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**LA THÈSE A ÉTÉ
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Flux Line Cutting and Flux Pinning Phenomena
in Hollow Cylinders of Type II Superconductors

by

Gilles Fillion

Thesis submitted to the School of Graduate Studies
of the University of Ottawa in partial fulfilment
of the requirements for the degree
of Doctor of Philosophy in Physics

Physics Department
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1986

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Abstract

We have investigated the effect of static azimuthal magnetic fields H_ϕ bathing the wall of a hollow cylinder of a hysteretic type II superconductor (Pb Bi) on its electrodynamic response to slow sweeps of an externally applied longitudinal magnetic field $H_{//}$. The specimen is made to become superconducting in the chosen H_ϕ and the initial $H_{//}$ by cooling from T_c to 4.2 K for all the measurements. The amplitude of the swings of $H_{//}$ extend far beyond the range required to cause changes in B_l , the longitudinal flux density in the hole of the tube. The magnitude of the critical induced azimuthal persistent current I_ϕ opposing flux entry into or exit from the hole is seen to be only slightly sensitive to the presence of H_ϕ over the range of values of the static H_ϕ available in these measurements ($\mu_0 H_\phi \lesssim 100$ mT).

Although H_ϕ is maintained fixed, the azimuthal flux permeating the wall is seen to vary dramatically and in an intricate manner during the sweeps of $H_{//}$. In particular, when $H_{//}$ is made to increase from zero, $\langle B_\phi \rangle$, the spatial average of the azimuthal flux, initially decreases, then traces two peaks in sequence. The first peak is appreciably higher and considerably narrower than the second and its descending slope is associated with the arrival, at the inner radius of the wall, of the advancing flux penetration. The magnitude of the two peaks is dictated by the strength of the static H_ϕ . The augmentation of azimuthal flux, generated by an increase in $H_{//}$ with H_ϕ kept constant, is called the azimuthal paramagnetic effect.

Conversely, $\langle B_\phi \rangle$ exhibits an appreciable diminution, in a static H_ϕ , when $H_{//}$ is reduced to zero. This behaviour is denoted the azimuthal

diamagnetic effect. In the range of weak H_ϕ , the azimuthal flux threading the wall is observed to have decreased by ~50% when the downsweep of $H_{//}$ attains zero.

Sweeps of $H_{//}$ from some large initial value through zero to a large value of opposite polarity, with H_ϕ kept fixed, consequently cause the azimuthal flux to display the diamagnetic effect and the paramagnetic effect in succession.

The variations of the azimuthal flux generated by changes in $H_{//}$ with H_ϕ kept fixed provide evidence for and data on the novel phenomenon of flux cutting in the presence of flux pinning behaviour.

The curves of I_ϕ and $\langle B_\phi \rangle$ versus $H_{//}$ increasing or decreasing in various static H_ϕ are analyzed to yield estimates of $j_\perp$ and $j_{//}$, the persistent current densities induced to flow $\perp$ and $//$ to $\vec{B}$ as a function of $|\vec{B}|$. In the analysis, we assume that j_ϕ and j_z , the azimuthal and longitudinal current densities are uniform and occupy the entire volume of the wall for those situations where a change in $H_{//}$ causes a change in $B_\perp$. Also, simple triangular configurations are conjectured for the $B_\phi(r)$ profiles which are consistent with the diamagnetic or paramagnetic behaviour exhibited. This analysis reveals that $j_\perp$ is critical but that $j_{//}$ is either subcritical or occupies only a fraction of the wall volume. Consequently, a reliable estimate is obtained for $j_{c\perp}$ versus B but only a lower limit is established for $j_{c//}$ versus B .

The general critical state theory developed by Clem is outlined and applied to the interpretation of our observations on the azimuthal paramagnetic and diamagnetic effects. For computational ease and economy, we exploited an approach where the sequences of flux density and flux orientation profiles are taken to exist in quasi static configurations. The

calculations yield good agreement with a variety of observations in those circumstances where the induced electric fields, $E_{//}$ and $E_{\perp}$ parallel and perpendicular to $\vec{B}(r)$, do not cause appreciable excursions of $j_{//}$ and $j_{\perp}$ beyond $j_{c//}$ and $j_{c\perp}$. Our method, however, is inappropriate for several crucial situations. In particular, the arrival of the flux disturbance at the inner surface of the wall cannot be treated in this manner and appears to require a relaxation or diffusion technique.

The "mirror" phenomena were also investigated. These are, (a) the influence of a static $H_{//}$ on the entry and exit of azimuthal flux in the wall of the hollow cylinder and (b), the evolution of the longitudinal flux permeating the wall, as H_{ϕ} is initially impressed or made to undergo a swing from some large initial value through zero to a comparable intensity of opposite direction. Again, the presence of $H_{//}$ is seen to exercise only a small influence on the response of the wall to sweeps of H_{ϕ} . Also, although $H_{//}$ is kept fixed, the longitudinal flux density, $\langle B_z \rangle_{\text{wall}}$, is seen to rise as H_{ϕ} is applied and to diminish when H_{ϕ} is decreased to zero, hence a longitudinal paramagnetic and a diamagnetic effect are now observed. As expected, the magnitudes of the paramagnetic and diamagnetic effects are dictated by the strength of the static $H_{//}$ and in the range of weak $H_{//}$, the longitudinal flux permeating the wall is observed to have decreased by ~50% when the downsweep of H_{ϕ} attains zero. Of course, the evolution of $\langle B_z \rangle_{\text{wall}}$ now exhibits a diamagnetic and then a paramagnetic magnetization during sweeps of H_{ϕ} from $+H_{\phi \text{max}}$ through zero to $-H_{\phi \text{max}}$.

A fascinating, although qualitatively anticipated, phenomenon has also been observed and examined in some detail. With a longitudinal flux ϕ_z initially threading the hole of the hollow cylinder, the nucleation of

helical flux lines at the inner surface of the wall and their entry into the wall, brought about by an increase in H_ϕ , is seen to cause ϕ_z , hence $B_1 = \phi_z / \pi R_1^2$, to decrease. In the range of weak $H_{//}$, the longitudinal flux is seen to be decreased to a small fraction of its initial value when H_ϕ attains the maximum strength available. Conversely, a decrease of H_ϕ in a static $H_{//}$, causes helical flux lines to emerge from the wall and generates a rise in ϕ_z , hence B_1 . These observations provide a simple confirmation and easy illustration on a macroscopic scale of prevailing concepts. Their analysis may yield useful insights and quantitative data on the flux cutting processes which presumably accompany the migration of a lattice of flux lines of spatially varying helicity.

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Chapter 1

Introduction

General Framework

Magnetic phenomena, critical current behaviour and A.C. losses in type II superconductors have been extensively investigated during the last 25 years for the situation where the current density $\vec{j}(x, y, z)$ is made to flow perpendicular to the magnetic flux density $\vec{B}(x, y, z)$.^{16,3970} Focusing on the evolution of the magnetic flux density spatially and with time, one can write, quite generally,

$$\vec{B}_f(x, y, z, t + \Delta t) = \vec{B}_i(x, y, z, t) + \Delta\vec{B}(x, y, z, t + \Delta t) \quad (1)$$

The situation just mentioned, has been denoted the collinear regime since it implies that the vector $\Delta\vec{B}$, the change in the magnetic induction, lies locally along $\vec{B}_i$, the existing magnetic induction in the element of volume under consideration. It is clear that the collinear regime constitutes only a very special case of the broad perspective expressed by equation 1. In general, $\Delta\vec{B}$ can be directed along any arbitrary direction with respect to the existing magnetic induction $\vec{B}_i$. The rich array of situations where $\Delta\vec{B}$ does not lie along $\vec{B}_i$ are then referred to as belonging to the noncollinear regime. In order that this nomenclature not lead to confusion it is important to bear in mind that the labels refer to the magnetic flux density. The reason for this precautionary warning is that the current density $\vec{j}$ in the noncollinear regime will have a component collinear to $\vec{B}$, whereas in the collinear regime $\vec{j}$ exists only transverse $\vec{B}$. In the foregoing and in this thesis we consider only isotropic materials.

In spite of its great generality, the noncollinear regime has been largely ignored and has only recently attracted significant experimental and theoretical attention. The interest and research activity has focused mainly on the collinear regime for several reasons. Probably foremost is the fact that this is the configuration encountered in the major applications of standard design (eg. superconducting coils, power transmission cables). This has guided the pertinent basic research and stimulated the applied research. Also, a conceptual framework for describing and understanding the observations, the critical state concept^{2,4,5,28,40,76} together with the picture of flux line displacements and the idea of flux flow voltages, was proposed at the time that the knowledge of ideal and nonideal type II superconductors burst on the scientific and technological scene in the early 60s. By contrast, no viable conceptual framework existed for interpreting and analyzing the phenomena in the noncollinear regime until the publication of the phenomenological theory developed by Clem and his collaborators.^{25,59}

The noncollinear regime is more complicated than the collinear but builds on the latter. Consequently we first outline the basic concepts developed for an understanding of the collinear situation and then describe the central ideas put forward to account for phenomena encountered in the noncollinear regime.

The Collinear Regime

Magnetic flux is visualized as existing in type II superconductors in the form of vortices or discrete flux lines each containing a quantum of flux $\phi_0 = h/2e$. Numerous experiments have now confirmed in some detail this picture first formulated by Abrikosov.¹ The basic picture is most easily

outlined by focusing on planar (infinite slab) geometry with the external magnetic field $\vec{H}_e$ parallel to the surfaces. Parallel flux lines repel each other and are restrained from pushing each other out of the specimen by the externally applied magnetic field and, in irreversible type II superconductors, by pinning sites due to imperfections and impurities. An increase in the externally applied magnetic field $\vec{H}_e$ will cause new flux lines to nucleate at the surface of the specimen when $H_e > H_{cl}$. Due to the increase in the flux line density at the surface, any adjacent sheet of flux lines will experience a net force inwards. This latter sheet will eventually be displaced inwards when the net force (the resultant of repulsive forces arising from neighbours on all sides) overcomes the pinning forces.

As the rise of H_e progresses, one can then readily picture a profile of the flux line density $B(x)$ advancing into the bulk of the slab from the surfaces. By Maxwell's equation, in the form $\nabla \times \vec{B} = \mu_0 \vec{j}$, the gradient of the flux line density in planar geometry, $dB_z/dx = \mu_0 j_y$, corresponds to a quasi-microscopic current density averaged over the cross-section of several flux lines. Here $\vec{B}$ and $\vec{H}_e$ are taken to be along the z-axis and x normal to the surfaces of the slab. The driving force on the flux lines is the Lorentz force $\vec{F}_L = \vec{j} \times \vec{B}$ and the critical state concept states that flux line displacement will take place when the equilibrium condition $\vec{F}_L + \vec{f}_p = 0$ is exceeded. Here f_p is the pinning force density characterizing the specimen. f_p is found to depend on the normalized flux line density, B/B_{c2} , and on the reduced temperature, T/T_c , in a manner which can be approximated by simple analytic functions for various categories of materials.

For slow changes of H_e , the profiles or configurations of the flux density (B profiles) evolve through a succession of quasi-static near

equilibrium critical states. Since this is the situation which applies in our work we will confine our discussion to these circumstances. We note simply that under rapid changes of H_e , non equilibrium or dynamic critical states will arise where viscous retarding forces proportional to the velocity of the flux lines, as well as the pinning forces, come into play.

For planar geometry, the critical state condition can be written

$$B \frac{dB}{dx} = \pm F_p \quad 2(a)$$

where $F_p = \mu_0 f_p$ and dB/dx is implicitly taken to represent a critical flux density gradient, $(dB/dx)_c$. Alternatively, since in this geometry, $(dB/dx)_c = \mu_0 j_{cl}$ where j_{cl} flows perpendicularly to $\vec{B}$ and corresponds to the critical value, equation 2(a) is frequently written,

$$\frac{dB}{dx} = \pm \mu_0 j_{cl} \quad 2(b)$$

Presuming $F_p(B, T)$, or equivalently, $j_{cl}(B, T)$ to be known, the evolution of the space averaged flux density, $\langle B(x, t) \rangle$ versus $H_e(t)$ has been investigated theoretically for a variety of situations. Conversely, authors have exploited equation 2(a) or 2(b) to determine $j_{cl}(B, T)$ and/or $F_p(B, T)$ for numerous materials from their observations of their magnetic response and critical current behaviour in a variety of arrangements using different experimental techniques. It is useful to remark at this point that hysteresis losses can readily be calculated, once $\langle B(x, t) \rangle$ versus $H_e(t)$ is known, by introducing these into Poynting's formula,

$$W = \oint H_e d\langle B \rangle \quad (3)$$

for the energy dissipation per cycle per unit volume.

A critical current $I_c = \int j_{cl} \cdot d\vec{s}$ is reached when the entire cross-section of a specimen (wire, ribbon or other geometry) is filled with current flowing at the critical current density j_{cl} . The magnetic induction

in which j_{c1} bathes can arise from the superposition of an externally applied magnetic field provided by an outside source (which could be neighbouring currents) and the magnetic induction generated by the current in the specimen itself. The spatial distribution of j_{c1} and B can be quite complex even for simple geometries. For instance, the configuration of j_{c1} over the cross-section of a long straight wire carrying a current $I = I_c$ in a uniform static transverse magnetic field³ presents a formidable computational problem. The problem is even more difficult when the magnetic history of the specimen in the superconducting state must be taken into account. By way of illustration we mention the computation of A.C. losses for a long straight wire where both I and H_e vary simultaneously with time as they will when the wire is part of the winding of a coil or solenoid being energized.

The essential physics of the collinear regime, regardless of the complications encountered in pursuing specific cases, resides in and emerges from the critical current density $j_{c1}(B, T)$ or equivalently, the bulk pinning function $F_p(B, T)$.

Finally, we note two features, which although of basic importance have played a negligible role in our preliminary investigation.

The local density of flux lines, $B(x, y, z)$ in type II superconductors whether ideal or nonideal, is regarded as in equilibrium with a local magnetic field $H(x, y, z)$ or vice versa. This relationship is written $H_{eq}(B)$ or $B_{eq}(H)$ and is denoted as the equilibrium or reversible or Abrikosov¹ diamagnetism. This relationship was first derived by Abrikosov and depends only on the Ginzburg-Landau parameter $\kappa = \lambda/\xi$, when B and H are normalized to B_{c2} and H_{c2} . Here λ is the low field (London) penetration

depth and ξ is the coherence length. For a reversible (ideal) specimen, with negligible demagnetization coefficient, this relationship can be determined experimentally by measuring $\langle B \rangle$ versus H , (i) by cooling from T_c to a chosen T in various static H_e or (ii) by sweeping H_e from zero to H_{c2} and vice versa. We stress that the collinear situation prevails in these measurements. For irreversible materials $B_{eq}(H)$ can, in principle, be extracted from measurements, again in the collinear regime, of $\langle B \rangle$ versus H_e where H_e is swept from $\pm H_{c2}$ to $\mp H_{c2}$. A careful analysis of the two sets of data obtained by this procedure can yield $j_{c1}(B/B_{c2})$ and $B_{eq}(H)$ for the specimen at the chosen temperature. Consequently, for materials exhibiting strong bulk pinning, it is important that the dimension orthogonal to H_e , the radius of the wire or the thickness of the ribbon, be small. The reason for this is that, the contribution of the bulk pinning to the magnetization (j_{c1} multiplied by the enclosed area integrated over the cross-section) increases as a power greater than one of the radius or thickness of the specimen. The reversible diamagnetism per unit volume is, however, size independent. In any case, demagnetization effects must be carefully avoided.

It is also now well known that type II superconductors can support an appreciable, persistent, irreversible, surface current,^{5,18,46-48} $I_s(B)$, particularly when H_e , hence B , are directed parallel to the surface of the specimen. The physics of this surface current is not well understood and this problem continues to pose a major theoretical and experimental challenge. This current, in contrast with the reversible diamagnetic surface current associated with $B_{eq}(H)$, flows to oppose changes in the magnetic flux threading the specimen, hence it is history dependent (irreversible). The

magnetic moment per unit volume generated by this current is size independent. As a consequence, although it is desirable to use a specimen of small cross section, to minimize the role of bulk pinning (j_{cl}), in the determination of $B_{eq}(H)$, there is no advantage in this approach when the specimen exhibits a large $I_S(B)$.

It has been shown by LeBlanc et al.⁴⁸ that $I_S(B)$ can be dramatically reduced when $\vec{H}$ is tilted so that a component pierces the major surfaces of the specimen. This stratagem, however, presents a dilemma. Demagnetization effects become significant when H_e is tilted as indicated. Alternatively, the sample can be cooled from T_c in the presence of a suitable but weak magnetic field H_1 directed perpendicular to the surface and maintained fixed as a magnetic field $H_{//}$ directed along the major surfaces is applied and cycled. The latter arrangement, however, belongs to the noncollinear regime where a variety of new behaviour appear which we will now address. In closing this section we note that Clem²³ has published an excellent synthesis of the phenomenological description of the magnetic response and of hysteresis losses in the collinear regime. In this work he also indentifies clearly the various contributions to the energy dissipation.

The Noncollinear Regime

A. Historical Review

The study of the noncollinear regime, although this label was introduced only recently, began with the observation in 1963 by Sekula et al.⁶⁰ that the lossless current carrying capacity, I_c , of straight wires of type II superconductors exhibited an impressive peak in the range of moderate

magnetic fields when immersed in a longitudinal magnetic field $H_{//}$. This is a noncollinear situation since the azimuthal magnetic field H_{ϕ} produced by the current I is orthogonal to $H_{//}$ with H_{ϕ} changing when I is impressed and varied. The following year Heaton and Rose-Innes³⁸ reported the same behaviour in nearly reversible NbTa wires. Also in that year, Cody and Cullen²⁶ found the same phenomenon in ribbons of Nb_3Sn when H_e was directed along the length of the specimen. Meanwhile Bergeron⁶ proposed that a force-free configuration, where $\vec{j}$ at I_c is everywhere parallel to $\vec{B}$, could account for this phenomenon and developed an expression, based on this idea, which reproduced the general features of the observations. He also indicated that the proposed force-free configuration would cause the longitudinal flux threading the wire at I_c to significantly exceed the externally applied longitudinal magnetic flux. LeBlanc et al.⁴⁴ pursued this feature and found that the prediction was verified for wires of the several materials they investigated. The phenomenon was labelled, the longitudinal paramagnetic effect. The observations, however, also showed major departures from the predictions of the force-free picture.

In 1967, Taylor⁶⁵ discovered that losses in straight wires of type II superconductors carrying an A.C. current of low frequency and large amplitude could be appreciably reduced by immersing the specimen in a modest stationary longitudinal magnetic field. Later Sugahara and Kato⁶⁴ (1971) and Nakayama et al.⁵⁵ (1974) monitored the variation of the azimuthal flux density, $\langle B_{\phi} \rangle$ and the longitudinal flux density, $\langle B_z \rangle$ in straight wires of NbZr and NbTi carrying A.C. currents of various amplitudes as a function of $H_{//}$. Gauthier and LeBlanc³⁴ (1977) showed that these data could be reproduced exploiting a simple empirical model where j_{c1} and $d\theta/dr$, the

change in orientation of the helical flux lines, were assumed to exist each in a critical state. We will see later in this thesis that $B d\theta/dx = \mu_c j_{//}$ where $j_{//}$ is a current density parallel to the flux line density.

The accurate measurement of $\langle B_{\phi} \rangle$ versus $I(t)$ in a straight solid wire poses severe problems, since this quantity must then be determined from the minute voltage induced across a pair of voltage leads attached to the sample. In effect, this method measures $d\langle B_{\phi} \rangle/dt$ threading a single turn pickup coil embracing the area circumscribed by the radius of the wire and the length between the two contacts to the specimen. To facilitate the measurement of the two orthogonal components of the magnetic flux density, $\langle B_{\phi} \rangle$ and $\langle B_z \rangle$ and, at the same time, to circumvent the difficulties involved in feeding a very large current to a sample of high current carrying capacity via current leads connected to a power supply, LeBlanc and his group adopted a novel approach and different geometry which we describe in the next section.

B. Noncollinear Regime in Ribbon Geometry with Induced Currents.

An externally applied magnetic field, $H_{\perp}$, oriented along the flat faces of a ribbon (thin compared to its length and width) and directed transverse to the length, is made to vary when the specimen is immersed in a stationary externally applied magnetic field, $H_{//}$, directed along the length. By the laws of induction, a change in $H_{\perp}$ will cause a persistent current to flow "up" on one side of the midplane of the ribbon and "down" the other side, the sense of circulation being determined by the sign of the increment $\Delta H_{\perp}$. In this manner, a very large current can be made to flow, although in opposite directions, in each half of the ribbon. Since the demagnetization

effects are negligible for any orientation of $\vec{H} = \vec{H}_\perp + \vec{H}_\parallel$ along the flat faces of the ribbon, the roles can be reversed with $H_\perp$ kept stationary and $H_\parallel$ made to vary. Evidently a variety of noncollinear and collinear schemes for the variation of $\vec{H}(t) = \vec{H}_\perp(t) + \vec{H}_\parallel(t)$ can be envisaged. The scheme adopted where $H_\parallel$ is kept fixed and $H_\perp$ is varied (or vice versa) is experimentally convenient. Further, each half of the ribbon, on either side of the midplane, when viewed by itself, can be regarded as a single isolated ribbon carrying a conduction current fed to it from an outside power supply. The disadvantage of the arrangement is that now a critical current I_c can be attained without manifesting itself by the appearance of a resistive transition or the onset of a flux flow voltage. In other words, dynamic equilibrium critical states (the flux flow regime) cannot be explored with the applied variables stationary, which is possible when I is fed through the ribbon via leads.

Two orthogonal pickup coils embrace the ribbon, one monitoring $\langle B_\parallel \rangle$ and the other $\langle B_\perp \rangle$ (or equivalently $\langle M_\parallel \rangle$ and $\langle M_\perp \rangle$, the components of the magnetization). Again focusing on the volume of ribbon on either side of the midplane, the quantity $\langle B_\perp \rangle$ when it is $H_\perp$ which is made to vary, can be regarded as corresponding to the azimuthal flux density B_ϕ in a wire carrying a current I . The advantage of the arrangement adopted by LeBlanc et al is that $\langle B_\perp \rangle$ can be easily and accurately monitored, in the steady state or at any frequency, by a multiturn pickup coil, whilst $\langle B_\parallel \rangle$ for the solid wire can only be measured accurately at appreciable frequencies and amplitudes of $I(t)$.

Exploiting the arrangement just described, LeBlanc and his group investigated all of the phenomena encountered in straight solid wires made to

carry a current $I(t)$ when immersed in a longitudinal magnetic field $H_{//}$, namely; (i) the peak in I_c , (ii) the longitudinal paramagnetic effect and (iii) the reduction in hysteresis (A.C.) losses ($I(t) V(t)$). The analogs of these phenomena now consist in the observation of, (i) an enhancement in $\langle M_L \rangle$ versus H_L caused by the presence of a stationary $H_{//}$, (ii) an increase in $\langle B_{//} \rangle$ (or $\langle B_z \rangle$), hence the appearance of a magnetic moment $\langle M_{//} \rangle$ along the stationary applied $H_{//}$ and (iii) hysteresis losses, namely the area enclosed by the locus of $\langle M_L \rangle$ versus $H_L(t)$ as the latter undergoes a cycle of arbitrary frequency (which can now be of the order of zero) and of various chosen amplitudes. It is noteworthy that behaviour "orthogonal" to that just listed can now be readily studied by keeping H_L fixed and causing $H_{//}$ to vary. In particular, a "transverse" paramagnetic effect can now be observed, i.e. an increase in $\langle B_L \rangle$, hence the appearance of a magnetic moment $\langle M_L \rangle$ along the stationary applied H_L . The label is evidently introduced in analogy with the title, "longitudinal" paramagnetic effect.

Finally, with this approach, a phenomenon which is a "mirror" of the longitudinal paramagnetic effect, is now revealed. It is found that as H_L is decreased, with $H_{//}$ kept fixed, the flux threading the ribbon along the stationary $H_{//}$ is seen to decrease, hence a diamagnetic moment $\langle M_{//} \rangle$ to appear. For obvious reasons, the behaviour has been called the longitudinal diamagnetic effect. Of course the "orthogonal" effect is now expected to occur and this is indeed the case; a diamagnetic moment $\langle M_L \rangle$ appears along the stationary H_L when it is $H_{//}$ which is made to decrease. It is important to note that the analogs of these diamagnetic effects are inaccessible when a wire or ribbon are carrying a current I fed into the specimen via connections to an external power supply. The reason is that the wire or ribbon does not cool below T_c and become superconducting while carrying an appreciable

current I because of the joule ($I^2 R$) heat.

The major features of a vast assortment of observations on ribbon samples have been quantitatively and qualitatively well reproduced exploiting the simple empirical double critical state model mentioned earlier. This model contains two adjustable parameters: the pinning strength coefficient linked with $j_{c\perp}$ and another parameter factoring $j_{c//}$, the critical current density parallel to the flux line density. These workers, however, have found it necessary to introduce an arbitrary functional dependence on θ the orientation of the flux lines with respect to an axis of the specimen (the length or width). Whether this complication is necessitated by anisotropy in the samples or some fundamental deficiency of the model, is not clear at present.

C. The Disk Geometry

For completeness we mention another simple arrangement which has been exploited by LeBlanc and his collaborators to reveal and probe the wealth of phenomena occurring in the noncollinear regime. A single disk, or a stack of electrically insulated thin disks, are made to undergo rotation about the axis $\perp$ to the plane of the specimen and passing through its center. The sample is immersed in a stationary externally applied field $\vec{H} = \hat{z} H_z$ directed along the flat faces of the disk(s). We note that this situation is equivalent to the rotation of an applied magnetic field $\vec{H}$ of constant magnitude in the plane of the disk. In either case, $\vec{H}$ changes direction with respect to any diameter of the disk and the noncollinear regime prevails. Again, two orthogonal pickup coils embrace the specimen with one coil monitoring $\Delta\langle B_z \rangle$, the change of the magnetic flux density along H_z and the other $\Delta\langle B_y \rangle$, the change of magnetic induction in the plane of the disk.

transverse to H_z . The disk may be made to undergo oscillations of various angular amplitudes θ_0 , hence the analog of A.C. conditions is encountered. Further, with this arrangement, the angle of rotation and the amplitude θ_0 are unrestricted. This is in contrast to the change of angle of the magnetic field $\vec{H} = \vec{H}_z + \vec{H}_\phi$ at the surface of a straight wire or a ribbon carrying a conduction current I in a static applied longitudinal magnetic field $\vec{H}_z$ where θ_0 must be less than $\pi/2$.

Again, a large variety of data and an extensive catalogue of intricate curves have been surprisingly well reproduced by an empirical two parameter model where $j_{c\perp}$ and $d\theta/dx$, hence $j_{c//}$, are taken to exist in critical states wherever they appear. An important feature of the data, which the model cannot generate, is that the locus of $\langle B_z \rangle$ and $\langle B_y \rangle$ versus θ cycled between chosen limits, will in some circumstances, trace loops which do not close. When this behaviour is encountered, the deviation from "closure" diminishes as the number of cycles progresses. The reason the model cannot account for such behaviour is precisely because it stipulates that both $j_{c\perp}$ and $j_{c//}$ must exist in critical states. Consequently, the model does not contain the flexibility required to account for open loops.

The observations⁹ reveal that, regardless of previous magnetic-temperature history, the magnetic flux threading the disk evolves toward a diamagnetic configuration as the rotation or oscillations progress. Diamagnetic here means that $j_{c\perp}$ circulates in a sense which tends to reduce the flux density. Since the magnitude of the applied magnetic field is maintained constant, this implies that either flux lines move out of the specimen against the flux density gradient adjacent to the surfaces of the disk or that flux is consumed by a process of flux line cutting inside the specimen.

Evidently a theory is required whose basic structure will dictate the observed open loops and the diamagnetism just mentioned. The general double critical state theory developed by Clem,²⁵ which we will outline shortly, accomplishes this task "naturally".

D. Hollow Cylinder Geometry

Hollow cylinder geometry for the study of the noncollinear regime was first exploited by Bussière.¹² In his experiment, a hollow tube of Nb was cooled from T_c to 4.2 K in a static longitudinal field $H_{//}$. Then with $H_{//}$ maintained fixed at the various chosen values, a current I was made to flow along the length of the hollow tube via leads attached to its ends and raised until I_c was attained. An axial pickup coil placed in the centre of the hollow tube monitored B_1 , the longitudinal flux density in the hole. As expected, a monotonic rise in B_1 was observed as I was increased. This indicates that the longitudinal flux permeating the wall is being displaced and compressed into the hole. This behaviour does not entail any new physics. The data, however, revealed an important and novel result. For a range of $H_{//}$, as I approached I_c , the longitudinal flux, $\Delta\phi_1 = \Delta B_1 \cdot \pi R_1^2$, added in the hole, exceeded the longitudinal flux originally permeating the wall. Evidently, helical flux lines are made to enter the hole while the longitudinal current has not yet reached a critical configuration throughout the wall of the tube. Gauthier has shown that these observations can be accounted for using the empirical critical state model. Calculations are in progress in our group to apply the Clem theory to these intriguing data. In our view, flux line cutting plays a crucial role in these results, but the conceptual connection is indirect and subtle.

In an effort to obtain unambiguous evidence of flux line cutting, we also adopted hollow cylinder geometry.³⁰ In our arrangement, "pure" azimuthal flux, provided by a steady current I flowing in a straight wire running along the axis of the tube, is trapped in the wall of the hollow cylinder. This is accomplished by letting the cylinder cool from T_c to 4.2 K while it is bathing in H_ϕ generated by I . Besides H_ϕ , only the earth's field was present, hence for practical purposes, $H_{//}$ was zero. Then I , hence the associated H_ϕ , are reduced to zero. This leaves the wall threaded by azimuthal flux lines (vortex rings) as verified by a toroidal pickup coil embracing the wall. Now a longitudinal magnetic field $H_{//}$ was applied while $\langle B_\phi \rangle$ in the wall and B_l in the hole were monitored by pickup coils. As expected, longitudinal flux appeared in the hole at $H_{//} \ll H_{c2}$ and increased in density until H_{c2} was attained. The crucial feature, however, is that, azimuthal flux continued to thread the wall, although with diminishing density, until $H_{//}$ attained H_{c2} . Since, outside the cylinder, $H_\phi = 0$ as $H_{//}$ was impressed, the flux lines entering the outer surface of the wall must be purely longitudinal. Eventually, longitudinal flux enters the hole while azimuthal flux continues to permeate the wall. This shows, rather dramatically, that the longitudinal flux lines entering the outer surface have, by some process, cut through the vortex rings.

We then undertook an extensive series of measurements on a hollow cylindrical specimen which are presented in this thesis. One of the purposes of this investigation was to explore in detail the analogs in this geometry of the several noncollinear phenomena already known to arise in ribbon and solid wire geometry. These are:

- (1) The enhancement in $\langle M_{//} \rangle$, the magnetic moment along $H_{//}$ when H_ϕ is kept fixed and conversely the enhancement in $\langle M_\phi \rangle$, the magnetic

moment along H_ϕ when $H_{//}$ is kept fixed. This, as noted above, is equivalent to the peak in I_c observed in wires in static $H_{//}$.

(ii) The reduction in hysteresis losses occasioned by the presence of a static component of $\vec{H}$ orthogonal to the varying component ($H_{//}$ fixed with H_ϕ oscillating and vice versa). We remark that there is a direct link between (i) and (ii).

(iii) The longitudinal and transverse paramagnetic and diamagnetic effects. The azimuthal coordinate now replaces the transverse direction.

A hollow cylinder with a longitudinal slit across its wall relates directly to ribbon geometry since the curvature plays no basic role in the phenomena of interest here. The only advantage of the slit hollow cylinder, however, would be that the demagnetization factor is truly zero in the azimuthal direction. Consequently we viewed a hollow slit cylinder as presenting no new challenge.

A hollow cylinder with a continuous wall constitutes a very simple configuration for investigations of flux cutting phenomena. Further the closed geometry of the "connected" wall gives rise to interesting behaviour as we have already seen in the experiment of Bussière and in our preliminary study of flux cutting. A more versatile arrangement would be one where the azimuthal magnetic field would be provided by two independent sources. (a) A toroidal magnet coil embracing the wall of the cylinder (or a single wire placed along the axis of the cylinder and carrying a suitably large current), plus, (b) a longitudinal current flowing along the length of the cylinder wall and fed into the wall via leads attached to its ends as in Bussière's work. The implementation of this combined scheme, however, presents severe experimental difficulties when one also envisages accommodating a toroidal

pickup coil embracing the wall in order to monitor $\langle B_\phi \rangle$. We elected to pursue scheme (a) alone for our investigation. Future work should probably endeavour to implement the combined arrangement.

Flux Line Cutting and $j_{c//}$

The voltage observed between probes attached along the length of a wire of a type II superconductor, carrying a steady current in zero externally applied magnetic field, is thought to arise from the continuous creation of the vortex rings at the surface which collapse by shrinking radially and self-annihilate at the axis of the wire. This dynamic model was first proposed by Gorter³⁵ to account for resistive phenomena in the intermediate state of type I superconductors. A paradox, as first noted by Walmsley,⁷² is encountered when the model is applied to the situations where $H_{//} \neq 0$, the noncollinear regime. Now the vortices nucleating at the surface of the wire are helical and their radial migration to the axis, instead of leading to annihilation, will cause a continuous accumulation of longitudinal flux in the specimen. Such an unrestricted growth is inconceivable. Walmsley⁷² studying the longitudinal paramagnetic effect in the flux flow (resistive) state of NbTa wires observed a stationary $\langle B_z \rangle$ as expected. To resolve the paradox, he proposed that, by a process of flux cutting, the azimuthal component of the helical flux lines could be transported to the axis across the stationary longitudinal component. The detailed picture is unclear.

Clem²² has developed a detailed picture for the flux flow voltage and the flux cutting process in these circumstances. Here a right handed

helical flux line in a longitudinal conduction current shrinks radially until it reaches the axis. Clem shows that a flux line, at the axis, undergoing fluctuations or perturbations to the left handed helicity is then unstable and will now expand until ejected at the surface. Thus two oppositely moving lattices of flux lines can be visualized as cutting through each other, each contributing to E_z the longitudinal electric field by the radial motion of their azimuthal component, according to the expression $E_z = -v_r B_\phi$.

The process of flux cutting is shown schematically in the sketches below. The solid lines in sketch (a) represent a sheet of flux lines (A), and the dashed lines another adjacent sheet of flux lines (B). The two, initially separate sheets are visualized as merging, either through some mechanism compressing them together or through mutual attraction, and temporarily forming a grid. (Brandt¹⁰ et al have shown that a critical angle θ_c exists where the mutual repulsion experienced by flux lines when $\theta < \theta_c$, vanishes and is replaced by mutual attraction when $\theta > \theta_c$). The overlap region of the grid network, containing two flux quanta, is energetically costly and this situation cannot persist. A new arrangement emerges where the segments of flux lines from sheet A are connected to the adjacent segments from sheet B as depicted in sketch (b).



The zigzag flux lines which ensue from the cross joining of the segments will now straighten, thereby reducing their energy which is proportional to the length. The resulting sheet of parallel flux lines is now much more densely packed than the two separate sheets at the outset and the lines exert considerable repulsive forces on each other. Consequently the sheet is unstable and will expand to occupy the available space by separating into two adjacent sheets. Pursuing this picture one can now visualize the penetration of an angular disturbance $\theta(x)$ from the surface the specimen into its volume just as we imagined the advance of a change in flux density into the bulk in the j_{cl} (collinear) regime.

In planar geometry, Maxwell's equation, $\nabla \times \vec{B} = \mu_0 \vec{j}$, leads to $\mu_0 j_{//} = B d\theta/dx$, hence the spatial variation or gradient of the orientation of the flux lines constitutes a current density flowing along the flux line density $\vec{B}$. We stress that the quantities $\vec{j}$ and $\vec{B}$ represent averages over the dimensions of several cross sections of flux lines. The detailed picture is beyond the scope of this work. Our primary concern in the interpretation of our observations is the existence of a critical angle θ_c , hence equivalently, a critical current density $j_{c//}$ for flux cutting. The concepts of j_{cl} and $j_{c//}$ alone, constituted the empirical double critical state model exploited so successfully by LeBlanc and his group. Besides the shortcoming of its empirical nature, this model failed when confronted by certain novel phenomena we encountered in our study of hollow cylinders. Fortunately, a fundamental model, the general critical state model of Clem, which we will outline below, came to the rescue.

The decrease of the magnetic flux density along a stationary component of $\vec{H}_e$, the externally applied field, as the latter is caused to

change direction and magnitude, is the signature of flux cutting. The evidence, of course, has more impact when the magnitude of $\vec{H}_e$ is increasing. Consequently, (i) a decrease in the longitudinal or transverse paramagnetic effect, (ii) a diamagnetic effect in $|\vec{H}_e|$ increasing and (iii) the longitudinal or transverse diamagnetic effects are all manifestations of flux cutting although (ii) clearly offers the most dramatic evidence. Further, although less direct and convincing evidence of flux cutting can emerge from a comparison of the growth of the paramagnetic effect with the much greater rise expected if flux transport at constant orientation were taking place. The message here is however less explicit and somewhat model dependent.

The General Critical State Model

We now present a qualitative description of the general critical state model developed by Clem and collaborators to account for phenomena in the noncollinear regime. In chapter 8 we will present the formalism of this theory in some detail and apply it to some of our observations in chapter 9.

The model is based on two basic ideas: the existence of a critical current density $j_{c\perp}$ perpendicular to the flux line density, and a critical current density $j_{c//}$ parallel to the flux line density. $j_{c\perp}$ is already well known in the collinear regime and is responsible via the action of the Lorentz force overcoming bulk pinning, for the displacement or transport of flux lines, hence the evolution of the B profiles. Clem introduces the crucial concept of a critical current density, $j_{c//}$, for the onset of flux cutting.

At a given time and place in the zone of action of a specimen, flux lines must be undergoing one of the following.

- (i) Displacement without change of orientation, hence compression or decompression of flux i.e. an increase or a decrease in flux density is taking place.
- (ii) Flux cutting without any displacement of the flux lines is taking place, hence the flux line density must be decreasing,
- (iii) both flux line cutting and flux line displacement are simultaneously taking place.

It is important to realize that depending on the circumstances, a specimen can be occupied by some combination of these three zones as well as possess an inert or passive region when no changes are taking place. The latter is denoted an O zone by Clem.

Campbell and Evetts have proposed that collinear phenomena could be accounted for by postulating flux transport at fixed orientation or constant helicity (scheme (i)). Lorrain et al⁵² have scrutinized this proposal in considerable detail and shown that this rigid picture leads to several unphysical consequences. The Clem theory allows for the appearance of such zones in conjunction however with the simultaneous or sequential existence of the two other zones.

The simple empirical double critical state model of LeBlanc et al did not envisage the possibility of pure flux cutting regions. Further, in their scheme, the boundary between zones (i) and (iii) was dictated solely by the interplay of j_{cl} and $j_{c//}$.

In the theory of Clem, the sequences of the spatial configurations of $\vec{j}$, $\vec{B}$ and of the electric field $\vec{E}$, are uniquely dictated by basic electrodynamics once the dependence of j_{cl} and $j_{c//}$ on B (and T) are taken to be known. Specifically, the components of $\vec{E}$ and $\vec{B}$ must be continuous. The

special or model constraints, introduced into the model are the following:

- (a) $j_{c\perp}$ and $j_{c//}$ must flow in the direction of $E_{\perp}$ and $E_{//}$ respectively, where $E_{\perp}$ and $E_{//}$ are the components of the electric field $\perp$ and $//$ to the flux density $\vec{B}$. This constraint is more stringent than required by the basic electrodynamic condition that $\vec{E}$ and $\vec{j}$ subtend an angle γ with $0 < \gamma < \pi/2$. The more rigid constraint imposed in the Clem theory is a consequence that both flux transport and flux cutting are dissipative processes and occur independently.
- (b) In the quasi-steady state, $j_{\perp}$ and $j_{//}$ cannot exceed $j_{c\perp}$ and $j_{c//}$ respectively. The former can, however, assume any value between 0 and the corresponding critical value in pure flux cutting and pure flux transport regions, hence subcritical B and θ profiles can occur in these zones. Here, however, $j_{\perp}$ and $j_{//}$ are not at all arbitrary but ensue from basic physics.

In this thesis we present extensive measurements of the azimuthal (transverse) and longitudinal paramagnetic and diamagnetic effects and our observations of some related phenomena. We estimate $j_{c\perp}$ and $j_{c//}$ versus B from some of our data. Then we present the Clem general critical state theory in some mathematical detail and finally apply this theory to the analysis of some of our results.

Chapter 2

Experimental Arrangement and Procedure

Introduction

Our purpose was to investigate magnetic phenomena in a hollow cylinder of a type II superconductor subjected to changes in an externally applied longitudinal magnetic field, $H_{//}$, while immersed in a stationary externally applied azimuthal magnetic field H_{ϕ} and vice versa. Hence, in brief, to determine the behaviour of the magnetic flux in the sample, (i) when $H_{//}$ is varied with H_{ϕ} fixed, and (ii) when H_{ϕ} is varied with $H_{//}$ fixed. These two sets of procedures were selected because they can provide considerable basic information while being simple in implementation and convenient for interpretation of the data. The magnetic response of the cylinder to changes in the externally applied magnetic field is monitored by simultaneously detecting any variations in, (a) the magnetic flux threading the entire sample (the combination of hole and wall) longitudinally, (b) the magnetic flux permeating the wall azimuthally, and (c) the longitudinal flux density in the hole.

Description of the Sample Assembly

A. The Specimen-Heater Unit

The eutectic alloy of PbBi was chosen for investigation. This material among the many candidates for investigation has many features which recommend it for a study of new phenomena in type II superconductors.^{49,50} It is superconducting at 4.2 K, the temperature of liquid helium at atmospheric pressure. This is clearly a great experimental convenience. Its

upper critical field H_{c2} at 4.2 K is of modest magnitude (≈ 1 Tesla), hence its superconducting behaviour can be investigated over a significant range without presenting severe problems. The superconducting hysteretic properties of this alloy have been well documented, and it can be easily "transformed" through a broad spectrum of bulk pinning behaviour ranging from strongly hysteretic to nearly reversible by simple annealing treatments. This feature will certainly prove highly beneficial for the long term objectives of our research program since one of our ultimate goals is to determine the dependence of $j_{c//}$ or equivalently, of the flux cutting process on the strength of the bulk pinning. Further, this alloy has a relatively low melting temperature, hence its preparation is quite easy. Finally, the components are inexpensive and can be obtained very pure at moderate cost.

The sample is a hollow cylinder 10 cm in length with an outer radius of 0.415 cm and inner radius of 0.345 cm of a $\text{Pb}_{0.45}\text{Bi}_{0.55}$ eutectic alloy. The sample was made by melting high purity lead and bismuth sealed in an evacuated glass tube at about 600 C. The liquid was kept at this temperature for a few hours to allow the two liquified metals to mix by convection and occasionally the sealed tube was shaken to release gas bubbles which may have formed in the liquid. At the end of this period the tube was dropped out of the bottom of the melting furnace and its contents were quenched by cooling in air at room temperature. The alloy ingot thus formed was then machined to the dimensions given above. No further mechanical or chemical treatment of the sample was performed.

In magnetic and critical current measurements of irreversible type II superconductors it is mandatory that the initial conditions be known and controlled. This requires that the temperature of the sample can be raised

above T_c in the chosen $H_{//}$ and H_ϕ . To accomplish this, a non-inductive single layer bifilar heater was wound around the circumference of the sample over its full length, making sure that no gaps were left between any turns so that a uniform temperature distribution is achieved along the length of the cylinder. A uniform temperature is important for eventual measurements to establish the temperature dependence of $j_{c//}$, hence of flux cutting in the range between ambient and T_c . Causing a current of the appropriate value to flow through the heater increases the temperature of the cylinder above T_c thereby erasing any previous magnetic history before a measurement. Insulated manganin wire (#38 gauge, Wylbur B Driver Company) was used for the heater since it has a high resistivity at low temperatures and negligible magnetic permeability.

This specimen and heater unit is inserted into a snug fitting thin walled cylindrical delrin sleeve. The latter serves several purposes. It protects the heater wire from physical damage during the winding of the toroidal pickup coil. It also assures the electrical insulation between the heater and the toroidal pickup coil and provides a smooth base on which to wind the latter. Finally, posts fitted to and extending from both ends of this sleeve are inserted in end plates thereby denying the unit longitudinal and azimuthal movement inside the sample holder.

B. The Pickup Coils

At least three different pickup coils are required to gather the desired signals. A toroidal pickup coil must embrace the wall of the cylinder to monitor the azimuthal flux permeating it. A longitudinal pickup coil must be placed in the hole of the cylinder to register the longitudinal flux density there. Another azimuthally wound, hence longitudinal pickup coil

must embrace the cylindrical sample to detect the longitudinal flux permeating the wall. Inevitably the latter coil also measures the longitudinal flux in the hole. The net longitudinal flux in the wall can be determined by subtracting the signals generated by the two longitudinal pickup coils, (i) electronically at the inputs after suitably amplifying that from the inner coil or, (ii) digitally at the outputs.

The cross-section of the hole must accommodate not only the central longitudinal pickup coil and the inner windings of the toroidal pickup coil but also the inner windings of the toroidal magnet coil. Care must be taken to eliminate any possibility of electrical contact between these three sets of windings crowded into the limited space of the hole so as to optimize the use of the available volume. Simple considerations dictate the geometric positioning of the various windings. Also, to maximize the number of turns of the pickup coils, small diameter, hence fragile wire must be employed.

A five layer toroidal pickup coil of approximately 2,000 turns embraces the cylinder wall and delrin sleeve lengthwise. The winding of this pickup coil is a somewhat tedious and delicate operation. A length of formvar insulated 45 B & S copper wire for this pickup coil is first wound onto a spindle. The wire is threaded through the hole of the cylinder by passing this long spindle through the hole, the wire being unwound from the spindle as needed for each circuit. Not enough wire, however, can be wound on the spindle for the construction of the toroidal coil in a single operation. The formvar insulation from the end of the section just wound must be removed and soldered to the bared end of the next section. These joints are insulated from each other by thin mylar strips glued into place.

The winding of the inner longitudinal pickup coil of 20,000 turns of formvar insulated 45 B & S copper wire occupies a length of 6.4 cm, and

has an inner radius of 0.10 cm and outer radius of 0.275 cm. The length of the winding is chosen significantly shorter than that of the cylinder to ensure that this coil measures the longitudinal field threading the "waist" of the hollow cylinder. The former or frame for this pickup coil comprises three sections glued together since it was not feasible to machine an integral unit of the required geometry, i.e. long thin walled central section. The end pieces each of 2.25 cm length are thickwalled and made of delrin. The central section which carries the winding is a very thin walled hollow cylinder (≈ 0.075 cm thick), made of orangewood which fits tightly inside inner circular shoulders bored into the portion of the end pieces facing each other. The outer ends of the end pieces are milled square and fit tightly into the support plates which keep this pickup coil as well as the sample-heater-toroidal pickup coil unit in place in a semi cylindrical bakelite holder. The frame for this pickup coil therefore extends the entire length of the hollow sample unit. The center of this frame is hollow and circular and will be threaded by the winding of the toroidal magnet coil.

No bucking coil was provided for the toroidal pickup coil. Penury of space to accommodate such a coil played an important role in this decision. Further the background signal arising from the azimuthal flux threading A'_0 , the average area embraced by this coil outside the wall of the cylinder is about one half of the signal from the flux threading the wall area A' and can readily be subtracted digitally from the measured signal.

The third pickup coil embraces the cylindrical sample-heater-toroidal pickup coil unit. It comprises 180,000 turns of formvar insulated 45 B & S copper wire wound on a bakelite spool. The winding occupies a volume of 5.7 cm length, 1.9 cm I.D. and 3.2 cm O.D. Only about 60,000 of

the turns however contribute to the net signal arising from changes in the longitudinal flux threading the cylindrical sample (wall and hole combined). The reason for this is that the coil is self-balanced, i.e. it consists of two concentric oppositely wound coils where $N_1 \langle A_1 \rangle = N_0 \langle A_0 \rangle$. Here N_1 and N_0 denote the number of turns and $\langle A_1 \rangle$ and $\langle A_0 \rangle$, the average area embraced by the turns of the inner and outer of the two concentric coils. The radius of the interface between the two concentric coils which satisfies this balanced condition was calculated and the winding direction reversed when this radius was reached. Actually, since it is more convenient to balance a coil by progressively removing rather than by adding turns, an estimated and significant excess number of turns was deliberately allowed in the winding of the outer coil. Very approximate balancing is first performed at room temperature in a copper wire wound long solenoid, then more accurate balancing, yet deliberately allowing for a few excess turns on the outer coil, carried out at liquid nitrogen temperature. The final balancing is done at liquid helium temperature and is accurate to the order of one to five turns.

The concentric balanced pickup coil arrangement devised by Dr. M.A. LeBlanc is chosen in preference to the standard and well known set up where the bucking coil is separate and located so as not to see the sample yet sense the applied magnetic induction. The latter arrangement requires a longer solenoid for the longitudinal magnetic field than our technique. More importantly perhaps is that the balance in the standard configuration is sensitive to possible small displacements of the two coils with respect to each other and to the ambient field. The disadvantage of the concentric balanced arrangement is that the effective number of turns is $N_1 - N_0$ hence only a fraction, $(N_1 - N_0)/(N_1 + N_0)$, of the total number of turns.

Indeed, it is often a surprise to scientists that the concentric balanced pickup coil yields a signal. This is made evident, however, when one considers that the desired signal S , by Faraday's law of induction, is determined by, A , the area of the sample and the resultant number of turns of the pickup coil embracing it, hence

$$S = \int \epsilon \, dt = (N_1 - N_0) A \Delta \langle B \rangle = N_0 \left(\frac{\langle A_0 \rangle}{\langle A_1 \rangle} - 1 \right) A \Delta \langle B \rangle \quad (1)$$

where we have introduced the balance condition,

$$N_1 \langle A_1 \rangle = N_0 \langle A_0 \rangle. \quad (2)$$

C. The $H_{//}$ and H_{ϕ} Magnet Coils

A longitudinal magnetic field $H_{//}$, uniform to $\leq 2\%$ over the volume of the sample, is provided by a superconducting solenoid fabricated using multifilamentary 0.040 cm diameter formvar insulated commercial NbTi wire. The winding of the solenoid occupies a volume 17.8 cm in length, 6.8 cm O.D., and 3.8 cm I.D. A battery driven, air cooled, transistorised power supply supplies ripple free magnetic induction of 0.0716 Tesla/Ampere. The applied field can be maintained steady to $\leq 1/1000$ at 0.6 Tesla for the duration of a measurement (several minutes).

The azimuthal magnetic field H_{ϕ} is provided by a seven turn toroidal coil made with NbTi wire identical to that used for the solenoid. The seven turns pass through the central hole of the inner pickup coil and form a tightly packed hexagon surrounding the central wire. Six of the windings return through grooves made in the outer radius of the frame of the outer pickup coil also forming a hexagonal arrangement. The ends of the first and seventh turn each terminate inside a sandwich of indium foil

pressed tightly with brass screws between massive copper blocks secured to the semi-cylindrical bakelite holder of the sample-heater-pickup coil unit. A set of four copper wires symmetrically placed outside the solenoid link the lower copper block to one of the main leads above the solenoid.

A large transistorised, water cooled, battery driven power supply can provide up to 400 Amps of ripple-free D.C. current for a few minutes without excessive current drift and helium boil off and 300 Amps of current, steady to better than $\pm 1\%$, for the several minutes required for those measurements where the H_ϕ must be held fixed. Applying Ampère's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ to this arrangement leads to

$$B_\phi = \frac{\mu_0 I}{2\pi r} \quad (3)$$

where $I = nI'$, with $n = 7$, the number of turns of the toroidal coil and I' , the current flowing in each wire. Thus at $I' = 100$ A, $B_\phi = 40.6$ mT and 33.7 mT at the inner and outer surfaces of the wall of the hollow cylinder sample.

D. Monitoring Set Up

The output of each of the three pickup coils is fed to an integrating amplifier (PAR model 215) each of which in turn drives the Y axis of an X-Y recorder. The X-axis of the recording instruments can be connected to a shunt monitoring the current flowing in the solenoid or to another shunt monitoring the current flowing in the toroidal coil. With this arrangement the locus of B_l , the longitudinal flux density in the hole, $\langle B_\phi \rangle$, the azimuthal flux density in the wall and $\Delta \langle B_z \rangle_{\text{cyl}}$, the change in the longitudinal flux density threading the combination of hole and wall can be

monitored and recorded simultaneously and continuously versus $H_{//}$ as $H_{//}$ is varied with H_{ϕ} kept fixed or versus H_{ϕ} as H_{ϕ} is swept with $H_{//}$ stationary.

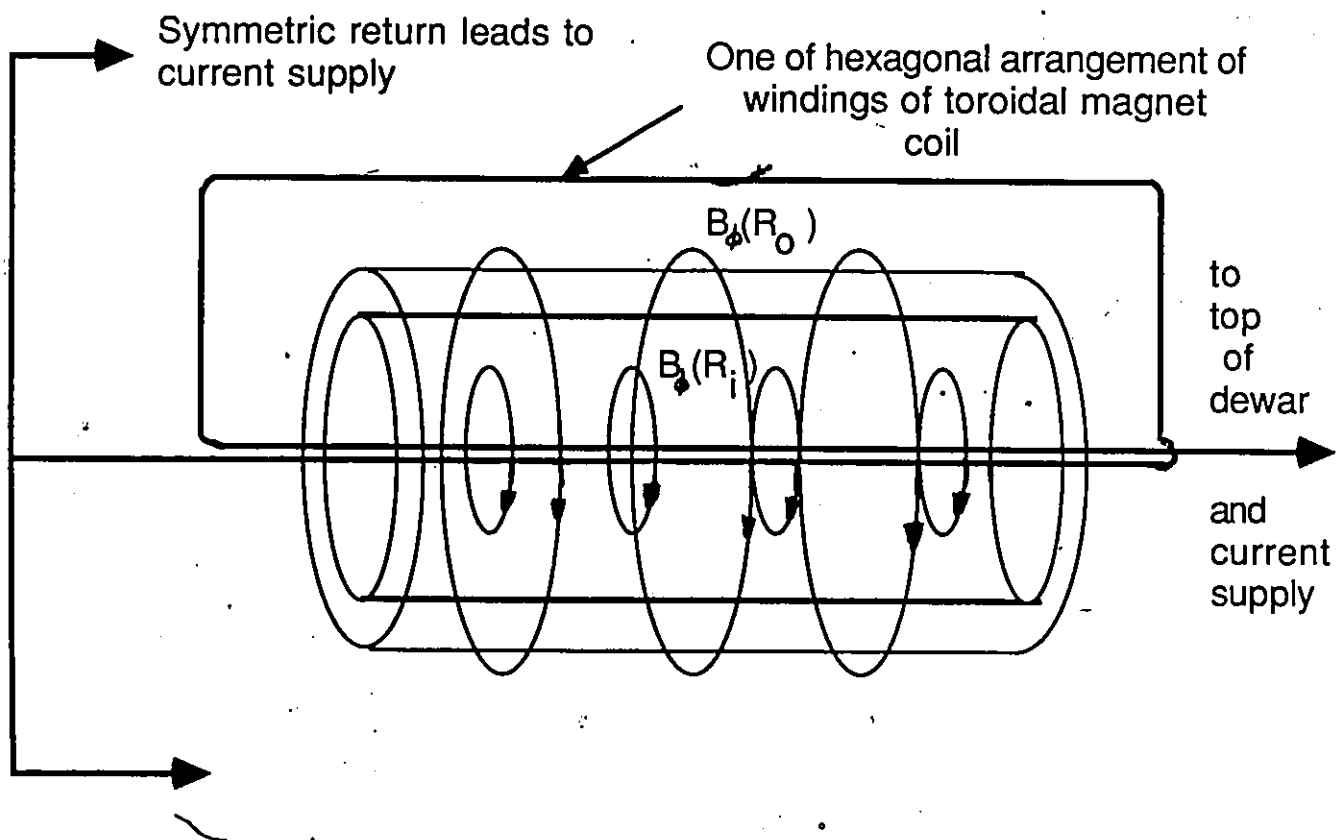
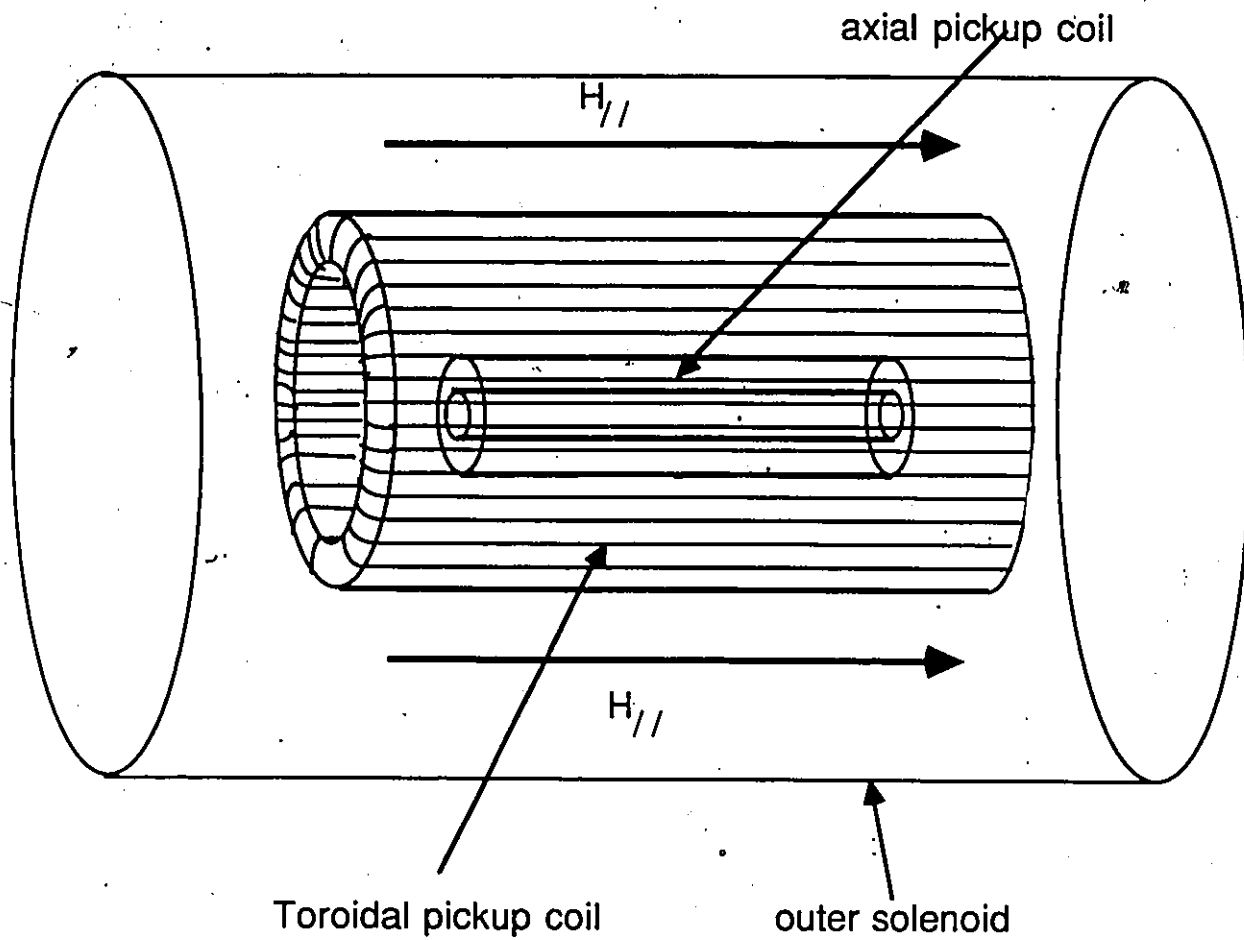
Measurement Procedure

Following established practice the signals are calibrated on the assumption that the specimen exhibits perfect shielding against an increase from zero of $H_{//}$ or H_{ϕ} in the range of weak fields, i.e. $H < H_{cl} \approx 40$ mT, with the specimen in the virgin state.

