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Three Essays on Population Ageing and Globalization
TITRE DE LA THÈSE / TITLE OF THESIS

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Three Essays on Population Ageing and Globalization

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Thesis submitted to the

Faculty of Graduate and Postdoctoral Studies

In partial fulfillment of the requirements

For the PhD degree in Economics

Department of Economics
Faculty of Social Science
University of Ottawa

May 2009

Your file Votre référence
ISBN: 978-0-494-59493-3
Our file Notre référence
ISBN: 978-0-494-59493-3

Acknowledgement

My deepest gratitude goes to my supervisor, Professor Marcel Mérette for his time and numerous strength devoted into helping me to capture all inspiring ideas and topics in relevant research fields, to initialize and polish the topics and to build the models in the thesis. An expert in CGE modeling, Professor Yazid Dissou has kindly offered me technical helps and insightful suggestions on my thesis ever since the very beginning. I also treasure the continuous support from Professor Zhiqi Chen and Professor Nguyen V. Quyen as committee members of this thesis. Special thanks to Professor John Whalley from the University of Western Ontario, the external examiner of the thesis, for his valuable comments and suggestions.

Maxime Fougère, together with his research team at Policy Research Directorate at the Department of Human Research and Skill Development Canada (HRSDC), has also shown tremendous understanding of and provided numerous conveniences in me writing the thesis. I have also significantly benefited from working on research projects at HRSDC and receiving expertise comments from the team.

Finally, I am obliged to my wife, Xiaochuan Wang and all family members and friends for their understanding and supports in the past several years. I also take the accomplishment of the thesis as the memorial of my father, Qichang Zhu, who passed away in his sleep on April 20 2009.

Writing a thesis is a huge project for most PhD Candidates. Supports from the university, the department, the family and the community have made this prolonged process pleasant and enjoyable. All of us share the success of this thesis.

Ottawa, Canada, May 2009

ABSTRACT

Chapter One: Revisiting the Issues: Free Trade and Demographic Transition. Over the next several decades, countries around the world will experience varied degrees of population ageing. Their population growth rates are also projected to be unequal across countries. Meanwhile, the accelerating process of globalization is making national markets more and more integrated through international trade. This paper uses an overlapping-generations computable general equilibrium model to investigate the evolution of trade and its welfare implications for two open economies during demographic transitions. Initially, the two economies are assumed to be identical except for the population growth rate. Two scenarios are considered for different patterns of demographic transition. Under both scenarios, one of the two economies becomes comparatively older than the other. The resulting changes of relative factor abundances give rise to different comparative advantages across economies and create incentives for trade. The Armington assumption is applied into the trade model and the two economies are assumed to export and import both capital-intensive and labour-intensive goods. Opposite to the result from related literature, our model shows that the older economy becomes better off and the younger economy becomes worse off after opening to trade. Furthermore, we find that the gap between the welfare levels of two economies is positively correlated with the steady-state population growth rate. Our results cast some light on the continued welfare gap between the developing and the developed world during globalization.

Chapter Two: International Migration, Skill Composition and Population Ageing: an OLG-CGE Study. Major industrial countries are facing varied degrees of slowing down of labour force growth resulted from population ageing. To fix this problem, many of them

are considering adopting more immigrants. Migration is also taken as an option to accommodate other negative impacts of ageing including to alleviate possible fiscal pressure. This paper uses an overlapping-generations general equilibrium model to analyze the impacts of different patterns of international migration on ageing economies. Based on the findings from recent literature, this paper introduces an aspect important in the analysis of migration: international trade is used to investigate the interactions between good mobility and labour mobility. The emphasis has also been put on the heterogeneity of labour and skill composition of immigrants. GTAP6.0 data is applied to calibrate the developed region, the developing region and the rest of the world. Simulation results in this paper suggest migration from the younger developing countries to the older developed countries may not be a win-win game for both of them. An older country could benefit from the inflow of immigrants with higher skill composition. However, the welfare gap between the developed and the developing world will be enlarged as the result of migration of skilled labour. In the paper, the effort of combining international trade and migration proves that good mobility is critical in analyzing international migration: the impacts of migration are well explained by the changes of comparative advantages across countries and terms of trade. This paper also shows that in the context of ageing, trade may be complementary, instead of being substitutable, to migration across countries. International trade increases with the scale and skill composition of migration.

Chapter three: An OLG-GE Modeling Framework for Endogenous Migration. The new era of international migration is accompanied with accelerating process of population ageing and deeper degree of globalization. Despite more aged developed countries' effort to adopt more immigrants to combat possible negative impacts of ageing, the demand from emerging economies for foreign intelligence is also increasing with the outspread of

multinational enterprises and outsourcing. Based on a two-country, three-input and three-generation OLG general equilibrium (GE) model, this paper introduces the concept of endogenous migration into a dynamic GE framework. In the model, the developed and the developing countries are calibrated based on real data and have different paths of population growth. Labour stock in each region is differentiated both by their skill levels and their countries of origin. For the first time in the dynamic GE modeling, we introduce a series of CET (constant elasticity of transformation) equations to capture the supply side of labour market. Population growth has also been endogenized in the model, and each region's population is recalculated based on bilateral migration in each period. Simulation results from our model suggest some stylized facts as observed in the process of globalization. For example, it is shown that more people migrate from the younger countries to the older countries. There will also be wage differentials between native and immigrant workers. Finally, there will be a more significant tendency for workers to move from the developed countries to the developing countries, as a larger economy, with the booming of bilateral trade.

CONTENTS

Introduction

Globalization and population ageing are two mostly explored fields of research for researchers, scholars and policy makers all over the world. International trade, migration and capital mobility, as three major components of globalization, could all be seen as the exchange and transactions of resources in the world market. Aside from numerous empirical studies on globalization since the invention of Heckscher-Ohlin theorem in the 1930s, computable general equilibrium models has been widely used to more efficiently explore both the micro and macro scopes of international mobility with the development of information and computer technologies in the past decade. Compared to globalization, population ageing is a relatively new research topic first been paid much attention to by many industrial countries recently. Varied sources (UN, OCED and government authorities) have already projected several negative impacts of population ageing on older economies in the world. This raised much of their concern on the continuous growth of labour force, sustainable economic growth and the fiscal soundness in supporting growing pension and health expenditures for the elderly in the economy.

Globalization and ageing are actually two highly correlated issues. Apparently, one of them focuses on the integrated global markets and transaction prices while the other is only a demographic shock on national labour markets. However, it is the projected uneven demographic growth paths for countries around the world that adds much more value to the exploration of the interactions between population ageing and globalization. Recall the contents of those fundamental trade theorems of comparative advantage and factor price equalization, it is not hard to find how important national demographic shocks are in determining the picture of international mobility. The typical questions we could raise include does trade benefits all trade partners, richer or poorer, older or younger? Is there any interactive relationship among trade, migration and demographic shock? Are trade and migration complements or substitutes for countries with different paths of population growth?

Inspired by the pioneering work by Sayan and his partners who first applied demographic shocks into a general equilibrium trade model in 1999, this thesis concentrates on extending their ideas to analyze interactions among trade, migration and ageing within more realistic model settings. First, in all three chapters of the thesis, international trade is treated as spirit of both our CGE modeling and our analysis. Most of our data sources are from GTAP6.0 which is most specialized in sectoral trade flows among countries in the world. In Chapters 2 and 3, all regions are calibrated based on this dataset. Second, in the trade models, the Armington assumption is a critical presumption of all our models because of its prevailing status and wide recognition by researchers in the field of international trade since its first invention in 1969. This application of the Armington assumption has been proved to have significantly affected our simulation results or even reversed changing directions of critical variables in the dynamic. In the first two chapters, two fundamental conclusions are interpreted using the Armington assumption in our trade model. In our attempts to build an endogenous-migration model in chapter three, the idea of tradable goods being differentiated by their countries of origin have successfully been applied into modeling the supply side of national labour market. Finally, our analysis elaborated the interactions among trade, migration and population ageing.

More specifically, our thesis started from chapter one in which a simple two country, two sector CGE model was developed to test the welfare predictions by Sayan (2005) who stated free trade eliminated the welfare gap between the South and the North. Many aspects of our own multiregional CGE model are identical to Sayan's model including initially symmetric two countries and the UN-projected population growth rates. It was found that in an Armington-type trade model, older and richer countries become richer in the long run but the poorer countries become poorer in terms of real GDP per capita and lifetime utilities for individuals. This could be simply explained by the fact that on one hand, the rule of differentiating tradable goods by countries of origin further enhanced the older countries' comparative advantage in capital-intensive goods. On the other hand, in order to meet greater demands from the more populated and larger developing markets, factor prices in the older countries move in favor of both capital and labour owners. A series of movements of factor prices in two countries determined the upwards or downwards trends of movements of lifetime wealth, lifetime utility and real GDP per capita. Our conclusion from Chapter one is right opposite to Sayan's predictions.

In chapter two, we moved forward to introduce labour mobility into a trade model with Armington assumption. Not only real data have been widely used in calibration, setting the reasonable scale of migration and simulating the predicted paths of demographic evolution, but also the interactions among trade, migration and ageing have been elaborated. It was shown that only migration with higher skill composition benefits both the destination and source countries. This benefit, however, is persistent for the developed world but diminishing for the developing world. The benefit or loss has been explained through changes in terms of trade for individual trade partners. Skill composition and factor intensity in production were found to be important in determining the economic impacts of immigration. Our conclusion on the welfare changes may be reversed in case of factor intensity reversal. In addition, we provided a new answer to the question of if trade and migration are complementary or substitutes. It was shown that in the context of population ageing, trade activities could be enhanced by migration between the South and the North. The scale of enhancement also increases with the skill composition of immigrants.

Finally, comparing with the model used in chapter two treating the net migration as exogenous shock, our chapter three can be seen as an extension to chapter two in order to capture a new trend of international migration: the bilateral labour mobility both from the developing to the developed world and from the developed to the developing world. This new trend has become clearer with the development of out-sourcing of multinationals in the developing countries and the faster paces of the national industries in emerging markets catching up with their developed trade partners. Foreign intelligence is highly demanded by the developed countries. The emphasis and contribution of our chapter three, however, was the modelling designs: we introduced a series of CET technologies, as used in modelling the supply of tradable goods in the world market, to model the supply of labour. Several obstacles have been overcome or substituted by alternative components in the model. Real data has also been used in calibration, and the simulation results seemed to have duplicated some stylized facts on international migration. Most importantly, it explained the wage differentials between native and immigrant workers at the same skill level. Foreign workers are overpaid compared to the natives in the South but underpaid in the North. Like we have found in chapter one, the larger scale of the developing economy takes effect again, and it helps to attract more people born in the developed region to relocate to the developing countries though labour is more scarce in their home country. There is also a clear trend for immigrants in the developed region to "re-migrate" to their home countries.

Although we have obtained varied "surprising" or "unsurprising" results in the thesis, we have fully realized the difficulty in conducting CGE studies to analyze trade, migration and ageing because of its more-than-usual requirement on varied data. In the process, we have been deterred by the lack of many important data. The robustness of some results has also become questionable because of incomplete usage of real data. For example, the quality of data for the developing countries is not as good as it is for the developed countries in GTAP dataset. In the field of migration, immigration data in the developing countries were very rare to find so that we have to assume some important share parameters. Many important elasticity values (the elasticity between immigrant and native workers, for example) are still to be tested by further empirical studies.

Further enhancements to the thesis require more complete datasets. There is also much room for us to conduct sensitivity tests in chapters two and three. In all three chapters, capital mobility was not considered in the dynamic model. Taking this into our modeling and analysis may generate more interesting results.

Chapter One

Revisiting the Issues: Free Trade and Demographic Transition

1.1. INTRODUCTION

The world population is experiencing a dramatic demographic transition: population growth is slowing down and the portion of the elderly keeps increasing. This process of ageing will further accelerate after 2010 when the baby boom generation[1] begins to retire (Mérette 2002, 2004). According to International Monetary Fund (IMF, 2004), the improvement in life expectancy and decrease of fertility rate are expected to drag down the overall population growth rate from around 1.25% in 2000 to 0.28% by 2050. Besides major OECD industrial countries, some developing countries are also projected to encounter a significant decline in population growth in the coming decades. Among them are developing economies in East Asia and East Europe. In other developing regions like West Asia and Africa, population growth is also slowing down but less rapidly. Overall, the world population will be in a process of ageing during which unequal population growth rates across countries are projected to be persistent over the next several decades. Figure 1 shows population growth rates for different regions projected by United Nations (UN, 2002).

1 The baby boom generation refers to the cohorts born between 1946 and 1966.

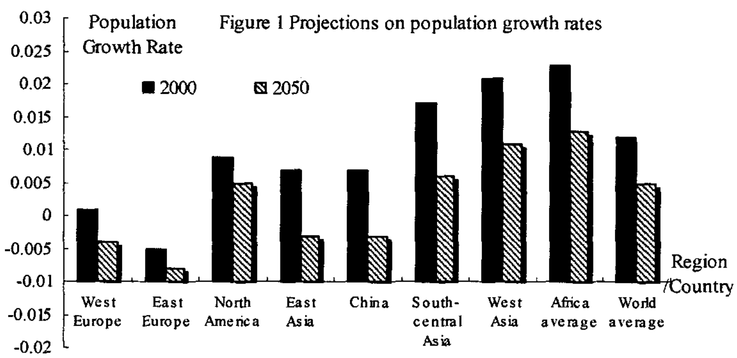


Figure 1 Projections on population growth rates

We also have observed a new phase of globalization over the past decades. National markets have become more and more integrated with the rest of the world and the share of international trade in Gross Domestic Product (GDP) has increased dramatically. Standard Heckscher-Ohlin (HO) model shows that given different factor intensities in the productions of tradable goods, a country produces more and exports the goods that use its abundant factors intensively. In the context of ageing, unequal population growth rates and the resulting divergence in the elderly dependency ratio (EDR)[2] have a direct impact on factor abundances across countries. On global market, the existing dissimilarities in relative factor abundance between capital-abundant economies in North America, Europe and East Asia and the rest of the world will continue to grow over the coming decades (UN 2001). Thus, it is highly relevant to investigate how and to what degree the demographic transition could affect the pattern of trade and the welfare implication of free trade for countries during the demographic transition.

Using a two-country, two-good, two-input and two-generation (2*2*2*2) overlapping generations computable general equilibrium (OLG-CGE) model, this paper investigates the evolution of trade and the welfare implications for two open economies during different

2 The EDR is defined as the ratio of people aged 65 and over to the working-age population (15 to 64 years old).

patterns of demographic transition. Initially, two countries are assumed to be identical in every aspect except for the population growth rate. During the demographic transition, one country grows into a relatively older and capital-abundant country. The Armington assumption is applied into our trade model to generate a more realistic picture of trade in which both capital-abundant and labour-abundant countries are trading both capital-intensive and labour-intensive goods. The simulation results of our model show that the older economy (with a lower initial population growth rate and a faster ageing process) gains from trade while the younger economy (with a higher population growth rate and a slower ageing process) loses from trade. The gap between the two countries' welfare is increasing during the demographic transition and persistent in the long run. This result is consistent with the prediction by Chen (1992) that "the world economy will approach a long-run equilibrium where the rich countries remain rich and the poor countries remain poor" (p.941). However, the directions of welfare changes for the older and the younger countries predicted by our trade model are opposite to those presented in Sayan (2005) which introduces demographic transition for two open economies within a HO framework. Moreover, the gap between the welfare levels of two countries is found to be positively correlated with the assumed long-run steady-state population growth rate. Our simulation results cast some light on the continued gap between the welfare levels of the developing and the developed world during globalization. We also provide a brief projection on the evolution of many variables for open economies during the demographic transition.

This paper is organized as follows. Section 2 reviews the related research papers. Section 3 describes the OLG-CGE model and Section 4 presents basic simulation results and comparisons. Section 5 concludes.

1.2. REVIEW ON RELEVANT LITERATURE

1.2.1 OLG researches on trade issues

According to the standard HO theorem, differences in factor abundance give rise to different comparative advantages across countries and thus create incentives for trade. Indeed, an economy has comparative advantage in producing and exporting the commodity that uses its abundant factor intensively. The welfare prediction of HO theorem is that all participating economies gain from trade given that resources are used more efficiently in production. Compared to autarky, trade partners benefit from the favourable terms of trade leading to a higher value of their export goods and less expensive imported goods for the consumers.

Over the past decades, many attempts have been made to test the predictions of the HO model using dynamic OLG frameworks. Buiter (1981) extended a Diamond type OLG model (Diamond 1965) into a two-country, one-sector dynamic model to investigate current account dynamics for two countries with different time preference rates. It was shown that an open economy with a higher time preference rate runs a current account deficit in the long run. The other country with a lower time preference rate, however, is always a lender on global financial market. The two countries' welfare levels (measured be stationary utility level for an individual) can be either higher or lower than their autarky levels which depends on the position of their initial capital-labour ratios compared to the golden rule capital-labour ratio.

Galor (1992) introduced two production sectors into a two-generation OLG growth model to investigate the global dynamics of perfect-foresight equilibria of capital-labour ratio. He found that multiple equilibria exist when consumption good is capital-intensive and income effect is sufficiently stronger than substitution effect when technologies are dissimilar across sectors. Galor and Lin (1994) developed a two-country, two-sector, two-input dynamic OLG model to investigate the relationship between terms of trade and current account dynamics. They concluded that when the change of terms of trade is no longer ignorable in the world market, world interest rate will be affected as well, which directly changes the dynamics of current accounts. Given a type of Hicks-neutral technological progress and the resulting terms-of-trade shock, a small open economy may experience dramatically opposing patterns of current account dynamic under different scenarios. Based on Galor's findings, Mountford (1997) used a two-country, two-good and two-input dynamic OLG HO model to examine the implications of multiple equilibria of steady-state capital-labour ratio for convergence literatures. According to him, international trade and the existence of multiple equilibria may generate some of the "stylized facts". Under free trade, two economies with an identical saving rate may end up with different GDP per capita levels. A poor economy (with a lower initial capital-labour ratio) may face an even lower capital-labour ratio. The existence of multiple equilibria under trade may reduce the welfare of one economy without improving the welfare of the other. Guillo (2001) also used a two-country, two-good and two-input dynamic OLG HO model to explain the relationship between relative price and trade balance. It was shown that for an open economy exporting only labour-intensive good, its trade balance is always positively related to relative price of export. If its export is capital intensive and consumption good is labour intensive, an inverse relationship is observed given the condition that there is a strong response of saving to changes in future prices. If both export and consumption goods are capital-intensive, an inverse relationship may also arise when the negative effect of labour income on the relative price of consumption is sufficiently large.

1.2.2 OLG researches on population ageing

Population ageing is a pressing issue in many countries, which has attracted the attention of economists around the world. Dynamic OLG-CGE models have been widely used in economic literatures to investigate the economic impacts of population ageing recently. OECD (1998) used a one-sector OLG-CGE model (the Minilink Model) to analyze domestic and international macro effects of varied degrees of ageing on international flows of good and capital. It showed that population ageing will substantially increase government pension and health expenditures. Several reforms, including public debt stabilization and encouraging labour participation, are suggested. It was stated that if those reforms are not adopted, there will be serious fiscal problems for ageing economies and economic growth will suffer. Miles (1999) developed a dynamic OLG-CGE model to simulate the evolution of saving patterns of forward looking people based on demographic projections on European countries. In this paper, age-specific saving rates are taken into account. He found that the private saving rate for Europe significantly drops as a result of population ageing in the long run. The impact of ageing on rate of return on capital, however, is moderate because lower saving rate is compensated by a smaller working force. INGENUE (2001) used a multi-regional OLG-CGE model to analyze capital accumulations induced by different demographic characteristics in the context of a single global financial market with free trade. It is shown that at the beginning of the projected demographic transition, a high current account deficit prevents a catching-up developing country from borrowing enough on global financial market. With the increase in the portion of young savers in the economy, a younger country turns to be a lender on global financial market in the long run. Based on simulation results from his OLG-CGE models, Mérette (2002) presented several mechanisms that could offset the negative effects of population ageing on public finance and hence the living standards of rich countries. He showed that a rise in the capital-labour ratio in an ageing economy encourages human capital accumulation. New investment in human capital significantly boosts the economic growth rates for seven OECD countries (Fougère and Mérette 1999). In the aspect of public finance, a generous tax-deferred saving plan, similar to the RRSP in Canada, will also offset some worrisome fiscal strains brought by population ageing.

Sayan and Uyar (2001) built a two-country, two-good, two-input, two-generation dynamic OLG-CGE model to investigate the welfare implications of free trade and full labour mobility for two countries with unequal population growth rates. It was shown that when free trade is allowed without labour mobility between countries, the welfare for an individual in the older economy deteriorates while the welfare for an individual in the younger economy improves compared to autarky. When full labour mobility is allowed without trade, an individual in the destination (older) economy loses while an individual in the sending (younger) economy gains in welfare. Based on a similar model, Sayan (2005) focused on the interactions between free trade and demographic transition applying population growth rates data projected by UN (2001)[3]. It was also shown that the older country with a faster ageing process loses from free trade. The younger country, with a slower ageing process, gains from free trade. Thus Sayan concluded that "Contrary to those

3 The unequal population growth rate levels are based on the 1950 values of average population growth rate for "more-developed regions" and "less developed regions" projected by UN (2001)

predictions (of the static HO model), however, trade would not necessarily represent a Pareto improvement over the state of autarky" (p.1491).

Both Sayan and Uyar (2001) and Sayan (2005) incorporate demographic transitions to test the welfare implications of free trade within a dynamic HO framework. Incentives for trade is generated by the fact that two countries become either relatively capital abundant or labour abundant with unequal population growth rates. The older country was assumed to specialize in producing and exporting the capital-intensive good when the younger country specialize in the labour-intensive good, as predicted by the HO model. The simulation results supported the previous argument that welfare implications predicted by standard HO model might not hold under different dynamic scenarios. In Sayan (2005), two demographic scenarios are considered. Under scenario one, country one has a lower and positive initial population growth rate which declines faster during the dynamic. Country two has a higher and positive initial population growth rate which declines at a lower speed. Under scenario two, both countries have a constant population growth rate but country one's population growth rate is lower[4].

In the aspect of welfare, the welfare loss for an individual in the older economy and the welfare gain for individuals in the younger economy were explained as "a result of these changes in relative commodity and factor prices" (Sayan 2005, p.1488). The driving forces of the welfare changes remain for further discussion.

1.3. MODEL DESCRIPTIONS

4 The author does not explain how the projected permanent difference in the population growth rates, as projected under scenario two, is sustainable in the long run. In other words, periods of high and positive population growth turns one country into a large economy. The resulting picture of international trade will definitely change in the long run when a large economy is trading with a small open economy.

This paper uses an OLG-CGE model to investigate the welfare implications for two open economies during demographic transitions in both the autarky and the trade case. In the model, we have two countries (*J1*, *J2*), two homogenous[5] goods (*S1*, *S2*), two inputs (capital and labour labelled as *Ld*, *Kd*) and two generations (the working generation *G1* and the retired generation *G2*) of population. The productions of two goods use different factor intensities of labour and capital inputs. Technologies are assumed to be identical across countries. No government sector is included. Thus there is not tax imposed on consumers or producers. Inheritance and bequest are not taken into account.

1.3.1 Firm's problem

In each country, there are two sectors producing each one of the two goods *s* (*S1* or *S2*) using different production technologies. In the production functions, we denote different parameters to differentiate technologies used by productions of the two goods: SP_s[6] is a scaling constant parameter and $\alpha^K{}_s$ is the expenditure share measuring the intensity of use of capital in production. *S1* is assumed to be a capital-intensive good and *S2* a labour-intensive good. Technologies are assumed to be identical across countries. A representative firm's problem is to maximize the profit subject to the embedded constraint that characterizes the firm's technology:

$$\underset{Kd_{j,s,t},Ld_{j,s,t}}{Maximize}\, Pq_{j,s,t}X_{j,s,t} - \mathrm{R}_{j,t}\,Kd_{j,s,t} + W_{j,t}Ld_{j,s,t} \qquad (1)$$

$$s.t.\ X_{j,s,t} = SP_s Kd^{\alpha_s^K}_{j,s,t} Ld^{(1-\alpha_s^K)}_{j,s,t} \qquad (2)$$

5 Homogenous in this paper means that the technologies employed in the production of each good are identical across countries.

6 In this paper, *SPs* are assumed to be one in all sectors.

In equation (1), $Pq_{j,s,t}$ is the production price of good *s* in country *j* at time *t*. $R_{j,t}$ and $W_{j,t}$ represent the rental price and wage rate in country *j* at time *t* respectively. $Kd_{j,s,t}$ is the capital demanded by production sector *s* in country *j* at time *t* and $Ld_{j,s,t}$ is the labour demanded. In equation (2), $X_{j,s,t}$ is the quantity of production of good *s* in country *j* at time *t*. The technology is represented by a Cobb-Douglas production function in which the sum of expenditure shares for capital and labour inputs is equal to one. We assume that input factors are perfectly mobile across sectors in each country. Consequently, the wage and rental rates are equalized across sectors. Moreover, we assume that all firms hire factors and sell outputs competitively on the markets, so the price of each factor is set to be equal to its the marginal product. Differentiating equation (1) with respect to equation (2), we get the following equation (3) and equation (4) as the first order conditions for producers in the economy:

$$R_{j,t} = \alpha_s^K \frac{Pq_{j,s,t} X_{j,s,t}}{Kd_{j,s,t}} \tag{3}$$

$$W_{j,t} = (1-\alpha_s^K) \frac{Pq_{j,s,t} X_{j,s,t}}{Ld_{j,s,t}} \tag{4}$$

1.3.2 Consumer's problem

An Allais-Samuelson overlapping generation framework characterizes individuals in the economy, so that this model is based on the life-cycle theory of savings. The determinants of saving include individual's labour income, demographic characteristics and rate of return on capital in the economy. In period *t*, each economy is populated by two generations *g* (*G1* and *G2*) of population. A representative cohort born in period *t* works in period *t* and retires in period *t+1*. Each period *t* approximately represents 30 years in our model.

The lifetime utility for a cohort is represented by a time-separable nested CES function. The optimization problem for a representative cohort can be divided into two steps in the autarky case and three steps in the trade case. First, immediately after birth, a representative cohort decides on the allocation of its labour income into either current or future aggregate consumption. In other words, part of its labour income is saved and the rest is consumed. In the second period of its life, this cohort spends all the wealth, which includes saving in the first period and its interest income, on consumption. On the second step of optimization, the aggregate consumption is allocated among different sectors. Finally, in the trade case, an individual decides on the composition of each consumption good s in terms of the origin of production. That is, in the trade case based on the Armington assumption, consumption of good s consists of both locally-produced products and imported products.

The intertemporal optimization problem for a representative cohort in country j is to maximize its lifetime utility subject to the following budget constraints:

$$\underset{C_{j,G1,t},C_{j,G2,t+1}}{Maximize}\, U(C_{j,g,t})_{j,t} = \frac{\sigma^{IT}-1}{\sigma^{IT}}\left[C_{j,G1,t}^{1-\frac{1}{\sigma^{IT}}} + \left(\frac{1}{1+\rho}\right)C_{j,G2,t+1}^{1-\frac{1}{\sigma^{IT}}}\right]^{\frac{\sigma^{IT}}{\sigma^{IT}-1}} \quad (5)$$

$$s.t. \quad W_{j,t}Ls_{j,G1} = PCon_{j,G1,t}C_{j,G1.t} + LD_{j,G2,t+1} \quad (6)$$

$$and \quad LD_{j,G2,t+1}RRT_{j,t+1} = PCon_{j,G2,t+1}C_{j,G2.t+1} \quad (7)$$

The aggregated lifetime utility for a representative cohort is shown by equation (5) and includes the present values of this cohort's consumptions in the two periods of its life. $U_{j,t}$ is the lifetime utility for a cohort born in country j at time t and $C_{j,g,t}$ is the aggregate consumption by an individual of age g at time t. σ^{IT} is the intertemporal elasticity of substitution which stands for the degree of substitutability of the cohort's consumptions across its two periods of life; ρ is the pure rate of time preference indicating the degree to which the cohort would prefer current consumption over future consumption. The larger ρ, the more of its lifetime resources a cohort would spend in the first stage of life and the less it saves. In both countries, each cohort supplies an identical quantity and quality of labour in the first half of its life.

Equation (6) shows that the wage income earned by a cohort in the first period of its life is allocated into either current consumption or saving (labelled as $LD_{j,G2,t+1}$). On the right hand side (RHS) of this equation, $PCon_{j,G1,t}$ is the aggregate consumption price for the first generation $G1$ in country j at time t. In period $t+1$, this cohort spends the principle of its saving at time t and the interest income on consumption, which is shown by equation (7). In this equation, $RRT_{j,t+1}$ equals to one plus the rate of return on capital in country j at time $t+1$. The value of $RRT_{j,t}$ is determined by the real rental price net of depreciation augmented by anticipated capital gains.

Differentiating a cohort's lifetime utility with respect to the budget constraint yields the following first order condition for the aggregate consumption $C_{j,g,t}$ (Equation 8):

$$C_{j,G2,t+1} = \left[\frac{RRT_{j,t+1} PCon_{j,G1,t}}{(1+\rho) PCon_{j,G2,t+1}} \right]^{\sigma^{IT}} C_{j,G1,t} \tag{8}$$

Equation (8) shows that the aggregate consumption for an individual of age g at time t is negatively correlated with the aggregate consumption price at time t and positively correlated with the aggregate consumption price at time $t+1$.

On the second step of optimization, an individual of age g allocates the optimal aggregate consumption, $C_{j,g,t}$, across sectors. The problem is to maximize the following CES utility function subject to budget constraint:

$$\underset{CS_{j,S1,g,t},CS_{j,S2,g,t}}{Maximize}\ C_{j,g,t} = (\delta_{S1}^{CS} CS_{j,S1,g,t}^{1-\frac{1}{\sigma^{CS}}} + \delta_{S2}^{CS} CS_{j,S2,g,t}^{1-\frac{1}{\sigma^{CS}}})^{\frac{\sigma^{CS}}{\sigma^{CS}-1}} \quad (9)$$

$$s.t.\ PCon_{j,g,t} C_{j,g,t} = PCons_{j,S1,t} CS_{j,S1,g,t} + PCons_{j,S2,t} CS_{j,S2,g,t} \quad (10)$$

The left hand side (LHS) of equation (9) is the aggregate consumption by generation g in country j at time t. This equation shows the aggregate consumption is a CES form of consumptions across sectors. We denote δ_s^{CS} as the parameter representing the preference of a consumer of age g on consumption goods across sectors. σ^{CS} is the elasticity of substitution for consumptions across sectors. $CS_{j,s,g,t}$ is the quantity of consumption of good s by an individual of age g in country j at time t. In equation (10), $PCons_{j,s,t}$ is consumption price of good s. The following equation (11) and (12) are the first order conditions for the second step of optimization:

$$CS_{j,s,g,t} = \delta_S^{CS^{\sigma^{CS}}} \left[\frac{PCon_{j,g,t}}{PCons_{j,s,t}} \right]^{\sigma^{CS}} C_{j,g,t} \quad (11)$$

$$PCon_{j,g,t}^{1-\sigma^{CS}} = \sum_s \delta_S^{CS^{\sigma^{CS}}} PCons_{j,s,t}^{1-\sigma^{CS}} \quad (12)$$

Equation (11) shows that the consumption of good s, $CS_{j,s,g,t}$, is positively correlated with preference parameter δ_s^{CS} and the aggregate consumption $C_{j,g,t}$. This level also increases with the ratio of aggregate consumption price, $PCon_{j,g,t}$, over the consumption price of good s, $PCons_{j,s,t}$. In other words, higher is the consumption price of good s, lower is the consumption of this good. Equation (12) shows that the aggregate consumption price is a weighted sum of the consumption prices of good s.

In the trade model, we apply the Armington assumption[7] (Armington 1969) so that a good (*S1* or *S2*) produced in *J1* is no longer a perfect substitute for this good produced in *J2*. When imported goods are available on the market, an individual in country *j* allocates the consumption expenditure on good *s* across countries, which is the third step of optimization. In other words, a decision has to be made on how much to spend on either locally-produced good *s* or imported good *s*. The following equations show the third step of consumer's optimization:

$$\underset{CSJ_{j,j,s,g,t},CSJ_{i,j,s,g,t}}{Maximize}\ CS_{j,s,g,t} = (\delta^{CE}_{j,j,s} CSJ_{j,j,s,g,t}^{1-\frac{1}{\sigma^{CE}}} + \delta^{CE}_{i,j,s} CSJ_{i,j,s,g,t}^{1-\frac{1}{\sigma^{CE}}})^{\frac{\sigma^{CE}}{\sigma^{CE}-1}} \qquad (13)$$

$$s.t.\ PCons_{j,s,t} CS_{j,s,g,t} = Pq_{j,s,t} CSJ_{j,j,s,g,t} + Pq_{i,s,t} CSJ_{i,j,s,g,t} \qquad (14)$$

Equation (13) shows that in country *j*, the consumption of good *s* by generation *g* at time *t* is a CES form of consumptions of good *s* produced in different countries. We denote $\delta_{i,j,s}{}^{CE}$ as the parameter representing the preference of a consumer on goods *s* produced in country *i*. σ^{CE} is the elasticity of substitution for consumption of good *s* across countries. $CSJ_{i,j,s,g,t}$ is defined as the quantity of consumption of good *s* produced in country *i* but consumed by country *j*'s consumer. In other words, set *i* shows the country of origin of a consumed good and set *j* shows where this good is consumed. Accordingly, $CSJ_{j,j,s,g,t}$ shows locally-produced consumption good *s* for generation *g* in country *j* at time *t*. The following equation (15) and equation (16) are the first order conditions for an individual's third step of optimization:

7 Tradable goods in the world market are differentiated not only by production technologies, but also by country of origin.

$$CSJ_{i,j,s,g,t} = \delta_{i,j,s}^{CE^{\sigma^{CE}}} \left[\frac{PCons_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{CE}} CS_{j,s,g,t} \tag{15}$$

$$and \quad PCons_{j,s,t}^{1-\sigma^{CE}} = \sum_i \delta_{i,j,s}^{CE^{\sigma^{CE}}} Pq_{i,s,t}^{1-\sigma^{CE}} \tag{16}$$

Equation (15) shows that the consumption of good s produced in country i is positively correlated with total consumption of good s and is negatively correlated with the production price of good s in country i. Equation (16) shows that the aggregate consumption price of good s is a weighted sum of the production prices of good s in all countries in which these goods are produced.

In our numerical model, consumers in *J1* and *J2* are assumed to have an identical utility function and an identical preference parameter, δ_s^{CS}, for each good s. In other words, the autarky consumption bundles for consumers in two perfectly symmetric countries consist of the same quantity of both locally-produced *S1* and *S2*. In the trade case, the preference parameter, $\delta_{i,j,s}^{CE}$, is also assumed to be identical for all four differentiated consumption goods on the market. In other words, at the initial steady state where two counties are perfectly symmetric and there is no difference among the prices for these four consumption goods, a consumer is likely to consume the same quantity of locally-produced *S1*, imported *S1*, locally-produced *S2* and imported *S2*.

1.3.3 Determinants of investment demand

In each period t, savings of the younger generation finance investment equipped by firms to increase the stock of physical capital in period $t+1$. The usual law of motion of capital stock can be represented by the following equation (17):

$$KS_{j,t+1} = I_{j,t} + (1 - DR)KS_{j,t} \tag{17}$$

In this equation, $I_{j,t}$ is the aggregate investment in country j at time t. The capital stock in the economy at time $t+1$, $KS_{j,t+1}$, augments with investment in the previous period t and declines with depreciation rate (DR). This equation shows the aggregate investment level is determined by the gap between demand of capital stock in the next period and current capital stock net of depreciation.

