UNIVERSITY OF OTTAWA

HYDRODYNAMIC RESISTANCE TO FALLING SPHERES IN LIQUID HELIUM II

by

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Submitted in partial fulfillment
of the requirements for the degree of
Master of Science.

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August 1962

ABSTRACT

The drag coefficient for a sphere falling freely in liquid helium has been measured in the range of Reynolds numbers, R, between 1.6×10^4 and 5×10^5. In the region from $R = 3 \times 10^4$ to 3×10^5 the values of the drag coefficient found for liquid helium are in good agreement with measurements made in other fluids. In particular the effective density of liquid helium II is found to be the total density ρ, in this range of R. The liquid is therefore behaving as a single fluid at these velocities. Measurements at higher Reynolds numbers than 3×10^5 are not considered reliable since it appears very doubtful that the spheres reached a terminal velocity. At Reynolds numbers below 3×10^4, corresponding to a velocity of 10 cm/sec, the experimental uncertainty has become too large for any significance to be attached to the results.

ACKNOWLEDGEMENT

I would like to express my gratitude to my supervisor, Dr. C.B. Benson, for his suggestion of the interesting research problem and his able guidance throughout.

I would also like to thank all the professors of the department and my student colleagues for their interest in this work and for their helpful discussions.

I am also grateful to all the technicians of the department for the construction of the cryostat.

Financial assistance through a National Research Council grant is also gratefully acknowledged.

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I. INTRODUCTION

In perfect potential flow, the hydrodynamic resistance to a sphere falling with a constant velocity is zero. In practice, that is in a normal fluid, this is not so and the resistance offered to the motion of a body is called hydrodynamic drag. At low velocities the drag is proportional to the velocity and the radius of the sphere (Stokes's law), whilst at higher velocities it is proportional to the velocity squared and to the radius squared, and it is usually written in the form $D = (1/2) C \pi \rho a^2 v^2$. The constant of proportionality, C, is called the drag coefficient, and is a function of a parameter called the Reynolds number. The graph of the drag coefficient as a function of the Reynolds number, R, is a universal curve for any sphere and for any fluid (fig.1). Near $R = 3 \times 10^5$ there is a sudden decrease in the drag coefficient; this is called the drag crisis, and it marks the onset of turbulence in the boundary layer.

Liquid helium below the λ-point (2.18 °K) is a quantum liquid, and as a consequence of this it has unusual physical properties. The most remarkable of the properties of liquid helium II is its superfluidity. Liquid helium II flows through very narrow gaps with an effective viscosity of less than 10^{-11} poise and independently of the pressure head; this vanishingly small viscosity should be compared with 2×10^{-5} poise for liquid helium I and 5.5×10^{-6} poise for helium gas at 1.64 °K (ref. 2). Using oscillating systems, the viscosity of liquid helium II is found to be much larger, of the ord r of 10^{-5} poise. An explanation of this dilemma is provided by the ' Two-Fluid ' theory, originated by Tisza (ref. 7) and London (ref.15), and developed by Landau (ref.8). On this theory, the liquid helium II may be considered as a mixture of a normal component and a superfluid component. The normal component is assumed to have a normal viscosity, while the superfluid has zero viscosity. The single 'Two-Fluid' theory is only valid for low velocity flow and it is found experimentally that above a certain critical velocity, energy may be dissipated in the superfluid. This energy dissipation takes the form of quantised vorticity, according to the theory of Feynman (ref.5), the energy being in the form of vortex rings or vortex lines in the superfluid. In addition to the dissipation of energy in the superfluid there is also, at high velocities, a force of interaction, or mutual friction, between the two fluids. In practice this means that at sufficiently high velocities liquid helium II may be treated as a single fluid, which is characterized by a definite kinematic viscosity, ν , and of density ρ equal to the total density of the fluid (see eg. Donnelly and Hallett, ref.11).

The drag force, proportional to the velocity squared and the radius squared, is also proportional to the density of the fluid. Since the superfluid behaves like an inviscid fluid, then the drag force at low velocities below the λ-point should be proportional to the density of the normal component ,

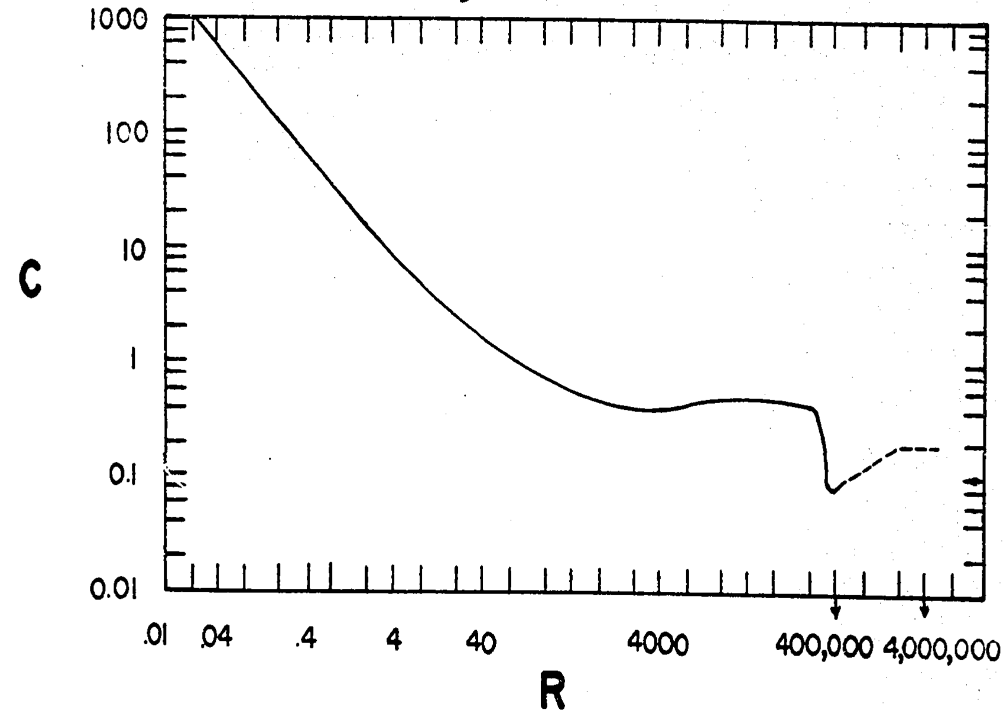

FIG. 1. DRAG COEFFICIENT FOR SPHERES AS A FUNCTION OF THE REYNOLDS NUMBER. (Ref. 1)

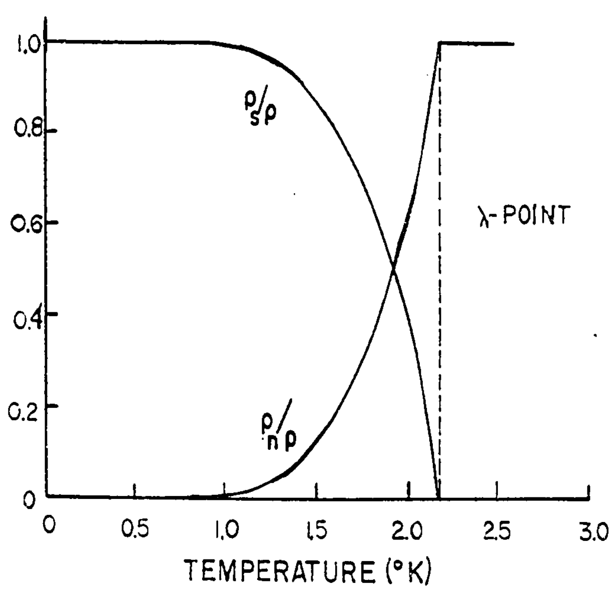

FIG. 2 THE RATIO ρ_n/ρ AS A FUNCTION OF TEMPERATURE (Ref. 2)

and at sufficiently high velocities it is expected to be proportional to the total density of the liquid. The following questions therefore arise: Will the drag coefficient curve for liquid helium II be similar to that for other fluids? What will happen to the drag coefficient when the sphere is falling at such low velocities (below the critical velocity) that the velocity field of the liquid helium can be separated into the velocity of the normal component of the liquid and the velocity of the superfluid component?

Two previous experiments have been reported on the drag coefficients of spheres in liquid helium. Dowley, Firth and Hallett (ref.16) measured the drag on spheres in a rotating vessel of liquid helium. They report drag coefficients lower by an order of magnitude than those found for normal fluids in wind tunnel experiments. Their Reynolds numbers lie in the range 10^3 to 10^4. Laing and Rorschach (ref.3) measured the terminal velocity of freely falling spheres at Reynolds numbers between 8×10^5 and 4×10^6, and reported drag coefficients higher than values found for other fluids by a factor varying between two and five at these Reynolds numbers. They also report other values determined by a different technique at Reynolds numbers, between 10^5 and 2×10^6, but only in liquid helium I at the normal boiling point The most remarkable feature of these latter measurements is the apparent absence of a drag crisis in liquid helium I at 4.2 °K. The significance of these results is discussed in section V.

In the present research, the terminal velocity of a sphere falling in liquid helium was measured by timing the transit of the sphere between two light beams, separated by a measured vertical distance. The drag coefficients and the corresponding Reynolds numbers have been calculated. The aim was to cover as wide a range of Reynolds numbers as possible. The results presented in this report cover the range of Reynolds numbers between 2×10^4 and 5×10^5. Between 3×10^4 and 3×10^5 the drag coefficient was found to be essentially constant and in good agreement with values reported for other fluids. There was no apparent difference between measurements in helium I and in helium II. The effective density of the liquid was taken as equal to the total density. Limitations in the present apparatus do not permit a thorough investigation of the drag crisis region when R is larger than 3×10^5. At the low Reynolds numbers ($R = 2 \times 10^4$), the error in the measured total contraction of the spheres produced a large uncertainty in the density of the spheres. For better results at these low Reynolds numbers, the error in the density of the spheres should be reduced to 0.001%.

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II. THEORETICAL STUDY

II.1. Hydrodynamic Resistance in Fluids.

While hydrodynamics based on an ideal, frictionless fluid predicts and explains many phenomena in real fluids, it fails completely when dealing with the hydrodynamic resistance, called drag, of bodies moving with a uniform velocity in a fluid (ref.4).

In an ideal fluid, two contacting layers of the fluid experience no tangential forces, that is, no shearing stress, when moved one relative to the other. In other words this perfect fluid offers no internal resistance to a change in shape. A calculation of drag, that is, of the force in the direction of motion, on a sphere due to inertia action in this completely frictionless fluid shows it to be (ref.4):

$$D = (2/3)\pi \rho a^3 \frac{dv}{dt} \qquad (1)$$

where a = radius of the sphere.

ρ = density of the fluid.

v = velocity of the sphere.

In case the sphere moves with constant velocity, that is:

$$\frac{dv}{dt} = 0$$

then the drag is zero, which is contrary to experiment. This defect of the theory arises from the fact that in a real fluid the tangential or frictional forces, connected with the property called viscosity, cannot be neglected, no matter how small they are.

At this point, it seems useful to define a parameter, the Reynolds number, which is very important in hydrodynamics because it is the criterion for mechanical similarity. The meaning of this is that it will tell us under what conditions a geometrically similar flow of a liquid or gas will occur around geometrically similar bodies. Assuming that all forces can be neglected except the inertia force and the frictional force, then the ratio of the inertia force to the frictional force is called the Reynolds number, and it is defined by the following relation:

$$R = \frac{va\rho}{\mu} \qquad (2)$$

where a = radius of sphere.

v = velocity of sphere.

ρ = density of fluid.

μ = coefficient of viscosity of the fluid.