From every measurement the sample is made to cool from T_c to 4.2 K in the chosen initial values for $H_{//}$ and H_{ϕ} . The "measurement" is repeated with the sample held in the normal state by maintaining its temperature above T_c . The latter measurement provides a background or reference curve. Unfortunately, in some circumstances, Nernst voltages can defeat the attempt to determine the corresponding reference curve. In these cases we rely on the most appropriate background curve. Alternatively we can exploit the more tedious procedure of establishing absolute reference values at various points along the desired measured curve. The latter is accomplished by driving the sample into the normal state by heating to T_c at the chosen "coordinates". Appreciable Nernst signals can arise at T greater than ambient for the following reason. At $T \gtrsim T_c >$ ambient temperature, the wire of the windings of each of the pickup coils bathes in a temperature gradient $\perp$ to the wire axis. A Nernst emf will develop when a magnetic field transverse to the wire axis is also present. Since the total length of wire in any of the pickup coils is large, significant Nernst voltages can be generated. This voltage is particularly troublesome in situations where a long time integral is required, such as in the measurement of a background curve, but can be

negligible in a one-shot or pulse measurement. Clearly, when the two procedures can be exploited it is useful to do so as a double check on the reliability of the net signal obtained.

Finally, at the end of each measurement, the amount of electronic drift can be ascertained by shorting the integrator - amplifier. With the applied field and temperature variables at the initial values, (hence $T = T_c$) the recording pen should return to its initial position. Data curves which deviate significantly from this requirement are rejected.



Chapter 3

The Azimuthal Paramagnetic Effect

Introduction

In this chapter we present our observations of the evolution of the magnetic flux permeating the wall of a hollow type II superconducting cylinder as a longitudinal magnetic field $H_{//}$ is impressed while an externally applied azimuthal magnetic field H_{ϕ} is kept fixed. An increase in the externally applied longitudinal magnetic field will, in keeping with the classical laws of induction, induce azimuthal screening currents in the wall of the cylinder. Since the material is superconducting these currents flowing around the circumference of the cylinder are persistent. Also, since the current density must be finite, hence have a critical value j_c , the screening will be imperfect and penetration of the applied longitudinal magnetic flux will occur along the periphery of the cylinder. The depth of this penetration is determined by the azimuthal current density $j_{\phi}(r)$, or equivalently, $j_c(B, T)$ for the particular material. Since j_c is generally much smaller than the current density associated with the London penetration depth λ and Meissner Effect currents, this penetration far exceeds λ and can readily be measured by undergraduate laboratory techniques (eg. pickup coils). The penetration advances as $H_{//}$ increases and eventually the entire width of the wall is filled with screening current.

At this juncture, the $B_z(r)$ profile varies from $\mu_0 H_{//}$ at the outer circumference to zero at the inner boundary if initially no longitudinal flux threaded the hole. In the language of irreversible type II superconductors, the flux disturbance or front of the B_z profile has advanced to the inner

surface and a saturated critical state now exists throughout the wall. Any further increase in $H_{//}$ will cause the longitudinal magnetic induction B_l in the hole to grow from zero. The difference between these two quantities can be written,

$$H_{//} - \frac{B_l}{\mu_0} = I_\phi = \int_{R_i}^{R_o} j_\phi(r) dr \quad (1)$$

where I_ϕ is the azimuthal current per unit length of the cylinder and R_o and R_i are the outer and inner radius respectively. Equation 1 is quite general since it expresses Ampère's law, but presumably $j_\phi(r) = 0$ over part of the inner volume of the cylinder wall until B_l begins to change. I_ϕ will diminish from the maximum attained at the onset of full penetration since in type II superconductors j_c decreases as B increases. (The peak effect is an exception to this rule but it is a special phenomenon encountered only in the vicinity of B_{c2}).

Many workers have used this approach^{42,29} and equation 1 to determine j_c as a function of B and T for a variety of materials and to study the effect of annealing, mechanical treatment and various types of irradiation and dosage levels on j_c for a chosen specimen. In these investigations it is frequently convenient to use thin walled cylinders. This helps to eliminate flux jumping and can simplify the analysis of the data. Generally, the linear approximation,

$$\langle B_z \rangle_{\text{wall}} = \frac{\mu_0 H_{//} + B_l}{2} \quad (2)$$

is introduced for the average magnetic induction permeating the wall. Caution must be exercised however, since in thin walled cylinders surface currents can play an important role and even dominate the bulk current, thereby invalidating equation 2 and the interpretation of I_ϕ .

In our work we monitor the "lag" between B_z and $\mu_0 H_{//}$ by means of an axial pickup coil placed inside the hollow of the cylinder. Further, we, in effect also measure $\langle B_z \rangle_{\text{wall}}$ since we also monitor, ϕ_{total} , the total longitudinal flux permeating the cylinder with a pickup coil embracing the latter. (Actually, as indicated in chapter 2, we monitor the difference between ϕ_{total} and $\phi_{\text{applied}} = \mu_0 H_{//} \pi R_0^2$, but this is immaterial here). The flux threading the hole, ϕ_{hole} , can be subtracted electronically from ϕ_{total} during the actual measurement by suitably combining these two signals, or subsequently by digitizing and subtracting the corresponding data traces.

The major new feature which was introduced in our work, is that the wall of the cylinder can bathe in an externally applied azimuthal magnetic field H_ϕ as the longitudinal field $H_{//}$ is applied. To simplify the physical situation and the consequent interpretation of the data we have elected to let the cylinder become superconducting in some chosen azimuthal field which is subsequently maintained fixed at the selected value throughout the measurement. This approach enables us to assume that when the measurement commences, i.e. when $\mu_0 H_{//}$ is increased, $B_\phi(r)$ can be reliably approximated by its configuration in the normal state. Our sample, like other hysteretic materials, exhibits negligible Meissner or Abrikosov diamagnetism during cooling from T_c in a magnetic field. Bulk pinning and possibly surface barriers inhibit the expulsion of flux expected for a reversible type II superconductor. In view of our arrangement for generating B_ϕ , this configuration is described by the standard expression,

$$B_\phi(r) = \frac{\mu_0 I}{2\pi r} \quad (3)$$

where $I = nI'$, with n , the number of wires or turns and I' , the steady current flowing in each wire threading the axis of the cylinder.

The presence of this stationary applied azimuthal magnetic induction orthogonal to the increasing longitudinal magnetic induction causes the magnetic flux permeating and penetrating the wall to display a variety of complex and fascinating behaviour.

Results and Discussion

A. Initially $\mu_0 H_{//} = 0$.

Firstly we examine the effect of the applied azimuthal magnetic field H_ϕ on the capacity of the induced azimuthally circulating persistent currents to screen the wall and the hole against penetrating by the increasing externally applied longitudinal magnetic field $H_{//}$. Figs. 1 and 2 compare the screening when $\langle B_\phi \rangle_{app} = 0$ and when $\langle B_\phi \rangle_{app} = 90$ mT. The latter is near the limit that our apparatus can provide without drift in our set up. This corresponds to supplying a steady current of 240 A to each wire of the 7 turn toroidal coil generating H_ϕ . Since the applied $B_\phi = \mu_0 H_\phi$ varies as $1/r$ we let $\langle B_\phi \rangle_{app}$ denote the spatial average of the applied azimuthal magnetic induction hence

$$\langle B_\phi \rangle_{app} = \frac{\mu_0 I}{2\pi (R_o - R_i)} \ln\left(\frac{R_o}{R_i}\right) \quad (4)$$

Our measurements with $0 < \langle B_\phi \rangle_{app} < 90$ mT show curves which lie between the two curves displayed in Fig. 1 and 2. We note that $\langle B_\phi \rangle_{app}$ has only a small influence on the screening current I_ϕ , reducing it slightly over a small range below the peak and enhancing it slightly in the vicinity of the peak and over a small range beyond it. Several workers in our laboratory have explored this analogous phenomenon in ribbon samples and shown that the B dependence of the pinning force density F_p , plays a dominant role in this

feature. Writing $F_p = \alpha B^n$, they find large enhancement when $n \approx 1$, the upper (Bean-London) limit of the physically allowed range, and small enhancement when $n \approx 0$, the lower (Kim) limit, of the reported values for n .

Fig. 1 is a direct measure of I_ϕ . Ideally we would expect a perfectly linear region followed by a monotonically descending curve as sketched in the inset. We note that the observed curves deviate from linear while still ascending, and trace a curved summit. Such behaviour is generally encountered in practice and is attributed to end effects and inhomogeneity of $\mu_0 H_{//}$ over the large dimensions of the sample. We adopt the usual and reasonable view that full penetration corresponds to the maximum. The curvature of the peak, however, introduces some uncertainty in the determination of the value of $\mu_0 H_{//}$ where the maximum occurs.

Fig. 2 displays $\mu_0 H_{//} - \langle B_z \rangle_{\text{cyl}}$, the difference between the applied longitudinal magnetic induction and $\langle B_z \rangle_{\text{cyl}}$, the average longitudinal magnetic flux density threading the entire cross-section of the cylinder, i.e. both the wall and the hole. It is this quantity which is directly monitored by the balanced pickup coil which embraces the cylinder. In the framework of the definition $\mu_0 M = B - \mu_0 H$, this is the average magnetization of the entire cylinder, (the hole included). This quantity also provides a reliable measure of I_ϕ although the relationship may not be exactly linear. Using Maxwell's equation in the form $dB_z/dr = -\mu_0 j_\phi$ leads to

$$B_z(r) = B_1 + \mu_0 \int_{R_1}^r j_\phi dr \quad (5)$$

Introducing this in the definition,

$$\langle B_z(r) \rangle_{\text{cyl}} = \frac{2}{R_o^2} \int_0^{R_o} B_z(r) r dr \quad (6a)$$

leads to

$$\langle B_z \rangle_{\text{cyl}} = B_1 + \frac{2\mu_0}{R_0} \int_{R_0}^{R_1} r dr \int_{R_1}^r j_\phi(r') dr' \quad (6b)$$

Consequently

$$\mu_0 H_{//} - \langle B_z \rangle_{\text{cyl}} = \mu_0 I_\phi - f(j_\phi) \quad (7)$$

where $f(j_\phi)$ can be regarded as a term amending I_ϕ and containing information on any deviation of the $B_z(r)$ profile from linearity. If j_ϕ is taken as uniform, hence $B_z(r)$ as linear, then

$$f(j_\phi) = \mu_0 I_\phi \left(\frac{2R_0^2 - R_0 R_1 - R_1^2}{3R_0^2} \right) \quad (8)$$

The factor in parentheses is 0.17 for our sample. In a later chapter we will exploit these data to estimate the magnitude and the B dependence of the critical current densities I and $//$ to the magnetic induction permeating the wall of the cylinder. We note that when an azimuthal field is present, j_ϕ is no longer purely transverse to the magnetic flux density but now flows with a component $//$ as well as a component $\perp$ to $\vec{B}$.

The most fascinating feature of the observations reported in this chapter is the evolution of the magnetic induction permeating the wall as $H_{//}$ is impressed although the applied azimuthal field H_ϕ bathing the wall is stationary. Fig. 3 presents a small selection of measured curves displaying the variation of

$$\Delta \langle B_\phi \rangle = \langle B_\phi \rangle - \langle B_\phi \rangle_{\text{app}} \quad (9)$$

versus $\mu_0 H_{//}$ in various fixed $\langle B_\phi \rangle_{\text{app}}$. Here $\langle B_\phi \rangle$ is the average azimuthal magnetic induction permeating the wall and $\langle B_\phi \rangle_{\text{app}}$ is the average applied magnetic induction. $\Delta \langle B_\phi \rangle$ is the quantity monitored by the toroidal pickup coil embracing the wall of the cylinder.

The occurrence of the quantity $\Delta\langle B_\phi \rangle$ indicates unambiguously that a longitudinal persistent current I_z is induced to flow along the outer volume of the cylinder wall with an equal return longitudinal current flowing along the inner volume of the wall as $H_{//}$ is increased. We stress that the appearance of such a current is unexpected in the framework of traditional electrodynamics since this current flows along the direction of the varying component of the magnetic field. This longitudinally circulating current is denoted as paramagnetic (diamagnetic) when the azimuthal flux it generates enhances (opposes) the applied azimuthal flux density. From examination of Fig. 3, it is clear that, except for a weak initial diamagnetic excursion, the major portion of the behaviour is paramagnetic, hence is referred to as the azimuthal paramagnetic effect, giving this chapter its title.

In this chapter we will focus on the paramagnetic behaviour because it is evidently dominant and intricate. The initial diamagnetic region however, although it represents a proportionately small effect, will emerge as quite important in the theoretical analysis of these curves. There we will see that this diamagnetic effect is the signature of the occurrence of a pure flux cutting zone in the vicinity of the front of the advancing flux disturbance.

We note that the locus of all the $\Delta\langle B_\phi \rangle$ versus $H_{//}$ curves display the same major features. After the weak diamagnetic "dip", the curves all trace a sharp peak, traverse a valley and then map out a second but very broad peak. From related work on ribbon samples we expect that the curve beyond the second summit will decline monotonically to zero as B approaches B_{c2} .

Fig. 4 displays $\Delta \langle B \rangle_{\phi \text{ peak}(1)}$, the height attained at the maximum of the sharp peak versus the stationary $\langle B_{\phi} \rangle_{\text{app}}$. The height of the paramagnetic peak is seen initially to rise rapidly as the value chosen for the fixed $\langle B_{\phi} \rangle_{\text{app}}$ which bathes the wall is larger. Then the rate of increase of $\Delta \langle B \rangle_{\phi \text{ peak}(1)}$ dwindles and the curve reaches a saturation level. Related measurements on ribbon samples indicate that the plateau we observe in Fig. 4 will be followed by a monotonically declining curve, with the descent commencing just beyond the present limit of the data.

It is useful to estimate and compare the magnitudes of the longitudinal and azimuthal current densities which are encountered in the phenomena just described. Assuming that the longitudinal current I_z has a uniform density j_z , then $j_z \approx 4(10^7) \text{ A/m}^2$ at the summit of the highest peak observed where $\Delta \langle B_{\phi} \rangle \approx 9 \text{ mT}$. This is to be compared with $j_{\phi} \approx 11(10^7) \text{ A/m}^2$ associated with the maximum $\mu_o I_{\phi} \approx 100 \text{ mT}$ (see Fig. 1); assuming a uniform j_{ϕ} . Also, in the framework of a uniform j_z occupying the entire width of the wall, it is deduced from the azimuthal paramagnetic effect that the B_{ϕ} profile rises from an initial value of 14.7 mT to 23.2 mT, hence increases by $\sim 60\%$ at the interface of the positively and negatively flowing I_z from its initial value when $\langle B_{\phi} \rangle_{\text{app}} = 15 \text{ mT}$. In our view such a large change is impressive indeed.

Considering Fig. 3 again, one notes that the position of the sharp peak does not appear to shift as its height is seen to vary under the influence of different choices for the stationary $\langle B_{\phi} \rangle_{\text{app}}$. Fig. 5 presents $\mu_o H_{\parallel \text{ peak}(1)}$, the value of $\mu_o H_{\parallel}$ where the first peak attains its summit, versus various fixed $\langle B_{\phi} \rangle_{\text{app}}$. Evidently, there is some uncertainty involved in the determination of the exact position of the summit, hence some scatter

is seen in the data points of Fig. 5. Nevertheless, this graph indicates quite convincingly that the position of the summit is insensitive to the magnitude selected for the stationary azimuthal field bathing the wall.

The position of the maxima in I_ϕ versus $\mu_0 H_{//}$ and of $\mu_0 H_{//} - \langle B_z \rangle_{cyl}$ versus $\mu_0 H_{//}$ were also seen to be insensitive to the choice of $\langle B_\phi \rangle_{app}$ (see Figs. 1 and 2). We note that the position of these maxima coincides within a few percent with the value of $\mu_0 H_{//}$ peak (1) ≈ 148 mT emerging from inspection of Fig. 5. Physically, this correspondence means that the azimuthal paramagnetic effect grows until the penetration of the invading longitudinal flux density profile attains the inner surface of the cylinder.

It is perhaps tempting to postulate that the azimuthal vortices initially permeating the wall and the successive generations of helical vortices introduced at the outer surface as $H_{//}$ is increased, migrate purely radially across the wall of the cylinder. This picture, first proposed by Campbell and Evetts, qualitatively and semi-quantitatively accounts for the initial rise in the paramagnetic effect. With this picture, the initial growth of the azimuthal flux can readily be understood from simple critical state considerations. The azimuthal flux initially permeating the outer region of the wall becomes compressed in a cylindrical zone which, at the same time, is displaced inward by the advance of the inner boundary of the successive generations of cylindrical sheets of helical vortices created at the surface. This process of compression and displacement continues until the front of the zone of compression attains the inner radius of the hollow cylinder. Until that happens, the azimuthal flux threading the wall has been continuously augmented by the addition of the azimuthal component of the

helical vortices injected into the wall at the outer surface. Meanwhile, the wall has retained its initial complement of vortex rings since no transport has taken place across the inner surface.

A fundamental problem, however, now arises with this picture once the front of the advancing zone of compression has reached the inner surface. We note that by Ampère's law, $B_{\phi}(R_i) = \mu_0 I / 2\pi R_i$, is and must remain fixed at the value set by the toroidal generating coil. In other words, the hole acts as a sink for azimuthal vortices (and for the azimuthal component of helical vortices) which cross the inner surface and enter the hole. Consequently any increase in the total magnetic induction in the hole must arise from the appearance and growth of a longitudinal flux density. The model we have been pursuing therefore leads to a contradiction at this juncture. The volume surrounding the immediate vicinity of the hole is filled with pure azimuthal vortex rings. Yet any further increase in $H_{//}$ must cause longitudinal flux to enter the hole. Where can this longitudinal flux come from? Such flux is available in the volume penetrated by helical vortices. This zone however is physically separated from the hole by the region of compressed azimuthal flux which it embraces. The model of flux conservation and radial displacement does not permit these helical vortices to cut through the "barrier" of azimuthal flux and appear in the hole. This impasse compels the rejection of this model. The paradox indicates that a model which allows for the juxtaposition of zones of flux transport and zones of flux cutting and the coexistence of flux transport and flux cutting in some regions must be invoked to avoid inconsistencies and anomalies. The model of pure radial migration of the flux lines can be useful however, in providing an upper limit for the initial evolution of the azimuthal paramagnetic effect.

In a framework where flux cutting, hence the consumption and reorientation of flux are allowed, it seems no longer possible, however, to predict intuitively whether a paramagnetic or diamagnetic effect will arise. Consequently, the influence of the hole on the evolution of the azimuthal flux in the wall is extremely difficult to visualize and account for.

Our observations show that before the flux front reaches the hole, the process of flux consumption and reorientation by flux cutting cannot compete with the rate of entry of azimuthal flux and the latter accumulates in the wall. Once the flux disturbance has attained the inner surface and flux is being fed into the hole, the ejection of azimuthal flux combined with flux cutting appear to strongly outweigh the rate of entry of azimuthal flux carried by helical vortices created at the outer surface and we witness a rapid decline in $\langle B_\phi \rangle$, the net amount of azimuthal flux permeating the wall. A priori, however, other possible behaviour can be visualized at this juncture. For instance one could expect a simple slowing down in rate of growth of $\langle B_\phi \rangle$ versus $H_{//}$ or the onset of a steady state where $\langle B_\phi \rangle$ would decrease monotonically with $H_{//}$. The reason for the massive and rapid reduction in $\langle B_\phi \rangle$ is not at all evident in our view. Further we cannot point to a simple explanation for the second accumulation of azimuthal flux (the second peak) after the abrupt descent. These complex features present a daunting challenge to any model and theory.

For completeness, we close this section by presenting Fig. 6 which displays $\Delta \langle B_\phi \rangle$ valley (1), the height observed for the bottom of the valley between the sharp summit and the broad peak versus various $\langle B_\phi \rangle_{app}$ (see Fig. 3). This quantity appears to be insensitive to the magnitude of $\langle B_\phi \rangle_{app}$ although $\langle B_\phi \rangle_{app}$ must be present for this behaviour to appear. When $\langle B_\phi \rangle_{app}$

is very small, the signal for $\Delta \langle B_\phi \rangle$ is correspondingly weaker and the percentage error in the data becomes appreciable. Hence, no particular significance is assigned to the points plotted for $\langle B_\phi \rangle_{app}$ small.

B. Initially $\mu_0 H_{//} \neq 0$

We now examine the magnetic response of the hollow cylinder to an increase in $H_{//}$ starting from some selected initial value $H_{//1}$ while the wall is bathing in a fixed azimuthal field. Since the cylinder becomes superconducting in both $\langle B_\phi \rangle_{app}$ and $\mu_0 H_{//1}$, it is permeated by cylindrical sheets of helical vortices when the increase in $H_{//}$ takes place. Our main purpose here was to explore the phenomena in the absence of vortex rings (pure azimuthal flux). For this series of measurements we selected the maximum H_ϕ which our apparatus could sustain without drift and excessive consumption of liquid helium, namely, $\langle B_\phi \rangle_{app} = 90 \text{ mT}$. Figs. 7 and 8 display the rise in the screening current to its maximum value, attained presumably when the current fills the entire cross-section of the wall, and show the subsequent decline in I_ϕ , reflecting the B -dependence of the critical current density. These two figures contain no surprises and present standard behaviour. The fact that, within our experimental accuracy, the various curves merge and trace a common envelope regardless of the starting point, indicates that, the spatial distribution of the azimuthal current density across the wall, after full penetration, is independent of or at least is insensitive to the degree of helicity of the initial configuration. The curve traced when the initial configuration is purely azimuthal appears however to deviate significantly from the common envelope traced by the other curves. This indicates that under these circumstances a larger azimuthal current can be induced.

Figs. 9 and 10 show the evolution of the excess azimuthal flux $\Delta\langle B_\phi \rangle$ versus $H_{//}$ starting with weak longitudinal fields $H_{//1}$ (Fig. 9) and stronger $H_{//1}$ (Fig. 10). We see no evidence that the various curves overlap and no tendency to trace a common envelope or master curve once full penetration has been achieved. Our observations therefore suggest that the configurations of the longitudinal currents and hence of the azimuthal magnetic flux is quite history dependent even when the entire wall is saturated with persistent current. These data indicate that a double critical state model will be inadequate. The reason is that a double critical state model will by its very nature generate previous history independent behaviour after full penetration (saturation) is established. Consequently these data demand a more versatile and flexible framework.

Another interesting feature which emerges from inspection of Figs. 9 and 10 is the following. We note that the initial peak in the paramagnetic effect is less pronounced and even vanishes when $H_{//1}$ is chosen larger and larger. Close examination, however, reveals that either a peak or an inflection point is traced. $\Delta H_{//} = H_{//} - H_{//1}$, the increment in $H_{//}$ needed to reach the summit or the inflection point is plotted versus $H_{//1}$ in Fig. 11. Also in Fig. 11 we plot $\Delta H'_{//}$, the increment in $\Delta H'_{//}$ required to generate the maximum I_ϕ versus $H_{//1}$. (see Fig. 7 and 8). We note that these two graphs correlate quite well. This correspondence again confirms that arrival of the flux disturbance at the hole precipitates an appreciable change in the rate of accumulation of azimuthal flux in the wall.

Conclusion

The magnetic phenomena when a hollow cylinder of a hysteretic type II superconductor is subjected to the application of a longitudinal field $H_{//}$

while immersed in a static azimuthal field H_ϕ have been investigated. In the material we have studied, the magnitude of the persistent azimuthal screening current I_ϕ induced to circulate around the cylindrical wall by an increase in $H_{//}$ appears not to be significantly affected by the presence of the azimuthal field over the range of H_ϕ available in the present set up.

The azimuthal flux permeating the wall, however, exhibits a complex and intriguing behaviour. An initial decrease in the azimuthal flux, although not appreciable, provides strong evidence that the initial increase in the externally applied field causes flux cutting to occur in the vicinity of the outside surface of the cylinder. The only mechanism presently known which accounts for the consumption of flux in type II superconductors is the process of flux cutting. Consequently the disappearance of some of the azimuthal flux permeating the wall in the circumstances existing here, where flux is being introduced into the wall at the outer surface and no flux is being extracted at the inner surface, can only be attributed to flux cutting.

The shallow diamagnetic valley is followed by a dramatic rise in azimuthal flux in the wall. This augmentation of azimuthal flux, however, is less than anticipated from considerations of flux conservation coupled with the postulate of pure radial displacement of cylindrical sheets of azimuthal and helical vortices. Consequently, in an indirect manner, the accumulation of azimuthal flux in the wall can also be interpreted as evidence that, during this phase, flux cutting is again taking place. The argument here is of course less convincing.

This accumulation of azimuthal flux, denoted the azimuthal paramagnetic effect, is seen not only to cease but to collapse abruptly, immediately after the front of the flux disturbance has attained the inner

surface of the cylinder. Experimentally, a decrease in the azimuthal screening current I_ϕ is taken as the criterion that the entire cross section of the wall has been filled with azimuthally circulating current, hence that the flux disturbance has penetrated to the inner radius.

The decline in the paramagnetic effect which can be quite steep when initially $H_{//}$ is small, also suggests that much of the accumulated azimuthal flux is made to vanish by flux cutting. The account here is no longer straightforward however, since the wall is now feeding flux into the hole. Finally as $H_{//}$ proceeds to larger values, a second intriguing accumulation of azimuthal flux is observed. All of these features present a variety of serious challenges which a model must meet to be accepted.

We have also observed that the evolution of the azimuthal flux, after full penetration, shows dependence on the initial configuration. This feature indicates that the double critical state model exploited by LeBlanc and his collaborators must be modified or set aside since it cannot account for history dependence of this nature.

Fig. 3-1. Compares the azimuthal current I_ϕ induced by an increasing longitudinal magnetic field $H_{||}$ when the static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$ is zero (curve A) and 90 mT (curve B).

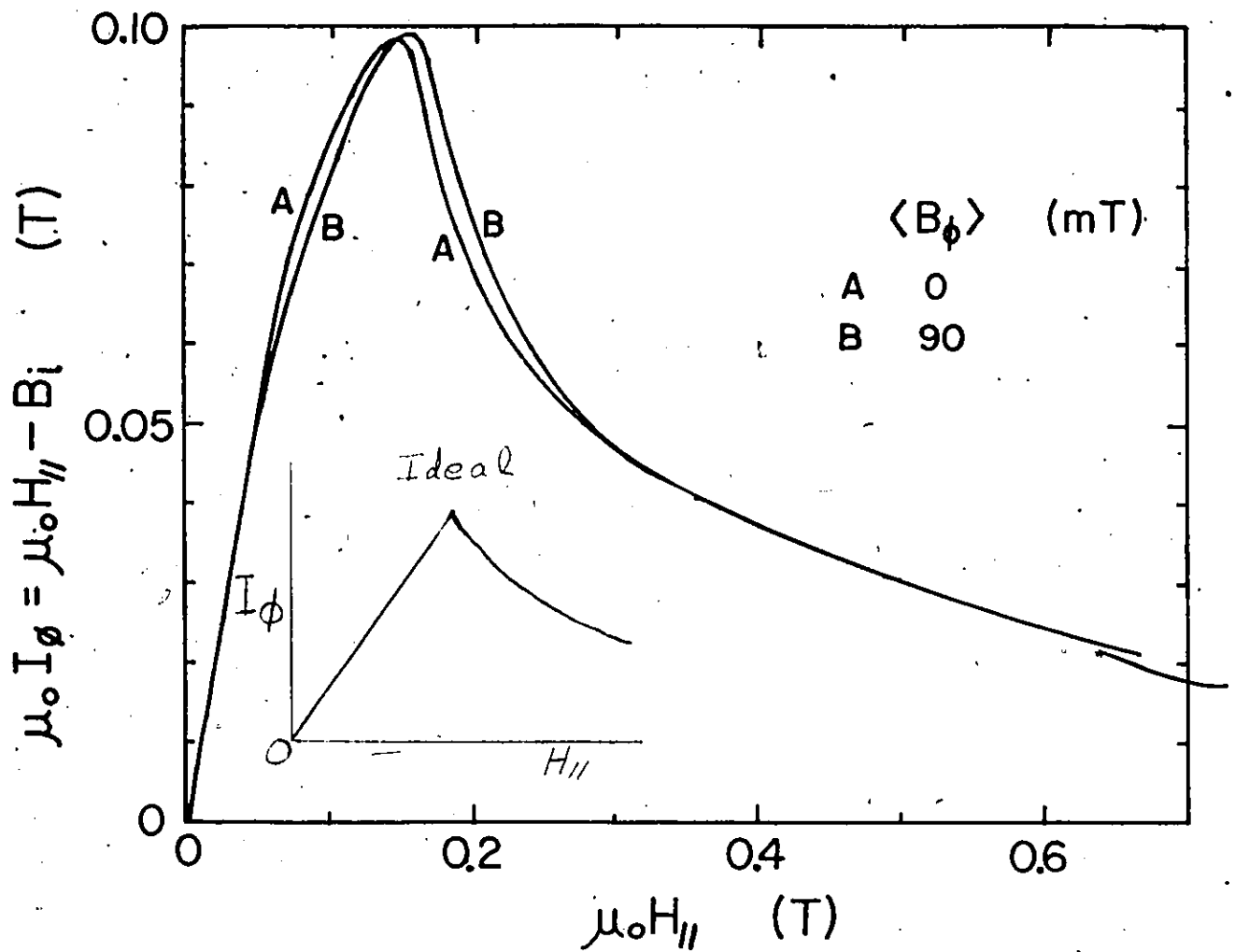


Fig. 3-2. Illustrates the influence (negligible) of the presence of a static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$ on the magnetic response of the hollow cylinder to the application of a longitudinal magnetic field $H_{//}$. Here $\Delta\langle B_z \rangle \equiv -\mu_0 \langle M_z \rangle$ is the deficit of flux in the cylinder averaged over the cross section of the combination of the wall and hole.

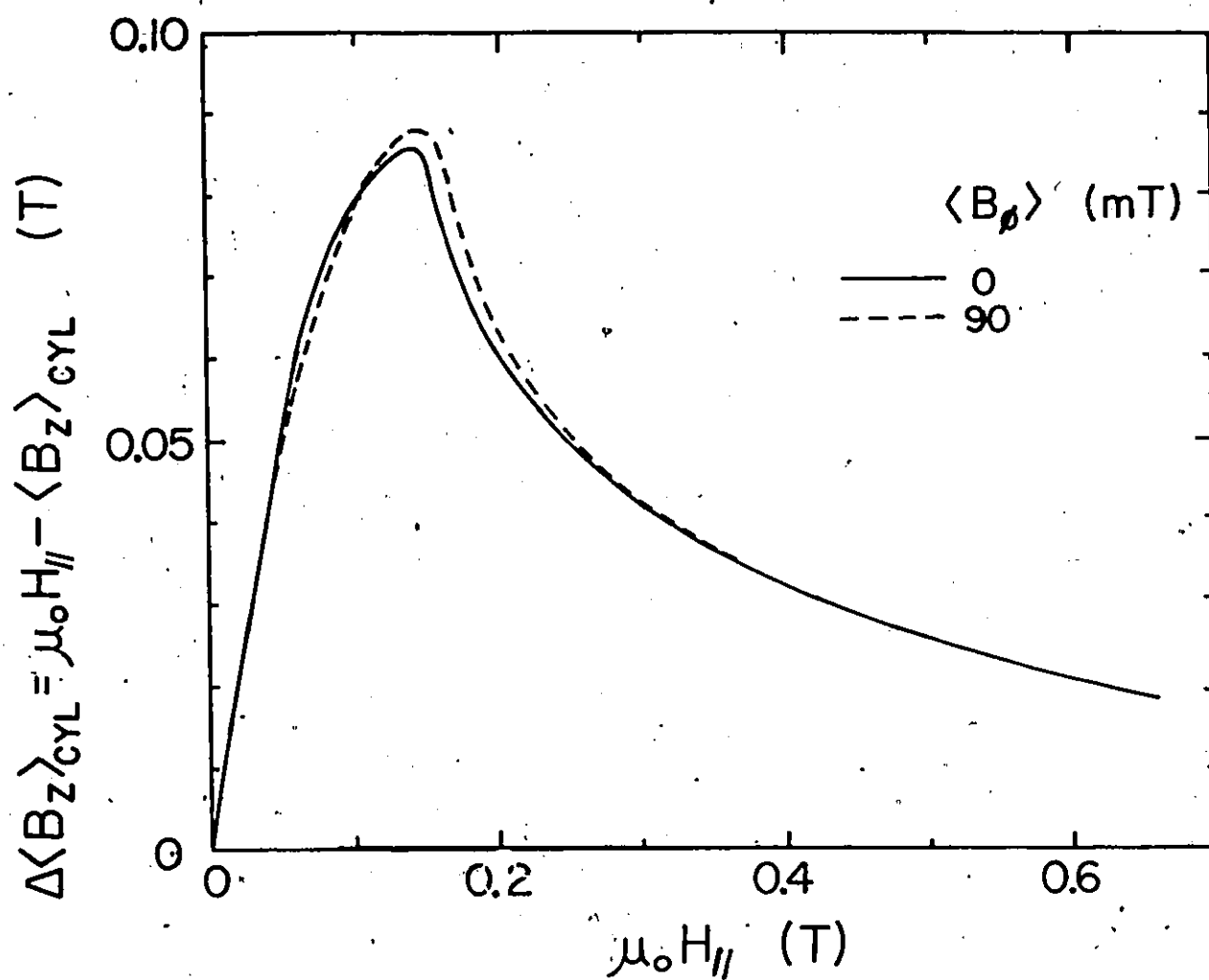


Fig. 3-3. Illustrates the evolution of the azimuthal magnetic induction permeating the wall with various static azimuthal magnetic fields present as $H_{//}$ is impressed. The initial negative dips and the valley constitute clear evidence of flux cutting. Only three representative curves are displayed from the several measured curves to avoid clutter. The observation, of $\Delta\langle B_{\phi} \rangle > 0$ is denoted the azimuthal paramagnetic effect.

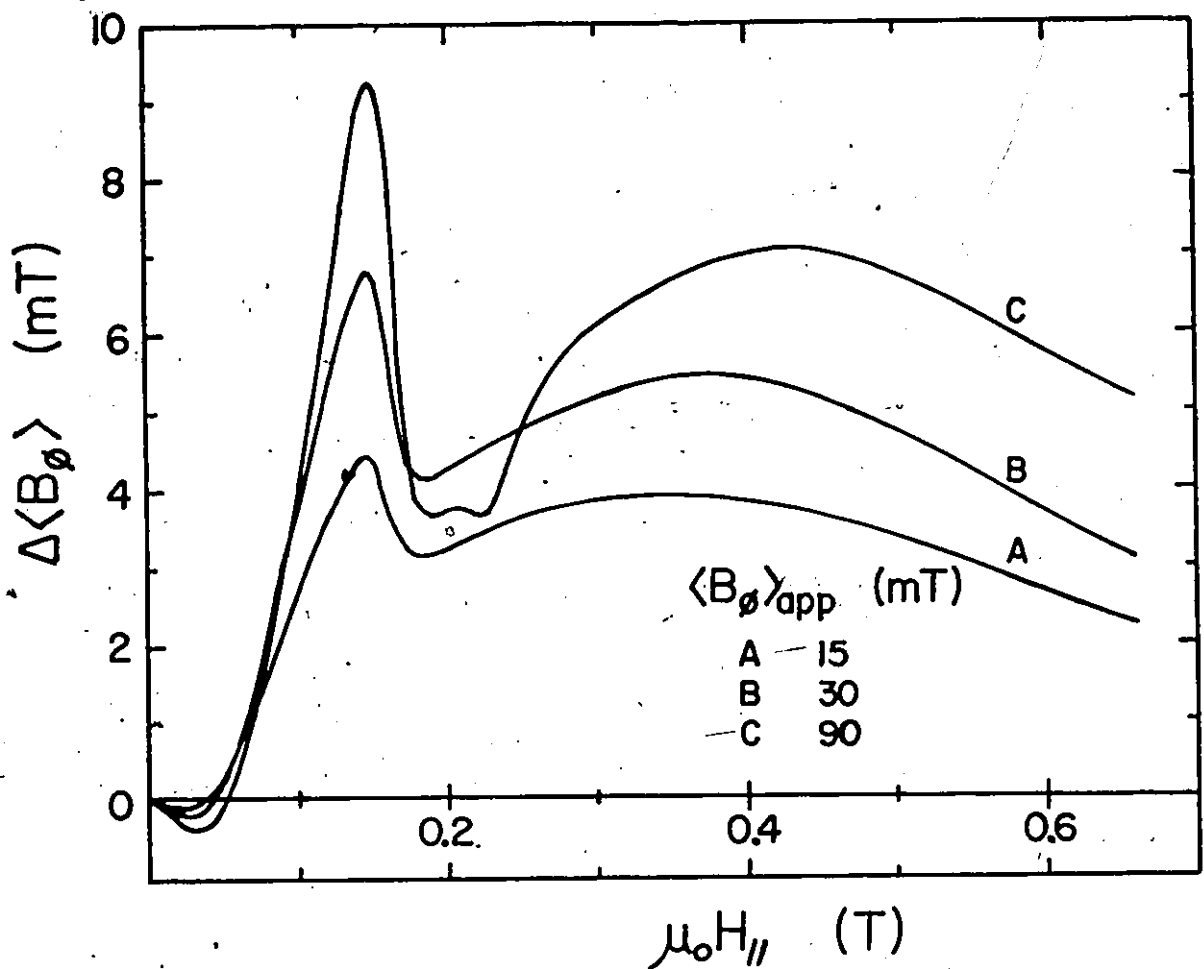


Fig. 3-4. Complements the previous figure and shows the dependence of the height of the first paramagnetic peak on the static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$.

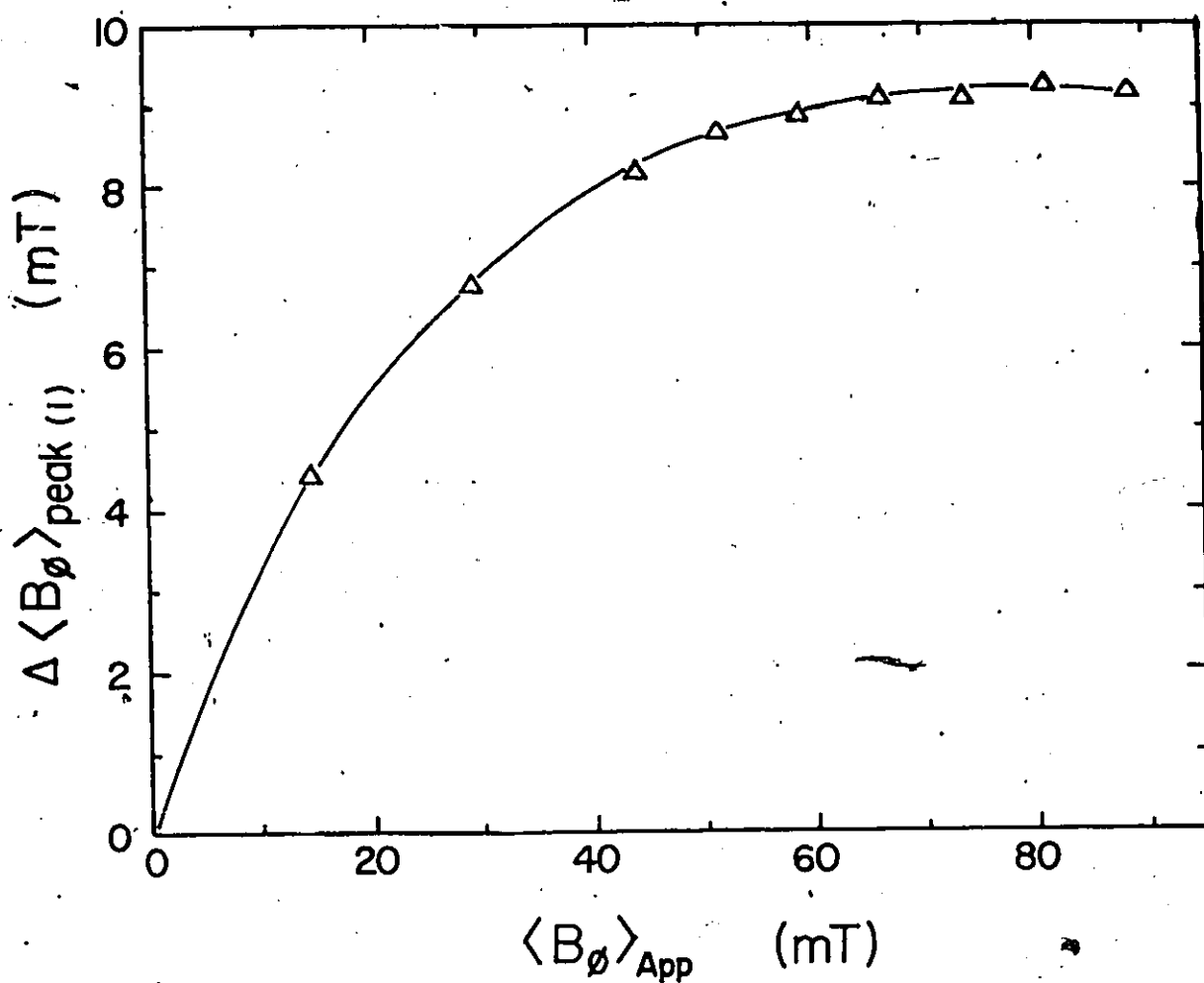


Fig. 3-5. Complements the two previous figures by displaying the value of $\mu_0 H_{//}$, where the maximum of the first peak is attained, as a function of the static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$. The position of the peak is seen to be insensitive to $\langle B_\phi \rangle_{app}$.

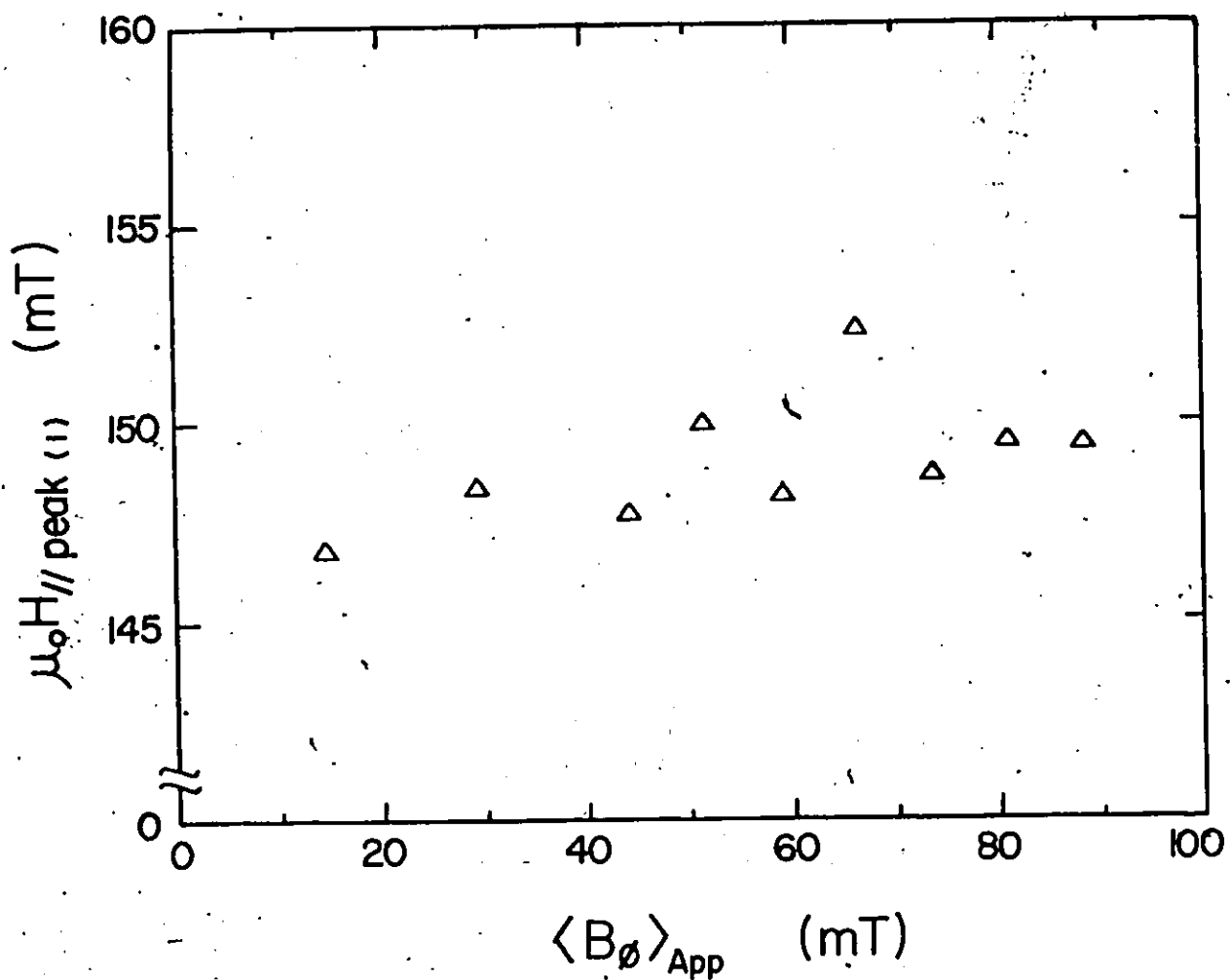


Fig. 3-6. Complements the three previous figures by showing the dependence of the height of the paramagnetic valley on the static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$.

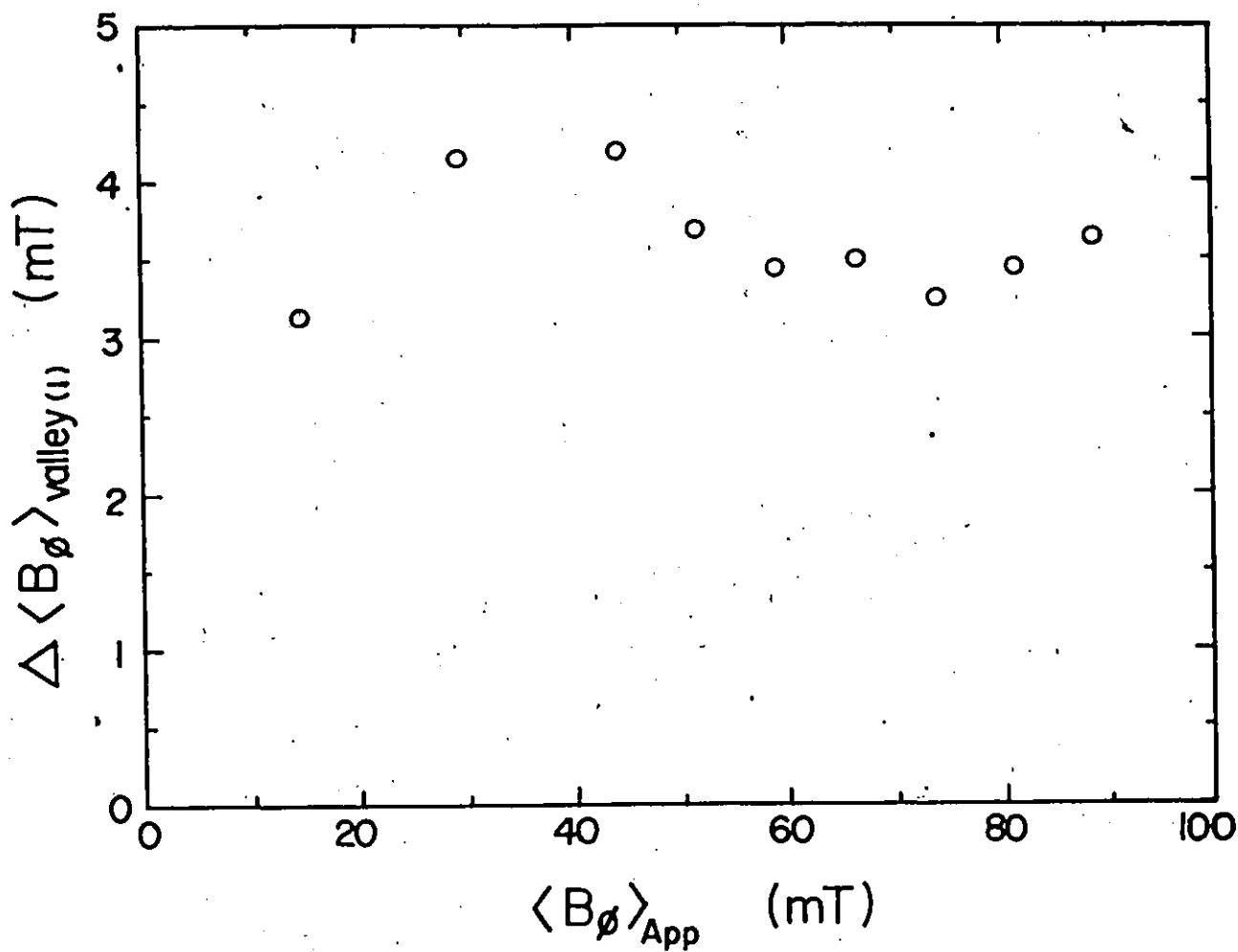


Fig. 3-7. Shows the evolution of the induced azimuthally circulating current I_ϕ as a longitudinal magnetic field $H_{||}$ is impressed starting from different initial values $H_{||i}$. A static azimuthal magnetic induction $\langle B_\phi \rangle_{app} = 90$ mT is present for all the data curves. The envelope is seen to be insensitive to the choice of starting point.

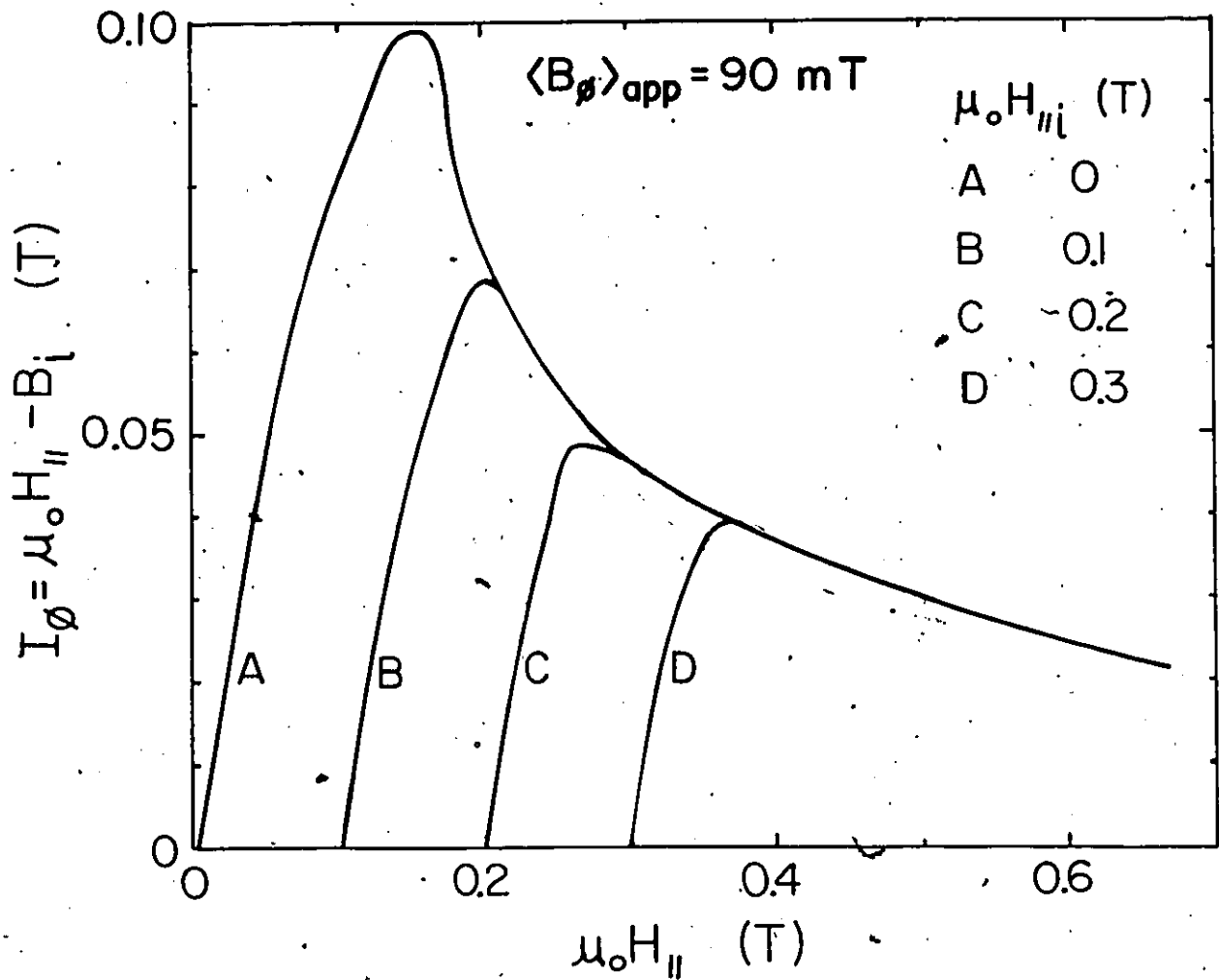


Fig. 3-8. Shows the magnetic response of the cylinder when an externally applied longitudinal magnetic field $H_{//}$ is impressed starting from different initial values $H_{//i}$. A static azimuthal magnetic induction $\langle B_{\phi} \rangle_{app} = 90$ mT is present for all the data curves. The envelope is seen to be insensitive to the choice of the starting point.

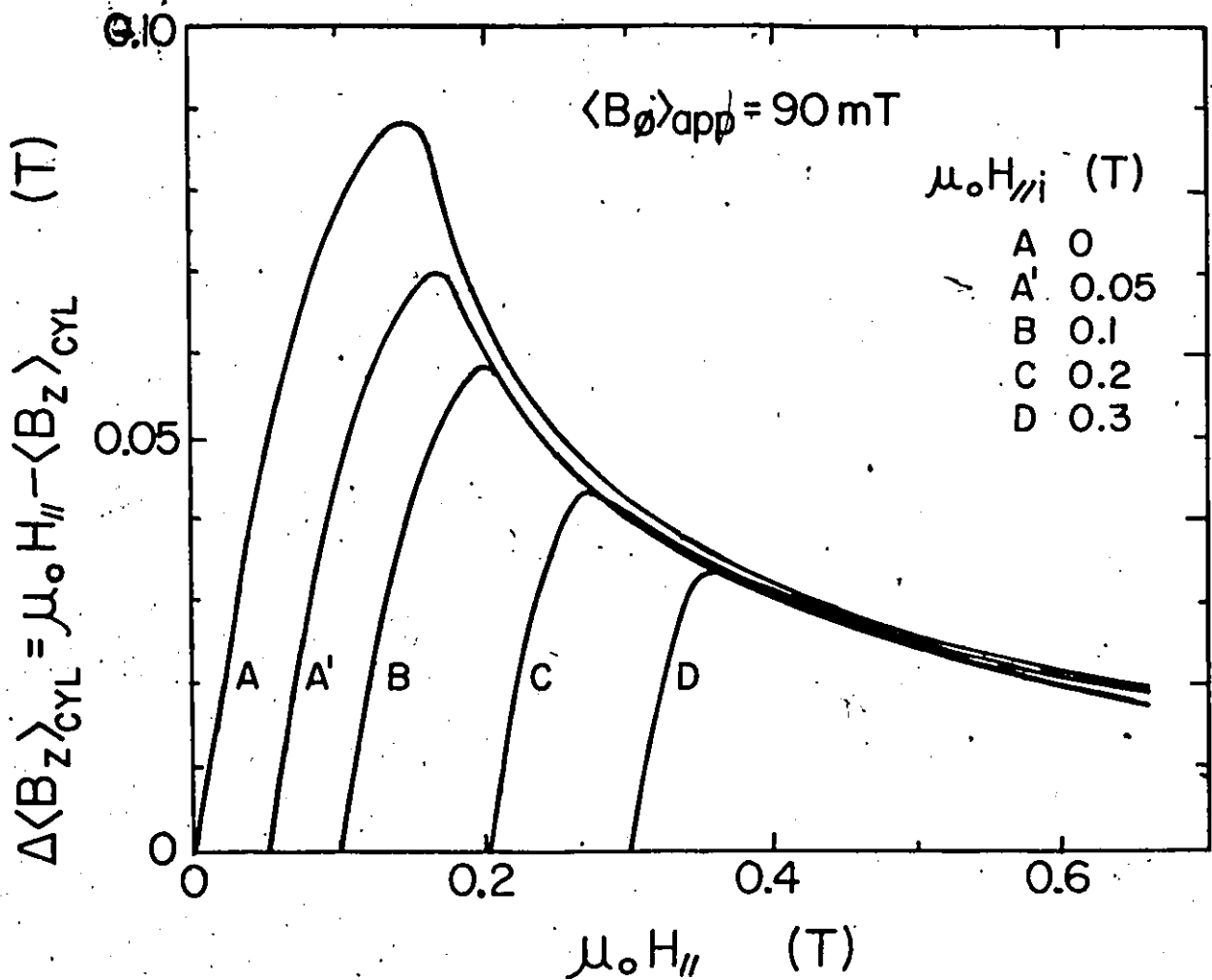


Fig. 3-9. Illustrates the evolution of the azimuthal magnetic induction permeating the wall as $H_{//}$ is impressed starting from different initial values $H_{//i}$. A static azimuthal magnetic induction $\langle B_{\phi} \rangle_{app} = 90$ mT is present for all the data curves. An increase in the choice of $H_{//i}$ causes the peak to shift to the right and diminish in height. Also, the locus of the second peak shows no overlap, hence the configurations of the magnetic induction and persistent currents must differ from one curve to the other.