The capital stock is a composite good composed of the available goods in the economy. Consequently, the capital stock in each economy is built using an investment technology requiring the use of both the capital-intensive good and the labour-intensive good. In the autarky case, this investment technology is represented by the following equation (18). In the trade case, this investment technology is represented by equation (18) and (19).

$$I_{j,t} = (\delta_{S1}^{IS} IS_{j,S1,t}^{1-\frac{1}{\sigma^{IS}}} + \delta_{S2}^{IS} IS_{j,S2,t}^{1-\frac{1}{\sigma^{IS}}})^{\frac{\sigma^{IS}}{\sigma^{IS}-1}} \tag{18}$$

$$IS_{j,s,t} = (\delta_{j,j,s}^{IE} ISJ_{j,j,s,t}^{1-\frac{1}{\sigma^{IE}}} + \delta_{i,j,s}^{IE} ISJ_{i,j,s,t}^{1-\frac{1}{\sigma^{IE}}})^{\frac{\sigma^{IE}}{\sigma^{IE}-1}} \tag{19}$$

$$PI_{j,t} I_{j,t} = PIS_{j,S1,t} IS_{j,S1,t} + PIS_{j,S2,t} IS_{j,S2,t} \tag{20}$$

$$PIS_{j,s,t} IS_{j,s,t} = Pq_{j,s,t} ISJ_{j,j,s,t} + Pq_{i,s,t} ISJ_{i,j,s,t} \tag{21}$$

Equation (18) shows that the aggregate investment, $I_{j,t}$, is built with two kinds of investment goods using a CES technology. δ_s^{IS} is the given share of good s in aggregate investment. σ^{IS} is the elasticity of substitution for investment goods which shows the degree of substitutability between investment goods across sectors. $IS_{j,s,t}$ is the investment good s used in country j at time t. In the trade case, equation (19) shows that the investment good s used in country j at time t is built with both locally-produced and imported investment good

s using a CES technology. $\delta_{i,j,s}{}^{IE}$ is the given share of investment good s produced in country i but used in country j. $ISJ_{i,j,s,t}$ is investment good s used in country j but produced in country i at time t. Accordingly, $ISJ_{j,j,s,t}$ is the locally-produced investment good s used in country j at time t. σ^{IE} represents the elasticity of substitution for investment good s produced in different countries. In equation (20), $PI_{j,t}$ represents the aggregate price of all investment goods in country j at time t and $PIS_{j,s,t}$ is the price of investment good s in country j at time t.

In the autarky case, an investor's problem is to minimize equation (21) subject to equation (19). The following equation (22) and equation (23) are the first order conditions:

$$IS_{j,s,t} = \delta_s^{IS^{\sigma^{IS}}} \left[\frac{PI_{j,t}}{PIS_{j,s,t}} \right]^{\sigma^{IS}} I_{j,t} \tag{22}$$

$$PI_{j,t}^{1-\sigma^{IS}} = \sum_s \delta_s^{IS^{\sigma^{IS}}} PIS_{j,s,t}^{1-\sigma^{IS}} \tag{23}$$

Equation (22) shows that the level of investment good s, $IS_{j,s,t}$, increases with the aggregate investment level and decreases with the price of investment good s. Equation (23) shows that the aggregate price of investment good is a weighted sum of prices for all investments goods in the economy.

When imported investment goods are available on the market in the trade case, an investor allocates its investment expenditure on investment good s across countries. The optimization problem is to minimize equation (21) subject to equations (19). The following equations (24) and (25) are the first order conditions:

$$ISJ_{i,j,s,t} = \delta_{i,j,s}^{IE^{\sigma^{IE}}} \left[\frac{PI_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{IE}} IS_{j,s,t} \tag{24}$$

$$PIS_{j,s,t}^{1-\sigma^{IE}} = \sum_i \delta_{i,j,s}^{IE^{\sigma^{IE}}} Pq_{i,s,t}^{1-\sigma^{IE}} \tag{25}$$

Equation (24) shows that the level of investment good s produced in country i, $ISJ_{i,j,s,t}$, increases with the aggregate investment level and decreases with the production price of good s in country i. Equation (25) shows that in the trade case, the price of investment good s in country j at time t is a weighted sum of production prices for good s in countries producing this investment good s at time t.

In our numerical model, we also assume that capital stock in both countries is built with an identical investment technology. The given share of good s in aggregate investment, δ_s^{IS}, is assumed to be identical across sectors. In other words, when two countries are perfectly symmetric, their capital stock is built with the same quantity of investment good *S1* and *S2*. In the trade case, we also assume that $\delta_{i,j,s}^{IE}$ is identical for all four differentiated investment goods on the market. That is, at the initial steady state where two counties are perfectly symmetric and there is no difference among prices for investment goods, the aggregate investment is built with the same quantity of locally-produced investment good *S1*, imported investment good *S1*, locally-produced investment good *S2* and imported investment good *S2*.

In the economy, the value of $RRT_{j,t}$ (one plus the rate of return on a unit of physical capital invested at time *t-1*) is determined by the following equation (26):

$$RRT_{j,t} = \frac{\mathrm{R}_{j,t} + (1-DR)PI_{j,t}}{PI_{j,t-1}} \qquad (26)$$

This equation shows that the expected rate of return on capital increases with the rental price and the aggregate price of investment good at time t and decreases with the price of investment good at time *t-1*.

1.3.4 Foreign trade with the rest of the world

In the trade case, we allocate demand by consumers in country j for goods produced in country i based on the Armington assumption. In other words, even though individual producers are microscopic price takers, good s are assumed to be differentiated in demand by country of origin. At the initial steady state where two countries are assumed to be perfectly symmetric, each country imports and exports the same quantity of *S1* and *S2*. Trade exists when two countries are perfectly symmetric because each of them demands both locally made and foreign made goods. During the demographic transition, changes in relative factor abundances bring changes to two countries' comparative advantages. Accordingly, demands for the four goods change overtime.

The aggregate level of one country's import or export of good s (denoted as $IMP_{j,s,t}$ and $EXP_{j,s,t}$) is determined by the above optimization problems for a consumer or an investor, which can be shown by the following equation (27) and equation (28):

$$IMP_{j,s,t} = \sum_{i,g} POP_{j,g,t} CSJ_{i,j,s,g,t} + ISJ_{i,j,s,t} \quad for\, i \neq j \tag{27}$$

$$EXP_{j,s,t} = \sum_{i,g} POP_{i,g,t} CSJ_{j,i,s,g,t} + ISJ_{j,i,s,t} \quad for\, i \neq j \tag{28}$$

where $POP_{j,g,t}$ is the population of generation g in country j at time t.

Equation (27) shows that the import of good s by country j at time t is sum of locally demanded consumption or investment good s produced in country i. The export of good s by country j is the sum of locally-produced consumption or investment good s demanded by country i.

1.3.5 Equilibrium conditions

A general equilibrium solution is one in which all economic behaviours (production, consumption and investment) are consistent with both current prices and future prices and all markets clear. In this model, four markets exist in the economy: the good market, the labour market, the capital market and the asset market.

The equilibrium condition for good market in each country requires that the quantity of good s produced in country j at time t must be equal to the quantity demanded by all countries:

$$X_{j,s,t} = \sum_{i,g} POP_{i,g,t} CSJ_{j,i,s,g,t} + ISJ_{j,i,s,t} \tag{29}$$

On labour market, we assume full employment and labour supply equals to labour demanded in each country at time t:

$$POP_{j,G1,t} Ls_{j,G1} = \sum_{s} Ld_{j,s,t} \tag{30}$$

where $Ls_{j,G1}$ is the unit labour supply provided by an individual in the working generation G*1* in country *j*.

In this paper, we assume that individual in all countries provide an identical level of unit labour supply. However, unequal population growth rates make the aggregate labour supply varies across countries overtime.

On capital markets, capital stock must be equal to capital demand in each country:

$$KS_{j,t} = \sum_{s} Kd_{j,s,t} \tag{31}$$

The equilibrium condition for the asset market requires that value of financial asset owned by old individuals at time $t+1$ equals to the value of capital stock in the economy of each country j at time $t+1$:

$$POP_{j,G2,t+1}LD_{j,G2,t+1} = PI_{j,t}KS_{j,t+1} \tag{32}$$

The financial asset owned by an individual in the old generation at time $t+1$ is saved by this individual at time t, which is expressed in equation (7).

1.3.6 To model the demographic transitions

The demographical transition is modeled as the following. In any period t, the fertility rate for the first generation in country j is labelled as $FR_{j,t}$. If we denote the number of young (old) individuals living in country j in period t as $POP_{j,G1,t}$ ($POP_{j,G2,t}$), the population of country j regenerates according to the following equation (33) and (34):

$$POP_{j,G1,t+1} = POP_{j,G1,t}(1+FR_{j,t}) \tag{33}$$
$$POP_{j,G2,t+1} = POP_{j,G1,t} \tag{34}$$

In this paper, we consider two scenarios of demographic transition. Under the first scenario (A), two countries start from different initial fertility rates labelled as $FR_{J1,t0}$ and $FR_{J2,t0}$. From the second period on, fertility rates of both countries decline towards zero but at different speed. Population of $J1$ with a lower initial fertility rate declines faster and this country turns to an older country. Under the second scenario (B), the younger generation in $J1$ also has a lower initial fertility rate and this rate is constant overtime. $J2$ starts from a higher fertility rate and its population grows at a constant rate before the fifth period ($t5$).

From the sixth period on, this rate begins to decline at a constant speed and converges to that of *J1*. In this paper, we use the same initial population growth rates for *J1* and *J2* as in Sayan (2005) to make the simulation results more comparable. Under scenario *B*, we assume that two counties' population growth rates converge to a higher steady-state level than it is under scenario *A*. This scenario is used to compare the evolutions of variables given different steady-state population growth rates. Under both scenarios, *J1* grows into an older[8] capital-abundant country. Table 1 and Figure 2 show the two scenarios of demographic transition for both countries.

Table 1 Evolutions of fertility rates

$FR_{J1,t}$	$FR_{J2,t}$	$FR_{J1,t}$	$FR_{J2,t}$
scenario *A*		scenario *B*	
$FR_{J1,t0}$=0.012	$FR_{J2,t0}$=0.0205	0.012 for all *t*	0.0205 for 0<*t*<5
$FR_{J1,t}$=0.5872$FR_{J1,t-1}$ For *t*>*0*	$FR_{J2,t}$=0.6258$FR_{J2,t-1}$ for *t*>0		$FR_{J2,t}$-0.012 =0.7*($FR_{J2,t-1}$ -0.012) for *t* >=5

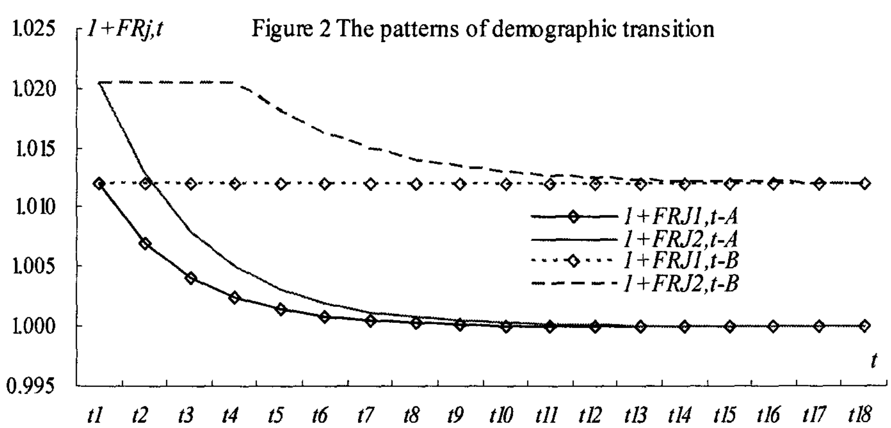


Figure 2 The patterns of demographic transition

Two countries are calibrated to be perfectly symmetric. In each of them, 0.5 units of *S1* and *S2* are produced, 0.4 units of *S1* and *S2* are consumed by 1 unit of individual (0.5 units

8 In the long run, the age structures of the two countries become identical as a result of demographic transition. In other words, country *J1* in our model can be viewed as a initially older economy or an economy with a faster ageing process. For convenience, we call *J1* the older country and *J2* the younger country in this paper.

of young individual and 0.5 units of old individual). The extra 0.2 units of *S1* and *S2* are used as investment goods to build capital stock demanded by productions. The effective labour supply (*Lsup*) in the economy is 0.5 units (the population of the young generation). Based on the above settings, we use some critical parameter values as shown in Table 2 to solve the autarky model.

Table 2 Critical parameter values

α^k_{S1}	α^k_{S2}	σ^{JT}	σ^{CS}	σ^{CE}	σ^{JS}	σ^{JE}	DR
share of capital in production		intertemporal EOS	EOS for consumption goods across sectors	EOS for consumption good s across countries	EOS for investment goods across sector	EOS for investment good s across countries	depreciation rate
0.6	0.4	1.5	3	3.5	3	3.5	0.6

On Table 2, α^K_s is the parameter for factor intensity in the production of good s. As shown by the parameter values, *S1* is a capital-intensive good and *S2* is a labour-intensive good. The elasticity of substitution (EOS) is represented by σ. The depreciation rate, DR, is set as 0.6. The intertemporal EOS and the depreciation rate may seem high but recall that in our simple OLG-CGE model, each period t is equivalent to about 30 years.

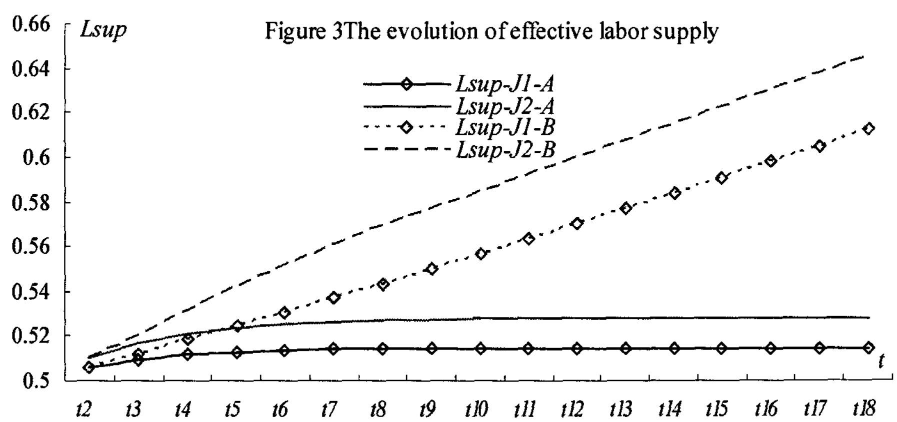
The model is solved based on the following strategy. Given some common parameters, we first solve the autarky model with no population growth. The calibrated values, including consumption by two generations, investment, rate of return on capital and discount rate, are used as initial steady-state values. Next we introduce the regional-specific population growth rates into the model and solve the autarky case for two counties. Then the trade case is simulated based on the Armington assumption.

1.4. SIMULATION RESULTS

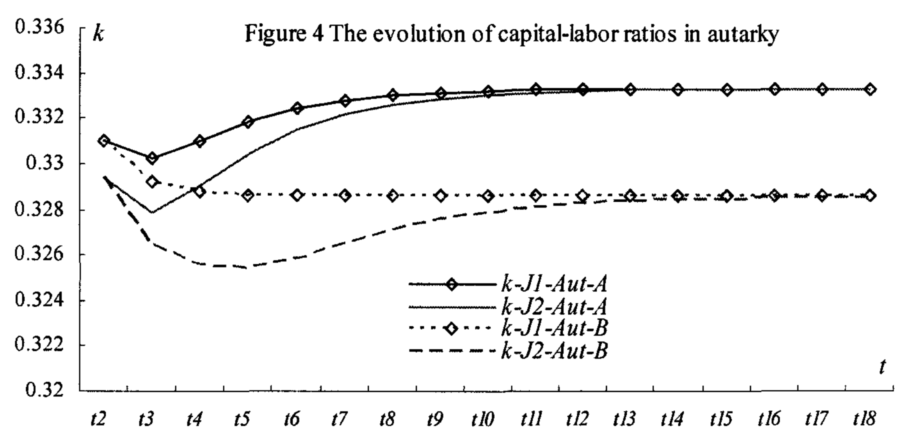
In this section, we present some important simulation results in both the autarky case and the trade case.

1.4.1 The autarky case

We first show the evolutions of effective labour supply in *J1* and *J2* under both scenarios. In our numerical model, both countries are assumed to be populated by one unit of people. As a result, the effective labour supply, which is equal to the population of the younger generation, is 0.5 units for both country *J1* and *J2* in the first period (*t0*). During the demographic transition, unequal population growth rates make the effective labour supply different across countries from the second period on. Under both scenarios, *J2* is a younger country than *J1*. As shown by the following Figure 3, the level of effective labour supply in *J2* dominates that in *J1* overtime.



With a positive rate of population growth, the aggregate demand in the economy will increase, which requires more capital stock used in the productions. As a result, the rate of return on capital (*RRT*) is increasing, which creates incentive for the younger generation to smooth more current consumption to its second half of life. *Ceteris paribus*, the steady-state capital-labour (k) level in an autarky economy is determined by the steady-state population growth rate as predicted by the Solow-Swan growth model of conditional convergence. In our model, population growth rates for both countries converge to an identical and constant level under both scenarios of demographic transition. Consequently, the autarky capital-labour ratios in both countries converge to a constant level after periods of gradual adjustment. The steady-sate population growth rate under scenario A is lower than it is under scenario B, so countries under scenario A have a higher autarky k level than it is under scenario B as shown by the following Figure 4.



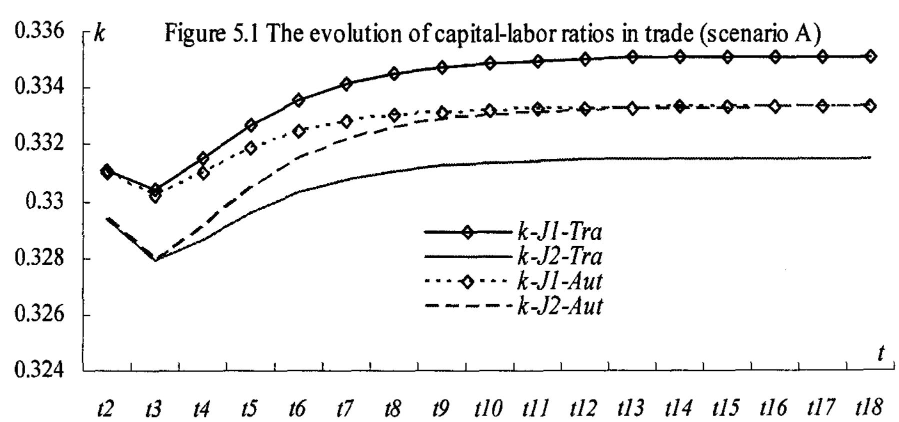
Results of other important variables in the autarky case are presented and compared to their simulation results in the trade case in the next section *4.2*.

1.4.2 The trade case

When Armington assumption is applied into our trade model, a tradable good is not only differentiated by the production technology but also by its country of origin. Thus, there are four differentiated tradable goods in the world market: *S1* produced in *J1*, *S1* produced in *J2*, *S2* produced in *J1* and *S2* produced in *J2*. The preference parameter for all these goods is set to be identical for consumers or investors across countries. With two perfectly symmetric countries, an individual in *J1* or *J2* demands 0.25 units of each of the four goods for either consumption or investment. During the demographic transitions, unequal population growth rates bring changes to the relative factor abundances of trade partners and their comparative advantages. This creates incentives for trade. International trade between these countries turns to be affected by each country's newly created comparative advantage. It is expected that the capital (labour)-abundant country exports relatively more of the capital (labour)-intensive good. This can be shown by the changes of the ratio of export of *S1* over export of *S2*.

As shown in section 3, *J1* turns into an older and capital-abundant country and *J2* a younger and labour-abundant country during the demographic transition. In a HO trade model with two homogenous goods and two inputs, *J1* (*J2*) specializes in the producing and exporting the capital (labour)-intensive *S1* (*S2*). After opening to free trade, locally-produced *S1* (*S2*) is a perfect substitute for imported *S1* (*S2*) in the world market and consumers are assumed to be indifferent to locally-produced and imported good *s*. According to the Factor Price Equalization Theorem, the marginal products of labour and capital and the factor prices (the wage rate and the rental price) turn to be equalized across countries with free trade. In this case, free trade successfully transmits the relative factor abundances across counties. The capital-labour ratios of the two countries share a unique evolutionary path which lies between the evolutionary paths of their autarky capital-labour ratios.

When the Armington assumption is applied into the trade model, the assumed identical preference parameter for both locally-produced and imported *S1* or *S2* means the homogenous good *S1* or *S2* produced in two countries are no longer perfect substitutes in the world market. As a result, marginal products of capital input and labour input are no longer equalized across countries, neither are factor prices in two countries. Different from the standard HO model, the capital-labour ratios of both countries are no longer equalized after opening to free trade. As shown by the following Figure 5, the simulated capital-labour ratios in *J1* and *J2* under trade converge to different steady-state values overtime. There is an increasing and persistent gap between the two countries' capital-labour ratios. In other words, the Armington assumption turns the older (younger) and capital (labour)-abundant country into a more capital (labour)-abundant economy. The differential in factor abundance across countries is amplified. It is also shown that the gap between their capital-labour ratios is larger under scenario *B* than it is under scenario *A*. This gap is positively correlated with the steady-state population growth rate.



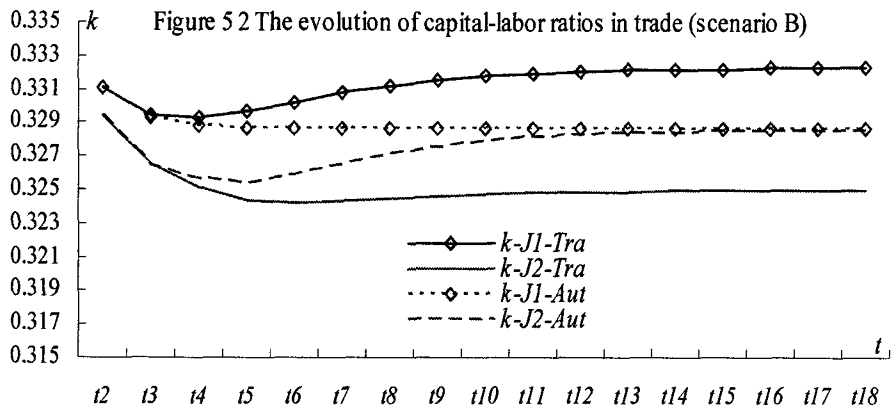


Figure 5 2 The evolution of capital-labor ratios in trade (scenario B)

With the changes of relative factor abundance overtime, the capital (labour)-abundant *J1* (*J2*) is expected to have the comparative advantage in producing the capital (labour)-intensive good, *S1* (*S2*). Within each economy, this comparative advantage can be shown by the level of relative production price ratios (the production price of *S1* over the production price of *S2*) under autarky.

As shown by Figure 6, this relative production price ratio under autarky is always lower in *J1* under both scenarios. In the long run, however, two countries' autarky production prices converge to an identical steady-state level because their age structures turn to be identical. When we examine the trade case based on the Armington assumption, it is shown that two countries' long-run relative production price ratios no longer converge. The comparative advantage of *J1* (*J2*) in producing *S1* (*S2*) is improving during the demographic transition and persistent in the long run when the age structures in two countries' population become identical.

It is also shown that under scenario *B* with a higher steady-state population growth rate, the gap between two countries' relative production price ratios is higher than it is under scenario *A*.

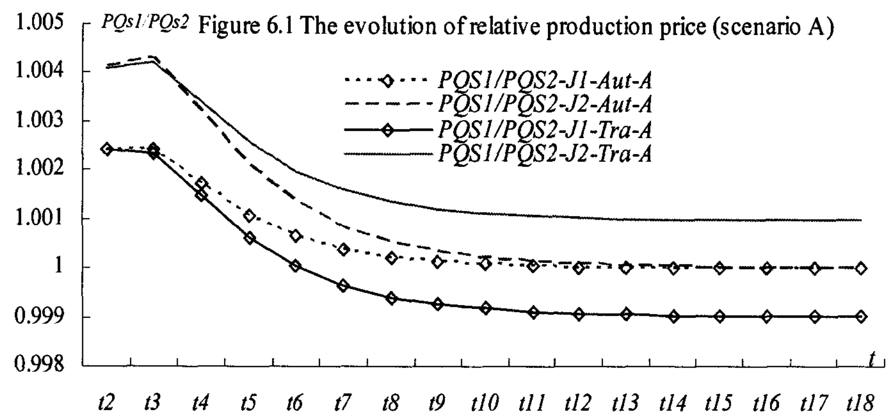


Figure 6.1 The evolution of relative production price (scenario A)

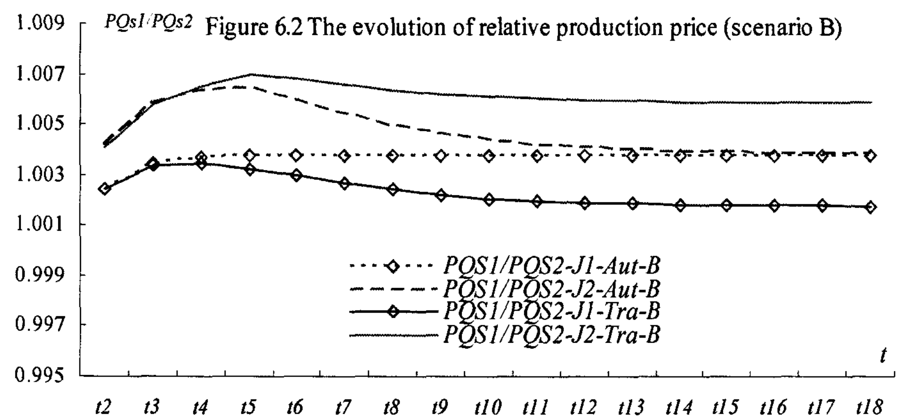
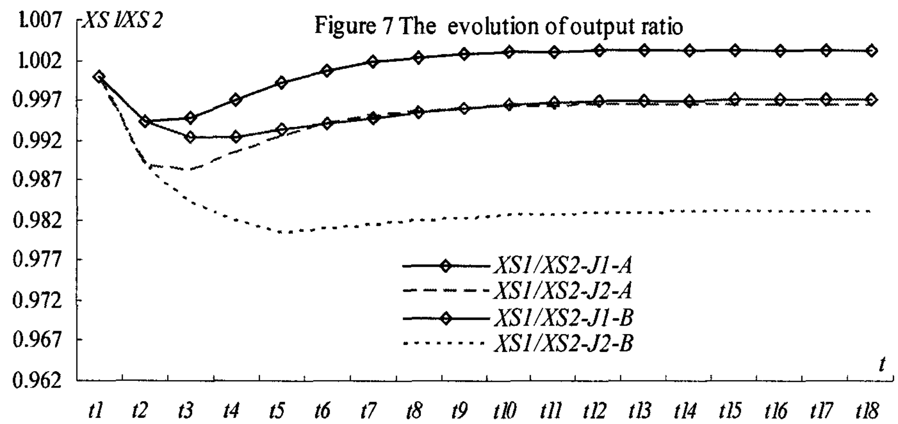


Figure 6.2 The evolution of relative production price (scenario B)

Consequently, the quantities of productions of the two goods will change according to each country's comparative advantage and demands from both the home and the foreign markets. Figure 7 shows that for the capital (labour)-abundant country *J1* (*J2*), the ratio of production of the capital (labour)-intensive good *S1* (*S2*) over the labour (capital)-intensive *S2* (*S1*) is higher than it is for the labour-abundant *J2* (*J1*) overtime.



In the standard HO model, countries export goods with comparative advantages and a capital (labour)-abundant country exports only capital (labour)-intensive good. In the Armington case, however, each country exports both locally-produced *S1* and *S2*. To maximize lifetime utility, a consumer tends to consume more imported goods with lower production prices. Clearly, after opening to trade, the quantity of import is not only determined by consumers' preference on imported or locally-produced goods, but also by different production prices across countries. Figure 8 shows that *J1* is exporting more *S1* than *S2* and its ratio of the export of *S1* over the export of *S2* dominates that for *J2* under both scenarios. Correspondingly, the ratio of export of *S2* over the export of *S1* is higher for the labour-abundant younger country *J2*. Comparing Figure 8 and Figure 6, we see that the export ratio is negatively correlated with the relative price of production. Figures 6, 7 and 8 confirm that country *J1* has a comparative advantage in producing *S1* and country *J2* has a comparative advantage in producing *S2*.

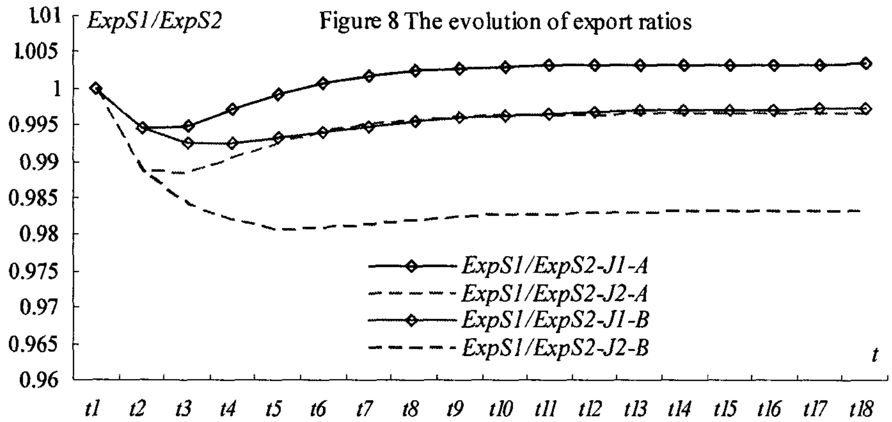
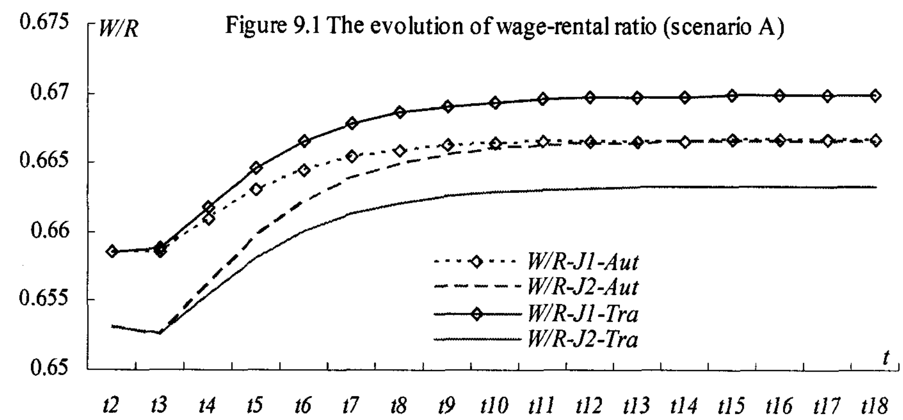


Figure 8 The evolution of export ratios

The welfare implication of trade under scenarios of demographic transition is the center question to answer in this paper. In our model, the welfare is measured by the lifetime utility for a representative cohort. As shown in section *3.2*, a cohort's lifetime utility is determined by its lifetime wealth and the unique lifetime utility function assumed for all cohorts in *J1* and *J2*. The lifetime wealth for a representative cohort consists of the present values of this cohort's labour income and interest income on its savings made in the first half of life. In our model, the labour income and interest income are determined by the factor prices in the economy. Higher is the wage rate, more labour income the young individuals receive. The rate of return on capital is determined by the rental price in the economy, so a higher rental price means the interest income for per unit of physical capital owned by the old individuals is higher.

Before presenting the evolutions of the absolute values of factor prices, we first show the evolutions of two countries' wage-rental ratios under scenario *A* and *B* (Figure 9):



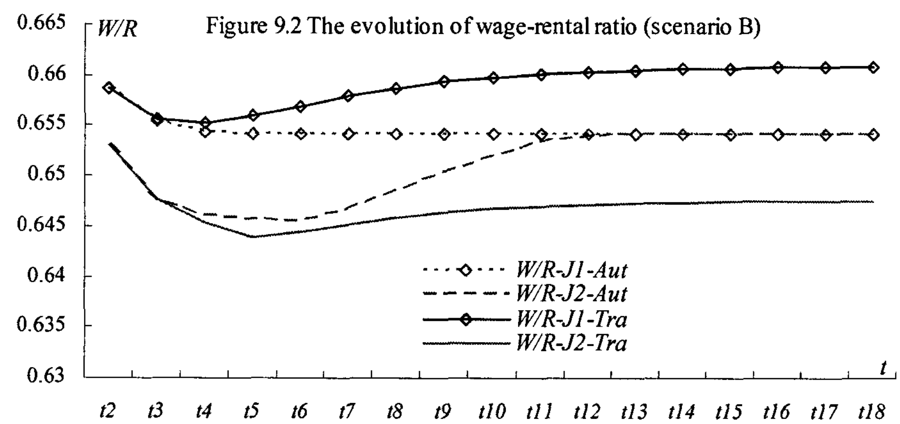


Figure 9 is correspondent to the above Figure 5 showing the evolution of capital-labour ratios for *J1* and *J2* under autarky and trade. In the autarky case, both countries' wage-rental ratios converge to an identical steady-state level when their capital-labour ratios converge. In the trade case, however, the wage-rental ratio in *J1* dominates that in *J2* under two scenarios of demographic transition, which reflects *J1* is a relatively capital-abundant country. The following Figure 10 shows the evolutions of the absolute values of factor prices in *J1* and *J2*. We observe that the wage level in the capital-abundant country *J1* dominates that in *J2* at steady state. In addition, the steady-state rental price in *J1* is also higher than it is in *J2*, which is observed under both scenarios

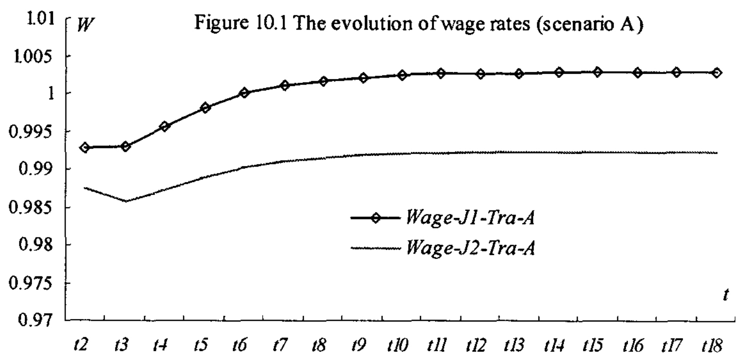


Figure 10.1 The evolution of wage rates (scenario A)

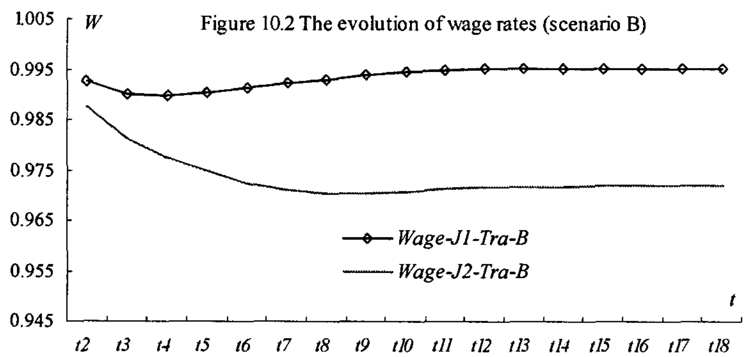


Figure 10.2 The evolution of wage rates (scenario B)

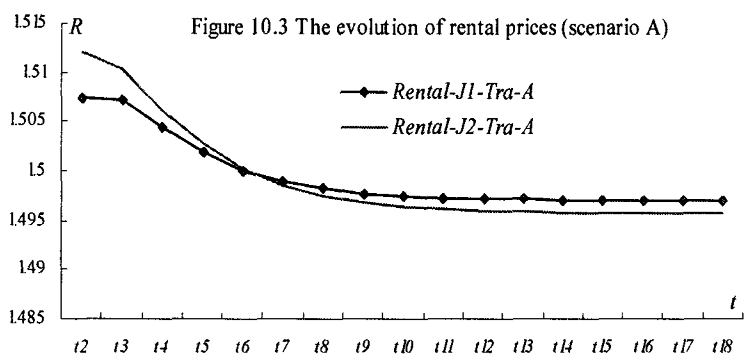


Figure 10.3 The evolution of rental prices (scenario A)

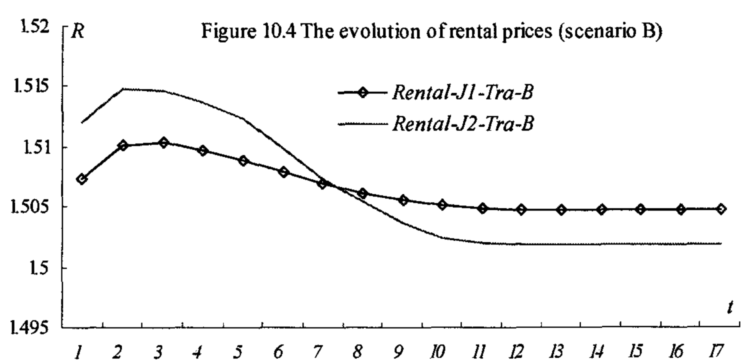


Figure 10.4 The evolution of rental prices (scenario B)

Indeed, in the capital-abundant country *J1*, wage rate is always higher than it is in the labour-abundant *J2* because of the labour scarcity in the economy. In addition, the demand from a consumer in *J2* for *S1* produced in *J1* is higher than the demand for *S2* produced in *J1* because *J1* has the comparative advantage in *S1*. Moreover, higher population growth rate changes *J2* into an economy larger than *J1* so that *J1* faces a much higher aggregate demand for *S1*. More production of the capital-intensive *S1* in *J1* requires more capital inputs. Consequently, the rental price in *J1* is slightly higher than it is in *J2* at steady state. With higher wage rate and rate of return on capital, an individual in the capital-abundant *J1* has a higher level of steady-state lifetime wealth (labelled as *LW* which is the wage and interest income in present values) as shown by Figure 11.

