SMALL AND LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES ON ELASTIC FOUNDATIONS

by

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ABSTRACT

The problems of small and large deflection of long rectangular plates resting on an elastic foundation with both sides clamped, both sides simply supported and one side clamped the other side simply supported subjected to various loading conditions. (uniformly distributed load, hydrostatic load, parabolic load and cosine load). All loads treated in this thesis are one dimensional in nature and are assumed to be continuous along the long direction of the plate.

The method of analysis is based on the small parameter pertubation technique to solve the governing nonlinear partial differential equations. Results are improved by successive approximations to the three displacement components of a point on the middle plane of the plate. Comparisons are made with existing results for uniformly loaded clamped long rectangular plates with small deflections as well as with results for uniformly loaded simply supported long plates with small and large deflections behaviour ; excellent

agreement is shown.

Design parameters such as the maximum deflection and maximum total stresses (maximum bending stress plus maximum membrane stress) for long rectangular plates subjected to various loading and boundary conditions are presented in the form of tables for the small deflection (linear) analysis and in the form of graphs for the large deflection nonlinear analysis. Based on the analytical results obtained, the effect of the elastic foundations on the small and large deflection behaviour of long rectangular plates is discussed.

ACKNOWLEDGEMENT

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NOTATION

a , b	dimensions of plate
D	flexural rigidity of plate $Eh^3/12(1-\nu^2)$
E	modulus of elasticity of plate material
G	shear modulus
h	thickness of plate
k	elastic foundation reaction per unit area per unit deflection
K	dimensionless foundation modulus
$M_{\bar{x}}$, $M_{\bar{y}}$, $M_{\bar{x}\bar{y}}$	bending and twisting moments per unit length of plate
M_x , M_y , M_{xy}	dimensionless bending and twisting moments per unit length of plate
$N_{\bar{x}}$, $N_{\bar{y}}$, $N_{\bar{x}\bar{y}}$	in-plane forces per unit length of plate
q	intensity of load
Q	dimensionless load
$Q_{\bar{x}}$, $Q_{\bar{y}}$	shearing forces per unit length of plate
R	plate aspect ratio a/b
u , v , w	displacement of point on middle plane of plate parallel to $\bar{x}$, $\bar{y}$, $\bar{z}$ axes respectively

U , V , W	dimensionless displacement components parallel to $\bar{x}$, $\bar{y}$, $\bar{z}$ axes respectively
W_o	dimensionless lateral displacement at centre of plate
$\bar{x}$, $\bar{y}$, $\bar{z}$	rectangular cartesian coordinates
x , y , z	dimensionless cartesian coordinates
ν	Poisson's ratio
γ_1 , γ_3 , γ_5	undetermined constants
∇	Laplacian operator
$\epsilon_{\bar{x}}$, $\epsilon_{\bar{y}}$, $\gamma_{\bar{xy}}$	longitudinal and shearing strain components
$\sigma''_{\bar{x}}$, $\sigma''_{\bar{y}}$, $\sigma''_{\bar{xy}}$	extreme fibre bending and shearing stresses
σ''_x , σ''_y , τ''_{xy}	dimensionless bending stresses and shearing stresses
$\sigma'_{\bar{x}}$, $\sigma'_{\bar{y}}$, $\sigma'_{\bar{xy}}$	menbrane stresses in middle surface of plate
σ'_x , σ'_y , σ'_{xy}	dimensionless stresses in middle surface of plate
A , B , C , T F , N , H , I J , L	unknown integration constants

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CHAPTER 1

INTRODUCTION

1.1. General

The bending of thin flat plates has attracted the attention of mathematicians and engineers since the formulation of the theory by Lagrange and Navier in the early nineteenth century. Though the literature since that time has been ponderous, it may be said that in the majority of cases it is impossible to find a relatively simple function which will simultaneously satisfy both the governing differential equation and the boundary conditions, even for the simple but important case of a uniform lateral load. Thus, various techniques have been developed for obtaining approximate solutions to these problems. From an engineering view point it would be desirable to :

(1) have solutions to all of these bending problems readily available based on a unified method of analysis regardless of the shape of the plate, the load distribution on the plate, or its boundary conditions,

(2) extend these solutions for the bending problems to the more difficult problems of combined bending and stretching.

In this investigation, the intent embodied in both of these statements is realized to a limited degree.

Solutions that consider only the bending of a plate subjected to lateral loading are in the category of " small deflection " theory and are characterized by their linear load-deflection relations ; solutions that consider bending as well as stretching of the middle-surface of the plate fall into the regime of " large-deflection " theory and are distinguished by their nonlinear load-deflection relations. The importance of the nonlinear theory can be attributed to the fact that when this condition of " large-deflection " is realized, the plate is stiffer than indicated by the classical theory. Thus, the design of load carrying members based on the linear theory can be highly conservative.

The long rectangular plate is a kind of plate which has certain width but extends to infinite length in the other two directions. They are frequently used as airport runways, concrete slabs and mat foundations.

1.2. The Present Investigation

The main objective of this thesis is to make a comprehensive study of the small and large deflection behavior of long rectangular plates with and without elastic foundations. A simple and yet sufficiently accurate method based on the small parameter pertubation technique is employed to analyze such plates with different loading and boundary conditions with and without elastic foundation.

The investigation can be divided into essentially five parts : the review of the work of previous investigators, the development and verification of the method of analysis, the application of the method of analysis to long rectangular plates, numerical example, results, comparison of results and conclusions.

A brief literature survey is presented in Chapter 2. The method of analysis used in this thesis is developed in Chapter 3. The major step in the analysis of long rectangular plates with different loading (uniformly distributed loading, hydrostatic loading, parabolic loading and cosine loading) and boundary conditions (clamped both sides case, simply supported both sides case and clamped one side and simply supported the other side case) are presented in Chapter 4. A numerical example of a long rectangular plate clamped at both sides and with uniformly distributed loading is solved in detail in Chapter 5. Results, comparisons and conclusions are presented in Chapter 6.

CHAPTER 2

REVIEW OF THE PREVIOUS WORK

The problem of bending of plates resting on an elastic foundation has received less attention than the corresponding problem of beams resting on elastic foundations. Any meaningful attempt to make a comprehensive analysis of the various nonlinear plate theories that are now appearing in the literature might well be the topic of a separate study. Here, we are content with restricting the discussion to the classical Von Karman theory, noting that in the past several years this theory has been extensively modified to include many effects of engineering interest, such as variable rigidity, small initial curvatures, large amplitude vibrations and special types of anisotropy. However, little numerical data exist for these extended cases as compared to the corresponding problems in the samll deflection regime.

Concerning the manner in which the original equations were obtained by Von Karman [1], Timoshenko [2] notes that Kirchhoff and Clebsch were the first investigators to recognize the importance of stretching of the middle surface of a laterally loaded thin plate. The concept of the membrane stress function was introduced by A. Foppl in 1907 who studied large deflections of " very thin " plates —— i.e., membranes while Von Karman in 1910 removed the " very thin " restriction and is responsible for the final form of the equation which bears his name. It appears that the only solution to these equations in rectangular cartesian coordinates, which in theory an exact solution* is that due to Levy [3] for uniformly loaded rectangular plates.

At present, a variety of approximate techniques are available for solving the two fourth order Von Karman equations. An extensive, though not up-to date, bibliography has been completed by Volmir [4] concerning this subject.

The following outline might result if one attempted to categorize the methods upon which approximate solutions to the equations governing the large deflection behavior of plates are based. Comments are made only when they might bear some relevance to the methods to be used in this investigation.

Approaches Based on Minimum Principles : Perhaps the most popular method in this category is the so called Ritz energy method used successfully by Way [5] for the uniformly loaded clamped rectangular plate and by Weil and Newmark [6]

* for large deflection problem with simply supported edges on two sides

for the uniformly loaded clamped elliptical plate. Application of this technique reduces the problem to finding the solution of simultaneously nonlinear algebraic equations.

Approaches Based on Finite Difference Equations : Wang [7,8] among others, has presented solution for simply supported rectangular plates of various aspect ratios using this method. He also has an extensive discussion of work done prior to 1948.

Approaches Based on Approximate Differential Equations : In this category, the method of Berger [9] for example in which the second invariant of middle surface strains is neglected in the expression for the total potential energy of the system has been applied to a variety of problems.

Approaches Based on Successive Linearization : In this category, the method normally chosen to obtain a solution to the Von Karman equations is the perturbation procedure, and it has been successfully applied in the case of a uniform load to a variety of plate problems. For the simply supported circular plate, Stippes and Havsrath [10] solve the governing fourth order equations using a nondimensionalized loading as the perturbation parameter. All other investigators, considering only clamped edges, have used a dimensionless central deflection as their perturbation parameter. In these instances, however, it is desirable to replace one of the fourth order Von Karman equations by an equivalent set of two second order equations involving the in-plane displacements.

CHAPTER 3

FORMULATION OF EQUATIONS

In the analysis of small deflection problems of elastic thin plates, the deflections of plates have been considered of such magnitude that strain due to stretching of the middle surface of the plate is negligible. When the deflections of plates are moderately large, that is, in the neighborhood of one half the plate thickness or more, the linear theory of thin plates is no longer applicable and the strain in the middle surface must be taken into account.

3.1. Introductory Comments on The Derivation of Governing Equations

The usual method of establishing the governing differential equations for the problems of solid mechanics consists of three steps :

(1) Formulate the geometrical relationship between the strains, the displacements and the derivations of displacements. The derivations of displacements can be physically interpreted as rotations or curvatures, but for brevity, relations of this type are normally referred to as strain displacement equations.

(2) Provide (or assume) a relationship between the stresses and strains. In the majority of cases this relation will be linear — i.e., a Hooke's Law — though this need not always be the case.

(3) Consider the equilibrium of all forces acting on an element of the body under investigation. For the static analysis of load carrying members, equilibrium consists of equation to zero the forces and moments in the coordinate directions.

Normally these three steps are combined to elimilate the components of stress and strain.

3.2. Assumptions

Thoughout the theoretical work of linear and nonlinear analyses, the following assumptions are made :

(1) Points which lie on a normal to the mid-plane of the undeflected plate lie on a normal to the mid-plane of the deflected plate.

(2) The stresses normal to the mid-plane of the plate, arising from the applied loading, are negligible in comparison with the stresses in the plane of the plate.

(3) The deflection, w , of the plate is small enough so that the first two assumptions still hold, and yet large enough so that the products of the in-plane forces or their derivatives and the derivatives of w are of the same order of maguitude as the derivatives of the shear forces.

(4) Throughout the following analysis, the plate is considered to be elastic and the foundation is considered to be of the Winkler type, i. e., foundation reaction is proportional to the deflection.

Also for the linear (small deflection) analysis the forces in the middle plane of the plate is neglected and this gives one additional assumption viz.

(5) The mid-plane of the plate is a neutral plane i.e., any mid-plane stresses arising from the deflection of the plate (into a non-developable surface) is ignored.

3.3. Derivation of The Governing Equations

The non-linear partial differential equations governing the large deflection behavior of thin elastic plates were first formulated by Von Karman. For plates resting on elastic foundations, these equations can be expressed in terms of the three displacement components u, v and w (parallel to the rectangular coordinate axes $\bar{x}$, $\bar{y}$ and $\bar{z}$ respectively) of a point in the middle surface of the plate.

(1) Equilibrium of the plate element in the $\bar{x}$ and $\bar{y}$ directions.

Consider the equilibrium of a small element cut out from the middle plane of the plate with sides $d\bar{x}$ and $d\bar{y}$ as shown in Figure 3-1. Let $N_{\bar{x}}$, $N_{\bar{y}}$ and $N_{\bar{x}\bar{y}}$ be the in-plane forces per unit length of the plate.

Neglecting body forces and since ϕ and ϕ' are small ($\cos\phi = \cos\phi' = 1$) the equilibrium of the plate element in the $\bar{x}$ and $\bar{y}$ directions yield respectively ,

$$N_{\bar{x},\bar{x}} + N_{\bar{x}\bar{y},\bar{y}} = 0. \qquad \text{......................(3 - 1)}$$

$$N_{\bar{x}\bar{y},\bar{x}} + N_{\bar{y},\bar{y}} = 0. \qquad \text{......................(3 - 2)}$$

where the comma notation signifies differentiation.

(2) Equilibrium of the plate element in the $\bar{z}$ direction

The equilibrium in the $\bar{z}$ direction is obtained by considering separately the in-plane forces and lateral loads acting along this direction.

(A) From Figure 3-1., the net contribution of the downward force by $N_{\bar{x}}$ and ($N_{\bar{x}} + N_{\bar{x},\bar{x}}\, d\bar{x}$) in the plate element is

$$- N_{\bar{x}}\, d\bar{y}\, \text{SIN}\, \phi + (N_{\bar{x}} + N_{\bar{x},\bar{x}}\, d\bar{x})\, d\bar{y}\, \text{SIN}\, \phi' \quad \ldots(3 - 3)$$

for small ϕ and ϕ' ,

$$\text{SIN}\, \phi \cong \phi \cong w_{,\bar{x}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3 - 4)$$

$$\text{SIN}\, \phi' \cong \phi' \cong \phi + \phi_{,\bar{x}}\, d\bar{x} = w_{,\bar{x}} + w_{,\bar{x}\bar{x}}\, d\bar{x} \quad \ldots\ldots(3 - 5)$$

Substitute equations (3 - 4) and (3 - 5) into equation (3 - 3) and neglect the higher-order terms, then we get

$$(N_{\bar{x}}\, w_{,\bar{x}\bar{x}} + N_{\bar{x},\bar{x}}\, w_{,\bar{x}})\, d\bar{x}\, d\bar{y} \quad \ldots\ldots\ldots\ldots(3 - 6)$$

Similarly, the net downward contribution of $N_{\bar{y}}$ and ($N_{\bar{y}} - N_{\bar{y},\bar{y}}\, d\bar{y}$) on the plate element is

$$(N_{\bar{y}}\, w_{,\bar{y}\bar{y}} + N_{\bar{y},\bar{y}}\, w_{,\bar{y}})\, d\bar{x}\, d\bar{y} \quad \ldots\ldots\ldots\ldots(3 - 7)$$

The net downward contribution for $N_{\bar{x}\bar{y}}$ and

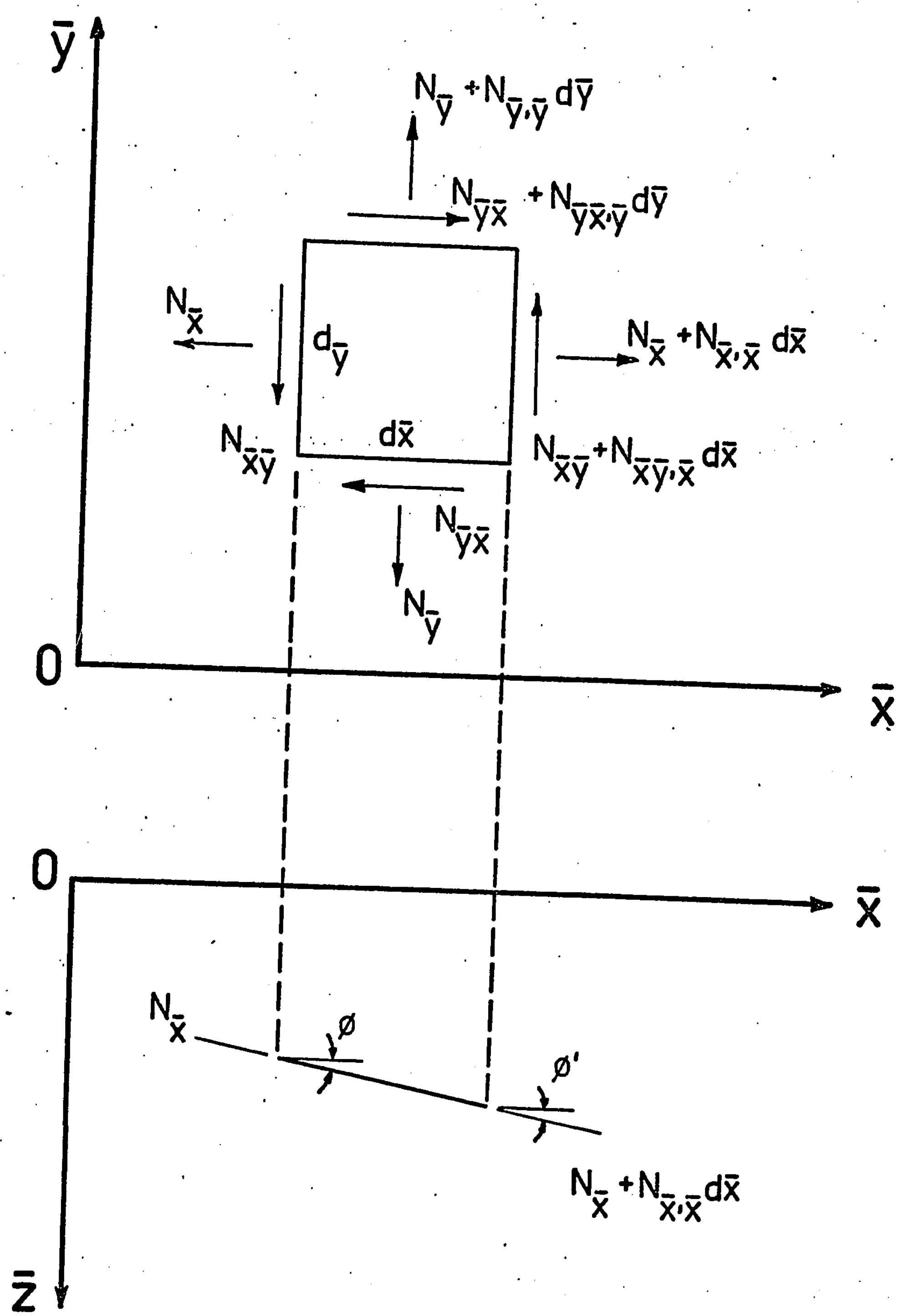


FIGURE 3-1 IN PLANE FORCES ON PLATE ELEMENT

$(N_{\bar{x}\bar{y}} + N_{\bar{x}\bar{y},\bar{x}} \, d\bar{x})$ on the plate element is (higher - order terms are neglected)

$$- N_{\bar{x}\bar{y}} \, d\bar{y} \, w_{,\bar{y}} + (N_{\bar{x}\bar{y}} + N_{\bar{x}\bar{y},\bar{x}} \, d\bar{x}) \, d\bar{y} \, (w_{,\bar{y}} + w_{,\bar{x}\bar{y}} \, d\bar{x})$$

or $$(N_{\bar{x}\bar{y}} \, w_{,\bar{x}\bar{y}} + N_{\bar{x}\bar{y},\bar{x}} \, w_{,\bar{y}}) \, d\bar{x} \, d\bar{y} \quad \ldots\ldots\ldots\ldots (3 - 8)$$

Similarly, the net downward contribution of $N_{\bar{y}\bar{x}}$ and $(N_{\bar{y}\bar{x}} + N_{\bar{y}\bar{x},\bar{y}} \, d\bar{y})$ is

$$(N_{\bar{y}\bar{x}} \, w_{,\bar{x}\bar{y}} + N_{\bar{y}\bar{x},\bar{y}} \, w_{,\bar{x}}) \, d\bar{x} \, d\bar{y} \quad \ldots\ldots\ldots\ldots (3 - 9)$$

Since $N_{\bar{x}\bar{y}} = N_{\bar{y}\bar{x}}$, and making use of equations (3 - 1) and (3 - 2) , the net downward contribution of all the in-plane forces can be obtained by adding equations (3 - 6) through (3 - 9) viz.

$$(N_{\bar{x}} \, w_{,\bar{x}\bar{x}} + 2 \, N_{\bar{x}\bar{y}} \, w_{,\bar{x}\bar{y}} + N_{\bar{y}} \, w_{,\bar{y}\bar{y}}) \, d\bar{x} \, d\bar{y} \quad \ldots (3 - 10)$$

(B) Forces in the $\bar{z}$ direction due to lateral loads

In Figure 3-2., let $Q_{\bar{x}}$, $Q_{\bar{y}}$ be the shear forces per unit length, $M_{\bar{x}}$, $M_{\bar{y}}$ and $M_{\bar{x}\bar{y}}$ be the bending and twisting moments per unit length, q the intensity of downward lateral load and k the

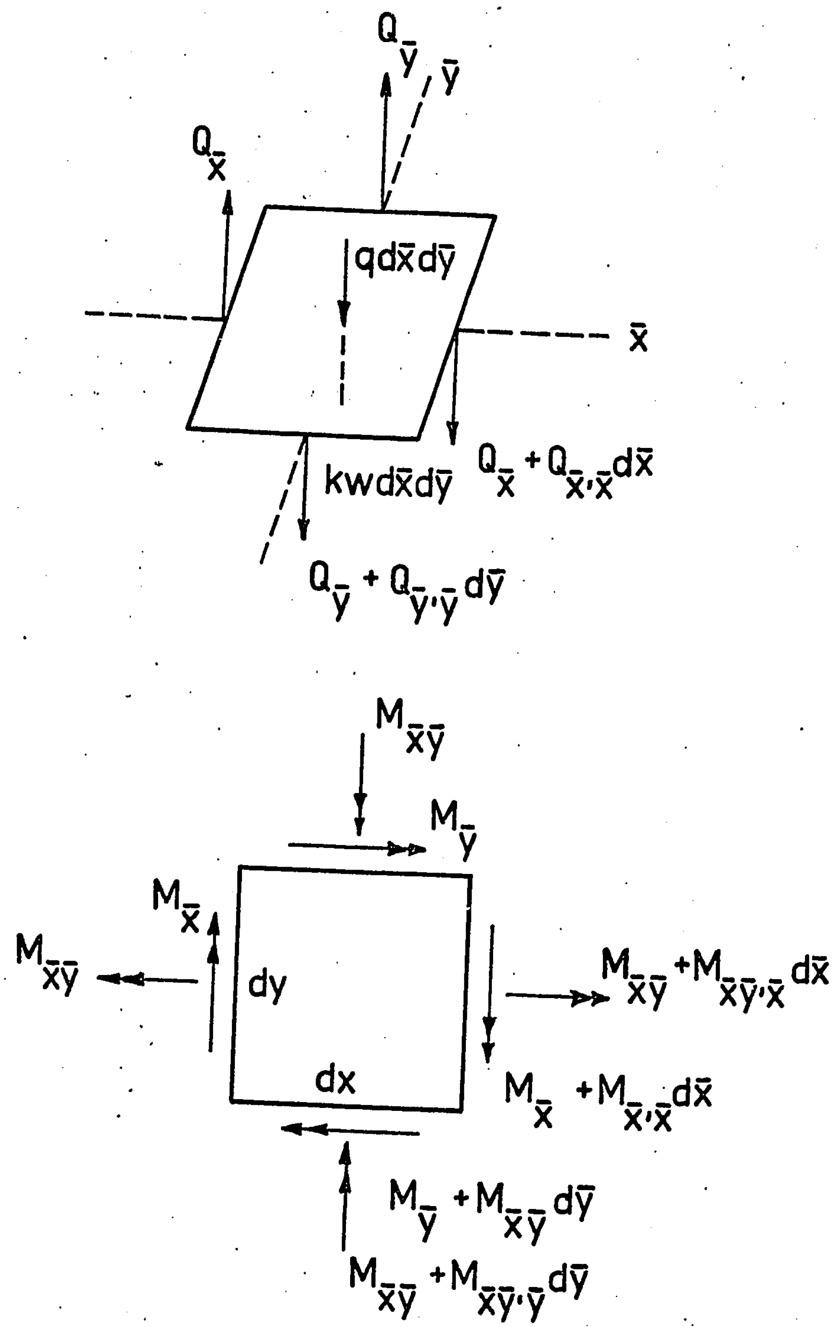


FIGURE 3-2 MOMENT AND SHEAR FORCES ON PLATE ELEMENT

foundation modulus the magnitude of which is proportional to the deflection.

Neglecting the body forces of the plate element, equilibrium in the downward direction(along the $\bar{z}$ axis) gives:

$$(Q_{\bar{x},\bar{x}} + Q_{\bar{y},\bar{y}} + q - kw) \, d\bar{x} \, d\bar{y} = 0. \qquad \text{....(3 - 11)}$$

By taking moments of all forces acting about an axis parallel to the $\bar{y}$ axis and neglecting higher-order terms gives :

$$Q_{\bar{x}} \, d\bar{x} \, d\bar{y} - M_{\bar{x},\bar{x}} \, d\bar{x} \, d\bar{y} + M_{\bar{x}\bar{y},\bar{y}} \, d\bar{x} \, d\bar{y} = 0.$$

or

$$M_{\bar{x},\bar{x}} - M_{\bar{x}\bar{y},\bar{y}} - Q_{\bar{x}} = 0. \qquad \text{.................... (3 - 12)}$$

Similarly by taking moments about an axis parallel to the $\bar{x}$ axis, we have

$$M_{\bar{y},\bar{y}} - M_{\bar{x}\bar{y},\bar{x}} - Q_{\bar{y}} = 0. \qquad \text{....................(3 - 13)}$$

Now making use of the moments curvature relationships [11] viz.

$$M_{\bar{x}} = - D \, (w,_{\bar{x}\bar{x}} + \nu \, w,_{\bar{y}\bar{y}})$$

$$M_{\bar{y}} = - D \, (w,_{\bar{y}\bar{y}} + \nu \, w,_{\bar{x}\bar{x}})$$

$$M_{\bar{x}\bar{y}} = D \, (1 - \nu) \, w,_{\bar{x}\bar{y}}$$

and substituting into expressions (3 - 12) and (3 - 13), we get

$$Q_{\bar{x}} = - D \left(w_{,\overline{xxx}} + w_{,\overline{xyy}} \right) \qquad \text{.............. (3 - 14)}$$

$$Q_{\bar{y}} = - D \left(w_{,\overline{yyy}} + w_{,\overline{yxx}} \right) \qquad \text{.............. (3 - 15)}$$

Now adding equations (3 - 10) to (3 - 11) gives the equilibrium equation in the vertical direction due to the combined action of the lateral and in-plane membrane forces.

$$Q_{\bar{x},\bar{x}} + Q_{\bar{y},\bar{y}} + q - kw + N_{\bar{x}} w_{,\overline{xx}} + 2 N_{\overline{xy}} w_{,\overline{xy}}$$

$$+ N_{\bar{y}} w_{,\overline{yy}} = 0. \qquad \text{........... (3 - 16)}$$

Substituting equations (3 - 14) and (3 - 15) into equation (3 - 16) gives

$$D \left(w_{,\overline{xxxx}} + 2 w_{,\overline{xxyy}} + w_{,\overline{yyyy}} \right)$$

$$= q - kw + N_{\bar{x}} w_{,\overline{xx}} + 2 N_{\overline{xy}} w_{,\overline{xy}} + N_{\bar{y}} w_{,\overline{yy}}$$

$$\text{........................ (3 - 17)}$$

In terms of stresses, Equations (3 - 1) , (3 - 2) and (3 - 17) can be written as

$$\sigma_{\bar{x},\bar{x}} + \tau_{\bar{x}\bar{y},\bar{y}} = 0. \quad \text{.......................}(3 - 18)$$

$$\sigma_{\bar{y},\bar{y}} + \tau_{\bar{x}\bar{y},\bar{x}} = 0. \quad \text{.................}(3 - 19)$$

and $$D \nabla^2 \nabla^2 w = q - kw + h (\sigma_{\bar{x}} w_{,\bar{x}\bar{x}} + \sigma_{\bar{y}} w_{,\bar{y}\bar{y}} + 2\tau_{\bar{x}\bar{y}} w_{,\bar{x}\bar{y}}) \quad \text{...................}(3 - 20)$$

To establish equations (3 - 18) , (3 - 19) and (3 - 20) in terms of the displacement u, v and w of the plate element, it is required to employ the equations of plane strain [12] viz.

$$\sigma_{\bar{x}} = \frac{E}{1 - \nu^2} (\epsilon_{\bar{x}} + \nu \epsilon_{\bar{y}}) \quad \text{............}(3 - 21)$$

$$\sigma_{\bar{y}} = \frac{E}{1 - \nu^2} (\epsilon_{\bar{y}} + \nu \epsilon_{\bar{x}}) \quad \text{............}(3 - 22)$$

$$\tau_{\bar{x}\bar{y}} = \frac{E}{2(1 + \nu)} \gamma_{\bar{x}\bar{y}} \quad \text{............}(3 - 23)$$

and the equations of compatibility [12] viz.

$$\epsilon_{\bar{x}} = u_{,\bar{x}} + \frac{1}{2} (w_{,\bar{x}})^2 \quad \text{...............}(3 - 24)$$

$$\epsilon_{\bar{y}} = v_{,\bar{y}} + \frac{1}{2} (w_{,\bar{y}})^2 \quad \text{...............}(3 - 25)$$

$$\tau_{\bar{x}\bar{y}} = u_{,\bar{y}} + v_{,\bar{x}} + w_{,\bar{x}} w_{,\bar{y}} \quad \text{............}(3 - 26)$$

Substituting equations (3 - 21) through (3 - 26) into equations (3 - 18), (3 - 19) and (3 - 20), the three general differential equations in terms of the displacements u, v and w in rectangular cartesian coordinates governing the large deflection of thin elastic plates are obtained:

$$u_{,\bar{x}\bar{x}} + w_{,\bar{x}} w_{,\bar{x}\bar{x}} + \nu (v_{,\bar{x}\bar{y}} + w_{,\bar{y}} w_{,\bar{x}\bar{y}}) + \frac{1}{2} (1 - \nu) (u_{,\bar{y}\bar{y}} + v_{,\bar{x}\bar{y}} + w_{,\bar{x}} w_{,\bar{y}\bar{y}} + w_{,\bar{y}} w_{,\bar{x}\bar{y}}) = 0. \qquad (3 - 27)$$

$$v_{,\bar{y}\bar{y}} + w_{,\bar{y}} w_{,\bar{y}\bar{y}} + \nu (u_{,\bar{x}\bar{y}} + w_{,\bar{x}} w_{,\bar{x}\bar{y}}) + \frac{1}{2} (1 - \nu) (v_{,\bar{x}\bar{x}} + u_{,\bar{x}\bar{y}} + w_{,\bar{y}} w_{,\bar{x}\bar{x}} + w_{,\bar{x}} w_{,\bar{x}\bar{y}}) = 0. \qquad (3 - 28)$$

$$D \nabla^2 \nabla^2 w = q - kw + h \left\{ \frac{E}{(1-\nu^2)} [u_{,\bar{x}} + \frac{1}{2} (w_{,\bar{x}})^2 + \nu (v_{,\bar{y}} + \frac{1}{2} (w_{,\bar{y}})^2)] w_{,\bar{x}\bar{x}} + \frac{E}{(1-\nu^2)} [v_{,\bar{y}} + \frac{1}{2} (w_{,\bar{y}})^2 + \nu (u_{,\bar{x}} + \frac{1}{2} (w_{,\bar{x}})^2)] w_{,\bar{y}\bar{y}} + \frac{E}{(1 + \nu)} [u_{,\bar{y}} + v_{,\bar{x}} + w_{,\bar{x}} w_{,\bar{y}}] w_{,\bar{x}\bar{y}} \right\} \qquad (3 - 29)$$

To render the above equations (3 - 27), (3 - 28) and (3 - 29) non-dimensional, the following dimensionless ratios are introduced

$$x = \frac{\bar{x}}{a} \quad , \quad y = \frac{\bar{y}}{b}$$

$$U = \frac{ua}{h^2} \quad , \quad V = \frac{va}{h^2}$$

$$W = \frac{w}{h} \quad , \quad R = \frac{a}{b}$$

$$Q = \frac{qa^4}{Dh} \quad , \quad K = \frac{ka^4}{D}$$

With these dimensionless ratios, equations (3 - 27), (3 - 28) and (3 - 29) become

$$2U_{,xx} + R^2(1 - \nu) U_{,yy} + R(1 + \nu) V_{,xy} + 2W_{,x} W_{,xx} + R^2(1 + \nu) W_{,y} W_{,xy} + R^2 (1 - \nu) W_{,x} W_{,yy} = 0. \qquad (3 - 30)$$

$$2R^2 V_{,yy} + (1 - \nu) V_{,xx} + R(1 + \nu) U_{,xy} + 2R^3 W_{,y} W_{,yy} + R(1 + \nu) W_{,x} W_{,xy} + R(1 - \nu) W_{,y} W_{,xx} = 0. \qquad (3 - 31)$$

$$W_{,xxxx} + 2R^2 W_{,xxyy} + R^4 W_{,yyyy}$$

$$= Q - KW + 12[U_{,x} + \frac{1}{2}(W_{,x})^2 + \nu (RV_{,y} + \frac{1}{2}(R W_{,y})^2)] W_{,xx} + 12[RV_{,y} + \frac{1}{2} (R W_{,y})^2 + \nu (U_{,x} + \frac{1}{2}(W_{,x})^2)] R^2 W_{,yy} + 12(1 - \nu)[R U_{,y} + V_{,x} + R W_{,x} W_{,y}] R W_{,xy} \qquad (3 - 32)$$

Hence, the problem of large deflections of plates on elastic foundations is reduced to finding the solutions to equations (3 - 30), (3 - 31) and (3 - 32).

3.4. Governing Differential Equation for Long Rectangular Plates on Elastic Foundations

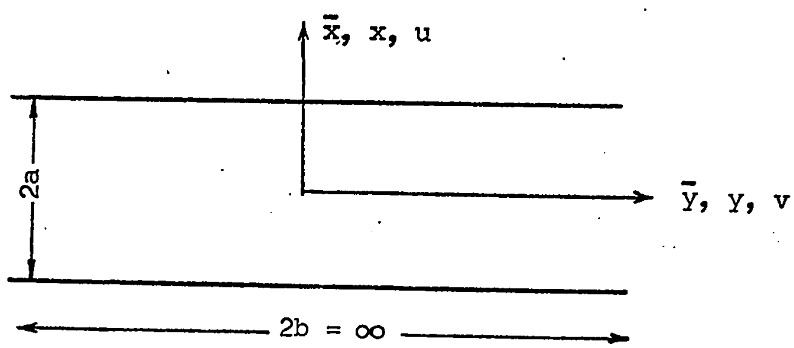


Fig.3-3 Long Rectangular Plate with Coordinate Axes

When the length of a rectangular plate approaches infinity as shown in Figure 3. then the ratio of the sides of the plate R becomes

$$R = \frac{2a}{2b} = \frac{a}{\infty} = 0.$$

Substituting R = 0. into equations (3 - 30), (3 - 31) and (3 - 32) . They become

$$U_{,xx} + W_{,x} W_{,xx} = 0. \quad \text{..................(3 - 33)}$$

$$V_{,xx} = 0. \quad \text{..............................(3 - 34)}$$

$$W_{,xxxx} + KW = Q + 12[U_{,x} + \frac{1}{2}(W_{,x})^2] W_{,xx}$$

$$\text{..................(3 - 35)}$$

Equations (3 - 33), (3 -34) and (3 - 35) are the governing differential equations for long rectangular plates on elastic foundations.

3.5. Governing Differential Equation for Long Rectangular Plates without Elastic Foundations

In this case, the ratio of the sides of the plate R ≐ 0. and the foundation modulus k = 0. (or K = 0.). with R = 0. and K = 0. equations (3 - 30), (3 - 31) and (3 - 32) can be written in the form

$$U_{,xx} + W_{,x} W_{,xx} = 0. \quad \text{.....................(3 - 36)}$$

$$V_{,xx} = 0. \quad \text{................................(3 - 37)}$$

$$W_{,xxxx} = Q + 12[U_{,x} + \frac{1}{2}(W_{,x})^2] W_{,xx}$$

$$\text{..................(3 - 38)}$$

Equations (3 - 36), (3 - 37) and (3 - 38) are the governing differential equations for long rectangular plates without elastic foundations.

Hence the problem of small and large deflections of long rectangular plates is reduced to finding a solution to the appropriate governing differential equations (3 - 33), (3 - 34), (3 - 35) or equations (3 - 36), (3 - 37) and (3 - 38).

CHAPTER 4

METHOD OF SOLUTION

4.1. The Perturbation Method

The small parameter perturbation method is now used to obtain the solutions to equations (3 - 33) , (3 - 34), (3 - 35) , (3 - 36), (3 - 37) and (3 - 38) for the linear and nonlinear deflections of long rectangular plates with different loading and boundary conditions with and without the elastic foundations.

This method requires the expansion of the displacement components and the dimensionless load quantity in a power seris of ascending powers of the dimensionless center deflection parameter W_o ,Thus let

$$U = S_2 W_o^2 + S_4 W_o^4 + \ldots\ldots \qquad (4-1)$$

$$V = t_2 W_o^2 + t_4 W_o^4 + \ldots\ldots \qquad (4-2)$$

$$W = w_1 W_o + w_3 W_o^3 + w_5 W_o^5 + \ldots \qquad (4-3)$$

$$Q = \gamma_1 F W_o + \gamma_3 F W_o^3 + \gamma_5 F W_o^5 + \ldots \qquad (4-4)$$

Where S_2 , S_4 , ... t_2 , t_4 , ... w_1 , w_3 , w_5 ...... are unknown functions of the dimensionless coordinates of x which satisfy the boundary conditions of the plate. γ_1 , γ_3 , γ_5 .... are unknown constants relating the dimensionless center deflection W_o to the dimensionless load Q , and such constants are a measure of the flexibility of the plate. The function F(x) , termed the load function is determined from the pressure distribution and is subject to the condition that F(0) = 1.

Interpreted physically, only even powers of W_o are required in Equations (4 - 1) and (4 - 2) because a change in sign of W_o , obtained by reversing the load, leaves u and v unaltered regardless of the load distribution. Similarly, only odd powers of W_o are required in Equations (4 - 3) and (4 - 4) because a change in sign of W_o corresponds to a change in sign of W and Q . Further, the function $S_o(x)$ and $t_o(x)$ are absent from Equations (4 - 1) and (4 - 2) since they correspond to in-plane forces. Since the only in-plane forces considered here are these that develop indirectly due to the membrane effect, we must have $u_o = v_o = 0$. These expansions, Equations (4 - 1) to

(4 - 4) are to be regarded as asymptotic series rather than as convergent power series. In practice, only the first two or three terms of the expansions are utilized, so that the series may even be ultimately divergent without being inefficient.

By definition, in order that the center deflection be W_o as defined, it is necessary to require that.

$$w_1 (0) = 1.$$

$$w_3 (0) = w_5 (0) = \ldots\ldots = 0.$$

4.2. Loading Conditions and Boundary Conditions

The four different loading conditions (Figure 4 - 1.) considered are :

(1) Uniformly distributed loading

(2) Hydrostatic loading

(3) Parabolic loading

(4) Cosine loading

The three different boundary conditions (Figure 4 - 2.) considered are :

(1) Clamped both sides

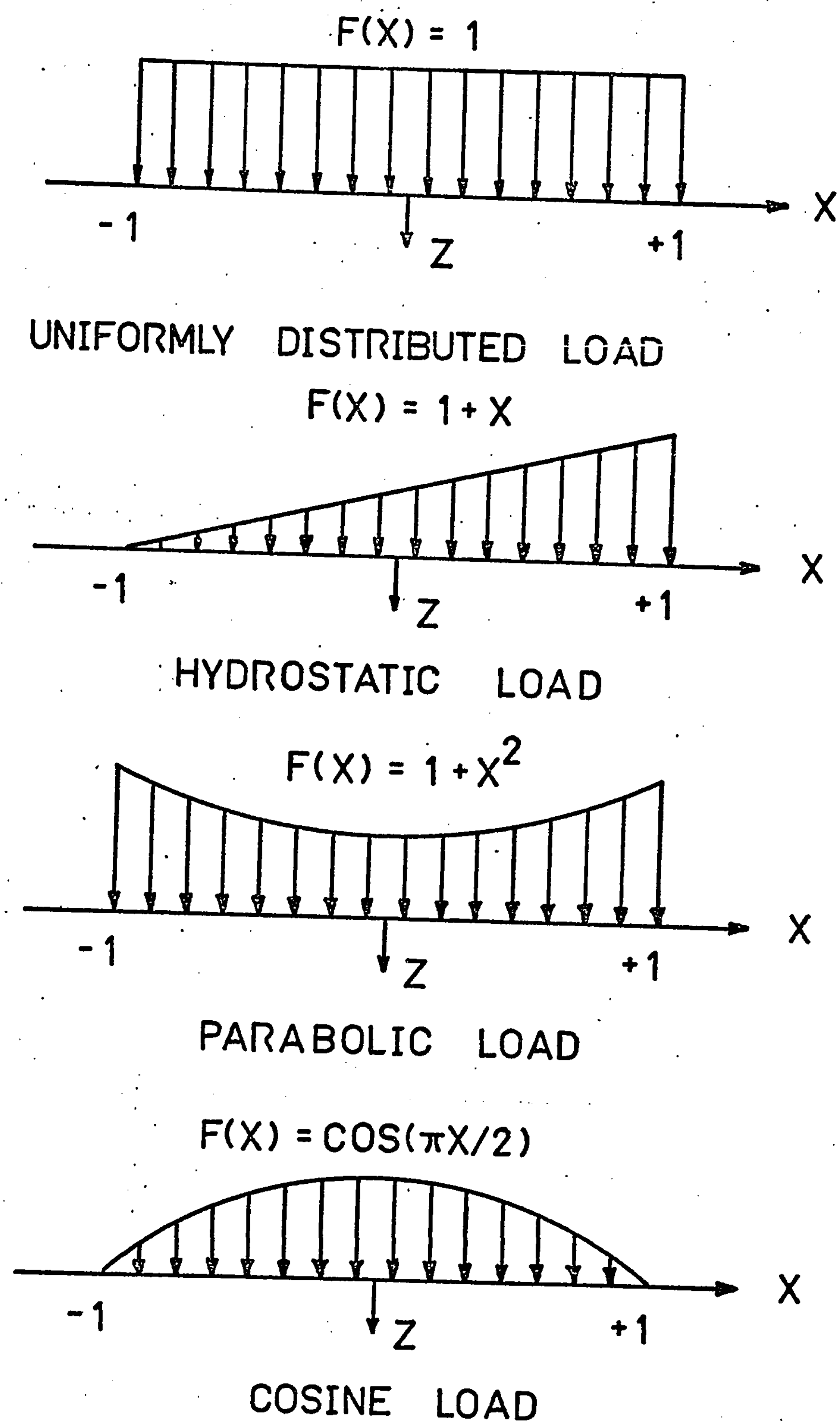
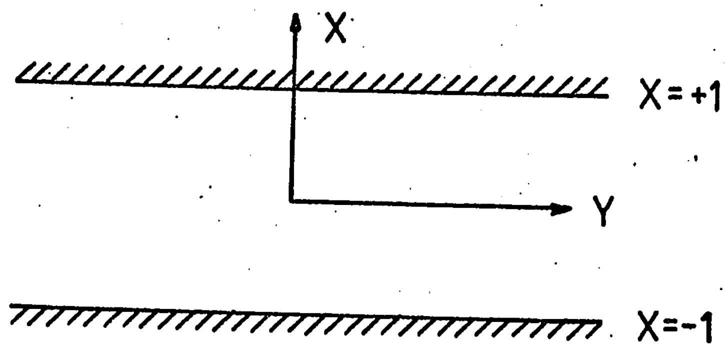
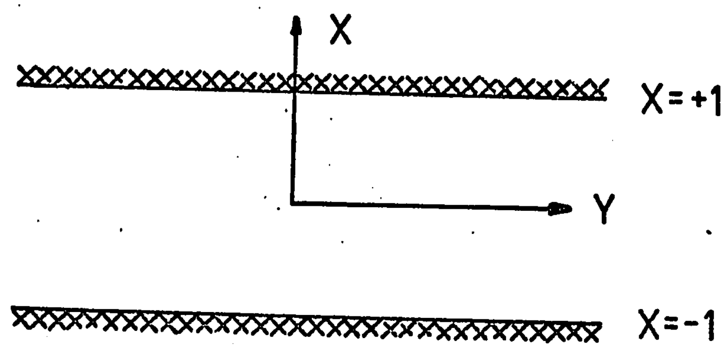


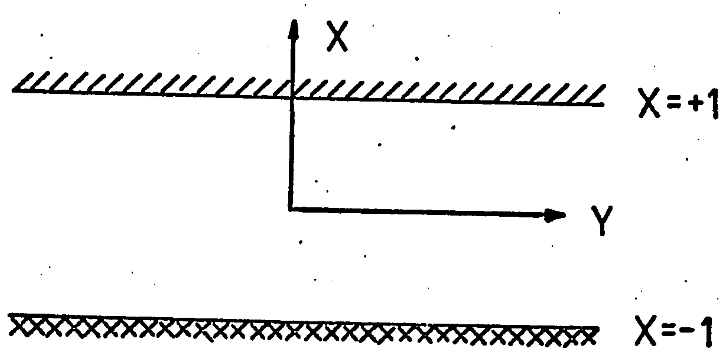
FIGURE 4-1 LOADING CONDITIONS



CLAMPED AT BOTH SIDES



SIMPLY SUPPORTED AT BOTH SIDES



CLAMPED AT X= +1
SIMPLY SUPPORTED AT X=-1

FIGURE 4-2 BOUNDARY CONDITIONS

(2) Simply supported both sides

(3) Clamped at $x = + 1$ and simply supported at $x = - 1$.

4.3. Solution to the Small (linear) Deflection Problems

For the linear small deflection problems, only the first order of W_o need be considered

(1) Without the elastic foundations

Substituting Equations (4 - 1) , (4 - 2) and (4 - 3) into Equations (3 - 36) , (3 - 37) and (3 - 38) and equating coefficients of the terms containing W_o , we obtain the following differential equation governing the small deflection problem of long rectangular plates without elastic support.

$$w_{1,xxxx} = F \gamma_1 \qquad \text{(4 - 5)}$$

To solve the linear small deflection problem of long rectangular plates without the elastic foundations, it is required to substitute the loading conditions into Equations (4 - 5) and integrate Equation (4 - 5) directly . The constants of integration are then evaluated by the substitution of the appropriate boundary conditions .

(A) Loading Conditions

(a) Uniformly distributed load

In this case F = 1.

Substituting F = 1. into equation(4 - 5) yields

$$w_{1,xxxx} = \gamma_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4 - 6)$$

By direct successive integrate Equation (4 -6), yields

$$w_1 = \frac{1}{24}\gamma_1 x^4 + \frac{1}{6} Ax^3 + \frac{1}{2} Bx^2 + Cx + T \quad \ldots\ldots\ldots\ldots\ldots (4 - 7)$$

(b) Hydrostatic load

In this case F = 1 + x

Substituting F = 1 + x into equation(4 - 5), yields

$$w_{1,xxxx} = (1 + x)\gamma_1 \quad \ldots\ldots\ldots (4 - 8)$$

By successive integrations Equation (4 - 8), yields

$$w_1 = \frac{1}{120}\gamma_1 x^5 + \frac{1}{24}\gamma_1 x^4 + \frac{1}{6} Ax^3 + \frac{1}{2} Bx^2 + Cx + T \quad \ldots\ldots (4 - 9)$$

(c) Parabolic load

In this case F = 1 + x^2

Substituting F = 1 + x^2 into equation (4 - 5), yields

$$w_{1,xxxx} = (1 + x^2) \gamma_1 \quad \text{...........(4 - 10)}$$

By successive integrations Equation (4 - 10),gives

$$w_1 = \frac{1}{360} \gamma_1 x^6 + \frac{1}{24} \gamma_1 x^4 + \frac{1}{6} Ax^3 + \frac{1}{2} Bx^2 + Cx + T \quad \text{........(4 - 11)}$$

(d) Cosine load

In this case $F = \cos(\pi/2)x$

Substituting $F = \cos(\pi/2)x$ into equation (4 - 5), yields

$$w_{1,xxxx} = \gamma_1 \cos(\pi/2)x \quad \text{........(4 -12)}$$

By successive integrations Equation (4 - 12),gives

$$w_1 = (2/\pi)^4 \cos(\pi/2)x + \frac{1}{6} Ax^3 + \frac{1}{2} Bx^2 + Cx + T \quad \text{.........(4 - 13)}$$

(B) Boundary Conditions of Long Rectangular Plates

(a) Clamped both sides (Associated boundary conditions)

$$w_{1,x} = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 1. \quad , \quad x = 0.$$

(b) Simply supported both sides (Associated boundary conditions)

$w_{1,xx} = 0.$, $x = \pm 1.$

$w_1 = 0.$, $x = \pm 1.$

$w_1 = 1.$, $x = 0.$

(c) Clamped at x = + 1. and simply supported at x = - 1. (Associated boundary conditions)

$w_{1,x} = 0.$, $x = + 1.$

$w_{1,xx} = 0.$, $x = - 1.$

$w_1 = 0.$, $x = \pm 1.$

$w_1 = 1.$, $x = 0.$

(2) With the elastic foundation modulus

Substituting Equations (4 - 1) , (4 - 2) , (4 - 3) and (4 - 4) into Equations (3 - 33) , (3 - 34) and (3 - 35) and equating coefficients of the terms containing W_o , then we obtain the following differential equation governing the small deflection problem of long plates with elastic support

$$w_{1,xxxx} + Kw_1 = \gamma_1 F \qquad \text{..................(4 - 14)}$$

Since Equation (4 - 14) is linear differential equation with constant coefficients, the general solution of the differential equation can easily be obtained.

(A) Loading Conditions

(a) Uniformly distributed load

In this case F = 1.

Substituting F = 1. into equation (4 - 14) yields

$$w_{1,xxxx} + k\, w_1 = \gamma_1 \qquad \text{..........(4 - 15)}$$

By successive integrations Equation (4 - 15), yields

$$\begin{aligned} w_1 &= e^{Qx}[\, A\cos(Qx) + B\sin(Qx)\,] \\ &\quad + e^{-Qx}[\, C\cos(Qx) + T\sin(Qx)\,] \\ &\quad + (\gamma_1 / K) \qquad \text{................(4 - 16)} \end{aligned}$$

(b) Hydrostatic load

In this case F = 1 + x

Substituting F = 1 + x into equation (4 -1 4) yields

$$w_{1,xxxx} + K\, w_1 = \gamma_1 (1 + x) \text{..(4 - 17)}$$

By successive integrations Equation (4 - 17), yields

$$w_1 = e^{Qx}[A\cos(Qx) + B\sin(Qx)] + e^{-Qx}[C\cos(Qx) + T\sin(Qx)] + \frac{\gamma_1}{K} x + \frac{\gamma_1}{K} \quad \text{............(4 - 18)}$$

(c) Parabolic load

In this case $F = 1 + x^2$

Substituting $F = 1 + x^2$ into Equation (4 - 14) yields

$$w_{1,xxxx} + K w_1 = \gamma_1 (1 + x^2) \quad \text{....(4 - 19)}$$

By successive integrations Equation (4 - 19), yields

$$w_1 = e^{Qx}[A\cos(Qx) + B\sin(Qx)] + e^{-Qx}[C\cos(Qx) + T\sin(Qx)] + \frac{\gamma_1}{k} x^2 + \frac{\gamma_1}{k} \quad \text{..........(4 - 20)}$$

(d) Cosine load

In this case $F = \cos (\pi/2)x$

Substituting $F = \cos (\pi/2)x$ into Equation (4 - 14), yields

$$w_1 + Kw_1 = \gamma_1 \cos(\pi/2)x \quad \text{.....(4 - 21)}$$

By successive integrations Equation (4 - 21), yields

$$w_1 = e^{Qx}[A\cos(Qx) + B\sin(Qx)] + e^{-Qx}[C\cos(Qx) + T\sin(Qx)] + \frac{\gamma_1}{[(\pi/2)^4 + K]} \cos(1.0708x) \qquad \text{.................... (4 - 22)}$$

(B) Boundary Conditions

(a) Clamped both sides (Associated boundary conditions)

$$w_{1,x} = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 1. \quad , \quad x = 0.$$

(b) Simply supported both sides (Associated boundary conditions)

$$w_{1,xx} = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 0. \quad , \quad x = \pm 1.$$

$$w_1 = 1. \quad , \quad x = 0.$$

(c) Clamped at $x = + 1$ and simply supported at $x = - 1$ (Associated Boundary conditions)

$$w_{1,x} = 0. \quad , \quad x = + 1.$$

$$w_{1,xx} = 0. \quad , \quad x = - 1.$$

$$w_1 = 1. \quad , \quad x = \pm 1.$$

$$w_1 = 1. \quad , \quad x = 0.$$

4.4. Solution to the Large Deflection Problems

For the nonlinear large deflection problem we have to consider the second and third order of W_o

(1) Without the elastic foundations

Substituting Equations (4 - 1) and (4 - 3) into Equations (3 - 36) and (3 - 37) and equating coefficients of the terms containing W_o^2 , we obtain the following differential equations governing the in-plane force without elastic foundations

$$S_{2,xx} + w_{1,x} w_{1,xx} = 0. \quad \text{.................}(4 - 23)$$

$$t_{2,xx} = 0. \quad \text{...........................}(4 - 24)$$

Substituting Equations (4 - 1) , (4 - 2) , (4 - 3) and (4 - 4) into Equations (3 - 38) and collecting coefficients of W_o^3 , we obtain the following differential equation governing the first nonlinear term of the lateral displacement for long plates without elastic foundations.

$$W_{3,xxxx} = \gamma_3 F + 12\, w_{1,xx}[\, S_{2,x} + \frac{1}{2}(\, w_{1,x})^2] \qquad \text{(4 - 25)}$$

To solve the nonlinear large deflection problem of long rectangular plates without elastic foundation, the loading conditions and the boundary conditions are substitute into Equations (4 - 23) and (4 - 25). Substitution of the boundary conditions into the Equations (4 - 23) and (4 - 25) results in a set of simultaneous equations from which all the constants of integration can be determined.

