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Analysis and Cancellation of Inter-carrier Interference for OFDM Systems over Time-variant Multipath Fading Channels

by

Yunjun Zhang, B.Eng., M.Eng.

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Ottawa-Carleton Institute for Electrical and Computer Engineering
School of Information Technology and Engineering
Faculty of Engineering, University of Ottawa
Ottawa, Ontario, Canada

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Abstract

The orthogonality among the subcarriers of OFDM systems may be impaired by the time-selectivity of the fading channels. The loss of the orthogonality results in ICI, and if it is not treated appropriately, the system performance may not be improved only by increasing the signal-to-noise ratio. In other words, ICI results in error floors. This research work concentrates on the ICI analysis and cancellation, and also on the effects of channel time-selectivity on the OFDM systems over frequency-selective time-variant mobile fading channels. In the first part of this study, a general time-variant frequency-selective WSSUS fading channel model is further characterized to support the OFDM ICI analysis, thus the obtained results are applicable for many specific channels. We then identify the cause of the ICI, i.e. how the orthogonality among the subcarriers of OFDM systems is impaired. The average ICI power and its distribution are obtained based on the general time-variant frequency-selective WSSUS fading channel model. To mitigate the ICI caused by the channel time-selectivity for OFDM systems, in the second part of this study, an efficient ICI cancellation scheme is designed based on the obtained ICI power distribution. The simulation results indicate that a significant performance improvement can be achieved. For OFDM systems, differential encoding can be performed not only between the information bits of the same subcarrier of the consecutive OFDM symbols (inter-frame differential encoding), but also between the information bits of the adjacent subcarriers of the same OFDM symbol (inter-carrier differential encoding). In the third part of this study, we compare the performance of inter-frame and inter-carrier differential detection for OFDM systems over multipath time-variant mobile fading channels. The objective is to identify which differential encoding scheme (inter-frame or inter-carrier) is more robust to the channel time-selectivity, when OFDM systems are experiencing both frequency-selective and time-selective fading. The conditions under which the inter-carrier differential encoding outperforms the inter-frame differential encoding and *vice-versa* are provided.

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List of Abbreviations

ACI	Adjacent Channel Interference
ADSL	Asymmetric Digital Subscriber Lines
ATM	Asynchronous Transfer Mode
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BEP	Bit Error Probability
BPSK	Binary Phase-Shift-Keying
CD	Coherent Detection
CIR	Channel Impulse Response
COFDM	Coded OFDM
CP	Cyclic Prefix
DAB	Digital Audio Broadcasting
DFT	Discrete Fourier Transform
DPSK	Differentially Coherent Phase-Shift-Keying
DQPSK	Differentially Coherent Quadrature Phase-Shift-Keying
DMT	Discrete Multi-Tone
DSP	Digital Signal Processing
DVB	Digital Video Broadcasting
DWMT	Discrete Wavelet Multi-Tone
ETSI	European Telecommunications Standards Institute
FDM	Frequency Division Multiplexing
FER	Frame Error Rate
FFT	Fast Fourier Transform
HDSL	High bit-rate Digital Subscriber Lines
HIPERLAN	High-performance European Radio LAN
HPA	High-Power-Amplifier
ICI	Inter-Carrier (Inter-Channel) Interference

IC-DD	Inter-Carrier Differential Detection
IDFT	Inverse Discrete Fourier Transform
IEEE	Institute of Electrical and Electronics Engineers
IF-DD	Inter-Frame Differential Detection
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbol Interference
LAN	Local Area Network
LMS	Least-Mean-Square
LOS	Line of Sight
MCM	Multi-Carrier Modulation
MMSE	Minimum Mean Square Error
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
OPS	Optimized Pilot Symbol
OQPSK	Offset Quadrature Phase-Shift-Keying
PAPR	Peak-to-Average Power Ratio
PCS	Personal Communication Services
PLL	Phase-Locked Loop
PSAM	Pilot Symbol Assisted Modulation
PSD	Power Spectral Density
PSK	Phase-Shift-Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase-Shift-Keying
RLS	Recursive Least-Squares
SER	Symbol Error Rate
SENR	Signal to Effective Noise Ratio
SINR	Signal to Interference plus Noise Ratio
SIR	Signal to Inter-carrier (Inter-channel) Interference Ratio
SNR	Signal to Noise Ratio
TCM	Trellis Coded Modulation
TWT	Traveling Wave Tube
WSSUS	Wide Sense Stationary Uncorrelated Scattering

List of Symbols

$[\mathbf{o}]^T$	Transpose of matrix $[\mathbf{o}]$
$[\mathbf{o}]^{-1}$	Inverse of matrix $[\mathbf{o}]$
a_{ij}	The puncturing table entries of rate-compatible punctured convolutional codes
a_n	Symbol modulated by the n^{th} subcarrier
$\mathbf{a}$	$[a_0, a_1, \dots, a_{N-1}]^T$
$\tilde{\mathbf{a}}$	Estimated $\mathbf{a}$ with the punctured ICI cancellation
β	Squared normalized spaced-time correlation factor
B	The bandwidth of uncoded OFDM system with cyclic prefix added
B_C	The bandwidth of coded OFDM system with cyclic prefix added
c	Speed of light
c_d	The distance spectra of convolutional code
c_l	ICI contributed to the l^{th} subcarrier by all the neighboring subcarriers
c_l^n	ICI contributed to the l^{th} subcarrier by the n^{th} subcarrier ($n \neq l$)
d_f	The free distance of convolutional code
$D_l^{(n-l)_N}$	The ratio of c_l^n to a_n
$\mathbf{D}$	Distortion matrix
$\tilde{\mathbf{D}}$	Punctured distortion matrix
$\delta(x)$	delta (Dirac impulse) function
E_b/N_0	Signal-to-noise ratio per bit
E_S/N_0	Signal-to-noise ratio per symbol
e_l	Effective additive noise of the l^{th} subcarrier
$\mathcal{F}_x$	<i>Fourier</i> transform relative to variable x
$\mathcal{F}_x^{-1}$	Inverse <i>Fourier</i> transform relative to variable x
$(\Delta f)_c$	Coherence bandwidth of the fading channel
$(\Delta t)_c$	Coherence time of the fading channel
η	System transmission efficiency with cyclic prefix added

$E\{xx^*\}$	Average power of random variable x
f_c	Carrier frequency
f_D	Maximum <i>Doppler</i> frequency shift
$f_D T_a$	Normalized <i>Doppler</i> frequency shift
$f(t)$	AWGN of the channel
$J_0(\bullet)$	The zeroth-order <i>Bessel</i> function of the first kind
K	The constraint length of convolutional codes
$h(\tau, t)$	Time-variant channel impulse response
$h(\tau)$	Time-invariant component of $h(\tau, t)$
$\Delta h(\tau, t)$	Time-variant component of $h(\tau, t)$
$h_{m,k}$	Discrete format of $h(\tau, t)$
h_m	Discrete format of $h(\tau)$
$\Delta h_{m,k}$	Discrete format of $\Delta h(\tau, t)$
γ	The average power ratio of $h(\tau)$ to $\Delta h(\tau, t)$ regardless of the time-selectivity
L_{ij}	The metric increment of rate-compatible punctured convolutional codes
m	The m^{th} path of the fading channel
M	The total number of paths of the fading channel
n_l	AWGN contributed to the l^{th} subcarrier
N	OFDM frame size, i.e. the number of subcarriers in the OFDM system
N_G	The number of cyclic prefix samples per OFDM symbol for uncoded system
N_G^C	The number of cyclic prefix samples per OFDM symbol for coded system
$\otimes$	Cyclic convolution
p	The puncturing period of rate-compatible punctured convolutional codes
P_b	Bit error probability of uncoded system
P_b^C	Bit error probability of coded system
P_d	The error event probability of rate-compatible punctured convolutional codes
$P_h(\Delta t, \tau)$	Delay cross-power spectral density
$P_H(\nu, \Delta f)$	<i>Doppler</i> cross-power spectral density
$P_s(\nu, \tau)$	Scattering function
$\Psi_T(\Delta t)$	Spaced-time correlation function
Q	Puncturing index
$R_T(\Delta t, \Delta f)$	Spaced-frequency spaced-time correlation function
T_a	Serial symbol duration before adding cyclic prefix
T_D	Maximum delay spread of the fading channel

T_r	The sampling interval of uncoded OFDM system after adding cyclic prefix
T_r^C	The sampling interval of coded OFDM system after adding cyclic prefix
T_S	OFDM symbol duration
ρ_e	Estimation error average power ratio of IF-DD to IC-DD
σ_h^2	Average power of $h(\tau)$
$\sigma_{\Delta h}^2$	Average power of $\Delta h(\tau, t)$ when $f_D \Delta t = 0$
σ_m^2	Average power of h_m
$\sigma_{\Delta m}^2$	Average power of $\Delta h_{m,k}$ when $f_D T_r(k - k') = 0$
σ_f^2	Average power of AWGN
v_s	Vehicle speed
$\mathbf{W}$	$[n_0, n_1, \dots, n_{N-1}]^T$
$\hat{\mathbf{W}}$	Effective additive noise with the full ICI cancellation
$\tilde{\mathbf{W}}$	Effective additive noise with the punctured ICI cancellation
$\mathbf{X}$	Puncturing matrix
Z_l	Demodulated symbol of the l^{th} subcarrier
$\mathbf{Z}$	$[Z_0, Z_1, \dots, Z_{N-1}]^T$
$\hat{\mathbf{Z}}$	Estimated $\mathbf{Z}$ with the full ICI cancellation
$\tilde{\mathbf{Z}}$	Estimated $\mathbf{Z}$ with the punctured ICI cancellation
ζ_l	Attenuation factor of the l^{th} subcarrier

Chapter 1

Introduction

1.1 Motivation

Traditional communication systems use serial transmission (single-carrier), where data are transmitted sequentially (symbol by symbol) through a communication channel. Even with low complexity, this scheme works well for both wired and wireless low speed communication systems, where the symbol period is much longer than the maximum excess delay¹ of the channel. With the increasing demand of high data rate to support multimedia, high quality audio and video services, high speed transmissions become necessary in order to provide such services. To achieve the required transmission rate, a serial transmission system has to either increase its signal constellation or reduce the symbol period. When the symbol interval is reduced, the maximum excess delay of the channel may become significant and cause intersymbol interference (ISI) [2]. When the maximum excess delay reaches more than 20% of the symbol period, the use of an equalizer becomes necessary in order to mitigate ISI [3], [4], otherwise, the system performance is constrained by an error floor (irreducible bit error rate). However, equalization can be computationally quite intensive.

Parallel transmission or multi-carrier modulation (MCM) scheme such as frequency division multiplexing (FDM) is an alternative method to achieve high speed and mitigate

¹*Maximum excess delay* is defined as the relative delay of the last received copy of the transmitted signal through a multipath channel compared to the first arriving copy of the transmitted signal [1].

the degradation caused by ISI. By transmitting a number of symbols from the same source simultaneously through several different subchannels obtained by dividing the overall available bandwidth, the symbol period for each subchannel² can be extended, making it less susceptible to the excess delay of the channel. However, because of its need for guard bands to separate the subchannels, FDM is not spectrally efficient. Orthogonal Frequency Division Multiplexing (OFDM), on the other hand, allows the spectra of subchannels to overlap, but remain orthogonal, making high spectrum efficiency achievable from the spectrum utilization point of view [5], [6].

Spreading in both time domain (due to multipath propagation) and frequency domain (due to the *Doppler* spreading) characterizes the transmission over wireless mobile channels [7]. The time domain dispersion of the channel impairs the orthogonality between successive OFDM symbols, causing ISI in OFDM transmissions. Meanwhile, the frequency domain dispersion of the channel mainly impairs the orthogonality among subcarriers and introduces inter-carrier (inter-channel) interference (ICI). Therefore, OFDM systems have to be sufficiently resistant to both time and frequency dispersion. By adding a cyclic prefix with the (minimum) length of the maximum excess delay of the channel to each OFDM symbol, ISI can be totally eliminated [6]. However, the sensitivity of OFDM to channel frequency dispersion (time-selectivity) remains unmitigated.

1.2 Research Objectives

The orthogonality among the subcarriers of OFDM could be impaired because of the channel time-selectivity. The loss of the orthogonality results in ICI, and if not treated appropriately, the system performance is constrained by an error floor. In the literature, the ICI analysis for fast-fading channels is widely reported. However, when the OFDM symbol duration is close to the channel coherence time, where it is not always sufficient to classify the channel to be fast or slow fading, would ICI exist? If so, how can one evaluate it? For fast fading channels, where ICI causes the most serious performance degradation,

²In this thesis, *subchannel* and *subcarrier*, *inter-channel* and *inter-carrier* are used interchangeably.

how can one design effective and efficient ICI cancellation schemes with a controllable complexity? In other words, what are the characteristics of the ICI caused by channel time-selectivity, based on which effective and efficient ICI cancellation schemes could be designed with a flexible complexity?

OFDM is a two-dimensional scheme: when performing the discrete *Fourier* transform based modulation, the data sequence is inserted along both the time axis (embedded to different OFDM symbols) and the frequency axis (inserted in different subcarriers). Thus differential encoding/decoding could be performed along either the time or the frequency axis with almost the same complexity. For a given system configuration and channel condition, how can one determine which differential encoding/decoding scheme would achieve the best performance? On the other hand, under which kind of system configurations and channel conditions, one differential encoding/decoding scheme would achieve a better performance than the other one? In this thesis, we try to answer these questions.

The major objectives of this research are the ICI analysis and cancellation for OFDM systems operating over frequency-selective time-variant fading channels, and the evaluation of the channel time-selectivity and frequency-selectivity effects on the performance of OFDM systems. In this research work, ISI free transmission is assumed. In other words, the length of the cyclic prefix added to each OFDM symbol is not shorter than the maximum excess delay of the fading channel. It is also assumed that the studied OFDM systems are fully loaded, i.e. all subcarriers are used to transmit data. The ultimate goal of this research is to make contributions to the evolution of OFDM for mobile applications. The application of OFDM to mobile communication systems constitutes the main subject of this thesis.

1.3 Thesis Outline

This thesis is organized as follows. In Chapter 2, we give some review and background information on OFDM systems. In Chapter 3, we present a doubly dispersive Wide

Sense Stationary Uncorrelated Scattering (WSSUS) fading channel model which is further characterized for the ICI analysis of the OFDM systems over fading channels. This general channel model leads to several widely used fading channel models, thus the results obtained based on this channel model are applicable to different types of fading channels. In Chapter 4, we present the ICI analysis for OFDM systems. In Chapter 5, we present the frequency-domain ICI cancellation equalizer and its performance. The designed ICI cancellation equalizer is based on the analysis presented in Chapter 4. In Chapter 6, we consider the case where the OFDM system experiences both channel time-selectivity and frequency-selectivity, and investigate the conditions under which the channel time-selectivity causes more performance degradation than the channel frequency-selectivity and *vice-versa*. This is performed by comparing the bit error performance of inter-frame differential detection and inter-carrier differential detection for both uncoded and coded OFDM systems using Quadrature Phase-Shift-Keying (QPSK) as the bit mapping scheme. Conclusions and recommendations for future research work are given in Chapter 7.

1.4 Major Contributions

The major contributions of this thesis include:

- A general time-variant frequency-selective WSSUS fading channels (not necessarily with zero mean) is characterized for channel time-selectivity evaluation purpose. Two parameters denoted as γ and β are defined to evaluate the channel time-selectivity for OFDM systems: γ represents the relative power of the time-variant component of the channel impulse response, and β is the square of the normalized spaced-time correlation function;
- ICI analysis is performed based on the general WSSUS fading channel model. The results are applicable to many specific fading channels, including the case that the channel coherence time is close to the symbol duration, where it is not always sufficient to characterize the channel to be fast or slow fading;

- An effective frequency-domain ICI cancellation equalizer is designed based on the ICI power distribution obtained through the ICI analysis. The simulation results indicate that the designed ICI cancellation equalizer can significantly reduce the error floor caused by ICI. The complexity of the developed ICI cancellation equalizer is controllable by choosing a variable *puncturing index* based on the required performance;
- The effect of both time-selective fading and frequency-selective fading on the performance of OFDM systems is investigated by evaluating the performance of inter-carrier differential detection and inter-frame differential detection;
- The impact of channel coding on the system performance is explored while the OFDM system is experiencing both time-selective fading and frequency-selective fading. The lowest code rate, which could be used in the coded OFDM systems, is also obtained for a given number of subcarriers and a given number of cyclic prefix samples used.

Chapter 2

System Overview and Background

This chapter presents an overview for OFDM: its evolution, applications, system model, advantages and disadvantages. The roles of channel coding in OFDM systems are also addressed, and the channel coding schemes widely used in OFDM systems are briefly discussed. The inherent orthogonality among the subcarriers of OFDM makes it vulnerable to many kinds of distortions. Channel time-selectivity is only one of the distortions introducing ICI. The ICI caused by the distortions other than channel time-selectivity for OFDM systems is also introduced in this chapter. We will discuss the ICI caused by channel time-selectivity in details in the following chapters.

2.1 The Evolution of OFDM

The first communication system utilizing parallel transmission could be dated back to the 1950's [8], the *Kineplex* system, where the data sequence is divided into several interleaved bit streams, and transmitted via several carriers. The subchannels (subcarriers) of the earliest multi-carrier systems do not overlap in frequency and are separated by band-pass filters at the receiver side. The parallel transmission system allowing subchannel spectra overlapping was not reported until 1966 by Chang in [9].

Transmitting data simultaneously through a linear bandlimited channel without ISI and ICI is possible if the overlapped subchannels meet a certain orthogonality condition which is based on the generalized *Nyquist* criterion [10]. Shortly after the publication of Chang's

work, the performance of the designed system was analyzed by Saltzberg [11], Chang and Gibby [12], where it was concluded that the main concern of designing an efficient parallel transmission system should be on reducing crosstalk instead of enhancing individual subchannels, because the dominant distortion is introduced by the crosstalk. Each subcarrier of the system designed by Chang in [9] is strictly bandlimited, and only the two nearest frequency neighboring subcarriers are overlapped. The orthogonality between the overlapped subcarriers is maintained by a time “offseted” (or staggered) Quadrature Amplitude Modulation (QAM). Such system was termed orthogonally multiplexed QAM system [13], which is a special form of OFDM. The complexity and cost on building oscillator (sinusoidal generator) bank for the subcarriers, transmitter and receiver filter banks constrain the application of the earliest MCM system to only high frequency military systems. When the number of subchannels is large, such a system becomes prohibitively complicated for practical applications.

A major breakthrough in the development of OFDM was contributed by Weinstein and Ebert. In their work [14], they proposed to use inverse discrete *Fourier* transform (IDFT) and discrete *Fourier* transform (DFT) to perform baseband modulation and demodulation. The utilization of IDFT/DFT in modulator/demodulator eliminates the subcarrier oscillators and the filter banks, and makes fully digital implementation possible. Instead of using an empty guard interval to eliminate ISI, Peled and Ruiz [15] proposed to use a *cyclic prefix* or *cyclic extension* which is part of each OFDM symbol as the guard interval. Adding the cyclic prefix converts the linear convolution of transmitted signal with the channel response into the cyclic convolution, at the same time, maintain the orthogonality between OFDM symbols (i.e. avoid ISI) when the length of the cyclic prefix is longer than the maximum excess delay of the channel [6].

The recent intensive research on OFDM started in the middle 1980's, when a number of European countries began to show increasing interest in offering digital audio broadcasting (DAB) services with compact disk quality designed particularly for mobile receivers [16], that is, with a targeted bit error rate (BER) in the order of 10^{-4} . Following the great

success of DAB systems, OFDM was proposed for terrestrial digital video broadcasting (DVB-T) systems [17], [18]. The DVB system has a similar system architecture as DAB, but with much lower BER requirement (in the order of 10^{-9}) [19], [20]. The OFDM scheme has also been considered as a candidate for digital transmission at high data rates over twisted pair telephone subscriber loops such as high bit-rate digital subscriber lines (HDSL) and asymmetric digital subscriber lines (ADSL), where the same transmission technique is adopted but termed DMT (Discrete Multi-Tone)¹ [21], [22].

The application of OFDM to wireless communication systems was first reported by Cimini in 1985 [5], and the research on this subject started to draw more and more attention at the beginning of the 1990's ([23] to [29], etc.). The OFDM technique has been under intensive investigation for its application to high speed wireless multiple access communication systems [30], [31], [32]. Recently, OFDM has been adopted in the IEEE 802.11 standard for high-speed wireless Local Area Networks (LANs) in the 5 GHz band (IEEE 802.11a) and European Telecommunications Standards Institute (ETSI) High-performance European Radio LAN (HIPERLAN, type 2) [33]. A more detailed introduction on OFDM and its applications may be found in [34].

2.2 OFDM System Model

If it is not specified, the pulse shape adopted by OFDM systems is the rectangular pulse, which makes the utilization of IDFT and DFT to perform modulation and demodulation straightforward [6]. Usually, such OFDM systems are classified as conventional OFDM systems. This section presents the system models of the conventional OFDM systems.

2.2.1 Analog Implementation

When the rectangular pulse is adopted by an OFDM system with N subchannels (subcarriers), the system block diagram of analog implementation may be presented by Figure 2.1 [5], where $f_n = f_0 + n\Delta f$, $\Delta f = \frac{1}{T_S}$, $0 \leq n \leq N - 1$, T_S stands for the transmitted

¹Typically in OFDM, the subchannels carry the same signal constellation, whereas in DMT usually the constellation sizes are different.

OFDM symbol duration, and $T_S = NT_a$, where T_a is the symbol duration of the serial input data sequence. The function of the serial to parallel conversion is to increase the symbol duration in order to make the transmitted signal less vulnerable to the channel frequency selectivity. The transmitted signal $s(t)$ may be written as [5]:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j(2\pi f_n t)} \quad \text{for } 0 \leq t < T_S \quad (2.1)$$

where $\frac{1}{\sqrt{N}}$ is a scaling factor.

2.2.2 Digital Implementation

By sampling the baseband (setting $f_0 = 0$) analog signal $s(t)$ given in (2.1) at discrete intervals $t = kT_a$ ($0 \leq k \leq N - 1$), we have,

$$\begin{aligned} S_k &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j(2\pi f_n kT_a)} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j2\pi n \left(\frac{kT_a}{T_S}\right)} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j2\pi n \left(\frac{k}{N}\right)} \\ &= IDFT\{a_n\} \end{aligned} \quad (2.2)$$

The corresponding system block diagram for baseband digital implementation is presented in Figure 2.2. Due to the high efficiency (or low complexity) of fast *Fourier* transform (FFT) and inverse FFT (IFFT), the IDFT/DFT is usually replaced by IFFT/FFT to perform modulation and demodulation.

The cyclic prefix is a copy of the last part of each OFDM symbol and it is prefixed to the transmitted OFDM symbol (see Figure 2.3). At the reception, the “smeared” cyclic prefix (which interferes with the previous OFDM symbol) is simply discarded. The addition of a cyclic prefix to each OFDM symbol is to avoid ISI, and it also converts the linear convolution of the data sequence with the channel impulse response into a circular convolution [6], [26]. ISI can be totally eliminated if the cyclic prefix is longer than the maximum excess delay of the channel. However, adding cyclic prefix reduces the

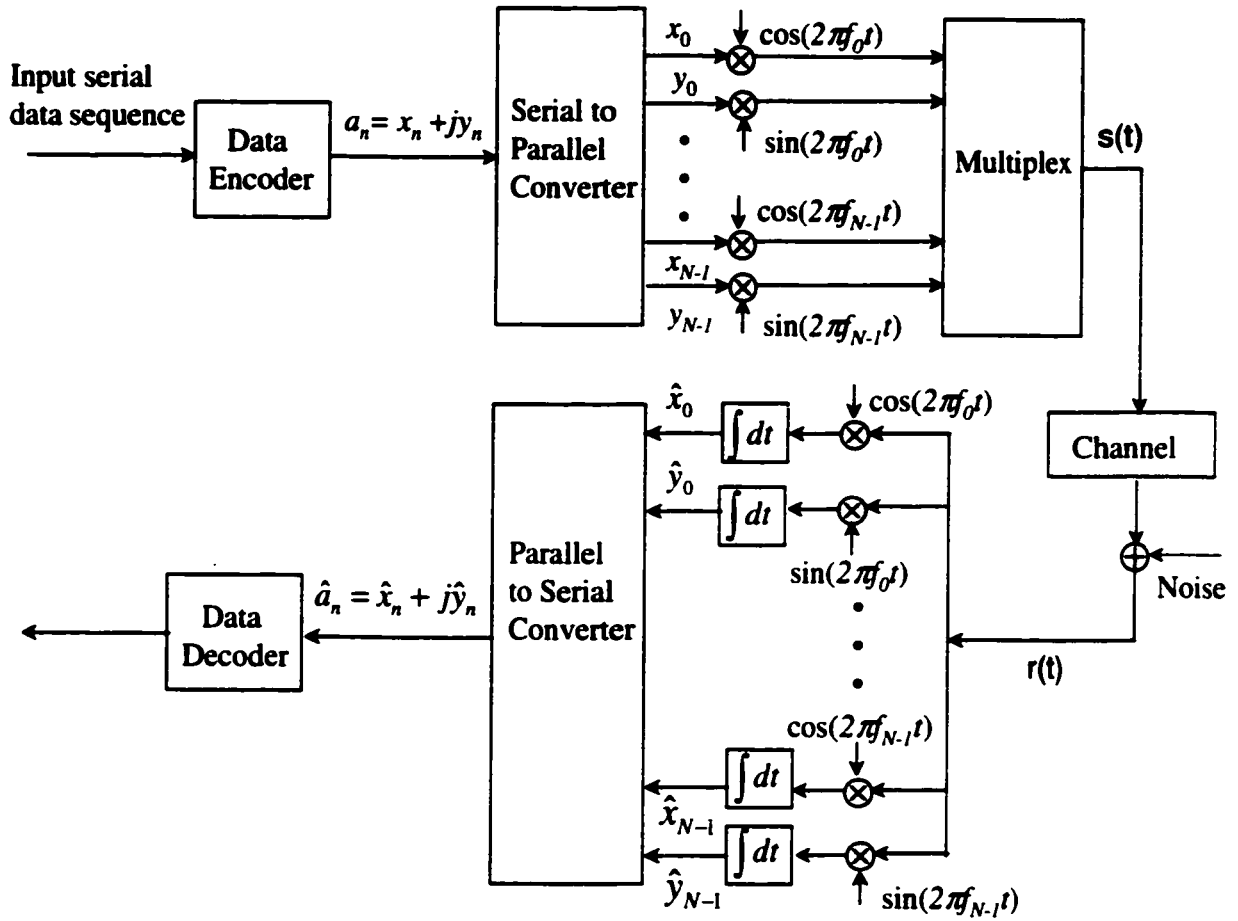


Figure 2.1: Block diagram of conventional OFDM systems. analog implementation.

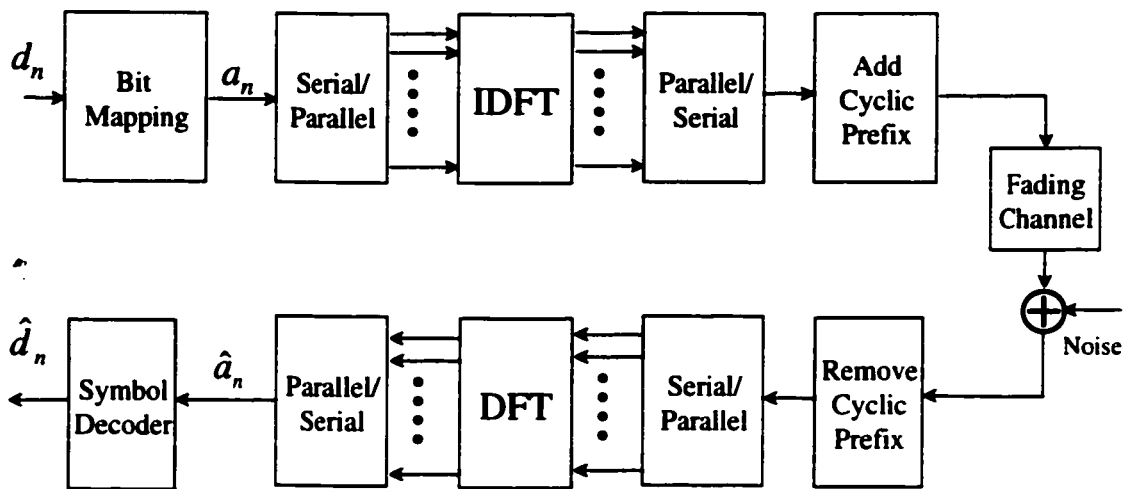


Figure 2.2: Block diagram of conventional OFDM systems, digital implementation.

transmission efficiency of the OFDM system. Using a smaller number of subcarriers in the system results in a lower transmission efficiency. Assuming the number of the cyclic prefix samples is N_G which is the equivalent number of samples for the maximum excess delay of the channel, the transmission efficiency may be written as:

$$\eta = \frac{N}{N + N_G} \quad (2.3)$$

For delay sensitive services, adding cyclic prefix should not change the OFDM symbol duration. This means that the spacing between the samples of the original OFDM symbol and the samples of the added cyclic prefix need to be reduced in order to keep the same information transmission rate. Assuming the reduced sample spacing is T_r , the relationship between T_r and T_a may be written as:

$$T_r = \frac{N}{N + N_G} T_a \quad (2.4)$$

Due to the reduction of the sample spacing, in order to maintain the same information transmission rate, the cyclic prefix added OFDM system increases the bandwidth, hence reducing the bandwidth efficiency. Adding the cyclic prefix also introduces energy loss proportional to the length of cyclic prefix, but eliminating ISI generally motivates accepting all the associated losses.

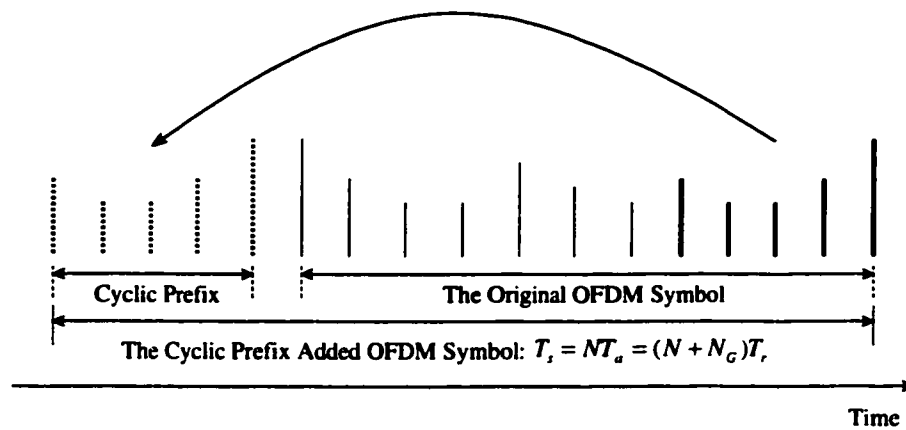


Figure 2.3: Adding a cyclic prefix.

2.2.3 System Parameters Used in the Studied OFDM Systems

The application of OFDM technique in wideband mobile communication systems is the main subject of this research work. Thus, the system parameters used in the analysis and simulations to present numerical results are chosen according to this application.

The information data rate is set to be 8 Mbps and QPSK is used as the bit mapping scheme. The carrier frequency is located at 2 GHz. To demonstrate the effects of the number of subcarriers on the system performance, four values ($N=128, 256, 512$ and 1024) are used when giving numerical results. The dynamic range of γ (as defined in equation (3.13)) is chosen to be in the range of $[10^{-3}, 10^3]$ to show how the value of γ affects the system performance. $\gamma = 0, 1$, and 9 are also used to give numerical results for a few particular scenarios, where using $\gamma = 0$ is mainly to make the channel time-selectivity more obvious. A wide range of normalized *Doppler* frequency $f_D T_a \in [10^{-6}, 1]$ is chosen to present an overview of the effects of *Doppler* frequency shift on the system performance even if some of them are not practical in actual real-life implementations. When evaluating the performance of the designed frequency-domain ICI cancellation equalizer, the maximum *Doppler* frequency shift is assumed to be 400 Hz, which is equivalent to a speed of 216 km/h for the given carrier frequency.

2.3 The Roles of Channel Coding in OFDM Systems

OFDM is robust to multipath propagation if a cyclic prefix (with the minimum length of the channel maximum excess delay) is added to each OFDM symbol. However, OFDM does not suppress fading. Generally, the fading experienced by each subcarrier of OFDM could be classified as Rician type of fading [16], under which the BERs decrease slowly with the increase of the signal-to-noise ratios if the specular (non-fading) component is not strong enough [2]. To achieve an acceptable performance with a reasonable signal power level, an effective channel coding scheme becomes essential. Furthermore, OFDM is a two-dimensional (time and frequency) system, which means that interleaving can

be performed in both time and frequency domains. Applying channel coding and time-frequency interleaving in OFDM has the function of averaging fading over the whole signal bandwidth and the time interleaving depth, which turns channel frequency-selectivity (a disadvantage for single-carrier systems) into an advantage for OFDM systems (known as frequency diversity [35]). This is the reason why in practice, all OFDM systems come with some form of channel coding and are called coded OFDM (COFDM) systems [35], [36]. Another role of channel coding (block codes are used mostly) in OFDM systems is to perform peak-to-average power ratio reduction for the transmitted signals [37], [38].

As one of the most active research fields in the area of OFDM research and development, the design, performance evaluation and implementation of channel coding schemes for OFDM systems cover a variety of error correction codes. The application of *convolutional codes* and its *rate-compatible punctured versions* in OFDM systems [39], [40], [41] are widespread, because of the availability of commercial Viterbi decoders: the same decoder can be used for different code rates obtained by means of puncturing. However, the complexity of a Viterbi decoder grows exponentially with the constraint length² K of the encoder. To achieve high coding gains with moderate decoding complexity, concatenated codes are attractive for OFDM systems [16], [42], [43]. Generally, the concatenated code used in OFDM (mainly DVB-T) systems is composed of an inner convolutional code serially concatenated with an outer Reed-Solomon code, but using trellis code as the inner code was also reported [44], [45]. *Turbo codes* [46], with high coding gain and reasonable decoding complexity, are also under intensive investigation for their application in OFDM systems [47], [48], [49]. To avoid bandwidth extension, the application of *trellis codes* in OFDM systems has also been reported [17], [20], [50], [51].

Channel coding plays a critical role in the standardization of the systems employing OFDM. The channel coding scheme standardized for DAB systems is based on a convolutional code with constraint length 7 and generator polynomials (in octal form) $g_0 = 133_8$, $g_1 = 171_8$, $g_2 = 145_8$ and $g_3 = 133_8$ [52] where XYZ_8 denotes the octal representation

²In this thesis, the constraint length K is defined as the number of memory elements in the encoder plus one (present encoder input).

of the number XYZ . With different puncturing vectors (puncturing patterns), the code rates of DAB systems range from $1/4$ to $8/9$. The standardized channel coding scheme for DVB-T systems is an inner convolutional code concatenated with an outer Reed-Solomon code [53]. The outer code is a Reed Solomon code $RS(204, 188, t=8)$, and the inner code is a punctured convolutional code with a rate $1/2$ mother code of constraint length 7 and generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$. There are 5 puncturing patterns for the mother code resulting in $1/2$, $2/3$, $3/4$, $5/6$ and $7/8$ as the possible code rates of the punctured convolutional code (the inner code) in DVB-T systems. A coding scheme based on the $1/2$ rate convolutional code with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$ is chosen as the standardized channel coding scheme by ETSI for HIPERLAN type 2 [54] and the LAN standard committee of the IEEE for high-speed wireless LANs [55]. The only difference is that different puncturing pattern sets are used in these two systems.

The research focus of this thesis is on the analysis and cancellation of the ICI caused by the channel time-selectivity over multipath time-variant fading channels instead of channel coding for OFDM systems. However, to reflect the impact of channel coding on OFDM systems when ICI exists, we choose a rate $1/2$ convolutional code with constraint length $K = 7$ and generator polynomials $g_0 = 133_8$, $g_1 = 171_8$ as the channel coding scheme in our OFDM systems. This channel coding scheme (including its rate-compatible punctured versions) is widely adopted in the standards of the systems using OFDM, and it is extensively studied by both industry and academia for its application in OFDM systems. Since its adoption in OFDM systems, its performance has been reported extensively in the literature. Many other coding schemes use this channel coding scheme as the benchmark for performance comparison (for example, the concatenated code reported in [16], the Turbo codes reported in [49] and [56], and the Walsh code reported in [57]).

2.4 Advantages and Disadvantages of OFDM

The distortion caused by multipath propagation has been one of the most challenging issues for wireless communication systems design [7]. Compared with many other

transmission schemes, OFDM has the robustness against frequency-selective multipath propagation which is the most prominent advantage of OFDM. Usually, this factor is the first motivation for adopting OFDM. Other major advantages of OFDM include [58]:

- Flexibility on channel assignment when used in a multiple access context;
- Equalization (if necessary) is simpler than that for single carrier transmissions over slow fading channels.

