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Flux Line Cutting and Cross-Flow in Tubes of High T_c Superconductors

by

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Thesis submitted

to

the School of Graduate Studies and Research

of

the University of Ottawa in partial fulfilment

of

the requirements for the degree

of

Doctor of Philosophy in Physics

Department of Physics, University of Ottawa,
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Abstract

We present strong evidence that helical flux lines can concurrently, enter and leave the wall of hollow cylinders of sintered high T_c superconductors at 77 K , hence cut and traverse each other. Tubes of two different materials , denoted BiSCCO and YBCO , have been studied.

The evidence for flux line cutting and cross-flow is examined in the perspective of basic concepts. The traffic of the flux lines is also described formally.

The penetration fields across the wall and into the grains of the sintered ceramic tubes are determined from their magnetic response to an applied axial field $H_{//}$ and from their flux trapping behaviour when subjected to two standard procedures , denoted H_{cool} and H_{cycle} . This provides information on $j_{c\perp m}$ and $j_{c\perp g}$, the intergrain and intragrain critical current densities for depinning of flux lines.

The penetration fields are seen to correlate with salient features (peaks and valleys) of the flux line cutting and cross flow phenomena. This enables us to claim that we are witnessing flux line cutting and cross flow in the weak link regime in the BiSCCO tube and in this regime as well as in the interior of the grains in the YBCO tube.

The Clem/Perez-Gonzalez phenomenological theory is exploited in a simplified framework to describe the crucial features of the data semi-quantitatively. This analysis confirms the above conclusions and provides estimates of $j_{c//m}$ and $j_{c//g}$, the critical current densities for intergranular and intragranular flux line cutting and their dependence on the magnetic flux density.

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I am deeply and sincerely grateful to my research supervisor, Prof. Dr. Marcel LeBlanc who trained and guided me through all the phases of the measurements, assisted me in their interpretation and provided extensive notes from internal seminars and background material which were invaluable in writing this thesis . Also I thank him for his patience and care in revising the successive drafts of the thesis . It is to recognize all of these contributions that the subject we (not the "royal" we) replaces the subject I in the text.

I am indebted to Sean X. Wang for assistance in developing the computer programme for the calculations exploiting the Clem/Perez-Gonzalez phenomenological theory.

My Ph.D. studies could not have been pursued without a leave of absence and a generous five year scholarship granted by Karadeniz Technical University in Turkey. My wife, my two children and I deeply appreciate this financial support and a partial waiver of tuition fees provided by the University of Ottawa.

I would like to thank to all my family and intimate friends , especially , my parents , my wife and my children for their patience , encouragement and moral support throughout our lives together.

I should not forget to thank all my teachers , friends and colleagues from my childhood till the present who contributed to my Ph.D. indirectly.

Dedication

I would like to dedicate my Ph.D. thesis to my father who passed away during my graduate studies in Canada.

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CHAPTER 1

Introduction

1.1. General Introduction

The property of type II superconductors, both classical and high critical temperature, to support large current densities in the presence of strong magnetic fields with negligible resistance has drawn considerable attention both for its technological importance and intrinsic interest. It is now well established that in these materials, electric fields arise and energy dissipation occurs when the driving Lorentz force $\vec{F}_L = \vec{J} \times \vec{B}$ causes the depinning and displacement of flux lines. It is visualized that, in the absence of thermal activation, a threshold or critical current density $j_{c\perp}(B, T)$ exists for the onset of depinning which can be written,

$$j_{c\perp} B = F_p(B, T) \quad (1.1)$$

where the subscript $\perp$ indicates that the current density flows perpendicularly to the local magnetic flux density $\vec{B}$ and $F_p(B, T)$ denotes the pinning force density characterizing the material. F_p is a function of the magnetic flux density and temperature. Flux line depinning and displacement gives rise to energy dissipation at a rate $dW_{\perp} / dt = E_{\perp} j_{c\perp}$ where the electric field is $\perp$ to $\vec{B}$.

It is also well established, although perhaps less well-known because of its secondary technological impact, that a threshold or critical current density

$j_{c//}(B,T)$ exists for the onset of contact and cross joining, hence "cutting" of neighbouring nonparallel flux lines. Various processes accompanying this phenomenon also give rise to an electric field $E_{//}$ and energy dissipation $dW_{//}/dt = E_{//}j_{c//}$. Here the subscript $//$ indicates that the electric field and current density are parallel to the flux line density.

$j_{c\perp}$ and $j_{c//}$, the critical current densities for flux line depinning and cutting, can be determined by the classical four probe technique and indirectly, but perhaps more accurately, from measurements of the magnetization of the specimen as an externally applied magnetic field $\bar{H}_a$ is impressed or removed. In the former approach, the voltage along the sample is monitored as a transport current I provided by an external current source is fed through the specimen via leads attached to its ends.

In the four probe arrangement, flux line cutting is made to occur when $\bar{H}_a$ is oriented with a component along the direction of transport current flow. In the magnetic method, the direction of the external field $\bar{H}_a$ is made to vary with respect to the coordinate axes of the specimen. The relative changes of orientation induce persistent currents to circulate with a component $j_{//}$ directed along the local magnetic flux density $\bar{B}$ in the body or bulk of the specimen. The changes of relative orientation can be accompanied by changes in the magnitude of $\bar{H}_a$. This is the situation which prevails in our work. We shall see that the simultaneous occurrence of changes in orientation and magnitude is experimentally convenient and simplifies the analysis of the phenomena.

Although an appreciable body of experimental evidence has been accumulated over the last two decades showing that flux line cutting is a real phenomenon, [4, 5, 6, 12, 13, 14, 27, 29, 30, 39, 40, 41, 42, 43, 56, 64, 67] the occurrence of this process remains controversial and not universally recognized [35, 47, 54, 55, 57]. Consequently it is desirable to provide additional

observations which support this idea. Further, various important ramifications of this concept remain to be explored. In this thesis we present new evidence confirming that flux line cutting takes place and examine some dramatic macroscopic consequences of this activity.

Our investigation has focused on high T_c superconductors since these materials are particularly fascinating but also because the phenomenon of flux line cutting in these materials remains largely unexplored. The granular nature of these ceramic materials however, introduces formidable complications in the quantitative analysis of the observations. Nevertheless, the qualitative messages emerge quite clearly from the data.

1.2. Outline of the Investigation

For our study we have selected hollow cylinder geometry. The principal reason for this choice is that the hole or cavity of a tube sample provides a finite reservoir. Inside this confined space, the longitudinal (axial) component of the magnetic flux density, denoted $\langle B_z \rangle_{\text{hole}}$ can be readily and continuously monitored. More importantly, the total magnetic flux

$$\Phi_{z,\text{hole}} = \langle B_z \rangle_{\text{hole}} \pi R_i^2 \quad (1.2)$$

embraced by the inner radius R_i of the wall of the tube is a quantity whose growth and diminution yields crucial information on the nature of the traffic of flux lines across the inner surface of the wall. Indeed it is the rise and fall of $\langle B_z \rangle_{\text{hole}}$ which will provide an unambiguous picture about the fascinating events taking place inside the wall.

In order to obtain a complete view of the cutting and cross flow of flux lines inside the wall we have also placed the "sample" tube concentrically (coaxially) inside another larger high T_c superconducting tube. In this arrangement, the annular space between the two tubes also constitutes a finite reservoir. In this second confined space, the longitudinal (axial) component of the magnetic flux density, $\langle B_z \rangle_{\text{annular}}$ can be readily and continuously monitored. More importantly again, the total magnetic flux,

$$\Phi_{z,\text{annular}} = \langle B_z \rangle_{\text{annular}} \pi(R_{i2}^2 - R_0^2) \quad (1.3)$$

enclosed in the annular space is also a quantity whose rise and fall gives vital information on the nature of the traffic of flux lines across R_0 , the outer surface of the "sample" (i.e. inner tube). Here R_{i2} is the inner radius of the larger tube.

To establish a simple initial configuration for the magnetic flux density $\bar{B}(r)$ permeating the sample we let it cool from T_c to the ambient temperature of 77 K in a stationary externally applied axial magnetic field $H_{//}$ (field cooling). In the arrangement where two coaxial tubes are present, the inner tube is made to cool first, followed by cooling of the outer tube.

Persistent currents are then induced to circulate in the wall of the inner tube with a component along the axial magnetic flux density $B_z(r)$ by applying an azimuthal magnetic field $H_\phi(r)$ via a toroidal magnet coil which embraces the wall of the inner tube. We stress that the outer tube does not experience this azimuthal magnetic field. $\langle B_\phi \rangle_{\text{wall}}$, the spatial average of the azimuthal magnetic flux density penetrating into the wall as H_ϕ is impressed, is continuously monitored. Also, as the application of H_ϕ induces persistent currents $\bar{j}(r)$ and attendant flux cutting activity in the wall when $j_{//}(r) \geq j_{c//}$,

we continuously record the concomitant evolution of $\langle B_z \rangle_{\text{wall}}$, the spatial average of the axial magnetic flux density in the wall.

The observation of a decrease of $\langle B_z \rangle_{\text{wall}}$ as H_ϕ is impressed is a clear message that flux line cutting is taking place in the wall. The concurrent appearance of a rise in $\langle B_z \rangle_{\text{hole}}$ (and a rise of $\langle B_z \rangle_{\text{annular}}$ if a second tube surrounds the first coaxially) is the signature of two-way flow of flux lines across each of the two surfaces (R_i and R_o) of the wall of the inner tube.

The Clem/Perez-Gonzalez phenomenological theory [18, 19, 59] is applied to the analysis of the evolution of $\langle B_z \rangle_{\text{wall}}$ and $\langle B_\phi \rangle_{\text{wall}}$ as H_ϕ is impressed. Although several simplifications and approximations are introduced in this formidable task, the model generates the main features of the observations and yields satisfactory quantitative agreement with the data. Consequently, an estimate of the intergrain $j_{c//}$ and its dependence on B can be extracted from our results. We believe that this is the first report on this quantity in granular high T_c superconductors.

After the experimental arrangement and procedure are described, we outline the background and conceptual framework of the investigation and sketch the existing theoretical picture. We then present and discuss our observations on tubes of two different high T_c superconductors separately. Finally, we summarize our results and conclusions and comment on various extensions of our preliminary study and new avenues of inquiry made accessible by our approach.

CHAPTER 2

Experimental Arrangement and Procedure

2.1. Introduction

Our purpose is to completely map out the evolution of magnetic flux configurations and the traffic of flux lines in the flux cutting regime in type II superconductors and particularly in high T_c superconductors. Singly connected, hollow cylinder geometry offers an ideal laboratory for such an investigation. Here the azimuthal demagnetization factor is zero and the hole of the tube acts as a finite reservoir for axial flux. To achieve our objective we need to measure several pertinent macroscopic quantities continuously and simultaneously.

Knowledge of the configuration of the magnetic flux in the wall of the tube and its evolution as the ambient magnetic fields are made to vary in magnitude and in direction requires the determination of two quantities. These are, $\langle B_z \rangle_{\text{wall}}$, the spatial average of $B_z(r)$, the axial magnetic flux density in the wall, and $\langle B_\phi \rangle_{\text{wall}}$, the spatial average of $B_\phi(r)$, the azimuthal magnetic flux density in the wall.

A complete accounting of the net magnetic migration of magnetic flux between the wall and the environment demands that the changes of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$, the spatial average of the flux density threading the inner and

annular reservoir be concurrently recorded together with the two quantities already mentioned. We describe below how these four measurements are accomplished.

To induce persistent currents $\bar{j}(r)$ to circulate with a component parallel to the flux density, $\bar{B}(r)$, in the wall, it is necessary that the orientation of the ambient magnetic fields $\bar{H}(R_i)$ and $\bar{H}(R_o)$ along the inner and outer surfaces of the inner tube be made to vary in direction. This is made to take place by superimposing azimuthal magnetic fields $H_\phi(R_i)$ and $H_\phi(R_o)$ on the axial magnetic fields $H_z(R_i)$ and $H_z(R_o)$. The former are provided by a toroidal magnet coil embracing the wall (see Fig. 2.1). The "baseline" for $H_z(R_i)$ and $H_z(R_o)$ is established by means of an externally applied static axial magnetic field denoted $H_{//}$.

We now describe the specimens, the monitoring systems, the sources of magnetic fields and the experimental procedures.

2.2. The samples

We have investigated flux line cutting and counterflow at 77 K in hollow cylinders of two different high T_c superconductors namely,



and



where $x=0.15$. These will be denoted BiSCCO and YBCO for brevity throughout our report. In our study on the first material the sample (i.e. the inner tube) was coaxially and concentrically placed inside a larger tube of the same material. Because we did not have second tube large enough to embrace the

YBCO tube (and the attendant paraphernalia), the measurements on this sample were performed on a "isolated" tube.

The BiSCCO tubes were provided by Dr. Vladimir Plechacek of the Institute of Physics, Academy of Sciences of the Czech Republic. The YBCO tube was purchased from HiTc Superconco. Two YBCO tubes of different sizes purchased from Seattle Specialty Ceramics (SSC Inc.) exhibited negligible intergrain persistent current at 77 K, hence trapped on insignificant amount of flux in the void. Consequently no useful investigation could be carried with these specimens.

The dimensions are listed below in centimeters.

<u>Sample</u>	<u>Inner diameter</u>	<u>Outer diameter</u>	<u>Length</u>
BiSCCO (1)	1.40	1.72	3.70
BiSCCO (2)	3.07	3.484	2.84
YBCO	1.50	3.00	5.00

2.3. Sources of $H_{//}$ and H_{ϕ}

The axial magnetic field $H_{//}$ provided by a copper wire solenoid 17.5 cm long, with inner and outer diameters of 4.0 and 5.5 cm for the winding (generating 377 G/A) is uniform within a few percent over the volume of the specimen.

The azimuthal magnetic field is generated by a toroidal magnet coil embracing the wall of the sample tube (see Fig 2.1). This coil consisted of 48 turns of 24 B&S copper wire for the BiSCCO sample and 36 turns of 24 B&S copper wire for the YBCO sample.

Care must be taken in constructing this toroidal coil to ensure that no net azimuthal turn is created in the process of winding. A resultant single azimuthal turn will generate an axial field component. This axial field , although relatively weak, will contribute (add or subtract) to $\langle B_z \rangle_{\text{hole}}$ and introduce an undesirable asymmetry in the observations. We circumvented this problem by winding half of the turns of the toroidal coil in a "clockwise" direction along the circumference of the wall of the tube and the other half of the turns in the opposite direction.

By Ampere's law,

$$H_\phi = \frac{nI}{2\pi r} \quad (2.1)$$

where n is the number of turns and I is the current carried by each turn. I is provided by a ripple-free battery-driven transistorized current source. The spatial average of the applied azimuthal magnetic field over the wall of the tube then reads,

$$\langle H_\phi \rangle = \frac{1}{(R_0 - R_i)} \int_{R_i}^{R_0} H_\phi(r) dr = \frac{nI}{2\pi(R_0 - R_i)} \ln \left[\frac{R_0}{R_i} \right] \quad (2.2)$$

We stress that by Ampere's law, the azimuthal field is confined to the annular volume embraced by an "ideal" toroidal coil. Consequently, $H_\phi(r) = 0$ outside the "ideal" toroidal coil. Hence, the outer tube which surrounds the "ideal" toroidal coil will not experience the azimuthal field. In practice, since the toroidal magnet coil is wound with circular wire and spaces exist between the windings, weak net "fringing" fields will exist in the close vicinity of the windings outside the toroidal coil. To minimize any effects of this leakage field on the wall of the outer tube, it is important that the inner surface of the latter be situated a distance Δr from the outer windings, where $\Delta r > d$, the

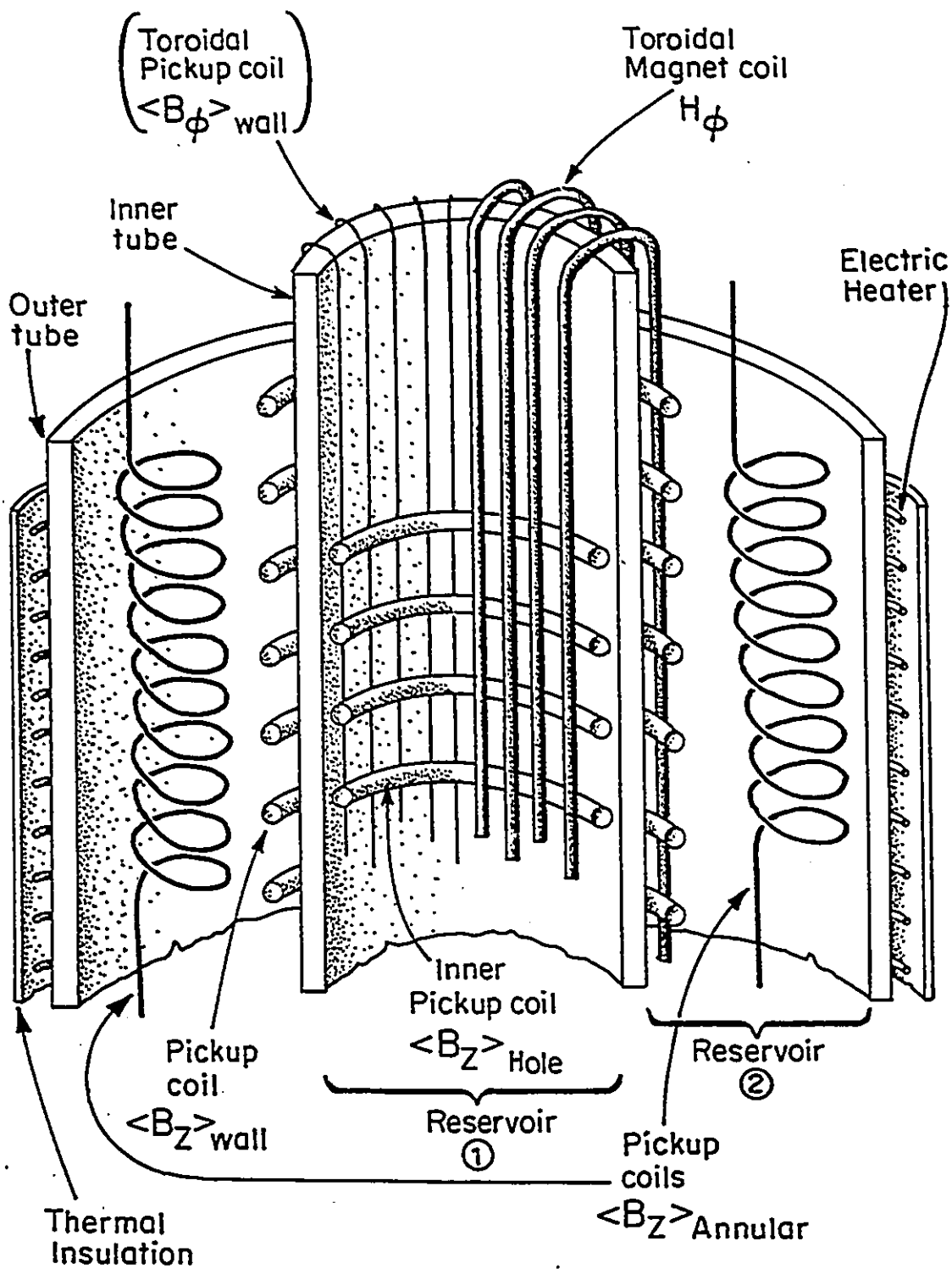


Figure 2.1 Schematic of experimental setup

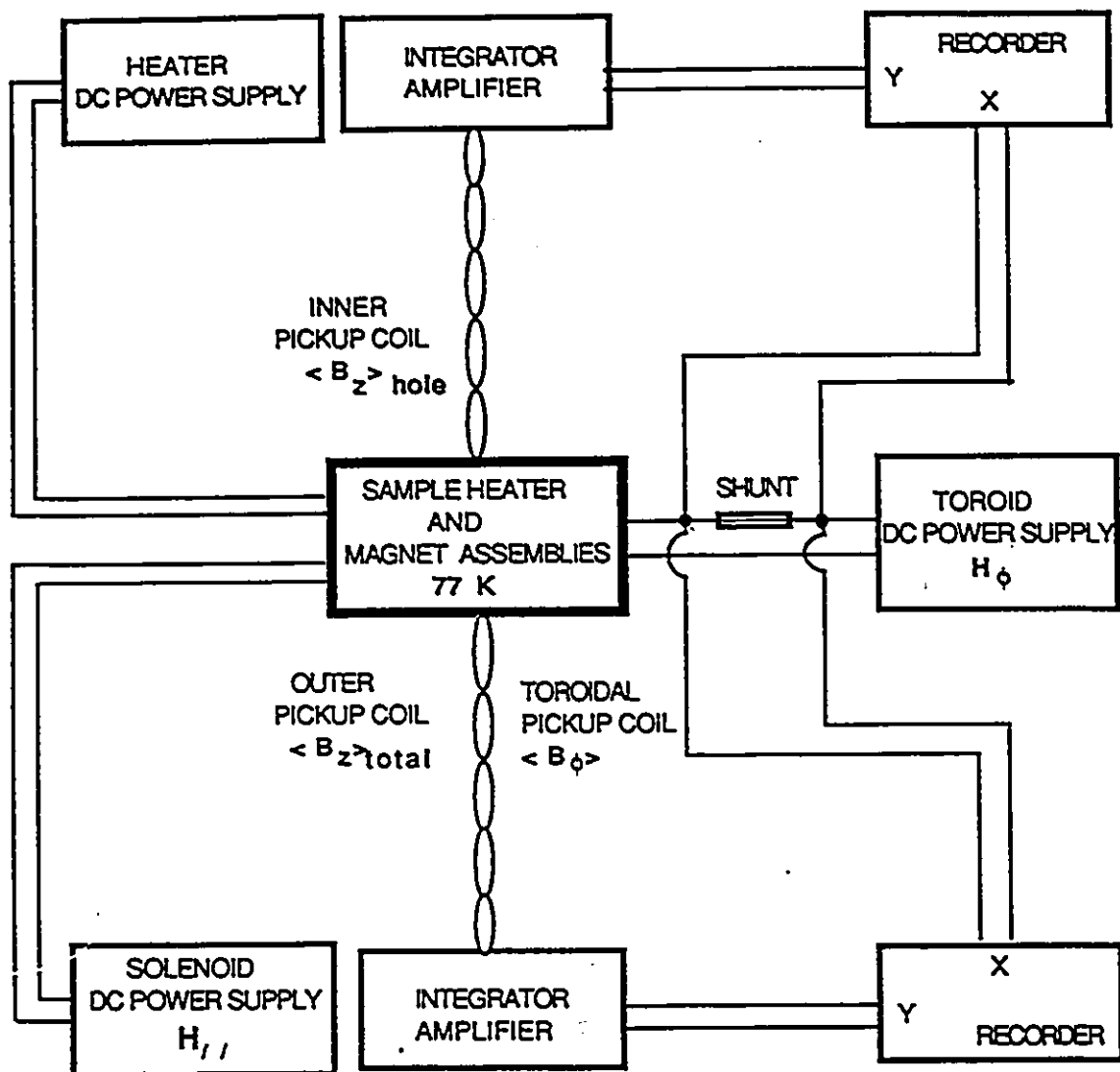


Figure 2.2 Schematic of the monitoring system.

separation between the wires of the toroidal coil. This requirement is well satisfied in our set up.

2.4. Monitoring of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{wall}}$, $\langle B_z \rangle_{\text{annular}}$ and $\langle B_\phi \rangle_{\text{wall}}$

The evolution of these four quantities is continuously monitored using four pickup coils which separately feed amplifier-integrators which drive the Y-axes of XY recorders (see Fig 2.2 for a schematic of the system). The X-axes can be driven by either by, (i) a signal proportional to H_ϕ or (ii) a signal proportional to $H_{//}$. These signals are each provided by a series shunt, (i) in the circuit of the toroidal magnet generating H_ϕ or, (ii) in the circuit of the solenoid generating $H_{//}$.

The location and "nesting" of these pickup coils and the toroidal magnet coil is shown schematically in Fig. 2.1.

The inner pickup coil monitoring $\langle B_z \rangle_{\text{hole}}$, the spatial average of the axial magnetic flux density threading the cavity of the inner tube, extends along the central half of the length of the inner tube. This coil comprised about 10,000 turns of 46 B & S copper wire

$\langle B_z \rangle_{\text{annular}}$, the spatial average of the axial magnetic flux density threading the annular space between the inner and outer tubes is detected by five series connected pickup coils extending along the entire length of the outer tube and arranged to optimally occupy the available annular space.

The determination of $\langle B_z \rangle_{\text{wall}}$, the axial magnetic flux density permeating the wall of the inner tube, is a composite measurement obtained as follows. A pickup coil embraces the waist of the inner tube and records $\Delta\Phi_{z,\text{total}}$, the change in the total axial magnetic flux threading this pickup coil. Since,

$$\begin{aligned}
\Phi_{z,\text{total}} &= \langle B_z \rangle_{\text{total}} \pi R_o^2 = \Phi_{z,\text{hole}} + \Phi_{z,\text{wall}} \\
&= \langle B_z \rangle_{\text{hole}} \pi R_i^2 + \langle B_z \rangle_{\text{wall}} \pi (R_o^2 - R_i^2)
\end{aligned} \tag{2.3}$$

(where R_i and R_o are the inner and outer radius of the inner tube), we can determine $\langle B_z \rangle_{\text{wall}}$ by subtracting the signal from the inner pickup coil, which detects $\Delta\Phi_{z,\text{hole}}$, from that of the outer pickup coil, either electronically or digitally.

A toroidal pickup coil, uniformly embracing the entire circumference of the wall of the inner tube monitors $\langle B_\phi \rangle_{\text{wall}}$, the spatial average of the azimuthal magnetic flux density permeating the wall. This pickup coil is surrounded by the toroidal magnet coil generating $\langle H_\phi \rangle$ (see Fig. 2.1). This coil comprised about 2,000 turns of 42 B & S copper wire.

2.5. Temperature Control

The data reported in this thesis was obtained with the specimens immersed in a bath of liquid nitrogen at atmospheric pressure hence at 77 K.

The temperature of the tubes can be raised from 77 K to T_c using electric heaters. Each heater is a noninductively (bifilar) wound single layer coil of 38 B&S manganin wire uniformly and directly embracing the entire external surface of the two concentric tubes. Where necessary, a thermal barrier of a few layers of masking tape was applied to minimize the heat input to the heaters needed to attain temperatures above T_c of the specimens. Because of lack of space and since these were not required in our work, no thermometric devices were employed to monitor the temperature. The heater currents necessary to raise the temperature of the specimens just above T_c was determined "magnetically". This means that we carefully established the

minimum electric heater current needed to completely destroy any flux shielding or trapping property of the specimens as either $H_{//}$ or H_{ϕ} were applied.

2.6. Calibration of the Detecting Systems

The signals from the inner and annular pickup coils monitoring $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ are calibrated by applying $H_{//}$ with the tube(s) maintained in the normal state.

The outer pickup coil monitoring $\langle B_z \rangle_{\text{wall}}$ is calibrated by applying $H_{//}$ to the virgin (zero field cooled) inner tube with the outer tube either absent or maintained above T_C . First, the range of $H_{//}$ for the initial linear regime of the magnetization of the inner tube is determined. According to accepted views, in this weak field range where $H_{//} < H_{C1}$, no magnetic flux is traversing the outer surface of the tube except into the relatively minute penetration depth region. With $H_{//}$ kept fixed at some arbitrary value in this range, the sample tube is driven into the normal state by means of the electric heater. The signal S_0 detected at this juncture by the outer pickup coil/integrator/amplifier corresponds to the entry of axial magnetic flux $\Phi_{z,\text{total}}$ into the wall and the hole, hence,

$$S_0 \propto \Phi_{z,\text{total}} = \mu_0 H_{//} \pi R_0^2 \quad (2.4)$$

The toroidal pickup coil monitoring $\langle B_{\phi} \rangle_{\text{wall}}$ is calibrated by applying $\langle H_{\phi} \rangle$ to the virgin (zero field cooled) inner tube. Again, here we determine the range of $\langle H_{\phi} \rangle$ below H_{C1} for the initial linear regime where no penetration of magnetic flux into the wall takes place except in the small penetration depth

region. Now with $\langle H_\phi \rangle$ held steady at some arbitrary value in this range, the wall is raised to T_c by means of the electric heater and is thereby penetrated by azimuthal flux, hence,

$$S_\phi \propto \Phi_\phi = \mu_0 H_\phi (R_o - R_i) L \quad (2.5)$$

where L is the length of the wall. The signal S_ϕ , detected at this juncture by the toroidal pickup coil/integrator/amplifier is proportional to Φ_ϕ and provides the desired calibration constant.

CHAPTER 3

The "Standard" Critical State Framework

3.1. Introduction

The properties of interest in our investigation of flux line cutting and cross-flow in high T_c type II superconductors are the following. (a) These materials can carry large persistent (or effectively persistent) currents, (b) they exhibit magnetic hysteresis and, (c) they expel some of the magnetic flux initially permeating their volume upon cooling from T_c in a static magnetic field (i.e. they show a small Meissner effect) . These properties of nonideal type II superconductors are linked. In this chapter we outline the basic picture of these properties and show how these phenomena are related.

3.2. The Meissner Effect in Ideal Type II Superconductors

The Meissner Effect in type II superconductors [1] is a consequence of the interaction of flux vortices (flux lines), (i) with each other and (ii) with an external magnetic field. We now examine these two interactions in a simple manner.

The superposition of the magnetic flux densities $\bar{B}_1(r)$ and $\bar{B}_2(r)$ and of the circulating vortex current densities $\bar{j}_1(r)$ and $\bar{j}_2(r)$ of two neighboring parallel flux lines augments the total magnetic and kinetic energies of the two isolated vortices by terms proportional to:

$$\int \bar{B}_1(r) \cdot \bar{B}_2(r) dv \quad \text{and} \quad \int \bar{j}_1(r) \cdot \bar{j}_2(r) dv \quad (3.1)$$

where the integrations extend over the volume of the specimen. Since these energies diminish as the separation of the flux lines increases, parallel flux lines repel each other. Consequently, in the absence of pinning and of an external magnetic field, flux lines initially threading a type II superconductor will expel each other from the specimen. Thus, the ideal (pinning free) type II superconductor traps no flux. Hence, the residual (remanent) magnetic moment is zero after an excursion of an externally applied field beyond H_{c1} .

An external magnetic field H_e tends to imprison flux lines inside a type II superconductor. This occurs because the interaction between the external magnetic field and the flux lines is one of mutual repulsion. This interaction can also be readily understood qualitatively. Let $B_M(r)$ denote the magnetic flux density in the penetration depth at the surface of the specimen and let $j_M(r)$ denote the Meissner current density associated with this flux penetration. The superposition of the magnetic flux and circulating vortex current densities of flux lines, $B_1(r)$, $B_2(r)$ etc. and $j_1(r)$, $j_2(r)$ etc. with $B_M(r)$ and $j_M(r)$ augments the total magnetic and kinetic energies of the system by terms proportional to:

$$\int \bar{B}_M(r) \cdot \bar{B}_1(r) dv \quad \text{and} \quad \int \bar{j}_M(r) \cdot \bar{j}_1(r) dv \quad (3.2)$$

As a consequence of these energy terms, the applied magnetic field H_e is said to exert a "pressure" on the lattice of flux lines and tends to confine the latter inside the specimen. The compression of the flux line lattice away from the surface by the action of this "pressure" reduces the overlap energy represented by eqn 3.2 but augments the superposition energy expressed by eqn 3.1. When $H \leq H_{c1}$, the mutual repulsion of the flux lines dominates and overcomes the effort at confinement exerted by the external magnetic field. Equilibrium is reached when the flux lines have all expelled each other from the pinning free specimen, i.e. when $\langle B \rangle = 0$ (perfect Meissner effect). Pinning, however, will assist H_e in imprisoning the flux lines in the material and in strong pinning materials, essentially no flux expulsion will occur. These materials will then exhibit negligible Meissner effect even when $H_e < H_{c1}$.

For the ideal (pinning free) specimen in an applied magnetic field in the range $H_{c1} < H_e < H_{c2}$, the equilibrium density of flux lines in the specimen is also determined by the balance between their mutual repulsion and the confining pressure exerted by H_e . This equilibrium is attained after a fraction of the flux permeating the sample in the normal state has been ejected. The fraction of flux lines which have to be expelled before equilibrium is reached diminishes as H_e approaches H_{c2} . Thus, the Meissner effect in the ideal, reversible specimen is always less than 100 % when $H_e > H_{c1}$ and diminishes monotonically to zero as H_e rises from H_{c1} to H_{c2} . When pinning is present, the Meissner effect is diminished from the ideal fraction and essentially vanishes in strong pinning samples.

The well known phase diagram and reversible magnetization curve displayed in Fig. 3.1 are macroscopic manifestations of the basic interactions outlined above.

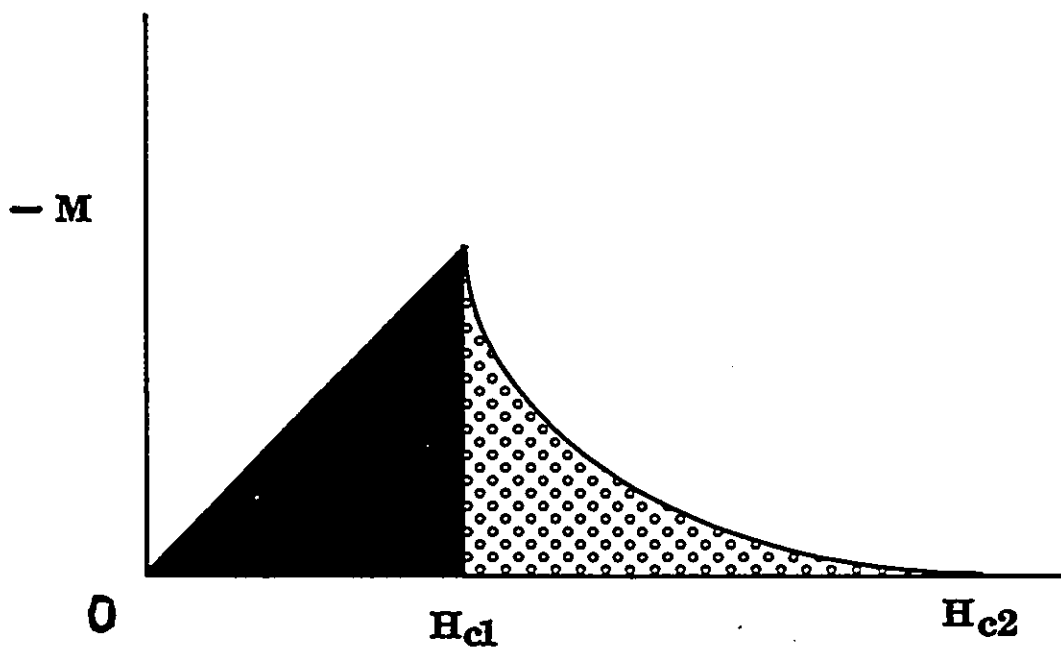
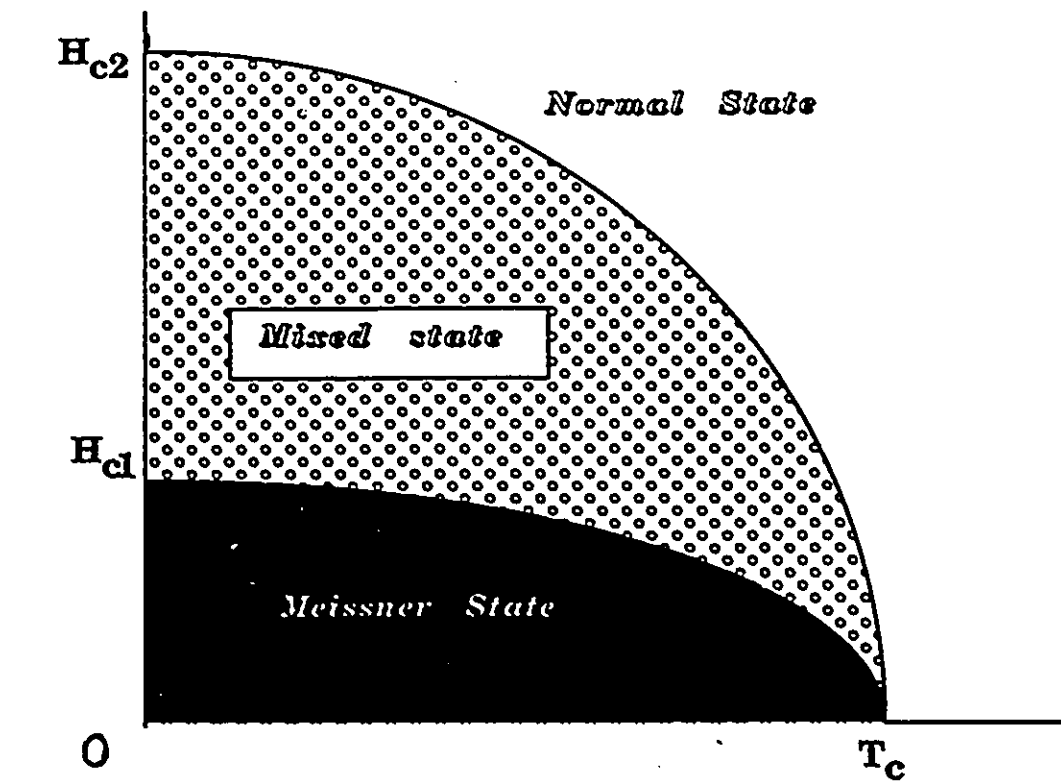


Fig 3.1 Phase diagram and magnetization curve of ideal type II superconductors.

Since in type II superconductors, the irreversibility (hysteresis) is a manifestation of pinning of flux lines, it can therefore serve as a measure of the pinning strength, i.e. of the density of the pinning sites or imperfections. Pinning is also intimately linked to the capacity of the material to support lossless persistent currents and hence a critical transport current I_c . We now outline the relationship between these various properties.

3.3. The Critical State Model

At equilibrium, a flux line in the interior of the pinning free material experiences no net force, being pushed equally in all directions by its neighbors. Here, interior means all of the volume beyond a few penetration depths from the surface. The flux lines adjacent to the surface also experience no net force, being repelled outwards by the flux lines in the interior but also by an equal but inwards force provided by the magnetic pressure exerted by the external magnetic field. In the event that the external magnetic field is now augmented, new flux lines are nucleated at the surface and penetrate the specimen until a new uniform equilibrium distribution is achieved. If, instead, the external field is lowered, thereby reducing the retaining magnetic pressure, flux lines migrate out of the sample until equilibrium is restored. We restrict our attention to rates of change such that viscous effects can be ignored and the adjustments in flux line density occur on a time scale that is negligible compared with that of the measurements.

To illustrate the effect of pinning on the scenario outlined above, we focus on several particular situations in succession. First, we consider that H_e is impressed from zero to a "virgin" specimen, i.e. a specimen containing no vortices, hence one that has just cooled from T_c to T in $H_e=0$. When H_e exceeds

H_{c1} , flux lines begin to nucleate and penetrate the specimen. If the specimen were pinning free (ideal), these flux lines would distribute uniformly throughout its cross-section and we would monitor $\langle M \rangle$ versus H_e displayed in Fig. 3.1 . Pinning forces however will oppose the penetration and migration of the flux lines. Consequently, the flux line distribution is not uniform and a critical gradient is established in the profile of the flux density.

It is instructive to examine the forces acting on a single flux line (or on a sheet of flux lines) in such a nonuniform configuration. We ignore the effect of the magnetic pressure from the external H_e since we regard the army of invading flux lines as having already penetrated a distance of several λ 's from the surface. Due to their mutual repulsion, each flux line experiences an inward (forward) push from the flux lines behind it and an outward (backward) push from the flux lines in front. A net force will arise however if the packing or concentration of flux lines varies along the path of advance, i.e. a flux line density gradient dB/dx develops. The net (repulsive) force acting on a stationary flux line which is on the threshold of being displaced is balanced by the maximum pinning force density, $F_p(B)$, furnished by the pinning sites. Any incremental advance by the army of flux lines requires that the pinning obstacles or impediments be surmounted over the entire volume occupied by the invading hordes of flux lines. Thus dB/dx must exist in a critical state over the entire space between the front and rear of the procession.

Alternatively and equivalently, the invasion of the flux lines can be viewed in the context of induced persistent currents. Over the range $0 < H_e < H_{c1}$, the Meissner screening current I_M restrains the entry of the applied magnetic induction to the penetration depth region. When H_e exceeds H_{c1} , the effective Meissner shielding current is now diminishing in magnitude and can no longer protect the interior against penetration by the applied

magnetic flux. Consequently, by Faraday's law of induction, $\epsilon = -d\Phi_M/dt$, persistent currents must now be generated in the body or bulk of the specimen to oppose the entry of the magnetic flux Φ_M . When the density j of such persistent currents exceeds a limit or ceiling, energy dissipation sets in. The induced current decays to the lossless critical value j_c . Concurrently, the induced emf ϵ with its associated electric field E diminish and disappear in accord with the Maxwell-Faraday equation:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3.3)$$

A configuration ensues where the profile of the magnetic flux density and the associated pattern of induced persistent currents are in a critical state. Thus the Maxwell-Ampere equation now reads:

$$\nabla \times \vec{B} = \mu_0 \vec{j}_c \quad (3.4)$$

which simplifies to

$$\frac{dB}{dx} = \pm \mu_0 j_c \quad (3.5)$$

for planar geometry where $\vec{B}$ is directed along the y-z plane.

In isotropic materials, the induced electric field $\vec{E}$ and the critical current density $\vec{j}_c$ are orthogonal to the flux line density $\vec{B}$, hence are denoted $E_{\perp}$ and $j_{c\perp}$. The critical current density $j_{c\perp}$ is linked to the threshold force for depinning of flux lines and is attained when the Lorentz driving force density,

$$\vec{F}_L = \vec{j} \times \vec{B} \quad (3.6)$$

surmounts the pinning force density $\vec{F}_p$.

The critical state can therefore be written

$$\bar{j}_c \times \bar{B} + \bar{F}_p = 0 \quad (3.7)$$

As the increase of H_e proceeds beyond H_{c1} , the advancing flux front eventually reaches the center of the specimen (the midplane for a slab or the axis for a cylinder). The corresponding H_e is called the first full penetration field and denoted H_* in this thesis. $\langle M \rangle_*$, the magnitude of the magnetization at this juncture as well as H_* provide complementary data from which $\langle j_{c\perp} \rangle$, the spatial average of the critical current density can be extracted.

The depinning critical currents dissipate energy at a rate, $\bar{E} \cdot \bar{j}_\perp$, when their density exceeds the critical value j_c . Their sense of circulation is entirely prescribed by the direction of the electric field as in normal metals. The crucial difference with normal metals is that these currents are lossless, hence persist, provided their density does not exceed $j_{c\perp}$.

We have now identified the three "ingredients" which govern the macroscopic magnetic behaviour of non ideal type II superconductors :

- The critical state concept that $j_\perp = j_{c\perp}$ wherever changes in magnetic flux density have been made to occur, hence electric fields have been generated.
- The laws of electromagnetic induction.
- The dependence of the magnitude of the diamagnetic Meissner current on H_e . In S.I. units, one can write,

$$I_M = \langle M \rangle_{ideal} \quad (3.8)$$

where I_M is the current per unit length of the surface along z and $\langle M \rangle_{\text{ideal}}$ is the reversible $\langle M \rangle$ versus H_e curve for the type II superconductor under scrutiny.

A careful application of these three prescriptions allows us to map out the vast spectrum of magnetization curves encountered in the study of real type II superconductors, be they classical or high T_c [2, 3, 10, 38].

3.4. Critical State in Granular High T_c Superconductors

Most bulk high T_c superconductors and the tubes we have studied are fabricated by sintering compressed powders of the chosen composition. These materials are not only polycrystalline but exhibit "granular" behaviour. It is now well established that in these materials, individual grains can carry large depinning critical current densities, denoted $j_{c\perp g}$. The subscript $\perp$ indicates that the depinning arises from the component of the current density $\vec{j}_{cg}$ which is transverse to the local magnetic flux density. The grains are very anisotropic and $\vec{j}_{cg}$ depends not only on its direction with respect to the crystal axes but also on the orientation of these axes with respect to $\vec{B}$.

