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HEAT TRANSFER AND PRESSURE DROP IN
INTERNALLY FINNED TUBES

By

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Submitted to the School of Graduate Studies
of the University of Ottawa
in partial fulfillment of the requirements
for the degree of
MASTER OF APPLIED SCIENCE
in Mechanical Engineering
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HEAT TRANSFER AND PRESSURE DROP IN
INTERNALLY FINNED TUBES

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J

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ABSTRACT

Heat transfer and pressure drop characteristics are investigated using finite difference technique for a laminar forced convection fully developed flow in internally finned circular tubes, assuming a fin shape very close to the real fin configuration. For the same mass flow rate, the larger the number of fins, their height and thickness the higher will be the flow friction. For a given fin configuration, there exists an optimum fin number for maximum heat transfer.

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NOMENCLATURE

A_f	flow area of the tube, dimensionless
A_f^*	flow area of the tube (ft^2)
C_1	constant parameter, defined by Eqn. (29)
C_2	K_f/K_s , constant parameter
C_p	specific heat at constant pressure ($\text{Btu}/\text{lb}_m\text{-}^\circ\text{F}$)
f	friction factor, dimensionless
g_c	conversion factor ($\text{lb}_m\text{-ft}/\text{lb}_f\text{-hr}^2$)
h	heat transfer coefficient ($\text{Btu}/\text{hr-ft}^2\text{-}^\circ\text{F}$)
K_f	thermal conductivity of fluid ($\text{Btu}/\text{hr-ft-}^\circ\text{F}$)
K_s	thermal conductivity of solid ($\text{Btu}/\text{hr-ft-}^\circ\text{F}$)
l	l^*/r_0^* , fin height, dimensionless
l^*	fin height (ft)
\dot{m}	mass flow rate (lb_m/hr)
M	number of fins
N_u	Nusselt number, defined by Eqn. (53)
p^*	pressure (lb_f/ft^2)
q	heat flux ($\text{Btu}/\text{ft}^2\text{-hr}$)
r	r^*/r_0^* , radial coordinate, dimensionless
r_i	the difference ($1-l$)
r^*	radial coordinate (ft)
r_0^*	inner radius of the tube (ft)
Re_e	Reynolds number, defined by Eqn. (46)
T	temperature defined by Eqn. (25-e), dimensionless
T^*	temperature ($^\circ\text{F}$)
T_b	bulk temperature defined by Eqn. (56), dimensionless

T_f^*	fluid temperature ($^{\circ}\text{F}$)
T_R^*	reference temperature ($^{\circ}\text{F}$)
T_S^*	solid temperature ($^{\circ}\text{F}$)
u	axial velocity defined by Eqn. (25-e), dimensionless
u^*	axial velocity (ft/hr)
u_b	bulk velocity defined by Eqn. (50), dimensionless
x^*	axial coordinate (ft)
$\Delta r, \Delta r_1$	grid sizes in the radial direction, dimensionless

Greek symbols:

ϕ	half the angle between the flanks of two adjacent fins
β	half the angle subtended by one fin
γ	Π/M , half the angle between the centre-lines of two adjacent fins
θ	angular coordinate
ρ	fluid density (lb_m/ft^3)
μ	dynamic viscosity of the fluid ($\text{lb}_m/\text{hr-ft}$)
α	$K/\rho C_p$, thermal diffusivity (ft^2/hr)
δ	δ^*/r_0^* , tube thickness, dimensionless
δ^*	tube thickness (ft)
$\Delta\beta, \Delta\theta$	grid sizes in the angular direction

Superscripts:

-	average value
*	dimensional quantity

Subscripts

b bulk value
i pertaining to the tip of the fins
w wall value

Chapter 1

INTRODUCTION

Internally finned tubes have the advantage, compared with the smooth tubes, of providing increased heat transfer area per unit length of the tube. Accordingly they are used in the manufacture of the compact heat exchange equipment where weight and size are of importance. The presence of fins in a tube will alter the flow pattern and hence the flow friction.

A few experimental investigations [1,2,3,4] have been reported in the literature concerning heat transfer and pressure drop characteristics of internally finned tubes with different fin designs during laminar and turbulent flows of different fluids. A survey carried out showed that the available data are limited to a certain range of the values of the independent parameters. For example, no experimental data exist for laminar flows, for continuous straight fins extended to about 40 percent or more of the tube radius. Since the use of internally finned tubes is not yet widespread, the cost of manufacturing a series of tubes according to given specifications would be high. Thus a purely experimental approach to the study of flow characteristics over a range of conditions would involve a considerable expenditure of time and money. An alternative way of tackling this problem is to solve numerically the equations which describe the transport of momentum and heat in an internally finned tube. The numerical procedures do not eliminate the need for experiments, but they diminish their extent and cost by highlighting the

most influential parameters governing the characteristics under given conditions.

The general trend of the experimental data showed enhancement in both heat transfer and pressure drop over smooth tube conditions.

A theoretical investigation of fully developed laminar forced convective heat transfer in internally finned tubes of zero thickness and with zero fin thickness, has been conducted by Hu and Chang [5]. They considered fully developed velocity and temperature profiles together with a uniform heat flux over the cylindrical surface and at each fin surface. They used a semianalytical approach to obtain the solution for friction factor and Nusselt number.

Masliyah and Nandakumar [6] presented results for internally finned tubes of zero thickness with fins of triangular shape (finite thickness), considering fully developed laminar flow. They assumed axially uniform heat flux with peripherally uniform temperature. The temperature along the fin was also assumed to be equal to the wall temperature. This paper utilized finite-element technique for the solution of the momentum and energy equations. It was shown that for circular sectors ($\ell=1$ and $\beta=0$), the results obtained by the finite element technique were in excellent agreement with that of the finite-difference method.

The objective of the present study is to investigate the fully developed laminar flow in internally finned tubes with fin shapes approximating as close as possible to the real fin configuration. Nusselt number and friction factor values are evaluated for a wide range of fin heights, numbers, and thickness, and the effects of these parameters are investigated.

Statement of the Problem

The problem to be investigated is concerned with the pressure drop and heat transfer in laminar flow through an internally finned tube of finite thickness as shown schematically in Fig. 1. The fin profile in this study is defined by two radial flanks encompassing a variable angle and having a variable height. The tip of the fin is a circular arc concentric with the tube. Fins are straight and equally distributed around the periphery of the tube. Properties of the fluid are assumed constant. The velocity and temperature profiles are assumed to be fully developed. A uniform heat flux is assumed at the outer surface of the tube. Finite-difference method is used for the solution of the momentum and energy equations.

The results are obtained considering a constant heat flux at the outer surface of the tube and they would be different for the case where the surface temperature is constant.

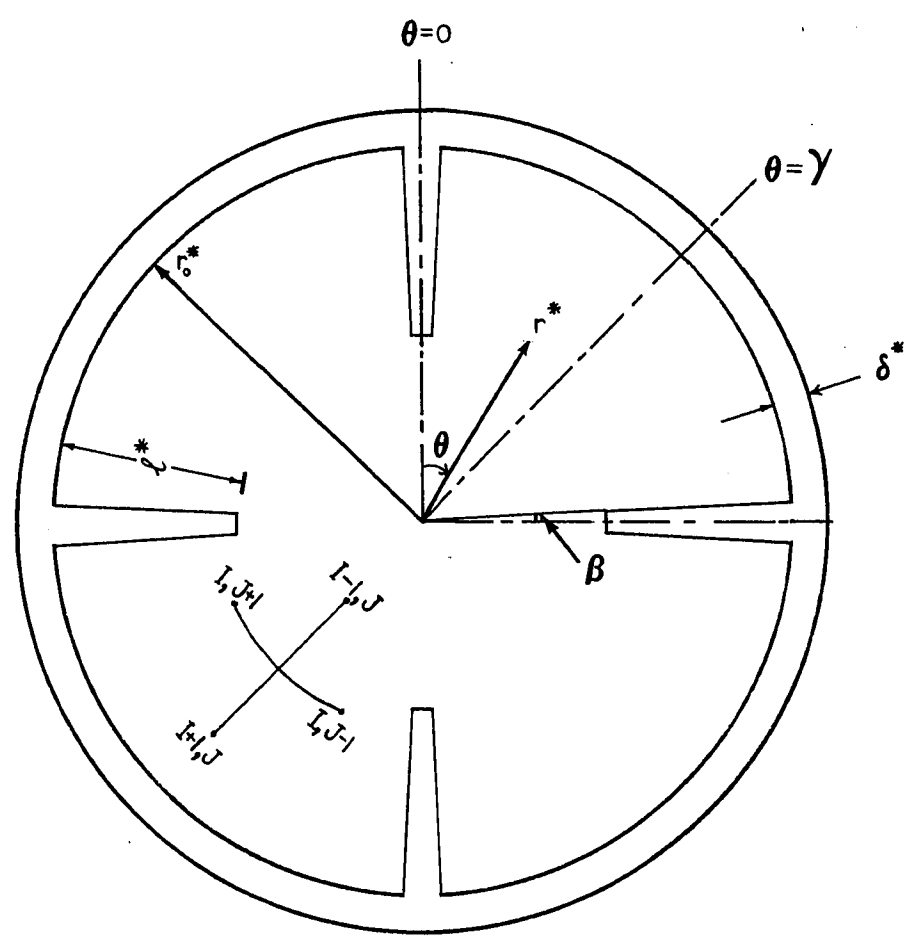


Fig. 1 Geometry of the internally finned tube

Chapter 2

THE MATHEMATICAL FORMULATION

2.1 The domain of interest

In the region of the fully-developed flow, the distributions of the velocities are independent of the axial distance along the tube; in other words, they are identical at every cross-section of the tube.

With the assumption of fully-developed flow, the only force acting on the fluid to cause a pressure drop is the viscous force. Since the viscous force is constant along x^* , we have

$$\frac{dp^*}{dx^*} = \text{constant} \quad (1)$$

The term fully-developed temperature profile implies that there exists a generalized temperature profile $(T^* - \bar{T}_w^*) / (T_b^* - \bar{T}_w^*)$, invariant with tube length, which can be expressed as

$$\frac{\partial}{\partial x^*} \left(\frac{T^* - \bar{T}_w^*}{T_b^* - \bar{T}_w^*} \right) = 0 \quad (2)$$

By carrying through the indicated differentiation of equation (2), and noting that \bar{T}_w^* and T_b^* are functions of x^* only, the following is obtained

$$\frac{\partial T^*}{\partial x^*} = \frac{d\bar{T}_w^*}{dx^*} - \frac{T^* - \bar{T}_w^*}{T_b^* - \bar{T}_w^*} \frac{d\bar{T}_w^*}{dx^*} + \frac{T^* - \bar{T}_w^*}{T_b^* - \bar{T}_w^*} \frac{dT_b^*}{dx^*} \quad (3)$$

If q is the uniform heat flux on the outer surface at every cross-section of the tube, then

$$q[2\pi(r_0^* + \delta^*)] = -\dot{m} C_p \frac{dT_b^*}{dx^*} \quad (4)$$

Consequently, if, q was assumed constant, then

$$\frac{dT_b^*}{dx^*} = \text{constant} \quad (5)$$

The convection rate equation is written as

$$q = h(T_b^* - \bar{T}_w^*) = \text{constant} \quad (6)$$

If h is constant, then

$$T_b^* - \bar{T}_w^* = \text{constant} \quad (7)$$

from which

$$\frac{dT_b^*}{dx^*} = \frac{d\bar{T}_w^*}{dx^*} \quad (8)$$

Thus substituting in equation (3), we get

$$\frac{\partial T^*}{\partial x^*} = \frac{dT_b^*}{dx^*} \quad (9)$$

2.2 Governing Equations

(a) Within the solid

$T_s^* = T_s^*(r^*, \theta)$, steady two-dimensional heat conduction with uniform properties and no heat generation. The governing equation for the heat conduction within the solid (tube and fins) will be

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_s^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 T_s^*}{\partial \theta^2} = 0 \quad (10)$$

The axial heat conduction within the solid is neglected.

(b) Within the fluid

$T_f^* = T_f^*(r^*, \theta)$, steady two-dimensional heat flow with uniform properties and no heat generation. The conservation equations for the momentum and energy within the fluids are

Momentum:

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial u^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 u^*}{\partial \theta^2} = \frac{1}{\mu} \frac{dp^*}{dx^*} \quad (11)$$

Energy:

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_f^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 T_f^*}{\partial \theta^2} = \frac{1}{\alpha} u^* \frac{\partial T_f^*}{\partial x^*} \quad (12)$$

Dissipation due to viscous effect in the fluid is neglected.

2.3 Boundary Conditions

Before stating the boundary conditions of this problem, it should be pointed out that the lines $\theta=0$ and $\theta=\gamma$ are symmetry lines, and consequently, the solution needs to be sought only within the domain limited by these two lines.

(a) Within the solid

$$\frac{\partial T_s^*}{\partial \theta} = 0 \quad \text{at} \quad \theta=0, \quad r_i^* \leq r^* \leq (r_0^* + \delta^*) \quad (13)$$

$$\frac{\partial T_s^*}{\partial \theta} = 0 \quad \text{at} \quad \theta=\gamma, \quad r_0^* \leq r^* \leq (r_0^* + \delta^*) \quad (14)$$

(b) Within the fluid

$$\frac{\partial u^*}{\partial \theta} = \frac{\partial T_f^*}{\partial \theta} = 0 \quad \text{at} \quad \theta=0, \quad 0 \leq r^* \leq r_i^* \quad (15)$$

$$\frac{\partial u^*}{\partial \theta} = \frac{\partial T_f^*}{\partial \theta} = 0 \quad \text{at} \quad \theta=\gamma, \quad 0 \leq r^* \leq r_0^* \quad (16)$$

$$\frac{\partial u^*}{\partial r^*} = \frac{\partial T_f^*}{\partial r^*} = 0 \quad \text{at} \quad r^* = 0, \quad \text{all } \theta \quad (17)$$

(c) At the solid-fluid interface

$$u^*=0 \quad \& \quad T_s^*=T_f^* \quad \text{at} \quad r^*=r_i^*, \quad 0 \leq \theta \leq \beta \quad (18)$$

$$K_s \frac{\partial T_s^*}{\partial r^*} = K_f \frac{\partial T_f^*}{\partial r^*} \quad \text{at} \quad r^*=r_i^*, \quad 0 \leq \theta \leq \beta \quad (19)$$

$$u^*=0 \quad \& \quad T_s^*=T_f^* \quad \text{at} \quad \theta=\beta, \quad r_i^* \leq r^* \leq r_0^* \quad (20)$$

$$K_s \frac{\partial T_s^*}{\partial \theta} = K_f \frac{\partial T_f^*}{\partial \theta} \quad \text{at} \quad \theta=\beta, \quad r_i^* \leq r^* \leq r_0^* \quad (21)$$

$$u^*=0 \quad \& \quad T_s^*=T_f^* \quad \text{at} \quad r^*=r_0^*, \quad \beta \leq \theta \leq \gamma \quad (22)$$

$$K_s \frac{\partial T_s^*}{\partial r^*} = K_f \frac{\partial T_f^*}{\partial r^*} \quad \text{at} \quad r^*=r_0^*, \quad \beta \leq \theta \leq \gamma \quad (23)$$

(d) At the outer surface of the tube

$$-K_s \frac{\partial T_s^*}{\partial r^*} = q \quad \text{at} \quad r^*=r_0^*+\delta^*, \quad \text{all } \theta \quad (24)$$

2.4 Dimensionless Equations

The following dimensionless quantities were introduced:

$$r = \frac{r^*}{r_0^*} \quad (25-a)$$

$$\ell = \frac{\ell^*}{r_0^*} \quad (25-b)$$

$$\delta = \frac{\delta^*}{r_0^*} \quad (25-c)$$

$$u = \frac{u^*}{\left[-r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*} \right]} \quad (25-d)$$

$$T = \frac{T^*-T_f^*}{(qr_0^*/K_f)} \quad (25-e)$$

The governing equations reduce to¹

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T_s}{\partial \theta^2} = 0 \quad (26)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = -1 \quad (27)$$

¹ The derivation of these equations is shown in the Appendix A.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T_f}{\partial \theta^2} = c_1 u \quad (28)$$

$$\text{where } c_1 = - \frac{2\Pi(1+\delta)}{A_f u_b} \quad (29)$$

The boundary conditions in terms of the dimensionless quantities are as follows:

$$\frac{\partial T_s}{\partial \theta} = 0 \quad \text{at } \theta=0, \quad r_i \leq r \leq (1+\delta) \quad (30-a)$$

$$\frac{\partial T_s}{\partial \theta} = 0 \quad \text{at } \theta=\gamma, \quad 1 \leq r \leq (1+\delta) \quad (30-b)$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial T_f}{\partial \theta} = 0 \quad \text{at } \theta=0, \quad 0 \leq r \leq r_i \quad (30-c)$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial T_f}{\partial \theta} = 0 \quad \text{at } \theta=\gamma, \quad 0 \leq r \leq 1 \quad (30-d)$$

$$\frac{\partial u}{\partial r} = \frac{\partial T_f}{\partial r} = 0 \quad \text{at } r=0, \quad \text{all } \theta \quad (30-e)$$

$$u=0 \quad \& \quad T_s=T_f \quad \text{at } r=r_i, \quad 0 \leq \theta \leq \beta \quad (30-f)$$

$$\frac{\partial T_s}{\partial r} = C_2 \frac{\partial T_f}{\partial r} \quad \text{at } r=r_i, \quad 0 \leq \theta \leq \beta \quad (30-g)$$

$$u=0 \quad \& \quad T_s=T_f \quad \text{at } \theta=\beta, \quad r_i \leq r \leq 1 \quad (30-h)$$

$$\frac{\partial T_s}{\partial \theta} = C_2 \frac{\partial T_f}{\partial \theta} \quad \text{at } \theta=\beta, \quad r_i \leq r \leq 1 \quad (30-k)$$

$$u=0 \quad \& \quad T_s=T_f \quad \text{at } r=1, \quad \beta \leq \theta \leq \gamma \quad (30-l)$$

$$\frac{\partial T_s}{\partial r} = C_2 \frac{\partial T_f}{\partial r} \quad \text{at } r=1, \quad \beta \leq \theta \leq \gamma \quad (30-m)$$

$$-\frac{\partial T_s}{\partial r} = C_2 \quad \text{at } r=(1+\delta), \quad \text{all } \theta \quad (30-n)$$

Chapter 3

METHOD OF SOLUTION

It was soon realized that because of the irregular nature of the flow boundaries it is unlikely that a closed-form solution can be found. Hence it was necessary to resort to one of the well-known numerical methods. One commonly used approach for such problems is the least-squares-matching technique. This method was successful in providing accurate solutions for problems such as the laminar flow inside ducts with triangular, sine, rhombic and trapezoidal cross-sections [15], and the laminar flow in polygonal shaped ducts with circular centered cores [16]. Nandakumar and Masliyah [7] found that this method was not successful for the values of ℓ greater than 0.1, for the laminar flow in internally finned tubes. Masliyah and Nandakumar [6] investigated the fully developed laminar flow in internally finned tubes using finite element method.

Another numerical technique applicable to the solution of partial differential equations is the finite difference method. A careful literature survey did not indicate that it has ever been employed for the calculation of the Nusselt number for the fully developed laminar flow in internally finned tubes. It was decided to make an attempt at adapting it to our circumstances.

3.1 Selection of Mesh Points

The accuracy of the friction factor and Nusselt number results

is mainly affected by the number and a judicial selection of mesh points. It is to be expected that an increase in the number of mesh points would increase the accuracy of the results but, in practice, it may not be true, because a very large matrix would suffer from computational errors outweighing the otherwise obvious advantages. The freedom of choice of the size of grid makes it possible to have smaller grid size in the critical region of the flow. The solutions were obtained with different grid sizes and, finally, a mesh with the optimum number of mesh points was selected, considering the accuracy and cost. Due to the large gradients that occur near the surface, more mesh points were taken in that region. In the region near the central core and far away from the fin tip, the variation in the velocity and temperature was very small in the angular direction and thus, less number of mesh points were taken in this region. Depending upon the length of the fin, two different arrangements were selected.

Mesh 1 for $0.2 \leq \ell \leq 0.5$, $8 \leq M \leq 32$, $1.5^\circ \leq \beta \leq 3.0^\circ$

Mesh 2 for $0.6 \leq \ell \leq 0.9$, $8 \leq M \leq 32$, $1.5^\circ \leq \beta \leq 3.0^\circ$

Let Δr and Δr_1 represent the grid sizes in the radial direction and $\Delta\beta$ and $\Delta\theta$ represent the grid sizes in the angular direction.