OFDM has several disadvantages as well. The major disadvantages of OFDM include:

- Sensitivity to synchronization errors;
- Sensitivity to nonlinear distortions, because transmitted signals exhibit high peak-to-average power ratio (PAPR);
- Sensitivity to *Doppler* spreading (frequency dispersion), which results in ICI (inter-carrier or inter-channel interference).

2.5 ICI Caused by Distortions other than Channel Time-Selectivity

Due to the inherent orthogonality among the overlapped subcarriers in OFDM systems (which makes high bandwidth efficiency achievable), ICI can be caused by distortions other than channel time-selectivity, which include: *nonlinear distortions* [59], [60], *phase noise* [61], [62], [63], *phase jitter* [64], [65], [66], *carrier frequency offset* [67], [68], [69], *symbol timing offset* [70], [71], [72], *timing jitter* [73], [74], and *sampling clock frequency error* [75], [76].

The sensitivity of OFDM signals to the nonlinearity can be reduced with the PAPR reduction schemes as presented in [37], [59], [77] and [78]. Sophisticated synchronization is usually required by OFDM systems as one of the most prominent tasks. An extensive discussion on the sensitivity of OFDM to synchronization errors may be found in [79]. When OFDM is used as a multiple access scheme, the behavior and analysis of

the synchronization errors on the performance of OFDM systems becomes more complicated as presented in [75]. Synchronization has a significant impact on the overall system design and performance of an OFDM system. The design of fast, accurate and reliable synchronization algorithms for wireless OFDM systems is even more challenging due to the characteristics of the time-variant multipath fading channels. Generally, the synchronization subsystems of OFDM systems are designed with a two-stage structure [80], [81], either with pilot symbols [82], or without pilot symbols [72], [83], [84]. The pilot symbol approach is usually adopted by the OFDM systems designed for (fast) fading channels to assist performing synchronization and channel estimation [85], [86]. Even though it sacrifices transmission or bandwidth efficiency, it makes synchronization and demodulation easier.

2.6 Summary

In this chapter, we presented the OFDM system model used in this thesis and provided necessary background information for the upcoming chapters. We also described the roles of channel coding in OFDM systems, and justified the channel coding scheme chosen in this thesis. Furthermore, we introduced the ICI caused by distortions other than channel time-selectivity as part of the overview on the subcarrier orthogonality impairment of OFDM systems.

Chapter 3

Doubly Dispersive Fading Channel Model

In this chapter, a general time-variant frequency-selective WSSUS fading channel model is presented and characterized. Two parameters, γ and β , are introduced to characterize the significance of the time-variant component of the channel impulse response for OFDM systems. Parameter γ represents the power ratio of the time-invariant component of the channel impulse response over its time-variant component. Parameter β is the square of the normalized spaced-time correlation function. Changes in the channel impulse response impair the orthogonality among the OFDM subchannels, thus causing ICI. It is observed that a higher rate in the channel's time variation results in a higher ICI power. The subset samples of the general channel model are also provided to demonstrate how to obtain the corresponding results for a specific channel from the results derived based on this channel model. The analytical results (presented in chapters 4 and 6) obtained from the channel model justify the necessity and usefulness of introducing parameters γ and β .

3.1 Wireless Communication Channels

Transmission in wireless communication systems is carried out in a radio wave propagation environment. The propagation conditions place fundamental limitations on the performance of wireless communication systems. An accurate characterization of the

propagation channel is an essential requirement for the successful design of reliable communication systems.

Multipath propagation is a very common phenomenon of radio communication channels and a major source of impairments for wireless communication systems. The signals received through a multipath channel experience a process known as *fading*. The origin of the fading mechanism for most of the wireless channels may be traced to the scattering of an electromagnetic wave by a random medium. Signals transmitted over the fading channel are affected by two general types of fading: short-term fading due to multipath propagation and long-term fading (large-scale signal variations or shadowing) due to the topographical variations along the transmission path. There is a general agreement in the literature that the shadowing effects can be modeled by a lognormal distribution [1], [87].

The short-term fading experienced by signals generally exhibits both delay and *Doppler* spreading. This type of fading is rapid: its effects on the signal vary over short distance. The fading rate is a function of the operating frequency and the relative velocity of the transmitter-receiver pair. The delay spreading results in time dispersion and frequency-selective fading. The *Doppler* spreading, on the other hand, results in frequency dispersion and time-selective fading. When the bandwidth of the transmitted signal is considerably smaller than the coherence bandwidth $(\Delta f)_c$ of the channel, the channel is classified as a *frequency-nonselective* (or *flat*) *fading channel* for the transmitted signals. When the bandwidth of the transmitted signals is considerably larger than the coherence bandwidth $(\Delta f)_c$ of the channel, the channel is classified as a *frequency-selective fading channel* [2], [88].

The ratio between the symbol duration T_S and the coherence time $(\Delta t)_c$ of the fading channel determines the rate of channel impulse response variation. Based on this ratio, a fading channel may be classified as a *time-selective (fast) fading channel* or *time-flat (slow) fading channel* [1], [2]. In the case of fast fading channel, T_S is considerably larger than $(\Delta t)_c$, and the channel impulse response changes rapidly within a symbol duration. On the other hand, the channel impulse response of a slow fading channel may be assumed

time-invariant during at least one symbol period¹. A channel is considered as a slow fading channel if T_S is considerably smaller than $(\Delta t)_c$.

A more precise classification for the fading channels is given in [88], where the difference between distortion and dispersion is emphasized². A systematic approach is presented in [89], where two sets of system functions³ and their corresponding correlation functions are proposed to describe the zero-mean linear time-variant channels. The correlation functions for the linear time-variant channels with non-zero means are presented in section 3.3.

3.2 Channel Characterization for OFDM ICI Analysis

For OFDM systems operating over time-invariant channels, it is observed that the orthogonality among the subchannels would be maintained [6]: there is no ICI. On the other hand, for fast fading channels, where the symbol duration is considerably longer than the channel coherence time ([2], [88]), ICI is introduced as indicated in [19], [90], [91], [92], [93] and [94]. In this case, the channel impulse response change could be significant within one symbol period. For the circumstance where the symbol duration is close to the channel coherence time, an extensive literature survey found no reported research work addressing the effect of ICI on OFDM systems.

Time-invariant and fast fading are only two limiting cases for the performance analysis of OFDM systems, as pointed out by Steendam and Moeneclaey in [95]. In the literature, the channel time-selectivity has been frequently ignored from the performance analysis and design of OFDM systems over broadband mobile channels [96]. For slow fading channels, the results are generally obtained with the time-invariant channel model [6], [16], [97]. Even if the channel impulse response of the slow fading channel does not change within the duration of tens or even hundreds of symbols, most likely, it could

¹Usually, the channel impulse response of a slow fading channel is assumed to be time-invariant over tens or even hundreds of symbols.

²Figure 2.18 of [88] gives a clear view on the channel classification according to this criterion.

³More details are presented later in section 3.3.

change significantly within a certain OFDM symbol and thus may impair the orthogonality among the subchannels and result in ICI. As it will be shown in chapter 4, if this significant change of the channel impulse response does not happen at the beginning or the end of an OFDM symbol, it introduces ICI to that OFDM symbol, thus affecting the overall performance of the system. In order to give a comprehensive ICI analysis for OFDM systems under general fading channels, we present a doubly dispersive WSSUS fading channel model and characterize it in this chapter. The main effort of characterizing this channel model is to emphasize the channel time-selectivity.

3.2.1 Analog Channel Model

As stated in [88] and [89], any linear time-variant fading channel can be modeled as the superposition of a deterministic channel and a purely random zero mean channel. The deterministic channel is usually the ensemble (statistical) average of the time-variant fading channel impulse responses. Based on the above statements, it is reasonable to assume that the channel impulse response of a linear time-variant fading channel regarding time-selectivity may be further divided into two components. A constant component which does not vary during the transmission, and a short-term-component which changes within one symbol duration. If the constant component is chosen as the ensemble average of the time-variant fading channel impulse responses, then the short-term-component is a zero mean stochastic process, which is assumed in this research work. The constant component can also be different from one transmission to another, that is, it is still time-variant on a long term observation. According to this assumption and presuming the channel is wide sense stationary, the time-variant channel impulse response $h(\tau, t)$, which is obtained by transmitting one impulse at time $t - \tau$, and receiving it at time t ([2], [88]), may be written as:

$$h(\tau, t) = h(\tau) + \Delta h(\tau, t) \quad (3.1)$$

where $h(\tau)$ is the time-invariant component, which is a function of τ . $\Delta h(\tau, t)$ represents the short term time-variant component, and is assumed to be a zero mean Gaussian stochastic process. The proposed channel impulse response model is supported by the

measurements presented in Section V (Fig. 3) of [98] and Section III (Fig. 2) of [99]. The time-invariant component $h(\tau)$ may be measured using the channel sounding techniques presented in [100].

A similar fading channel model is presented in [101], where the channel impulse response is expressed as:

$$h(\tau, t) = \bar{h}(\tau, t) + \tilde{h}(\tau, t) \quad (3.2)$$

where $\bar{h}(\tau, t)$ is a slowly varying mean response component, and $\tilde{h}(\tau, t)$ is a zero-mean fast-fading component. $\bar{h}(\tau, t)$ and $\tilde{h}(\tau, t)$ are then modeled as the first-order and $\mathcal{R}$ th-order autoregressive processes to perform channel estimation. No further characterization on $\bar{h}(\tau, t)$ and $\tilde{h}(\tau, t)$ is provided, except that $\tilde{h}(\tau, t)$ can be assumed as a Gaussian process. Therefore the channel can exhibit Rayleigh (when $\bar{h}(\tau, t)$ has a zero mean) and Rician (when $\bar{h}(\tau, t)$ has a nonzero mean) fading.

To further characterize the channel model, it is assumed that $h(\tau)$ and $\Delta h(\tau, t)$ are uncorrelated, i.e.,

$$E\{h(\tau)\Delta h^*(\tau', t)\} = E\{h(\tau)\}E\{\Delta h^*(\tau', t)\} = 0 \quad (3.3)$$

and

$$E\{h^*(\tau')\Delta h(\tau, t)\} = E\{h^*(\tau')\}E\{\Delta h(\tau, t)\} = 0 \quad (3.4)$$

where y^* denotes the complex conjugate of y . In addition, we assume that both $h(\tau)$ and $\Delta h(\tau, t)$ are still belong to the WSSUS model [87], [88], that is,

$$E\{h(\tau)h^*(\tau')\} = \sigma_h^2\delta(\tau - \tau') \quad (3.5)$$

and

$$E\{\Delta h(\tau, t)\Delta h^*(\tau', t')\} = \sigma_{\Delta h}^2 J_0(2\pi f_D(t - t'))\delta(\tau - \tau') \quad (3.6)$$

where $\delta(x)$ is the delta function and f_D is the maximum *Doppler* frequency defined as:

$$f_D = \frac{v_s}{c}f_c \quad (3.7)$$

where $c = 3.0 \times 10^8$ m/s is the speed of light, v_s is the vehicle speed in m/s, f_c is the carrier frequency in Hertz, and $J_0(\bullet)$ is the zeroth-order Bessel function of the first kind. These channel characteristics are needed in our ICI analysis presented in the following chapters.

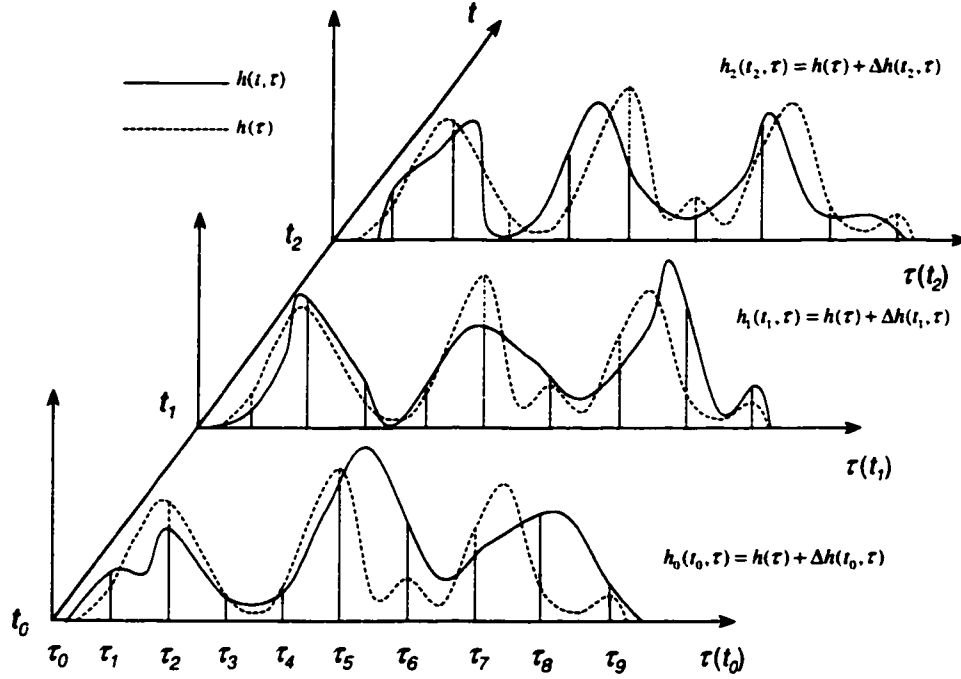


Figure 3.1: Superimposed channel impulse response model (analog and discrete) for OFDM ICI analysis.

3.2.2 Discrete Channel Model

The corresponding discrete channel impulse response of $h(\tau, t)$, obtained by sampling $h(\tau, t)$ at observation time $t = kT_r$ and excitation time (delay) $\tau = mT_r$, may be written as:

$$h_{m,k} = h_m + \Delta h_{m,k} \quad (3.8)$$

where $0 \leq k \leq N - 1$, $0 \leq m \leq M - 1$, h_m is the time-invariant discrete component, $\Delta h_{m,k}$ is the short term time-variant discrete component and is still assumed to be a zero mean Gaussian random process.

Figure 3.1 depicts several impulse response realizations of the channel model presented above, where $h(\tau)$ is represented by the dashed-lines and $h(\tau, t)$ is represented by the solid lines. The corresponding discrete channel is obtained by sampling $h(\tau, t)$ at $\tau = \tau_0, \tau_1, \tau_2, \dots$ and $t = t_0, t_1, t_2, \dots$

Equations (3.3), (3.4), (3.5) and (3.6) may be expressed in discrete form as:

$$E\{h_m \Delta h_{m,k}^*\} = E\{h_m\} E\{\Delta h_{m,k}^*\} = 0 \quad (3.9)$$

$$E\{h_m^* \Delta h_{m,k}\} = E\{h_m^*\} E\{\Delta h_{m,k}\} = 0 \quad (3.10)$$

$$E\{h_m h_{m'}^*\} = \sigma_m^2 \delta(m - m') \quad (3.11)$$

$$E\{\Delta h_{m,k} \Delta h_{m',k'}^*\} = \sigma_{\Delta m}^2 J_0(2\pi f_D T_\tau(k - k')) \delta(m - m') \quad (3.12)$$

3.3 Channel Time-selectivity Evaluation

The channel model presented in the previous section is applicable to OFDM ICI analysis when the symbol duration is close to the coherence time of the fading channel, where it is not always sufficient to classify the channel as either slow or fast fading. The importance of the time-variant component of the channel impulse response may be specified by the average power ratio of $h(\tau)$ and $\Delta h(\tau, t)$ (h_m and $\Delta h_{m,k}$ in discrete format), as defined below:

$$\gamma = \frac{\sigma_h^2}{\sigma_{\Delta h}^2} = \frac{\sum_{m=0}^{M-1} \sigma_m^2}{\sum_{m=0}^{M-1} \sigma_{\Delta m}^2} \quad (0 \leq \gamma < +\infty) \quad (3.13)$$

As will be seen in the later chapters 4 and 6, the average power ratio between the time-invariant component and the time-variant component of the channel impulse response plays a critical role on determining the average ICI power and thus the OFDM system performance.

To characterize the linear random time-variant channels, Bello [89] proposed two sets of system functions: the *input delay spread function* based set (the first set of Bello

functions), the *input Doppler spread function* based set (the second set of Bello functions), and there are four Bello functions in each set. A complete description of the channel is embodied in each system function. Having the full knowledge of one system function in each set allows obtaining other system functions through direct and inverse *Fourier* transforms⁴ [89]. The relationships among the four Bello functions of the first set are depicted in Figure 3.2, and the second set Bello functions are related with the first set Bello functions through the *duality* principle [102]. In the literature, the first set Bello functions and their correlation functions are widely used for channel characterization and simulation [88].

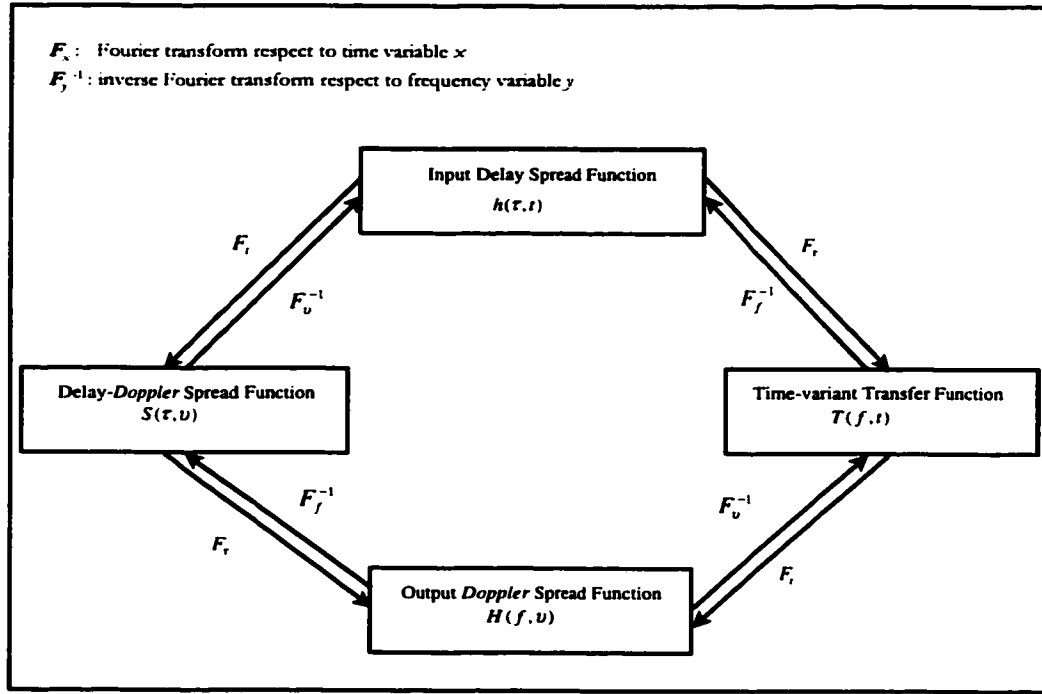


Figure 3.2: Relationships of the input delay spread function based (first set) Bello functions.

⁴In the literature, there are two different definitions used for the *Fourier* transform when handling the Bello functions (correspondingly, two different definitions for the inverse *Fourier* transform as well). One approach (for example [88]) follows the definition of pure mathematics, that is, the *Fourier* transform and the inverse *Fourier* transform are defined precisely following the mathematical definitions, no matter what are the variables under operation. Another approach naturally follows the *physics* connotation: the *Fourier* transform is defined as the transform from time domain variable to frequency domain variable, and the inverse *Fourier* transform is defined as the transform from frequency domain variable to time domain variable, no matter which mathematics expression is used to perform the transform. The *Fourier* transform and the inverse *Fourier* transform used in this thesis follow the second definition, which was adopted by Bello in his original work [89].

The statistical characterization of a random time-variant linear channel is further pursued through the correlation functions of the various system functions [89]. For simplicity, the discussion on the correlation properties presented in [89] was limited to those system functions with zero ensemble average only. However, it is stated in [89] (page 367) that: *“In general, a randomly time-variant channel has a mixed deterministic and random behavior. Thus, for example, the input delay-spread function $g(t, \xi)$ ⁵ may be separated into the sum of a purely random part and a deterministic part [equal to the ensemble average of $g(t, \xi)$]. This separation implies a representation of the channel as the parallel combination of a deterministic channel and a purely random channel.”* This is the foundation of equation (3.1).

The channel time-selectivity causes ICI in OFDM systems, but the ICI is introduced only by the time-variant component of the channel impulse response. However, the signal to interference ratio (thus the system performance) is also related to the time-invariant component of the channel impulse response, because it contributes to the total signal power. Thus for OFDM systems over time-variant fading channels, the correlation functions of the various system functions should be evaluated including the time-invariant (deterministic) component of the channel impulse response.

Based on equations (3.3), (3.4), (3.5) and (3.6), the correlation function of $h(\tau, t)$ (WS-SUS is assumed) may be written as:

$$\begin{aligned} E\{h(\tau, t)h^*(\tau', t')\} &= E\left\{\left(h(\tau) + \Delta h(\tau, t)\right)\left(h(\tau') + \Delta h^*(\tau', t')\right)\right\} \\ &= \left(\sigma_h^2 + \sigma_{\Delta h}^2 J_0\left(2\pi f_D(t - t')\right)\right)\delta(\tau - \tau') \end{aligned} \quad (3.14)$$

Thus the *delay cross-power spectral density* [89] may be written as:

$$P_h(\Delta t, \tau) = \sigma_h^2 + \sigma_{\Delta h}^2 J_0(2\pi f_D \Delta t) \quad (3.15)$$

where $\Delta t = t - t'$. From the *Fourier* transform relationships given in Figure 3.3 [88], the *time-frequency correlation function* [89] (i.e. the *spaced-frequency spaced-time correlation*

⁵This is equivalent to channel impulse response $h(\tau, t)$.

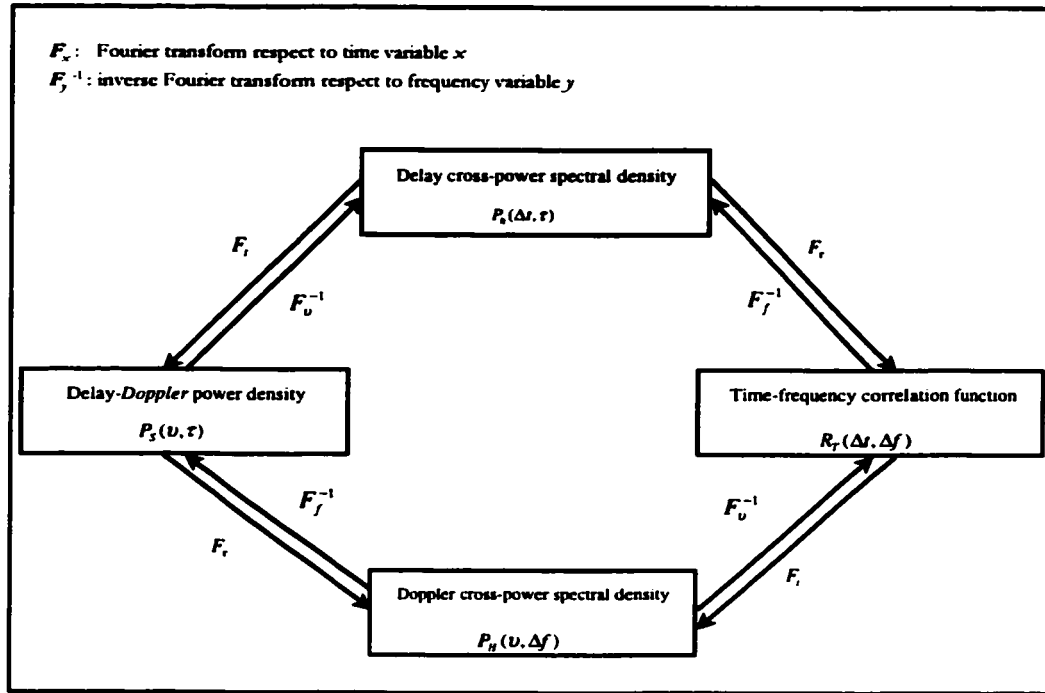


Figure 3.3: Relationships of the correlation functions of the input delay spread function based (first set) Bello functions for WSSUS channels.

function [2]) may be written as:

$$\begin{aligned}
 R_T(\Delta t, \Delta f) &= \mathcal{F}_\tau \{ P_h(\Delta t, \tau) \} \\
 &= \left(\sigma_h^2 + \sigma_{\Delta h}^2 J_0(2\pi f_D \Delta t) \right) \delta(\Delta f)
 \end{aligned} \tag{3.16}$$

where $\mathcal{F}_\tau \{ P_h(\Delta t, \tau) \}$ stands for the *Fourier* transform of $P_h(\Delta t, \tau)$ relative to variable τ . From equations (3.15) and (3.16), it can be observed that when the time-invariant component of the channel impulse response exists (i.e. $h(\tau) \neq 0$), its contribution to the *delay cross-power spectral density* and the *time-frequency correlation function* equals to its mean power σ_h^2 .

The *delay-Doppler power density function* (or simply *scattering function*) $P_s(\nu, \tau)$ [89] may be written as:

$$\begin{aligned}
 P_s(\nu, \tau) &= \mathcal{F}_t \{ P_h(\Delta t, \tau) \} \\
 &= \sigma_h^2 \delta(\nu) + \sigma_{\Delta h}^2 P_J(\nu)
 \end{aligned} \tag{3.17}$$

where $P_J(\nu)$ is defined as:

$$P_J(\nu) = \mathcal{F}_t \left\{ J_0(2\pi f_D \Delta t) \right\} \quad (3.18)$$

From [103], we have:

$$P_J(\nu) = \begin{cases} \frac{1}{\pi f_D} \frac{1}{\sqrt{1-(\frac{\nu}{f_D})^2}} & (\text{if } |\nu| < f_D) \\ 0 & (\text{otherwise}) \end{cases} \quad (3.19)$$

which is the classical U-shape *Doppler* spectrum.

The *Doppler cross-power spectral density* [89] may be obtained by performing either $\mathcal{F}_t \{ R_H(\Delta t, \Delta f) \}$ or $\mathcal{F}_\tau \{ P_s(\nu, \tau) \}$ [88], which is:

$$P_H(\nu, \Delta f) = \left(\sigma_h^2 \delta(\nu) + \sigma_{\Delta h}^2 P_J(\nu) \right) \delta(\Delta f) \quad (3.20)$$

Equations (3.17) and (3.20) demonstrate that when the *Doppler* frequency $\nu = 0$ only, the time-invariant component of the channel impulse response affects the *scattering function* and the *Doppler cross-power spectral density* with its mean power σ_h^2 .

The channel time-selectivity may still be evaluated through the *time-frequency correlation function* even when $h(\tau) \neq 0$ by setting $\Delta f = 0$, which becomes the *spaced-time correlation function* [2]:

$$\begin{aligned} \Psi_T(\Delta t) &\equiv R_T(\Delta t, 0) \\ &= \sigma_h^2 + \sigma_{\Delta h}^2 J_0(2\pi f_D \Delta t) \\ &= \sigma_{\Delta h}^2 \left(\gamma + J_0(2\pi f_D \Delta t) \right) \end{aligned} \quad (3.21)$$

For the given values of γ and f_D , in order to evaluate the change of $\Psi_T(\Delta t)$ as a function of Δt , we normalize $\Psi_T(\Delta t)$ with its own maximum value, and define a parameter β , which we call the *squared normalized spaced-time correlation factor*, as:

$$\begin{aligned} \beta &= \left(\frac{\Psi_T(\Delta t)}{\max \{ \Psi_T(\Delta t) \}} \right)^2 \\ &= \left(\frac{\gamma + J_0(2\pi f_D \Delta t)}{\gamma + 1} \right)^2 \end{aligned} \quad (3.22)$$

where $\max\{\Psi_T(\Delta t)\}$ stands for the maximum value of $\Psi_T(\Delta t)$ with variable Δt . It should be noted that, when $\gamma = 0$, β becomes

$$\beta|_{\gamma=0} = J_0^2(2\pi f_D \Delta t) \quad (3.23)$$

which is the *envelope correlation with time separation* (or the *time correlation function*) in [88] (see equation (2.103) of page 123). This function has been widely used as the conventional method of channel time-selectivity evaluation [87], [103].

Numerical values of β are presented in Figure 3.4, where the effects of γ can be observed clearly. It shows that with the same *Doppler* frequency, fading channels with larger values of γ appear to be less time-selective because β has larger values, which means that the channel has a higher correlation. Thus, for performance analysis purposes of OFDM systems over fading channels, the evaluation of the channel time-selectivity must include the time-invariant component of the channel impulse response.

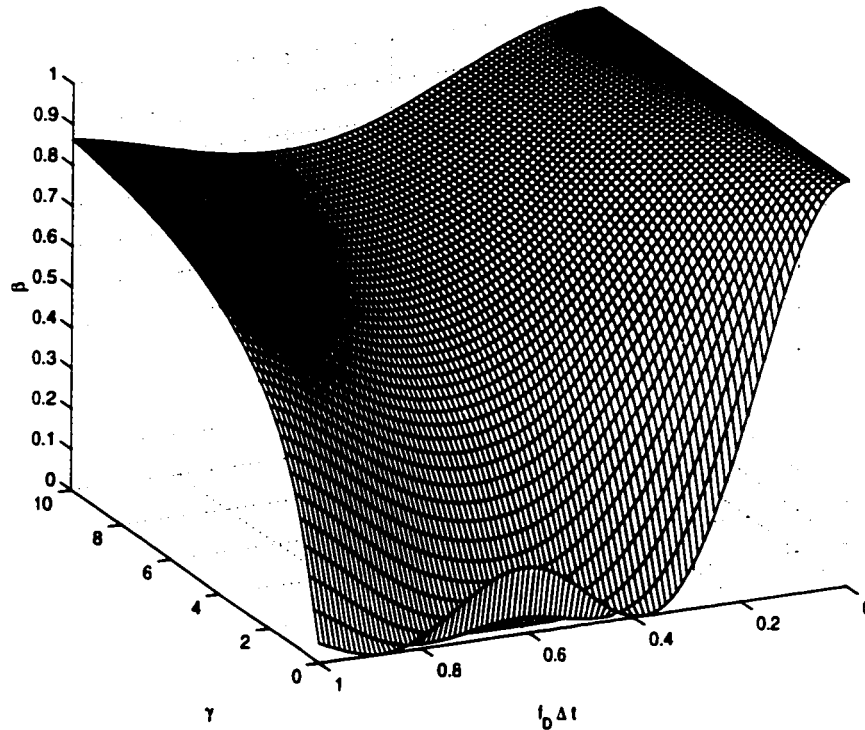


Figure 3.4: Channel time-selectivity β evaluation as a function of the ratio γ and $f_D \Delta t$.

Three threshold values of the *time correlation function* (equivalent to the *spaced-time correlation function* given in equation (3.21)) can be used to evaluate the coherence time of fading channels [88]. These values are 0.9, 0.5 and $1/e$ (≈ 0.3679), among which the value 0.5 is widely used [87], [103]. A high threshold value stands for a tight requirement to classify the channel as a slow fading channel. The *contours* corresponding to $\beta = 0.9$, $\beta = 0.5$ and $\beta = 1/e$, which are obtained based on Figure 3.4 with extended γ values, are denoted by curves A, B and C in Figure 3.5 respectively. The left hand-side area (lower γ value area) of the curve A is the region where the value of β is below 0.9. The same notation is applicable for curves B and C, but the value of β is below 0.5 and $1/e$ respectively.

From Figure 3.5, it is observed that two different $f_D \Delta t$ values may have the same value of β , which means that the coherence time (if defined according to the value of the *time correlation function*) may have two values under the same condition. However, the smaller one shall be chosen as the coherence time in order to make it valid for general cases. It is also observed that when the value of γ is large enough, the value of β keeps above the threshold of defining the channel coherence time for any given $f_D \Delta t$. This means that the channel may appear to be slow fading for certain applications even with high *Doppler* frequency. As an example, when the threshold is set to be $1/e$, 0.5 and 0.9, the corresponding value of γ is 2.56, 3.79 and 26.34 respectively, and the corresponding values of β are presented in Figure 3.6. However, as shown in chapter 4, for OFDM systems, the evaluation of channel time-selectivity is very tight. More numerical results of channel time-selectivity evaluation for OFDM systems are presented in chapter 4.

3.4 Relationship with Widely Used Channel Models

The channel model presented by equations (3.1) (analog model) and (3.8) (discrete model) is a general time-variant frequency-selective WSSUS fading channel model. It can be modified to represent many specific channel models, and the corresponding results can be obtained from the results obtained with the general channel model by modifying $h(\tau)$

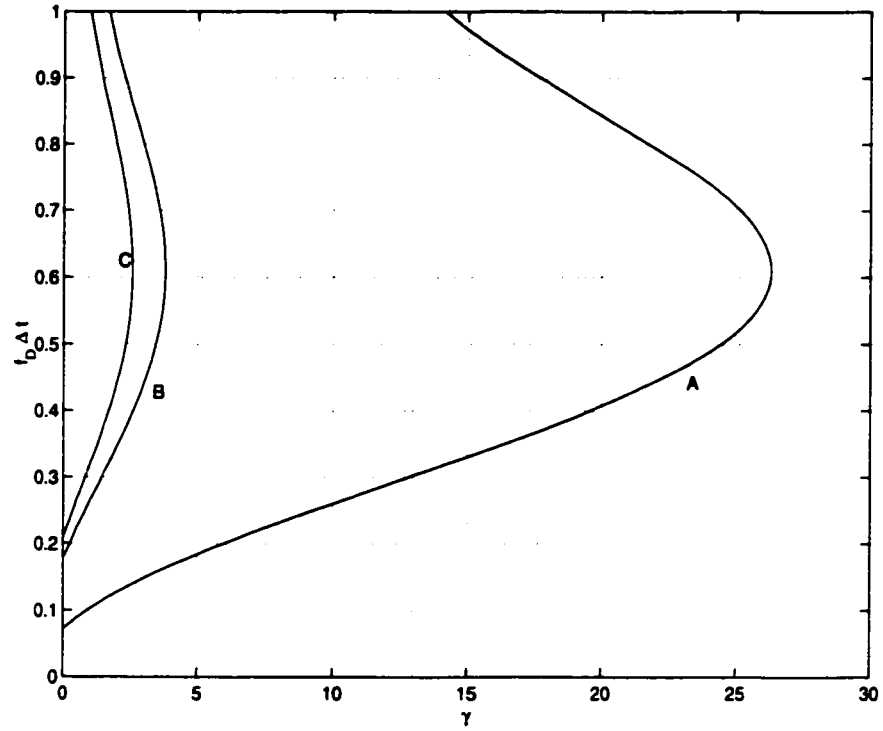


Figure 3.5: The contours for $\beta = 0.9$ (A), $\beta = 0.5$ (B) and $\beta = 1/e$ (C) as a function of γ and $f_D \Delta t$.

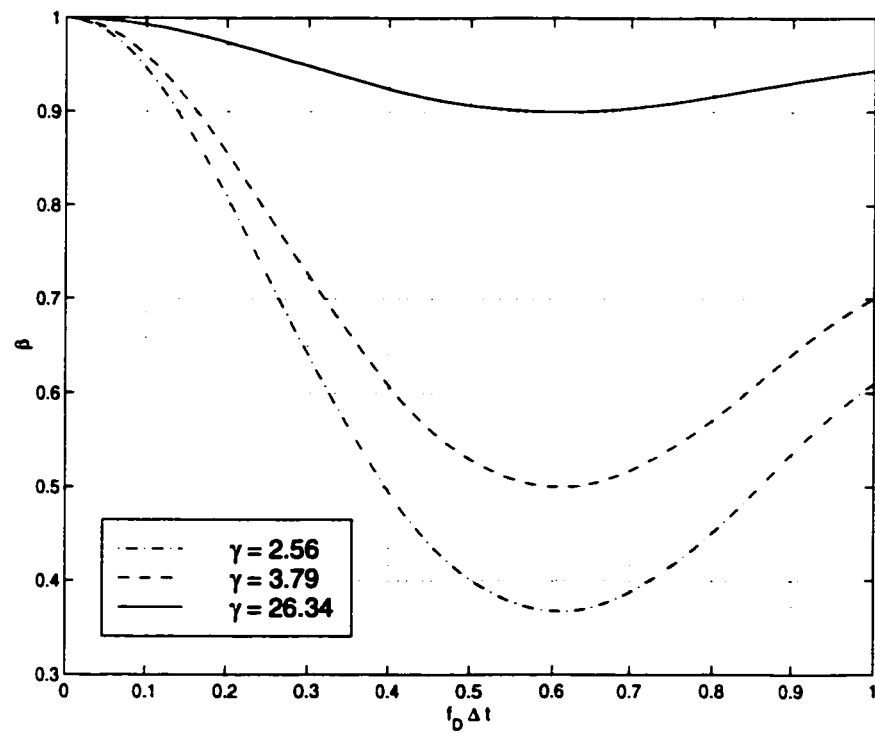


Figure 3.6: The values of β as a function of $f_D \Delta t$ when $\gamma = 2.56, 3.79$ and 26.34 .

and $\Delta h(\tau, t)$ accordingly. We show how the proposed channel model can be modified to represent several widely used channel models.

3.4.1 AWGN Channel Model

Setting $h(\tau) = \delta(\tau)$ and $\Delta h(\tau, t) \equiv 0$, the proposed channel model becomes an additive white Gaussian noise (AWGN) channel [104]. The value of γ is therefore infinite and β is undefined.