Since the above is a ratio of forces, it is dimensionless. It is also useful to define the kinematic viscosity as:

$$\nu = \frac{\mu}{\rho} \qquad (3)$$

Thus, two flows about geometrically similar bodies (e.g. about two spheres) with different fluids, different velocities and different linear dimensions will be similar when the Reynolds number R is equal for both. This is the principle of similarity.

From experience, the motion of a fluid around a body has velocities of the same order of magnitude as the velocity at infinity. But very close to the body there is a large velocity gradient. The transition from zero velocity at the body to the velocity near the body takes place in a very thin layer, called the boundary layer (ref.6). Hence the field splits into two regions.

1. A boundary layer where the velocity gradient is very large and the shear stress cannot be neglected.

2. The region outside this boundary layer where the effect of viscosity is negligible. Hence the streamline picture is determined by the action of pressure and the picture is that of potential flow.

An important characteristic of the boundary layer is that when $R > 1$ a back flow takes place in it which leads to the creation of vortices and to a complete change in the flow pattern. Because of the influence of the boundary layer and of the vortices, the potential pressure distribution at the surface of the body is changed to such an extent that the resultant of the pressure forces becomes different from zero. The component of this resultant in the direction of the motion is the pressure drag. Hence the effect of viscosity is :

(1) there are frictional forces tangent to the surface of the body, the resultant of which is the friction drag.

(2) the viscosity causes a change in the geometry of the streamline picture which in turn causes a change in the pressure field and consequently leads to a pressure drag. When the Reynolds number is very small, the influence of the viscosity forces on the geometry of the motion and consequently on the drag becomes of much greater importance than the influence of inertia. The body pushes itself through the

fluid which is deformed by it. The resistance caused by this is due primarily to the forces necessary for the deformation of the various fluid particles. This is called deformation drag and is represented by Stokes's law:

$$D = 6\pi\mu va \tag{4}$$

The drag on a moving body is usually written in the form

$$D = \frac{1}{2} CA\rho v^2 \tag{5}$$

where C = drag coefficient.

A = area of the body projected in the direction of the motion.

ρ = density of the fluid.

v = velocity of the body with respect to the liquid.

It is found experimentally that the drag coefficient is not a constant but that it varies with the Reynolds number.

$$C = f(R) \tag{6}$$

The importance of the similarity principle given in equation (6) is very great in fluid mechanics. First, the dimensionless coefficients C and R are independent of the system of units. Secondly, their use leads to a considerable simplification in the extent of the experimental work. Supposing that it is desired to determine the drag coefficient for a sphere, then without the application of the principle of similarity it would be necessary to carry out drag measurements for four independent variables v, ρ, a , and μ, and this would constitute a tremendous programme of work.

It follows, however, that the drag coefficient for spheres of different diameters with different stream velocities and different fluids depends solely on one variable, the Reynolds number (fig.1).

The drag crisis, that is, the sudden decrease in the value of the drag coefficient at $R = 3 \times 10^5$, is due to the fact that the laminar boundary layer becomes turbulent before it separates from the sphere. The turbulent boundary layer can sustain considerably larger adverse pressure gradients without separation than a laminar boundary layer, owing to the turbulent eddy motions and to the consequent transfer of momentum. The fact that the boundary layer becomes turbulent causes the point of separation to move downstream. In turn, the dead area (the area behind the point of breaking away of the flow from the sphere) decreases considerably. Since the pressure drag of a body is determined primarily by the kinetic energy in the eddies of the wake, it is clear that a decrease of the size in this region causes a smaller resistance.

II.2. Hydrodynamic Resistance in Liquid Helium II.

Liquid helium is a very striking fluid because, to the best of our knowledge, it exists as a fluid down to absolute zero. The reason for this is the fact that the van der Waals forces in helium are weak and that the zero point energy is large. This keeps the atoms apart and hence it remains a fluid. Because of this it is called a quantum liquid. The Two - Fluid Model, proposed independently by

Tisza (ref.7) and Landau (ref.8) considers the fluid below the λ- transition (2.18 K) as a mixture of two fluids, each possessing separate properties. On this model helium II consists of a normal component and a superfluid component. Landau shows that the thermal energy will exist in the form of phonons (quantised sound waves) and rotons. These elementary excitations are the normal component, and the superfluid component is the "background" in which the excitations are embedded. In flow through narrow slits, the excitations (the rotons and phonons) collide with the walls and cannot pass through it; the superfluid flows through easily. At absolute zero, there are no thermal excitations, and all the fluid is superfluid. As the temperature is raised thermal excitations appear thus forming the normal component with an effective density $\rho_n < \rho$. Mathematically this can be described by the following relation:

$$\rho = \rho_n + \rho_s \qquad (7)$$

where ρ = total density of liquid He II.

ρ_s = density of superfluid component

ρ_n = effective density of the thermal excitations.

Also,

$\rho_s = \rho \qquad \rho_n = 0 \qquad T = 0$ K.

$\rho_s = 0 \qquad \rho_n = \rho \qquad T = \lambda$-point ($^\circ$K) (8)

The viscosity of the superfluid is vanishingly small. The normal component behaves as any other fluid with finite viscosity. Equation (7) can be rewritten as

$$\frac{\rho_n}{\rho} = 1 - \frac{\rho_s}{\rho} \qquad (9)$$

This function depends on the temperature (fig.2) and the following occurs: as the temperature is raised, excitations (phonons and rotons) are created thus increasing the concentration of the normal component and decreasing that of the superfluid.

The superfluid may be considered as a perfect inviscid fluid only for sufficiently low velocities. Hence for low enough velocities we expect the drag forces to be zero in the case of the superfluid, and the only contribution to the drag will be from the normal component. This will occur if the fluid is moving at velocities lower than a certain critical velocity, because above some finite velocity the superfluid begins to show a resistance to flow, and a force of mutual friction between the two fluids also comes into play. The drag contribution will then be from the normal component and from the superfluid. Hence measurements of drag in liquid helium II are of interest because they could tell us some of the differences in hydrodynamical behaviour between liquid helium II and other fluids. This could provide information on the validity of the law of similarity in liquid helium II and tell us how good a parameter is the Reynolds number for helium II. Also it could provide information on the critical velocity in the superfluid.

The drag on a sphere is usually written in the form

$$D = \frac{1}{2} C \pi \rho a^2 v^2 \qquad (10)$$

This equation shows that the drag is proportional to the density of the fluid. Below the critical velocity it is expected that the drag will be proportional to the density of the normal component. Therefore,

$$D = \tfrac{1}{2} C \pi \rho_n a^2 v^2 \qquad v < v_{s,c} \qquad (11)$$

because the superfluid offers no drag. However above the critical velocity, when the superfluid is no longer inviscid, the drag is expected to be proportional to the total density of the liquid. Hence,

$$(4/3) a^3 \pi (\rho_1 - \rho) g = \tfrac{1}{2} C \pi \rho_n a^2 v^2 \qquad (12)$$

$$C = 8/3 \left(\frac{\rho_1 - \rho}{\rho_n}\right) \frac{ag}{v^2} \quad \text{if } v < v_{s,c} \qquad (13)$$

$$\text{and } C = 8/3 \left(\frac{\rho_1 - \rho}{\rho}\right) \frac{ag}{v^2} \quad \text{if } v > v_{s,c} \qquad (14)$$

where $v_{s,c}$ = the critical velocity

ρ_1 = density of the sphere

g = acceleration due to gravity.

This is assuming that C is constant, which is approximately true from $R = 10^3$ to 3×10^5. The interesting relation is that of the drag coefficient as a function of the Reynolds number (fig.1). It is to be noted that equation (12) is valid if $R \gg 1$. For $R < 1$, we obtain Stokes's law:

$$D = 6 \pi \mu v a$$

and the drag is not proportional to the density of the fluid. Hence we are interested in the region $R \gg 1$ where the drag will be proportional to the density of the fluid.

The kinematic viscosity used in the Reynolds number (equation 2) is for the normal component if $v < v_{s,c}$, and if $v > v_{s,c}$ the total density is used to calculate the kinematic viscosity and hence R.

If measurements of this drag are made at various temperatures, its variation would give directly the relative fraction of the normal component in liquid helium II. Assuming that the drag coefficient is the same for liquid helium II as for liquid helium I at the same Reynolds number, then

$$D_T \propto \rho_n a^2 v^2 \qquad v < v_{s,c}$$

$$D_\lambda \propto \rho a^2 v^2$$

Therefore,

$$\frac{D_T}{D_\lambda} = \frac{\rho_n}{\rho} \tag{15}$$

where D_T = drag at any temperature below the λ-point.

D_λ = drag at the λ-point.

This provides us a means of determining the concentration of the normal component, if measurements are made below the critical velocity (ref.9).

II.3. Equations of Motion.

This research is concerned with the free fall of a sphere under the action of gravity and it involves measurements of the terminal velocity acquired by the sphere when the gravitational pull, the buoyancy of the fluid, and the drag are in equilibrium. Let us consider the equation of motion of the freely falling sphere in liquid helium II.

$$(m + b)\frac{dv}{dt} = m'g - (1/2)C\pi\rho a^2 v^2 \tag{16}$$

where m = mass of sphere

b = carried mass of the liquid (see section II.1.)

m' = effective mass of sphere in the fluid.

Integrating equation (16) we obtain the velocity of the sphere at a time t.

$$v = v_0 \tanh qt \qquad (17)$$

$$\text{where } v_0 = \left(\frac{2m'g}{C\pi\rho a^2}\right)^{\frac{1}{2}} \qquad (18)$$

$$q = \left(\frac{m'g\frac{1}{2}C\pi\rho a^2}{m+b}\right)^{\frac{1}{2}} \qquad (19)$$

The term v_0 is called the terminal velocity. Integrating equation (17) with respect to time, we obtain the distance travelled by the sphere after a time t.

$$x = \frac{v_0}{q} \ln \cosh qt \qquad (20)$$

Combining equation (17) and equation (20), a relation between the velocity and the displacement is obtained:

$$v^2 = v_0^2 \left(1 - e^{-\frac{qx}{v_0^2}}\right) \qquad (21)$$

This is true provided the drag coefficient C is constant.

III. Description of the Apparatus.

III.1. The Cryostat.

III.1.1. Remarks.

We are interested in the relation between the drag coefficient C and the Reynolds number R. According to equation (14) all that we need to measure basically, in the cryostat in order to determine C, is the terminal velocity v and the corresponding temperature. The cryostat was designed bearing this in mind.

The cryostat was built (fig.3) in such a way as to perform the following functions:

(1) to determine the velocity of the falling sphere.

(2) to verify if the terminal velocity is reached.

(3) to have an indication whether the sphere was falling straight.

(4) to drop the sphere as many times as required and from variable positions.

(5) to store the sphere and to use others of different density.

(6) to vary the temperature of the liquid helium and to keep it constant at any particular value.

