

THREE ESSAYS ON THE ECONOMICS OF CLIMATE CHANGE AND THE ELECTRICITY SECTOR

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ABSTRACT

This doctoral thesis contains three essays on the economics of climate change and the electricity sector. The first essay deals with the subject of greenhouse gas (GHG) emissions and economic growth. The second essay addresses the issues of climate change policies, especially the role of the emergent innovative technologies, and the restructuring of the electricity sector. The third essay presents a model of transmission investments in electric power networks.

Chapter One studies the impacts of climate change on economic growth in the world economies. The paper contains explicit formalization of the depletion process of exhaustible fossil fuels and the phase of technology substitution. The impacts of climate change on capital flows and welfare across countries are also investigated.

The restructuring of the electricity sector is studied in Chapter Two. It also analyzes how climate change policies can benefit from emergent innovative technologies and how emergent innovative technologies can lower GHG emissions. It is shown that the price of electricity is strictly rising before emergent innovative firms with zero GHG emissions enter the market, but strictly declining as the entry begins.

In Chapter Three, a model of electricity transmission investments from the perspective of the regulatory approach is formulated. The Mid-West region of Western Australia, a subsystem of the South West Interconnected System is considered. In contrast with most models in the literature that deal only with network deepening, this model deals with both network deepening and network widening. Moreover, unlike the conventional investment models which are static and deal only with the long run, this model is dynamic and focuses on the timing of the infrastructure investments. The paper is a study of an optimal transmission investment program which is part of the optimal investment program for an integrated model in which investments in transmission and investments in generation are made at the same time.

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INTRODUCTION TO THE THESIS

It is widely recognized that the accumulation of greenhouse gases (GHG) is likely to lead to significant climatic change which concerns not only environmental scientists, but also economists. The impacts of climate change on the environment, public health and productivity are becoming increasingly understood, and increasingly problematic. The main characteristics of climate change are increases in average global temperature (global warming); changes in cloud cover and precipitation patterns; melting of ice caps and glaciers and reduced snow cover; rising sea levels; and increases in ocean temperatures, ocean acidity. According to World Health Organization (WHO), although global warming may bring some localized benefits, such as fewer winter deaths in temperate climates and increased food production in certain areas, the overall effects of a changing climate are likely to be overwhelmingly negative. Extreme weather events, such as high temperature, heavy storms, or droughts, are becoming more intense and frequent, which disrupt crop production. Climate change affects the fundamental requirements for health – clean air, safe drinking water, sufficient food and secure shelter.¹ The burning of fossil fuels, which is a necessary input of economic activities, is the main source of GHG emissions. On the other hand, the ultimate stock of fossil fuels is finite. The accumulation of backstop capital is essential to make it possible for the world economies to evolve along a path of sustainable development after the stock of fossil fuels has been depleted. Although there is a large literature on natural resources and on the economics of climate change, studies on the depletion of exhaustible resources incorporating the impact of climate change are almost non-existent.

To stabilize the greenhouse gas concentration in the atmosphere, an international environmental treaty - the United Nations Framework Convention on Climate Change (UNFCCC) - was produced at the United Nations Conference on Environment and Development in June 1992 in Rio de Janeiro. The Kyoto Protocol² - the protocol to the

¹ WHO – Climate Change and Health, 2010

² The protocol was initially adopted on 11 December 1997 in Kyoto, Japan and entered into force on 16

UNFCCC - establishes legally binding commitments for the reduction of GHG by an average of five per cent against 1990 levels over the five-year period 2008-2012. Under the agreement, nations that emit less than their quotas will be able to sell emissions credits to nations that exceed their quotas.³

For compliance with the Kyoto emission objectives, The European Union has implemented the European Union emission trading scheme (EU ETS) and pursued an increasing target share of renewable energy in overall energy consumption. Canada ratified the treaty in December 2002 that came into force in February 2005, requiring it to reduce emissions to 6% below 1990 levels during the period between 2008 and 2012. However, the Canadian federal government has failed to meet its Kyoto Protocol obligations to cut greenhouse gas emissions. Australia ratified the Kyoto Protocol in December 2007, and released the Carbon Pollution Reduction Scheme (CPRS) White Paper in December 2008 together with a commitment of the national Renewable Energy Target (RET) of 20% share of electricity supply in Australia by 2020. Yet, the Australian government has announced a delayed start of the CPRS. Although the United States have not ratified the Kyoto Protocol, the Obama government plans the New Energy for America to invest in renewable energy to meet the twin challenges of energy security and climate change. China and India - the major polluters in the world - maintain that the major responsibility of curbing emission rests with the developed countries. A difficulty with the Kyoto Protocol is that it does not appear to lay the groundwork for a self-enforcing treaty– i.e., every country will be in its self-interest to abide by the treaty.

To fulfill the emission reduction target commitment, the large emitters are the industries that would be required to achieve a significant reduction. These sectors consist of thermal electricity; oil and gas production; mining; and manufacturing such as cement, pulp and paper, aluminum, potash, nitrogen fertilizer, lime. Many sectors have raised their concerns about the impact of climate change policies on their own competitiveness. For example, in Australia, the majority of its energy comes from coal, and Australia is also

February 2005.

³ Emissions Trading, UNFCCC

the world's largest coal exporter. In addition, its electricity industry is heavily dependent on the burning of fossil fuels. It has been warned that Australia will face problems with price and reliability caused by the breakdown of the national electricity market triggered by the departure of big coal-fired power stations if stringent climate change policies are implemented. In the transition to a low-emission electricity system, the Carbon Reduction Scheme White Paper proposed a price cap on emissions of AUD 40 per tonne of carbon dioxide equivalent. Furthermore, about 130.7 million permits would be provided to coal-fired generators. Free permits are also allocated to emissions-intensive, trade-exposed businesses - such as aluminum producers, iron and steel makers, petrol refiners and LNG⁴ producers, initially totaling 25% to 33% of permits and rising to 45% by 2020. The national climate change adviser, Professor Ross Garnaut condemned the Rudd Government's carbon policy on this over-compensation of coal fired electricity generators leading to windfall gains for this sector, but a potential threat to public finances.

Most electricity today is generated by burning fossil fuels. Fossil fuel power stations are major emitters of GHG. The power generation sector is therefore the main focus of a nation's efforts to reduce its greenhouse gas emissions. For most of the twentieth century, consumers bought electricity from vertically integrated utilities which were monopolies in their own geographical areas. The philosophy underlying this market structure is that generation, transmission, and distribution all exhibit economies of scale, and thus have the characteristics of natural monopolies. Consequently, the market structure adopted in most countries was either public ownership or regulated monopolies. Much of the motivation behind the restructuring effort taken place in many countries for over a decade is to permit competition from more energy-efficient technologies, especially gas turbines, which promotes generation at lower costs. The restructuring of the electricity sector is driven by the technological progress in generation technologies which make the competition among electricity producers possible, in addition to the imperative of lowering greenhouse gases emissions generated by coal-fired electric generating plants. More importantly, in many cases the all-in cost of a new plant was lower than what customers were paying for the sunk costs of old plants. These developments have reduced

⁴ Liquefied Natural Gas

the minimum efficient scale to the point where the position that generation is a natural monopoly is no longer tenable. Furthermore, technological progress has made it possible to construct transmission lines that carry electric currents at 1000 kV and that span thousands of kilometers. It is also possible to build generating plants that have a capacity of more than 1000 MW. These developments in transmissions and generation mean that competition in generation can come from many producers from far away; such as hydroelectric and wind farms.

Under the restructuring, the vertically linked structures of electric utilities were broken up into separate and independent components. Generation, transmission, and distribution are no longer under the control of a single firm, and the reform led to the establishment of dozens of competing generators and retailers in the market. The transmission component of an electricity network exhibits economies of scale and remains a natural monopoly. Transmission networks are capital intensive. Transmission assets are long-lived – from 20 to 40 years – and once installed, are sunk. Because electricity transmission networks are considered as natural monopolies, the appropriate market structure for transmission service is either public ownership or regulated monopolies. To ensure open access to electric producers and purchasers, the transmission system is often run by an Independent System Operator (ISO) or an Independent Market Operator (IMO). The rates are regulated and set at an appropriate level that allows investors to recover the investments and earn an acceptable rate of return. From the perspective of the regulatory approach, an overwhelming majority of transmission investments are remunerated on the regulatory basis at the present time. Although the regulator can only control the transmission investments, not the investments made by electric generators who are free to set prices or to enter the market, an optimal transmission investment program must be part of the optimal investment program for an integrated model in which investments in transmission and investments in generation are made at the same time. At the distribution level, economies of scale in covering the territory argue for the distribution segment of an electricity network being a natural monopoly because there is only one set of wires going into a building and because there is only room for one set of wires on the street.

The restructuring of the electricity industry has been discussed by many economists, and questions arise when concerns about climate change should also be taken into account. First, it is questioned if deregulation of the electricity markets, which can lead to higher efficiencies, is always favourable in terms of emissions. Market deregulation contributing to increased electricity output and trade may result in higher emissions due to the increasing use of burning fossil fuels. As a result, there may be a conflict between the electricity industry restructuring and the national commitments for greenhouse gases emissions reduction. Second, a power station investment appears to be locked into a technology over 40 or 50 years, and changes in the capital stock occur slowly. The adoption of new technologies, together with changes required in the energy resource mix for the electricity industry, with low or near-net-zero greenhouse gas emissions takes time. It does not allow for a rapid change in the mix of energy resources without a stranding of power station assets and the ensuing financial loss. If significant changes are required in the energy resource mix for the electricity industry, actions must be taken that take into account the resulting stranded assets.

The question of climate change has moved from a purely scientific matter to a real threat for the prosperity of the economy, and has serious practical consequences for governments, incumbent generators, and potential entrants in the electricity sector. Potential investors need to add to their considerations the risk associated with the uncertainty involved in climate change policy when making decision on investments in long-lived and sector-specific assets. As for incumbent generators – especially old coal-fired generators – they now face the prospect of being unable to achieve the earnings expected of the coal-fired power plants constructed under the previous regulatory regime, when other types of electricity generation would have been cheaper if they had known that carbon emissions reduction would be committed by the government. The country may face problems with price and reliability caused by the breakdown of the national electricity market triggered by the departure of big coal-fired power stations. The outcome depends a great deal on how the restructuring process, together with other complementary policies are actually carried out. Furthermore, with gain in popularity and stringency within climate change policies in many countries, the effects of combining

black (GHG emissions) and green (renewable energies) quotas raise the current policy question for governments of “what is the impact of the overlapping regulation through both renewable energy and GHG emissions quotas?”.

The thesis focuses on the intersection between the economics of Climate Change and the Electricity Industry, and offers some original contributions to the economic analysis of the exploitation of exhaustible resources, climate change policies, and electricity industry regulation. In the thesis, three subjects are studied: (1) economic growth with the exploitation of an exhaustible resource incorporating the impact of climate change; (2) the innovation in the electric sector and the impact of the overlapping regulation through both renewable energy and GHG emissions quotas; and (3) the transmission investments in the electricity sector.

Chapter One is about economic growth with the exploitation of an exhaustible resource incorporating the impact of climate change. The essay contributes to the literature on the depletion of exhaustible resources. The modeling strategy has the advantage of shedding light on the questions of intergenerational equity and a call for government intervention. The excessive burning of fossil fuels benefits the current generation at the expense of future generations. A slightly reduction in oil input and transferred to the following period will be a Pareto improvement. The paper also provides some baseline justifications for an international environmental treaty negotiation for future research. More specifically, Chapter One depicts the status quo in the global environment debate, and it can serve as the baseline for negotiations on a global environment treaty. In the paper, economic growth for a multi-country world is analyzed under the overlapping-generations framework. In the model, an individual agent has a two-period life caring for her young and old age consumptions, and transferring her wealth over time through either oil or capital, or both. This modeling strategy is thus somewhat more “market oriented” than the central planning model with perfect foresight in the Ramsey-Solow tradition. The burning of fossil fuels - the main source of greenhouse gases - and the substitution of renewable energy for a sustainable development are formulated. The paper contains explicit formalization of the depletion of fossil fuels and the phase of technology

substitution. A competitive equilibrium consists of three phases. In the first phase, fossil fuels provide all the energy needs of the world economy. During this phase, the price of oil rises steadily at the rate of interest. The second phase – which might or might not exist – begins when the price of oil has risen to the level of the interest rate. In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In the third phase of the competitive equilibrium – the post fossil fuel phase – the backstop completely takes over and provides all the energy needs of the world economy.

The paper also studies the impact of climate change on capital flows and welfare across countries. The findings are that for each generation that is born after the world's stock of fossil fuels has been depleted, the welfare of each of its members varies across countries inversely with the negative impact of climate change on its own country of origin. In addition, in any period after the world's stock of fossil fuels has been depleted, a country that is severely affected by climate change will import capital, while a country that is impervious to climate change will export capital. These results provide powerful arguments for an international treaty negotiation on equity grounds. The excessive burning of fossil fuels is more costly in the long run for countries severely affected by climate change.

Chapter Two is about the innovation in the electric sector. In this chapter, we formulate a Cournot model of competition to examine the effects of restructuring, technological changes, together with climate change policies: cap and trade and renewable energy target policy on the largest emitter - the electric power industry. A dynamic theoretical analysis is provided to answer the current policy question of what impacts of the overlapping regulation through both renewable energy and GHG emissions quotas have on the electric power industry. The essay contributes to the study on the effects of combining tradable black (GHG emissions) and green (renewable energies) quotas. Furthermore, the paper presents a revised expression for measuring stranded costs when the present value of the stream of revenues derived from the sales of the emissions permits given by the government as a form of compensation is taken into account.

Chapter Two also studies the role of innovative generation technologies in reducing GHG emissions, and its impacts on the price of emissions permits, the price of electricity as well as electricity output. In the presence of caps on carbon emissions, the entry of advanced generation technologies constrains the output and *a fortiori* the greenhouse gas emissions of the less advanced technologies. It is shown that the price of electricity is strictly rising before emergent innovative firms with zero GHG emissions enter the market, but strictly declining as the entry begins. Although further analysis is required to examine the effect of the climate change policies on the national greenhouse gas emissions, we demonstrate a decline in the greenhouse gas emissions from the electricity sector while its output rises. In addition, the paper discusses the impact of the government subsidy on the exact time entry begins and the total capacity installed by the emergent innovative firms. The analysis can be applied to the case of Australia where question on the efficiency of the Mandatory Renewable Energy Target (MRET) in parallel with the proposed emissions trading scheme arises.

In Chapter Three, a dynamic model of electricity transmission investments from the perspective of the regulatory approach is formulated. The model we formalize is that of a sub-network within a global network in which how the various components of the network, i.e., the topology of the network, interact with each other to generate the network externalities – positive or negative – is taken into account. The transmission investments in a specific sub-network, i.e., the Mid-West region of Western Australia – a sub-system of the South West Interconnected System (SWIS), with its own topology and its own constraints is, therefore, considered for our purpose. Although lack of data prevents us from calibrating the model and deriving some practical policy recommendations, the paper provides a qualitative analysis to exploit the structure and the characteristics of the sub-network in the Mid-Western region of Australia. Our model can be looked at as a first step in the analysis of the investments in larger transmission systems of more complicated topology electric power networks. In contrast with most models in the literature that deal only with network deepening, this model deals with both network deepening and network widening. Furthermore, unlike the conventional investment models which are static and deal only with the long run, this model is

dynamic and focuses on the timing of the infrastructure investments. Indeed, Chapter Three is a study of an optimal transmission investment program which is part of the optimal investment program for an integrated model in which investments in transmission and investments in generation are made at the same time. Economically speaking, the investments in generation depend on the availability of transmission capacity. Therefore, the investments in transmission should be analyzed under the perspective of a dynamic Stackellberg game in which the Transco acts as the leader and generation investors are the followers. Another important feature of the paper is that because the transmission segment is regulated while competition is encouraged in the generation segment, the optimal pattern of transmission investments will maximize social welfare, taking into account the reaction of merchant electric generation and giving a decent rate of return to the owners of the transmission assets. Since this chapter of the thesis focuses on constructing and analyzing an original model of electricity transmission investments, climate change is not mentioned. Environmental issues could be incorporated into the model in future research.

CHAPTER ONE

CLIMATE CHANGE AND ECONOMIC GROWTH

1. INTRODUCTION

The accumulation of greenhouse gases in the Earth's atmosphere is likely to lead to significant climatic change, and this has serious consequences for human health and production productivity. The actions taken on climate change require international co-operation, and the economic analysis must be global and encompasses a long time horizon. To stabilize the greenhouse gas concentration in the atmosphere, the United Nations Framework Convention on Climate Change (UNFCCC)¹ has set out the objective of reducing global greenhouse gas emissions to a certain level. The Stern Review (2007), the most comprehensive and most widely known report, indicates that one percent of global GDP per annum should be invested to avoid the worst effects of climate change. As of January 2009, 183 parties have ratified the Kyoto Protocol.²

The question is whether nations are actively cooperative in taking actions on climate change or national governments just implement policies to maximize political support, not economic efficiency. In reality, to achieve political goals, governments often carry out distortionary policies. Canada ratified the treaty in December 2002, which was supposed to come into effect in February 2005, to reduce emissions by 6% below 1990 levels during the period between 2008 and 2012. However, the Canadian federal government has failed to meet its Kyoto Protocol obligations to cut greenhouse gas emissions. China and India - the two major polluters in the world - maintain that the main responsibility of curbing emission rests with the industrialized. The United States also refuses to take actions. The former Australian government, which declined to ratify the

¹ An international environmental treaty was produced at the United Nations Conference on Environment and Development in June 1992 in Rio de Janeiro

² A protocol to the UNFCCC, which was initially adopted on 11 December 1997 in Kyoto, Japan and entered into force on 16 February 2005

Kyoto Protocol, argued that the Kyoto Protocol would cost Australians jobs as most of Australia's energy comes from coal and Australia is also the world's largest coal exporter.³ Thus, it is a remarkable move when the Rudd government ratified the Kyoto Protocol in December 2007. The proposed Carbon Pollution Reduction Scheme is a cap and trade scheme with a medium-term target range to reduce emissions between 5 and 15 per cent below 2000 levels by 2020.⁴ However, the Senate failed to pass the Emissions Trading Scheme (ETS) in December 2009, causing a potential double dissolution election over the ETS. Moreover, even if it were implemented, the proposed target of emission reductions would still be criticized on the grounds that it is entirely inadequate for an equitable global response to the threat of global warming.⁵

Another controversial issue concerning the Kyoto Protocol is that it is based on equity, rather than on efficiency grounds. Nordhaus and Yang (1996); Nordhaus and Boyer (2000) discussed this issue in the RICE⁶ (1996) and the revised RICE (2000) models built to study the environmental and economic impacts of alternative approaches of international collaboration on climate change policy. Nordhaus and Yang (1996) showed that a global cooperative strategy on climate change requires China to implement a higher reduction rate than other countries and that the US will incur significant costs, but receive few benefits in return for its cooperation. At the present time, it is the non-cooperative strategy that is adopted by most nations. In the absence of a world government, an effective agreement to control emissions should be self-enforcing; that is, it is in its own self-interest for every country to abide by the treaty. A difficulty with the Kyoto Protocol is that it does not appear to lay the groundwork for a self-enforcing treaty, as discussed by Finus (2001), Barrett (2003), Dutta and Radner (2004).

There is a large literature on natural resources and on the economics of climate change. Yet, there is scarcely any literature on the depletion of exhaustible resources

³ "Howard rejects emissions targets". *BBC News Website*. 2006-08-16

⁴ Carbon Reduction Scheme White Paper, Volume 1, 2008. Yet, the government has announced a delayed start of the Scheme with a deeper conditional target (25% by 2020, in the event of a global agreement aiming at 450 ppm)

⁵ Australian Science Media Centre, Rapid Roundup: Carbon Pollution Reduction Scheme - White Paper - experts respond

⁶ RICE is the acronym for Regional Integrated Model of Climate and the Economy

incorporating the impact of climate change. Howarth (1991) studied intertemporal resource allocation with overlapping-generations, under the constraint of a non-renewable resource. This researcher modeled natural resources as inputs in the production processes, rather than as consumption goods. However, the pollution caused by the burning of fossil fuels was not modeled in the paper. Environmental degradation is a fundamental challenge to sustainability in Pearce and Atkinson (1993). These authors assumed the possibility of substitution between “natural” and “man-made” capital, and proposed a measure of sustainable development. Weak and strong sustainability rules were presented in their paper. Weak sustainability rule states that an economy is sustainable if it saves more than the combined depreciation on the “natural” and “man-made” capital. Strong sustainability rule requires that “natural” capital be held constant (or increasing) within the more general constraint that capital be constant (or increasing). Chen (1997) analyzed a two-country bargaining model on global warming. In the model formulated by this researcher, labor is the only input in the production process, and the mean global temperature depends on the level of economic activities. The burning of fossil fuels, which is the main source of GHG emissions, was not modeled.

An overlapping-generations framework is useful in modeling intergenerational distributional issues, which is important in the economics of climate change. John and Pecchenino (1994) formulated an overlapping-generations model to analyze the potential conflict between economic growth and the maintenance of environmental quality. These researchers focused on consumption externalities, and showed that increased savings by the current generations – made possible by more production – raise the capital stock and thus benefit future generations, but lower the welfare of future generations through reduced environmental quality maintenance. Higher savings are desirable if the first effect dominates. Thus, over-maintenance of the environment, analogous to dynamically inefficient over-accumulation of capital, may emerge. Our model, on the other hand, shows that the excessive burning of fossil fuels benefits the current generation at the expense of future generations. The cut in current oil input reduces current output; however, the better environmental quality in the following period raises output in that period. A slightly reduction in oil input in the current period coupled with a reduction in

consumption of a young individual in the current period to be compensated for by a rise in the old-age consumption of such an individual will be a Pareto improvement. This result reinforces the argument for government intervention on intergenerational equity grounds.

In general, climate change could affect both welfare and productivity. More specifically, climate change could lead to a large number of public health problems, such as heat-induced mortality and the spread of malaria and dengue fever. These consumption externalities cause disutility for consumers. On the other hand, climate change could cause loss of biodiversity, a rise in the sea level, infrastructure damages. These result in loss of production infrastructures. Climate change is likely to have different impacts on different sectors and in different countries. Countries like Japan or the United States are relatively insulated from the negative impacts of climate change, while developing countries like China or India are more vulnerable.

In this paper, we investigate the economic impacts of climate change on different countries under the overlapping-generations framework. The choice of the overlapping-generations framework as a modeling strategy – in contrast with the traditional model of economic growth of the Ramsey-Solow tradition – allows us to address explicitly the issues of intergenerational equity and makes it possible to compare welfare across nations at any point in time. The model we formulate is an extension into a multi-country world of the one-country model of Tang (2007); who treats the world as a single country.

In the model, a consumption good is produced from three essential inputs – labor, capital, and energy. The energy input in the production process comes either from fossil fuels or a backstop technology. Because the ultimate stock of fossil fuels is finite, backstop capital must be accumulated to provide the energy needed in the production process after the stock of fossil fuels has dwindled. The introduction of a backstop technology into the model makes it possible for the world economies to evolve along a path of sustainable development after the stock of fossil fuels has been depleted. In our model, an agent lives two periods, working when she is young, and retiring when she is old. A young

individual has one unit of time that she supplies in-elastically on the labor market. Part of the labor income is used for current consumption; the remaining part is saved under the form of capital and oil to provide for old-age consumption. There is free trade in the consumption good, oil, and renewable energy. Also, there is perfect capital mobility between sectors and across countries.

As already mentioned, the negative impacts of climate change affect both productivity and human health. Most theoretical studies on the economics of climate change model the negative impacts of climate change as consumption externalities. However, we eschew consumption externalities, and follow Nordhaus and Yang (1996) by focusing on the production externalities caused by an increase in the stock of greenhouse gases in the atmosphere. In our paper, the burning of fossil fuels which is the energy input of the production process is the main source of GHG emissions. By modeling the impacts of climate change as production externalities, the impacts of burning of fossil fuels can be fully explained since a country is now facing two apparently opposite effects. An increase in the burning of fossil fuels as an input of the production process may increase its current output to some extent. On the other hand, it has negative impacts on future productions since the negative impacts of climate change affect productivity. Indeed, this modeling choice makes an important contribution to the investigation of impacts of climate change on capital flows and welfare across countries. The main contrast between our model and that of Nordhaus (1994), Nordhaus and Yang (1996) is that these researchers solved a central planner's problem, and invoked the second theorem of welfare economics to assert that the solution of the central planner's problem is also the competitive equilibrium, while we analyze the decisions of economic agents in a decentralized setting.

The findings of the paper can be summarized as follows. Under a competitive equilibrium, extraction activities are terminated in finite time. Furthermore, the stock of fossil fuels might or might not be completely depleted. The possibility of partial depletion, i.e., when part of the stock of fossil fuels is left in situ unexploited, depends on the values of the parameters of the model. The equilibrium under incomplete oil

exhaustion is clearly inefficient. Third, if the world's initial stock of fossil fuels is large, then the initial global demand for oil will be high, and the backstop sector in each country will not be active in period 0. However, it will be brought into use in finite time.

In light of the results just mentioned, a competitive equilibrium might consist of three phases. In the first phase, fossil fuels meet all the energy needs of the world economy. During this phase, the price of oil rises steadily at the rate of interest. The second phase – which might or might not exist – begins when the price of oil has risen to the level of the interest rate. In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In the third phase of the competitive equilibrium – the post fossil fuel phase – the backstop completely takes over and provides all the energy needs of the world economy. We show that the price of oil cannot be too high during the first and the second phases of the competitive equilibrium. Furthermore, the price of oil rises steadily at the rate of interest, and is higher than the rental rate of capital after the stock of fossil fuels has been depleted.

As for a comparison across countries, for each generation that is born after the world's stock of fossil fuels has been depleted, the welfare of each of its members varies across countries inversely with the degree of vulnerability to the impact of climate change on its own country of origin. In any period after the world's stock of fossil fuels has been depleted, a country that is more vulnerable to climate change will import capital, while a country that is impervious to climate change will export capital. These results provide powerful arguments for an international treaty negotiation on equity grounds. The excessive burning of fossil fuels is more costly in the long run for countries severely affected by climate change.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the temporary equilibria. The characterization of the competitive equilibria is analyzed in Section 4. Numerical examples to illustrate these results are presented in Section 5. Section 6 contains some concluding remarks.

2. THE MODEL

Time is discrete and denoted by $t, t = 0, 1, \dots$. Countries are indexed by $i, i \in I$, with I representing the set of countries. In each period, four classes of economic agents co-exist in each country: a young generation, an old generation, competitive firms producing a consumption good, and competitive firms producing renewable energy. The consumption good is produced using capital, labor, and energy. We shall assume that the consumption good can also be used as investment goods to accumulate capital. The energy inputs come from two sources: fossil fuels, say oil, and a backstop. Furthermore, the renewable energy produced by the backstop also uses capital, say solar collectors.

The consumption good, oil, and renewable energy are freely traded. Capital is also perfectly mobile between sectors and across countries, so that at any instant there is a single interest rate on the global capital market. An individual lives two periods, working when she is young and retiring when she is old. In each period the real assets that belong to a country are owned by the old generation of that country. A young individual in a country in any period owns nothing except for a unit of time that she supplies inelastically in the labor market of her country of origin. Part of her wage is spent on current consumption; the remaining part is saved to provide for her old-age consumption.

In any period, the state of country i is represented by the list $(X_{i,t}, K_{i,t}, N_{i,t}^0, N_{i,t}^1)$, where $X_{i,t}$, $K_{i,t}$, $N_{i,t}^0$, and $N_{i,t}^1$ represent, respectively, its stock of oil, its own capital stock, its number of young individuals, and its number of old individuals. The population in each country is assumed to grow at the constant rate $n \geq 0$ per period, so that $N_{i,t+1}^0 = (1+n)N_{i,t}^0, t = 0, 1, \dots$. We assume that $(X_{i,0}, K_{i,0}, N_{i,0}^0, N_{i,0}^1)$, the initial state of country i , is known. The consumption good is taken to be the numéraire in each period. For any $t \geq 0$, the rental rate of capital, the price of oil, and the price of renewable energy – all in period t – are denoted, respectively, by ρ_t , ϕ_t , and φ_t . Also, the wage rate in period t in country i is denoted by $\omega_{i,t}$. The list $(\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t)$ is called a price system in period t . A price system is an infinite sequences $\mathbf{P} = (\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t)_{t=0}^{\infty}$.