3.4.2 Time-invariant Multipath Channel Model

The proposed channel model can produce a time-invariant channel model by simply setting $\Delta h(\tau, t) \equiv 0$ for the analog channel model, and $\Delta h_{m,k} \equiv 0$ ($0 \leq m \leq M - 1$) for the discrete channel model [104]. Again, here γ is infinite and β is not defined.

3.4.3 Frequency-nonselective Fading Channel Model

Assuming $M = 1$, the discrete channel given in equation (3.8) becomes a frequency non-selective fading channel. For the analog channel (equation (3.1)), setting $h(\tau) = \alpha\delta(\tau)$ and $\Delta h(\tau, t) = \Delta h(t)\delta(\tau)$ makes the channel to be frequency non-selective [104]. When α is zero for the analog channel (h_0 is zero for the discrete channel), the channel becomes a frequency non-selective Rayleigh fading channel, where $\gamma = 0$ and β as given in equation (3.23). If $\alpha \neq 0$, the channel is a frequency non-selective Rician fading channel [2], [105]. In this case, the ratio γ describes the relative importance of the time-varying component to the time-invariant impulse response component, and β indicates the time variability of the fading channel.

3.4.4 Frequency-selective Rayleigh Fading Channel Model

Setting $h(\tau) \equiv 0$ for the analog channel model, and $h_m \equiv 0$ ($0 \leq m \leq M - 1$) for the discrete channel model, the proposed channel model reduces to a Rayleigh fading channel ($\gamma = 0$). The Rayleigh fading channel model is the most widely used model for ICI analysis of OFDM systems. However, as stated earlier, the results are valid only for

the case where the symbol duration is considerably larger than the coherence time of the fading channel, i.e. for fast fading conditions.

3.4.5 Frequency-selective Rician Fading Channel Model

Generally, a Rician channel is referred to a fading channel which has a specular (non-fading) component besides a Rayleigh component (or multipath components) [105]. When we set $h(\tau) = \alpha\delta(\tau - \tau_0)$ for the analog channel model, or $h_m = \alpha\delta[m - m_0]$ ($0 \leq m, m_0 \leq M - 1$, where $\delta[m]$ is the discrete delta function) for the discrete channel model, our proposed channel model becomes a frequency-selective Rician fading channel model. When $\tau_0 = 0$ ($m_0 = 0$ for the discrete channel model), the specular component usually results from a line-of-sight propagation [1]. Otherwise, a fixed scatterer is generally assumed. This channel impulse response is widely used in analyses when the channel is modeled as a frequency-selective Rician fading channel [106], [107]. However, one extreme case might exist, where there are a few fixed scatterers, that means, there are a few specular components besides the Rayleigh component. In this case, the channel could be modeled with our proposed channel model by setting all the values of $h(\tau)$ and h_m ($0 \leq m \leq M - 1$) to be zero, except of those values corresponding to the specular components. Thus, our proposed channel model may be classified as an extended frequency-selective Rician fading channel model.

3.5 Summary

In this chapter, we presented and characterized a general time-variant frequency-selective WSSUS fading channel model. It will be used in later chapters to obtain ICI and performance analysis for OFDM systems operating over time-variant multipath fading channels. Examples were provided to show how we can deduce several specific widely used channel models by using this model. The performance results for those specific channels can be obtained from the general results by modifying the time-invariant and time-varying components of the channel impulse response accordingly. To evaluate the significance of time-variant component of the channel impulse response, in this chapter, we defined two

parameters, γ and β . We also re-evaluated the correlation functions of the Bello system functions (the first set) when the time-invariant component of the channel impulse response exists for WSSUS channels.

Chapter 4

ICI Analysis for OFDM Systems

In this chapter, using the general time-variant frequency-selective WSSUS fading channel model characterized in chapter 3, we analyze the ICI caused by transmitting OFDM symbols over the time-variant multipath fading channels. The ICI power distribution, its effects on the signal to effective noise ratio, the performance degradation caused by ICI, the effect of γ (defined in equation (3.13)) on the system performance, and the channel time selectivity evaluation with β (defined in equation (3.22)) are presented. The ICI analysis performed in this chapter is essential for the ICI cancellation scheme design and the performance analysis of OFDM systems over time-variant fading channels, which are presented in chapter 5 and chapter 6 respectively.

4.1 Introduction

In [11], Saltzberg observed that, “..., *the strategy of designing an efficient parallel system should concentrate more on reducing crosstalk between adjacent channels than on perfecting the individual channels themselves, since the distortions due to crosstalk tend to dominate.*” This is one very important guideline for the design of any parallel transmission systems, including OFDM.

The spectra of OFDM subcarriers are overlapped, but remain orthogonal. However, if the orthogonality among the subchannels is impaired, for example because of channel time selectivity, ICI is introduced as indicated in [19], [91], [92], [93], [94] and [108]. If ICI

is not treated appropriately, the system performance can not be improved by increasing SNR only. In other words, ICI results in error floor, which can be expressed as a function of the *Doppler* frequency and the number of subcarriers in the OFDM system [91].

Only recently, researchers began to pay more attention to the time-selectivity of fading channels on OFDM systems ([96], [109], [110] and [111]). In [109], the ICI analysis presented in [19] is further developed for infinite number of subcarriers. The tight and universal bounds of the average ICI power presented in Robertson and Kaiser's work [109] are derived by Li and Cimini [111] to give an explicit view of relationship between *Doppler* spread and ICI. The ICI caused by the channel time-selectivity is further discussed in [110] when OFDM is used as a multiple access scheme. However, even with channel time-selectivity in mind for the design of realistic OFDM systems [112], the relationship between the number of subcarriers and ICI power must be known.

In this chapter, we generalize the ICI analysis presented in [19] for OFDM systems over time-variant frequency-selective fading channels, with the emphasis on the ICI power distribution and the relationship between the average ICI power and the number of subcarriers in the system. The obtained results are applicable to general time-variant frequency-selective WSSUS fading channels, including the case where the channel coherence time $(\Delta t)_c$ is close to the symbol duration T_S , where is not always sufficient to classify the channel to be slow or fast fading. In the literature, it has been widely believed that the central subcarrier (located at the center of the frequency band) receives most of the ICI power, and the edge subcarriers experience less ICI [109]. Instead, our analysis shows that most of the ICI power of the desired subcarrier is contributed by few closest neighboring subcarriers in a *cyclic* fashion, and the total ICI power received by each subcarrier is the same, i.e. it is independent of frequency location.

4.2 Transmission over Fading Channels

4.2.1 Transmitted Signal

Assume that the complex information sequence in one OFDM symbol is a_n ($0 \leq n \leq N - 1$), where N is the number of subcarriers in the system. When a rectangular pulse is employed, the baseband transmitted signal (one symbol) may be written as [6]:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j(2\pi f_n t)} \quad (0 \leq t \leq T_S) \quad (4.1)$$

where $j = \sqrt{-1}$, a_n is the information symbol modulated by the n^{th} subcarrier, f_n is the carrier frequency of the n^{th} subcarrier, and T_S is one OFDM symbol duration. If we sample the baseband analog OFDM signal at $t = kT_a$ ($0 \leq k \leq N - 1$), where T_a is the symbol duration of the serial sequence a_n ($T_S = NT_a$), one gets the transmitted signal in digital format:

$$S_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n e^{j\frac{2\pi nk}{N}} \quad (4.2)$$

which converts the OFDM modulation process into a N point IDFT [14]. After adding N_G cyclic prefix samples (see Figure 2.3), the transmitted signal with cyclic prefix may be written as [19]:

$$S_{k_g}^g = S_{(k_g - N_G)_N} \quad (0 \leq k_g \leq N + N_G - 1) \quad (4.3)$$

where $(k_g - N_G)_N$ is the residue of $(k_g - N_G)$ modulo N . After digital to analog conversion, the digital sequence $S_{k_g}^g$ is converted to $s^g(t)$, and transmitted through the channel.

4.2.2 Received Signal

Assuming that the impulse response of the fading channel is $h(\tau, t)$, which is time-variant, the received signal may be written as:

$$r^g(t) = h(\tau, t) * s^g(t) + f(t) \quad (4.4)$$

where $*$ denotes the linear convolution operation, and $f(t)$ is the AWGN of the channel, which is a zero mean Gaussian random process with mean power σ_f^2 .

With the assumption that the maximum delay spread of the fading channel is T_D , and it is not longer than the length of the added cyclic prefix, the received signal may be written as:

$$r^g(t) = \int_0^{T_D} h(\tau, t) s^g(t - \tau) d\tau + f(t) \quad (4.5)$$

Assuming that the sampled channel impulse response¹ is h_{m,k_g} , which is obtained by sampling $h(\tau, t)$ at observation time $t = k_g T_r$ and excitation time (delay) $\tau = m T_r$ ($0 \leq k_g \leq N + N_G - 1$, $0 \leq m \leq M - 1$), the received digital signal (obtained by sampling the $r^g(t)$ given in equation (4.5)), which does not include the cyclic prefix samples, may be written as:

$$R_{k_g}^g = h_{m,k_g} \otimes S_{k_g}^g + f_{k_g} = \sum_{m=0}^{M-1} h_{m,k_g} S_{(k_g-m)_N}^g + f_{k_g} \quad (4.6)$$

where $\otimes$ stands for the cyclic convolution [15].

4.2.3 Demodulation

At the receiver side, the first N_G samples of the received sequence $R_{k_g}^g$, which are the cyclic prefix samples, are discarded. The remaining sequence may be written as:

$$\begin{aligned} R_k &= R_{k+N_G}^g \quad (0 \leq k \leq N - 1) \\ &= \sum_{m=0}^{M-1} h_{m,k+N_G} S_{(k-m)_N} + f_{k+N_G} \end{aligned} \quad (4.7)$$

Therefore, for one OFDM symbol, the demodulated sequence Z_l ($0 \leq l \leq N - 1$) is the DFT of the received sequence R_k ($0 \leq k \leq N - 1$), that is:

$$\begin{aligned} Z_l &= DFT\{R_k\} \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} R_k e^{-j \frac{2\pi l k}{N}} \\ &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n \left(\frac{1}{N} \sum_{k=0}^{N-1} h_{m,k+N_G} e^{j \frac{2\pi k(n-l)}{N}} \right) e^{-j \frac{2\pi n m}{N}} + \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f_{k+N_G} e^{-j \frac{2\pi l k}{N}} \end{aligned} \quad (4.8)$$

Defining $H_m(n-l)$ as :

$$H_m(n-l) = \frac{1}{N} \sum_{k=0}^{N-1} h_{m,k+N_G} e^{j \frac{2\pi k(n-l)}{N}} \quad (4.9)$$

¹The sampling rates before and after adding cyclic prefix are slightly different. If they are assumed to be $\frac{1}{T_a}$ and $\frac{1}{T_r}$ respectively, their relationship is $T_r = \frac{N}{N+N_G} T_a$. Results given in this thesis are mainly based on the value of T_a .

and

$$n_l = DFT\{f_{k+N_G}\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f_{k+N_G} e^{-j\frac{2\pi}{N}lk} \quad (4.10)$$

then, equation (4.8) may be expressed as:

$$Z_l = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n H_m(n-l) e^{-j\frac{2\pi}{N}nm} + n_l \quad (4.11)$$

Since DFT is an orthonormal linear transform, n_l has the same statistical characteristics as f_{k+N_G} , that is, n_l is also a zero mean Gaussian random process with mean power σ_f^2 [6].

4.3 Inter-carrier Interference Analysis

4.3.1 Averaged ICI Power

Substituting (3.8) into (4.9), we have:

$$\begin{aligned} H_m(n-l) &= \frac{1}{N} \sum_{k=0}^{N-1} h_{m,k+N_G} e^{j\frac{2\pi}{N}k(n-l)} \\ &= \frac{1}{N} h_m \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}k(n-l)} + \frac{1}{N} \sum_{k=0}^{N-1} \Delta h_{m,k+N_G} e^{j\frac{2\pi}{N}k(n-l)} \\ &= h_m \delta(n-l) + \Delta H_m(n-l) \end{aligned} \quad (4.12)$$

where

$$\Delta H_m(n-l) = \frac{1}{N} \sum_{k=0}^{N-1} \Delta h_{m,k+N_G} e^{j\frac{2\pi}{N}k(n-l)} \quad (4.13)$$

Extracting a_l in (4.11) from other items, one obtains:

$$\begin{aligned} Z_l &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n \left(h_m \delta(n-l) + \Delta H_m(n-l) \right) e^{-j\frac{2\pi}{N}nm} + n_l \\ &= a_l \sum_{m=0}^{M-1} \left(h_m + \Delta H_m(0) \right) e^{-j\frac{2\pi}{N}ml} + \sum_{m=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} a_n \Delta H_m(n-l) e^{-j\frac{2\pi}{N}nm} + n_l \\ &= \zeta_l a_l + c_l + n_l = \zeta_l a_l + e_l \end{aligned} \quad (4.14)$$

where $e_l = c_l + n_l$ is defined as the *effective* additive noise. c_l is the ICI:

$$c_l = \sum_{m=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} a_n \Delta H_m(n-l) e^{-j\frac{2\pi}{N}nm} \quad (4.15)$$

and ζ_l is the attenuation factor of the desired subchannel:

$$\begin{aligned}\zeta_l &= \sum_{m=0}^{M-1} \left(h_m + \Delta H_m(0) \right) e^{-j \frac{2\pi m l}{N}} \\ &= \sum_{m=0}^{M-1} \left(h_m + \frac{1}{N} \sum_{k=0}^{N-1} \Delta h_{m,k+N_G} \right) e^{-j \frac{2\pi m l}{N}}\end{aligned}\quad (4.16)$$

If a_l is assumed phase-shift-keying (PSK) modulated and the signal power is normalized to unity, then the signal to noise ratio (SNR , i.e. E_S/N_0 where E_S is the average power per symbol and N_0 is the channel AWGN average power which is σ_f^2) is:

$$SNR = E_S/N_0 = \frac{E\{\zeta_l \zeta_l^*\}}{E\{n_l n_l^*\}} \quad (4.17)$$

the signal to inter-carrier interference ratio (SIR) is:

$$SIR = \frac{E\{\zeta_l \zeta_l^*\}}{E\{c_l c_l^*\}} \quad (4.18)$$

and the signal to effective noise ratio² ($SENR$) is:

$$SENR = \frac{E\{\zeta_l \zeta_l^*\}}{E\{c_l c_l^*\} + E\{n_l n_l^*\}} \quad (4.19)$$

The relationship among SNR , SIR and $SENR$ is:

$$\frac{1}{SENR} = \frac{1}{SIR} + \frac{1}{SNR} \quad (4.20)$$

The average signal power is obtained as:

$$\begin{aligned}E\{\zeta_l \zeta_l^*\} &= E\left\{ \sum_{m=0}^{M-1} \left(h_m + \Delta H_m(0) \right) e^{-j \frac{2\pi m l}{N}} \times \sum_{m'=0}^{M-1} \left(h_{m'}^* + \Delta H_{m'}^*(0) \right) e^{j \frac{2\pi m' l}{N}} \right\} \\ &= \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \left\{ E\{h_m h_{m'}^*\} + E\{h_m \Delta H_{m'}^*(0)\} + E\{h_{m'}^* \Delta H_m(0)\} \right. \\ &\quad \left. + E\{\Delta H_m(0) \Delta H_{m'}^*(0)\} \right\} e^{-j \frac{2\pi l(m-m')}{N}}\end{aligned}\quad (4.21)$$

²The term *signal to interference plus noise ratio* ($SINR$) is also used in the literature [113].

Based on (3.3) and (3.4), we know that $E\{h_m \Delta H_{m'}^*(0)\} = 0$ and $E\{h_{m'}^* \Delta H_m(0)\} = 0$. Since

$$\begin{aligned}
 E\{\Delta H_m(0) \Delta H_{m'}^*(0)\} &= E\left\{\frac{1}{N} \sum_{k=0}^{N-1} \Delta h_{m,k+N_G} \frac{1}{N} \sum_{k'=0}^{N-1} \Delta h_{m',k'+N_G}^*\right\} \\
 &= \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} E\{\Delta h_{m,k+N_G} \Delta h_{m',k'+N_G}^*\} \\
 &= \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} \sigma_{\Delta m}^2 J_0\left(2\pi f_D T_r(k - k')\right) \delta(m - m')
 \end{aligned} \tag{4.22}$$

with (3.11), we have

$$E\{\zeta_l \zeta_l^*\} = \sum_{m=0}^{M-1} \left(\sigma_m^2 + \Omega \sigma_{\Delta m}^2 \right) \tag{4.23}$$

where Ω is defined as:

$$\Omega = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0\left(2\pi f_D T_r(k - k')\right) \tag{4.24}$$

The mean ICI power is

$$\begin{aligned}
 E\{c_l c_l^*\} &= E\left\{\left(\sum_{m=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} a_n \Delta H_m(n-l) e^{-j \frac{2\pi n m}{N}}\right) \times \left(\sum_{m'=0}^{M-1} \sum_{\substack{n'=0 \\ n' \neq l}}^{N-1} a_{n'}^* \Delta H_{m'}^*(n'-l) e^{j \frac{2\pi n' m'}{N}}\right)\right\} \\
 &= E\left\{\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} \sum_{\substack{n'=0 \\ n' \neq l}}^{N-1} a_n a_{n'}^* \Delta H_m(n-l) \Delta H_{m'}^*(n'-l) e^{-j \frac{2\pi}{N}(mn - m'n')}\right\} \\
 &= \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} \sum_{\substack{n'=0 \\ n' \neq l}}^{N-1} E\{a_n a_{n'}^*\} E\{\Delta H_m(n-l) \Delta H_{m'}^*(n'-l)\} e^{-j \frac{2\pi}{N}(mn - m'n')}
 \end{aligned} \tag{4.25}$$

Using (4.13), we have:

$$\begin{aligned}
 &E\{\Delta H_m(n-l) \Delta H_{m'}^*(n'-l)\} \\
 &= E\left\{\left(\frac{1}{N} \sum_{k=0}^{N-1} \Delta h_{m,k+N_G} e^{j \frac{2\pi k(n-l)}{N}}\right) \times \left(\frac{1}{N} \sum_{k'=0}^{N-1} \Delta h_{m',k'+N_G}^* e^{-j \frac{2\pi k'(n'-l)}{N}}\right)\right\} \\
 &= \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} E\{\Delta h_{m,k+N_G} \Delta h_{m',k'+N_G}^*\} e^{j \frac{2\pi}{N}[k(n-l) - k'(n'-l)]} \\
 &= \frac{\sigma_{\Delta m}^2}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0\left(2\pi f_D T_r(k - k')\right) \delta(m - m') e^{j \frac{2\pi}{N}[k(n-l) - k'(n'-l)]}
 \end{aligned} \tag{4.26}$$

Without loss of generality, we assume that there is no correlation among data a_n , that is

$$E\{a_n a_{n'}^*\} = \delta(n - n') \quad (4.27)$$

Then one obtains,

$$E\{c_l c_l^*\} = \frac{1}{N^2} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0(2\pi f_D T_r(k - k')) e^{j\frac{2\pi}{N}[(k-k')(n-l)]} \quad (4.28)$$

Evaluating (4.28) including $n = l$, we have Λ equal to:

$$\begin{aligned} \Lambda &= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \left\{ \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0(2\pi f_D T_r(k - k')) e^{j\frac{2\pi}{N}[(k-k')(n-l)]} \right\} \\ &= \frac{1}{N^2} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \left\{ \left(\sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0(2\pi f_D T_r(k - k')) e^{-j\frac{2\pi}{N}l(k-k')} \right) \left(\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(k-k')} \right) \right\} \\ &= \frac{1}{N^2} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \left\{ \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0(2\pi f_D T_r(k - k')) e^{-j\frac{2\pi}{N}l(k-k')} N \delta(k - k') \right\} \\ &= \frac{1}{N^2} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \sum_{k=0}^{N-1} N \\ &= \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \end{aligned} \quad (4.29)$$

Neglecting the contribution of the component at $n = l$, we have

$$E\{c_l c_l^*\} = (1 - \Omega) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \quad (4.30)$$

where Ω is defined in equation (4.24). When there is no movement between the transmitter and the receiver, i.e. $f_D = 0$, $\Omega = 1$, it is clear to see that there is no ICI.

Substituting (4.23) into (4.17), one gets:

$$SNR = \frac{\sum_{m=0}^{M-1} \sigma_m^2 + \Omega \sum_{m=0}^{M-1} \sigma_{\Delta m}^2}{\sigma_f^2} = \frac{\gamma + \Omega}{\Gamma} \quad (4.31)$$

where Γ is defined as:

$$\Gamma = \frac{\sigma_f^2}{\sum_{m=0}^{M-1} \sigma_{\Delta m}^2} \quad (4.32)$$

and γ is defined in equation (3.13). Substituting (4.23) and (4.30) into (4.18), we get:

$$SIR = \frac{\sum_{m=0}^{M-1} \sigma_m^2 + \Omega \sum_{m=0}^{M-1} \sigma_{\Delta m}^2}{(1 - \Omega) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2} = \frac{\gamma + \Omega}{1 - \Omega} \quad (4.33)$$

Substituting (4.23) and (4.30) into (4.19), we have,

$$SENR = \frac{\sum_{m=0}^{M-1} \sigma_m^2 + \Omega \sum_{m=0}^{M-1} \sigma_{\Delta m}^2}{(1 - \Omega) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 + \sigma_f^2} = \frac{\gamma + \Omega}{1 - \Omega + \Gamma} \quad (4.34)$$

The fading channel model developed in chapter 3 could be widely used in scenarios where the symbol duration is close to the coherence time of the fading channel. The corresponding SNR , SIR and $SENR$ for time-invariant [6], and fast fading channels [19], [91] can be obtained by setting $\Omega = 1$ and $\gamma = 0$ respectively.

4.3.2 ICI Power Distribution

In order to design efficient ICI cancellation schemes, obtaining the ICI power distribution is the first step. From equation (4.15), we know that the interference from n^{th} subcarrier ($n \neq l$) is:

$$c_l^n = \sum_{m=0}^{M-1} a_n \Delta H_m(n-l) e^{-j \frac{2\pi n m}{N}} \quad (4.35)$$

then, its mean power is

$$\begin{aligned} E\{c_l^n (c_l^n)^*\} &= E\left\{\left(\sum_{m=0}^{M-1} a_n \Delta H_m(n-l) e^{-j \frac{2\pi n m}{N}}\right) \times \left(\sum_{m'=0}^{M-1} a_n^* \Delta H_{m'}^*(n-l) e^{j \frac{2\pi n m'}{N}}\right)\right\} \\ &= E\left\{\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} a_n a_n^* \Delta H_m(n-l) \Delta H_{m'}^*(n-l) e^{-j \frac{2\pi n}{N} (m-m')}\right\} \\ &= \sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} E\left\{\Delta H_m(n-l) \Delta H_{m'}^*(n-l)\right\} e^{-j \frac{2\pi n}{N} (m-m')} \\ &= \Omega_{[n-l]} \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \end{aligned} \quad (4.36)$$

where (4.26) and (4.27) are used, and $\Omega_{[n-l]}$ is defined as:

$$\Omega_{[n-l]} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0\left(2\pi f_D T_r(k-k')\right) e^{j 2\pi (k-k') \left(\frac{n-l}{N}\right)} \quad (4.37)$$

then, based on (4.30), we have,

$$\rho_{[n-l]} = \frac{E\{c_l^n (c_l^n)^*\}}{E\{c_l c_l^*\}} = \frac{\Omega_{[n-l]}}{1 - \Omega} \quad (4.38)$$

From equation (4.37), we know that, given N and $f_D T_r$, the distribution of the ICI power depends only on $(n - l)_N$ (the residue of $n - l$ modulo N) or $|n - l|$ (the absolute value of $n - l$). In addition, the ICI power distribution is independent of γ .

4.3.3 Slow Fading Channel with ICI

As a special case of the ICI analysis, we derive the *SIR* and *SENR* when the channel impulse response changes significantly only once during one OFDM symbol, to show the relevance of developing the channel model presented in chapter 3. The results obtained for this case could be used to analyze the overall system performance over slow fading channels.

Assuming that the channel impulse response changes significantly at observation time $t = (i + N_G)T_r$ ($0 \leq i \leq N - 1$, and $N \geq 2$), the channel impulse response may be written as:

$$h_{m,k+N_G} = \begin{cases} h_m & (\text{when } 0 \leq k \leq i - 1) \\ h_m + \Delta h_m & (\text{when } i \leq k \leq N - 1) \end{cases} \quad (4.39)$$

where Δh_m is assumed to be a zero mean Gaussian random process, and the variances of h_m and Δh_m are assumed to be σ_m^2 and $\sigma_{\Delta m}^2$ respectively.

With the channel impulse response given in equation (4.39), the demodulated sequence Z_l^i ($0 \leq l \leq N - 1$) may be written as:

$$Z_l^i = \zeta_l^i a_l + c_l^i + n_l \quad (4.40)$$

where c_l^i is the ICI:

$$c_l^i = \sum_{m=0}^{M-1} \sum_{\substack{n=0 \\ n \neq l}}^{N-1} a_n \Delta H_m^i(n - l) e^{-j \frac{2\pi n m}{N}} \quad (4.41)$$

and ζ_l^i is the attenuation factor of the desired subchannel:

$$\zeta_l^i = \sum_{m=0}^{M-1} \left(h_m + \Delta H_m^i(0) \right) e^{-j \frac{2\pi m l}{N}} \quad (4.42)$$

and

$$\Delta H_m^i(n-l) = \frac{\Delta h_m}{N} \sum_{k=i}^{N-1} e^{j \frac{2\pi k(n-l)}{N}} \quad (4.43)$$

Repeating the derivations of $E\{\zeta_l \zeta_l^*\}$ and $E\{c_l c_l^*\}$, the average signal power is:

$$E\{\zeta_l^i \zeta_l^{i*}\} = \sum_{m=0}^{M-1} \left\{ \sigma_m^2 + \sigma_{\Delta m}^2 \left(1 - \frac{i}{N} \right)^2 \right\} \quad (4.44)$$

and the mean ICI power:

$$E\{c_l^i c_l^{i*}\} = \frac{i}{N} \left(1 - \frac{i}{N} \right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \quad (4.45)$$

From (4.45), it can be seen that there is no ICI when $i = 0$ and $i = N$, that is, there is no ICI if the channel impulse response changes significantly at the beginning or end of one OFDM symbol. In addition, the maximum mean ICI power is obtained when

$$\begin{cases} i = \frac{N}{2} & (\text{when } N \text{ is even}) \\ i = \frac{N \pm 1}{2} & (\text{when } N \text{ is odd}) \end{cases} \quad (4.46)$$

The SIR and the $SENR$ may be written as:

$$SIR^i = \frac{\sum_{m=0}^{M-1} \sigma_m^2 + \left(1 - \frac{i}{N} \right)^2 \sum_{m=0}^{M-1} \sigma_{\Delta m}^2}{\frac{i}{N} \left(1 - \frac{i}{N} \right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2} = \frac{\gamma + \left(1 - \frac{i}{N} \right)^2}{\frac{i}{N} \left(1 - \frac{i}{N} \right)} \quad (4.47)$$

and

$$SENR^i = \frac{\sum_{m=0}^{M-1} \sigma_m^2 + \left(1 - \frac{i}{N} \right)^2 \sum_{m=0}^{M-1} \sigma_{\Delta m}^2}{\frac{i}{N} \left(1 - \frac{i}{N} \right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 + \sigma_f^2} = \frac{\gamma + \left(1 - \frac{i}{N} \right)^2}{\frac{i}{N} \left(1 - \frac{i}{N} \right) + \Gamma} \quad (4.48)$$

It is reasonable to assume that the location (denoted by i) where the channel impulse response changes significantly in one OFDM symbol duration is uniformly distributed

(randomly across $0 \leq i \leq N - 1$) with equal probability. Based on this assumption, the overall averaged signal power and overall averaged ICI power may be written as:

$$\begin{aligned}
 \overline{E\{\zeta_l^i \zeta_l^{i*}\}} &= \frac{1}{N} \sum_{i=0}^{N-1} E\{\zeta_l^i \zeta_l^{i*}\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} \sum_{m=0}^{M-1} \left\{ \sigma_m^2 + \sigma_{\Delta m}^2 \left(1 - \frac{i}{N}\right)^2 \right\} \\
 &= \sum_{m=0}^{M-1} \sigma_m^2 + \left(\frac{1}{3} + \frac{1}{2N} + \frac{1}{6N^2}\right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2
 \end{aligned} \tag{4.49}$$

and

$$\begin{aligned}
 \overline{E\{c_l^i c_l^{i*}\}} &= \frac{1}{N} \sum_{i=0}^{N-1} E\{c_l^i c_l^{i*}\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} \left\{ \frac{i}{N} \left(1 - \frac{i}{N}\right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 \right\} \\
 &= \frac{1}{6} \left(1 - \frac{1}{N^2}\right) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2
 \end{aligned} \tag{4.50}$$

Then the overall averaged SIR^i and $SENR^i$ are:

$$\overline{SIR^i} = \frac{\overline{E\{\zeta_l^i \zeta_l^{i*}\}}}{\overline{E\{c_l^i c_l^{i*}\}}} = \frac{\gamma + \left(\frac{1}{3} + \frac{1}{2N} + \frac{1}{6N^2}\right)}{\frac{1}{6} \left(1 - \frac{1}{N^2}\right)} \tag{4.51}$$

and

$$\overline{SENR^i} = \frac{\overline{E\{\zeta_l^i \zeta_l^{i*}\}}}{\overline{E\{c_l^i c_l^{i*}\}} + \sigma_f^2} = \frac{\gamma + \left(\frac{1}{3} + \frac{1}{2N} + \frac{1}{6N^2}\right)}{\frac{1}{6} \left(1 - \frac{1}{N^2}\right) + \Gamma} \tag{4.52}$$

When $N \rightarrow +\infty$, (4.51) and (4.52) become:

$$\lim_{N \rightarrow +\infty} \overline{SIR^i} = 6\gamma + 2 \tag{4.53}$$

$$\lim_{N \rightarrow +\infty} \overline{SENR^i} = \frac{6\gamma + 2}{1 + 6\Gamma} \tag{4.54}$$

From (4.51) and (4.53), we know that, the $\overline{SIR^i}$ is a monotonously decreasing function of N .

4.4 Analytical Results

In order to provide some numerical results on SIR , $SENR$ and ICI power distribution, analytical results are presented with the assumption that the number of cyclic prefix samples is $N_G = 20$, and $N = 128, 256, 512$ and 1024 .

4.4.1 ICI Power Distribution

When $f_D T_a \leq 10^{-4}$, it is found that $\rho_{[\pm 1]}$ have the highest ratio, that is, the two immediately adjacent subchannels contribute the highest ICI power among all the subchannels. and the ICI mean power mainly comes from the few closest adjacent subchannels. In addition, for the 1^{st} subchannel, we found that the 2^{nd} and N^{th} subchannel contribute the highest ICI power. For the N^{th} subchannel, the $(N-1)^{th}$ and 1^{st} subchannel contribute the highest ICI power. Thus, the interference pattern may be illustrated by ‘warping’ the frequency band (where the OFDM subcarriers are located) into a ring, as shown in Figure 4.1. The main ICI power of all the subchannels marked with “X” comes from the two immediately adjacent subchannels marked with “O”, and *vice versa*. It is also found that the ICI power distribution is only slightly different for different N , and the ICI power distribution is independent of the value of γ . The results for $N = 256$ and $f_D T_a = 10^{-4}$ are chosen as the typical values which are given in Table 4.1.

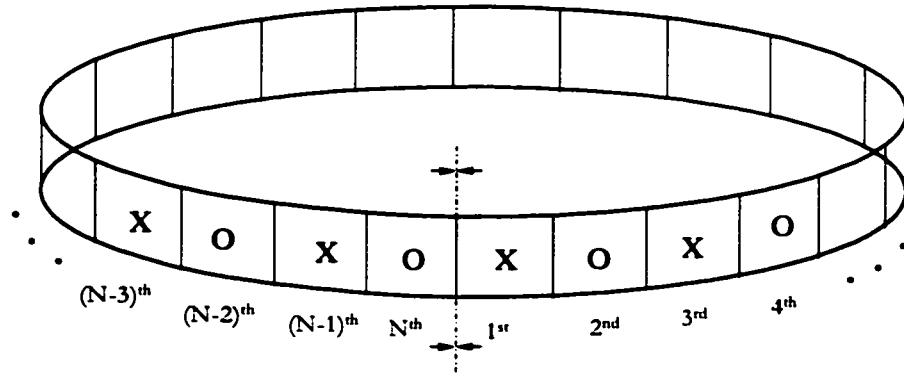


Figure 4.1: ICI pattern within one OFDM symbol ($f_D T_a \leq 10^{-4}$).

Table 4.1: ICI power distribution ($N = 256$, $f_D T_a = 10^{-4}$).

$n - l$	$\rho_{[n-l]} + \rho_{[-(n-l)]}$	$\Sigma(\rho_{[n-l]} + \rho_{[-(n-l)]})$
1	60.82%	60.82%
2	15.19%	76.01%
3	6.75%	82.76%
4	3.80%	86.56%
5	2.43%	88.99%
6	1.69%	90.68%
7	1.24%	91.92%
8	0.95%	92.87%
9	0.75%	93.62%
10	0.61%	94.23%
11	0.50%	94.97%
12	0.42%	95.15%

Table 4.2: Approximate normalized diverging *Doppler* frequencies of ICI power distribution.

N	128	256	512	1024
$f_D T_a$	2×10^{-2}	1×10^{-2}	4×10^{-3}	2×10^{-3}

When the value of $f_D T_a$ is larger than 10^{-4} , the distribution of the ICI power starts to change. For a given N , with the increase of the value of $f_D T_a$, at the beginning, the percentage of the ICI power contributed by the two immediately adjacent subchannels begins to increase, but after $f_D T_a$ reaches a certain value, the two subchannels which contribute the highest ICI power start to *drift* from the two immediately adjacent subchannels to the two second immediately adjacent subchannels, then third, then fourth, and so on. We call the subchannel, which contributes the highest ICI power, *dominant disturbing subchannel*, and the $f_D T_a$ value, where the ICI power distribution starts drifting from the two immediately adjacent subchannels, *normalized diverging Doppler frequency*. This *drifting* phenomenon of the *dominant disturbing subchannel* is caused by the fact that the *Doppler* frequency shift corresponding to the *normalized diverging Doppler frequency* tends to be close to the subcarrier bandwidth. When the *Doppler* frequency shift is larger than the subcarrier bandwidth, the highest ICI power is contributed by the subcarrier

with frequency spacing from the desired subcarrier is closest to the *Doppler* frequency shift.

The approximate values of the normalized diverging *Doppler* frequency for $N = 128$, 256, 512 and 1024 are given in Table 4.2. From this table, it can be seen that, a larger N yields a smaller value of the normalized diverging *Doppler* frequency, which means the system with a larger N is more sensitive to *Doppler* spreading. However, it is found that the main ICI power is still contributed by the few closest adjacent subchannels (the two dominant disturbing subchannels and the subchannels located between them). As an example, choosing $N = 1024$, the ICI power distribution is given in Figure 4.2, where $10^{-6} \leq f_D T_a \leq 10^0$. From this figure, the *diverging* phenomenon of the ICI power distribution can be observed. The same conclusion is reached when N is an odd number, where $N = 127$, 255, 511 and 1023 are chosen.

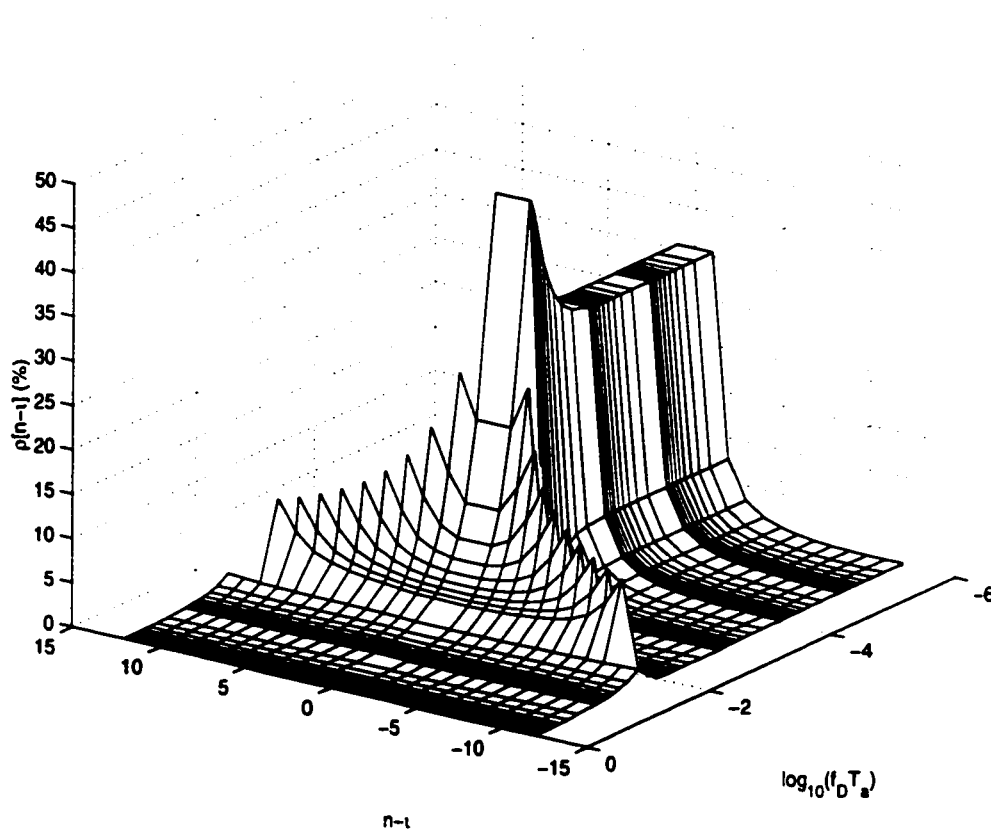


Figure 4.2: ICI power distribution when $N = 1024$, $|n - l| \leq 12$ and $10^{-6} \leq f_D T_a \leq 10^0$.

