Space-Time ARIMA and Transfer Function-Noise Modeling of Rainfall-Runoff Process

by

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ABSTRACT

Two broad classes of aggregate regional forecasting models are defined. The autoregressive models belong to the general family of space-time autoregressive integrated moving average (STARIMA) process, and the explanotary models belong to the class of space-time transfer function noise process. These models are characterized by autoregressive and moving average terms lagged in both time and space.

This thesis develops and demonstrates a three-stage iterative procedure for building STARIMA models of precipitation series and space-time transfer function noise models of rainfall-runoff process. For a given data set, the degree of non-stationarity, as well as the type and the order of the model can be decided by studying the shape of the appropriate space-time correlation functions. The relative spatial locations of the sites in the system can explain the observed correlative structure since an hierarchical spatial ordering of the neighbors of each site is used.

Different types of stochastic models are developed for rainfall and runoff stations located in the Watershed in Southern Ontario. These models are used to forecast rainfall and runoff for the selected Watershed. The forecasting results indicated that the space-time models produce reasonably reliable forecasts.

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Chapter I

INTRODUCTION

1.1 PROBLEM STATEMENT

Rainfall is the most important input to a simulation model of the land phase of the hydrological cycle. It takes the form of rain, snow, hail or dew and can possess an extreme distribution in magnitude in time and over space. Once rainfall reaches the ground, some of it may fill depressions, part may penetrate the ground to replenish soil moisture and ground reservoirs, and some may become surface runoff that flows over the earth's surface to a defined channel such as a stream.

Information regarding flow rates at any point of interest along a stream is necessary in the analysis and design of many types of water projects. Engineers are sometimes faced with the problem of little or no streamflow records in designing of flood control reservoirs, canals and water supply systems. This caused a need for development of simulation models to aid engineers in expanding their knowledge of the hydrologic regime of a catchment.

There are two major approaches to simulation modeling: Optimization and nonoptimization methods. Optimization models consider the element of selecting the best set of conditions to satisfy a set of objectives for a given set of constraints. Nonoptimization methods are generally associated with assessment of hydrologic data and are used to quantify the physical process and can be separated into statistical and physical/deterministic models.

The deterministic simulation techniques are categorized into empirical and conceptual methods. The former describes the hydrological process based on an assumption that the input and output observations do not contain errors and a unique transformation from input to output exists. This type of model, being governed by known physical laws, does not include probabilistic interpretation of the results.

Conceptual models are intended to incorporate within their structure the general physical mechanism which governs the hydrological cycle. In this approach the internal description of the various subprocesses are modeled according to empirically determined functions which are linked in their conceptualized logical order.

In order to account for uncertainties in the data, model and parameters, many statistical models have been introduced. They include regression and correlation techniques, probabilistic methods and stochastic methods. All these approaches tend to treat a limited, or specific sample of data to make predictions.

Stochastic models are constructed from random data and allow for memory effects so that each observation is dependent to some degree upon its predecessors in time. Therefore, statistical properties of observed time series provide a mean for identifying stochastic processes. This identification approach consists of a priori model postulation and acceptance or rejection of the model in terms of statistical tests. The stochastic methods of time series analysis with a representation in the form of an input-output system will be considered further in this thesis, since they are related to simulation problems.

Rainfall is a space-time phenomenon and its variability possess one of the major modeling challenges in hydrology. Specifically Linsley, Kohler and Paulhus (48) observe that 'Among climatic factors that establish the hydrologic features of a region are the amount and distribution of rainfall'. Space-time modeling and forecasting have been approached by many researchers. Chisolm, Frey and Hagget (18) have shown the importance of modeling geographical processes and how the space-time modeling and forecasting could be applied to urban and regional analysis.

The space-time forecasting methods adapted from Box and Jenkins classical time series analysis can be used to produce much information about the behavior of rainfall-runoff data in time and space.

1.2 OBJECTIVES OF THE RESEARCH

The objectives of this thesis are:

1. To extend the identification, estimation and diagnostic stages of Box and Jenkins (ARIMA) models development into the spatial domain by using a hierarchical ordering of the spatial neighbors of each location of raingage site. This results in a general model class of space-time autoregressive integrated moving average (STARIMA) models.
2. To investigate the Box and Jenkins transfer function-noise models which have been successfully applied to hydrological systems for flow simulation and forecasting with rainfall as input data. This transfer function-noise model is extended into the spatial domain by using a hierarchical ordering of the spatial neighbors of each location of rainfall and runoff gage sites. This procedure leads to the class of space-time transfer function- noise (STTFN) process.
3. To apply the STARIMA and STTFN modeling methodologies to a set of rainfall and runoff time series to provide runoff regional forecasts. The selected Watershed is located in Southern Ontario, Canada.

1.3 THESIS ORGANIZATION

The work presented in this thesis may be divided into two parts.

1.

a) The development of a methodology, to describe the extention of the three-stage iterative model building procedure of ARIMA models by Box and Jenkins into spatial domain to accommodate the space-time model class of space -time autoregressive integrated moving average (STARIMA) models.

b) The development of a mathematically efficient technique to extend the Box and Jenkins transfer function-noise model into the spatial domain. This leads to the class of space-time transfer function-noise process.

2. The application of the above methodologies to a realistic hydrological system, studying different aspects of modeling rainfall and rainfall runoff processes.

In what follows, the description of the research work compiled in different chapters of this thesis is described.

In chapter I, the background of this research and the review of this thesis are discussed.

In chapter II, the general properties of time series and stochastic processes are first presented. A brief theoretical description of Box-Jenkins ARIMA models are proposed in terms of linear difference equations. A systematic approach to time series modeling of ARIMA process is introduced. The approach is composed of three main phases, identification, estimation and diagnostic checks which are performed to test the adequacy of the fitted model and to give direction for updating the fitted model if it is inadequate. The focus of Section 2.3 is to review a number of mathematical models that have been adapted to represent different aspects of rainfall-runoff process. Finally, a comprehensive class of space-time models that have been refered to as STARIMA and STTFN models is proposed. A brief review of contributions made by researchers within the general area of space-time modeling is presented.

In chapter III, features of the sampling stations in the selected watershed and data selection for the numerical analysis are investigated.

Chapter IV includes a comprehensive STARIMA and space-time TFN model building procedure. In Section 4.1 the general framework for model identification in terms of space-time autocorrelation (STACF) and space-time partial autocorrelation (STPACF) functions is developed. The second stage of the modeling procedure is estimation of the model parameters and a variety of linear and nonlinear optimization techniques are reported. The last stage, diagnostic checking of the fitted model to reveal any inadequacies, is also the subject of Section 4.1. The basic properties of STTFN models and development of general framework for model identification based on space-time cross correlation functions (STCCF) is proposed in Section 4.2. The extention of transfer function-noise model procedure developed by Box and Jenkins to accommodate the space-time is presented. This section also deals with the estimation of unknown parameters of the selected STTFN models and diagnostic checks applied to test the adequacy of the models and to suggest potential improvements.

Chapter V includes the application of the methods proposed in chapter IV to a realistic hydrologic system studying aspects of STARIMA modeling of rainfall series and STTFN modeling of rainfall-runoff process.

Chapter VI of the thesis provides the conclusions of this research work.

The suggestions for future investigations are given in Chapter VII.

Chapter II

LITERATURE REVIEW

In this chapter the general properties of time series and stochastic processes are first discussed. This introduces basic concepts and properties of various types of stationary Box and Jenkins processes(9). A general appraisal is given regarding the stochastic rainfall-runoff models. Following this, some statistical properties for examining STARIMA modeling in the time domain are pointed out.

What follows is an account of time series methods of forecasting and an attempt to adapt them in the spatial domain.

2.1 UNIVARIATE TIME SERIES

Time series analysis is concerned with extracting information from the data about the studied phenomenon and where the relations between consecutive observations are of interest.

A number of stochastic modeling techniques have been adapted to establish a dynamic linkage between observed time series, namely rainfall and runoff. In most cases the parameters of such models have limited physical significance and are chosen entirely on the basis of optimization of some statistical criterion. Such models have been found to work extremely well in many hydrological applications(62). Their use in surface water hydrology depends on the ability to satisfy the critical assumptions related to mathematical theory.

The most popular approaches in modeling of hydrological time series have been the Box and Jenkins ARIMA and TFN modeling approaches. Some of the uses of Box and Jenkins models in water resources include(40);

1. Simulation: Since historical data will probably never exactly repeat itself in the future, synthetic data can be generated to test possible designs and operating policies of pipelines, dams, sewage treatment plants, etc... Simulation can also be used to determine an empirical distribution for a statistic such as the Hurst coefficient.
2. Forecasting: Based on the most recent data and the particular model being used, minimum mean square error forecast can be made for future values of the time series.
3. Describing dynamic relationships: Box and Jenkins transfer function -noise models(9) describe the dynamic relationship between the dependent or response variable (ex. river flow) and the independent variables or inputs (ex. precipitation) in the presence of autocorrelated noise. The resulting finite differ-

ence equation is analogous to a linear differential equation.

4. Control problems: Two reservoirs on a river may be operated in a series to control downstream flow based on inflows and the current water level of each reservoir.
5. Efficient summarization of historical statistics: The historical data is summarized in a few model parameters that are estimated from the data. If the proper model passes all diagnostic checks then the statistical properties of the historical data will be preserved.

2.1.1 ARIMA (p,d,q) process

Let $Y_1, Y_2, \ldots, Y_{t-1}, Y_t, Y_{t+1} \ldots Y_T$ be a discrete time series measured at equal time intervals. The Box and Jenkins method for analysing time series uses the backshift operator, B, defined by

$$BY_t = Y_{t-1}$$

and the difference operator, ∇, defined by

$$\nabla Y_t = Y_t - Y_{t-1}$$

which can be used to remove the nonstationarity and the resulting differenced data can be modeled as a stationary series. The backshift and the difference operators are connected by the relation

$$\nabla = 1 - B$$

The general form for an autoregressive process of order p, an AR(p) process, is

$$Y_t = \phi_1 Y_{t-1} + \ldots\ldots + \phi_P Y_{t-P} + a_t \qquad (2.1)$$

where a_t, are normally independently distributed white noise residual with mean 0 and variance σ_a^2 NID$(0,\sigma_a^2)$, and ϕ are unknown parameters. The current value of the process is expressed as a weighted sum of past values plus current shock. Thus Y_t can be considered to be regressed on the previous Y's.

Equation(2.1) can be written as

$$\phi(B)Y_t = a_t \qquad (2.2)$$

where

$\phi(B)=1-\phi_1 B-\phi_2 B^2-\ldots\ldots-\phi_P B^P$, is the autoregressive (AR) operator or polynomial of order p such that the roots of the characteristics equation $\phi(B)=0$ lie outside the unit circle for stationarity and the ϕ_i, i=1,2,3,...., p are the AR parameters.

The moving average model of order q, the MA(q) process is given by

$$Y_t = a_t - (\theta_1 a_{t-1} + ... + \theta_q a_{t-q}) \tag{2.3}$$

where a_t are normally independently distributed white noise residuals with mean 0 and variance σ_a^2 (written as NID(0,σ_a^2)). This can be written

$$Y_t = \theta(B)a_t \tag{2.4}$$

where

$\theta(B)=1-\theta_1 B-\theta_2 B^2-....-\theta_q B^q$, is the moving average (MA) operator or polynomial of order q such that the roots of $\theta(B)=0$ lie outside the unit circle for stationarity and the θ_i, i=1,2,...,q are the MA parameters.

if a process consists of both AR and MA parameters, it is called an ARMA (p,q) process of the form

$$Y_t = \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-P} + a_t + \theta_1 a_{t-1} + \dots + \theta_q a_{t-q} \quad (2.5)$$

or an ARIMA(p,d,q) model, and written as

$$\Phi(B)Y_t = \Theta(B)a_t \quad (2.6)$$

where d specifies the number of differences needed to induce stationarity in the original series. Stationarity and invertibility require the zeros of $\phi(B)$ and $\Theta(B)$ to lie outside the unit circle. As ARMA(p,q) process contains both the AR and MA processes, an AR(p) process is equivalent to an ARMA(p,0) process while an MA(q) process is the same as an ARMA(0,q) process.

So far the discussion has been restricted to stationary process, and the ARMA model have had all the zeros of $\Theta(B)$ outside the unit circle. A series is said to be stationary if the probability distribution of the variable is constant over time. Under this condition, such quantities as the mean $E(Y_t) = \mu_t$ and the variance $\sigma_t^2 = E(Y_t - \mu_t^2)$ will remain constant. A nonstationary process can be dealt with by taking the model with first, or higher difference, such as

$$\nabla Y_t = Y_t - Y_{t-1} \; ; \; \nabla^2 Y_t = \nabla Y_t - \nabla Y_{t-1} \; \dots\dots \text{ etc.}$$

Usually first or second difference is efficient to approximate this state.

The three-stage iterative building procedure developed by Box and Jenkins for ARIMA modeling is as following (Figure 1):

1. Identification: The type and the order of a tentative model are decided by studying the shape of the appropriate correlation functions.
2. Estimation: The parameters of the selected model are estimated.
3. Diagnostic checks: The adequacy of the selected model is tested through several checks. If any inadequacy is found, the three-stage iterative cycle of identification, estimation and diagnostic checks is repeated for updating the model.

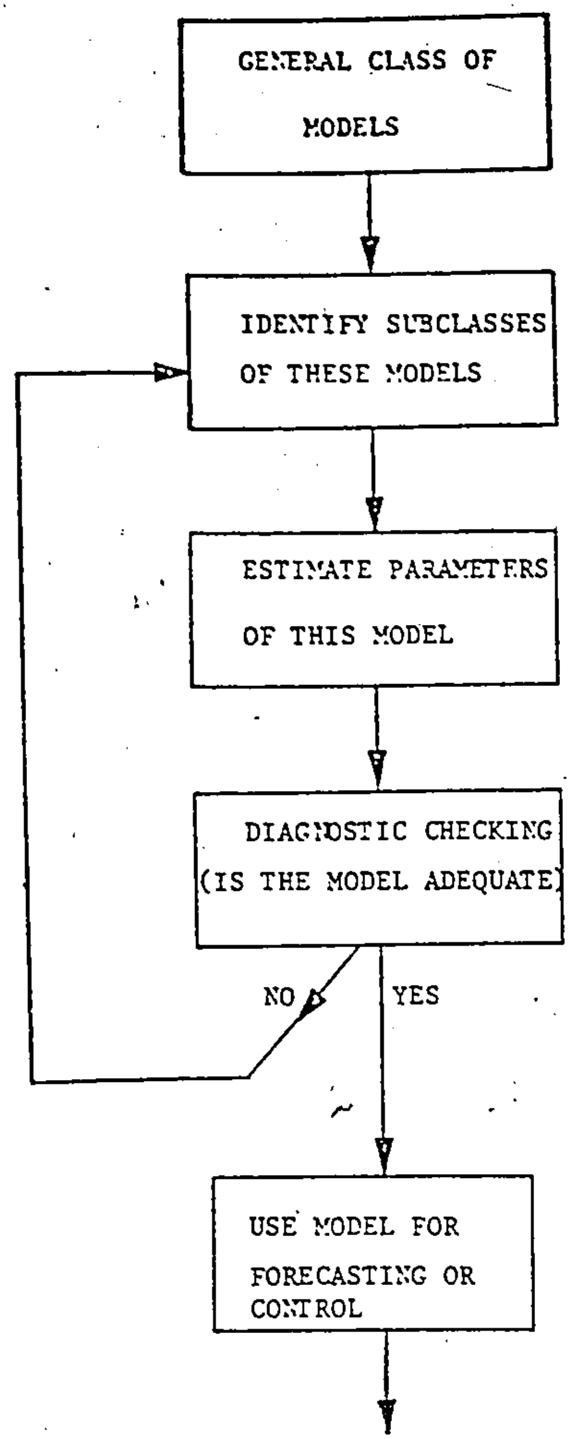


Figure 1: Stages in the iterative approach to model building

2.1.2 Identification

Identification of a tentatively selected model to be used in forecasting a time series is done through the analysis of historical data. The purpose of the identification is to determine the orders of AR and MA operators for the time series. The sample autocorrelation functions (ACF) the sample partial autocorrelation functions (PACF), the sample inverse autocorrelation functions (IACF)(37), the sample inverse partial autocorrelation functions (IPACF)(37) and R and S functions(33) can be used to identify the particular model that adequately describes an observed time series.

2.1.2.1 Time Plot

The first step in analysis of a time series is to plot the observations against time. This will show up important features such as trends in the mean or in the variance of the series, seasonality, outliers, persistence and discontinuities.

2.1.2.2 The Computation of Autocorrelation Functions (ACF)

The ACF r_k measures the linear relationships between observations in a time series that are separated by a lag of K time units. The autocorrelation coefficients are usually calculated by computing the series of autocovariance coefficients(40). The autocorrelation functions ρ_k at lag k in time is defined by

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

The estimate for r_k is given by

$$r_k = \frac{C_k}{C_o} \tag{2.7}$$

where,

$$C_k = \frac{1}{N} \sum_{t=1}^{N-K} (Y_t - \bar{Y})(Y_{t+k} - \bar{Y})$$

C_o is the autocovariance coefficient at lag k=0, $\bar{Y}$ is the mean of the time series and N is the length of the time series. The ACF for MA(q) process cuts off at lag q in time. The ACF of an AR(p) process is a mixture of damped exponentials and sinusoids, and dies out slowly.

2.1.2.3 The Computation of Partial Autocorrelations (PACF)

PACF ϕ_{kk} for an AR, autoregressive process of order K satisfies the Yule-Walker equation(9),

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \vdots & & \ddots & & \vdots \\ \rho_{k-1} & \rho_{k-2} & \cdots & & 1 \end{bmatrix} \begin{bmatrix} \phi_{k1} \\ \phi_{k2} \\ \vdots \\ \phi_{kk} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_k \end{bmatrix} \tag{2.8}$$

where,

ρ_k , the ACF of the Y_t series;

ϕ_{kj}, jth coefficient in an AR process of order K such that ϕ_{kk} is the last coefficient;

$$\sigma_a^2 = \left(1 - \rho_1\phi_{k1} - \rho_2\phi_{k2} - \cdots - \rho_k\phi_{kk}\right)\sigma_Y^2,$$

is the Yule-Walker equation used to determine σ_a of the white noise and σ_Y is the variance of Y_t series.

To get an estimate $\hat{\phi}_{KK}$ for ϕ_{KK} simply replace ρ_K by the sample ACF and solve the Yule-Walker equation for $\hat{\phi}_{KK}$. Substitute the autocovariance coefficient C_o at lag $k = 0$ for σ_Y in order to estimate σ_a^2 by $\hat{\sigma}_a^2$. The possible values of ϕ_{KK} range from -1 to 1. The partials tail off for an MA process and cut off for an AR process.

2.1.2.4 The Computation of Inverse Autocorrelation Functions (IACF)

IACF is used in conjuction with the other identification procedures to determine the orders of AR and MA parameters for the Y_t time series(40).

To obtain an estimate ri_K for ρi_K at lag K in time, the first step is to model the Y_t series by a finite AR process of order r given by,

$$Y_t = a_t' + \sum_{i=1}^{r} \pi_i Y_{t-r} \qquad (2.9)$$

where π_i is the ith AR parameter when the model (2.6) is written in invert form. For an AR process of order r the IACF is given by,

$$ri_k = \frac{-\pi_k + \pi_1 \pi_{k+1} + \ldots + \pi_{r-k}\pi_r}{1+\pi_1 + \ldots + \pi_r^2} \qquad (2.10)$$

An estimate ri_K for the IACF ρi_K can be obtained by calculating π_i, i=1, 2,....r for π_i from equation (2.8). Then substitute the $\hat{\pi}_i$ into (2.10) for π_i to estimate ri_K.

When the process is an AR model, ri_K cuts off and is not significantly different from zero after lag p. If ri_K tails off, this suggests the presence of an MA component.

2.1.2.5 The Computation of Inverse Partial Autocorrelations (IPACF)

The IPACF is developed in a manner similar to PACF(39). The inverse Yule-Walker equations are given by,

$$\begin{bmatrix} 1 & \rho i_1 & \rho i_2 & \cdots & \rho i_{k-1} \\ \rho i_1 & 1 & \rho i_2 & \cdots & \rho i_{k-2} \\ \vdots & & \ddots & & \vdots \\ \rho i_{k-1} & \rho i_{k-2} & \cdots & & 1 \end{bmatrix} \begin{bmatrix} \phi i_{k1} \\ \phi i_{k2} \\ \vdots \\ \phi i_{kk} \end{bmatrix} = \begin{bmatrix} \rho i_1 \\ \rho i_2 \\ \vdots \\ \rho i_k \end{bmatrix} \qquad (2.11)$$

where ρi_K is the IACF of the time series and ϕi_{Kj} is the jth coefficient in an MA process of order K such that ϕi_{KK} is the last coefficient. The coefficient ϕi_{KK} is called the IPACF. To get an estimate $\hat{\phi} i_{KK}$ for ϕi_{KK}, replace ρi_K by the sample IACF $r i_K$ and then solving the inverse Yule-Walker equation (2.11) for $O i_{KK}$. If the process is an MA model, IPACF is not significantly different from zero after lag q. If IPACF fails to cut off at time lags, this suggests the need for an AR model.

2.1.2.6 The Computation of R and S Functions

An identification technique based on using R and S functions was proposed by Gray(33). This technique can improve the identification of the order of ARIMA models. It is based on transforming the ACF ρi_K into the two functions R and S. The functions can be written as,

$$R_n(\rho_k) = \frac{H_n(\rho_{k-n+1})}{H_n(1,\rho_{k-n+1})} \qquad (2.12)$$

and

$$S_n(\rho_k) = \frac{H_{n+1}(1,\rho_{k-n+1})}{H_n\ (\rho_{k-n+1})} \tag{2.13}$$

where ρ_K is the ACF of the ARIMA process and the H functions are,

$$H_n(\rho_k) = \begin{vmatrix} \rho_k & \rho_{k+1} & \cdots\cdots & \rho_{k+n-1} \\ \rho_{k+1} & \rho_{k+2} & \cdots\cdots & \rho_{k+n} \\ \vdots & & & \vdots \\ \rho_{k+n-1} & \cdots\cdots & \cdots\cdots & \rho_{k-2n-2} \end{vmatrix} \tag{2.14}$$

and

$$H_{n+1}\ (1,\rho_k) = \left| \begin{array}{c|c} 1 \quad 1 \cdots\cdots & 1 \\ \hline & \rho_{k+n} \\ H_n(\rho_k) & \vdots \\ & \rho_{k+2n-1} \end{array} \right| \tag{2.15}$$

where $H_o(\rho_K) = 1$, K is an integer and n is a positive integer.

The following recursive expressions for estimation of R and S functions have been introduced by Gray(33),

$$R_{n+1}(\rho_k) = R_n(\rho_k)\left[\frac{S_n(\rho_k)}{S_n(\rho_{k-1})}\right] \tag{2.16}$$

and

$$S_n(\rho_k) = S_{n-1}(\rho_k)\left[\frac{R_n(\rho_{k+1})}{R_n(\rho_k)}\right] \tag{2.17}$$

for n = 1, 2, ... and K = 0, 1, 2, .. with $S_o(\rho_K) = 1$ and $R_1(\rho_K) = \rho_K$.

The ARIMA model identification procedure based on R and S functions require the estimation of the two functions by using (2.16) and (2.17), and displaying the functions in tabular form with n values as columns and K values as rows. Then the order of an ARMA (p,q) model is defined based on recognizing certain patterns which are characteristics of such functions.

The R function of Table 1 shows that:

1. The (p+1) column has zero values in all but (2q+1) rows centered around row zero.
2. The row q+1 has also zero values for n > p and undefined values appear for rows q+1 < k < -q and columns n > p+1.

The S function of Table 2 shows that:

1. There are two sets of constants in column p (C2 up to C1 down) around the nonconstants. The constant C2 above the center line starts in row -q-1 and continues up for any k < -q.
2. The row -q-1 has a series of values beyond the column p. All the elements above these infinite values are undefined.
3. Similarly, the constant C1 in column p below the center line starts at row q and continues down for any k > q. The same constant C1 appears in row q beyond the column p but with alternating sign.

4. The elements to the right of column p and below row q are all undefined.

It is clear from the above information that if the auto-correlation function ρ_K is known, the R and S functions identify the order p and q of the ARMA (p,q) model. A detailed analysis of R and S functions has been presented in(33).

TABLE 1

Display of the R function for an ARMA (p,q) model

		n		
k	1 . . . . . . :	p+1	p+2	p+i
-1	.	0	u	u
.	.	.	.	.
-q-2	Rl (-q-2)	0	u	u
-q-1	Rl (-q-1)	0	u	u
-q	Rl (-q)			
.				
-2	Rl (-2)			
-1	Rl (-1)			
0	Rl (0) 2q+1 nonzero elements			
1	Rl (1)			
2	Rl (2)			
.	.			
q	Rl (q)			
q+1	Rl (q+1)	0	0	0
q+2	Rl (q+2)	0	u	u
.		.	.	.
j	Rl (j)	0	u	u
.	.	.	.	.

u is undefined

TABLE 2

Display of the S function for an ARMA (p,q) model

n				
k	1	p	p+1	p+i
-1	S1 (-1)	C2	u	u
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
-q-2	.	C2	u	u
-q-1		C2	$\pm\infty$	$\pm\infty$
-q	S1 (-q)			
.	.			
.	.			
.	.			
-3	S1 (-3)			
-2	S1 (-2)			
-1	S1 (-1)			
0	S1 (0)	2q nonzero constants		
1	S1 (1)			
2	S1 (2)			
3	S1 (3)			
.				
.				
.				
.				
q-1	S1 (q-1)			
q	.	C1	-C1	(-1)C1
q+1	.	C1	u	u
q+2	.	C1	u	u
.		.	.	.
.		.	.	.
.		.	.	.
j		C1	u	u
.		.	.	.
.		.	.	.
.		.	.	.

u is undefined

2.1.3 Estimation

The identification process leads to a tentative formulation for the model and the model parameters have to be estimated.

Box and Jenkins(9) suggest the approximate maximum likelihood estimates for the ARMA parameter estimation. The parameters are obtained by minimizing the residual sum of squares

$$S(\phi, \theta) = \sum_{t=1}^{T} \hat{a}^2(t) \tag{2.18}$$

where, $S(\phi,\theta)$ is the residual sum of squares, T is the sample length, $\hat{a}(t)$ is the white noise series and ϕ and θ are parameters.

Mcleod(51) demonstrates a modified sum of squares method which provides parameter estimates that are closer approximations than those of Box and Jenkins to the exact maximum likelihood estimates and the advantages are:

1. the modified method gives better parameter estimates for shorter series;
2. the modified method is more efficient;
3. it is the exact maximum likelihood estimates (MLE) procedure for an AR process; and

4. improved parameter estimates are obtained for an MA process.

At the estimation stage, estimates are obtained for AR and MA parameters unless the exact value of a parameter is known in advance. If this is the case, the known parameter can be fixed and the remaining parameters are estimated. The prameters with values not significantly different from zero can be deleted from the model.

2.1.4 Diagnostic Checks

Often the identification stage yields a few tentative models to fit the time series. Therefore, it is important to perform diagnostic checks to test the adequacy of the model and, if needed to suggest potential improvement. Such checks are performed by various tests, which assume that the possible types of model inadequacies are known in advance.

Most diagnostic tests deal with the analysis of the residuals. In other words, the focus is to examine the differences between the observed data and the predictions given by the identified model and to determine whether the residuals are independent and normally distributed. The following tests were used in this thesis.

2.1.4.1 Overfitting

The procedure of overfitting consists of adding a parameter and testing the hypothesis that the added parameter is equal to zero, provided that the new parameter is not redundant. For example, if the proposed model is an ARMA(1,1)

$$Y_t = \phi_1 Y_{t-1} + a_t - \theta_1 a_{t-1}$$

one could test the ARMA(2,2) model

$$Y_t = \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

The new residual variance for the ARMA(2,2) model corrected for the additional degree of freedom due to the additional parameters may be calculated by

$$(\hat{\sigma}_a^2)_c = \frac{1}{N-n} (\Phi, \hat{\theta}) \tag{2.19}$$

where $S(\phi,\hat{\theta})$ is the sum of the squares of the residuals, N is the numbers of observations and n is the number of parameters (ϕ and θ). A decrease in the corrected residual variance provides a measure of the improvement due to the added parameters.