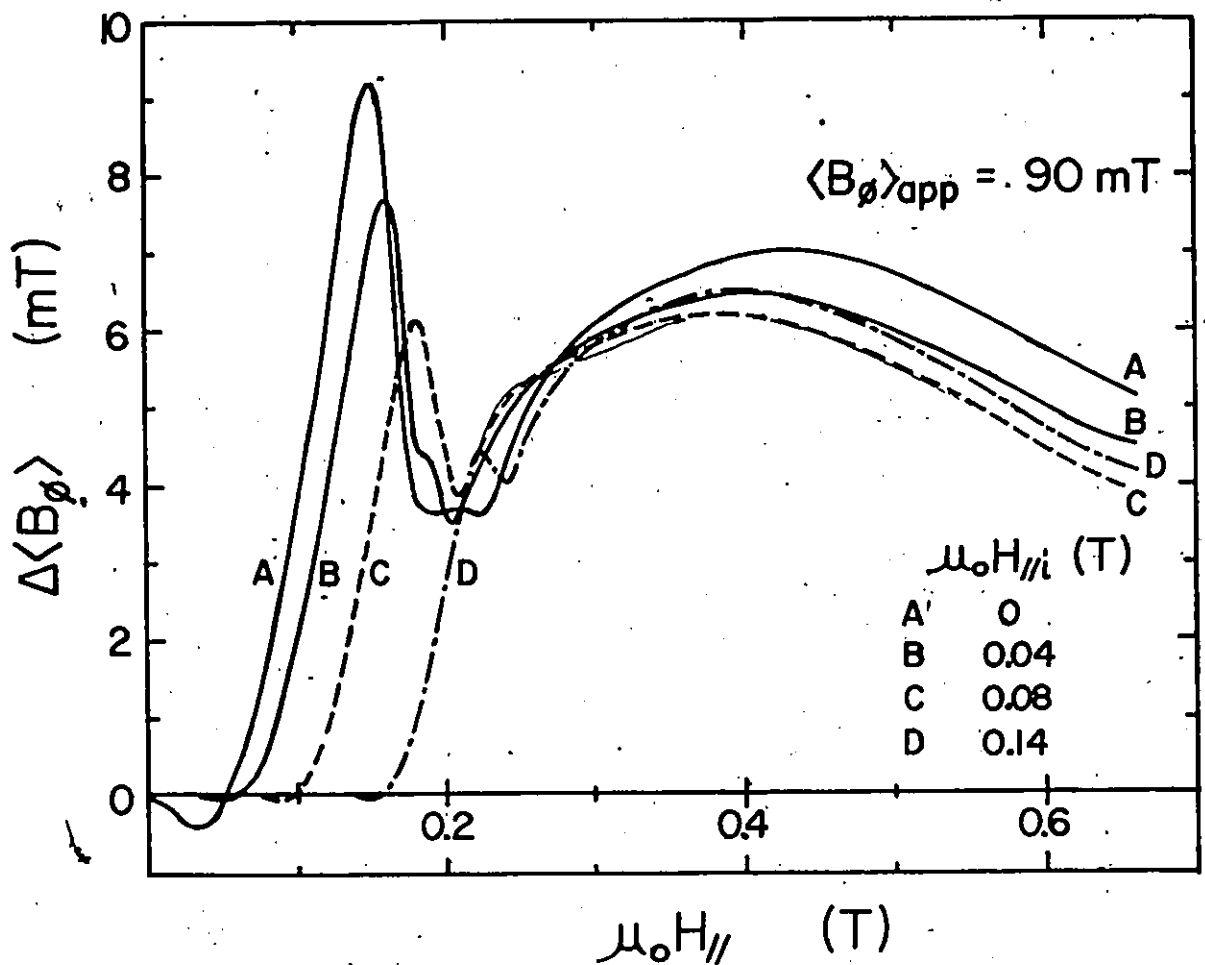


Fig. 3-10. Same as the previous figure but shows the behaviour over a wider range of $H_{//}$. For larger $H_{//}$, the paramagnetic peak and associated valley have gradually vanished and have been replaced by an inflected curve (see curves C and D).

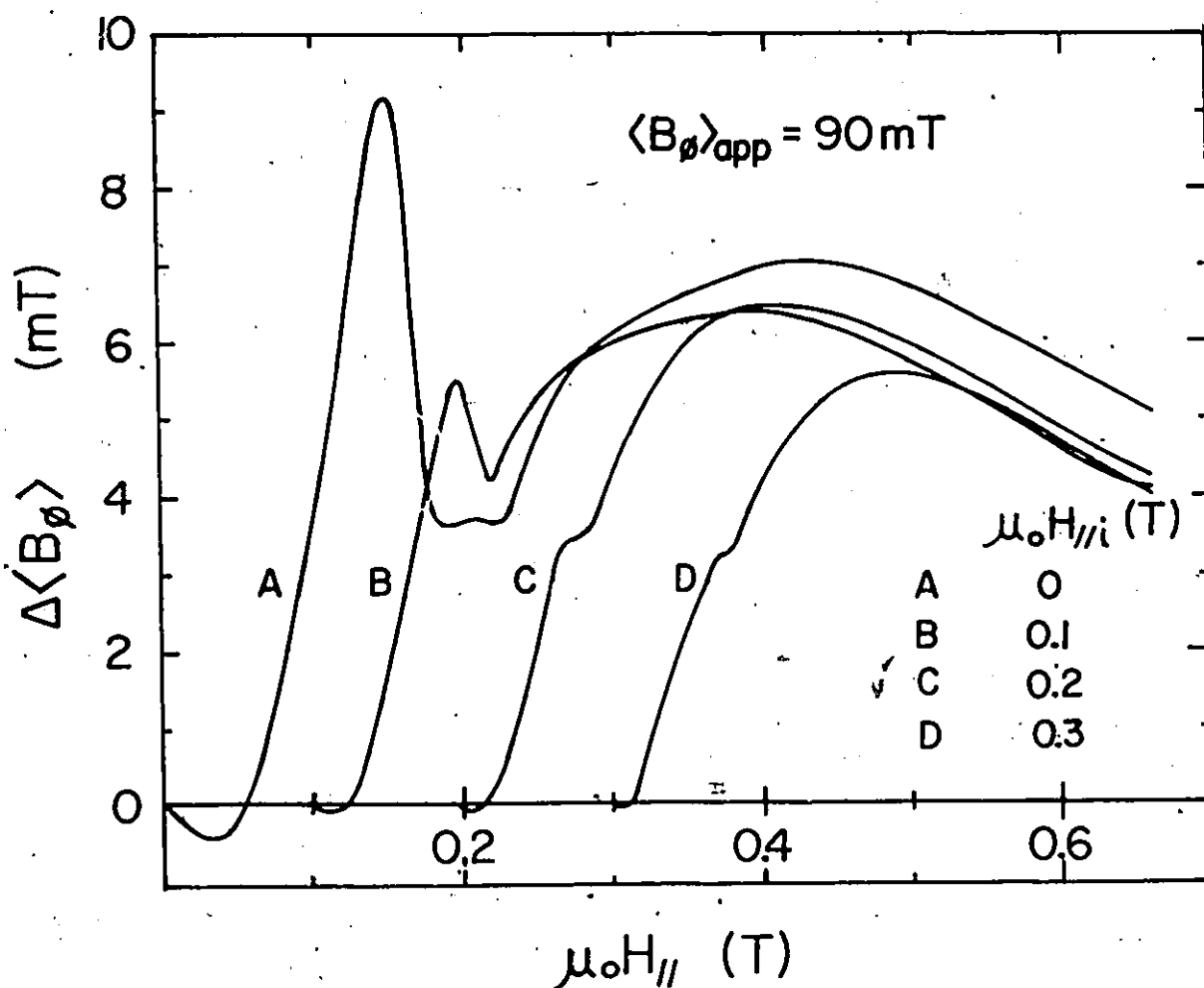
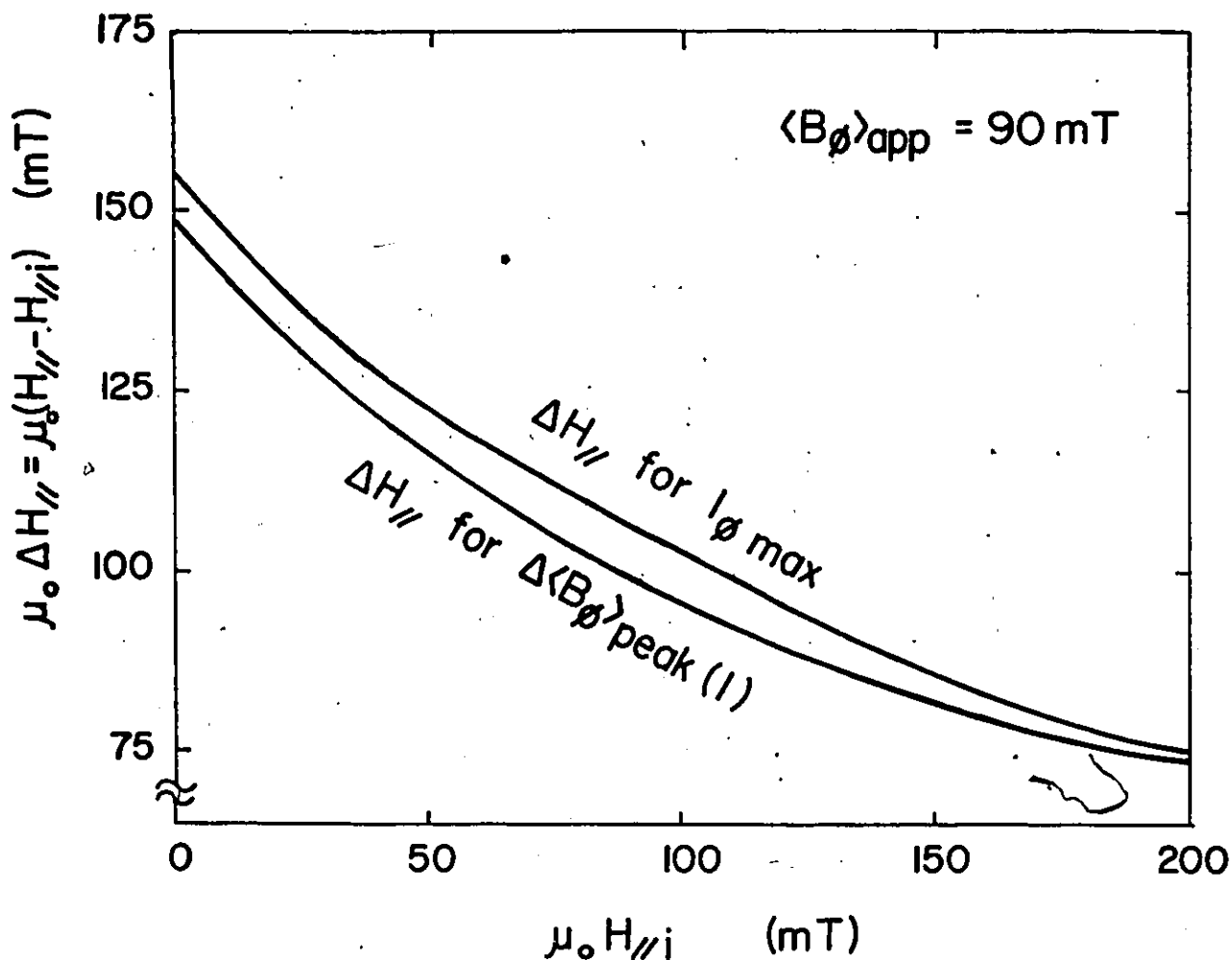


Fig. 3-11. The upper data curve is extracted from the measurements illustrated in Figs. 3-7 and 3-8. Here the increment, $\Delta H_{//} = H_{//} - H_{//i}$, i.e. the sweep of $H_{//}$ starting from $H_{//i}$ needed to attain the summits in these figures, is plotted versus $H_{//i}$. At the summits I_{ϕ} fills the entire width of the wall. The lower data curve is extracted from the measurements illustrated in Figs. 9 and 10. Here $\Delta H_{//}$, the sweep of $H_{//}$ starting from $H_{//i}$ needed to attain the top of the first peak or the start of the inflection, (see Fig. 10), is plotted versus $H_{//i}$. The close proximity of the two curves shown above is evidence that the penetration of the flux disturbance into the hole initiates a considerable amount of flux cutting in the wall.



Chapter 4

Diamagnetic to Paramagnetic Azimuthal Effect

Introduction

In this chapter we report on our observations of the magnetic behaviour of a hollow cylinder of a type II superconductor subjected to a sweep of an externally applied longitudinal magnetic field $H_{//}$ starting from some appreciable value $H_{// \text{max}}$ in which it has become superconducting by cooling through T_c to 4.2 K. The sweep of $H_{//}$ traverses the zero value and continues to a comparable intensity of opposite polarity. The new element in our investigation is that the sweep can be made to occur with the wall of the cylinder immersed in various selected stationary azimuthal magnetic fields H_ϕ .

It is useful to first review the sequence of events with $H_\phi = 0$. By the classical laws of induction, a decrease of $H_{//}$ from some initial value $H_{// \text{max}}$ will generate a persistent current, ΔI_ϕ , circulating azimuthally around the periphery of the cylinder. Since the current density j_ϕ must be finite, this current occupies a measurable fraction, $\Delta R / (R_o - R_i)$, of the wall thickness. Within this volume the magnetic induction decreases spatially from $\mu_o H_{// \text{max}}$ at the inner boundary of the region occupied by ΔI_ϕ , to $\mu_o H_{//}$ at the outer surface of the cylinder. The detailed spatial variation of B_z with radius r is determined by $j_\phi(r)$, hence by the dependence of the critical current density $j_c = j_\phi$ on $B(r) = B_z(r)$. It is useful to contrast the present situation with that examined in the previous chapter where $H_{//}$ was taken as increasing thereby inducing a screening or diamagnetic current

ΔI_ϕ which opposed the penetration of the applied magnetic flux. Here, the sense of the circulation of ΔI_ϕ must oppose the exit of the flux permeating the cylinder and is therefore referred to as a flux retaining or paramagnetic current.

In the previous case we visualized a profile of flux encroaching upon and penetrating the wall of the cylinder. Now we must consider a profile of flux undergoing a reduction in density in the annular region of thickness ΔR adjacent to the outer surface of the cylinder. The flux embraced by the inner radius of the annular region remains undisturbed, however, since it is shielded from any knowledge of the decrease in $H_{//}$ by the induced current ΔI_ϕ flowing in the annular region ΔR . It is convenient and meaningful to continue to refer to the annular region as penetrated by a flux disturbance and to its inner boundary as a flux front. This disturbance advances into the wall as $H_{//}$ continues to decrease and eventually attains the inner radius. We can then write

$$I_\phi = \frac{B_1}{\mu_0} - H_{//} = \int_{R_i}^{R_o} j_\phi(r) dr \quad (1)$$

where B_1 , the longitudinal magnetic induction in the hole is equal to $\mu_0 H_{//max}$ until the decrease in $H_{//}$ has induced an azimuthal current density in the entire thickness of the wall. Any further reduction of $H_{//}$ will cause B_1 to diminish from $\mu_0 H_{//max}$ and, consequently, the magnetic flux threading the hole to decrease. The flux leaving the hole first nucleates into vortices at the inner surface of the wall. These cylindrical sheets of vortices subsequently migrate across the wall and may eventually arrive at and exit through the outer surface.

A portion of the magnetic flux threading the hole and permeating the wall can be regarded as excess flux, and is defined by

$$\phi_{\text{excess}} = (\langle B_z \rangle_{\text{cyl}} - \mu_0 H_{//}) \pi R_0^2 \quad (2)$$

It is viewed as excess because it is not sustained by the externally applied magnetic field. Alternatively this excess flux can be regarded as trapped. Unfortunately, there is some confusion in the literature in this area and many authors refer to the entire amount of flux permeating the sample, (i) after cooling in a field from T_c , or (ii) after some decrease of the applied field, as trapped flux. Adopting the nomenclature of magnetic materials, the excess flux, when $H_{//}$ has been reduced to zero, is referred to as the residual or the remanent flux. Evidently when $H_{//}$ is being increased, and $\langle B_z \rangle_{\text{cyl}}$ lags behind $\mu_0 H_{//}$, equation 2 can be used to define a deficit of flux. Also, equation 2 can be seen as describing the magnetic moment of a unit length of the cylindrical sample. In this context, $\phi_{\text{excess}}/\pi R_0^2$ (or $\phi_{\text{deficit}}/\pi R_0^2$) are equivalent to the magnetisation or magnetic moment per unit volume.

In principle, for a homogeneous cylinder, the saturated flux retaining current I_ϕ , measured when $H_{//}$ has been reduced to zero, should coincide in magnitude with the flux screening I_ϕ observed at full penetration upon applying $H_{//}$ starting from zero. Indeed, for a hollow cylinder, we expect an exact correspondence between flux screening and flux retaining saturated states. This correspondence or symmetry can be written

$$I_\phi^\uparrow = H_{//}^\uparrow - \frac{B_1^\uparrow}{\mu_0} = I_\phi^\downarrow = \frac{B_1^\downarrow}{\mu_0} - H_{//}^\downarrow \quad (3)$$

with,

$$\mu_0 H_{//}^\uparrow = B_1^\uparrow \quad \text{and} \quad \mu_0 H_{//}^\downarrow = B_1^\downarrow \quad (4)$$

where the vertical arrow $\uparrow(\downarrow)$ indicates that $\mu_0 H_{//}$ has been increased.

(decreased). It is important to stress that equation 3 is valid even when equilibrium (reversible) currents associated with $B_{eq}(H)$ and irreversible surface currents are taken into account, provided that the outer and inner surfaces are not physically different. The reason for this is the reversal of the roles played by the outer and inner surfaces during the up and the down swing of $H_{//}$. When one surface opposes the entry of flux into the wall, the other impedes exit of flux from the wall. A comparison of our measurements of $I_{\phi}^{\uparrow}$ and $I_{\phi}^{\downarrow}$ confirms the validity of equation 2 with an experimental accuracy of a few percent. The $I_{\phi}^{\uparrow}$ and $I_{\phi}^{\downarrow}$ data (when $H_{\phi} = 0$) can therefore be regarded as mirror images of each other. Several workers have exploited these measurements to determine $j_c(B, T)$ for various materials. This induced current technique is particularly convenient for materials with high critical currents.

The sense of circulation of I_{ϕ} remains unchanged when the sweep of $H_{//}$ traverses zero and continues its swing in the direction opposite to the starting value. I_{ϕ} can however, now be regarded as divided into two concentric annular regions with an outer zone occupied by a flux screening current opposing the entry of the applied magnetic induction $|\mu_0 H_{//}|$ and an inner zone filled with a flux retaining current impeding the escape of the original flux still remaining in the hole and in the wall. The $B_z(r)$ profile is now somewhat more complex, decreasing in density from $|\mu_0 H_{//}|$ at the outer surface of the cylinder to zero at the interface between the two current zones, then changing polarity and rising in density to B_1 at the surface of the hole. The boundary where $B_z(r) = 0$ will move inward as $|H_{//}|$ is increased. The vortices of opposite polarity meeting at the moving interface undergo mutual annihilation with consequent dissipation of energy. Clem has

examined this process phenomenologically in some detail in his analysis of the various contributions to energy dissipation in hysteretic type II superconductors.

The current I_ϕ will display a maximum when the interface is situated at $r = (R_1 + R_0)/2$, the midpoint between the inner and outer radii (see Fig. 1). This follows from the symmetry of the $B_z(r)$ profile at this juncture (when $H_\phi = 0$) together with the fact that $j_c(B,T)$ decreases monotonically with B .

As the magnitude of the sweep of $H_{//}$ progresses after the reversal of polarity, the moving interface eventually reaches the surface of the hole. At this juncture, ignoring any surface barrier, and always in the context of a homogeneous infinite cylinder, the hole presumably contains no longitudinal flux and the wall is permeated with straight vortices of the polarity of $H_{//}$. The configurations of $B_z(r) = B(r)$ and $j_\phi(r) = j_c(B,T)$ should now be identical with that encountered when full penetration is attained after starting in zero applied field with no flux initially in the sample as discussed in the introduction to the previous chapter.

The continuation of the swing of $H_{//}$ will now recreate the sequence of events already encountered during the first application of $H_{//}$ and outlined earlier in this thesis.

The important new element we have introduced in these studies is the presence of a stationary azimuthal magnetic field H_ϕ as $H_{//}$ is decreased from $H_{//max}$ through zero and then swings to a comparable intensity of opposite sign. Due to the presence of H_ϕ , the induced currents now have a component $//$ to the total magnetic induction. This generates unusual and intricate flux configurations which we now describe. These observations can

only be accounted for by invoking the flux cutting phenomenon.

Results and Discussion

A. Initial decrease of $H_{//}$ from $H_{//max}$ to zero.

Fig. 1 displays I_{ϕ} and Fig. 2, the excess or deficit of longitudinal magnetic flux density, namely $\mu_0 H_{//} - \langle B_z \rangle_{cyl}$ versus $H_{//}$ both measured with $H_{\phi} = 0$ and $\mu_0 H_{\phi} = 90$ mT. Within the experimental accuracy and reproducibility of the data, the family of curves measured for I_{ϕ} versus $H_{//}$ appear largely unaffected by the presence of H_{ϕ} over the range of $\mu_0 H_{\phi}$ available in our set up (90 mT) (see Fig. 1). It is then not surprising that the locus of the excess flux and flux deficit versus $H_{//}$ also displays no significant dependence on H_{ϕ} (see Fig. 2). This insensitivity to the presence of H_{ϕ} is particular to the material under study. In some materials (eg. NbZr, NbTi, NbTa) these quantities can be appreciably enhanced by the presence of a static field orthogonal to the varying magnetic field.

The evolution of the azimuthal flux permeating the wall of the cylinder as $H_{//}$ is swept from a large positive value, through zero, to a comparable negative value, with H_{ϕ} kept fixed, is displayed in Fig. 3. Although measurements were made in numerous H_{ϕ} , only three representative curves have been selected for display in this figure to avoid clutter. The quantity of interest is the change in the azimuthal flux, namely

$$\Delta \langle B_{\phi} \rangle = \langle B_{\phi} \rangle - \langle B_{\phi} \rangle_{app} \quad (5)$$

where $\langle B_{\phi} \rangle$ is the average azimuthal magnetic induction and $\langle B_{\phi} \rangle_{app}$ is the average applied azimuthal magnetic induction permeating the wall in the normal state. By Ampère's law, $B_{\phi} = \mu_0 I / 2\pi r$, in the normal state, hence

$$\langle B_{\phi} \rangle_{app} = \frac{\mu_0 I}{2\pi(R_0 - R_1)} \ln\left(\frac{R_0}{R_1}\right) \quad (6)$$

Since the expulsion of flux is negligible in this material during cooling from T_c to 4.2 K, this also represents the average azimuthal flux density in the superconducting state just before the decrease of $H_{//}$ commences. The toroidal pickup coil monitors $\Delta\langle B_{\phi} \rangle$ directly.

First we focus on the behaviour when $H_{//}$ is decreased from $\mu_0 H_{//max}$ to zero. We note in Fig. 3 that, although the applied azimuthal magnetic field is stationary, the amount of azimuthal flux permeating the wall is decreasing dramatically. This means that the change in the longitudinal applied magnetic field, besides generating a flux retaining azimuthally circulating current ΔI_{ϕ} , in harmony with Faraday's law, also, somehow, induces a persistent current ΔI_z to flow along the length of the cylinder wall, presumably near its outer surface. By conservation of current, an equal, oppositely flowing, return current must also be present. Presumably, again in keeping with the laws of induction, these counterflowing longitudinal currents occupy concentric and adjacent cylindrical shells. This longitudinally circulating current is designated as diamagnetic since it causes a decrease in the azimuthal flux threading the wall. In other words, ΔI_z and its return companion, generate an azimuthal magnetic field which opposes the externally applied stationary azimuthal magnetic field H_{ϕ} . As the decrease in $H_{//}$ progresses, $\pm\Delta I_z$ grow in magnitude and eventually the two cylindrical shells they occupy expand to fill the entire cross section of the wall.

For obvious reasons, the phenomenon we have just described, is labelled the azimuthal diamagnetic effect. It can be viewed as a mirror

image of the azimuthal paramagnetic effect encountered in the previous chapter. Consequently, both phenomena can be intuitively understood in the framework of conservation of flux combined with a picture of pure radial displacement of helical vortices. In this context, when $H_{//}$ is decreased, helical flux lines adjacent to the outer surface of the cylinder, are made to exit via the action of their mutual repulsion since they are less effectively sustained in the wall when the externally applied helical field $H = (H_{//}^2 + H_{\phi}^2)^{1/2}$ is reduced in strength. Each helical flux line which leaves, however, carries an azimuthal component, hence the amount of azimuthal flux remaining in the wall diminishes. This simple picture, unfortunately, cannot be retained. Detailed considerations reveal that it leads to arbitrary and untenable discontinuities in the components of the magnetic induction at the surfaces (and also in the body) of the specimen. This can be appreciated by noting that the helical configuration possessed initially by a given cylindrical sheet of flux lines cannot match that existing at the surface when the chosen sheet reaches the surface to exit there. We must bear in mind that the orientation or helicity of the external magnetic field has changed, essentially in an arbitrary manner, during the radial migration of any specific cylindrical sheet of flux lines to the surface

The only general way the boundary conditions can be satisfied is to envisage that the cylindrical shells undergo changes of helicity as they are compelled to migrate radially. There are however, no electromagnetic forces acting azimuthally and longitudinally on the cylindrical sheets to accomplish this. Consequently the process of flux cutting must be invoked to achieve the required changes of orientation (helicity) of the flux lines. The

azimuthal and longitudinal magnetic flux permeating the wall can consequently decrease by consumption of flux as flux cutting takes place, and by the departure of flux lines at the outer surface as $H_{//}$ is decreased. Eventually, when the flux disturbance has reached the inner surface, the decrease can be mitigated and even overwhelmed by the entry of flux from the hole into the wall. From inspection of Fig. 3, we note that the azimuthal flux does indeed cease to diminish and starts to increase as $H_{//}$ approaches zero when the stationary H_{ϕ} is strong.

The magnitude of $\Delta\langle B_{\phi} \rangle$ is seen in Fig. 3 to increase, the stronger the stationary H_{ϕ} is chosen. Fig. 4 displays $\Delta\langle B_{\phi} \rangle$ at $H_{//} = 0$, denoted $\Delta\langle B_{\phi} \rangle_{res}$ (since it pertains to the residual or remanent azimuthal flux in the wall), plotted versus $\langle B_{\phi} \rangle_{app}$. Fig. 4 constitutes a vertical cut taken at

$H_{//} = 0$ through the entire family of curves of $\Delta\langle B_{\phi} \rangle$ versus $H_{//}$ we have catalogued and reproduced only in part in Fig. 3.

It is noteworthy that the height of the plateau of $\Delta\langle B_{\phi} \rangle_{res}$ versus $\langle B_{\phi} \rangle_{app}$ seen in Fig. 4 is twice the height of the plateau of $\Delta\langle B_{\phi} \rangle_{peak(1)}$ versus $\langle B_{\phi} \rangle_{app}$, the sharp paramagnetic peak reported in the previous chapter (see Fig. 4 of chapter 3). The values are ~ 18 mT and ~ 9 mT respectively. The difference in the magnitudes of $\Delta\langle B_{\phi} \rangle$ between these two situations signifies that $|I_z|$ hence $|j_z|$ are correspondingly different. The difference can therefore be attributed to the fact that the local magnetic flux density

$B = (B_z^2 + B_{\phi}^2)^{1/2}$ averaged over the wall thickness is appreciably smaller for the azimuthal diamagnetic effect ($B_{\phi}(r) < B_{\phi}(r)_{app}$) than for the azimuthal paramagnetic effect ($B_{\phi}(r) > B_{\phi}(r)_{app}$). This weaker flux density leads to higher critical current densities in the "diamagnetic" situation compared with that prevailing in the "paramagnetic" case. (We should recall

that I_ϕ , hence the $B_z(r)$ profiles, in the two situations under scrutiny, are very similar). The increase in the total critical current density with decrease of B suggests that $j_{c//}$, the critical current // to $\vec{B}$ (the flux cutting critical current density) also increases as B decreases. It is already well established that $j_{c\perp}$, the critical current density transverse to $\vec{B}$ (the bulk pinning or flux transport critical current density) rises as B diminishes.

It is useful to estimate and compare $\langle j_z \rangle$ and $\langle j_\phi \rangle$ as $H_{//}$ is decreased. We focus on the situation where $H_{//}$ has been reduced to zero. Here $\mu_0 I_\phi = 110$ mT and is insensitive to H_ϕ (see Fig. 1). Assuming a uniform j_ϕ , then $\langle j_\phi \rangle = I_\phi / \mu_0 (R_o + R_i) = 12.5 (10^7) \text{ A/m}^2$. At the plateau in Fig. 4, $\Delta \langle B_\phi \rangle_{\text{res}} = 17$ mT. Assuming j_z uniform, then $\langle j_z \rangle = 7.8 (10^7) \text{ A/m}^2$, hence $\langle j_z \rangle / \langle j_\phi \rangle = 0.6$.

It is also of interest to explore the B_ϕ profile, especially when $\langle B_\phi \rangle_{\text{app}}$ is weak. By way of illustration we take $\langle B_\phi \rangle_{\text{app}} = 15$ mT. Here, from Fig. 4, $\Delta \langle B_\phi \rangle_{\text{res}} = 7$ mT, hence $\langle j_z \rangle = 3.2 (10^7) \text{ A/m}^2$. At the interface r_i between $+I_z$ and $-I_z$,

$$B_\phi(r_i) = \frac{\mu_0}{2\pi r_i} (I - \langle j_z \rangle \pi (\frac{R_o^2 - R_i^2}{2})) \quad (7)$$

where $r_i = (R_o^2 + R_i^2)^{1/2} / 2 = 3.82 (10^{-3})$ meter. Consequently $B_\phi(r_i) = 0.5$ mT.

The $B_\phi(r)$ profile is seen to vary from 16.2 mT at R_i to ≈ 0.5 mT at the interface to 13.5 mT at R_o . In the normal state $B_\phi(r_i) = 14.7$ mT. This indicates that the azimuthal diamagnetic effect has essentially caused the azimuthal flux density to vanish in the vicinity of the interface when $\langle B_\phi \rangle_{\text{app}} \approx 15$ mT. This, in our view, is an impressive result.

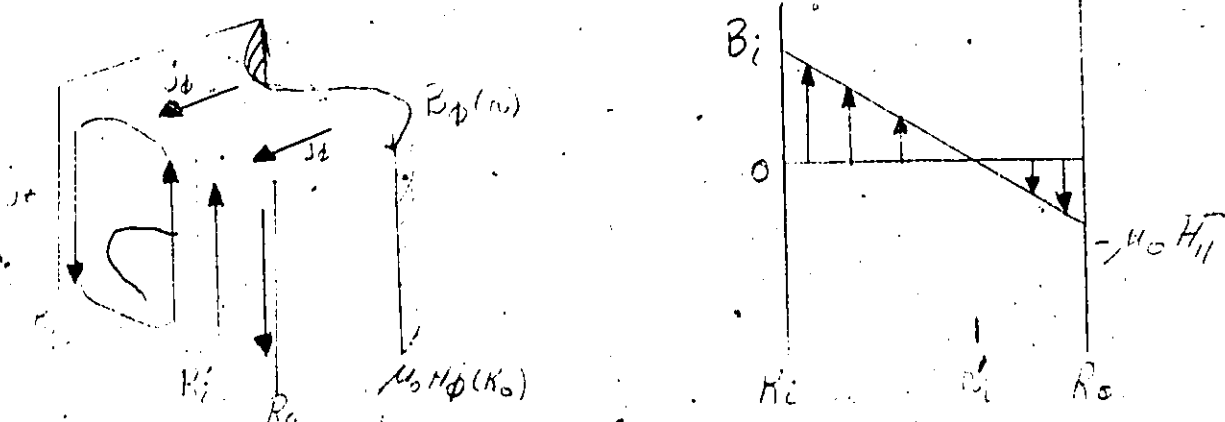
B. Second part of the swing of $H_{//}$ from 0 to $-H_{//\max}$

The externally applied $H_{//}$ having traversed zero now continues its swing, hence increases in magnitude but with its polarity opposed to that existing during the first half of the sweep. The radial pattern of the induced persistent current I_ϕ is qualitatively simple, since its sense of circulation must remain unchanged as the sweep progresses, and it continues to occupy the entire cross section of the wall. Also, the magnitude of I_ϕ is seen to be insensitive to the presence of H_ϕ , (see Figs. 1 and 2).

The configuration of the total magnetic induction $\vec{B}(r)$, is complex however. Taking H_ϕ to be positive, then next to the hole and the wall is initially permeated by right-handed helical flux lines while left-handed helical flux lines are now made to enter at the outer surface and penetrate into the wall. We can then visualize two zones separated by an inward moving cylindrical interface of radius r'_1 where $B_z = 0$, but $B_\phi \neq 0$ (presumably $B_\phi(r'_1) \approx B_\phi(r'_1)_{\text{app}}$).

In the inner zone, $B_z(r)$ varies spatially from B_1 at R_1 to 0 at r'_1 . As r'_1 moves inward, this zone shrinks in size. Also B_1 and $B_z(r)$ are decreasing. In this zone, the configurations of $\vec{B}(r)$ and $\vec{j}_z(r)$ should continue to be similar to that just encountered when $H_{//}$ was reduced to zero. This situation was discussed in part A of this chapter. There, on the basis of the observations, we visualized a deep trench gouged out of the azimuthal flux profile. Taking $\vec{B}_\phi$ as directed into the paper, we can picture j_z circulating longitudinally in a counterclockwise (diamagnetic) sense in the inner zone, thereby producing this deep depression in $B_\phi(r)$ by partially cancelling the applied azimuthal magnetic field there. We refer the reader

to the sketch below which depicts the situation schematically.



Sheets of right-handed helical flux lines entering the wall at R_i , the surface of the hole, are subsequently consumed by flux cutting in the inner zone and at its moving boundary r_1' .

In the outer zone, $B_z(r)$ varies spatially from 0 at r_1' to $-\mu_o H_{//}$ at R_o . Also, $|B_z(r)|$ increases as $|\mu_o H_{//}|$ increases and the zone expands as r_1' moves inward. Here j_z is circulating longitudinally in a clockwise (paramagnetic) sense, thereby producing a hump in $B_\phi(r)$ by aiding the applied azimuthal magnetic field there. The situation in the outer zone, except for the change of sign for $B_z(r)$, is identical to that envisaged in the previous chapter during the initial penetration of the externally applied longitudinal magnetic induction $\mu_o H_{//}$ into the wall.

As the outer zone expands, the inner zone shrinks correspondingly with the diamagnetic and paramagnetic azimuthal effects coexisting side by side and sharing the thickness of the wall. The observed $\Delta\langle B_\phi \rangle$ is the resultant expression of these "competing" phenomena. Initially, the azimuthal diamagnetic zone is dominant. Then at some $|H_{//}|$, the two effects cancel exactly and the net $\Delta\langle B_\phi \rangle = 0$. Subsequently, the paramagnetic effect reigns supreme and a peak in $\Delta\langle B_\phi \rangle$, denoted $\Delta\langle B_\phi \rangle_{\text{peak}(2)}$ is attained when the

longitudinal magnetic induction in the hole B_1 vanishes (see Fig. 3).

Fig. 5 displays $\Delta\langle B_\phi \rangle$, the height observed at the summit of this sharp peak versus various $\langle B_\phi \rangle_{app}$. This figure should be compared with Fig. 4 of Chapter 3. It is remarkable that $\Delta\langle B_\phi \rangle_{peak(2)}$ is larger than $\Delta\langle B_\phi \rangle_{peak(1)}$ for all corresponding $\langle B_\phi \rangle_{app}$, the ratio of the former to the latter being ~ 1.3 . In particular, at the plateaus, we find $\Delta\langle B_\phi \rangle \sim 12$ and 9 mT for peak 2 and 1 respectively. We can provide no qualitative or intuitive explanation for this unexpected result. This feature is surprising indeed since for a chosen $\langle B_\phi \rangle_{app}$ the boundary conditions ($B_1 = 0$ and $\mu_o H_{//}$) correspond closely when these summits are traced.

Fig. 6 displays $\mu_o H_{//peak(2)}$, the swing of $|\mu_o H_{//}|$ from zero required to reach the summit of the sharp paramagnetic peak for various $\langle B_\phi \rangle_{app}$ as represented in Fig. 3. We note that $\mu_o H_{//peak(2)}$ is ~ 142 mT and insensitive to the magnitude of $\langle B_\phi \rangle_{app}$. The latter must however be present for the phenomenon under consideration to occur. Comparing this quantity with the equivalent quantity, $\mu_o H_{//peak(1)}$ (see Fig. 5 of Chapter 3), we find that the former is $\sim 5\%$ smaller, indicating a corresponding diminution in I_ϕ . This diminution, as well as the associated higher azimuthal paramagnetic peaks, present a formidable challenge to a definitive model.

Fig. 7 displays $\Delta\langle B_\phi \rangle_{valley(2)}$, the height observed at the valley versus various $\langle B_\phi \rangle_{app}$ as represented in Fig. 3. It is noteworthy that the height of the valley is greater under the circumstances under consideration than for the case where the cylinder became superconducting in $H_{//}$ examined in the previous chapter. Fig. 7 should be compared with Fig. 12 of chapter 3. Indeed the ratio $\Delta\langle B_\phi \rangle_{valley(2)} / \Delta\langle B_\phi \rangle_{valley(1)} \sim 1.3$ for any chosen

$\langle B_\phi \rangle_{app}$, corresponds to that encountered when comparing the sharp peaks. All of these features are presently unaccounted for.

Conclusion

We have examined the magnetic behaviour of a hollow cylinder of a type II superconductor subjected to a swing of $H_{//}$ from some initial value through zero to a comparable magnitude of opposite polarity while immersed in various stationary H_ϕ .

The longitudinal response of the cylinder to the sweep of $H_{//}$ is classical and is seen to be insensitive to the presence of H_ϕ . This independence on H_ϕ is thought to be particular to the sample under study. Analogous experiments on ribbons of other materials suggest that I_ϕ , hence $j_\phi(r)$, in these materials would be appreciably enhanced by the presence of a stationary H_ϕ of appropriate strength.

The decrease of $H_{//}$ causes an appreciable fraction of the azimuthal magnetic flux permeating the wall to vanish. This feature provides solid confirmation that flux cutting is made to occur and consumes much of the azimuthal component of the flux. A corresponding consumption of the longitudinal component of the longitudinal flux is presumably also taking place but is masked by the concomitant transport of cylindrical sheets of helical flux lines out of the specimen as $H_{//}$ is made to decrease.

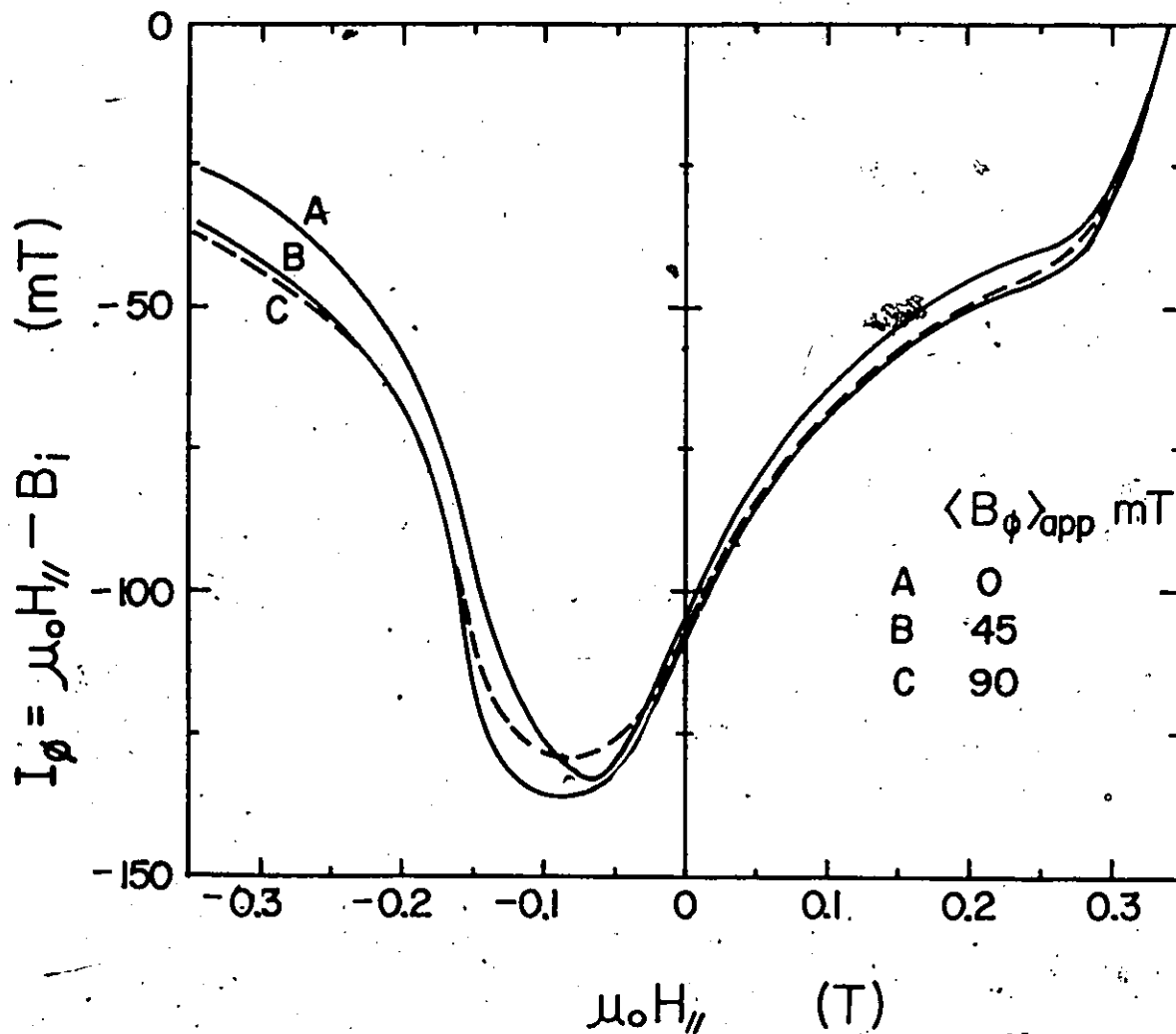
The large residual azimuthal diamagnetism observed at $H_{//} = 0$ after the decrease of $H_{//}$ from $H_{//max}$, indicates that $B_\phi(r) = 0$ in the wall, in a region straddling the radius $r_1 = (R_1^2 + R_0^2)^{1/2}$.

The coexistence of a shrinking annular zone depleted of azimuthal

magnetic flux surrounded by an expanding annular region rich in azimuthal flux is surmised from the locus of $\Delta\langle B_\phi \rangle$ as the sweep of $H_{//}$ continues after crossing zero.

The location along the $H_{//}$ axis of the sharp azimuthal paramagnetic peak which ensues indicates that it is attained when B_1 traverses zero and the $B_z(r)$ profile has just been converted to the polarity of $\mu_0 H_{//}$. The height of the azimuthal paramagnetic summits, however, and of the subsequent valley is ⁴ seen to be some 30% greater than the corresponding features generated by an initial sweep of $H_{//}$ from zero with no previous magnetic history in the superconducting state other than cooling from T_c to 4.2 K in the chosen $\langle B_\phi \rangle_{app}$. These appreciable changes in the magnitude of the azimuthal paramagnetic effect are fascinating but baffling.

Fig. 4-1. Compares the azimuthal current induced to circulate around the hollow cylinder by a sweep of $H_{//}$ from $+H_{//\max}$ to $-H_{//\max}$ when the static applied azimuthal magnetic induction $\langle B_{\phi} \rangle_{app} = 0, 45$, and 90 mT.



1 Fig. 4-2. Illustrates the influence (negligible) of the presence of a static applied azimuthal magnetic induction $\langle B_\phi \rangle_{app}$ on the magnetic response of the hollow cylinder to a sweep of $H_{//}$ from $+H_{//max}$ to $-H_{//max}$.

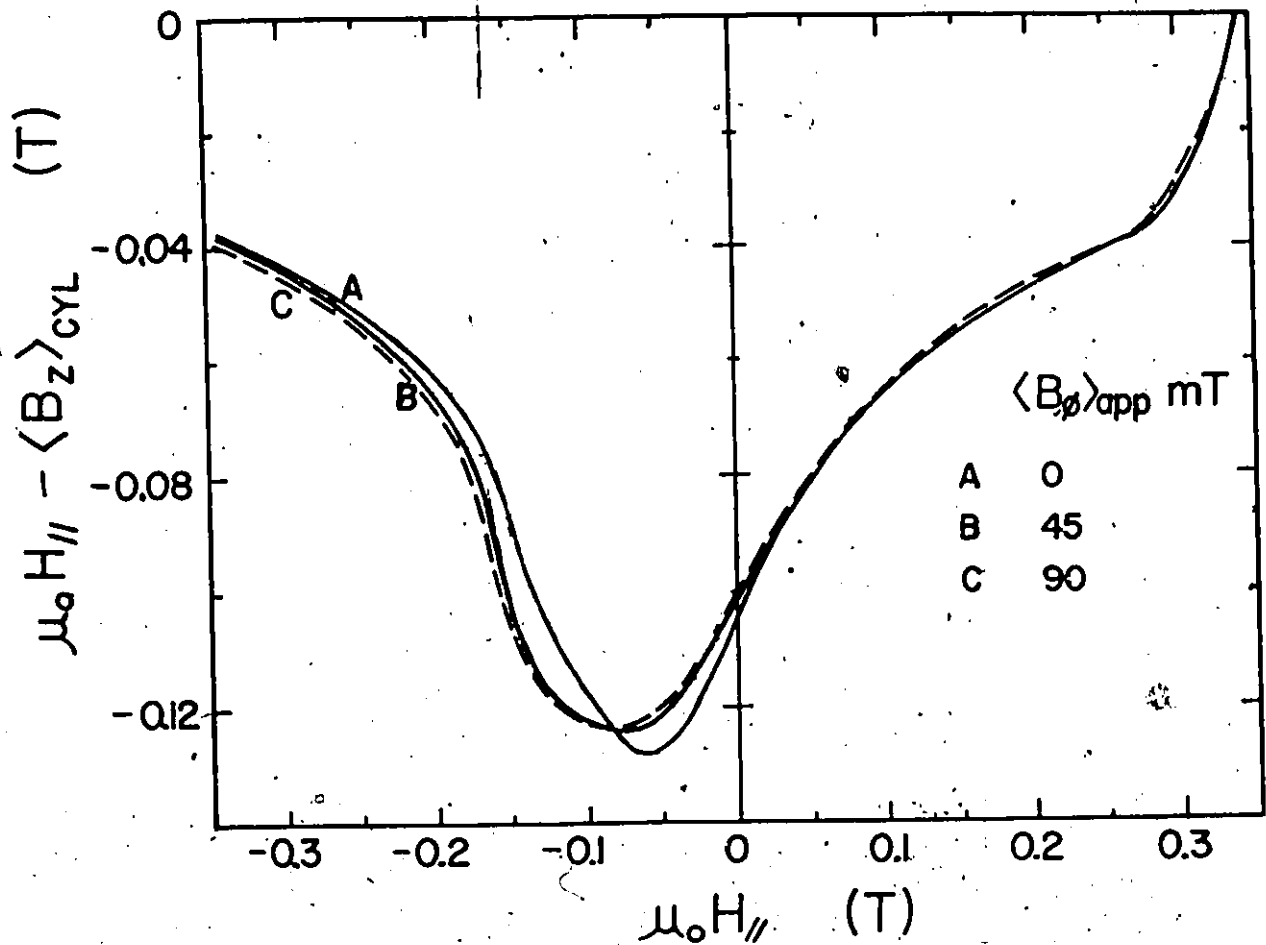


Fig. 4-3. Illustrates the evolution of the azimuthal magnetic induction permeating the wall with various static azimuthal magnetic fields present as $H_{//}$ is swept from $+H_{//max}$ to $-H_{//max}$. The dramatic display of flux cutting although the interpretation requires careful consideration. We note the correspondence of the behaviour displayed in the left-hand quadrant and that presented in Fig. 3-3. Only three representative curves are displayed from the several measured curves to avoid clutter.

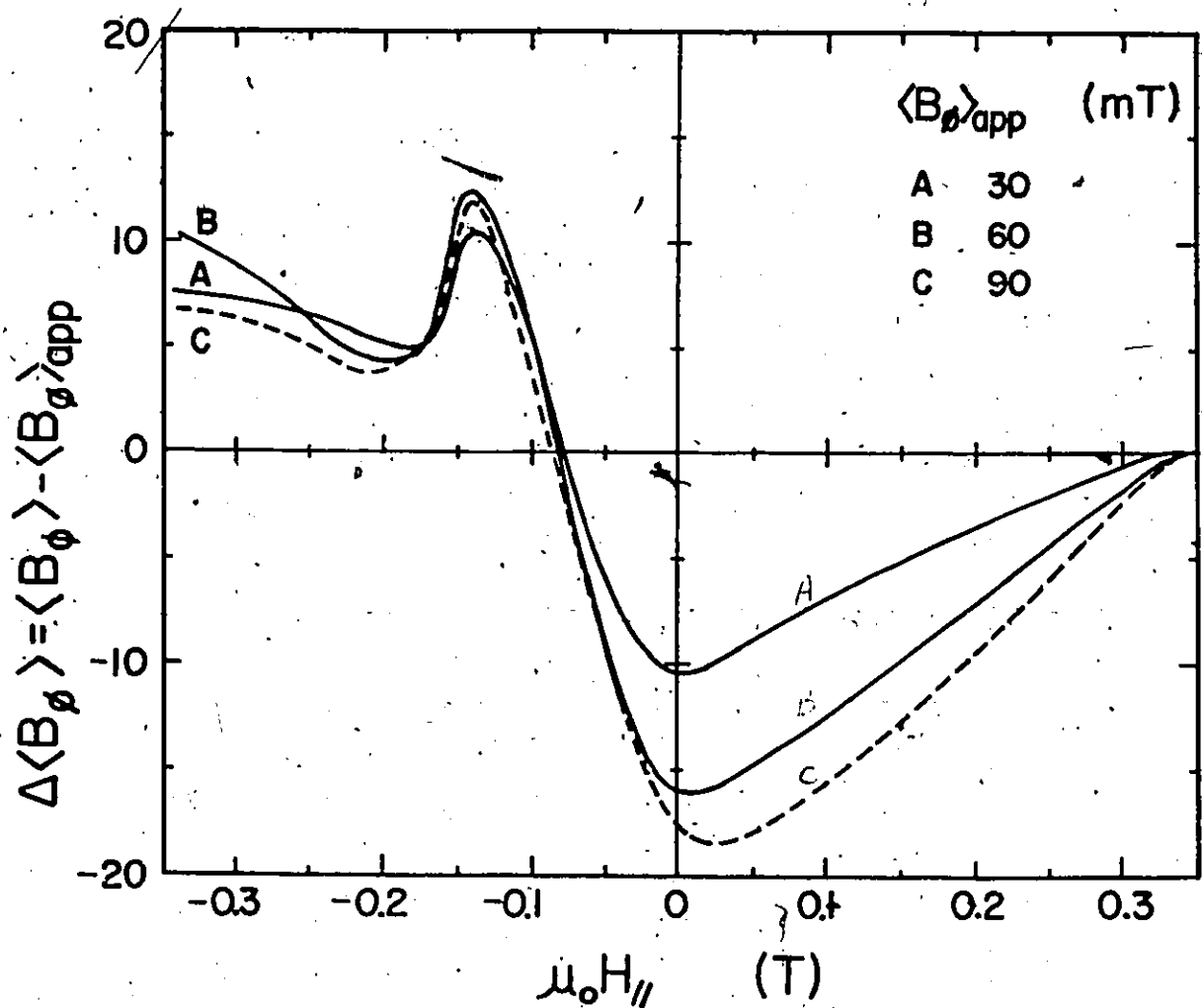


Fig. 4-4. Complements the previous figure and shows a "vertical" cut through the data points at $H_{//} = 0$ in that figure. This plot displays the dependence of the "residual" azimuthal flux on the static azimuthal magnetic field present during the sweeps of $H_{//}$.

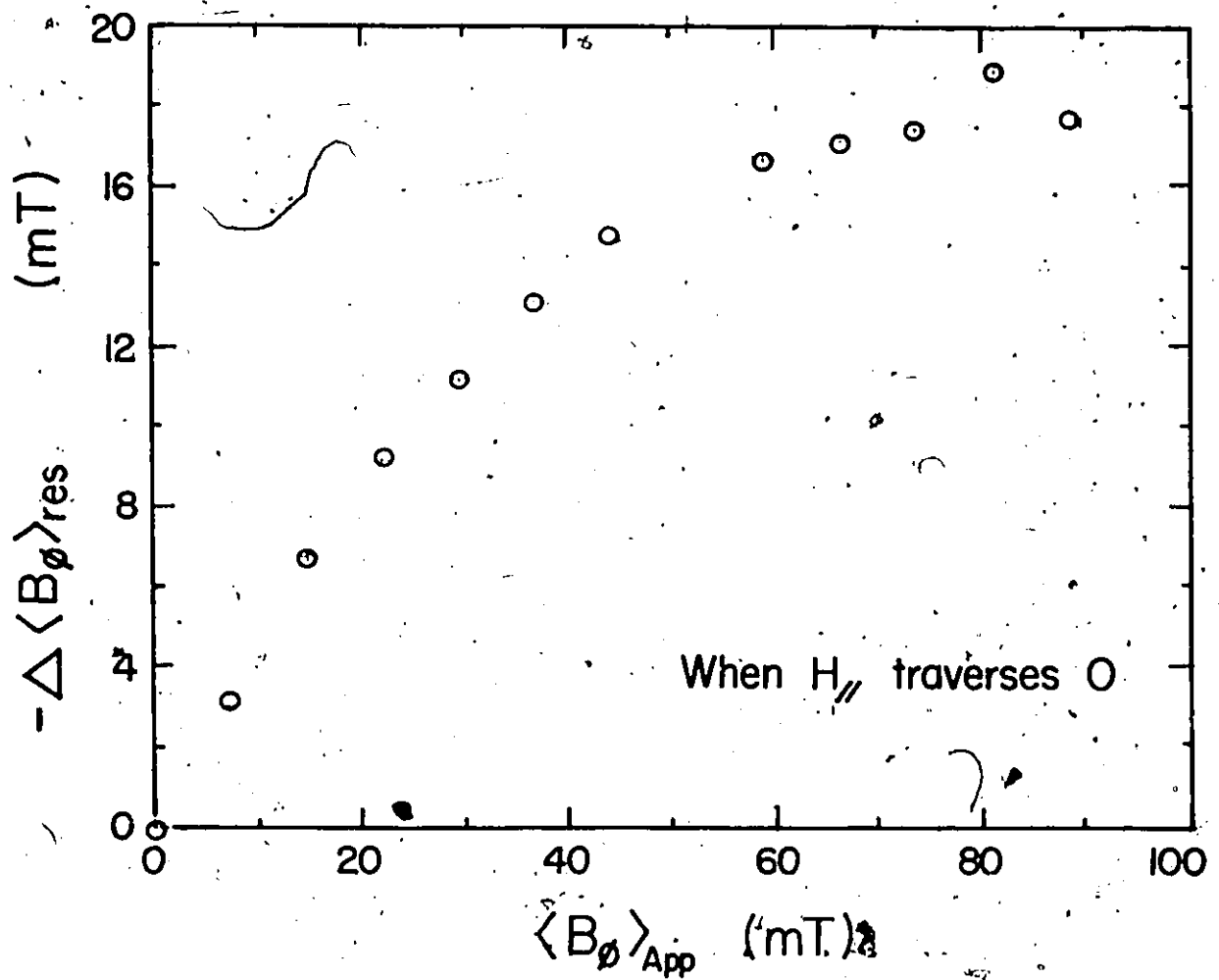


Fig. 4-5. Complements the two previous figures and shows the dependence of the height of the paramagnetic peak on the static azimuthal magnetic field present during the sweep of $H_{//}$ from $H_{//\max}$ to $-H_{//\max}$ (see Fig. 4-3). It is remarkable that these paramagnetic peaks are higher than that observed upon an initial application of $H_{//}$ from zero (see Fig. 3-3 and 3-4.).

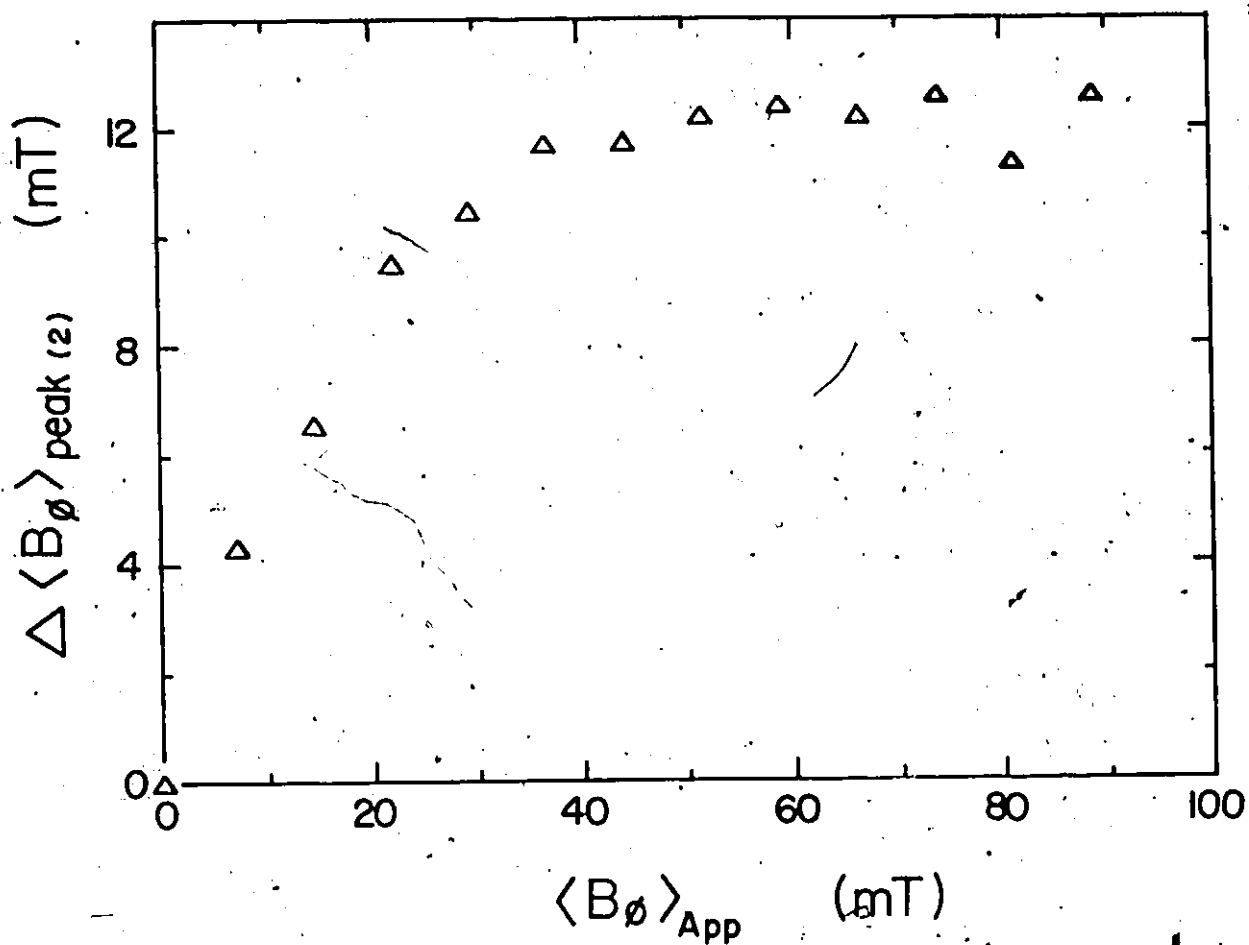


Fig. 4-6. Complements the three previous figures, and displays the value of $\mu_0 H_{\parallel}$ where the maximum of the paramagnetic peak is attained (see Fig. 3) as a function of the static applied azimuthal magnetic induction $\langle B_{\phi} \rangle_{\text{app}}$. The position of the peak is seen to be insensitive to $\langle B_{\phi} \rangle_{\text{app}}$. These data correspond closely to that of Fig. 3-5 and communicate the same message.