Mesh 1

In this mesh, 73 points were selected in the flow region, out of the total 115 mesh points as shown in Fig. 2. The grid sizes are described as follows:

$$\Delta r = \ell/5$$

$$\Delta r_1 = r_1/8$$

$$\Delta\beta = \beta/4$$

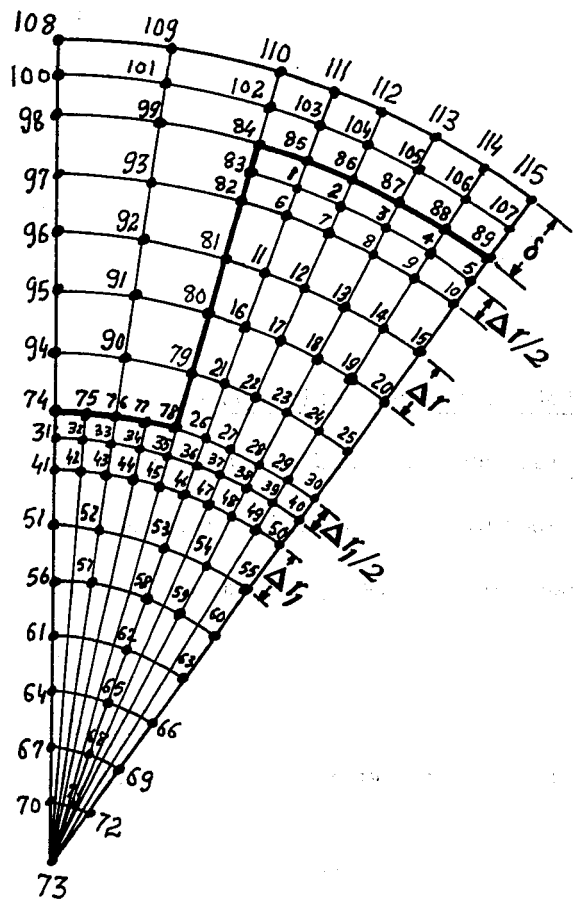


Fig. 2 Finite Difference Grid for Mesh 1

$$\Delta\theta = \phi/5$$

Mesh 2

Out of the total 123 points in the mesh, 72 points were taken in the flow region. This mesh is shown in Fig. 3. The grid sizes were selected as follows:

$$\Delta r = l/8$$

$$\Delta r_1 = r_i/4$$

$$\Delta\beta = \beta/4$$

$$\Delta\theta = \phi/5$$

3.2 The Finite-Difference Equations and their Solutions

The governing equations and boundary conditions were represented at each mesh point by difference equations which were obtained by using finite-difference techniques. These equations were formulated as follows:

Momentum equation

The momentum equation in the dimensionless form is written as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = -1 \quad (27)$$

This equation can be expressed in the finite difference form by using Taylor's expansion.

Points within the flow domain:

By employing Taylor series expansion, we can write

$$u(r+\Delta r, \theta) = u(r, \theta) + \frac{\partial u}{\partial r} \Delta r + \frac{\partial^2 u}{\partial r^2} \frac{(\Delta r)^2}{2} + \mathcal{O}(\Delta r^3) \quad (31)$$

$$u(r-\Delta r, \theta) = u(r, \theta) - \frac{\partial u}{\partial r} \Delta r + \frac{\partial^2 u}{\partial r^2} \frac{(\Delta r)^2}{2} - \mathcal{O}(\Delta r^3) \quad (32)$$

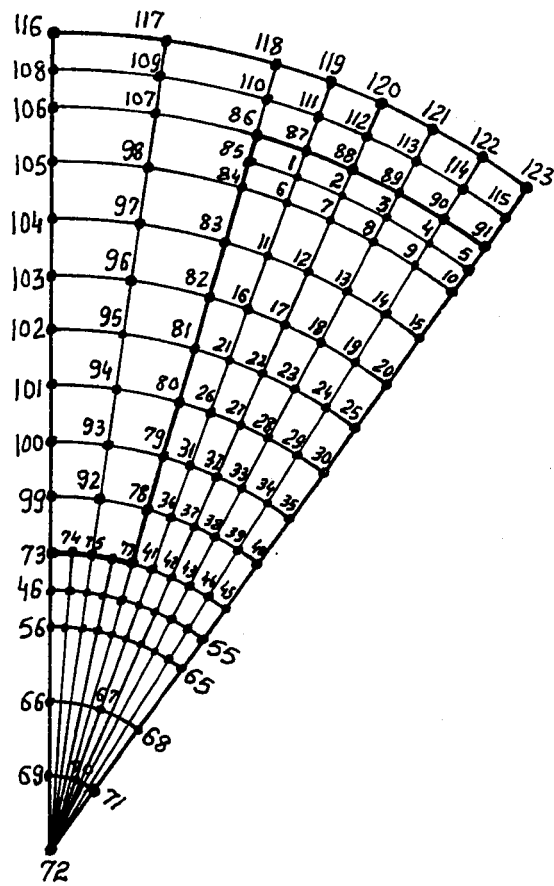


Fig. 3 Finite Difference Grid for Mesh 2

The notation $\mathcal{O}(\Delta r^3)$ is used to indicate that subsequent terms are of the order of (Δr^3) and higher. If terms of the order of (Δr^2) , and greater, are neglected, the following central finite difference equations for the first and second derivatives were obtained.

$$\frac{\partial u}{\partial r} = \frac{u(r+\Delta r, \theta) - u(r-\Delta r, \theta)}{2\Delta r} \quad (33)$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{u(r+\Delta r, \theta) + u(r-\Delta r, \theta) - 2u(r, \theta)}{(\Delta r)^2} \quad (34)$$

A similar development yields

$$\frac{\partial^2 u}{\partial \theta^2} = \frac{u(r, \theta+\Delta \theta) + u(r, \theta-\Delta \theta) - 2u(r, \theta)}{(\Delta \theta)^2} \quad (35)$$

If the derivatives in Eqn. (27) are replaced by the finite difference equations, the following expression results, which expresses the momentum equation in the finite difference form:

$$\begin{aligned} & \left(1 + \frac{\Delta r}{r}\right)u(I+1, J) + \left(1 - \frac{\Delta r}{r}\right)u(I-1, J) + \left(\frac{\Delta r}{r\Delta \theta}\right)^2 u(I, J+1) \\ & + \left(\frac{\Delta r}{r\Delta \theta}\right)^2 u(I, J-1) - \left[2 + 2\left(\frac{\Delta r}{r\Delta \theta}\right)^2\right]u(I, J) = -(\Delta r)^2 \quad (36) \end{aligned}$$

Linear interpolation was used wherever found necessary.

Points on the symmetry lines:

The following boundary conditions were imposed on the momentum equation for the points on the symmetry lines.

$$\frac{\partial u}{\partial \theta} = 0 \quad \text{at} \quad \theta=0, \quad 0 \leq r \leq r_i$$

$$\frac{\partial u}{\partial \theta} = 0 \quad \text{at} \quad \theta=\gamma, \quad 0 \leq r \leq 1$$

The Eqn. (36) was used together with the following relationship for writing finite difference equations at points on symmetry

lines:

$$u(I,J+1) = u(I,J-1)$$

At the axis of the tube:

Because of symmetry,

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r=0 \quad (37)$$

The forward finite difference expression for the first derivative is given by

$$4u(I+1,J) - u(I+2,J) - 3u(I,J) = 0 \quad (38)$$

The momentum equation was represented at each mesh point by finite-difference equation. The resulting system of linear algebraic equations was solved on a digital computer using a subroutine employing Gauss-Jordan reduction. Thus the dimensionless axial velocities at the mesh points were obtained.

The following values for the various constants were used in this study.

$$K_f = 0.074 \text{ Btu/hr-ft-}^\circ\text{F}$$

$$K_s = 220 \text{ Btu/hr-ft-}^\circ\text{F}$$

$$\delta^* = 0.1 r_0^*$$

Energy equation:

The energy equation was expressed in the finite difference form in a manner as discussed below.

Points within the fluid:

The finite difference equations for energy equation were developed in the similar way as for the momentum equation in the region of flow.

Points at the solid-fluid interface:

For points at $\theta = \beta$ and $r_i < r < 1$, the following finite difference equation was developed

$$\begin{aligned} & -4(\Delta\theta)T(I,J-1) + (\Delta\theta)T(I,J-2) - 8C_2(\Delta\beta)T(I,J+1) \\ & + 2C_2(\Delta\beta)T(I,J+2) + [3(\Delta\theta) + 6(\Delta\beta)C_2]T(I,J) = 0 \quad (39) \end{aligned}$$

For points at $r=r_i$ and $0 \leq \theta < \beta$, the finite difference is as follows

$$\begin{aligned} & 4T(I+1,J) - T(I+2,J) + 8\frac{(\Delta r)}{(\Delta r_1)} C_2 T(I-1,J) - 2\frac{(\Delta r)}{(\Delta r_1)} C_2 \\ & T(I-2,J) - (3 + 6\frac{(\Delta r)}{(\Delta r_1)} C_2) T(I,J) = 0 \quad (40) \end{aligned}$$

For points at $r=1$ and $\beta < \theta \leq \gamma$, the finite difference equation can be written as

$$\begin{aligned} & 4(\Delta r)T(I+1,J) - (\Delta r)T(I+2,J) + 4C_2\delta T(I-1,J) - \\ & C_2\delta T(I-2,J) - [3(\Delta r) + 3C_2\delta]T(I,J) = 0 \quad (41) \end{aligned}$$

The derivation of Eqn. (39) is shown in the Appendix B and Eqn. (40) and (41) were derived in the same fashion.

At two corner points, $T(r_i, \beta)$ and $T(1, \beta)$, the energy equation represented by Eqn. (26) were written in the finite difference form. The derivation for Eqn. (42) is shown in the Appendix B.

The finite difference equation for point $T(r_i, \beta)$ is

$$\begin{aligned} & (-2 + \frac{2(\Delta r)}{r})T(I+1,J) + (1 - \frac{\Delta r}{2r})T(I+2,J) + [-2(\frac{\Delta r}{r\Delta\beta})^2]T(I,J-1) \\ & + (\frac{\Delta r}{r\Delta\beta})^2 T(I,J-2) + [1 - \frac{3\Delta r}{2r} + (\frac{\Delta r}{r\Delta\beta})^2]T(I,J) = 0 \quad (42) \end{aligned}$$

For point $T(1, \beta)$, the difference equation is as follows:

$$\begin{aligned}
& \left[\frac{8}{\delta(\Delta r + \delta)} + \frac{2(\Delta r)}{r\delta(\Delta r + \delta)} \right] T(I+1, J) + \left[\frac{8}{\Delta r(\Delta r + \delta)} - \frac{2}{r\Delta r(\Delta r + \delta)} \right] T(I-1, J) \\
& + \left[\frac{2}{r^2 \Delta \theta (\Delta \theta + 2\Delta \beta)} \right] T(I, J+1) + \left[\frac{1}{r^2 \Delta \beta (\Delta \theta + 2\Delta \beta)} \right] T(I, J-1) \\
& - \left[\frac{8}{\Delta r \delta} + \frac{2(r-\delta)}{r\Delta r \delta} + \frac{1}{r^2 \Delta \beta \Delta \theta} \right] T(I, J) = 0 \tag{43}
\end{aligned}$$

Points within the solid:

The energy equation was represented by finite difference equations, developed in a manner similar to the one previously shown for the momentum equation.

Points at the outer surface of the tube:

The boundary condition to be satisfied for points at the outer surface of the tube is

$$-\frac{\partial T}{\partial r} = C_2$$

Substituting the value of the first derivative in terms of the backward finite difference expression, we get

$$4T(I-1, J) - T(I-2, J) - 3T(I, J) = \delta C_2 \tag{44}$$

As described above, the finite-difference equation was written at each mesh point. This resulted in a system of linear algebraic equations which was solved by means of Gauss-Jordan reduction on a digital computer to determine the dimensionless temperature at each mesh point.

3.3 Friction Factor

The friction factor is defined as follows:

$$f = \frac{\Pi^2 \rho r^{*5} g_c}{m^2} \left(- \frac{dp^*}{dx^*} \right) \tag{45}$$

This definition was adopted because it provides values for f that could be taken as a dimensionless measure of the pressure gradient dp^*/dx^* inside different finned tubes with the same radius r_0^* , for the same density ρ , and the same mass flow rate \dot{m} . Accordingly, the value of f would reflect the effect of the number of fins M , their height l , and their half-angle β on the pressure drop.

In order to agree with its definition for a finless tube, the Reynolds number is herein described as

$$R_e = \frac{2\dot{m}}{\pi r_0^* \mu} \quad (46)$$

From Eqns. (45) and (46), and knowing that

$$\dot{m} = \rho A_f^* u_b^* \quad (47)$$

where u_b^* is the overall bulk velocity, and A_f^* is the flow area, we could write

$$fR_e = \frac{2}{\left[\frac{g_c}{\mu} r_0^{*2} \left(-\frac{dp^*}{dx^*} \right) \right]} \frac{\pi r_0^{*2}}{A_f^*} \quad (48)$$

Substituting, $A_f^* = r_0^{*2} [\pi - M\beta(1-r_i^2)]$, we get

$$fR_e = \frac{\pi}{\pi - M\beta(1-r_i^2)} \left(\frac{2}{u_b^*} \right) \quad (49)$$

The dimensionless bulk velocity u_b is defined as

$$u_b = \frac{1}{A_f} \left[\int_0^\beta \int_0^{r_i} u_r dr d\theta + \int_\beta^Y \int_0^1 u_r dr d\theta \right] \quad (50)$$

where A_f is the dimensionless flow area given by

$$A_f = A_f^*/r_0^{*2}$$

The integral for dimensionless bulk velocity was evaluated by means of a two-dimensional extension of Simpson's rule.

3.4 Heat Transfer Coefficients

The heat transfer coefficient is defined as

$$q2\pi(r_0^*+\delta^*) = h2\pi r_0^*(T_b^*-\bar{T}_w^*) \quad (51)$$

leading to

$$h = \frac{q(r_0^*+\delta^*)}{r_0^*(T_b^*-\bar{T}_w^*)} \quad (52)$$

The Nusselt number is then given by

$$N_u = \frac{2hr_0^*}{K_f} \quad (53)$$

Substituting the value of h, we get

$$N_u = \frac{2q(r_0^*+\delta^*)}{K_f(T_b^*-\bar{T}_w^*)} \quad (54)$$

and in terms of the dimensionless quantities, Nusselt number is expressed as

$$N_u = \frac{2(1+\delta)}{T_b - \bar{T}_w} \quad (55)$$

This method of presentation was employed because it indicates the improvement to be expected on the inner side of an existing tubular heat exchanger if the smooth tubes originally in the exchanger are replaced by finned tubes. Such improvement is indicative of the compactness of a heat exchanger. For a given rate of heat transfer it determines the average temperature difference between the fluid and the solid surface, an important factor in the design of heat exchanger.

The dimensionless bulk temperature T_b used in Eqn. (55) is defined as

$$T_b = \frac{\left[\int_0^\beta \int_0^{r_i} u T r dr d\theta + \int_\beta^\gamma \int_0^1 u T r dr d\theta \right]}{u_b A_f} \quad (56)$$

The integrations in the above equations were evaluated by employing two-dimensional extension of the Simpson's rule.

Chapter 4

DISCUSSION AND CONCLUSION

There are three independent parameters the length of the fin, the number of fins and the thickness of the fin. They were varied within the following ranges in this investigation

$$8 \leq M \leq 32$$

$$0.2 \leq \ell \leq 0.9$$

$$1.5^\circ \leq \beta \leq 3^\circ$$

4.1 Velocity Distribution

The dimensionless equi-velocity lines are shown in Fig. 4 for $M=8$ and $\ell=0.2$, and in Fig. 5 for $M=8$ and $\ell=0.8$ with $\beta=1.5^\circ$. From these figures it is clear that the velocity gradient in both the radial and angular directions becomes steeper as we approach solid boundaries. For $M=8$ and $\ell=0.8$, two types of loops occur: one at the centre and the other in the region formed by any two adjacent fins. Hu and Chang [5] also obtained such types of loops for long fins and called these loops primary and secondary loops, respectively. They noted that for short fins ($\ell < 0.32$) such secondary loops did not exist. Fin height was the only parameter reported by Hu and Chang [5] affecting secondary loops. It was found in the present study that the formation of secondary loops is affected by all the three parameters M , ℓ and β . For every combination of M and β , there is a critical value of ℓ above which secondary loops are

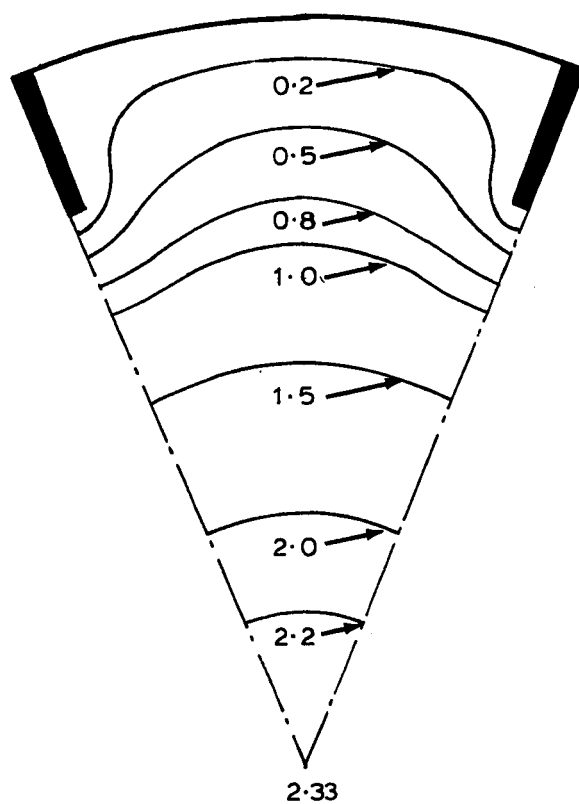


Fig. 4 Equi-velocity lines u/u_b for $M=8$, $l=0.2$ and $\beta=1.5^\circ$

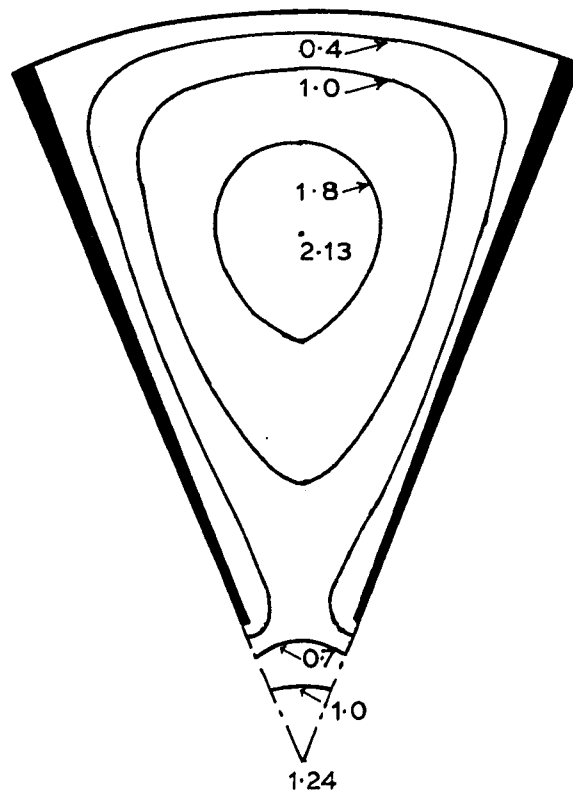


Fig. 5 . Equi-velocity lines u/u_b for $M=8$, $l=0.8$ and $\beta=1.5^\circ$

formed. The critical value of ℓ is affected by β only at high values of M . An increase of β tends to decrease the critical value of ℓ for high values of M . If β is kept constant, the increase of M decreases the critical value of ℓ . For instance, with $\beta=3^\circ$, the secondary loops started to occur at a fin height within the range $0.5 < \ell < 0.6$ when $M=8$, while they would form at a fin height as low as $0.2 < \ell < 0.3$ when $M=28$. As shown in Fig. 5, for $M=8$ and $\ell=0.8$, the highest velocity occurs in the eyes of the secondary loops. For $\ell=0.8$, the increase of M tends to shift secondary-loop eyes towards the tube wall, but the highest velocity is shifted from the secondary to the primary loop. Secondary loops are one of the important features of the flow in internally finned tubes because their existence would affect the temperature distribution, and consequently the rate of heat transfer from these tubes.

It can be seen that the axial velocity varies in the angular direction in region near the fin and the variation is very small in the region near the central core of tube and far away from the fin tip.

Dimensionless velocity distributions for $\beta=3^\circ$ at $0 < r < r_1$ and $\theta=0^\circ$ are shown in Fig. 6 for various values of M and ℓ . It was found that for all values of M and ℓ , the dimensionless velocity at any location within the tube decreased as β increased. This is due to the increase in pressure drop accompanying the increase in β .

