Three Essays on Modeling Aging Population

by

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Abstract

Chapter 1: Interregional Transfers through Public Pension in Canada

In this chapter, I build a regional computable general equilibrium model with an overlapping generations (OLG) structure of the Canadian economy to analyse population aging dynamics and public pensions. Canada is divided into three regions: Atlantic, Quebec and Rest of Canada (ROC). The impact of population aging is investigated on each of three regions' pension systems. The results confirm that as a result of aging all regions are affected negatively if they choose to have an independent pension system. Under a joint pension system most of the pressure of the provision of the pension system is on the ROC. Atlantic region benefits the most from a joint pension plan as the implicit funds flow from ROC to Atlantic region. Quebec benefits from having its own program, but the benefits disappear slowly in future years.

Chapter 2: Age-Variable Rate of Time Preference in CGE-OLG Model

Contrary to the mainstream studies in the area of intertemporal optimization that assume a constant rate of time preference over individuals' life cycles, in this chapter I propose a new approach to measure the rate of time preference by assuming that the rate of time preference evolves by age. I construct an overlapping generations model (OLG) and calibrate rate of time preference. The age-variable rate of time preference would permit to capture many other elements that affect the life cycle profile of consumption as observed in the data. The results show that rate of time preference exhibits three phases and is different for young versus old.

Chapter 3: Computing Demographic Change Simulation under Constant and Age-variable Rate of Time Preference

This chapter simulates the impact of an aging population on various macroeconomic variables and calculates the cohort welfare as well as social welfare. The outcomes from simulations are dependent on the choice of rate of time preference as well as the structure of the model. The results in this chapter provide a new approach to determining the impact of aging population. The choice of a realistic rate of time preference, which allows its variability by age, affects the cohort welfare noticeably.

Acknowledgement

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Introduction to Thesis

As people are living longer and baby-boomers are retiring, we observe significant demographic changes in many countries of the world. This represents important challenges for economic policy. Aging of the population is expected to affect economic growth and many other issues, including intergenerational welfare, the ability of states to provide resources for elderly and international trade. It is crucial for better policy decisions to be able to develop appropriate models on demographic changes in order to assess properly the impact of aging populations.

The purpose of this thesis is to contribute to the compelling picture of the impact of population aging on economic performance. Given the enormous breadth of issues that aging population generally encompass, in the present context, I narrow down the horizon by focusing on issues relevant to modeling the demographic changes in an overlapping generations (OLG) framework. OLG models are considered the most appropriate to investigate demographic changes since it takes into consideration the age of the representative agents.

In chapter one, I begin by developing an OLG model to investigate the impact of aging on public pension in Canada. A considerable share of seniors approach their retirement over the next few decades, while the share of people aged less than 30 years old, will drop.

This will put pressure to the provision of the pension program. Demographic changes are not the same across Canadian provinces. Atlantic Provinces have higher share of elderly population compared to the rest of the Canada. Quebec has its own pension plan. Thus, I focus on this issue by dividing Canada into three regions and develop a multi-region Computable General Equilibrium (CGE) with OLG framework. The results derived from this exercise highlight the importance of developing a multi-region model to analyze aging and pension in Canada.

In chapter two of this thesis, I develop an approach to calibrate OLG models with the rate of time preference allowed to vary by age. In the conventional calibration of the discounted utility function, the evolution of consumption over life cycle is derived as a function of the inter-temporal elasticity of substitution, the market interest rate, and the rate of time preference. A typical assumption is to impose a constant rate of time preference. This implies upward sloping consumption profile (if the rate of interest is larger than rate of time preference). Recent empirical studies by National Transfer Accounts (NTA) have demonstrated that the age profile of consumption is bell shaped. This permits for new calibration procedure in which the rate of time preference varies by age. The advantage of the new procedure is that it respects consumption behavior observed in data. By letting the rate of time preference to vary by age, this chapter provides a new calibration procedure that satisfies life-cycle profiles observed in the data.

In chapter three of this thesis, long-run impact of aging population is simulated, using the calibration results obtained from the innovative approach developed in chapter two. Results derived from simulating an OLG model shows that the impact of aging population is sensitive to the choice of rate of time preference.

Chapter 1

Interregional Transfers through Public Pension in Canada

1.1 Introduction

Over the next few decades there will be significant demographic changes in Canada. A considerable share of seniors approach their retirement, while the share of people aged less than 30 years old will drop. This demographic change will put pressure on pension plans for many years to come. This issue has generated debate over the feasible reforms for pension plans (Baldwin [10], Baldwin [12], Baker et al. [9]). This chapter's objectives are twofold: first, to investigate and measure the impact of population aging on Canada's public pension contribution rate in a multi-regional computable general equilibrium model, second, to measure the implicit transfers of resources from the west part of Canada to the east, generated by the asymmetric population dynamic. In addition, I estimate the implicit cost or benefit for Quebec for having its own pension plan. In this model Canada is divided into three regions: Atlantic, Quebec and Rest of Canada (ROC). The population is aging faster in the Atlantic Provinces and in Quebec. Atlantic and ROC retirees receive benefits from the Canadian pension plan (CPP), but Quebec pension benefits are paid under the Quebec pension plan (QPP).

While there are some studies that investigate the implications of the aging population for pension plans, there are no studies that have taken into consideration the two separate pension plans, in Canada's income security system. Since population dynamics are different across Canadian provinces, the consequences of the aging population will differ. Heterogeneity in the composition of the population (young versus old) across Canada's regions has a critical role in the impact of pension reforms and speed of the population aging. In the presence of regional differences, there exist diverse regional effects based on the economic and demographic structures. Wage, employment and productivity effects of any reforms across regions may not be the same as those observed at the national level.

There are transfers of implicit funds from provinces with implicit pension surplus to those with implicit pension deficit under the common pension plan. I use a general equilibrium model with an overlapping generations (OLG) structure to run three sets of simulations to examine the impact of population dynamics. First, I consider the current situation in which ROC and Atlantic share the same pension system, while Quebec has its own plan. In the second scenario, I assume that one pension plan exists for all regions. In the third scenario, I assume that each region has its own pension plan.

