

The Market Volatility Risk Premium and Transaction Costs

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## **Abstract**

Bakshi and Kapadia (2003) support a negative market volatility risk premium with the profit statistics of simulated purchasing and delta hedging S&P 500 options using historical price data. They show that certain profit statistics must be due to a negative volatility risk premium under very general process assumptions. First, without questioning their assumptions, or challenging their theoretical conclusions based on their assumptions, but substantially challenging and changing their statistical methods due to autocorrelation, I still support their overall conclusion of a negative volatility risk premium by roughly replicating a portion of their study with a sample of British FTSE100 options. Second, motivated by their regressions on Vega, a known proxy for transaction costs, I question their assumption that there are no transaction costs. I do so by considering a model that allows for transaction costs but is otherwise far more restrictive, that of Leland (1985), as an explanation for the profit statistics. I am able to reject transaction costs conforming to Leland (1985) as causing the large delta hedged gains observed under fairly liberal transaction cost related assumptions. This provides some support for a negative volatility risk premium under a hypothetical model that would extend the assumptions of BK to include transaction costs.

**Keywords:** volatility risk premium, transaction costs, FTSE, options

**JEL classification:** G13

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# 1. Introduction

Modern option pricing began with Black and Scholes (1973) [Hereafter BS]. The BS model has been successfully tested and is still widely used but it does have known (generally small) biases as noted in Rubinstein (1985). Recent evidence from Bakshi, Cao and Chen (2000) specifically rejects one dimensional (single stochastic variable) models (of which BS is an example) as being able to completely explain option prices. Option pricing theory has developed since BS partly to try to account for these experimental results. To develop the theory, two assumptions (among others) that BS make, that there are no transaction costs and that the process of the underlying is geometric brownian motion (a.k.a. lognormal random walk), have been modified.

The core of virtually all option pricing models is dynamic delta hedging. This means that negative 'delta' (the first derivative of the option price with respect to the underlying) amount of the underlying is added to the option to reduce the risk of the combined portfolio. In the case of BS assumptions, the two item portfolio is riskless. Moving to a second stochastic variable means a second "underlying" or asset is required to make a riskless hedge. In the case of stochastic volatility, if volatility was a traded asset it would suffice as that second asset. However it is not so instead of hedging the second risk (stochastic volatility) to compute a price based on riskless arbitrage, the risk must in general be valued with utility. Theory reduces utility to a risk premium, or rate over the risk free interest rate. If the risk premium is positive then an investor must be compensated for taking on the risk. If it is negative then an investor is willing to pay to

take on the risk. It might seem counterintuitive that the premium would be negative for anything. One way to see how it could be negative for stochastic volatility is to observe that typically a drop in prices in the stock market results in a significant increase in volatility so that purchased options are a hedge against market declines as shown by French, Schwert, and Stambaugh (1987) and Glosten, Jagannathan, and Runkle (1993). It would be ideal to nail down one feature of option pricing models (the sign of the volatility risk premium) in a very general setting so that feature can be built on. To this end Bakshi and Kapadia (2003) [hereafter BK] support a negative market volatility risk premium with very general assumptions. One restrictive assumption they make (without supporting it) is that there are no transaction costs in the underlying.

This assumption is not innocuous in this case as profits proportional to product of the option Vega and the underlying volatility in general (only Vega in the cross section and only volatility in the time series) are precisely what one would expect under plausible transaction cost assumptions and the otherwise much more specific model of Leland (1985). That model is a modification of BS based on the idea that the dynamic delta hedging results in transactions in the underlying which may be costly. That these transactions are likely to have some cost can be seen in Knez and Ready (1996) and Harris and Hasbrouck (1996).

First, without questioning BK's assumptions, or challenging their theoretical conclusions based on their assumptions, but substantially challenging and changing their statistical methods due to autocorrelation, I still support their overall conclusion of a negative volatility risk premium by roughly replicating a portion of their study with a sample of British FTSE100 options. Second, motivated by their regressions on Vega, a

known proxy for transaction costs, I question their assumption that there are no transaction costs by considering Leland (1985) as an alternative explanation for the observed British profit statistics. I reject Leland (1985) as an explanation for these profit statistics. Without extending the model of BK to include transaction costs, it is not possible to formally support negative stochastic volatility risk premium in such a general case. However, a reasonable first order approximation to such a model would show the delta hedged profits as the linear sum of Leland (1985) size transaction costs and a volatility risk premium term. Therefore since the profits are shown to be too large in magnitude to be explained by plausible transaction costs, they must be due to the negative volatility risk premium, and a negative volatility risk premium is given some support under assumptions of BK modified to include transaction costs.

Section 2 will review the literature. Its subsections, 2.1 to 2.3, will review the literature of Black and Scholes (1973), transaction costs and stochastic volatility respectively. Section 3 will cover the methodology of this paper and its differences from BK. 3.1 will cover the basic idea common to BK and its rough replication in this paper, which is that if you hedge against changes in the underlying then if the hedged portfolio has profits, it must be due to the unhedged part – the volatility. 3.2 will cover assumptions and derived propositions which are basically that if there is no volatility risk premium then the delta hedged gains should be zero, and if there is, it should cause certain statistics in those gains. 3.3 will cover the rough data description necessary for the remainder of the methodology section which is basically that tick data is turned into daily data by taking the last quote of the day and then further filtering is done to tune the data to the papers needs. 3.4 will cover the methodology for the summary statistics section of

the results. 3.5 will cover the methodology of the cross sectional test which groups options by volatility at the time they are initially hedged and regresses their gains over the cross section of moneyness on a measure of the option vega. 3.6 will cover the methodology of time series tests which take only at the money (strike near current price) options and regress their delta hedged gains on a volatility measure. 3.7 will cover the methodology of testing that transaction costs could be an alternate explanation for all the profit statistics seen here. Section 4 will cover the results. Section 4.1 will cover the simulations (with in only this case *simulated underlying prices*) which show that options with fairly near strikes have significantly correlated gains which leads to autocorrelation that must be dealt with when doing the cross section regressions. Section 4.2 and its subsections will cover the results of simulations using British price data with the methodology of section 3. Section 5 will conclude.

## **2. Literature Review**

### **2.1. The Black Scholes Model**

Modern option pricing began with Black and Scholes (1973) [hereafter BS]. BS make several assumptions including the following:

- (A) The underlying (stock or index) follows a continuous lognormal random walk.
- (B) There are no transaction costs.
- (C) All securities are perfectly divisible.
- (D) Trading can be continuous

Assumption (A) can be restated mathematically as follows:

$$dS = \mu S dt + \sigma S dz$$

where  $S$  is the underlying price,  $\mu$  is the constant drift of the underlying,  $\sigma$  is the *constant* standard deviation of the underlying, and  $dz$  is a Weiner process. By Ito's lemma the process of an option on this underlying can be written. BS show that a portfolio (sum of two processes) of 1 option and (a continuously changing)  $\Delta$  amount of the underlying, can be created whose coefficient on the Weiner term in the stochastic process is zero. The delta ( $\Delta$ ) is defined by the first derivative of the option price with respect to the price of the underlying. This portfolio must then earn the risk free rate and hence the option price (call or put) is determined completely by this riskless arbitrage. Therefore unlike previous attempts to model option prices, utility is completely removed from the calculation because there is no risk. In other words this arbitrage occurs by *continuously dynamically* hedging the option with  $\Delta$  amount of the underlying. The equation that results from setting the rate of return on this dynamically hedged portfolio equal to the risk free rate is a differential equation whose key boundary condition is that at expiration (for a call option)  $C = \max(S-K, 0)$  where  $C$  is the call price and  $K$  is the strike price. For a call option the solution to this differential equation is:

$$(0A) C = S_0 N(d_1) - K e^{-rT} N(d_2) \quad \text{where}$$

$$(0B) d_1 = [ \ln(S_0/K) + (r + \sigma^2/2)T ] / [ \sigma (T)^{1/2} ]$$

$$(0C) d_2 = d_1 - \sigma (T)^{1/2} \text{ and } N \text{ is the cumulative normal distribution and } r \text{ is the risk free rate and } S_0 \text{ is the initial underlying.}$$

The BS model has been successfully tested and is still widely used but it does have known (generally small) biases as noted in Rubinstein (1985). Recent evidence from Bakshi, Cao and Chen (2000) specifically rejects one dimensional (single stochastic

variable) models (of which BS is an example) as being able to completely explain option prices. Option pricing theory has developed since BS partly to try to account for these experimental results. To develop the theory, two assumptions (among others) that BS make, that there are no transaction costs and that the process of the underlying is geometric brownian motion (a.k.a. lognormal random walk), have been modified. Section 2.2 will introduce the relevant history of papers modifying the one dimensional BS process to a two dimensional process where volatility  $\sigma$  is allowed to be stochastic. Section 2.3 will introduce the relevant history of papers that allow for transaction costs. The models resulting after modification of BS assumptions still virtually always require dynamic delta hedging. However, this hedging or arbitrage does not result in a riskless portfolio, and utility must re-enter in the general case, though some assumptions can remove it again.

## **2.2. Literature Review: Transaction Costs**

Virtually all theoretical and practical option hedging requires some form of dynamic delta hedging. Since deltas change as other variables change - namely the price of the underlying in one dimensional models - there is turnover in the hedged amount of the underlying. Whether the underlying is a stock or future, there are practical costs such as the bid-ask spread and exchange and regulatory fees. The largest of these is the bid-ask spread, and in some cases the others are so small that they could be ignored. Then the question is does a firm making a market in options have to pay the spread in the underlying ? What if they are a market maker in the underlying as well ? These questions can be examined by briefly looking at the economic theory of market microstructure.