4.2 Temperature Distribution

Loops of isotherms $(T - \bar{T}_w) / (T_b - \bar{T}_w)$ over the cross-section of

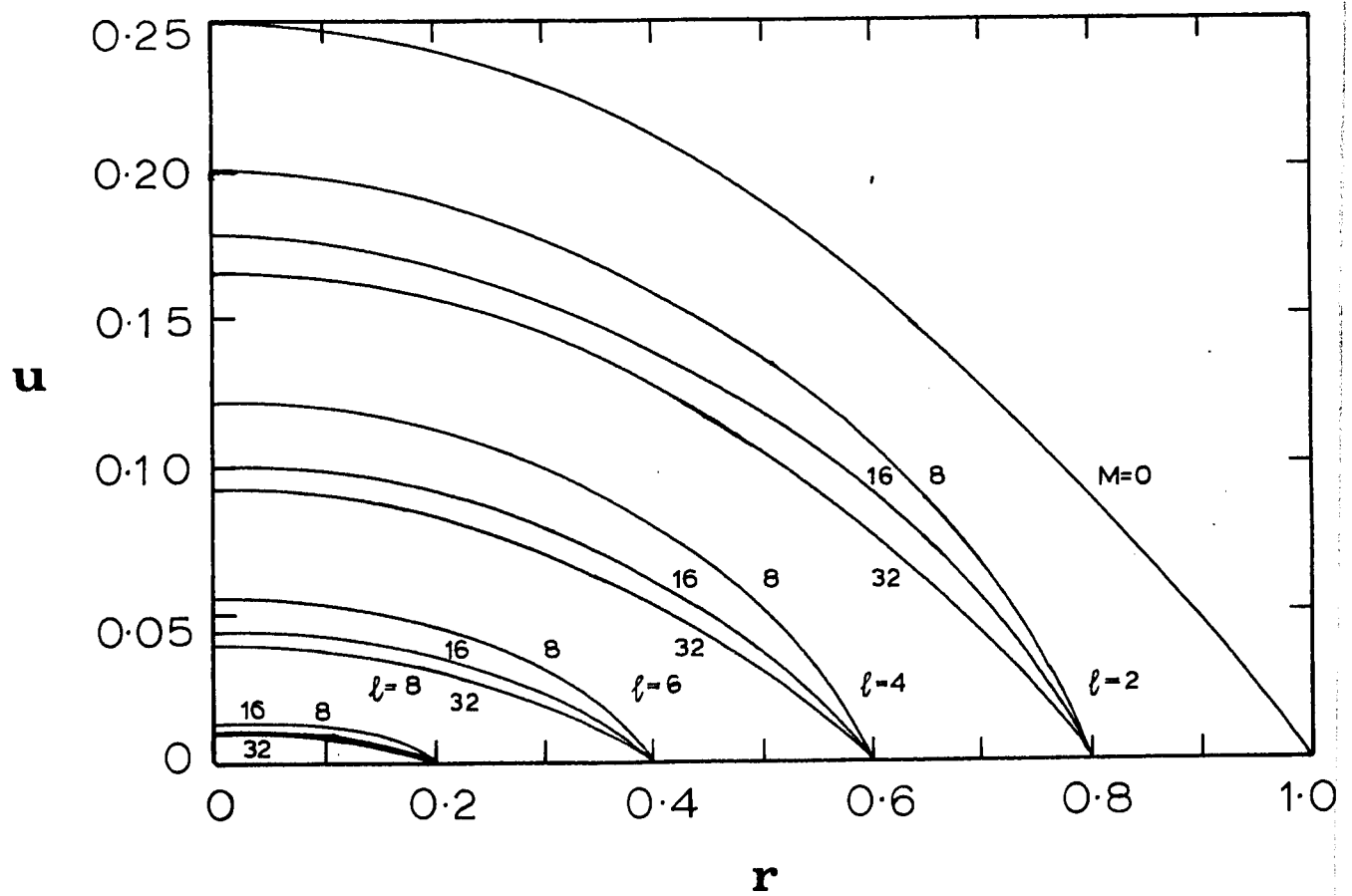


Fig. 6 Dimensionless velocity distributions at $0 < r < r_i$ and $\theta=0^\circ$ for various values of M and l with $\beta=1.5^\circ$

the tube are shown in Fig. 7 for $M=8$ and $\ell=0.2$, and in Fig. 8 for $M=8$ and $\ell=0.8$ with $\beta=1.5^\circ$. As in the case of equi-velocity lines, secondary loops were also formed for isotherms. Also there exists a critical value of ℓ for every combination of M and β above which secondary loops are formed. It was found that this critical fin height decreased as the number of fins increased, keeping the value of β constant. For example, with $\beta=3^\circ$, secondary loops were formed at a fin height within the range $0.6 < \ell < 0.7$ when $M=8$ and this range becomes $0.5 < \ell < 0.6$ for $M=28$. At high values of M , the value of β had an effect on the critical value of ℓ . In these cases, an increase of β causes the secondary loops to appear at higher values of ℓ .

The variation of the dimensionless temperature $(T - \bar{T}_w) / (T_b - \bar{T}_w)$ with radial position along $\theta = \gamma$, for various number of fins, is shown in Figs. 9 and 10 for $\ell=0.4$ and $\ell=0.8$ respectively. Fig. 9 shows that for $\ell=0.4$ the dimensionless temperature decreases with the increase of the number of fins, within the fin region as well as in the region near the tube centre. However, for $\ell=0.8$ the dimensionless temperature first increases with an increase in M but decreases with further increase in the number of fins, in both regions.

4.3 Effect of fin-parameter on friction factor

Friction factor results based on Eqn. (49) are shown in Fig. 11 for $\beta=1.5^\circ$ and a wide range of fin numbers and fin heights. Similar results, but for $\beta=3^\circ$, are shown in Fig. 12. For a given combina-

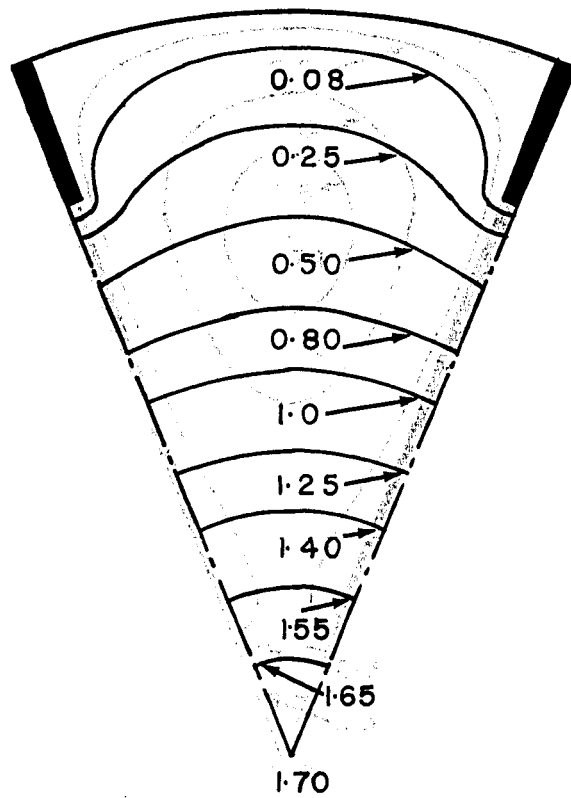


Fig. 7 Isotherms $(T - \bar{T}_w) / (T_b - \bar{T}_w)$ for $M=8$, $\lambda=0.2$ and $\beta = 1.5^\circ$

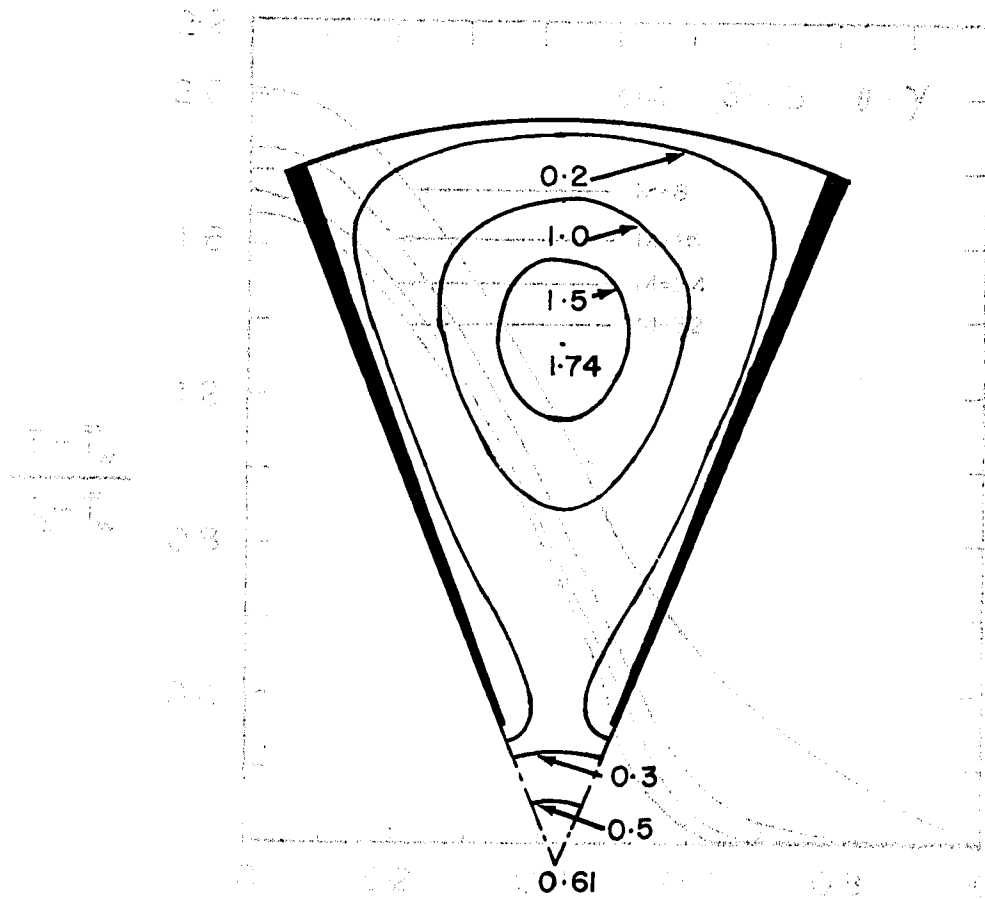


Fig. 8 Isotherms $(T - \bar{T}_w) / (T_p - \bar{T}_w)$ for $M=8$, $l=0.8$ and $\beta=1.5^\circ$

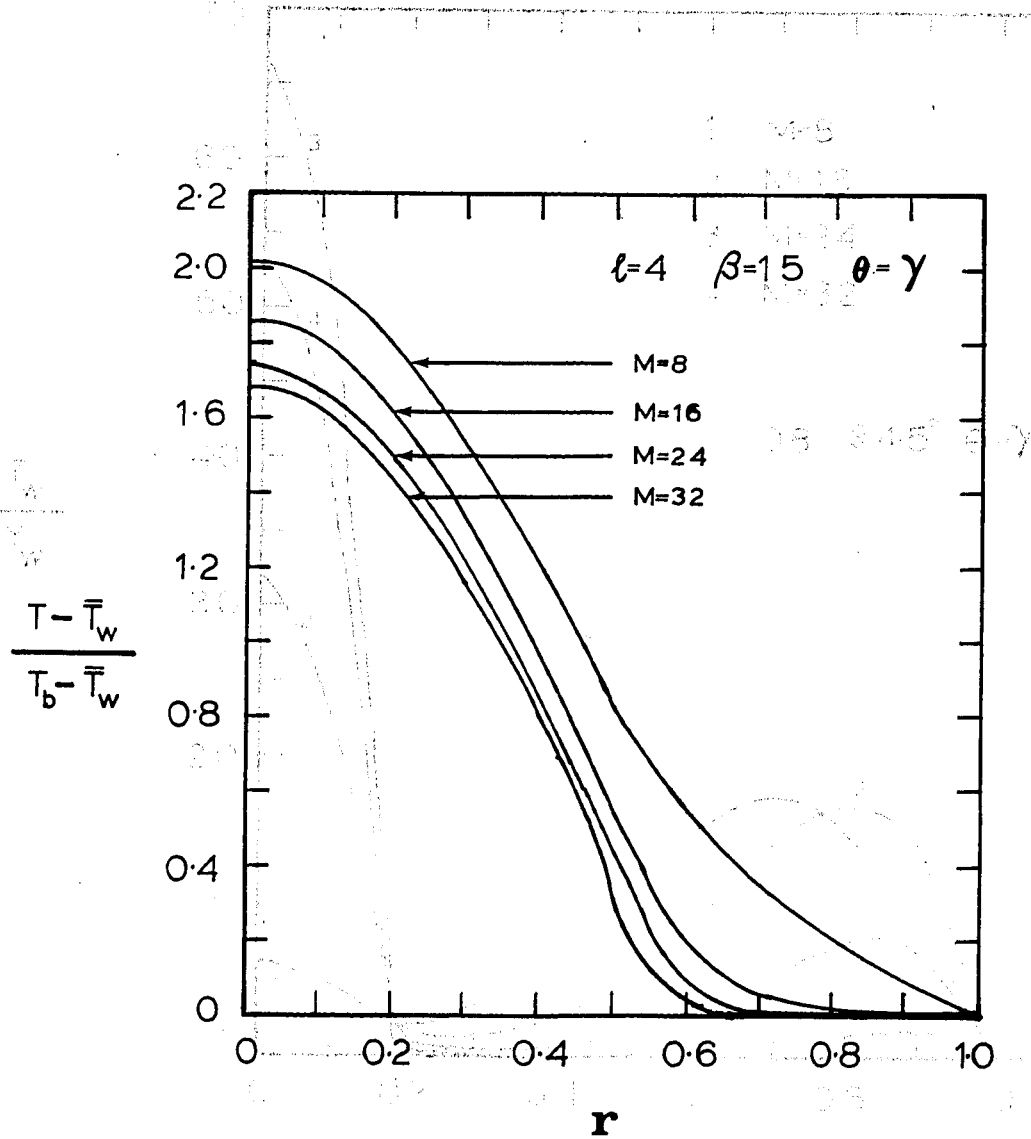


Fig. 9 Dimensionless temperature variation in radial direction
 for $l=0.4$ and $\beta=1.5^\circ$ along $\theta=\gamma$.

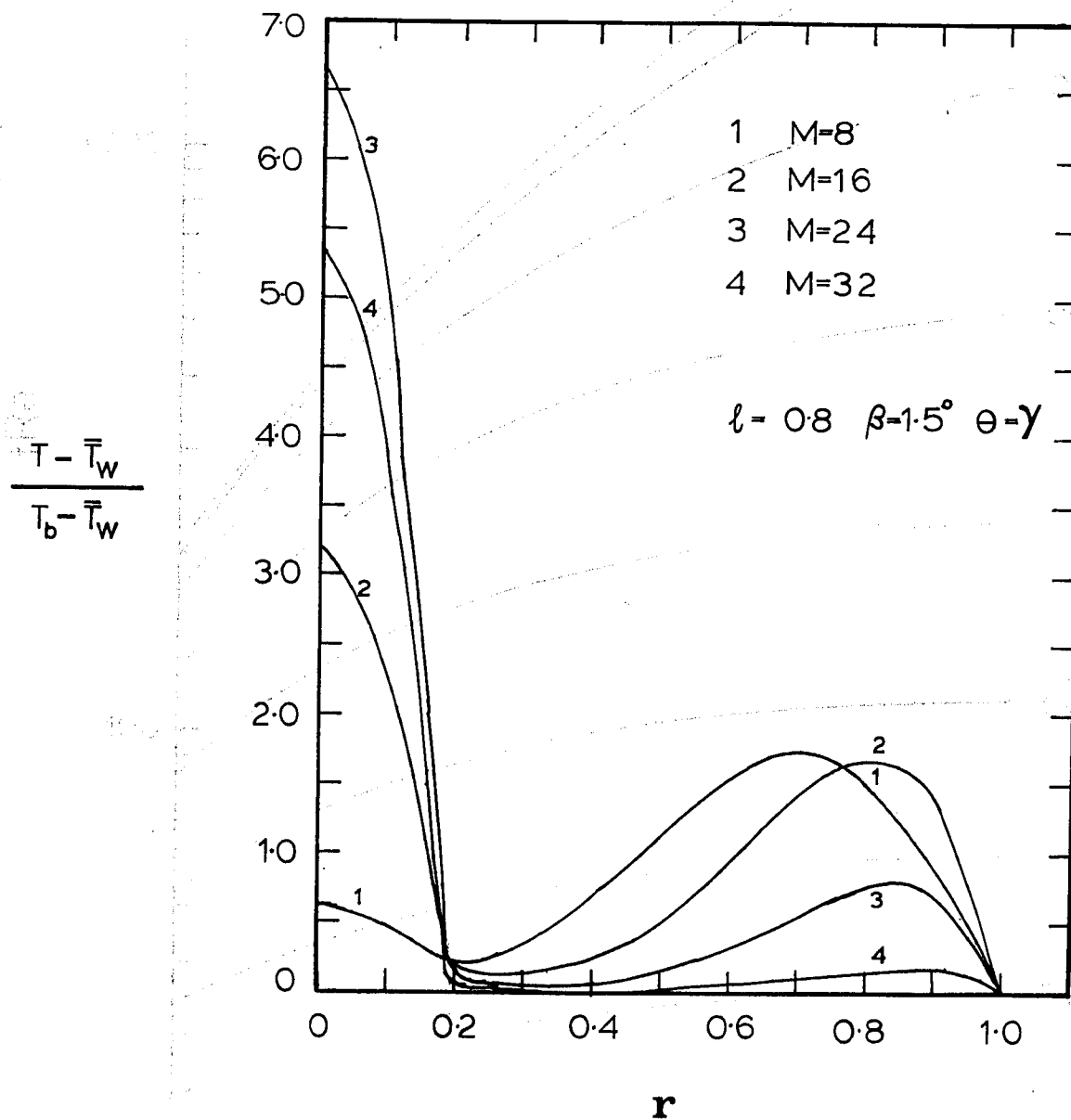


Fig. 10 Dimensionless temperature variation in radial direction
for $l=0.8$ and $\beta=1.5^\circ$ along $\theta=\gamma$