2.1. The Production Technologies and Profit Maximization

In each period t , the representative firm that produces the consumption good in country $i, i \in I$, uses the following Cobb-Douglas technology:

$$(1) \quad Y_{i,t} = A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^\beta (Q_{i,t} + B_{i,t})^{1-\alpha-\beta}.$$

In (1) $Y_{i,t}$, $K_{i,t}$, $L_{i,t}$, $Q_{i,t}$, and $B_{i,t}$ denote, respectively, the output net of depreciation, the capital input, the labor input, the oil input, and the renewable energy input – all in period t . As for A , it represents the technological level, while α and β are two positive parameters satisfying $\alpha + \beta < 1$. Note that $\Omega_i(H_t)$ is a scaling factor representing the impact of climate change on the production of the consumption good in country i , and this is the only factor that differentiates countries. We shall assume that the scaling factor for country i has the following functional form:

$$\Omega_i(H_t) = \begin{cases} 1 & \text{if } \underline{H} \leq H_t \leq H^\#, \\ e^{-\gamma_i(H_t - H^\#)^2} & \text{if } H^\# < H_t, \end{cases}$$

where $\gamma_i > 0$ is a parameter specific to country i ; H_t is the stock of greenhouse gases in the atmosphere in period t ; and $\underline{H} > 0$ is the natural level of the stock of greenhouse gases in the atmosphere in steady state if there were a negligible amount of greenhouse emissions. Also, $H^\#$ is a constant greater than \underline{H} . As specified, the scaling factor for each country is 1 if the stock of greenhouse gases in the atmosphere is above its natural level, but not exceeding the critical level $H^\#$. Only when the stock of greenhouse gases in the atmosphere exceeds the critical level $H^\#$ will the negative impact of climate change begin to exert its influence. The specification is adopted so that the impact of a small rise in the stock of greenhouse gases above its natural level has no impact on the technology used in the production of the consumption good, and this means that when the stock of greenhouse gases in the atmosphere is below $H^\#$, the benefits obtained by

burning a small amount of fossil fuels will not induce any nefarious effects due to a small change in the climate. The parameter γ_i in the scaling factor characterizes the negative impact of climate change on the consumption good technology of country i , and can be interpreted as the degree of vulnerability of this country as far as the impact of climate change is concerned. Note that if $\gamma_i > \gamma_j$, then $\Omega_i(H_t) \leq \Omega_j(H_t)$, with strict inequality holding if H_t is above the threshold level $H^\#$.

Given a price system $\mathbf{P} = (\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t)_{t=0}^\infty$, the representative firm that produces the consumption good in country i solves the following profit maximization in each period:

$$(2) \quad \max_{(K_{i,t}, L_{i,t}, Q_{i,t}, B_{i,t})} \left[\begin{array}{l} A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^\beta (Q_{i,t} + B_{i,t})^{1-\alpha-\beta} \\ - \rho_t K_{i,t} - \omega_{i,t} L_{i,t} - \phi_t Q_{i,t} - \varphi_t B_{i,t} \end{array} \right].$$

The following first-order conditions characterize the solution of (2).

$$(3) \quad \alpha A\Omega_i(H_t)K_{i,t}^{\alpha-1}L_{i,t}^\beta(Q_{i,t} + B_{i,t})^{1-\alpha-\beta} - \rho_t = 0,$$

$$(4) \quad \beta A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^{\beta-1}(Q_{i,t} + B_{i,t})^{1-\alpha-\beta} - \omega_{i,t} = 0,$$

$$(5a) \quad (1 - \alpha - \beta)A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^\beta Q_{i,t}^{-\alpha-\beta} - \phi_t = 0, \text{ if } \phi_t < \varphi_t,$$

$$(5b) \quad (1 - \alpha - \beta)A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^\beta B_{i,t}^{-\alpha-\beta} - \varphi_t = 0, \text{ if } \phi_t > \varphi_t.$$

When $\phi_t = \varphi_t$, the mix of energy input $Q_{i,t} + B_{i,t}$ is indeterminate; their sum, however, is determinate, and satisfies the following relation:

$$(5c) \quad (1 - \alpha - \beta)A\Omega_i(H_t)K_{i,t}^\alpha L_{i,t}^\beta (Q_{i,t} + B_{i,t})^{-\alpha-\beta} = \phi_t = \varphi_t.$$

In any period, the energy input – measured in Btus – used by the competitive firms that produce the consumption good come either from a stock of fossil fuels or the backstop.

While oil can be extracted at negligible cost, its ultimate stock is limited. The backstop, on the other hand, can produce a perpetual flow of energy. However, harnessing energy from the Sun requires capital, say solar collectors. We shall assume that renewable energy is produced in the backstop sector in each country from capital according to a linear technology, with one unit of capital producing one Btu. In each period, the representative firm in the backstop sector of country i solves the following profit maximization:

$$(6) \quad \max_{K_{i,t,0}} (\varphi_t - \rho_t) K_{i,t,0}.$$

If $\rho_t < \varphi_t$, then $K_{i,t,0} = \infty$. If $\rho_t > \varphi_t$, then $K_{i,t,0} = 0$. When $\rho_t = \varphi_t$, $K_{i,t,0}$ is indeterminate, and must adjust to meet demand for renewable energy by the consumption good sector. Because renewable energy is produced by a linear technology that uses only capital, the capital input used to produce energy is only well determined at the global level, not at the country level. Without loss of generality, we can suppose that each country produces its own renewable energy either from its own capital or imported capital.

2.2. The Evolution of the Stock of Greenhouse Gases

The evolution of the stock of greenhouse gases is assumed to be governed by the following differential equation:

$$(7) \quad H_{t+1} = H_t + \sum_{i \in I} Q_{i,t} - \varepsilon(H_t - \underline{H}),$$

where, we recall, $\underline{H} > 0$ is the natural level of the stock of greenhouse gases in the atmosphere in steady state if there were negligible amount of greenhouse emissions, and $\varepsilon > 0$ is a parameter representing the rate of decay of the excess stock of greenhouse gases above its natural level. The initial stock of greenhouse gases in the atmosphere H_0 is assumed to be known.

2.3. Preferences and Utility Maximization

An old individual of period t in country i owns $x_{i,t} = X_{i,t} / N_{i,t}^1$ units of oil and $k_{i,t} = K_{i,t} / N_{i,t}^1$ units of capital. Her old-age income is thus given by $\phi_t x_{i,t} + (1 + \rho_t)k_{i,t}$. We shall assume that she makes no bequest. Her old-age consumption is then given by $c_{i,t}^1 = \phi_t x_{i,t} + (1 + \rho_t)k_{i,t}$.

For a young individual of period t , a lifetime plan is a list $(c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})$, where $c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}$, and $k_{i,t+1}$ represent, respectively, her current consumption, her old-age consumption, her oil investment, and her capital investment. We shall suppose that her single-period utility function of consumption is logarithmic and that she uses $\delta, 0 < \delta < 1$, as the factor to discount future utilities. Her problem of maximizing lifetime utility can be stated formally as follows:

$$(8) \quad \max_{(c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})} \text{Log} c_{i,t}^0 + \delta \text{Log} c_{i,t+1}^1$$

subject to the following two budget constraints:

$$(9) \quad c_{i,t}^0 + \phi_t x_{i,t+1} + k_{i,t+1} = \omega_{i,t},$$

$$(10) \quad c_{i,t+1}^1 = \phi_{t+1} x_{i,t+1} + (1 + \rho_{t+1})k_{i,t+1}.$$

Now if we let

$$(11) \quad r_{t+1} = \max \left\{ \frac{\phi_{t+1}}{\phi_t}, 1 + \rho_{t+1} \right\},$$

then r_{t+1} represents the rate of return that a young individual of period t in any country earns on her savings. Using r_{t+1} , we can restate the preceding lifetime utility maximization problem in the following simpler form:

$$(12) \quad \max_{c_{i,t}^0} \text{Log} c_{i,t}^0 + \delta \text{Log} [(1 + r_{t+1})(\omega_{i,t} - c_{i,t}^0)].$$

The solution of the preceding lifetime utility maximization problem is given by

$$(13) \quad c_{i,t}^0 = \frac{1}{1 + \delta} \omega_{i,t},$$

and the saving of a young individual of period t in country i is

$$(14) \quad s_{i,t} = \frac{\delta}{1+\delta} \omega_{i,t}.$$

How the saving represented by (14) is divided between oil and capital investments depends on the rates of return earned by these two assets. If $\phi_{t+1}/\phi_t > 1 + \rho_{t+1}$, then the individual will put all her savings in oil; that is, $x_{i,t+1} = s_{i,t}/\phi_t$ and $k_{i,t+1} = 0$. If $\phi_{t+1}/\phi_t < 1 + \rho_{t+1}$, then the individual will put all her savings in capital; that is, $x_{i,t+1} = 0$ and $k_{i,t+1} = s_{i,t}$. If $\phi_{t+1}/\phi_t = 1 + \rho_{t+1}$, then the individual will be indifferent between oil and capital investments. In this case, $k_{i,t+1}$ can assume any value between 0 and $s_{i,t}$.

2.4. Definition of Competitive Equilibrium

Let $P = (\rho_t, (\omega_{i,t})_{i \in I}, \phi_t, \varphi_t)_{t=0}^{\infty}$ be a price system. An allocation induced by the price system

P is a list of infinite sequences, say

$\Lambda = (c_{i,0}^1, (c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})_{t=0}^{\infty}, (K_{i,t,0})_{t=0}^{\infty}, (K_{i,t,1}, L_{i,t}, Q_{i,t}, B_{i,t}, Y_{i,t})_{t=0}^{\infty}, (X_{i,t}, K_{i,t}, N_{i,t}^0, N_{i,t}^1)_{t=0}^{\infty})_{i \in I}$,
with the following properties: Under the price system P ,

- (i) $c_{i,0}^1 = [\phi_0 X_{i,0} + (1 + \rho_0) K_{i,0}] / N_{i,0}^1$;
- (ii) $(c_{i,t}^0, c_{i,t+1}^1, x_{i,t+1}, k_{i,t+1})$ is the optimal lifetime plan for a young individual of period t in country i ;
- (iii) $K_{i,t,0}$ is the demand for capital by the representative firm in the backstop sector in country i in period t ;
- (iv) $(K_{i,t,1}, L_{i,t}, Q_{i,t}, B_{i,t}, Y_{i,t})$ is an optimal production plan of the representative firm that produces the consumption good in country i in period t ;
- (v) $(X_{i,t}, K_{i,t}, N_{i,t}^0, N_{i,t}^1) = N_{i,t-1}^0 (x_{i,t}, k_{i,t}, (1+n)N_{i,t-1}^0, 1)$.

The pair (P, Λ) is said to constitute a *competitive equilibrium* if the following market-clearing conditions are satisfied for each $t = 0, 1, \dots, \infty$

$$(vi) \quad \sum_{i \in I} X_{i,t+1} + \sum_{i \in I} Q_{i,t} = \sum_{i \in I} X_{i,t},$$

$$(vii) \quad \sum_{i \in I} B_{i,t} = \sum_{i \in I} K_{i,t,0},$$

$$(viii) \quad \sum_{i \in I} [K_{i,t,0} + K_{i,t,1}] = \sum_{i \in I} K_{i,t},$$

$$(ix) \quad L_{i,t} = N_{i,t}^0, \text{ for all } i.$$

Observe that (vi), (vii), and (viii) represent, respectively, the equilibrium conditions on the world oil market, the world market for renewable energy, and the world market for capital. As for (ix), it represents the equilibrium conditions for the labor market in country i .

The existence of competitive equilibrium for an overlapping-generations model, such as the one just formulated is a delicate question. There is a sparse literature on the existence of competitive equilibrium for an overlapping-generations model of an exchange economy associated with Balasko and Shell (1980, 1981a, 1981b), Balasko, Cass, and Shell (1980), and Wilson (1981). Compared to the model formulated by these researchers, the model formulated in Section 2 is much more complex. It has a production structure, capital, and an exhaustible resource. Furthermore, the climate change that is induced by the burning of fossil fuels also generates one-way temporal production externalities – in the direction of the arrow of time – between one period and the periods that follow. Hence it is not possible to invoke the results discovered by these researchers to assert that the model formulated in Section 2 has a competitive equilibrium. For the model we have just formulated, the existence of a competitive equilibrium can be established by using the technique by Hung and Quyen (2010). For technical details of this technique, we refer the reader to Hung and Quyen (2010). In short, the existence of a competitive equilibrium of our model can be established in two stages. First, it can be shown that if the model is truncated after a finite number of periods, then the truncated economy has a competitive equilibrium. Second, take any sequence of truncated

economies with the time horizon of a truncated economy being strictly longer than that of the preceding truncated economy in the sequence, and then apply Cantor's diagonal trick to the sequence. In the limit, we obtain a competitive equilibrium for the overlapping-generations model under infinite time horizon.

3. THE TEMPORARY EQUILIBRIA: INTEREST RATES AND WAGES

In an arbitrary period $t \geq 0$, one of the following three possibilities occurs: (i) no oil is used as part of the energy input in the production the consumption good in any country; (ii) both oil and renewable energy are used in one country to produce the consumption good; and (iii) oil is the only source of energy used to produce the consumption good in each country.

3.1. *The Energy Inputs Consist Solely of Renewable Energy*

If no oil is used in period t in country i to produce the consumption good, then renewable energy constitutes the only source of energy used in the production of the consumption good. In equilibrium, the first-order conditions (3) and (5b) now assume the following forms, respectively,

$$(15) \quad \alpha A \Omega_i(H_t) K_{i,t,1}^{\alpha-1} [N_{i,t}^0]^\beta B_{i,t}^{1-\alpha-\beta} = \rho_t,$$

and

$$(16) \quad (1 - \alpha - \beta) A \Omega_i(H_t) K_{i,t,1}^\alpha [N_{i,t}^0]^\beta B_{i,t}^{-\alpha-\beta} = \varphi_t.$$

Furthermore, the zero profit condition in the backstop sector implies that the price of renewable energy is equal to the rental rate of capital, i.e., $\varphi_t = \rho_t$, and this result allows us to rewrite (16) as

$$(17) \quad (1 - \alpha - \beta) A \Omega_i(H_t) K_{i,t,1}^\alpha [N_{i,t}^0]^\beta B_{i,t}^{-\alpha-\beta} = \rho_t.$$

Dividing (17) by (15), then rearranging the result, we obtain

$$(18) \quad \frac{1-\alpha-\beta}{\alpha} K_{i,t,1} = B_{i,t}.$$

Summing (18) over $i \in I$, we obtain

$$(19) \quad \begin{aligned} \frac{1-\alpha-\beta}{\alpha} \sum_{i \in I} K_{i,t,1} &= \sum_{i \in I} B_{i,t} \\ &= \sum_{i \in I} K_{i,t} - \sum_{i \in I} K_{i,t,1}. \end{aligned}$$

It follows immediately from (19) that the part of the world's capital stock used in the production of the consumption good is given by

$$(20) \quad \sum_{i \in I} K_{i,t,1} = \frac{\alpha}{1-\beta} \sum_{i \in I} K_{i,t}.$$

Now using (18), we can rewrite (15) as follows

$$\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \Omega_i(H_t) K_{i,t,1}^{-\beta} [N_{i,t}^0]^\beta = \rho_t,$$

from which we obtain

$$(21) \quad \rho_t^{\frac{1}{\beta}} K_{i,t,1} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0.$$

Summing (21) over $i \in I$, and using (20), we obtain

$$(22) \quad \begin{aligned} \rho_t^{\frac{1}{\beta}} \sum_{i \in I} K_{i,t,1} &= \rho_t^{\frac{1}{\beta}} \frac{\alpha}{1-\beta} \sum_{i \in I} K_{i,t} \\ &= \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} \sum_{i \in I} \left[[\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \right]. \end{aligned}$$

It follows from the second equality in (22) that

$$(23) \quad \begin{aligned} \rho_t &= \alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \left(\frac{1-\beta}{\alpha} \right)^\beta \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]^{-\beta} \\ &= \sigma_1 \left(\sum_{i \in I} \eta_{i,t} K_{i,t} \right)^{-\beta}, \end{aligned}$$

where we have let

$$(24) \quad \sigma_1 = \alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \left(\frac{1-\beta}{\alpha} \right)^\beta,$$

$$(25) \quad \eta_{i,t} = \frac{N_{i,0}^0}{\sum_{j \in I} [\Omega_j(H_t)]^\beta N_{j,0}^0},$$

and $\kappa_{i,t} = K_{i,t} / N_{i,t}^0$, as well as have used $N_{i,t}^0 = (1+n)^t N_{i,0}^0, i \in I, t = 0, 1, \dots$

Equation (23) expresses the equilibrium interest rate in a period when no oil is used in any country to produce the consumption good as a function of the weighted average of the capital endowments per young individual of the countries that make up the world economy, with the weight of a country depending on its initial young population and the impact of climate change on its economy at the time in question.

Using (23) in (21), we obtain the following expression for the equilibrium capital input in the consumption good sector at time t in country i :

$$(26) \quad K_{i,t,1} = \frac{\alpha}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]$$

$$= \frac{\alpha}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j \in I} \eta_{j,t} \kappa_{j,t}.$$

Using (26) in (18), we obtain the following expression for the demand of renewable energy at time t in country i :

$$(27) \quad B_{i,t} = \frac{1-\alpha-\beta}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]$$

$$= \frac{1-\alpha-\beta}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j \in I} \eta_{j,t} \kappa_{j,t}.$$

The demand for capital at time t by the backstop sector in country i is then given by

$$\begin{aligned}
(28) \quad K_{i,t,0} &= \frac{1-\alpha-\beta}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right] \\
&= \frac{1-\alpha-\beta}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \sum_{j \in I} \eta_{j,t} \kappa_{j,t}.
\end{aligned}$$

Using (26), (27), and (4), we obtain the following expression for the equilibrium wage rate in period t in country i :

$$\begin{aligned}
(29) \quad \omega_{i,t} &= \sigma_2 [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]^{1-\beta} \\
&= \sigma_2 [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\sum_{j \in I} \eta_{j,t} \kappa_{j,t} \right]^{1-\beta},
\end{aligned}$$

where we have let

$$(30) \quad \sigma_2 = \beta A \left[\frac{\alpha}{1-\beta} \right]^{\alpha} \left[\frac{1-\alpha-\beta}{1-\beta} \right]^{1-\alpha-\beta}.$$

Equation (29) expresses the equilibrium wage rate in a country in a period in which oil does not constitute part of the energy input used in the production of the consumption good.

3.2. The Energy Inputs Consist Solely of Fossil Fuels

In any period $t \geq 0$, if oil is the only source of energy used in the production of the consumption good, then $\phi_t \leq \rho_t$. The following first-order conditions characterize, respectively, the equilibrium demand for capital and the equilibrium demand for oil by the representative firm that produces the consumption good in country i :

$$(31) \quad \alpha A \Omega_i(H_t) K_{i,t,1}^{\alpha-1} [N_{i,t}^0]^{\beta} Q_{t,t}^{1-\alpha-\beta} = \rho_t,$$

and

$$(32) \quad (1-\alpha-\beta) A \Omega_i(H_t) K_{i,t,1}^{\alpha} [N_{i,t}^0]^{\beta} Q_{t,t}^{-\alpha-\beta} = \phi_t.$$

It follows from (31) and (32) that the equilibrium demand for oil and the equilibrium demand for capital in period t by the representative firm in the consumption good sector in country i are linked by the following relation:

$$(33) \quad Q_{i,t} = \frac{\rho_t(1-\alpha-\beta)}{\alpha\phi_t} K_{i,t,1}.$$

Using (33) in (31), we obtain

$$(34) \quad K_{i,t,1} = \frac{(\alpha A)^{\frac{1}{\beta}} \Omega_i(H_t) N_{i,t}^0 \left(\frac{1-\alpha-\beta}{\alpha} \right)^{\frac{1-\alpha-\beta}{\beta}}}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}} \\ = \sigma_3 \frac{\Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}},$$

where we have let

$$(35) \quad \sigma_3 = (\alpha A)^{\frac{1}{\beta}} \left(\frac{1-\alpha-\beta}{\alpha} \right)^{\frac{1-\alpha-\beta}{\beta}}.$$

Using (34) in (33), we obtain the following expression for the demand for oil at each instant by the representative firm producing the consumption good in country i :

$$(36) \quad Q_{i,t} = \sigma_4 \frac{\Omega_i(H_t) N_{i,t}^0}{\rho_t^{\frac{\alpha}{\beta}} \phi_t^{\frac{1-\alpha}{\beta}}},$$

where we have let

$$(37) \quad \sigma_4 = (\alpha A)^{\frac{1}{\beta}} \left(\frac{1-\alpha-\beta}{\alpha} \right)^{\frac{1-\alpha}{\beta}}.$$

In period t , the equilibrium condition on the world capital market is given by

$$(38) \quad \sum_{i \in I} K_{i,t} = \sum_{i \in I} K_{i,t,1} \\ = \frac{\sigma_3}{\phi_t^{\frac{1-\alpha-\beta}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}} \sum_{i \in I} \Omega_i(H_t) N_{i,t}^0,$$

where the second equality in (38) has been obtained by summing (34) over $i \in I$.

Equation (38) can be rewritten as follows:

$$\begin{aligned}
(39) \quad \rho_t &= \sigma_3^{\frac{\beta}{\alpha+\beta}} \frac{1}{\phi_t^{\frac{\alpha+\beta}{1-\alpha-\beta}}} \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} \Omega_i(H_t) N_{i,t}^0} \right]^{\frac{\beta}{\alpha+\beta}} \\
&= \sigma_3^{\frac{\beta}{\alpha+\beta}} \frac{1}{\phi_t^{\frac{\alpha+\beta}{1-\alpha-\beta}}} \left[\sum_{i \in I} \eta_{i,t} \kappa_{i,t} \right]^{\frac{\beta}{\alpha+\beta}}.
\end{aligned}$$

Using (39) in (36), we obtain

$$\begin{aligned}
(40) \quad Q_{i,t} &= \sigma_4 \frac{\Omega_i(H_t) N_{i,t}^0}{\rho_t^{\frac{\alpha}{\beta}} \phi_t^{\frac{1-\alpha}{\beta}}} \\
&= \sigma_4 \sigma_3^{\frac{\alpha}{\alpha+\beta}} \frac{\Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1}{\alpha+\beta}}} \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} \Omega_i(H_t) N_{i,t}^0} \right]^{\frac{\alpha}{\alpha+\beta}}.
\end{aligned}$$

Summing (40) over $i \in I$, we obtain the following expression of the world demand for oil in period t

$$(41) \quad \sum_{i \in I} Q_{i,t} = \sigma_4 \sigma_3^{\frac{\alpha}{\alpha+\beta}} \frac{\sum_{i \in I} \Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{1}{\alpha+\beta}}} \left[\frac{\sum_{i \in I} K_{i,t}}{\sum_{i \in I} \Omega_i(H_t) N_{i,t}^0} \right]^{\frac{\alpha}{\alpha+\beta}}.$$

Using (34) and (36) in the first-order condition (4), we obtain the following expression for the equilibrium wage rate in period t in country i when only oil is used in the production of the consumption good

$$\begin{aligned}
(42) \quad \omega_{i,t} &= \beta A \Omega_i(H_t) \left[\sigma_3 \frac{\Omega_i(H_t) N_{i,t}^0}{\phi_t^{\frac{\alpha}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}} \right]^{1-\beta} [N_{i,t}^0]^{\beta-1} \left[\frac{\rho_t(1-\alpha-\beta)}{\alpha \phi_t} \right]^{1-\alpha-\beta} \\
&= \beta A \left[\frac{1-\alpha-\beta}{\alpha} \right]^{1-\alpha-\beta} \sigma_3^{1-\beta} [\Omega_i(H_t)]^{2-\beta} \frac{1}{\phi_t^{\frac{\alpha}{\beta}} \rho_t^{\frac{\alpha+\beta}{\beta}}}.
\end{aligned}$$

3.3. The Energy Inputs Consist of both Oil and Renewable Energy

In any period $t \geq 0$, if both fossil fuels and the backstop provide energy inputs into the production of the consumption good, then $\rho_t = \phi_t = \varphi_t$. The first-order conditions (3) and (5c) then become, respectively,

$$(43) \quad \alpha A \Omega_i(H_t) K_{i,t,1}^{\alpha-1} [N_{i,t}^0]^\beta (Q_{i,t} + B_{i,t})^{1-\alpha-\beta} = \rho_t,$$

and

$$(44) \quad (1 - \alpha - \beta) A \Omega_i(H_t) K_{i,t,1}^\alpha [N_{i,t}^0]^\beta (Q_{i,t} + B_{i,t})^{-\alpha-\beta} = \varphi_t = \phi_t = \rho_t.$$

Because $\phi_t = \varphi_t$, the mix of energy input $Q_{i,t} + B_{i,t}$ is indeterminate; their sum, however, is determinate, and satisfies the following relation:

$$(45) \quad \frac{1 - \alpha - \beta}{\alpha} K_{i,t,1} = Q_{i,t} + K_{i,t,0}.$$

Summing (45) over $i \in I$, we obtain

$$\frac{1 - \alpha - \beta}{\alpha} \sum_{i \in I} K_{i,t,1} = \sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t} - \sum_{i \in I} K_{i,t,1},$$

which can be rewritten as

$$(46) \quad \sum_{i \in I} K_{i,t,1} = \frac{\alpha}{1 - \beta} \left[\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t} \right]$$

Using (45) in (43), we obtain

$$\alpha A \left(\frac{1 - \alpha - \beta}{\alpha} \right)^{1-\alpha-\beta} \Omega_i(H_t) K_{i,0,1}^{-\beta} [N_{i,0}^0]^\beta = \rho_t,$$

from which we obtain

$$(47) \quad \rho_t^{\frac{1}{\beta}} K_{i,t,1} = \left[\alpha A \left(\frac{1 - \alpha - \beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} [\Omega_i(H_t)]^\beta N_{i,t}^0.$$

Summing (47) over $i \in I$, and using (46), we obtain

$$\begin{aligned} \rho_t^{\frac{1}{\beta}} \sum_{i \in I} K_{i,t,1} &= \rho_t^{\frac{1}{\beta}} \frac{\alpha}{1-\beta} \left[\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t} \right] \\ &= \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} \sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0, \end{aligned}$$

which can be rewritten as

$$(48) \quad \rho_t^{\frac{1}{\beta}} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} \frac{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0}{\frac{\alpha}{1-\beta} \left[\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t} \right]}.$$

Using (48) in (47), we obtain the following expression for the demand for capital in the consumption good sector of country i in period t :

$$(49) \quad K_{i,t,1} = \frac{\alpha}{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0 \frac{\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0}.$$

Using (49) and (45) in the first-order condition (4), we obtain the following expression for the wage rate in period t in country i when both fossil fuels and renewable energy are used in the production of the consumption good:

$$(50) \quad \omega_{i,t} = \beta A \left[\frac{1-\alpha-\beta}{\alpha} \right]^{1-\alpha-\beta} \left[\frac{\alpha}{1-\beta} \right]^{1-\beta} [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\frac{\sum_{i \in I} Q_{i,t} + \sum_{i \in I} K_{i,t}}{\sum_{i \in I} [\Omega_i(H_t)]^{\frac{1}{\beta}} N_{i,t}^0} \right]^{1-\beta}.$$

4. A CHARACTERIZATION OF THE COMPETITIVE EQUILIBRIA

To study the climate change induced by the burning of fossil fuels, we shall assume that $\sum_{i \in I} X_{i,0}$, the world's initial stock of fossil fuels, is much greater than $\sum_{i \in I} K_{i,0}$, the world's initial capital stock.⁷ Intuitively, we expect that if the world's initial stock of fossil fuels is large, but the world's initial capital stock is not, then the oil input into the production of the consumption good will be large, and the backstop sector in each country will not be active. The following lemma confirms this intuition.

LEMMA 1: *If the world's initial stock of fossil fuels is large, then the initial global demand for oil will be high, and the backstop sector in each country will not be active in period 0.*

PROOF: There are three possibilities to consider: (i) no oil is used as part of the energy input used to produce the consumption good in any country in period 0, (ii) both oil and renewable energy are used in one country to produce the consumption good in period 0, and (iii) oil is the only source of energy used to produce the consumption good in period 0.

If no oil is used in any country to produce the consumption good in period 0, then we must have $\phi_0 \geq \rho_0$. Furthermore, applying (23) for $t = 0$, we can assert that

$$(51) \quad \rho_0 = \sigma_1 \left(\sum_{i \in I} \eta_{i,0} \kappa_{i,0} \right)^{-\beta} \leq \phi_0,$$

i.e., the price of oil is bounded below by $\sigma_1 \left(\sum_{i \in I} \eta_{i,0} \kappa_{i,0} \right)^{-\beta}$. On the other hand, according to (29), the wage rate in period 0 in country i is given by

$$(52) \quad \omega_{i,0} = \sigma_2 [\Omega_i(H_0)]^{\frac{1}{\beta}} \left[\sum_{j \in I} \eta_{j,0} \kappa_{j,0} \right]^{1-\beta},$$

⁷ The burning of a small stock of fossil fuels has a negligible impact on the climate.

which is bounded above. Thus if the world's initial stock of fossil fuels is large and is not exploited in period 0, the value of this stock will exceed the labor income of all the young individuals of period 0 in the whole world. Hence if the world's initial stock of fossil fuels is large, it will be exploited in period 0.

