducing these normalized quantities into eqn 5.10 and 5.12 and noting the important feature that,

$$h_r = \frac{H_r}{I_{c0}} = \frac{I_{c0}/2}{I_{c0}} = \frac{1}{2} \text{ when } i_{c0} = 1 \quad \text{C-1}$$

hence at the "boundary" where both eqns 5.10 and 5.12 are valid, we see that both eqns lead to

$$(h_0 + 1)^{n+1} - h_0^{n+1} = k^{n+1} \quad \text{C-2}$$

Consequently eqn 5.10 normalized reads,

$$\left(h_0 + h_s + \frac{i_c}{2}\right)^{n+1} + \left(h_0 - h_s + \frac{i_c}{2}\right)^{n+1} = (h_0 + 1)^{n+1} + h_0^{n+1} \quad \text{C-3}$$

valid when $i_c \geq 1$ and $h_s < 1/2$.

Eqn 5.12 normalized reads,

$$\left(h_0 + h_s + \frac{i_c}{2}\right)^{n+1} - \left(h_0 + h_s - \frac{i_c}{2}\right)^{n+1} = (h_0 + 1)^{n+1} - h_0^{n+1} \quad \text{C-4}$$

valid when $0 \leq i_c \leq 1$ and $h_s > 1/2$.

A very useful advantage of these two formulae is that the critical state curves of i_c versus h_s with different values for h_0 and n all intersect at $i_c = 1$ and $h_s = 0.5$, hence their qualitative behaviour can readily be compared.

Appendix 5.D

Field Profiles and M_z versus H_a : Kim $J_{eq}(H)$

For idealized slab and cylinder geometry, Maxwell's eqn $\nabla \times \vec{H} = \vec{J}$ and the critical

state assumption that, $J = \pm J_{eq} = \frac{J_{eq} H_1^n}{(H + H_o)^n}$ D-1

leads to,

$$\frac{dH}{dr} = \pm J_{eq} = \pm \frac{J_{eq} H_1^n}{(H + H_o)^n} \quad \text{D-2}$$

and the integral form,

$$\int_{H_s(r)}^{H_1} (H + H_o)^n dH = \pm J_{eq} H_1^n \int_r^R dr \quad \text{D-3}$$

Hence, when H_a is first ascending,

$$H_s(r)_1 = \left[(H_s + H_o)^{n+1} - (n+1) J_{eq} R \left(1 - \frac{r}{R} \right) H_1^n \right]^{1/(n+1)} - H_o \quad \text{D-4}$$

which applies over the range $r_o \leq r \leq R$ as illustrated in Fig 5.17a, where,

$$\left(1 - \frac{r_o}{R} \right) = \frac{(H_s + H_o)^{n+1} - H_o^{n+1}}{(n+1) J_{eq} R H_1^n} \quad \text{D-5}$$

When H_a descends from H_{max} , eqn D-3 leads to,

$$H_s(r)_2 = \left[(H_s + H_o)^{n+1} + (n+1) J_{eq} R \left(1 - \frac{r}{R} \right) H_1^n \right]^{1/(n+1)} - H_o \quad \text{D-6}$$

which applies over the range $r_i \leq r \leq R$ as illustrated in Fig 5.17b.

The continuity of $H_s(r)$ requires that,

$$H_s(r_i)_1 = H_s(r_i)_2 \quad \text{D-7}$$

Hence introducing eqn D-4 where $H_a = H_{\max}$ and eqn D-6 into eqn D-7 leads to,