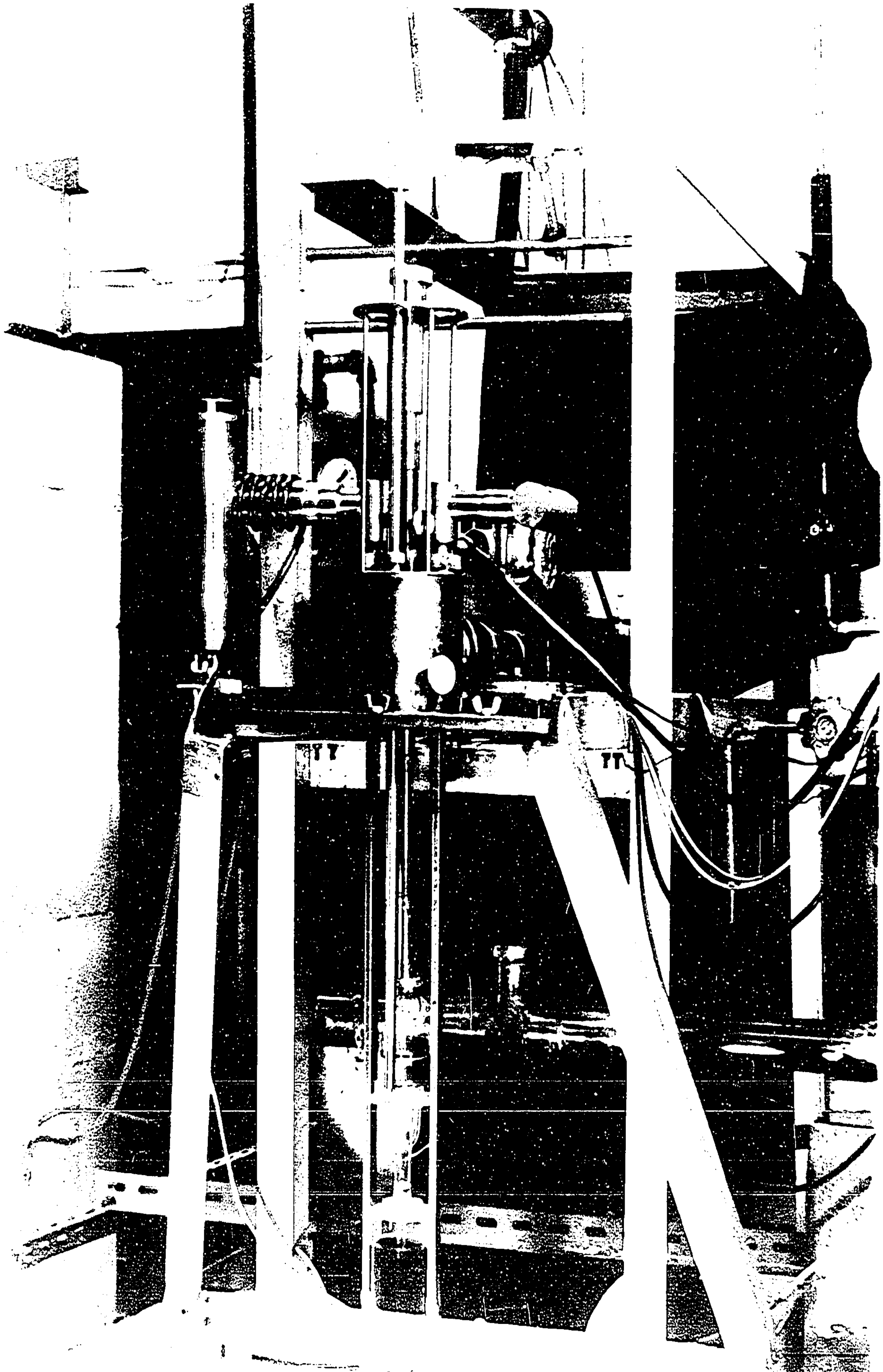
(7) to retransfer more liquid helium without stopping the run.

(8) to drop eight spheres in a single run.

(9) to study the wall effects on the results.

(10) to measure the temperature of the liquid helium bath.

FIG.3. THE CRYOSTAT.



III.1.2. Velocity of the Falling Sphere.

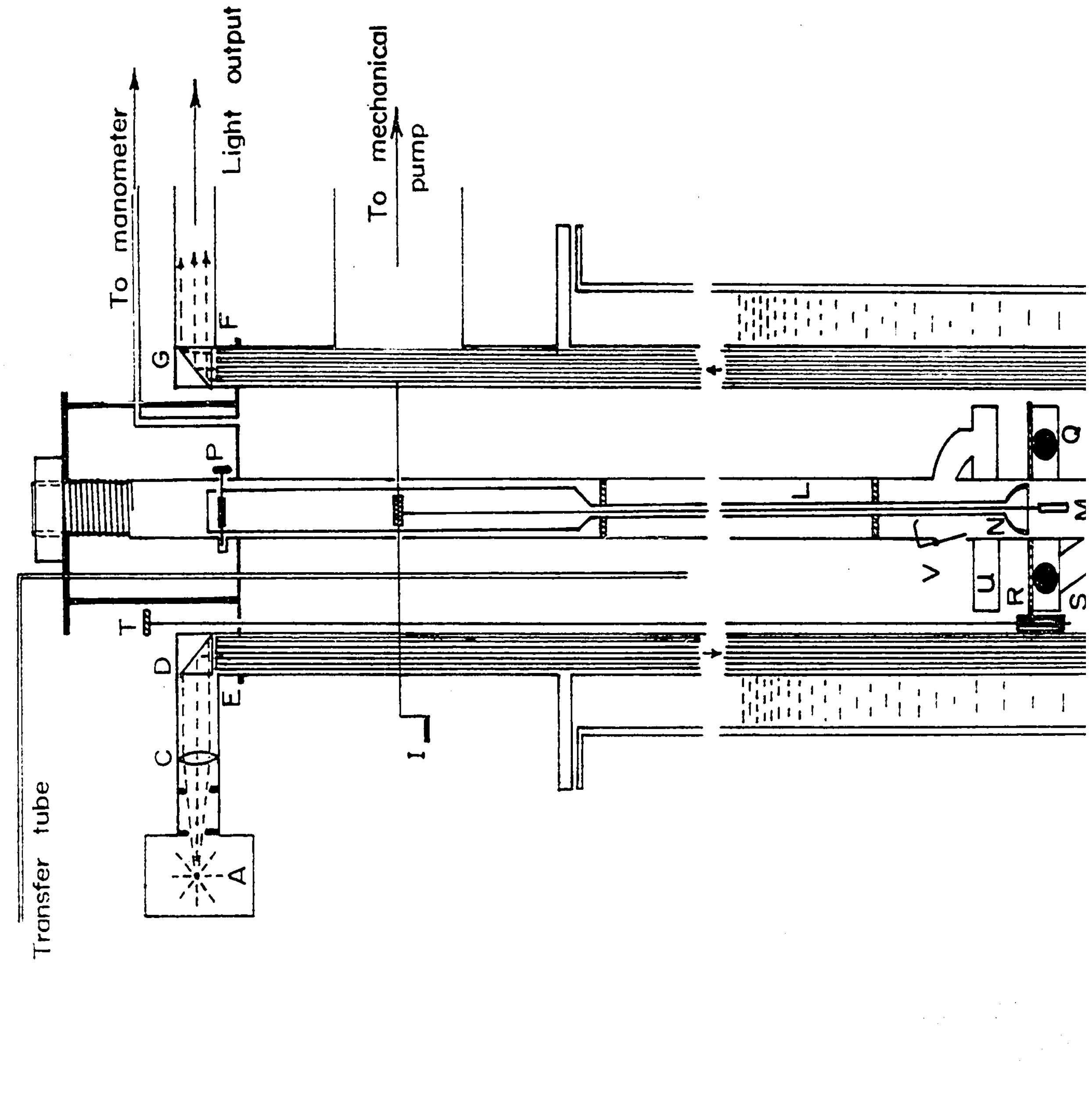
The velocity of the falling sphere is calculated from the relation:

$$v = \frac{\Delta s}{\Delta t} \qquad (22)$$

where Δs is the distance between two fixed points and Δt is the time taken to travel between these points

A Pointolite source (A) is placed at the focus of an achromatic doublet (C) in order to obtain a collimated beam of light (figure 4). By means of a prism (D) this beam of light is sent down into four light guides (E), made from Plexiglass, each one being of different length. The end of each light guide is cut at 45°; this will cause total internal reflection to the light beam, and now each light beam will come out perpendicularly to the light guides Since the light guides are placed parallel to the path of the falling spheres, the light beams coming out of the light guides will be perpendicular to the path of the sphere. Another set of light guides (F) is placed opposite the first set in such a manner that the light beams, emerging from the latter, are picked up by these light guides. After being internally reflected again, they are sent through a prism (G) and into a photomultiplier.

Now, as the sphere is falling it will cut the first beam of light; this will produce a pulse. As the sphere keeps on falling it will cut the next beam of light, producing another pulse. If the distance between these two beams of light is known and if the pulses produced are used


To manometer
Light output
To mechanical pump
Transfer tube
A
C
D
E
F
G
I
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M
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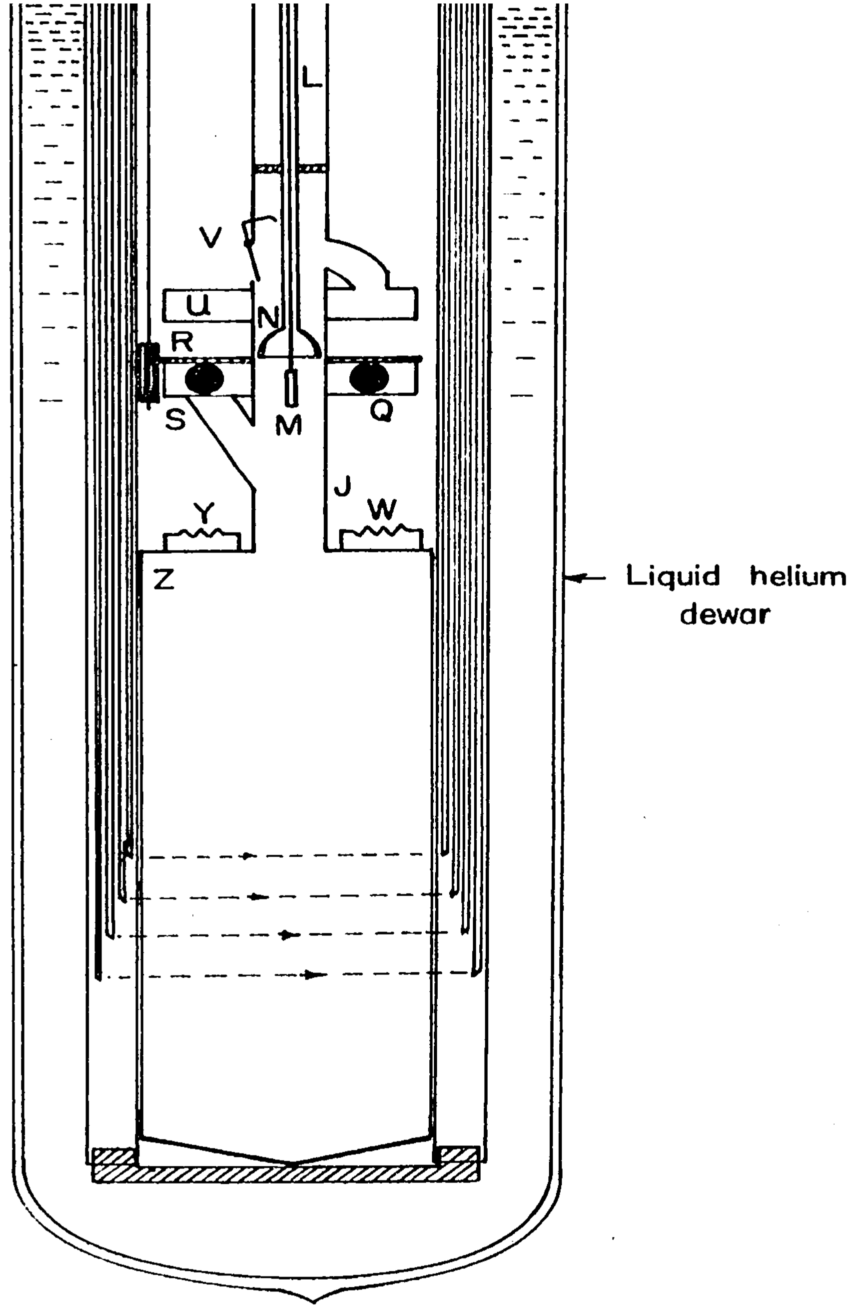

Fig. 4. Sketch of the cryostat

to tell us the time of fall between these two light beams, then the velocity of the falling sphere can be determined using equation (22). In the same way the velocity is determined with the next two light beams.

