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Computing Business-as-Usual with a Representative Agent and a Pollution Externality

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Abstract

Computing the no-policy equilibrium (business-as-usual) in a representative-agent (RA) model is complicated by the presence of a pollution externality, since simple optimization internalizes the pollution cost. Many researchers use ad hoc methods, but there is no way to know how reliable these are. A solution is presented in which the RA model is divided into N identical components, each identified with its own agent. Agents play a dynamic game, leading to a Nash equilibrium. For sufficiently large N, this approach keeps most of the pollution cost external, and in the limit it is equivalent to a myopic-firms model, in which the entire cost is external. This approach has the advantage of theoretical consistency, and empirical applications indicate that it is easily implemented.

(JEL D62, Q5; keywords: representative agent, externality, pollution)

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I. Introduction

One result of the computing revolution has been the emergence of computer-based simulation modelling as an important tool for analyzing all varieties of policy problems. The present paper focuses on the use of representative agent structures in dynamic models to address problems of pollution externalities. These models are relevant to the analysis of stock or fund pollutants, for which the effects last over time. A pre-eminent example concerns greenhouse gas emissions, which are hypothesized to be a contributing factor in global climate change. The well-known DICE and RICE models (Nordhaus 1994, Nordhaus and Yang 1996, Nordhaus and Boyer 2000) fall into this category, and they have spawned numerous variations, including ENTICE (Popp 2004). These three models will be discussed in some detail below.

Practicality requires a certain amount of aggregation in empirical work, and in the present context aggregation takes the form of identifying regional or national outcomes with the decisions of a single, infinitely-lived agent. In world models, such as DICE and its successors, there is one region – the world – and therefore one representative agent whose maximizing behaviour determines the evolution of the major endogenous variables. In multi-region models, such as RICE, there are multiple representative agents, one for each region.

There are a number of ways in which one might interpret the concept of a representative agent. In a static context, Gorman (1953) specifies the conditions under which the aggregate behaviour of heterogeneous consumers will be consistent with the behaviour generated by a single consumer facing the same prices and enjoying the aggregated level of income. These conditions amount to restrictions on individual preferences, of which homothetic preferences are a special case. In this context, then, a representative agent can be viewed as a mathematical expedient for representing a disaggregated population of consumers.

In a dynamic context, Barro (1974) observes that a succession of overlapping generations will behave like a single, infinitely-lived agent provided that, among other things, the generations are linked by altruism and that inter-generational gifts are chosen optimally – i.e. chosen to maximize the donor's utility. In this context, an infinitely-lived agent can be justified as an expedient for representing overlapping generations linked by altruistic gifts, a structure which is often referred to as the dynasty model.

Both of these interpretations are positive in nature in that they justify the use of a representative agent on the basis of an underlying economic reality. In contrast, a normative interpretation views the representative agent as equivalent to an ideal social planner. In this case, the agent maximizes a social welfare function (static) or functional (dynamic) which embodies certain desirable ethical properties, concerning for example the weights placed on different generations. Ramsey (1928) falls into this category, as do Arrow and Kurz (1970) with their concept of a "publicly optimal policy."

If one intends a positive interpretation of the representative agent, then it follows by definition that the solution of the associated model represents economic equilibrium. Even if one intends a normative interpretation, it is essential to ask whether the optimal policy can in fact be implemented in a decentralized economy. If the response is yes, then the policy is deemed controllable, to use the language of Arrow and Kurz, and it is then useful to compare the paths of key variables between the optimal and baseline (i.e. no-policy) scenarios. Thus, in all cases, it is important to consider the relationship between the representative agent and equilibrium.

Blanchard and Fischer (1989, pp.48-51) show the equivalence of a representative-agent (RA) optimization and decentralized equilibrium in the context of a single-good model with competitive markets, constant returns to scale in production, identical public and private values (objective functions), and no externalities or public goods. This equivalence between a centralized optimization problem and equilibrium has a long tradition, running through Samuelson (1949) and Negishi (1960), and no doubt it has contributed to the view that an RA framework can be justified as a proxy for an underlying decentralized economy. Nonetheless, it is important to note that the equivalence only holds under the assumption of perfect markets. Add in a public good or an externality, such as pollution, and the framework reverts to that of Arrow and Kurz, in which the RA solution must be interpreted as the social optimum. The problem remains, then, of how to compute the no-policy equilibrium – frequently referred to as business-as-usual (BAU) – using a representative agent.

The most direct – and obvious – solution to this problem would be to build and solve a decentralized model. But the cost of such an approach can be high in terms of both computational complexity and transparency. Faced with this dilemma, many modellers have resorted to ad hoc methods for approximating BAU while retaining the simple RA structure. These methods are not always clearly described – indeed in some cases it is necessary to delve into the computer code to learn the exact methods employed to approximate general equilibrium. And there is usually no way to tell how accurate these methods are.

The present paper is intended to address this shortcoming. The essential problem is how to keep an externality external while optimizing in an RA model. By definition, the representative agent represents the whole economy for the corresponding region, and therefore simple optimization of the agent's utility will internalize the external cost associated with the pollutant. In the context of a world model, the resulting solution is the social optimum, in which the externality is fully internalized. In a multi-region model, the RA-based solution is the Nash equilibrium of a non-cooperative game in which each agent takes into account self-inflicted damages but ignores the effect of emissions on other agents. In this case, the externality is partially internalized.

The solution to this dilemma is to view the problem as a dynamic game involving not one agent per region but many identical agents (henceforth referred to as sub-agents). Within each region, each sub-agent allocates its resources to maximize its own utility, taking as given the allocations of all other sub-agents. The solution of the intra-region game is a Nash equilibrium, in which all allocations are mutually consistent.

This approach adds very little in terms of extra complexity, since each sub-agent is identical. In practice, the modeller specifies a new parameter, N, which represents the number of sub-agents. Initial values of private variables taken from the RA model – e.g. output, consumption, investment, capital, labour – are divided by N to give the sub-agents' values. Thus, this approach maintains the essential structure of the RA model, as the economy is in effect sliced into N identical pieces per region. Each sub-agent has its own utility function, its own production function, and its own accumulation of private stocks.

In this approach, each $\frac{1}{N}^{\text{th}}$ piece of the economy is hermetically sealed – there is no trade or "foreign" investment – with the exception of the public bad of pollution which aggregates globally and affects all agents. Each sub-agent takes into account the cost of self-inflicted damages when choosing its own level of pollution emissions but ignores the cost of these emissions for the other N-1 agents. Thus, the proportion of total pollution cost which is external is $\frac{N-1}{N}$, or equivalently $1-\frac{1}{N}$. Clearly, this externality proportion rises in N, and in the limit all pollution costs are external. In practice, one must employ a finite value of N. Fortunately, this value does not need to be very large for the externality proportion to be high. In two of the three cases tested, we employ a value of $N=100$, which corresponds with an externality proportion of 0.99, and in one case we employ $N=300$ for an externality proportion of 0.997.

We refer to this method as the N-agent (NA) approach. It is intended as a tool for preserving the externality in business-as-usual without introducing any other effects. Thus, it is desirable that the NA approach should generate the same solution as the RA approach when the pollution damage function is not operative. It is shown that this requirement translates into specific rules for converting constraints and utility functions from the RA model to the NA model. In particular, equivalence requires that all constraint functions be linearly homogeneous in the scaling factor $\frac{1}{N}$, and that RA utility is an affine transformation of NA utility.

