

CRITICAL VELOCITIES IN HELIUM II

by

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ABSTRACT

An attempt is made to investigate the wave form of a torsionally oscillating system in helium II. Such a system is known to follow normal damped S.H.M. below a certain critical amplitude  $\varphi_c$ . Above  $\varphi_c$  excess damping occurs as a result of the breakdown of the frictionless character of the superfluid component of helium II. This breakdown is attributed to the wall velocity of the oscillator being greater than a critical velocity  $v_{sc}$ . It is not known how the excess energy is extracted in any one cycle as  $v_{sc}$  is exceeded, but the possibility of large amounts of energy being involved is evident from previous experiments. The experiments described in this work indicate that a large amount of energy is extracted from the oscillator as its velocity rises through  $v_{sc}$  and that this energy is later returned to the oscillator as its velocity falls through  $v_{sc}$ . An attempt is made to relate these observations to the Onsager-Feynman vortex hypothesis.

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TABLE OF CONTENTS

Abstract .....	iii
Acknowledgements .....	iv
List of Illustrations .....	vi
Chapter 1 - Introduction .....	1
Chapter 2 - Design of the Experiment .....	5
Chapter 3 - Apparatus .....	8
Chapter 4 - Experimental Results .....	13
Chapter 5 - Conclusions .....	18
Appendix A .....	24
Appendix B .....	26
References .....	27

List of Illustrations

Fig. 1 - Cryostat Assembly .....	28
Fig. 2 - The Oscillator and Cam .....	29
Fig. 3 - Schematic Arrangement of Torsion Assembly and Optical Parts .....	30
Fig. 4 - Detailed Diagram of Optical Head .....	31
Fig. 5 - Differentiating Circuit .....	32
Fig. 6 - Tracing of the $\theta$ -t curve .....	33
Fig. 7 - Calibration of Photomultiplier Output .....	34
Fig. 8 - Cryostat Assembly .....	35
Fig. 9 - Torsion Head .....	36
Fig. 10 - Overall View of Instrumentation ..	37

## CHAPTER I

### INTRODUCTION

The concept of a critical velocity in helium II first arose out of a consideration of the results of such experiments as those of Daunt and Mendelssohn (1) in which the velocity of helium in a superfluid film reached a definite maximum value independent of pressure head. Such a maximum velocity should not have been expected on the basis of the simple two fluid model, the superfluid component of the HeII having a frictionless character. Daunt and Mendelssohn, however, assumed that at a certain critical velocity ( $v_{sc}$ ), the superfluid suddenly lost its frictionless character. Hence, in this experiment,  $v_{sc}$  is a limiting velocity for frictionless flow of the superfluid.

Many experiments have been designed and performed in which a "critical velocity" has been measured (directly or indirectly) which characterises a transition between two rates of energy dissipation. The lower dissipation has been attributed to frictional dissipation in the normal component alone and the higher to dissipation in the total fluid which occurs when the superfluid ceases to be frictionless.

The hydrodynamical experiments from which  $v_{sc}$  has been measured can be divided into two major categories: (a) the oscillating solid type and (b) the flow of helium in narrow channels.

The former is illustrated by the work of Hallett (2) and the latter by Atkins (3).

In Landau's theory of helium II (4) a critical velocity appears which is about two orders of magnitude greater than experiments indicate. This velocity is obtained by finding the minimum velocity required to generate roton excitations. However, the minimum velocity required to generate macroscopic quantized vortices of the Onsager-Feynman type (5) comes much closer to experimentally obtained values. The mechanism whereby vortices are generated is not clearly understood but the energy required to generate them will evidently account for the excess dissipation which occurs above  $v_{sc}$ . In experiments which involve the flow of helium II through narrow channels, the translational kinetic energy of the superfluid must under suitable conditions convert into vortex energy. However, in the case of the oscillating solid type of experiment, the energy required for the creation of vorticity, assuming a vortex free liquid at the start of the experiment, cannot come from the kinetic energy of the superfluid because there is none. It can only come from the oscillator itself by way of some interaction between the wall of the oscillator and the superfluid. In order to investigate the nature of such an interaction, observations should be made of the oscillator motion at the instant it passes from sub-critical to super-critical velocities.

It should be noted that in the oscillating type experiments, thus far performed, only the overall features of the oscillatory motion have been measured [e.g. maximum displacement in each cycle by Hallett (2), and the maximum velocity in each cycle by Benson (6)] and, as these features have been measured as the energy of the oscillator decays, the transition observed in these experiments has in all cases been from supercritical to sub-critical conditions. In particular, the logarithmic decrement below a certain critical amplitude  $\varphi_c$  is attributed to frictional dissipation in the normal fluid whilst the excess decrement which occurs above  $\varphi_c$  is attributed to the frictional behaviour of the superfluid. This behaviour may be the result of either the energy required for the creation of vortices, or to the mutual friction between the superfluid and the normal component of the helium II occurring as the result of the presence of vortices previously formed. Thus the maximum speed of the oscillator wall at  $\varphi_c$  may well be the critical velocity. If however, the decay time of the vortex motion is long compared with the periodic time of the oscillator, the occurrence of  $\varphi_c$  may only indicate that most of the vorticity created at higher velocities has now decayed and the effect of mutual friction has become negligible.

The value of  $v_{sc}$  obtained from these experiments may not therefore represent a true critical velocity of the superfluid.

From such considerations it was decided that an attempt must be made to observe the transition from sub-critical to supercritical conditions when it first occurs during the first cycle after release of a suitable oscillator from a sufficiently large static displacement. This experiment necessitates obtaining a displacement-time ( $\theta - t$ ) curve and noting the value of  $\theta$  at which the onset of any departure from damped S.H.M. occurs, This value of  $\theta$  is the critical displacement ( $\theta_c$ ) which occurs when the angular velocity ( $\dot{\theta} = d\theta/dt$ ) of the oscillator reaches a critical value  $\dot{\theta}_c$  corresponding to a critical wall velocity  $v_{sc}$ .

An attempt will also be made to obtain the velocity function ( $\dot{\theta}-t$  curve) of the oscillator, as such a curve would be more sensitive to any anomalous behaviour in the region of  $\dot{\theta}_c$ .

CHAPTER 2

DESIGN OF THE EXPERIMENT

2.1. Discussion of previous experiments.

The results of Hallett (2) indicate that the logarithmic decrement,  $\delta$ ,  $\left[ \delta = \frac{1}{2\pi} \frac{d(\ln \varphi_n)}{dn} \right]$  has a constant value only when  $\varphi < \varphi_c$ . This constant value will be denoted by  $\Delta$ . The excess decrement  $(\delta - \Delta)$  which occurs above  $\varphi_c$  is dependent on the amplitude and on the temperature of the helium for a given oscillator. The excess decrement can be related to the excess energy ( $\Delta E_s$ ) extracted in one cycle as follows:

$$\Delta E_s/E = 4\pi(\delta - \Delta) \quad (\text{appendix A}) \quad - - - 2-1$$

It is found that the excess decrement for a pile of discs of the type used by Hallett ranges from zero to  $6 \times 10^{-2}$  depending on the temperature of the helium and the amplitude of the oscillation [Hallett (2)]. Hence  $\Delta E_s/E$  ranges up to  $4\pi \times 6 \times 10^{-2}$  or approximately 0.75. This means that in one cycle, when the amplitude is sufficiently greater than  $\varphi_c$ , as much as 75% of the total energy of the oscillator can be extracted by the superfluid alone. As this energy is extracted by a non-linear frictional force, the departures from damped S.H.M. could be considerable. Furthermore, there is no way of knowing for certain whether this energy is extracted over a short period in the cycle or whether the extraction time extends over most of the cycle. Owing to the quantum nature of the vortex (5), it would

seem that some sort of discontinuous extraction should take place.

### 2.3. Selection of Oscillator.

The previous experiments of Hallett (2) and Eisele (7) were performed with piles of discs of the Andronikashvili type. Such a pile of discs was obtained from Professor Hallett and mounted in a similar manner, thus enabling previous values of  $\varphi_c$  and excess decrement to be used in the design of the experiment.

A smooth walled cylinder was also used for which the values of  $\varphi_c$  were obtained from the work of Anglin and Benson (6). Experiments were also conducted using a grooved cylinder (Fig. 2b) for reasons which will be discussed later (5,1)

### 2.4. Estimate of range of starting amplitudes.

The critical displacement  $\theta_c$  can be obtained from the critical amplitude  $\varphi_c$  as follows:

Assuming S.H.M.

$$\dot{\theta} = \frac{2\pi}{T} \left[ \varphi^2 - \theta^2 \right]^{\frac{1}{2}} \quad (2-2)$$

where T is the period of oscillation.

$$\text{Thus } \dot{\theta}_c = \frac{2\pi}{T} \left[ \varphi^2 - \theta_c^2 \right]^{\frac{1}{2}} \quad (2-3)$$

And assuming  $\dot{\theta}_c$  is the maximum angular velocity in the cycle when  $\varphi = \varphi_c$ ,

$$\dot{\theta}_c = \frac{2\pi}{T} \varphi_c \quad (2-4)$$

combining (2-3) and (2-4)

$$\theta_c = (\varphi^2 - \varphi_c^2)^{\frac{1}{2}} \quad (2-5)$$

Hence  $\theta_c$  for a given amplitude can be obtained for a particular oscillator for which  $\varphi_c$  is known.