Under possibility (ii), we have $\phi_0 = \rho_0$, and applying (49) for $t = 0$, we can write

$$(53) \quad K_{i,0,1} = \frac{\alpha}{1-\beta} [\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0 \frac{\sum_{i \in I} Q_{i,0} + \sum_{i \in I} K_{i,0}}{\sum_{i \in I} [\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0} < \sum_{i \in I} K_{i,0}.$$

The strict inequality in (53) implies that if possibility (ii) holds, then $\sum_{i \in I} Q_{i,0}$ is bounded above when the world's initial stock of fossil fuels becomes indefinitely large. Applying (48) and (50), respectively, for $t = 0$, we obtain

$$(54) \quad \rho_0^{\frac{1}{\beta}} = \left[\alpha A \left(\frac{1-\alpha-\beta}{\alpha} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\beta}} \frac{\sum_{i \in I} [\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0}{\frac{\alpha}{1-\beta} \left[\sum_{i \in I} Q_{i,0} + \sum_{i \in I} K_{i,0} \right]}$$

and

$$(55) \quad \omega_{i,0} = \beta A \left[\frac{1-\alpha-\beta}{\alpha} \right]^{1-\alpha-\beta} \left[\frac{\alpha}{1-\beta} \right]^{1-\beta} [\Omega_i(H_0)]^{\frac{1}{\beta}} \left[\frac{\sum_{i \in I} Q_{i,0} + \sum_{i \in I} K_{i,0}}{\sum_{i \in I} [\Omega_i(H_0)]^{\frac{1}{\beta}} N_{i,0}^0} \right]^{1-\beta}.$$

Note that if possibility (ii) holds, then (54) is bounded below, and (55) bounded above, which together imply that the wages earned by all the young individuals of period 0 in the world cannot afford to buy the world's remaining stock of fossil fuels $[\sum_{i \in I} X_{i,0}] - [\sum_{i \in I} Q_{i,0}]$ for investment purposes. Hence possibility (ii) cannot arise in equilibrium.

Having proved that possibility (iii) will prevail if the world's initial stock of fossil fuels is large, let us now show that the amount of oil extracted for use in the production of the

consumption good in period 0 will be large. Indeed, if there exists a number $M > 0$ such that $\sum_{i \in I} Q_{i,0} < M$ no matter how large $\sum_{i \in I} X_{i,0}$ is, then we must have

$$(56) \quad \begin{aligned} \phi_0([\sum_{i \in I} X_{i,0}] - M) &< \phi_0([\sum_{i \in I} X_{i,0}] - [\sum_{i \in I} Q_{i,0}]) < \sum_{i \in I} Y_{i,0} \\ &< \sum_{i \in I} A\Omega_i(H_0) [\sum_{i \in I} K_{i,0}]^\alpha [N_{i,t}^0]^\beta [M + \sum_{i \in I} K_{i,0}]^{-\alpha-\beta}. \end{aligned}$$

In (56), the strict inequality between the expression on the left-hand side of the first inequality and the expression on the last line implies that ϕ_0 will be small when $\sum_{i \in I} X_{i,0}$ is large. Applying (40) for $t = 0$, we can then assert that $Q_{i,0}$ will be large when $\sum_{i \in I} X_{i,0}$ is large, contradicting the premise that $\sum_{i \in I} Q_{i,0}$ is bounded above by M . ■

Now along a competitive equilibrium path, the resource price must appreciate at the rate of interest in order to induce a young individual to put part of her savings in oil. Furthermore, whenever oil is extracted for use in the production of the consumption good, its price is constrained not to exceed the rental rate of capital. Hence extraction activities cannot be expected to go on forever. The following lemma confirms this intuition.

LEMMA 2: *Under a competitive equilibrium, the exploitation of the world's stock of fossil fuels is terminated in finite time.*

PROOF: If the exploitation of the world's stock of fossil fuels is not terminated in finite time, then there exists a strictly increasing natural numbers, say $\tau(m), m = 0, 1, \dots$, such that the world's stock of fossil fuels is exploited in period $\tau(m)$ to meet the world's demand, i.e., $\sum_{i \in I} Q_{i,\tau(m)} > 0$. Because the successive young generations of periods $t = 0, 1, \dots$ invest in both oil and capital, the price of oil must rise geometrically through time at the rate of interest. Hence $\phi_{\tau(m)} = \phi_0 \prod_{t=1}^{\tau(m)} (1 + \rho_t)$, and this means $\phi_{\tau(m)}$ is strictly

increasing with m . We claim that $\lim_{m \rightarrow \infty} \phi_{\tau(m)} = \infty$. Indeed, if $\lim_{m \rightarrow \infty} \phi_{\tau(m)} < \infty$, then $\lim_{m \rightarrow \infty} \rho_{\tau(m)} = 0$, which implies $\rho_{\tau(m)} < \phi_{\tau(m)}$ for large m , and this last strict inequality means that oil will not constitute part of the energy input used to produce the consumption good in period $\tau(m)$, contradicting the reductio ad absurdum hypothesis. The claim is now proved. To complete the proof of Lemma 2, pick a positive integer m such that $\phi_{\tau(m)} > 1$, then note that

$$\phi_{\tau(m+1)} = \phi_{\tau(m)} \prod_{t=\tau(m)+1}^{\tau(m+1)} (1 + \rho_t) > (1 + \rho_{\tau(m+1)}) > \rho_{\tau(m+1)},$$

which means that oil does not constitute part of the energy input used to produce the consumption good in period $\tau(m+1)$, contradicting the premise of the reductio ad absurdum argument. ■

In light of Lemmas 1 and 2, we can characterize a competitive equilibrium as consisting of three phases. In the first phase, fossil fuels provide all the energy needs of the world economy. During this phase, the price of oil rises steadily at the rate of interest. The second phase – which might or might not exist – begins when the price of oil has risen to the level of the interest rate. In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In the third phase of the competitive equilibrium – the post fossil fuel phase – the backstop completely takes over and provides all the energy needs of the world economy. We shall now analyze these three phases in the reverse order.

4.1. *The Post Fossil Fuel Phase*

Let T denote the time period that follows immediately the period in which the world's stock of fossil fuels is last exploited. Then we have $\sum_{i \in I} Q_{i,T-1} > 0$ and $\sum_{i \in I} Q_{i,t} = 0, t \geq T$. While Lemma 2 asserts that extraction activities are terminated in finite time, it is silent about whether the world's stock of fossil fuels is depleted when extraction activities are terminated. Depending on the values of the parameters, it might

be the case that $\sum_{i \in I} X_{i,T} > 0$, i.e., the world's stock of fossil fuels is only partially depleted. Thus there are two possibilities to consider: complete oil exhaustion and partial oil exhaustion.

4.1.1. Complete Oil Exhaustion

After the world's stock of fossil fuels has been depleted, the energy input used to produce the consumption good in each country in each period consists only of renewable energy. According to (29), the equilibrium wage rate in period $t \geq T$ in country i is given by

$$(57) \quad \omega_{i,t} = \sigma_2 [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\sum_{j \in I} \eta_{j,t} \kappa_{j,t} \right]^{1-\beta}, \quad (i \in I, t \geq T).$$

The capital endowment of a young individual of period $t, t \geq T$, in country i is then given by

$$(58) \quad \kappa_{i,t+1} = \frac{\delta \sigma_2}{(1+\delta)(1+n)} [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\sum_{j \in I} \eta_{j,t} \kappa_{j,t} \right]^{1-\beta}.$$

Multiplying (58) by $\eta_{i,t+1}$ then summing $\eta_{i,t+1} \kappa_{i,t+1}$ over $i, i \in I$, we obtain

$$(59) \quad \sum_{i \in I} \eta_{i,t+1} \kappa_{i,t+1} = \frac{\delta \sigma_2}{(1+\delta)(1+n)} \left(\sum_{i \in I} \eta_{i,t+1} [\Omega_i(H_t)]^{\frac{1}{\beta}} \right) \left[\sum_{j \in I} \eta_{j,t} \kappa_{j,t} \right]^{1-\beta}.$$

Now note that when $t \rightarrow \infty$, we have $H_t \rightarrow \underline{H}$, which implies that when t is large enough so that $H_t < H^\#$, we have $\sum_{i \in I} \eta_{i,t+1} [\Omega_i(H_t)]^{\frac{1}{\beta}} = 1$, and $\sum_{i \in I} \eta_{i,t+1} \kappa_{i,t+1}$ must converge. More specifically, we have

$$(60) \quad \lim_{t \rightarrow \infty} \sum_{i \in I} \eta_{i,t+1} \kappa_{i,t+1} = \bar{\kappa} = \left[\frac{\delta \sigma_2}{(1+\delta)(1+n)} \right]^{\frac{1}{\beta}}.$$

Using (60) in (59), we obtain

$$(61) \quad \lim_{t \rightarrow \infty} \kappa_{i,t} = \bar{\kappa}, \quad (i \in I).$$

Equation (61) asserts that in the long run – after the negative impact of climate change has dissipated – and under the scenario of complete oil exhaustion the capital endowment per young individual in each country converges to the same steady-state level, and this steady-state level depends on the production technologies, the preferences, and the population growth rate.

4.1.2. *Incomplete Oil Depletion*

Suppose that at the end of the period in which the world's stock of fossil fuels is last exploited the remaining stock of oil is positive, i.e., $\sum_{i \in I} X_{i,T} > 0$. Under this scenario, the remaining stock in situ $\sum_{i \in I} X_{i,t} > 0$ is passed on from one generation to the next for $t \geq T$. Although now the expressions for the equilibrium interest rate and the equilibrium wage rate in each country are still the same as under complete oil exhaustion, the savings of a young individual of each period $t \geq T$ must include both capital and oil, and the capital endowment per young individual in country i in the next period is given by

$$(62) \quad \kappa_{i,t+1} = \frac{1}{1+n} \left[\frac{\delta \sigma_2}{(1+\delta)} [\Omega_i(H_t)]^{\frac{1}{\beta}} \left[\sum_{j \in I} \eta_{j,t} \kappa_{j,t} \right]^{1-\beta} - \phi_t x_{i,t+1} \right].$$

For a period t far into the future, we have $H_t < H^\#$, and all the negative impact of climate change has dissipated. In such a period, we have $\Omega_i(H_t) = 1$, $\eta_{i,t} = 1$, and this means that the capital endowment per young individual, the oil endowment per young individual, and the wage rate are identical across countries. Thus when t is large, the evolution of the world economy can be studied by focusing on an arbitrary country, say country i . Furthermore, equation (62) is now reduced to

$$(63) \quad \kappa_{i,t+1} = \frac{1}{1+n} \left[\frac{\delta \sigma_2}{(1+\delta)} \kappa_{i,t}^{1-\beta} - \phi_t x_{i,t+1} \right].$$

Let us consider an arbitrary country i and an arbitrary distant future period t . We have $x_{i,t} = [\sum_{i \in I} X_{i,t}] / \sum_{i \in I} N_{i,t-1}$, which implies that $x_{t+1} = x_t / (1+n)$. If the system converges to a steady state, let

$$(64) \quad \hat{\kappa} = \lim_{t \rightarrow \infty} \kappa_{i,t},$$

and

$$(65) \quad z = \lim_{t \rightarrow \infty} \phi_t x_{i,t+1}.$$

Because the value of the oil investment made by a young individual in steady state remains the same, and because the amount of oil in the investment portfolio of a young individual declines geometrically at the rate of population growth, the price of oil must rise geometrically in steady state at rate n , which in turn implies that the rate of interest in steady state must be equal to n . That is, $\sigma_1 \hat{\kappa}^{-\beta} = n$, or equivalently

$$(66) \quad \hat{\kappa} = \left[\frac{\sigma_1}{n} \right]^{\frac{1}{\beta}}.$$

Using (66) in the steady-state version of (63), we obtain

$$(67) \quad z = \left[\frac{\sigma_1}{n} \right]^{\frac{1}{\beta}} \left(\frac{\delta \sigma_2}{1 + \delta} \left[\frac{\sigma_1}{n} \right]^{-1} - (1+n) \right).$$

In order for z to be positive, the following condition must be satisfied:

$$(68) \quad \frac{n}{n+1} > \frac{(1+\delta)\sigma_1}{\delta\sigma_2}.$$

Note that (68) cannot be satisfied if $[\sigma_1(1+\delta)]/[\sigma_2\delta] \geq 1$ or if the population growth rate is small.

Now recall that under complete oil exhaustion the steady-state capital labor ratio is given by

$$(69) \quad \bar{\kappa} = \left[\frac{\delta\sigma_2}{(1+\delta)(1+n)} \right]^{\frac{1}{\beta}}$$

$$= \left[\frac{\sigma_1}{n} \right]^{\frac{1}{\beta}} \left[\frac{n\delta\sigma_2}{\sigma_1(1+\delta)(1+n)} \right]^{\frac{1}{\beta}} > \left[\frac{\sigma_1}{n} \right]^{\frac{1}{\beta}} = \hat{\kappa},$$

where the strict inequality in (69) has been obtained with the help of (68). The steady-state capital labor ratio is thus lower under incomplete than under complete oil exhaustion.

The equilibrium under incomplete oil exhaustion is clearly inefficient: the part of the world's stock of fossil fuels left in situ can be used judiciously to increase the output of the consumption good without inducing any negative impact of climate change after the stock of greenhouse gases has fallen below its threshold level.

4.1.3. Capital Flows and Welfare

According to (57), the wage rate earned by a young individual after oil exhaustion is lower in a country which is more adversely affected (higher value of γ_i) by climate change. On the other hand, free capital mobility means that a young individual – regardless of country of origin – earns the same rate of return for her savings. Hence the lifetime utility of an individual varies inversely with the negative impact of climate change on her country of origin. This result is formally stated in the following proposition.

PROPOSITION 1: *For each generation that is born after the world's stock of fossil fuels has been depleted, the welfare of each of its members varies across countries inversely with the degree of vulnerability to climate change of its own country of origin.*

To study the impact of climate change on the direction of capital flows, let $\hat{k}_{i,t} = (K_{i,t,0} + K_{i,t,1}) / N_{i,t}^0, i \in I, t \geq T$, denote the capital demand young individual ratio of country i in a period after the world's stock of fossil fuels has been depleted, then:

$$(70) \quad \hat{k}_{i,t+1} - \kappa_{i,t+1} = \frac{\delta \sigma_2}{(1 + \delta)(1 + n)} e^{\frac{\gamma_i(H_{t+1}-H)^2}{\beta}} [\sum_{j \in I} \eta_{j,t} k_{j,t}]^{1-\beta} \left[\frac{\sum_{i \in I} N_{i,0}^0 e^{\frac{\gamma_i(H_t-H)^2}{\beta}}}{\sum_{j \in I} e^{\frac{\gamma_j(H_{t+1}-H)^2}{\beta}} N_{j,0}^0} - e^{\frac{\gamma_i(H_t-H_{t+1})^2}{\beta}} \right].$$

The sign of (70) is the same as the sign of

$$\frac{\sum_{i \in I} N_{i,0}^0 e^{\frac{\gamma_i(H_t-H)^2}{\beta}}}{\sum_{j \in I} e^{\frac{\gamma_j(H_{t+1}-H)^2}{\beta}} N_{j,0}^0} - e^{\frac{\gamma_i(H_t-H_{t+1})^2}{\beta}},$$

an expression that depends on the stock of greenhouse gases in the atmosphere, the parameters that characterize the negative impact of climate change on the production of the consumption good, the initial populations of the countries that make up the world economy, and the factor share of labor in national income of each country.

Note that

$$0 < \frac{\sum_{i \in I} N_{i,0}^0 e^{\frac{\gamma_i(H_t-H)^2}{\beta}}}{\sum_{j \in I} e^{\frac{\gamma_j(H_{t+1}-H)^2}{\beta}} N_{j,0}^0} < 1.$$

and $e^{\frac{\gamma_i(H_t-H_{t+1})^2}{\beta}}$ is close to 0 if γ_i is large, and is close to 1 if γ_i is small. Hence $\hat{k}_{i,t} - \kappa_{i,t}$ is positive (negative) if γ_i is large (small). The following proposition is now immediate:

PROPOSITION 2: *In any period after the world's stock of fossil fuels has been depleted, a country that is more vulnerable to climate change will import capital, while a country that is more impervious to climate change will export capital.*

4.2. The Fossil Fuel Phase

During this phase, oil is used in the production of the consumption good. According to Lemma 1, if the world's initial stock of fossil fuels is large, then there will be excessive burning of fossil fuels under the competitive equilibrium. The excessive burning of fossil fuels leads to a dramatic rise in the stock of greenhouse gases in the atmosphere in period 1, which shifts downward the production function of the consumption good in every country in period 1, with the ensuing result of a drastic reduction in the output of the consumption good in each country in period 1. The following proposition, which is a generalization of Tang (2007, Proposition 3) to a multi-country world, confirms this result.

PROPOSITION 3: *If the initial world's stock of fossil fuels is large, then under the competitive equilibrium there will be excessive burning of fossil fuels in period 0, with the ensuing consequence of a drastic reduction in the output of the consumption good in each country in period 1. More precisely, $Y_{i,1} \rightarrow 0$ for each $i \in I$ when $\sum_{i \in I} X_{i,0} \rightarrow +\infty$.*

PROOF: The saving of an individual in country i in period 0 is

$$(71) \quad s_{i,0} = \frac{\delta}{1+\delta} \omega_{i,0} = \frac{\delta}{1+\delta} \beta A \Omega_i (H_0) K_{i,0,1}^\alpha L_{i,0}^{\beta-1} (Q_{i,0} + B_{i,0})^{1-\alpha-\beta}$$

Because the saving of a young individual in country i of period 0 must be at least sufficient to purchase the oil stock remaining at the end of period 0, the following inequality must hold

$$(72) \quad Q_{i,0} \geq \frac{(1+\delta)(1-\alpha-\beta)X_0 - \delta\beta L_{i,0}^{-1}B_{i,0}}{\delta\beta L_{i,0}^{-1} + (1+\delta)(1-\alpha-\beta)}$$

It follows from (72) that the remaining oil stock at the beginning of period 1 satisfies the following inequality

$$(73) \quad X_{i,1} = X_{i,0} - Q_{i,0} \leq \frac{\delta \beta L_{i,0}^{-1}}{\delta \beta L_{i,0}^{-1} + (1 + \delta)(1 - \alpha - \beta)} (X_{i,0} + B_{i,0})$$

The capital investment of a young individual of country i of period 0 was given by

$$(74) \quad \begin{aligned} K_{i,1} &= s_{i,0} - \phi_0 X_{i,1} \\ &= A \Omega_i (H_0) K_{i,0,1}^\alpha L_{i,0}^\beta (Q_{i,0} + B_{i,0})^{-\alpha - \beta} \left[\begin{aligned} &\left(\frac{\delta}{1 + \delta} \beta L_{i,0}^{-1} + 1 - \alpha - \beta \right) Q_{i,0} \\ &+ \frac{\delta}{1 + \delta} \beta L_{i,0}^{-1} B_{i,0} - (1 - \alpha - \beta) X_{i,0} \end{aligned} \right] \end{aligned}$$

Now the output of the consumption good of country i in period 1 was

$$Y_{i,1} = A e^{-\gamma_i \left[(1 - \varepsilon) H_0 + \sum_{i \in I} Q_{i,0} \right]^2} K_{i,1,1}^\alpha L_{i,1}^\beta (Q_{i,1} + B_{i,1})^{1 - \alpha - \beta}, \quad \text{assuming } \underline{H} = 0.$$

Because $Q_{i,1} \leq X_{i,1}$ and $B_{i,1} \leq K_{i,1}$ the output of the consumption good in period 1 satisfied:

$$(75) \quad Y_{i,1} \leq A e^{-\gamma_i \left[(1 - \varepsilon) H_0 + \sum_{i \in I} Q_{i,0} \right]^2} K_{i,1,1}^\alpha L_{i,1}^\beta \left[\begin{aligned} &\frac{\delta \beta L_{i,0}^{-1}}{\delta \beta L_{i,0}^{-1} + (1 + \delta)(1 - \alpha - \beta)} (X_{i,0} + B_{i,0}) + \\ &+ A \Omega_i (H_0) K_{i,0,1}^\alpha L_{i,0}^\beta (Q_{i,0} + B_{i,0})^{-\alpha - \beta} \left[\begin{aligned} &\left(\frac{\delta}{1 + \delta} \beta L_{i,0}^{-1} + 1 - \alpha - \beta \right) Q_{i,0} + \\ &+ \frac{\delta}{1 + \delta} \beta L_{i,0}^{-1} B_{i,0} - (1 - \alpha - \beta) X_{i,0} \end{aligned} \right] \end{aligned} \right]^{1 - \alpha - \beta}$$

and $Q_{i,0} \leq X_{i,0}$, it followed from the preceding inequality that

$$(76) \quad Y_{i,1} \leq A e^{-\gamma_i \left[(1 - \varepsilon) H_0 + \sum_{i \in I} Q_{i,0} \right]^2} \times K_{i,1,1}^\alpha L_{i,1}^\beta \left[\begin{aligned} &\frac{\delta \beta L_{i,0}^{-1}}{(1 + \delta)(1 - \alpha - \beta)} (Q_{i,0} + B_{i,0}) + \\ &+ \frac{\delta}{1 + \delta} \beta A \Omega_i (H_0) K_{i,0,1}^\alpha L_{i,0}^{\beta - 1} (Q_{i,0} + B_{i,0})^{1 - \alpha - \beta} \end{aligned} \right]^{1 - \alpha - \beta}$$

Now when $Q_{i,0}$ was large, the exponential term $e^{-\gamma_i \left[(1-\varepsilon)H_0 + \sum_{i \in I} Q_{i,0} \right]^2}$ would dominate the expression

$$\left[\frac{\delta \beta L_{i,0}^{-1}}{(1+\delta)(1-\alpha-\beta)} (Q_{i,0} + B_{i,0}) + \frac{\delta}{1+\delta} \beta A \Omega_i (H_0) K_{i,0,1}^\alpha L_{i,0}^{\beta-1} (Q_{i,0} + B_{i,0})^{1-\alpha-\beta} \right]^{1-\alpha-\beta}$$

Therefore, the output of the consumption good of each country in period 1 under the competitive equilibrium would tend to 0 when its initial oil stock tended to infinity. ■

According to Proposition 3, the excessive burning of fossil fuels benefits the current generation at the expense of future generations. The cut in current oil input reduces current output; however, the better environmental quality in the following period raises output in that period. The lifetime utility of the current young generation experiences a net gain without lowering the utility of any other generation. Thus, a slightly reduction in oil input and transferred to the following period will be a Pareto improvement. The following proposition, which is a generalization to a multi-country world, is now immediate:

PROPOSITION 4: *Suppose that the world governments do not implement any policy on climate change. Consider a competitive equilibrium under which oil constitutes part of the energy input used in the production of the consumption good in country i in at least one period. Then the competitive equilibrium is not Pareto optimal.*

PROOF: Without loss of generality, suppose that $Q_{i,t} > 0$ for $t = 0$. For country i , if we cut back the oil input in period 0, say by μ and transfer it to period 1, we will be able to lower the stock of greenhouse gases at the beginning of period 1. Of course, the output of the consumption good in country i in period 0 will be lower. The variation in the output of the consumption good in country i in period 0 is given by

$$(77) \quad \Delta Y_{i,0}(\mu) = A\Omega_i(H_0)K_{i,0,1}^\alpha [N_{i,0}^0]^\beta (Q_{i,0} - \mu + B_{i,0})^{1-\alpha-\beta} - A\Omega_i(H_0)K_{i,0,1}^\alpha [N_{i,0}^0]^\beta (Q_{i,0} + B_{i,0})^{1-\alpha-\beta} .$$

If we maintain the same capital investment as the one made by the young generation of period and raises the oil input in period 1 above its competitive equilibrium, then the output of the consumption good in country i in period 1 will raise by

$$(78) \quad \Delta Y_{i,1}(\mu) = A\Omega_i \left(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{j \neq i} Q_{j,0} + Q_{i,0} - \mu \right) K_{i,1,1}^\alpha [N_{i,1}^0]^\beta (Q_{i,1} + \mu + B_{i,1})^{1-\alpha-\beta} \\ - A\Omega_i \left(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{i \in I} Q_{i,0} \right) K_{i,1,1}^\alpha [N_{i,1}^0]^\beta (Q_{i,1} + B_{i,1})^{1-\alpha-\beta} .$$

The lifetime utility for a young individual of period 0 in country i under this intervention is

$$(79) \quad u_i(\mu) = \text{Log}[c_{i,0}^0 + \Delta Y_{i,0}(\mu)] + \delta \text{Log}[c_{i,1}^1 + \Delta Y_{i,1}(\mu)] .$$

Differentiating (79) with respect to μ , then evaluating the result at $\mu = 0$, we obtain

$$\begin{aligned}
u'(0) &= -\frac{(1-\alpha-\beta)A\Omega_i(H_0)K_{i,0,1}^\alpha [N_{i,0}^0]^\beta (Q_{i,0} + B_{i,0})^{-\alpha-\beta}}{c_{i,0}^0} + \\
&\quad + \delta \left[\frac{(1-\alpha-\beta)A\Omega_i(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{i \in I} Q_{i,0})K_{i,1,1}^\alpha [N_{i,1}^0]^\beta (Q_{i,1} + B_{i,1})^{-\alpha-\beta}}{c_{i,1}^1} + \right. \\
&\quad \left. + \frac{A\Omega_i'(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{i \in I} Q_{i,0})K_{i,1,1}^\alpha [N_{i,1}^0]^\beta (Q_{i,1} + B_{i,1})^{1-\alpha-\beta}}{c_{i,1}^1} \right] \\
(80) \quad &= -\frac{\phi_0}{c_{i,0}^0} + \\
&\quad + \delta \left[\frac{\phi_1 + 2\gamma_i(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{i \in I} Q_{i,0} - H^\#)A\Omega_i(H_0 - \varepsilon(H_0 - \underline{H}) + \sum_{i \in I} Q_{i,0})K_{i,1,1}^\alpha [N_{i,1}^0]^\beta (Q_{i,1} + B_{i,1})^{1-\alpha-\beta}}{\delta c_{i,0}^0 \frac{\phi_1}{\phi_0}} \right] \\
&= \frac{\phi_0}{c_{i,0}^0} \left[\frac{2\gamma_i(H_1 - H^\#)(Q_{i,1} + B_{i,1})}{1-\alpha-\beta} \right] > 0.
\end{aligned}$$

For country i , because $u'(0) > 0$, the intervention just described raises the lifetime utility of the young generation of period 0 without lowering the old-age utility of the old generation of period 0 and the lifetime utilities of the generations to come. ■

4.2.1. The Phase of Technology Substitution

In this phase, the two technologies – fossil fuels and backstop – co-exist, and the energy inputs used in the production of the consumption good consist of both oil and renewable energy. In order to induce a young individual to put part of her savings in oil, the price of oil must rise steadily but not exceed the rental rate of capital. Moreover, due to the ultimate finite stock of fossil fuels, it is necessary to look for an everlasting source of energy input for substitution in the long run. Hence when the price of oil has risen to the level of the interest rate the backstop will be brought into use. The transition phase in which both fossil fuels and backstop technologies co-exist might arise. If fossil fuels and

the backstop are both used in a period, say period $t + 1$, then $\rho_{t+1} = \phi_{t+1} = \varphi_{t+1}$ and the individual will be indifferent between oil and backstop capital investments. Therefore, the following condition holds:

$$(81) \quad \phi_{t+1} = (1 + \rho_{t+1})\phi_t = \rho_{t+1},$$

which implies that $\phi_t < 1$, that is, the price of oil cannot be too high.

4.2.2. *The Age of Fossil Fuels*

During this phase, fossil fuels provide all the energy needs of the world economy. Along the equilibrium path the rate of return to capital must be greater than or equal to the rate of return to oil investment. If oil is the only source of energy used in the production of the consumption good in a period, say period $t + 1$, then $\phi_{t+1} < \rho_{t+1}$ and the two rates of return are equal. Hence, the following condition satisfies:

$$(82) \quad \phi_{t+1} = (1 + \rho_{t+1})\phi_t < \rho_{t+1}$$

(81) and (82) show that the price of oil cannot be too high during the first and the second phases of the competitive equilibrium. Furthermore, the price of oil has risen steadily at the rate of interest to and then has exceeded the level of the rental rate of capital when the oil stock has been depleted.