The first restriction is weaker than it seems, for it does not require that constraint functions be linearly homogeneous in general, but only with respect to the particular factor $\frac{1}{N}$. A simple adjustment can be employed to make any homogeneous function linearly homogeneous in this factor. Similarly, there is no problem satisfying the restriction on utility transformations, since the modeller is free to choose NA utility as a function of RA utility.

Like the RA model, the NA model is still somewhat artificial in that, within each $\frac{1}{N}^{th}$ of the economy, all consumption, production and investment decisions are vested in a single, forward looking agent. This feature leads to the internalization of self-inflicted damages. In contrast, it is probably more natural to think of externality problems arising from the failure of productive agents – i.e. firms – to take account of the costs of their emissions on consuming agents and other firms. This view requires modeling firms and consumers as distinct entities with independent decision making processes.[2] In an extreme version, we may think of firms as myopic, maximizing static profits with no regard for future consequences while consumers are represented by a dynamically optimizing representative consumer. This model will be referred to as Myopic Firms (MF). It is shown that the NA model is asymptotically equivalent to MF.

The paper is divided into two sections: the first explores the issues in the context of analytical models, while the second discusses the implementation of the NA method in three models: DICE (Nordhaus 1994, Nordhaus and Boyer 2000), ENTICE (Popp 2004), and RICE 99 (Nordhaus and Boyer 2000). DICE and ENTICE are world models, while RICE 99 is a multi-region model.

DICE and ENTICE use ad hoc or approximative methods to estimate BAU. The comparison with the NA approach indicates that the numerical differences are not large.

[2] But note that individual firms would still internalize self-inflicted damages if (i.) emissions had a negative effect on production and (ii.) managers were forward-looking.

Nonetheless, the theoretical consistency of the NA approach remains compelling, since one can only assess the reliability of the ad hoc methods by comparing them with a theoretically consistent method.

For RICE 99, Nordhaus and Boyer do indeed provide a theoretically consistent method for estimating BAU. However, the programming structure of their method is distinct from what is required to solve the non-cooperative abatement game, and indeed they make no mention of a non-cooperative solution for RICE 99. In contrast, the programming structure of BAU and the non-cooperative abatement game is identical under the NA approach, the only difference being the value of N. Thus the remarkable ease of transition from one solution concept to another recommends the NA approach in this case.

II. Analytical Models

This section explores the theoretical relationships between a representative agent, an N-agent approach, and myopic firms in the computation of business-as-usual with a stock externality. For simplicity, we assume there is only one region.

1. Representative Agent Model

The agent produces output, Y, from inputs of capital, K, labour, L, and fuel, F, according to a gross production function, $Q(K, L, F)$. Time subscripts are suppressed except where necessary for clarity.

The combustion of fuel creates pollution, which accumulates as a stock, S. The accumulation relationship is

$$\dot{S} = F - \delta_s S \tag{1}$$

where the dot notation indicates the time derivative and δ_s is the decay rate of the stock. The pollution stock has a negative effect on production, represented by the damage function, $D(S)$, with $0 < D(S) \leq 1$, $D(0) = 1$, and $D'(S) < 0$. There is no consumption externality in the model, although it would be straightforward to add one. Output net of damages is given by $Y = D(S) Q(K, L, F)$.

In each period, the agent allocates output between consumption, C, capital investment, $\dot{K}$, and the purchase of fuel. Labour is supplied inelastically and is fully employed. The unit price of fuel, P_F, is assumed constant, and capital does not depreciate. Thus

$$\dot{K} = D(S)\,Q(K, L, F) - P_F F - C. \tag{2}$$

The agent seeks to maximize an intertemporal welfare function subject to constraints (1) and (2). The welfare function is $\int_0^\infty L u\left(\frac{C}{L}\right) e^{-\delta t} dt$, where labour L is assumed equal to population, C/L represents per capita consumption, $u(\cdot)$ represents a concave utility function, and δ represents a constant rate of pure time preference. If a solution exists, it can be found using optimal control theory, with control variables F and C and state variables K and S. The current-value Hamiltonian is

$$H = Lu\left(\tfrac{C}{L}\right) + \theta\left[D(S)Q(K, L, F) - P_F F - C\right] + \gamma\left[F - \delta_s S\right]$$

The first-order necessary conditions are[3]

$$\frac{\partial H}{\partial C} = u'\left(\tfrac{C}{L}\right) - \theta = 0 \tag{3}$$

$$\frac{\partial H}{\partial F} = \theta\left[D(S)Q_F - P_F\right] + \gamma = 0 \tag{4}$$

$$\dot{\theta} - \delta\theta = -\frac{\partial H}{\partial K} \quad \Rightarrow \quad \dot{\theta} - \delta\theta = -\theta D(S) Q_K \tag{5}$$

$$\dot{\gamma} - \delta\gamma = -\frac{\partial H}{\partial S} \quad \Rightarrow \quad \dot{\gamma} - \delta\gamma = \gamma\delta_s - \theta\, D'(S)\, Q(K, L, F) \tag{6}$$

It will be useful in what follows to have the solution for $\gamma(t)$. To find the solution, integrate equations (5) and (6) to obtain

$$\theta(v) = \theta(t) \exp\left\{\int_t^v \left(\delta - D(S(q))Q_K(q)\right) dq\right\}, \; v \geq t \tag{7}$$

and

$$\gamma(t) = \int_t^\infty \exp\left\{-(\delta + \delta_s)(v - t)\right\} \theta(v)\, D'(S(v))\, Q(K(v), L(v), F(v))\, dv. \tag{8}$$

Substituting (7) into (8) and simplifying yields

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[3] The notation f_i is used to denote the partial derivative of a function f with respect to argument i.

$$\frac{\gamma(t)}{\theta(t)} = \int_t^{\infty} \exp\left\{ -\int_t^{v} \left(\delta_s + D(S(q))Q_K(q)\right)dq \right\} D'(S(v))\, Q(K(v), L(v), F(v))\, dv$$

This expression indicates that the shadow value of the pollution stock, in terms of the consumption numeraire, is equal to the present discounted value of the damage resulting from a marginal unit of emissions. The discount rate consists of two components: δ_s, the decay rate of the pollution stock, and $D(S)\,Q_K$, the marginal product of capital. The representative agent takes into account the total effect of emissions on future production, and therefore the solution to this problem represents the first-best social optimum.

2. N-Agent Model

Now consider an economy consisting of N identical sub-agents. These sub-agents make both consumption and production decisions for their portion of the economy. Their decisions are modeled as a standard RA inter-temporal optimization.

In general, upper-case letters denote economy-wide aggregates whereas lowercase letters denote values for a specific sub-agent. Thus for example $F = \sum_{n=1}^{N} f_n$. Since sub-agents are identical, they have identical labour supply, $\ell = \frac{L}{N}$, and identical solution values, $k = \frac{K}{N}$, $f = \frac{F}{N}$ and $c = \frac{C}{N}$.

The law of motion for capital is

$$\dot{k} = D(S)\,Q(k, \ell, f) - P_F f - c \tag{9}$$

and the law of motion for the pollution stock is

$$\dot{S} = \sum_{n=1}^{N} f_n - \delta_s S \tag{10}$$

Equations (2) and (9) are equivalent under the assumption of constant-returns-to-scale production, i.e. $Q(K, L, F)$ linearly homogeneous which implies $Q(k, \ell, f) = \frac{1}{N} Q(K, L, F)$ and $Q_f(k, \ell, f) = Q_F(K, L, F)$.

Each sub-agent seeks to maximize an inter-temporal utility function $\int_0^\infty \ell u\left(\frac{c}{\ell}\right)e^{-\delta t}dt$ subject to (9) and (10). If a solution exists, it can be found using optimal control theory. The sub-agent's control variables are c and f, and the state variables are k and S.