The structure of this chapter is as follows: in section 1.2, I briefly review the income security system in Canada and the previous reforms. In section 1.3, I describe the population dynamics and projections for Canada over the next few decades. In section 1.4, I review the relevant literature. In sections 1.5 and 1.6, I present the model and the required data and in section 1.7, I discuss the main results concerning the overall dynamic effects of population aging in the two scenarios. Section 1.8 concludes.

1.2 Income Security System in Canada

The national pension system of Canada is primarily an extension of the welfare state from the 1950s to the 1970s (Beland et al. [19]) [1]. Canada's income security system has three pillars (Horner [71])[2]. The first pillar is a flat benefit (Old Age Security) with a supplement (Guaranteed Income Supplement (GIS)) and is financed through general tax revenues. The eligibility age for receiving the OAS is 65 years old for Canadians who meet residence requirements [3]. GIS is an income-tested supplement that is allocated to people who do not receive any income except for the OAS. For income received from sources other than OAS, the GIS will be "taxed away".

The second pillar is the Canada/Quebec pension plans (CPP/QPP), which are income-related and employment-based pensions. CPP and QPP are the outcome of long bargaining between the federal government and all ten provinces. It was Quebec's campaign for "greater provincial autonomy" that led to the creation of two separate plans that are highly coordinated. Both plans are financed by employees, employers and those who are self-employed. People aged between 18 and 70 who have an income that exceeds the minimum level of earnings contribute to the plan. CPP/QPP contribution rate is 9.9

[1] The structure of the national pension system of the 60's takes root in legislative procedures that date back as far as the 1920s. The first national pension legislation, the Old Age Pension Act (OAP), was legislated in 1927 (Beland et al. [19]). It assigned 20 dollars to each Canadian over the age of 70. With time, legislatures evolved, and in 1951, the Old Age Security Act extended the benefit to 40 dollars while the Old Age Assistance Act (OAS) lowered the admissibility age to 65. The latter remained the status quo until 1970. By the 1980s, OAS along with GIS could provide an "income floor equal to 50 percent of average earnings for an elderly couple and 31 percent for a single individual" (Beland et al. [19]).

[2] Contribution rate of employers and employees is 4.95 percent each. Self-employed workers pay the full 9.9 percent.

[3] Since 1989, OAS recipients with income above a threshold ($66,733 in 2010) are exposed to a special tax (Baldwin [10]).

percent of earnings between $3,500 dollars and the year's maximum pensionable earnings [4]. The benefit payment is 25 percent of the average monthly income (up to Year's Maximum Pensionable Earning (YMPE)) during the contributions years (from age 18 to retirement), with allowance for some years of low earnings to be dropped from the average. Benefit payments are adjusted to changes in the CPI (Consumer Price Index). The minimum age for eligibility for CPP/QPP benefits dropped from 68 to 65 between 1966 and 1970. Since 1987, benefits can be accessed at age 60 [5]; however, benefit payments are reduced by 0.5 percent per month between the age at which a person begins receiving benefits and age 65 [6]. CPP/QPP benefits, including survivor and disability benefits, were paid to 7.3 million people in December 2010 [Horner [71]].

The third pillar of Canada's income security system is workplace pension and savings plans (Registered pension plans (RPPs), Registered Retirement Saving Plans (RRSPs)). Together, these three pillars formed the national pension system at the end of 1960.

The pension plan was established to supplement income after retirement and aims to replace approximately 25% of pre-retirement earnings. People aged 18 and older contribute to the plan[7]. The CPP was initially established as a PAYG (Pay as you go) plan with a small reserve and with an employer-employee contribution rate of 3.6%. Since the early 1970s, numerous changes have been made in CPP/QPP; however, reforms made in the late

[4] YMPE, set at $48,300 in 2011, Horner [72].

[5] In the 2012 government budget, OAS benefits eligibility age in Canada was raised from 65 to 67, starting progressively in year 2023 (HRSDC [74], Budget [32]).

[6] From 2011 to 2013 the government will gradually increase the percentage from 0.5 percent per month to 0.7 percent. So, if the contributors delay the receipt of their CPP until age 70, the amount receivable will be 42% more than the amount taken at age 65 (HRSDC [73]).

[7] Except for people whose earnings are less than the Year's Basic Exemption (YBE), members of certain religious groups.

1990s had the most significant impact. These changes transformed the pension plan from a PAYG system to a hybrid of PAYG financing and full funding [8].

In the next section, I briefly review the main pension reforms during the last few decades.

1.2.1 Reforms to the Pension Plan

The so called "great-pension-reform" occurred between the mid 1970s and the early 1980s and took place during two periods: first, from 1984 to 1993 and second, since 1993 (Beland et al. [19]). The major concern was the expansion of the second tier of the pension plan; however, in both periods, the flat part of the pension (OAS) was targeted. By 2001, less than 5 percent of the elderly were affected by the reforms introduced in 1989.

In 1993, there were concerns about the deficit and demographic issues, which made the Liberals create five principles to reform the pension plan, but no changes were made to CPP/QPP. Eventually, after the publication of the Fifteenth Actuarial Report of CPP in 1995, it became clear that if the contributions did not change by 2015, CPP would not collect enough revenue to pay all the benefits and the contribution rate must increase to 14.2% by 2030. In response to this issue, the governing Liberals initiated a consultative process to reform the pension plan.

In February 1996, the Ministry of Finance published a joint report evaluating the "long-term financial" situation of CPP and setting the agenda for consensual reform. This

[8] In the Actuarial Report of 1997, it was called "steady-state funding".

consultation aimed to review the CPP by federal and provincial governments. At the same time, the government of Quebec was conducting its own consultancy concerning QPP.