5. NUMERICAL EXAMPLES

In the numerical examples, the following values for the parameters of the model are assumed:

$$I = 2; A = 4; \alpha = 0.25; \beta = 0.65; \varepsilon = 0.05; \delta = 0.8; \\ \gamma_1 = 0.25; \gamma_2 = 0.75; n = 0.01; \underline{H} = 0.01; H^\# = 0.25$$

A numerical illustration of a competitive equilibrium consisting of three phases with complete oil exhaustion is presented in the following table:

TABLE I

A Competitive Equilibrium Consisting of Three Phases with Complete Oil Exhaustion

Period	X_t	K_t	$N_{1,t}^0$	$N_{2,t}^0$	H_t	ϕ_t	ρ_t	$\omega_{1,t}$	$\omega_{2,t}$	$\kappa_{1,t}$	$\kappa_{2,t}$
0	0.879	0.014	0.98	1.225	0.01	0.376	38.7	0.64	0.64		
1	0.299	0.277	0.99	1.238	0.59	0.603	0.603	1.2	1.1		
2	0.11	1.07	1	1.25	0.75	1.518	1.518	1.64	1.35	0.5	0.455
3	0	1.48	1.01	1.263	0.82	3.445	1.269	1.75	1.36	0.721	0.595
4	0	1.548	1.02	1.275	0.78	7.811	1.267	1.78	1.44	0.77	0.598
...
100	0	5.314	2.65	3.314	0.01	8.37×10^{33}	1.224	2.03	2.03	0.891	0.891
101	0	5.371	2.68	3.348	0.01	1.86×10^{34}	1.224	2.03	2.03	0.891	0.891

A numerical example of a competitive equilibrium without the phase of technology substitution with complete oil exhaustion is illustrated in the following table:

TABLE II

A Competitive Equilibrium without the Phase of Technology Substitution with Complete Oil Exhaustion

Period	X_t	K_t	$N_{1,t}^0$	$N_{2,t}^0$	H_t	ϕ_t	ρ_t	$\omega_{1,t}$	$\omega_{2,t}$	$\kappa_{1,t}$	$\kappa_{2,t}$
0	0.91	1.56×10^{-6}	0.98	1.23	0.01	0.053	34688	0.064	0.064		
1	0.5	0.07	0.99	1.24	0.42	0.64	11.16	0.95	0.93		
2	0	1.25	1	1.25	0.9	1.5	1.34	1.64	1.18	0.6	0.52
3	0	1.38	1.01	1.26	0.86	3.45	1.3	1.7	1.28	0.72	0.521
4	0	1.49	1.02	1.28	0.81	7.86	1.28	1.75	1.37	0.75	0.565
...
100	0	5.31	2.65	3.31	0.01	8.57×10^{33}	1.224	2.03	2.03	0.891	0.891
101	0	5.37	2.68	3.35	0.01	1.91×10^{34}	1.224	2.03	2.03	0.891	0.891

Note that the initial distribution of wealth (global capital stock and global oil stock) among the old generations of period 0 only affects the welfare of the old generations of period 0. A redistribution of capital and oil between the old generations of the two countries in period 0 has no impact on the equilibrium price system, and a fortiori no impact on the welfare of generations of period 1, 2, 3, Moreover, the price of oil is monotonically increasing while the convergence of rental rate of capital is not monotonic.

In the above numerical simulation, each period represents a period of 50 years. As we can see the interest rate is quite large and volatile initially and fossil fuels extraction stops after 2-3 periods. These results are not puzzling when $\sum_{i \in I} X_{i,0}$, the world's initial stock of fossil fuels, is much greater than $\sum_{i \in I} K_{i,0}$, the world's initial capital stock. If the world's initial stock of fossil fuels were not much greater than the world's initial capital stock the burning of a small stock of fossil fuels would have a negligible impact on the climate and then we do not need to study the climate change induced by the burning of fossil fuels. Intuitively, we expect that if the world's initial stock of fossil fuels is large, but the world's initial capital stock is not, then the price of oil is much lower than the interest rate (the rental rate of capital). With a scarcity of capital, the interest rate would go to infinity. Consequently, the oil input into the production of the consumption good will be large, and the backstop sector in each country will not be active. However, in contrast to an increasing trend of capital stock due to R&D with increasing returns to scales in production, non-renewable stock of fossil fuels is decreasing in its size and becoming more and more insufficient to meet our demand. Therefore, the interest rate drops drastically after some period. When the price of oil has risen to the level of the interest rate the backstop will be brought into use. Table 1 shows that the transition phase in which both fossil fuels and backstop technologies co-exist arises in period 1 and period 2 with $\rho_{t+1} = \phi_{t+1} = \varphi_{t+1}$, $t = 0,1$ and the individual will be indifferent between oil and backstop capital. In Table 2, results show that this phase does not arise and the interest rate has dropped below the price of fossil fuels since period 2.

6. CONCLUSION

In this paper, we have formulated and analyzed a multi-country model of economic growth in which the production technology in each country is negatively affected by the impact of climate change. Furthermore, in contrast with most theoretical models in the literature on climate change, ours contains an explicit modeling of the exploitation of the world's stock of fossils. A backstop provides a substitute for oil, and allows the world economies to evolve along a path of sustainable development in the long run – after the stock of fossil fuels has been depleted or part of the stock of fossil fuels is left in situ forever unexploited.

As a modeling strategy, we have chosen to formalize our ideas under the framework of an overlapping-generations model. The adoption of the overlapping-generations framework offers several advantages over the representative agent approach of the Ramsey-Solow tradition. First, it depicts in a more realistic manner how agents with finite lifetime make their decisions in a decentralized setting. Second, unlike the representative agent approach in which intergenerational equality concerns and welfare comparison cannot be addressed, the overlapping-generations model allows for an explicit analysis of these issues. Another advantage of the overlapping-generations model over the representative agent approach is that unexpected results might occur under the former approach, but not under the latter approach. An example of such unexpected results is the possible incomplete depletion of the stock of fossil fuels under the overlapping-generations approach, a scenario which cannot occur when the problem is formulated under the perspective of a central planner.

The model we have formulated depicts the status quo in the global environment debate, and it can serve as the baseline for negotiations on a global environment treaty. The next step is to explain how a self-enforcing international environmental treaty should be negotiated and examine the impacts of environment policies on economic growth.

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CHAPTER TWO

THE RESTRUCTURING OF THE ELECTRICITY SECTOR AND THE ROLE OF THE EMERGENT INNOVATIVE TECHNOLOGIES IN CLIMATE CHANGE POLICIES

1. INTRODUCTION

Fundamental changes in the electricity sector have taken place in many countries for over a decade. These changes are driven by the technological progress in generation technologies, which make the competition among electricity producers possible. Another driving force behind the restructuring of the electricity sector in many countries is the imperative of lowering greenhouse gases (GHG) emissions generated by coal-fired electric generating plants. The United Kingdom restructured and privatized its electricity industry in 1990. In Australia, the restructuring of the electricity sector began in 1991, and the privatization of the state-owned industry has been taking place across the country since 1995. With the passage of The Energy Policy Act in 1992, restructuring in the electricity industry has spread across the United States since the late 1990s. Restructuring took place in states with the highest retail electricity prices, such as California, Massachusetts, Rhode Island, New York, New Jersey, Maine and Pennsylvania.¹

Before its restructuring, the electricity sector in a country was made up of vertically integrated utilities either under government ownership or subject to public regulation. Under monopoly regulation, the electric utility industry had little incentive to take advantage of technological advances. Much of the motivation behind the restructuring effort is to permit competition from more energy-efficient technologies, especially gas turbines, which promotes generation at lower costs. Under the restructuring, the vertically

¹ See Joskow (2003).

linked structures of electric utilities were broken up into separate and independent components. Generation, transmission, and distribution are no longer under the control of a single firm, and the reform led to the establishment of many competing generators and retailers in the market. The reform left intact the transmission and distribution sectors because these sectors are considered to be natural monopolies. The transmission systems in the UK, US, and Australia are all regulated. To ensure open access to electric producers and purchasers, the transmission system is often run by an Independent System Operator (ISO) or an Independent Market Operator (IMO). The rates are regulated and set at an appropriate level that allows investors to recover the costs of infrastructure investments and earn an acceptable rate of return.

Electricity has certain economic and physical characteristics that determine the processes by which it is traded. Firstly, because it cannot be economically stored for future use, electricity must be instantaneously produced to satisfy demand at any time. Secondly, as it is not possible to distinguish one unit of electricity from another, determining which generator produces which particular unit of electricity is not feasible. These characteristics mean that electricity is an ideal commodity to be traded using pool arrangements. Therefore, the establishment of a wholesale electricity market was another important area in the reform of the electricity sector. The IMO also manages and operates the market.

The accumulation of greenhouse gases is likely to lead to global warming and other significant climatic changes. Climate change is global in its causes and consequences, and is currently a serious threat to the global environment. Its impact on the environment, human health, and productivity are now well understood, and is becoming increasingly problematic. Therefore, the actions on climate change require broad international co-operation, and the economic analysis must be global and deal with long time horizons. To stabilize the greenhouse gas concentration in the atmosphere, the United Nations Framework Convention on Climate Change (UNFCCC)² has set out the objective of

² An international environmental treaty was produced at the United Nations Conference on Environment and Development in June 1992 in Rio de Janeiro

reducing global greenhouse gas emissions to a certain target level.³ The Kyoto Protocol⁴ – a global environment treaty initiated by the UNFCCC – establishes legally binding commitments for the reduction of GHG's by an average of five per cent against 1990 levels over the five-year period 2008-2012.⁵ Under the agreement, nations that emit less than their quotas will be able to sell emissions credits to nations that exceed their quotas.⁶

Most electricity today is generated by burning fossil fuels. Fossil fueled power stations are major emitters of GHG. The restructuring of the electricity industry has been discussed by many economists, and questions arise when concerns about climate change should also be taken into account. Papadopoulos (2001) discussed the case in Europe, and pointed out that deregulation of the electricity markets can lead to higher efficiencies, but is not always favourable in terms of emissions unless the GHG emissions are regulated. The power generation sector is therefore the main focus of a nation's efforts to reduce its greenhouse gas emissions. In response to the Kyoto emission objectives, The European Union has implemented the European Union emission trading scheme (EU ETS), which caps the overall level of emissions, but, within that limit, allows participants to buy and sell emissions allowances as they require, so as to cut emissions cost-effectively. The scheme covers all power plants with capacity of over 20 MW thermal inputs. Along with emission trading program, the EU pursues a substantial increase in the share of renewable energy with the target of 20% by 2020. Although the United States have not ratified the Kyoto Protocol, the Obama government's New Energy for America plans to invest in renewable energy to meet the twin challenges of energy security and climate change. The plan has for renewable energy a target of 10 percent share of the United States' electricity by 2012, and 25 percent by 2025.⁷ Australia ratified the Kyoto Protocol in December 2007, and released The Carbon Pollution Reduction Scheme White Paper in December

³ Article 2, UNFCCC

⁴ The Kyoto Protocol was initially adopted on 11 December 1997 in Kyoto, Japan and entered into force on 16 February 2005.

⁵ Kyoto Protocol, UNFCCC

⁶ Emissions Trading, UNFCCC

⁷ http://change.gov/agenda/energy_and_environment_agenda/

2008 together with a commitment of the national Renewable Energy Target (RET) of 20% share of electricity supply in Australia by 2020.⁸

Before restructuring, electric utilities were required to provide universal service at a fixed price, regardless of the true cost of service. Public utilities were also required to act as the carrier of last resort; or to adopt production processes mandated by regulators that are not cost efficient, but serve other social objectives, such as use of more costly renewable energy sources. In return for these commitments, regulators allowed electric utilities to earn a reasonable rate of return on their investments. Such an arrangement is known as a regulatory contract. Although no formal contract exists between public utilities and regulators, a regulated firm expects that it would be able to recover at least its investments, based on the representations made by regulators.

To establish a network, power generators must make substantial market specific investments which are known as sunk costs. A power station investment appears to be locked into a technology over 40 or 50 years, and changes in the capital stock occur slowly. This allows for long-term repayment of debt and effective utilization of the asset. A potential entrant must anticipate earnings before it attempts to enter the market, so that sunk costs would be recovered. The benefits that competition brings to the market of electricity include improvements in operating efficiencies, competitive prices, efficient investment decisions, technological innovation, and product variety. However, in opening regulated markets to competition, regulators would reduce the earnings of incumbent utilities. The capital equipments and other facilities of these utilities may not be suited to the changing requirements of competitive markets. Those changes in regulatory policy can reduce the regulated firm's net revenues and deny its investors the opportunity to earn a fair rate of return on the investments made under the previous regulatory regime. That inability of utility shareholders to secure the expected returns on their investment gives rise to the condition known as *stranded costs*. Therefore, the regulatory treatment of stranded costs has important distributional consequences, and affects the incentives for the utility to make future investments in highly specific assets.

⁸ Australia's Renewable Energy Target, Department of Climate Change

In that respect, neglect of cost recovery would profoundly affect the outcome of the repeated game between investors and regulators. Gorte, Kaarsberg and Skip Laitner (1999) also raised this question, but left it unsolved. Sidak and Spulber (1998) discussed the measurement of stranded costs in only a single period and stated the difficulties of the calculation when the assets of the deregulated utility have long lives. In this paper, we propose an expression for computing them in dynamic time. We define stranded cost as the appropriate damage measure corresponding to the difference between the present value of the stream of profits – with the stream beginning at the time of restructuring – of the firm under regulation and under the post-restructuring scenario. In the new competitive environment, the incumbent generators should have an opportunity to achieve for its investors the expected earnings associated with the former regulatory regime under which heavy investments in long-lived facilities and other specialized assets were made.

The generation sector uses a variety of fuel sources to produce electricity. Coal-fired technologies have been the dominant technology for electricity generation for more than fifty years. An electric generating plant that uses brown coal as fuel has a thermal efficiency of about 0.28, i.e., only 28% of the energy content of the fuel input is transformed into electricity. If the plant uses black coal, then its thermal efficiency rises to 37.6%. Coal-fired technologies are also highly polluting. One MWh of electricity produced by an electric generating plant that uses brown coal as fuel emits 1.20 tonnes of carbon dioxide equivalents. The number for an electric generating plant that uses black coal is 0.86 tonnes of carbon dioxide equivalents.

For an electric generating plant that uses the natural gas simple cycle technology, the thermal efficiency factor is 38% and the greenhouse gas emissions in carbon dioxide equivalents per MWh of electricity produced is 0.49 tonnes. For the natural gas combined cycle technology, the thermal efficiency is 53.4%, and the amount of greenhouse gas emissions in carbon dioxide equivalents per MWh of electricity produced stands at 0.35 tonnes of carbon dioxide equivalents.⁹ Because of their recent improvements in capital and operating costs and ultra-low emissions, electric power plants that use the natural gas

⁹ The number is computed according to the greenhouse gas calculator of NEMSIM.

combined cycle technology will be the generating plants of the future. Hydro-electric, solar electric, and wind generation still account for a small market share of electricity sector because outputs depend on prevailing weather conditions.

The reason why coal-fired technologies are still the dominant technologies for generating electricity is that coal is the least expensive electric generation fuel, at least for old plants anyway. Moreover, the adoption of new technologies, together with the changes required in the energy resource mix for the electricity industry, with low or near-net-zero greenhouse gas emissions takes time. Therefore, it does not allow for a rapid change in the mix of energy resources without a stranding of power station assets and the ensuing financial loss. If significant changes are required in the energy resource mix for the electricity industry, actions must be taken that take into account the resulting stranded assets. In the United States and the United Kingdom, stranded costs revolve around the thorny problem of how to pay for the already incurred costs of the construction of nuclear power plants that are no longer desired by consumers. This problem was largely unimportant in Australia when competition was introduced in the Australian electricity industry in 1990. The reason is that Australia has a very competitive coal industry, and has never developed a nuclear power industry.

However, the policy is beginning to change with the publication of the Garnaut Report; the Carbon Pollution Reduction Scheme White Paper; and the announcement of a national Mandatory Renewable Energy Target (MRET) of 20% share of total electricity supply in Australia by 2020. The question of climate change has moved from a purely scientific matter to a real threat for the prosperity of the Australian economy, and has serious practical consequences for governments, incumbent generators, and potential entrants in the electricity sector. Potential investors need to add to their considerations the risk associated with the uncertainty involved in climate change policy when making decision on investments in long-lived and sector-specific assets. As for incumbent generators – especially old coal-fired generators – they now face the prospect of being unable to achieve the earnings expected of the coal-fired power plants constructed under the previous regulatory regime, when other types of electricity generation would have

been cheaper if they had known that carbon emissions reduction would be committed by the government. In their submission to the Garnaut climate change review, generators have warned that Australia will face problems with price and reliability caused by the breakdown of the national electricity market triggered by the departure of big coal-fired power stations. Consequently, the outcome depends a great deal on how the restructuring process together with other complementary policies, are actually carried out as well as how utilities are allowed to recover stranded costs. Our paper presents a revised expression for measuring stranded costs when the present value of the stream of revenues derived from the sales of the emissions permits given by the government as a form of compensation is taken into account.

Emergent innovative technologies – wind, biomass, geothermal, and photovoltaic – are small-scale renewable energy technologies. They emit no greenhouse gases, and once installed use no fuels. The main drawback of emergent innovative technologies is their high capital cost. With the passage of time, R&D efforts will reduce the capital costs of these technologies substantially and enable them to be more competitive with the fossil fuels technologies. At the present time, the damages to the global environment caused by greenhouse gas emissions from fossil fuels technologies are not reflected in the price of electricity. If the price of electricity represents the true social cost of its generation, then the emergent innovative technologies might be on an equal footing with the fossil fuels technologies.

Joskow (1998a) discussed the effects of the restructuring of the electricity sector on the environment. Palmer and Burtraw (2005) pointed out that policies other than electricity restructuring will play a larger role in influencing the emissions from this sector. Naughten (2003) carried out an economic assessment of natural gas combined cycle generation in Australia to evaluate its competition position to coal-fired steam turbines. The paper, however, was silent on the important role of natural gas technologies in reducing greenhouse gas emissions. The main goal of our paper is to incorporate climate change policies – cap and trade, and renewable energy target policies; together with electricity sector reform, and technological change, to analyze the role of innovative

generation technologies in reducing greenhouse gas emissions as well as their impacts on the generation output and on the price of electricity. Our paper contributes to the study on the effects of combining black (GHG emissions) and green (renewables) quotas. A theoretical analysis is provided to answer the current policy question: “What is the impact of the overlapping regulation through both renewable energy and GHG emissions quotas?”. Böhringer and Rosendahl (2010) considered a static, partial equilibrium model of a closed power market, and showed that a green quota imposed on top of a black quota in energy markets promotes power production from the dirtiest technologies. Our paper, on the other hand, builds a dynamic theoretical model with some R&D efforts are made. The results are thus different with theirs. We show that, in the presence of caps on carbon emissions, the entry of advanced generation technologies constrains the output and *a fortiori* the greenhouse gas emissions of the less advanced technologies.

Furthermore, the Australian government’s leading think tank, the Productivity Commission, claimed the MRET would push up energy prices and would do nothing to cut greenhouse gas emissions. It has questioned the efficiency of the proposed MRET in parallel with the emissions trading scheme.¹⁰ Our analysis suggests that the price of electricity is strictly rising before emergent innovative firms with zero greenhouse gas emissions enter the market, but is strictly declining as the entry begins. Although further analysis is required to examine the effect of the proposed climate change policies on the national greenhouse gas emissions, we assert a decline in the greenhouse gas emissions from the electricity sector while its output rises. The paper also discusses the impact of the government subsidy on the exact time entry begins and the total capacity installed by the emergent innovative firms.

In the literature on an electricity market with imperfect competition, three approaches are used to model the behavior of the oligopolistic firms: competition à la Cournot, competition à la Bertrand, and supply functions. Proponents of the Bertrand model of competition argue that because electricity is not storable, price competition is intense in

¹⁰ Kevin Rudd's energy strategy 'flawed' says Productivity Commission , The Australian, May 23, 2008

the short run. However, empirical evidence suggests that prices in some markets with imperfect competition are well above marginal costs, and this contradicts the prediction of the Bertrand model of competition. Proponents of the Cournot model of competition, on the other hand, argue that long-term contracts constitute a large proportion of electricity transactions, and so competition is mostly in quantities. In addition, the Cournot model of competition can be recast as a two-stage game. In the first stage of the game, firms choose production capacities, and in the second stage they compete in price as in the Bertrand competition (Hildebrand, 2009). Yet, others have argued that models that use supply functions are more appropriate in describing oligopolistic behavior. In models with supply functions, generators submit bids under the form of a pair of price and quantity supplied at the bid price every half hour. However, the equilibrium of a model with imperfect competition is difficult to compute, and this is the main criticism leveled against the supply function approach. For a more extensive discussion of the merits of the three approaches, the reader can consult Batstone (2000). The model we formulate adopts the Cournot model of competition.

The paper is organized as follows. In Section 2, the model is presented. In Section 3, we discuss the issue of stranded costs and present an expression for computing them. Some comparative static results of a change in the price of emissions permits are given in Section 4. The equilibrium price of emissions permits is determined in Section 5. The emergence of innovative generation technologies is formalized and analyzed in Section 6. Some concluding remarks are given in Section 7.

2. THE MODEL

In the model, time is continuous and denoted by t , with $t = 0$ as the present time at which the restructuring of the electricity takes place and at which the policy of cap and trade of greenhouse gas emissions permits is implemented.

2.1. *The Generation Technologies*

In the model, electricity is produced by burning fossil fuels, and four generation technologies are considered: conventional brown coal (technology 1), conventional black coal (technology 2), natural gas simple cycle (technology 3), and natural gas combined cycle (technology 4). Each technology is represented in the market by a single firm, and for each $i = 1, \dots, 4$, the firm that uses technology i is referred to as firm i . Furthermore, firms 1 and 2 are assumed to be the incumbents, while firms 3 and 4 are the entrants that enter the market at time 0.

Each of the technologies produces electricity according to a Leontief technology: More specifically, for each $i = 1, \dots, 4$, one unit of electricity can be produced by using k_i units of capital, f_i units of fuels, and ℓ_i units of labor. In electricity generation, labor is required for operating, repairing, and maintaining electric generating plants. The list (k_i, f_i, ℓ_i) thus represents the production process of generation technology $i, i = 1, \dots, 4$. For an electricity generating plant that uses generation technology i and that has a capital stock of size K_i , its capacity, i.e., the maximum electricity output that it can produce, is thus given by K_i / k_i .

Unlike the other air pollutants that can be scrubbed from smokestacks, the only cost-effective way to reduce carbon emissions is to burn less fossil fuels. Hence the carbon dioxide emissions generated by an electricity generating plant depend on the amount of fossil fuel it burns. For our purpose, we shall assume that the amount of carbon dioxide emissions that are generated by producing one unit of electricity according to generation technology i is assumed to be given by $e_i = \varepsilon_i f_i$, where ε_i is a parameter that represents

the emission factor of the fuel used by this generation technology. Because brown coal is the most polluting fuel, followed successively by black coal, natural gas simple cycle, and natural gas combined cycle, the following chain of inequalities characterizes the ordering of the emissions per unit of output for the technologies considered in this paper:

$$e_1 > e_2 > e_3 > e_4.$$

For each $i = 1, 2$, let \bar{E}_{it} denote the number of greenhouse gas emissions permits given to firm i – as compensation for its stranded costs – at time $t, t \geq 0$, where $\bar{E}_{it} = 0$ for $t \geq \theta$, with $\theta > 0$ being the time at which the compensation is terminated. The entrants, on the other hand, do not qualify for the benefits of the grandfather clause, and thus do not receive any greenhouse gas emissions permits. Thus, we set $\bar{E}_{it} = 0, i = 3, 4, t \geq 0$.

Let q_{it} and ω_t denote, respectively, the price of the fuel used in the production of electricity by generation technology i and the wage rate – both at time t . Also, let ϕ_t denote the price of a greenhouse gas emissions permit at time t . The unit variable cost of producing electricity at time t by generation technology i is then given by

$$(1) \quad c_{it} = q_{it}f_i + \omega_t\ell_i + \phi_t e_i, \quad (i = 1, \dots, 4).$$

2.2. The Regulatory Contract

Suppose that at some time $\tau < 0$ in the past firms 1 and 2 each signed a regulatory contract to supply electricity during a time interval of length T . Furthermore, suppose that at each instant $t, \tau \leq t \leq \tau + T$, during the contract, the producer must supply an electricity output level \bar{X}_{it} at the regulated price of \bar{p}_t . For each $i = 1, 2$, the capital investment that the i^{th} electricity producer must make at time τ to fulfill its obligations under the regulatory contract is $K_i = k_i \max_{\tau \leq t \leq \tau + T} \bar{X}_{it}$, and the following participation constraint must be satisfied to induce the electricity producer into signing the contract.

$$(2) \quad \int_{\tau}^{\tau+T} e^{-r(t-\tau)} (\bar{p}_t - c_{it}) \bar{X}_{it} dt - K_i \geq 0.$$

2.3. The Post-Restructuring Equilibrium

Suppose that the inverse market demand for electricity at each instant $t \geq 0$ is linear and given by $p = a_t - b_t X$, where p is the price and X is the market demand at price p . Also, a_t and b_t are two positive parameters. Let $X_{it}, i = 1, \dots, 4, t \geq 0$, denote the electricity output of firm i . Under the combination of strategies (X_{1t}, \dots, X_{4t}) , the profit made by firm i is

$$(3) \quad \Pi_{it}[(X_{jt})_{j=1}^4] + \phi_t \bar{E}_{it}, \quad (i = 1, \dots, 4),$$

where we have let

$$(4) \quad \Pi_{it}[(X_{jt})_{j=1}^4] = \left(a_t - b_t \sum_{j=1}^4 X_{jt} - c_{it} \right) X_{it}$$

denote the profit earned in the electricity market by firm $i, i = 1, \dots, 4$, at time t .

The first-order condition that characterizes the best response of each firm at time $t, t \geq 0$, is

$$(5) \quad \frac{\partial \Pi_{it}[(X_{jt})_{j=1}^4]}{\partial X_{it}} = 0, \quad (i = 1, \dots, 4).$$

We shall let $X_{it}^*, i = 1, \dots, 4$, denote the solution of the system of best responses constituted by (4). It is simple to show that

$$(6) \quad X_{it}^* = \frac{1}{5b_t} \left(a_t - 4c_{it} + \sum_{j \neq i} c_{jt} \right), \quad (i = 1, \dots, 4).$$

As can be seen from (6), the output of each firm depends on the demand parameters a and b , on its own variable cost, and on the variable costs (which include the price of emissions permits) of all the other firms. Furthermore, the output of each incumbent firm is independent of the emissions permits given by the government. The emissions permits

that the government gives to the incumbent generating plants are just windfalls; they only affect the overall profits of these firms without influencing their production plans.

The industry output at each instant after the restructuring is

$$(7) \quad \sum_{i=1}^4 X_{it}^* = \frac{1}{5b_t} \left(4a_t - \sum_{i=1}^4 c_{it} \right)$$

The total volume of greenhouse gas emissions of the electricity sector is

$$(8) \quad \begin{aligned} E_t^0[\phi_t] &= \sum_{j=1}^4 e_j X_{jt}^* \\ &= \frac{1}{5b_t} \sum_{j=1}^4 e_j \left(a_t - 4c_{it} + \sum_{j \neq i} c_{jt} \right). \end{aligned}$$

3. STRANDED COSTS

When the regulators breached the regulatory contract by allowing new firms to enter the market, the difference between the present value of the stream of profits that an incumbent firm would earn – counting from the time the restructuring begins – and the present value of the stream of profits it actually earn (the revenues from the sales of emissions permits given as compensation by the government not included in the calculations) represents the firm's stranded cost. More precisely, the stranded cost of an incumbent electricity generator is given by

$$(9) \quad \int_0^{\tau+T} e^{-rt} (\bar{p}_t - c_{it}) \bar{X}_{it} dt - \int_0^{\tau+T} e^{-rt} \Pi_{it} [(X_{jt}^*)_{j=1}^4] dt, \quad (i=1,2).$$

When the present value of the stream of revenues derived from the sales of the emissions permits given by the government is taken into account, the actual stranded cost incurred by an incumbent firm is

$$(10) \quad \int_0^{\tau+T} e^{-rt} (\bar{p}_t - c_{it}) \bar{X}_{it} dt - \int_0^{\tau+T} e^{-rt} \Pi_{it} [(X_{jt}^*)_{j=1}^4] dt - \int_0^{\tau+T} e^{-rt} \phi_t \bar{E}_{it} dt, \quad (i=1,2).$$

Given that the incumbent electricity generators only receive free emissions permits for 5 years as proposed in the case of Australia, it is questionable that this form of compensation will lower the effective stranded cost, as represented by (10), substantially.