Bombardment of the grains by fast neutrons, gamma rays and charged particles has been shown to appreciably enhance $j_{c\perp g}$, presumably by causing a variety of defects hence introducing suitable pinning sites for the flux lines.

The lossless current carrying capacity of the bulk specimens, hence the resultant critical conduction current density, denoted $j_{c\perp m}$ in this thesis, is considerably smaller than $j_{c\perp g}$ measured for the individual grains. This sad state of affairs is attributed to the interfaces, contacts and connections between the grains which are then regarded as weak links and bottlenecks to the flow

of current from grain to grain. Consequently $j_{c\perp m}$ is referred to as the intergrain critical current density . Considerable progress has already been achieved by materials scientists in enhancing $j_{c\perp m}$ by several orders of magnitude . This is accomplished in a variety of ways , generally by "texturing" the bulk material and by various schemes for augmenting the contact areas and improving the pinning properties of the interfaces .

The interface grid or contact network or interconnection structure of the agglomeration of grains need not be continuous. The net conduction or transport current as it flows along the bulk specimen meanders (percolates) from grain to grain across some of the weak link "cement". In any case the current need not penetrate deeply into the volume of the grains since $j_{c\perp g} \gg j_{c\perp m}$. Thus the "grid" , "network" , "skeleton" or "matrix" of the entire specimen may be filled with a critical current density $j_{c\perp m}$ while most of the interior volume of the grains does not participate in supporting the flow of the conduction current .

We can visualize such a situation in a variety of ways. For instance we can regard the specimen as a sponge . This analogy was exploited in a different perspective by Mendelssohn before the discovery of type II superconductivity . In our picture, the body of the sponge consists of the superficial volume of the grains together with their interconnections . The interior volume of the grains are associated with the pores or hollows of the sponge . In this thesis we will generally use the expression "matrix" to refer to the skeleton or body of this sponge , hence the subscript "m" in our notation.

We now apply the critical state model to the specimen we have just described , i.e. a matrix in which grains (interior volume of the grains) are embedded . We note that the basic ideas we outline below are prevalent

throughout the pertinent literature and have been independently proposed and developed by numerous workers [16, 17, 23, 24, 25, 28, 31, 32, 51, 75].

A magnetic field applied to the external boundaries of a virgin (zero field cooled) specimen will penetrate into the matrix in the manner we have already sketched in the above discussion of the critical state. We will denote this magnetic field and magnetic flux density,

$$H_m(x) = B_m(x)/\mu_0 \quad (3.9)$$

For planar geometry, Maxwell's eqn incorporating the critical state then reads,

$$\frac{dH_m}{dx} = \pm j_{c1m}(B(x)) \quad (3.10)$$

The new feature which will enter in our discussion in later chapters relates to the response of the individual grains (i.e. the interior volume of the grains) to the changes of the magnetic field $H_m(x)$ along their surfaces. Since $H_m(x)$ varies slowly on the scale of the lateral dimension of the grains, its strength is generally regarded uniform along the boundaries of the individual grain.

The critical state ideas outlined above are then applied separately to the individual grains. Clearly to pursue these concepts quantitatively, several approximations and simplifications will have to be introduced.

We hope that this brief introduction to the basic framework will prove useful to the reader when these ideas are applied to specific situations later in this thesis.

CHAPTER 4

Flux Line Cutting and Cross-Flow. Qualitative Picture

4.1. Introduction

The concept of flux line cutting was introduced to account for the observation of a flux-flow voltage along a straight wire of a type II superconductor carrying a steady current $I > I_c$ in a static longitudinal magnetic field $H_{//}$ [56]. When $H_{//}=0$, the azimuthal flux lines generated at the surface of the wire by the longitudinal current I close on themselves and form vortex rings. Under the influence of the Lorentz force and line tension energy, the rings shrink radially and collapse on the axis.

The vector superposition of an externally applied axial magnetic field $H_{//}$ and the azimuthal magnetic field H_ϕ generated by the conduction current $I > I_c$ causes helical flux lines to nucleate and enter at the surface of the wire. A radially inward migration of these helical flux lines is required to account for the generation of a longitudinal electric field E_z , hence a flux flow voltage when the static current exceeds I_c . The Josephson eqn, $\vec{E} = -\vec{v} \times \vec{B}$, now reads,

$$E_z = -v_r(r)B_\phi(r) \quad (4.1)$$

where v_r is the radial velocity of the flux lines and $B_\phi(r)$ is the azimuthal magnetic flux density.

A continuous creation and inward radial migration of helical flux lines will , however , lead to an increasing accumulation of longitudinal magnetic flux at the core of the wire , in disagreement with observations and the steady-state condition .

The idea of flux line cutting was put forward by several workers to resolve this paradox. Josephson in 1966 [35] and recently, Nelson and co-workers [47, 54, 55] and Obukhov and Rubinstein [57] , have however estimated flux line cutting to be too energetically costly. Consequently these authors have focused on various other schemes whereby entangled configuration of flux lines can disentangle . The debate on the energy barrier for this process continues [7, 11, 44, 60, 62, 66]. Meanwhile evidence that this phenomenon indeed takes place continues to grow . The observations we report in this thesis provide additional evidence that this mechanism operates both in the "matrix" and in the interior of the grains of polycrystalline high T_c superconductors.

Theoretical investigations by Brandt et. al [9] and Clem and Yeh [20] of the basic interactions between flux lines reveals that as the angle $\Delta\theta$ subtended by adjacent sheets of flux lines is augmented , their mutual repulsion diminishes monotonically and an attractive interaction sets in when a critical angle $\Delta\theta_c$ is attained . This turn of events should not be surprising since it is well known that anti-parallel flux lines attract and will annihilate each other if unrestrained by pinning forces [10] . Clem [13] has also shown that a flux line , $\hat{z}\Phi_0$ immersed in a conduction current I , flowing along its axis and in the $\hat{z}$ direction is unstable. If any segment of the flux line experiences a fluctuation or perturbation towards a left-handed helix , the flux line will expand radially to the surface of the wire carrying the current .

These "ingredients" provide the basis for two complementary scenarios proposed by Brandt [8] and Clem [18] for flux line cutting and cross-flow on a macroscopic scale . These two scenarios have been labelled , the single and double flux line cutting regimes and are now outlined.

4.2. Single Flux Line Cutting and Cross-Flow Scenario

We consider two adjacent sheets of flux lines subtending an angle $\Delta\theta_{12} > \Delta\theta_c$ with respect to each other. In order that such a configuration occurs the two sheets must be immersed in a pattern of current $j/(x)$ flowing along the flux lines . The sequence of events is portrayed schematically in Fig. 4.1. The two sheets are drawn side by side in Fig. 4.1(a) but should be visualized as superimposed and lying one above and one below the plane of the paper. Since $\Delta\theta_{12} > \Delta\theta_c$, the two sheets attract each other. Consequently, they will move towards each other under the influence of their mutual attraction, making contact at an array of points of intersection as displayed in Fig. 4.1(b). At these points of contact, the flux lines temporarily fuse and form doubly quantized regions. The resulting regions are energetically expensive, hence unstable and cannot persist. Consequently the unions will break up with segments from one sheet now attached to the adjacent segments from the other sheet. This situation is sketched in Fig. 4.1(c). The process where two lattices of flux lines intersect and cross join is called flux line cutting or vortex reconnection.

The immediate result of the vortex reconnection is a dense sheet of zig zag flux lines as depicted in Fig. 4.1(c). This zig zag configuration is energetically unfavorable and the pattern straightens out to reduce the line tension energy (see Fig. 4.1(d)). Since the flux lines become shorter as they straighten, the

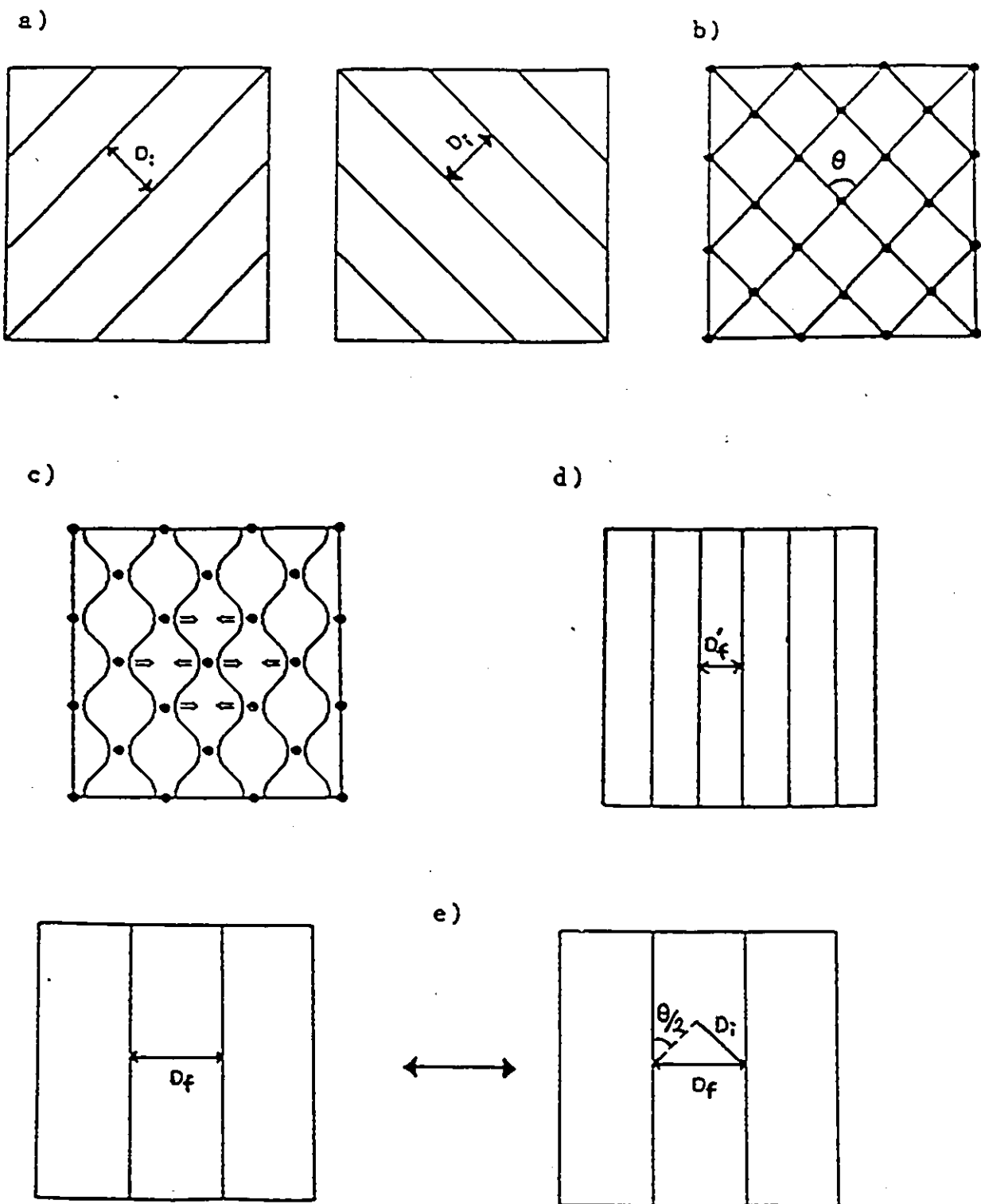


Fig 4.1 Schematic of single flux line cutting process.

volume of flux, $\Phi_v = \int \Phi_0 dL$ is consequently diminished. Here Φ_0 is the flux quantum and dL is an element of length along the flux line. Flux is said to be consumed by the process of flux line cutting since the total volume amount of flux is reduced. In another perspective, we note that it is the transverse component of the flux line density of the initially separate sheets which has been made to vanish. The number of flux lines, however, is conserved in the process just described.

The ensuing densely packed sheet of flux lines shown in Fig. 4.1(d) is unstable because of the large mutual repulsion. Consequently, these separate into two adjacent sheets which occupy the volume initially filled by the two sheets before their merger. The separated sheets are displayed side by side in Fig. 4.1(e) but should again be viewed in three dimensions, with one sheet above and the other below the plane of the paper.

We note that D_f , the final separation between the flux lines in each of the resulting two sheets is larger than the original separation D_i , although the total number of flux lines is conserved. Simple trigonometric considerations lead to the relation,

$$D_f = \frac{D_i}{\cos(\Delta\theta/2)} \quad (4.2)$$

The motion of flux lines or of flux line segments dissipates energy since a force $\bar{F}$ (whether it is repulsive or attractive) acts along the displacements $d\bar{s}$. Therefore energy is dissipated when, (i) the sheets of flux lines approach each other, (ii) the zig zag configuration straightens out and (iii) the ensuing densely packed sheet of flux lines separates into two sheets. Process (i) can be shown to generate a net electric field $E_{//}$ directed upwards in Fig. 4.1, hence

parallel to $j_{//}$. The energy dissipation can also be viewed in terms of $\int dt \int E_{//} j_{//} dv$.

We now visualize that the two sheets of flux lines whose interaction history we have just chronicled are situated at the edge or frontier of a lattice of flux lines consisting of sheets initially subtending an angle $\Delta\theta_{ij} = \Delta\theta_c$ with respect to their nearest neighbours. Clearly then after sheets $i=1$ and $j=2$ which we have just looked at, have undergone the process described above, $\Delta\theta_{23}$, the angle subtended by sheets 2 and 3 is now overcritical since it has increased from $\Delta\theta_c$ to $3/2\Delta\theta_c$. Hence, sheets 2 and 3 will go through the ritual just outlined. As a consequence, $\Delta\theta_{34}$ now becomes overcritical and so on. The disturbance in orientation of the flux lines thereby propagates throughout the volume of a lattice initially at the critical angular threshold, causing a change of orientation of the flux lines throughout the critical region. After the passage of each "wave" of angular adjustment, things quiet down until a new perturbation is introduced at the surface of the specimen by an increment $\Delta\bar{H}_c$.

In our experiment, the external helical magnetic field,

$$\bar{H}_c = \bar{z}H_{//} + \hat{\phi}H_{\phi} \quad (4.3)$$

is made to change both its magnitude and orientation by slowly varying H_{ϕ} . We note that an increment ΔH_{ϕ} will induce persistent currents to flow along $H_{//}$, hence with a component along $\bar{H}_c$.

When the events we have described take place along a surface of the specimen, the sheet of flux lines on the left of Fig 4.1 (a) enters the specimen whereas that on the left of Fig 4.1(e) may exit from the surface. This state of affairs has been labelled "breathing" mode.[8, 18] The entry (nucleation) of the flux lines constitutes the "breathing in" phase and the release of the flux lines

corresponds to the "breathing out" phase. The observations we report later in this thesis provide strong evidence that a breathing mode , hence two way or counterflow of flux lines does indeed occur as $\bar{H}_c$ is varied .

4.3. Double Flux Line Cutting and Cross-Flow Scenario

For aid in visualizing this process we refer the reader to Fig 4.2 where steps (a) , (b) and (c) are identical to that described in (a) , (b) and (c) of Fig 4.1. [18]

The crucial difference is that in Fig 4.2 (c) the current density $j_{//}$ in which the flux lines are immersed is sufficiently strong so as to trigger the helical instability we have mentioned in the introduction above. Careful consideration of the direction of the Lorentz Force $\vec{j} \times \hat{\Phi}_0$ on the different segments of the zig zag flux lines shows that the flux lines will now distort to form left-handed helices (see Fig 4.2 (d)) which will then expand radially also under the influence of the Lorentz force as shown in Fig 4.2(e). As neighboring helices expand they eventually touch , fuse at contact points and cross-join i.e. undergo vortex reconnection again. Hence, the label double flux line cutting.

The subsequent straightening out of the two sheets of "corrugated" flux lines ensuing from Fig. 4.2(e) leads to the configuration depicted in Fig. 4.2(f). A comparison of the initial situation (Fig 4.2(a)) and the final situation (Fig 4.2(f)) reveals that two adjacent sheets have exchanged position via the double flux cutting mechanism.

Again when these events take place at the surface of the specimen we can envisage a "breathing" mode comprising a "breathing in" phase (Fig 4.2(a)) and a "breathing out" phase (Fig 4.2(e)).

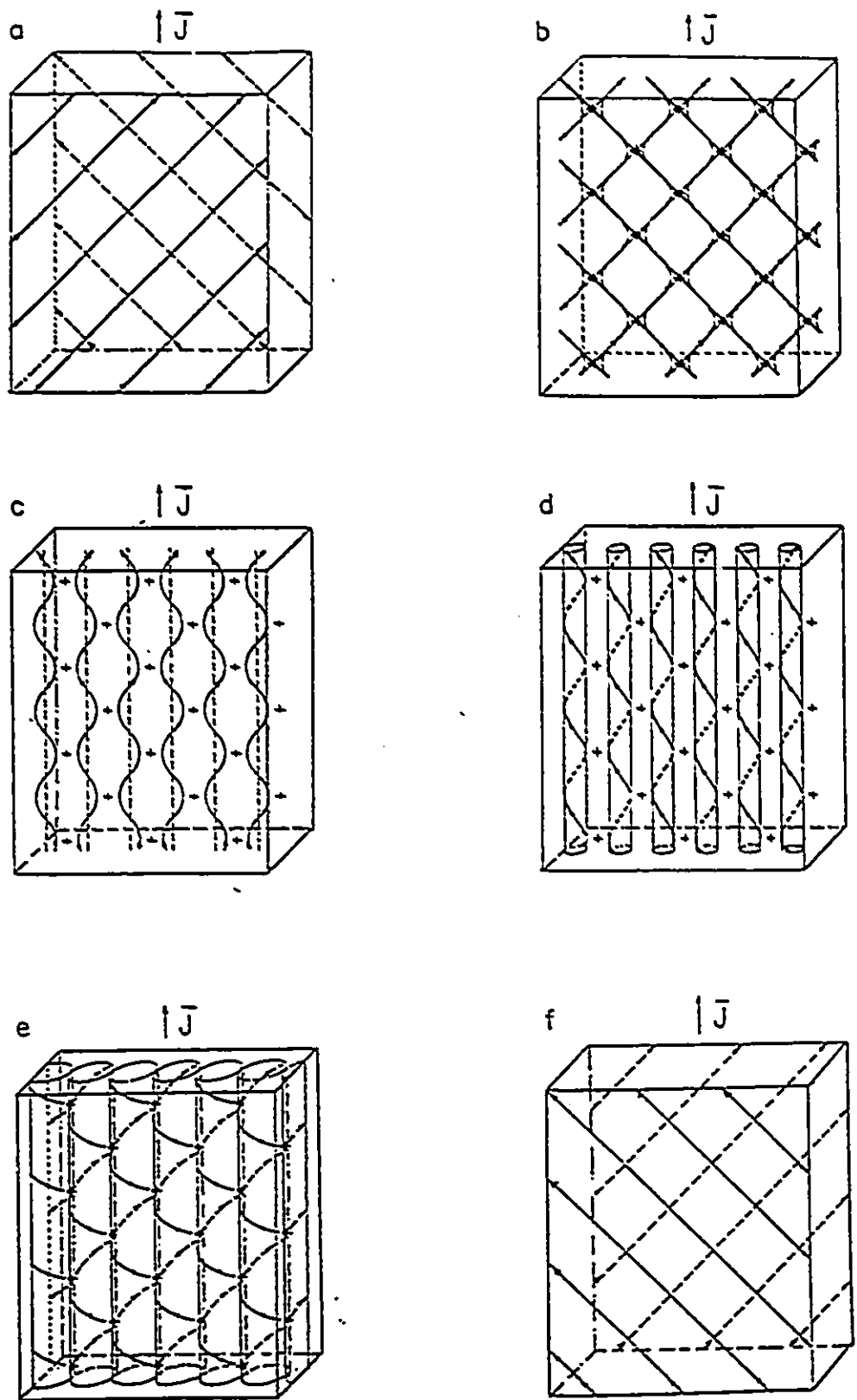


Fig 4.2 Schematic of double flux line cutting process.

In our experiments we cannot distinguish between these two proposed scenarios. Our observations however , show unambiguously that in some circumstances , the reservoirs and wall of the tube are "exchanging" flux lines . In some instances this "exchange" or two-way flow or cross flow or counter-flow is seen to cause a drop in the axial flux density in the wall although the increase in the magnitude of $\bar{H}_{\text{ext}}$ is causing the azimuthal magnetic flux density in the wall to increase.

CHAPTER 5

The Generalized Critical State Model

5.1. Introduction

Clem and Perez-Gonzalez [19] [59] have developed a generalized critical state model which incorporates flux line depinning , hence the classical Bean-London-Kim-Anderson [2, 3, 38, 46] "standard" critical state concept , as well as flux line cutting. This phenomenological framework is based on Maxwell's equations. In this chapter we present a brief outline of this theory.

5.2. The Clem/Perez-Gonzalez Phenomenological Theory

Two critical currents are now envisaged to play a role in type II superconductors:

- $j_{c\perp}$, flowing transverse to the flux lines is needed to overcome the opposition of pinning forces to the migration of the flux lines,
- $j_{c//}$, flowing along the flux lines is needed to drive non parallel flux lines to merge and intersect.

When $j_{\perp} > j_{c\perp}$ an electric field, denoted $E_{\perp}$, is generated by the transport of the flux lines. This feature was already envisaged in the classical critical

state concept. When $j_{//} > j_{c//}$, and flux cutting occurs, an electric field, denoted $E_{//}$, is generated along the flux lines. This electric field arises from the displacement, straightening and other contortions experienced by the flux lines as the sequence of events associated with the flux cutting process are unfolding. Here $E_{//}$ is regarded as a net electric field averaged over the duration of the flux cutting scenario and over the volume of the flux cutting event.

Introducing these ideas into Maxwell's equations, $\nabla \times \bar{B} = \mu_0 \bar{j}$ and $\nabla \times \bar{E} = -\partial \bar{B} / \partial t$, for infinite cylindrical geometry leads to:

$$\mu_0 j_{\perp} = - \left(\frac{\partial B}{\partial r} + B \frac{\sin^2 \theta}{r} \right) \quad (5.1)$$

$$\mu_0 j_{//} = B \left(\frac{\partial \theta}{\partial r} + \frac{\sin \theta \cos \theta}{r} \right) \quad (5.2)$$

$$\frac{\partial B}{\partial t} = - \frac{\partial E_{\perp}}{\partial r} - E_{\perp} \frac{\cos^2 \theta}{r} - E_{//} \left(\frac{\partial \theta}{\partial r} + \frac{\sin \theta \cos \theta}{r} \right) \quad (5.3)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{B} \left[\frac{\partial E_{//}}{\partial r} + E_{//} \frac{\sin^2 \theta}{r} - E_{\perp} \left(\frac{\partial \theta}{\partial r} - \frac{\sin \theta \cos \theta}{r} \right) \right] \quad (5.4)$$

where $\theta(r)$ is the angle between the flux density $\bar{B}(r)$ and the z axis. We stress that $E_{\perp} = 0$ when $0 \leq j_{\perp} \leq j_{c\perp}$ and $E_{//} = 0$ when $0 \leq j_{//} \leq j_{c//}$.

In this scheme, energy dissipation is visualized to occur through flux migration, $E_{\perp} j_{\perp}$, and through flux cutting, $E_{//} j_{//}$. Regions where only the former is occurring are denoted **T** zones (flux transport zones), and where only the latter is occurring are denoted **C** zones (flux cutting zones). Regions where both processes are taking place simultaneously are denoted **CT** zones (flux cutting and flux transport zones). Regions where no energy dissipation occurs

($E_{\perp}=0$ and $E_{\parallel}=0$) are called **O** zones. We stress that in **T** zones, $j_{\parallel}$ need not vanish but can lie in the range $0 \leq j_{\parallel} \leq j_{c\parallel}$ and can vary with time. Here flux lines are migrating while retaining their orientation. The important and novel features which now emerge are the following:

- the appearance of **CT** zones,
- the occurrence of pure flux cutting domains (**C** zones), and
- the possibility that in **C** zones $j_{\perp}$ does not have to vanish but can lie in the range $0 \leq j_{\perp} < j_{c\perp}$ and be allowed to vary with time.

Regions where $j_{\perp}$ was subcritical ($0 < j_{\perp} < j_{c\perp}$) were forbidden in the framework of the classical critical state.

In chapter 7 and 9 we present our observations of flux line cutting and cross-flow and examine these in the framework of the theory we have outlined above.

CHAPTER 6

Properties of the BiSCCO Tube

6.1. Introduction

The most dramatic evidence for flux line cutting and cross-flow in the high T_c materials we have investigated emerged from our study of these phenomena in the BiSCCO tube, hence we will present our observations on this specimen first. In a subsequent chapter we will describe our observations on the YBCO tube where the behaviour is very similar to that of the BiSCCO tube but where "end" or "geometry" effects combined with the granular behaviour obscure some aspects of the flux cutting and cross-flow responses.

For each specimen, before we proceed to report on flux line cutting and cross-flow behaviour, it is useful however to first determine the standard properties of these granular materials, namely, $j_{c\perp m}$ and $j_{c\perp g}$, the intergrain and intragrain critical current densities for flux line depinning and their dependencies on the magnetic flux density B . We obtain estimates of these quantities and insight into their variation with B from various complementary and related magnetic measurements. In one approach we monitor the evolution of the magnetic flux in the tube versus slow sweeps of $H_{\parallel}$. In another approach we focus on flux trapping behaviour. The latter method provides data only on the low field values of $j_{c\perp m}$ and $j_{c\perp g}$. That is sufficient for our purpose, since we examine flux line cutting and cross-flow in the low field range only.

6.2. Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{total}}$ vs $H_{//}$

For a hollow cylinder, $j_{c\perp m}$, the intergrain critical current density for depinning, is traditionally and most readily and unambiguously measured by monitoring the evolution of $\langle B_z \rangle_{\text{hole}}$, the axial magnetic flux density in the cavity as the tube is subjected to slow sweeps and cycles of an applied axial magnetic field $H_{//}$. Typical traces of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$ for the BiSCCO tube at 77 K are displayed in Fig 6.1 for the weak field range of interest in our study.

As $H_{//}$ is first impressed to the "virgin" (zero field cooled) tube in the superconducting state, $\langle B_z \rangle_{\text{hole}}$ remains "zero" since the persistent currents, induced to circulate azimuthally around the circumference of the wall by Faraday's law, completely screen the cavity from penetration by the applied magnetic flux. We refer the reader to Fig 6.2 for corresponding sketches of the advancing $B(r)$ profiles. (The vertical or z component of the earth's field threading the hole of the tube is negligible on the scale of these measurements).

The rise of $H_{//}$ eventually generates critical intergrain screening currents which fill the entire thickness of the wall. Concomitantly, the front of the applied magnetic flux penetrates from R_0 , the outer radius, to R_i , the inner radius of the wall of the tube as shown in Fig 6.2. $\langle j_{c\perp m} \rangle$, the spatial average of the intergrain depinning critical current density is obtained by exploiting Ampere's law

$$\oint \vec{H} \cdot d\vec{L} = IL \quad (6.1)$$

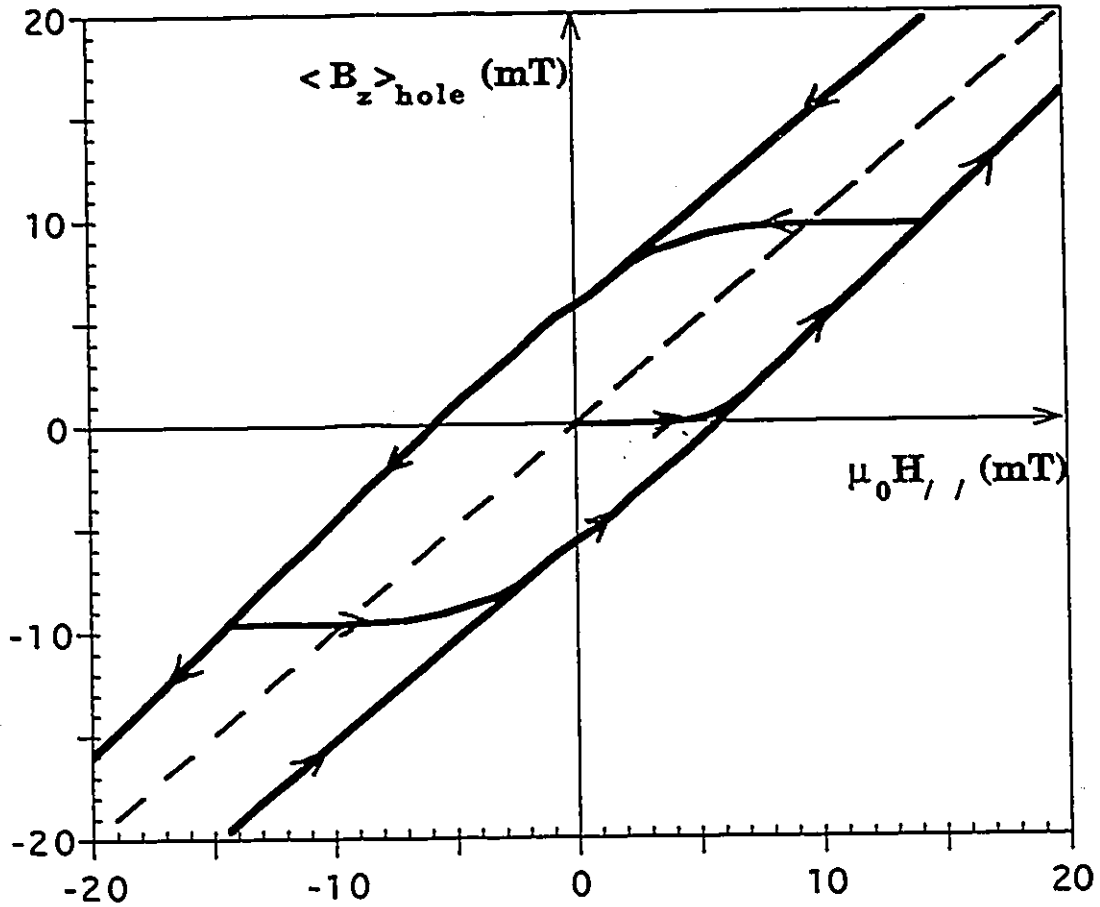


Fig 6.1 Evolution of the axial magnetic flux density in the cavity of the BiSCCO tube as $H_{//}$ is impressed and varied along and between the "envelopes" of the magnetization curve. The dashed (45°) line is for the normal state

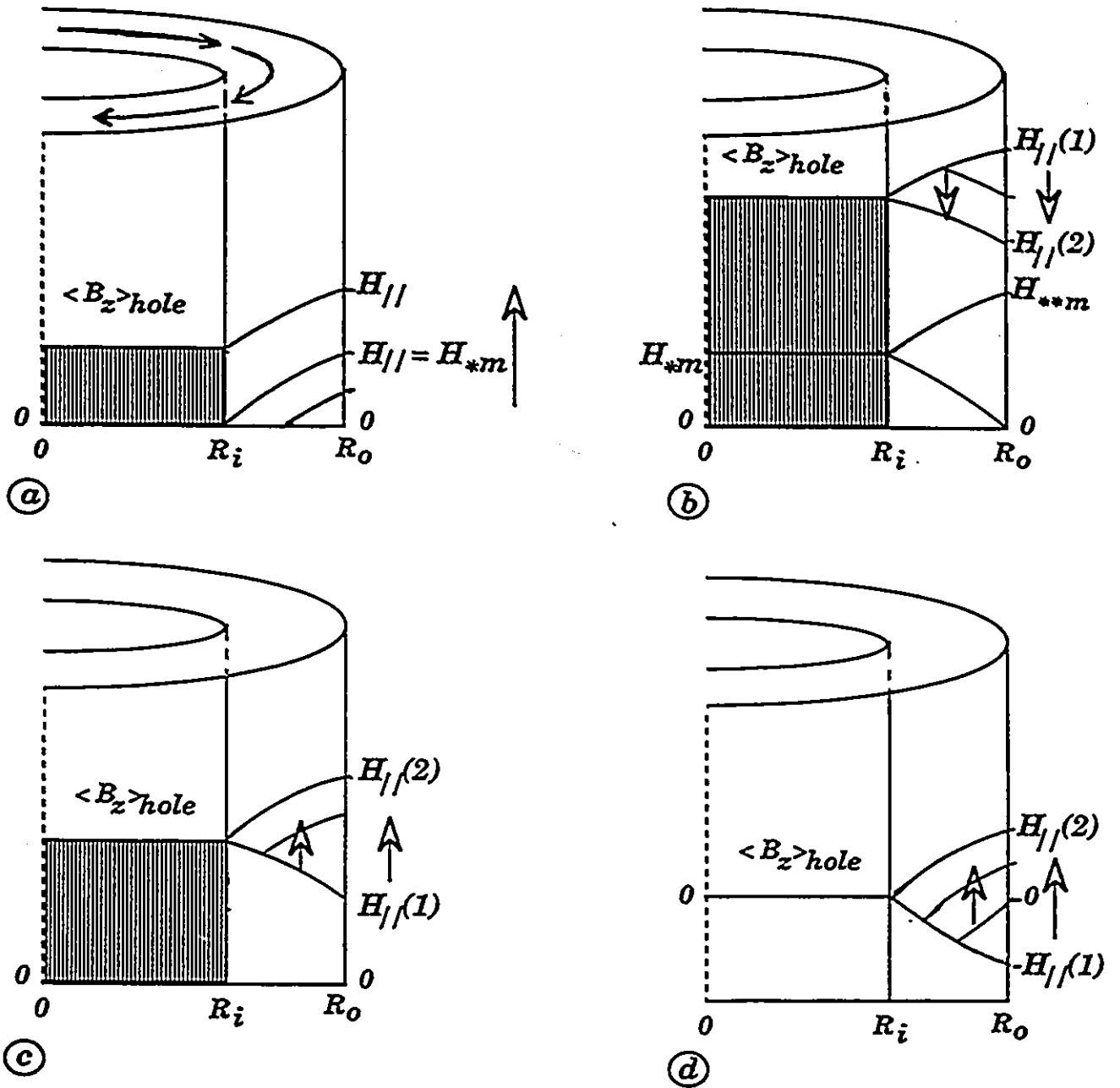


Fig 6.2 Schematics of the $B(r)$ profiles in the wall of a tube when , (a) $H_{||}$ is first impressed and raised through H_{*m} , the first penetration field , (b) $H_{||}$ is decreased from $H_{||}(1) > H_{*m}$ to $H_{||}(2)$ (upper curve) or decreased from the "double" penetration field H_{**m} to 0. (c) reversed from a saturation (critical) profile , and (d) varied from $H_{||}(1) = -H_{*m}$ to $H_{||}(2) = +H_{*m}$.

in an "idealized" context. Here I is the current flowing azimuthally per unit length L of the wall. In this idealized picture, the external return flux associated with the azimuthally circulating current I is considered to be negligible, hence end effects are ignored and the tube is regarded as "infinitely" long.(i.e. very long compared with its outer diameter). With an aspect ratio (length/outer diameter) $L/2R_o = 2.15$, the BiSCCO can be regarded as approximately satisfying these conditions at least with respect to configurations of magnetic flux in the vicinity of its "waist". Consequently $B_z(r)$, in the cavity, in the wall and outside the tube in the volume adjacent to the mid plane can be regarded as undistorted and axially uniform. We note that the pickup coil monitoring $\langle B_z \rangle_{\text{hole}}$ occupies only the central part of the length of the tube.

We let H_{cm} denote the field for first penetration of magnetic flux in the cavity and I_{cm}^* denotes the corresponding critical current per unit length. Then by Ampere's law in the idealized framework outlined above, we can write,

$$H_{//} = H_{\text{cm}} = I_{\text{clm}}^* = \langle j_{\text{clm}} \rangle (R_o - R_i) \quad (6.2)$$

It is noted that $\langle B_z \rangle_{\text{hole}}$ does not exhibit a sharp rise from zero in Fig 6.1. The observed curvature in the initial rise is attributed to the "end" effects just mentioned. Consequently H_{cm} is estimated by backward extrapolation of the linear region of the trace of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$. From Fig 6.1 we estimate $\mu_0 H_{\text{cm}} \approx 6.0\text{mT}$, hence $\langle j_{\text{clm}} \rangle \approx 3(10^6)\text{A/m}^2$.

When $H_{//}$ is increased beyond H_{cm} , magnetic flux enters into the cavity as depicted in the uppermost curve of Fig 6.2(a). Again by Ampere's law ,

$$H_{//} - \frac{\langle B_z \rangle_{\text{hole}}}{\mu_0} = I_{\text{clm}} = \langle j_{\text{clm}} \rangle (R_o - R_i) \quad (6.3)$$

We note from Fig 6.1 that $I_{c\perp m}$ is approximately constant as $H_{//}$ increases beyond H_{*m} over the range illustrated, which is the range of interest in our study. Consequently,

$$j_{c\perp m}(B) \approx \langle j_{c\perp m} \rangle \quad (6.4)$$

over this range. This insensitivity of $j_{c\perp m}$ on the magnetic flux density is referred to as the Bean approximation (Bean like, for brevity).

The magnetic flux which has entered the cavity remains completely trapped in that reservoir when $H_{//}$ is subsequently removed provided that $H_{//}$ does not exceed a value denoted H_{*m} . From a family of "half cycles" of $H_{//}$, commencing at $H_{//}=0$ after zero field cooling we determine that $H_{*m} \approx 12\text{mT}$. A typical member of such a family of curves is displayed in Fig 6.1. After applying and removing $H_{//} > H_{*m}$, it is observed that the magnetic flux density trapped in the cavity is $\approx 6\text{ mT}$ (see Fig 6.1). (We note that in the Bean approximation, $H_{*m} = 2H_{*c}$ since here $j_{c\perp m}$ is independent of B). The corresponding $B_z(r)$ profile is sketched in Fig 6.2(b) (lowermost curve). We note that as expected from the critical state concept, the trapped flux density corresponds to $\mu_0 H_{*m}$, the applied magnetic flux density for first penetration into the cavity.

For completeness we display the evolution of $\langle B_z \rangle_{\text{total}}$ vs $H_{//}$ in Fig 6.3. $\langle B_z \rangle_{\text{total}}$ denotes the spatial average of the axial magnetic flux density over the entire cross section of the tube , i.e. hole plus wall. This is the quantity measured directly by the outer pickup coil/amplifier/integrator set up .

The sweeps of $H_{//}$ from "large" positive to negative values and vice versa, generate critical state configurations of induced persistent currents in the wall and trace boundaries or envelopes of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$ such as displayed in Fig

6.1 and of $\langle B_z \rangle_{\text{total}}$ vs $H_{//}$ presented in Fig 6.3 . Along the envelopes, the induced persistent currents not only fill the entire width of the wall but circulate only in one sense , i.e. "clockwise" when $H_{//}$ is swept from negative to positive, "counterclockwise" when $H_{//}$ is swept from positive to negative.

The evolution of the $B_z(r)$ profiles in the wall of the tube after the direction of the sweep of $H_{//}$ is reversed is sketched in Figs. 6.2 (b), (c) and (d). These reversals of the sense of the sweep of $H_{//}$ cause $\langle B_z \rangle_{\text{wall}}$ and $\langle B_z \rangle_{\text{total}}$ to travel from one envelope or boundary to the other as displayed in Figs 6.1 and 6.3. Ideally in Fig 6.1 the traversal should be horizontal from one boundary across to the other boundary. Again we attribute the curvature when the traversal approaches the opposite side to arise from the finite geometry (end effects) of the tube.

Again from Ampere's law ,

$$\frac{H_{//}(1) - H_{//}(2)}{2} = \pm I_{c\perp m} = \langle j_{c\perp m} \rangle (R_o - R_i) \quad (6.4)$$

Where $H_{//}(1)$ and $H_{//}(2)$ denote the magnetic fields at the extremities of the traversals (see Fig 6.2). From inspection of the data illustrated in Fig 6.1 and 6.3 we note that the magnitude of the traversals is not sensitive to the choice of the reversal points, hence on the position of the extremities. Consequently, we again conclude that $j_{c\perp m}(B) \approx \langle j_{c\perp m} \rangle$ over the range displayed. In other words $\langle j_{c\perp m} \rangle$ is Bean like for this specimen.

6.3. The Intragrain Critical Current Density, $j_{c\perp g}$

The grains of sintered high T_c superconductors generally possess a critical current density for depinning denoted $j_{c\perp g}$, which is appreciably larger

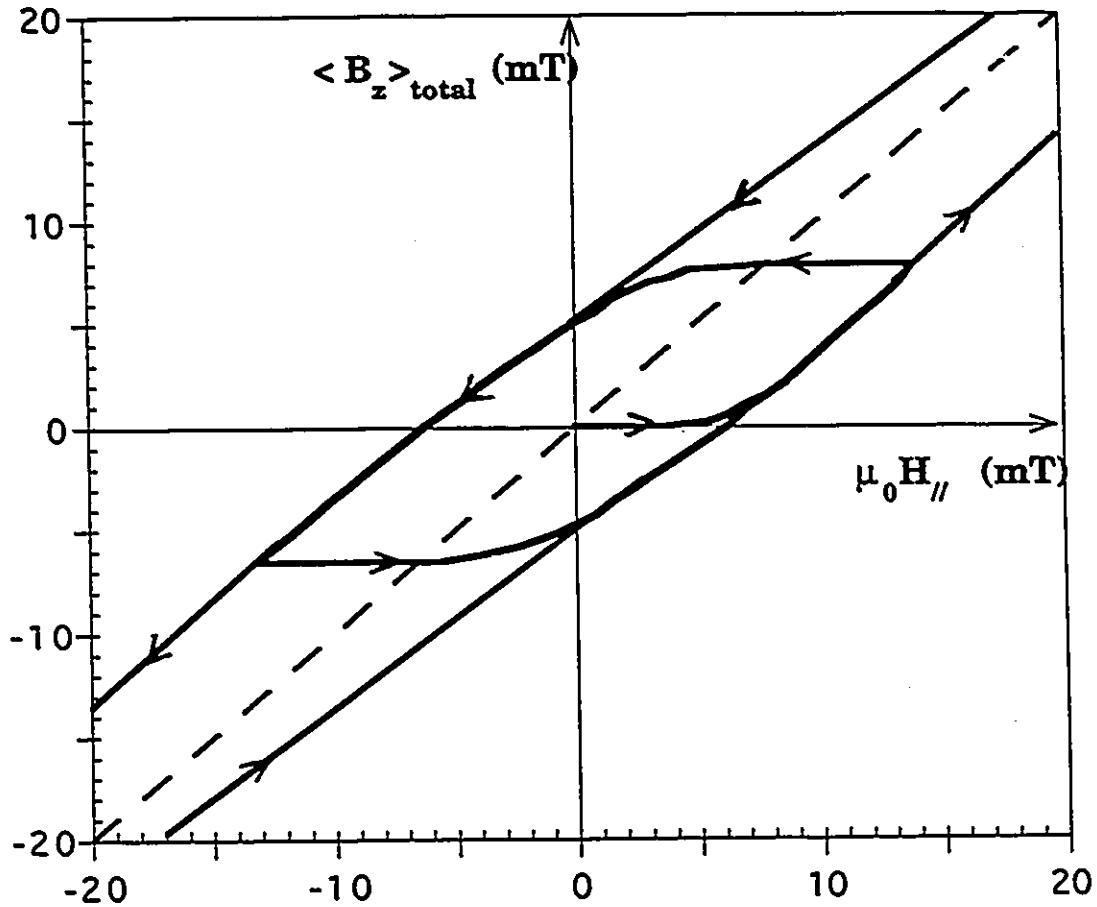


Fig 6.3 Evolution of $\langle B_z \rangle_{\text{total}}$, the spatial average of the axial magnetic flux density in the wall plus cavity as $H_{//}$ is impressed and varied along and across the boundaries (envelopes) of the magnetization curve of the BiSCCO tube.

than that of the "matrix" or "links" between the grains . Evidently we could determine $j_{c\perp g}$ by grinding the tube or a portion of the tube and carrying out four probe or magnetization measurements on the isolated grains. We wish however to keep our tube specimen intact and avoid such a destructive approach. We proceed to obtain an estimate of $\langle j_{c\perp g} \rangle$ for the low field range of our investigation by using two complementary approaches for trapping magnetic flux in the tubes, denoted H_{cool} and H_{cycle} procedures [26, 33, 37, 45, 61, 63, 65, 69, 73] . We now describe these procedures and our results in detail.