Now the thickness of the light guides is 1.6mm. Since the sphere is about 10mm in diameter, this will produce slow-rising pulses. To overcome this, the light guides were placed in a stainless steel rectangular tube with wide slits opposite the place where the light will come out from the input side, and where it will come in at the output side. Then, very fine slits (0.1mm by 16mm) were made in a rectangular piece of brass in such a way as to be able to slide on the stainless steel tubing and to be fixed to it at the desired place in front of the beams of light. A slit like this was made for each light beam, for the input side and for the output side. Thus the light beams had a width of only 0.1mm and since the diameter of the sphere was 1cm a fast-rising pulse was obtained.

This arrangement caused trouble. It was found that the relative contraction between Plexiglass and stainless steel was very important at liquid air and liquid helium temperatures. This caused the slits to move in such a way as to not be opposite the beams of light at these low temperatures.

To overcome this, the following alteration was made. A strip of Plexiglass was placed along two opposite sides of the rectangular stainless steel tubing containing the light guides. These strips were fixed to this tubing at one end only, the end closest to the prism. New slits were made but to tell us the time of fall between these two light beams, then the velocity of the falling sphere can be determined using equation (22). In the same way the velocity is determined with the next two light beams.

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The reason for having four beams of light is to verify if the terminal velocity is reached. The first two beams of light give one velocity, the next two beams give another velocity (fig.5). If these velocities are the same or approximately the same, then the velocity of the sphere is constant, that is, the terminal velocity is reached.

This arrangement can tell us if the sphere is falling straight, or whether it has completely swerved away from its vertical course. If the sphere is falling straight, or almost straight, then it will cut the four beams of light in one fall, and produce four pulses. The width of each light beam is such that if the sphere swerves away from its straight course, then no pulse will be produced unless it remains in the plane of the light beams. Hence, if there are less than four pulses it mean that the sphere has gone astray.

III.1.3. <u>Electronics Used in Velocity Measurements.</u>

The falling sphere produces 4 negative pulses by cutting the beams of light. We are interested in the time between these pulses. The pulses of light are fed into a photomultiplier. The output goes to an amplifier, an

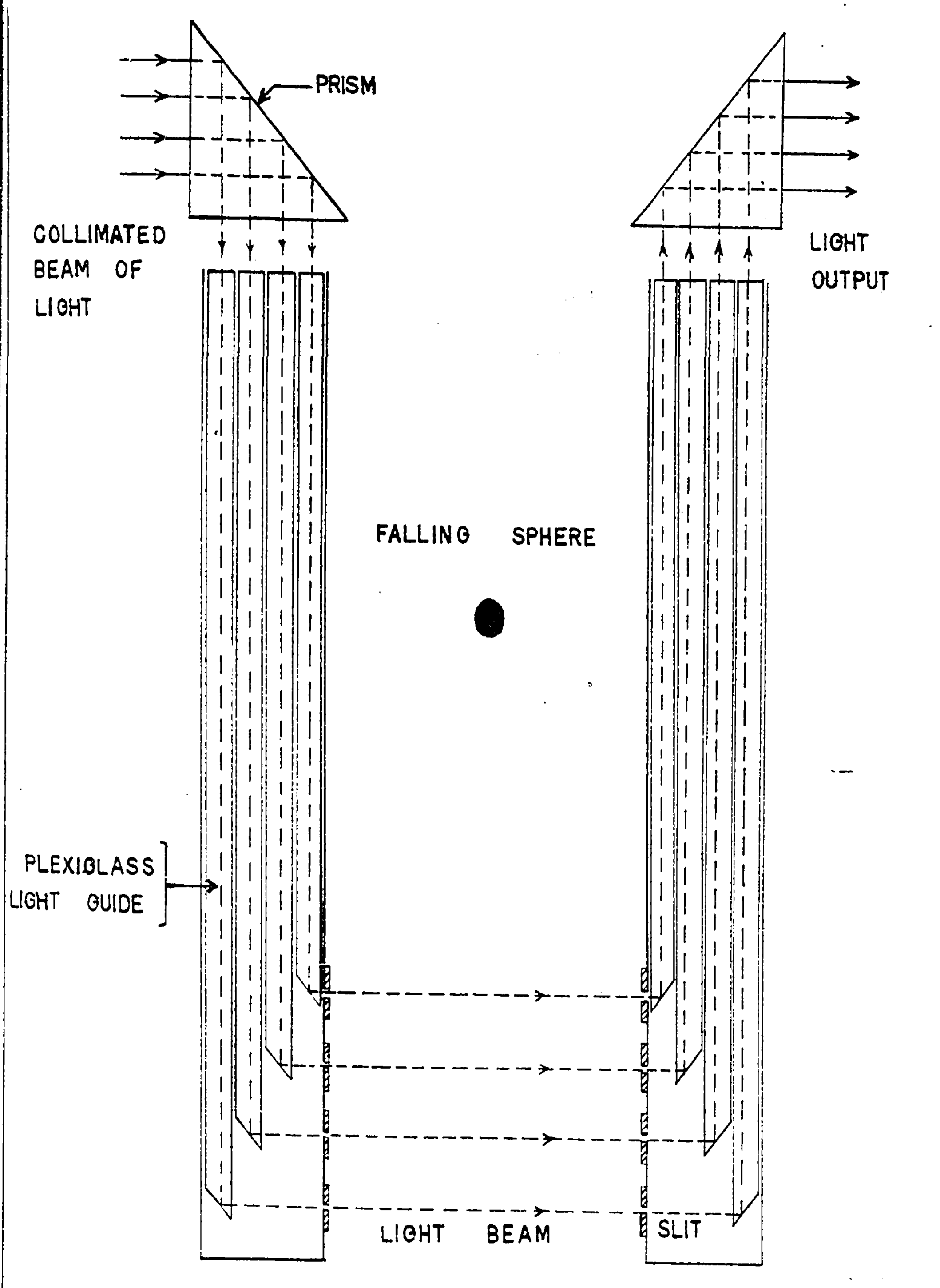


FIG. 5. SKETCH SHOWING THE PRINCIPLE BY WHICH THE VELOCITY OF THE SPHERE IS DETERMINED.

oscilloscope, a timer and a marker generator (figure 6).

The first pulse triggers the counter and the second one stops it. In order to have a positive visual indication of the points on the waveform at which the counter is starting and stopping its time-interval measurements, a marker generator circuit is used. This marker is used in the following way: the pulses under measurement are connected to the counter and to the vertical deflection system of the oscilloscope. The marker circuit is inserted between the counter and the Z-axis modulation terminal on the oscilloscope. The points at which the counter is started and stopped are shown as bright spots on the waveforms, seen on the oscilloscope.

If the display time on the counter is set at zero, then two possible things can happen. If the sphere is falling down slowly, then the first two beams of light give one time interval which is displayed on the counter. The next two beams of light give another time interval which erases the previous reading and displays itself on the counter. But since the sphere is falling slowly there is sufficient time to read both displays on the counter. If the spheres are falling fast, there is no time to read the first time interval, and only the second time interval, which now remains displayed, can be read. By setting the display time to infinity, the first time interval only is read.

The measurements from the counter give us the time interval Δt. Using a cathetometer, the distance interval Δs is measured at room temperature. Corrections will be necessary for contraction at helium temperatures. Now using

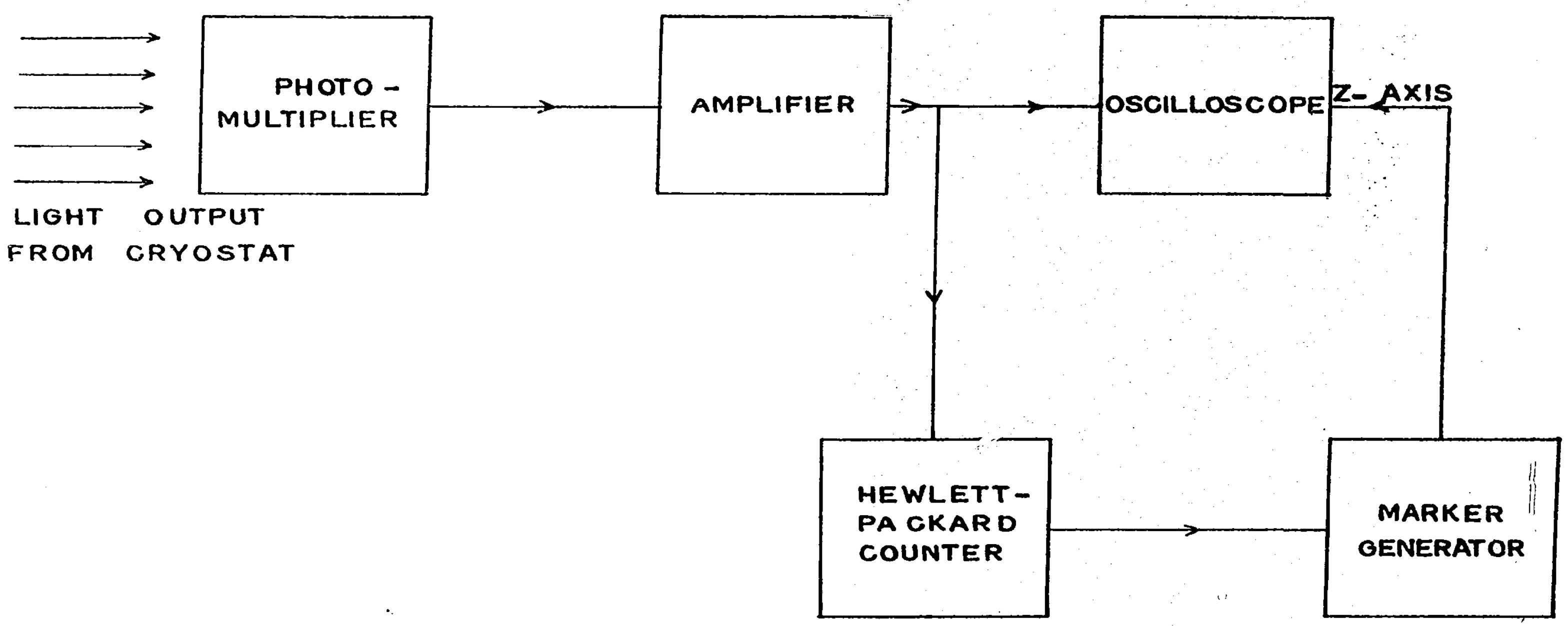


FIG. 6. ELECTRONICS FOR MEASURING THE VELOCITY OF THE FALLING SPHERE IN LIQUID HELIUM.

equation (22), the velocity can be calculated.

III.1.4. Releasing the Sphere.

All the spheres used are magnetic. This provides a means of picking them up. The fallen sphere is picked up by a small magnet (M) attached to a silk thread (fig.4). A little crank (I) located in the head of the cryostat raises this up. The thread passes in a stainless steel tube (L), which has at one of its ends a funnel-shaped opening (N) made from Plexiglass. As the thread and magnet with the sphere are cranked up, the magnet, being small, will pass through this funnel-shaped opening and will go into the tube, but the sphere will be retained by this opening. As soon as the magnet is pulled higher up the sphere will fall. By means of the crank, the magnet can be lowered to pick up the sphere, and the same thing is repeated. This permits the sphere to be dropped as many times as required.