In a Walrasian auction with a large number of competitive traders, no frictions and symmetric information, the market would be efficient and there would be no need for a bid-ask spread. Demsetz (1968) showed that the addition of the time dimension to the market mechanism (as exists on the NYSE) adds a price of immediacy seen in the bid-ask spread. Basically he supposes that there are two supply and two demand curves from those who wish to sell and buy immediately and those who are content to wait. This results in two equilibrium prices. At any given time if there are more immediate buyers than sellers they must raise the price to attract the waiting sellers. In many markets a dealer (or many dealers) commit to a price for immediate buy or sell. If there are many dealers, the closest bid and ask are considered to represent the spread. There is usually a quantity quoted for any bid and ask called depth representing the amount the dealer is willing to trade on each side

Stoll (1978) theoretically analyzed the dealer's problem of maximizing utility given the 3 costs in providing this immediacy. These 3 costs are inventory (holding a suboptimal portfolio), order processing costs like exchange fees, and asymmetric information. Stoll (1989) and Huang and Stoll (1997) empirically determined the portion of the costs attributable to the 3 components for the Nasdaq and NYSE using two different methodologies. The papers differ in the weights they find attributable to each component.

Regardless of the weights of the 3 components, it is clear that trading immediately is costly. Knez and Ready (1996) experimentally try to determine the functional form of price improvement. Price improvement is the difference between the execution price of an order and the prevailing bid or ask. They find that there is some price improvement for

market (immediate) orders less than the quoted depth and steep negative price improvement for market orders greater than the quoted depth.

There is some evidence that in some situations lack of need for an immediate trade can reduce transaction costs. Harris and Hasbrouck (1996) show that someone without NYSE floor access can do better than market orders if they use limit orders. However, they also show that such an individual could not profitably act as an off floor dealer.

In our specific context of option market makers, it is unclear how time sensitive the orders for the underlying would be and therefore unclear what kinds of orders would be used. The dynamic hedging could perhaps force only very long time frames to trade. Suppose the hedge is only rebalanced every day. Then trades could be spread out over a day. This would offer some benefit by allowing the use of limit orders. Further an option market making firm might very well be a market maker in the underlying as well. Market makers in the underlying profit on the average trade in the underlying. However they do so by being a *passive* investor accepting others trading needs. The option market maker (or the portion of the firm that makes a market in the options) must trade a certain quantity in a given direction when the underlying changes price and therefore the delta of their hedged portfolio changes. They often have to make large trades which is what leads to possibly large transaction costs despite the possible use of limit orders or market making privileges.

The microstructure research discussed above leads one to conclude that transaction costs are certain to play some role in option pricing but it is uncertain how large a role. It is because these costs are certain to play some role, rather than that specific

kinds of pricing errors were observed, that many modifications to BS were proposed which all used assumed transaction cost parameters.

Three general approaches have been developed to price options under discrete trading and transaction costs: super replication, utility maximization, and imperfect replication. Super replication strategies are those that don't replicate the option return exactly, but dominate it almost surely and only set bounds on the option prices. Utility maximization, recognizes that with transaction costs and discrete trading there is risk for reasonable pricing models and therefore, risk neutral valuation is no longer appropriate. The disadvantage of this method is now investor preferences are taken into account. Other names for this method are stochastic optimal control or dynamic programming. It uses the Hamilton, Jacobi, Bellman equation to find the function that represents the optimal trading strategy for a seller or buyer of an option (they may produce different prices). A key paper in this area was by Hodges and Neuberger (1989). Imperfect replication allows for deviation from BS hedging and introduces criteria for tracking the hedging error. Risk neutrality is assumed. The most widely cited paper in transaction costs, Leland (1985), uses imperfect replication and builds on Boyle and Emanuel (1980).

Boyle and Emanuel (1980) consider discrete hedge trading with continuous stock prices without allowing for transaction costs. They show that if trading takes place frequently then the hedging errors may be small. They also argue that those errors will be uncorrelated with the market return. Leland (1985) considers discrete hedge trading and proportional transaction costs. Virtually all option pricing models which include transactions costs model them as linear. Most further assume that there is no fixed cost only a cost proportional to the value of the underlying traded like Leland (1985) did. He

argues that with transaction costs, discrete hedging like Boyle and Emanuel (1980) will no longer have costs uncorrelated with the market, and to achieve reasonable hedging errors before transaction costs may require unreasonably short revision intervals and high transaction costs (more than the price of the stock itself for example). He then goes on to derive an option pricing model in continuous time, with discrete hedging and transaction costs that does not have these problems. His model assumes that rebalancing of the hedge occurs discretely, at a fixed frequency over the life of the option. The higher the frequency, the higher are the transaction costs, and the lower the variance in the error of the options replication. He derives a formula relating the transaction costs to the Vega of the option, the transaction cost rate and the root of the time interval between rebalancing. His model prices options as though they were priced by the BS formula with a higher volatility.

Since Leland (1985) papers have been written that have somewhat more optimal solutions as stated by Martellini, and Priaulet (2000). They empirically test (with simulated data) various models that incorporate transaction costs. However all these other models result in equations that require numerical solutions. None has a simple expression for transaction costs as a function of other variables. Further no papers that I am aware of use real options/underlying data to empirically test which transaction cost model might be in use by real market makers. Perhaps this is because it is thought that other factors dominate the deviations from BS and these must be nailed down first. This paper is the first known challenge to that implicit assumption. To begin to challenge this assumption we first have to consider the primary other factor that would cause deviations from BS, namely stochastic volatility.

### 2.3. Stochastic Volatility

The change from one dimensional diffusion to two dimensional diffusion means that a second asset is required to perform a riskless hedge. In the case of stochastic volatility, if volatility was a traded asset it would suffice as that second asset. However it is not available so the market is then known as incomplete. Since it is not the risk premium for volatility which incorporates utility is usually required to solve for the option price as done in closed form in Heston (1993). If volatility is assumed to be uncorrelated with aggregate consumption (and therefore the volatility risk premium is zero) then risk neutral valuation can be used to solve for the option price as in Hull and White (1987). Another method of solving for the option price is called super replication. Super replication can only establish bounds on the option price. However without having a priori bounds on the volatility, typically only trivial bounds can be established as Frey (1996) did for the model of Hull and White (1987). Assuming volatility is stochastic, but that volatility is *not-necessarily* uncorrelated with aggregate consumption then there may be a volatility risk premium.

The addition of stochastic volatility to option pricing has improved the fit of observed option prices considerably over BS. This was shown empirically using a somewhat more general model than that of Heston (1993) in Bakshi, Cao and Chen (1997). This was also shown using Heston's (1993) model in Bakshi, Cao and Chen (2000). These papers among others have lead researchers to conclude that stochastic volatility *in some form* should be part of the specification of the underlying process when pricing options. However both those papers among others still note anomalous differences between the model predictions and observed option prices.

Given that stochastic volatility *in some form* should be used in any option pricing model, volatility risk may be compensated since that it can't be hedged and therefore utility must come in to play. That is, assuming volatility is stochastic, but that volatility is *not-necessarily* uncorrelated with aggregate consumption then there may be a volatility risk premium. Given that specific forms of stochastic volatility have resulted in less than perfect option pricing models as noted above, it would be desirable to "nail-down" one aspect of option pricing by at least determining if there is a risk premium and its sign using the most general assumptions possible. To this end BK supported their hypothesis that the volatility risk premium is negative while making few assumptions.

### **3. Methodology**

For one part of this paper, the methodology of BK is roughly replicated with British index option data instead of US index option data. In sections 3.1 to 3.6 below I describe BK's methodology and its differences with this paper. In section 3.7 below I describe my transaction cost methodology. One theme of the differences between the papers is that BK often deals with autocorrelation in the data incompletely or even surprisingly improperly.

#### **3.1. Basic Idea – no changes from BK**

BK simulate the process of purchasing and delta hedging using real data. That is, if they purchase an option on a given day, they immediately delta hedge it and then *adjust that hedge every trading day* till expiration. The purchase is simulated with a payment of the midpoint of the bid-ask spread *of the option* at a given time. The sale of the underlying, is simulated by selling short the index (value probably computed with

transaction prices of the constituents) at approximately the same time and investing the proceeds at the risk free rate. If the price process followed BS assumptions and the hedge was adjusted continuously, then the profit at option expiration would be precisely zero. BK show that under discrete rebalancing and very general assumptions, certain properties of these profits imply a negative volatility risk premium. The *key* idea of BK is that by delta hedging, all risks are removed *except* volatility risk, and if that remaining risk is compensated, the delta hedged gain (profits) will have certain properties. Following the above discussion the discrete delta hedged gains of a given single call option can be calculated with the following equation:

$$(0) \pi_{i,t+\tau} = \max(S_{t+\tau} - K, 0) - C_{K,\tau,t} - \sum_{\theta=0}^{N-1} \Delta_{i(n)} (S_{i(n+1)} - S_{i(n)}) - \sum_{\theta=0}^{N-1} r_{i(n),\tau} (C_{K,\tau,t} - \Delta_{i(n)} S_{i(n)}) \tau / N$$

where  $\pi_{i,t+\tau}$  are the discrete gains for the given call option from time  $t$  to expiration  $t+\tau$ ,  $S$  is the underlying price,  $K$  is the strike price,  $C_{K,\tau,t}$  is the call price,  $\Delta$  is the estimated call delta,  $r$  is the interest rate,  $N$  is the number of trading days to expiration,  $t$  is the trading date in the sample that the option is bought,  $n$  is used to index the trading days till expiration and  $t(n)$  is a trading date of the option between the time it is bought  $t(0)$  (the same as  $t$ ) and the time it expires  $t(N)$  (the same as  $t+\tau$ ). In all cases below  $C_t$  refers to  $C_{K,\tau,t}$  with a fixed unspecified  $K$  and  $\tau$ .

### 3.2. Assumptions and Derived Properties - slight change from BK

The assumptions BK make follow. The four key properties they derive follow after.