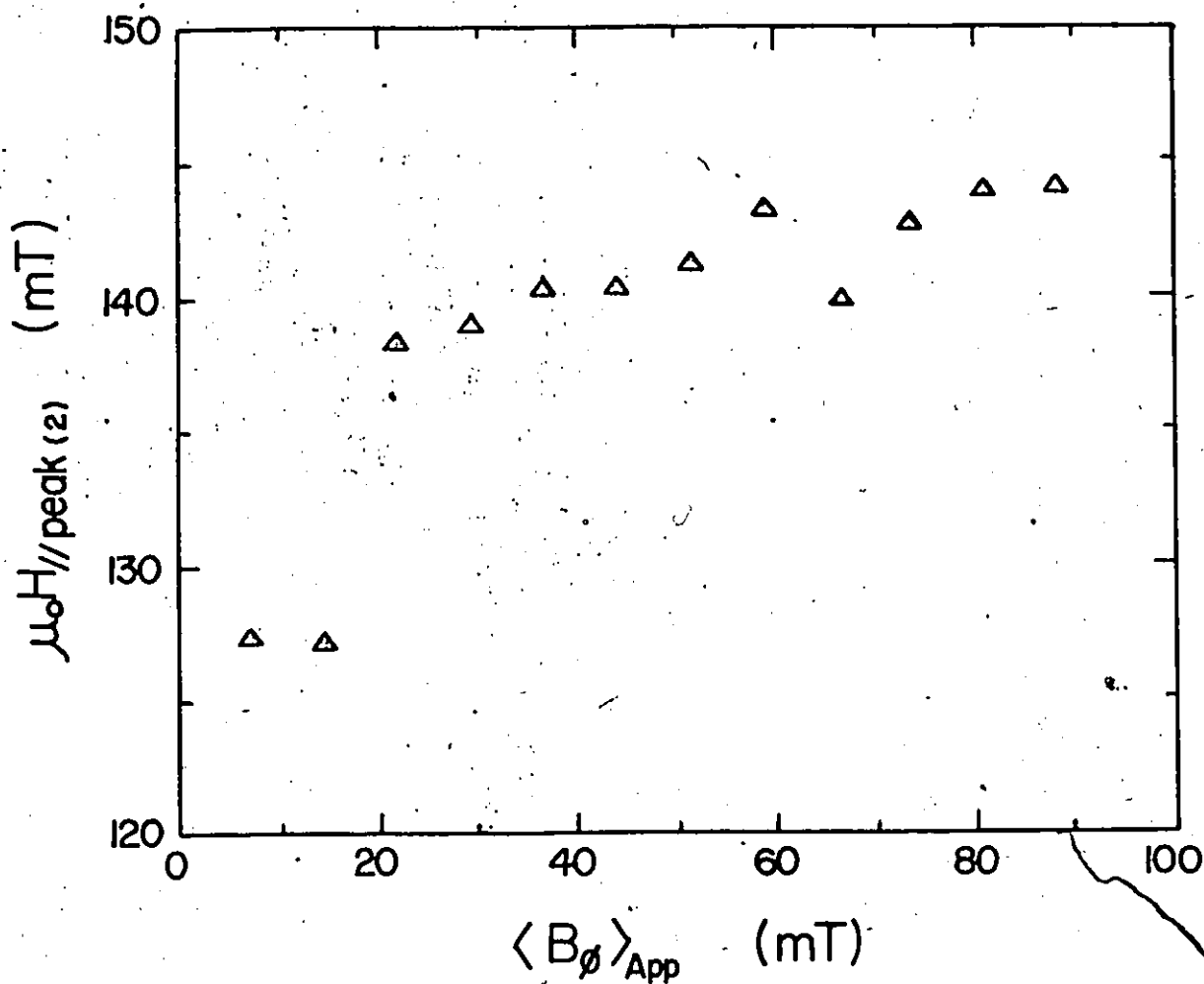
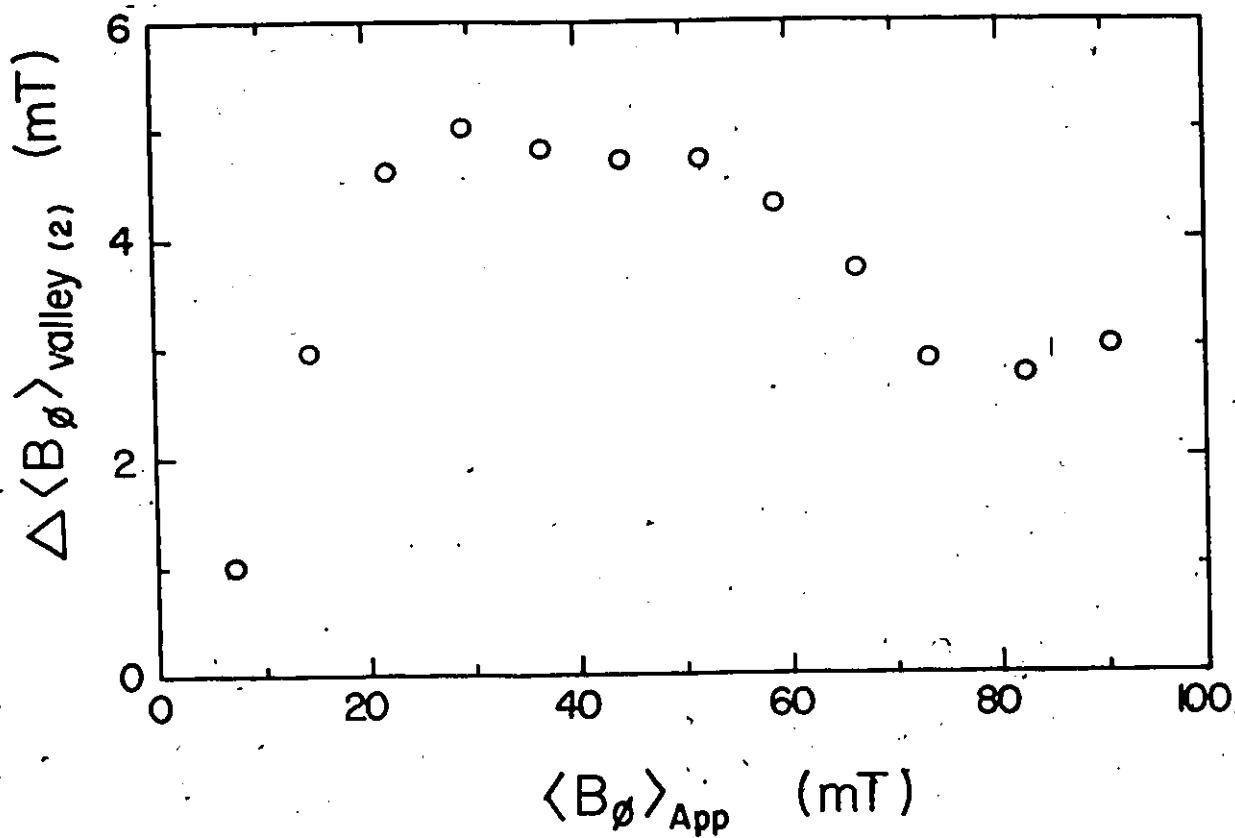


Fig. 4-7 Complements the four previous figures, and displays the dependence of the height of the paramagnetic valleys seen in Fig. 4-3 on the static azimuthal magnetic field present during the sweep of $H_{//}$. This figure should be compared with Fig. 3-6.



Chapter 5

Estimates of j_{cl} and $j_{c//}$ versus B

Introduction

Soon after and over the years since the discovery of high current, high field type II superconductors, workers have exploited hollow cylinder geometry to obtain data on the dependence of the critical current density j_{cl} on magnetic flux density B , and temperature T , in these materials. All of these investigations have focused on the collinear regime. As a consequence $j_{cl}(B, T)$ is a relatively well known quantity. In contrast, to date, only one curve of $j_{c//}$ versus B has been published in the literature. This solitary result appears in a paper by Perez-Gonzalez and Clem⁵⁹ who analyze, by way of illustration, measurements of LeBlanc and Lorrain of hysteresis losses in a ribbon of $V_{0.24}Ti_{0.76}$ in the noncollinear regime.

In the collinear regime, the procedure and the arrangement for the determination of j_{cl} are quite straightforward. A pickup coil centrally positioned in the hole of the cylinder monitors, B_1 , the internal longitudinal flux density, as an externally applied longitudinal magnetic field $H_{//}$ is impressed and raised to H_{c2} , if that limit can be attained. The curves of B_1 versus $\mu_0 H_{//}$, hence, $\mu_0 I_\phi = \mu_0 H_{//} - B_1$, readily yield curves of $j_\phi = j_{cl}$ versus B for the range $\mu_0 H_{//}^* \lesssim B \lesssim \mu_0 H_{//max}$, where $H_{//}^*$ is the applied field $H_{//}$ when longitudinal flux begins to appear in the hole. Curves of B_1 versus $\mu_0 H_{//}$ decreasing from $H_{//max}$ to zero complement the first set of data and should confirm the results obtained there. Furthermore, in these measurements, there exists no ambiguity that j_ϕ , hence j_l exists in

critical states, i.e. $j_\phi = j_1 = j_{c1}$. Variations on the experimental theme just outlined substantiate this identification.

In this chapter we extract estimates of $j_{c//}$ as well as data on j_{c1} versus B for our sample from the measurements of $\Delta\langle B_\phi \rangle$ and B_1 (or, equivalently $\langle B_z \rangle_{cyl}$) versus $H_{//}$ increasing from zero, presented in chapter 3, and versus $H_{//}$ decreasing from $H_{//max}$ to zero, presented in chapter 4. The procedure for obtaining these quantities is, inevitably, appreciably more complicated in the noncollinear regime than in the collinear regime. Nevertheless, hollow cylinder geometry and the types of measurements we have carried out probably provide the most direct method of obtaining data on $j_{c//}$ in bulk materials.

Description of the Approach

The method we have exploited is relatively unsophisticated and constitutes a first approximation approach. Refinements of the approach, however, pose severe problems, in our view.

Firstly, we focus on only those parts of the data where I_ϕ is flowing throughout the volume of the wall. This restriction is necessary to ensure that we are dealing with situations where flux is being transported across the wall from the exterior into the hole when $H_{//}$ is increasing from zero and from the hole to the outside world when $H_{//}$ is decreasing from $H_{//max}$ to zero. The configurations of the currents and magnetic induction in the wall when the sweep of $H_{//}$ continues through zero into the region of opposite polarity are more intricate and complicated and will not be dealt with in our analysis. By assuring that flux transport is occurring across the width of the wall we are ensuring that j_{c1} exists in a critical state

everywhere. This certitude follows from the concept, that flux line displacement can only take place when $j_{\perp} = j_{c\perp}$. Also, the occurrence of flux transport across the entire wall sets the stage for the appearance throughout the volume of the wall of a current density, $j_{//}$, flowing along $\vec{B}$. However any $j_{//}$, if present, need not exist in a critical state $j_{c//}$ and zones where $j_{//} < j_{c//}$ may arise. A priori, even if the behaviour of $\langle B_{\phi} \rangle$ versus $H_{//}$ attests that flux cutting is taking place, this process may be localized in annular regions, hence $j_{//} < j_{c//}$ in the remainder of the volume.

To unravel $j_{c\perp}$ and $j_{//}$ from the measurements of I_{ϕ} , and $\Delta\langle B_{\phi} \rangle$ we must first estimate $j_{\phi}(B)$ and $j_z(B)$. Following a standard approach, we assume j_{ϕ} and j_z to be uniform across the width of the wall although these two quantities are allowed to vary with B . These two suppositions are evidently inconsistent but are generally accepted in the spirit of this approximation.

With the stipulation of a uniform j_{ϕ} , any measurement of I_{ϕ} versus $H_{//}$ then yields,

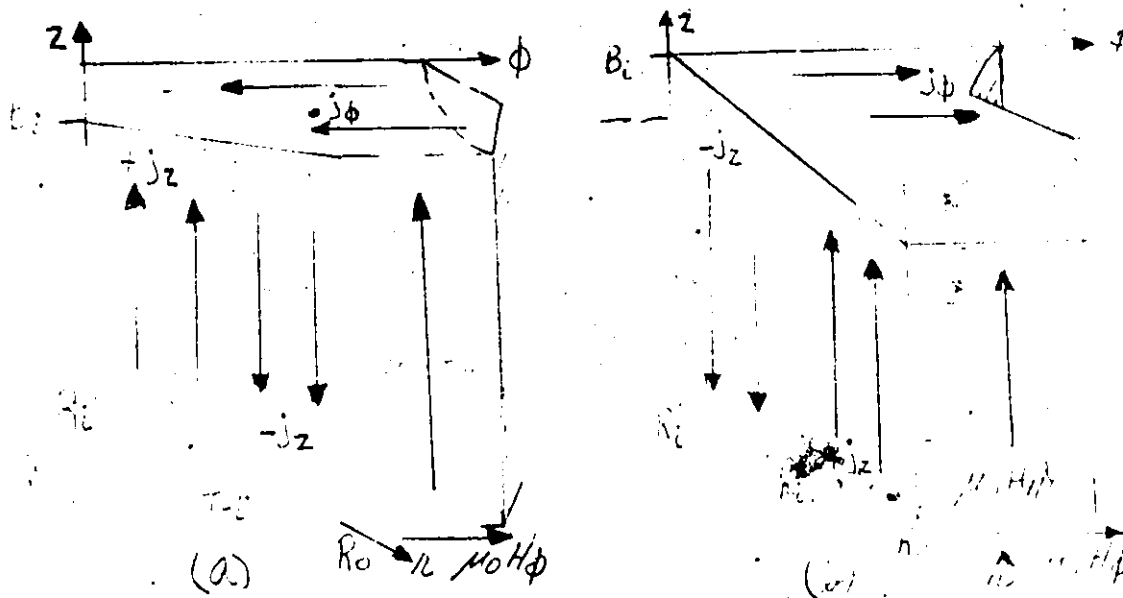
$$j_{\phi} = \frac{I_{\phi}}{R_o - R_i} \quad (1)$$

Also the stipulation of a uniform j_z combined with simple configurations for $B_{\phi}(r)$ when the azimuthal paramagnetic (chapter 3) and azimuthal diamagnetic effects (chapter 4) occupy the width of the wall as shown in the sketches below, leads to

$$\Delta\langle B_{\phi} \rangle = \mu_o j_z C \quad (2)$$

as shown in the appendix, where

$$C = \frac{(R_o^2 + R_i^2) \ln \left\{ \frac{\sqrt{2} R_o}{\sqrt{R_o^2 + R_i^2}} \right\} - R_i^2 \ln \left(\frac{R_o}{R_i} \right)}{2(R_o - R_i)} \quad (3)$$



The sketches show schematically the configuration of j_z , j_ϕ , $B_\phi(r)$ and $B_z(r)$ which correspond to our model when, (a) $\mu_0 H_{//}$ is increasing and, (b) $\mu_0 H_{//}$ is decreasing, with the applied azimuthal field kept constant. Clearly two regions must be identified in both situations (a) and (b) since j_z reverses direction at an interface of radius r_1 . We must therefore treat the outer and inner regions separately.

Applying equations 1 and 2, it is a trivial exercise to determine the direction and magnitude of the current vector

$$\vec{j} = \hat{z} j_z + \hat{\phi} j_\phi \quad (4)$$

for each region from the family of curves of I_ϕ and $\Delta\langle B_\phi \rangle$ versus $H_{//}$ illustrated in Fig. 3-1, 3-3, 4-1 and 4-3. The magnitude and direction of an "average" $\vec{B}$ associated with each region and each $\vec{j}$ vector is obtained as follows. We introduce the linear approximations,

$$B_z(r_1) = \mu_0 H_{//} \pm \frac{\mu_0 I_\phi}{2} \quad (5)$$

$$\text{and } B_\phi(r_1) = \frac{\mu_0}{2\pi r_1} \{I \pm j_z \pi(r_1^2 - R_1^2)\} \quad (6)$$

as shown in the appendix. Here I is the total current in the toroidal coil

where $r_1 = (R_o^2 + R_i^2)^{1/2} \approx (R_o + R_i)/2$.

It is now straightforward to determine the magnitude and direction of the vectors $\vec{B}_o$ and $\vec{B}_i$ whose components are given by the linear approximations;

$$B_{zo} = \frac{\mu_o H_{//} + B_z(r_1)}{2}, \quad B_{zi} = \frac{B_i + B_z(r_1)}{2} \quad (7)$$

$$B_{\phi o} = \frac{\mu_o H_{\phi o} + B_{\phi}(r_1)}{2}, \quad B_{\phi i} = \frac{\mu_o H_{\phi i} + B_{\phi}(r_1)}{2} \quad (8)$$

where the subscripts o and i denote the outer and inner regions. $H_{\phi i}$ and $H_{\phi o}$ denote the azimuthal field at the outer and inner surfaces of the wall.

Finally, the $\vec{j}$ vectors are then decomposed into components $//$ and $\perp$ to the corresponding $\vec{B}$ vectors thereby yielding $j_{//}$ and $j_{\perp}$.

Results and Discussion

Some of the results of the analysis of our data according to the method just described are displayed in Figs. 5-1 through 5-6. Curves of $j_{\perp}$ and $j_{//}$ were determined for each choice of a stationary applied azimuthal magnetic induction $\langle B_{\phi} \rangle_{app}$ as $H_{//}$ was made to increase or to decrease. The vertical arrow along the symbol $\langle B \rangle$ for the "average" magnetic induction bathing the region, denotes whether $H_{//}$ was made to increase ($\uparrow$) or to decrease ($\downarrow$). We only present a few representative curves selected to span the range of $\langle B_{\phi} \rangle_{app}$ of the measurements.

Figs. 1 and 2 display $j_{\perp}$ versus $\langle B \rangle \uparrow$ for the outer and inner regions. The curve labelled A belongs to the collinear regime since, here $H_{\phi} = 0$. We note that with larger $\langle B_{\phi} \rangle_{app}$, the curves are displaced downwards in the outer region and upwards in the inner region, hence the deviations from curve A cancel out when an average over the entire wall is taken. This is as expected since $j_{\perp}$ must be in a critical state under the circumstances under

scrutiny. The deviations arise from the linear approximations introduced. This view is in keeping with the trends of the displacements.

The values of $j_{//}$ coexisting with the j_{cl} displayed in the previous figures are presented in Figs. 3 and 4. We note that these curves exhibit a considerable spread indicating that $j_{//}$ does not exist in a critical state over a significant fraction of the volume of the wall. Since our method calculates spatial averages, $j_{//} < j_{c//}$ can represent either, (i) the actual state of affairs over the region or, (ii) an average of $j_{//} = j_{c//}$ over part of the region plus $0 < j_{//} < j_{c//}$ over the rest of the region. The rise in $j_{//}$ with $\langle B \rangle_{app}$ seen in Fig. 3 suggests that either (i) $j_{//}$ is approaching $j_{c//}$ everywhere in the outer region or (ii) $j_{//} = j_{c//}$ is occupying an increasing fraction of the region. Presumably the uppermost curve yields a good estimate of $j_{c//}$.

Figure 4, indicates that, as expected, $j_{//}$ changes direction with respect to $\vec{B}$ at an interface in the inner region in proceeding from the low to the high range of B . Here, however, our coarse averaging procedure and crude approximations do not lead to meaningful information.

Figs. 5 and 6 display j_{cl} and $j_{//}$ for $H_{//}$ decreasing where we only present the curves for one region in each case. The comments made earlier again apply here. We note that the upper curves for $j_{//}$ in the range of low $\langle B \rangle$ for Figs. 2 and 6 correspond closely.

Conclusion

A straightforward analysis of our measurements on the hollow cylinder show that $j_{//}$ does not, as a spatial average exist in a critical state in the wall although $j_{\perp}$ is in a critical state. Nevertheless, the data provide a good estimate of $j_{c//}$ versus B .

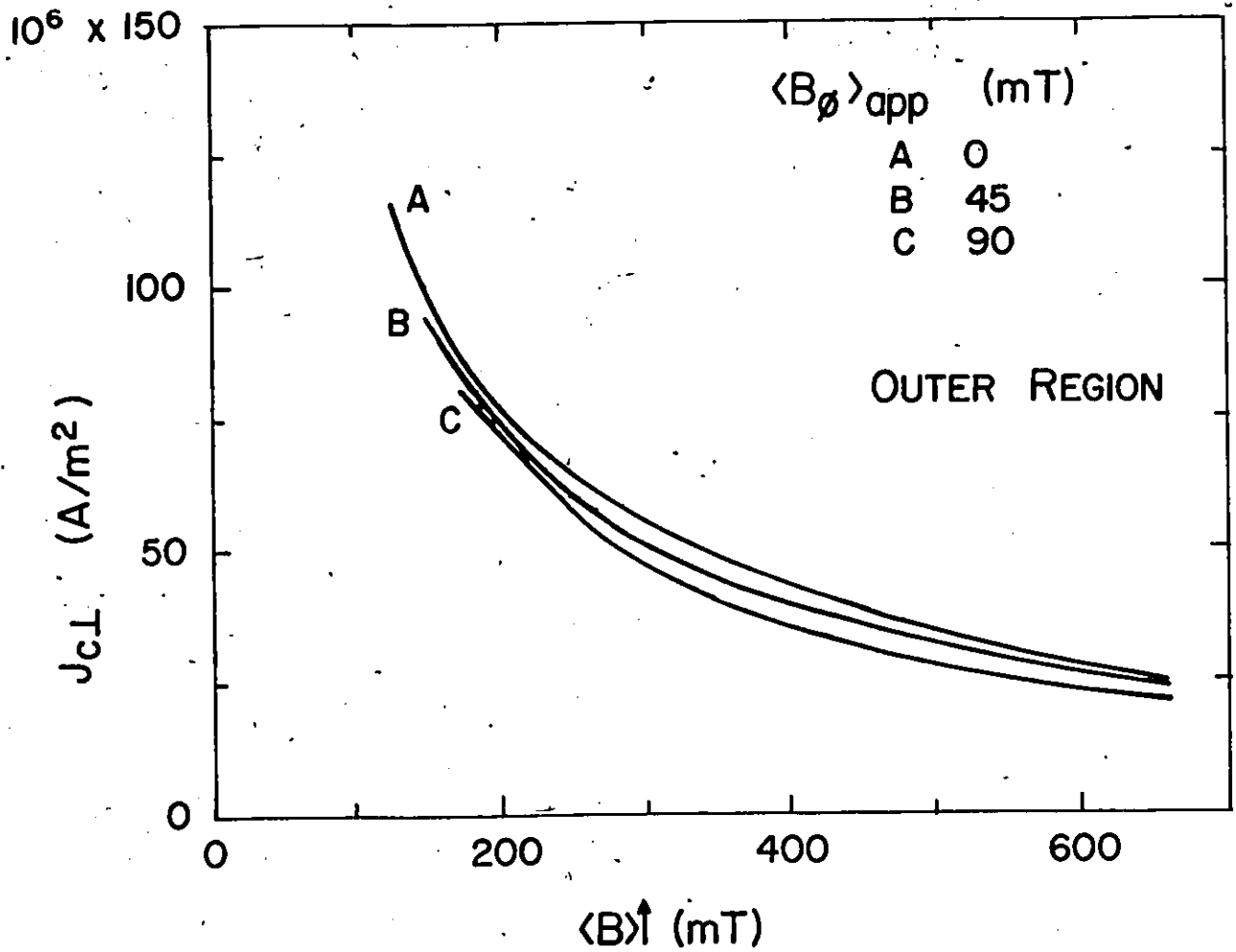


Fig. 5-1. The critical current density J_{c1} versus $\langle B \rangle$ in the outer region for increasing $H_{//}$. The sample was cooled in a static $\langle B_{\phi} \rangle_{app}$ of 0, 45 and 90 mT for A, B and C.

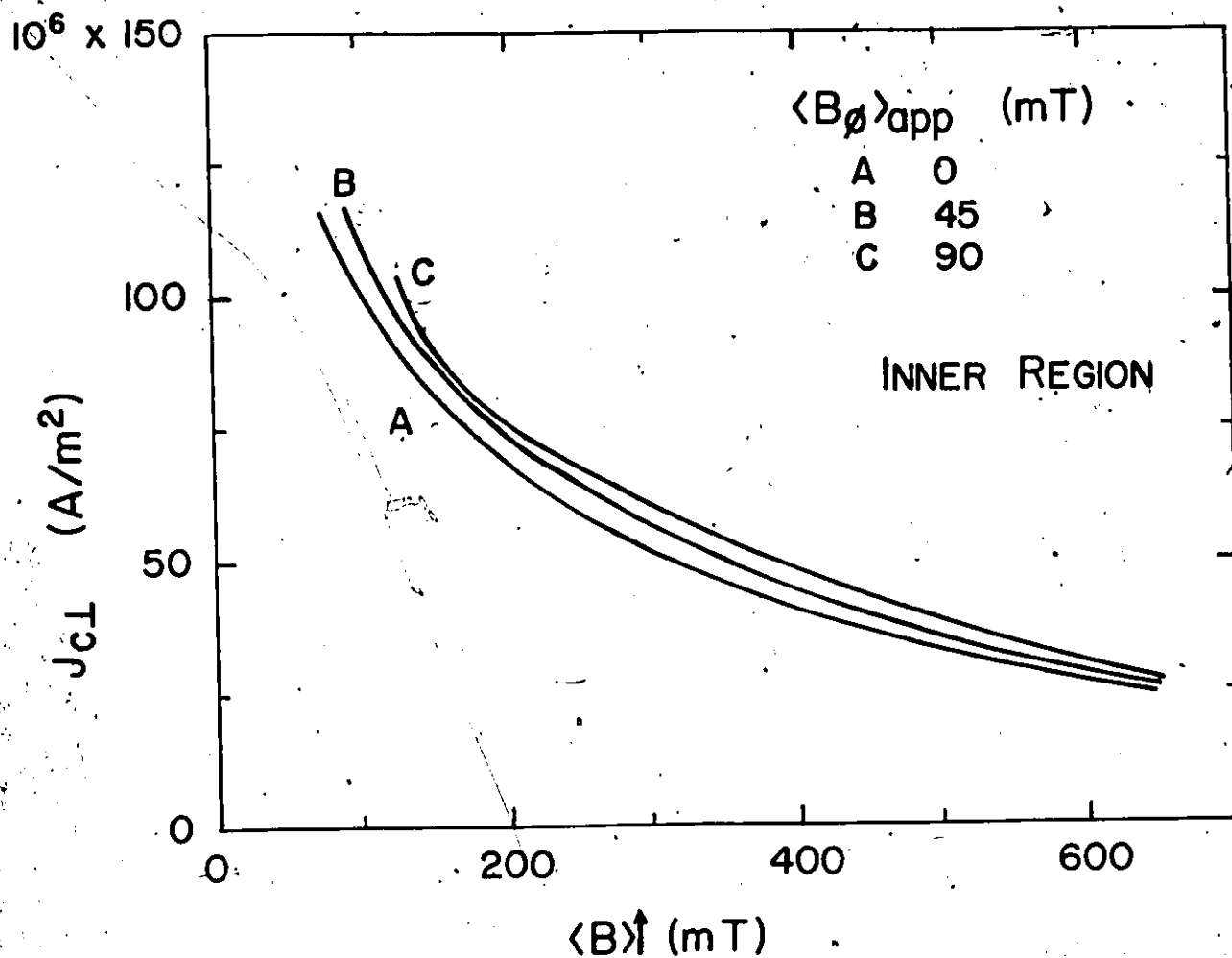


Fig. 5-2. Complements the previous figure and shows the critical current density J_{c1} vs. $\langle B \rangle$ in the inner region of the wall with $H_{//}$ increasing. The sample was cooled in static $\langle B_{\phi} \rangle_{app}$ of 0, 45 and 90 mT for A, B and C.

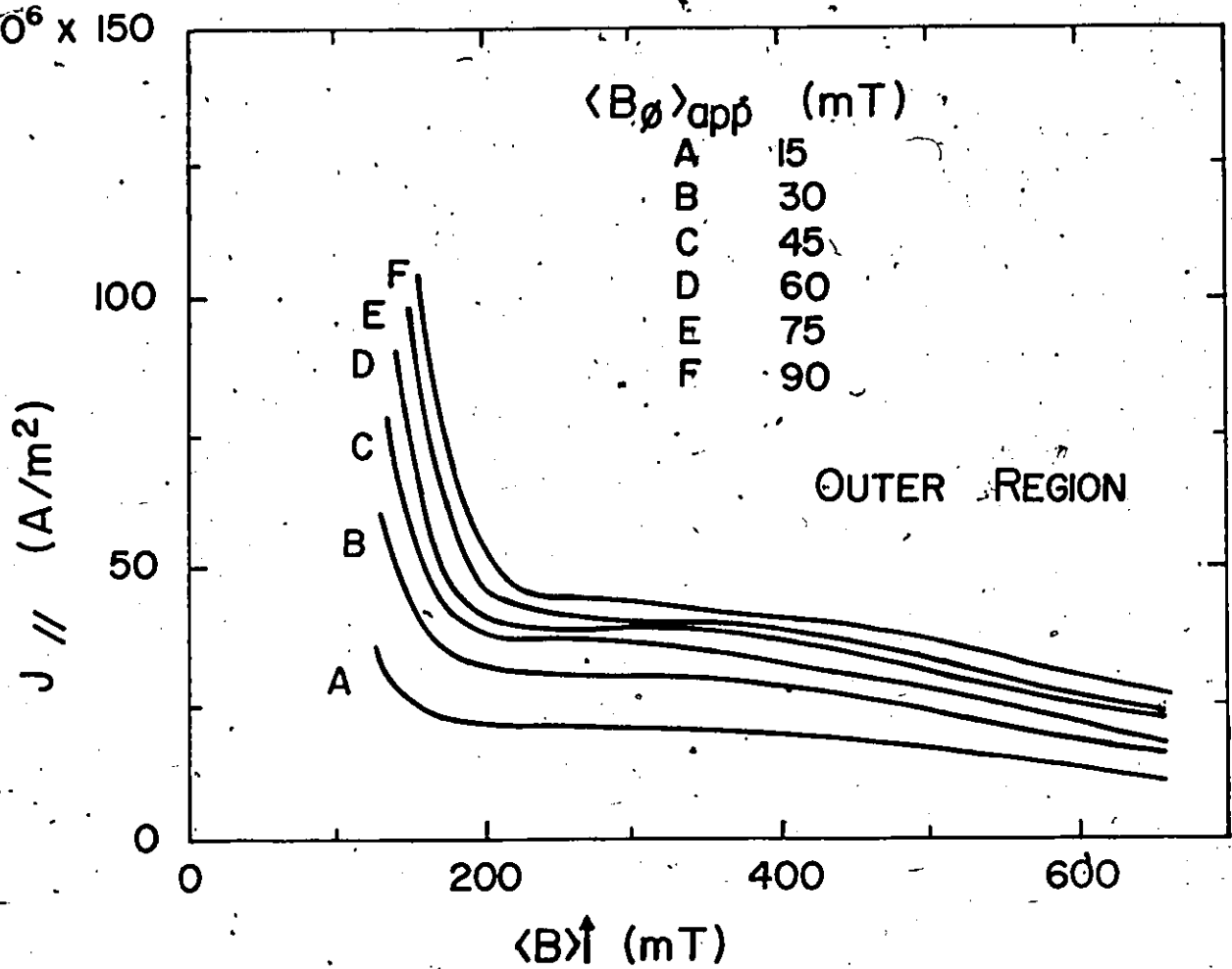


Fig. 5-3. The parallel current density $J_{//}$ vs. $\langle B \rangle$ in the outer region of the wall of the cylinder with $H_{//}$ increasing. The sample was cooled in static $\langle B_0 \rangle_{app}$ of 15, 30, 45, 60, 75 and 90 mT for A, B, C, D, E and F.

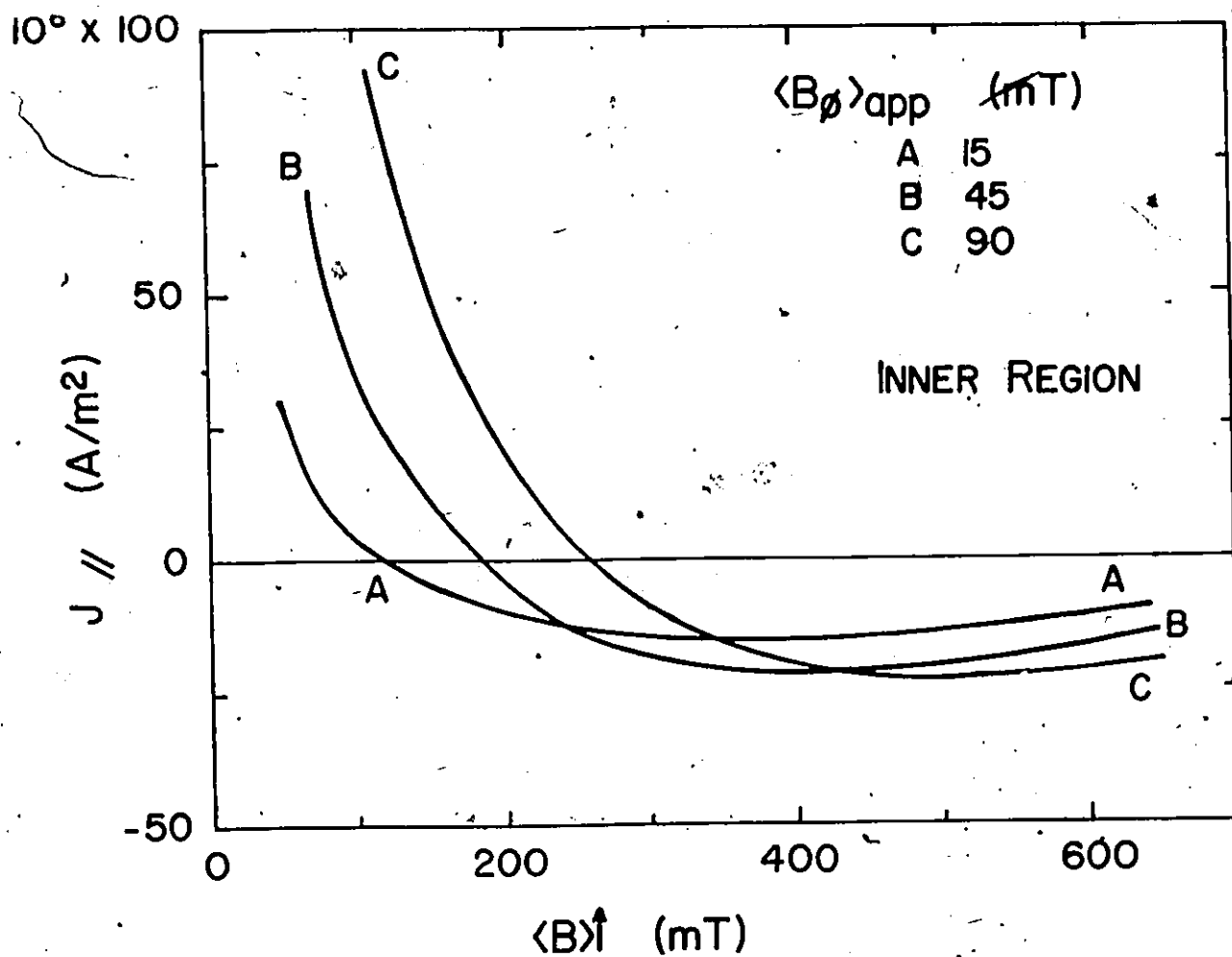


Fig. 5-4. Complements the previous figure and shows the parallel current density $J_{//}$ vs. $\langle B \rangle$ in the inner region of the cylinder wall with $H_{//}$ increasing. The sample was cooled in static $\langle B_0 \rangle_{app}$ of 15, 45 and 90 mT for A, B, and C.

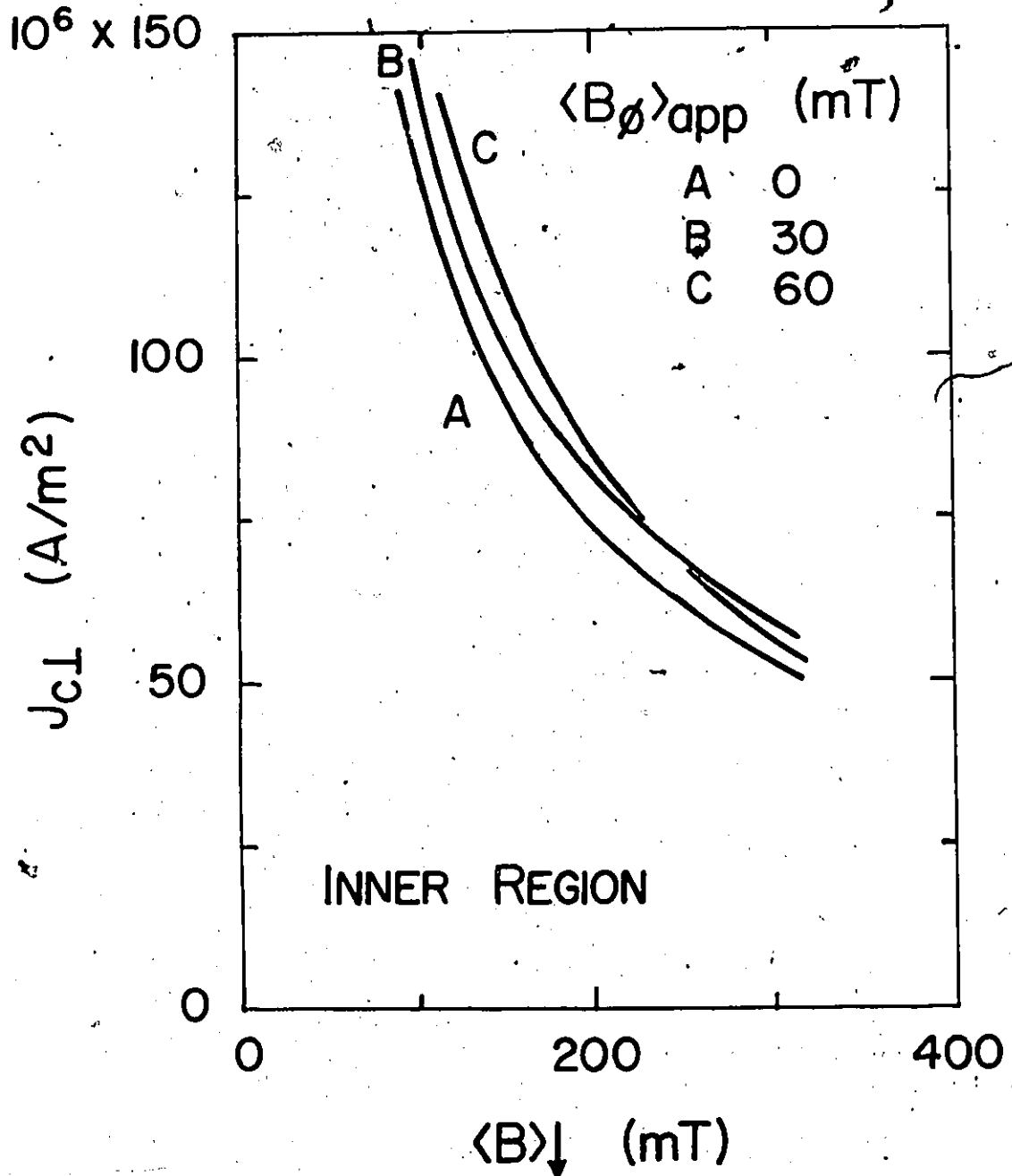


Fig. 5-5. The perpendicular critical current density $J_{c\perp}$ vs. $\langle B \rangle$ in the inner region of the cylinder wall for $H_{//}$ decreasing. The sample was cooled in static $\langle B_{\phi} \rangle_{\text{app}}$ of 0, 30 and 60 mT for A, B and C.

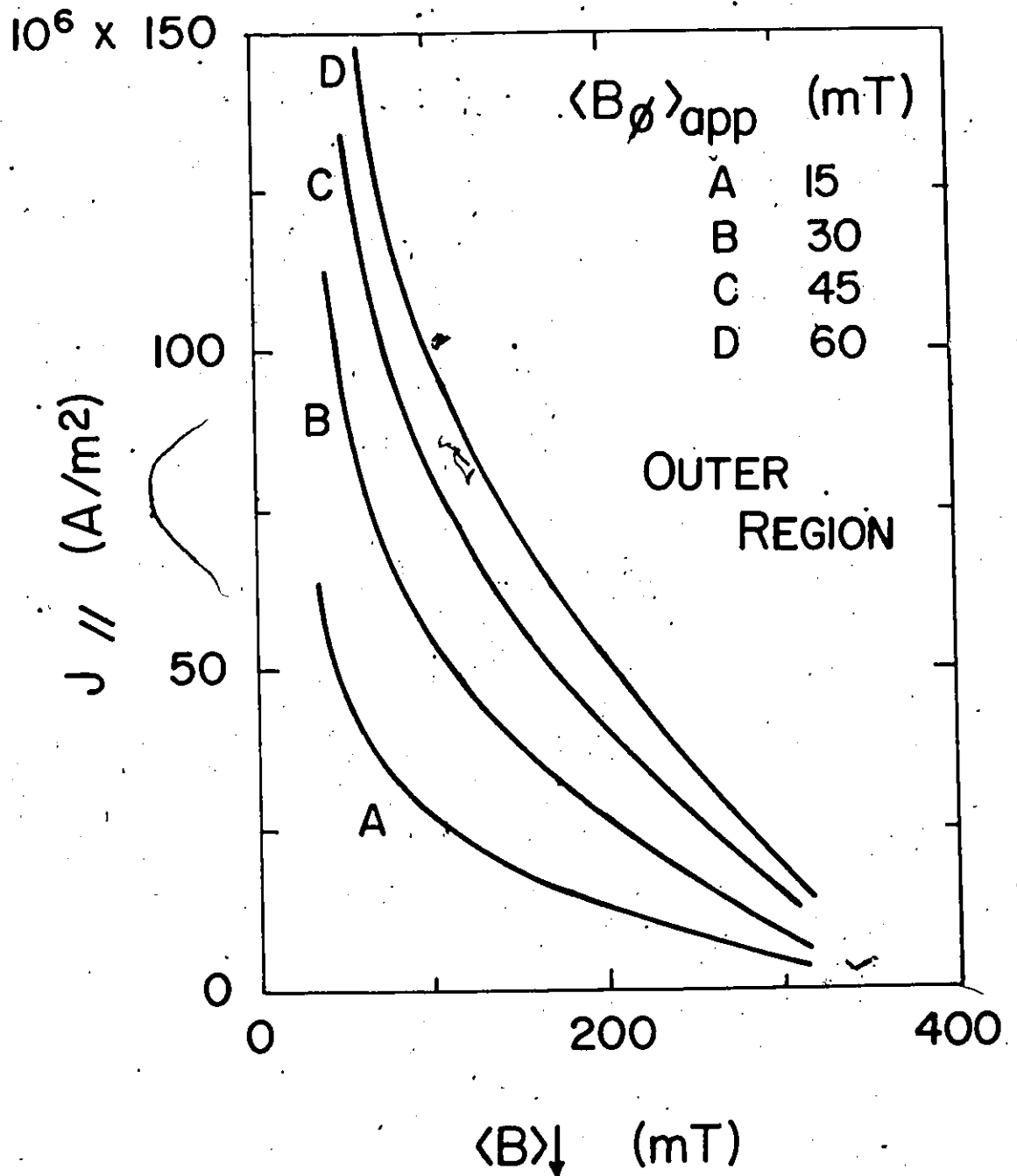


Fig. 5-6. The parallel current density $J_{//}$ versus $\langle B \rangle$ in the outer region of the cylinder wall for $H_{//}$ decreasing. The sample was cooled in static $\langle B_{\phi} \rangle_{app}$ of 15, 30, 45 and 60 mT for A, B, C and D.

Chapter 6

Longitudinal Paramagnetic Effect

Introduction

In this chapter we report on our observations of the evolution of the magnetic flux permeating the wall of a hollow cylinder of a type II superconductor as an externally applied azimuthal magnetic field H_ϕ is impressed while an externally applied longitudinal magnetic field $H_{//}$ is maintained stationary. In chapter 3 we examined the behaviour when $H_{//}$ was impressed while H_ϕ initially present during the transition to the superconducting state was kept fixed. Now the cylinder becomes superconducting in the chosen $H_{//}$ as it cools through T_c to 4.2 K.

Several workers have examined the magnetic response of ribbon samples subjected to an externally applied magnetic field $\vec{H}$ directed along the broad faces of the ribbon. In these measurements it is convenient to orient $\vec{H}$ either along the length or the width of the ribbon. In either case, the demagnetization factor is negligible provided that the length and width are large compared with the thickness of the ribbon. Consequently, $\vec{H}$ will be spatially uniform over the surfaces of the specimen and the latter can then be treated as an infinite slab. Further, the B profiles are symmetric with respect to the midplane of the ribbon in a homogeneous applied field. The components of $\vec{H}$ along the length and width of the ribbon are denoted $H_{//}$ and $H_\perp$ respectively. Several workers have examined the magnetic behaviour with $H_{//}$ kept fixed while $H_\perp$ is varied and vice versa. This arrangement has several advantages for the study of the critical current, flux cutting

phenomena and hysteresis losses in ribbons with large current carrying capacity.

Azimuthally, the wall of the cylinder has a zero demagnetization factor and, in this respect, the wall can be regarded as an ideal infinite slab with H_ϕ playing a role equivalent to H_L for ribbon geometry. A complication arises, however, with a thick walled hollow cylinder, since $H_\phi = I/2\pi r$ and therefore varies radially, being appreciably larger by $\approx 20\%$ for our specimen, at the inner surface of the wall than at its outer surface. Also because of the cylindrical geometry, the mathematical description of the profile of the azimuthal flux density $B_\phi(r)$ is more difficult than in the case of the corresponding quantity, $B_L(x)$ for the wide ribbon. The reason for this is that Maxwell's equation, $\nabla \times \vec{B} = \mu_0 \vec{j}$ which reads $dB_L/dx = \mu_0 j_z$ for the infinite slab, now reads

$$\mu_0 j_z = \frac{dB_\phi}{dr} + \frac{B_\phi}{r} \quad (1)$$

with the second term on the right hand side not negligible in our arrangement.

When H_ϕ is impressed, a persistent screening current $+\Delta I_z$ is induced to flow longitudinally along the outer surface of the cylinder with an equal oppositely directed (return) current $-\Delta I_z$ flowing along the inner surface of the wall. The current $+\Delta I_z$ can be regarded as opposing the penetration of the applied azimuthal flux into the outer surface of the cylinder while the current $-\Delta I_z$ as shielding the wall against invasion of the applied azimuthal flux from the hole. Since the longitudinal current density is finite, the applied azimuthal flux cannot be completely excluded and penetration takes place on a macroscopic scale compared with London's penetration depth λ . Azimuthal flux therefore enters through both the inner

and outer surfaces of the wall. Eventually, the fronts of the two advancing B_ϕ profiles meet. At this juncture, the cross section of the wall is filled by the two oppositely flowing longitudinal currents $\pm I_z$ and the situation is denoted as saturated.

Initially, after the hollow cylinder has cooled from T_c to 4.2 K, in $\mu_0 H_{//}$, the longitudinal magnetic induction in the hole, $B_1 = \mu_0 H_{//}$. There is no requirement however, that B_1 will remain at this level subsequently as H_ϕ is varied although the externally applied longitudinal magnetic induction at the outer surface of the cylinder is maintained fixed at the chosen value for $\mu_0 H_{//}$. Indeed our observations reveal the occurrence of an extremely interesting phenomenon under these circumstances. We discover that as H_ϕ is impressed, the level of the longitudinal flux density in the hole is diminished. The longitudinal flux that vanishes from the hole is presumably transferred to the wall. This is thought to take place through the creation of helical vortices at the inner surface of the wall which then penetrate into it carrying each their component of longitudinal flux.

The transfer of flux from the hole to the wall will not generate any signal in the external pickup coil embracing the cylinder. We find, however, that the longitudinal magnetic flux threading the specimen grows as H_ϕ is increased. This indicates that some longitudinal flux is also being deposited into the wall through the outer surface of the cylinder as H_ϕ is impressed. The increase in $\langle B_z \rangle_{\text{wall}}$, the longitudinal flux density permeating the wall, induced by an increase in H_ϕ , is denoted the longitudinal paramagnetic effect. We now present our observations of these phenomena.

Results and Discussion

Adapting the classical relationship, $B = \mu_0 H + \mu_0 M$, to the present situation where azimuthal flux permeates the wall of a hollow cylinder we write

$$-\mu_0 \langle M_\phi \rangle = \langle B_\phi \rangle_{app} - \langle B_\phi \rangle \quad (2)$$

where $\mu_0 \langle M_\phi \rangle$ and $\langle B_\phi \rangle$ are spatial averages of the azimuthal magnetization and azimuthal flux density in the wall and

$$\langle B_\phi \rangle_{app} = \frac{\mu_0 I}{2\pi(R_0 - R_1)} \ln\left(\frac{R_0}{R_1}\right) \quad (3)$$

is the applied azimuthal flux density in the absence of the specimen but averaged over the dimensions of its wall. We can regard a negative $\mu_0 \langle M_\phi \rangle$ as a deficit or lag in flux density.

Fig. 1 displays the influence of a stationary $H_{//}$ on the entry of azimuthal flux into the wall as H_ϕ is impressed. We note that this influence, over the range shown (≈ 100 mT) is not significant. The range of H_ϕ available in our arrangement appears only sufficient to just attain full penetration of the wall by the two advancing B_ϕ profiles. Consequently, the tantalizing panorama of behaviour lying beyond full penetration which we were able to explore in the case of the azimuthal paramagnetic effect (chapter 3) remains beyond the reach of the present study. Nevertheless the results we have obtained and will describe shortly in some detail, provide new information and insights of considerable interest.

Assuming a uniform current density j_z , and that the two advancing B_ϕ profiles have just met leads to the expression (see Appendix)

$$\mu_0 \langle M_\phi \rangle = \frac{\mu_0 j_z}{2(R_2 - R_1)} \left\{ (R_2^2 + R_1^2) \ln\left(\frac{R_2}{R_1}\right) - R_1^2 \ln\left(\frac{R_2}{R_1}\right) \right\} \quad (4)$$

where $R_1 = (R_2^2 + R_1^2)^{1/2}$. Taking $\mu_0 \langle M_\phi \rangle = 70$ mT (the maximum value attained

in Fig. 1) leads to $j_z = 32(10^7) \text{ A/m}^2$.

Fig. 2 displays the evolution of the longitudinal flux threading the entire cylindrical sample in various stationary $H_{//}$ as H_ϕ is impressed. It is more convenient to present $\Delta \langle B_z \rangle_{\text{cyl}} = \langle B_z \rangle_{\text{cyl}} - \mu_0 H_{//}$, the change in the spatial average of the longitudinal flux threading the combination of wall plus hole, hence permeating the entire cross section of the hollow cylinder and denoted $\langle B_z \rangle_{\text{cyl}}$. This is the quantity directly monitored by the pickup coil embracing the cylindrical sample.

Firstly we note that, initially, the longitudinal flux permeating the sample is seen to diminish. This decrease, although not considerable, is the signature that flux cutting must be taking place since the externally applied longitudinal field $H_{//}$ is kept fixed and the applied azimuthal field H_ϕ at both the inner and outer surfaces of the wall is made to increase. We draw the attention of the reader to an analogous behaviour encountered in Chapter 3.

As the rise in H_ϕ progresses, the longitudinal flux ceases to diminish and then shows an appreciable increase. This phenomenon is denoted the longitudinal paramagnetic effect. Historically, this expression was first introduced to describe a related phenomenon, namely, the rise in longitudinal flux permeating a wire in a static $H_{//}$ as a transport current I was impressed. The magnitude of the longitudinal paramagnetic effect is seen to depend on the strength of $H_{//}$, showing a greater increase where the stationary $H_{//}$ is chosen stronger. The influence of $H_{//}$ on the longitudinal paramagnetic effect is presented in Fig. 3. This figure complements Fig. 2 and displays vertical cuts through the measured family of curves of $\Delta \langle B_z \rangle_{\text{cyl}}$ versus $\langle B_\phi \rangle_{\text{app}}$, represented in part in Fig. 2. Vertical cuts for three

values of $\langle B_\phi \rangle_{app}$ are shown in Fig. 3. We note that the influence of $H_{//}$ "saturates". The saturation exhibited here should not be interpreted as showing that the wall is now filled with induced persistent current. Rather, the plateaus here only show that the manifestation of the paramagnetic effect has been optimized for a chosen increase in $\langle B_\phi \rangle_{app}$. Analogous measurements on ribbon samples and our observations of the azimuthal paramagnetic effect (see Fig. 4 of Chap. 3) indicate, however, that this family of curves is approaching an upper limit. Further the uppermost set of data points in Fig. 3 are close to this "saturation" curve.

Initially, the longitudinal flux density in the hole, $B_1 = \mu_0 H_{//}$, the externally applied longitudinal magnetic induction present when the hollow cylinder cooled from T_c to 4.2 K. The longitudinal flux density B_1 is seen to vary as an azimuthal flux is impressed. A few typical traces of this evolution are depicted in Fig. 4 for various stationary $H_{//}$. For convenience, $\Delta B_1^k = B_1 - \mu_0 H_{//}$, the change of B_1 with respect to the initial strength $\mu_0 H_{//}$ is displayed. A remarkable large scale phenomenon is now observed for the first time.

As the increase of H_ϕ progresses, the longitudinal flux density in the hole is seen to diminish dramatically. This decrease can be readily understood in the accepted framework of flux line nucleation and migration. Helical vortices are created in the wall at the surface of the hole as the total flux density $B_t = \sqrt{(B_1^2 + \mu_0 H_\phi^2)}$ in the hole is made to increase. At the same time, helical vortices are being created and entering the wall at the external surface of the cylinder as we have seen above in Fig. 2. The focus now, however is on the behaviour in the hole and in the adjacent volume of

the wall. Each helical vortex introduced into the wall from the hole carries with it a quantity of longitudinal flux whose source can only be the finite reservoir of longitudinal flux contained by the hole. The "pumping" of helical flux lines from the hole into the wall therefore lowers the level of longitudinal flux in the hole. By contrast, the azimuthal flux is sustained by its source, the toroidal generating coil. The hole is effectively isolated by the wall from the source of longitudinal flux, the external solenoid. This isolation will persist until the helical flux lines penetrating from the outside interact in the wall with those penetrating from the hole. Only at this juncture do we expect B_1 to cease decreasing and exhibit an increase towards $\mu_0 H_{//}$, presumably attaining this value when $B_1 = B_{c2}$. Unfortunately, this intriguing expected behaviour is not accessible with our present arrangement.

Fig. 5 presents various vertical cuts through the measured family of curves of which only a small sampling is displayed in Fig. 4. The two figures therefore complement each other. Again we note the saturation behaviour and refer the reader to the pertinent comments made a few paragraphs earlier.

The pickup coil embracing the cylinder azimuthally, hence monitoring $\langle B_z \rangle_{cyl}$, the magnetic flux density threading the combination of hole and wall longitudinally, will record the net change in $\phi_{cyl} = \langle B_z \rangle_{cyl} \pi R_o^2$. Any redistribution of flux between the hole and the wall and in particular, the transfer of flux from the hole to the wall will not be registered by this sensor. This external pickup coil will however measure the resultant of various competing processes. (1) Any augmentation of ϕ_{cyl} arising from the entry of longitudinal flux through the external surface and

(ii) any decrease of ϕ_{cyl} due to consumption of longitudinal flux by flux cutting. We note that flux lines that have entered the wall through either the outside or the inner surface as well as flux lines initially present in the wall, can experience flux line cutting and hence be "depleted" by the attendant flux consumption.

As a first step towards identifying the actual net augmentation of the longitudinal flux in the wall, it is appropriate to "normalize" the raw data displayed in Fig. 4, namely $\Delta\phi_{cyl} = \Delta\langle B_z \rangle_{cyl} \pi R_o^2$ and recast it in terms of the dimensions of the wall, using the following identity,

$$\Delta\phi_{cyl} = \Delta\langle B_z \rangle_{cyl} \pi R_o^2 = \Delta\langle B_z \rangle'_{wall} \pi (R_o^2 - R_i^2) \quad (5a)$$

which can be rewritten to read,

$$\Delta\langle B_z \rangle'_{wall} = \Delta\langle B_z \rangle_{cyl} \left(\frac{R_o^2}{R_o^2 - R_i^2} \right) \quad (5b)$$

The above quantity, however, does not provide a true measure of the change of longitudinal flux density in the wall since it takes no account of the fact that longitudinal flux has been transferred from the hole into the wall. The amount of flux thus displaced can be written,

$$\Delta\phi_{hole} = (\Delta B_i) \pi R_i^2 = (\mu_o H_{//} - B_i) \pi R_i^2 \quad (6)$$

Normalizing this transferred flux to the cross section of the wall leads to the expression

$$\Delta\langle B_z \rangle''_{wall} \pi (R_o^2 - R_i^2) = (\Delta B_i) \pi R_i^2 \quad (7a)$$

which can be rewritten to read,

$$\Delta\langle B_z \rangle''_{wall} = (\Delta B_i) \left(\frac{R_i^2}{R_o^2 - R_i^2} \right) \quad (7b)$$

It is important to bear in mind, that a portion of $\Delta\phi_{hole}$ will surely be made to disappear in the wall as flux cutting, hence flux

consumption, takes place. Unfortunately our sets of pickup coils cannot determine directly what fraction disappears.