Experimental values of  $\varphi_c$  for a pile of discs range from 0.1 to about 0.4 rads depending on the temperature of the helium. In order that  $\theta_c$  appear at a suitable position on the displacement-time curve, the condition

$$\theta_c \sim \varphi_0/2 \quad (2-6)$$

was chosen, where  $\varphi_0$  is the starting displacement. Inserting this condition in equation (2-5):

$$\varphi_0 \cong \frac{2}{\sqrt{3}} \varphi_c \quad (2-7)$$

Hence, in order to ensure that the probable values of  $\theta_c$  be adequately covered, the value of  $\varphi_0$  for the pile of discs should range from

approximately 0.15 to 0.5 radian.

The experiments described ranged from 0.25 to about 1.5 rads in starting displacement.

## CHAPTER 3

### APPARATUS

#### 3.1 The Oscillator.

The main oscillator used consisted of a pile of mica discs constructed as shown in Figure 2a. Two other oscillators were used, both hollow cylinders of the type shown in Figure 2b. One of these cylinders had smooth walls and the other was grooved as shown in the inset (Figure 2b).

A small magnet was mounted transversally through the shaft of each oscillator (Figure 2a and R in Figure 3) to provide a means of deflection. A 1/2 mm hole was drilled coaxially down the top of each oscillator shaft as a means of attaching to the torsion assembly.

#### 3.2 The Torsion Assembly. (Figure 3).

The oscillator was attached coaxially to the lower end of a one metre long 1/2 mm diameter quartz rod. (B, Fig. 3). The upper end of this rod was attached coaxially to a 10 cm long quartz fibre (E) which was in turn held rigidly to the top of the cryostat head. A pair of helmholtz coils (s) were mounted outside the dewar. These coils were capable of being rotated about the torsion axis, thus rotating the oscillator (A) to some suitable initial displacement by means of the torque

produced on the magnet (R). The magnitude of this initial displacement was measured by rotating the protractor (P) until the pin (Q) was observed to be in line with the centre of the mirror (C) and the image of the pin as observed in this mirror. The angle through which the protractor had been rotated from the rest position was a direct measure of the initial deflection angle ( $\varphi_0$ ) of the oscillator. The torsion system was then set oscillating by switching off the current in the helmholtz coils.

### 3.3 Instrumentation.

A cam-shaped disc (D) (and Figure 2c) having a profile of the form  $r = k\theta$  was mounted at the top of the quartz rod (B). A miniature lamp (F) was placed at the focal point of the achromatic doublet (G) and the collimated light emerging from this lens was allowed to pass into the cryostat head through the optical window (H). The beam was then bent through two right angles by the prisms(J) and allowed to fall onto a ground-glass window (K). As the collimated beam passed between the prisms, it was interrupted by the 'cam'. A 1/2 mm uniform slit (L) was placed close to the ground glass surface and the scattered light emerging from this slit was guided to the photocathode of a photomultiplier tube (N) by means of the parallel mirrors (M). Thus the output of the photomultiplier

was directly related to the angular position of the cam (D).

The photomultiplier output was then fed directly to a Leeds & Northrup potentiometer pen recorder which had a chart speed of one inch per second and a full chart width response time of 0.4 seconds.

A differentiating circuit (Figure 5a) was constructed and connected up as shown (Figure 5b) in order that the displacement function could be converted into a velocity function.

A small protuberance was built onto the cam (Fig. 2c) in order that the zero-deflection position would register on the chart. In order to locate the position on the chart at which the oscillator was released, a small secondary coil (T) (Figure 3) was placed at the centre of one member of the helmholtz pair. The decay of the field in the helmholtz coils produced a small pulse in this secondary coil and this in its turn was fed directly into the pen recorder circuit showing up as a short duration pulse on the chart. Figure 6 shows the manner in which these two locating pulses show up on a typical curve of the first quarter cycle.

### 3.4 Displacement calibration.

The angular displacement of the cam was measured as a function of photomultiplier output. This was done by deflecting and holding the suspension at about  $100^\circ$  with the helmholtz coils. The deflection angle was measured with the

protractor as previously described and the corresponding position of the pen was recorded by running the chart momentarily. The deflection of the suspension was then reduced by small intervals and the above process repeated each time.

A graph of deflection (in degrees) against photo-multiplier output (in mv) is shown in Figure 7. This graph indicates only slight departures from linearity over the range -80 degrees to +80 degrees. It can be estimated from this graph that the pen recorder is capable of linearly following the displacement of the oscillator to an accuracy of about  $\pm 1$  degree.

However, this whole system was capable of responding to a swinging motion of the oscillator as well as to the torsional motion. No attempt was made to prevent the swinging motion from being recorded but by suitably isolating the cryostat from local vibrations, such unwanted effects were kept at a minimum.

### 3.5 Cryostat Assembly.

In order to isolate the torsion assembly from disturbing vibrations, the cryostat was rigidly attached to a 3000 lb concrete block which sat on four rubber ("isomode") feet (Figure 1). This arrangement had a natural period of oscillation of approximately a half second, thus providing isolation from local disturbances of shorter periods. As a further precaution against disturbances, all experimental runs

were performed at times when local vibrations were at a minimum.

The helium dewar was connected to the four-inch laboratory pumping line by way of four one-inch flexible "tygon" tubes lined with steel springs for support of the walls.

The temperature of the helium was measured with mercury and oil manometers connected directly to the helium vapour region of the dewar. Below the lambda temperature, the oil manometer only was used, the levels in the two arms being read accurately with a cathetometer.

The temperature of the helium was held constant by means of the following control mechanism: A carbon resistance (10 ohms at room temperature) which constituted one arm of an A.C. wheatstone bridge was immersed in the helium. At a given pumping speed the bridge was first balanced and then unbalanced in such a manner that the out of balance bridge current, after suitable amplification, could be supplied to a heating element immersed in the helium. Any slight change in the pumping speed would then be compensated for by a corresponding reverse change in the power dissipation in the heating element. By this means, the temperature of the helium was held constant to a few millidegrees.

EXPERIMENTAL RESULTS

4.1 The displacement-time curves obtained with cylinders.

Preliminary experiments carried out using a smooth walled stainless steel cylinder showed no measurable departures from damped S.H.M.

In an attempt to increase the overall sensitivity of the apparatus, aluminum cylinders were constructed having about one third the moment of inertia of those previously used. Furthermore, the walls of these new cylinders were grooved as shown in Figure 2b. It was believed that the grooves would aid the onset of vorticity in the form of vortex rings whose energies could be readily calculated from the dimensions of the grooves.

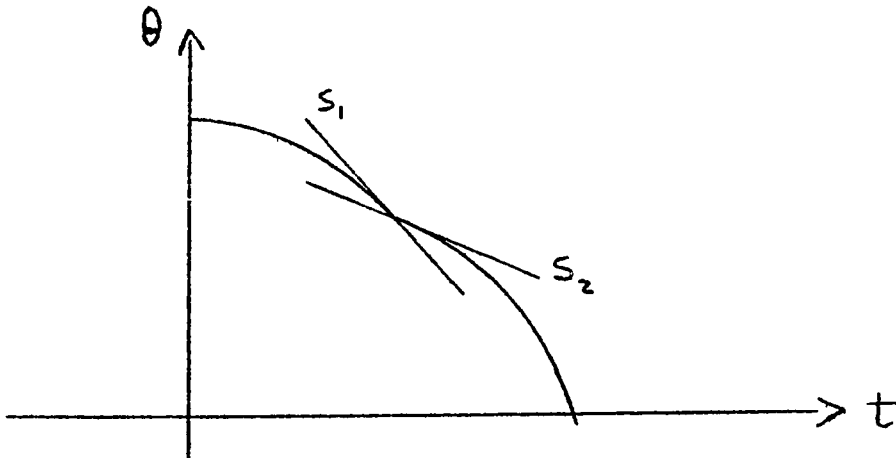
It was assumed that a cylinder of this sort would behave in much the same manner as a pile of discs; but whereas the onset of vorticity associated with the discs occurs in a region which, at any instant, has a wide range of velocities, that associated with a grooved cylinder occurs in a region where the wall velocity is essentially a single value at any instant. Thus, the term 'wall velocity' would have a more exact meaning for such a cylinder than for a pile of discs.

Despite the large number of experiments performed with these grooved cylinders, no evidential departures from damped S.H.M. were observed. On a number of occasions however,

departures were observed but were subsequently ascribed to poorly aligned elements in the oscillating system. Henceforth, extreme care was taken to achieve a sufficiently well aligned system to ensure that any departures from a smooth  $\theta$ - $t$  curve were not due to causes other than the onset of vorticity in HeII,

4.2 The displacement-time curves obtained with discs.

A final series of experiments were conducted using a pile of discs and it is believed that a real effect was observed during this series. Several traces were obtained on which a sudden change in the slope of the  $\theta$ - $t$  curve occurred shortly after the discs had been released. A tracing of one of these curves is shown in Figure 6, where the time interval over which the change from a slope  $S_1$  to a slope  $S_2$  is seen to be very short (perhaps about 0.1 seconds).



Sketch showing the change in slope from  $S_1$  to  $S_2$ .