4. COMPARATIVE STATICS

An increase in the price of emissions permits raises the variable cost of every firm, and this means, according to (7), that industry output of electricity will decline. To find the effect on the total greenhouse gas emissions of the electricity industry, differentiate (8) with respect to ϕ_i to obtain

$$(11) \quad \frac{\partial E_t^0[\phi_i]}{\partial \phi_i} = -\frac{1}{5b_i} \left(\begin{array}{l} 4e_1^2 + 4e_2^2 + 4e_3^2 + 4e_4^2 \\ -2e_1e_2 - 2e_1e_3 - 2e_1e_4 - 2e_2e_3 - 2e_2e_4 - 2e_3e_4 \end{array} \right)$$

$$= -\frac{1}{5b_i} \left(\begin{array}{l} (e_1 - e_2)^2 + (e_1 - e_3)^2 + (e_1 - e_4)^2 \\ + (e_2 - e_3)^2 + (e_2 - e_4)^2 + (e_3 - e_4)^2 \\ + e_1^2 + e_2^2 + e_3^2 + e_4^2 \end{array} \right) < 0.$$

Although the effects of a rise of the price of emissions permits at the industry level are as expected – a decline in industry output and a reduction in the greenhouse gas emissions of the whole industry – the results are a little surprising at the firm level. To find the effect of a rise in the price of emissions permits on the output of a firm, differentiate (6) with respect to ϕ_i to obtain the following result:

$$(12) \quad \frac{\partial X_{it}^*}{\partial \phi_i} = -\frac{1}{5b_i} \left(4e_i - \sum_{j \neq i} e_j \right), \quad (i = 1, \dots, 4).$$

To determine the sign of (12), recall that $e_1 > e_2 > e_3 > e_4$. Hence the output of firm 1, the firm that uses brown coal as the generating fuel, will decline when the price of emissions permits rises. As for the influence of the price of emissions permits on the outputs of the other firms, it is not possible to determine the result from (12) alone, without appealing to the numerical values of $e_1 = 1.202, e_2 = 0.861, e_3 = 0.489, e_4 = 0.348$, which have been

computed with the help of the greenhouse gas calculator. When these numerical values are used, we obtain the following results:

$$(13) \quad \begin{aligned} \frac{\partial X_{1t}^*}{\partial \phi_t} &= -\frac{0.62}{5b_t} < 0, \\ \frac{\partial X_{2t}^*}{\partial \phi_t} &= -\frac{0.28}{5b_t} < 0, \\ \frac{\partial X_{3t}^*}{\partial \phi_t} &= \frac{0.09}{5b_t} > 0, \\ \frac{\partial X_{4t}^*}{\partial \phi_t} &= \frac{0.23}{5b_t} > 0. \end{aligned}$$

As can be seen from (13), the outputs of firms 1 and 2 and a fortiori their greenhouse gas emissions decline when the price of emissions permits rises. For firms 3 and 4, the opposite results occur. The output of firm 3 as well as the output of firm 4 – and a fortiori their greenhouse gas emissions – both rise with the price of emissions permits. The explanation for the result at the firm level is rather subtle. A rise in the price of emissions permits affects the more polluting firms – the firm using brown coal and the firm using black coal – more than the firms using natural gas, and thus puts them in a more disadvantageous position relative to the latter firms when the price of emissions permits rises. The natural gas technologies thus can compete more effectively with the coal-fired technologies, and this is why their outputs both rise when the price of emissions permits rises. Also, it can be seen from the four inequalities in (13) that the impact of a rise in the price of emissions permits is more pronounced for the more polluting technology. More specifically, the fall in output of the firm using brown coal is more pronounced than that of the firm using black coal. As for the natural gas technologies, which are less polluting, more electricity is produced when the price of emissions permits rises. In particular, a rise in the price of emissions permits has a stronger impact on the natural gas combined cycle technology than the natural gas simple cycle because the former technology is more efficient and less polluting than the latter technology.

We summarize the results just obtained in the following proposition:

PROPOSITION 1: *Ceteris paribus, a rise in the price of emissions permits has the following impacts:*

(i) *At the industry level, the electricity output and the greenhouse gas emissions both decline.*

(ii) *At the firm level, the outputs and a fortiori the greenhouse gas emissions of the firms using the coal-fired technologies both decline, and the impact of the rise in the price of emissions permits is more pronounced on the brown coal technology (the most polluting technology) than the black coal technology. As for the natural gas technologies, their outputs as well as their greenhouse gas emissions both rise, and the impact is stronger for the natural gas combined cycle, the most efficient and least polluting technology.*

(iii) *Although the rise in the price of emissions permits induces a rise in the outputs and GHG emissions of the two natural gas technologies, these impacts are more than offset by the reduction in outputs and in greenhouse gas emissions of the two coal-fired technologies. The net impact is a decline in both industry output and in the total volume of the greenhouse gas emissions emanating from the electricity sector.*

The following proposition gives the comparative static results of a rise in the thermal efficiency of a generation technology, say the natural gas combined cycle, that is the fruit of an R&D program.

PROPOSITION 2: *Suppose that R&D efforts result in a rise in the thermal efficiency of the natural gas combined cycle. More specifically, suppose that the improved technology now can produce the same electricity output using less fuel. The impact on the industry of the rise in thermal efficiency of the natural gas combined cycle can be described as follows:*

(i) *The output of all the other technologies and a fortiori their greenhouse gas emissions all decline. As for the natural gas combined cycle technology, its output and its greenhouse gas emissions both rise with the technological improvement.*

(ii) *At the industry level, total output rises, but total greenhouse gas emissions fall.*

PROOF: (i) Differentiating (6) with respect to f_4 , we obtain

$$(14) \quad \frac{\partial X_{it}^*}{\partial f_4} = \frac{1}{5b_t}(q_4 + \varepsilon_4\phi_t) > 0, \quad (i = 1,2,3),$$

$$(15) \quad \frac{\partial X_{4t}^*}{\partial f_4} = -\frac{4}{5b_t}(q_4 + \varepsilon_4\phi_t) < 0.$$

It follows from (14) that a decline in f_4 will induce a fall in the output and a fortiori the greenhouse gas emissions of each of the other technologies. It follows from (15) that a rise in the thermal efficiency of the natural gas combined cycle induces a rise in its output. To obtain the impact on the greenhouse gas emissions of a rise in the thermal efficiency of this technology, differentiate $e_4X_{4t}^*$ with respect to f_4 to obtain

$$(16) \quad \frac{\partial}{\partial f_4} e_4X_{4t}^* = -\frac{4e_4}{5b_t}(q_4 + \varepsilon_4\phi_t) < 0.$$

It follows from (16) that an improvement in the thermal efficiency of the natural gas combined cycle induces a rise in its greenhouse gas emissions.

(ii) The impact on the industry output of the rise in the thermal efficiency of the natural gas combined cycle can be obtained by summing $\frac{\partial X_{it}^*}{\partial f_4}$ over $i = 1, \dots, 4$. The result is

$$(17) \quad \sum_{i=1}^4 \frac{\partial X_{it}^*}{\partial f_4} = -\frac{1}{5b_t}(q_4 + \varepsilon_4\phi_t) < 0,$$

which implies that industry output rises with a rise in the thermal efficiency of the natural gas combined cycle. The impact on the total greenhouse gas emissions of the industry that is induced by a rise in the thermal efficiency of the natural gas combined cycle can be obtained as follows.

$$\begin{aligned}
 (18) \quad \sum_{i=1}^4 \frac{\partial}{\partial f_4} e_i X_{it}^* &= \frac{1}{5b_t} (e_1 + e_2 + e_3 - 4e_4)(q_4 + \varepsilon_4 \phi_t) \\
 &= \frac{0.232204}{b_t} (q_4 + \varepsilon_4 \phi_t) > 0,
 \end{aligned}$$

from which it follows that a rise in the thermal efficiency of the cleanest technology currently available induces a fall in the industry's total greenhouse gas emissions. ■

5. THE EQUILIBRIUM PRICE OF EMISSIONS PERMITS

Greenhouse gas emissions permits are also demanded by the other sectors of the economy, but we shall not model this demand, and simply assume that at each instant $t, t \geq 0$, the demand for greenhouse gas emissions permits outside the electricity sector, say $E_t^1[\phi_t]$, is a decreasing function of the price of permits. This assumption can certainly be justified if we are willing to assume that all the sectors other than the electricity sector are perfectly competitive. The global demand for greenhouse gas emissions permits at each instant $t, t \geq 0$, is then given by $E_t[\phi_t] = E_t^0[\phi_t] + E_t^1[\phi_t]$. The following market-clearing condition represents the equilibrium in the market of greenhouse gas emissions permits at each instant:

$$(19) \quad E_t[\phi_t] = \bar{E}_t,$$

where \bar{E}_t is the number of greenhouse gas emissions permits issued by the government at time $t, t \geq 0$. Because the global demand for emissions permits is downward-sloping, the market-clearing condition (19) determines the price of emissions permits uniquely.

Intuitively, one might think that as more emissions permits are allocated to coal-fired electricity generators, they might produce more electricity output, and thus emit more greenhouse gases. However, we have already shown in Sub-section 2.3 that the output of each firm and a fortiori its greenhouse gas emissions depend only on the price of emissions permits, not on the number of emissions permits allocated by the government. Hence, the price of emissions permits is completely determined by their supply, i.e., by

the total number of emissions permits issued by the government, and how part of the emissions permits is allocated among firms in the economy is immaterial as far as climate change policy is concerned. Of course, how emissions permits are allocated among firms in the economy have distributional consequences as far as the firms that are given these emissions permits consider them as windfalls.

6. EMERGENT INNOVATIVE TECHNOLOGIES

6.1. *The Perfect-Foresight Equilibrium*

Suppose that at some time in the future a number of firms – each using an emergent innovative generation technology – enter the market. Because the scale of an emergent innovative technology is small, each of these firms has no market power. Together, the new entrants constitute the competitive fringe of the market. In what follows, we treat the number of emergent innovative firms as a continuous variable, and assume that if an emergent innovative firm chooses to enter the market, then it installs exactly one unit of capacity. This assumption means that at any instant the number of the emergent innovative firms that have entered the market is equal to the total capacity already installed by the competitive fringe.

Let γ_s be the cost of one unit of capacity that an emergent innovative firm must incur to enter the market at time $s, s \geq 0$. To reflect the improvement through time of the emergent innovative technologies, we shall assume that γ_s is a decreasing and convex function of s . More specifically, we assume that $d\gamma/ds < 0, d^2\gamma/ds^2 > 0$, and $\lim_{s \rightarrow \infty} \gamma_s = \bar{\gamma} > 0$. We shall also assume that in its efforts to encourage the development of renewable energy sources, the government chooses to subsidize new sources of renewable energy at a specific rate of σ per unit of output.

Let $(p_t)_{t \geq 0}$ be the time path of the price of electricity anticipated by the emergent innovative firm. By an entry process we mean an ordered pair $(s, (K_{st})_{t \geq 0})$, where s represents the exact time that emergent innovative firms begin to enter the market and $K_{st}, t \geq 0$, represents the number of emergent innovative firms present on the market at time t . The curve $t \rightarrow K_{st}, t \geq 0$, is assumed to be continuous, non-decreasing, and satisfies the following conditions:

$$(20) \quad \begin{aligned} K_{st} &= 0, t \leq s, \\ &> 0, t > s. \end{aligned}$$

Presumably, we expect K_{st} to be strictly increasing from 0 at $t = s$, and then slopes upward as more and more emergent innovative firms enter the market.

The present value of the stream of profits earned by an emergent innovative firm that enters the market at time $h, h \geq s$, is

$$(21) \quad \int_h^{\infty} e^{-rt} (p_t + \sigma) dt - e^{-rh} \gamma_h.$$

Entry by emergent innovative firms will drive the present value of the stream of profits that each of them earns down to 0. Thus, if the emergent innovative firms anticipate correctly the time path of the price of electricity, then the following zero-profit condition must hold for the competitive fringe:

$$(22) \quad \begin{aligned} \int_h^{\infty} e^{-rt} (p_t + \sigma) dt - e^{-rh} \gamma_h &< 0, h < s, \\ &= 0, h \geq s. \end{aligned}$$

If $K_{st}, t \geq 0$, is the number of emergent innovative firms in the market at time t , then the inverse residual market demand shared by all the fossil fuel technologies at that instant is

$$(23) \quad p_t = a_t - b_t \left(\sum_{i=1}^4 X_{it} + K_{st} \right)$$

and the time paths of the outputs of the firms using the fossil fuels technologies at time t are given by

$$(24) \quad X_{it} = \frac{1}{5b_t} (a_t - 4c_{it} + \sum_{j \neq i} c_{jt} - b_t K_{st}), \quad (i = 1, \dots, 4, t \geq 0).$$

The electricity price at each instant is then given by

$$(25) \quad \begin{aligned} p_t &= a_t - b_t \left(\sum_{i=1}^4 \left(\frac{1}{5b_t} (a_t - 4c_{it} + \sum_{j \neq i} c_{jt} - b_t K_{5t}) \right) + K_{5t} \right) \\ &= \frac{1}{5} \left(a_t + \sum_{i=1}^4 c_{it} - b_t K_{5t} \right). \end{aligned}$$

We call $(p_t)_{t \geq 0}$ the time path of the price of electricity induced by $(K_{5t})_{t \geq 0}$.

DEFINITION: A perfect-foresight equilibrium is a list $((s, (K_{5t})_{t \geq 0}), ((X_{it})_{t \geq 0})_{i=1}^4, (p_t)_{t \geq 0})$ with the following properties:

- (i) s is the exact time emergent innovative firms begin to enter the market, and K_{5t} is the total capacity of the competitive fringe at time t . That is K_{5t} satisfies condition (20).
- (ii) X_{it} is the output of firm $i, i = 1, \dots, 4$, at time $t, t \geq 0$, as given by (24).
- (iii) $(p_t)_{t \geq 0}$ is the time path of the electricity price induced by $(K_{5t})_{t \geq 0}$. That is, $p_t, t \geq 0$, is given by (25).
- (iv) The zero-profit condition (22) holds for each time $h \geq 0$.

We shall assume that at the present time the unit capacity cost of the emergent innovative technologies are too high to induce entry by firms using these technologies if renewable energies are not subsidized. However, improvements in these technologies will reduce the unit capacity cost sufficiently at some time in the future to make them competitive with the fossil fuel technologies at some time in the future. Mathematically, these assumptions are captured, respectively, by the following two conditions:

$$(26) \quad \int_0^{\infty} e^{-rt} \frac{1}{5} \left(a_t + \sum_{i=1}^4 c_{it} \right) dt < \gamma_0,$$

and

$$(27) \quad \int_s^{\infty} e^{-rt} \frac{1}{5} \left(a_t + \sum_{i=1}^4 c_{it} \right) dt > \gamma_s,$$

for some time $s > 0$.

To capture the growth in demand, we shall assume that the choke price a_t is rising and the slope b_t of the inverse market demand curve declines over time. Also, we assume that the variable costs of the fossil fuel technologies are non-decreasing over time. The following proposition gives a characterization of the exact time entry occurs and the price of electricity under a perfect-foresight equilibrium.

PROPOSITION 3: *Let $((s, (K_{5t})_{t \geq 0}), ((X_{it})_{t \geq 0})_{i=1}^4, (p_t)_{t \geq 0})$ be a perfect-foresight equilibrium.*

Then the exact time emergent innovative firms begin to enter the market is the value of s that solves the following equation:

$$(28) \quad \frac{1}{5} \left(a_s + \sum_{i=1}^4 c_{is} \right) + \sigma = r\gamma_s - \frac{d\gamma}{ds}.$$

Furthermore, the price of electricity at each instant $t \geq 0$ is given by

$$(29) \quad \begin{aligned} p_t &= \frac{1}{5} \left(a_t + \sum_{i=1}^4 c_{it} \right), t < s, \\ &= r\gamma_s - \frac{d\gamma}{ds} - \sigma, t \geq s. \end{aligned}$$

That is, the price of electricity is strictly rising before entry begins, but is strictly declining as more and more emergent innovative firms enter the market.

PROOF: For any time $h \geq s$, the zero profit condition (22) must hold; that is,

$$(30) \quad \int_h^\infty e^{-rt} (p_t + \sigma) dt - e^{-rh} \gamma_h = 0.$$

Differentiating (30) with respect to h , we obtain

$$-e^{-rh} (p_h + \sigma) + r e^{-rh} \gamma_h - e^{-rh} \frac{d\gamma}{dh} = 0,$$

which can be simplified to

$$p_h + \sigma = r\gamma_h - \frac{d\gamma}{dh}.$$

For $h = t \geq s$, the preceding equation takes on the following form:

$$(31) \quad p_t = r\gamma_t - \frac{d\gamma}{dt} - \sigma,$$

which is (29). Setting $t = s$ in (31), we obtain (28). As can be seen from (28), the price of electricity must be rising before the first emergent innovative firms enter the market because of the rising choke price and the rising costs of the fossil fuel technologies. After entry has started, the price of electricity is given by $r\gamma_s - \frac{d\gamma}{ds} - \sigma, t \geq s$, which is strictly decreasing in t . ■

6.2. Properties of the Perfect-Foresight Equilibrium

PROPOSITION 4: *Suppose that the time path of the emissions permits issued by the government is given. The entry of the representative emergent innovative firm induces a fall in the price of emissions permits at every instant after entry has occurred. Furthermore, the total greenhouse gas emissions from the electricity sector decline while the industry output rises.*

PROOF: For any given level of the price of emissions permits, the output of each of the firms that burn fossil fuels to generate electricity declines. The decline in output in turn implies a decline in greenhouse gas emissions and a fortiori a decline in the demand for emissions permits by each of these firms. That is, the entry of emergent innovative firms induces a downward shift in the demand curve for greenhouse gas emissions by the electricity sector. Given the number of emissions permits issued by the government, and given the demand curve for emissions permits by the rest of the economy, the new equilibrium price of permits will be lower. A lower price of emissions permits induces a rise in their demand by the rest of the economy. Given the number of emissions permits issued at each instant, the rise in the demand for emissions permits by the rest of the economy must be completely offset by the fall in their demand by the electricity sector, and this means a decline in the total greenhouse gas emissions by this sector.

The decline in the price of emissions permits reduces the variable cost of each of the firms generating electricity by burning fossil fuels, lessening somewhat the impact of the entry by emergent innovative firms. The net impact is a rise in the industry output at each instant after entry has occurred. ■

The following proposition gives the impact of the subsidy on the exact time entry begins and the total capacity installed by the emergent innovative firms.

PROPOSITION 5: *An increase in σ , the subsidy for one unit of electricity produced from emergent innovative technologies induces an earlier entry of these technologies. Furthermore, the installed capacity of the competitive fringe is also higher*

PROOF: The exact time that emergent innovative firms begin to enter the market is the

time, say s , at which the curve $t \rightarrow \frac{1}{5} \left(a_t + \sum_{i=1}^4 c_{it} \right), t \geq 0$, and the curve

$t \rightarrow r\gamma_t - \frac{d\gamma}{dt} - \sigma, t \geq 0$, cross each other. Because the former curve is upward-sloping

while the latter curve is downward-sloping, and because entry will occur at some time in the future even without subsidy, these two curves cross each other only once, determining

the exact time at which emergent innovative firms begin to enter the market. A rise in σ shifts the latter curve downward without affecting the former curve, and this induces an

earlier time of entry. Because the latter curve represents the equilibrium price of electricity after emergent innovative firms begin to enter the market, the price of

electricity after entry has begun is also lower with a higher subsidy. The lower electricity price, according to the second line of (25), in turn implies a rise in $b_t K_{s_t}$ and a fortiori a

higher value of K_{s_t} for each instant after entry has begun, i.e., more emergent innovative firms are in the market after entry has begun. ■

7. CONCLUSION

The model we formulate and analyze in this paper represents an attempt to study the impacts on the electricity industry of the proposed climate change policies together with the electricity sector reform and technological change. We show that, in the presence of caps on carbon emissions, the entry of advanced generation technologies constrains the output and a fortiori the greenhouse gas emissions of the old technologies. Furthermore, the output of each firm and a fortiori its greenhouse gas emissions depend only on the price of emissions permits, not on the number of emissions permits allocated by the government to incumbents. Hence, the price of emissions permits is completely determined by their supply, i.e., by the total number of emissions permits issued by the government, and how part of the emissions permits is allocated among firms in the economy is immaterial as far as climate change policy is concerned. How emissions permits are allocated among firms in the economy has distributional consequences as far as the firms that are given these emissions permits consider them as windfalls.

Our paper also analyzes the emergence of innovative technologies that have zero greenhouse gas emissions. In its efforts to encourage the development of renewable energy sources, an increase in the government subsidy per unit of output produced from emergent innovative technologies induces an earlier entry of these technologies, and their installed capacity is also higher. On the other hand, the entry induces a fall in the price of emission permits at every instant after entry has occurred. Furthermore, we found that the total greenhouse gas emissions from the electricity sector decline while the industry output rises and the price of electricity is strictly declining as more and more emergent innovative firms enter the market.

The model can be extended in several directions. First, the time path of the emission permits issued by the government is taken as exogenous. The model is silent on how these numbers are arrived at. The next step is to endogenize these variables. Second, because there are powerful special-interest groups that benefit from the status quo, the competition among the various special-interest groups for the implementation of an

environmental policy that is favorable to its own through lobbying activities can be introduced. Third, as special physical characteristics of electricity and electric power networks require non-market mechanisms to meet specific physical criteria governing network frequency, voltage and stability, these non-market mechanisms can be incorporated into the model. Finally, to respond efficiently to greenhouse gas mitigation goals reflected in policies to promote renewable energy, very significant investments in new long-distance transmission facilities will be required as the most efficient sites for renewable energy facilities, especially wind and large scale solar facilities, are often located far from load centers – on-shore and off-shore. An economic assessment of new investments in transmission network can be presented.

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CHAPTER THREE

TRANSMISSION INVESTMENTS IN ELECTRIC POWER NETWORKS

THE CASE OF THE MID-WESTERN REGION OF WESTERN AUSTRALIA

1. INTRODUCTION

The electricity sector is made up of three vertically related components: generation, transmission, and distribution¹. Electricity is generated by rotating a turbine. The generator's rotor can be turned by high-pressure steam obtained from boiling water, and the energy needed to boil the water might come from burning hydrocarbons, from fissioning atoms, or from concentrated sunlight. The rotor of the turbine might also be turned by water running downhill. The electricity generated by power plants is then transported through transmission lines to load centers before being distributed to end-users in these load centers.

For most of the twentieth century, consumers bought electricity from vertically integrated utilities which were monopolies in their own geographical areas. The philosophy underlying this market structure is that generation, transmission, and distribution all exhibit economies of scale, and thus have the characteristics of natural monopolies. Competition was not thought to be viable, and the market structure adopted in most countries was either public ownership or regulated monopolies. The conventional view stood unchallenged until the 1980's when economists began to question the orthodoxy. The new view argued that public ownership or regulated utilities do not have the

¹ See Hausman and Newfeld (2004) for a readable introduction to the economics of electricity networks and the history of the electricity industry in the US.

incentives to operate efficiently and they often make unnecessary investments. On another front, technologies imported from materials science and the space program have made turbines more efficient than they had ever been. Another factor that contributed to the erosion of the conventional wisdom was the decline in the price of gas. It is now possible to build smaller and cheaper generating plants. More importantly, in many cases the all-in cost of a new plant was lower than what customers were paying for the sunk costs of old plants. These developments have reduced the minimum efficient scale to the point where the position that generation is a natural monopoly is no longer tenable. Technological progress has made it possible to construct transmission lines that carry electric currents at 1000 kV and that span thousands of kilometers. It is also possible to build generating plants that have a capacity of more than 1000 MW. These developments in transmissions and generation mean that competition in generation can come from many producers from far away; such as hydroelectric and wind farms. Competition in the wholesale market for electricity is now viable.

On the other hand, the transmission component of an electricity network remains a natural monopoly. At the present time, it is inconceivable that a group of investors will build a new transmission line next to an existing transmission line in direct competition with the latter. Transmission networks are capital intensive. To transport electricity securely and efficiently over great distances requires expensive equipments – transmission lines, transformers, switchgears, and reactive compensation devices. Maintaining the reliability of a transmission network also requires costly communication equipments as well as sophisticated control centers. Transmission assets are long-lived – from 20 to 40 years – and once installed, are sunk. Another important feature of transmission assets is that they are only manufactured for a small number of standardized voltage and MVA (power rating). For example, transmission lines are only manufactured to carry electric currents at 66 kV, 132 kV, 330 kV, 500 kV, and up to 1000 kV. Thus, transmission investments are lumpy. Also, transmission networks exhibit economies of scale. Obviously the costs involved in building transmission lines can be taken to be proportional to the distance covered by the lines. On the other hand, new sub-stations must be built at either end of the line, and the costs of the sub-stations are significant.

Because the costs of the sub-stations are independent of the amount of power that the network can transport, the average cost of transporting electricity declines with the amount of power transported; that is, transmission networks exhibit economies of scale.

Because electricity transmission networks are considered as natural monopolies, the appropriate market structure for transmission service is either public ownership or regulated monopolies. The transmission systems in the US, the UK and Wales, and Australia are all regulated. To ensure open access to electric producers and purchasers, the transmission system is often run by an Independent System Operator (ISO) or an Independent Market Operator (IMO). The rates are regulated and set at an appropriate level that allows investors to recover their investments and earn an acceptable rate of return. Although some researchers, such as Hogan (1992) and Chao and Peck (1996), have attempted to introduce competition into the market for transmission services by proposing some market mechanism that can support competition, the research in this subject has not made inroads into the policy debates.

At the distribution level, local delivery of electricity to end-users is usually considered to be a natural monopoly because there is only one set of wires going into a building and because there is only room for one set of wires on the street. Although it might be physically possible to string two sets of wires on the street, economies of scale in covering the territory argue against the duplication of a delivery system. Competition can be introduced at the distribution level by separating the local transport of electricity from retail activities. In this manner, end-users can choose to purchase electricity from any producer and use the local delivery system as a common carrier. In New Zealand, there is competition at the generation as well as the distribution level. At the distribution level, there are choices for retailers and end-users.

There are two approaches to modeling transmission investments: the merchant transmission investment approach and the regulatory transmission investment approach. Underlying the regulatory transmission investments approach is the premise that transmission is a natural monopoly and should be regulated. In exchange for being

granted the status of a regional monopoly, a transmission firm must accept the authorities of a regulatory body to set the price that it can charge users of the transmission network. The revenues collected should allow the transmission company to recover its investments and earn a decent rate of return on its investments. For a flavor of this approach, the reader can consult, for example, Gans and King (1999), Léautier (2000), and Tanaka (2005). Underlying the merchant transmission investment approach is the premise that investments in transmissions should be treated like all the other kinds of investments and should be left to market forces. Rather getting a sure, but modest rate of return on their investments, merchant transmission investors pay for the construction of transmission links with their own funds and hope to obtain much greater revenues from operating these links without being subjected to any constraint on how they can set the price of the transmission service these links provide. Of course, merchant transmission investors must also bear all the risks associated with the investments. For an authoritative and comprehensive analysis of the merits and problems of the merchant transmission investment approach, the reader can consult Joskow and Tirole (2005). Both approaches of transmission investments are still being developed. At the present time, an overwhelming majority of transmission investments are remunerated on the regulatory basis, and this reality is also reflected in the economic literature with relatively more research efforts being devoted to the regulatory transmission investments approach than the merchant transmission investment approach.

In this paper, we formulate a model of electricity transmission investments from the perspective of the regulatory approach. The model we formalize is that of a sub-network within a global network. In the economic literature on networks, the network effects are captured by taking the overall size of the network. How the various components of the network, i.e., the topology of the network, interact with each other to generate the network externalities – positive or negative – is most often not modeled. When the topology of the network is considered, it is either too simple or too general to capture adequately these externalities. The network considered by Joskow and Tirole, *op cit.*, has only two nodes linked by one transmission lines. Léautier, *op cit.*, described a network in an abstract manner as a set of nodes and a set of links that connect these nodes. In this

paper we formalize a dynamic model of transmission investments in a sub-network of an electric grid that takes into account the topology of the sub-network. For this purpose, we eschew an abstract specification of the sub-network and formalize our ideas in the context of a specific sub-network, with its own topology and its own constraints. More specifically, we consider the transmission investments in the Mid-West region of Western Australia, a sub-system of the South West Interconnected System (SWIS). This subsystem is known as the North Country Region network. Given the complexity – both physical and economic – of a vast electric power grid that interconnects generation, transmission, and distribution, our model can be looked at as a first step in the analysis of the investments in larger transmission networks. Lack of data prevents us from calibrating the model and deriving some practical policy recommendations. Our qualitative analysis exploits the structure and the characteristics of the sub-network in the Mid-Western region of Australia. It is hoped that the analysis can be applied in studying the transmission investments of more general electric networks.