BK assume the following:

$$(1) \quad dS = \mu[S, \sigma] S dt + \sigma S dz^1$$

$$(2) \quad d\sigma = \theta[\sigma]dt + \eta[\sigma]dz^2$$

where  $S$  is the underlying price,  $\mu[S,\sigma]$  is the drift of the underlying,  $\sigma$  is the *stochastic* standard deviation of the underlying, and  $dz^1$  and  $dz^2$  are Weiner processes with constant correlation coefficient  $\rho$ ,  $\theta[\sigma]$  is the volatility drift coefficient,  $\eta[\sigma]$  is the volatility diffusion coefficient. The subscript  $t$  was dropped from all preceding variables except  $\rho$  for readability.  $\theta[\sigma]$ ,  $\eta[\sigma]$ , and  $\mu[S,\sigma]$  are unspecified except for their functional form.

BK also assumes zero transaction costs and that the volatility risk premium is of the general form  $\lambda[\sigma]$  with the  $t$  subscripts again dropped. One more definition is useful for the discussion below which is moneyness  $\gamma \equiv Se^{(r-z)\tau}/K$  where  $z$  is the dividend yield from  $t$  to  $t+\tau$ . The interest rate and dividends are assumed to be known with certainty for the life of the option and up to 70 days after expiration due to the need to calculate a futures implied underlying as described in section 4 below.

Using the preceding assumptions BK derive the following key results:

Proposition 0: If the volatility risk premium is zero then the expectation of the continuously rebalanced delta-hedged gains,  $\Pi$ , are zero. (from equation 3 below)

Proposition 1: If the volatility risk premium is zero then the expectation of the *discrete* delta-hedged gains or profits,  $\pi$ , are of order  $1/N$  where  $N$  is the number of times the option is re-hedged in its lifetime.

Proposition 2: If the volatility risk premium is a single sign for all  $\sigma$  then there is a one-to-one correspondence between the sign of the volatility risk premium and the sign of the mean delta hedged gains. This comes from the following equation:

$$(3) \quad E_t(\Pi_{t,t+\tau}) = \int_t^{t+\tau} E_t(\lambda_u[\sigma_u] * (\partial C_u / \partial \sigma_u)) du$$

Where  $\tau$  is the time to expiration,  $\Pi_{t,t+\tau}$  is the continuous profits or delta hedged gains from  $t$  to  $t+\tau$ ,  $\partial C_u/\partial \sigma_u$  is the vega of the call option, and  $E_t(\cdot)$  is the expectation operator. The equation implies proposition 2 because the vega is always positive. The “single sign” condition above was not in the wording of the BK paper, which I believe was an oversight or that they implicitly assumed that it must be a single sign (*slight change from BK here is the only change in this subsection*)

Proposition 3: In the cross section (keeping volatility fixed) we can reject the hypothesis of zero volatility risk premium if we find that the absolute value of the mean delta hedged gains are maximized for at-the-money options and decreases for strikes away from the money like the option vega does.

Proposition 4: In the time series for at-the-money options (moneyness is obviously fixed) we can reject the hypothesis of a zero-volatility risk premium if we find a relation between mean delta-hedge gains and *any* function of physical volatility.

Under the far more specific Heston (1993) stochastic volatility model, they derive a corollary to proposition 4.

Proposition 4b: In the time series for at-the-money options the delta-hedged gains normalized by stock price should be approximately linear in volatility.

Some of these propositions may seem rather vague. One might think this is due to the use of wording rather than mathematics to describe the proposition. This is not the case. It is because so many functions in the stochastic processes are in a general unrestrictive form that the propositions are general as to seem vague.

The only theoretical result that BK derive for *discrete* delta-hedged gains assuming a stochastic volatility model is what I call Proposition 1 above. All the other

propositions *strictly speaking* apply only to continuously hedged gains. Probably because of this they also derived and referred to results done under the BS constant volatility (one dimensional) assumptions. They related the results of Bertsimas, Kogan, and Lo (2000) [here after BKL] who derived the full distribution of the profits under those assumptions. Those results suggest that the distribution of the mean *discrete* delta-hedged gains are centered about zero for a wide range of parameters and for low values of N (the number of times the option is re-hedged) of about 10 (the minimum used in this study since 14 days equals 10 trading days usually). To further support the idea that *discrete* hedging is not skewing their results, they simulate a stock conforming to the specific stochastic volatility model of Heston (1993) where they further set the volatility risk premium to zero. These simulations show that the gains are practically zero and of a far smaller magnitude than the observed losses both in BK's paper and in this one. The main purpose of all of this is to suggest, which I do not question, that the other propositions above can be taken to be *true for discrete gains as well*. The only other thing the simulations are used for is to find better critical points for the time series tests under the small sample bias of the tests they use and given that they use BS hedge ratio as an approximation of the true hedge ratio.

### **3.3. Rough Data Description and Filters - same as BK**

The propositions above all refer to profits from hedging a given single purchased option. The statistical tests are of course performed with the profits from many different purchased and individually hedged options. The time period and complete details of the data, which differ from that of BK, are detailed in a separate section below. However the rough outline of the data is presented now to help illustrate the methodology.

The clearest way to see the what data is used for both papers is to consider a *sequence of filters* on the raw data of all bid-ask European option quotes for a time period of years on a given underlying (each matched with the contemporaneous value of that underlying). The reasons for each filter are discussed after.

DailyFilter: Every day of the sample the *last* bid ask quote before a specific time (near or at the option market closing) for every strike and every expiration is selected.

TimeToExpirationFilter: Only those of the remaining sample which have between 14 and 60 days to expiration are kept.

MoneynessFilter: Only those of the remaining sample which have moneyness  $\gamma$  between .9 and 1.1

BadDataFilter: Call options which were outside the hard arbitrage bounds [ $\max(S - Ke^{-rt}, 0), S$ ] are discarded. Second, all options with Black BS implied volatilities exceeding 100% or less than 1% are discarded.

The reason for the DailyFilter is that there would be a near 100% correlation in the delta hedged gains of two options with the same strike price and time to expiration within a day (they are really the same option at different times). In fact as expanded on below the same options on adjacent trading days are expected to have a very high correlation. One new definition to keep in mind from now on is time value and intrinsic value. Intrinsic value is  $\max(S - Ke^{-rt}, 0)$  or the lower hard arbitrage bound noted above. Time value is the current call price minus the intrinsic value. The reason for the TimeToExpirationFilter is that options with very short time to expiration very low time values which make studying volatility changes difficult while options with a long time to expiration have too much of

a dependence on interest rate and dividend volatility which are assumed known for the life of the option. The MoneynessFilter is used because options which are outside the noted range have very low time values which again makes the impact of volatility on these time values difficult to study. A second reason is that the option vega is also very small outside of the moneyness bounds of .9 and 1.1 which in equation (3) above it can be seen would shrink the profits and make it difficult to detect the volatility premium  $\lambda[\sigma]$  using the propositions above. The BadDataFilter is used to just remove data errors. BS implied volatilities of near 80% occurred on the day of the 87 crash but that was not in either BK's or our sample and 100% is double the observed maximum daily standard deviation in our sample detailed below (and 4 times the maximum of BK's sample). All of these filters and reasons are the same for this paper and BK.

#### **3.4. Summary Statistics – different from BK**

BK and this paper report the delta hedged gains for the full sample categorized by groups of moneyness and two groups of times to expiration. The table is useful to get some idea of the distribution of the gains. However BK make more of this table than I think is warranted. They use it and implicitly Proposition 1 and 2 to prove that the volatility risk premium is negative. At best the negative gains reported there and here below suggest that. They report standard errors for each of these categories. Even if the standard errors properly took account of the immense autocorrelation between the data within each category, those errors don't mean much unless the correct model for each category is a constant. A standard error, or a t-test, which is OLS, produces a biased estimate when variables are omitted. In subsequent sections at least 2 variables are shown

in BK's and this paper to be significant in explaining the errors. So the reported tables are really just summary statistics.

Further BK reports standard errors which ignore the large autocorrelation within each sample moneyness/time to expiration category. Part of this is due to *overlap*. For example 14 and 15 day options will have delta hedged returns that are almost the same except for 1 day (they aren't quite just additive but can be approximated as such). The other part of the autocorrelation is due to close strikes (close moneyness) with the same number of days to expiration being very highly correlated. There will be only a few such in each group since the grouping is partly by moneyness. This second source of autocorrelation will be explained in significant detail in the next sub section.

BK do state that "these standard errors may not account for the fact that the theoretical distribution of  $\pi_{t,t+\tau}$  depends on option moneyness and maturity". They mention overlap later in their paper and so must understand the autocorrelation at least caused by the overlap. So they produce series of delta hedged gains to account for this. Namely they restrict the data to only those options which are closest to the money and those which are exactly 30, 44 and 58 days in time to expiration and test the means of these series. The 44 and 58 days do have *overlap* however, though it is a different kind than that mentioned above. Since the expirations are placed about 1 month apart the 44 and 58 day options will overlap with the 44 and 58 day options from the prior expiration by about  $\frac{1}{2}$  and 1 month respectively. They ignore/don't mention this. The 30 day series does have some slight overlap as detailed in the next subsection below so this paper uses 14 and 28 day at the money (ATM) options to provide a test of the means that has no autocorrelation but may have dependence on volatility (as shown later). So the statistical

test that needs to be performed here is that the means of the 14 and 28 day samples are negative. Further BK show that for the model of Heston (1993) the absolute value of the profits should be monotonically decreasing in  $\tau$ . Therefore another statistical test that needs to be performed is that the 14 day mean is less than the 28 day mean. Because the samples will have significant unspecified correlation due to overlap, no attempt will be made to actually perform a test, rather the differences of means will just be noted. One last note is that BK use the same ATM data above and essentially the same regression with more regressors, also feel the need to introduce a lag of the profits and use the Newey-West correction. If they are needed in the later tests, they are needed for the simple mean test as well (I find they are not needed for the later tests as will be shown).