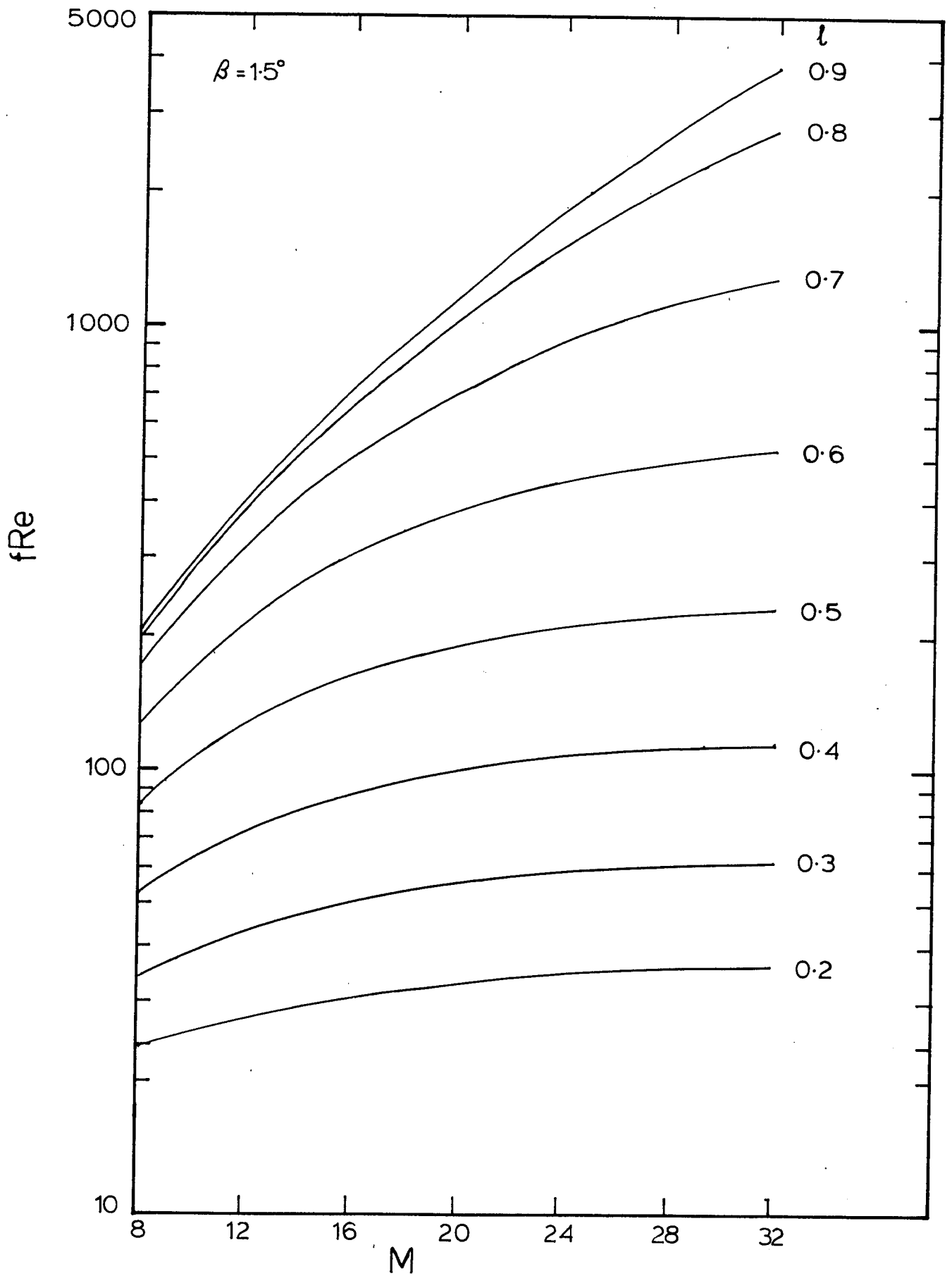


Fig. 11 Variation of fRe with fin number for $\beta=1.5^\circ$

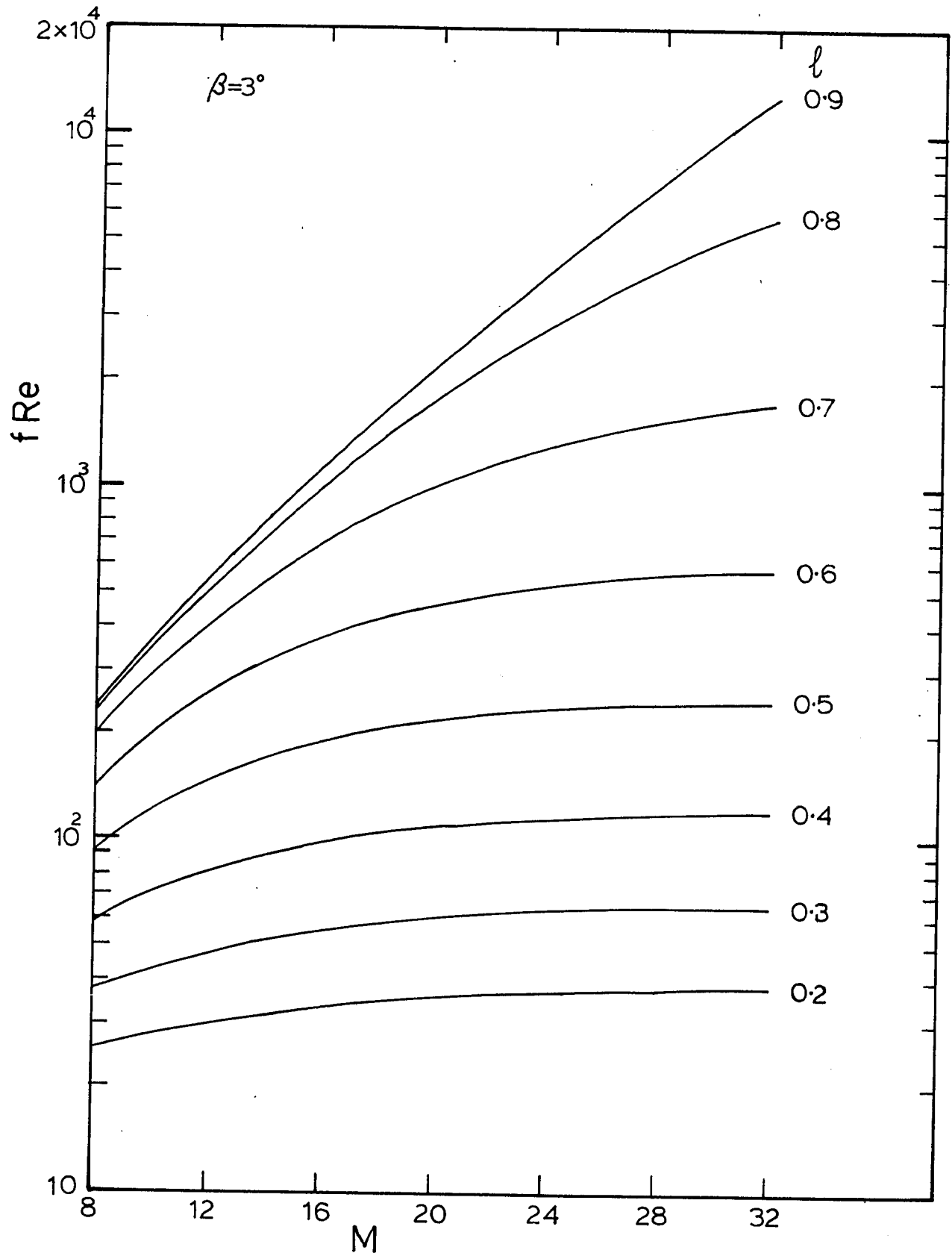


Fig. 12 Variation of fRe with fin number for $\beta = 3^\circ$

tion of β and M , the value of fRe increases with the increase in the value of l . Similarly, if β and l are kept constant, an increase of M would increase the value of fRe . It was also found that the value of fRe increased consistently with the increase in β for any value of M and l . However, it was observed that the effect of β is much more appreciable for longer fins than it is for shorter fins. This is attributed to the fact that for long fins, the value of β has a strong effect on the flow area of the tube.

4.4 Effect of fin-parameter on Nusselt number

The variation of the Nusselt number with the number of fins is shown in Fig. 13 for $\beta=1.5^\circ$ and 3° . The Nusselt number is a strong function of the fin length, fin thickness and the number of fins. For a given value of l and β , the maximum Nusselt number occurs at a certain value of M . For a given value of $8 < M < 16$ and $\beta=1.5^\circ$, the Nusselt number first increases and then decreases with the increase in the value l . However for $16 < M < 32$, the Nusselt number increases with the increase in l . For a given value of M , the Nusselt number increases with decrease in β up to certain value of l , after which the effect of β on the Nusselt number depends upon the value of M also. The Nusselt number increases with increase in β up to certain value of M for a given value of l and then the Nusselt number increases with decrease in β . However, after the maximum value of Nusselt number for a given value of l , the Nusselt number always increases with decrease in β .

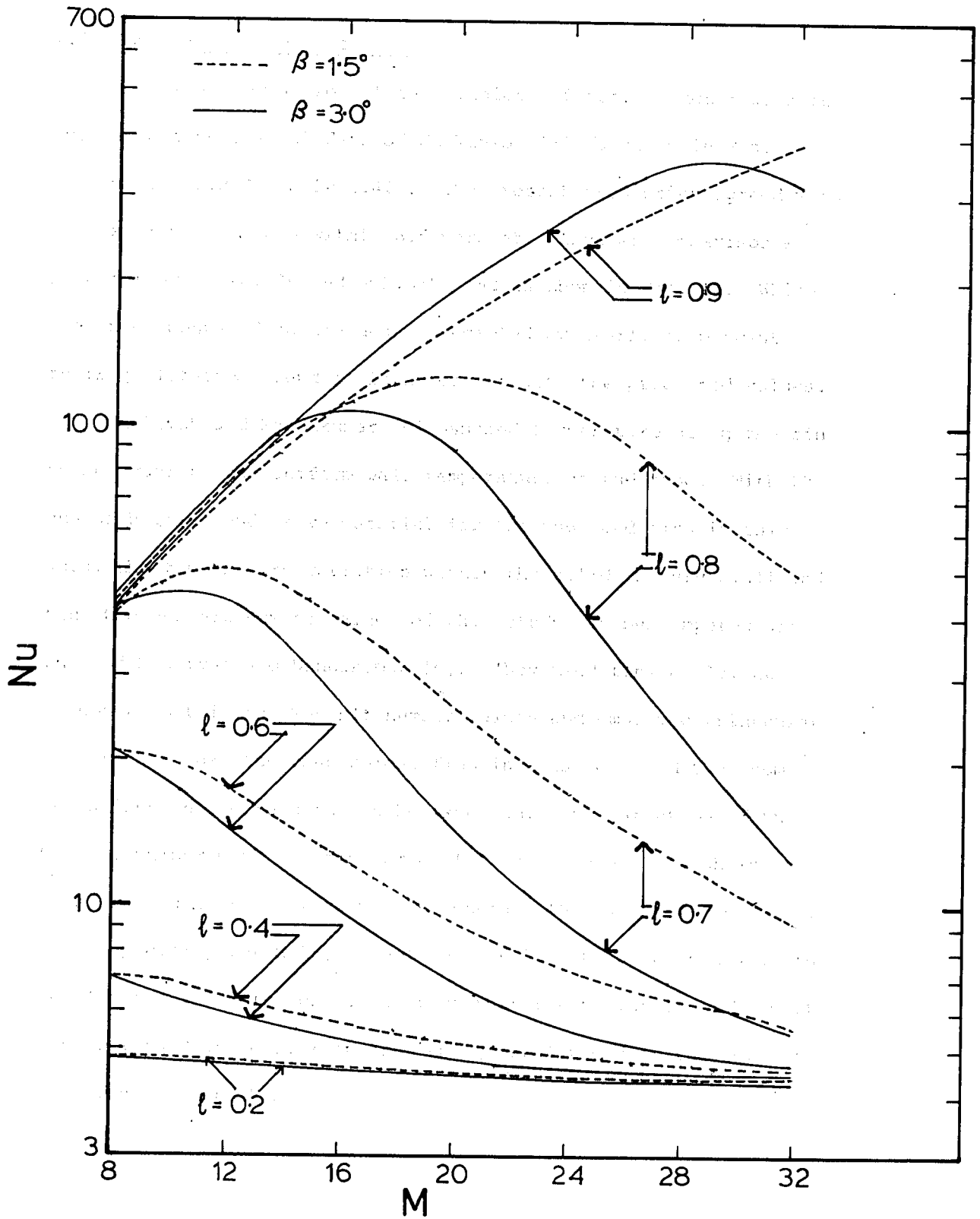


Fig. 13 Nusselt number variation with fin number

4.5 Comparison with other data

A typical comparison of the predicted friction factor with the available experimental data of Watkinson [3] is shown in Fig. 14 for $M=10$, $\ell=0.222$ and $\beta=4.42^\circ$. The present prediction agreed with the experimental data within 6.5 percent. Another comparison was made for $M=6$, $\ell=0.296$ and $\beta=2.56^\circ$ and is shown in Fig. 15. While the experimental data are under-predicted by nearly 18 percent, their qualitative trend is in agreement with the predicted values.

Masliyah and Nandakumar [6] assumed temperature along the fin to be equal to the uniform wall temperature of the tube. With the use of highly conductive material for the tube and fins in this work, the temperature variation within the solid is very small and thus the Nusselt number values of this study can be compared with that of Masliyah and Nandakumar [6]. They used finite element method to obtain the Nusselt number values and employed triangular fins rather than the trapezoidal fins in this work and thus some deviations in the results can be expected. The variation of the Nusselt number with the fin number for $\beta=1.5^\circ$ and 3° is shown in Figs. 16 and 17 respectively. In general, the agreement is fairly good especially for $M < 16$, except for $\ell=0.7$. The discrepancy in the results for $\ell=0.7$ is due to the fact that the variation of Nusselt number with M was different from the general trend in the work of Masliyah and Nandakumar.

4.6 Conclusions

An analysis of fully developed laminar flow in internally

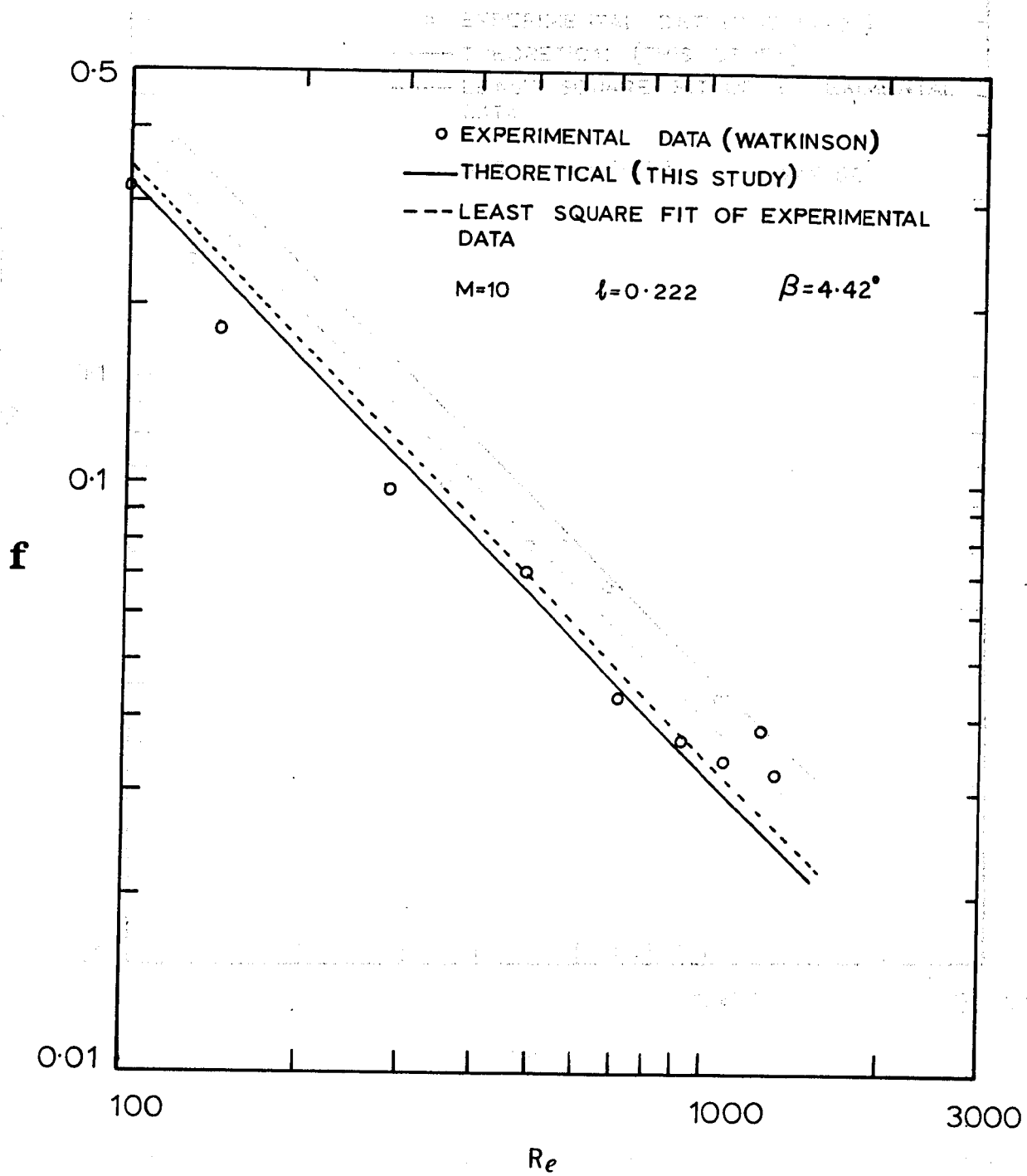


Fig. 14 Comparison of predicted and experimental friction factors

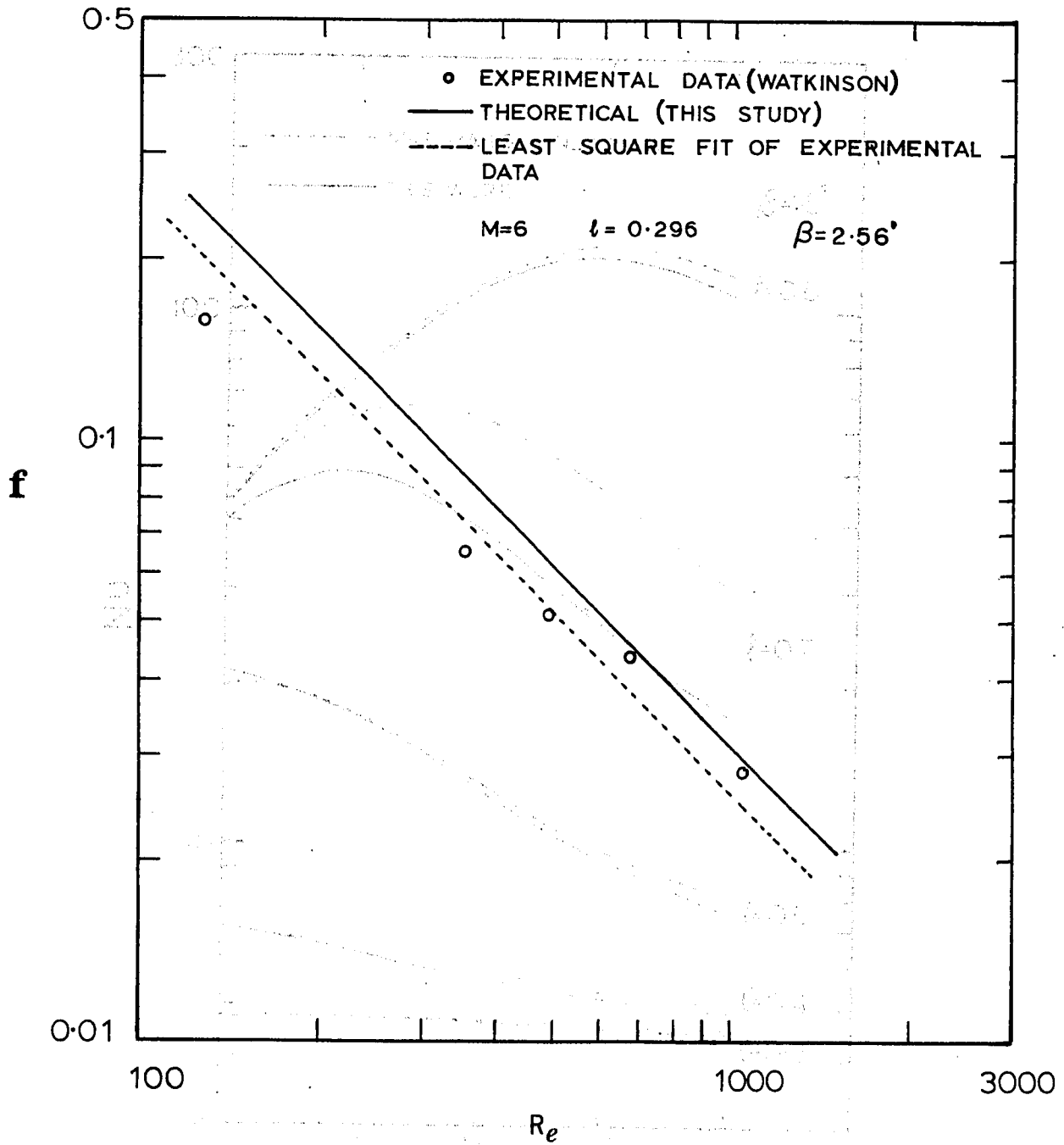


Fig. 15 Comparison of predicted and experimental friction factors.