2.1.4.2 Checking the Statistical Significance of the Parameters

This procedure tests the hypothesis that the additional parameters are zero. The hypothesis test is based on the fact that each of the set of R parameter is equal to zero(26)

$$\left[\frac{TN-k}{R}\right] \frac{S(\hat{\phi}_*,\hat{\theta}_*) - S(\hat{\phi},\hat{\theta})}{S(\hat{\phi},\hat{\theta})} \qquad (2.20)$$

where, $S(\hat{\Phi},\hat{\Theta})$ is the conditional maximum likelihood estimate of the full parameter vector which contains the parameter to be tested and $S(\hat{\Phi}_*,\hat{\Theta}_*)$ is the conditional maximum likelihood estimate of the parameters vector with the appropriate R parameter constrained to be zero.

The above statistics is approximately distributed as an $F_{R,TN-R}$ under the null hypothesis. Any estimated parameter found to be insignificant should be removed. The model building procedure then returns to the estimation stage.

2.1.4.3 Portmanteau Test

The whiteness of the estimated residuals from the fitted model is tested according to(9),

$$Q = T \sum_{k=1}^{K} r_k^2(\hat{a}) \qquad (2.21)$$

where $(\hat{a})$ is the ACF of the residuals a_t, T is the sample size and K is the maximum lag. The statistic Q is approximately chi-square distributed with K - p - q degree of freedom, where p is the AR order and q is the MA order. The adequacy of the ARMA model may be checked by comparing the statistic Q with the chi-square value $\chi_{K,0.05}$ (K-p-q) of a given significance level. If $Q < \chi_{K,0.05}$ (K-p-q), a_t is an independent series and so the model is adequate, otherwise the model is inadequate.

2.1.4.4 The Cumulative Periodogram Test

The cumulative periodogram provides an effective means for the detection of periodic nonrandomness(9).

The periodogram of a white noise series a_t, t=1,2,...,N is (9)

$$I(F_j) = \frac{N}{2} \left| \sum_{t=1}^{N} a_i e^{2\pi F_j t} \right|^2$$

$$= \frac{N}{2} \left[\left(\sum_{t=1}^{N} a_t \cos 2\pi F_j t \right)^2 + \left(\sum_{t=1}^{N} a_t \sin 2\pi F_j t \right)^2 \right]$$

where, F = j/N the jth frequency. The normalized cumulative periodogram is given by

$$P(F_k) = \frac{1}{N\sigma_a^2}\sum_{j=1}^{K} I(F_j) \qquad (2.22)$$

where σ_a is the variance of the residuals. If the series is white noise then the plot of $P(F_K)$ vs. F_K would be close to straight line joining the points (0,0) and (0.5,1). The cumulative periodogram of a nonrandom series would show significant deviation from this line. For seasonal data with seasonal length SL, significant deviation may appear in $P(F_K)$ at frequencies 1/SL, 1/2SL, 1/3SL,....etc.

Confidence limits for white noise may be drawn on the cumulative periodogram plot parallel to the line from (0,0) to (0,0.5). They are drawn at distances $\pm H_\epsilon/\sqrt{K}$ around the white noise line. Typical values of H_ϵ are given in Table 3.

TABLE 3

Coefficients for calculating approximate probability limits for cumulative periodogram test

ϵ	0.01	0.05	0.10	0.25
H	1.63	1.36	1.22	1.02

The values of K is equal to

$$\frac{(N-2)}{2}$$

for N even, and

$$\frac{(N-1)}{2}$$

for N odd.

2.1.4.5 Akaike Information Criterion (AIC)

A mathematical formulation which considers the principle of parsimony in model building is the AIC proposed by Akaike(40). For comparison ARIMA(p,d,q) models he used

$$AIC(P,d,q) = N \ln (\hat{\sigma}_a^2) + 2(P+q) \tag{2.23}$$

where N is the sample size and σ_a is the maximum likelihood estimate of the residuals variance. Akaike suggested this criterion to select the correct model among competing ARIMA models. Under this criterion the model which gives the minimum AIC is the one to be selected.

2.1.4.6 Autocorrelation Check

This test is useful for checking the statistical significance of departures of the residuals autocorrelations from zero(9). If the residuals are white noise the autocorrelations should have zero mean and variance-covariance matrix equal to $\sigma^2 I_n$ and all autocovariances at nonzero lags equal to zero. If any of the residuals autocorrelation is significantly different from zero another model is selected and the model building stages are repeated. A model can then be identified to represent the residuals, which could be incorporated into the original model to obtain a better updated model for the space-time series.

Once an appropriate ARIMA model has been adopted, it is used to forecast future values of the time series.

2.2 TRANSFER FUNCTION NOISE MODELING OF RAINFALL-RUNOFF PROCESS

The focus of this review is on the on-line estimation of streamflow discharges and will address the models that have been developed for the purpose of river flow forecasting. In recent years a large number of mathematical models of many types have been adopted to represent different aspects of rainfall-runoff process(62). This has resulted in the development of two types of models, deterministic and stochastic rainfall-runoff models. In this review only stochastic modeling will be considered.

The most popular approaches in modeling of hydrologic time series have been the Box-Jenkins transfer function noise (TFN) modeling techniques. The rainfall-runoff models can be used to simulate streamflows based on past flow records or relating streamflow to one or more hydrologic time series(e.g. past flow data and rainfall series). In general the Box-Jenkins TFN models have been found effective in modeling of weekly, monthly and yearly flows. They have been ineffective in the modeling of hourly and daily flows(62).

Kitanidis and Brass(45) suggested that in order to account for the time varying nature of precipitation process the parameters of a linear model should be allowed to vary with time.

In cases of changing hydrologic conditions the forecasts of the model are not accurate because the assumption of time varying parameters takes into account only the most recent input/output behavior of the model.

Mao and Rao(50) examined the implication of transforming the precipitation series into a modified rainfall process by accounting for evaporation and soil moisture storage. The cross correlation coefficients were used to study the relationships between modified rainfall-runoff process and the original rainfall-runoff series. The coefficients were reported to be higher for the modified case than those for the original time series. The residuals of the models were found to be white noise and free of periodicity. It was concluded that the ordinary least squares parameter estimates(OLS) were accurate even though biased.

Several studies in the hydrologic literature have been performed to determine the impulse response function of linear system via input-output analysis. Multiple frequency response analysis provided one way to establish a relationship between inputs and outputs of a multiple input-single output system set up for surface water relationship. Cross correlation and cross spectral analysis were employed in the analysis of rainfall and runoff to explain the relationship in this multiple input-single output system(62).

Delleur(27) formulated three stochastic models, two for the simulation of daily precipitation series and one daily rainfall-runoff transfer function model to produce daily runoff process from the above daily precipitation models. The models were successfully applied to five Indiana watersheds in U.S.A.

Yakowitz(64) studied the suitability of a nonparametric model of time series. He presented a nonparametric Markov modeling approach for daily river flow forecasting. It was pointed out that the ARMA parameter estimation involves some nonlinear programming problems and the length of the time series plays an important role in the accuracy of the ARMA parameters. The length of the data to estimate the parameters of an ARMA(2,2) with a relative error of less than 0.3 is at least a thousand observations. Therefore the accuracy of the parameter estimate decreases as the order of the model and the record length decreases.

Miller(55) provided a comprehensive case study in which he fit transfer function-noise models to a number of watersheds. He used the log-transformation of the flows and the inclusion of second degree polynomial precipitation terms to decrease the skewness of the residuals and to improve the forecasting ability of the model. The work indicated that significant care is needed in the model selection and the verification of modeling assumptions.

Dunn(28) suggested that much closer cooperation between field work and modeling efforts is needed. Such cooperation can lead to the formulation of better and more relevant field experiments. Sophisticated field experiments can assist the hydrologist to "discover unexpected hydrologic phenomena, to develop new concepts about familiar processes and to guide the development of mathematical models based on sound physical insights". It would also discourage much of the aimless collection of data. This will improve the quantity and the quality of the data for the purpose of model identification, generation and forecasting.

In the work presented the emphasis has been on the various contributions made by researchers on the developments of stochastic models which establish relationship between rainfall and runoff. It is evident that the emphasis was mainly concentrated on single input-single output systems for rainfall-runoff modeling. The models capability was mostly limited to predicting runoff generation at a point. The methods vary in their complexity of estimation schemes and forecasting performance.

Remote sensing techniques promise to provide valuable spatial information to the hydrologist interested in rainfall and rainfall-runoff forecasting. Therefore it is important to extend the existing modeling techniques of hydrological system into spatial domain. This could lead to more accurate forecasting models of both rainfall and runoff processes.

This thesis is extending the Box and Jenkins ARIMA and TFN models into the spatial domain to model and forecast rainfall and runoff over a region.

2.3 SPACE-TIME MODELING

It is often important to predict the values of hydrological process at some point, which has not been observed yet within the area of observation, or to predict the value of the variable at some point outside the area of observation. There are many possible reasons for doing this. Examples are the following:

1. Data which have been observed in past time periods may not cover the exact point which is being investigated. The value of the variable at this point could be predicted by using the available data.
2. In weather forecasting only a set number of recording stations could be possible due to economical reasons. Due to this, the forecasting of various aspects of the weather within the region covered by the stations or outside the region is often desirable.

A detailed formulation of the space-time rainfall modeling using a point process approach was recently reported by Wymire and Gupta(63). In this method a suitable mathematical framework was built upon a combination of observed empirical (hydrologic) behavior of space-time rainfall and physical relations. Therefore, the analytical structure provides a natural background for simulation of rainfall process.

Martin and Oeppen(54) recently reported the extension of Box and Jenkins univariate ARIMA time series and TFN modeling approach into the spatial domain. These techniques lead to the formulation of STARIMA and STTFN models.

The objective of this review is to address STARIMA and STTFN models that have been developed for the purpose of forecasting. These spatial forecasting models are attractive and can be used to provide much information about the behavior of hydrological variable over an area.

2.3.0.7 STARIMA Process

These models apply to a random variable observable at N fixed locations or sites in space over T time periods, t=1,2,...T. They are of value for forecasting purposes when the observed system exhibits spatial autocorrelation, defined by Cliff and Ord(20) as follows:'If the presence of some quality in a county of a country makes its presence in neighboring counties more or less likely, we say the phenomenon exhibits spatial correlation'.

Martin and Oeppen(54) presented two broad classes of regional forecasting models. The models belong to the general family of STARMA models,

$$Y_{it} = a_{it} + \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{s} Y_{i,t-k} - \sum_{s=0}^{\lambda} \sum_{k=1}^{q} \theta_{sk} L^{s} a_{i,t-k} \qquad (2.24)$$

where p is the AR order, q is the MA order, M and λ are the spatial orders of autoregressive and moving average terms respectively, L^s is the space lag operator, a_{it}'s are random normal errors and θ and ϕ are parameters. An explanatory model that constitutes the class of STTFN process of order s in space and k in time is,

$$Y_{it} = \sum_{s=0}^{S} \sum_{k=1}^{K} \omega_{sk} L^s X_{i,t-k} + a_{it} \tag{2.25}$$

where ω_{sk} are unknown parameters.

The STARMA model class collapses into the ARMA model class in the absence of spatial correlation. Two special subclasses of the STARMA model are of note. Models that contain only moving average term (p=0) are refered to as space-time moving average (STMA) process of order λ in space and q in time,

$$Y_{it} = a_{it} - \sum_{s=0}^{\lambda} \sum_{k=1}^{q} \theta_{sk} L^s a_{i,t-k} \tag{2.26}$$

When q=0, only autoregressive terms remain and the class is space-time autoregressive (STAR) process of order M in space and p in time,

$$Y_{it} = a_{it} + \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{s} Y_{i,t-k} \tag{2.27}$$

2.3.0.8 Applications of STARIMA Models

Martin and Oeppen(54) introduced an identification technique based on the estimation of space-time autocorrelation (STACF) and space-time partial autocorrelation (STPACF) functions for identifying STARMA models. The data employed in the analysis consisted of the number of fowl-pest outbreaks in eastern England (1970-77) by 15 day periods for different locations. The identification of space-time transfer function process required pre-whitening the input series. A STARMA model was identified for the input series, which was then used to transform the correlated input series to the uncorrelated series. The same transformation was applied to the output series and the space-time cross correlation function was then computed between these two new series. Martin and Oeppen introduced the spatial difference operators which can be used to achieve stationarity like the temporal difference operators. No attempts were made to estimate the parameters and calibrate the identified models.

Pfeifer and Deutsch(26) presented a comprehensive study for building space-time autoregressive integrated moving average (STARIMA) models. The identification tools were the STACF and STPACF. The maximum likelihood method was applied to estimate the parameters for the STARIMA model. Since only STAR models are linear in form, it was necessary to estimate STMA parameters based on nonlinear optimization technique. To illustrate the space-time modeling procedure, arrest data from 822 areas of the city of Boston were combined to give monthly figures for 14 areas of Northeast Boston. The records were observed for 72 time periods. The models were subjected to diagnostic checks via the examination of the residuals from the fitted models and by testing the statistical significance of the estimated parameters. All parameters were proven statistically significant and the models were used as forecasting tools for the number of arrests in the 14 districts of Northeast Boston.

Recently, Hooper and Hewings(38) reported studies on the basic theory of STARMA models comparing it to the simpler time series framework. The main application of the work was to the problem of identifying an appropriate type order of model(for a given spatial lag operator) by referring to the shape of the sample STACF and STPACF. It was indicated that STACF and STPACF for a given space-time model do not behave in a manner similar to the classical time series. It was also indicated that the assumption of stationarity especially across space is the greatest problem for the space-time modeling. Therefore, some transformation must be applied to the time series in order to obtain the stationarity in time and across space. As the choice of a spatial lag operator affects the form of the STACF and STPACF, care must be taken in defining the spatial lag operator so that useful models would not be eliminated from consideration. Thus it is important to realize that the space-time correlation functions do not behave in a manner completely similar to the univariate classical time series. The following rules for STACF and STPACF are in effect:

1. for STMA models the cut off point for STACF in space after a certain number of lags in time and space depends on the lag structure and the order of the model; and
2. the cut off in the STPACF for STAR process after a certain number of lags in time and space depends only on the order of the model and not on the lag structure.

The STACF tailed off faster in space than in time for the STARMA and STAR models than for the STMA model. The moving average coefficients were smaller than the autoregressive coefficients.

In 1974 Cliff and Ord(20) published a paper considering autoregressive, moving average and regression forecasting models which may be applied to space-time processes. They indicated that such models in a forecasting role, have been useful for helping governments and planners in policy formulation and decision making. They Incorporated spatial autoregressive, moving average and regressive components into one model(SARMAR) and combined these features with the Box-Jenkins ARIMA model. The basic structure of the model was illustrated by an application to the diffusion of tractor ownerships in 25 states of the Central U.S.A. from 1920-64. In the analytical study it was concluded that the principal difficulty with regional modeling is the crucial space-time interaction which is swamped by major regional or temporal differences on nonhomogeneity in space. Cliff and Ord suggested an analysis of covariance system to avoid the problem. This is performed by breaking down the time series into spatial, temporal and interaction components. Each component should be analysed separately. The main reasons for this separation are:

1. the outliers are easier to be detected;
2. since the components are uncorrelated sets of variables, they could be modeled separately;
3. it allows the assessment of the properties of the total variance attributable to each component. This helps to decide which components are important for further modeling;
4. computational problems are often reduced.

The percentage of farms using tractors in each of 25 states which form the central farm production region of the United States were analysed. The innovation centre was North Dakota. This system implied that the percentage of farms in a state using tractors at a given date was a function of the distance of that state from North Dakota.

Bennet(14) carried out simulation studies to formulate a set of mathematical representations which are capable of describing the past and present landscapes. The paper has also sought to describe the identification problems that result from the application of autocorrelation processes to the description of real world process. The extension of the Box-Jenkins statistical technique to space-time modeling was explained using the autocorrelated diffusion process describing the spread and growth of populations in North-West England over the period 1891-1971. The identification methods which were adopted by using STACF and STPACF have suggested that the North-West population series under study was governed by space-time autoregressive process. The results of applying ordinary least squares (OLS) to the space-time parameter estimation problem with the model were consistent although inefficient. For general mixed STARMA or STMA process in which moving average elements are present, a more efficient parameter estimator must be used, since it is a nonlinear problem. This estimator was introduced by Bennet as a research area that should be investigated in order to get more reliable parameters. Bennett considered four major types of space-time processes:

1. The autocorrelated process: it relates the changes in a certain region to the changes in neighboring regions. These changes are dependent on many factors, such as distance decay or lags and urban hierarchy. These factors determine levels of structural dependence upon sets of control variables which are not or cannot be observed.
2. The interaction process: it is introduced as the evolution of a given zone to be governed by an interaction matrix which controls the dependence of one zone upon the rest of the zones in the system. It is the extension of the weight matrix proposed by Martin and Oeppen(54).
3. The non-stationary process: Bennett suggested that the non-stationarity in space-time processes across time and space via trends might be a crucial factor in constructing any space-time model.

Therefore, differencing in either time or space independently might be undertaken in this study.

In conclusion, this review has identified three important facts concerning space-time modeling. First, STARIMA models are useful toward modeling systems that exhibit spatial correlation. Secondly the identification of space-time TFN model relating changes in runoff to rainfall in time and space based on cross correlation functions could lead to spurious results. Third, building a dynamic space-time TFN model based on prewhitening the rainfall series by a STARIMA model and then following Box and Jenkins TFN modeling approach is an attractive model building approach on theoretical grounds. Therefore, this study will focus on the TFN modeling approach of Box and Jenkins(9) with the purpose of extending the approach into a space-time system. This is important since rainfall is a space-time phenomenon with a high impact on runoff over an area.

Chapter III
RESEARCH DATA

3.1 THE WATERSHED

The Grand River Basin above Galt (Figure 2) is selected for this study. The watershed, 3480 square kilometers in size, is located in Southern Ontario, west of Toronto. The town of Galt, near the outlet of the basin, often faces flood problems mainly from spring runoff. There is, therefore, a need for a water management system.

For the numerical analysis 4 and 11 raingage sites are used, which are located in the watershed. In this procedure the area is subdivided into polygonal subareas using rain gages as centers following Thiessen method. The subareas are used to formulate a weighting scheme. To illustrate the space-time TFN modeling procedure for rainfall-runoff process, only four discharge stations within the four subareas are considered. This is due to the availability of only 4 runoff sites which correspond to the same subareas where the 4 raingages are located. Additional features of precipitation and discharge stations relevant to this study are shown in Tables 4 and 5.

3.2 . DATA SELECTION

For the numerical analysis the daily discharge in cubic meters per second at four locations for the period of records from 1966 to 1980 have been collected by the Inland Waters Directorate, Water Resources Branch, Environment Canada. Precipitation data was obtained from the Atmospheric Environment Service, Environment Canada. The daily discharge data used in this research was sampled every 15 days, starting on July 1966, and continuing until June 1980, giving a total of 336 data points. The precipitation data is in mm accumulated over 15 days total for the same periods as the discharge data and giving the same record length of 336 data points. For the space-time modeling procedure of parameter estimation only the first 192 data points are used. The rest of the data, 144 data points, are used for testing the forecasting performance of the selected space-time models. The selection of the rainfall data was based on sampling period of 15 days total. This was the case for the following reasons: first, in order to have a record length enough for the parameter estimation stage of the model building procedure; second, sampling record of 15 days total for rainfall time series preserves the characteristics of rainfall storms.

3.3 ESTIMATING MISSING RAINFALL DATA

Some of the rainfall stations had missing values in their records. It was necessary to estimate this missing data in order to establish a consistent and complete data base. This was done by applying the normal-ratio method(48). In this method, the rainfall data from three nearby stations were required. The estimate of the missing rainfall data were based on a ratio of the normal annual rainfall given by

$$P_x = \frac{1}{3}\left(\frac{N_x}{N_a} P_a + \frac{N_x}{N_b} P_b + \frac{N_x}{N_c} P_c\right)$$

where P_a, P_b and P_c are the measurements of the rainfall at the nearby stations a, b and c respectively, N_a, N_b and N_c are the mean rainfall for stations a, b and c respectively, N_x is the annual rainfall mean at the station with missing data, and P_x is the unknown rainfall value at the station with missing data. The missing data points of the rainfall series are shown in Table 6.

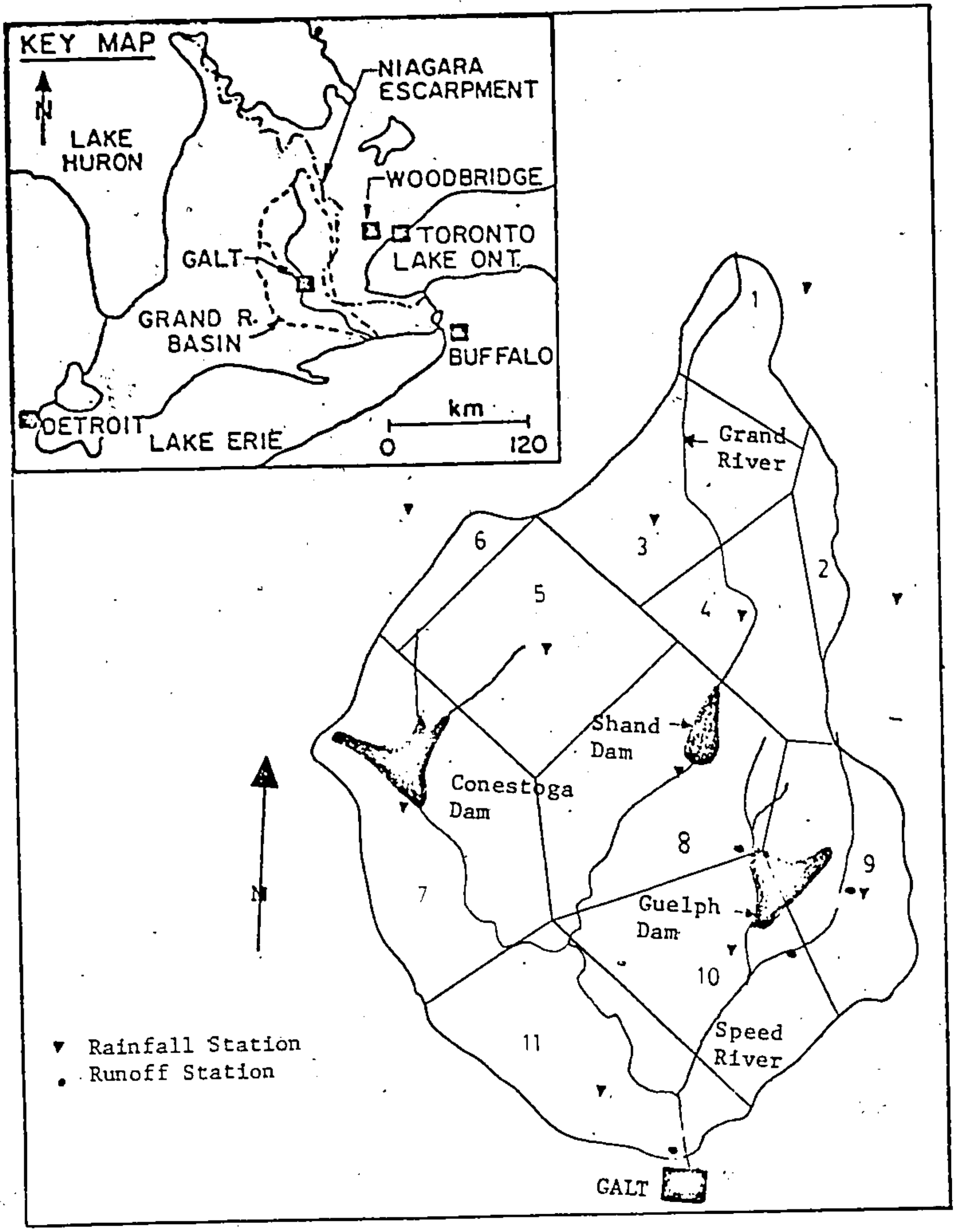


Figure 2: The Watershed and key map

TABLE 4

Features of rainfall stations in the selected Watershed

Station Name	No.	Period of Records
Redkvile	6146939	1966-1980
Orangeville	6155790	1966-1980
Monticello	6145267	1966-1980
Waldemar	6149205	1966-1980
Arthur	6140348	1966-1980
Mount Forest	6145503	1966-1980
Glen Allan	6142803	1966-1980
Fergus Shand	6142400	1966-1980
Blue Spring Creek	6150818	1966-1980
Guelph O.A.C.	6143088	1966-1980
Preston	6146711	1966-1980

TABLE 5

Features of runoff stations in the selected Watershed

Station Name	No.	Location	Period of Records
Luteral Creek near Oustic	02GA033	43 39 46 80 15 19	1966-1980
Blue Spring Creek near Eden Mills	02GA031	43 34 35 80 06 36	1966-1980
Eramosa River above Guelph	02GA029	43 32 52 80 10 59	1966-1980
Grand River at Galt	02GA003	43 21 10 80 19 10	1966-1980

TABLE 6

Rainfall Stations with Missing Data

Station No.	6150818	6142400	6143088	6145503	6149205
Date	10 1976	03 1971	12 1973	03 1980	07 1966
	11 1976	06 1971	.		08 1966
	10 1978	09 1971	.		01 1967
	12 1978		.		01 1968
	01 1979		.		04 1968
	02 1979		.		06 1968
	03 1979		.		05 1969
	04 1979		.		02 1971
	12 1979		.		10 1971
	01 1980		.		11 1971
	02 1980		06 1975		12 1971
	03 1980				01 1972
	04 1980				02 1972
					03 1972
					05 1976
					.
					.
					12 1976
					01 1977
					.
					.
					12 1977
					02 1978
					03 1978
					04 1978

Chapter IV

THEORETICAL DEVELOPMENTS

4.1 SPACE-TIME ARIMA MODELING

The first step in space-time ARMA modeling is to identify the model that might be worth considering. Following this, the model parameters are estimated using any of a variety of linear or nonlinear optimization techniques. Then the model is checked for possible inadequacies. If the diagnostic checks reveal serious inadequacies, then appropriate model modifications can be made by repeating the identification and estimation stages.

To assist in the models that follow, the concept of the spatial lag operator L is presented. This concept of spatial lag order is incorporated into the model form via a matrix of scaled weights for sth spatial lag(54).

let L be a spatial lag operator,

$$L^{0}Y_{it} = Y_{it}$$

and

$$L^{s}Y_{it} = \sum_{j=1}^{N} W_{ij}^{s} Y_{it} \qquad \text{for any } s > 0 \qquad (4.1)$$

where W_{ij} are weights scaled so that

$$\sum_{j=1}^{N} W_{ij}^{s} = 1 \qquad (4.2)$$

For any s and T If Y_{it} is an (NX1) column vector of observations i=1,2,.....,N, then

$$L^{s}Y_{it} = \sum W^{s}Y_{it} \qquad \text{for any } s > 0 \qquad (4.3)$$

where W^{s} is the (NXN) matrix of scaled weights for the sth spatial lag with each row summing to one. The choices for the weights are made to reflect physical properties of the observed system, such as:

1. length of the common boundary between subareas;
2. barriers between subareas, such as rivers, mountains etc.

These weights must reflect a hierarchical ordering of spatial neighbors. The definition of spatial order represents an ordering in terms of distances of all sites surrounding the location of interest. First order neighbors are those closest to the zone of interest. Second order neighbors should be farther away from first order but closer than third order neighbors. A definition of spatial order in two and one dimensional systems is illustrated in Figure 3.