6.4. Flux Trapping by H_{cool} Procedure

In the first approach, the tube is field cooled from T_c to 77 K in a static axial field $H_{//}$, denoted H_{cool} , which is subsequently removed . Both $\Phi_{z,hole}$, the axial magnetic flux trapped in the cavity after H_{cool} has been removed , and, $\Phi_{z,total}$, the axial magnetic flux trapped in the entire cross section of the tube, i.e. in the hole plus the wall are readily determined separately but simultaneously by means of the pickup coil in the cavity and the pickup coil embracing the outer surface of the wall.

Fig 6.4 (a) displays the growth of $\Phi_{z,hole} = \langle B_z \rangle_{hole} \pi R_i^2$, the axial flux trapped in the cavity of the tube, or equivalently its density $\langle B_z \rangle_{hole}$, as a function of H_{cool} . We note that $\langle B_z \rangle_{hole}$ vs H_{cool} exhibits a "kink" followed at higher fields by a plateau . The kink is observed when $\mu_0 H_{cool} \approx 6mT$ and the threshold of the plateau is situated in the vicinity of $\mu_0 H_{cool} \approx 30mT$.

By Faraday's law of induction , the removal of H_{cool} induces currents to circulate around the circumference of the tube to retain the total flux,

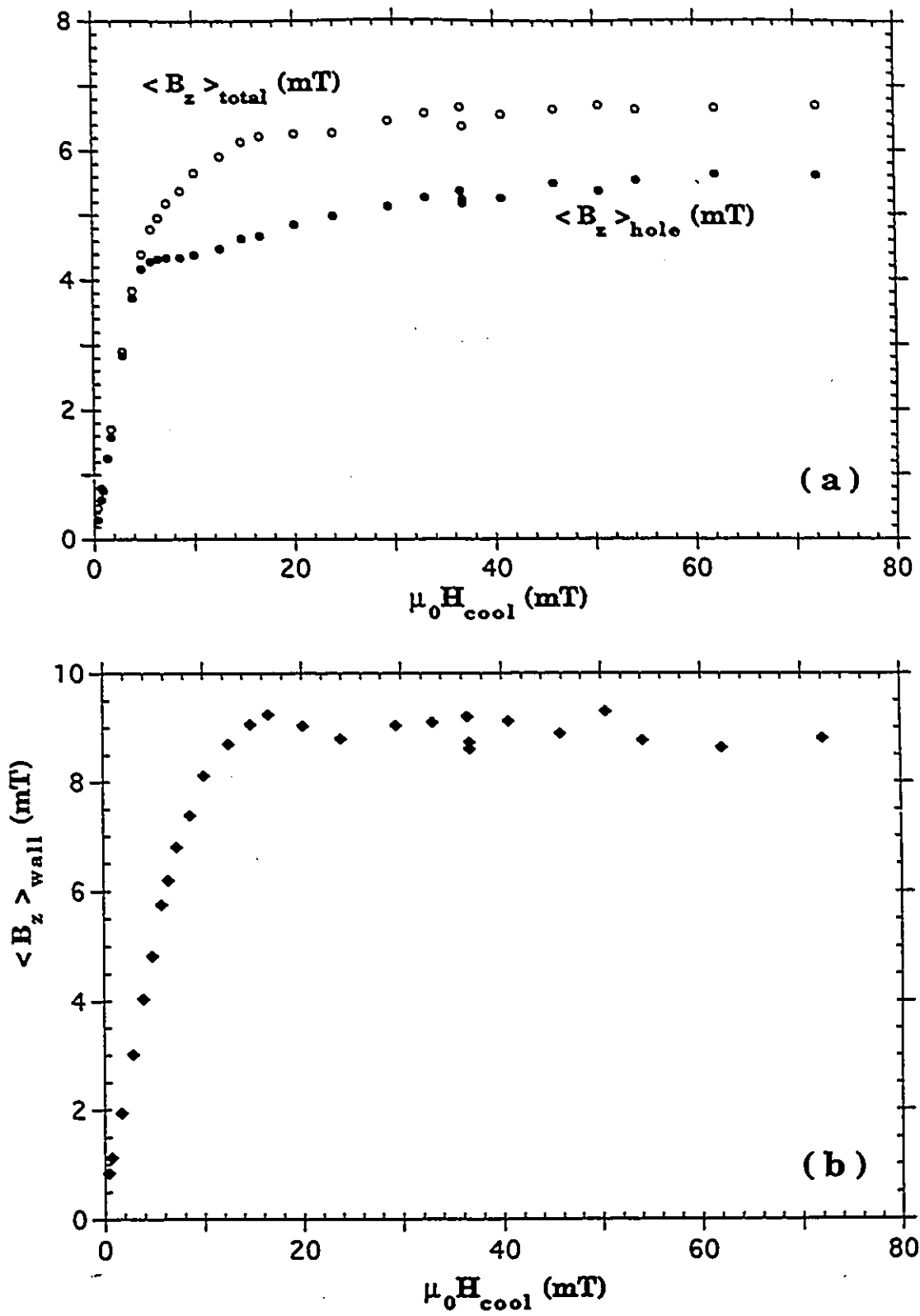


Fig 6.4 (a) $\langle B_z \rangle_{hole}$ and $\langle B_z \rangle_{total}$, and (b) $\langle B_z \rangle_{wall}$ trapped in the BiSCCO tube by H_{cool} procedure.

Fig 6.5 Schematics of B profiles after H_{cool} has been removed, (a) from a tube and (b) from an isolated grain of "idealized" geometry. (c) Displays expectations for a monolithic and (e) for a granular tube where B_{return} from the magnetized grains μ_g shown in (d) play a role.

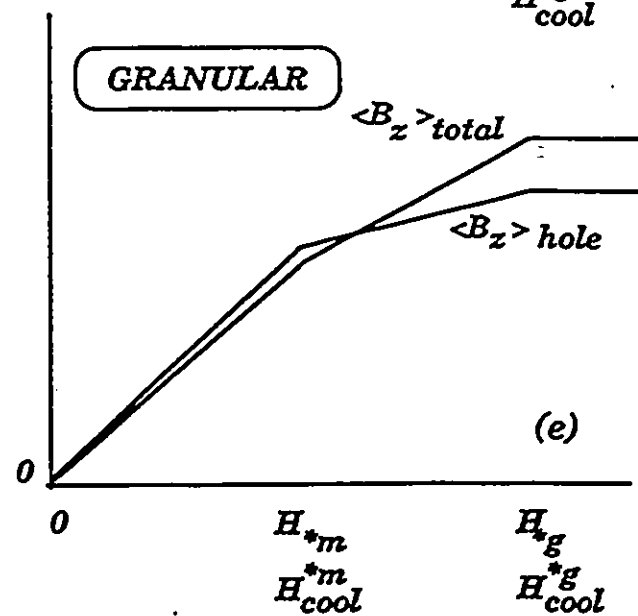
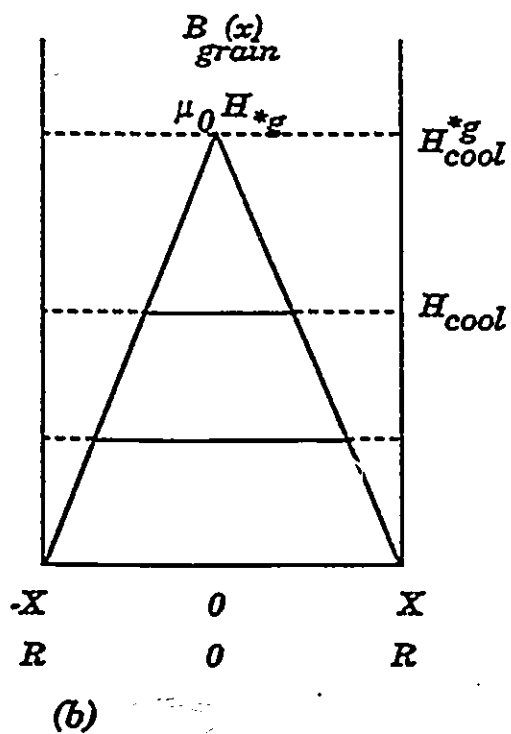
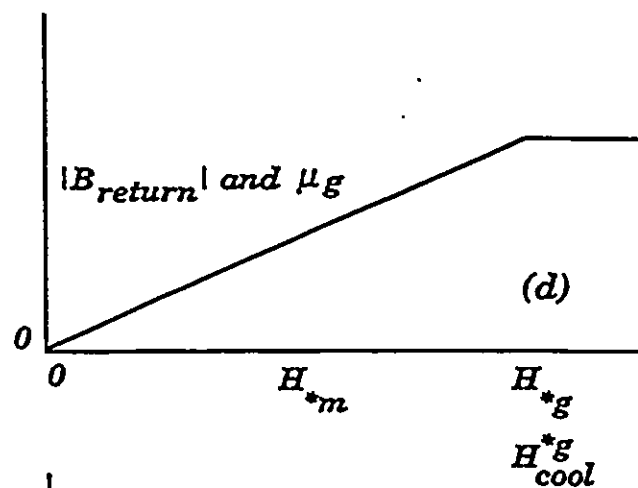
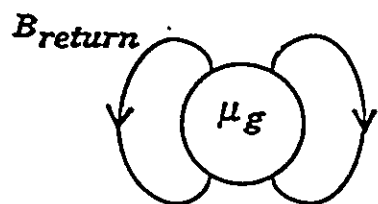
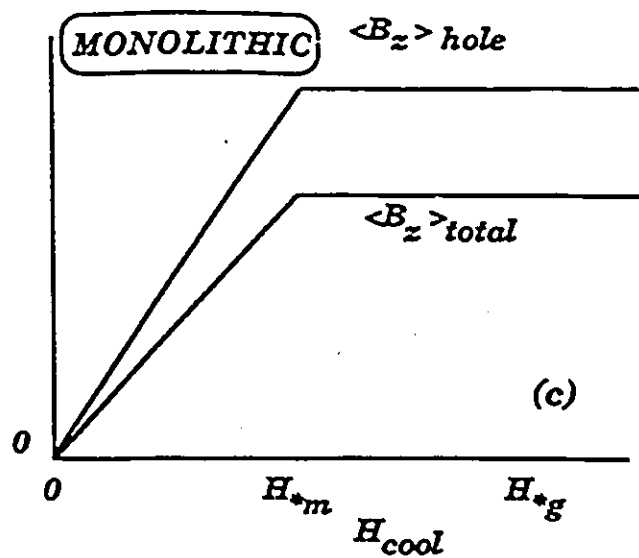
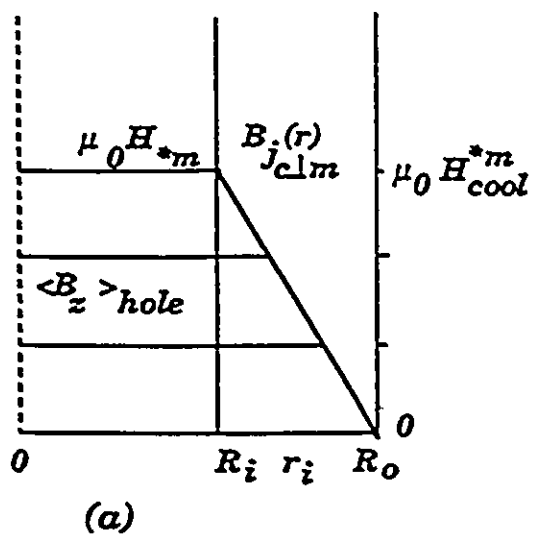


Figure 6.5

$$\Phi_{z, \text{total}} = (\mu_0 H_{\text{cool}}) \pi R_0^2 \quad (6.5)$$

initially permeating the cavity and the wall of the tube. For low H_{cool} this is achieved by an induced persistent current, I_{total} , which occupies only an outer annular portion of the volume of the wall of the tube (see lowermost curve of Fig 6.5(a)). Thus,

$$I_{\text{total}} = \langle j_{cLw} \rangle L (R_0 - r_i) \quad (6.6)$$

where r_i is the inner boundary of the space filled by the induced current. By Ampere's law, $H_{\text{cool}} = I_{\text{total}} / L$, hence r_i depends on H_{cool} as depicted schematically in Fig 6.5(a) .

Let I_{cLm}^* denote the saturation critical current per unit length flowing around the circumference of the wall when r_i has advanced to R_i . Let H_{cool}^{*m} denote the corresponding H_{cool} and $\langle B_z \rangle_{\text{hole}}^*$ denote the corresponding trapped flux density in the cavity. At this juncture, from Ampere's law and the critical state picture ,

$$\frac{\langle B_z \rangle_{\text{hole}}^*}{\mu_0} = H_{\text{cool}}^{*m} = H_{sm} = I_c^* = \langle j_{cLm} \rangle (R_o - R_i) \quad (6.7)$$

We believe that the kink in Fig 6.4(a) corresponds to this situation. This interpretation is consistent with the value of the corresponding quantities since here $\mu_0 H_{\text{cool}}^* \approx 6\text{mT}$ and $\langle B_z \rangle_{\text{hole}}^* \approx 4.5\text{mT}$. This agreement is satisfactory although "ideally" the latter two quantities should be equal .

The picture we have just developed envisaged that the wall of the tube was homogeneous or monolithic . Under these circumstances $\langle B_z \rangle_{\text{hole}}^*$ vs H_{cool} would display a plateau immediately beyond the kink as shown in Fig 6.5(c).

We note in Fig 6.4(a) that $\langle B_z \rangle_{\text{hole}}$ continues to grow, albeit slowly, as H_{cool} is chosen greater than H_{cool}^m and eventually attains a plateau. The onset of this plateau is attributed to saturation remanent magnetization of the grains and provides an estimate of $\langle j_{c\perp g} \rangle$, the intragrain critical current density for depinning. We now develop this idea.

Many workers using the four probe technique for measuring the critical depinning current $I_{c\perp}$ have observed that this quantity is enhanced when the externally applied field H_a has been decreased in comparison with the data for the case where H_a has been increased [21, 22, 25, 34, 36, 48, 49, 50, 52, 58, 68, 69, 74]. The prevailing account for this behaviour is now outlined and applied to our situation. In the regime where H_a has been decreased, the individual grains of the sintered granular material become saturated with flux retaining circulating currents. Thus each grain acquires a magnetic moment $\bar{\mu}$, hence becomes a small magnet which is magnetized along $\bar{H}_a$ as sketched in Fig 6.5(b). (The grains are assumed isotropic for simplicity). Let $\bar{B}_{\text{return}} = \mu_0 \bar{H}_{\text{return}}$, denote the "return" magnetic flux density associated with $\langle M_g \rangle$, the magnetization of the grains. Some of this return flux will thread the intergrain volume (the matrix). The matrix however is already permeated by magnetic flux, denoted $B_{j_{c\perp m}}(r)$, generated by the intergrain currents of density $j_{c\perp m}(r)$ circulating around the circumference of the wall.

In this picture, $\bar{B}_{\text{total}}(r)$, the resultant magnetic flux density in the "matrix" (i.e. the intergrain regions) is the superposition of these contributions and can be written,

$$\bar{B}_{\text{total}}(r) = \mu_0 \bar{H}_a + \bar{B}_{j_{c\perp m}}(r) + \bar{B}_{\text{return}} \quad (6.8)$$

We confine our attention to the special case where $H_a = H_{cool}$ has been reduced to zero, hence eqn 6.8 simplifies to,

$$\bar{B}_{total}(r) = |\bar{B}_{j_{c,lm}}(r)| - |\bar{B}_{return}| \quad (6.9)$$

where we have explicitly incorporated the feature that here, for the data of Fig 6.4(a), $\bar{B}_{return}$ is directed opposite to $\bar{B}_{j_{c,lm}}$ inside the matrix. We stress that the higher the value of H_{cool} which is then removed, the stronger the final magnetization of the grains which ensues and hence the greater the corresponding return flux B_{return} as depicted in Fig. 6.5 (b) and (d).

Consequently as H_{cool} is augmented and the ensuing B_{return} generated by the removal of H_{cool} is increased, we anticipate a rise in the level of trapped flux in the cavity. This rise will cease when B_{return} attains its maximum value. This is achieved when the grains are filled with intragrain critical currents, hence when, as depicted schematically in Figs 6.5(b) and (d),

$$H_{cool} = H_{cool}^{*g} = \langle j_{c,lg} \rangle R = H_{*g} \quad (6.10)$$

where $\langle R \rangle$ denotes an effective average radius for the grains. Rigorously, eqn 6.10 is valid only for grains in the form of long isotropic cylinders whose length is // to H_{cool} . It is a useful frequently exploited approximation.

In practice, since the grains are irregular in shape and span a range of dimensions, the approach to the plateau in Fig 6.4(a) will be smeared out (asymptotic). Nevertheless from the data displayed in this figure we can estimate $\mu_0 H_{cool}^{*g} = 30\text{mT}$. We stress that this quantity also corresponds physically to H_{*g} the first penetration field for the grains. The central feature pertinent to our analysis of flux cutting and cross-flow in the tube is that,

$$\mu_0 H_{*g} \approx 30 \text{mT} \gg \mu_0 H_{*m} \approx 5 \text{mT} \quad (6.11)$$

The magnitude of the rise of $\langle B_z \rangle_{\text{hole}}$ observed as H_{cool} progresses from H_{cool}^m to H_{cool}^* and displayed in Fig 6.4(a) is consistent with our earlier observations. We have already established that $j_{c\perp m}$ is only weakly dependent on B in this range. Finally, it is useful to emphasize, that it is the magnetic field H_{cool} for the onset of the plateau that provides a measure of the penetration field into the grains. The magnitude of the rise of $\langle B_z \rangle_{\text{hole}}$ from the kink to the plateau does not affect the estimate of H_{*g} . The model we have outlined is drawn from the literature and serves to account for the appearance of a kink followed by a plateau in the data of Fig 6.4(a). It also links the edge of the plateau with H_{*g} . We note that in this picture, the plateau will approach the kink if H_{*g} is made comparable to H_{*m} and these will merge or coalesce when $H_{*g} \leq H_{*m}$.

The growth of $\Phi_{z,\text{total}} = \langle B_z \rangle_{\text{total}} \pi R_o^2$, the total magnetic flux trapped in the hole plus the wall of the tube after $H_{\parallel} = H_{\text{cool}}$ has been removed is also displayed as a function of H_{cool} in Fig 6.4(a). This trapped flux is monitored directly by the outer pickup coil embracing the tube when the tube is heated from 77 K to T_c after H_{cool} has been removed. As the specimen is driven into the normal resistive state by the heating process, the persistent currents circulating around the wall and inside the grains are destroyed and the magnetic flux which they sustain is released

We note that $\Phi_{z,\text{total}}$, hence $\langle B_z \rangle_{\text{total}}$ rises to a plateau whose edge is situated in the vicinity of $\mu_0 H_{\text{cool}} \approx 30 \text{mT}$ and corresponds to the threshold of the plateau for $\langle B_z \rangle_{\text{hole}}$ which we have just discussed. This correspondence is not surprising since as can be seen from the sketches of Fig 6.5, the two sets of measurements complement each other. We now elaborate on this feature.

It is useful at this juncture to review the "context" of $\Phi_{z,\text{total}}$. We again refer the reader to Fig 6.5 for aid in visualizing the various quantities involved. By definitions,

$$\Phi_{z,\text{total}} = \Phi_{z,\text{hole}} + \Phi_{z,\text{wall}} = \quad (6.12(a))$$

$$\langle B_z \rangle_{\text{total}} \pi R_o^2 = \langle B_z \rangle_{\text{hole}} \pi R_i^2 + \langle B_z \rangle_{\text{wall}} \pi (R_o^2 - R_i^2) \quad (6.12(b))$$

If the wall consisted of a monolithic (i.e. homogeneous and nongranular) type II superconducting material, $\langle B_z \rangle_{\text{total}}$ would be smaller than $\langle B_z \rangle_{\text{hole}}$ throughout Fig 6.4(a), (and as illustrated in Fig. 6.5 (c)), although $\Phi_{z,\text{total}}$ must be larger than $\Phi_{z,\text{hole}}$. The reason that $\langle B_z \rangle_{\text{total}}$ must be smaller than $\langle B_z \rangle_{\text{hole}}$ is that $\langle B_z \rangle_{\text{wall}}$ for the homogeneous tube is less than $\langle B_z \rangle_{\text{hole}}$ (see Fig 6.5(a)). We note however that $\langle B_z \rangle_{\text{total}}$ rises above $\langle B_z \rangle_{\text{hole}}$ in the data presented in Fig 6.4. This occurs because in granular sintered high T_c materials, the "isolated" grains can trap a higher density of magnetic flux than the matrix (i.e. the intergranular volume) of the wall. To bring out this feature, in Fig 6.4(b) we have plotted,

$$\langle B_z \rangle_{\text{wall}} = \frac{\langle B_z \rangle_{\text{total}} R_o^2}{(R_o^2 - R_i^2)} - \frac{\langle B_z \rangle_{\text{hole}} R_i^2}{(R_o^2 - R_i^2)} \quad (6.13)$$

which is rearrangement of eqn 6.12 and contains both sets of data of Fig 6.4(a). It is clear from Fig 6.4 that $\langle B_z \rangle_{\text{wall}}$ rises above $\langle B_z \rangle_{\text{hole}}$ as H_{cool} is augmented. Further $\langle B_z \rangle_{\text{wall}}$ attains a maximum when $\mu_0 H_{\text{cool}} \approx 30\text{mT}$. This correspondence confirms the account we have given for the occurrence of the plateaus and the link we have made between H_{cool}^* the threshold of the plateaus with the intragrain penetration field H_{*g} .

Fig 6.5(e) is obtained by superposition of Fig 6.5(c) and (d) in accordance with the model just described. Fig 6.5(e) , although schematic, should be compared with the actual data displayed in Fig 6.4(a) . The model qualitatively reproduces the salient features of the data and identifies the two pertinent quantities , H_{*m} and H_{*g} , hence the corresponding average critical current densities $\langle j_{c\perp m} \rangle^*$ and $\langle j_{c\perp g} \rangle^*$.

6.5. Flux Trapping by H_{cycle} Procedure

We have also exploited another standard procedure for trapping magnetic flux in type II superconductors. In this approach , the tube in the virgin state (i.e. zero field cooled from T_c to 77 K) is subjected to a slow sweep of an axial magnetic field $H_{//}$ which is then gradually removed. Let H_{cycle} denote the maximum value attained by $H_{//}$ during such a "half cycle" . $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{total}}$, the magnetic flux densities trapped in the hole and in the entire cross-section (hole plus wall) are plotted vs H_{cycle} in Fig 6.6(a). Fig 6.6(b) displays $\langle B_z \rangle_{\text{wall}}$, the corresponding magnetic flux density trapped in the wall (see eqn 6.19).

We note from inspection of Fig 6.6(a) that again , as observed when the H_{cool} procedure was used for flux trapping , (i) $\langle B_z \rangle_{\text{hole}}$ exhibits a "kink" followed by a plateau , (ii) $\langle B_z \rangle_{\text{total}}$ also traces a plateau , (iii) the threshold of the plateau for $\langle B_z \rangle_{\text{hole}}$ approximately corresponds to the onset of the plateau for $\langle B_z \rangle_{\text{total}}$, (iv) $\langle B_z \rangle_{\text{total}}$ rises above $\langle B_z \rangle_{\text{hole}}$ in the range of large H_{cycle} , and (v) the rise of $\langle B_z \rangle_{\text{total}}$ above $\langle B_z \rangle_{\text{hole}}$ commences in the vicinity of the kink in $\langle B_z \rangle_{\text{hole}}$ vs H_{cycle} .

All of these features have already been encountered for flux trapping using the H_{cool} procedure . The model already outlined to account for these

behaviours in the H_{cool} case can again be applied to the H_{cycle} context . The new element which needs to be explained is the fact that the onset of the plateaus and the maximum for $\langle B_z \rangle_{\text{wall}}$ are now shifted to larger values of $H_{\parallel}$. These shifts are not only consistent with the model but provide information on the dependence of $\langle j_{c\perp m} \rangle$ and $\langle j_{c\perp g} \rangle$ on B .

We refer the reader to Fig 6.7 which illustrates the pertinent quantities for assistance in following our account of the observed shifts .

Let H_{cycle}^m denote $H_{\parallel}$ whose application to the virgin tube generated the flux density configuration in the wall and cavity sketched by the upper curve of Fig 6.7(a). The crucial characteristics we wish to portray in this sketch are that, (i) all of the magnetic flux previously threading the cavity when H_{cycle}^m was present remains completely trapped in the cavity when H_{cycle}^m is reduced to zero, and, (ii) when H_{cycle} is selected higher than H_{cycle}^m , some of the flux initially threading the cavity is released during the removal of H_{cycle} . The latter behaviour is displayed by the uppermost curves in Fig 6.7(a). In other words and as illustrated in Fig 6.7(a), H_{cycle}^m is the minimum H_{cycle} for complete retention in the cavity of the flux threading it at the maximum of the half cycle. We stress that the same residual flux configuration is attained by cooling in $H_{\text{cool}} = H_{\text{cool}}^m = H_{\text{m}}$ (see Fig 6.5(a)) and then removing H_{cool} .

As a consequence of the critical state concept, the ratio, $H_{\text{cycle}}^m / H_{\text{cool}}^m$ is independent of the wall thickness and is completely dictated by the dependence of the intergrain critical current density $j_{c\perp m}$ on B .

It is useful at this juncture to dwell upon and develop a basic feature of saturated critical state flux density configurations which we are encountering here when we focus on the wall and which will appear again shortly in our examination of the effect of the magnetized grains. Therefore we now introduce a sub-section on this topic.

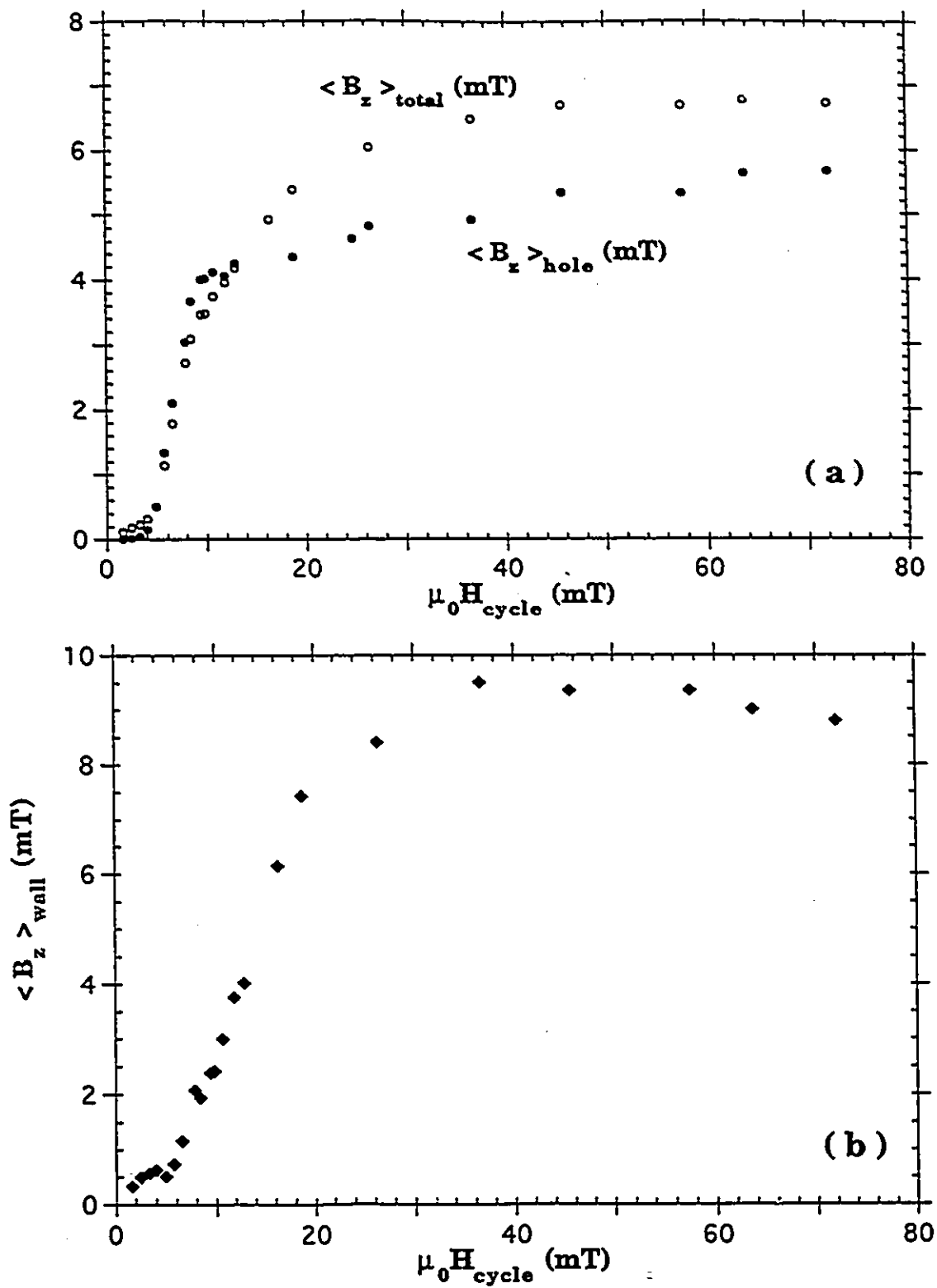
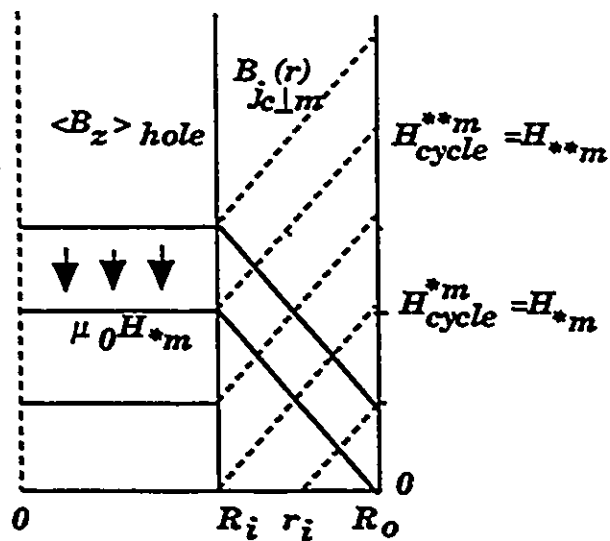


Fig 6.6 (a) $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{total}}$, and (b) $\langle B_z \rangle_{\text{wall}}$ trapped in the BiSCCO tube by H_{cycle} procedure.

Fig 6.7 Schematics of B profiles when H_{cycle} is impressed and has been removed, (a) from a tube and (b) from a grain of "ideal" geometry. (c) Displays expectations for a monolithic and (e) for a granular tube where B_{return} and μ_g shown in (d) play a role.



(a)

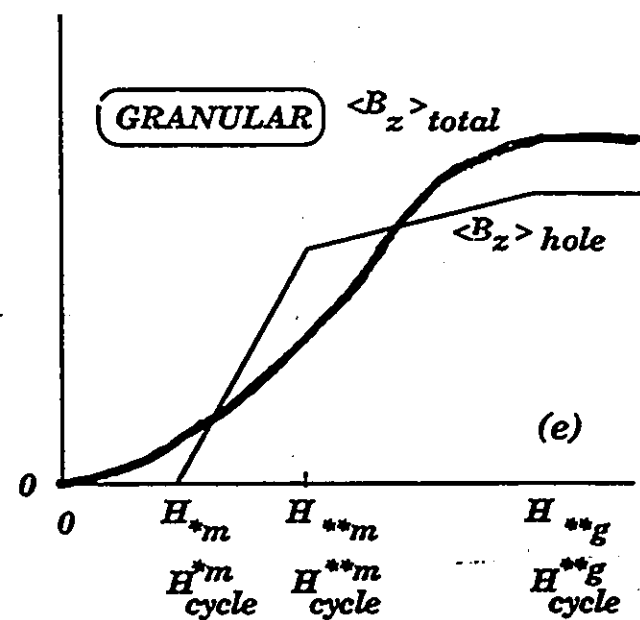
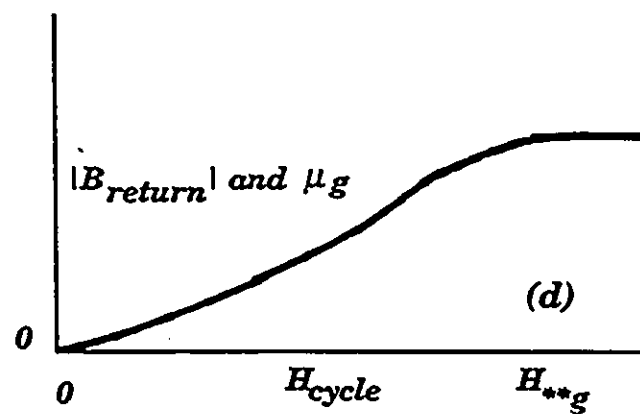
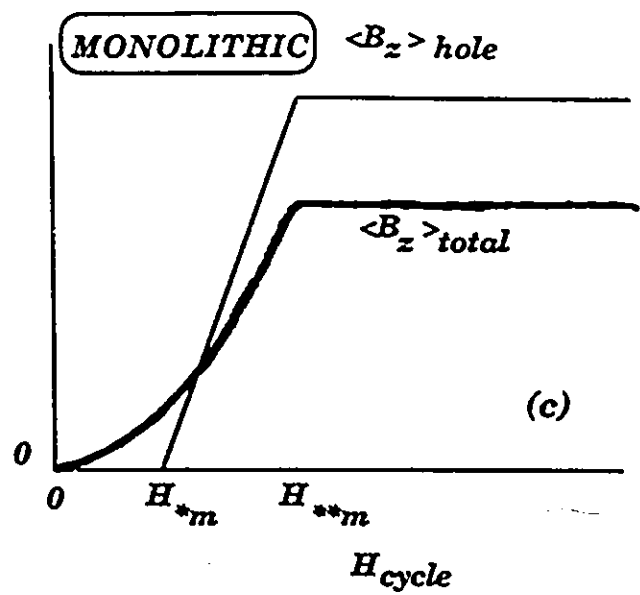


Figure 6.7

6.6. General Relationships for Critical State Configurations

The development envisages infinite slab (planar geometry) and cylinder geometry where $\mu_0 \bar{H} = \bar{B}$ is $\perp$ to the space variables x or r which we will denote by x . We refer the reader to the sketch below. Maxwell's eqn $\nabla \times (\bar{B}/\mu_0) = \nabla \times \bar{H} = \bar{j}$ together with the critical state assumption that $j=j_c$, then reads for the idealized situations under consideration,

$$\frac{1}{\mu_0} \frac{dB}{dx} = \frac{dH}{dx} = \pm j_c \quad (6.14)$$

Consequently we can write the following series of definitions,

$$\int_0^{H_*} dH = H_* = \int_0^X j_c dx = \langle j_c \rangle_0^* X \quad (6.15)$$

Here and in the following expressions, the subscript and superscript indicate the limits of the range of the quantity.

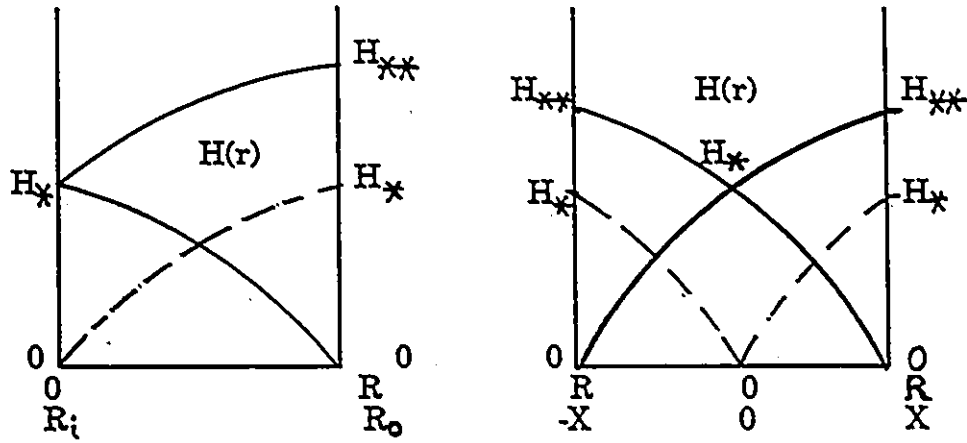
$$\int_0^{H_{**}} dH = H_{**} = \int_0^{2X} j_c dx = \langle j_c \rangle_0^{**} 2X \quad (6.16(a))$$

The above can be rewritten,

$$\begin{aligned} \int_0^{H_{**}} dH &= \int_0^{H_*} dH + \int_{H_*}^{H_{**}} dH = H_* + H_{**} \\ &= \int_0^X j_c dx + \int_X^{2X} j_c dx = \langle j_c \rangle_0^* X + \langle j_c \rangle_*^{**} X \end{aligned} \quad (6.16(b))$$

Introducing these statements into ratios leads to,

$$\frac{H_{\infty}}{H_*} = \frac{2 \langle j_c \rangle_0^{\infty}}{\langle j_c \rangle_0^{\infty}} = 1 + \frac{\langle j_c \rangle_0^{\infty}}{\langle j_c \rangle_0^{\infty}} \quad (6.17)$$



To further illustrate the content of these relationships and definitions it is instructive to introduce a critical current density dependent on $H=B/\mu_0$ of the frequently exploited form

$$j_c = \frac{\alpha}{H^n} \quad (6.18)$$

Eqn 6.14 now reads

$$\frac{dH}{dx} = \pm \frac{\alpha}{H^n} \quad (6.19(a))$$

$$H^n dH = \pm \alpha dx \quad (6.19(b))$$

Integrating from 0 to X and taking H to be zero at one of the limits and $H_* = B_*/\mu_0$ at the other limit leads to,

$$H_* = \{(n+1)\alpha X\}^{1/(n+1)} \quad (6.20)$$

Integrating eqn 6.19(b) from 0 to 2X and taking H to be zero at one of the limits and $H_{..} = B_{..} / \mu_0$ (by definition) at the other limit leads to,

$$H_{..} = \{2(n+1)\alpha X\}^{1/(n+1)} \quad (6.21(a))$$

$$H_{..} = 2^{1/(n+1)} H_{.} \quad (6.21(b))$$

Hence for the Bean approximation where $n=0$ in eqn 6.18, we obtain,

$$\frac{H_{..}}{H_{.}} = 2 \quad (6.22)$$

The simple Kim approximation where $n=1$ in eqn 6.18, leads to,

$$\frac{H_{..}}{H_{.}} = \sqrt{2} \quad (6.23)$$

Hence,

$$\frac{\langle j_c \rangle_0^{**}}{\langle j_c \rangle_0} = \frac{\sqrt{2}}{2} \quad , \quad \frac{\langle j_c \rangle_0^{**}}{\langle j_c \rangle_0} = \sqrt{2} - 1 \quad (6.24)$$

Equations 14 through 24 are "generic" and can be applied either to the matrix of the wall . Here then ,

$$j_c = j_{cLm} \quad , \quad B_{.}/\mu_0 = H_{.} = H_{.m} \quad , \quad B_{..}/\mu_0 = H_{..} = H_{..m} \quad (6.25)$$

or to the isolated isotropic grain of idealized geometry . Here then,

$$j_c = j_{cIg} \quad , \quad B_{.}/\mu_0 = H_{.} = H_{.g} \quad , \quad B_{..}/\mu_0 = H_{..} = H_{..g} \quad (6.26)$$

We note that the H_{cool} and H_{cycle} procedure generate the same final or residual saturated critical state configurations (compare uppermost solid curves of Fig 6.5(a) and (b) with that of Fig 6.7(a) and (b)). The difference is that to achieve these identical final B profiles in the matrix (wall) or in the individual grains, H_{cycle} must be larger than H_{cool} , and the ratios H_{cycle}/H_{cool} corresponding to various salient features of the graphs of trapped flux vs H_{cycle} and H_{cool} yield insight on the dependence of $j_{c\perp m}$ and $j_{c\perp g}$ on B.

6.7. Application of these Relations to the BiSCCO Tube

From the data for $\langle B_z \rangle_{hole}$ in Figs 6.4 and 6.6 we estimate the values of $\mu_0 H_{cool}$ and $\mu_0 H_{cycle}$ for the kinks to be ~ 6 and 12 mT respectively. Since the model identifies this feature as arising from critical intergrain configurations we write,

$$\mu_0 H_{cool}^{*m} = \mu_0 H_{*m} = \langle j_{c\perp m} \rangle_0^* (R_o - R_i) = 6 \text{ mT} \quad (6.27)$$

$$\mu_0 H_{cycle}^{**m} = \mu_0 H_{**m} = \langle j_{c\perp m} \rangle_0^{**} (R_o - R_i) = 12 \text{ mT} \quad (6.28)$$

and note that here the ratio $H_{**m}/H_{*m} = 2$ confirms the Bean like dependence of $j_{c\perp m}$ on B in this low field range.

The other salient features namely,

- (i) the rise of $\langle B_z \rangle_{hole}$ to a plateau after the appearance of the kink,
- (ii) the growth of $\langle B_z \rangle_{total}$ to a plateau which lies above that of $\langle B_z \rangle_{hole}$ and,

(iii) the correspondence of the threshold of these two plateaus, are all attributed , as in the H_{cool} procedure for flux trapping to the role played by the magnetization of the grains and to the effect of their return flux on the intergrain critical current. The relationship between these various features and the characteristic penetration fields into the grains are illustrated in Fig 6.7 (b), (c), (d), and (e). From the data for H_{cycle} (see Fig 6.6) we estimate,

$$\mu_0 H_{cycle}^{**g} = \mu_0 H_{*g} \approx 50 \text{mT} \quad (6.29)$$

Earlier from the data for H_{cool} (see Fig 6.4) we had estimated,

$$\mu_0 H_{cool}^{*g} = \mu_0 H_{*g} \approx 30 \text{mT} \quad (6.30)$$

From the ratio $H_{*g}/H_{*g} = 1.6$ we conclude that $j_{c \perp g}$ is perhaps slightly more dependent on B than $j_{c \perp m}$.

Because of the approach to the plateaus is asymptotic, the determination of their thresholds is inevitably ambiguous and inexact. We will see however that the important feature for our understanding of the flux line cutting and cross-flow behaviour of the BiSCCO tube is that the penetration fields H_* and H_{**} for the grains are appreciably larger than that for the "matrix" (i.e. the intergrain structure of the wall).