Each sub-agent considers the emissions of other sub-agents as fixed. In this manner, he takes into account the effect of his own emissions on his future production, through S, but ignores the effect on the production of other agents. Thus, in the absence of regulation, there is a pollution externality at play in the economy. If each sub-agent represents only a small portion of the total economy – i.e. if N is large – then the externality will be significant.

The current-value Hamiltonian for the sub-agent's problem is

$$H = \ell u\left(\tfrac{c}{\ell}\right) + \lambda\left[D(S)Q(k,\ell,f) - P_F f - c\right] + \mu\left[\sum_{n=1}^{N} f_n - \delta_s S\right].$$

The first-order conditions are

$$\frac{\partial H}{\partial c} = u'\left(\tfrac{c}{\ell}\right) - \lambda = 0 \tag{11}$$

$$\frac{\partial H}{\partial f} = \lambda\left[D(S)Q_f - P_F\right] + \mu = 0 \tag{12}$$

$$\dot{\lambda} - \delta\lambda = -\frac{\partial H}{\partial k} \quad \Rightarrow \quad \dot{\lambda} - \delta\lambda = -\lambda D(S)Q_k \tag{13}$$

$$\dot{\mu} - \delta\mu = -\frac{\partial H}{\partial S} \quad \Rightarrow \quad \dot{\mu} - \delta\mu = \mu\delta_s - \lambda D'(S)Q(k,\ell,f) \tag{14}$$

Since all sub-agents are identical, they will all be characterized by the same set of optimality conditions – i.e. the FOC's (11) – (14) and the laws of motion (9) – (10). Nash equilibrium consists of time paths $c(t), f(t), k(t), S(t)$ which satisfy these conditions.[4] There is no trade in goods in this model; only the public-bad pollution is exchanged.

[4] Technically speaking, the solution is an open-loop Nash equilibrium. Open-loop strategies are characterized by an infinite horizon of commitment on the part of agents, starting in the initial period. In contrast, closed-loop strategies allow agents to change plans in response to unanticipated changes in the path of the state variables. In models of perfect foresight, in which there are no unanticipated changes in state variables, this distinction is unimportant. See Basar and Olsder (1982) for an authoritative discussion of the relevant concepts. A succinct overview is provided in Xepapadeas (1997, pp.102-108). In an interesting paper, Yang (2003) shows that any closed-loop solution can be constructed from a sequence of open-loop solutions.

It will be useful in what follows to have the solution for $\mu(t)$. To find the solution, integrate equations (13) and (14) to obtain

$$\lambda(v) = \lambda(t)\exp\left\{\int_t^v (\delta - D(S(q))Q_k(q))dq\right\},\ v \geq t \tag{15}$$

and

$$\mu(t) = \int_t^\infty \exp\{-(\delta+\delta_s)(v-t)\}\lambda(v)\,D'(S(v))\,Q(k(v),\ell(v),f(v))\,dv\ . \tag{16}$$

Substituting (15) into (16) and drawing upon the properties of linearly homogeneous functions yields

$$\frac{\mu(t)}{\lambda(t)} = \frac{1}{N}\int_t^\infty \exp\left\{-\int_t^v (\delta_s + D(S(q))Q_K(q))dq\right\}D'(S(v))\,Q(K(v),L(v),F(v))\,dv\,. \tag{17}$$

This result indicates that the shadow value of the pollution stock for the sub-agent, in terms of the consumption numeraire, is equal to $\frac{1}{N}$ th of the present value of aggregate damage resulting from a marginal unit of emissions. The result seems logical, since it refers to a single sub-agent, of which there are N.

A comparison of the solutions of the RA and NA models is facilitated by the common occurrence of P_F in (4) and (12). Solving these equations for P_F and equating yields

$$D(\hat{S})Q_F(\hat{K},L,\hat{F}) + \frac{\mu}{\lambda} = D(S^*)Q_F(K^*,L,F^*) + \frac{\gamma}{\theta} = P_F, \tag{18}$$

where the hat notation indicates solutions in the NA model and the asterisk denotes solutions in the RA model. Now substituting from (17) yields

$$D(\hat{S})Q_F(\hat{K},L,\hat{F}) + \frac{1}{N}\int_t^\infty \exp\left\{-\int_t^v (\delta_s + D(\hat{S}(q))Q_K(q))dq\right\}D'(\hat{S}(v))\,Q(\hat{K}(v),L(v),\hat{F}(v))\,dv$$

$$= D(S^*)Q_F(K^*,L,F^*) + \frac{\gamma}{\theta} = P_F\,. \tag{18$'$}$$

The right-hand side of the equality is the net marginal product of fuel at the social optimum; i.e. the marginal product in production, $D(S^*)Q_F(K^*,L,F^*)$, less the present value of the marginal pollution damages, $\frac{\gamma}{\theta}$.[5] This value is equal to the marginal product in production from the NA model, $D(\hat{S})Q_F(\hat{K}, L, \hat{F})$, less $\frac{1}{N}^{th}$ of the present value of marginal damages.

The expectation of different solutions for NA and RA complicates the interpretation of (18) and $(18')$. However, if $\frac{\mu}{\lambda} > \frac{\gamma}{\theta}$, as seems likely (i.e. $\frac{\mu}{\lambda}$ smaller in absolute value), it follows that $D(\hat{S})Q_F(\hat{K}, L, \hat{F}) < D(S^*)Q_F(K^*, L, F^*)$. Higher fuel-use and emissions in the NA model are consistent with this condition, given diminishing marginal productivity. In the limit, as $N \to \infty$, pollution damages are completely ignored in the NA model, with inputs adjusted such that $D(\hat{S})Q_F(\hat{K}, L, \hat{F}) = P_F$. In contrast, in the RA model, there is always a wedge between $D(S^*)Q_F(K^*, L, F^*)$ and P_F, due to the marginal pollution damages, γ/θ.

This result confirms our intuition that the externality remains external in the NA model, with the exception of self-inflicted damages. In the limit, even self-inflicted damages are ignored in terms of the aggregate result, as they diminish to insignificance. In practice, the value of N does not need to be very large to ensure that most pollution damages remain external. Equation $(18')$ indicates that only $\frac{1}{N}^{th}$ of the present value of damages are accounted for in the solution of the NA model. Thus, the proportion of pollution cost which is external is $\frac{N-1}{N}$. For $N = 100$, this proportion equals 0.99; for $N = 300$, it is 0.997.

3. Equivalence Conditions

An important requirement is that the NA and RA models must generate the same aggregate solution when the pollution externality is not present. The intuition here is that the NA model is just a mathematical tool for keeping pollution costs external; it is not desired that it should introduce any other changes to economy-wide aggregates. The equivalence requirement is satisfied if and only if (i) all constraint functions are linearly homogeneous for the scaling factor $\frac{1}{N}$, and (ii) instantaneous utility in the RA model is an affine transformation of instantaneous utility in the NA model, i.e. $u_{RA} = a + b u_{FN}$, where a and b are constants and $b > 0$.[6]

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[5] Note that $\frac{\gamma}{\theta}$ has a negative sign.

[6] This condition could just as easily be formulated as u_{FN} an affine transformation of u_{RA}.

Linear homogeneity is necessary to ensure that the constraints are equivalent between the two models, since in this case dividing both sides of constraints (1) and (2) by N will yield constraints (9) and (10). This condition is intuitive, since if all transformation functions exhibit constant returns to scale, then it will not matter whether output is produced by one large agent (RA model) or many small sub-agents (NA model). In contrast, if some of the functions exhibit economies of scale (or diseconomies), then dividing production among more agents will have an impact on economy-wide aggregates even in the absence of the pollution externality.