The STARMA model family is expressed in equation (2.24). When q=0, only the autoregressive terms remain and the model is space-time autoregressive (STAR),

$$Y_{it} = a_{it} + \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{s} Y_{i,t-k} \tag{4.4}$$

Models that contain no autoregressive terms (p=0) are referred to as space-time moving average (STMA),

$$Y_{it} = a_{it} - \sum_{s=0}^{\lambda} \sum_{k=1}^{q} \theta_{sk} L^{s} a_{i,t-k} \tag{4.5}$$

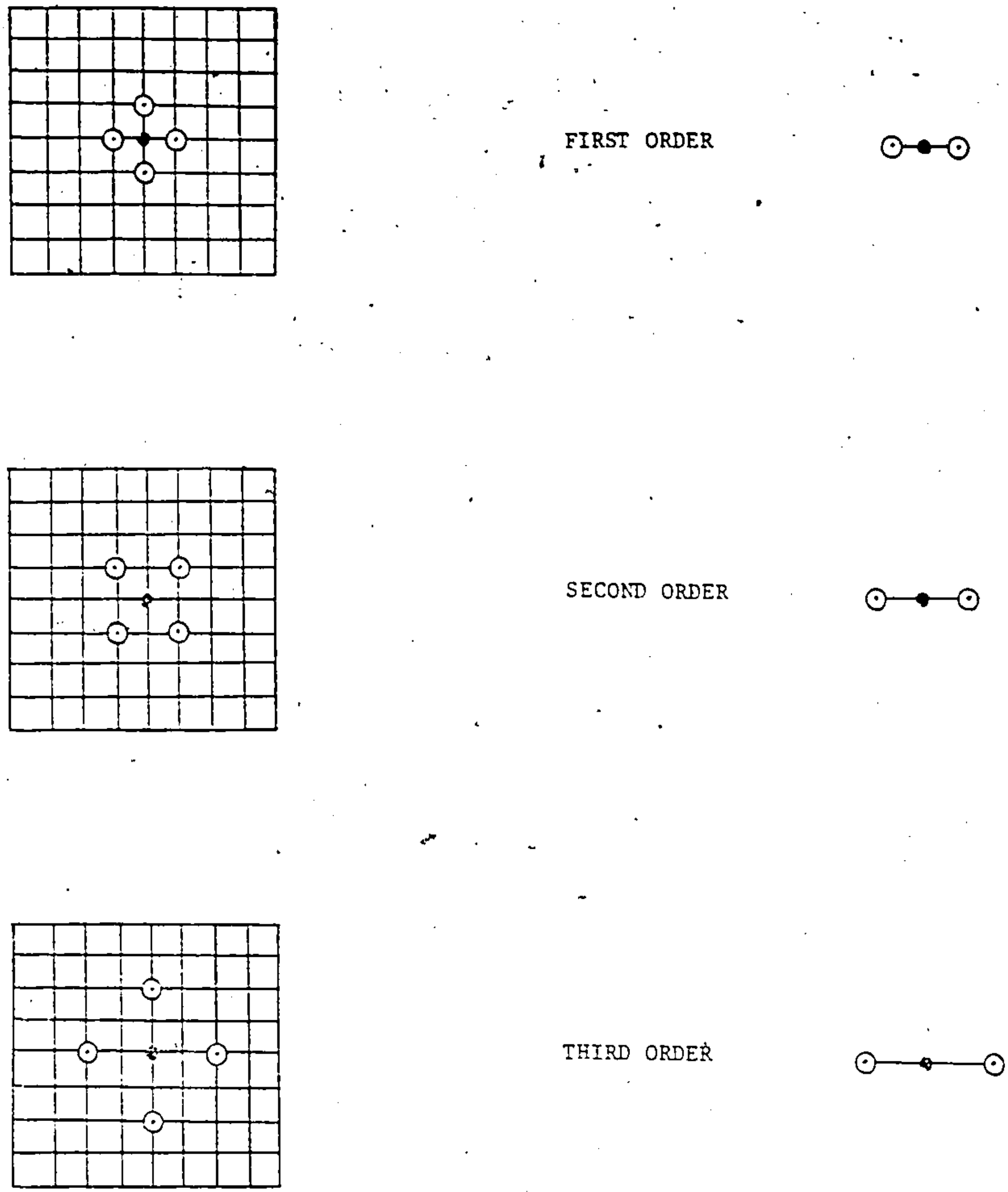


Figure 3: Spatial order in two- and one-dimensional systems

4.1.1 Identification

Identification of a tentative model to be used in forecasting a time series is done through the analysis of historical data. In Section 4.1.1.1 the concepts of space-time ACF will be discussed. In Section 4.1.1.2 the concepts of space-time PACF will be presented. In Section 4.1.2 characteristics of the theoretical STACF and STPACF for identifying STAR, STMA and STARIMA models will be presented in detail.

4.1.1.1 Space-Time Autocorrelation Functions (STACF)

Using the definition of the spatial lag operator presented previously in equation(4.1), let $L^s Y_{i,t-k}$ denotes the kth temporally and sth spatially lagged value for ith site(54). The STACF expressing the covariance between random variables lagged both in time and space is,

$$\gamma_{sk} = \left[\mathrm{COV}\ Y_{it}\ ,\ L^s Y_{i,t-k} \right]$$

$$= E\left[(Y_{it} - \mu)(L^s Y_{i,t-k} - \mu) \right] \qquad (4.6)$$

where μ is the grand mean of the space-time series

$$\mu = E\left[Y_{it}\right] \tag{4.7}$$

The variance of the series is

$$\begin{aligned} \sigma^2{}_{00} &= \mathrm{var}\left[Y_{it}\right] \\ &= E\left[(Y_{it} - \mu)^2\right] \end{aligned} \tag{4.8}$$

The STACF at spatial lag s and time lag K is given by:

$$\begin{aligned} \rho_{sk} &= \frac{\gamma_{sk}}{\sigma_{00}{}^2} \\ &= \frac{E\left[(Y_{it} - \mu)(L^s Y_{i,t-k} - \mu)\right]}{E\left[(Y_{it} - \mu)^2\right]} \end{aligned} \tag{4.9}$$

The estimate for ρ_{sk} is

$$r_{sk} = \frac{C_{sk}}{C_{00}{}^2} \tag{4.10}$$

where C_{oo}^2 is the estimate for the variance σ_{oo}^2. Thus, the STACF at spatial lag s and time lag K for the observed time series Y_{it} at N locations in space over T time periods via (4.9) and (4.10) is

$$r_{sk} = \frac{\sum_{t=k+1}^{T} \sum_{i=1}^{N} (Y_{it} - \bar{Y})(L^s Y_{i,t-k} - \bar{Y})}{\sum_{t=1}^{T} \sum_{i=1}^{N} (Y_{it} - \bar{Y})^2} \qquad (4.11)$$

where $\bar{Y}_{it}$ is the grand mean estimate of μ

$$\bar{Y}_{it} = \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} Y_{it} \qquad (4.12)$$

Martin and Oeppen(54) suggested a second procedure for estimating the STACF. The variance of the series Y_{it} is defined by:

$$\left[\mathrm{var}\,(Y_{it})\right]^{1/2}\left[\mathrm{var}(L^{s}Y_{it})\right]^{1/2} < \mathrm{var}(Y_{it}) \qquad (4.13)$$

Thus, the STACF at spatial lag s and time lag K is

$$\rho_{sk} = \frac{\gamma_{sk}}{(\sigma_{00}{}^{2})^{1/2}\,(\sigma_{sk}{}^{2})^{1/2}}$$

or

$$\rho_{sk} = \frac{E\left[(Y_{it} - \mu)(L^{s}Y_{i,t-k} - \mu)\right]}{\left(E\left[(Y_{it} - \mu)^{2}\right]\right)^{1/2}\left(E\left[(L^{s}Y_{it} - \mu)^{2}\right]\right)^{1/2}} \qquad (4.14)$$

and the estimate for ρ_{sk} from the time series Y_{it} is,

$$r_{sk} = \frac{\sum_{t=k+1}^{T}\sum_{i=1}^{N}(Y_{it} - \bar{Y})(L^{s}Y_{i,t-k} - \bar{Y})}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N}(Y_{it} - \bar{Y})^{2}\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}(L^{s}Y_{it} - \bar{Y})^{2}\right]^{1/2}} \qquad (4.15)$$

A third alternative for developing the STACF was also introduced by Martin and Oeppen(54). This is done by setting the problem in a multivariate framework. The definition of the STACF at spatial lag s and time lag K is provided by

$$\rho_{sk} = \frac{\gamma_{sk}}{(\sigma_{00}^2 \sigma_{sk}^2)^{1/2}}$$

or

$$\rho_{sk} = \frac{\left[E\,(Y_{it} - \mu)(L^s Y_{it} - \mu_s)\right]}{\left(E\left[(Y_{it} - \mu)^2\right]\right)^{1/2} \left(E\left[(L^s Y_{i,t-k} - \mu_s)^2\right]\right)^{1/2}} \qquad (4.16)$$

where

$$\mu_s = E\left[L^s Y_{it}\right]$$

and the estimates for ρ_{sk} from the time series Y_{it} is given by,

$$r_{sk} = \frac{\sum_{t=k+1}^{T} \sum_{i=1}^{N} \left[Y_{it} - Y\right]\left[L^s Y_{i,t-k} - (\overline{L^s Y_{it}})\right]}{\left(\sum_{t=k+1}^{T} \sum_{i=1}^{N} \left[Y_{it} - \overline{Y}\right]^2\right)^{1/2} \left(\sum_{t=k+1}^{T} \sum_{i=1}^{N} \left[L^s Y_{i,t-k} - \overline{(L^s Y_{it})}\right]^2\right)^{1/2}} \qquad (4.17)$$

where

$$\overline{(L^sY_{it})} = \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} L^sY_{it} \tag{4.18}$$

is an estimate of the grand space-time mean. Martin and Oeppen indicated that using alternative one for computing the STACF at spatial lag s and time lag K via equation(4.11) produces unsatisfactory estimates. This is due to the smoothing effect caused by the spatial lag operator L^s. The L^s produces the form of a weighted average of $L^s\ Y_{it}$ with a variance normally less than the variance of Y_{it},

$$\mathrm{var}(L^sY_{it}) < \mathrm{var}(Y_{it})$$

Alternative three for the definition of the STACF via equation(4.17) does not produce satisfactory estimates. This is due to the following two reasons. The first reason is that two means are used in the calculations with one of the means changing with the spatial lag. The second reason is the variation of the normalizing factor with time lag K. Alternative two is the recommended one for the estimation of the

STACF at spatial lag s and time lag K via equation(4.15). This equation could be rewritten in much simpler form for computational purposes:

$$r_{sk} = \frac{\sum_{t=k+1}^{T} \sum_{i=1}^{N} z_{it} L^s z_{i,t-k}}{\left\{ \left[\sum_{t=1}^{T} \sum_{i=1}^{N} z_{it}^2 \right] \left[\sum_{t=1}^{T} \sum_{i=1}^{N} (L^s z_{it})^2 \right] \right\}^{1/2}} \tag{4.19}$$

where

$$z_{it} = Y_{it} - \overline{Y}$$

4.1.1.2 Space-Time Partial Autocorrelation Functions (STPACF)

The STPACF is defined as follows. The STAR model is

$$Y_{it} = \sum_{s=0}^{M} \sum_{K=1}^{P} \phi_{sk} L^s Y_{i,t-k} + a_{it} \tag{4.20}$$

Let $Z_{it} = Y_{it} - \overline{Y}_{it}$, where $\overline{Y}_{it}$ is the grand space-time mean, then the STAR model becomes:

$$Z_{it} = \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{s} Z_{i,t-k} + a_{it} \qquad (4.21)$$

Multiplying equation(4.21) by $L^{h} Z_{i,t-j}$ gives

$$L^{h} Z_{i,t-j} Z_{it} = \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{h} Z_{i,t-j} L^{s} Z_{i,t-k} + L^{h} Z_{i,t-j} a_{it} \qquad (4.22)$$

where, h, s=0,1,2,......M are lags in space and j,k=1,2,....p are the lags in time. Taking expected values,

$$\gamma_{hj} = \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} \gamma_{h-s,j-k} \qquad (4.23)$$

since

$$\left[E\, L^h Z_{i-j} a_{it}\right] = 0 \qquad \text{for } j > 0$$

Dividing equation(4.23) by $(\sigma_{oo}^2)^{1/2}\,(\sigma_{so}^2)^{1/2}$ to obtain a set of spatial- temporal Yule-Walker equation,

$$\rho_{hj} = \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk}\rho_{h-s,j-k} \qquad \text{for } j > 0 \qquad (4.24)$$

Replacing the theoretical autocorrelation functions ρ_{hj} by the estimated autocorrelation r_{hj} to obtain a system of equations

$$\begin{bmatrix} r_{01} \\ r_{02} \\ \vdots \\ r_{MP} \end{bmatrix} = \begin{bmatrix} 1 & r_{01} & r_{02} & \cdots & r_{MP-1} \\ r_{01} & 1 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ r_{MP-1} & r_{MP-2} & r_{MP-3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \phi_{01} \\ \phi_{02} \\ \vdots \\ \phi_{MP} \end{bmatrix} \qquad (4.25)$$

Denoting by ϕ_{MsPk} so that the last coefficient is ϕ_{MMPP}. From equation (4.24) and (4.25), ϕ_{MMPP} satisfies a set of equations for s=0,1,2,..... and k=1,2,.... which is the space-time analog of the Yule-Walker equation for univariate time series,

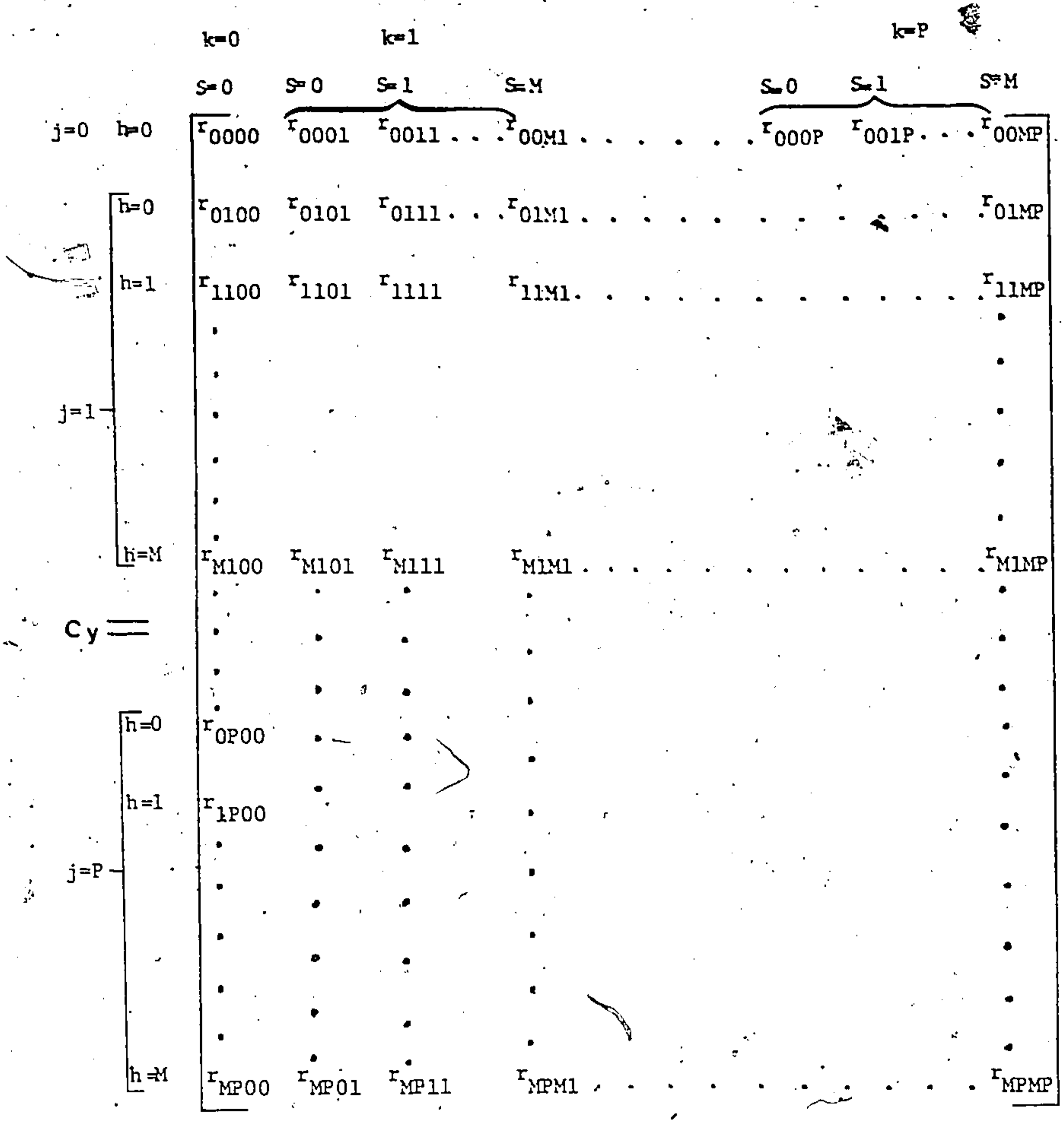
$$
\begin{aligned}
r_{sk} = {} & \phi_{M0P1} r_{sk-1} + \phi_{M0P2} r_{sk-2} + \cdots\cdots + \phi_{M0PP} r_{sk-P} + \\
& + \phi_{M1P1} r_{s-1k-1} + \phi_{M1P2} r_{s-1k-2} + \cdots\cdots + \phi_{M1PP} r_{s-1k-P} \\
& \vdots \\
& + \phi_{MMP1} r_{s-Mk-1} + \phi_{MMP2} r_{s-Mk-2} + \cdots\cdots + \phi_{MMPP} r_{s-Mk-P}
\end{aligned} \tag{4.26}
$$

ϕ_{MMPP} is the partial correlation for M=1,2,... for each p=0,1,2.. holding all intermediate lagged terms fixed. ϕ_{MsPk} represents the space-time autoregressive coefficients s,k for the M,p order process.

Kendall and Stuart (46) adapted a method for estimation of partial correlation function. With this method, each partial autocorrelation is computed holding all other lagged terms constant; i.e, conditionally upon those other variables taking certain fixed values. To apply this analysis to space-time modeling, the order of a priori specification of the space-time interactions is not required. Let Cy be the intercorrelation matrix between all pairs of lagged variables, $L^s Y_{i,t-k}$ and $L^h Y_{i,t-j}$, where h,s=0,1,......,M are the lags in space and j,k=1,2,....,p are the lags in time. equation(4.15) for the STACF estimation may be modified in the following form:

$$r_{hjsk} = \frac{\sum_{t=\upsilon+1}^{T} \sum_{i=1}^{N} (L^h Y_{i,t-j} - \bar{Y})(L^s Y_{i,t-k} - \bar{Y})}{\left[\sum_{t=1}^{T} \sum_{i=1}^{N} (L^h Y_{it} - \bar{Y})^2\right]^{1/2} \left[\sum_{t=1}^{T} \sum_{i=1}^{N} (L^s Y_{it} - \bar{Y})^2\right]^{1/2}} \qquad (4.27)$$

to build up the rows of the symmetric autocorrelation matrix Cy (Display 1). In equation(4.27) v=max(j,k) over the range of the indices h,j,s, and k. The first column and row of Cy contains the estimated STACF r_{oosk}. For computational purposes equation(4.27) may be rewritten in the following form:



Display 1. Structure of the symmetrical space-time autocorrelation matrix C_Y.

$$r_{hjsk} = \frac{\sum_{t=\upsilon+1}^{T}\sum_{i=1}^{N}(L^h Z_{i,t-j})(L^s Z_{i,t-k})}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N}(L^h Z_{it})^2\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}(L^s Z_{it})^2\right]^{1/2}} \quad (4.28)$$

where,

$$Z_{it} = Y_{it} - \overline{Y}$$

The partial correlation between variable I and any other variable such as d with the effect of the other p-2 variables held constant was defined by Kendall and Stuart (46) as,

$$\rho_{1d-2,\ldots,d-1,d+1\ldots,P} = \frac{-C_{1d}}{(C_{11}C_{dd})^{1/2}} = \psi_{1d} \quad (4.29)$$

where C_{1d} is the cofactor of ρ_{1d} in

$$|C| = \begin{vmatrix} 1 & \rho_{1d} & \rho_{13} & \cdots & \rho_{1P} \\ & 1 & \rho_{23} & \cdots & \rho_{2P} \\ & & 1 & \cdots & \rho_{3P} \\ & & & & \vdots \\ \text{symmetric} & & & & 1 \end{vmatrix} \tag{4.30}$$

Equation(4.29) is the general definition of the partial correlation coefficient. This could be extended to estimate the partial correlation ψ_{oosk} between Y_{it} and $L^S Y_{i,t-k}$,

$$\psi_{00sk} = \frac{-C_{00sk}}{(C_{0000}C_{sksk})^{1/2}} \tag{4.31}$$

where C_{oosk} is the cofactor of r_{oosk}, the correlation between Y_{it} and $L^S Y_{i,t-k}$ in $|C|$.

Martin and Oeppen(54) mentioned the possibility of unsatisfactory estimates of the STPACF via the Yule-Walker equations (4.25) and (4.26) due to the following reasons:

1. The partial function $\mathcal{O}_{MMPP}$ is conditional for time lag M and spatial lag 1 holding all intermediate lagged terms fixed. The equations (4.25) are ordered by time

lag for each successive spatial lag. These equations could be also ordered by spatial lag for each successive time lag. This will cause different estimates of the space-time partial autocorrelation coefficients. Therefore, a priori specification of the structure of space-time interactions is required.

2. As long as estimates of the space-time autocorrelation functions ρ_{sk} at spatial lag s and time lag k are defined via equation(4.9), the spatial-temporal Yule-Walker type equations(4.24) are valid. Alternatively, if the autocorrelations are provided by far better equation(4.14), the set of linear equations for autocorrelation coefficients(4.25) might be no longer positive.

The approach adopted by Kendall and Stuart for the estimation of the space-time partial autocorrelation functions is more straight forward and provides resonable estimates and therefore will be used in this thesis.

4.1.2 Identifying a Stationary Time Series Model

Once the conclusions have been made concerning the theoretical space-time partial autocorrelation and theoretical space-time autocorrelation functions of a stationary time series, the description of the behavior of STACF and STPACF may be used to identify a particular stationary time series model. This method is an extention of the classical Box and

Jenkins time series identification technique. The main point is that each particular model in the class of models is characterized by the behavior of its theoretical autocorrelation functions and its theoretical partial autocorrelation functions.

If a time series is nonstationary, it has no constant mean. This would require temporal and/or spatial differencing to achieve stationarity,

$$\nabla_t Y_{it} = Y_{it} - Y_{i,t-1}$$

$$\nabla_s Y_{it} = Y_{it} - \sum \omega_{ij} Y_{it} \qquad (4.32)$$

In a manner analogous to stationary time series, STARMA processes are characterized by distinct space-time partial and autocorrelation functions. STAR (M,p) model exhibits autocorrelations that decay in space and time, and the partial autocorrelation functions that cut off after p lags in time and M lags in space. STMA(λ,q) model exhibits autocorrelation that cut off after λ lags in space and q lags in time and partials that decay exponentially both in space and time. The STARMA (M,p, λ,q) models are characterized by space-time autocorrelation and partial autocorrelation functions that both tail off in time and space. Table 7 summarizes the characteristics of the theoretical STACF and STPACF.

TABLE 7

Characteristics of the theoretical STACF and STPACF for STAR, STMA and STARMA models

Model form	STACF	STPACF
STAR (M,P)	Tails off	Cuts off after P time lags, M spatial lags.
STMA (λ,q)	Cuts off after q time lags, λ spatial lags.	Tails off
STARMA (M,P,λ,q)	Tails off	Tails off

4.1.3 Parameter Estimation

After the identification process has led to a tentative formulation for the model, it is necessary to estimate the parameters.

Approximate estimates for the parameters ϕ and θ are estimated by minimizing the residual sum of squares:

$$S(\phi\ \theta) = a'a = \sum_{i=1}^{N} \sum_{t=1}^{T} a_i(t)^2 \qquad (4.33)$$

The determination of optimal values for parameters in a system model is often of crucial importance in both the formulation of mathematical model for systems and in their subsequent use in simulation studies. Optimization subproblems are therefore associated with model based studies. Many methods for solving nonlinear unconstrained problems have been suggested over the years(12). The objective of this work was to find values for a set of parameters of a model, which causes its behavior to best approximate a time series. Some of the optimization techniques available to minimize errors in equation(4.33) include:

1. Optimal-step steepest descent algorithm.
2. A variation on the conjugate gradient method suggested by Polack.
3. Conjugate gradient algorithm proposed by Fletcher and Reeves.
4. Fletcher-Powell variation on the Davidon algorithm.
5. Broydon, Fletcher, Shanno, Goldfarb algorithm.
6. An algorithm not requiring gradient information, proposed by Powell.
7. A variation on Powells method suggested by Zangwill.
8. A basic gradient independent method based on the classical univariate approach.
9. The gradient independent simplex method proposed by Nelder and Mead.

The above mentioned nonlinear optimization techniques were developed by Dr. L. Birta as a package for use on the central computer facility operated by the Computing Centre of the University of Ottawa and were used in this thesis. In order to use the package, it was necessary to make some adjustments which were developed in the form of a subroutine. The subprogram which is mandotary is a DOUBLE PRECISION FUNCTION subprogram called F which serves to specify the criterion function (the residual sum of squares) to be minimized(12).

The appropriate equation for estimating errors is

$$a_{it} = Z_{it} - \sum_{s=0}^{M} \sum_{k=1}^{P} \phi_{sk} L^{s} Z_{i,t-k} + \sum_{s=0}^{\lambda} \sum_{k=1}^{q} \theta_{sk} L^{s} a_{i,t-k} \quad (4.34)$$

for t=1,2,..... T.

The exact estimates of the parameters are not easy to obtain if the moving average terms are present in the candidate model. This is due to the fact that the residuals, a vector, are functions of observations Z(t), and errors a, at times before time 1. The solution for this problem is by setting Z(t) and a(t) equal to zero for t<1. As an example, consider the STMA process of order k=1 for s=0, k=1 for s=1 and k=1 for s=2, that is,

$$Z_{it} = a_{it} - \sum_{s=0}^{2} \theta_{sk} L^{s} a_{i,t-k} \qquad (4.35)$$

In general, the nonlinear model form of (4.35) can be written as shown in Figure 4. The parameter vector that solves this system of equations is the conditional maximum likelihood estimate of the parameters of the STMA model.

The estimation of the STAR model parameters is based on standard linear regression theory(26). As an example, consider the STAR process of order k=1 for s=0, k=1 for s=1 and k=2 for s=0, that is,

$$Z_{it} = \phi_{10} Z_{i,t-1} + \phi_{11} L^{s} Z_{i,t-1} + \phi_{20} Z_{i,t-2} \qquad (4.36)$$

$$
\begin{bmatrix} Z_1(1) \\ Z_2(1) \\ \vdots \\ Z_N(1) \\ \hline Z_1(2) \\ Z_2(2) \\ \vdots \\ Z_N(2) \\ \hline \vdots \\ \hline Z_1(T) \\ Z_2(T) \\ \vdots \\ Z_N(T) \end{bmatrix}
=
\begin{bmatrix} a_1(1) \\ a_2(1) \\ \vdots \\ a_N(1) \\ \hline a_1(2) \\ a_2(2) \\ \vdots \\ a_N(2) \\ \hline \vdots \\ \hline a_1(T) \\ a_2(T) \\ \vdots \\ a_N(T) \end{bmatrix}
-
\left[\begin{array}{c|c|c}
0 & 0 & 0 \\ \hline
\begin{matrix} a_1(1) \\ a_2(1) \\ \vdots \\ a_N(1) \end{matrix} &
\begin{bmatrix} W^1_{1,1} & \dots & W^1_{1,N} \\ W^1_{2,1} & \dots & W^1_{2,N} \\ \vdots & & \vdots \\ W^1_{N,1} & \dots & W^1_{N,N} \end{bmatrix} \begin{bmatrix} a_1(1) \\ a_2(1) \\ \vdots \\ a_N(1) \end{bmatrix} &
\begin{bmatrix} W^2_{1,1} & \dots & W^2_{1,N} \\ W^2_{2,1} & \dots & W^2_{2,N} \\ \vdots & & \vdots \\ W^2_{N,1} & \dots & W^2_{N,N} \end{bmatrix} \begin{bmatrix} a_1(1) \\ a_2(1) \\ \vdots \\ a_N(1) \end{bmatrix} \\ \hline
\vdots & \vdots & \vdots \\ \hline
\begin{matrix} a_1(T-1) \\ a_2(T-1) \\ \vdots \\ a_N(T-1) \end{matrix} &
\begin{bmatrix} W^1_{1,1} & \dots & W^1_{1,N} \\ W^1_{2,1} & \dots & W^1_{2,N} \\ \vdots & & \vdots \\ W^1_{N,1} & \dots & W^1_{N,N} \end{bmatrix} \begin{bmatrix} a_1(T-1) \\ a_2(T-1) \\ \vdots \\ a_N(T-1) \end{bmatrix} &
\begin{bmatrix} W^2_{1,1} & \dots & W^2_{1,N} \\ W^2_{2,1} & \dots & W^2_{2,N} \\ \vdots & & \vdots \\ W^2_{N,1} & \dots & W^2_{N,N} \end{bmatrix} \begin{bmatrix} a_1(T-1) \\ a_2(T-1) \\ \vdots \\ a_N(T-1) \end{bmatrix}
\end{array}\right]
$$

Figure 4: The STMA (1_2) in general model form

This model is shown in Figure 5 and can be written in general linear form as

$$Y = XB + a$$

where B represents the number of parameters and a's are the residuals. The procedure is to minimize the square of a which equals,

$$(Y - XB)'(Y - XB)$$

where ' signifies transpose. By differencing, one can obtain the least squares normal equations,

$$(X'X)B = X'Y$$

or

$$(X'X)\Phi = X'Z$$

and B can be estimated by,

$$B = (X'X)^{-1}X'Z$$

The conditional least squares normal equations for this model are shown in Figure 6.

