

The Role of Thermodynamics in the Economic Theory of the Environment: Should the Thermodynamic Laws Be Explicitly Incorporated in the Theories of Renewable and Non-Renewable Natural Resources and in Environmental Economics?

by

Eun Sook Ellen Song-Maurice

(500477)

Major Paper presented to the

Department of Economics of the University of Ottawa

in partial fulfillment of the requirements of the M.A. Degree

Supervisor: Professor Philippe J. Crabbé

ECO 7997

Ottawa, Ontario

November, 1996

Contents

1.0 Introduction	2
1.1 Interdisciplinary Research in Economics	19
2.0 History of Energy	21
2.1 Entropy	24
3.0 Energy Valuation	25
4.0 The Importance of Thermodynamics in Economics	29
4.1 Incorporating Thermodynamics in Economics	30
4.2 Survey of Thermodynamic Production Functions	36
5.0 Synthesis of Amir's Objectives	39
6.0 Amir's Model	44
6.1 Critiques of the Amir Model	48
6.2 Changes and Modifications to the Amir Model	58
7.0 Summary and Conclusions	59
Appendix	65
References	79

1.0 Introduction

This paper introduces a topic that has yet to be applied explicitly to mainstream economics: The role of thermodynamics in economics. Thermodynamics is a branch of physics that studies the flows and transformations of energy. More specifically, thermodynamics “pertains to the study of heat and motion, the relations between heat and mechanical, electrical, and other forms of energy or work” (Blatt 1983, p.276). According to Amir (1991), the focus of thermodynamics is on the direction of physical processes rather than with their speed, and this is the foremost reason for the connection between thermodynamics and neoclassical economic theory.

Classical thermodynamics consists mainly of *two* laws that govern the behaviour of energy. The first law is sometimes referred to as the conservation law of energy, and the second law as the maximum entropy law (Nicolis and Prigogine 1977). Simply put, the first law states that “the energy of the universe¹ is constant”, and the second law states that “the entropy of the universe is constantly increasing” (Commoner 1976, p.28). Commoner believes that this leads to a paradox: “while energy cannot be lost, merely possessing it is of no value”.² He explains that, “energy is valuable only insofar as it is used to generate work, but in that process, some of its ability to do work is necessarily lost”(Ibid.). Therefore, “what inevitably diminishes is not the world’s constant stock of energy, but it’s ability to do what we value-work”(Ibid.). These laws of equilibrium thermodynamics apply to isolated systems (Nicolis and Prigogine 1977).

The key issues concerning thermodynamics in economics are whether to

¹Note that the universe being unbounded cannot be taken strictly as an isolated system.

²What this means is that if we don’t use energy, what value does it have (to society)?

apply the thermodynamic laws directly to economics or whether it is already implicitly in the existing theoretical models of economics. Amir (1991, p.2) says, "nobody claims that economic or ecological systems operate in violation of thermodynamic laws. Rather, the claim is that constraints imposed on the system by these laws are not taken into consideration by economic and policy theorists, and this results in excessive use of natural resources". Amir in his 1991 paper gives a brief history of the relationship that economics has had with thermodynamics.

Amir (1991) believes that the interface between economics and thermodynamics has diverged into two different directions. First is the analytical direction, which according to Amir, has been little explored; it includes works by Davis (1942), Lisman (1949), and Samuelson (1983). The work done by Lisman led him to wonder whether establishing analogies between thermodynamics and economics produces any practical results. Then, Samuelson tried to expose the similarities of the two theories and to delineate the existing relations that exist between them. Although the study succeeded in identifying analytical overlaps, it has not generated the debates or enthusiasm for further study. Furthermore, according to Amir (1991), recent developments in economic theory have not concerned physicists (found in Faber & Proops 1985).

The second direction, according to Amir (1991), is the tendency to blame economic theory for neglecting absolute scarcity in economic analysis as dictated by the laws of thermodynamics (Underwood & King 1989) and for ignoring the laws of thermodynamics totally (Soddy 1912, 1933, 1934; Boulding, 1966; Georgescu-Roegen, 1971; Odum, 1971; Daly, 1973). This direction also believes that the physical and biophysical foundations of economic theory have been ignored by researchers (see Ayres & Nair, 1984; Faber, 1985;

Faber et al., 1987; Daly, 1987) and Ibid..

This belief—that thermodynamics could have led economics to generate more efficient policies—has not gone unchallenged. However, according to Amir (1991, p.2), the “critics did not assume a formal position either (Solow 1972; Nordhaus & Tobin 1973; Simon 1980, 1981)”. In that the critics, while challenging the above belief, did not assert clearly whether the laws of thermodynamics are or are not relevant in economics. So the debate over whether the thermodynamic laws should be incorporated in to economic theory is not over.

Amir gives two reasons for the relation between economic theory and thermodynamics, *first*, both economic theory and thermodynamics use optimization, though not in the same degree; but one can say that “economics and thermodynamics are analogous methodologically” (Amir 1991, p.2).³ The use of optimization methods in economics, such as the Lagrange method, to determine optimal levels of consumption or production for rational economic agents, also has an analog in thermodynamics. A method of optimization in thermodynamics, developed by Hatsopoulos and Keenan (1965), assumes systems are changing up to a point when the (constrained) state function reaches a stable position. At the point of a stable position, the maximal or minimal values of the constrained function can be determined. Thus, as Amir claims, optimization methodology, implicit in the thermodynamic system, and explicit in the economic system, are similar. *Second*, “prices and quantities of commodities in economic systems are the analogs of the intensive and extensive variables, respectively, of a thermodynamic system” (Amir

³Using the same methods does not imply that they are similar methodologically, what Amir is trying to say is that the two fields share the same objectives in terms of optimization, but the methods they use are dissimilar.

1991, pp.2-3). To the best of Amir's knowledge, "no study...has been able to elucidate fully the analogy between a market and a gas cylinder or a laboratory beaker.⁴ Hence, none could have addressed properly the question of relevancy of the laws of thermodynamics to economic theory..."(Amir 1991, p.3).

Amir (1991, p.5) goes on further to explain that "in addition to its boundaries, the definition of a system includes its macroscopic coordinates (variables)", which are observable and "measurable properties of the system used to define the system's state". There are two categories of coordinates, they are intensive coordinates and extensive coordinates. Examples of *intensive coordinates* are: mass, volume and the flows and stocks of different commodities. Examples of *extensive coordinates* are: temperature, pressure, density and prices. Amir states that "intensive coordinates of the system are those variables that characterize the state of the system rather than its size", while "extensive coordinates of the system are those cardinal variables that directly depend on the system mass" (Ibid.).

It's only under certain conditions that a system possesses intensive coordinates. According to Amir, the "intensive variables are the emergent properties of the whole conceived as a single unit, and their existence is intimately related with the whole being in a state of equilibrium" (Ibid.). A system is in *equilibrium* "when its intensive properties do not change with time, and no flows exist within the system or across its boundaries" (Amir 1991, p.6). Amir states that "these conditions are related because spatial and temporal

⁴The connection between a laboratory beaker or a gas cylinder and a market becomes relevant because thermodynamics is concerned with the transfer of heat and work, and converting one form of energy into another (Rogers and Mayhew 1980). Thus if one could, carefully, argue that the market has similar analogues (to heat and work and converting one form of energy into another) this would strengthen the connection between thermodynamics and economics.

differences in the intensive variables always cause internal flows” (Ibid.). If a system’s “intensive properties do not change with time (they may vary spatially), and no flows exist within the system”, then the system is said to be in *steady state* (Ibid.). However, according to Amir, “time-independent flows may cross the system boundaries and pass through it in steady state” (Ibid.). Amir concludes that “when a system is not in either of these states it is said to be *disequibrated*”, and in such a state, intensive properties are not defined (Ibid.).

Amir (1991, p.4) states, “thermodynamic theory starts by dividing the world of discussion into a system and its surroundings (environment). These parts are separated by well defined boundaries (walls), and the walls provide conceptual and practical control over any interaction between the system and its surroundings”. Thermodynamic theory deals with three kinds of bounded systems. First, an *isolated system*: “an isolated system is one which is surrounded by a ‘wall’ which prevents interaction of any kind (matter or energy) between the system and its surroundings” (Crabbé 1992, p.1). Thus isolated systems are “completely independent of their surroundings” (Amir 1991, p.4). Isolated systems are either near equilibrium or far from equilibrium.⁵ Near equilibrium, they are subject to the two first laws of thermodynamics.⁶ Mathematically, the motion equations can be linearised.⁷ Second, a *closed system*: “a system is closed if it is isolated with respect to matter but not energy, the Earth is a closed system” (Crabbé 1992, p.1). Third, an *open system*: “a system is open if it is isolated with respect to neither matter nor energy, a living organism (including humans) is an open system” (Crabbé 1992, p.1).

⁵These comments result from discussions with Professor Crabbé

⁶Ibid.

⁷Ibid.

Open systems are either in steady-state or not.⁸ In steady state, they minimize entropy by importing maximum negentropy.⁹ Open systems far-from-equilibrium are subject to a restated Second Law of Thermodynamics, found in Schneider and Kay (1994).¹⁰ Far-from-equilibrium systems are discussed further, in this section. They cannot be linearised, and when non-linear, they may exhibit chaotic behaviour.¹¹ Amir (1991) believes that many biological and economic systems are semipermeable: They are surrounded by boundaries that are selective in exchanging certain forms of matter. So the question arises: what type of a system is the economic system? The aforementioned definitions, of isolated, closed and open systems are summarized in Table 1.

⁸Ibid.

⁹Ibid. Negentropy can be defined in the following statement: some organisms consume *negentropy* from their surroundings; which is meant to say that organisms, in a system which has decreasing levels of entropy, ingest low entropy foods and excrete high entropy wastes (Proops 1985).

¹⁰Ibid.

¹¹Ibid. Chaotic behaviour implies a "sensitive dependence on initial conditions" (Gregori et al. 1984, p.261), which means that changing the initial conditions of a system by a *very small* amount can cause the system to diverge exponentially. In other words, a system moving through time may seem stable but at sometime, diverge exponentially.

Table 1

		Matter	Energy
Universe ...	Isolated System	No Exchange	No Exchange
Earth ...	Closed System	No Exchange	Exchange
Living Organism ...	Open System	Exchange	Exchange
Economic System ...	?	?	?

Note: The Universe being unbounded cannot be taken strictly as an isolated system.

Note that, in Table 1, the state of the economic system is unknown. Amir (1991, p.4) believes that “most economic systems, starting from the individual consumer and producer and culminating in the world economy, are open systems in the thermodynamic sense”. His belief is that this openness of the system is why several economists (Ayres & Kneese 1969; Kneese et al. 1970; Converse 1971; Noll & Trijonis 1971), studied “the economy as a thermodynamically closed system¹² by requiring it to obey a mass balance constraint”. Yet, Ayres and Nair (1984, p.64) state that the “economic system cannot be a closed system¹³,...unless one includes within the system the global environment and the Sun itself”. They go on to say that “even if one does consider such a large closed system¹⁴ it cannot be in thermodynamic equilibrium: A closed system¹⁵ in thermodynamic equilibrium is necessarily passive and inert, without flows of matter or energy.” Furthermore, “the flow

¹²Strictly speaking, this is an isolated system. This comment results from discussions with Professor Crabbé.

¹³Ibid.

¹⁴Ibid.

¹⁵Ibid.

of matter and energy through the system, in turn, precludes the existence of an economic equilibrium except in the special case of zero growth". Ayres and Nair (1984, p.64) conclude by stating that "it is clearly more realistic to regard the economy, like the Earth itself, as an open system through which materials and energy continuously flow". Ayres and Nair's view of the Earth as an open system differs from Umaña (in Daly and Umaña 1981). Umaña believes that the Earth is a closed system, and he goes on to say that "open systems are important because all living organisms and economies exchange matter and energy with the environment" (in Daly and Umaña 1981, p.32). Likewise, Boulding and Georgescu-Roegen also say that the economic system is an open system near equilibrium. From all this, what is certain is that the economic system is not an isolated system (Crabbé 1992, p.2).

The applicability of Thermodynamic Laws for the above mentioned systems are summarized in Table 2. This information can be found, albeit in a non-tabular form, in Crabbé (1992).

Table 2: Systems Near Thermodynamic Equilibrium

	Isolated System	Open System	Closed System
Laws of Equilibrium Thermodynamics ...	applies		
Conservation Law for Matter ...	applies	does not apply	applies
Conservation Law of Energy (1st Law of Thermodynamics ...)	applies	does not apply	does not apply
Maximum Entropy Law Second Law of Thermodynamics ...	applies		
Entropy Concept ...	applies	applies	applies

As previously noted, the First Law of Thermodynamics deals with the conservation of energy, and it states that *energy can neither be created nor destroyed—only transformed*. Thus, we can say that the energy of the universe¹⁶ is constant (Daly and Umaña 1981). This notion of limited energy, ties scarcity¹⁷ and the production process in economics to the First Law of Thermodynamics. This relation is due to the observation that *inputs* in a production process are limited to the availability of those resources. So it is not in our power to control or create these resources, just transform them through a production, or some other, process.

Mathematically, the First Law of Thermodynamics can be stated as:

$$\Delta Q = \Delta U + \Delta W$$

¹⁶Note again that the universe is unbounded.

¹⁷Scarcity results from the lack of substitutability for some forms of natural capital. From discussions with Professor Crabbé.

where ΔQ is heat (energy) supplied to the system, ΔU is the change in internal energy of the system, and ΔW is the work done by the system (Blatt 1983). ΔW and ΔQ “may be either positive or negative”, and ΔU may be “positive for some processes and negative for others” (Sears, Zemansky and Young 1985, p.357). Thus $\Delta Q > 0$ is heat received and $\Delta Q < 0$ is heat released, while $\Delta W > 0$ is work received and $\Delta W < 0$ is work performed (Burness et al. 1980). The above equation can be read as follows: “when heat(energy) is added to a system, some of the energy remains within the system, increasing its internal energy by amount ΔU while the remainder leaves the system as the system does work against its surroundings” (Sears, Zemansky and Young 1985, p.356).

Jørgensen (1992, p.97) uses a slightly different definition of the First Law of Thermodynamics, applied to ecosystems, he states *energy and matter are neither created nor destroyed*. He qualifies that the expression ‘energy and matter’ are used, since energy can be transformed into matter and matter into energy. He further states that the unification of the two concepts is possible by the use of Einstein’s law: $E = mc^2$, where E is energy, m mass and c the velocity of electromagnetic radiation in a vacuum ($= 3 \cdot 10^8$ m/sec). Jørgensen (1992, p.102) also states that “the transformation of solar energy to chemical energy by plants (photosynthesis) conforms with the First Law of Thermodynamics: solar energy assimilated by plants = chemical energy of plant tissue + heat energy of respiration”.

The Second Law of Thermodynamics deals with entropy. It states that *the entropy of the universe is always increasing* (Daly and Umaña 1981). Entropy can be loosely defined as the measure of the ‘mixed-upness’ of a system (Faber and Proops 1990, p.76). Commoner (1976, p.23) states that “the Second Law of Thermodynamics binds together a very fundamental body of

knowledge-about the spontaneity and irreversibility of natural processes, the degree of order and disorder in the universe, and the meaning of probability and information". Thus, the "central assertion of the Second Law is that the spontaneous processes that are the actual events of the real world always lead to states that are less ordered, more probable, and represent less information than the states in which they began". According to Commoner, the second law tells us that this process can be reversed, with the input of energy, but again this leads to the paradox mentioned before. This reversal is only done "at the expense of further decay in the overall order of the world" (Commoner 1976, p.23). In Blatt (1983, p.285) the Second Law of Thermodynamics is given in three formulations.

First, heat does not, of itself, flow from a cooler to a hotter body. Second, it is impossible to take heat from a reservoir and convert it completely to work if no other changes are taking place in the system or its environment. Third, in any process taking place in an isolated system, the entropy of the system either remains fixed (a reversible process) or increases. The entropy of an isolated system and of the entire universe tends toward a maximum.