4.4.2 SIR and SENR

When $\gamma = 0$ and $SNR=10$ dB are assumed, the relationship between SIR and $f_D T_a$, and the relationship between $SENR$ and $f_D T_a$ are presented in Figure 4.3 and Figure 4.4 respectively. It can be observed that, both SIR and $SENR$ are decreasing rapidly with the increase of $f_D T_a$. Even with high SNR , the $SENR$ could be quite low (below 0 dB) when $f_D T_a$ reaches a certain value (which depends on the number of subcarriers N in the system). In order to maintain a high SIR and therefore $SENR$, either f_D or T_a has to be small or both. A larger N may achieve a higher transmission efficiency, but at the same time, it results in a lower SIR and $SENR$ (worse performance). The results presented in Figure 4.3 and Figure 4.4 also show that, with the increase of $f_D T_a$, ICI becomes the dominant noise, and therefore the system performance can not be improved by increasing SNR .

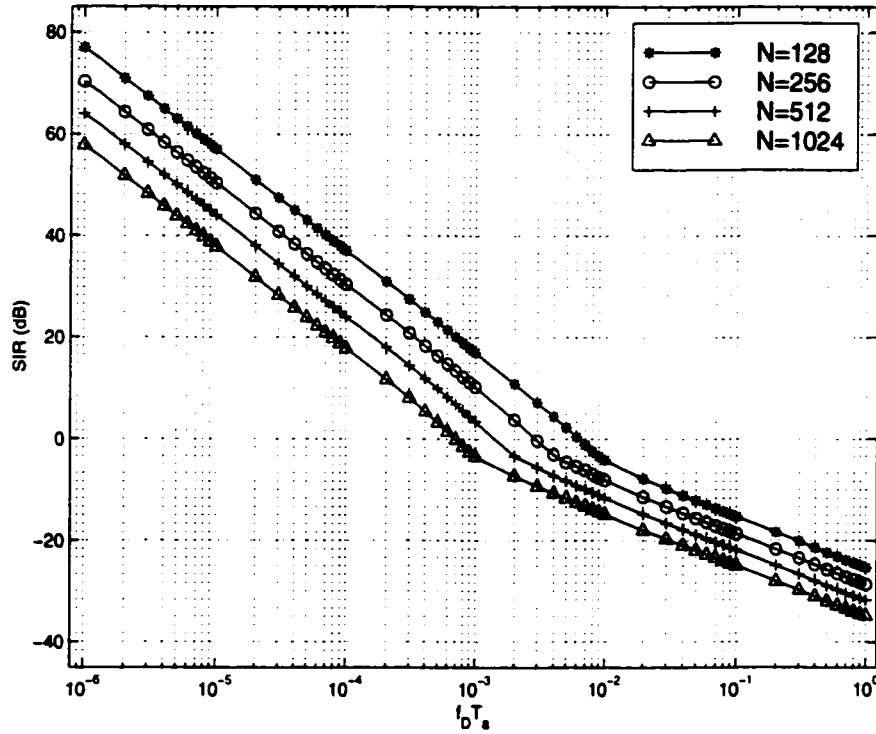


Figure 4.3: SIR when $\gamma = 0$.

Choosing $\gamma = 9$ (that is $\gamma = 9.54$ dB) and $SNR = 10$ dB, Figure 4.5 presents the relationship between SIR and $f_D T_a$, and Figure 4.6 presents the relationship between

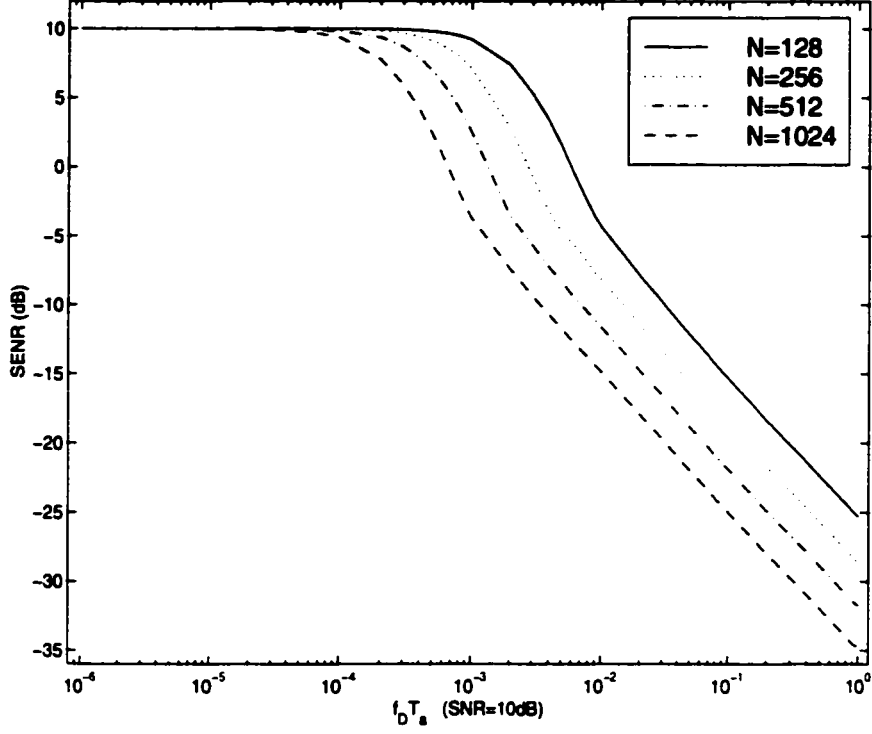


Figure 4.4: SEN when $\gamma = 0$ and $SNR = 10$ dB.

SEN and $f_D T_a$. Comparing the SIR of $\gamma = 9$ with that of $\gamma = 0$, it is observed that a large SIR improvement (near 10 dB) can be achieved when $f_D T_a$ is smaller than 10^{-3} . The value of SIR approaches 9.54 dB after $f_D T_a$ is larger than 10^{-2} . When $SNR = 10$ dB and $\gamma = 9$, the SEN results, as presented in Figure 4.6, are interesting: it is shown that the value of SEN decreases after $f_D T_a$ exceeds 10^{-4} , but all SEN values approaches 6.8 dB when $f_D T_a$ approaches 10^{-1} . Compared with the *time-invariant* channel, the performance degradation when $\gamma = 9$ in terms of SEN is about 3.2 dB. The same phenomena with different scales are also observed for the other values of SNR and γ .

The general results of SIR and SEN are given by choosing $\gamma \in [10^{-3}, 10^3]$ and $f_D T_a \in [10^{-6}, 10^0]$. SIR is given in Figure 4.7 for $N = 128$, and Figure 4.8 for $N = 1024$. From these two figures, it can be observed that SIR drops rapidly with either the increase of the normalized *Doppler* frequency, or the decreasing of γ , or both. Also, it is found that a larger N results in a lower SIR .

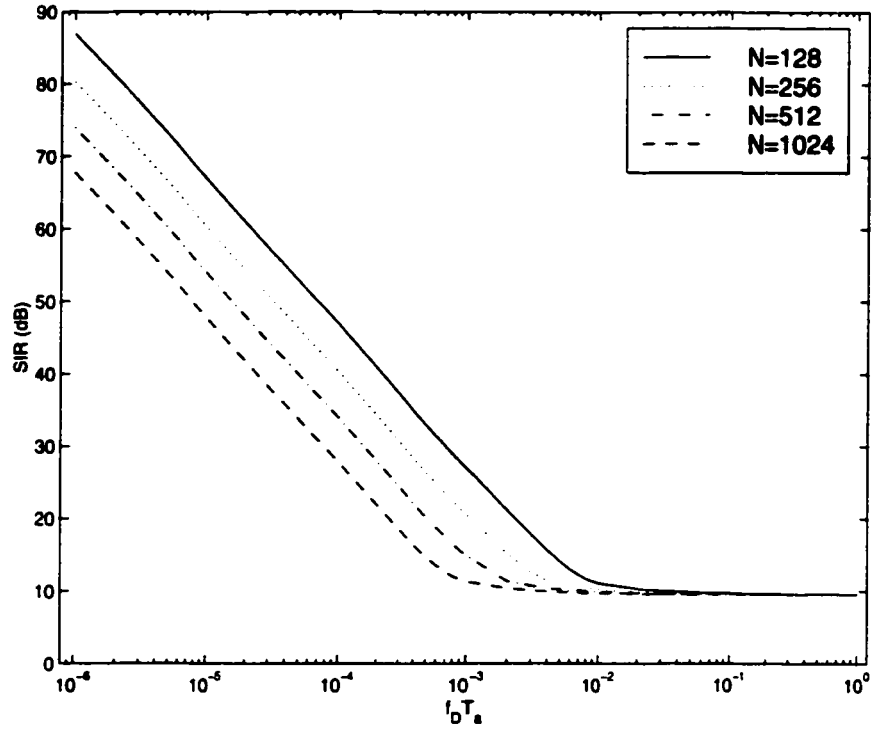


Figure 4.5: Signal to inter-channel interference ratio when $\gamma = 9$.

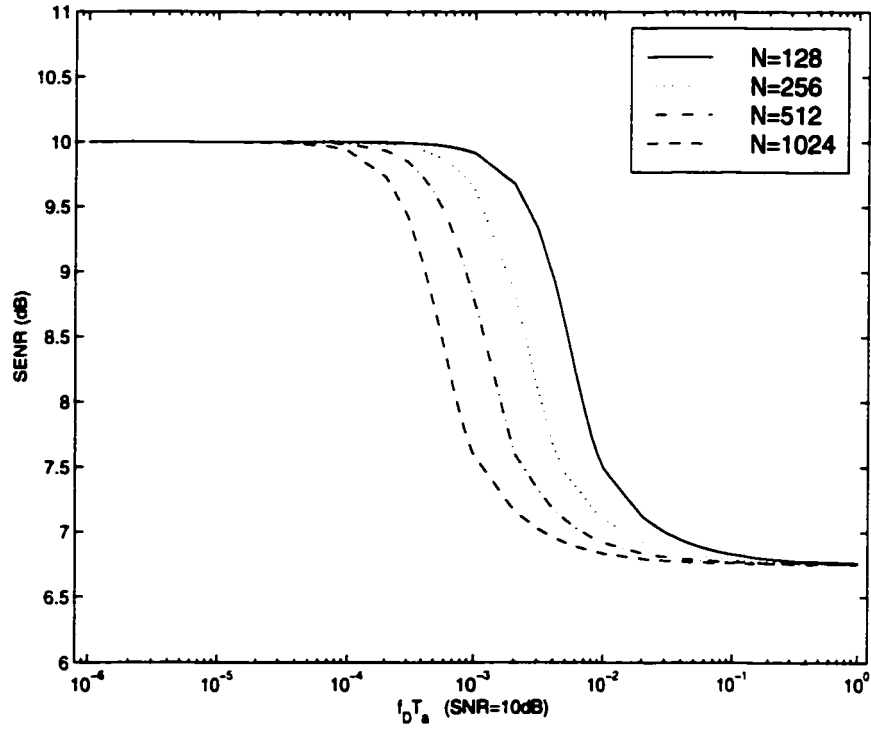


Figure 4.6: SEN when $\gamma = 9$ and $SNR = 10$ dB.

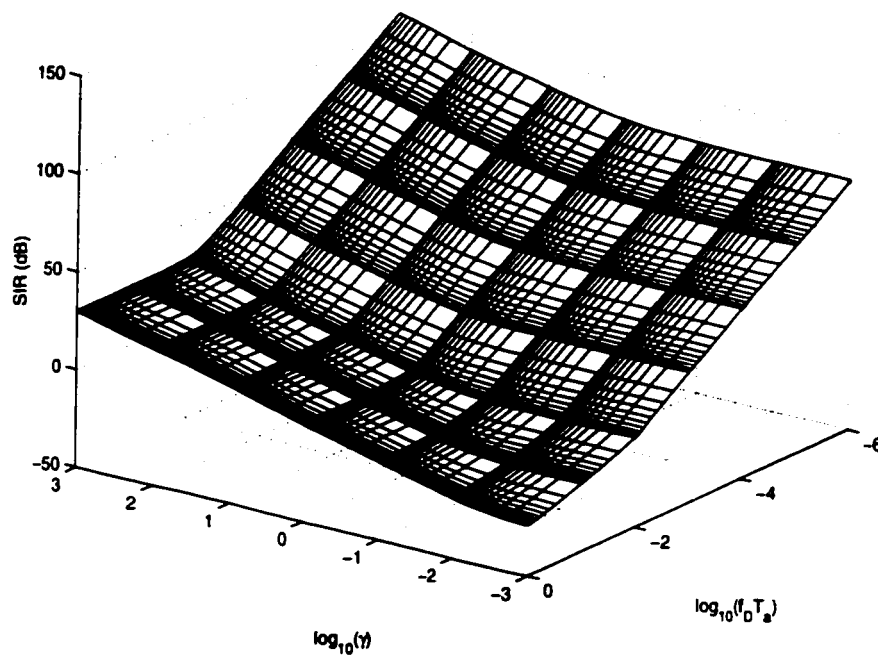


Figure 4.7: SIR with $N = 128$, $\gamma \in [10^{-3}, 10^3]$ and $f_D T_a \in [10^{-6}, 10^0]$.

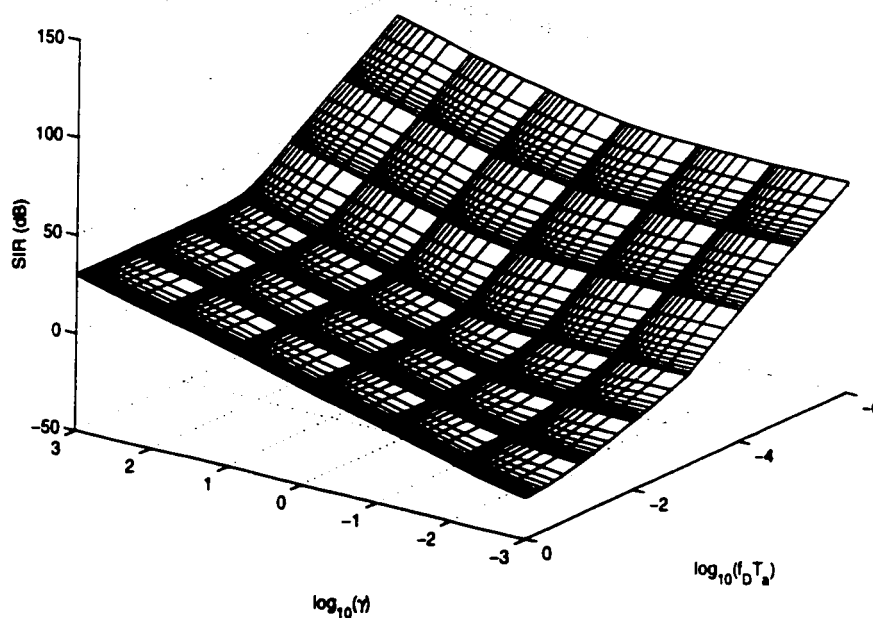


Figure 4.8: SIR with $N = 1024$, $\gamma \in [10^{-3}, 10^3]$ and $f_D T_a \in [10^{-6}, 10^0]$.

When $N = 128$, choosing $\gamma \in [10^{-3}, 10^3]$, $f_D T_a \in [10^{-6}, 10^0]$, and $SNR = 10$ dB, 30 dB and 50 dB, some $SEN R$ results are given in Figure 4.9, Figure 4.10, and Figure 4.11 respectively. These figures show that, a higher SNR results in a larger size of the area where $SEN R$ drops (compared with SNR) in the $f_D T_a - \gamma$ plane. We call the area, where $SEN R$ drops, *SEN R dropping zone*. To see the change of the *SEN R dropping zone* with the change of N , $SNR = 10$ dB, 30 dB and 50 dB are chosen for $N = 1024$, and the corresponding $SEN R$ results are given in Figure 4.12, Figure 4.13 and Figure 4.14. Comparing the figures for $N=128$ and 1024, it can be observed that, a larger N results in a larger size of the *SEN R dropping zone*. The system performance would be unacceptable if the system is operating in the high $f_D T_a$ and low γ area. On the other hand, no performance degradation would be observed if the OFDM system is operating in the complementary part of *SEN R dropping zone*, where the effects of ICI on OFDM system performance could be ignored.

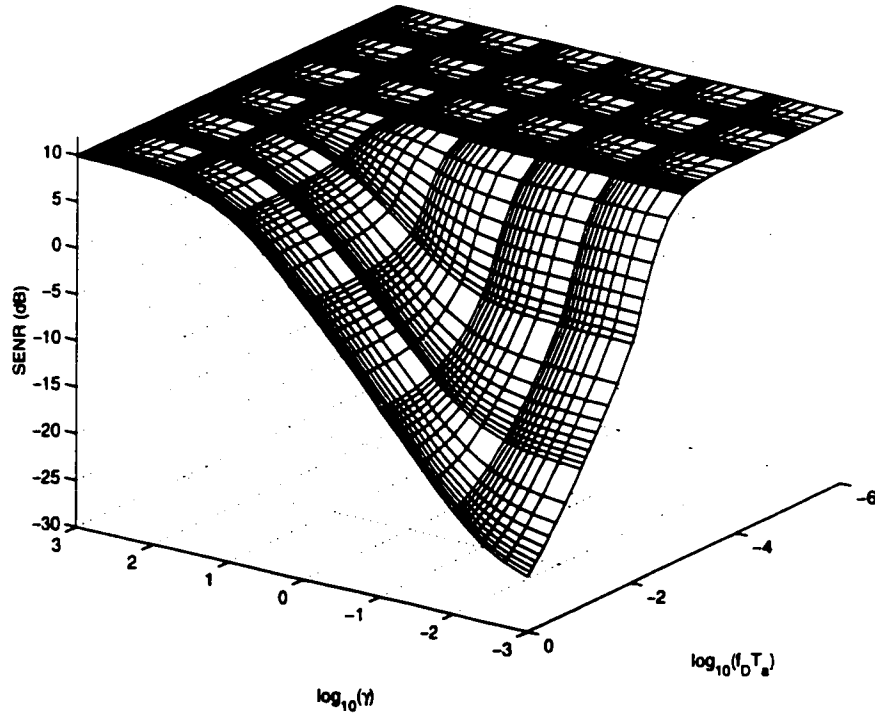


Figure 4.9: $SEN R$ when $N = 128$ and $SNR = 10$ dB.

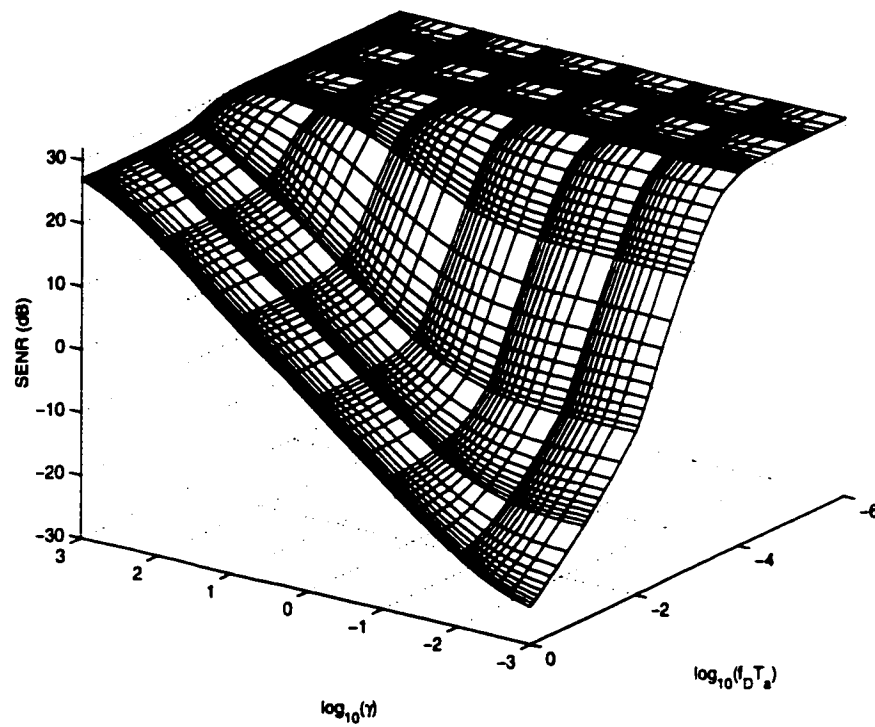


Figure 4.10: $SENR$ when $N = 128$, $SNR = 30$ dB.

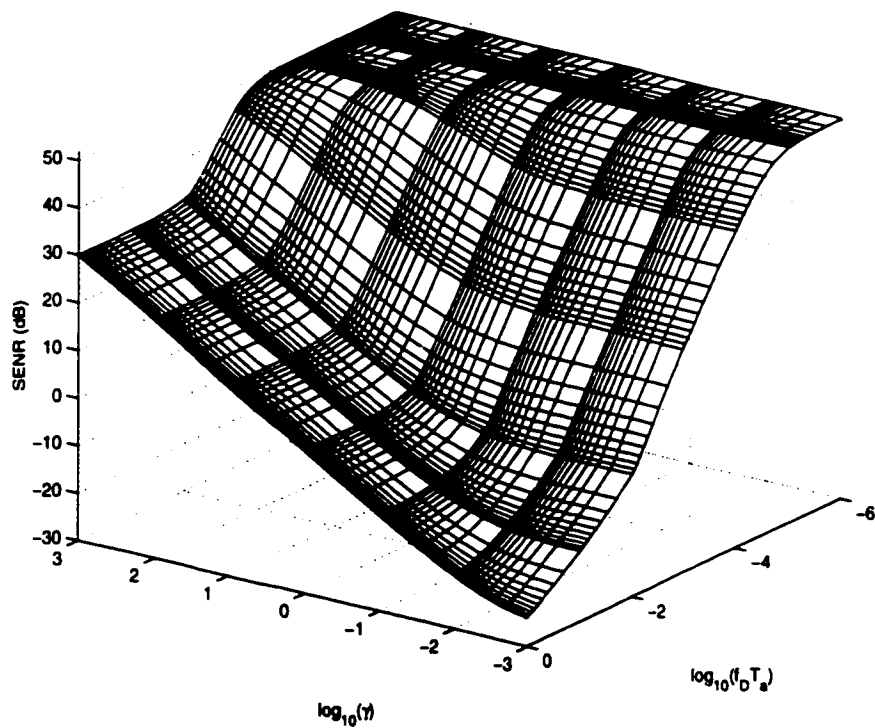


Figure 4.11: $SENR$ when $N = 128$, $SNR = 50$ dB.

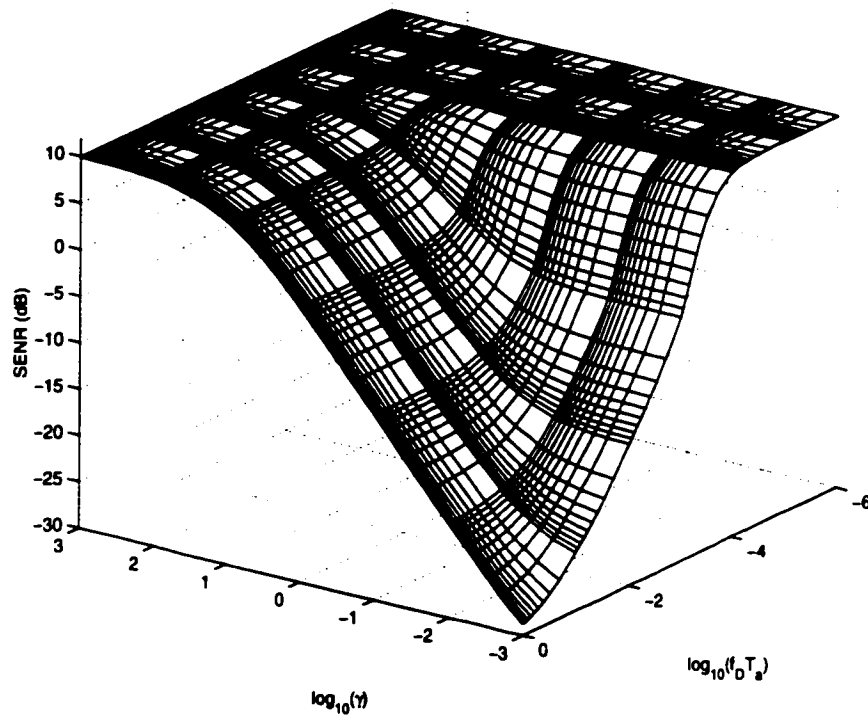


Figure 4.12: $SENR$ when $N = 1024$, $SNR = 10$ dB.

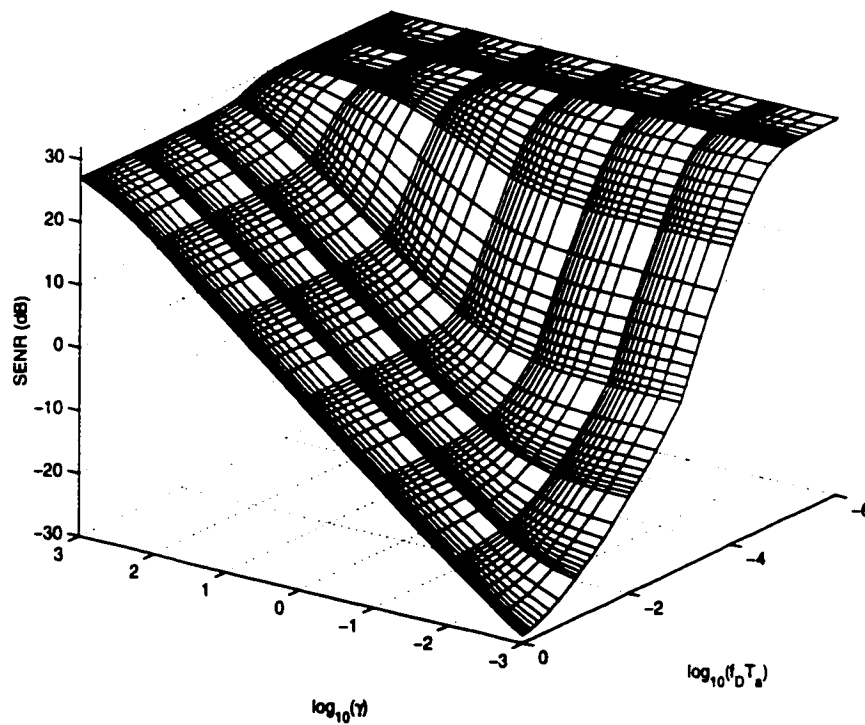


Figure 4.13: $SENR$ when $N = 1024$, $SNR = 30$ dB.

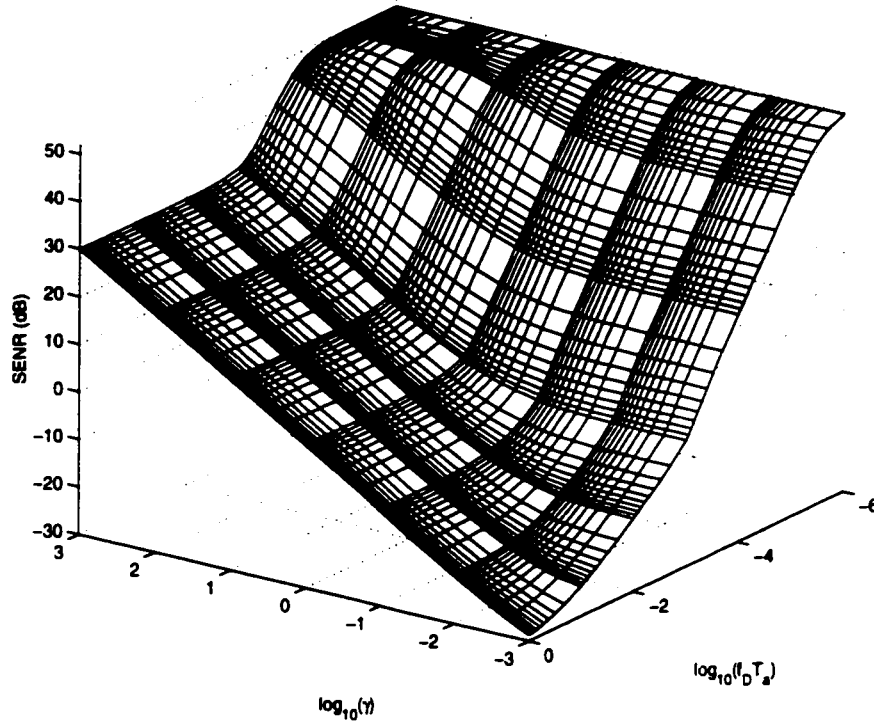
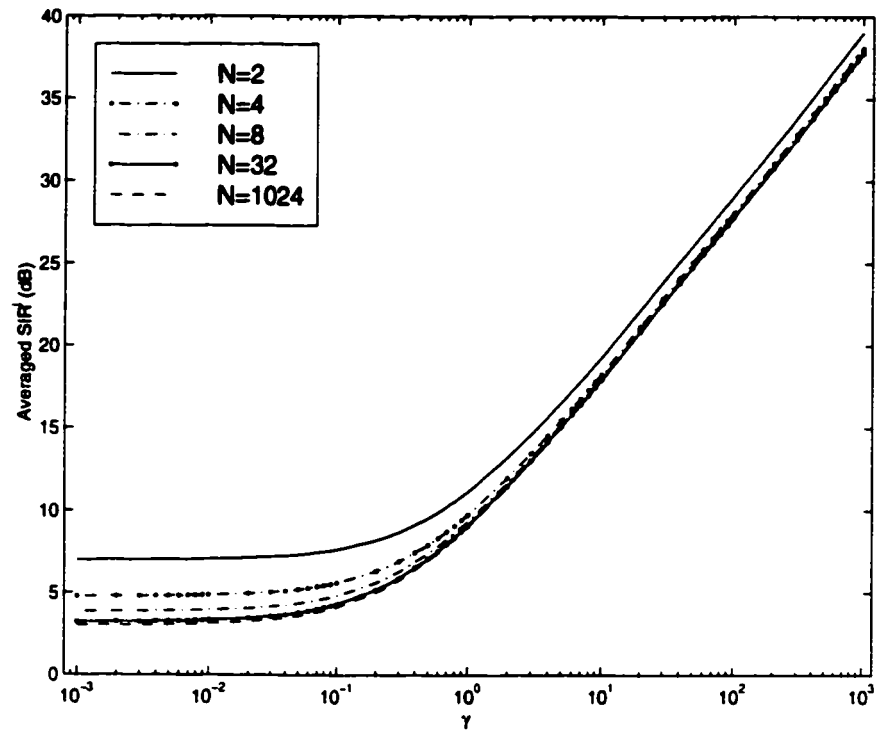
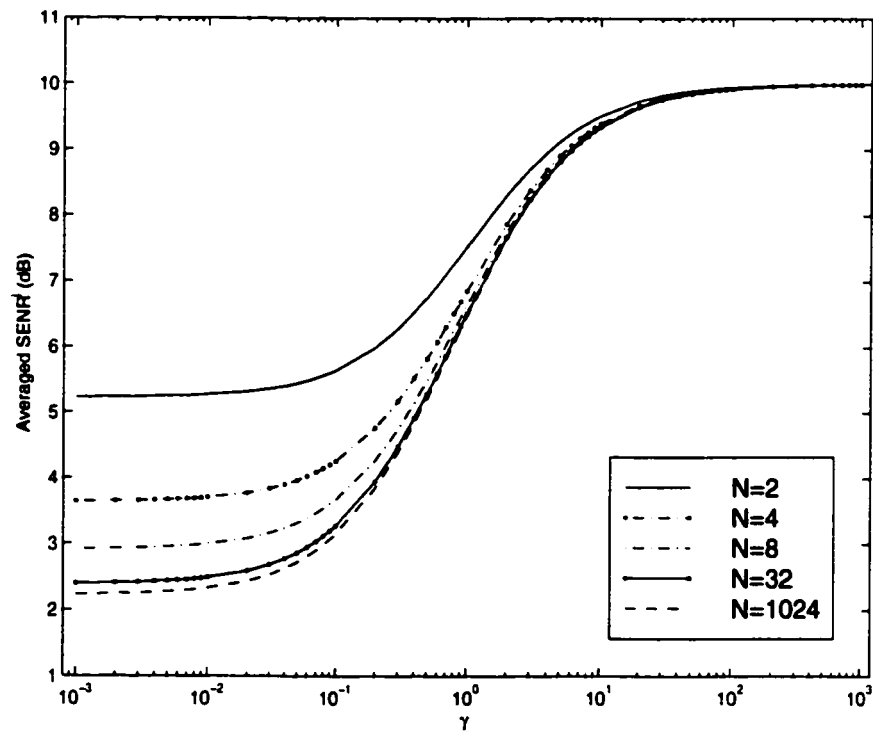


Figure 4.14: SEN when $N = 1024$, $SNR = 50$ dB.

Some results of $\overline{SIR^i}$ and $\overline{SEN R^i}$ are presented in Figure 4.15 and Figure 4.16 respectively, with the assumption that $SNR = 10$ dB. The results show that when $\gamma \geq 1$, $\overline{SIR^i}$ increases with the increment of γ , and when $\gamma \leq 10^{-2}$, it is small enough to simply set γ to be zero. With the increase of γ , all $\overline{SEN R^i}$'s for different N values approach 10 dB which is the value of SNR . Thus when $\gamma \geq 10^2$, the effects of ICI on OFDM system performance could be ignored (or the channel could be classified as time-invariant). It is also found that when $N \geq 32$, the corresponding values of $\overline{SIR^i}$ and $\overline{SEN R^i}$ may be calculated with (4.53) and (4.54), and there is almost no difference between the values of $\overline{SIR^i}$ and $\overline{SEN R^i}$ obtained with $N = 1024$ and N approaches $+\infty$. The results presented above show that the slow fading channel may not be replaced by the time-invariant channel model for the performance analysis of OFDM systems.

4.4.3 Effects of γ and Doppler Spread on System Performance

To investigate the effects of γ and Doppler frequency shift on system performance, a QPSK modulated OFDM system with coherent detection and perfect channel state


 Figure 4.15: $\overline{SIR^i}$ when $\gamma \in [10^{-3}, 10^3]$.

 Figure 4.16: $\overline{SENR^i}$ when $\gamma \in [10^{-3}, 10^3]$ and $SNR = 10$ dB.

information is chosen as the example. The example values of γ are chosen as 1 (0 dB) and 9 (9.54 dB), and the example values of $f_D T_a$ are chosen as 10^{-4} and 10^{-3} . $\gamma = 1$ represents the case where the time-invariant component and the time-variant component of the channel impulse response have the same average power. Bit error probability³ (BEP) vs E_S/N_0 (i.e. SNR per symbol) results are given in Figure 4.17 where the E_S/N_0 is defined in equation (4.17), and the relationship (in dB) between the E_S/N_0 and E_b/N_0 (i.e. SNR /bit [2]) for QPSK (2bits/symbol) is:

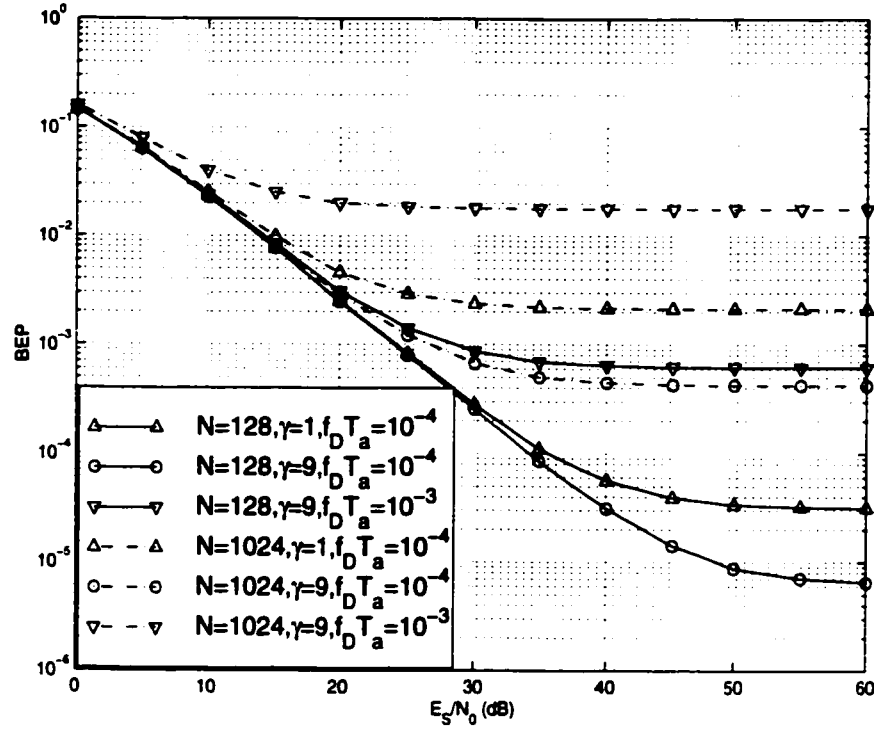
$$E_b/N_0 = E_S/N_0 - 3 = SNR - 3 \quad (\text{dB}) \quad (4.55)$$

From the results presented in the last subsection, we know that, for a given value of γ , the $SENR$ decreases with the increase of the value of $f_D T_a$; for a given value of $f_D T_a$, the $SENR$ value increases with the increase of the value of γ . From Figure 4.17, we see that for a given $f_D T_a$, the performance improves with the increase of γ ; then for a fixed γ , the performance degradation with the increase of $f_D T_a$, can be observed. Also from the results presented in the last subsection, we know that a larger N results in a higher average ICI power, thus worse performance. This is verified by the results presented in Figure 4.17 as well.