The tube (L) can be lowered or raised by means of a rack and pinion, the latter being fixed in the head of the cryostat (P). This permits the sphere to be released from any desired height, since the tube (L) can be moved about 20 cm.

III.1.5. Storage.

Eight spheres are stored initially in a container, made from Plexiglass which has a separate compartment for each sphere (Q). To the top of this compartment is fixed a gear. This is arranged in such a way that the tube (J) passes through its centre. This compartment rests on a piece of Plexiglass (S), fixed to the tube (J). Another gear (R) is so placed that it engages into the gear fixed to the sphere compartment. By means of a rod (T) which goes from the head of the cryostat to the gear (R), the sphere compartment can be made to turn around the tube (J). At one place in the Plexiglass (S) there is an opening and a passageway to the tube (J). Now the spheres are in the compartment (Q); turning the rod (T) the compartment turns also until a sphere is just above the opening in (S), where it falls down into (J), and from there down, past the light beams, to the bottom of the cryostat. Measurements of the terminal velocity of the sphere can now be made. When enough measurements are taken on this sphere, the sphere can be stored away in the following manner. A circular box (U) is attached to the tube (J). An opening and a passage way is made leading from the tube (J) to this box. Opposite this opening in the tube is a little swinging trap door (V). By raising (L) and the sphere with the magnet, the trap door will swing, when the sphere passes it, in such a way as to block the tube and to guide the sphere into the opening, and from there it will fall into the box made for its storage. By lowering the tube (L), the trap door will swing back to its original place leaving the tube unblocked.

This allows us to make measurements on eight spheres, each of different density, in a single run.

These measurements may require a long time and the helium level in the cryostat will probably be low. More helium can be siphoned into the cryostat by means of a transfer tube, which has a valve at its lower end. The liquid helium storage flask is connected to the cryostat during the run.

III.1.6. Temperature Control.

The temperature is controlled by an a.c. Wheatstone bridge. One arm of the bridge is a carbon resistor (Y) in the cryostat. The off-balance current is amplified and sent to a heater (W) in the liquid helium bath. This will tend to raise the temperature of the helium bath until the Wheatstone bridge is balanced. The pumping rate on the helium bath is adjusted manually to give a suitable input voltage to the heater. The off-balance amplified voltage is seen with an oscilloscope (figure 7).

The temperature of the liquid helium bath is found from the vapour pressure of the liquid which is measured with a mercury manometer and an oil manometer (ref.10).

III.2. Spheres.

III.2.1. Requirements.

We are interested in measurements of the drag in the region of Reynolds number $R \gg 1$, where the drag is proportional to the density of the fluid. The kinematic viscosity of liquid helium is approximately 10^{-4} cgs units (ref.11). This means that (va) should lie somewhere between 1 and 10 cgs units. It is expected that the critical velocity is of the order of 1cm/sec. If we choose spheres of very small diameter this will throw us in the Stokes region. Hence we choose $a=\frac{1}{2}$cm.

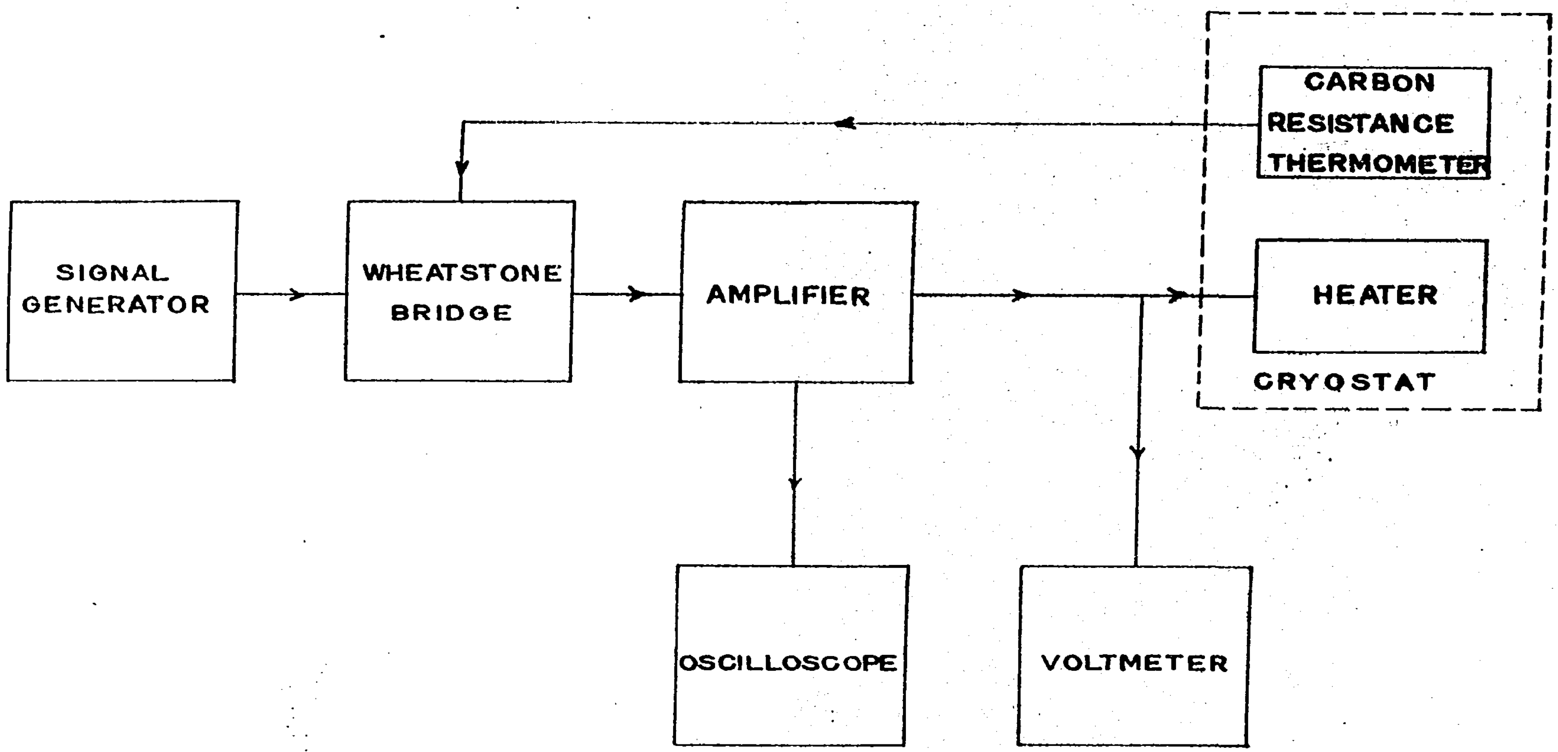


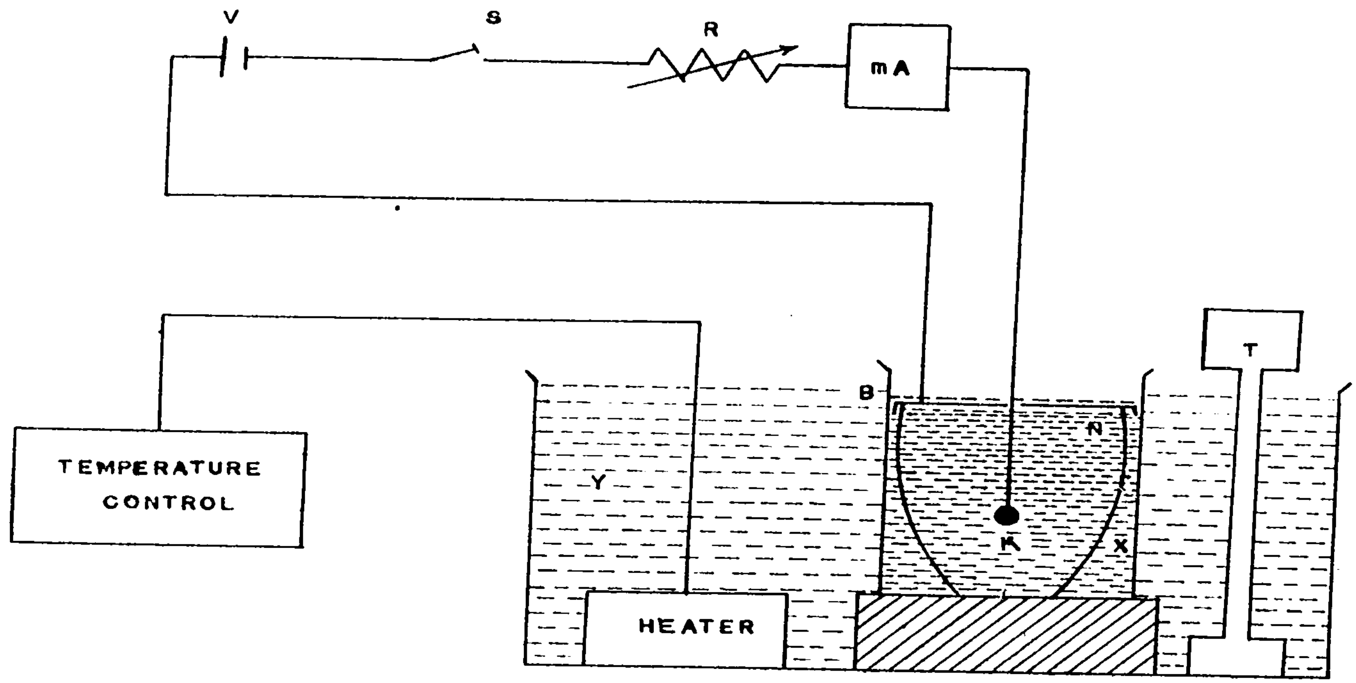
FIG. 7. TEMPERATURE CONTROL FOR THE LIQUID HELIUM BATH.

In order to obtain low velocities, the spheres must be of small density, that is, they must be just a little denser than liquid helium which is about 0.145 gm/cm^3. To achieve this the sphere must be hollow. For this the spheres with a radius $a=0.5$cm must have a mass from about 0.075gms up. This presents some difficulties. One of them is that the walls of the spheres will be very thin. The superfluid having a viscosity less than its own gas will pass through any small pore, and thus increase the density of the sphere. This means that the spheres must have very thin walls and at the same time they must be non porous to the superfluid.

III.2.2. Making the Spheres.

It was found that some beads from a cheap necklace were sufficiently spherical (0.5%). The beads were cleaned and then silvered by a chemical reduction process to give them a conducting surface. In order to make them magnetic, they were nickel-plated. To improve the quality of the deposit, the nickel-plating was done in a constant temperature bath and at 50°C.(fig.8). The plastic was dissolved in a solution of amyl acetate; a hollow nickel sphere was left. However, the nickel is probably porous to the superfluid. To overcome this, the sphere was coated with an epoxy resin and cured; this was repeated a few times.

There was no guarantee that the spheres were leakproof. In order to check this, they were dropped in liquid helium II. By means of a small magnet each one was lifted above the level of the liquid helium II. The ones that leaked



T = STIRRER
V = 1.5 VOLT BATTERY
R = RHEOSTAT
S = SWITCH

B = BEAKER
N = NICKEL BEAKER
X = NICKEL SULPHATE SOLUTION
K = SPHERE
Y = CONSTANT TEMPERATURE BATH

FIG. 8. APPARATUS FOR MAKING NICKEL SPHERES.

badly dripped for a few seconds, the ones that had small leaks blew up when taken out from the Dewar; this was caused by the superfluid which was trapped inside the sphere and which changed to helium gas, and the great overpressure did the rest. The spheres that passed this test were used. Also, during each run the spheres were tested for leaks by this method.

IV. EXPERIMENTAL STUDY

IV.1. Determination of Terminal Velocities.