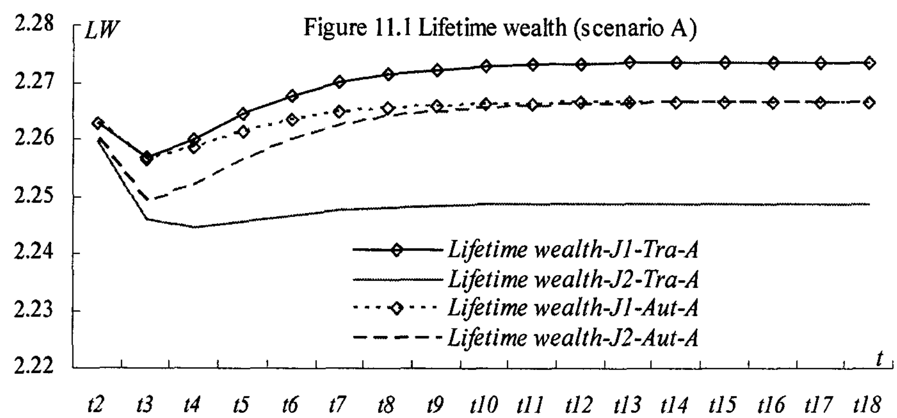


Figure 11.1 Lifetime wealth (scenario A)

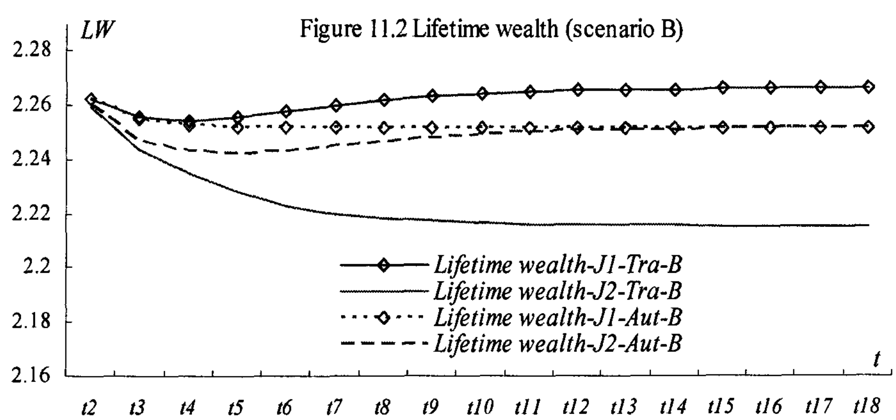


Figure 11.2 Lifetime wealth (scenario B)

In this paper, the welfare for individuals in the two countries is measured by the lifetime utility of a representative cohort in each country. To compare our results to the results presented in Sayan (2005), we also present the evolutions of the GDP per capita levels for the two countries under both autarky and trade. The lifetime wealth for a representative cohort living in *J1* is always higher than it is for a representative cohort living in *J2* (Figure 11), so a cohort in *J1* (*J2*) has a higher (lower) lifetime utility in the trade case, as shown by the following Figure 12. Under both scenarios, the gap between two countries' welfare levels is increasing during the demographic transition and persistent in the long run.

Figure 12.1 The evolution of lifetime utility (scenario A)

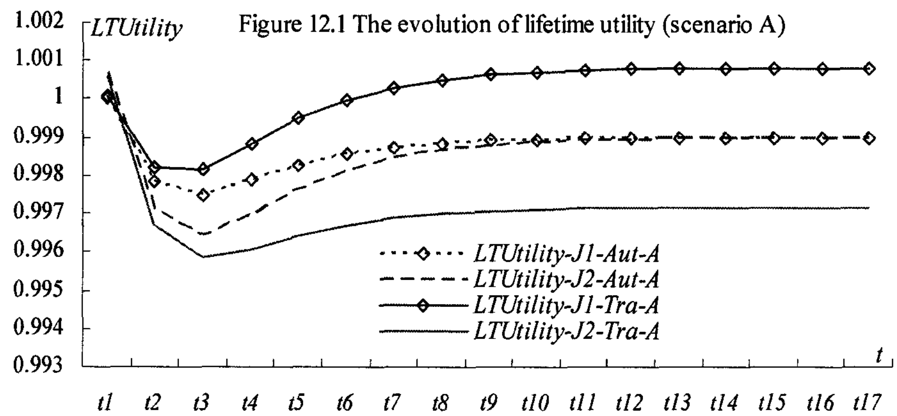


Figure 12.2 The evolution of lifetime utility (scenario B)

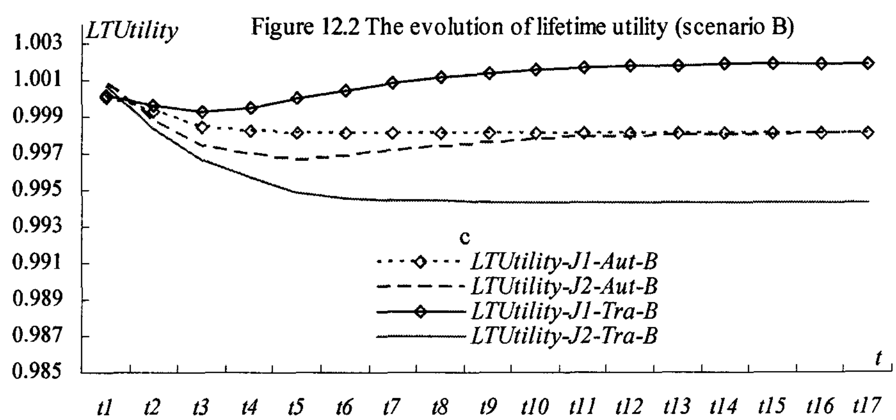
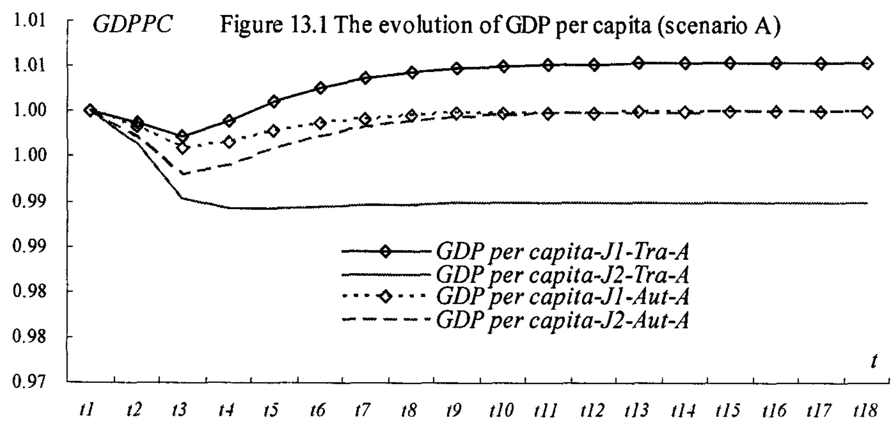
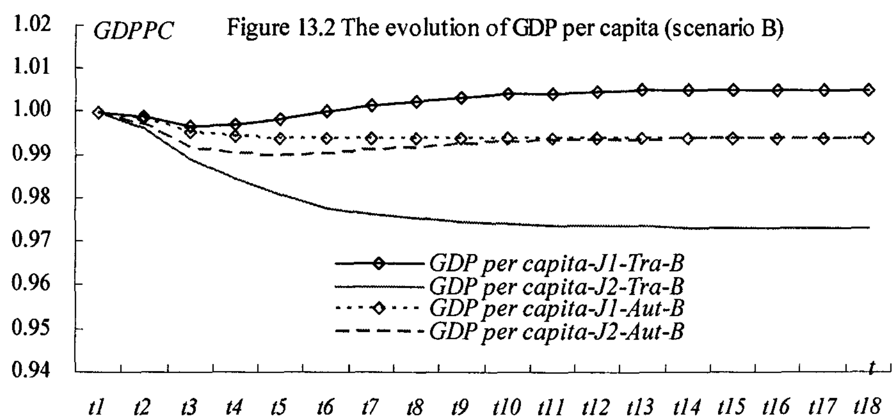


Figure 13 reports the evolutions of GDP per capita levels as approximations of the standard of living for both countries. In the autarky case, the GDP per capita levels for *J1* and *J2* converge to an identical steady-state level. This steady-state level of GDP per capita is 1 under scenario *A* and 0.994 under scenario *B* with a higher steady-state population growth rate. In the trade case, however, the steady-state GDP per capita level is 1.0054 for *J1* and 0.9898 for *J2* under scenario *A*. Under scenario *B*, this level is 1.0053 for *J1* and 0.9731 for *J2*. It is shown that at each time *t*, the GDP per capita level for *J1* is higher than it is for *J2* under trade. The gap between them is increasing during the demographic transitions and persistent in the long run.





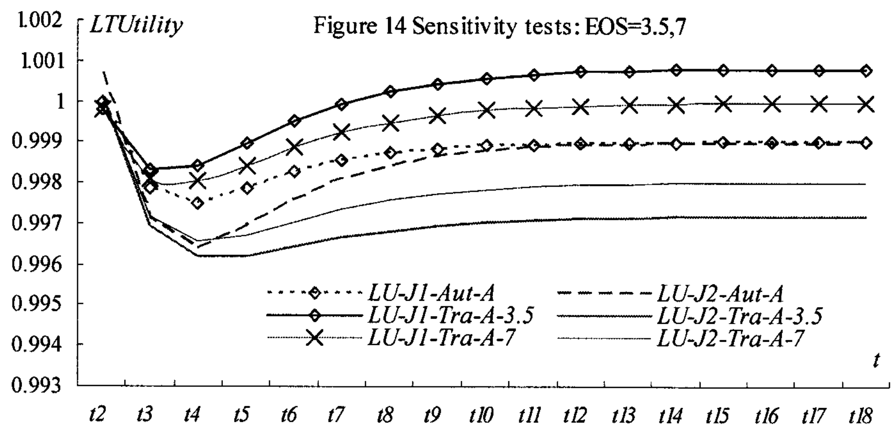
Comparing Figures 12 and 13, we find that the evolutionary paths of lifetime wealth and GDP per capita are very similar. Thus, we conclude that for two open economies during demographic transitions, the older and capital-abundant country gains from trade, and the younger and labour-abundant country loses from trade, which is predicted by our trade model based on the Armington assumption.

Moreover, both figures show that the gap between the two countries' welfare levels is increasing with the steady-state population growth rate. Under scenario *A*, the steady-state population growth rate is 1, which is lower than it is under scenario *B* (1.012). Correspondingly, the gap between the steady-state lifetime utility (GDP per capita) levels is enlarged from 0.15% to 0.32% (1.58% to 3.58%).

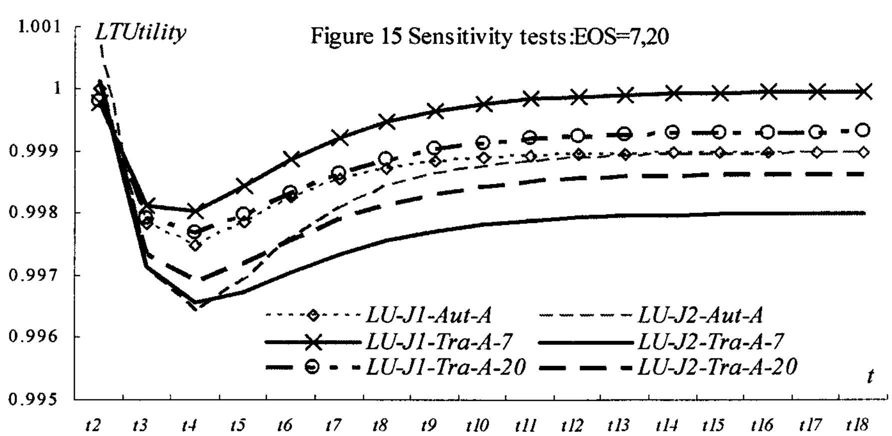
1.4.3 Sensitivity analysis

The sensitivity analysis in this paper aims to test how well our CGE model is replicating the Heckscher-Ohlin model. In our CGE framework based on the Armington assumption, the value of elasticity of substitution between imported good and locally-produced good (the Armington elasticity), σ^{CE}, is critical in determining the allocation of consumption expenditure according to the countries of origin. The higher the value of σ^{CE}, the smaller is the degree of products differentiation and the larger is the effect of terms of trade on consumption. In other words, with the increase of the value of σ^{CE}, the CGE model based on the Armington assumption approaches the Heckscher-Ohlin model in which imported good and locally-produced good are perfect substitutes in the world market.

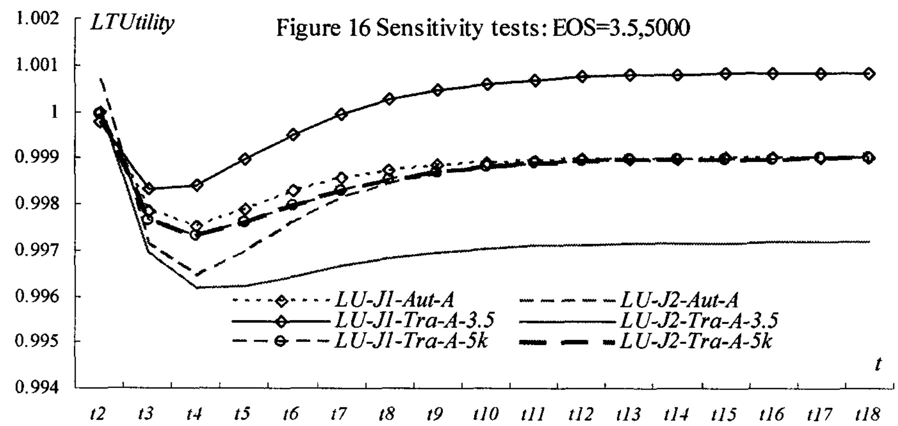
The following, we present results of the sensitivity tests in which we increase the value of σ^{CE} without changing any other assumed parameter values. Figure 14 to Figure 16 show the change of the evolutionary paths of the lifetime utility levels for cohorts living in country *J1* and *J2* when we applying different values of σ^{CE}. We compare the simulation results when the value of σ^{CE} equals to 3.5, 7, 20 and 5000 with the autarky case.



As shown by Figure 14, the lifetime utility for a representative cohort living in the older country *J1* is decreasing in each period *t* when we double the value of σ^{CE} from 3.5 to 7. In contrast, the lifetime utility for a cohort living in the younger country *J2* improves with the increase of the value of σ^{CE}. This trend can also be observed in the following Figure 15 showing the comparisons of the simulation results when we increase the value of σ^{CE} from 7 to 20.



From the above two figures, we also see that when we increase the value of σ^{CE} to a higher level, a cohort in country *J2* will benefit from international trade in the short run. Accordingly, a cohort in country *J1* loses in the short run. Although there is still a persistent gap between the welfare levels for cohorts living in the two countries, the gap is decreasing with the increase of the σ^{CE} value.



The gap between the older and the younger countries' welfare levels is decreasing with the increase of the σ^{CE} value so that this gap will be eliminated if we apply a very high level of σ^{CE} into our model. This is shown by Figure 16 in which we compare the autarky case, the original trade case (σ^{CE} equals to 3.5) and the case in which we assume σ^{CE} equals to 5000. With a very high value of σ^{CE}, the lifetime utility levels for cohorts in the two countries become identical in each period *t*. This finding suggests that the older and capital-abundant country loses from trade, but the younger and labour-abundant country gains from trade. This result is opposite to the conclusion we present in section *4.2*, but identical to the conclusion of Sayan (2005).

The above sensitivity tests results suggest that the CGE model used in this paper is a generalization of the Heckscher-Ohlin type CGE framework. Without changing the structure of our model, the simulation results suggested by a Heckscher-Ohlin framework could be well replicated by our model. This replication is achieved by adjusting the value of the Armington elasticity we are applying in this paper.

1.5 CONCLUSION REMARKS

Using an OLG-CGE model, this paper revisits the effects of different types of demographic transitions on the evolution of trade and the welfare implications. We assume two economies are perfectly symmetric in every aspect except for the population growth rate. We compare the time paths and the steady-state values of critical variables for both countries in the autarky case and the trade case. In our model, a nested CES utility function is introduced into the intertemporal maximization of life time utility for a representative cohort. In the real world, tradable goods are differentiated not only by factor intensities in productions, but also by country of origin. Thus the Armington assumption is applied into our trade model to generate a more realistic picture of international trade.

First, our findings for the autarky case are similar to those presented in related literatures. Many variables, including the EDRs, capital-labour ratios, wage-rental ratios in the two economies, converge to identical levels in the long run. In the aspect of welfare, the autarky lifetime utility levels for a representative cohort in the two economies also converge to an identical level. After opening to trade, our simulation results suggest that a representative cohort in the older economy has a higher level of welfare than in autarky and gains from trade. The welfare for a representative cohort in the younger economy, however, deteriorates. The gap between their welfare levels is increasing during the dynamic transition and persistent in the long run. This result supports the argument that international trade would not necessarily represent a Pareto improvement over the state of autarky. In addition, the directions of welfare change for the older and the younger economy are opposite to those shown by Sayan (2005). The economy with a faster ageing process gains from trade and the economy with a slower ageing process loses, which is predicted by our trade model based on the Armington assumption.

Sensitivity tests in this paper suggest that our OLG model based on the Armington assumption is a generalization of the Heckscher-Ohlin model. By increasing the value of the Armington elasticity, the simulation results suggested by a Heckscher-Ohlin model, as used by Sayan (2005), could be replicated by the CGE model used in this paper.

Furthermore, we provide some explanations of the gap between the welfare levels for representative cohorts living in the two economies. When demographic shocks are introduced into the standard HO model, it has been shown by Sayan (2005) that most trade-case variables for two economies, with unequal population growth rates, are equalized after opening to free trade. The values of these variables, including the lifetime utility for a cohort and the GDP per capita, are between their autarky levels in the capital-abundant economy and the labour-abundant economy. In each period *t*, free trade successfully transmits the factor abundances across economies.

In our trade model based on the Armington assumption, however, the gap between the relative factor abundances of the capital-abundant economy and the labour-abundant economy is persistent overtime. The older (younger) economy always has a comparative advantage in the capital (labour)-intensive good. As a result, the demand from the younger and larger economy for capital-intensive good produced in the older economy is higher so that the steady-state rate of return on capital is higher in the older economy. For the younger economy where the labour-intensive good is more efficiently produced, the positive effect of higher demand for labour-intensive good on its wage level is limited because the older economy is comparatively smaller. The gap between the lifetime wealth levels for a representative cohort in the two economies is increasing during the demographic transition and persistent in the long run, which generates the increasing and persistent gap between their welfare levels.

Finally, it is shown that the gap between the welfare levels for cohorts in the older economy and the younger economy is positively correlated with their steady-state population growth rate. In this model, we assume that the steady-state population growth rate is lower under scenario *A* than it is under scenario *B*, so the gap between their cohorts' welfare levels under scenario *A* is smaller than it is under scenario *B*.

This model can be applied into studies on trade issues between the younger and labour-abundant economies and the older and capital-abundant economies. Our simulation results suggest that as the population growth rate of a labour-abundant developing economy is projected to be persistently higher than it is in its developed trade partners, the gap between their cohorts' welfare levels is expected to be persistent overtime. It seems that in an ageing context with free trade, it is better to be a member of the older and hence a relatively smaller economy.

The model used in this paper can be extended for many future studies. A multi-regional model will be built to investigate the evolution of trade among many economies with different patterns of demographic transition. In this paper, we assume two economies have an identical initial capital-labour ratio before calibration, which doesn't reflect the actual relative factor abundances for the developing and the developed economies. Real data will be collected and applied into this model. The robustness of our results remains to be tested in the future work.

Chapter Two

International Migration, Skill Composition and Population Ageing: an OLG-CGE Study

2.1 INTRODUCTION

The world population is experiencing a dramatic process of population ageing. Demographic projections show that all developed countries will be in the process of fast ageing after 2010 when the baby-boom generation (those born between 1946 and 1966) begins to retire (Mérette 2002). In the developing world, there is a considerable divergence in the degrees of population ageing across countries. Many countries in the East Europe and East Asia are already on the way of ageing. Fertility rates of developing countries in Africa, the Middle East and the South Asia, however, are projected to be much higher than the rest of the world. Consequently, these countries' age structures are expected to be much younger. Overall, the process of ageing in the developing world is less rapid than it is for the developed world. Figure 1 shows the projected trends of population ageing for countries around the world (UN 2002).

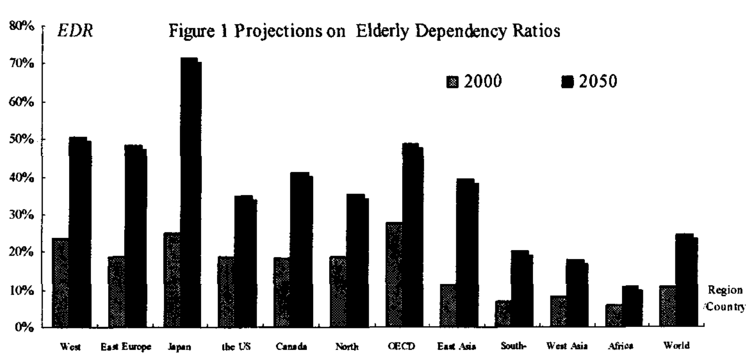


Figure 1 Projections on Elderly Dependency Ratios

Population ageing is widely viewed as a threat to the sustainability of social welfare system and economic growth of developed countries, as shown by World Bank (1994) and OECD (1998). These pessimistic views urged many rich and ageing countries to find solutions to problems caused by populations ageing. Besides some internal reforms, such as debt stabilization and delayed retirement, suggested by OECD (1998), adopting more immigrants is considered as an option to accommodate the negative impacts of ageing by opening door to international labour markets and improving the age structure of an ageing economy. International migration has become a hot issue in almost all industrialized countries over the past decades. According to UN statistics, foreign-born residents around the world rose from 75 million to 120 million in the period between 1965 and 1990 (Hatton and Williamson 2002). In OECD countries, this number has increased from 44 million in 1989 to 58 million in 1999, accounting for 8% of their total population (OECD 2001). Different from the European mass emigration to the new continents before 1914, the post-WWII wave of world migration is accompanied with a deeper integration of national markets (globalization) and an accelerating process of population ageing. In addition, with the development of information and communication technologies (ICTs), we observed a greater demand for foreign skilled labour from the developed world. The improvement of education infrastructure in many developing countries also provides sources of more educated labour. It is highly relevant to study the migration of different skill levels of labour across countries and its economic implications.

This paper uses a 3*2*2*7 [9] type overlapping-generations computable general equilibrium (OLG-CGE) model to investigate the impacts of different patterns of migration

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9 3*2*2*7 refers to three-country, two- good, two- input, and seven- generation.

on countries with different degrees of ageing. In this three-region model, real data is used to calibrate the developed, the developing world and the rest of the world with different demographic characteristics. The decision making of migration is based on projected paths of demographic transition and the resulted wage differentials across countries. Different from recent literature on this issue, international migration is analyzed using a trade model into which the Armington assumption (Armington 1969) is applied, and the welfare implication of migration are explained by the changes of comparative advantages across countries and terms of trade resulted from labour mobility.

Section 2 reviews recent CGE researches on the economic impacts of international migration. Section 3 describes the OLG-CGE model and data sets and section 4 presents simulation results. Section 5 concludes and shows some future research topics suggested by this paper.

2.2 CGE RESEARCHES IN INTERNATIONAL MIGRATION

The importance of CGE approach in analyzing migration is being recognized by more and more researchers. Storesletten (2000) used an OLG-CGE model to show that an increase of income tax rate from 28.2% to 32.6% is necessary to eliminate the existing fiscal imbalance in US. However, such a tax increase can be substituted by an increase of 50% in immigration rate on condition that all future immigrants are selected by age and skill and there is no increase in low-skill immigrants. Fehr *et al.* (2003) also built a multi-country, overlapping-generation CGE model to analyze the role of migration in alleviating fiscal stresses caused by population ageing and the capital shortage brought by a possible tax hike in developed countries. They found that doubling present scale of immigration does little to increase the payroll tax base so as to mitigate the fiscal stresses. This finding is explained as immigrants to the US and European Union (EU) always arrive with less human capital and contribute less to the effective labour supply in destination countries. In addition, lower wage workers also tend to claim and receive more benefit from the destination countries.

To our knowledge, Sayan and Uyar (2001) was the first CGE research paper to combine the shock of population ageing and an endogenized decision making of international migration. They developed a 2*2*2*2 type dynamic OLG-CGE model, with homogenous labour supply, to investigate the welfare implication of labour mobility for two countries with different demographic characteristics. International migration was assumed to be driven by wage differentials between cohorts living in the two countries. It was shown that compared with autarky, a representative cohort in the destination country loses from this country adopting immigrants but a cohort in the source (younger) country gains. Iregui (2003) used a static multiregional one-sector GE model to analyze the impacts of international migration on global output. In this paper, labour heterogeneity was taken into account and labour stock in each region was classified into skilled and the unskilled labour according to real data. Wage differential was assumed to be the only driving force of international migration. It was shown that in the source regions, migration benefits (hurts) workers whose skill is substitutable (complementary) to that of migrating labour. In the destination regions, workers whose skill level is complementary (substitutable) to the skill level of immigrants benefit (lose). Iregui also suggested a large gain ranging from 15% to 67% of world GDP as a result of international migration[10]. In other words, a full integration

10 The contributions to world GDP are large because with perfect labour mobility, 37 to 53% of labour endowments in the developing regions will migrate to those developed regions with higher productivity.

of national labour markets has positive effects on world output, and the degree of contribution depends on the skill composition of migrating people.

Sayan and Uyar (2001) and Iregui (2003) are two representative papers using the CGE approach to investigate the welfare implication of international migration. In both of them, workers migrate according to wage differentials. Sayan and Uyar (2001) precisely modeled the decision making of migration using a migration coefficient showing the percentage of migrating people out of total population of the source country. In Iregui (2003), international migration was simply modeled as the integration of national labour markets around the world. Demographic shock was not included in her static model, but skill differential was taken into account based on real data. In both papers, perfect labour market mobility is assumed and a large number of people migrate from less developed region(s) to developed region(s).

In this paper, we develop a dynamic OLG-CGE model comprising both the shock of population ageing and labour heterogeneity. In addition, we allow only a reasonable portion of workers migrate from the developing region to the developed region for a limited period of time. The scale of labour mobility is calculated based on UN (2004) data on international migration in the base year of 2000. Thus, international migration acts as a temporary shock on the future upward trend in the ageing process for older and developed region.

2.3 MODEL DESCRIPTIONS

In the model, we calibrate the developed world (*DD*), the developing world (*DN*) and the rest of the world (*ROW*) based on real data. The developed region covers major industrial and most aged countries around world (US, Canada, EU and Japan); the developing region includes China and India, the two most populated developing countries. The age structures of China and India are comparatively younger than the developed region but older than the rest of the world in the dynamic, as projected by UN (2004). China and India are also the largest emerging economies characterized by their labour-abundant economy, export-oriented industry structures, increasing openness to the world market and high economy growth rate. On global labour market, these two countries are also the two largest source countries of high-skilled immigrants: in 2000, people migrating from China and India to other countries accounts for more than half of the total mobile people among countries around the world (UN2004). A large portion of their immigrants are highly educated.

In each of the three regions, two homogenous goods (*S1*, *S2*) are produced and two major inputs (capital and labour labelled as *Kd* and *Ld*) are used in the production of each good. In this paper, factor intensity of each good is calculated based on GTAP6.0 data on 56 sectors. We first treat the world as one single economy and classify the 56 sectors into two categories (capital-intensive and labour-intensive) according to the share of capital input in total value added. The sectors with more than 50% share of capital input in total value added are classified as capital-intensive good, so the capital-intensive good in our model includes 31 sectors and the labour-intensive good includes 25 sectors. Then we aggregate data for 56 sectors in each region according to the above two categories. Thus, capital intensity in the production of two goods is calibrated to be different across regions.

Labour input used in the production of each good is an aggregation of two kinds of labour differentiated by their skill levels. The unit labour endowment of these two kinds of labour is also calibrated based on GTAP6.0 data on the value added of "skilled" and "unskilled" labour in total output and total numbers of skilled and unskilled workers. Technologies are no longer identical across countries, and we calibrate individual production function for each good produced in each region based on the above data on capital, skilled labour and unskilled labour inputs.

In the model, each region is populated with seven generations (the working generation *G1* to *G5*, or *gj*, and the retired generation *G6*, *G7*, or *gm*) of population. *G1* is the only fertile generation. At initial steady state, we assume population of each of the seven generations equals to 1/7 of total population, and the shares of skilled and unskilled workers in each generation are fixed. Thus, the age structures of two kinds of labour are identical based on the above model settings. In our model, government behaviour is taken into account but bequest and heritance are not included.

2.3.1 Producer behaviour

In each region, there are two sectors producing each one of the two goods s (*S1* or *S2*) using Cobb-Douglas (CD) production technologies. A representative firm minimizes the production cost subject to the embedded constraint that characterizes the firm's technology:

$$\underset{Kd_{j,s,t},Ld_{j,s,t}}{Maximize}\, Pq_{j,s,t}X_{j,s,t} - R_{j,t}\,Kd_{j,s,t} + W_{j,t}Ld_{j,s,t} \tag{1}$$

$$s.t.\ X_{j,s,t} = SP_s Kd_{j,s,t}^{\alpha_s^K} Ld_{j,s,t}^{(1-\alpha_s^K)} \tag{2}$$

In equation (1), $Pq_{j,s,t}$ is the production price of good s in country j at time t. $R_{j,t}$ and $W_{j,t}$ represent the rental price and wage rate in country j at time t respectively. $Kd_{j,s,t}$ is the capital demanded by production sector s in country j at time t. $Ld_{j,s,t}$ is the aggregate labour demanded.

In equation (2), SP_s is a scaling constant parameter and $\alpha^K{}_s$ is the expenditure share measuring the intensity of use of capital in production. *S1* is capital-intensive good and *S2* is labour-intensive good. The technology is represented by a CD production function in which the sum of expenditure shares for capital and labour inputs is equal to one.

In this model, we assume that the aggregate labour supply, $Ld_{j,s,t}$, is a CES mixture of two kinds of labour supply (*qual*) in the economy: the skilled labour and the unskilled labour (equation 3).

$$Ld_{j,s,t} = (\delta^Q_{s,skill} Lsq_{j,s,skill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}} + \delta^Q_{s,unskill} Lsq_{j,s,unskill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}})^{\frac{\sigma_s^{LD}}{\sigma_s^{LD}-1}} \quad (3)$$

Skilled and unskilled labour is differentiated by the quantity of labour supply they could provide per unit of time. In equation (3), $\delta^Q{}_{s,qual}$ is the share of labour at skill level *qual* demanded by the production of good *s*, and the sum of $\delta^Q{}_{s,skill}$ and $\delta^Q{}_{s,unskill}$ is equal to one. $\sigma_s{}^{Ld}$ is the elasticity of substitution between different kinds of labour in sector *s*. The value of $\delta^Q{}_{s,qual}$ is calibrated to be different both across sectors and across countries.

Differentiating equation (1) with respect to equation (2), we get the following equations (4) and (5) as the first order conditions for producers in the economy:

$$\mathrm{R}_{j,t} = \alpha_s^K \frac{Pq_{j,s,t} X_{j,s,t}}{Kd_{j,s,t}} \quad (4)$$

$$W_{j,t} = (1-\alpha_s^K) \frac{Pq_{j,s,t} X_{j,s,t}}{Ld_{j,s,t}} \quad (5)$$

After the rental price and the aggregate wage rate are determined, a firm's optimization problem turns to be minimizing its expenditure on labour input. That is, it minimizes the following equation (6) subject to the above equation (3):

$$W_{j,t}Ld_{j,s,t} = wage_{j,skill,t}Lsq_{j,s,skill,t} + wage_{j,unskill,t}Lsq_{j,s,unskill,t} \tag{6}$$

where $wage_{j,qual,t}$ is the wage rate paid for labour input at skill level *qual* in country *j* at time *t*. The following equation (7) and equation (8) are the first order conditions:

$$Lsq_{j,s,qual,t} = \delta_{s,qual}^{Q^{\sigma_s^{LD}}} \left[\frac{W_{j,t}}{wage_{j,qual,t}} \right]^{\sigma_s^{LD}} Ld_{j,s,t} \tag{7}$$

$$W_{j,t}^{1-\sigma_s^{LD}} = \sum_s \delta_{s,qual}^{Q^{\sigma_s^{LD}}} wage_{j,qual,t}^{1-\sigma_s^{LD}} \tag{8}$$

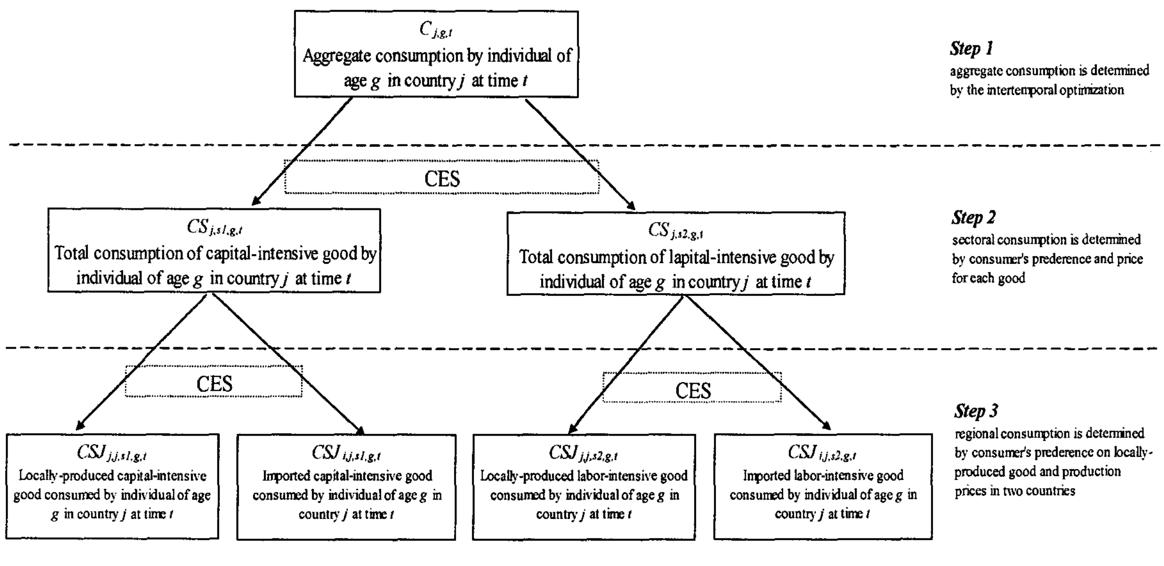
Equation (7) shows the demand for labour input at skill level *qual*, $Lsq_{j,s,qual,t}$, increases with the aggregate labour demanded, the share of this kind of labour in the technology and decreases with the wage rate paid for this kind of labour. Equation (8) shows the aggregate wage in region *j* is a weighted sum of wage rate paid for each kinds of labour.