The model we formalize differs from the traditional models in the literature in several respects. First, in contrast with most models in the literature that deal only with network deepening, our model deals with transmission investments that involve both network deepening and network widening. Second, unlike with the conventional investment models which are static and deal only with the long run, our model is dynamic and focuses on the timing of the infrastructure investments. Our model share one common feature with most models in the literature in that the size of the investments are taken to be continuous variables and that the investment in infrastructure is made only once. This modeling strategy is only adopted for convenience of exposition. The model can readily be modified to deal with repeated lumpy investments; the reader can consult, for example, Chapter 2 in Sydee (2010) to see how this can be done.

Because transmission lines are constructed to transport electricity, making investments in transmission in isolation without any consideration of investments in generation will not result in an efficient program of transmission investments. An optimal transmission investment program is part of the optimal investment program for an integrated model in

which investments in transmission and investments in generation are made at the same time. Both Western Power² and CRA International³ justified the reinforcements of the North Country network by appealing to the growth in demand – natural load growth and block loads. The logic is somewhat circular. Economically speaking, the investments in generation depend on the availability of transmission capacity. Hence, the investments in transmission should be analyzed under the perspective of a dynamic Stackellberg game in which Western Power acts as the leader and generation investors as the followers. Another point worth mentioning is that Western Power's proposed reinforcements of the North Country Region network represent the least-cost response to the growth in demand in this region. The causality is only one-way; no consideration is given to the impact that the transmission investments might have on demand. Furthermore, because the transmission segment is regulated while competition is encouraged in the generation segment, the regulator can only control the transmission investments made by Western Power, not the investments made by electric producers who are free to set prices or to enter the market. The optimal pattern of transmission investments will maximize social welfare, taking into account the reaction of merchant electric generation and giving a decent rate of return to the owners of the transmission assets who are represented by Western Power.

The paper is organized as follows. In Section 2, a brief description of the electric transmission network in the Mid-West region of Western Australia – also known as the North Country Region network or simply as the North Country network – is described. The constraints on the North Country network and Western Power's options for relaxing these constraints are presented in Section 3. A mathematical model of this network is presented in Section 4. In Section 5, we perform a power flow analysis of the North Country network in 2005 to gain some more understanding of the constraints on the network. The behavior of the network – in particular, the evolution of the prices of electricity in the network – is analyzed in Section 6 for the scenario that the network is not reinforced. In Section 7, our model of the reinforcements of the North Country

² The SWIS is run by a public corporation known as Western Power.

³ A consulting firm.

network – network deepening and network widening – which is patterned after the option preferred by Western Power, is presented. Section 8 contains some concluding remarks.

2. THE TRANSMISSION SYSTEM IN THE MID-WEST REGION OF WESTERN AUSTRALIA

The existing transmission network in the Mid-West region of Western Australia – usually referred to as the North Country Region network, or simply as the North Country network – extends from Pinjar and Muchea in the South to Geraldton and Chapman in the North.⁴ It is a radial system⁵ of transmission lines of 132 kV that extends approximately 400 km from its Southern to its Northern extremities. Figure 1 depicts load areas, substations, and transmission lines of The North Country network.⁶

⁴ The materials on the transmission system in the Mid-Western region of Western Australia are drawn from the following sources: (i) “Proposed Improvements to the Mid-West region transmission network,” 22 March 2007, Western Power; (ii) “2008 Transmission and Distribution Annual Report,” Western Power, p. 83-88; and (iii) Final Report V2: “Reinforcement Options for the North Country Region: Public Version,” 30 March 2007, CRA International. All these reports can be found on the website of Western Power.

⁵ In an electric network, the flow of electricity is governed by the laws of physics, namely Kirchoff current law (KCL) and Kirchoff voltage law (KVL). A network has a *radial structure* if for any two different locations within the network, there is exactly one route along which electricity can be transported. In a radial configuration, transmission lines branch out sequentially, and power flows strictly in one direction. In general, electric transmission networks are not radial, and have many interconnections, and any two points in a network are connected by more than one path; that is, some lines form *loops* within the network. If there are two distinct paths linking two points in the network, then the electricity that flows from one point to the other will flow along both paths, and the power that flows along a path will be inversely related to the impedance of the path. The phenomenon is known in electric power systems as *loop flows*, and is very difficult to handle, especially in large networks. In a radial system, there is no loop flow, and this makes the power flow analysis much simpler.

⁶ Our Figure 1 is Figure 46 of The 2008 Transmission and Distribution Annual Report of Western Power and is imported from this report.

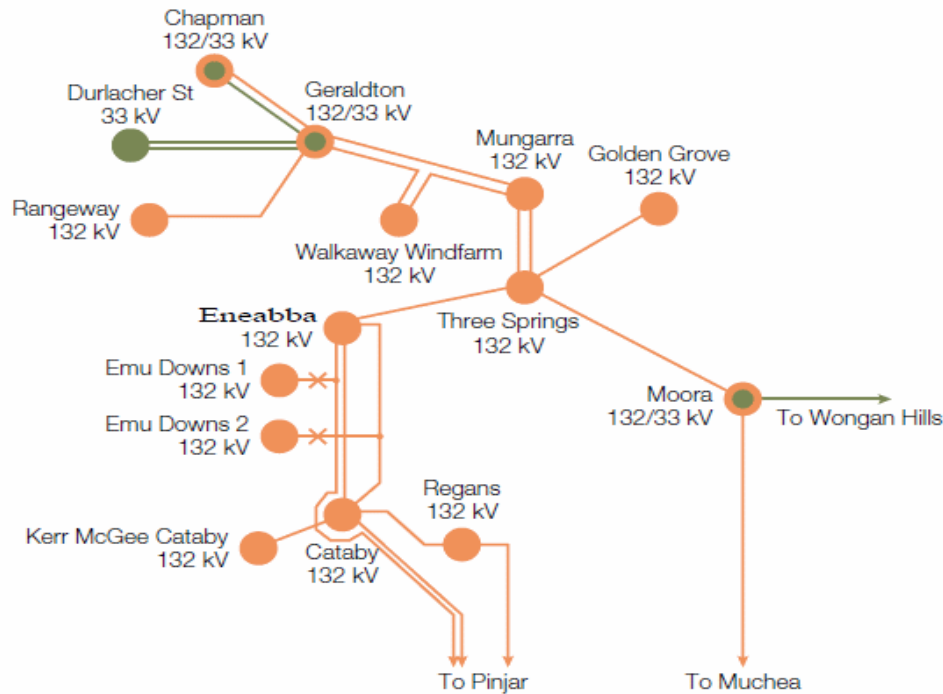


Figure 1. □ The North Country network

The North Country network serves a range of mining and industrial loads as well as numerous rural areas and the major population center of Geraldton. There are five main sources of generation in the North Country Region:

- (i) The Mungarra power station, which is owned by Verve Energy, is connected to the 132 kV transmission lines at approximately 40 km south of Geraldton. This power station supplies about 85 MW in the summer.
- (ii) The Geraldton power station, which is connected to the 33 kV system at Geraldton, has a 20 MW gas turbine. This power station runs on expensive distillates, and provides additional power during peak periods. However, it creates much noise disturbances to the houses nearby.
- (iii) The Walkaway Wind Farm, which is located close to Geraldton, was commissioned in 2005. It was built for a notional capacity of 90 MW, but its contribution to peak summer capacity is much lower than the notional

capacity because output depends on prevailing winds.⁷ Western Power estimates that the Walkaway Wind Farm only has a firm supply of 5 MW.

- (iv) The Emu Downs Wind Farms – Emu Downs 1 and Emu Downs 2 – which are located near Cataby. These wind farms were commissioned in 2006, and each of them has a notional capacity of 80 MW.
- (v) Imports from The South West Interconnected System (SWIS) via Pinjar and Muchea. According to Western Power (2007), the North Country network allows for about 65 MW to be imported into the NCR from the SWIS.

Total diversifiable peak load (without losses) in the NCR was 140.4 MW in 2005, with approximately one third of the load accounted for by the three major mines at Cataby, Eneabba, and Golden Grove. The following table gives a breakdown – according to the load areas – of the summer diversified peak load in the North Country region in 2005.⁸

TABLE I
Summer Diversified Peak Load in the NCR in 2005

Substation	Peak (MW)	Types of customers
Geraldton 33kV	34.4	Industrial, rural, residential
Durlacher	24.4	CBD, residential, commercial
Eneabba	21.5	Mining
Cataby	12.3	Mining
Golden Grove	12.1	Mining
Moora	11.1	Rural
Chapman	10.4	Residential, commercial
Regans 33/22 kV	7.7	Rural
Three Springs	6.5	Rural
Total	140.4	

⁷ The average supply factor of a wind farm is from 30 to 35 per cent of the notional capacity.

⁸ Reinforcement Options for the North Country Region: Public Version, Final Report V2, CRA International, 30 March, 2007.

3. THE CONSTRAINTS ON THE NORTH COUNTRY NETWORK AND THE OPTIONS CONSIDERED BY WESTERN POWER

CRA International (2007) cited a report of Western Power that gives an estimate of a 6% per annum natural load growth rate at Geraldton for the period 2000-06, and an estimate of a 4.6% per annum natural load growth rate for the area above Eneabba and Muchea during the period from 1998 to 2006. According to Western Power's,⁹ forecast of natural load growth, demand will exceed capacity (Eneabba and Muchea excepted) in 2010.¹⁰ For 2010, the forecast loads¹¹ for Chapman, Durlacher, Geraldton, Rangeway, and Three Springs are 18.01 MW, 18.35 MW, 43.71 MW, 23.93 MW, and 8.43 MW, respectively. These forecast loads amount to a total of 112.33 MW for these sub-stations, and this does not include the demand by the North Country contract customers. This total load forecast clearly exceeds the local generation capacity provided by the Mungarra power station and the Walkaway Wind Farm combined. A second source of demand growth is the expected block load growth generated by major industrial development projects – mainly mining – which dwarfs the existing load in the NCR. It is forecast that this block load demand will reach 314 MW in 2012.¹²

The North Country network was designed to supply relatively small loads distributed over a large geographical area. It is not capable of transporting large amounts of power due to thermal, voltage, and synchronous stability limitations.¹³ Heavy reliance is placed on the Mungarra power station and the Geraldton power station. However, operation of the gas turbines at Mungarra runs the risk of synchronous instability for faults on the

⁹ Figure 2: Supply and Demand Based on Natural Load Growth from 1997/98 to 2015/16, Proposed Improvements to the Mid-West region's Transmission Network, 22 March 2007, Western Power.

¹⁰ At present there are constraints within the network as well as within the broader electricity system. In addition, it is forecast that supply capacity to the region north of Eneabba and Muchea will be constrained from 2011/12 onwards for natural load growth conditions.

¹¹ These are the 2010 load forecasts for the North Country found on page.113 of the 2008 Transmission and Distribution Annual Planning Report, Western Power.

¹² Table 3, Final Report V2, Reinforcement Options for the North Country: Public Version, CRA International, 30 March 2007.

¹³ The amount of power that a transmission line can transport depends on its length. For a short transmission line, the heating up of the line sets a thermal limit on how much power the line can transport. If too much power flows along the line, it heats up and sags. For a line of intermediate length – on the order of 100 km – the limit is set by the voltage drop at the end of the line. For longer AC lines, system stability constrains the amount of power that can be carried by a line.

lines in the NCR or in the SWIS. The potential risk of synchronous instability sets a limit on the power transfers in the North Country network, and the limit is not relaxed until the network is reinforced.

Western Power has identified a number of constraints in the North Country network. First, the transmission capacity constraints are particularly severe on the Eneabba-Three Springs, Muchea-Three Springs and Three Springs-Geraldton sections of the NCR network. Second, there is constraint on the import capability of the NCR network from the SWIS. The South to North power transfer is very complex. It depends on local generation, availability of reactive power support, regional load (North of Eneabba and Muchea), and the thermal ratings of the transmission lines. Third, the North to South power transfer in the NCR is also significantly constrained. Construction of the 132 kV line between Pinjar and Eneabba in 2004 has temporarily eased some of these constraints. However, the recent connection of the Emu Downs wind farms has exhausted transmission capacity available to connect new generation between Pinjar and Eneabba. New generation in the Geraldton area cannot be accommodated due to existing thermal limits on the 132 kV network. Constraints in the northern part of the Mid-West region are the main reasons why the North Country network needs reinforcements.

According to a review of the load forecasts and the adequacy of the 132 kV transmission systems by Western Power, the region above Eneabba and Muchea only has a firm spare supply of slightly above 20 MW. This constraint on the northern part of the North Country network is the main reason why the system needs reinforcements, and if this issue is not addressed, congestion will set in by summer 2009/10.

Currently, there are numerous connection enquiries from industrial and mining loads, and new generation. The loads coming from potential industrial and mining interests amount to about 314 MW.¹⁴ New generation – gas fired and coal-fired as well as wind farms – amounts to 1000 MW. These connections are only possible if reinforcements of the North Country are made. Under such a scenario, there will be rigorous competition among local

¹⁴ CRA International (2007), Table 3.

electric producers in the NCR, and the new capacities will serve not only the NCR but also the metropolitan area of Perth in the South.

To meet the natural load growth and the expected block loads, Western Power has identified 12 options. These options include additional generation, demand-side management, and transmission reinforcements. The options that involve additional generation are ruled out as being not technically feasible. As for demand-side management, there are limits on what this strategy can achieve. The only options that are viable involve transmission reinforcements. For example, the transmission line between Mungarra and Geraldton can be reinforced with another 132 kV line to carry more power to Geraldton and its surrounding areas. Another strategy is to build a 330 kV transmission line from Pinjar to Geraldton. The options that can be considered under both of the transmission reinforcement strategies all achieve the objective of meeting the demand loads in the NCR for the foreseeable future. A high-level review of the options that involve transmission reinforcements by CRA International (2007) suggests that the construction of a 330 kV transmission line from Pinjar to Geraldton dominates all the other transmission reinforcement option in terms of costs. This option – labeled option 1 – is the option most preferred by Western Power (2007).

4. A MATHEMATICAL MODEL OF THE NORTH COUNTRY NETWORK

If we replace all the parallel lines that link two nodes in Figure 1 by a single line, we obtain a one-line representation of the North Country network.¹⁵ In the one-line depiction of the network, the power that is transported by the single line is equal to the sum of the power carried by all the constituent lines that it replaces. Figure 2 below is the output of the *Mathematica* program that represents the North Country network at the present time.

¹⁵In electric power engineering texts, transmission lines that link two busbars are represented by a single line in a diagram that depicts the electric circuit in question. The diagram is referred to as a one-line diagram.

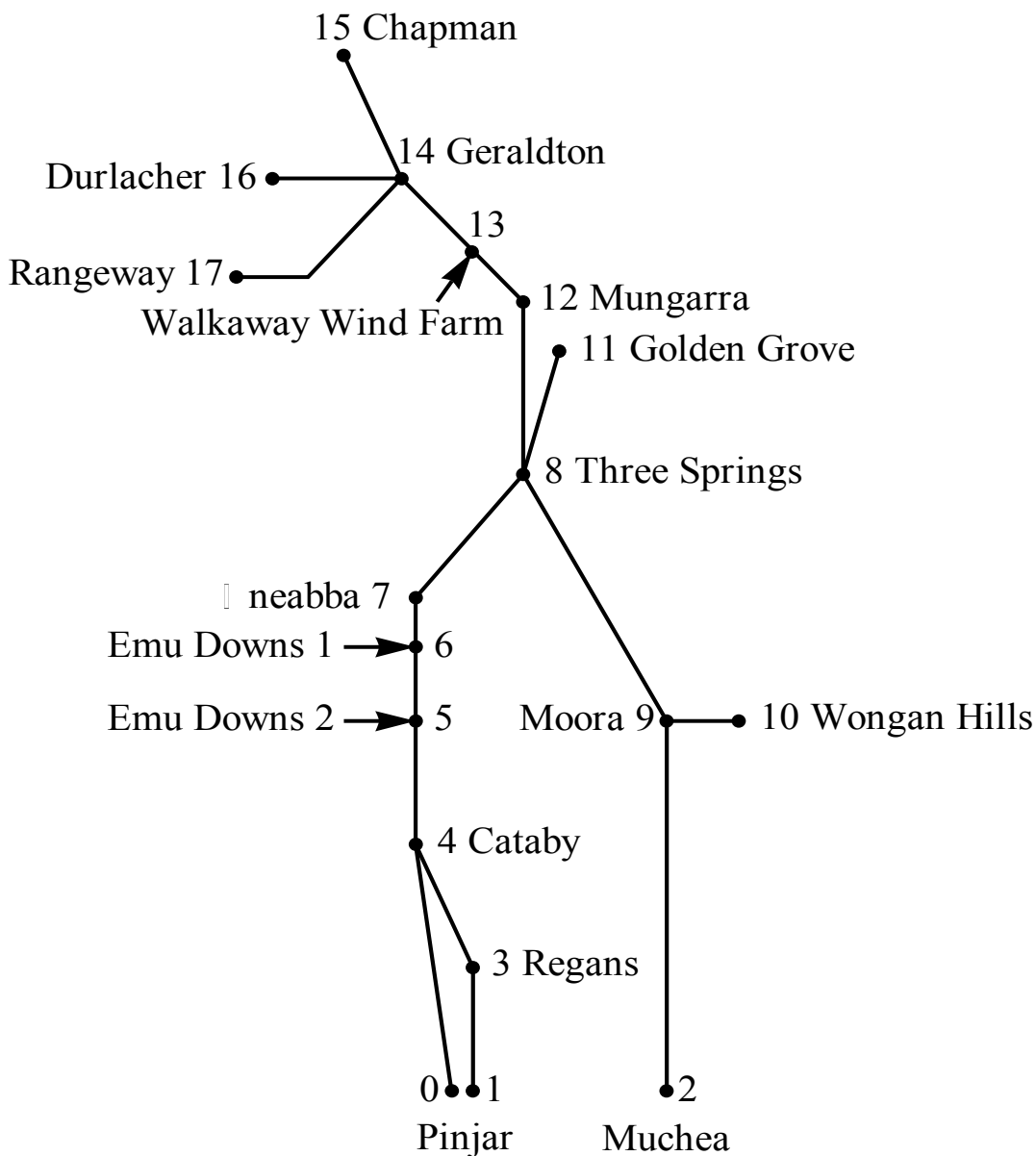


Figure 2. □ The one-line representation of the North Country network

Note that in the one-line representation of the North Country network, all the nodes have been assigned a numerical identification in addition to its geographical name. Let $N = \{0,1, \dots,17\}$ denote the set of nodes in the North Country network.

A link in a transmission network is a transmission line that connects two nodes, say n and n' , in N , and that along this transmission line there is no other nodes. A link that connects two nodes, say n and n' is denoted by $\{n, n'\}$ or $\{n', n\}$. As written, a link is represented

by the set that contains its two end-points, and no direction is specified. For example, $\{7,8\}$ and $\{8,7\}$ both represent the link between node 7 (Eneabba) and node 8 (Three Springs). To capture the direction of the power flows along a link, the concept of a directed link is needed. In what follows, we write $[n, n']$ for the link $\{n, n'\}$ endowed with the direction from n to n' . Note that the directed link $[n, n']$ is expressed as a closed interval, with n as the initial node and n' as the end node. Also, note that the directed link $[n', n]$ is not the same as the directed link $[n, n']$.

A route from node j to node i is a series of nodes n_0, \dots, n_k , with $n_0 = j, \dots, n_k = i$, such that $[n_0, n_1], \dots, [n_{k-1}, n_k]$ are directed links. A route thus defined also has a direction. In the model, the route from one node (a supply node) to another node (a load area) is the directed path used by a producer to send part of the power it generates to a load area. Because the North Country network is a radial system, there is exactly one route that connects any two nodes of the network.

The set of links in the North Country network is

$$L = \{\{0,4\}, \{1,3\}, \{3,4\}, \{4,5\}, \{5,6\}, \{6,7\}, \{7,8\}, \{2,9\}, \{9,10\}, \{9,8\}, \{8,11\}, \{8,12\}, \\ \{12,13\}, \{13,14\}, \{14,15\}, \{14,16\}, \{14,17\}\}.$$

The ordered pair (N, L) is a mathematical representation of the topology of the North Country network. In the one-line diagram, nodes 0 and 1 represent the interface – through Pinjar – between the North Country network and the SWIS, while node 2 represents another interface – through Muceha – between the NCR network and the SWIS. Through these three nodes power can be imported into the NCR or exported from the NCR to the SWIS. For simplicity, the power imported at each of the interface nodes 0, 1, and 2 is assumed to originate from a different producer in the SWIS. Each of these producers sells the power it generates in both the SWIS and the NCR.

The load areas are the substations at nodes 3 (Regans), 4 (Cataby), 7 (Eneabba), 8 (Three Springs), 9 (Moora), 10 (Wongan Hills), 11 (Golden Grove), 14 (Geraldton), 15 (Chapman), 16 (Durlacher), and 17 (Rangeway). The set of nodes at which power is extracted is $I = \{3,4,7,8,9,10,11,14,15,16,17\}$.

In what follows, time is continuous and denoted by $t, t \geq 0$. For each $i \in I$, let

$$(1) \quad x_i[t] = a_i[t] - b_i[t]p_i[t]$$

denote the market demand curve for electricity at node i at time t . Here $a_i[t]$ and $b_i[t]$ are two positive parameters that vary with time and that capture the growth in demand through time. The growth in demand can be represented by a rise in $a_i[t]$ or in a decline in the slope $b_i[t]$.

Within the North Country region, generation is found at nodes 5 (Emu Downs 1), 6 (Emu Downs 2), 12 (Mungarra power station), 13 (Walkaway Wind Farm), and 14 (Geraldton power station). In what follows, we shall refer to the electric producers inside the NCR as local producers, and the power they produce as local generation. For each $j \in \{5,6,12,13,14\}$, let c_j and \bar{y}_j denote, respectively, the marginal generation cost and the capacity of the local generator located at node j . Because the marginal generation costs of wind farms are zero, we shall set $c_5 = c_6 = c_{13} = 0$.

Power is imported into the NCR at nodes 0, 1, and 2. The power imports are sent by competitive producers in the SWIS. For consistency of notation, we shall refer to the competitive producers from the SWIS who collectively inject power at each of the import node $j, j = 0,1,2$, as producer 0, producer 1, and producer 2, respectively. The set of nodes at which power is injected into the NCR is $J = \{0,1,2,5,6,12,13,14\}$. In what follows, we shall refer to the producers who inject power at node j as producer $j, j \in J$.

For each $j \in J$ and each $i \in I$, let $y_{ji}[t]$ denote the amount of power that producer j ships to node i at time t . The following capacity constraint must hold for a generator located inside the NCR:

$$(2) \quad \sum_{i \in I} y_{ji}[t] \leq \bar{y}_j, \quad (j \in \{5,6,12,13,14\}).$$

For $j = 0,1,2$, $y_{ji}[t]$ represents the total amount of power imported into the NCR at the interface node j and sent to node i at time t . Because there are many competitive generators in the SWIS, there is no capacity constraint for the total volume of power that can be injected at such an interface node. That is, the constraints (2) do not exist at the interface nodes. Any constraint on the volume of imports, when it exists, is dictated by the requirement of synchronous stability of the North Country network as a whole. For our purpose, we assume that the following imports constraint holds at each of the interface nodes:

$$(3) \quad \sum_{i \in I} y_{ji}[t] \leq \frac{\bar{I}m}{3}, \quad (j = 0,1,2).$$

where $\bar{I}m$ denotes the limit of the total power imports from the SWIS. Note that in (3) we have assumed that the total power imports from the SWIS are equally divided among the three interface nodes.

For each directed link, say $[n, n']$, the projection of $y_{ji}[t]$ on $[n, n']$ that we write as $pr_{[n, n']} [y_{ji}[t]]$, represents the amount of power flow induced by $y_{ji}[t]$ along $[n, n']$. If the directed link $[n, n']$ is not on the route from j to i , then $pr_{[n, n']} [y_{ji}[t]] = 0$. The total power flow induced on the directed link $[n, n']$ by a combination of shipment plans $((y_{ji}[t])_{i \in I})_{j \in J}$ is then given by

$$(4) \quad \sum_{j,i} pr_{[n, n']} [y_{ji}[t]].$$

Similarly, the total power flow induced on the directed link $[n', n]$ by the combination of shipment plans $((y_{ji}[t])_{i \in I})_{j \in J}$ is given by

$$(5) \quad \sum_{j,i} pr_{[n', n]} [y_{ji}[t]].$$

The power flow represented by (5) is known as the counter-flow to the power flow represented by (4). The absolute value of the difference between (4) and (5) is the net power flow on the link $\{n, n'\}$ that is induced by the combination of shipment plans $((y_{ji}[t])_{i \in I})_{j \in J}$.

A combination of shipment plans $((y_{ji}[t])_{i \in I})_{j \in J}$ is said to be feasible if the net power flow on each link does not exceed the transmission capacity of that link, i.e., if

$$(6) \quad \sum_{j,i} pr_{[n,n']} [y_{ji}[t]] - \sum_{j,i} pr_{[n',n]} [y_{ji}[t]] \leq k_{\{n,n'\}}, \quad (\{n, n'\} \in L)$$

In (6), $k_{\{n,n'\}}$ represents the transmission capacity of the link $\{n, n'\}$.

5. THE CONSTRAINTS ON THE NORTH COUNTRY NETWORK: A POWER FLOW ANALYSIS

For each link between two nodes, say i and j , let $f_{[j,i]}[t]$ denote the power flow along the directed link $[j, i]$ at time t . Using the data in Table I, we obtain the following power flows along each of the links in the North Country network for 2005.

TABLE II

The Power Flows in the North Country Region in 2005

Link $[j, i]$	$f_{[j,i]}$ (Directed Flow from j to i)
[0,4]	16.8
[1,3]	16.8
[3,4]	$f_{[1,3]} - x_4 = 16.8 - 7.7 = 9.1$
[4,5]	$f_{[0,4]} + f_{[3,4]} - x_4 = 16.8 + 9.1 - 12.3 = 13.6$
[5,6]	$f_{[4,5]} + y_5 = 13.6 + 0 = 13.6$
[6,7]	$f_{[5,6]} + y_6 = 13.6 + 0 = 13.6$
[7,8]	$f_{[6,7]} - x_7 = 13.6 - 21.5 = -7.9$
[2,9]	16.8
[9,10]	$x_{10} = 0$
[9,8]	$f_{[2,9]} - x_9 - x_{10} = 16.8 - 11.1 - 0 = 5.7$
[8,11]	$x_{11} = 12.1$
[12,8]	$y_{12} - f_{[12,13]} = 85 - 64.2 - 20.8$
[12,13]	$f_{[13,14]} - y_{13} = 69.2 - 5 = 64.2$
[13,14]	$x_{14} + x_{15} + x_{16} + x_{17}$ $= 34.4 + 10.4 + 24.4$ $= 69.2$
[14,15]	$x_{15} = 10.4$
[14,16]	$x_{16} = 24.4$
[14,17]	$x_{17} = 0$

In computing the power flows in Table II, we have assumed that the power supply provided by the Mungarra power station is 85 MW and that of the Walkaway Wind Farm is 5 MW. Given that the total diversified load in 2005 was 140.4 MW, the amount of power imported into the North Country Region from the SWIS in 2005 was 50.4 MW.

Without access to more detailed information from Western Power, we shall assume that the amount of power imported from the SWIS is divided equally among the three interface nodes 0, 1, and 2. That is $y_0 = y_1 = y_2 = 16.8$. Also, because the Emu Downs Wind Farms, the Rangeway substation, and the Wongan Hills substation were not yet constructed in 2005, we set $y_5 = y_6 = 0$, $x_{17} = 0$, and $x_{10} = 0$.

According to the power flows presented in Table II, the traffic is heavy on the route from the Mungarra power station to Geraldton. For a 138 kV transmission line, the amount of power that the line can transport is 75 MW.¹⁶ The amount of power transported along the route from the Mungarra power station to Geraldton is 69.2 MW. This segment of the North Country network had little spare capacity in 2005, and would experience congestion due to the natural demand growth in Geraldton and the surrounding areas if there is no reinforcement in this part of the North Country network.