In November 1996, following the consultation between the federal government and provincial governments, a joint report was published which contained nine principles regarding reforming the CPP (Beland et al. [19]). Among them was the creation of the CPP fund to ensure a more efficient fund management. Investing the CPP fund was a response to the existence of the Caisse de Depot et Placement du Quebec, which is a provincial investment board that has successfully invested QPP's money in equities since the 1960s.

In February 1997, new CPP legislation was proposed by the Ministry of Finance. The new reforms increased CPP contributions from 5.6 to 9.9 by 2003, in order to accumulate funds. The fund value is equal to two years of benefits. The CPP Investment Board was created in order to invest surplus reserve funds in diversified investment portfolios.

Over the last decade, the federal government and the four provinces of Alberta, British Columbia, Ontario and Nova Scotia have started important pension reviews. In November 2009, the government of Ontario released the report of the Ontario Expert Commission on Pension (OECP), providing a number of recommendations for improving the pension system. Soon after the release of the OECP, the Alberta/British Colombia Joint Expert Panel on Pension Standards (JEPPS), released a report providing recommendations to improve the pension standards in order to facilitate establishment of an independent, "widely accessible", "privately-run" and voluntary Defined Contribution (DC) pension plan to deal with the problem of low pension coverage. In January 2009, Nova Scotia's Pension Review recommended a model similar to the JEPPS model.

In 2009, the Ministry of Finance of the government of Ontario provided a report containing the options available for pension plan reforms (Baldwin [10]). Since then, several reports on pension plan reform options have been released either by federal or provincial governments or by private research institutions, which accentuate the necessity for pension plan reforms (Baldwin [12], Horner [72], Baldwin [11]).

At the time of the inception of the pension plan, due to the economic conditions, a PAYG pension plan was appropriate and the total contribution of the workforce was sufficient to cover the pension benefits; however, since then demographic conditions have changed. An ever decreasing mortality rate and a declining fertility rate manifest themselves in increasing the percentage of the elderly in the total population. Thus, with a growing share of the population 65 years old and over, the cost of the PAYG pension system will continue to increase. In order to ensure the sustainability of the pension plan for future generations, including changes in the population structure should be an important part of any pension plan reform. The next section describes the demographic dynamics in Canada over the past four decades.

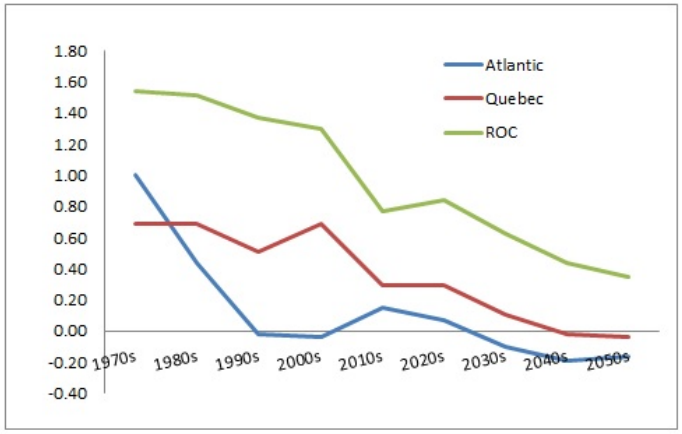
1.3 Population Dynamics in Canada

On July 1, 2012, Canada's population was estimated at 34,880,500 with a 1.1 percent increase compared to the previous year. This rate is slightly larger than the average growth rate for the past 30 years (1%). The highest population growth rate that Canada experienced was 1.8 percent in 1988/1989. The variation of the growth rate for the past 30 years was between 0.8 and 1.2 percent.

Population growth varies across Canada's provinces and territories. Figure 1.1 shows the estimated and projected growth rate across Canada's three regions. As is shown, the population growth is lower in the Atlantic provinces and Quebec compared to the rest of the Canada. The projected growth rate will become negative for Quebec in 2040 and for the Atlantic provinces in early 2025. Compared with the ROC, population growth steeply declines in Quebec and the Atlantic provinces. In Quebec, the growth rate moved from 0.7 in 1999 to 0.3 in 2011. In the past 30 years, the population growth rate of the western provinces has been higher than the Atlantic provinces and Quebec. The decline in the birth rate and the mortality rate and, the increase in life expectancy are contributing factors to the lower growth rate in Canada.

Figure 1.2 shows the population pyramid for Canada as of July 1, 2010. The large cohort of baby boomers, born between 1946 and 1965, are currently in the 45 to 64 year age range. By comparing the 2010 pyramid to that of 1971, we can see the progression of the baby boomers through the age structure. This cohort makes the population pyramid more rectangular-shaped. The lower birth rate has narrowed the lower part of the pyramid, which clearly shows low fertility at the base of the pyramid and increasing life expectancy

Figure 1.1: Population Growth for Regions, Decade Yearly Average



Source: Statistics Canada, CANSIM, Table 051-0001

at old ages.

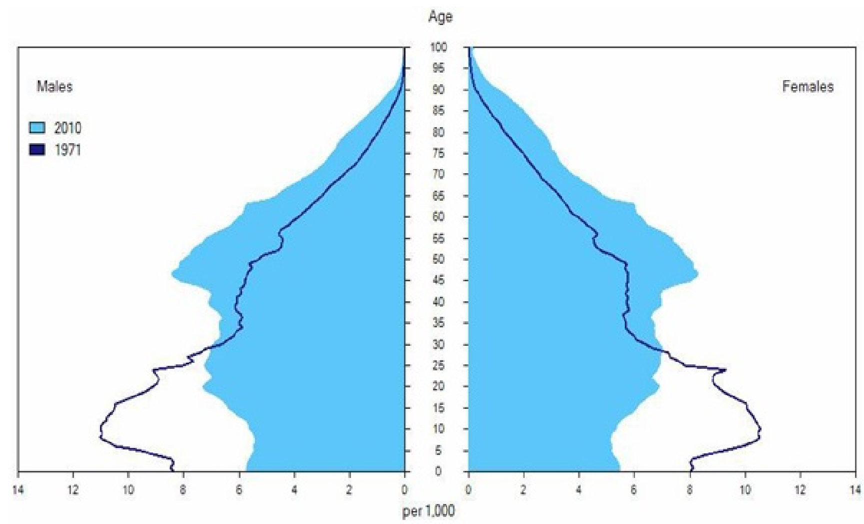
The aging of the Canadian population is shown in Figure 1.3[9]. These figures show the proportion of children aged 14 years and under and persons 65 and over. Older age groups have larger shares in the population and the proportions of children are decreasing.