2.3.2 Household behaviour

An Allais-Samuelson overlapping generation framework characterizes households in the economy, so that this model is based on the life-cycle theory of savings. In period *t*, each economy is populated by seven generations *g* (*G1* to *G7*) of population. A representative cohort born in period *t* works in the first five periods of its life (from 16 to 65 years of age), retires in period *t+6* and lives another two periods of time (from 66 to 85 years of age). Thus each period *t* approximately represents 10 years in our model.

The lifetime utility for a representative cohort is represented by a time-separable nested CES function with the elasticity different from one. The optimization problem for a cohort at skill level *qual* can be divided into three steps in an open economy with international trade. First, immediately after birth, a cohort decides on the allocation of its labour income on either current or future consumptions. On the second step of optimization, the aggregate consumption expenditure is allocated among different kinds of consumption goods. Finally, on the third step of optimization, an individual decides on the composition of each consumption good *s* in terms of the country of origin. In other words, consumption of good *s* is an aggregation of both locally- produced products and imported products following the Armington assumption. The following Figure 2 shows the relationships among the three steps of optimization.

Figure 2 The optimizing steps for a consumer



The intertemporal optimization problem for a representative cohort, at the skill level *qual* in country *j*, is to maximize its lifetime utility subject to the budget constraint:

$$\underset{C_{j,g,t}}{Maximize}\, U(C_{j,qual,g,t})_{j,qual,t} = \frac{\sigma^{IT}}{\sigma^{IT}-1}\sum_{g=1}^{7}(\frac{1}{1+\rho})^{g} C_{j,qual,g,t+g-1}^{\frac{\sigma^{IT}-1}{\sigma^{IT}}} \tag{9}$$

$$s.t.\quad LW_{j,qual,t} = \sum_{g=1}^{7} PCon_{j,g,t+g-1}(1+\tau^{C})(C_{j,qual,g,t+g-1}) \tag{10}$$

The CES-form utility function is shown by equation (9), and the lifetime utility is an aggregation of the present values of current consumption and future consumptions. In each country j, $U(C_{j,qual,g,t})_{j,qual,t}$ is the lifetime utility for a representative cohort at skill level *qual* and $C_{j,qual,g,t}$ is the quantity of aggregate consumption. σ^{IT} is the intertemporal elasticity of substitution, and ρ is the pure rate of time preference. The larger ρ, the more of its lifetime resources a cohort would spend in the first stage of life and the less it saves. As shown in equation (10), the lifetime wealth for an individual at the skill level *qual*, $LW_{j,qual,t}$, is allocated into current consumption and future consumptions. In this equation, $PCon_{j,g,t}$ is the aggregate consumption price for an individual of age g at time t. τ^c is the consumption tax rate. $LW_{j,qual,t}$ is represented by the following equation (11):

$$LW_{j,qual,t} = \sum_{g=1}^{7} \left\{ \frac{1}{1+Rint_{j,t+g-1}(1-\tau^K)} \right\}^t [LInc_{j,qual,g,t+g-1}(1-\tau^W - CtR_t) + Pens_{j,qual,g,t+g-1}] \quad (11)$$

where $LInc_{j,qual,g,t}$ is the labour income for an individual at skill level *qual* of age g at time t. $Rint_{j,t}$ is the interest rate in region j at time t. τ^k and τ^w are capital and wage tax rates respectively. CtR_t represents the contribution rate of pension out of total labour income; $Pens_{j,qual,g,t}$ is the pension income received by the last two generations.

In all countries, an individual's labour income depends both on the individual's age-dependent productivity and the skill level. Thus, the labour income for an individual in the working generation gj at time t can be represented by the following equations (12) and (13):

$$LInc_{j,qual,gj,t} = wage_{j,qual,t} EP_{j,g,qual} \quad (12)$$

$$EP_{j,g,qual} = \alpha^{Q}_{qual}(\gamma + \lambda g - \psi g^2), \quad \gamma, \lambda, \psi, \geq 0 \quad (13)$$

In equation (12), $EP_{j,g,qual}$ is the country-specific productivity coefficient to differentiate the quantities of labour supplied by individuals living in different working generations. In equation (13), $\alpha^{Q}{}_{qual}$ is the parameter to differentiate the quality of the skilled and the unskilled labour. It is assumed to be 1.2 for skilled labour and 1 for unskilled labour in our model. The productivity coefficient $EP_{j,g,qual}$ is calibrated based on the GTAP6.0 data on the value added for workers at skill level *qual* and the total number of this kind of workers in the economy. Following Fougère *et al.* (2007), we assume that an individual in the first generation contribute the least quantity of labour supply and an individual works at the maximum capacity during the ten years between 46 and 55 years old. Since we don't take into account different professions of labour supply, it is hard to classify the skill levels of workers into skilled and unskilled by profession. Thus, we assume the quantity of skilled labour supply accounts for 20% of the total labour force in the developed world, but only 10% in the developing world and the rest of the world. The following Table 1 shows the unit labour supply calibrated using GTAP6.0 data.

Table 1 Calibrated unit labour supply by skill and generation

	DD		*DN*		*ROW*	
	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled
G1	56.9858	6.1654	4.4844	0.6496	5.5946	1.2896
G2	63.3803	6.8572	4.9876	0.7225	6.2224	1.4343
G3	66.2013	7.1624	5.2096	0.7547	6.4993	1.4981
G4	65.4490	7.0811	5.1504	0.7461	6.4255	1.4811
G5	61.1234	6.6131	4.8105	0.6968	6.0008	1.3832

The intertemporal maximization problem is as follows. Households in all countries are forward looking and have perfect foresight. A representative cohort is born in the first period and works in the first five periods of its life as the labour force in the economy. Part of its labour income is saved as investment in capital stock and the rest is consumed. In the last two periods, this cohort spends all its wealth, including investment in the first period and interest income, on consumption. Immediately after birth, an individual decides on the allocation of his or her expected labour income into current consumption and future consumptions. Differentiating the above equation (9) with respect to the budget constraint (10) yields the following first order condition for the aggregate consumption, $C_{j,qual,g,t}$, consumed by an individual at skill level *qual* of age *g* (Equation 14):

$$C_{j,qual,g+1,t+g} = \left[\frac{(1+Rint_{t+g}(1-\tau^K))(1+\tau^C)PCon_{g,t+g-1}}{(1+\rho)(1+\tau^C)PCon_{g+1,t+g}} \right]^{\sigma^{IT}} C_{j,qual,g,t+g-1} \qquad (14)$$

Equation (14) shows the ratio of aggregate consumption by individual of age *g* at skill level *qual* at time *t+1* to it is at time *t* increases with the ratio of aggregate consumption price at time *t* to the aggregate consumption price at time *t+1*. In other words, higher are future consumption prices, higher is current consumption and lower is future consumption.

On the second step of optimization, an individual of age *g* allocates the expenditure on the optimal aggregate consumption, $C_{j,qual,g,t}$, across sectors. The problem is to maximize the following CES utility function subject to budget constraint:

$$\underset{CS_{j,S1,qual,g,t},CS_{j,S2,qual,g,t}}{Maximize} \; C_{j,qual,g,t} = (\delta_{S1}^{CS} CS_{j,S1,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}} + \delta_{S2}^{CS} CS_{j,S2,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}})^{\frac{\sigma^{CS}}{\sigma^{CS}-1}} \qquad (15)$$

$$s.t. \; PCon_{j,g,t} C_{j,qual,g,t} = PCons_{j,S1,t} CS_{j,S1,qual,g,t} + PCons_{j,S2,t} CS_{j,S2,qual,g,t} \qquad (16)$$

In equation (15), δ_s^{CS} as the parameter representing the preference of a consumer of age *g* on consumption goods across sectors. σ^{CS} is the elasticity of substitution for consumptions across sectors. $CS_{j,s,qual,g,t}$ is the quantity of consumption of good *s* by an individual of age *g* at skill level *qual* living in country *j* at time *t*. In equation (16), $PCons_{j,s,t}$ is consumption price of good *s*. The following equations (17) and (18) are the first order conditions for the second step of optimization:

$$CS_{j,qual,s,g,t} = \delta_S^{CS^{\sigma^{CS}}} \left[\frac{PCon_{j,g,t}}{PCons_{j,s,t}} \right]^{\sigma^{CS}} C_{j,qual,g,t} \tag{17}$$

$$PCon_{j,g,t}^{1-\sigma^{CS}} = \sum_s \delta_S^{CS^{\sigma^{CS}}} PCons_{j,s,t}^{1-\sigma^{CS}} \tag{18}$$

Equation (17) shows the consumption of good *s* by individual of age *g* at skill level *qual* at time *t* increases with the aggregate consumption but decreases with the consumption price of good *s* on the market. Equation (18) shows the aggregate consumption price is a weighted sum of the consumption prices for different good *s*.

In our Armington-type trade model, an individual in country *j* allocates the consumption expenditure on good *s* by country of origin, which is the third step of optimization. In other words, a decision is made on how much to spend on either locally-produced good *s* or imported good *s*. The following equations show the third step of consumer's optimization:

$$\underset{CSJ_{j,j,s,qual,g,t},CSJ_{i,j,s,qual,g,t}}{Maximize} \quad CS_{j,s,qual,g,t} = \left(\delta_{j,j,s}^{CE} CSJ_{j,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}} + \delta_{i,j,s}^{CE} CSJ_{i,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}}\right)^{\frac{\sigma^{CE}}{\sigma^{CE}-1}} \tag{19}$$

$$s.t.\ PCons_{j,s,t} CS_{j,s,qual,g,t} = Pq_{j,s,t} CSJ_{j,j,s,qual,g,t} + Pq_{i,s,t} CSJ_{i,j,s,qual,g,t} \tag{20}$$

In equation (19), $\delta_{i,j,s}{}^{CE}$ as the parameter representing the preference of a consumer on goods *s* produced in country *i*, and σ^{CE} is the elasticity of substitution for locally-produced and imported consumption good *s*. $CSJ_{i,j,qual,s,g,t}$ is the quantity of consumption of good *s* produced in country *i* but consumed by country *j*'s consumer. In other words, set *i* shows the country of origin of a consumed good and set *j* shows where this good is consumed.

Accordingly, $CSJ_{j,j,qual,s,g,t}$ represents locally-produced consumption good s consumed by generation g in country j at time t. The following equation (21) and equation (22) are the first order conditions for the third step of optimization:

$$CSJ_{i,j,s,qual,g,t} = \delta_{i,j,s}^{CE^{\sigma^{CE}}} \left[\frac{PCons_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{CE}} CS_{j,s,qual,g,t} \tag{21}$$

$$PCons_{j,s,t}^{1-\sigma^{CE}} = \sum_{i} \delta_{i,j,s}^{CE^{\sigma^{CE}}} Pq_{i,s,t}^{1-\sigma^{CE}} \tag{22}$$

In the numerical model, the parameter values used in the above optimization problems for households are all calibrated according to real data.

2.3.3 Government behaviour

One feature of this model is we take into account government behaviour. The government revenue comes from three taxable sources: wage income, capital income and consumption expenditure. τ_t^W, τ^K, *and* τ^C are the wage tax rate, the constant capital income tax rate and consumption tax rate respectively. In our model, we assume the wage tax rate is endogenous. Government expenditure is dominated by government consumption, $Gov_{j,t}$, and this aggregate government consumption is a CES form of two kinds of government consumption of good s. When tax revenue does not offset expenditure, a government issues bond and pays interest on the debt it owns to the public in each period. The social welfare system is characterized by a pay-as-you-go pension system, and pension benefits for the elderly are financed by contributions by the working generations. The budget constraint for a government is as the following equation (23):

$$PGov_{j,t}Bond_{j,t+1}+\sum_{g}Pop_{j,g,t}\left\{\begin{array}{l}\tau^{W}(Linc_{j,g,t}+Pens_{j,g,t})+\tau^{C}PCon_{j,t}C_{j,g,t}+\\ \tau^{K}\left(\dfrac{R\text{int}_{j,t-1}PGov_{j,t}}{PGov_{j,t-1}}-1\right)PGov_{j,t+1}Bij_{j,g,t}+\tau^{K}(Rret_{j,t-1}-1)PI_{j,t-1}Kij_{j,g,t}\end{array}\right\}$$

$$=PGov_{j,t}Gov_{t}+\left(\frac{R\text{int}_{j,t-1}PGov_{j,t}}{PGov_{j,t-1}}\right)PGov_{j,t}Bond_{t} \qquad (23)$$

where $PGov_{j,t}$ is the price of government bond and $Bond_{j,t}$ is the quantity of government bonds issued at time t. $Pop_{g,t}$ represents population of generation g living in country j at time t. $Bij_{j,g,t}$ and $Kij_{j,g,t}$ are individuals' ownerships of foreign bonds and capital stock respectively. $Pens_{g,t}$ is the pension benefit received by an elderly of generation g at time t. More specifically, the total pension benefit received by the elderly in region j at time t is financed by part of the labour income of the working generations, as shown by the following equation (24). As the age structures turn to be older in the economy, the contribution rate, CtR_t, is adjustable to finance higher demand for pension benefit from the economy.

$$\sum_{gm}Pop_{j,iqual,gm,t}Pens_{j,gm,t}=\sum_{gj}CtR_{t}Linc_{j,g,t} \qquad (24)$$

In our model, government consumption in country j is assumed to be increasing at a same rate as the population growth rate of this country in each period. Thus, the aggregate government consumption, $Gov_{j,t}$, is exogenous in our model. A government also needs to solve some optimization problems which are similar to those depicted in sector *3.2*: after the $Gov_{j,t}$ is determined, it allocates this aggregate consumption expenditure across sectors and then allocates the expenditure on good s by country of origin, which is depicted by the following four equations:

$$\underset{GovSJ_{i,j,S1,t},GovSJ_{j,j,S2,t}}{Maximize} \; Gov_{j,t} = (\delta_{S1}^{GS} GovS_{j,S1,t}^{1-\frac{1}{\sigma^{GS}}} + \delta_{S2}^{GS} GovS_{j,S2,t}^{1-\frac{1}{\sigma^{GS}}})^{\frac{\sigma^{GS}}{\sigma^{GS}-1}} \tag{25}$$

$$s.t. \quad GovS_{j,s,t} = (\delta_{j,j,s}^{GE} GovSJ_{j,j,s,t}^{1-\frac{1}{\sigma^{GE}}} + \delta_{i,j,s}^{GE} GovSJ_{i,j,s,t}^{1-\frac{1}{\sigma^{GE}}})^{\frac{\sigma^{GE}}{\sigma^{GE}-1}} \tag{26}$$

$$and \quad PGov_{j,t} Gov_{j,t} = PGovS_{j,S1,t} GovS_{j,S1,t} + PGovS_{j,S2,t} GovS_{j,S2,t} \tag{27}$$

$$and \quad PGovS_{j,s,t} GovS_{j,s,t} = Pq_{j,s,t} GovSJ_{j,j,s,t} + Pq_{i,s,t} GovSJ_{i,j,s,t} \tag{28}$$

The government behaves as the following. As $Gov_{j,t}$, the government expenditure, is known, it minimizes equation (27) subject to equation (25), then minimizes equation (28) subject to equation (26). Maximizing equation (25) subject to equation (27), we get equations (29) and (30) as the first order conditions for a government's second step of optimization:

$$GovS_{j,s,t} = \delta_s^{GS^{\sigma^{GS}}} \left[\frac{PGov_{j,t}}{PGovS_{j,s,t}} \right]^{\sigma^{GS}} Gov_{j,t} \tag{29}$$

$$PGov_{j,t}^{1-\sigma^{GS}} = \sum_s \delta_s^{GS^{\sigma^{GS}}} PGovS_{j,s,t}^{1-\sigma^{GS}} \tag{30}$$

On the third step of optimization, a government allocates the expenditure on good s across countries, which is to maximize equation (26) subject to equation (28). The following equations (31) and (32) are the first order conditions:

$$GovSJ_{i,j,s,t} = \delta_{i,j,s}^{GE^{\sigma^{GE}}} \left[\frac{PGovS_{j,t}}{Pq_{i,s,t}} \right]^{\sigma^{GE}} GovS_{j,s,t} \tag{31}$$

$$PGovS_{j,t}^{1-\sigma^{GE}} = \sum_i \delta_{i,j,s}^{GE^{\sigma^{GE}}} Pq_{i,s,t}^{1-\sigma^{GE}} \tag{32}$$

2.3.4 Determinants of investment demand

In each period t, savings of the younger generations finance investment equipped by firms to increase and maintain the stock of physical capital in period $t+1$. The usual law of motion of capital stock can be represented by the following equation (33):

$$KS_{j,t+1} = I_{j,t} + (1-DR)KS_{j,t} \tag{33}$$

In equation (33), $I_{j,t}$ is the aggregate investment in country j at time t. The capital stock in the economy at time $t+1$, $KS_{j,t+1}$, augments with investment in the previous period t and declines with depreciation rate (DR). This equation shows the aggregate investment level is determined by the gap between demand of capital stock in the next period and current capital stock net of depreciation.

The capital stock is a composite good composed of all available goods in the economy. Consequently, the capital stock in each economy is built using an investment technology with both the capital-intensive good and the labour-intensive good. After the aggregate investment level is determined, there are also two steps of optimization for the investors in the economy, which are depicted by the following four equations:

$$I_{j,t} = (\delta_{S1}^{IS} IS_{j,S1,t}^{1-\frac{1}{\sigma^{IS}}} + \delta_{S2}^{IS} IS_{j,S2,t}^{1-\frac{1}{\sigma^{IS}}})^{\frac{\sigma^{IS}}{\sigma^{IS}-1}} \tag{34}$$

$$IS_{j,s,t} = (\delta_{j,j,s}^{IE} ISJ_{j,j,s,t}^{1-\frac{1}{\sigma^{IE}}} + \delta_{i,j,s}^{IE} ISJ_{i,j,s,t}^{1-\frac{1}{\sigma^{IE}}})^{\frac{\sigma^{IE}}{\sigma^{IE}-1}} \tag{35}$$

$$PI_{j,t} I_{j,t} = PIS_{j,S1,t} IS_{j,S1,t} + PIS_{j,S2,t} IS_{j,S2,t} \tag{36}$$

$$PIS_{j,s,t} IS_{j,s,t} = Pq_{j,s,t} ISJ_{j,j,s,t} + Pq_{i,s,t} ISJ_{i,j,s,t} \tag{37}$$

In equation (34), δ_s^{IS} is the given share of good s in aggregate investment, and σ^{IS} is the elasticity of substitution for investment goods. $IS_{j,s,t}$ is the investment good s used in country

j at time t. In equation (35), $\delta_{i,j,s}{}^{IE}$ is the given share of investment good s produced in country i but used in country j. $ISJ_{i,j,s,t}$ is the investment good s used in country j but produced in country i at time t. Accordingly, $ISJ_{j,j,s,t}$ is the locally-produced investment good s used in country j at time t. σ^{IE} represents the elasticity of substitution for locally-produced and imported investment good s. In equations (36) and (37), $PI_{j,t}$ represents the aggregate price of all investment goods in country j at time t and $PIS_{j,s,t}$ is the price of investment good s in country j at time t.

On the first step of optimization, an investor's problem is to minimize equation (36) subject to equation (34). The following equations (38) and (39) are the first order conditions:

$$IS_{j,s,t} = \delta_s^{IS^{\sigma^{IS}}} \left[\frac{PI_{j,t}}{PIS_{j,s,t}} \right]^{\sigma^{IS}} I_{j,t} \tag{38}$$

$$PI_{j,t}^{1-\sigma^{IS}} = \sum_s \delta_s^{IS^{\sigma^{IS}}} PIS_{j,s,t}^{1-\sigma^{IS}} \tag{39}$$

After the $IS_{j,s,t}$ is determined, on the second step of optimization, an investor allocates its investment expenditure on investment good s across countries. The optimization problem is to minimize equation (37) subject to equations (35). The following equations (40) and equation (41) are the first order conditions:

$$ISJ_{i,j,s,t} = \delta_{i,j,s}^{IE^{\sigma^{IE}}} \left[\frac{PI_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{IE}} IS_{j,s,t} \tag{40}$$

$$PIS_{j,s,t}^{1-\sigma^{IE}} = \sum_i \delta_{i,j,s}^{IE^{\sigma^{IE}}} Pq_{i,s,t}^{1-\sigma^{IE}} \tag{41}$$

2.3.5 Foreign trade with the rest of the world

As explained in the above sections, we allocate demand in country *j* for goods produced in country *i* based on the Armington assumption. In other words, even though individual producers are microscopic price takers, good *s* are assumed to be differentiated in demand by country of origin. At the initial steady state where all countries are assumed to be perfectly symmetric, each country imports and exports the same quantity of *S1* and *S2*. Trade exists when countries are perfectly symmetric because each of them demands both locally-made and foreign-made goods. During the processes of population ageing, changes in relative factor abundances bring changes to two countries' comparative advantages. Accordingly, demands for the four goods change overtime.

The aggregate level of one country's import or export of good *s* (denoted as $IMP_{j,s,t}$ and $EXP_{j,s,t}$) is determined by the above optimization steps for agents (consumer, investor and government), which can be shown by the following equation (42) and equation (43):

$$IMP_{j,s,t} = \sum_{i,qual,g} POP_{j,qual,g,t} CSJ_{i,j,qual,s,g,t} + ISJ_{i,j,s,t} + GovSJ_{i,j,s,t} \quad for\, i \neq j \quad (42)$$

$$EXP_{j,s,t} = \sum_{i,qual,g} POP_{i,qual,g,t} CSJ_{j,i,qual,s,g,t} + ISJ_{j,i,s,t} + GovSJ_{j,i,s,t} \quad for\, i \neq j \quad (43)$$

2.3.6 Equilibrium conditions

A general equilibrium solution is one in which all economic behaviour are consistent with both current prices and future prices and all markets clear. In this model, we impost the following four equilibrium conditions for good market, labour market, capital market and the market for financial asset respectively:

$$X_{j,s,t} = \sum_{i,g} POP_{i,g,t} CSJ_{j,i,s,g,t} + ISJ_{j,i,s,t} + GovSJ_{j,i,s,t} \quad (44)$$

$$\sum_{g} POP_{j,qual,gj,t} LS_{j,qual,gj,t} = \sum_{s} Lsq_{j,qual,s,t} \quad (45)$$

$$KS_{j,t} = \sum_{s} Kd_{j,s,t} \quad (46)$$

$$\sum_{g,qual} POP_{j,qual,gm,t+1} LD_{j,qual,gm,t+1} = PI_{j,t} KS_{j,t+1} + PGov_{j,t} Bond_{j,t+1} \quad (47)$$

In equation (44), $LS_{qual,g,t}$ is the unit labour supply provided by an individual at skill level *qual* in the working generation *gj* in country *j*. In equation (47), $LD_{j,qual,g,t}$ is the financial asset owned by the all generations (except the youngest one) living in country *j* at time *t*.

2.3.7 To model population ageing

In any period *t*, the fertility rate in country *j* is labelled as $FR_{j,t}$. The population of country *j* regenerates according to the following equation (48) and (49):

$$POP_{j,qual,G1,t+1} = POP_{j,qual,G1,t}(1 + FR_{j,t}) \quad (48)$$

$$POP_{j,qual,g+1,t+1} = POP_{j,qual,g,t} \quad (49)$$

In this paper, the modeling of population ageing is to apply a certain series of fertility rate so that we can replicate the baby boom after the WWII and the process of ageing starting from the 1990s. Population ageing is modeled according to the age structures of the developed, the developing world and the rest of the world from 2000 to 2100 projected by UN (2004). The evolution of the resulted *EDR*s for the three countries is depicted by the following Figure 3. It is shown that the *EDR* for the developed world is significantly higher than the rest of the world after 2000. In addition, the degree of ageing for the developing region is also much higher than it is for the rest of the world. This is because that the fertility rate of Chinese population is projected to significantly drop over the next several decades. In the long run, however, the fertility rates for all regions converge to zero, so their levels of *EDR*s also converge to the initial level of 0.4.

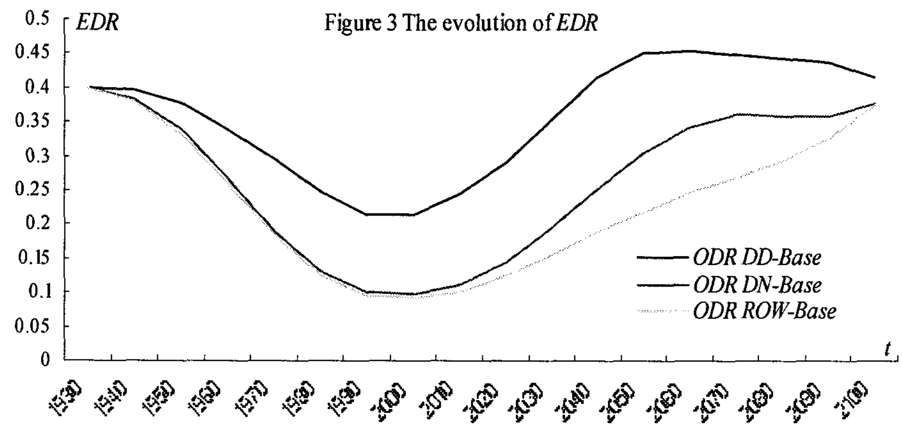


Figure 3 The evolution of *EDR*

2.3.8 To model the labour mobility

In the simulation experiments, we investigate the following scenarios of international migration and compare them with the baseline scenario under which there is no migration. Under scenario A, we assume that both the skilled labour and the unskilled labour migrate from region *DN* to region *DD*. Following Sayan and Uyar (2001), we also assume a migration coefficient, $MigCoe_{i,j,qual,t,g}$, representing the ratio of the migrating population, at skill level *qual*, from the source country (*i*) to the destination country (*j*) to the total population. This coefficient is calculated according to the following equation (50):

$$MigCoe_{i,j,qual,t+1,g} = Z\left[\frac{Max(Wage_{i,qual,t}, Wage_{j,qual,t}) - Min(Wage_{i,qual,t}, Wage_{j,qual,t})}{Wage_{i,qual,t}}\right]^{\gamma} \quad (50)$$

In equation (50), γ is the elasticity showing the responsiveness of the portion of migrating people out of total population to the wage differential across countries, and its value is set to be 1.5 (Sayan & Uyar 2001). Z is a value selected so that we can calculate a reasonable portion of migrating people. According to UN (2004), the inflow of labour accounts for 0.056% of the total population of the developed world, so the value of Z is set to be 0.006 in our model. It is shown that the portion of migrating people at time $t+1$ increases with the value of Z, the wage differential in the previous period t and decreases with the value of γ. Given the wage levels in all three countries, we could calculate a positive value of $MigCoe_{i,jqual,t,g}$ for each period t given that there are wage differentials across countries. Moreover, for country i and j, the value of $MigCoe_{i,j}$ and $MigCoe_{j,i}$ are different positive numbers according to the above equation (50). Under the migration scenario A, the percentage of unskilled labour migrating from the developing region to the developed region is slightly higher than it is of skilled labour in 2020.

Migration between country i and j is modeled as the following: in any period t, if the wage in country i is lower than it is in country j, part of country i's population in $G1$ (the first generation) migrates to country j at the beginning of the next period $t+1$. Then the total population of country i turns to be:

$$POP_{i,qual,t+1} - MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1}$$

and the total population in country j turns to be:

$$POP_{j,qual,t+1} + MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1}$$

Under scenario B, we only allow migration among the skilled labour across countries, and the scale of labour mobility across countries is set to be identical to it is under scenario A. To equalize the scales of migration under two scenarios, we assign a higher value of Z

(0.186) for scenario *B*. Thus, the impacts of migration on the dynamics of two regions' population are identical. The only difference between two scenarios is the skill composition of immigrants. Under both scenarios, we only allow migration for a limited period of time. In this paper, we use the baseline data in 2000 and investigate the impacts of migration shock on the source and destination regions. Thus we assume that after observing the wage differential between two regions in previous period 2010, people in region *DN* decide on migrating to region *DD* at time 2020. In this manner, international migration acts as a temporary shock on the future upward trend in the ageing process for older countries.

2.3.9 Calibration

In our numerical model, variables in all three economies are calibrated according to their real situation in the base year of 2000 according to GTAP 6.0 data set. The following Table 2 presents values of critical elasticity values we assumed in our model and Table 3 presents calibrated values of the parameters.

Table 2 Elasticity values assumed in calibration

	DD	*DN*	*ROW*
σ_S^{LD}	0.7	0.7	0.7
σ^{IT}	2.5	1.5	2.5
σ^{CS}	2.5	2.5	2.5
σ^{CE}	3	3	3
σ^{GS}	3.5	3.5	3.5
σ^{GE}	3	3	3
σ^{IS}	3	3	3
σ^{IE}	3	3	3
DR	0.6	0.6	0.6

On Table 2, most of the elasticity values are following Fougère *et al.* (2007). The value of intertemporal elasticity of substitution, σ^{IT}, follows Auerbach and Kotlikoff (1987) who assumed this value to be 0.25 on a one-year base. According to Cashin and McDermott

(2003), the rich countries (consumers) have a higher value of intertemporal elasticity of substitution than poor countries (consumers), as shown by empirical evidences by Atkeson and Ogaki (1996). Thus, we assume a higher value of σ^{IT} for the developed region in our model. In the literature, estimations of the intertemporal elasticity of substitution for developing countries are very limited. In their OLG-CGE research on the fiscal policy of China, Lin *et al.* (2005) assumed this elasticity value to be 1.5 for China. This value was claimed to be within the range considered by real business cycle literature (see Kydland and Prescott, 1982; Lucas, 1990). In our model, we also use 1.5 as the value of intertemporal elasticity of substitution for the developing region.

The following Table 3 presents share parameters calculated based on the assumed elasticity values on Table 2 and real data. The values of α^{K}_{s} show the capital intensity in different sectors across countries[i]. It is found the production of capital-intensive good uses the highest portion of capital input in the developed region. Our calibration results also show that the developed region has the highest value of time preference rate, ρ. This means consumers in the developed region are the most impatient group of individuals who are the least likely to smooth their current consumption by saving part of their labour income.

Regarding the skill intensities in the production of each good, we observe that for the developed region, the shares of skilled labour in the production of both kinds of good are higher than they are for the developing region and the rest of the world. In the developed region, the capital-intensive good is more skill-intensive than the labour-intensive good. However, for the developing region and the rest of the world, the skilled labour is comparatively more intensively used in the production of labour-intensive good *S2*. In other words, the capital-intensive good *S1* is more skill intensive for the developed region than it is for the developing region. This finding is critical in our future analysis.

On the consumption side, the shares of the labour-intensive good *S2* in consumption, government expenditure and investment are higher than of the capital-intensive good *S1*. In other words, a consumer, an investor and a government in each economy tends to demand higher quantity of labour-intensive good than capital-intensive good.

Table 3 Calibrated parameters

	DD	*DN*	*ROW*
$\alpha^K{}_{S1}$	0.632	0.599	0.642
$\alpha^K{}_{S2}$	0.277	0.418	0.430
ρ	0.463	0.447	0.421
$\delta_{s1,skill}{}^{LD}$	0.469	0.148	0.256
$\delta_{s2,skill}{}^{LD}$	0.403	0.206	0.345
$\delta_{s1}{}^{CS}$	0.428	0.451	0.447
$\delta_{s2}{}^{CS}$	0.571	0.548	0.552
$\delta_{s1}{}^{IS}$	0.330	0.258	0.273
$\delta_{s2}{}^{IS}$	0.670	0.742	0.777
$\delta_{s1}{}^{GS}$	0.059	0.097	0.041
$\delta_{s2}{}^{GS}$	0.941	0.903	0.959

The model is solved based on the following strategy. Given the assumed elasticity values and calibrated share parameters, we first solve the trade model with zero population growth. The calibrated values of variables are used as initial steady-state values. Then we introduce the regional-specific shock of population ageing, projected by UN (2004), into the model and solve the baseline scenario without migration for all counties. Finally, we introduce the shock of labour mobility into the model and solve two scenarios of international migration.

2.4 SIMULATION RESULTS

2.4.1 The baseline scenario

The baseline scenario is characterized by three open economies with different processes of population ageing trading with each other. In this paper, we introduce a persistent shock of population ageing since the end of the 20th century and analyze the impacts of migration on the developed destination region and the developing source region between 2000 and 2100. Important simulation results for this scenario are presented and compared with the following two scenarios A and B of international migration.

2.4.2 Scenarios of international migration

Under scenario A, we assume both skilled and unskilled labour migrates from the younger developing country *DN* to the older developed country *DD*. The driving force of international migration is the differentials between the wage rates paid for skilled or unskilled labour in the two countries. At time 2020, the portion of migrating people out of the total population of *DN* ($MigCoe_{DN,DD,qual,2020,g}$) is calculated based on the above equation (50). The absolute values of this migration coefficient for skilled and unskilled labour at time 2020 are 0.1642% and 0.1655% respectively. This means unskilled workers have a higher tendency to migrate from the developing region to the developed region in the context of ageing. Thus, under scenario A, there will be 0.00088 units of skilled labour and 0.00835 units of unskilled labour migrating from region *DN* to region *DD*. Migrating people account for around 0.2% of the net increase of the source region's population at time 2020. However, it accounts for around 2% of the net increase of the destination region's population in the same period.

Under scenario B, it is assumed that all the migrating people under scenario A are skilled labour from region *DN*. Thus, under both scenarios, migration has identical impact on the total population and overall age structures of the two regions. However, different skill compositions of migrating people change the skill composition of the aggregate labour stock in both regions.