6.8. Flux Expulsion (Meissner Effect)

For simplicity we have so far ignored the complications brought about by the partial expulsion of some of the magnetic flux threading the wall during the process of cooling from T_c to 77 K in H_{cool} . Some of this flux is expelled into the cavity of the tube and some into the "environment". The amount

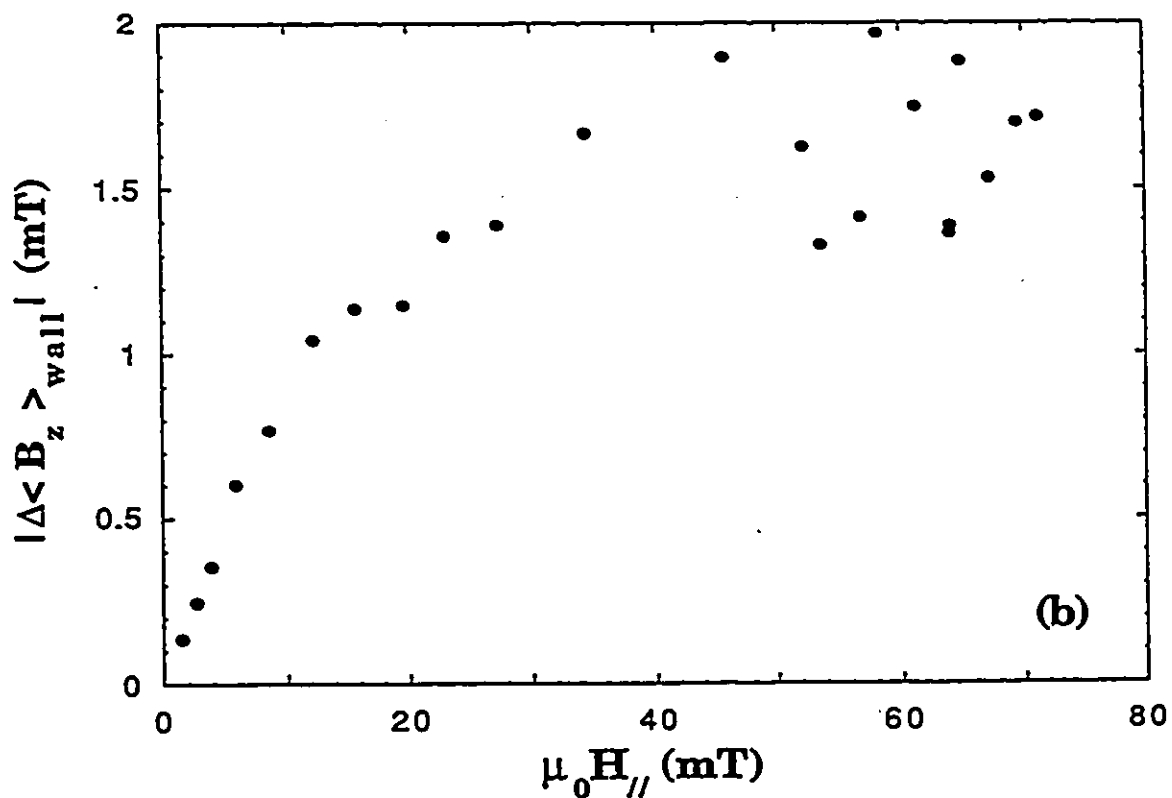
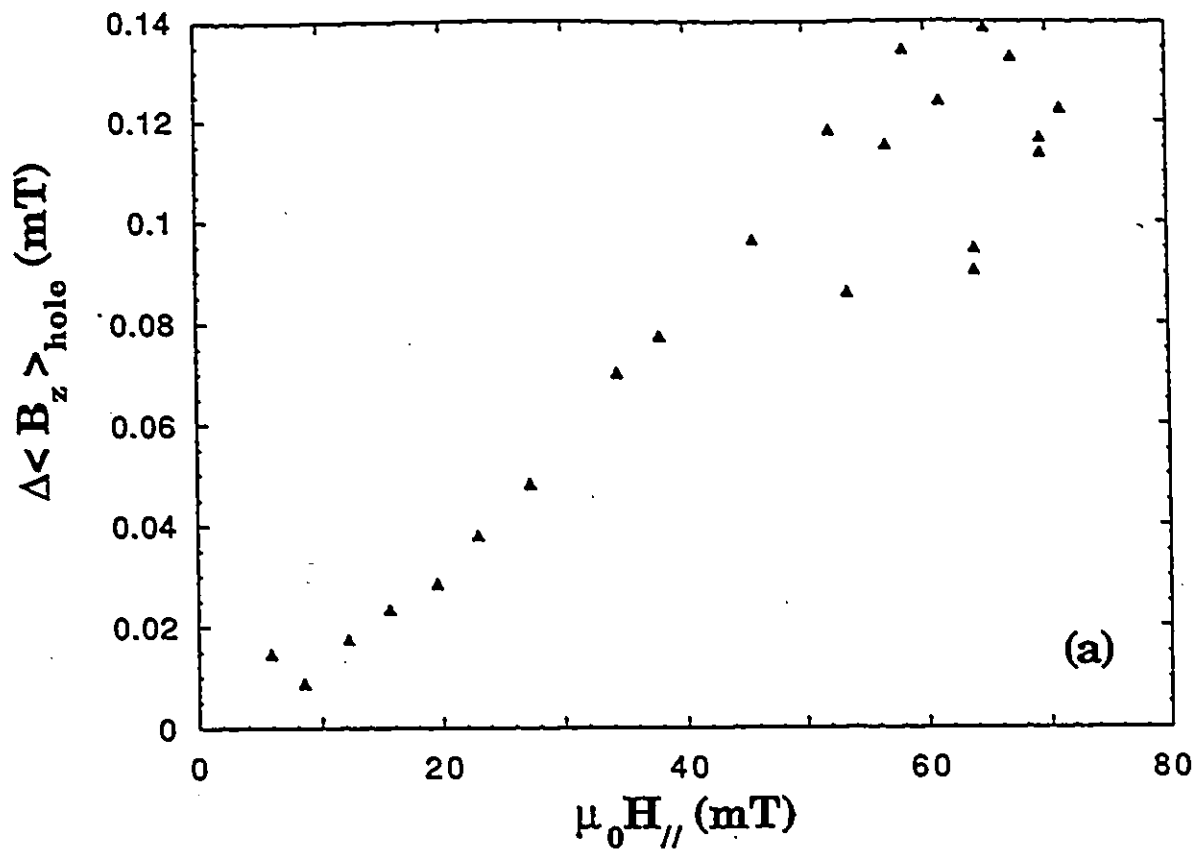


Fig 6.8 (a) Rise of axial flux density in the cavity and (b) decrease of axial flux density in the wall during cooling from T_c in various static $H_{//}$ (Meissner effect).

ejected into the cavity causes a rise in the axial flux density $\mu_0 H_{\text{cool}}$ initially threading this space. This rise, $\Delta \langle B_z \rangle_{\text{hole}}$ is displayed in Fig 6.8(a) versus $H_{\text{cool}}/\mu_0 H_{\text{cool}}$. We note that the rise is negligible relative to the initial flux density since $\Delta \langle B_z \rangle_{\text{hole}}/\mu_0 H_{\text{cool}} \approx 0.002$. Also we note that the rise is approximately linear, then plateaus out when $\mu_0 H_{\text{cool}} \geq 50\text{mT}$.

$\Delta \langle B_z \rangle_{\text{wall}}$, the decrease of the axial flux density in the wall during the field cooling process is shown in Fig 6.8(b). We note that the behaviour is approximately linear in the low field range $0 < H_{\text{cool}} \leq 10\text{mT}$ and that the ratio $\Delta \langle B_z \rangle_{\text{wall}}/\mu_0 H_{\text{cool}} \approx 0.1$ in this range.

For a homogeneous, monolithic hence nongranular specimen, we might expect, from symmetry considerations that approximately half of the magnetic flux expelled by the wall during the field cooling process would be ejected into the cavity. Thus for the BiSCCO tube since the cross-sections of the wall and of the hole, A_w and A_h have a ratio $A_w/A_h = 1/2$ we anticipate that,

$$\Delta \langle B_z \rangle_{\text{hole}} = \frac{1}{2} |\Delta \langle B_z \rangle_w| \frac{A_w}{A_h} = \frac{1}{4} |\Delta \langle B_z \rangle_w| \quad (6.31)$$

The ratios of $\Delta \langle B_z \rangle_{\text{hole}}/|\Delta \langle B_z \rangle_w|$ we observe are however an order of magnitude smaller than this. Although it is not germane to our investigation, this discrepancy warrants some discussion.

Our "naive" assumption that the magnetic flux expelled by the wall would be equally ejected into the cavity and into the environment may be valid for idealized circumstances. In such an idealized situation, $T(r)$, the initial temperature profile across the wall should be uniform and the cooling procedure should ensure that this uniform temperature distribution is approximately maintained as $T(r)$ descends from T_c to the ambient temperature $T_f(r) = 77\text{K}$.

In our setup, however, an appreciable temperature gradient initially exists across the thickness of the wall prior to cooling since the heater is in intimate contact with the outer surface only. Further, in our work, the rate of cooling along the two surfaces of the wall is probably unequal. Indeed in our arrangement the cooling rate is probably larger along R_i than along R_o . Both of these circumstances may cause the superconducting state to first appear along R_i and expand towards R_o . This may lead to a preferential expulsion of flux lines into the "environment".

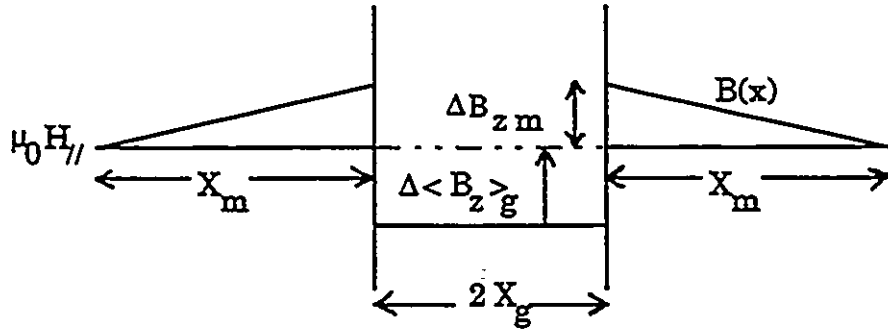
The fact that the wall is not a monolithic homogeneous entity but possesses a granular structure is now taken into consideration in our examination of the flux expulsion behaviour. For purpose of discussion we assume that $T(r)$ is uniform throughout the cooling history.

First we focus on a special "extreme" or limiting situation. Let us regard the superconducting grains as embedded in a matrix which is a normal metal at all temperatures. For simplicity, we view the electrically isolated grains as long rods or bars parallel to the length of a wall of infinite length, hence ignore demagnetization factors (end effects). For such a specimen, $\langle B_z \rangle_{\text{hole}}$ would remain constant and equal to $\mu_0 H_{//}$ as the "grains" cool from T_c to T_f and expel magnetic flux. An outer pickup coil embracing this "special" tube would detect the expulsion of flux from the assembly of uncoupled grains. The inner pickup coil in the cavity of the tube would, however, record no change in the flux threading it. We note that these result corresponds closely to our observations.

We know however, that the wall can sustain weak circumferential intergrain persistent currents of density $j_{c\perp m}$ at T_f . Therefore the intergrain matrix may be regarded as a multiply connected network or "sponge" of zero resistance material which can attempt, albeit feebly, to imprison the flux ejected by the grains embedded in it. The intergrain matrix is not capable of

effectively confining or containing the flux expelled by the grains because $j_{c\perp m}$ and the cross-section of the intergrain network are too small. This statement is now illustrated in a simple model.

We visualize, for simplicity a grain in the form of a long thin slab embedded in an infinite matrix of uniform critical current density $j_{c\perp m}$. Upon field cooling in $H_{//}$, the Meissner effect has caused a magnetic flux $\Delta\Phi_g = 2X_g Y_g |\Delta\langle B_z \rangle_g|$ to be expelled from the grain. Here $2X_g$ is the thickness of the grain of width $Y_g \gg X_g$. By Faraday's Law of induction, the magnetic flux expelled from the grain is trapped by the persistent currents induced to circulate around the grain in the surrounding matrix or medium. Integrating Maxwell's eqn $dB_z/dx = \pm\mu_0 j_{c\perp m}$ leads to $\Delta B_{zm} = \pm\mu_0 j_{c\perp m} X_m$ where X_m is the trapping depth in the matrix as shown in the accompanying sketch



By conservation of flux,

$$\Delta\Phi_g = 2X_g Y_g |\Delta\langle B_z \rangle_g| = \Delta\Phi_m = 2X_m Y_m \frac{\Delta B_{zm}}{2} = X_m^2 Y_m \mu_0 j_{c\perp m} \quad (6.32)$$

where the 2 in the denominator appears from averaging the linear $B(x)$ profile over the distance X_m . We recall that our measurements show $B_m = \mu_0 H_m = \mu_0 j_{cm} (R_o - R_i) = 5 \text{ mT}$.

$$X_m^2 = \frac{2X_g |\Delta < B_z >_g| (R_o - R_i)}{\mu_0 j_{cm} (R_o - R_i)} = \frac{2X_g |\Delta < B_z >_g| (R_o - R_i)}{B_m} \quad (6.33)$$

With $\Delta < B_z >_g = 1 \text{ mT}$, $X_g = 10^{-5}$ meter eqn 6.33 leads to $X_m = 10^{-4}$ meter hence an order of magnitude greater than X_g .

Thus in order that the magnetic flux expelled by the grains remain trapped by the intergrain network and not be allowed to escape from the wall, the ratio of matrix to grain volume should be ≈ 10 . Since the grains occupy the most of the volume of the wall it is not surprising that magnetic flux they expel is released by the wall essentially unopposed by the matrix.

In our measurements, the outer tube was field cooled after the inner tube was field cooled. Consequently, the axial flux density permeating the annular space and the wall of the outer tube starts at $\mu_0 H_{//}$ when the outer tube begins to cool. We observe only a negligible rise in $< B_z >_{\text{annular}}$ when the cooling takes place. This is as expected in view of the foregoing.

The preceding detailed discussion of the flux expulsion behaviour upon field cooling and the consequent configurations of the $B(r)$ profiles serves to set the stage for our central enterprise. Thus we face the following scenario when we initiate flux line cutting and cross-flow. The flux densities in the cavity, and in the annular space are essentially equal to each other and equal to the static applied flux density $\mu_0 H_{//}$. Because of the Meissner effect, The flux density in the grains is fractionally lower than $\mu_0 H_{//}$. The detailed flux density distribution in the matrix (intergrain space) is probably complicated but its spatial average is probably close to $\mu_0 H_{//}$.

6.9. Summary

From a variety of magnetic and flux trapping measurements on the BiSCCO inner tube we have extracted the following information.

The intergrain depinning critical current density,

$$\langle j_{c\perp m} \rangle = \frac{H_{*m}}{(R_o - R_i)} = 2.5(10^6) \frac{A}{m^2} \quad (6.34)$$

and is not sensitive to the flux density B in the low field range of our measurements.

The penetration field into the grains, H_{*g} is a factor of ≈ 5 larger than H_{*m} , the penetration field across the wall. Taking the average radius R of the grains as 10 microns this penetration field yields an intragrain depinning critical current density,

$$\langle j_{c\perp g} \rangle = \frac{H_{*g}}{R} = 2.5(10^9) \frac{A}{m^2} \quad (6.35)$$

We also find that $j_{c\perp g}$ is not sensitive to B in the low field range.

CHAPTER 7

Flux Line Cutting and Cross Flow in the BiSCCO Tube

7.1. Introduction

From our measurements of the magnetic behaviour of the BiSCCO tube as the axial magnetic field $H_{//}$ is impressed and reversed and from the data on its flux trapping properties by the H_{cool} and H_{cycle} procedures we have been able to estimate the penetration fields H_{*m} and H_{**m} across the wall and the penetration fields H_{*g} and H_{**g} into the grains. We have seen that these parameters, in the framework of the critical state model, can provide estimates of the critical current densities for depinning of the flux lines in the intergrain network (matrix) ($j_{c\perp m}$) and inside the grains ($j_{c\perp g}$). Further the ratios H_{**m}/H_{*m} and H_{**g}/H_{*g} indicate that both $j_{c\perp m}$ and $j_{c\perp g}$ are not sensitive to the magnitude of the magnetic flux density B in the low field range we have explored.

From previous investigations of flux line cutting in monolithic classical low T_c type II superconductors carried out in our laboratory it has been established that in order to obtain a clear signature of the occurrence of this phenomenon, it is necessary that ΔH_{total} , the increment in the magnitude of the applied field should approach and, better yet, exceed the penetration field H_* , thus,

$$\Delta H_{\text{total}} = \sqrt{H_{\phi}^2 + H_{//}^2} - H_{//} \geq H. \quad (7.1)$$

with the static axial field $H_{//} = H$.

The toroidal magnet coil embracing the wall of the tube must consequently be designed to achieve this goal. The reader will appreciate however that the space available inside the cavity of the tube imposes severe constraints . This space must accommodate both, (i) a crucial inner pickup coil of adequate area-turn product and (ii) a toroidal magnet coil with sufficient number of windings of appropriate cross section. Clearly favoring one of these entities entails sacrifices for the other, hence a judicious compromise must be struck. The reader will judge from the results reported in this chapter that our design and construction attained the main objective. We will present clear evidence of cutting and cross-flow of flux lines in the weak link regime of the BiSCCO tube . Our toroidal magnet coil however was incapable of effectively probing flux line cutting and cross-flow in the interior or bulk of the grains . Stated mathematically, in our work in the BiSCCO tube ,

$$\Delta H_{\text{total}} \geq H_{\bullet m} \quad \text{but} \quad \Delta H_{\text{total}} < H_{\bullet g} \quad (7.2)$$

We wish to mention now however that in our investigation of the YBCO tube , to be reported later in this thesis, both of these goals were attained .

7.2. Results and Discussion

7.2.1. Experimental Basis for the Main Message

The evolution of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{annular}}$ and $\langle B_z \rangle_{\text{wall}}$, the spatial averages of the magnetic flux density in the cavity, the annular space between the two tubes , and in the wall of the inner tube , as an azimuthal magnetic field $\langle H_\phi \rangle$ is impressed and removed in static ambient axial magnetic fields $H_{//}$, is displayed in Figs 7.1 , 7.2 and 7.3 for various $H_{//}$. These data curves were selected from our inventory of measured curves to illustrate the observed phenomena over the range we have explored .

The rises observed for $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ as H_ϕ is impressed both carry the clear signature of cross-flow of flux lines . The increase of axial flux in these two reservoirs indicate that the wall is releasing flux lines into the these enclosures . The concomitant drop in the axial flux density in the wall also testifies that some of these quantity is vanishing in that medium as the adjacent reservoirs are experiencing a growth of axial flux.

However , these events are taking place while the magnetic fields,

$$\bar{H}_{\text{total}}(R_i) = \hat{\phi} H_\phi(R_i) + \hat{z} \frac{\langle B_z \rangle_{\text{hole}}}{\mu_0} \quad (7.3)$$

$$\bar{H}_{\text{total}}(R_o) = \hat{\phi} H_\phi(R_o) + \hat{z} \frac{\langle B_z \rangle_{\text{annular}}}{\mu_0} \quad (7.4)$$

along the inner and outer surfaces of the wall are increasing in magnitude , thereby nucleating and "pumping" helical flux lines into the wall. By conservation of flux , every flux line introduced into the wall by an increment of $|\bar{H}_{\text{total}}(R_i)|$ and $|\bar{H}_{\text{total}}(R_o)|$ removes a quantum of axial flux from the corresponding reservoir. Further, as shown in Fig 7.4 , azimuthal flux is indeed entering and accumulating in the wall as $\langle H_\phi \rangle$ is impressed . We note that the evolution of the $\langle B_\phi \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ is not only "traditional" but also relatively insensitive to the presence of $H_{//}$ and the dramatic events just

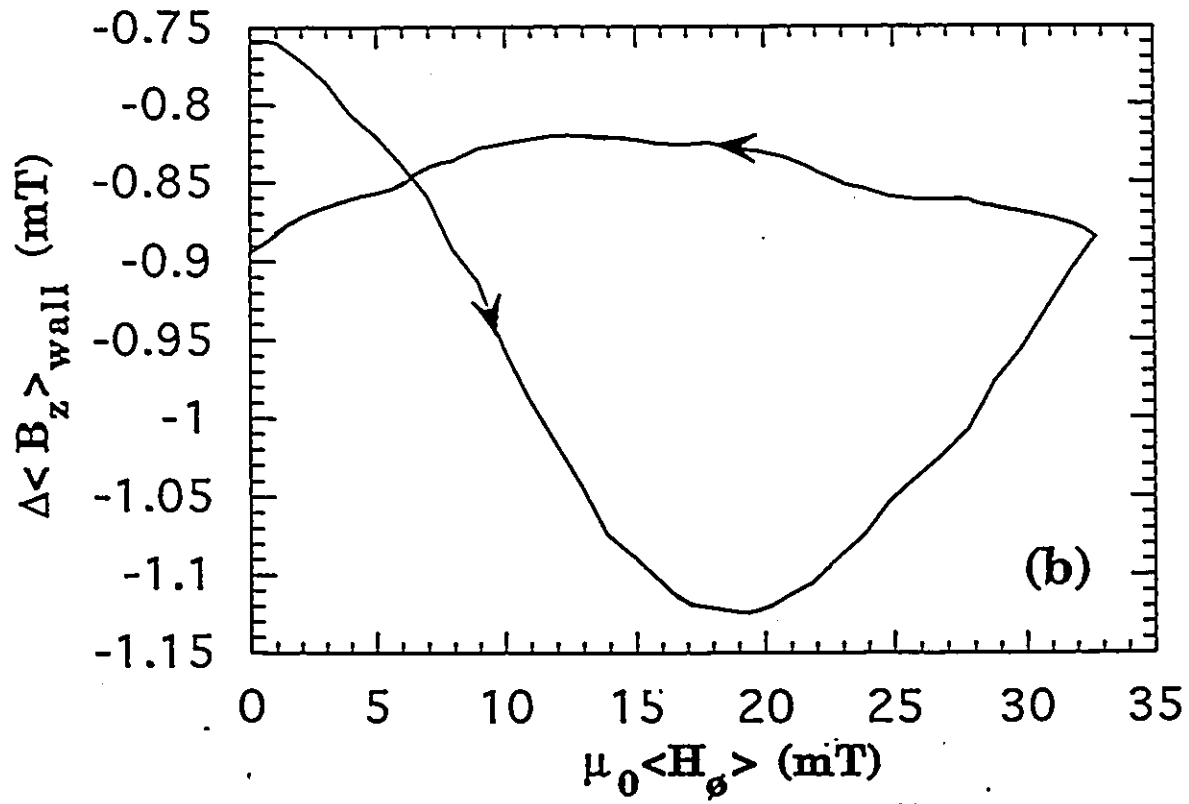
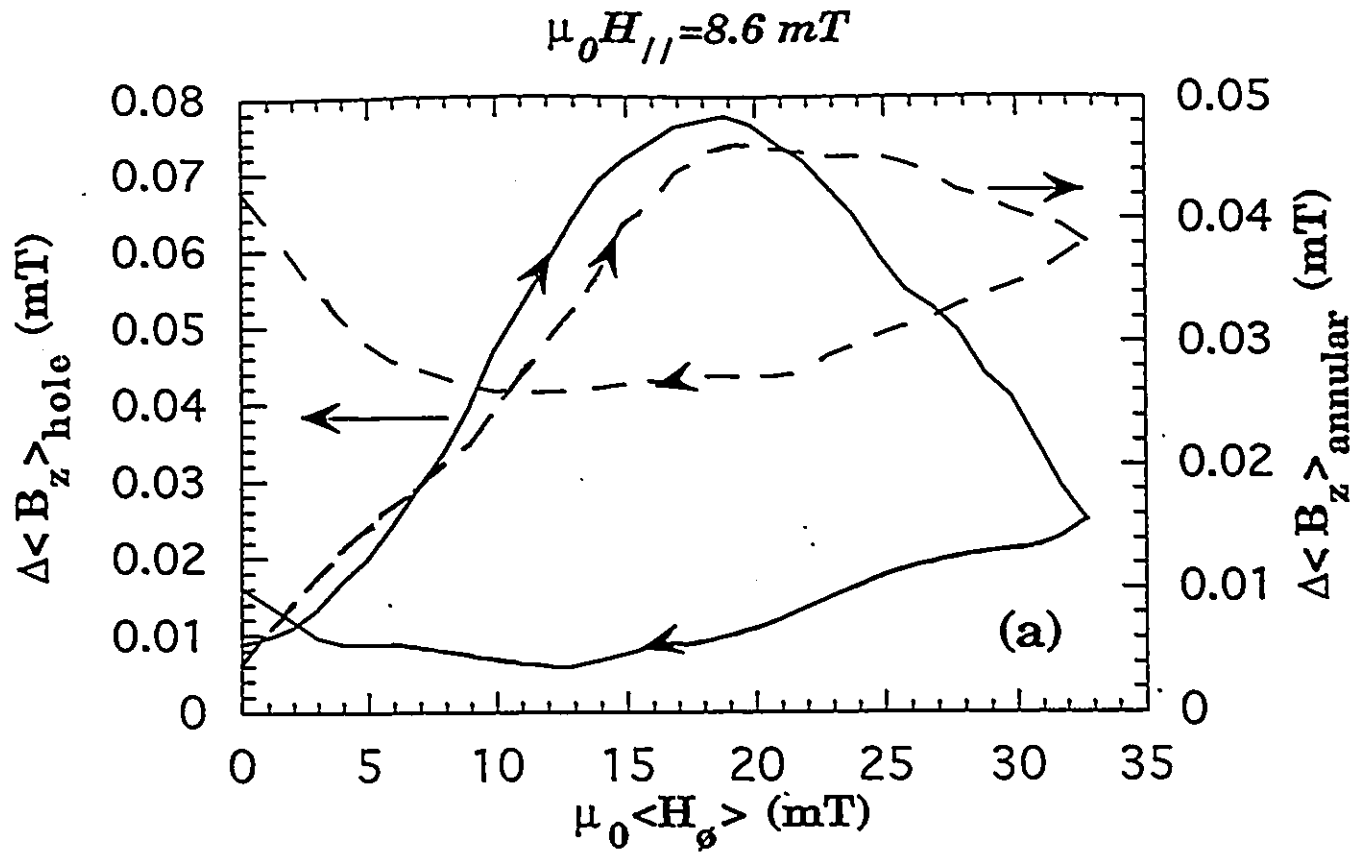


Fig 7.1 Evolution of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{annular}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ when $\mu_0 H_{//} = 8.6 \text{ mT}$.

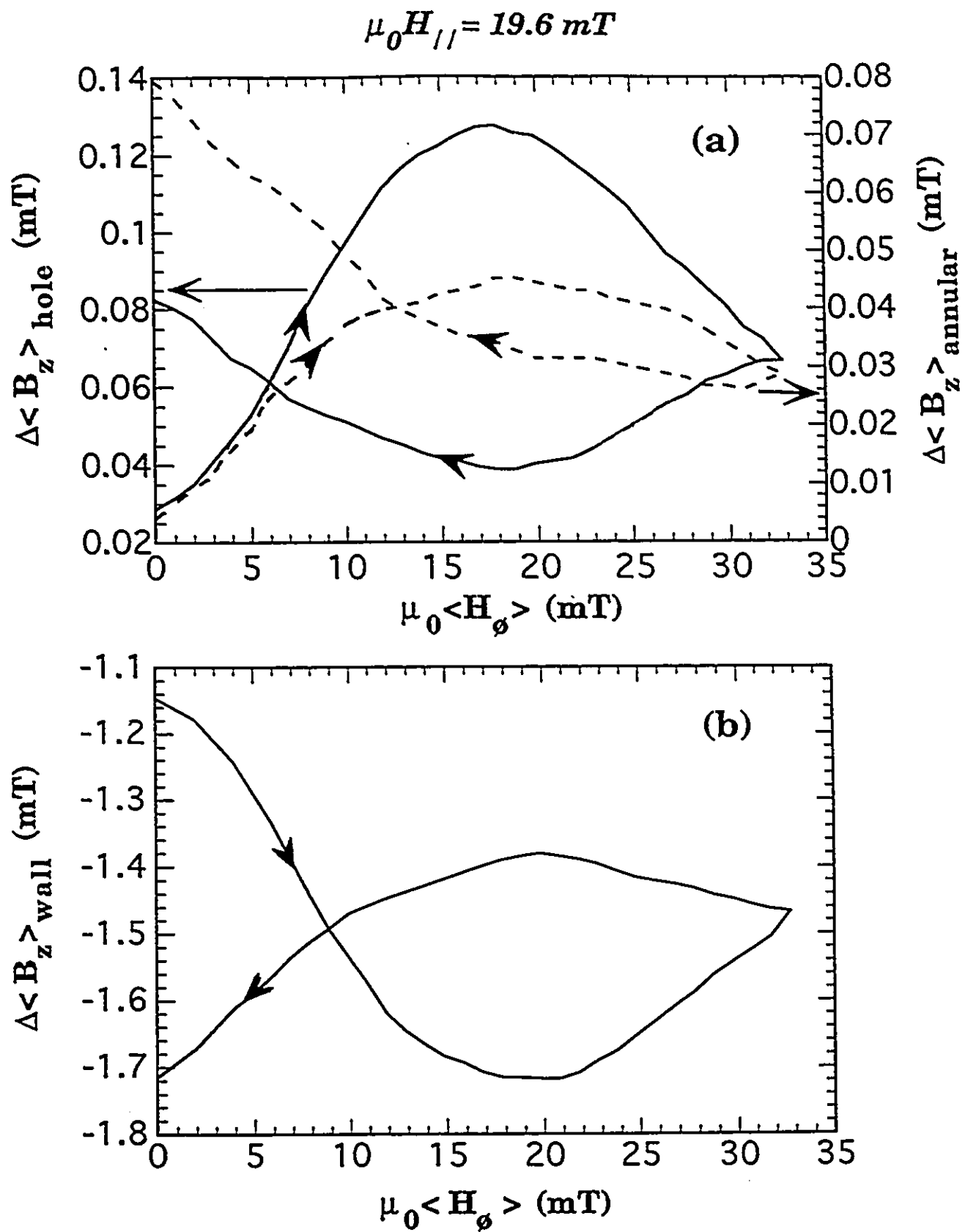


Fig 7.2 Evolution of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{annular}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ when $\mu_0 H_{//} = 19.6 \text{ mT}$.

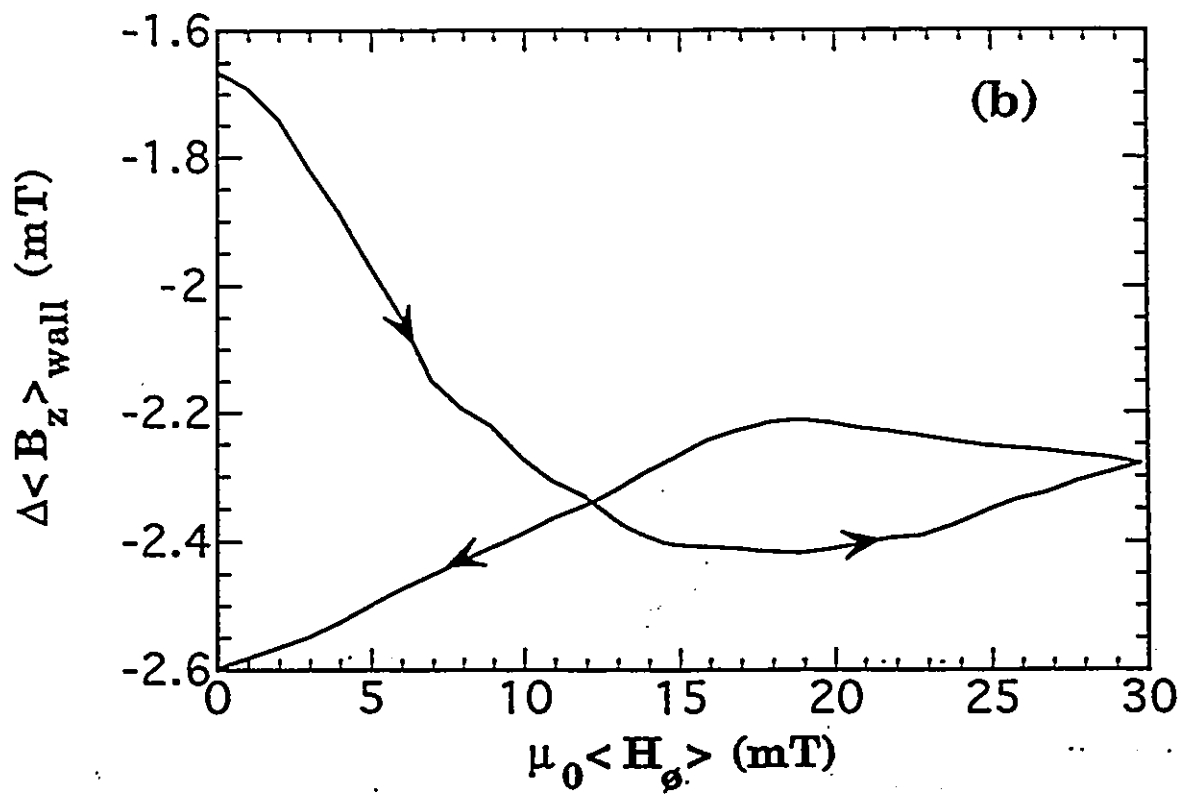
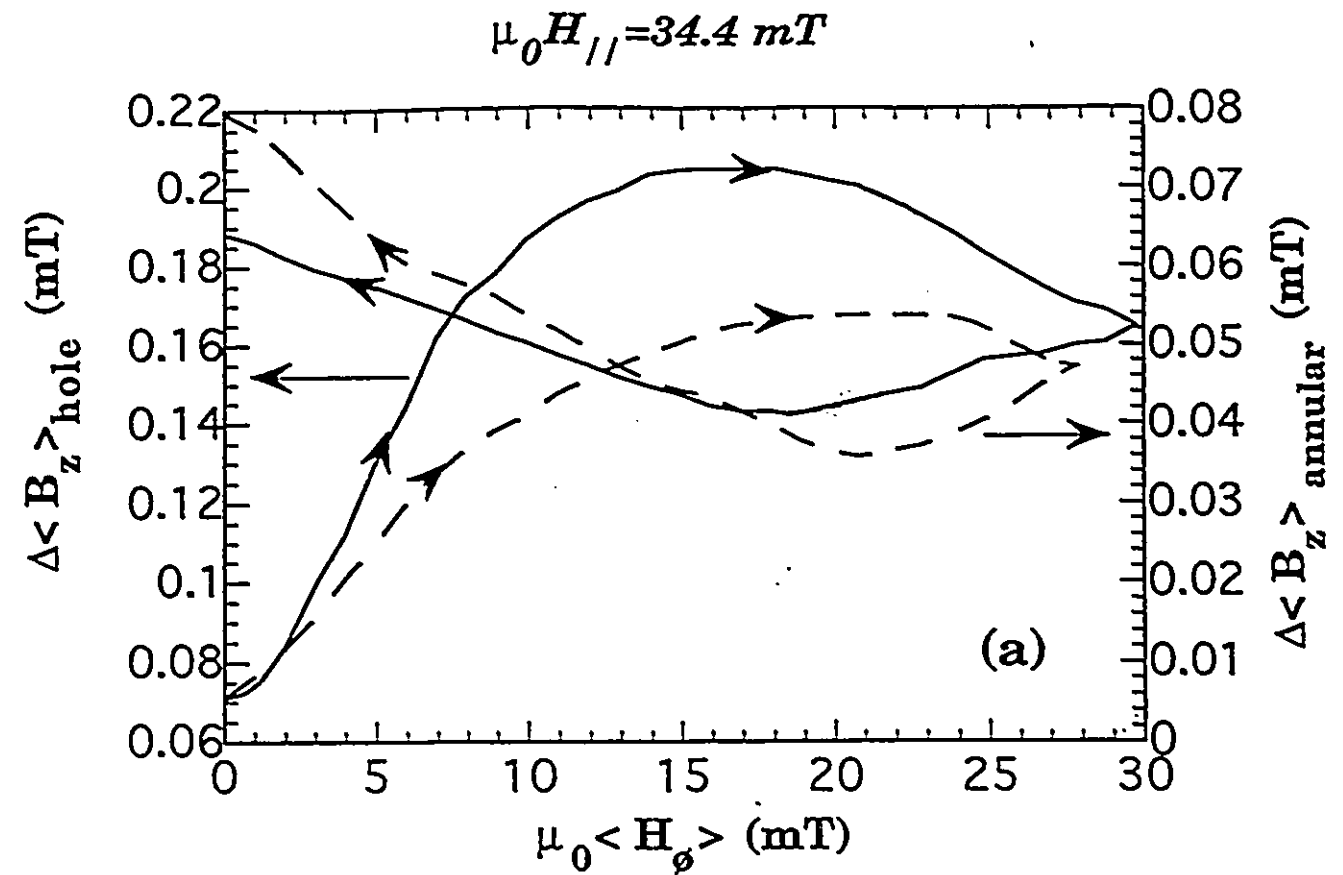


Fig 7.3 Evolution of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{annular}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ when $\mu_0 H_{//} = 34.4 \text{ mT}$.

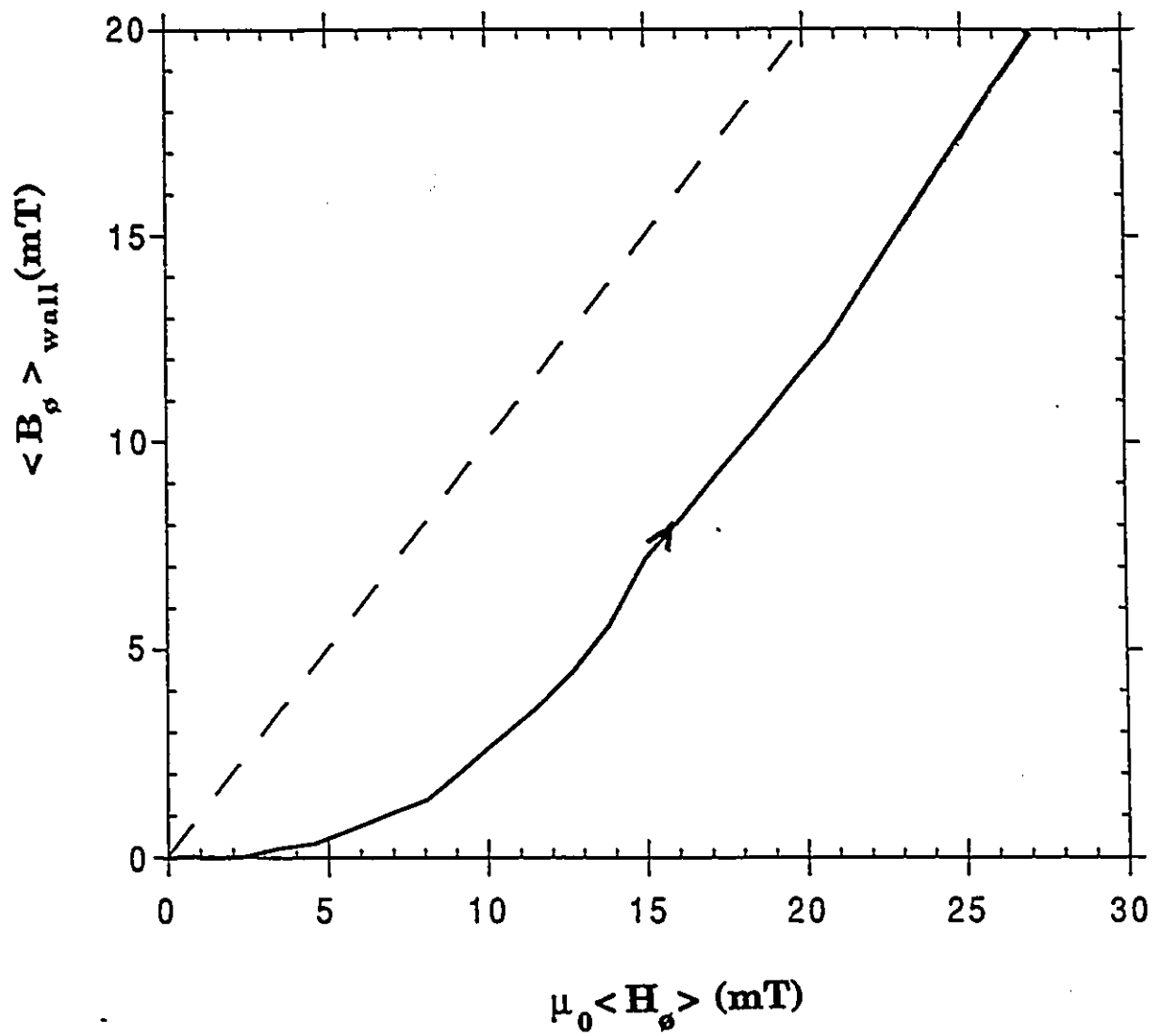


Fig 7.4 Typical evolution of azimuthal magnetic flux density in the wall vs $\langle H_\phi \rangle$ in $0 \leq \mu_0 H_{//} \leq 60 \text{ mT}$.

described. Indeed the presence of $H_{//}$ facilitates the penetration of the azimuthal field into the wall.

The behaviour of $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{annular}}$, $\langle B_z \rangle_{\text{wall}}$ and $\langle B_\phi \rangle_{\text{wall}}$ as H_ϕ is subsequently reduced to zero is also "traditional" , hence not especially remarkable . Here, the evolution of these four quantities corresponds qualitatively to the expectations from straight-forward migration of flux line populations out of the wall into the adjacent reservoirs as the ambient magnetic "pressures" (i.e. $|\bar{H}_{\text{total}}(R_i)|$ and $|\bar{H}_{\text{total}}(R_o)|$) are diminished and released. A careful quantitative analysis of these "return" curves can however , reveal that flux line cutting and cross-flow is also taking place under these circumstances. The agreement here is less convincing since it emerges from elaborate computations influenced by the approximations introduced, of necessity, in these computations. Consequently we will focus on the phenomena where H_ϕ is impressed since here the message needs only careful consideration of basic physics to be read. The theoretical calculations will merely provide a quantitative perspective and confirmation for the central theme which we now present.

7.2.2. Main Message of the Thesis

It is useful at this juncture to identify the basic "tenets" or "axioms" for our "theorem".

First it is crucial to emphasize that the walls of the tubes are each continuous superconducting entities although their detailed structure is known to be intricate , inhomogenous , anisotropic and granular. The evidence for the continuity or connectedness of the walls comes from the observations reported in the previous chapter. There we noted that persistent currents, denoted I_{cm}^* ,

can be made to circulate around the circumference of the wall and thereby establish a difference, $\Delta H_{//} = I_{cm}^*(B)$ between the axial field in the cavity of the inner tube and the ambient axial field $H_{//}$, viz. ,

$$\Delta H_{//} = I_{cm}^*(B) = \frac{\langle B_z \rangle_{hole}}{\mu_0} - H_{//} \quad (7.5)$$

The following statements are universally accepted. Magnetic flux in type II superconductors exists in the form of discrete entities each carrying a unit or quantum of flux, $\Phi_0 = h/2e$. Further, flux lines can be added to or subtracted from a volume , already superconducting, only via entry or exit through the surfaces.

Every helical flux line nucleated along the inner surface R_i of the inner tube removes a quantum of axial flux from the cavity (reservoir 1) and transfers it to the wall . This helical flux line also injects an amount ,

$$\Delta\Phi_\phi(\bar{R}_i) = n_c(\bar{R}_i)\Phi_0 Z \quad (7.6)$$

of azimuthal flux into the wall. Here $n_c(\bar{R}_i)$ denotes the number of the circuits the helical flux line makes per unit length of the wall of radius R_i and length Z . The superscript arrows indicate the direction of the displacement of the flux line away from the cavity hence outward from the axis of the tube.

Every helical flux line exiting from the wall through its inner surface removes a quantum of axial flux from the wall and releases it into the cavity. This helical flux line also removes an amount,

$$\Delta\Phi_\phi(\bar{R}_i) = n_c(\bar{R}_i)\Phi_0 Z \quad (7.7)$$

of azimuthal flux from the wall. Here $n_c(\vec{R}_i)$ has the same meaning as above but the superscript arrow indicates that the displacement is towards the axis of the tube.

Every helical flux line nucleated along the outer surface R_o of the inner tube removes a quantum of axial flux from the annular space (reservoir 2) and transfers it to the wall. This helical flux line also injects an amount,

$$\Delta\Phi_\phi(\vec{R}_o) = n_c(\vec{R}_o)\Phi_0 Z \quad (7.8)$$

of azimuthal flux into the wall. Again the superscript arrows indicate the direction of displacement of the flux line towards the axis of the tube

Finally, every helical flux line exiting from the wall of the inner tube through its outer surface removes a quantum of axial flux from the wall and releases it into the annular space. This helical flux line also removes an amount,

$$\Delta\Phi_\phi(\vec{R}_o) = n_c(\vec{R}_o)\Phi_0 Z \quad (7.9)$$

of azimuthal flux from the wall. Again the superscript arrows indicate the direction of displacement of the flux line away from the axis of the tube.

An increment, $\Delta < B_\phi >_{\text{wall}}$, of the azimuthal flux density permeating the wall produced by an increment in the applied azimuthal fields $H_\phi(R_i)$ and $H_\phi(R_o)$ can then be written,

$$\begin{aligned} \Delta\Phi_{\phi, \text{wall}} &= \Delta < B_\phi >_{\text{wall}} Z(R_o - R_i) \\ &= \Phi_0 Z \{ [\Delta N(\vec{R}_i)n_c(\vec{R}_i) - \Delta N(\vec{R}_i)n_c(\vec{R}_i)] \} \end{aligned}$$

$$+[\Delta N(\bar{R}_o)n_c(\bar{R}_o)-\Delta N(\bar{R}_o)n_c(\bar{R}_o)] \} \quad (7.10)$$

where $\Delta N(\bar{R}_i)$, $\Delta N(\bar{R}_i)$, $\Delta N(\bar{R}_o)$, $\Delta N(\bar{R}_o)$ denote the number of flux lines traversing the corresponding surfaces and the superscript arrow pointing to the right (left) indicates displacement away from (towards) the axis of the tube.