The *first* implication of the entropy law is that the entropy of the universe is increasing, (this is also known as the *First arrow of time*¹⁸, i.e., time flows in one direction only, which implies it is irreversible; it describes how systems have a tendency over time to disorder (Faber and Proops 1990)). This can be demonstrated by two experiments. Number one: using two identical containers that are joined by a stopper, one filled with inert gas and the other a vacuum, when the stopper is released the gas expands to fill

¹⁸Crabbé (1992) notes that the First Law of Thermodynamics holds irrespective of time.

the available volume, and subsequently entropy has increased. Number two: when an ice cube is placed in a glass of water, the ice cube melts and cools the entire glass and this also is an increase of entropy. As a result, the entropy of an isolated system (as opposed to an open system, which exchanges heat and energy with its surroundings) must also be increasing (Daly and Umaña 1981).

From Jørgensen (1992), the entropy law can also be expressed mathematically in the following formula:

$$dS = \frac{\partial Q}{T},$$

where S is entropy, ∂Q is an incremental infinitesimal heat flow, and T is the absolute temperature of the system (Proops 1987). Entropy can be expressed as follows: $S = k \ln W$, “where k is the Boltzmann’s constant¹⁹ and W is the number of microstates that make up the macrostate of entropy S ” (Blatt 1983, p.293). In an isolated system, entropy has the following property:

$$dS \geq 0,$$

where “>” refers to non-equilibrium processes, and “=” refers to equilibrium processes (Jørgensen 1992).

Secondly, entropy is also a measure of the *unavailable* energy of a system (Ruth 1993). Unavailable energy is energy that is bound, and bound energy is energy that can not be used to perform work. We can encapsulate the concept of bound energy in the following statement: If you use energy then that energy is no longer *re-usable* in the same form. Also, if energy is no longer *re-usable* in the same form then that energy has been used. Equivalently, we

¹⁹ $k = 1.38 \times 10^{-23}$ in joules per calvin energy units: converts S to energy equivalents.

can say that using energy is a necessary and sufficient condition for not re-using that energy in the same form.

Lastly, the Second Law of Thermodynamics is also what determines evolutionary time (Eddington's arrow of time, or the *Second arrow of time*). As noted earlier, the maximum entropy law is the First arrow of time, where isolated systems in equilibrium tend toward disorder; the Second arrow of time "reflects the tendency over time of certain systems towards a greater organization and complexity" (Faber and Proops 1990, p.76). The Second arrow of time applies to far-from-equilibrium systems, "these systems are open and are moved away from equilibrium by the fluxes of material and energy across their boundary" (Schneider and Kay 1994). This tendency towards organization in far-from-equilibrium systems has been extensively studied by Prigogine and his colleagues (Nicolis, Babloyantz and Stengers) as well as by Kay and Schneider.

Prigogine (1972, p.24) inquired about "the feasibility of extending the concept of order to nonequilibrium situations, to systems in which the appearance of ordered structures, in thermodynamic equilibrium, would be very unlikely." One of the main conclusions of his inquiry was that "there exists a class of systems showing two kinds of behaviour: a tendency to the state of maximum disorder from one type of situation, and coherent behaviour for a second type" (Ibid). He noted that the tendency towards maximum disorder always prevails in the neighbourhood of a thermodynamic equilibrium, and that the creation of order, or the tendency toward coherent behaviour, may occur far-from-equilibrium. "Traditionally, thermodynamics has dealt with the first type of behaviour, but an extension of irreversible thermodynamics that permits treating the other aspects as well as this one has been developed recently" (Ibid).

According to Jørgensen (1992, p.147), “Prigogine and his colleagues have shown that open systems that are exposed to an energy through-flow exhibit self-organization behaviour and are known as dissipative structures.” This is demonstrated in the following experiment described by Jørgensen (1992) and Faber and Proops (1990): when a temperature gradient is applied to a layer of water between two horizontal parallel plates, initially heat is transported through the liquid via conduction, but as the temperature gradient is increased to a critical level, the system becomes a dissipative structure, spontaneously generating macroscopic convection cells, which allow a more rapid transport of heat through the system. This is also known as a Bénard cell experiment. Prigogine (1972, p.25) believes that these “structures are created by the continuous flow of energy and matter from the outside world; their maintenance requires a critical distance from equilibrium, that is, a minimum level of dissipation.”

The work done by Kay and Schneider, has supported the findings of Prigogine and his co-workers. Kay (1989, p.1) notes that “living systems...have evolved in accordance with the second law so as to decrease the overall effect of incoming energy by increasing throughput and degrading the exergy²⁰ content of the mass and energy flow through the system.” He goes on to say that “this is the overall thermodynamic direction of evolution, to dissipate and degrade the energy flowing into the system, and this exergy reduction is accomplished via the development of highly organized structures”, which can join together to form what Kay calls “super systems”(Ibid.). Kay believes that “the cornerstone of the paradigm is to view living systems as the solutions to the thermodynamic problem of maximizing the degradation of the

²⁰Exergy is the qualitative energy content, i.e. maximum amount of work which can be extracted from a source of energy (Crabbé et al.).

incoming solar energy in a changing and sometimes unpredictable environment”(Ibid.). Schneider and Kay (1994), also conclude that the “emergence of the coherent self-organizing structures are the expected response of systems as they attempt to resist and dissipate the external gradients that are moving them away from equilibrium.” In fact, they go as far as restating the Second Law of Thermodynamics to account for this behaviour. Schneider and Kay use the work of Kestin (1976), in the Unified Principle of Thermodynamics, to derive the following restated Second Law of Thermodynamics:

“The thermodynamic principle which governs the behaviour of systems is that, as they are moved away from equilibrium, they will utilize all avenues as available to counter the applied gradients. As the applied gradients increase, so does the system’s ability to oppose further movement from equilibrium”(Schneider and Kay 1994, p.9).

Schneider and Kay claim that this restatement “allows for the discussion of system behaviour in nonequilibrium situations”, and it “overcomes the difficulty of describing nonequilibrium systems in terms of entropy, which can only be defined in the equilibrium state”(Ibid).

Therefore, far-from-equilibrium systems are systems under continuous energy and material flow, able to maintain their systems by developing complex internal structures. They are able to “pull energy and matter into themselves and export it in degraded forms to an external sink as a condition of maintaining a changing pattern of internal self-organization, which they continue to exhibit until and unless the flow of energy and matter falls below a crucial threshold”(Depew and Weber, p.333, in Weber, Depew and Smith eds. 1988).

The continued work on far-from-equilibrium thermodynamics and its application to ecosystems, may lead one to question our decision to study the work of Amir. The reason being Amir views the economic system as an open-near-equilibrium system, in steady-state, where entropy is minimized and negentropy is imported into the system. He may have chosen to do this because economic theory is in general an equilibrium theory. But more specifically, "Classical thermodynamics applies only to systems in...equilibrium and to systems that change so slowly that they may be regarded as making gradual transitions from one equilibrium state to a nearby equilibrium state" (Blatt 1983, p.292). Steady state is attained when the system's "intensive properties do not change with time (they may vary spatially), and no flows exist within the system" (Amir 1991, p.6). Amir also believes that intensive variables, such as prices, do not exist unless the system is in equilibrium (steady state). Amir states that even though the system is continuously disturbed by externalities, it is always able to return to equilibrium, as does the above described classical thermodynamic system. He further states that "during a quasistatic process the system is assumed to be passing through a succession of equilibria (or steady states)" (Amir 1989, p.214), again, as does the classical thermodynamic system.

Most of the work that has been done on far-from-equilibrium systems has been done on ecosystems. According to Jean and Auger (1993, p.3), "living systems are neither isolated nor close to equilibrium,...a living system must exchange exergy (closed systems) or both exergy and matter (open systems) over its boundary,...and these attributes of living systems are typical for all far-from-equilibrium dissipative systems". Depew and Weber (in Weber, Depew and Smith eds., p.348) believe that " an evolutionary theory based on nonequilibrium thermodynamics will advance the study of human

society considerably”, and thus economics as well. Also, Depew and Weber (Ibid.) believe that the development of “an ecologically, and hence thermodynamically based, economic science that is at home with irreversibility and complexity” will solve the problems of the lack of “predictive reliability of equilibrium models in economics”.²¹ Ayres and Nair (1984, p.71, taken from Nicolis and Prigogine, 1977) state that the “economic system as a whole is evidently a stable dissipative structure far from (thermodynamic) equilibrium, in much the same sense that living systems may be considered to be stable dissipative systems far from thermodynamic equilibrium”. Therefore, further study has to be done on the analogy between ecosystems and economic systems, before we can fully embrace economic systems as far-from-equilibrium systems.

Jørgensen (1992, p.123) makes the following observations about the Second Law of Thermodynamics and ecosystems, “in nature we can distinguish two processes: spontaneous processes, which occur naturally without an input of energy from outside, and non-spontaneous processes, which require an input of energy from outside. These facts are included in the second law of thermodynamics, which states that processes involving energy transformations will not occur spontaneously, unless a degradation of energy from a non-random to a random form occurs; or from a concentrated into a dispersed form. In other words, all energy transformations will involve energy of high quality being degraded to energy of lower quality.” This ‘quality’ of energy is measured by the thermodynamic state variable ‘entropy’ (men-

²¹How? is a good question. The authors are not very clear on this question. However, they elude to another study which examines British history in terms of energy flows, on the assumption that the economy is a dissipative system: examining cities as dissipative structures suggests the predictive role of an evolutionary and ecologically based social science (Depew and Weber).

tioned earlier). Also, it should be noted that when we say low entropy we are referring to high quality energy. According to Jørgensen (1992, p.124), “from a physical stand point the environmental crisis is an entropy crisis,²² as pollution creates disorder,” and in an attempt to maintain order, the more energy we require, the more stress (entropy) we put on the environment.

1.1 Interdisciplinary Research in Economics

Economics arose as a study of human behaviour: the behaviour of buying and selling goods. While natural sciences, such as physics and astronomy, study the material world around us (see section on Energy Prices). As these disciplines have evolved and information has become more readily available, there has been an increasing exchange of ideas between disciplines.

Economics has applied many ideas from the study of human behaviour to the field of rational expectations and firm behaviour, such as the predator-prey models. Faber and Proops (1990, p.218) have commented that “if economics is to make good progress, it must open itself to a greater extent to work in other disciplines, be it politics, law, sociology, psychology, physics, chemistry or biology”. This, according to them, being especially true for environmental economics.

Faber and Proops (1990, pp.222-223) discuss difficulties that arise in interdisciplinary research²³. They are as follows. First, interdisciplinary research may be regarded as less than serious work and often faces discouragement from peers. Second, the criteria which are used to evaluate research in the

²²Some may disagree on this point. But, what should be clear is that pollution increases entropy.

²³Ideally, interdisciplinarity means the development of a set of axioms common to several disciplines e.g., ecosystem health. (Comment results from discussions with Professor Crabbé).

already established and specialized fields are used for the interdisciplinary work. As a result, critiques are often harsher than in disciplinary work, and one is bound to make certain errors. One must be prepared for the critiques of one's own discipline but also that of others. Third, it is difficult to find those who want to and are willing to cooperate in interdisciplinary research. Fourth, difficulty arises in the language used; the same word in one discipline can mean something quite different in another. Fifth, it is difficult to get work published, since established fields have established journals and these journals may feel that interdisciplinary work is inappropriate for their readership. Lastly, difficulty arises from the fact that the researcher must be thoroughly familiar with more than one discipline. Many researchers spend a life time mastering their one chosen field, but, when one takes on interdisciplinary research, that researcher must now master the other also. But as N. Gregory Mankiw states (1996, p.16) "...broad interests give you more opportunities for success."

In conclusion, thermodynamics in economics should be pursued to a point where its benefit (opening new areas for research) outweighs its cost (i.e. erroneous modifications to existing models to incorporate thermodynamic laws) to economics. It is hoped that readers gain a deeper understanding of thermodynamics in economics and determine for themselves whether it is an area that deserves further exploration. While this paper does not claim to be a complete discussion of thermodynamics in economics, it, nonetheless, attempts to heighten awareness and stimulate discussion in the area.

The above introduction has touched upon the technical and definitional sides of thermodynamics, while important, the purpose of this paper is to give one a working knowledge of thermodynamics in economics. Moreover, focus will be on the weaknesses and strengths of thermodynamics in eco-

nomics. Sections 2.0, 2.1 present a history of the energy concept. Section 3.0 presents a discussion on energy valuation. Sections 4.0, 4.1, and 4.2, discuss the concerns of incorporating thermodynamics in economics, and how thermodynamics has been applied to economics. We then present a model developed by Shmuel Amir (1991), which demonstrates the importance of thermodynamics in economics. Section 5.0, discusses the synthesis of Amir's objectives, and section 6.0 presents Amir's model. Section 6.1 criticizes his model, and Section 6.2 discusses possible changes to his model. Finally, Section 7.0 presents a summary and conclusion that will end this paper.

2.0 History of Energy

The notion of energy will seem familiar to many. Indeed, people use energy on a day-to-day basis. Thus the relevance of energy to our life needs no explanation. What needs explanation is to understand what energy is from a historical point of view.

Energy concepts are a commonplace in the natural sciences, mainly physics. It is in physics that energy takes its form as a workable theoretical concept. The word energy comes from the Greek word *energia* (Mirowski 1989). There are several notions of energy: it can be thought of as activity; or, as a way of transforming the potential (work) into the actual (work). If these notions seem metaphysical or philosophical, it is because they are. Aristotle used the notion of energy in his *Ethics* (Ibid.), along with other philosophers. Thus energy concepts have been around for some time. The conservation principle is one aspect that is intertwined with the meaning of energy. The conservation principle by definition "is the rule that some particular aspect of a phenomenon remains invariant or unaltered while the greater

phenomenon undergoes certain specified transformations” (Mirowski, p.13). In other words, components of an activity need not be altered as the activity undergoes changes. Energy can also be thought of as the process by which work is done (Mirowski). Work has a precise mathematical definition which is (Anton 1995, p.338)

$$Work = Force \times Distance.$$

The precise meaning given to work in the early physics is of *lifting a known weight through a certain distance*. The concept of energy conservation was apparent in the idea of force. Mathematically force is mass times acceleration. Upon impact, force given up by one body was gained by another (Mirowski). The body that gained the force, *conserved* it. One objection to the conservation of living force was that many physical changes seemed irreversible (Mirowski, p.20). This meant that when force is transferred by an object or activity, does the object or activity, that receives it, have equal amount of force as the object or activity expending it? Much debate and research over this question remained (Mirowski). As a result, mathematical techniques that linked variational principles (as in Calculus) to conservation principles were developed (Mirowski, p.22). Techniques of finding maximum and minimum values of functions were used to explain systems undergoing change; these techniques were rooted in the science of dynamics (Mirowski, p.23).

The concept of work was further demonstrated by Sadi Carnot, who is credited with being the founder of thermodynamics (Mirowski, p.25). Carnot set out to show that energy moved from a warmer body to a colder body. Specifically, he concluded that “Only the existence of a difference of temperature allows the production of ‘moving power’ and the ideal efficiency of

a machine employing heat to produce power depends solely on the temperature differential between source and sink, and not upon the nature of the working substance heated or cooled" (in Mirowski, p.25). Apart from the remarkable nature of this result, it set forth the idea that perpetual motion was attainable. The impossibility of this idea, perpetual motion, did not convince everyone (Mirowski).

The people who made clear the notion of the conservation of energy were Julius Mayer, James Joule, Hermann von Helmholtz, and Ludwig Colding. The most important of the four is Helmholtz; it is his work that will be briefly analysed. In one of his important works, Helmholtz makes the statement that "...it is impossible to create force continually out of nothing" (in Mirowski, p.46). Moreover, he claims that the impossibility of perpetual motion is abstractly equal to the conservation of living force (Mirowski). In his beliefs, "energy, first intended to reduce all life to mechanism, later seemed to undermine the existence of an independent inert matter" (in Mirowski, p.49). This shift in belief raised additional doubts concerning the notion of energy.

The conservation of energy raises four points (Mirowski, p.50): 1) developing a concept of energy; 2) the claim that there was energy "out there" and "in here" waiting to be found; 3) the statement that energy is neither created nor destroyed; and 4) some procedure of justification of ideas 2) and 3). The concept of energy was still unclear: force (mass times acceleration) had to be differentiated from energy (Mirowski, p.51). The definition of irreversibility, among others, needed to be clarified. These notions were made clearer by Helmholtz. Mirowski colorfully states that "...Helmholtz presided over the wedding of the variational principle and the conservation principle, which marks the beginning of the energy revolution in physics" (Mirowski,

p.52).