(A) Loading Conditions

(a) Uniformly distributed load

In this case F = 1.

Substituting F = 1. into Equation (4 - 25) yields

$$w_{3,xxxx} = \gamma_3 + 12 w_{1,xx}[\, S_{2,x} + \frac{1}{2}(w_{1,x})^2] \qquad \text{(4 - 26)}$$

By successive integration give

$$w_3 = \frac{1}{60} (N^* + \frac{1}{2}c^2) \gamma_1 x^6 + \frac{1}{10} (N + \frac{1}{2}c^2) Ax^5 + [\frac{1}{2} (N + \frac{1}{2}c^2)B + \frac{1}{24} \gamma_3]x^4 + \frac{1}{6}Hx^3 + \frac{1}{2}Ix^2 + Jx + L \qquad \text{(4 - 27)}$$

(b) Hydrostatic load

In this case F = 1 + x

Substituting F = 1 + x into Equation (4 - 24) yields

$$w_{3,xxxx} = \gamma_3 (1 + x) + 12w_{1,xx}[S_{2,x} + \frac{1}{2} (w_{1,x})^2] \qquad \text{(4 - 28)}$$

By successive integration Equation (4 - 28) gives

$$w_3 = \frac{1}{420} (N + \frac{1}{2}c^2) \gamma_1 x^7 + \frac{1}{60} (N + \frac{1}{2}c^2) \gamma_1 x^6 + [\frac{1}{10} (N + \frac{1}{2}c^2)A + \frac{1}{120} \gamma_3]x^5 + [\frac{1}{2} (N + \frac{1}{2}c^2)B + \frac{1}{24} \gamma_3]x^4 + \frac{1}{6}Hx^3 + \frac{1}{2}Ix^2 + Jx + L \qquad \text{(4 - 29)}$$

(c) Parabolic load

* N,H,I,J are constants of integration.

In this case $F = 1 + x^2$

Substituting $F = 1 + x^2$ into Equation (4 – 25) yields

$$w_{3,xxxx} = \gamma_3(1 + x^2) + 12w_{1,xx}[S_{2,x} + \frac{1}{2}(w_{1,x})^2] \quad \text{.............(4 – 30)}$$

By successive integration Equation (4 – 30) give

$$w_3 = \frac{1}{1680}(N + \frac{1}{2}c^2)\gamma_1 x^8 + [\frac{1}{60}(N + \frac{1}{2}c^2)\gamma_1 + \frac{1}{360}\ {}_3]x^6 + \frac{1}{10}(N + \frac{1}{2}c^2)Ax^5 + [\frac{1}{2}(N + \frac{1}{2}c^2)B + \frac{1}{24}\gamma_3]x^4 + \frac{1}{6}Hx^3 + \frac{1}{2}Ix^2 + Jx + L \quad \text{..................(4 – 31)}$$

(d) Cosine load

In this case $F = \cos(\pi/2)x$

Substituting $F = \cos(\pi/2)x$ + $12\, w_{1,xx}[S_{2,x}$

$$+ \frac{1}{2}(w_{1,x})^2] \quad \text{.............(4 – 32)}$$

By successive intergration Equation (4 – 32) give

$$w_3 = \Big\{ \gamma_3 (2/3.1416)^4 + 12\,\gamma_1 (2/3.1416)^6 \left[\gamma_1^2 (16/3.1416^6) + \frac{1}{2} C^2 + N \right] \Big\} \cos(\pi/2) x + \frac{1}{10} \left[\gamma_1^2 (16/3.1416^6) + \frac{1}{2} C^2 + N \right] A x^5 + \frac{1}{2} \left[\gamma_1^2 (16/3.1416^6) + \frac{1}{2} C^2 + N \right] B x^4 + \frac{1}{6} H x^3 + \frac{1}{2} I x^2 + J x + L \qquad \text{.................(4 - 33)}$$

(B) Boundary Conditions

(a) Clamped both sides (Associated boundary conditions)

$S_2 = 0.$* , $x = \pm 1.$

$w_{3,x} = 0.$, $x = \pm 1.$

$w_3 = 0.$, $x = \pm 1.$

$w_3 = 0.$, $x = 0.$

(b) Simply supported both sides (Associated boundary conditions)

$S_2 = 0.$, $x = \pm 1.$

$w_{3,xx} = 0.$, $x = \pm 1.$

* i.e. u=0. from Equ.(4-23) $U=S_2(x)W_o^2$

$$w_3 = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = 0.$$

(c) Clamped at $x = + 1.$ and simply supported at $x = - 1.$ (Associated boundary conditions)

$$S_2 = 0. \quad , \quad x = \pm 1.$$

$$w_{3,x} = 0. \quad , \quad x = + 1.$$

$$w_{3,xx} = 0. \quad , \quad x = - 1.$$

$$w_3 = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = 0.$$

(2) With the elastic foundations

Substituting Equations (4 - 1), (4 - 2), (4 - 3) and (4 - 4) into Equations (3 - 33) and (3 - 34) and by following much the same procedure as outlined for without elastic foundations the following differential equations governing the in-plane displacement components of the long rectangular plates with elastic support are obtained

$$S_{2,xx} + w_{1,x}w_{1,xx} = 0. \qquad \text{(4 - 34)}$$

$$t_{2,xx} = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4-35)$$

Also, the differential equation governing the first nonlinear term of the lateral displacement of the long plates with elastic foundations can be obtained as follows :

$$w_{3,xxxx} + Kw_3 = \gamma_3 F + 12\, w_{1,xx}[\, S_{2,x} + \frac{1}{2}(w_{1,x})^2] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4-36)$$

Substitute the boundary conditions into the general solution of the differential equations results in a set of simultaneous equations from which the constants of integration namely G,H,I,J,L can be determined

(A) Loading Conditions

(a) Uniformly distributed load

In this case F = 1.

Substituting F = 1. into Equation (4 - 36) yields

$$w_{3,xxxx} + K\,w_3 = \gamma_3 + 12\, w_{1,xx}[\, S_{2,x} + \frac{1}{2}(\, w_{1,x}\,)^2] \quad \ldots\ldots (4-37)$$

By successive integrations Equation (4 - 37) yields

(Here $Y = 24[\,(\,AT - BC\,)Q^2 + N\,]/\,Q$)

$$w_3 = e^{Qx}[\,H\cos(Qx) + I\sin(Qx)\,]$$
$$+ e^{-Qx}[\,J\cos(Qx) + L\sin(Qx)\,]$$
$$+ \frac{1}{16}\,Yx\,[(B + A)e^{Qx}\sin(Qx) - (B - A)e^{Qx}\cos(Qx)$$
$$+ (C - T)e^{-Qx}\sin(Qx) - (C + T)e^{-Qx}\cos(Qx)] + \frac{\gamma_3}{k}$$

..........................(4 - 38)

(b) Hydrostatic load

In this case $F = 1 + x$

Substituting $F = 1 + x$ into Equation (4 - 36) yields

$$w_{3,xxxx} + K\,w_3 = \gamma_3(1 + x) + 12\,w_{1,xx}[\,S_{2,x}$$
$$+ \frac{1}{2}(\,w_{1,x}\,)^2]$$

.............(4 - 39)

By successive integrations Equation (4 - 39), gives

$$w_3 = e^{Qx}[\,H\cos(Qx) + I\sin(Qx)\,]$$
$$+ e^{-Qx}[\,J\cos(Qx) + L\sin(Qx)\,]$$
$$+ \frac{1}{16}\,Yx\,[(B + A)e^{Qx}\sin(Qx) - (B - A)e^{Qx}\cos(Qx)$$
$$+ (C - T)e^{-Qx}\sin(Qx) - (C + T)e^{-Qx}\cos(Qx)\,]$$
$$+ \frac{\gamma_3}{k}x + \frac{\gamma_3}{k}$$

............(4 - 40)

Here $Y = 24[\,(AT - BC)\,Q^2 + N + \frac{1}{2}(\frac{\gamma_1}{k})^2]/\,Q$

(c) Parabolic load

In this case $F = 1 + x^2$

Substituting $F = 1 + x^2$ into Equation (4 - 36) yields

$$w_{3,xxxx} + K w_3 = \gamma_3 (1 + x^2) + 12 w_{1,xx} \left[S_{2,x} + \frac{1}{2} (w_{1,x})^2 \right] \quad \text{......(4 - 41)}$$

By successive integrations Equation (4 - 41) yields

$$\begin{aligned} w_3 = {} & e^{Qx}[H\cos(Qx) + I\sin(Qx)] \\ & + e^{-Qx}[J\cos(Qx) + L\sin(Qx)] \\ & + \frac{1}{16} Yx[(B + A)e^{Qx}\sin(Qx) \\ & - (B - A)e^{Qx}\cos(Qx) + (C - T)e^{-Qx}\sin(Qx) \\ & - (C + T)e^{-Qx}\cos(Qx)] + \frac{\gamma_3}{k}x^2 + \frac{\gamma_3}{k} \\ & + YQ \frac{\gamma_1}{K^2} \quad \text{..................(4 - 42)} \end{aligned}$$

Here $Y = 24[(AT - BC) Q^2 + N] / Q$

(d) Cosine load

In this case $F = \cos(1.0708X)$

Substituting $F = \cos(1.0708x)$ into Equation (4 - 36) yields

$$w_{3,xxxx} + K w_3 = \gamma_3 \cos(1.0708x) + 12 w_{1,xx} \left[S_{2,x} + \frac{1}{2}(w_{1,x})^2 \right] \quad \ldots\ldots(4-43)$$

By successive integrations Equation (4 -43) give

$$w_3 = e^{Qx}[H\cos(Qx) + I\sin(Qx)] + e^{-Qx}[J\cos(Qx) + L\sin(Qx)] + \left\{ \frac{\gamma_3}{[(3.1416/2)^4 + K]} - \frac{3[(AT - BC)Q^2 + N]}{[(3.1416/2)^4 + K]}(3.1416^2\gamma_1) \right\} \cos(1.0708x) + \frac{1}{16} Yx[(B + A)e^{Qx}\sin(Qx) - (B - A)e^{Qx}\cos(Qx) + (C - T)e^{-Qx}\sin(Qx) - (C + T)e^{-Qx}\cos(Qx)] \quad \ldots\ldots\ldots(4-44)$$

Here $Y = 24[(AT - BC)Q^2 + N] / Q$

(A) Boundary Conditions

(a) Clamped both sides (Associated boundary conditions)

$$S_2 = 0. \quad , \quad x = \pm 1.$$

$$w_{3,x} = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = 0.$$

(b) Simply supported both sides (Associated boundary conditions)

$$S_2 = 0. \quad , \quad x = \pm 1.$$

$$w_{3,xx} = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = 0.$$

(c) Clamped at $x = + 1$ and simply supported at $x = - 1$ (Associated boundary conditions)

$$S_2 = 0. \quad , \quad x = \pm 1.$$

$$w_{3,x} = 0. \quad , \quad x = + 1.$$

$$w_{3,xx} = 0. \quad , \quad x = -1.$$

$$w_3 = 0. \quad , \quad x = \pm 1.$$

$$w_3 = 0. \quad , \quad x = 0.$$

Closed form solutions for various loading and boundary conditions of long rectangular plates considered in this thesis ; i.e., the complete expressions of w_1 , S_2 and w_3 are listed for reference in Appendix A and the complete expressions of γ_1 and γ_3 are listed for reference in Appendix B.

4.5. Displacements and Stress Relationships

It is well known that, in the theory of thin plates the bending moment and stresses can be expressed in terms of the displacements. In rectangular coordinates $\bar{x}$ and $\bar{y}$, the relation between displacements u, v and w and the plate moments can be expressed as :

(1) Bending Moments :

$$M_{\bar{x}} = - D(w_{,\bar{x}\bar{x}} + \nu w_{,\bar{y}\bar{y}}) \qquad \text{................(4 - 45)}$$

$$M_{\bar{y}} = - D(w_{,\bar{y}\bar{y}} + \nu w_{,\bar{x}\bar{x}}) \qquad \text{................(4 - 46)}$$

$$M_{\bar{x}\bar{y}} = M_{\bar{y}\bar{x}} = D(1 - \nu) w_{,\bar{x}\bar{y}} \qquad \text{................(4 - 47)}$$

(2) Stresses Due to Bending :

$$\sigma''_{\bar{x}} = - \frac{Eh}{2(1-\nu^2)}(w_{,\bar{x}\bar{x}} + \nu w_{,\bar{y}\bar{y}}) \quad ......(4 - 48)$$

$$\sigma''_{\bar{y}} = - \frac{Eh}{2(1-\nu^2)}(w_{,\bar{y}\bar{y}} + \nu w_{,\bar{x}\bar{x}}) \quad ......(4 - 49)$$

$$\tau''_{\bar{x}\bar{y}} = \frac{Eh}{2(1-\nu^2)}(1 - \nu) w_{,\bar{x}\bar{y}} \quad ......(4 - 50)$$

(3) Membrane Stresses :

$$\sigma'_{\bar{x}} = \frac{E}{1-\nu^2} \Big\{ u_{,\bar{x}} + \frac{1}{2}(w_{,\bar{x}})^2 + \nu [v_{,\bar{y}} + \frac{1}{2}(w_{,\bar{y}})^2] \Big\} \quad ....(4 - 51)$$

$$\sigma'_{\bar{y}} = \frac{E}{1-\nu^2} \Big\{ v_{,\bar{y}} + \frac{1}{2}(w_{,\bar{y}})^2 + \nu [u_{,\bar{x}} + \frac{1}{2}(w_{,\bar{x}})^2] \Big\} \quad ....(4 - 52)$$

$$\tau'_{\bar{x}\bar{y}} = \frac{E}{2(1-\nu)} u_{,\bar{y}} + v_{,\bar{x}} + w_{,\bar{x}} w_{,\bar{y}} \quad ....(4 - 53)$$

By using the dimensionless ratio previously outlined and the following additional dimensionless ratios for moments, bending stresses and membrane stresses, viz.

$$M_x = \frac{12(1 - \nu^2)\, a^2}{Eh^4} \, M_{\bar{x}} \qquad \ldots\ldots\ldots\ldots (4-54)$$

$$M_y = \frac{12(1 - \nu^2)\, a^2}{Eh^4} \, M_{\bar{y}} \qquad \ldots\ldots\ldots\ldots (4-55)$$

$$M_{xy} = \frac{6a^2}{Gh^4} \, M_{\overline{xy}} \qquad \ldots\ldots\ldots\ldots (4-56)$$

$$\sigma''_x = \frac{(1 - \nu^2)\, a^2}{Eh^2} \, \sigma''_{\bar{x}} \qquad \ldots\ldots\ldots\ldots (4-57)$$

$$\sigma''_y = \frac{(1 - \nu^2)\, a^2}{Eh^2} \, \sigma''_{\bar{y}} \qquad \ldots\ldots\ldots\ldots (4-58)$$

$$\tau''_{xy} = \frac{a^2}{Gh^2} \, \tau''_{\overline{xy}} \qquad \ldots\ldots\ldots\ldots (4-59)$$

$$\sigma'_x = \frac{(1 - \nu^2)\, a^2}{Eh^2} \, \sigma'_{\bar{x}} \qquad \ldots\ldots\ldots\ldots (4-60)$$

$$\sigma'_y = \frac{(1 - \nu^2)\, a^2}{Eh^2} \, \sigma'_{\bar{y}} \qquad \ldots\ldots\ldots\ldots (4-61)$$

$$\tau'_{xy} = \frac{a^2}{Gh^2} \, \tau'_{\overline{xy}} \qquad \ldots\ldots\ldots\ldots (4-62)$$

Equations (4 - 45) to (4 - 53) can be rewrite in the dimensionless forms as follows :

$$M_x = -[W_{,xx} + \nu R^2 W_{,yy}] \quad \text{........(4 - 63)}$$

$$M_y = -[R^2 W_{,yy} + \nu W_{,xx}] \quad \text{........(4 - 64)}$$

$$M_{xy} = \frac{6(1-\nu)}{Gh^3} R W_{,xy} \quad \text{........(4 - 65)}$$

$$\sigma''_x = -\frac{1}{2}[W_{,xx} + \nu R^2 W_{,yy}] \quad \text{....(4 - 66)}$$

$$\sigma''_y = -\frac{1}{2}[R^2 W_{,yy} + \nu W_{,xx}] \quad \text{....(4 - 67)}$$

$$\tau''_{xy} = R W_{,xy} \quad \text{....................(4 - 68)}$$

$$\sigma'_x = U_{,x} + \frac{1}{2}(W_{,x})^2 + \nu[R V_{,y} + \frac{1}{2} R^2 (W_{,y})^2] \quad \text{.............(4 - 69)}$$

$$\sigma'_y = R V_{,y} + \frac{1}{2} R^2 (W_{,y})^2 + \nu[U_{,x} + \frac{1}{2}(W_{,x})^2] \quad \text{.............(4 - 70)}$$

$$\tau'_{xy} = R U_{,y} + V_{,x} + R W_{,x} W_{,y} \quad \text{......(4 - 71)}$$

For long rectangular plates $b = \infty$ and the ratio of the plate becomes R = 0. Substituting R = 0. into Equations (4 - 63) through (4 - 71)

$$M_x = - W_{,xx} \quad \text{.......................} (4-72)$$

$$M_y = - \nu W_{,xx} \quad \text{.......................} (4-73)$$

$$M_{xy} = 0. \quad \text{.......................} (4-74)$$

$$\sigma''_x = - \frac{1}{2} W_{,xx} \quad \text{..................} (4-75)$$

$$\sigma''_y = - \frac{1}{2} \nu W_{,xx} \quad \text{..................} (4-76)$$

$$\tau''_{xy} = 0. \quad \text{..................} (4-77)$$

$$\sigma'_x = U_{,x} + \frac{1}{2} (W_{,x})^2 \quad \text{..........} (4-78)$$

$$\sigma'_y = \nu [U_{,x} + \frac{1}{2} (W_{,x})^2] \quad \text{......} (4-79)$$

$$\tau'_{xy} = V_{,x} \quad \text{.......................} (4-80)$$

Substituting Equations (4 - 1) , (4 - 2) and (4 - 3) into Equations (4 - 72) through (4 - 80) yield the relationships among the moments, bending stresses, membrane stresses and the perturbation parameter W_o

$$M_x = -[\, w_{1,xx} W_o + w_{3,xx} W_o^3 + \ldots\ ] \quad \ldots (4-81)$$

$$M_y = -\nu[\, w_{1,xx} W_o + w_{3,xx} W_o^3 + \ldots\ ] \quad \ldots (4-82)$$

$$M_{xy} = 0. \quad \ldots\ldots\ldots\ldots (4-83)$$

$$\sigma''_x = -\frac{1}{2}[\, w_{1,xx} W_o + w_{3,xx} W_o^3 + \ldots] \quad \ldots (4-84)$$

$$\sigma''_y = -\frac{1}{2}\nu[\, w_{1,xx} W_o + w_{3,xx} W_o^3 + \ldots] \quad \ldots (4-85)$$

$$\tau''_{xy} = 0. \quad \ldots\ldots\ldots\ldots (4-86)$$

$$\sigma'_x = S_{2,x} W_o^2 + \frac{1}{2} (\, w_{1,x})^2 W_o^2 \ldots\ldots] \quad \ldots (4-87)$$

$$\sigma'_y = \nu [\, S_{2,x} W_o^2 + \frac{1}{2} (\, w_{1,x})^2 W_o^2 \ldots] \quad \ldots (4-88)$$

$$\tau'_{xy} = 0. \quad \ldots\ldots\ldots\ldots (4-89)$$

CHAPTER 5

NUMERICAL EXAMPLE AND USE THE DEFLECTION AND STRESS CURVES FOR DESIGN

5.1. A Typical Way to Find w_1, S_2, and w_3

A long rectangular thin plate clamped at both sides with uniformly distributed loading and resting on an elastic foundation is used as a numerical example to show in detail the step refined to calculate the displacement functions w_1, S_2, and w_3 .

(1) From Equation (4 - 14) Chapter 4

$$w_{1,xxxx} + K w_1 = \gamma_1 F$$

For uniformly distributed loading F = 1.

$$w_{1,xxxx} + K w_1 = \gamma_1 \qquad \text{.................(5 - 1)}$$

This is a nonhomogeneous differential equation with constant coefficients and the right hand side is a constant γ_1

Rewrite Equation (5 - 1)

$$(D^4 + K) w_1 = \gamma_1 \qquad \text{.................(5 - 2)}$$

The complete solution of Equation (5 - 2) will be

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$$w_1 = w_{1c} + w_{1p}$$

Here w_{1c} is complementary function

w_{1p} is particular integral

(A) Find out the complementary function w_{1c}

The associated homogeneous equation of Equation (5 - 2) is

$$(D^4 + K) w_1 = 0. \qquad \text{.......................(5 - 3)}$$

or

$$D^4 + K = 0. \qquad \text{.......................(5 - 4)}$$

let $q^4 = K$

then Equation (5 - 4) becomes

$$D^4 + q^4 = 0.$$

$$D^4 = - q^4 = (- 1) q^4 = i^2 q^4 \qquad \text{........(5 - 5)}$$

$$\because \quad i = e^{1.0708i}$$

$$\therefore \quad i^2 = e^{3.1416i} = e^{(6.2832n - 3.1416)i}$$

Substituting $i^2 = e^{(6.2832n - 3.1416)i}$ into Equation (5 - 5), yields

$$D^4 = e^{(6.2832n - 3.1416)i} q^4$$

or

$$D = e^{[(6.2832n-3.1416)/4]i} q \qquad \text{.............(5 - 6)}$$

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When n = 0 , 1 , 2 , 3

n = 0.

$$D_1 = e^{(\pi / 4)\, i}\, q$$

$$= [\cos(\pi / 4) + i \sin(\pi / 4)]q$$

$$= (\sqrt{2} / 2)q - i(\sqrt{2} / 2)q$$

n = 1.

$$D_2 = e^{(3\pi / 4)\, i}\, q$$

$$= [\cos(3\pi / 4) + i \sin(3\pi / 4)]q$$

$$= -(\sqrt{2} / 2)q + i(\sqrt{2} / 2)q$$

n = 2.

$$D_3 = e^{(5\pi / 4)\, i}\, q$$

$$= [\cos(5\pi / 4) + i \sin(5\pi / 4)]q$$

$$= -(\sqrt{2} / 2)q - i(\sqrt{2} / 2)q$$

n = 3.

$$D_4 = e^{(7\pi / 4)\, i}\, q$$

$$= [\cos(7\pi / 4) + i \sin(7\pi / 4)]q$$

$$= (\sqrt{2} / 2)q - i(\sqrt{2} / 2)q$$

Then the two sets of conjugate complex roots of Equation (5 - 6) are

$$\begin{cases} (\sqrt{2}/2)q + i(\sqrt{2}/2)q \\ (\sqrt{2}/2)q - i(\sqrt{2}/2)q \end{cases}$$

$$\begin{cases} -(\sqrt{2}/2)q + i(\sqrt{2}/2)q \\ -(\sqrt{2}/2)q - i(\sqrt{2}/2)q \end{cases}$$

The complementary function is

$$w_{1c} = e^{\sqrt{2}qx/2}[\, A\cos(\sqrt{2}qx/2) + B\sin(\sqrt{2}qx/2)\,]$$
$$- e^{-\sqrt{2}qx/2}[\, C\cos(\sqrt{2}qx/2) + T\sin(\sqrt{2}qx/2)\,]$$

Let $Q = \sqrt{2}q/2$

$$w_{1c} = e^{Qx}[\, A\cos(Qx) + B\sin(Qx)\,]$$
$$+ e^{-Qx}[\, C\cos(Qx) + T\sin(Qx)\,] \quad \text{..........(5 - 7)}$$

(B) Find out the particular integral

Assume the particular integral

$w_{1p} = ç$ (ç is a constant)

Substituting $w_{1p} = ç$ into Equation (5 - 2), yields

$$(D^4 + L)\, ç = \gamma_1$$
$$K\, ç = \gamma_1$$
$$ç = \gamma_1 / K$$

The particular integral is

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$$w_{1p} = \gamma_1 / K \quad \text{.......................(5 - 8)}$$

From Equation (5 - 7) and (5 - 8) , the complete solution is

$$w_1 = w_{1c} + w_{1p}$$

$$= e^{Qx}[A\cos(Qx) + B\sin(Qx)]$$

$$-e^{+Qx}[C\cos(Qx) + T\sin(Qx)]$$

$$- \gamma_1 / K \quad \text{.......................(5 - 9)}$$

From Equation (5 - 9), the following derivatives yield

$$w_{1,x} = e^{Qx}Q[\cos(Qx) - \sin(Qx)] A$$

$$+ e^{Qx}Q[\cos(Qx) + \sin(Qx)] B$$

$$-e^{-Qx}Q[\cos(Qx) + \sin(Qx)] C$$

$$+e^{-Qx}Q[\cos(Qx) - \sin(Qx)] T$$

$$\text{.......................(5 - 10)}$$

$$w_{1,xx} = -2e^{Qx}Q^2A \sin(Qx) + 2e^{Qx}Q^2B \cos(Qx)$$

$$+2e^{-Qx}Q^2C \sin(Qx) - 2e^{-Qx}Q^2 \cos(Qx)$$

$$\text{.......................(5 - 11)}$$

(2) From Equation (4 - 23) Chapter 4

$$S_{2,xx} = - w_{1,x}\, w_{1,xx}$$

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Substituting Equations (5 - 10) and (5 - 11) into the above Equation , yields

$$
\begin{aligned}
S_{2,xx} = 2Q^3 \Big\{ & A^2[\, e^{2Qx}\cos(Qx)\sin(Qx) - e^{2Qx}\sin^2(Qx) \,] \\
& -AB[\, e^{2Qx}\cos^2(Qx) - e^{2Qx}\cos(Qx)\sin(Qx) \,] \\
& -AC[\, \cos(Qx)\sin(Qx) - \sin^2(Qx) \,] \\
& +AT[\, \cos^2(Qx) - \cos(Qx)\sin(Qx) \,] \\
& +AB[\, e^{2Qx}\cos(Qx)\sin(Qx) + e^{2Qx}\sin^2(Qx) \,] \\
& -B^2[\, e^{2Qx}\cos^2(Qx) + e^{2Qx}\cos(Qx)\sin(Qx) \,] \\
& -BC[\, \cos(Qx)\sin(Qx) + \sin^2(Qx) \,] \\
& +BT[\, \cos^2(Qx) + \cos(Qx)\sin(Qx) \,] \\
& -AC[\, \sin^2(Qx) + \cos(Qx)\sin(Qx) \,] \\
& +BC[\, \cos(Qx)\sin(Qx) + \cos^2(Qx) \,] \\
& +C^2[\, e^{-2Qx}\sin^2(Qx) + e^{-2Qx}\cos(Qx)\sin(Qx) \,] \\
& -CT[\, e^{-2Qx}\cos(Qx)\sin(Qx) + e^{-2Qx}\cos^2(Qx) \,] \\
& +AT[\, \cos(Qx)\sin(Qx) - \sin^2(Qx) \,] \\
& -BT[\, \cos^2(Qx) - \cos(Qx)\sin(Qx) \,] \\
& -CT[\, e^{-2Qx}\cos(Qx)\sin(Qx) - e^{-2Qx}\sin^2(Qx) \,] \\
& +T^2[\, e^{-2Qx}\cos^2(Qx) - e^{-2Qx}\cos(Qx)\sin(Qx) \,] \Big\}
\end{aligned}
$$

..........................(5 - 14)

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By direct successive integrate Equation (5 - 14), yields

$$S_2 = -(Q/8)(A^2 + 2AB - B^2)\, e^{2Qx} \cos(2Qx)$$
$$+ (Q/8)(C^2 - 2CT - T^2)\, e^{-2Qx} \cos(2Qx)$$
$$- (Q/4)(A^2 + B^2)\, e^{2Qx}$$
$$+ (Q/4)(C^2 + T^2)\, e^{-2Qx}$$
$$+ (Q/8)(A^2 - 2AB - B^2)\, e^{2Qx} \sin(2Qx)$$
$$+ (Q/8)(C^2 + 2CT - T^2)\, e^{-2Qx} \sin(2Qx)$$
$$+((Q/2)(AC - BT) \sin(2Qx)$$
$$- (Q/2)(AT + BC) \cos(2Qx)$$
$$+ Ex + N \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5 - 15)$$

(3) From Equation (4 -25) Chapter 4

$$w_{3,xxxx} + K w_3 = \gamma_3 F + 12\, w_{1,xx}[S_{2,x} + \frac{1}{2}(w_{1,x})^2] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5 - 16)$$

This is a nonhomogeneous differential equation with constant coefficients and the right hand side is a function of x. So the complementary function will be the same form as Equation (5 - 1), but the particular integral will not be the same.

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(A) The complementary function is

$$w_{3c} = e^{Qx}[\, H\cos(Qx) + I\sin(Qx)\,] + e^{-Qx}[\, J\cos(Qx) + L\sin(Qx)\,] \qquad \text{.....................(5 - 16)}$$

(B) Find out the particular integral

From Equations (5 - 10) and (5 - 14)

$$S_{2,x} + \frac{1}{2}(w_{1,x})^2 = (AT - BC)Q^2 + N \qquad \text{.....................(5 - 17)}$$

Substituting Equations (5 - 11) and (5 - 17) into Equation (5 - 16), yields

$$w_{3,xxxx} + Kw_3 = \gamma_3 + 24[(AT - BC)Q^2 + N]Q^2[\, -Ae^{Qx}\sin(Qx) + Be^{Qx}\cos(Qx) + Ce^{-Qx}\sin(Qx) - Te^{-Qx}\cos(Qx)] \qquad \text{..........(5 - 18)}$$

Assume the particular integral as

$$w_{3p} = x\,[\, SS\, e^{Qx}\sin(Qx) + TT\, e^{Qx}\cos(Qx) + UU\, e^{-Qx}\sin(Qx) + VV\, e^{-Qx}\cos(Qx)\,] + PP \qquad \text{.....................(5 - 19)}$$

let

$$w_s = SS\, e^{Qx}\sin(Qx) + TT\, e^{Qx}\cos(Qx) + UU\, e^{-Qx}\sin(Qx) + VV\, e^{-Qx}\cos(Qx)$$

Substituting w_s into Equation (5 - 19), we get

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$$w_{3p} = x\, w_s + PP$$

$$D\, w_{3p} = x\, D\, w_s + w_s$$

$$D^2\, w_{3p} = x\, D^2\, w_s + 2\, D\, w_s$$

$$D^3\, w_{3p} = x\, D^3\, w_s + 3\, D^2\, w_s$$

$$D^4\, w_{3p} = x\, D^4\, w_s + 4\, D^3\, w_s$$

or

$$\begin{aligned} D^4\, w_{3p} = & -4Q^4 x[\, SS\, e^{Qx}\sin(Qx) + TT\, e^{Qx}\cos(Qx) \\ & + UU\, e^{-Qx}\sin(Qx) + VV\, e^{-Qx}\cos(Qx)] \\ & +8Q^3[\, -SS\, e^{Qx}\sin(Qx) + SS\, e^{Qx}\cos(Qx) \\ & - TT\, e^{Qx}\cos(Qx) - TT\, e^{Qx}\sin(Qx) \\ & + UU\, e^{-Qx}\sin(Qx) + UU\, e^{-Qx}\cos(Qx) \\ & + VV\, e^{-Qx}\cos(Qx) - VV\, e^{-Qx}\sin(Qx)] \end{aligned} \qquad \text{.................... (5 - 20)}$$

Substituting Equation (5 - 20) into Equation (5 - 16) yields

$$\begin{aligned} & -4Q^4 x[\, SS\, e^{Qx}\sin(Qx) + TT\, e^{Qx}\cos(Qx) \\ & + UU\, e^{-Qx}\sin(Qx) + VV\, e^{-Qx}\cos(Qx)] \\ & +8Q^3[(\, -SS - TT\,)e^{Qx}\sin(Qx) - (\, SS - TT\,)e^{Qx}\cos(Qx) \\ & - (\, UU - VV\,)e^{-Qx}\sin(Qx) - (\, UU + VV\,)e^{-Qx}\cos(Qx)] \\ & - Kx[\, SS\, e^{Qx}\sin(Qx) + TT\, e^{Qx}\cos(Qx) \\ & + UU\, e^{-Qx}\sin(Qx) + VV\, e^{-Qx}\cos(Qx)] + K\, PP \\ = & \, \gamma_3 + 24[(AT - BC)Q^2 + N]Q^2[\, -Ae^{Qx}\sin(Qx) \\ & + Be^{Qx}\cos(Qx) + Ce^{-Qx}\sin(Qx) - Te^{-Qx}\cos(Qx)] \end{aligned} \qquad \text{..(5-21)}$$

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Let $Y = 24[(AT - BC)Q^2 + N] / Q$

For all real numbers x, if and only if

$$-YA = 8(-SS - TT)$$

$$YB = 8(SS - TT)$$

$$YC = 8(UU - VV)$$

$$-YT = 8(UU + VV)$$

$$KPP = \gamma_3$$

Solving these equations, we get

$$SS = \frac{1}{16} Y(B + A) \qquad \text{.................. (5 - 22)}$$

$$TT = -\frac{1}{16} Y(B - A) \qquad \text{.................. (5 - 23)}$$

$$UU = \frac{1}{16} Y(C - T) \qquad \text{.................. (5 - 24)}$$

$$VV = -\frac{1}{16} Y(C + T) \qquad \text{.................. (5 - 25)}$$

$$PP = \frac{\gamma_3}{K} \qquad \text{.................. (5 - 26)}$$

Substituting Equations (5 - 22), (5 - 23), (5 - 24), (5 - 25) and (5 - 26) into Equations (5 - 19), we get the particular integral as following

$$w_{3p} = \frac{\gamma_3}{K} + \frac{1}{16} Yx[(B + A)e^{Qx}\sin(Qx)$$

$$- (B - A)e^{Qx}\cos(Qx) + (C - T)e^{-Qx}\sin(Qx)$$

$$- (C + T)e^{-Qx}\cos(Qx)] \quad \ldots\ldots\ldots\ldots\ldots(5 - 27)$$

From Equation (5 - 16) and (5 - 27) we get the complete solution of Equation (5 - 16)

$$w_3 = e^{Qx}[H\cos(Qx) + I\sin(Qx)]$$

$$+ e^{-Qx}[J\cos(Qx) + L\sin(Qx)]$$

$$+ \frac{1}{16} Yx[(B + A)e^{Qx}\sin(Qx) - (B - A)e^{Qx}\cos(Qx)$$

$$+ (C - T)e^{-Qx}\sin(Qx) - (C + T)e^{-Qx}\cos(Qx)]$$

$$+ \frac{\gamma_3}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5 - 28)$$

From Equation (5 - 28), the following derivatives yield

$$w_{3,x} = Q[e^{Qx}H\cos(Qx) - e^{Qx}H\sin(Qx)$$

$$+ e^{Qx}I\cos(Qx) + e^{Qx}I\sin(Qx)$$

$$- e^{-Qx}J\cos(Qx) - e^{-Qx}J\sin(Qx)$$

$$+ e^{-Qx}L\cos(Qx) - e^{-Qx}L\sin(Qx)]$$

$$+ \frac{1}{16} Y \Big\{ 2Qx[e^{Qx}A\cos(Qx) + e^{Qx}B\sin(Qx)$$

$$+ e^{-Qx}C\cos(Qx) + e^{-Qx}D\sin(Qx)]$$

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$$+ [(B + A)e^{Qx}\sin(Qx) \;-\; (B - A)e^{Qx}\cos(Qx)$$

$$+ (C - T)e^{-Qx}\sin(Qx) \;-\; (C + T)e^{-Qx}\cos(Qx)] \Big\}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots (5 - 29)$$

$$w_{3,xx} = 2Q^2[-e^{Qx}H\sin(Qx) + e^{Qx}I\cos(Qx)$$

$$+ e^{-Qx}J\sin(Qx) - e^{-Qx}L\sin(Qx)]$$

$$+ \frac{1}{8}Y \Big\{ Q^2x[e^{Qx}A\cos(Qx) - e^{Qx}A\sin(Qx)$$

$$+ e^{Qx}B\cos(Qx) + e^{Qx}B\sin(Qx)$$

$$- e^{-Qx}C\cos(Qx) - e^{-Qx}C\sin(Qx)$$

$$+ e^{-Qx}T\cos(Qx) - e^{-Qx}T\sin(Qx)]$$

$$+ 2Q[e^{Qx}A\cos(Qx) + e^{Qx}B\sin(Qx)$$

$$+ e^{-Qx}C\cos(Qx) + e^{-Qx}\sin(Qx)]$$

$$\dots\dots\dots\dots\dots\dots\dots\dots (5 - 30)$$

(4) Using the boundary concitions to solve A,B,C,T γ_1,N,G,H,I,J,L, and γ_3

Boundary Condition 1.

$$w_{1,x} = 0. \quad , \quad x = + 1.$$

$$e^{Q}(\cos Q - \sin Q)A + e^{-Q}(\cos Q + \sin Q)B$$

$$- e^{-Q}(\cos Q + \sin Q)C + e^{-Q}(\cos Q - \sin Q)T = 0.$$

$$\dots\dots\dots\dots\dots\dots\dots\dots (5 - 31)$$

Boundary Condition 2.

$$w_{1,x} = 0. \quad , \quad x = -1.$$

$$e^{-Q}(\cos Q + \sin Q)A + e^{-Q}(\cos Q - \sin Q)B$$
$$-e^{Q}(\cos Q - \sin Q)C + e^{Q}(\cos Q + \sin Q)T$$
$$= 0. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5-32)$$

Boundary Condition 3.

$$w_1 = 0. \quad , \quad x = +1.$$

$$Ae^{Q}\cos Q + Be^{Q}\sin Q + Ce^{-Q}\cos Q + Te^{-Q}\sin Q + \frac{\gamma_1}{K}$$
$$= 0. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5-33)$$

Boundary Condition 4.

$$w_1 = 0. \quad , \quad x = -1.$$

$$Ae^{-Q}\cos Q - Be^{-Q}\sin Q + Ce^{Q}\cos Q - Te^{Q}\sin Q + \frac{\gamma_1}{K}$$
$$= 0. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5-34)$$

Boundary Condition 5.

$$w_1 = 1. \quad , \quad x = 0.$$

$$A + C + \frac{\gamma_1}{K} = 1. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5-35)$$

Boundary Condition 6.

$$S_2 = 0. \quad , \quad x = +1.$$

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$$
\begin{aligned}
& N + G \\
= & \frac{1}{8}Q(A^2 + 2AB - B^2)e^{2Q}\cos(2Q) \\
& - \frac{1}{8}Q(C^2 - 2CT - D^2)e^{-2Q}\cos(2Q) \\
& + \frac{1}{4}Q(A^2 + B^2)e^{2Q} - \frac{1}{4}Q(C^2 + T^2)e^{-2Q} \\
& - \frac{1}{8}Q(A^2 - 2AB - B^2)e^{2Q}\sin(2Q) \\
& - \frac{1}{8}Q(C^2 + 2CT - T^2)e^{-2Q}\sin(2Q) \\
& - \frac{1}{2}Q(AC - BT)\sin(2Q) + \frac{1}{2}Q(AT + BC)\cos(2Q)
\end{aligned}
\qquad \text{.....................(5 - 36)}
$$

Boundary Condition 7.

$S_2 = 0. \quad , \quad x = -1.$

$$
\begin{aligned}
& -N + G \\
= & \frac{1}{8}Q(A^2 + 2AB - B^2)e^{2Q}\cos(2Q) \\
& - \frac{1}{8}Q(C^2 - 2CT - T^2)e^{-2Q}\cos(2Q) \\
& + \frac{1}{4}Q(A^2 + B^2)e^{2Q} - \frac{1}{4}Q(C^2 + T^2)e^{-2Q} \\
& + \frac{1}{8}Q(A^2 - 2AB - B^2)e^{2Q}\sin(2Q) \\
& + \frac{1}{8}Q(C^2 + 2CT - T^2)e^{-2Q}\sin(2Q) \\
& - \frac{1}{2}Q(AC - BT)\sin(2Q) + \frac{1}{2}Q(AT + BC)\cos(2Q)
\end{aligned}
\qquad \text{.....................(5 - 37)}
$$

Boundary Condition 8.

$w_{3,x} = 0. \quad , \quad x = +1.$

$$e^{Q}(\cos Q - \sin Q)H + e^{Q}(\cos Q + \sin Q)I$$

$$- e^{-Q}(\cos Q + \sin Q)J + e^{-Q}(\cos Q - \sin Q)L$$

$$= - \frac{1}{8}Y[e^{Q}(A\cos Q + B\sin Q) + e^{-Q}(C\cos Q + T\sin Q)]$$

$$- \frac{1}{16Q}Y\left\{e^{Q}[(B + A)\sin Q - (B - A)\cos Q]\right.$$

$$\left. + e^{-Q}[(C - T)\sin Q - (C + T)\cos Q]\right\}$$

......................(5 - 38)

Boundary Condition 9.

$w_{3,x} = 0.$, $x = -1.$

$$e^{-Q}(\cos Q + \sin Q)H + e^{-Q}(\cos Q - \sin Q)I$$

$$- e^{Q}(\cos Q - \sin Q)J + e^{Q}(\cos Q + \sin Q)L$$

$$= \frac{1}{8}Y[e^{-Q}(A\cos Q - B\sin Q) + e^{Q}(C\cos Q - T\sin Q)]$$

$$+ \frac{1}{16Q}Y[e^{-Q}(B + A)\sin Q + e^{-Q}(B - A)\cos Q$$

$$+ e^{Q}(C - T)\sin Q + e^{Q}(C + T)\cos Q].$$

......................(5 - 39)

Boundary Condition 10.

$w_3 = 0.$, $x = +1.$

$$e^{Q}H\cos Q + e^{Q}I\sin Q + e^{-Q}J\cos Q + e^{-Q}L\sin Q + \frac{\gamma_3}{K}$$

$$= - \frac{1}{16} Y \left\{ e^{Q}[(B + A)\sin Q - (B - A)\cos Q] \right.$$

$$\left. + e^{-Q}[(C - T)\sin Q - (C + T)\cos Q] \right\}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5 - 40)$$

Boundary Condition 11.

$$w_3 = 0. \quad , \quad x = -1.$$

$$e^{-Q}H\cos Q - e^{-Q}I\sin Q + e^{Q}J\cos Q - e^{Q}L\sin Q + \frac{\gamma_3}{K}$$

$$= - \frac{1}{16} Y \left\{ e^{-Q}[(B + A)\sin Q - (B - A)\cos Q] \right.$$

$$+ e^{Q}[(C - T)\sin Q - (C + T)\cos Q]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5 - 41)$$

Boundary Condition 12.

$$w_3 = 0. \quad , \quad x = 0.$$

$$H + J + \frac{\gamma_3}{K} = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5 - 42)$$

(5) Computer Analysis

Equations (5 - 31) through (5 - 42) are written in matrix form and designated as the [A] matrix, and [C] matrix with the following elements

$$A(1,1) = e^{Q}(\cos Q - \sin Q)$$

$$A(1,2) = e^{Q}(\cos Q + \sin Q)$$

$$A(1,3) = -e^{-Q}(\cos Q + \sin Q)$$

$$A(1,4) = e^{-Q}(\cos Q - \sin Q)$$

$$A(2,1) = e^{-Q}(\cos Q + \sin Q)$$

$$A(2,2) = e^{-Q}(\cos Q - \sin Q)$$

$$A(2,3) = -e^{Q}(\cos Q - \sin Q)$$

$$A(2,4) = e^{Q}(\cos Q + \sin Q)$$

$$A(3,1) = e^{Q}\cos Q$$

$$A(3,2) = e^{Q}\sin Q$$

$$A(3,3) = e^{-Q}\cos Q$$

$$A(3,4) = e^{-Q}\sin Q$$

$$A(3,5) = 1 / K$$

$$A(4,1) = e^{-Q}\cos Q$$

$$A(4,2) = -e^{-Q}\sin Q$$

$$A(4,3) = e^{Q}\cos Q$$

$$A(4,4) = -e^{Q}\sin Q$$

$$A(4,5) = 1 / K$$

$$A(5,1) = 1.$$

$$A(5,2) = 0.$$

$$A(5,3) = 1.$$

$$A(5,4) = 0.$$

$$A(5,5) = 1 / K$$

$$A(6,6) = 1.$$

$$A(6,7) = 1.$$

$$A(7,6) = -1.$$

$$A(7,7) = 1.$$

$$A(8,8) = e^{Q}(\cos Q - \sin Q)$$

$$A(8,9) = e^{Q}(\cos Q + \sin Q)$$

$$A(8,10) = -e^{-Q}(\cos Q + \sin Q)$$

$$A(8,11) = e^{-Q}(\cos Q - \sin Q)$$

$$A(9,8) = e^{-Q}(\cos Q + \sin Q)$$

$$A(9,9) = e^{-Q}(\cos Q - \sin Q)$$

$$A(9,10) = -e^{Q}(\cos Q - \sin Q)$$

$$\bar{A}(9,11) = e^{Q}(\cos Q + \sin Q)$$

$$\bar{A}(10,8) = e^{Q}\cos Q$$

$$\bar{A}(10,9) = e^{Q}\sin Q$$

$$\bar{A}(10,10) = e^{-Q}\cos Q$$

$$\bar{A}(10,11) = e^{-Q}\sin Q$$

$$\bar{A}(10,12) = 1 / K$$

$$\bar{A}(11,8) = e^{-Q}\cos Q$$

$$\bar{A}(11,9) = -e^{-Q}\sin Q$$

$$\bar{A}(11,10) = e^{Q}\cos Q$$

$$\bar{A}(11,11) = e^{Q}\sin Q$$

$$\bar{A}(11,12) = 1 / K$$

$$\bar{A}(12,8) = 1.$$

$$\bar{A}(12,9) = 0.$$

$$\bar{A}(12,10) = 1.$$

$$\bar{A}(12,11) = 0.$$

$$\bar{A}(12,12) = 1 / K$$

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Note : A(1,5) through A(1,12), A(2,5) through A(2,12), A(3,6) through A(3,12), A(4,6) through A(4,12), A(5,6) through A(5,12), A(6,1) through A(6,5), A(6,8) through A(6,12), A(7,1) through A(7,5), A(7,8) through A(7,12), A(8,1) through A(8,7), A(9,1) through A(9,7), A(10,1) through A(10,7), A(11,1) through A(11,7), A(12,1) through A(12,7) are all zero.

$$C(1) = 0.$$

$$C(2) = 0.$$

$$C(3) = 0.$$

$$C(4) = 0.$$

$$C(5) = 1.$$

$$\begin{aligned} C(6) = {} & (1/8)Q(A^2+2AB-B^2)e^{2Q}\cos(2Q) \\ & - (1/8)Q(C^2-2CT-T^2)e^{-2Q}\cos(2Q) \\ & + (1/4)Q(A^2+B^2)e^{2Q} - (1/4)Q(C^2+T^2)e^{-2Q} \\ & - (1/8)Q(A^2-2AB-B^2)e^{2Q}\sin(2Q) \\ & - (1/8)Q(C^2+2CT-T^2)e^{-2Q}\sin(2Q) \\ & - (1/2)Q(AC-BT)\sin(2Q) \\ & + (1/2)Q(AT+BC)\cos(2Q) \end{aligned}$$

$$\begin{aligned} C(7) = {} & (1/8)Q(A^2+2AB-B^2)e^{-2Q}\cos(2Q) \\ & - (1/8)Q(C^2-2CT-T^2)e^{2Q}\cos(2Q) \\ & + (1/4)Q(A^2+B^2)e^{-2Q} - (1/4)Q(C^2+T^2)e^{2Q} \\ & + (1/8)Q(A^2-2AB-B^2)e^{-2Q}\sin(2Q) \end{aligned}$$

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$$+ \ (1/8)Q(C^2+2CT-T^2)e^{2Q}\sin(2Q)$$

$$+ \ (1/2)Q(AC-BT)\sin(2Q)$$

$$+ \ (1/2)Q(AT+BC)\cos(2Q)$$

$$C(8) = -(1/8)Y[e^{Q}(A\cos Q+B\sin Q)+e^{-Q}(C\cos Q+T\sin Q)]$$
$$-(1/16)Y(1/Q) \ e^{Q}[(B+A)\sin Q-(B-A)\cos Q]$$
$$+e^{-Q}[(C-T)\sin Q-(C+T)\cos Q]$$

$$C(9) = (1/8)Y[e^{-Q}(A\cos Q+B\sin Q)-e^{Q}(C\cos Q-T\sin Q)]$$
$$+(1/16)Y(1/Q) \ e^{-Q}[(B+A)\sin Q-(B-A)\cos Q]$$
$$+e^{Q}[(C-T)\sin Q-(C+T)\cos Q]$$

$$C(10) = -(1/16)Y \ e^{Q}[(B+A)\sin Q-(B-A)\cos Q]$$
$$-e^{-Q}[(C-T)\sin Q-(C+T)\cos Q \ ]$$

$$C(11) = -(1/16)Y \ e^{-Q}[(B-A)\sin Q+(B-A)\cos Q]$$
$$+e^{Q}[(C-T)\sin Q-(C+T)\cos Q \ ]$$

$$C(12) = 0.$$

(36) Answer : Substituting the values of $A,B,C,T,\gamma_1,N,G,H,I,J,L,\gamma_3$ back into Equations (5- 9),(5 - 15) and (5 - 28), we get (for K = 20.)

$$w_1 = -0.49e^{1.49x}\cos(1.49x)$$
$$-0.42e^{1.49x}\sin(1.49x)$$

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$$-0.49e^{-1.49x}\cos(1.49x) + 0.42e^{-1.49x}\sin(1.49x)$$
$$+1.98$$

$$\begin{aligned} S_2 = {} & -0.08e^{2.98x}\cos(2.98x) + 0.08e^{-2.98x}\cos(2.98x) \\ & -0.15e^{2.98x} + 0.15e^{-2.98x} - 0.06e^{2.98x}\sin(2.98x) \\ & -0.06e^{2.98x}\sin(2.98x) + 0.31\sin(2.98x) + 1.54x \end{aligned}$$

$$\begin{aligned} w_3 = {} & -0.44e^{1.49x}\cos(1.49x) + 0.45e^{1.49x}\sin(1.49x) \\ & -0.44e^{-1.49x}\cos(1.49x) - 0.45e^{-1.49x}\sin(1.49x) \\ & -0.56xe^{1.49x}\sin(1.49x) - 0.03xe^{1.49x}\cos(1.49x) \\ & -0.56xe^{-1.49x}\sin(1.49x) - 0.03xe^{-1.49x}\cos(1.49x) \\ & -0.03xe^{-1.49x}\cos(1.49x) + 0.89 \end{aligned}$$

(7) For the same loading and boundary conditions but without elastic foundation

This is the limiting case where the foundation modulus K is reduced to zero. Substitute K = 0. into all the above equations and follow the same proceduce, w_1, S_2, and w_3 for plates without elastic foundations are obtained as follows :

$$w_1 = x^4 - 2x^2 + 1$$
$$S_2 = -1.14x^7 + 3.2x^5 - 2.62x^3 + 0.61x$$
$$w_3 = 0.24x^6 - 0.48x^4 + 0.24x^2$$

5.2. Applications

Once the three functions w_1, S_2 and w_3 (and the constants γ_1 and γ_3) have been determined, they are most conveniently used when they are substituted into the original power series expansions i.e.,

$$\frac{ua}{h^2} = S_2\left(\frac{w_o}{h}\right)^2 + \ \cdots\cdots \qquad (5-49)$$

$$\frac{w}{h} = w_1\left(\frac{w_o}{h}\right) + w_3\left(\frac{w_o}{h}\right)^3 + \cdots \qquad (5-50)$$

$$\frac{q_o a^4}{Dh} = \gamma_1\left(\frac{w_o}{h}\right) + \gamma_3\left(\frac{w_o}{h}\right)^3 + \cdots \qquad (5-51)$$

Where q_o represents the load intensity at the origin of the coordinate system. Equation (5 - 51) is the nonlinear load deflection relation which characterize the large deflection problem. Each term in the right hand side of this equation may be thought of as a resistance to deflection on the z direction. The first term represents flexural resistance ; the second term represents the restraining action due to the membrane effect. For small deflections, the membrane term is negligible, for large enough deflections, this term is predominant.