Since , as expected , $\langle B_\phi \rangle_{\text{wall}}$ is observed to increase when $\langle H_\phi \rangle$ is impressed , eqn 7.10 will obviously be satisfied, when both quantities in square brackets are positive or only one is positive and the other zero. Alternatively , if one of the square bracketed quantities is negative , then the other must compensate to yield a net positive total.

It is useful to express the condition that a square bracketed quantity in eqn 7.10 be positive by the following corresponding inequalities ,

$$\frac{n_c(\bar{R}_i)}{n_c(\bar{R}_i)} > \frac{\Delta N(\bar{R}_i)}{\Delta N(\bar{R}_i)} \quad (7.11)$$

and

$$\frac{n_c(\bar{R}_o)}{n_c(\bar{R}_o)} > \frac{\Delta N(\bar{R}_o)}{\Delta N(\bar{R}_o)} \quad (7.12)$$

We now turn our attention to the "traffic" of the axial component of the flux lines across the inner and outer surfaces of the wall of the inner tube .

We visualize that a change , $\Delta \langle B_z \rangle_{\text{hole}}$, of the axial flux density in the cavity caused by an increment $\Delta H_\phi(R_i)$ of the azimuthal field is the net result of ,

(i) the nucleation of $\Delta N(\bar{R}_i)$ flux lines along the surface R_i and their penetration into the wall and , concurrently or in alternation,

(ii) a release of $\Delta N(\bar{R}_i)$ flux lines by the wall into the cavity. Thus the increase of axial flux density in the cavity can be written,

$$\Delta \Phi_{z, \text{hole}} = \Delta \langle B_z \rangle_{\text{hole}} \pi R_i^2 = \Phi_0 \{ \Delta N(\bar{R}_i) - \Delta N(\bar{R}_i) \} \quad (7.13)$$

We stress that the observed increase of $\langle B_z \rangle_{\text{hole}}$, signifies that ,

$$\frac{\Delta N(\bar{R}_i)}{\Delta N(\bar{R}_i)} > 1 \quad (7.14)$$

In other words , more flux lines are leaving the wall than entering it across the surface R_i . In the absence of flux line cutting and cross-flow we would expect the opposite to occur since an increment of $|\bar{H}_{\text{wall}}(R_i)|$ associated with an increment $\Delta H_\phi(R_i)$ will inject or pump flux lines into the wall thereby tending to cause $\langle B_z \rangle_{\text{hole}}$, the level in the inner reservoir to drop.

Similarly we visualize that a change , $\Delta \langle B_z \rangle_{\text{annular}}$, of the axial flux density in reservoir 2 brought about by an increment $\Delta H_\phi(R_o)$ of the azimuthal field is the net result of,

- (i) the nucleation of $\Delta N(\bar{R}_o)$ flux lines along the surface R_o and their penetration into the wall of the inner tube and concurrently or in alternation,
- (ii) a release of $\Delta N(\bar{R}_o)$ flux lines by the wall into the annular space. Thus the increase of axial flux density in reservoir 2 can be written,

$$\Delta \Phi_{z, \text{annular}} = \Delta \langle B_z \rangle_{\text{annular}} \pi (R_{i2}^2 - R_o^2) = \Phi_0 \{ \Delta N(\bar{R}_o) - \Delta N(\bar{R}_o) \} \quad (7.15)$$

We stress that the observed increase of $\langle B_z \rangle_{\text{annular}}$ requires that ,

$$\frac{\Delta N(\bar{R}_o)}{\Delta N(\bar{R}_o)} > 1 \quad (7.16)$$

In eqn 7.15 we are neglecting the minute amount of axial flux "pushed" across the inner surface R_{i2} of the outer tube by the rise of $\langle B_z \rangle_{\text{annular}}$. (It may also be appropriate to again indicate that the outer tube does not experience the azimuthal field.)

The above inequality states that more flux lines are leaving the wall than entering it across the surface R_o . Again we emphasize that, in the absence of flux line cutting and cross-flow, the opposite behaviour is expected to take place since the increments $|\bar{H}_{\text{wall}}(R_o)|$ associated with the increments of $\Delta H_\phi(R_o)$ are injecting or pumping flux lines into the wall thereby tending to cause the level of axial flux $\langle B_z \rangle_{\text{annular}}$ in the outer reservoir to drop.

Introducing the inequalities (eqns) 7.14 and 7.16, (which formally express the crucial and novel aspect of the observations displayed in Figs 7.1, 7.2 and 7.3) into the inequalities (eqns) 7.11 and 7.12 (which express the standard behaviour displayed in Fig 7.4) we obtain, the important statements that,

$$\frac{n_c(\bar{R}_i)}{n_c(\bar{R}_i)} > \frac{\Delta N(\bar{R}_i)}{\Delta N(\bar{R}_i)} > 1 \quad (7.17)$$

and

$$\frac{n_c(\bar{R}_o)}{n_c(\bar{R}_o)} > \frac{\Delta N(\bar{R}_o)}{\Delta N(\bar{R}_o)} > 1 \quad (7.18)$$

We emphasize that in the "construction" of "eqns" 7.17 and 7.18 we used only universally accepted concepts and definitions plus the following experimental observations that as $\langle H_\phi \rangle$ is initially impressed after field cooling of the tubes in a static $H_{//}$,

- (a) $\langle B_z \rangle_{\text{hole}}$ rises, (yields eqn 7.11)
- (b) $\langle B_z \rangle_{\text{annular}}$ rises (yields eqn 7.12), and

(c) $\langle B_\phi \rangle_{\text{wall}}$ increases (yields eqn 7.10)

"Eqn " 7.17 states that, (i) the number of flux lines leaving the wall at R_i is greater than that entering it, and (ii) the number of the circuits of the flux lines entering the wall at R_i is greater than that of the flux lines leaving it. For traffic of the nature of (ii) to take place, the flux lines entering the wall at R_i must traverse, hence, cut through the flux lines leaving through that surface.

"Eqn " 7.18 states that, (i) the number of flux lines leaving the wall at R_o is greater than that entering it, and (ii) the number of the circuits of the flux lines entering the wall at R_o is greater than that of the flux lines leaving it. For traffic of the nature of (ii) to take place, the flux lines entering the wall at R_o must traverse, hence, cut through the flux lines leaving through that surface.

Eqn 7.10 requires that cross flow must be taking place either at both surfaces concurrently or at least one of the surfaces.

It is perhaps illuminating to cast the argument in a different way. In order that two populations of helical flux lines countermove through a given surface without cutting as they traverse each other, it is necessary that the number of circuits $n_c(R)$ possessed by the members of both families, be equal, hence we write,

$$n_c(\bar{R}_i) = n_c(\bar{R}_i) = n_c(R_i) \quad (7.19)$$

$$n_c(\bar{R}_o) = n_c(\bar{R}_o) = n_c(R_o) \quad (7.20)$$

Equation 7.10 now reads,

$$\Delta \langle B_z \rangle_{\text{wall}} \frac{(R_o - R_i)}{\Phi_o} = n_c(R_i) [\Delta N(\bar{R}_i) - \Delta N(\bar{R}_i)] \\ + n_c(R_o) [\Delta N(\bar{R}_o) - \Delta N(\bar{R}_o)] \quad (7.21)$$

We have seen however that the square brackets in eqn 7.21 are negative (see eqns 7.14 and 7.16) when $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ are observed to increase. Consequently, the assumption that there is no flux cutting and cross-flow at both surfaces (eqns 7.19 and 7.20) is untenable. Thus our central message, that flux line cutting and cross-flow is a real process and is encountered in our experiments, is validated.

In the next subsection we present an inventory of the salient features attesting to the phenomenon of flux cutting and cross-flow in the BiSCCO tube. Then we proceed to show that the flux line cutting we observed in the BiSCCO tube is quantitatively associated with the intergrain structure.

7.2.3. Inventory of Salient Features

In Fig 7.5 curves of the evolution of $\langle B_z \rangle_{\text{wall}}$ as H_ϕ is impressed are juxtaposed in a family picture to illustrate the role of $H_{//}$ on certain salient and important features of these curves, namely on,

- (i) the depth of the valley traced by the locus of $\langle B_z \rangle_{\text{wall}}$ and,
- (ii) the horizontal position (value of $\langle H_\phi \rangle$) when the bottom or minimum of the valley is traversed.

Note that in Fig 7.5 the baselines have been shifted so that all the curves have the same vertical starting level. This is done to better bring out the effect of $H_{//}$. In this format, the depth of the valleys is measured relative to the

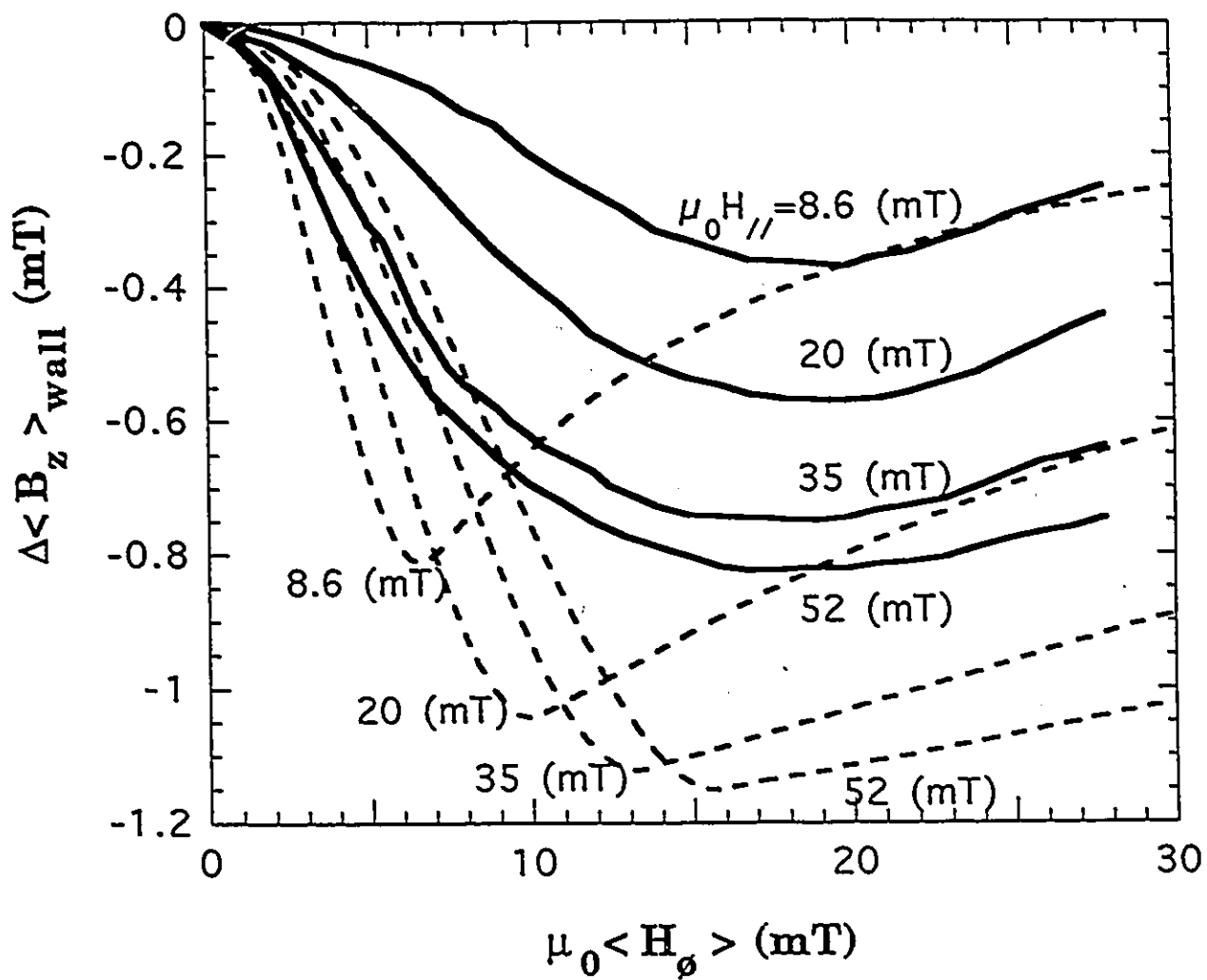


Fig 7.5 Displays a family of measured (solid lines) and corresponding calculated (dashed lines) curves of the evolution of $\langle B_z \rangle_{\text{wall}}$ as $\langle H_\phi \rangle$ is impressed.

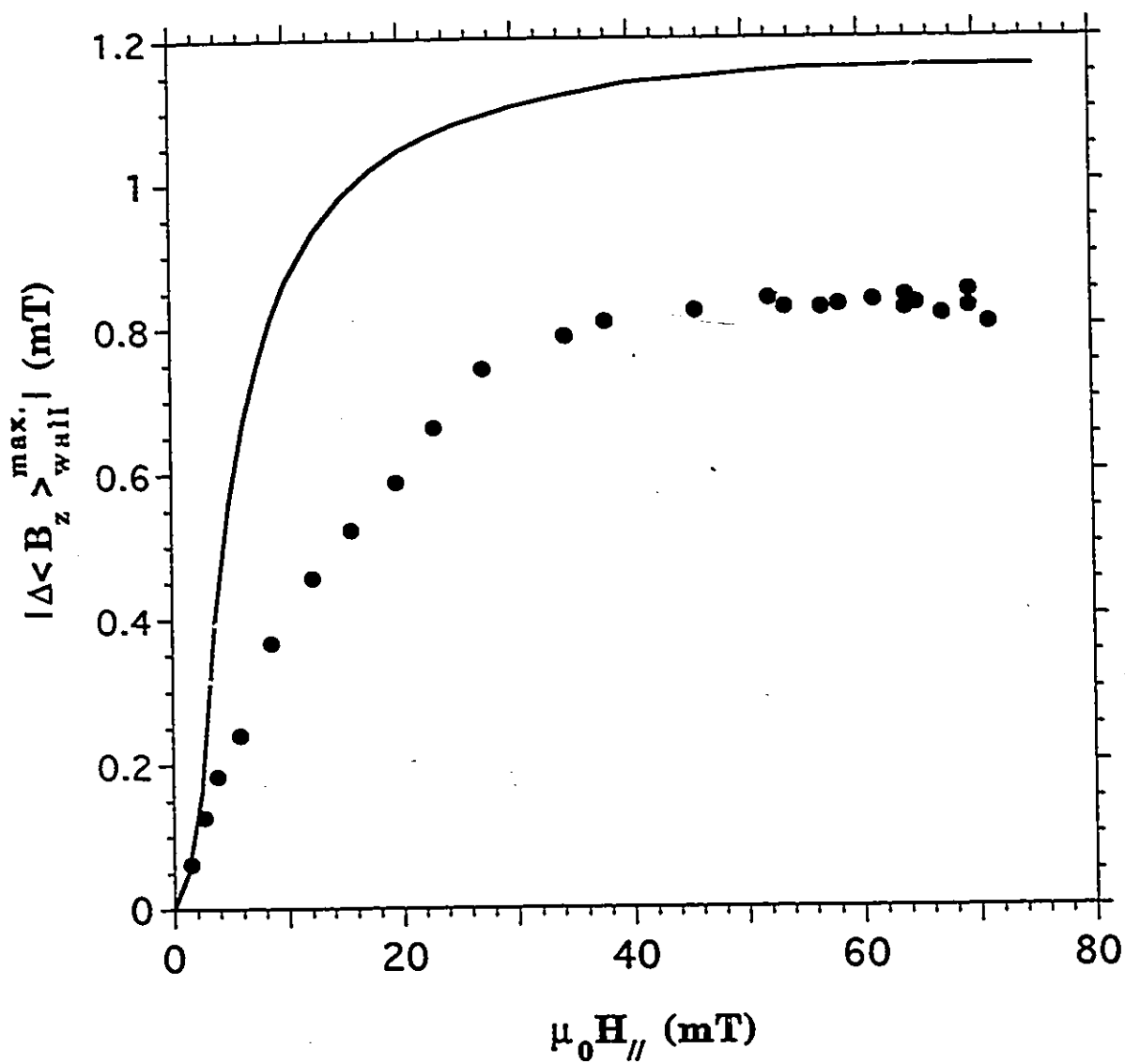


Fig 7.6 The data points display the maximum decrease of $\langle B_z \rangle_{wall}$ ("depth of valleys ") vs the static $H_{//}$ present as $\langle H_\phi \rangle$ was impressed. The solid curve is theoretical.

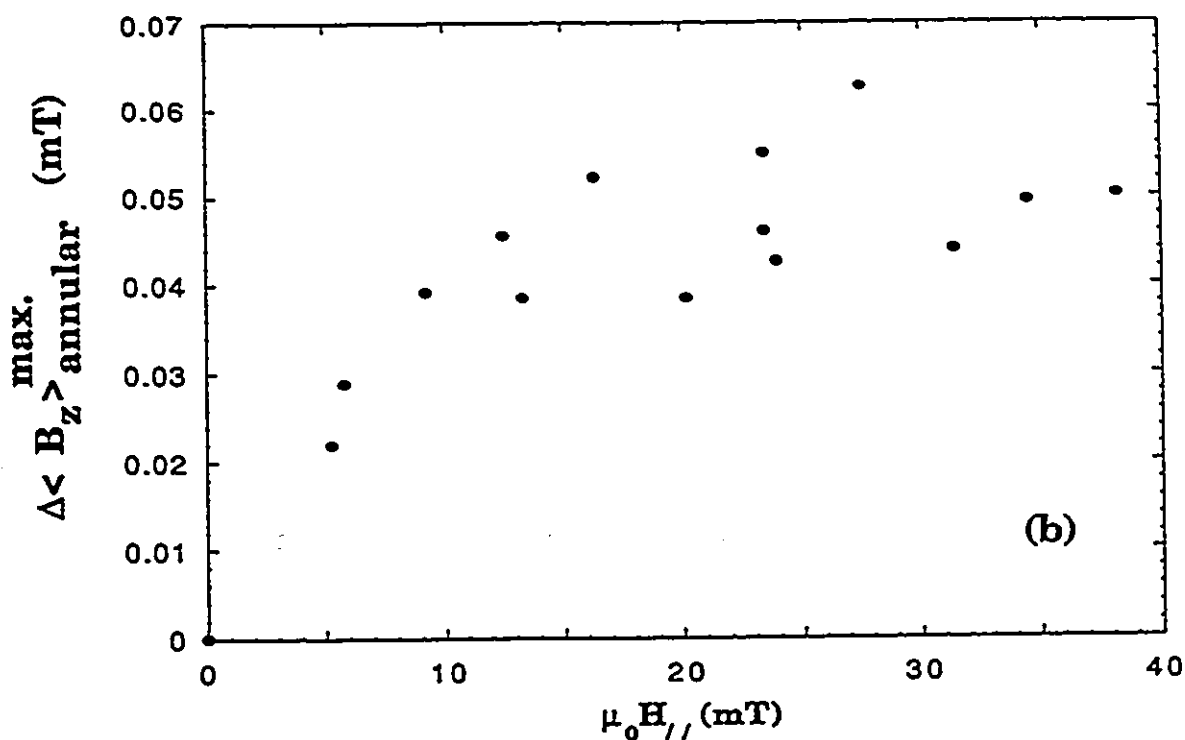
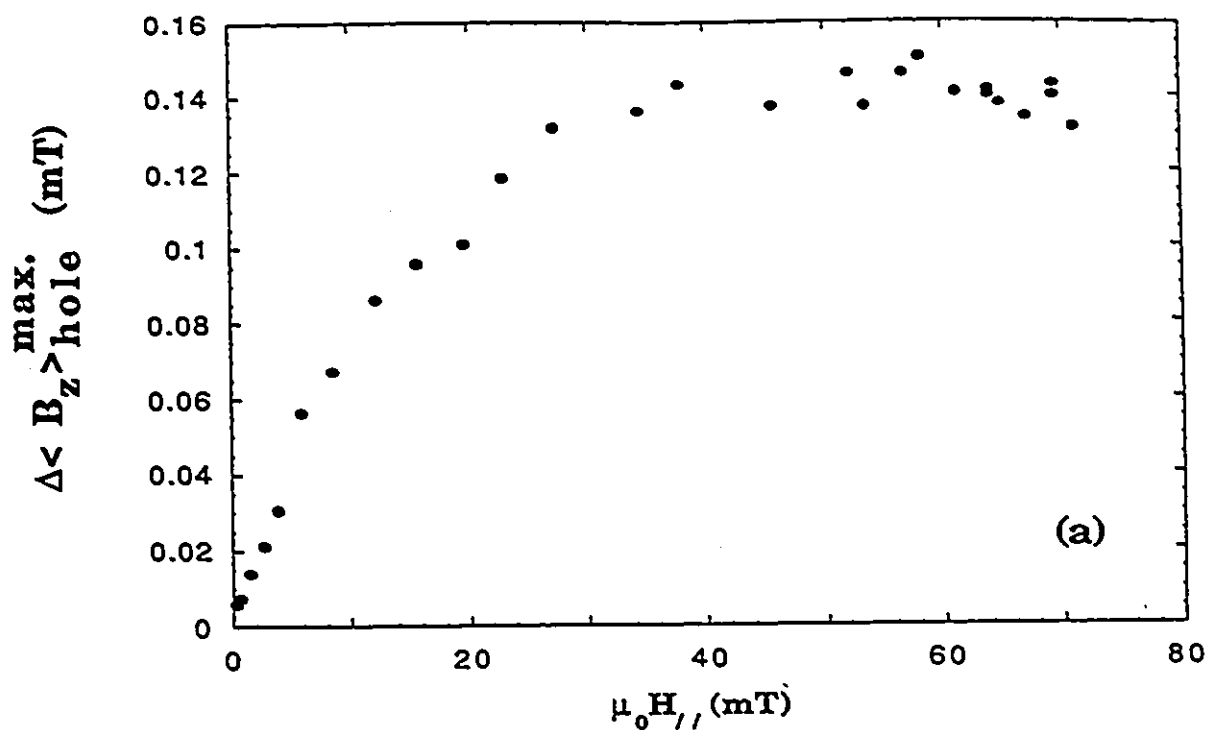


Fig 7.7 Displays the maximum increase of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ ("height of peaks") vs the static $H_{//}$ as $\langle H_{\perp} \rangle$ was impressed.

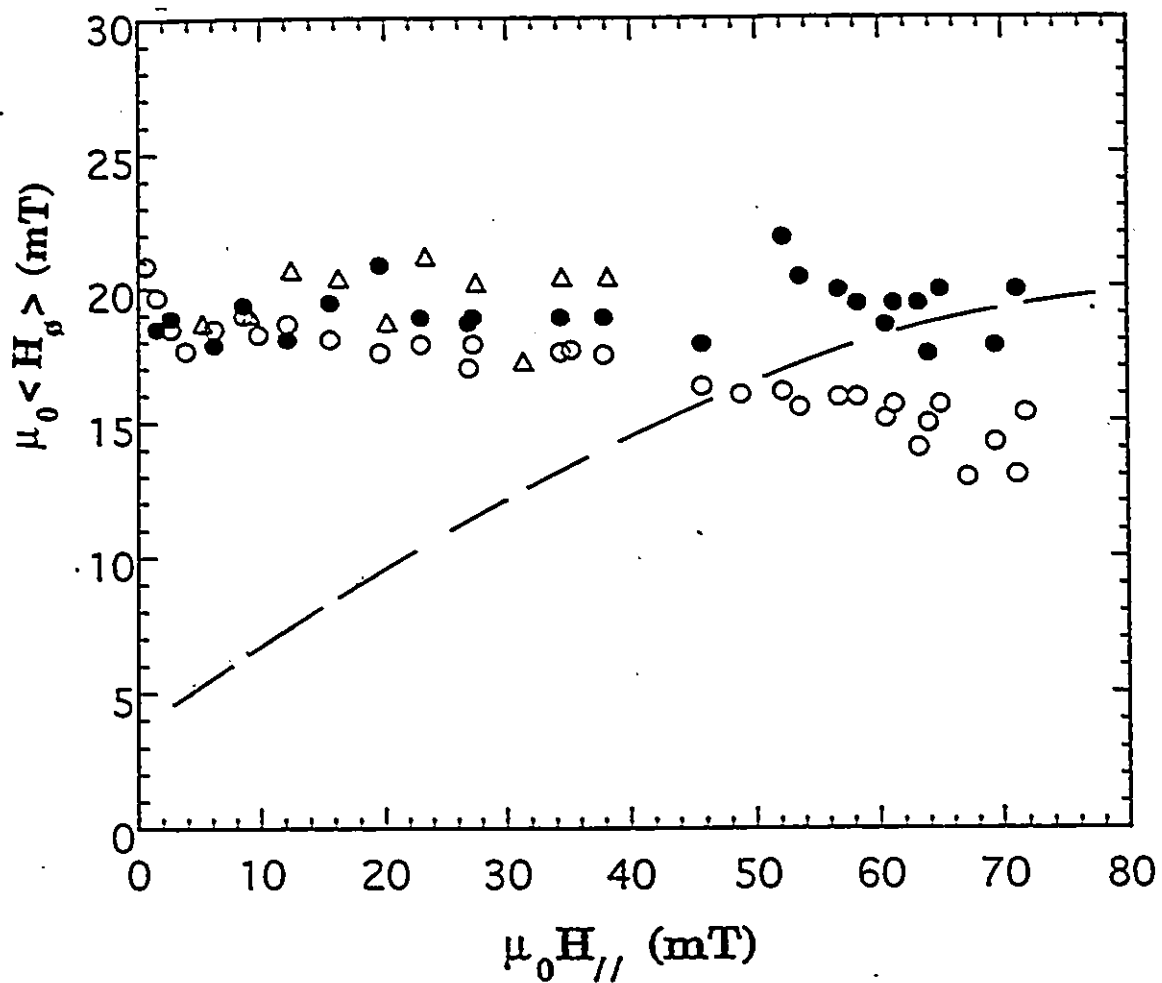


Fig 7.8 The O , Δ and $\bullet$ data points display the value of $\langle H_\phi \rangle$ (the "position") of the peaks for $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ and the valley in $\langle B_z \rangle_{\text{wall}}$ vs the static $H_{//}$ present when these are generated by impressing $\langle H_\phi \rangle$. The dashed curve is theoretical.

initial superconducting state , hence after the field cooling and after the accompanying Meissner effect has occurred. We note that , consequently, the starting level or baseline is not $\mu_0 H_{//}$ but ,

$$\langle B_z \rangle_{\text{wall}}^i = \mu_0 H_{//} - \Delta \langle B_z \rangle_{\text{wall}}^{\text{F.C.}} \quad (7.22)$$

since the axial flux density permeating the wall diminished by an amount, $\Delta \langle B_z \rangle_{\text{wall}}^{\text{F.C.}}$, due to the Meissner effect during field cooling in $H_{//}$. (see Fig. 6.8(b))

$|\Delta \langle B_z \rangle_{\text{wall}}^{\text{max}}|$, the magnitude of the depth of the valleys (relative to $\langle B_z \rangle_{\text{wall}}^i$) is plotted vs the corresponding static $H_{//}$ in Fig 7.6 . This quantity can readily be evaluated in the theoretical curves where it is seen to be determined by the appropriate first penetration field H_c . Consequently, it is important in linking these observations to the intergrain critical current densities.

The height of the peaks traced by $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ as $\langle H_\phi \rangle$ is impressed is plotted vs the corresponding static $H_{//}$ in Fig 7.7 (a) and (b) and denoted $\Delta \langle B_z \rangle_{\text{hole}}^{\text{max}}$ and $\Delta \langle B_z \rangle_{\text{annular}}^{\text{max}}$. The height of the former is measured relative to the starting value ,

$$\langle B_z \rangle_{\text{hole}}^i = \mu_0 H_{//} + \Delta \langle B_z \rangle_{\text{hole}}^{\text{F.C.}} \quad (7.23)$$

where $\Delta \langle B_z \rangle_{\text{hole}}^{\text{F.C.}}$ denotes the rise of the axial flux density in the cavity observed during field cooling (see Fig 6.8 (a)). This rise occurs because some of the flux expelled by the wall during field cooling (Meissner effect) is ejected into the cavity. The starting level for the evolution of $\langle B_z \rangle_{\text{annular}}$ is $\mu_0 H_{//}$ since the outer tube field cooled after field cooling of the inner tube .

Recalling that the cross sections have the following ratios ,

$$A_{\text{hole}}/A_{\text{wall}} = 2 \quad , \quad A_{\text{annular}}/A_{\text{wall}} = 5.5$$

it is of interest to note from Figs 7.6 and 7.7 that approximately one half of the axial flux vanishing from the wall appears in the cavity reservoir , and approximately one half appears in the surrounding annular reservoir.

The values of $\langle H_\phi \rangle$ when the peaks of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{annular}}$ are traced and when the minimum of the valley of $\langle B_z \rangle_{\text{wall}}$ is traversed , are plotted versus the corresponding static $H_{//}$ in Fig 7.8. These quantities are intimately related to the appropriate first penetration field H_* . Hence their value is important in our attribution of these phenomena to flux line cutting and cross-flow in the intergrain network.

7.3.Semi-Quantitative Theoretical Analysis

The main purpose of our theoretical analysis was to determine whether the phenomena we observed arose from flux line cutting and cross-flow inside the grains or in the weak link structure of the sintered BiSCCO tube. (i.e. intragrain or intergrain behaviour). Of necessity, numerous approximations and simplifications have been introduced in this exercise . The reader should be able to judge from our account whether our central conclusion is valid , namely, that we are witnessing in the data presented in this chapter manifestations of $j_{c\perp m}$ and $j_{c//m}$, the intergrain critical current densities.

We investigate the evolution of the magnetic flux density in the wall of the inner tube as H_ϕ is impressed in the framework of the Clem/Perez-Gonzalez phenomenological theory [19, 59] . This theoretical study is exploratory. A more thorough application of the theory to our (and other) observations is being pursued by other workers in our laboratory.

The first approximation we introduce in this exercise is geometric. The development of the Clem equations for cylindrical geometry presents a formidable computational problem. Clem and his collaborators , skilled theoreticians and experienced programmers have addressed flux line cutting in cylindrical geometry either in the limit of weak penetration or focused on the special case of thin walled tubes, hence converted these problems to the more tractable planar geometry . We also regard the wall as a slab of thickness $R_o - R_i = 2X$, infinite along the azimuthal direction , hence replace the unit vector $\hat{\phi}$ by the unit vector $\hat{y}$. The axial field $H_{//}$ is directed along the z axis . This approximation is quite acceptable since the ratios of wall thickness to radius ratios for the BiSCCO tube ,

$$\frac{R_o - R_i}{R_i} = 0.230 \quad , \quad \frac{R_o - R_i}{R_o} = 0.186 \quad (7.24)$$

are small.

In the same spirit , we take the applied azimuthal fields $H_{\phi}(R_i)$ and $H_{\phi}(R_o)$ as equal to $\langle H_{\phi} \rangle$ their spatial average across the wall thickness . Again this is reasonable since , for the BiSCCO tube,

$$\frac{H_{\phi}(R_i)}{\langle H_{\phi} \rangle} = 1.110 \quad , \quad \frac{H_{\phi}(R_o)}{\langle H_{\phi} \rangle} = 0.904 \quad (7.25)$$

The axial magnetic flux densities in the cavity and the annular space as H_{ϕ} is impressed are taken as static and equal to each other . For convenience we also ignore the rise of the axial flux density in the cavity due to the Meissner effect during field cooling , and take ,

$$\langle B_z \rangle_{\text{hole}} = \mu_0 H_{//} = \langle B_z \rangle_{\text{annular}} \quad (7.26)$$

Since the maximum excursions of these boundary values relative to their initial values are small, except in the range of very weak $H_{//}$, these approximations are valid. After the variation of $\langle B_z \rangle_{\text{wall}}$ vs H_0 has been evaluated in the framework of these static axial boundary conditions, this information can be exploited to estimate the corresponding ebb and flow of the axial flux density in the two reservoirs.

The wall is regarded as isotropic and homogenous (monolithic) hence the granularity is ignored. Also since the Clem/Perez-Gonzalez theory does not incorporate the concept of the intrinsic (equilibrium) Abrikosov diamagnetism, we also take

$$\bar{H} = \bar{B}/\mu_0 \quad (7.27)$$

The Clem/Perez-Gonzalez formulation of the Maxwell-Ampere eqn $\nabla \times \bar{B} = \mu_0 \bar{j}$, in our planar geometry, reads,

$$\frac{\partial \bar{B}}{\partial x} = -\mu_0 j_{\perp} \quad (7.28)$$

and

$$\bar{B} \frac{\partial \theta}{\partial x} = \mu_0 j_{//} \quad (7.29)$$

where

$$\bar{B} = \hat{y}B_y(x,t) + \hat{z}B_z(x,t) \quad (7.30)$$

and θ is the angle subtended by $\bar{B}$ and the z axis. Their formulation of the Maxwell-Faraday eqn $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$, in this geometry, reads,

$$\frac{\partial E_{\perp}}{\partial x} = -E_{\parallel} \frac{\partial \theta}{\partial x} - \frac{\partial B}{\partial t} \quad (7.30)$$

$$\frac{\partial E_{\parallel}}{\partial x} = B \frac{\partial \theta}{\partial t} + E_{\perp} \frac{\partial \theta}{\partial x} \quad (7.31)$$

The boundaries of the wall of infinite height along z and infinite width along y are situated at $R_i = -X$ and $R_o = X$, hence the midplane of the infinite slab ($x=0$) is situated at $(R_i + R_o)/2$

Since we take $H_{\parallel}(R_i) = H_{\parallel}(R_o) = H_{\parallel}$ and $H_{\phi}(R_i) = H_{\phi}(R_o) = \langle H_{\phi} \rangle = H_y(-X) = H_y(X)$, the magnetic flux configurations $\bar{B}(x,t)/\mu_0$ are symmetric with respect to the midplane ($x=0$).

The relaxation or diffusion method of treating the above coupled differential equations requires considerable computer time beyond the scope of this thesis. We have therefore adopted an alternate approach.

The sequences of configurations of the magnetic flux density as H_{ϕ} (hence $H_y(X) = H_y(-X)$) is impressed are calculated on the assumption that they correspond to critical configurations and hence, the induced current densities, $j_{\perp}(x,t)$ and $j_{\parallel}(x,t)$ exist in the critical state. Thus eqns 7.28 and 7.29 read,

$$\frac{dB}{dx} = -\mu_0 j_{c\perp}(B(x)) \quad (7.32)$$

$$B \frac{d\theta}{dx} = \mu_0 j_{c\parallel}(B(x)) \quad (7.33)$$

In effect then, in the nomenclature introduced by Clem, we envisage the existence of CT (cutting-transport), T (transport) and O (passive) zones but ignore the possible appearance of C (pure cutting) zones.

To verify whether this major simplification leads to significant distortion or perturbation of the quasi-static ("quasi-final") sequences of the profiles generated as H_0 is slowly impressed , we proceed as follows.

Eqns 7.30 and 7.31 are rewritten to read,

$$\frac{\partial E_{\perp}(x)}{\partial x} = -E_{\parallel}(x) \frac{\partial \theta(x)}{\partial x} - \frac{\partial B(x)}{\partial B_s} \quad (7.34)$$

$$\frac{\partial E_{\parallel}(x)}{\partial x} = B(x) \frac{\partial \theta(x)}{\partial B_s} + E_{\perp}(x) \frac{\partial \theta(x)}{\partial x} \quad (7.35)$$

since in the framework we have adopted,

$$E_{\perp}(x,t) = E_{\perp}(x) \frac{dB_s}{dt} , \quad E_{\parallel}(x,t) = E_{\parallel}(x) \frac{dB_s}{dt} \quad (7.36)$$

$$\frac{\partial B}{\partial t} = \frac{\partial B}{\partial B_s} \frac{dB_s}{dt} , \quad \frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial \theta_s} \frac{d\theta_s}{dB_s} \frac{dB_s}{dt} \quad (7.37)$$

where B_s is the total magnetic flux density at the surface of the slab and θ_s is the angle B_s subtends with respect to the z axis. We note that the rates of changes with time have canceled out in eqns 7.34 and 7.35 since the sets of configurations of $B(x)$ and $\theta(x)$ as B_s and θ_s are incremented are regarded as quasi-static (critical).

The approach we have adopted is frequently exploited in the calculations of hysteresis losses in the pure flux line depinning regime, a computationally much simpler problem.

The boundary conditions on $E_{\perp}(x)$ and $E_{\parallel}(x)$ are the following . After full penetrations of the B and θ "disturbances" , $E_{\perp}=0$ and $E_{\parallel}=0$ at the midplane , by symmetry. Also $E_{\perp}(x)=0$ in the region where the advancing flux density disturbance (B gradient) has not penetrated and $E_{\parallel}(x)=0$ in the region where

the advancing angular disturbance (θ gradient) has not entered. Our main purpose in this exercise is to determine whether $E_{\perp}(x)$ and $E_{\parallel}(x)$ have the correct sign , hence satisfy the separate energy dissipation requirements that ,

$$E_{\perp}(x)j_{c\perp}(x) > 0 \quad (7.38(a))$$

$$E_{\parallel}(x)j_{c\parallel}(x) > 0 \quad (7.38(b))$$

Our calculations show that, after penetration of the B and θ gradients to the midplane has occurred , eqns 7.32 and 7.33 which envisage CT (cutting-transport) zones provide a correct description of all the cases we have examined. In some circumstances , the θ gradient gains on and eventually precedes the B gradient during their advance to the midplane. In the latter situations , the electric field $E_{\perp}(x)$ fails to satisfy eqn 7.36(a) over a small region of the slab. Fillion [27] and Lalonde [40] have examined these "pathological" cases and developed computational schemes to rectify the problem. This requires that a C (pure cutting) zone be introduced just behind the advancing θ disturbance. In this zone $j_{\parallel}(x)=j_{c\parallel}(x)$ but $j_{\perp}(x)<j_{c\perp}(B(x))$ and $E_{\perp}(x)=0$. Now eqns 7.28, 7.33, 7.34 and 7.35 must be exploited in order to obtain solutions.

These workers found that the evolution of $\langle B_z \rangle$ and $\langle B_{\phi} \rangle$ vs $\langle H_{\phi} \rangle = H_y(x)$, ensuing from these intricate and time consuming "corrections" was not substantially different from that obtained by using approximate "uncorrected" profiles of $B(x)$ and $\theta(x)$. Elaborate calculations by Clem [19] and Perez-Gonzalez [59] using relaxation (diffusion) techniques to solve eqns 7.28 through 7.31 have also led to results corresponding closely to that obtained by Fillion who exploited the simple approach we have described above. We

emphasize that the purpose of our semi-quantitative analysis was the evaluation of the depth and the position of the valleys, in the evolution of $\langle B_z \rangle$ vs H_ϕ , with sufficient accuracy to determine whether this feature is a property of the matrix or the grains. Consequently, in view of the foregoing, we have not pursued the calculations beyond the first but adequate approximation. Thus in our calculations, the possible appearance of a C zone at the front of the advancing B and θ gradients has been ignored.

The sequences of $B(x)$ and $\theta(x)$ profiles generated by numerical solutions of eqns 7.32 and 7.33 as $\langle H_\phi \rangle$ is impressed (i.e. incremented over small intervals) are introduced into the definitions ,

$$\langle B_z \rangle = \frac{1}{X} \int_0^X B_z(x) dx = \frac{1}{X} \int_0^X B(x) \cos \theta(x) dx \quad (7.39)$$

and

$$\langle B_y \rangle = \langle B_\phi \rangle = \frac{1}{X} \int_0^X B_y(x) dx = \frac{1}{X} \int_0^X B(x) \sin \theta(x) dx \quad (7.40)$$

where the integrations (summations) are performed numerically .

We confined our analysis to dependencies of $j_{c\perp}$ and $j_{c\parallel}$ on $H=B/\mu_0$ of the simple "traditional" form,

$$j_{c\perp} = \frac{\alpha_\perp}{H^n} \quad (7.41)$$

and

$$j_{c\parallel} = \frac{\alpha_\parallel}{H^p} \quad (7.42)$$

with $0 \leq n \leq 1$ and $0 \leq p \leq 1$

Introducing eqn 7.41 into 7.32 and integrating leads to

$$\int_x^X H^n dH = \alpha_{\perp} \int_x^X dx = H_s^{n+1} - H^{n+1}(x) = H_s^{n+1} \left(1 - \frac{x}{X}\right) \quad (7.43)$$

where $H(X)$, the surface total field is denoted H_s and ,

$$H_s = \{(n+1)\alpha_{\perp} X\}^{1/(n+1)} \quad (7.44)$$

is the first penetration field for the B gradient (B disturbance) .

Introducing eqn 7.42 and 7.43 into 7.33 and integrating leads to,

$$\int_x^X d\theta = \theta_s - \theta(x) = \alpha_{\parallel} \int_x^X \frac{dx}{\left\{ H_s^{n+1} - H_s^{n+1} \left(1 - \frac{x}{X}\right) \right\}^{\frac{p+1}{n+1}}} \quad (7.45)$$

where the right-hand side again yields a closed form expression. These analytic expressions for $B(x)$ and $\theta(x)$ are useful for testing the computer program developed to solve eqns 7.32 and 7.33 numerically.

In the calculations we have taken $n=0$ in eqn 7.41 , hence $j_{c\perp} = \alpha_{\perp}$, since the observations presented in the previous chapter show that both $j_{c\perp m}$ and $j_{c\perp g}$, the depinning critical current densities for the matrix and the grains are insensitive to B (i.e. Bean like). We have investigated the behaviour predicted by three choices for p in eqn 7.42 (e.g. 0, 0.5 and 1) hence over the range from the Bean approximation to the simple Kim approximation. We have also examined the predictions for ratios of $j_{c\parallel}(H_s)/j_{c\perp}(H_s)$ ranging from 1/4 to 2. The salient features for comparison between theory and observations are the maximum depth of the valleys and the value of $\langle H_{\phi} \rangle$ (the "position") for the minimum in the valleys. The limit values for these quantities change by less than a factor of two over the entire range of our scan.

The best overall correspondence between data and calculations was obtained using $p=1.0$ and $j_{c||}(H_*)/j_{c\perp}(H_*)=1.0$. The theoretical results are displayed in Fig 7.5 7.6 and 7.8. The agreement between theory and the observations although not impressive is encouraging in view of the numerous approximations. Most importantly, the central verdict we sought emerges clearly as we now indicate. In the computations we introduced,

$$j_{c\perp} = j_{c\perp m} = 2.5(10^6) \text{ A/m}^2 \quad (7.46)$$

hence,

$$\mu_0 H_* = \mu_0 j_{c\perp} X = \mu_0 j_{c\perp} \frac{(R_o - R_i)}{2} = 2.5 \text{ mT} \quad (7.47)$$

In the previous chapter we found that the penetration field across the wall,

$$\mu_0 I_{cm}^* = \mu_0 H_{*m} = \mu_0 j_{c\perp m} (R_o - R_i) = 5 \text{ mT} = 2\mu_0 H_* \quad (7.48)$$

Clearly then, our observations indicate that the flux line cutting and cross flow we have witnessed and documented is associated with the intergrain or weak link critical current densities and not with the intragrain penetration field which is almost an order of magnitude larger than the value used in the calculations (eqn 7.47).

7.4. Summary and Conclusion

We report on measurements of the evolution of the magnetic flux density in the singly connected wall of a BiSCCO tube surrounded by a second tube of the same material. We observe that initially, the nucleation of helical flux

lines along the surfaces of the wall instead of removing axial flux from the finite reservoirs inside and outside the inner tube and adding this axial flux to its wall as "traditionally" expected , leads to the opposite behaviour.

These phenomena indicate unambiguously that flux lines are released by the wall in greater number than received by the nucleation process as $|\bar{H}_{\text{total}}(R_i)|$ and $|\bar{H}_{\text{total}}(R_o)|$ are augmented . To account for the concomitant increase of the azimuthal flux in the wall , it is necessary that the number of circuits possessed by the helical flux lines entering the wall be greater than that of the exiting flux lines . Since the flux lines entering must then differ in pitch from that departing , it is clear that the countermoving flux lines must touch and cut through each other inside the wall.

This scenario for the traffic of helical flux lines across the surfaces of the wall into and out of the two adjacent finite reservoirs (the cavity of the inner tube and its surrounding annular volume) is formulated in some mathematical detail.

The basic phenomenological theory of Clem/Perez-Gonzalez is applied to the quantitative analysis of our observations. The central purpose of this exercise is to determine whether the cutting and cross flow of flux lines is associated with the intergrain or the intragrain critical current densities . The main features of the data are adequately reproduced when the intergrain j_{cLm} , extracted from the penetration field H_{*m} across the wall , is introduced into the calculations . Since the intragrain penetration field H_{*g} is almost an order of magnitude greater than $H_{*m}/2$ we conclude that the events we witness are determined by the weak link structure of the material.