The energy concept was further refined by Max Planck who held close the fact that energy must be a changing state. His belief is best read in his statement (in Mirowski, p.56)

“[A]n energy is a quantity which depends only on the instantaneous state of the system, not on the manner in which the system reached this state or on the manner in which it later changes its state. The whole importance of the concept of energy rests on this property; without it the principle of the conservation of energy would be illusionary.”

Thus energy depends on the state of system at a point in time. Energy does not depend on the nature of this system through time.

2.1 Entropy

Entropy has been defined above in this paper. The whole idea of entropy however remains elusive. Introduced by Rudolph Clausius in 1865 (Mirowski), entropy had the properties of a variable state in a reversible system (just like energy) (Mirowski, p.61). This led Clausius to found the science of thermodynamics on two laws (Ibid.): 1) the universe held constant energy; and 2) that the entropy of the universe is bounded. Classical thermodynamics did present some problems. Mainly, that when a system sets out by initial conditions, it is unable to return to it. In that, it cannot trace back to the same place where it started from, in other words, the system forgets its initial conditions. Furthermore, classical thermodynamics also cannot tell you how fast it will take to get where you want to go (Mirowski, p.61). The reason for this is that entropy is not really a conserved quantity like energy (Ibid.).

Entropy conforms to an “as-if” conservation which requires the system to be reversible (Mirowski). And so, only in systems that are fully reversible can the system trace back its path to the initial conditions, and thus, entropy, can be maximized. The second law was reinterpreted to mean time irreversibility. This *constraint* brought new notions into physical theory such as probability and random behaviour. Boltzman however suggested *fluctuations* in the evolution of entropy—in that sometimes time could reverse itself—but that these fluctuations were faint (Mirowski, p.65).

Entropy concept remains a *subset* of the energy concept. However, this is not to say that the former is less important than the latter, rather, the energy concept has been around longer. The abstract nature of entropy still needs to be clarified, and through this clarification we may learn more about what energy entails.

3.0 Energy Valuation

The discussion in this section will touch on issues concerned with ways energy is valued. Energy enters almost all economic processes at the consumer and production levels. Thus it becomes relevant to understand how to value it. To value energy we must understand how to measure it, and what units to use. There are several: *joule*, *Calorie*²⁴, BTU (British Thermal Units), etc. However, Calorie and the joule are the most common. Heat is one form of energy which allows for the conversion of all other forms of energy (Odum & Odum 1981, p.18). Therefore, the measure of heat, the Calorie, allows us to quantify the amount of energy: One calorie is equivalent to 4186 joules (Ibid.). The amount of heat that is expended during some process is difficult

²⁴The word Calorie means a kilocalorie *if* spelled with a capital C, and is equal to 1000 small calories (Odum & Odum, p.27).

to harness because the molecular motions are highly nonlinear (Ibid.). Heat is degraded energy which cannot do any work if all processes are at the same temperature. What this means is that heat remains constant and cannot *flow*. So that if temperature differed, the nonlinear motions are able to spread themselves between the higher and lower temperatures (Ibid.); in this situation, heat is able to flow.

Energy has a low- and a high-quality: Heat is low quality energy, while gasoline, dynamite, and high-voltage are high-quality. To convert from a low quality energy to a high quality, there must be a degradation of a considerable part of the energy (Odum & Odum, p.25). For example, four Calories of coal are required for a Calorie of household electricity; 1000 Calories of sunlight are required to make up one Calorie of wood (Ibid.)²⁵.

If energies differ in quality then they differ in their ability to do work (Ibid.). One calorie of dispersed heat is useless. Whereas, highly concentrated energy can do much work. This simple difference in the quality of energy is the basis for valuing energy.

Efficiency of energy is the ratio of energy flows: desired energy output to all of energy input (Ibid.). The efficiency of conversion of Calories from low- to high-quality is a measure of the usefulness of the higher-quality type of energy (Ibid.). For example, if 1 Calorie of electricity is generated for every 3.9 Calories of coal burned in a power plant, the conversion efficiency is 25.6 per cent. This efficiency is important because it tells us how well processes are working to generate energy. An efficiency comparison, between processes that generate energy, may isolate the best process.

As mentioned above, the law of conservation of energy, says that energy

²⁵These conversion factors include the energy cost of running machinery required by the conversion process (Odum & Odum, p.25).

is neither created nor destroyed (Odum & Odum, p.29). In that energy enters a system from several sources, and is accounted for as degraded heat energy (Ibid.). The second law is the law of degradation of energy: In all processes that use energy, some of the energy loses its ability to do work (Odum & Odum). According to this second law, the amount of high grade-energy leaving a process is always less than the energy going into it (Odum & Odum). Any process that uses energy degrades some this energy to build high-quality energy (Odum & Odum). In this way, the second law becomes apparent. A third principle is the maximum-power principle: A process that gets most energy and uses it in an efficient manner is able to defeat all other competing processes (Odum & Odum). This is an obvious result, and highlights the fact that having the most energy is not enough, it must be used in a way that gets most out of it.

The link between energy and money is believed to be strong. Energy is an input into the economic system and therefore must be accounted for in a process that analyses the economic system. Money has a circular flow, while energy flows in a linear way through a system: energy flows through a system and out as (degraded) heat (Odum & Odum). Thus, "We must understand something about money and energy and their relationship in order to understand the economic system and the way energy affects it" (Odum & Odum, p.41). Energy and money flow in opposite directions (Ibid.). It is through work the gives way to energy use. For example, economic transactions such as business activity, paperwork, banking, etc. involve work and so involve energy which allows work to be done. Human activity is ultimately dependent on the world's energy (Odum & Odum). "It is, therefore, a mistake to measure everything by money. Instead, we should use energy as the measure, since only in this way can we account for the contribution of nature" (Odum

& Odum, p.42). Money flows in a cycle and is heavily dependent on material cycles and energy flows (Odum & Odum). Therefore, if energy flows were to slow down, then the economic system (which is dependent on money) also slows down. Note however, that energy flows do not require money to flow. Since by the first law, energy is neither created nor destroyed and so remains in the system so long as the sun is shining, literally.

Money can circulate only if energy flows through the system to support the work money buys (Odum & Odum). At any time, there is an average ratio of energy flow to money. This is represented as the ratio of the total Calories in the system to the overall money flow (Gross National Product (GNP)). For example, the GNP in 1980 for the US was \$2.5 trillion, and the energy flow was 28×10^{15} Calories, thus the ratio of energy flow to a dollar flow, in that year, was 11,000 Calories per dollar. Inflation is inversely related to this ratio: with high inflation the ratio falls, with low inflation the ratio rises (Odum & Odum).

Capital assets are defined to be internal storages (in the system) of structure that include buildings, people, food stocks, information, culture, education, memories and things that are regarded to be useful and valuable and subject to depreciation (Odum & Odum, p.52). It is from capital assets that old activities can be continued and new activities started by pumping in more energy (Odum & Odum). So generating more new activities adds to the capital assets which allow more energy to be pumped in and on it goes in a cycle.

Prices in the economic system fail to recognize the valuable contribution the environment makes (Odum & Odum). It fails to indicate how much the wealth of the economy is due to external inflows. "When the inflow from the environment is greatest, contributing most to the economy, the price may be

least, since the source is so rich that little is fed back to process it" (Odum & Odum, p.54): Thus excessive supply reduces its economic price. Only through energy evaluation can the real value of the external input be determined (Odum & Odum). For example, the dollar effect of an input should use the ratio of energy flow to a dollar flow to determine the proportionate dollar effect this input will have (Odum & Odum). The price of oyster and fish from an estuary before accounting for the value of energy is about \$20, but if the full value of energy is taken into consideration the price jumps to \$272 (Odum & Odum, pp.54-55). Therefore, "Using prices to evaluate contribution of an externality to the economy is incorrect" (Odum & Odum, p.55).

4.0 The Importance of Thermodynamics in Economics

The questions to be analysed in this section are why should we be concerned about thermodynamics in economics? What advantages and disadvantages does thermodynamics, in economics, present to modeling both economic behaviour and policy? All these questions will be answered, either directly or indirectly, in this section.

Economics, we can say, deals with the physical world around us and since the physical world is under the domain of the thermodynamic laws, we must understand and incorporate that knowledge. Thus, if we are to correctly model the real economy, we must understand the physical underpinnings of economic actions. This will also help us (society) understand the real costs of our actions. We are assuming that the present models used in economics do not explicitly incorporate the thermodynamic laws.

The thermodynamic laws indirectly give us the scarcity constraint and the recycling constraint. Specifically, the first law says matter and available energy stocks are limited (scarcity constraint), but does not make explicit how many times they can be re-used. Therefore the first law holds irrespective of time and gives us a sense of reversibility. The second law says there is a limit to the number of times matter and energy can be re-used (recycling constraint). Thus giving us the sense of irreversibility. Recall that once energy is spent it can not be recaptured and re-used in the same form as before. Therefore, the amount of materials that we recycle can never be 100 percent of its original form. Thus, we must decide carefully what we use up, and what we leave for future generations—assuming the system is isolated.

So, assuming that economic models have failed to incorporate the laws of thermodynamics (constraints), as some authors believe they have (Amir 1991; Ruth 1993; Goergescu-Roegen 1971, 1979), then, it is believed, their integration into economics will be advantageous in aiding economists to make better decisions, and to better understand the world around us. The main disadvantage is that economists are not as familiar with thermodynamics, as say physicists. This presents a challenge, for economists, in finding the correct way to incorporate the thermodynamic laws into economic models and decision making; and at the same time ensuring that the properties of the models do not contradict established economic theory.

4.1 Incorporating Thermodynamics in Economics

The application of thermodynamics in economics becomes particularly relevant in areas of resource use. The inputs into economic processes are matter

and energy (Georgescu-Roegen 1979), and so their presence should be explicitly incorporated into economic models (Burness et al. 1980, p.1). The issue at hand is whether market solutions ignore physical laws, and if so, what damage does that cause? The entropy law, which provides an upper bound on work obtained in any transformation process that uses energy, is ignored by market solutions according to Georgescu-Roegen, 1971, Burness et al., 1980, and Young, 1991. While there is agreement on this, distance still remains between market solutions that ignore this law, and the implications that result from ignoring thermodynamic laws. Burness et al. state that it is in the concept of *value* i.e., determining appropriate prices for resources for efficient allocation, that brings forth the need to consider thermodynamic concepts in resource use. However, they state that “it is simply not clear as to how thermodynamic concepts—the entropy and energy ‘theories’ of value—are to be used in enriching the promulgation of public policy”. What is known is that economic growth is not limited by the technical process of production, but by the physical and ecological laws (Cleveland 1987). Boulding (1970) also makes the point that the presence many of the immediate problems of pollution of the atmosphere or of bodies of water are due to the failure of the price system to allocate and value resources correctly. He further states that many of the present environmental problems can be solved through the proper use of taxes. This begs the question of whether incorporating thermodynamic laws in economic models can lead to valuing resources more accurately? The belief of some (Ruth 1993; Ayres and Kneese 1989) that this can be done is promising. Samuelson (1983) sheds light on thermodynamics in economics mainly through dealing with equilibrium relations.

At the heart of the issue is that the entropy law absolutely restricts infinite use of resources that can't be circumvented by technological change, and

further, by exploration or substitution (Young 1991). The effect of this on any measure of value of a resource becomes relevant in this context. The explicit realization of the entropy law in economic models has both supporters and detractors. The supporters are several (Georgescu-Roegen, Burness et al., Daly 1991), some detractors make strong statements such as “the entropy law does not add anything which is not already considered in economic models of long-run economic growth in relation to the availability of environmental resources” (Young 1991, p.169). Young (1991) has been criticized heavily by Crabbé (1992, also by Daly 1991), mainly on (accepted) definitional grounds. While there is still on going debate as to the relevance of thermodynamics in economics, what we know for sure is that energy is involved, in some way or form, in an economic process.

A concrete example of the relationship between thermodynamics in economics can be found in Daly (1985) and is summarized in Cleveland (1987). Daly criticizes the flows of goods and services from firms to households that we call national product; and the reverse flow of labour services from homes to firms. The *circular* nature of this flow is criticized. Specifically, Daly states that the circular nature of this economic flow is implicitly conjoined with a physical flow of matter and energy. Since goods and services are borne in nature, and since the matter-energy flow is linear: beginning with the depletion of the resource at the beginning of the economic process and ending, ultimately, with the waste of heat and by products from that process; the environment becomes *both* the source (low entropy) and sink (pollution-high entropy) of the economic process (Cleveland, p.63). Therefore, the circular flow diagram, so fundamental in elementary economics, in Daly’s view, should be extended to account for this physical flow process.

The notion of time, in an economic process has thermodynamic relevance.

At issue is the nature of time: whether time is reversible or irreversible. Reversible time implies that time is bi-directional, and thus both the future and the past are treated symmetrically (Faber and Proops 1990). The more important notion of time irreversibility raises some important concerns for economic analysis. Time irreversibility implies that past events are known with certainty, and future events are ambiguous. Adaptive expectations and rational expectations, are two areas of theory that presuppose the time irreversibility of economic processes; in that economists try to predict the future using past data or particular case-specific information. The irreversibility of time can be understood through the “Two Arrows of Time” concept (Ibid.). This concept says that the “First Arrow of Time” refers to the tendency of systems over time to disorder (which is encapsulated in the Second Law of Thermodynamics (see Georgescu-Roegen 1971)); while the “Second Arrow of Time” reflects the tendency over time of systems towards greater efficiency and complexity (Faber and Proops 1990). The production process is time irreversible, which means that while outputs can be produced from inputs, inputs *cannot* be produced from outputs (Koopmans 1951)²⁶ This an example of the Second Law of Thermodynamics at work in the production process. But, while there is the presence of the second Law in the production process, its recognition in models of production behaviour is not explicit (Faber and Proops 1990).

Thermodynamics can be used to steer economic processes towards higher efficiency in material use and energy transformations (Ruth 1993). Ruth discusses the role of thermodynamics in the production process. Specifically,

²⁶One immediate example is that capital (K) and labour (L) inputs used to produced output (Y) can not, in turn, be produced from Y. For example, let $f : (K, L) \subset \mathfrak{R}^2 \rightarrow Y \subset \mathfrak{R}$ then $Y = f(K, L)$, but it is *not* true that $f^{-1}(Y) = (K, L)$.

the laws of thermodynamics have implications on the production process. Ayres and Nair (1984) claim that exergy of the total output of a particular economic sector, must be less than the exergy of the inputs. And at each stage of the production process, the intrinsic nature of the inputs change, and the entropy level increases through the release of heat and waste from the physical production process. As such, economic models of production do not account for this. Moreover, since production processes are carried out in a closed interval of time, assuming output is fixed before the production processes begin, in this case, finite-time thermodynamics should be applied (Ruth 1993). The objectives of finite-time thermodynamics are to evaluate trade-off's between the speed of a processes and energy transformation (Ruth 1993), subject to constraints on the rate at which process are performed. Along this line of thinking, the First Law of Thermodynamics implies a materials balance constraint on the production process (Daly 1987). In that, matter-energy can neither be created nor destroyed, only transformed (Ibid.); and since the production process is a transformation of inputs into outputs, the materials content of the unit output can never exceed the material input. For this reason, the Cobb-Douglas type production functions are invalid for analysing economy-environment interactions because they allow for the unbounded substitution between capital or labour for resources (Ibid.) ²⁷.

The concept of entropy, as posited in the Second Law of Thermodynamics, remains most relevant for economic analysis. It becomes, therefore, crucial for one to understand what applications this law has in economics. Proops (1987) gives a nice summary of the uses of the entropy measure, and its

²⁷We are not saying that the Cobb-Douglas function implies perfect substitutes, rather in the case of a materials balance constraint, the Cobb-Douglas function allows for unbounded substitution between capital or labour for resources because the marginal rate of substitution (between the inputs) is infinite in this case.

many interpretations in the social sciences. In economics, the concept of the entropy measure is most applicable in issues concerning concentration of manufacturing; while in environmental studies, it is a measure of the increase in degradation of the environment (Proops, p.225). Economic studies have used the entropy measure to measure the change in the concentration of manufacturing in the brewing industry (Horowitz and Horowitz 1967, 1970) and the change in income inequality (Theil 1967). Entropy measure has also been used to investigate the functioning of the anti-trust legislation in the US (Finkelstein and Friedburg 1966). Further studies have also been done in integrating entropy measure in spatial economics (Semple and Gauthier 1972; Semple 1973; Semple and Demko 1977).