### 3.5. Cross Sectional Test – significant differences from BK

This section describes a statistical test using Proposition 3. The sample is divided into sub samples based on small volatility ranges. The idea is have volatility approximately “fixed” within a sub sample so that Proposition 3 is applicable. Then proposition 3 could be tested with the following regression:

$$(4) \text{ GAINS}_i^i = \Psi_0 + \Psi_1 \text{ VEGA}_i^i + e_i^i \quad i = 1, \dots, I \text{ indexed by option}$$

where  $\text{GAINS}_i^i \equiv \pi_{i,t+\tau}^i / S_i$  and  $\text{VEGA}_i^i$  is an approximation of the vega of the call option  $i$  and  $e$  is the error term. More will be said about this error term shortly. If the coefficient on vega is significantly negative for each volatility category then by proposition 3 we can *reject the hypothesis* of a zero volatility risk premium in favor of a negative volatility risk premium.

If we did not further filter the sample beyond the procedure in 3.3 and volatility range sub samples we would have an enormous autocorrelation problem from *overlap* as explained above. BK's solution to this is to only use 30 or 44 day options. Their reasoning is as follows. Options expire on the third Friday of the month. As a result 30 day options will have no overlap with the next and previous months 30 day options. Actually this is not quite true since third Fridays can be either 28 or 35 days from each other and BK seem to overlook or ignore this. For this reason this paper uses 28 day options. BK uses 44 day options for an additional set of samples. However BK make no effort to deal with the overlap of a minimum of 11 days and the resulting autocorrelation, another example of BK partially not dealing with an autocorrelation.

There is still actually a significant autocorrelation problem even if using only 28 day options. Since obviously options bought on a given day will all be in the same volatility category or group, they will all be in the same regression, equation 4, as expected to test in the "cross section" for different moneyness. So while some options in a volatility category will be from different expiration cycles, there will be many groups of 5-15 options with the same expiration date but different moneyness. 28 day options from the one month should have completely uncorrelated delta hedged gains with all other months in the sample. However options on a given day with different moneyness will have somewhat correlated delta hedged gains. This within expiration correlation will be detailed and explained more below. BK's solution to these groups of gains with common expirations is to use date (expiration) specific random effects and FGLS regression (as in Greene 1997 or 2003) without accounting for autocorrelation within the groups (or expirations) [If they did so, they did not mention it, which is one reason that it is highly

unlikely that they did so. Either way such a correction should be described in this paper]. This omission is a serious problem. Fortunately the regressions are significant either way as will be shown in the results section. This methodology section will contain the intuition for theory of the form of this autocorrelation, an explanation as to why random effects alone is a poor solution and an explanation of better solutions. The results section below will statistically verify the form of the autocorrelation and the random effects and one better solution. The simulation Section 4.1 below will verify the existence of the autocorrelation and demonstrate its form.

Intuitively two options with very close strike prices are virtually the same option. The options referred to in this paper (similarly for the BK paper) have available (but sometimes there is no quote or no data) strikes at a minimum of .5% to 1.5% (of the underlying) apart. With a 10% annualized volatility (standard deviation) an underlying index (on the low end of the data considered in this paper), the volatility over 4 weeks (28 days) would be  $10\%/\sqrt{52/4}$  or about 2.8%. The likelihood of the underlying moving  $x\%$  and  $x\%+.5\%$  are not very different in that time. Therefore, 4 week call options with strikes .5% apart should be priced fairly similarly. Further, options with such close strikes should have similar delta hedged profits as well. The larger the volatility, the more true this is. In BK's case the largest volatility is 20%, and in this paper (over a different time period in addition to a different index) the largest volatility is 50%. Alternatively, the larger the difference in strikes, and the smaller the volatility, the less correlated the two profits would be. Beyond this general statement, it is difficult to intuit anything. So I run a simulation to approximate the correlation among strikes using BS assumptions. We don't know the true model so that is the reason for calling it an approximation. The

simulation below shows strong correlation for close strikes, falling off quickly and then becoming negative for a while before zeroing out. As expected, higher volatility results in the correlation falling off less quickly. Ideally this could also be verified by extending the results of BKL with a formula for joint distribution of hedging profits for options on the same underlying.

Given these correlations, using random effects does not make much sense at least assuming the underlying roughly follows BS. One solution is to use Newey-West with standard OLS which is what is done in this paper. It would be nice to be able to use both random effects and Newey-West since it is possible that the BS simulated distribution is significantly different from the true distribution. This could be true because delta hedging precisely misses hedging other things than stock changes (like volatility changes) and therefore all options could be affected by this jointly. However the results below verify that implementing equation (4) with random effects results in significant autocorrelation of the residuals and that there is not too much more volatility than predicted by simulations suggesting that random effects would not add much to the regressions. Another regression method would be deriving a distribution as just mentioned and using FGLS or ML. However again, the true distribution may differ significantly from a BS derived distribution.

So in closing OLS and FGLS with date specific random effects (to demonstrate the autocorrelation within) will be run using equation (4) for separate volatility groups. This is a cross section and not a panel test even though multiple time periods are involved because different time periods within a sample are unrelated. In the case of OLS the error term in equation (4) is assumed to have zero mean and a multivariate normal distribution

with unspecified correlations. In the case of FGLS  $e_i^t = u_t + v_i^t$  where  $u_t$  is normal with zero mean and unknown variance and uncorrelated with anything and  $v_i^t$  is normal with zero mean and unknown variance and uncorrelated with anything.  $VEGA_i^t$  is set to  $\exp(-d_1^2)$  which is within a constant the BS vega/S for a call option where  $d_1$  is as BS define it stated in equation 0B above.

### 3.6. Time Series Test – significant differences from BK

This section describes statistical test using proposition 4 and 4b. Here again autocorrelation results in difference in this papers and BK's methodology. To satisfy the conditions of proposition 4 and 4a we need to look at only ATM calls. To reduce autocorrelation only 30, 44 and 58 day options are chosen by BK. They agree that the 44 and 58 day options will have overlap. They claim that the 30 day options have no overlap, but as discussed above there will be some overlap. To be more clear about  $\frac{1}{2}$  the time there will be a 2 trading day overlap when the difference between expirations is 28 days. So this is about a 5% overlap overall (with about 20 trading days per 28 day period) which is fairly small so it doesn't seem that this would cause too much autocorrelation. Nevertheless this may be the cause of the autocorrelation that BK see that perhaps caused them to include a lag of the gains. They never mention a theoretical reason why there would be any autocorrelation for the 30 day options and they refer to it as "nonoverlapping". There is no reason why it would be. The market itself should be some sort of martingale process, but we are measuring something like the first difference of such a process (the gains). Looking at equation (3), a lag in the profits of nonoverlapping options could be caused by the  $\lambda[\sigma]$  being equal to a constant plus a lagged error term but it is hard to understand why it would be so.

This paper chooses to use 28 day ATM calls to rule out any overlap. With that choice no autocorrelation is ever seen using 3 different testing methods as detailed in the results section below. This simplifies the methodology for this section over BK's similar section and reduces the possibility for error. The following OLS regression is run:

$$(5) \text{ GAINS}_t^i = \Psi_0 + \Psi_1 \text{ VOL}_t^i + e_t^i \quad i = 1, \dots, I$$

where  $\text{GAINS}_t^i \equiv \pi_{t,t+\tau}^i / S_t$ ,  $\text{VOL}_t^i$  is a measure of volatility of the underlying, and  $e_t^i$  is assumed to be a mean zero, normally distributed error term. Whites correction is performed to allow for heteroskedasticity, but no autocorrelation is assumed and this is tested. So assuming we can show that the coefficient on the volatility proxy is significantly negative then we can reject the hypothesis of zero volatility risk premium in favor of negative volatility risk premium following Proposition 4 or 4a. One problem with the very general proposition 4 is that it makes a statement for "any function of physical volatility". The problem with this will be expanded on in the next methodology section. If you make certain fairly reasonable assumptions about transaction costs, there is certain to be *some* functional dependence on volatility for (regression) equation 5. It is an open question how much of the regression could be due to be transaction costs however because a number of parameters relating to transaction costs are unknown.

### 3.7. Transaction Costs – ignored in BK

Suppose that options are priced according to the transaction cost model of Leland (1985). Since BK and this paper delta hedge without taking into account underlying transaction costs, there should be losses *at least if* the index price really follows the one dimensional model of Leland (1985). The question is then could the losses observed in BK and this paper be caused by transaction costs rather than a volatility risk premium.

There are several questions that must be answered to begin this analysis. First, what are the round trip transaction costs ? Second, how frequently are the hedges be rebalanced (controlled by the tolerance for risk [hedging error] of the market makers) ? Third, what does the portfolio of options that market makers (delta-hedgers) actually hedge look like? Only the last of these needs explanation. Suppose a market maker sells a call of a given maturity and strike and shortly later buys it back. Then they have no inventory. Therefore they would not even have to hedge at all. If the distribution of inventory was known (how much market makers are hedging on average for each strike and maturity) the appropriate model to use would be an extension of Leland (1985) to a portfolio of options in Hoggard, Whalley and Wilmot (1994).

The weakness of my alternative hypothesis is that none of these three questions is known. More research must be done to try to answer these questions before the issue of the importance of transaction costs relative to the volatility risk premium can be answered definitively. This is why I make two sets of assumptions related to transaction costs that I call E (extreme) and R (reasonable) respectively. It turns out that even under the extreme (extremely large) transaction costs assumptions, the profits are too negative to be explained by Leland (1985). However it is still possible that the E assumptions do not assume large enough transaction costs.