CHAPTER 8

Properties of the YBCO Tube

8.1. Introduction

Before presenting our observations and analysis of flux line cutting and cross-flow in the YBCO tube in the next chapter it is important that its "standard" magnetic properties be mapped out and that the intergrain and intragrain depinning critical current densities, j_{cLm} and j_{cLg} and their dependence on the magnetic flux density B be estimated. For this purpose we have carried out the same variety of magnetic and flux trapping measurements on the YBCO tube that were performed on the BiSCCO tube. We now describe and discuss the results of this exploration. The major new feature that will emerge throughout this survey will be the prominent role played by demagnetization effects. It is useful to list and compare the pertinent quantities which control these "end effects" before we proceed.

Table 8.1

	<u>Length/Outer diameter</u>	<u>Wall area/Cavity area</u>
YBCO	1.67	3.0
BiSCCO	2.15	0.5

The background and the concepts already reviewed in our discussion of the phenomena encountered in the study of the BiSCCO tube (see Chapter 6) will not be outlined again in this chapter.

8.2. Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{total}}$ vs $H_{//}$

Typical traces of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$ for the YBCO tube at 77 K are displayed in Fig 8.1 for the weak field range. The reader should compare Fig 8.1 with the corresponding measurements on the BiSCCO tube presented in Fig 6.1.

We note three major differences in these magnetic responses.

(a) The first penetration field, $\mu_0 H_{*m}$ into the cavity of the virgin (zero field cooled) tube is ≈ 1.2 mT hence appreciably smaller than the value of 6 mT observed for the BiSCCO tube although the former has a wall thickness of 0.75 cm, large compared with 0.16 cm for the latter. From Ampere's law for idealized geometry, $H_{*m} = \langle j_{cLm} \rangle (R_o - R_i)$ and we estimate $\langle j_{cLm} \rangle = 5.5(10^4) \text{ A/m}^2$ for the YBCO tube. This is minute compared with the value of $\approx 2.5(10^6) \text{ A/m}^2$ for the BiSCCO specimen.

(b) In the YBCO tube the horizontal traversals diminish appreciably in extent as the reversal or end fields are chosen larger. This indicates that j_{cLm} is quite sensitive to the strength of B in this weak field range, i.e. here j_{cLm} is Kim like rather than nearly constant as in the Bean approximation.

(c) For the YBCO tube the curves $\langle B_z \rangle_{\text{hole}}$, as $H_{//}$ increases in magnitude, approach and eventually crossover the normal state 45° line (dashed line in Fig. 8.1). Since, as seen in (a), j_{cLm} diminishes rapidly with B increasing, we expect $\langle B_z \rangle_{\text{hole}}$ to approach but not to touch and crossover the normal 45° . In our view the latter behaviour arises from demagnetization effects which we now examine.

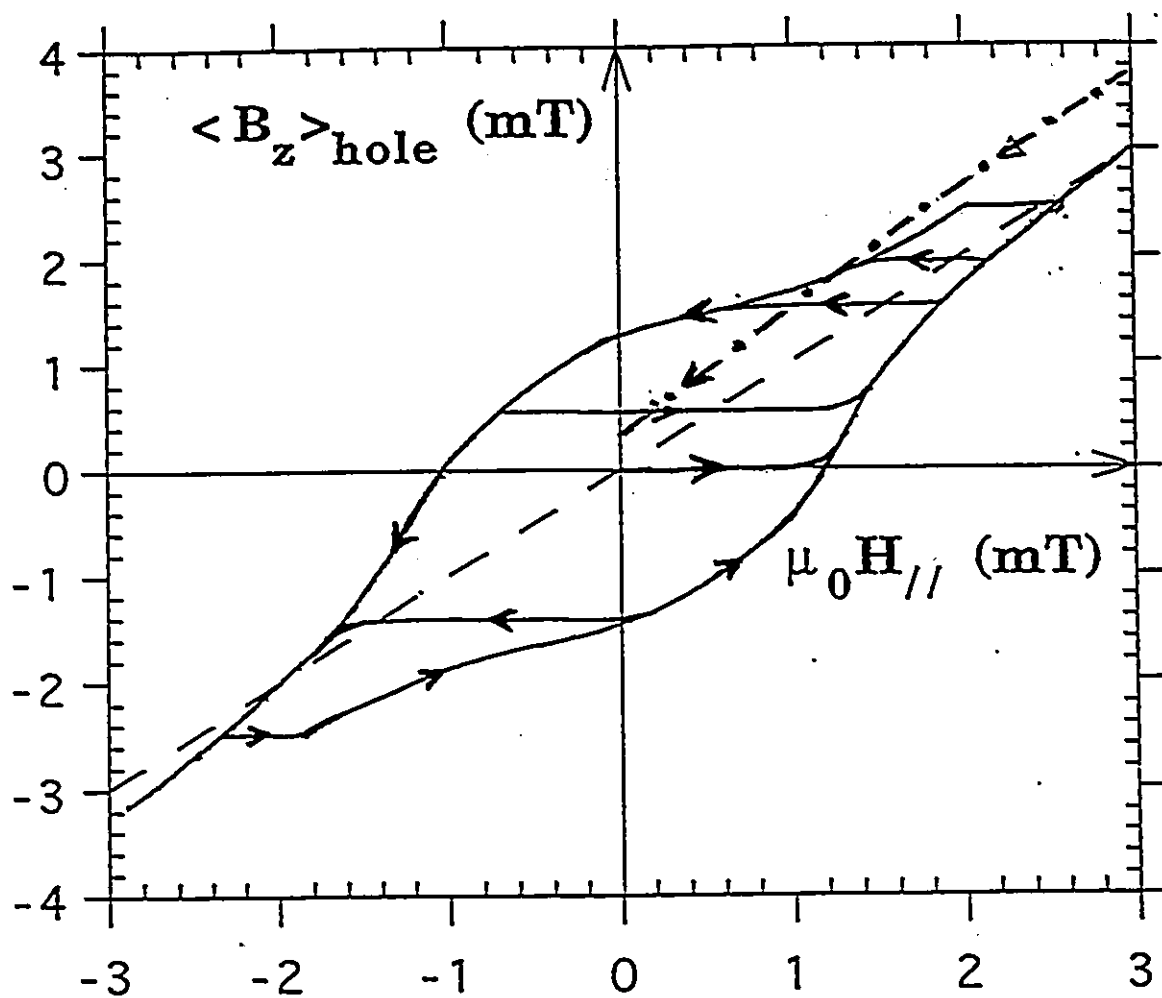


Fig 8.1 Evolution of $\langle B_z \rangle_{\text{hole}}$ when $H_{//}$ is impressed and varied along and between the boundaries (envelopes). The dashed (45°) line is for the normal state. The dash-dot curve is observed after an excursion to "large" fields.

As can be seen from table 8.1 , the geometry of the YBCO tube is far "less" ideal than that of the BiSCCO specimen. As a consequence, the magnetized grains in the thick wall of the relatively short YBCO tube generate a return magnetic flux outside the tube and through the cavity which is not negligible, indeed, since H_m and hence the intergrain critical current density $\langle j_{cm} \rangle$ are significantly smaller in the YBCO tube, the role of the return flux of the magnetized grains becomes correspondingly very noticeable.

To estimate the strength of the return flux we exploit the "text book" picture of a magnetized medium consisting of the juxtaposition of magnetic dipoles in the form of closed amperian current elements. Here the magnetic dipoles are generated by field shielding or flux retaining persistent currents which have been induced to circulate around the periphery and throughout the volume of individual grains by changes in the applied magnetic field [15, 16, 17, 23, 24, 25, 31, 32, 51, 53] . Consequently we can write,

$$\nabla \times \vec{M}_g = \vec{J}_g \quad (8.1)$$

and,

$$\vec{M}_g \times \hat{r} = \vec{K}_{gw} \quad (8.2)$$

where $\vec{M}_g$ is the magnetization associated with the agglomeration of grains viewed on a quasi-macroscopic scale encompassing several grains. $\hat{r}$ is a unit radial vector. $\vec{J}_g$ is a resultant current density which will appear in the volume of the wall when the magnetization of the juxtaposition of grains is nonuniform. $\vec{K}_{gw}$ is a pseudo surface current flowing along the inner and outer circumferences of the wall. As in the text book treatment of eqn 8.2 it is a "localized" current confined to the individual grains but is viewed as equivalent

to a continuous current flowing circumferentially around the inner and outer boundaries of the wall.

We assume, for simplicity, that $\bar{M}_g$ is uniform and directed along the z axis, i.e. along the length L of the wall. Then,

$$\bar{K}_{gw}(R_o) = \hat{\phi} M_g \quad , \quad \bar{K}_{gw}(R_i) = -\hat{\phi} M_g \quad (8.3)$$

The collection of magnetized grains in the wall is thus replaced by two "thin" solenoids of equal length L, carrying counter circulating currents of magnitude K_{gw} per unit length. Applying the textbook formula for thin solenoids of finite length, the net magnetic flux density, $\bar{B}_{return}$ generated at the center of the cavity by these two counter circulating pseudo surface currents then reads,

$$\begin{aligned} \bar{B}_{return} &= -\mu_0 M_g \left\{ \frac{L/2}{\sqrt{(L/2)^2 + R_i^2}} - \frac{L/2}{\sqrt{(L/2)^2 + R_o^2}} \right\} \\ &= -\mu_0 M_g k = -\mu_0 M_g (0.100) \end{aligned} \quad (8.4)$$

Where we have introduced the dimensions of the YBCO tube in the evaluation of the coefficient k. The negative sign indicates that B_{return} is directed opposite to the magnetization M_g of the grains. For the BiSCCO tube, $k=0.028$, hence B_{return} can be neglected for that sample.

It is instructive at this juncture to examine the consequences of eqn 8.4 in an illustrative example. We refer the reader to Fig. 8.2 for a schematic display of the concepts and quantities.

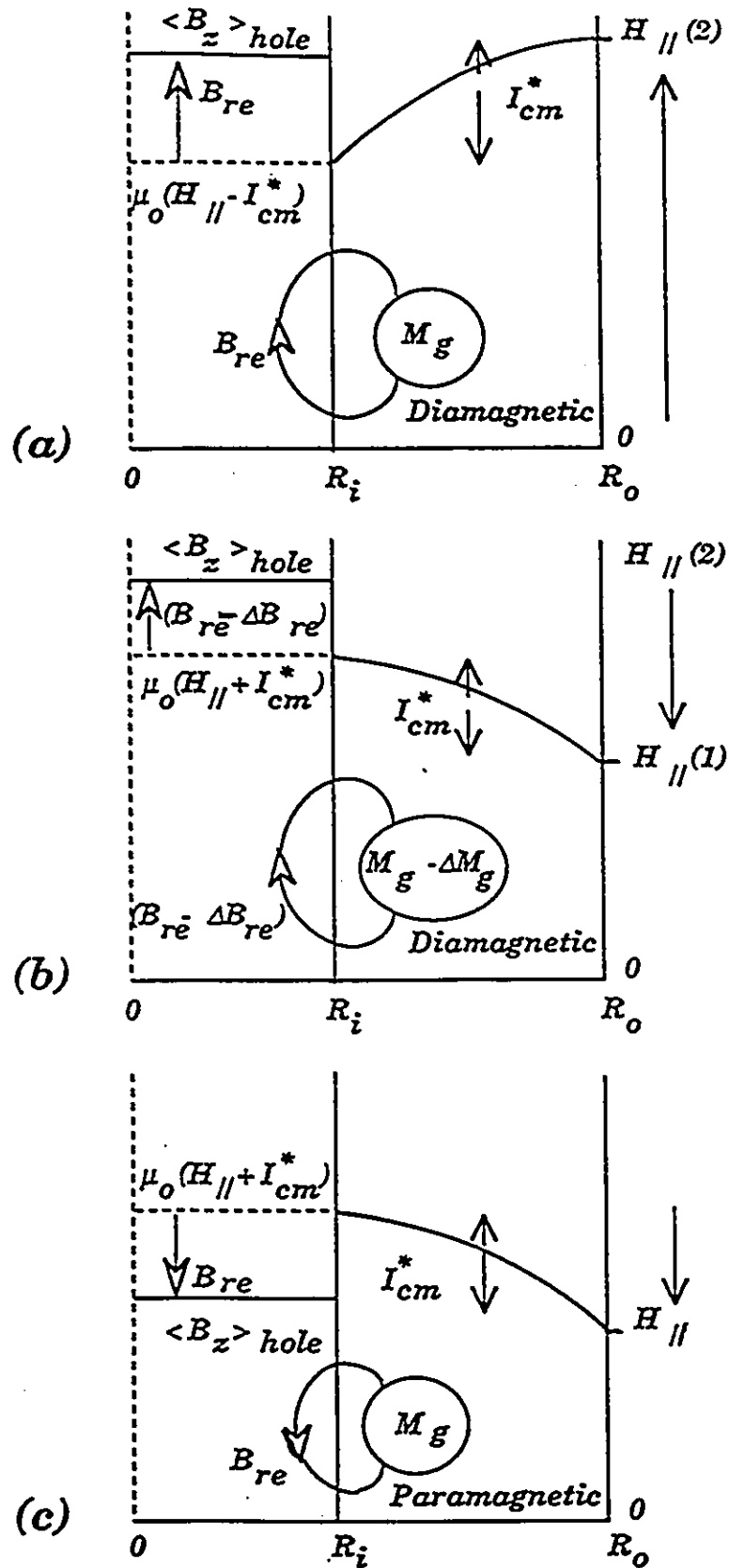


Fig 8.2 Schematics displaying the various contributions to the net magnetix flux density in the cavity, (a) after an increase to $\mu_o H_{//}(2) \approx 2.5$ mT , (b) the subsequent decrease to $\mu_o H_{//}(1) \leq 2.0$ mT , and (c) after a decrease of $H_{//}$ from a very high to a very small value.

The width of the horizontal traversal commencing at $\mu_0 H_{//}(2) = 2.5\text{mT}$ in Fig 8.1 and terminating at $\mu_0 H_{//}(1) = 2\text{mT}$ is given by

$$\mu_0 H_{//}(2) - \mu_0 H_{//}(1) = 0.5\text{mT} \quad (8.5)$$

Therefore, the critical intergrain (matrix) current I_{cm}^* circulating azimuthally per unit length of the wall and opposing entry of this field into the cavity when $H_{//}(2)$ is present, generates a flux density difference across the wall,

$$\mu_0 \Delta H_z = \mu_0 I_{cm}^* = \frac{\mu_0 H_{//}(2) - \mu_0 H_{//}(1)}{2} = 0.25\text{mT} \quad (8.6)$$

The applied field $H_{//}(2)$ is therefore only fractionally diminished in the volume of the wall by the weak shielding effect of the intergrain current I_{cm}^* (see Fig. 8.2(a)). Consequently, all of the grains have effectively experienced almost the full applied field $\mu_0 H_{//}(2) = 2.5\text{mT}$. As we shall see later in this chapter, this applied field is much smaller than the first full penetration field into the grains which is $\mu_0 H_{*g} = 12\text{mT}$. Thus the diamagnetic magnetization of the grains in $\mu_0 H_{//} = 2.5\text{mT}$ is nearly "perfect". Hence $|\mu_0 M_g| \leq 2.5\text{mT}$ since the intragrain currents shielding the interior of the grains against entry of $\mu_0 H_{//}$ essentially occupy only the peripheral volume of the grains. Introducing this value into the eqn 8.4 leads to $B_{return} \leq 0.25\text{mT}$ which is comparable but opposite to $\mu_0 \Delta H_z = \mu_0 I_{cm}^* = 0.25\text{mT}$.

Thus when $H_{//}$ ascending in magnitude attains the range under scrutiny (see Fig. 8.2(a) for a schematic display),

$$\langle B_z \rangle_{hole} = \mu_0 H_{//}(2) - \mu_0 I_{cm}^* + B_{return} \leq \mu_0 H_{//}(2) \quad (8.7)$$

where, for clarity, all the quantities are absolute values. Thus $\langle B_z \rangle_{\text{hole}}$ touches or crosses over the 45° normal line in Fig 8.1. Since here,

$$\mu_0 I_{\text{cm}}^* \approx B_{\text{return}} = \mu_0 |M_s| (0.100) \quad (8.8)$$

During the horizontal traversal commencing at $\mu_0 H_{//}(2) = 2.5 \text{ mT}$ and terminating at $\mu_0 H_{//}(1) = 2 \text{ mT}$, the magnitude of the diamagnetic magnetization of the grains is diminished approximately by the width of the traversal. Consequently, $|M_s|$ is only fractionally decreased by an amount $|\Delta M_s| \ll |M_s|$ and the associated B_{return} correspondingly reduced to $(B_{\text{return}} - \Delta B_{\text{return}})$, where $B_{\text{return}} \gg \Delta B_{\text{return}}$. I_{cm}^* however reverses its sense of circulation hence its sign as a result of the traversal. Now, (see Fig 8.2(b) for a schematic illustration),

$$\langle B_z \rangle_{\text{hole}} = \mu_0 H_{//}(1) + \mu_0 I_{\text{cm}}^* + (B_{\text{return}} - \Delta B_{\text{return}}) \quad (8.9)$$

Again, all the quantities are absolute values. Thus $\langle B_z \rangle_{\text{hole}}$ lies above the normal 45° line.

Finally, we note that in the very low range of $H_{//}$, when $H_{//}$ is descending after an excursion to large values, the locus of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$ moves closer to the normal line as shown by the dash-dot curve in Fig. 8.1. Flux retaining amperian currents induced in the grains during the decrease of $H_{//}$ have overwhelmed the Meissner (Abrikosov) diamagnetism and endowed the grains with a saturated paramagnetic magnetization. Hence eqn 8.7 now reads,

$$\langle B_z \rangle_{\text{hole}} = \mu_0 H_{//} + \mu_0 I_{\text{cm}}^* - B_{\text{return}} \geq \mu_0 H_{//} \quad (8.10)$$

where both I_{cm}^* and B_{return} have now changed sign (see Fig. 8.2(c)).

The foregoing semi-quantitative account of the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $H_{//}$ ascending or descending could, in principle, be developed in some mathematical detail. Since the magnitude of I_{cm}^* and M_g (hence B_{return}) depend in a fairly complicated way on $H_{//}$ and its previous history, this task is somewhat formidable and outside of the scope of this thesis. We will however quantitatively pursue the model just outlined to analyze the flux trapping behaviour of the tube, since here the variation of I_{cm}^* and M_g as a function of H_{cool} and H_{cycle} is much simpler.

8.3. Flux Trapping by H_{cool} Procedure

$\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{total}}$ and $\langle B_z \rangle_{\text{wall}}$, the spatial averages of the magnetic flux densities trapped in the cavity, in the entire cross section of the tube (the hole plus wall) and in the wall alone are plotted versus H_{cool} in Figs 8.3 and 8.4. Fig 8.3 focuses on the behaviour at the low fields whereas Fig. 8.4 displays the data over a broader range. We recall that $\langle B_z \rangle_{\text{total}}$ is monitored directly by the outer pickup coil and $\langle B_z \rangle_{\text{wall}}$ is obtained from the other two quantities using the definition that,

$$\langle B_z \rangle_{\text{wall}} = \frac{\langle B_z \rangle_{\text{total}} R_o^2 - \langle B_z \rangle_{\text{hole}} R_i^2}{(R_o^2 - R_i^2)} \quad (8.11)$$

Figs 8.3 and 8.4 should be compared with Fig 6.4 which displays the corresponding data for the BiSCCO tube. We note that for both specimens, $\langle B_z \rangle_{\text{total}}$ rises to a plateau which lies above $\langle B_z \rangle_{\text{hole}}$. This crossover or "inversion" is again attributed to the residual magnetization of the grains which increases to a saturation level when $H_{\text{cool}} = H_{\text{cool}}^* = H_g$.

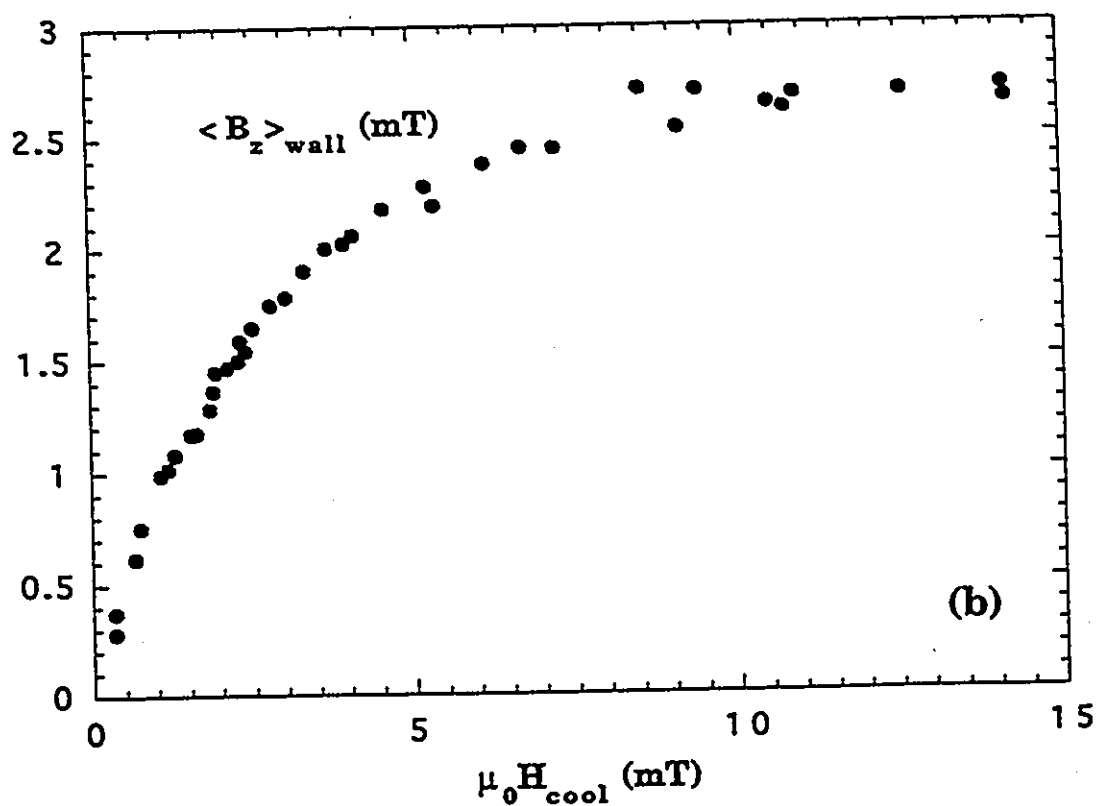
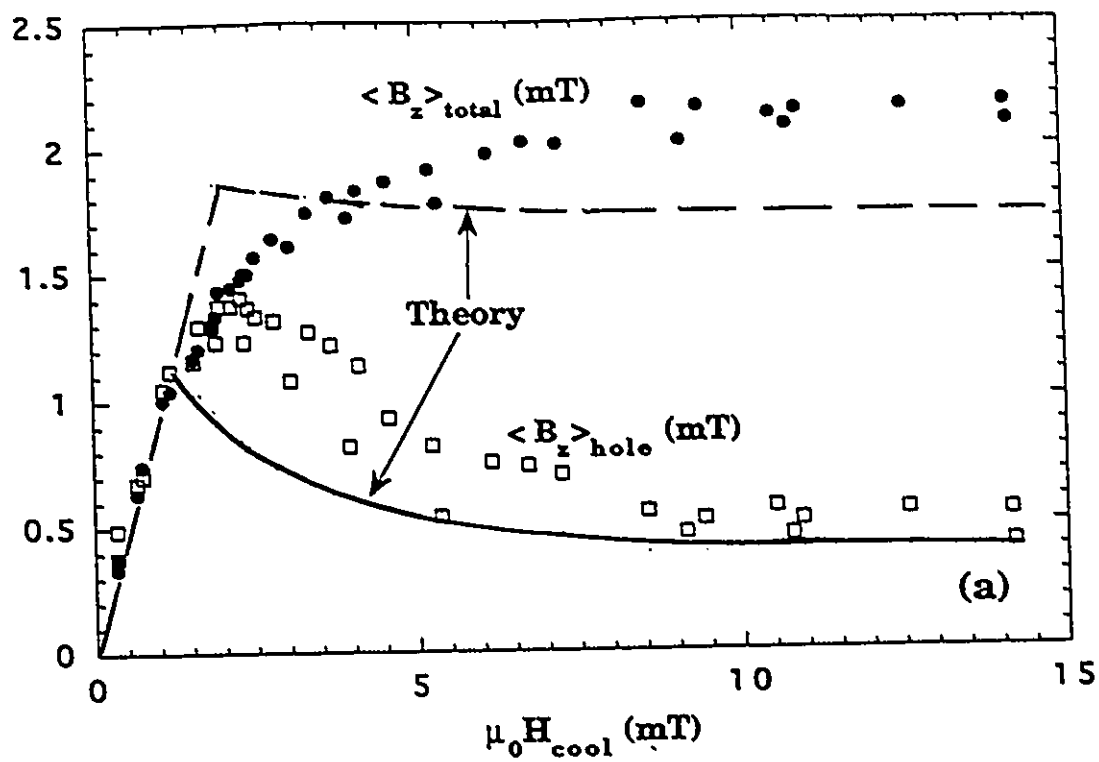


Fig 8.3 $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{total}}$ and $\langle B_z \rangle_{\text{wall}}$ trapped in the YBCO tube by the H_{cool} procedure. The dashed and solid curves display theoretical calculations for $\langle B_z \rangle_{\text{hole}}$ described in the text.

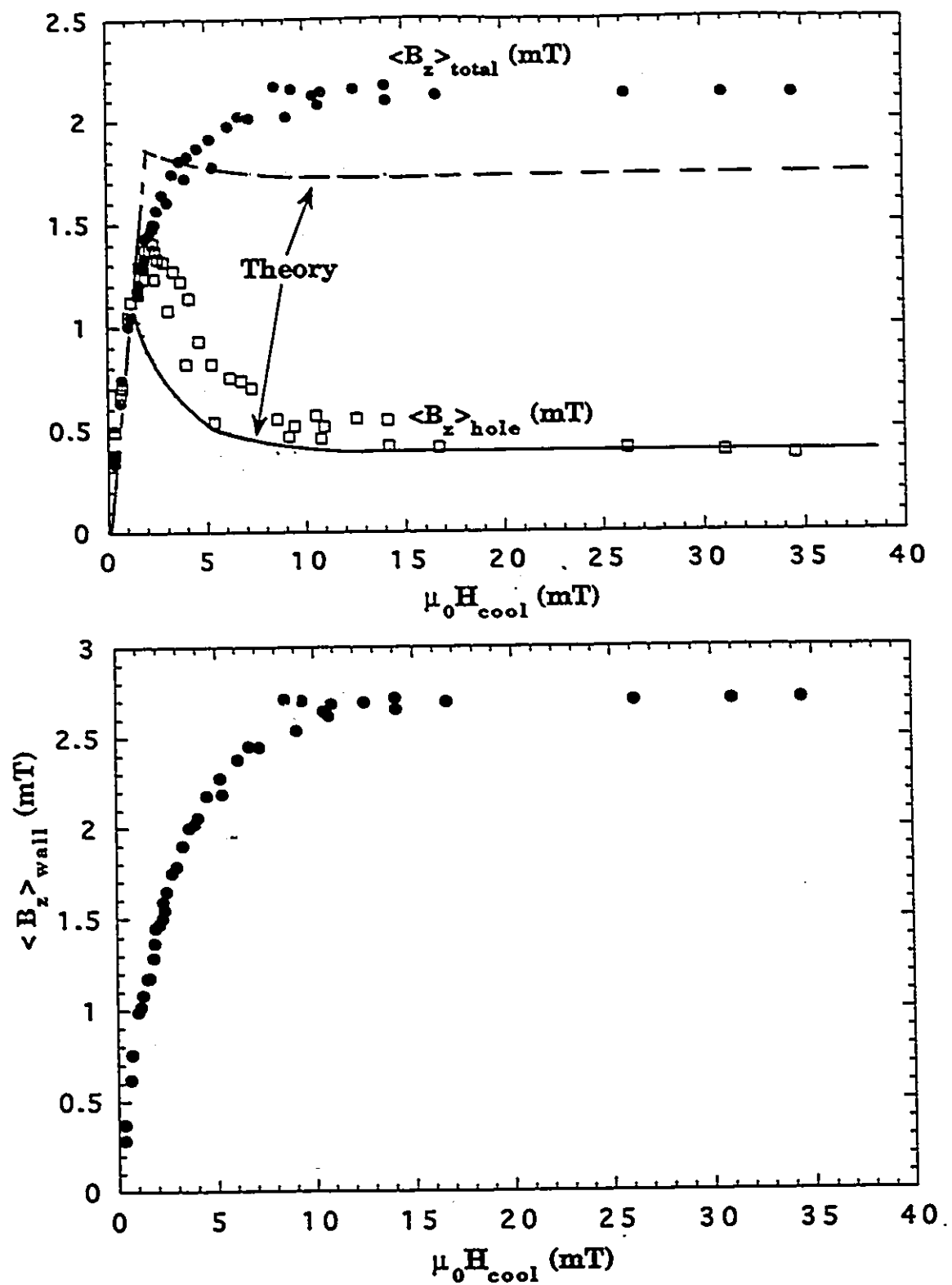


Fig 8.4 Same as the previous figure but over a broader range of H_{cool} .

$\langle B_z \rangle_{\text{hole}}$ vs H_{cool} for the YBCO tube is seen to trace a peak instead of a kink and then descends to a plateau. The threshold of the two plateaus again correspond closely confirming their identification with saturation remanent magnetization of the grains and the associated parameter H_{*g} . From these two sets of data we estimate $\mu_0 H_{*g} \approx 12 \text{ mT}$.

The summit of the peak is tentatively associated with $I_{\text{cm}}^* (M_g = 0)$, the generation of an intergrain flux retaining critical current induced, by the removal of H_{cool} , to circulate around the circumference of the wall and occupying its entire cross section. In the simple idealized model, (i.e. monolithic homogeneous wall, infinite tube length), we expect that,

$$I_{\text{cm}}^* = H_{*m} = H_{\text{cool}}^* = \frac{\langle B_z \rangle_{\text{hole}}}{\mu_0} \quad (8.12)$$

From Fig 8.3 we estimate $\langle B_z \rangle_{\text{hole}}$ at the peak $\approx 1.3 \text{ mT}$ and $\mu_0 H_{\text{cool}}^* \approx 2 \text{ mT}$. This discrepancy is not surprising since the situation we are confronting hardly corresponds to the simple idealized case.

The central new feature which we now address is the fact that the YBCO tube displays a peak in $\langle B_z \rangle_{\text{hole}}$ vs H_{cool} and a dramatic descent before reaching a plateau. We recall that the BiSCCO tube exhibited a "kink" followed by a slow rise to a plateau.

The reader will have correctly anticipated that the "novel" behaviour of $\langle B_z \rangle_{\text{hole}}$ vs H_{cool} for the thick walled YBCO tube is attributed to, (i) the effect of the magnetization of the grains on $j_{\text{cLm}}(r)$, hence on I_{cm}^* , (ii) the magnetic flux of the magnetized grains returning through the cavity.

We now develop the first of these statements in some detail.

Maxwell's eqn, $\nabla \times \vec{H} = \vec{j}$ applied to the circumferentially circulating intergrain critical current density $j_{\text{cLm}}(B(r))$ reads,

$$\frac{dH_m(r)}{dr} = -j_{c\perp m}(B(r)) \quad (8.13)$$

where $H_m(r)$ is the magnetic field generated in the "matrix" by $j_{c\perp m}$.

We exploit the simple "traditional" analytic formula for the dependence of $j_{c\perp m}$ on the magnetic flux density, $B_{total} = \mu_0 H_{total}(r)$ of the form,

$$j_{c\perp m}(B(r)) = \frac{\alpha}{\{H_{total}(r)\}^n} \quad (8.14)$$

We stress that $j_{c\perp m}$ depends on the total magnetic flux density (magnetic field) permeating the "matrix". $\bar{H}_{total}(r)$ is the superposition of three magnetic fields from different sources.

a) the field $H_m(r)$ produced by the intergrain (matrix) current

$$I_{cm}(r) = \int_r^{R_0} j_{c\perp m}(r) dr \quad (8.15)$$

b) The "external" magnetic field (return field) $\bar{H}_g(r)$ of the magnetized grains, denoted H_g , and

c) The externally applied magnetic field $H_{//}$.

We let $H_{//}=0$ since we focus on situations where $H_{//}=H_{cool}$ or $H_{//}=H_{cycle}$ have been removed.

$\bar{H}_g(r)$ has obviously an extremely complicated configuration. For simplicity we consider $\bar{H}_g(r)$ to be uniform and directed to aid $H_m(r)$. Thus we write

$$H_g = f < M_g > \quad (8.16)$$

where f is an "arbitrary" geometric coefficient and $< M_g >$ is the spatial average of the magnetization of the grains. Consequently, we write

$$H_{\text{total}}(r) = H_m(r) + H_g = H_m(r) + f < M_g > \quad (8.17)$$

Introducing eqn 8.17 into 8.14 and the latter into 8.13 leads to,

$$\int_{R_i}^{R_o} \{H_m(r) + H_g\}^n d\{H_m(r) + H_g\} = -\alpha \int_{R_i}^{R_o} dr \quad (8.18)$$

We stress that, $dH_g/dr = 0$ since H_g is taken uniform and H_g is not "linked" to j_{cLm} .

Equation 8.18 yields ,

$$\{H_m(R_o) + H_g\}^{n+1} - \{H_m(R_i) + H_g\}^{n+1} = -(n+1)\alpha(R_o - R_i) = -H_{*m}^{n+1} \quad (8.19)$$

where H_{*m} is the first penetration field across the wall of the tube.

$H_m(r)$ is continuous across the boundaries R_i and R_o since we envisage no true surface currents to exist in this scenario. Thus,

$$H_m(R_o) = H_{//} = 0 \quad (8.20)$$

From Ampere's law, $\int \tilde{H} \cdot d\vec{L} = IL$, we can write ,

$$H_m(R_i) = I_{cm}^*(M_g) = \int_{R_i}^{R_o} j_{cLm}(r) dr \quad (8.21)$$

Eqn 8.19 now reads

$$\begin{aligned} H_m(R_i) = I_{cm}^*(M_g) &= \{H_g^{n+1} + H_{*m}^{n+1}\}^{1/n+1} - H_g \\ &= \{f < M_g >^{n+1} + I_{cm}^*(0)^{n+1}\}^{1/n+1} - f < M_g > \end{aligned} \quad (8.22)$$

We stress that on the right hand side of eqn 8.22 we have introduced the feature that, the critical intergrain current I_{cm}^* when the grains are "not magnetized" , therefore denoted $I_{cm}^*(0)$, corresponds to the first penetration field H_m .

Eqn 8.22 applies when H_{cool} was sufficiently large so that its removal induced intergrain critical currents across the entire wall (see Fig. 8.2(c)), hence when,

$$H_{cool} \geq I_{cm}^*(M_g) \quad (8.23)$$

We note that,

$$H_m(R_i) = H_{cool} < I_{cm}^*(M_g) \quad (8.24)$$

traces a straight line .

Eqns 8.22 and 8.23 describe the effect of the magnetization of the grains on the amount of remanent flux the tube can trap. Eqn 8.4 describes the magnitude of the return flux of the magnetized grains at the center of the cavity. Combining these two contributions , the net magnetic flux density at the center of the cavity reads,

$$\langle B_z \rangle_{hole} = \mu_0 I_{cm}^*(M_g) - |B_{return}| = \mu_0 I_{cm}^*(M_g) - \mu_0 k \langle M_g \rangle \quad (8.25)$$

When $H_{cool} < I_{cm}^*(M_g)$, this picture leads to ,

$$\langle B_z \rangle_{hole} = \mu_0 H_{cool} - |B_{return}| = \mu_0 H_{cool} - \mu_0 k \langle M_g \rangle \quad (8.26)$$

We stress that , $\langle M_g \rangle$, the remanent magnetization of the grains is a monotonically increasing function of H_{cool} , rising almost linearly in the low

range of H_{cool} and asymptotically attaining a plateau when $H_{cool} = H_g = H_{cool}^*$. We also note that B_{return} is proportional to $\langle M_g \rangle$ (see eqn 8.4). Eqn 8.26 predicts a nearly linear rise of $\langle B_z \rangle_{hole}$ vs H_{cool} but with a slope significantly smaller than unity. Consequently as observed in Fig 8.3(a), $\langle B_z \rangle_{hole}$ at the peak is smaller than the corresponding $\mu_0 H_{cool}$.

The detailed variation of $\langle M_g \rangle$ as a function of H_{cool} can readily be modeled for ideal geometry (i.e. long cylinders, rods or slabs of uniform cross section). Gandolfini in his thesis and others have developed analytic expressions for these ideal geometries exploiting the Bean, Kim and other simple formulae for the dependence of $j_{c\perp g}$ on B . These calculations reveal that the evolution of $\langle M_g \rangle$ vs H_{cool} is not sensitive to either the choice of geometry or the dependence of $j_{c\perp g}$ on B . In view of this we decided to ignore these detailed prescriptions and to trace a smooth curve through our data for $\langle B_z \rangle_{wall}$ vs H_{cool} displayed in Figs 8.3(b) and 8.4(b). The structure of $\langle B_z \rangle_{wall}$ vs H_{cool} corresponds to the model calculations of $\langle M_g \rangle$ vs H_{cool} by Gandolfini [29] and others [70, 71, 72]. Since the remanent magnetization of the grains is the dominant contribution to $\langle B_z \rangle_{wall}$, we use, for simplicity, the approximation that,

$$\langle B_z \rangle_{wall} = \mu_0 \langle M_g \rangle \quad (8.27)$$

The smoothed measured values of $\langle B_z \rangle_{wall}$ vs H_{cool} are thus introduced into eqns 8.25 and 8.26. In the calculations $n=1$, hence $j_{c\perp m}(B) = \alpha/H$, i.e. Kim like. This leads to the solid curve displayed in Figs 8.3 (a) and 8.4 (a). The good correspondence of this curve with the data provides strong support for the self-consistency of the model.

Eqn 8.25 contains both , (i) the effect of the magnetization of the grains on $j_{c\perp m}(r)$, hence on I_{cm}^* , and , (ii) the contribution of the return flux of the magnetized grains to the diminution of $\langle B_z \rangle_{hole}$ vs H_{cool} . We note that the former plays the dominant role in causing $\langle B_z \rangle_{hole}$ to descend to a low lying plateau. To illustrate this feature we display the evolution of $\langle B_z \rangle_{hole}$ vs H_{cool} when I_{cm}^* is regarded to be unaffected by the magnetization of the grains (see the dashed curve in Figs 8.3(a) and 8.4(a)). In our development , this is accomplished by taking , $H_g=0$, in eqn 8.17 , hence writing , $H_{total}(r)=H_m(r)$. Consequently , eqn 8.25 then reads ,

$$\langle B_z \rangle_{hole} = \mu_0 I_{cm}^*(0) - |B_{return}| = \mu_0 I_{cm}^*(0) - \mu_0 k |\langle M_g \rangle| \quad (8.28)$$

which incorporates only the effect of the return flux through the cavity via $\langle M_g \rangle$. The dependence of the latter on H_{cool} remains as indicated above.

8.4. Flux Trapping by H_{cycle} Procedure

The evolution of $\langle B_z \rangle_{hole}$, $\langle B_z \rangle_{total}$ and $\langle B_z \rangle_{wall}$ vs H_{cycle} is displayed in Fig 8.5 for the low field range and in Fig 8.6 over a broader field range. These data should be compared with the corresponding measurements for the thin walled BiSCCO tube (see Fig 6.6). The salient new feature, emerging from this comparison , is the broad peak traced by the locus of $\langle B_z \rangle_{hole}$ followed by a descent to a plateau.

In our discussion of the BiSCCO flux trapping behaviour we have already examined the cause for the displacement or shift of various "benchmarks" to higher fields in the H_{cycle} procedure relative to their appearance in the H_{cool} procedure. The reason for this shift was illustrated in Figs 6.5 and 6.7. In this

scenario, the threshold of the plateau for the H_{cycle} procedure (denoted H_{cycle}^{**g}) is a measure of the "double" penetration field into the grains, H_{**g} . From the data we estimate $\mu_0 H_{\text{cycle}}^{**g} = \mu_0 H_{**g} = 25\text{mT}$ for the YBCO tube, hence evaluate the ratio $H_{**g}/H_{*g} = 2$ for this specimen. ($\mu_0 H_{*g} = 13\text{mT}$ was obtained from the H_{cool} procedure). This indicates that $j_{c\perp g}$ is insensitive to B (Bean like) in the corresponding low field range.

The "kinks" in the curves of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{total}}$ occurring when $\mu_0 H_{\text{cycle}} = 2.0\text{mT}$ are associated with H_{**m} , the "double" penetration field across the wall. From the H_{cool} data we also estimated the first penetration field $\mu_0 H_{*m} = 2.0\text{mT}$. Thus the ratio $H_{**m}/H_{*m} \geq 1$ indicates a rapid decrease of $j_{c\perp m}$ with increasing B . This conclusion is in harmony with the behaviour of the virgin curve for $\langle B_z \rangle_{\text{hole}}$ vs $H_{\parallel}$ (see Fig 8.1) and also with the initial rapid rise of $\langle B_z \rangle_{\text{hole}}$ vs H_{cycle} observed in Fig 8.6(a).

We now turn our attention to the theme that the magnetization of the grains is responsible for the structure of the curve of $\langle B_z \rangle_{\text{hole}}$ vs H_{cycle} . It is difficult to make valid approximations in modeling the initial rapid rise of $\langle B_z \rangle_{\text{hole}}$. Consequently we focus on the behaviour commencing just after this initial steep rise has occurred hence examine the situations where $\mu_0 H_{\text{cycle}} > \mu_0 H_{\text{cycle}}^{**m} = \mu_0 H_{**m} = 2.0\text{mT}$.