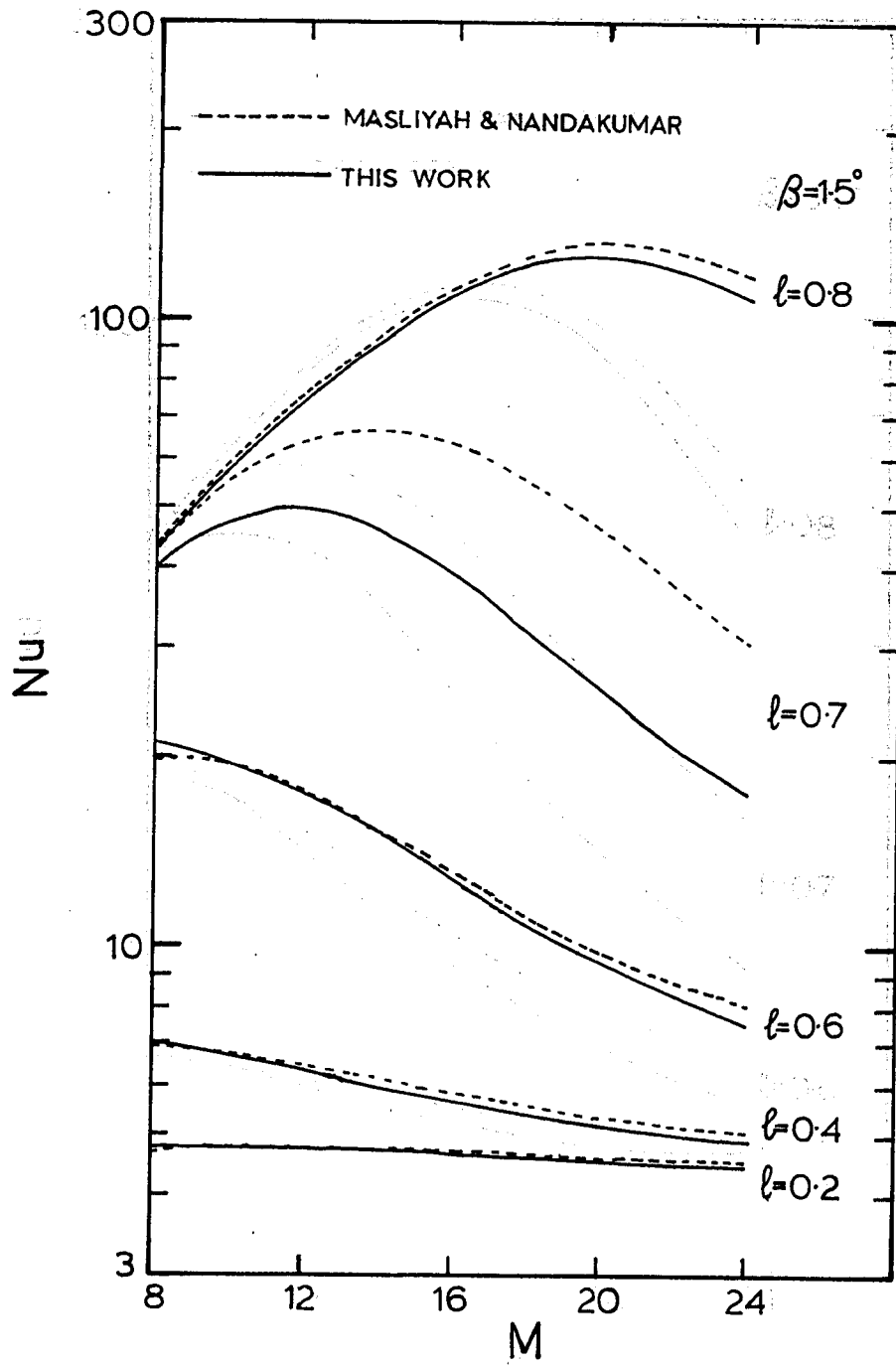


Fig. 16 Nusselt number variation with fin number

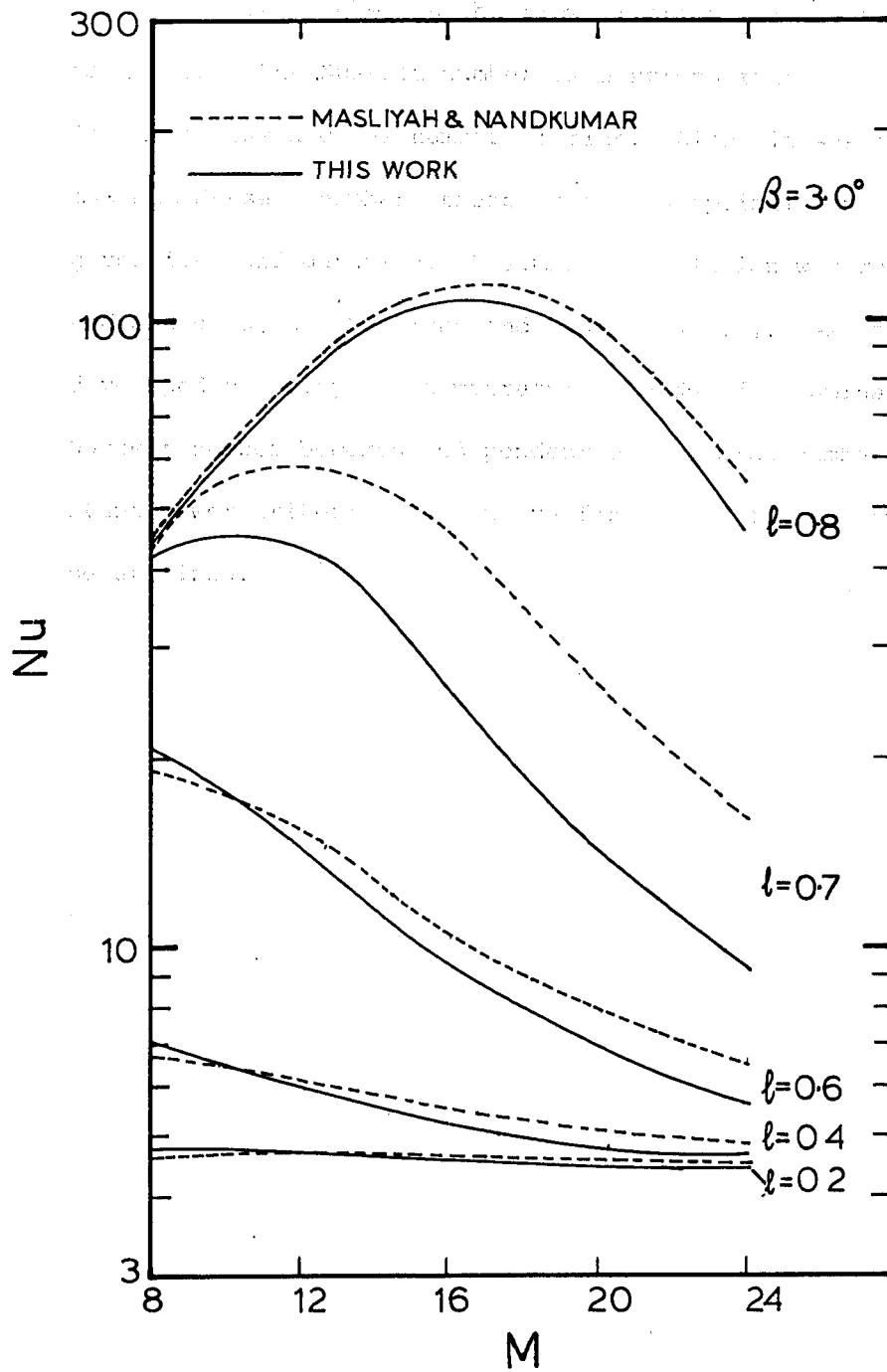


Fig. 17 Nusselt number variation with fin number

finned tubes with fin shapes very close to real fin configuration was performed, using finite difference technique. The analysis indicates that the larger the number of fins, their height and thickness, the higher is the flow friction for the same rate of mass flow. The Nusselt number is a strong function of fin length, fin thickness and the number of fins. Also, it was found that for maximum Nusselt number, there exists an optimum fin number for a given fin configuration. A similar conclusion was reported by Hu and Chang [5] and Masliyah and Nandakumar [6]. As in fully developed velocity and temperature fields of a laminar flow, the Nusselt number becomes independent of Reynolds number, the constant power criterion to compare finned and finless tubes cannot be utilized.

APPENDIX A

Derivation of the governing equations in terms of the dimensionless quantities.

1. Energy equation in solid:

Differentiating Eqn. (25-e) w.r.t. r , we get

$$\frac{\partial T_s}{\partial r} = \frac{\partial T_s^*}{\partial r} \cdot \frac{K}{qr_0^*}$$

or

$$\frac{\partial T_s^*}{\partial r} = \frac{\partial T_s}{\partial r} \cdot \frac{qr_0^*}{K} \quad (A-1)$$

In similar fashion, the second differentiation of Eqn. (25-e) w.r.t. θ yields

$$\frac{\partial^2 T_s^*}{\partial \theta^2} = \frac{\partial^2 T_s}{\partial \theta^2} \cdot \frac{qr_0^*}{K} \quad (A-2)$$

Substituting the above derivatives in Eqn. (10) and knowing that $r^* = r r_0^*$, we get

$$\frac{1}{r r_0^{*2}} \frac{\partial}{\partial r} \left(r r_0^* \frac{q}{K} \frac{\partial T_s}{\partial r} \right) + \frac{1}{r^2 r_0^{*2}} \frac{qr_0^*}{K} \frac{\partial^2 T_s}{\partial \theta^2} = 0$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T_s}{\partial \theta^2} = 0$$

2. Momentum Equation:

Differentiating Eqn. (25-d) w.r.t. r , we get

$$\frac{\partial u}{\partial r} = \frac{1}{(-r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*})} \frac{\partial u^*}{\partial r}$$

or

$$\frac{\partial u^*}{\partial r} = -r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*} \frac{\partial u}{\partial r}$$

Similarly, the second differentiation of Eqn. (25-d) w.r.t. θ gives

$$\frac{\partial^2 u^*}{\partial \theta^2} = -r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*} \frac{\partial^2 u}{\partial \theta^2}$$

Substituting the above derivatives in Eqn. (11) and knowing that $r^* = r r_0^*$, we get

$$\begin{aligned} \frac{1}{r r_0^{*2}} \frac{\partial}{\partial r} \left[r r_0^* \left(-r_0^* \frac{g_c}{\mu} \frac{dp^*}{dx^*} \right) \frac{\partial u}{\partial r} \right] \\ + \frac{1}{r^2 r_0^{*2}} \left(-r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*} \right) \frac{\partial^2 u}{\partial \theta^2} = \frac{1}{u} \frac{dp^*}{dx^*} \end{aligned}$$

The above equation can be written as

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = -1$$

3. Energy equation in fluid:

The energy equation in fluid is

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_f^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 T_f^*}{\partial \theta^2} = \frac{1}{\alpha} u^* \frac{\partial T_f^*}{\partial x^*} \quad (12)$$

As shown in section 2.1, for fully developed laminar flow

$$\frac{\partial T_f^*}{\partial x^*} = \frac{dT_b^*}{dx^*}$$

Thus, Eqn. (12) can be written as

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T_f^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 T_f^*}{\partial \theta^2} = \frac{1}{\alpha} u^* \frac{dT_b^*}{dx^*} \quad (A-3)$$

The Eqns. (A-1) and (A-2) are rewritten as

$$\frac{\partial T_f^*}{\partial r} = \frac{\partial T_f}{\partial r} \cdot \frac{qr_0^*}{K}$$

and

$$\frac{\partial^2 T_f^*}{\partial \theta^2} = \frac{\partial^2 T_f}{\partial \theta^2} \cdot \frac{qr_0^*}{K}$$

From Eqn. (4), and knowing that

$$\dot{m} = \rho A_f^* u_b^*$$

where u_b^* is the overall bulk velocity, and A_f^* is the flow area,

we could write

$$\begin{aligned} \frac{1}{\alpha} u^* \frac{dT_b^*}{dx^*} &= \frac{-2\pi q(r_0^* + \delta^*) u^* \rho C_p}{\rho A_f^* u_b^* C_p K_f} \\ &= \frac{-2\pi q r_0^* (1+\delta) u (-r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*})}{r_0^{*2} A_f u_b (-r_0^{*2} \frac{g_c}{\mu} \frac{dp^*}{dx^*}) K_f} \\ &= \frac{-2\pi q (1+\delta)}{r_0^* A_f u_b K_f} u \end{aligned}$$

The dimensionless bulk velocity u_b , and the dimensionless flow area A_f are defined in section 3.3.

Substituting the above values in Eqn. (A-3), we get

$$\frac{1}{r r_0^{*2}} \frac{\partial}{\partial r} \left(r \frac{q r_0^*}{K_f} \frac{\partial T_f}{\partial r} \right) + \frac{1}{r^2 r_0^{*2}} \frac{q r_0^*}{K_f} \frac{\partial^2 T_f}{\partial \theta^2} = \frac{-2\pi q (1+\delta)}{r_0^* A_f u_b K_f} u$$

The above equation can be written as

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T_f}{\partial \theta^2} = C_1 u$$

where $C_1 = \frac{-2\pi(1+\delta)}{A_f u_b}$

APPENDIX B

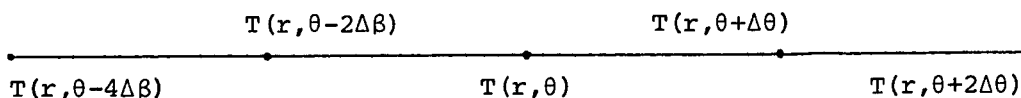
Derivation of the finite-difference equations

1. Points at the solid-fluid interface:

For points at $\theta = \beta$ and $r_1 < r < 1$, the Eqn. (39) was derived as follows:

The boundary condition to be satisfied is

$$\frac{\partial T_s}{\partial \theta} = C_2 \frac{\partial T_f}{\partial \theta} \quad (B-1)$$



By employing Taylor series expansion, we can write

$$T(r, \theta + \Delta \theta) = T(r, \theta) + \frac{\partial T}{\partial \theta} \Delta \theta + \frac{\partial^2 T}{\partial \theta^2} \frac{(\Delta \theta)^2}{2} + \xi (\Delta \theta)^3$$

$$T(r, \theta + 2\Delta \theta) = T(r, \theta) + \frac{\partial T}{\partial \theta} 2\Delta \theta + \frac{\partial^2 T}{\partial \theta^2} \frac{(2\Delta \theta)^2}{2} + \xi (\Delta \theta)^3$$

Neglecting terms of the order of $(\Delta \theta)^2$ and greater, the forward finite difference expression for the first derivative is given by

$$\frac{\partial T}{\partial \theta} = \frac{4T(r, \theta + \Delta \theta) - T(r, \theta + 2\Delta \theta) - 3T(r, \theta)}{2(\Delta \theta)}$$

In like fashion, backward finite difference expression was obtained

$$\frac{\partial T}{\partial \theta} = \frac{-4T(r, \theta - 2\Delta \beta) + T(r, \theta - 4\Delta \beta) + 3T(r, \theta)}{4\Delta \beta}$$

Substituting above expressions in Eqn. (B-1), we get

$$\begin{aligned} & -4(\Delta \theta)T(r, \theta - 2\Delta \beta) + (\Delta \theta)T(r, \theta - 4\Delta \beta) - 8C_2(\Delta \beta)T(r, \theta + \Delta \theta) \\ & + 2C_2(\Delta \beta)T(r, \theta + 2\Delta \theta) + [3(\Delta \theta) + 6(\Delta \beta)C_2]T(r, \theta) = 0 \end{aligned}$$

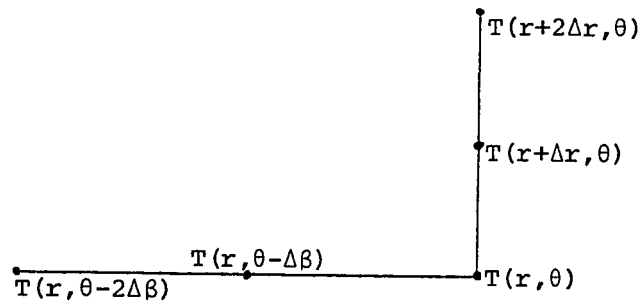
The above equation can be written as

$$-4(\Delta\theta)T(I,J-1) + (\Delta\theta)T(I,J-2) - 8C_2(\Delta\beta)T(I,J+1) \\ + 2C_2(\Delta\beta)T(I,J+2) + [3(\Delta\theta)+6(\Delta\beta)C_2]T(I,J) = 0$$

2. Point at $r=r_i$ and $\theta=\beta$

The energy equation at this point is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (B-2)$$



By employing Taylor series expansion, we can write

$$T(r+\Delta r, \theta) = T(r, \theta) + \frac{\partial T}{\partial r} (\Delta r) + \frac{\partial^2 T}{\partial r^2} \frac{(\Delta r)^2}{2} + \mathcal{O}(\Delta r)^3$$

$$T(r+2\Delta r, \theta) = T(r, \theta) + \frac{\partial T}{\partial r} (2\Delta r) + \frac{\partial^2 T}{\partial r^2} \frac{(2\Delta r)^2}{2} + \mathcal{O}(\Delta r)^3$$

Neglecting terms of the order of $(\Delta r)^2$ and greater, the following forward finite difference expressions for the first and second derivatives were obtained.

$$\frac{\partial T}{\partial r} = \frac{4T(r+\Delta r, \theta) - T(r+2\Delta r, \theta) - 3T(r, \theta)}{2\Delta r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{-2T(r+\Delta r, \theta) + T(r+2\Delta r, \theta) + T(r, \theta)}{(\Delta r)^2}$$

A similar development yields

$$\frac{\partial^2 T}{\partial \theta^2} = \frac{-2T(r, \theta-\Delta\beta) + T(r, \theta-2\Delta\beta) + T(r, \theta)}{(\Delta\beta)^2}$$

Replacing the derivatives in Eqn. (B-2) by the finite difference

expressions, the following equation was obtained.

$$\begin{aligned} & \left(-2 + \frac{2(\Delta r)}{r}\right)T(r+\Delta r, \theta) + \left(1 - \frac{\Delta r}{2r}\right)T(r+2\Delta r) + \left[-2\left(\frac{\Delta r}{r\Delta\beta}\right)^2\right]T(r, \theta-\Delta\beta) \\ & + \left(\frac{\Delta r}{r\Delta\beta}\right)^2T(r, \theta-2\Delta\beta) + \left[1 - \frac{3\Delta r}{2r} + \left(\frac{\Delta r}{r\Delta\beta}\right)^2\right]T(r, \theta) = 0 \end{aligned}$$

The above equation can be written as

$$\begin{aligned} & \left(-2 + \frac{2(\Delta r)}{r}\right)T(I+1, J) + \left(1 - \frac{\Delta r}{2r}\right)T(I+2, J) + \left[-2\left(\frac{\Delta r}{r\Delta\beta}\right)^2\right]T(I, J-1) \\ & + \left(\frac{\Delta r}{r\Delta\beta}\right)^2T(I, J-2) + \left[1 - \frac{3\Delta r}{2r} + \left(\frac{\Delta r}{r\Delta\beta}\right)^2\right]T(I, J) = 0 \end{aligned}$$

PROGRAM FOR MESH I

THE VARIABLES ARE DEFINED AS FOLLOWS :

XM NU. OF FINS
XL DIMENSIONLESS FIN LENGTH
BETA HALF THE ANGLE SUBTENDED BY ONE FIN
DELTA DIMENSIONLESS TUBE THICKNESS
DELRI GRID SIZE IN THE RADIAL DIRECTION
GAMMA HALF THE ANGLE BETWEEN THE CENTRE-LINES
OF TWO ADJACENT FINS
DTHETA GRID SIZE IN THE ANGULAR DIRECTION
DBETA GRID SIZE IN THE ANGULAR DIRECTION
XKF THERMAL CONDUCTIVITY OF THE FLUID
XKS THERMAL CONDUCTIVITY OF THE SOLID
D(I) DIMENSIONLESS VELOCITY
H(I) DIMENSIONLESS TEMPERATURE

DIMENSION R(14),A(14),B(14),X(14),G(14),E(14),C(73,74)
DIMENSION DD(38),QA(10),QB(9),Q(14),F(115,116),H(115)
DIMENSION HH(38),OUT(14),D(73)

XM=8.
XL=0.4
PI=3.14159265
BETA=1.5*PI/180.

DELTA=0.1
DELRI=XL/5.
GAMMA=(1.-XL)/8.
THETA=GAMMA-BETA
DTHETA=THETA/5.
DBETA=BETA/4.
XKF=0.074
XKS=220.
C2=XKF/XKS
R(1)=DELRI
DO 1 I=2,7
J=I-1
R(I)=R(J)+DELRI
CONTINUE
R(8)=R(7)+DELRI/2.
R(9)=8.*DELRI
DO 2 I=10,13
J=I-1
R(I)=R(J)+DELRI

0001
0002
0003

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0024
0025
0026
0027

0028 CONTINUE
 0029 R(14)=R(13)+DEL R/2.
 0030 DO 3 I=1,8
 0031 A(I)=DEL R1/(2.*R(I))
 0032 B(I)=((DEL R1/(R(I)*DTHETA))**2.)*100.
 0033 X(I)=(1.+A(I))*100.
 0034 G(I)=(1.-A(I))*100.
 0035 E(I)=-200.-2.*B(I)
 0036 CONTINUE
 0037 DO 4 I=10,14
 0038 A(I)=DEL R/(2.*R(I))
 0039 B(I)=((DEL R/(R(I)*DTHETA))**2.)*100.
 0040 X(I)=(1.+A(I))*100.
 0041 G(I)=(1.-A(I))*100.
 0042 E(I)=-200.-2.*B(I)
 0043 CONTINUE
 0044
 0045 DO 5 I=1,73
 0046 DO 5 J=1,74
 0047 C(I,J)=0.0
 CONTINUE
 PROGRAM FOR EQS. FROM 1 TO 5
 0048 DO 6 I=1,5
 0049 C(I,I)=-800.-2.*B(14)
 0050 J=I+5
 0051 C(I,J)=(4.-DEL R/R(14))*100.
 0052 CONTINUE
 0053 DO 7 I=2,4
 0054 J=I+1
 0055 C(I,J)=B(14)
 0056 K=I-1
 0057 C(I,K)=B(14)
 0058 CONTINUE
 0059 C(1,2)=B(14)
 0060 C(5,4)=2.*B(14)
 PROGRAM FOR EQS. FROM 6 TO 25
 0061 I=1
 0062 K=14
 0063 DO 8 N=1,4
 0064 I=I+5
 0065 K=K-1
 0066 I1=I-1
 0067 DO 9 M=1,5
 0068 I1=I1+1
 0069 C(I1,I1)=E(K)
 0070 J1=I1+5
 0071 C(I1,J1)=G(K)
 0072 CONTINUE
 0073 I2=I

 PROGRAM FOR DIMENSIONLESS VELOCITIES

```

0074 DO 10 L=1,3
0075 I2=I2+1
0076 I3=I2-1
0077 C(I2,I3)=B(K)
0078 I4=I2+1
0079 C(I2,I4)=B(K)
0080 CONTINUE
0081 I6=I+1
0082 C(I,I6)=B(K)
0083 I7=I+4
0084 I8=I7-1
0085 C(I7,I8)=2.*B(K)
0086 CONTINUE
0087 DO 11 I=11,15
0088 J=I-5
0089 C(I,J)=X(12)
0090 CONTINUE
0091 DO 12 I=16,20
0092 J=I-5
0093 C(I,J)=X(11)
0094 CONTINUE
0095 DO 13 I=21,25
0096 J=I-5
0097 C(I,J)=X(10)
0098 CONTINUE

```

PROGRAM FOR EQS. FROM 26 TO 30

```

0099 Z=DELR*DELR1*(DELR1+2.*DELR)
0100 P=4.*DELR1/Z+(DELR1*DELR1)/(R(9)*Z)
0101 Y=8.*DELR/Z-((2.*DELR)**2.)/(R(9)*Z)
0102 W=(1./(R(9)*DTHETA))**2.
0103 V=PT+Y+2.*W
0104 DO 14 I=26,30
0105 C(I,I)=-V
0106 J=I-5
0107 C(I,J)=P
0108 K=I+10
0109 C(I,K)=Y
0110 CONTINUE
0111 DO 15 I=27,29
0112 J=I+1
0113 C(I,J)=W
0114 K=I-1
0115 C(I,K)=W
0116 CONTINUE
0117 C(26,27)=W
0118 C(30,29)=2.*W