The senior population is continuously growing. As of July 1, 2010, seniors aged 65 years and over accounted for 14.1% of the Canadian population, in 1971, 8 percent of the population were seniors and this percentage has been continuously increasing for the past forty years. As these figures suggest, in 2011, the population aged 65 years and above in Quebec and the Atlantic provinces surpassed the population aged 14 years and less; however, in ROC, it is projected that this intersection will occur in 2017. The baby boomers began to enter their retirement in 2011.

As demonstrated in Figure 1.3, the share of the population aged 65 years and over

[9]Source: Statistics Canada, CANSIM, Table 051-0001.

Figure 1.2: Age Pyramid for Canada

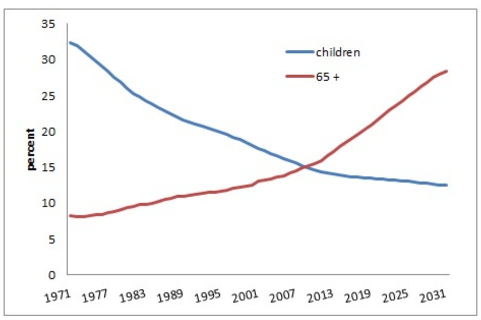


Source: Statistics Canada,2010, Annual Demographic Estimates: Canada, Provinces and Territories, No 91-215x.

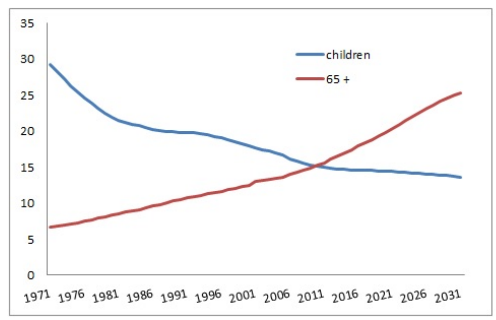
in Quebec and the Atlantic provinces will increase from 15% in 2011 to 25% and 27% respectively in 2031; however, in the ROC, it is projected to increase from 15% in 2017 to 22% in 2031. In 2011, the share of people aged 65 years and above continues to be lower than those aged 14 years and less for the ROC. The share of this group was the same for all three regions in 1971. The age gap between these two age groups increases faster for Quebec and the Atlantic provinces compared to the ROC.

The economic implications of the population dynamics appear in the old age Dependency Ratio (DR). Figure 1.4 shows the projected old age dependency ratio for Canada's three regions. The old age DR is the number of persons aged 65 years and above, over persons aged between 15 and 64. All of Canada's three regions have almost the same DR in 2006; however, in 2031, there is a large difference in the DR among regions. Between 2006 and 2031, the DR triples for the Atlantic region and increases more than 100% in

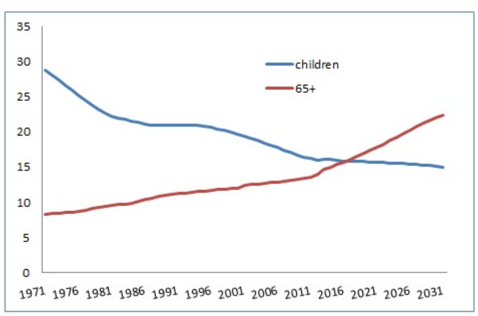
Figure 1.3: Proportion of Persons Aged 65 Years and Over and Children aged 14 Years and Less, Canada, 1971 to 2031



(a) Atlantic

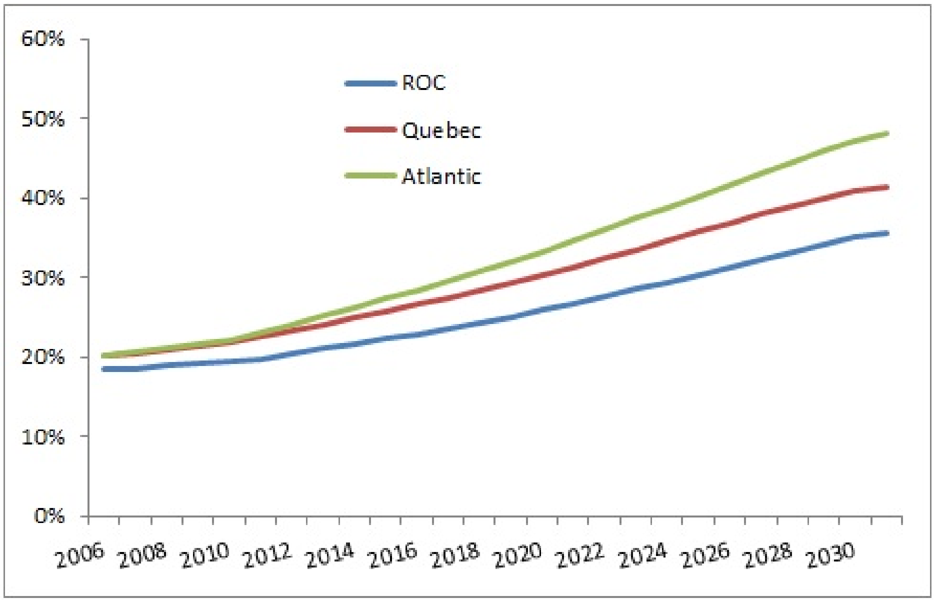


(b) Quebec



(c) ROC

Figure 1.4: Dependency Ratio



Source: Statistics Canada, CANSIM, Table 051-0001

Quebec. In the ROC, it only increases about 60%. The DR provides an insight into the future financial burden for elderly pensions. The increase in the senior population relative to the working-age population has implications for Canada's economy.