Eqn 8.25 which we repeat here,

$$\langle B_z \rangle_{\text{hole}} = \mu_0 I_{cm}^* (M_g) - |B_{\text{return}}| = \mu_0 I_{cm}^* (M_g) - \mu_0 k | \langle M_g \rangle |$$

continues to apply to this regime bearing in mind that $\langle M_g \rangle$, the magnetization of the grains is now a function of H_{cycle} . Gandolfini in his thesis [29] and other workers [70, 71, 72] have developed detailed expressions

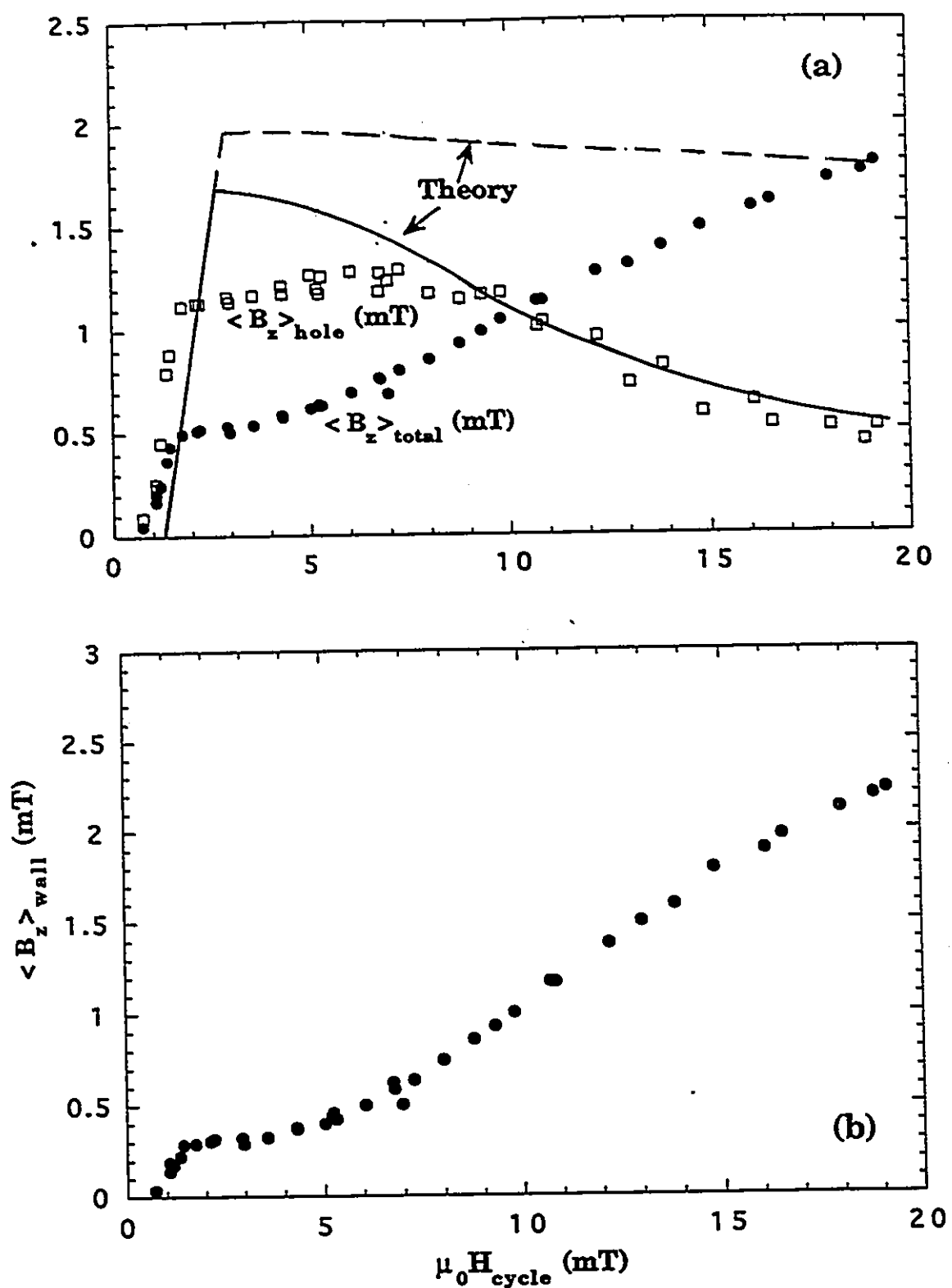


Fig 8.5 $\langle B_z \rangle_{\text{hole}}$, $\langle B_z \rangle_{\text{total}}$ and $\langle B_z \rangle_{\text{wall}}$ trapped in the YBCO tube by the H_{cycle} procedure. The dashed and solid curves display theoretical calculations for $\langle B_z \rangle_{\text{hole}}$ described in the text.

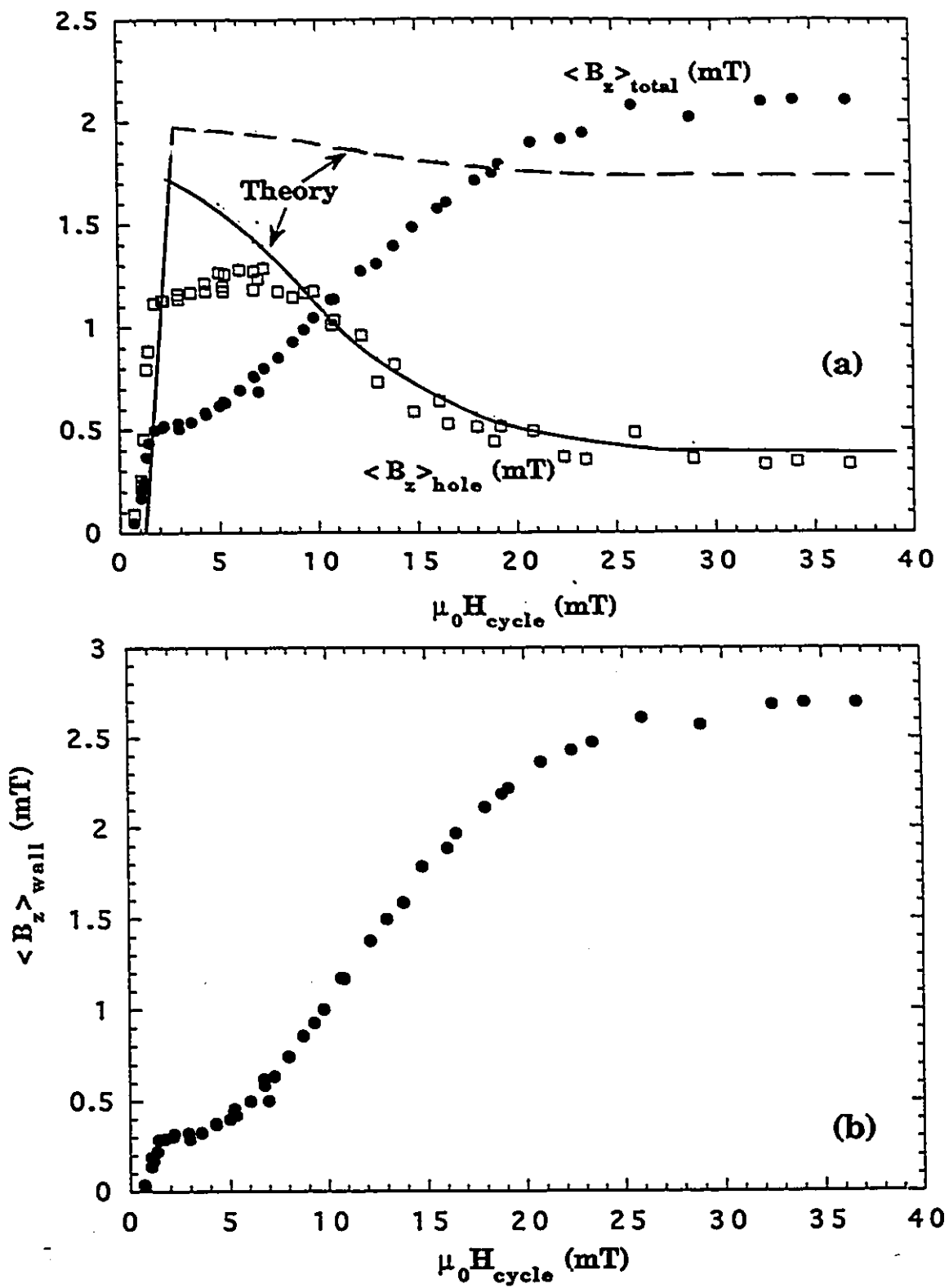


Fig 8.6 Same as the previous figure but over a broader range of H_{cycle} .

Meissner Effect

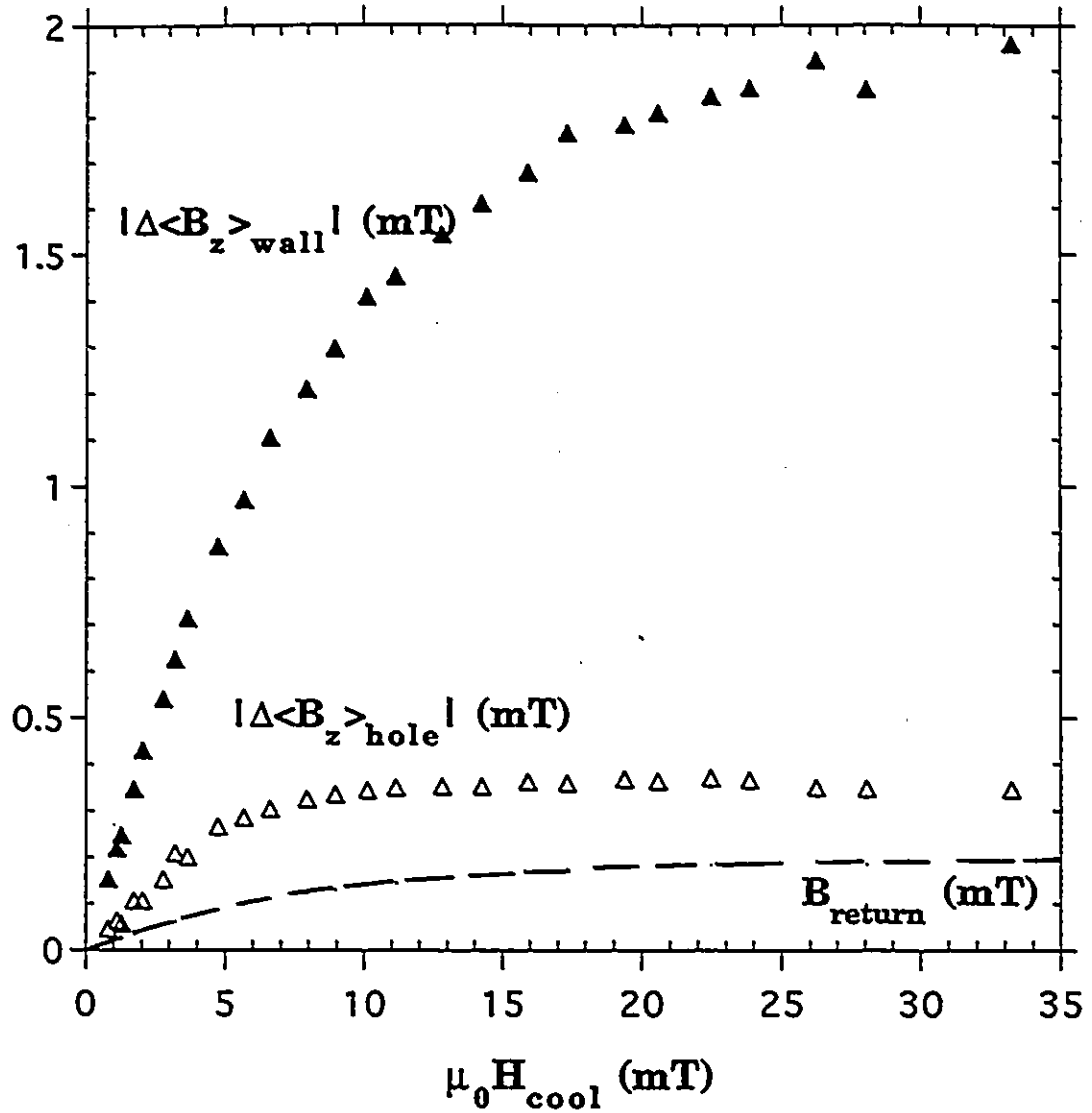


Fig 8.7 Displays the expulsion of axial magnetic flux from the wall into the cavity during cooling from T_c in various $H_{\parallel}$. The dashed curve is an estimate of the return flux density at the center of the cavity.

for the remanent magnetization, (i.e. $\langle M_g \rangle$) vs H_{cycle} for idealized geometries (long cylinders , rectangular rods, thin sheets of uniform cross section) and for various dependencies of $j_{c\perp g}$ on B . These calculations show that the salient features of $\langle M_g \rangle$ vs H_{cycle} are not sensitive to the choice of geometry or the formulae for $j_{c\perp g}$ vs B . In other words the standard model is "generic". Consequently, we again elect to exploit our data for $\langle B_z \rangle_{\text{wall}}$ vs H_{cycle} in testing the validity of eqn 8.25 and the model upon which this equation is based. We note again that taking $\langle B_z \rangle_{\text{wall}} = \mu_0 \langle M_g \rangle$ rests on the stipulation that the magnetized grains make the major contribution to the residual trapped flux in the wall. The validity of this approximation improves as H_{cycle} is augmented and approaches the threshold of the plateaus. We also stress that the structure of $\langle B_z \rangle_{\text{wall}}$ vs H_{cycle} corresponds to the model calculations of $\langle M_g \rangle$ vs H_{cycle} by Gandolfini and others.

The dashed and solid curves in Figs 8.5 and 8.6 display $\langle B_z \rangle_{\text{hole}}$ vs H_{cycle} resulting from this exercise when, (i) I_{cm}^* is taken independent of $\langle M_g \rangle$ and when, (ii) I_{cm}^* is assumed to depend on $\langle M_g \rangle$. Again we take $j_{c\perp m} = \alpha/H_{\text{total}}$, hence Kim-like. The correspondence between calculations and data again fortify our confidence in the intricate framework we have described.

8.5. Flux expulsion (Meissner Effect)

The decrease of the magnetic flux density in the wall, $\Delta \langle B_z \rangle_{\text{wall}}$ (the Meissner effect) , and the corresponding rise of the flux density in the cavity, $\Delta \langle B_z \rangle_{\text{hole}}$ as the tube cools from T_c to 77 K in various static $H_{\parallel}$ are displayed in Fig 8.7

The rise of the magnetic flux density in the YBCO tube is attributed to two mechanisms. Firstly, as for the BiSCCO tube, we expect that the mutual

expulsion of the flux vortices (the cause of the Meissner effect in Type II superconductors) will lead to the expulsion of flux lines into the cavity where they are then imprisoned by the simply connected superconducting environment, i.e. intergrain volume or matrix. The return flux of the diamagnetically (Meissner effect) magnetized grains will, however, also thread the cavity and cause an increase in $\langle B_z \rangle_{\text{hole}}$ above the ambient $\mu_0 H_{//}$. We can readily estimate the latter contribution to $\Delta \langle B_z \rangle_{\text{hole}}$ using eqn 8.4 reproduced here,

$$B_{\text{return}} = \mu_0 k |M_g| = k \langle B_z \rangle_{\text{wall}} = 0.100 \langle B_z \rangle_{\text{wall}}$$

This is shown by the dashed curve in Fig 8.7, and is seen to provide $\approx 50\%$ of the rise of the axial flux density in the cavity during field cooling.

8.6. Summary and Conclusions

From measurements of, (a) the magnetic behaviour of the YBCO tube as $H_{//}$ is impressed and varied and from, (b) the evolution of the magnetic flux trapped in the cavity and in the total cross section by the H_{cool} and H_{cycle} procedures, we obtain estimates of, (i) $j_{c\perp m}$, the depinning intergrain critical current density and its dependence on B and, (ii) the penetration field into the grains and the dependence of $j_{c\perp g}$ on B .

We find $\langle j_{c\perp m} \rangle \approx 2.5(10^6) \text{ A/m}^2$ and sensitive to B (Kim like), $j_{c\perp g}$ insensitive to B (Bean like) and $H_{*g}/H_{*m} \approx 6$

Because the YBCO tube is thick walled and "stout", the return flux from the magnetization of the grains makes a significant contribution to the net magnetic flux threading the cavity. Because in the YBCO tube, $j_{c\perp m}$, is

sensitive to B (Kim like), the magnetization of the grains has an important influence on the amount of flux trapped in the cavity by the H_{cool} and H_{cycle} procedures.

A simple model incorporating both of these effects successfully describes the observations.

CHAPTER 9

Flux Line Cutting and Cross Flow in the YBCO Tube

9.1. Introduction

We believe that the flux cutting and cross-flow phenomena we observed in the BiSCCO tube and reported in chapter 7 are "generic" and "universal" to all type II superconductors whether of the classical or high T_c variety. It is therefore important to verify whether the behaviour we encountered in the BiSCCO tube is also exhibited by other high T_c superconductors. For this reason we undertook the study of an YBCO hollow cylinder in similar circumstances. The two sintered YBCO hollow cylinders we obtained from SSC corporation however trapped only a maximum of 0.1 and 0.3 millitesla of axial magnetic flux in their cavity at 77 K. This negligible flux trapping capacity made these specimens essentially useless for our purpose since this indicates that their depinning intergrain critical current density $\langle j_{c\perp m} \rangle$ is negligible. Fortunately we secured (at some expense) a sintered hollow tube from HiTc Superconco which trapped about 1.2 mT in the cavity at 77 K. From published reports on flux trapping by YBCO tubes at 77 K this level seems quite typical and standard for the "first" generation of sintered YBCO tubes. Recent progress in fabrication techniques and procedures have since raised this

quantity by better than an order of magnitude. It will therefore be of much interest for future work to pursue flux cutting and cross-flow research on these improved second and third generation materials.

We will see that the YBCO tube we have examined exhibits the same salient flux line cutting and cross-flow features observed in the BiSCCO specimen. The signature of the flux line cutting in the intergrain matrix is smudged in the YBCO sample data by significant background contributions which complicate the quantitative analysis of the behaviour. We will discuss the origin of this background, and indicate why it was not noteworthy in the BiSCCO data. In the YBCO tube, however, we were able to witness, "separately", the flux line cutting in the interior of the grains.

The measurements on the YBCO sample were performed on an "isolated" tube since no high T_c tube of sufficiently large diameter to "comfortably" embrace this specimen coaxially was available when the investigation was carried out. We stress that an outer tube must not only contain the inner tube but provide sufficient annular space to accommodate the several components indicated in Fig 2.1.

9.2. Results

The evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$, the axial magnetic flux densities in the cavity and in the wall of the tube, as an azimuthal magnetic field $\langle H_\phi \rangle$ is impressed and removed in static ambient axial magnetic fields $H_{//}$, is displayed in Figs 9.1 through 9.4 for various $H_{//}$. These data curves were selected from a catalogue of measured curves to illustrate the observed phenomena over the low field range $0 < \mu_0 H_{//} \leq 3\text{mT}$ where the evidence of cross-flow of flux lines in the weak link regime is quite dramatic. The rise of

$\langle B_z \rangle_{\text{hole}}$ as $\langle H_\phi \rangle$ is applied carries the clear signature of the cross-flow of flux lines since here the wall is releasing axial flux into the cavity although the rise in the magnitude of the total applied field,

$$\vec{H}_{\text{total}} = \hat{\phi} H_\phi (R_i) + \hat{z} \frac{\langle B_z \rangle_{\text{hole}}}{\mu_0} \quad (9.1)$$

is nucleating and "pumping" helical flux lines into the wall. This "pumping" action removes axial flux from the cavity.

The behaviour as $\langle H_\phi \rangle$ is reduced to zero is "traditional" in the sense that it qualitatively corresponds to expectations from simple migration of flux line populations. Here helical magnetic flux lines are being released by the wall into the cavity (and the environment) thereby causing $\langle B_z \rangle_{\text{hole}}$ to rise and $\langle B_z \rangle_{\text{wall}}$ to fall.

In Fig 9.5 curves of the evolution of $\langle B_z \rangle_{\text{hole}}$ $\langle B_z \rangle_{\text{wall}}$ as H_ϕ is impressed are juxtaposed in a family picture to display the effect of the augmentation of the static $H_{//}$ on the salient and important features, namely;

- (i) the height of the peak traced by $\langle B_z \rangle_{\text{hole}}$,
- (ii) the horizontal position (value of $\langle H_\phi \rangle$), for the summit of the peak,
- (iii) the depth of the valley traced by $\langle B_z \rangle_{\text{wall}}$, and ,
- (iv) the horizontal position (value of $\langle H_\phi \rangle$) for the bottom of the valley.

Note that in Fig 9.5 the baselines have been shifted so that all the curves have the same vertical starting level in order to better illustrate the effect of $H_{//}$ on the height of the peak and the depth of the valley.

Fig 9.6 presents the dependence of $\Delta \langle B_z \rangle_{\text{hole}}$, the height of the peaks on the static $H_{//}$. The height of each peak is measured relative to the superconducting state starting level. We note that the starting level or baseline is not $\mu_0 H_{//}$.

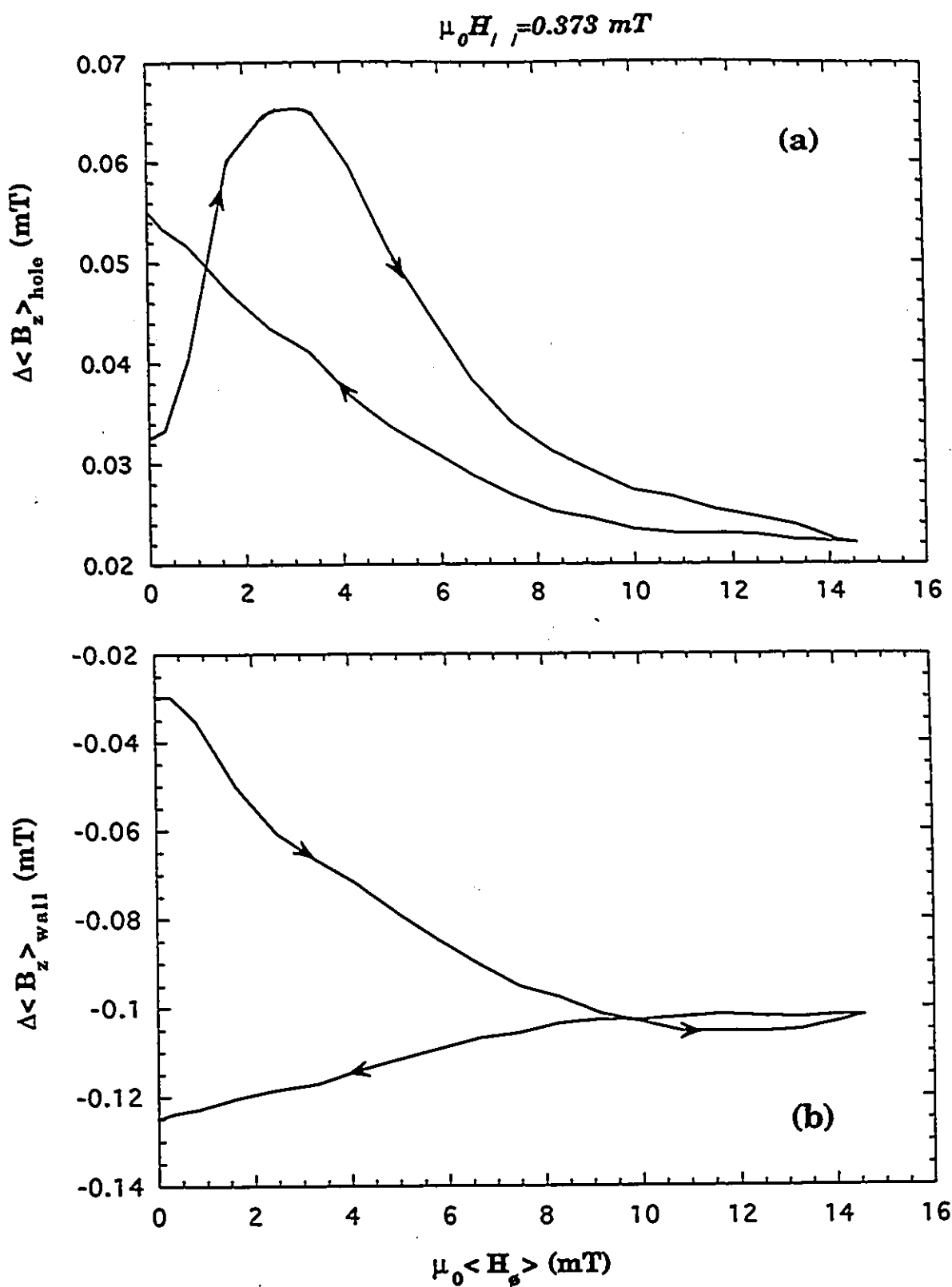


Fig 9.1 Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ in a static axial field $\mu_0 H_{\parallel} = 0.373 \text{ mT}$.

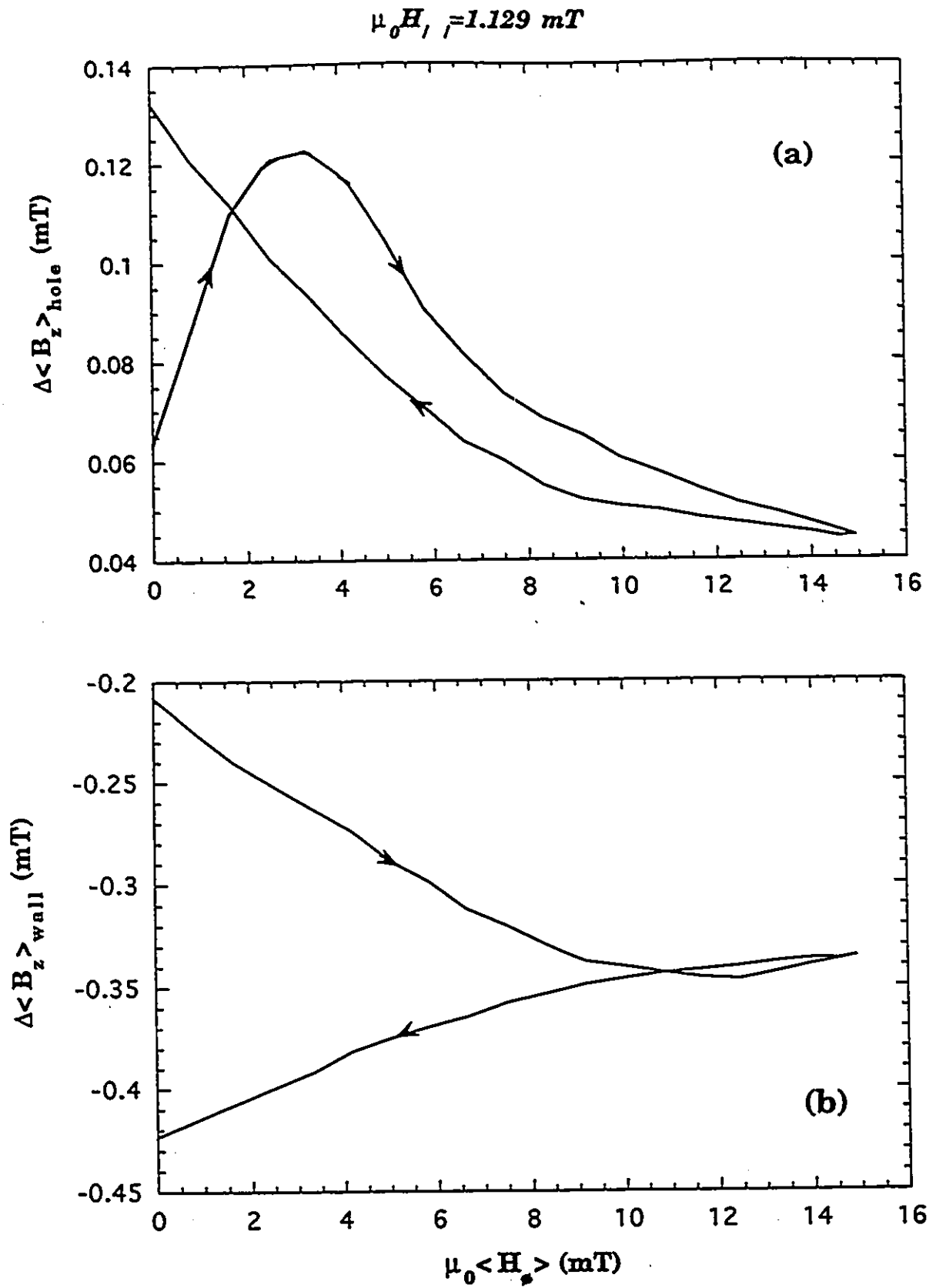


Fig 9.2 Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_z \rangle$ in a static axial field $\mu_0 H_{\parallel} = 1.129 \text{ mT}$.

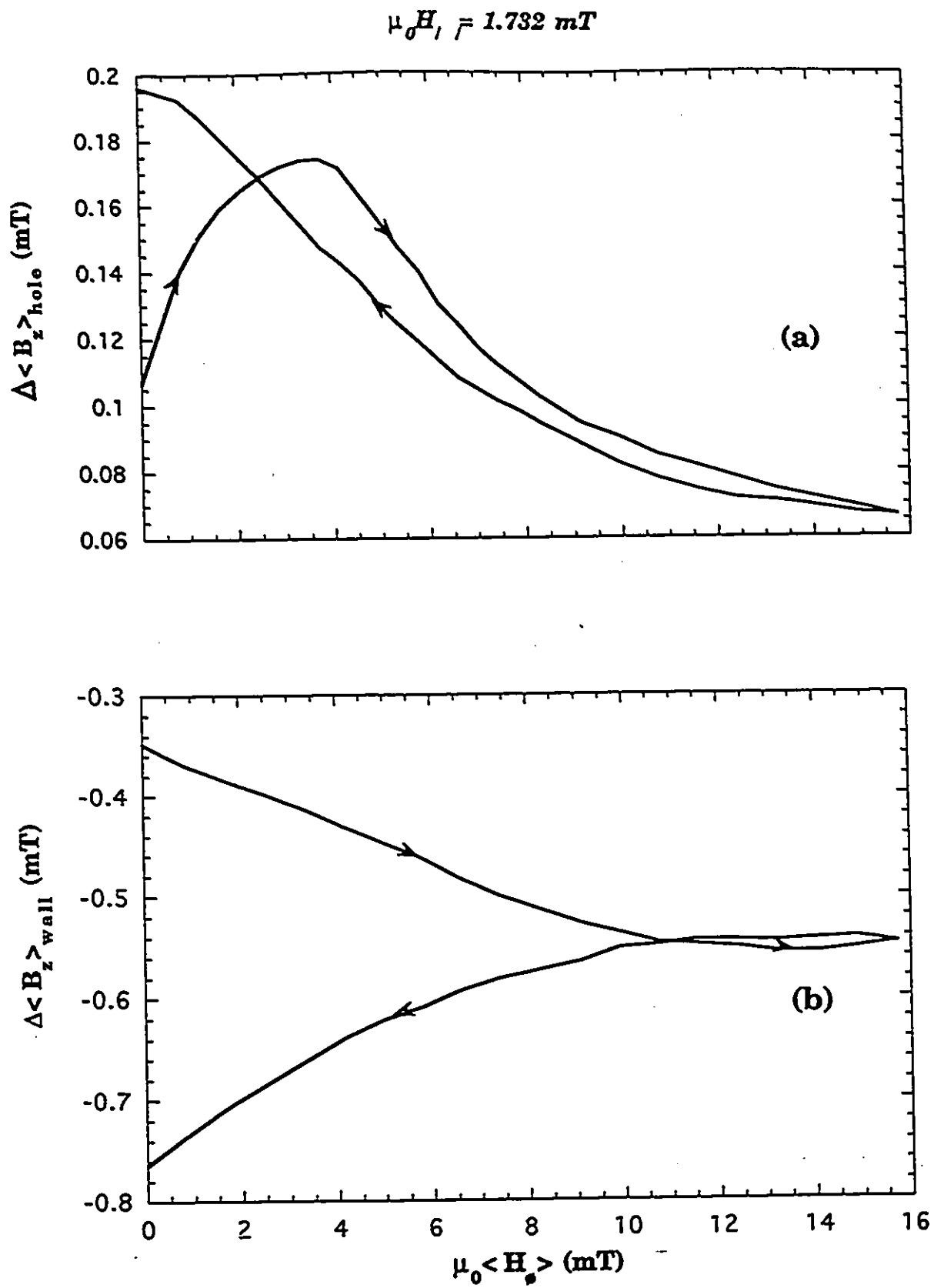


Fig 9.3 Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ in a static axial field $\mu_0 H_{\parallel} = 1.732 \text{ mT}$.

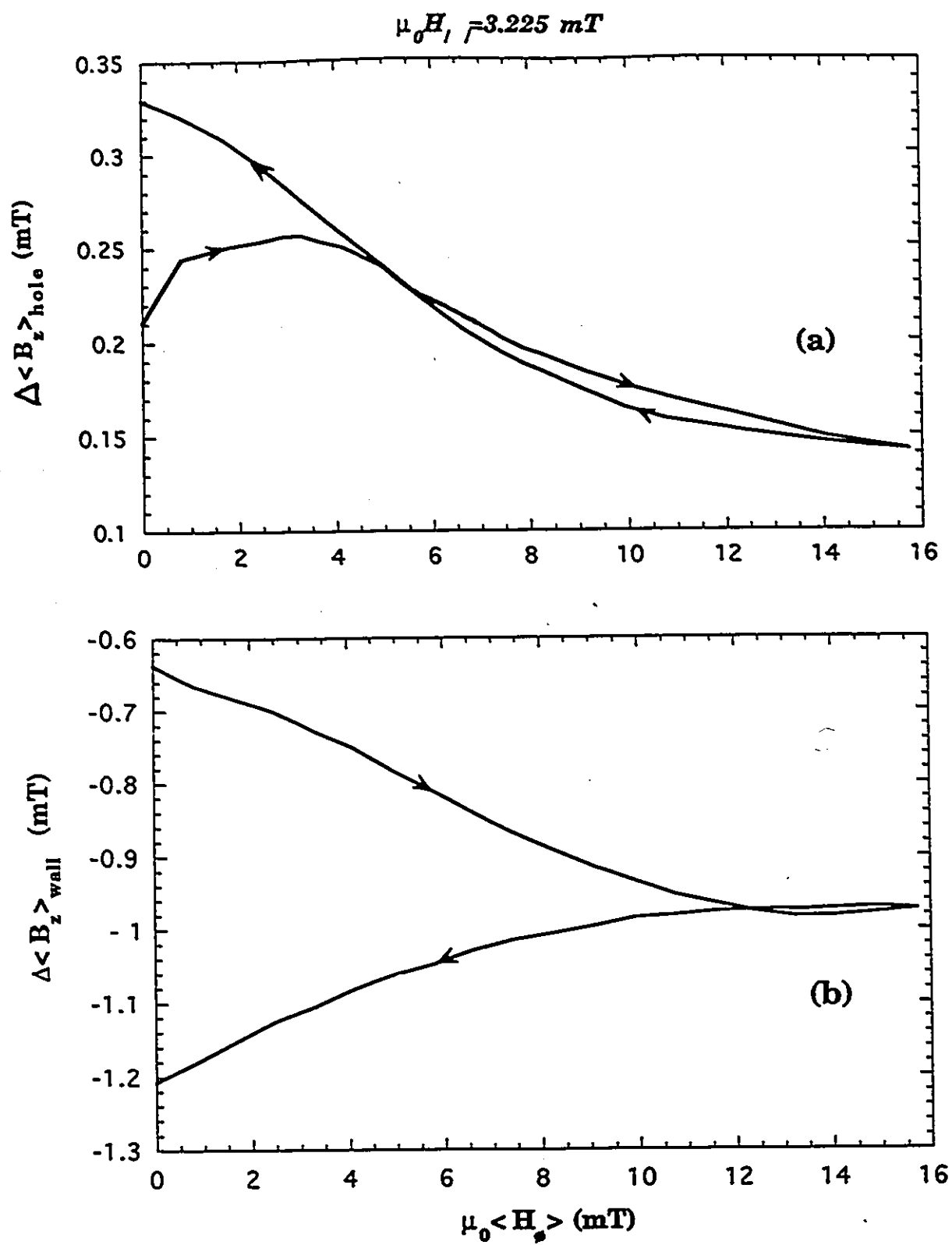


Fig 9.4 Evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ in a static axial field $\mu_0 H_{\parallel} = 3.225 \text{ mT}$.

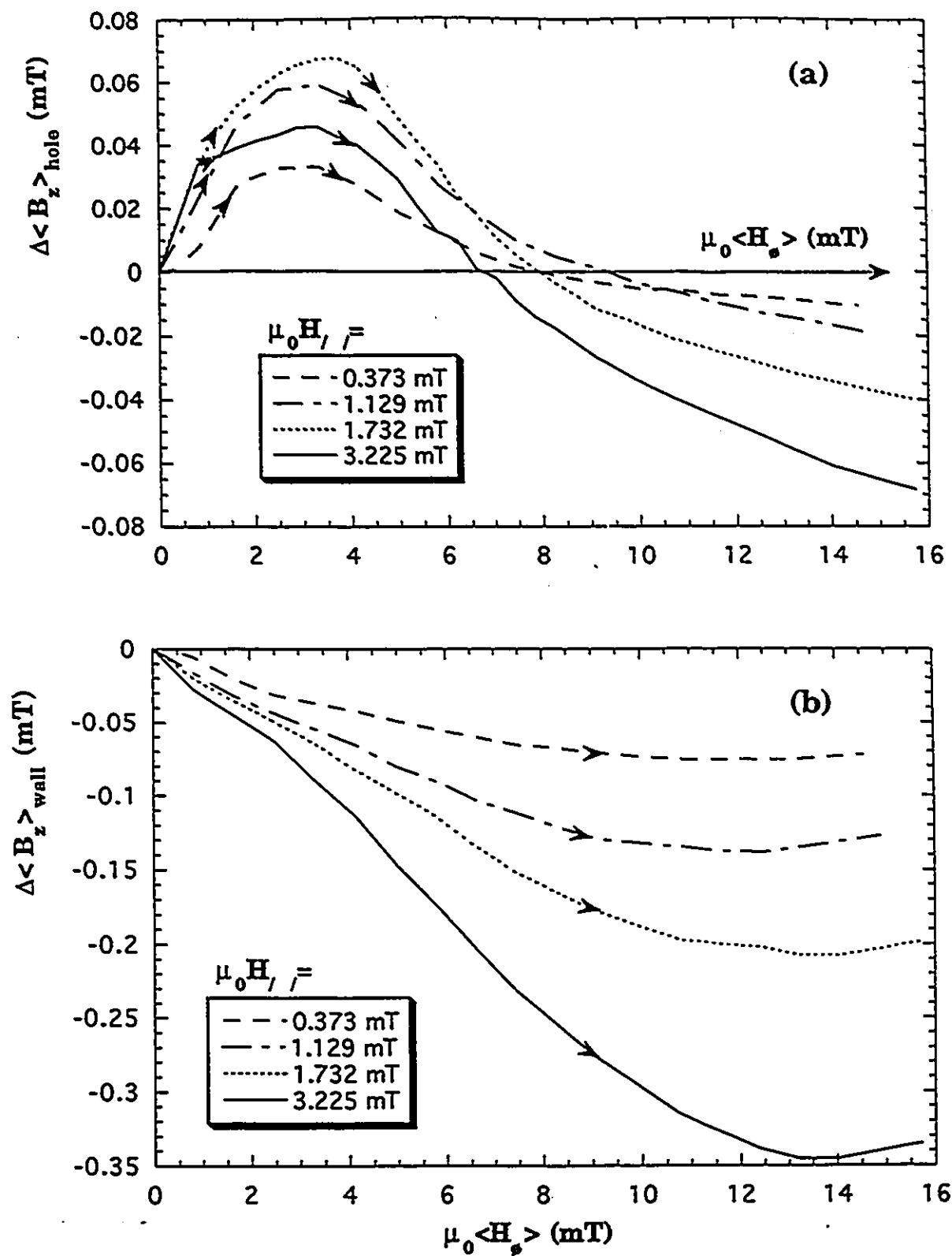


Fig 9.5 A superposition (family picture) of curves of the evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ when $\langle H_\phi \rangle$ is impressed.

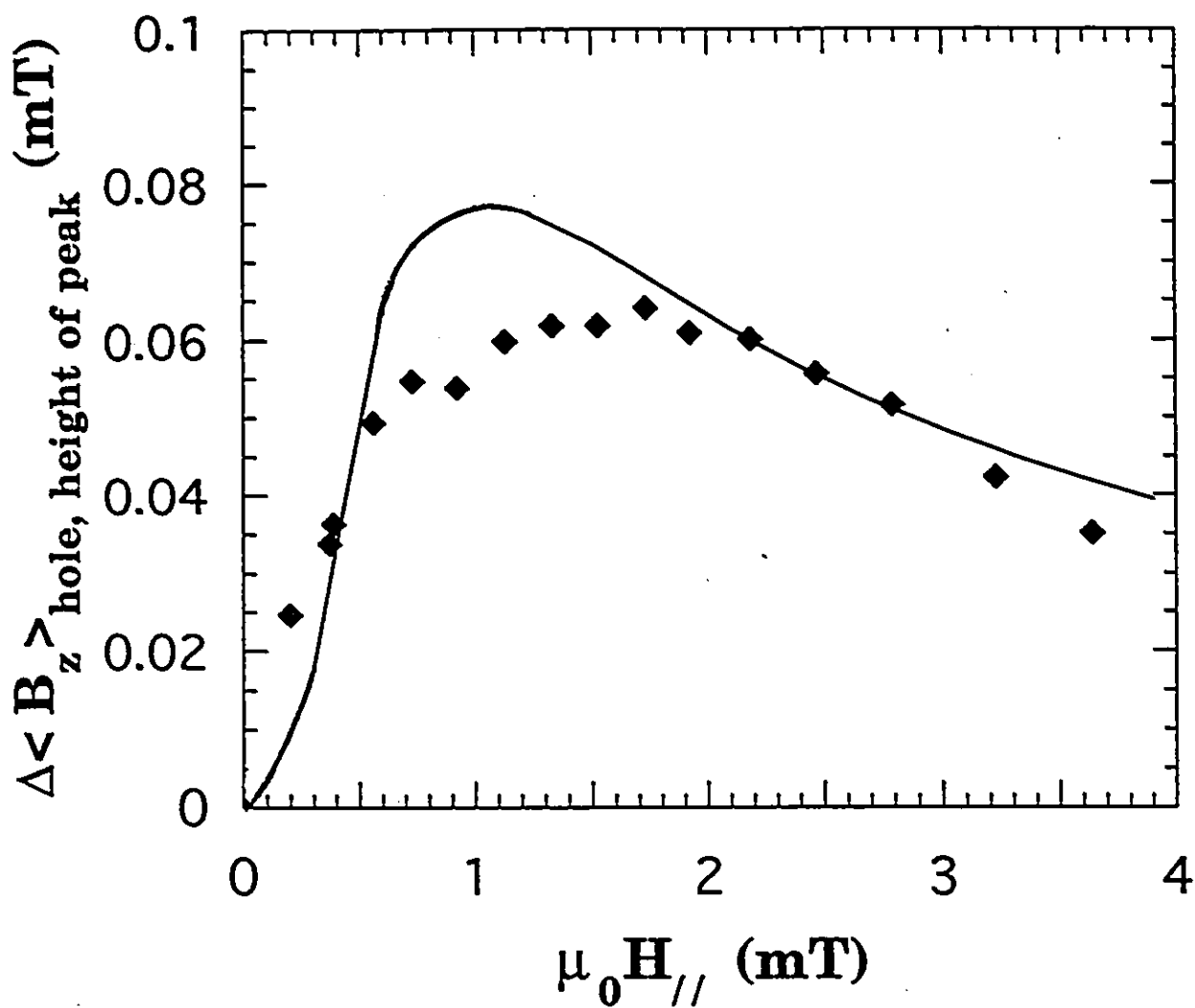


Fig 9.6 The data points display the maximum height of the peaks of $\Delta \langle B_z \rangle_{\text{hole}}$ vs $\langle H_{\phi} \rangle$ (see the previous figure) vs the static $H_{//}$ present during their evolution. The solid curve is theoretical.

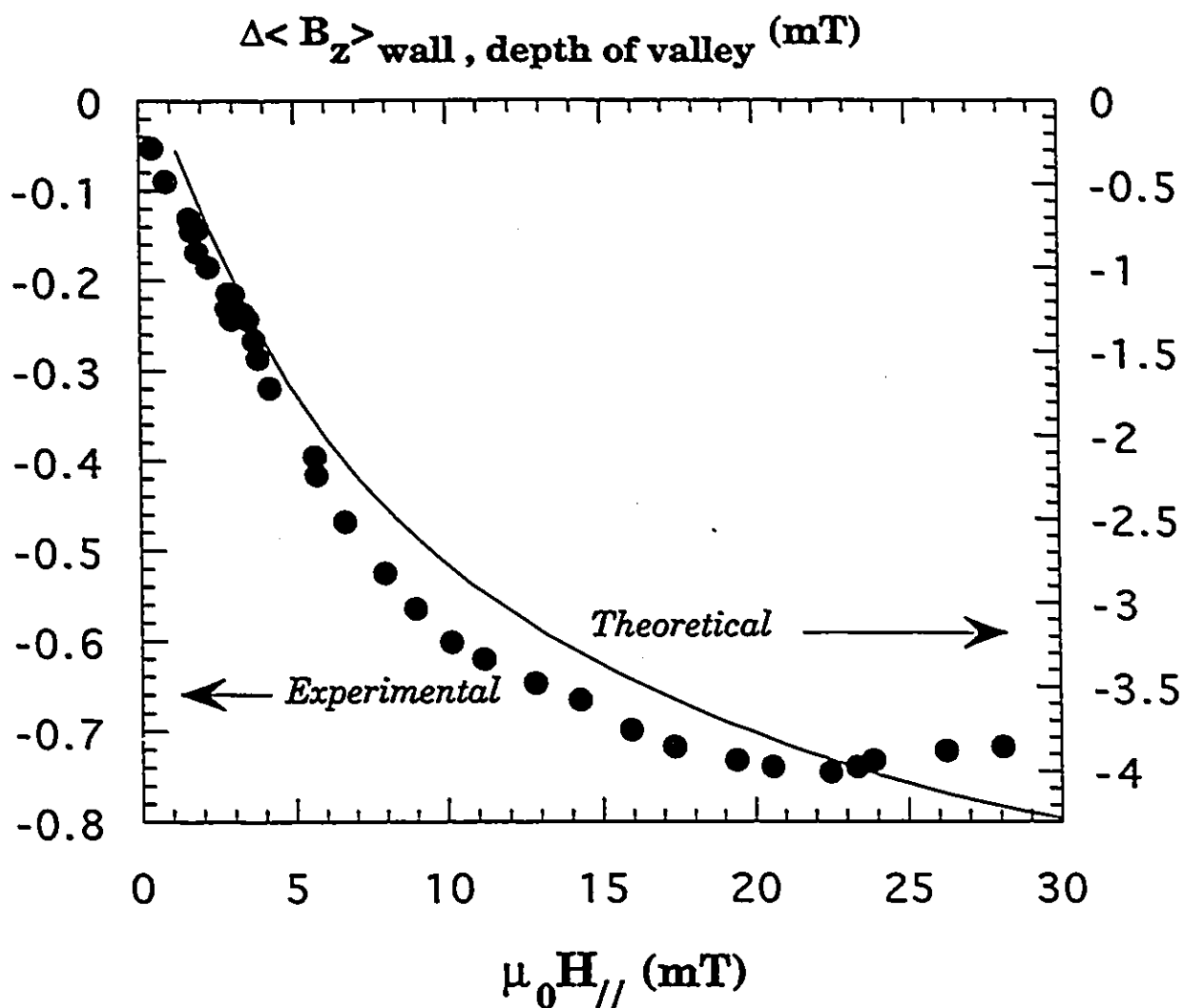


Fig 9.7 The data points display the maximum depth of the valleys of $\Delta \langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ (see figure 9.5) vs the static $H_{//}$ present during their evolution. The solid curve is theoretical.