While a plot of the load - deflection relation is

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useful in that it shows at what point the nonlinear theory deviates from the classical theory, it is normally the stress that governs the design of a load carrying member. For" long " rectangular plates, the shear stress vanishes everywhere and the dimensionless membrane and bending stress components are obtained from

$$\sigma'_x = \left(\frac{w_o}{h} \right)^2 \left[S_{2,x} + \frac{1}{2} (w_{1,x})^2 \right] \quad \ldots\ldots (5 - 52)$$

$$\sigma''_x = \frac{1}{2} \left\{ \left(\frac{w_o}{h} \right) w_{1,xx} + \left(\frac{w_o}{h} \right)^3 w_{3,xx} \right\} \ldots\ldots (5 - 53)$$

While the stresses in the Y direction are related to these above through the Poisson's ratio.

The membrane stress, always tensile, is a constant for any value of (w_o / h). This fact is most easily seen by noting that in Equation (5 - 52), the quantity inside the square brackets represents a first integral of

$$S_{2,xx} + w_{1,x} w_{1,xx}$$

which from Equ.(4-34) is equal to zero. Thus the quantity inside the square brackets in Equ.(5-52) is independent of coordinate x.

Also, this first integral is a measure of the strain of the middle surface of the " long " rectangular plate ;

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the larger this first integral is , the more the plate is stretched.

The bending stress, as expected, depends on the coordinate x. The largest value of the bending stress may be determined by the usual procedures governing maxima and minima of functions of a single variable.

The total stress, the sum of the membrane and bending stresses will attain its maximum value on the convex side of the plate when the bending stress is tensile.

5.3. Typical Numerical Example

Consider a long rectangular steel plate 50 in. wide and half in. thick having the material properties $\nu = 0.3$ and E = 30,000,000 p.s.i. A total load of 500 lb/in corresponds to q_o = 10 p.s.i. for both the uniform and hydrostatic case.* For the nondimensionlized loading,

$$\frac{q_o a^4}{Dh} = \frac{10(25)^4(12)(1 - 3^2)}{30(10)^6(1/2)^4} = 22.75$$

From the curves, first read the dimensionless central deflection w_o/h and the dimensionless total stress S_x and then computes the actural total stress σ_x. The results are

* For boundary conditions, see the table on page 79.

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summarized as follows (with the stress magnitudes obtained from the linear theory appearing in parathese)

	$\frac{w_o}{h}$	S_x	σ_x p.s.i.
Case 1 F = 1 Clamped both sides	0.69	3.39	44,700 (50,000)
Case 2 F = 1 Simply supported both sides	1.07	1.85	24,400 (75,000)
Case 3 F = 1 Clamped at x = +1 & Simply supported at x = − 1	0.86	3.99	52,600 (75,000)
Case 4 F = 1 + x Clamped both sides	0.69	4.07	53,600 (60,000)
Case 5 F = 1 + x Simply supported both sides	1.07	2.11	27,800 (77,000)

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Here, use has been made of the relation

$$\sigma_x = \frac{Eh^2 s_x}{a^2(1 - \nu^2)} = \frac{30(10)^6(1/2)^2 s_x}{(25)^2(1 - 0.3^2)}$$

The same numerical example has been worked by Timoshenko and Woinowsky-Krieger (p. 16) for case 1 (the plate with both sides clamped subjected to a uniform load). The value of the maximum stress obtained from the solution of the perturbed equations (44,700 p.s.i.) is within 1 % of the value reported by these authors (45,000 p.s.i.).

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CHAPTER 6

RESULTS AND COMPARISON OF RESULTS

The problem of small and large deflections and total stresses of long rectangular plates with or without elastic support has been solved by using the small parameter perturbation technique.

All necessary computational work was programmed in Fortran IV for the IBM 360/65 . To guard against round-off errors, double precision arithmatic was used in all computations. A typical computer program coded in Fortran IV is given in the Appendix C.

Numerical results for maximum deflections and stresses are presented in the form of tables for the linear analysis. Such results, wherever possible are compared with available data in the technical literature.

6.1. Linear Analysis —— Deflections and Stresses

For clamped both sides, simply supported both sides and clamped one side, simply supported the other side long

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rectangular plates with uniformly distributed loads ($F = 1$), hydrostatic loads ($F = 1 + x$), parabolic loads ($F = 1 + x^2$) and cosine loads $F = \cos(\pi/2)x$ respectively, with or without the elastic foundations, the coefficients of the maximum deflection are tabulated in Table 6 - 1. and the dimensionless stress coefficients for center and maximum points are tabulated in Table 6 - 2.

Discussion and comparison of results

From Table 6 - 1., it can be seen that the influence of elastic support in reducing the magnitude of the maximum deflection of the long rectangular plate decreases as the foundation modulus increases. Furthermore, the results also show that the elastic support is more effective in reducing the maximum deflection for long rectangular plates simply supported both sides case than in the other two boundary conditions (clamped both sides, clamped one side and simply supported the other side), when the dimensionless elastic foundation modulus increases, for all loading conditions considered. For example, for simply supported long rectangular plates, when the foundation modulus K changes from 0. to 20., with parabolic load ($F = 1 + x^2$) the reduction in the

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maximum deflection is 81.2 %, whereas the corresponding reduction in the maximum deflection with the same loading condition, but clamped at both sides is only 39.6 %.

It can also be observed that, the location of the maximum deflection points obtained from the small deflection analysis is always at the center (x = 0.) of the plate, for plates clamped on both sides and simply supported on both sides with uniformly distributed, parabolic and cosine loadings. The reason is because the uniformly distributed loading, parabolic loading and cosine loadings are all symmetrical with respect to the Y axis. With the same boundary conditions but with hydrostatic loadings, the location of points of maximum deflection are displaced towards x = + 1 side when foundation modulus increases. In the case of long plates clamped at one side and simply supported on the other side, the maximum deflection points are not at the center when K is equal to zero and they shift toward x = + 1 side when the dimensionless foundation modulus K increases for all the four kinds of loading conditions considered. For example, for clamped (x = + 1) and simply supported (x = - 1) long rectangular plates

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with hydrostatic loadings the maximum deflection point is at $x = -0.11$ when the dimensionless foundation modulus $K = 0.$ and the maximum deflection point is at $x = +0.13$ when the dimensionless foundation modulus $K = 100.$

For long rectangular plates clamped on both sides or simply supported on both sides subjected to a uniformly distributed load, it can be seen that results for maximum deflections in Table 6 - 1. are in exact agreement with those obtained by Timoshenko [2] using a series solution.

It can be observed from Table 6 - 2. that, the influence of elastic support in reducing the magnitude of the bending stresses of long rectangular plates decreases as the foundation modulus increases. Furthermore, the results also show that the elastic support is more effective in reducing the bending stresses at the center than the maximum bending stresses. This is to be expected since the elastic support reaction is proportional to the lateral deflection and hence most effective at the plate center. For instance, for the clamped long rectangular plate, when the dimensionless foundation modulus K changes from 0. to 100. with parabolic load ($F = 1 + x^2$), the reduction in the center bending

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stress is 84 %, whereas the corresponding reduction in the maximum bending stress is 65.4 %. But there is an exception in simply supported both sides case with cosine loading because the maximum bending stress is at the center.

For the clamped both sides and simply supported both sides long rectangular thin plate with uniformly distributed load, results for maximum stresses in Table 6 - 2. are in exact agreement with those obtained by Timoshenko [2] using a series solution.

6.2. Nonlinear Analysis —— Deflection and Total Stresses

For long rectangular plates clamped at both sides and with uniformly distributed loads, hydrostatic loads, parabolic loads and cosine loads with and without the elastic foundation modulus, the variation of the dimensionless maximum deflection with the dimensionless load are plotted in Figure 6 - 1., Figure 6 - 2., Figure 6 - 3., Figure 6 - 4. respectively. Also the variation of the maximum total dimensionless stresses produced by the effect of both bending and stretching of the plate with the dimensionless load are plotted in Figure 6 - 5., Figure 6 - 6., Figure 6 - 7.

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and Figure 6 - 8. respectively.

For simply supported both sides long rectangular thin plate with uniformly distributed load, hydrostatic load, parabolic load and cosine load with and without the elastic foundation modulus, the variation of the dimensionless maximum deflection with the dimensionless load are plotted in Figure 6 - 9., Figure 6 - 10., Figure 6 - 11. and Figure 6 - 12. respectively. Also the variation of the maximum total dimensionless stresses produced by the effect of both bending and stretching of the plate with the dimensionless load are plotted in Figure 6 - 13., Figure 6 - 14., Figure 6 - 15., and Figure 6 - 16. respectively.

For clamped at one side and simply supported at the other side long rectangular plate with uniformly distributed load, hydrostatic load, parabolic load and cosine load with and without the elastic foundation support, the variation of the dimensionless maximum deflection with the dimensionless load are plotted in Figure 6 - 17., Figure 6 - 18., Figure 6 - 19. and Figure 6 - 20. respectively. Also the variation of the maximum total dimensionless stresses produced by the

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effect of both bending and stretching of the plate with the dimensionless load are plotted in Figure 6 - 21., Figure 6 - 22., Figure 6 - 23. and Figure 6 - 24. respectively.

Discussion and comparison of results.

From the deflection against load curves (Figure 6 - 1. through Figure 6 - 4., Figure 6 - 9. through Figure 6 - 12., and Figure 6 - 17. through Figure 6 - 20.), it can be seen that the maximum deflection starts to deviate notably from linear theory, when the dimensionless deflection exceeds 0.25 . This means that the elementary linear plate theory would not be sufficiently accurate once the ratio of the maximum deflection to the thickness of the plate exceeds this value. For a simply supported both sides long rectangular plate with uniformly distributed load and with the different values of the dimensionless elastic foundation modulus, K = 0., K = 20., K = 40. and K = 80. Comparison is made with the results obtained by S. N. Sinha [12]. Curves plotted in Figure 6 - 9. show very good agreement.

For a given loading and boundary conditions, the maximum deflection decreases as the foundation modulus

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increases. This is to be expected since the effect of the elastic support is to reduce the lateral pressure on the plate. Also, as the foundation modulus increases, the influence of the nonlinear term for deflection decreases. Hence the large deflection curves tend to become relatively more linear as the values of the foundation modulus become higher.

No data in the literature are as yet available for stresses of long rectangular plates with the same loading and boundary concitions and hence comparison is not possible for stress values.

From the total stress against load curves (Figure 6 - 5. through Figure 6 - 8., Figure 6 - 21. through Figure 6 - 24.), for all plates clamped on both sides, one side clamped and the other side simply supported, are for all loading conditions and foundation moduli considered, the maximum total stress occurs at the clamped side acting in the $\bar{x}$ direction (Figure 3 - 3.). This stress being much larger than the total stress at the center of the plate would therefore govern the relevant design requirements.

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On the other hand, in simply supported both sides case, the maximum total stress points do not occur at the same points; for uniform distributed load (Figure 6 - 13) they change from x = 0. to x = $\pm$ 0.65 , when K incresed from 0. to 100., for hydrostatic loading (Figure 6 - 14), they change from x = + 0.14 to x = + 0.31 , when K incresed from 0. to 100., for parabolic loading (Figure 6 - 15), they change from x = $\pm$ 0.43 to x = $\pm$ 0.70 , when K incresed from 0. to 100., for cosine loading they always occur at the center of the plate.

It may be noted that the curves representing the total stress in clamped both sides, simply supported both sides and simply supported one side clamped the other side are essentially linear until the dimensionless total stress ratio reaches about 3.0 , 1.20 and 3.0 respectively. Beyond such ratios, the relationships become noticeably non-linear; thus the range of applicability of the small deflection theory can be readily predicted from such graphs.

The effect of the foundation modulus on the bending stresses is significant, especially in reducing the bending

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stress in simply supported both sides case.

6.3. Conclusions

From the analysis of long rectangular plates with or without elastic support by the successive approximations method, the following conclusions may be drawn :

(1) For clamped both sides and simply supported both sides long rectangular plates with no elastic support, the perturbation method yields results which closely agree with existing values by other investigators.

(2) Maximum deflection of the long rectangular plate decreases with an increases in the foundation modulus.

(3) The elastic support is more effective in reducing the maximum deflection in simply supported both sides case than in the other two boundary conditions considered viz., —— clamped both sides case and clamped at $x = +1$, simply supported at $x = -1$ case.

(4) When the loading function are symmetrically placed with respect to the axis of the long rectangular plate, the maximum deflection points from both the small and large

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deflection analysis are always on the axis of the long rectangular plate.

(5) When the loading function are not symmetrically placed with respect to the axis of the long rectangular plate, the maximum deflection points always shift to x = + 1 side when the foundation modulus K increases.

(6) The maximum total stress of the long rectangular plate decreases with an increases in the foundation modulus.

(7) The large deflection of long rectangular plates becomes increasing linear as foundation modulus increases.

(8) The total stress for long rectangular plates occurs at x = + 1 side when the boundary conditions are clamped both sides and clamped at x = + 1 , simply supported at x = -1 .

(9) The total stress of long rectangular plates becomes increasing linear as foundation modulus increases.

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Table 6-1. Coefficient for maximum deflection for long rectangular plate from small deflection analysis

$$W_{max} = \omega \frac{qa^4}{D} (10^{-2})$$

Boundary Conditions	Loading Conditions	Uniformly Loading F = 1	Hydrostatic Loading F=1+x	Parabolic Loading $F=1+x^2$	Cosine Loading $F=\cos(\pi/2)x$
Clamped both sides	K = 0.	4.16667* x = 0.	4.18733 x=0.05	4.72222 x = 0.	3.52490 x = 0.
	K = 20.	2.52303 x = 0.	2.55250 x=0.08	2.85025 x = 0.	2.14473 x = 0.
	K = 40.	1.80229 X = 0.	1.83837 x=0.10	2.02968 x = 0.	1.53917 x = 0.
	K = 60.	1.39782 x = 0.	1.43875 x=0.12	1.56941 x = 0.	1.19908 x = 0.
	K = 80.	1.13908 x = 0.	1.18361 x=0.14	1.27514 x = 0.	0.98133 x = 0.
	K = 100.	0.95944 x = 0.	1.00666 x=0.16	1.07097 x = 0.	0.830013 x = 0.
Simply supported both sides	K = 0.	20.8333** x = 0.	20.8710 x=0.04	24.7222 x = 0.	16.4254 x = 0.
	K = 20.	4.80351 x = 0.	4.91798 x=0.14	5.65825 x = 0.	3.83317 x = 0.
	K = 40.	2.68866 x = 0.	2.83685 x=0.21	3.14499 x = 0.	2.16976 x = 0.
	K = 60.	1.85546 x = 0.	2.01704 x=0.25	2.15601 x = 0.	1.53313 x = 0.

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Boundary Conditions	Loading Conditions	Uniformly Loading $F = 1$	Hydrostatic Loading $F=1+x$	Parabolic Loading $F=1+x^2$	Cosine Loading $F=\cos(\pi/2)x$
	K = 80.	1.41052 x = 0.	1.57588 x=0.29	1.62869 x = 0.	1.16160 X ≑ 0.
	K =100.	1.13416 x = 0.	1.29863 x=0.31	1.30176 x = 0.	0.942613 x = 0.
Clamped ((x=+1.)& simply supported ((x=-1)	K = 0.	8.66566 x=-0.16	7.63224 x=-0.11	10.1418 x=-0.16	6.98673 x=-0.15
	K = 20.	3.65684 x=-0.15	3.24812 x=-0.04	4.26561 x=-0.17	2.96833 x=-0.13
	K = 40.	2.30219 x=-0.15	2.07910 x= 0.02	2.67668 x=-0.18	1.88222 x=-0.11
	K = 60.	1.67250 x=-0.15	1.54102 x= 0.06	1.93866 x=-0.19	1.37738 x=-0.10
	K = 80.	1.30916 x=-0.14	1.23170 x= 0.10	1.51331 x=-0.20	1.08586 x=-0.09
	K =100.	1.07291 x=-0.14	1.03034 x= 0.13	1.23723 x=-0.21	0.896093 x=-0.08

* Timoshenko 4.16

** Timoshenko 20.832[illegible]

K Dimensionless Foundation Modulus

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Table 6-2. Dimensionless stress coefficient c and m for long rectangular plate from small deflection analysis

σ''_x at center $= c \frac{qa^2}{h^2}(10^{-2})$ $\quad\quad$ $\sigma''_{x(max)} = m \frac{qa^2}{h^2}(10^{-2})$

Boundary Conditions	Loading Conditions	Uniformly Loading F = 1		Hydrostatic Loading F=1+x		Parabolic Loading $F=1+x^2$		Cosine Loading $F=\cos(\pi/2)x$	
		c	m	c	m	c	m	c	m
Clamped both sides	K = 0.	100. x = 0.	-200.* x = -1	100. x = 0.	-240. x = -1	110. x = 0.	-240. x = -1	88.3635 x = 0.	-154.807 x = -1
	K = 20.	57.9588 x = 0.	-130.161 x = -1	57.9588 x = 0.	-167.814 x = -1	62.6075 x = 0.	-160.27 x = -1	52.9608 x = 0.	-96.3692 x = -1
	K = 40.	39.6176 x = 0.	- 99.3408 x = -1	39.6176 x = 0.	-134.979 x = -1	41.4110 x = 0.	-125.001 x = -1	37.4555 x = 0.	- 70.6745 x = -1
	K = 60.	29.3933 x = 0.	- 81.9016 x = -1	29.3933 x = 0.	-115.791 x = -1	29.8580 x = 0.	-104.981 x = -1	28.7677 x = 0.	- 56.2037 x = -1
	K = 80.	22.9056 x = 0.	- 70.6347 x = -1	22.9056 x = 0.	-102.988 x = -1	22.5578 x = 0.	- 92.000 x = -1	23.2206 x = 0.	- 46.9072 x = -1
	K = 100.	18.4437 x = 0.	- 62.7241 x = -1	18.4437 x = 0.	- 93.719 x = -1	17.5618 x = 0.	- 82.8478 x = -1	19.3780 x = 0.	- 40.4217 x = -1

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Boundary Conditions	Loading Conditions	Uniformly Loading F = 1		Hydrostatic Loading F=1+x		Parabolic Loading $F=1+x^2$		Cosine Loading $F=\cos(\pi/2)x$	
		c	m	c	m	c	m	c	m
Simply supported both sides	K = 0.	300.** x = 0.	300. x = 0.	300. x = 0.	307.913 x= 0.15	350. x = 0.	350. x = 0.	243.170 x = 0.	243.170 x = 0.
	K = 20.	63.0738 x = 0.	63.0738 x = 0.	63.0741 x = 0.	87.9824 x= 0.47	68.5057 x = 0.	72.1349 x=±0.43	56.7479 x = 0.	56.7479 x = 0.
	K = 40.	32.1209 x = 0.	34.4931 x= 0.46	32.1209 x = 0.	61.6642 x= 0.56	31.9780 x = 0.	46.0949 x=±0.60	32.1221 x = 0.	32.1221 x = 0.
	K = 60.	20.1157 x = 0.	25.6744 x= 0.57	20.1158 x = 0.	49.8707 x= 0.60	17.9060 x = 0.	37.2409 x=±0.65	22.4011 x = 0.	22.4011 x = 0.
	K = 80.	13.8348 x = 0.	21.3926 x= 0.62	13.8348 x = 0.	43.1949 x=-0.63	10.7433 x = 0.	32.6181 x=±0.68	17.1969 x = 0.	17.1969 x = 0.
	K = 100.	10.0283 x = 0.	18.8138 x= 0.65	10.0283 x = 0.	38.6525 x=-0.65	6.4465 x = 0.	29.6694 x=±0.70	13.9599 x = 0.	13.9599 x = 0.

* Timoshenko -199.92

** Timoshenko 300.

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Boundary Conditions	Loading Conditions	Uniformly Loading $F = 1$		Hydrostatic Loading $F=1+x$		Parabolic Loading $F=1+x^2$		Cosine Loading $F=\cos(\pi/2)x$	
		c	m	c	m	c	m	c	m
Clamped (x= 1.)& simply supported (x=-1.)	K = 0.	150. x = 0.	-300. x = 1.	140. x = 0.	-320. x = 1.	170. x = 0	-360.001 x = 1.	127.064 x = 0.	-232.210 x = 1.
	K = 20.	60.2245 x = 0.	-145.01 x = 1.	59.5691 x = 0.	-178.368 x = 1.	64.9971 x = 0.	-178.555 x = 1.	54.6385 x = 0.	-107.364 x = 1.
	K = 40.	35.9786 x = 0.	-102.241 x = 1.	37.2841 x = 0.	-136.840 x = 1.	36.8323 x = 0.	-128.650 x = 1.	34.8666 x = 0.	- 72.737 x = 1.
	K = 60.	24.7521 x = 0.	- 81.663 x = 1.	26.6726 x = 0.	-115.766 x = 1.	23.9089 x = 0.	-104.927 x = 1.	25.5827 x = 0.	- 56.174 x = 1.
	K = 80.	18.3145 x = 0.	- 69.7672 x = 1.	20.4174 x = 0.	-102.518 x = 1.	16.5781 x = 0.	- 90.870 x = 1.	20.1718 x = 0.	- 46.331 x = 1.
	K = 100.	14.1654 x = 0.	- 61.6724 x = 1.	16.2795 x = 0.	- 93.1867 x = 1.	11.9110 x = 0.	- 81.459 x = 1.	16.6210 x = 0.	- 39.744 x = 1.

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FIGURE 6-1
UNIFORM LOADING (F=1)
CLAMPED BOTH SIDES
-------- LINEAR THEORY
———— NONLINEAR THEORY

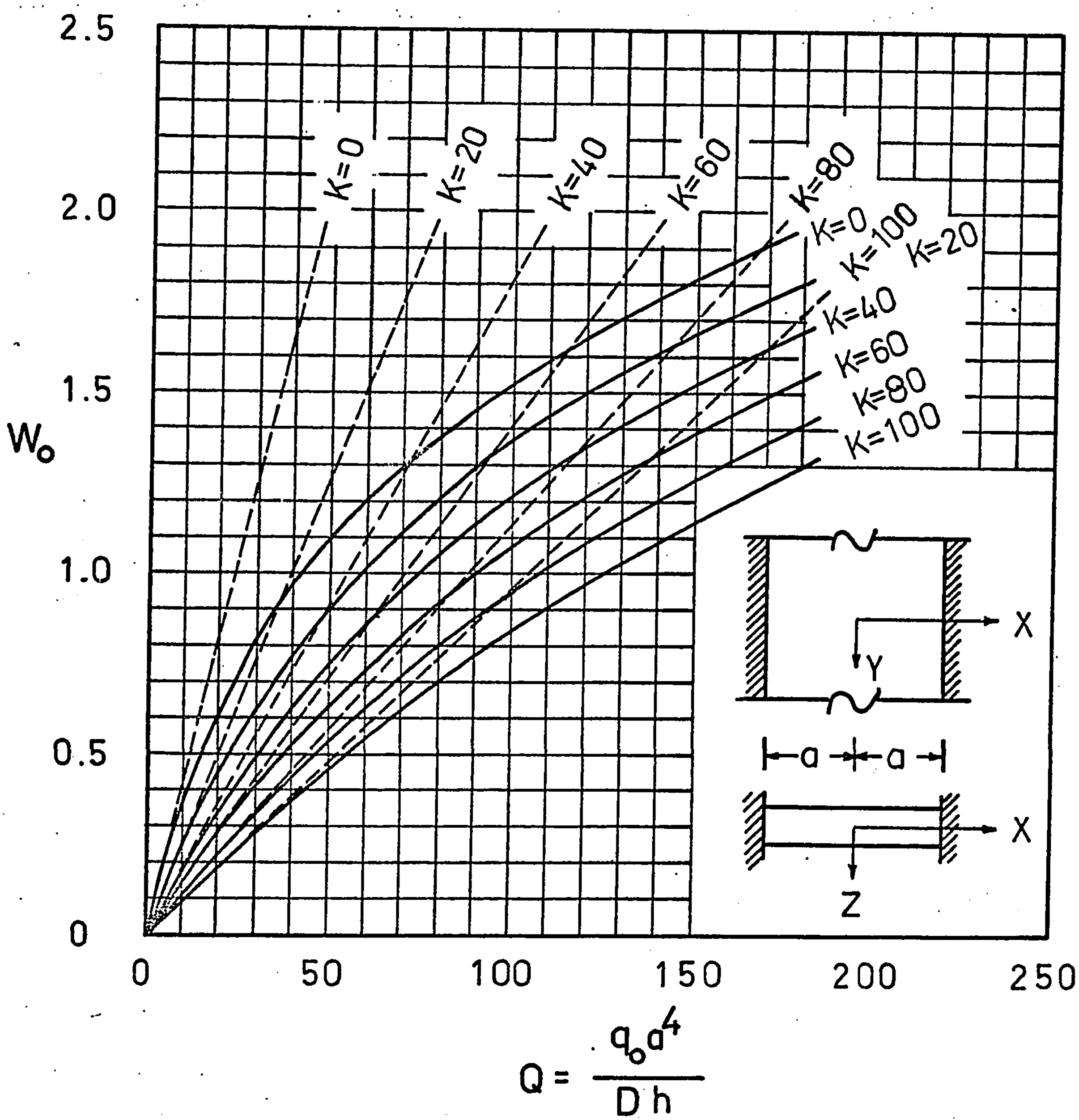

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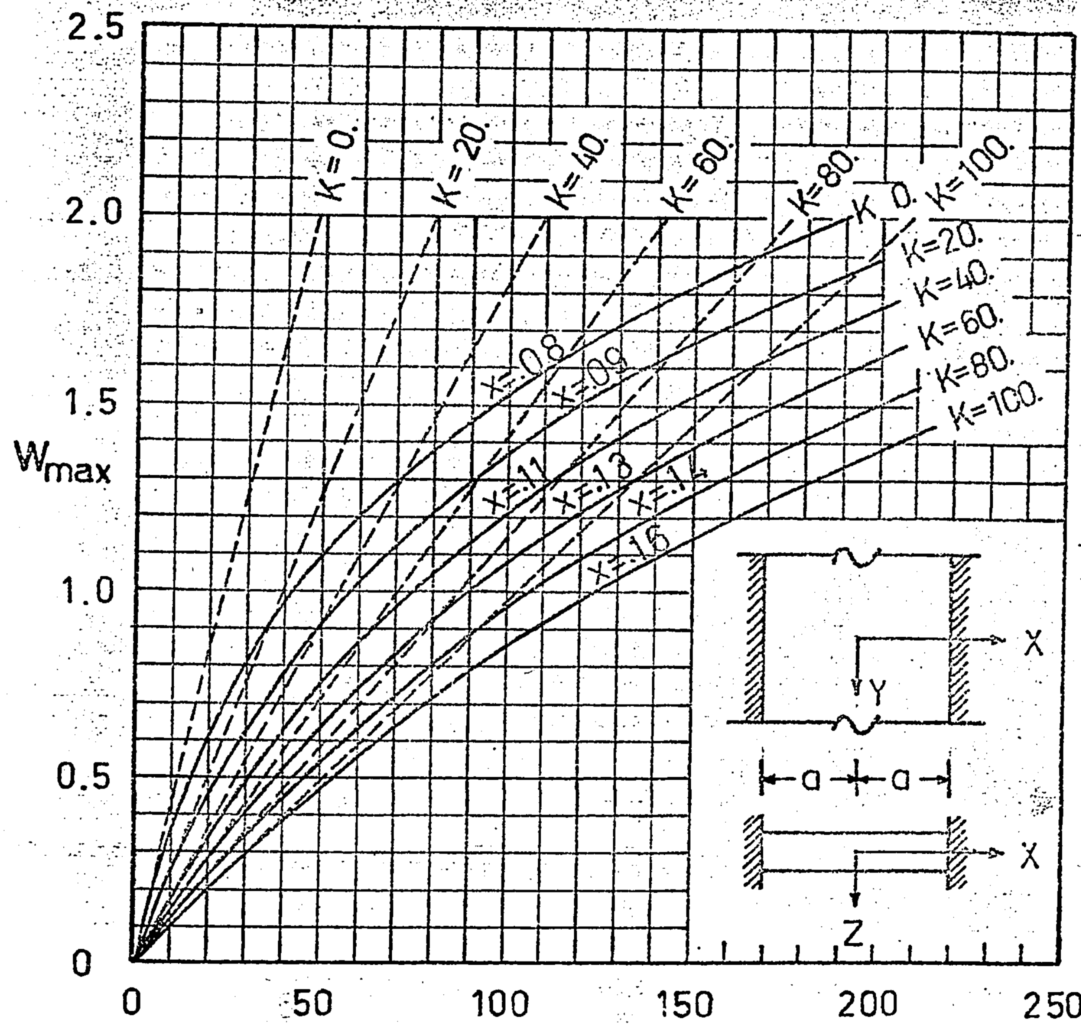
FIGURE 6-2
HYDROSTATIC LOADING(F=1+X)
CLAMPED BOTH SIDES
-------- LINEAR THEORY
———— NONLINEAR THEORY
x max. deflection point



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6-3

PARABOLIC LOADING (F 1 + X^2)

CLAMPED BOTH SIDES

---------- LINEAR THEORY

———— NONLIAR THEORY

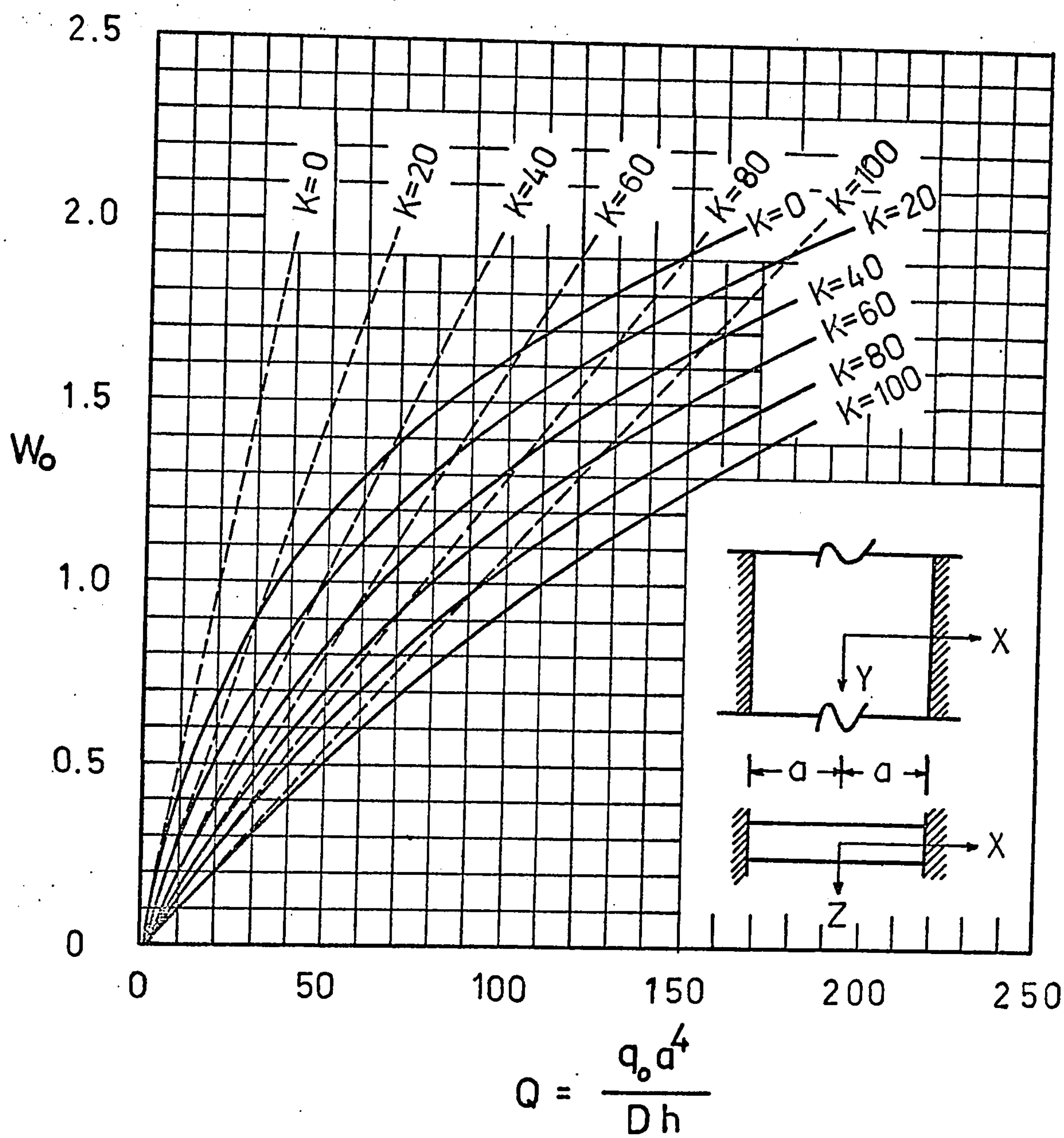

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FIGURE 6-4
COSINE LOADING (F = COSπx/2)
CLAMPED BOTH SIDES
---------- LINEAR THEORY
———— NONLINEAR THEORY

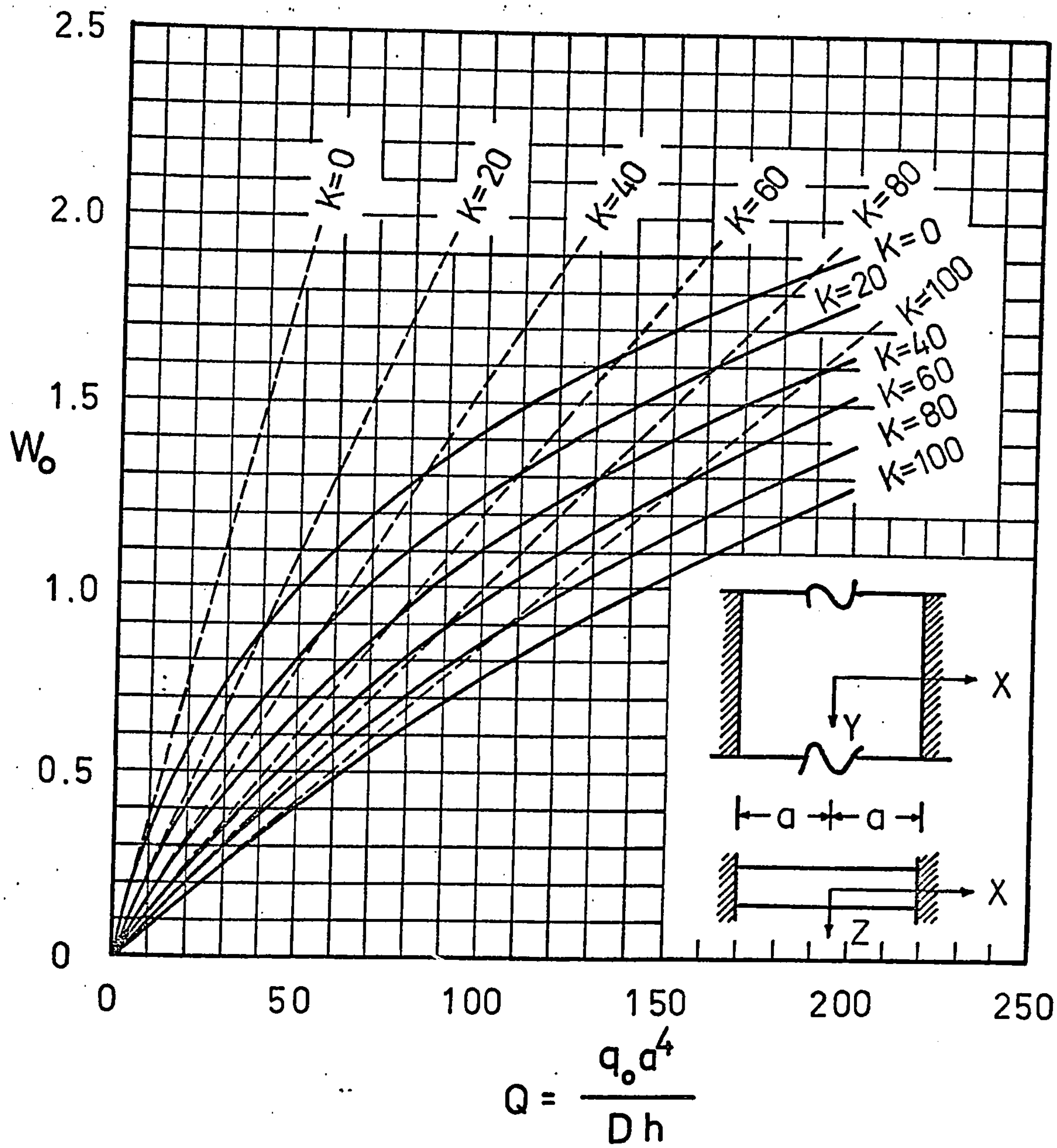

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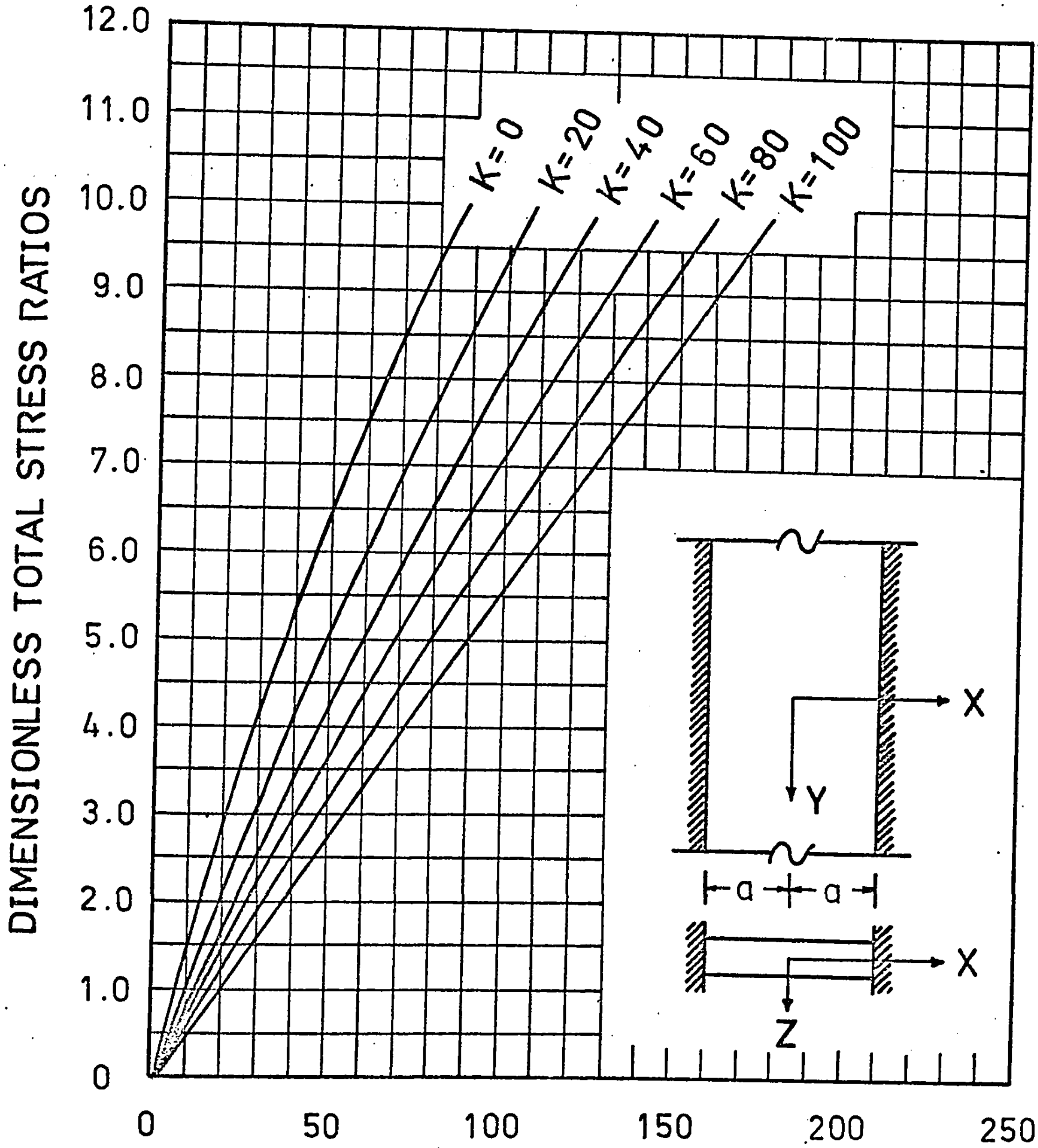
FIGURE 6-5
UNIFORM LOADING (F=1)
CLAMPED BOTH SIDES



$$Q = \frac{q_o a^4}{D h}$$

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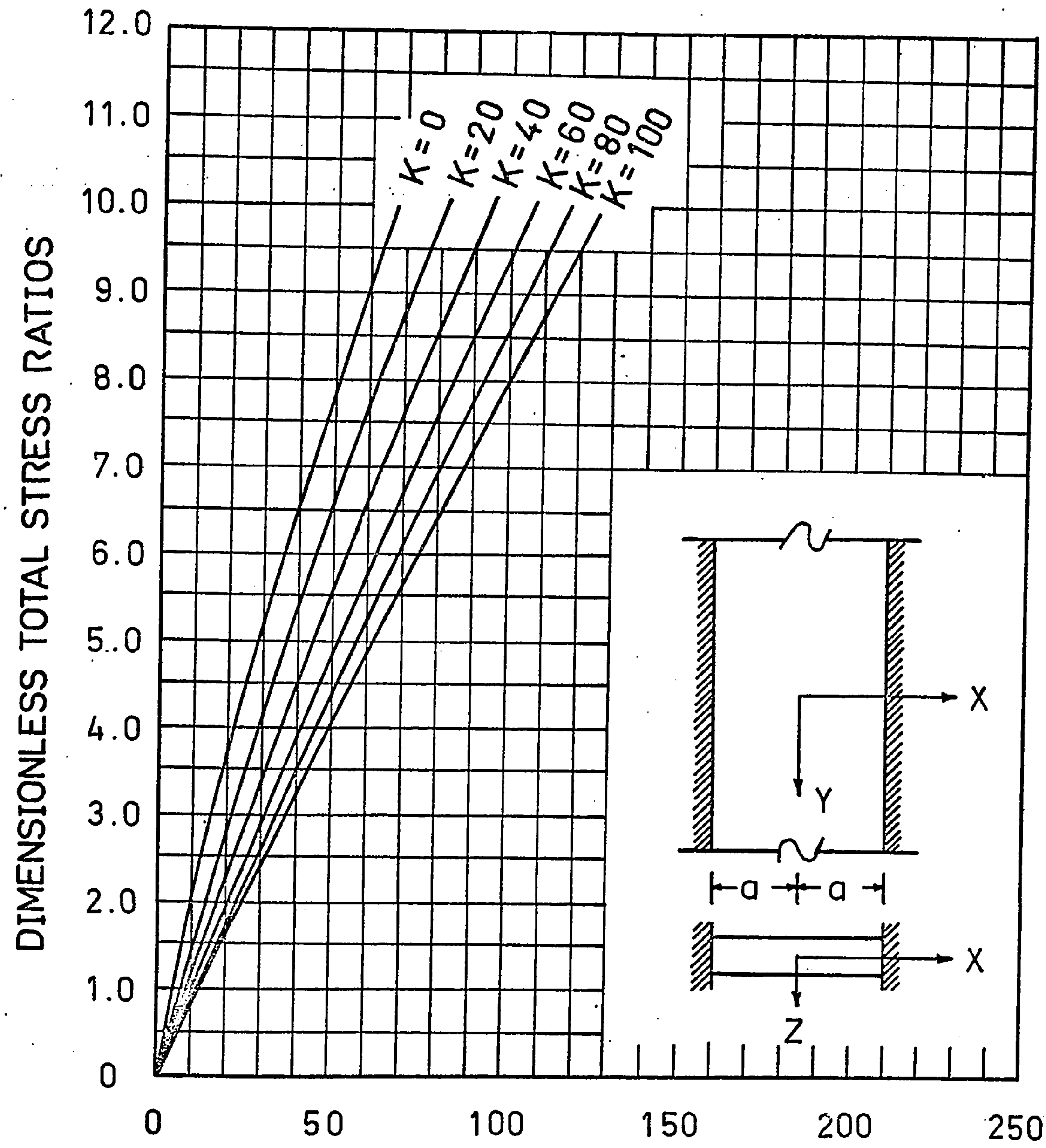
FIGURE 6-6
HYDROSTATIC LOADING (F=1+X)
CLAMPED BOTH SIDES



$$Q = \frac{q_o a^4}{D h}$$

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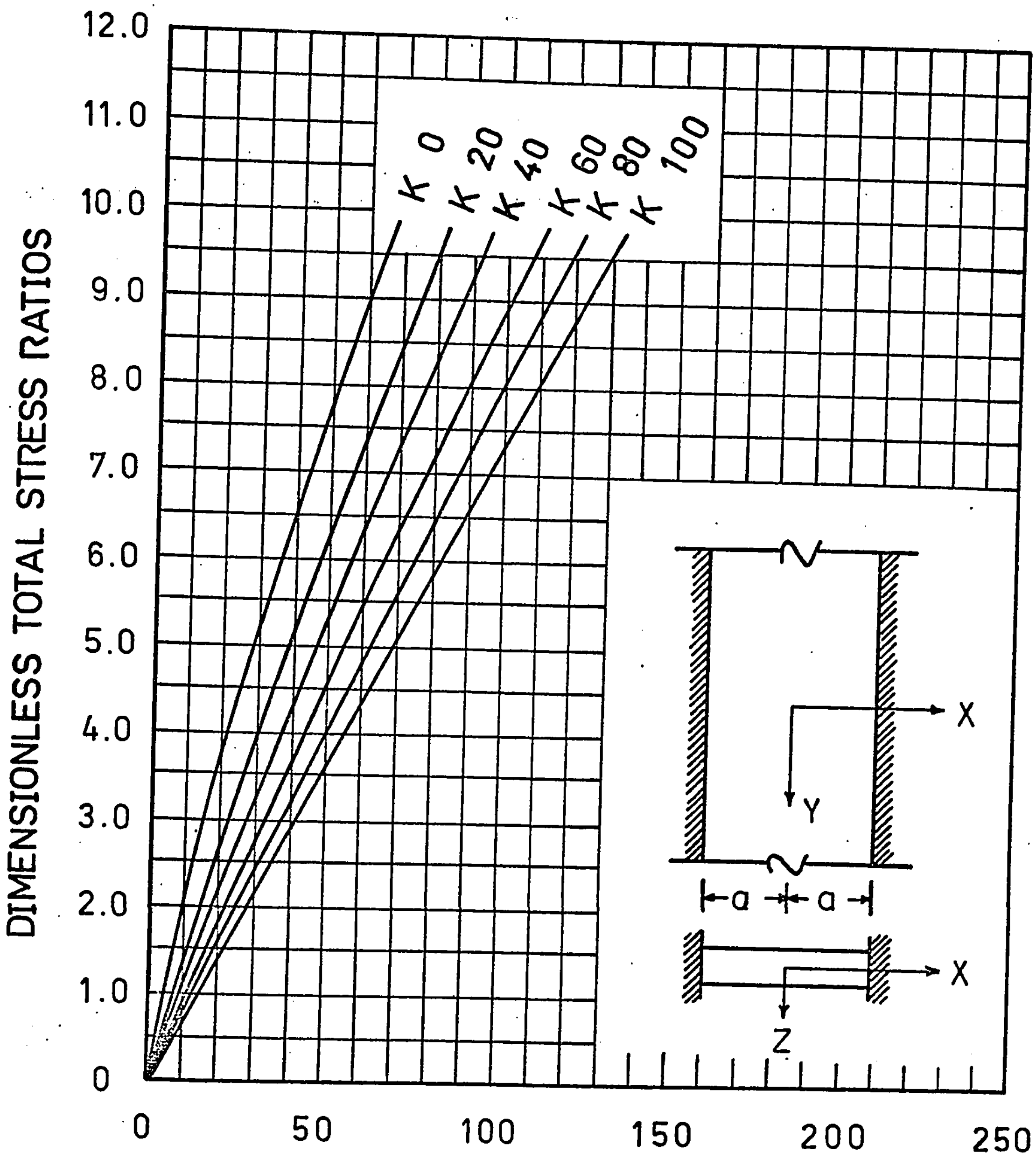
FIGURE 6-7
PARABOLIC LOADING ($F = 1 + X^2$)
CLAMPED BOTH SIDES



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6 - 8

COSINE LOADING ($F = COS \pi x/2$)

CLAMPED BOTH SIDES

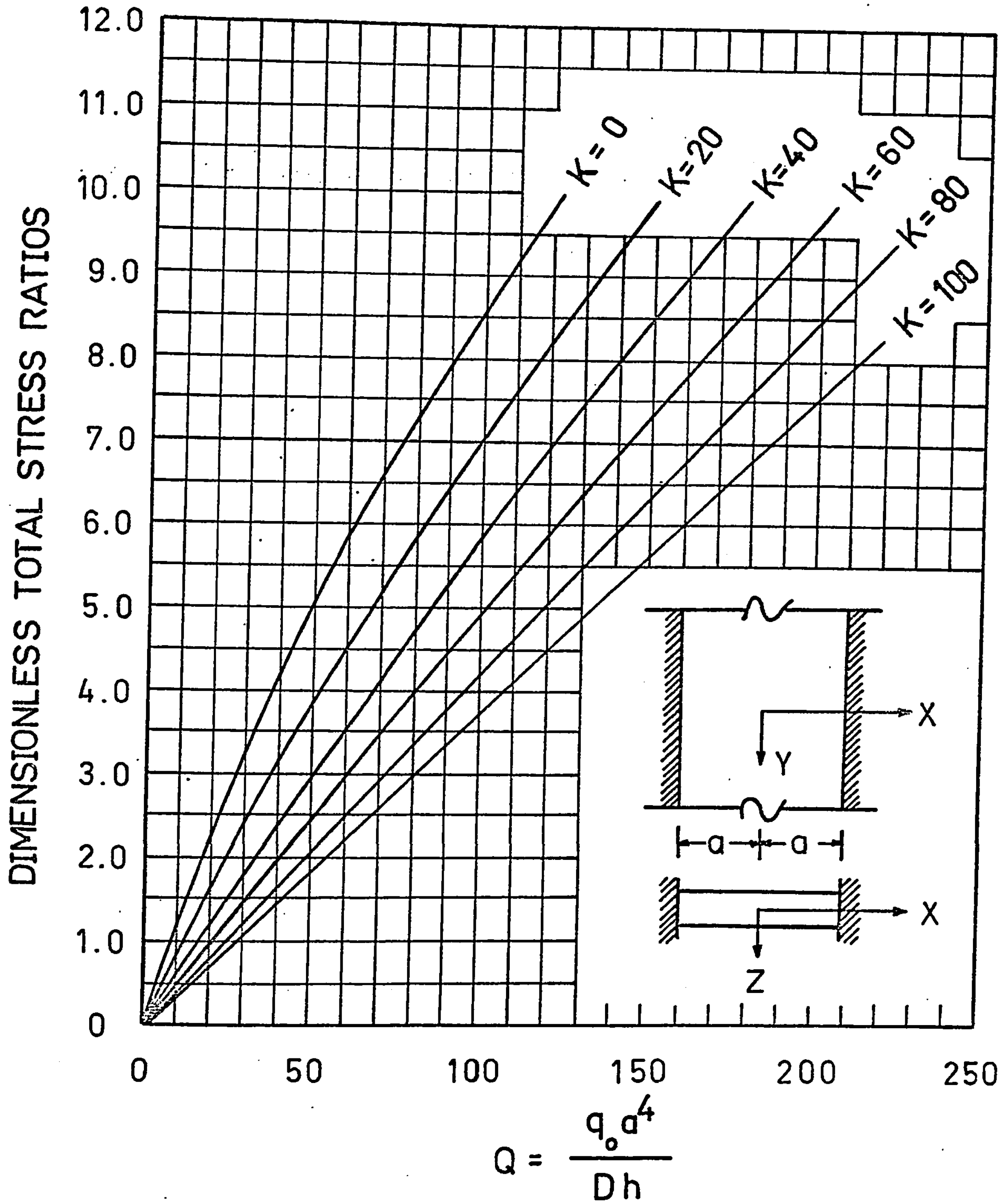

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FIGURE 6-9
UNIFORM LOADING (F = 1)
SIMPLY SUPPORTED BOTH SIDES
---------- LINEAR THEORY
———— NONLINEAR THEORY
—·—·— S.N. SINHA [12]