What are the round trip transaction costs? The literature review of transaction costs in section 2.2 above deals with this issue in detail. The result is that it is unknown what these costs are. For the R assumptions, a reasonable estimate is that the bid-ask spread has to be paid which is typically .03% for the British underlying considered here. Since market makers might be making very large trades to rebalance it is possible they

need much more than the standard quoted depth and have to pay a premium for that. So for the E assumptions this paper assumes that round trip transaction costs are about 3 times the typical bid-ask spread .1% for the E assumptions.

The frequency with which option market makers re-hedge is unknown. There is some anecdotal evidence that they re-hedge daily, so I assume that for the R assumptions. For the E assumptions I make the convenient assumption (fits the data but also fits a reasonable market maker behavior assumption) that the square root of the rebalance frequency is proportional to volatility. This is motivated by the distributional results of BKL. BKL show that the RMSE of the hedging error is proportional to  $g/\sqrt{N}$ , where  $g$  is a constant for a given option and  $N$  is the number of rebalancings in the option life.  $N=T/dt$  where  $dt$  is the rebalance frequency. The 'constant'  $g$  is approximately proportional to  $\sigma$  (stdev). Therefore if one assumes that under different volatility regimes that market makers vary their hedging frequency to maintain the same level of risk, some algebra shows that transaction costs under Leland (1985) would be proportional to  $\text{Vega} \cdot \sigma$  and the square root of the rebalance frequency would be proportional to  $\sigma$ . This relation could be tested with a regression equation in the time series and cross section simultaneously. However in the cross section alone, the profits would be proportional to Vega alone, which results in the regression already required for methodology section 3.5. Further, in the time series alone the profits would be proportional to  $\sigma$ , which is already done with the methodology of section 3.6. That is, these precise relations are predicted both by Leland (1985) and BK for different reasons. These were the primary motivating factors for considering transaction costs. One question is how to distinguish the statistics that the two models would produce. It turns out that for at least part of the sample the

profits are too negative to have been caused even by the extreme 'E' assumptions as will be seen in the results section 4.2.5 below.

Finally I will ignore the portfolio effects on the  $\Delta$  (delta) of the options for both the E and R assumptions. The portfolio effects are as follows. If the average hedger/market maker is short call and put options, then since the  $\Delta$  are positive for calls and negative for puts, they could have a net zero or very small delta at times which means that a change in the stock price does not require much change in the futures position for hedging. Determining the distribution of option market maker inventory is not as simple as using open interest of the options even if I had this at my disposal. There are many cases where trades may be between non market makers. For example a portfolio manager sells a covered call on their near index portfolio. Another manager buys the call. A new call is created (opened) and it is not hedged. Only recently have data sets with information necessary to determine the hedger open interest been made available and I don't have access to them. However by ignoring any portfolio effects, I will estimate larger (perhaps much larger) transaction costs than hedgers actually experience because portfolios can only decrease the average  $\Delta$ . Ignoring portfolio effects really is an extreme assumption since it seems clear that they could and do have a significant effect in *reducing* transaction costs. It is the primary reason for considering the E assumptions "extreme". Since, it turns out that even under the extreme assumptions the profits can't be explained by transaction costs, I don't worry about trying to approximate the true portfolio with the R assumptions.

Before turning all of the above in to tests we have the following summary for the R (reasonable) assumptions: .03% round trip transaction costs, and costs which are just

like Leland (1985) and proportional to Vega and hedging occurs daily. The formula for transaction costs in Leland (1985) is

$$(6) \text{ Transaction costs} = k * S * N'(d_1) * \text{sqrt}(\tau) / \text{sqrt}(2 * \pi * \Delta t)$$

where  $k$  is the round trip transaction costs in the underlying,  $N'()$  is the derivative of the normal CDF and  $\Delta t$  is the time between rehedging. One simple test that can be performed is to test at the money gains. At the money the percent transaction cost / underlying simplifies to :

$$(7) k * \text{sqrt}(\tau / \Delta t) / 2\pi$$

So with the null hypothesis that gains are explained by (7), we can reject the delta hedged gains being caused by transaction costs if they are significantly different from (7) with  $k=.03\%$  under the R assumptions.

A summary of the E (extreme) assumptions is underlying transaction costs of .1%, and costs proportional to  $\sigma * \text{Vega}$  with unspecified periods between rebalancing. But we should probably add one more assumption to the E assumptions, which is that rebalancing would occur at a period larger than .5 days. The reason for this is that already hedgers have to accept overnight risks when they can't rebalance. It does not seem reasonable that they should hedge too much more frequently than daily. It turns out that with this assumption we can use the same test as above for the R assumptions to reject transaction costs as causing the effects seen at least in the 1998 to 2002 period as I will show below in the results section 4.2.5. The same test can be performed because if one uses the highest rebalance frequency of .5 days (the most extreme) then costs are no longer proportional to  $\sigma * \text{Vega}$  but only to Vega when volatility increases.

## 4. Results

This section will implement the methodology described in section 3. First, because it can be most independently presented, a simulation demonstrating correlation in the cross section (referred to in section 3.5 but vaguely described) is presented in Section 4.1. Section 4.1 not only does simulated hedging but also uses *simulated index* (or underlying) prices to show that correlation would occur when using the methodology of section 3.5 with *real index* prices. Section 4.2 will describe the results when implementing the methodology described in section 3 (using real British pricing data instead of the US pricing data that BK use).

#### **4.1. Simulated Hedging With Simulated Prices Shows Autocorrelation**

In section 3.5 a simulation was referred to as one source of the knowledge that there is autocorrelation between the delta hedged gains for call options with the same time to maturity but different strikes (the cross section). This section will detail the simulation using *simulated index* prices (under BS assumptions) and its prediction of significant correlation in the cross section when using *real index* prices from British or US data. (Since BS is a very good first order approximation to the correct model, there is likely correlation in the cross section no matter what the true model is.) BS assumptions are used to randomly generate delta hedged gains with different strikes. The interest rate is assumed to be zero because it wouldn't make much difference in the results. The mean return is assumed to be 12% in the underlying. Standard deviation is assumed to be 10%, 20%, or 40%. The underlying price is assumed to start at 100. Delta hedging is simulated for 20 trading periods in 28 days with strikes from 90 to 110 (to correspond with +-10% moneyness used elsewhere in this paper) are simulated at intervals of 1. That is every pair of strike in that range 21\*21 is simulated using the *same* random stock data so that the

correlations can be measured. Obviously the diagonal (both strikes equal) will always be 1 and there will be two results for every pair since ordering is irrelevant. 1000 runs are simulated for each ordered pair, with each ordered pair reported separately on the graph so the extent of symmetry can be used as a gauge of the error in each measurement.

Graphs 1, 2 and 3 show that there is significant correlation approaching 1 when close strikes are hedged. It is relatively unimportant that those close strikes are in the money (around 90) or out of the money (around 110). The correlations become actually negative (min about -.4) within the  $\pm 10\%$  moneyness range as the pair of strikes gets farther away. Since 20% standard deviation means that twice the distance of a move would happen in the same time as it would for 10%, it is not a surprise that the graph is about 2 times as wide measured from the diagonal. The same is true for 40% relative to 20% assumed standard deviation.

It is clear that autocorrelation is an issue when considering the profits of delta hedged calls with the same expiration but different strikes as is done in the cross section tests.

## **4.2. Results of Simulated Hedging Using British Data**

This section will implement the methodology of section 3. These results are statistics of profits from simulated delta hedging using real British index price data (instead of US price data as BK use). These results support the conclusion of BK using British instead of US data and using an overall somewhat different methodology described in section 3. Section 4.2.1 will describe the British data and variables related to the rest of section 4.2. Section 4.2.2 will report the summary statistics described in methodology section 3.4. Section 4.2.3 will report the cross sectional test described in

methodology section 3.5. Section 4.2.4 will report the time series test described in methodology section 3.6. Section 4.2.5 will report the transaction cost test described in methodology section 3.7.

#### 4.2.1. Description of the Data and Variable Definitions

All the propositions in section 3.2 and all the resulting regressions described in section 2 and in the results section (5) below use a dependent variable of the delta hedged gains of a given call option. For readability the equation (0) for those profits of a *given single hedged call option* is repeated below.

$$(0) \pi_{t,t+\tau} = \max(S_{t+\tau} - K, 0) - C_{K,\tau,t} \\ - \sum_0^{N-1} \Delta_{t(n)}(S_{t(n+1)} - S_{t(n)}) - \sum_0^{N-1} r_{t(n),\tau} (C_{K,\tau,t} - \Delta_{t(n)} S_{t(n)}) \tau / N$$

where  $\pi_{t,t+\tau}$  are the discrete gains for the given call option from time  $t$  to expiration  $t+\tau$ ,  $S$  is the underlying price,  $K$  is the strike price,  $C_{K,\tau,t}$  is the call price,  $\Delta$  is the estimated call delta,  $r$  is the interest rate,  $N$  is the number of trading days to expiration,  $t$  is the trading date in the sample that the option is bought,  $n$  is used to index the trading days till expiration and  $t(n)$  is a trading date of the option between the time it is bought  $t(0)$  (the same as  $t$ ) and the time it expires  $t(N)$  (the same as  $t+\tau$ ).

The data item above that can be the most independently described is the interest rate  $r_{t(n),\tau}$ . The interest rate used was the linear interpolation of the 7 day ( $r_{t,7}$ ), 1 month ( $r_{t,30}$ ), and 3 month ( $r_{t,90}$ ) LIBOR GBP middle rates where the 1 month and 3 month rates are taken to be 30 and 90 days respectively. These rates were available every trading day.

A data item not directly referred to above is the dividend yield between a pair of days ( $d_{s,t}$ ). This was computed in a straightforward way from a daily (closing) price index and daily total return index of the FTSE100 made available by the FTSE.