This condition is not as restrictive as it may seem, since it only applies for the particular scaling factor $\frac{1}{N}$. Thus, if a constraint function is homogeneous of degree $m \neq 1$, it is possible to make a simple adjustment to make it homogeneous of degree 1 for $\frac{1}{N}$ but not for other scaling factors. To demonstrate, consider a production function that is homogeneous of degree $m \neq 1$; i.e. $Q\left(\frac{K}{N}, \frac{L}{N}, \frac{F}{N}\right) = \left(\frac{1}{N}\right)^m Q(K, L, F)$. Then define an adjusted production function

$\hat{Q}(K, L, F) = \left(\frac{1}{N}\right)^{1-m} Q(K, L, F)$.

This function is homogeneous of degree one for the scaling factor $\frac{1}{N}$, as can be readily verified, and it equals the original function when $N = 1$.

Provided the linear homogeneity condition is satisfied, the restriction to affine transformations of utility is necessary and sufficient for the equivalence of the NA and RA solutions. This result is formally identical to the well-known invariance requirement for utilitarian welfare functions, as shown in Sen (1977), and therefore we state it without proof.

Both conditions – linear homogeneity and affine transformations – were satisfied in sections II.1 and II.2 above. In section III.2, we consider the ENTICE model in which one of the constraint functions is not linearly homogeneous. The problem is rectified with the adjustment discussed above. Satisfying the requirement for affine transformations of utility is straightforward, since the modeller is free to specify utility as desired.

4. Myopic Firms

This model separates decision making between a representative consumer and a representative firm, which is perhaps more natural for considering problems of pollution externalities than the NA and RA models, in which both consumption and production decisions are vested in one agent. The consumer chooses time paths of consumption and savings to maximize an intertemporal utility function. In contrast, the firm is myopic, choosing fuel use – and thus emissions – in each period to maximize static profits. Thus, all of the pollution effect is external in this model.

The representative consumer seeks to maximize an intertemporal utility function $\int_0^\infty Lu\left(\tfrac{C}{L}\right)e^{-\delta t}dt$ subject to the savings equation

$$\dot{K} = \pi + rK + wL - C \ , \qquad (27)$$

where π is the profit of the representative firm, which is distributed to the consumer, r is the rental rate of capital, and w is the equilibrium wage rate. As before, we assume labour is supplied inelastically and is fully employed. The consumer considers π, r and w exogenous. The consumer's choice variable is C and his state variable is K. The current-value Hamiltonian is

$$H = Lu\left(\tfrac{C}{L}\right) + \psi\left[\pi + rK + wL - C\right] .$$

The first-order conditions are

$$\frac{\partial H}{\partial C} = u'\left(\tfrac{C}{L}\right) - \psi = 0$$

$$\dot{\psi} - \delta\psi = -\frac{\partial H}{\partial K} \quad \Rightarrow \quad \dot{\psi} - \delta\psi = -\psi r$$

The firm's static profit function is defined

$$\pi = D(S)\,Q(K, L, F) - P_F F - rK - wL . \qquad (28)$$

The firm chooses F and K in each period to maximize π. Thus the first-order conditions are

$$\frac{\partial \pi}{\partial F} = D(S)Q_F - P_F = 0$$

$$\frac{\partial \pi}{\partial K} = D(S)Q_K - r = 0$$

Combining (27) and (28) and dividing through by N yields

$$\dot{k} = D(S)\,Q(k, \ell, f) - P_F f - c .$$

Furthermore, combining the first-order conditions for the consumer and the firm, and noting that $\tfrac{C}{L} = \tfrac{c}{\ell}$, $Q_F = Q_f$, and $Q_K = Q_k$, we obtain

$$u'\left(\tfrac{c}{\ell}\right) = \psi$$

$$D(S)Q_f = P_F$$

$$\dot{\psi} - \delta\psi = -\psi D(S)Q_k$$

Comparison of these conditions with (9) and (11) – (14) indicates that the MF model is equivalent to the NA model in the limiting case when $\mu = 0$, i.e. as N becomes arbitrarily large and all pollution damages are external. Now, as argued above, N need not be very large for most pollution damages to be external. Therefore, for reasonable N, we may view the two models as almost equivalent, and NA may be used as a proxy for MF, notwithstanding the formal presence of self-inflicted damages.

III. Applications

The present section will assess how much difference the NA approach makes in practice, compared with ad hoc approaches, in the context of some well-known dynamic simulation models. The externality problem concerns global climate change, resulting from the emission of greenhouse gases, primarily CO_2. Three models are assessed: DICE (Nordhaus 1994, Nordhaus and Boyer 2000), ENTICE (Popp 2004), and RICE 99 (Nordhaus and Boyer 2000). The first two are single-region models of the world, whereas the third is multi-regional.

The Nash equilibria were computed using an iterative procedure identical to Nordhaus and Yang (1996) and Shiell (2003).[7] Initial values of private variables taken from the corresponding RA model – i.e. output, consumption, investment, capital, labour – are divided by N to give the values of the individual agents. Each agent treats the values of the other N-1 agents as exogenous. One agent "goes first", optimizing utility conditional on values of zero for the exogenous variables. Then the exogenous variables are updated, with values equal to $N-1$ times the values just chosen by the first agent. The process is iterated until no further changes are observed. In the models tested, convergence was obtained very quickly – usually within seven iterations. All simulations were conducted on a personal computer, using the GAMS modeling software and the CONOPT2 optimizer.[8]

The theory indicates that we should choose as large a value of N as possible, in order to maximize the externality proportion. The practical limitation on this choice arises from the scaling requirements of the optimization software. Scaling of models is time consuming, and in

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[7] Nordhaus and Yang (1996) use the algorithm to find equilibrium in a non-cooperative game of emission control. However, they do not use it to compute business-as-usual.

[8] For more information, consult the GAMS website <www.gams.com>. We would be happy to provide our codes to interested readers.

any case there are diminishing returns to increasing N in terms of the resulting increase in the externality proportion. Thus it falls to the modeller to choose a value which she feels represents a reasonable trade-off between computational ease and high externality proportion. Fortunately, this trade-off is not usually severe, as discussed previously. In two of the three applications we use a value of $N = 100$ (ENTICE and RICE 99) while in one of the applications we use $N = 300$ (DICE).

1. DICE

DICE, or Dynamic Integrated Model of the Climate and Economy, is a single-region model of the world's economic and climate system. As the first dynamically optimizing, fully integrated model developed for the assessment of global climate policy, DICE has undergone several changes from its first to its latest versions, referred to respectively as DICE 94 (Nordhaus 1994) and DICE 99 (Nordhaus and Boyer 2000).

The main differences between these two versions concern discounting, the modeling of carbon flows, and the damage function. Regarding discounting, DICE 94 assumes a constant rate of pure time preference of 3 % per year, whereas DICE 99 assumes a geometrically declining rate, starting at 3% per year in 1995 and declining "to 2.3 percent per year in 2100 and 1.8 percent per year in 2200" (Nordhaus and Boyer 2000, p.16). This approach is intended to capture the phenomenon of declining impatience, consistent with the recent literature on hyperbolic discounting.[9] Regarding carbon flows, DICE 94 posits a single relationship characterizing atmospheric retention of carbon. In contrast, DICE 99 specifies a three-reservoir model, corresponding with the atmosphere, a quickly mixing zone consisting of the biosphere and the upper oceans, and finally the deep oceans. Regarding damages, DICE 99 builds upon DICE 94 by taking account of the possibility of low probability, catastrophic outcomes.