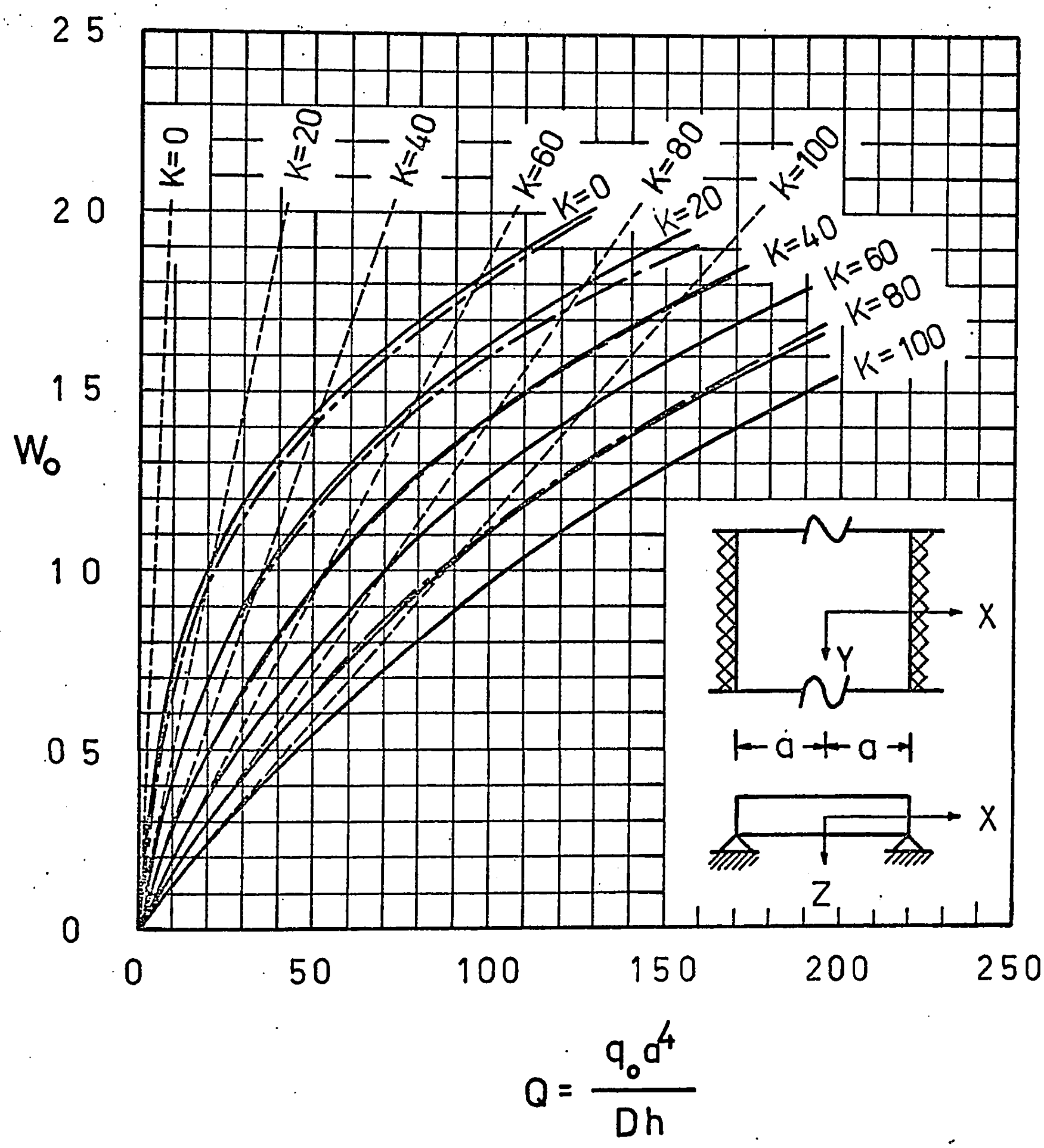

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FIGURE 6-10

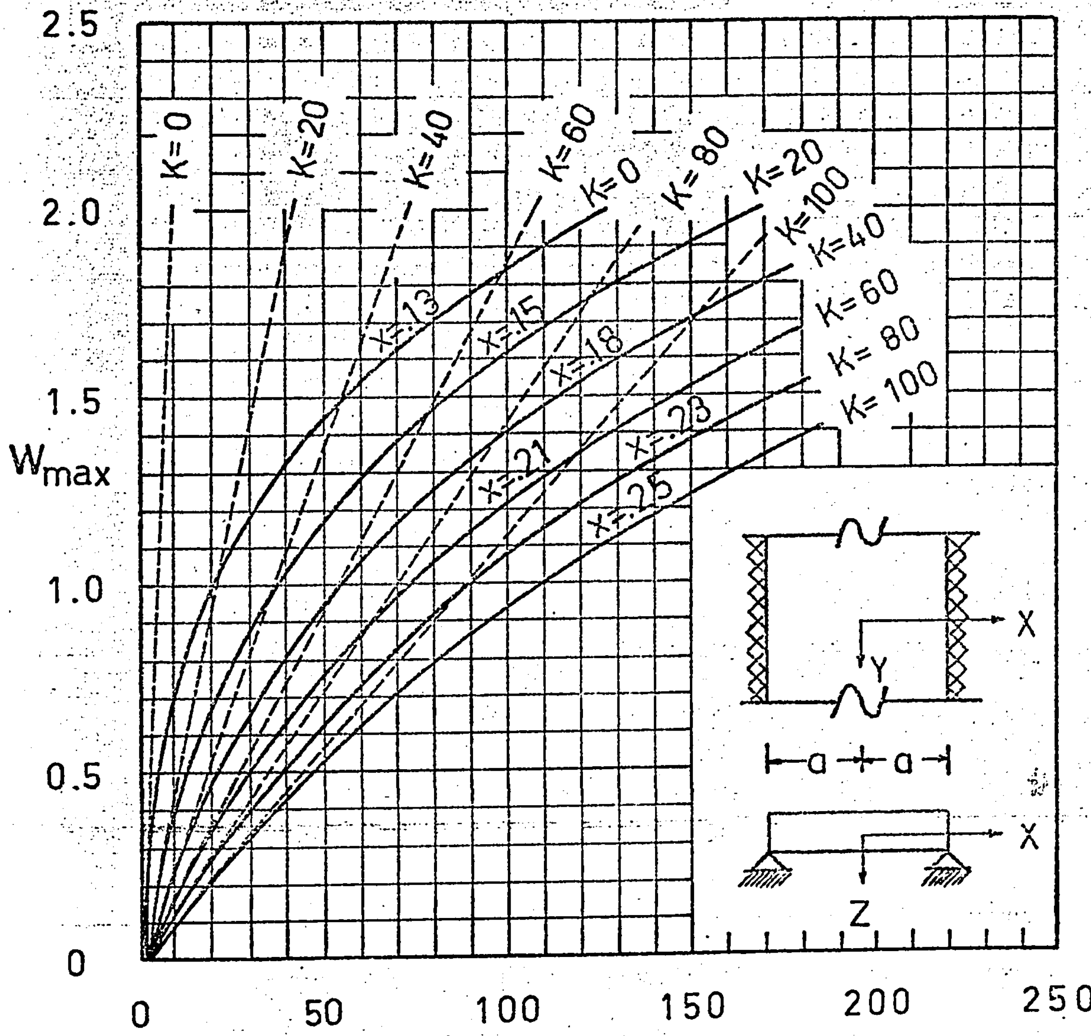
HYDROSTATIC LOADING (F = 1+X)

SIMPLY SUPPORTED BOTH SIDES

_ _ _ _ _ _ _ LINEAR THEORY

_________ NONLINEAR THEORY

x max. deflection point



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6-11
PARABOLIC LOADING ($F = 1 + X^2$)
SIMPLY SUPPORTED BOTH SIDES
--------- LINEAR THEORY
_________ NONLINEAR THEORY

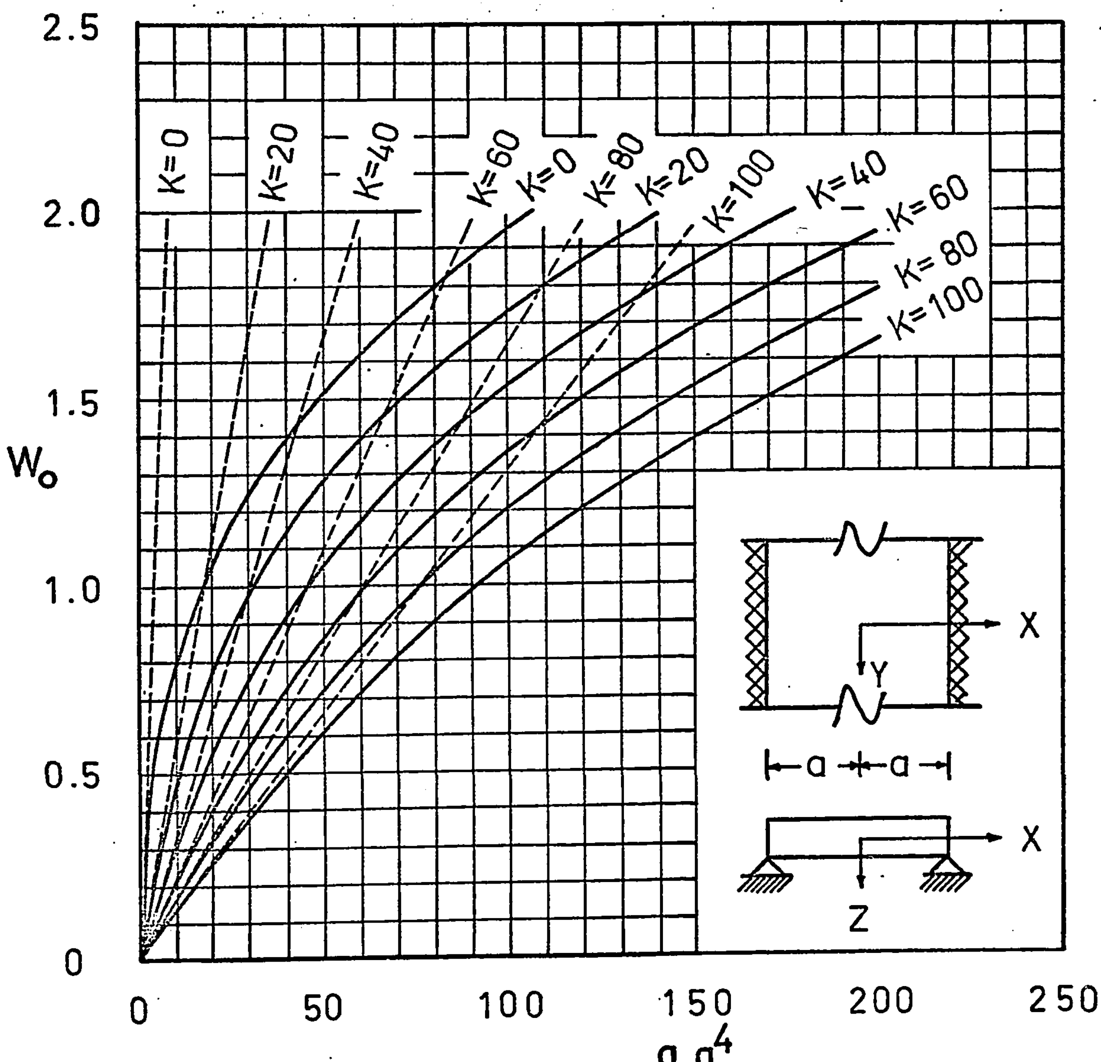

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FIGURE 6-12

COSINE LOADING ($F = \cos \pi x/2$)

SIMPLY SUPPORTED BOTH SIDES

---------- LINEAR THEORY

———— NONLINEAR THEORY

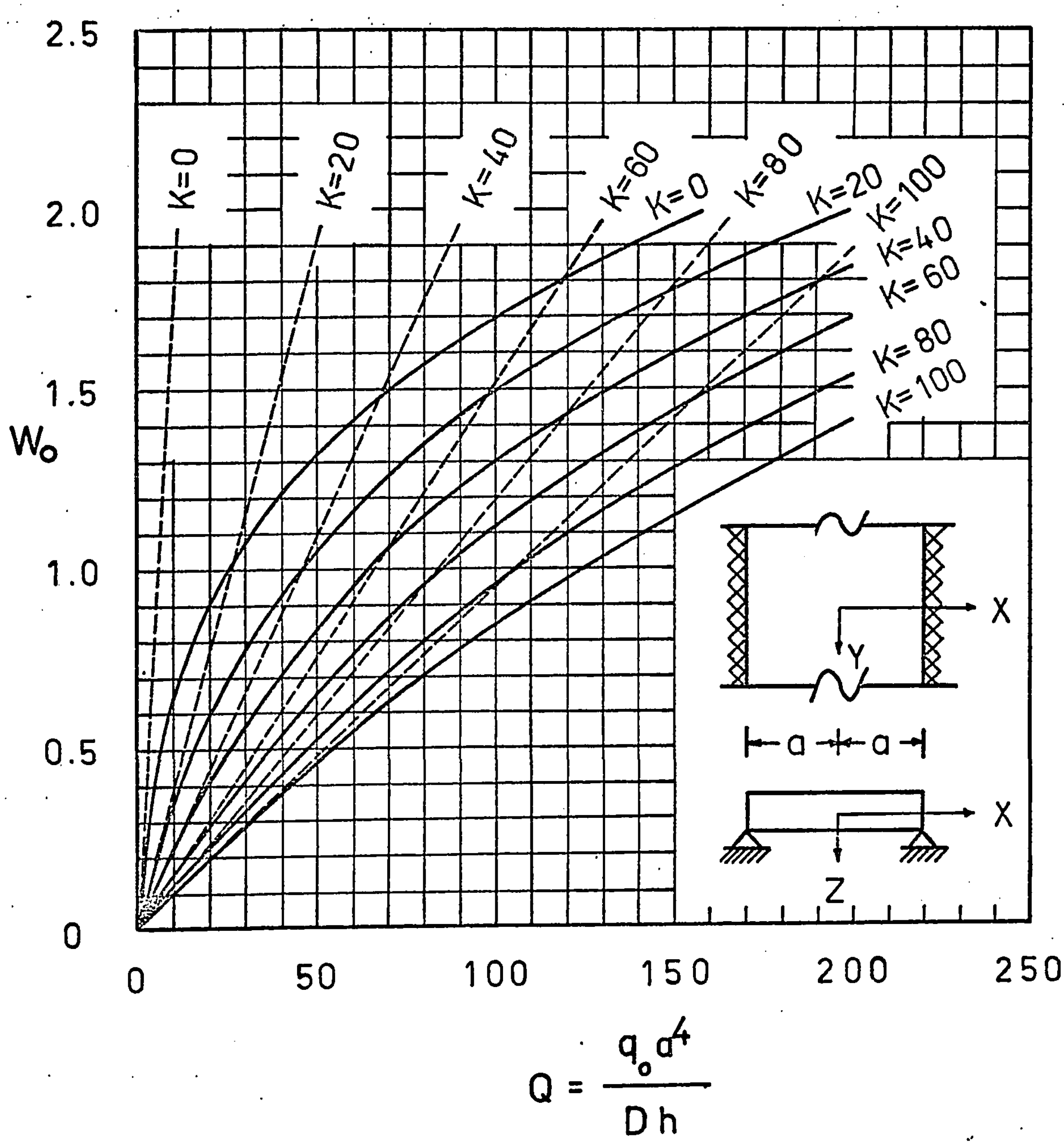

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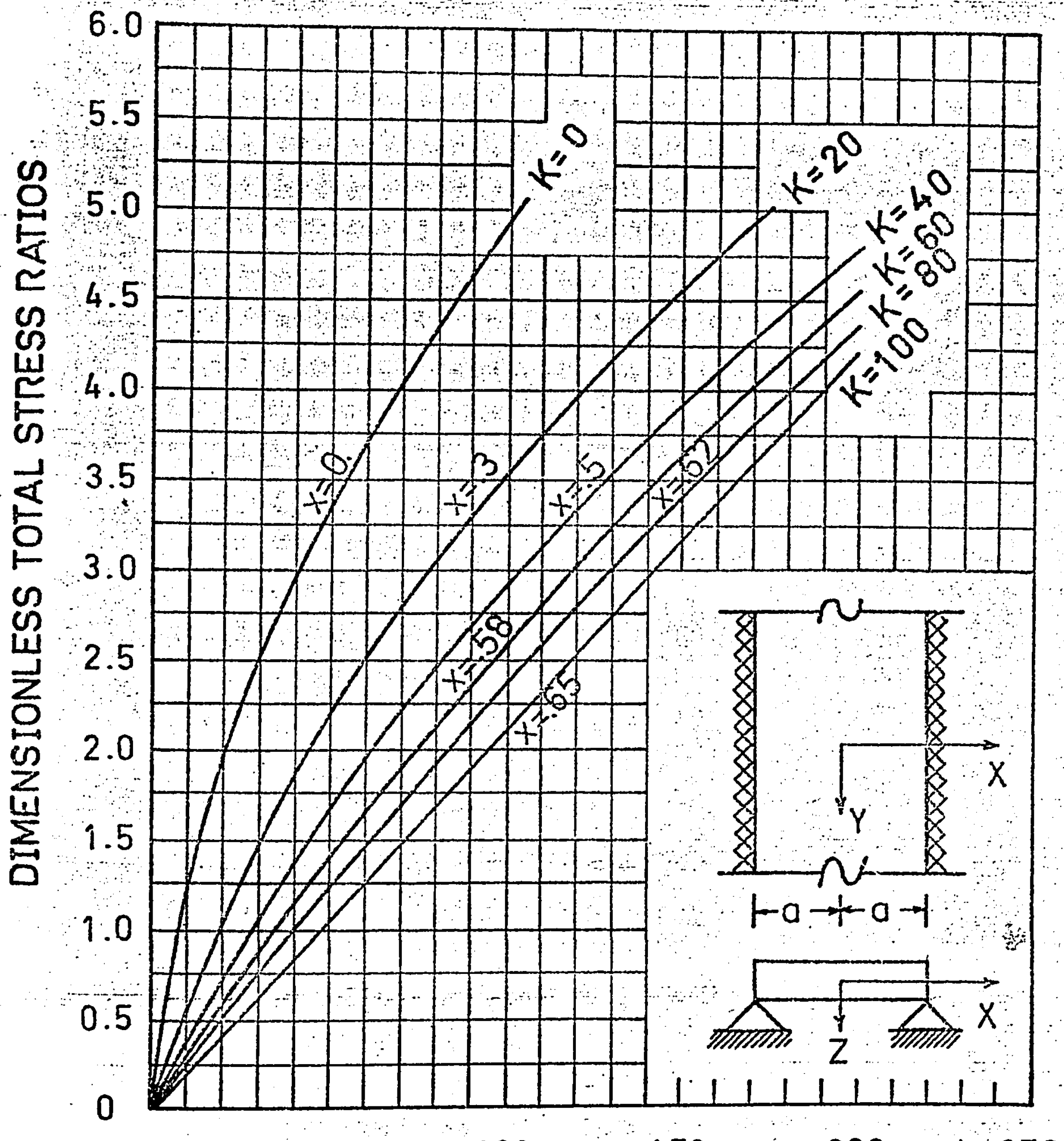
FIGURE 6-13
UNIFORM LOADING (F = 1)
SIMPLY SUPPORTED BOTH SIDES
x mamax. stress point



$$Q = \frac{q_o a^4}{D\,h}$$

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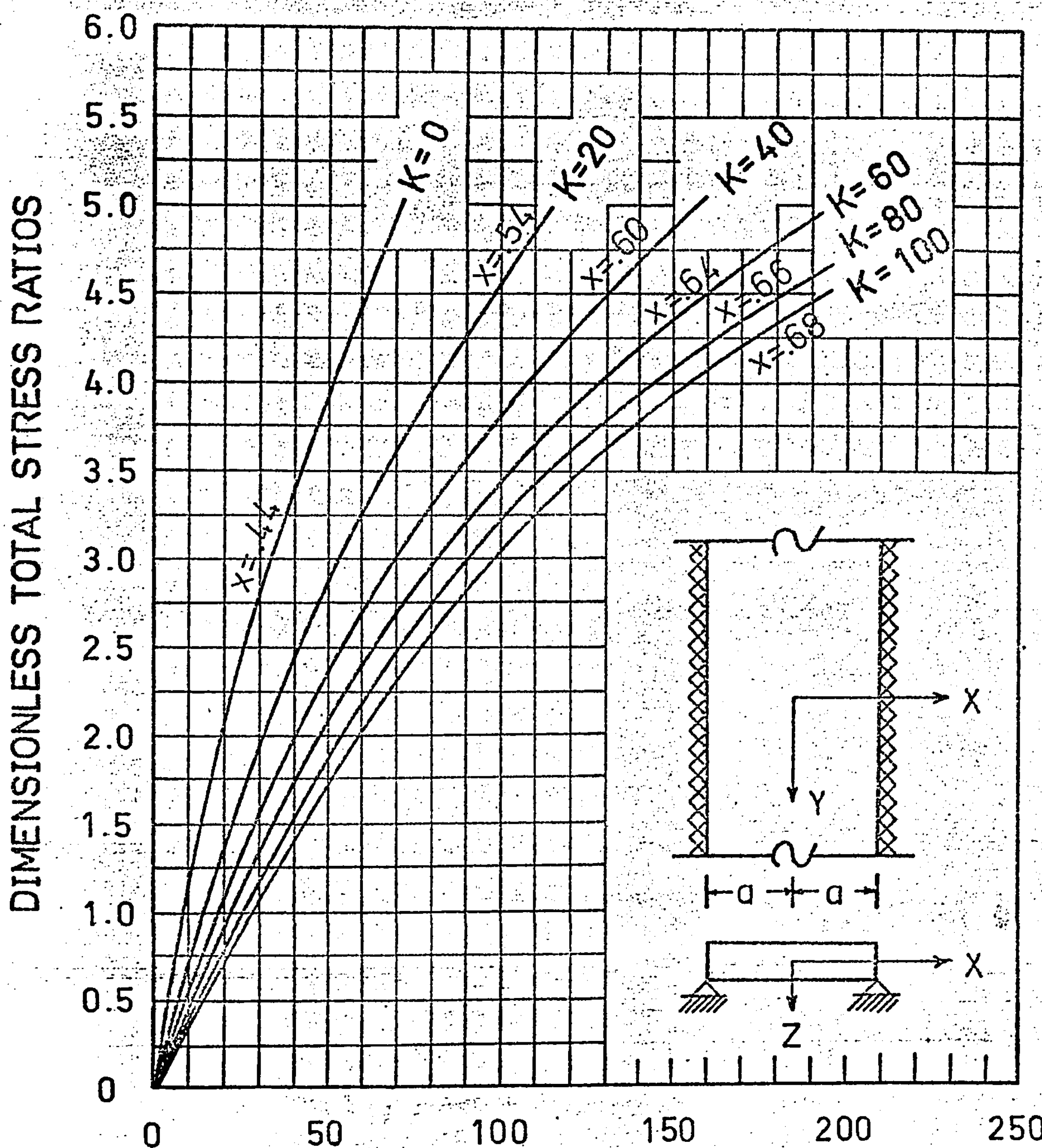
FIGURE 6-14
HYDROSTATIC LOADING (F=1+X)
SIMPLY SUPPORTED BOTH SIDES
x = max. stress point



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6 - 15

PARABOLIC LOADING ($F = 1 + X^2$)

SIMPLY SUPPORTED BOTH SIDES

x max. stress point

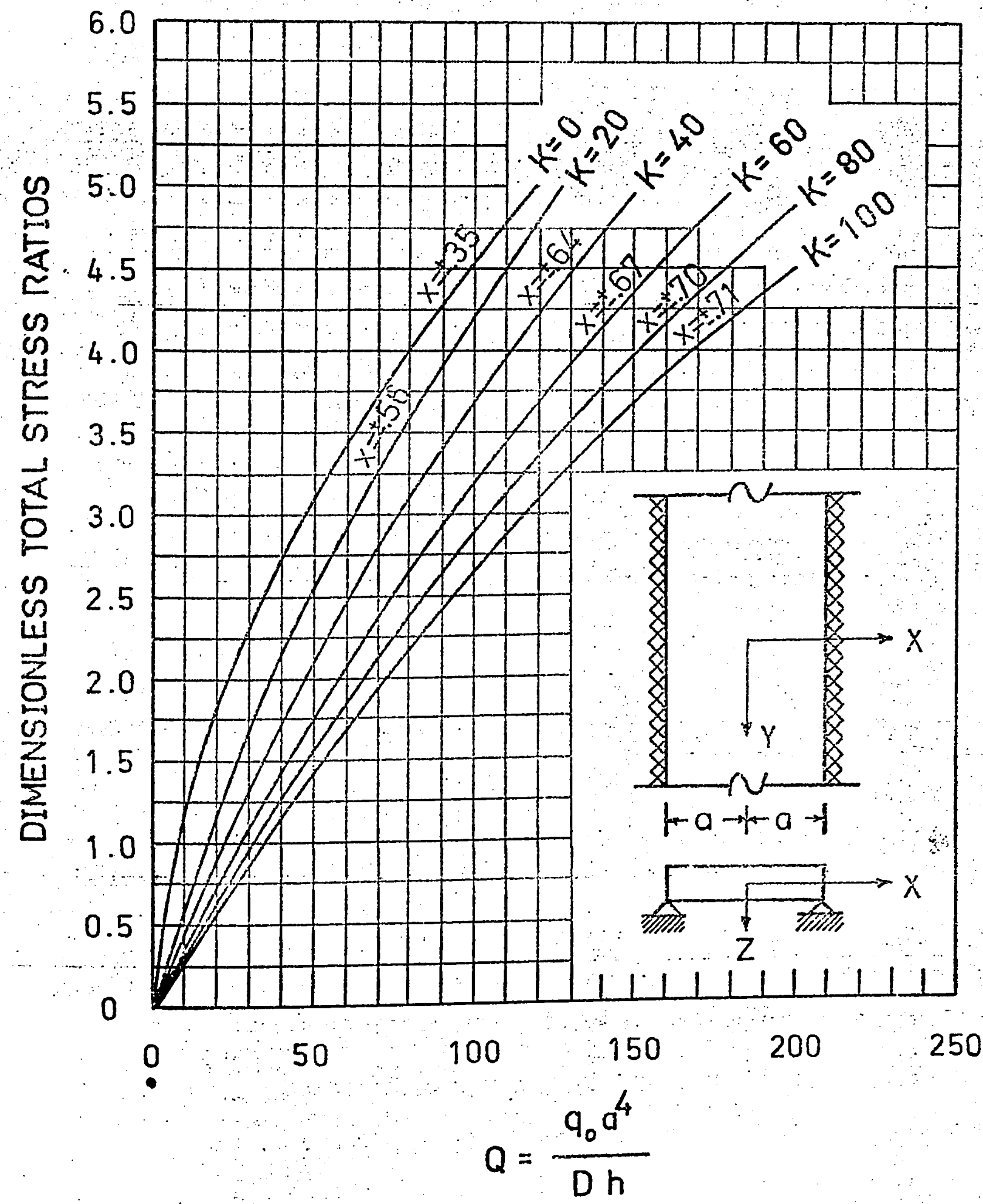

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FIGURE 6-16
COSINE LOADING (F = COS πx/2)
SIMPLY SUPPORTED BOTH SIDES

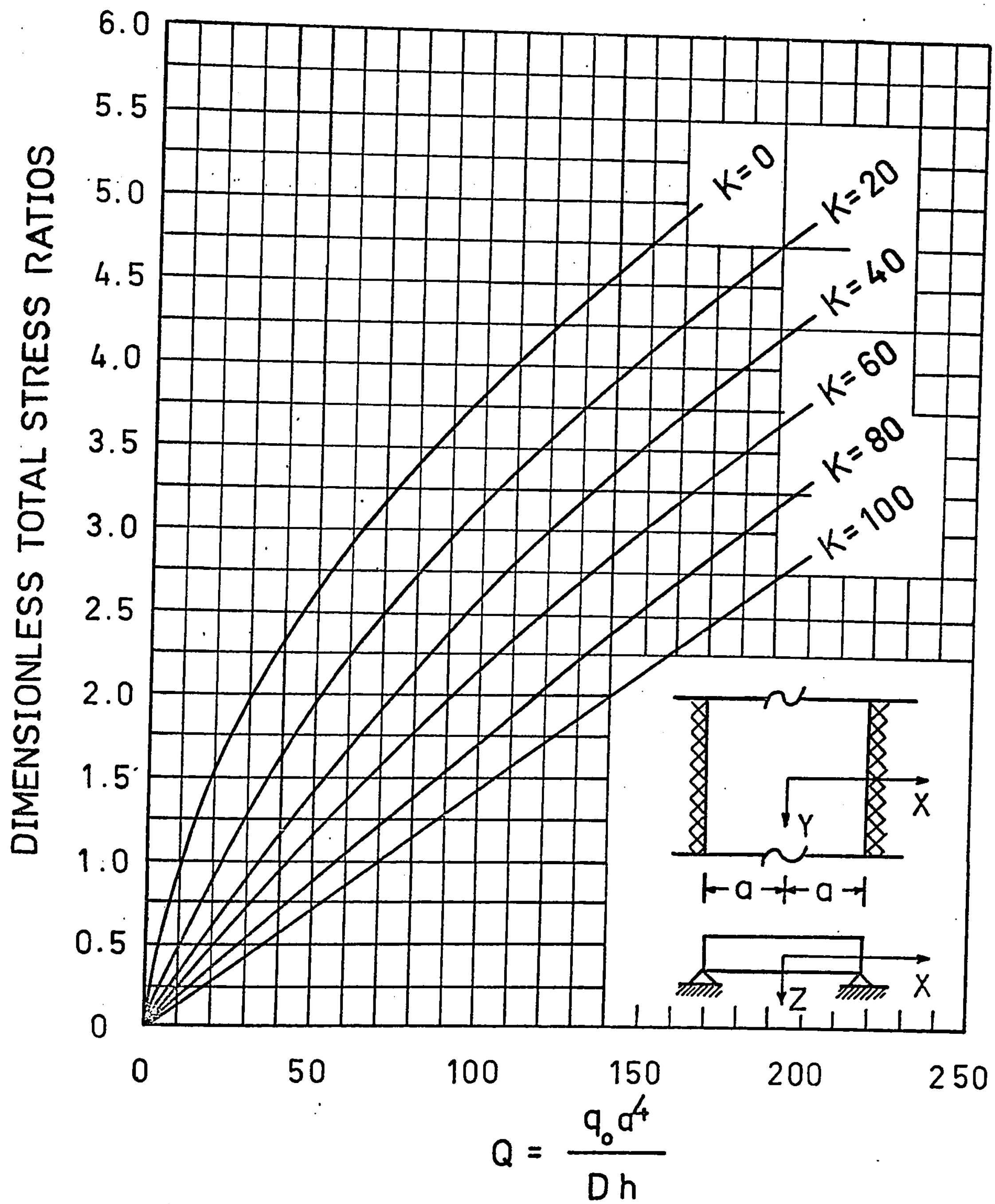

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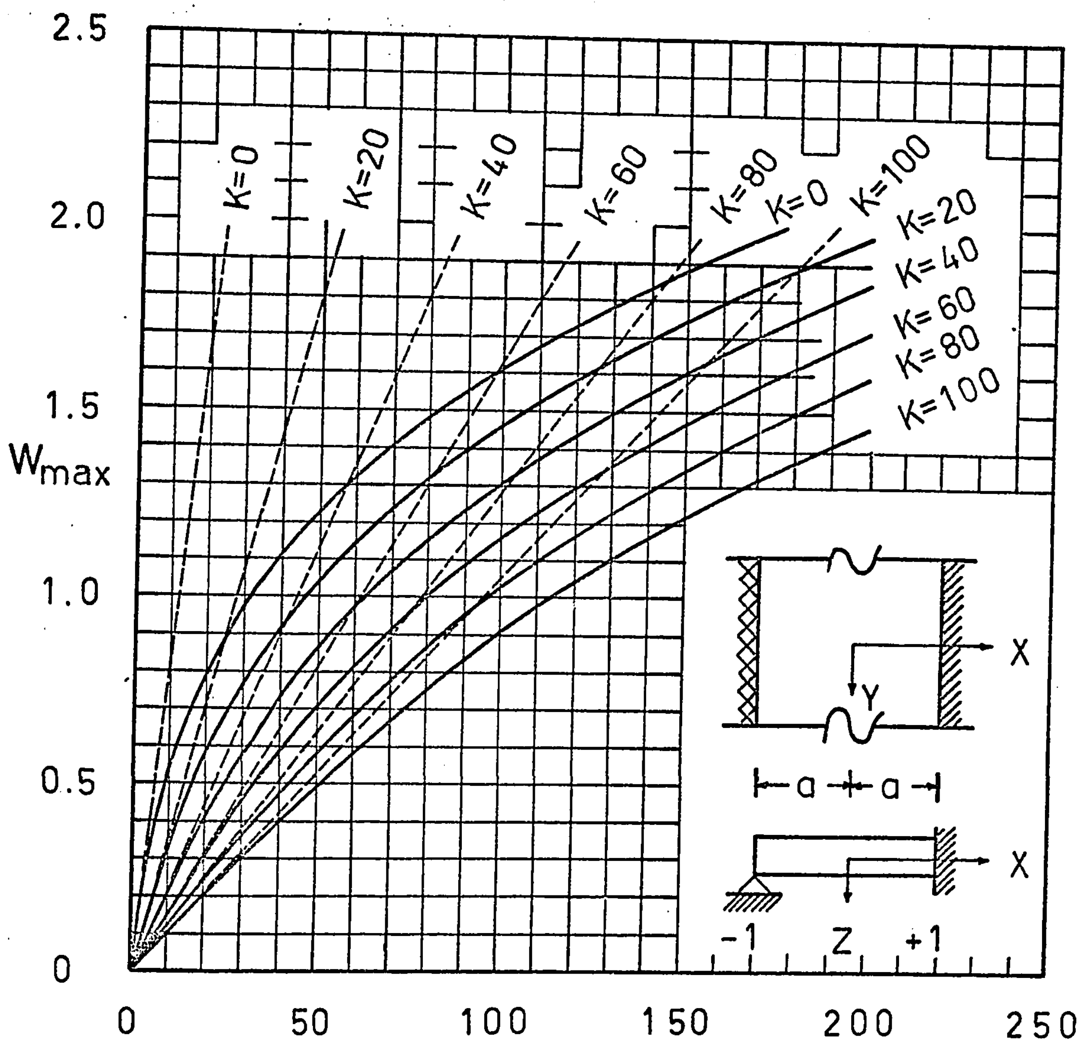
FIGURE 6-17

CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

UNIFORM LOADING (F=1)

---------- LINEAR THEORY

———— NONLINEAR THEORY



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6-18

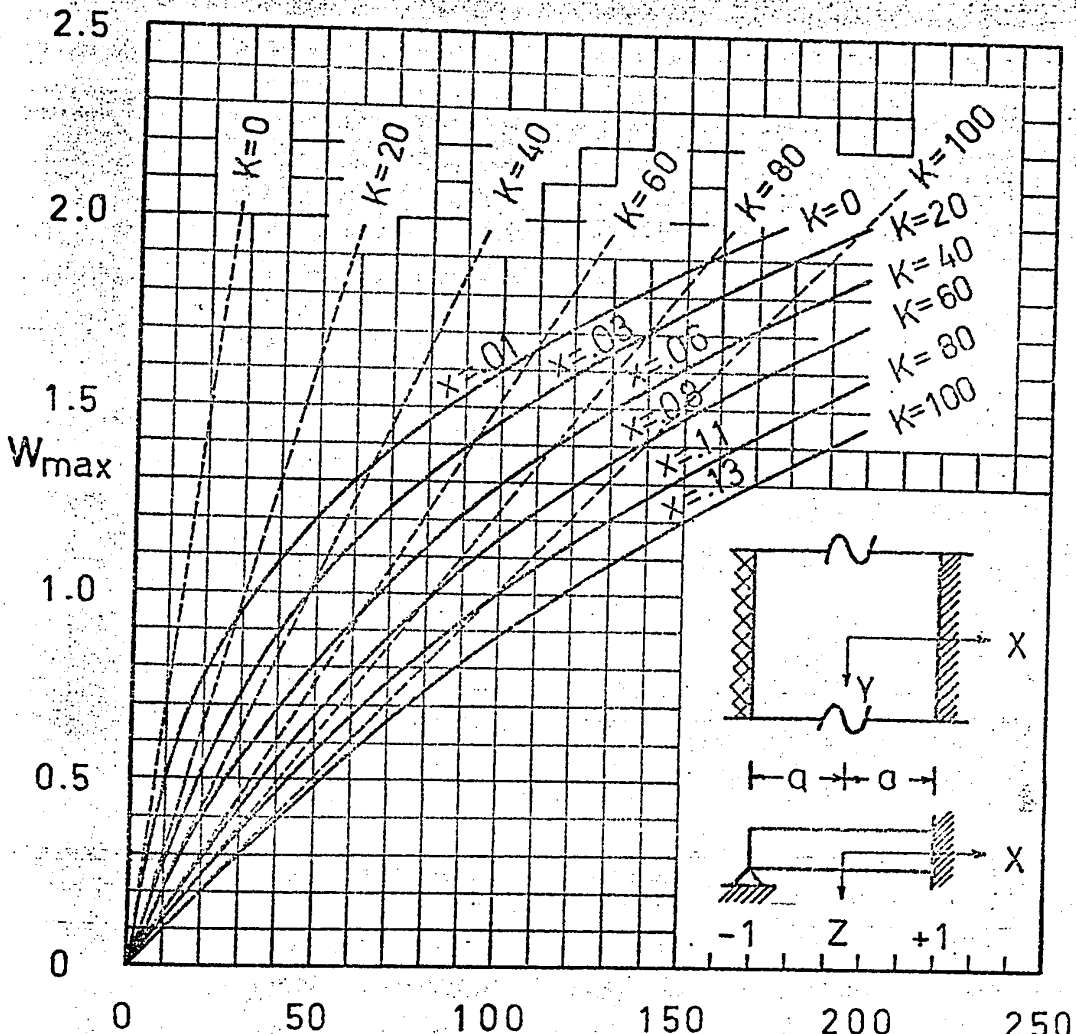
HYDROSTATIC LOADING (F = 1 + X)

CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

---------- LINEAR THEORY

———— NONLINEAR THEORY

x max. deflection point



$$Q = \frac{q_o a^4}{D h}$$

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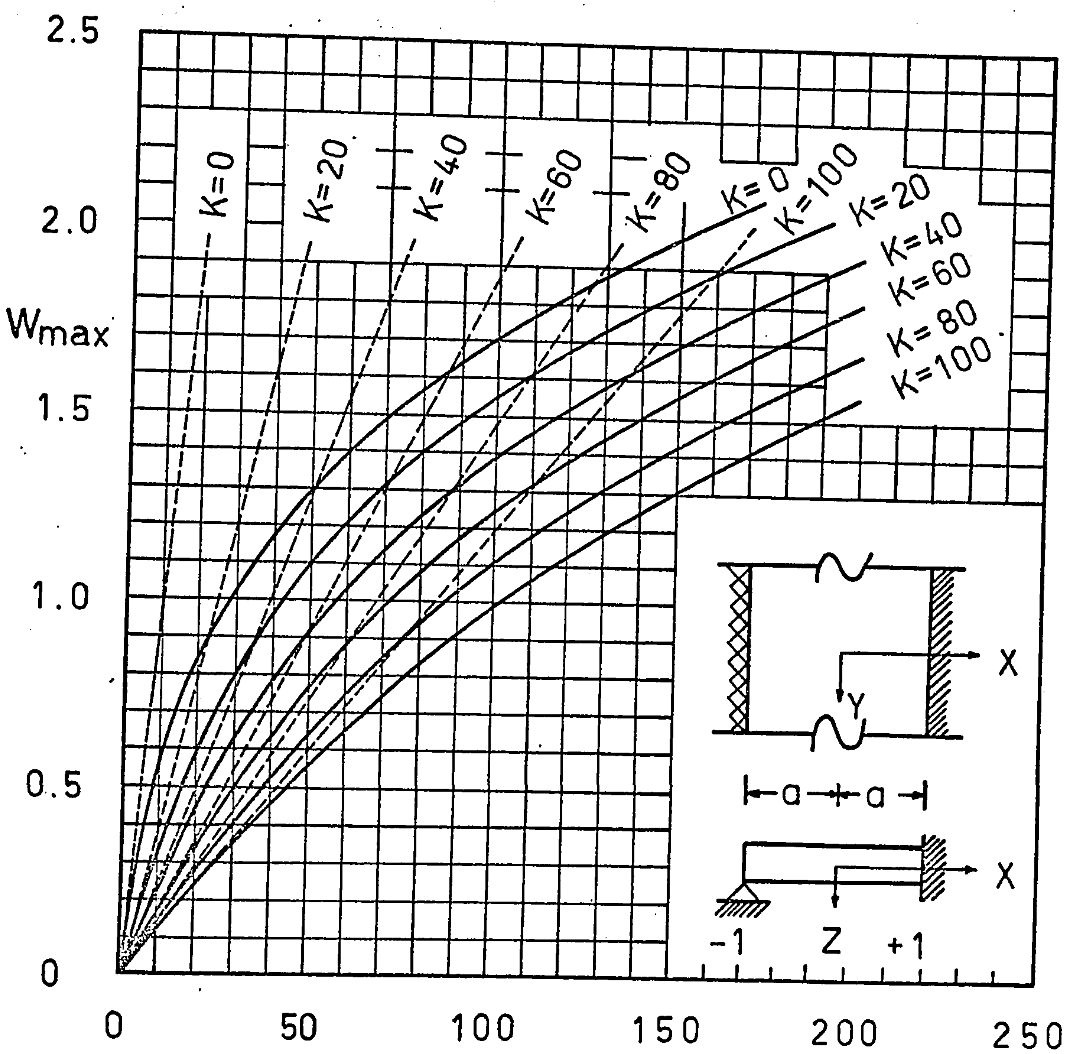
FIGURE 6-19

PARABOLIC LOADING ($F = 1 + X^2$)

CLAMPED AT X = 1, SIMPLY SUPPORTED AT X = -1

---------- LINEAR THEORY

———————— NONLINEAR THEORY



$$Q = \frac{q_o a^4}{D h}$$

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FIGURE 6-20

COSINE LOADING ($F = \cos \pi x/2$)

CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

---------- LINEAR THEORY

———— NONLINEAR THEORY

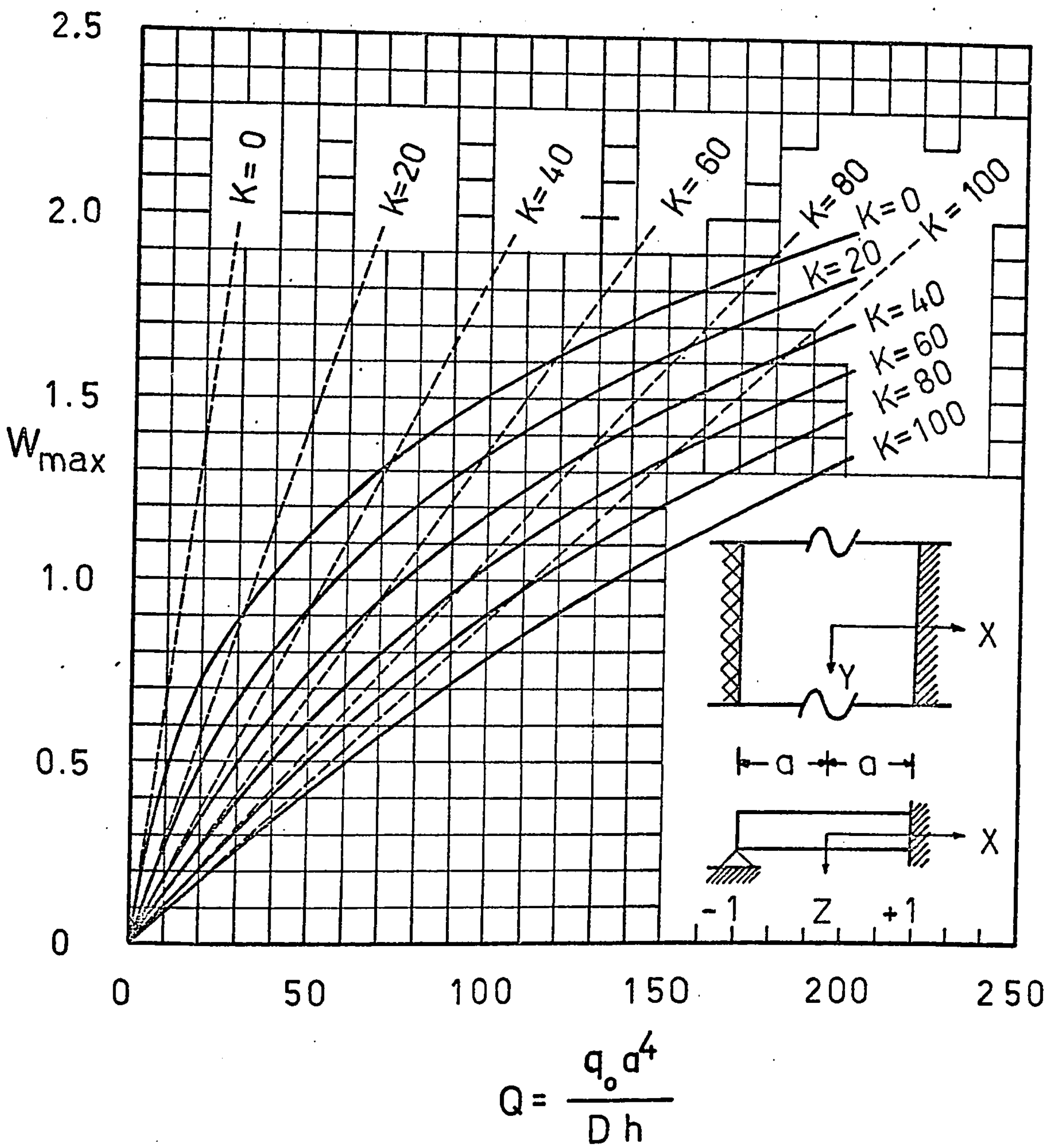

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FIGURE 6-21
UNIFORM LOADING (F = 1)
CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

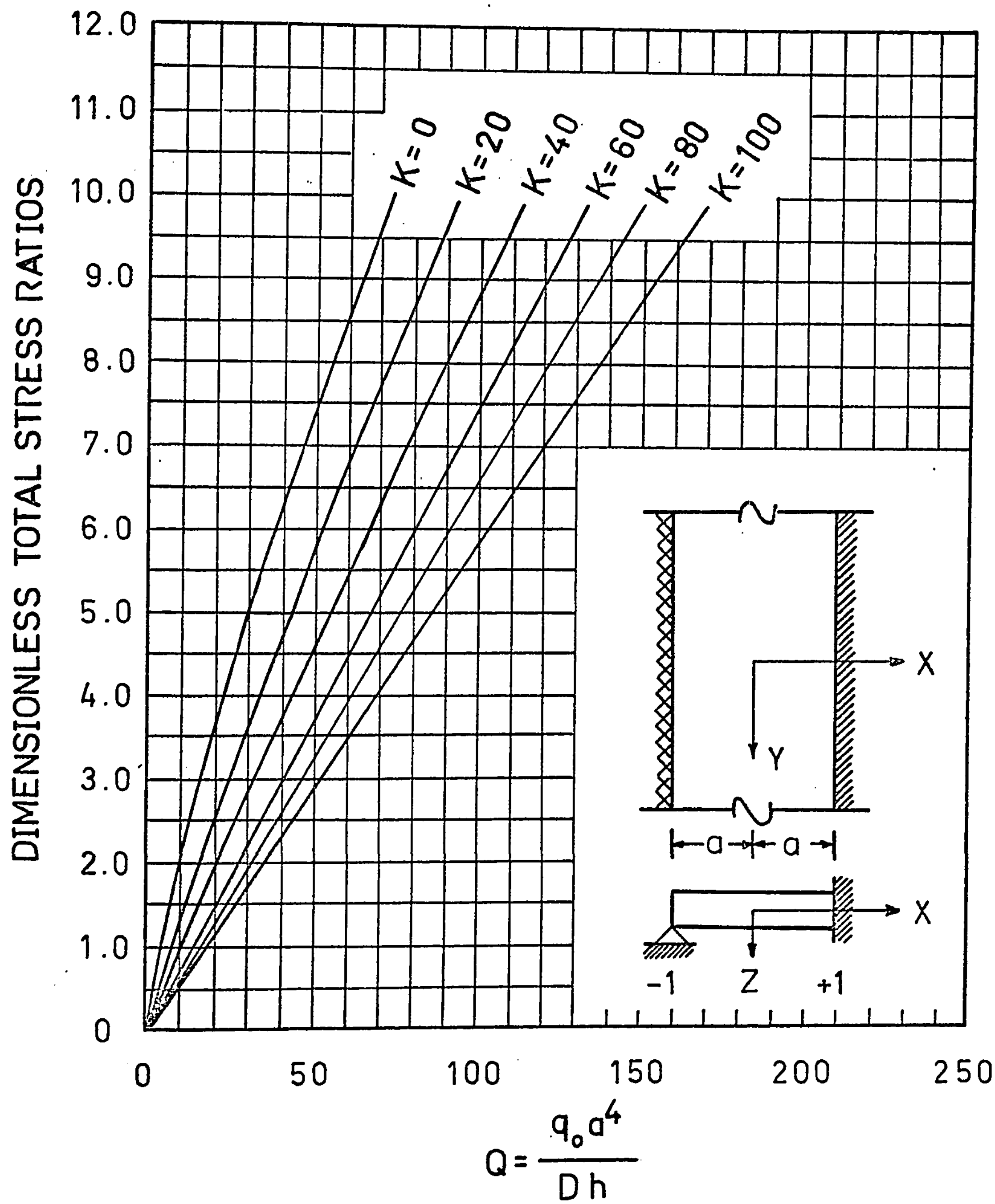

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FIGURE 6-22
HYDROSTATIC LOADING (F = 1 + X)
CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

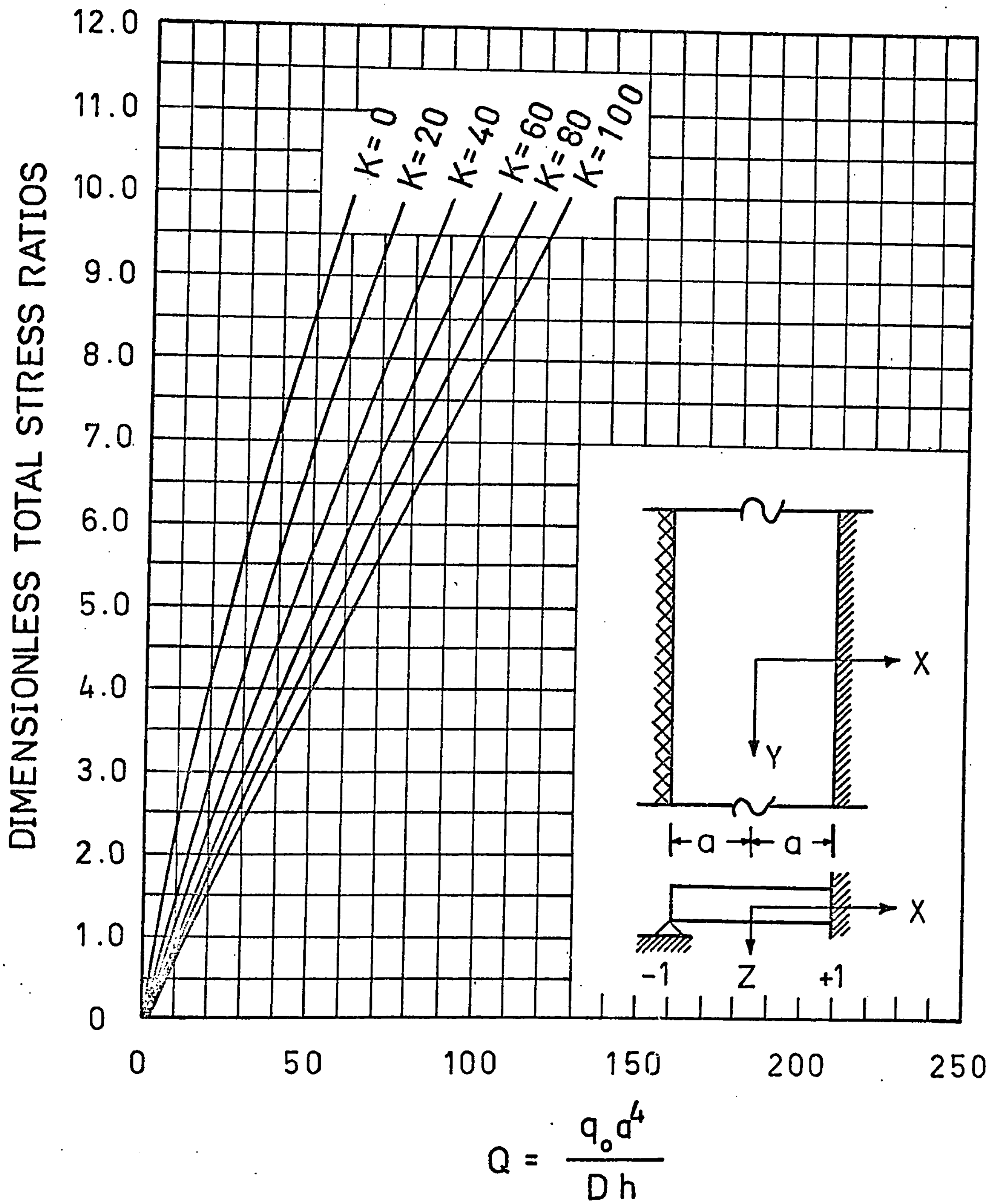

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FIGURE 6-23

PARABOLIC LOADING ($F = 1 + X^2$)

CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1

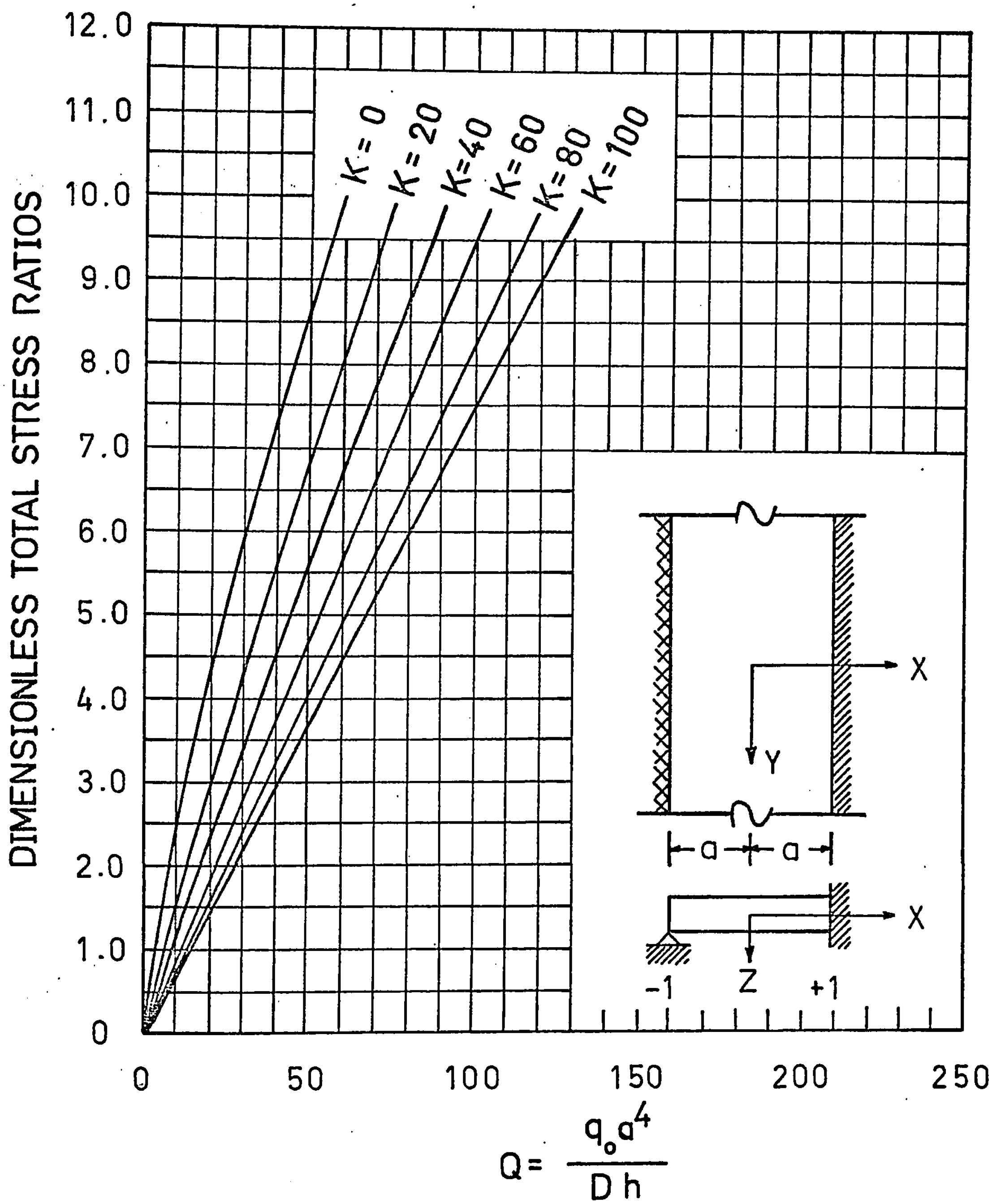
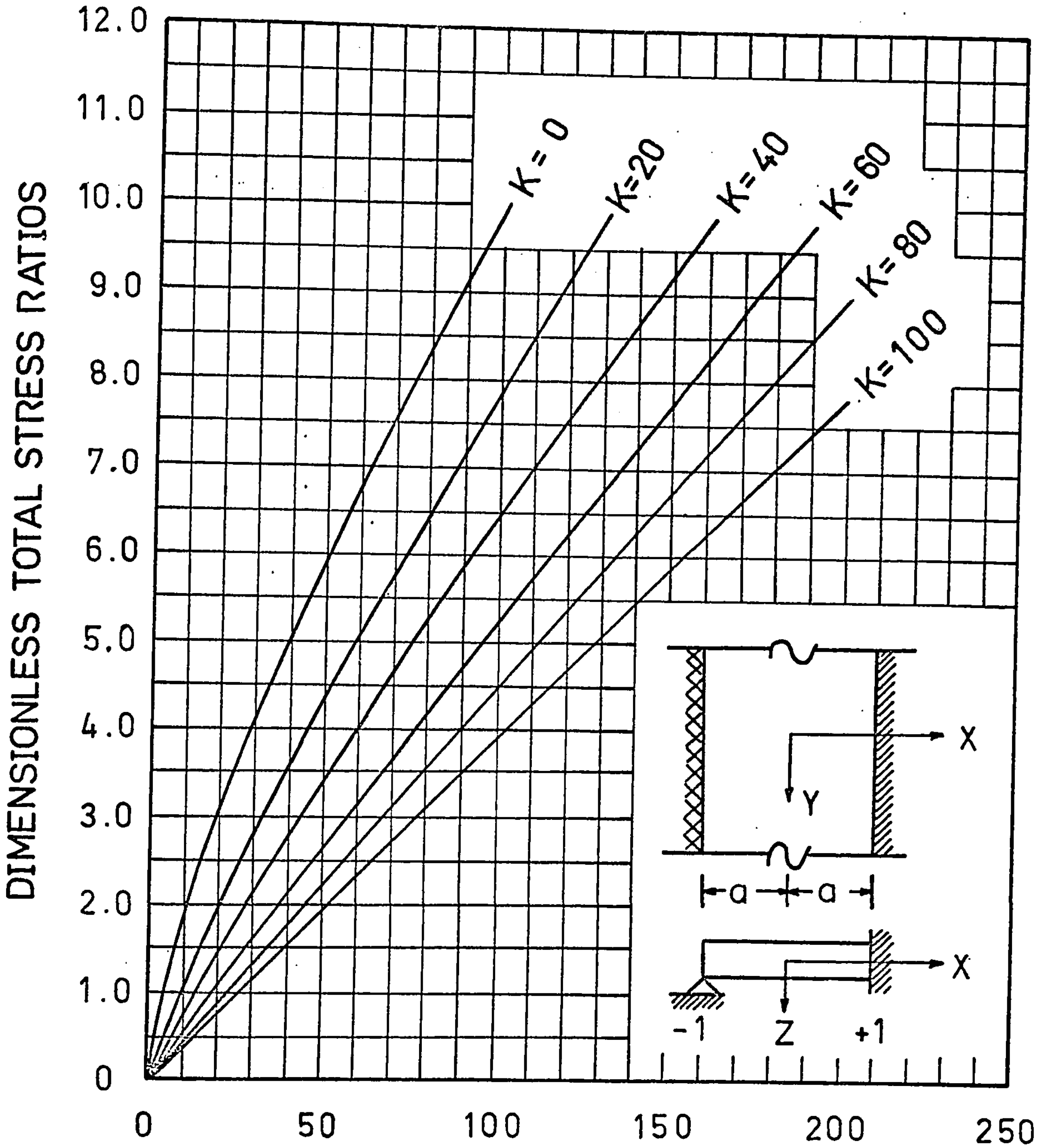


FIGURE 6-24
COSINE LOADING ($F = \cos \pi x/2$)
CLAMPED AT X=1, SIMPLY SUPPORTED AT X=-1



$$Q = \frac{q_o a^4}{D h}$$

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APPENDIX A

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1. Loading Condition : F=1

Boundary Condition : Clamped both sides

K = 0.

$$w_1 = X^4 - 2X^2 + 1$$

$$s_2 = - 1.14X^7 + 3.2X^5 - 2.67X^3 + 0.61X$$

$$w_3 = 0.24X^6 - 0.48X^4 + 0.24X^2$$

K = 20.

$$w_1 = - 0.49e^{1.49X}COS(1.49X) - 0.42e^{1.49X}SIN(1.49X)$$
$$- 0.49e^{-1.49X}COS(1.49X) + 0.42e^{-1.49X}SIN(1.49X)$$
$$+ 1.98$$

$$s_2 = -0.08e^{2.98X}COS(2.98X) + 0.08e^{-2.98X}COS(2.98X)$$
$$- 0.15e^{2.98X} + 0.15e^{-2.98X} -0.06e^{2.98X}SIN(2.98X)$$
$$- 0.06e^{2.98X}SIN(2.98X) + 0.31SIN(2.98X) + 1.54X$$

$$w_3 = -0.44e^{1.49X}COS(1.49X) + 0.45e^{1.49X}SIN(1.49X)$$
$$-0.44e^{-1.49X}COS(1.49X) - 0.45e^{-1.49X}SIN(1.49X)$$
$$- 0.56Xe^{1.49X}SIN(1.49X) - 0.03Xe^{1.49X}COS(1.49X)$$
$$- 0.56Xe^{-1.49X}SIN(1.49X) - 0.03Xe^{-1.49X}COS(1.49X)$$
$$+ 0.03Xe^{-1.49X}COS(1.49X) + 0.89$$

K = 40.

$$w_1 = - 0.19e^{1.77X}COS(1.77X) - 0.28e^{1.77X}SIN(1.77X)$$
$$-0.19e^{-1.77X}COS(1.77X) + 0.28e^{-1.77X}SIN(1.77X)$$
$$+ 1.38$$

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$$s_2 = -0.01e^{3.54X}\cos(3.54X) + 0.01e^{-3.54X}\cos(3.54X)$$

$$-0.05e^{3.54X} + 0.05e^{-3.54X} - 0.03e^{3.54X}\sin(3.54X)$$

$$-0.03e^{3.54X}\sin(3.54X) - 0.03e^{-3.54X}\sin(3.54X)$$

$$+ 0.1\sin(3.54X) + 0.1\sin(3.54X) + 0.96X$$

$$w_3 = -0.22e^{1.77X}\cos(1.77X) + 0.14e^{1.77X}\sin(1.77X)$$

$$-0.22e^{-1.77X}\cos(1.77X) - 0.14e^{-1.77X}\sin(1.77X)$$

$$-0.24Xe^{1.77X}\sin(1.77X) + 0.04Xe^{1.77X}\cos(1.77X)$$

$$-0.24Xe^{-1.77X}\sin(1.77X) - 0.04Xe^{-1.77X}\cos(1.77X)$$

$$+ 0.45$$

$K = 60.$

$$w_1 = -0.09e^{1.96X}\cos(1.96X) - 0.22e^{1.96X}\sin(1.96X)$$

$$-0.09e^{-1.96X}\cos(1.96X) + 0.22e^{-1.96X}\sin(1.96X)$$

$$+ 1.19$$

$$s_2 = -0.02e^{3.92X} + 0.02e^{-3.92X} - 0.02e^{3.92X}\sin(3.92X)$$

$$-0.02e^{-3.92X}\sin(3.92X) + 0.05\sin(3.92X) + 0.78X$$

$$w_3 = -0.15e^{1.96X}\cos(1.96X) + 0.06e^{1.96X}\sin(1.96X)$$

$$-0.15e^{-1.96X}\cos(1.96X) - 0.06e^{-1.96X}\sin(1.96X)$$

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$$- 0.15Xe^{1.96X}SIN(1.96X) + 0.06Xe^{1.96X}COS(1.96X)$$

$$- 0.15Xe^{-1.96X}SIN(1.96X) - 0.06Xe^{-1.96X}COS(1.96X)$$

$$- 0.06Xe^{-1.96X}COS(1.96X) + 0.30$$

K = 80.

$$w_1 = - 0.04e^{2.11X}COS(2.11X) - 0.18e^{2.11X}SIN(2.11X)$$

$$- 0.04e^{-2.11X}COS(2.11X) + 0.18e^{-2.11X}SIN(2.11X)$$

$$+ 1.09$$

$$s_2 = - 0.01e^{4.22X} + 0.01e^{-4.22X} - 0.01e^{4.22X}SIN(4.22X)$$

$$- 0.01e^{-4.22X}SIN(4.22X) + 0.03SIN(4.22X) -$$

$$+ 0.69$$

$$w_3 = - 0.11e^{2.11X}COS(2.11X) + 0.03e^{2.11X}SIN(2.11X)$$

$$- 0.11e^{-2.11X}COS(2.11X) - 0.03e^{-2.11X}SIN(2.11X)$$

$$- 0.1Xe^{2.11X}SIN(2.11X) + 0.06Xe^{2.11X}COS(2.11X)$$

$$- 0.1Xe^{-2.11X}SIN(2.11X) - 0.06Xe^{-2.11XCOS(2.11X)}$$

$$+ 0.23$$

K = 100.

$$w_1 = - 0.02e^{2.23X}COS(2.23X) - 0.16e^{2.23X}SIN(2.23X)$$

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$$- 0.02e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.16e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 1.04$$

$$s_2 = -0.01e^{4.46X} + 0.01e^{-4.46X} + 0.02\mathrm{SIN}(\ 4.46X\ ) + 0.65X$$

$$w_3 = - 0.09e^{2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.02e^{2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$- 0.09e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) -0.02e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$- 0.07Xe^{2.23X}\mathrm{SIN}(\ 2.23X\ ) + 0.05Xe^{2.23X}\mathrm{COS}(\ 2.23X\ )$$

$$- 0.07Xe^{-2.23X}\mathrm{SIN}(\ 2.23X\ ) - 0.05Xe^{-2.23X}\mathrm{COS}(\ 2.23X\ )$$

$$+ 0.18$$

2. Loading Condition : F = 1

Boundary Condition : Simply supported both sides

$$K = 0.$$

$$w_1 = 0.2X^4 - 1.2X^2 + 1$$

$$s_2 = - 0.04X^7 + 0.38X^5 - 0.96X^3 + 0.62X$$

$$w_3 = 0.04X^6 - 0.13X^4 + 0.08X^2$$

$$K = 20.$$

$$w_1 = - 0.02e^{1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.24e^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$- 0.02e^{-1.49X}\mathrm{COS}(\ 1.49X\ ) + 0.24e^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

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$$+ 1.04$$

$$s_2 = - 0.02e^{2.98X} + 0.02e^{-2.98X} - 0.01e^{2.98X}SIN(2.98X)$$

$$- 0.01e^{-2.98X}SIN(2.98X) + 0.04SIN(2.98X) + 0.66X$$

$$w_3 = -0.38e^{1.49X}COS(1.49X) + 0.02e^{1.49X}SIN(1.49X)$$

$$- 0.38e^{-1.49X}COS(1.49X) - 0.02e^{-1.49X}SIN(1.49X)$$

$$- 0.16Xe^{1.49X}SIN(1.49X) + 0.14Xe^{1.49X}COS(1.49X)$$

$$- 0.16Xe^{-1.49X}SIN(1.49X) - 0.14Xe^{-1.49X}COS(1.49X)$$

$$+ 0.76$$

$K = 40.$

$$w_1 = 0.03e^{1.77X}COS(1.77X) - 0.15e^{1.77X}SIN(1.77X)$$

$$+ 0.03e^{-1.77X}COS(1.77X) + 0.15e^{-1.77X}SIN(1.77X)$$

$$+ 0.93$$

$$s_2 = - 0.01e^{3.54X} + 0.01e^{-3.54X} + 0.02SIN(3.54X) + 0.62X$$

$$w_3 = -0.19e^{1.77X}COS(1.77X) - 0.01e^{1.77X}SIN(1.77X)$$

$$- 0.19e^{-1.77X}COS(1.77X) + 0.01e^{-1.77X}SIN(1.77X)$$

$$- 0.06Xe^{1.77X}SIN(1.77X) + 0.1Xe^{1.77X}COS(1.77X)$$

$$- 0.06Xe^{-1.77X}SIN(1.77X) - 0.1Xe^{-1.77X}COS(1.77X)$$

$$+ 0.39$$

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$K = 60.$

$$w_1 = 0.05e^{1.96X}\mathrm{COS}(1.96X) - 0.11e^{1.96X}\mathrm{SIN}(1.96X)$$
$$+ 0.05e^{-1.96X}\mathrm{COS}(1.96X) + 0.11e^{-1.96X}\mathrm{SIN}(1.96X)$$
$$+ 0.89$$

$$s_2 = 0.01\mathrm{SIN}(3.92X) + 0.62X$$

$$w_3 = -0.13e^{1.96X}\mathrm{COS}(1.96X) - 0.02e^{1.96X}\mathrm{SIN}(1.96X)$$
$$- 0.13e^{-1.96X}\mathrm{COS}(1.96X) + 0.02e^{-1.96X}\mathrm{SIN}(1.96X)$$
$$- 0.03Xe^{1.96X}\mathrm{SIN}(1.96X) + 0.08Xe^{1.96X}\mathrm{COS}(1.96X)$$
$$- 0.03Xe^{-1.96X}\mathrm{SIN}(1.96X) - 0.08Xe^{-1.96X}\mathrm{COS}(1.96X)$$
$$+ 0.27$$

$K = 80.$

$$w_1 = 0.05e^{2.11X}\mathrm{COS}(2.11X) - 0.09e^{2.11X}\mathrm{SIN}(2.11X)$$
$$+ 0.05e^{-2.11X}\mathrm{COS}(2.11X) + 0.09e^{-2.11X}\mathrm{SIN}(2.11X)$$
$$+ 0.88$$

$$s_2 = 0.01\mathrm{SIN}(4.22X) + 0.64X$$

$$w_3 = -0.1e^{2.11X}\mathrm{COS}(2.11X) - 0.03e^{2.11X}\mathrm{SIN}(2.11X)$$
$$- 0.1e^{-2.11X}\mathrm{COS}(2.11X) + 0.03e^{-2.11X}\mathrm{SIN}(2.11X)$$

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$$- 0.01Xe^{2.11X}\mathrm{SIN}(\ 2.11X\ ) + 0.07Xe^{2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$- 0.01Xe^{-2.11X}\mathrm{SIN}(\ 2.11X\ ) - 0.07Xe^{-2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$+ 0.20$$

$K = 100.$

$$w_1 = 0.05e^{2.23X}\mathrm{COS}(\ 2.23X\ ) - 0.07e^{2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.05e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.07e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.88$$

$$s_2 = 0.66X$$

$$w_3 = - 0.08e^{2.23X}\mathrm{COS}(\ 2.23X\ ) - 0.03e^{2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$- 0.08e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.03e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.06Xe^{2.23X}\mathrm{COS}(\ 2.23X\ ) - 0.06e^{-2.23X}\mathrm{COS}(\ 2.23X\ )$$

$$+ 0.16$$

3. Loading Condition : $F = 1$

Boundary Condition : Clamped at $X=+1$ and Simply supported at $X=-1$

$K = 0.$

$$w_1 = 0.5X^4 + 0.5X^3 - 0.1X^2 - 0.5X + 1$$

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$$s_2 = -0.28X^7 - 0.5X^6 + 0.97X^5 + 1.37X^4 - 1.25X^3 - 0.75X^2$$

$$+ 0.56X - 0.12$$

$$w_3 = 0.13X^6 + 0.2X^5 - 0.24X^4 - 0.44X^3 + 0.1X^2 + 0.24X$$

$$K = 20.$$

$$w_1 = -0.37e^{1.49X}\mathrm{COS}(1.49X) - 0.3e^{1.49X}\mathrm{SIN}(1.49X)$$

$$-0.04e^{-1.49X}\mathrm{COS}(1.49X) + 0.32e^{-1.49X}\mathrm{SIN}(1.49X)$$

$$+ 1.41$$

$$s_2 = -0.05e^{2.98X}\mathrm{COS}(2.98X) - 0.01e^{-2.98X}\mathrm{COS}(2.98X)$$

$$- 0.08e^{2.98X} + 0.04e^{-2.98X} - 0.03e^{2.98X}\mathrm{SIN}(2.98X)$$

$$- 0.02e^{-2.98X}\mathrm{SIN}(2.98X) + 0.08\mathrm{SIN}(2.98X)$$

$$+ 0.08\mathrm{COS}(2.98X) + 0.98X - 0.09$$

$$w_3 = -0.42e^{1.49X}\mathrm{COS}(1.49X) + 0.31e^{1.49X}\mathrm{SIN}(1.49X)$$

$$- 0.51e^{-1.49X}\mathrm{COS}(1.49X) - 0.1e^{-1.49X}\mathrm{SIN}(1.49X)$$

$$- 0.46Xe^{1.49X}\mathrm{SIN}(1.49X) - 0.04Xe^{1.49X}\mathrm{COS}(1.49X)$$

$$- 0.25Xe^{-1.49X}\mathrm{SIN}(1.49X) - 0.19Xe^{-1.49X}\mathrm{COS}(1.49X)$$

$$+ 0.93$$

$$K = 40.$$

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$$w_1 = -\ 0.15e^{1.77X}COS(\ 1.77X\ ) - 0.23e^{1.77X}SIN(\ 1.77X\ )$$

$$+\ 0.03e^{-1.77X}COS(\ 1.77X\ ) + 0.19e^{-1.77X}SIN(\ 1.77X\ )$$

$$+\ 1.12$$

$$s_2 = -\ 0.01e^{3.54X}COS(\ 3.54X\ ) - 0.03e^{3.54X} + 0.01e^{-3.54X}$$

$$-\ 0.02e^{3.54X}SIN(\ 3.54X\ ) + 0.03SIN(\ 3.54X\ )$$

$$+\ 0.03COS(\ 3.54X\ ) + 0.75X - 0.12$$

$$w_3 = -\ 0.22e^{1.77X}COS(\ 1.77X\ ) + 0.1e^{1.77X}SIN(\ 1.77X\ )$$

$$-\ 0.24e^{-1.77X}COS(\ 1.77X\ ) - 0.22Xe^{1.77X}SIN(\ 1.77X\ )$$

$$+\ 0.04Xe^{1.77X}COS(\ 1.77X\ ) - 0.08Xe^{-1.77X}SIN(\ 1.77X\ )$$

$$-\ 0.13Xe^{-1.77X}COS(\ 1.77X\ ) + 0.46$$

$K = 60.$

$$w_1 = -0.08e^{1.96X}COS(\ 1.96X\ ) - 0.19e^{1.96X}SIN(\ 1.96X\ )$$

$$+\ 0.05e^{-1.96X}COS(\ 1.96X\ ) + 0.13e^{-1.96X}SIN(\ 1.96X\ )$$

$$+\ 1.02$$

$$s_2 = -\ 0.02e^{3.92X} + 0.01e^{-3.92X} - 0.01e^{3.92X}SIN(\ 3.92X\ )$$

$$+\ 0.02SIN(\ 3.92X\ ) + 0.02COS(\ 3.92X\ ) + 0.68X - 0.12$$

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$$w_3 = -0.15e^{1.96X}\text{COS}(\ 1.96X\ ) + 0.05e^{1.96X}\text{SIN}(\ 1.96X\ )$$

$$- 0.16e^{-1.96X}\text{COS}(\ 1.96X\ ) + 0.02e^{-1.96X}\text{SIN}(\ 1.96X\ )$$

$$- 0.14Xe^{1.96X}\text{SIN}(\ 1.96X\ ) + 0.05Xe^{1.96X}\text{COS}(\ 1.96X\ )$$

$$- 0.04Xe^{-1.96X}\text{SIN}(\ 1.96X\ ) - 0.1Xe^{-1.96X}\text{COS}(\ 1.96X\ )$$

$$+ 0.31$$

$K\ = 80.$

$$w_1 = -\ 0.04e^{2.11X}\text{COS}(\ 2.11X\ ) - 0.16e^{2.11X}\text{SIN}(\ 2.11X\ )$$

$$+ 0.06e^{-2.11X}\text{COS}(\ 2.11X\ ) + 0.1e^{-2.11X}\text{SIN}(\ 2.11X\ )$$

$$+ 0.98$$

$$s_2 = -\ 0.01e^{4.22X} - 0.01e^{4.22X}\text{SIN}(\ 4.22X\ ) + 0.01\text{SIN}(\ 4.22X\ )$$

$$+ 0.01\text{COS}(\ 4.22X\ ) + 0.66X - 0.13$$

$$w_3 = -\ 0.11e^{2.11X}\text{COS}(\ 2.11X\ ) + 0.03e^{2.11X}\text{SIN}(\ 2.11X\ )$$

$$- 0.11e^{-2.11X}\text{COS}(\ 2.11X\ ) + 0.03e^{-2.11X}\text{SIN}(\ 2.11X\ )$$

$$- 0.1Xe^{2.11X}\text{SIN}(\ 2.11X\ ) + 0.06Xe^{2.11X}\text{COS}(\ 2.11X\ )$$

$$- 0.01Xe^{-2.11X}\text{SIN}(\ 2.11X\ ) - 0.08Xe^{-2.11X}\text{COS}(\ 2.11X\ )$$

$$+ 0.23$$

$K\ = 100.$

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$$w_1 = -0.01e^{2.23X}\mathrm{COS}(2.23X) - 0.14e^{2.23X}\mathrm{SIN}(2.23X) + 0.06e^{-2.23X}\mathrm{COS}(2.23X) + 0.08e^{-2.23X}\mathrm{SIN}(2.23X) + 0.95$$

$$s_2 = -0.01e^{4.46X} + 0.01\mathrm{SIN}(4.46X) + 0.01\mathrm{COS}(4.46X) + 0.66X - 0.13$$

$$w_3 = -0.09e^{2.23X}\mathrm{COS}(2.23X) + 0.01e^{2.23X}\mathrm{SIN}(2.23X) - 0.09e^{-2.23X}\mathrm{COS}(2.23X) + 0.03e^{-2.23X}\mathrm{SIN}(2.23X) - 0.07Xe^{2.23X}\mathrm{SIN}(2.23X) + 0.05Xe^{2.23X}\mathrm{COS}(2.23X) - 0.06Xe^{-2.23X}\mathrm{COS}(2.23X) + 0.18$$

4. Loading Condition : $F = 1 + X$

Boundary Condition : Clamped both sides

$$K = 0.$$

$$w_1 = 0.2X^5 + X^4 - 0.4X^3 - 2X^2 + 0.2X + 1$$

$$s_2 = -0.05X^9 - 0.5X^8 - 0.97X^7 + 1.46X^6 + 3.01X^5 - 1.4X^4 - 2.58X^3 + 0.4X^2 + 0.59X + 0.03$$

$$w_3 = 0.03X^7 + 0.24X^6 - 0.49X^4 - 0.1X^3 + 0.24X^2 + 0.07X$$

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$K = 20.$

$$w_1 = -0.63e^{1.49X}\mathrm{COS}(1.49X) - 0.84e^{1.49X}\mathrm{SIN}(1.49X)$$
$$- 0.34e^{-1.49X}\mathrm{COS}(1.49X) + 0.01e^{-1.49X}\mathrm{SIN}(1.49X)$$
$$+ 1.98X + 1.98$$

$$s_2 = -0.14e^{2.98X}\mathrm{COS}(2.98X) + 0.02e^{-2.98X}\mathrm{COS}(2.98X)$$
$$- 0.47e^{2.98X} + 0.04e^{-2.98X} - 0.25e^{2.98X}\mathrm{SIN}(2.98X)$$
$$+ 0.02e^{-2.98X}\mathrm{SIN}(2.98X) + 0.17\mathrm{SIN}(2.98X)$$
$$- 0.2\mathrm{COS}(2.98X) + 1.26e^{1.49X}\mathrm{COS}(1.49X)$$
$$+ 1.66e^{1.49X}\mathrm{SIN}(1.49X) - e^{-1.49X}\mathrm{COS}(1.49X)$$
$$+ 0.61e^{-1.49X}\mathrm{SIN}(1.49X) - 0.66X + 1.18$$

$$w_3 = -e^{1.49X}\mathrm{COS}(1.49X) + 0.61e^{1.49X}\mathrm{SIN}(1.49X)$$
$$+ 0.09e^{-1.49X}\mathrm{COS}(1.49X) - 0.31e^{-1.49X}\mathrm{SIN}(1.49X)$$
$$- 0.93Xe^{1.49X}\mathrm{SIN}(1.49X) + 0.12Xe^{1.49X}\mathrm{COS}(1.49X)$$
$$- 0.22Xe^{-1.49X}\mathrm{SIN}(1.49X) + 0.2Xe^{-1.49X}\mathrm{COS}(1.49X)$$
$$+ 0.91X + 0.91$$

$K = 40.$

$$w_1 = -0.23e^{1.77X}\mathrm{COS}(1.77X) - 0.53e^{1.77X}\mathrm{SIN}(1.77X)$$

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$$- 0.15e^{-1.77X}COS(1.77X) + 0.04e^{-1.77X}SIN(1.77X)$$

$$+ 1.37X + 1.37$$

$$s_2 == - 0.14e^{3.54X} + 0.01e^{-3.54X} - 0.1e^{3.54X}SIN(3.54X)$$

$$+ 0.05SIN(3.54X) - 0.06COS(3.54X)$$

$$+ 0.32e^{1.77X}COS(1.77X) + 0.73e^{1.77X}SIN(1.77X)$$

$$- 0.47e^{-1.77X}COS(1.77X) + 0.19e^{-1.77X}SIN(1.77X)$$

$$- 0.02X - 0.26$$

$$w_3 = - 0.47e^{1.77X}COS(1.77X) + 0.19e^{1.77X}SIN(1.77X)$$

$$- 0.11e^{-1.77X}SIN(1.77X) - 0.41Xe^{1.77X}SIN(1.77X)$$

$$+ 0.16Xe^{1.77X}COS(1.77X) - 0.11Xe^{-1.77X}SIN(1.77X)$$

$$+ 0.05Xe^{-1.77X}COS(1.77X) + 0.47X + 0.47$$

$$K = 60.$$

$$w_1 = -0.1e^{1.96X}COS(1.96X) - 0.4e^{1.96X}SIN(1.96X)$$

$$-0.09e^{-1.96X}COS(1.96X) + 0.04e^{-1.96X}SIN(1.96X)$$

$$+ 1.19X + 1.19$$

$$s_2 = 0.01e^{3.92X}COS(3.92X) - 0.08e^{3.92X} - 0.05e^{3.92X}SIN(3.92X)$$

$$+ 0.02SIN(3.92X) - 0.03COS(3.92X) + 0.12e^{1.96X}COS(1.96X)$$

$$+ 0.48e^{1.96X}SIN(1.96X) - 0.31e^{-1.96X}COS(1.96X)$$

$$+ 0.09e^{-1.96X}SIN(1.96X) + 0.1X - 0.04$$

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$$w_3 = -0.31e^{1.96X}COS(1.96X) + 0.09e^{1.96X}SIN(1.96X)$$
$$-0.01e^{-1.96X}COS(1.96X) - 0.05e^{-1.96X}SIN(1.96X)$$
$$-0.25Xe^{1.96X}SIN(1.96X) + 0.15Xe^{1.96X}COS(1.96X)$$
$$-0.06e^{-1.96X}SIN(1.96X) + 0.02e^{-1.96X}COS(1.96X)$$
$$+0.32X + 0.32$$

$K = 80.$

$$w_1 = -0.03e^{2.11X}COS(2.11X) - 0.33e^{2.11X}SIN(2.11X)$$
$$-0.06e^{-2.11X}COS(2.11X) + 0.04e^{-2.11X}SIN(2.11X)$$
$$+ 1.09X + 1.09$$

$$s_2 = 0.02e^{4.22X}COS(4.22X) - 0.05e^{4.22X} - 0.03e^{4.22X}SIN(4.22X)$$
$$+ 0.01SIN(4.22X) - 0.01COS(4.22X)$$
$$+ 0.04e^{2.11X}COS(2.11X) + 0.35e^{2.11X}SIN(2.11X)$$
$$- 0.24e^{-2.11X}COS(2.11X) + 0.04e^{-2.11X}SIN(2.11X)$$
$$+ 0.17X + 0.06$$

$$w_3 = - 0.24e^{2.11X}COS(2.11X) + 0.04e^{2.11X}SIN(2.11X)$$
$$-0.01e^{-2.11X}COS(2.11X) - 0.03e^{-2.11X}SIN(2.11X)$$
$$-0.17Xe^{2.11X}SIN(2.11X) - 0.14e^{2.11X}COS(2.11X)$$
$$-0.04e^{-2.11X}SIN(2.11X) + 0.25X + 0.25$$

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$$K = 100.$$

$$w_1 = -\ 0.28e^{2.23X}SIN(\ 2.23X\ ) - 0.04e^{-2.23X}COS(\ 2.23X\ )$$

$$+\ 0.03e^{-2.23X}SIN(\ 2.23X\ ) + 1.04X + 1.04$$

$$s_2 = 0.02e^{4.46X}COS(\ 4.46X\ ) - 0.04e^{4.46X} - 0.02e^{4.46X}SIN(\ 4.46X\ )$$

$$+0.01COS(\ 4.46X\ ) - 0.29e^{2.23X}SIN(\ 2.23X\ )$$

$$+\ 0.19e^{-2.23X}COS(\ 2.23X\ ) - 0.02e^{-2.23X}SIN(\ 2.23X\ )$$

$$+\ 0.21X + 0.12$$

$$w_3 = -\ 0.19e^{2.23X}COS(\ 2.23X\ ) + 0.02e^{2.23X}SIN(\ 2.23X\ )$$

$$-\ 0.01e^{-2.23X}COS(\ 2.23X\ ) - 0.02e^{-2.23X}SIN(\ 2.23X\ )$$

$$-\ 0.13Xe^{2.23X}SIN(\ 2.23X\ ) + 0.13Xe^{2.23X}COS(\ 2.23X\ )$$

$$-\ 0.03Xe^{-2.23X}SIN(\ 2.23X\ ) + 0.2X + 0.2$$

VANIER LIBRARY

5. Loading Condition : $F = 1 + X$

Boundary Condition : Simply supported both sides

$$K = 0.$$

$$w_1 = 0.04X^5 + 0.2X^4 - 0.13X^3 - 1.2X^2 + 0.09X + 1$$

$$s_2 = -\ 0.002X^9 - 0.02X^8 - 0.34X^7 + 0.13X^6 + 0.36X^5 - 0.25X^4$$

$$-\ 0.94X^3 + 0.11X^2 + 0.61X + 0.03$$

$$w_3 = 0.007X^7 + 0.04X^6 + 0.07X^5 - 0.13X^4 - 0.28X^3 + 0.08X^2$$
$$+ 0.21X$$

$$K = 20.$$

$$w_1 = - 0.03e^{1.49X}\mathrm{COS}(1.49X) - 0.46e^{1.49X}\mathrm{SIN}(1.49X)$$
$$+ 0.02e^{-1.49X}\mathrm{SIN}(1.49X) + 1.04X + 1.04$$

$$s_2 = 0.03e^{2.98X}\mathrm{COS}(2.98X) - 0.08e^{2.98X} - 0.04e^{2.98X}\mathrm{SIN}(2.98X)$$
$$+ 0.03e^{1.49X}\mathrm{COS}(1.49X) + 0.48e^{1.49X}\mathrm{SIN}(1.49X)$$
$$- 0.73e^{-1.49X}\mathrm{COS}(1.49X) + 0.01e^{-1.49X}\mathrm{SIN}(1.49X)$$
$$+ 0.13X + 0.13$$

$$w_3 = -0.73e^{1.49X}\mathrm{COS}(1.49X) + 0.01e^{1.49X}\mathrm{SIN}(1.49X)$$
$$-0.06e^{-1.49X}\mathrm{COS}(1.49X) - 0.04e^{-1.49X}\mathrm{SIN}(1.49X)$$
$$-0.33Xe^{1.49X}\mathrm{SIN}(1.49X) + 0.28Xe^{1.49X}\mathrm{COS}(1.49X)$$
$$- 0.01Xe^{-1.49X}\mathrm{SIN}(1.49X) - 0.01Xe^{-1.49X}\mathrm{COS}(1.49X)$$
$$+ 0.79X + 0.79$$

$$K = 40.$$

$$w_1 = 0.06e^{1.77X}\mathrm{COS}(1.77X) - 0.3e^{1.77X}\mathrm{SIN}(1.77x)$$
$$+ 0.93X + 0.93$$

VANIER LIBRARY

$$s_2 = 0.02e^{3.54X}COS(3.54X) - 0.04e^{3.54X} - 0.01e^{3.54X}SIN(3.54X)$$
$$- 0.06e^{1.77X}COS(1.77X) + 0.28e^{1.77X}SIN(1.77X)$$
$$- 0.41e^{-1.77X}COS(1.77X) - 0.04e^{-1.77X}SIN(1.77X)$$
$$+ 0.29X + 0.29$$

$$w_3 = -0.41e^{1.77X}COS(1.77X) - 0.04e^{1.77X}SIN(1.77X)$$
$$-0.02e^{-1.77X}COS(1.77X) - 0.14Xe^{1.77X}SIN(1.77X)$$
$$+ 0.22Xe^{1.77X}COS(1.77X) + 0.43X + 0.43$$

$$K = 60.$$

$$w_1 = 0.09e^{1.96X}COS(1.96X) - 0.23e^{1.96X}SIN(1.96X)$$
$$+ 0.89X + 0.89$$

$$s_2 = 0.02e^{3.92X}COS(3.92X) - 0.03e^{-3.92X} -0.08e^{1.96X}COS(1.96X)$$
$$+ 0.2e^{1.96X}SIN(1.96X) - 0.3e^{-1.96X}COS(1.96X)$$
$$- 0.06e^{-1.96X}SIN(1.96X) + 0.38X + 0.38$$

$$w_3 = -0.3e^{1.96X}COS(1.96X) - 0.06e^{1.96X}SIN(1.96X)$$
$$-0.01e^{-1.96X}COS(1.96X) - 0.08Xe^{1.96X}SIN(1.96X)$$
$$+ 0.19Xe^{1.96X}COS(1.96X) + 0.31X + 0.31$$

VANIER LIBRARY

$K = 80.$

$$w_1 = 0.11e^{2.11X}\cos(2.11X) - 0.18e^{2.11X}\sin(2.11X)$$
$$+ 0.88X + 0.88$$

$$s_2 = 0.01e^{4.22X}\cos(4.22X) - 0.02e^{4.22X} - 0.09e^{2.11X}\cos(2.11X)$$
$$+ 0.16e^{2.11X}\sin(2.11X) - 0.25e^{-2.11X}\cos(2.11X)$$
$$- 0.07e^{-2.11X}\sin(2.11X) + 0.46X + 0.46$$

$$w_3 = -0.25e^{2.11X}\cos(2.11X) - 0.07e^{2.11X}\sin(2.11X)$$
$$-0.04Xe^{2.11X}\sin(2.11X) + 0.17Xe^{2.11X}\cos(2.11X)$$
$$+ 0.25X + 0.25$$

$K = 100.$

$$w_1 = 0.11e^{2.23X}\cos(2.23X) - 0.14e^{2.23X}\sin(2.23X)$$
$$+ 0.88X + 0.88$$

$$s_2 = 0.01e^{4.46X}\cos(4.46X) - 0.01e^{4.46X}$$
$$- 0.1e^{2.23X}\cos(2.23X) + 0.13e^{2.23X}\sin(2.23X)$$
$$- 0.21e^{-2.23X}\cos(2.23X) - 0.08e^{-2.23X}\sin(2.23X)$$
$$+ 0.52X + 0.52$$

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$$w_3 = -0.21e^{2.23X}COS(\ 2.23X\ ) - 0.08e^{2.23X}SIN(\ 2.23X\ )$$

$$-0.01Xe^{2.23X}SIN(\ 2.23X\ ) + 0.16Xe^{2.23X}COS(\ 2.23X\ )$$

$$+ 0.21X + 0.21$$

6. Loading Condition : F = 1 + X

Boundary Condition : Clamped at X = +1 and Simply supported at X = -1

$K = 0.$

$$w_1 = 0.11X^5 + 0.55X^4 + 0.22X^3 - 1.55X^2 - 0.33X + 1$$

$$s_2 = -0.01X^9 - 0.15X^8 - 0.4X^7 + 0.04X^6 + 1.37X^5 + 0.7X^4$$

$$-1.53X^3 - 0.51X^2 + 0.58X - 0.07$$

$$w_3 = 0.02X^7 + 0.14X^6 - 0.24X^5 - 0.2X^4 - 0.63X^3 + 0.06X^2 + 0.36X$$

$K = 20.$

$$w_1 = -0.51e^{1.49X}COS(\ 1.49X\ ) - 0.65e^{1.49X}SIN(\ 1.49X\ )$$

$$-0.03e^{-1.49X}COS(\ 1.49X\ ) + 0.02e^{-1.49X}SIN(\ 1.49X\ )$$

$$+ 1.54X + 1.54$$

$$s_2 = -0.09e^{2.98X}COS(\ 2.98X\ ) - 0.25e^{2.98X} - 0.15e^{2.98X}SIN(2.98X)$$

$$+ 0.02SIN(\ 2.98X\ ) + 0.79e^{1.49X}COS(\ 1.49X\ ) + 1.01e^{1.49X}$$

VANIER LIBRARY

$\mathrm{SIN}(1.49X) - 0.83e^{-1.49X}\mathrm{COS}(1.49X) + 0.39e^{-1.49X}\mathrm{SIN}(1.49X)$

$- 0.49X - 0.49$

$$w_3 = -0.83e^{1.49X}\mathrm{COS}(1.49X) + 0.39e^{1.49X}\mathrm{SIN}(1.49X)$$
$$- 0.03e^{-1.49X}\mathrm{COS}(1.49X) - 0.09e^{-1.49X}\mathrm{SIN}(1.49X)$$
$$- 0.72Xe^{1.49X}\mathrm{SIN}(1.49X) + 0.08Xe^{1.49X}\mathrm{COS}(1.49X)$$
$$- 0.03Xe^{-1.49X}\mathrm{SIN}(1.49X) + 0.87X + 0.87$$

$K = 40.$

$$w_1 = -0.2e^{1.77X}\mathrm{COS}(1.77X) - 0.45e^{1.77X}\mathrm{SIN}(1.77X)$$
$$+ 0.01e^{-1.77X}\mathrm{SIN}(1.77X) + 1.2X + 1.2$$

$$s_2 = -0.11e^{3.54X} - 0.07e^{3.54X}\mathrm{SIN}(3.54X)$$
$$+ 0.24e^{1.77X}\mathrm{COS}(1.77X) + 0.55e^{1.77X}\mathrm{SIN}(1.77X)$$
$$- 0.39e^{-1.77X}\mathrm{COS}(1.77X) + 0.13e^{-1.77X}\mathrm{SIN}(1.77X)$$
$$- 0.1X - 0.1$$

$$w_3 = -0.39e^{1.77X}\mathrm{COS}(1.77X) + 0.13e^{1.77X}\mathrm{SIN}(1.77X)$$
$$- 0.02e^{-1.77X}\mathrm{COS}(1.77X) - 0.01e^{-1.77X}\mathrm{SIN}(1.77X)$$
$$- 0.33Xe^{1.77X}\mathrm{SIN}(1.77X) + 0.13Xe^{1.77X}\mathrm{COS}(1.77X) + 0.42X$$
$$+ 0.42$$

VANIER LIBRARY

$K = 60.$

$$w_1 = -0.09e^{1.96X}\mathrm{COS}(\ 1.96X\ ) - 0.36e^{1.96X}\mathrm{SIN}(\ 1.96X)$$

$$+ 1.08X + 1.08$$

$$s_2 = 0.01e^{3.92X}\mathrm{COS}(\ 3.92X\ ) - 0.07e^{3.92X} - 0.04e^{3.92X}\mathrm{SIN}(\ 3.92X\ )$$

$$+ 0.09e^{1.96X}\mathrm{COS}(\ 1.96X\ ) + 0.4e^{1.96X}\mathrm{SIN}(\ 1.96X\ )$$

$$- 0.27e^{-1.96X}\mathrm{COS}(\ 1.95X\ ) + 0.07e^{-1.96X}\mathrm{SIN}(\ 1.96X\ )$$

$$+ 0.02X + 0.02$$

$$w_3 = -0.27e^{1.96X}\mathrm{COS}(\ 1.96X\ ) + 0.07e^{1.96X}\mathrm{SIN}(\ 1.96X\ )$$

$$- 0.01e^{-1.96X}\mathrm{COS}(\ 1.96X\ ) - 0.21Xe^{1.96X}\mathrm{SIN}(\ 1.96X\ )$$

$$+ 0.12Xe^{1.96X}\mathrm{COS}(\ 1.96X\ ) + 0.28X + 0.28$$

$K = 80.$

$$w_1 = -0.03e^{2.11X}\mathrm{COS}(\ 2.11X\ ) - 0.31e^{2.11X}\mathrm{SIN}(\ 2.11X\ )$$

$$+ 1.03X + 1.03$$

$$s_2 = 0.01e^{4.22X}\mathrm{COS}(\ 4.22X\ ) - 0.05e^{4.22X} - 0.03e^{4.22X}\mathrm{SIN}(\ 4.22X)$$

$$-0.03e^{2.11X}\mathrm{COS}(\ 2.11X\ ) + 0.32e^{2.11X}\mathrm{SIN}(\ 2.11X\ )$$

$$-0.2e^{-2.11X}\mathrm{COS}(\ 2.11X\ ) + 0.04e^{-2.11X}\mathrm{SIN}(\ 2.11X\ )$$

$$+ 0.09X + 0.09$$

VANIER LIBRARY

$$w_3 = -0.2e^{2.11X}COS(\ 2.11X\ ) + 0.04e^{2.11X}SIN(\ 2.11X\ )$$

$$-0.15Xe^{2.11X}SIN(\ 2.11X\ ) + 0.12Xe^{2.11X}COS(\ 2.11X\ )$$

$$+ 0.21X + 0.21$$

$K = 100.$

$$w_1 = -0.26e^{2.23X}SIN(\ 2.23X\ ) + 0.99X + 0.99$$

$$s_2 = 0.02e^{4.46X}COS(\ 4.46X\ ) - 0.04e^{4.46X}$$

$$- 0.02e^{4.46X}SIN(\ 4.46X\ ) + 0.26e^{2.23X}SIN(\ 2.23X\ )$$

$$- 0.17e^{2.23X}COS(\ 2.23X\ ) + 0.02e^{-2.23X}SIN(\ 2.23X\ )$$

$$+ 0.14X + 0.14$$

$$w_3 = -0.17e^{2.23X}COS(\ 2.23X\ ) + 0.02e^{2.23X}SIN(\ 2.23X\ )$$

$$-0.11Xe^{2.23X}SIN(\ 2.23X\ ) + 0.11Xe^{2.23X}COS(\ 2.23X\ )$$

$$+ 0.17X + 0.17$$

VANIER LIBRARY

7. Loading Condition : $F = 1 + X^2$

Boundary Condition : Clamped both sides

$K = 0.$

$$w_1 = 0.05X^6 + 0.88X^4 - 1.94X^2 + 1$$

$$s_2 = -0.13X^9 - 0.69X^7 + 2.74X^5 - 2.51X^3 + 0.6X$$

$$w_3 = 0.04X^6 + 0.64X^4 + 0.27X^2$$

$$K = 20.$$

$$w_1 = -0.37e^{1.49X}COS(\ 1.49X\ ) - 0.79e^{1.49X}SIN(\ 1.49X\ )$$

$$-0.37e^{-1.49X}COS(\ 1.49X\ ) + 0.79e^{-1.49X}SIN(\ 1.49X\ )$$

$$+ 1.75X^2 + 1.75$$

$$s_2 = -0.01e^{2.98X}COS(\ 2.98X\ ) + 0.01e^{-2.98X}COS(\ 2.98X\ )$$

$$-0.29e^{2.98X} + 0.29e^{2.98X} - 0.29e^{-2.98X}$$

$$-0.2e^{2.98X}SIN(\ 2.98X\ ) + 0.2e^{-2.98X}SIN(\ 2.98X\ )$$

$$+0.58SIN(\ 2.98X\ ) - 0.44e^{1.49X}[\ COS(\ 1.49X\ ) + SIN(\ 1.49X)]$$

$$-0.93e^{1.49X}[\ SIN(\ 1.49X\ ) - COS(\ 1.49X\ )]$$

$$+0.44e^{-1.49X}[\ COS(\ 1.49X\ ) - SIN(\ 1.49X\ )]$$

$$-0.93e^{-1.49X}[\ SIN(\ 1.49X\ ) + COS(\ 1.49X\ )]$$

$$+1.32[\ Xe^{1.49X}COS(\ 1.49X\ ) + (e^{1.49X}\ /\ 1.49\ )[\ COS(\ 1.49X\ )$$

$$-\ SIN(\ 1.49X\ )]] + 2.8[\ Xe^{1.49X}SIN(\ 1.49X\ ) + (e^{1.49X}/1.49)$$

$$[\ SIN(\ 1.49X\ ) - COS(\ 1.49X\ )]] + 1.32[\ Xe^{-1.49X}COS(\ 1.49X\ )$$

$$+\ (e^{-1.49X}/1.49)[\ COS(\ 1.49X\ ) - SIN(\ 1.49X\ )]] - 2.8[$$

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$$Xe^{-1.49X}\text{SIN}(1.49X) + (e^{-1.49X}/1.49)[\text{SIN}(1.49X)$$

$$+ \text{COS}(1.49X)]] - 2.05X^3 + 1.95X$$

$$w_3 = -1.03e^{1.49X}\text{COS}(1.49X) + 0.18e^{1.49X}\text{SIN}(1.49X)$$

$$- 1.03e^{-1.49X}\text{COS}(1.49X) - 0.18e^{-1.49X}\text{SIN}(1.49X)$$

$$+ 0.78X^2 - 0.72Xe^{1.49X}\text{SIN}(1.49X) + 0.25Xe^{1.49X}\text{COS}(1.49X)$$

$$- 0.72Xe^{-1.49X}\text{SIN}(1.49X) - 0.25Xe^{-1.49X}\text{COS}(1.49X)$$

$$+ 2.07$$

$$K = 40.$$

$$w_1 = -0.11e^{1.77X}\text{COS}(1.77X) - 0.46e^{1.77X}\text{SIN}(1.77X)$$

$$- 0.11e^{-1.77X}\text{COS}(1.77X) + 0.46e^{-1.77X}\text{SIN}(1.77X)$$

$$+ 1.23X^2 + 1.23$$

$$s_2 = 0.02e^{3.54X}\text{COS}(3.54X) - 0.02e^{-3.54X}\text{COS}(3.54X)$$

$$-0.1e^{3.54X} + 0.06e^{3.54X}\text{SIN}(3.54X) - 0.06e^{-3.54X}\text{SIN}(3.54X)$$

$$+ 0.2\text{SIN}(3.54X) - 0.08e^{1.77X}[\text{COS}(1.77X) + \text{SIN}(1.77X)]$$

$$- 0.32e^{1.77X}[\text{SIN}(1.77X) - \text{COS}(1.77X)] + 0.08$$

$$e^{-1.77X}[\text{COS}(1.77X) - \text{SIN}(1.77X)] - 0.32e^{-1.77X}$$

$$[\text{SIN}(1.77X) + \text{COS}(1.77X)] + 0.28[Xe^{1.77X}\text{COS}(1.77X)$$

$$- (e^{1.77X}/1.77)[\text{COS}(1.77X) + \text{SIN}(1.77X)]]$$

$$+ 1.14[Xe^{1.77X}\text{SIN}(1.77X) - (e^{1.77X}/1.77)$$

VANIER LIBRARY

$$[\ \mathrm{SIN}(1.77X) - \mathrm{COS}(1.77X)]] + 0.28[\ Xe^{-1.77X}\mathrm{COS}(1.77X)$$

$$+ (e^{-1.77X}/1.77)[\ \mathrm{COS}(1.77X) - \mathrm{SIN}(1.77X)]]$$

$$-1.14[\ Xe^{-1.77X}\mathrm{SIN}(1.77X) + (e^{-1.77X}/1.77)$$

$$[\ \mathrm{SIN}(1.77X) + \mathrm{COS}(1.77X)]] - 1.01X^3 + 0.95$$

$$w_3 = -0.42e^{1.77X}\mathrm{COS}(1.77X) + 0.03e^{1.77X}\mathrm{SIN}(1.77X)$$

$$-0.42e^{-1.77X}\mathrm{COS}(1.77X) - 0.03e^{-1.77X}\mathrm{SIN}(1.77X)$$

$$+0.39X^2 - 0.3Xe^{1.77X}\mathrm{SIN}(1.77X) + 0.18Xe^{1.77X}\mathrm{COS}(1.77X)$$

$$-0.3Xe^{-1.77X}\mathrm{SIN}(1.49X) - 0.18Xe^{-1.77X}\mathrm{COS}(1.77X) + 0.85$$