The Simplex method of Nelder and Mead(58) (more commonly referred to now as the polytope method to distinguish it from the Simplex method of linear programming) was found to

$$
\begin{bmatrix} Z_1(1)\\ Z_2(1)\\ \vdots\\ Z_N(1)\\ \hline Z_1(2)\\ Z_2(2)\\ \vdots\\ Z_N(2)\\ \hline Z_1(3)\\ Z_2(3)\\ \vdots\\ Z_N(3)\\ \hline \vdots \\ \hline Z_1(T)\\ Z_2(T)\\ \vdots\\ Z_N(T) \end{bmatrix}
=
\left[\begin{array}{c|c|c|c}
0 & 0 & 0 & \\
\hline
\begin{matrix} Z_1(1)\\ Z_2(1)\\ \vdots\\ Z_N(1)\end{matrix} & \begin{matrix} Z_1(1)\\ Z_2(1)\\ \vdots\\ Z_N(1)\end{matrix} & \begin{bmatrix} W^1_{1,1} & \cdots & W^1_{1,N}\\ W^1_{2,1} & \cdots & W^1_{2,N}\\ \vdots & & \vdots\\ W^1_{N,1} & \cdots & W^1_{N,N}\end{bmatrix}\begin{bmatrix} Z_1(1)\\ Z_2(1)\\ \vdots\\ Z_N(1)\end{bmatrix} & \\
\hline
\begin{matrix} Z_1(2)\\ Z_2(2)\\ \vdots\\ Z_N(2)\end{matrix} & \begin{matrix} Z_1(2)\\ Z_2(2)\\ \vdots\\ Z_N(2)\end{matrix} & \begin{bmatrix} W^1_{1,1} & \cdots & W^1_{1,N}\\ W^1_{2,1} & \cdots & W^1_{2,N}\\ \vdots & & \vdots\\ W^1_{N,1} & \cdots & W^1_{N,N}\end{bmatrix}\begin{bmatrix} Z_1(2)\\ Z_2(2)\\ \vdots\\ Z_N(2)\end{bmatrix} & \begin{matrix} Z_1(1)\\ Z_2(1)\\ \vdots\\ Z_N(1)\end{matrix} \\
\hline
\vdots & \vdots & \vdots & \vdots \\
\hline
\begin{matrix} Z_1(T-1)\\ Z_2(T-1)\\ \vdots\\ Z_N(T-1)\end{matrix} & \begin{matrix} Z_1(T-1)\\ Z_2(T-1)\\ \vdots\\ Z_N(T-1)\end{matrix} & \begin{bmatrix} W^1_{1,1} & \cdots & W^1_{1,N}\\ W^1_{2,1} & \cdots & W^1_{2,N}\\ \vdots & & \vdots\\ W^1_{N,1} & \cdots & W^1_{N,N}\end{bmatrix}\begin{bmatrix} Z_1(T-1)\\ Z_2(T-1)\\ \vdots\\ Z_N(T-1)\end{bmatrix} & \begin{matrix} Z_1(T-2)\\ Z_2(T-2)\\ \vdots\\ Z_N(T-2)\end{matrix}
\end{array}\right]
\begin{bmatrix} \phi_{10}\\ \phi_{11}\\ \phi_{20}\end{bmatrix}
+
\begin{bmatrix} a_1(1)\\ a_2(1)\\ \vdots\\ a_N(1)\\ \hline a_1(2)\\ a_2(2)\\ \vdots\\ a_N(2)\\ \hline a_1(3)\\ a_2(3)\\ \vdots\\ a_N(3)\\ \hline \vdots \\ \hline a_1(T)\\ a_2(T)\\ \vdots\\ a_N(T) \end{bmatrix}
$$

Figure 5: The STAR (2_{10}) in general linear model form

$$\begin{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' z_i(t) & \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' W^{1} z_i(t) & \sum_{i=1}^{N}\sum_{t=2}^{T-1} z_i(t)' z_i(t-1) \\ & \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' W^{1'} W^{1} z_i(t) & \sum_{i=1}^{N}\sum_{t=2}^{T-1} z_i(t)' W^{1} z_i(t-1) \\ \text{symmetric} & & \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' z_i(t) \end{bmatrix} \begin{bmatrix} \phi_{10} \\ \phi_{11} \\ \phi_{20} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' z_i(t+1) \\ \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' W^{1} z_i(t+1) \\ \sum_{i=1}^{N}\sum_{t=1}^{T-1} z_i(t)' z_i(t+2) \end{bmatrix}$$

Figure 6: The conditional least squares normal equations for the STAR (2_{10}) model

offer the advantage of not requiring gradient information about the objective function and not having a dependence on a linear search subproblem(12). It has been found also useful when measurements of the objective function which has to be minimized are subject to random errors. Therefore, the polytope method is used in this study.

What follows is a brief discussion regarding basic concepts of the gradient independent polytope method for parameter estimation. A Simplex in an n-dimensional space is the convex hull of any set of n+1 points which do not all lie on one hyperplane. The formation of a simplex in two dimensional space is shown in Figure 8. It is formed by any three points which do not lie on the same straight line. In three dimensional space, the simplex is formed by four points. The four points do not lie on the same plane. This is shown in Figure 7.

The computations start by setting up a regular simplex in n-dimensional space and evaluating the objective function at the vertices. The simplex then proceeds by reflecting the maximum vertex in the centroid of the other vertices. If at any stage the new vertex has the largest value then the evaluation proceeds by reflecting the next largest value.

Figure 8 shows in two dimensions the procedure of steps which are made by simplex method until point 10 is reached. The next vertex has a larger value than point 10. This causes the Simplex to move on by using the next largest value.

Point 10 has approximately the minimum value for the objective function. Therefore the Simplex revolves around point 10 to make some progress in minimizing the objective function.

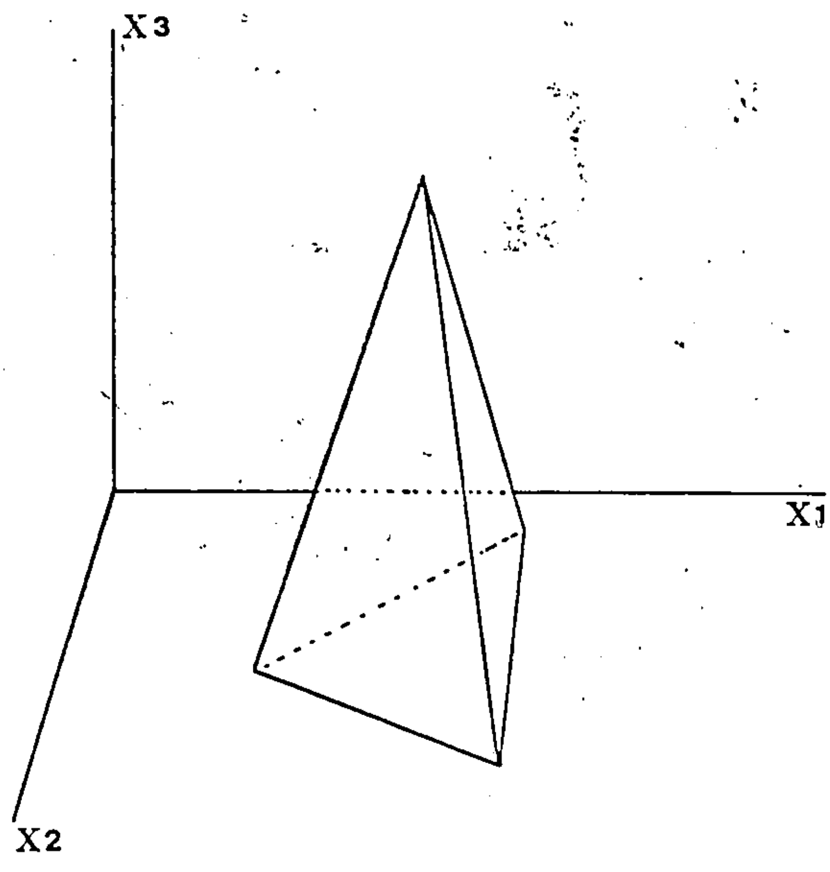


Figure 7: Three Dimensional example of Simplex method

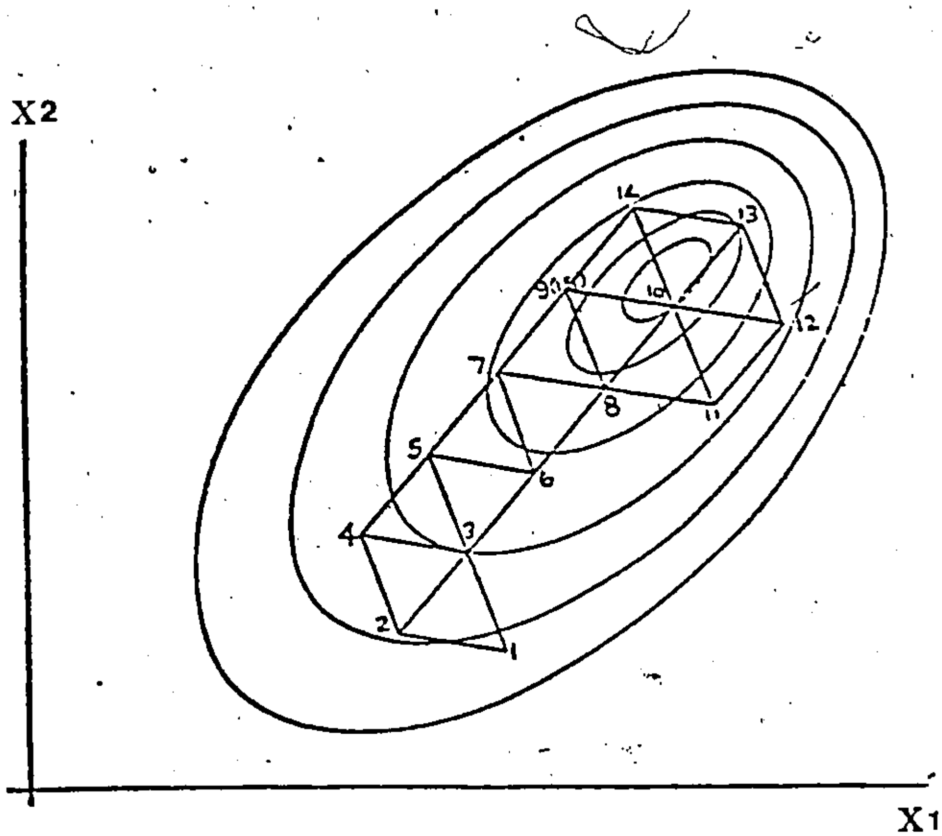


Figure 8: Two dimensional example of Simplex Method

4.1.4 Diagnostic Checks

It is necessary to apply diagnostic tests after model identification and estimation of its parameters to see whether there is any serious inadequacy in the selected model. The first phase of the diagnostic checking stage is the examination of the residuals from the fitted model. If the residuals are not random and show marked correlation patterns, this suggests model inadequacy and the tentative model is updated. If the model does adequately represent the space-time series the analysis moves to the second phase of the diagnostic tests. This phase involves checking the statistical significance of the estimated parameters. The diagnostic checking tools are discussed in Chapter II.

4.2 SPACE-TIME TRANSFER FUNCTION-NOISE MODELING

In an extension of space-time modeling of rainfall, a space-time transfer function-noise (STTFN) model is developed which relates the input of rainfall with runoff time series. This section presents the statistical procedures for answering dynamic relationships between two time series. Then it is explained how a cross-correlation analysis can be used to construct a space-time transfer function-noise model to link between two time series. The method of Box-Jenkins for designing a STTFN model is explained in detail, which is followed by the estimation and diagnostic checking stages that are discussed.

Suppose an input, i.e., rainfall, to a system influences another variable as the output, i.e., runoff, of a system. It will usually be the case where a change in the input from one level to another will have no immediate effect on the output, instead will produce delayed response with the output coming to equilibrium at a new level. This is due to the inertia of the system. Such changes are referred to as dynamic response. A model which describes this dynamic response is called a transfer function model. This kind of model when designed to mathematically model the connections between two time series with the knowledge that one data set definitely causes another, takes into the account not only the dynamic relationship but also the noise infecting the system. Such models are obtained by combining a deterministic transfer function with a stochastic noise model(9).

4.2.1 Identification

In this section the basic properties of the STTFN models are discussed and it is explained how a space-time cross correlation analysis of the residual from the model fitted to time series can be applied to detect dynamic relationships. The information from the space-time cross correlation can then be used to design a STTFN model to describe the mathematical relationship between the input and the output data sequences.

In space-time TFN model, the output Y_{it} from $i=1,2,\ldots,N$ zones over $t=1,2,\ldots T$ time periods is assumed to be linearly dependent upon values of the input $X_1, X_2, \ldots$ etc. There are two classes of this model. The first class may be expressed as

$$Y_{it} = E\left[Y_{it} \middle| L^s X_{i,t-k}\right] \quad , \; s \geq 0 \text{ and } k \geq 1$$

where s is the number of lags in space, K is the number of lags in time and L^S is the spatial lag operator. Thus the STTFN model is

$$Y_{it} = \sum_{s=0}^{S} \sum_{k=1}^{K} \omega_{sk} L^s X_{i,t-k} + a_{it} \qquad (4.37)$$

where ω_{sk} are unknown parameters and a_{it} is noise at the output. The second class of the STTFN model can be defined by

$$Y_{it} = \left[Y_{it} \middle| L^s Y_{i,t-k} \text{ and } L^s X_{i,t-k}\right] \quad , \; s \geq 0 \text{ and } k \geq 1 .$$

$$Y_{it} = \sum_{s=0}^{M} \sum_{k=1}^{p} \phi_{sk} L^s Y_{i,t-k} - \sum_{s=0}^{g} \sum_{k=1}^{h} \theta_{sk} L^s X_{i,t-k} + a_{it} \quad (4.38)$$

The term $L^S X_{i,t-k}$ represents a deterministic sequence, where M and g are the lags in space p and h are the lags in time and ϕ and θ are parameters. The noise term a_{it} must satisfy the following assumptions:

$$E\left[a_{it} L^s X_{i,t-k}\right] = 0 \quad \text{for all } s \text{ and } k$$

$$E\left[a_{it}\right] = 0$$

and

$$E\left[a_{it} a_{i,t-k}\right] = \begin{cases} \sigma^2 I & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

These models have received attention in the literature by Martin and Oeppen(54). They indicated that a relationship exists between model (4.37) and (4.38). If B^k is the backward shift operator in time so that $B^k Y_{it} = Y_{i,t-k}$, then equation (4.37) may be given in the following form:

$$Y_{it} = \sum_{s=0}^{S} \sum_{k=1}^{K} \gamma_{sk} B^{k-b} L^{s+j} X_{it} + a_{it} \tag{4.39}$$

and equation (4.38) by

$$Y_{it} = \frac{\sum_{s=0}^{g} \sum_{k=1}^{h} \omega_{sk} B^k L^s}{(1 - \sum_{s=0}^{M} \sum_{k=1}^{P} \delta_{sk} B^k L^s)} B^b L^j X_{it} + a_{it} \tag{4.40}$$

where g,h,M and p are the orders and b and j are defining an initial period of pure delay or dead time before the response to a given input change begins to take effect, a_{it} is noise at the output and independent of X_{it} and γ , δ and ω are the coefficients of the model. Martin and Oeppen(54) assumed that equation (4.39) is a finite distributed lag function capable of being presented by a stationary rational process given by equation (4.40), then the sequence γ_{sk} may be given by

$$\sum_{s=0}^{S}\sum_{k=1}^{K}\gamma_{sk}B^{k}L^{s} = \frac{\sum_{s=0}^{g}\sum_{k=1}^{h}\omega_{sk}B^{k}L^{s}}{1-\sum_{s=0}^{M}\sum_{k=1}^{P}\delta_{sk}B^{k}L^{s}}\,B^{b}L^{j} \qquad (4.41)$$

4.2.1.1 Space-Time Cross Correlation Functions (STCCF)

In the same way that the STACF and the STPACF are used to identify STARMA models, the data analysis tool employed for the identification of STTFN model is the STCCF between the input and the output(54).

Let the expected value of the space-time series X_{it} and Y_{it} be

$$\mu_x = E\left[X_{it}\right] \qquad (4.42)$$

and

$$\mu_Y = E\left[Y_{it}\right] \tag{4.43}$$

The cross covariance between Y_{it} and $L^s X_{i,t-k}$ can be defined by

$$\gamma_{YX} = E\left[(Y_{it} - \mu_Y)(L^s X_{i,t-k} - \mu_X)\right] \tag{4.44}$$

The cross variate of the series is,

$$\sigma_{00}^2 = E\left[(Y_{it} - \mu_Y)^2 (L^s X_{it} - \mu_X)^2\right] \tag{4.45}$$

Thus, the STCCF at spatial lag s and time lag K is given by.

$$\rho_{YX}(s,k) = \frac{E\left[(Y_{it} - \mu_Y)(L^s X_{i,t-k} - \mu_x)\right]}{\left(E\left[(Y_{it} - \mu_Y)^2\right]\right)^{1/2}\left(E\left[(L^s X_{it} - \mu_X)^2\right]\right)^{1/2}} \tag{4.46}$$

The estimated STCCF from the observed series Y_{it} , X_{it} (i=1,2,....N and t=1,2,......,T) is

$$r_{YX}(s,k) = \frac{\sum_{t=k+1}^{T} \sum_{i=1}^{N} (Y_{it} - \bar{Y})(L^s X_{i,t-k} - \bar{X})}{\left[\sum_{t=1}^{T} \sum_{i=1}^{N} (Y_{it} - \bar{Y})^2\right]^{1/2} \left[\sum_{t=1}^{T} \sum_{i=1}^{N} (L^s X_{it} - \bar{X})^2\right]^{1/2}} \quad (4.47)$$

where the grand means are estimated from

$$\bar{Y} = \frac{1}{N\,T} \sum_{t=1}^{T} \sum_{i=1}^{N} Y_{it} \quad (4.48)$$

and

$$\bar{X} = \frac{1}{N\,T} \sum_{t=1}^{T} \sum_{i=1}^{N} X_{it} \quad (4.49)$$

For computational purposes the STCCF equation (4.47) may be written in the following form

$$r_{YX}(s,k) = \frac{\sum_{t=k+1}^{T}\sum_{i=1}^{N} U_{it}L^{s}Z_{i,t-k}}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N} U_{it}^{2}\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}(L^{s}Z_{it})^{2}\right]^{1/2}} \tag{4.50}$$

where

$$z_{it} = X_{it} - \overline{X}$$

and

$$U_{it} = Y_{it} - \overline{Y}$$

4.2.2 Box and Jenkins Identification Method

Consider a hydrological system with input X_{it} and output Y_{it} and that a STTFN model has to be fitted to the input and the output. Box and Jenkins(9) identified a transfer function model that relates X_{it} and Y_{it} series as,

$$\begin{aligned} Y_{it} &= \delta^{-1}(B)\omega(B)X_{i,t-b} + a_{it} \\ &= \upsilon(B)X_{it} + a_{it} \end{aligned} \tag{4.51}$$

where

$V(B) = \omega(B) / \delta(B)$

is the transfer function with weights $\upsilon_0, \upsilon_1, \upsilon_2, \ldots$ which are called the impulse response functions of the system,

$$\delta(B) = (I - \delta_1 B - \delta_2 B^2 - \ldots\ldots - \delta_r B^r)$$

and

$$\omega(B) = (\omega_0 - \omega_1 B - \omega_2 B^2 - \ldots\ldots\ldots - \omega_s B^s)$$

are polynomials of order r and s, respectively.

The identification consists of the following steps:

1. Based upon the characteristics of the STACF and the STPACF, identify the type of a STARMA model to fit the input series X_{it}. Estimate the unknown parameters of the identified STARMA model and finally test the adequacy of the model. Use the STARMA model to estimate white noise residuals for the input X_{it},

$$\phi_X(B)\theta_X^{-1}(B)X_{it} = \alpha_{it} \tag{4.52}$$

where B is the backward shift operator defined by $BX_{it} = X_{i,t-1}$, and ϕ_X and θ_X are parameters.

2. Apply the ϕ_X and θ_X obtained in step 1 to Y_{it} and develop a transformed output series β_{it} given by

$$\beta_{it} = \phi_x(B)\ \theta_x^{-1}(B) Y_{it} \tag{4.53}$$

Then the model of equation (4.51) may be written as

$$\beta_{it} = \upsilon(B)\alpha_{it} + n_{it} \qquad (4.54)$$

where a_{it} is the transformed noise series defined by

$$n_{it} = \phi_X^{-1}(B)\theta_X(B)a_{it} \qquad (4.55)$$

The model of equation (4.39) may be written as

$$\beta_{it} = \sum_{s=0}^{S}\sum_{k=1}^{K}\omega_{sk}B^kL^s\alpha_{it} + n_{it} \qquad (4.56)$$

3. Use equation (4,50) to calculate the STCCF between the uncorrelated series α_{it} and the transformed series β_{it}.
4. The impulse response weights υ_{sk} (the coefficients of $\upsilon(\mathbf{B})$) are given by the formula

$$\upsilon_{sk} = r_{\alpha\beta}(s,k) \frac{SD_{\beta}}{SD_{\alpha}} \tag{4.57}$$

where SD_{β} and SD_{α} are the transformed deviations of the β_{it} and α_{it}, respectively. Note that equation (4.57) is identical to the formula for the regression coefficient of simple regression which is expressed as

$$R_c = r_{XY} \frac{SD_Y}{SD_X} \tag{4.58}$$

5. The estimated STCCF between the new series α_{it}, β_{it} and the coefficients υ_{sk} of the impulse response functions can be used to gain some idea of the orders of g,h,M,p and the pure delay elements b and j in equation (4.59) using the following:
   a) Zero or near zero correlation values up to spatial lag j-1 and time lag b-1, followed by
   b) Irregular or rising values up to spatial lag j+g-M and time lag b+h-p (no such values occur if $g<M$ and $b<p$), and
   c) Correlation, $r_{\beta\alpha}(s,k)$, $s>j+g-M+1$, $K>b+h-p+1$ which decay exponentially in time and space. Once the

values of g,h,M,p,j and b have been determined then the starting values of the coefficients ω_{sk} and δ_{sk} can be determined from

$$Y_{it} = \frac{\sum_{s=0}^{g} \sum_{k=1}^{h} \omega_{sk} B^k L^s}{(1 - \sum_{s=0}^{M} \sum_{k=1}^{P} \delta_{sk} B^k L^s)} B^b L^j X_{it} + a_{it} \qquad (4.59)$$

Note that equation (4.59) is identical to the equations proposed by Box and Jenkins(9) to solve the parameters ω_s and δ_s of the transfer function model

$$Y_t = \frac{(\omega_0 - \omega_1 B \ldots\ldots - \omega_s B^s)}{(1 - \delta_1 B - \delta_2 B^2 \ldots\ldots - \delta_r B^r)} B^b X_t + a_t \qquad (4.60)$$

6. The next step is to estimate the noise series a's. The output Y_{it} from the transfer function model may be provided by

$$\hat{Y}_{it} = \frac{\sum_{s=0}^{g} \sum_{k=1}^{h} \omega_{sk} B^k L^s}{(1 - \sum_{s=0}^{M} \sum_{k=1}^{P} \delta_{sk} B^k L^s)} B^b L^j X_{it}. \quad (4.61)$$

Having estimated the $\hat{Y}_{it}$ series. The noise series $\hat{a}_{it}$ can be calculated from

$$\hat{a}_{it} = Y_{it} - \hat{Y}_{it} \quad (4.62)$$

By examining graphs such as the STACF and STPACF of $\hat{a}_{it}$ series, identify the STARMA model needed to fit the noise series

$$n_{it} = \phi^{-1}(B)\theta(B)a_{it} \quad (4.63)$$

7. The space-time transfer function and the noise models are combined in the final form as follows:

$$Y_{it}' = \frac{\sum_{s=0}^{g} \sum_{k=1}^{h} \omega_{sk} B^k L^s}{(1 - \sum_{s=}^{M} \sum_{k=1}^{P} \delta_{sk} B^k L^s)} L^j X_{i,t-b} + \phi^{-1}(B)\theta(B)a_{it} \quad (4.64)$$

4.2.3 Estimation of Parameters

Following the identification of the STTF model(4.61) and the noise model (4.63) for the rainfall-runoff series, a nonlinear least squares algorithm is used to obtain the parameters δ, ω, Φ and Θ. The polytope method is used in this study to estimate the parameters of the identified space-time models, which has been previously explained.

4.2.4 Diagnostic Checks

It is necessary to apply diagnostic tests after the parameters of the selected STTFN model of the rainfall-runoff process have been estimated. This is performed to see whether there is any inadequacy in the selected model. The checks include residuals STACF and STPACF tests to assess the statistical significance of departure from zero of both STACF and STPACF. If the residuals autocorrelation functions show a marked correlation pattern, this suggests model inadequacy. A pattern of non zero cross correlation between the residuals and the prewhitened input series suggests inadequacy of the space-time TFN model.

Only the space-time cross corelation check will be reported in the following section. The rest of the diagnostic checking tools were discussed in Chapter II.

4.2.4.1 Space-Time Cross Correlation Check

The STCCF (4.47) could indicate the inadequacy in the selected STTFN model. The test is performed by obtaining the STCCF between the prewhitened input series and estimated residuals from the fitted model(9). Confidence limits for the STCCF are determined from

$$\text{C.L.}(95\%) = 0 \pm 1.96 \frac{1}{\sqrt{N - k}} \tag{4.65}$$

where N is the sample size and K is number of lags in time. The standard error associated with the cross correlation is estimated from

$$ER = \frac{1}{\sqrt{N - k}} \tag{4.66}$$

Once an appropriate space-time TFN model has been selected to describe the rainfall-runoff process, it can be used to provide runoff regional forecasts.

Chapter V

DATA ANALYSIS

5.1 STARIMA MODELS OF RAINFALL PROCESS

The rainfall data employed in the numerical analysis and application of the developed model from the selected areas of four raingage sites of the watershed are shown in Figure 9. P1, P2, P3 and P4 represent the following rainfall stations: Fergus Shand, Blue Spring Creek, Guelph O.A.C. and Preston, respectively, as shown in Figure 9.

Equal weights were assigned for this spatial system. The weights for this system are specified to agree with the spatial order of all sites surrounding the location of interest.

Table 8 defines the spatial ordering with the neighbors at each spatial order for each of the four sites. Spatial orders 1 and 2 are presented in Figure 10. In this figure each row sums to one which agree with the equally weighted formulation presented in Chapter IV.

The observed time series over all 4 sites of rainfall data are shown in Figure 11. A visual inspection of the plots of the rainfall time series suggest that there are no trends in the time series.

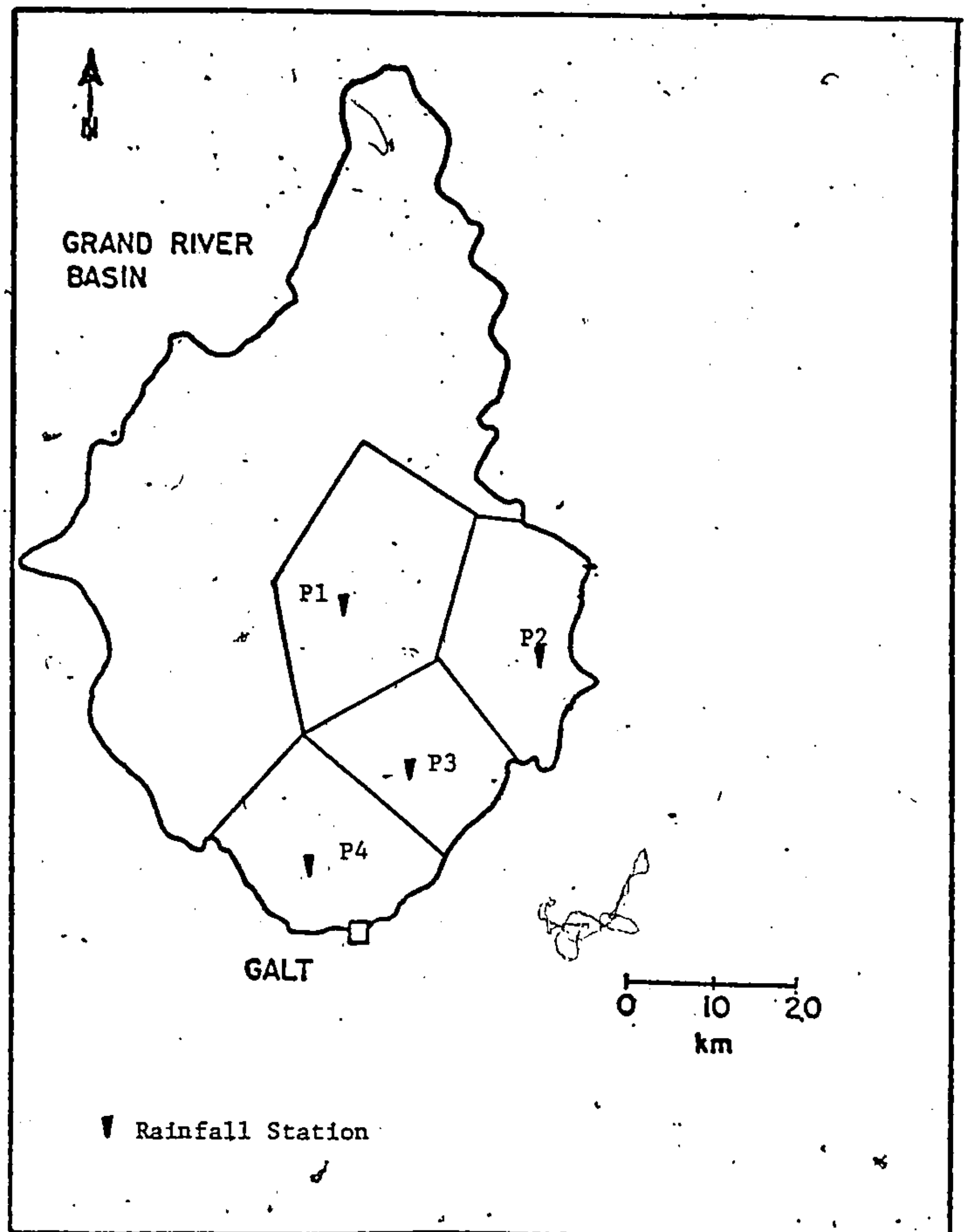


Figure 9: Location of raingage sites

W^1

	1	2	3	4
1		0.5	0.5	
2	0.5		0.5	
3	0.33	0.33		0.33
4			1.0	

W^2

	1	2	3	4
1				1.0
2				
3				
4	0.5	0.5		

Figure 10: W for the 4-raingage system

TABLE 8

Neighbors of each site for each spatial order

	ORDER	0	1	2
Site	1	1	2,3	4
	2	2	1,3	4
	3	3	1,2,4	
	4	4	3	1,2

The correlation matrix for the rainfall data is shown in Table 9. The high correlation values for the time series indicate the possibility of building an adequate STARMA model to represent the precipitation series. This is confirmed by the fact that the rainfall possesses an extreme distribution in magnitude over both time and space.