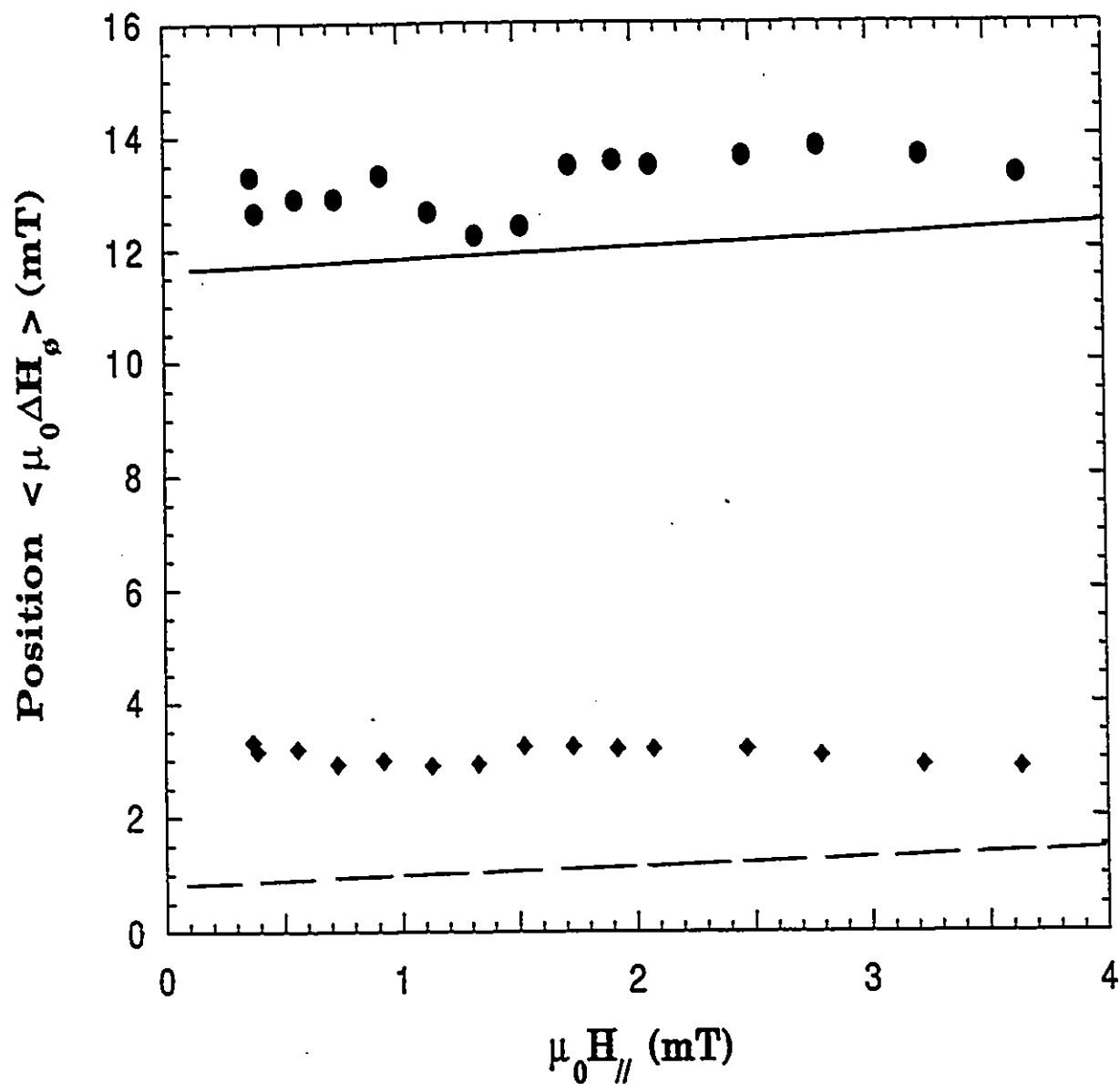


Fig 9.8 ● and ♦ data points display the "horizontal position" (the value of $\langle H_{\phi} \rangle$) when the valleys and peaks of curves such as displayed in Fig 9.5 are traversed vs the static $H_{//}$. The solid and dashed curves are theoretical results .

Since flux was expelled by the wall into the cavity during field cooling in $H_{//}$, the baseline is given by ,

$$\Delta < B_z >_{\text{hole}}^{\text{F.C.}} + \mu_0 H_{//} \quad (9.2)$$

where $\Delta < B_z >_{\text{hole}}^{\text{F.C.}}$ denotes the rise of the axial flux density occurring in the cavity during field cooling in $H_{//}$. Similarly, the baseline for the determination of the depth of the valleys is not $H_{//}$, but the superconducting state level.

$$\mu_0 H_{//} - \Delta < B_z >_{\text{wall}}^{\text{F.C.}} \quad (9.3)$$

where $\Delta < B_z >_{\text{wall}}^{\text{F.C.}}$ denotes the magnitude of the decrease of the axial magnetic flux density occurring in the wall during field cooling in $H_{//}$ because of the Meissner effect (flux expulsion).

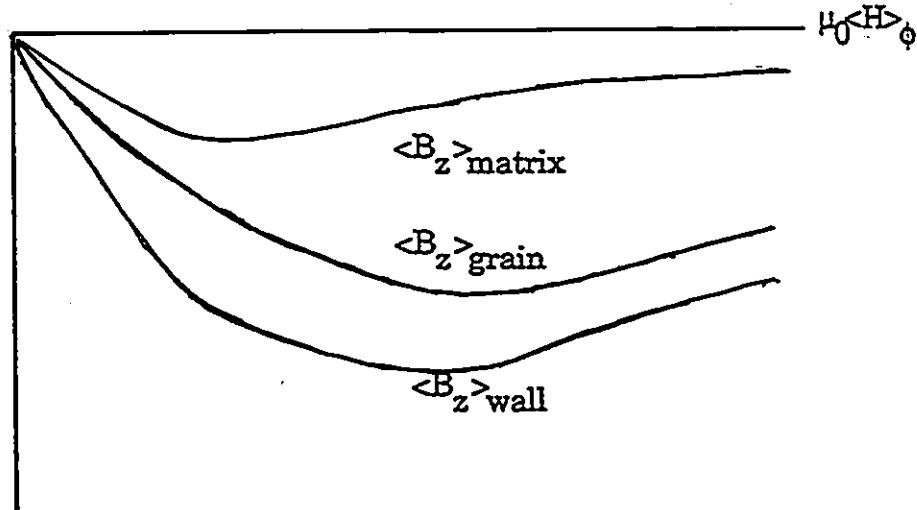
The values attained by $< H_{\phi} >$ when , (a) the summit of the peaks is attained and, (b) the bottom of the valleys is traversed , are displayed as a function of the static $H_{//}$ in Fig. 9.8 .

It is evident from a glance at Figs 9.1 through 9.5 and Fig 9.8 that the peaks and valleys are widely separate occurrences. This behaviour contrasts markedly with that encountered in the BiSCCO tube where these two sets of events nearly coincided. In the BiSCCO tube, the location of the summits of the peaks and the bottom of the valleys not only corresponded closely with each other but coincided approximately with $H_{*m}/2$, the first full penetration field into the wall from both surfaces (hence the factor 1/2) . For these reasons , these signatures of flux line cutting and cross-flow were identified with the intergrain (weak link) structure and the associated critical current densities j_{c1m} and $j_{c//m}$.

It is also noteworthy that in the YBCO sample , the location of the summits of the peaks also corresponds to H_{*m} for this specimen whereas the position of the valley bottoms is related to H_{*g} , the first penetration field into the grains.

In view of the foregoing , we propose the following scenario. The peak observed in the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$ arises , as in the BiSCCO tube, from flux line cutting and cross-flow in the matrix (intergrain volume) causing the release of axial flux into the cavity. This process, as in the BiSCCO tube, is accompanied by a decrease in $\langle B_z \rangle_{\text{wall}}$ which we will denote $\langle B_z \rangle_{\text{matrix}}$ to indicate its origin explicitly . Concurrently, flux line cutting and cross-flow in the grains is also causing a diminution in $\langle B_z \rangle_{\text{wall}}$, which we denote $\langle B_z \rangle_{\text{grain}}$, again to indicate its origin explicitly. The valley which is observed in $\langle B_z \rangle_{\text{wall}}$ results from the superposition of these two concomitant events , as sketched below . Thus we can write,

$$\langle B_z \rangle_{\text{wall}} = \langle B_z \rangle_{\text{matrix}} (H_\phi) + \langle B_z \rangle_{\text{grain}} (H_\phi) \quad (9.4)$$



The shallow valley associated with H_{*m} and the corresponding weak intergrain $j_{c\perp m}$ and $j_{c//m}$ traverses its minimum well before the deeper and more extensive valley associated with $H_{*g} \gg H_{*m}$ and the larger $j_{c\perp g}$ and $j_{c//g}$ of the grains . Consequently , the intragrain valley overwhelms and masks the small intergrain valley.

We stress that the flux released by the agglomeration of grains "vanishes" since the continuous matrix does not possess the current carrying capacity to confine this large amount of flux in the wall and in the cavity , especially when $\langle H_\phi \rangle$ becomes large . Therefore for an infinitely (very) long tube , the valley of $\langle B_z \rangle_{\text{grain}}$ vs $\langle H_\phi \rangle$ would not generate a corresponding peak in $\langle B_z \rangle_{\text{hole}}$. Because of the aspect ratio of the thick walled "stout" YBCO tube , some of the return flux of the magnetization of the grains through the cavity will however cause a modest rise in $\langle B_z \rangle_{\text{hole}}$ by an amount $k|\Delta \langle B_z \rangle|_{\text{grain}} = \mu_0 k |M_g| \approx 0.1 \mu_0 |M_g|$. We will ignore this perturbation or background in our semi-quantitative calculation of the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$.

9.3.Semi-Quantitative Analysis

In the foregoing we have outlined a specific scenario for the behaviour of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ as $\langle H_\phi \rangle$ is impressed in various static $H_{//}$.Our objective in this analysis is to examine this scenario quantitatively exploiting the Clem/Perez-Gonzalez phenomenological theory of flux line cutting and transport.

Although the YBCO tube is thick walled, hence the ratios $(R_o - R_i)/R_o = 1/2$ and $(R_o - R_i)/R_i = 1$ are far from small, we will set aside the formidable mathematical complications encountered in cylindrical geometry.

Further, although the YBCO tube is "stout" , we will view it as infinitely long , hence ignore end effects , return flux , demagnetization factors, etc.

We stress that the two "parameters" which enter into the application of the Clem/Perez-Gonzalez framework are the critical current densities for flux line depinning and cutting , $j_{c\perp}$, $j_{c//}$. The qualitative explanation just outlined for our observations on $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ for the YBCO tube took into account two such pairs of "parameters" or functions, namely $j_{c\perp m}(B)$, $j_{c//m}(B)$ and $j_{c\perp g}(B)$, $j_{c//g}(B)$. Consequently, we will examine separately and compare the quantitative predictions of these two sets of ingredients with our observations.

The Clem/Perez-Gonzalez framework can be exploited to describe the solution of the magnetic flux density configurations , hence $\langle B_z \rangle_{\text{wall}}$ and $\langle B_\phi \rangle_{\text{wall}}$, as a function of the evolution of the boundary conditions ($B_z(R_i)$, $B_z(R_o)$, $B_\phi(R_i)$ and $B_\phi(R_o)$).

As in the analysis of the phenomena in the BiSCCO tube , besides regarding the tube as very (infinitely) long , we further simplify the problem in the following ways.

We consider $B_z(R_i)$ as stationary and take $B_z(R_i) = B_z(R_o) = \mu_0 H_{//}$.

Also we introduce the same approximations with respect to the applied azimuthal fields that,

$$H_\phi(R_o) \equiv H_\phi(R_i) = \langle H_\phi \rangle \quad (9.5)$$

although these simplifications are less valid for the thick walled YBCO tube since here,

$$H_\phi(R_o) = 0.7 \langle H_\phi \rangle \quad \text{and} \quad H_\phi(R_i) = 1.4 \langle H_\phi \rangle \quad (9.6)$$

The first new feature now introduced in the quantitative interpretation of the phenomena is to regard the weak link network (the matrix) and the agglomeration of grains as two independent entities. We now endeavor to justify this crude but crucial and radical simplification of the entangled reality.

In the low field range where the peak in $\langle B_z \rangle_{\text{hole}}$ appears, the grains, because of their high critical currents densities $j_{c\perp g}(B)$ and $j_{c//g}(B)$ are effectively impervious to the changes occurring in the configurations of the magnetic flux density permeating the matrix as $\langle H_\phi \rangle$ is impressed. It is these changes however which cause flux line cutting and cross-flow to take place in the matrix since here, $j_{c\perp m}$ and $j_{c//m}$ are very small. In other words the interior or bulk of the grains does not experience the drama taking place around them.

In the higher field range, the basically weak $j_{c\perp m}$ and $j_{c//m}$ are essentially suppressed since both of these quantities are highly sensitive to the strength of the magnetic flux density. Consequently in the higher field range of $\langle H_\phi \rangle$ where the valley in $\langle B_z \rangle_{\text{wall}}$ becomes deep, both $j_{c\perp m}$ and $j_{c//m}$ can be ignored. Hence now the grains can be regarded as a collection of electrically isolated entities embedded in an essentially normal state medium. The valley in $\langle B_z \rangle_{\text{wall}}$, then, in the high field range, can be attributed to the occurrence of flux line cutting and cross-flow in the collection of separate grains where the pertinent pair of parameters are $j_{c\perp g}(B)$ and $j_{c//g}(B)$.

9.4. Valley in $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ (Property of the Grains)

Consequently our theoretical approach to the analysis of the valleys in the evolution of $\langle B_z \rangle_{\text{wall}}$ is identical to that which we applied to the study of the

BiSCCO tube. Indeed , after some exploratory calculations with various dependences of $j_{c\perp g}$ and $j_{c\parallel g}$ on B we selected the same simple functions introduced earlier , namely,

$$j_{c\perp g} = \alpha_{\perp} \quad (\text{Bean approximation}) \quad (9.7)$$

$$j_{c\parallel g} = \frac{\alpha_{\parallel}}{H} \quad (\text{Simple Kim approximation}) \quad (9.8)$$

The choice for $j_{c\perp g}$ is suggested by the flux trapping behaviour of the tube in large H_{cool} and H_{cycle} as we have seen in the previous chapter where we found $H_{cool}/H_{*g} = 2$, hence Bean like ,

The choice for $j_{c\parallel g}$ is made partly because of its simplicity, partly because it corresponds to that introduced in the description of the BiSCCO tube and also because it generates an overall description of the observations somewhat superior than that provided by other analogous simple approximations (i.e. of the form $j_{c\parallel g} = \alpha_{\parallel}/H^n$ where $0 \leq n \leq 1$).

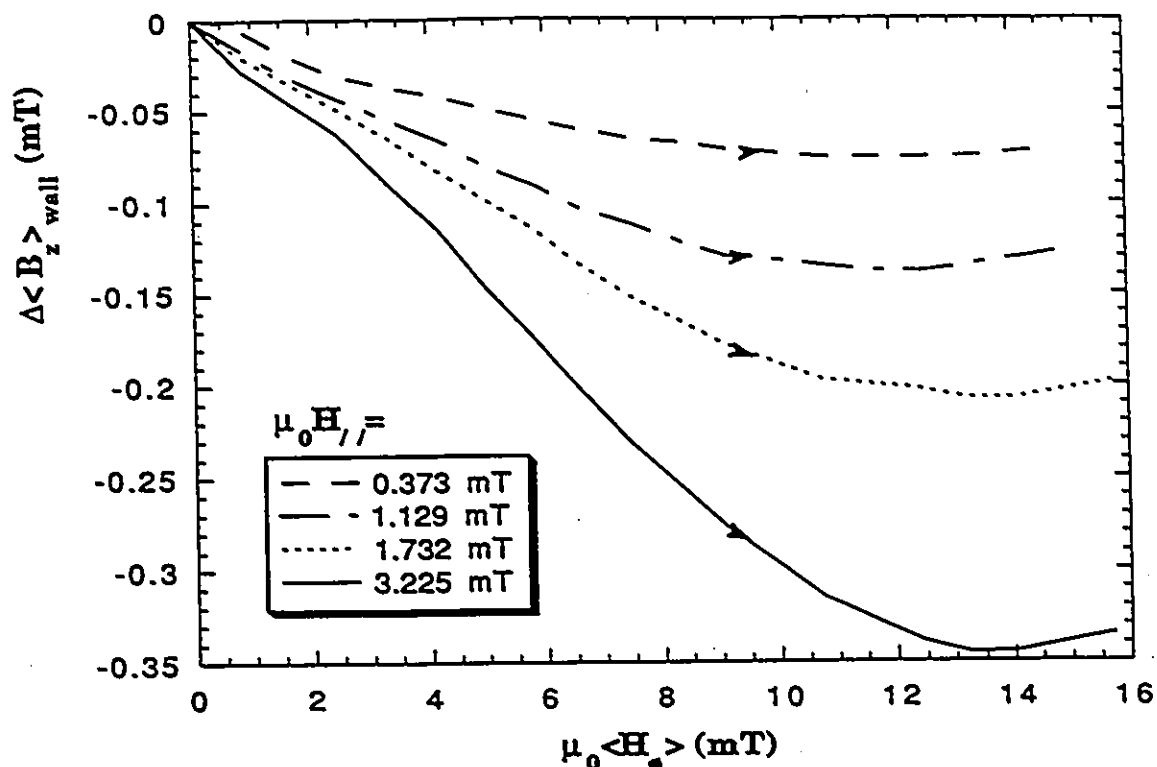
In the calculations , we let ,

$$\mu_0 H_{*g} = 12 \text{mT} \quad (9.9)$$

in harmony with the data on flux trapping in the grains presented of the previous chapter. After some exploration we selected ,

$$\frac{\alpha_{\parallel}}{\alpha_{\perp}} = 0.05 H_{*g} \quad (9.10)$$

Illustrative theoretical curves are compared with corresponding typical measured curves in Fig 9.9 . The depth of the valleys generated by the



Bean-Kim

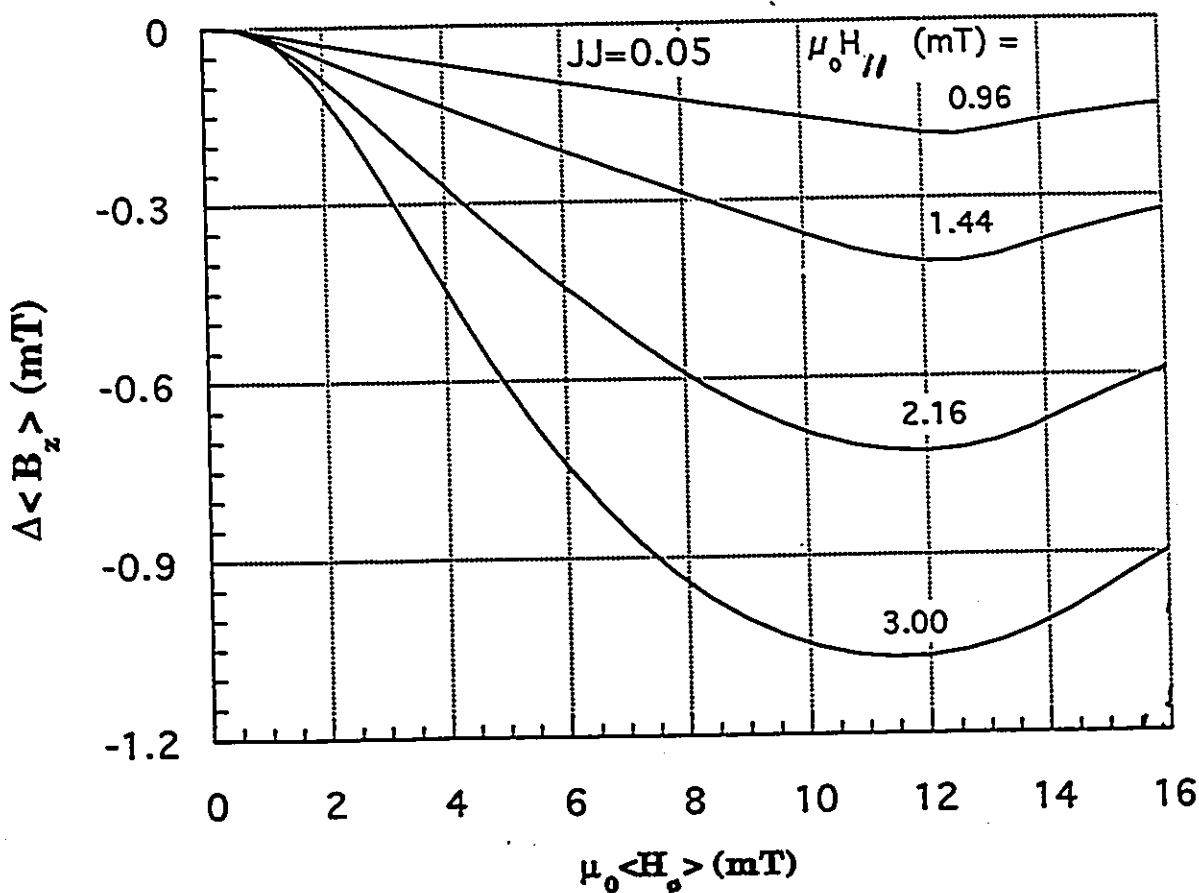


Fig 9.9 Comparison of, (a) a measured and (b) calculated families of curves of $\Delta \langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ in various static $H_{//}$. $j_{c//}/j_{c\perp}$ at $H_* = 0.05$

calculations and the location of the minima are compared with the data in Figs 9.7 and 9.8.

The depths of the theoretical valleys are appreciably greater than that observed. This is perhaps not surprising since in the experiment, $\langle B_z \rangle_{\text{wall}}$ is an average of the axial magnetic flux density over the cross section of the wall whereas the grains occupy only a fraction of the volume of the wall. Indeed if we consider the Clem/Perez-Gonzalez phenomenological theory as "true" and ignore the several simplifications and approximations introduced in its application, then we can estimate f_g , the volume fraction of the grains from the ratio of observed and calculated depths of valleys. This yields $f_g=0.25$ in agreement with estimates from other measurements in sintered YBCO slabs carried out in our laboratory.

9.5. General Comment on the Theoretical Framework

At this juncture it is useful to pause and dwell on a general feature of the calculations. Although the Clem formulation of the Maxwell-Ampere eqn, $\nabla \times \vec{H} = \vec{j}$ contains $j_{\perp}$ and $j_{\parallel}$ explicitly, (e.g. for planar geometry),

$$-j_{\perp} = \frac{dH}{dx} \quad , \quad j_{\parallel} = H \frac{d\theta}{dx} \quad (9.11)$$

it is useful and convenient, in the development of these equations to relate the critical current densities $j_{c\perp}(B)$, $j_{c\parallel}(B)$ to the first penetration field H_1 . For instance, when the dependence of $j_{c\perp}$ on $H=B/\mu_0$ is prescribed by a simple analytical formula of the form,

$$j_{c\perp} = \frac{\alpha_{\perp}}{H^n} \quad (9.12)$$

it is readily found that

$$H_c = \{(n+1)\alpha_{\perp}X\}^{1/(n+1)} \quad (9.13)$$

As a consequence , a knowledge of both the current density $j_{c\perp}$ (i.e. $\alpha_{\perp}$) and the dimension X is not required for the analysis of data and a comparison of theoretical predictions with observations. The determination of H_c for the specimen is sufficient. Subsequently, the magnitude of $j_{c\perp}$ is fixed, if desired , when X is specified . When it is the intergrain critical current density that is under scrutiny, the thickness of the wall fixes X . In scrutinizing the behaviour of the independent grains , X corresponds to the average radius of the equivalent circular rod or half thickness of the equivalent platelets (visualized as oriented with their long dimension along $H_{//}$). To estimate $j_{c\perp g}$ we need to introduce some realistic value for X . A macroscopic examination of the material suggests $X \approx 10^{-5}$ meter as a crude average with an appreciable margin of uncertainty. (Since the grains of the sintered tube possess a variety of shapes , sizes and orientations it is indeed surprising that we obtain a good description of a variety of complicated observations).

In all cases , $j_{c//}$ is scaled relative to $j_{c\perp}$ at H_c regardless of the choice of dependence of $j_{c//}$ on B . In our calculations, for simplicity and convenience , we confined our choice to formulae of the same general form as that exploited for $j_{c\perp}$, namely

$$j_{c//} = \frac{\alpha_{//}}{H^p} \quad (9.14)$$

where p may be equal to or different from n . The parameters $\alpha_{//}$ and p are selected from exploratory calculations in order to identify a pair which generates a family of theoretical curves which provide an optimum fit to the corresponding set of experimental curves.

The curves of $\langle B_\phi \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ are "traditional" and resemble that displayed for the BiSCCO tube. The evolution of $\langle B_\phi \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ is weakly influenced by the presence of $H_{//}$. Indeed the presence of $H_{//}$, facilitates the entry and exit of azimuthal flux as H_ϕ is varied. This behaviour corresponds to that observed in weak pinning monolithic low T_c type II superconductors [29]. Because of their plain structure and poor sensitivity to $H_{//}$, the behaviour of the family of curves of $\langle B_\phi \rangle$ vs $\langle H_\phi \rangle$ has not been studied in detail, since they do not provide a good test and guide on the role of $j_{C//}$, or its magnitude and dependence on B . The qualitative behaviour $\langle B_\phi \rangle$ vs $\langle H_\phi \rangle$ is important however in that these data testify clearly that azimuthal flux is entering the specimen at a rate,

$$\frac{\Delta \langle B_\phi \rangle}{\mu_0 \Delta \langle H_\phi \rangle} \approx 1 \quad (9.15)$$

when the peaks in $\langle B_z \rangle_{\text{hole}}$ and valleys in $\langle B_z \rangle_{\text{wall}}$ are being traced.

9.6. Peaks of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$ (Weak Link Phenomenon)

We believe that the peaks observed in the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$ are a manifestation of flux line cutting and cross flow in the matrix (intergrain network, weak link structure). We support this assertion by showing that the

magnitudes and position of the peaks is dictated by H_{*m} , the first penetration field into the wall , hence $j_{c\perp m}$.

As noted earlier , the Clem/Perez-Gonzalez phenomenological theory provides a framework for predicting the evolution of the magnetic flux configurations in a specimen as a function of the changes of the magnetic fields at its boundaries i.e. $B_z(R_i)=<B_z>_{hole}$, $B_z(R_o)$, $B_\phi(R_i)$ and $B_\phi(R_o)$. We can again "predict" the behaviour of a valley in $<B_z>_{matrix}$ proceeding as we have done previously but introducing the parameter H_{*m} for the matrix in these computations . We note however that in our data , the valley associated with the intergrain structure is overwhelmed and masked by the grains. Consequently we cannot make a direct comparison between these theoretical predictions and observations.

In order to relate our calculations of a hypothesized valley in the matrix vs $<H_\phi>$ to the concomitant evolution of the boundary condition $B_z(R_i)=<B_z>_{hole}$ vs $<H_\phi>$ we proceed as follows. We assume that half of the axial flux which is removed from (or added to) the matrix , as the hypothetical valley is traced , is transferred into the cavity thereby causing a rise (or drop) in $B_z(R_i)=<B_z>_{hole}$.

Thus on one hand we first regard the boundary value $B_z(R_i)=<B_z>_{hole}$ as stationary in order to obtain a first approximation to the evolution of the valley in $<B_z>_{matrix}$ as $<H_\phi>$ is impressed . Then we exploit these results on the evolution of the valley to determine the changes in the boundary condition $B_z(R_i)=<B_z>_{hole}$. This approach is valid since generally , the maximum variation $\Delta <B_z>_{hole} < \mu_0 H_{//}$, the initial ambient field , except in the range of very weak $H_{//}$. We could , of course, then introduce the computed variation of $<B_z>_{hole}=B_z(R_i)$ vs $<H_\phi>$ into a second evaluation of $<B_z>_{matrix}$ and exploit the latter to improve our initial estimate of $<B_z>_{hole}$ vs $<H_\phi>$ and so on . In

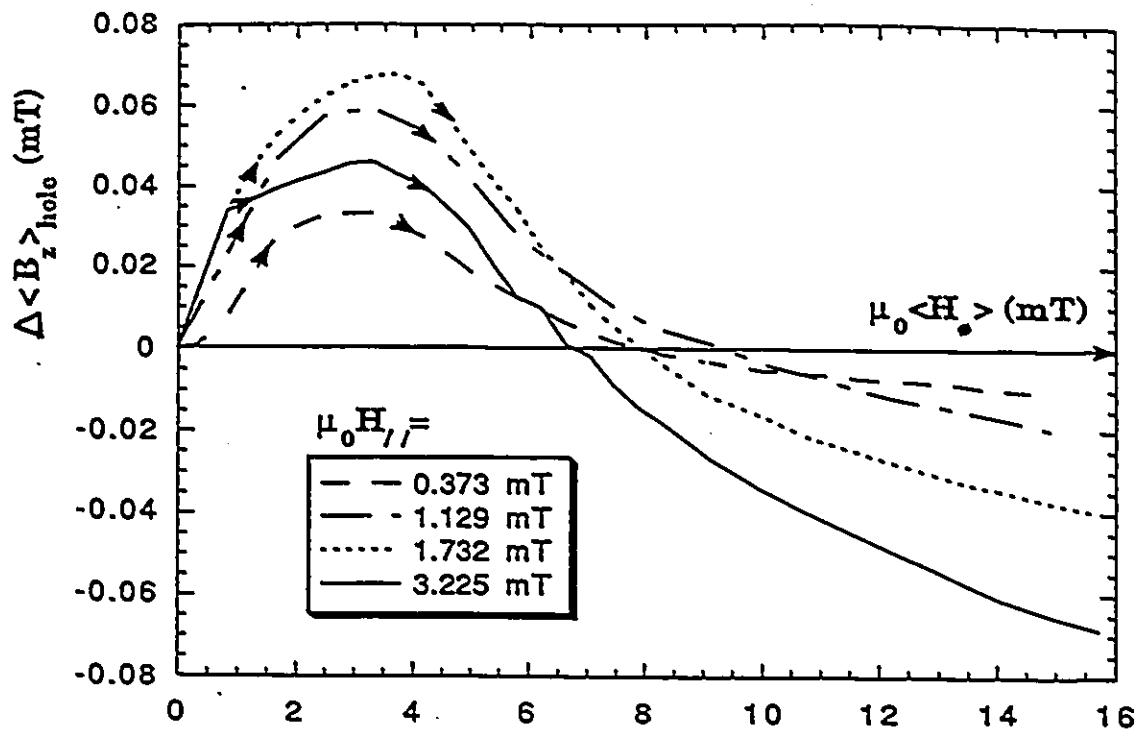
view of the many crude approximations already introduced into the initial calculation of $\langle B_z \rangle_{\text{matrix}}$, we have not deemed this complicated sequential exercise worthwhile pursuing. We present results obtained by a straightforward first approximation (single round) in Fig. 9.10.

The height and the position of the peaks from the calculations are compared with the data in Figs 9.6 and 9.8. We stress that the height of the peaks is proportional to α , hence sensitive to α , the choice of the fraction f of axial flux removed from the wall and transferred into the cavity. In contrast, their location is independent of the choice of f . Thus, agreement between data and calculation in the latter case provides better confirmation of our account. Then, aimed with faith in the validity of the results, we can adjust f to yield agreement between theory and observations with respect to the height of the peaks. This approach therefore yields useful information on the parameter f .

In these computations we have taken $j_{c\perp} = \alpha_{\perp}/H$ and $j_{c\parallel} = \alpha_{\parallel}/H$ where $H_* = H_{*m}/2 = 0.6\text{mT}$ and $j_{c\parallel}(H_{*m})/j_{c\perp}(H_{*m}) = 0.5$ and assumed that one half of the axial flux "vanishing" from the matrix was transferred to the cavity.

9.7. Behaviour in Large $H_{\parallel}$

For completeness, in Fig 9.11(a) we have displayed typical curves of the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_{\phi} \rangle$ in the range of large $H_{\parallel}$ where the return flux of the grains magnetized by flux line cutting and cross-flow (as shown in Fig. 9.11(b)) makes a major contribution to the changes in $\langle B_z \rangle_{\text{hole}}$. We note that as $H_{\parallel}$ is chosen larger, the low field peak in $\langle B_z \rangle_{\text{hole}}$ gradually vanishes and is transformed into a valley. This valley "is traditional" in the sense that is in harmony with the classical picture where flux lines nucleating along the



Kim-Kim.

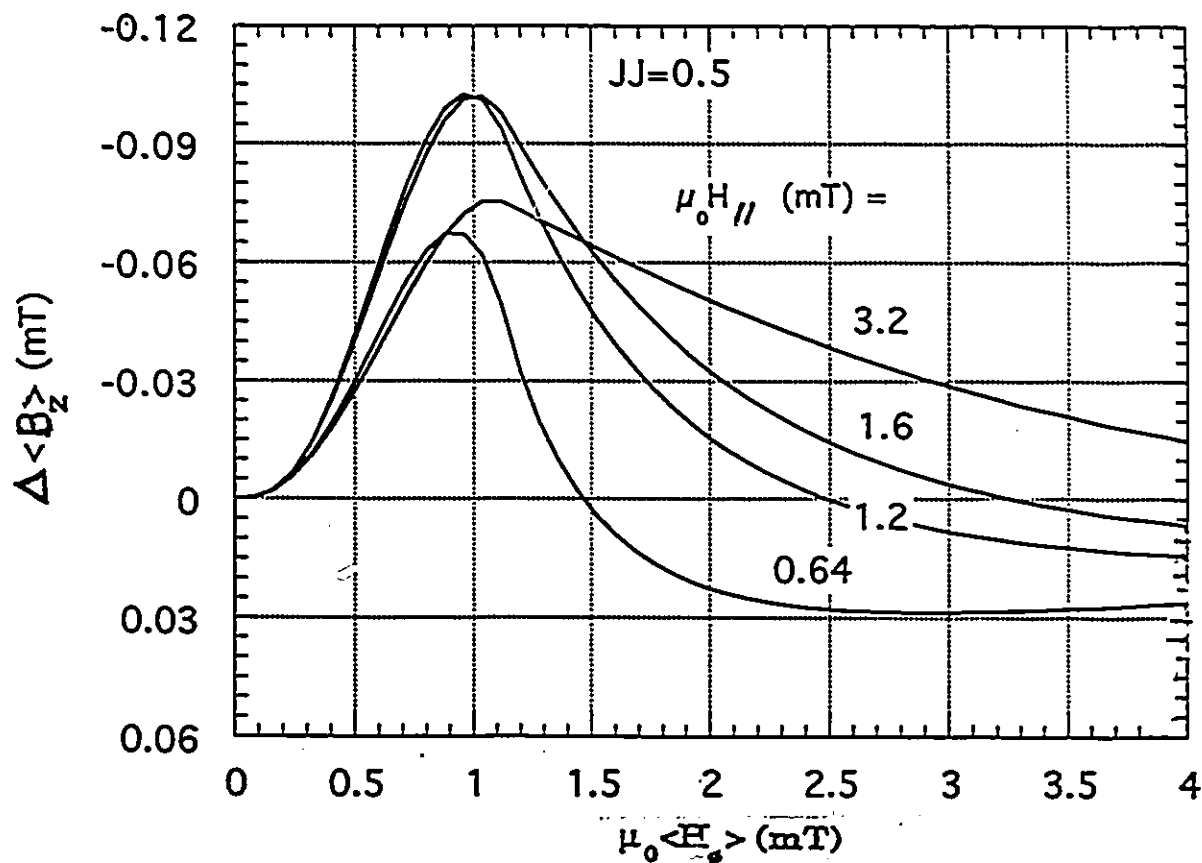


Fig 9.10 Comparison of, (a) a measured and (b) calculated families of curves of $\Delta \langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$ in various static $H_{//}$. $j_{c//}/j_{c\perp}$ at $H_* = 0.5$

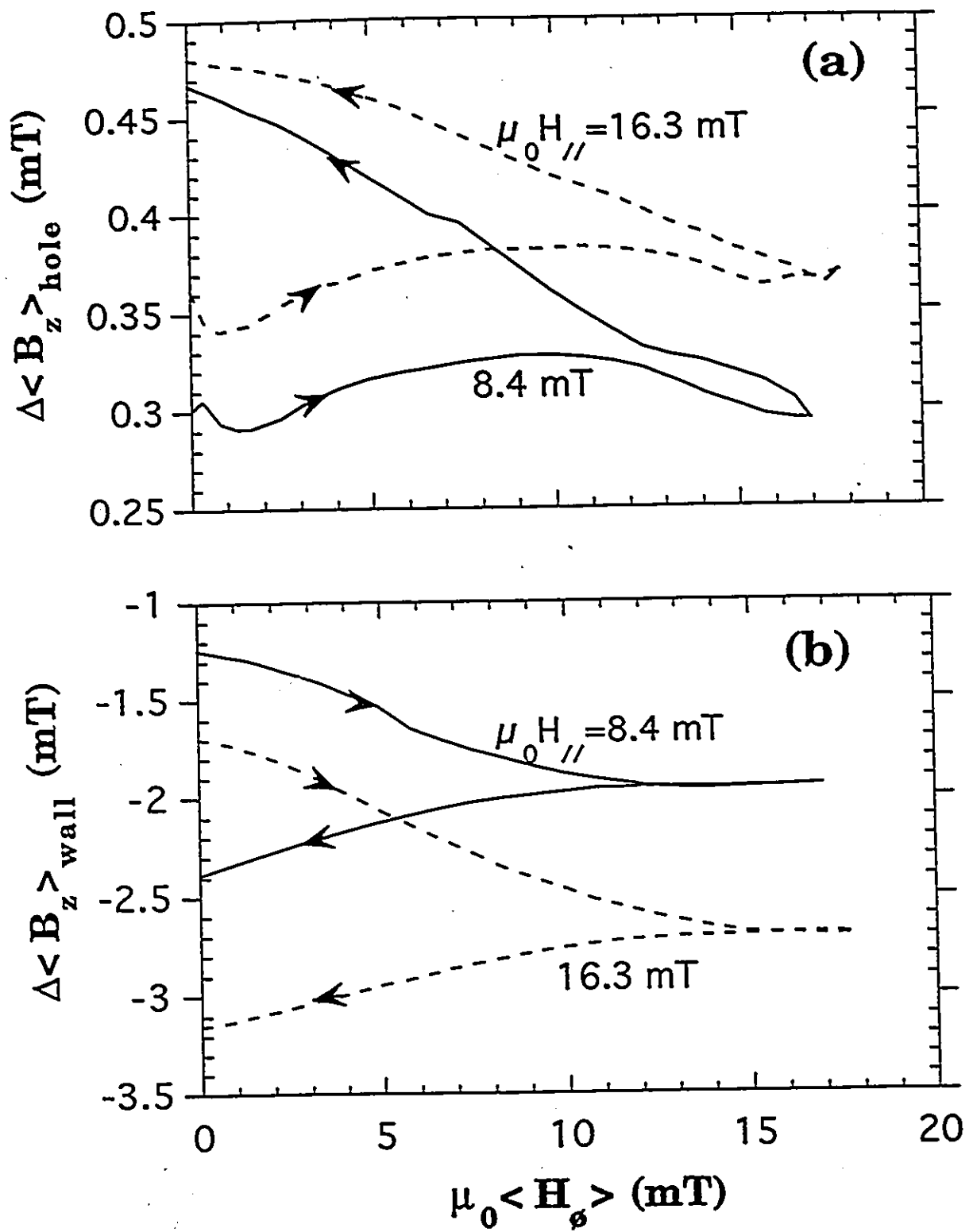


Fig 9.11 Illustrates the evolution of $\langle B_z \rangle_{\text{hole}}$ and $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ in the range of "large" $H_{//}$.

surface R_i because of the increase of $|\vec{H}_{\text{total}}| = \hat{\phi}H_{\phi}(R_i) + 2H_z(R_i)$, deplete the cavity of one quantum of axial flux . The low field peak (or the valley which replaces it) , is followed by broad summit which appears to grow in height as $H_{//}$ is augmented .

We attribute the broad summit to return flux through the cavity from the grains diamagnetizally magnetized by flux line cutting and cross-flow . In other words the valley in $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_{\phi} \rangle$ illustrated in Fig 9.11(b) also manifests itself , via its return flux , in the summits of $\langle B_z \rangle_{\text{hole}}$ (Fig 9.11(a)). The grains in the YBCO tube are not uniformly magnetized since $H_{\phi}(r)$ varies significantly from R_i to R_o . The grains near R_i experience a larger H_{ϕ} than that in the vicinity of R_o . The observed $\langle B_z \rangle_{\text{wall}}$ represents an average over the distribution of the grains along the radius r hence over $H_{\phi}(r)$. $\langle B_z \rangle$ for the grains near R_i is rising from its minimum when $\langle B_z \rangle$ for the grains near R_o is still tracing a minimum. In the cavity, however, the return flux from the grains near R_i overwhelms that from the grains farther away. Hence $\langle B_z \rangle_{\text{hole}}$, which is dominated by the return flux of the magnetized grains in the range of large $H_{//}$, can diminish as seen at the extreme right in Fig 9.9(a) while $\langle B_z \rangle_{\text{wall}}$, which averages over all the grains , is still tracing a minimum.

Because of the many complications which are encountered in developing theoretical curves for $\langle B_z \rangle_{\text{hole}}$ in the range of large $H_{//}$ we have not pursued this enterprise . Further , we anticipate scant benefit occurring from this effort since it would provide no new insights even if agreement were obtained between theory and observations.

9.8. Conclusion and Summary

The behaviour observed in the YBCO hollow cylinder is qualitatively similar to that encountered in BiSCCO tube . Now, however, the valley observed in the evolution of the $\langle B_z \rangle_{\text{wall}}$ as $\langle H_\phi \rangle$ is impressed traverses a minimum at a much higher value of $\langle H_\phi \rangle$ than observed for the corresponding peak in $\langle B_z \rangle_{\text{hole}}$. Consequently the valley in the locus of $\langle B_z \rangle_{\text{wall}}$ vs $\langle H_\phi \rangle$ is attributed to flux line cutting and cross-flow in the grains . As for the BiSCCO tube , the peak in the evolution of $\langle B_z \rangle_{\text{hole}}$ vs $\langle H_\phi \rangle$ is associated with flux line cutting and cross-flow in the matrix or weak link network.

Application of the Clem/Perez-Gonzalez phenomenological theory to the description of the two sets of families of curves yield good agreement in each situation provided that the first penetration field into the grains (H_{*g}) is introduced into the analysis of the valleys and the first penetration field into the matrix (H_{*m}) is introduced into the computations for the peaks.

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