$K = 60.$

$$w_1 = -0.03e^{1.96X}\mathrm{COS}(1.96X) - 0.34e^{1.96X}\mathrm{SIN}(1.96X)$$

$$-0.03e^{-1.96X}\mathrm{COS}(1.96X) + 0.34e^{-1.96X}\mathrm{SIN}(1.96X)$$

$$+1.06X^2 + 1.06$$

$$s_2 = 0.02e^{3.92X}\mathrm{COS}(3.92X) - 0.02e^{-3.92X}\mathrm{COS}(3.92X)$$

$$-0.05e^{3.92X} + 0.03e^{3.92X}\mathrm{SIN}(3.92X)$$

$$-0.03e^{-3.92X}\mathrm{SIN}(3.92X) - 0.11\mathrm{SIN}(3.92X)$$

$$+0.01e^{1.96X}[\ \mathrm{COS}(1.96X) + \mathrm{SIN}(1.96X)]$$

$$-0.18e^{1.96X}[\ \mathrm{SIN}(1.96X) - \mathrm{COS}(1.96X)]$$

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$$+ 0.01e^{-1.96X}[\cos(1.96X) - \sin(1.96X)]$$

$$- 0.18e^{-1.96X}[\sin(1.96X) + \cos(1.96X)]$$

$$+ 0.06[Xe^{1.96X}\cos(1.96X) - (e^{1.96X}/1.96)[\cos(1.96X)$$

$$+ \sin(1.96X)]] + 0.72[Xe^{1.96X}\sin(1.96X)$$

$$- (e^{1.96X}/1.96)[\sin(1.96X) - \cos(1.96X)]]$$

$$+ 0.06[Xe^{-1.96X}\cos(1.96X) + (e^{-1.96X}/1.96)[\cos(1.96X)$$

$$- \sin(1.96X)]] - 0.72[Xe^{-1.96X}\sin(1.96X)$$

$$+ (e^{-1.96X}/1.96)[\sin(1.96X) + \cos(1.96X)]]$$

$$- 0.75X^3 + 0.7$$

$$w_3 = -0.26e^{1.96X}\cos(1.96X) - 0.26e^{-1.96X}\cos(1.96X)$$

$$+0.17Xe^{-1.96X}\sin(1.96X) - 0.14Xe^{-1.96X}\cos(1.96X)$$

$$+ 0.53$$

$K = 80.$

$$w_1 = - 0.27e^{2.11X}\sin(2.11X) + 0.27e^{-2.11X}\sin(2.11X)$$

$$+ 0.98X^2 + 0.98$$

$$s_2 = 0.02e^{4.22X}\cos(4.22X) - 0.02e^{-4.22X}\cos(4.22X)$$

$$- 0.03e^{4.22X} + 0.03e^{-4.22X} - 0.01e^{4.22X}\sin(4.22X)$$

$$- 0.01e^{-4.22X}\sin(4.22X) + 0.07\sin(4.22X)$$

VANIER LIBRARY

$$- 0.12e^{2.11X}[\ \mathrm{SIN}(\ 2.11X\ ) + \mathrm{COS}(\ 2.11X\ )]$$

$$- 0.12e^{-2.11X}[\ \mathrm{SIN}(\ 2.11X\ ) + \mathrm{COS}(\ 2.11X\ ) - 0.01$$

$$[\ Xe^{2.11X}\mathrm{COS}(\ 2.11X\ ) - (\ e^{2.11X}/2.11\ )[\ \mathrm{COS}(\ 2.11X\ )$$

$$+ \mathrm{SIN}(\ 2.11X\ )]] + 0.53[\ Xe^{2.11X}\mathrm{SIN}(\ 2.11X\ ) - (\ e^{2.11X}/2.11)$$

$$[\ \mathrm{SIN}(\ 2.11X\ ) - \mathrm{COS}(\ 2.11X\ )]] - 0.01[\ Xe^{-2.11X}$$

$$\mathrm{COS}(\ 2.11X\ ) + (\ e^{-2.11X}/2.11\ )[\ \mathrm{COS}(\ 2.11X\ )$$

$$- \mathrm{SIN}(\ 2.11X\ )]] - 0.53[\ Xe^{-2.11X}\mathrm{SIN}(\ 2.11X\ )$$

$$+ (\ e^{-2.11X}/2.11\ )[\ \mathrm{SIN}(\ 2.11X\ ) + \mathrm{COS}(\ 2.11X\ )]]$$

$$- 0.64X^3 + 0.6$$

$$w_3 = -0.19e^{2.11X}\mathrm{COS}(\ 2.11X\ ) - 0.19e^{-2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$+0.2X^2 - 0.11Xe^{2.11X}\mathrm{SIN}(\ 2.11X\ ) + 0.12Xe^{2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$-0.11Xe^{-2.11X}\mathrm{SIN}(\ 2.11X\ ) - 0.12Xe^{-2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$+ 0.38$$

$K = 100.$

$$w_1 = 0.03e^{2.23X}\mathrm{COS}(\ 2.23X\ ) - 0.23e^{2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.03e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.23e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.93X^2 + 0.93$$

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$$
\begin{aligned}
s_2 = {} & 0.01e^{4.46X}\cos(4.46X) - 0.01e^{-4.46X}\cos(4.46X) \\
& -0.03e^{4.46X} + 0.03e^{-4.46X} - 0.01e^{4.46X}\sin(4.46X) \\
& -0.01e^{4.46X}\sin(4.46X) + 0.06e^{-4.46X}\sin(4.46X) \\
& + 0.01\cos(4.46X) - 0.13e^{2.23X}[\cos(2.23X) \\
& + \sin(2.23X)] - 0.09e^{2.23X}[\sin(2.23X) \\
& - \cos(2.23X)] - 0.01e^{-2.23X}[\cos(2.23X) \\
& - \sin(2.23X)] - 0.09e^{-2.23X}[\sin(2.23X) \\
& + \cos(2.23X)] - 0.06[Xe^{2.23X}\cos(2.23X) \\
& - (e^{2.23X}/2.23)[\cos(2.23X) + \sin(2.23X)]] \\
& + 0.42[Xe^{2.23X}\sin(2.23X) - (e^{2.23X}/2.23) \\
& [\sin(2.23X) - \cos(2.23X)]] - 0.06[X \\
& e^{-2.23X}\cos(2.23X) + (e^{-2.23X}/2.23)[\cos(2.23X) \\
& - \sin(2.23X)]] + 0.42[Xe^{-2.23X}\sin(2.23X) \\
& + (e^{-2.23X}/2.23)[\sin(2.23X) + \cos(2.23X)]] \\
& - 0.58X^3 + 0.55
\end{aligned}
$$

$$
\begin{aligned}
w_3 = {} & -0.15e^{2.23X}\cos(2.23X) - 0.01e^{2.23X}\sin(2.23X) \\
& - 0.15e^{-2.23X}\cos(2.23X) + 0.01e^{-2.23X}\sin(2.23X) \\
& + 0.16X^2 - 0.08Xe^{2.23X}\sin(2.23X)
\end{aligned}
$$

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$$+ 0.11Xe^{-2.23X}\cos(2.23X) - 0.30$$

8. Loading Condition : $F = 1 + X^2$

Boundary Condition : Simply supported both sides

$K = 0.$

$$w_1 = 0.01X^6 + 0.16X^4 - 1.17X^2 + 1$$

$$s_2 = 0.31X^5 - 0.92X^3 + 0.62X$$

$$w_3 = 0.03X^6 + 0.51X^4 + 0.14X^2$$

$K = 20.$

$$w_1 = 0.05e^{1.49X}\cos(1.49X) - 0.42e^{1.49X}\sin(1.49X)$$
$$+ 0.05e^{-1.49X}\cos(1.49X) + 0.42e^{-1.49X}\sin(1.49X)$$
$$+ 0.88X^2 + 0.88$$

$$s_2 = 0.04e^{2.98X}\cos(2.98X) - 0.04e^{-2.98X}\cos(2.98X)$$
$$- 0.06e^{2.98X} + 0.06e^{-2.98X} - 0.02e^{2.98X}\sin(2.98X)$$
$$- 0.02e^{-2.98X}\sin(2.98X) + 0.13\sin(2.98X)$$
$$+ 0.03e^{1.49X}[\cos(1.49X) + \sin(1.49X)]$$

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$$- 0.25e^{1.49X}[\ SIN(\ 1.49X\ ) - COS(\ 1.49X\ )]$$

$$- 0.03e^{-1.49X}[\ COS(\ 1.49X\ ) - SIN(\ 1.49X\ )]$$

$$- 0.25e^{-1.49X}[\ SIN(\ 1.49X\ ) + COS(\ 1.49X\ )]$$

$$- 0.1[\ Xe^{1.49X}COS(\ 1.49X\ ) - (\ e^{1.49X}/1.49\ )$$

$$[\ COS(\ 1.49X\ ) + SIN(\ 1.49X\ )]] + 0.74[\ Xe^{1.49X}$$

$$SIN(\ 1.49X\ ) - (\ e^{1.49X}/\ 1.49\ )[\ SIN(\ 1.49X\ )$$

$$- COS(\ 1.49X\ )]] - 0.1[\ Xe^{-1.49X}COS(\ 1.49X\ )$$

$$+ (\ e^{-1.49X}/1.49\ )[\ COS(\ 1.49X\ ) - SIN(\ 1.49X\ )]]$$

$$- 0.74[\ Xe^{-1.49X}SIN(\ 1.49X\ ) + (\ e^{-1.49X}/1.49\ )$$

$$+ COS(\ 1.49X\ )]] - 0.52X^3 + 0.54X$$

$$w_3 = -0.67e^{1.49X}COS(\ 1.49X\ ) - 0.18e^{1.49X}SIN(\ 1.49X\ )$$

$$- 0.67e^{-1.49X}COS(\ 1.49X\ ) + 0.18e^{-1.49X}SIN(\ 1.49X\ )$$

$$+ 0.66X^2 - 0.23Xe^{1.49X}SIN(\ 1.49X\ )$$

$$+ 0.31Xe^{1.49X}COS(\ 1.49X\ ) - 0.23Xe^{-1.49X}SIN(\ 1.49X\ )$$

$$- 0.31Xe^{-1.49X}COS(\ 1.49X\ ) + 1.35$$

$K = 40.$

$$w_1 = +\ 0.1e^{1.77X}COS(\ 1.77X\ ) - 0.25e^{1.77X}SIN(\ 1.77X\ )$$

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$$+ 0.1e^{-1.77X}COS(1.77X) + 0.25e^{-1.77X}SIN(1.77X)$$

$$+ 0.79X^2 + 0.79$$

$$s_2 = 0.02e^{3.54X}COS(3.54X) - 0.02e^{-3.54X}COS(3.54X)$$

$$- 0.03e^{3.54X} + 0.03e^{-3.54X} + 0.06SIN(3.54X)$$

$$+ 0.04e^{1.77X}[COS(1.77X) + SIN(1.77X)]$$

$$- 0.11e^{1.77X}[SIN(1.77X) - COS(1.77X)]$$

$$- 0.16[Xe^{1.77X}COS(1.77X) - (e^{1.77X}/1.77)$$

$$[COS(1.77X) + SIN(1.77X)]] + 0.41[Xe^{1.77X}$$

$$SIN(1.77X) - (e^{1.77X}/1.77)[SIN(1.77X)$$

$$- COS(1.77X)]] - 0.16[Xe^{-1.77X}COS(1.77X)$$

$$+ (e^{-1.77X}/1.77)[COS(1.77X) - SIN(1.77X)]]$$

$$- 0.41[Xe^{-1.77X}SIN(1.77X) + (e^{-1.77X}/1.77)$$

$$[SIN(1.77X) + COS(1.77X)]] - 0.42X^3 + 0.51X$$

$$w_3 = - 0.34e^{1.77X}COS(1.77X) - 0.11e^{1.77X}SIN(1.77X)$$

$$- 0.35X^2 + 0.09Xe^{1.77X}SIN(1.77X) + 0.2Xe^{1.77X}COS(1.77X)$$

$$- 0.2Xe^{-1.77X}COS(1.77X) + 0.68$$

$K = 60.$

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$$w_1 = 0.11e^{1.96X}\cos(1.96X) - 0.18e^{1.96X}\sin(1.96X)$$

$$+ 0.11e^{-1.96X}\cos(1.96X) + 0.18e^{-1.96X}\sin(1.96X)$$

$$+ 0.77X^2 + 0.77$$

$$s_2 = 0.01e^{3.92X}\cos(3.92X) - 0.01e^{-3.92X}\cos(3.92X)$$

$$- 0.02e^{3.92X} + 0.02e^{3.92X} + 0.02e^{-3.92X} + 0.04\sin(3.92X)$$

$$+ 0.04e^{1.96X}[\cos(1.96X) - \sin(1.96X)]$$

$$- 0.07e^{1.96X}[\sin(1.96X) - \cos(1.96X)]$$

$$- 0.04e^{-1.96X}[\cos(1.96X) - \sin(1.96X)]$$

$$- 0.07e^{-1.96X}[\sin(1.96X) + \cos(1.96X)]$$

$$- 0.17[Xe^{1.96X}\cos(1.96X) - (e^{1.96X}/1.96)$$

$$[\cos(1.96X) + \sin(1.96X)]] + 0.29[Xe^{1.96X}$$

$$\sin(1.96X) - (e^{1.96X}/1.96)[\sin(1.96X)$$

$$- \cos(1.96X)]] - 0.17[Xe^{-1.96X}\cos(1.96X)$$

$$+ (e^{-1.96X}\cos(1.96X) - (e^{-1.96X}/1.96)$$

$$[\cos(1.96X) - \sin(1.96X)]] - 0.29[Xe^{-1.96X}\sin(1.96X)$$

$$+ (e^{-1.96X}/1.96)[\sin(1.96X) + \cos(1.96X)]]$$

$$- 0.39X^3 + 0.55X$$

$$w_3 = - 0.23e^{1.96X}\cos(1.96X) - 0.1e^{1.96X}\sin(1.96X)$$

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$$- 0.23e^{-1.96X}\cos(1.96X) + 0.1e^{-1.96X}\sin(1.96X)$$

$$+ 0.24X^2 - 0.04Xe^{1.96X}\sin(1.96X) + 0.16Xe^{1.96X}\cos(1.96X)$$

$$- 0.04Xe^{-1.96X}\sin(1.96X) - 0.16Xe^{-1.96X}\cos(1.96X) + 0.47$$

K = 80.

$$w_1 = 0.11e^{2.11X}\cos(2.11X) - 0.14e^{2.11X}\sin(2.11X)$$

$$+ 0.11e^{-2.11X}\cos(2.11X) + 0.14e^{-2.11X}\sin(2.11X)$$

$$+ 0.76X^2 + 0.76$$

$$s_2 = 0.01e^{4.22X}\cos(4.22X) - 0.01e^{-4.22X}\cos(4.22X)$$

$$- 0.01e^{4.22X} + 0.01e^{4.22X} + 0.03\sin(4.22X)$$

$$+ 0.04e^{2.11X}[\cos(2.11X) + \sin(2.11X)]$$

$$- 0.05e^{2.11X}[\sin(2.11X) - \cos(2.11X)]$$

$$- 0.04e^{-2.11X}[\cos(2.11X) + \sin(2.11X)]$$

$$- 0.05e^{-2.11X}[\sin(2.11X) - \cos(2.11X)]$$

$$+ 0.17[Xe^{2.11X}\cos(2.11X) + (e^{2.11X}/2.11)$$

$$[\cos(2.11X) - \sin(2.11X)]] - 0.22[Xe^{2.11X}$$

$$\sin(2.11X) - (e^{2.11X}/2.11)[\sin(2.11X)$$

$$- \cos(2.11X)]] - 0.17[Xe^{-2.11X}\cos(2.11X)$$

$$+ (e^{-2.11X}/2.11)[\cos(2.11X) - \sin(2.11X)]]$$

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$$- 0.22[Xe^{-2.11X}\mathrm{SIN}(2.11X) + (e^{-2.11X}/2.11)$$

$$[\mathrm{SIN}(2.11X) + \mathrm{COS}(2.11X)]] - 0.39X^3 + 0.6X$$

$$w_3 = -0.18e^{2.11X}\mathrm{COS}(2.11X) - 0.09e^{2.11X}\mathrm{SIN}(2.11X)$$

$$-0.18e^{-2.11X}\mathrm{COS}(2.11X) + 0.09e^{-2.11X}\mathrm{SIN}(2.11X)$$

$$+0.19X^2 - 0.01Xe^{2.11X}\mathrm{SIN}(2.11X) + 0.14Xe^{2.11X}\mathrm{COS}(2.11X)$$

$$-0.14Xe^{-2.11X}\mathrm{COS}(2.11X) + 0.37$$

$K = 100.$

$$w_1 = 0.11e^{2.23X}\mathrm{COS}(2.23X) - 0.11e^{2.23X}\mathrm{SIN}(2.23X)$$

$$+ 0.11e^{-2.23X}\mathrm{COS}(2.23X) + 0.11e^{-2.23X}\mathrm{SIN}(2.23X)$$

$$+ 0.76X^2 + 0.76$$

$$s_2 = -0.01e^{4.46X} + 0.01e^{-4.46X} + 0.03\mathrm{SIN}(4.46X)$$

$$+ 0.03e^{2.23X}[\mathrm{COS}(2.23X) + \mathrm{SIN}(2.23X)]$$

$$- 0.04e^{2.23X}[\mathrm{SIN}(2.23X) - \mathrm{COS}(2.23X)]$$

$$- 0.03e^{-2.23X}[\mathrm{COS}(2.23X) - \mathrm{SIN}(2.23X)]$$

$$- 0.04e^{-2.23X}[\mathrm{SIN}(2.23X) + \mathrm{COS}(2.23X)]$$

$$- 0.17[Xe^{2.23X}\mathrm{COS}(2.23X) - (e^{2.23X}/2.23)$$

$$[\mathrm{COS}(2.23X) + \mathrm{SIN}(2.23X)]] + 0.18[Xe^{2.23X}$$

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$$\text{SIN}(2.23X) - (e^{2.23X}/2.23)[\text{SIN}(2.23X)$$

$$+ \text{COS}(2.23X)]] - 0.17[Xe^{-2.23X}\text{COS}(2.23X)$$

$$+ (e^{-2.23X}/2.23)[\text{COS}(2.23X) - \text{SIN}(2.23X)]]$$

$$- 0.18[Xe^{-2.23X}\text{SIN}(2.23X) + (e^{-2.23X}/2.23)$$

$$[\text{SIN}(2.23X) + \text{COS}(2.23X)]] - 0.39X^3 + 0.66X$$

$$w_3 = -0.15e^{2.23X}\text{COS}(2.23X) - 0.08e^{2.23X}\text{SIN}(2.23X)$$

$$-0.15e^{-2.23X}\text{COS}(2.23X) + 0.08e^{-2.23X}\text{SIN}(2.23X)$$

$$+ 0.16X^2 + 0.12Xe^{2.23X}\text{COS}(2.23X) - 0.12Xe^{-2.23X}$$

$$\text{COS}(2.23X) + 0.30$$

9. Loading Condition : $F = 1 + X^2$

Boundary Condition : Clamped at $X = +1$ and simply supported at $X = -1$

$K = 0.$

$$w_1 = 0.02X^6 + 0.42X^4 + 0.51X^3 - 1.45X^2 - 0.51X + 1$$

$$s_2 = -0.03X^9 - 0.03X^8 - 0.13X^7 - 0.42X^6 + 0.76X^5 + 1.34X^4$$

$$- 1.15X^3 - 0.74X^2 + 0.56X - 0.13$$

$$w_3 = 0.04X^6 + 0.68X^4 - 0.44X^3 + 0.15X^2 + 0.23X$$

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$$K = 20.$$

$$w_1 = -\ 0.28e^{1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.55e^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+\ 0.06e^{-1.49X}\mathrm{COS}(\ 1.49X\ ) + 0.58e^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+\ 1.22X^2 + 1.22$$

$$s_2 = -\ 0.01e^{2.98X}\mathrm{COS}(\ 2.98X\ ) - 0.07e^{-2.98X}\mathrm{COS}(\ 2.98X\ )$$

$$-\ 0.14e^{2.98X} + 0.12e^{-2.98X} - 0.1e^{2.98X}\mathrm{SIN}(\ 2.98X\ )$$

$$-\ 0.04e^{-2.98X}\mathrm{SIN}(\ 2.98X\ ) + 0.22\mathrm{SIN}(\ 2.98X\ )$$

$$+\ 0.14\mathrm{COS}(\ 2.98X\ ) - 0.23e^{2.98X}[\ \mathrm{COS}(\ 2.98X\ )$$

$$+\ \mathrm{SIN}(\ 2.98X\ )] - 0.45e^{2.98X}[\ \mathrm{SIN}(\ 2.98X\ )$$

$$-\ \mathrm{COS}(\ 2.98X\ )] - 0.05e^{-2.98X}[\ \mathrm{COS}(\ 2.98X\ )$$

$$-\ \mathrm{SIN}(\ 2.98X\ )] - 0.47e^{-2.98X}[\ \mathrm{SIN}(\ 2.98X\ )$$

$$+\ \mathrm{COS}(\ 2.98X\ )\ ] + 0.69[\ Xe^{1.49X}\mathrm{COS}(\ 1.49X\ )$$

$$-\ (\ e^{1.49X}/1.49\ )[\ \mathrm{COS}(\ 1.49X\ ) + \mathrm{SIN}(\ 1.49X\ )]]$$

$$+\ 1.36[\ Xe^{1.49X}\mathrm{SIN}(\ 1.49X\ ) - (\ e^{1.49X}/1.49\ )$$

$$[\ \mathrm{SIN}(\ 1.49X\ ) - \mathrm{COS}(\ 1.49X\ )]] - 0.15[\ Xe^{-1.49X}$$

$$\mathrm{COS}(\ 1.49X\ ) + (\ e^{-1.49X}/1.49\ )[\ \mathrm{COS}(\ 1.49X\ )$$

$$-\ \mathrm{SIN}(\ 1.49X\ )]] - 1.41[\ Xe^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+\ (\ e^{-1.49X}/1.49\ )[\ \mathrm{SIN}(\ 1.49\ ) + \mathrm{COS}(\ 1.49X\ )]]$$

$$- 0.99X^3 + 0.99X + 0.11$$

$$w_3 = - 0.89e^{1.49X}\cos(1.49X) + 0.05e^{1.49X}\sin(1.49X)$$
$$- 0.98e^{-1.49X}\cos(1.49X) + 0.16e^{-1.49X}\sin(1.49X)$$
$$+ 0.83X^2 - 0.59Xe^{1.49X}\sin(1.49X) + 0.19Xe^{1.49X}$$
$$\cos(1.49X) - 0.36Xe^{-1.49X}\sin(1.49X) - 0.45Xe^{-1.49X}$$
$$\cos(1.49X) + 1.87$$

$K = 40.$

$$w_1 = 0.09e^{1.77X}\cos(1.77X) - 0.36e^{1.77X}\sin(1.77X)$$
$$+ 0.12e^{-1.77X}\cos(1.77X) + 0.32e^{-1.77X}\sin(1.77X)$$
$$+ 0.97X^2 + 0.97$$

$$s_2 = 0.01e^{3.54X}\cos(3.54X) - 0.03e^{-3.54X}\cos(3.54X)$$
$$-0.06e^{3.54X} + 0.05e^{-3.54X} - 0.04e^{3.54X}\sin(3.54X)$$
$$+ 0.09\sin(3.54X) + 0.06\cos(3.54X)$$
$$- 0.05e^{1.77X}[\cos(1.77X) + \sin(1.77X)]$$
$$- 0.19e^{1.77X}[\sin(1.77X) - \cos(1.77X)]$$
$$- 0.06e^{-1.77X}[\cos(1.77X) - \sin(1.77X)]$$
$$- 0.17e^{-1.77X}[\sin(1.77X) + \cos(1.77X)]$$

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$$+ 0.18[Xe^{1.77X}\cos(1.77X) - (e^{1.77X}/ 1.77)[\cos(1.77X)$$

$$+ \sin(1.77X)]] + 0.7[Xe^{1.77X}\sin(1.77X) - (e^{1.77X}/1.77)\cdot$$

$$[\sin(1.77X) - \cos(1.77X)]] - 0.23[Xe^{-1.77X}$$

$$\cos (1.77X) + (e^{-1.77X}/1.77)[\cos(1.77X)$$

$$- \sin(1.77X)]] - 0.62[Xe^{-1.77X}\sin(1.77X)$$

$$+ (e^{-1.77X}/1.77)[\sin(1.77X) + \cos(1.77X)]]$$

$$- 0.63X^3 + 0.67X - 0.09$$

$$w_3 = -0.4e^{1.77X}\cos(1.77X) - 0.43e^{-1.77X}\cos(1.77X)$$

$$+ 0.12e^{-1.77X}\sin(1.77X) + 0.42X^2 - 0.27Xe^{1.77X}\sin(1.77X)$$

$$+ 0.16Xe^{1.77X}\cos(1.77X) - 0.12Xe^{-1.77X}\sin(1.77X)$$

$$- 0.26Xe^{-1.77X}\cos(1.77X) + 0.84$$

$K = 60.$

$$w_1 = -0.02e^{1.96X}\cos(1.96X) - 0.28e^{1.96X}\sin(1.96X)$$

$$+ 0.13e^{-1.96X}\cos(1.96X) + 0.22e^{-1.96X}\sin(1.96X)$$

$$+ 0.89X^2 + 0.89$$

$$s_2 = 0.01e^{3.92}\cos(3.92X) - 0.02e^{-3.92X}\cos(3.92X)$$

$$- 0.04e^{3.92X} + 0.03e^{-3.92X} - 0.02e^{3.92X}\sin(3.92X)$$

$$+ 0.05\mathrm{SIN}(3.92X) + 0.04\mathrm{COS}(3.92X)$$

$$- 0.01e^{1.96X}[\mathrm{COS}(1.96X)) + \mathrm{SIN}(1.96X)]$$

$$- 0.12e^{1.96X}[\mathrm{SIN}(1.96X) - \mathrm{COS}(1.96X)]$$

$$- 0.05e^{-1.96X}[\mathrm{COS}(1.96X) - \mathrm{SIN}(1.96X)]$$

$$- 0.1e^{-1.96X}[\mathrm{SIN}(1.96X) + \mathrm{COS}(1.96X)]$$

$$+ 0.04[Xe^{1.96X}\mathrm{COS}(1.96X) - (e^{1.96X}/1.96)$$

$$[\mathrm{COS}(1.96X) + \mathrm{SIN}(1.96X)]] + 0.5[X e^{1.96X}$$

$$\mathrm{SIN}(1.96X) - (e^{1.96X}/ 1.96)[\mathrm{Sin}(1.96X)$$

$$- \mathrm{COS}(1.96X)]] - 0.23[Xe^{-1.96X}\mathrm{COS}(1.96X)$$

$$+ (e^{-1.96X}/1.96)[\mathrm{COS}(1.96X) - \mathrm{SIN}(1.96X)]]$$

$$- 0.39[Xe^{-1.96X}\mathrm{SIN}(1.96X) + (e^{-1.96X}/1.96)$$

$$[\mathrm{SIN}(1.96X) + \mathrm{COS}(1.96X)]] - 0.53X^3 + 0.61X - 0.61$$

$$w_3 = - 0.27e^{1.96X}\mathrm{COS}(1.96X) - 0.01e^{1.96X}\mathrm{SIN}(1.96X)$$

$$- 0.28e^{-1.96X}\mathrm{COS}(1.96X) + 0.11e^{-1.96X}\mathrm{SIN}(1.96X)$$

$$+ 0.28X^2 - 0.17Xe^{1.96X}\mathrm{SIN}(1.96X)$$

$$+ 0.14Xe^{1.96X}\mathrm{COS}(1.96X) - 0.05Xe^{-1.96X}\mathrm{SIN}(1.96X)$$

$$- 0.19Xe^{-1.96X}\mathrm{COS}(1.96X) + 0.55$$

$$K = 80.$$

$$
\begin{aligned}
w_1 = {} & 0.01e^{2.11X}\mathrm{COS}(\ 2.11X\ ) - 0.23e^{2.11X}\mathrm{SIN}(\ 2.11X\ ) \\
& + 0.13e^{-2.11X}\mathrm{COS}(\ 2.11X\ ) + 0.16e^{-2.11X}\mathrm{SIN}(\ 2.11X\ ) \\
& + 0.86X^2 + 0.86
\end{aligned}
$$

$$
\begin{aligned}
s_2 = {} & 0.01e^{4.22X}\mathrm{COS}(\ 4.22X\ ) - 0.01e^{-4.22X}\mathrm{COS}(\ 4.22X\ ) \\
& - 0.02e^{4.22X} + 0.02e^{-4.22X} - 0.01e^{4.22X}\mathrm{SIN}(\ 4.22X\ ) \\
& + 0.04\mathrm{SIN}(\ 4.22X\ ) + 0.03\mathrm{COS}(\ 4.22X\ ) \\
& - 0.09e^{2.11X}[\ \mathrm{SIN}(\ 2.11X\ ) - \mathrm{COS}(\ 2.11X\ )] \\
& - 0.05e^{-2.11X}[\ \mathrm{COS}(\ 2.11X\ ) - \mathrm{SIN}(\ 2.11X\ )] \\
& - 0.06e^{-2.11X}[\ \mathrm{SIN}(\ 2.11X\ ) - \mathrm{COS}(\ 2.11X\ )] \\
& - 0.01[\ Xe^{2.11X}\mathrm{COS}(\ 2.11X\ ) - (\ e^{2.11X}/2.11\ )[\ \mathrm{COS}(\ 2.11X\ ) \\
& + \mathrm{SIN}(\ 2.11X\ )]] + 0.4[\ Xe^{2.11X}\mathrm{SIN}(\ 2.11X\ ) - (e^{2.11X}/2.11\ ) \\
& [\ \mathrm{SIN}(\ 2.11X\ ) - \mathrm{COS}(\ 2.11X\ )]] - 0.22[\ Xe^{-2.11X} \\
& \mathrm{COS}(\ 2.11X\ ) + (\ e^{-2.11X}/2.11\ )[\ \mathrm{COS}(\ 2.11X\ ) \\
& - \mathrm{SIN}(\ 2.11X\ )]] - 0.28[\ Xe^{-2.11X}\mathrm{SIN}(\ 2.11X\ ) \\
& + (\ e^{-2.11X}/2.11\ )[\ \mathrm{SIN}(\ 2.11X\ ) + \mathrm{COS}\ (\ 2.11X\ )]] \\
& - 0.49X^3 + 0.6X - 0.19
\end{aligned}
$$

$$
\begin{aligned}
w_3 = {} & - 0.2e^{2.11X}\mathrm{COS}(\ 2.11X\ ) - 0.01e^{2.11X}\mathrm{SIN}(\ 2.11X\ ) \\
& - 0.2e^{-2.11X}\mathrm{COS}(\ 2.11X\ ) - 0.1e^{-2.11X}\mathrm{SIN}(\ 2.11X\ )
\end{aligned}
$$

$$+ 0.22X^2 - 0.12Xe^{2.11X}\mathrm{SIN}(\ 2.11X\ ) + 0.13Xe^{2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$- 0.01Xe^{-2.11X}\mathrm{SIN}(\ 2.11X\ ) - 0.15Xe^{-2.11X}\mathrm{COS}(\ 2.11X\ )$$

$$+ 0.41$$

$K = 100.$

$$w_1 = 0.03e^{2.23X}\mathrm{COS}(\ 2.23X\ ) - 0.2e^{2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.12e^{-2.23X}\mathrm{COS}(\ 2.23X\ ) + 0.13e^{-2.23X}\mathrm{SIN}(\ 2.23X\ )$$

$$+ 0.84X^2 + 0.84$$

$$s_2 = 0.01e^{4.46X}\mathrm{COS}(\ 4.46X\ ) - 0.02e^{4.46X} + 0.01e^{-4.46X}$$

$$+ 0.02\mathrm{COS}(\ 4.46X\ ) + 0.01e^{2.23X}[\ \mathrm{COS}(\ 2.23X\ ) + \mathrm{SIN}(\ 2.23X\ )]$$

$$- 0.07e^{2.23X}[\ \mathrm{SIN}(\ 3.23X\ ) - \mathrm{COS}(\ 2.23X\ )]$$

$$- 0.04e^{-2.23X}[\ \mathrm{COS}(\ 2.23X\ ) - \mathrm{SIN}(\ 2.23X\ )]$$

$$- 0.04e^{-2.23X}[\ \mathrm{SIN}(\ 2.23X\ ) + \mathrm{COS}(\ 2.23X\ )]$$

$$- 0.05[\ Xe^{2.23X}\mathrm{COS}(\ 2.23X\ ) - (\ e^{2.23X}/2.23\ )$$

$$[\ \mathrm{COS}(\ 2.23X\ ) + \mathrm{SIN}(\ 2.23X\ )]] + 0.34[\ Xe^{2.23X}$$

$$\mathrm{SIN}(\ 2.23X\ ) - (\ e^{2.23X}/2.23\ )[\ \mathrm{SIN}(\ 2.23X\ )$$

$$- \mathrm{COS}(\ 2.23X\ )]] - 0.21[\ Xe^{-2.23X}\mathrm{COS}(\ 2.23X\ )$$

$$+ (\ e^{-2.23X}/2.23\ )[\ \mathrm{COS}(\ 2.23X\ ) - \mathrm{SIN}(\ 2.23X\ )]]$$

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$$- 0.21[\, Xe^{-2.23X}\mathrm{SIN}(\, 2.23X\,) + (\, e^{-2.23X}/2.23\,)$$

$$[\, \mathrm{SIN}(\, 2.23X\,) + \mathrm{COS}(\, 2.23X\,)]] - 0.47X^3 + 0.62X$$

$$- 0.21$$

$$w_3 = - 0.17e^{2.23X}\mathrm{COS}(\, 2.23X\,) - 0.02e^{2.23X}\mathrm{SIN}(\, 2.23X\,)$$

$$- 0.16e^{-2.23X}\mathrm{COS}(\, 2.23X\,) + 0.09e^{-2.23X} + 0.17X^2$$

$$- 0.09Xe^{2.23X}\mathrm{SIN}(\, 2.23X\,) + 0.12Xe^{2.23X}\mathrm{COS}(\, 2.23X\,)$$

$$- 0.13e^{-2.23X}\mathrm{COS}(\, 2.23X\,) + 0.33$$

10. Loading Condition : F = COS(1.0708X)

Boundary Condition : Clamped both sides

K = 0.

$$w_1 = 4.65\mathrm{COS}(\, 1.0708X\,) + 3.65X^2 - 3.65$$

$$s_2 = 4.26\mathrm{SIN}(\, 3.1416\,) - 34.1X\mathrm{COS}(\, 1.0708X\,)$$

$$+ 21.71\mathrm{SIN}(\, 1.0708X\,) - 8.92X^3 - 12.78X$$

$$w_3 = - 10.4\mathrm{COS}(\, 1.0708X\,) + 2.23X^4 - 12.63X^2 + 10.4$$

K = 20.

$$w_1 = - 0.39e^{1.49X}\mathrm{COS}(\, 1.49X\,) + 0.03e^{1.49X}\mathrm{SIN}(\, 1.49X\,)$$

$$- 0.39e^{-1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.03e^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+ 1.78\mathrm{COS}(\ 1.0708X\ )$$

$$s_2 = - 0.02e^{2.98X}\mathrm{COS}(\ 2.98X\ ) + 0.02e^{-2.98X}\mathrm{COS}(\ 2.98X\ )$$

$$- 0.05e^{2.98X} + 0.05e^{-2.98X} + 0.03e^{2.98X}\mathrm{SIN}(\ 2.98X\ )$$

$$+ 0.03e^{-2.98X}\mathrm{SIN}(\ 2.98X\ ) + 0.11\mathrm{SIN}(\ 2.98X\ )$$

$$+ 0.62e^{1.49X}\mathrm{COS}(\ 1.0708 - 1.49\ )X$$

$$- 0.62e^{-1.49X}\mathrm{COS}(\ 1.0708 - 1.49\ )X$$

$$+ 0.08e^{1.49X}\mathrm{COS}(\ 1.0708 + 1.49\ )X$$

$$- 0.08e^{-1.49X}\mathrm{COS}(\ 1.0708 \pm 1.49\ )X$$

$$- 0.47e^{1.49X}\mathrm{SIN}(\ 1.0708 - 1.49\ )X$$

$$- 0.47e^{-1.49X}\mathrm{SIN}(\ 1.0708 - 1.49\ )X$$

$$- 0.33e^{1.49X}\mathrm{SIN}(\ 1.0708 + 1.49\ )X$$

$$- 0.33e^{-1.49X}\mathrm{SIN}(\ 1.0708 + 1.49\ )X$$

$$+ 12.38\mathrm{SIN}(\ 1.0708X\ )\ \mathrm{COS}(\ 1.0708X\ ) - 1.41X$$

$$w_3 = 0.21e^{1.49X}\mathrm{COS}(\ 1.49X\ ) + 0.24e^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+ 0.21e^{-1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.24e^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$- 0.42\mathrm{COS}(\ 1.0708X\ ) - 0.22Xe^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$- 0.26Xe^{1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.22Xe^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+ 0.26Xe^{-1.49X}\cos(1.49X)$$

$$K = 40.$$

$$w_1 = - 0.2e^{1.77X}\cos(1.77X) - 0.04e^{1.77X}\sin(1.77X)$$

$$- 0.2e^{-1.77X}\cos(1.77X) + 0.04e^{-1.77X}\sin(1.77X)$$

$$+ 1.4\cos(1.0708X)$$

$$s_2 = -0.01e^{3.54X}\cos(3.54X) + 0.01e^{-3.54X}\cos(3.54X)$$

$$-0.01e^{3.54X} + 0.01e^{-3.54X} + 0.03\sin(3.54X) -$$

$$+0.14e^{1.77X}\cos(1.0708-1.77)X - 0.14e^{-1.77X}\cos(1.0708-1.77)X$$

$$+0.07e^{1.77X}\cos(1.0708 + 1.77)X$$

$$- 0.07e^{-1.77X}\cos(1.0708 + 1.77)X$$

$$- 0.29e^{1.77X}\sin(1.0708 - 1.77)X$$

$$- 0.29e^{-1.77X}\sin(1.0708 - 1.77)X$$

$$- 0.13e^{1.77X}\sin(1.0708 + 1.77)X$$

$$- 0.13e^{-1.77X}\sin(1.0708 + 1.77)X$$

$$+ 7.7\sin(1.0708X) \cos(1.0708X) - 0.55X$$

$$w_3 = 0.04e^{1.77X}\cos(1.77X) + 0.12e^{1.77X}\sin(1.77X)$$

$$+0.04e^{-1.77X}\cos(1.77X) - 0.12e^{-1.77X}\sin(1.77X)$$

$$- 0.08\text{COS}(\ 1.0708X\ ) - 0.12Xe^{1.77X}\text{SIN}(\ 1.77X\ )$$

$$- 0.08Xe^{1.77X}\text{COS}(\ 1.77X\ ) - 0.12Xe^{-1.77X}\text{SIN}(\ 1.77X\ )$$

$$+ 0.08Xe^{-1.77X}\text{COS}(\ 1.77X\ )$$

$$K = 60.$$

$$w_1 = - 0.13e^{1.96X}\text{COS}(\ 1.96X\ ) - 0.05e^{1.96X}\ \text{SIN}(\ 1.96X\ )$$

$$- 0.13e^{-1.96X}\text{COS}(\ 1.96X\ ) + 0.05e^{-1.96X}\text{SIN}(\ 1.96X\ )$$

$$+ 1.26\text{COS}(\ 1.0708X\ )$$

$$s_2 = - 0.01e^{3.92X} + 0.01e^{-3.92X} + 0.02\text{SIN}(\ 3.92X\ )$$

$$+ 0.03e^{1.96X}\text{COS}(\ 1.0708 - 1.96\ )X$$

$$- 0.03e^{-1.96X}\text{COS}(\ 1.0708 - 1.96\ )X$$

$$+ 0.06e^{1.96X}\text{COS}(\ 1.0708 + 1.96\ )X$$

$$- 0.06e^{-1.96X}\text{COS}(\ 1.0708 + 1.96\ )X$$

$$- 0.19e^{1.96X}\text{SIN}(\ 1.0708 - 1.96\ )X$$

$$- 0.19e^{-1.96X}\text{SIN}(\ 1.0708 - 1.96\ )X$$

$$- 0.07e^{1.96X}\text{SIN}(\ 1.0708 + 1.96\ )X$$

$$- 0.07e^{-1.96X}\text{SIN}(\ 1.0708 + 1.96\ )X -$$

$$+ 6.17\text{SIN}(\ 1.0708X\ )\text{COS}(\ 1.0708X\ ) - 0.31X$$

$$w_3 = 0.08e^{1.96X}SIN(1.96X) - 0.08e^{-1.96X}SIN(1.96X)$$
$$-0.01COS(1.0708X) - 0.08Xe^{1.96X}SIN(1.96X)$$
$$-0.03Xe^{1.96X}COS(1.96X) - 0.08X\,e^{-1.96X}SIN(1.96X)$$
$$+0.03Xe^{-1.96X}COS(1.96X)$$

$K = 80.$

$$w_1 = -0.09e^{2.11X}COS(2.11X) - 0.05e^{2.11X}SIN(2.11X)$$
$$-0.09e^{-2.11X}COS(2.11X) + 0.05e^{-2.11X}SIN(2.11X)$$
$$+1.18COS(1.0708X)$$

$$s_2 = 0.01SIN(4.22X) + 0.05e^{2.11X}COS(1.0708 + 2.11)X$$
$$-0.05e^{-2.11X}COS(1.0708 + 2.11)X$$
$$-0.13e^{2.11X}SIN(1.0708 - 2.11)X$$
$$-0.13e^{-2.11X}SIN(1.0708 - 2.11)X$$
$$-0.04e^{2.11X}SIN(1.0708 + 2.11)X$$
$$-0.04e^{-2.11X}SIN(1.0708 + 2.11)X$$
$$+5.43SIN(1.0708X)COS(1.0708X) - 0.2X$$

$$w_3 = 0.05e^{2.11X}SIN(2.11X) - 0.05e^{-2.11X}SIN(2.11X)$$
$$-0.06Xe^{2.11X}SIN(2.11X) - 0.01Xe^{2.11X}COS(2.11X)$$

$$- 0.06Xe^{-2.11X}\mathrm{SIN}(2.11X) + 0.01Xe^{-2.11X}\mathrm{COS}(2.11X)$$

$K = 100.$

$$w_1 = - 0.06e^{2.23X}\mathrm{COS}(2.23X) - 0.05e^{2.23X}\mathrm{SIN}(2.23X)$$

$$- 0.06e^{-2.23X}\mathrm{COS}(2.23X) + 0.05e^{-2.23X}\mathrm{SIN}(2.23X)$$

$$+ 1.13\mathrm{COS}(1.0708X)$$

$$s_2 = - 0.01e^{2.23X}\mathrm{COS}(1.0708 - 2.23)X$$

$$- 0.01e^{-2.23X}\mathrm{COS}(1.0708 - 2.23)X$$

$$+ 0.04e^{2.23X}\mathrm{COS}(1.0708 + 2.23)X$$

$$- 0.04e^{-2.23X}\mathrm{COS}(1.0708 + 2.23)X$$

$$- 0.1e^{2.23X}\mathrm{SIN}(1.0708 - 2.23)X$$

$$- 0.1e^{-2.23X}\mathrm{SIN}(1.0708 - 2.23)X$$

$$- 0.03e^{2.23X}\mathrm{SIN}(1.0708 + 2.23)X$$

$$- 0.03e^{-2.23X}\mathrm{SIN}(1.0708 + 2.23)X$$

$$+ 4.99\mathrm{SIN}(1.0708X)\mathrm{COS}(1.0708X) - 0.14X$$

$$w_3 = 0.04e^{2.23X}\mathrm{SIN}(2.23X) - 0.04e^{-2.23X}\mathrm{SIN}(2.23X)$$

$$+ 0.01\mathrm{COS}(1.0708X) - 0.05Xe^{2.23X}\mathrm{SIN}(2.23X)$$

$$- 0.05Xe^{-2.23X}\mathrm{SIN}(2.23X)$$

11. Loading Condition : F = COS(1.0708X)

Boundary Condition : Simply supported both sides

K = 0.

w_1 = COS(1.0708X)

s_2 = 0.19SIN(3.1416X)

w_3 = 0.

K = 20.

w_1 = COS(1.0708X)

s_2 = 3.87SIN(1.0708X)COS(1.0708X)

w_3 = 0.

K = 40.

w_1 = COS(1.0708X)

s_2 = 3.87SIN(1.0708X)COS(1.0708X)

w_3 = 0.

K = 60.

w_1 = COS(1.0708X)

s_2 = 3.87SIN(1.0708X)COS(1.0708X)

w_3 = 0.

K = 80.

w_1 = COS(1.0708X)

s_2 = 3.87SIN(1.0708X)COS(1.0708X)

w_3 = 0.

K = 100.

w_1 = COS(1.0708X)

s_2 = 3.87SIN(1.0708X)COS(1.0708X)

w_3 = 0.

12. Loading Condition : F = COS(1.0708X)

Boundary Condition : Clamped at X = +1 and simply supported at X = -1

K = 0.

$$w_1 = 2.43\cos(1.0708X) + 0.47X^3 + 1.43X^2 - 0.47X - 1.43$$

$$s_2 = 1.16\sin(3.1416X) - 3.84X^2\cos(1.0708X)$$
$$+ 4.44X\sin(1.0708X) + 2.82\cos(1.0708X)$$
$$- 6.97X\cos(1.0708X) + 4.44\sin(1.0708X)$$
$$+ 1.16\cos(1.0708X) - 0.2X^5 - 1.02X^4 - 1.14X^3$$
$$+ 0.68X^2 - 3.09X - 4.09$$

$$w_3 = -4.23\cos(1.0708X) + 0.19X^5 + 0.96X^4 - 0.44X^3$$
$$-5.18X^2 + 0.25X + 4.22$$

$K = 20.$

$$w_1 = -0.3e^{1.49X}\cos(1.49X) + 0.02e^{1.49X}\sin(1.49X)$$
$$-0.01e^{-1.49X}\cos(1.49X) + 1.32\cos(1.0708X)$$

$$s_2 = -0.01e^{2.98X}\cos(2.98X) - 0.03e^{2.98X} + 0.02e^{2.98X}\sin(2.98X)$$
$$+ 0.36e^{1.49X}\cos(1.0708 - 1.49)X -$$
$$- 0.02e^{-1.49X}\cos(1.0708 - 1.49)X$$
$$+ 0.05e^{1.49X}\cos(1.0708 + 1.49)X$$
$$- 0.27e^{1.49X}\sin(1.0708 - 1.49)X$$
$$- 0.01e^{-1.49X}\sin(1.0708 - 1.49)X$$
$$- 0.19e^{1.49X}\sin(1.0708 + 1.49)X$$

$$- 0.01e^{-1.49X}\mathrm{SIN}(\ 1.0708 + 1.49\ )X$$

$$+ 6.8\mathrm{SIN}(\ 1.0708X\ )\ \mathrm{COS}(\ 1.0708X\ )$$

$$- 0.42X - 0.42$$

$$w_3 = 0.11e^{1.49X}\mathrm{COS}(\ 1.49X\ ) + 0.19e^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$+0.03e^{-1.49X}\mathrm{COS}(\ 1.49X\ ) - 0.02e^{-1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$-0.15\mathrm{COS}(\ 1.0708X\ ) - 0.18Xe^{1.49X}\mathrm{SIN}(\ 1.49X\ )$$

$$-0.21Xe^{1.49X}\mathrm{COS}(\ 1.49X\ )$$

K = 40.

$$w_1 = - 0.17e^{1.77X}\mathrm{COS}(\ 1.77X\ ) - 0.03e^{1.77X}\mathrm{SIN}(\ 1.77X\ )$$

$$+ 1.17\mathrm{COS}(\ 1.0708X\ )$$

$$s_2 = - 0.01e^{3.54X} + 0.1e^{1.77X}\mathrm{COS}(\ 1.0708 - 1.77\ )X$$

$$+ 0.05e^{1.77X}\mathrm{COS}(\ 1.0708 + 1.77\ )X$$

$$- 0.2e^{3.54X}\mathrm{SIN}(\ 1.0708 - 1.77\ )X$$

$$- 0.09e^{3.54X}\mathrm{SIN}(\ 1.0708 + 1.77\ )X$$

$$+ 5.35\mathrm{SIN}(\ 1.0708X\ )\ \mathrm{COS}(\ 1.0708X\ )$$

$$- 0.2X - 0.2$$

$$w_3 = 0.02e^{1.77X}\mathrm{COS}(\ 1.77X\ ) + 0.1e^{1.77X}\mathrm{SIN}(\ 1.77X\ )$$

$$- 0.01e^{-1.77X}SIN(1.77X) - 0.02COS(1.0708X)$$

$$- 0.11Xe^{1.77X}SIN(1.77X) - 0.07Xe^{1.77X}COS(1.77X)$$

K = 60.

$$w_1 = -0.11e^{1.96X}COS(1.96X) - 0.04e^{1.96X}SIN(1.96X)$$

$$+1.11COS(1.0708X)$$

$$s_2 = 0.02e^{1.96X}COS(1.0708 - 1.96)X$$

$$+0.04e^{1.96X}COS(1.0708 + 1.96)X$$

$$-0.14e^{1.96X}SIN(1.0708 - 1.96)X$$

$$-0.05e^{1.96X}SIN(1.0708 + 1.96)X$$

$$+4.82SIN(1.0708X) COS(1.0708X)$$

$$-0.12X - 0.12$$

$$w_3 = 0.06e^{1.96X}SIN(1.96X) - 0.07Xe^{1.96X}SIN(1.96X)$$

$$-0.03Xe^{1.96X}COS(1.96X)$$

K = 80.

$$w_1 = - 0.08e^{1.96X}COS(1.96X) - 0.05e^{1.96X}SIN(1.96X)$$

$$+ 1.08COS(1.0708X)$$

$$S_2 = 0.04e^{1.96X}\text{COS}(\ 1.0708 + 1.96\ )X$$

$$-0.11e^{1.96X}\text{SIN}(\ 1.0708 - 1.96\ )X$$

$$-0.04e^{1.96X}\text{SIN}(\ 1.0708 + 1.96\ )X$$

$$+4.54\text{SIN}(\ 1.0708X\ )\text{COS}(\ 1.0708X\ )$$

$$-0.08X - 0.08$$

$$w_3 = 0.04e^{1.96X}\text{SIN}(\ 1.96X\ ) - 0.05Xe^{1.96X}\text{SIN}(\ 1.96X\ )$$

$$-0.01Xe^{1.96X}\text{COS}(\ 1.96X\ )$$

$$K = 100.$$

$$w_1 = -0.06e^{2.23X}\text{COS}(\ 2.23X\ ) - 0.04e^{2.23X}\text{SIN}(\ 2.23X\ )$$

$$+1.06\text{COS}(\ 1.0708X\ )$$

$$s_2 = -0.01e^{2.23X}\text{COS}(\ 1.0708 - 2.23\ )X$$

$$+0.03e^{2.23X}\text{COS}(\ 1.0708 + 2.23\ )X$$

$$-0.08e^{2.23X}\text{SIN}(\ 1.0708 - 2.23\ )X$$

$$-0.02e^{2.23X}\text{SIN}(\ 1.0708 + 2.23\ )X$$

$$+4.37\text{SIN}(\ 1.0708X\ )\ \text{COS}(\ 1.0708X\ )$$

$$-\ 0.06X - 0.06$$

$$w_3 = 0.03e^{2.23X}\text{SIN}(\ 2.23X\ ) - 0.04Xe^{2.23X}\text{SIN}(\ 2.23X\ )$$

APPENDIX B

1. Loading Condition : F = 1

Boundary Condition : Clamped both sides

K = 0.	γ_1 =	24.
	γ_3 =	17.5543
K = 20.	γ_1 =	39.6349
	γ_3 =	17.8289
K = 40.	γ_1 =	55.4849
	γ_3 =	18.0721
K = 60.	γ_1 =	71.5399
	γ_3 =	18.2864
K = 80.	γ_1 =	87.7905
	γ_3 =	18.4741
K = 100.	γ_1 =	104.227
	γ_3 =	18.6369

2. Loading Condition : F = 1

Boundary Condition : Simply supported both sides

K = 0.	γ_1 =	4.8
	γ_3 =	14.5630

K = 20.	γ_1 = 20.8181
	γ_3 = 15.2119
K = 40.	γ_1 = 37.1933
	γ_3 = 15.7608
K = 60.	γ_1 = 53.8949
	γ_3 = 16.2151
K = 80.	γ_1 = 70.8956
	γ_3 = 16.5791
K = 100.	γ_1 = 88.1706
	γ_3 = 16.8576

3. Loading Condition : F = 1

Boundary Condition : Clamped one side and Simply supported the other side

K = 0.	γ_1 = 12.
	γ_3 = 18.9257
K = 20.	γ_1 = 28.3031
	γ_3 = 18.7757

K = 40. γ_1 = 44.7921

γ_3 = 18.7296

K = 60. γ_1 = 61.4719

γ_3 = 18.7350

K = 80. γ_1 = 78.3412

γ_3 = 18.7624

K = 100. γ_1 = 95.3956

γ_3 = 18.7941

4. Loading Condition : F = 1 + x

Boundary Condition : Clamped both sides

K = 0. γ_1 = 24.