Figure 12 shows the plots of the estimated autocorrelation functions for the rainfall data and the 95% confidence intervals. Initial identification of the autocorrelations suggests that the data are stationary as the original values of the series do seem to fluctuate around a constant mean that is roughly equal to 0. The rainfall series can be considered uncorrelated sequence.

Because the STACF is symmetric about lag zero in time, it is only required to plot the sample STACF for positive lags except for lag zero, to a maximum lag of about N/4. Therefore, the number of K lags in time in this thesis are selected to be 20. Tables 10 and 11 show the STACF and STPACF

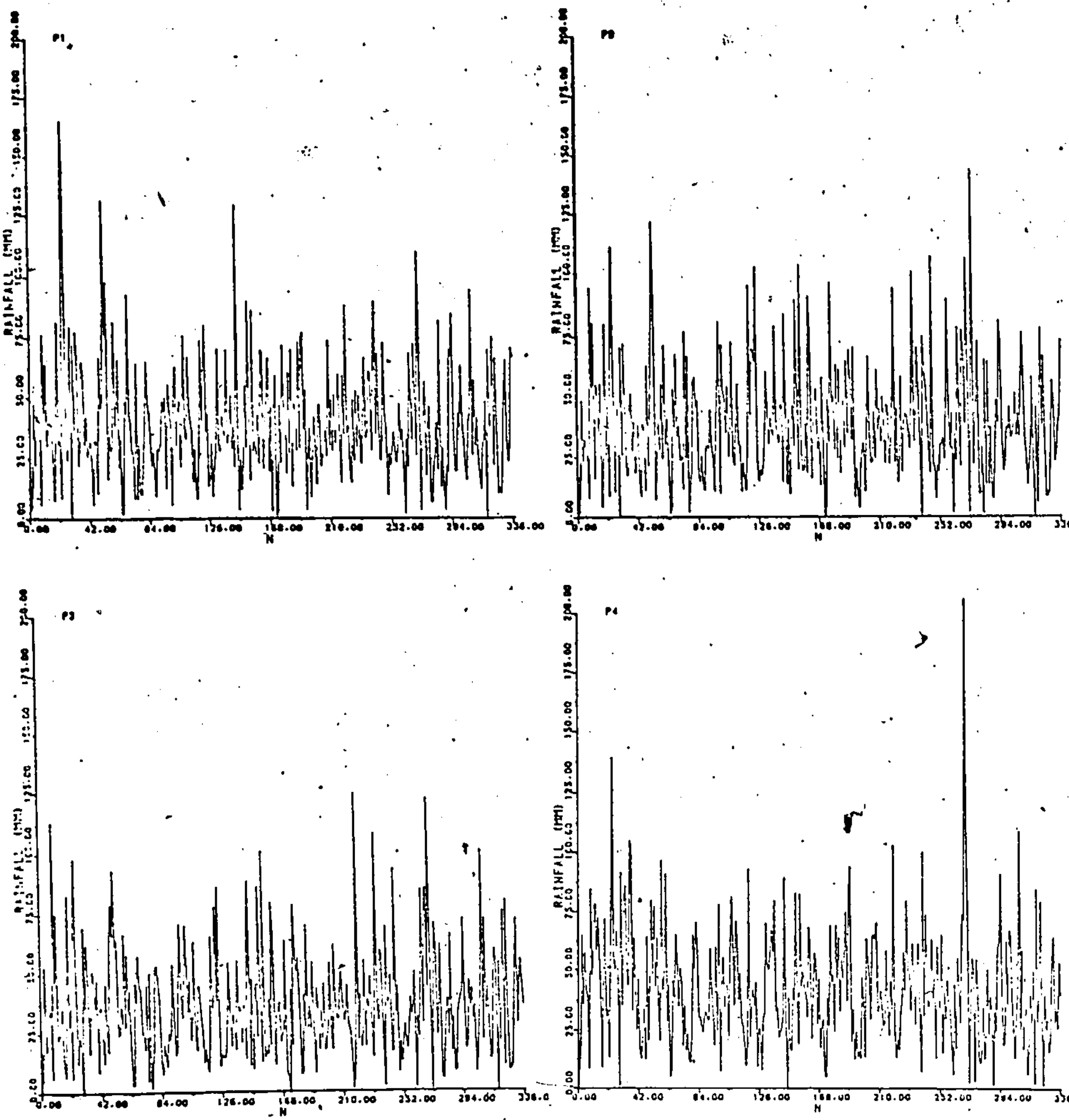


Figure 11: Original values of the rainfall data .

TABLE 9

Correlation matrix for the rainfall series

Stations	P1	P2	P3	P4
P1	1.00	0.80	0.74	0.84
P2	0.80	1.00	0.82	0.78
P3	0.74	0.82	1.00	0.83
P4	0.84	0.78	0.83	1.00

for the original precipitation series up to spatial lag 2 and time lag 20 for the STACF and spatial lag 2 and time lag 6 for the STPACF. The initial identification of the precipitation time series suggests that the space-time precipitation system is nonstationary. First differencing in time is then applied to the data sets to achieve stationarity.

The sample STACF and STPACF for first difference in time of the precipitation series are shown in Tables 12 and 13. The general pattern is one of decay for the STPACF in time and cut off for the STACF after one lag in time for spatial order S=0,1, and 2. These results suggesting that the precipitation series might be presented by STMA model of order 1 in time (K=1) for S=0,1, and 2 in space, STMA(1_2), that is,

$$\nabla_T X_{it} = a_{it} - \sum_{s=0}^{2} \theta_{s1} W^s a_{i,t-1}$$

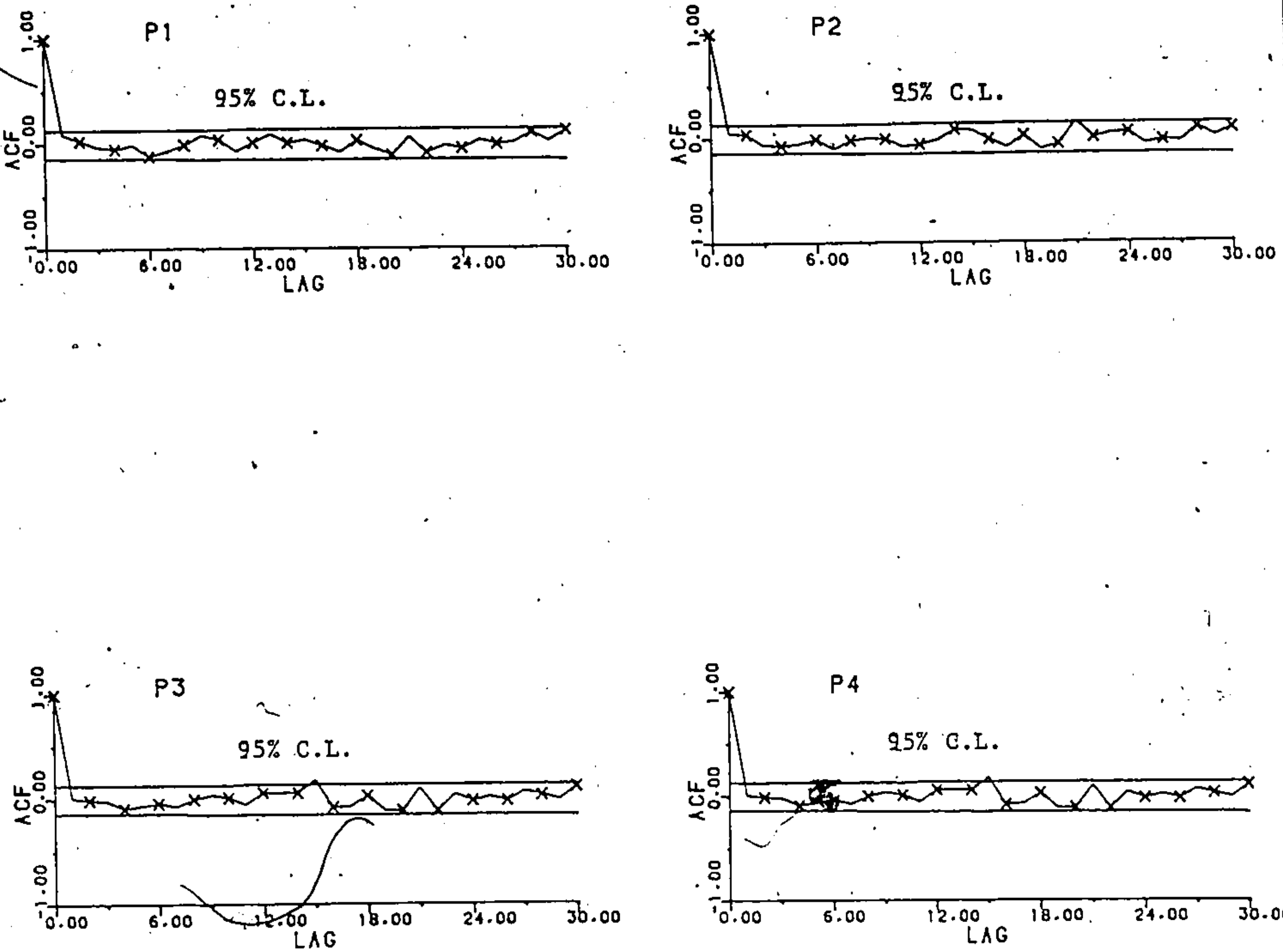


Figure 12: ACF of the observed rainfall data

TABLE 10

STACF of the original rainfall series

Spatial lag (S) / Time lag (K)	0	1	2
1	0.0694	0.0654	0.0567
2	0.0340	0.0335	0.0016
3	-0.0374	-0.0414	-0.0420
4	-0.0727	-0.0856	-0.0619
5	-0.0523	-0.0717	-0.0275
6	-0.0468	-0.0523	-0.0605
7	-0.0707	-0.0889	-0.0385
8	-0.0049	-0.0265	-0.0122
9	0.0495	0.0387	0.0263
10	0.0265	0.0185	0.0354
11	-0.0588	-0.0629	-0.0748
12	0.0011	0.0029	0.0190
13	0.0399	0.0361	0.0614
14	0.0456	0.0451	0.0457
15	0.0935	0.0840	0.0895
16	-0.0413	-0.0525	-0.0328
17	-0.0644	-0.0690	-0.0640
18	0.0303	0.0195	0.0264
19	-0.0820	-0.0935	-0.0816
20	-0.0799	-0.0746	-0.0831

TABLE 11

STPACF of the original rainfall series

Spatial lag (S) / Time lag (K)	0	1	2
1	-0.0234	-0.0036	-0.0062
2	-0.0247	-0.0191	-0.0420
3	0.0033	0.0075	0.0163
4	0.0050	0.0384	-0.0043
5	-0.0011	0.0464	-0.0332
6	-0.0007	-0.0029	0.0397

TABLE 12

STACF of the differenced rainfall series

Spatial lag (S) / Time lag (K)	0	1	2
1	-0.4797	-0.3988	-0.3243
2	0.0227	0.0274	-0.0038
3	-0.0256	-0.0242	-0.0179
4	-0.0250	-0.0269	-0.0236
5	0.0060	-0.0036	0.0335
6	0.0187	0.0321	-0.0262
7	-0.0551	-0.0630	-0.0081
8	0.0216	0.0180	0.0055
9	0.0361	0.0377	0.0121
10	0.0347	0.0392	0.0630
11	-0.0835	-0.0883	-0.1111
12	0.0096	0.0166	0.0247
13	0.0210	0.0135	0.0346
14	-0.0249	-0.0169	-0.0342
15	0.0974	0.0940	0.0873
16	-0.0672	-0.0721	-0.0544
17	-0.0552	-0.0484	-0.0565
18	0.1182	0.1171	0.1090
19	-0.0656	-0.0765	-0.0577
20	-0.1020	-0.0974	-0.0935

TABLE 13

STPACF of the differenced rainfall series

Spatial lag (S) / Time lag (K)	0	1	2
1	0.3889	-0.0508	-0.0027
2	0.2582	-0.0818	0.0336
3	0.2023	-0.0762	0.0353
4	0.1590	-0.0489	0.0246
5	0.1205	-0.0240	-0.0072
6	0.0777	-0.0478	0.0336

The comparison of the estimates for the STMA(1_2) model parameters from various estimation algorithm techniques are shown in Table 14. The results suggest that the performance of all methods are quite similar. Therefore, the simplex method was employed to estimate the model parameters. The STMA(1_2) model is estimated and the estimated MA terms with residual sum of squares are listed in Table 15. The STMA (1_2) model can be given by

$$\nabla_T X_{it} = a_{it} - 0.9448\, a_{i,t-1} + 0.0672 W^1\, a_{i,t-1} - 0.004 W^2\, a_{i,t-1}$$

TABLE 14

Estimated parameters from various optimization techniques

Method	Parameter	Guess	Estimate	Initial-F	Final-F
Powell	θ_{01}	0.000	0.9453		
	θ_{11}	0.000	-0.0666		
	θ_{21}	0.000	0.0041	0.8397×10^{6}	0.4977×10^{6}
Zang	θ_{01}	0.000	0.9449		
	θ_{11}	0.000	-0.0667		
	θ_{21}	0.000	0.0052	0.8397×10^{6}	0.4977×10^{6}
Simplex	θ_{01}	0.000	0.9448		
	θ_{11}	0.000	-0.0667		
	θ_{21}	0.000	0.0040	0.8397×10^{6}	0.4977×10^{6}
Esq	θ_{01}	0.000	0.9470		
	θ_{11}	0.000	-0.0657		
	θ_{21}	0.000	0.0020	0.8397×10^{6}	0.4977×10^{6}

F is the residual sum of squares

TABLE 15

Estimated parameters and residual sum of squares for STMA (1_2) model

Run	Parameter	Guess	Estimate	Initial-F	Final-F
1	θ_{01}	0.000	0.9448		
	θ_{11}	0.000	-0.0672		
	θ_{21}	0.000	0.0040	0.8397×10^6	0.4977×10^6
2.	θ_{01}	0.100	0.9448		
	θ_{11}	0.009	-0.0672		
	θ_{21}	0.009	0.0040	0.7660×10^6	0.4977×10^6
3	θ_{01}	0.200	0.9448		
	θ_{11}	0.009	-0.0672		
	θ_{21}	0.009	0.0040	0.9757×10^6	0.4977×10^6
4	θ_{01}	0.000	0.9448		
	θ_{11}	0.009	-0.0672		
	θ_{21}	0.200	0.0040	0.9757×10^6	0.4977×10^6

F is the residual sum of squares

Diagnostic checks are applied to insure the adequacy of the identified model by checking the assumptions of STARMA modeling such as independence and normality of the residuals. The sample STACF and STPACF of the generated residuals are tabulated in Tables 16 and 17 together with the upper bound $1/\sqrt{N}$ for their standard error. From these tables it is clear that the STACF and the STPACF show a lack of structure in these residuals. The generated residuals from the model can be considered to have characteristics of a white noise sequence as they are highly uncorrelated. There is no evidence of the STMA(1_2) model inadequacy from the behavior of the STACF and the STPACF. Portmanteau test (Table 18) demonstrates that the residuals from the selected model can be considered uncorrelated. The cumulative periodograms for the rainfall series in Figure 13 indicate that the generated errors are white noise. All the calculated values lie close to the white noise line from (0,0) to (1,0.5) and are well within the 95% confidence limits drawn parallel to this line.

TABLE 16

STACF of the residuals from the STMA (1_2) model

Spatial lag (S) Time lag (K)	0	1	2
1	0.0628	0.0683	0.0528
2	0.0352	0.0426	0.0035
3	-0.0452	-0.0449	-0.0463
4	-0.0644	-0.0713	-0.0494
5	-0.0472	-0.0597	-0.0193
6	-0.0280	-0.0304	-0.0406
7	-0.0583	-0.0740	-0.0224
8	0.0195	0.0045	0.0097
9	0.0613	0.0547	0.0413
10	0.0257	0.0275	0.0378
11	-0.0629	-0.0620	-0.0732
12	-0.0041	0.0010	0.0158
13	0.0431	0.0403	0.0677
14	0.0477	0.0514	0.0499
15	0.1044	0.0993	0.0998
16	-0.0441	-0.0477	-0.0325
17	-0.0424	-0.0384	-0.0410
18	0.0508	0.0482	0.0479
19	-0.0709	-0.0740	-0.0643
20	-0.0628	-0.0496	-0.0601

Upper bound to standard error $\pm$ 0.07

TABLE 17

STPACF for the residuals from the STMA (1_2) model

Spatial lag (S) / Time lag (K)	0	1	2
1	-0.0097	-0.0199	-0.0011
2	-0.0141	-0.0348	0.0462
3	0.0086	0.0057	0.0156
4	0.0112	0.0280	-0.0108
5	0.0049	0.0368	-0.0358
6	-0.0030	-0.0073	0.0319
Upper bound to standard error $+0.07$			

TABLE 18

Results from Portmanteau test on the generated errors

Spatial lag (S)	Portmanteau statistic	Chi-square statistic $\alpha = 0.05$	Decision
0	10.64	27.6	accepted
1	11.19	27.6	accepted
2	9.07	27.6	accepted

Since the generated residuals are shown to be white noise and normally distributed, the subroutine Gauss was used to generate 144 data points in order to test the forecasting performance of the STMA(1_2) model. The subroutine computes a normally distributed random number t_i with a given mean μ and standard deviation σ,

$$t_i = \mu + \sigma(A_i - 6.0)$$

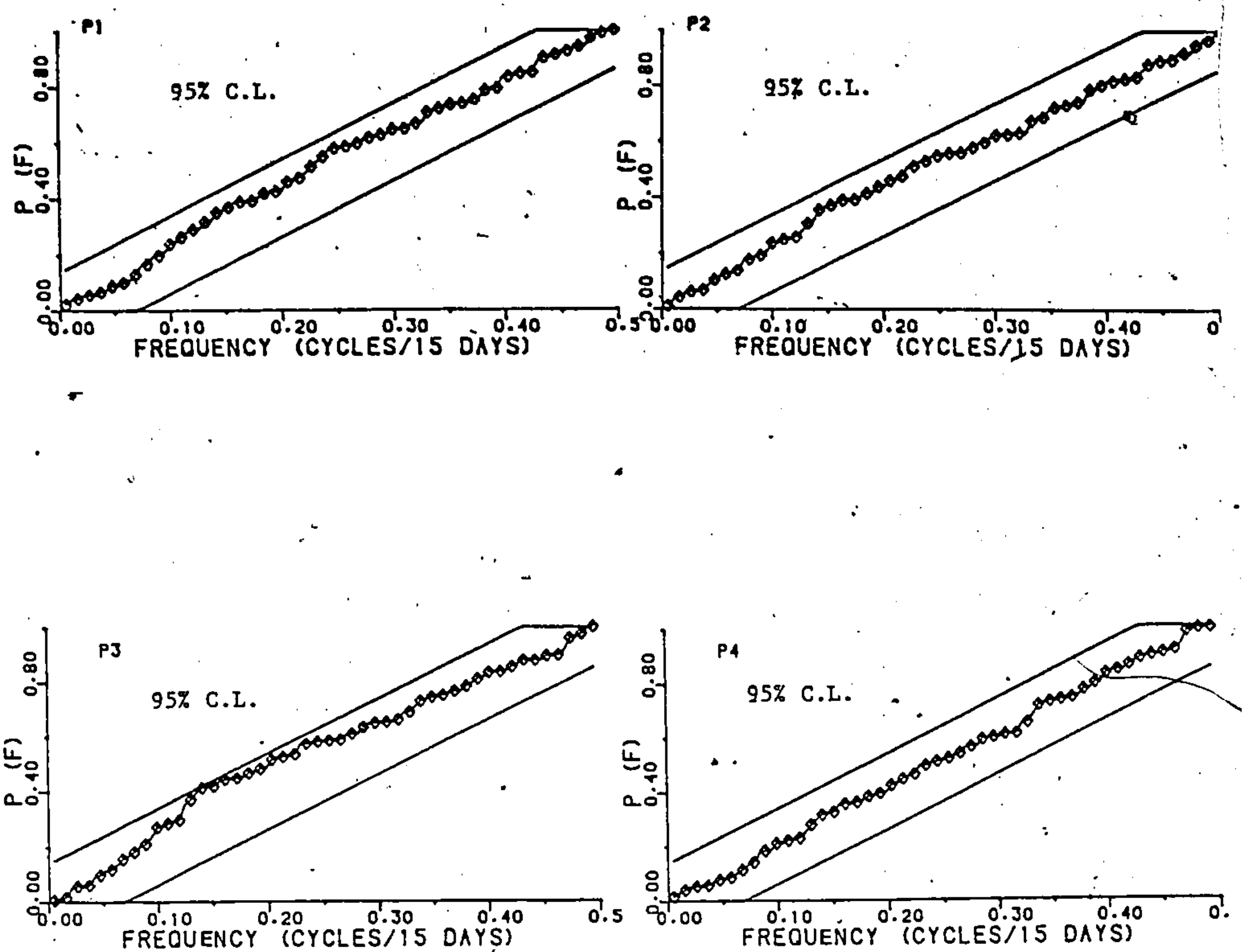


Figure 13: Cumulative periodograms of the generated errors

where A_i is the sum of twelve uniform random numbers used to compute normal random numbers by central limit theorem. The results are then adjusted to match the given mean and standard deviation.

The forecasted errors are then tested for whiteness and presence of any periodicities. The autocorrelations are well within the confidence limits(Figure 14). The results from Portmanteau test (Table 19) and the cumulative periodograms shown in Figure 15 indicate that the forecasted errors are uncorrelated and do not contain any periodicities. Thus the errors are white noise.

A goodness of fit test was performed between the observed, the forecasted and the generated rainfall values. The special correlation coefficient (r_s) and the integral square error (ISE) were calculated from(6),

$$r_s = \frac{2\sum_{i=1}^{N} O_i F_i - \sum_{i=1}^{N} F_i^2}{\sum_{i=1}^{N} O_i^2} \qquad (5.1)$$

and

$$ISE = \frac{\sum_{i=1}^{N} (O_i - F_i)^{1/2}}{\sum_{i=1}^{N} O_i} \times 100 \qquad (5.2)$$

The r_s is equal to 1 if all $O_i = F_i$ and the ISE is equal to 0% if all $F_i = O_i$. Table 20 shows the special correlation

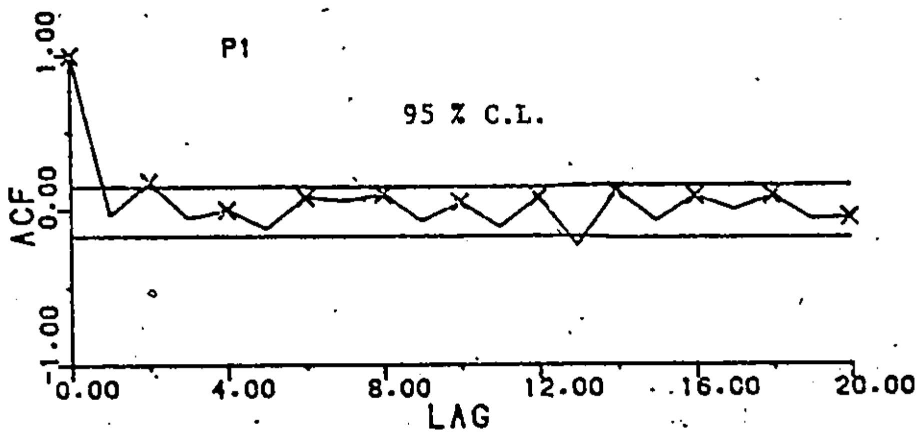


Figure 14: ACF of the forecasted errors

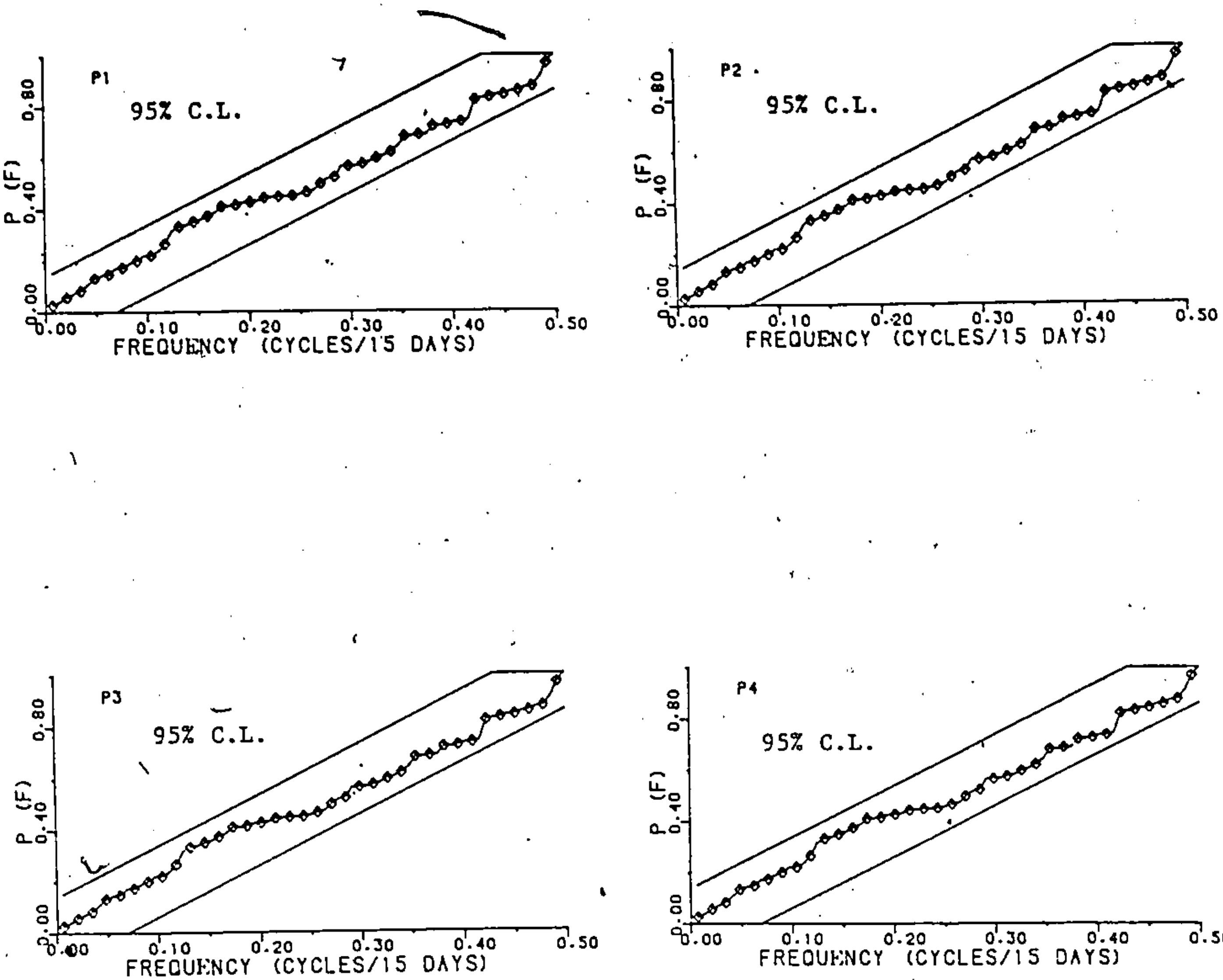


Figure 15: Cumulative periodograms of the forecasted errors

TABLE 19

Results from Portmanteau test on the forecasted errors

	Value
Test statistic	27.25
Critical value of $\alpha = 0.05$	27.60
Decision	O.K

coefficient (r_s) and the integral square error (ISE) for forecasted and generated rainfall data. The results show reasonable correlation between the observed and estimated rainfall data.