A high dependency ratio is likely to reduce productivity growth. Since the retired population will make a bigger share of the population, the productive share of the labour market will diminish and lead to lower economic growth. In a PAYG pension system, the seniors' pension benefits have to be financed by the working age population. The increase in the senior population relative to the working age population will put pressure on the provision of pension benefits.

As of July 1, 2010, the working-age population, aged 15 to 64 years, accounted for 69.4% of the total population of Canada. This proportion was 62.7% in 1971. As Figure

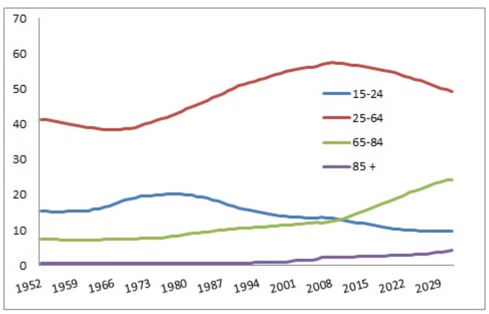
1.4[10] shows, Quebec and the Atlantic provinces have a higher dependency ratio compared to ROC and it is expected to increase substantially over the next 20 years. The increase is less dramatic in the ROC. The increase is projected to be 140 percent in the Atlantic provinces, 110 percent in Quebec, and 100 percent in the ROC. As of July 1, 2010, people aged 80 years and over in Canada represent 3.9% of the total population. By the year 2031 this will double and by 2061, the end of the most recent projection, there could be 5.1 million people aged 80 and above.

The structure of the working population is also very crucial. As Figure 1.5[11] shows, the share of the younger part of the working population, people aged between 15 and 24, has been decreasing since the 1990s which also affects the share of the working age population between 25 and 64. In the same period the share of the old age group is increasing. There is an increase in the ratio of those aged 15 to 24 years old in all three regions when the baby boomers enter the labour market. This increase is later shifted to the second part of the working age population, people aged 25 to 64. Since 2005, in the Atlantic region, the share of people aged 25 to 64 is decreasing rapidly while for Quebec it is decreasing gradually since 1995. In ROC, the decrease has started since 2012.

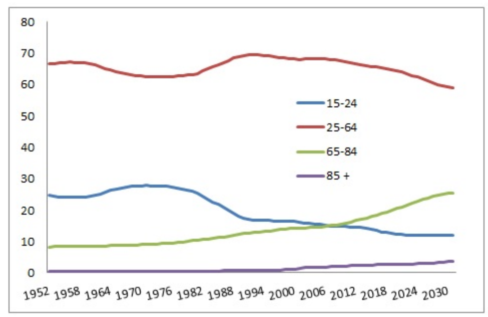
The economic role and the contributions of people vary by age. Adults (people aged between 25 to 64) are net producers and savers and the elderly are net consumers. The declining share of individuals in the labour force has discouraging effects on productivity growth. Population aging could affect not only labour supply but also capital deepening and total factor productivity, because economic output will be achieved by a contracted

[10] Statistics Canada, CANSIM, Table 051-0001.
[11] Statistics Canada, CANSIM, Table 051-0001.

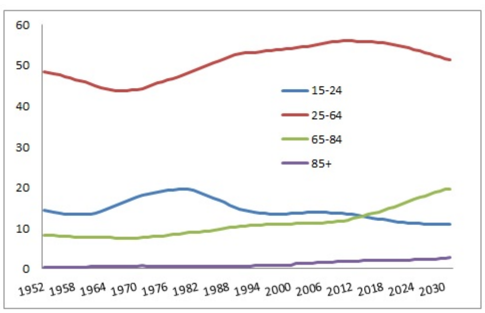
Figure 1.5: Proportion of Age Groups in Total Population



(a) Atlantic



(b) Quebec



(c) ROC

and older labour force.

1.4 Review of Relevant Literature

1.4.1 Population Dynamics and the Economy's Performance

Investigating the impact of population dynamics in an economy was first studied in the Life Cycle model by Modigliani [97]. Modigliani [97] probes the relationship between private and public saving with population structure. Since then, many researchers study the link between aggregate saving and demographic structure (Attanasio and Weber [5], Graham et al. [59], Masson et al. [93]). Numerous studies have investigated the impact of population structure and its dynamics on different aspects of an economy, such as government spending, tax revenue, economic growth and labour market (Leibfritz et al. [90], Disney [39], Fougere and Merette [54], Fougere et al. [53]).

Leibfritz et al. [90] explore the effects of demographic changes on government budget and national savings and find that government fiscal position is important to better handle the impact of the demographic changes. Also, increasing the retirement age will increase the working-age population and therefore the GDP.

In Canada, King and Jackson [82] investigate the effects of population aging on economic growth and government revenues and expenditures, and conclude that an aging population alone should not cause major problems for public finances.

Fougere and Merette [54] examine the prediction of the life cycle model that an aging population puts downward pressure on private savings. They estimate the aggregate personal savings rate for Canada that captures population aging, then simulate the effects of age structure of the population on private savings and find a co-integration relation. By using demographic projection until 2050, their simulation results indicate that aging population decreases personal saving rate up to 50% by 2050, which supports the life-cycle model.

Bloom et al. [22] show that the pace and extent of demographic changes is different between developed and developing countries. The population aging does not impede economic growth in developing countries in the near future; however, developed countries are immediately faced with decline of economic growth.