γ_3 = 17.7883

K = 20. γ_1 = 39.6349

γ_3 = 18.3786

K = 40. γ_1 = 55.4849

γ_3 = 19.0085

K = 60. γ_1 = 71.5399

γ_3 = 19.6502

K = 80. γ_1 = 87.7905

γ_3 = 20.2857

K = 100. γ_1 = 104.227

γ_3 = 20.9062

5. Loading Condition : F = 1 + x

Boundary Condition : Simply supported both sides

K = 0. γ_1 = 4.8

γ_3 = 14.6201

K = 20. γ_1 = 20.8181

γ_3 = 15.9653

K = 40. γ_1 = 37.1933

γ_3 = 17.5341

K = 60. γ_1 = 53.8949

γ_3 = 19.0602

K = 80. γ_1 = 70.8956

γ_3 = 20.4417

K = 100. γ_1 = 88.1706

γ_3 = 21.6404

6. Loading Condition : F = 1 + x

Boundary Condition : Clamped one side and simply supported the other side

K = 0. γ_1 = 13.3333

γ_3 = 19.0805

K = 20. γ_1 = 30.8551

γ_3 = 17.5189

K = 40. γ_1 = 48.1189

γ_3 = 17.0331

K = 60. γ_1 = 65.2728

γ_3 = 17.0729

K = 80. γ_1 = 82.4038

γ_3 = 17.3827

K = 100. γ_1 = 99.5646

γ_3 = 17.8311

7. Loading Condition : $F = 1 + x^2$

Boundary Condition : Clamped both sides

K = 0. γ_1 = 21.1765

γ_3 = 15.4614

K = 20. γ_1 = 35.0847

γ_3 = 15.7406

K = 40. γ_1 = 49.2689

γ_3 = 15.9954

K = 60. γ_1 = 63.7183

γ_3 = 16.2276

K = 80. γ_1 = 78.4229

γ_3 = 16.4380

K = 100. γ_1 = 93.3729

γ_3 = 16.6269

8. Loading Codition : $F = 1 + x^2$

Boundary Condition : Simply supported both sides

K = 0.	γ_1 =	4.04494
	γ_3 =	12.3579
K = 20.	γ_1 =	17.6733
	γ_3 =	13.6895
K = 40.	γ_1 =	31.7966
	γ_3 =	14.1434
K = 60.	γ_1 =	46.3819
	γ_3 =	14.9074
K = 80.	γ_1 =	61.3989
	γ_3 =	15.5672
K = 100.	γ_1 =	76.8193
	γ_3 =	16.1077

9. Loading Condition : $F = 1 + x^2$

Boundary Condition : Clamped one side and simply supported the other side

K = 0.	γ_1 = 10.2857
	γ_3 = 16.5093
K = 20.	γ_1 = 24.4256
	γ_3 = 12.7850
K = 40.	γ_1 = 38.8945
	γ_3 = 17.0725
K = 60.	γ_1 = 53.6808
	γ_3 = 17.3510
K = 80.	γ_1 = 68.7717
	γ_3 = 17.6062
K = 100.	γ_1 = 84.1539
	γ_3 = 17.8273

10. Loading Condition : $F = COS(\pi/2)X$

Boundary Condition : Clampled both sides

K = 0.	γ_1 = 28.3696
	γ_3 = 20.8264

K = 20.	γ_1 = 42.6260
	γ_3 = 21.1073
K = 40.	γ_1 = 64.9702
	γ_3 = 21.3665
K = 60.	γ_1 = 83.3971
	γ_3 = 21.6020
K = 80.	γ_1 = 101.902
	γ_3 = 21.8158
K = 100.	γ_1 = 120.480
	γ_3 = 22.0098

11. Loading Condition : $F = COS(\pi/2)X$

Boundary Condition : Simply supported both sides

K = 0.	γ_1 = 6.08812
	γ_3 = 18.2644
K = 20.	γ_1 = 26.0881
	γ_3 = 18.2644

K = 40. γ_1 = 46.0881

γ_3 = 18.2644

K = 60. γ_1 = 66.0881

γ_3 = 18.2644

K = 80. γ_1 = 86.0881

γ_3 = 18.2644

K = 100. γ_1 = 106.088

γ_3 = 18.2644

12. Loading Condition : $F = COS(\pi/2)X$

Boundary Condition : Clamped one side and simply supported the other side

K = 0. γ_1 = 14.8148

γ_3 = 22.7531

K = 20. γ_1 = 34.5706

γ_3 = 21.8153

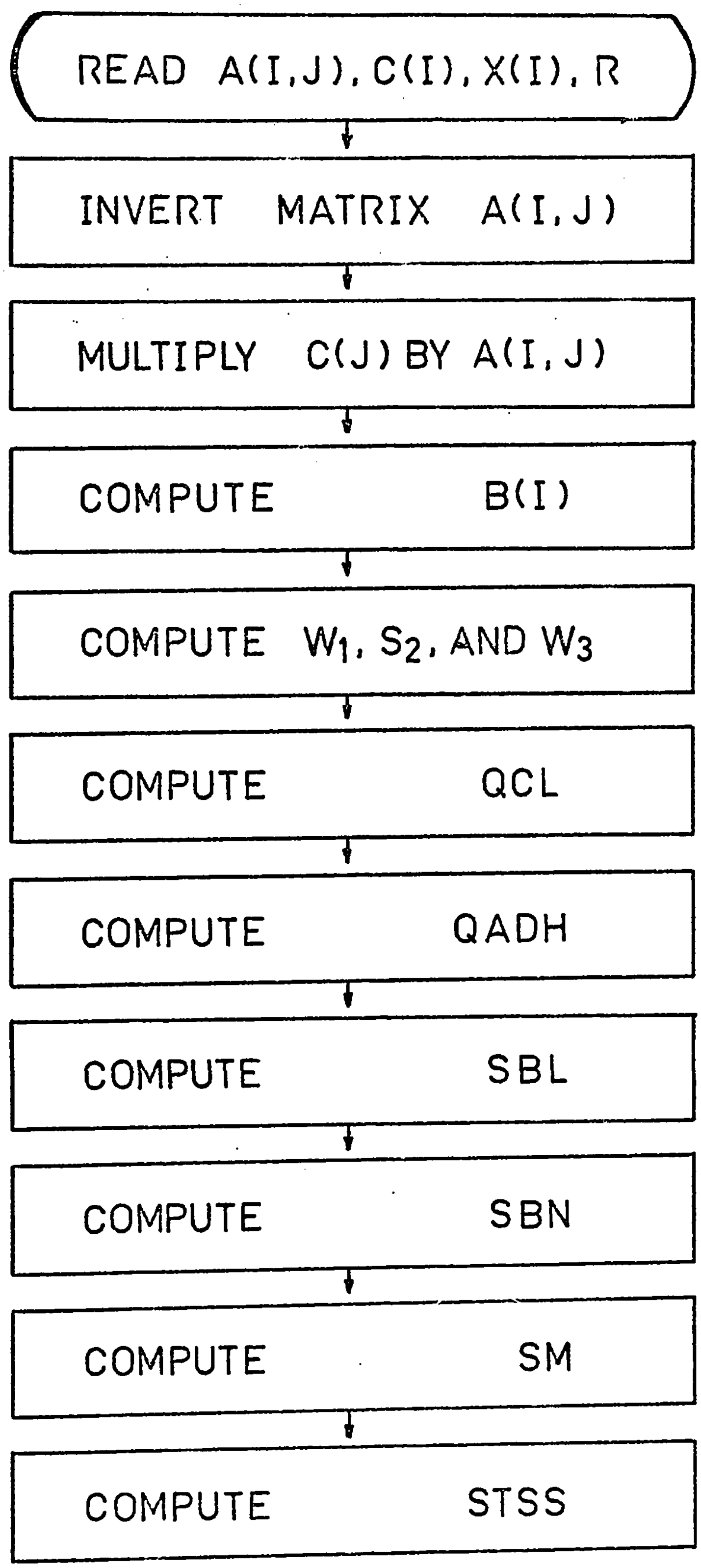
K = 40. γ_1 = 54.1930

γ_3 = 21.2783

K = 60.	γ_1 = 73.7361
	γ_3 = 20.9506
K = 80.	γ_1 = 93.2335
	γ_3 = 20.7556
K = 100.	γ_1 = 112.707
	γ_3 = 20.6432