On the following Table 4, we present the changes of elderly dependent ratios compared with the 1930 base-year level (0.4) for all regions under the baseline and migration scenarios:

Table 4 Changes of EDRs

(unit: 1%)

	EDR-DD-Base	*EDR-DD-Mig*	*EDR-DN-Base*	*EDR-DN-Mig*	*EDR-ROW-Base*	*EDR-ROW-Mig*
2020	-27.41	-27.53	-63.78	-63.77	-69.09	-69.09
2030	-11.59	-11.88	-51.21	-51.17	-61.14	-61.14
2040	4.15	3.63	-36.94	-36.88	-52.77	-52.77
2050	12.53	11.76	-24.05	-23.95	-45.42	-45.42
2060	13.51	12.53	-14.51	-14.36	-38.37	-38.37
2070	12.11	11.62	-9.95	-9.88	-32.70	-32.70
2080	10.72	10.72	-10.56	-10.56	-26.70	-26.70
2090	9.63	9.63	-10.35	-10.35	-18.19	-18.19
2100	4.10	4.10	-5.51	-5.51	-6.33	-6.33

In the first chapter of this dissertation, it is shown that based on our Armington-type OLG-CGE model, the older country with a faster process of population ageing gains from international trade. This gain stems from a persistent gap between two countries' comparative advantages. Under the baseline scenario, the comparative advantages in capital-intensive and labour-intensive goods for the developed and the developing regions are shown by the following Figure 3. On this figure, we show the evolution of the percentage deviation of the capital-labour ratios from their base-year levels in 1930 (Figure 4).

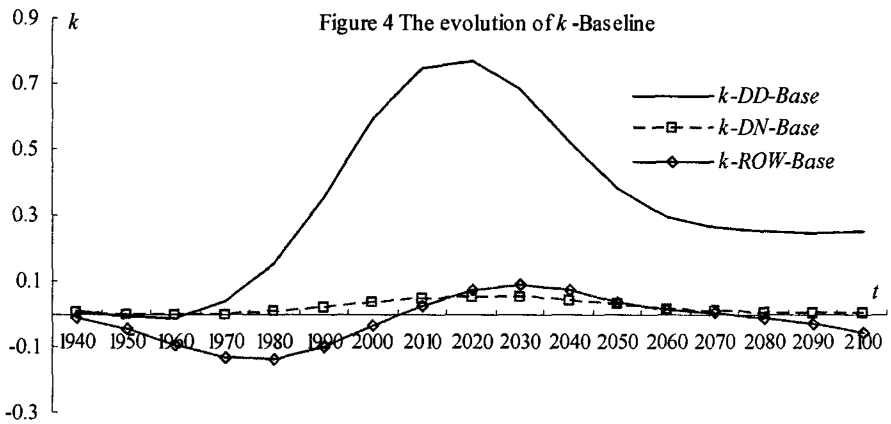


Figure 4 The evolution of k -Baseline

Our Armington-type trade model based on real data also shows that with the population ageing around the world, the capital-labour ratio in the developed region is much larger than other regions. This suggests the developed (developing) region always has comparative advantages in capital-intensive (labour-intensive) good. Under the baseline scenario, as the degrees of ageing diverge across regions, producers in each region adjust their production according to the changes of market prices and factor prices in the economy. In the several periods before 2000, the age structures of all regions turn to be younger than previous periods (Table 4), more effective labour supply reduces wage rates and more labour input is employed during the baby-boom periods. Simultaneously, the economies have to expand so that it could "feed" more people. This requires higher quantity of capital input in the economy. A larger number of young savers in the economy make investment more attractive. During these periods, it is shown the speed of capital stock accumulating is faster than the growth of population because of the presence of a larger number of young savers in the economy. As their age structures become older after 2000, labour becomes more expensive and less labour input is employed in production. The scale of economy also shrinks as the result of population ageing. An ageing society also means relatively smaller number of savers in the economy, so it is shown these regions' capital-labour ratios begin to fall to their long-run steady-state levels. Comparing the evolutionary paths of this ratio in all regions, we see the developed region has the highest steady-state k value. This finding is consistent with that in the first chapter of this dissertation: an Armington-type trade model suggests the capital-labour ratio for an older capital-abundant country is persistently higher than it is for a younger labour-abundant country.

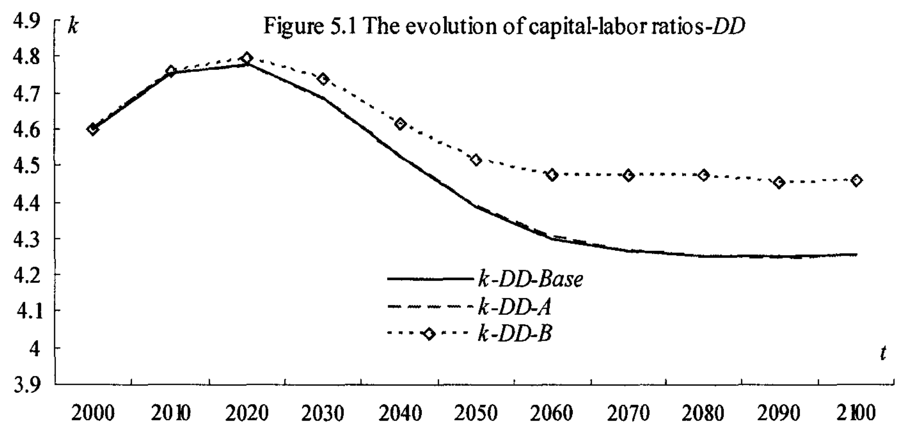


Figure 5.1 The evolution of capital-labor ratios-*DD*

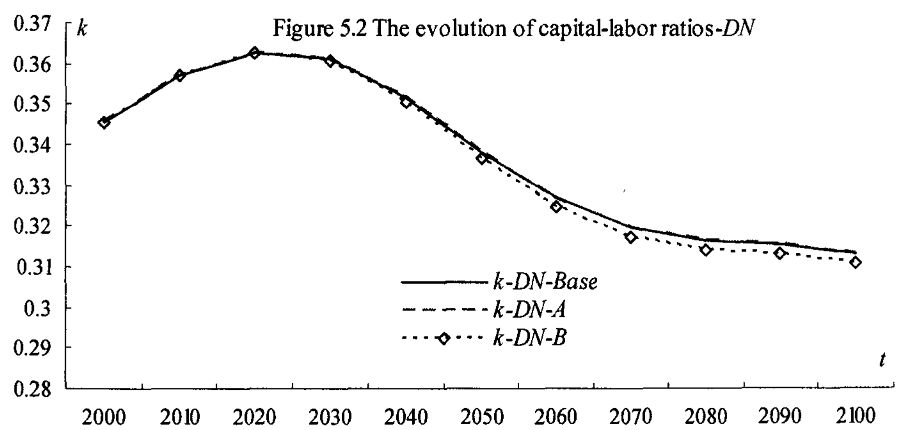


Figure 5.2 The evolution of capital-labor ratios-*DN*

Under the migration scenario A, it is shown by Figure 5 that the inflow of both kinds of labour does not have significant effect on the capital-labour ratios in the developed and developing regions. The absolute value of region *DD*'s steady-state capital-labour ratio under this scenario turns to be slightly lower than the baseline scenario. This is because the wage differential between unskilled labour across countries turns to be slightly higher than it is between skilled labour, so comparatively higher portion of unskilled labour migrates to the developed region. In other words, based on equation (50) used to calculate the migration coefficients, the migration of skilled and unskilled labour is very close to be proportionate to the labour stock in the source region. This is because as the developed region turns to be older than the developing region, the wage differential between unskilled labour in the two countries is found to be higher than it is between the skilled labour under scenario A. In this paper, the emphasis is put on the impacts of increasing skill composition of immigrants on destination and source regions, so we call this "close-to-proportionate" migration as proportionate migration for convenience. Consequently, the post-migration skill composition of labour turns to be lower than the baseline scenario, so is the capital stock demanded by the production. This finding is consistent with that by Fehr *et al.* (2003) who showed doubling present scale of immigration to ageing and developed countries could lower those countries' capital-labour ratios.

Under scenario B, however, inflow of skilled labour could significantly improve the capital-labour ratio in the developed region. In this paper, we assume that there is no mismatching problem for immigrants. In other words, skilled immigrants immediately find a job matching his or her skill level upon arrival. Thus, the same scale of migration of skilled labour is accompanied by a higher purchasing power than it is under scenario A because immigrants could work at a higher level of productivity. This requires the destination region to expand its economy even more because of higher demand from the economy. Consequently, it has to accumulate higher quantity of capital stock. Figure 5.1 shows the speed of capital stock accumulating is faster than the speed of population increasing, so the capital-labour ratio turns to be higher under scenario B. For the developing region, its capital-labour ratio moves in the opposite way. The outflow of labour frustrates local purchasing power and demand for locally-produced goods. Consequently, the size of economy is reduced because the developing region turns to be a smaller economy than it is under the baseline scenario and less capital stock is demanded by production. Under scenario A, the decrease of capital stock is proportionate to the decrease of labour stock in the economy and this region's capital-labour ratio is least affected. Under scenario B, the speed of capital stock decreasing is faster than it is of population deceasing, so the capital-labour ratio in the developing region is lower than the baseline scenario. Fehr *et al.* (2003) found capital-labour ratios in the destination developed countries are reduced after immigration. They explained the so-called "capital shortage" to landed immigrants are endowed with lower level of human capital compared with native workers. In contrast, findings in this paper suggest that increasing skill composition of immigrants could significantly improve the post-migration capital-labour ratio for the destination region.

The changes of factor abundance are consistent with the changes of two regions' factor prices. As shown by the following Figure 6, the inflow of both kinds of labour under scenario A has a very limited impact on the wage rates for both the skilled and unskilled labour in the developed region. Under scenario B, however, the inflow of foreign skilled labour frustrates the wage rate for local skilled worker, but makes the unskilled labour more expensive in the destination country *DD*. This is because cheaper skilled labour makes producers produce more capital-intensive good and thus has a positive effect on the wage rate for the unskilled labour whose skill level is complementary.

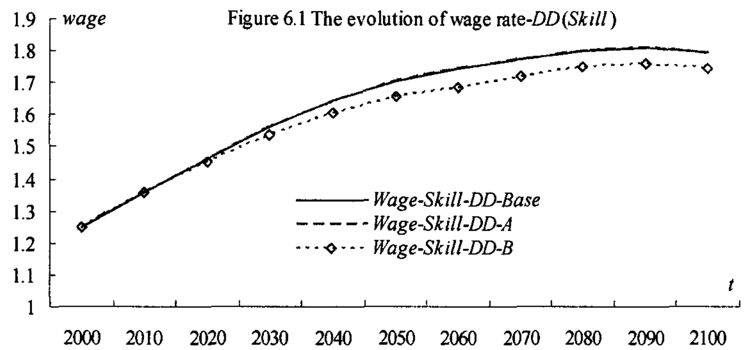


Figure 6.1 The evolution of wage rate-*DD*(*Skill*)

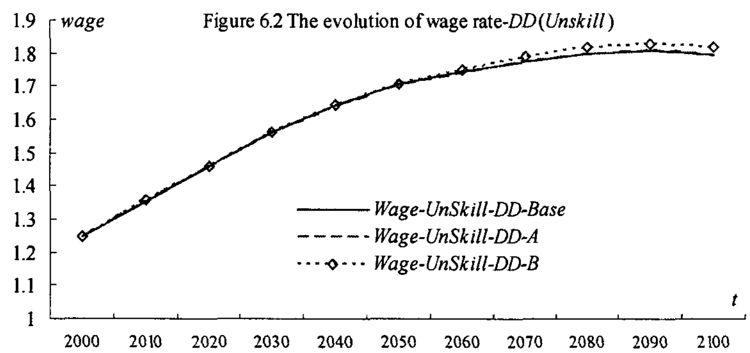


Figure 6.2 The evolution of wage rate-*DD*(*Unskill*)

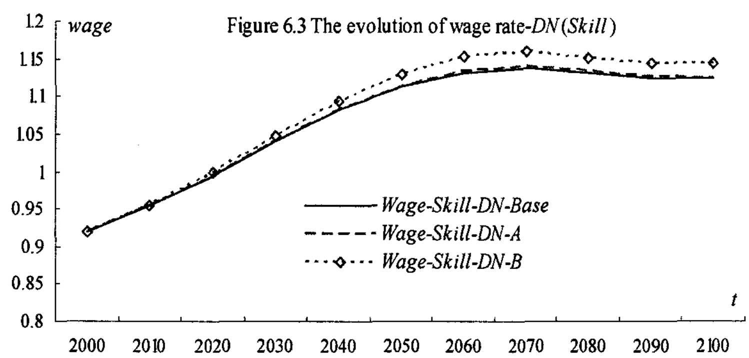


Figure 6.3 The evolution of wage rate-*DN*(*Skill*)

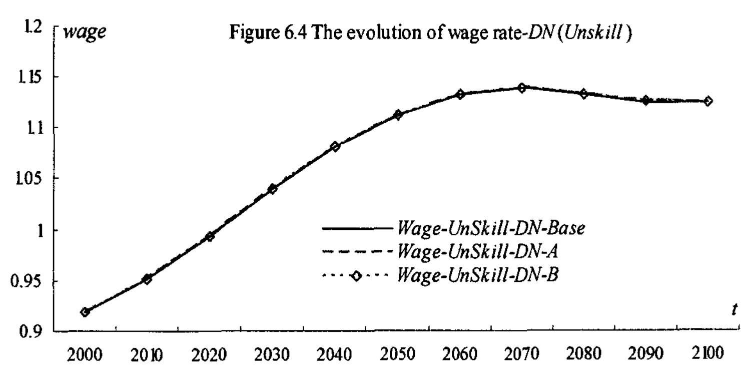
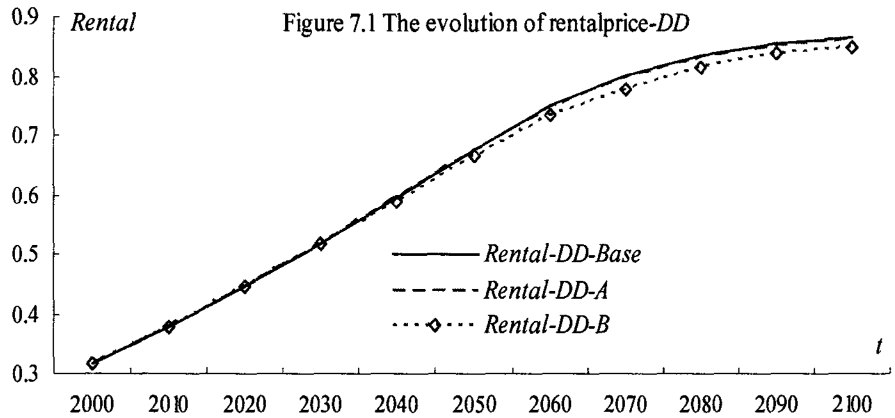
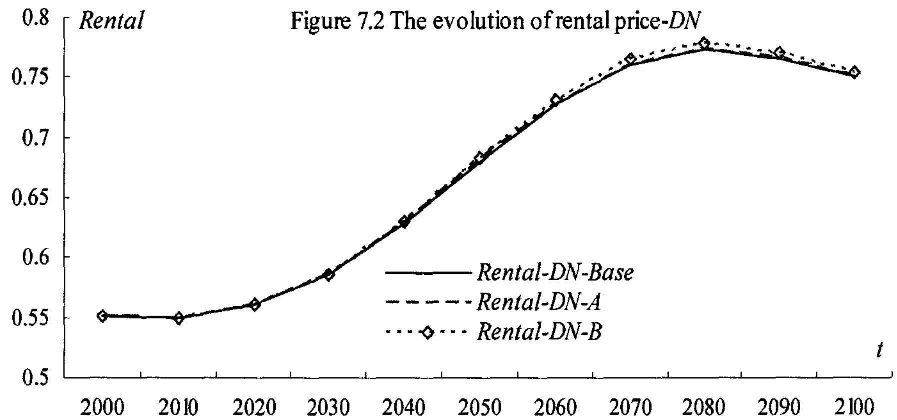


Figure 6.4 The evolution of wage rate-*DN*(*Unskill*)

For the source region *DN*, the outflow of both kinds of labour also has least effect on the wage rates under scenario A. The loss of skilled labour (scenario B), however, makes skilled labour more expensive in the economy. It is also shown that the wage rate for the unskilled labour turns to be slightly higher than both the baseline scenario and the migration scenario A. The outflow of skilled labour has two effects on the developing region's comparative advantage. On one hand, the lower capital-labour ratio after migration makes the developing region to produce more labour-intensive good. On the other hand, higher wage rate for skilled labour increases the relative production cost of the labour-intensive good because the production of this good uses a higher share of skilled labour in the developing region according to real data. As shown by the following Figure 8.2, the ratio of output of capital-intensive good to output of labour-intensive good is higher under the migration scenario B. This suggests the developing region produces relatively higher quantity of capital-intensive good than it does under other scenarios based on the above two effects. As mentioned in section 3.9, in the developing region and the rest of the world, the production of capital-intensive good requires relatively smaller portion of skilled labour than the labour-intensive good. Thus, more production of capital-intensive good demands more unskilled labour and has a positive effect on its wage rate in the developing region.

In the following Figure 7 we present the percentage deviation of rental prices from their initial steady-state values in the base year of 1930. It shows that under the migration scenarios, the rental prices in the destination and source regions move in directions opposite to the directions of movements for unskilled labour's wage prices. The changes of rental prices are consistent with the changes of capital-labour ratios in the two regions: a region with higher (lower) capital-labour ratio has a lower (higher) rental price.





Producers in the two regions adjust their outputs of two kinds of good in correspondence to the changes of comparative advantages and factor prices. For the destination (source) region *DD* (*DN*), the economy becomes larger (smaller) as a result of the increase (decrease) of population. However, the ratio of its output of capital-intensive *S1* to the output of labour-intensive *S2* (the output ratio) increase after migration, as shown by the following Figure 7. That is, the older destination region produces a higher quantity of *S1* relative to *S2* under scenario A and even higher quantity of *S1* under scenario B. For the developing region, on one hand, its capital-labour ratio turns to be lower under scenario B, so producers in the economy switch more resources into the production of the labour-intensive good. On the other hand, the scarcity of skilled labour prevents producers from employing more skilled labour and producing a higher quantity of labour-intensive good. In Figure 8.2, we see that the output ratio for the developing region turns to be higher than it is under the baseline scenario and scenario A. In other words, the developing region turns to produce more capital-intensive good after sending out part of its skilled labour. The direction of change of output ratio is opposite to it is of the change of capital-labour ratio. This suggests that in our trade model where skill differential is introduced, each region's comparative advantage is not only determined by its abundances in capital and labour, but also by its abundances in skilled and unskilled labour.

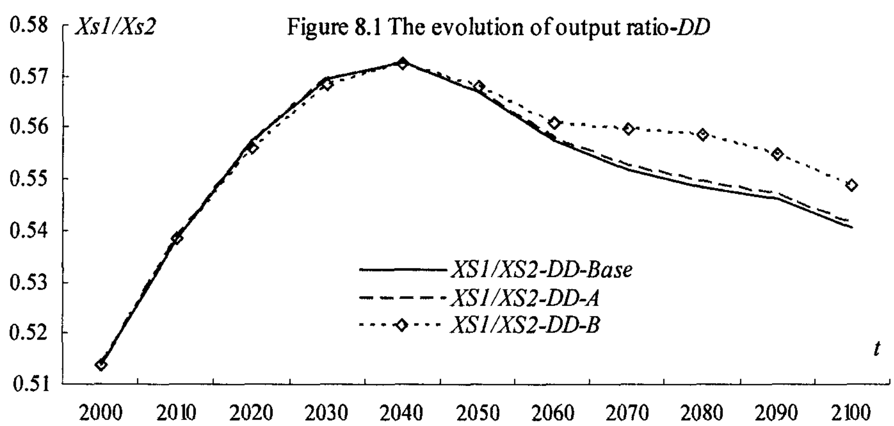


Figure 8.1 The evolution of output ratio-*DD*

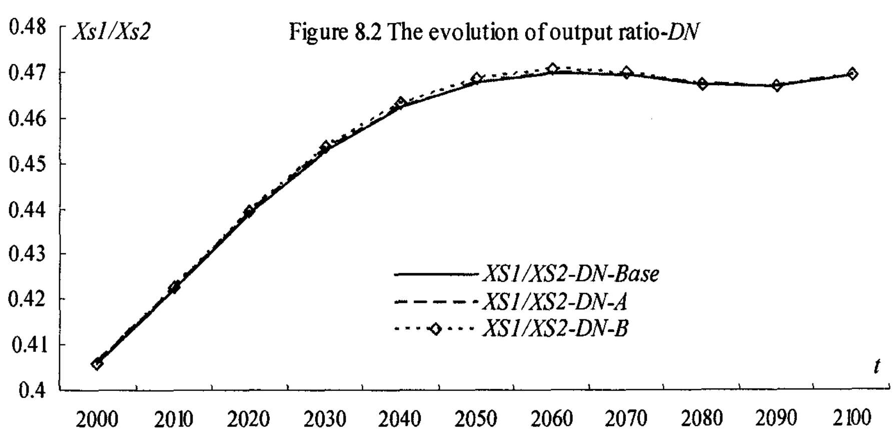
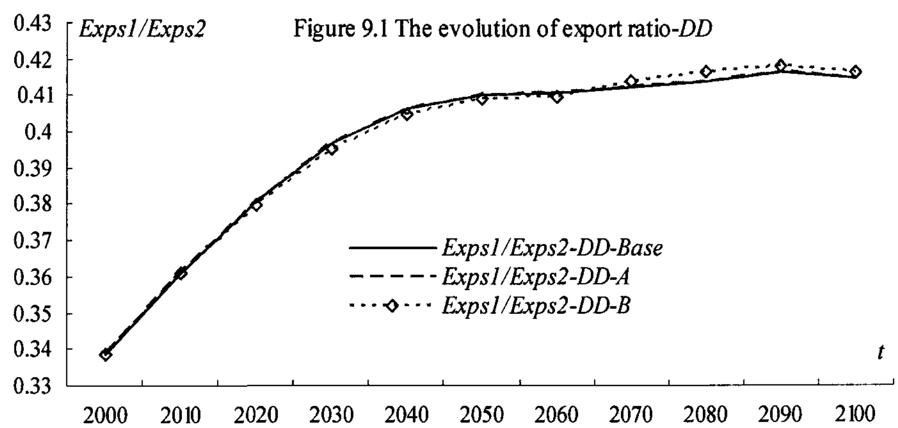


Figure 8.2 The evolution of output ratio-*DN*

Under the migration scenarios, the quantities of export of the two goods also change according to two countries' new comparative advantages. As shown by the following Figure 9, under the baseline case, the older developed region exports relatively more capital-intensive good than the younger developing country does. Under scenario A, these two region' export ratios (the export of *S1* over the export of *S2*) are not affected much because their comparative advantages do not change much as a result of the proportionate migration of both kinds of labour. Under scenario B, however, the developed region is observed to export a higher quantity of *S1* relative to *S2* than it does under other scenarios because producing capital-intensive good becomes even cheaper in this region. For the developing region, its export ratio also increases because the production cost of *S1* also turns to be relatively lower than *S2*'s production cost. This is consistent with our analysis of the changes of factor prices and output ratios for the developing region.



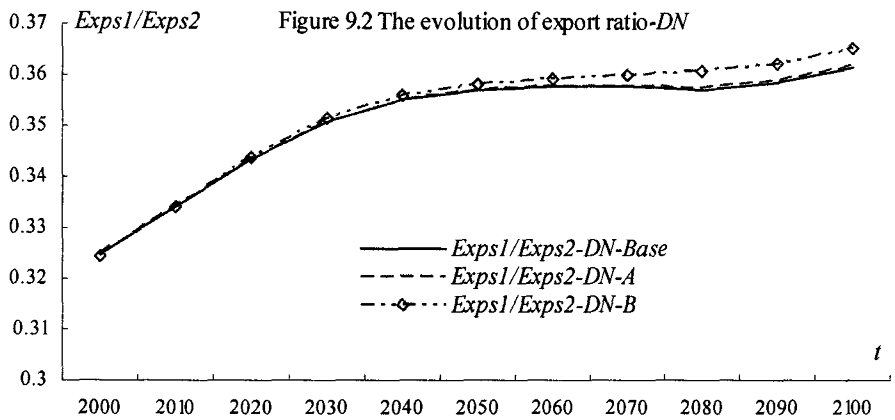


Figure 9.2 The evolution of export ratio-*DN*

The following we present the directions changes of welfare indicators for the destination and the source regions before and after migration. In this paper, we take into account skill differentials in labour stock. Thus, we first present the simulation results of lifetime utility for the skilled and unskilled labour respectively. In addition, to measure the overall impacts of migration on the source and receiving regions, we also present the changes of real GDP per capita levels as the approximation of the changes of living standards in each country.

In this paper, lifetime utility for a representative cohort at skill level *qual* is defined as the utility a new-born cohort in the economy could have in its whole life. The level of lifetime utility is determined by this cohort's utility function and constrained by the present value of real lifetime income this cohort receives. Real lifetime income is defined as the present values of a new-born cohort's real wage income in the first five periods of its life and real interest income on the saving it receives after retirement.

The following Figure 10 shows the percentage deviation of real lifetime income for a cohort at skill level *qual* from its baseline level in 1930. Real lifetime income is calculated as the lifetime income for a cohort born at time *t* divided by the consumption price index at time *t*, and it shows the real purchasing power for cohorts across countries. Under the baseline scenario, the unit labour supply and wage rates for both kinds of labour in the developed region are higher than they are for labour in the developing region, so is the real lifetime income for cohorts in region *DD*. In addition, our Armington-type trade model suggests an increasing and persistent gap between lifetime income for a skilled or unskilled labour in the two regions. A proportionate migration of both kinds of labour under scenario A does not have a significant impact on lifetime income for labour in the source and destination regions. However, the inflow of skilled labour reduces (increases) real lifetime income for a skilled cohort in the developed region (developing region).

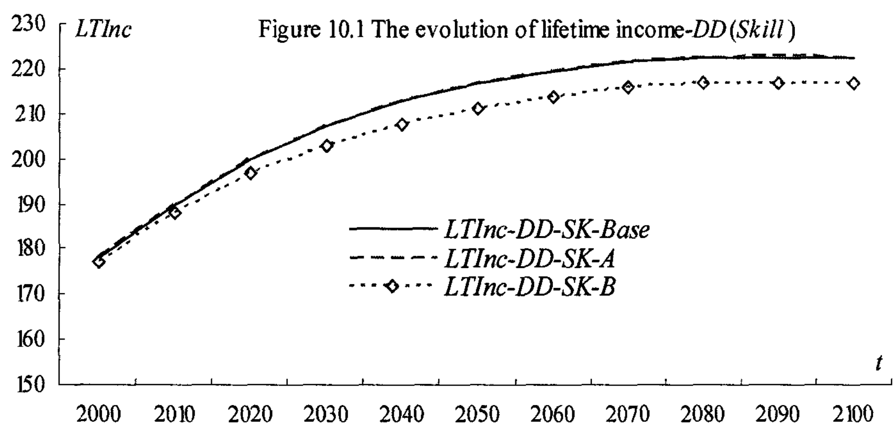


Figure 10.1 The evolution of lifetime income-*DD* (*Skill*)

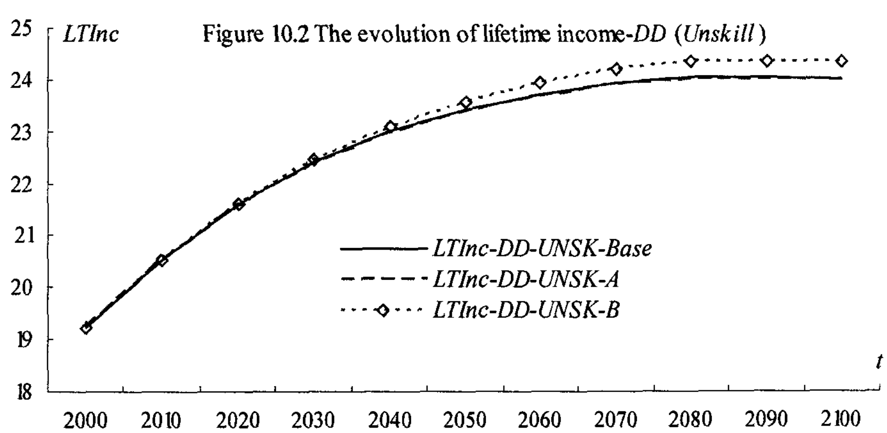
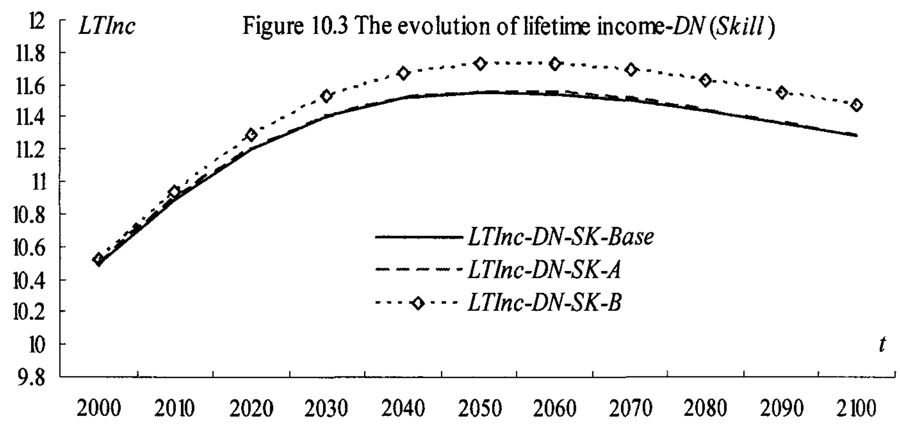
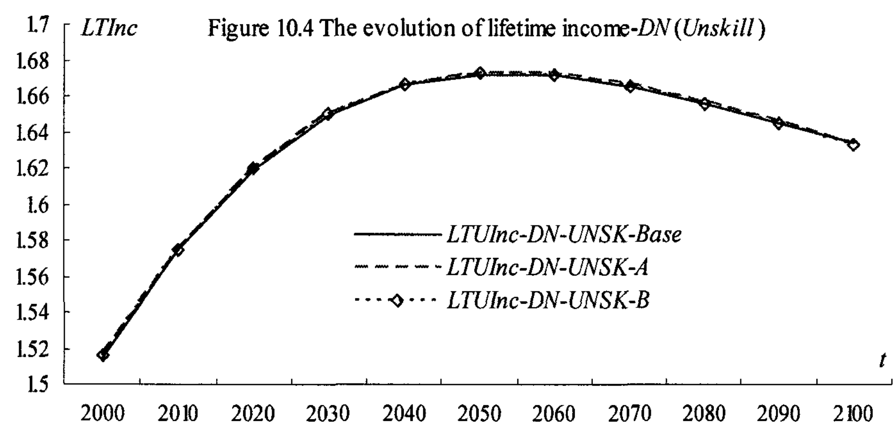


Figure 10.2 The evolution of lifetime income-*DD* (*Unskill*)





The resulted lifetime utility for a representative cohort at skill level *qual* is presented in the following Figure 11. This figure shows the percentage deviation of a cohort's welfare relative from its 1930 baseline level. We see that in the developed region, the lifetime utility levels for both the skilled and unskilled cohorts do not change much after migration under scenario A. Under scenario B, however, a skilled cohort loses from inflow of skilled immigrants but an unskilled cohort gains. In the developing region, only the skilled cohort benefits from migration under scenario B. The welfare levels for the unskilled cohorts are slightly affected by migration under both scenarios. These results support the finding by Iregui (2003) that when the heterogeneity of labour endowment is taken into account, those people whose skill level is complementary to the skill level of immigrants gain from the inflow of foreign labour. For those people whose skill level is substitutable to the skill level of immigrants, they lose from migration. Comparing Figures 10 and 11, we see that the growth rates of real lifetime income and lifetime utility turn to be smaller in the long run.

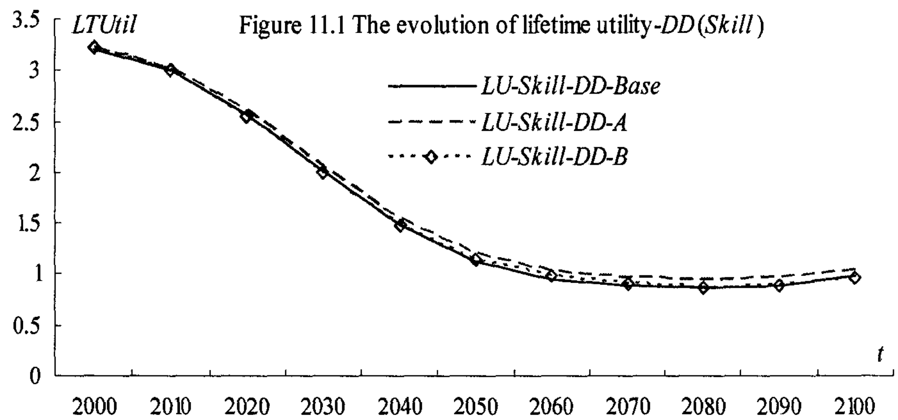


Figure 11.1 The evolution of lifetime utility-*DD*(*Skill*)

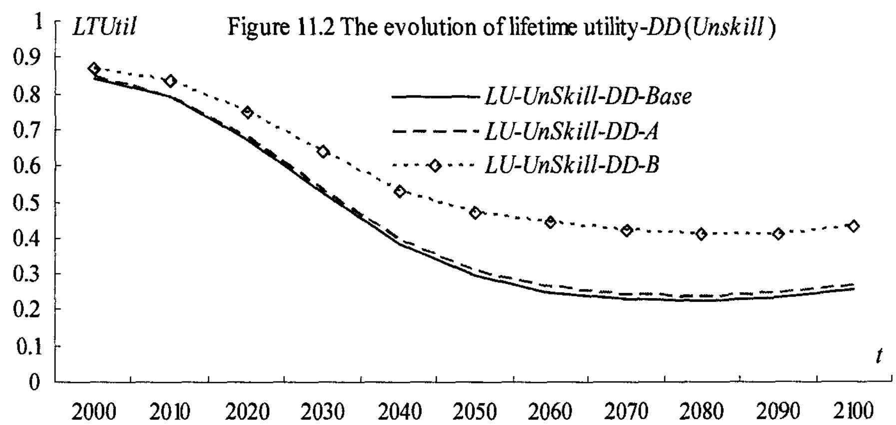


Figure 11.2 The evolution of lifetime utility-*DD*(*Unskill*)

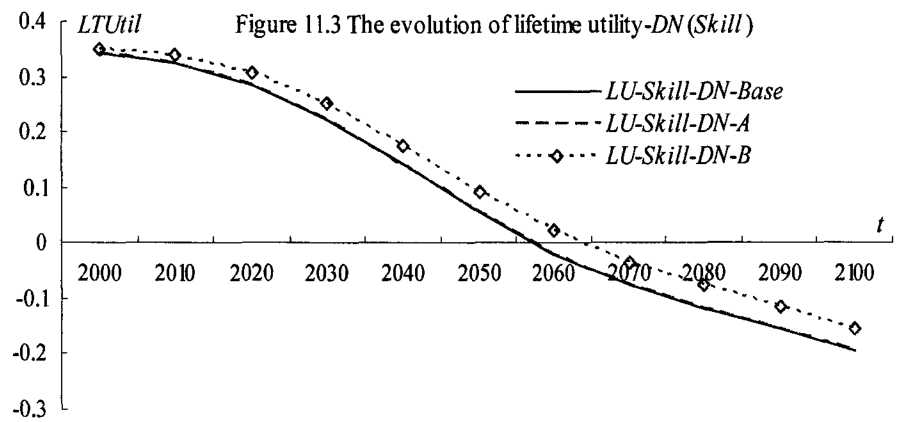


Figure 11.3 The evolution of lifetime utility-*DN* (*Skill*)

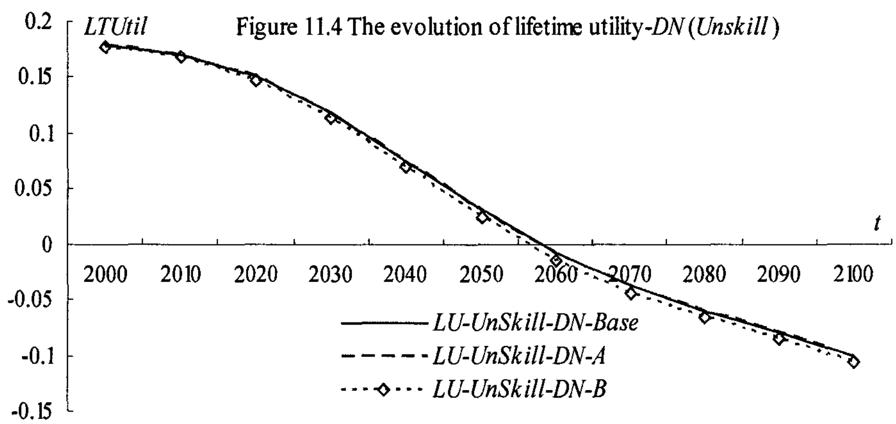


Figure 11.4 The evolution of lifetime utility-*DN* (*Unskill*)

To show the impacts of migration on productivity and standard of living of each country, we also present the percentage deviation of real GDP per capita level from its baseline level in 2000 in the following Figure 12. It is shown that under the baseline scenario, the real GDP per capita levels for both the developed and developing regions decrease in the first half of the 21st century. The growth of living standard for more aged developed region starts to decline in the mid of the 2010s, and the turning point for the developing regions' growth is in the mid of the 2020s. This shows there is a ten years' lag between the timing when negative effect of ageing on economic growth begins to take effect on the developed region and it is for the developing region. After 2050, however, the growth of living standard turns to be more positive for the developed region but negative for the developing region. This creates a persistent and increasing gap for both the two regions' growth rates and their real GDP per capita levels. This finding is consistent with our conclusion in the first chapter of the dissertation: an Armington-type trade model suggests an increasing and persistent welfare gap between the older region (country) and the younger region (country). Similar results are also shown by Batini *et al.* (2006) who used a MSG3 model and found real GDP per adult for the US, Japan, other industrial and other developing regions first increasing and then decreasing in the first half of the 21st century.