4.4.4 Channel Time Selectivity Evaluation for OFDM Systems

The values of β as a function of $f_D \Delta t$ when $\gamma = 0, 1$ and 9 (which are chosen to give examples for specific γ values in section 4.4.2 and section 4.4.3) are presented in Figure 4.18. To give some general numerical results of β corresponding to the presented SIR and $SENR$ results, β is re-evaluated choosing $\gamma \in [10^{-3}, 10^3]$ and $f_D \Delta t \in [10^{-2}, 10^0]$, as shown in Figure 4.19. It should be noted that to evaluate the channel time-selectivity of the OFDM systems presented in section 4.4.1, section 4.4.2 and section 4.4.3, the value of $f_D \Delta t$ corresponds to $N \times f_D T_a$ (i.e. $f_D T_S$ because $N \times T_a = T_S$ which is the OFDM symbol duration), where N is the number of subcarriers used in the system.

³In the literature, the term *probability of error* [114] is also widely used.

Figure 4.17: Effects of γ on OFDM systems.

When the normalized *Doppler* frequency $f_D T_S \in [0, 1]$ (which is generally satisfied in mobile communication systems), the value of $f_D T_a$ is approximately in $[0, 10^{-3}]$ for $N = 1024$; $[0, 2 \times 10^{-3}]$ for $N = 512$; $[0, 4 \times 10^{-3}]$ for $N = 256$ and $[0, 8 \times 10^{-3}]$ for $N = 128$. This means that among the values of $f_D T_a \in [10^{-6}, 10^0]$ which are used to give numerical results in section 4.4.1, section 4.4.2 and section 4.4.3, the values of $f_D \Delta t$ presented in Figure 4.18 roughly corresponds up to $f_D T_a \in [10^{-6}, 10^{-2}]$.

Computing $N \times f_D T_a$ with the values of N and $f_D T_a$ given in Table 4.2 of section 4.4.1, it is found that the values of $N \times f_D T_a$ (i.e. $f_D T_S$) are very close. This indicates that the normalized diverging *Doppler* frequencies of ICI power distribution is in fact determined by the value of $f_D T_S$ for OFDM systems, which may be used as one of the criteria to choose the number of subcarriers by the OFDM system designers for mobile applications.

Figure 4.18 shows that when $f_D T_S \in [0, 10^{-1}]$, the value of β is not less than 0.8 even for $\gamma = 0$, which means that the channel impulse response between two consecutive symbols

is highly correlated, and the channel shall appear to be slow fading. However, from the results presented in section 4.4.2, both the *SIR* and *SENR* still drop dramatically even for $\gamma = 9$ (see Figure 4.5 and Figure 4.6) when $f_D T_a \in [10^{-6}, 10^{-2}]$, and more *SENR* results can be found in Figures 4.9 to 4.14. It can be observed that the degradation caused by channel time-selectivity is more serious for the OFDM system with a larger number of subcarriers than those with a smaller N , because of a larger normalized *Doppler* frequency $f_D T_s$. All these results indicate that OFDM is extremely sensitive to channel time-selectivity compared with single-carrier systems.

From Figure 4.19, it can be observed that when $\gamma \geq 10^2$, the channel time-selectivity may be ignored even at high *Doppler* frequency, because $\beta \rightarrow 1$ for the shown $f_D \Delta t$ values (up to $f_D \Delta t = 1$). The same conclusion is reached in section 4.4.2, where the analysis is performed for slow fading channels. Another condition under which the channel time-selectivity may also be ignored is $f_D T_s \leq 10^{-2}$ simply because $\beta \rightarrow 1$ as well. When expressed in $f_D T_a$, the value of $f_D T_s = 10^{-2}$ may be approximated as $f_D T_a = 7.8 \times 10^{-5}$ ($N = 128$), $f_D T_a = 3.9 \times 10^{-5}$ ($N = 256$), $f_D T_a = 1.9 \times 10^{-5}$ ($N = 512$) and $f_D T_a = 9.7 \times 10^{-6}$ ($N = 1024$) respectively. However, based on the presented *SENR* results in Figures 4.9 to 4.14, and the BER presented in Figure 4.17, it indicates that the *SENR* may be more precise than the conventional coherence time $(\Delta t)_c$ (even it is defined with a high threshold) when used to evaluate the channel time-selectivity for OFDM systems.

4.5 Summary

In this chapter, we performed ICI analysis using the general time-variant frequency-selective WSSUS fading channel model characterized in chapter 3. The results are applicable to many specific channels. It was observed that the changes in the channel impulse response during one OFDM symbol duration cause ICI, and it is introduced in a *cyclic* fashion. We also defined the *SENR dropping zone*, where the BER error floor would be introduced. The average ICI power and its distribution were obtained. The required

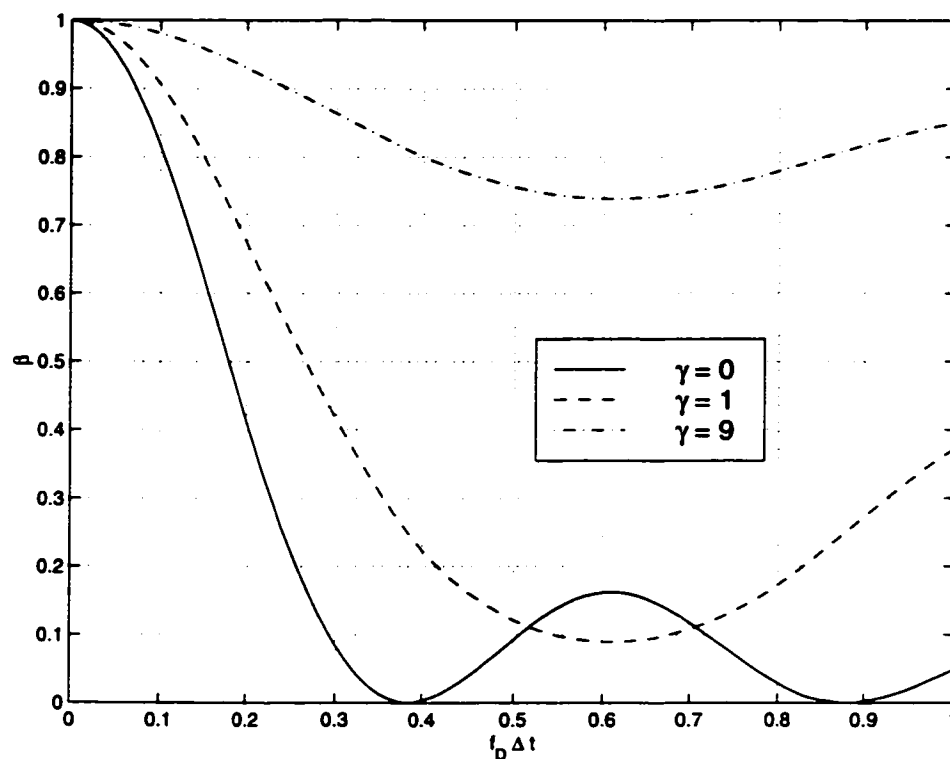


Figure 4.18: The values of β as a function of $f_D \Delta t$ when $\gamma = 0, 1$ and 9 .

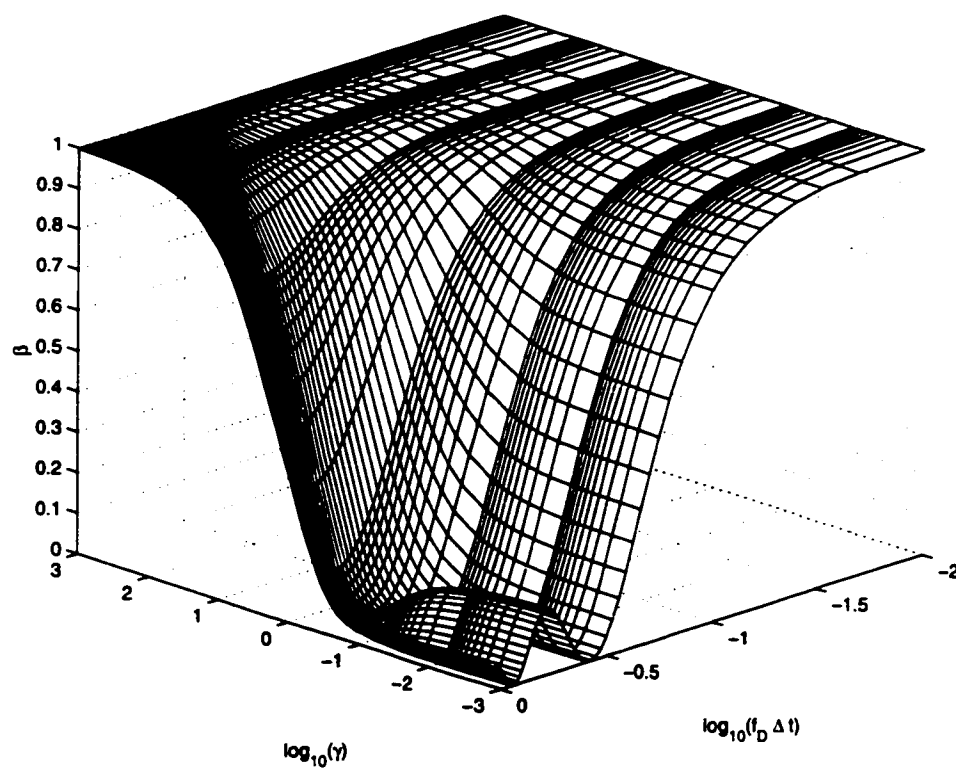


Figure 4.19: The values of β when $\gamma \in [10^{-3}, 10^3]$ and $f_D \Delta t \in [10^{-2}, 10^0]$.

values of γ and $f_D T_a$ with certain SNR to make the channel time-selectivity ignorable were also provided. In addition, we investigated the effects of γ on OFDM system performance, and demonstrated the channel time selectivity evaluation with β . The numerical results indicated that OFDM is very sensitive to channel time-selectivity, and $SENR$ may be a more appropriate parameter to evaluate the channel time-selectivity compared with the conventional channel coherence time. Based on the obtained ICI analysis results, we provided guidelines not only for the design of OFDM systems, but also for efficient ICI cancellation schemes (if it is necessary) for OFDM systems over multipath time-variant fading channels.

Chapter 5

Inter-carrier Interference Cancellation for OFDM Systems

From the ICI analysis presented in chapter 4, we know that, if ICI is caused by channel time-selectivity, most of the ICI power is contributed by a few closest neighboring subcarriers in a *cyclic* fashion (see Figure 4.1). This ICI power distribution is critical for the design of efficient ICI cancellation schemes, which is the subject of this chapter.

5.1 Introduction

To mitigate the ICI caused by the fast change of channel impulse responses, Leung and Ho proposed in [91] a successive (decision feedback) cancellation scheme designed for coherent QPSK OFDM systems. However, this scheme comes with high complexity and the performance improvement is not significant unless with perfect channel estimation. Inspired by the pilot symbol aided linear channel estimation technique proposed by Lau and Cheung [115], and assuming also that the channel impulse response is linear time-variant within one OFDM symbol period, a frequency-domain equalizer is proposed by Jeon, Chang and Cho [116] to cancel the ICI caused by the rapidly varying channel impulse response. The equalizer complexity is reduced by ignoring most of the interferences contributed by the subcarriers located far from the desired subcarrier, but to achieve a given performance gain, no guideline is given on how to determine the number of the neighboring subcarriers whose interference contribution should be accounted for.

The performance results of [115] are given by canceling the ICI contributed by 1, 3 and 5 neighboring subcarriers without specific reason. In addition, for the boundary subcarriers (located at the highest and lowest frequencies), only the interferences contributed by the subcarriers located at one side of the subcarrier are accounted for while those located on the other side are not, resulting in *partial* ICI cancellation. Figure 4.1 and Table 4.1 show that most of the ICI power is contributed by a few closest neighboring subcarriers on both sides of the desired subcarrier in a *cyclic* fashion.

In this chapter, we improve the frequency-domain equalizer proposed in [116] by canceling the ICI introduced by the neighboring interferences from both sides. We also provide guidelines on how to determine the number of neighboring subcarriers whose ICI contribution should be accounted in order to achieve a given performance gain. Based on the ICI analysis presented in chapter 4, we observed that ICI cancellation also may be performed with a time-domain equalizer when the time-variant channel impulse response can be estimated with high accuracy. However, an extensive literature survey shows that this work has already been done as reported in [117].

5.2 Pilot Symbol Assisted Channel Estimation for OFDM Systems

To cancel the ICI caused by the channel time-selectivity for OFDM systems, we need to estimate the channel impulse response, with successive ICI cancellation [91], frequency-domain equalization [116], or time-domain equalization [117]. Due to the fact that coherent detection has typically a 2-3 dB *SNR* advantage over differential detection, and because we need to perform channel estimation, we assume coherent detection for the studied OFDM systems.

Among all the channel estimation schemes proposed for OFDM systems, most of them use pilot symbol (data aided) estimation ([86], [118], [119], [120], [121]). Although using pilot symbols in system reduces the overall system transmission efficiency, it is a very

promising technique to combat the degradation caused by fast time-variant fading channels [122]. By periodically inserting known symbols in the transmitted sequence, the fading channel may be estimated by examining the distortion on the known pilot symbols, and then the channel state information may be estimated by using interpolation techniques. In order to determine the limitations of the proposed ICI cancellation equalizers, estimation and interpolation is performed using a *Wiener* filter [122]: the variance of the estimation error is assumed to be the mean power of the additive noise [123], which is denoted by σ_f^2 . As a benchmark reference, the performance obtained with perfect channel estimation is given as well.

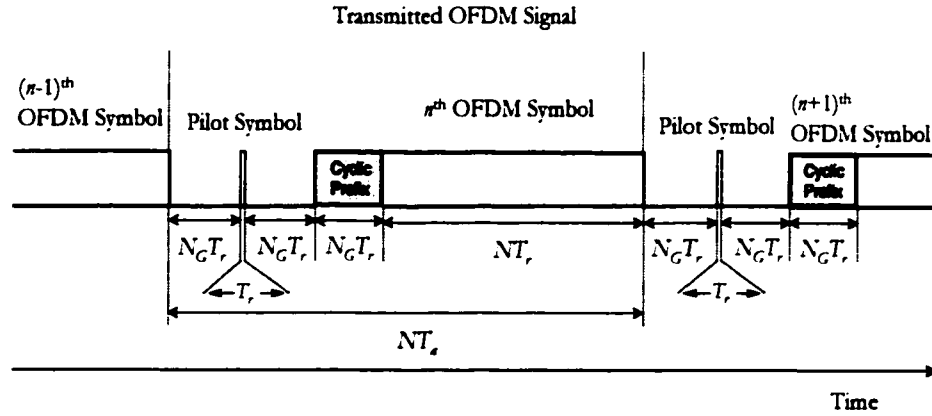


Figure 5.1: Pilot symbol assisted channel impulse response estimation for OFDM systems.

To estimate the rapidly varying impulse response of the fading channel for OFDM systems, a similar scheme with low complexity was developed in [91] and [116] independently, which is adopted in this chapter. With the assumption that the maximum excess delay of the channel is not longer than $N_G T_r$, Figure 5.1 (see also [91], [116]) illustrates how the known pilot symbols are inserted into the transmission stream to allow channel impulse response estimation at the reception. In the $(2N_G + 1)T_r$ period, the transmission signal is null except for one pilot symbol located at the center of the guard interval. The pilot symbol is chosen to be a time domain pulse with unit energy and one sample duration long. The first $N_G T_r$ null interval is used to avoid the interference caused by the previous OFDM symbol to the present channel delay power profile. The pilot symbol and the

second $N_G T_r$ null interval are used to obtain the delay power profile of the channel. The cyclic prefix is preserved simply to convert the linear convolution between the transmitted signal and the channel impulse response into a cyclic convolution [6], which makes the DFT demodulation process straightforward. It also should be noted that after adding the channel impulse response estimation interval, the sampling rate becomes:

$$T_r = \frac{N}{N + 3N_G + 1} T_a \quad (5.1)$$

5.3 Frequency-domain ICI Cancellation Equalizer

The system block diagram given in Figure 5.2 depicts the frequency-domain equalizer employed to perform ICI cancellation with the channel impulse response estimation, where those dashed-line blocks denote the components of the designed frequency-domain ICI cancellation equalizer. To avoid extra ICI or distortion to be introduced by the channel impulse response estimation error, we use the zero-forcing algorithm [2], [123] in our ICI cancellation equalizer design. This method is also used by Jeon, Chang and Cho in [116].

For the demodulated sequence given in equation (4.11), the ICI contributed by the n^{th} subcarrier (given by equation (4.35)) to the data symbol a_l ($n \neq l$) may be rewritten as:

$$c_l^n = a_n D_l^{(n-l)_N} \quad (5.2)$$

where the superscript $(n-l)_N$ stands for the residue of $(n-l)$ modulo N , and $D_l^{(n-l)_N}$ is defined as:

$$\begin{aligned} D_l^{(n-l)_N} &= \sum_{m=0}^{M-1} \Delta H_m(n-l) e^{-j \frac{2\pi n m}{N}} \\ &= \frac{1}{N} \sum_{m=0}^{M-1} \left(\sum_{k=0}^{N-1} \Delta h_{m,k+3N_G+1} e^{j \frac{2\pi k(n-l)}{N}} \right) e^{-j \frac{2\pi n m}{N}} \end{aligned} \quad (5.3)$$

Then the demodulated sequence given by equation (4.14) may be further expressed in matrix form:

$$\mathbf{Z} = \mathbf{D}\mathbf{a} + \mathbf{W} \quad (5.4)$$

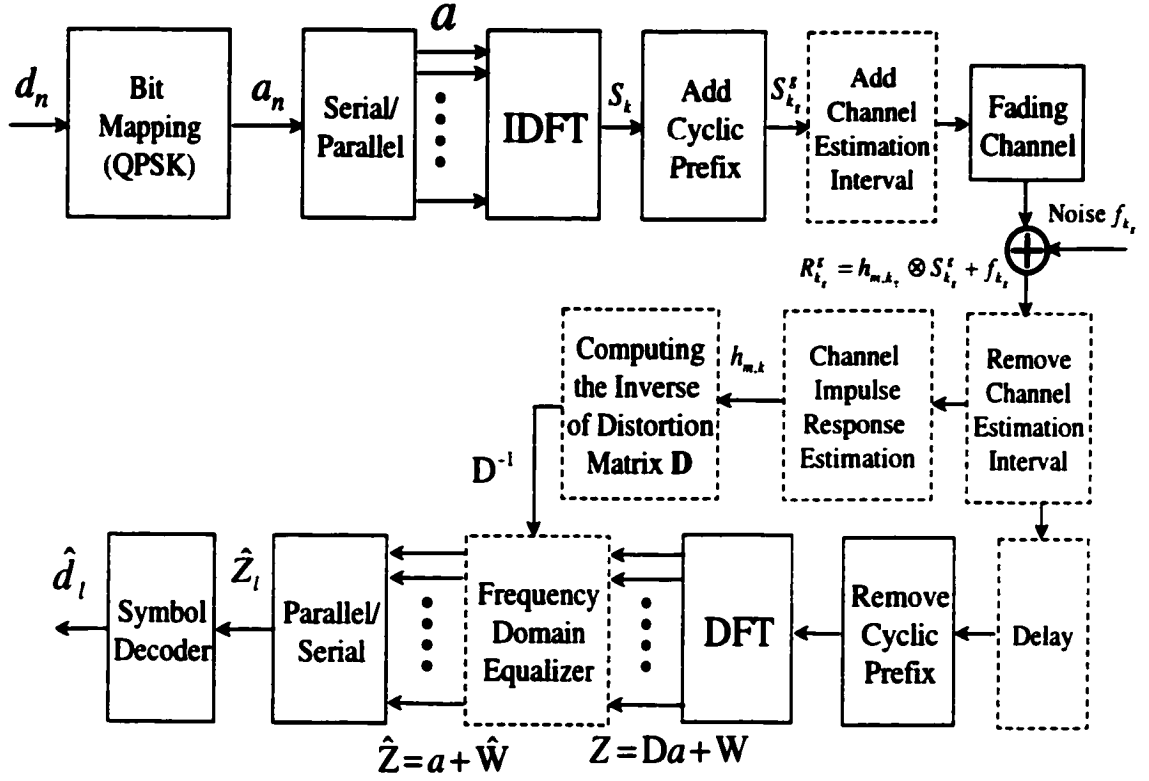


Figure 5.2: OFDM systems with frequency-domain ICI cancellation equalizer.

where $\mathbf{Z} = [Z_0, Z_1, \dots, Z_{N-1}]^T$ (Here the superscript T stands for the transpose of a matrix), $\mathbf{a} = [a_0, a_1, \dots, a_{N-1}]^T$, $\mathbf{W} = [w_0, w_1, \dots, w_{N-1}]^T$, and

$$\mathbf{D} = \begin{bmatrix} \zeta_0 & D_0^1 & D_0^2 & D_0^3 & \dots & D_0^{N-3} & D_0^{N-2} & D_0^{N-1} \\ D_1^{N-1} & \zeta_1 & D_1^1 & D_1^2 & \dots & D_1^{N-4} & D_1^{N-3} & D_1^{N-2} \\ D_2^{N-2} & D_2^{N-1} & \zeta_2 & D_2^1 & \dots & D_2^{N-5} & D_2^{N-4} & D_2^{N-3} \\ D_3^{N-3} & D_3^{N-2} & D_3^{N-1} & \zeta_3 & \dots & D_3^{N-6} & D_3^{N-5} & D_3^{N-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ D_{N-3}^3 & D_{N-3}^4 & D_{N-3}^5 & D_{N-3}^6 & \dots & \zeta_{N-3} & D_{N-3}^1 & D_{N-3}^2 \\ D_{N-2}^2 & D_{N-2}^3 & D_{N-2}^4 & D_{N-2}^5 & \dots & D_{N-2}^{N-1} & \zeta_{N-2} & D_{N-2}^1 \\ D_{N-1}^1 & D_{N-1}^2 & D_{N-1}^3 & D_{N-1}^4 & \dots & D_{N-1}^{N-2} & D_{N-1}^{N-1} & \zeta_{N-1} \end{bmatrix} \quad (5.5)$$

where ζ_n is the desired subchannel attenuation factor defined in equation (4.16), and $\mathbf{D}$ is called the *distortion matrix*. When the time-variant channel impulse response $h_{m,k+3N_G+1}$ ($0 \leq m \leq M-1$ and $0 \leq k \leq N-1$) is estimated, $\mathbf{D}$ may be obtained from (4.16) and (5.3). After getting $\mathbf{D}$, by detecting $\hat{\mathbf{Z}} = [\hat{Z}_0, \hat{Z}_1, \dots, \hat{Z}_{N-1}]^T$ (by multiplying $\mathbf{D}^{-1}$, the

inverse of $\mathbf{D}$ with $\mathbf{Z}$), the transmitted data sequence may be obtained:

$$\begin{aligned}
 \hat{\mathbf{Z}} &= \mathbf{D}^{-1}\mathbf{Z} \\
 &= \mathbf{D}^{-1}(\mathbf{D}\mathbf{a} + \mathbf{W}) \\
 &= \mathbf{a} + \mathbf{D}^{-1}\mathbf{W} \\
 &= \mathbf{a} + \hat{\mathbf{W}}
 \end{aligned} \tag{5.6}$$

where $\hat{\mathbf{W}}$ is defined as $\mathbf{D}^{-1}\mathbf{W}$. Thus, ICI is eliminated if the channel estimation is perfect.

With an accurate estimation of the channel impulse response, based on (5.6), the data sequence $\mathbf{a}$ may be successfully recovered in additive colored noise $\hat{\mathbf{W}}$. However, the computation of $\mathbf{D}$ and its inverse (both are $N \times N$ matrixes) could be quite time consuming and high speed devices are needed to perform the equalization, especially for those systems with a large N . Based on the analysis presented in chapter 4, we know that most of ICI power is contributed by a very few closest neighboring subcarriers in a cyclic fashion. that is, a large number of distortion elements $D_l^{(n-l)_N}$ in $\mathbf{D}$ could be set to zero, which substantially reduces the computation load of the equalization process.

Assuming that most of the ICI power is contributed by Q closest neighboring subcarriers, that is:

$$D_l^{(n-l)_N} = 0 \quad \text{when } (n-l)_N > Q \tag{5.7}$$

then $\mathbf{D}$ may be approximated by $\tilde{\mathbf{D}}$, which may be expressed as:

$$\tilde{\mathbf{D}} = \mathbf{D}\mathbf{X} \tag{5.8}$$

where we call $\mathbf{X}$ the *puncturing matrix*, and its puncturing pattern is given in Figure 5.3. Correspondingly, we call Q the *puncturing index*. Due to the fact that $Q \ll N$ (as the example given in Table 4.1), $\tilde{\mathbf{D}}$ is a sparse matrix (in each row, only $2Q + 1$ out of N elements are nonzero), the computation load is much smaller than that of $\mathbf{D}$. Also, because $\tilde{\mathbf{D}}$ is a sparse matrix, its inverse is much simpler to compute as well [116], [124]. In the frequency-domain equalizer designed in [116], the ICI contributions by the subcarriers located in the upper triangular and lower triangular of $\mathbf{X}$ are not taken into account (see

Eq. (9) of [116]). This means that, for those boundary subcarriers, significant amount of ICI power still exists after equalization. Our proposed ICI cancellation equalizer is therefore an improved version of [116].

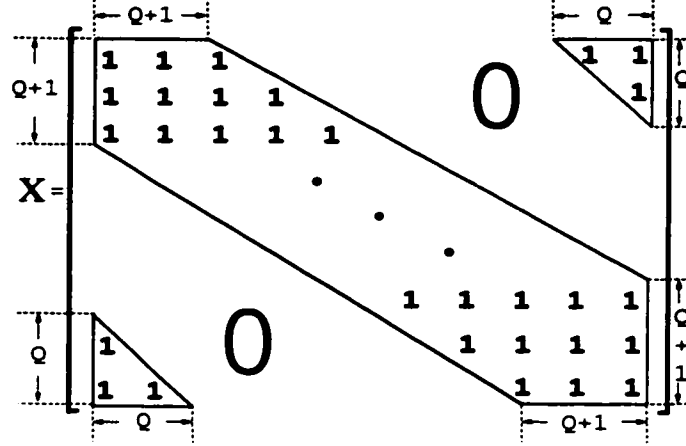


Figure 5.3: Puncturing pattern for matrix $\mathbf{D}$.

After obtaining the inverse of $\tilde{\mathbf{D}}$, which is denoted by $\tilde{\mathbf{D}}^{-1}$, the detection is performed by

$$\begin{aligned}
 \tilde{\mathbf{Z}} &= \tilde{\mathbf{D}}^{-1} \mathbf{Z} \\
 &= \tilde{\mathbf{D}}^{-1} \mathbf{D} \mathbf{a} + \tilde{\mathbf{D}}^{-1} \mathbf{W} \\
 &= \tilde{\mathbf{a}} + \tilde{\mathbf{W}}
 \end{aligned} \tag{5.9}$$

with $\tilde{\mathbf{W}} = \tilde{\mathbf{D}}^{-1} \mathbf{W}$ and the estimated data sequence $\tilde{\mathbf{a}} = \tilde{\mathbf{D}}^{-1} \mathbf{D} \mathbf{a}$. By choosing the value of Q large enough, most of the ICI power may be canceled out, and a much lower error floor may be achieved. For instance, with the assumption that the channel impulse response is perfectly known at the receiver, based on the results given in Table 4.1, for the OFDM system with $N=256$ and $f_D T_a = 10^{-4}$, up to 80% ICI power may be canceled with a puncturing index of $Q = 3$ only. Simulation results are given in the following section to show the performance of the proposed frequency-domain equalizer with different puncturing indexes.

5.4 Simulation Results

5.4.1 Assumptions and System Parameters

As in [91] and [116], our simulated system is an uncoded OFDM system¹. QPSK modulation is used as the bit mapping scheme. Perfect synchronization (no timing error, phase shift or frequency offset) is also assumed at the receiver side. The fading channel model used is a 2-path fast fading Rayleigh channel, which is widely used to investigate the performance of mobile communication systems (also in [91] and [116]). In addition, it is assumed that the average powers of the two fading paths are the same (equal power split) [91], and that the two paths are separated by $2\mu s$ [116] which is the maximum excess delay of the channel.

Aiming at offering interactive broadband wireless services [33], we assume that the system throughput is 8 Mbps (4 Mbauds for QPSK, i.e. $T_a = 0.25\mu s$), and operating at the 2 GHz frequency band [1]. To make the channel time-selectivity obvious, $\gamma = 0$ is assumed, and the *Doppler* frequency is assumed to be $f_D = 400$ Hz, which corresponds to vehicle speed roughly $v_s = 216$ km/h (e.g. high speed train communications).

$N = 256$ is chosen for a moderate computation complexity. Thus $f_D T_a = 10^{-4}$, and the normalized *Doppler* frequency $f_D T_S = 2.56\%$. After adding the channel impulse response estimation period and the cyclic prefix, the OFDM symbol period should still be unchanged and thus the sampling rate has to be increased. The number of samples needed for cyclic prefix is $N_G = 9$, the sampling interval $T_r \approx 0.2254\mu s$, and the required system bandwidth is 4.5 MHz.

Based on Figure 4.3, when $N = 256$ and $f_D T_a = 10^{-4}$, the $SIR \approx 30$ dB. This means that when $SNR < 30$ dB, ICI cancellation would not be necessary. We thus start our

¹As will be observed in chapter 6, the error floor caused by channel time-selectivity for OFDM is not obvious when channel coding is applied in the system when $BER \geq 10^{-6}$. From the simulation point of view, any simulation for $BER < 10^{-6}$ becomes practically infeasible because of the prohibitively long simulation time. To demonstrate the effectiveness of the designed ICI cancellation scheme, we choose uncoded OFDM systems as an illustration example.

simulations from $SNR = 25$ dB. From Table 4.1, we know that, for *puncturing indexes* $Q = 1, 3, 6$ and 12 , the total amount of ICI power contributed by those neighboring subcarriers for $N=256$ is 60.82%, 82.76%, 90.68% and 95.15% respectively. These four *puncturing indexes* are used to show the performance of the designed ICI cancellation equalizer and how the *puncturing index* affects the performance. If ICI could be ideally canceled by 60%, 80%, 90% and 95%, the corresponding *SIR* improvement would be roughly 4 dB, 7 dB, 10 dB and 13 dB respectively. For comparison purposes, the BER curves of without ICI cancellation (obtained by simulation) and doubly-flat (i.e. frequency and time non-selective) Rayleigh fading channel [2] (analytical results, which have no ICI [6]) are also presented.

BER results presented in this chapter are based on the E_S/N_0 (i.e. SNR per symbol) defined in equation (4.17). According to the relationship between the E_S/N_0 and the E_b/N_0 of QPSK (see equation (4.55)), E_b/N_0 based BER results are simply the 3 dB shifted E_S/N_0 based BER results.

5.4.2 Simulation Procedure

The simulations of the designed frequency domain ICI cancellation equalizer are performed based on equation (5.9), and the flowchart of the simulation procedure is given in Figure 5.4. There are six major parts in the simulation software: *System Initialization*, *Rayleigh Channel Generator*, *Distortion Matrix Computation*, *OFDM Transmission*, *ICI Cancellation Equalizer*, and *Error Bit Detection and Accumulation*.

The function of *System Initialization* is setting values to parameters such as the number of subcarriers N in the system, the maximum *Doppler* frequency shift f_D , the puncturing index Q , the minimum number of error bits to be accumulated, the minimum number of information bits to be tested, the SNR range, etc..

As the name indicates, the *Rayleigh Channel Generator* generates the coefficients of the Rayleigh channel impulse response according to the required channel characterization (i.e.

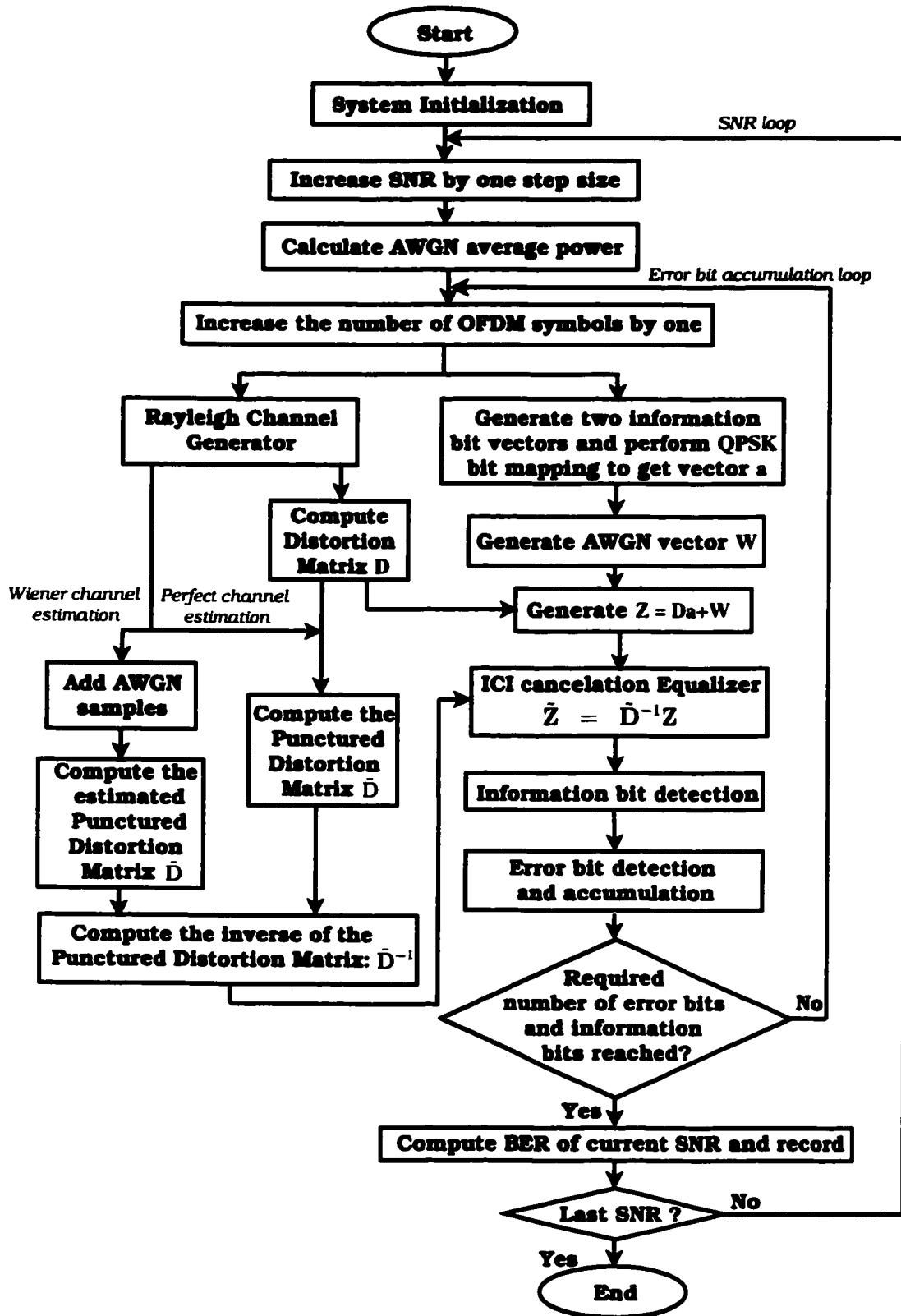


Figure 5.4: The simulation flowchart of the designed frequency domain ICI cancellation equalizer.

number of paths, *Doppler* frequency, etc.). The Clarke's model with spectrum shaping ([1], [125]) is used to generate those coefficients. The generated coefficients of the Rayleigh channel impulse response are used as the inputs of the *Distortion Matrix Computation*, which computes the distortion matrix $\mathbf{D}$. It should be noted that, to simulate the fading channel exactly, a full size ($N \times N$) distortion matrix $\mathbf{D}$ is computed.