The terminal velocity of the spheres was measured in the following manner. Eight spheres were loaded into the cryostat and, after pumping on the system for a few hours at room temperature in order to get rid of most of the water vapour and air, liquid helium was transferred. The temperature control was set at some particular temperature, usually below the λ temperature, and, after equilibrium was reached, the first sphere was dropped and recovered. It was found necessary to wait for about ten minutes between drops, otherwise the sphere would swerve from its straight downward path and no pulses would be produced. This was repeated several times at the same temperature, the time of fall between the beams of light being registered by the counter. The temperature was changed and the measurements were repeated. The first sphere was stored away, as described in section (III.1.5), when sufficient measurements had been taken with it, and the process was repeated with the next sphere.

The distance between the slits was measured at room temperature by means of a cathetometer. Because all the measurements were done at liquid helium temperatures and because this means a change of roughly 300 degrees from room temperature, the corrections for the thermal expansion of the materials cannot be neglected. The slits are fixed to Plexiglass strips and so the corrections must be applied to the Plexiglass. The coefficient of expansion for Plexiglass is taken from the work of Giauque (ref. 13). For the distance, $\Delta s = 1.8cm$, the correction is 1.0% The velocity of each sphere was then calculated using equation (22).

For each sphere it was necessary to verify whether terminal velocity had been reached; this was done by means of the two pairs of slits, as described in the previous section. The velocities given in table I (Appendix) are the average of between 5 to 10 separate measurements in each case.

IV.2. Determination of Sphere Densities.

The room temperature density of each sphere was calculated directly from measurements of its mass and diameter. The mass of the sphere was measured on an automatic balance reading to 0.01 mgms. The diameter used was the average of about forty different readings on each sphere; the scatter was 0.5%. The density of the sphere at liquid helium temperature was found by correcting the room temperature density for the total contraction of the sphere on cooling to $4.2^{o}K$.

It was necessary to measure this total contraction, in a separate experiment, since the sphere was made of a double layer of nickel and epoxy resin for which no value would be given in the literature. The contraction of the spheres was observed in an unsilvered Dewar vessel, measurements of the diameter of the sphere being made from outside by means of a cathetometer reading to $\pm 0.001cm$. An average of the measurements gave a total contraction of (0.9 ± 0.1) %. Since all the spheres were made of the same material the correction was applied to the diameter of each sphere.

While this gives a relatively unimportant correction to the radius of the spheres, it becomes very important in determining the value of $(\rho_1 - \rho)$ in equation (14), particularly for the lighter spheres. In fact, in the case of the lightest sphere the uncertainty in $(\rho_1 - \rho)$ can be as much as 40% at lower temperatures and up to 1000 % at temperatures very close to the λ temperature. For this reason the calculated drag coefficients with the lightest sphere have been omitted from figure(9). It is hoped that the total contraction of the spheres will soon be obtained with greater accuracy.

IV.3. <u>Drag Coefficient and Errors</u>.

Using the terminal velocity determined in section IV.1., the drag coefficient was calculated by means of equation (14), involving the total density of the liquid helium II. The densities used for liquid helium II were taken from the work of Kerr (ref.12). The drag coefficient was then plotted against the Reynolds number (fig.9). The dotted curve on this graph corresponds to the drag coefficient curve for other fluids, taken from reference (1).

The main source of errors arises from the fact that the terminal velocity varies with each fall. Because of this, from 5 to 10 measurements were made for each sphere, and the average of these velocities was taken. The deviations from the mean vary up to 5%. Since the drag depends on the velocity squared, this will cause a maximum error of 10% in the drag coefficient. These deviations in the velocity are

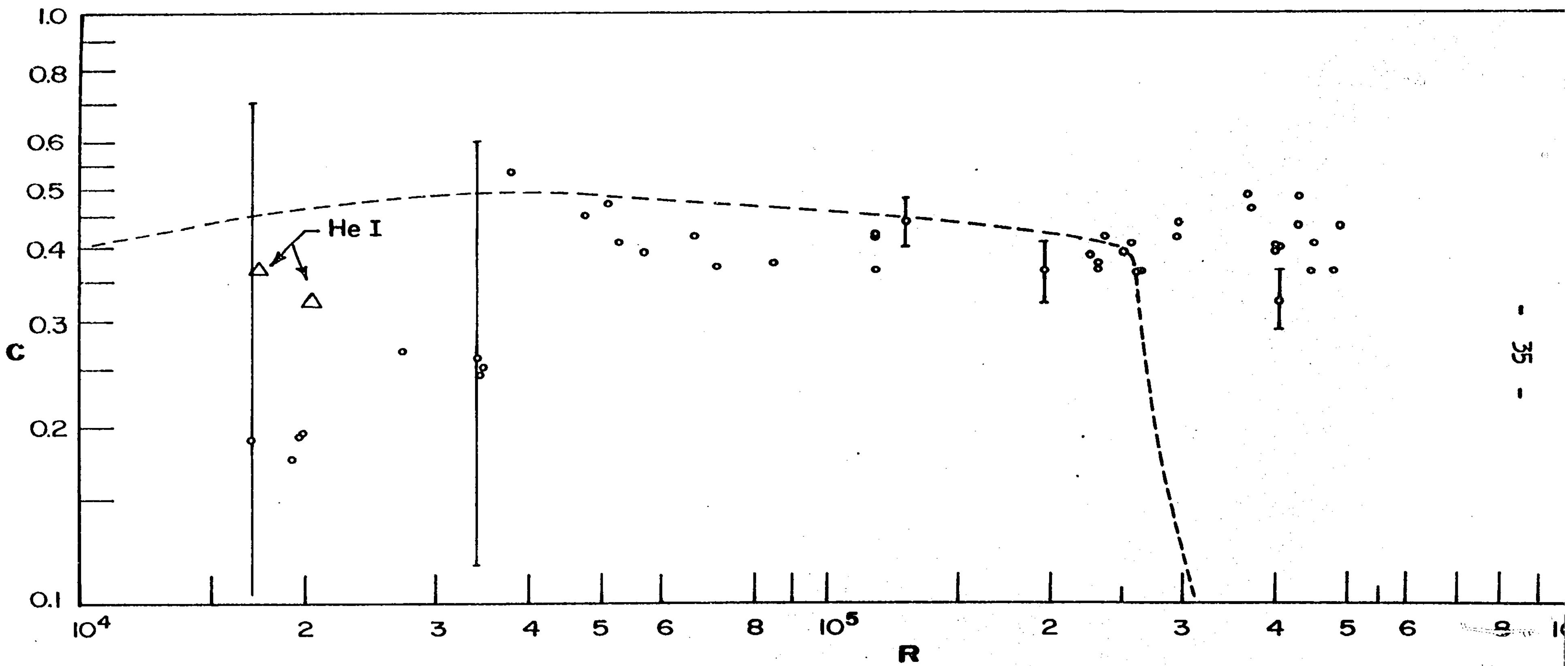


Fig. 9. DRAG COEFFICIENT AS A FUNCTION OF REYNOLD NUMBER IN LIQUID He II COMPARED WITH THE CURVE FOR OTHER LIQUIDS.

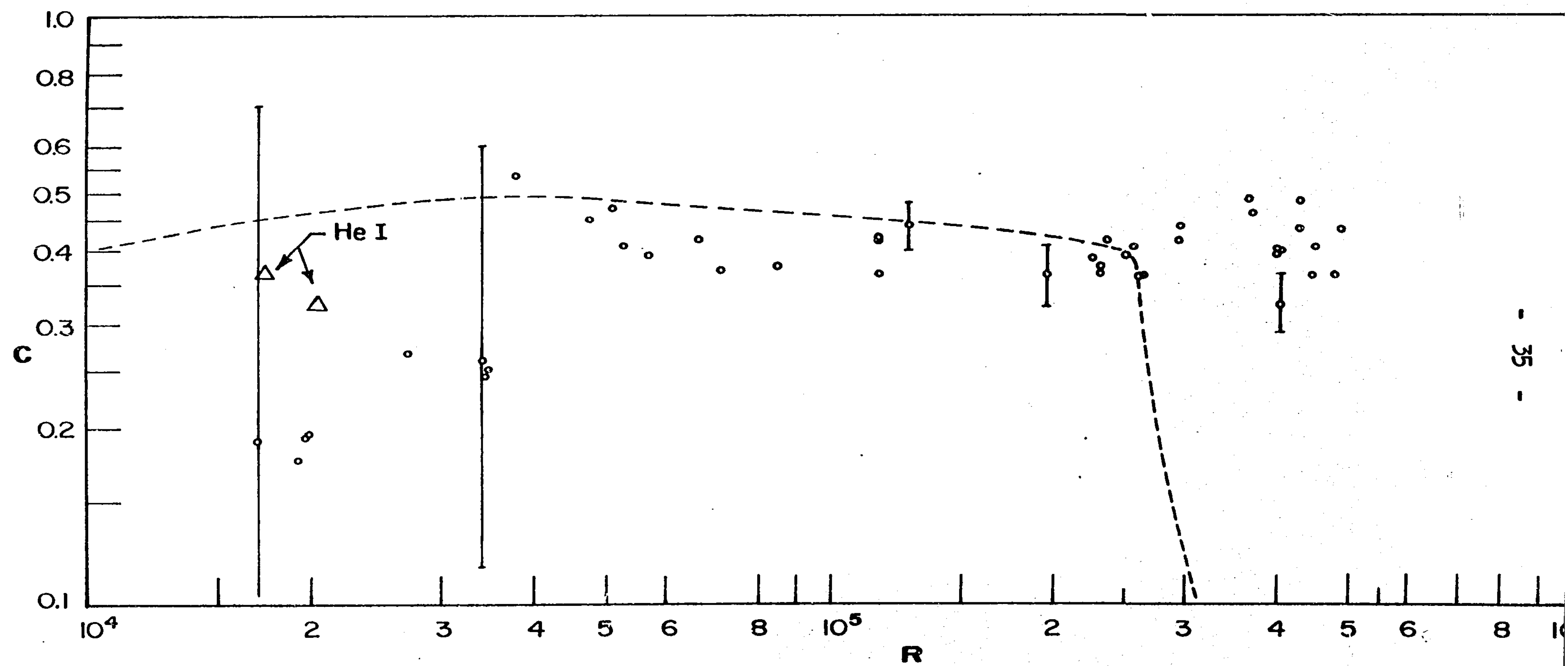


Fig. 9. DRAG COEFFICIENT AS A FUNCTION OF REYNOLD NUMBER IN LIQUID He II COMPARED WITH THE CURVE FOR OTHER LIQUIDS.

due to the fact that the vortices, created behind the sphere, leaves it in a more or less irregular fashion (ref.17). The lateral movements of the sphere are accompanied by changes in the vertical velocity. This may be the cause of the slightly curved and spiral path, which is always followed by a sphere falling through liquid.

The temperature of the bath is kept constant to 0.001 deg. K. The determination of the temperature from the manometer system gives a temperature reading [illegible] accurate to 0.001 deg. K. The density of the liquid helium (ref.12) is accurate to 0.01%.

These errors accumulate to a maximum error of 11% in the drag coefficient. This is true for all the spheres used except for the two lightest ones, where the errors become extremely large, for reasons discussed previously.

When (va) is plotted against the downward force D, a straight line is obtained (fig.10). This graph shows that the drag coefficient is constant in the range of Reynolds numbers of this experiment.

IV.4. <u>Reynolds Number and Errors</u>.