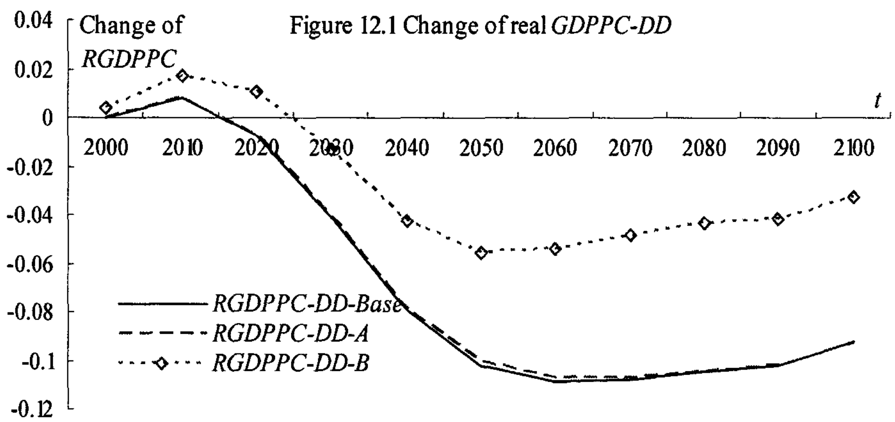
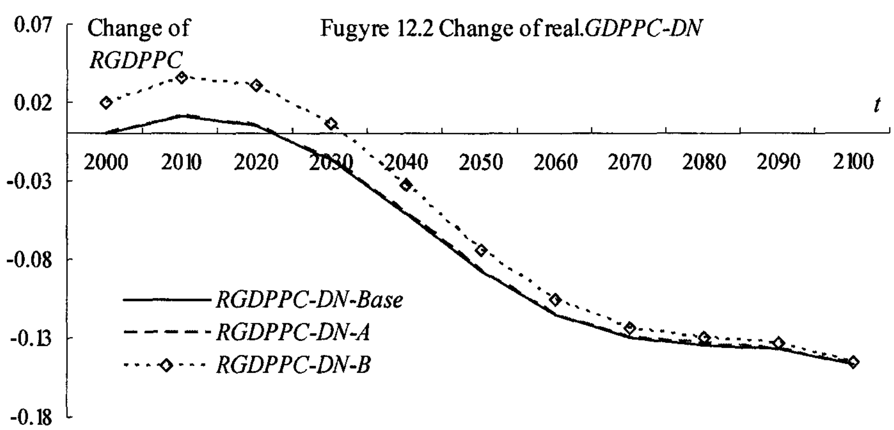


Figure 12.1 Change of real *GDPPC-DD*

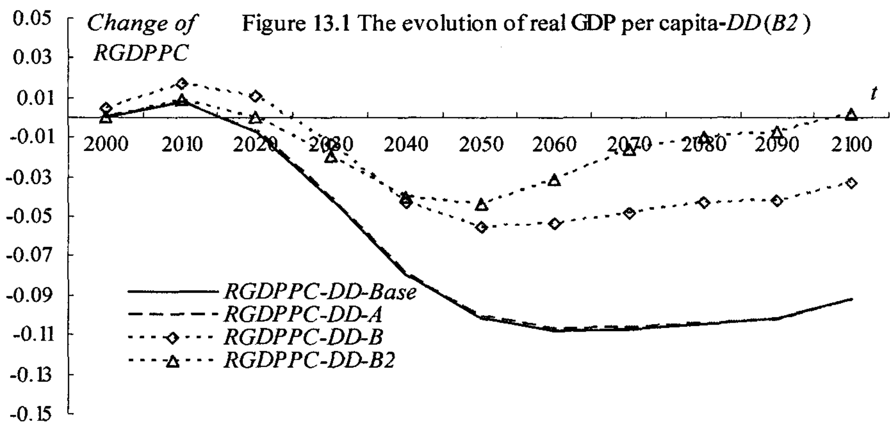


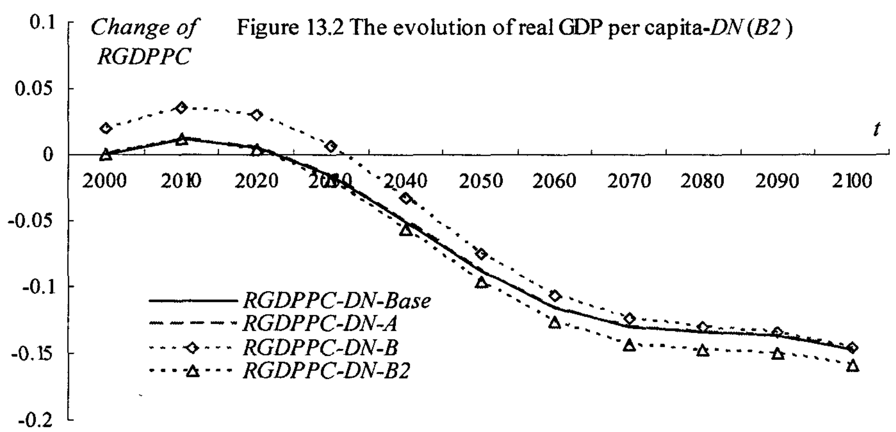
Fugyre 12.2 Change of real.*GDPPC-DN*

Under the migration scenarios, real GDP per capita for the developed region is significantly improved after this region adopts skilled immigrants under scenario B. For the developing region, it has a short-run gain from sending out its skilled labour in the several periods around 2020. However, this positive impact of migration is diminishing in the long run and the absolute value of real GDP per capita for the developing region converges to its baseline level. Migration of skilled labour could have a persistent positive effect on the real GDP per capita for the older destination region, but only a temporary positive effect on it is for the developing region.

In addition, it is shown that in the developed region, cohorts born in several periods before migration benefit least from inflow of skilled immigrants, but the positive effect of migration on living standard turns to be more and more significant for future-born cohorts in this region. For the developing region, those benefit most from sending out skilled labour are cohorts living in the several periods before the migration shock in 2020. Benefit received by future-born cohorts is found to be decreasing overtime and finally diminishes. Overall, migration of skilled labour could benefit both the source and destination regions. However, we observe a persistent and increasing gap between these two regions' living standards.

To show a better picture of the impacts of skilled immigrants on living standards, we do experiment for another scenario (B2) under which we assume skilled labour keeps migrating from the developing to the developed region for two periods (2020 and 2030). It is shown that the steady-state welfare level for the developed region turns to be even higher than it is under scenario B. In addition, cohorts living in periods close to when migration shock is applied benefit less and future-born cohorts benefit more from the continuous inflow of skilled immigrants. For the developing region, sending out its skilled labour for continuous two periods leads to deterioration of local living standard compared with scenario B, as shown by the following Figure 13. The welfare levels for cohorts born in all periods go below their baseline levels. Comparing Figures 12 and 13, we see that migration of skilled labour may not be a win-win game for the source and destination regions. A comparatively small scale of migration could benefit both regions (countries). Migration of skilled labour from the developing countries to the developed countries may enlarge the gap between the two regions' national income in the context of globalization.





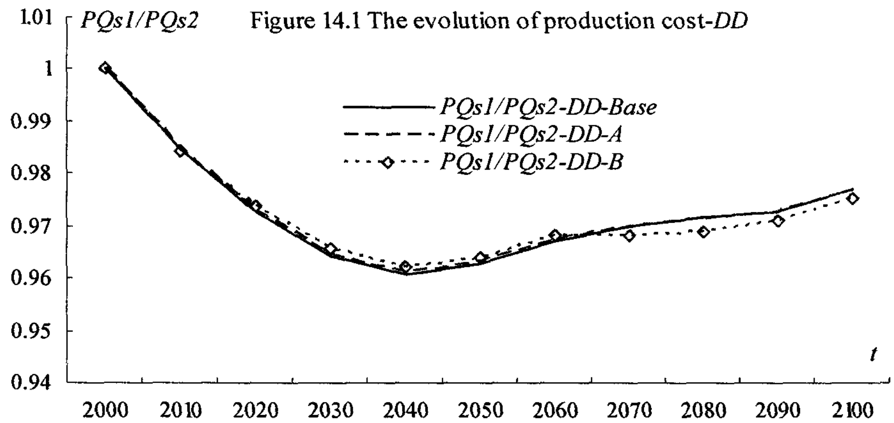
2.4.3 Trade and migration

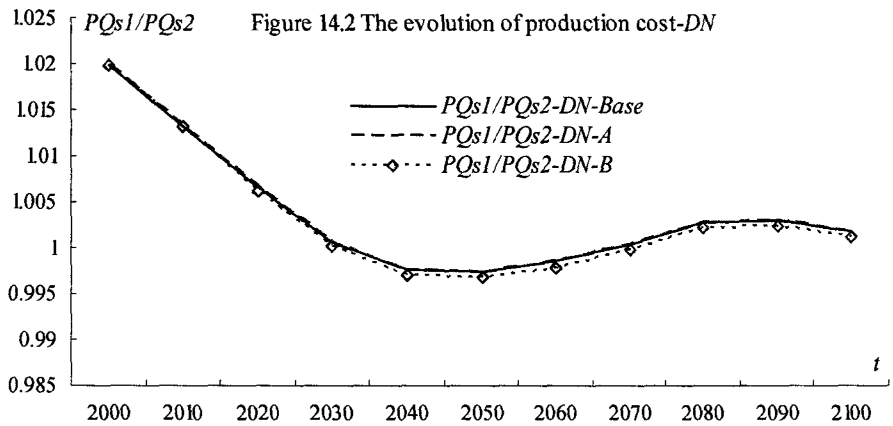
International trade and migration are two most important elements of globalization. In the context of population ageing, on one hand, different age structures across countries change the comparative advantages across countries and thus have impacts on the picture of international trade. Wage or lifetime utility differentials also could generate incentives for labour mobility across borders. On the other hand, cross-border good and labour mobility also have important impacts on trade partners and the source and destination countries. Thus, we believe it is critical to introduce trade and labour mobility into the analysis of population ageing and study the interactions among them.

This dissertation is the first to combine the above important issues: trade, migration and population ageing. As concluded in the first chapter, international trade benefits the older country, but makes cohorts living in the younger country worse off. An Armington-type OLG-CGE model shows that the older country's comparative advantage in the capital-intensive good is enhanced as a result of population ageing. In addition, higher demand for locally-produced capital-intensive good from the larger and younger country makes the older country benefit from the improvement of terms of trade. We explained the welfare changes for two ageing countries as the result of changes of comparative advantages, production costs and terms-of-trade.

In this chapter in which skill differential is introduced into our OLG-CGE model, there are three kinds of factor inputs in the production of each good: capital, skilled labour and unskilled labour. Thus, comparative advantage in this paper is defined not only based on each region's capital abundance, but also on each country's endowments of different kinds of labour. Real data shows at the initial steady state, the developed region is endowed with more capital than labour. In addition, the share of skilled labour in total labour stock is also higher in this region. Thus, region *DD* has comparative advantage in both capital-intensive good and the good uses skilled labour more intensively. For the developing region *DN*, it has relatively more labour endowment and the comparative advantage in labour-intensive good. However, GTAP6.0 data shows the production of labour-intensive good requires higher share of skilled labour in the developing region. In this aspect, the developing region's comparative advantage in labour-intensive good is reduced to some degree.

When the shock of migration is introduced, the inflow of both kinds of labour (scenario A) does not have significant impacts on the source and destination regions' comparative advantages. Under scenario B, however, the developed region's comparative advantage in capital-intensive good is enhanced by the inflow of skilled immigrants because capital-intensive good in this region is also skill intensive, as shown by Figure 5 in section *4.2.* A lower production cost of capital-intensive good attracted more resources in the economy compared with the baseline case (Figure 14.1). Accordingly, this region produces and exports more capital-intensive good than labour-intensive good (Figures 8 and 9). For the developing region, its capital-labour ratio deceases compared with its baseline level after sending out skilled labour under scenario B (Figure 5). Its comparative advantage in labour-intensive good is enhanced. However, GTAP data shows the production of labour-intensive good *S2* in the developing region is more skill intensive than capital-intensive good *S1*. On one hand, lower capital-labour ratio makes producers produce more labour-intensive good in the developing region. On the other hand, higher wage rate for skilled labour under scenario B increases the production costs of labour-intensive good. The following Figure 14.2 shows the production price of *S1* relative to production price of *S2* turns to be cheaper than it is under baseline scenario for the developing region. This suggests the developing region also produces and exports more capital-intensive good than labour-intensive good under scenario B.





In this paper, terms of trade for region j at time t ($TOT_{j,t}$) is calculated based on the Laspeyres index according the following equation (51):

$$TOT_{j,t} = (\sum_{i,s} Exp_{j,i,s,1930} PCons_{i,s,t} / \sum_{i,s} Exp_{j,i,s,1930} PCons_{i,s,1930})$$
$$/(\sum_{j,s} \mathrm{Im}\, p_{i,j,s,1930} PCons_{j,s,t} / \sum_{j,s} \mathrm{Im}\, p_{i,j,s,1930} PCons_{j,s,1930}) \quad (51)$$

In equation (51), the numerator is the Laspeyres export index showing the change of export revenue on an identical basket of export goods in the current year and the denominator is the Laspeyres export index in the base year. This method is widely used in the calculation of terms of trade. In the following Figure 15 we present the evolutions of terms of trade for the source and destination regions overtime.

It is shown that under the migration scenario B, the terms of trade change in favor of the developed region. Although the relative production costs turns to be slightly higher than the baseline scenario in the several periods after migration, this region still gains from trade after adopting foreign skilled labour. In the long run, with the decrease of production cost and terms of trade turning to be more favourable, the developed region benefits more from migration, as shown by the above Figure 12. For the developing region, the terms of trade turn to be unfavourable after this region send out part of its skilled labour. However, in the short run, the negative effect of terms of trade is offset by lower relative production cost of capital-intensive good. In the long run, however, as its terms of trade keep deteriorating, the developing region's gain turns to be smaller and finally diminishes in 2100. The above analysis explains the directions of welfare change for the source and destination regions before and after migration (Figure 12). Overall, in this paper, welfare level for each region after migration depends on the size of improvement of terms of trade relative to the changes of production costs. In this manner, the economic impacts of migration are well explained through the changes of each region's comparative advantage, production cost and terms of trade.

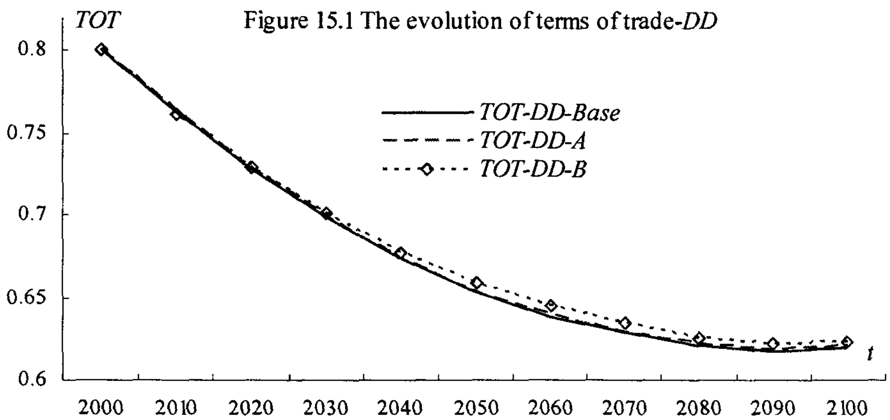


Figure 15.1 The evolution of terms of trade-*DD*

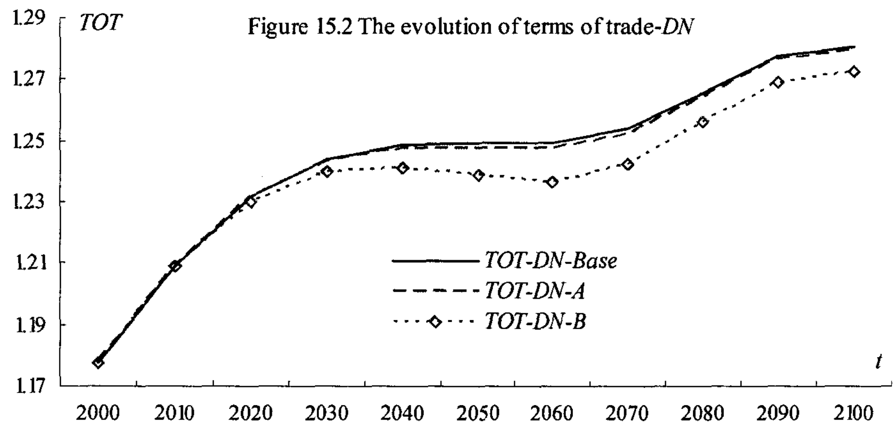
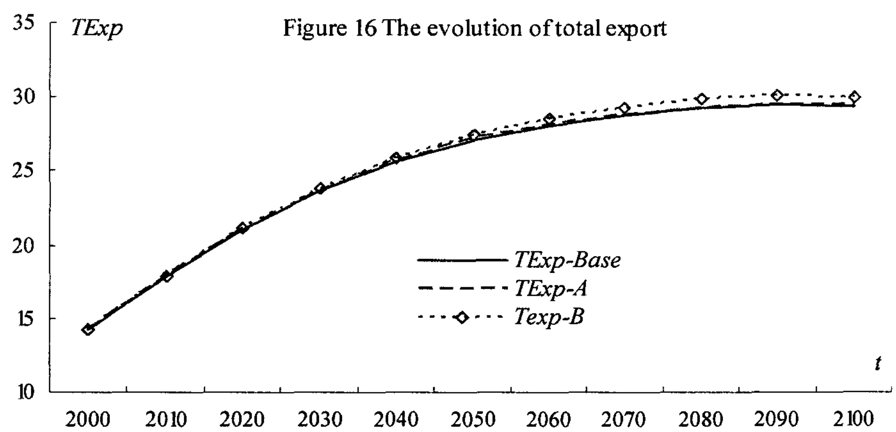


Figure 15.2 The evolution of terms of trade-*DN*

Another important finding of this paper is that trade and migration are complements, instead of being substitutes, as suggested by our Armington-type trade model. In the literature, Mundell (1957) showed that the increase in trade barriers stimulates movements of capital and labour while restrictions on factor mobility stimulate trade. Faini *et al.* (1993) also suggested that more liberalized trade policy among industrialized economies may reduce migration. Recent developments in trade theory, however, give rise to the argument that international trade and migration are complements instead of substitutes. When crucial assumptions for Heckscher-Ohlin theorem (perfect competition, constant return to scale, full employment) are relaxed, pressures to migrate can easily increase with free trade. Based on our Armington-type trade model, simulation results in this paper shows the migration of both kinds of labour under scenario A does not have significant impact on the scale of world trade. This is because a proportionate migration of both kinds of labour does not change each region's comparative advantage. The migration of skilled labour from the developing region to the developed region, however, provides higher incentives for international trade. As shown by the following Figure 16, the total quantity of international trade increases under the migration scenario B. This is because migration of skilled labour across border enhances comparative advantages for both the developed and the developing regions. Consequently, the scale of international trade increases as all regions are willing to take advantage of more favourable terms of trade.

Figure 16 The evolution of total export



The increase of world export is accompanied by a higher level of world output in each period after migration (Figure 17). This finding is consistent with Iregui (2003) who assumed a full integration of world labour market and concluded that a large scale of migration could significantly increase world output. She explained the contribution of migration to world output as immigrants from developing countries could work at a high productivities in the developed country. Our dynamic model, however, suggests that an equal scale of migration of skilled labour could have a larger positive effect on world output than the proportionate migration of both kinds of labour does. In addition, this positive effect of migration of skilled labour on world output is persistent in the long run.

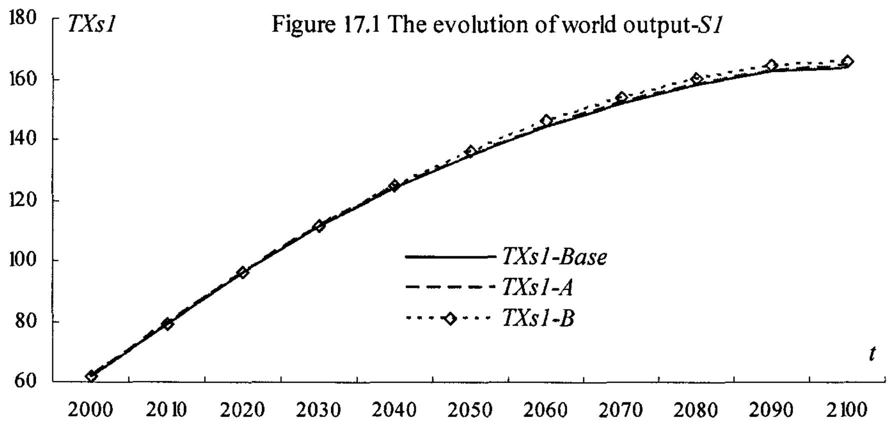


Figure 17.1 The evolution of world output-*S1*

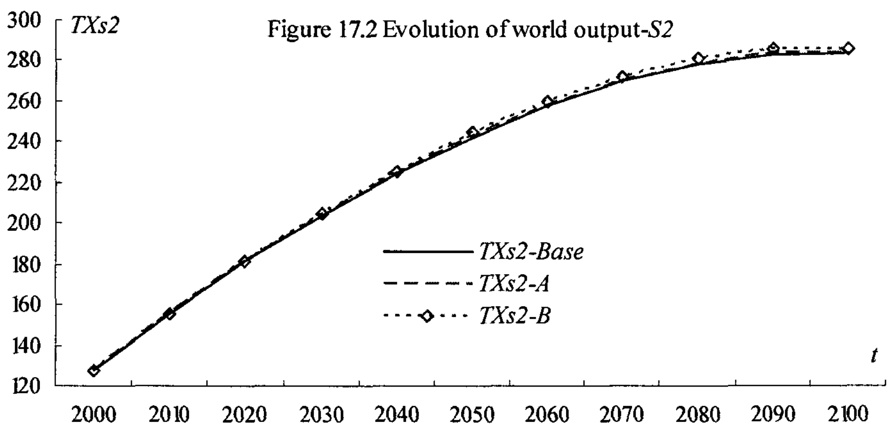


Figure 17.2 Evolution of world output-*S2*

2.5 CONCLUSION

Using a 3*2*2*7 OLG-CGE model, this paper investigates the impacts of different patterns of international migration on the destination and source regions with different degrees of population ageing. The world is classified into three regions according to real data and each of them has different projected demographic characteristics, factor abundances and factor intensities in production. Different from recent literature, this paper is the first to combine three important issues in world economy: trade, migration and population ageing. A trade model has been built to simulate several economies under the baseline and migration scenarios. The welfare impacts of migration are all analyzed and explained through the changes of each country's comparative advantages, production costs and terms of trade. In our trade model, we apply the Armington assumption so that the simulated picture of trade is more realistic and identical to real data. In the paper, we take into account the heterogeneity of labour and both countries are assumed to be populated with both skilled labour and unskilled labour. The decision making of international migration is modeled based on the differentials between the wage rates paid for a skilled worker or an unskilled worker in different countries. The parameters used to calculate the migration coefficient are selected so that the scale of labour mobility in the model is in accord with UN statistics.

Overall, simulation results in this paper suggest the following meaningful findings:

First, the skill composition of immigrants is proved to be very important in determining the economic impacts of international migration, as shown by many studies over the past decades. In the context of population ageing, the migration of labour from a younger and labour-abundant region to an older and capital-abundant region may have opposite impacts if we apply different skill compositions of immigrants. It is shown that a close-to-proportionate migration of both skilled and unskilled labour does not have significant impact on the source and destination regions. In contrast, an equal size of migration of skilled labour from the developing region significantly improves the overall living standard for the destination region. For the developing region, our simulation results suggest that it could benefit from sending out part of its skilled labour. The inflow of immigrants could offset the shortage of effective labour supply in the older and developed region. However, inflow of foreign labour does not necessarily mean a welfare improvement. A skill-oriented immigration policy should be more effective in improving the destination region's living standard.

Furthermore, the OLG-CGE model used in this paper provides us a picture on the interaction between good mobility and labour mobility in the context of population ageing. On one hand, population ageing generates differentials of factor abundances across countries, which provides incentives for both international trade and international migration. On the other hand, the cross-border movements of labour change the age structure and factor abundances across countries. As a result, production costs and terms of trade change according to the trade partners' new factor abundances. This has a direct effect on goods motility across countries. Based on the above interactions among ageing, trade and migration, we explain the economic impacts of migration using our Armington-type trade model: the welfare impacts of migration on regions with different demographic characteristics are well explained through the changes of comparative advantages, production costs and terms of trade.

In our analysis, different factor intensities in the production of capital or labour-intensive good across regions are found important in determining the economic impacts of migration. For the developing region, it could gain from sending out its skilled labour because the labour-intensive good in this region is more skill intensive. With the changes of skill compositions of immigrants, the changes of terms of trade may not be in favor of each region's change of comparative advantage. Either the source or the destination region may lose from international migration.

Finally, it was shown that in the context of population ageing, good mobility and labour mobility are complements, instead of being substitutes, as suggested by our Armington-type trade model. This finding is contrary to the conclusion suggested by a HO model, but similar to those by researchers who relaxed critical assumption of HO trade model and analyzed the interactions between trade and migration.

Although our model provides a picture on the interactions among trade, migration and ageing and a detailed explanation of the welfare impacts of migration, there are still many important issues worth further researching:

First, the emphasis of this paper has been put on the skill composition of immigrants on the economic performances of source and destination regions. However, the rest of the world does play a role in the picture of world trade and thus plays a role in our explanation of the impacts of migration. In addition, labour market in the rest of the world is assumed to be closed under our migration scenarios. In the context of globalization, migration among source and destination countries could have significant impacts on those countries absent from international labour market. Overall, the role of the rest of the world needs to be further analyzed.

Second, in section 4.2, we showed that migration for continuous two periods could reverse the impacts of labour mobility for the developing region. That is, the source region could lose from sending out a larger number of skilled workers. Thus, it is highly relevant to study on the optimal scale of migration for the source and destination countries, and the best timing of migration.

Third, all scenarios of migration in this paper are assumed to be a one-way movement from the younger developing region to the older developed region. This setting is unrealistic because over the past several decades, we have observed people migrating both from the developing countries to the developed countries and from the developed countries to the developing countries. With national markets become more and more integrated and the development of multinational enterprises (MNEs) in the context of globalization, this double-way migration turns to be increasingly important in the analysis of international migration. Obviously, the structure of model provided by this paper could not capture this double-way migration. A new type of CGE model will be built to endogenize the decision making of migration.

Finally, as in the first chapter, capital mobility is not taken into account in the model. However, good, capital and labour mobility are the three most important components of globalization. To introduce capital mobility must have varied effects on our simulation results as well as our explanations. In order to elaborate the interactions among trade, migration and demographic changes, we ignore many roles of capital (mobility, adjustment or installation cost, etc.) in the analysis. To take into account these factors creates good extensions of this thesis.

Chapter three

An OLG-GE Modeling Framework for Endogenous Migration

3.1 INTRODUCTION

Over the next several decades, countries around the world will experience different degrees of population ageing. Demographic projections show that all developed countries will be in the process of fast ageing after 2010 when the baby-boom generation (those born between 1946 and 1966) begins to retire (Mérette 2002).

Population ageing is widely viewed as a threat to living standard and economic growth, especially for the ageing developed countries, as shown by related researches by World Bank (1994) and OCED (1998). Pessimistic views urged these countries to find solutions to accommodate possible negative effects of ageing. Besides some internal reform scenarios, such as debt stabilization and delayed retirement (OECD 1998), adopting more immigrants is considered by many countries as an option to reduce the pressure of ageing on labour market by opening door to international labour market so as to improve the age structure of an ageing economy. International migration had already become a hot issue in almost all industrial countries.

Different from the European mass emigration to the new continents before 1914, the recent wave of world migration is accompanied with a deeper integration of national markets (globalization) and an accelerating process of population ageing. On one hand, developed countries have incentives of attracting immigrants to overcome the slowing down of labour force growth and to alleviate possible fiscal stresses as the result of population ageing. With the development of information and telecommunication (ICT) industry, many developed countries have also put emphasis on attracting more high-skilled foreign workers from the developing world. This is a new characteristic of the 21^{st} Century wave of international migration. On the other hand, deeper integration of national markets provides incentives for a large scale of industry out-sourcing, from the developed world to the developing world, and more and more multinational enterprises (MNEs) are operating overseas. Consequently, many emerging developing economies (China, India, Brazil and South Africa, for example) are demanding more and more foreign intelligence, especially those from the advanced countries, as a means of enhancing efficiency and productivities, upgrading local industries and filling the technology gaps between them and the developed world.

Overall, in the past decade, we have observed a large scale of migration of labour, at all skill levels, from the developing countries to the developed countries. In addition, more and more people from the developed world (mostly high-skill workers) have moved to emerging economies. Thus, it is highly relevant to study bilateral migration between the North and the South.

This model uses a 2*2*3*2 type overlapping-generation computable general equilibrium (OLG-CGE) model to incorporate the concept of "endogenous migration" and study the bilateral migration between two regions. The developed and the developing regions have different initial steady states calibrated based on real data. Working individuals in the two regions are allowed to make decisions on their labour supply allocation into either working in his or her homeland or working abroad. In the dynamic, population growth rates of two countries are assumed to be different so that the developed region grows into an older economy. Changes of age structures differentiate these two regions' labour markets, and the resulting wage differentials create further incentives for people to move. A series of CET functions are used to model the supply side of labour market so that working individuals in each region could make migration decision based on both their preferences in working locations and changes of their expected earnings.

The paper is organized as follows. There will be no section of literature review because we could hardly find relevant literature on the dynamic GE modeling on endogenous migration. Dynamic GE models with migration in the literature use ad hoc migration functions. I discussed that literature in the second chapter of the thesis. Section 2 describes our dynamic model and datasets. Section 3 presents simulation results and section 4 concludes.

3.2 MODEL DESCRIPTIONS

In the 2*2*3*2 type OLG-GE model, the world economy has been divided into two regions (developed region and developing region labelled as *J1* and *J2*). In each of the them, two sectors of goods (*S1*, *S2*) are produced and two major inputs (capital and labour, labelled as *Kd* and *Ld*) are used in the production of each good. Labour input is a constant elasticity of substitution (CES) aggregation of two kinds of labour differentiated by their skill levels (*Skilled and unskilled*).

In the model, each country is populated with three generations (the working generations *G1* and *G2, or gj*, and the partly retired generation *G3*, or *gm*) of population, and *G1* is assumed to be the only fertile generation. At the initial steady state, we assume population of each of the three generations equals to 1/3 of total population, and the two countries have different shares of skilled and unskilled workers in each generation. 30% (20%) of the developed (developing) region's population are skilled workers. Also, at the initial steady state, a small portion of each skill level of workers in both countries work and live in the foreign country. For example, in the developing (developed) country, 8.8% (1.5%) of the skilled or unskilled worker work in the developed (developing) country. The rest of the workers works in their home country.

The above percentage determines the preference of workers to relocate that will be specified in the CET labour supply function for a representative cohort. In the dynamic as the degrees of ageing diverge across countries, the resulting wage differentials between the two countries' labour markets generates incentives for both the skilled and the unskilled workers to move in both directions (from the developing region to the developed region and vice versa). Consequently, the shares of skilled and unskilled workers in each region' labour force vary overtime, as a result of bilateral migration.

To simplify our model and put emphasis on the decision making of different types of labour, government behaviour is not taken into account. We also ignore the existence of pensions, bequest and heritance in the optimization process for a typical individual.

3.2.1 Producer behaviour

In each region, there are two sectors producing each one of the two goods *s* (capital-intensive *S1* or labour-intensive *S2*) using Cobb-Douglas (CD) production technologies. A representative firm's problem is to minimize the production cost subject to the embedded constraint that characterizes the firm's technology:

$$\underset{Kd_{j,s,t},Ld_{j,s,t}}{Maximize}\, Pq_{j,s,t}X_{j,s,t} - R_{j,t}Kd_{j,s,t} + W_{j,t}Ld_{j,s,t} \tag{1}$$

$$s.t.\ X_{j,s,t} = SP_s Kd_{j,s,t}^{\alpha_s^K} Ld_{j,s,t}^{(1-\alpha_s^K)} \tag{2}$$

In equation (1), $Pq_{j,s,t}$ is the production price of good s in country j at time t. $R_{j,t}$ and $W_{j,t}$ represent the rental price and wage rate in country j at time t respectively. $Kd_{j,s,t}$ is the capital demanded by production sector s in country j at time t. $Ld_{j,s,t}$ is the aggregate labour demanded.

In equation (2), SP_s is a scaling constant parameter and $\alpha^K{}_s$ is the expenditure share measuring the intensity of use of capital in production. $S1$ is capital-intensive good and $S2$ is labour-intensive good. The technology is represented by a CD production function in which the sum of expenditure shares for capital and labour inputs is equal to one.