If perfect channel estimation is assumed, the *punctured distortion matrix* $\tilde{\mathbf{D}}$ is obtained with the given *puncturing index* Q according to equation (5.8). If a *Wiener* filter is used to estimate the channel, the generated channel coefficients (channel impulse response) are added with the white Gaussian noise samples. The power of the AWGN samples added to the generated channel coefficients equals to the power of AWGN (denoted by $\mathbf{W}$) in the system [123]. Then the estimated distortion matrix is computed with the given *puncturing index* Q according to equations (5.3) and (5.5). After this, the inverse of the estimated distortion matrix, i.e. $\tilde{\mathbf{D}}^{-1}$, is computed.

To simulate the OFDM transmissions, two random information bit vectors are generated according to the given value of N for each transmitted OFDM symbol. Then with the bit-mapping scheme, the QPSK symbol vector $\mathbf{a}$ is obtained. Based on the current *SNR*, the AWGN vector $\mathbf{W}$ is also generated randomly as the one part of the OFDM transmission. The demodulated sequence $\mathbf{Z}$ is then obtained according to equation (5.4).

With the generated $\mathbf{Z}$ and $\tilde{\mathbf{D}}^{-1}$, now we are ready to perform the designed frequency-domain ICI cancellation according to equation (5.9). This task is performed by the *ICI Cancellation Equalizer*. As for the input of the *Error Bit Detection*, the equalized sequence $\tilde{\mathbf{Z}}$ is separated into two sequences: one sequence is the real part of the $\tilde{\mathbf{Z}}$, and the other sequence is the imaginary part of the $\tilde{\mathbf{Z}}$. Then a hard decision is performed according to the sign of the elements in each sequence: the elements with a positive sign are detected as 1 and those elements with a negative sign are detected as -1. By comparing these two sequences with the generated two random information bit sequences, the number of error bits for this equalized OFDM symbol is obtained.

The procedure of the simulation is as follows. After the system is initialized, the simulation goes into the *SNR loop* (from the lowest *SNR* to the highest *SNR* with a given step size). Then the power of AWGN is computed based on the assumption that the signal power is normalized to unity. Next, the simulation program goes into the second loop: the *error bit accumulation loop* which simulates one OFDM symbol at each iteration. To simulate each transmitted OFDM symbol, the Rayleigh channel impulse response is first generated and then the distortion matrix $\mathbf{D}$ is computed. The next step is to compute the *punctured distortion matrix* $\tilde{\mathbf{D}}$ and its inverse $\tilde{\mathbf{D}}^{-1}$ assuming that the channel is perfect, or estimated with a *Wiener* filter. After generating two random information bit vectors and one AWGN vector, ICI cancellation is performed and then error bits are counted. The *error bit accumulation loop* ends when both the required minimum error bits and minimum information bits are reached. Then the program starts another iteration for next *SNR* value. The program ends when all the interested *SNR* values are simulated. For more details on the simulation procedure of the designed frequency domain ICI cancellation equalizer, please refer to Figure 5.4.

5.4.3 BER of Frequency-domain ICI Cancellation Equalizer

The BER simulation results of the designed frequency-domain ICI cancellation equalizer with perfect channel estimation are presented in Figure 5.5. From this figure, the performance improvement with an increase of *puncturing index* Q can be observed clearly. The error floors caused by channel time-selectivity can be effectively lowered by using the designed frequency-domain ICI cancellation equalizer, and the BER of the doubly-flat fading channel can be approached by increasing the value of the *puncturing index*. However, these results are achievable only theoretically since perfect channel estimation is not possible in reality.

Results are presented in Figure 5.6 when the channel estimation is performed with a *Wiener* filter [122]. As expected, with *Wiener* channel estimation, the proposed ICI cancellation equalizer does not work efficiently at low *SNR* range. Unlike perfect channel estimation, using *Wiener* channel estimation causes more errors at low *SNR* values, and

using a larger *puncturing index* results in a higher BER. This is because of the channel estimation error of the *Wiener* filter, which has the same power as the additive noise [122]. However, at high *SNR* range, the designed frequency-domain ICI cancellation equalizer works effectively. At high *SNR* range, the BER curves with *Wiener* channel estimation are very close to those when perfect channel estimation is assumed. Compared with the error floor without ICI cancellation at the corresponding error floors, the performance improvement (lowered error floor) of using $Q = 1, 3, 6$ and 12 at high *SNR* range is roughly 2 dB, 5 dB, 7 dB and 10 dB respectively. When compared with ideal cancellation, this stands for a performance loss of only 2 dB for $Q = 1$ and 3 , and a performance loss of 3 dB for $Q = 6$ and 12 .

To evaluate the performance improvement of the proposed ICI cancellation equalizer over the one given in [116], we compare their performance with the same assumptions and parameters. In order to make the comparison easier, we choose $Q = 6$ and 12 with perfect channel estimation. Simulation results are presented in Figure 5.7. Compared with our ICI *punctured* cancellation method, when $Q = 6$ and 12 , the *partial* cancellation of [116] results in higher error floors. The performance improvement of our ICI *punctured* cancellation over the *partial* cancellation at the corresponding error floor BER is 1.8 dB and 2 dB for $Q=6$ and 12 respectively. This demonstrates that our ICI cancellation scheme is an improved version over the *partial* cancellation scheme presented in [116].

Based on the BER results presented above, we know that Table 4.1 indeed provides a guideline on choosing the value of Q to achieve a given performance for the designed frequency-domain ICI cancellation equalizer. For instance, if we need a 8 dB performance gain compared with the results without ICI cancellation, the *SIR* gain for the ideal cancellation is nearly 11 dB (assume the loss is 3 dB), which corresponds to a 92% ICI cancellation. From Table 4.1, this means that the value of Q needs to be around 8. Thus, in this section, we not only demonstrated the effectiveness of our designed ICI cancellation scheme, but also confirmed that we provided a guideline on choosing the value of the *puncturing index* Q to achieve a given performance gain with the designed

frequency-domain ICI cancellation equalizer.

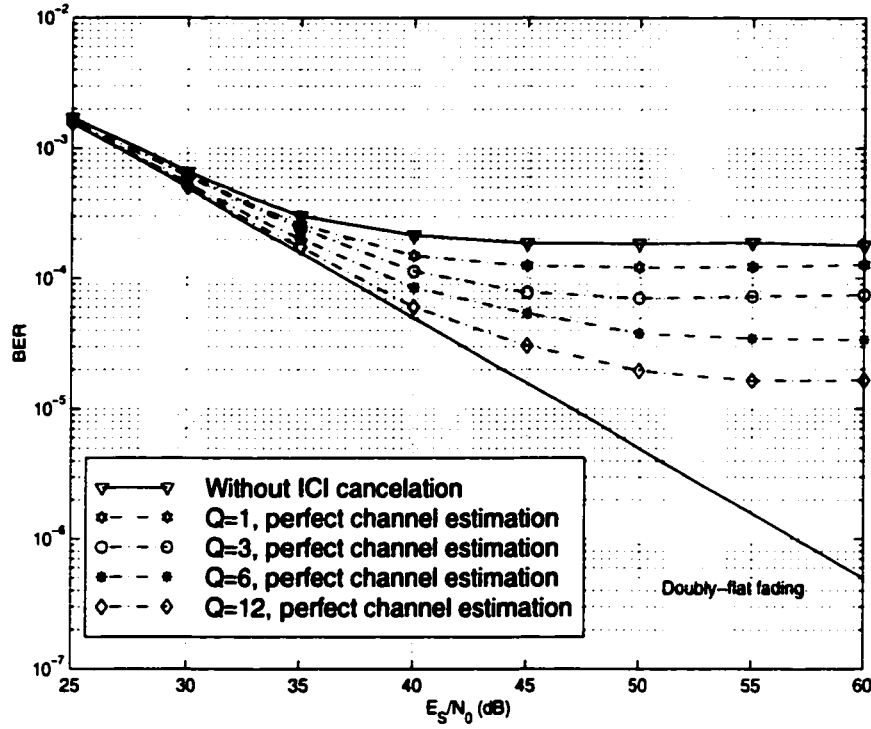


Figure 5.5: BER with perfect channel estimation when $f_D T_a = 10^{-4}$ and $N = 256$.

5.5 Summary

In this chapter, we developed a frequency-domain ICI cancellation equalizer and investigated its performance. The simulation results show that the error floor caused by ICI can be effectively lowered. The designed ICI cancellation equalizer has a flexible complexity, which is determined by the required performance by choosing a different *puncturing index* based on Table 4.1. We also provided a guideline on choosing the *puncturing index* value for the designed ICI cancellation equalizer, and it is confirmed by the presented simulation results. It was also demonstrated that our designed ICI *punctured* cancellation equalizer is an improved version of the *partial* cancellation equalizer presented in [116].

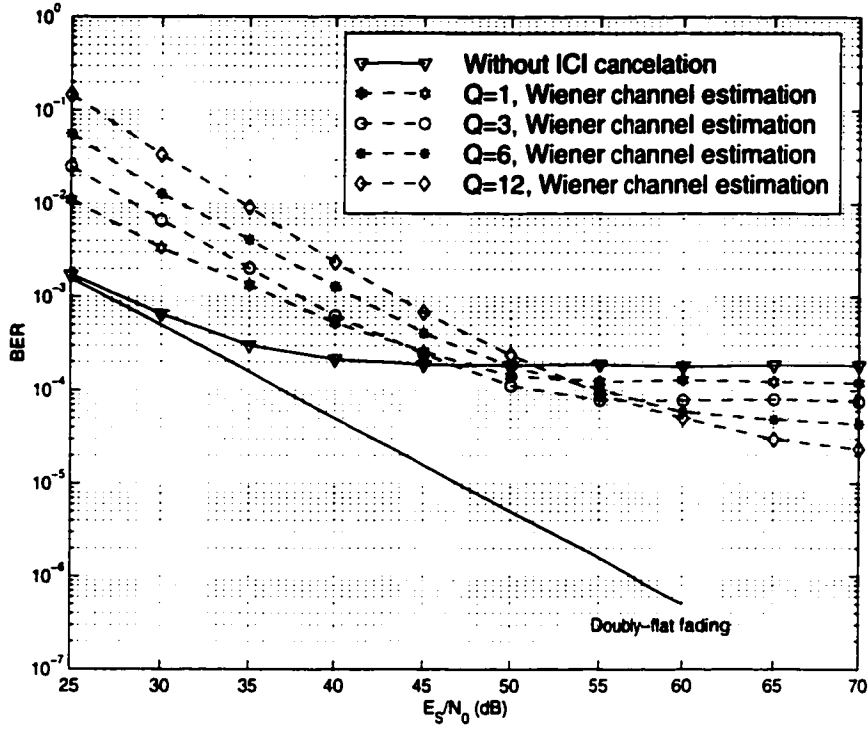


Figure 5.6: BER with *Wiener* channel estimation when $f_D T_a = 10^{-4}$ and $N = 256$.

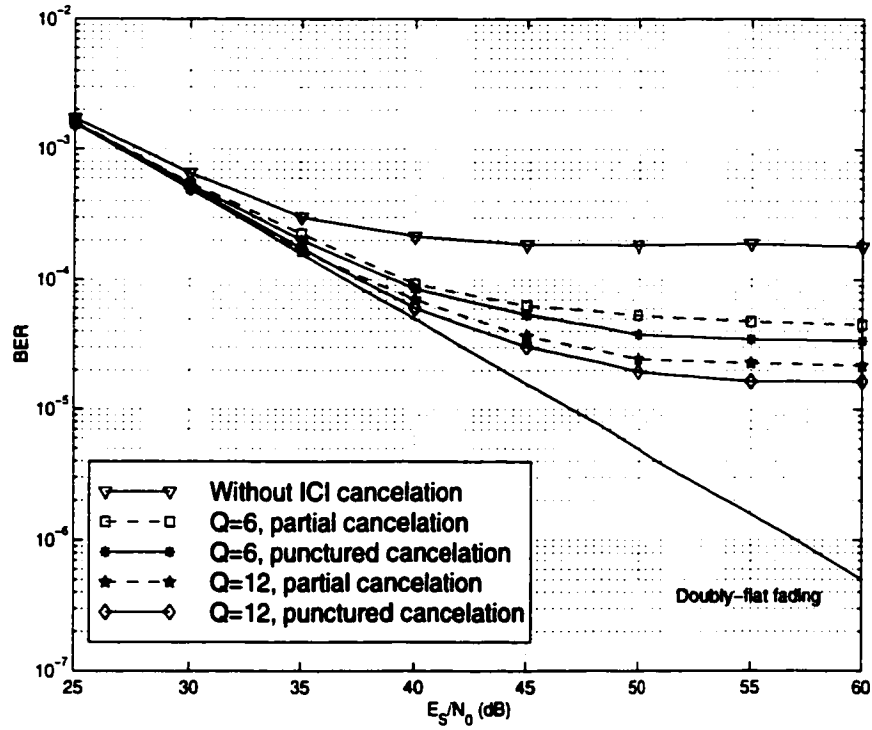


Figure 5.7: BER comparison of *punctured* and *partial* ICI cancellations with perfect channel estimation when $f_D T_a = 10^{-4}$ and $N = 256$.

Chapter 6

Inter-frame and Inter-carrier Differential Detection for OFDM Systems

Since OFDM is a two dimensional modulation scheme, differential modulation (encoding) may be implemented on either inter-frame¹ or inter-carrier basis with essentially the same complexity. In this chapter, based on the ICI analysis results derived in chapter 4, the performance of inter-frame differential detection (IF-DD) and inter-carrier differential detection (IC-DD) for QPSK modulated OFDM systems (both coded and uncoded) is compared over general time-variant multipath fading channels. The effects of the time-selectivity and frequency-selectivity of the fading channel on the performance are investigated. The conditions under which one differential detection scheme outperforms the other one are provided as the guideline for OFDM system design over multipath time-variant fading channels, and the lowest code rate of employing channel coding in OFDM system to transmit the same information bit rate is also given.

¹In this thesis, one OFDM frame is defined as the data sequence $a_0, a_1, \dots, a_{N-1}$ assigned to N OFDM subcarriers (see Figure 6.2), and N is also called the *frame size*. After IDFT (modulation), one OFDM frame is transformed into its corresponding OFDM symbol.

6.1 Introduction

Due to its low complexity, differential detection of QPSK [126] is widely employed in wireless communication systems. The main disadvantage associated with differential detection is the performance loss in the terms of SNR compared with coherent detection, which requires an accurate phase reference. The signal constellation of QPSK is depicted in Figure 6.1. Although not essential, QPSK is always differentially *Gray* encoded [2] to make both coherent and differential detection applicable at the receiver.

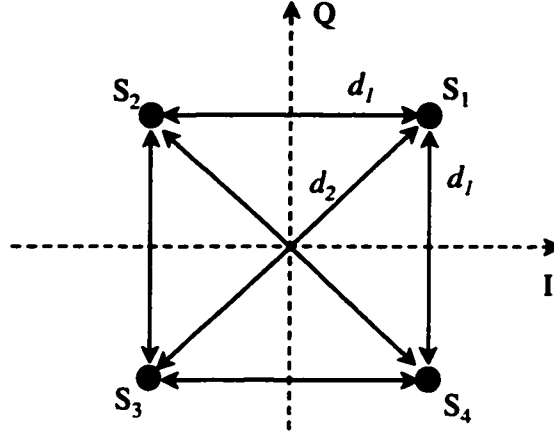


Figure 6.1: Signal constellation of QPSK.

Traditionally, the differential modulation (encoding) of multicarrier (including OFDM) systems is performed between the information bits of the same subcarrier but consecutive frames, we call it *inter-frame differential modulation (encoding)*. The corresponding demodulation (decoding) is called *inter-frame differential detection*. The term *time domain differential modulation (encoding)* is also used in the literature [127]. The resulting multicarrier system may be viewed as a parallel transmission system composed by a number of differentially encoded single carrier systems. However, for multicarrier systems, differential operation may also be performed between the information bits of the adjacent subcarriers of the same frame. This results in *inter-carrier differential modulation*

(*encoding*), and the corresponding demodulation (decoding) is termed *inter-carrier differential detection*. The same scheme is termed *frequency domain differential modulation* (*encoding*) in [127].

Some primary performance results of inter-carrier and inter-frame differential detection for OFDM are reported in [127] where the results are obtained for one-path time-variant and stationary multipath Rayleigh fading channels. Recently, a two-dimensional (combining inter-frame and inter-carrier) differential demodulation algorithm for OFDM is reported in [128], and simulation results for AWGN channel and Rician channel are provided, which show significant performance improvement compared with conventional one-dimensional differential demodulation. The two-dimensional differential demodulation for OFDM developed in [128] may be classified as an extension of the one-dimensional multiple-symbol differential detection [129].

In this chapter, we investigate the effects of channel time-selectivity and frequency-selectivity on the performance of inter-carrier and inter-frame differential detection for OFDM over general frequency-selective time-variant fading channels. The impact of channel coding on system performance is also investigated. The major objective of comparing the performance of IF-DD and IC-DD is to find out the conditions where one differential detection outperforms the other, and analytical results are provided.

6.2 Inter-frame and Inter-carrier Differential Detection

The difference between IF-DD and IC-DD is depicted in Figure 6.2. The symbol a_0^J could be modulated (encoded) using a_{N-1}^{J-1} as the reference, but in order to reduce the errors caused by error propagation, which usually results in decoding errors over two consecutive signaling intervals [2], in our inter-carrier differentially modulated (encoded) OFDM systems, each OFDM frame is isolated from the other OFDM frames. The differential modulation (encoding) begins with a_0 using S_1 as the reference and ends at a_{N-1} .

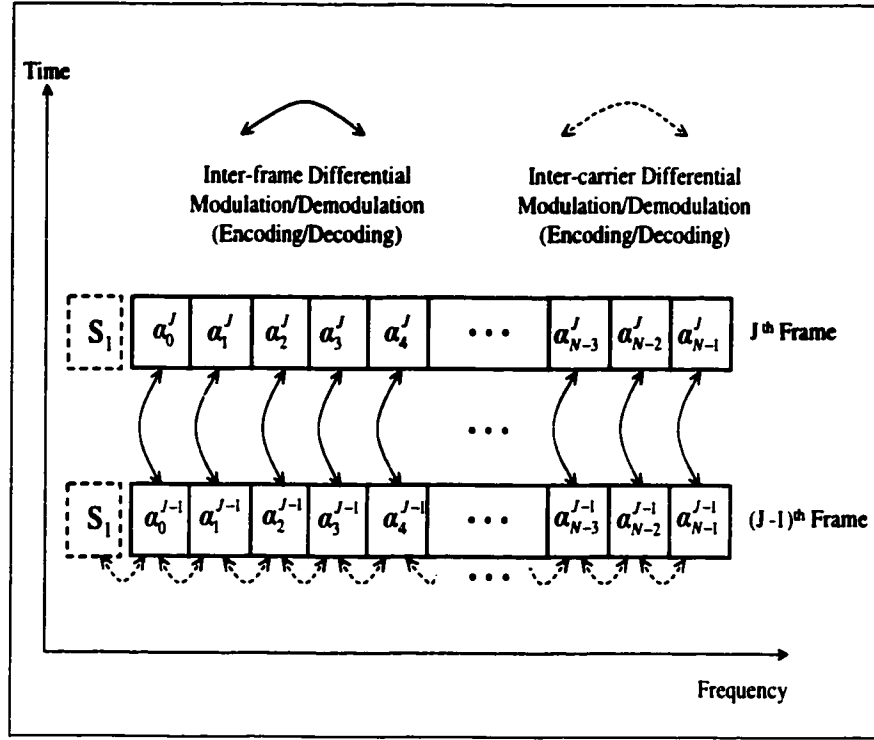


Figure 6.2: Inter-frame and inter-carrier differential encoding and decoding.

If the J^{th} OFDM symbol is currently received by the receiver, then the demodulated symbol of the l^{th} subcarrier ($0 \leq l \leq N - 1$) is (see equation (4.14)):

$$Z_l^J = \zeta_l^J a_l^J + c_l^J + n_l^J = \zeta_l^J a_l^J + e_l^J \quad (6.1)$$

where ζ_l^J is the attenuation factor, c_l^J is the ICI, n_l^J is the additive noise with mean power σ_f^2 and $e_l^J = c_l^J + n_l^J$ is the *effective* additive noise.

When the OFDM system is inter-frame differentially modulated (encoded), assuming that $a_l^{J-1} = 1$, then the estimation of ζ_l is:

$$v_{IF} = Z_l^{J-1} = \zeta_l^{J-1} + c_l^{J-1} + n_l^{J-1} = \zeta_l^{J-1} + e_l^{J-1} \quad (6.2)$$

Similarly, for inter-carrier differentially modulated (encoded) OFDM systems, with the assumption that $a_{l-1}^J = 1$, the estimation of ζ_l is:

$$v_{IC} = Z_{l-1}^J = \zeta_{l-1}^J + c_{l-1}^J + n_{l-1}^J = \zeta_{l-1}^J + e_{l-1}^J \quad (6.3)$$

It should be noted that when $l = 0$, we have $Z_{-1} = S_1$. S_1 is chosen as the reference for all the data symbols a_0^J ($-\infty \leq J \leq +\infty$) assigned to the first subcarrier of all the frames.

Using differential detection as the channel estimator works well only when the received symbol has a high correlation with the previous received symbol [2], and this applies to both inter-carrier and inter-frame differential detection. Based on this observation, we know that, for inter-frame differentially encoded OFDM systems, the BEP decreases with an increase of N (the number of subcarriers in the system). The reason for this is: for a given *Doppler* frequency, as shown in Figure 3.4, a larger N results in a larger $f_D T_S$ value, thus a lower correlation between Z_l^J and Z_l^{J-1} (two demodulated symbols of the same subcarrier but from adjacent OFDM frames). On the other hand, compared with the correlation between Z_l^J and Z_l^{J-1} , the correlation between Z_l^J and Z_{l-1}^J (two demodulated symbols of the same OFDM frame but from adjacent subcarriers) is less affected by N (i.e. channel time-selectivity). However, the correlation between Z_l^J and Z_{l-1}^J tends to be more sensitive to the channel frequency-selectivity.

With the same *Doppler* frequency and under certain circumstances, inter-carrier differential detection should outperform inter-frame differential detection from the channel time-selectivity point of view. On the other hand, for a given frequency-selective fading channel, inter-frame differential detection should outperform inter-carrier differential detection when the channel time-selectivity is not significant. When the channel is neither time-variant nor frequency-selective, a similar performance would be observed. For one-path time-variant Rayleigh fading channel and stationary multipath channels, some of these statements are demonstrated in [127] where the results are obtained with computer simulations. In this chapter, analytical results for both uncoded and coded OFDM systems are given for general time-variant frequency-selective WSSUS fading channels.

6.3 Performance of Uncoded OFDM Systems

For the received signal in the form of equation (6.1), the pairwise error analysis can be used to obtain the union bound of BEP [130] or approximate BEP [131] with high accuracy for a broad range of SNR values. This technique is used to analyze the effects of channel time-selectivity and frequency-selectivity on the performance of uncoded OFDM systems.

6.3.1 Pairwise Error Event Probability

The pairwise error event probability for the received signal given in equation (6.1) is obtained as (see also equation (A.19) of appendix A):

$$P(x_k \rightarrow \hat{x}_k) = \int_{-\infty}^0 p(D) dD = \frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \frac{\Phi_D(s)}{s} ds \quad (6.4)$$

6.3.2 Bit Error Probability

When an equiprobable source is assumed, and *Gray* encoding is adopted by both IF-DD and IC-DD, based on Figure 6.1, the BEP may be approximated as [131]:

$$P_b \approx \frac{1}{2} P(S_1 \rightarrow S_2) + \frac{1}{2} P(S_1 \rightarrow S_3) \quad (6.5)$$

6.3.3 Mean and Variance Computations

In order to obtain the union bound presented in appendix A, we need to calculate the means and variances of some random variables. The mean of e_l is $\bar{n} = 0$. The mean of ζ_l , which is equivalent to the $\bar{u}$ in equation (A.4), is:

$$\bar{u} = E\{\zeta_l\} = \sum_{m=0}^{M-1} h_m e^{-j \frac{2\pi m l}{N}} \quad (6.6)$$

With $E\{\zeta_l \zeta_l^*\}$ given by (4.23), the variance of ζ_l becomes:

$$\sigma_u^2 = E\{(\zeta_l - \bar{u})(\zeta_l - \bar{u})^*\} = \sum_{m=0}^{M-1} \left(\sigma_m^2 + \Omega \sigma_{\Delta m}^2 \right) - |\bar{u}|^2 \quad (6.7)$$

where Ω is given in equation (4.24). Similarly, with $E\{c_l c_l^*\}$ given by equation (4.30), the variance of e_l is obtained as:

$$\sigma_n^2 = E\{(e_l - \bar{n})(e_l - \bar{n})^*\} = (1 - \Omega) \sum_{m=0}^{M-1} \sigma_{\Delta m}^2 + \sigma_f^2 \quad (6.8)$$

The variance of v (which is defined as the estimation of ζ_l), and the covariance between u (replacing ζ_l for simplicity) and v depend on the estimation method used. The computation presented hereafter is based on both inter-frame and inter-carrier differential detections. The variances of v_{IF} and v_{IC} are obtained as:

$$\begin{aligned} \sigma_{v|IF}^2 &= E\{(v_{TM} - \bar{u})(v_{TM} - \bar{u})^*\} \\ &= E\{\zeta_l^{J-1}(\zeta_l^{J-1})^*\} + E\{c_l^{J-1}(c_l^{J-1})^*\} + \sigma_f^2 - |\bar{u}|^2 \end{aligned} \quad (6.9)$$

$$\begin{aligned} \sigma_{v|IC}^2 &= E\{(v_{FQ} - \bar{u})(v_{FQ} - \bar{u})^*\} \\ &= E\{\zeta_{l-1}^J(\zeta_{l-1}^J)^*\} + E\{c_{l-1}^J(c_{l-1}^J)^*\} + \sigma_f^2 - |\bar{u}|^2 \end{aligned} \quad (6.10)$$

where *unconditional unbiased* estimation [132] is assumed, that is, $\bar{v}_{IF} = \bar{v}_{IC} = \bar{u}$. Based on the derivation of (4.23) and (4.30), we have:

$$\sigma_{v|IF}^2 = \sigma_{v|IC}^2 = \sum_{m=0}^{M-1} (\sigma_m^2 + \sigma_{\Delta m}^2) - |\bar{u}|^2 + \sigma_f^2 \quad (6.11)$$

The covariance between u and v_{IF} is obtained as:

$$\sigma_{uv|IF}^2 = E\{(u - \bar{u})(v_{IF} - \bar{u})^*\} = \sum_{m=0}^{M-1} (\sigma_m^2 + \Omega_{N_G} \sigma_{\Delta m}^2) - |\bar{u}|^2 \quad (6.12)$$

where Ω_{N_G} is defined as:

$$\Omega_{N_G} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{k'=0}^{N-1} J_0(2\pi f_D T_r(k - k' - (N + N_G))) \quad (6.13)$$

The covariance between u and v_{IC} is:

$$\sigma_{uv|IC}^2 = E\{(u - \bar{u})(v_{IC} - \bar{u})^*\} = \sum_{m=0}^{M-1} (\sigma_m^2 + \Omega \sigma_{\Delta m}^2) e^{-j\frac{2\pi m}{N}} - |\bar{u}|^2 \quad (6.14)$$

6.3.4 Mean Squared Estimation Errors

The mean squared estimation errors of inter-frame and inter-carrier differentially encoded OFDM systems are, respectively,

$$\begin{aligned}
\sigma_{e|IF}^2 &= E\left\{(u - v_{IF})(u - v_{IF})^*\right\} \\
&= \sum_{m=0}^{M-1} \left(2\sigma_m^2 + (1 + \Omega)\sigma_{\Delta m}^2\right) + \sigma_f^2 - 2\text{Re}\left\{\sum_{m=0}^{M-1} (\sigma_m^2 + \Omega_{N_G}\sigma_{\Delta m}^2)\right\} \quad (6.15)
\end{aligned}$$

and

$$\begin{aligned}
\sigma_{e|IC}^2 &= E\left\{(u - v_{IC})(u - v_{IC})^*\right\} \\
&= \sum_{m=0}^{M-1} \left(2\sigma_m^2 + (1 + \Omega)\sigma_{\Delta m}^2\right) + \sigma_f^2 - 2\text{Re}\left\{\sum_{m=0}^{M-1} (\sigma_m^2 + \Omega\sigma_{\Delta m}^2)e^{-j\frac{2\pi m}{N}}\right\} \quad (6.16)
\end{aligned}$$

To find out the conditions where IC-DD outperforms IF-DD, we only need to evaluate their mean squared estimation errors, i.e. $\sigma_{e|IF}^2$ and $\sigma_{e|IC}^2$. IC-DD outperforms IF-DD as long as $\sigma_{e|IC}^2 < \sigma_{e|IF}^2$, that is

$$\rho_e = 10\log_{10}\left(\frac{\sigma_{e|IF}^2}{\sigma_{e|IC}^2}\right) > 0 \quad (\text{dB}) \quad (6.17)$$

6.3.5 Numerical Results

• Assumptions and Parameters

Instead of giving specific values of f_D (the maximum *Doppler* frequency), $f_D T_a$ is used to evaluate the detection performance. To give a complete view how the value of $f_D T_a$ affects the system performance, $f_D T_a \in [10^{-6}, 10^0]$ is chosen, even some of the values appear to be higher than reality. The channel parameters are chosen by modifying the 6-tap typical urban channel model used by Russell and Stüber in [19]. In order to make it suitable for the channel model presented in chapter 3, the values of $\sigma_m^2 + \sigma_{\Delta m}^2$ are chosen as given in Table 6.1 ($M = 6$). For simplicity, it is also assumed that $\sigma_m^2 = \gamma\sigma_{\Delta m}^2$ ($0 \leq m \leq M - 1$), where the value of γ is chosen from $[10^{-3}, 10^3]$ and two extreme cases, $\gamma = 0$ (the fast fading Rayleigh channel) and $\gamma \rightarrow +\infty$ (the time-invariant channel). The length of cyclic prefix is chosen as $N_G = 6$, the minimum length required to make the system ISI-free. $N = 128$ and 1024 are chosen.

• Inter-frame versus Inter-carrier

Evaluating the value of ρ_e as expressed in equation (6.17), it is found that inter-carrier differential detection outperforms inter-frame differential detection only in part of the

Table 6.1: Modified 6-tap urban channel model.

m	0	1	2	3	4	5
$\sigma_m^2 + \sigma_{\Delta m}^2$	0.19	0.38	0.24	0.09	0.06	0.04

$f_D T_a - \gamma$ plane, where $\gamma \in [10^{-3}, 10^3]$ and $f_D T_a \in [10^{-6}, 10^0]$. We define the area of the $f_D T_a - \gamma$ plane where $\rho_e > 3$ dB as the $\mathcal{F}$ -zone; the area where $\rho_e < -3$ dB as the $\mathcal{T}$ -zone; the area where $-3 \text{ dB} \leq \rho_e \leq 3 \text{ dB}$ as the $\mathcal{E}$ -zone. The curve, which is parallel to the γ axis and where $\sigma_{e|IF}^2$ achieves the highest ratio over $\sigma_{e|IC}^2$, is defined as the *crest* of the $\mathcal{F}$ -zone. The reason of choosing 3 dB to define these *zones* is that, perceptible performance differences between IC-DD and IF-DD are expected for the ρ_e with values above 3 dB and below -3 dB. For instance, ρ_e for $N=128$ is plotted in Figure 6.3 when $E_S/N_0=15$ dB and Figure 6.4 when $E_S/N_0=40$ dB, where E_S/N_0 is defined in equation (4.17). For comparison purposes, the same set of results for ρ_e when $N=1024$ are given in Figure 6.5 and Figure 6.6. The comparison shows that: for a given N , the performance difference between IF-DD and IC-DD is more obvious with a higher E_S/N_0 ; for a given E_S/N_0 , the performance difference between inter-carrier differential detection and inter-frame differential detection is more obvious for the OFDM system with a smaller N .

The analytical results of (6.17) show that, for a given N , with the increase of E_S/N_0 , both of the ranges of the $\mathcal{F}$ -zone and the $\mathcal{T}$ -zone increase, while the range of $\mathcal{E}$ -zone decreases. For a given E_S/N_0 , increase of N leads to the increase of both the ranges of the $\mathcal{E}$ -zone and $\mathcal{F}$ -zone, while the range of the $\mathcal{T}$ -zone decreases. The *crest* of the $\mathcal{F}$ -zone is shifting to lower $f_D T_a$ values with the increase of N , and the range of the $\mathcal{F}$ -zone is getting larger with the increase of E_S/N_0 along γ axis but mainly $f_D T_a$ axis. To give an insight view of the effects of E_S/N_0 on ρ_e , some results are presented in Figure 6.7, Figure 6.8, Figure 6.9 and Figure 6.10 for $\gamma=0$, $N = 128, 256, 512$ and 1024 respectively. According to these four figures, the operating zones of the systems with $N = 128, 256, 512$ and 1024 when $\gamma = 0$ for $E_S/N_0 \geq 10$ dB, $f_D T_a=10^{-6}, 10^{-5}$ and 10^{-4} are as indicated

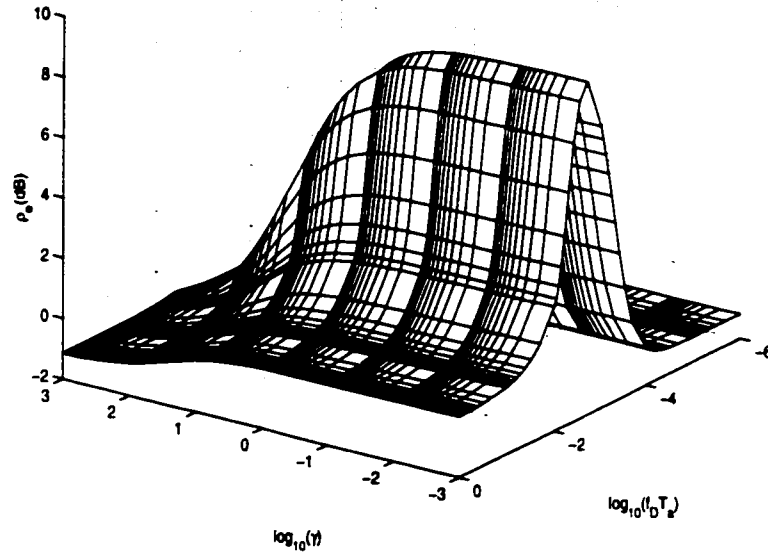


Figure 6.3: Distribution of ρ_e when $N = 128$ and $E_S/N_0 = 15$ dB.

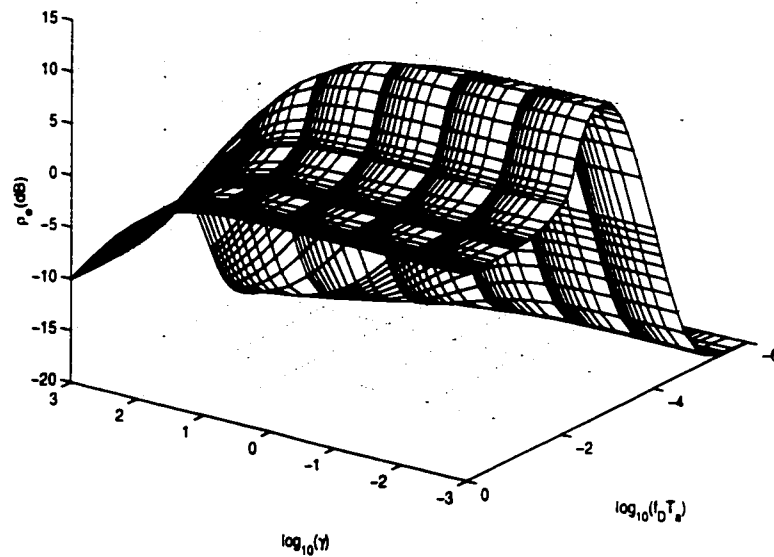


Figure 6.4: Distribution of ρ_e when $N = 128$ and $E_S/N_0 = 40$ dB.

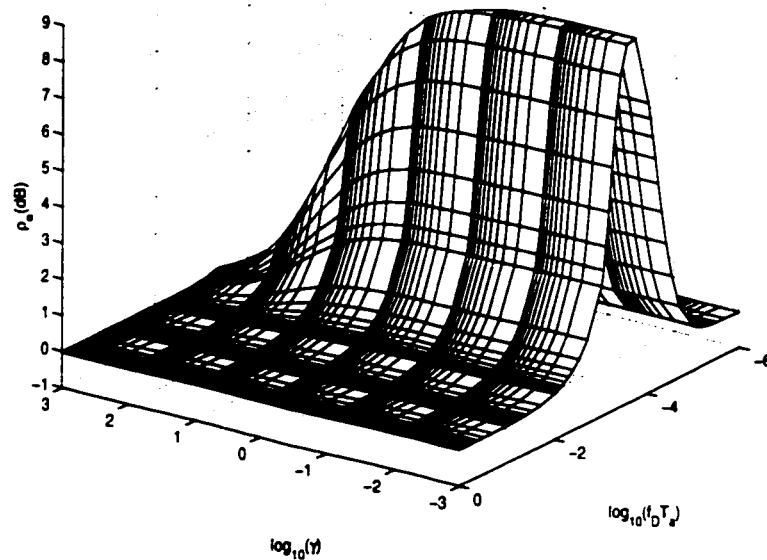


Figure 6.5: Distribution of ρ_e when $N = 1024$ and $E_S/N_0 = 15$ dB.