Both DICE 94 and DICE 99 differ from the RA model presented above in the absence of an explicit fuel input. Rather, output is produced from inputs of capital and labour according to a gross production function $Q(K, L)$. Emissions of greenhouses gases, E, are related to gross output by a baseline carbon-GDP ratio, σ, and an emission-control variable, $\mu \in [0,1]$. In particular,

[9] Cropper and Laibson (1999) provide an overview of this literature.

$$E = (1-\mu)\sigma Q(K,L). \tag{29}$$

The value of μ is treated as a policy choice, whereas σ reflects the exogenous evolution of technology.

Net production reflects damages from climate change. Damages are modelled as a function of global mean temperature, TE. In addition, net production also reflects the cost of climate control policies, which are modelled as a function of μ. In particular,

$$Y = \Omega(\mu, TE) Q(K,L) \tag{30}$$

where the adjustment factor $\Omega(\mu, TE) \in (0,1]$ summarizes the effect of damages and control costs on output.

Nordhaus' ad hoc approach to business-as-usual consists of maximizing the inter-temporal utility function of a representative agent subject to the constraint $\mu = 0$. This approach seems quite persuasive on the surface, since μ is the only explicit policy variable. Nonetheless, even with $\mu = 0$, the representative agent is aware of the costs of pollution damages and will adjust its choices of other variables in response. In particular, since the choice of K affects emissions and damages, through (29), as well as the level of output available for consumption and investment, through (30), one would expect the agent to choose the time profile of K to optimally balance the cost of damages against the benefit of consumption. In other words, the externality is internalized.

This discussion highlights that there are two avenues through which pollution is controlled in DICE. It follows therefore that μ is probably mislabelled. Rather than referring to it as the emission-control rate, it would be more appealing to view it as representing end-of-tailpipe abatement efforts, or abatement due to fuel-switching. Both types of activities typically entail recurring costs, an interpretation which is consistent with the cost structure in (30). In contrast, an alteration in the time profile of K may be interpreted as a behavioural response to pollution. For example, a downward adjustment of K over time (the result obtained below) would reflect a decision to reduce production of a polluting good.

The comparison between an NA approach and Nordhaus' ad hoc approach can be undertaken for the long-run equilibrium of the model as well as the transitional path. The long-run equilibrium represents the limiting or steady state solution of the representative agent's problem; thus it can be obtained as the solution of the model equations and first order conditions under steady state. The appendix describes our method. Results based on the long-run equilibrium of DICE are presented in Table 1, and, as discussed below, the differences between the NA and RA approaches are not large. It follows that one should not expect the differences to be large for the transitional path either, and therefore no such comparison has been undertaken.

The results presented in Table 1 are based on the reduced-form model of carbon flows and the damage function from DICE 94. It was decided that these features were better suited to the task at hand than those of DICE 99, since the latter did not yield a "realistic" long-run solution under business-as-usual. In particular, the variable TE, which measures the increase in the average atmospheric temperature compared with the pre-industrial level, takes on a value of 133 degrees Celsius in the RA long-run equilibrium in DICE 99, based on a 3% rate of pure time preference. In contrast, for the same assumptions DICE 94 yields a value of 7.9 degrees.

For the purpose of this comparison, the different treatments of discounting in DICE 94 and DICE 99 reduce to different values of the rate of pure time preference in the long-run. Table 1 presents data for two cases: $\delta = 0.03$, which corresponds with DICE 94, and $\delta = 0.01$, which was chosen to represent the limit of a geometrically declining rate of time preference, as in DICE 99. This choice of the limiting value was arbitrary, the point of the exercise being to choose some value significantly less than 0.03 to demonstrate the influence of the discount rate on the comparison.[10]

The results in Table 1 indicate that, in the case of a constant rate of pure time preference of 3 percent (DICE 94), the differences in steady-state solution values between Nordhaus' ad hoc approach, based on an RA model, and the NA approach are very small. All the differences are less than 1 percent, with a high of 0.91 percent for K and a low of 0.07 percent for C. Nonetheless, the signs of the differences are plausible, with the NA solution calling for more capital, and therefore more production, more consumption, more emissions, higher mean temperature, and greater damages. This result indicates that the representative agent responds to climate change damages, even though constrained by the requirement that $\mu = 0$, by moderating the level of K.

The small magnitude of this response may be attributed to two factors: the rate of pure time preference and the relatively low elasticity of emissions with respect to capital formation.

[10] One could have chosen $\delta = 0$, in which case the relevant steady state would have corresponded with the golden rule.

The positive value of the rate of pure time preference indicates a bias toward the present in the weighting of costs and benefits by the agent. In this and other models of global climate change, the benefits from accumulating capital are enjoyed as soon as the next period, while the damages from emissions are spread over the distant future. Therefore, the higher the rate of pure time preference, the less the representative agent cares about future damages and the lower the response. In contrast, with a lower rate of pure time preference, the representative agent cares more about future damages, and therefore we would expect to see a greater difference between the RA and NA models in this case. This expectation is borne out in the case of $\delta = 0.01$. Here the long-run value of capital is 2.79 percent higher in the NA model, with pollution costs external, than in the RA model, with pollution costs internal. Thus, we would expect the differences between NA and RA to be more pronounced in models such as DICE 99, which are characterized by declining rates of pure time preference, than in models such as DICE 94, which are characterized by a high and constant value.

Notwithstanding the more pronounced difference in K when $\delta = 0.01$, the differences in the other variables remain small. This result is explained by the fact that the elasticity of emissions with respect to capital formation in DICE – at 0.25 – is relatively low. When $\mu = 0$, the only option for the representative agent to respond to climate damages is to moderate K. Yet given the low elasticity, the agent must sacrifice a lot of capital to achieve even a modest reduction in emissions – in particular a 4 percent reduction in K for each 1 percent reduction in E. In Table 1, the 2.79 percent reduction in K, which results from using the RA model rather than NA, translates into a 0.71 percent reduction in E.[11] Given the high cost of abatement implied by this trade-off, it does not pay for the representative agent to be more aggressive in controlling pollution.

2. ENTICE

The previous discussion suggests that differences between the RA and NA models will be greater when abatement cost is lower. Such is the case in the ENTICE model of Popp (2004). ENTICE builds on DICE by (i) adding an energy supply module corresponding with a fuel input,

[11] Rounding and approximation error account for the slight discrepancy between the value obtained for E and what would be predicted based on the elasticity of 0.25 and $\Delta K = 2.79\,\%$.

F, and (ii) adding an endogenous component of technological change, corresponding with an energy input, EN.

The energy supply module is based on RICE 99 (Nordhaus and Boyer 2000). In particular, the price of fuel, P_F, increases as a function of cumulative fuel consumption, reflecting scarcity. This structure is similar to the one presented in section II above, with the exception that P_F is not constant here.

With the addition of energy-related technical change, output becomes a function of capital, labour and energy services, EN. The net output function is

$$Y = D(TE)Q(K, L, EN)$$

with damages, D(TE), a function of the global mean temperature. Energy services are provided by both fuel, F, and energy-related human capital, H, according to a function

$$EN = \varphi(F, H).$$

Additions to the stock of human capital are produced by the flow input of research, R, according to an innovation function

$$H_{t+1} = h(R_t, H_t). \tag{31}$$

In ENTICE, agents can invest in either K or H, with the latter serving as a substitute for F in the production of EN. Emissions, E, are directly proportional to F. Thus, agents can deliberately reduce the emissions content of output through their investment decisions. Stated another way, with three variable inputs (K,F,R) instead of one (K), it is possible to substitute away from the polluting input, F, toward pollution-free capital, K, or knowledge, H. In contrast, in DICE, with $\mu = 0$, the only option for reducing emissions is to sacrifice capital and output. As a consequence, the abatement cost is lower in ENTICE. Furthermore, there is a second inducement to substitute away from F, as increasing scarcity leads to increasing P_F. For both these reasons, we should expect the representative agent to be more aggressive in controlling emissions in ENTICE than in DICE (with $\mu = 0$), with the result that difference between the NA and RA approaches should be more pronounced.