Miles [96] analyses the three existing methods for studying the impact of population changes on saving and capital accumulation. The first method is the time series analysis that studies the correlation between saving rate and population changes, second is household micro surveys on age and saving, and the third one is simulation models. The time series analysis uses a series of aggregate factors such as wage, wealth, inflation, public debt and demographic changes in saving regression. The results of this study show that the young population (less than 30) and the elderly (over 60) have lower saving rates, which is consistent with the life-cycle hypothesis. The survey data have the advantage of clarifying whose behaviour is studied; however, the scale of the demographic changes in the survey data is small.

Fougere et al. [53] investigate the impact of aging population on Canada's economy and quantify sectoral and transitional dynamics of the aging population, taking into account both the supply-side and demand-side of the economy. Their paper shows that two important ”structural changes”, negative labour supply shock and change in the composition of consumption demands, affect the Canadian economy and its labour market. Their paper utilizes a sectoral and occupational computable general equilibrium model with overlapping generations. Their study finds that the negative labour supply has a larger effect. Also, it finds that due to final-demand changes, there are significant structural shifts. The GDP share of some sectors increases up to 50%; however, the contribution of some other sectors falls significantly.

1.4.2 Population Dynamics and Pension Plan

Some studies not only investigate the impact of population dynamics in an economy but they also study the implications of the aging population on the pension plan. Studying and modeling the impact of pension reforms in the context of an aging population was first investigated in 1987 (Auerbach and Kotlikoff [7]). Since then, studying pension reforms considering population dynamics has been the interest of many researchers (Hachon [63], Gora [58], Disney [39]).

Auerbach and Kotlikoff [7] were the first to develop a dynamic general equilibrium with an overlapping generations structure to investigate the impacts of demographic changes on the social security system. The demographic changes are studied in response to four social security policies, such as ”reduction in benefit replacement rates, advances in social security retirement age, taxation of the social security benefits and the accumulation of the social security trust fund”. The study finds that demographic changes have major impacts on factor returns over the long run and bring abrupt changes in saving rates in the short run. The choice of the social security policy changes the size of the impact of demographic changes on intergenerational welfare.

Auerbach et al. [8] modify the 75 overlapping generations model in [7] and include bequest, technological change and international trade, to study the impact of demographic changes on the economy of four OECD countries. Their paper simulates the effects of three different policy options: "freezing non-pension government expenditure", a "2-year increase in the retirement age", and a 20 percent cut in pension benefits. The result indicates that demographic changes affect national saving rate, wage rate and current account.

In the European context there are numerous studies that probe the pension reforms (Willets [129], Blanchet [21]). Most of these papers focus on evaluating the reforms already executed in the EU since 1990s and propose options for improving the pension system considering the changes in population structure. For example, Hachon [63] investigates pension reforms in France and evaluates some of the reforms such as increasing the required number of years working to be eligible to receive pension and financial incentives to postpone retirements. Hachon [63] uses an overlapping generations model with a closed economy and heterogenous agents and shows that when the population ages, the link between pension contributions and pension benefits has an impact on the level of capital.

In the United Kingdom, Disney et al. [40] illustrate the path of pension reforms that focuses on keeping the cost of public pension low, while maintaining the minimum level of income security for low-income individuals. Tax incentives have been used as an instrument to encourage individuals to increase private retirement savings. Their paper illuminates the effectiveness of tax incentives on increasing private savings.

In Poland, Gora [58] describes the design of the new Polish old-age (OA) pension system that creates a balance between intergenerational transfers and the welfare of each generation. The traditional pension system supports the welfare of the current generation at the expense of future generations; however, due to the new pension system, old age expenditure will fall and the burden of the pensions on the current generation will weaken.

In Germany, Borsch-Supan and Wilke [29] illustrate and assess the transition of the German pension system toward a sustainable multi-pillar system in order to accommodate the demographic pressures.

Deger [38] presents an OLG model to analyse the impact of change in pension plan replacement rate. Households are heterogenous because they belong to one of the three existing social security systems, which have different contribution and benefit rates. The results show that the change in the replacement rate decreases the benefit payments and changes the consumption path for households. The magnitude and direction of the changes depend on their involvement in either of the social security systems. Consequently, welfare will increase for some households and decrease for others. Thus, while some households are better off, some are worse off, so Pareto optimum is not reached.

Adema [1] analyses the impact of the aging population on portfolio choice and risk premium using a stochastic two-period overlapping-generation general equilibrium model with PAYG pension. The results indicate that a pension reform that reduces the PAYG benefit makes people become more risk averse and increases the demand for safe assets (government bonds), therefore the risk premium will increase. With the aging of the population, people perceive that the PAYG system is less sustainable and less safe, so they reduce the riskiness of their asset portfolio and the risk premium of stocks over bonds will increase. Adema [1] argues that as longevity rises, people increase their savings and consequently increase the risk premium on stocks over bonds.

In the Canadian context, there are very few papers that model pension reforms and the aging population (Fougere et al. [53]). Most of the studies for Canada either assess the options for pension reforms (Gruber [61], Gruber and Wise [62], Horner [72], Beland et al. [19], Baker et al. [9]) or are federal government proposals (Baldwin [10], Baldwin [12]).

1.4.3 Population Dynamics, Pension Plan and Multi-region Model

There are several studies that investigate the impact of the aging population and pension reforms using multi-country or multi-region models. Most of these studies compare the response of a group of countries (mostly OECD countries) to population dynamics and pension reforms (Auerbach et al. [8], Leibfritz et al. [90], Hviding and Merette [76]).

Hviding and Merette [76] investigate the impacts of potential pension reforms in the context of the aging population for seven OECD countries using a general equilibrium model with overlapping generations. Their paper analyses four reforms: "a gradual privatization" of the public pension system in 52 years; an "across-the-board cut in the replacement rate" of 20 percent; "fiscal consolidation"; and, finally an increase in the effective retirement age of four years. Their simulation results suggest that the benefits from pension plan reforms do not offset the macroeconomic effects of the aging population. The gradual removal of public pension is the most effective reform in the long run; however, increasing the retirement age is the most effective reform in the short run or medium run.