TABLE 20

Results from Goodness of Fit Test on the Rainfall Data

Rainfall Series	Statistical Test	P1	P2	P3	P4
Generated	r_s ISE	0.52 65.93	0.52 65.24	0.48 68.60	0.56 60.10
Forecasted	r_s ISE	0.60 56.05	0.61 55.51	0.66 54.43	0.68 55.30

All these tests indicate that the generated and forecasted errors from the STMA(1_2) are white noise and free from any periodic components. Therefore, the model passes the diagnostic checking stage of the three-stage model development.

The observed and computed rainfall of a portion of the data during both generation(1-192) and forecasting(193-336) periods are shown in Figures 16 and 17. The generated and forecasted rainfall compare well with the corresponding observed rainfall.

Examining Table 14 reveals small value for θ_{21} suggesting that a second model, STMA of order 1 in time (K=1) for S=0 and 1 in space, STMA(1_1), that is

$$\nabla_T X_{it} = a_{it} - \sum_{s=0}^{1} \theta_{s1} W^s a_{i,t-1}$$

can be selected for the rainfall series. Estimating the model gave θ_{01} to be 0.9464, θ_{11} to be -0.0658 and the residual sum of squares is 0.4977 X 10^6 (Table 21). The resulting alternative STMA(1_1) for the rainfall process is expressed by

$$\nabla_T X_{it} = a_{it} - 0.9464\, a_{i,t-1} + 0.0659\, W^1 a_{i,t-1}$$

$$X_{it} - X_{i,t-1} = a_{it} - 0.9464\, a_{i,t-1} + 0.0659\, W^1 a_{i,t-1}$$

$$X_{it} = X_{i,t-1} + a_{it} - 0.9464\, a_{i,t-1} + 0.0659\, W^1 a_{i,t-1}$$

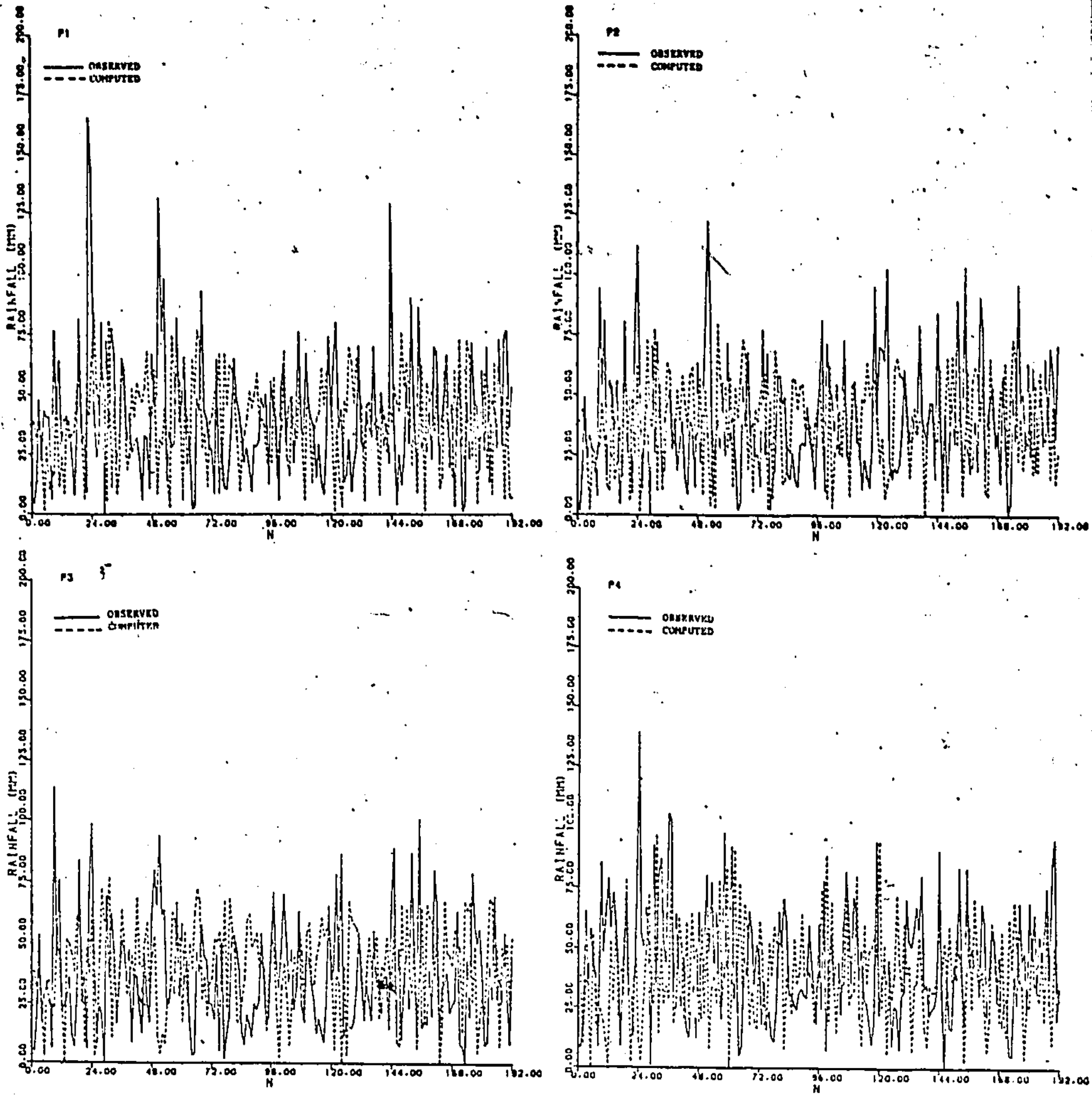


Figure 16: Observed and generated rainfall values

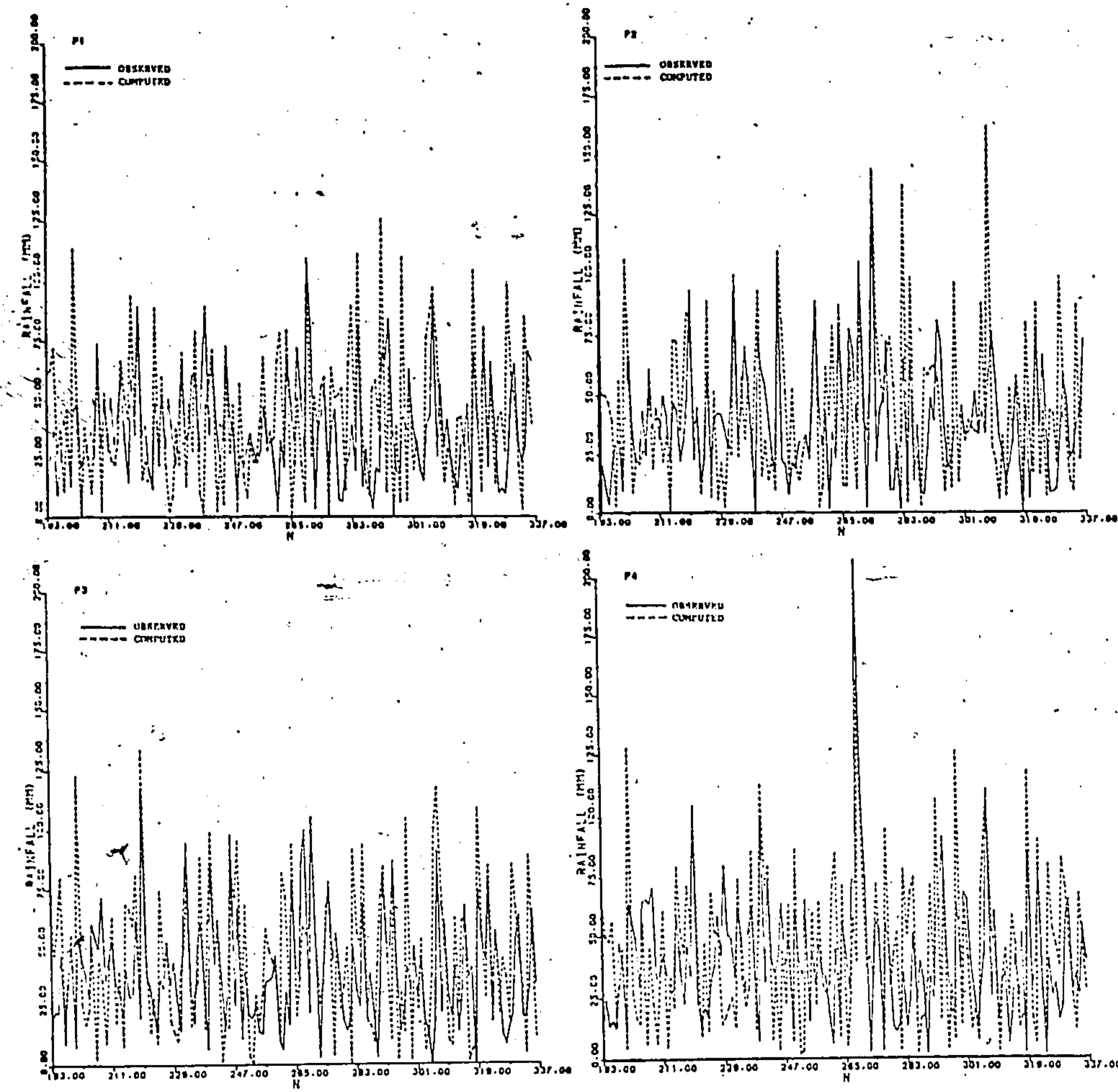


Figure 17: Observed and forecasted rainfall values

TABLE 21

Parameter estimates of the STMA (1_1) model

Run	Parameter	Guess	Estimate	Initial-F	Final-F
1	θ_{01} θ_{11}	0.000 0.000	-0.9468 -0.0659	0.839×10^6	0.497×10^6
2	θ_{01} θ_{11}	0.200 -0.059	0.9464 -0.0659	0.733×10^6	0.497×10^6
3	θ_{01} θ_{11}	0.500 0.200	0.9464 -0.0658	0.547×10^6	0.497×10^6
4	θ_{01} θ_{11}	0.400 -0.500	0.9464 -0.0659	0.588×10^6	0.497×10^6
F is the residual sum of squares					

The STACF of the residuals from this model are shown in Table 22 together with upper bound $1/\sqrt{N}$ for their standard error. There is no evidence of the model inadequacy from the behavior of autocorrelations as the residuals are uncorrelated and show a lack of structure. A number of individual values of the STACF of the residuals appear large compared to their standard error of 0.07. The value of the autocorrelation at time lag 15 and spatial lag 0 is large. However, this can be attributed to chance. Diagnostic checking stage of the model building procedure indicated that the STMA(1_1) model is adequate.

In summary, the residual sum of squares from both models, STMA(1_1) and STMA(1_2) are equal since the estimate θ_{21}=0.004 is very small. This estimate can be omitted from the mod-

TABLE 22

STACF of the residuals from the STMA (1_1) model

Spatial lag (S) / Time lag (K)	0	1	2
1	0.0580	0.0637	0.0495
2	0.0305	0.0385	0.0028
3	-0.0422	-0.0398	-0.0410
4	-0.0698	-0.0752	-0.0530
5	-0.0443	-0.0558	-0.0143
6	-0.0347	-0.0338	-0.0426
7	-0.0589	-0.0704	-0.0219
8	0.0196	0.0071	0.0132
9	0.0566	0.0531	0.0365
10	0.0293	0.0298	0.0425
11	-0.0689	-0.0674	-0.0776
12	-0.0037	0.0029	0.0204
13	0.0378	0.0395	0.0647
14	0.0444	0.0508	0.0502
15	0.0962	0.0939	0.0976
16	-0.0382	-0.0397	-0.0249
17	-0.0474	-0.0418	-0.0432
18	0.0555	0.0553	0.0533
19	-0.0716	-0.0722	-0.0652
20	-0.0662	-0.0516	-0.0626
Upper bound to standard error ± 0.07			

el without affecting the remaining parameters. The STMA(1_2) model can be selected for the rainfall process and the spatial structure of the system. This model for the rainfall series is expressed by

$$\nabla_T X_{it} = a_{it} - 0.9448\, a_{i,t-1} + 0.0672\, W^1 a_{i,t-1} - 0.004\, W^2 a_{i,t-1}$$

$$X_{it} = X_{i,t-1} + a_{it} - 0.9448\, a_{i,t-1} + 0.0672\, W^1 a_{i,t-1} - 0.004\, W^2 a_{i,t-1}$$

The model will be now employed in the STTFN modeling of the rainfall-runoff process.

To illustrate the behavior of a STARIMA model with spatial orders 1, 2 and 3 eleven raingage sites were used in this analysis (Figure 2). Table 23 gives the spatial ordering scheme for this system. The definition of spatial order 1 for this system is presented in Figure 18.

TABLE 23

Neighbors of each site for each spatial order

ORDER	1	2	3
Site 1	3	2	4,6
2	3,4	1	5,8,9
3	1,2,4,5	6,8	7,9
4	2,3,8	5,9	1,6,7,10
5	3,6,7,8	4	2,9,10,11
6	5	3,7	1,4,8
7	8,11,5	6,10	3,4
8	4,5,7,10	3,11,9	2,6
9	10	4,8	3,5,2,11
10	8,9,11	7	4,5
11	7,10	8	5,9

The correlation matrix for this system is shown in Table 24 and it can be observed that the rainfall series shows the existence of high correlation for the time series.

Initial identification of the rainfall data suggested that the space-time system is nonstationary. This is indicated by the STACF and STPACF shown in Tables 25 and 26. Therefore, the space-time system required a first difference in time.

	1	2	3	4	5	6	7	8	9	10	11
1			1.0								
2			0.50	0.50							
3	0.25	0.25		0.25	0.25						
4		0.33	0.33					0.33			
5			0.25			0.25	0.25	0.25			
6					1.0						
7					0.33			0.33			0.33
8				0.25	0.25		0.25			0.25	
9										1.0	
10								0.33	0.33		0.33
11							0.50			0.50	

Figure 18: W^1 for the 11-raingage system

TABLE 24

Correlation matrix for the rainfall series

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
P 1	1.0	0.66	0.79	0.61	0.65	0.67	0.65	0.63	0.52	0.59	0.59
P 2	0.66	1.00	0.78	0.77	0.68	0.79	0.76	0.77	0.77	0.69	0.69
P 3	0.79	0.78	1.00	0.74	0.82	0.83	0.79	0.72	0.62	0.67	0.66
P 4	0.61	0.77	0.74	1.00	0.65	0.64	0.67	0.71	0.67	0.57	0.57
P 5	0.65	0.68	0.82	0.65	1.00	0.75	0.82	0.77	0.67	0.69	0.69
P 6	0.67	0.79	0.83	0.64	0.75	1.00	0.85	0.77	0.70	0.77	0.77
P 7	0.65	0.76	0.79	0.67	0.82	0.85	1.00	0.85	0.78	0.85	0.85
P 8	0.63	0.77	0.72	0.71	0.77	0.77	0.85	1.00	0.81	0.84	0.84
P 9	0.52	0.77	0.62	0.67	0.67	0.70	0.78	0.81	1.00	0.82	0.77
P10	0.59	0.69	0.67	0.57	0.69	0.77	0.85	0.84	0.82	1.00	0.83
P11	0.59	0.69	0.66	0.57	0.69	0.77	0.85	0.84	0.77	0.83	1.00

The STACF and STPACF of the differenced series are presented in Tables 27 and 28. The general pattern is one of decay for the STPACF and the STACF cut off in time after one lag. These results suggesting that the differenced series might be identified as STMA(1_3) process,

$$\nabla_T X_{it} = a_{it} - \sum_{s=0}^{3} \theta_{s1} W^s a_{i,t-1}$$

TABLE 25

STACF of the original rainfall series

Spatial lag (S) / Time lag (K)	0	1	2	3
1	0.0866	0.0728	0.0756	0.0752
2	0.0338	0.0134	-0.0029	0.0029
3	0.0298	-0.0010	-0.0112	0.0097
4	-0.0269	-0.0378	-0.0442	-0.0468
5	-0.0412	-0.0514	-0.0401	-0.0471
6	-0.0113	-0.0320	-0.0352	-0.0249
7	-0.0696	-0.0720	-0.0682	-0.0725
8	-0.0218	-0.0400	-0.0344	-0.0379
9	0.0695	0.0640	0.0528	0.0638
10	0.0218	0.0291	0.0360	0.0305
11	-0.0381	-0.0394	-0.0415	-0.0430
12	0.0102	0.0079	0.0198	-0.0009
13	0.0388	0.0616	0.0620	0.0466
14	0.0223	0.0197	0.0235	0.0085
15	0.0332	0.0299	0.0477	0.0197
16	-0.0330	-0.0435	-0.0291	-0.0380
17	-0.0916	-0.0953	-0.0858	-0.0966
18	0.0319	0.0170	0.0267	0.0244
19	-0.0536	-0.0713	-0.0561	-0.0583
20	-0.0417	-0.0735	-0.0554	-0.0607

TABLE 26

STPACF of the original rainfall series

Spatial lag (S) / Time lag(K)	0	1	2	3
1	-0.0381	0.0008	-0.0098	0.0060
2	-0.0478	0.0158	-0.0063	0.0307
3	-0.0280	-0.0174	0.0244	0.0178
4	-0.0308	0.0097	0.0156	0.0302
5	0.0159	0.0087	0.0173	-0.0226
6	-0.0272	0.0292	0.0025	-0.0025

The parameters of this model were estimated by simplex method and are shown with the residual sum of squares in Table 29.

TABLE 27

STACF of the differenced rainfall series

Spatial lag (S) / Time lag (K)	0	1	2	3
1	-0.4711	-0.3660	-0.3971	-0.3490
2	-0.0207	-0.0180	-0.0320	-0.0358
3	0.0251	0.0081	0.0097	0.0302
4	-0.0220	-0.0112	-0.0192	-0.0298
5	-0.0241	-0.0181	-0.0005	-0.0121
6	0.0488	0.0327	0.0204	0.0384
7	-0.0648	-0.0460	-0.0420	-0.0517
8	-0.0101	-0.0247	-0.0157	-0.0219
9	0.0702	0.0692	0.0503	0.0672
10	0.0101	0.0222	0.0366	0.0256
11	-0.0652	-0.0696	-0.0819	-0.0697
12	0.0116	-0.0026	0.0116	-0.0020
13	0.0265	0.0542	0.0462	0.0492
14	-0.0189	-0.0303	-0.0358	-0.0290
15	0.0412	0.0433	0.0523	0.0352
16	-0.0091	-0.0159	-0.0146	-0.0039
17	-0.0957	-0.0852	-0.0881	-0.0936
18	0.1218	0.1176	0.1141	0.1196
19	-0.0602	-0.0547	-0.0529	-0.0518
20	-0.0898	-0.1083	-0.0971	-0.1127

The residuals were tested for whiteness and presence of any periodicities. The STACF and STPACF of the generated residuals are given in Tables 30 and 31, respectively, along with the upper bound $1/\sqrt{N}$ for the residuals standard error. The STACF and STPACF show that the generated errors are uncorrelated with a lack of structure among these errors. The results from Portmanteau test and the corresponding variate

TABLE 28

STPACF of the differenced rainfall series

Spatial lag (S) / Time lag (K)	0	1	2	3
1	0.3736	-0.0092	-0.0212	-0.0067
2	0.2316	0.0001	-0.0299	0.0104
3	0.1646	-0.0155	-0.0139	0.0125
4	0.1111	-0.0109	-0.0081	0.0259
5	0.1008	-0.0133	0.0011	-0.0009
6	0.0549	-0.0009	-0.0004	-0.0148

TABLE 29

Parameter estimates of the STMA (1_3) model

Run	Parameter	Guess	Estimate	Initial-F	Final-F
1	θ_{01}	0.000	0.9455		
	θ_{11}	0.000	-0.0385		
	θ_{21}	0.000	-0.0417		
	θ_{31}	0.000	0.0007	0.219×10^6	0.130×10^6
2	θ_{01}	0.400	0.9455		
	θ_{11}	0.100	-0.0385		
	θ_{21}	0.200	-0.0417		
	θ_{31}	0.100	0.0007	0.148×10^6	0.130×10^6
3	θ_{01}	0.200	0.9455		
	θ_{11}	-0.100	-0.0385		
	θ_{21}	-0.100	-0.0417		
	θ_{31}	-0.100	0.0007	0.198×10^6	0.130×10^6
F is the residual sum of squares					

at a 5% level of significance are summarized in Table 32. These results indicate that the generated errors are uncorrelated. The cumulative periodograms of the generated errors are shown in Figure 19 along with the confidence limits at a 5% level of significance. Since the periodograms fall within these limits, the errors do not contain periodicities. Based on the above tests, the generated errors are shown to be uncorrelated and white noise. Therefore, the STMA(1_3) model passes the diagnostic checking stage of the three-stage iterative procedure for space-time modeling.

TABLE 30

STACF of the generated errors

Spatial lag (S) / Time lag (K)	0	1	2	3
1	0.0572	0.0575	0.0620	0.0605
2	0.0128	0.0060	-0.0079	-0.0026
3	0.0084	-0.0093	-0.0171	0.0022
4	-0.0427	-0.0404	-0.0456	-0.0487
5	-0.0514	-0.0501	-0.0385	-0.0441
6	-0.0182	-0.0267	-0.0292	-0.0190
7	-0.0778	-0.0676	-0.0639	-0.0678
8	-0.0147	-0.0203	-0.0156	-0.0171
9	0.0677	0.0728	0.0618	0.0735
10	0.0167	0.0340	0.0404	0.0361
11	-0.0548	-0.0466	-0.0486	-0.0505
12	-0.0018	0.0063	0.0172	-0.0035
13	0.0314	0.0635	0.0618	0.0487
14	0.0111	0.0180	0.0196	0.0067
15	0.0249	0.0318	0.0462	0.0216
16	-0.0390	-0.0386	-0.0288	-0.0326
17	-0.0856	-0.0774	-0.0727	-0.0787
18	0.0494	0.0460	0.0517	0.0540
19	-0.0518	-0.0584	-0.0463	-0.0456
20	-0.0370	-0.0573	-0.0431	-0.0457
Upper bound to standard error $\pm$ 0.07				

TABLE 31

STPACF of the generated errors

Spatial lag (S) / Time lag (K)	0	1	2	3
1	-0.0115	0.0060	-0.0186	-0.0093
2	-0.0216	-0.0138	0.0230	0.0128
3	-0.0282	0.0177	0.0301	-0.0232
4	0.0121	-0.0141	0.0093	0.0189
5	0.0218	0.0148	-0.0154	-0.0013
6	-0.0006	0.0113	0.0110	-0.0123
Upper bound to standard error $\pm$ 0.07				

TABLE 32

Results from Portmanteau test on the generated errors

Spatial lag (S)	Portmanteau statistic	Chi-square statistic $\alpha = 0.05$	Decision
0	7.52	26.3	O.K
1	8.21	26.3	O.K
2	7.50	26.3	O.K
3	7.5	26.3	O.K

The estimate θ_{31} =0.0007 (Table 29) is very small and suggests that this parameter may be omitted. This indicates that a second model STMA(1_2),

$$\nabla_T X_{it} = a_{it} - \sum_{s=0}^{2} \theta_{s1} W^s a_{i,t-1}$$

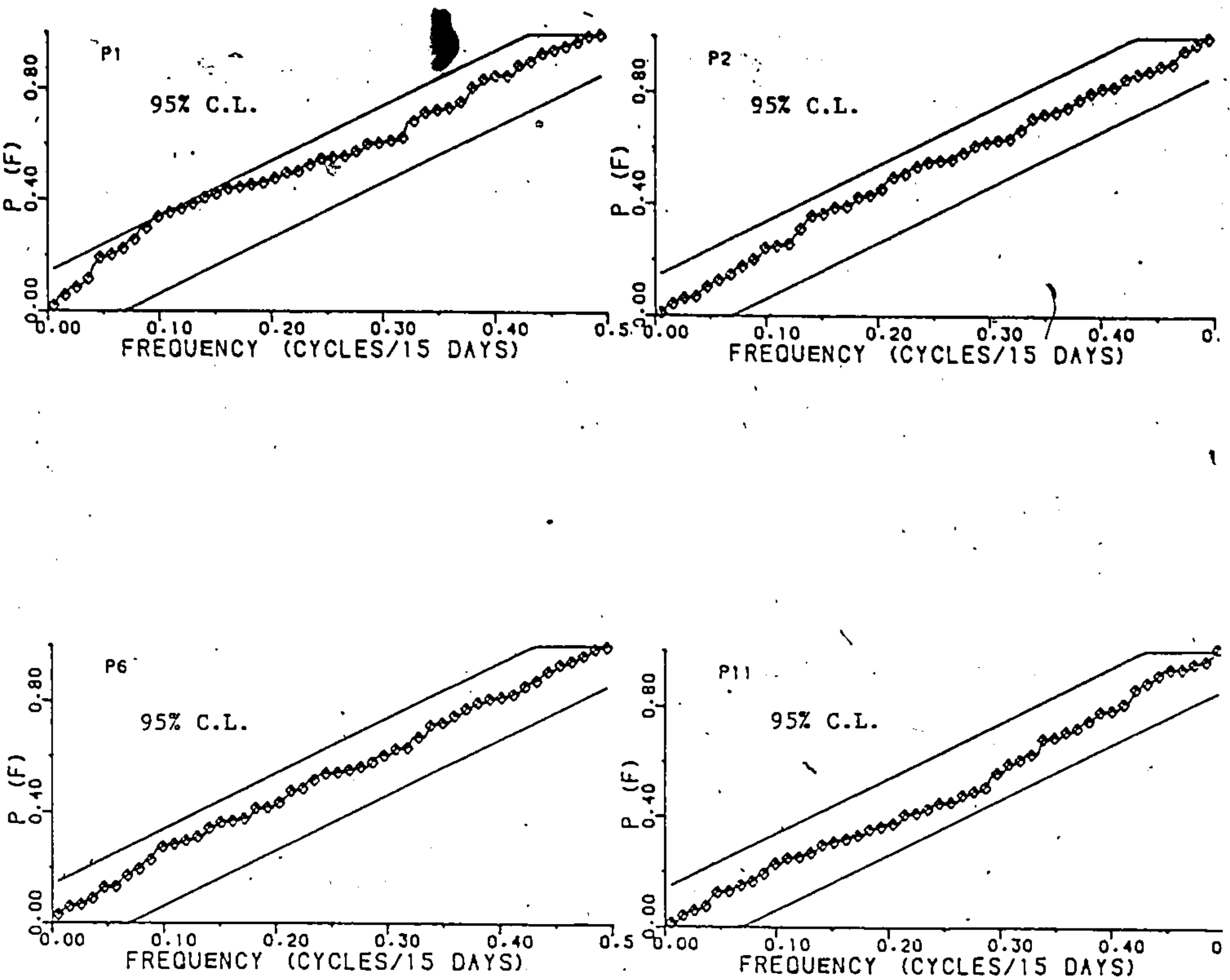


Figure 19: Cumulative periodogram of generated errors

can be considered for the rainfall series. Estimating the model parameters gave θ_{01} =0.9460, θ_{11} =-0.03896, θ_{21} =-0.04127 and the residual sum of squares is 0.1306×10^{6}. Examining the estimates of the STMA(1_2) and STMA(1_3) models reveal an equal values for, the residual sum of squares for both models. Based on this fact, there is no evidence of one of the model giving better forecasted estimates of the rainfall time series. The STMA(1_3) model was selected for the spatial structure of the eleven raingages.

Subroutine Gauss was used to forecast 144 data points in order to test the forecasting performance of the STMA(1_3) model. The forecasted errors are then tested for whiteness and presence of any periodicities. The results from autocorrelation functions, Portmanteau test and the cumulative periodograms indicated that the errors do not contain any periodicities, therefore, they were considered to be white noise. Table 33 shows the estimates of r_s and ISE estimated from (5.1) and (5.2) for the forecasted and generated rainfall data. The estimates show satisfactory correlation coefficients for the data.