Note that $f_{[7,8]} = -7.9 < 0$ indicates that power flows from node 8 (Three Springs) to node 7 (Eneabba). Also, $f_{[9,8]} = 5.7 > 0$ indicates that power flows from node 9 (Moora) to node 8 (Three Springs). These results can be interpreted in the following manner. The power generated at the Mungarra power station serves both the part of the network above Mungarra and the part of the network below Mungarra and above Three Springs, and still has $7.9 - 5.7 = 2.2$ MW left to send to the branch of the network that links Pinjar and Eneabba. Also, the power imported along the other branch of the network (linking Muchea and Three Springs) exceeds the demand located on this branch, and the excess supply of 5.7 MW is sent – via Three Springs – to the other branch (Pinjar to Eneabba) of the network to meet demand on this branch.

The power flows on the links Eneabba-Three Springs and Muchea-Three Springs seem to be low relative to the power flow into the part of the network above the Mungarra power station. Although these low-levels power flows do not openly suggest that this part of the network will soon experience congestion, they are imposed by the synchronous stability

¹⁶ Hadley et al. (2001): The Potential Economic Impact of Electricity Restructuring in Oklahoma: Phase II Report, p.12.

requirement, according to the technical analysis of Western Power. Thus unless reinforcements are made to the North Country network, the expected loads from future industrial developments and mining interests cannot be accommodated.

6. THE PRICES OF ELECTRICITY IN THE NORTH COUNTRY IF THE NETWORK IS NOT REINFORCED

Suppose that the North Country network is not reinforced. This is the option of doing nothing. Because the NCR is small relative to the SWIS, we shall assume that as far as electricity is concerned the NCR is a small economy linked to the rest of the world – the SWIS – through the points of imports – nodes 0, 1, and 2. Furthermore, because there are many generators in the SWIS, we assume that competition is perfect in that region. Also, because the generation segment is deregulated in Western Australia, electric producers are allowed to compete in the NCR as well as in the SWIS.

Let $p_{-1}[t]$ denote the competitive price of electricity that prevails at time t in the SWIS. In Australia, the postage-stamp method is used to set the transmission tariff for transporting power. That is, the access price for transmission service is uniform in each region. For our purpose, we shall let $\phi[t]$ denote the transmission fee paid by a producer in the NCR or in the SWIS to send power to any load area in these two regions. The marginal delivery cost for a producer, say $j, j \in J$, to send power to any load area in the NCR or in the SWIS is then given by $c_j + \phi[t]$.

6.1. *The Prices of Electricity in the NCR before Congestion Sets in*

Before congestion sets in, the net power flow along any link in the North Country network is below the capacity of the link. Thus power generated in the SWIS can flow freely into the NCR via the interface nodes 0, 1, and 2. Competition then ensures that the price of electricity that prevails in the SWIS also prevails in each of the load areas in the NCR; that is,

$$(7) \quad p_i[t] = p_{-1}[t], \quad (i \in I).$$

6.2. *The Prices of Electricity in the NCR after Congestion has Set in*

Under the option of doing nothing, there is no improvement in the transmission capacity of the North Country network, and congestion will set in after 2010. In our analysis of the power flows carried out in Section 5, we found out that there is a potential bottleneck in the North Country network on the directed link [13,14] along which power flows into the Northern part – node 14 (Geraldton), node 15 (Chapman), node 16 (Durlacher), and node 17 (Rangeway) – of the NCR network. Recall that in Section 2, where the NCR network is described, we mentioned that there is a limit of 65 MW for the power imports from the SWIS due to the synchronous system stability requirements. In this sub-section, we study the evolution of the electricity prices in the NCR network that is induced by the natural load growth at the various load areas under the option of doing nothing.

Because the directed link [13,14] can only transfer a limited amount of power to the sub-region above Geraldton, and because natural demand growth in this part of the network is high, we can expect that the price of electricity is higher in this part of the network than in the remaining part of the network. That is, the link [13,14] separates the set of demand load areas I into two distinct sub-regions, say I_0 and I_1 , where $I_0 = \{3,4,7,8,9,10,11\}$ (the subset of load areas in the Northern part of the network below the Mungarra power station) and $I_1 = \{14,15,16,17\}$ (the subset of load areas above the Mungarra power station). Because within each sub-region the amount of power that has entered it can flow from one load area to another without being subject to any transmission constraint, the optimal dispatch, which maximizes social welfare subject to (i) the shipment plans submitted to the IMO by electric producers, and (ii) the transmission capacity constraints, should equalize prices on all markets in each sub-region. Thus, in what follows we shall denote by $p_{I_0}[t]$ and $p_{I_1}[t]$ the prices of electricity at time t in I_0 and I_1 , respectively, under the optimal dispatch. If the North Country network is not reinforced, then we expect that in the near future the congestion on the link [13,14] will create a *price differential* between the two sub-regions, and the price differential represents the *congestion charge* that a producer must pay at time t to Western Power to have one unit of power transported to the load areas in sub-region I_1 .

Given the small number of generators inside the NCR (the Mungarra power station, the Walkaway Wind Farm, the Geraldton power station, and the Emu Downs Wind Farms) and the transmission constraints of the North Country network, it is reasonable to expect that these generators will exploit their locational market power by raising their prices above the competitive level that prevails in the SWIS, assuming, of course, that they compete according to the Bertrand model of competition.

6.2.1. *The Optimal Dispatch*

For each $j \in J$ and each $i \in I$, let $p_{ji}[t]$ and $y_{ji}[t]$ denote, respectively, the unit price at or above which producer j is willing to sell part of the power it produces in load area i at time t and the maximum amount of power this producer is willing to offer in this market at that time if there is enough demand to absorb such a quantity. For each $j \in J$, the list $(p_{ji}[t], y_{ji}[t])_{i \in I}$ thus represents a plan of prices cum shipments that producer j submits to the Independent Market Operator (IMO). The ordered pair $(p_{ji}[t], y_{ji}[t])$ represents the supply curve of producer j at node i at time t , which can be expressed under the following conventional form of a Marshallian supply curve:

$$(8) \quad s_{ji}[p_i[t]] = \begin{cases} 0, & p_i[t] < p_{ji}[t], \\ s_{ji}, & 0 \leq s_{ji} \leq y_{ji}[t], \\ y_{ji}[t], & p_i[t] > p_{ji}[t]. \end{cases}$$

In (8), $p_i[t]$ denotes the price set by the IMO in market i at time t .

A combination of prices cum shipments plans $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$ for the producers – in the NCR and in the SWIS – who sell electricity to end-users in the NCR is said to be technically feasible if the shipments component $((y_{ji}[t])_{i \in I})_{j \in J}$ satisfies the following technical conditions. First, the capacity constraint (2) holds for each generator located inside the NCR. Second, the import constraint (3) holds at each of the interface nodes. Third, the transmission constraint (6) holds for each of the links in the North Country network.

Let $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$ be a technically feasible combination of prices cum shipments plans submitted by the producers to the IMO. The optimal dispatch that is induced by $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$ can be computed as follows.

First, let $p_{I_k}[t], k = 0,1$, denote the uniform price of electricity in sub-region k at time t . The aggregate supply curve in a sub-region I_k is obtained by summing horizontally over $j \in J$ and $i \in I_k$ all the individual supply curves (8). More precisely, the aggregate supply curve in sub-region k is given by

$$(9) \quad s_{I_k}[p_{I_k}[t]] = \sum_j \sum_{i \in I_k} s_{ji}[p_{I_k}[t]], \quad (k = 0,1).$$

As for the aggregate demand curve for each sub-region, it is obtained by first setting $p_i[t] = p_{I_k}[t]$ in (1), and then summing the result over $i \in I_k$ to obtain

$$(10) \quad \begin{aligned} x_{I_k}[p_{I_k}[t]] &= \sum_{i \in I_k} (a_i[t] - b_i[t]p_{I_k}[t]) \\ &= \sum_{i \in I_k} a_i[t] - (\sum_{i \in I_k} b_i[t])p_{I_k}[t] \end{aligned} \quad (k = 0,1).$$

The equilibrium price of electricity in each sub-region under the optimal dispatch is the value of $p_{I_k}[t]$ that equates aggregate demand and aggregate supply in the sub-region. More precisely, $p_{I_k}[t]$ is defined implicitly by

$$(11) \quad x_{I_k}[p_{I_k}[t]] = s_{I_k}[p_{I_k}[t]], \quad (k = 0,1).$$

Let $q_{j,I_k}[t]$ denote the amount of electricity that producer j is allowed to sell in sub-region $k, k = 0,1$, under the optimal dispatch. In a sub-region, if the aggregate demand curve crosses the aggregate supply curve on a vertical segment of the latter curve, then the optimal dispatch permits each producer to sell the maximum amount of power that it submits for this sub-region. In this case, we have

$$(12) \quad q_{j,I_k}[t] = \sum_{i \in I_k} s_{ji}[p_{I_k}[t]], \quad (j \in J, k = 0,1).$$

On the other hand, if the intersection is on a horizontal segment of the aggregate supply curve, then not all the producers can sell the maximum amounts that they wish to sell, and *an allocation rule* must be used by the IMO to decide how much power each producer is allowed to sell in the sub-region in question. A *simple allocation rule* is to allow the local producer with the lowest index to serve first. If the maximum amount of power that this local producer is willing to sell in the sub-region more than meets aggregate demand at the equilibrium price, then the amount of power that it is allowed to sell is equal to $x_{I_k} [p_{I_k} [t]]$. The amounts of power that each of the other producers are allowed to sell is then set equal to zero. If the maximum amount of power that the local producer with the lowest index is willing to sell in the sub-region falls short of $x_{I_k} [p_{I_k} [t]]$, then the local producer with the second lowest index is allowed to serve the unmet demand. This process continues until all the local producers have been allowed to serve the sub-region. At this stage, if the total maximum amounts that all the local producers are willing to sell still falls short of aggregate demand, then imports are allowed to serve the remaining unmet demand. Mathematically, this allocation rule assumes the following form.

$$\begin{aligned}
 (13) \quad q_{5,I_k} [t] &= \min \left\{ \sum_{i \in I_k} s_{5,i} [p_{I_k} [t]], x_{I_k} [p_{I_k} [t]] \right\}, \\
 q_{6,I_k} [t] &= \min \left\{ \sum_{i \in I_k} s_{6,i} [p_{I_k} [t]], x_{I_k} [p_{I_k} [t]] - q_{5,I_k} [t] \right\} \\
 q_{12,I_k} [t] &= \min \left\{ \sum_{i \in I_k} s_{12,i} [p_{I_k} [t]], x_{I_k} [p_{I_k} [t]] - q_{5,I_k} [t] - q_{6,I_k} [t] \right\} \\
 q_{13,I_k} [t] &= \min \left\{ \sum_{i \in I_k} s_{13,i} [p_{I_k} [t]], x_{I_k} [p_{I_k} [t]] - q_{5,I_k} [t] - q_{6,I_k} [t] - q_{12,I_k} [t] \right\} \\
 q_{14,I_k} [t] &= \min \left\{ \sum_{i \in I_k} s_{14,i} [p_{I_k} [t]], x_{I_k} [p_{I_k} [t]] - q_{5,I_k} [t] - q_{6,I_k} [t] - q_{12,I_k} [t] - \right. \\
 &\quad \left. q_{13,I_k} [t] \right\},
 \end{aligned}$$

$$q_{0,I_k} [t] = \min \left\{ \begin{array}{c} \sum_{i \in I_k} s_{0,i} [p_{I_k} [t]], \\ x_{I_k} [p_{I_k} [t]] - q_{5,I_k} [t] - q_{6,I_k} [t] - q_{12,I_k} [t] - q_{13,I_k} [t] - q_{14,I_k} [t] \end{array} \right\}$$

$$q_{1,I_k}[t] = \min \left\{ \begin{array}{c} \sum_{i \in I_k} s_{1,i} [p_{I_k}[t]], \\ x_{I_k} [p_{I_k}[t]] - q_{5,I_k}[t] - q_{6,I_k}[t] - q_{12,I_k}[t] - q_{13,I_k}[t] - q_{14,I_k}[t] - q_{0,I_k}[t] \end{array} \right\}$$

$$q_{2,I_k}[t] = \min \left\{ \begin{array}{c} \sum_{i \in I_k} s_{2,i} [p_{I_k}[t]], \\ x_{I_k} [p_{I_k}[t]] - q_{5,I_k}[t] - q_{6,I_k}[t] - q_{12,I_k}[t] - q_{13,I_k}[t] - q_{14,I_k}[t] - q_{0,I_k}[t] \\ - q_{1,I_k}[t] \end{array} \right\}$$

The optimal dispatch induced by the combination of offers $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$ submitted to the IMO can be described as follows: The IMO instructs producer $j, j \in J$, to inject an amount of power equal to $\sum_{k=0}^1 q_{j,I_k}[t]$ into the network and then allows each load area to extract an amount of power equal to $a_i[t] - b_i[t]p_{I_k}[t], i \in I_k, k = 0, 1$.

If the optimal dispatch involves congestion on the directed link [13,14], then we expect that $p_{I_1}[t] > p_{I_0}[t]$. In this case *the congestion charge* for transporting one unit of power into the sub-region I_1 is given by the *price differential*

$$(14) \quad \eta[t] = p_{I_1}[t] - p_{I_0}[t].$$

Under the optimal dispatch induced by $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$, the profit earned by a producer other than the Geraldton power station is given by

$$(15) \quad \pi_j \left[\left((p_{ji}[t], y_{ji}[t])_{i \in I} \right)_{j \in J} \right] = q_{j,I_0} (p_{I_0}[t] - c_j - \phi[t]) + q_{j,I_1} (p_{I_1}[t] - c_j - \phi[t] - \eta[t]),$$

$$(j \in (J - \{14\})).$$

As for the Geraldton power station, which is located upstream of the congested link [13,14], it does not have to pay the congestion charge for the shipments to the load areas inside sub-region I_1 , and thus the profits it makes are given by

$$(16) \quad \pi_{14} \left[\left((p_{ji}[t], y_{ji}[t])_{i \in I} \right)_{j \in J} \right] = q_{14, I_0} (p_{I_0}[t] - c_{14} - \phi[t]) + q_{14, I_1} (p_{I_1}[t] - c_{14} - \phi[t]).$$

6.2.2. The Nash Equilibrium

A technically feasible combination of prices cum shipments plans $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$ is said to constitute a Nash equilibrium if for each $j' \in \{5, 6, 12, 13, 14\}$ and each feasible prices cum shipments plan $(p'_{j'i}[t], y'_{j'i}[t])_{i \in I}$ for this local producer such that the combination of prices cum shipments plans

$$\left((p'_{j'i}[t], y'_{j'i}[t])_{i \in I}, ((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J, j \neq j'} \right)$$

is technically feasible, the following inequality holds

$$(17) \quad \pi_{j'} \left[\left((p'_{j'i}[t], y'_{j'i}[t])_{i \in I}, ((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J, j \neq j'} \right) \right] \leq \pi_{j'} \left[\left((p_{ji}[t], y_{ji}[t])_{i \in I} \right)_{j \in J} \right].$$

That is, under a Nash equilibrium no producer can increase her profit by unilaterally deviating, given that the combination of strategies that results from the deviation must be technically feasible.

6.2.3. The Prices of Electricity in the North Country region under the Option of Doing Nothing after Congestion has Set in

The inverse demand curve at time t for sub-region k , $k = 0, 1$, is given by

$$(18) \quad f_{I_k}[x_{I_k}, t] = \alpha_{I_k}[t] - \beta_{I_k}[t] x_{I_k}$$

where we have let

$$(19) \quad \alpha_{I_k}[t] = \frac{\sum_{i \in I_k} a_i[t]}{\sum_{i \in I_k} b_i[t]}, \beta_{I_k}[t] = \frac{1}{\sum_{i \in I_k} b_i[t]}.$$

The total revenue curve and the marginal revenue curves associated with the inverse market demand curve (18) are given, respectively, by

$$(20) \quad g_{I_k}[x_{I_k}, t] = f_{I_k}[x_{I_k}, t]x_{I_k},$$

and

$$(21) \quad h_{I_k}[x_{I_k}, t] = \alpha_{I_k}[t] - 2\beta_{I_k}[t]x_{I_k}.$$

At an instant t if the supply in sub-region I_1 consists of \bar{y}_{14} (the capacity of the Geraldton power station) and $k_{\{13,14\}}$, the maximum amount of power that can flow into this sub-region through the link $\{13,14\}$, then the price of electricity in this sub-region under the optimal dispatch is given by

$$(22) \quad f_{I_k}[\bar{y}_{14} + k_{\{13,14\}}, t] = \alpha_{I_1}[t] - \beta_{I_1}[t](\bar{y}_{14} + k_{\{13,14\}}).$$

Furthermore, if all the remaining power supplies are dispatched to sub-region I_0 , then the price of electricity in that sub-region is given by

$$(23) \quad f_{I_k}[\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}, t] \\ = \alpha_{I_0}[t] - \beta_{I_0}[t](\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}).$$

According to the forecasts of Western Power, the natural growth of the load in sub-region I_1 is higher than that in sub-region I_0 . Hence we can expect that (22) will be higher than (23) after some point of time in the near future. For our purpose, we shall assume that there exists an instant, say t_c , such that (22) is greater than (23) for $t > t_c$. We can interpret t_c as the forecast time congestion first sets in on the link $\{13,14\}$ if the North Country network is not reinforced. Also, we can expect that the marginal revenue in sub-region I_1 at the market demand level $\bar{y}_{14} + k_{\{13,14\}}$ is higher than the marginal revenue in sub-region I_0 at the market demand level $\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}$. In what follows, we shall presume that this is indeed the case after time t_c .

A number of factors make electricity markets especially vulnerable to the exercise of market power. First, electricity is extremely costly to store. Second, an imbalance of real-time supply and demand in combination of inelastic short-run demand and supply threaten the stability of the entire grid and can disrupt delivery of electricity for all suppliers and consumers on the grid. The result is that firms with even modest market shares have the ability to exercise market power. Intuitively, we expect that a firm with market power might find it profitable to raise price by restricting output. A number of empirical studies have shown that sellers with a comparatively small market share have exercised significant market power in California's wholesale electricity market (Borenstein, 2002). However, long-term contracts can provide less incentive for a firm to restrict its output in the spot market in an attempt to push up prices in that market, as it does not receive the higher spot price on the output it has already sold through a forward contract. Therefore, long-term contract can be viewed as part of the solution to the market power problem of electricity markets. The difference between the SWIS's electricity market and the California's wholesale electricity market is that electricity in the SWIS is mainly traded through bilateral contracts outside the pool market, which covers over 95% of electricity sold.¹⁷ Beyond bilateral contracts, the short term energy market and a balancing market are used to trade wholesale electricity. The short term energy market supports bilateral trades by allowing market participants to trade around their net contract volumes a day before energy is delivered. A market participant's actual supply or consumption of electricity during a trading interval may deviate from the sum of their bilateral volumes and short term energy market trades due to unexpected deviations in demand and unplanned plant outages. The shortfall or surplus is traded on the balancing market. As a result, a firm in the NCR, with a small relative capacity to the size of the market, having sold much of its output under forward contracts would have much less incentive to restrict its output to increase the spot price. The result is confirmed by the following lemma.

¹⁷ State of the Energy Market, Australian Competition & Consumer Commission (ACCC), 2008

LEMMA 1: Consider a Nash equilibrium, say $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$. Let $q_{j,I_k}[t], j \in J, k = 0,1$, denote the amount of electricity that producer j is allowed to sell in sub-region $k = 0,1$, under the optimal dispatch induced by this Nash equilibrium. For each local producer, we have

$$(24) \quad \sum_{k=0}^1 q_{j,I_k}[t] = \bar{y}_j, \quad (j \in \{5,6,12,13,14\}).$$

For each representative producer in the SWIS that injects power at an interface node, we have

$$(25) \quad \sum_{k=0}^1 q_{j,I_k}[t] = \frac{\bar{m}}{3}, \quad (j = 0,1,2).$$

PROOF: To prove the lemma, suppose that one local producer, say j , does not produce at capacity. That is

$$(26) \quad \sum_{i \in I} q_{ji}[t] < \bar{y}_j.$$

Now consider sub-region I_1 . Under the optimal dispatch, the following condition must hold for this sub-region:

$$(27) \quad \sum_{j' \in J} q_{j',I_1}[t] \leq \sum_{j' \in J} \sum_{i \in I_1} y_{j'i}.$$

If equality holds in (27), then the maximum amounts that this local producer would like to sell in the load areas in sub-region I_1 can be realized under the optimal dispatch. In this case, if this local producer modifies her offers in sub-region I_0 so that the residual amount of power $\bar{y}_j - \sum_{i \in I_1} q_{ji}[t]$ is offered in sub-region I_0 , then the deviation will bring her more profits. Indeed, because the market in this sub-region, with the natural load growth, is large relative to the capacities of all the local producers plus the allowed power imports, the revenue obtained from each of the additional unit of the extra offer in sub-region I_0 will exceed marginal delivery cost. Furthermore, the deviation induces a fall in the price of electricity in sub-region I_0 , and this reduces the amount of electricity that all the other producers are allowed to sell in this sub-region, which in turn raises the marginal revenue of each of the unit that the deviating producer sold before the deviation. The deviation thus raises the profits of this local producer, contradicting the assumption

that the combination of prices cum shipments plans being considered is a Nash equilibrium.

If strict inequality holds in (27), then some of the offers submitted by a producer for sub-region I_1 will not materialize. Under this scenario, such a producer can increase the amount of power she can sell in this sub-region by lowering the bid prices in the load areas where she is not allowed to sell under the optimal dispatch. The argument in the preceding can then be used to show that the deviation is profitable for this producer; again, contradicting the assumption that the combination of prices cum shipments plans being considered is a Nash equilibrium. ■

LEMMA 2: *Consider an instant t and a Nash equilibrium, say $((p_{ji}[t], y_{ji}[t])_{i \in I})_{j \in J}$. Let $q_{j,I_k}[t], j \in J, k = 0,1$, denote the amount of electricity that producer j is allowed to sell in sub-region $k = 0,1$, at time t under the optimal dispatch induced by this Nash equilibrium. If at this instant congestion has set in on the link $\{13,14\}$, then the load areas in sub-region I_1 is served by the Geraldton power station at capacity and the maximum amount of power that can flow into this sub-region along the directed link $[13,14]$. More precisely, we have*

$$(28) \quad \sum_{j \in J} q_{j,I_1}[t] = \bar{y}_{14} + k_{\{13,14\}}.$$

As for the total amount of power sent to sub-region I_0 , it is given by

$$(29) \quad \sum_{j \in J} q_{j,I_0}[t] = \bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}.$$

PROOF: Note that $\bar{y}_{14} + k_{\{13,14\}}$ represents the maximum amount of power that can be supplied to sub-region I_1 . If $\sum_{j \in J} q_{j,I_1}[t] < \bar{y}_{14} + k_{\{13,14\}}$, then there is no congestion – and a fortiori no congestion charge – on the link $\{13,14\}$. Furthermore, according to Lemma 1, each producer will be able to inject the maximum amount of power that it can, and this means that the total amount of power sent to sub-region I_0 will be greater than $\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}$, with the ensuing consequence that the marginal revenue is higher in the former sub-region than in the latter sub-region. Hence some producers will find it profitable to transfer part of the offers submitted for the latter sub-

region to the former sub-region, as argued in the proof of Lemma 1, and this contradicts the assumption that the combination of prices cum shipments plans being considered is a Nash equilibrium. ■

The following proposition, which describes the equilibrium prices of electricity in the NCR, follows immediately from Lemmas 1 and 2.

PROPOSITION 1: *After congestion has set in, i.e., for each instant $t > t_c$, the equilibrium price of electricity in sub-region I_1 , which encompasses all the load areas above the Mungarra power stations, is given by*

$$(30) \quad p_{I_1}[t] = \alpha_{I_1}[t] - \beta_{I_1}[t](\bar{y}_{14} + k_{\{13,14\}}),$$

and the equilibrium price of electricity in sub-region I_0 , which encompasses all the load areas below the Mungarra power station, is given by

$$(31) \quad p_{I_0}[t] = \alpha_{I_0}[t] - \beta_{I_0}[t](\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}).$$

Furthermore, congestion on the directed link [13,14], which transports power into sub-region I_1 , results in a price of electricity that is higher in sub-region I_1 than in sub-region I_0 , and the price differential

$$(32) \quad \eta[t] = \alpha_{I_1}[t] - \beta_{I_1}[t](\bar{y}_{14} + k_{\{13,14\}}) - (\alpha_{I_0}[t] - \beta_{I_0}[t](\bar{y}_5 + \bar{y}_6 + \bar{y}_{12} + \bar{y}_{13} + \bar{I}m - k_{\{13,14\}}))$$

represents the congestion charge on top of the basic access price $\phi[t]$ that a producer must pay to the Transco for shipping one unit of power into sub-region I_1 .

7. THE REINFORCEMENTS OF THE NORTH COUNTRY NETWORK

Because it is beyond the scope of the paper to analyze all the 12 options identified by Western Power, especially when several of them are more of a technical nature, we choose to formalize our model of transmission investments for the NCR by taking as the starting point the option most preferred by Western Power, namely Option 1. The questions we are interested in include (i) the timing and the size of the investments, (ii)

the entry of local generators, and (iii) the prices of electricity in the NCR after the network reinforcements have been made.

7.1. Description of the Reinforced North Country Network

Under the option preferred by Western Power, the reinforcements of the North Country network involve both network deepening and network widening. The network deepening involves upgrading the transmission lines between node 1 (interface node) and node 7 (Eneabba), while the network widening involves the construction of a new transmission line between node 7 (Eneabba) and node 18 (Moonyoonooka), east of Geraldton. The Moonyoonooka station is near Geraldton and is meant to serve Geraldton and the load areas around it. The new line allows for more power than the maximum amount of power that can be transported by the link {13,14} to enter the load areas above the Mungarra power station. As for the upgrade of the links between node 1 and node 7 (Eneabba), the upgraded lines help to raise the limits on the power flows below the Mungarra power station that are imposed by the synchronous stability requirements of the North Country network. The reinforcements proposed by Western Power can thus be justified from the economic point of view, Figure 3 represents the reinforced North Country network, as envisioned by Western Power. In Figure 3, the new line and the upgraded lines are depicted as thicker lines.

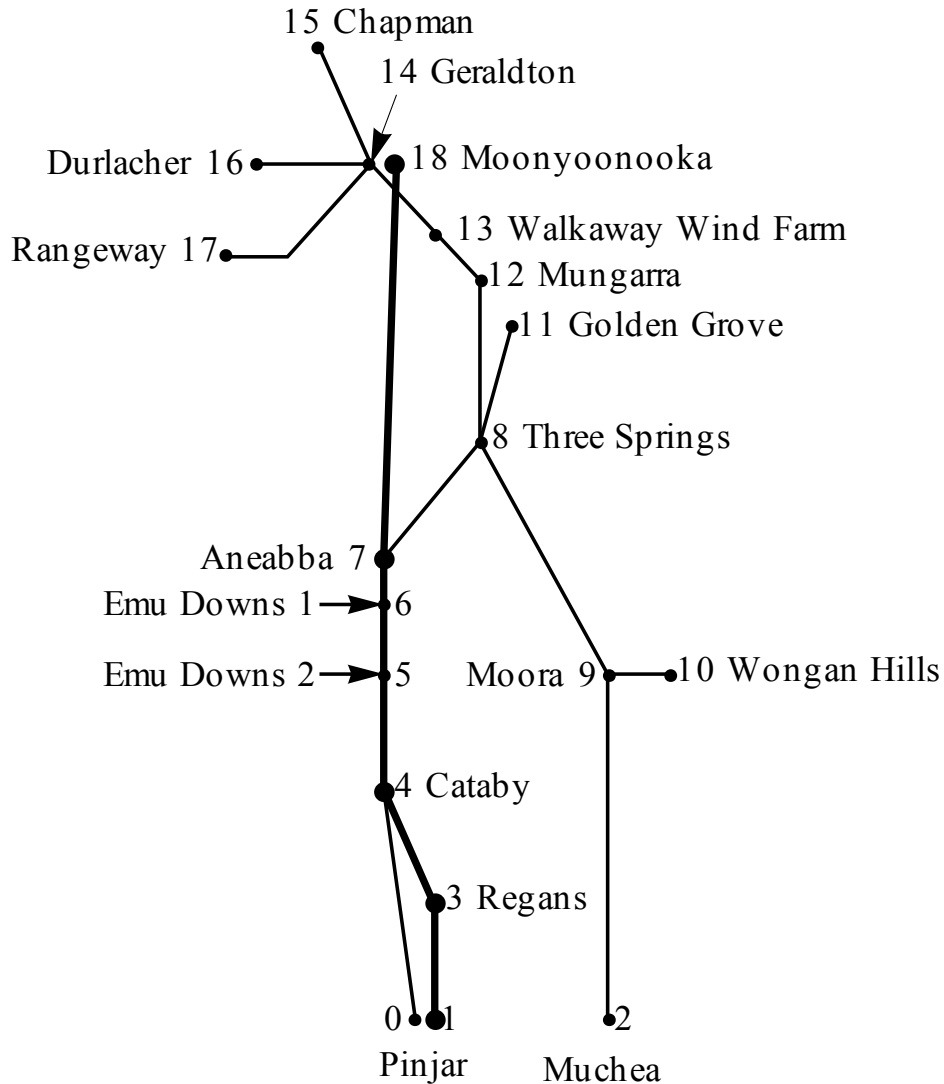


Figure 3. □ The North Country network with the reinforcements

The reinforcements of the North Country network add one more node, node 18, to the current network, and the set of nodes in the reinforced North Country network is $N^+ = \{0, 1, \dots, 18\}$. The set of links in the reinforced North Country network is $L^+ = L \cup \{7, 18\}$. Because the Moonyoonooka station is intended to serve Geraldton, Chapman, Durlacher, and Rangeway, we shall consider this station as one of the load areas above the Mungarra power station. In order not to clutter the Northern extremity of the North Country network, we have chosen not to draw the short transmission lines linking Moonyoonooka and the load areas above the Mungarra power station. The set of demand loads in the reinforced North Country network is then $I^+ = I \cup \{18\}$. In the reinforced North Country

network, the set of load areas above the Mungarra power station is now $I_1^+ = I_1 \cup \{18\}$, while the set of load areas below the Mungarra power station is still the same, namely $I_0^+ = I_0$.