Ayres and Kneese (1989) claim, as does Georgescu-Roegen (1971), that the real economy is a dissipative system, not a self-sustaining one. Ayres and Kneese (1989, p.91) further claim that the use of energy and matter by the economic system is similar, albeit partially, to an ecosystem. They assure us that thermodynamic concepts can be integrated into economics to yield insights into optimality conditions for resource and environmental management. The most important economic implication of the Second Law of Thermodynamics – given the fact that the economic system is open, dissipative, dependent on external sources for energy, raw materials and environmental resources – is the need for ever changing technology (Ayres and Kneese 1989, p.108).

This section has discussed the most relevant issues concerning thermodynamics in economics. The reasons to integrate one with the other are, I feel, compelling. As one engineer states in Koopmans (1979, p.12), “Economics is the thermodynamics of the Social Sciences”. While a bit strong, this statement does clearly indicate the belief that integrating thermodynamics

in economics should be a real concern. Whether this means reformulating old models, or developing new ones is open for debate. Either way, it presents exciting ground for research.

4.2 Survey of Thermodynamic Production Functions

Thermodynamic production functions all have a commonality: they take into consideration nature's constraints. The extensions on thermodynamic production functions stem from their *awareness* of the environment around them, and of natural laws. The Bergstrom production function (Crabbé et al.)

$$Q = R[1 - \exp(-F(K, L)/R)]$$

where Q is output, R is an exhaustible resource, K is capital, and L is labour, satisfies the first law of thermodynamics (conservation of mass (Gross et al. 1990, p.78)). The Cobb-Douglas production would also satisfy the first law if at least one of the factors were "resource wasted in production" (Crabbé et al., p.6). This would lead to the elasticity of substitution, for the static Cobb-Douglas function between capital and the resource, to be less than one. However, with an exhaustible resource as an input, as in the case of a Bergstrom production function, resources are essential for the production process to take place; this is the case of explicitly incorporating materials-balance (which says that material goods can not be created without natural resources) in the production process. As Gross et al. state, without resources no production can occur, but if capital and labour are used in large amounts, the amount of natural resource input in the production process can be arbitrarily small. In this case, a constant level of consumption can be maintained

under three conditions (Gross et al., p.79): 1) a constant elasticity of substitution (CES) production function with the elasticity of substitution between capital and resources greater than one; 2) a Cobb-Douglas aggregate production function with the share of capital in the production process exceeding the share of resources; or 3) continuous resource augmenting technical progress.

Gross et al. (p.80) show a production function, they call a materials-balance Cobb-Douglas (MBCD), that incorporates materials-balance:

$$Q = K^{\alpha_1} L^{\alpha_2} (R - Q)^{\alpha_3} \quad \alpha_1 + \alpha_2 + \alpha_3 = 1 \quad (1)$$

which is a modified version of the Cobb-Douglas production function, where Q is output, K is capital, L is labour, R is resources and $X = R - Q$ is a non-negative variable representing the amount of excess resources or resources wasted in production. The above function incorporates waste into the production process. One question that came to my mind is: does minimizing waste necessarily lead to the efficient levels of capital and labour? Gross et al. answer this question partially and indirectly. Through their equation for waste, $X = R - Q$, they state that with little capital, resource waste is high. They fail to indicate the optimal level of capital that minimizes waste. Another difficulty I have with their statement is that waste, to some degree, is unobservable, while capital and labour are not. Therefore, for them to say that with little capital, resource waste is high, presupposes that waste can be *fully* measured in the production process. This seems unlikely to me, especially in a production process that operates on a very large scale. Also in their model, capital is a good substitute for resources only if the capital-resource ratio is low, but substitution becomes more difficult as the capital-resource ratio increases (Gross et al., p.82).

Another model, proposed by Anderson (1987), incorporates the concepts

of material processing, energy use, and waste. Anderson views the production process as that made up of two relationships: one is the materials requirement of the output; and two, is the ability of agents (labourers) to act upon that material to produce the desired amount of output (Anderson, p.3). Another relationship is the one between the agents (labour input) in the production process and the waste that is produced. Since waste comes about primarily from the incomplete utilization of material inputs (this may be an example of the relevance of my question of whether minimizing waste necessarily leads to efficient levels of capital and labour or material inputs), Anderson claims that in some cases, employing more capital and/or labour may reduce the amount of material waste.

The production relationship Anderson (p.3) defines when waste is zero is:

$$Q' = R/m$$

where Q' is maximum output, R is a material input, and m is mass per unit of output. The isoquants that represent this production plane is a constant proportion technology or a Leontief technology. When waste is not zero, the production function becomes (Anderson, p.5)

$$Q' = vR/m \tag{2}$$

where $0 \leq v \leq 1$ is the proportion of the mass of material input which is useable. If v is less than one, or a fraction, then less output will be produced than before from the same amount of material inputs. The isoquants from this production function resemble the Cobb-Douglas isoquants, or isoquants where the elasticity of substitution is one. A class of functions that Anderson favors to have the required properties of satisfying the mass balance constraint, and diminishing marginal productivities, are CES production functions.

While the above functions differ, both in meaning and functional form, from traditional production functions in economics, they are deemed to be representations of a production process that recognizes its effect on its surroundings. The incorporation of waste is one example of this. The authors show that their production functions are consistent with both economic theory, in terms of the first and second derivative tests on marginal productivities, and with the constraints imposed by natural law. It is unclear why these types of production functions have not entered the mainstream of production economics.

5.0 Synthesis of Amir's Objectives

The purpose of this section is to establish the evolutionary development of Amir's model of economy-environment interactions. The series of past papers that become relevant are Amir 1989, 1987, and 1979. Starting with 1979, Amir concerns himself with the concept of equilibrium in economics; as well as attempting to tie occurrences in the social and natural sciences with principles operating in nature. Amir argues that a Nash equilibrium is the most plausible equilibrium among various steady states (p.293). The thermodynamic and economic concepts of prices are integrated to determine the emergence of the intensive variables of the system, in an equilibrium state, as they result from the exchange relations taking place in the system. The relevance of this is to aid in developing policies that can manage the environment in the most efficient way and achieve optimal resource allocations. The definition of prices is quite different. Amir defines prices as the "expenses in terms of energy incurred during the exchange process in which the commodity is involved" (p.300). Thus prices are dependent on the sup-

ply and demand of those commodities in particular ecological markets. The ecological system analysed is thermodynamically closed. The price system based on the energy valuation will be proportional to the price system based on a dollar valuation. This means that the units in which these prices are measured in is irrelevant. As a result, the steady state movements of the system will be insensitive to the change in valuation units (p.312).

Steady states are important in the sense that they are the only states that possess prices that reflect the intensive variables of the system (Ibid.). The movements of the system are not valid in the long-run and the results of the model are heavily dependent on the steady state. As Amir believes, the real world is not constant but ever changing in an evolutionary fashion. Prices at which commodities are transacted are also fluctuating. For this reason, presenting change by a moving equilibrium is the relevant way to evaluate evolutionary motion analytically (Ibid.). Therefore, Amir makes the (strong) statement that conventional utility and production functions, and maximizing assumptions fail to describe individual behaviour in both the ecological or the economic areas (Ibid.). Evolutionary motions are the difference to any equilibrium in ecology and economics.

Amir's 1987 paper attempts to present an analogy between economic and ecological systems by treating their components as thermodynamically open systems, which face severe resource constraints. He uses a linear production model to discuss the allocation of scarce resources.²⁸ Efficiency prices are also derived from this model. These prices are state dependent and are the intensive properties of the system (Amir 1987, p.53). The dual nature of these prices is discussed in the sense that these prices not only represent the

²⁸The use of a linear production model is not a restriction on the analysis rather it is a method to simplify the (equilibrium) analysis.

value of scarce resources, but also play the role of energy prices. A model that is integrated with respect to ecological and economic activities can be used to calculate a single set of prices. In addition, he claims that the model can be used for analysing resource allocations and deriving systems of “shadow” prices. The whole idea of finding nature’s worth is key to Amir’s analysis, and the implications this *worth* presents to policy and decision makers for managing the environment. Amir’s belief in this paper that economics and ecology are analogous is readily apparent. And so the theories that explain these systems should, as he believes, also be analogous. Resource scarcity is an important motivation for this paper.

Integrating prices in ecological theory has been hampered by the mere difference between ecological and economic systems. In economic systems money and commodities flow in opposite directions (in most cases); while in natural systems, matter and energy flow unidirectionally (p.66). Another reason for not introducing prices into ecological theory is that it can handle optimization models without taking into consideration efficiency prices (p.67). As he defines it, efficiency prices reflect the notion of optimization subject to constraints (p.65). While shadow prices reflect the scarcity of the resources that otherwise would not be reflected, as precisely, by the market.

The paper develops a generalized production, linear activity type model, to consider approaches to resource allocation. The basic result is that the value of the stocks is equal to the value of the flows²⁹

$$\dot{p}K^* = u^*Y$$

where \dot{p} is the flow of the change in $W = W(K)$, which is linear homogeneous in K , with respect to K , K is the stock of capital, u 's are the shadow prices,

²⁹From the following equation the reader may recall Hotelling’s rule (discussed in the critiques), which Amir does not use.

and Y represents commodity flows. The feasible u 's must be proportional to the energy embodied in heat and light (p.78). What Amir claims in this paper is that energy is accounting money, and so its value to the economic process should be recognized and incorporated. The steady state prices in economic systems are the intensive variables (variables that are only present in an equilibrium or steady state) of the system (p.83). Moreover, prices, that allocate resources efficiently, are derived from behaviour that recognizes resource scarcity. These prices are termed as "efficient ecological prices". These ecological prices are deemed to better represent the problem of resource scarcity than market prices. Moreover, they reflect the "idea that the value of the product is already embodied in the value of whatever is required to make it" (p.93). The same can not be said of economic prices (Amir 1987).

The last paper is Amir 1989. This paper again continues with the theme of ecological prices. Its purpose is to look "for a theory that can offer quantitative indicators for *nature's worth*" (Amir 1989, p.204). While Crabbé et al. present a nice critique of this paper, I will only highlight the relevant points.

Amir attempts to define ecological prices and other system-wide indicators of the ecosystem and examines the limits to their applicability. The analogy between thermodynamics and economics is based on the fact that the systems concerned are open thermodynamically and face problems of resource allocations (Amir 1989, 1987, 1979). Amir shows that economic and ecological systems are insensitive to the methodology –economic or thermodynamic– used to study them (Amir 1989, p.204). After showing this he develops system-wide-indicators for *nature's worth* from an integrated system (one which combines the economy and the ecosystem as a single complex system). Pricing nature's value to Man is an important objective in this paper. The

“services rendered by any given ecosystem may be dear but they are not priceless” (p.204). Amir represents the value of nature to Man as a scalar value. This he derives from the estimation of entirely different costs, benefits, and their proper weights, as found in an integrated, system-wide and state-dependent indicators (p.204); the pricing of non-marketed resources is done in this realm.

The model used by Amir is again a generalized production, linear-activity-type model to which a nonlinear objective function is attached (p.206). The model derives the condition that the change in the value of the stocks is equal to the value of the flows. Two system wide indicators are found. The first is called ecological capital, i.e., the ecological value of a standing crop (p.211). And the second indicator is called the gross ecological product of steady-state ecosystems. Furthermore, it is found again that biological or economic behaviour that is optimal in viewing limiting resources gives rise to a set of prices that efficiently allocate resources (Amir 1989). Efficient ecological prices are central once again to this study. However, the problem with the objective of finding efficient ecological prices is that no one knows the functional form of the objective function of the ecosystem. Thus in economics where a explicit function is given, which allows us to find equilibrium quantities and prices; the task is not so easy when dealing with the environment. The ecosystem is dynamic and not static. Therefore, it can not aid in defining, unequivocally, the rate of exchange between productivity and viability (Amir 1989).

The general sense of Amir’s thinking is that economic theory and/or methodology is insensitive to the integration of thermodynamics. Meaning that economics and thermodynamics can be integrated in a way that is consistent with respective theories. And, through this integration, the true

costs and/or benefits of environmental policies can be evaluated through a set of ecologically efficient prices.³⁰ Valuing nature's worth to man through ecologically efficient prices aids in better understanding the true severity of problems such as natural resource exploitation. Amir's methodology is fairly consistent throughout, and is implemented in his 1991 paper.

6.0 Amir's Model

The model developed by Amir (1991) attempts to reconcile the two fields of thermodynamics and economics. He attempts to do this by pointing out the similarities in theory and methodology, between the two fields. For example, prices and quantities of commodities in economic systems are analogous to the intensive and extensive variables³¹ in a thermodynamic system (Amir 1991, pp.2-3). The use of optimization methods in economics, such as the Lagrange method, to determine optimal levels of consumption or production for rational economic agents, also has an analog in thermodynamics. A method of optimization in thermodynamics, developed by Hatsopoulos and Keenan (1965), assumes systems are changing up to a point when the (constrained) state function reaches a stable position. At the point of a stable position, the maximal or minimal values of the constrained function can be determined. Thus, as Amir claims, optimization methodology, implicit in the thermodynamic system, and explicit in the economic system, are similar.

Another objective of Amir is to see whether there are thermodynamic

³⁰Ecologically efficient prices are derived from the assumption that ecosystems allocate available resources efficiently among their components (Amir 1989).

³¹The intensive variables are: temperature, pressure, density, and prices. The extensive variables are: mass, volume, and the flows and stocks of different commodities. The extensive coordinates depend on the size of the system, while intensive coordinates characterise the state, rather than the size, of the system (Amir 1991, p.5).

laws that are neglected by neoclassical economics and “which must be incorporated into economic theory to generate effective policies of natural resource use and intertemporally efficient states of resource allocation” (p.3). Amir’s intent is to derive shadow prices for nature (i.e. ecologically efficient prices), and to show that thermodynamics does not hinder the allocation of resources in the economic sense. Thermodynamics, however, does constrain the rate at which resources are used.

To understand Amir’s model, we must understand its component parts. This can be done by going through and deriving each equation fully— which Amir does not do. While the derivations (see the Appendix) add to the understanding of the Amir model, it is best to explain his model in a qualitative fashion. The reader should, however, supplement the discussion to follow by referring to the Appendix.

The model presents an open system and represents any economic or ecological system starting from an individual agent (organism) and ending with the global economy (ecosystem) (Amir 1991). Thus the interlink between the individual agent as part of the ecosystem is established. Amir assumes that the system is linear and is represented by a generalized-production-, linear-activity- type model. There are m commodities, n subsystems, J activities, and he assumes J is strictly greater than n , and n is greater than m . Activities are a linear way of transforming commodities into other commodities. Subsystems are any part of the original *open* systems: individuals, firms, industries, economic sectors, national economies. The definition of the variables used by Amir can be seen in the Appendix, along with a discussion of their role in the model.

The operation of the system is dependent on net output, change in stocks, and commodity flows. Net output must be at least greater than or equal

to the change in stocks and commodities associated with the system. The wealth function used by Amir is a function of the stocks of the system and time. This function is linear homogeneous in the stocks. The wealth function ($W(K,t)$) is constrained by the fact that it cannot flow through the system boundaries because it is merely an accounting medium (Amir). Thus wealth may increase or decrease as the system passes from one state to another but its nature (in terms of representing value) is well defined. Profits are defined by taking the derivative of wealth with respect to the stocks (see equations (15)-(19) in the Appendix).³² Profits are defined for a steady state where the change in the stocks is equal to zero. The wealth function increases with the stocks and is to be maximized subject to the steady state constraint which says that the change in stocks (\dot{K}) equal zero. Thus to maximize the stocks one could simply maximize profits with respect to the stocks and set this equation equal to zero.