```

PROGRAM FOR EQS. FROM 31 TO 34

```

0119 Z=((DELR1/(R(8)*DBETA))**2.)*100.
0120 DO 16 I=31,34
0121 C(I,I)=-800.-2.*Z
0122 J=I+10
0123 C(I,J)=(4.-DELR1/R(8))*100.
0124 CONTINUE
0125 DO 17 I=32,34

```

```

0126 J=I+1
0127 C(I,J)=Z
0128 K=I-1
0129 C(I,K)=Z
0130 CONTINUE
0131 C(31,32)=2.*Z
C
C PROGRAM FOR EQS. FROM 36 TO 40
C
0132 DO 18 I=36,40
0133 C(I,I)=-800.-2.*B(8)
0134 J=I-10
0135 C(I,J)=(4.+DELRI/R(8))*100.
0136 K=I+10
0137 C(I,K)=(4.-DELRI/R(8))*100.
0138 CONTINUE
0139 DO 19 I=36,39
0140 J=I+1
0141 C(I,J)=B(8)
0142 K=I-1
0143 C(I,K)=B(8)
0144 CONTINUE
0145 C(40,39)=2.*B(8)
C
C PROGRAM FOR EQ. NO. 35
C
0146 Z=((DELRI/R(8))*2.)*100.
0147 C(35,35)=-800.-2.*Z/(DTHEIA*DBETA)
0148 C(35,34)=2.*Z/(DBETA*(DTHEIA+DBETA))
0149 C(35,36)=2.*Z/(DTHEIA*(DTHEIA+DBETA))
0150 C(35,45)=(4.-DELRI/R(8))*100.
C
C PROGRAM FOR EQS. FROM 41 TO 44
C
0151 W=((DELRI/(R(7)*DBETA))*2.)*100.
0152 DO 20 I=41,44
0153 C(I,I)=-200.-2.*W
0154 DO 21 I=42,44
0155 J=I-1
0156 C(I,J)=W
0157 K=I+1
0158 C(I,K)=W
0159 CONTINUE
0160 C(41,42)=2.*W
0161 C(41,51)=G(7)
0162 C(42,51)=G(7)/2.
0163 C(42,52)=G(7)/2.
0164 C(43,52)=G(7)
0165 Z=DBETA/(2.*DBETA+DTHEIA)
0166 C(44,52)=(1.-Z)*G(7)
0167 C(44,53)=Z*G(7)
C
C PROGRAM FOR EQS. FROM 45 TO 50
C
0168 DO 31 I=46,49
0169 C(I,I)=E(7)
0170 J=I+1
0171 C(I,J)=B(7)

```

```

0172 K=I-1
0173 C(I,K)=B(7)
0174 L=I-20
0175 C(I,L)=X(7)
0176 CONTINUE
0177 C(50,50)=E(7)
0178 C(50,49)=2.*R(7)
0179 C(50,55)=G(7)
0180 C(50,30)=X(7)
0181 C(46,53)=G(7)
0182 C(47,53)=G(7)/2.
0183 C(47,54)=G(7)/2.
0184 C(48,54)=G(7)
0185 C(49,54)=G(7)/2.
0186 C(49,55)=G(7)/2.

```

31

```

C PROGRAM FOR EQ. NO. 45
C
0187 Z=((DELRI/R(7))**2.)*100.
0188 C(45,45)=-200.-2.*Z/(DTHEA*DBETA)
0189 C(45,44)=2.*Z/(DBETA*(DTHEA+DBETA))
0190 C(45,46)=2.*Z/(DTHEA*(DTHEA+DBETA))
0191 W=(2.*DBETA)/(2.*DBETA+DTHEA)
0192 C(45,52)=(1.-W)*G(7)
0193 C(45,53)=W*G(7)

```

C C

```

C PROGRAM FOR EQS. NO. 51 & 56
C
0194 Z=((DELRI/(R(6)*2.*DBETA))**2.)*100.
0195 W=((DELRI/(R(5)*2.*DBETA))**2.)*100.
0196 C(51,51)=-200.-2.*Z
0197 C(56,56)=-200.-2.*W
0198 C(51,52)=2.*Z
0199 C(56,57)=2.*W
0200 C(51,56)=G(6)
0201 C(51,41)=X(6)
0202 C(56,61)=G(5)
0203 C(56,51)=X(5)

```

C C

```

C PROGRAM FOR EQS. NO. 54, 55, 59 & 60
C
0204 C(54,54)=-200.-B(6)/2.
0205 C(54,53)=B(6)/4.
0206 C(54,55)=B(6)/4.
0207 C(54,59)=G(6)
0208 C(54,48)=X(6)
0209 C(59,59)=-200.-B(5)/2.
0210 C(59,58)=B(5)/4.
0211 C(59,60)=B(5)/4.
0212 C(59,62)=G(5)/2.
0213 C(59,63)=G(5)/2.
0214 C(59,54)=X(5)
0215 C(55,55)=-200.-B(6)/2.
0216 C(55,54)=B(6)/2.
0217 C(55,60)=G(6)
0218 C(55,50)=X(6)
0219 C(60,60)=-200.-B(5)/2.
0220 C(60,59)=B(5)/2.

```

```

0221 C(60,63)=G(5)
0222 C(60,55)=X(5)

```

```

C
C PROGRAM FOR EQS. NO. 52 & 57

```

```

0223 Y=2.*DBETA+DTHETA
0224 Z=4.*DBETA+DTHETA
0225 W6=((DELRI/R(6))*2.)*100.
0226 W5=((DELRI/R(5))*2.)*100.
0227 C(52,52)=-200.-W6/(DBETA*Y)
0228 C(52,51)=W6/(DBETA*Z)
0229 C(52,53)=(W6*2.)/(Y*Z)
0230 C(52,57)=G(6)
0231 C(52,43)=X(6)
0232 C(57,57)=-200.-W5/(DBETA*Y)
0233 C(57,56)=W5/(DBETA*Z)
0234 C(57,58)=(W5*2.)/(Y*Z)
0235 W=(2.*DBETA)/(4.*DBETA+DTHETA)
0236 C(57,61)=(1.-W)*G(5)
0237 C(57,62)=W*G(5)
0238 C(57,52)=X(5)

```

```

C
C PROGRAM FOR EQS. NO. 53 & 58

```

```

0239 Y=2.*DBETA+3.*DTHETA
0240 Z=2.*DBETA+DTHETA
0241 C(53,53)=-200.-W6/(DTHETA*Z)
0242 C(53,52)=(2.*W6)/(Y*Z)
0243 C(53,54)=W6/(DTHETA*Y)
0244 C(53,58)=G(6)
0245 C(53,46)=X(6)
0246 C(58,58)=-200.-W5/(DTHETA*Z)
0247 C(58,57)=(2.*W5)/(Y*Z)
0248 C(58,59)=W5/(DTHETA*Y)
0249 C(58,62)=G(5)
0250 C(58,53)=X(5)

```

```

C
C PROGRAM FOR EQS. NO. 61, 64, 67 & 70

```

```

0251 W=4.*DBETA+DTHETA
0252 K=5
0253 I=58
0254 DO 22 N=1,4
0255 I=I+3
0256 K=K-1
0257 C(I,I)=-200.*((DELRI/(R(K)*W))*2.)
0258 J=I+1
0259 C(I,J)=200.*((DELRI/(R(K)*W))*2.)
0260 L=I+3
0261 C(I,L)=G(K)
0262 CONTINUE
0263 C(61,56)=X(4)
0264 C(64,61)=X(3)
0265 C(67,64)=X(2)
0266 C(70,67)=X(1)

```

```

C
C PROGRAM FOR EQS. NO. 63, 66, 69 & 72

```

```

0267 K=5
0268 I=60
0269 DO 23 N=1,4
0270 I=I+3
0271 K=K-1
0272 C(I,I)=-200.*((DELRI/R(K))*4.*DTHETA)**2.)
0273 J=I-1
0274 C(I,J)=200.*((DELRI/R(K))*4.*DTHETA)**2.)
0275 L=I-3
0276 C(I,L)=X(K)
0277 CONTINUE
0278 C(63,65)=G(4)
0279 C(66,69)=G(3)
0280 C(69,72)=G(2)
0281 C(72,73)=G(1)

```

23

```

C PROGRAM FOR EQS. NO. 62, 65, 68 & 71
C

```

```

0282 W=200./((4.*DTHETA*(4.*DBETA+5.*DTHETA))
0283 Y=200./((4.*DBETA+DTHETA)*(4.*DBETA+5.*DTHETA))
0284 Z=200./((4.*DBETA+DTHETA)*4.*DTHETA)
0285 K=5
0286 I=59
0287 DO 24 N=1,4
0288 I=I+3
0289 K=K-1
0290 C(I,I)=-200.*Z*((DELRI/R(K))**2.)
0291 J=I-1
0292 C(I,J)=Y*((DELRI/R(K))**2.)
0293 L=I+1
0294 C(I,L)=W*((DELRI/R(K))**2.)
0295 CONTINUE
0296 C(62,65)=G(4)
0297 C(62,58)=X(4)
0298 C(65,68)=G(3)
0299 C(65,62)=X(3)
0300 C(68,71)=G(2)
0301 C(68,65)=X(2)
0302 C(71,73)=G(1)
0303 C(71,68)=X(1)

```

24

```

C PROGRAM FOR EQ. NO. 73
C

```

```

0304 C(73,73)=-3.
0305 C(73,68)=-1.
0306 C(73,71)=4.

```

```

C PROGRAM FOR COLUMN NO. 74
C

```

```

0307 DO 25 I=1,25
0308 C(I,74)=-DEL R#100.
0309 CONTINUE
0310 DO 26 I=26,30
0311 C(I,74)=-1.
0312 CONTINUE
0313 DO 27 I=31,72
0314 C(I,74)=-DELRI#100.
0315 CONTINUE

```

25
26
27

PROGRAM FOR FIRST 73 EQS. FOR DIMENSIONLESS TEMPERATURES

```

0316 DO 500 I=1,115
0317 DO 500 J=1,116
0318 F(I,J)=0.0
0319 CONTINUE
0320 500
0321 DO 501 I=1,73
0322 DO 501 J=1,73
0323 F(I,J)=C(I,J)
0324 CALL GAJO (C,D,73,74)
0325 WRITE (6,100)
0326 WRITE (6,1233) XM,XL
0327 WRITE (6,100)
0328 WRITE (1H1)
0329 WRITE (6,101)
0330 WRITE (4X,'DIMENSIONLESS VELOCITIES',/)
0331 WRITE (6,102) (D(I),I=1,73)
0332 FORMAT (,' ',10F13.8/)
0333 Z=DBETA/(4.*DBETA+THETA)
0334 DO 40 K=1,4
0335 DD(K)=D(70)+(K*Z)*(D(71)-D(70))
0336 I=K+7
0337 DD(I)=D(67)+(K*Z)*(D(68)-D(67))
0338 J=K+14
0339 DD(J)=D(64)+(K*Z)*(D(65)-D(64))
0340 M=K+21
0341 DD(M)=D(61)+(K*Z)*(D(62)-D(61))
0342 CONTINUE
0343 I=4
0344 DO 41 K=1,3
0345 I=I+1
0346 DD(I)=D(71)+(K/4.)*(D(72)-D(71))
0347 J=I+7
0348 DD(J)=D(68)+(K/4.)*(D(69)-D(68))
0349 M=I+14
0350 DD(M)=D(65)+(K/4.)*(D(66)-D(65))
0351 N=I+21
0352 DD(N)=D(62)+(K/4.)*(D(63)-D(62))
0353 CONTINUE
0354 DD(29)=(D(56)+D(57))/2.
0355 Z=DBETA/(2.*DBETA+THETA)
0356 DD(30)=D(57)+Z*(D(58)-D(57))
0357 DD(31)=D(57)+2.*Z*(D(58)-D(57))
0358 DD(32)=(D(58)+D(59))/2.
0359 DD(33)=(D(59)+D(60))/2.
0360 DD(34)=(D(51)+D(52))/2.
0361 DD(35)=D(52)+Z*(D(53)-D(52))
0362 DD(36)=D(52)+2.*Z*(D(53)-D(52))
0363 DD(37)=(D(53)+D(54))/2.
0364 DD(38)=(D(54)+D(55))/2.

```

```

*****
PROGRAM FOR FRICTION FACTOR
*****

```

```

0365 J=40
0366 DO 43 I=1,5
0367 J=J+1
0368 QA(I)=(DELRI/(2.*3.))*D(J)*R(7)+4.*D(J-10)*R(8)
0369 Q(1)=(DELRI/3.)*(4.*D(70)*R(1)+2.*D(67)*R(2)+4.*D(64)*R(3)+D(61))*
43 1*(R(4))+3.*DELRI/8.)*(D(61)*R(4)+3.*D(56)*R(5)+3.*D(51)*R(6)+D(41))*
J=0
0370 DO 44 I=2,5
0371 J=J+1
0372 QB(I)=(DELRI/3.)*(4.*DD(J)*R(1)+2.*DD(J+7)*R(2)+4.*DD(J+14)*R(3)+
0373 1DD(J+21)*R(4))
44 CONTINUE
0374 Q(2)=(3.*DELRI/8.)*(DD(22)*R(4)+3.*DD(29)*R(5)+3.*DD(34)*R(6)+
0375 1D(42)*R(7))+QA(2)+QB(2)
0376 Q(3)=(3.*DELRI/8.)*(DD(23)*R(4)+3.*D(57)*R(5)+3.*D(52)*R(6)+D(43)
0377 1*(R(7))+QA(3)+QB(3)
0378 Q(4)=(3.*DELRI/8.)*(DD(24)*R(4)+3.*DD(30)*R(5)+3.*DD(35)*R(6)+
1D(44)*R(7))+QA(4)+QB(4)
0379 Q(5)=(3.*DELRI/8.)*(DD(25)*R(4)+3.*DD(31)*R(5)+3.*DD(36)*R(6)+
0380 1D(45)*R(7))+QA(5)+QB(5)
0381 DO 45 I=6,10
0382 J=J+1
45 QA(I)=(DELRI/(3.*2.))*D(J)*R(7)+4.*D(J-10)*R(8)+D(J-20)*R(9)+
1(DEL/3.)*(D(J-20)*R(9)+4.*D(J-25)*R(10)+2.*D(J-30)*R(11))+4.*
2D(J-35)*R(12)+D(J-40)*R(13))+DELRI/(2.*3.))*D(J-40)*R(13)+4.*
3D(J-45)*R(14))
0383 CONTINUE
0384 Q(6)=(DELRI/3.)*(4.*D(71)*R(1)+2.*D(68)*R(2)+4.*D(65)*R(3)+D(62)
1*(R(4))+3.*DELRI/8.)*(D(62)*R(4)+3.*D(58)*R(5)+3.*D(53)*R(6)+D(46)
2*(R(7))+QA(6)
J=4
0385 DO 46 I=7,9
0386 J=J+1
0387 QB(I)=(DELRI/3.)*(4.*DD(J)*R(1)+2.*DD(J+7)*R(2)+4.*DD(J+14)*R(3)
0388 1+DD(J+21)*R(4))
46 CONTINUE
0389 Q(7)=(3.*DELRI/8.)*(DD(26)*R(4)+3.*DD(32)*R(5)+3.*DD(37)*R(6)+
0390 1D(47)*R(7))+QA(7)+QB(7)
0391 Q(8)=(3.*DELRI/8.)*(DD(27)*R(4)+3.*D(59)*R(5)+3.*D(54)*R(6)+D(48)
0392 1*(R(7))+QA(8)+QB(8)
0393 Q(9)=(3.*DELRI/8.)*(DD(28)*R(4)+3.*DD(33)*R(5)+3.*DD(38)*R(6)+
1D(49)*R(7))+QA(9)+QB(9)
0394 U(10)=(DELRI/3.)*(4.*D(72)*R(1)+2.*D(69)*R(2)+4.*D(66)*R(3)+D(63)
0395 1*(R(4))+3.*DELRI/8.)*(D(63)*R(4)+3.*D(60)*R(5)+3.*D(55)*R(6)+D(50)
2*(R(7))+QA(10))
0396 Q(11)=(DBETA/3.)*(Q(1)+4.*Q(2)+2.*Q(3)+4.*Q(4)+Q(5))
0397 Q(12)=(DTHETA/3.)*(Q(5)+4.*Q(6)+Q(7))
0398 Q(13)=(3.*DTHETA/8.)*(Q(7)+3.*Q(8)+3.*Q(9)+Q(10))
0399 Q(14)=Q(11)+Q(12)+Q(13)
0400 Z1=(1.-XL)*#2.*BETA/2.
0401 Z2=THETA/2.
0402 Z=Z1+Z2
0403 AREA=Z
DBULK=Q(14)/AREA
W=BETA/(GAMMA-BETA)

```

```

0404 FRE=(2.*(1.+W))/(DBULK*(1.+W*((1.-XL)**2.)))
0405 WRITE (6,109)
109  FORMAT (//,3X,'FRICTION FACTOR',/)
0407 WRITE (6,110) FRE
110  FORMAT (2X,F13.8)
0408 WRITE (6,107)
107  FORMAT (//,3X,'BULK VELOCITY',/)
0410 WRITE (6,110) DBULK
C
C *****
C PROGRAM FOR DIMENSIONLESS TEMPERATURES *****
C *****
C AF=XM*(GAMMA-BETA*(1.-((1.-XL)**2.)))
C CI=-2.*PI*(1.+DELTA)/(AF*DBULK)
C
C PROGRAM FOR COLUMN NO. 116
C
C DO 502 I=1,25
C F(I,116)=DELR*DELR*100.*CI*D(I)
502
C DO 503 I=26,30
C F(I,116)=CI*D(I)
503
C DO 504 I=31,72
C F(I,116)=DELR*DELR*100.*CI*D(I)
504
C DO 505 I=108,115
C F(I,116)=DELTA*C2
505
C
C J=84
C DO 506 I=1,5
C J=J+1
506
C F(I,J)=(4.+DELR/R(14))*100.
C DO 507 I=6,10
C J=I+79
507
C F(I,J)=X(13)
C F(1,83)=B(14)
C F(6,82)=B(13)
C F(11,81)=B(12)
C F(16,80)=B(11)
C F(21,79)=B(10)
C F(26,78)=(1./R(9)*DTHETA)**2.
C DO 508 I=31,35
C J=I+43
508
C F(I,J)=(4.+DELR/R(8))*100.
C DO 509 I=41,45
C J=I+33
509
C F(I,J)=X(7)
C
C PROGRAM FOR EQS. FROM 74 TO 77
C
C Z=DELR/DELR1
C DO 510 I=74,77
C F(I,I)=-3.-6.*Z*C2
C J=I-33
C F(I,J)=-2.*Z*C2
C K=I-43
C F(I,K)=8.*Z*C2
510
C CONTINUE
C F(74,94)=4.
0449
    
```

```

0450 F(74,95)=-1.
0451 F(75,94)=2.
0452 F(75,90)=2.
0453 F(75,95)=-1./2.
0454 F(75,91)=-1./2.
0455 F(76,90)=4.
0456 F(76,91)=-1.
0457 F(77,90)=2.
0458 F(77,79)=2.
0459 F(77,91)=-1./2.
0460 F(77,80)=-1./2.

```

```

C C C PROGRAM FOR EQ. NO. 78
C W=(DEL R/(R(9)*DBETA))**2.
C F(78,78)=1.-(.3.*DEL R)/(2.*R(9))+W
C F(78,76)=W
C F(78,77)=-2.*W
C F(78,80)=1.-DEL R/(2.*R(9))
C F(78,79)=-2.+2.*DEL R/R(9)

```

```

C C C PROGRAM FOR EQS. FROM 79 TO 83

```

```

0467 J=27
0468 DO 512 I=79,83
0469 F(I,I)=3.*DTHE TA+6.*DBETA*C2
0470 J=J-5
0471 F(I,J)=2.*C2*DBETA
0472 K=J-1
0473 F(I,K)=-8.*C2*DBETA
0474 CONTINUE
0475 DO 513 I=79,82
0476 J=I+15
0477 F(I,J)=DTHE TA
0478 K=I+11
0479 F(I,K)=-4.*DTHE TA
0480 CONTINUE
0481 F(83,93)=-2.*DTHE TA
0482 F(83,99)=-2.*DTHE TA
0483 F(83,97)=DTHE TA/2.
0484 F(83,98)=DTHE TA/2.