Such a change would occur if, at this moment, an amount of energy ( $\Delta E$ ) were extracted from the total kinetic energy of the oscillator given by

$$\frac{\Delta E}{E} = \frac{s_1^2 - s_2^2}{s_1^2} \quad (4-1)$$

From the curve shown, this extraction amounts to:

$$\frac{\Delta E}{E} = 50 \pm 15\%$$

Assuming that the displacement at which this takes place is the true critical displacement ( $\theta_c$ ) from which can be calculated a critical angular speed ( $\dot{\theta}_c$ ) and a critical peripheral speed ( $V_{sc}$ ), the following results were obtained:

$$\theta_c = 0.31 \text{ rads.}$$

$$\dot{\theta}_c = 0.08 \text{ rads. sec}^{-1}$$

$$V_{sc} = 0.12 \text{ cm sec}^{-1}$$

The value of  $V_{sc}$  found here is the peripheral speed of the discs at the instant  $\theta = \theta_c$ . This figure is in close agreement with that given for a single disc by Atkins (9).

The fractional energy  $\left(\frac{\Delta E}{E}\right)_t$  extracted during the whole cycle can be calculated from the initial amplitude ( $\varphi_0$ )

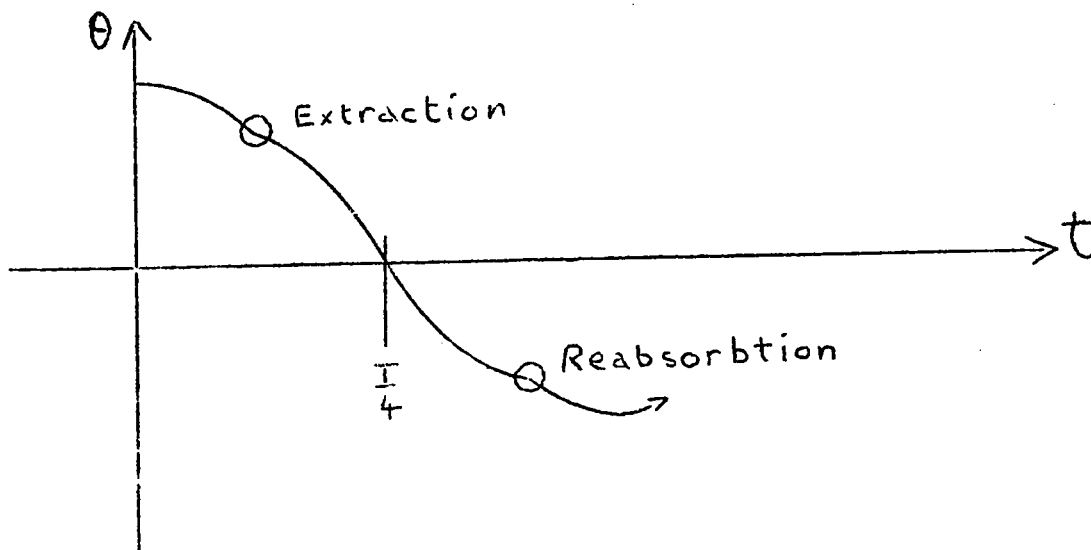
and the amplitude ( $\varphi_1$ ) at the end of the cycle:

$$\left(\frac{\Delta E}{E}\right)_t = \frac{\varphi_0^2 - \varphi_1^2}{\varphi_0^2}$$

And, using the values of  $\varphi_0$  and  $\varphi_1$  taken from the curves in Figure 6:

$$\left(\frac{\Delta E}{E}\right)_t \cong 7\%$$

Thus, even though it appears that 50% of the energy was extracted at one point in the cycle, yet only 7% is extracted over the whole cycle. This result might be explained if the 50% extracted during the first quarter cycle was largely reabsorbed in the second quarter cycle as the oscillator passed from super-critical to sub-critical velocities. Some evidence for this reabsorption can be seen in the original  $\theta$ - $t$  curves.



Sketch showing extraction and reabsorption.

The data obtained from the curves in which the effect described occurred are shown in the following table.

T	$\varphi_0$	$\theta_c$	$\dot{\theta}_c$	$v_{sc}$	$\Delta E/E$	$(\Delta E/E)t$
$^{\circ}K$	Rad	Rad	Rad $sec^{-1}$	cm $sec^{-1}$	%	%
2.13	0.35	0.29	0.096	0.14+0.01	40+15	34
2.13	0.24	6.17	0.077	0.12	40	41
1.73	0.35	0.30	0.072	0.11	40	15
1.42	0.35	0.31	0.077	0.11	50	8
1.20	0.35	0.31	0.081	0.12	50	7

#### 4.2 The velocity-time curves.

Velocity-time curves were obtained on several occasions. However, the extreme sensitivity of the differentiating circuit to small unwanted mechanical vibrations has so far resulted in curves of no real value. The sensitivity of the circuit to these vibrations can only be reduced at the cost of sensitivity to the effects being looked for. However, more elaborate attempts to isolate the oscillator from its surroundings may well be made prior to embarking on a new series of experiments.

CHAPTER 5

CONCLUSION

5.1 Introductory Remarks.

The sparsity of results in this work has been largely due to the difficulty of determining and then obtaining the necessary conditions required to make sensible observations of the predicted effects. It is understood now that the minimum requirements are: (a) that the amplitude of the oscillator be sufficiently close to the critical amplitude in order that the cusp associated with the critical velocity be at the most sensitive part of the  $\theta$ - $t$  curve; (b) that the sources of unwanted vibration be effectively eliminated; (c) that the oscillator be well aligned; (d) that the oscillator be held in its initial deflection position for a sufficiently long time (i.e. about twenty minutes) prior to release, in order to ensure a vortex free superfluid. These requirements have now all been realized.

The most suitable oscillator for observing the effect seems to be the pile of discs, though this may be solely due to its low moment of inertia. Further experiments will be performed using other oscillators after a more complete experimental series has been carried out with the discs.

Meanwhile, some discussion of the theoretical aspects of the present observations will be attempted. The

next section of this chapter suggests, rather crudely, a mechanism based on the Onsager-Feynman vortex theory by which energy is extracted from the discs to generate vorticity.

## 5.2 Calculation of $\frac{\Delta E_s}{E}$ .

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The creation of vorticity in HeII requires the expenditure of energy, and the Onsager-Feynman vortex theory yields for the energy  $\epsilon$  of one vortex ring formed when  $V > V_{sc}$  [Atkins (11)]:

$$\epsilon \simeq 2\pi^2 R \rho_s \frac{\hbar^2}{m^2} \ln \left( \frac{R}{a} \right) \quad (5-1)$$

where  $\rho_s$  is the density of the superfluid,  $m$  is the mass of the helium atom,  $a$  is the atomic spacing and  $R$  is the radius of the vortex ring.

In order to calculate how much energy would be required to completely fill the volume of space between the discs with vortex rings, it is necessary to make certain assumptions about the radius,  $R$ , of the vortex rings. Assuming that  $R$  is governed by the dimensions of the region between the discs where the vortices probably form, it has been shown, Atkins (8), that fair agreement with observation is obtained if  $R = d/4$ , where  $d$  is the width of the region. In this case  $d \simeq 0.05$  cm.

Selecting a temperature of about  $1.2^{\circ}\text{K}$  where  $\rho_s/\rho \simeq 0.98$  and  $\rho_s \simeq 0.14 \text{ gm cm}^{-3}$

$$\begin{aligned}\epsilon &= 1.6 \times 10^{-7} R \log_{10} \left( \frac{R}{2} \times 10^8 \right) \\ &= 1.3 \times 10^{-8} \text{ ergs/ring}\end{aligned}$$

From a consideration of the size of the vortex rings and the total space between the discs, it can be easily shown (appendix B) that the number of rings required to fill this region is approximately  $2.6 \times 10^4$ . Thus the total energy required to fill the region is given by

$$\Delta E_s = \epsilon \times 2.6 \times 10^4 = 3.6 \times 10^{-4} \text{ ergs.}$$

Assuming that this energy is suddenly extracted from the discs kinetic energy, the moment vorticity commences, then the kinetic energy ( $E$ ) of the discs at this moment would be required in order to estimate the magnitude of the fractional energy extraction  $\frac{\Delta E_s}{E}$ .

The angular velocity  $\dot{\theta}_c$  of the discs when the extraction is about to take place is equal to the slope  $S_1$  (Figure 6) and is about 0.11 rads/sec and the moment of inertia of the discs, Hallett (10), is  $1.97 \text{ gm cm}^2$ . Consequently the kinetic energy of the discs is

$$E = \frac{1}{2} I \dot{\theta}_c^2 \simeq 1.2 \times 10^{-2} \text{ ergs.}$$

$$\frac{\Delta E_s}{E} \cong \frac{3.6 \times 10^{-4}}{1.2 \times 10^{-2}} = 3 \times 10^{-2}$$

i.e.  $\frac{\Delta E_s}{E} \cong 3\%$

This figure then is derived using a model whereby the energy extracted from the discs is simply that required to fill the region between the discs with vorticity. If, however, the rate of propagation of vorticity through the liquid is sufficiently great, then the value  $\Delta E$  may be that required to fill some of the surrounding volume with vorticity, then  $\frac{\Delta E_s}{E}$  could become considerably larger than 3%.

However, this calculated value is considerably smaller than the observed figure of 50% indicating the incompleteness of the suggested mechanism.

### 5.3 Calculation of critical velocity.

The use of the Onsager-Feynman vortex theory to calculate the critical velocity ( $V_{sc}$ ) for the discs used yields a close agreement with that calculated from observations.