Another data item not directly referred to above is a measure of the true volatility ( $\sigma_t$ ). This is calculated in two ways both from the daily FTSE 100 total return index.  $VOL_t^h$  is simply the sample standard deviation calculated over the previous 20 trading days.  $VOL_t^g$  is the forecast  $\sigma_t$  calculated by assuming the volatility and underlying follow a GARCH(1,1) model.

The only call option considered here is the European call on the FTSE100 called the ESX. European means that the call option can only be exercised at expiration (expirations exist on the third Friday of every month) and is type of options the propositions in section 2 refer to. The raw data sample from the LIFFE exchange consists of all consists of all bids, asks and trades recorded from Jan 1, 1993 to Dec 31, 2002. There were about 1,000,000 records from 1993 to 1997 and about 30,000,000 from 1998 to 2002 due to progressive introduction of electronic trading and some increase in volume as well. While tick data was the raw data used, it was used just to extract daily quotes from all the unique option strike ( $K$ ) and time to expiration ( $\tau$ ) combinations for a given day. For every such combination for every trading day the last bid and ask recorded at the same second and before 4:09:30 pm was extracted to form the raw daily data. So price of a call option (with a given unspecified moneyness) on a given day referred to above in equation 0 as  $C_{K,\tau,t}$  is the average of the bid,  $C_{K,\tau,t}^b$ , and the ask,  $C_{K,\tau,t}^a$ , that were quoted at the same second. So the daily filtered sample of the data will contain on every trading day of the sample will contain call prices  $C_{K,\tau,t}$  for as many as dozens of different strikes  $K$  with a few different times to expiration  $\tau$ .

We also need values of the underlying,  $S_{t(n)}$ , to calculate equation 0 for a given call. For  $S_{t(N)}$  (on expiration days of which there are 12 per year),  $S_{t(N)}$  is calculated

simply as the historical EDSP provided by the exchange. Two of these historical EDSP values the exchange made available were actually significantly wrong and had to be approximated using their stated procedure using the (available) minute by minute underlying between 10:10am and 10:30am on expiration days.

On a day that the option is purchased the value of  $S_{t(0)}$  or  $S_t$  must be at exactly the same time as the option purchase. So more completely this value could use the symbol  $S_{K,\tau,t}$  to correspond to  $C_{K,\tau,t}$ . That is there are as many of them as there are daily call prices. From 1993 to 1997 the underlying value was made available in the tick data simultaneous with each record. The value was updated at the beginning of every minute so it might not change over a sequence of a few options quotes. So in those years  $S_{K,\tau,t}$  could be

$$(8) S_{K,\tau,t} = U_{K,\tau,t} * \text{Exp}(-d_{i,t+\tau} * (\tau+t))$$

where  $U_{K,\tau,t}$  is the raw underlying value recorded at the same time as  $C_{K,\tau,t}$ . The reason for this transformation is that dividends are assumed to be constant for the period until expiration so their value (which causes the index and its option to fall in value on each dividends ex-date) can be subtracted out beforehand.

Because simultaneous underlying values were not available from 1998 to 2002, a futures implied underlying was calculated for that period (and the whole sample) from LIFFE tick futures data. Using the futures implied underlying is consistent with the fact that options traders are known to use the futures to hedge index options because it has lower transaction costs than the underlying and tracks the underlying closely. The FTSE100 raw tick futures data consists of all bids, asks and trades for the sample time period. There were about 1,000,000 records per year till 1999. In the years 2000, 2001 and 2002 there were about 4,000,000, 11,000,000, and 14,000,000 records respectively.

For every daily options quote as described above, a futures bid and ask which were each within 30 seconds of the time stamp on the options quote were found. The futures is the contract with the closest (quarterly) expiration on or after the options (monthly) expiration. If either a bid or ask on the futures could not be found within this period, the options quote was discarded from the sample. The implied underlying at that time is computed with the following formula for futures/underlying arbitrage:

$$(9) \underline{U}_{K,\tau,t} = F_{K,\tau,t} * \text{Exp}(d_{t,t+\tau} - r_{t,t+\tau})$$

where  $\tau$  is the time to the *futures expiration*,  $F_{K,\tau,t}$  is the futures price (average of bid and ask) for within 30 seconds of the call price ( $C_{K,\tau,t}$ ) time stamp,  $d$  is the dividend yield till futures expiration, and  $r$  is the interest rate till futures expiration. Then to calculate  $S_{K,\tau,t}$  from  $\underline{U}_{K,\tau,t}$  we substitute  $\underline{U}_{K,\tau,t}$  for  $U_{K,\tau,t}$  in equation (8) above.

For the years 1993 to 1997 both the futures and underlying source for  $S_{K,\tau,t}$  produced about the same results so only the results using the futures were reported in this paper.

We also need values of the underlying,  $S_{t(n)}$ , on days other than the day of purchase and the expiration day to calculate equation 0 for a given call. In this case there is no reason to have a unique value for each  $C_{K,\tau,t}$  since hedging can take place at any time of the day. Since we restrict the sample  $C_{K,\tau,t}$  to  $14 \leq \tau \leq 60$  for reasons described in section 3.3, there is only a need for two unique values of  $S_{t(n)}$  per day, one for an expiration up to 30 days ahead, and another for an expiration up to 60 days ahead. A time of day can be picked and any underlying contemporaneous with some options quote near that time can be used to calculate equation (8). Alternatively a contemporaneous futures quote could be used and then equations (8) and (9) can be used. Alternatively even the

closing underlying can be used in equation (8) Some limited testing showed little differences in the results depending on these choices so the average paired futures quote for the entire futures trading day was used to simulate hedging actions averaged throughout the day and reduce the impact of any data errors.

As touched on just above and described roughly in section 3.3, 3 more filters were applied to the daily data. Only those  $C_{K,\tau,t}$  with  $14 \leq \tau \leq 60$  are kept in the sample. Only those with initial moneyness  $\gamma$  between .9 and 1.1 are kept in the sample where moneyness for a given option  $C_{K,\tau,t}$  is

$$(10) \quad \gamma_{K,\tau,t} \equiv Se^{(r-z)\tau}/K \quad \text{where } z = d_{t,t+\tau}.$$

Finally those call options whose price  $C_{K,\tau,t}$  is outside of the hard arbitrage bounds  $[\max(S_t - Ke^{-r\tau}, 0), S_t]$  are discarded and all options with BS implied volatilities exceeding 100% or less than 1% are discarded (i.e. the BS model is numerically solved for volatility given  $C_{K,\tau,t}$ ).

Finally a formula is needed for the estimated delta  $\Delta_{I(n)}$  or estimated first derivative of option price change with respect to a change in the underlying. This is set to be the BS delta:

$$(11) \quad \Delta = N(d_I) \quad \text{where } N \text{ is the cumulative normal distribution and } d_I \text{ is defined by equation 0B above}$$

The equation for  $d_I$  depends on the all the values in the call price  $C_{K,\tau,t}$  and since it is different for different  $K$ , it must be computed separately for each option on all the days it is hedged/rehedged. As a result the total number of delta calculations is equal to the number of call options in the total sample times the average number of days they have to expiration or the sum of all the number of days to expiration. The estimate of  $\sigma$  that is

used for  $d_t$  is either  $VOL_t^h$  or  $VOL_t^g$ . Which one is used for a given test is specified in the results section below.

#### 4.2.2. Summary Statistics

Tables 1 reports the mean delta hedged gains for the full sample categorized by moneyness and two ranges of time to expiration. The full sample is described in Section 4.2.1 The standard deviation used in constructing the hedges was the Garch derived one. All categories are negative. As discussed in the methodology section because of the large autocorrelation in each category with an unknown distribution standard deviations are not reported. However these means are consistent with there being a negative volatility risk premium. Tables 2 and 3 show the same data segregated into two sub samples by date, the first and second 5 years respectively of the total sample. The second 5 years had more negative gains than the first. The second 5 years had much higher volatility on average 27.6% vs 10.0% measured with  $VOL_t^g$ . So since a negative volatility premium would roughly cause bigger losses when volatility rises, this is consistent with a negative volatility risk premium.

To produce basic statistics with no autocorrelation the average delta hedged gains divided by the index price of 14 and 28 day at the money options are produced. There is still possible dependence on other variables (which is shown to be true below) and strictly speaking would invalidate these results because of the missing variables bias in OLS. The average for 14 and 28 days respectively are -.17 (-3.51) and -.32% (-3.28) where the number in parenthesis are the t-statistics. That these numbers are negative supports the hypothesis of a negative volatility risk premium according the exposition in 3.4. The fact that the absolute value of the 14 day profits are smaller than the 28 day profits is also

consistent with a negative volatility risk premium. No statistical test is performed because of unknown high autocorrelation due to overlap between the samples.

### 4.2.3. Cross Section Test

Following the methodology discussion in section 3.5 a sub sample of all 28 day options was created from the primary sample described in section 4.2.1. That is all  $C_{K,\tau,t}$  remaining after the filters described in section 4.2.1, with the additional filter that  $\tau=28$ . For all the regressions  $VOL_t^h$  is used because it keeps the distribution of volatilities tighter. No attempt was made with the Garch measure of volatility. The remaining sample was then divided into volatility categories in such a way as to balance the goal of having the volatility in the sub sample be approximately constant with the need for each sub sample to be large enough to run a regression. The volatility categories are as follows (defined by their lower end standard deviation so that shown sequentially the categories are clear) .06, .08, .10, .12, .14, .16, .18, .20, .24, .30, .40 and the last category goes up to .52. These categories happen to be a superset of those that BK use. BK does not have a category for 6%-8% standard deviation or any category above 18%.