Nevertheless we can define a quantity,

$$\Delta \langle B_z \rangle_{\text{wall}} = \Delta \langle B_z \rangle'_{\text{wall}} + \Delta \langle B_z \rangle''_{\text{wall}} \quad (8a)$$

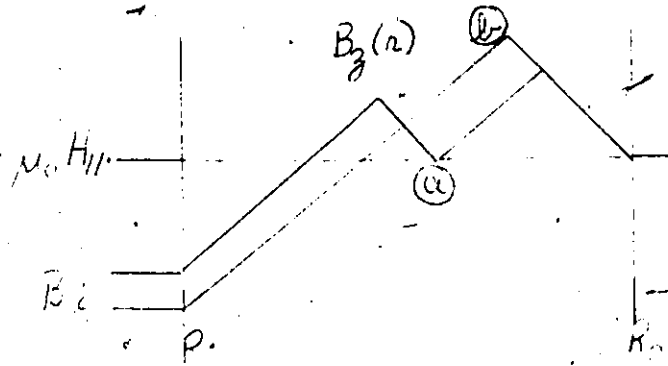
which is a measure of the actual net increase of the longitudinal flux density in the wall. Any longitudinal flux which has been made to disappear has contributed to the signal detected by the external coil and is therefore included in $\Delta \langle B_z \rangle'_{\text{wall}}$. Introducing equations 5b and 7b into 8a yields,

$$\Delta \langle B_z \rangle_{\text{wall}} = \frac{\Delta \langle B_z \rangle_{\text{cyl}} R_o^2 + (\Delta B_1) R_1^2}{R_o^2 - R_1^2} \quad (8b)$$

The marriage of the observations depicted in Figs. 2 and 4 and announced through equation 8b is displayed in Fig. 6. We find that $\Delta \langle B_z \rangle'_{\text{wall}}$ and $\Delta \langle B_z \rangle''_{\text{wall}}$ make comparable contributions to $\Delta \langle B_z \rangle_{\text{wall}}$. We had expected the latter to be somewhat smaller since it represents flux ingress diminished by the flux consumption in the wall whereas the former measures flux ingress only.

Two sets of B_z profiles consistent with our observations and with critical state concepts are sketched below depicting limiting cases in the regime of saturated states. For simplicity, all of the slopes are taken to be constant and identical, hence j_ϕ is assumed uniform and fixed in magnitude. In the appendix we develop analytic expressions for $\Delta \langle B_z \rangle_{\text{cyl}}$ and equivalently, for $\Delta \langle B_z \rangle_{\text{wall}}$ in terms of ΔB_1 and j_ϕ in the framework illustrated by these sketches. Introducing the maximum values observed for ΔB_1 and $\Delta \langle B_z \rangle_{\text{cyl}}$ (or equivalent $\Delta \langle B_z \rangle_{\text{wall}}$) into these expressions namely $\Delta B_1 = 2$ mT and $\Delta \langle B_z \rangle_{\text{cyl}} = 1.2$ mT yields $j_\phi = 4.3 (10^7)$ and $8 (10^7)$ A/m²

depending on whether configuration (b) or (a) is stipulated. These values are comparable to j_z estimated in a similar context from the azimuthal paramagnetic effect (see Chap 3).



To further compare the magnitudes of the longitudinal and azimuthal paramagnetic effects we can inspect the uppermost curve of Fig. 7 side by side with Fig. 4 of Chap. 3. We note that the height of the plateau for the former case is $\approx 2/3$ that of the latter. This may be expected, however, since one of the boundary fields (B_1) has decreased by ≈ 2 mT during the generation of the longitudinal paramagnetic effect. Also, in our arrangement, the peak of the latter phenomenon may not have been attained.

Conclusion

The magnetic flux density permeating the wall of a hollow cylinder cooled into the superconducting type II state in a fixed externally applied longitudinal magnetic field $H_{//}$ has been monitored as an azimuthal field H_{ϕ} was impressed. Although $H_{//}$ is maintained stationary, the longitudinal flux threading the wall is seen to increase as H_{ϕ} is applied. This phenomenon is labelled the longitudinal paramagnetic effect. It is evidently analogous to the observation reported in Chapter 3 of an augmentation in the azimuthal

flux permeating the wall of a hollow cylinder bathing in a stationary applied azimuthal field as a longitudinal field is impressed. In the framework of the two coordinates involved in these experiments, namely, the azimuthal and longitudinal unit direction vectors $\hat{\phi}$ and $\hat{z}$, these two effects can be regarded as a single phenomenon, viewed in a frame of reference in one instance and then in this frame of reference rotated by 90° in the other case. In this context, the occurrence of the longitudinal paramagnetic effect could be readily predicted once its "twin", the azimuthal paramagnetic effect was observed and vice versa.

There is however, no longitudinal cut across the wall of our cylinder, hence its wall is continuous along the ϕ coordinate. As a consequence, this perspective is not entirely appropriate. Indeed important new behaviour has been encountered in our experiment, precisely because the longitudinal flux in the hole is prevented by the wall from communicating with the longitudinal magnetic field outside the cylinder. In particular we observe that the longitudinal flux in the hole diminishes as H_ϕ is increased. This behaviour like the paramagnetic effects, is fully consistent with the accepted picture of flux lines nucleating at the surface of type II superconductors from flux existing in empty space and then migrating into the bulk of the material. This large scale phenomenon, although predictable, has not been previously reported and provides a useful laboratory demonstration of macroscopic flux dynamics in type II superconductors.

The observation of the displacement of flux from the hole into the wall yields, by itself, no information on flux cutting processes. One can however, design experimental arrangements where this transfer of flux from the hole into the wall can yield important data on the occurrence of flux

cutting and the attendant flux consumption in the wall. The key to these investigations will be the important feature that the quantity of flux entering the wall can be measured by monitoring the longitudinal flux in the hole.

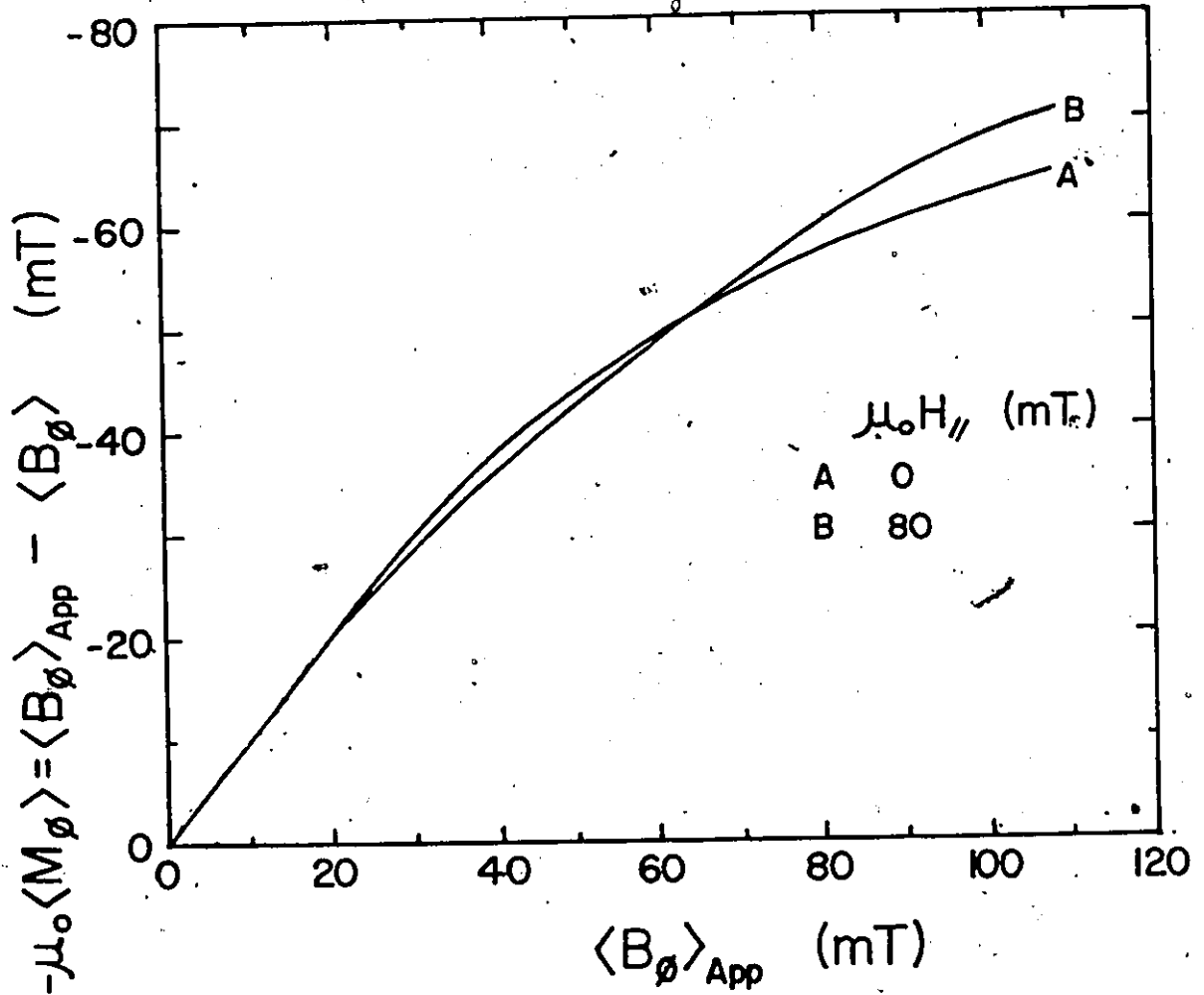


Fig. 6-1. Illustrates the influence of a static longitudinal magnetic field $H_{//}$ on the magnetic response (shielding against flux entry) of the wall to an impressed azimuthal field by comparing the behaviour when $\mu_0 H_{//} = 0$ and 80 mT. For intermediate values of $\mu_0 H_{//}$, the observed curves lie in sequence between the two curves displayed above.

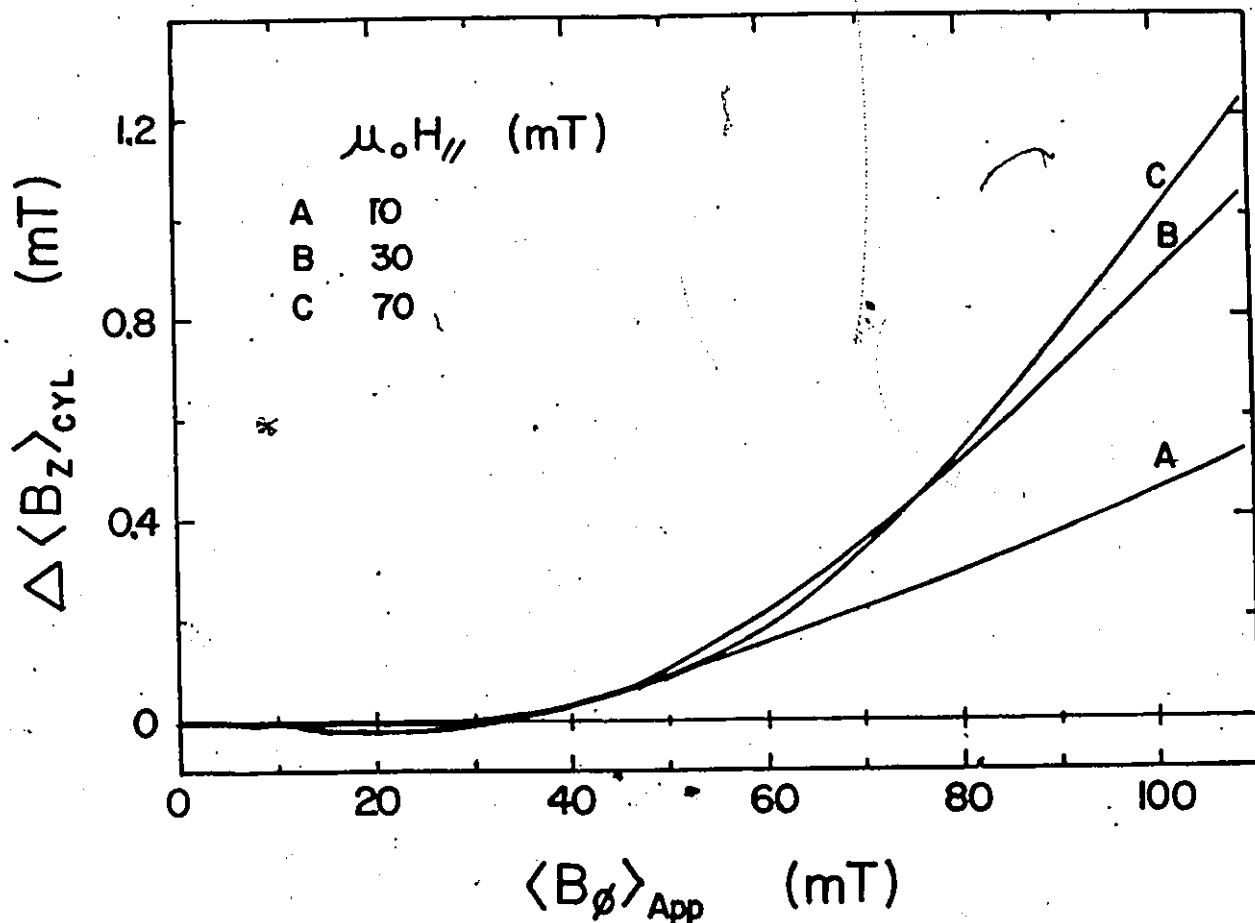


Fig. 6-2. The curves illustrate the evolution of, $\langle B_z \rangle_{\text{cyl}}$, the average longitudinal magnetic flux threading the cross section of the cylinder (hole plus wall) in various stationary externally applied $H_{||}$ as an azimuthal magnetic induction $\langle B_\phi \rangle_{\text{app}}$ is impressed. Only three representative curves are displayed from the several measured curves to avoid clutter. The phenomenon that $\Delta \langle B_\phi \rangle > 0$ is denoted the longitudinal paramagnetic effect. The maximum azimuthal magnetic field we can generate with the toroidal magnet coil in our experiment is not strong enough to trace the peaks expected for these curves.

Fig. 6-3. Complements the previous figure by displaying data points taken along vertical cuts through the family of measured curves represented in that figure. The vertical cuts shown were taken at $\langle B_\phi \rangle_{app} = 50, 77$ and 100 mT. Since the maximum available azimuthal magnetic field was inadequate to trace the expected peaks in Fig. 6-2, we can only presume that the upper curve above corresponds to "saturation".

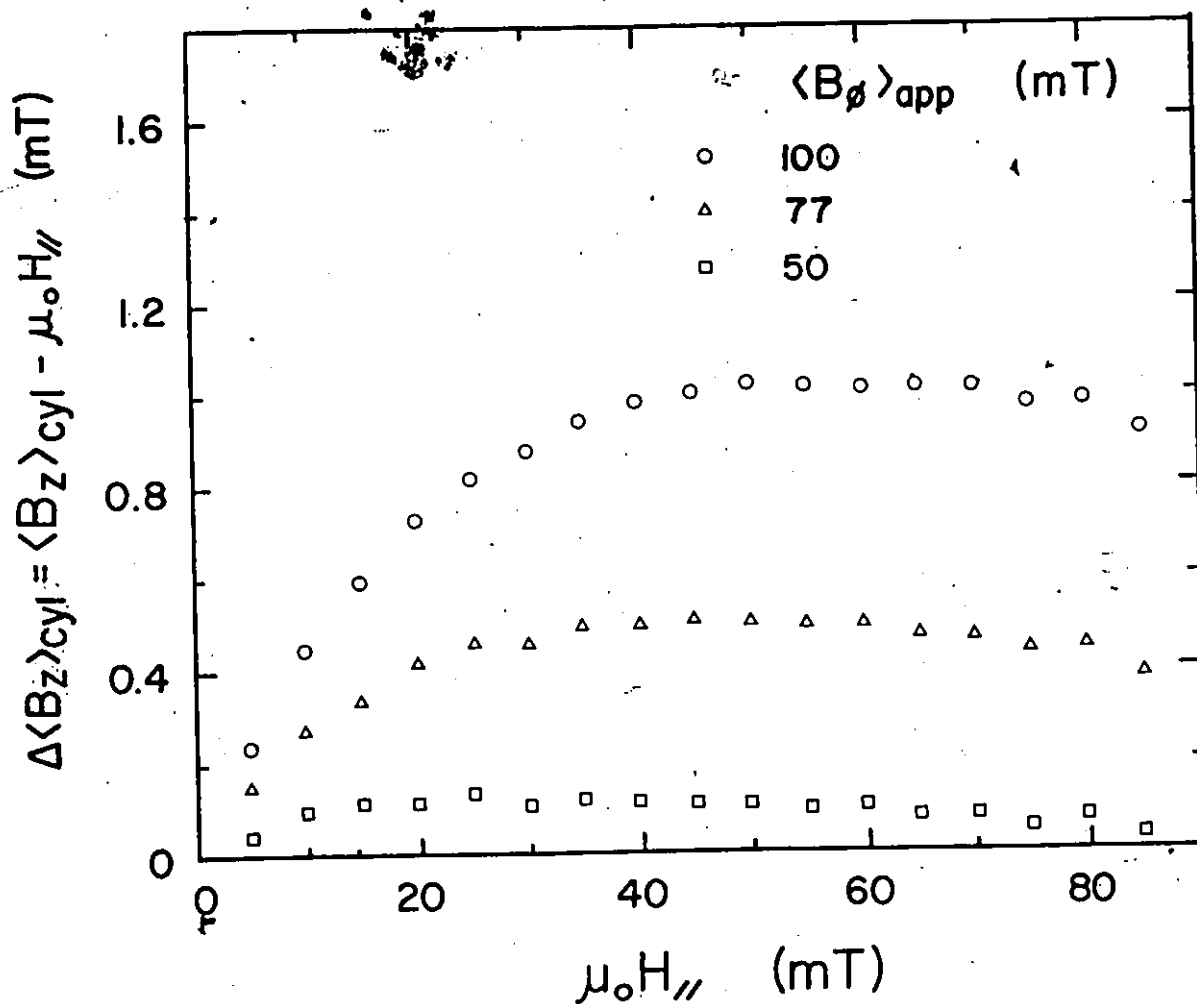


Fig. 6-4. Displays the decrease of the longitudinal magnetic flux density in the hole in various stationary externally applied longitudinal magnetic fields $H_{||}$ as an azimuthal magnetic induction $\langle B_{\phi} \rangle_{app}$ is impressed. Only three representative curves are presented from the several measured curves to avoid clutter. The flux disappearing from the hole must be transferred to the wall.

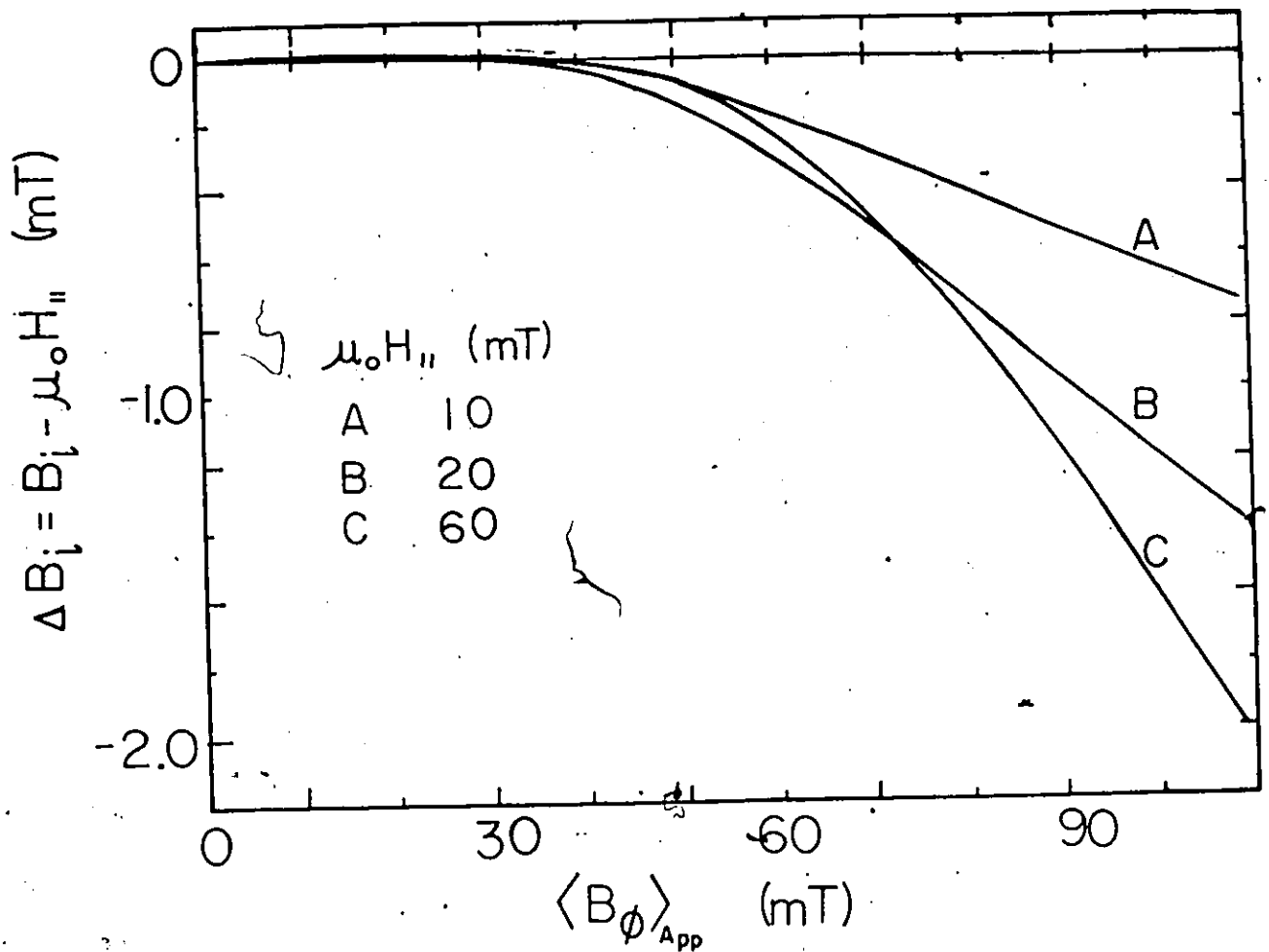


Fig. 6-5. Complements the previous figure by displaying data points taken along vertical cuts through the family of measured curves represented in that figure. The vertical cuts shown were taken at $\langle B_\phi \rangle_{app} = 50, 77$ and 100 mT.

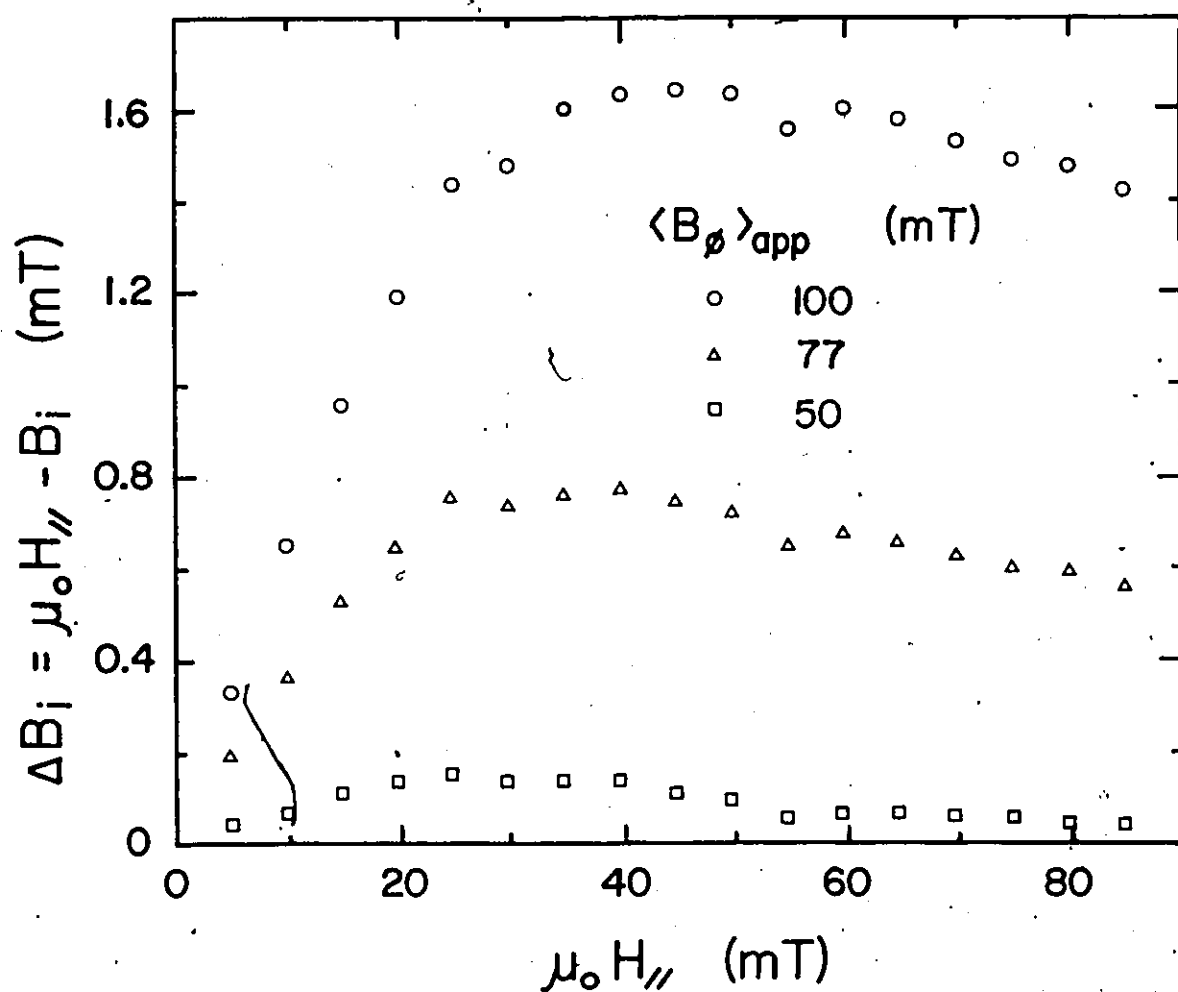


Fig. 6-6. The curves illustrate the evolution of the average longitudinal magnetic induction permeating the wall in various stationary externally applied $H_{//}$ as an azimuthal magnetic field is impressed. These data curves are constructed, as described in the text, from the family of measured curves displayed in Fig. 6-2 and 6-4. Only three representative curves are displayed from the several data curves to avoid clutter. The phenomenon that $\Delta \langle B_z \rangle_{\text{wall}} > 0$ is labelled the longitudinal paramagnetic effect.

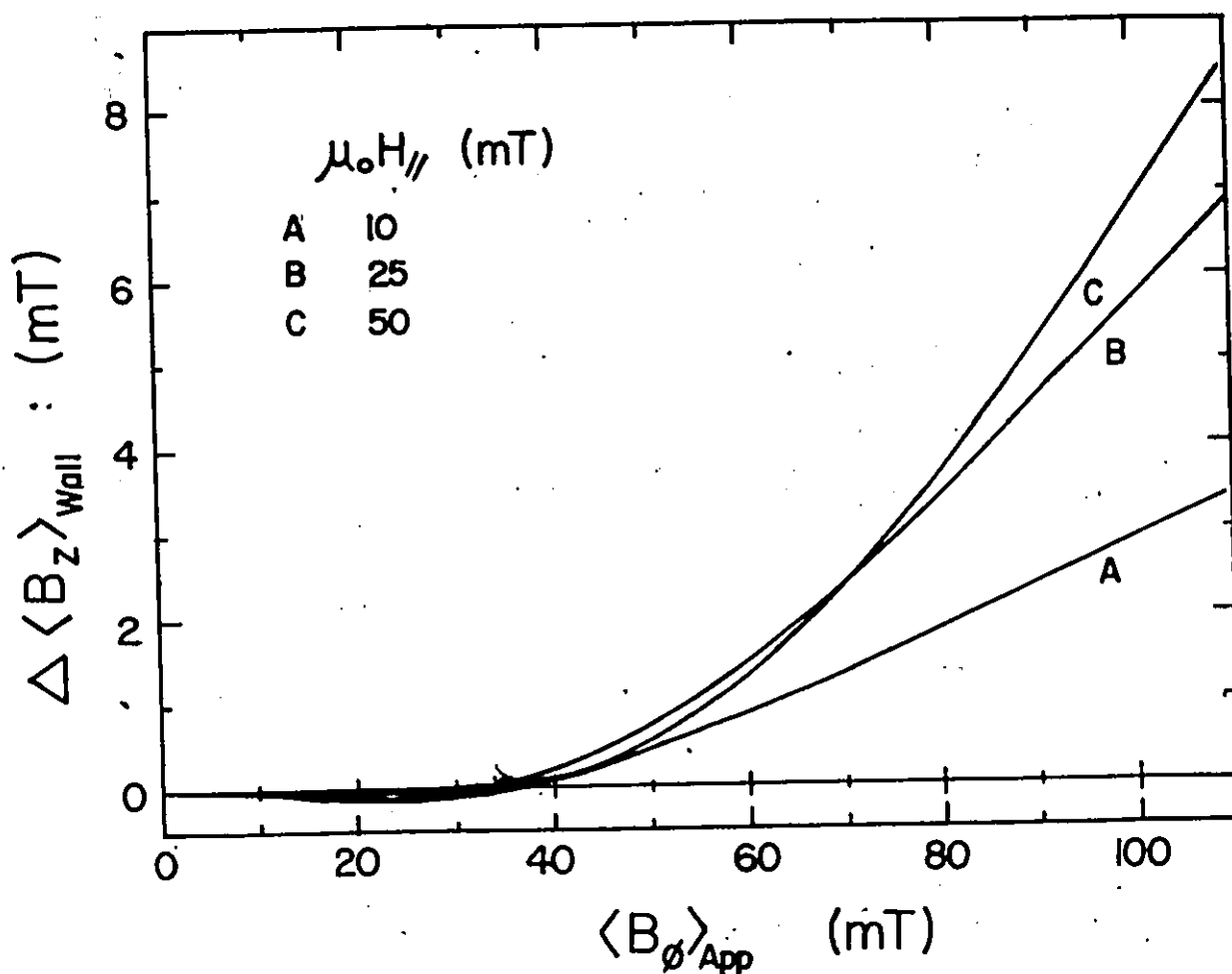
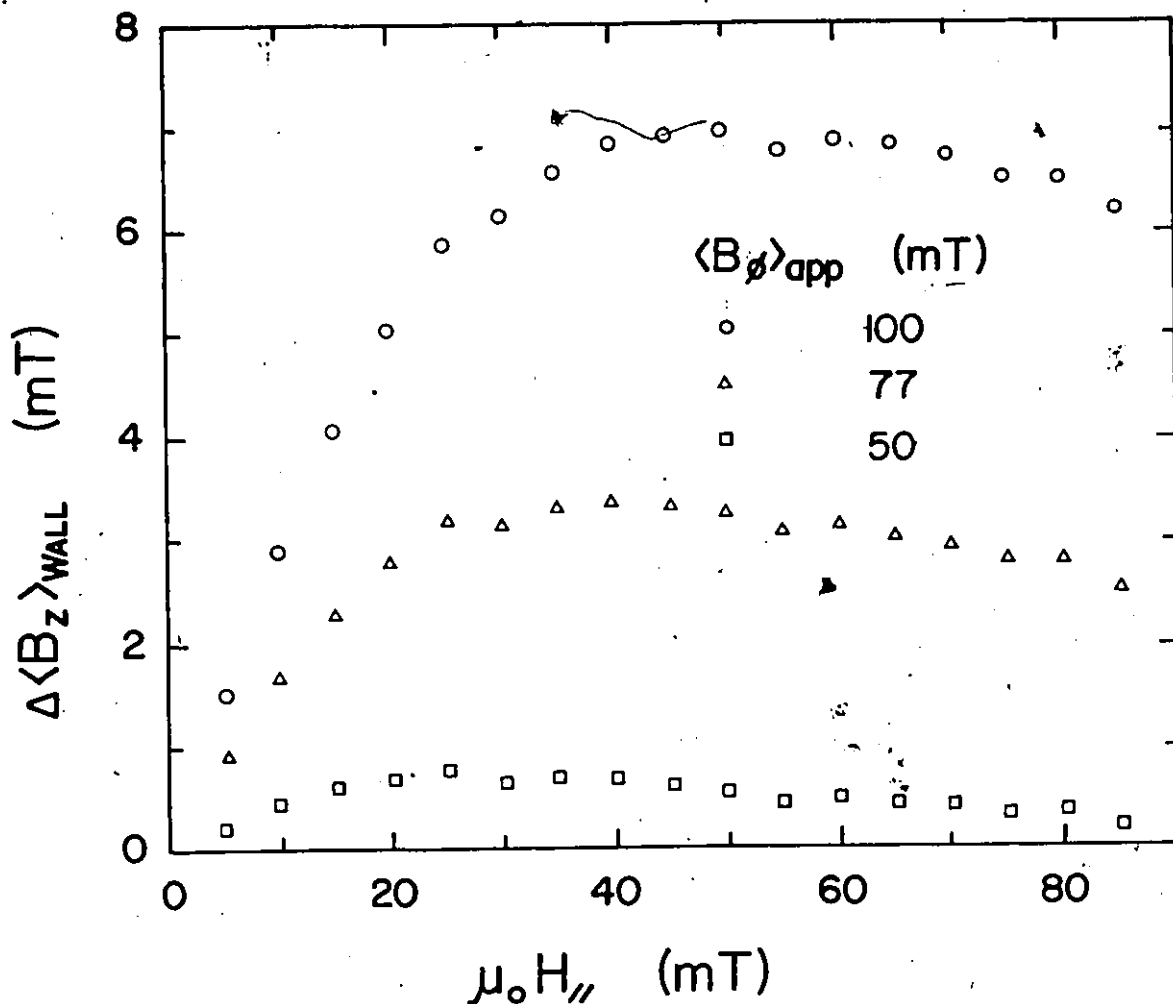


Fig. 6-7. Complements the previous figure by displaying data points taken along vertical cuts through the family of curves represented in that figure. The upper curve presumably approaches the saturated critical state curve and can be compared with the analogous plot for the azimuthal paramagnetic effect shown in Fig. 3-4.



Chapter 7

Longitudinal Diamagnetic-Paramagnetic Effect

Introduction

We have seen in the preceding Chapter that, subjecting a hollow cylinder to an increasing externally applied azimuthal magnetic field H_ϕ , while the cylinder is immersed in a stationary externally applied longitudinal magnetic field $H_{//}$ causes $\langle B_z \rangle_{\text{wall}}$ the longitudinal flux density in the wall to increase and, B_1 , the longitudinal flux density in the hole to diminish. These two concomitant phenomena can readily be understood with the picture of helical flux lines being made to enter the wall at its inner and outer surfaces as the total applied field $H = (H_{//}^2 + H_\phi^2)^{1/2}$ is caused to increase since each flux line carries a component of longitudinal flux. The longitudinal flux entering the wall from the hole must be extracted from the supply of longitudinal flux initially stored in the hole thereby causing the level of B_1 there to drop. In light of these observations we expect the opposite behaviour to occur when H_ϕ instead of being increased from zero to some maximum value $H_{\phi \text{ max}}$, is made to decrease from $H_{\phi \text{ max}}$ to zero. The observations we report in this Chapter indeed fulfill these expectations. $\langle B_z \rangle_{\text{wall}}$ is seen to diminish and B_1 is observed to rise as helical flux lines are made to exit from the wall by the decrease in the externally applied magnetic field. The decrease of $\langle B_z \rangle_{\text{wall}}$ in a static $H_{//}$ is, for obvious reasons, labelled the longitudinal diamagnetic effect.

The swing of H_ϕ from $H_{\phi \text{ max}}$ is made to traverse zero and allowed to continue on into the region of opposite polarity until a value of comparable magnitude is attained. From our experience with an analogous situation recounted in Chapter 4, and from the picture of helical flux lines exploited already, we anticipate that the evolution of both $\langle B_z \rangle_{\text{wall}}$ and B_i will now change direction. The observations we now present are seen to justify all of these expectations.

Results and Discussion

Proceeding as in previous Chapters we first compare the magnetic behaviour of the wall to a swing of H_ϕ from $H_{\phi \text{ max}}$ to $-H_{\phi \text{ max}}$ in the absence and in the presence of a static $H_{//}$. This comparison is depicted in Fig. 1 where, as previously, we note a small enhancement in the azimuthal magnetic moment of the wall, hence in $\langle j_z \rangle$, when $H_{//} = 70$ mT is present. The behaviour exhibited by this material is consequently rather pedestrian in this respect.

The evolution of $\Delta \langle B_z \rangle_{\text{cyl}}$, the change in the average longitudinal flux threading the entire cylinder (hole and wall combined) during the swing of H_ϕ from $+H_{\phi \text{ max}}$ to $-H_{\phi \text{ max}}$ in various static $H_{//}$ is displayed in Fig. 2. Again, to avoid clutter only a small sampling of the measured curves is presented. To provide a more complete picture of the behaviour of the entire family of curves as a function of the several choices for the fixed $H_{//}$, we have plotted, in Fig. 3, a selection of vertical cuts through our full collection of such curves.

During the first half of the sweep of H_ϕ , as H_ϕ varies from $H_{\phi \text{ max}}$

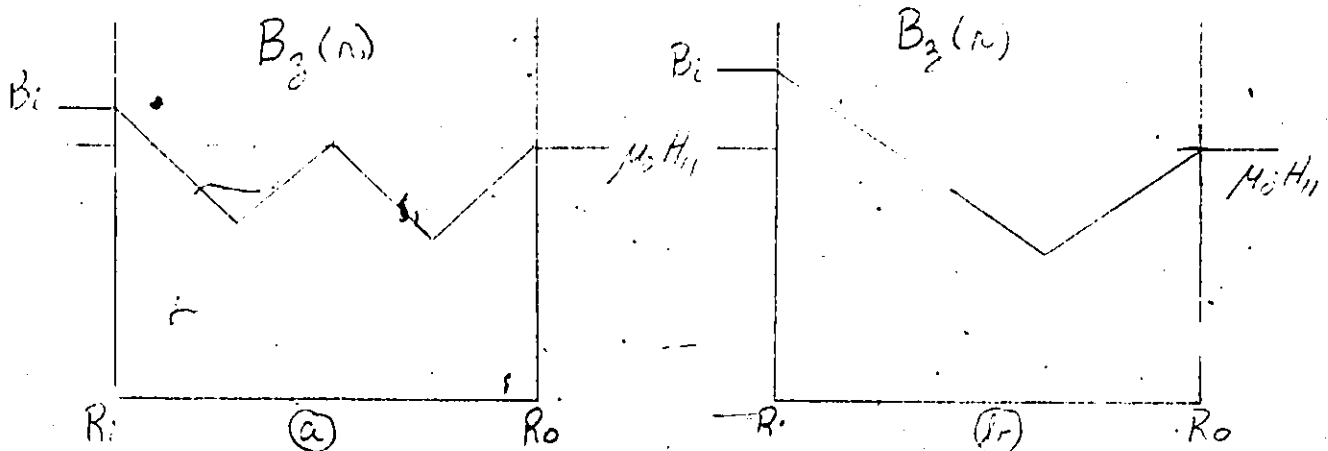
to zero, $\langle B_z \rangle_{\text{cyl}}$ is seen to decrease. This decrease indicates that the exit of flux lines, occasioned by the decrease of the applied magnetic field, removes longitudinal flux from the wall although the externally applied $H_{//}$ is stationary. That this disappearance of longitudinal flux in a static $H_{//}$ must involve flux cutting is perhaps not evident. To appreciate this feature, it is useful to first note that the helicity of the magnetic induction in free space at the surface of the cylinder, cannot correspond, during the decrease of H_{ϕ} , to that initially existing in the wall, since the former has been made to change. Two possibilities can now be envisaged. The direction of $\vec{B}$ just inside the surface is taken either as, (i) permanently fixed once the superconducting state has been established or, (ii) as capable of changing in step with the change of orientation of $\vec{B}$ outside the surface. The first hypothesis leads to a discontinuity in the orientation of $\vec{B}$ across the surface. Such a discontinuity can, of course, be accounted for in terms of a surface current flowing parallel to $\vec{B}$ at the surface. This explanation, however, relies on the ad hoc existence of a surface current of arbitrary and unrestricted magnitude. Physically, a limiting value for such a current must eventually be attained. When this limit is exceeded, the orientation of $\vec{B}$ inside will vary in unison with that outside. Consequently, we must finally address the fact that the helicity of the flux lines in the bulk must change from the existing configuration towards that prevailing outside. In order that these changes of helicity take place, the process of flux cutting and attendant flux consumption must now come into play. Consequently, in view of the considerations just outlined, the longitudinal diamagnetic effect can be regarded as intimately linked to and a manifestation of the occurrence of flux cutting in the wall.

We cannot, however, from our present data, identify the relative contributions of flux exit and flux consumption to $\Delta \langle B_z \rangle_{\text{cyl}}$.

The exit of helical flux lines from the wall is made dramatically evident by the rise of B_1 , the longitudinal flux density in the hole during the decrease of H_ϕ , displayed in Fig. 4.

As in the previous Chapter, we have combined the two sets of data represented by Figs. 2 and 4, to obtain an accurate picture of the evolution of $\Delta \langle B_z \rangle_{\text{wall}}$, the actual decrease of the longitudinal flux density in the wall. As described there, $\Delta \langle B_z \rangle_{\text{wall}}$ takes into account that flux is displaced from the wall into the hole. This is measured by the inner pickup coil monitoring B_1 . Also flux is consumed in the wall and exits from the outer surface of the cylinder. This is monitored by the outer pickup coil which measure $\Delta \langle B_z \rangle_{\text{cyl}}$, the change of flux density averaged over the entire cross section of the cylinder (the wall and the hole included). The evolution of $\Delta \langle B_z \rangle_{\text{wall}}$, the deficit of longitudinal flux in the wall as H_ϕ descends from $H_{\phi \text{ max}}$ through zero into the region of opposite polarity is displayed in Fig. 6 and complementary data on the numerous family of measured curves, provided by Fig. 7.

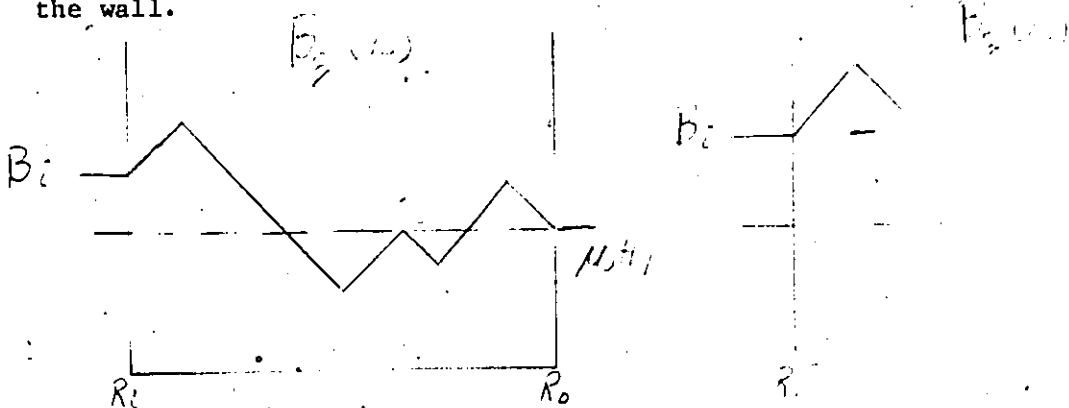
The limiting B_z profiles in the regime of saturated states are depicted schematically in the sketches below. Introducing



the values observed at $H_\phi = 0$ and $\mu_0 H_{//} = 15$ mT, namely $\Delta \langle B_z \rangle_{\text{wall}} = 3.5$ mT, (or equivalently $\Delta \langle B_z \rangle_{\text{cyl}} = 0.5$ mT), and $\Delta B_1 = 7.5$ mT, into the expression derived in the appendix, we find that $B_z = 0$ at the minimum of the B_z profiles. The longitudinal diamagnetic effect is therefore seen to cause a dramatic reduction in the strength of the B_z profile.

A comparison of Figs. 6 and 7 of this chapter with Figs. 3 and 4 of Chapter 4, indicates that $\Delta \langle B_z \rangle_{\text{wall}} / \Delta \langle B_\phi \rangle$ is ≈ 0.7 . Part of the deviation of this ratio from unity can be ascribed to the fact that the occurrence of the decrease in $\langle B_z \rangle_{\text{wall}}$ is accompanied by a competing rise in B_1 which will tend to impede the exit of flux.

As the swing of H_ϕ proceeds to higher magnitudes in the region of "negative" polarity after traversing zero, the longitudinal paramagnetic effect discussed in the previous chapter (and the associated decrease in B_1) initially coexists in the wall with the "remains" of the longitudinal diamagnetic effect as depicted schematically in the sketch below. Eventually, the former dominates and finally overtakes the entire volume of the wall.



From an inspection of Figs. 2 through 7 of this Chapter alone or of these together with Figs. 2 through 7 of Chapter 6 we can compare the relative magnitudes of the longitudinal paramagnetic and diamagnetic

effects under corresponding circumstances. This comparison indicates that these phenomena are quantitatively very similar. We view this as surprising since flux consumption which occurs in both cases will tend to diminish the paramagnetic effect whereas it should assist the diamagnetic phenomenon.

Conclusion

The magnetic flux permeating the wall and the hole of a hollow cylinder of a type II superconductor bathing in a stationary applied longitudinal field $H_{//}$ has been monitored as the applied azimuthal field H_{ϕ} swings from $H_{\phi \text{ max}}$ to $-H_{\phi \text{ max}}$. The longitudinal flux in the wall is seen to diminish appreciably and the longitudinal flux in the hole to grow during the descent of H_{ϕ} from $H_{\phi \text{ max}}$ to zero. As the swing of H_{ϕ} continues from 0 to $-H_{\phi \text{ max}}$, the reverse phenomena appear and the longitudinal flux in the wall is observed to increase while that in the hole decreases.

This set of observations completes our survey of the magnetic response of a hollow cylinder subjected to two related regimes of variations in the applied magnetic field, namely: (i) where H_{ϕ} is held fixed while $H_{//}$ is applied from zero or made to undergo a full sweep from positive to a comparable negative value and (ii) where $H_{//}$ is held fixed while H_{ϕ} is varied in the manner just described. We have seen that in these circumstances, the component of the flux density in the wall along the stationary field increases (decreases) although it is the applied field component orthogonal to that direction which is made to increase (decrease). All of these phenomena can readily be understood exploiting a picture of helical vortices being added to or subtracted from the wall. More careful considerations show, however, that these observations are basically manifestations of flux cutting processes.

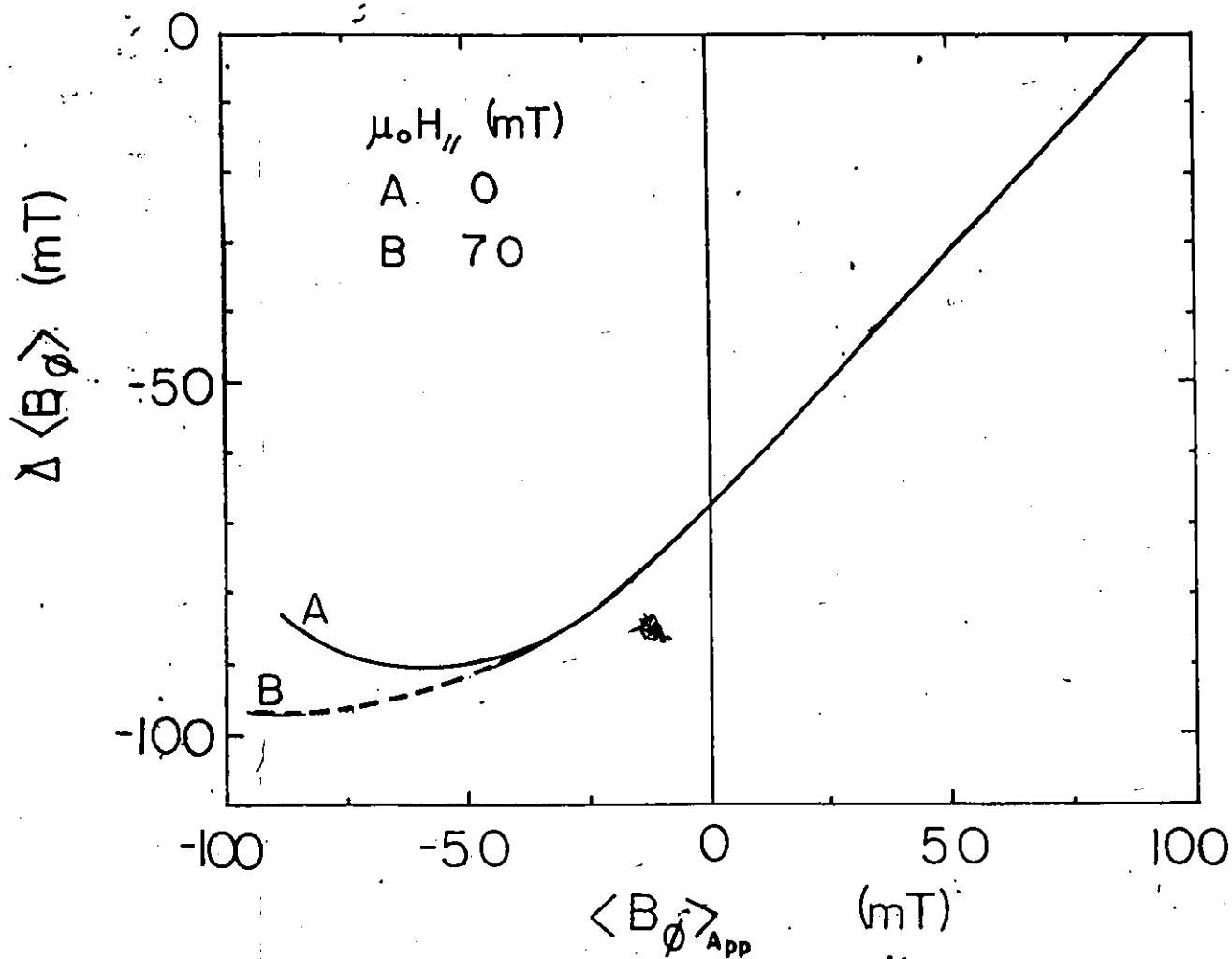


Fig. 7-1. Illustrates the influence (seen to be negligible) of the presence of a stationary externally applied $H_{||}$ on the magnetic response of the wall of the cylinder to a sweep of an azimuthal magnetic field swinging from a "positive" to a "negative" value. The curves compare the behaviour when $\mu_0 H_{||} = 0$ and 70 mT. For intermediate $H_{||}$, the curves lie between the two shown above.

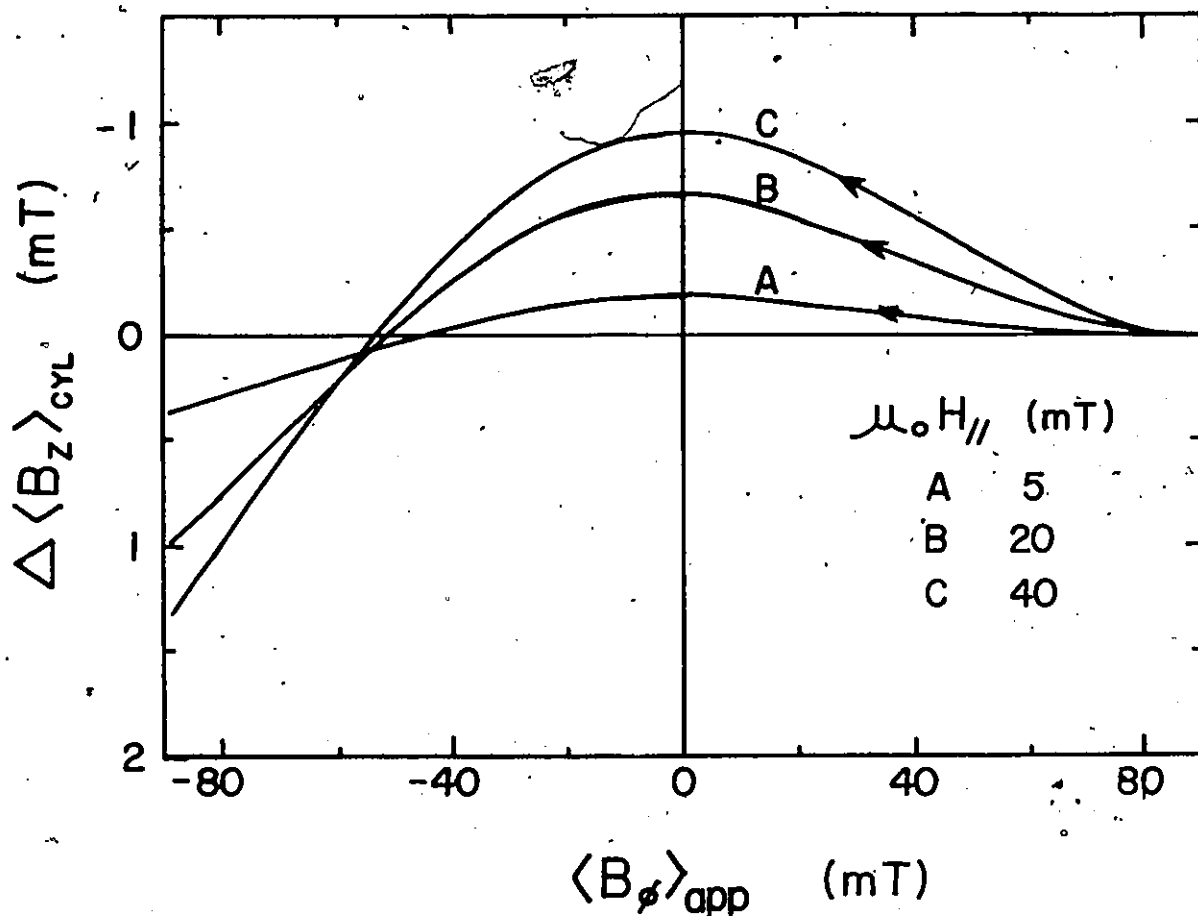


Fig. 7-2. The curves illustrate the evolution of $\langle B_z \rangle_{\text{cyl}}$, the average longitudinal magnetic flux threading the cross section of the cylinder (hole plus wall) in various stationary externally applied $H_{||}$ as an externally applied azimuthal magnetic field is made to swing from a "positive" to a "negative" value. The disappearance of longitudinal flux exhibited by the curves in the first quadrant is called the longitudinal diamagnetic effect and constitutes an impressive manifestation of flux cutting although the interpretation requires careful consideration. The effect evolves from diamagnetic to paramagnetic as the sweep of $\langle B_\phi \rangle_{\text{app}}$ continues into the second quadrant. (See Fig. 4-3 for the analogous azimuthal effects). Only three representative curves are displayed from the several measured curves to avoid clutter.

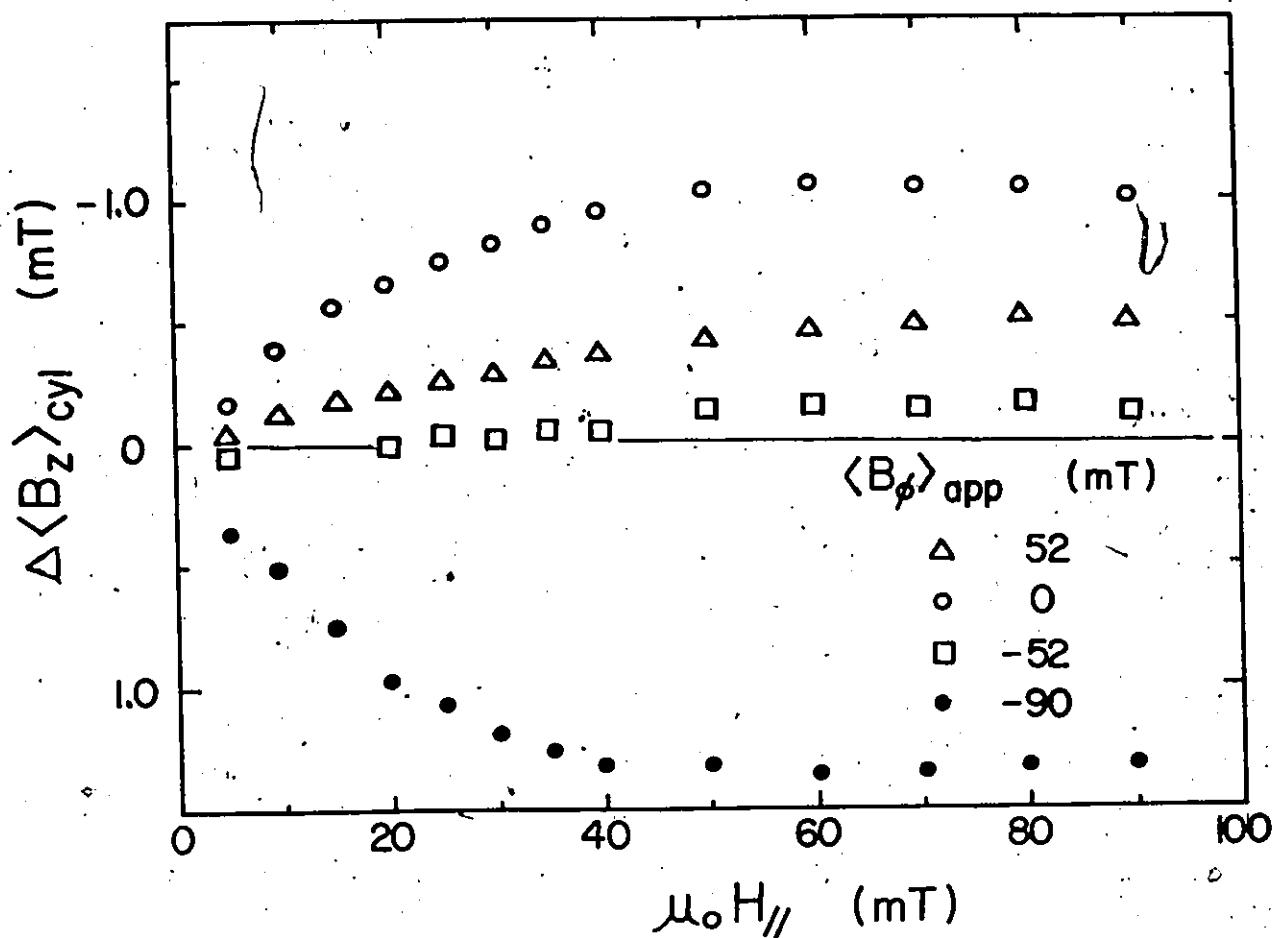


Fig. 7-3. Complements the previous figure by displaying data points taken along vertical cuts through the family of measured curves represented in that figure. The vertical cuts shown were taken at $\langle B_\phi \rangle_{\text{app}} = 52, 0, -52$ and -90 mT. The reason for these choices becomes evident from inspection of that figure.