For consistency of notation, we shall assume that the demand for power at node 18 is given by

$$(33) \quad x_{18}[t] = a_{18}[t] - b_{18}[t]p_{18}[t],$$

where $a_{18}[t] = b_{18}[t] = 0$ for all time t .

With this convention, the aggregate demand for electricity in the two sub-regions of the reinforced North Country network is given by

$$(34) \quad \begin{aligned} x_{I_k^+} \left[p_{I_k^+}[t] \right] &= \sum_{i \in I_k^+} \left(a_i[t] - b_i[t]p_{I_k^+}[t] \right) \\ &= \sum_{i \in I_k^+} a_i[t] - \left(\sum_{i \in I_k^+} b_i[t] \right) p_{I_k^+}[t] \end{aligned} \quad (k = 0, 1).$$

We shall assume that the natural load growth in each sub-region tapers off in the long run, and let

$$(35) \quad a_i = \lim_{t \rightarrow \infty} a_i[t], b_i = \lim_{t \rightarrow \infty} b_i[t], \quad (i \in I^+).$$

7.2. The Upgrades and the New Line

The capacity of the new link is $k_{\{7,18\}}$. Because all the existing links between node 1 and node 7 (Eneabba) are 132 kV lines, and presuming that they are equally upgraded, their capacities after being upgraded are all the same and are given by

$$(36) \quad k_{\{1,3\}}^+ = k_{\{1,3\}} + \Delta k, k_{\{3,4\}}^+ = k_{\{3,4\}} + \Delta k, k_{\{4,7\}}^+ = k_{\{4,7\}} + \Delta k.$$

Under the option preferred by Western Power, the upgraded line (which runs from node 1 to node 7) and the new line are both 330 kV double circuits. In our model, both Δk and $k_{\{7,18\}}$ are endogenous, and that the capacity of the new line is not necessarily the same as the capacity of the upgraded line.

The total cost of the transmission investments consists of the cost of the upgrade, say $\Gamma_0[\Delta k]$, and the cost of the new line, say $\Gamma_1[k_{\{7,18\}}]$. The sum of these two cost components gives the total cost of reinforcing the North Country network, and is denoted by

$$(37) \quad \Gamma[\Delta k, k_{\{7,18\}}] = \Gamma_0[\Delta k] + \Gamma_1[k_{\{7,18\}}].$$

We shall assume that the cost of the upgrade satisfies the following conditions: $\Gamma_0[0] = 0, \Gamma_0'[\Delta k] > 0, \Gamma_0''[\Delta k] > 0$. As for the cost of the new line, we shall assume that $\Gamma_1[0] = 0, \Gamma_1'[k_{\{7,18\}}] > 0, \Gamma_1''[k_{\{7,18\}}] > 0$.

7.3. *The Entry of New Generators*

After the North Country network has been reinforced, we can expect that new generators will enter the NCR. To fully specify the entry of new generators, we need to describe their number, their capacities, the generation fuels they use, and their marginal costs. Furthermore, for each entrant, we also need to specify the point of the electric grid at which the entrant injects power. We eschew a detailed description of the entry process, and simply assume that all the entrants inject power at various points on the upgraded line running from node 1 to node 7 (Eneabba). To fix ideas, we shall assume that the entrants inject the power they generate at node 4 (Cataby) and node 7 (Eneabba).¹⁸ We shall refer to the entrants at node 4 and the entrants at node 7, respectively, as producer 4 and producer 7. The set of producers in the reinforced North Country network is now $J^+ = J \cup \{4,7\}$.

For $j = 4,7$, let $\gamma[\bar{y}_j]$, with $\gamma[0] = 0, \gamma' > 0, \gamma'' > 0$, denote the cost of building the capacity level \bar{y}_j . With technological progress in generation, we expect that the unit capacity cost and the marginal cost of generation both fall through time, and this means that the entrants will be able to compete rigorously with the existing producers, and might

¹⁸ According to CRA International (2007), Table 4, AVIVA proposed to build a coal-fired generating plant with 400 MW capacity at Eneabba. The generating plant is expected to be operating in 2011.

be able to displace inefficient existing generators, especially the highly inefficient Geraldton power station. Furthermore, if the new capacities installed are large, we can expect that the entrants will be able to send part of the power they generate down to the South, after meeting the needs of the end-users in the NCR. Furthermore, to capture the diversity of the entrants at each of these two nodes – wind farms, coal-fired generators, gas-fired generators – we shall assume that collectively the entrants at each of these two nodes has a marginal cost curve $c_j[y_j]$, $j = 4,7$, with y_j representing their collective output. The total cost that gives rise to $c_j[y_j]$ will be denoted by $C_j[y_j]$. The curve $c_j[y_j]$ can be expected to be upward-sloping, with several horizontal segments along which marginal cost is constant.

7.4. *The Reinforcements of the North Country Network*

The main objective of the infrastructure investments is to alleviate the supply shortage in the North Country network. Adopting Western Power's perspective that the reinforcements would eliminate the potential congestion in the network, we shall interpret Western Power's objective as making the North Country network and the SWIS an integrated system. That is, after the North Country network has been reinforced, the price of electricity at each instant will be uniform over the integrated system.

For the infrastructure investments problem, we shall let T denote its time horizon. The problem faced by Western Power is to determine the time, say τ , at which the upgrades Δk and the new transmission line with capacity $k_{\{7,18\}}$ are carried out. Thus, an investment program to reinforce the North Country network is a list $(\tau, \Delta k, k_{\{7,18\}})$, where τ is the date the infrastructure investment is carried out; Δk is the upgrade of the transmission capacity of the links between node 1 and node 7; and $k_{\{7,18\}}$ is the capacity of the link between node 7 (Eneabba) and node 18 (Moonyoonooka).

As far as the date of the infrastructure investments is concerned, the benefits will accrue sooner if the investments are made earlier. However, the present value of the investments

costs are also higher in this case, and probably for some time after the investments are made the new line and the upgraded lines will be operated below capacity. On the other hand, if the investments are pushed into the distant future, this action lowers the present value of the investments costs, but the benefits will accrue much later. Hence the infrastructure investments should only be made when the congestion in the North Country network is just severe enough. That is, the optimal date the infrastructure investments should satisfy the inequality $\tau > t_c$, where t_c , we recall, is the exact time congestion will set in if the North Country network is not reinforced.

As for the size of the upgrades and the capacity of the new line, if the investments are low, they will not be able to alleviate the congestion. On the other hand, if the upgrades and the size of the new line are large, the network will run at excess capacity, and this is a waste of scarce resources. Thus, the optimal reinforcements should allow for the North Country network to be operated with congestion at some future time.

For the entrants, the generating capacity they build will depend on the size of the infrastructure investments. The entrants will not build a capacity level that the transmission system cannot support. For simplicity, we shall assume that entry occurs only once and at the same time as the infrastructure investments are carried out. The entry process is represented by a list $(\tau, \bar{y}_4, \bar{y}_7)$, where τ is the exact date the new generating capacities are built; \bar{y}_4 is the generating capacity built by the representative entrant at node 4 (Cataby) and \bar{y}_7 is the generating capacity built by the representative entrant at node 7 (Eneabba).

7.5. *The Entrants' Investments in Generation*

If the upgrade and the capacity of the new line are high enough, the North Country network and the SWIS will become a fully integrated system, and at each instant a uniform price of electricity will prevail over the entire integrated system. Under such a scenario, the demand for electricity in a load area, say $i, i \in I^+$, at an instant t is given by

$x_i[t] = a_i[t] - b_i[t]p_{-1}[t]$. We can expect that the demand for electricity in each load area to be rising through time, and will let

$$(38) \quad \xi_i[T] = a_i[T] - b_i[T]p_{-1}[T], \quad (i \in I^+),$$

denote the demand for electricity at node i at the end of the time horizon. Under such a scenario, and presuming that the Geraldton power station will be retired after the North Country network has been reinforced, the capacity of the new line that meets the objective of Western Power is given by

$$(39) \quad k_{\{7,18\}} = \sum_{i \in I^+} \xi_i [T] - k_{\{13,14\}}.$$

To determine the upgrade Δk , it is necessary to find the best responses of the entrants to $(\tau, \Delta k, k_{\{7,18\}})$, Western Power's program of reinforcements of the NRC network. To this end, let \bar{y}_4 and \bar{y}_7 denote, respectively, the capacity of the entrant at node 4 and the capacity of the entrant at node 7, in response to $(\tau, \Delta k, k_{\{7,18\}})$.

7.5.1. The Best Response of the Entrant at Node 7

Presuming that the output of the Walkaway Wind Farm and part of the output of the Mungarra power station is sent through the link $\{13,14\}$ at the capacity of this link, the residual output of the Mungarra power station to be shipped to sub-region below this power station is $\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13}$. This residual output is sent to serve some of the load areas, such as node 8 (Three Springs) and node 11 (Golden Grove) – below the Mungarra power station. Because the transmission lines between node 2 (Mucnea) and node 8 (Three Springs) are not upgraded, we shall assume that the power import at node 2 (Mucnea) remains at the level $\bar{m}/3$ that we assumed before. Presuming that the power import from node 2 (Mucnea) and the residual output of the Mungarra power station are sent to the part of the network that encompasses nodes 8 (Three Springs), 9 (Moora), 10 (Wongan Hills), and 11 (Golden Grove), the shortfall – at each instant t after the reinforcements – of the total supplies at each instant t over this part of the network is

$$(40) \quad \sum_{i \in \{8,9,10,11\}} (a_i[t] - b_i[t]p_{-1}[t]) - \left(\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13} + \frac{\bar{m}}{3} \right).$$

This shortfall in supplies must be filled by part of the new generation at node 7 (Eneabba). At each instant t , the residual generation at node 7 (Eneabba) – after the representative entrant at this node has (i) met the demand at node 7 (Eneabba), (ii) sent part of her output through the new line to meet part of the demand at the load areas above the Mungarra power station, and (iii) sent part of her output up the Eneabba-Three Springs link to fill the shortfall in supplies represented by (40) – is then given by

$$(41) \quad \zeta_7[t] = y_7[t] - (a_7[t] - b_7[t]p_{-1}[t]) - k_{\{7,18\}} \\ - \left(\sum_{i \in \{8,9,10,11\}} (a_i[t] - b_i[t]p_{-1}[t]) - \left(\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13} + \frac{\overline{Im}}{3} \right) \right).$$

In (41), $y_7[t]$ represents the output at time t of the entrant at node 7. The residual new generation represented by (41) will be sent down to the Perth metropolitan area in the South as exports to the SWIS. Presuming that the outputs of the Emu Downs Wind Farms are sent down the link $\{4,7\}$, the following transmission capacity constraint must hold after the lines between node 4 (Cataby) and node 7 (Eneabba) have been upgraded:

$$(42) \quad \bar{y}_5 + \bar{y}_6 + \zeta_7[t] \leq \kappa_{\{4,7\}} + \Delta k.$$

Note that $y_7[t]$ is the output that maximizes the profit at time t of the entrant at node 7 subject to its generation capacity constraint and the transmission capacity constraint of the reinforced link $\{4,7\}$. That is,

$$(43) \quad y_7[t] = \operatorname{argmax}_{y_7} (p_{-1}[t] y_7 - C_7[y_7])$$

subject to

$$(44) \quad y_7 \leq \bar{y}_7,$$

$$(45) \quad \bar{y}_5 + \bar{y}_6 + y_7 - (a_7[t] - b_7[t]p_{-1}[t]) - k_{\{7,18\}}$$

$$- \left(\sum_{i \in \{8,9,10,11\}} (a_i[t] - b_i[t]p_{-1}[t]) - \left(\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13} + \frac{\overline{Im}}{3} \right) \right) \leq \kappa_{\{4,7\}} + \Delta k.$$

Note that $y_7[t]$ depends on \bar{y}_7 and $(\tau, \Delta k, k_{\{7,18\}})$. and we shall write it as $y_7[t](\tau, \Delta k, k_{\{7,18\}}, \bar{y}_7)$ to make the dependence explicit.

The generation capacity of the entrant at node 7 that is a best response to Western Power's program of reinforcements is then obtained by solving the following discounted profit maximization problem:

$$(46) \quad \max_{\bar{y}_7} \int_{\tau}^T e^{-rt} \left(\begin{array}{l} p_{-1}[t] y_7[t](\tau, \Delta k, k_{\{7,18\}}, \bar{y}_7) \\ -C_7[y_7[t](\tau, \Delta k, k_{\{7,18\}}, \bar{y}_7)] \end{array} \right) dt - e^{-r\tau} \gamma[\bar{y}_7] \\ = \Pi_7[\tau, \Delta k, k_{\{7,18\}}].$$

Note that in (46), r is the market interest rate. In what follows, we shall denote by $\bar{y}_7[(\tau, \Delta k, k_{\{7,18\}})]$ the generation capacity of the entrant at node 7, as a best response to $(\tau, \Delta k, k_{\{7,18\}})$. Whenever there is no possible confusion, we shall simply write this best response simply as \bar{y}_7 .

7.5.2. The Best Response of the Entrant at Node 4

Because the line between node 0 and node 4 (Cataby) is not upgraded, we shall assume that its transmission capacity remains at $\bar{I}m/3$. Furthermore, at each instant t if we let $y_4[t]$ denote the output at time t of the entrant at node 4 then its residual generation, after meeting the load at node 4 (Cataby), is $y_4[t] - (a_4[t] - b_4[t]p_{-1}[t])$. Hence at each instant t the power flow down South on the link $\{3,4\}$, after allowing for the amount of power $\bar{I}m/3$ to be shipped down South to the SWIS through the interface node 0, is

$$(47) \quad \zeta_{\{3,4\}}[t] = \bar{y}_5 + \bar{y}_6 + \zeta_7[t] + y_4[t] - (a_4[t] - b_4[t]p_{-1}[t]) - \bar{I}m/3.$$

The constraint on the transmission capacity of the upgraded link between node 3 and node 4 is then given by

$$(48) \quad \zeta_{\{3,4\}}[t] \leq k_{\{3,4\}} + \Delta k.$$

Using (41) and (47), we can express the transmission capacity constraint (48) as follows:

$$(49) \quad \bar{y}_5 + \bar{y}_6 + y_7[t] - (a_7[t] - b_7[t]p_{-1}[t]) - k_{\{7,18\}} \\ - \left(\sum_{i \in \{8,9,10,11\}} (a_i[t] - b_i[t]p_{-1}[t]) - \left(\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13} + \frac{\overline{Im}}{3} \right) \right) \\ + y_4[t] - (a_4[t] - b_4[t]p_{-1}[t]) - \overline{Im}/3 \leq k_{\{3,4\}} + \Delta k.$$

Part of the power flow represented by $\zeta_{\{3,4\}}[t]$ will be used to meet the load at node 3 (Regans), the remaining part will leave the North Country network at the interface node 1 as export to the SWIS, and the export is given by

$$(50) \quad \zeta_{\{1,3\}}[t] = \zeta_{\{3,4\}}[t] - (a_3[t] - b_3[t]p_{-1}[t]).$$

The following transmission capacity constraint of the upgraded line between node 1 and node 3 (Regans) must hold:

$$(51) \quad \zeta_{\{1,3\}}[t] \leq \kappa_{\{1,3\}} + \Delta k.$$

That is,

$$(52) \quad \bar{y}_5 + \bar{y}_6 + \zeta_7[t] + y_4[t] - (a_4[t] - b_4[t]p_{-1}[t]) - \frac{\overline{Im}}{3} \\ - (a_3[t] - b_3[t]p_{-1}[t]) \leq \kappa_{\{1,3\}} + \Delta k.$$

Using the expression for $\zeta_7[t]$, which is given by (41), we can rewrite the transmission capacity constraint (52) as

$$(53) \quad \bar{y}_5 + \bar{y}_6 + y_7[t] - (a_7[t] - b_7[t]p_{-1}[t]) - k_{\{7,18\}} \\ - \left(\sum_{i \in \{8,9,10,11\}} (a_i[t] - b_i[t]p_{-1}[t]) - \left(\bar{y}_{12} - k_{\{13,14\}} + \bar{y}_{13} + \frac{\overline{Im}}{3} \right) \right) \\ + y_4[t] - (a_4[t] - b_4[t]p_{-1}[t]) - \frac{\overline{Im}}{3} \\ - (a_3[t] - b_3[t]p_{-1}[t]) \leq \kappa_{\{1,3\}} + \Delta k.$$

Note that $y_4[t]$ is the output that maximizes the profit at time t of the entrant at node 4 subject to its generation capacity constraint and the transmission capacity constraints of the reinforced link $\{3,4\}$. More precisely,

$$(54) \quad y_4[t] = \operatorname{argmax}_{y_4} (p_{-1}[t] y_4 - C_4[y_4])$$

subject to (49), (53), and $y_4 \leq \bar{y}_4$.

Note that $y_4[t]$ depends on \bar{y}_4 and $(\tau, \Delta k, k_{\{7,18\}})$. and we shall write it as $y_4[t | (\tau, \Delta k, k_{\{7,18\}}), \bar{y}_4]$ to make the dependence explicit. The generation capacity of the entrant at node 4 that is a best response to Western Power's program of reinforcements is then obtained by solving the following discounted profit maximization problem:

$$(55) \quad \max_{\bar{y}_4} \int_{\tau}^T e^{-rt} \left(\begin{array}{l} p_{-1}[t] y_4[t | (\tau, \Delta k, k_{\{7,18\}}), \bar{y}_4] \\ - C_4[y_4[t | (\tau, \Delta k, k_{\{7,18\}}), \bar{y}_4]] \end{array} \right) dt - e^{-r\tau} \gamma[\bar{y}_4]$$

$$= \Pi_4[\tau, \Delta k, k_{\{7,18\}}].$$

In what follows, we shall denote by $\bar{y}_4[(\tau, \Delta k, k_{\{7,18\}})]$ the generation capacity of the entrant at node 4, as a best response to $(\tau, \Delta k, k_{\{7,18\}})$. Whenever there is no possible confusion, we shall simply write this best response simply as \bar{y}_4 .

7.6. Formal Statement of the Infrastructure Investments Problem

Let $(\tau, \Delta k, k_{\{7,18\}})$ be the infrastructure investments program chosen by Western Power. The social welfare for the NCR that this program of transmission investments is given by

$$\begin{aligned}
 (56) \quad W[\tau, \Delta k, k_{\{7,18\}}] &= \int_0^\tau e^{-rt} \left(\sum_{i \in I} \frac{1}{2} \left(\frac{a_i[t]}{b_i[t]} - p_i[t] \right) (a_i[t] - b_i[t] p_i[t]) \right) dt \\
 &+ \int_\tau^T e^{-rt} \left(\sum_{i \in I} \frac{1}{2} \left(\frac{a_i[t]}{b_i[t]} - p_{-1}[t] \right) (a_i[t] - b_i[t] p_{-1}[t]) \right) dt \\
 &+ \int_0^\tau e^{-rt} \left(\sum_{j \in \{5,6,12,13\}} (p_{I_0}[t] - c_j - \phi[t]) \bar{y}_j \right) dt \\
 &+ \int_\tau^T e^{-rt} \left(\sum_{j \in \{5,6,12,13\}} (p_{-1}[t] - c_j - \phi[t]) \bar{y}_j \right) dt \\
 &+ \int_0^\tau e^{-rt} (p_{I_1}[t] - c_{14} - \phi[t]) \bar{y}_{14} dt \\
 &+ \int_0^\tau e^{-rt} \eta[t] k_{\{13,14\}} dt \\
 &+ \int_0^\tau e^{-rt} \left(\sum_{j \in \{0,1,2\}} (p_{I_0}[t] - c_j - \phi[t]) \frac{\bar{I}m}{3} \right) dt \\
 &+ \Pi_4[\tau, \Delta k, k_{\{7,18\}}] + \Pi_7[\tau, \Delta k, k_{\{7,18\}}]
 \end{aligned}$$

In (56), $p_i[t]$ is the price of electricity at time t in load area i , and $p_{I_k}[t]$, $k = 0,1$, is the price of electricity in sub-region k of the North Country network. Also, $p_{-1}[t]$ is the price of electricity at time t in the integrated system.

Observe that on the right side of (56) the first line and second lines represent, respectively, the present value of the stream of consumer surplus before and after the NCR network has been reinforced. The third and fourth lines represent, respectively, the present value of the stream of profits earned by the original local generators before and after the NCR network has been reinforced. The fifth line represents the present value of the stream of profits earned by the Geraldton power station before the reinforcements.

This power station is presumed to be shut down after the reinforcements. The sixth line represents the present value of the stream of congestion rents collected by Western Power before the reinforcements. The seventh line represents the present value of the stream of profits earned by the generators from the SWIS who sell part of the electricity they produce in the NRC. The two expressions on the last line represents, respectively, the present value of the stream of profits earned by the entrant at node 4 and the entrant at node 7.

The problem of the IMO is to solve the following constrained maximization problem:

$$(57) \quad \max_{(\tau, \Delta k, k_{\{7,18\}})} W[\tau, \Delta k, k_{\{7,18\}}].$$

subject to the following revenue constraint

$$(58) \quad \int_0^\tau e^{-rt} \phi[t] \left(\sum_{j \in \{5,6,12,13,14\}} \bar{y}_j + \bar{I}m \right) dt + \int_\tau^T e^{-rt} \phi[t] \left(\sum_{j \in \{4,5,6,7,12,13\}} \bar{y}_j \right) dt \\ \geq e^{-r\tau} (\Gamma_0[\Delta k] + \Gamma_1[k_{\{7,18\}}]) + \bar{R}.$$

On the left side of (58), the two integrals represent, respectively, the present value of the stream of transmission fees earned by Western Power before and after the reinforcements. On the right side of this inequality, the first expression represents the present value of the transmission investments, while \bar{R} represents the present value of the revenues requirements needed for recovering past infrastructure investments. The constraint (58) ensures that Western Power recovers its infrastructure investments through the access fees.

To solve the constrained welfare maximization problem constituted by (57) and (58), we can apply the Kuhn Tucker theorem. The Lagrangian is

$$(59) \quad \Omega = W[\tau, \Delta k, k_{\{7,18\}}] - \lambda \left(\begin{array}{c} e^{-r\tau} (\Gamma_0[\Delta k] + \Gamma_1[k_{\{7,18\}}]) + \bar{R} \\ - \int_0^\tau e^{-rt} \phi[t] \left(\sum_{j \in \{5,6,12,13,14\}} \bar{y}_j + \bar{I}m \right) dt \\ - \int_\tau^T e^{-rt} \phi[t] \left(\sum_{j \in \{4,5,6,7,12,13\}} \bar{y}_j \right) dt \end{array} \right).$$

Recall that (39) fixes the capacity of the new line, given Western Power's objective of integrating the North Country network with the SWIS. Hence the only relevant variables for the welfare maximization problem are τ and Δk . Differentiating (59) with respect to τ , we obtain the following first-order condition that characterizes the optimal date of the transmission investments:

$$(60) \quad D_1 W[\tau, \Delta k, k_{\{7,18\}}] - \lambda \left(\begin{array}{l} -re^{-r\tau}(\Gamma_0[\Delta k] + \Gamma_1[k_{\{7,18\}}]) \\ -e^{-r\tau}\phi[\tau] \left(\sum_{j \in \{5,6,12,13,14\}} \bar{y}_j + \bar{m} \right) \\ +e^{-r\tau}\phi[\tau] \left(\sum_{j \in \{4,5,6,7,12,13\}} \bar{y}_j \right) \end{array} \right) = 0.$$

The first expression on the left of (60) gives the marginal loss in discounted welfare if the transmission investments are delayed. In the grand parentheses, the first line weighted by the multiplier gives the marginal reduction in the cost of the reinforcements obtained by delaying their implementation. The second line weighted by the multiplier gives the marginal access revenues obtained before the reinforcements. The last line inside the grand parentheses weighted by the multiplier gives the marginal loss in access fees due to the delay. What (60) asserts is that at the time the transmission investments are made the social benefits of a delay in the investments are completely offset by the marginal social loss.

Differentiating (59) with respect to Δk , we obtain the following first-order condition that characterizes the optimal upgrade:

$$(61) \quad D_2 \Pi_4[\tau, \Delta k, k_{\{7,18\}}] + D_2 \Pi_7[\tau, \Delta k, k_{\{7,18\}}] - \lambda e^{-r\tau} \Gamma_0'[\Delta k] = 0.$$

In (61), the first two expressions represent, respectively, the marginal gain in discounted profits of the two entrants, while the last expression represents the marginal cost of the upgrade. What (61) asserts is that the upgrades are for the benefits of the entrants, and at the margin, the combined marginal benefits of the two entrants are equal to the marginal cost of the upgrade weighted by the multiplier.

Now if the price of electricity in the integrated system is stable or rising moderately relative to the growth in electricity demand, then we can expect that the power sent from local generators to the South will decline through time, and we have the following proposition gives some properties of the optimal program for reinforcing the North Country network.

PROPOSITION 2: *Suppose that the objective of Western Power is to reinforce the North Country network so that it becomes fully integrated with the SWIS.*

- (i) *The entry of new generators will more than meet the needs of the end-users in the NCR, and the NCR now exports power to the SWIS.*
- (ii) *At the time the investments are made, the new line is not used at full capacity. Its capacity is only fully exploited at the end of the time horizon.*
- (iii) *At the time the investments are made, the upgraded lines operate at capacity. As time goes on and demand for power grows at each load area in the NCR, more and more power will be retained to serve end-users in the NCR, leaving less and less power to be exported to the South, and this means that the upgraded lines will run below capacity most of the time.*

8. CONCLUDING REMARKS

In this paper, we have formalized a model of transmission investments for an electric power network. To fix ideas, we have chosen to formulate the model in the context of the North Country network in Western Australia. In the model, the topology of the North Country network is formalized and analyzed. We believe that our model can serve as a stepping stone for electric power networks with more complicated topologies.

Our modeling strategy stands in contrast with the strategy adopted in most conventional models in which the network analyzed contains only two nodes, with a third node occasionally thrown in to illustrate the method or to illustrate loop flows. When the network is modeled with the help of graph theory, no specific structure for the graph is given, and thus the model can only handle network deepening, not network widening. To analyze network widening, one must know where to build a new transmission line, and this is not possible without a definite structure for the network. By focusing on a particular network – the North Country network – we are able to analyze both network deepening and network widening.

Another important feature of our model is that we formulated a model dealing with both investments in transmission and investments in generation. Our model captures the present state of deregulation in the electric sector in countries, such as the US, the UK and Wales, and Australia. In these countries, the transmission segment is regulated while competition is encouraged in the generation segment. Our model has been formulated as a leader-follower game in which the Transco is the leader and electric producers are the followers. The Transco carries out infrastructure investments under the guidance of the regulator, and is allowed to recover its investments as well as to earn a decent rate of return on the investments.

Finally, our model of transmission investments deals with a radial network with a rather simple topology. The next stage in the research is to consider more complex topologies and networks with loop flows.

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