The observed and computed rainfall values of a portion of the time series during generation period (1-192) are shown in Figure 20. The observed and computed rainfall during forecasting period (193-336) are shown in Figure 21. The generated and forecasted rainfall data compare well with the corresponding observed rainfall. The STMA (1_3) model is now

TABLE 33

Results from Goodness of Fit Test on the Rainfall Data

Rainfall Series	Statistical Test	P1	P2	P6	P11
Generated	r_s ISE	0.48 66.80	0.48 66.69	0.56 62.40	0.55 61.57
Forecasted	r_s ISE	0.61 56.81	0.59 59.40	0.65 55.18	0.67 52.10

ready to be employed as a forecasting tool for the rainfall series in the selected Watershed.

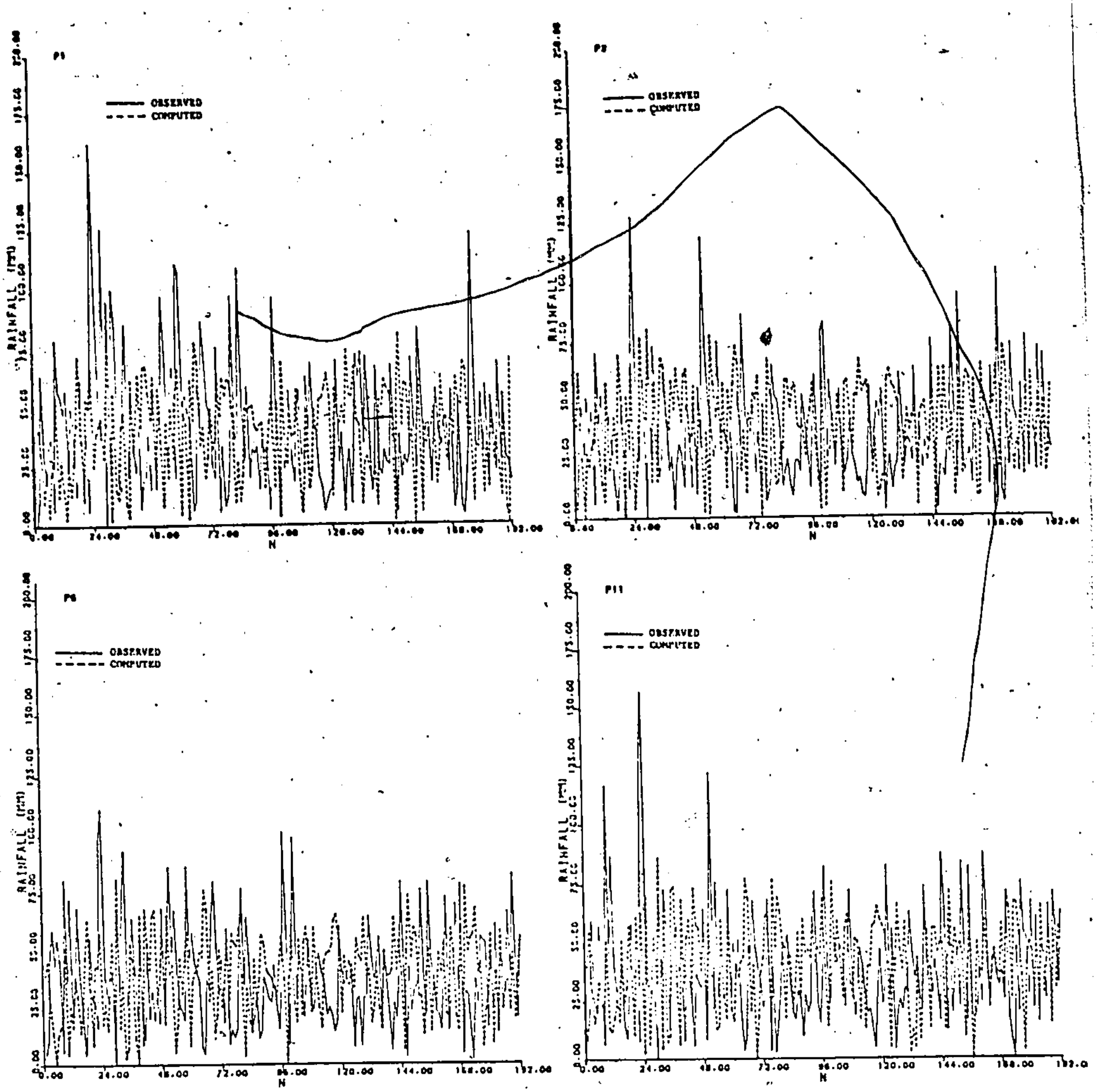


Figure 20: Observed and generated rainfall values

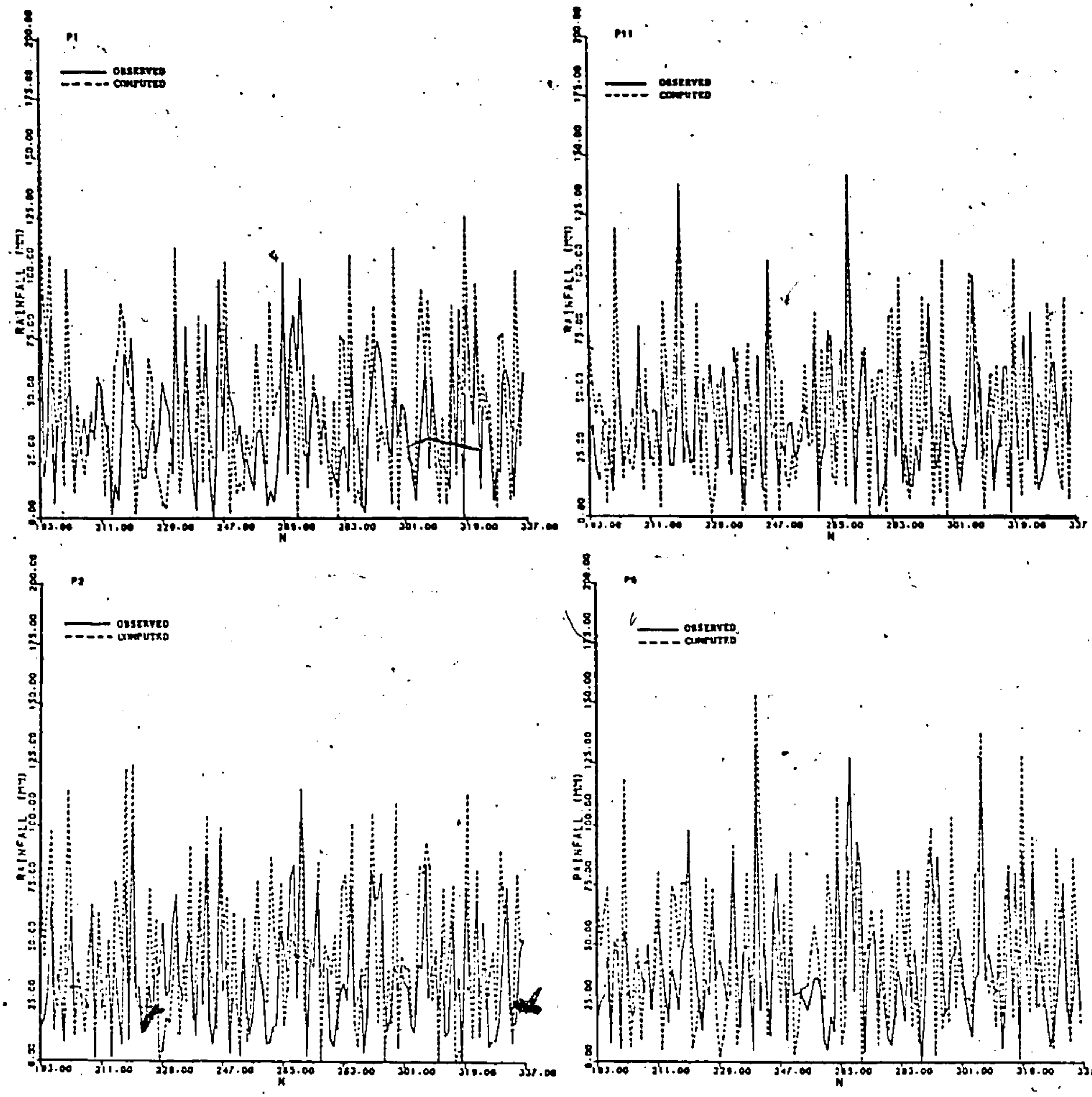


Figure 21: Observed and forecasted rainfall values

5.2 SPACE-TIME TRANSFER FUNCTION NOISE MODELS OF RAINFALL-RUNOFF PROCESS

The runoff data used in the analysis were selected in a manner that the chosen four runoff stations are located along with the rainfall stations within the same four zones of the selected watershed. The location of the runoff and rainfall stations are shown in Figure 22, where R1, R2, R3 and R4 represent the four different runoff stations, respectively.

Equal weights were selected for this analysis. They have the same spatial ordering as the rainfall series since the runoff stations are assumed to be at the same locations as the rainfall gages. The spatial ordering of the system is limited to spatial order 0, 1 and 2. Table 8 defines the spatial ordering. Figure 23 presents spatial orders 1 and 2 as an example of the weighting scheme.

Table 34 shows the correlations between the runoff stations. The results show the existence of high correlation among the runoff stations in space.

The autocorrelations of the original runoff series in Figure 24 indicate the existence of significant correlation sequence. Removing the seasonality from the runoff series show a change in the structure of the autocorrelations, some high correlation values become small and the series is considered uncorrelated. Seasonality was removed from the runoff data due to the nonstationarity nature of the autocorrelations. The seasonal components were removed by

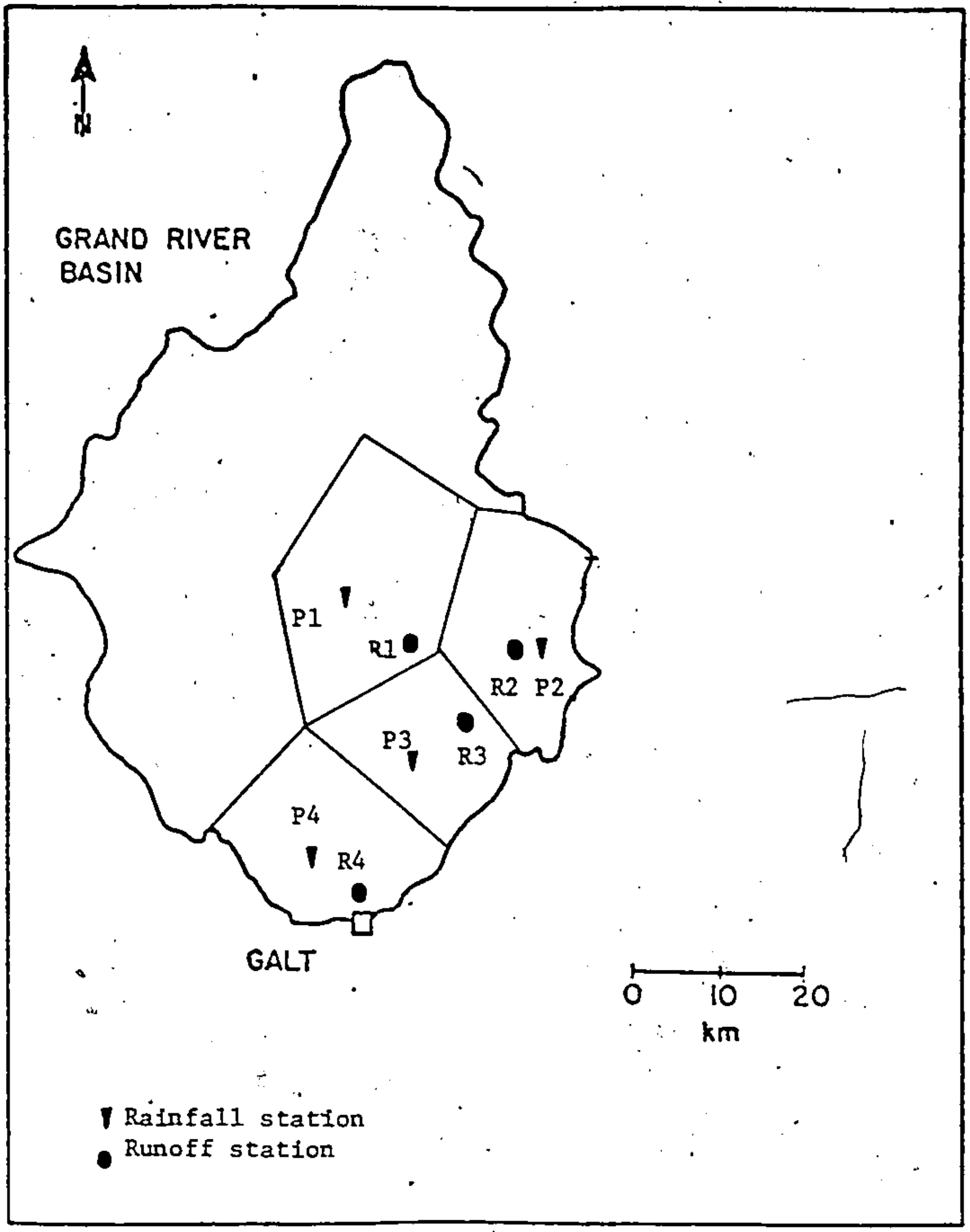


Figure 22: Location of the rainfall and runoff stations

W^1

	1	2	3	4
1		0.5	0.5	
2	0.5		0.5	
3	0.33	0.33		0.33
4			1.0	

W^2

	1	2	3	4
1				1.0
2				
3				
4	0.5	0.5		

Figure 23: W for the 4-rainfall runoff spatial system

TABLE 34

Correlation matrix for the runoff series

	R1	R2	R3	R4
R1	1.00	0.93	0.80	0.92
R2	0.93	1.00	0.96	0.79
R3	0.80	0.96	1.00	0.82
R4	0.92	0.79	0.82	1.00

subtracting the computed mean values for each time period of 15 days from the original series, provided that the standard deviations remain unchanged for all time periods. Therefore,

$$E_t = Y_t - \bar{Y}$$

where E_t is the deseasonalized series, and $\bar{Y}$ is the mean.

The STACF and STPACF for the original and deseasonalized runff series, respectively, are shown in Tables 35 and 36. The original data are nonstationary in time since the autocorrelations fail to tail off quickly at all spatial lags. The deseasonalized data are stationary in time. The decay with spatial lags is much steeper than that with time lags.

Table 37 shows the STCCF for the original rainfall and original runoff. In general there is a lack of structure in these functions despite the stationarity of the rainfall series. Table 38 shows the STCCF for the original and desea-

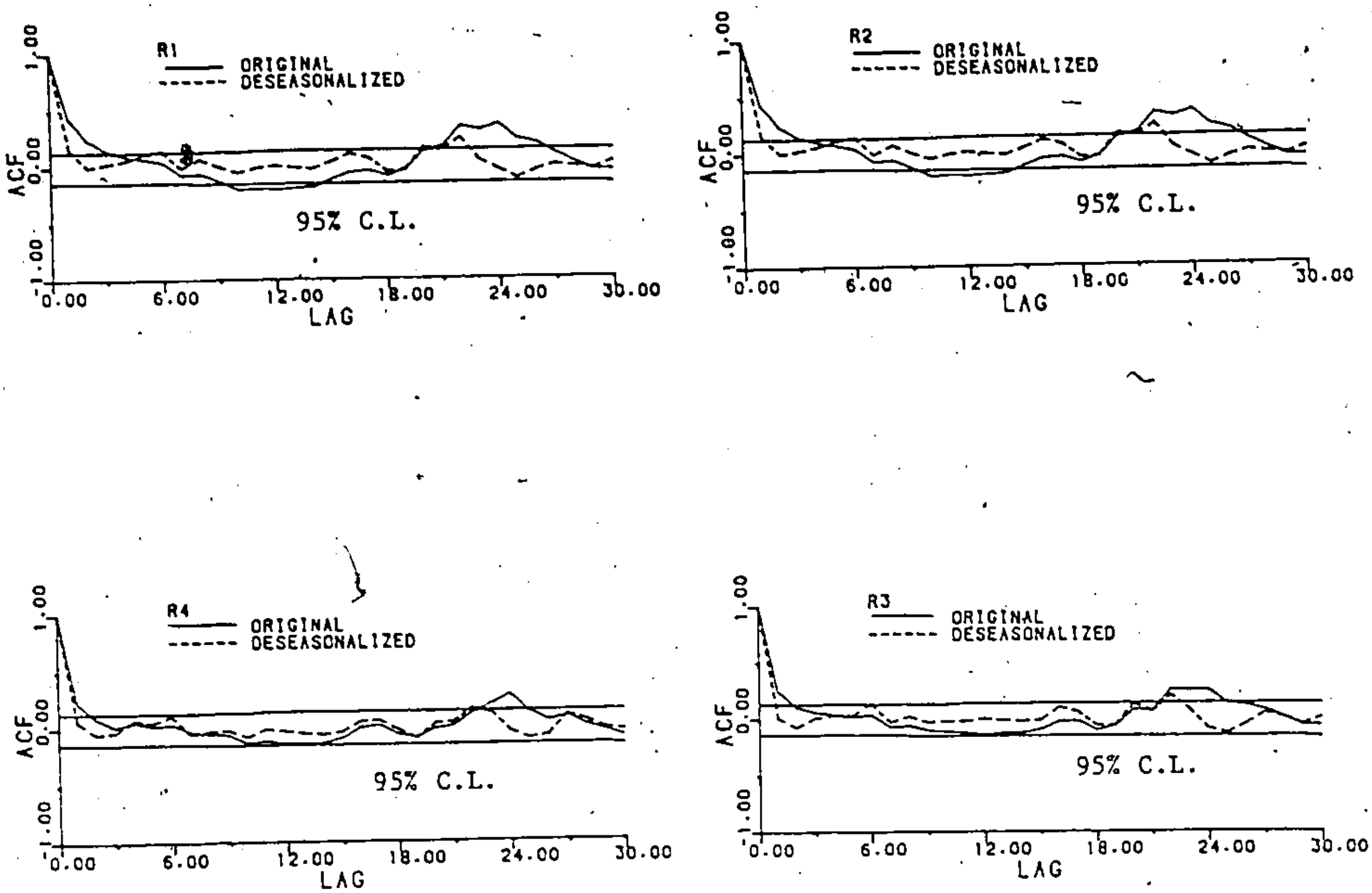


Figure 24: Autocorrelation functions of the observed and deseasonalized runoff values

TABLE 35

STACF of the original runoff series

Spatial lag (S) / Time lag (K)	0	1	2
1	0.5236	-0.0228	-0.2168
2	0.4367	-0.0533	-0.2203
3	0.3805	-0.0588	-0.2163
4	0.4055	-0.0554	-0.2143
5	0.3760	-0.0585	-0.2113
6	0.3845	-0.0587	-0.2096
7	0.3382	-0.0690	-0.2105
8	0.3258	-0.0736	-0.2097
9	0.3179	-0.0741	-0.2098
10	0.2744	-0.0814	-0.2091
11	0.2881	-0.0778	-0.2084
12	0.2670	-0.0827	-0.2078
13	0.2628	-0.0831	-0.2077
14	0.2679	-0.0817	-0.2067
15	0.2917	-0.0747	-0.2042
16	0.3482	-0.0626	-0.2009
17	0.3565	-0.0547	-0.2000
18	0.3062	-0.0715	-0.2011
19	0.2760	-0.0650	-0.1991
20	0.3277	-0.0493	-0.1961

sonalized runoff. The STCCF between the two space-time series, Y_{it} and X_{it} are difficult to interpret. This is because when both the X_{it} and Y_{it} series are autocorrelated, the estimates of the STCCF can have high variance and the estimates of the STCCF at different time lags can be highly correlated with one another. Therefore, the input (rainfall) series would require prewhitening, by fitting a model to the time series. Same structure is then used to transform the output (runoff) series, and compute STCCF between the prewhitened and transformed series.

TABLE 36

STPACF of the deseasonalized runoff series

Spatial lag (S) / Time lag (K)	0	1	2
1	0.0643	0.0088	0.0003
2	-0.0505	-0.0309	-0.0033
3	-0.0297	-0.0087	-0.0012
4	0.0724	0.0115	0.0023
5	0.0504	0.0126	0.0016
6	0.1115	0.0303	0.0039
7	-0.0524	-0.0024	-0.0017
8	-0.0344	-0.0034	-0.0002
9	-0.0223	-0.0010	-0.0010
10	-0.0760	-0.0102	-0.0019
11	-0.0140	0.0023	-0.0006
12	-0.0259	0.0005	-0.0003
13	-0.0467	-0.0057	-0.0011
14	-0.0630	-0.0088	-0.0021
15	-0.0381	-0.0054	-0.0010
16	0.0482	0.0162	0.0025
17	0.0640	0.0281	0.0023
18	-0.0160	-0.0107	-0.0009
19	-0.0953	-0.0225	-0.0035
20	0.0335	0.0195	0.0017

The three-stage iterative procedure for space-time modeling applied to the rainfall series X_{it} indicated that it is described by the STMA(1_2) model,

$$X_{it} = X_{i,t-1} + a_{it} - 0.9448\, a_{i,t-1} + 0.0672\, W^1 a_{i,t-1} - 0.004\, W^2 a_{i,t-1}$$

therefore the transformation

$$a_{it} = \alpha_{it} = X_{it} - X_{i,t-1} + 0.9448\, a_{i,t-1} + 0.0672\, W^1 a_{i,t-1} + 0.004\, W^2 a_{i,t-1} \quad (5.3)$$

$$a_{it} = \beta_{it} = E_{it} - E_{i,t-1} + 0.9448\, a_{i,t-1} + 0.0672\, W^1 a_{i,t-1} + 0.004\, W^2 a_{i,t-1} \quad (5.4)$$

TABLE 37

STCCF for the original rainfall and runoff series

Spatial lag (S) / Time lag (K)	0	1	2
0	-0.0085	-0.0221	0.0824
1	0.0525	0.0255	0.1341
2	0.0029	-0.0165	0.0861
3	-0.0020	-0.0181	0.0894
4	-0.0096	-0.0234	0.0745
5	-0.0060	-0.0146	0.0765
6	0.0155	-0.0076	0.1128
7	0.0040	0.0019	0.0990
8	0.0413	0.0105	0.1181
9	0.0281	0.0201	0.1103
10	0.0655	0.0504	0.1374
11	0.0207	0.0205	0.1100
12	0.0413	0.0268	0.1153
13	0.0330	0.0086	0.0974
14	-0.0054	-0.0071	0.0691
15	-0.0158	-0.0194	0.0653
16	0.0217	0.0143	0.0894
17	-0.0164	-0.0185	0.0574
18	0.0377	0.0285	0.1107
19	0.0140	-0.0088	0.0885
20	-0.0041	-0.0149	0.0748

are applied to determine the prewhitened series α_{it} and the transformed series β_{it}.

The STCCF for the α_{it} and β_{it} series are shown in Table 39. The cross correlations are cut off after 1 lag in time (K=1) for spatial lag S=0,1 and 2. There are large values only at lag 1 in time for the three orders in space.

TABLE 38

STCCF for original rainfall and deseasonalized runoff

Spatial lag (S) Time lag (K)	0	1	2
0	0.0002	-0.0022	0.0204
1	0.0748	0.0601	0.0758
2	0.0192	0.0117	0.0204
3	0.0174	0.0146	0.0340
4	0.0066	0.0074	0.0097
5	0.0045	0.0145	0.0065
6	0.0331	0.0316	0.0524
7	0.0061	0.0238	0.0183
8	0.0391	0.0151	0.0325
9	0.0006	0.0004	0.0079
10	0.0608	0.0546	0.0583
11	-0.0093	-0.0032	0.0066
12	0.0305	0.0256	0.0270
13	0.0414	0.0266	0.0279
14	-0.0071	-0.0062	-0.0179
15	-0.0151	-0.0156	-0.0314
16	0.0243	0.0263	-0.0025
17	-0.0344	-0.0253	-0.0361
18	0.0451	0.0385	0.0452
19	0.0039	-0.0187	0.0128
20	-0.0241	-0.0098	-0.0200

The impulse response functions are estimated from

$$\nu_{s,k} = r_{\alpha\beta}(s,k)\,\frac{S_\beta}{S_\alpha}$$

where $r_{\alpha\beta}(s,k)$ are the estimated cross correlations, S_α and S_β are the standard deviations of the α_{it} and β_{it} respectively. Therefore,

$$\nu_{s,k} = r_{\alpha\beta}(s,k)\,\frac{17.917}{25.487}\ .$$

TABLE 39

STCCF of the α_{it} series leading the β_{it} series

Spatial lag (S) / Time lag (K)	0	1	2
0	-0.0186	-0.0236	-0.0076
1	0.1357	0.1000	0.1312
2	0.0275	0.0030	0.0237
3	0.0220	0.0078	0.0490
4	0.0015	-0.0073	0.0029
5	-0.0044	0.0041	-0.0058
6	0.0522	0.0421	0.0857
7	0.0012	0.0237	0.0214
8	0.0620	0.0103	0.0486
9	-0.0110	-0.0207	0.0028
10	0.1053	0.0910	0.1026
11	-0.0278	-0.0231	0.0063
12	0.0489	0.0363	0.0468
13	0.0736	0.0420	0.0579
14	-0.0184	-0.0230	-0.0334
15	-0.0330	-0.0419	-0.0598
16	0.0443	0.0495	-0.0050
17	-0.0679	-0.0623	-0.0708
18	0.0847	0.0662	0.0861
19	0.0065	-0.0429	0.0255
20	-0.0488	-0.0306	-0.0386

The estimated impulse response functions $\upsilon_{s,k}$ with their approximate standard error from $1/\sqrt{N}$ are shown in Table 40. The impulse response functions with values significantly different from zero are only at lag K=1 in time for lag 0, 1 and 2 in space. This indicates the dependence of runoff on rainfall and that the output (runoff) lags behind the input (rainfall) by 15 days. Therefore, the memory of the rainfall-runoff process ,b, can be assumed to be 15 days or 1 lag in time.

TABLE 40

Impulse response functions with their approximate standard errors

Spatial lag (S) / Time lag (K)	0	1	2
0	-0.014	-0.016	-0.005
1	0.095	0.070	0.092
2	0.019	0.002	0.016
3	0.015	0.005	0.034
4	0.001	-0.005	0.002
5	-0.003	0.003	-0.004
6	0.036	0.029	0.060
7	0.000	0.016	0.015
8	0.042	0.007	0.034
9	-0.007	-0.014	0.001
10	0.074	0.064	0.072
11	-0.019	-0.016	0.004
12	0.034	0.025	0.032
13	0.051	0.029	0.041
14	-0.013	-0.016	-0.023
15	-0.021	-0.029	-0.042
16	0.031	0.028	-0.003
17	-0.047	-0.043	-0.049
18	0.059	0.046	0.060
19	0.004	-0.030	0.017
20	-0.034	-0.021	-0.027

Upper bound to standard error : 0.07

Using the model building procedure of Box and Jenkins (9), the preliminary identification suggests a space-time transfer function model, with (P,M,g,h,b) equal to (0,0,2,1,1) or (2,1,2,1,1).

The first space-time transfer function model (0,0,2,1,1) may be written in the form,

$$E_{it} = \sum_{s=0}^{2} \omega_{0s} W^{s} X_{i,t-1} + a_{it}$$

The Simplex method was used to estimate the parameters of the identified STTFN models. The estimated parameters with the residual sum of squares are given in Table 41.

TABLE 41

Parameter estimates of space-time transfer function model (0,0,2,1,1)

Run	Parameter	Guess	Estimate	Initial-F	Final-F
1	ω_{00} ω_{01} ω_{02}	0.00 0.00 0.00	-0.0547 0.0157 -0.0243	0.238×10^6	0.235×10^6
2	ω_{00} ω_{01} ω_{02}	-0.05 0.30 0.40	-0.0547 0.0157 -0.0243	$0.330\ 10^6$	$0.235\ 10^6$
3	ω_{00} ω_{01} ω_{02}	-0.30 0.60 -0.01	-0.0547 0.0157 -0.0243	0.338×10^6	0.235×10^6
F is the residual sum of squares					

Now the space-time transfer function model has been fitted by Simplex algorithm and the generated error a_{it} have been calculated from

$$a_{it} = E_{it} + 0.0547 \nabla X_{i,t-1} - 0.0157\, W^1 \nabla X_{i,t-1} + 0.0243\, W^2 \nabla X_{i,t-1}$$

Then the STACF of these errors are computed to check the adequacy of the model. The STACF of the generated errors are tabulated in Table 42 together with the upper bound $1/\sqrt{N}$ for their standard error. The autocorrelations for lag 0, 1 and 2 in space do not show any correlation patterns. Thus the generated errors are white noise.