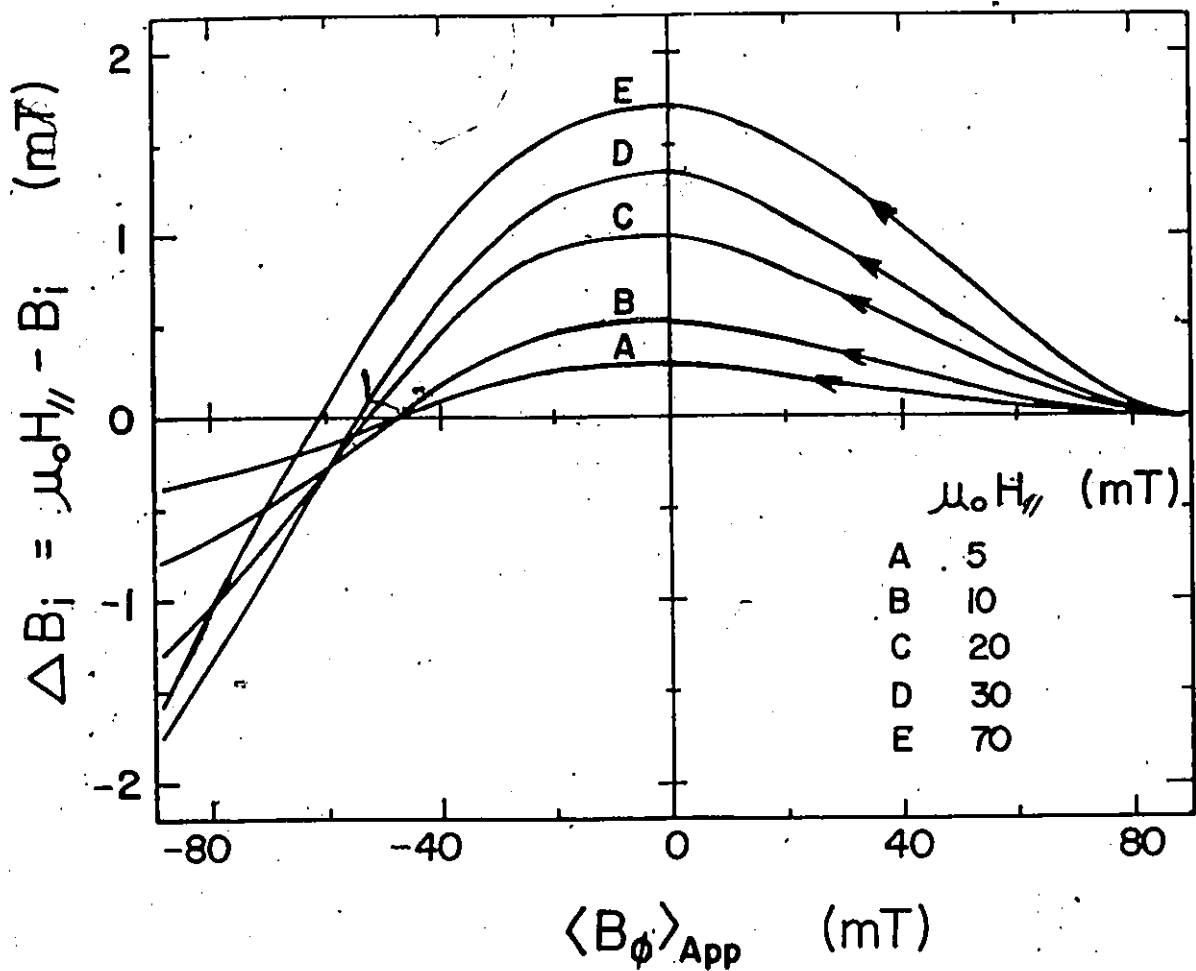


Fig. 7-4. The curves illustrate the evolution of the longitudinal flux density in the hole in various stationary externally applied $H_{//}$ as an applied azimuthal magnetic field is made to swing from a "positive" to a "negative" value. In the first quadrant, the increase of the longitudinal flux in the hole must originate from helical flux in the wall. Longitudinal flux is returned to the wall throughout the second quadrant, the density of longitudinal flux in the hole equalling the externally applied but stationary $\mu_0 H_{//}$ at the cross over of the curves with the horizontal axis. Again only representative measured curves are displayed to avoid cluttering the figure.

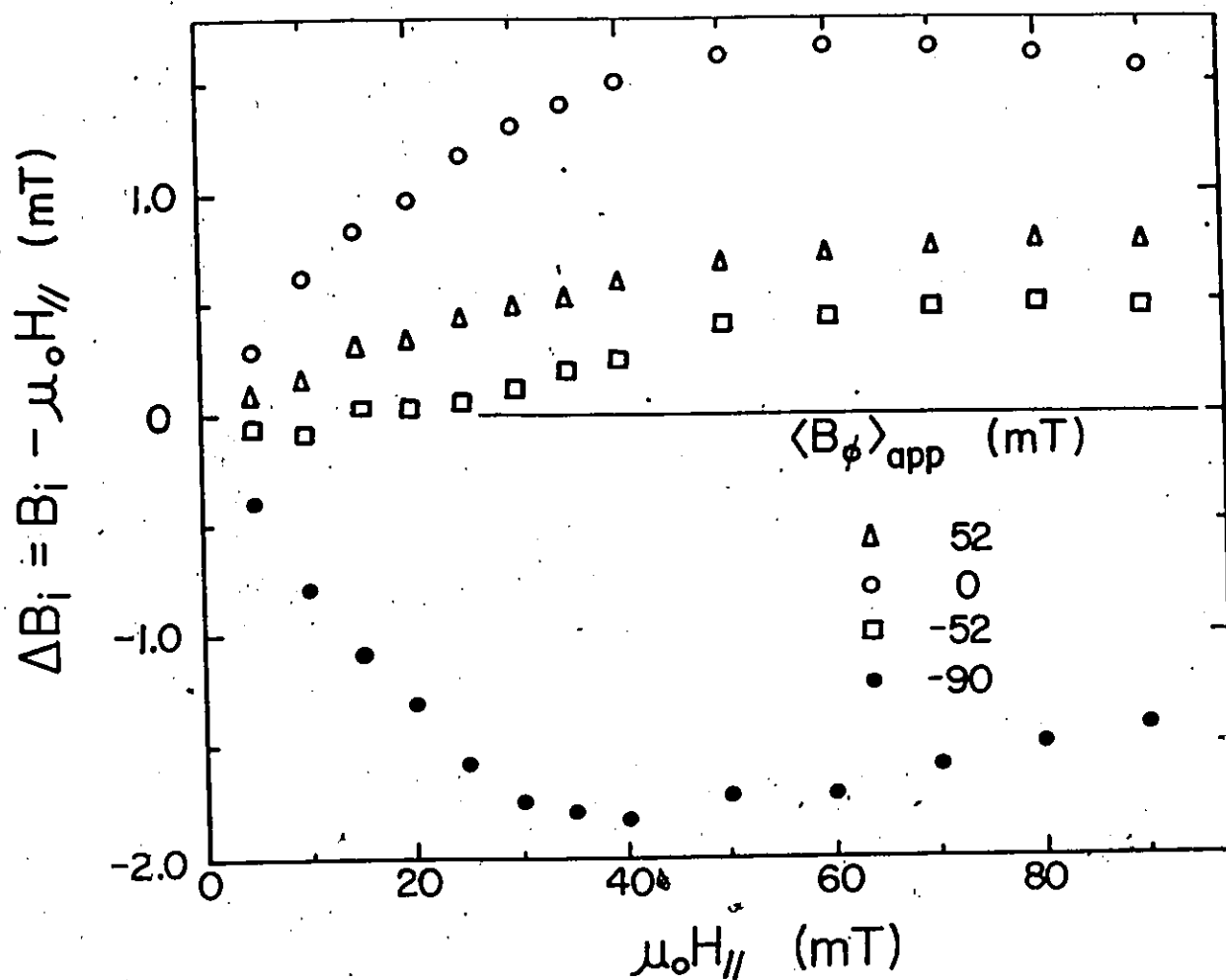


Fig. 7-5. Complements the previous figure by displaying data points taken along vertical cuts through the family of measured curves represented in that figure. The vertical cuts shown were taken at $\langle B_\phi \rangle_{app} = 52, 0, -52$ and -90 mT. The reason for these choices becomes evident from inspection of that figure.

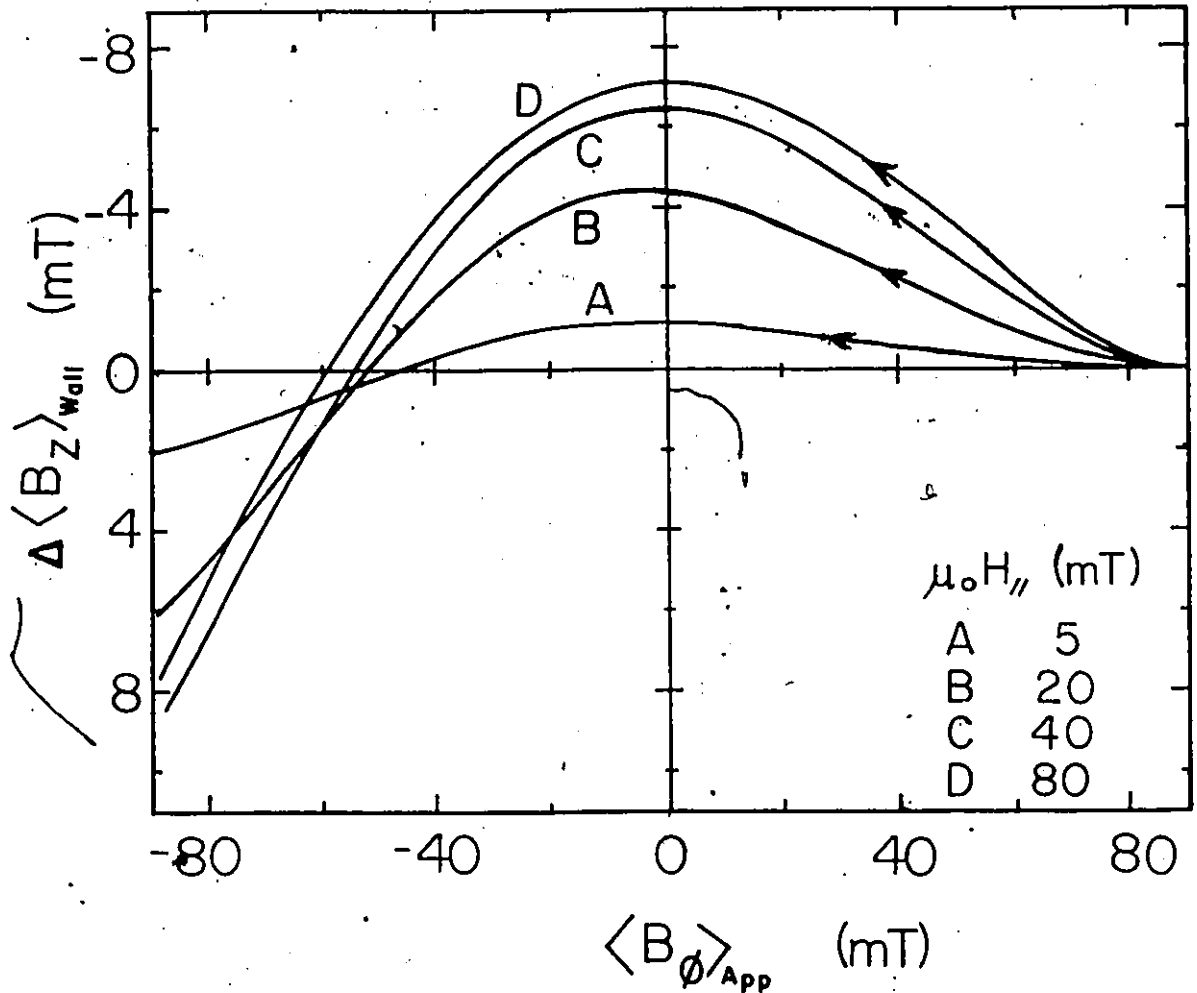


Fig. 7-6. The curves illustrate the evolution of the average longitudinal magnetic induction permeating the wall in various stationary externally applied $H_{||}$ as an applied azimuthal magnetic field is made to swing from a "positive" to a "negative" value. These data curves are constructed, as described in the text, from the family of measured curves presented in Figs. 7-2 and 7-4. Only four representative curves thus constructed are displayed to avoid clutter. The phenomenon that $\Delta\langle B_z \rangle_{\text{wall}}$ is negative in the first quadrant is called the diamagnetic longitudinal effect and constitutes strong evidence of flux cutting. The behaviour evolves from diamagnetic to paramagnetic as the sweep of $\langle B_\phi \rangle_{\text{app}}$ continues into the second quadrant. (See Fig. 4-3 for the analogous azimuthal effects).

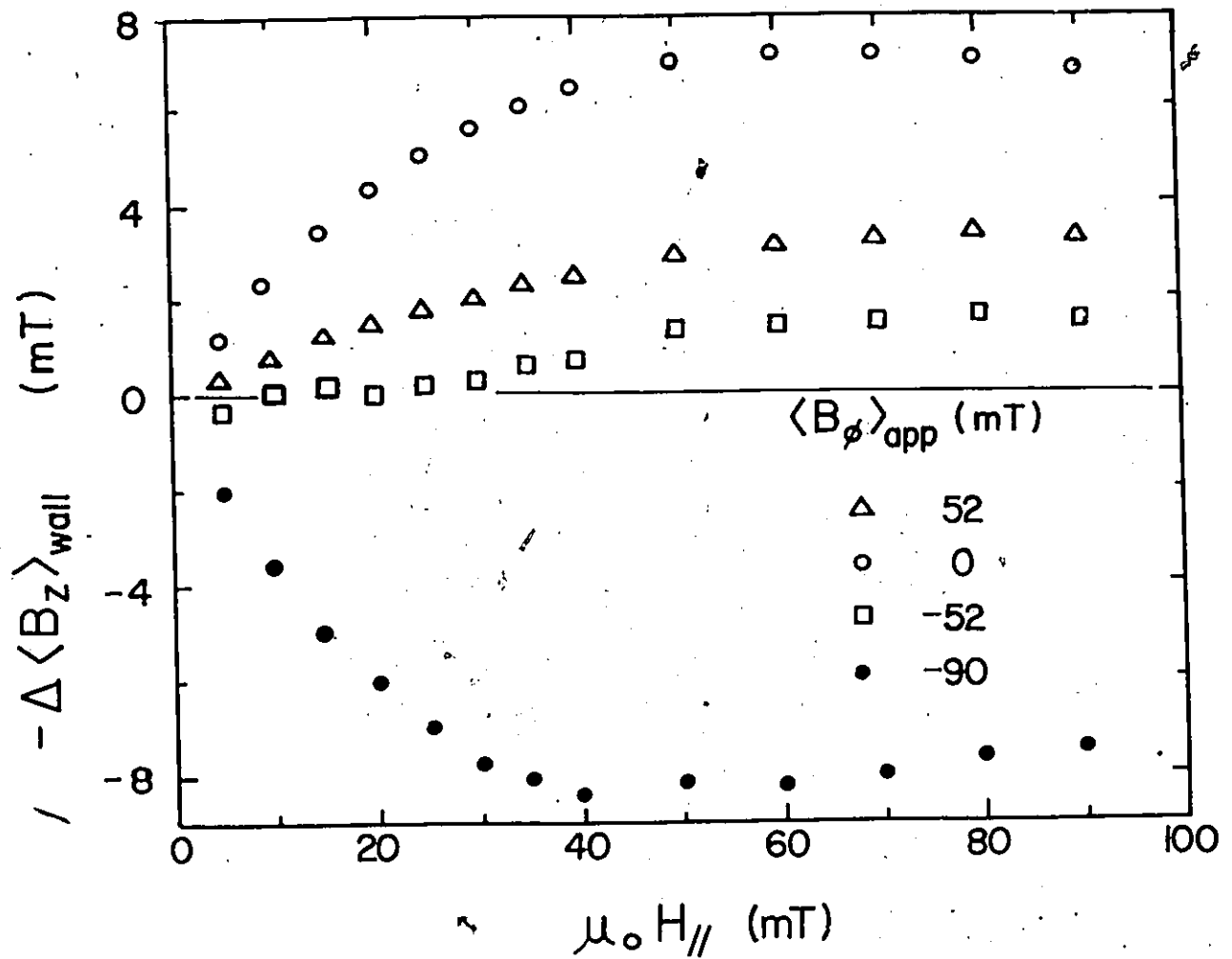


Fig. 7-7. Complements the previous figure by displaying data points taken along vertical cuts through the family of curves represented in that figure. The vertical cuts shown were taken at $\langle B_\phi \rangle_{\text{app}} = 52, 0, -52$ and -90 mT. The reason for these choices should become clear from inspection of that figure.

Chapter 8

The General Critical State Model

Introduction

Our measurements monitor the evolution of the average magnetic induction along the z and ϕ coordinates of a hollow cylinder as $H_{//}$ is varied with H_{ϕ} kept fixed or vice versa. These data readily provide some insight into the general patterns of the persistent currents which then circulate longitudinally and azimuthally. Vis à vis the physical mechanisms, the important quantities, however, are $j_{\perp}$ and $j_{//}$, the current densities flowing perpendicular and parallel to the local flux density $\vec{B}(r)$. The appropriate frame of reference then is the vector $\vec{B}$. The direction of this vector, however, varies with radial position in the specimen, hence with respect to the sample or laboratory coordinates. We will refer to the latter as global coordinates and to the frame of reference defined by the vector $\vec{B}$ as the local coordinates. The realization that Maxwell's equations should be expressed in the local coordinates rather than the global coordinates constitutes the crucial first step of the theory developed by Clem and which we now describe.

Maxwell's Equations in the Frame of Reference of $\vec{B}(r,t)$.

With reference to the cylindrical coordinates $\hat{\phi}$ and $\hat{z}$ of the specimen, the magnetic flux density can be written,

$$\vec{B}(r,t) = \hat{\phi} B_{\phi}(r,t) + \hat{z} B_z(r,t) \quad (1)$$

Alternatively, visualizing a unit vector $\hat{\alpha}$, locally parallel to the flux density $\vec{B}$, we write

$$\vec{B} = \hat{\alpha} B(r, t) \quad (2)$$

where, the unit vector,

$$\hat{\alpha} = \hat{\phi} \sin \alpha + \hat{z} \cos \alpha \quad (3)$$

is constructed by combining the components of the unit vectors $\hat{\phi}$ and $\hat{z}$ along $\hat{\alpha}$, and defines this unit vector with respect to the global coordinates. Another unit vector $\hat{\beta}$, orthogonal to $\hat{\alpha}$ and $\hat{r}$ can be defined by the vector product,

$$\hat{\beta} = \hat{\alpha} \times \hat{r} \quad (4)$$

Hence,

$$\hat{\beta} = \hat{\phi} \cos \alpha - \hat{z} \sin \alpha \quad (5)$$

The converse transformation relations can be obtained by multiplying equations 3 and 4 by $\sin \alpha$ (or $\cos \alpha$), combining and making use of the well known trigonometric relation, $\sin^2 \alpha + \cos^2 \alpha = 1$. This yields,

$$\hat{\phi} = \hat{\alpha} \sin \alpha + \hat{\beta} \cos \alpha \quad (6)$$

$$\hat{z} = \hat{\alpha} \cos \alpha - \hat{\beta} \sin \alpha \quad (7)$$

The electric field vector $\vec{E}(r, t)$ reads

$$\vec{E} = \hat{\phi} E_{\phi} + \hat{z} E_z \quad (8)$$

in global coordinates and,

$$\vec{E} = \hat{\alpha} E_{//} + \hat{\beta} E_{\perp} \quad (9)$$

in local coordinates where $E_{//}$ denotes the component of $\vec{E}$ along $\hat{\alpha}$, hence along $\vec{B}$, and $E_{\perp}$, the component perpendicular to $\vec{B}$. Introducing equations 6 and 7 into equation 8 leads to,

$$\vec{E} = \hat{\alpha}(E_{\phi} \sin \alpha + E_z \cos \alpha) + \hat{\beta}(E_{\phi} \cos \alpha - E_z \sin \alpha) \quad (10)$$

In view of equation 9, the component $E_{//}$ and $E_{\perp}$ then read,

$$E_{//} = E_{\phi} \sin \alpha + E_z \cos \alpha \quad (11)$$

$$E_{\perp} = E_{\phi} \cos \alpha - E_z \sin \alpha \quad (12)$$

Alternatively, introducing equations 3 and 5 into equation 9

leads to,

$$\vec{E} = \hat{\phi}(E_{//} \sin \alpha + E_{\perp} \cos \alpha) + \hat{z}(E_{//} \cos \alpha - E_{\perp} \sin \alpha) \quad (13)$$

In view of equation 8, the components E_{ϕ} and E_z then read,

$$E_{\phi} = E_{//} \sin \alpha + E_{\perp} \cos \alpha \quad (14)$$

$$E_z = E_{//} \cos \alpha - E_{\perp} \sin \alpha \quad (15)$$

The components of the Maxwell-Faraday equation,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (16)$$

in global cylindrical coordinates read,

$$\frac{\partial E_{\phi}}{\partial r} + \frac{E_{\phi}}{r} = -\frac{\partial B_z}{\partial t} \quad (17)$$

and

$$\frac{\partial E_z}{\partial r} = \frac{\partial B_{\phi}}{\partial t} \quad (18)$$

We note that,

$$B_z = B \cos \alpha, \quad B_{\phi} = B \sin \alpha \quad (19)$$

Introducing these expressions as well as equations 14 and 15

into equation 17 leads to,

$$\begin{aligned} \sin \alpha \frac{\partial E_{//}}{\partial r} + \cos \alpha \frac{\partial E_{\perp}}{\partial r} + (E_{//} \cos \alpha - E_{\perp} \sin \alpha) \frac{\partial \alpha}{\partial r} \\ + \frac{E_{//} \sin \alpha + E_{\perp} \cos \alpha}{r} = -\cos \alpha \frac{\partial B}{\partial t} + B \sin \alpha \frac{\partial \alpha}{\partial t} \end{aligned} \quad (20)$$

Now, introducing equations 19, 14 and 15 into equation 18 yields,

$$\begin{aligned} \cos \alpha \frac{\partial E_{//}}{\partial r} - \sin \alpha \frac{\partial E_{\perp}}{\partial r} - (E_{//} \sin \alpha + E_{\perp} \cos \alpha) \frac{\partial \alpha}{\partial r} \\ = \sin \alpha \frac{\partial B}{\partial t} + B \cos \alpha \frac{\partial \alpha}{\partial t} \end{aligned} \quad (21)$$

Multiplying equation 20 by $-\cos \alpha$ and equation 21 by $\sin \alpha$

and combining leads to,

$$\frac{\partial B}{\partial t} = \frac{-\partial E_{\perp}}{\partial r} - E_{\perp} \frac{\cos^2 \alpha}{r} - E_{//} \left(\frac{\partial \alpha}{\partial r} + \frac{\sin \alpha \cos \alpha}{r} \right) \quad (22)$$

Multiplying equation 20 by $\sin \alpha$ and equation 21 by $\cos \alpha$ and combining leads to,

$$B \frac{\partial \alpha}{\partial t} = \frac{\partial E_{//}}{\partial r} + E_{//} \frac{\sin^2 \alpha}{r} - E_{\perp} \left(\frac{\partial \alpha}{\partial r} - \frac{\sin \alpha \cos \alpha}{r} \right) \quad (23)$$

The components of the Maxwell-Ampère equation,

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad (24)$$

in global cylindrical coordinates read,

$$\frac{-\partial B}{\partial r} z = \mu_0 j_{\phi} \quad (25)$$

$$\frac{\partial B}{\partial r} \phi + \frac{B}{r} \phi = \mu_0 j_z \quad (26)$$

In analogy with equations 11, 12, 14 and 15 or proceeding as we have done in obtaining these expressions, we write,

$$j_{//} = j_{\phi} \sin \alpha + j_z \cos \alpha \quad (27)$$

$$j_{\perp} = j_{\phi} \cos \alpha - j_z \sin \alpha \quad (28)$$

$$j_{\phi} = j_{//} \sin \alpha + j_{\perp} \cos \alpha \quad (29)$$

$$j_z = j_{//} \cos \alpha - j_{\perp} \sin \alpha \quad (30)$$

where $j_{//}$ and $j_{\perp}$ are components of the current density $\vec{j}$, along $\hat{\alpha}$ and $\hat{\beta}$, hence parallel and perpendicular to $\vec{B}(r,t)$. Introducing equations 19, 29 and 30 into equation 25 yields,

$$B \sin \alpha \frac{\partial \alpha}{\partial r} - \cos \alpha \frac{\partial B}{\partial r} = \mu_0 (j_{//} \sin \alpha + j_{\perp} \cos \alpha) \quad (31)$$

Now introducing equations 19, 29 and 30 into equation 26 yields,

$$B \cos \alpha \frac{\partial B}{\partial r} + \sin \alpha \frac{\partial B}{\partial r} + \frac{B \sin \alpha}{r} = \mu_0 (j_{//} \cos \alpha - j_{\perp} \sin \alpha) \quad (32)$$

Multiplying equation 31 by $\sin \alpha$ and equation 32 by $\cos \alpha$ and combining yields,

$$\mu_0 j_{//} = B \left(\frac{\partial \alpha}{\partial r} + \frac{\sin \alpha \cos \alpha}{r} \right) \quad (33)$$

Multiplying equation 31 by $\cos \alpha$ and equation 32 by $-\sin \alpha$ and combining yields,

$$\mu_0 j_{\perp} = -\frac{\partial B}{\partial r} - \beta \frac{\sin^2 \alpha}{r} \quad (34)$$

Equations 22, 23, 33 and 34 constitute general formulations of the two pertinent Maxwell's equations for an infinite cylinder in the frame of reference of the flux density. The appropriate expressions for infinite slab geometry can readily be obtained by letting r go to infinity.

Since $\alpha + \beta = \pi/2$, then

$$\frac{\partial \alpha}{\partial t} = -\frac{\partial \beta}{\partial t} \quad \text{and} \quad \frac{\partial \alpha}{\partial r} = -\frac{\partial \beta}{\partial r} \quad (35)$$

Substituting the latter into equations 22, 23, 33 and 34 expresses these in terms of the complementary angle β .

Application of the Clem Equations to Type II Superconductors.

We refer to equations 22, 23, 33 and 34 as the Clem equations and now examine the application of these general formulae to phenomena encountered in type II superconductors subjected to the noncollinear regime.

Firstly we note that the formulae incorporated the "free space" relation,

$$\vec{B} = \mu_0 \vec{H} \quad (36)$$

hence, these do not explicitly take equilibrium (intrinsic or Abrikosov) diamagnetism into account. We will therefore proceed, in this discussion and in the next chapter, in a framework where the role of $B_{eq}(H)$ is taken as negligible and regard equation 36 as an acceptable approximation.

The effect of flux pinning is introduced phenomenologically by postulating the existence of a critical current density $j_{cl}(B, T)$ perpendicular to $\vec{B}$ which characterizes the chosen material. When $j_{\perp} > j_{cl}$, the Lorentz driving force exceeds the pinning forces and the flux lines are made to migrate, thereby generating an electric field,

$$E_{\perp} = v_{\perp} B = \rho_{\perp} (j_{\perp} - j_{cl}) \quad (37)$$

where $v_{\perp}$ is the flux line velocity perpendicular to the flux density and $\rho_{\perp}$ is denoted the flux flow resistivity. So far then, the Clem theory builds on the critical state concept enshrined in the literature for two decades and exploited in hundreds of papers addressing phenomena in the collinear regime. In our work we confine our attention to sequences of quasi-static configurations of $\vec{j}$ and $\vec{B}$, hence we will henceforth limit our discussion to flux transport critical state situations where $j_{\perp} = j_{cl}$. Equation 37 accordingly plays no explicit role in our analysis. The electric field $E_{\perp}$ which arises as the B profile changes from one metastable configuration to another, is obtained from Maxwell's equation as formulated above by Clem. In harmony with established views, the first ansatz of the Clem theory is that zones can appear in the specimen where,

$$E_{\perp} \neq 0, \quad j_{\perp} > j_{cl}, \quad \frac{\partial B}{\partial t} \neq 0 \quad (38)$$

These are denoted pure flux transport regions (T zones). Since the displacement of flux lines is a dissipative process,

$$E_{\perp} \text{ must be in the same direction as } j_{cl} \quad (39)$$

The effect of flux cutting is introduced phenomenologically by postulating the existence of a critical current density $j_{c//}(B, T)$ flowing parallel to $\vec{B}$ which characterizes the chosen material. When

$j_{//} > j_{c//}$, the process of flux cutting takes place, thereby generating an electric field $E_{//}$ directed along the local flux density $\vec{B}$. Any consumption of flux which ensues from the operation of flux cutting is embodied in and emerges from the pertinent formulae (equations 22 and 23). Consequently, the detailed mechanisms of flux cutting and flux consumption play no direct role in the implementation of the theory. A flux cutting resistivity $\rho_{//}$ can be introduced via the relation

$$E_{//} = \rho_{//} (j_{//} - j_{c//}) \quad (40)$$

We will, however, not pursue this important equation further since in our work we have dealt only with sequences of quasi-static configurations where $j_{//}$ is only infinitesimally greater than $j_{c//}$ when the changes in the externally applied magnetic field are taking place.

The crucial ansatz of the Clem theory can then be summarized in the following statement. Regions can appear in the specimen in the noncollinear regime where,

$$E_{//} \neq 0, \quad j_{//} > j_{c//}, \quad \frac{\partial B}{\partial t} \neq 0, \quad \frac{\partial \alpha}{\partial t} \neq 0 \quad (41)$$

Two possibilities now arise.

a) Regions can exist where both flux cutting and flux transport are taking place. These are denoted CT zones. Consequently,

$$E_{//} \neq 0, \quad E_{\perp} \neq 0, \quad j_{//} > j_{c//}, \quad j_{\perp} > j_{c\perp}, \quad \frac{\partial B}{\partial t} \neq 0, \quad \frac{\partial \alpha}{\partial t} \neq 0 \quad (42)$$

b) Regions can exist where only flux cutting is occurring and no flux transport is taking place. These pure flux cutting regions are denoted C zones. Consequently,

$$E_{//} \neq 0, \quad E_{\perp} = 0, \quad j_{//} > j_{c//}, \quad 0 < j_{\perp} < j_{c\perp}, \quad \frac{\partial B}{\partial t} \neq 0, \quad \frac{\partial \alpha}{\partial t} \neq 0 \quad (43)$$

The first possibility had been envisaged, in an unsophisticated way, in the empirical double critical state model of LeBlanc

and his group. - The second situation constitutes a major departure from the "classical" critical state picture. The classical critical state concept, applicable to the treatment of the collinear regime, allowed for the occurrence of $0 < j_{\perp} < j_{c\perp}$ only in passive (inert) zones where $\partial B/\partial t = 0$. Using the prevailing parlance or jargon, one can say that the Clem theory allows for the generation of subcritical B profiles in active ($\partial B/\partial t \neq 0$ and/or $\partial \alpha/\partial t \neq 0$ regions).

Since flux cutting is a dissipative process, it is also required that $E_{//}$ have the same sign (i.e. the same direction) as $j_{//}$. The Clem theory consequently imposes a more stringent set of constraints than is normally applied usually to energy consuming interactions. In the latter case, it is generally only required that the angle between $\vec{E}$ and $\vec{j}$ be less than 90° .

It is useful to return now to the pure flux transport region (T zones) and examine in more detail the variety of situations which this can encompass. We can envisage a region permeated by planar or cylindrical sheets of flux lines where the orientation or helicity varies spatially with $0 < j_{//} < j_{c//}$. These flux lines can be made to undergo displacement, compression or decompression with no change in their orientation or helicity. This is the scheme proposed by Campbell and Evetts to account for noncollinear phenomena. In their scheme, however, no limit $j_{c//}$ is imposed on $j_{//}$. Consequently cutting-transport (CT) zones and, of course, pure cutting (C) zones do not occur in their approach. As a result, this model leads to unphysical situations and sharp disagreement with observations.

Finally we note that the solutions to the Clem equations must

satisfy the requirements that $E_{//}(r)$, $E_{\perp}(r)$ and $B(r)$ be continuous.

The Clem equations, together with the several constraints just enumerated, provide a versatile and powerful framework to predict new phenomena and to account for a vast assortment behaviour. In the next chapter we apply this theory to the analysis of some of our more salient observations on the magnetic response of a hollow cylinder in the noncollinear regime.

Chapter 9

Application of the Clem Theory

Introduction

In this chapter we exploit the general critical state theory (Clem theory) to describe our observations of the azimuthal paramagnetic effect presented in chapter 3 and of the azimuthal diamagnetic effect presented in chapter 4. We analyze the electromagnetic behaviour of the specimen in the context where the spatial and temporal variations of the magnetic flux density (B profiles) and of α , the orientation of the magnetic induction $\vec{B}(r,t)$ with respect to the z -axis (α profiles) consist of sequences of metastable configurations. Consequently, the "dynamic" equations,

$$E_{\perp} = \rho_{\perp} (j_{\perp} - j_{c\perp}), \quad E_{//} = \rho_{//} (j_{//} - j_{c//}) \quad (1)$$

do not enter into our analysis. Instead we write,

$$\frac{\partial B}{\partial t} = \frac{\partial B}{\partial B_s} \frac{dB_s}{dt}, \quad \frac{\partial \alpha}{\partial t} = \frac{\partial \alpha}{\partial \alpha_s} \frac{d\alpha_s}{dB_s} \frac{dB_s}{dt} \quad (2)$$

$$\vec{E}(r,t) = \vec{E}(r) \frac{dB_s}{dt} \quad (3)$$

where,

$$B_s = ((\mu_0 H_{//})^2 + B_{\phi}^2(R_o))^{\frac{1}{2}} \quad (4)$$

is the total externally applied magnetic induction at the outer surface of the cylinder, and,

$$\alpha_s = \sin^{-1} \left(\frac{B_{\phi}(R_o)}{B_s} \right) \quad (5)$$

is the direction of $\vec{B}_s$ with respect to the z -axis. We note that dB_s/dt then cancels out of the Clem formulae for $\partial B/\partial t$ and $\partial \alpha/\partial t$ given in the previous chapter.

We have developed a computer program which solves the coupled Clem equations with arbitrary analytic functions for $j_{cl}(B)$ and $j_{c//}(B)$ as $\mu_0 H_{//}$, hence B_s and α_s , are incremented (increased or decreased) with the azimuthal magnetic induction $B_\phi(R_0)$ and $B_\phi(R_1)$, at the outer and inner surfaces of the wall of a hollow cylinder, maintained stationary. The solutions consist firstly of sequences of $B(r)$ and $\alpha(r)$ profiles corresponding to the incremental evolution of B_s in unison with α_s . We stress that the latter are not independent in our experiments and in our analysis but are linked by equation 5 where $B_\phi(R_0)$ is held constant. The $B(r)$ and $\alpha(r)$ profiles must, of course, satisfy boundary conditions at both R_0 and R_1 . This feature, may appear rather pedestrian, at first glance, but it contains a most severe test for theories endeavouring to account for electromagnetic phenomena in the noncollinear regime.

It is appropriate, useful and indeed necessary to expand briefly on this last statement at this juncture. In all other experimental arrangements exploited previously to investigate noncollinear behaviour, namely; (i) ribbons in orthogonal externally applied magnetic fields, (ii) solid wires carrying a varying current in a stationary or varying $H_{//}$ and (iii) rotating or oscillating disks in a stationary or varying $\vec{H}$ directed along the flat surfaces, only one set of boundary conditions enter into the picture. These are B_s and α_s . The set B and α at the midplane of the ribbon or disk and at the axis of the wire are not only not measured but exercise no constraint on the allowed profiles. For the solid wire, symmetry requires that $\alpha = 0$ at the axis, but $B_z(r)$ at the axis is unconstrained. The absence

of a set of "internal" constraints allows the modelling of observations considerably more freedom to generate plausible configurations and to avoid the important verification of whether the values calculated for α and B at the inner boundary are correct.

In our geometry, besides satisfying the outer and inner boundary conditions, the solutions to Clem's equations must satisfy the physically imposed requirements that,

- (i) $B(r)$, $\alpha(r)$, $E_{\perp}(r)$ and $E_{//}(r)$ be continuous, and
- (ii) $E_{\perp}j_{\perp} > 0$ and $E_{//}j_{//} > 0$, hence $E_{\perp}$ has the same sign as $j_{\perp}$ and $E_{//}$ that of $j_{//}$.

In the process of solving Clem's equations, the computer programme must therefore compute sequences of $B(r)$, $\alpha(r)$, $E_{\perp}(r)$, $E_{//}(r)$, $j_{\perp}(r)$ and $j_{//}(r)$. To relate these results to observations, the $B(r)$ and $\alpha(r)$ profiles are converted to $B_z(r)$ and $B_{\phi}(r)$ profiles which are integrated over $R_0 - R_1$ to yield the spatial averages $\langle B_z \rangle$ and $\langle B_{\phi} \rangle$. It is also of interest to follow the evolution of the $j_z(r)$, $j_{\phi}(r)$, $E_z(r)$ and $E_{\phi}(r)$ profiles. Computer plots of these various data as the phenomena evolve provide fascinating insights.

Results and Discussion

A. Choice of j_{cl} and $j_{c//}$.

The determination of $j_{cl}(B)$ from magnetization or critical current curves is a standard exercise in the classical collinear regime. Curve A of Figs. 1, 2 and 6 of chapter 5 display a good approximation for j_{cl} versus B extracted from the magnetic response of the hollow cylinder to $H_{//}$ increasing or decreasing in the absence of

H_ϕ . For computational and theoretical convenience we select a simple analytic expression which provides a satisfactory description of j_{c1} , namely

$$j_{c1} = \frac{C}{B^{\frac{1}{2}}} \left(1 - \frac{B}{B_{c2}} \right) \quad (6)$$

where the coefficient $C = 3.65(10^7) \text{ A T}^{\frac{1}{2}}/\text{m}^2$. We have also taken $B_{c2} = 1 \text{ T}$. This choice is somewhat arbitrary, since our measurements do not extend to B_{c2} . The success of this prescription is reproducing the behaviour of the hollow cylinder to an increase or a decrease of $H_{//}$ in the collinear regime ($H_\phi = 0$) can be assessed by comparing curves A and A' in Figs. 9-1 and 9-7.

The phenomenon of particular interest in our work is the evolution of the azimuthal flux permeating the wall with H_ϕ kept fixed while $H_{//}$ is swept up or down. It is the behaviour of this quantity which signals and manifests the occurrence of flux cutting most dramatically. We stress that flux cutting occurs when $j_{//}$ attains and exceeds $j_{c//}$, however infinitesimally. As expected then, $j_{c//}$ will play a dominant role in dictating the locus of $\langle B_\phi \rangle$ versus $H_{//}$. A "correct" prescription for $j_{c//}(B)$ is consequently quite important in order that the model reproduce the intricate features and trends of the families of measured curves. The choice of $j_{c1}(B)$, however, also impacts on the onset and extent of flux cutting since $j_{//}$ and j_1 are coupled in the Clem equations in cylindrical geometry via their interdependence on $\alpha(r)$ and $B(r)$. Also, the latter are coupled in planar as well as in cylindrical geometry through the Maxwell-Ampère equation $\nabla \times \vec{E} = - \partial B / \partial t$. At any rate, having fixed on a choice for $j_{c1}(B)$ we now seek a suitable expression for $j_{c//}(B)$.

The estimates of $j_{//}$ from the magnetic response in the noncollinear regime obtained and described in chapter 5, yield only lower limits for $j_{c//}$ since $j_{//}$ appears to exist in a critical state over only a fraction of the wall thickness in the situations under scrutiny. Again, we have deemed it desirable to introduce a simple analytic function and relying on the data of chapter 5 and other information we have selected,

$$j_{c//} = \frac{D}{B} \left(1 - \frac{B}{B_{c2}} \right) \quad (7)$$

and set the parameter $D = 1.53(10^7) \text{ AT/m}^2$. We illustrate the effect of varying this parameter in Figs. 3 and by a comparison of Fig. 8 with Fig. 10. It is evident that the azimuthal paramagnetic effect ($H_{//}$ increasing) is significantly more sensitive to the choice of this parameter than the azimuthal diamagnetic effect ($H_{//}$ decreasing). In the course of our calculations we have also examined the effect of increasing or decreasing the power of B in the denominator of equation 7. We find that some degree of latitude arises in this respect. This arbitrariness or uncertainty could be expressed by writing B^n with $n = 1 \pm \frac{1}{2}$. For each choice of n , there corresponds, of course, an "optimum" value for the parameter D .

The outer and inner radii, R_0 and R_1 , introduced in the calculations are those of the sample.

B. Evolution of $B(r)$ and $\alpha(r)$ for $H_{//}$ increasing.

Although equations 22, 23, 33 and 34 of chapter 8 were written in terms of $\alpha(r)$, the angle subtended between $\vec{B}$ and the z axis, it is more convenient in our case to portray the behaviour of its

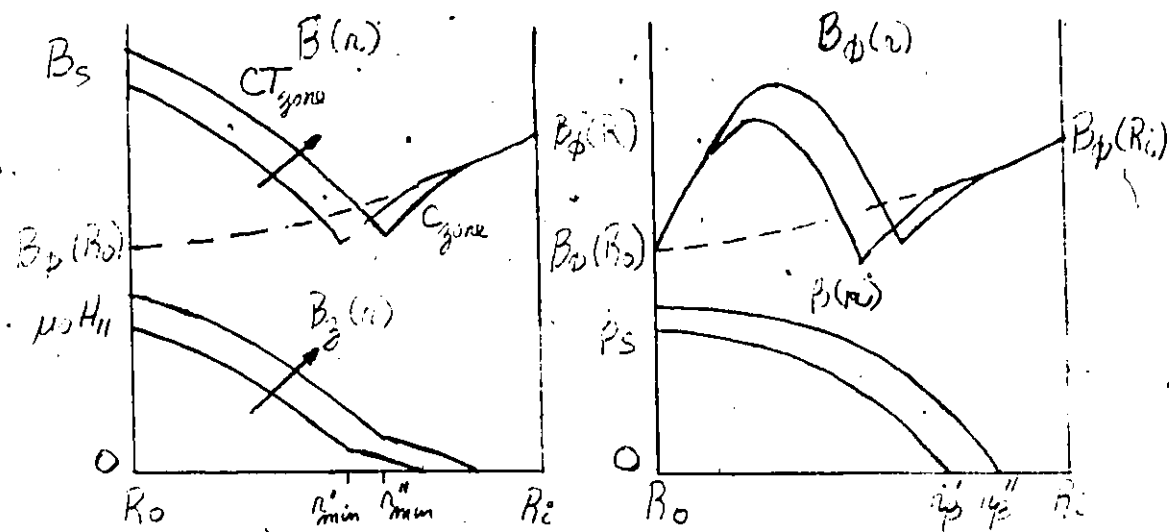
complementary angle $\beta(r) = \pi/2 - \alpha(r)$, subtended by $\vec{B}$ and the ϕ axis.

In the calculations B_s has usually been incremented in steps of 25 mT over the range $B_\phi(R_0) < B_s < 200$ mT, and in steps of 50 mT above 300 mT. The wall is divided into 1000 elements. The results we compute are insensitive to a diminution of the increments for B_s to 10 mT and an increase of the spatial grid to 4,000 elements changed the calculated $\Delta\langle B_\phi \rangle$ by $\leq 1\%$.

Figs. 2 and 3 display the evolution of $\Delta\langle B_\phi \rangle$, the change of $\langle B_\phi \rangle$ from the initial value, $\langle B_\phi \rangle_{app}$, the static azimuthal magnetic induction in which the sample become superconducting, versus $H_{//}$ increasing from zero. Fig. 2 shows the effect of different static applied azimuthal magnetic fields on the behaviour. Fig. 3 illustrates the sensitivity of the calculated curves to the choice of the parameter D controlling the magnitude of $j_{c//}$. To assist the reader in comparing these calculated curves with actual observations we have reproduced corresponding experimental curves from Chapter 3 in Fig. 4.

It is evident that the overall agreement between the calculated and the measured $\Delta\langle B_\phi \rangle$ versus $H_{//}$ curves is disappointing. Nevertheless it is encouraging to note that the calculations successfully generate some major features of the families of intricate data curves:

- (i) An initial shallow diamagnetic (pure flux cutting) valley.
- (ii) A steep rise mapping out a paramagnetic peak.
- (iii) A rapid descent and,
- (iv) A subsequent gradual and monotonic rise to the paramagnetic region after the abrupt descent, hence the locus of $\Delta\langle B_\phi \rangle$



versus $H_{//}$ traces a second wide valley.

We examine each of these features and their physical implications in succession.

(i) Initial Diamagnetic Valley.

The depth and breadth of this valley is determined by the chosen ratio for $j_{c//}(B)/j_{cl}(B)$ over the low range of B . This valley can become very pronounced when $j_{c//}$ and j_{cl} are chosen to be comparable over this range. With our choice of the parameters C and D in equations 6 and 7 for j_{cl} and $j_{c//}$, the depth of the valley is relatively small compared with the height of the subsequent peak since in our case $j_{c//}$ is appreciably larger than j_{cl} over the applicable range of B . We note that there $j_{c//} > j_{cl}$ when $B < (D/C)^{1/2} = 17.6$ mT.

An examination of the sequences of $B(r)$, $\beta(r)$ (instead of $\alpha(r)$), $E_{\perp}(r)$ and $E_{//}(r)$ profiles, shows clearly that this diamagnetic valley arises from the appearance of a pure flux cutting zone in the vanguard section of the advancing B and β profiles. This C zone precedes an expanding CT zone. The C zone is evident in the schematic sketches above which display a typical sequence of $B(r)$, $B_\phi(r)$, $B_z(r)$

and $\beta(r)$ profiles associated with the evolution of this diamagnetic valley. We note that the front of the "new" $B(r)$ and $B_\phi(r)$ profiles dip below the corresponding "old" profiles. The movement of all the profiles is towards the right in all these sketches since the penetration progresses from the outer radius to the inner radius as $H_{//}$ increases. The C zone occupies the space, $|r'_\beta - r'_{\min}|$ (or $|r''_\beta - r''_{\min}|$), between the front of the leading $\beta(r)$ profile and the minimum in the $B(r)$ profile. In this zone, $E_\perp = 0$, $0 < j_\perp < j_{cl}$ and flux is being consumed by pure flux cutting.

The trench gouged in the $B_\phi(r)$ profile by the flux cutting, initially widens and deepens. As the advance progresses, these two trends cease, albeit independently, and reverse. The contour of the diamagnetic valley in $\Delta\langle B_\phi \rangle$, is determined, however, by the competition between two concurrent processes. These are, (i) the growth in the rate of destruction of azimuthal flux by pure flux cutting in the C zone and, (ii) the rate of growth of the "bulge" in the azimuthal flux in the CT zone, occupying the space between $r'_{\min}$ (or $r''_{\min}$) and the outer radius R_0 . In the latter zone, although consumption of azimuthal flux may be occurring, this process is overwhelmed by the continuous entry of "fresh" azimuthal flux at the outer surface.

The evolution of the characteristics of this diamagnetic valley which emerges from the calculations is in good agreement with observations. Specifically, (i) the depth of the valley becomes greater as the static $\langle B_\phi \rangle_{app}$ is chosen larger (see Fig. 5) and (ii) the valley for a chosen $\langle B_\phi \rangle_{app}$ becomes shallower and disappears quickly as the initial $H_{//}$ is chosen larger. This diamagnetic valley has vanished when $H_{//1} \approx 100$ mT.

We stress that this valley carries the clear signature of flux cutting events.

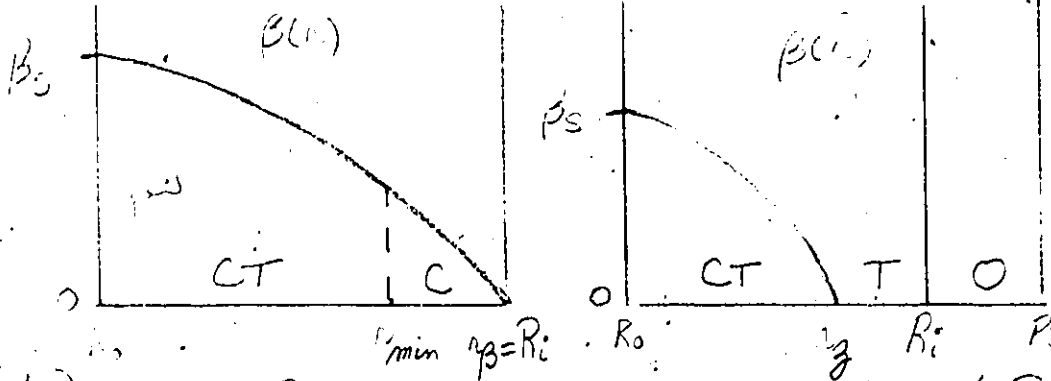
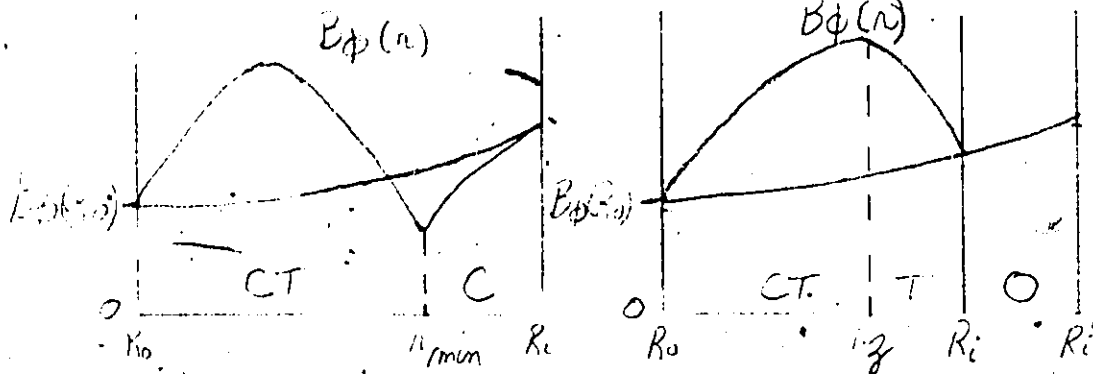
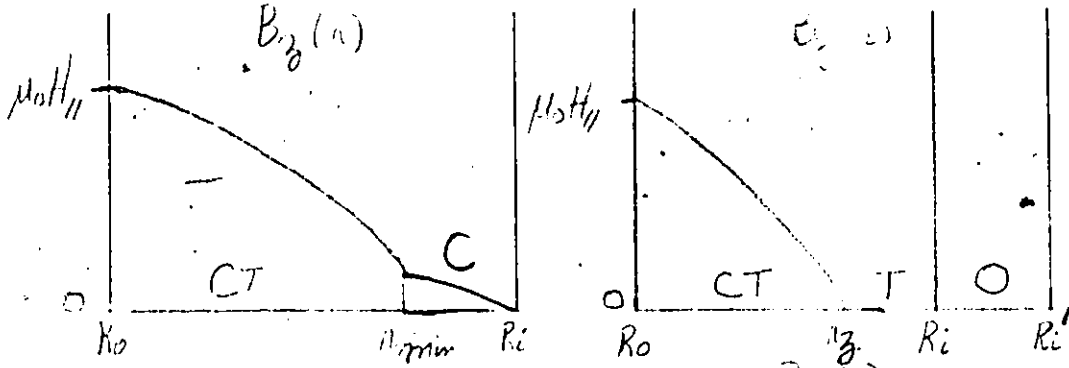
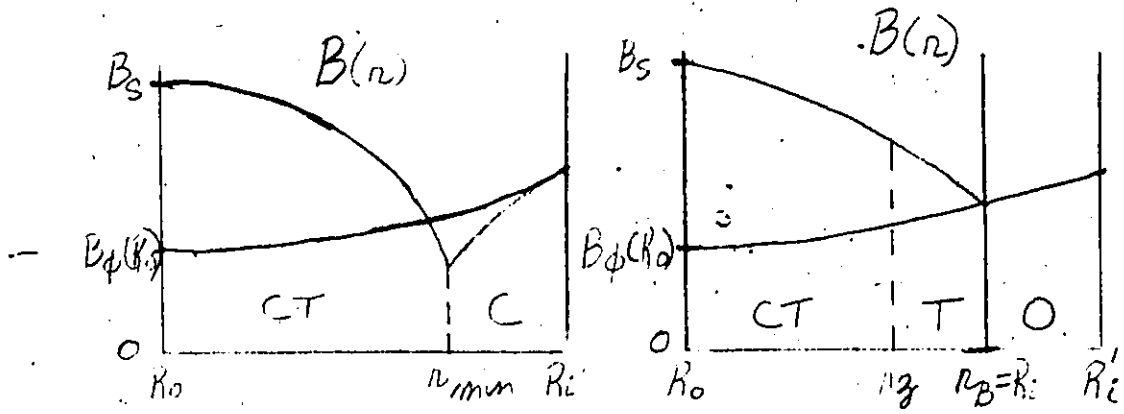
(ii) Growth of the Paramagnetic Peak.

Depending on the parameters, $j_{cl}(B)$ and $j_{c//}(B)$ and the initial conditions (i.e. the choice of $\langle B_\phi \rangle_{app}$ and $H_{//}$ in which the hollow cylinder becomes superconducting), two situations or modes of behaviour arise.

A) The C zone, although diminishing in extent, after its initial growth, during its advance, has survived throughout the transit of the $\beta(r)$ profile across the wall. This situation is illustrated in Fig.(A) of the sketches on page 143.

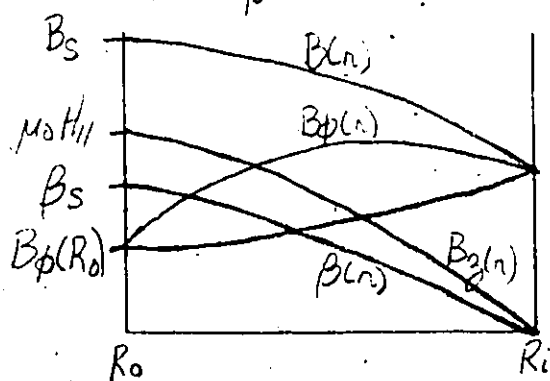
B) The C zone has shrunk in breadth during progress across the wall and has vanished before the front of the $B(r)$ profile attains the inner radius. This situation is displayed schematically in Fig. (B) where it can be compared to and contrasted with the former case.

We encounter both of these regimes in our analysis of our data depending on the strength of the static applied azimuthal field H_ϕ and the choice of the initial $H_{//}$. For $H_{//}$ increasing from zero case (B) occurs with $\langle B_\phi \rangle_{app} < 30$ mT and case (A) with $\langle B_\phi \rangle_{app} > 30$ mT. With $\langle B_\phi \rangle_{app} \approx 30$ mT, the disappearance of the diminishing C zone coincides with the arrival of the fronts of the $B(r)$, $B_z(r)$, $B_\phi(r)$ and $\beta(r)$ profiles at the inner radius. This therefore constitutes a special transition case at the boundary between the two regimes just described and is displayed schematically in sketch (A/B)-on page 143. Here, the rate of advance of the minimum in the $B(r)$ profile, although



(A)

(B)



(A | B)

slower at the outset than that of the $\beta(r)$ profile, has experienced an acceleration during the transit and $r_{\min}$ of the $B(r)$ profile has just caught up in its race with r_β , the front of the leading $\beta(r)$ profile as they reach the R_1 boundary. The transition from one mode of behaviour to the other is not abrupt, however, but occurs over a small range of stationary $\langle B_\phi \rangle_{\text{app}}$.

We now examine the behaviour in regime (B) before the fronts of the disturbance in the $B(r)$ and $B_\phi(r)$ profiles have attained the inner radius. The set of sketches (B) can still serve to display the pertinent profiles and types of zones if the boundary labelled R_1 in the sketches is viewed as indicating an interface within the wall, and R'_1 is now regarded as the applicable inner radius. The space between R_1 and R'_1 is now viewed as an inert or passive region (O zone) where no changes in the flux and current configurations are taking place ($E_1 = E_{//} = 0$). The crucial feature is that r_β , the front of the advancing $B(r)$ profile leads r_z , the front of the $B_z(r)$ profile. The differences between two possible cases, with respect to the space between r_β and r_z , need to be brought out, namely where, (i) $H_{//1} = 0$ and, (ii) $H_{//1} \neq 0$. In case (i) pure azimuthal flux is undergoing compression in this region, hence $j_{//} = 0$, $\beta(r) = 0$. In case (ii) initially $j_{//} = 0$ but $\beta(r) \neq 0$. Helical flux is being compressed in this region, hence $j_{//}$ and $\beta(r)$ are changing as the situation evolves but $j_{//}$ remains less than $j_{c//}$. In both cases flux is being displaced radially inwards, hence $E_1 \neq 0$ but $E_{//} = 0$. This is consequently a pure flux transport region (T zone).

This T zone appears immediately after the shrinking C zone

has vanished and now expands as its two boundaries (r_z and r_B) advance towards the inner radius. It is of considerable interest to note that at the "instant" the C zone vanishes and the T zone is born, the $B(r)$ and $\beta(r)$ profiles both exist in critical CT states over the entire active expanse of the wall. This is a unique configuration in the sequence of configurations where the profiles can, in principle, be readily determined by solving the two critical state equations, only,

$$-\mu_o j_{cl}(B) = \frac{\partial B}{\partial r} + B \frac{\sin^2 \alpha}{r} \quad (8)$$

$$\mu_o j_{c//}(B) = B \left(\frac{\partial \alpha}{\partial r} + \frac{\sin \alpha \cos \alpha}{r} \right) \quad (9)$$

for any appropriate $H_{//}$, $H_{//1}$ and $H_\phi(R_o)$. We stress that here, the fronts of the $B(r)$, $B_z(r)$ and $\beta(r)$ profiles must coincide, hence the "inner" boundary conditions are prescribed.

This crucial occurrence provides a "standard" or checkpoint for verifying and comparing the evolution of the outputs of different computer programmes which may be devised to solve the coupled Clem equations. For corresponding choices of $H_{//}$, $H_{//1}$ and $H_\phi(R_o)$, all solutions must agree at this juncture. In general the solution of the Clem equations appears to require a sequential procedure where previous solutions must first be determined in succession starting from a known state of affairs. The occurrence of this double critical state configuration may thus also provide a useful intermediate starting point for computation.

The T zone which replaces the vanished C zone at the vanguard of the $B(r)$ and $\beta(r)$ profiles, now expands as its boundaries advance across the wall. In other words, r_B progresses more rapidly than r_z .