Amir defines a methodology for production to handle the problem of resource allocation constrained by thermodynamic laws. The production problem maximizes profits over the stocks of resources, change in resources plus commodity flows, and the level of an activity, subject to the condition that net output be greater than or equal to the change in stocks and commodities associated with the system; and that the total amount of resources be no greater than what is available. This problem can also be re-expressed in its dual form which minimizes the value of the commodities subject to the similar constraints as in the maximization problem. This dual expression also introduces shadow prices, u and v . u is equal to the change in the value

³²Note that the properties of this profit function satisfy the properties defined for an economic profit function (personal communication with prof. Crabbé) as shown in Varian (1992)

of the commodities given a slight change in the commodity flows; and v is equal to a change in the value of the commodities given a slight change in the stocks. Note that the stocks absorb the difference between net output and the commodities associated with the system, but in an optimal situation, the stocks will tend to zero, when this happens, the value of the stocks will equal the value of the flows.

The consumption problem is to maximize wealth subject to the value of the stocks being greater than or equal to the value of the flows. Wealth is maximized over the stocks. Re-expressed in its dual form, the solution to this minimization problem leads to the first order conditions which say that the change in the wealth with respect to the stocks be no greater than the product of the turnover time and the change in the systems wealth. The inverse of this turnover time can be interpreted as the discount rate (λ) (Amir). Two important features of the discount rate are: 1) it is not necessarily zero in a steady state; and 2) many discount rates may satisfy the conditions of the steady state (Ibid.) The latter feature determines how the system is configured in a steady state, this is possible only if the system is observable and only then can we specify a particular function. Also the reason why the discount rate need not, or rather, is not zero in a steady state because by its definition, $\beta = \frac{1}{\lambda}$ and we know $1 > 0$ and λ must be different from zero for β to be defined.

It is hoped that the above analysis of the Amir model sheds some light on the approach and methodology employed by Amir. The reader is encouraged to scan the appendix for any finer points that may clarify the above discussion.

6.1 Critiques of the Amir Model

It is not the intention of this section to target mathematical errors in Amir's model. Rather, it is best to see whether Amir's model and paper are relevant from a theoretical and applied perspective. And, to see whether the paper advances knowledge in the area of thermodynamics and economics. The advance of knowledge is relative, hence readers should recall section 2, of this paper, which discusses the link between thermodynamics and economics.

This section will present five critiques which try to point out both theoretical and applied difficulties in Amir's paper. As will be seen, Amir's paper falls short of actually showing how his model can be applied to a real world economic-environmental problem. I found it difficult to see how thermodynamics can actually aid in analysing real world problems and still be consistent with economic theory. Nonetheless, I believe that thermodynamics can be relevant for production and consumption problems. This is because thermodynamics puts a natural constraint on economic behaviour. How this natural constraint is imposed on production and consumption problems is demonstrated by several authors (Amir 1987, 1989, 1991; Ruth 1993, for production examples).

Critique 1: Value Condition

In Amir's 1987 paper, the value of stocks is equal to the value of the flows, mathematically it is defined as (Amir 1987, p.75)

$$\dot{p}K^* = u^*Y \quad (3)$$

In contrast, in his 1991 paper the same statement is used to define the fol-

lowing mathematical expression (Amir 1991, p.13)

$$\dot{p}K^* = -u^*Y \quad (4)$$

The above statement says that the value of the stocks is *negatively* related to the value of the flows. These two equations are inconsistent and add ambiguity to Amir's analysis.

Moreover, if we integrate Hartwick's rule³³ then eq.(4) should read

$$\dot{p}K^* = C - u^*Y$$

where C is a non-negative constant such that the left-hand side of the above equation is always non-negative, and uY is profit. At equilibrium, we must have an interior solution for uY . Specifically, \dot{p} , must be non-negative because it must grow with the rate of interest.

Critique 2: Indeterminate Path

The issue here concerns the cases that result from the maximization and minimization of the wealth function. These cases can be seen in the Appendix (following eq.(46)); they deal with the following intervals:

$$0 \leq p_i(K, t) \leq \lambda(K, M)p_i(K, M)$$

and

$$0 \geq p_i(K, t) \geq -\lambda(K, M)p_i(K, M)$$

for $i = 1, \dots, m$, and $\lambda \geq 0$. The cases result from the " \geq " and " \leq " weak inequalities.³⁴ Amir does not indicate which case is likely to result when

³³This rule says that optimal investment along an efficient time path is equal to the rents on resources (Common and Perrings, p.7).

³⁴For example, if a and b are any real numbers and $a \geq b$, we know then that $a > b$ or $a = b$; this statement is true when either $a > b$ is true, or $a = b$ is true, or both $a > b$ and $a = b$ are true.

$\lambda > 0$. He says “where $K_i = 0$ whenever the right hand side of equation (18) is satisfied as a strict inequality” (Amir 1991, p.16). But *when* will the right hand side of equation (18) be satisfied as a strict inequality? What situations help to classify when cases 1, 2, 3, or 4, will occur? Its important to know this because

$$p_i(K, t) < \lambda(K, M)\dot{p}_i(K, M)$$

or

$$p_i(K, t) = \lambda(K, M)\dot{p}_i(K, M)$$

and

$$p_i(K, t) > -\lambda(K, M)\dot{p}_i(K, M)$$

or

$$p_i(K, t) = -\lambda(K, M)\dot{p}_i(K, M)$$

have different implications.

$$p_i(K, t) < \lambda(K, M)\dot{p}_i(K, M)$$

says that the change in wealth with respect to the change in the systems stocks, $p_i(K, t)$, is strictly *less than* the flows of the change in wealth with respect to the change in the systems stocks, $\dot{p}_i(K, M)$, multiplied by the change in wealth with respect to the change in the value of the flows (λ). Note that since λ is a constant, its role is to simply change the magnitude of \dot{p}_i . So the magnitude (or length) of \dot{p}_i is dependent on $\lambda \in \mathfrak{R}^+ \cup \{0\}$. It is conceivable that the present value of \dot{p}_i , p_i , be less than $\lambda\dot{p}_i$ because maximizing wealth subject to $\dot{p}K \leq M$ implies that stocks can be less than or equal to $\frac{-u^*Y}{\dot{p}}$ i.e. the ratio of the value of the flows to the flows of marginal wealth with respect to stocks, when wealth is maximized. And so

the maximum value K can take is of course equal to

$$K = \frac{-u^*Y}{\dot{p}}$$

and thus if $p_i < \lambda \dot{p}_i$, in present value terms, K will still be less than or equal to $\frac{M}{p_i}$ because $p_i < \lambda \dot{p}_i$ implies that $\frac{M}{p_i} > \frac{M}{\dot{p}_i}$ for $\lambda = 1$.

The case when

$$0 \leq p_i(K, t) = \lambda(K, M)\dot{p}_i(K, M)$$

implies that the present value of the change in marginal wealth with respect to the change in stocks is equal to the flows. Thus

$$pK \leq M,$$

says that present wealth is less than or equal to the value of the flows. From eq.(72), in the appendix, we know that at the optimum $pK^* = (\frac{1}{\beta})\dot{p}K^* \leq M$, which says that wealth is the present value of future flows of $\dot{p}K^*$, which is less than or equal to the value of the flows, M , giving the constraint in the maximization problem. Since $p_i = \lambda \dot{p}_i$, and $\dot{p}_i > 0$, implies that

$$\lambda = \frac{p_i}{\dot{p}_i} \leq p_i(K, t) \tag{5}$$

but $\lambda = \frac{\partial W(k, t)}{\partial M}$ and so

$$\frac{\partial W(k, t)}{\partial M} = \frac{p_i}{\dot{p}_i} \tag{6}$$

which may be (more easily) satisfied given the assumption that the system is *linear*, than if the system is *nonlinear*. Because with a linear system we are moving through time without any (or very small) shocks hitting the system, whereas in a nonlinear system, there maybe shocks to the system causing abrupt changes to the (optimal) path, and as a result, it is not

entirely sure whether the magnitude of the direction of the right hand side will exactly equal left the hand side of eq.(6), in order for the equality to be satisfied. However, in a linear system, as Amir assumes, the possibility of eq.(6) being satisfied is higher due to the fact that in a linear system the “amplitude of fluctuations depends upon the initial displacement; no intrinsic amplitude –as between full employment and zero employment – is involved.” (Samuelson 1965, p.288). In other words, the optimal path of a linear system is *dependent* on the initial conditions, where as in a nonlinear system, this may not necessarily be the case because the system may start off from a non-optimal path and still be forced into an optimal path (Samuelson 1965).

We can apply the same arguments, as we did above, to situations when

$$p_i(K, t) > -\lambda(K, M)\dot{p}_i(K, M)$$

and

$$p_i(K, t) = -\lambda(K, M)\dot{p}_i(K, M)$$

resulting when wealth is minimized. Again, it is not sure what cases are likely to result and when. It would seem to me that the assumption of linear systems, as opposed to a nonlinear one, is very important for Amir’s results to go through. Unfortunately, the assumption of a linear system is rarely applicable to the real world and does not relate well to economics, where systems are highly nonlinear, unpredictable, and, frequently, out of equilibrium. Consider this quote from Blanchard and Fischer (1989, p.1)

“Although developed economies are characterized by growth, this growth is far from steady. Expansions and recessions alternate over time, associated with movements in unemployment.”

Economic systems are routinely hit by shocks which may cause changes in the (optimal) path of the system over time. One doesn’t need to look far for an

example, such as financial markets, international trade, etc., all contribute to shocks to economic systems through the changes in behaviour of participants.

An important point to note is that Common and Perrings state

$$\frac{\dot{p}}{p} = r - \frac{\dot{K}}{K}.$$

This is the asset equilibrium price condition: the rate of return on an asset, $\frac{\dot{K}}{K}$, plus capital gains, $\frac{\dot{p}}{p}$, must equal the rate of interest. Thus all capital is like non-renewable resources if the stock of capital is equal to the sum of its services. Hence, this confirms that $\dot{p} > 0$. Along these lines, Amir does not use Hotelling's rule. The rule is defined as

$$\dot{p}_i(t)/p_i(t) = r,$$

where $p_i(t)$ is the price of the i 'th resource at time t , and says that the efficient intertemporal allocation of exhaustible resources occurs when the price of an exhaustible resource rises at the rate of interest—assuming zero extraction costs (Common and Perrings, p.4). The interest rate must remain constant over time for Hotelling's rule to have any significance. Common and Perrings derive optimality conditions that use Hotelling's rule, for resource allocation. First, let $\mathbf{x}(t)$ (K in Amir) be an n -dimensional vector of resources available to a global system which includes produced capital, renewable natural resources, exhaustible resources, and both intermediate and final consumption goods (Common and Perrings, p.6). $\dot{\mathbf{x}}$ is the change in the level of the components. Let $\mathbf{p}(t)$ denote an n -dimensional vector of prices corresponding to $\mathbf{x}(t)$. By denoting the rate of discount by r , the price path $[\mathbf{p}(t), \dot{\mathbf{p}}(t)]_{t=0}^{\infty}$ is said to be efficient if and only if (Common and Perrings, p.7)

$$\dot{p}_i/p(t) = r - \dot{x}_i(t)/x_i(t)$$

for all t and i or, equivalently if

$$\dot{p}_i/p(t) + \dot{x}_i(t)/x_i(t) = r.$$

A time path of resources $[\mathbf{x}(t), \mathbf{x}(\dot{t})]$ is said to be efficient at prices $[\mathbf{p}(t), \mathbf{p}(\dot{t})]_{t=0}^{\infty}$ if and only if, it maximizes instantaneous profit, defined by

$$\pi = \mathbf{p}'(t)\dot{\mathbf{x}}(t) + \dot{\mathbf{p}}'(t)\mathbf{x}(t)$$

where $\mathbf{p}'(t)$ is a row vector, the transpose of $\mathbf{p}(t)$. Then, by maximizing π , $\frac{d[\pi]}{dt} = 0$, we get

$$\frac{d[\mathbf{p}'(t)\dot{\mathbf{x}}(t)]}{dt} = -\frac{d[\dot{\mathbf{p}}'(t)\mathbf{x}(t)]}{dt}$$

where dt represents the differential of time, which is necessary for profit maximization and efficiency. The above equation reflects the sense of Hartwick's rule (Common and Perrings, p.7).

To relate these conditions to the Amir model one finds that the efficiency conditions are applicable to Amir's model, if prices are defined by the Hotelling rule. Since profits, in Amir, are defined by

$$\pi = dW/dt + \sum_i \dot{p}_i K_i$$

then maximizing π we get

$$d[\dot{W}] + \frac{d[\sum_i \dot{p}_i K_i]}{dt} = 0$$

results in

$$d[\dot{W}] = -\frac{d[\sum_i \dot{p}_i K_i]}{dt}$$

where $\dot{W} = \frac{dW}{dt}$. Since every resource has a shadow price, as Amir believes, defining prices of exhaustible resources by the Hotelling rule, can be done. And the efficiency conditions above may apply. In addition, the constant

nature of the stock of capital, as in a steady state, lends to the applicability of the efficiency conditions to Amir's model. The immediate advantage being that the results of Amir will have a stronger theoretical basis.

Critique 3: Theoretical Justification

Is Amir's model justified on theoretical grounds? While the choice of a linear system, represented by a generalized-production, linear-activity type model has been used by von Neumann, 1945, Koopmans, 1951, Malinvaud, 1953, Gale, 1960, the exact arguments of the wealth function, $W=W(K,t)$, seem to be ad hoc.³⁵ Amir is not clear in his reasons of why the wealth function is defined on the stocks of commodities and time alone (Amir, 1991, p.10). It is unsure whether his results are sensitive to these particular arguments. For example, should risk enter into this wealth function. If so, will his results still carry through?

Further note, that Amir does not maximize the wealth function directly. He maximizes profit and wealth through a two-stage procedure, subject to the maximum profit condition.³⁶ This two-stage procedure is not equivalent to the problem³⁷

$$\max_K \{W(K,t) : BX - AX \geq Y; CX \leq K\},$$

which is broken into two independent problems: a linear production problem and a nonlinear consumption problem (Amir 1991, p.12). Because optimal wealth may not result, automatically, from optimal profits (since profits are not a function of wealth alone), explains Amir's two step approach.

Furthermore, Amir states that "Observed behaviour rather than theory

³⁵However, this problem is minimized since maximizing wealth makes economic sense.

³⁶This comment results from discussions with Professor Crabbé.

³⁷Ibid.

tests the validity of a proposed function in its way to becoming the ‘objective’ function”; it is unsure what Amir means by this statement. Does he mean that theory is not important in determining an objective function? What determines rational behaviour from irrational behaviour, if theory is not used? Do we disregard theory all together and develop “objective functions” solely on observed behaviour? In my view, any theory that is relevant, predicts, and (should) predict well, the behaviour of economic agents. Any theory that fails to predict, has minimal value to society and to researchers.

Critique 4: The Applicability of Amir’s Model

This is one of the important, yet very basic, critique to Amir’s model. First, it is interesting to note the following quote from Ruth (1993, p.64) who states

“...it has not yet been shown convincingly, whether the laws of thermodynamics impose constraints that are significant enough to be considered explicitly in economic analysis. Rather, the sometimes inaccurate adoption of thermodynamic concepts in economic theory led, time and again, to considerable confusion among economists.”

It seems to me that Amir’s paper falls victim to such an observation. The reason being is that Amir does not answer one of the main questions of whether incorporating thermodynamic laws make much difference to existing models in economics. And, how can the implementation of thermodynamic laws in economic models be tested to see whether they are actually making a difference or leading to an efficient allocation of resources? Along these lines is the question of how should we correctly estimate (if even possible) a thermodynamically consistent production or consumption function? Amir

does not remedy these questions.

Assuming it is possible to estimate a thermodynamically consistent production or consumption function, what variables and functional form should one use? For example, in equations (46) and (51), is it possible to get an estimate of λ , the Lagrange multiplier, and \dot{p}_i , using real world data? It would seem to me that it would be difficult to get an estimate for λ or \dot{p}_i for any first order equations because they are represented as (continuous) functions through time; in order for us to get any kind of estimates we must use a discrete approximation to these equations. Amir (pp.34-35), however, has reconciled, through integration, the *continuous* flow of "value", $\dot{p}_i K^*$, with the *finite* stocks of the system (see eq.'s (65) and (76) in the Appendix).

The above concerns about the applicability, in terms of estimation, of Amir's model are important because it is mainly through estimation that we validate theoretical propositions. It remains to be seen whether the results in Amir's paper can be validated through empirical estimation of real world data. This, empirical estimation of Amir's model, would also suggest an area for further research.