The equation 5-1 gives the energy  $\epsilon$  of the vortex ring and its momentum perpendicular to the plane of the ring can be shown to be [Atkins (11)]:

$$p \cong 2\pi^2 R^2 \rho_s \frac{h}{m} \quad (5-2)$$

and the critical velocity is given by [Atkins (11)]:

$$V_{sc} = \frac{\epsilon}{p} = \frac{h}{mR} \ln\left(\frac{R}{a}\right) \quad (5-3)$$

and using  $R = \frac{d}{4}$  as in the previous section:

$$V_{sc} \approx \frac{4h}{m} \ln\left(\frac{d}{4a}\right) \quad (5-4)$$

Using this equation, the value of  $V_{sc}$  becomes:

$$V_{sc} = 0.15 \text{ cm sec}^{-1}$$

which compares favourably with the values found from observation (page 15).

This calculation of  $V_{sc}$  shows that the use of the Onsager-Feynman vortex theory is not wholly unrealistic.

#### 5.4 Further Experiments.

The effect described in this thesis is believed to be a genuine phenomenon associated with vortex formation in the superfluid. In order to gain more detailed information about the nature & magnitude of the effect, a series of experiments is planned. With only minor modifications to the apparatus, an attempt will be made to obtain a set of  $\theta$ - $t$  curves covering a wide range of small amplitudes and temperatures. It also seems that some successful results might be obtained varying the distance between the oscillator and the surrounding walls.

The present work has been essentially an attempt to observe a dissipative effect at the instant  $V_{sc}$  is exceeded. It is hoped that these observations will ultimately elucidate the mechanism of the critical velocity though at this stage insufficient quantitative data have been collected.

However, now that the effect is believed to have been observed, the necessity of continuing these experiments is apparent.

APPENDIX A

The decrement  $\delta$  is defined as

$$\delta = \frac{1}{2\pi} \left[ \frac{d(\ln \varphi_n)}{dn} \right] \quad (1)$$

where  $n$  is the number of oscillations. When the amplitude is sufficiently small that there exists no vorticity ( $\varphi < \varphi_c$ ), then the decrement is a constant ( $\Delta$ ) defined as:

$$\Delta = \frac{1}{2\pi} \left[ \frac{d(\ln \varphi_n)}{dn} \right]_0 \quad (2)$$

Expressing  $\delta$  and  $\Delta$  in terms of the change in amplitude ( $d\varphi_n$ ) per cycle providing this is small:

$$\delta = \frac{1}{2\pi} \left( \frac{d\varphi_n}{\varphi_n} \right)$$

$$\Delta = \frac{1}{2\pi} \left( \frac{d\varphi_n}{\varphi_n} \right)_0$$

$$\text{And } (\delta - \Delta) = \frac{1}{2\pi} \left[ \left( \frac{d\varphi_n}{\varphi_n} \right) - \left( \frac{d\varphi_n}{\varphi_n} \right)_0 \right]$$

And the energy ( $E_n$ ) of the oscillator when its amplitude is  $\varphi_n$  is given by

$$E_n \propto \varphi_n^2$$

$$\text{Thus: } \frac{dE_n}{E_n} = 2 \frac{d\varphi_n}{\varphi_n}$$

$$\therefore (\delta - \Delta) = \frac{1}{4\pi} \left[ \left( \frac{dE_n}{E_n} \right) - \left( \frac{dE_n}{E_n} \right)_0 \right]$$

$\left( \frac{dE_n}{E_n} \right)_0$  is a constant at all amplitudes, provided there is no

turbulence, and represents the normal fluid contribution to the decrement. Thus we may write

$$(\delta - \Delta) = \frac{1}{4\pi} \left[ \frac{dEn - (dEn)_o}{En} \right]$$

the quantity  $[dEn - (dEn)_o]$  representing the energy  $(\Delta E)_s$  extracted in a given cycle by the superfluid.

$$\text{Thus } (\delta - \Delta) = \frac{1}{4\pi} \frac{(\Delta E)_s}{En}$$

$$\text{And } \frac{(\Delta E)_s}{En} = 4\pi(\delta - \Delta).$$

APPENDIX B

The number of vortex rings which can be generated in the inter-disc region will be given by

$$N = \frac{\text{volume of region}}{\text{volume of one ring}}$$

It is suggested that one ring will occupy a volume  $\simeq d^3$  where  $d$  is the disc spacing.

$$\text{Hence } N = \frac{\pi(R_d^2 - R_s^2)n}{d^3}$$

where  $R_d$  is the radius of the discs = 1.52 cm,

$R_s$  is the radius of the spacers = 0.609 cm,

$n$  is the number of spacers = 13,

$d = 0.553$  cm.

From which  $N = 2.6 \times 10^4$ .

REFERENCES

1. J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. A, 170, 423, 439 (1939).
2. A. C. Hollis-Hallett, Proc. Roy. Soc. A, 210, 404 (1952).
3. K. R. Atkins, Proc. Phys. Soc. A, 64, 833 (1951).
4. L. Landau, J. Phys. (U.S.S.R.), 5, 71, (1941).
5. R. P. Feynman, Progress in Low Temperature Physics, Vol. 1, (Edited by C. J. Gorter) Chapter II.
6. F. Anglin and C. B. Benson, VIIth International Conference on Low Temperature Physics, P. 558 (Toronto 1960).
7. K. M. Eisele, Ph.D. Thesis, University of Toronto (1958).
8. K. R. Atkins, 'Liquid Helium' (Cambridge 1959) P. 201.
9. K. R. Atkins, 'Liquid Helium' P. 199.
10. A. C. Hollis-Hallett, Ph.D. Thesis, University of Cambridge (1951).
11. K. R. Atkins, 'Liquid Helium', P. 116.

# CRYOSTAT ASSEMBLY

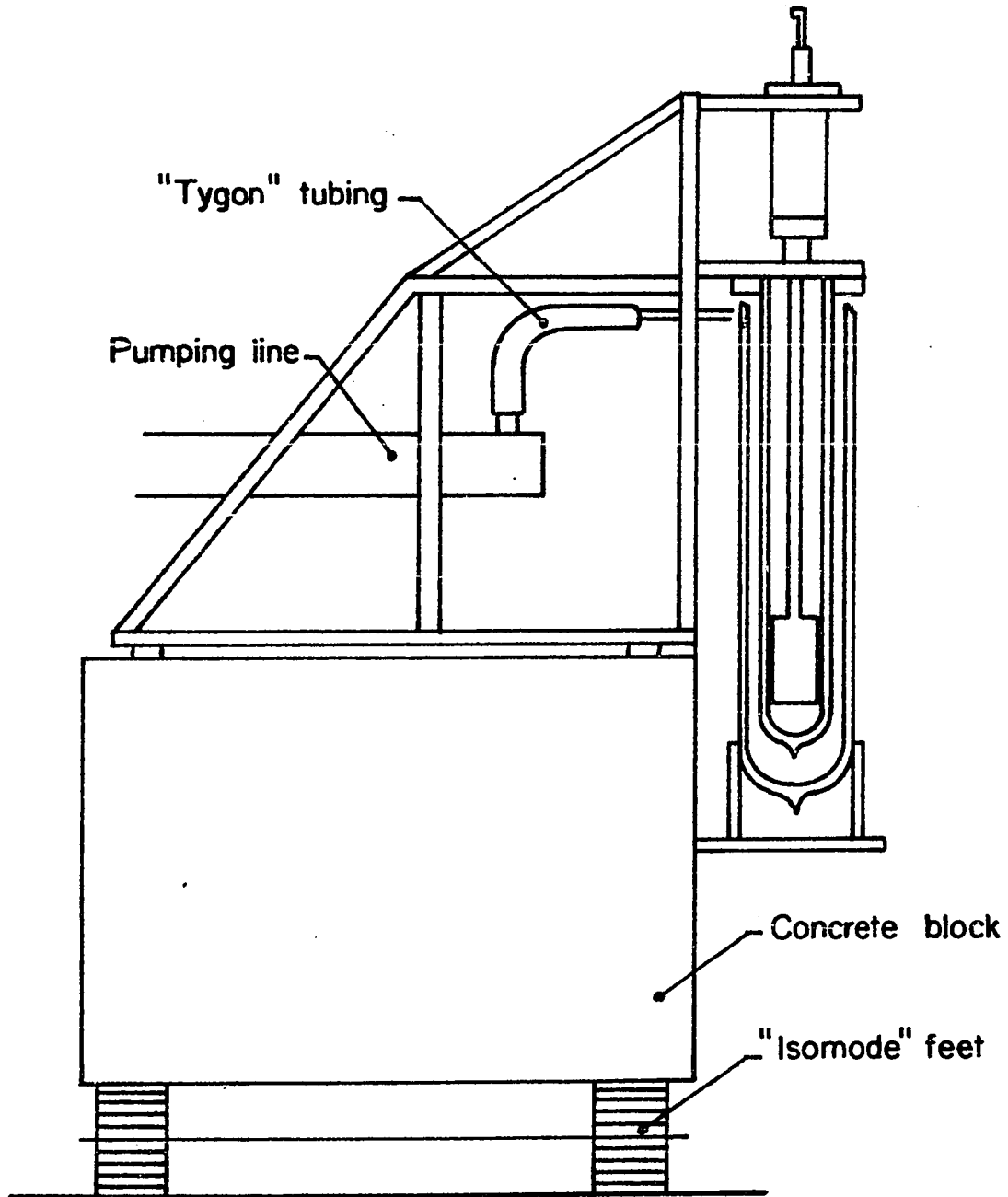
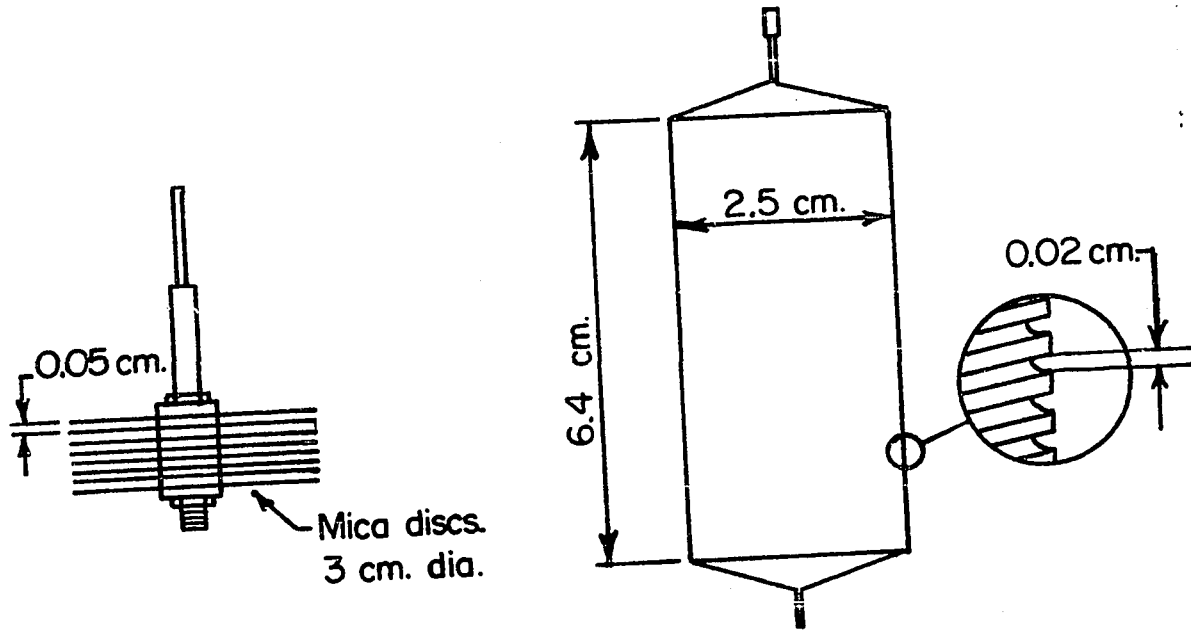