Appendix C

FLOW CHART


READ A(I,J), C(I), X(I), R
INVERT MATRIX A(I,J)
MULTIPLY C(J) BY A(I,J)
COMPUTE B(I)
COMPUTE W_1, S_2, AND W_3
COMPUTE QCL
COMPUTE QADH
COMPUTE SBL
COMPUTE SBN
COMPUTE SM
COMPUTE STSS

```
C
C
C     ************************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *
C     *         WITH OR WITHOUT ELASTIC FOUNDATION               *
C     *                                                          *
C     * (1) LOADING  CONDITION      F=1.                         *
C     * (2) BOUNDARY CONDITION      CLAMPED BOTH SIDES           *
C     ************************************************************
C
C
C     R             ELASTIC FOUNDATION MODULUS
C     Q             A FUNCTION OF R
C     A             COEFFICIENT MATRIX
C     B             INTEGRATION CONSTANT MATRIX
C     C             CONSTANT MATRIX
C     B(5),B(12)    UNDETERMINED FOEFFICIENT
C     X             DEFLECTION
C     QCL           LOAD (LINEAR ONLY )
C     QADH          LOAD (LINEAR AND NONLINEAR )
C     SBL           BENDING STRESS ( LINEAR ONLY )
C     SBN           BENDING STRESS ( LINEAR A-D NONLINEAR )
C     W1XX          SECOND DERIVATIVE OF W1
C     W3XX          SECOND DERIVATIVE OF W3
C     SM            MEMBRANE STRESS
C     STSS          TOTAL STRESS
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(12,12),B(12),C(12),W1(20),S2(20),W3(20),T(12,12),
      DIMENSION T5(5,5),T7(7,7),T12(12,12),U5(5,5),U7(7,7),U12(12,12),
      DIMENSION QCL(50),QADH(50),Z(202),W1CC(202),W1CXX(202),X(50),
      DIMENSION W1W3CC(202),W1W3XX(202),SBL(50),SBN(50),SM(50),STSS(50)
C
      R=20.
    5 Q=((1./4.)*R)**(0.25)
      WRITE (3, 10) R
   10 FORMAT (1H1,5X,'ELASTIC FOUNDATION MODULUS R=',F10.2///)
      WRITE (3, 15) Q
   15 FORMAT (17X,' THE EQUIVALENT Q= ', E18.6/)
      QQ=2.*Q
      WRITE (3, 20) QQ
   20 FORMAT (17X,' THE           QQ= ', E18.6/)
      XQ=-Q
```

```
      XQQ=-QQ
C
C     1.   FIND OUT B(1)TO B(12)
C
      DO 30 I=1,12
      DO 30 J=1,12
   30 A(I,J)=0.
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))
      A(2,1)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))
      A(2,2)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))
      A(2,3)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))
      A(2,4)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(XQ))
      A(3,4)= (DSIN(Q))*(DEXP(XQ))
      A(3,5)=1./R
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(4,5)=1./R
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)=1./R
      A(6,6)=1.
      A(6,7)=1.
      A(7,6)=-1.
      A(7,7)=1.
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))
      A( 9, 8)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))
      A( 9, 9)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))
      A( 9,10)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))
      A( 9,11)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))
      A(10, 8)= (DCOS(Q))*(DEXP( Q))
      A(10, 9)= (DSIN(Q))*(DEXP( Q))
      A(10,10)= (DCOS(Q))*(DEXP(XQ))
      A(10,11)= (DSIN(Q))*(DEXP(XQ))
```

```
      A(10,12)=1./R
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))
      A(11,10)= (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(11,12)=1./R
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)=1./R
C
      WRITE (3, 35)
   35 FORMAT (1H1,30X,' MATRIX A BEFORE INVERSION '/)
      DO 40 I=1,12
   40 WRITE (3, 45) (A(I,J),J=1,12)
   45 FORMAT (5X,6E18.6,/5X,6E18.6//)
      DO 50 I=1,12
      DO 50 J=1,12
   50 T(I,J)=A(I,J)
      C(1)=0.
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      WRITE (3,55)
   55 FORMAT (1H1,40X,' MATRIX A( 5, 5) BEFORE INVERSION '///)
      DO 60 I=1,5
   60 WRITE (3,65) (A(I,J),J=1,5)
   65 FORMAT (10X,5F18.6/)
      CALL MATIN (A,5)
      WRITE (3,70)
   70 FORMAT (////40X,' MATRIX A( 5, 5) AFTER INVERSION '///)
      DO 75 I=1,5
   75 WRITE (3,65) (A(I,J),J=1,5)
      DO 80   I=1,5
      DO 80   J=1,5
   80 T5(I,J)=A(I,J)
      CALL MATMUL (T,T5,U5,5,5,5)
      WRITE (3,85)
   85 FORMAT (1H1,40X,' THE FOLLOWING MATRIX U5( 5, 5) SHOULD BE A UNIT
     &MATRIX ' ///)
      DO 90 I=1,5
   90 WRITE (3,65)    (U5(I,J),J=1,5)
      DO 100 I=1,5
      B(I)=0.
```

```
      DO 100 J=1,5
100   B(I)=B(I)+A(I,J)*C(J)
      WRITE (3,105)
105   FORMAT(/////30X,'THE FIRST CHECK FOR THE VALUES OF B(1)TOB(5)'///)
      WRITE (3,110)(B(I),I=1,5)
110   FORMAT (40X,'B(1)=',E18.6,/40X,'B(2)=',E18.6,/40X,'B(3)=',
     &E18.6,/40X,'B(4)=',E18.6,/40X,'B(5)=',E18.6///)
C                                     *********************
      C(6)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
     1*(DEXP( QQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
     3*(DEXP(XQQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
C                                     *********************
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
     1*(DEXP(XQQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
     3*(DEXP( QQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))
     6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
     9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))
     1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
C                                     *********************
      DO 115 I=1,12
      DO 115 J=1,12
115   A(I,J)=T(I,J)
      WRITE (3,120)
120   FORMAT (1H1,40X,' MATRIX A( 7, 7) BEFORE INVERSION '///)
      DO 125 I=1,7
125   WRITE (3,130)   (A(I,J),J=1,7)
130   FORMAT (5X,7E18.6/)
      CALL MATIN (A,7)
      WRITE (3,135)
```

```
135   FORMAT (////40X,' MATRIX A( 7, 7) AFTER INVERSION '///)
      DO 140  I=1,7
140   WRITE (3,130)   (A(I,J),J=1,7)
      DO 145  I=1,7
      DO 145  J=1,7
145   T7(I,J)=A(I,J)
      CALL MATMUL (T,T7,U7,7,7,7)
      WRITE (3,150)
150   FORMAT (1H1,40X,' THE FOLLOWING MATRIX U7( 7, 7) SHOULD BE A UNIT
     &MATRIX ' ///)
      DO 155  I=1,7
155   WRITE (3,130)   (U7(I,J),J=1,7)
      DO 160 I=1,7
      B(I)=0.
      DO 160 J=1,7
160   B(I)=B(I)+A(I,J)*C(J)
      WRITE(3,165)
165   FORMAT(///30X, 'THE  SECOND CHECK FOR THE VALUES B(1) TO B(7)'//)
      WRITE (3,170)(B(I),I=1,7)
170   FORMAT (40X,'B(1)=',E18.6,/40X,'B(2)=',E18.6,/40X,'B(3)=',
     &E18.6,/40X,'B(4)=',E18.6,/40X,'B(5)=',E18.6,/40X,'B(6)=',
     &E18.6,/40X,'B(7)=',E18.6///)
C                                  *********************
      Y=24.*((B(1)*B(4)-B(2)*B(3))*Q**2+B(6))/Q
      WRITE (3,175) Y
175   FORMAT (/// 17X,'                  Y= ', E18.6///)
C                                  *********************
      C(8)=-(1./8.)*Y*  ((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
     &                   +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
     &-(1./16.)*(Y/Q)*
     &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
      C(9)=  (1./8.)*Y*  ((DEXP(XQ))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
     &                   +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
     &+(1./16.)*(Y/Q)*
     &((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
      C(10)=-(1./16.)*Y*
     &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
      C(11)=-(1./16.)*Y*
     &((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
      C(12)=0.
```

```
C                                  ********************
      DO 180 I=1,12
      DO 180 J=1,12
  180 A(I,J)=T(I,J)
      WRITE (3,185)
  185 FORMAT (1H1,40X,' MATRIX A(12,12) BEFORE INVERSION '//)
      DO 190  I=1,12
  190 WRITE (3,195)   (A(I,J),J=1,12)
  195 FORMAT (5X,6E18.6,/5X,6E18.6//)
      CALL MATIN (A,12)
      DO 200 I=1,12
      B(I)=0.
      DO 200 J=1,12
  200 B(I)=B(I)+A(I,J)*C(J)
      WRITE (3,205)
  205 FORMAT (1H1,40X,' MATRIX A(12,12) AFTER INVERSION '/)
      DO 210  I=1,12
  210 WRITE (3,195)   (A(I,J),J=1,12)
      DO  215 I=1,12
      DO  215 J=1,12
  215 T12(I,J)=A(I,J)
      CALL MATMUL (T,T12,U12,12,12,12)
      WRITE (3,220)
  220 FORMAT (1H1,40X,' THE FOLLOWING MATRIX U12(12,12) SHOULD BE A UNIT
     &MATRIX '/)
      DO  225 I=1,12
  225 WRITE (3,195)   (U12(I,J),J=1,12)
      WRITE (3,230) R
  230 FORMAT(1H1,///10X,'ANSWER 1-1 THE VALUES OF THE CONSTANS A,B,C,D,
     &R1,E,G,H,I,J,L,R3  R=',F10.2//)
      WRITE (3,235)(B(I),I=1,12)
  235 FORMAT (30X,'B(1)=',E18.6,/30X,'B(2)=',E18.6,/30X,'B(3)=',
     &E18.6,/30X,'B(4)=',E18.6,/30X,'B(5)=',E18.6,/30X,'B(6)=',
     &E18.6,/30X,'B(7)=',E18 .6,/30X,'B(8)=',E18.6,/30X,'B(9)=',E18.6
     &,/30X,'B(10)=',E18.6/30X,'B(11)=',E18.6,/30X,'B(12)=',E18.6///)
C
C     2.   FIND OUT THE EXPRESSION FOR W1,S2 & W3
C
      S2(1)=-(1./8.)*Q*(B(1)*B(1)+2.*B(1)*B(2)-B(2)*B(2))
      S2(2)= (1./8.)*Q*(B(3)*B(3)-2.*B(3)*B(4)-B(4)*B(4))
      S2(3)=-(1./4.)*C*(B(1)*B(1)+B(2)*B(2))
      S2(4)= (1./4.)*Q*(B(3)*B(3)+B(4)*B(4))
      S2(5)= (1./4.)*Q*((1./2.)*B(1)*B(1)-B(1)*B(2)-(1./2.)*B(2)*B(2))
      S2(6)= (1./4.)*C*((1./2.)*B(3)*B(3)+B(3)*B(4)-(1./2.)*B(4)*B(4))
```

```
      S2(7)= (1./2.)*Q*(B(1)*B(3)-B(2)*B(4))
      S2(8)=-(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))
      S2(9)=B(6)
      S2(10)=B(7)
      W3(1)=B(8)
      W3(2)=B(9)
      W3(3)=B(10)
      W3(4)=B(11)
      W3(5)=(1./16.)*Y*  (B(2)+B(1))
      W3(6)=-(1./16.)*Y*  (B(2)-B(1))
      W3(7)=(1./16.)*Y*  (B(3)-B(4))
      W3(8)=-(1./16.)*Y*  (B(3)+B(4))
      W3(9)=(1./8)*B(12)
      WRITE (3,250)
  250 FORMAT (/10X,'ANSWER 2-1 WE CAN WRITE W1 DIRECTLY AFTER WE GET THE
     1VALUES OF ',/17X,'Q & A,B,C,D,R1 USING THE EQU.(2-5)ON PAGE 3'///)
      WRITE (3,255)
  255 FORMAT (/10X,'ANSWER2-2 THE VALUES OF S2(1)TO S2(10)'/)
      WRITE (3,260)  (S2(I),I=1,10)
  260 FORMAT (30X,'S2(1)=',E18.6,/30X,'S2(2)=',E18.6,/30X,'S2(3)=',E18.6
     &,/       30X,'S2(4)=',E18.6,/30X,'S2(5)=',E18.6,/30X,'S2(6)=',E18.6
     &,/       30X,'S2(7)=',E18.6,/30X,'S2(8)=',E18.6,/30X,'S2(9)=',E18.6
     &,/       30X,'S2(10)=',E18.6//)
      WRITE (3,265)
  265 FORMAT (10X,'ANSWER 2-3 THE VALUES OF W3(1) TO W3(9)  '/)         00002940
      WRITE (3,270)  (W3(I),I=1,9)
  270 FORMAT (30X,'W3(1)=',E18.6,/30X,'W3(2)=',E18.6,/30X,'W3(3)=',E18.6
     &,/       30X,'W3(4)=',E18.6,/30X,'W3(5)=',E18.6,/30X,'W3(6)=',E18.6
     &,/       30X,'W3(7)=',E18.6,/30X,'W3(8)=',E18.6,/30X,'W3(9)=',E18.6
     &///)
C
C     3.   FIND OUT THE RELATIONSHIP BETWEEN LOAD & DEFLECTION W/H
      X(1)=0.0
      X(2)=0.05                                                         00000050
      X(3)=0.10                                                         00000060
      X(4)=0.15                                                         00000070
      X(5)=0.20                                                         00000080
      X(6)=0.25                                                         00000090
      X(7)=0.30                                                         00000100
      X(8)=0.35                                                         00000110
      X(9)=0.40                                                         00000120
      X(10)=0.45                                                        00000130
      X(11)=0.50                                                        00000140
      X(12)=0.55                                                        00000150
                                                                        00000160
```

```
      X(13)=0.60
      X(14)=0.65
      X(15)=0.70
      X(16)=0.75
      X(17)=0.80
      X(18)=0.85
      X(19)=0.90
      X(20)=0.95
      X(21)=1.00
      X(22)=1.05
      X(23)=1.10
      X(24)=1.15
      X(25)=1.20
      X(26)=1.25
      X(27)=1.30
      X(28)=1.35
      X(29)=1.40
      X(30)=1.45
      X(31)=1.50
      X(32)=1.55
      X(33)=1.60
      X(34)=1.65
      X(35)=1.70
      X(36)=1.75
      X(37)=1.80
      X(38)=1.85
      X(39)=1.90
      X(40)=1.95
      X(41)=2.00
      WRITE (3,275) R
275   FORMAT (1H1,15X,' 3-1 THE RELATIONS BETWEEN LOAD & CENTER DEFLECTI
     &ON (LINEAR PART ONLY) R=',F10.2/)
      WRITE (3,280)
280   FORMAT (10X,'ANSWER 3-1.',14X,'CENTER DEFLECTION ',15X,'LOAD'/)
      DO 285 I=1,41
285   QCL(I)=B(5)*X(I)
      WRITE (3,290) (X(I),QCL(I),I=1,41)
290   FORMAT (40X,F5.2,15X,E18.6)
      WRITE (3,295) R
295   FORMAT (1H1,15X,' 3-2 THE RELATIONS BETWEEN LOAD & CENTER DEFLECTI
     &ON (LINEAR & NONLINEAR)  R=',F10.2/)
      WRITE (3,300)
300   FORMAT (10X,'ANSWER 3-2.',14X,'CENTER DEFLECTION ',15X,'LOAD'/)
      DO 305 I=1,41
```

```
305   QADH(I)=B(5)*X(I)+B(12)*X(I)**3
      WRITE (3,290) (X(I),QADH(I),I=1,41)
      WRITE (3,310) R
310   FORMAT (1H1,15X,'3-3 FIND THE MAXI DEFLECTION POINT (LINEAR PART O
     &NLY)  R=',F10.2/)
      WRITE (3,315)
315   FORMAT (35X,'DISTANCE FROM X=-1.SIDE',10X,'DEFLECTION'/)
      DO 320 I=1,201
      Z(1)=-1.01
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
320   W1CC(J)= (DEXP( S))*(B(1)*(DCOS(S))+B(2)*(DSIN(S)))
     &        +(DEXP(-S))*(B(3)*(DCOS(S))+B(4)*(DSIN(S)))
     &       +(B(5)/R)
      WRITE (3,290) (Z(J),W1CC(J),J=2,202)
      WRITE (3,325)
325   FORMAT (/////10X,'ANSWER 3-3. FROM THE ABOVE OUT PUT THE MAXI DEFL
     &ECTION POINT IS AT X=0.(AT THE CENTER)(LINEAR PART ONLY)')
      WRITE (3,330)
330   FORMAT (////10X,'ANSWER 3-4. FROM THE ANSWER 3-3. THE RELATIONS BE
     &TWEEN LOAD & MAXI DEFLECTION (LINEAR PART ONLY)'/17X,' WILL BE THE
     &SAME AS IN ANSWER 3-1.'/)
      WRITE (3,335) R
335   FORMAT (1H1,15X, ' 3-5 FIND THE MAXI DEFLECTION POINT (LINEAR & NO
     &NLINEAR)  R=',F10.2/)
      DO 340 I=1,100
      Z(1)=-1.01
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
340   W1W3CC(J)= (DEXP( S))*(B(1)*(DCOS(S))+B(2)*(DSIN(S)))
     &        +(DEXP(-S))*(B(3)*(DCOS(S))+B(4)*(DSIN(S)))
     &       +(B(5)/R)
     &   +(DEXP( S))*(B(8)*(DCOS( S))+B(9)*(DSIN( S)))+
     &(DEXP(- S))*(B(10)*(DCOS( S))+B(11)*(DSIN( S)))
     &-(1./16.)*Y*(DABS(Z(J)))*
     &((B(2)+B(1))*(DEXP(   S))*(DSIN(   S))
     &-(B(2)-B(1))*(DEXP(   S))*(DCOS(   S))
     &+(B(3)-B(4))*(DEXP(-  S))*(DSIN(   S))
     &-(B(3)+B(4))*(DEXP(-  S))*(DCOS(   S)))
     &+(B(12)/R)
      DO 345 I=101,201
      Z(1)=-1.01
```

```
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
345   W1W3CC(J)= (DEXP( S))*(B(1)*(DCOS(S))+B(2)*(DSIN(S)))
     &          +(DEXP(-S))*(B(3)*(DCOS(S))+B(4)*(DSIN(S)))
     &         +(B(5)/R)
     &    +(DEXP( S))*(B(8)*(DCOS( S))+B(9)*(DSIN( S)))+
     &(DEXP(- S))*(B(10)*(DCOS( S))+B(11)*(DSIN( S)))
     &+(1./16.)*Y*(DABS(Z(J)))*
     &((B(2)+B(1))*(DEXP(   S))*(DSIN(   S))
     &-(B(2)-B(1))*(DEXP(   S))*(DCOS(   S))
     &+(B(3)-B(4))*(DEXP(- S))*(DSIN(   S))
     &-(B(3)+B(4))*(DEXP(- S))*(DCOS(   S)))
     &+(B(12)/R)
      WRITE (3,315)
      WRITE (3,290)  (Z(J),W1W3CC(J),J=2,202)
      WRITE (3,350)
350   FORMAT (/////10X,'ANSWER 3-5. FROM THE ABOVE OUT PUT THE MAXI DEFL
     &ECTION POINT IS AT X=0.(AT THE CENTER)(LINEAR & NONLINEAR)')
      WRITE (3,355)
355   FORMAT (/////10X,'ANSWER 3-6. FROM THE ANSWER 3-5. THE RELATIONS BE
     &TWEEN LOAD & MAXI DEFLECTION (LINEAR & NONLINEAR)'/21X,          '
     &WILL BE THE SAME AS IN ANSWER 3-2.'/)
C
C     ******************************************************************
C     4. FIND OUT THE RELATIONSHIPS BETWEEN LOAD & TOTAL STRESS
C     ******************************************************************
C
      WRITE (3,360) R
360   FORMAT(1H1,10X,'4-1 FIND OUT THE POINTS WHERE THE MOMENT HAS THE
     &MAXI VALUES (LINEAR PART ONLY)    R=',F10.2/)
      DO 365 I=1,201
      Z(1)=-1.01
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
365   W1CXX(J)= (2.*G**2)*(-(DSIN( S))*(DEXP( S))*B(1)
     1                    +(DCOS(S  ))*(DEXP( S  ))*B(2)
     2                    +(DSIN(S  ))*(DEXP(-S  ))*B(3)
     3                    -(DCOS(S  ))*(DEXP(-S  ))*B(4))
      WRITE (3,290)  (Z(J),W1CXX(J),J=2,202)
      WRITE(3,370)   R
370   FORMAT (1H1,10X,'4-2 FIND OUT THE POINTS WHERE THE MOMENT HAS THE
     &MAXI VALUES (LINEAR & NONLINEAR)    R=',F10.2/)
```

```
      DO 371 I=1,100
      Z(1)=-1.01
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
371   W1W3XX(J)=(2.*Q**2)*(-(DSIN( S))*(DEXP( S))*B(1)
     1                        +(DCOS(S  ))*(DEXP(  S  ))*B(2)
     2                        +(DSIN(S  ))*(DEXP(-S  ))*B(3)
     3                        -(DCOS(S  ))*(DEXP(-S  ))*B(4))
     4        +(2.*Q**2)*(-(DSIN(S  ))*(DEXP(  S  ))*B(8)
     5 +(DCOS(  S))*(DEXP(  S))*B(9)+(DSIN(  S))*(DEXP(-S))*B(10)
     6 -(DCOS(  S))*(DEXP(-S))*B(11))-(1./3.)*(DABS(Z(J)))*Y*Q**2*
     7(((B(2)+B(1))*(DCOS(S  ))+(B(2)-B(1))*(DSIN(S  )))*(DEXP(  S  ))
     8+((B(4)-B(3))*(DCOS(S  ))-(B(4)+B(3))*(DSIN(S  )))*(DEXP(-S  )))
     9+(1./4.)*Y*Q*    ((B(1)*(DCOS(S  ))+B(2)*(DSIN(S  )))*(DEXP(  S  ))
     1+(B(3)*(DCOS(S  ))+B(4)*(DSIN(S  )))*(DEXP(-S  )))
      DO 372 I=100,201
      Z(1)=-1.01
      J=I+1.
      Z(J)=Z(I)+0.01
      S=Q*Z(J)
372   W1W3XX(J)=(2.*Q**2)*(-(DSIN(  S))*(DEXP(  S))*B(1)
     1                        +(DCOS(S  ))*(DEXP(  S  ))*B(2)
     2                        +(DSIN(S  ))*(DEXP(-S  ))*B(3)
     3                        -(DCOS(S  ))*(DEXP(-S  ))*B(4))
     4        +(2.*Q**2)*(-(DSIN(S  ))*(DEXP(  S  ))*B(8)
     5 +(DCOS(  S))*(DEXP(  S))*B(9)+(DSIN(  S))*(DEXP(-S))*B(10)
     6 -(DCOS(  S))*(DEXP(-S))*B(11))+(1./8.)*(DABS(Z(J)))*Y*Q**2*
     7(((B(2)+B(1))*(DCOS(S  ))+(B(2)-B(1))*(DSIN(S  )))*(DEXP(  S  ))
     8+((B(4)-B(3))*(DCOS(S  ))-(B(4)+B(3))*(DSIN(S  )))*(DEXP(-S  )))
     9+(1./4.)*Y*Q*    ((B(1)*(DCOS(S  ))+B(2)*(DSIN(S  )))*(DEXP(  S  ))
     1+(B(3)*(DCOS(S  ))+B(4)*(DSIN(S  )))*(DEXP(-S  )))
      WRITE (3,315)
      WRITE(3,290) (Z(J),W1W3XX(J),J=2,202)
      WRITE (3,375) R
375   FORMAT (////10X,'ANSWER 4-1. FROM THE ABOVE OUT PUT WE KNOW X=+1.
     1        WHERE THE MOMENT HAS THE MAXI VALUE THE MAXI STRESS'/21X,'
     2WILL BE AT THE SAME POINT X=+1.    R=',F10.2/////)
      S1=Q*(+1.)
      W1XX = (2.*Q**2)*(-(DSIN(S1))*(DEXP( S1))*B(1)
     &                   +(DCOS(S1))*(DEXP( S1))*B(2)
     &                   +(DSIN(S1))*(DEXP(-S1))*B(3)
     &                   -(DCOS(S1))*(DEXP(-S1))*B(4))
      W3XX = (2.*Q**2)*(-(DSIN(S1))*(DEXP( S1))*B(8)
```

```
     &                        +(DCOS(S1))*(DEXP( S1))*B(9)
     &                        +(DSIN(S1))*(DEXP(-S1))*B(10)
     &                        -(DCOS(S1))*(DEXP(-S1))*B(11))
     &+(1./8.)*(+1.  )*Y*Q**2*
     &(((B(2)+B(1))*(DCOS(S1))+(B(2)-B(1))*(DSIN(S1)))*(DEXP( S1))
     &+((B(4)-B(3))*(DCOS(S1))-(B(4)+B(3))*(DSIN(S1)))*(DEXP(-S1)))
     &+(1./4.)*Y*Q*    ((B(1)*(DCOS(S1))+B(2)*(DSIN(S1)))*(DEXP( S1))
     &+(B(3)*(DCOS(S1))+B(4)*(DSIN(S1)))*(DEXP(-S1)))
      WRITE (3,380)  W1XX,W3XX
380   FORMAT (30X,'W1XX='   ,E18.6,/30X,'W3XX=',E18.6/// )
      WRITE (3,385) R
385   FORMAT (1H1,10X,'ANSWER 4-2. THE FOLLOWING ARE X(I) & SBL(I), SBL
     &IS THE STRESS FROM BENDING , LINEAR PART ONLY R=',F10.2//)
      DO 390  I=1,41
390   SBL(I)= (1./2.)* (DABS(W1XX))*X(I)
      WRITE (3,290)  (X(I),SBL(I),I=1,41)
      WRITE (3,395) R
395   FORMAT (1H1,10X,'ANSWER 4-3. THE FOLLOWING ARE X(I) & SBN(I), SBN
     &IS THE STRESS FROM BENDING, LINEAR & NONLINEAR  R=',F10.2//)
      DO 400  I=1,41
400   SBN(I)=(1./2.)*((DABS(W1XX))*X(I)+(DABS(W3XX))*(X(I)**3))
      WRITE (3,290)  (X(I),SBN(I),I=1,41)
      WRITE (3,405) R
405   FORMAT (1H1,10X,'ANSWER 4-4. THE FOLLOWING ARE X(I) & SM(I), SM
     &IS THE STRESS FROM MEMBERANE  R=',F10.2//)
      DO 410  I=1,41
410   SM(I)=    (X(I)**2)*((B(1)*B(4)-B(2)*B(3))*Q**2+B(6))
      WRITE (3,290)  (X(I),SM(I),I=1,41)
      DO 415 I=1,41
415   STSS(I)= (X(I)**2)*((B(1)*B(4)-B(2)*B(3))*Q**2+B(6))
     &+(1./2.)*((DABS(W1XX))*X(I)+(DABS(W3XX))*(X(I)**3))
      WRITE (3,420) R
420   FORMAT (1H1,10X,'ANSWER 5. FIND OUT THE RELATIONSHIPS BETWEEN
     &STRESS(STSS) AND DEFLECTION (W/H)  R=',F10.2//)
      WRITE (3,425)  (X(I),STSS(I),I=1,20)
425   FORMAT (30X,'X( 1)=',   F5.2   ,10X,'STSS( 1)=',E18.6,   /
     &             30X,'X( 2)=',   F5.2   ,10X,'STSS( 2)=',E18.6,   /
     &             30X,'X( 3)=',   F5.2   ,10X,'STSS( 3)=',E18.6,   /
     &             30X,'X( 4)=',   F5.2   ,10X,'STSS( 4)=',E18.6,   /
     &             30X,'X( 5)=',   F5.2   ,10X,'STSS( 5)=',E18.6,   /
     &             30X,'X( 6)=',   F5.2   ,10X,'STSS( 6)=',E18.6,   /
     &             30X,'X( 7)=',   F5.2   ,10X,'STSS( 7)=',E18.6,   /
     &             30X,'X( 8)=',   F5.2   ,10X,'STSS( 8)=',E18.6,   /
     &             30X,'X( 9)=',   F5.2   ,10X,'STSS( 9)=',E18.6,   /
```

```
     &          30X,'X(10)=',     F5.2    ,10X,'STSS(10)=',E18.6,    /
     &          30X,'X(11)=',     F5.2    ,10X,'STSS(11)=',E18.6,    /
     &          30X,'X(12)=',     F5.2    ,10X,'STSS(12)=',E18.6,    /
     &          30X,'X(13)=',     F5.2    ,10X,'STSS(13)=',E18.6,    /
     &          30X,'X(14)=',     F5.2    ,10X,'STSS(14)=',E18.6,    /
     &          30X,'X(15)=',     F5.2    ,10X,'STSS(15)=',E18.6,    /
     &          30X,'X(16)=',     F5.2    ,10X,'STSS(16)=',E18.6,    /
     &          30X,'X(17)=',     F5.2    ,10X,'STSS(17)=',E18.6,    /
     &          30X,'X(18)=',     F5.2    ,10X,'STSS(18)=',E18.6,    /
     &          30X,'X(19)=',     F5.2    ,10X,'STSS(19)=',E18.6,    /
     &          30X,'X(20)=',     F5.2    ,10X,'STSS(20)=',E18.6)
      WRITE (3,430)  (X(I),STSS(I),I=21,31)
430   FORMAT (30X,'X(21)=',     F5.2    ,10X,'STSS(21)=',E18.6,    /
     &          30X,'X(22)=',     F5.2    ,10X,'STSS(22)=',E18.6,    /
     &          30X,'X(23)=',     F5.2    ,10X,'STSS(23)=',E18.6,    /
     &          30X,'X(24)=',     F5.2    ,10X,'STSS(24)=',E18.6,    /
     &          30X,'X(25)=',     F5.2    ,10X,'STSS(25)=',E18.6,    /
     &          30X,'X(26)=',     F5.2    ,10X,'STSS(26)=',E18.6,    /
     &          30X,'X(27)=',     F5.2    ,10X,'STSS(27)=',E18.6,    /
     &          30X,'X(28)=',     F5.2    ,10X,'STSS(28)=',E18.6,    /
     &          30X,'X(29)=',     F5.2    ,10X,'STSS(29)=',E18.6,    /
     &          30X,'X(30)=',     F5.2    ,10X,'STSS(30)=',E18.6,    /
     &          30X,'X(31)=',     F5.2    ,10X,'STSS(31)=',E18.6)
      WRITE (3,435)  (X(I),STSS(I),I=32,41)
435   FORMAT (30X,'X(32)=',     F5.2    ,10X,'STSS(32)=',E18.6,    /
     &          30X,'X(33)=',     F5.2    ,10X,'STSS(33)=',E18.6,    /
     &          30X,'X(34)=',     F5.2    ,10X,'STSS(34)=',E18.6,    /
     &          30X,'X(35)=',     F5.2    ,10X,'STSS(35)=',E18.6,    /
     &          30X,'X(36)=',     F5.2    ,10X,'STSS(36)=',E18.6,    /
     &          30X,'X(37)=',     F5.2    ,10X,'STSS(37)=',E18.6,    /
     &          30X,'X(38)=',     F5.2    ,10X,'STSS(38)=',E18.6,    /
     &          30X,'X(39)=',     F5.2    ,10X,'STSS(39)=',E18.6,    /
     &          30X,'X(40)=',     F5.2    ,10X,'STSS(40)=',E18.6,    /
     &          30X,'X(41)=',     F5.2    ,10X,'STSS(41)=',E18.6//)
      R=R+20.
      IF(R-100.) 5,5,500
500   STOP
      END
      SUBROUTINE MATIN(A,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION INDEX(12,2),A(12,12)
      DO 108 I=1,N
108   INDEX (I,1) = 0
      II= 0
```

```
109   AMAX=-1.
      DO 110 I=1,N
      IF (INDEX(I,1)) 110,111,110
111   DO 112 J=1,N
      IF (INDEX(J,1)) 112,113,112
113   TEMP=DABS(A(I,J))
      IF (TEMP-AMAX) 112,112,114
114   IROW=I
      ICOL=J
      AMAX=TEMP
112   CONTINUE
110   CONTINUE
      IF (AMAX) 225,115,116
116   INDEX (ICOL,1)=IROW
      IF (IROW-ICOL) 119,118,119
119   DO 120 J=1,N
      TEMP=A(IROW,J)
      A(IROW,J)=A(ICOL,J)
120   A(ICOL,J)=TEMP
      II=II+1
      INDEX (II,2)=ICOL
118   PIVOT=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      PIVOT=1./PIVOT
      DO 121 J=1,N
121   A(ICOL,J)=A(ICOL,J)*PIVOT
      DO 122 I=1,N
      IF (I-ICOL) 123,122,123
123   TEMP=A(I,ICOL)
      A(I,ICOL)=0.
      DO 124 J=1,N
124   A(I,J)=A(I,J)-A(ICOL,J)*TEMP
122   CONTINUE
      GO TO 109
125   ICOL=INDEX(II,2)
      IROW=INDEX(ICOL,1)
      DO 126 I=1,N
      TEMP=A(I,IROW)
      A(I,IROW)=A(I,ICOL)
126   A(I,ICOL)=TEMP
      II=II-1
225   IF (II) 125,127,125
115   WRITE (3,1001)
1001  FORMAT (//2X,'ZERO PIVO')
```

```
127   CCNTINUF
      RFTURN
      END
      SUBPCUTINE MATMUL (A,B,C,M,N,K)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(12,12),B(12,12),C(12,12)
      DC 21 I=1,M
      DO 21 J=1,K
      C(I,J)=0.
      DC 20 L=1,N
 20   C(I,J)=C(I,J)+A(I,L)*B(L,J)
 21   CCNTINUF
      RFTURN
      END
```

```
C*****************************************************************
C* LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES          *
C*        WITH OR WITHOUT ELASTIC FOUNDATION                     *
C*                                                               *
C* (1) LOADING  CONDITION      F=1.                              *
C* (2) BOUNDARY CONDITION      SIMPLY SUPPORTED                  *
C*                             BOTH SIDES                        *
C*****************************************************************
      A(1,1)=-(DSIN(Q))*(DEXP( Q))
      A(1,2)= (DCOS(Q))*(DEXP( Q))
      A(1,3)= (DSIN(Q))*(DEXP(-Q))
      A(1,4)=-(DCOS(Q))*(DEXP(-Q))
      A(1,5)=0.
      A(2,1)= (DSIN(Q))*(DEXP(-Q))
      A(2,2)= (DCOS(Q))*(DEXP(-Q))
      A(2,3)=-(DSIN(Q))*(DEXP( Q))
      A(2,4)=-(DCOS(Q))*(DEXP( Q))
      A(2,5)=0.
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(-Q))
      A(3,4)= (DSIN(Q))*(DEXP(-Q))
      A(3,5)=1./R
      A(4,1)= (DCOS(Q))*(DEXP(-Q))
      A(4,2)=-(DSIN(Q))*(DEXP(-Q))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(4,5)=1./R
      A(5,1)=1.
      A(5,2)=0.
      A(5,3)=1.
      A(5,4)=0.
      A(5,5)=1./R
      A(6,6)=1.
      A(6,7)=1.
      A(7,6)=-1.
      A(7,7)=1.
      A(8,8)=-(DSIN(Q))*(DEXP( Q))
      A(8,9)= (DCOS(Q))*(DEXP( Q))
      A(8,10)= (DSIN(Q))*(DEXP(-Q))
```

```
      A(8,11)=-(DCOS(Q))*(DEXP( -Q))
      A(9,8)= (DSIN(Q))*(DEXP( -Q))
      A(9,9)= (DCOS(Q))*(DEXP( -Q))
      A(9,10)=-(DSIN(Q))*(DEXP( Q))
      A(9,11)=-(DCOS(Q))*(DEXP(Q))
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                   00000520
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                   00000530
      A(10,10)= (DCOS(Q))*(DEXP(-Q))                                   00000540
      A(10,11)= (DSIN(Q))*(DEXP(-Q))                                   00000550
      A(10,12)=1./R                                                    00000560
      A(11, 8)= (DCOS(Q))*(DEXP(-Q))                                   00000570
      A(11, 9)=-(DSIN(Q))*(DEXP(-Q))                                   00000580
      A(11,10)= (DCOS(Q))*(DEXP( Q))                                   00000590
      A(11,11)=-(DSIN(Q))*(DEXP( Q))                                   00000600
      A(11,12)=1./R                                                    00000610
      A(12,8)=1.                                                       00000620
      A(12,10)=1.                                                      00000630
      A(12,12)=1./R                                                    00000640

      C(1)=0.                                                          00000980
      C(2)=0.                                                          00000990
      C(3)=0.                                                          00001000
      C(4)=0.                                                          00001010
      C(5)=1.                                                          00001020
      C(6)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)   00001320
     1*(DEXP( QQ))*(DCOS(QQ))                                          00001330
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)        00001340
     3*(DEXP(XQQ))*(DCOS(QQ))                                          00001350
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))         00001360
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))         00001370
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)            00001380
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))                         00001390
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)            00001400
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))                         00001410
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                      00001420
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))                      00001430
                                       ***********************         00001440
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)   00001450
     1*(DEXP(XQQ))*(DCOS(QQ))                                          00001460
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)        00001470
     3*(DEXP( QQ))*(DCOS(QQ))                                          00001480
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( XQQ))        00001490
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))         00001500
```

```
6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
 C(8)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DCOS(Q))+(B(2)-B(1))*(DSIN(Q)))
&-(DEXP(-Q))*((B(3)-B(4))*(DCOS(Q))+(B(3)+B(4))*(DSIN(Q))))
&-(1./(8.*Q))*Y*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
&              +(DEXP(-Q))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
 C(9)= (1./16.)*Y*
&((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
&-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
&-(1./(8.*Q))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
&              +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
 C(10)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
 C(11)=-(1./16.)*Y*
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
 C(12)=0.
 END
```

```
C     ***************************************************************   00000090
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *          00000040
C     *       WITH OR WITHOUT ELASTIC FOUNDATION                    *   00000050
C     *                                                             *   00000060
C     * (1) LOADING  CONDITION      F=1                             *
C     * (2) BOUNDARY CONDITION      CLAMPED X=+1.                   *   00000080
C     *                             SIMPLY SUPPORTED X=-1.          *   00000080
C     ***************************************************************   00000030

      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))                              00000150
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))                              00000160
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(-Q))                              00000170
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(-Q))                              00000180
      A(1,5)=0.                                                         00000190
      A(2,1)= (DSIN(Q))*(DEXP(-Q))
      A(2,2)= (DCOS(Q))*(DEXP(-Q))
      A(2,3)=-(DSIN(Q))*(DEXP( Q))
      A(2,4)=-(DCOS(Q))*(DEXP( Q))
      A(2,5)=0.                                                         00000240
      A(3,1)= (DCOS(Q))*(DEXP( Q))                                      00000250
      A(3,2)= (DSIN(Q))*(DEXP( Q))                                      00000260
      A(3,3)= (DCOS(Q))*(DEXP(-Q))                                      00000270
      A(3,4)= (DSIN(Q))*(DEXP(-Q))                                      00000280
      A(3,5)=1./R                                                       00000290
      A(4,1)= (DCOS(Q))*(DEXP(-Q))                                      00000300
      A(4,2)=-(DSIN(Q))*(DEXP(-Q))                                      00000310
      A(4,3)= (DCOS(Q))*(DEXP( Q))                                      00000320
      A(4,4)=-(DSIN(Q))*(DEXP( Q))                                      00000330
      A(4,5)=1./R                                                       00000340
      A(5,1)=1.                                                         00000350
      A(5,2)=0.                                                         00000360
      A(5,3)=1.                                                         00000370
      A(5,4)=0.                                                         00000380
      A(5,5)=1./R                                                       00000390
      A(6,6)=1.                                                         00000400
      A(6,7)=1.                                                         00000410
      A(7,6)=-1.                                                        00000420
      A(7,7)=1.                                                         00000430
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))                            00000440
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))                            00000450
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(-Q))                            00000460
```

```
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(-Q))
      A(9,8)= (DSIN(Q))*(DEXP( -Q))
      A(9,9)= (DCOS(Q))*(DEXP( -Q))
      A(9,10)=-(DSIN(Q))*(DEXP( Q))
      A(9,11)=-(DCOS(Q))*(DEXP(Q))
      A(10, 8)= (DCOS(Q))*(DEXP( Q))
      A(10, 9)= (DSIN(Q))*(DEXP( Q))
      A(10,10)= (DCOS(Q))*(DEXP(-Q))
      A(10,11)= (DSIN(Q))*(DEXP(-Q))
      A(10,12)=1./R
      A(11, 8)= (DCOS(Q))*(DEXP(-Q))
      A(11, 9)=-(DSIN(Q))*(DEXP(-Q))
      A(11,10)= (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(11,12)=1./R
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)=1./R
C
      C(1)=0.
C
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      C(6)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
     1*(DEXP( QQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
     3*(DEXP(XQQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2 +(DABS(B(2)))**2)*(DEXP( QQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( XQQ))
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
C                                  *********************
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
     1*(DEXP(XQQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
     3*(DEXP( QQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2 +(DABS(B(2)))**2)*(DEXP(XQQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))
```

```
6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
 C(8)=-(1./8.)*Y*  ((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
&                  +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
&-(1./16.)*(Y/Q)*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
 C(9)= (1./16.)*Y*
&((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
&-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
&-(1./(8.*Q))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
&              +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
 C(10)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
 C(11)=-(1./16.)*Y*
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
 C(12)=0.
 END
```

```
C     ********************************************************    00000090
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *    00000040
C     *       WITH OR WITHOUT ELASTIC FOUNDATION              *    00000050
C     *                                                       *    00000060
C     * (1) LOADING   CONDITION     F=1 + X                   *
C     * (2) BOUNDARY CONDITION      CLAMPED BOTH SIDES        *    00000080
C     ********************************************************    00000030
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q                       00000420
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q                       00000430
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q                       00000440
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q                       00000450
      A(1,5)=1./R
      A(2,1)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q                       00000470
      A(2,2)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q                       00000480
      A(2,3)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q                       00000490
      A(2,4)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q                       00000500
      A(2,5)=1./R
      A(3,1)= (DCOS(Q))*(DEXP( Q))                                 00000520
      A(3,2)= (DSIN(Q))*(DEXP( Q))                                 00000530
      A(3,3)= (DCOS(Q))*(DEXP(XQ))                                 00000540
      A(3,4)= (DSIN(Q))*(DEXP(XQ))                                 00000550
      A(3,5)=2./R
      A(4,1)= (DCOS(Q))*(DEXP(XQ))                                 00000570
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))                                 00000580
      A(4,3)= (DCOS(Q))*(DEXP( Q))                                 00000590
      A(4,4)=-(DSIN(Q))*(DEXP( Q))                                 00000600
      A(5,1)=1.                                                    00000620
      A(5,3)=1.                                                    00000640
      A(5,5)=1./R                                                  00000660
      A(6,6)=1.                                                    00000670
      A(6,7)=1.                                                    00000680
      A(7,6)=-1.                                                   00000690
      A(7,7)=1.                                                    00000700
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))                       00000710
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))                       00000720
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))                       00000730
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))                       00000740
      A(8,12)=1./(R*Q)
      A( 9, 8)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))                       00000750
      A( 9, 9)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))                       00000760
```

```
      A( 9,10)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))                            00000770
      A( 9,11)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))                            00000780
      A(9,12)=1./(R*Q)
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                    00000790
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                    00000800
      A(10,10)= (DCOS(Q))*(DEXP(XQ))                                    00000810
      A(10,11)= (DSIN(Q))*(DEXP(XQ))                                    00000820
      A(10,12)=2./R                                                     00000830
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))                                    00000840
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))                                    00000850
      A(11,10)= (DCOS(Q))*(DEXP( Q))                                    00000860
      A(11,11)=-(DSIN(Q))*(DEXP( Q))                                    00000870
      A(12,8)=1.                                                        00000890
      A(12,10)=1.                                                       00000900
      A(12,12)=1./R                                                     00000910
C
C
      C(1)=0.                                                           00000980
      C(2)=0.                                                           00000990
      C(3)=0.                                                           00001000
      C(4)=0.                                                           00001010
      C(5)=1.                                                           00001020
      C(6)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    000013
     1*(DEXP( QQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         0000
     3*(DEXP(XQQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))          00
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)             00
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
     3+(B(5)/R)*B(1)*(DEXP( Q))*(DCOS( Q))
     3+(B(5)/R)*B(2)*(DEXP( Q))*(DSIN( Q))
     3+(B(5)/R)*B(3)*(DEXP(-Q))*(DCOS( Q))
     3+(B(5)/R)*B(4)*(DEXP(-Q))*(DSIN( Q))
C                                       *********************           00001440
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    00001450
     1*(DEXP(XQQ))*(DCOS(QQ))                                           00001460
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         00001470
     3*(DEXP( QQ))*(DCOS(QQ))                                           00001480
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))          00001490
```

```
5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))        00001500
6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)           00001510
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))                         00001520
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)           00001530
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))                         00001540
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                      00001550
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))                      00001560
&+(B(5)/R)*B(1)*(DEXP(-Q))*(DCOS( Q))
&-(B(5)/R)*B(2)*(DEXP(-Q))*(DSIN( Q))
&+(B(5)/R)*B(3)*(DEXP( Q))*(DCOS( Q))
&-(B(5)/R)*B(4)*(DEXP( Q))*(DSIN( Q))
 C(8)=-(1./8.)*Y*  ((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))   00001950
&                  +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))   00001960
&-(1./16.)*(Y/Q)*                                                 00001970
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))        00001980
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))       00001990
 C(9)= (1./8.)*Y*  ((DEXP(XQ))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))   00002000
&                  +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))   00002010
&+(1./16.)*(Y/Q)*                                                 00002020
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))        00002030
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))       00002040
 C(10)=-(1./16.)*Y*                                               00002050
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))        00002060
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))       00002070
 C(11)=-(1./16.)*Y*                                               00002080
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))        00002090
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))       00002100
 C(12)=0.                                                         00002110
 END
```

```
C     ****************************************************************   00000090
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *          00000040
C     *      WITH OR WITHOUT ELASTIC FOUNDATION
C     *                                                              *   00000060
C     * (1) LOADING  CONDITION      F=1 + X                          *
C     * (2) BOUNDARY CONDITION      SIMPLY SUPPORTED                 *   00000080
C     *                             BOTH SIDES                       *   00000080
C     ****************************************************************   00000030
      A(1,1)=-(DSIN(Q))*(DEXP( Q))                                       00000420
      A(1,2)= (DCOS(Q))*(DEXP( Q))                                       00000430
      A(1,3)= (DSIN(Q))*(DEXP(-Q))                                       00000440
      A(1,4)=-(DCOS(Q))*(DEXP(-Q))                                       00000450
      A(2,1)= (DSIN(Q))*(DEXP(-Q))                                       00000470
      A(2,2)= (DCOS(Q))*(DEXP(-Q))                                       00000480
      A(2,3)=-(DSIN(Q))*(DEXP( Q))                                       00000490
      A(2,4)=-(DCOS(Q))*(DEXP( Q))                                       00000500
      A(3,1)= (DCOS(Q))*(DEXP( Q))                                       00000520
      A(3,2)= (DSIN(Q))*(DEXP( Q))                                       00000530
      A(3,3)= (DCOS(Q))*(DEXP(-Q))                                       00000540
      A(3,4)= (DSIN(Q))*(DEXP(-Q))                                       00000550
      A(3,5)=2./R
      A(4,1)= (DCOS(Q))*(DEXP(-Q))                                       00000570
      A(4,2)=-(DSIN(Q))*(DEXP(-Q))                                       00000580
      A(4,3)= (DCOS(Q))*(DEXP( Q))                                       00000590
      A(4,4)=-(DSIN(Q))*(DEXP( Q))                                       00000600
      A(5,1)=1.                                                          00000620
      A(5,3)=1.                                                          00000640
      A(5,5)=1./R                                                        00000660
      A(6,6)=1.                                                          00000670
      A(6,7)=1.                                                          00000680
      A(7,6)=-1.                                                         00000690
      A(7,7)=1.                                                          00000700
      A(8,8)=-(DSIN(Q))*(DEXP( Q))                                       00000710
      A(8,9)= (DCOS(Q))*(DEXP( Q))                                       00000720
      A(8,10)= (DSIN(Q))*(DEXP(-Q))                                      00000730
      A(8,11)=-(DCOS(Q))*(DEXP( -Q))                                     00000740
      A(9,8)= (DSIN(Q))*(DEXP( -Q))                                      00000750
      A(9,9)= (DCOS(Q))*(DEXP( -Q))                                      00000760
      A(9,10)=-(DSIN(Q))*(DEXP( Q))                                      00000770
      A(9,11)=-(DCOS(Q))*(DEXP(Q))                                       00000780
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                     00000790
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                     00000800
```

```
      A(10,10)= (DCOS(Q))*(DEXP(-Q))
      A(10,11)= (DSIN(Q))*(DEXP(-Q))
      A(10,12)=2./R
      A(11, 8)= (DCOS(Q))*(DEXP(-Q))
      A(11, 9)=-(DSIN(Q))*(DEXP(-Q))
      A(11,10)= (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)=1./R
C
C
      C(1)=0.
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      C(6)= (1./8.)*Q*(B(1)**2+2.*B(1)*B(2)-B(2)**2)
     1*(DEXP( QQ))*(DCOS(QQ))
     2-(1./8.)*Q*(B(3)**2-2.*B(3)*B(4)-B(4)**2)
     3*(DEXP(XQQ))*(DCOS(QQ))
     4+(1./4.)*Q*(B(1)**2+B(2)**2)*(DEXP( QQ))
     5-(1./4.)*Q*(B(3)**2+B(4)**2)*(DEXP(XQQ))
     6-(1./4.)*Q*((1./2.)*B(1)**2-B(1)*         B(2)-(1./2.)*B(2)**2)
     7*(DEXP( QQ))*(DSIN(QQ))
     8-(1./4.)*Q*((1./2.)*B(3)**2+B(3)*B(4)-(1./2.)*B(4)**2)
     9*(DEXP(XQQ))*(DSIN(QQ))
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))
     &+(B(5)/R)*B(1)*(DEXP( Q))*(DCOS( Q))
     &+(B(5)/R)*B(2)*(DEXP( Q))*(DSIN( Q))
     &+(B(5)/R)*B(3)*(DEXP(-Q))*(DCOS( Q))
     &+(B(5)/R)*B(4)*(DEXP(-Q))*(DSIN( Q))
C                          *********************
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
     1*(DEXP(XQQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
     3*(DEXP( QQ))*(DCOS(QQ))
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))
     6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
     9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))
```

```
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                          00001620
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))                          00001630
&+(B(5)/R)*B(1)*(DEXP(-Q))*(DCOS( Q))                                 00001640
&-(B(5)/R)*B(2)*(DEXP(-Q))*(DSIN( Q))                                 00001650
&+(B(5)/R)*B(3)*(DEXP( Q))*(DCOS( Q))                                 00001660
&-(B(5)/R)*B(4)*(DEXP( Q))*(DSIN( Q))                                 00001670
 C(8)=-(1./16.)*Y*                                                    00001940
&((DEXP( Q))*((B(2)+B(1))*(DCOS(Q))+(B(2)-B(1))*(DSIN(Q)))            00001950
&-(DEXP(-Q))*((B(3)-B(4))*(DCOS(Q))+(B(3)+B(4))*(DSIN(Q))))           00001960
&-(1./(8.*Q))*Y*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))           00001970
&          +(DEXP(-Q))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))               00001980
 C(9)= (1./16.)*Y*                                                    00001990
&((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))            00002000
&-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))           00002010
&-(1./(8.*Q))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))           00002020
&          +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))               00002030
 C(10)=-(1./16.)*Y*                                                   00002160
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))            00002170
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))           00002180
 C(11)=-(1./16.)*Y*                                                   00002190
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))            00002200
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))           00002210
 C(12)=0.                                                             00002220
 END
```

```
C ***************************************************************
C * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES        *
C *        WITH OR WITHOUT ELASTIC FOUNDATION                   *
C *                                                             *
C *                                                             *
C * (1) LOADING  CONDITION      F=1 + X                         *
C * (2) BOUNDARY CONDITION      CLAMPED X=+1.                   *
C *                             SIMPLY SUPPORTED X=-1.          *
C *                                                             *
C ***************************************************************
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(-Q))*Q
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(-Q))*Q
      A(1,5)=1./R
      A(2,1)= (DSIN(Q))*(DEXP(-Q))
      A(2,2)= (DCOS(Q))*(DEXP(-Q))
      A(2,3)=-(DSIN(Q))*(DEXP( Q))
      A(2,4)=-(DCOS(Q))*(DEXP( Q))
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(-Q))
      A(3,4)= (DSIN(Q))*(DEXP(-Q))
      A(3,5)=2./R
      A(4,1)= (DCOS(Q))*(DEXP(-Q))
      A(4,2)=-(DSIN(Q))*(DEXP(-Q))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)=1./R
      A(6,6)=1.
      A(6,7)=1.
      A(7,6)=-1.
      A(7,7)=1.
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(-Q))
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(-Q))
      A(8,12)=1./(R*Q)
      A(9,8)= (DSIN(Q))*(DEXP( -Q))
      A(9,9)= (DCOS(Q))*(DEXP( -Q))
      A(9,10)=-(DSIN(Q))*(DEXP( Q))
```

```
      A(9,11)=-(DCOS(Q))*(DEXP(Q))                                      00000790
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                    00000800
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                    00000810
      A(10,10)= (DCOS(Q))*(DEXP(-Q))                                    00000820
      A(10,11)= (DSIN(Q))*(DEXP(-Q))                                    00000830
      A(10,12)=2./R                                                     00000840
      A(11, 8)= (DCOS(Q))*(DEXP(-Q))                                    00000850
      A(11, 9)=-(DSIN(Q))*(DEXP(-Q))                                    00000860
      A(11,10)= (DCOS(Q))*(DEXP( Q))                                    00000870
      A(11,11)=-(DSIN(Q))*(DEXP( Q))                                    00000880
      A(12,8)=1.                                                        00000900
      A(12,10)=1.                                                       00000910
      A(12,12)=1./R                                                     00000920
C
C
      C(1)=0.                                                           00001010
      C(2)=0.                                                           00001020
      C(3)=0.                                                           00001030
      C(4)=0.                                                           00001040
      C(5)=1.                                                           00001050
      C(6)= (1./8.)*Q*(B(1)**2+2.*B(1)*B(2)-B(2)**2)                    00001350
     1*(DEXP( QQ))*(DCOS(QQ))                                           00001360
     2-(1./8.)*Q*(B(3)**2-2.*B(3)*B(4)-B(4)**2)                         00001370
     3*(DEXP(XQQ))*(DCOS(QQ))                                           00001380
     4+(1./4.)*Q*(B(1)**2+B(2)**2)*(DEXP( QQ))                          00001390
     5-(1./4.)*Q*(B(3)**2+B(4)**2)*(DEXP(XQQ))                          00001400
     6-(1./4.)*Q*((1./2.)*B(1)**2-B(1)*          B(2)-(1./2.)*B(2)**2)  00001410
     7*(DEXP( QQ))*(DSIN(QQ))                                           00001420
     8-(1./4.)*Q*((1./2.)*B(3)**2+B(3)*B(4)-(1./2.)*B(4)**2)            00001430
     9*(DEXP(XQQ))*(DSIN(QQ))                                           00001440
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                       00001450
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))                       00001460
     &+(B(5)/R)*B(1)*(DEXP( Q))*(DCOS( Q))                              00001470
     &+(B(5)/R)*B(2)*(DEXP( Q))*(DSIN( Q))                              00001480
     &+(B(5)/R)*B(3)*(DEXP(-Q))*(DCOS( Q))                              00001490
     &+(B(5)/R)*B(4)*(DEXP(-Q))*(DSIN( Q))                              00001500
                                   ********************                 00001510
      C(7)= (1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    00001520
     1*(DEXP(XQQ))*(DCOS(QQ))                                           00001530
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         00001540
     3*(DEXP( QQ))*(DCOS(QQ))                                           00001550
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))          00001560
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))          00001570
     6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)             00001580
```

```
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))                                 00001590
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)                     00001600
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))                                  00001610
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                               00001620
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ))                               00001630
&+(B(5)/R)*B(1)*(DEXP(-Q))*(DCOS( Q))                                      00001640
&-(B(5)/R)*B(2)*(DEXP(-Q))*(DSIN( Q))                                      00001650
&+(B(5)/R)*B(3)*(DEXP( Q))*(DCOS( Q))                                      00001660
&-(B(5)/R)*B(4)*(DEXP( Q))*(DSIN( Q))                                      00001670
 C(8)=-(1./8.)*Y*  ((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))             00002060
&                  +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))            00002070
&-(1./16.)*(Y/Q)*                                                          00002080
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))                 00002090
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))                00002100
 C(9)= (1./16.)*Y*
&((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))                 00002000
&-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))                0000201
&-(1./(8.*Q))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))                00002020
&              +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))                00002030
 C(10)=-(1./16.)*Y*                                                        00002160
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))                 00002170
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))                00002180
 C(11)=-(1./16.)*Y*                                                        00002190
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))                 00002200
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))                00002210
 C(12)=0.                                                                  00002220
 END
```

```
C
C      ******************************************************************
C      * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *        00000040
C      *        WITH OR WITHOUT ELASTIC FOUNDATION                *        00000050
C      * (1) LOADING  CONDITION       F=1 + X**2                  *
C      * (2) BOUNDARY CONDITION       CLAMPED BOTH SIDES          *        00000080
C      ******************************************************************
C
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(1,5)=2./R
      A(2,1)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A(2,2)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(2,3)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(2,4)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(2,5)=-(2./R)
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(XQ))
      A(3,4)= (DSIN(Q))*(DEXP(XQ))
      A(3,5)=2./R
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(4,5)=2./R
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)=1./R
      A(6,6)=Q*(R**2)
      A(6,7)=Q*(R**2)
      A(7,6)=-Q*(R**2)
      A(7,7)=Q*(R**2)
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(8,12)=2./R
      A( 9, 8)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A( 9, 9)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
```

```
      A( 9,10)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A( 9,11)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(9,12)=-2./R
      A(10, 8)= (DCOS(Q))*(DEXP( Q))
      A(10, 9)= (DSIN(Q))*(DEXP( Q))
      A(10,10)= (DCOS(Q))*(DEXP(XQ))
      A(10,11)= (DSIN(Q))*(DEXP(XQ))
      A(10,12)=2./R
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))
      A(11,10)= (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(11,12)=2./R
      A(12,8)=R**2
      A(12,10)=R**2
      A(12,12)=R
C
C
      C(1)=0.                                                           00000980
      C(2)=0.                                                           00000990
      C(3)=0.                                                           00001000
      C(4)=0.                                                           00001010
      C(5)=1.                                                           00001020
      C(6)=((1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    000013
     1*(DEXP( QQ))*(DCOS(QQ))
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         0000
     3*(DEXP(XQQ))*(DCOS(QQ))                                           00001350
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))          00
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))                          00001390
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)
     3+     (R)*B(5)*((DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(1)+(DSIN(Q)
     4-DCOS(Q))*B(2))-(DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(3)+(DSIN(Q)+
     5 DCOS(Q))*B(4)))+       B(5)*((DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+
     6DSIN(Q)))*B(1)+(DEXP( Q))*(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(2)
     7+(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)-DSIN(Q)))*B(3)+(DEXP(-Q))*
     8(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(4))*(2.*Q*R)
     9+(2.*Q/3.)*B(5)**2
C                         *********************                         00001440
      C(7)=((1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    00001450
```

```
1*(DEXP(XQQ))*(DCOS(QQ))                                                      00001460
2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)                    00001470
3*(DEXP( QQ))*(DCOS(QQ))                                                      00001480
4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( XQQ))                    00001490
5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))                     00001500
6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)                        00001510
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))                                     00001520
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)                        00001530
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))                                     00001540
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)
3+     (R)*B(5)*((DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(1)-(DSIN(Q)
4+DCOS(Q))*B(2))-(DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(3)-(DSIN(Q)-
5 DCOS(Q))*B(4)))+       B(5)*(-(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)
6-DSIN(Q)))*B(1)+(DEXP(-Q))*(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(2)
7-(DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+DSIN(Q)))*B(3)+(DEXP( Q))*
8(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(4))*(2.*Q*R)
9-(2.*C/3.)*B(5)**2
 C(8)=-(1./8.)*Y*Q*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))                000019
&               +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))                  00001960
&-(1./16.)*(Y   )*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))                    00001980
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))                   00001990
 C(9)= (1./8.)*Y*Q*((DEXP(XQ))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))                00002000
1               +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))                  00002010
2+(1./16.)*(Y   )*
3((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))                    00002030
4+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))                   00002040
 C(10)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))                    00002060
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))                   00002070
&-Y*Q*(B(5)/R**2)
 C(11)=-(1./16.)*Y*
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))                    00002090
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))                   00002100
&-Y*Q*(B(5)/R**2)
 C(12)=-Y*Q*(B(5))
 END
```

```
C     ****************************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *
C     *        WITH OR WITHOUT ELASTIC FOUNDATION
C     * (1) LOADING  CONDITION      F=1 + X**2                     *
C     * (2) BOUNDARY CONDITION      SIMPLY SUPPORTED               *
C     *                             BOTH SIDES                     *
C     ****************************************************************
      A(1,1)=-(DEXP( Q))*(DSIN(Q))*Q**2
      A(1,2)= (DEXP( Q))*(DCOS(Q))*Q**2
      A(1,3)= (DEXP(-Q))*(DSIN(Q))*Q**2
      A(1,4)=-(DEXP(-Q))*(DCOS(Q))*Q**2
      A(1,5)=1./R
      A(2,1)= (DEXP(-Q))*(DSIN(Q))*Q**2
      A(2,2)= (DEXP(-Q))*(DCOS(Q))*Q**2
      A(2,3)=-(DEXP( Q))*(DSIN(Q))*Q**2
      A(2,4)=-(DEXP( Q))*(DCOS(Q))*Q**2
      A(2,5)=1./R
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(XQ))
      A(3,4)= (DSIN(Q))*(DEXP(XQ))
      A(3,5)=2./R
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(4,5)=2./R
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)=1./R
      A(6,6)=Q*(R**2)
      A(6,7)=Q*(R**2)
      A(7,6)=-Q*(R**2)
      A(7,7)=Q*(R**2)
      A(8,8)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(8,9)= (DCOS(Q))*(DEXP( Q))*Q**2
      A(8,10)= (DSIN(Q))*(DEXP(-Q))*Q**2
      A(8,11)=-(DCOS(Q))*(DEXP( -Q))*Q**2
      A(8,12)=1./R
      A(9,8)= (DSIN(Q))*(DEXP( -Q))*Q**2
```

```
      A(9,9)= (DCOS(Q))*(DEXP( -Q))*Q**2
      A(9,10)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(9,11)=-(DCOS(Q))*(DEXP(Q))*Q**2
      A(9,12)=1./R
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                    00000790
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                    00000800
      A(10,10)= (DCOS(Q))*(DEXP(XQ))                                    00000810
      A(10,11)= (DSIN(Q))*(DEXP(XQ))                                    00000820
      A(10,12)=2./R                                                     00000830
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))                                    00000840
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))                                    00000850
      A(11,10)= (DCOS(Q))*(DEXP( Q))                                    00000860
      A(11,11)=-(DSIN(Q))*(DEXP( Q))                                    00000870
      A(11,12)=2./R
      A(12,8)=R**2
      A(12,10)=R**2
      A(12,12)=R
C
C
      C(1)=0.                                                           00001010
      C(2)=0.                                                           00001020
      C(3)=0.                                                           00001030
      C(4)=0.                                                           00001040
      C(5)=1.                                                           00001050
      C(6)=((1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    00
     1*(DEXP( QQ))*(DCOS(QQ))                                           00001360
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         00001370
     3*(DEXP(XQQ))*(DCOS(QQ))                                           00001380
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))          00001390
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))          00001400
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)             00001410
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))                          00001420
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)             00001430
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))                          00001440
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                       00001450
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)
     3+     (R)*B(5)*((DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(1)+(DSIN(Q)       00
     4-DCOS(Q))*B(2))-(DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(3)+(DSIN(Q)+      00001480
     5 DCOS(Q))*B(4)))+       B(5)*((DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+00001490
     6DSIN(Q)))*B(1)+(DEXP( Q))*(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(2) 00001500
     7+(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)-DSIN(Q)))*B(3)+(DEXP(-Q))*   00001510
     8(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(4))*(2.*Q*R)
     9+(2.*Q/3.)*B(5)**2
C                        ********************                           00001530
```

```
 C(7)=((1./8.)*C*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)
1*(DEXP(XQQ))*(DCOS(QQ))
2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)
3*(DEXP( QQ))*(DCOS(QQ))
4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))
5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))
6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)
7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))
8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)
9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))
1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))
2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)
3+     (R)*B(5)*((DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(1)-(DSIN(Q)
4+DCOS(Q))*B(2))-(DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(3)-(DSIN(Q)-
5 DCOS(Q))*B(4)))+          B(5)*(-(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)
6-DSIN(Q)))*B(1)+(DEXP(-Q))*(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(2)
7-(DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+DSIN(Q)))*B(3)+(DEXP( Q))*
8(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(4))*(2.*Q*R)
9-(2.*Q/3.)*B(5)**2
 C(8)=-(1./16.)*Y*(Q**2)*
&((DEXP( Q))*((B(2)+B(1))*(DCOS(Q))+(B(2)-B(1))*(DSIN(Q)))
&-(DEXP(-Q))*((B(3)-B(4))*(DCOS(Q))+(B(3)+B(4))*(DSIN(Q))))
&-(Q /(8.   ))*Y*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
&            +(DEXP(-Q))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
 C(9)= (1./16.)*Y*(Q**2)*
&((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
&-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
&-(Q /(8.   ))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
&            +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
 C(10)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
&-Y*Q*(B(5)/R**2)
 C(11)=-(1./16.)*Y*
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
&-Y*C*(B(5)/R**2)
 C(12)=-Y*Q*(B(5))
 END
```

```
C
C     ***********************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *     00000040
C     *       WITH OR WITHOUT ELASTIC FOUNDATION
C     * (1) LOADING   CONDITION      F=1 + X**2              *
C     * (2) BOUNDARY CONDITION       CLAMPED X=+1.           *     00000080
C     *                              SIMPLY SUPPORTED X=-1.  *     00000080
C     ***********************************************************
C
C
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q                       00000420
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q                       00000430
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q                       00000440
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q                       00000450
      A(1,5)=2./R                                                  00000460
      A(2,1)= (DEXP(-Q))*(DSIN(Q))*Q**2
      A(2,2)= (DEXP(-Q))*(DCOS(Q))*Q**2
      A(2,3)=-(DEXP( Q))*(DSIN(Q))*Q**2
      A(2,4)=-(DEXP( Q))*(DCOS(Q))*Q**2
      A(2,5)=1./R
      A(3,1)= (DCOS(Q))*(DEXP( Q))                                 00000520
      A(3,2)= (DSIN(Q))*(DEXP( Q))                                 00000530
      A(3,3)= (DCOS(Q))*(DEXP(XQ))                                 00000540
      A(3,4)= (DSIN(Q))*(DEXP(XQ))                                 00000550
      A(3,5)=2./R                                                  00000560
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(4,5)=2./R
      A(5,1)=1.                                                    00000620
      A(5,3)=1.                                                    00000630
      A(5,5)=1./R                                                  00000640
      A(6,6)=Q*(R**2)                                              00000650
      A(6,7)=Q*(R**2)                                              00000660
      A(7,6)=-Q*(R**2)                                             00000670
      A(7,7)=Q*(R**2)                                              00000680
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q                     00000690
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q                     00000700
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q                     00000710
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q                     00000720
      A(8,12)=2./R                                                 00000730
      A(9,8)= (DSIN(Q))*(DEXP( -Q))*Q**2
```

```
      A(9,9)= (DCOS(Q))*(DEXP( -Q))*Q**2
      A(9,10)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(9,11)=-(DCOS(Q))*(DEXP(Q))*Q**2
      A(9,12)=1./R                                                      00000790
      A(10, 8)= (DCOS(Q))*(DEXP( Q))                                    00000800
      A(10, 9)= (DSIN(Q))*(DEXP( Q))                                    00000810
      A(10,10)= (DCOS(Q))*(DEXP(XQ))                                    00000820
      A(10,11)= (DSIN(Q))*(DEXP(XQ))                                    00000830
      A(10,12)=2./R                                                     00000840
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))                                    00000850
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))                                    00000860
      A(11,10)= (DCOS(Q))*(DEXP( Q))                                    00000870
      A(11,11)=-(DSIN(Q))*(DEXP( Q))                                    00000880
      A(11,12)=2./R                                                     00000890
      A(12,8)=R**2                                                      00000900
      A(12,10)=R**2                                                     00000910
      A(12,12)=R
C
C
      C(1)=0.                                                           00001010
      C(2)=0.                                                           00001020
      C(3)=0.                                                           00001030
      C(4)=0.                                                           00001040
      C(5)=1.                                                           00001050
      C(6)=((1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)    00001350
     1*(DEXP( QQ))*(DCOS(QQ))                                           00001360
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)         00001370
     3*(DEXP(XQQ))*(DCOS(QQ))                                           00001380
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP( QQ))          00001390
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP(XQQ))          00001400
     6-(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)             00001410
     7*(DABS(B(2)))**2)*(DEXP( QQ))*(DSIN(QQ))                          00001420
     8-(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)             00001430
     9*(DABS(B(4)))**2)*(DEXP(XQQ))*(DSIN(QQ))                          00001440
     1-(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                       00001450
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)             00001460
     3+      (R)*B(5)*((DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(1)+(DSIN(Q)      00001470
     4-DCOS(Q))*B(2))-(DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(3)+(DSIN(Q)+      00001480
     5 DCOS(Q))*B(4)))+      B(5)*((DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+00001490
     6DSIN(Q)))*B(1)+(DEXP( Q))*(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(2) 00001500
     7+(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)-DSIN(Q)))*B(3)+(DEXP(-Q))*   00001510
     8(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(4))*(2.*Q*R)                 00001520
     9+(2.*Q/3.)*B(5)**2                                                00001530
C                    *******************                                00001540
```

```
      C(7)=((1./8.)*Q*((DABS(B(1)))**2+2.*B(1)*B(2)-(DABS(B(2)))**2)      00001550
     1*(DEXP(XQQ))*(DCOS(QQ))                                              00001560
     2-(1./8.)*Q*((DABS(B(3)))**2-2.*B(3)*B(4)-(DABS(B(4)))**2)           00001570
     3*(DEXP( QQ))*(DCOS(QQ))                                              00001580
     4+(1./4.)*Q*((DABS(B(1)))**2+(DABS(B(2)))**2)*(DEXP(XQQ))            00001590
     5-(1./4.)*Q*((DABS(B(3)))**2+(DABS(B(4)))**2)*(DEXP( QQ))            00001600
     6+(1./4.)*Q*((1./2.)*(DABS(B(1)))**2-B(1)*B(2)-(1./2.)               00001610
     7*(DABS(B(2)))**2)*(DEXP(XQQ))*(DSIN(QQ))                             00001620
     8+(1./4.)*Q*((1./2.)*(DABS(B(3)))**2+B(3)*B(4)-(1./2.)               00001630
     9*(DABS(B(4)))**2)*(DEXP( QQ))*(DSIN(QQ))                             00001640
     1+(1./2.)*Q*(B(1)*B(3)-B(2)*B(4))*(DSIN(QQ))                          00001650
     2+(1./2.)*Q*(B(1)*B(4)+B(2)*B(3))*(DCOS(QQ)))*Q*(R**2)               00001660
     3+     (R)*B(5)*((DEXP(-Q))*((DCOS(Q)-DSIN(Q))*B(1)-(DSIN(Q)          00001670
     4+DCOS(Q))*B(2))-(DEXP( Q))*((DCOS(Q)+DSIN(Q))*B(3)-(DSIN(Q)-         00001680
     5 DCOS(Q))*B(4)))+      B(5)*(-(DEXP(-Q))*(DCOS(Q)+(1./Q)*(DCOS(Q)00001690
     6-DSIN(Q)))*B(1)+(DEXP(-Q))*(DSIN(Q)+(1./Q)*(DSIN(Q)+DCOS(Q)))*B(2)00001700
     7-(DEXP( Q))*(DCOS(Q)-(1./Q)*(DCOS(Q)+DSIN(Q)))*B(3)+(DEXP( Q))*   00001710
     8(DSIN(Q)-(1./Q)*(DSIN(Q)-DCOS(Q)))*B(4))*(2.*Q*R)                   00001720
     9-(2.*Q/3.)*B(5)**2                                                   00001730
      C(8)=-(1./8.)*Y*Q*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))      00002120
     &                +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q)))))       00002130
     &-(1./16.)*(Y   )*                                                    00002140
     &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))          00002150
     &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))         00002160
      C(9)= (1./16.)*Y*(Q**2)*
     &((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
     &-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
     &-(Q /(8.   ))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
     &          +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
      C(10)=-(1./16.)*Y*                                                   00002220
     &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))          00002230
     &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))         00002240
     &-Y*Q*(B(5)/R**2)                                                     00002250
      C(11)=-(1./16.)*Y*                                                   00002260
     &((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))          00002270
     &+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))         00002280
     &-Y*Q*(B(5)/R**2)                                                     00002290
      C(12)=-Y*Q*(B(5))                                                    00002300
      END
```

```
C
C
C     ***************************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *      00000040
C     *       WITH OR WITHOUT ELASTIC FOUNDATION
C     * (1) LOADING   CONDITION      F= COS(3.1416X/2)         *
C     * (2) BOUNDARY CONDITION      CLAMPED BOTH SIDES         *    00000080
C     ***************************************************************
C
C
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(1,5)=-(V1/(V4+R))
      A(2,1)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A(2,2)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(2,3)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(2,4)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(2,5)=(V1/(V4+R))
      A(3,1)= (DCOS(Q))*(DEXP( Q))                                  00000500
      A(3,2)= (DSIN(Q))*(DEXP( Q))                                  00000510
      A(3,3)= (DCOS(Q))*(DEXP(XQ))                                  00000520
      A(3,4)= (DSIN(Q))*(DEXP(XQ))                                  00000530
      A(4,1)= (DCOS(Q))*(DEXP(XQ))                                  00000550
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))                                  00000560
      A(4,3)= (DCOS(Q))*(DEXP( Q))                                  00000570
      A(4,4)=-(DSIN(Q))*(DEXP( Q))                                  00000580
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)= 1./(V4+R)
      A(6,6)= 1.
      A(6,7)= 1.
      A(7,6)=-1.
      A(7,7)= 1.
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A( 8,12)=-(V1/(V4+R))
      A( 9, 8)= (DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A( 9, 9)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A( 9,10)=-(DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A( 9,11)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
```

```
      A( 9,12)= (V1/(V4+R))
      A(10, 8)= (DCOS(Q))*(DEXP( Q))
      A(10, 9)= (DSIN(Q))*(DEXP( Q))
      A(10,10)= (DCOS(Q))*(DEXP(XQ))
      A(10,11)= (DSIN(Q))*(DEXP(XQ))
      A(11, 8)= (DCOS(Q))*(DEXP(XQ))
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))
      A(11,10)= (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)= (1./(V4+R))
C
C
      C(1)=0.
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      C61=-Q*(-(1./8.)*(T1+T8-T2)*U3*D5+(1./8.)*(T3-T6-T4)*U4*D5
     1-(1./4.)*(T1+T2)*U3+(1./4.)*(T3+T4)*U4
     2+(1./8.)*(T1-T8-T2)*U3*U6+(1./8.)*(T3+T6-T4)*U4*U6
     3+(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
     4*D5)
C
      C62=-((V2*Q*B(5))/(2.*(V4+R)))*
     1(+SS1*(+(SS5*U8)-(SS7*D7))*B(1)+SS2*(-(SS5*U8)+(SS8*D7))*B(1)
     2+SS1*(-(SS5*D7)-(SS7*U8))*B(2)+SS2*(+(SS5*D7)+(SS8*U8))*B(2)
     3+SS3*(-(SS5*U8)-(SS7*D7))*B(3)+SS4*(+(SS5*U8)+(SS8*D7))*B(3)
     4+SS3*(+(SS5*D7)-(SS7*U8))*B(4)+SS4*(-(SS5*D7)+(SS8*U8))*B(4))
C
      C63=-((V1*V5*B(5))/(V4+R))*
     1(+SS1*(+(SA1*D7)-(SA3*U8))*B(1)+SS2*((SA2*D7)+(SA4*U8))*B(1)
     2+SS1*((SA1*U8)+(SA3*D7))*B(2)+SS2*((SA2*U8)-(SA4*D7))*B(2)
     3+SS3*((SA1*D7)+(SA3*U8))*B(3)+SS4*((SA2*D7)-(SA4*U8))*B(3)
     4+SS3*((SA1*U8)-(SA3*D7))*B(4)+SS4*((SA2*U8)+(SA4*D7))*B(4))
C
      C64=((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
      C(6)=C61+C62+C63+C64
C
C
      C71=-Q*(-(1./8.)*(T1+T8-T2)*U4*D5+(1./8.)*(T3-T6-T4)*U3*D5
     1-(1./4.)*(T1+T2)*U4+(1./4.)*(T3+T4)*U3
     2-(1./8.)*(T1-T8-T2)*U4*U6-(1./8.)*(T3+T6-T4)*U3*U6
```

```
3-(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
4*D5)

 C72=-((V2*Q*B(5))/(2.*(V4+R)))*
1(+SS3*(+(SS5*U8)+(SS7*D7))*B(1)+SS4*(-(SS5*U8)-(SS8*D7))*B(1)
2+SS3*(+(SS5*D7)-(SS7*U8))*B(2)+SS4*(-(SS5*D7)+(SS8*U8))*B(2)
3+SS1*(-(SS5*U8)+(SS7*D7))*B(3)+SS2*(+(SS5*U8)-(SS8*D7))*B(3)
4+SS1*(-(SS5*D7)-(SS7*U8))*B(4)+SS2*(+(SS5*D7)+(SS8*U8))*B(4))

 C73=-((V1*V5*B(5))/(V4+R))*
1(+SS3*(-(SA1*D7)-(SA3*U8))*B(1)+SS4*(-(SA2*D7)+(SA4*U8))*B(1)
2+SS3*(+(SA1*U8)-(SA3*D7))*B(2)+SS4*(+(SA2*U8)+(SA4*D7))*B(2)
3+SS1*(-(SA1*D7)+(SA3*U8))*B(3)+SS2*(-(SA2*D7)-(SA4*U8))*B(3)
4+SS1*(+(SA1*U8)+(SA3*D7))*B(4)+SS2*(+(SA2*U8)-(SA4*D7))*B(4))

 C74=-((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
 C(7)=C71+C72+C73+C74

 C(8)=-(1./8.)*Y*Q*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
&                    +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q)))))
&-(1./16.)*(Y   )*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
&-(1.5*((3.1416)**3)*B(5)*((Y*Q)/24.))/((V4+R)*(V4+R))

 C(9)= (1./8.)*Y*Q*((DEXP(XQ))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
1                    +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q)))))
2+(1./16.)*(Y   )*
3((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
4+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
5+(1.5*((3.1416)**3)*B(5)*((Y*Q)/24.))/((V4+R)*(V4+R))

 C(10)=-(1./16.)*Y*
&((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
&+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))

 C(11)=-(1./16.)*Y*
&((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
&+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))

 C(12)= (3.*(3.1416**2)*B(5)*((Y*Q)/24.))/((V4+R)*(V4+R))

 END
```

```
C
C
C     *************************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *
C     *          WITH OR WITHOUT ELASTIC FOUNDATION
C     * (1) LOADING  CONDITION     F= COS(3.1416X/2)           *
C     * (2) BOUNDARY CONDITION     SIMPLY SUPPORTED            *
C     *                            BOTH SIDES                  *
C     *************************************************************
C
      A(1,1)= -(DSIN(Q))*(DEXP( Q))
      A(1,2)=  (DCOS(Q))*(DEXP( Q))
      A(1,3)=  (DSIN(Q))*(DEXP(-Q))
      A(1,4)= -(DCCS(Q))*(DEXP(-Q))
      A(2,1)=  (DSIN(Q))*(DEXP(-Q))
      A(2,2)=  (DCOS(Q))*(DEXP(-Q))
      A(2,3)= -(DSIN(Q))*(DEXP( Q))
      A(2,4)= -(DCOS(Q))*(DEXP( Q))
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(XQ))
      A(3,4)= (DSIN(Q))*(DEXP(XQ))
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)=1./(V4+R)
      A(6,6)= 1.
      A(6,7)= 1.
      A(7,6)=-1.
      A(7,7)= 1.
      A(8,8)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(8,9)= (DCOS(Q))*(DEXP( Q))*Q**2
      A(8,10)= (DSIN(Q))*(DEXP(-Q))*Q**2
      A(8,11)=-(DCOS(Q))*(DEXP( -Q))*Q**2
      A(9,8)= (DSIN(Q))*(DEXP( -Q))*Q**2
      A(9,9)= (DCOS(Q))*(DEXP( -Q))*Q**2
      A(9,10)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(9,11)=-(DCOS(Q))*(DEXP(Q))*Q**2
      A(10, 8)= (DCOS(Q))*(DEXP( Q))
      A(10, 9)= (DSIN(Q))*(DEXP( Q))
```

```
      A(10,10)=  (DCOS(Q))*(DEXP(XQ))
      A(10,11)=  (DSIN(Q))*(DEXP(XQ))
      A(11, 8)=  (DCOS(Q))*(DEXP(XQ))
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))
      A(11,10)=  (DCOS(Q))*(DEXP(  Q))
      A(11,11)=-(DSIN(Q))*(DEXP(  Q))
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)= 1./(V4+R)
C
C
      C(1)=0.
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      C61=-Q*(-(1./8.)*(T1+T8-T2)*U3*D5+(1./8.)*(T3-T6-T4)*U4*D5
     1-(1./4.)*(T1+T2)*U3+(1./4.)*(T3+T4)*U4
     2+(1./8.)*(T1-T8-T2)*U3*U6+(1./8.)*(T3+T6-T4)*U4*U6
     3+(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
     4*D5)
C
      C62=-((V2*Q*B(5))/(2.*(V4+R)))*
     1(+SS1*(+(SS5*U8)-(SS7*D7))*B(1)+SS2*(-(SS5*U8)+(SS8*D7))*B(1)
     2+SS1*(-(SS5*D7)-(SS7*U8))*B(2)+SS2*(+(SS5*D7)+(SS8*U8))*B(2)
     3+SS3*(-(SS5*U8)-(SS7*D7))*B(3)+SS4*(+(SS5*U8)+(SS8*D7))*B(3)
     4+SS3*(+(SS5*D7)-(SS7*U8))*B(4)+SS4*(-(SS5*D7)+(SS8*U8))*B(4))
C
      C63=-((V1*V5*B(5))/(V4+R))*
     1(+SS1*(+(SA1*D7)-(SA3*U8))*B(1)+SS2*((SA2*D7)+(SA4*U8))*B(1)
     2+SS1*((SA1*U8)+(SA3*D7))*B(2)+SS2*((SA2*U8)-(SA4*D7))*B(2)
     3+SS3*((SA1*D7)+(SA3*U8))*B(3)+SS4*((SA2*D7)-(SA4*U8))*B(3)
     4+SS3*((SA1*U8)-(SA3*D7))*B(4)+SS4*((SA2*U8)+(SA4*D7))*B(4))
C
      C64=((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
      C(6)=C61+C62+C63+C64
C
C
      C71=-Q*(-(1./8.)*(T1+T8-T2)*U4*D5+(1./8.)*(T3-T6-T4)*U3*D5
     1-(1./4.)*(T1+T2)*U4+(1./4.)*(T3+T4)*U3
     2-(1./8.)*(T1-T8-T2)*U4*U6-(1./8.)*(T3+T6-T4)*U3*U6
     3-(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
     4*D5)
C
```

```
      C72=-((V2*Q*B(5))/(2.*(V4+R)))*
     1(+SS3*(+(SS5*U8)+(SS7*D7))*B(1)+SS4*(-(SS5*U8)-(SS8*D7))*B(1)
     2+SS3*(+(SS5*D7)-(SS7*U8))*B(2)+SS4*(-(SS5*D7)+(SS8*U8))*B(2)
     3+SS1*(-(SS5*U8)+(SS7*D7))*B(3)+SS2*(+(SS5*U8)-(SS8*D7))*B(3)
     4+SS1*(-(SS5*D7)-(SS7*U8))*B(4)+SS2*(+(SS5*D7)+(SS8*U8))*B(4))
C
      C73=-((V1*V5*B(5))/(V4+R))*
     1(+SS3*(-(SA1*D7)-(SA3*U8))*B(1)+SS4*(-(SA2*D7)+(SA4*U8))*B(1)
     2+SS3*(+(SA1*U8)-(SA3*D7))*B(2)+SS4*(+(SA2*U8)+(SA4*D7))*B(2)
     3+SS1*(-(SA1*D7)+(SA3*U8))*B(3)+SS2*(-(SA2*D7)-(SA4*U8))*B(3)
     4+SS1*(+(SA1*U8)+(SA3*D7))*B(4)+SS2*(+(SA2*U8)-(SA4*D7))*B(4))
C
      C74=-((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
      C(7)=C71+C72+C73+C74
C
      C(8)=-(1./16.)*Y*(V5)*
     &((DEXP( Q))*((B(2)+B(1))*(DCOS(Q))+(B(2)-B(1))*(DSIN(Q)))
     &-(DEXP(-Q))*((B(3)-B(4))*(DCOS(Q))+(B(3)+B(4))*(DSIN(Q))))
     &-(Q /(8.   ))*Y*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
     &           +(DEXP(-Q))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
      C(9)= (1./16.)*Y*(V5)*
     &((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
     &-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
     &-(Q /(8.   ))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
     &           +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
      C(10)=-(1./16.)*Y*
     &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
      C(11)=-(1./16.)*Y*
     &((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
     &+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
      C(12)=(3.*(3.1416*3.1416)*B(5)*((Y*Q)/24.))/((V4+R)*(V4+R))
      END
```

```
C
C     ***************************************************************
C     * LARGE DEFLECTION ANALYSIS OF LONG RECTANGULAR PLATES *
C     *        WITH OR WITHOUT ELASTIC FOUNDATION
C     * (1) LOADING  CONDITION      F= COS(3.1416X/2)          *
C     * (2) BOUNDARY CONDITION      CLAMPED X=+1.              *
C     *                             SIMPLY SUPPORTED X=-1.     *
C     ***************************************************************
C
      A(1,1)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A(1,2)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A(1,3)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A(1,4)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A(1,5)=-V1/(V4+R)
      A(2,1)=  (DSIN(Q))*(DEXP(-Q))
      A(2,2)=  (DCOS(Q))*(DEXP(-Q))
      A(2,3)= -(DSIN(Q))*(DEXP( Q))
      A(2,4)= -(DCOS(Q))*(DEXP( Q))
      A(3,1)= (DCOS(Q))*(DEXP( Q))
      A(3,2)= (DSIN(Q))*(DEXP( Q))
      A(3,3)= (DCOS(Q))*(DEXP(XQ))
      A(3,4)= (DSIN(Q))*(DEXP(XQ))
      A(4,1)= (DCOS(Q))*(DEXP(XQ))
      A(4,2)=-(DSIN(Q))*(DEXP(XQ))
      A(4,3)= (DCOS(Q))*(DEXP( Q))
      A(4,4)=-(DSIN(Q))*(DEXP( Q))
      A(5,1)=1.
      A(5,3)=1.
      A(5,5)= 1./(V4+R)
      A(6,6)= 1.
      A(6,7)= 1.
      A(7,6)=-1.
      A(7,7)= 1.
      A( 8, 8)= (DCOS(Q)-DSIN(Q))*(DEXP( Q))*Q
      A( 8, 9)= (DCOS(Q)+DSIN(Q))*(DEXP( Q))*Q
      A( 8,10)=-(DCOS(Q)+DSIN(Q))*(DEXP(XQ))*Q
      A( 8,11)= (DCOS(Q)-DSIN(Q))*(DEXP(XQ))*Q
      A( 8,12)=-V1/(V4+R)
      A(9,8)= (DSIN(Q))*(DEXP( -Q))*Q**2
      A(9,9)= (DCOS(Q))*(DEXP( -Q))*Q**2
      A(9,10)=-(DSIN(Q))*(DEXP( Q))*Q**2
      A(9,11)=-(DCOS(Q))*(DEXP(Q))*Q**2
```

```
      A(10, 8)=  (DCOS(Q))*(DEXP( Q))
      A(10, 9)=  (DSIN(Q))*(DEXP( Q))
      A(10,10)=  (DCOS(Q))*(DEXP(XQ))
      A(10,11)=  (DSIN(Q))*(DEXP(XQ))
      A(11, 8)=  (DCOS(Q))*(DEXP(XQ))
      A(11, 9)=-(DSIN(Q))*(DEXP(XQ))
      A(11,10)=  (DCOS(Q))*(DEXP( Q))
      A(11,11)=-(DSIN(Q))*(DEXP( Q))
      A(12,8)=1.
      A(12,10)=1.
      A(12,12)= 1./(V4+R)
C
C
      C(1)=0.
      C(2)=0.
      C(3)=0.
      C(4)=0.
      C(5)=1.
      C61=-Q*(-(1./8.)*(T1+T8-T2)*U3*D5+(1./8.)*(T3-T6-T4)*U4*D5
     1-(1./4.)*(T1+T2)*U3+(1./4.)*(T3+T4)*U4
     2+(1./8.)*(T1-T8-T2)*U3*U6+(1./8.)*(T3+T6-T4)*U4*U6
     3+(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
     4*D5)
C
      C62=-((V2*Q*B(5))/(2.*(V4+R)))*
     1(+SS1*(+(SS5*U8)-(SS7*D7))*B(1)+SS2*(-(SS5*U8)+(SS8*D7))*B(1)
     2+SS1*(-(SS5*D7)-(SS7*U8))*B(2)+SS2*(+(SS5*D7)+(SS8*U8))*B(2)
     3+SS3*(-(SS5*U8)-(SS7*D7))*B(3)+SS4*(+(SS5*U8)+(SS8*D7))*B(3)
     4+SS3*(+(SS5*D7)-(SS7*U8))*B(4)+SS4*(-(SS5*D7)+(SS8*U8))*B(4))
C
      C63=-((V1*V5*B(5))/(V4+R))*
     1(+SS1*(+(SA1*D7)-(SA3*U8))*B(1)+SS2*((SA2*D7)+(SA4*U8))*B(1)
     2+SS1*((SA1*U8)+(SA3*D7))*B(2)+SS2*((SA2*U8)-(SA4*D7))*B(2)
     3+SS3*((SA1*D7)+(SA3*U8))*B(3)+SS4*((SA2*D7)-(SA4*U8))*B(3)
     4+SS3*((SA1*U8)-(SA3*D7))*B(4)+SS4*((SA2*U8)+(SA4*D7))*B(4))
C
      C64=((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
      C(6)=C61+C62+C63+C64
C
C
      C71=-Q*(-(1./8.)*(T1+T8-T2)*U4*D5+(1./8.)*(T3-T6-T4)*U3*D5
     1-(1./4.)*(T1+T2)*U4+(1./4.)*(T3+T4)*U3.
     2-(1./8.)*(T1-T8-T2)*U4*U6-(1./8.)*(T3+T6-T4)*U3*U6
     3-(1./2.)*(B(1)*B(3)-B(2)*B(4))*U6-(1./2.)*(B(1)*B(4)+B(2)*B(3))
```

```
      4*D5)
C
       C72=-((V2*Q*B(5))/(2.*(V4+R)))*
      1(+SS3*(+(SS5*U8)+(SS7*D7))*B(1)+SS4*(-(SS5*U8)-(SS8*D7))*B(1)
      2+SS3*(+(SS5*D7)-(SS7*U8))*B(2)+SS4*(-(SS5*D7)+(SS8*U8))*B(2)
      3+SS1*(-(SS5*U8)+(SS7*D7))*B(3)+SS2*(+(SS5*U8)-(SS8*D7))*B(3)
      4+SS1*(-(SS5*D7)-(SS7*U8))*B(4)+SS2*(+(SS5*D7)+(SS8*U8))*B(4))
C
       C73=-((V1*V5*B(5))/(V4+R))*
      1(+SS3*(-(SA1*D7)-(SA3*U8))*B(1)+SS4*(-(SA2*D7)+(SA4*U8))*B(1)
      2+SS3*(+(SA1*U8)-(SA3*D7))*B(2)+SS4*(+(SA2*U8)+(SA4*D7))*B(2)
      3+SS1*(-(SA1*D7)+(SA3*U8))*B(3)+SS2*(-(SA2*D7)-(SA4*U8))*B(3)
      4+SS1*(+(SA1*U8)+(SA3*D7))*B(4)+SS2*(+(SA2*U8)-(SA4*D7))*B(4))
C
       C74=-((V3*B(5)*B(5))/((V4+R)*(V4+R)))*(1./(2.*3.1416))
       C(7)=C71+C72+C73+C74
C
       C(8)=-(1./8.)*Y*Q*((DEXP( Q))*(B(1)*(DCOS(Q))+B(2)*(DSIN(Q)))
      &                +(DEXP(XQ))*(B(3)*(DCOS(Q))+B(4)*(DSIN(Q))))
      &-(1./16.)*(Y   )*
      &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
      &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
      & -(1.5*((3.1416)**3)*B(5)*(Y*Q/24.))/((V4+R)*(V4+R))
       C(9)= (1./16.)*Y*(Q**2)*
      &((DEXP(-Q))*((B(2)+B(1))*(DCOS(Q))-(B(2)-B(1))*(DSIN(Q)))
      &-(DEXP( Q))*((B(3)-B(4))*(DCOS(Q))-(B(3)+B(4))*(DSIN(Q))))
      &-(Q /(8.   ))*Y*((DEXP(-Q))*(B(1)*(DCOS(Q))-B(2)*(DSIN(Q)))
      &           +(DEXP( Q))*(B(3)*(DCOS(Q))-B(4)*(DSIN(Q))))
       C(10)=-(1./16.)*Y*
      &((DEXP( Q))*((B(2)+B(1))*(DSIN(Q))-(B(2)-B(1))*(DCOS(Q)))
      &+(DEXP(XQ))*((B(3)-B(4))*(DSIN(Q))-(B(3)+B(4))*(DCOS(Q))))
       C(11)=-(1./16.)*Y*
      &((DEXP(XQ))*((B(2)+B(1))*(DSIN(Q))+(B(2)-B(1))*(DCOS(Q)))
      &+(DEXP( Q))*((B(3)-B(4))*(DSIN(Q))+(B(3)+B(4))*(DCOS(Q))))
       C(12)= (3.*((3.1416)**2)*B(5)*(Y*Q/24.))/((V4+R)*(V4+R))
       END
```