Critique 5: The Wealth Function

The exact nature of the wealth function, $W = W(K, t)$, where K is the capital stock, and t is time, used by Amir is ambiguous. Amir (1991, p.10) states

"The important point is that any state function may become a wealth function, provided maximization of this function does not give rise to behavioural patterns in contradiction with observed (economic) behaviour."

So the choice of a wealth function is only contingent on the fact that the maximization of this function does not contradict rational (economic) behaviour. In essence, a particular wealth function should be consistent with economic theory. So, it is not clear how one would go about choosing an appropriate wealth function, if indeed one exists.

In addition, the choice of a linear or a nonlinear function may have implications on the results presented by Amir. If wealth is a nonlinear function, the path that p takes ($\frac{dW}{dK} = p$) to achieve an optimum will be largely indeterminate in a dynamic context. But, if wealth is linear homogeneous (as Amir assumes) in the capital stock, this greatly enhances the predictability of the function. However, the linear homogeneity constraint adds further difficulties. Linear homogeneity, in the economic context, implies simply constant returns to scale, which is at times a very restrictive assumption on firm behaviour. Likewise, linear homogeneity of W on K may be equally restrictive in a systems context and may limit the situations for W 's applications in short-run cases. However, in long-run equilibrium cases, it seems to present no difficulties. Rather, linear homogeneity is a logical assumption in a long-run equilibrium model. In addition, the linear system is justified near equilibrium.

6.2 Recommended Changes and Modifications to the Amir Model

The changes that can be made to Amir's model are several. But the most important one would be to present a particular example, i.e. a particular functional form, and then see how his results would effect this functional form. What would be interesting to show is the cause and effect of a thermo-

dynamic model(s). In other words, using his results, Amir could have shown what causes have what effects in his (thermodynamically consistent) economic model. Thus, the abstract nature of Amir's paper could be minimized by the use of particular examples.

My overall impression of the paper is that it is too general and too abstract. It is hard to see the relevance of the results presented by Amir because he does not clearly show the impact of his results on real world economic situations. He could have done this through data analysis or through estimation of a model using real world data. In addition, he could show how existing environmental-economic policies would change by his results; through welfare analysis, it may be possible to determine the gainers and the losers from a particular policy, that incorporates Amir's results. This, evaluating the changes in policy as a result of thermodynamic constraints, through welfare analysis, is an area for further research.

7.0 Summary and Conclusions

This paper has presented methodologies of incorporating thermodynamics in economics. It also presented views that argue in favour of constraining economic behaviour by the Laws of Thermodynamics. And it discussed difficulties involved in incorporating these laws in economics.

An important focus of this paper was to present and critique a model developed by Shmuel Amir (1991). Several critiques were pointed out. The main critique of Amir's paper was the applicability of his results. In that, Amir did not present a convincing argument in showing how his results affect existing models in economics. Also, he failed to show how the distributional effect of policy (in terms of income, for example)—that recognized thermody-

dynamic constraints—would be affected. In addition, who would be the gainers and the losers, via welfare analysis, of a thermodynamically consistent policy? All these considerations are important from an economic (policy) point of view. Also, Amir could have made his paper seem less abstract if he included particular examples from economics and used them to demonstrate some of the main results of his paper. In all due fairness, Amir's work in the area of thermodynamics and economics should be commended. His analysis was inspiring.

Furthermore, economics has mimicked 19th century physics by focusing on equilibrium systems and reversible systems. Rather it should focus on 20th century physics and ecology and look at far-from-equilibrium systems which are irreversible according to the extended second law developed by Schneider and Kay, 1994. Since the system is open, the extended second law is not a foundation for scarcity. The current focus is no longer on entropy but on organization i.e., technological progress which is resource augmenting, see Gross and Veendrop, i.e. on knowledge.³⁸

Why should economists care about thermodynamics either in microeconomics or macroeconomics? A static microeconomic equilibrium system is a thermodynamically isolated system. For example, in a simple exchange economy, initial resource endowments (say, matter and energy), are given and allocated as consumption between the exchangers. Since no production has occurred, the first law applies trivially: what is consumed in the first period can be returned as resources at the beginning of the second period to allow the game to be repeated indefinitely. If production is allowed, the secondary chips, (say again, matter and energy), essentially the same process occurs and the economic system is still a thermodynamically isolated

³⁸These comments result from discussions with Professor Crabbé.

system.³⁹

A fundamental axiom of production theory is that production cannot operate in reverse; one cannot produce inputs from outputs. Therefore, once matter and energy have been transformed into capital and labour, one cannot recover that matter and energy for the next round. Thermodynamics agrees with this axiom since it says that you cannot reverse a thermodynamic process such as production without adding exergy. However, the first law requires that the mass of the output equals the mass of the inputs. Here is where the definition of economic output and thermodynamic output differ. Economic output will not be produced unless it provides utility to an economic agent, but thermodynamic output includes both economically desirable and undesirable products, such as waste. Thermodynamic closure requires that waste be accounted for in the economic model as output or as negative input which leads to further economic costs.⁴⁰

Consumption is sometimes defined in economics as destruction of goods or services. This thermodynamic interpretation is not compatible with the static economic, thermodynamically isolated, model referred to above; since what has been consumed must be able to be re-used in the second period. Once a non-durable good has been destroyed by consumption, it has become economically inexistant, this is inconsistent with the First Law of Thermodynamics. According to the first law, a consumed good may have lost its utility but it has not disappeared; it has simply been transformed. Therefore, thermodynamic accounting requires that one take this transformation into account. Transformation may turn one good into another like when one eats food, it turns in part into investment in human capital (survival),

³⁹Ibid.

⁴⁰Ibid.

and part into waste. Economically, the consumer must depreciate their human capital, which is the cost for one's body of not eating, and include the depreciation cost and the cost of waste in their budget equation as a cost.⁴¹

In an isolated system if production cannot be reversed, therefore irreversible, this is a manifestation of the Second Law of Thermodynamics. With the First Law of Thermodynamics alone, the exchange game can be repeated forever. But with production irreversibility, the exchange game cannot be repeated. Production irreversibility is then an extreme manifestation of the second law: exergy is degraded fully after one use. If the economic system is a near equilibrium open system, the first and second laws apply within the walls of the system, but not to the inputs from outside and the outputs to outside the system. Open systems near equilibrium import exergy to minimize entropy i.e. to compensate for maximum entropy within the walls.⁴²

Production irreversibility satisfies the second law but not necessarily the first. Given inputs, output is being maximized by a production function. The production entity is essentially isolated and must thus satisfy the first law. Production functions which exhibit infinite average or marginal utilities when inputs go to zero violate the first law. Essential resources have that property. Not only is output nil without an essential input, the latter's productivity is unbounded when the input tends to zero. And so do production functions with elasticities of substitution larger than one.⁴³

At the macroeconomic level, exponential growth is acceptable only if technological progress is matter and energy resource saving. Thermodynamics

⁴¹Ibid.

⁴²Ibid.

⁴³Ibid.

does not apply to information or knowledge.⁴⁴

Amir's analysis has been concerned with the intrinsic or ecocentric value of ecological entities, including economic ones on the basis of near-equilibrium thermodynamic "as if" optimization. This is a very reductionist procedure. It is tantamount to assuming that only thermodynamics matters in order to determine the value of ecological entities. This reduces all ecosystems, including economic agents, to their capacity to degrade exergy. Though the ecosystemic and specific functions of ecological individuals i.e., the so-called emergent properties of the ecosystem, can be identified independently of the optimization model, (through structural analysis of constraint parameters), shadow prices will always be dependent upon the objective function.⁴⁵

The advantage of using far-from-equilibrium thermodynamics is to recognize at least that efficient exergy degradation leads to more organized structures including ecological structures. Their thermodynamic value, resides in their capacity to degrade exergy. But, concomitant to this, valuation may be attributed to increased organization even though this valuation will have to come from outside thermodynamics, i.e. from environmental ethics. Non-linear models will have to be used, and the realization of their attractors whether they are the result of optimization or not, will fundamentally be unpredictable. Any attempt at valuing ecological functions through optimization, even through "as if" optimization, is likely to be anthropocentric.⁴⁶

Economic theory should be consistent with thermodynamic laws; it is not clear that it is not, except in the case of some frequently used production functions. Exergy is not a limiting factor of the economic system and is unlikely to

⁴⁴Ibid.

⁴⁵Ibid.

⁴⁶Ibid.

become one in the foreseeable future. Far-from-equilibrium thermodynamics, by emphasizing complexity, is certainly closer to environmental ethics than is near equilibrium thermodynamics. However, any attempt to find valuation exclusively on thermodynamics is mechanistic and non-operational.⁴⁷

So, where do we want to (or should) go with this knowledge of thermodynamics in economics? Thermodynamics has its advantages and disadvantages in better understanding environmental problems. One of the biggest disadvantage of thermodynamics in economics is due to its lack of accessibility to economists. Thermodynamics is based in the natural sciences not the social sciences. This may be one of the reasons why it has not been picked up by many economists; this is not to say that thermodynamics is not relevant to environmental research—it is.

The future of thermodynamics in economics is not absolutely sure. Its greatest use is in dealing with environmental problems, but is not entirely limited to it. Areas such as production economics, I feel, offer one area of applicability. Especially in agricultural production. The reason is that agricultural production processes can be seen as open systems. In order to maximize, for example, crop production, insecticides, weather, land, all play a crucial role in this process. The question then becomes, how can we apply the First and Second Law of Thermodynamics to agricultural problems: maximizing crop production. Recalling the first and second laws, in order to maximize crop production, we know from these laws that once energy is used it can not be re-used; thus while farmers would like to maximize crop production or livestock production, they should not ignore their surroundings. The problem of top soil erosion and methane gas from cattle production also lends itself to future environmental problems. So, in maximizing farm pro-

⁴⁷Ibid.

duction, one should take into consideration the effects of the above to future farming operations.

Thermodynamics should be given more serious considerations in economics. Interdisciplinary research has possibilities of opening up new frontiers for future research. Economists should be open to new and different ideas that, in the long run, will not only strengthen economics but other disciplines as well, leaving all better off!

Appendix

This appendix will present the Amir model from a slightly technical angle. It should be used to supplement the qualitative explanation given above. Amir's model has m commodities, n subsystems, J activities, and assumes $J \gg n > m$. Activities are a linear way of transforming commodities into other commodities. Subsystems are any part of the original *open* systems: individuals, firms, industries, economic sectors, national economies. The variables used in the Amir model are:

Variables	Definitions
X_j	Level of the j th activity or the production technology
b_{ij}	The amount of commodity i produced by the j th activity
Y	flow of commodities associated with the given system
C	The elements in this matrix, c'_{ij} 's, are the amounts of commodity i needed per unit of process j .
A	Input matrix of dimension m by j . The elements in A , a_{ij} 's, are the amounts of commodity i consumed by the j 'th activity (per unit of time and activity).
B	Output matrix of dimension m by j . The elements of B , b_{ij} 's, are the amounts of commodity i produced by the j 'th activity (per unit of time and activity).
K	stock of commodities held within the boundaries of the system

If $Y_i > 0$, the i 'th commodity flows out of the *open* system; if $Y_i < 0$ the i 'th commodity flows in to the *open* system. dK is the change in the stock of commodities in the system: if $dK > 0$ implies an increase in the stock of commodities, if $dK < 0$ implies a decrease in the stock of commodities. If activities are viewed as chemical reactions, where products (outputs) are formed from reactants (inputs), the process of associating the inputs to the reaction with stocks and the inflows during the period of reaction is given by the following weak inequality

$$AX\delta t \leq -Y^-\delta t + K_t \quad (7)$$

where Y^- = commodity inflows, δt is the change in time, and K_t is the stock of commodities in the t ' th period. Amir expresses the association of outputs with the final stocks and the outflows as

$$BX\delta t \geq Y^+\delta t + K_{t+1}. \quad (8)$$

The operation of the system, which involves net output, change in stocks and commodity flows, is expressed as

$$BX\delta t - AX\delta t \geq \delta K + Y\delta t. \quad (9)$$

Expression (9) is derived by adding (7) and (8) (with (7) being multiplied by -1) as follows

$$-AX\delta t \geq Y^-\delta t - K_t \quad (10)$$

$$BX\delta t + (-AX\delta t) \geq Y^+\delta t + K_{t+1} + (Y^-\delta t - K_t) \quad (11)$$

$$BX\delta t - AX\delta t \geq (Y^+ + Y^-\delta t) + (K_{t+1} - K_t) \quad (12)$$

setting $Y = Y^+ + Y^-$, and letting $\delta K = K_{t+1} - K_t$ we get

$$BX\delta t - AX\delta t \geq \delta K + Y\delta t \quad (13)$$

which is expression (9). Now dividing (13) by δt we get

$$BX - AX \geq \frac{\delta K}{\delta t} + Y \quad (14)$$

this is equation (3') in Amir 1991, which says that net output must be at least equal to the change in stocks and flows of commodities associated with the system.

The wealth function (W) is $W=W(K,t)$, where t is time, and W is linear homogeneous in K . The wealth function is constrained by the fact that it

cannot flow through the system boundaries because it is merely an accounting medium (Amir 1991, p.10). Thus, wealth may increase or decrease as the system passes from one state to another but its nature (in terms of representing value) is well defined. Profits are defined by taking the derivative of W with respect to time, and the derivative of W with respect to K_i , is denoted by $\pi(K, t)$:

$$\pi(K, t) = \frac{dW}{dt} + (K_1 \frac{dp_1}{dt} + K_2 \frac{dp_2}{dt} + \dots + K_n \frac{dp_n}{dt}) \quad (15)$$

$$+ (p_1 \frac{dK_1}{dt} + p_2 \frac{dK_2}{dt} + \dots + p_n \frac{dK_n}{dt}) + \frac{dp_1}{dK_1} dK_1 \frac{dK_1}{dt} \quad (16)$$

$$+ \frac{dp_1}{dK_2} dK_1 \frac{dK_2}{dt} + \dots + \frac{dp_1}{dK_n} dK_1 \frac{dK_n}{dt} \quad (17)$$

$$\pi(K, t) = \frac{dW}{dt} + \sum_i K_i \frac{dp_i}{dt} + \sum_i \left[p_i + \sum_1 \left(\frac{dp_1}{dK_i} dK_1 \right) \right] \frac{dK_i}{dt} \quad (18)$$

(18) above is (6') in Amir 1991. Note that (18) is derived using the product rule and the chain rule of calculus, and the assumption that p_i (marginal wealth associated with a change in K_i) depends on K and t and thus a particular path for wealth can be found. Now setting $\frac{dK_i}{dt} = 0$ (or equivalently, $\dot{K} = 0$), which represents a steady state scenario, and assuming that any path is possible ensures that wealth will be time independent (since one will choose to maximize wealth and thus choose the most profitable path) we get:

$$\pi(K, t) = \frac{dW}{dt} + \sum_{i=1}^n K_i \frac{dp_i}{dt}. \quad (19)$$

Consider the conventional approach to solving this problem of resource allocation. The wealth function, $W(K, t)$, increases with the K_i 's and is to be maximized subject to the steady state constraint ($\frac{dK_i}{dt} = 0$):

$$\max_K \{W(K, t) : BX - AX \geq Y; CX \leq K\}. \quad (20)$$

This maximization problem can be broken down into a (linear) production problem, and a (nonlinear) consumption problem. The production problem is to maximize the value of net product

$$BX\delta t - AX\delta t \geq \delta K + Y\delta t \quad (21)$$

noting that $\delta K = K_{t+1} - K_t$, and defining p_t and p_{t+1} as price vectors for the current and next period, we get

$$p_t BX\delta t - p_t AX\delta t \geq p_{t+1} K_{t+1} - p_t K_t + p_t Y\delta t \quad (22)$$

letting $p_{t+1} = p_t + \delta p_t = p + \delta p$, we get

$$p_t BX\delta t - p_t AX\delta t \geq (p_t + \delta p_t) K_{t+1} - p_t K_t + p_t Y\delta t \quad (23)$$

multiplying both sides by $\frac{1}{\delta t}$ (since $\frac{1}{\delta t} > 0$, the inequality is preserved)

$$p_t BX - p_t AX \geq \frac{(p_t + \delta p_t) K_{t+1}}{\delta t} - \frac{p_t K_t}{\delta t} + \frac{p_t Y \delta t}{\delta t} \quad (24)$$

$$p_t BX - p_t AX \geq \frac{p_t K_{t+1}}{\delta t} + \frac{\delta p_t K_{t+1}}{\delta t} - \frac{p_t K_t}{\delta t} + p_t Y \quad (25)$$

since $\delta K = K_{t+1} - K_t$

$$p_t BX - p_t AX \geq \frac{p_t}{\delta t} (K_{t+1} - K_t) + \frac{\delta p_t K_{t+1}}{\delta t} + p_t Y \quad (26)$$

$$p_t BX - p_t AX \geq p_t \frac{\delta K}{\delta t} + p_t Y + \frac{\delta p}{\delta t} K_{t+1} \quad (27)$$

$$p_t BX - p_t AX \geq p_t \left(\frac{\delta K}{\delta t} + Y \right) + \frac{\delta p}{\delta t} K_{t+1} \quad (28)$$

$$p BX - p AX \geq p \left(\frac{\delta K}{\delta t} + Y \right) + \frac{\delta p}{\delta t} K \quad (29)$$

dropping the time subscripts indicates that both K and p are constant in every time period due to the nature of the steady state analysis. Thus an equality may be used in equation (29).