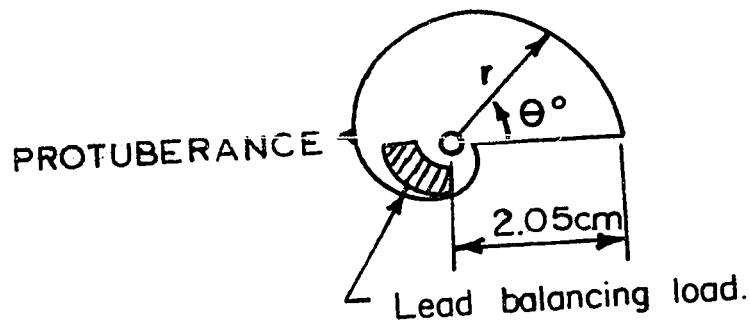
FIG. 1

# THE OSCILLATOR AND CAM



(a) ANDRONIKASHVILI PILE (b) GROOVED CYLINDER  
Aluminum with brass end cap.

Cut from 0.025 cm.  
aluminum sheet.  
 $r = 2.05 - 0.01 \theta$



(c) THE "CAM"



DETAILED DIAGRAM OF OPTICAL HEAD

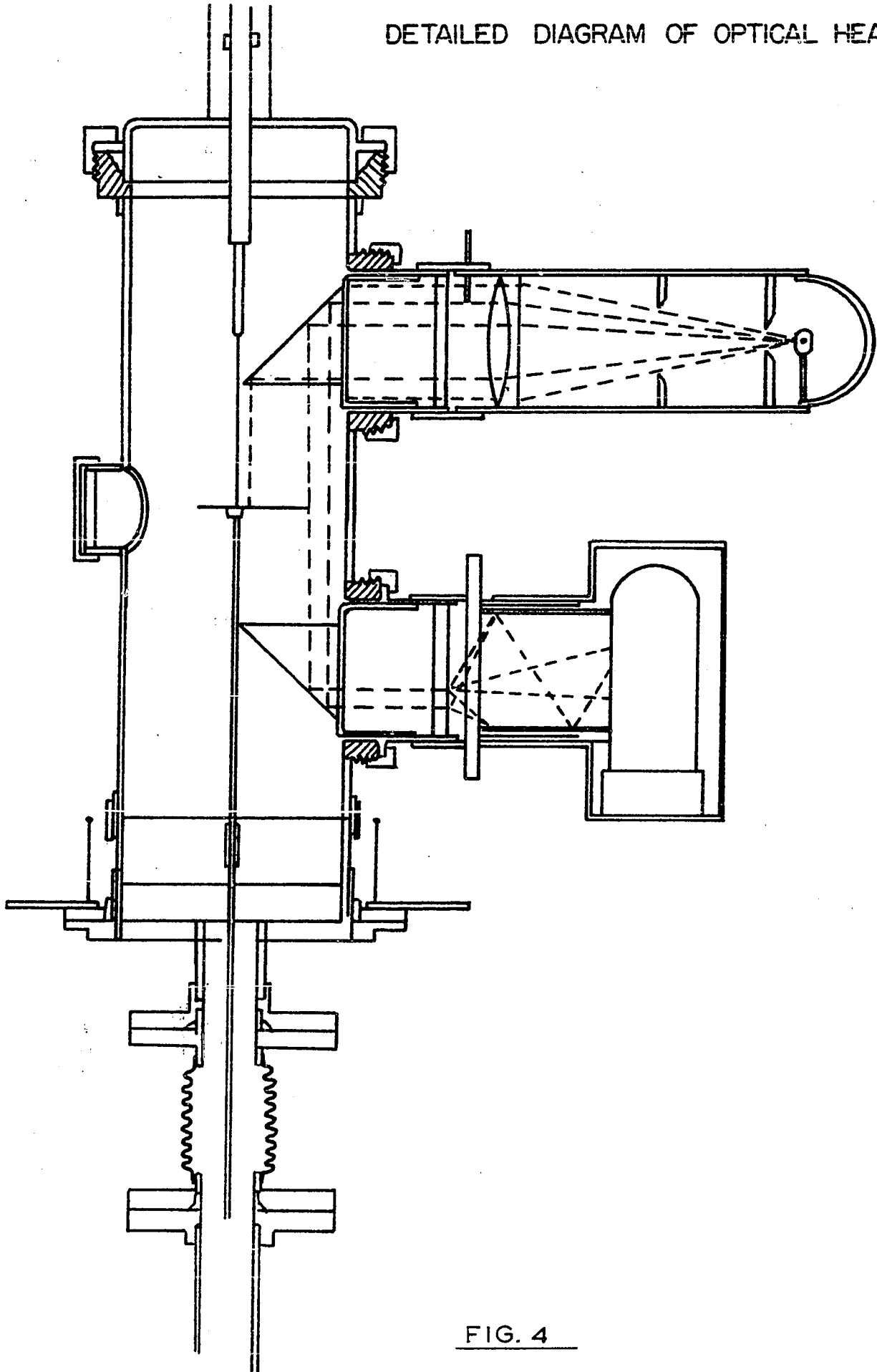
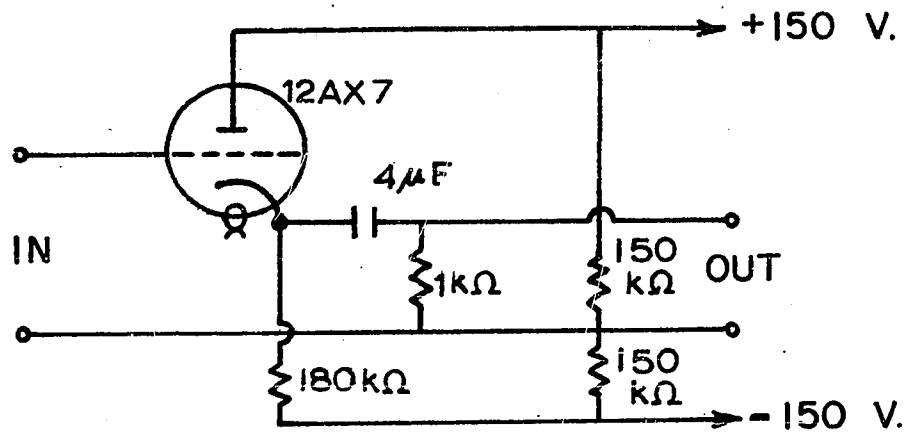
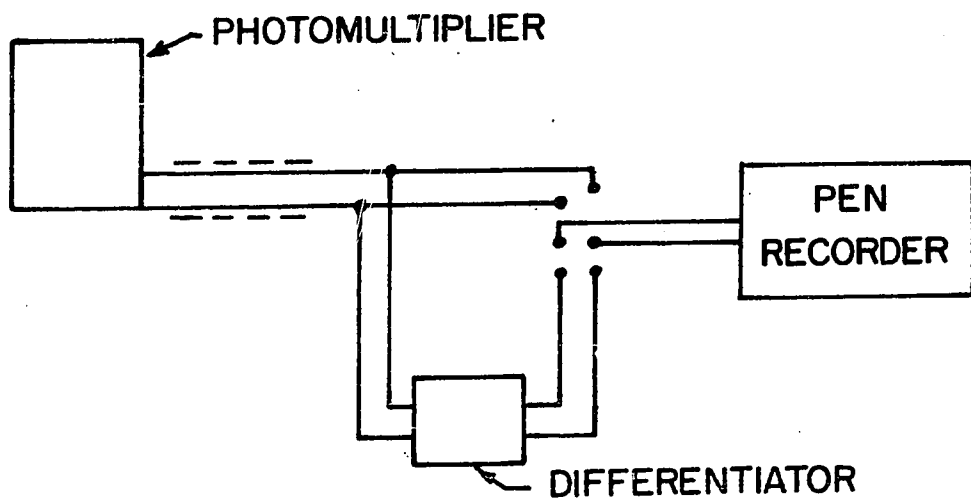