This result is straightforward if one compares RA and NA approaches, since RA yields the social optimum. However, RA is obviously not an acceptable proxy for BAU. To calculate BAU, Popp employs an ad hoc approach consisting of three steps. First, he solves the model without pollution damages. Second, he calculates the trajectory of the savings rate for this solution. Third, he uses this savings rate to calculate adjusted trajectories of Y, C and K, iterating forward from the first period in the presence of pollution damages.

This post-hoc adjustment for damages works remarkably well for Y, C and K. In our comparison, the trajectories of these variables were virtually identical under NA $(N = 100)$ and Popp's BAU. Unfortunately, there appears to be no coherent way to extend the adjustment to the remaining variables. Thus, the values of F, R, H, TE and any other variables of interest are based on the assumption of no damages.

Figure 1 presents our comparison of NA and Popp's approach for the variables F, R, H, and TE for the time frame 2000-2200. Following DICE, each period is one decade. All of the equations in ENTICE are linearly homogeneous, with the exception of the innovation function, (31). In particular, the innovation function takes the form

$$H_{t+1} = aR_t^{0.20}H_t^{0.55}$$

which is homogenous of degree 0.75. In order to satisfy the RA-NA equivalence conditions, we employ the revised form

$$H_{t+1} = N^{-0.25}aR_t^{0.20}H_t^{0.55}$$

which is linearly homogeneous in N and equals the original form when $N = 1$.

We present the results in deviation form using NA as the benchmark, since it is the preferred approach on theoretical grounds. ENTICE is calibrated to DICE 99 and certain other empirical values, as discussed by Popp. For the purpose of our comparison, we have performed independent calibrations for the NA method and Popp's BAU.

Figure 1 clearly reveals a divergence in terms of the consumption of F, with Popp's method generating a consistently higher level than the NA method, and the gap increasing over time. The other main feature is a quadratic path of deviations in spending on energy-related research, R. During the first century (2000-2100), Popp's approach generates progressively less spending on R. A sharp reversal of trend follows, such that Popp's approach generates progressively more spending on R from 2130 onward.[12]

These features reflect the influence of damages, a scale effect, and the increasing fuel price. Damages from greenhouse gas accumulations depress the marginal product of K ceteris paribus, and therefore investment in K is relatively less attractive than investment in H (through

[12] Although not shown, this trend continues unabated for the next century (2200 – 2300) as well.

research spending R) when the damage function is operative. This phenomenon explains the progressively lower spending on R during the first century (2000 – 2100) under Popp's approach, relative to the NA method, since the damage function is not operative in Popp's approach.[13]

The lack of damages in Popp's approach also contributes to a scale effect, as there is more available to spend on everything. This effect explains in part the consistently higher consumption of F noted in the figure. The scale effect compounds over time, through the accumulation of capital, which explains the progressive nature of this trend.

Higher consumption of F under Popp's approach has its cost, however, as the price of fuel rises more quickly. This phenomenon explains the abrupt reversal of the trend in research spending, after 2100, as the representative agent in Popp's approach seeks to substitute human capital, H, for fuel in the provision of energy services. The delay of this response is due to the highly elastic specification of the fuel supply function.[14]

Finally, the trend of deviations for H follows that of R, with a long lag owing to the low elasticity of the innovation function with respect to R (0.20). Similarly, the trend of deviations for TE follows F with a long lag.

3. RICE 99

RICE 99 (Nordhaus and Boyer 2000) is a regionalized version of DICE, which accounts for regional differences in damage sensitivity, abatement costs, productivity growth, and initial capital-labour ratios. It includes an energy supply module, which was described above in the context of ENTICE. Both energy supply and the global climate represent global processes which are common to all regions. Except for the global bad of pollution, there is no inter-regional trade in the model. There are eight regions: USA, Other High Income, OECD Europe, Russia and Eastern Europe, Middle Income, Low Middle Income, China, and Low Income.

Nordhaus and Boyer's (2000) method of estimating BAU involves holding damages constant and updating among regions in an iterative fashion. The initial value of damages is chosen arbitrarily, and the solution is obtained as the fixed point of the iterative process. This approach is theoretically consistent for BAU, as each agent regards pollution damages as

[13] For the same reason, we should expect to find progressively higher relative investment in K under Popp's approach. However, this result is effectively offset by his post hoc adjustment for damages.
[14] See Nordhaus and Boyer (2000, 54-55) for a discussion of the specification.

exogenous. The disadvantage of this approach is that it cannot be readily adapted to solve for a non-cooperative abatement game in which each agent takes account of self-inflicted damages. Instead, the modeller must write a new program. Indeed, the authors do not provide a solution to a non-cooperative game.[15]

In contrast, the NA approach internalizes self-inflicted damages. The only difference then between BAU and the non-cooperative game is the choice of N: high for BAU, in order to maximize the externality proportion, and $N = 1$ for the non-cooperative game (the RA solution). Thus the modeller only needs to write one program.

Under the NA method, inter-regional and intra-regional externalities are blended in the solution algorithm. One sub-agent goes first ($\frac{1}{N}^{th}$ of a regional agent), optimizing utility conditional on values of zero for the variables under the control of the other sub-agents in its region and the other regions. A sub-agent for a different region goes next, taking as exogenous the choices of the first sub-agent grossed up by a factor of N. A third sub-agent, from a third region, goes next, taking as exogenous the choices of the first two sub-agents, all grossed up by a factor of N, and so on until all regions have been represented once. Then the process iterates until a fixed point is achieved. From the second iteration on, there are two sources of exogenous values: first, as above, the aggregation of choices of sub-agents from other regions, all grossed up by a factor of N; second, the choices made by the same agent in the previous iteration, grossed up by a factor of N-1, representing the other sub-agents within the same region. When $N = 1$, the distinction between agent and sub-agent vanishes and only the inter-regional externalities are operative in the algorithm.

In Figure 2 we present a comparison of aggregate fuel use (emissions) under NA $(N = 100)$, RA $(N = 1)$, and the Pareto optimal scenario.[16] As expected, aggregate fuel use varies directly with the externality proportion: less fuel is used in the non-cooperative game than in BAU, and less again in the Pareto scenario. In Table 2, we present carbon taxes for both the non-cooperative (RA) and Pareto optimal scenarios. The carbon taxes, which are equivalent to

[15] In contrast, in an earlier version, Nordhaus and Yang (1996) provide solutions for BAU, non-cooperative and socially optimal scenarios. Like DICE, this earlier version employs an emissions control rate, μ. As observed above, the authors' approach to BAU using this parameter is not theoretically consistent, as some of the externality is internalized.

[16] Following Nordhaus and Boyer (2000), the Pareto optimum here is conditioned on zero transfers, such as would obtain under a carbon tax in which the amount paid by each region was returned in a lump-sum manner, or alternatively under a tradable permit system in which each region was grandfathered an amount of permits equal to its final emissions.

marginal abatement costs in equilibrium, are significantly higher in the Pareto scenario than in the non-cooperative game, which reflects the higher abatement levels under Pareto.