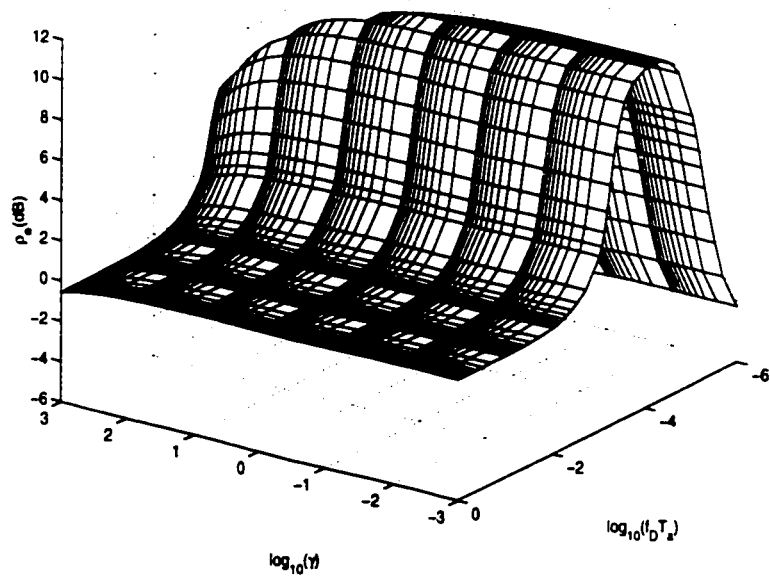


Figure 6.6: Distribution of ρ_e when $N = 1024$ and $E_S/N_0 = 40$ dB.

in Table 6.2. If the information bit rate is assumed to be 8 Mbps (same as in chapter 5), that is $T_a = 0.25\mu s$ because of using QPSK, the corresponding value of f_D is 4Hz, 40Hz and 400Hz for $f_D T_a = 10^{-6}$, 10^{-5} and 10^{-4} respectively.

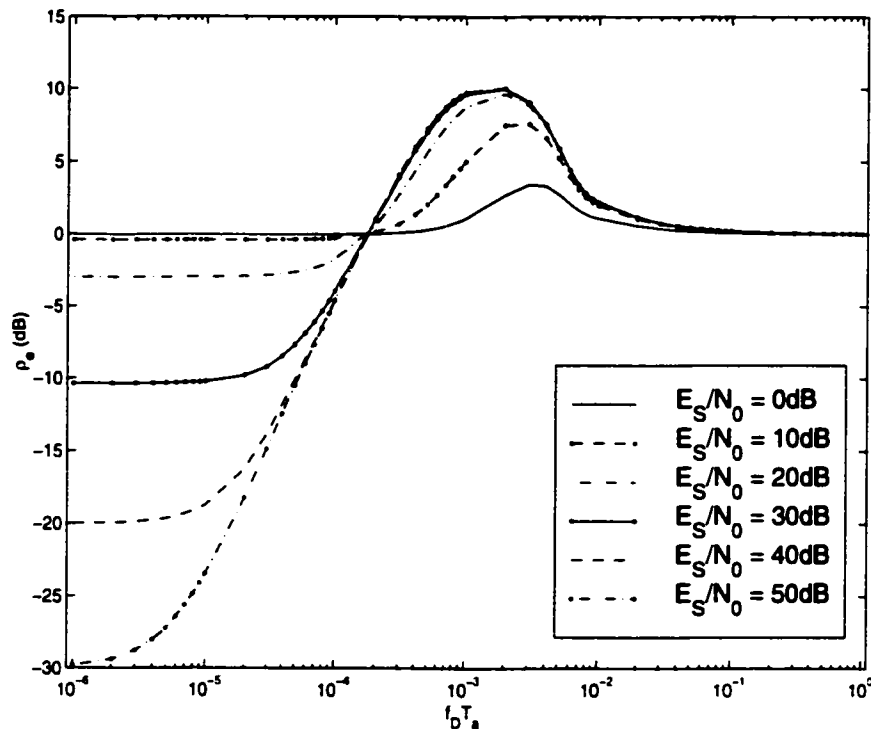
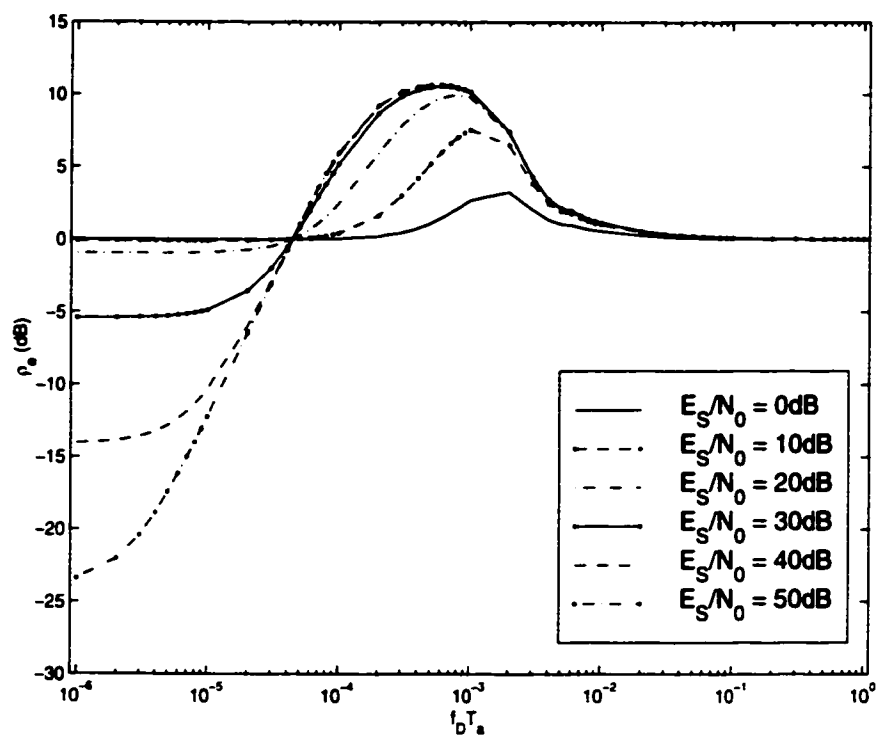
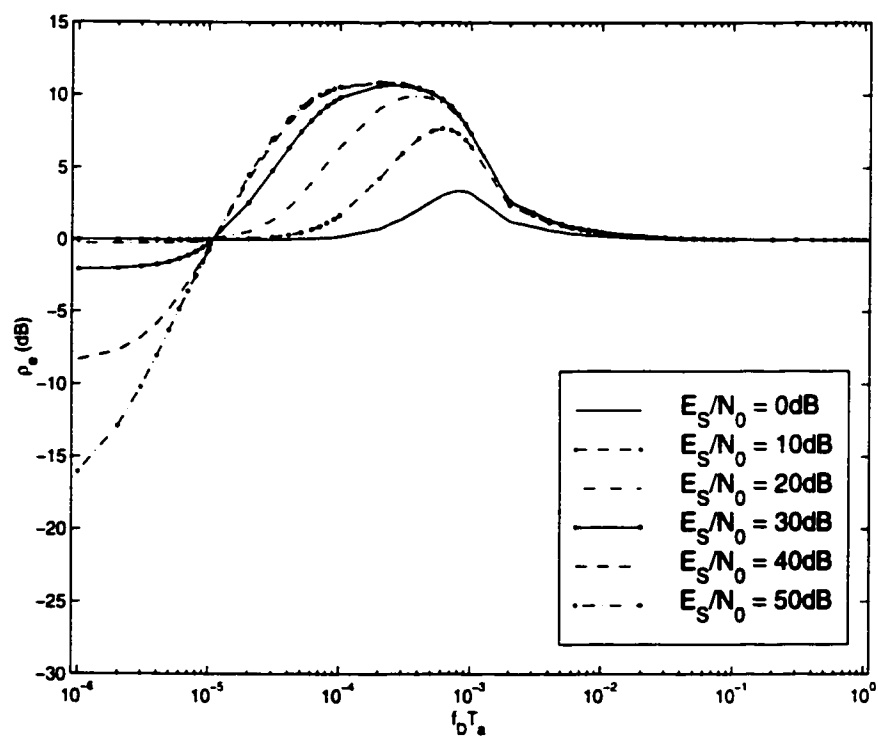


Figure 6.7: Distribution of ρ_e when $N = 128$ and $\gamma = 0$.

Table 6.2: System operating zones ($\gamma = 0$).

$f_D T_a$	$N=128$	$N=256$	$N=512$	$N=1024$
10^{-6}	$\mathcal{T}$ -zone	$\mathcal{T}$ -zone	$\mathcal{T}$ -zone	$\mathcal{T}$ -zone
10^{-5}	$\mathcal{T}$ -zone	$\mathcal{T}$ -zone	$\mathcal{E}$ -zone	$\mathcal{F}$ -zone
10^{-4}	$\mathcal{T}$ -zone	$\mathcal{F}$ -zone	$\mathcal{F}$ -zone	$\mathcal{F}$ -zone

To verify the results given in Table 6.2, the uncoded QPSK BEP results for the corresponding values of $f_D T_a$ listed in Table 6.2 are presented in Figure 6.11 ($f_D T_a = 10^{-6}$), Figure 6.12 ($f_D T_a = 10^{-5}$), and Figure 6.13 ($f_D T_a = 10^{-4}$). The results given in Figure 6.11 show that, as indicated in Table 6.2, when $f_D T_a = 10^{-6}$, all the systems ($N=128, 256, 512$ and 1024) are operating in the $\mathcal{T}$ -zone, because IF-DD always achieves better performance


 Figure 6.8: Distribution of ρ_e when $N = 256$ and $\gamma = 0$.

 Figure 6.9: Distribution of ρ_e when $N = 512$ and $\gamma = 0$.

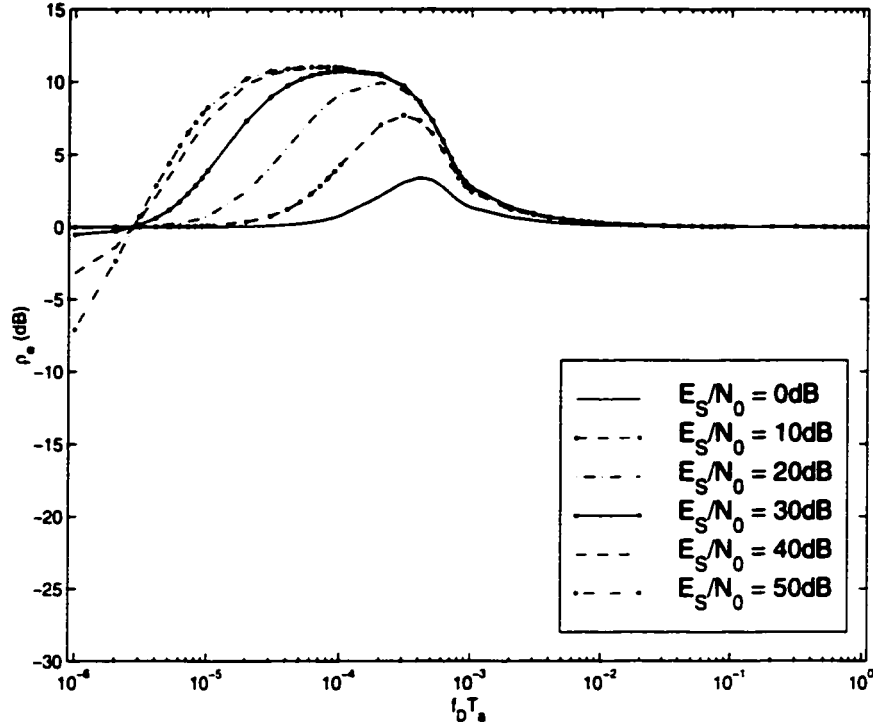


Figure 6.10: Distribution of ρ_e when $N = 1024$ and $\gamma = 0$.

than IC-DD. Figure 6.12 and Figure 6.13 show that for the system with $N = 256$, with the increasing of the value of $f_D T_a$ from 10^{-5} to 10^{-4} , IC-DD gives a better performance than IF-DD, that is, the system operating zone is shifting towards the $\mathcal{F}$ -zone from the $\mathcal{T}$ -zone. For the system with $N=512$, when $f_D T_a = 10^{-5}$, IF-DD has almost the same performance as IC-DD. When the value of $f_D T_a$ is 10^{-4} , IC-DD outperforms IF-DD. This indicates that the system operating zone is shifting towards the $\mathcal{F}$ -zone from the $\mathcal{E}$ -zone. For $N=1024$, when $f_D T_a \in [10^{-5}, 10^{-4}]$, IC-DD always has better performance than IF-DD: this means, the system is operating in the $\mathcal{F}$ -zone. From these three figures, we know that for the given values of $f_D T_a$, the system with $N = 128$ is operating in the $\mathcal{T}$ -zone. All the results presented in these three figures agree with that given in Table 6.2.

In general, for IF-DD, irrespective of operating in the $\mathcal{F}$ -zone or the $\mathcal{T}$ -zone, a smaller N achieves a better BEP performance. For IC-DD, this conclusion is applicable only for those systems operating in the $\mathcal{F}$ -zone. For systems operating in the $\mathcal{T}$ -zone, a larger N leads to a better performance. However, if both IF-DD and IC-DD are available at

the receiver, or if the transmitter and receiver are designed such that they could always switch to the differential detection scheme giving the best performance, then the system with the smallest N achieves the best performance for a given value of normalized *Doppler* frequency $f_D T_a$.

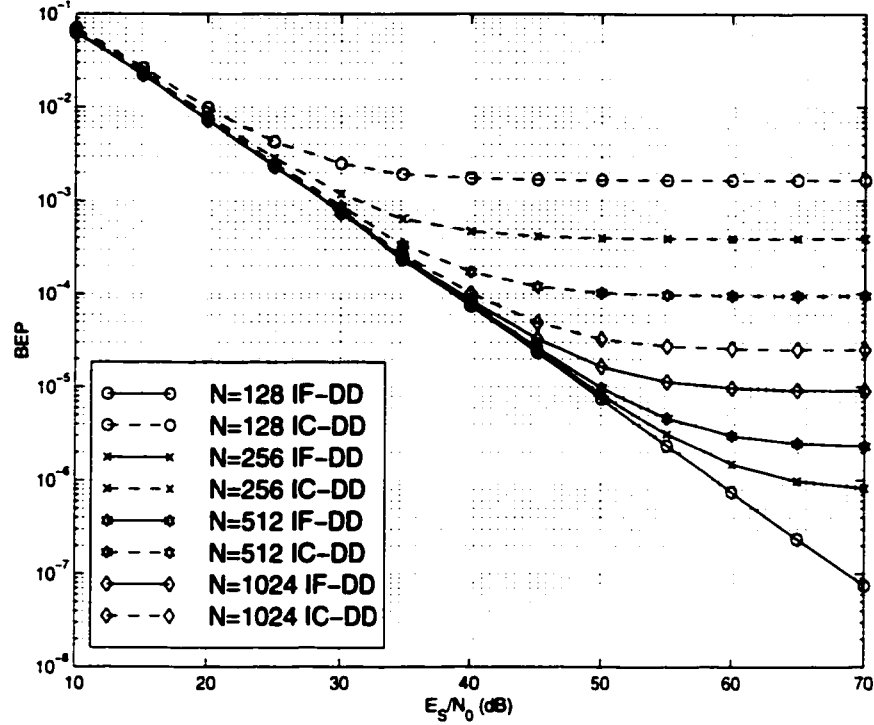


Figure 6.11: Uncoded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-6}$.

• Effects of Channel Time-selectivity and Frequency-selectivity

Figure 6.14 ($N = 128$) and Figure 6.15 ($N = 1024$) present some BEP results of uncoded QPSK inter-frame differentially modulated OFDM system when $\gamma = 0$. The results presented in these two figures show that, for inter-frame differential detection, the error floor is caused by the channel time-selectivity (which results in ICI), and a higher *Doppler* frequency results in a higher error floor. Figures 6.14 and 6.15 also show that, for inter-frame differential detection, a larger N results in a higher BEP and earlier the error floor is reached with the increase of $f_D T_a$. This means, for inter-frame differential detection, the OFDM system with a larger N is more sensitive to the channel time-selectivity.

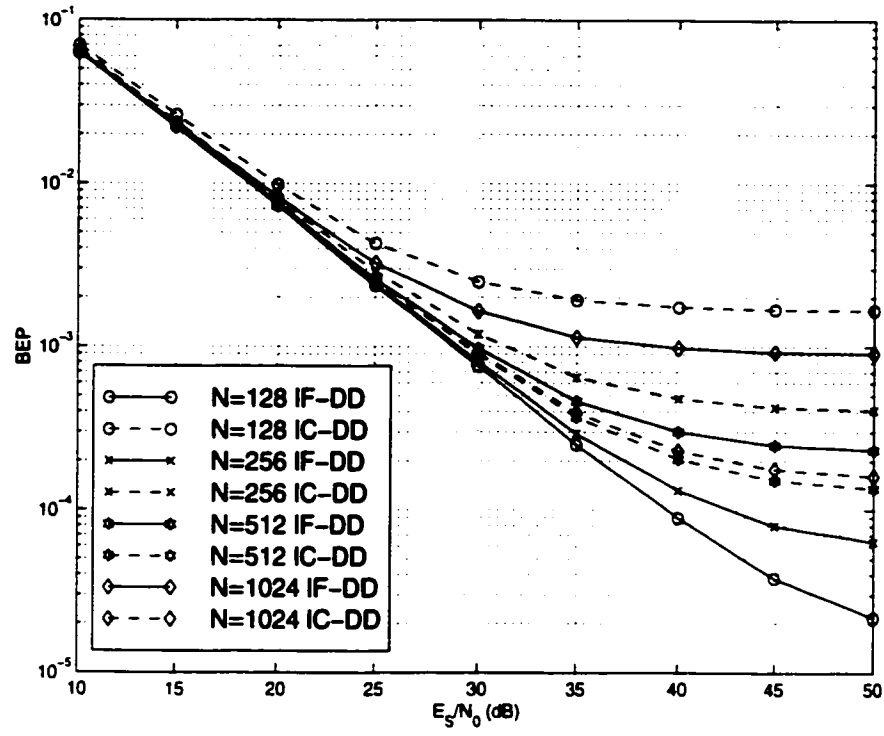


Figure 6.12: Uncoded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-5}$.

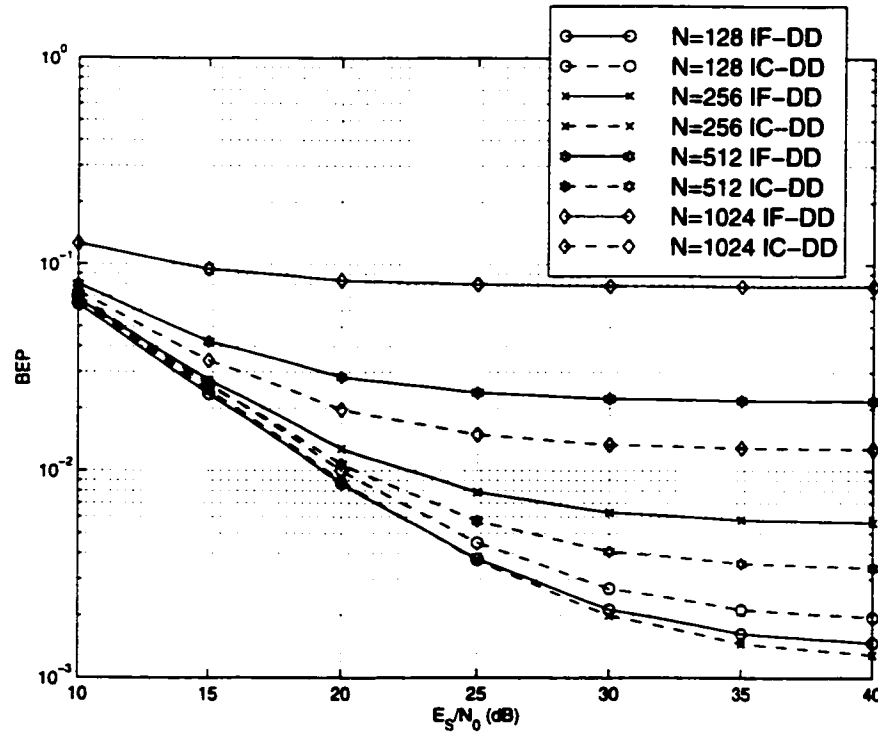


Figure 6.13: Uncoded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-4}$.

For inter-carrier differentially modulated QPSK uncoded OFDM systems, some performance results are presented in Figure 6.16 ($N = 128$) and Figure 6.17 ($N = 1024$). From these two figures, it is observed that inter-carrier differential detection leads to an error floor along the E_S/N_0 axis even with small $f_D T_a$ values, and a smaller N results in a more serious error floor problem. The main reason resides in the channel frequency-selectivity. When $f_D T_a \rightarrow 0$, $\Omega \rightarrow 1$ and $\Omega_{NG} \rightarrow 1$ (a smaller N results in a faster approach for $\Omega \rightarrow 1$ and $\Omega_{NG} \rightarrow 1$), thus equation (6.15) and equation (6.16) may be approximated as:

$$\sigma_{e|IF}^2 \approx \sigma_f^2 \quad (6.18)$$

and

$$\sigma_{e|IC}^2 \approx \sigma_f^2 + 2Re\left\{ \sum_{m=0}^{M-1} (\sigma_m^2 + \sigma_{\Delta m}^2)(1 - e^{-j\frac{2\pi m}{N}}) \right\} \quad (6.19)$$

For high E_S/N_0 (small σ_f^2) values, the second item of the right hand side of (6.19) becomes the dominant and thus the channel estimation (and so is the system performance) may not be improved by increasing E_S/N_0 . From (6.19), we know that, a smaller N results in a larger $\sigma_{e|IC}^2$ value. However, when the channel is frequency-nonselective ($M = 1$), the second item of the right hand side of (6.19) becomes zero, then $\sigma_{e|IF}^2 \approx \sigma_{e|IC}^2$, i. e. they have almost the same performance when the value of $f_D T_a$ is small. Thus we demonstrated that inter-carrier differential detection and inter-frame differential detection have a similar performance if the channel is neither time-selective nor frequency-selective.

6.4 Performance of Coded OFDM Systems

6.4.1 Bit Error Probability

To reflect the impact of channel coding on the OFDM systems experiencing both channel time-selectivity and frequency-selectivity, we choose a 1/2 rate convolutional code with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$ as the channel coding scheme in our studied OFDM systems using inter-carrier and inter-frame differential detection. The reason

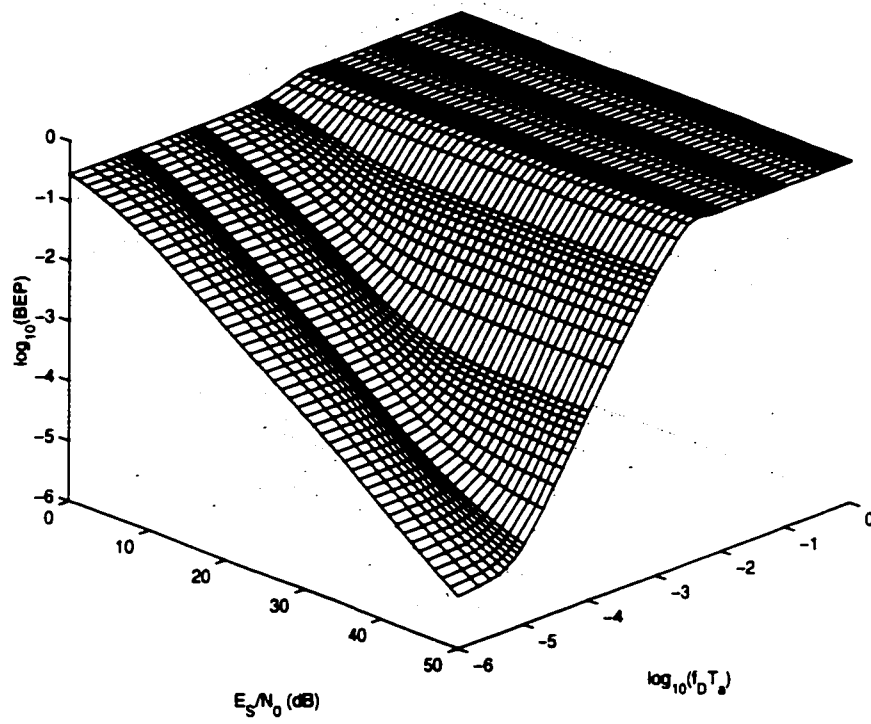


Figure 6.14: Uncoded performance of IF-DD when $\gamma = 0$ and $N = 128$.

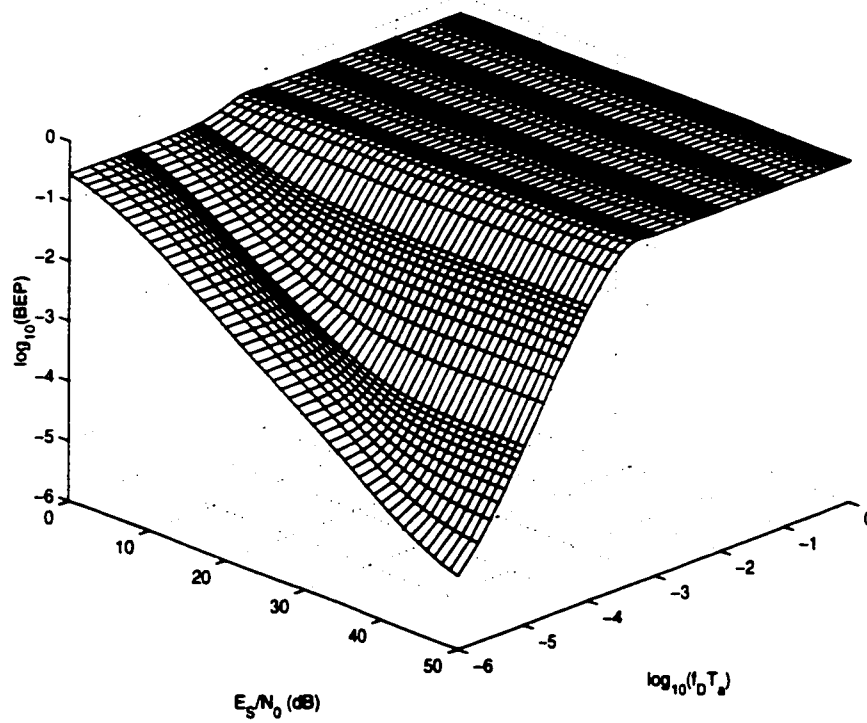


Figure 6.15: Uncoded performance of IF-DD when $\gamma = 0$ and $N = 1024$.

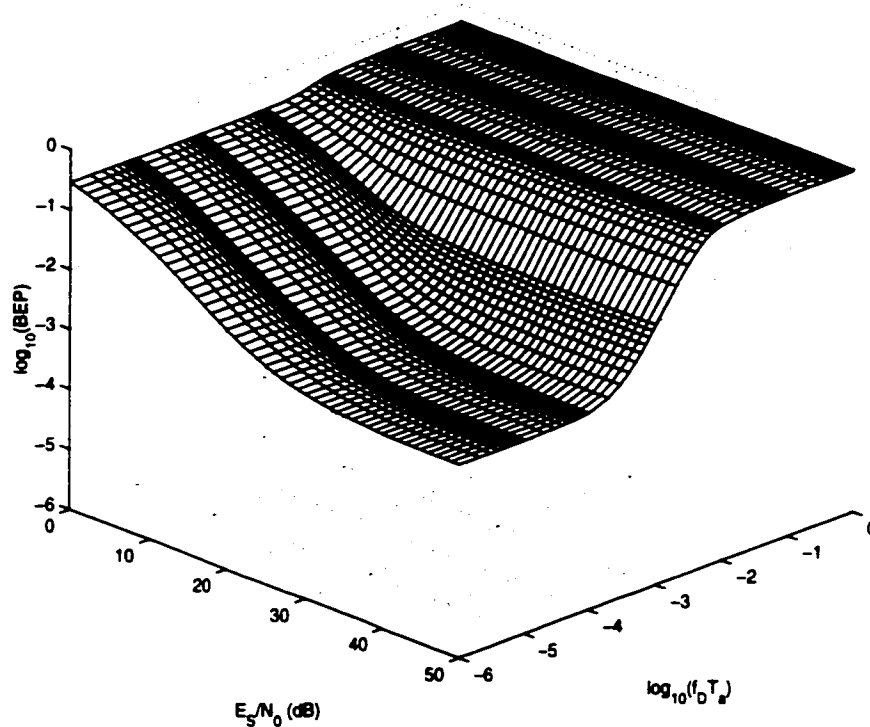


Figure 6.16: Uncoded performance of IC-DD when $\gamma = 0$ and $N = 128$.

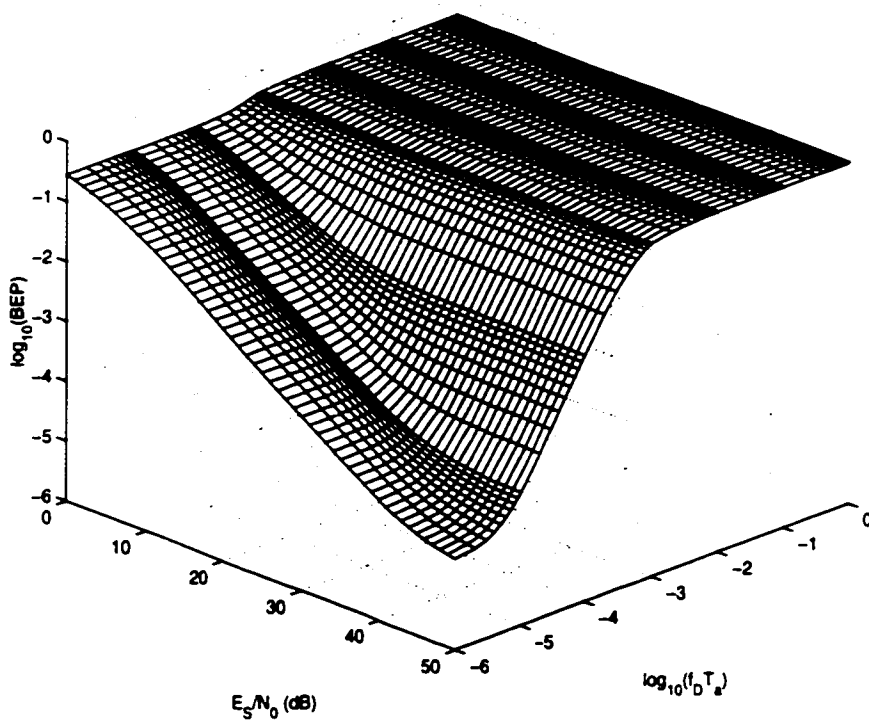


Figure 6.17: Uncoded performance of IC-DD when $\gamma = 0$ and $N = 1024$.

(see also section 2.3 in chapter 2) is because this channel coding scheme and its rate-compatible punctured versions [133] are widely studied and many other channel coding schemes use them as the benchmark for performance comparison purpose.

The technique presented in [133] and [134] is used to obtain the BEP in the studied OFDM systems. The simulation results given in [133] and [134] indicate that the analytical results obtained with this technique are very accurate, especially when the E_b/N_0 is higher than 10 dB. The BEP after Viterbi decoding is obtained based on:

$$P_b^C \leq \frac{1}{p} \sum_{d=d_f}^{\infty} c_d \times P_d \quad (6.20)$$

where p is the puncturing period, d_f stands for the free distance of the rate-compatible punctured convolutional code, c_d is the distance spectra and P_d is the error event probability given in equation (B.5). More details of deriving the BEP of the systems coded with rate-compatible punctured convolutional code over fading channels are presented in appendix B.

6.4.2 Numerical Results

• Convolutional Code and Related Parameters

As discussed in section 2.3 of chapter 2, convolutional code and its rate-compatible punctured versions are widely used in all kinds of digital communication systems. The rate 1/2 convolutional code with generator polynomials $g_0 = 133_8$, $g_1 = 171_8$ (constraint length $K = 7$) is adopted as the *mother code* of the high-speed wireless LANs by both ETSI [54] and IEEE [55]. It is also the *mother code* of DAB systems when code rate is not less than 1/2 [52]. Its performance is always used as the *benchmark* for comparison purpose of many other coding schemes ([16], [49], [56], [57], etc.). This is the reason why we use this specific channel coding scheme in our OFDM systems to show the impact of channel coding on the performance of IF-DD and IC-DD. In addition, to make the channel time-selectivity more obvious, $\gamma = 0$ is assumed. The uncoded results presented in Figure 6.11, Figure 6.12 and Figure 6.13 are chosen as the benchmark to compare with the coded results.

To obtain the BEP with equation (6.20), the values of the puncturing period p , the convolutional code's free distance d_f , and the convolutional code's distance spectra c_d are needed. For the chosen rate 1/2 convolutional code with generator polynomials $g_0 = 133_8$, $g_1 = 171_8$, $p = 8$ [133], $d_f = 10$ [135], and the values of c_d are given in the Table II of [133]. P_d is calculated based on equation (B.5), and P_0 (the channel bit error rate before decoding) is obtained with equation (6.5).

• Impact of Using Channel Coding

The same information bit rate is assumed in both uncoded and coded OFDM systems, that is, the transmission data rate is doubled after channel encoding due to the rate 1/2 convolutional code. Other parameters need to be changed accordingly are: the sampling rate, the number of cyclic prefix samples added to each OFDM symbol and the system bandwidth. If assuming the added cyclic prefix (guard interval) still equals to the maximum excess delay of the fading channel, we have:

$$N_G T_r = N_G^C T_r^C = T_D \quad (6.21)$$

where T_D is the maximum excess delay of the fading channel, N_G is the number of cyclic prefix samples of the uncoded system, T_r is the sampling interval of the uncoded system, N_G^C and T_r^C are the corresponding symbols of N_G and T_r for the coded system. To reach the same information transmission rate with channel coding having code rate R , we need:

$$R(N + N_G)T_r = (N + N_G^C)T_r^C \quad (6.22)$$

From equation (6.21) and (6.22), we have²:

$$N_G^C = \left(\frac{N}{NR - (1 - R)N_G} \right) N_G \quad (6.23)$$

$$T_r^C = \left(R - (1 - R) \frac{N_G}{N} \right) T_r \quad (6.24)$$

and

$$B_C = \left(\frac{N}{NR - (1 - R)N_G} \right) B \quad (6.25)$$

²Practically, $N_G^C = \left\lceil \left(\frac{N}{NR - (1 - R)N_G} \right) N_G \right\rceil$, where $\lceil x \rceil$ is the *ceiling* function which gives the closest integer larger than the real-value variable x .

where B and B_C indicate the bandwidth of the uncoded system and the bandwidth of the coded system respectively, and both with cyclic prefix added. For the chosen convolutional code ($R = 1/2$), we have:

$$N_G^C = \left(\frac{2N}{N - N_G} \right) N_G \quad (6.26)$$

$$B_C = \left(\frac{2N}{N - N_G} \right) B \quad (6.27)$$

$$T_r^C = \left(\frac{N - N_G}{2N} \right) T_r = \left(\frac{N - N_G}{2(N + N_G)} \right) T_a \quad (6.28)$$

From equations (6.23), (6.24) and (6.25), to physically implement channel coding in OFDM systems, we need:

$$R - (1 - R) \frac{N_G}{N} > 0 \quad (6.29)$$

that is:

$$R > \frac{N_G}{N + N_G} \quad (6.30)$$

This condition is originated by the fact that the length of the added cyclic prefix must not be less than the maximum excess delay of the channel and the information bit rate must be the same as the uncoded system.

For performance comparison purpose, the energy per information bit is also assumed to be the same for both coded and uncoded systems, i.e. the energy per coded bit is 3 dB lower than the energy per uncoded bit because of the chosen rate 1/2 convolutional code. Generally, the power loss due to adding more cyclic prefix samples compared with uncoded system (see equation (6.23)) shall be considered as well, especially when this power loss is significant compared with the power loss caused by channel coding. However, for our coded systems, the power loss caused by adding extra cyclic prefix samples due to channel coding is very small (as indicated in Table 6.3) when compared with the power loss caused by channel coding (3 dB), thus it is ignored when giving numerical results. The number of cyclic prefix samples $N_G^C = 13$ when the rate 1/2 convolutional code is applied for the chosen values of N and N_G .

Table 6.3: E_b/N_0 loss caused by adding extra cyclic prefix samples due to channel coding when compared with the uncoded system ($N_G = 6$, $N_G^C = 13$).