FIG. 4



(a)



(b)

THE DIFFERENTIATOR CIRCUIT

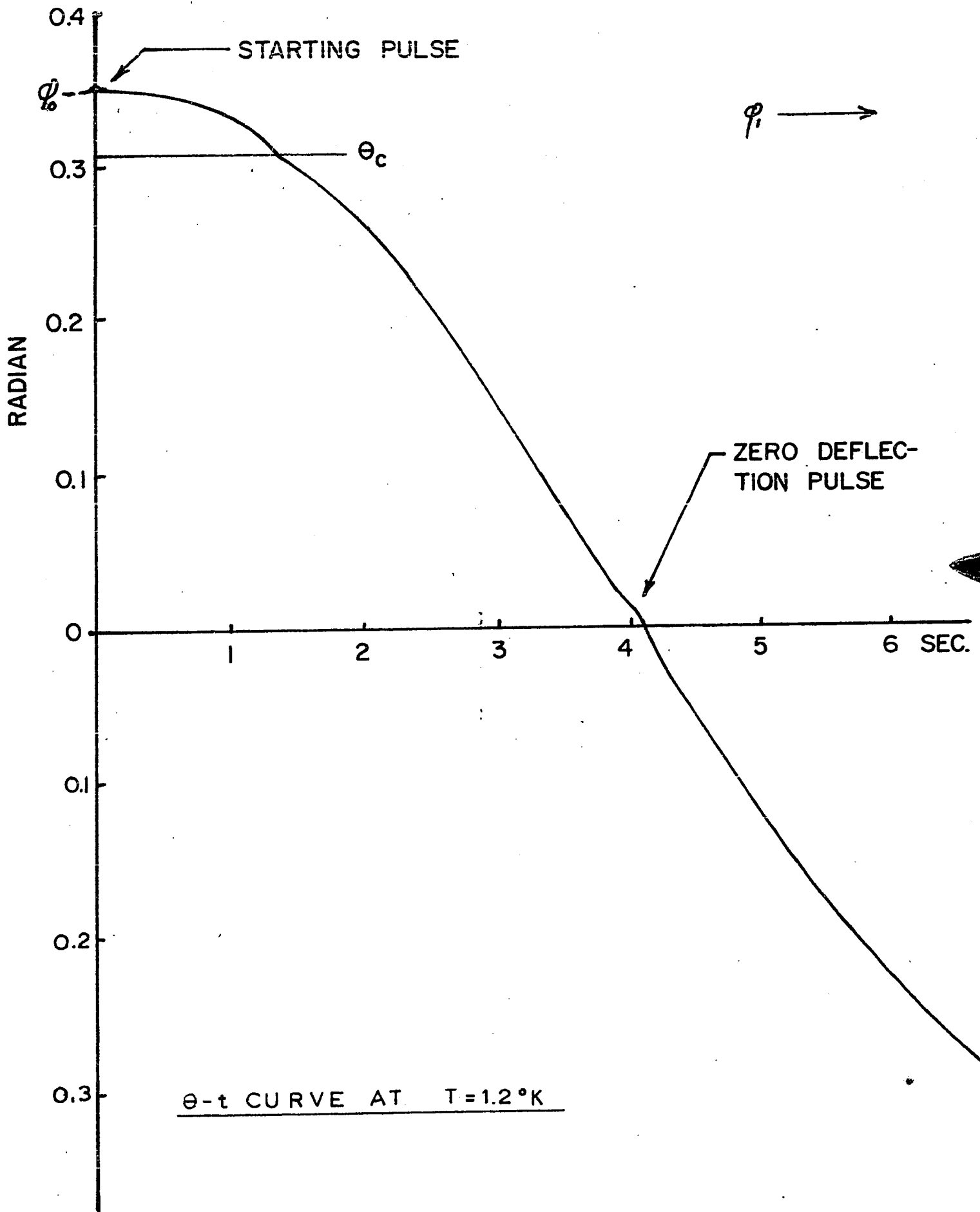


FIG. 6

CALIBRATION OF THE PHOTOMULTIPLIER OUTPUT.