TABLE 42

STACF of generated errors from STTF (0,0,2,1,1) model

Spatial lag (S) Time lag (k)	0	1	2
1	0.0646	0.0088	-0.0040
2	-0.0553	-0.0286	-0.0014
3	-0.0277	-0.0098	0.0026
4	0.0778	0.0134	0.0083
5	0.0512	0.0120	0.0009
6	0.1057	0.0214	-0.0074
7	-0.0555	0.0065	-0.0012
8	-0.0276	-0.0019	0.0067
9	-0.0343	-0.0169	-0.0057
10	-0.0727	0.0035	-0.0054
11	-0.0147	-0.0044	0.0056
12	-0.0289	0.0027	-0.0022
13	-0.0403	0.0004	0.0017
14	-0.0605	-0.0081	0.0015
15	-0.0359	-0.0097	0.0054
16	0.0455	0.0223	-0.0092
17	0.0588	0.0153	0.0028
18	-0.0067	0.0125	0.0033
19	-0.0929	-0.0350	0.0015
20	0.0284	0.0150	-0.0027
Upper to standard error $\pm$ 0.07			

The results of Portmanteau test and the corresponding variate at a 5% level of significance are shown in Table 43. The results indicate that the generated errors are uncorrelated.

TABLE 43

Results from Portmanteau test on the generated errors

Spatial lag (S)	Portmanteau test	Chi-square statistic $\alpha = 0.05$	Decision
0	11.59	27.6	accepted
1	8.97	27.6	accepted
2	6.68	27.6	accepted

The STPACF of the generated errors are shown in Table 44. In general there is a lack of structure in these errors for lag 0, 1 and 2 in space. Thus the generated errors are uncorrelated and white noise. The STCCF between the inputs α_{it} in prewhitened form obtained from (5.3) and the residuals α_{it} estimated from (5.4) are given in Table 45 together with the upper bound $1/\sqrt{N}$ for their standard error. The cross correlations for the three spatial lags are not large compared with the upper bound of their standard error and they are uncorrelated.

The cumulative periodograms of the generated errors (Figure 25 with the confidence limits at a 5% level of significance) show the points of the cumulative periodograms falling closely within these limits. Therefore the errors do not contain periodicities and are considered white noise. These checks based on the study of the STACF, STPACF, Portmanteau test, cumulative periodogram and STCCF conclude that accep-

TABLE 44

STPACF of generated errors from STTF (0,0,2,1,1) model

Spatial lag (S) Time lag (K)	0	1	2
1	-0.0613	0.0058	0.0028
2	0.0599	0.0173	-0.0024
3	0.0245	-0.0004	-0.0013
4	-0.0856	0.0071	-0.0074
5	-0.0306	-0.0046	0.0002
6	-0.1148	-0.0008	0.0080

TABLE 45

Space-time CCF for $\tilde{\alpha}_{it}$ series and generated errors from TFN model (0,0,2,1,1)

Spatial lag (S) Time lag (K)	0	1	2
1	0.0383	0.0256	0.0425
2	0.0421	0.0301	0.0477
3	0.0096	-0.0057	0.0042
4	0.0017	-0.0056	0.0300
5	-0.0150	-0.0161	-0.0118
6	-0.0159	-0.0002	-0.0166
7	0.0383	0.0382	0.0717
8	-0.0136	0.0171	0.0118
9	0.0593	0.0178	0.0462
10	-0.0157	-0.0149	-0.0001
11	0.0984	0.0942	0.0996
12	-0.0367	-0.0216	-0.0032
13	0.0498	0.0475	0.0539
14	0.0765	0.0531	0.0628
15	-0.0173	-0.0121	-0.0331
16	-0.0320	-0.0333	-0.0586
17	0.0295	0.0337	-0.0225
18	-0.0758	-0.0627	-0.0843
19	0.0919	0.0809	0.0897
20	-0.0019	-0.0440	0.0126
Upper bound to standard error $\pm$ 0.07			

table generated results of runoff values can be obtained by using the space-time TFN model (0,0,2,1,1).

In order to test the forecasting performance of the model it was necessary to generate 144 random data points based on the mean and standard deviation of the generated residuals. Subroutine Gauss was used to generate the random numbers. The forecasted errors were then tested for whiteness and presence of periodicities. The cumulative periodograms of the forecasted errors (Figure 26) show that the points of cumulative periodograms are within the 95% confidence limit. This indicates that the errors do not contain any periodicities. The Portmanteau test results (Table 46) indicate that the forecasted errors are uncorrelated. The ACF of the forecasted errors (Figure 27) together with the 95% confidence limits show that the errors are uncorrelated due to lack of structure and therefore are white noise.

The estimates for r_s and ISE for the generated and forecasted runoff values are shown in Table 47. This table shows high estimates of r_s and ISE for stations R3 and R4 compared to R1 and R2. This might be due to the fact that R1 and R2 do not belong to the same stream as R3 and R4. This caused a reduction of the adequacy of the forecasted runoff values for the selected model.

The STTFN model (0,0,2,1,1) is given by

$$E_{it} = -0.0547 \nabla X_{i,t-1} + 0.0157\, W^1 \nabla X_{i,t-1} - 0.0243\, W^2 \nabla X_{i,t-1} + a_{it}$$

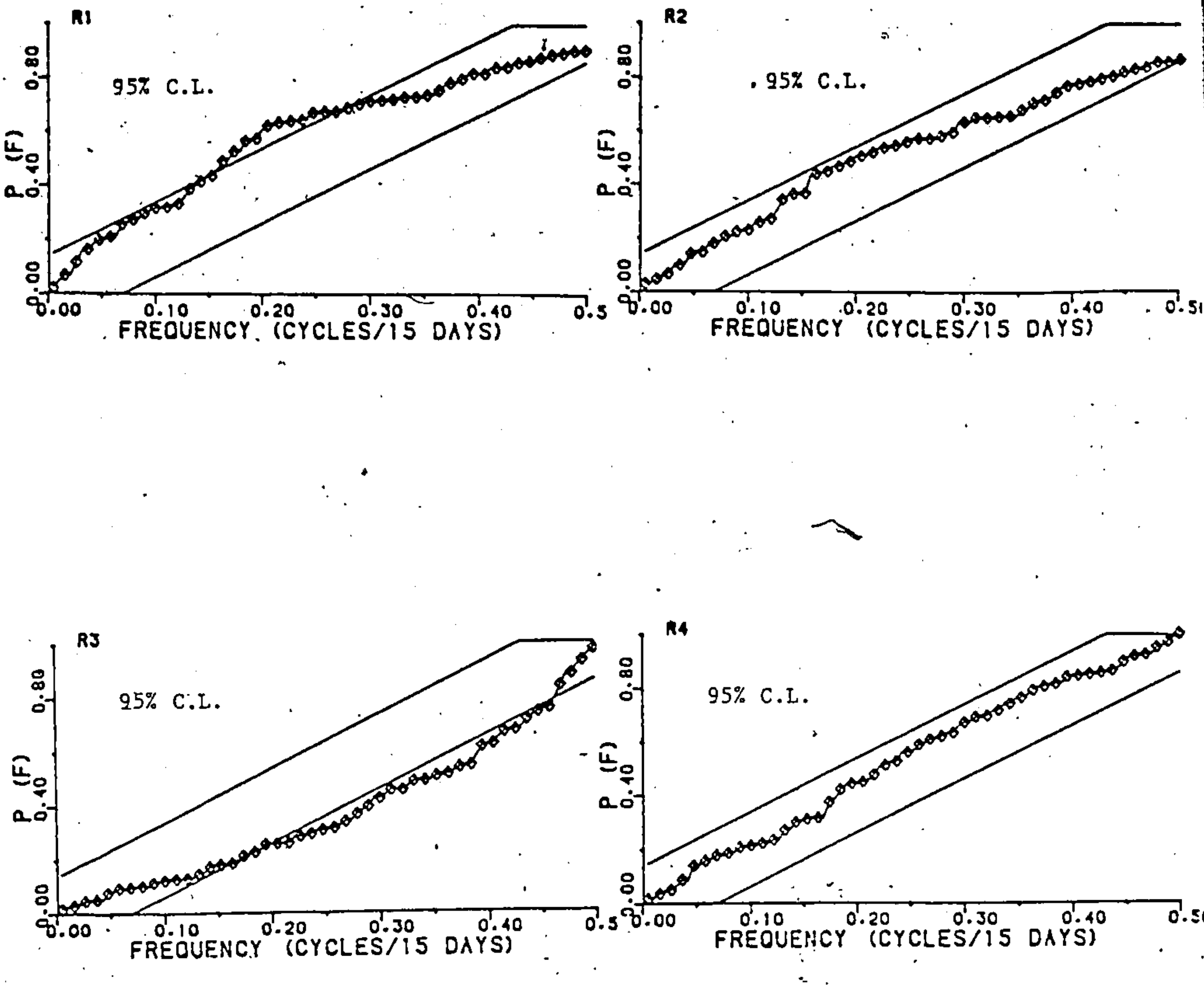


Figure 25: Cumulative periodograms of generated errors

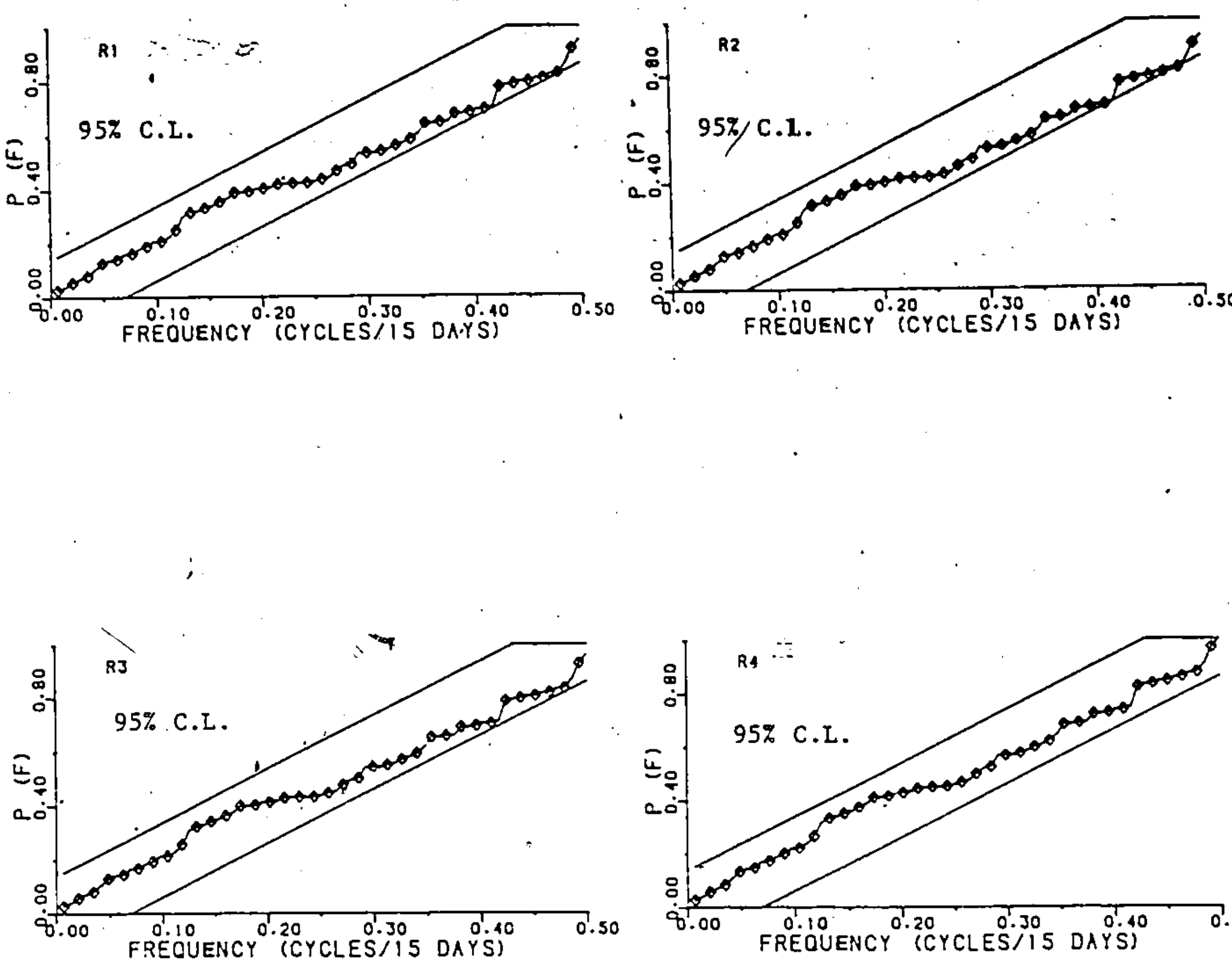


Figure 26: Cumulative periodograms of forecasted errors

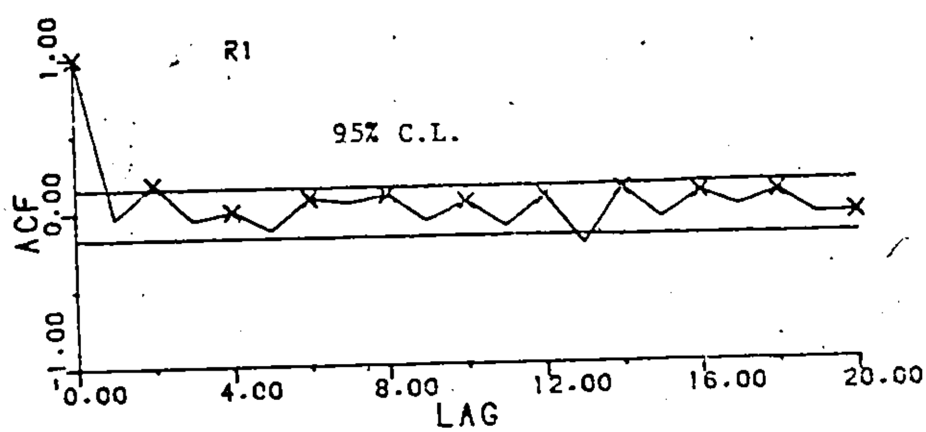


Figure 27: ACF of the forecasted errors

TABLE 46

Results from Portmanteau test on the forecasted errors

	Value
Test statistic	42.48
Critical value of $\alpha = 0.05$	40.10
Decision	O.K

TABLE 47

Results from Goodness of Fit Tests on the Runoff Data

Runoff Series	Statistical Test	R1	R2	R3	R4
Generated	r_s ISE	-1.36 201	-4.2 220	0.77 51.39	0.75 42.52
Forecasted	r_s ISE	-1.45 187	-2.52 171	0.64 64.85	0.69 65.20

and passes the diagnostic checking step of the three stage model building.

Consider the case that the rainfall runoff process is presented by a space-time transfer function model with (P,M,g,h,b) equal to (2,1,2,1,1) and written in the following form ,

$$E_{it} = \frac{\sum_{s=0}^{2} \omega_{0s} W^s}{1 - \sum_{s=0}^{2} \delta_{ks} B^k W^s} X_{i,t-1} + a_{it}$$

The estimated parameters with the residual sum of squares are given in Table 48.

TABLE 48

Parameter estimates of space-time transfer function (2,1,2,1,1) model

Run	Parameter	Guess	Estimate	Initial-F	Final-F
1	ω_{00}	0.00	-0.05725		
	ω_{01}	0.00	0.01699		
	ω_{02}	0.00	-0.02132		
	δ_{10}	0.00	0.06290		
	δ_{11}	0.00	-0.02156		
	δ_{12}	0.00	-0.00502	0.238×10^{6}	0.234×10^{6}
2	ω_{00}	0.2	-0.05725		
	ω_{01}	0.1	0.01699		
	ω_{02}	0.4	-0.02132		
	δ_{10}	-0.3	0.06290		
	δ_{11}	-0.1	-0.02156		
	δ_{12}	-0.5	-0.00502	0.769×10^{6}	0.234×10^{6}

F is the residual sum of squares

The STACF, STPACF and Portmanteau test results indicated that the generated errors are uncorrelated. The cumulative periodograms for the runoff series indicated that the generated errors for runoff station R1 and R2 are white noise. On the other hand, the generated errors for runoff stations R3 and R4 were shown with a marked departure from linearity in the periodograms. The periodograms fall outside the confidence limits at a 5% level of significance. This indicated that the errors from R3 and R4 contained some periodicities and were not white noise. Therefore, the STTFN (2,1,2,1,1) model was considered inadequate. Improvements in the model could be expected by repeating the iterative cycle of identification, estimation, and diagnostic checking. The STTFN (0,0,2,1,1) model is then selected and used to produce forecasts of the runoff data.

The observed and generated runoff values for the period 1-192 for the STTFN (0,0,2,1,1) model are shown in Figure 28. The generated runoff compare well with corresponding observed runoff. The peak runoff values are preserved during generation for station R3. The peak runoff values for the rest of the stations are not well presented. The generated runoff at peak times are higher than the observed runoff. This suggests the need for a form of seasonal STTFN model for the rainfall-runoff process.

The observed and forecasted runoff values for the period 193-336 for the STTFN (0,0,2,1,1) model are shown in Figure 29. The forecasted runoff compare well to the observed runoff for all the runoff stations. The estimation values of the spatial correlation coefficient (r_s) shown in Table 47 indicate the fact that runoff stations R1 and R2 do not belong to the same stream where the corresponding rainfall stations P1 and P2 are located. This caused a reduction of the adequacy of the forecasted runoff values for the selected space-time transfer function model. It is a hydrological fact that rainfall causes runoff, therefore the forecasted performance of space-time TFN models describing rainfall-runoff relation can be improved by carefull selection of spatial lag system to represent hydrological characteristics of the rainfall- runoff process.

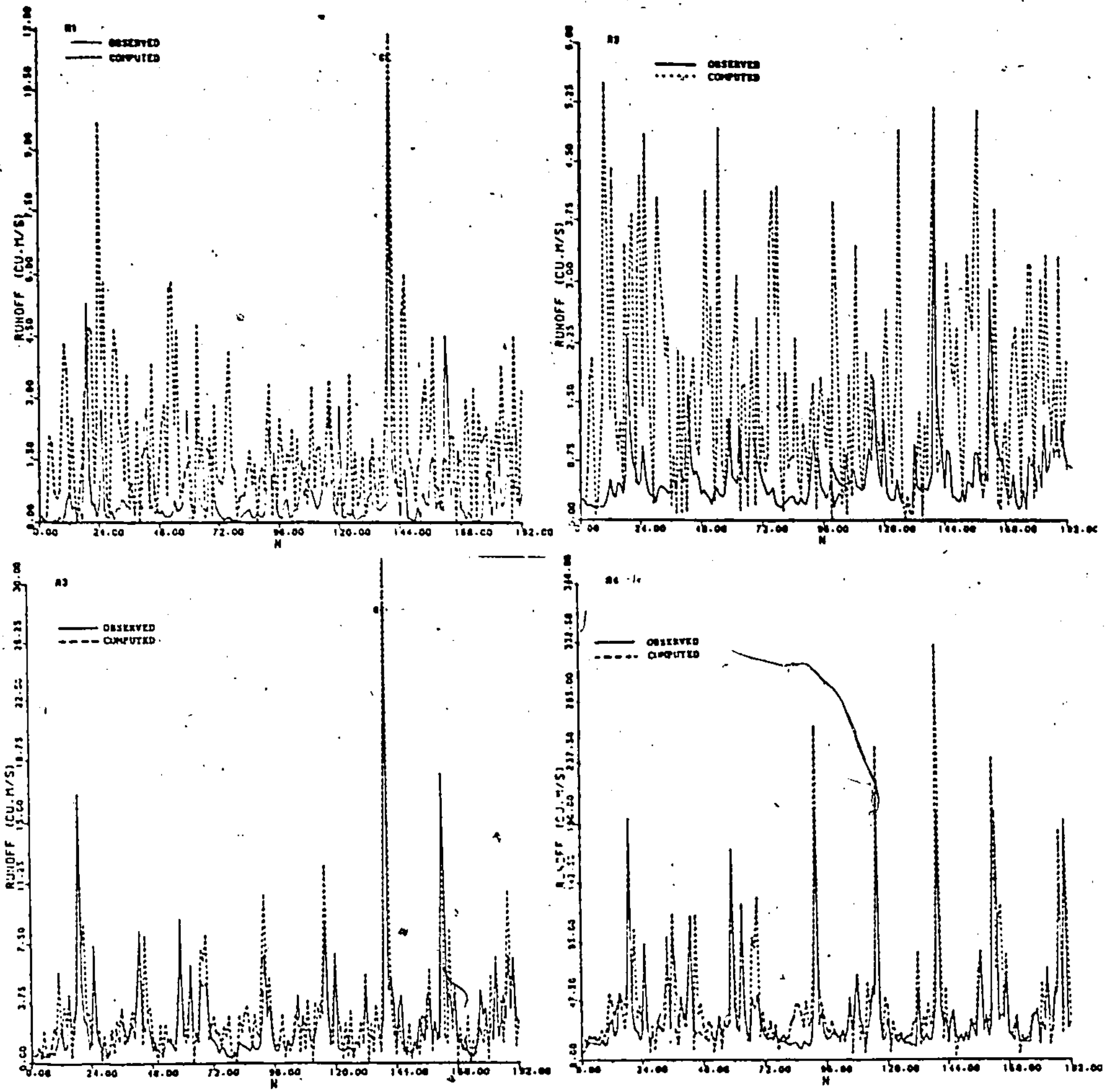


Figure 28: Observed and generated runoff values

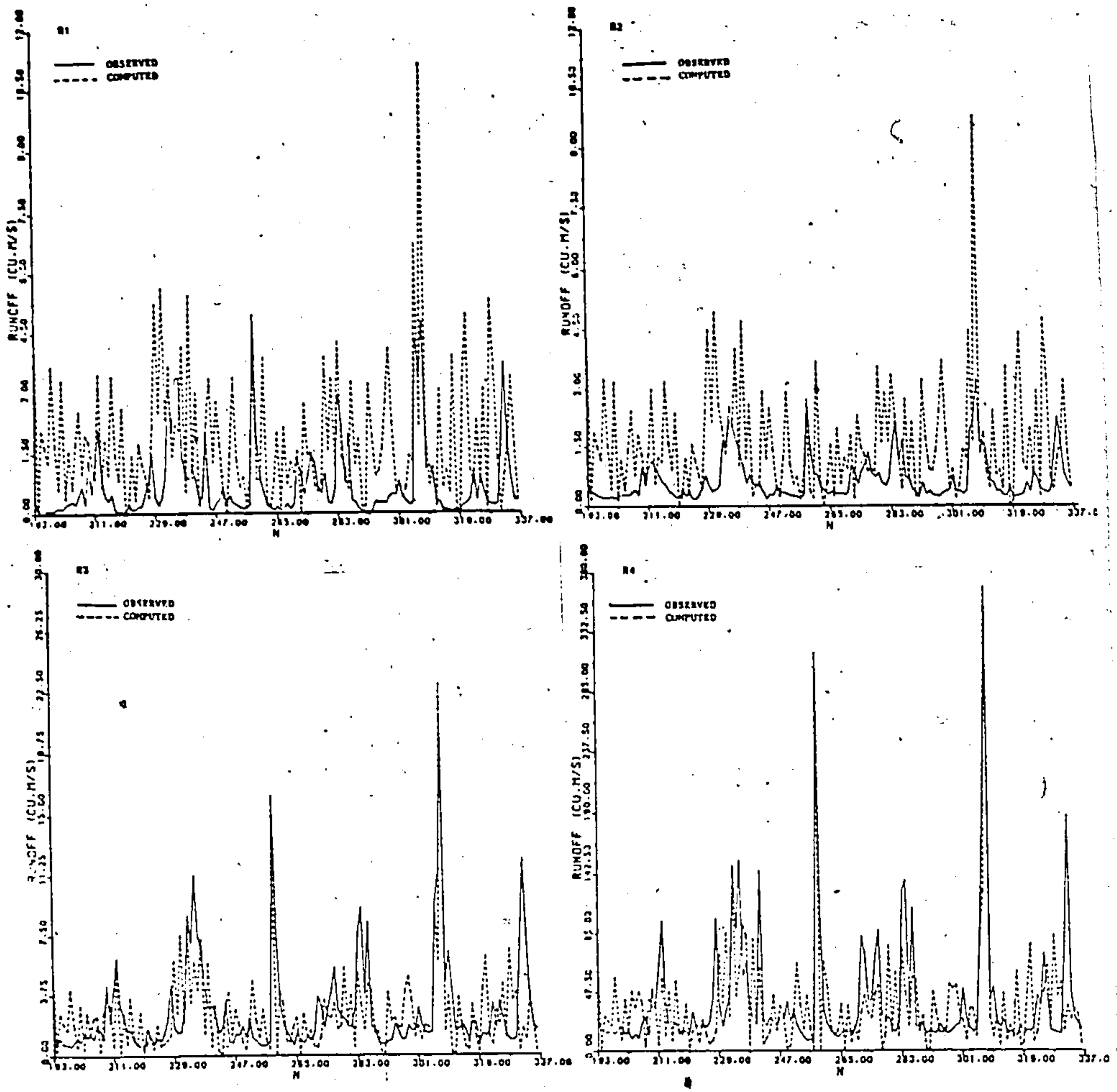


Figure 29: Observed and forecasted runoff values

Chapter VI

CONCLUSIONS

The general class of STARIMA models and the three iterative stages of the model building procedure (identification, estimation, and diagnostic checking) presented in this thesis performed ~~well in~~ describing the spatial characteristics of the rainfall data. They are useful towards modeling of hydrologic time series that exhibit spatial correlation. The STARIMA model building procedures are illustrated with spatial data for a system of four and eleven raingage sites located in the selected Watershed in Southern Ontario.

The comprehensive procedure presented in this work for building STTFN models is useful for detecting dynamic relationships between hydrological time series. It can be used as a forecasting tool in the field of water resources engineering. The STTFN modeling techniques were applied to model the rainfall-runoff process for a system of four rainfall and four runoff sites located in the Watershed in Southern Ontario.

Based on the results presented in this thesis, the following conclusions can be made:

1. The choice of spatial lag structure to reflect the influence of one zone on another can deeply affect

the form of the STACF and STPACF. This could lower the adequacy of the identified space-time model by producing spurious results and might also eliminate useful models from consideration.

2. Care should be given to the selection of rainfall and runoff series from the same watershed in order to avoid low correlations among the zones over space.
3. Although the rainfall series are stationary in time, the STACF and STPACF indicated the nonstationarity of the system in space. Therefore, the space-time system required a first difference in time.
4. The runoff series have shown the seasonality in the means, therefore the seasonal components were removed. Alternatively a general class of seasonal space-time model can be introduced to represent the seasonality of the runoff series.
5. The optimization methods of estimating parameters included in the OPTPAK package and specifically the Simplex method are a collection of relatively well-developed and computationally effective in estimating parameters in simulation studies.
6. For the selected watershed, the output (runoff) lags behind the input (rainfall) and the delay parameter was found to be 15 days or 1 lag in time.
7. The generated noise components present in the STMA and STTFN models are found to possess the character-

istics of a white noise. The forecasted noise were tested for whiteness and periodicities and found to be white and free of any periodic components.

8. The forecasting and generating performance of the STMA and STTFN models are found to compare well with the corresponding observed rainfall and runoff data.
9. The original contribution of the thesis is the development of a space-time precipitation system, which is STMA (1_2). Also the rainfall-runoff process was modeled by a STTFN (0,0,2,1,1) model.
10. 15-day data could be applied (or the developed models) to any simulation studies, design studies of water resources, synthetic data generation and forecasting of any selected steps ahead.

In conclusion, the forecasting performance of the STARIMA and the STTFN models could be improved by selecting good quality data with similar hydrological characteristics and belonging to the same watershed.

Chapter VII

SUGGESTIONS FOR FUTURE RESEARCH

The proposed method of constructing weighting schemes in this thesis is based on equal weights to agree with ordering the sites surrounding the location of interest. This system has been found to be efficient. An improvement could be expected if the weighting structure is selected based on physical and hydrological properties of the observed zones. However, this would require spatially collected data.

In this thesis, the method of Box and Jenkins(9) has been used for STTFN modeling of rainfall-runoff process. Further investigations can be made considering the method of L. Haugh (41). The basic idea of this method is to fit a different STARMA model to both the input and output separately and then follow the model building stages which are described in this thesis.

Finally, further studies can be made in the field of space-time modeling of rainfall-runoff process by considering a multi input-single output system. This approach will consist of fitting a STARMA model to the input. Then following the Box and Jenkins(9) combined transfer function-noise modeling technique.

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