(When $H_{//1} \neq 0$, the $B(r)$ profile rises but remains subcritical in the T zone occupying the space between r_B^* and r_2^*). $E_{//}$ and $j_{//}$ in the vicinity of the front of the CT zone rise considerably in magnitude as the growth of the T zone takes place. Meanwhile, in the vicinity of R_0 , the outer radius, $E_{\perp}$ is rising in intensity whereas $|E_{\chi//}|$ is diminishing to zero. Now, either, (i) the front of the T zone reaches the inner radius R_1 before $E_{//}$ becomes zero at the outer radius, or (ii) $E_{//}$ becomes zero at the outer radius before the front of the T zone reaches R_1 . In the first eventuality, the summit of the paramagnetic peak has been reached and a decline in $\Delta\langle B \rangle$ now ensues. In the second instance, our computer programme fails to find further solutions to the Clem equations. The trend of the sequence of $E_{//}(r)$ profiles in the vicinity of R_0 indicates that a T zone should now appear and develop adjacent to R_0 . Presumably, simultaneously with this, the T zone at the front of the B profile should begin to shrink. Our programme, however, is now unable to generate any profiles which satisfy the several constraints and boundary conditions.

The reason for this failure appears to reside in the physics of the method we have elected to exploit. In our approach, as indicated earlier, the sequences of $B(r)$ and $\beta(r)$ profiles are taken to be quasi-stationary or metastable configurations where $j_{//} \leq j_{c//}$ and $j_{\perp} \leq j_{c\perp}$. In other words we have not allowed for situations where temporarily, $j_{//}$ (or $j_{\perp}$) can significantly exceed and relax to $j_{c//}$ (or $j_{c\perp}$). As a consequence, our technique is unable to cope with the situations which we now encounter. Here as the T zone, at the front of the B profile, grows, $|E_{//}|$ is seen to become very large over part of

the CT zone and also $|E_{\perp}|$ near the outer radius. These strong electric fields will assuredly generate current densities which make transient excursions beyond the critical thresholds. In the light of the failure of our approach in these circumstances, we are developing a computer program which allows for such excursions by incorporating the pertinent prescriptions, $E_{\perp} = \rho_{\perp} (j_{\perp} - j_{cl})$ and $E_{//} = \rho_{//} (j_{//} - j_{cl})$, which we had set aside. The results of this effort, however, are beyond the scope of this thesis.

Unfortunately, the range of the upswing of $H_{//}$ where our approach produces solutions, diminishes as the chosen static $\langle B_{\phi} \rangle_{app}$ is taken to be smaller. We are consequently unable to trace out the full growth of the paramagnetic peak when $\langle B_{\phi} \rangle_{app} < 30$ mT. Hence we cannot compare predictions of the theory with observations for $\Delta \langle B_{\phi} \rangle$ versus $H_{//}$ over the locus of the entire first paramagnetic peak when $\langle B_{\phi} \rangle_{app} < 30$ mT. Further, our calculated curves presumably begin to deviate from the "correct" results before the catastrophic computational limit is encountered. Nevertheless, the evolution of $\Delta \langle B_{\phi} \rangle$ versus $H_{//}$ for various $\langle B_{\phi} \rangle_{app} < 30$ mT reveals good qualitative agreement with observations over the corresponding ranges of $H_{//}$ increasing. As noted above, the unique configuration encountered upon the disappearance of the C zone and the birth of a T zone at the front of the advancing CT profile, provides an intermediate test and check. We can consequently be confident that our results are reliable and valid up to this juncture. Faith in the reliability of the output of our program becomes less certain beyond this crucial event.

(iii) The Descent from the Paramagnetic Summit.

As noted in the previous section, two configurations can exist when the inner surface of the wall comes into play. We refer the reader to the sketches on page 143. In (A), a C zone preceding the CT zone reaches R_1 , while in (B), a T zone preceding the CT zone reaches R_1 . As a consequence, an increment in B_S , hence β_S , however minute, must now cause $B_z(R_1)$ to increase, hence the longitudinal flux density in the hole to rise, regardless of which of the two situations exists. (Surface barriers are ignored and, in any case, do not alter the eventual state of affairs).

To appreciate that B_z must rise at R_1 in both sets of circumstances it is useful to consider each case separately. In (A), the $\beta(r)$ profile is in a critical state over the entire wall. Consequently, a rise in β at the outer surface, must lead to a rise in β at the inner radius. Since,

$$\frac{B_z(R_1)}{B_\phi(R_1)} = \tan \beta(R_1) \quad (10)$$

and $B_\phi(R_1)$ is stationary, clearly then, this requires $B_z(R_1)$ to increase. In (B), the $B(r)$ profile is in a critical state over the entire wall. Consequently, an increase in B at the outer surface, must lead to an increase in B at the inner radius, hence an increase in $B_z(R_1)$ since,

$$B(R_1) = (B_z^2(R_1) + B_\phi^2(R_1))^{1/2} \quad (11)$$

and $B_\phi(R_1)$ is fixed. Further, in view of equation 10, $\beta(R_1)$ must consequently also rise. We now consider both regimes separately.

(I). CT zone is initially preceded by a T zone which extends to R_1 (Fig.(B) on page 143).

A T zone separates the front of the $B_z(r)$ and $\beta(r)$ profiles

from R_1 . We consider two sub-cases separately, (a) $H_{//1} = 0$, and (b) $H_{//1} \neq 0$.

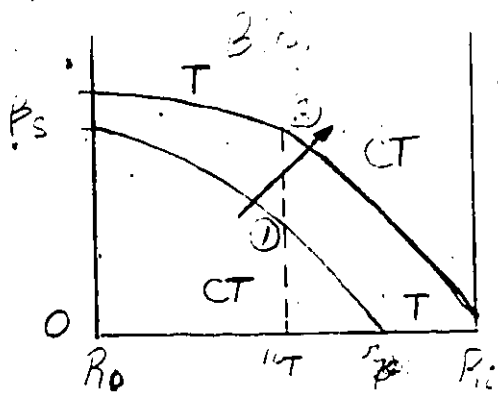
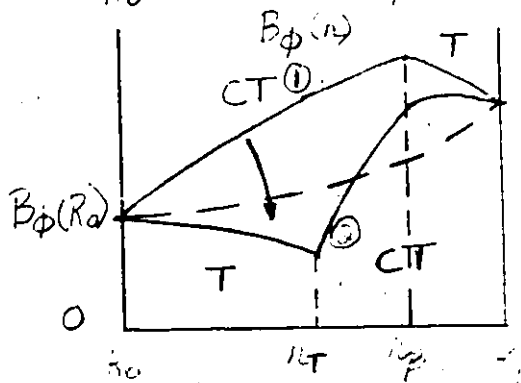
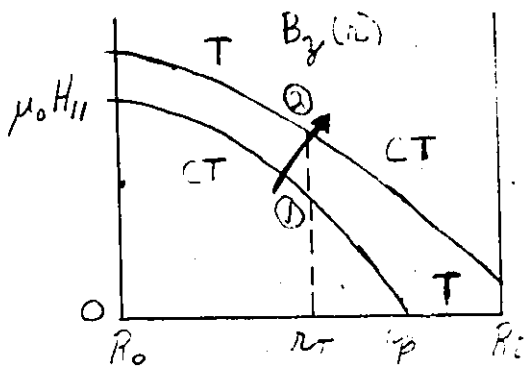
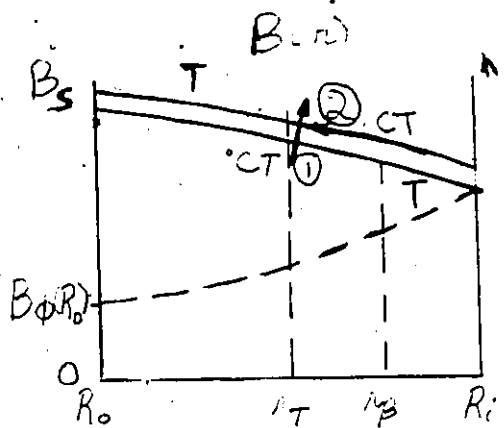
(a) $H_{//1} = 0$.

Here, the T zone is filled with some portion of the pure azimuthal flux originally permeating the wall and now compressed into the space between r_β and R_1 by radially inward displacement. Since longitudinal flux must now appear in the hole, the front of the $B_z(r)$ and $\beta(r)$ profiles must now suddenly advance across the zone of pure azimuthal flux. The concomitant increase in the longitudinal flux in the hole from 0 to $B_1 \pi R_1^2$ will generate a strong azimuthal electrical field,

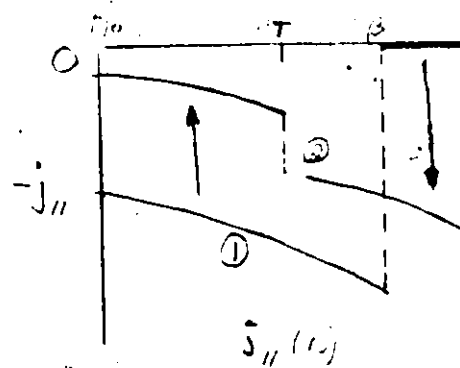
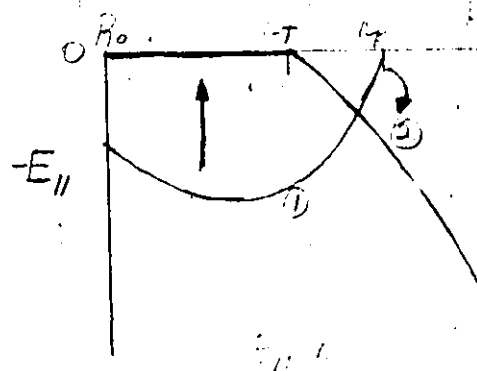
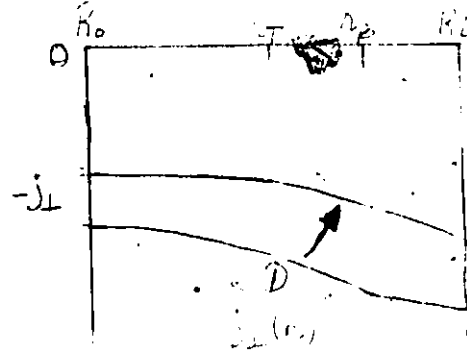
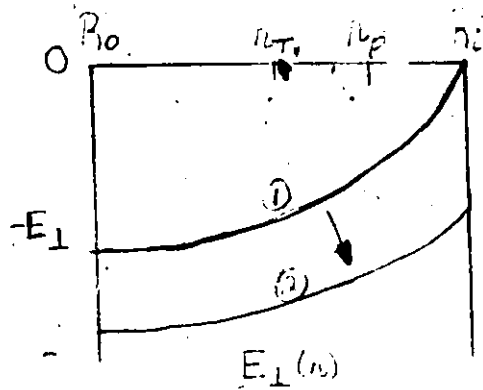
$$E_\phi(R_1) = - \frac{R_1}{2} \frac{dB_1}{dB_S} \quad (12)$$

Since, $B_\phi(R_1) \gg B_z(R_0)$, at this juncture, $E_{//} \approx E_\phi$. As a consequence, the T zone transforms into a CT zone. Simultaneously with this occurrence, the various constraints and electrodynamic requirements dictate the appearance of a T zone adjacent to the outer radius. The sequence of events is displayed schematically in the sketches on page 150, where most of the features are exaggerated for clarity of presentation. The abrupt collapse of the $B_\phi(r)$ profile is associated with the sudden increase in the $\beta(r)$ profile away from the azimuthal and towards the longitudinal direction.

Alternatively, one can say that the amount of azimuthal flux carried by the helical flux lines entering at the surface and spreading into the wall is appreciably smaller, for comparable B , than the azimuthal content of the helical flux lines already permeating the



(a)



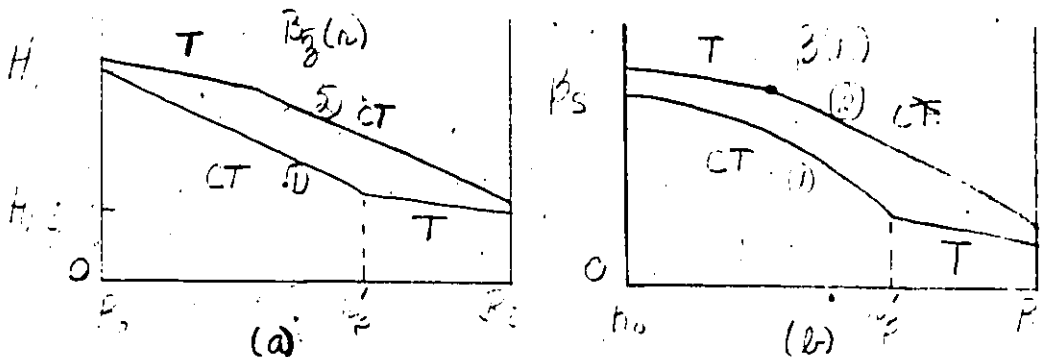
(br)

wall. Further the latter are undergoing displacement into the new CT zone where some of their azimuthal component undergoes consumption faster than it is being replenished. As a consequence of all these happenings, $\langle B_\phi \rangle$ exhibits a precipitous descent, at this juncture, from the summit it had just attained. Indeed, in our account, the decline in $\langle B_\phi \rangle$ is so severe that a strong diamagnetic azimuthal magnetization, $-\Delta \langle B_\phi \rangle$, ensues (see Figs. 2 and 3).

In our approach for solving the Clem equations, which focused on sequences of metastable, quasi-static configurations where $j_{//}$ and $j_\perp$ are not allowed to exceed $j_{c//}$ and $j_{c\perp}$, the events we have just outlined, unfold in steps where the changes are exaggerated and abrupt. Unfortunately, a resort to smaller increments for B_s and a finer spatial grid does not significantly attenuate the rapid changes in profiles. Consequently, although our program finds solutions satisfying Clem's equations and the several constraints of the theory, throughout the large excursions we have just described, these solutions presumably lose contact with the physics of the situation. As mentioned earlier, a "relaxation" or "diffusion" technique which permits excursions of $j_{//}$ and $j_\perp$ beyond $j_{c//}$ and $j_{c\perp}$ would be more compatible with the events and consistent with the large electric fields generated by the appreciable changes in the $B(r)$ and $\phi(r)$ profiles which are taking place. We are confident that the program under development and incorporating these crucial elements will prove more successful in quantitatively reproducing the phenomena encountered and yield results in better accord with observations.

(b) $H_{//1} \neq 0$.

When the wall has become superconducting in a longitudinal field as well as an azimuthal field, the T zone at the vanguard of the B profile is filled with some fraction of the helical flux originally permeating the wall and now compressed into the space between r'_p and R_1 . The interface, r'_p , between the T zone and the advancing CT zone is now made evident by an abrupt change of slope in the $\beta(r)$ profile since $d\beta/dr$ is critical in the CT zone and subcritical in the T zone. The $B_z(r)$ profile also reflects the transition from one zone to the other by also showing a sudden change of slope at r'_p as depicted schematically in the sketches below.



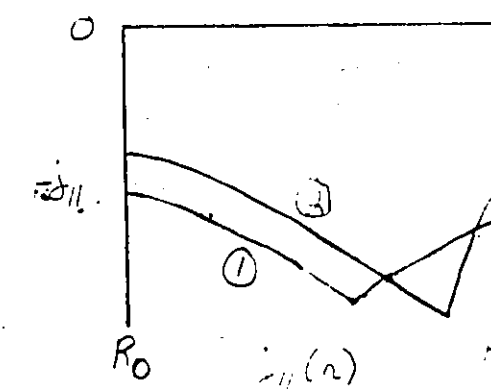
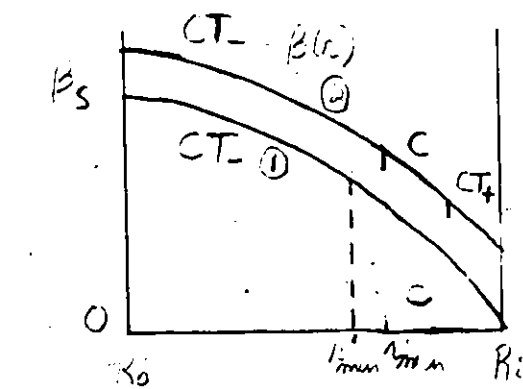
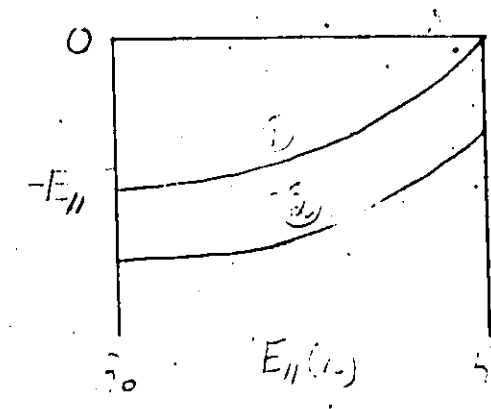
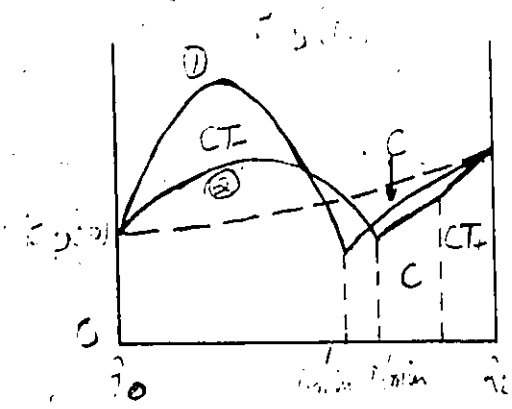
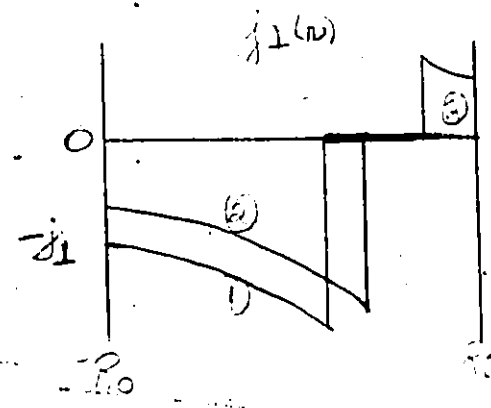
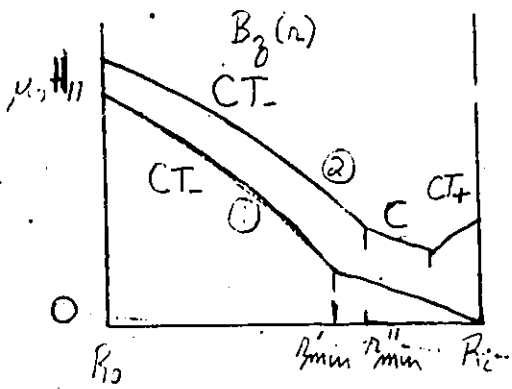
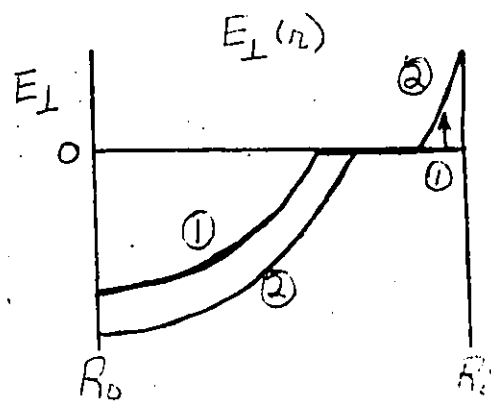
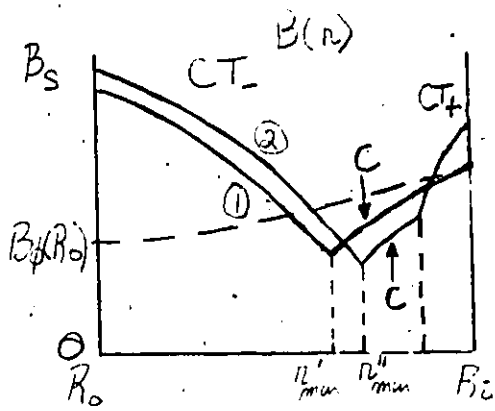
In contrast with the case where $H_{//1} = 0$, hence the T zone contained only azimuthal flux, one might now expect the sequence of events to be less dramatic, since here displacement of helical flux from the T zone into the hole should allow for a gradual rise in $B_z(R_1)$ and a smooth evolution of the profiles. In particular, we anticipate the T zone to shrink while the CT zone continues to advance and to witness the appearance of a growing T zone adjacent to the outer surface. Our computer program, however, does not generate this

expected solution. Again, as when $H_{//1} = 0$, the configuration of a T zone leading a CT zone transforms suddenly to that of a CT zone leading a T zone. In both instances therefore, a precipitous drop in $\langle B_\phi \rangle$ occurs at this juncture. Also, in both cases, the continuation of the descent into the diamagnetic quadrant (see Fig. 2) is caused by an expansion of the outer T zone, hence a reduction of the CT zone. Also, once more, it is the appearance of a strong electric field $E_{//}$ at R_1 which is responsible for triggering the abrupt transformation.

II. CT zone is initially preceded by a C zone which extends to R_1 (see Fig. (A) on page 143).

Because flux cutting has occurred in the C zone, the flux permeating it is helical, regardless of whether longitudinal flux already exists in the hole or not (i.e. whether $H_{//1} = 0$ or $H_{//1} \neq 0$). The rise of the longitudinal flux in the hole, however gradual, generated an appreciable azimuthal electric field, $E_\phi(R_1) = R_1 dB_z(R_1)/2dB_S$, around the circumference of the hole. This electric field has a component $\perp$ to the helical flux lines in the wall adjacent to the hole. This feature introduced into the Clem equations, and coupled with the several constraints and requirements of the theory, now dictates extremely fascinating behaviour.

A CT_+ zone now appears adjacent to the hole (we now distinguish the zones by introducing the subscripts + and - to indicate whether the flux transport is radially outward, or inward). Consequently an unusual three active zone structure is now encountered as depicted schematically in the sketches on page 154. We stress that



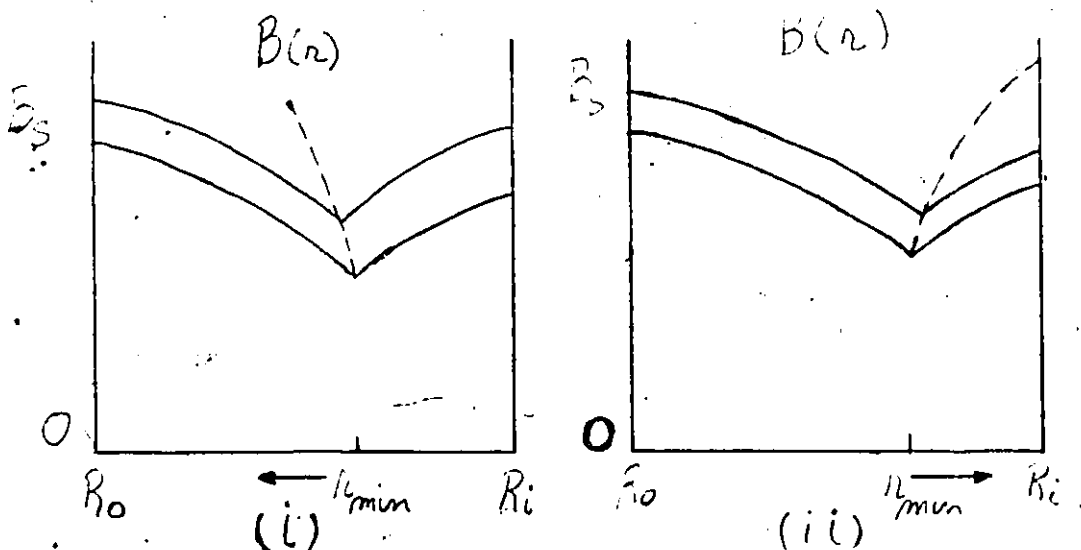
the sign of $E_{\perp}$ in the inner CT_{+} zone is opposite to that in the outer CT_{-} zone. Both CT zones expand by invading the territory of the C zone which is consequently gradually "annexed" and obliterated.

The nature of the inner CT_{+} zone is particularly intriguing. A pure flux cutting zone is interposed between this inner CT_{+} region and the outer CT_{-} zone where flux transport is radially inwards. The C zone, in effect, thus constitutes a "barrier" against the penetration of flux from the outer CT_{-} zone into the inner CT_{+} zone. Yet, the flux density both in the hole and, in the part of the inner CT_{+} zone adjacent to the hole, is increasing. Because of the C zone "obstacle", the longitudinal flux appearing in the hole and the rise of flux density in the inner CT_{+} zone cannot originate from the outer zone. The question then arises, where does this flux come from? The sign of $E_{\perp}$ in the inner CT_{+} zone indicates that, here, the flux transport is radially outwards. Evidently, the flux cutting and flux transport processes in the inner CT_{+} zone "enrich" the longitudinal component of the helical flux there at a rate sufficient to cause $B(r)$ to rise. Presumably, the azimuthal flux component of the helical flux lines which is consumed and tilted towards the z direction by flux cutting is replenished by the azimuthal component of flux traversing R_1 and supplied by the toroidal magnet coil which maintains $B_{\phi}(R_1)$ constant. In a strict sense, the rise of the flux density in the hole and its longitudinal component there is dictated by the requirement that $\vec{B}(r)$ be continuous across the boundary R_1 , while the rise of $B(r)$ and $B_z(r)$ in the inner CT_{+} zone ensues from flux cutting and radially outward flux transport. This picture, however, although self consistent has

the appearance of a bootstrap mechanism for lifting the flux density in the hole. One could regard the flux cutting process as an agent for the conversion of azimuthal flux generated by the toroidal magnet coil into longitudinal flux.

Sequences of CT_- and CT_+ profiles.

Two routes can now be travelled by r_{min} , the position of the minimum in the $B(r)$ profile, once the C zone has vanished and the CT_- and CT_+ zones have met or "collided". The interface r_{min} also defines the boundary between the two CT zones and may now migrate either (i) outwards or, (ii) inwards (see sketches below). The route followed at this crossroads is uniquely determined by $j_{cl}(B)$, $j_{c//}(B)$ and the boundary conditions $H_{//}(R_0)$, $H_{//}(R_1)$, $H_\phi(R_0)$ and $H_\phi(R_1)$. We stress that in our work, $H_\phi(R_0)$ and $H_\phi(R_1)$ are stationary and the sequences of $H_{//}(R_0)$ are specified. The computational exercise therefore consists in finding r_{min} , hence $H_{//}(R_1)$, which satisfy the requirements that $B_z(r)$, $B_\phi(r)$, hence $B(r)$, be continuous. Concurrently, the computer program ascertains that the sequence of solutions for r_{min} , hence $H_{//}(R_1)$ meet the requirements that, (a) $E_{//}j_{c//} > 0$, $E_{\perp}j_{cl} > 0$ (i.e.



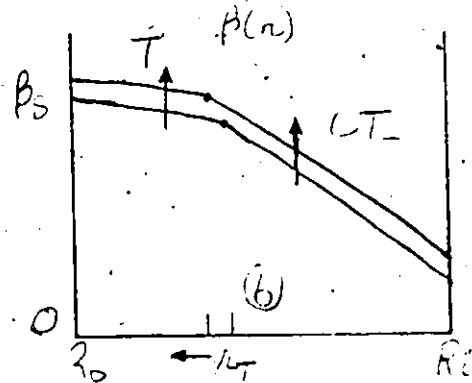
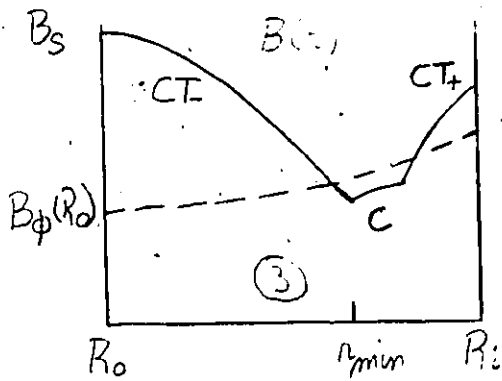
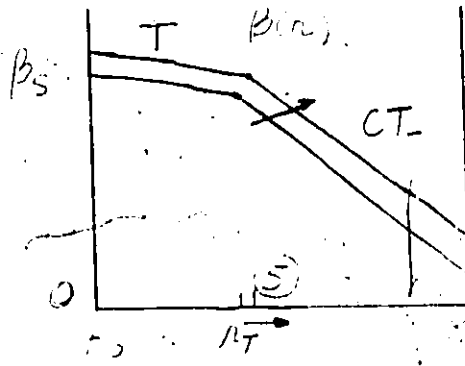
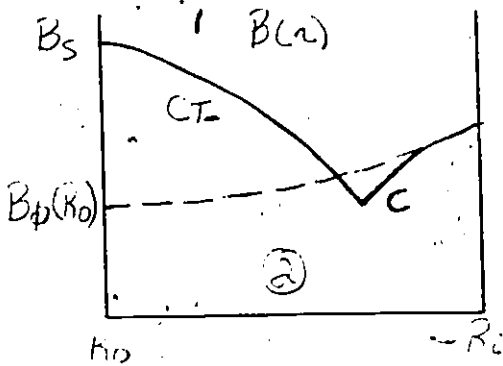
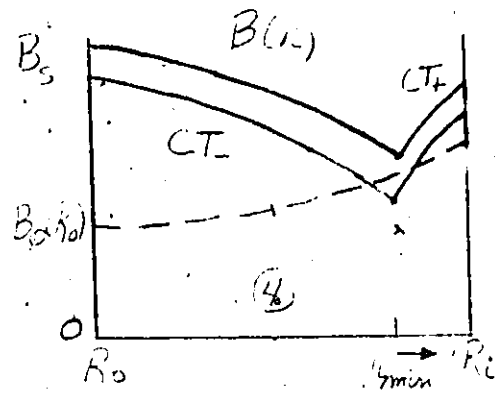
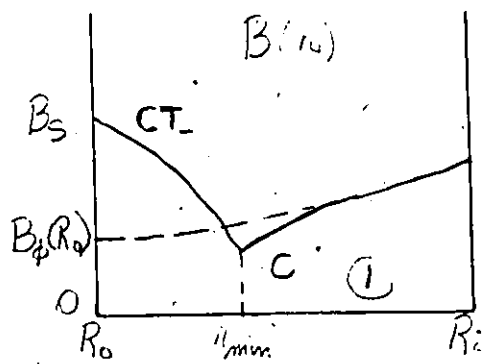
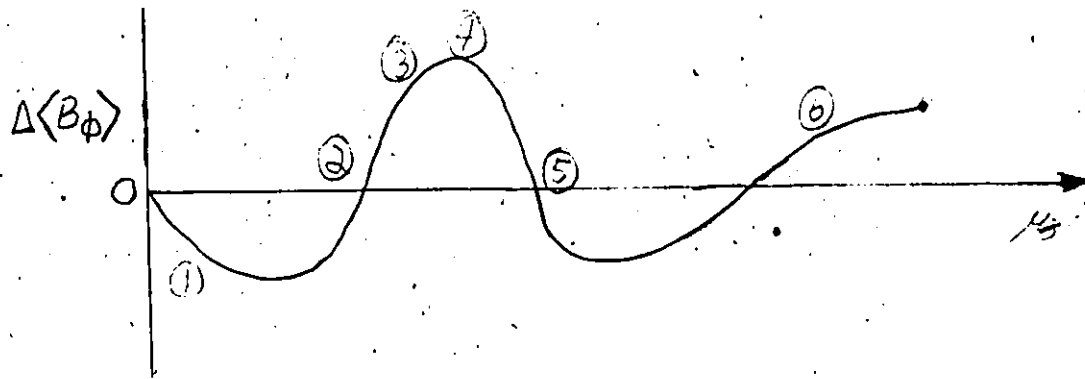
that the components of the electric field have the same sign as the corresponding component of the current density), and (b) $E_{//}(r)$ and $E_{\perp}(r)$ be continuous. Condition (a) implies here that $E_{\perp} > 0$ in the CT_+ zone and $E_{\perp} < 0$ in the CT_- zone. A variety of complicated migrations of $r_{\min}$ can be envisaged, eg. outwards then inwards, inwards then outwards. Presumably such elaborate paths could be "fabricated" by various sophisticated sets of functions for $j_{c\perp}(B)$ and $j_{c//}(B)$ over the applicable range of B . We have investigated only the two simple paths, (i) and (ii), and outline our computational results for each case.

(i) Outward migration of $r_{\min}$.

$\langle B_{\phi} \rangle$ is initially diamagnetic in this case and its locus versus $H_{//}$ remains in the diamagnetic region throughout the computational exercise. A very gradual evolution towards zero magnetization is exhibited in some instances. Consequently, the situation does not correspond to any of our experimental observations. Eventually, the sequences of $E_{\perp}(r)$ and $E_{//}(r)$ profiles cease to display a smooth progression and start to display aperiodic and erratic increases and decreases. As a consequence, very soon, reliable solutions can no longer be determined.

(ii) Continuous inward migration of $r_{\min}$.

It is of interest to describe the evolution of $\langle B_{\phi} \rangle$ prior to the meeting of the CT_- and CT_+ zones. As $H_{//}$ increased from $H_{//1}$ with $H_{\perp}$ held constant, $\langle B_{\phi} \rangle$ traced a rather deep diamagnetic valley, then climbed gradually into the paramagnetic quadrant (see sketches on page 158). All this took place while r_g , the front of the $B(r)$ profile, advanced across the wall to the inner radius R_1 (see Fig. A, page 143).



While the C zone shrank (see sketches on page 158), $\langle B_\phi \rangle$ did not vary significantly. We note that the range of increase of $H_{//}$, during which this event occurred, is not appreciable. The continuous migration of $r_{\min}$ inwardly to R_1 is accompanied by a small decrease in $\langle B_\phi \rangle$.

Again, we stress that the set of $B(r)$ and $B(r)$ profiles encountered as $r_{\min}$ progresses inwards until R_1 is attained, constitute unique, history independent, configurations. Here $j_{//}$ and $j_\perp$ are not only critical throughout the entire specimen but also, only one type of CT zone exists. Indeed, the entire sequence of events from the disappearance of the C zone to the present juncture, is independent of the Clem equations for $\partial B/\partial t$ and $\partial \beta/\partial t$ and is dictated solely by the critical state equations for $j_{c\perp}(B)$ and $j_{c//}(B)$ (equations 8 and 9). This feature, as we have remarked earlier in a similar situation, presents several tantalizing opportunities.

iv) Rise of $\langle B_\phi \rangle$ to a second peak.

A. First we continue the development of the situation under scrutiny in the previous paragraph.

Upon further increase of $H_{//}$, a T zone appears at the outer radius and rapidly penetrates and occupies an appreciable fraction of the outer volume of the hollow cylinder. The growth of this zone causes $\langle B_\phi \rangle$ to decline steeply from the paramagnetic to diamagnetic quadrant.

The decrease of $\langle B_\phi \rangle$ associated with the advance of the T zone can be qualitatively understood from a consideration of the sequence of $B(r)$ profiles (see sketches on page 150) and the fact that the corresponding $B(r)$ profiles are critical throughout at every stage.

In view of the latter, the rate of growth of the $B(r)$ profiles remains essentially steady. The rapid rise in the $\beta(r)$ profiles caused by the entry of the T zone, indicates, however, that the inward transport of helical flux brings about a reorientation of $\vec{B}(r)$ towards the z axis and hence away from the azimuthal direction.

The large $E_1(r)$ generated by the rapid advance of this T zone is certainly inconsistent with the quasi-static approach we have adopted and indicate that a programme where $j_{\perp}$ and $j_{//}$ are allowed excursions beyond $j_{c\perp}$ and $j_{c//}$ should be utilised to treat the situation now encountered. Such an effort is under way. We emphasize, however, that our method yields solutions consistent with the governing equations and the several constraints.

The T zone, after its blitzkrieg invasion of the outer volume, now collapses gradually, its front retreating slowly towards R_0 . This retreat is accompanied by a slow rise of $\langle B_{\phi} \rangle$ from the diamagnetic quadrant into the paramagnetic region. The picture just exploited to account for the decrease in $\langle B_{\phi} \rangle$ during the invasion by the T zone can also now be applied to explain the rise in $\langle B_{\phi} \rangle$ associated with the retreat of this zone.

The diminution of the T zone and the concomitant expansion of the $CT_{\perp}$ zone are linked to the relative magnitudes and B dependences of $j_{c\perp}(B)$ and $j_{c//}(B)$. Our choices for these two functions and the parameters C and D which they contain, play a major role in the migration of r_T , the interface between the two zones. The choice we have made leads to a gradual but continuous migration of r_T towards R_0 and finally to a situation where r_T reaches R_0 . As a consequence, a

single zone (a CT zone) now occupies the entire specimen. At this juncture, with our choice of $j_{c1}(B)$ and $j_{c//}(B)$ no further solutions to the Clem equations can be found as the increase in $H_{//}$ continues.

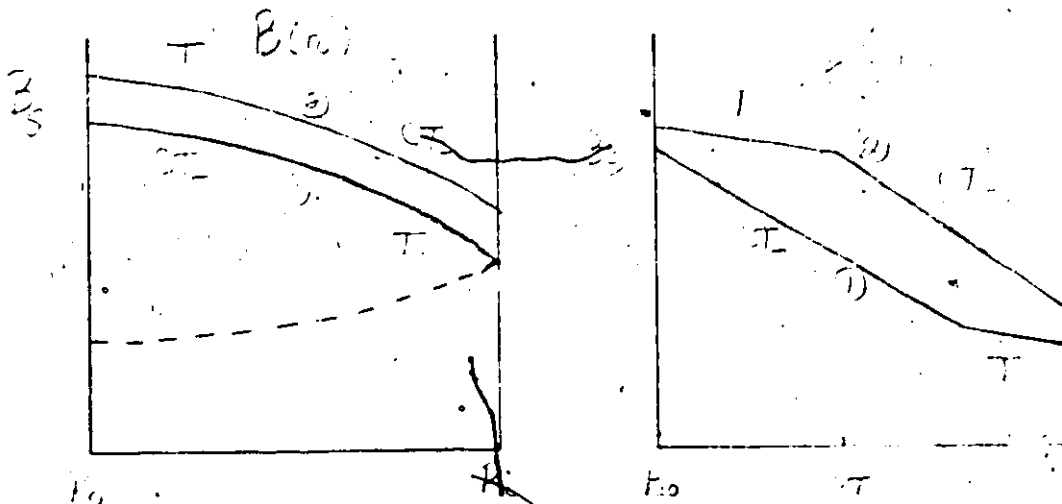
We note that a unique configuration of $B(r)$ and $\beta(r)$ solely determined by $j_{c1}(B)$, $j_{c//}(B)$ and the boundary conditions $H_{//}(R_0)$, $H_{\phi}(R_0)$ and $H_{\phi}(R_1)$ is now again encountered. As a consequence; maintaining our prescriptions for $j_{c//}(B)$ and $j_{c1}(B)$, leads to an impasse. No combination of T and CT zone structures can yield solutions. We emphasize that C zones can no longer appear since $B(r)$ must increase with time (with B_s) throughout the wall.

At this juncture, solutions to the Clem equations can be obtained by introducing amendments to $j_{c1}(B)$ and/or $j_{c//}(B)$ in the range of B just below and beyond the threshold value, B_T just reached. The migration of r_T to R_0 in our exercise arose basically because our chosen $j_{c//}(B)$ falls below $j_{c1}(B)$ in the range $B > 2T$. We note that the threshold B_T is not fixed by this crossover of $j_{c//}$ and j_{c1} but is also affected by the initial conditions, namely, $H_{//i}$ and $\langle B_{\phi} \rangle_{app}$, hence $H_{\phi}(R_0)$ and $H_{\phi}(R_1)$. The dotted line in Fig. 2 illustrates the effect of replacing $j_{c//}(B)$ of equation 7, by $j_{c//} = \text{constant} = 6(10^7) \text{ A/m}^2$ over the range $B > 2.5 T$.

The physical implications of our observation that, allowing $j_{c//}(B)$ to fall below $j_{c1}(B)$ in the range of large values of B , leads to a situation where no solutions can be found, are not clear. Since the evolution of the several profiles is a continuous, history dependent, process and we have drastically interfered in this process by not allowing excursions of $j_{//}$ and $j_{\perp}$ beyond $j_{c//}$ and j_{c1} during

this evolution, the configurations our program has developed, although satisfying the equations, requirements and constraints, must certainly deviate from the correct configurations, particularly subsequent to the abrupt changes in the zone structures which we have noted in our discussion. As a consequence, the impasse we confront because $j_{c//}$ falls below $j_{c\perp}$ may have been avoided by a more appropriate relaxation or diffusion approach throughout the development of the profiles and, especially, when dramatic changes in zone structure are encountered.

B). We now return to the situation discussed earlier in this chapter, where the arrival of the $B(r)$ profile at the surface of the hole preceded the penetration of the $B(r)$ disturbance (see Fig. B of page 143). This state of affairs led to a reversal of the zone structure upon further increase of $H_{//}$. The pertinent features are again sketched below for convenience. The zone structure changed abruptly from, (i) a T zone preceding a CT_{-} zone to, (ii) a CT_{-} zone preceding a T zone. Basically, then, the situation which ensues from this transformation is now identical to that we have just been examining.



The difference lies in the detailed histories leading to the "final" configuration. The evolution of the profiles now must proceed as previously, i.e., r_T migrates gradually to R_0 and solutions vanish unless we amend $j_{c//}(B)$ and/or $j_{c\perp}(B)$ in the appropriate range of B so that $j_{c//}$ does not fall below $j_{c\perp}$.

The large electric fields associated with the change in zone structure and the rapid advance of the T zone, most probably invalidate our quasi-static approach in these circumstances. Consequently, the details of the profiles during this sequence of events are probably unreliable. We recall that, the rapid advance of the T zone, generated a precipitous drop in $\langle B_\phi \rangle$ from the paramagnetic summit deep into the diamagnetic quadrant. As a consequence we strongly suspect that the large diamagnetism obtained by our method may be either fictitious or excessive. It is then perhaps not too surprising that the migration of the front of the T zone to R_0 does not succeed in causing $\langle B_\phi \rangle$ to rise appreciably into the paramagnetic region and trace a second broad peak (see Figs. 2 and 3). An encouraging feature of our results after the "catastrophic" diamagnetic collapse is that the governing equations seem capable of recovery from a disaster and of guiding the system back to a "healthy" paramagnetic climate.

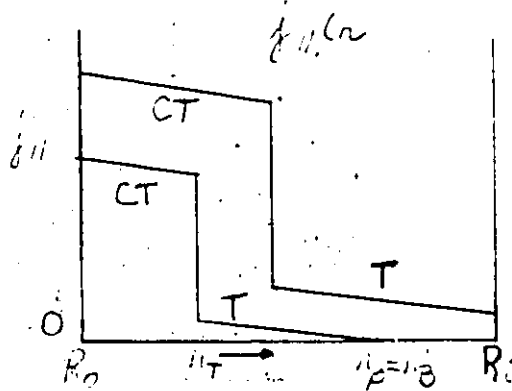
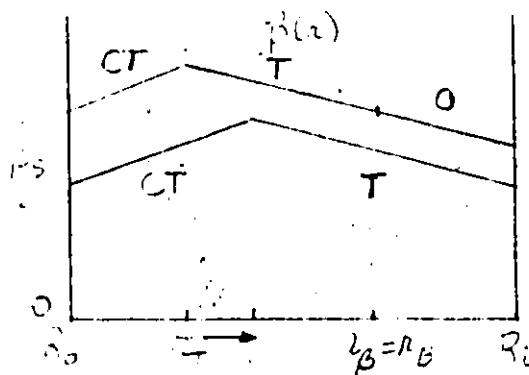
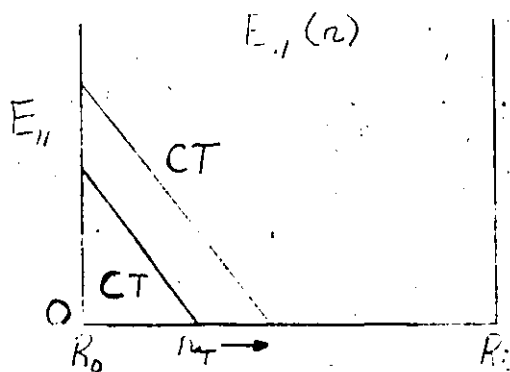
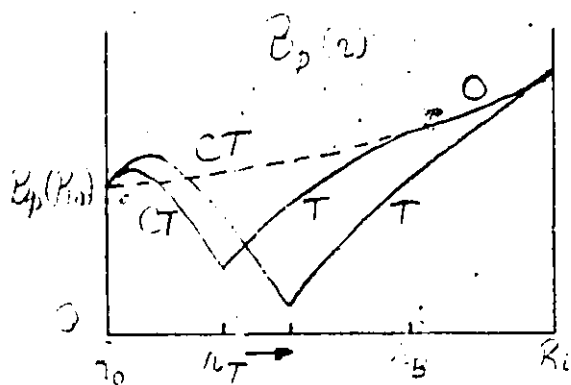
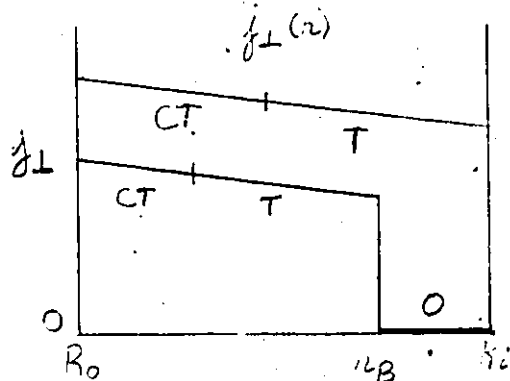
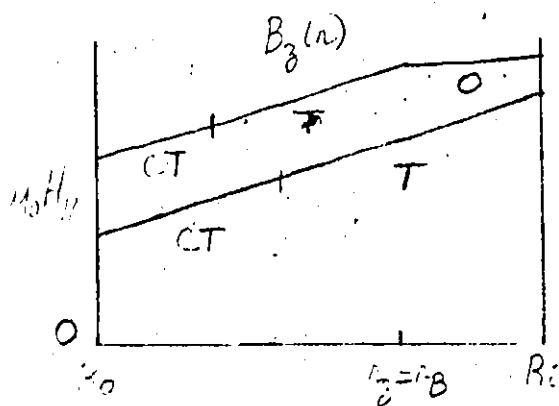
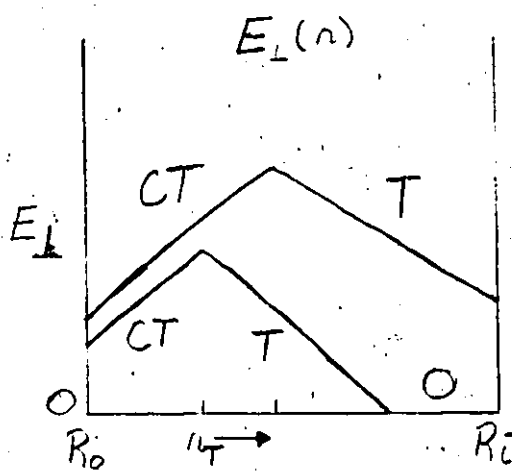
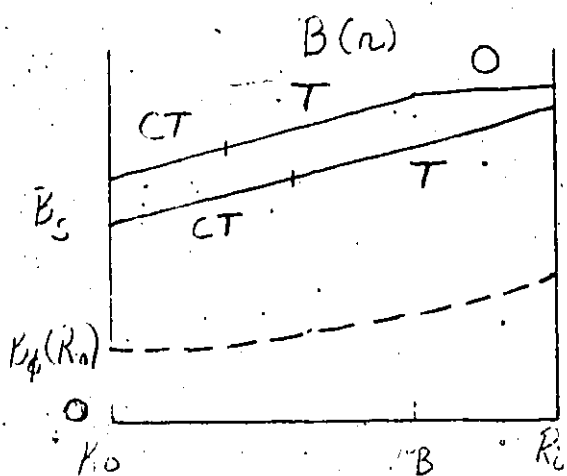
Finally we note that the calculated curves of the longitudinal magnetization of the cylinder versus $H_{//}$ are insensitive to the presence of a fixed azimuthal field, in agreement with our observations for this material. Further, as can be seen by inspection of Fig. 1, the magnitude and the direction of the change effected by the presence of an azimuthal field is well reproduced by the

theoretical curves. The theoretical curves, deal with an idealized situation where demagnetization effects are ignored, hence magnetic fields along the wall are uniform over the entire inner and outer surfaces. As a consequence, the peak is "mathematically" sharp, while experimentally, it is inevitably rounded.

C. Evolution of $B(r)$ and $\beta(r)$ for $H_{//}$ decreasing from $H_{//max}$ to zero.

Fig. 7 displays the calculated and observed longitudinal magnetization of the cylinder versus $H_{//}$ decreasing in the presence and absence of a stationary azimuthal field. These data are insensitive to the presence of an azimuthal field for the material under study and the theoretical results reflect this feature remarkably well. The small influence of $\langle B_{\phi} \rangle_{app}$ is of the right magnitude and correct direction in the range of small $H_{//}$ but it has the wrong direction in the range of large $H_{//}$.

Figs. 8 and 10 allow the reader to compare the theoretical and experimental evolution of $\langle B_{\phi} \rangle$ versus $H_{//}$ decreasing in various static azimuthal fields. The theoretical curves are not very sensitive to the magnitude of $j_{c//}(B)$ in the range of high fields. This feature is illustrated by comparing Figs. 8 and 10 where the parameter D in the expression $j_{c//} = D(1 - (B/B_{c2})/B)$ was allowed to vary from $2.3(10^7)$ to $1.53(10^7)$ A/m². We note that the latter value corresponds to that chosen in the analysis of the data for $H_{//}$ increasing. This insensitivity to $j_{c//}$ arises because a major fraction ($\approx 80\%$) of the wall thickness is occupied by a T zone, where $j_{//} < j_{c//}(B)$, during the sweep of $H_{//}$ from $H_{//max}$ to zero.



The evolution of the configuration of $B(r)$ and $\beta(r)$ and the associated $B_z(r)$, $B_\phi(r)$, $E_\perp(r)$, $E_{//}(r)$, $j_\perp(r)$ and $j_{//}(r)$ profiles is remarkably simple in these circumstances. This feature emerges readily from inspection of the sketches on page 165, which display schematics of typical profiles in the evolutionary sequence. Again some features are exaggerated for clarity of presentation.

A narrow $C_{++}T_{++}$ zone develops adjacent to the outer surface of the cylinder and embraces an appreciably larger T_{+} zone. The subscript $+$ indicates that $E_{//}$, $E_\perp$, $j_{//}$ and $j_\perp$ are positive. This is to be expected since both $B(r)$ and $\beta(r)$ are decreasing. Previously, with $H_{//}$ increasing, the opposite signs appeared but were ignored until a special case arose where a CT zone appeared, adjacent to the hole, where the flux transport was outwards. Since now the $+$ sign prevails throughout the history of the descent of $H_{//}$, the subscript will henceforth be set aside.

Both the CT and T zones expand until the front r_B of the T zone reaches the inner radius R_1 . The interface, r_T , of the CT and T zones subsequently remains essentially static until the range of low fields $H_{//} \lesssim H_\phi(R_0)$ is attained. As $H_{//}$ is reduced further, in the range $0 < H_{//} \lesssim H_\phi(R_0)$, the interface r_T now migrates towards R_1 , hence the CT zone expands and, consequently, the T zone shrinks. It is this inward migration of r_T which causes $\langle B_\phi \rangle$ versus $H_{//}$ to exhibit a valley in the vicinity of $H_{//} = 0$ (see Figs. 8 and 10). The valley displayed by the measured curves is less pronounced than that calculated and occurs at corresponding larger $\langle B_\phi \rangle_{app}$ in the data than in the theoretical curves (compare Figs 9 and 4-3 with 8 and 10). These

deviations between theory and observations are not surprising nor disturbing. The reasons for this equanimity are the following. The structure of the valley is quite sensitive to the extent of the displacement of r_T . For instance, a fractional displacement, $\Delta r_T/r_T \approx 20\%$, is responsible for the valley in curve C of Fig. 8. Further, the displacement Δr_T is very dependent on the relative magnitudes of $j_{c//}(B)$ and $j_{cl}(B)$ in the low field range of, eg. $0 < B < 150$ mT. Unfortunately, neither of these quantities is reliably known in this low field range. It is well established that $j_{cl}(B)$ varies extremely rapidly in this range but an accurate picture of its dependence on B and of its magnitude is notoriously difficult to obtain in this range (as we have seen in chapter 5) and has eluded most workers.

In our view, the overall correspondence between the large family of calculated and measured curves is quite satisfactory. A frustrating feature of this good agreement, however, is that $j_{c//}(B)$ does not play an important role in the behaviour. Consequently, these data and the calculations do not provide a sturdy vehicle for extracting good information on this crucial quantity.

Conclusion

Our application of the general critical state theory to our observations of the evolution of $\langle B_\phi \rangle$ versus $H_{//}$ increasing and decreasing, in various static azimuthal fields, has provided considerable insight into the sequences of intricate patterns of flux cutting, flux transport and flux cutting-flux transport zone structures which arise within this theoretical framework. We have shown that the

theory reproduces several salient features of the observations. We have identified the occurrence of several complicated features with specific arrangements and histories of zone structures. Our quasi-static approach for solving the equations of the theory has shown that several circumstances arise, however, where the technique we have adopted is inappropriate. Nevertheless since the analysis shows clearly where and why our method is unsuitable, the path to a solution of the problems has been well charted.

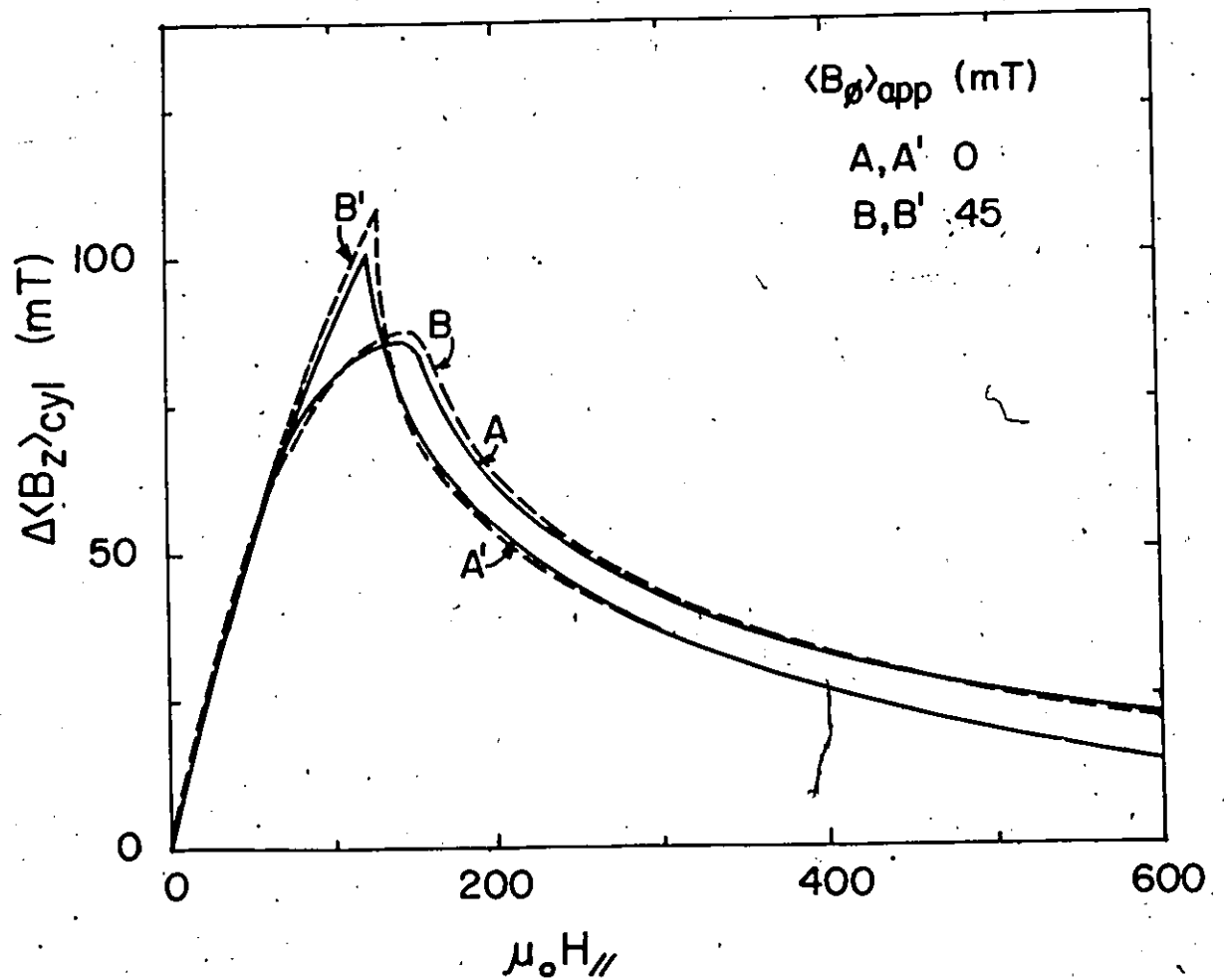


Fig. 9-1 Magnetization of the entire hollow cylinder (hole combined with the wall) with $H_{//}$ increasing. Curves A and A' are measured and calculated for $H_\phi = 0$, curves B and B' for $\langle B_\phi \rangle_{\text{app}} = 45$ mT.

Fig. 9-2 Typical theoretical results for the evolution of the azimuthal flux in the wall versus $\mu_0 H_{//}$ in various stationary azimuthal fields $\langle B_\phi \rangle_{app} = 30, 45$ and 60 mT for curves A, B and C. These curves should be compared with Fig. 9-4. Curves B, B' and B'' were calculated with $H_{//1} = 0, 80$ and 150 mT for $\langle B_\phi \rangle_{app} = 45$ mT. These curves should be examined together with Fig. 3-9 and 3-10. The rapid descent and subsequent rise of B' and B'' are not shown to avoid clutter. $D = 1.53(10^7)$ AT/m² for all the curves. Curves terminate when solutions disappear. $j_{q1} = C(1-(B/B_{c2})^2)/B^2$ and $j_{c//} = D(1-(B/B_{c2}))^2/B$ with $C = 3.65(10^7)$ AT³/m² and $B_{c2} = 1$ T.