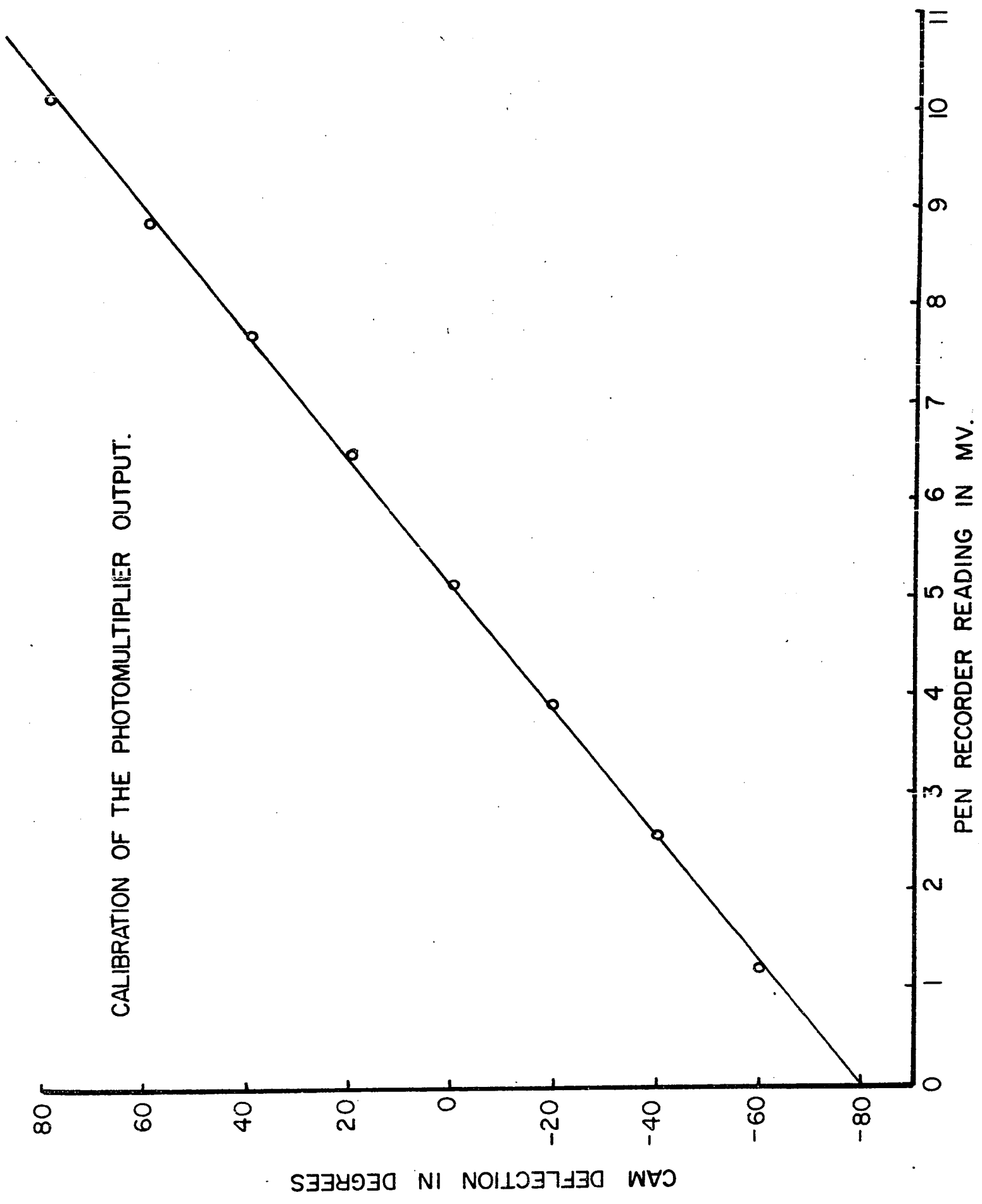
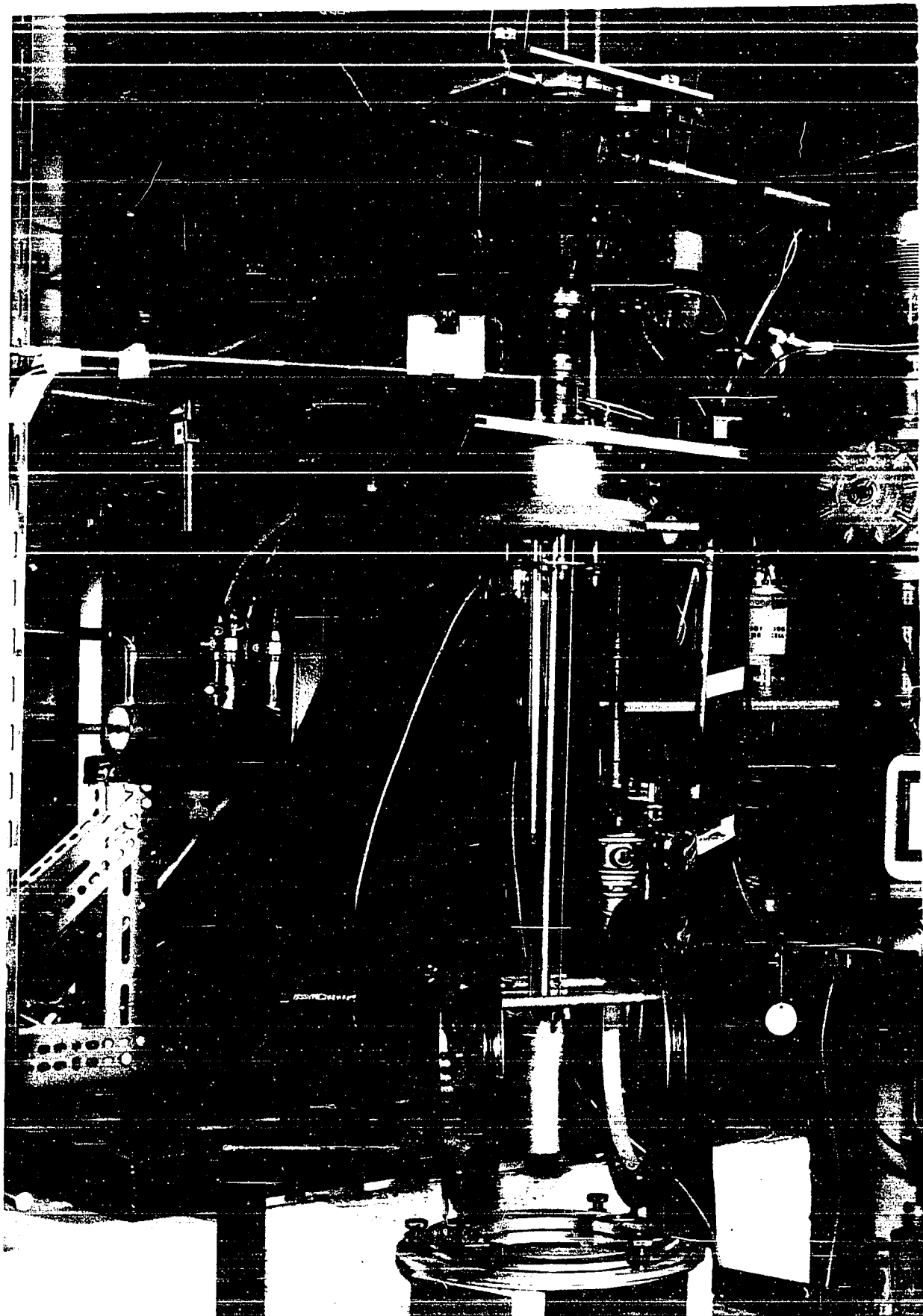
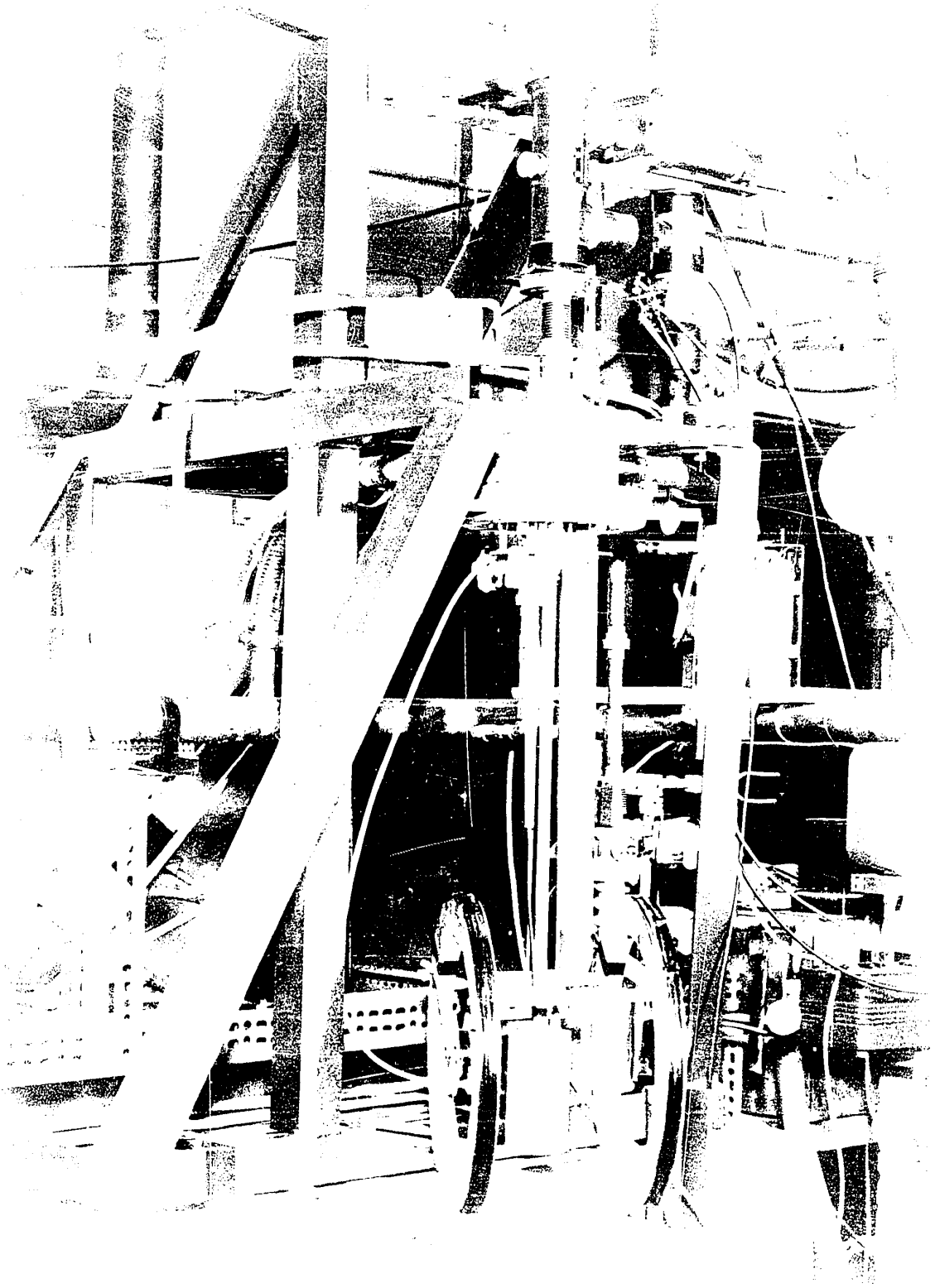