Continuing following the methodology in section 3.5 we can reject a zero volatility risk premium in favor of a negative premium if (regression) equation 4 has a significant negative coefficient on the vega proxy for most volatility categories. First we demonstrate that using date specific random effects and FGLS results in clear autocorrelation. The data values were sorted by moneyness so that the Durbin-Watson statistic would show results confirming the most damaging predictions of 3.5 and the simulation section 3. Table 4 shows the results of the FGLS regressions for each volatility category. The bolded t-stats and Durbin-Watson statistics are significant at 5%. Every

category had significant Durbin-Watson statistics (often very significant) except one and another where no regression could be run because of the small sample size. Excluding the regression with too little data, 7/10 have significantly negative t-stats for the vega coefficient. However we can't know if these are spurious because of the autocorrelation.

Continuing following the methodology in section 3.5 we perform the same regressions with OLS and Newey-West autocorrelation correction. One issue here is what value to use for L (the maximum assumed length of autocorrelations). I chose the conservative choice which is to use the maximum number of unique strikes for any expiration of 21. Perhaps some alteration of the simulation in Section 4.1 combined with the actual separation of strikes in each expiration in each volatility category could result in the choice of much smaller choices of L (Interestingly, using the Eviews fixed values  $[L=\text{floor}(4*(T/100)^{2/9})]$  in regressions not reported results in fairly similar statistics as those reported below for larger L using Shazam, though it is unclear how much autocorrelation remained). Before continuing I note that using the OLS specification Eviews allowed autocorrelation tests on the samples (they are still sorted by moneyness within an expiration). All but one of the 11 volatility categories had a significant first order autocorrelation. Often the 2<sup>nd</sup> third and fourth order were significantly positive as well. Around the 10<sup>th</sup> order(varies), typically the correlations become significantly negative for a while then become insignificant again. This is a second (and stronger because of the negative values as well) confirmation of the theory in 3.5 and especially the simulations in section 3. were

The results using OLS with L=21 are shown in Table 5. Fortunately for BK's methodology all the same vega coefficients are significantly negative with the Newey-

West correction (as with FGLS previously) with the exception of the one NA regression which now can be estimated. I think having 8/11 of the regressions have significantly negative vega coefficients and none of them positive is significant. So using the methodology in section 3.5 we can formally reject the hypothesis that the volatility risk premium is zero in favor of it being negative.

#### 4.2.4. Time Series Test

Following the methodology of section 3.6 the sample detailed in section 4.2.1 was sub sampled by choosing those  $C_{K,\tau,t}$  such that  $\tau = 28$  and then further taking the single option in each expiration that is closest to at the money meaning  $\gamma$  is about 1.0. The result is a single time series with 91 values. The results of many runs of regression 5 are shown in Table 6. In no case did one of those regressions show autocorrelation according to the Durbin-Watson test, autocorrelation tables, or the serial correlation LM test. Regression 5 was tried with both volatility measures as the volatility proxy (and delta was measured with the corresponding volatility measure in the delta hedged gain simulations). It was tried with the  $\frac{1}{2}$  sub samples used in section 4.2.2. The volatility proxy was consistently significant and negative in the whole sample and the first sub sample. However, the second sub sample consistently was insignificant.

Many other variables were tried as a second regressor. Naturally the square of the volatility proxy is highly correlated with itself (.97 in the case of the simple standard deviation and .93 in the case of the Garch forecasts) so it doesn't make much difference which is used but both can't be. The only other variable which was significant was also fairly consistently significant which is the return of the index (underlying). This is very surprising. The whole point of delta hedging was to remove the dependence of the hedge

on changes in the underlying. I artificially increased the Garch volatility fed into the delta hedging simulation by 1% and decreased it by 2% to try to cause over or under delta hedging. There was not much change in the results which was also somewhat surprising. It is hard to imagine a form of the volatility risk premium that would cause the significance of the return variable.

Leaving aside the significance of the return variable, I think that with the full sample and the first half being consistently significant, we can reject the hypothesis of a zero volatility risk premium in favor of a negative volatility risk premium under BK assumptions with these time series results. This is especially true because the full sample has a much wider range of volatility which helps the test be significant. The significance of the market variable is puzzling though only intuition of the meaning of the hedging suggests it should not be significant. Perhaps my knowledge is limited or more research needs to be done in this area.

#### **4.2.5. Transaction Cost Test**

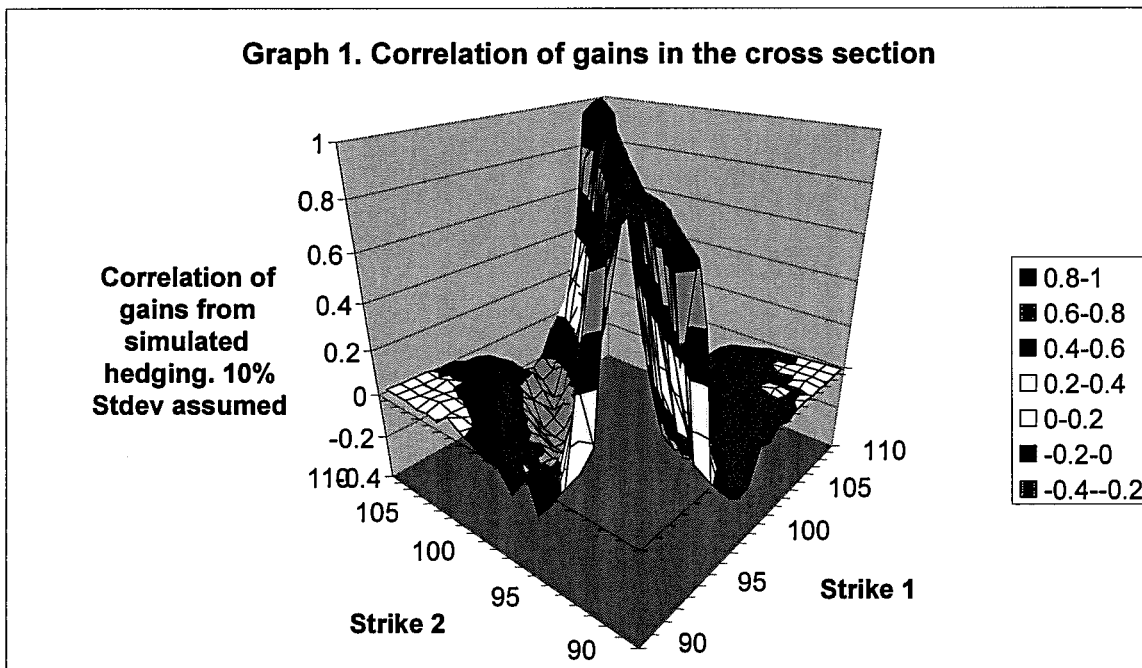
Using the methodology of section 3.7 one test we can perform is on the at the money delta hedged gains. From 1998 to 2002 the gains for 28 day at the money options averaged  $-.56\%$  of the underlying with a standard error of  $.17$  when using the Garch forecast volatility to hedge (the results were even more significant for the simple historical standard deviation). From 1993 to 1997 the mean of  $-.03$  were not even significantly different from zero given a standard error of  $.07$ . Plugging the 98-02 values into equation 7 for the R assumptions we get about  $.025\%$ . The t-stat is over 3 so we can reject transaction costs as causing the delta hedged gains under the R assumptions. Under the E assumptions the value for equation 7 changes to  $.12\%$ . The t-stat is 2.6 and we can

reject transaction costs under the E assumptions as causing the delta hedged gains seen. If future research found a reason to increase the rebalance frequency beyond  $\frac{1}{2}$  a day or that there are larger round trip transaction costs than .1% (perhaps due to extremely large trade sizes of hedgers) then transaction costs could again be considered a cause of observed delta hedged losses.

## 5. Conclusions

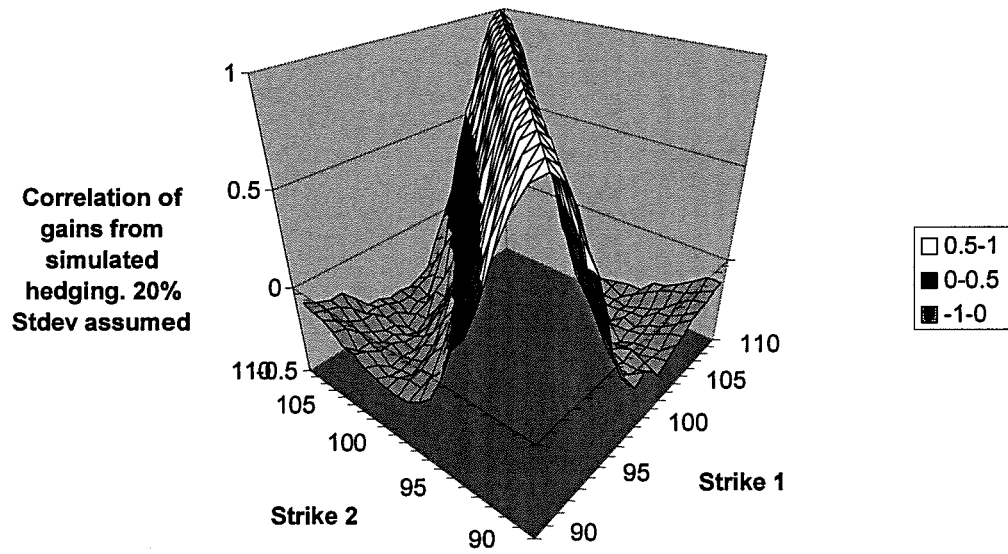
Despite causing, missing and ignoring some autocorrelation in delta hedged gains BK's hypothesis of a negative volatility risk premium is still supported and a zero premium rejected after that autocorrelation is corrected for and BK's results are roughly replicated with British FTSE100 data under BK assumptions. BK ignore the possibility of transaction costs due to turnover in the underlying causing the delta hedged gains. I show that assuming transaction costs conform to Leland (1985) and making related liberal assumptions, transaction costs could not have caused the delta hedged gains (actually large losses) seen from 1998 to 2002. Without extending the model of BK to include transaction costs, it is not possible to formally support negative stochastic volatility risk premium in such a general case. However, a reasonable first order approximation to such a model would show the delta hedged profits as the linear sum of Leland (1985) size transaction costs and a volatility risk premium term. Therefore since the profits are shown to be too large in magnitude to be explained by plausible transaction costs, they must be due to the negative volatility risk premium, and a negative volatility risk premium is given some support under assumptions of BK modified to include transaction costs.