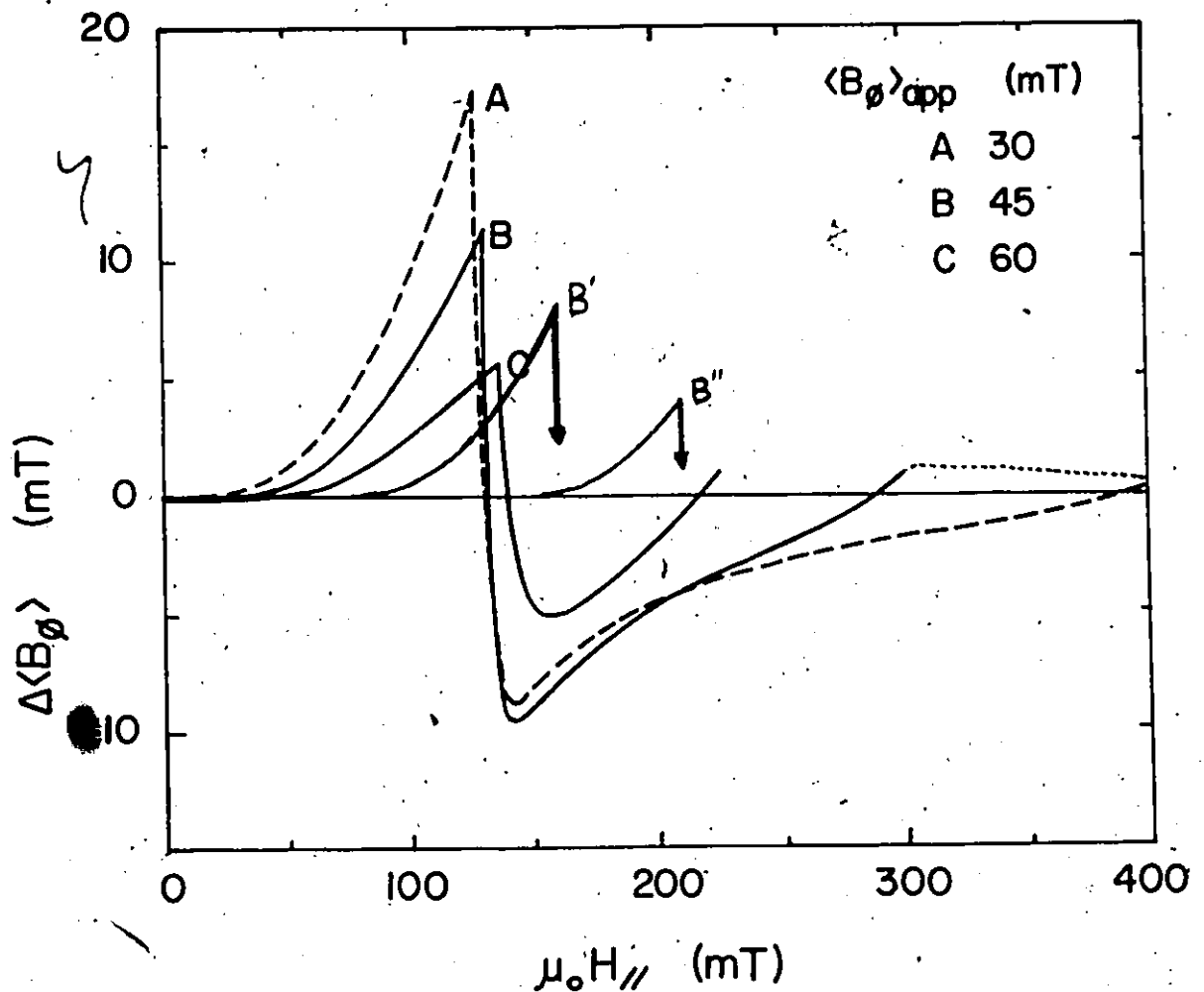


Fig. 9-3 Illustrates the effect of varying the magnitude of $j_{c//}$ on the evolution of the calculated curves of $\langle B_{\phi} \rangle$ versus $H_{//}$. The parameter $D = 0.92(10^7)$, $1.53(10^7)$ and $1.91(10^7)$ AT/m² for curves A', B', and C'. $\langle B_{\phi} \rangle_{app} = 45$ mT for the three curves. The curves are interrupted when solutions disappear. $j_{c\perp} = C(1-(B/B_{c2}))/B^{1/2}$ where $C = 3.65(10^7)$ AT^{1/2}/m² and $j_{c//} = D(1-(B/B_{c2}))/B$, with $B_{c2} = 1$ T.

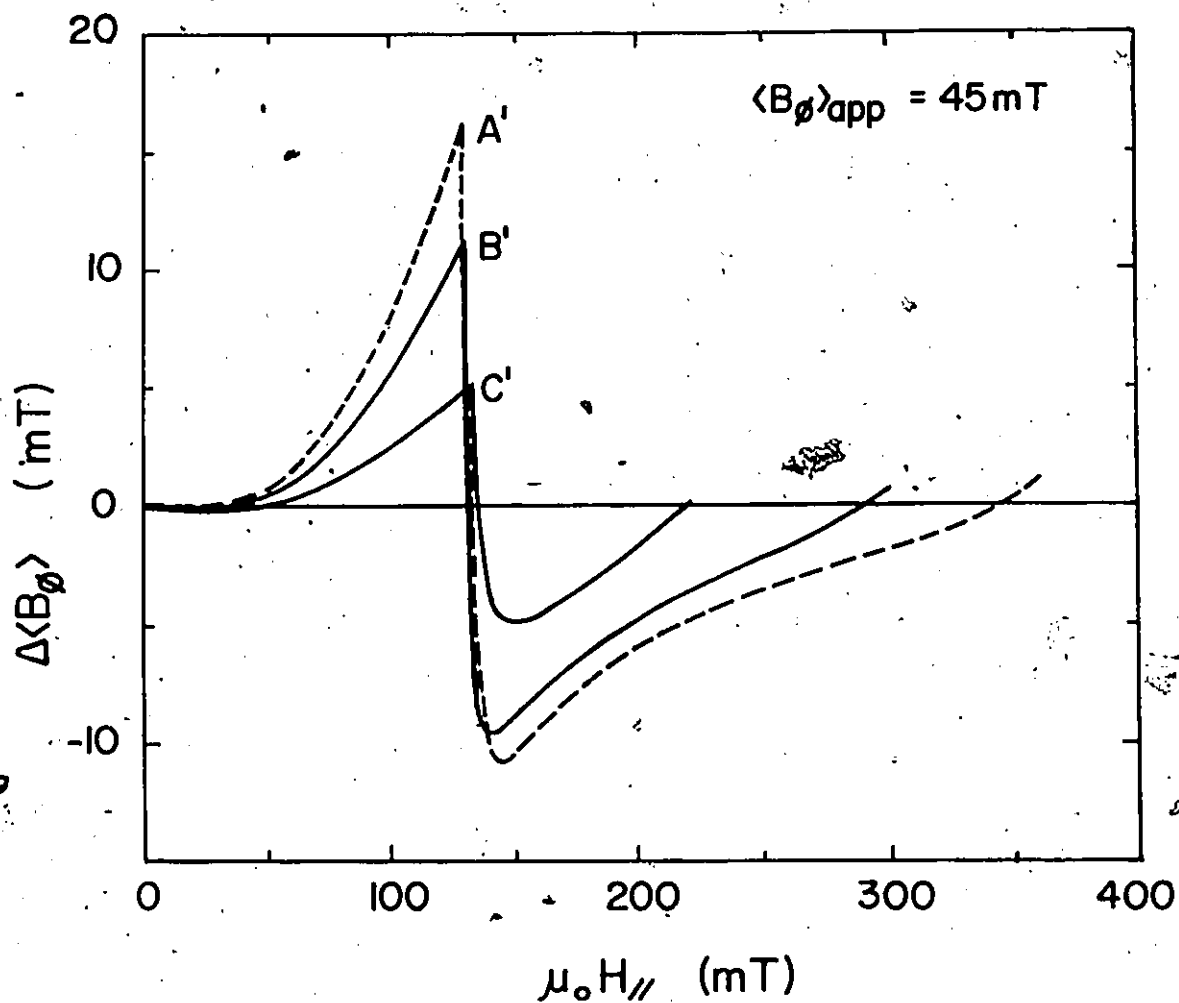


Fig. 9-4 Fig. 3 of chapter 3 is reproduced here for convenience. Shows three representative curves from that measured at several different $\langle B_\phi \rangle_{app}$.

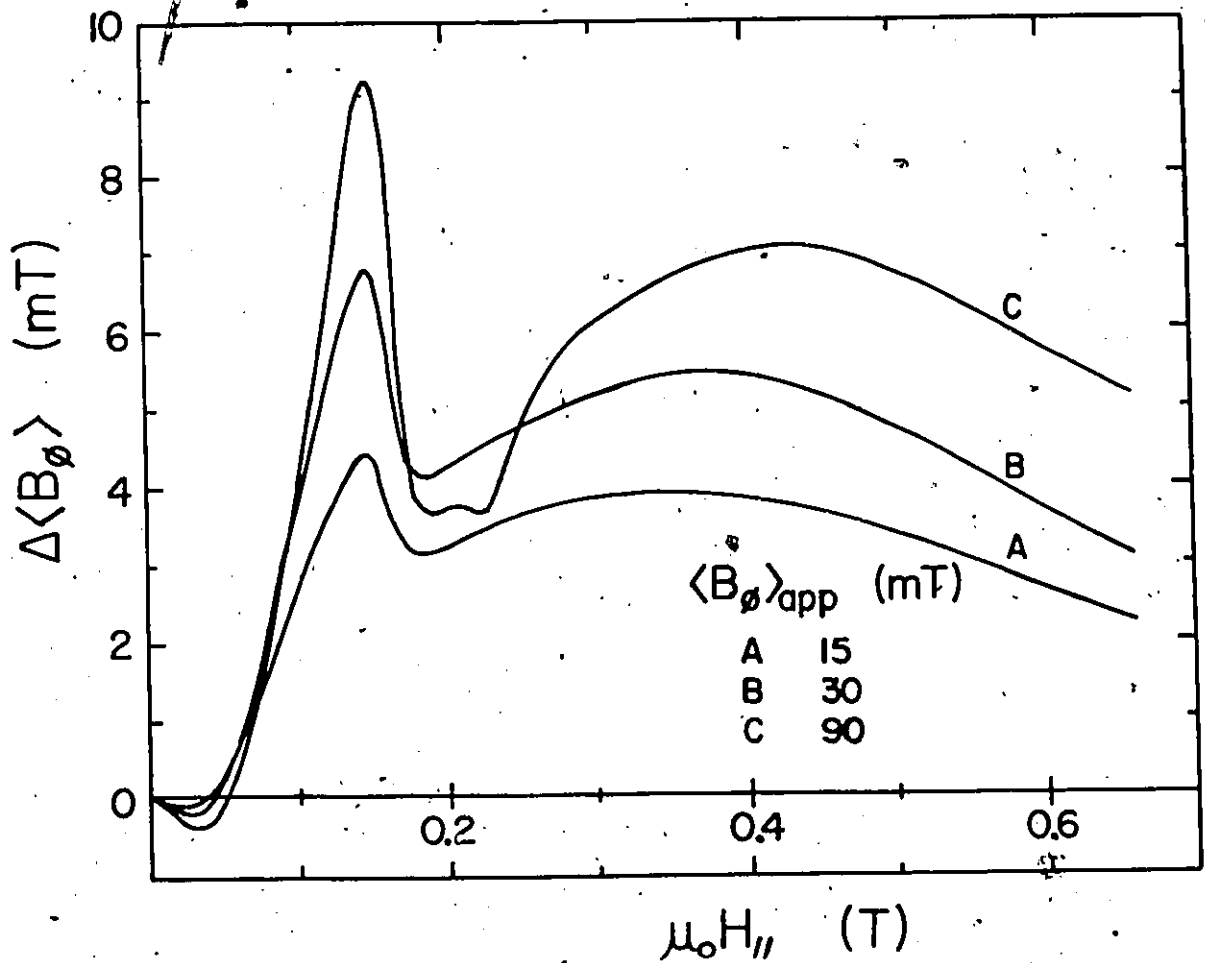


Fig. 9-5 The data points (Δ) are taken from the several measured curves illustrated in the previous figure. These indicate the depth of the initial diamagnetic valley in the locus of $\langle B_\phi \rangle$ versus $H_{//}$ corresponding to a chosen stationary $\langle B_\phi \rangle_{app}$. The solid curve is calculated using equations 6 and 7 with $C = 3.65(10^7) \text{ AT}^{\frac{1}{2}}/\text{m}^2$ and $D = 0.92(10^7) \text{ AT}/\text{m}^2$.

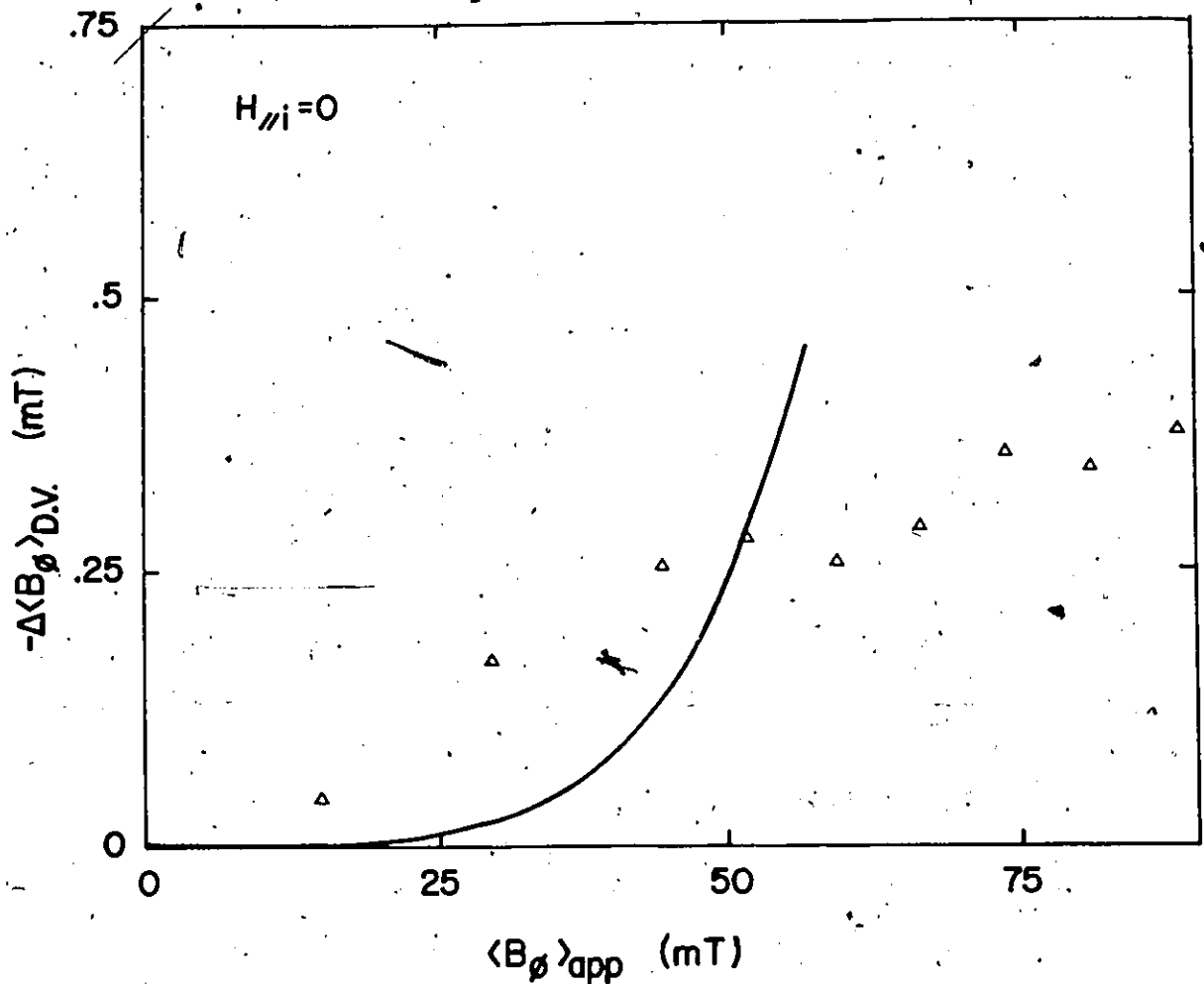


Fig. 9-6 Compares the calculated and measured dependences on $H_{//i}$ for the height of the sharp paramagnetic peak (see Figs 9-2, 3-9, and 3-10). The data points (o) were taken from curves measured at $\langle B_{\phi} \rangle_{app} = 90$ mT. The calculations (solid curve) were performed for $\langle B_{\phi} \rangle_{app} = 45$ mT using j_{c1} , $j_{c//}$ and the parameters given in the caption to Fig. 9-2.

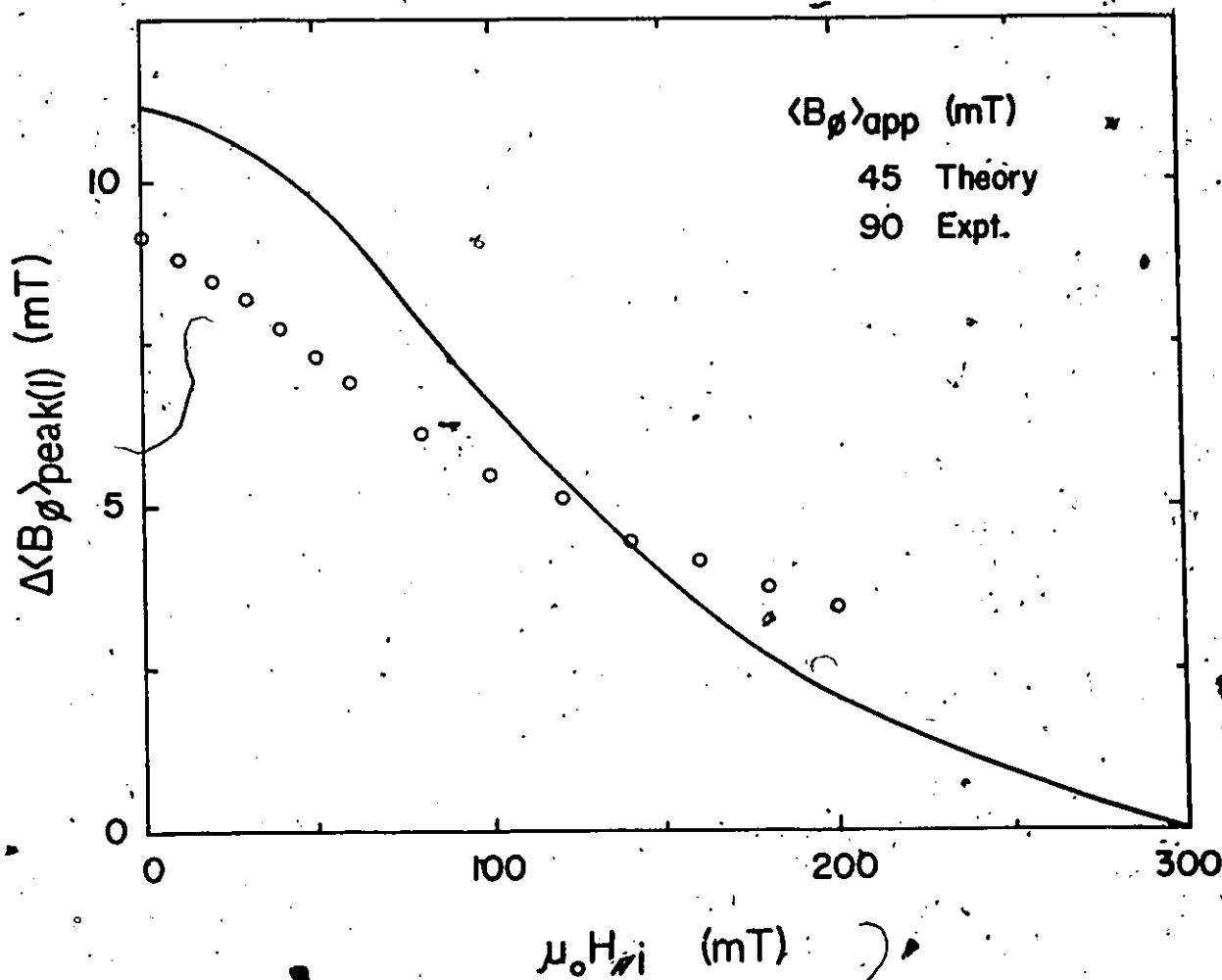


Fig. 9-7 Magnetization of the entire hollow cylinder (combination of hole and wall) with $H_{//}$ decreasing. Curves A and A' are measured and calculated for $H_{\phi} = 0$, curves B and B' for $\langle B_{\phi} \rangle_{app} = 45$ mT.

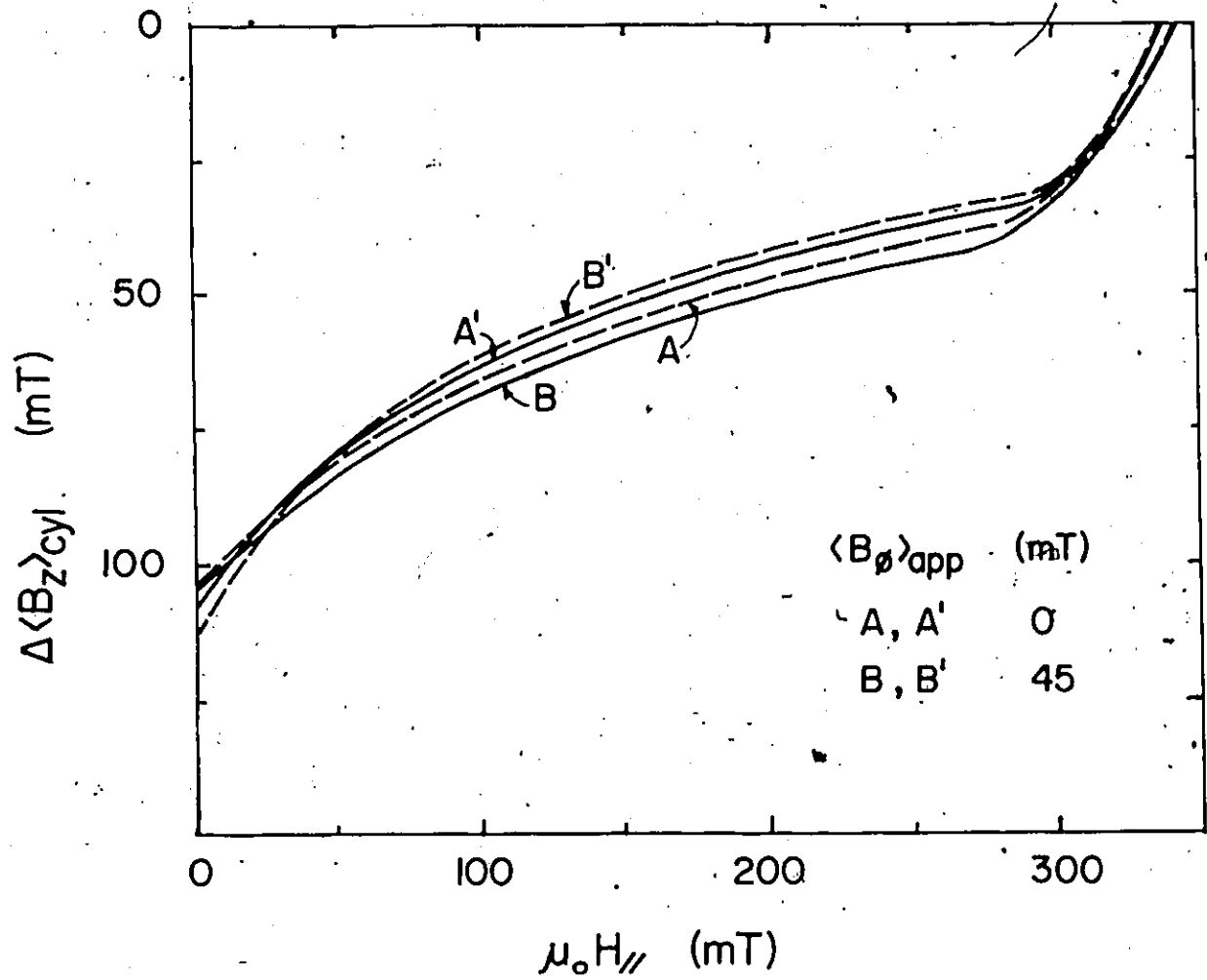


Fig. 9-8 Evolution of the azimuthal diamagnetic effect calculated for various stationary $\langle B_\phi \rangle_{app}$ using equations 6 and 7 with $C = 3.65(10^7) \text{ AT}^2/\text{m}^2$, $D = 2.30(10^7) \text{ AT}/\text{m}^2$ and $B_{c2} = 1 \text{ T}$. These curves should be compared with the corresponding measured curves presented in the next figure.

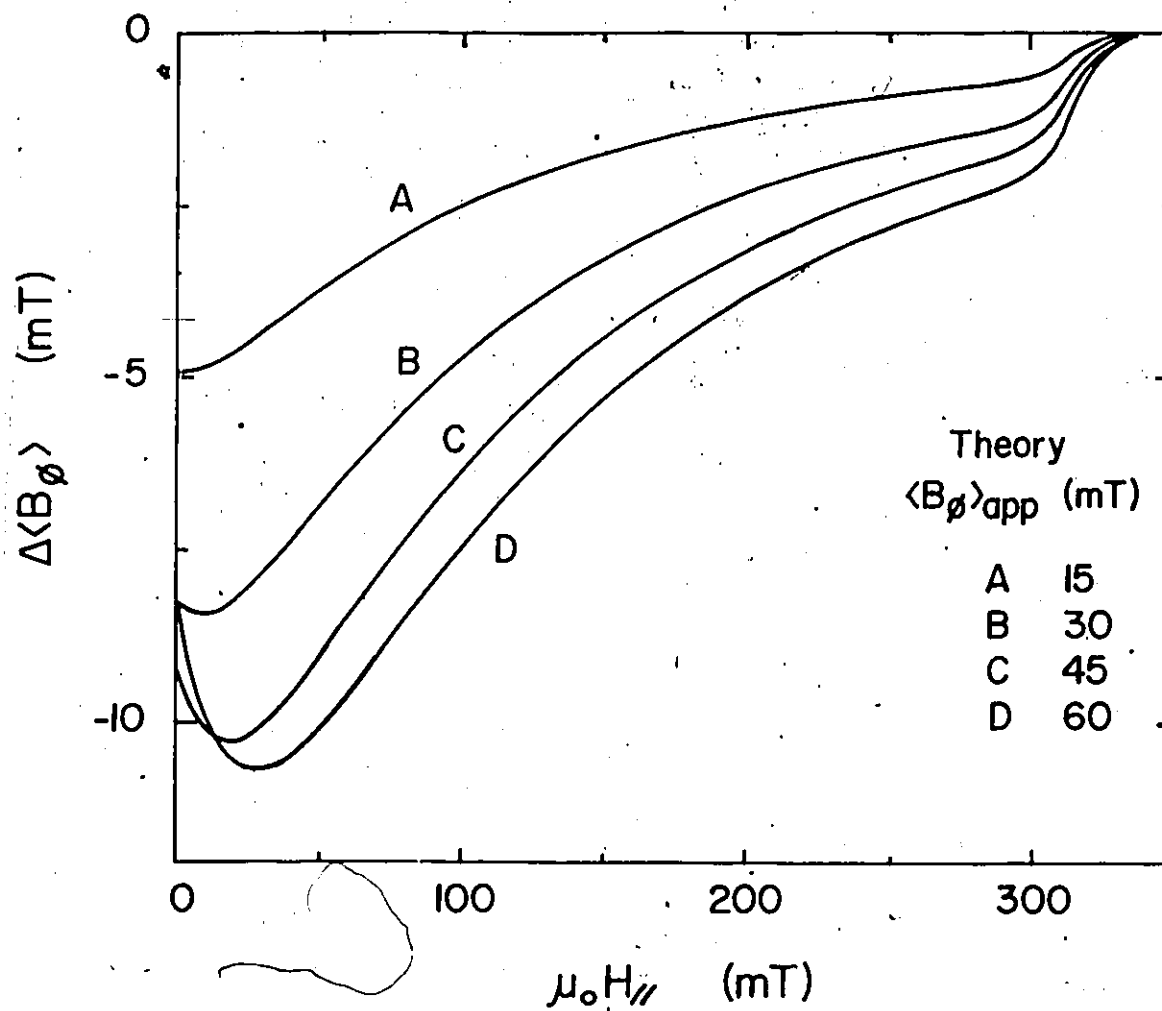


Fig. 9-9 Evolution of the azimuthal diamagnetic effect observed in various stationary azimuthal fields as $H_{//}$ is decreased from $H_{// \text{ max}}$ to zero.

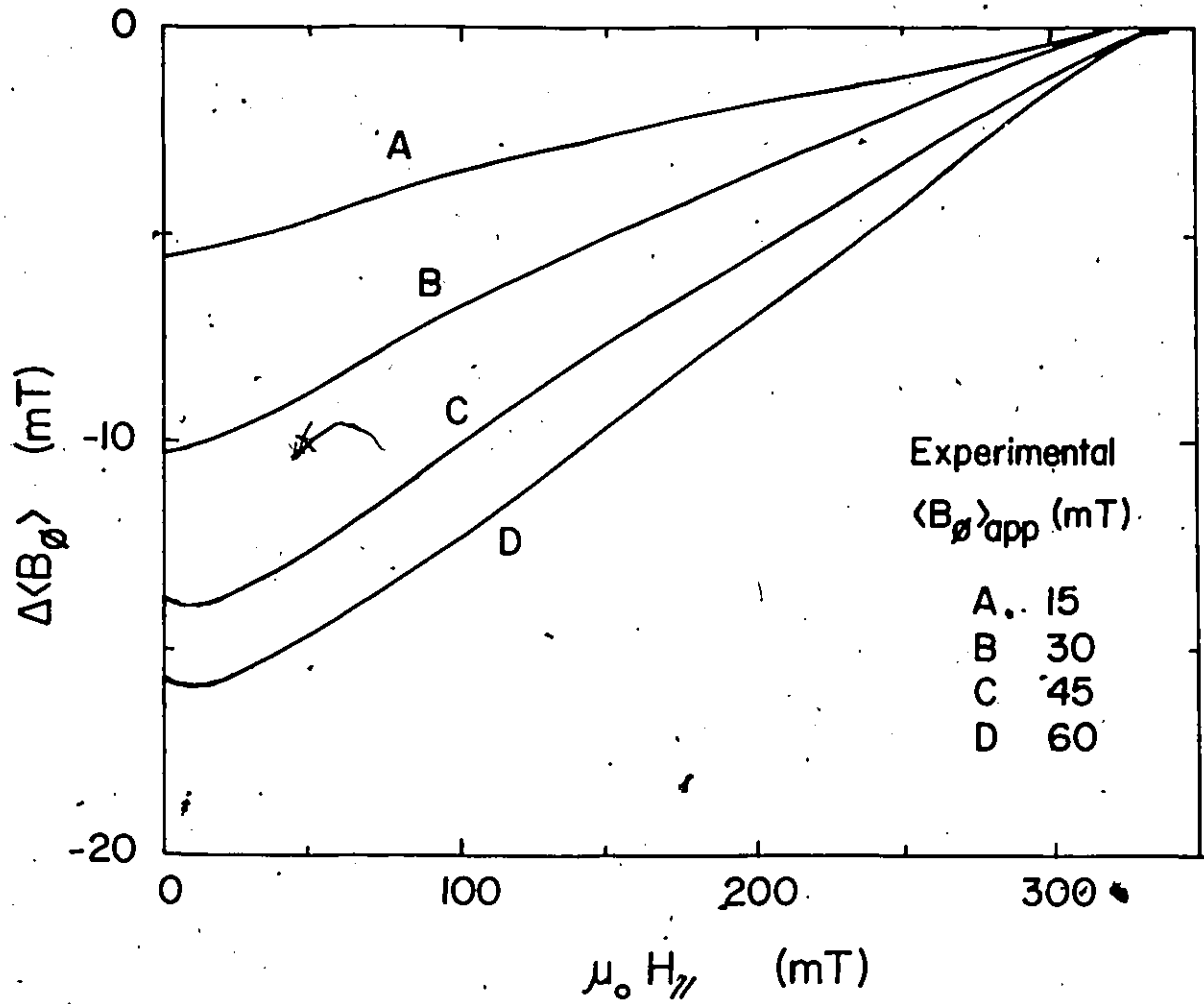
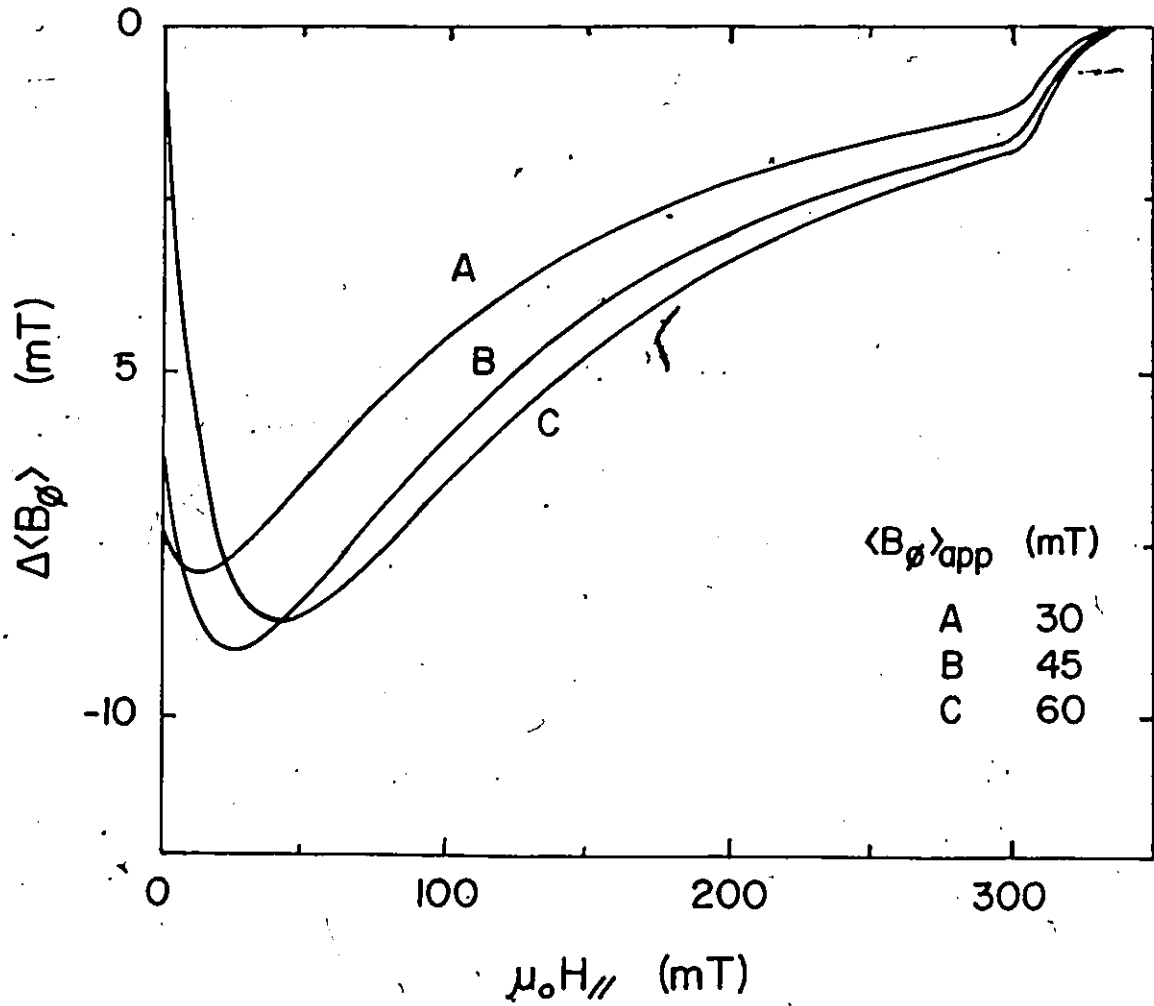


Fig. 9-10 Illustrates the effect of varying the magnitude of $j_{c//}$ on the calculations of the evolution of the azimuthal diamagnetic effect. Here the parameter D in the expression for $j_{c//}$ is $1.53(10^7)$ compared to $2.30(10^7)$ AT/m² for Fig. 9-8.



APPENDIX A

Azimuthal Paramagnetic or Diamagnetic Effect

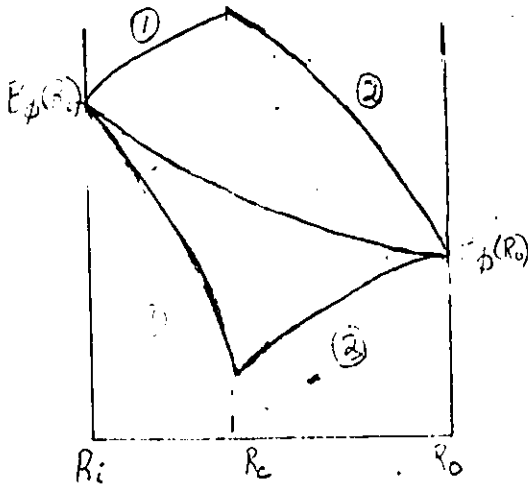
We develop an expression for the excess or deficit of azimuthal flux in the wall of the hollow cylinder bathing in a stationary externally applied azimuthal field H_ϕ . This configuration would arise when the penetration (exit) of helical flux at the outer surface generated by the increase (decrease) of $H_{//}$ reaches the inner radius. We assume,

- (i) a uniform longitudinal current density j_z and
- (ii) that the entire thickness of the wall, $R_o - R_i$, is occupied by persistent circulating j_z .

The paramagnetic and diamagnetic $B_\phi(r)$ profiles are sketched below. Ampère's circuital law, $\oint \vec{B} \cdot d\vec{l} = B_\phi 2\pi r = \mu_o I_{encl}$ where I_{encl} is the enclosed current, leads to,

$$B_{\phi \text{ app}} = \frac{\mu_o I}{2\pi r} \quad (1)$$

when the wall is in the normal state and I is the total current in the toroidal magnet coil.



$$B_{\phi 1} = \frac{\mu_o I}{2\pi r} + \frac{\mu_o j_z}{2r} (r^2 - R_i^2), \quad R_i < r < R_c \quad (2)$$

$$B_{\phi 2} = \frac{\mu_o I}{2\pi r} + \frac{\mu_o j_z}{2r} (R_c^2 - R_i^2) - \frac{\mu_o j_z}{2r} \left(\frac{r^2 - R_c^2}{2} \right), \quad R_c < r < R_o \quad (3)$$

Since the net longitudinal current I_z must be zero, then,

$$R_c = \left(\frac{R_i^2 + R_o^2}{2} \right) \quad (3)$$

Introducing, the above into the definitions,

$$\Delta \langle B_\phi \rangle = \langle B_\phi \rangle - \langle B_\phi \rangle_{app} \quad (4)$$

where

$$\langle B_\phi \rangle_{app} = \frac{1}{(R_o - R_i)} \int_{R_i}^{R_o} B_{\phi app} dr = \frac{\mu_o I}{2\pi} \ln\left(\frac{R_o}{R_i}\right) \quad (5)$$

$$(R_o - R_i) \langle B_\phi \rangle = \int_{R_i}^{R_c} B_{\phi 1} dr + \int_{R_c}^{R_o} B_{\phi 2} dr \quad (6)$$

leads to

$$\Delta \langle B_\phi \rangle = \frac{\pm \mu_o j_z}{2(R_o - R_i)} \left\{ (R_o^2 + R_i^2) \ln\left(\frac{R_2}{R_c}\right) - R_i^2 \ln\left(\frac{R_o}{R_i}\right) \right\} \quad (7)$$

where the + sign applies to the paramagnetic and the - sign to the diamagnetic azimuthal effect.

APPENDIX B

Longitudinal Paramagnetic or Diamagnetic Effect

We develop expressions for the excess or deficit of longitudinal magnetic flux in the wall of a hollow cylinder in a stationary externally applied $H_{//}$ for various simple configurations of $B_z(r)$. These configurations are associated with the penetration (or exit) of longitudinal flux at the inner and outer surfaces of the wall. The situations we examine arise once the two advancing (or receding) flux fronts meet and coalesce. We assume that,

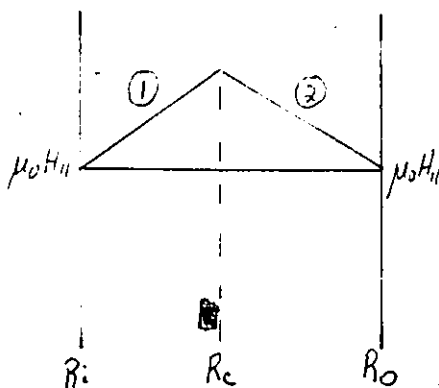
- (i) the azimuthal current density j_ϕ is uniform and,
- (ii) j_ϕ fills the entire width, $R_o - R_i$, of the wall.

R_o and R_i denote the outer and inner radii of the hollow cylinder.

We examine four types of B_z profiles. In each case the diamagnetic profile is the mirror image of the paramagnetic profile with the horizontal axis as the mirror.

Case 1: Single Peak.

The paramagnetic profile is sketched below.



In region 1,

$$B_{z1}(r) = \mu_o H_{//} + \mu_o j_\phi (r - R_i) \quad (1)$$

In region 2,

$$B_{z2}(r) = \mu_o H_{//} + \mu_o j_\phi (R_o - r) \quad (2)$$

Since $B_{z1}(R_c) = B_{z2}(R_c)$ then

$$R_c = \frac{R_1 + R_o}{2} \quad (3)$$

Introducing these relations into the definitions,

$$\Delta \langle B_z \rangle_{\text{wall}} = \langle B_z \rangle_{\text{wall}} - \mu_o H // \quad (4)$$

where

$$\langle B_z \rangle_{\text{wall}} = \frac{2}{(R_o^2 - R_1^2)} \left\{ \int_{R_1}^{R_c} B_{z1} r dr + \int_{R_c}^{R_o} B_{z2} r dr \right\} \quad (5)$$

leads to,

$$\Delta \langle B_z \rangle_{\text{wall}} = \frac{\mu_o j \phi (R_1^3 - 2R_c^3 + R_o^3)}{3 (R_o^2 - R_1^2)} \quad (6)$$

which simplifies to,

$$\Delta \langle B_z \rangle_{\text{wall}} = \frac{\pm \mu_o j \phi (R_o - R_1)}{4} \quad (7)$$

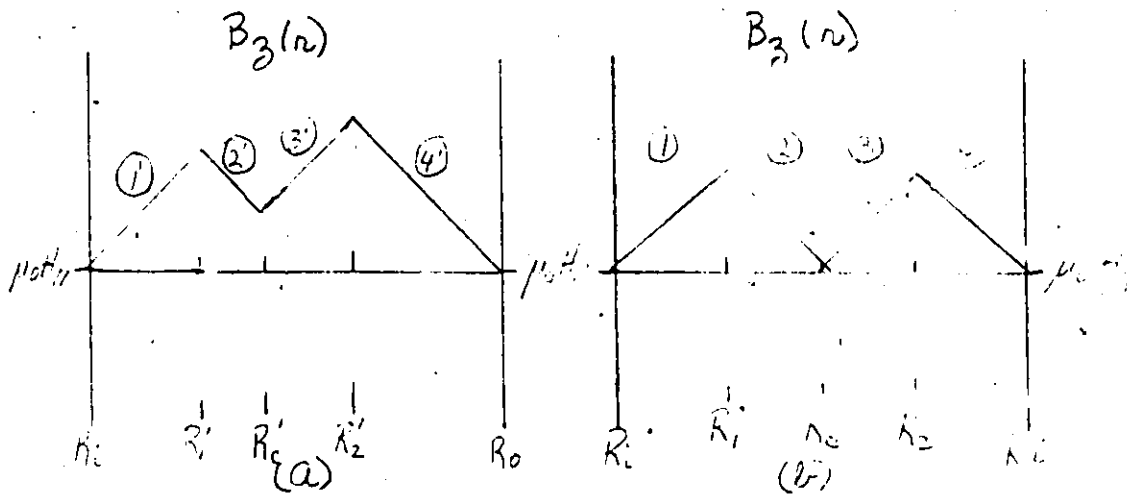
A surprisignly simple result in view of the cylindrical $2\pi r$ factor in the integrals of equation 5. The equivalent expression for an infinite slab of thickness X is essentially identical and reads,

$$\Delta \langle B_z \rangle_{\text{slab}} = \frac{\pm \mu_o j X}{4} \quad (8)$$

The + sign applies to the paramagnetic and the - sign to the diamagnetic situation.

Case 2: Two Symmetric Peaks. $B_z(R'_c) = \mu_o H //$

One can envisage a spectrum of possible two peak $B_z(r)$ configurations as sketched in (a) below. We focus on the simple symmetric configuration sketched in (b) where $R'_c = R_c$ and $B_z(R'_c) = \mu_o H //$.



Using a numerical subscript to indicate $B_z(r)$ in the various regions we write

$$B_{z1}(r) = \mu_o H_{//} + \mu_o j_\phi (r - R_1), \quad (9)$$

$$B_{z2}(r) = \mu_o H_{//} + \mu_o j_\phi (R_c - r) \quad (10)$$

$$B_{z3}(r) = \mu_o H_{//} + \mu_o j_\phi (r - R_c), \quad (11)$$

$$B_{z4}(r) = \mu_o H_{//} + \mu_o j_\phi (R_o - r) \quad (12)$$

Since,

$$B_{z1}(R_1) = B_{z2}(R_1), B_{z2}(R_c) = B_{z3}(R_c) = \mu_o H_{//}, B_{z3}(R_2) = B_{z4}(R_2) \quad (13)$$

Then,

$$R_1 = \frac{R_i + R_c}{2}, \quad R_c = \frac{R_i + R_o}{2}, \quad R_2 = \frac{R_c + R_o}{2} \quad (14)$$

Introducing these expressions into the definition,

$$\langle B_z \rangle_{\text{wall}} = \frac{2}{(R_o^2 - R_i^2)} \int_{R_1}^{R_o} B_z(r) r dr \quad (15)$$

Integrating and simplifying leads to

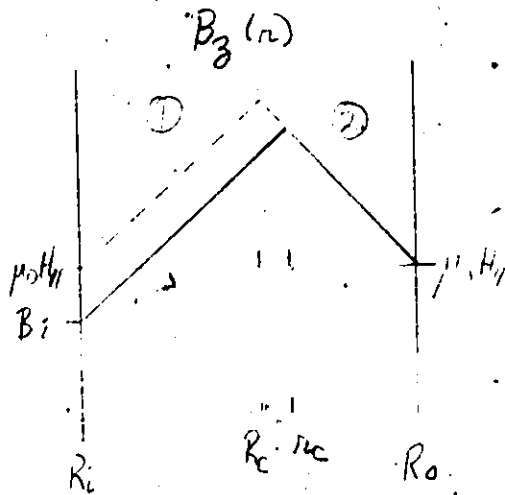
$$\Delta \langle B_z \rangle_{\text{wall}} = \frac{\pm \mu_o j_\phi}{8} (R_o - R_i) \quad (16)$$

Alternatively, this result can be obtained by considering the wall as consisting of two concentric walls in contact at $r = R_c$ and exploiting equation 8.

Case 3: Single Peak. $B_i < \mu_o H_{//}$

Flux has been transferred from the hole to the wall or vice

versa, hence $B_1 \neq \mu_0 H_{//}$. We refer the reader to the sketch depicting a



paramagnetic $B_z(r)$ profile.

$$B_{z1}(r) = B_1 + \mu_0 j_\phi (r - R_1) \quad (17)$$

$$B_{z2}(r) = \mu_0 H_{//} + \mu_0 j_\phi (R_o - r) \quad (18)$$

$$\text{Since } B_{z1}(r_c) = B_{z2}(r_c),$$

Then,

$$r_c = \frac{\mu_0 H_{//} - B_1}{2\mu_0 j_\phi} + \frac{R_1 + R_o}{2} = \Delta + R_c \quad (19)$$

Introducing these expressions into the definition,

$$\Delta \langle B_z \rangle_{\text{cyl}} = \langle B_z \rangle_{\text{cyl}} - \mu_0 H_{//} \quad (20)$$

Where,

$$\langle B_z \rangle_{\text{cyl}} = \frac{2}{R_o^2} \int_0^{R_o} B_z(r) r dr \quad (21)$$

where

$$B_z(r) = B_1 \quad 0 < r < R_1 \quad (22)$$

Integrating and simplifying leads to,

$$\Delta \langle B_z \rangle_{\text{cyl}} = \frac{\pm \mu_0 j_\phi}{3 R_o^2} \{ R_1^3 - 2(R_c + \Delta)^3 + R_o^3 \} \quad (23)$$

where $\Delta = (\mu_0 H_{//} - B_1)/2 \mu_0 j_\phi$

Equation 23 reduces, after some algebra, to equation 6, hence equation 7, as it must, when $B_1 = \mu_0 H_{//}$, hence $\Delta = 0$, and we note that by definition,

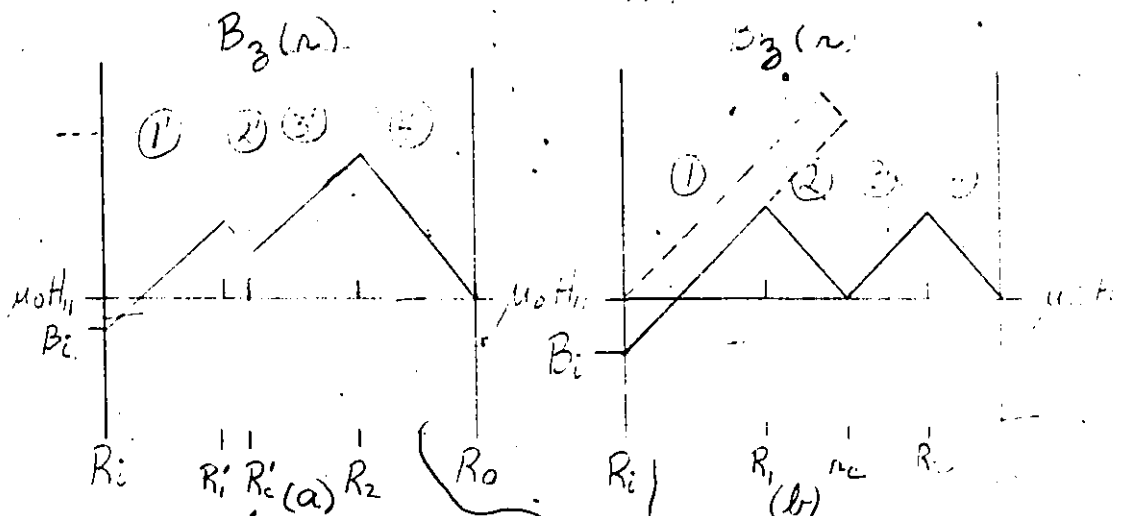
$$\Delta \langle B_z \rangle_{\text{wall}} = \Delta \langle B_z \rangle_{\text{cyl}} \frac{R_o^2}{(R_o^2 - R_1^2)} \quad (24)$$

Again we can envisage a spectrum of possible two peak configurations as sketched in (a) below. We focus on the simple configuration sketched in (b) where,

$$B_z(R_1) = B_z(R_2), \quad (25)$$

and

$$B_z(r_c) = \mu_o H_{//} \quad (26)$$



$$B_{z1}(r) = B_1 + \mu_o j_\phi (r - R_1) \quad (27)$$

$$B_{z2}(r) = \mu_o H_{//} + \mu_o j_\phi (r_c - r) \quad (28)$$

$$B_{z3}(r) = \mu_o H_{//} + \mu_o j_\phi (r - r_c) \quad (29)$$

$$B_{z4}(r) = \mu_o H_{//} + \mu_o j_\phi (R_0 - r) \quad (30)$$

The symmetry constraint (equation 25), together with equation 26 and the choice of linear profiles leads to equation 19 again,

$$r_c = \frac{\mu_o H_{//} - B_1}{2 \mu_o j_\phi} + \frac{R_1 + R_0}{2} = \Delta + R_c \quad (19)$$

Since

$$B_{z1}(R_1) = B_{z2}(R_1) \text{ and } B_{z3}(R_2) = B_{z4}(R_2)$$

$$R_1 = \Delta + \frac{r_c + R_1}{2}, \text{ and } R_2 = \frac{r_c + R_0}{2} \quad (31)$$

Introducing these into the definitions (equations 20 and 21), integrating and simplifying leads to,

$$\Delta \langle B_z^2 \rangle_{\text{cyl}} = \frac{\pm \mu_o j_\phi}{3 R_0^2} \{ R_1^3 + 2(R_c + \Delta)^3 + R_0^3 - R_1^3 - R_2^3 \} \quad (32)$$

which, after some manipulation, can be seen to reduce to equation 16 when $B_1 = \mu_o H_{//}$, hence $\Delta = 0$, and we make use of equation 24.

APPENDIX C

Relationship Between I_ϕ and $\Delta\langle B_z \rangle_{\text{cyl}}$

The assumption of a uniform azimuthal current density j_ϕ filling the thickness, $R_o - R_i$, of the wall of the hollow cylinder leads to a simple ratio for I_ϕ and $\Delta\langle B_z \rangle_{\text{cyl}}$. We recall that by definition;

$$I_\phi = \pm (H_{//} - \frac{B_1}{\mu_o}) \quad (1)$$

where B_1 is the longitudinal flux density in the hole, and,

$$\Delta\langle B_z \rangle_{\text{cyl}} = \pm (\mu_o H_{//} - \langle B_z \rangle_{\text{cyl}}) \quad (2)$$

where

$$\langle B_z \rangle_{\text{cyl}} = \frac{2}{R_o^2} \int_0^{R_o} B_z(r) r dr \quad (3)$$

Maxwell's equation $dB_z/dr = -\mu_o j_\phi$ leads to,

$$B_z(r) = \mu_o H_{//} \pm \mu_o j_\phi (r - R_o), R_i < r < R_o \quad (4)$$

The + and - signs depend on whether the current is flowing in a flux shielding (diamagnetic) or flux retaining (paramagnetic) sense. Introducing equation 4 into the definitions, integrating and simplifying leads to

$$\frac{\Delta\langle B_z \rangle_{\text{cyl}}}{\mu_o I_\phi} = \frac{1}{3} \left(1 + \left(\frac{R_i}{R_o} \right) + \left(\frac{R_i}{R_o} \right)^2 \right) \quad (5)$$

This ratio is 0.84 for our sample.

Summary and Conclusions

We have documented the results of extensive measurements on the evolution of the azimuthal flux density, $\langle B_\phi \rangle$, in the wall of a hollow cylinder of a type II superconductor immersed in a static azimuthal field, H_ϕ , as an externally applied longitudinal magnetic field, $H_{//}$, is increased from zero and also when $H_{//}$ is swept from $H_{//max}$ through zero, to $-H_{//max}$. The measurements were performed for several static H_ϕ in the range $0 < H_\phi \lesssim 100$ mT. Concurrently with these observations, we have monitored the effect of H_ϕ on I_ϕ , the capacity of the wall of the cylinder to shield against the entry of the applied longitudinal flux and its ability to oppose the exit of flux from the hole of the specimen.

Assuming j_z and j_ϕ , the longitudinal and azimuthal current densities to be uniform throughout the wall and postulating the simplest triangular B_ϕ profiles consistent with the observations of paramagnetic or diamagnetic behaviour of $\langle B_\phi \rangle$ in the chosen static H_ϕ , we have extracted estimates of $j_{//}$ and $j_\perp$ versus B from these data. The results show that $j_\perp$ is critical, as expected, when the increase (decrease) of $H_{//}$ causes, $B_\perp$, the longitudinal flux density in the hole to increase (decrease). The behaviour obtained for $j_{//}$ versus B in the various fixed H_ϕ , reveal that it is not in a critical configuration over the volume of the wall for any of the situations which we have explored. This conclusion is in harmony with the results of our computer analysis of the data exploiting the coupled equations developed by Clem in his formulation of a general critical state theory. Our calculations reveal that a flux transport zone, where $E_{//} = 0$ and $0 < j_{//} < j_{c//}$, occupies an appreciable fraction of the wall cross-section

during both the increase of $H_{//}$ from zero and its decrease from $H_{//\max}$ to zero.

The display of an increase in $\langle B_{\phi} \rangle$, in a static H_{ϕ} , during the upsweep of $H_{//}$ is called the azimuthal paramagnetic effect. The simple picture, proposed by many workers, of helical flux lines nucleating at the outer surface of the cylinder then advancing in the wall as the total surface field grows, while preserving their pitch of creation as they migrate, appears at first glance, to account for this phenomenon. This picture has also been invoked to explain the decrease of $\langle B_{\phi} \rangle$ when $H_{//}$ is reduced to zero with H_{ϕ} kept fixed. The latter behaviour is denoted, the azimuthal diamagnetic effect. This naive intuitive model however has been shown to be fundamentally flawed when carefully scrutinized. It violates the required boundary conditions when the flux disturbance reaches the inner surface of the wall. Further it leads to discontinuities in the B_{ϕ} and B_z components of the $\vec{B}(r)$ profiles at the outer surface when $H_{//}$ is diminished.

A shallow diamagnetic valley observed in $\langle B_{\phi} \rangle$ versus $H_{//}$ increasing in static H_{ϕ} , provides direct and striking evidence that flux cutting processes must be taking place. A sharp descent in $\langle B_{\phi} \rangle$ when the growth in $H_{//}$ has caused the flux front to travel across the width of the wall provides a further dramatic manifestation that the novel phenomenon of flux cutting is occurring extensively at this juncture. Finally, the decline of $\langle B_{\phi} \rangle$ after tracing a second summit, again attests that this mechanism must be playing a major role in causing the redistribution of flux which is being witnessed. The decrease of $\langle B_{\phi} \rangle$ as $H_{//}$ is being reduced to zero with H_{ϕ} fixed, presents additional and unambiguous evidence for and information on the mechanism of flux cutting.

Exploiting the concept of sequences of quasi steady flux configurations, we have applied Clem's theory to the interpretation of families of curves for $\langle B \rangle_\phi$ and I_ϕ versus $H_{//}$ decreasing from $H_{//\max}$ to zero in various static H_ϕ . The families of calculated and measured curves are seen to be in excellent agreement.

Our approach however is inadequate for a quantitative account of several important features observed during the upsweep of $H_{//}$. We note that the arrival of the flux disturbance at the inner surface of the wall generates large electric fields. This causes both $j_{//}$ and $j_\perp$ to significantly exceed $j_{c//}$ and $j_{c\perp}$ and invalidates our assumption of quasi-static configurations. This failure of our simple computational framework indicates that a relaxation or diffusion method should be introduced to treat these difficult situations. Such work is in progress.

We have also monitored and catalogued the "mirror" behaviour, i.e. the evolution of $\langle B \rangle_{z \text{ wall}}$ in a static $H_{//}$ as H_ϕ is impressed and swept from $H_{\phi\max}$ through zero to $-H_{\phi\max}$. As now anticipated from the previous work, we observe a paramagnetic (diamagnetic) magnetization of the wall along the static $H_{//}$ as H_ϕ is applied (reduced). Further, we confirm with detailed quantitative data, the expectation that the longitudinal flux density in the hole, (i) will be depleted and transferred to the wall when H_ϕ is made to increase and (ii) will be augmented when the decrease of H_ϕ releases helical flux lines from the wall into the hole. The numerous and fascinating data mentioned in this paragraph remain to be subjected to theoretical analysis.

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