The production problem becomes

$$\max_{(X, K, \frac{dK}{dt} + Y)} \left\{ \frac{dp}{dt} K + p \left(\frac{dK}{dt} + Y \right) : BX - AX \geq \frac{dK}{dt} + Y; CX \leq K \right\}. \quad (30)$$

Equivalently, we can express (26) as a primal problem

$$\min_{(X, K, \frac{dK}{dt})} \left\{ -\frac{dp}{dt} K - p \frac{dK}{dt} - pY : (B - A)X - \frac{dK}{dt} \geq Y; CX \leq K \right\} \quad (31)$$

(multiplying (30) by -1, and since we are minimizing over X, K, and dK/dt, we set pY=0 for simplification, since we are not maximizing with respect to this variable). The dual problem is

$$\max_{(u, v)} \{ uY : (B - A)'u \leq -vC'; -\frac{dp}{dt}; u \geq p \} \quad (32)$$

where $(B - A)' = (B - A)_{j \times m}$ is the transpose of $(B - A)_{m \times j}$ and $C' = C_{j \times m}$ is the transpose of $C_{m \times j}$, and uY is profit. In equilibrium we have

$$-\frac{dp}{dt} K^* - p \frac{dK^*}{dt} = u^* Y \quad (33)$$

where u's and v's are shadow prices, u_i and v_i are equal to a change in

$$-\frac{dp}{dt} K^* - p \frac{dK^*}{dt} \quad (34)$$

given a slight change in Y_i or K_i respectively. This can be seen more clearly as

$$\Delta Y_i \rightarrow \Delta(-\dot{p}K^* - pK^*) \rightarrow u_i \quad (35)$$

$$\Delta K_i \rightarrow \Delta(-\dot{p}K^* - pK^*) \rightarrow v_i \quad (36)$$

Y_i affects K_i because a change in commodity flows affects inventory. Moreover, whenever $K \rightarrow 0$ (i.e., K absorbs the difference between $(B-A)X - Y$)

$$-\frac{dp}{dt} K^* = u^* Y \quad (37)$$

(37) shows that the change in the value of the stocks is equal to the value of the flows. If (31) and (32) possess feasible solutions, then the value of a change in stocks is equal to zero, when the value of net production is maximized in a steady state.

The consumption problem is where M is the (negative) value of the flows.

$$\max_K \{W(K, t) : \frac{dp}{dt} K \leq M\} \quad (38)$$

$$\frac{dp}{dt} > 0, \quad M = -u^*Y > 0. \quad (39)$$

Amir maximizes consumption given that profits are maximized for a given Y . This is the meaning of the constraint in eq.(38).⁴⁸ If the change in wealth with respect to the stocks (K) of the system is negative, then $W(K,t)$ is to be minimized with respect to K and the maximization problem (in eq. (38)) is replaced with the following minimization problem

$$\min_K \{W(K, t) : \dot{p}K \geq M\} \quad (40)$$

where again $\frac{dp}{dt} > 0$ and $M = -u^*Y > 0$. Note that wealth here is a free-energy like function. To derive the first order conditions, we use the method developed by Lagrange for maximizing a function. The Lagrange method for maximizing or minimizing an objective function, $f : \mathfrak{R}^n \rightarrow \mathfrak{R}$ subject to the constraint $g : \mathfrak{R}^n \rightarrow \mathfrak{R}$, says that

$$\nabla f(\mathbf{x}) = \lambda \nabla g(\mathbf{x}) \quad (41)$$

where ∇ is a gradient vector, \mathbf{x} is a vector of the arguments in \mathfrak{R}^n for the respective functions, and $\lambda \in \mathfrak{R}^+ \cup \{0\}$ is the Lagrange multiplier. The formal problems and the accompanying first order conditions can now be outlined.

$$\max_K \{W(K, t) : \dot{p} \leq M\}, \quad (42)$$

⁴⁸This observation results from discussions with prof. Crabbé.

by the Lagrange method we get

$$\nabla W(K, t) \leq \lambda \nabla g(K, M) \quad (43)$$

where $g(K, M) = \dot{p}K - M \leq 0$, the first order conditions (FOCs) become

$$\frac{\partial W}{\partial K_i} \leq \lambda \frac{\partial g}{\partial K_i} \quad i = 1, \dots, m \quad (44)$$

and so

$$\frac{\partial W}{\partial K_i} \leq \lambda \dot{p}_i \quad i = 1, \dots, m \quad (45)$$

equivalently, we can write that

$$\frac{\partial W}{\partial K_i} - \lambda \dot{p}_i \leq 0 \quad (46)$$

where $\lambda = \frac{\partial W}{\partial M}$. For a solution to eq.(46) we note two cases: either $\frac{\partial W}{\partial K_i} - \lambda \dot{p}_i < 0$ or $\frac{\partial W}{\partial K_i} - \lambda \dot{p}_i = 0$.

Case 1: Suppose $\frac{\partial W}{\partial K_i} - \lambda \dot{p}_i < 0$ this is true *only if* $\frac{\partial W}{\partial K_i} < \lambda \dot{p}_i$, which says that the change in wealth with respect to a change in the systems stocks (marginal wealth) is less than the *flows* of marginal wealth. Since $\lambda \in \mathfrak{R}^+ \cup \{0\}$ it can be zero and so it is not possible to divide by λ , but if $\lambda > 0$ we are led to the following Sub-case.

Sub-Case 1: If $\lambda > 0$ we can divide both sides of the above inequality by λ to get

$$\frac{\frac{\partial W}{\partial K_i}}{\lambda} < \dot{p}_i \quad (47)$$

since λ is equal to the change in wealth with respect to the change in the value of the flows, upon substitution we get

$$0 < \frac{\frac{\partial W}{\partial K_i}}{\frac{\partial W}{\partial M}} < \dot{p}_i \quad (48)$$

for ease of exposition we can write

$$0 < \frac{p_i}{\lambda} < \dot{p}_i \quad (49)$$

this last inequality is bounded, but not closed, and says that the ratio of marginal wealth with respect to the stocks and marginal wealth with respect to the value of the flows is less than the flows of marginal wealth with respect to the stocks. Next, consider case 2.

Case 2: $\frac{\partial W}{\partial K_i} - \lambda \dot{p}_i = 0$ only if $\frac{\partial W}{\partial K_i} = \lambda \dot{p}_i$, which says that marginal wealth is equal to the *flows* of marginal wealth. Likewise, we analyse the following sub-case.

Sub-Case 2: If $\lambda > 0$ we have $\frac{\frac{\partial W}{\partial K_i}}{\lambda} = \dot{p}_i$ or $\frac{p_i}{\lambda} = \dot{p}_i$ which says that the ratio of marginal wealth with respect to stocks and marginal wealth with respect to the value of the flows is *equal* to the flows of marginal wealth with respect to the stocks.

Similarly, we can analyse the problem of minimizing

$$\min_K \{W(K, t) : \dot{p}K \geq M\}. \quad (50)$$

The Lagrange method gives

$$\nabla W(K, t) \geq -\lambda \nabla g(K, M).$$

The FOCs are:

$$\frac{\partial W}{\partial K_i} + \lambda \dot{p}_i \geq 0 \quad (51)$$

Case 3: $\frac{\partial W}{\partial K_i} + \lambda \dot{p}_i > 0$ is true only if $\frac{\partial W}{\partial K_i} > -\lambda \dot{p}_i$ which says that marginal wealth is greater than the *flows* of marginal wealth.

Sub-Case 3: $\lambda > 0 \Rightarrow -\lambda < 0$ so dividing by λ we get

$$-\frac{\frac{\partial W}{\partial K_i}}{\lambda} < \dot{p}_i \quad (52)$$

and so

$$-\frac{p_i}{\lambda} < \dot{p}_i, \quad (53)$$

which says that the negative of the ratio of marginal wealth with respect to stocks and marginal wealth with respect to the value of the flows is less than the flows of marginal wealth with respect to stocks. In case 4 we have the following situation.

Case 4: $\frac{\partial W}{\partial K_i} = -\lambda \dot{p}_i$ which says that marginal wealth is equal to the negative of the flows of marginal wealth.

Sub-Case 4: If $\lambda > 0$ we get $\frac{\frac{\partial W}{\partial K_i}}{-\lambda} = \dot{p}_i$ upon substitution we get

$$-\frac{p_i}{\lambda} = \dot{p}_i \quad (54)$$

which says that the negative of the ratio of marginal wealth with respect to stocks and marginal wealth with respect to the value of the flows is equal to the flows of marginal wealth with respect to the stocks.

Moreover, $\lambda(K, M)$ is the turnover time, p_i is the present value of \dot{p}_i , and $1/\lambda$ is the discount rate. By integrating eq.(46) and eq.(51) it is possible to find the present value of the stocks. First note that wealth is an entropy like function, $u^*Y \leq 0$, if $p < 0$ and $u^*Y \geq 0$ if $p > 0$. Wealth is "generated" in the system and is dissipated into the environment – so that surrounding wealth increases with time. The signs of p and dp/dt are opposite: if we define the discount rate as

$$\beta = \frac{d}{dW} \left(\frac{dW}{dt} \right) > 0 \quad (55)$$

we can derive an expression for the present value of wealth (pK^*) (which is the future value of past flows.) First, recall that $\frac{dW}{dt} = pY$

$$\beta = \frac{d}{dW}(pY) > 0 \quad (56)$$

$$= \left(p \frac{dY}{dW} + Y \frac{dp}{dW} \right) > 0. \quad (57)$$

Now,

$$pK^* = \int_{-\infty}^0 -\dot{p}K^* e^{\beta t} dt \quad (58)$$

$$= -\dot{p}K^* \int_{-\infty}^0 e^{\beta t} dt \quad (59)$$

let $f = \beta t$, $df = \beta dt$, and so $dt = \frac{1}{\beta} df$

$$= \left(-\frac{1}{\beta} \right) \dot{p}K^* \int_{-\infty}^0 e^f df \quad (60)$$

$$= \left(-\frac{1}{\beta} \right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} \int_v^0 e^f df \right) \quad (61)$$

$$= \left(-\frac{1}{\beta} \right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} (e^f) \Big|_v^0 \right) \quad (62)$$

$$= \left(-\frac{1}{\beta} \right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} (e^0 - e^v) \right) \quad (63)$$

$$= \left(-\frac{1}{\beta} \right) \dot{p}K^* (1 - 0) \quad (64)$$

$$pK^* = \left(-\frac{1}{\beta} \right) \dot{p}K^* \quad (65)$$

Thus wealth at present value is equal to $pK^* = \left(-\frac{1}{\beta} \right) \dot{p}K^*$ or $p = \left(-\frac{1}{\beta} \right) \dot{p}$

In the case when 'wealth' is a free energy-like function, environmental wealth always decreases in steady state. In this case p and dp/dt have the same sign, and the discount rate is defined as

$$-\beta = \frac{d}{dW} \left(\frac{dW}{dt} \right) < 0 \quad (66)$$

we can now derive an expression for wealth as the present value of *future* flows. Again, $\frac{dW}{dt} = pY$ and we get

$$-\beta = \frac{d}{dW}(pY) < 0 \quad (67)$$

$$-\beta = \left(p \frac{dY}{dW} + Y \frac{dp}{dW}\right) < 0 \quad (68)$$

or multiplying both sides of eq.(68) by -1, we get

$$\beta = \left(-p \frac{dY}{dW} - Y \frac{dp}{dW}\right) > 0. \quad (69)$$

Now,

$$pK^* = \int_0^{\infty} \dot{p}K^* e^{-\beta t} dt \quad (70)$$

let $g = -\beta t$, $dg = -\beta dt$, and so $dt = -\frac{1}{\beta} dg$

$$= \left(-\frac{1}{\beta}\right) \dot{p}K^* \int_{-\infty}^0 e^g dg \quad (71)$$

noting that as $t \rightarrow \infty$, $g \rightarrow -\infty$,

$$= \left(-\frac{1}{\beta}\right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} \int_v^0 e^g dg\right) \quad (72)$$

$$= \left(-\frac{1}{\beta}\right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} (e^g) \Big|_v^0\right) \quad (73)$$

$$= \left(-\frac{1}{\beta}\right) \dot{p}K^* \left(\lim_{v \rightarrow -\infty} (e^0 - e^v)\right) \quad (74)$$

$$= \left(-\frac{1}{\beta}\right) \dot{p}K^* (0 - 1) \quad (75)$$

$$pK^* = \left(\frac{1}{\beta}\right) \dot{p}K^*. \quad (76)$$

So the value of wealth as a present value of future flows is $p = \left(\frac{1}{\beta}\right) \dot{p}$. The above analysis shows that free energy-like criteria and entropy-like do not

differ by much when in a steady state (Amir 1991, p.17). Free energy-like criteria evaluate future streams of value, whereas entropy-like criteria evaluate past streams of value (Ibid.).