```

```

512 CONTINUE
513 CONTINUE

```

```

C C C PROGRAM FOR EQ. NO. 84

```

```

0485 W=DEL R*DELTA*(DEL R+DELTA)
0486 V=2.*DBETA*DTHE TA*(DTHE TA+2.*DBETA)/2.
0487 F(84,84)=-3.*(DEL R+DELTA)/W-2.*(DEL R**2.-DELTA**2.)/W-(2.*DBETA
I+DTHE TA)/V
0488 F(84,99)=DTHE TA/V
0489 F(84,85)=2.*DBETA/V
0490 F(84,83)=8.*DBETA/W-2.*(DELTA**2.)/W
0491 F(84,102)=8.*DEL R/W+2.*(DEL R**2.)/W

```

```

C C C PROGRAM FOR EQS. FROM 85 TO 89

```

```

0492 DO 515 I=85,89
0493 F(I,I)=-3.*DEL R-3.*C2*DELTA
0494 J=I-79

```

```

0450 F(I,J)=-C2*JLLTA
0456 K=J-5
0457 F(I,K)=4.*C2*DELTA
0458 L=I+26
0459 F(I,L)=-DELTA
0500 M=I+18
0501 F(I,M)=4.*DELTA
0502 CONTINUE
    
```

515

C C

```

PROGRAM FOR EQS. FROM 90 TO 93
    
```

```

0503 DO 516 I=90,93
0504 K=K+1
0505 F(I,I)=-2.*((DELTA/(2.*R(K)*DBETA))**2.)
0506 J=I+4
0507 F(I,J)=(DELTA/(2.*R(K)*DBETA))**2.
0508 L=I-11
0509 F(I,L)=(DELTA/(2.*R(K)*DBETA))**2.
0510 CONTINUE
0511 F(90,76)=1.-DELTA/(2.*R(10))
0512 F(90,91)=1.+DELTA/(2.*R(10))
0513 F(91,92)=1.+DELTA/(2.*R(11))
0514 F(91,90)=1.-DELTA/(2.*R(11))
0515 F(92,91)=1.-DELTA/(2.*R(12))
0516 F(92,93)=1.+DELTA/(2.*R(12))
0517 F(93,99)=1.+DELTA/(2.*R(13))
0518 F(93,92)=1.-DELTA/(2.*R(13))
0519
    
```

516

C C C

```

PROGRAM FOR EQS. FROM 94 TO 97
    
```

```

0520 K=9
0521 DO 517 I=94,97
0522 K=K+1
0523 F(I,I)=-2.*((DELTA/(2.*R(K)*DBETA))**2.)
0524 J=I-4
0525 F(I,J)=2.*((DELTA/(2.*R(K)*DBETA))**2.)
0526 L=I+1
0527 F(I,L)=1.+DELTA/(2.*R(K))
0528 CONTINUE
0529 F(94,74)=1.-DELTA/(2.*R(10))
0530 F(95,94)=1.-DELTA/(2.*R(11))
0531 F(96,95)=1.-DELTA/(2.*R(12))
0532 F(97,96)=1.-DELTA/(2.*R(13))
    
```

517

C C C

```

PROGRAM FOR EQS. NO. 98 & 99
    
```

```

0533 W=DELTA*(DELTA+2.*DELTA)
0534 V=(1./((2.*DBETA))**2.)
0535 F(98,98)=-4.*((2.*DELTA+DELTA)/W-(4.*(DELTA**2.))-DELTA**2.)/W-2.*V
0536 F(99,99)=F(98,98)
0537 F(98,99)=2.*V
0538 F(99,98)=V
0539 F(99,84)=V
0540 F(98,97)=4.*DELTA/W-(DELTA**2.)/W
0541 F(99,93)=4.*DELTA/W-(DELTA**2.)/W
0542 F(98,100)=8.*DELTA/W+4.*(DELTA**2.)/W
0543 F(99,101)=3.*DELTA/W+4.*(DELTA**2.)/W
    
```

```

0544      R16=1.+DELTA/2.
0545      W=(DELTA/(2.*R16*DBETA))**2.
0546      F(100,100)=-8.-2.*W
0547      F(101,101)=-8.-2.*W
0548      F(100,101)=2.*W
0549      F(101,100)=W
0550      F(101,102)=W
0551      F(100,98)=4.-DELTA/R16
0552      F(101,99)=4.-DELTA/R16
0553      F(100,108)=4.+DELTA/R16
0554      F(101,109)=4.+DELTA/R16

C
C
C      PROGRAM FOR EQ. NU. 100 & 101

0555      V=2.*DBETA*DTHEA*(DTHETA+2.*DBETA)/2.
0556      W=R16**2.
0557      Z=DELTA**2.
0558      F(102,102)=-8./Z-(2.*DBETA+DTHETA)/(W*V)
0559      F(102,101)=DTHETA/(W*V)
0560      F(102,103)=(2.*DBETA)/(W*V)
0561      F(102,84)=4./Z-1./(R16*DELTA)
0562      F(102,110)=4./Z+1./(R16*DELTA)

C
C
C      PROGRAM FOR EQS. FROM 103 TO 107

0563      W=(DELTA/(R16*DTHEA))**2.
0564      DO 520 I=103,107
0565      F(I,I)=-8.-2.*W
0566      J=I-18
0567      F(I,J)=4.-DELTA/R16
0568      K=I+8
0569      F(I,K)=4.+DELTA/R16
0570      CONTINUE
0571      DO 521 I=103,106
0572      J=I-1
0573      F(I,J)=W
0574      K=I+1
0575      F(I,K)=W
0576      CONTINUE
0577      F(107,106)=2.*W

C
C
C      PROGRAM FOR EQS. FROM 108 TO 115

0578      DO 523 I=108,115
0579      F(I,I)=-3.
0580      J=I-8
0581      F(I,J)=4.
0582      F(108,98)=-1.
0583      F(109,99)=-1.
0584      DO 524 I=110,115
0585      J=I-26
0586      F(I,J)=-1.

C
C
C      CALL GAJD (F,H,115,116)
0587

```

```

0588 WRITE (6,150)
0589 FORMAT (//,3X,'DIMENSIONLESS TEMPS.',/,)
0590 WRITE (6,151) (H(I),I=1,115)
0591 FORMAT ('.',10F13.8/)
0592 Z=DBETA/(4.*DBETA+DTHETA)
0593 DO 600 K=1,4
0594 HH(K)=H(70)+(K*Z)*(H(71)-H(70))
0595 I=K+7
0596 HH(I)=H(67)+(K*Z)*(H(68)-H(67))
0597 J=K+14
0598 HH(J)=H(64)+(K*Z)*(H(65)-H(64))
0599 M=K+21
0600 HH(M)=H(61)+(K*Z)*(H(62)-H(61))
0601 CONTINUE
0602 I=4
0603 DO 601 K=1,3
0604 I=I+1
0605 HH(I)=H(71)+(K/4.)*(H(72)-H(71))
0606 J=I+7
0607 HH(J)=H(68)+(K/4.)*(H(69)-H(68))
0608 M=I+14
0609 HH(M)=H(65)+(K/4.)*(H(66)-H(65))
0610 N=I+21
0611 HH(N)=H(62)+(K/4.)*(H(63)-H(62))
0612 CONTINUE
0613 HH(29)=(H(56)+H(57))/2.
0614 Z=DBETA/(2.*DBETA+DTHETA)
0615 HH(30)=H(57)+Z*(H(58)-H(57))
0616 HH(31)=H(57)+2.*Z*(H(58)-H(57))
0617 HH(32)=(H(58)+H(59))/2.
0618 HH(33)=(H(59)+H(60))/2.
0619 HH(34)=(H(51)+H(52))/2.
0620 HH(35)=H(52)+Z*(H(53)-H(52))
0621 HH(36)=H(52)+2.*Z*(H(53)-H(52))
0622 HH(37)=(H(53)+H(54))/2.
0623 HH(38)=(H(54)+H(55))/2.
0624 J=40
0625 DO 80 I=1,5
0626 J=J+1
0627 QA(I)=(DELRI/(2.*3.))*D(J)*H(J)*R(7)+4.*D(J-10)*H(J-10)*R(8)
0628 QUT(1)=(DELRI/3.)*4.*D(70)*H(70)*R(1)+2.*D(67)*H(67)*R(2)+4.*
1D(64)*H(64)*R(3)+D(61)*H(61)*R(4)+3.*DELRI/8.*D(61)*H(61)*R
2(4)+3.*D(56)*H(56)*R(5)+3.*D(51)*H(51)*R(6)+D(41)*H(41)*R(7))+QA(
31)
J=0
0629 DO 81 I=2,5
0630 J=J+1
0631 QB(I)=(DELRI/3.)*4.*D(J)*HH(J)*R(1)+2.*DD(J+7)*HH(J+7)*R(2)+4.*
0632 1DD(J+14)*HH(J+14)*R(3)+DD(J+21)*HH(J+21)*R(4))
CONTINUE
0633 QUT(2)=(3.*DELRI/8.)*(DD(22)*HH(22)*R(4)+3.*DD(29)*HH(29)*R(5)+3.
0634 1*DD(34)*HH(34)*R(6)+D(42)*H(42)*R(7))+QA(2)+QB(2)
0635 QUT(3)=(3.*DELRI/8.)*(DD(23)*HH(23)*R(4)+3.*D(57)*H(57)*R(5)+3.*
0636 1D(52)*H(52)*R(6)+D(43)*H(43)*R(7))+QA(3)+QB(3)
0637 QUT(4)=(3.*DELRI/8.)*(DD(24)*HH(24)*R(4)+3.*DD(30)*HH(30)*R(5)+3.
0638 1*DD(35)*HH(35)*R(6)+D(44)*H(44)*R(7))+QA(4)+QB(4)
0639 QUT(5)=(3.*DELRI/8.)*(DD(25)*HH(25)*R(4)+3.*DD(31)*HH(31)*R(5)+3.
0640 1*DD(36)*HH(36)*R(6)+D(45)*H(45)*R(7))+QA(5)+QB(5)

```



```

*****
SUBROUTINE GAJO(A,S,N,NI)
*****

```

```

THIS SUBROUTINE SOLVES N SIMULTANEOUS LINEAR ALGEBRAIC
EQUATIONS IN N UNKNOWNS USING THE GAUSS-JORDAN REDUCTION

```

```

DIMENSION A(N,NI),S(N)

```

```

DO 200 J=1,N
IF(A(J,J).NE.0.)GO TO 300
JI=J+1
DO 302 K=JI,N
IF(A(K,J).NE.0.)GO TO 303
CONTINUE
302 DO 304 I=J,N
DUMMY=A(K,I)
A(K,I)=A(J,I)
304 A(J,I)=DUMMY
300 DIV=A(J,J)
SA=1./DIV
DO 201 K=J,NI
A(J,K)=A(J,K)*SA
201 DO 202 I=1,N
IF(I-J)203,202,203
203 AIJ=-A(I,J)
204 A(I,K)=A(I,K)+AIJ*A(J,K)
202 CONTINUE
200 DO 400 I=1,N
S(I)=A(I,NI)
400 RETURN
END

```

```

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C
C
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C

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```

*****
PROGRAM FOR MESH 2
*****

```

THE VARIABLES ARE DEFINED AS FOLLOWS :

```

XM      NO. OF FINS
XL      DIMENSIONLESS FIN LENGTH
BETA    HALF THE ANGLE SUBTENDED BY ONE FIN
DELTA   DIMENSIONLESS TUBE THICKNESS
DELRI   GRID SIZE IN THE RADIAL DIRECTION
GAMMA   HALF THE ANGLE BETWEEN THE CENTRE-LINES
        OF TWO ADJACENT FINS
DTHETA  GRID SIZE IN THE ANGULAR DIRECTION
DBETA   GRID SIZE IN THE ANGULAR DIRECTION
XKF     THERMAL CONDUCTIVITY OF THE FLUID
XKS     THERMAL CONDUCTIVITY OF THE SOLID
D(I)    DIMENSIONLESS VELOCITY
H(I)    DIMENSIONLESS TEMPERATURE
*****

```

```

DIMENSION R(13),A(13),B(13),X(13),G(13),E(13),C(72,73),D(72)
DIMENSION DD(14),QA(10),Q(14),F(123,124),H(123),HH(14),OUT(14)

XM=8.
PI=3.14159265
XL=0.6
BETA=(1.5*PI)/180.

```

```

DELTA=0.1
DELR=XL/8.
DELRI=(1.-XL)/4.
GAMMA=(360.*PI)/(2.*XM*180.)
THETA=GAMMA-BETA
DTHETA=THETA/5.
DBETA=BETA/4.
XKS=220.
XKF=0.074
C2=XKF/XKS
R(1)=DELRI
R(2)=2.*DELRI
R(3)=3.*DELRI
R(4)=3.5*DELRI
R(5)=4.*DELRI
J=4
DO 1 I=6,12
J=J+1
R(I)=R(J)+DELR
CONTINUE
R(13)=R(12)+DELR/2.

```

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```

```

0028 DO 2 I=1,4
0029 A(I)=DELRI/(2.*R(I))
0030 B(I)=((DELRI/(R(I)*DTHETA))**2.)*100.
0031 X(I)=(1.+A(I))*100.
0032 G(I)=(1.-A(I))*100.
0033 E(I)=-200.-2.*B(I)
0034 CONTINUE
0035 DO 3 I=6,13
0036 A(I)=DELRI/(2.*R(I))
0037 B(I)=((DELRI/(R(I)*DTHETA))**2.)*100.
0038 X(I)=(1.+A(I))*100.
0039 G(I)=(1.-A(I))*100.
0040 E(I)=-200.-2.*B(I)
0041 CONTINUE

```

```

C *****
C PROGRAM FOR DIMENSIONLESS VELOCITIES
C *****

```

```

0042 DO 4 I=1,72
0043 DO 4 J=1,73
0044 C(I,J)=0.0
0045 CONTINUE

```

```

C *****
C PROGRAM FOR EQS. FROM 1 TO 5
C *****

```

```

0046 DO 5 I=1,5
0047 C(I,1)=-800.-2.*B(13)
0048 J=I+5
0049 C(I,J)=(4.-DELRI/R(13))*100.
0050 CONTINUE
0051 DO 6 I=2,4
0052 J=I+1
0053 C(I,J)=B(13)
0054 K=I-1
0055 C(I,K)=B(13)
0056 CONTINUE
0057 C(1,2)=B(13)
0058 C(5,4)=2.*B(13)

```

```

C *****
C PROGRAM FOR EQS. FROM 6 TO 10
C *****

```

```

0059 DO 10 I=6,10
0060 C(I,1)=E(12)
0061 J=I+5
0062 C(I,J)=G(12)
0063 CONTINUE
0064 DO 11 I=7,9
0065 J=I+1
0066 C(I,J)=B(12)
0067 K=I-1
0068 C(I,K)=B(12)
0069 CONTINUE
0070 C(6,7)=B(12)
0071 C(10,9)=2.*B(12)

```

```

C *****
C PROGRAM FOR EQS. FROM 11 TO 40
C *****

```

```

0072 I=6

```

```

0073 K=12 7 N=1,6
0074 DO
0075 I=I+5
0076 K=K-1
0077 I1=I-1
0078 DO 8
0079 I1=I1+1
0080 C(I1,I1)=E(K)
0081 J1=I1+5
0082 C(I1,J1)=G(K)
0083 K1=I1-5
0084 C(I1,K1)=X(K)
0085 CONTINUE
0086 I2=I
0087 DO 9 L=1,3
0088 I2=I2+1
0089 I3=I2-1
0090 C(I2,I3)=B(K)
0091 I4=I2+1
0092 C(I2,I4)=B(K)
0093 CONTINUE
0094 I6=I+1
0095 C(I,I6)=B(K)
0096 I7=I+4
0097 I8=I7-1
0098 C(I7,I8)=2.*B(K)
0099 CONTINUE

```

3

9

7

C
C
C

PROGRAM FOR EOS. FROM 41 TO 45

```

0100 Z=DEL R*DEL R1*(DEL R1+2.*DEL R)
0101 P=4.*DEL R1/Z+((DEL R1**2.)/(R(S)*Z)
0102 Y=8.*DEL R/Z-((2.*DEL R)**2.)/(R(S)*Z)
0103 W=(1./R(S)*DTHETA)**2.
0104 V=4.*((DEL R1+2.*DEL R)/Z+(DEL R1**2.-
0105 (2.*DEL R)**2.)/(R(S)*Z)+2.*W
0106 DO 12 I=41,45
0107 C(I,I)=-V
0108 J=I+10
0109 C(I,J)=Y
0110 K=I-5
0111 C(I,K)=P
0112 CONTINUE
0113 DO 13 I=42,44
0114 J=I+1
0115 C(I,J)=W
0116 K=I-1
0117 C(I,K)=W
0118 CONTINUE
0119 C(41,42)=W
0120 C(45,44)=2.*W

```

12

13

C
C
C

PROGRAM FOR EOS. FROM 46 TO 49

```

0120 Z=((DEL R1/(R(4)*DBETA)**2.)*100.
0121 DO 14 I=46,49
0122 C(I,I)=-300.-2.*Z
0123 J=I+10
0124 C(I,J)=(4.-DEL R1/R(4))*100.

```

```

0125 CONTINUE
0126 DO 15 I=47,49
0127 J=I+1
0128 C(I,J)=Z
0129 K=I-1
0130 C(I,K)=Z
0131 CONTINUE
0132 C(46,47)=2.*Z
C
C PROGRAM FOR EQ. NU. 50
0133 Z=((DELRI/R(4))*2.)*100.
0134 C(50,50)=-800.-(2.*Z)/(DIHETA*DBETA)
0135 C(50,49)=(2.*Z)/(DBETA*(DIHETA+DBETA))
0136 C(50,51)=(2.*Z)/(DIHETA*(DIHETA+DBETA))
0137 C(50,60)=(4.-DELRI/R(4))*100.
C
C PROGRAM FOR EQS. FROM 51 TO 55
0138 DO 16 I=51,55
0139 C(I,I)=-800.-2.*B(4)
0140 J=I+10
0141 C(I,J)=(4.-DELRI/R(4))*100.
0142 K=I-10
0143 C(I,K)=(4.+DELRI/R(4))*100.
0144 CONTINUE
0145 DO 17 I=51,54
0146 J=I+1
0147 C(I,J)=B(4)
0148 K=I-1
0149 C(I,K)=B(4)
0150 CONTINUE
0151 C(55,54)=2.*B(4)
C
C PROGRAM FOR EQS. FROM 55 TO 59
0152 W=((DELRI/(R(3)*DBETA))*2.)*100.
0153 DO 18 I=56,59
0154 C(I,I)=-200.-2.*W
0155 DO 19 I=57,59
0156 J=I+1
0157 C(I,J)=W
0158 K=I-1
0159 C(I,K)=W
0160 CONTINUE
0161 C(56,57)=2.*W
0162 C(56,66)=G(3)
0163 Z=DBETA/(4.*DBETA+DIHETA)
0164 C(57,66)=G(3)*(1.-Z)
0165 C(57,67)=G(3)*Z
0166 C(58,66)=G(3)*(1.-2.*Z)
0167 C(58,67)=G(3)*2.*Z
0168 C(59,66)=G(3)*(1.-3.*Z)
0169 C(59,67)=G(3)*3.*Z
C
C PROGRAM FOR EQS. FROM 61 TO 65
0170 DO 20 I=61,64

```

```

0171 C(I,I)=E(3)
0172 J=I-1
0173 C(I,J)=B(3)
0174 K=I+1
0175 C(I,K)=H(3)
0176 L=I-20
0177 C(I,L)=X(3)
0178 CONTINUE
0179 C(65,65)=E(3)
0180 C(65,64)=2.*B(3)
0181 C(65,68)=G(3)
0182 C(65,45)=X(3)
0183 C(61,67)=G(3)
0184 K=0
0185 DO 21 I=62,64
0186 K=K+1
0187 C(I,67)=G(3)*(1.-K/4.)
0188 C(I,68)=G(3)*(K/4.)
0189 CONTINUE

```

20

21

PROGRAM FOR EQ. NU. 60

```

0190 Z=((DELRI/R(3))**2.)*100.
0191 C(60,50)=-200.-(2.*Z)/(DIHETA*DBETA)
0192 C(60,59)=(2.*Z)/(DBETA*(DIHETA+DBETA))
0193 C(60,61)=(2.*Z)/(DIHETA*(DIHETA+DBETA))
0194 Y=(4.*DBETA)/(4.*DBETA+DIHETA)
0195 C(60,66)=G(3)*(1.-Y)
0196 C(60,67)=G(3)*Y

```

PROGRAM FOR EQS. NU. 66 & 69

```

0197 Z=2.*((DELRI/(R(2))*(4.*DBETA+DIHETA))**2.)*100.
0198 Y=2.*((DELRI/(R(1))*(4.*DBETA+DIHETA))**2.)*100.
0199 C(66,66)=-200.-Z
0200 C(66,67)=Z
0201 C(66,69)=G(2)
0202 C(66,56)=X(2)
0203 C(69,69)=-200.-Y
0204 C(69,70)=Y
0205 C(69,72)=G(1)
0206 C(69,66)=X(1)

```

PROGRAM FOR EQS. NU. 68 & 71

```

0207 Z=2.*((DELRI/(R(2))*4.*DIHETA)**2.)*100.
0208 Y=2.*((DELRI/(R(1))*4.*DIHETA)**2.)*100.
0209 C(68,68)=-200.-Z
0210 C(68,67)=Z
0211 C(68,71)=G(2)
0212 C(68,65)=X(2)
0213 C(71,71)=-200.-Y
0214 C(71,70)=Y
0215 C(71,72)=G(1)
0216 C(71,66)=X(1)

```

PROGRAM FOR EQS. NU. 67 & 70

MAIN

```

0217 Z2=((DELRI/R(2))*#Z.)#100.
0218 Z1=((DELRI/R(1))*#2.)#100.
0219 Y=2./((4.*DBETA+DTHETA)*#100.
0220 W=2./((4.*DTHETA*(4.*DBETA+5.*DTHETA))
0221 V=2./((4.*DTHETA*(4.*DBETA+DTHETA))
0222 C(67,67)=-200.-W#Z2
0223 C(70,70)=-200.-W#Z2
0224 C(67,66)=Y#Z2
0225 C(70,69)=Y#Z1
0226 C(67,68)=V#Z2
0227 C(70,71)=V#Z1
0228 C(67,70)=G(2)
0229 C(67,61)=X(2)
0230 C(70,72)=G(1)
0231 C(70,67)=X(1)

```

C PROGRAM FOR EQ. NO. 72

```

0232 C(72,72)=-3.
0233 C(72,67)=-1.
0234 C(72,70)=4.