But the lessons of this section do not lie in the numbers, since Nordhaus and Boyer already employ theoretically consistent methods for BAU and Pareto. Rather, the main lessons concern the ease of the transition from BAU to the non-cooperative game and the ease of solution under the NA approach. The ease of transition flows from the structural similarity of the modelling between BAU and the non-cooperative scenario under the NA approach – one only needs to change the value of N. Moreover, the structural similarity can be carried over to the Pareto solution as well if one employs, as we have, an iterative algorithm with carbon tax equal to the aggregate of the regional shadow values of atmospheric carbon.

The ease of solution of the NA approach is reflected in the speed. In these applications, convergence to a solution was obtained very quickly – usually within seven iterations. Both our BAU and non-cooperative abatement programs for RICE 99 solved in just over one minute (to be precise 1:14 and 1:27 respectively) on an Intel Pentium III processor with 662 MHz and 256 MB RAM. Our Pareto abatement program, which includes an initial run of the non-cooperative scenario, solved in 1:58. In contrast, Nordhaus and Boyer's BAU program (pp.189-205) took 7 minutes on the same computer, while their Pareto program, which includes an initial solve of BAU, took 15 minutes. One must be cautious, however, in attributing this improvement in speed to the NA approach entirely, as their programs differ from ours in three other ways: (1) aggregate welfare optimization with time-variant Negishi weights, (ii.) use of the MINOS5 optimizer, and (iii.) lack of programmer scaling. Nonetheless, one may legitimately conclude that the NA approach does not introduce any undo complication into the model.

IV. Conclusion

The paper has shown how an N-agent approach can be used to estimate the no-policy or business-as-usual equilibrium in the context of a pollution externality, while otherwise preserving the essential structure of a representative-agent model. This approach keeps virtually all of the pollution cost external, and in the limit it is equivalent to the Myopic Firms model, in which the entire cost is external.

The solution concept involves a Nash equilibrium of a dynamic game. Because the players are identical, and because there is no trade apart from the public bad of pollution, the equilibrium is easily solved with a simple iterative algorithm. The additional complexity, compared with a standard representative-agent model, is minimal. Indeed, in some of our trials, the NA method solved more quickly than the original methods.

An invariance requirement is proposed in which the NA method replicates the solution of the RA method in the absence of pollution damages. This equivalence requirement places restrictions on the transformation of constraint and utility functions from RA to the NA framework, restrictions which are easily met in practice.

The NA method is applicable to both single-region and multi-region models. In the former, the RA approach corresponds with the social optimum, and therefore authors have resorted to ad hoc methods to estimate BAU. In the latter, the RA approach corresponds with a non-cooperative game in which each region takes into account self-inflicted damages and chooses abatement optimally while ignoring the effects on other regions. Again, BAU must be estimated in a different fashion, as well as the social optimum.

Comparison of NA and ad hoc methods for BAU in the DICE, ENTICE and RICE models suggests that the NA method is very flexible in terms of the transition from one scenario to another (e.g. BAU to non-cooperative game) and it compares favourably with ad hoc methods in terms of solution speed. Moreover, theoretical consistency recommends it, since one cannot know the efficacy of ad hoc methods except by comparison with such an approach.

Appendix – Long-Run Equilibrium of the DICE Model

The equations of the DICE model are

$$Y(t) = \left(1 - b_1\mu^{b_2}\right)\left(A(t)K(t)^{\gamma} L(t)^{1-\gamma}\right)/\left(1 + A_1 TE(t)^2\right) \tag{A1}$$

$$E(t) = \sigma(t)\left[1 - \mu\right]A(t)K(t)^{\gamma} L(t)^{1-\gamma} \tag{A2}$$

$$FORC(t) = 4.1\left[\ln(M(t)/590)/\ln 2\right] + FORCOTH(t) \tag{A3}$$

$$K(t+1) - K(t) = 10\left[Y(t) - C(t) - \delta_K K(t)\right] \tag{A4}$$

$$M(t+1) - M(t) = 10\beta E(t) - \delta_M\left(M(t) - 590\right) \tag{A5}$$

$$TE(t+1) - TE(t) = C1[FORC(t) - (\lambda + C3)TE(t) + C3TL(t)] \tag{A6}$$

$$TL(t+1) - TL(t) = C4[TE(t) - TL(t)] \tag{A7}$$

Definitions of variables and parameter values are given in Nordhaus (1994). In BAU, $\mu(t) = 0 \ \forall t$. The representative agent's problem is:

$$\max \sum_{t=1}^{T} L(t)\frac{\ln\left(C(t)/L(t)\right)}{(1+10\delta)^{t-1}} \quad \text{subject to (A1) – (A7).}$$

The control variables are C, Y, E, and FORC. The state variables are K, M, TE, and TL. We define current-value dual variables p^Y, p^E, p^F, p^K, p^M, p^{TE} and p^{TL}, corresponding with (A1) – (A7) respectively.

By means of the discrete-time maximum principle, we obtain the first order conditions which must be satisfied in all periods $t = 1,\ldots,T$:

$$10P^K(t) = L(t)/C(t)$$

$$\left(1 + 10\gamma\frac{Y(t+1)}{K(t+1)} - 10\delta_K\right)P^K(t+1) - (1+10\delta)P^K(t) - 10\beta\gamma\frac{E(t+1)}{K(t+1)}P^M(t+1) = 0$$

$$(1-\delta_M)P^M(t+1) - (1+10\delta)P^M(t) + \frac{4.1}{\ln 2}\frac{C_1}{M(t+1)}P^{TE}(t+1) = 0$$

$$\left(C_1(\lambda + C_3) - 1\right)P^{TE}(t+1) + (1+10\delta)P^{TE}(t) - C_4 P^{TL}(t+1) - \frac{10P^K(t+1)Y(t+1)}{\left(1 + A_1 TE(t+1)^2\right)}2A_1 TE(t+1) = 0$$

$$C_1 C_3 P^{TE}(t+1) + (1 - C_4)P^{TL}(t+1) - (1+10\delta)P^{TL}(t) = 0$$

plus the constraints (A1) – (A7).[17]

[17] The original problem yields eight first-order conditions. We have combined some of them to yield the five equations shown here. In doing so, we have substituted out the dual variables p^Y, p^E and p^F.

In the steady state, $C(t) = C(t+1)$, $Y(t) = Y(t+1)$, etc. Thus the constraints and first-order conditions reduce to

$$Y = \left(AK^{\gamma}L^{1-\gamma}\right)/\left(1 + A_1 TE^2\right) \tag{A8}$$

$$E = \sigma AK^{\gamma}L^{1-\gamma} \tag{A9}$$

$$FORC = 4.1\left[\ln(M/590)/\ln 2\right] + FORCOTH \tag{A10}$$

$$Y = C + \delta_K K \tag{A11}$$

$$10\beta E = \delta_M \left(M(t) - 590\right) \tag{A12}$$

$$FORC = (\lambda + C_3)TE - C_3 TL \tag{A13}$$

$$TE = TL \tag{A14}$$

$$10P^K = L/C \tag{A15}$$

$$10\left(\gamma\frac{Y}{K} - \delta_K - \delta\right)P^K = 10\beta\gamma\frac{E}{K}P^M \tag{A16}$$

$$(\delta_M + 10\delta)P^M = \frac{4.1}{\ln 2}\frac{C_1}{M}P^{TE} \tag{A17}$$

$$\left(C_1(\lambda + C_3) + 10\delta\right)P^{TE} = C_4 P^{TL} + \frac{10P^K Y}{\left(1 + A_1 TE^2\right)}2A_1 TE \tag{A18}$$

$$C_1 C_3 P^{TE} = (C_4 + 10\delta)P^{TL} \tag{A19}$$

In addition, we require long-run values for the exogenous parameters L, A, σ, and FORCOTH. Based on the dynamics presented in Nordhaus (1994), we obtain $\lim_{t\to\infty} A(t) = 0.038$, $\lim_{t\to\infty} L(t) = 10571.965$, $\lim_{t\to\infty} \sigma(t) = 0.179$, and $\lim_{t\to\infty} FORCOTH(t) = 1.420$.