In this model, we assume that the aggregate labour supply, $Ld_{j,s,t}$, is a CES mixture of two kinds of labour supply (*qual*) in the economy: the skilled labour and the unskilled labour (equation 3).

$$Ld_{j,s,t} = (\delta^Q_{s,skill} LdQ_{j,s,skill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}} + \delta^Q_{s,unskill} LdQ_{j,s,unskill,t}^{\frac{\sigma_s^{LD}-1}{\sigma_s^{LD}}})^{\frac{\sigma_s^{LD}}{\sigma_s^{LD}-1}} \tag{3}$$

Skilled and unskilled labour is differentiated by the quantity of labour supply they could provide per unit of time. In equation (3), $\delta^Q{}_{s,qual}$ is the share of labour at skill level *qual* demanded by the production of good s and the sum of $\delta^Q{}_{s,skill}$ and $\delta^Q{}_{s,unskill}$ is equal to one. $\sigma_s{}^{Ld}$ is the elasticity of substitution between different kinds of labour in sector s. The value of $\delta^Q{}_{s,qual}$ is calibrated to be different both across sectors and across countries.

Differentiating equation (1) with respect to equation (2), we get the following equations (4) and (5) as the first order conditions for a producer:

$$R_{j,t} = \alpha_s^K \frac{Pq_{j,s,t} X_{j,s,t}}{Kd_{j,s,t}} \tag{4}$$

$$W_{j,t} = (1-\alpha_s^K)\frac{Pq_{j,s,t} X_{j,s,t}}{Ld_{j,s,t}} \tag{5}$$

After the rental price and the aggregate wage rate are determined, a firm's optimization problem turns to be minimizing its expenditure on labour input. That is, it minimizes the following equation (6) subject to the above equation (3):

$$W_{j,t} Ld_{j,s,t} = WdQ_{j,skill,t} LdQ_{j,s,skill,t} + WdQ_{j,unskill,t} LdQ_{j,s,unskill,t} \tag{6}$$

where $WdQ_{j,qual,t}$ is the wage rate paid for labour input at skill level *qual* in country *j* at time *t*.

The following equation (7) and equation (8) are the first order conditions:

$$LdQ_{j,s,qual,t} = \delta_{s,qual}^{Q^{\sigma_s^{DQ}}} \left[\frac{W_{j,t}}{WdQ_{j,qual,t}} \right]^{\sigma_s^{DQ}} Ld_{j,s,t} \tag{7}$$

$$W_{j,t}^{1-\sigma^{DQ}} = \sum_s \delta_{s,qual}^{Q^{\sigma_s^{DQ}}} WdQ_{j,qual,t}^{1-\sigma_s^{DQ}} \tag{8}$$

Equation (7) shows the demand for labour input at skill level *qual*, $LdQ_{j,s,qual,t}$, increases with the aggregate labour demanded, the share of this kind of labour in the technology and decreases with the wage rate paid for this kind of labour. Equation (8) shows the aggregate wage in country *j* is a weighted sum of wage rate paid for all kinds of labour.

In this model, migration is introduced so that the aggregate labour demand at skill level *qual* by sector *s*, $LdQ_{j,s,qual,t}$, could be supplied by workers from both the home and foreign countries. In other words, this labour demand is a CES aggregation of labour demand differentiated by their countries of origin ($LdM_{j,j,s,qual,t}$ and $LdM_{i,j,s,qual,t}$), as shown by the following equation (9):

$$LdQ_{j,s,qual,t} = (\delta^{Ldm}_{j,j,s,qual} LdM_{j,j,s,qual,t}^{\frac{\sigma_j^{DM}-1}{\sigma_j^{DM}}} + \delta^{Ldm}_{i,j,s,qual} LdM_{i,j,s,qual,t}^{\frac{\sigma_j^{DM}-1}{\sigma_j^{DM}}})^{\frac{\sigma_j^{DM}}{\sigma_j^{DM}-1}} \qquad (9)$$

where $\delta_{j,j,s,qual}{}^{Ldm}$ is the share of labour supplied by workers born in country j to sector s in country j and $\delta_{i,j,qual,s}{}^{Ldm}$ is the share of labour supplied by workers born in country i to sector s in country j. $LdM_{j,j,qual,t}$ ($LdM_{i,j,qual,t}$) is the quantity of labour supplied by individuals born in country j (i) and working in country j. In other words, $LdM_{j,j,s,qual,t}$ and $LdM_{i,j,s,qual,t}$ are labour supplied by native and foreign workers respectively. $\sigma_j{}^{DM}$ is the elasticity between native labour demand and foreign labour demand in sector s.

At time t, a producer in sector s in country j minimizes its aggregate wage expenditure on native and foreign workers subject to the above equation (9). The aggregate wage expenditure is determined by the following equation (10):

$$Wdq_{j,qual,t} LdQ_{j,s,qual,t} = WdM_{j,j,qual,t} LdM_{j,j,qual,t} + WdM_{i,j,qual,t} LdM_{i,j,qual,t} \qquad (10)$$

where $WdM_{j,j,s,qual,t}$ and $WdM_{i,j,s,qual,t}$ are demand wage rates paid by producers in country j to native and immigrant workers at skill *qual* working in sector s in country j.

The first order conditions for this stage of optimization are the following equations (11) and (12):

$$LdM_{i,j,s,qual,t} = \delta_{i,j,s,qual}^{Ldm^{\sigma_j^{DM}}} \left[\frac{WdQ_{j,qual,t}}{WdM_{i,j,qual,t}} \right]^{\sigma_j^{DM}} LdQ_{j,s,qual,t} \tag{11}$$

$$WdQ_{j,qual,t}^{1-\sigma_j^{DM}} = \sum_i \sum_s \delta_{i,j,s,qual}^{Ldm^{\sigma_j^{DM}}} WdM_{i,j,qual,t}^{1-\sigma_j^{DM}} \tag{12}$$

Equation (11) shows the demand for native (immigrant) workers increases with the ratio of aggregate wage rate for workers at skill level *qual* to the wage rate for native (immigrant) workers, the share parameter of native (immigrant) workers in total labour force and the aggregate demand for workers at skill level *qual* in sector *s*. Equation (12) shows the aggregate demand wage rate is a weighted sum of demand wage rates for native and immigrant workers working in country *j*.

3.2.2 Household behaviour

An Allais-Samuelson overlapping generation framework characterizes households in the economy, so that this model is based on the life-cycle theory of savings. In period *t*, each economy is populated by three generations *g* (*G1* to *G3*) of population. A representative cohort born in period *t* works full time in the first and the second periods of its life (from 16 to 55 years of age), partly retires in period *t+3* and lives another period of time (from 56 to 75 years of age). Thus, each period *t* approximately represents 20 years in our model.

The lifetime utility for a representative cohort is represented by a time-separable nested CES function with the elasticity different from one. The optimization problem for a cohort at skill level *qual* can be divided into three steps in an open economy with international trade. First, immediately after birth, a cohort decides on the allocation of its labour income on either current or future consumptions. On the second step of optimization, the aggregate consumption expenditure is allocated among different kinds of consumption goods. Finally, on the third step of optimization, an individual decides on the composition of each consumption good *s* in terms of the country of origin. In other words, consumption of good *s* is an aggregation of both locally-produced products and imported products following the Armington assumption.

The intertemporal optimization problem for a representative cohort at the skill level *qual* in country *j* is to maximize its lifetime utility subject to the budget constraint:

$$\underset{C_{j,qual,g,t}}{Maximize}\; U(C_{j,qual,g,t})_{j,qual,t} = \frac{\sigma^{IT}}{\sigma^{IT}-1}\sum_{g=1}^{7}\left(\frac{1}{1+\rho_j}\right)^{g} C_{j,qual,g,t+g-1}^{\frac{\sigma^{IT}-1}{\sigma^{IT}}} \qquad (13)$$

$$s.t.\;\; LW_{j,qual,t} = \sum_{g=1}^{3} PCon_{j,g,t+g-1}(C_{j,qual,g,t+g-1}) \qquad (14)$$

The CES-form utility function is shown by equation (13), and the lifetime utility is an aggregation of the present values of current consumption and future consumptions. In each country *j*, $U(C_{j,qual,g,t})_{j,qual,t}$ is the lifetime utility for a representative cohort born in country *j*, at skill level *qual*, and $C_{j,qual,g,t}$ is the quantity of aggregate consumption. σ^{IT} is the intertemporal elasticity of substitution, and ρ_j is the pure rate of time preference. The larger ρ, the more of its lifetime resource a cohort would spend in the first two stages of life and the less it saves. As shown in equation (14), the lifetime wealth for an individual at the skill level *qual*, $LW_{j,qual,t}$, is allocated into current consumption and future consumptions. In this equation, $PCon_{j,g,t}$ is the aggregate consumption price for an individual of age *g* at time *t*. $LW_{j,qual,t}$ is represented by the following equation (15):

$$LW_{j,qual,t} = \sum_{g=1}^{7}\left\{\frac{1}{1+Rint_{t+g-1}}\right\}^{t} LInc_{j,qual,g,t+g-1} \qquad (15)$$

where $LInc_{j,\, qual,g,t}$ is the labour income for an individual at skill level *qual* of age *g* at time *t*. $Rint_t$ is the interest rate in country *j* at time *t*

In both countries, an individual's labour income depends both on the individual's age-dependent productivity and the skill level. Thus, the labour income for an individual in the working generation *gj* at time *t* can be represented by the following equations (16) and (17):

$$LInc_{j,qual,gj,t} = wage_{j,qual,t} EP_{j,g,qual} \tag{16}$$

$$EP_{j,g,qual} = \alpha^{Q}_{qual}(\gamma + \lambda g - \psi g^2), \quad \gamma, \lambda, \psi, \geq 0 \tag{17}$$

The quadratic function in (17) implies that the maximum labour earnings of a cohort occur at middle-age (*G2*). The intertemporal maximization problem is as follows. Households in all countries are forward looking and have perfect foresight. A representative cohort is born in the first period and starts to work as the labour force. Part of its labour income is saved as investment in capital stock and the rest is consumed. In the last period of its life, this cohort spends all its wealth, including investment in the first two periods and interest income, on consumption. Immediately after birth, an individual decides on the allocation of his or her expected labour income into current consumption and future consumptions. Differentiating the above equation (13) with respect to the budget constraint (14) yields the following first order condition for the aggregate consumption, $C_{j,qual,g,t}$, consumed by an individual at skill level *qual* of age *g* (Equation 18):

$$C_{j,qual,g+1,t+g} = \left[\frac{(1+Rint_{t+g})PCon_{g,t+g-1}}{(1+\rho_j)PCon_{g+1,t+g}} \right]^{\sigma^{IT}} C_{j,qual,g,t+g-1} \tag{18}$$

Equation (18) shows the ratio of aggregate consumption by individual of age g at skill level *qual* at time $t+1$ to it is at time t increases with the ratio of aggregate consumption price at time t to the aggregate consumption price at time $t+1$. In other words, higher are future consumption prices, higher is current consumption and lower is future consumption.

On the second step of optimization, an individual of age g allocates the expenditure on the optimal aggregate consumption, $C_{j,qual,g,t}$, across sectors. The problem is to maximize the following CES utility function subject to budget constraint:

$$\underset{CS_{j,S1,qual,g,t},CS_{j,S2,qual,g,t}}{Maximize} \; C_{j,qual,g,t} = (\delta_{S1}^{CS} CS_{j,S1,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}} + \delta_{S2}^{CS} CS_{j,S2,qual,g,t}^{\frac{\sigma^{CS}-1}{\sigma^{CS}}})^{\frac{\sigma^{CS}}{\sigma^{CS}-1}} \tag{19}$$

$$s.t.\; PCon_{j,g,t} C_{j,qual,g,t} = PCons_{j,S1,t} CS_{j,S1,qual,g,t} + PCons_{j,S2,t} CS_{j,S2,qual,g,t} \tag{20}$$

In equation (19), δ_s^{CS} is the parameter representing the preference of a consumer of age g on consumption goods across sectors. σ^{CS} is the elasticity of substitution for consumptions across sectors. $CS_{j,s,qual,g,t}$ is the quantity of consumption of good s by an individual of age g at skill level *qual* living in country j at time t. In equation (20), $PCons_{j,s,t}$ is consumption price of good s. The following equations (21) and (22) are the first order conditions for the second step of optimization:

$$CS_{j,qual,s,g,t} = \delta_S^{CS^{\sigma^{CS}}} \left[\frac{PCon_{j,g,t}}{PCons_{j,s,t}} \right]^{\sigma^{CS}} C_{j,qual,g,t} \tag{21}$$

$$PCon_{j,g,t}^{1-\sigma^{CS}} = \sum_s \delta_S^{CS^{\sigma^{CS}}} PCons_{j,s,t}^{1-\sigma^{CS}} \tag{22}$$

Equation (21) shows the consumption of good s by individual of age g at skill level *qual* at time t increases with the aggregate consumption but decreases with the consumption price of good s on the market. Equation (22) shows the aggregate consumption price is a weighted sum of the consumption prices for different good s.

In our Armington-type trade model, all foreign born and native individuals living in country j allocates the consumption expenditure on good s by country of origin, which is the third step of optimization. In other words, a decision is made on how much to spend on either locally-produced good s or imported good s. The following equations show the third step of consumer's optimization:

$$\underset{CSJ_{j,j,s,qual,g,t},CSJ_{i,j,s,qual,g,t}}{Maximize} \quad CS_{j,s,qual,g,t} = (\delta_{j,j,s}^{CE} CSJ_{j,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}} + \delta_{i,j,s}^{CE} CSJ_{i,j,s,qual,g,t}^{\frac{\sigma^{CE}-1}{\sigma^{CE}}})^{\frac{\sigma^{CE}}{\sigma^{CE}-1}} \quad (23)$$

$$s.t.\ PCons_{j,s,t} CS_{j,s,qual,g,t} = Pq_{j,s,t} CSJ_{j,j,s,qual,g,t} + Pq_{i,s,t} CSJ_{i,j,s,qual,g,t} \quad (24)$$

In equation (23), $\delta_{i,j,s}{}^{CE}$ is the parameter representing the preference of a consumer on goods s produced in country i, and σ^{CE} is the elasticity of substitution for locally-produced and imported consumption good s. $CSJ_{i,j,qual,s,g,t}$ is the quantity of consumption of good s produced in country i but consumed by country j's consumer. In other words, set i shows the country of origin of a consumed good and set j shows where this good is consumed. Accordingly, $CSJ_{j,j,qual,s,g,t}$ represents locally-produced consumption good s consumed by generation g in country j at time t. The following equation (25) and equation (26) are the first order conditions for the third step of optimization:

$$CSJ_{i,j,s,qual,g,t} = \delta_{i,j,s}^{CE^{\sigma^{CE}}} \left[\frac{PCons_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{CE}} CS_{j,s,qual,g,t} \quad (25)$$

$$PCons_{j,s,t}^{1-\sigma^{CE}} = \sum_i \delta_{i,j,s}^{CE^{\sigma^{CE}}} Pq_{i,s,t}^{1-\sigma^{CE}} \quad (26)$$

3.2.3 Labour supply, migration and demographics

In this sub-section, we put emphasis on our contribution: the modeling of the supply side of labour market in a dynamic CGE model as a necessity to analyze endogenous migration. Since the evolution of each region's population is no longer exogenously determined and becomes more dependent on the results of bilateral migration, we also introduce how the post-migration population evolutes.

In each country, the aggregate labour supply is provided by workers either born in this country or born abroad. Thus, there are four types of labour on the market in each country: native skilled workers, immigrant skilled worker, native unskilled workers and immigrant unskilled workers. The decision making of labour supply is closely correlated to household behaviour. At time *t*, each individual in country *j* is endowed with a certain quantity of labour. Since there is no constraint on the movement of labour across borders, this individual has the freedom to allocate his or her unit of labour supply into either working in the home country or working abroad.

In the model, the labour supply at skill level *qual* in region *j* at time *t*, $LsQ_{j,qual,t}$, is provided by the productivity profile at skill level *qual* ($EP_{j,qual,}$), who are born in this country:

$$LsQ_{j,qual,t} = Pop_{j,qual,t} EP_{j,gj,qual} \tag{27}$$

where $LsQ_{j,qual,t}$ is a CET aggregation of labour supplied to home labour market and foreign labour market ($LsM_{j,j,qual,t}$ and $LsM_{j,i,qual,t}$), as shown by the following equation (28):

$$LsQ_{j,qual,t} = (\delta^{Lsm}_{j,j,qual} LsM_{j,j,qual,t}^{\frac{\sigma^{SM}+1}{\sigma^{DM}}} + \delta^{Lsm}_{j,i,qual} LsM_{j,i,qual,t}^{\frac{\sigma^{SM}+1}{\sigma^{SM}}})^{\frac{\sigma^{SM}}{\sigma^{SM}+1}} \tag{28}$$

In equation (28), $\delta_{j,j,qual}^{Lsm}$ is the share of time allocated by an individual born in country j into working in country j, and $\delta_{j,i,qual}^{Lsm}$ is the share of time allocated by an individual, born in country j, into working in country i. $LsM_{j,j,qual,t}$ ($LsM_{j,i,qual,t}$) is the quantity of labour supplied by an individual born in country j and work in country $j(i)$. σ^{SM} is the elasticity of transformation between labour supplied to production in country j by workers born in country j and labour supplied to production in country i by workers born in country j. This elasticity value shows the responsiveness of labour born in country j to the wage rates paid for them by producers in country j and country i.

As mentioned above, an individual born in country i allocates its unit of labour supply into either working in his or her home country or abroad. The optimization problem for this individual is to maximize its labour income gained from both working in his or her home country and abroad. In the model, the problem is to maximize the following equation (29) subject to equation (28):

$$WsQ_{j,qual,t}LsQ_{j,qual,t} = WsM_{j,j,qual,t}LsM_{j,j,qual,t} + WsM_{j,i,qual,t}LsM_{j,i,qual,t} \qquad (29)$$

where $WsQ_{j,qual,t}$ is the aggregate wage rate paid for an individual, at skill level $qual$, born in country j at time t. $WsM_{j,j,qual,t}$ ($WsM_{j,i,qual,t}$) is the wage rate paid by producers in country j (i) for native (immigrant) workers born in country j.

The following two equations (30) and (31) are the first order conditions:

$$LsM_{j,i,qual,t} = \delta_{j,i,qual}^{Lsm^{\sigma^{SM}}} \left[\frac{WsM_{j,i,qual,t}}{WsQ_{j,qual,t}} \right]^{\sigma^{SM}} LsQ_{j,qual,t} \qquad (30)$$

$$WsQ_{j,qual,t}^{1+\sigma^{SM}} = \sum_i \delta_{j,i,qual}^{Lsm^{\sigma^{SM}}} WsM_{j,i,qual}^{1+\sigma^{SM}} \qquad (31)$$

Equation (30) suggests the quantity of labour provided by native (immigrant) workers born in country *j* increases with the ratio of wage rate for native (immigrant) workers in home (destination) country to the aggregate supply wage rate for workers, at skill level *qual*, born in country *j*. It also augments with the share of labour supplied to home or foreign labour market in the aggregate labour supply and the aggregate quantity of labour supply provided by workers at skill level *qual* and born in country *j*. Equation (31) suggested the aggregate supply wage rate for an individual born in country *j* is the weighted sum of supply wage rate paid by producers in country *j* to native workers and supply wage rate paid by producers in country *i* to immigrant workers born in country *j*.

In the first chapter of this thesis, we introduce the demographic shock as given the projected fertility rates, we calculate each region's population and effective labour supply for each period. The changes of all economic variables in the dynamic are based on this demographic shock. In chapter 2, we introduced a one-period exogenous shock of unilateral migration so that the population of the source country and the destination country will be recalculated for all periods after the timing of migration, as shown by the following equations (32) and (33):

$$POP_{i,qual,t+1} - MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1} \tag{32}$$

$$POP_{j,qual,t+1} + MigCoe_{i,j,t+1,g} Pop_{i,qual,gi,t+1} \tag{33}$$

where $MigCoe_{i,j,qual,t,g}$ is the migration coefficient calculated based on wage differentials across countries.

The economic implication of this migration coefficient is the percentage of emigrating people out of the population of the source countries, so we only need to adjust the population for all countries for one period and allow their population to regenerate based on the adjusted population in the following periods. However, with this setting, we are unable to model the bilateral migration as this migration coefficient only shows the net migration rate based on regional wage differentials.

In this paper, modeling of the supply side of the labour market enables us to differentiate native and immigrant workers. In addition, we no longer rely on the migration coefficient to exogenously determine the net percentage of migrating people. In other words, for maybe the fist time in the dynamic CGE modeling, we treat labour stock in each country as commodities on the good markets based on the rationale of Armington assumption. How much of his or her labour supply unit an individual allocates into working in the home country and working abroad are determined by both the preferences to relocate and the endogenously determined changes of wage rates for native and immigrant workers.

Consequently, the population of both countries is also endogenously determined in each period. This raises a problem of the population of both countries at each time period could be either the number of people before migration or the number of people after migration. Another obstacle is the timing for us to impose the exogenous fertility rates.

To solve these problems, we first assume that both native and immigrant workers follow the fertility rates in their countries of origin. That is, an individual born in country j always follows $NN_{j,t}$ though part of the labour supply unit will be allocated into working in country i. Then, we introduce two types of population (the native population $PopM_{j,j,qual,t,g}$, and the migrating population $PopM_{j,i,qual,t,g}$) into the model.

The following equations (34) and (35) are used to calculate native and migrating population at each time t:

$$PopM_{j,i,qual,t,g} = \delta_{j,i,qual}^{Pop^{\sigma^{SM}}} \left[\frac{LuM_{j,i,qual,t}}{LuM0_{j,i,qual}} \right] Pop_{j,qual,t,g} \tag{34}$$

$$LuM_{j,i,qual,t} = \delta_{j,i,qual}^{Lsm^{\sigma^{SM}}} \left[\frac{WsM_{j,i,qual,t}}{WsQ_{j,qual,t}} \right]^{\sigma^{SM}} LuQ_{j,qual} \tag{35}$$

$$Pop_{j,qual,t+1,gi} = NN_{j,t} Pop_{j,qual,t+1,gi} \tag{36}$$

$$Pop_{j,qual,t+1,g+1} = Pop_{j,qual,t,g} \tag{37}$$

In equation (34), $PopM_{j,i,qual,t,g}$ is the population of people, at skill level *qual*, born in country j but working in country i at time t. ${\delta_{j,i,qual}}^{Pop}$ is share parameter of migrating population out of total population of country j. $LuM_{j,i,qual,t}$ and $LuM0_{j,i,qual}$ are the unit labour supplied by this individual in working in country i at time t and at the initial steady state respectively. $Pop_{j,qual,t,g}$ is the population born in country j at time t. In equation (35), $LuQ_{j,qual}$ is the unit labour supply in country i by individuals at skill level *qual*.

Equation (34) shows the mobile people increases with the preference parameter and the ratio of unit labour supply at time t to the initial unit labour supply of this migrating people. Equation (35) shows this native worker or migrating worker's labour supply is derived from the exogenous unit of labour supply he or she is endowed ($LuQ_{j,qual}$). The level increases with the wage rate paid by county i to immigrant workers born in country j but decreases with the aggregate wage rate in the home country j.

Equations (36) and (37) show that we apply the exogenous fertility rates in country j to the native born population of country j in the first generation *gi*. In this manner, the country-specific fertility rates affect the number of natives born. The amount of immigrants is determined by (34).

3.2.4 Determinants of investment demand

In each period t, savings of the younger generations finance investment equipped by firms to increase and maintain the stock of physical capital in period $t+1$. The usual law of motion of capital stock can be represented by the following equation (38):

$$KS_{j,t+1} = I_{j,t} + (1 - DR)KS_{j,t} \tag{38}$$

In equation (38), $I_{j,t}$ is the aggregate investment in country j at time t. The capital stock in the economy at time $t+1$, $KS_{j,t+1}$, augments with investment in the previous period t and declines with depreciation rate (DR). This equation shows the aggregate investment level is determined by the gap between demand of capital stock in the next period and current capital stock net of depreciation.

The capital stock is a composite good composed of the available goods in the economy. Consequently, the capital stock in each economy is built using an investment technology with both the capital-intensive good and the labour-intensive good. After the aggregate investment level is determined, the optimal constituting mix of market goods for investment is determined by two steps of optimization, which are depicted by the following four equations:

$$I_{j,t} = (\delta_{S1}^{IS} IS_{j,S1,t}^{1-\frac{1}{\sigma^{IS}}} + \delta_{S2}^{IS} IS_{j,S2,t}^{1-\frac{1}{\sigma^{IS}}})^{\frac{\sigma^{IS}}{\sigma^{IS}-1}} \tag{39}$$

$$IS_{j,s,t} = (\delta_{j,j,s}^{IE} ISJ_{j,j,s,t}^{1-\frac{1}{\sigma^{IE}}} + \delta_{i,j,s}^{IE} ISJ_{i,j,s,t}^{1-\frac{1}{\sigma^{IE}}})^{\frac{\sigma^{IE}}{\sigma^{IE}-1}} \tag{40}$$

$$PI_{j,t} I_{j,t} = PIS_{j,S1,t} IS_{j,S1,t} + PIS_{j,S2,t} IS_{j,S2,t} \tag{41}$$

$$PIS_{j,s,t} IS_{j,s,t} = Pq_{j,s,t} ISJ_{j,j,s,t} + Pq_{i,s,t} ISJ_{i,j,s,t} \tag{42}$$

In equation (39), δ_s^{IS} is the given share of good s in aggregate investment, and σ^{IS} is the elasticity of substitution for investment goods. $IS_{j,s,t}$ is the investment good s used in country j at time t. In equation (40), $\delta_{i,j,s}^{IE}$ is the given share of investment good s produced in country i but used in country j. $ISJ_{i,j,s,t}$ is the investment good s used in country j but produced in country i at time t. Accordingly, $ISJ_{j,j,s,t}$ is the locally-produced investment good s used in country j at time t. σ^{IE} represents the elasticity of substitution for locally-produced and imported investment good s. In equations (41) and (42), $PI_{j,t}$ represents the aggregate price of all investment goods in country j at time t and $PIS_{j,s,t}$ is the price of investment good s in country j at time t.

On the first step of optimization, an investor minimizes equation (41) subject to equation (39). The following equations (43) and (44) are the first order conditions:

$$IS_{j,s,t} = \delta_s^{IS^{\sigma^{IS}}} \left[\frac{PI_{j,t}}{PIS_{j,s,t}} \right]^{\sigma^{IS}} I_{j,t} \tag{43}$$

$$PI_{j,t}^{1-\sigma^{IS}} = \sum_s \delta_s^{IS^{\sigma^{IS}}} PIS_{j,s,t}^{1-\sigma^{IS}} \tag{44}$$

After the $IS_{j,s,t}$ is determined, on the second step of optimization, an investor allocates its investment expenditure on investment good s across countries. The optimization problem is to minimize equation (42) subject to equations (40). The following equations (45) and equation (46) are the first order conditions:

$$ISJ_{i,j,s,t} = \delta_{i,j,s}^{IE^{\sigma^{IE}}} \left[\frac{PI_{j,s,t}}{Pq_{i,s,t}} \right]^{\sigma^{IE}} IS_{j,s,t} \tag{45}$$

$$PIS_{j,s,t}^{1-\sigma^{IE}} = \sum_i \delta_{i,j,s}^{IE^{\sigma^{IE}}} Pq_{i,s,t}^{1-\sigma^{IE}} \tag{46}$$

3.2.5 Foreign trade with the rest of the world

As explained in the above sections, we allocate demand in country j for goods produced in country i based on the Armington assumption. In other words, even though individual producers are microscopic price takers, good s are assumed to be differentiated in demand by country of origin. At the initial steady state where all countries are assumed to be perfectly symmetric, each country imports and exports the same quantity of *S1* and *S2*. Trade exists when countries are perfectly symmetric because each of them demands both locally-made and foreign-made goods. During the processes of population ageing, changes in relative factor abundances bring changes to two countries' comparative advantages. Accordingly, demands for the four goods change overtime.

The aggregate level of one country's import or export of good s (denoted as $IMP_{j,s,t}$ and $EXP_{j,s,t}$) is determined by the above optimization steps for agents (consumer, investor and government), which can be shown by the following equation (47) and equation (48):

$$IMP_{j,s,t} = \sum_{i,qual,g} PopM_{i,j,qual,g,t} CSJ_{i,j,qual,s,g,t} + ISJ_{i,j,s,t} \quad for\, i \neq j \qquad (47)$$

$$EXP_{j,s,t} = \sum_{i,qual,g} PopM_{j,i,qual,g,t} CSJ_{j,i,qual,s,g,t} + ISJ_{j,i,s,t} \quad for\, i \neq j \qquad (48)$$

3.2.6 Equilibrium conditions

A general equilibrium solution is one in which all economic behaviours are consistent with both current prices and future prices and all markets clear. In this model, we impose the following four equilibrium conditions for good market, labour market, capital market and the market for financial asset respectively:

$$X_{j,s,t} = \sum_{i,g} PopM_{i,j,qual,g,t} CSJ_{i,j,qual,s,g,t} + IS_{j,s,t} \quad (49)$$

$$\sum_{i,gj} PopM_{i,j,qual,gj,t} LuM_{i,j,qual,gj,t} = \sum_{s} LdQ_{j,s,qual,t} \quad (50)$$

$$KS_{j,t} = \sum_{s} Kd_{j,s,t} \quad (51)$$

$$\sum_{j,g,qual} POP_{j,qual,gm,t+1} LD_{j,qual,gm,t+1} = \sum_{j} PI_{j,t} KS_{j,t+1} \quad (52)$$

In equation (52), $LD_{j,qual,g,t}$ is the financial asset owned by the old generations *G3* living in country *j* at time *t*.

3.2.7 Model solving strategy

From a microeconomic consistency point of view, the above equation (18) describing the intertemporal choice of the representative household is correct. As we know, the representative household works at homeland but abroad as well. Consequently, in the numerical model, it is important distinguish how much the household is consuming and saving at home and abroad to be able to determine the accounting at the countries' level. The challenge is to find a way so that we still satisfy the dynamic consistency of the household problem.

The first initiative is to expand the budget constraint of the household by clearly identifying what is consumed and saved at home and abroad. We can start by rewriting the budget constraint (equation 14) of a household born in country *j* in the dynamic transition way:

$$PCon\ C_{j,qual,g,t} + \left(Wealth_{j,qual,g+1,t+1} - Wealth_{j,qual,g,t}\right) =$$
$$WLdem_{j,i,qual,g,t} L\sup_{j,i,qual,t} + WLdem_{j,j,qual,g,t} L\sup_{j,j,qual,t}$$
$$+R\text{int}_t\left(Wealth_{j,qual,g,t}\right) \tag{53}$$

The wage revenue is the sum of what the household is earning working at home and working abroad (in country *i*). Savings is the difference of the stock of wealth accumulated at time *t+1* and at time *t*: ($Wealth_{j,qual,g+1,t+1}$ − $Wealth_{j,qual,g,t}$). Next, we expand further the budget by distinguishing what is consumed and saved at home and abroad:

$$PCon_i C_{j,i,qual,g,t} + PCon_j C_{j,j,qual,g,t} +$$
$$\left(Wealth_{j,i,qual,g+1,t+1} - Wealth_{j,i,qual,g,t}\right) + \left(Wealth_{j,j,qual,g+1,t+1} - Wealth_{j,j,qual,g,t}\right) =$$
$$WLdem_{j,i,qual,g,t} L\sup_{j,i,qual,t} + WLdem_{j,j,qual,g,t} L\sup_{j,j,qual,t} +$$
$$R\text{int}_t\left(Wealth_{j,i,qual,g,t}\right) + R\text{int}_t\left(Wealth_{j,j,qual,g,t}\right) \tag{54}$$

In the equation above, the price index of the consumption activity at home is $PCon_j$ and abroad $PCon_i$. This implies the implicit assumption we are making here is that for intra-temporal consumption allocation across goods, the foreigner behaves like the local people. As each period in our model is equivalent to about 20 years, assuming that foreigners adopt local preferences over goods seems to be reasonable. Using budget constraint (54) and solving the intertemporal problem, we come up with the following equation:

$$\left[PCon_{i,g+1,t+g}{}^{\sigma} C_{j,i,qual,g+1,t+g} + PCon_{j,g+1,t+g}{}^{\sigma} C_{j,j,qual,g+1,t+g}\right] =$$
$$\left[\frac{\left(1+R\text{int}_{t+g}\right)}{\left(1+\rho\right)}\right]^{\sigma} \left[PCon_{i,g,t+g-1}{}^{\sigma} C_{j,i,qual,g,t+g-1} + PCon_{j,g,t+g-1}{}^{\sigma} C_{j,j,qual,g,t+g-1}\right] \tag{55}$$

We can split this equation into two parts as the following:

$$\left[PCon_{i,g+1,t+g}{}^{\sigma}C_{j,i,qual,g+1,t+g}\right]=\left[\frac{\left(1+R\text{int}_{t+g}\right)}{\left(1+\rho\right)}\right]^{\sigma}\left[PCon_{i,g,t+g-1}{}^{\sigma}C_{j,i,qual,g,t+g-1}\right] \quad (56)$$

$$\left[PCon_{j,g+1,t+g}{}^{\sigma}C_{j,j,qual,g+1,t+g}\right]=\left[\frac{\left(1+R\text{int}_{t+g}\right)}{\left(1+\rho\right)}\right]^{\sigma}\left[PCon_{j,g,t+g-1}{}^{\sigma}C_{j,j,qual,g,t+g-1}\right] \quad (57)$$

Indeed, if we sum equation (56) and (57), we come up with equation (55). In the numerical model, we use equations (56) and (57) for the intertemporal first-order conditions rather than equation (18).[11]

The budget constraint in (54) can also be split into two parts as shown below:

$$\begin{aligned}&PCon_iC_{j,i,qual,g,t}+\left(Wealth_{j,i,qual,g+1,t+1}-Wealth_{j,i,qual,g,t}\right)=\\&WLdem_{j,i,qual,g,t}L\sup_{j,i,qual,t}+R\text{int}_t\left(Wealth_{j,i,qual,g,t}\right)\end{aligned} \quad (58)$$

$$\begin{aligned}&PCon_jC_{j,j,qual,g,t}+\left(Wealth_{j,j,qual,g+1,t+1}-Wealth_{j,j,qual,g,t}\right)=\\&WLdem_{j,j,qual,g,t}L\sup_{j,j,qual,t}+R\text{int}_t\left(Wealth_{j,j,qual,g,t}\right)\end{aligned} \quad (59)$$

Again, adding the last two equations, we come up with equation (54). In the numerical model we use (58) and (59) as part of the budget constraint.

Implicitly this avenue means that the intertemporal decisions are based on the preferences of origin country, whereas for intertemporal decisions the household adopt the preferences of the working location. By using equations (56), (57), (58) and (59), it is as if the household born in country j optimizes two household problems. One using the wage earnings arising in country j, and the other using the wage earnings arising for his work abroad in country i. This numerical approach is consistent with the aggregate problem and helps us to distinguish clearly what is consumed and saved in each of the country in which

11 Remark that the fact that capital is perfectly mobile across regions is very convenient since the rate of interest is the same whatever the location of the household.

the household is working. Consequently, it clarifies as well the GDP and GNP accounting in each country.

To make our simulation results more straightforward, we conduct two scenarios: the case of controlled labour mobility (scenario A) and the case of uncontrolled labour mobility (scenario B). In scenario A, we prohibit new migration of both skill levels of workers between the developed and the developing countries in the dynamic though these workers do allocate part of their time into working abroad. This scenario is represented by the following equation (60) as the equilibrium condition for labour market.

$$\sum_{i,gj} PopM_{i,j,qual,gj,t} LuM0_{i,j,qual,gj,t} = \sum_{s} LdQ_{j,s,qual,t} \qquad (60)$$

The application of equation (60) suggests the unit labour supply for native and foreign workers $LuM0_{i,j,qual,g,t}$ does not change overtime. In other words, share of immigrating population in total population remains constant in the entire horizon of the model. Accordingly, equations (34) and (35) are not included and supply side of labour market is missing in our scenario A model. In scenario B, people are allowed to move across borders based on supply and demand on home and foreign labour markets.