For each drag coefficient the corresponding Reynolds number was calculated according to equation (2) using the terminal velocity of the sphere and the radius of that sphere. The kinematic viscosities are taken from a paper by Donnelly and Hallett (ref. 11) in which a number of different experiments have been analyzed, most of periodic boundary layer

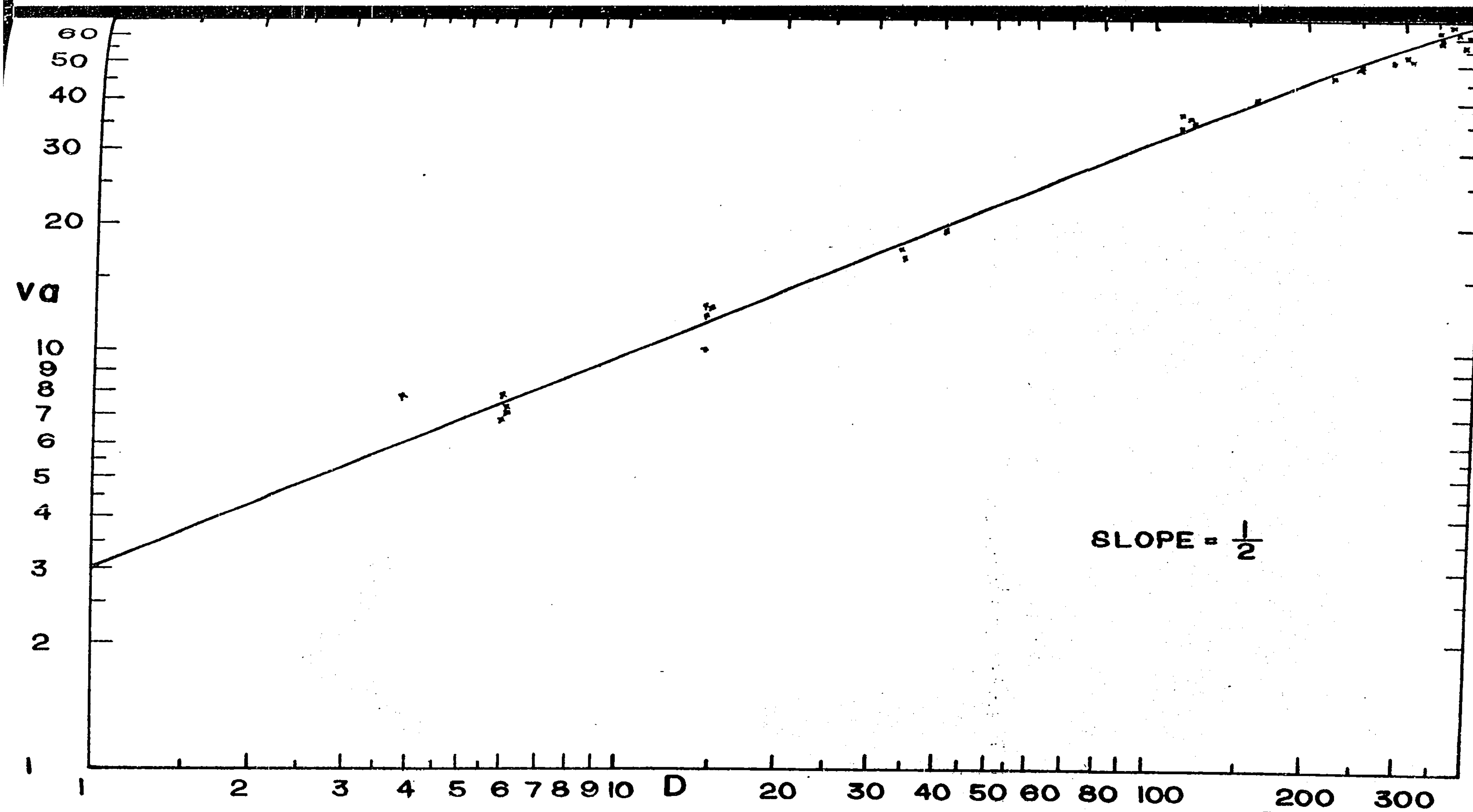


FIG. 10 GRAPH SHOWING THAT D IS PROPORTIONAL TO $(va)^2$ IN LIQUID HELIUM.

experiments, and the kinematic viscosities calculated for that part of the data where it is believed that liquid helium II is behaving as a single fluid. It is assumed here that these kinematic viscosities are applicable to the present experiment in the region where single fluid behaviour is found.

Since the terminal velocity also enters into the calculation of R the uncertainty in the quoted Reynolds numbers will be due to the uncertainty in the measured terminal velocity and the uncertainty in the values of the kinematic viscosity calculated by Donnelly and Hallett. This may amount to a possible error of ± 10% in the Reynolds numbers.

IV.5. Wall Effects.

The variation of the drag coefficient with Reynolds number for a sphere shown in figure (1) is that found when the effect of the walls of the containing vessel is negligible. It had to be determined what effect, if any, the walls of the plastic cylinder (diameter 6.6 cm) had on the measured drag coefficients of the spheres in this experiment (diameter of spheres is 1cm). No theoretical formula for this effect is known which has any validity in the range of Reynolds numbers above about 10^3. It was therefore decided to make an experimental study of the effect of the walls. A set of four transparent plastic cylinders was made to fit inside the original cylinder, the diameters being 5 cm, 4cm, 3cm, and 2.5cm respectively. An experimental run was made with each of these smaller cylinders. The drag coefficients from these runs are shown in figure (11), where it may be seen that the drag

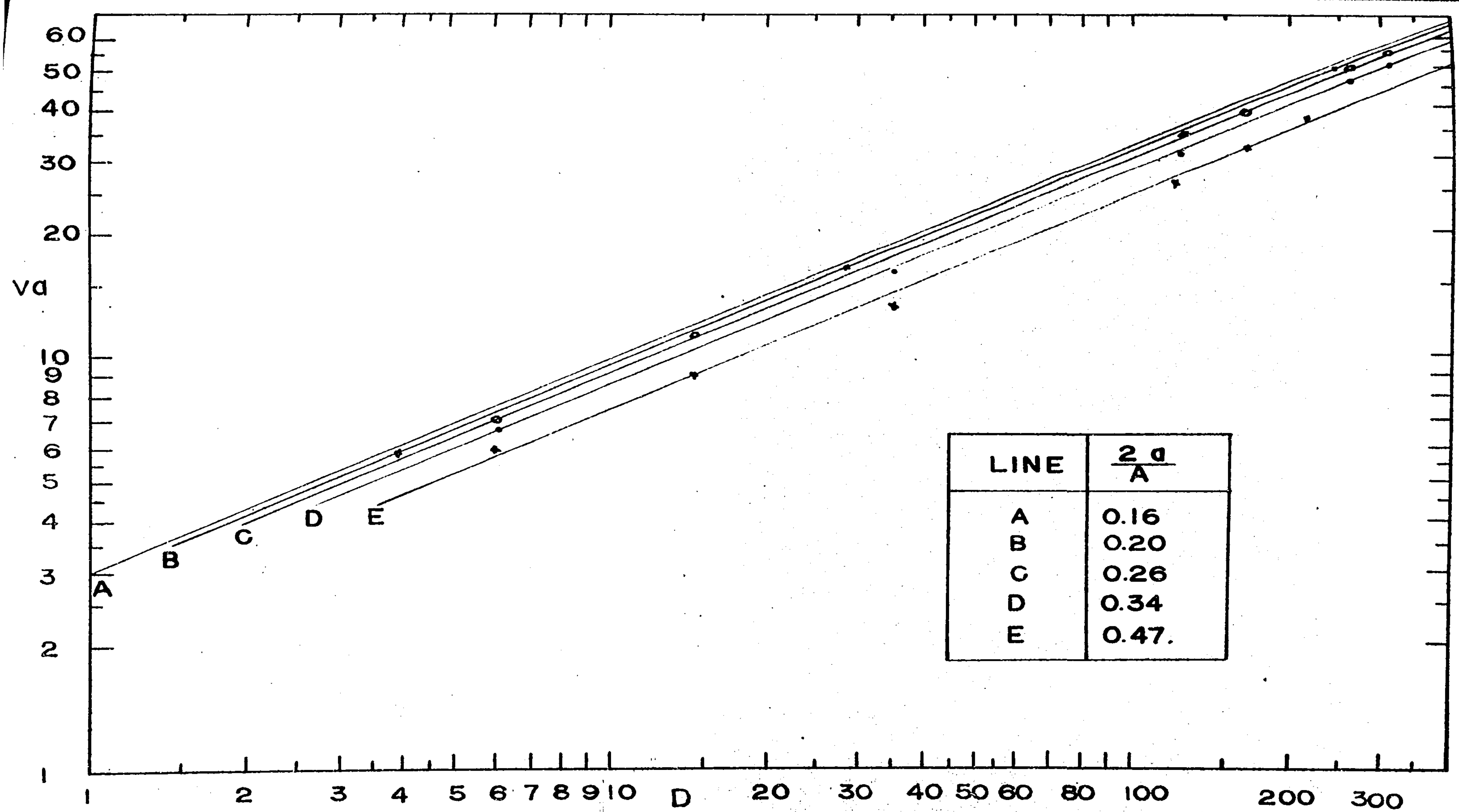


FIG. II WALL EFFECTS.

coefficient is affected by the ratio of the sphere's diameter to the diameter of the cylinder($\frac{2a}{A}$). The fact that the lines are parallel in this graph shows that only the drag coefficient is affected by the walls. Line A corresponds to the results obtained in the usual cylinder (6.6 cm) (see also figure 10). The values of the drag coefficient were calculated for the 6.6 cm cylinder ($\frac{2a}{A} = 0.16$), no extrapolation being made to the case of an infinite bath ($\frac{2a}{A} \to 0$). The necessary correction would seem to be only about 5 % from a few values extrapolated to $\frac{2a}{A} = 0$ graphically.

V. DISCUSSION OF RESULTS

V.1. Results in the Range of R from 3×10^4 to 3×10^5 .

The experimentally determined drag coefficients, plotted as a function of Reynolds number (fig.9), are in agreement with the drag coefficient curve found for normal fluids between the Reynolds numbers 3×10^4 and 3×10^5. Since the drag coefficients are calculated from equation (14) using the total density of the fluid as the effective density, it is clear that liquid helium II under these conditions is behaving as a single fluid. In this range of Reynolds numbers the drag coefficient is effectively constant so that the drag force is proportional to $(va)^2$. This is shown very clearly in figure (10) where the drag force is plotted against (va) on a log-log plot. The slope of the best straight line through the experimental points is $1/2 \pm 6\%$. Experimental points for both liquid helium I and liquid helium II are included on this graph. The single fluid behaviour in helium II has also been found in other experiments at sufficiently high velocities, see eg. Donnelly and Hallett (ref.11) for a discussion of some of them.

The measurements of Laing and Rorschach (ref.3) on the drag coefficients for a sphere in liquid helium I at 4.2°K were made using a counterbalance sphere. Their drag coefficients are higher by a factor of two or three than those found in the present experiment, and those found for all other normal fluids. It seems likely that some additional frictional term

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V.2. High Reynolds Numbers Measurements.

The apparent absence of a drag crisis at a Reynolds number of about 3×10^5 is puzzling although Laing and Rorschach (ref.3) also report this. It is possible that the true terminal velocity has not been reached in these measurements at high Reynolds numbers. If this is so then, the measured velocity being too small, a drag coefficient will be calculated which is greater than its actual value. The same arguments will apply in the experiment of Laing and Rorschach. There seems to be no reason why the drag crisis should not occur, particularly in helium I, since other experiments, such as the oscillating sphere experiment by Benson and Hallett (ref.18) and the U-tube experiment by Donnelly and Penrose (ref.19), have shown that the single fluid behaviour does break down at sufficiently high Reynolds numbers to give turbulent flow. The drag crisis is caused by the onset of turbulence in the boundary layer as explained in section II.1.