The solution to (A8) – (A19) yields the long-run equilibrium of the DICE model. We obtained the solution in GAMS as a pseudo-optimization utilizing a dummy objective function unrelated to the equations.

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Table 1
DICE 94: Long-Run Equilibrium

	$\delta = 0.03$			$\delta = 0.01$		
	RA	NA	% dev	RA	NA	% dev
Y	159.45	159.79	0.21	170.47	171.59	0.65
C	132.37	132.46	0.07	134.33	134.41	0.06
K	416.67	420.49	0.91	555.96	571.92	2.79
E	31.69	31.76	0.23	34.06	34.30	0.71
TE	7.86	7.87	0.10	8.11	8.13	0.30
DAM	14.57	14.63	0.41	16.57	16.77	1.24

Y,C,K,DAM are measured in $US trillions (1990).

E is measired in billion tons carbon.

TE is measured in degrees C (deviation from pre-indusrial average).

NA is based on N = 300.

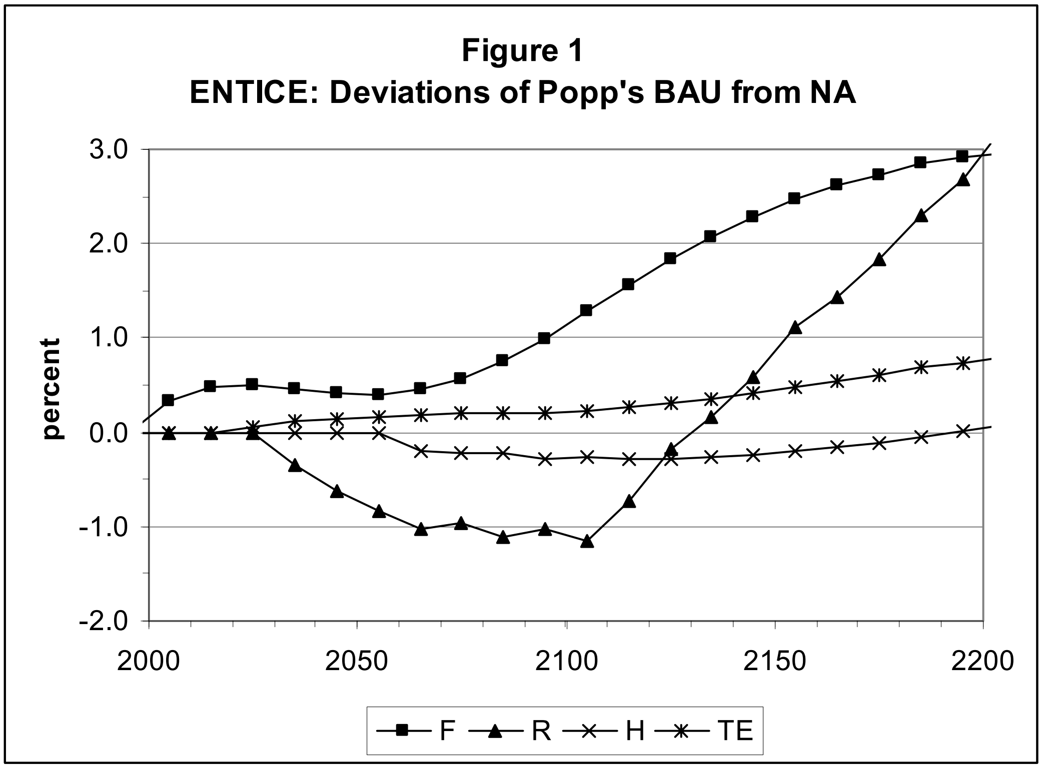
Figure 1
ENTICE: Deviations of Popp's BAU from NA
3.0
2.0
1.0
0.0
-1.0
-2.0
percent
2000
2050
2100
2150
2200
F
R
H
TE

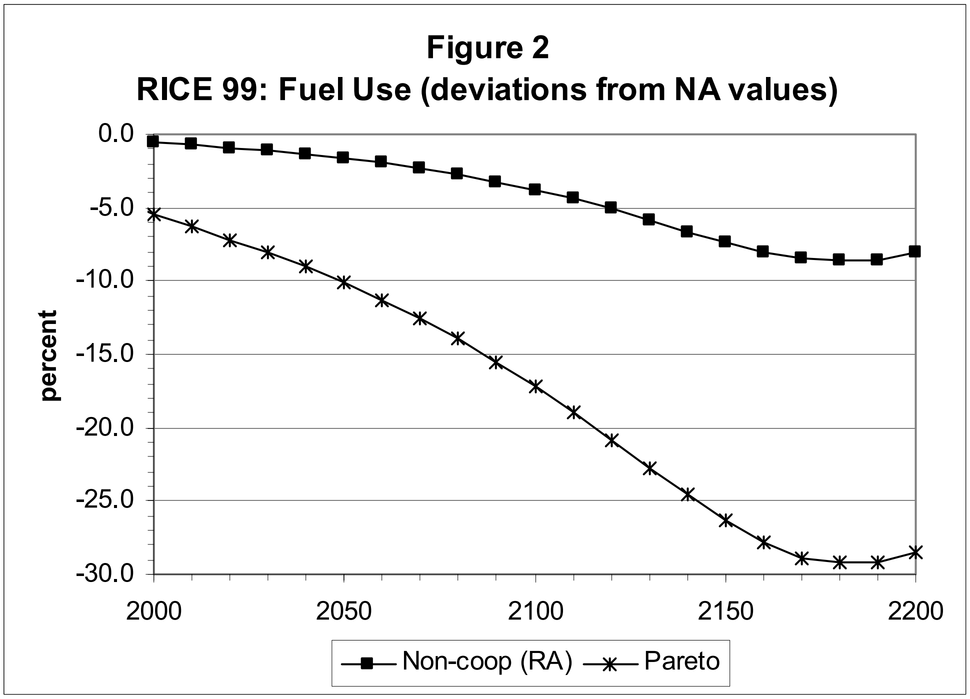
Figure 2
RICE 99: Fuel Use (deviations from NA values)
percent
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
2000
2050
2100
2150
2200
Non-coop (RA)
Pareto

Table 2
RICE 99: Carbon Taxes ($US 1990 per tonne)

Year	Pareto	Non-cooperative (RA) solution							
		USA	OHI	Europe	EE	MI	LMI	China	LI
2000	9.77	0.88	0.33	4.30	0.00	1.06	1.10	0.08	2.18
2010	13.61	1.31	0.69	5.48	0.04	1.40	1.58	0.17	3.12
2020	17.91	1.77	1.08	6.70	0.12	1.76	2.14	0.29	4.23
2030	22.68	2.28	1.51	7.93	0.23	2.17	2.79	0.45	5.51
2040	28.00	2.81	1.96	9.19	0.36	2.62	3.55	0.67	7.00
2050	33.93	3.37	2.44	10.51	0.53	3.12	4.41	0.95	8.71

OHI = Other High Income, EE = Russia & Eastern Europe, MI = Middle Income, LMI = Low Middle Income, LI = Low Income