N	128	256	512	1024
E_b/N_0 loss (dB)	0.209	0.103	0.051	0.026

• Numerical BEP Results

Some of the coded BEP results of IF-DD and IC-DD for the chosen $f_D T_a$ values are presented in Figure 6.18 ($f_D T_a = 10^{-6}$), Figure 6.19 ($f_D T_a = 10^{-5}$) and Figure 6.20 ($f_D T_a = 10^{-4}$), where results are obtained with the rate 1/2 convolutional code with generator polynomials $g_0 = 133_8$, $g_1 = 171_8$, hard decision *Viterbi* decoding, and ideal interleaving is assumed. The corresponding uncoded results can be found in Figure 6.11, Figure 6.12 and Figure 6.13 respectively.

Figure 6.18 shows that at $f_D T_a = 10^{-6}$, for IC-DD, the system with a larger number of subcarriers has a better performance, and all the systems employing IF-DD have almost the same performance which is better than using IC-DD. This indicates that all the system are operating in the $\mathcal{T}$ -zone. The results presented in Figure 6.19 indicate that with increasing $f_D T_a$ from 10^{-6} to 10^{-5} , for the system with $N = 1024$, the IC-DD tends to outperform IF-DD (operating in the $\mathcal{F}$ -zone). It also shows that for the system with $N = 128$, the IF-DD still outperforms IC-DD (operating in the $\mathcal{T}$ -zone). Error floors are still visible when $f_D T_a = 10^{-4}$ as shown in Figure 6.20. If 10^{-6} is set to be the required BER of the mobile system, the system using IF-DD with 1024 may not be acceptable. The results also show that, when $f_D T_a = 10^{-4}$, only the system with $N = 128$ is operating in the $\mathcal{T}$ -zone, all the other 3 systems are operating in the $\mathcal{F}$ -zone where IC-DD outperforms IF-DD.

As presented in chapter 5, if the information bit rate is assumed to be 8 Mbps, the values of f_D used are 4 Hz (Figure 6.11 and Figure 6.18), 40 Hz (Figure 6.12 and Figure 6.19), and 400 Hz (Figure 6.13 and Figure 6.20). The comparison between the uncoded results and the coded results with the same f_D value demonstrates that a significant performance

improvement can be achieved if channel coding is employed in the system. The presented coded BEP results indicate that for a small value of $f_D T_a$ (such as $f_D T_a \leq 10^{-5}$ for the presented system configurations), using a channel coding scheme with a moderate error correction capability (such as the chosen rate 1/2 convolutional code) shall be able to lead the system (with N up to 1024) to achieve the required BER performance instead of being constrained by an error floor above the required BER (such as 10^{-10}).

The given examples are for the systems with N up to 1024. For those systems with a larger number of subcarriers³, the channel selectivity may be a more serious problem even for a moderate *Doppler* frequency. When the channel time-selectivity is getting more and more serious (i.e. $f_D T_a$ is getting larger and larger), to mitigate the degradation caused by the channel time-selectivity, either the number of subcarriers N used in the system needs to be reduced, or the error correction capability of the used channel coding scheme needs to be increased. Using a small N mitigates the channel time-selectivity, but at the same time, it reduces the system transmission efficiency [91]. On the other hand, a channel coding scheme with a stronger error correction capability usually requires either a lower code rate (which results in a lower transmission efficiency as well), or a decoder with a higher complexity (which may increase exponentially with the increase of the error correction capability, such as the *Viterbi* decoder). As a tradeoff, ICI cancellation could be used with a channel coding scheme having a moderate error correction capability.

6.5 Summary

In this chapter, we presented the performances of inter-frame and inter-carrier differential detection techniques for both uncoded and coded OFDM systems, and derived the minimum code rate to apply channel coding in OFDM system with the same information bit rate. The results show that, without channel time-selectivity, inter-frame differential detection always outperforms inter-carrier differential detection if the channel is frequency selective, and they have the same performance if the channel is frequency nonselective.

³Such as the DVB system which has two modes with 1705 (2K mode) and 6817 (8K mode) subcarriers in the system [53] and requires BER to be no more than 10^{-9} [20].

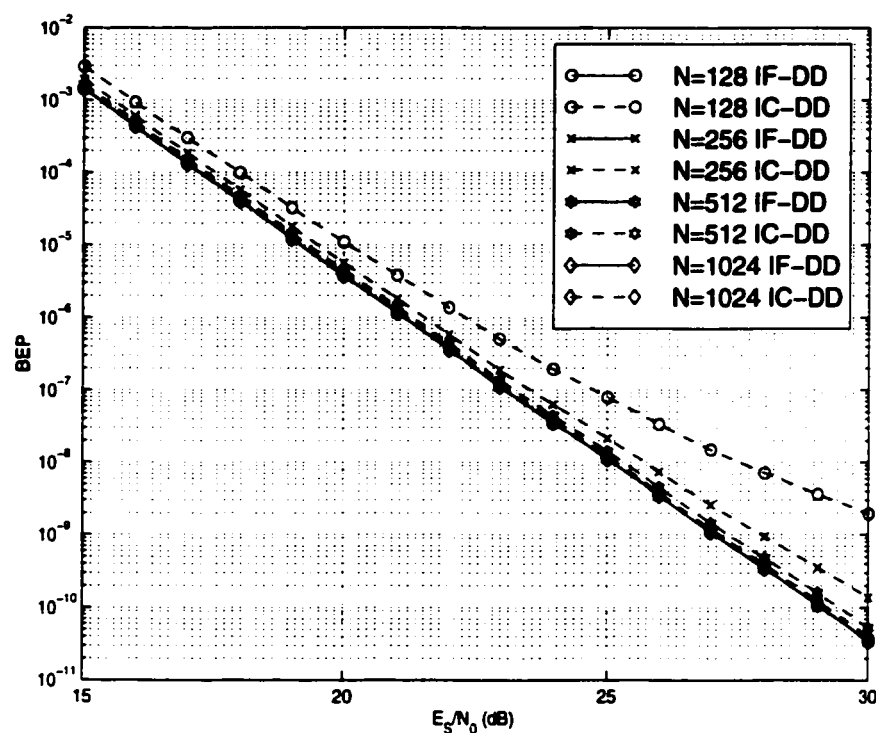


Figure 6.18: Coded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-6}$.

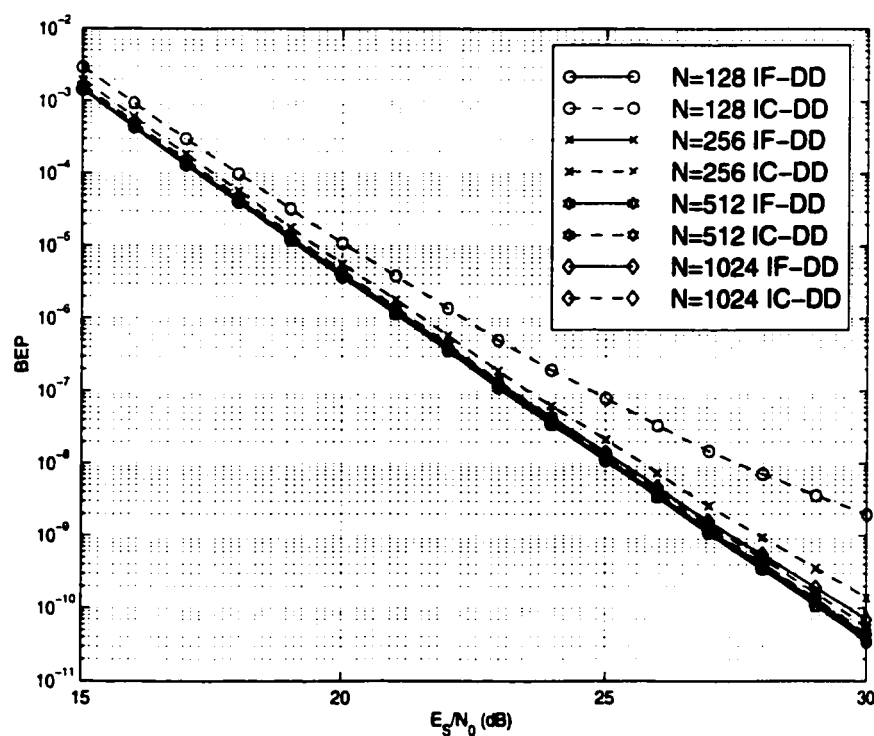


Figure 6.19: Coded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-5}$.

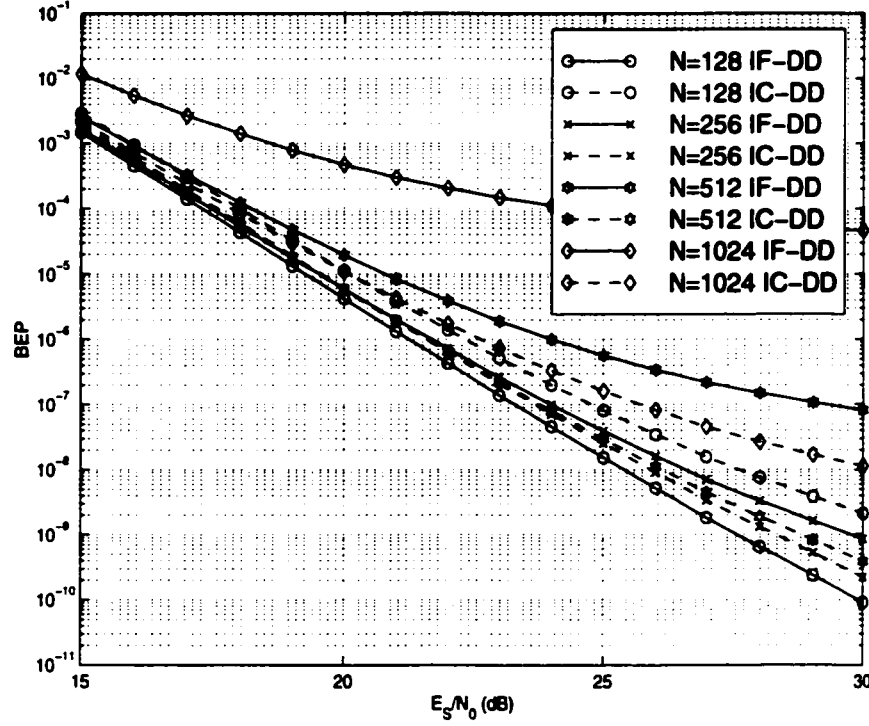


Figure 6.20: Coded performance of IF-DD and IC-DD when $\gamma = 0$ and $f_D T_a = 10^{-4}$.

However, with channel time selectivity, inter-carrier differential detection may outperform inter-frame differential detection under certain conditions no matter the channel is frequency selective or nonselective. In other words, the channel time selectivity may cause more serious performance degradation for inter-frame differential detection than that for inter-carrier differential detection. The performance investigation of inter-frame and inter-carrier differential detections lead to the definitions of the $\mathcal{F}$ zone, the $\mathcal{T}$ zone, and the $\mathcal{E}$ zone. The analytical results shows that, if the system is operating in the $\mathcal{F}$ zone, inter-carrier differential detection outperforms inter-frame differential detection. In the $\mathcal{T}$ zone, inter-frame differential detection outperforms inter-carrier differential detection, and in the $\mathcal{E}$ zone, two differential detection schemes have a similar performance. Thus we provided guidelines for the design of OFDM systems over multipath time-variant fading channels.

Chapter 7

Conclusion

7.1 Summary

The research described in this thesis had the objectives of investigating the ICI caused by channel time-selectivity, designing effective and efficient ICI cancellation schemes, and exploring the effects of channel time-selectivity and frequency-selectivity on the system performance. This was accomplished by:

- characterizing a general time-variant frequency-selective WSSUS fading channel model;
- evaluating channel time-selectivity with the normalized spaced-time correlation function including the time-invariant component of the channel impulse response;
- performing ICI analysis with the characterized general WSSUS fading channel model and obtaining the ICI power distribution pattern;
- designing the frequency-domain ICI cancellation equalizer with a controllable complexity based on the obtained ICI power distribution pattern, and providing the numerical performance results of the designed ICI cancellation equalizer through simulations;
- investigating the effects of channel time-selectivity and frequency-selectivity on OFDM systems by obtaining and comparing the bit error performance of inter-frame and inter-carrier differential detection;

- studying the impact of channel coding on the system performance, and obtaining the lowest code rate that could be used in the coded OFDM systems for a given system configuration and channel condition.

The ICI caused by channel time-selectivity was well analyzed in the thesis, and the obtained ICI power distribution was utilized to design the frequency-domain ICI cancellation equalizer. The simulation results obtained for the designed ICI cancellation scheme are very encouraging as they indicate that the error floor caused by ICI can be lowered significantly and the designed ICI cancellation scheme has a controllable complexity. When dedicated to mobile applications, OFDM systems are generally affected by both channel time-selectivity and frequency-selectivity, which should be carefully explored by the system designers. The best system designs would be those which could utilize both channel time-selectivity and frequency-selectivity.

7.2 Thesis Contributions

The contributions made in this thesis could be classified into five categories: channel time-selectivity evaluation, OFDM ICI analysis, ICI cancellation scheme design, effects of channel time-selectivity and frequency-selectivity on OFDM systems, and the impact of channel coding on system performance.

7.2.1 Channel Time-selectivity Evaluation

To evaluate the channel time-selectivity of fading channels, we characterized a general time-variant frequency-selective WSSUS fading channel model, and also defined the selective channel parameters γ and β for this purpose. It was demonstrated that the time-invariant component of the channel impulse response should also be included in the *Bello* system functions when evaluating the channel time-selectivity, instead of limiting the application of *Bello* system functions only to those channels with a zero mean.

7.2.2 OFDM ICI Analysis

Because of using the characterized general time-variant frequency-selective WSSUS fading channel model in the OFDM ICI analysis, the obtained results are applicable to a wide variety of fading channels, even when the channel coherence time is close to the symbol duration.

It was observed that ICI is caused only by the time-variant component of the channel impulse response. The results also indicate that the signal to inter-carrier interference ratio is a function of parameter γ and the normalized *Doppler* frequency, while the average ICI power distribution is independent of the value of γ . Thus the work presented in [19] was further developed for general WSSUS fading channels.

The total ICI power suffered by each subcarrier was obtained. In the literature, it was reported that the central subcarrier (located at the center of the frequency band) receives most of the ICI, and the edge subcarriers suffer less ICI [109]. Our analysis showed that the total ICI power suffered by each subcarrier is the same. In other words, the total ICI power for each subcarrier is independent of its frequency location. In addition, it was observed that most of the ICI power is contributed by a few closest neighboring subcarriers in a *cyclic* fashion.

One systematic method, which evaluates the defined signal-to-effective noise ratio based on γ and *Doppler* frequency, was provided to determine whether the channel time-selectivity could be ignored or not, and the effects of γ on the system performance was studied. The *signal-to-effective noise ratio dropping zone*, where an error floor could be introduced, was defined and investigated for various transmission scenarios. Due to the high sensitivity of OFDM systems to the channel time-selectivity, signal-to-effective noise ratio may become a more precise parameter than the traditionally used coherence time to evaluate the channel time-selectivity.

7.2.3 ICI Cancellation Scheme Design

It was demonstrated that effective and efficient ICI cancellation schemes can be designed based on the obtained ICI power distributions. Our simulation results indicated that the error floors caused by the channel time-selectivity can be significantly lowered with the designed frequency domain ICI cancellation equalizer for the studied OFDM systems. By using different puncturing indexes, the complexity of the designed frequency domain ICI cancellation equalizer can be adapted to the required bit error performance.

7.2.4 Channel Time-selectivity and Frequency-selectivity Effects

Due to the fact that OFDM is a two-dimensional system (i.e. time and frequency domains), differential encoding/decoding can be implemented based on either inter-frame or inter-carrier basis with almost the same complexity. When such OFDM systems are operating over time-variant multipath fading channels, the channel time-selectivity and the channel frequency-selectivity affect these two differential detection schemes quite differently. The inter-frame differential detection is more vulnerable to the channel time-selectivity. On the other hand, the inter-carrier differential detection is more sensitive to the channel frequency-selectivity.

When these two differential detection schemes are experiencing both time-selective and frequency-selective fading, we defined three *zones* to reflect their bit error performances: in the $\mathcal{F}$ zone, the inter-carrier differential detection has a better performance; in the $\mathcal{T}$ zone, the inter-frame differential detection outperforms the inter-carrier differential detection; in the $\mathcal{E}$ zone, these two differential detection schemes have comparable performances. For a given scenario (i.e. with the given fading channel model, *Doppler* frequency, number of subcarriers, etc.), the zone where the inter-frame and inter-carrier differential detection are operating can be predicted accurately by evaluating the mean squared estimation errors of inter-frame differential detection and inter-carrier differential detection.

7.2.5 Impact of Channel Coding

OFDM systems generally come with channel coding schemes, especially for those systems operating over Rayleigh fading type channels where the bit error rate decreases slowly with the increase of SNR . Channel coding provides a very effective way to lower the error floors caused by the channel time-selectivity. Our analysis indicated that a channel coding scheme with a moderate complexity may achieve an acceptable bit error performance for mobile OFDM systems with the number of subcarriers up to 1024 and experiencing a moderate *Doppler* frequency. However, with the increase of the *Doppler* frequency, or if the number of subcarriers in the system has to be large in order to achieve a higher transmission efficiency, either the error correction capability of channel coding scheme needs to be increased (which means a higher complexity) or ICI cancellation could be utilized combined with a channel coding scheme with a moderate error correction capability. Tradeoffs need to be made based on the required system bit error performance and the overall system complexity.

7.3 Recommendations for Future Research

With respect to the subject of this thesis, several aspects would be of interest for further investigation:

- In this thesis, ICI analysis was performed based on the assumption that the transmitted symbols a_n are uncorrelated. In reality, this assumption may not be valid when the information bit stream is encoded with some specific channel coding schemes. In order to analyze the performance of such coded OFDM systems and design effective ICI cancellation schemes, it would be critical to obtain the total ICI power suffered by each subcarrier and the ICI power distribution according to the correlation of a_n introduced by the specific channel coding scheme used in the system;
- The channel estimation method used was based on the use of *Wiener* filter in chapter 5. Other methods, such as the linear interpolation reported in [115], [116], the

lowpass filter used in [91], [136], and the approximate Gaussian filter presented in [137], could be evaluated for performance *versus* complexity purpose;

- Zero-forcing algorithm was used in our designed frequency domain ICI cancellation equalizer. For performance, complexity and stability comparison purpose, equalization algorithms such as Least-Mean-Square (LMS) and Recursive Least-Squares (RLS) ([1], [2], [123]) could also be used to design ICI cancellation equalizer based on the obtained ICI power distribution pattern;
- As shown in chapter 6, the error floor caused by channel time-selectivity can be significantly lowered by using a channel coding scheme in the system. For OFDM systems over time-variant frequency-selective fading channels, there are always two approaches to achieve the required bit error rate from lowering the error floor point of view. One approach is using a channel coding scheme (including the concatenated coding schemes) with a strong or moderate error correction capability¹. The other approach could be using a channel coding scheme with a moderate or low error correction capability combining with an ICI cancellation scheme². It would be very useful in practice to compare the implementation complexities of these two approaches for different scenarios to determine which one to deploy.

¹Depends mainly on the channel condition, the system configuration and the required bit error rate.

²Such as the frequency-domain ICI cancellation equalizer presented in the chapter 5 of this thesis or the time-domain ICI cancellation equalizer reported in [117]. The estimation scheme of the channel impulse response could be chosen from linear interpolation, lowpass filter, approximate Gaussian filter, or *Wiener* filter.

Appendix A

Characteristic Function Based Bit Error Probability Analysis

In this appendix, we introduce a characteristic function based bit error probability analysis technique. This method is accurate and effective to obtain the performance of both uncoded and Trellis coded systems over fading channels [131], [138].

For a fading channel with received signal in the form of

$$r_k = u_k x_k + n_k \quad (\text{A.1})$$

where x_k is the transmitted symbol, u_k is the attenuation factor of the channel, and n_k is the AWGN of the channel which is Gaussian distributed but not necessary to have a zero mean, the characteristic function based pairwise error analysis can be used to obtain bit error probability [139], [140]. This error analysis technique is well documented in [139], [140], as well as the appendix 4B of [114]. Typical applications for this technique can be found in [131] and [138], where an exact expression for the pairwise error event probability of Trellis-coded modulation signals transmitted over Rayleigh fading channels is obtained. This technique is particularly suitable for the performance analysis of PSK modulation schemes, both coded and uncoded [131], [138]. It is also adopted by Leung and Ho to derive the bit error probability of QPSK modulated conventional OFDM systems over fast fading channels, for both differential detection [141] and coherent detection [91].

A.1 Mean and Variance

It is assumed that of u_k and n_k are uncorrelated, with mean $\bar{u}$ and $\bar{n}$ respectively, and the variances of u_k and n_k are:

$$\sigma_u^2 = E\left\{(u_k - \bar{u})(u_k - \bar{u})^*\right\} \quad (\text{A.2})$$

and

$$\sigma_n^2 = E\left\{(n_k - \bar{n})(n_k - \bar{n})^*\right\} \quad (\text{A.3})$$

where y^* denotes the complex conjugate of y . Given x_k is transmitted, the mean of r_k is

$$\begin{aligned} \bar{r}_k &= E\left\{r_k\right\} \\ &= E\left\{u_k x_k + n_k\right\} \\ &= \bar{u} x_k + \bar{n} \end{aligned} \quad (\text{A.4})$$

and the variance of r_k can be written as:

$$\begin{aligned} \sigma_{r|x_k}^2 &= E\left\{(r_k - \bar{r}_k)(r_k - \bar{r}_k)^*\right\} \\ &= E\left\{(u_k x_k + n_k - \bar{u} x_k - \bar{n})(u_k x_k + n_k - \bar{u} x_k - \bar{n})^*\right\} \\ &= E\left\{\left((u_k x_k - \bar{u} x_k) + (n_k - \bar{n})\right)\left((u_k x_k - \bar{u} x_k)^* + (n_k - \bar{n})^*\right)\right\} \\ &= E\left\{(u_k x_k - \bar{u} x_k)(u_k x_k - \bar{u} x_k)^*\right\} + E\left\{(n_k - \bar{n})(n_k - \bar{n})^*\right\} \\ &\quad + E\left\{(u_k x_k - \bar{u} x_k)(n_k - \bar{n})^*\right\} + E\left\{(u_k x_k - \bar{u} x_k)^*(n_k - \bar{n})\right\} \\ &= |x_k|^2 \sigma_u^2 + \sigma_n^2 + x_k E\left\{u_k - \bar{u}\right\} E^*\left\{n_k - \bar{n}\right\} + x_k^* E^*\left\{u_k - \bar{u}\right\} E\left\{n_k - \bar{n}\right\} \\ &= |x_k|^2 \sigma_u^2 + \sigma_n^2 \end{aligned} \quad (\text{A.5})$$

where $|x_k|^2$ is the *Euclidean* distance of x_k from the origin in the signal constellation (see Figure 6.1). If the estimation of u_k is assumed as v_k , its variance is defined as:

$$\sigma_v^2 = E\left\{(v_k - \bar{v})(v_k - \bar{v})^*\right\} \quad (\text{A.6})$$

Without loss of generality, it is assumed that the estimation of v_k is *unconditional unbiased* [132], that is, the mean of v_k is the same as the mean of u_k , i.e. $\bar{v} = \bar{u}$. The covariance of u_k and v_k is defined as:

$$\sigma_{uv}^2 = E\left\{(u_k - \bar{u})(v_k - \bar{v})^*\right\} \quad (\text{A.7})$$

Given x_k is transmitted, r_k and v_k are correlated random variables with a covariance of

$$\begin{aligned}
 \sigma_{rv|x_k}^2 &= E\left\{(r_k - \bar{r}_k)(v_k - \bar{v})^*\right\} \\
 &= E\left\{(u_k x_k + n_k - \bar{u} x_k - \bar{n})(v_k - \bar{v})^*\right\} \\
 &= E\left\{\left((u_k x_k - \bar{u} x_k) + (n_k - \bar{n})\right)(v_k - \bar{v})^*\right\} \\
 &= E\left\{(u_k x_k - \bar{u} x_k)(v_k - \bar{v})^*\right\} + E\left\{(n_k - \bar{n})(v_k - \bar{v})^*\right\} \\
 &= x_k \sigma_{uv}^2 + E\left\{n_k - \bar{n}\right\} E^*\left\{v_k - \bar{v}\right\} \\
 &= x_k \sigma_{uv}^2
 \end{aligned} \tag{A.8}$$

A.2 Decision Metric

Given that the received signal is in the form presented in (A.1), the decision metric can be written as:

$$M(\hat{x}_k) = |r - v \hat{x}_k|^2 \tag{A.9}$$

where $\hat{x}_k$ is the estimation of x_k , and the random variable

$$D = M(x_k) - M(\hat{x}_k) \tag{A.10}$$

can be written in a quadratic form as [140], [114]:

$$D = A|r|^2 + B|v|^2 + Crv^* + C^* r^* v \tag{A.11}$$

where, for PSK modulation, A, B, C can be chosen as (see [131], [138])

$$A = 0 \tag{A.12}$$

$$B = 0 \tag{A.13}$$

$$C = (x_k^* - \hat{x}_k^*) \tag{A.14}$$

A.3 Pairwise Error Event Probability

The pairwise error event probability is the probability when $D < 0$ and can be expressed as ([140], or appendix 4B of [114]),

$$P(x_k \rightarrow \hat{x}_k) = P(D < 0) = \int_{-\infty}^0 p(D) dD \tag{A.15}$$

where $p(D)$ is the probability density function of D .

Traditionally, the characteristic function of a random variable (for example D) is defined as [2]

$$\Psi_D(j\omega) = \int_{-\infty}^{+\infty} p(D)e^{j\omega D} dD \quad (\text{A.16})$$

which is the complex conjugate of the *Fourier* transform of $p(D)$ where $p(D)$ is real. However, a more concise expression of the characteristic function is using the bilateral *Laplace* transform [142],

$$\Phi_D(s) = \int_{-\infty}^{+\infty} p(D)e^{-sD} dD \quad (\text{A.17})$$

Thus, the $p(D)$ is related with the characteristic function $\Phi_D(s)$ by the inverse *Laplace* transform [142],

$$p(D) = \frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \Phi_D(s)e^{sD} ds \quad (\text{A.18})$$

where δ is chosen to be the value that the integration contour, which is a straight line in the complex plane, parallel to $j\omega$ -axis, and determined by the value of δ , is in the region of convergence. The convergence region is the region between the poles of the rightmost left plane and leftmost right plane. Therefore, the pairwise error even probability, $P(x_k \rightarrow \hat{x}_k)$, is equal to

$$\begin{aligned} P(x_k \rightarrow \hat{x}_k) &= \int_{-\infty}^0 p(D)dD \\ &= \int_{-\infty}^0 \left(\frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \Phi_D(s)e^{sD} ds \right) dD \\ &= \frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \Phi_D(s) \left(\int_{-\infty}^0 e^{sD} dD \right) ds \\ &= \frac{1}{2\pi j} \int_{\delta-j\infty}^{\delta+j\infty} \frac{\Phi_D(s)}{s} ds \end{aligned} \quad (\text{A.19})$$

A.4 Characteristic Function

Based on the characteristic function given by the equation (5) of [140], or the equation (4B.5) of [114] which is in the form of *Fourier* transform complex conjugate, and the relationship between equation (A.16) and equation (A.17), by setting

$$v = js \quad (\text{A.20})$$

$$v_1 = -P_L \quad (\text{A.21})$$

$$v_2 = P_R \quad (\text{A.22})$$

the characteristic function $\Phi_D(s)$ can be obtained as

$$\Phi_D(s) = \frac{P_L P_R}{(s - P_L)(s - P_R)} \exp\left\{ \frac{P_L P_R}{(s - P_L)(s - P_R)} s(s\alpha_1 - \alpha_2) \right\} \quad (\text{A.23})$$

where P_L and P_R are the poles of left and right planes, which are expressed as

$$P_L = w - \sqrt{w^2 - P_L P_R} \quad (\text{A.24})$$

$$P_R = w + \sqrt{w^2 - P_L P_R} \quad (\text{A.25})$$

where

$$\begin{aligned} P_L P_R &= -\frac{1}{(\sigma_{r|x_k}^2 \sigma_v^2 - |\sigma_{rv|x_k}^2|^2)(|C|^2 - AB)} \\ &= -\frac{1}{|x_k - \hat{x}_k|^2 (|x_k|^2 \sigma_u^2 \sigma_v^2 + \sigma_n^2 \sigma_v^2 - |x_k|^2 |\sigma_{uv}|^4)} \\ &= -\frac{1}{|x_k - \hat{x}_k|^2 \left(|x_k|^2 (\sigma_u^2 \sigma_v^2 - |\sigma_{uv}|^4) + \sigma_n^2 \sigma_v^2 \right)} \end{aligned} \quad (\text{A.26})$$

and

$$\begin{aligned} w &= \frac{A\sigma_{r|x_k}^2 + B\sigma_v^2 + C^* \sigma_{rv|x_k}^2 + C(\sigma_{rv|x_k}^2)^*}{2(\sigma_{r|x_k}^2 \sigma_v^2 - |\sigma_{rv|x_k}^2|^2)(|C|^2 - AB)} \\ &= \frac{\text{Re}\{C^* \sigma_{rv|x_k}^2\}}{|x_k - \hat{x}_k|^2 \left(|x_k|^2 (\sigma_u^2 \sigma_v^2 - |\sigma_{uv}|^4) + \sigma_n^2 \sigma_v^2 \right)} \\ &= \frac{\text{Re}\{\sigma_{uv}^2 x_k (x_k - \hat{x}_k)\}}{|x_k - \hat{x}_k|^2 \left(|x_k|^2 (\sigma_u^2 \sigma_v^2 - |\sigma_{uv}|^4) + \sigma_n^2 \sigma_v^2 \right)} \end{aligned} \quad (\text{A.27})$$

The corresponding α_1 and α_2 are

$$\begin{aligned} \alpha_1 &= (|C|^2 - AB) \left(|\bar{r}_k|^2 \sigma_v^2 + |\bar{v}_k|^2 \sigma_{r|x_k}^2 - \bar{r}_k^* \bar{v}_k \sigma_{rv|x_k}^2 - \bar{r}_k \bar{v}_k^* (\sigma_{rv|x_k}^2)^* \right) \\ &= |x_k - \hat{x}_k|^2 \left(|\bar{u}x_k + \bar{n}|^2 \sigma_v^2 + |\bar{v}|^2 (|x_k|^2 \sigma_u^2 + \sigma_n^2) \right. \\ &\quad \left. - 2\text{Re}\{(\bar{u}x_k + \bar{n})^* \bar{v}x_k \sigma_{uv}^2\} \right) \\ &\stackrel{\bar{v}=\bar{u}}{=} |x_k - \hat{x}_k|^2 \left(|\bar{u}x_k + \bar{n}|^2 \sigma_v^2 + |\bar{u}|^2 (|x_k|^2 \sigma_u^2 + \sigma_n^2) \right. \\ &\quad \left. - 2\text{Re}\{\sigma_{uv}^2 (|\bar{u}|^2 |x_k|^2 + x_k \bar{u} \bar{n}^*)\} \right) \end{aligned} \quad (\text{A.28})$$

and

$$\begin{aligned}
\alpha_2 &= A|\bar{r}_k|^2 + B|\bar{v}_k|^2 + C\bar{r}_k^* \bar{v}_k + C^* \bar{r}_k \bar{v}_k^* \\
&= 2\text{Re}\left\{C^* \bar{r}_k \bar{v}_k^*\right\} \\
&= 2\text{Re}\left\{(x_k - \hat{x}_k)(\bar{u}x_k + \bar{n})\bar{v}^*\right\} \\
&\stackrel{\bar{v}=\bar{u}}{=} 2\text{Re}\left\{(x_k - \hat{x}_k)(\bar{u}x_k + \bar{n})\bar{u}^*\right\} \\
&= 2\text{Re}\left\{(x_k - \hat{x}_k)(|\bar{u}|^2 x_k + \bar{n} \bar{u}^*)\right\}
\end{aligned} \tag{A.29}$$

All the parameters derived above are for the characteristic function given in equation (A.23) which is in the form of bilateral *Laplace* transform, and the derivation is based on the corresponding parameters given by the equation (6) of [140], or the equation (4B.6) of [114]. The exact error probability that the transmitted symbol x_k will be detected as symbol $\hat{x}_k$ can be obtained by calculating equation (A.19).

A.5 Bit Error Probability

To evaluate the performance of a digital communication system, the average bit error probability is more important than the pairwise error event probability. In this thesis, it is assumed that *Gray* encoding is adopted for all the differential encoding, which means that the symbols stands for the codewords with the largest *Hamming* distances are separated the furthest in the signal constellation. Assume S_i is transmitted for coherent detection, or S_i is the correct current state for differential detection, where $1 \leq i \leq N_S$ and N_S is the number of possible symbols in the signal constellation, the union bound of the bit error probability can be obtained as the summation of the pairwise error probabilities as [130],

$$P_b \leq \frac{1}{n_s} \sum_{i=1}^{N_S} \sum_{\substack{j=1 \\ j \neq i}}^{N_S} m_{ij} P(S_i) P(S_i \rightarrow S_j) \tag{A.30}$$

where n_s is the number of information bits per symbol ($N_S = 2^{n_s}$), $P(S_i)$ is the a priori probability of transmitting symbol S_i , and m_{ij} is the *Hamming* distances between the bit pairs represented by symbol S_i and S_j . When equal a priori probability of transmission

is assumed, that is, $P(S_i) = \frac{1}{N_s}$, equation (A.30) changes to:

$$P_b \leq \frac{1}{n_s} \sum_{\substack{j=1 \\ j \neq i}}^{N_s} m_{ij} P(S_i \rightarrow S_j) \quad (\text{A.31})$$

Appendix B

Bit Error Probability Analysis of Rate-Compatible Punctured Convolutional Codes

In this appendix, we introduce an analytical method to obtain the error performance of rate-compatible punctured convolutional codes when Viterbi decoder is utilized for the systems over binary symmetric channels. The major advantage of this approach is that the error performance is obtained through the bit error rate before decoding (i.e. the raw bit error rate), and it is applicable to many scenarios. The material presented in this appendix is for systems with hard decision which is chosen for our studied OFDM systems. More details and the results with soft decision are available in [133] and [134].

B.1 Rate-compatible Punctured Convolutional Codes

Rate-compatible punctured convolutional codes [133] were extended from the punctured convolutional codes ([143], [144]) originally introduced to simplify Viterbi decoding by reducing the number of the branches arriving at each node in the Viterbi decoder while having maximal free distances. Compared with the punctured convolutional codes, the rate-compatible punctured convolutional codes have a rate-compatibility rule which requires that the puncturing tables preserve all the code bits of higher rate codes in the lower rate codes. This feature makes flexible channel encoding and adaptive decoding

simpler: the same encoder and decoder can be used for many different code rates without changing their basic structures. The transmissions with different error protection needs and the time-variant nature of the wireless communication channels require flexible channel encoding and adaptive decoding to achieve the required efficiency and bit error rate.

B.2 Bit Error Probability Analysis

The Viterbi decoder is a maximum likelihood decoder [2] which performs maximum likelihood decision on the m th code sequence $\mathbf{x}^{(m)}$:

$$Pr(\mathbf{y}|\mathbf{x}^{(m)}) > Pr(\mathbf{y}|\mathbf{x}^{(m')}) \quad (\text{for all } m \neq m') \quad (\text{B.1})$$

where $\mathbf{y}$ is the received real value sequence ($y \in \pm 1$ for hard decision). With the assumption that statistical independence is achieved with sufficient interleaving, the Viterbi decoder maximizes [134]:

$$\max_m \sum_j \sum_{i=1}^n a_{ij} x_{ij}^{(m)} L_{ij} \quad (\text{B.2})$$

where $j = 1, 2, 3, \dots$ indicates the decoding time steps and the decoded bits, $i = 1, 2, 3, \dots, n$ indicates the code bits transmitted per information bit and n is the total code bits for each information bit, a_{ij} are the puncturing table entries, and L_{ij} is the metric increment defined as:

$$L_{ij} = \log \frac{Pr(y_{ij}|x_{ij}^{(m)} = +1)}{Pr(y_{ij}|x_{ij}^{(m)} = -1)} \quad (\text{B.3})$$

The bit error probability after decoding can be estimated as [134]:

$$P_b^C \leq \frac{1}{p} \sum_{d=d_f}^{\infty} c_d \times P_d \quad (\text{B.4})$$

where p is the puncturing period ($a_{ij+p} = a_{ij}$ due to periodical puncturing), d_f is the free distance of the rate-compatible punctured convolutional code, c_d is the distance spectra and P_d is the error event probability. For the Viterbi decoder with hard decision, the optimum metric is $L_{ij} = y_{ij} = \pm 1$, which results in [133]:

$$P_d = \begin{cases} \sum_{e=(d+1)/2}^d \binom{d}{e} P_0^e (1 - P_0)^{d-e} & (\text{d odd}) \\ P_{d-1} & (\text{d even}) \end{cases} \quad (\text{B.5})$$

where P_0 is the channel bit error rate before decoding. The values of c_d and d_f for some widely used rate-compatible punctured convolutional codes with different puncturing vectors are provided in the Table II and Table III of [133] respectively.

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