Borsch-Supan et al. [28] study the impact of the aging population and pension reform on the international capital market, using an overlapping generation model for seven world regions. The results indicate that the demographic changes affect saving within each country. This effect increases when the pension reform is moving from pure PAYG to a more pre-funded system. Taking into account the timing and initial condition of demographic changes, the aging population encourages capital flow between countries. Capital exporting countries, such as EU countries, are more affected by aging, while capital importer countries, such as other OECD countries and the United States, are less affected by aging. When the baby boomers start to "decumulate" assets, the fast-paced aging countries become capital importers. There is also an interaction between saving and labour supply. The saving rate, rate of return and international capital flow respond less to demographic changes when households work longer hours in response to demographic shocks.

Aidt et al. [2] study the financial stability of the pension in nine OECD countries and compare the efficiency of a PAYG system with the funded pension system [12]. The "method of calculation of benefits", the "use of indexation", "age of pension entitlement", "generosity of the basic state pension" and differences in the retirement age (ranging from 58 in Italy to 68 in Japan) are the major differences in pension systems across OECD countries. These differences between countries determine the source of disposable income. In most OECD countries, the main source of income after retirement is public pension; however, in countries like Canada, the UK and the US, private pensions are also important. Their paper argues that the aging of the population and an increase in the dependency ratio justifies moving away from a PAYG system toward a funded system.

[12]PAYG pension system is the most common pension system in OECD countries

Merette and Georges [95] investigate the impact of "demographic changes" on an economy in a multi-country overlapping generation model. Although demographic pressures lower the GDP across countries, the "intertemporal gains" from globalization improve the terms of trade and maintains real consumption per capita while stimulating capital accumulation. The paper's model consists of seven regions. It predicts that when countries are interdependent, an aging population will have large economic and fiscal impacts.

To the best of my knowledge there is no study for the Canadian economy that takes into consideration the existence of the two pension plans, CPP and QPP, while investigating the impact of the aging population on the economy. This chapter contributes in this area by introducing a three-region OLG model with different pension plans across regions. While it investigates the impact of population dynamics on each of three regions' economies, it also derives implicit payment transfers across regions as a result of the asymmetric aging process.

1.5 Model

This section provides a detailed analytical description of the OLG model. This OLG model is a modified version of the model developed in Auerbach and Kotlikoff [7] and Auerbach et al. [8] with exogenous labour supply. The model represents the economy of Canada and consists of three regions: Atlantic region (Newfoundland, New Brunswick, Nova Scotia and Prince Edward Island), Quebec and the Rest of Canada (Ontario, Manitoba, Saskatchewan, Alberta, British Colombia, Yukon, Northwest Territories and Nunavut).

1.5.1 Firm Behaviour

The Production Function

The representative firm in each region, i, produces one imperfectly substitutable good, using labour and capital. It is assumed that capital is homogeneous and depreciating. Labour differs in its efficiency because individuals of different ages provide different amounts of labour input. The production function is assumed to be of the Cobb-Douglas form with constant return to scale.

$$Q_{i,t} = A_{i,t}(K_{i,t})^{\alpha_i}(L_{i,t})^{1-\alpha_i}, i \in (ROC, MARI, QC) \tag{1.1}$$

where, $Q_{i,t}$ represents the output of region i at time t, K is the capital stock, L, labour, A represents the scaling constant and α measures the capital intensity in production. A perfectly competitive firm maximizes its profit to derive the factor demands.

The Demand for Labour

I assume firms employ labour without any costs. As mentioned above, the competitive behaviour assumption leads to marginal product of labour equal to wage, w_i; so, given the Cobb-Douglas production function, we have:

$$w_{i,t} = \alpha_i A_{i,t} \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\alpha_i} \tag{1.2}$$

The Investment Decision

I also assume that capital is adjusted costlessly and the firm sets the marginal product of capital equal to the interest rate, r_i:

$$r_{i,t} = (1 - \alpha_i) A_{i,t} \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\alpha_i - 1} \tag{1.3}$$

The above two equations give the wage rate and interest rate as a function of capital stock and labour.

The firm in each region invests in the regional goods to acquire the optimal level of investment. It uses a CES investment function to minimize its spending on regional goods and derive the investment demand for each good[13]. The price of investment is a composite

[13] The price of goods is the same whether they are used for consumption or investment purposes.

of the three final goods' prices.

$$min \quad I_{i,t}PI_{i,t} = \sum_{j} i_{j,i,t}P_{j,t} \tag{1.4}$$

$$Subject\ to \quad I_{i,t} = \left[\sum_{j} \alpha_{I,j,i} i_{j,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right]^{\frac{\sigma_i^I}{\sigma_i^I - 1}} \tag{1.5}$$

where, $I_{i,t}$ is the composite investment, $PI_{i,t}$ is the investment price. $i_{j,i,t}$ represents region-i investment demand for region-j good, σ_i^I is the elasticity of substitution and $\alpha_{I,j,i}$ are the investment technology parameters that differ across regions. Minimization of the investment spending equation (1.4) subject to the investment technology equation (1.5) yields first order conditions with respect to $i_{k,i,t}$:

$$P_{k,i,t} = \omega \left[\sum_{j} \alpha_{I,j,i} i_{j,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right]^{\frac{\sigma_i^I}{\sigma_i^I - 1} - 1} \alpha_{I,k,i} i_{k,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I} - 1} \tag{1.6}$$

where, ω is the lagrangian multiplier. Multiplying both sides by $i_{k,i,t}$:

$$i_{k,i,t}P_{k,i,t} = \omega \left[\sum_{j} \alpha_{I,j,i} I_{j,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right]^{\frac{\sigma_i^I}{\sigma_i^I - 1} - 1} \alpha_{I,k,i} i_{k,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \tag{1.7}$$

Taking the sum over all goods gives:

$$\sum_{k} i_{k,i,t}P_{k,i,t} = \omega \left[\sum_{j} \alpha_{I,j,i} i_{j,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right]^{\frac{\sigma_i^I}{\sigma_i^I - 1} - 1} \left[\sum_{k} \alpha_{I,k,i} i_{k,i,t}^{\frac{\sigma_i^I - 1}{\sigma_i^I}} \right] \tag{1.8}$$