```

C PROGRAM FOR COLUMN NO. 73

```

0235 DO 25 I=1,40
0236 C(I,73)=-DELRI*DELRI*100.
0237 CONTINUE
0238 DO 26 I=41,45
0239 C(I,73)=-1.
0240 CONTINUE
0241 DO 27 I=46,71
0242 C(I,73)=-DELRI*DELRI*100.
0243 CONTINUE
0244

```

C PROGRAM FOR FIRST 72 EQS. FOR DIMENSIONLESS TEMPS.

```

0244 DO 200 I=1,123
0245 DO 200 J=1,124
0246 F(I,J)=0.0
0247 CONTINUE
0248 DO 201 I=1,72
0249 DO 201 J=1,72
0250 F(I,J)=C(I,J)

```

C CALL GAJO (C,D,72,73)

```

0251
0252 WRITE (6,1233) XM,XL
0253 FORMAT (F6.1,F10.2)
0254 WRITE (6,100)
0255 FORMAT (IHI)
0256 WRITE (6,101)
0257 FORMAT (4X,'DIMENSIONLESS VELOCITIES',/)
0258 WRITE (6,102) (C(I),I=1,72)
0259 FORMAT (I,10F13.6/)
0260 Z=DBETA/(4.*DBETA+DTHETA)
0261 DO 30 K=1,4
0262 DD(K)=D(59)+(K*Z)*(D(70)-D(59))
0263 LEKT

```

```

0264 30 DD(I)=D(66)+(K*Z)*(D(67)-D(66))
0265 I=4
0266 DO 31 K=1,3
0267 I=I+1
0268 DD(I)=D(70)+(K/4.)*(D(71)-D(70))
0269 J=I+7
0270 DD(J)=D(67)+(K/4.)*(D(68)-D(67))
C
C *****
C PROGRAM FOR FRICTION FACTOR
C *****
0271 J=55
0272 DO 40 I=1,5
0273 J=J+1
0274 QA(I)=(DELRI/(2.*3.))*(D(J)*R(3)+4.*D(J-10)*R(4))
0275 Q(1)=(3.*DELRI/8.)*(3.*D(69)*R(1)+3.*D(66)*R(2)+D(56)*R(3))+QA(I)
0276 DO 41 I=2,5
0277 K=I-1
0278 Q(I)=(3.*DELRI/8.)*(3.*DD(K)*R(1)+3.*DD(K+7)*R(2)+D(I+55)*R(3))+
1QA(I)
0279 CONTINUE
0280 J=60
0281 DO 45 I=6,10
0282 J=J+1
0283 QA(I)=(DELRI/(2.*3.))*(D(J)*R(3)+4.*D(J-10)*R(4)+D(J-20)*R(5))+
1(DELRI/3.)*(D(J-20)*R(5)+4.*D(J-25)*R(6)+2.*D(J-30)*R(7)+4.*D(J-35)
2*(R(8)+D(J-40)*R(9))+3.*DELRI/8.)*(D(J-40)*R(9)+3.*D(J-45)*R(10)+
33.*D(J-50)*R(11)+D(J-55)*R(12))+((DELRI/(2.*3.))*(D(J-55)*R(12))+4.
4*(D(J-60)*R(13))
0284 CONTINUE
0285 Q(6)=(3.*DELRI/8.)*(3.*D(70)*R(1)+3.*D(67)*R(2)+D(61)*R(3))+QA(6)
0286 K=4
0287 DO 46 I=7,9
0288 K=K+1
0289 Q(I)=(3.*DELRI/8.)*(3.*DD(K)*R(1)+3.*DD(K+7)*R(2)+D(K+57)*R(3))+
1QA(I)
0290 CONTINUE
0291 Q(10)=(3.*DELRI/8.)*(3.*D(71)*R(1)+3.*D(68)*R(2)+D(65)*R(3))+QA(10)
1)
0292 Q(11)=(DBETA/3.)*(Q(1)+4.*Q(2)+2.*Q(3)+4.*Q(4)+Q(5))
0293 Q(12)=(DTHETA/3.)*(Q(5)+4.*Q(6)+Q(7))
0294 Q(13)=(3.*DTHETA/8.)*(Q(7)+3.*Q(8)+3.*Q(9)+Q(10))
0295 Q(14)=Q(11)+Q(12)+Q(13)
0296 Z1=((1.-XL)*2.)*BETA/2.
0297 Z2=THETA/2.
0298 Z=Z1+Z2
0299 DBULK=G(14)/Z
0300 W=BETA/(GAMMA-BETA)
0301 FRE=(2.*(1.+W))/(DBULK*(1.+W*(1.-XL)**2.))
0302 WRITE (6,109)
0303 FORMAT (//,,3X,'FRICTION FACTOR',/)
0304 WRITE (6,110) FRE
0305 FORMAT (2X,F17.8)
0306 WRITE (6,107)
0307 FORMAT (//,,3X,'BULK VELOCITY',/)
0308 WRITE (6,110) CBULK
C

```

```

*****
PROGRAM FOR DIMENSIONLESS TEMPERATURES
*****

```

```

0309 AF=XM*(GAMMA-BETA*(1.-((1.-XL)**2.)))
0310 C1=-2.*PI*(1.+DELTA)/(AF*DBJLK)

```

```

C
C
C
PROGRAM FOR COLUMN NO. 124
C

```

```

0311 DO 202 I=1,40
0312 F(I,124)=DELTA*DELTA*100.*C1*D(I)
0313 DO 203 I=41,45
0314 F(I,124)=C1*D(I)
0315 DO 204 I=46,71
0316 F(I,124)=DELTA*DELTA*100.*C1*D(I)
0317 DO 210 I=116,123
0318 F(I,124)=DELTA*C2
C

```

```

0319 J=86
0320 DO 205 I=1,5
0321 J=J+1
0322 F(I,J)=(4.+DELTA/R(13))*100.
0323 DO 206 I=6,10
0324 J=I+81
0325 F(I,J)=X(12)
0326 J=41
0327 K=5
0328 DO 207 I=78,85
0329 K=K+1
0330 J=J-5
0331 F(J,I)=B(K)
0332 F(41,77)=(1./R(5)*DIHETA)**2.
0333 DO 208 I=46,50
0334 J=I+27
0335 F(I,J)=(4.+DELTA/R(4))*100.
0336 DO 209 I=55,60
0337 J=I+17
0338 F(I,J)=X(3)
C
C

```

```

PROGRAM FOR EQS. NO. 73 & 75
C
C

```

```

0339 Z=DELTA/DELTA
0340 F(73,73)=-3.-6.*Z*C2
0341 F(73,56)=-2.*Z*C2
0342 F(73,46)=8.*Z*C2
0343 F(73,100)=-1.
0344 F(73,99)=4.
0345 F(75,75)=-3.-6.*Z*C2
0346 F(75,58)=-2.*Z*C2
0347 F(75,48)=8.*Z*C2
0348 F(75,93)=-1.
0349 F(75,92)=4.
C
C

```

```

PROGRAM FOR EQS. NO. 74 & 76
C
C

```

```

0350 F(74,74)=-3.-6.*Z*C2
0351 F(74,57)=-2.*Z*C2
0352 F(74,47)=8.*Z*C2

```

0353 F(74,100)=-1./2.
 0354 F(74,93)=-1./2.
 0355 F(74,99)=2.
 0356 F(74,92)=2.
 0357 F(76,76)=-3.-0.*Z*C2
 0358 F(76,59)=-2.*Z*C2
 0359 F(76,49)=8.*Z*C2
 0360 F(76,93)=-1./2.
 0361 F(76,79)=-1./2.
 0362 F(76,92)=2.
 0363 F(76,78)=2.

C
 C

PROGRAM FOR EQ. NO. 77
 W=(DEL R(R(5)*DBETA))*2.
 F(77,77)=1.-(3.*DEL R)/(2.*R(5))+W
 F(77,75)=W
 F(77,76)=-2.*W
 F(77,79)=1.-DEL R/(2.*R(5))
 F(77,78)=-2.+2.*DEL R/R(5)

C
 C

PROGRAM FOR EQS. FROM 78 TO 84

0370 J=42
 0371 DO 220 I=78,84
 0372 F(I,I)=3.*DTHETA+6.*C2*DBETA
 0373 J=J-5
 0374 F(I,J)=2.*C2*DBETA
 0375 K=J-1
 0376 F(I,K)=-8.*C2*DBETA
 0377 L=I+21
 0378 F(I,L)=DTHETA
 0379 M=I+14
 0380 F(I,M)=-4.*DTHETA
 0381 CONTINUE

220
 C
 C

PROGRAM FOR EQ. NO. 85
 F(85,85)=3.*DTHETA+6.*C2*DBETA
 F(85,2)=2.*C2*DBETA
 F(85,1)=-8.*C2*DBETA
 F(85,105)=DTHETA/2.
 F(85,106)=DTHETA/2.
 F(85,98)=-2.*DTHETA
 F(85,107)=-2.*DTHETA

C
 C

PROGRAM FOR EQ. NO. 86
 W=DEL R*DELTA*(DEL R+DELTA)
 V=2.*DBETA*DTHETA*(DTHETA+2.*DBETA)/2.
 F(86,86)=-8.*(DEL R+DELTA)/W-2.*(DEL R**2.-DELTA**2.)/W-(2.*DBETA+
 1DTHETA)/V
 F(86,107)=DTHETA/V
 F(86,87)=2.*DBETA/V
 F(86,85)=8.*DELTA/W-2.*(DELTA**2.)/W
 F(86,110)=8.*DEL R/W+2.*(DEL R**2.)/W

C
 C

PROGRAM FOR EQS. FROM 87 TO 91

```

0356 DO 221 I=87,91
0397 F(I,I)=-J.*DEL R-3.*C2*DELTA
0355 J=I-81
0359 F(I,J)=-C2*DELTA
0400 K=J-5
0401 F(I,K)=4.*C2*DELTA
0402 L=I+32
0403 F(I,L)=-DEL R
0404 M=I+24
0405 F(I,M)=4.*DEL R
0406 CONTINUE
221
C
C PROGRAM FOR EQS. FROM 92 TO 98
C
0407 K=5
0408 DO 222 I=92,98
0409 K=K+1
0410 F(I,I)=-2.*-2.*((DEL R/(2.*R(K)*DBETA))**2.)
0411 J=I+7
0412 F(I,J)=(DEL R/(2.*R(K)*DBETA))**2.
0413 L=I-14
0414 F(I,L)=(DEL R/(2.*R(K)*DBETA))**2.
0415 CONTINUE
222
C
C DO 223 I=93,97
C
0416 K=K+1
0417 J=I-1
0418 F(I,J)=1.-DEL R/(2.*R(K))
0419 L=I+1
0420 F(I,L)=1.+DEL R/(2.*R(K))
0421 F(92,75)=1.-DEL R/(2.*R(6))
0422 F(92,93)=1.+DEL R/(2.*R(6))
0423 F(98,97)=1.-DEL R/(2.*R(12))
0424 F(98,107)=1.+DEL R/(2.*R(12))
0425
0426
C
C PROGRAM FOR EQS. FROM 99 TO 105
C
0427 K=5
0428 DO 224 I=99,105
0429 K=K+1
0430 F(I,I)=-2.*-2.*((DEL R/(2.*R(K)*DBETA))**2.)
0431 J=I-7
0432 F(I,J)=2.*((DEL R/(2.*R(K)*DBETA))**2.)
0433 L=I+1
0434 F(I,L)=1.+DEL R/(2.*R(K))
0435 K=6
0436 DO 225 I=100,105
0437 K=K+1
0438 J=I-1
0439 F(I,J)=1.-DEL R/(2.*R(K))
0440 F(99,73)=1.-DEL R/(2.*R(6))
C
C PROGRAM FOR EQS. NO. 106 & 107
C
0441 V=(1./(2.*DBETA))**2.
0442 W=DEL R*DELTA*(DELTA+2.*DEL R)
0443 F(106,106)=-4.*(2.*DEL R+DELTA)/W-(4.*(DEL R**2.)-DELTA**2.)/W-2.*V

```

```

0444 F(107,107)=F(106,106)
0445 F(106,107)=2.*V
0446 F(107,106)=V
0447 F(107,86)=V
0448 F(106,105)=4.*DELTA/W-(DELTA**2.)/W
0449 F(107,98)=F(106,105)
0450 F(106,108)=8.*DELR/W+4.*(DELR**2.)/W
0451 F(107,109)=F(106,108)

```

C C PROGRAM FOR EQS. NO. 108 & 109

```

0452 R15=1.+DELTA/2.
0453 W=(DELTA/(2.*R15*DBETA))**2.
0454 F(108,108)=-8.-2.*W
0455 F(109,109)=-8.-2.*W
0456 F(108,109)=2.*W
0457 F(109,108)=W
0458 F(109,110)=W
0459 F(108,106)=4.-DELTA/R15
0460 F(109,107)=4.-DELTA/R15
0461 F(108,116)=4.+DELTA/R15
0462 F(109,117)=4.+DELTA/R15

```

C C C PROGRAM FOR EQ. NO. 110

```

0463 W=R15**2.
0464 V=2.*DBETA*DTHETA*(DTHETA+2.*DBETA)/2.
0465 Z=DELTA**2.
0466 F(110,110)=-8./Z-(2.*DBETA*DTHETA)/(W*V)
0467 F(110,109)=DTHETA/(W*V)
0468 F(110,111)=2.*DBETA/(W*V)
0469 F(110,86)=4./Z-1./(R15*DELTA)
0470 F(110,118)=4./Z-1./(R15*DELTA)

```

C C C PROGRAM FOR EQS. FROM 111 TO 115

```

0471 W=(DELTA/(R15*DTHETA))**2.
0472 DO 226 I=111,115
0473 F(I,I)=-8.-2.*W
0474 J=I-24
0475 F(I,J)=4.-DELTA/R15
0476 K=I+8
0477 DO 227 I=111,114
0478 F(I,K)=4.+DELTA/R15
0479 J=I+1
0480 F(I,J)=W
0481 K=I-1
0482 F(I,K)=W
0483 F(115,114)=2.*W

```

C C C PROGRAM FOR EQS. FROM 116 TO 123

```

0484 DO 228 I=116,123
0485 F(I,I)=-3.
0486 J=I-8
0487 F(I,J)=4.
0488 F(116,106)=-1.
0489 F(117,107)=-1.

```

```

0490 DO 229 I=118,123
0491 J=I-32
0492 F(I,J)=-1.
C
0453 CALL GAJO (F,H,123,124)
C
0494 WRITE (6,150)
0495 FORMAT (///,3X,'DIMENSIONLESS TEMPS.',/)
0496 WRITE (6,151) (H(I),I=1,123)
0497 FORMAT (' ',10F13.8/)
0498 Z=DBETA/(4.*DBETA+DTHEIA)
0499 DO 300 K=1,4
0500 HH(K)=H(69)+(K*Z)*(H(70)-H(69))
0501 I=K+7
0502 HH(I)=H(66)+(K*Z)*(H(67)-H(66))
0503 I=4
0504 DO 301 K=1,3
0505 I=I+1
0506 HH(I)=H(70)+(K/4.)*(H(71)-H(70))
0507 J=I+7
0508 HH(J)=H(67)+(K/4.)*(H(68)-H(67))
0509 J=55
0510 DO 80 I=1,5
0511 J=J+1
0512 QA(I)=(DELRI/(2.*3.))*(D(J)*H(J)*R(3)+4.*D(J-10)*H(J-10)*R(4))
0513 QUT(I)=(3.*DELRI/8.)*(3.*D(69)*H(69)*R(1)+3.*D(66)*H(66)*R(2)+D
1(56)*H(56)*R(3))+QA(I)
0514 DO 81 I=2,5
0515 K=I-1
0516 QUT(I)=(3.*DELRI/8.)*(3.*DD(K)*HH(K)*R(1)+3.*DD(K+7)*HH(K+7)*R(2)
1+D(I+55)*H(I+55)*R(3))+QA(I)
0517 CONTINUE
0518 DO 85 I=6,10
0519 J=J+1
0520 QA(I)=(DELRI/(2.*3.))*(D(J)*H(J)*R(3)+4.*D(J-10)*H(J-10)*R(4)+D
1(J-20)*H(J-20)*R(5))+D(DELRI/3.)*(D(J-20)*H(J-20)*R(5)+4.*D(J-25)*H
2(J-25)*R(6)+2.*D(J-30)*H(J-30)*R(7)+4.*D(J-35)*H(J-35)*R(8)+D(J-40
3)*H(J-40)*R(9))+3.*DELRI/8.)*(D(J-40)*H(J-40)*R(9)+3.*D(J-45)*H
4(J-45)*R(10)+3.*D(J-50)*H(J-50)*R(11)+D(J-55)*H(J-55)*R(12))+D
5DELRI/(2.*3.))*(D(J-55)*H(J-55)*R(12)+4.*D(J-60)*H(J-60)*R(13))
0522 CONTINUE
0523 QUT(6)=(3.*DELRI/8.)*(3.*D(70)*H(70)*R(1)+3.*D(67)*H(67)*R(2)+D(61
1)*H(61)*R(3))+QA(6)
0524 K=4
0525 DO 86 I=7,9
0526 K=K+1
0527 QUT(I)=(3.*DELRI/8.)*(3.*DD(K)*HH(K)*R(1)+3.*DD(K+7)*HH(K+7)*R(2)
1+D(K+57)*H(K+57)*R(3))+QA(I)
0528 CONTINUE
0529 QUT(10)=(3.*DELRI/6.)*(3.*D(71)*H(71)*R(1)+3.*D(68)*H(68)*R(2)+
1D(65)*H(65)*R(3))+QA(10)
0530 QUT(11)=(DBETA/3.)*(QUT(1)+4.*QUT(2)+2.*QUT(3)+4.*QUT(4)+QUT(5))
0531 QUT(12)=(DTHEIA/3.)*(QUT(5)+4.*QUT(6)+QUT(7))
0532 QUT(13)=(3.*DTHEIA/8.)*(QUT(7)+3.*QUT(8)+3.*QUT(9)+QUT(10))
0533 QUT(14)=QUT(11)+QUT(12)+QUT(13)
C

```

C AVERAGE NUSSELT NO. AT THE INNER TUBE SURFACE
C *****
C *****

```

0534 SUM=0.0
0535 DO 299 I=73.91
0536 SUM=SUM+H(I)
0537 CONTINUE
0538 TWBAR=SUM/19.
0539 XNU=2.*((1.+DELTA)/((OUT(14)/Q(14))-TWBAR))
0540 WRITE (6,172)
0541 FORMAT (///,3X,'AVERAGE NUSSELT NO. AT THE INNER TUBE SURFACE',
1,/,)
0542 WRITE (6,173) XNU
0543 FORMAT (F13.8)
0544 STOP
0545 END

```

```

0001 C
0002 C
0003 C
0004 C
0005 C
0006 C
0007 C
0008 C
0009 C
0010 C
0011 C
0012 C
0013 C
0014 C
0015 C
0016 C
0017 C
0018 C
0019 C
0020 C
0021 C
0022 C
0023 C
0024 C
0025 C
0026 C
0027 C

SUBROUTINE GAJO(A,S,N,NI)
  THIS SUBROUTINE SOLVES N SIMULTANEOUS LINEAR ALGEBRAIC
  EQUATIONS IN N UNKNOWNNS USING THE GAUSS-JORDAN REDUCTION

  DIMENSION A(N,NI),S(N)

  DO 200 J=1,N
  IF(A(J,J).NE.0.)GO TO 300
  J1=J+1
  DO 302 K=J1,N
  IF(A(K,J).NE.0.)GO TO 303
  302 CONTINUE
  DO 304 I=J,N
  DUMMY=A(K,I)
  A(K,I)=A(J,I)
  A(J,I)=DUMMY
  304 DIV=A(J,J)
  SA=1./DIV
  DO 201 K=J,NI
  A(J,K)=A(J,K)*SA
  DO 202 I=1,N
  IF(I-J)203,202,203
  203 AIJ=-A(I,J)
  DO 204 K=J,NI
  A(I,K)=A(I,K)+AIJ*A(J,K)
  204 CONTINUE
  202 CONTINUE
  200 CONTINUE
  DO 400 I=1,N
  S(I)=A(I,NI)
  400 RETURN
  END

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