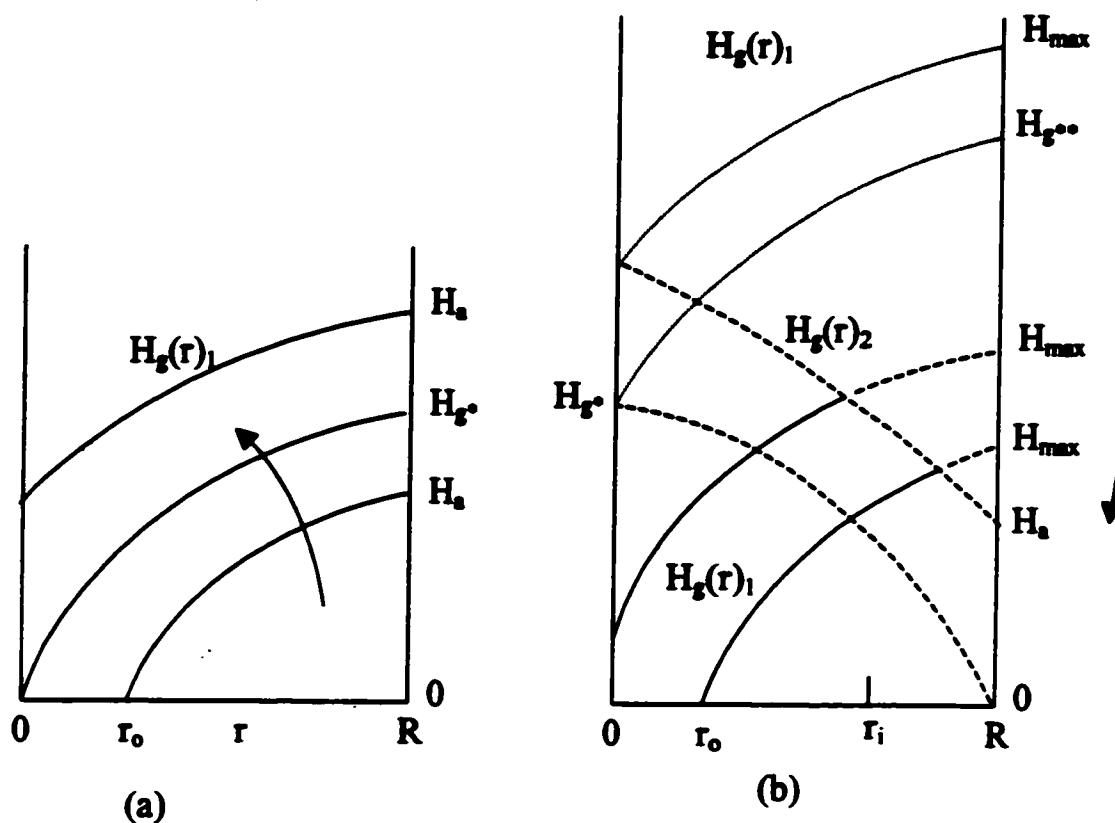


Fig 5.17. Displays the field profiles $H_g(r)$ for idealized cylinder and slab geometry. (a) Sequence of $H_g(r)$ when H_a is impressed to a virgin (zero field cooled) specimen. H_{g*} denotes H_a when r_0 attains 0 . (b) curves show sequence of $H_g(r)$ as H_a increases beyond $H_a = H_{g*}$ where $H_g(r)_1 = H_{g*}$ at $r_0 = 0$. Dashed curves, labeled $H_g(r)_2$ display sequences of profiles when H_a is made to descend from various $H_{\max}$ values. Note that with $H_{\max} > H_{g*}$, then when H_a descends to zero, $H_g(r)_2 = H_{g*}$ at $r = 0$. r_i denotes the intersections of the flux trapping induced currents with the previously generated field shielding induced currents

$$\left(1 - \frac{r_i}{R}\right) = \frac{(H_{\max} + H_o)^{n+1} - (H_i + H_o)^{n+1}}{2(n+1)J_{\text{eq}}RH_i^n} \quad \text{D-8}$$

Let H_{g^*} denote the full penetration field H_a for the situation where, $H_g(r)_1 = 0$, at $r = r_o$ = 0. Then eqn D-4 leads to,

$$H_{g^*} = \left[(n+1)J_{\text{eq}}RH_i^n + H_o^{n+1} \right]^{1/(n+1)} - H_o \quad \text{D-9}$$

Let $H_{g^{**}}$ denote the "double" penetration field H_a for the situation where $H_g(r)_1 = H_{g^*}$ at, $r = 0$. Then eqn D-4 leads to,

$$\begin{aligned} H_{g^{**}} &= \left[(H_{g^*} + H_o)^{n+1} + (n+1)J_{\text{eq}}RH_i^n \right]^{1/(n+1)} - H_o \\ &= \left[2(n+1)J_{\text{eq}}RH_i^n + H_o^{n+1} \right]^{1/(n+1)} - H_o \end{aligned} \quad \text{D-10}$$

where in the 2nd line, we have introduced eqn D-9 for $H_{g^*} + H_o$ appearing in the 1st line.

The variety of flux profiles are displayed in Fig 5.17 where the quantities H_{g^*} and $H_{g^{**}}$ are also identified. The grains are diamagnetically magnetized to saturation when H_a ascends beyond H_{g^*} . In order to magnetize the grains to saturation when H_a is descending from $H_{\max}$ requires that previously H_a ascending, hence $H_{\max}$ exceeded $H_{g^{**}}$.

The magnetization of a grain visualized as an infinite slab or cylinder is calculated using the definitions,

$$M_s = \frac{1}{X} \int_0^x H_s(x) dx - H_a = \frac{2}{R^2} \int_0^R H_s(r) r dr - H_a \quad \text{D-11}$$

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