3.2.8 Parameters and elasticities

The following Tables 2 and 3 list critical parameter and elasticity values we apply into the model. On Table 2, most of the elasticity values are following Fougère *et al.* (2007) and Auerbach and Kotlikoff (1987). σ^{JT} is assumed to be 3 because in this three-generation model, each period contains 20 years. σ_j^{DM} is set to be lower in the developed region based on the assumption that producers in the more aged region are less sensitive to changes in wage rates for immigrant workers.

Table 2 Elasticity values assumed in calibration

	$J1$	$J2$
σ_S^{LD}	3.0	3.0
σ^{IT}	3	3
σ^{CS}	2.5	2.5
σ^{CE}	2.5	2.5
σ^{IS}	2.5	2.5
σ^{IE}	2.5	2.5
σ_j^{DM}	1.5	2.5
σ^{SM}	2	2
DR	0.8	0.8

Table 3 Preference parameters

	$J1$	$J2$				
$\alpha^K{}_{S1}$	0.632	0.644	$\delta_{j1,j1,qual}{}^{LSM}$	0.095	$\delta_{j1,j1,s1}{}^{CE}$	0.956
$\alpha^K{}_{S2}$	0.277	0.436	$\delta_{j1,j2,qual}{}^{LSM}$	0.100	$\delta_{j2,j1,s1}{}^{CE}$	0.044
ρ	0.463	0.234	$\delta_{j2,j2,qual}{}^{LSM}$	0.900	$\delta_{j1,j1,s2}{}^{CE}$	0.957
$\delta_{s1,skill}{}^{LD}$	0.469	0.148	$\delta_{j2,j1,qual}{}^{LSM}$	0.050	$\delta_{j2,j1,s2}{}^{CE}$	0.043
$\delta_{s2,skill}{}^{LD}$	0.403	0.206	$\delta_{j1,j2,s,skill}{}^{LDM}$	0.025	$\delta_{j2,j2,s1}{}^{CE}$	0.932
$\delta_{s1}{}^{CS}$	0.428	0.373	$\delta_{j1,j2,s,unskill}{}^{LDM}$	0.035	$\delta_{j1,j2,s1}{}^{CE}$	0.068
$\delta_{s2}{}^{CS}$	0.572	0.627	$\delta_{j2,j1,s,skill}{}^{LDM}$	0.088	$\delta_{j2,j2,s2}{}^{CE}$	0.900
$\delta_{s1}{}^{IS}$	0.330	0.333	$\delta_{j2,j1,s,unskill}{}^{LDM}$	0.088	$\delta_{j1,j2,s2}{}^{CE}$	0.100
$\delta_{s2}{}^{IS}$	0.670	0.667				

3.3 SIMULATION RESULTS

Since migration and its interactions with trade and demographic changes have already been explored in previous chapters, analysis in this paper focuses on new quantitative findings suggested by our endogenous migration model. The findings permit to analyse the migration issue from both the origin and destination country. It does also offer an international perspective in terms of the usual economic indicators and welfare. From the international perspective, one would not be surprise that when the demographic shift differs across countries, free migration (uncontrolled scenario) would be preferable as it leads to a better allocation of human resources.

3.3.1Changes of demographic and labour market variables

In the paper, fertility rates ($NN_{j,t}$) is assumed to be 1.03(1.01), 1.06(1.06), 1.04(1.04), 1.01(1.03), 0.97(1.01), 0.93(1.01), 0.97(1) for the developed (developing) region between period *t4* to *t10*. In the rest periods, fertility rates for all countries are assumed to be 1. It is clear that both the developed and the developing regions experience baby boom and baby bust periods but the developed countries' baby bust implies a fertility below replacement rates and its population is growing at a much lower than for the developing country.

The following Figure 1 shows the changes of elderly dependent ratios (EDR) for both regions in the controlled-migration and uncontrolled-migration scenarios that result from simulations. First, notice that under migration-controlled scenario, the EDR is much higher in country *J1* than in country *J2*. This reflects the demographic shock that is imposed on the initial steady state, which implies that the country *J1* is assumed to represent somewhat the North with a smaller, but more rapidly ageing population. The population in country *J2* (the South) is much more populated and is younger on average as the ageing process is much slower. In the new long-run equilibrium, the total population of *J1* is about the same as the baby boom and the baby bust offset each other. The total population in *J2* is, however, much larger in the long run as the baby boom is smoothed over time that is, not followed by an important baby bust. This difference in the demographic shock is going to generate results that were not expected.

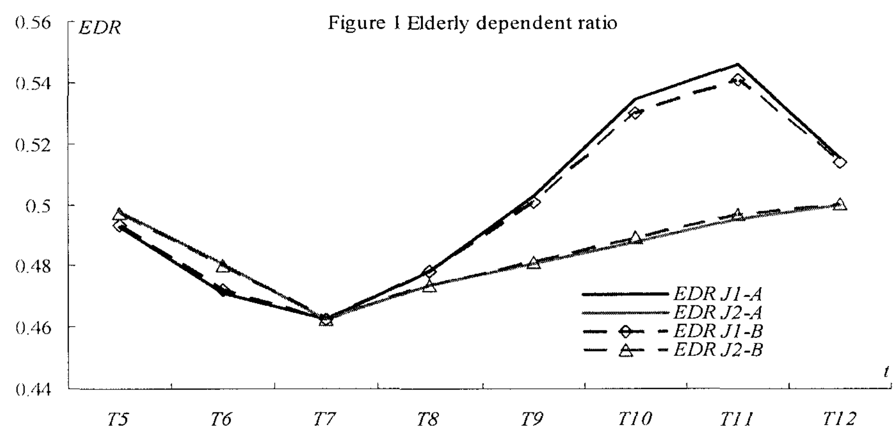


Figure 1 also shows that in the migration scenario B, elderly dependent ratios across countries are smoothed. This suggests that at least during some part of the baby bust period (*t7* to *t11*), migration occurs from *J2* to *J1*. With the absence of barrier to move across borders, the inflow of foreign workers in the developed region has not only a positive impact on the population growth rate in this region, but also reduces EDR in *J1* by making the population in *J1* younger in general.

Table 4 Percentage change rates of immigrant population

(unit: 1%)

	Scenario A				Scenario B			
	Skilled Immigrant (*J1*)	Unskilled Immigrant (*J1*)	Skilled Immigrant (*J2*)	Unskilled Immigrant (*J2*)	Skilled Immigrant (*J1*)	Unskilled Immigrant (*J1*)	Skilled Immigrant (*J2*)	Unskilled Immigrant (*J2*)
T3	0.000	0.000	0.000	0.000	0.001	0.001	-0.008	-0.008
T4	0.000	0.000	0.000	0.000	0.003	0.004	-0.034	-0.035
T5	1.000	1.000	3.000	3.000	1.017	1.021	2.940	2.934
T6	7.060	7.060	9.180	9.180	7.109	7.121	8.896	8.869
T7	11.342	11.342	13.547	13.547	11.418	11.437	13.025	12.986
T8	14.683	14.683	14.683	14.683	14.736	14.749	14.291	14.263
T9	16.976	16.976	11.242	11.242	16.963	16.962	11.089	11.075
T10	18.146	18.146	3.455	3.455	17.999	17.969	3.676	3.678
T11	18.146	18.146	0.352	0.352	17.843	17.780	1.124	1.133
T12	18.146	18.146	0.352	0.352	17.724	17.639	1.733	1.742
T13	18.146	18.146	0.352	0.352	17.693	17.603	1.910	1.921
T14	18.146	18.146	0.352	0.352	17.694	17.604	1.902	1.915
T15	18.146	18.146	0.352	0.352	17.697	17.607	1.874	1.887

The above Table 4 compares the percentage change rates of population of immigrant workers, relative to initial population of each category, in both scenarios. In scenario A when further immigration is prohibited in the dynamic, this percentage stabilized after *t10* when the fertility rates of both regions return to one. In scenario B, however, this rate changes in every single period because the shares of native and immigrant individuals are determined endogenously by supply and demand situations on the labour market in each period. As long as people are allowed to move and the age structure varies across regions, individuals make decisions to move or stay in order to maximize lifetime utility.

In addition, the percentage of individuals moving from the developing to the developed region is higher (lower) in scenario B (A) at the beginning of the ageing transition (*t5* to *t9*). This suggests that more people are willing to move from the developing region to the developed region at the beginning the ageing transition because the labour market in *J1* becomes more attractive. These results are in accord with the changes of unit labour supply, as presented by the following Table 5.

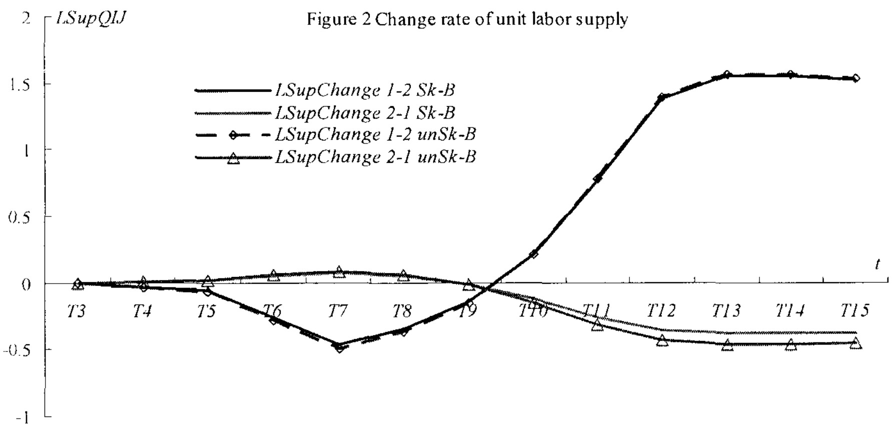


Table 5 Percentage change rates of unit labour supply in scenario B

(unit: 1%)

	Skilled		Unskilled	
	Immigrants in *J1*	Immigrants in *J2*	Immigrants in *J1*	Immigrants in *J2*
T3	0.001	-0.008	0.001	-0.008
T4	0.003	-0.034	0.004	-0.035
T5	0.017	-0.058	0.020	-0.064
T6	0.045	-0.260	0.057	-0.285
T7	0.068	-0.460	0.085	-0.494
T8	0.047	-0.341	0.058	-0.366
T9	-0.011	-0.138	-0.013	-0.150
T10	-0.124	0.214	-0.150	0.215
T11	-0.257	0.770	-0.310	0.779
T12	-0.357	1.377	-0.429	1.386
T13	-0.383	1.553	-0.460	1.564
T14	-0.382	1.545	-0.459	1.558
T15	-0.380	1.517	-0.456	1.530

As introduced in model descriptions (section *2.3*), how willingly an individual would like to relocate is represented by the distributions of unit labour supply $LuM_{j,i,qual,t}$ (equation 51). The unit labour supply keeps constant so that the percentage change rates are all zero in scenario A. In scenario B, the asymmetry in the trend of immigrants between country *J1* and

J2 is clear. It is shown that from *t3* to *t9*, individuals born in the developing region are more likely to work in the developed region than scenario A, since the labour supply of immigrants in *J1* increases. On the contrary, labour supply of immigrants in *J2* declines during that period, which means that individuals born in the developed region but working in country *J2* are going back home. For individuals born in the developing region, their willingness to relocate keeps decreasing after period *t9*. In addition, the unskilled individuals are more (less) willing to work in their home (foreign) country. This finding is correspondent to the changes of immigrant populations as shown by the Table 4. Compared to scenario A, the percentage increase of people working in *J1* but born in *J2* increases in scenario B at the beginning of the ageing process, but the degree of this increase is higher for unskilled individuals from the developing region. The model thus suggests that at the beginning of the baby bust, the immigrant stock of a region will rise because of scarcity of working-age generations.

However, at the end of the ageing process, the developed region's demands for both the skilled and unskilled foreign workers are reduced, this can be shown by the negative numbers for immigrants in *J1* after *t9* (Table 5). This finding is unexpected and would not occur in a single-country analysis. It is unexpected because an older region is deemed to attract more foreign labour to overcome the slowing down of labour force growth. However, our simulation results show that in the part of the baby bust, not only workers born in the developing countries are more willing to stay home, but also more people from the developed region are willing to work in the developing countries. This can be explained by the change in total population and hence regional GDP. As noted above, total population does not increase much in the long run in *J1*, while it does significantly in *J2*. With consumption strongly bias in favour of local good (see Table 3 for details), GDP in *J2* is called to increase much more than in *J1*. As the production technology assumes that domestic workers are not perfect substitute for foreign workers, the demand in *J2* for native and immigrant workers increase substantially (which reverse the immigration trend after *t9*). Also note that both the downward and upward trends for immigrants to stay in the developed and the developing region are more significant for unskilled workers. This can be explained by our analysis in previous two chapters: the labour-abundant country *J2* produces and exports more unskilled-labour-intensive products so that demands for this kind of labour tend to be higher than for the skilled labour.

The supply and demand of labour markets in the two countries are directly indexed by wage rates for workers of different skill levels and countries of origin (Figures 3.1 to 3.4). Among them, Figures 3.1 and 3.2 show the evolutions of wage rates paid for immigrant workers. Figure 3.3 and Figure 3.4 compare the wage rates for immigrant workers and for native workers in both regions. The trends in these figures confirm the analysis above.

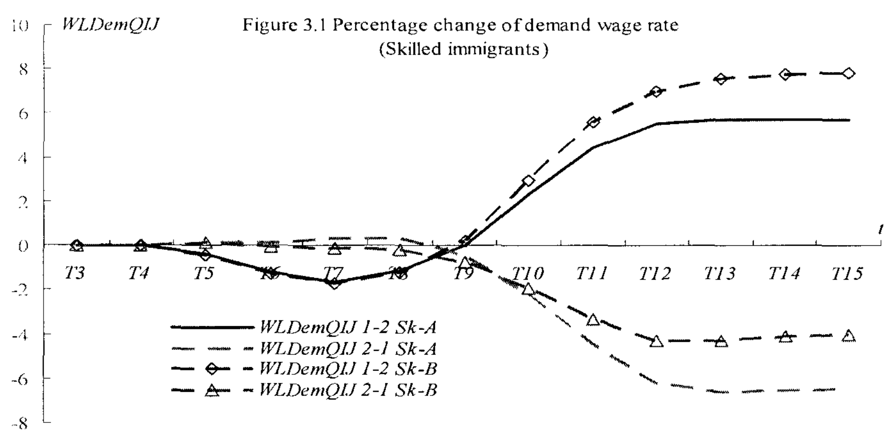


Figure 3.1 Percentage change of demand wage rate (Skilled immigrants)

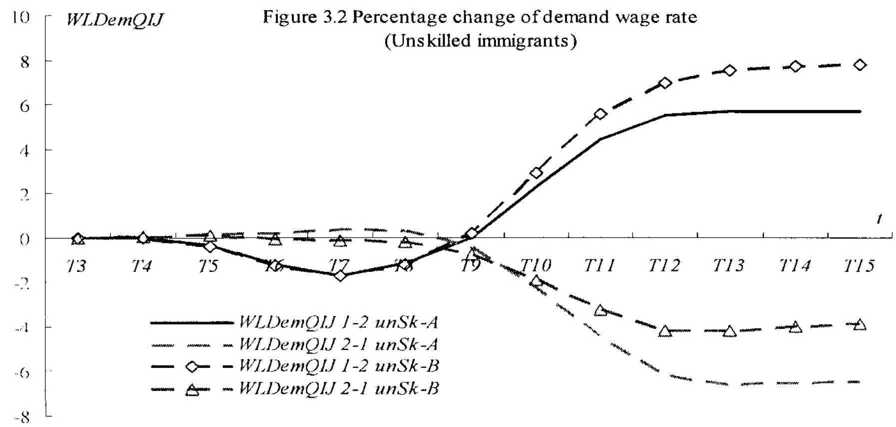


Figure 3.2 Percentage change of demand wage rate (Unskilled immigrants)

Indeed, in the absence of barriers for migration (scenario B), wage rates paid by the developed regions to immigrant workers increase if the first half of the ageing process with respect to the baseline level but starts to decline in the second half of the process for both skilled and unskilled workers. After *t9*, the percentage change rates become negative until the long run. In contrast, wage rates paid to foreign workers born in the developed region increase after *t9*. These also show the downward (upward) trend in the developed (developing) labour market in demanding foreign labour at the end of the baby bust. As for migration results, it seems that after *t9*, the size of GDP in *J2* is dominating the EDR factor in *J1* to determine the net impact on wage rates.

Compared with scenario A, migration seems have added to the demand for both skilled and unskilled foreign workers as shown that all wage rates become more positive in scenario B. However, the gap between the demand by the developed labour market and by the developing labour market is actually reduced because we also observe that the wage gaps between the two regions have been smoothed by migration.

Comparing the wage rates paid to native and immigrant workers provides us another interesting finding. That is, there are wage differentials between native workers and immigrant workers, as shown by the following Figures 3.3 and 3.4.

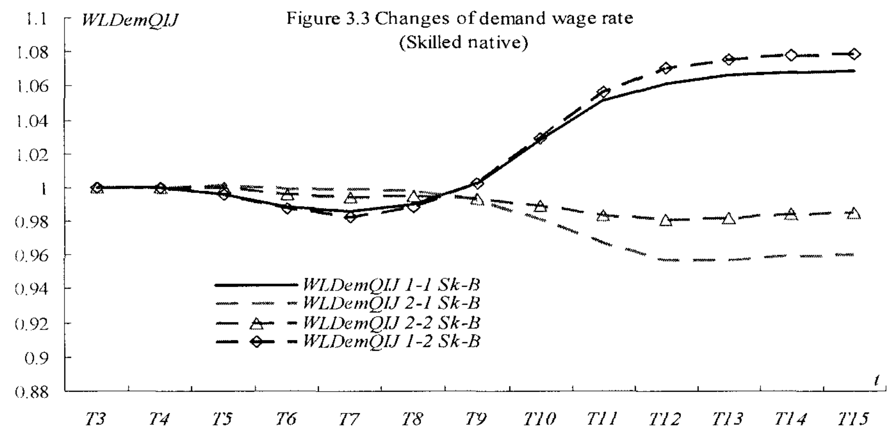


Figure 3.3 Changes of demand wage rate (Skilled native)

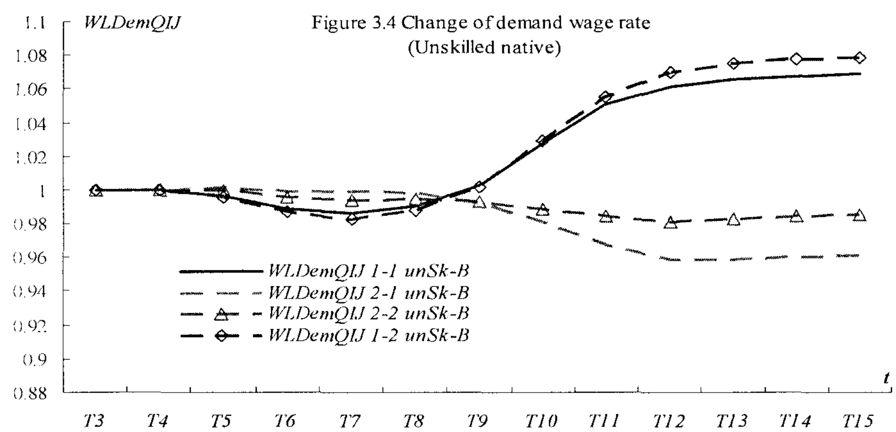


Figure 3.4 Change of demand wage rate (Unskilled native)

For example, in the developed region, there are higher demands for foreign labour at the beginning of the ageing process, which leads to higher wage rates for immigrant workers as shown by the above figures. At the end of the ageing process, however, lower demand for immigrant workers significantly reduces the wage rate for immigrants compared with it is for the natives. In addition, the wage rates for native workers turn to be increasingly higher than for immigrant workers, this creates wage differentials for same skill levels of labour on the labour market.

The above findings suggest that in addition to efficiency indexes differentiating native and immigrant workers, demand and supply situations could, to some degree, explain the wage gaps between native and immigrant workers on the labour markets of many destination countries of international migration (Canada, for example). In addition, the wage differential varies across different skill levels of labour.

3.3.2 Changes of welfare indicators

Overall, findings from our endogenous migration model suggest that cross-border movement of labour benefits both the developed and the developing regions in terms of per capita revenue. Consequently, world economy also gains from bilateral migration. However, this gain from immigration varies both across countries and across different time spans.

Table 5 Percentage change rates of per capita revenue

(unit: 1%)

	Per capita revenue *J1* (A)	Per capita revenue *J1* (B)	Per capita revenue *J2* (A)	Per capita revenue *J2* (B)	Per capita worldrevenue (A)	Per capita worldrevenue (B)
T1	0.000	0.000	0.000	0.000	0.000	0.000
T2	0.000	0.000	0.000	0.000	0.000	0.000
T3	-0.010	0.015	-0.011	0.016	-0.008	0.006
T4	0.005	0.031	-0.007	0.023	0.004	0.007
T5	-0.904	-0.937	-0.056	-0.083	0.231	0.131
T6	-3.408	-3.643	-1.531	-1.758	0.533	0.165
T7	-7.284	-7.766	-4.464	-4.946	1.062	0.407
T8	-10.793	-11.302	-8.640	-9.159	0.728	0.145
T9	-11.323	-11.670	-11.972	-12.330	-0.234	-0.385
T10	-7.991	-7.807	-15.017	-14.872	-2.219	-1.379
T11	-2.532	-1.487	-17.455	-16.595	-4.503	-2.387
T12	2.295	4.247	-19.092	-17.566	-6.064	-2.859
T13	4.030	6.449	-19.963	-18.112	-6.365	-2.684
T14	4.187	6.772	-20.323	-18.353	-6.322	-2.499
T15	4.242	6.898	-20.352	-18.328	-6.294	-2.430

For individual regions, we first calculate the revenue and per capita revenue based on the following equations (61) and (62):

$$\mathrm{Re}\,v_{j,t} = \sum_{i,g,qual} PopM_{j,i,qual,gj,t} LuM_{j,i,qual,gj,t} WsM_{j,i,qual,t} + LD_{j,i,qual,gm,t}\,\mathrm{R}\,ret_i) \quad (61)$$

$$PC\,\mathrm{Re}\,v_{j,t} = \mathrm{Re}\,v_{j,t} / Tpop_{j,t} \quad (62)$$

The variable $Rev_{j,t}$ is the sum of all individuals', born in country j, wage income in the working age and interest income on their financial asset. $PCRev_{j,t}$ is the per capita revenue for country j. Equations (61) and (62) show that this indicator can be seen as per capita national income for country j.

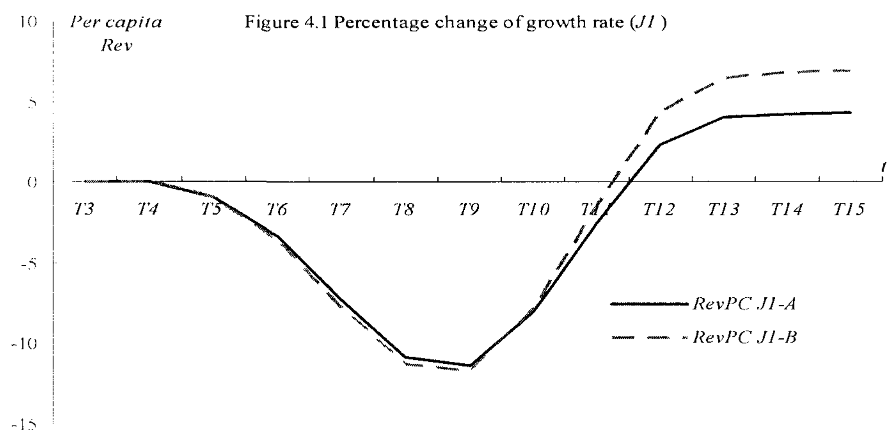


Figure 4.1 Percentage change of growth rate (*J1*)

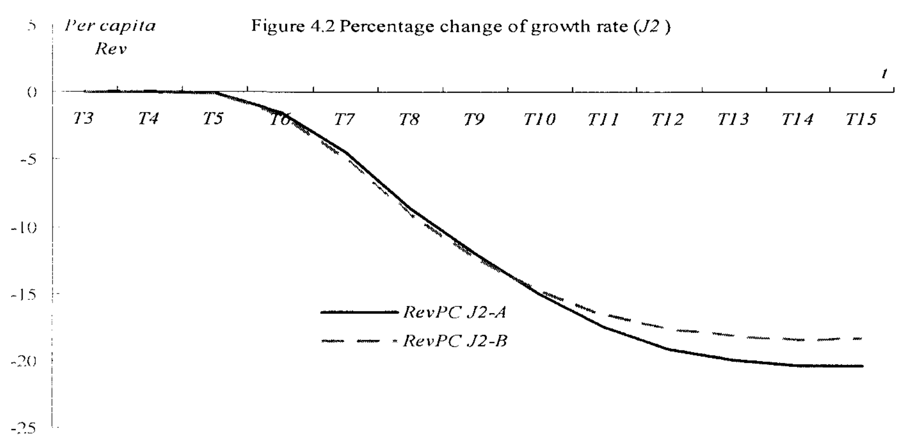
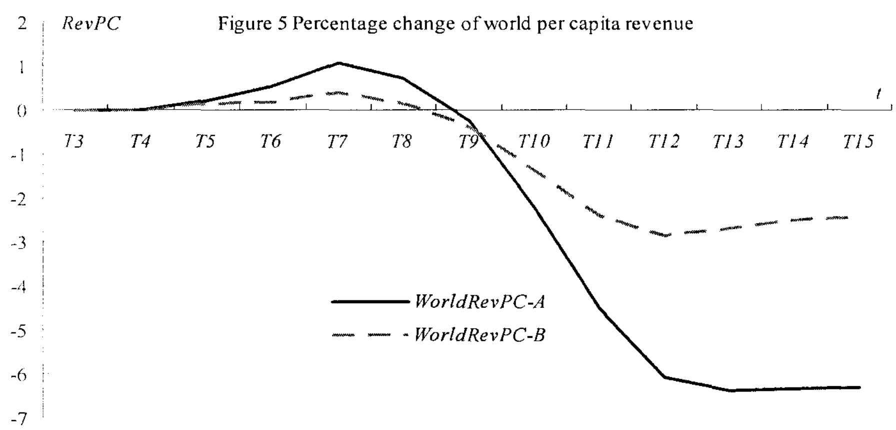


Figure 4.2 Percentage change of growth rate (*J2*)

In scenario B, individuals in both regions experience small reduction in per capita revenue in the short run during the baby boom and baby bust. This revenue loss turns to be increasing during the baby boom but diminishing during the baby bust after period *t5*. After the 9th period, revenue turns to be higher as the result of uncontrolled migration. This could be explained by wage rates for both the skill and the unskilled workers turn higher in scenario B than in scenario A (Figures 3.1-3.4). In both regions, free labour mobility benefits individuals in the context of population ageing (the mid run and long run). This trend is easier to observe from the following Figure 5 on the worldwide gain of per capita revenue.



The following we analyze the impacts on national and worldwide production.

Table 6 Percentage change of production and per capita production

(unit: 1%)

	Scenario A						Scenario B					
	Q *J1*	Q *J2*	QPC *J1*	QPC *J2*	WorldQ	WorldQPC	Q *J1*	Q *J2*	QPC *J1*	QPC *J2*	WorldQ	WorldQPC
T3	-0.012	0.000	-0.012	0.000	-0.008	-0.008	0.013	-0.009	0.013	-0.009	0.006	0.006
T4	0.000	0.015	0.000	0.015	0.004	0.004	0.038	-0.066	0.038	-0.066	0.007	0.007
T5	0.777	0.403	-0.221	0.070	0.665	0.231	0.785	0.050	-0.213	-0.283	0.564	0.131
T6	3.782	2.642	-0.267	-0.044	3.440	0.533	3.649	1.689	-0.395	-0.972	3.061	0.165
T7	8.482	6.597	-0.087	0.122	7.916	1.062	8.115	5.119	-0.424	-1.266	7.217	0.407
T8	12.495	11.024	0.022	-0.004	12.053	0.728	12.038	9.927	-0.384	-0.992	11.405	0.145
T9	13.836	14.022	0.599	-0.272	13.892	-0.234	13.491	14.250	0.295	-0.073	13.719	-0.385
T10	11.867	15.715	1.889	-0.761	13.021	-2.219	12.021	18.594	2.029	1.709	13.992	-1.379
T11	8.280	16.146	3.108	-1.368	10.639	-4.503	9.146	22.297	3.933	3.856	13.091	-2.387
T12	5.483	15.989	4.041	-1.826	8.634	-6.064	6.998	24.809	5.535	5.640	12.340	-2.859
T13	4.830	15.871	4.463	-1.925	8.142	-6.365	6.679	25.729	6.305	6.418	12.393	-2.684
T14	4.898	15.876	4.530	-1.922	8.190	-6.322	6.882	25.968	6.507	6.621	12.606	-2.499
T15	4.943	15.878	4.575	-1.920	8.223	-6.294	6.984	25.995	6.609	6.643	12.686	-2.430

In the literature on international migration, Iregui (2003) used a static multiregional one-sector GE model to analyze the impacts of international migration on global output. In her paper, welfare impacts of migration are solely analyzed based on changes in output. Her paper shows that the world output as a whole benefit from uncontrolled migration from the developing region to the developed region, and there will be a large gain from 15% to 67%. The degrees of benefit vary across different assumed scenarios of migration.

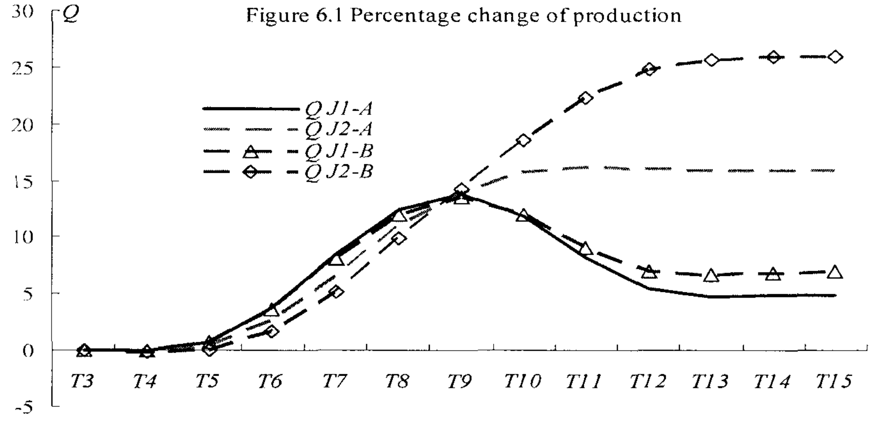
In contrast to Iregui (2003) who used a static model to compare the no-migration and uncontrolled migration cases, this paper uses our endogenous migration model to compare controlled migration and uncontrolled migration cases. The above Table 6 shows total and per capita output increases by 4.463% and 3.864%, compared with scenario A, respectively when we don't control worldwide migration. This level is calculated at the long-run steady state at *t15*.

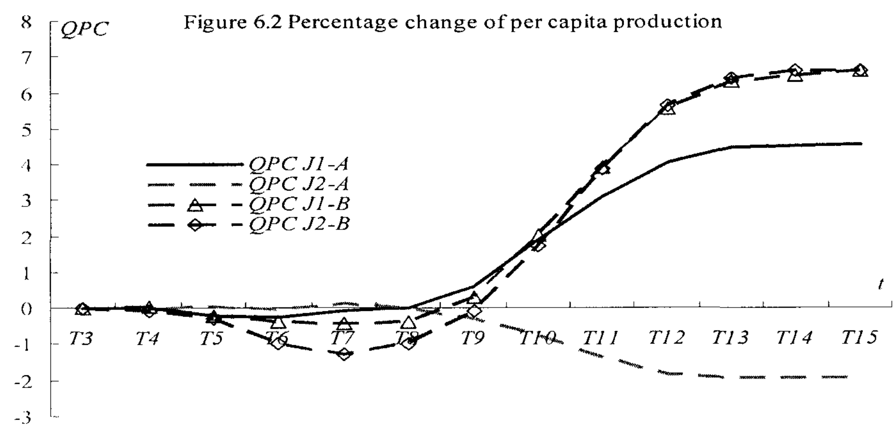
The magnitudes of output gains are smaller than those suggested by Iregui (2003) because migration was purely designed as a united global labour market and a huge number of people will migrate from the poor countries to the rich countries. Migration in this model, however, is designed based on initial preferences on relocate and changes of supply and demand situations on global labour markets in the dynamic context of demographic transition. Although there is no barrier to migrate, the scale of migration is found to be very limited: migrating people account for around 3% of world population in scenario A and 3.5% in scenario B.

For national products, the changing paths of output and per capita output are consistent with it is of revenue and per capita revenue: in scenario B, uncontrolled migration has small negative impacts on output during the baby boom but large positive impacts in the mid and long run.

Compared with the developed region, gross output of the developing countries increases much more because its population growth has been positive and its economy has been expanding. The gross output of the developed region increases moderately because on one hand, its economy turns to be smaller than the developing region as the result of population ageing. On the other hand, however, there are more trade activities with the developing world, which raises its production of tradable goods. Relevant analysis could be found in chapter two of this thesis.

Figure 6.1 Percentage change of production





3.4 CONCLUSION

Using a 2*2*3*2 type dynamic CGE model incorporating the idea of "endogenous migration", this paper investigates the impacts of eliminating barriers to international migration on the developing and developed regions. In order to facilitate a multiregional model with the ability to endogenize bilateral migration, a series of CET functions have been introduced to model the supply side of labour markets. In the modelling, we also differentiated the two regions by their demographic characteristics: the developed region turns to be an older region after the baby boom periods. This is consistent with our post WWII observations and UN demographic projections. For the first time in CGE modeling, the supply side of labour market has been incorporated in a dynamic version. Labour has been treated as tradable goods on global labour markets. International migration has been modelled following the rationale of international flows of tradable goods: the direction and scale of migration are determined by both preferences on relocate and differentiated wage rates for immigrating workers.

Several interesting findings are suggested from our simulation results:

First of all, a united labour market benefits both developed and developing regions. The growth rates of national income, gross and per capita output in both of them are all found to be positively impacted by bilateral migration. However, these positive impacts could only be observed in the mid and long run after the baby boom. Despite the small negative impacts of uncontrolled migration during the baby-boom periods, the world economy benefits much from a united labour market.

Second, our endogenous migration model suggested some stylized facts we have observed on the global labour markets. Wage differentials exist between native and immigrant workers in both regions. A native worker turns to be more expensive than an immigrant worker of both skill levels in the developed region. In contrast, immigrant workers from the developed region are found to be better treated than native workers in the developing labour market. This is consistent with several important empirical findings. In addition, a stronger tendency was found among workers born in the developed region to relocate to and work in the developing economies. This is also contrasted by less workers from the developing labour market are willing to work in the developed region during and after the baby bust. This shows a clear tendency of younger economies in the world growing into more important players in the world market, which creates demands for both native and foreign labour forces.

Overall, this paper is mainly an attempt to introduce a new modeling framework to deal with demand and supply of labour markets, labour mobility and population ageing. Although we have made solid progresses in providing the design of the model, the robustness of our simulation results still needs further investigation and may be constrained by lack of data on international labour mobility. For example, the shares of labour supplied by immigrant workers in production are assumed in both regions. The initial shares of native and immigrant populations in each region are also assumed due to our lack of data on the developing labour market. Some of the elasticity values used in analyzing labour mobility need to be tested because relevant studies on this topic are very limited in the literature. A series of sensitivity tests are to be carried out as our extensions to this paper.

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