An attempt was made to determine the true terminal velocity by dropping the sphere from different heights above the light beams, and extrapolating the velocity curve by means of equation (21) which relates the instantaneous velocity with distance of fall. The extrapolation proved to be too complicated, however, and did not lead to any definite terminal velocity. The complication arises because equation (21) is derived on the assumption of a constant drag coefficient and this is not true in practice. Two factors will cause sudden changes in the drag coefficient. First, the diameter of the cylindrical tube (J) in which the sphere falls (fig.4) changes from about 2 cm to 6.6 cm (Z). And secondly, if a drag crisis does occur this in itself will cause a sudden decrease in the drag coefficient at a sphere velocity determined by the Reynolds number for the onset of turbulence in the boundary layer.

A possible solution to this difficulty would be to give the sphere an additional acceleration at the start of its fall so as to reduce the distance necessary to reach a terminal velocity. It would be possible to do this with a suitably situated superconducting coil, since the spheres are magnetic.

Another possible explanation of the absence of a drag crisis in the present measurements may be that the actual Reynolds numbers are not reliable and that a sufficiently high Reynolds number has not been attained. It has been assumed that the kinematic viscosities given by Donnelly and Hallett (ref.11) are applicable to this experiment, whereas this may not be so. It seems unlikely, however, that this can be the explanation particularly in liquid helium I, which is expected to behave as an ordinary fluid in all these hydrodynamic experiments. Further work is evidently necessary to decide this matter.

V.3. Low Velocity Measurements.

At low Reynolds numbers, when the difference ($\rho_1 - \rho$) becomes small, the possible error obscures the trend of the results. This error arises from the fact that the correction for the contraction of the sphere is known to ± 10% ,as discussed in section IV.2. The values of ρ are known sufficiently accurately (ref.12), but unless the density of the sphere ρ_1 is known with the same accuracy, the results with the very light spheres are almost meaningless. To improve the accuracy of the low velocity results, it will be necessary to measure the total contraction of the sphere to helium temperatures more accurately. This contraction can be measured by interferometric means, which would involve the construction of a special cryostat. It may be possible to make some accurate measurements at low velocities with the present apparatus if a sphere is made which has a density at helium temperatures which lies in the range 0.1452 to 0.1467 gm/c.c.. Under these conditions the sphere can be made to fall at zero velocity by adjusting the temperature of the helium bath below the λ-point and, since the density of liquid helium is known accurately as a function of temperature, the density of the sphere is known with the same accuracy.

The experiment of Dowley, Firth, and Hallett (ref.16) on the drag coefficient of spheres in a rotating beaker experiment are in the range of Reynolds number just below the lowest recorded in the present experiment (1.6×10^4). Their drag coefficients are lower by a factor of about five than those found in the present research. The difference in the nature of their experiment is however rather fundamental since they are measuring the drag on a stationary sphere in a moving fluid. Moreover since the liquid helium in their experiment is rotating, the superfluid has a large number of vortex lines associated with it (Feynman ref.20). Even at very low velocities therefore the experiment would not show a single two fluid behaviour since the vortex lines will contribute to the drag on the sphere. The effective density in their experiment is also unknown and would be neither ρ_n nor ρ. A comparison between these two experiments is therefore very difficult.

V.4. The Critical Velocity.

The lowest velocity reached was about 6 cm/sec, but the measurements at velocities below about 10 cm/sec (R = 3×10^4) are unreliable. Since liquid helium II at these Reynolds numbers is behaving as a single fluid it is clear that, if a critical velocity for the superfluid can be derived for an experiment of this type, it must be less than 10 cm/sec. The only experiment, sufficiently similar to the present one which has yielded a critical velocity, is the superfluid wind tunnel experiment of Craig and Pellam (ref.14). They measured the lift on a stationary airfoil in a moving stream of superfluid. The normal fluid was prevented from moving. Below a superfluid velocity of 0.6 cm/sec the lift was zero but above this velocity the lift increased with velocity. This value is certainly not in conflict with $v_{s,c} < 10$ cm/sec as found in the falling sphere experiment.

V.5. Conclusion.

It has been shown that at Reynolds numbers in the range 3×10^4 to 3×10^5 liquid helium II behaves as a single fluid of density equal to the total hydrodynamic density. This corresponds to a range of velocities from 10 cm/sec to 100 cm/sec . The existence or not, of a drag crisis is left as open question. The critical velocity above which the two fluid model breaks down has been shown to be less than 10 cm/sec in this type of experiment. It is hoped that some of the remaining questions will be answered in further work on this project.

REFERENCES

(1) Birkhoff, G., Hydrodynamics, Dover Publications, Inc., New York, 1955.

(2) Atkins, K. R., Liquid Helium, University Press, Cambridge, 1959.

(3) Laing, R. A. and Rorschach, Jr., H. E., The Physics of Fluids, 1961, 4, 564.

(4) Prandtl, L. and Tietjins, O. G., Applied Hydro-and Aeromechanics, Dover Publications, Inc., New York, 1957, p.105.

(5) Feynman, R. P., Physical Review, 1953, 91, 1301. see ref.(20)

(6) Schlichting, H., Boundary Layer Theory, Mc Graw-Hill Book Co. Inc., New York, 1955.

(7) Tisza, L., Physical Review, 1947, 72, 838.

(8) Landau, L., Journal of Physics, 1941, V, 71.(U.S.S.R.)

(9) Bhagat, S. M., and Pathria, R. K., The Physical Review, 1957, 106, 199.

(10) National Bureau of Standards, The "1958 He^4 Scale of Temperatures.", U. S. Government Printing Office, Washington, 1960.

(11) Donnelly, and Hallett, Annals of Physics, 1958, 3, 320.

(12) Kerr, E. C., The Journal of Chemical Physics, 1957, 26, 511.

(13) Giauque, W. F., Geballe, T. H., Lyon, D. N., and Fritz, J. J., The Review of Scientific Instruments 1952, 23, 169.

(14) Craig, P. P., and Pellam, J. R., Physical Review., 1957, 108 , 1109.

(15) London, F., Superfluids, John Wiley and Sons, Inc., New York, vol. I , 1950 ; vol. II , 1954.

(16) Dowley , M. W., Firth , and Hallett, A. C., Proceedings of the Fifth International Conference on Low Temperature Physics and Chemistry, ed. Dillinger, University of Wisconsin Press, Madison, 1957 , p. 19.

(17) Lunnon , R. G., Royal Society, 1928 , A 118 , 680.

(18) Benson, C. B., and Hallett, A. C. H., Canadian Journal of Physics, 1956, 34, 668.

(19) Donnelly, R. J. and Penrose, O., Physical Review, 1956, 103 ,1137.

(20) R. P. Feynman, Progress in Low Temperature Physics, Interscience Publishers, Inc., New York, 1955, vol.1, p. 45.

APPENDIX

Drag Coefficient Data for a Sphere in Liquid Helium.

It seems useful to include the calculated drag coefficients. The velocity in each case is an average of 5 to 10 readings. Also in the calculations, the diameter for each sphere is an average of forty measurements on that sphere and it is corrected for contraction. The velocities are at liquid helium temperatures.

TABLE I.

Mass of Sphere (mgm)	Velocity (cm/sec)	Temp. (°K)	Drag Coefficient C	Reynolds Number R
111.93 ±.01	36.8 ± 5%	1.73 ±.001	0.44 ±11%	1.26×10^5 ±10%
111.93 "	37.8 "	1.60 "	0.42 "	1.16×10^5 "
111.93 "	37.9 "	1.40 "	0.42 "	1.16×10^5 "
111.93 "	40.8 "	1.30 "	0.36 "	1.16×10^5 "
90.44 "	26.6 "	1.30 "	0.37 "	7.53×10^4 "
90.44 "	25.1 "	1.20 "	0.42 "	6.61×10^4 "
90.44 "	26.3 "	1.50 "	0.38 "	8.45×10^4 "
79.36 "	16.7 "	1.80 "	0.39 ± 12%	5.69×10^4 "
79.36 "	16.5 "	1.50 "	0.41 "	5.24×10^4 "
79.36 "	15.3 "	1.60 "	0.47	5.08×10^4 "
79.36 "	15.7 "	1.40 "	0.45 "	4.74×10^4 "
79.36 "	14.4 "	1.20 "	0.54 "	3.76×10^4 "
199.57 "	75.1 "	1.40 "	0.38 ± 11%	2.29×10^5 "
199.57 "	76.1 "	1.73 "	0.36 "	2.61×10^5 "

TABLE I. (continued)

Mass of Sphere (mgm)		Velocity (cm/sec)		Temp. (°K)		Drag Coefficient C		Reynolds Number R	
199.57	±.01	76.6	±5%	1.60	±.001	0.36	±11%	2.56×10^5	± 10%
199.57	"	73.9	"	1.40	"	0.39	"	2.25×10^5	"
246.67	"	82.8	"	1.40	"	0.41	"	2.55×10^5	"
246.67	"	81.7	"	1.30	"	0.42	"	2.35×10^5	"
196.46	"	72.9	"	1.73	"	0.39	"	2.49×10^5	"
196.46	"	75.0	"	1.20	"	0.37	"	1.98×10^5	"
196.46	"	75.5	"	1.40	"	0.36	"	2.30×10^5	"
312.64	"	94.1	"	1.40	"	0.44	"	2.92×10^5	"
333.21	"	102.2	"	1.30	"	0.42	"	2.93×10^5	"
392.76	"	106.7	"	1.30	"	0.46	"	3.69×10^5	"
392.76	"	103.7	"	1.80	"	0.49	"	3.61×10^5	"
447.67	"	134.2	"	1.80	"	0.36	"	4.73×10^5	"
447.67	"	127.1	"	1.70	"	0.40	"	4.44×10^5	"
447.67	"	122.9	"	1.60	"	0.43	"	4.21×10^5	"
447.67	"	128.8	"	1.50	"	0.39	"	3.98×10^5	"
447.67	"	128.5	"	1.40	"	0.39	"	4.00×10^5	"
519.62	"	144.5	"	1.20	"	0.32	"	4.06×10^5	"
77.08	"	10.15	"	1.80	"	0.25	±40%	3.49×10^4	"
77.08	"	10.07	"	1.70	"	0.26	"	3.44×10^4	"
77.08	"	10.60	"	1.50	"	0.24	"	3.41×10^4	"
77.08	"	10.22	"	1.20	"	0.27	"	2.69×10^4	"
75.54	"	6.45	"	1.20	"	0.19	±300%	1.70×10^4	"
75.54	"	5.62	"	1.80	"	0.18	"	1.93×10^4	"
75.54	"	5.95	"	1.60	"	0.20	"	1.99×10^4	"
75.54	"	6.15	"	1.50	"	0.19	"	1.97×10^4	"

TABLE I (continued)

Mass of Sphere (mgm)		Velocity (cm/sec)		Temp. (^oK)		Drag Coefficient C		Reynolds Number R	
447.67	±.01	134.3	± 5%	1.50	±.001	0.36	± 11%	4.42 x 10^5	± 10%
620.68	"	147.1	"	1.50	"	0.44	± [illegible]%	4.84 x 10^5	"
77.08	"	7.14	"	2.20	"	0.37	± 20%	1.69 x 10^4	"
77.08	"	8.55	"	2.30	"	0.32	± 20%	2.03 x 10^4	"
447.67	"	132.	"	4.2	"	0.44	± 11%	4.00 x 10^5	"
392.76	"	114.9	"	1.87	"	0.40	"	3.92 x 10^5	"
392.76	"	113.4	"	1.97	"	0.41	"	3.83 x 10^5	"