FIG. 7



THE CRYOSTAT ASSEMBLY

FIG. 8





THE TORSION HEAD

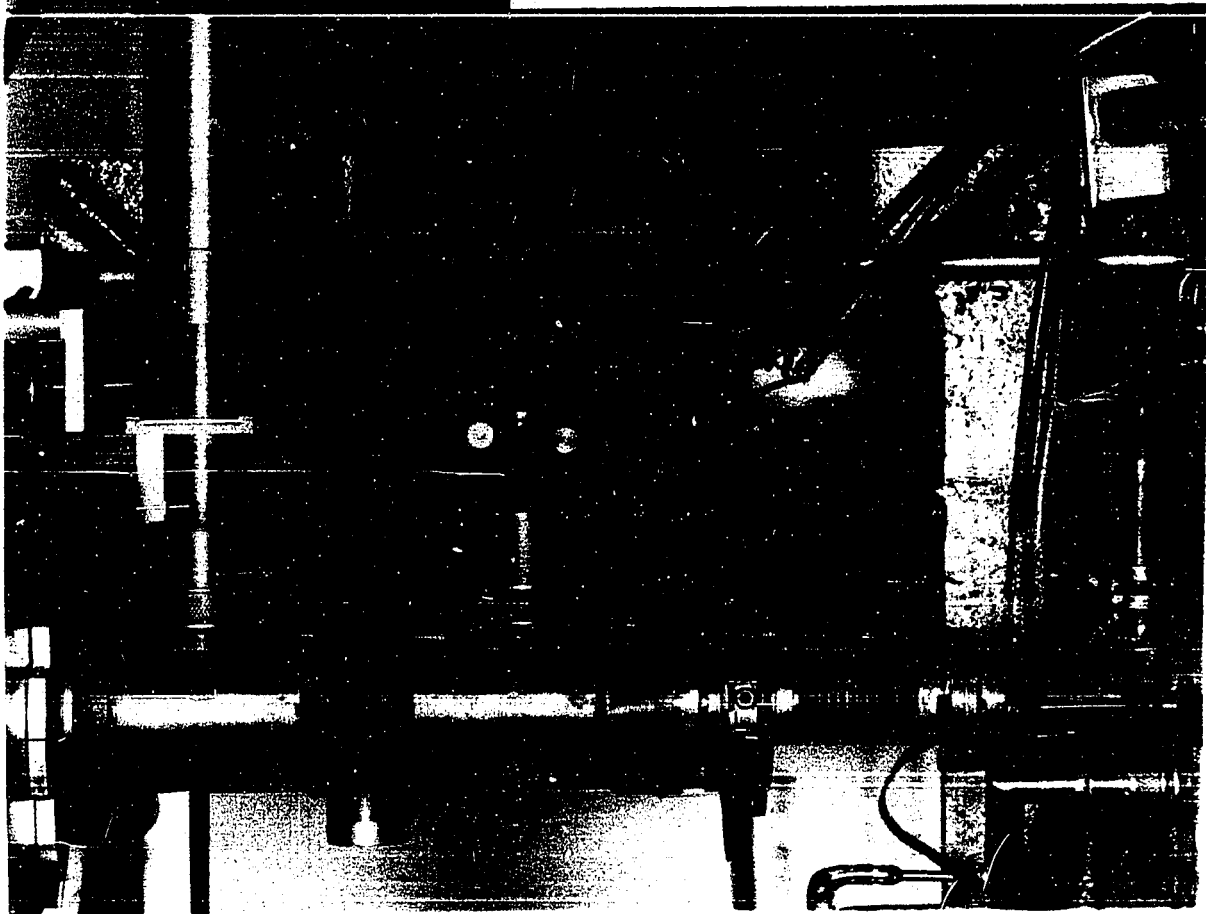
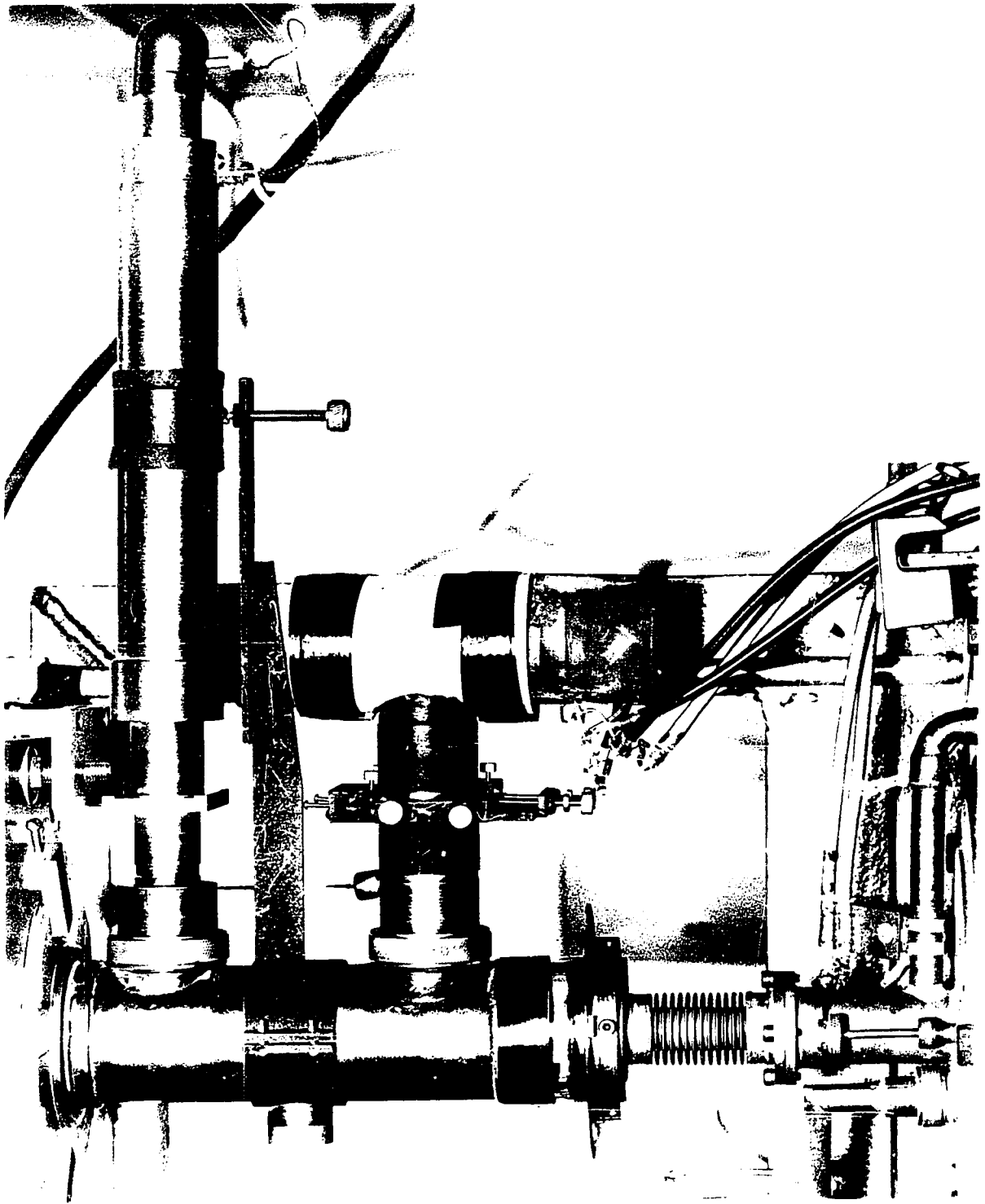
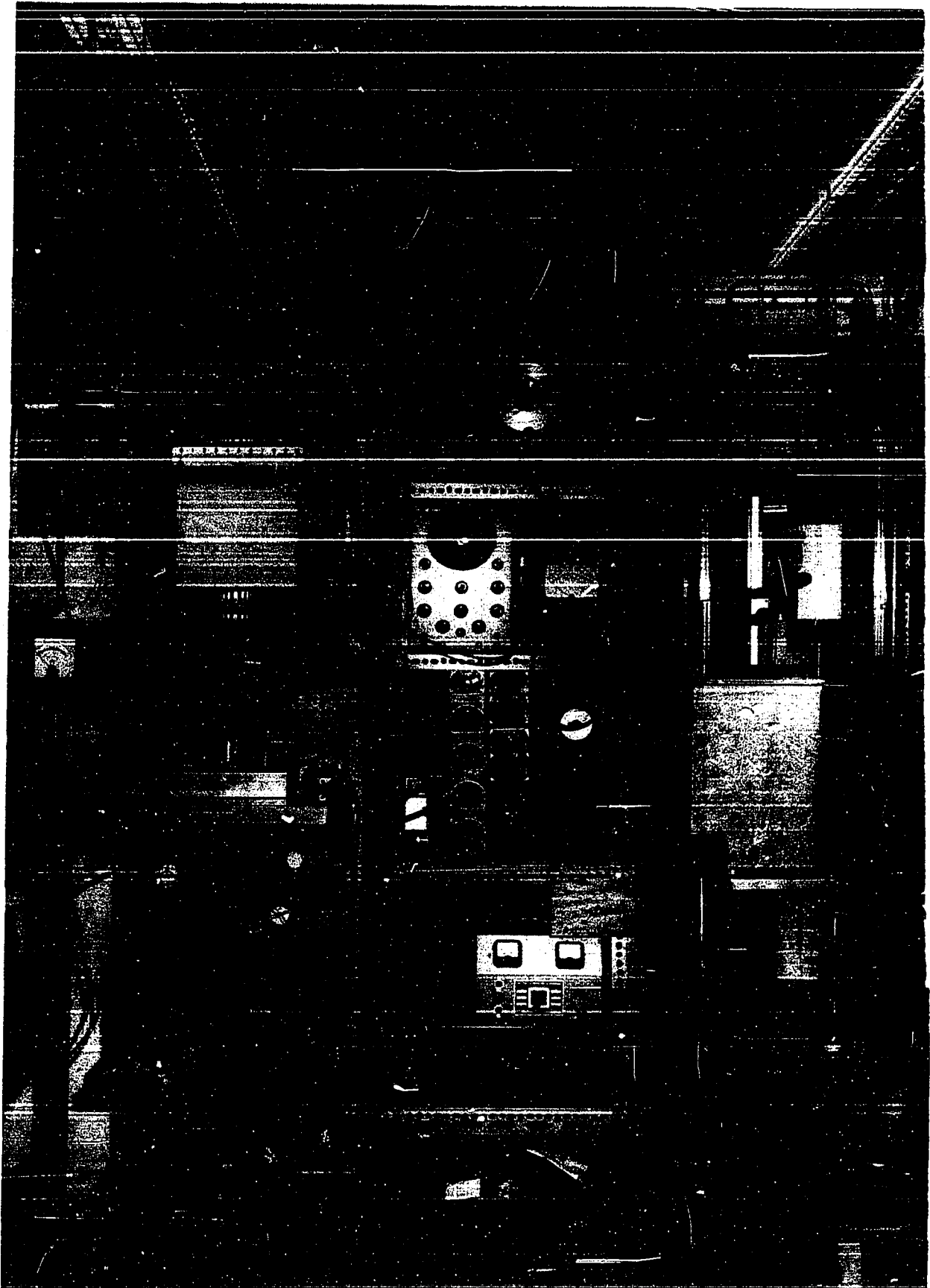


FIG. 9





AN OVERALL VIEW OF THE INSTRUMENTATION

FIG. 10