# Appendix



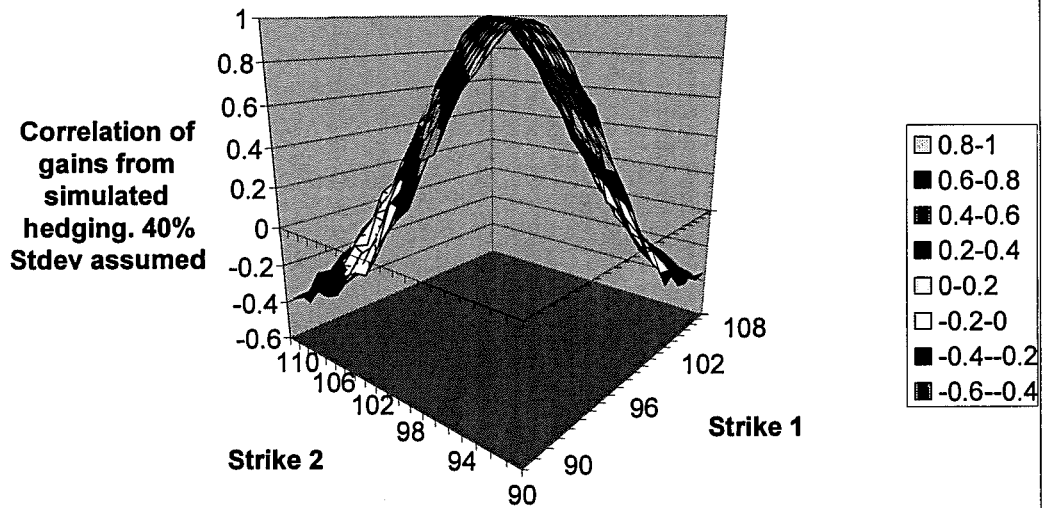
See section 4.1 for a full description of the source.

**Graph 2. Correlation of gains in the cross section**



See section 4.1 for a full description of the source.

**Graph 3. Correlation of gains in the cross section**



See section 4.1 for a full description of the source.

For Table 1, 2 and 3 below S is the underlying (FTSE 100) at the beginning of the hedging for each option. C is the call price. N is the number of data points. Moneyness is  $y$  and T is time to expiration. The procedure to generate the tables / the sample is detailed in the methodology and results sections above.

**Table1 – Delta hedged gains by moneyness category**

T	min(y-1)	max(y-1)	% Profit/S	% Profit/C	N
14-30	-0.1	-0.08	-0.16	-58.36	1024
14-30	-0.08	-0.05	-0.15	-64.76	1882
14-30	-0.05	-0.03	-0.18	-21.8	2595
14-30	-0.03	0	-0.25	-12.91	2571
14-30	0	0.03	-0.23	-7.44	2004
14-30	0.02	0.05	-0.16	-3.06	1114
14-30	0.05	0.08	-0.16	-2.14	559
14-30	0.08	0.1	-0.16	-1.76	340
31-60	-0.1	-0.08	-0.2	-32.91	2134
31-60	-0.08	-0.05	-0.25	-24.37	3043
31-60	-0.05	-0.03	-0.31	-16.17	3470
31-60	-0.03	0	-0.35	-10.07	3464
31-60	0	0.03	-0.33	-7.14	2667
31-60	0.02	0.05	-0.25	-3.76	1415
31-60	0.05	0.08	-0.11	-0.78	824
31-60	0.08	0.1	-0.01	0.13	542

**Table 2-Delta hedged gains by moneyness 1993-1997**

T	min(y-1)	max(y-1)	% Profit/S	% Profit/C	N
14-30	-0.1	-0.08	-0.06	-75.99	208
14-30	-0.08	-0.05	-0.03	1.25	528
14-30	-0.05	-0.03	-0.03	5.97	1070
14-30	-0.03	0	-0.07	0.83	1157
14-30	0	0.03	-0.05	-0.88	878
14-30	0.02	0.05	-0.04	-0.49	474
14-30	0.05	0.08	-0.04	-0.29	187
14-30	0.08	0.1	0.08	0.95	103
31-60	-0.1	-0.08	-0.1	-82.91	460
31-60	-0.08	-0.05	-0.06	8.08	932
31-60	-0.05	-0.03	-0.02	8.61	1452
31-60	-0.03	0	0.02	7.36	1539
31-60	0	0.03	0.06	4.1	1234
31-60	0.02	0.05	0.09	2.3	656
31-60	0.05	0.08	0.27	4.4	395
31-60	0.08	0.1	0.37	4.45	262

**Table 3 – Delta hedged gains by moneness 1998-2002**

<b>T</b>	<b>min(y-1)</b>	<b>max(y-1)</b>	<b>% Profit/S</b>	<b>% Profit/C</b>	<b>N</b>
14-30	-0.1	-0.08	-0.18	-53.86	816
14-30	-0.08	-0.05	-0.2	-90.5	1354
14-30	-0.05	-0.03	-0.28	-41.29	1525
14-30	-0.03	0	-0.39	-24.14	1414
14-30	0	0.03	-0.38	-12.56	1126
14-30	0.02	0.05	-0.24	-4.97	640
14-30	0.05	0.08	-0.21	-3.07	372
14-30	0.08	0.1	-0.26	-2.94	237
31-60	-0.1	-0.08	-0.23	-19.17	1674
31-60	-0.08	-0.05	-0.34	-38.7	2111
31-60	-0.05	-0.03	-0.52	-34	2018
31-60	-0.03	0	-0.64	-24	1925
31-60	0	0.03	-0.67	-16.82	1433
31-60	0.02	0.05	-0.54	-9	759
31-60	0.05	0.08	-0.46	-5.56	429
31-60	0.08	0.1	-0.37	-3.91	280

**Table 4. Cross section regressions using FGLS random date effects**

<b>stdev category</b>	<b>Sample Size</b>	<b>constant coeff</b>	<b>tstat</b>	<b>vega coeff</b>	<b>tstat</b>	<b>Durbin- Watson</b>
0.06	80	0.07	0.50	0.01	0.41	<b>0.99</b>
0.08	109	0.05	0.18	0.00	0.11	<b>1.26</b>
0.1	120	0.05	0.30	-0.14	<b>-6.68</b>	<b>0.88</b>
0.12	167	0.00	-0.01	-0.17	<b>-7.22</b>	<b>0.71</b>
0.14	110	0.02	0.20	-0.07	<b>-3.36</b>	<b>0.97</b>
0.16	102	-0.06	-0.24	-0.15	<b>-6.71</b>	<b>0.68</b>
0.18	155	-0.09	-0.21	-0.02	-0.58	<b>0.46</b>
0.2	132	0.21	0.69	-0.27	<b>-3.23</b>	1.79
0.24	47	-0.24	-1.04	-0.32	<b>-3.21</b>	<b>1.53</b>
0.3	42	0.45	0.79	-0.34	<b>-3.02</b>	<b>0.58</b>
0.4	27	NA	NA	NA	NA	NA

**Table 5. Cross section regressions using OLS and Newey-West with L=21**

stdev category	Sample Size	constant coeff	tstat	vega coeff	tstat	r squared	stdev of profit
0.06	80	0.09	<b>3.03</b>	0.00	0.01	0.000	0.250
0.08	109	0.03	0.70	0.06	0.42	0.003	0.364
0.1	120	0.03	1.06	-0.30	<b>-3.43</b>	0.130	0.280
0.12	167	-0.09	-0.86	-0.34	<b>-2.76</b>	0.060	0.470
0.14	110	0.02	0.62	-0.15	<b>-3.66</b>	0.060	0.190
0.16	102	-0.06	-1.60	-0.38	<b>-2.19</b>	0.130	0.320
0.18	155	-0.12	-0.85	-0.01	-0.04	0.000	0.570
0.2	132	0.38	<b>2.76</b>	-0.91	<b>-3.26</b>	0.080	0.700
0.24	47	0.34	1.32	-1.90	<b>-5.40</b>	0.390	0.540
0.3	42	0.05	0.09	-0.49	-0.61	0.007	0.780
0.4	27	3.97	<b>7.98</b>	-5.34	<b>-7.88</b>	0.490	0.750

**TABLE 6 - time series at the money  
regression using 28 day options**

	Simple standard deviation	garch forecast vol			
		<b>1993-2002</b>			
	coef	tstat	coef	tstat	
constant	4.83	<b>2.44</b>	6.40	<b>2.44</b>	
volatility proxy	-3.66	<b>-3.03</b>	-1.43	<b>-2.21</b>	
index return	-4.54	<b>-2.34</b>	-6.40	<b>-2.43</b>	
		<b>1993-1997</b>			
constant	3.94	1.59	4.86	1.75	
volatility proxy	-3.95	<b>-2.06</b>	-5.28	<b>-2.39</b>	
index return	-3.49	-1.45	-4.36	-1.63	
		<b>1998-2002</b>			
constant	5.74	<b>2.23</b>	8.02	<b>2.29</b>	
volatility proxy	-0.78	-0.74	-0.41	-0.66	
index return	-6.17	<b>-2.31</b>	-8.47	<b>-2.36</b>	

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