The physical meaning of the discounting process is explained by Amir (1991, p.18) as follows. Assume $W(K,t)$ is a thermodynamic function and consider the economic (ecological) system as either a cyclic engine or a steady flow. The system produces outputs from inputs and returns to the input stage after the output stage is completed. During this cycle, the system is absorbing material resources and light which are emitted as different material resources and heat. Efficiency (ϵ) is a measure of the initial value that has been transformed successfully into final value, by definition

$$\epsilon = \frac{uY^+}{-uY^-} = \frac{vK - uY^-}{-uY^-} \quad (77)$$

efficiency of the system increases with the length of the cycle. Optimal turnover time is determined at a point where $-vK$ is minimized, $-uY$ is maximized, mathematically $\min -vK = \max -uY$. This can be seen in Figure 1 (Amir 1991, p.19). Optimal efficiency (ϵ^*) is also determined at λ^* , such that

$$\epsilon^* = \frac{u^*Y^*}{-u^*Y^-} \quad (78)$$

What this figure says is that if "wealth" is any general function, and the $\dot{p}_i = 0$ in a steady state, we can still identify β with $\frac{\partial^2 W}{\partial W \partial t} = \frac{\partial M}{\partial W}$, but in this case the present value of \dot{p}_i , p_i , cannot differ from zero if $\beta > 0$. In other words, $\beta = 0$ is the only solution to eq.(46) when the \dot{p}_i 's are equal to zero and the p_i 's differ from zero. As mentioned above, $\beta = 0 \Rightarrow \lambda \rightarrow \infty$ ($\beta = \frac{1}{\lambda} = 0$, solving for λ we get $\lambda = \frac{1}{0}$, so λ is undefined). This is the case when wealth is an internal energy-like function. $\lambda \rightarrow \infty$ is done by passing from one chemical reaction to another along a definite sequence, as shown for

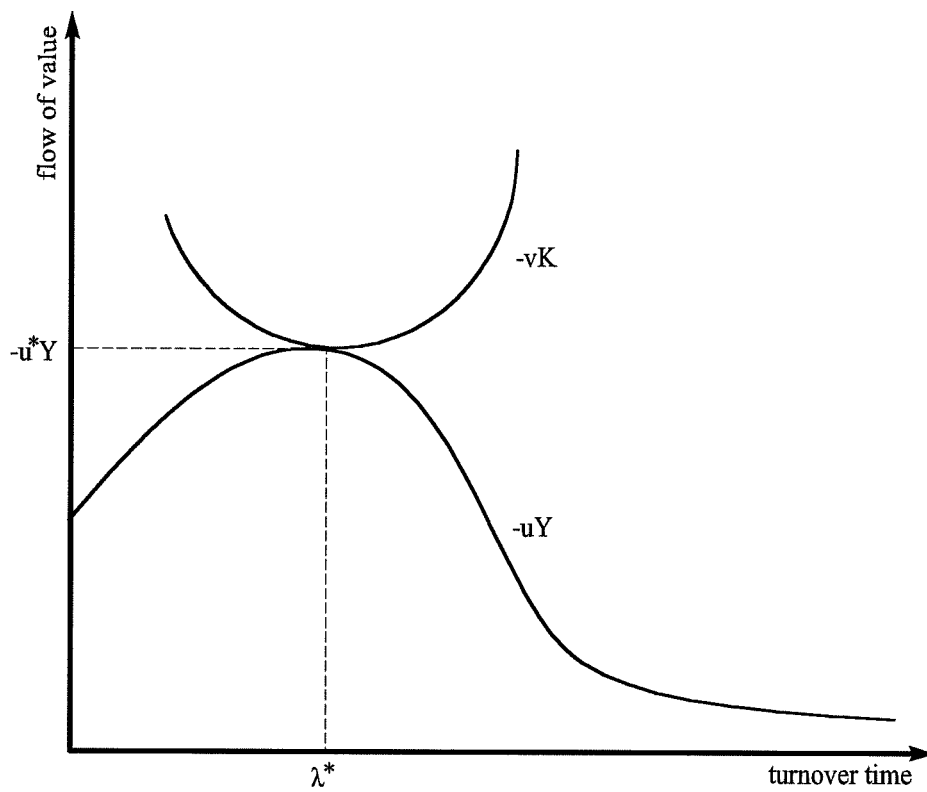


Figure 1: The Optimal Turnover Time (1/discount rate) of a Steady State Economy

heat engines by Ondrechen et al. (1981). Since each chemical reaction takes infinite time to be completed, the process is thermodynamically *reversible* and does not affect its surroundings (Amir 1991, p.20). When the cycle is completed, both internal energy, free energy, and entropy are globally conserved. In this situation, $u^* > 0$ and $u^*Y = 0$, for a solution, it must be the case that $Y = 0$. In other words, commodity flows are constant in the case when $\lambda \rightarrow \infty$. Hence, free energy-like prices (proportional to entropy-like prices) will also be proportional to internal energy-like prices (Amir 1991, p.20).

In real cases, $\beta \neq \infty$, rather $\beta > 0$, which indicates that each reaction is completed in a finite time. The optimization problem is solved by maximizing the internal energy-like function subject to the value of the stocks, pK^* . Since the process is irreversible, the change in free energy (entropy) is not zero, when the entire cycle is completed. Thus, free energy-like criteria and internal energy-like criteria cannot be proportional. In other words, the maximization of wealth as an internal energy-like function, in a steady state, does not result in the minimization of free energy-like consumption flow, and entropy-like production flow, as a result, thermodynamic-like efficiency is not achieved.

References

- [1] Amir, Shmuel (1991), *Economics and Thermodynamics: An exposition and Its Implications for Environmental Economics*, Resources for the Future, Quality of the Environment Division, Discussion Paper No. QE92-04.
- [2] Amir, Shmuel (1989), "On the use of Ecological Prices and System-Wide

Indicators derived therefrom to quantify man's impact on the Ecosystem", *Ecological Economics* 1, pp.203-231.

- [3] Amir, Shmuel (1987), *Energy pricing, biomass accumulation, and project appraisal: A thermodynamic approach to the economics of ecosystem management* In: Pillet, G. and Murota, T (eds.) *Environmental Economics: The Analysis of a Major Interface* (Geneva: R. Leimgruber). pp.53-108.
- [4] Amir, Shmuel (1979), "Economic Interpretation of equilibrium concepts in ecological systems", *Journal of Social & Biological Structures* 2, pp.293-314.
- [5] Anderson, Curt L. (1987), "The Production Process: Inputs and Wastes", *Journal of Environmental Economics and Management* 14, pp.1-12.
- [6] Anton, H. (1995), *Calculus with Analytical Geometry: Brief Edition* (New York: John Wiley & Sons).
- [7] Ayres, R.U. and I. Nair (1984), "Thermodynamics and Economics", *Physics Today* 37, 62-71.
- [8] Ayres, R.U. and A.V. Kneese (1989), "Externalities: Economics & Thermodynamics", in *Economy and Ecology: Towards Sustainable Development*, Archibugi, F. and Nijkamp, P. (eds.) (Boston: Kluwer Academic Publishers).
- [9] Blanchard, O. and S. Fischer (1989), *Lectures on Macroeconomics* (Cambridge, Mass.: The MIT Press).

- [10] Blatt, Frank J. (1983), *Principles of Physics* (Boston: Allyn and Bacon, Inc.).
- [11] Boulding, K.E. (1966), "The Economics of the Coming Spaceship Earth", in *Environmental Quality in a Growing Economy*, Jarrett, H. (ed.) (Baltimore: The Johns Hopkins Press).
- [12] Boulding, Kenneth E. (1970), *The economics of the coming spaceship earth* In: Jarrett, Henry (ed.) *Environmental Quality In a Growing Economy* (Baltimore: The Johns Hopkins University Press). pp.3-14.
- [13] Burness, S., R. Cummings, Glen Morris, Inja Paik (1980), "Thermodynamic and Economic Concepts as Related to Resource-Use Policies", *Land Economics* 56(1), pp.1-9.
- [14] Cleveland, C.J. (1987), "Biophysical Economics: Historical Perspective and Current Research Trends", *Ecological Modelling* pp.47-73.
- [15] Common, Mick and Charles Perrings, *Towards an Ecological Economics of Sustainability*, Centre for Resources and Environmental Studies, Australian National University, unpubl. no date.
- [16] Crabbé, P.J. (1992), *Entropy and Economics: A Comment* (mnss).
- [17] Crabbé, P.J., M. Mueller, and N.V. Quyen (Working Paper), *Asymptotic Growth, Ecology and Thermodynamics*, Preliminary draft. University of Ottawa, Institute for Research on Environment and Economy.
- [18] Daly, Herman and Alvaro Umaña, eds. (1981), *Energy Economics and the Environment* (Boulder, Colorado: Westview Press Incorporated).

- [19] Daly, H.E. (1973), *Towards a Steady-State Economy*(San Francisco: W.H. Freeman).
- [20] Daly, H.E. (1985), "The circular flow of exchange value and the linear throughput of matter and energy: a case of misplaced concreteness", *Review of Social Economy*, pp.279-297.
- [21] Daly, H.E. (1987a), "The Economic Growth debate: What some economists have learned but many have not", *Journal of Environmental Economics and Management* 14, pp.323-336.
- [22] Daly, H.E. (1987b), "Filters Against Folly in Environmental Economics-The Impossible, the Undesirable, and the Uneconomic", in *Environmental Economics-The Analysis of a Major Interface*, Pillet and Murota, eds.
- [23] Daly, H.E. (1992), "Comment: Is the Entropy Law Relevant to the Economics of Natural Resource Scarcity?—Yes, Of Course it is!", *Journal of Environmental Economics and Management* 23, pp.91-95.
- [24] Davis, H.T. (1942), *The Theory of Econometrics*(Bloomington, IN.:Principia).
- [25] Depew, D.J. and B.H. Weber (1988), "Consequences of Nonequilibrium Thermodynamics for the Darwinian Tradition", in *Entropy, Information, and Evolution: New Perspective on Physical and Biological Evolution*, Weber, B.H. and D.J. Depew (eds.) (Cambridge: The MIT Press).
- [26] Faber, M. (1985), "A biophysical approach to the economy: entropy, environment and resources", pp.315-337 in *Energy and Time in Economics*

and *Physical Sciences* van Gool, W. and J. Buggink (eds.) (Amsterdam: North Holland).

- [27] Faber, Malte and John L.R. Proops (1985), "Interdisciplinary Research Between Economists and Physical Scientists: Retrospect and Prospect", *Kyklos* 38, Fasc.4, pp.599-616.
- [28] Faber, M., H. Niemes and G. Stephan (1987), *Entropy, Environment and Resources: An essay in Physico-Economics* (Berlin: Springer-Verlag).
- [29] Faber, Malte and John L.R. Proops (1990), *Evolution, Time, Production and the Environment* (Berlin: Springer-Verlag).
- [30] Finkelstein, M.O. and R.M. Friedburg (1966), "The application of an entropy theory of concentration to the Clayton Act", *Yale Law Journal* 76, p.677.
- [31] Gale, D. (1960), *Theory of Linear Economic Models*(New York: McGraw-Hill).
- [32] Georgescu-Roegen, N. (1971), *The Entropy Law and Economic Process*(Cambridge, Mass.: Harvard University Press).
- [33] Georgescu-Roegen, N. (1979), "Energy Analysis and Economic Valuation", *Southern Economic Journal* 45(4), pp.1023-1057.
- [34] Grebogi, Celso, Edward Ott, Steven Pelikan, and James Yorke (1984), "Strange Attractors that are Not Chaotic", *Physica* 13D, pp.261-268.
- [35] Gross, L.S. and E.C.H. Veendrop (1990), "Growth with Exhaustible Resources and a Materials-Balance Production Function", *Natural Resource Modeling* 4(1), pp.77-94.

- [36] Hatsopoulos, G.N. and J.H. Keenan (1965), *Principles of General Thermodynamics* (New York: J. Wiley & Sons).
- [37] Horowitz, A. and L. Horowitz (1967), "Entropy, Markov Processes and competition in the brewing industry", *Journal of Industrial Economics* 16, p.196.
- [38] Horowitz, I. (1970), "Employment concentration in the Common Market: an entropy approach", *Journal of the Royal Statistical Society* part 3, p.463.
- [39] Jean, R.V. and P.M. Auger (September, 1993), *Biological Systems* 1(3), (London: World Scientific).
- [40] Jørgensen, Sven Erik (1992), *Integration of Ecosystem Theories: A Pattern* (Netherlands: Kluwer Academic Publishers).
- [41] Kay, James J. (1989), "A Thermodynamic Perspective of the Self-Organization of Living Systems" in P.W.J. Ledington, Ed. *Proceedings of the 33rd Annual Meeting of the International Society for the System Sciences* (Edinburgh) 3, pp.24-30.
- [42] Kay, James J. (1991), "The Concept of Ecological Integrity, Alternative Theories of Ecology, and Implications for Decision-Support Indicators", in P. A. Victor, J.J. Kay, and H.J. Ruitenbeek, *Indicators of Ecologically Sustainable Development* (Canadian Environment Advisory Council) pp.23-58.
- [43] Kestin, J. (1976), "The Second Law of Thermodynamics", in Dowden, Hutchinson and Ross, *Benchmark Papers on Energy* v.5.

- [44] Koopmans, T. (1951), "Analysis of Production as an efficient combination of activities", in T.C. Koopmans (eds.) *Activity Analysis of Production and Allocation*(New York: J. Wiley & Sons).
- [45] Koopmans, T. (1979), "Economics Among the Sciences", *American Economic Review* 69, 1, pp.1-13.
- [46] Lisman, J.H.C. (1949), "Econometrics and Thermodynamics: A remark on Davis' theory of budgets", *Econometrica* 12, pp.59-62.
- [47] Malinvaud, E. (1953), "Capital accumulation and efficient allocation of resources", *Econometrica* 21, pp.233-269.
- [48] Mankiw, G. N. (1996), "My Rules of Thumb", *The American Economist* 40(1), pp.14-19.
- [49] Mirowski, Philip (1989), *Economics as Social Physics: Physics as Nature's Economics* (Cambridge, Mass.: Cambridge University Press).
- [50] Nicolis, G. and I. Prigogine (1977), *Self Organization in Non-Equilibrium Systems: From Dissipative Structures to Order through Fluctuations* (New York: Wiley & Sons).
- [51] Nordhaus, W.D. and J. Tobin (1973), "Is Growth obsolete?", in *The measurement of economic and social performance*, Moss, M. (ed.)(New York: Columbia University Press).
- [52] Odum, H.T. and E.C. Odum (1981), *Energy Basis for Man and Nature* (New York: McGraw-Hill Book Company).

- [53] Ondrechen, M.J., B. Andresen, M. Mozurkewich, and R.S. Berry (1981), "Maximum work from a finite reservoir by sequential Carnot cycles", *American Journal of Physics* 49, pp.681-685.
- [54] Perrings, C. (1991), "Ecological Sustainability and Environmental Control", *Structural Change and Economic Dynamics* vol.2, pp.275-295.
- [55] Prigogine, Ilya, G. Nicolis, and A. Babloyantz (1972), "Thermodynamics of evolution: Living systems defy the Second Law by staying far from equilibrium", *Physics Today* vol.25, no.11, pp.23-28.
- [56] Proops, J.L.R. (1985), "The Theory of Economic Growth and Thermodynamical Laws", in *Energy and Time in the Economic and Physical Sciences*, van Gool, W. and Bruggink, J.J.C. (eds.), (New York: North-Holland).
- [57] Proops, J.L.R. (1987), "Entropy, Information and Confusion in the Social Sciences", *The Journal of Interdisciplinary Economics* 1, pp.225-242.
- [58] Rogers, G.F.C. and Y.R. Mayhew (1980), *Engineering, Thermodynamics, Work, & Heat Transfer: Third edition* (New York: Longman).
- [59] Ruth, Mathias (1993), *Integrating, Economics, Ecology, and Thermodynamics* (The Netherlands: Kluwer Academic Publishers).
- [60] Samuelson, Paul A. (1965), *Foundations of Economic Analysis* (New York: Atheneum).
- [61] Samuelson, P.A. (1983), "Rigorous Observational Positivism: Klein's Envelope Aggregation; Thermodynamics and Economic Isomorphisms",

- in *Global Econometrics*, Adams, F.G. and Hickman, B.G. (eds.), (Cambridge, Mass.: The MIT Press).
- [62] Schneider, Eric D. and James J. Kay (1994), "Life as a Manifestation of the Second Law of Thermodynamics", *Math. Compt. Modelling* 19, pp.5-48.
- [63] Sears, F.W., M.W. Zemansky and H.D. Young (1985), *College Physics* (Massachusetts: Addison-Wesley Publishing Company).
- [64] Semple, R.K. (1973), "Recent trends in the concentration of corporate headquarters", *Economic Geography* 49, p.309.
- [65] Semple, R.K. and H.L. Gauthier (1972), "Spatial temporal trends in income inequalities in Brazil", *Geographical Analysis* 4, p.169.
- [66] Semple, R.K. and G.K. Demko (1977), "An Information theoretic analysis: an application to Soviet-COMECON trade flows", *Geographical Analysis* 9, p.51.
- [67] Simon, J.L. (1980), "Resources, population, environment: an oversupply of false bad news", *Science* 208, pp.1431-1437.
- [68] Soddy. F. (1912), *Matter and Energy*(London: Routledge).
- [69] Soddy. F. (1933), *Wealth, Virtual Wealth and Debt*(London: Allen & Unwin).
- [70] Soddy. F. (1934), *The Role of Money*(London: Routledge).
- [71] Solow, R.M. (1972), "Notes on Doomsday Models", *Proceedings of Natural Academy of Science* 69, pp.3832-3833.

- [72] Theil, H. (1967), *Economics and Information Theory*(Amsterdam: North Hollan).
- [73] Underwood, D.A. and P.G. King (1989), "The ideological foundations of environmental policy", *Ecological Economics* 1, pp.315-334.
- [74] Varian, Hal (1992), *Microeconomic Analysis, third edition* (W.W. Norton & Company: New York).
- [75] von Neumann, J. (1945), "A model of general economic equilibrium", *Review of Economic Studies* 13, pp.1-9.
- [76] Young, J.T. (1991), "Is the Entropy Law Relevant to the Economics of Natural Resource Scarcity?", *Journal of Environmental Economics and Management* 21, pp.169-179.