Substituting equation (1.5) into the above equation yields the following form for the Lagrangian multiplier:

$$I_{i,t,g}PI_{i,t} = \omega I_{i,t,g} \Rightarrow \omega = PI_{i,t} \tag{1.9}$$

Replacing ω by its value into the equation (1.7) gives the final investment demand of region-i for good produced in region-k at period t:

$$i_{k,i,t}P_{k,t} = I_{i,t}^{\frac{1}{\sigma_i^I}} PI_{i,t}\alpha_{I,k,i} i_{k,i,t}^{\frac{\sigma_i^I-1}{\sigma_i^I}} \tag{1.10}$$

$$i_{k,i,t} = \alpha_{I,k,i}^{\sigma_i^i} \left(\frac{PI_{i,t}}{P_{k,t}}\right)^{\sigma_i^I} I_{i,t} \tag{1.11}$$

Then using the constraint function, equation (1.5), and substituting $i_{k,i,t}$, I derive the composite investment price index:

$$I_{i,t} = \left[\sum_j \alpha_{I,j,i} I_{i,t}^{\frac{\sigma_i^I-1}{\sigma_i^I}} PI_{i,t}^{\sigma_i^I-1} \alpha_{I,j,i}^{\sigma_i^I-1} P_{j,i,t}^{1-\sigma_i^I}\right]^{\frac{\sigma_i^I}{\sigma_i^I-1}} \tag{1.12}$$

Since the summation is over j in the above equation, the $I_{i,t}$ cancel out from both side of the equation, therefore we have:

$$PI_{i,t} = \left[\sum_j \alpha_{I,j,i}^{\sigma_i^I} P_{j,t}^{1-\sigma_i^I}\right]^{\frac{1}{1-\sigma_i^I}} \tag{1.13}$$

The composite investment price index is a non-linear weighted average of regional prices.

Capital Accumulation

Accumulation of the capital stock is given in equation (1.14). Physical assets and government bonds are perfect substitutes, so the expected rate of return on capital should be equal to the expected rate of return on government bonds. Since financial capital is perfectly mobile across regions, the rate of returns on bonds is equalized across regions.

$$Kstock_{t+1} = I_t + (1+\delta)Ktock_t \tag{1.14}$$

where, I represents investment and $Kstock$ and δ are respectively the capital stock and the depreciation rate of capital.

1.5.2 Household Behaviour

At any given time the household sector consists of seven overlapping generations that live side by side. Each generation lives seven periods of ten years. Individuals in an age-cohort are identical and they have identical tastes. An individual is born at the age 15, retires at age 65 and dies at age 84. Individuals are assumed to be forward looking with perfect foresight. Households make lifetime decisions on consumption based on life-cycle behaviour and they do not have altruistic behaviour. Therefore, they leave no bequest and receive no inheritance. Following Barro and Friedman [15], when households encounter uncertain lifetimes, they might leave "unintentional bequests". Assuming "perfect annuity market", the unintentional bequests are evenly distributed across the living generations [14].

[14] The unintentional bequest approach was first developed by Yaari [130], for a continuous time model. Application of this theory to the OLG model was implemented by Borsch-Supan et al. [28].

The households optimization consists of two steps. In the first step, the households choose between consumption and saving. Once the households determine the aggregate consumption path over lifetime, they allocate the consumption expenditure among differentiated regional goods [15] using a CES function.

Preferences

The household preferences are represented by a constant elasticity of substitution (CES) function with current and future consumption. Households maximize the intertemporal utility function with respect to the budget constraint to drive the consumption demand. The inter-temporal preferences of an individual are given by:

$$Max \quad U_{it} = \frac{1}{1-\gamma_i} \sum_{k=0}^{k=6} \left[\left(\frac{1}{1+\varphi_i} \right)^{k+1} q_{t+k,g+k} (C_{i,t+k,g+k})^{1-\gamma_i} \right], 0 < \gamma_i < 1, \tag{1.15}$$

where, C is aggregate consumption and γ is the inverse of intertemporal elasticity of substitution between consumption in different years that shows the percentage change in the ratio of consumption between two years with respect to percentage change in the price of consumption between the same two years. φ denotes the rate of time preference and is often referred to the degree to which the households prefer current over future consumption during their life-time. A large φ indicates that a household will spend more of its resources early in lifetime. $q_{t+k,g+k}$ represents probability of survival and denotes as follow:

[15] Home produced goods and imported goods from other regions.

$$q_{t+k,g+k} = \prod_{n=0}^{n=k} SR_{t+n,g+n} \tag{1.16}$$

where, $SR_{t+n,g+n}$ indicates an exogenous survival rate between two consecutive generations and time periods. This form of utility function imposes some constraints on preferences. The intertemporal elasticity of substitution, which expresses the degree of consumption substitutability across time, is fixed. Also, at any point in time the individual decisions depend only on future consumption. Past levels of consumption affect households' current wealth.

Budget Constraint

At each period, the household earns income from labour and capital, decides how much to spend on consumption and will save the remaining and add to its lifetime stock of asset. The present value of lifetime consumption should be equal to the present value of lifetime earnings. The household dynamic budget constraint is as follow:

$$\begin{aligned} A_{i,t+k+1,g+k+1} = \frac{1}{SR_{t+k,g+k}}[(1-\tau_i^l - CTR_i)Y^l_{i,t+k,g+k} + (1+(1-\tau_i^K)r_{i,t+k})A_{i,t+k,g+k} \\ +Pen_{i,t+k,g+k} - (1-\tau_i^c)C_{i,t+k,g+k}PC_{i,t+k,g+k}] \end{aligned} \tag{1.17}$$

$$Y^l_{i,t,g} = w_{i,t}.EP_{i,t,g}.LS_{i,g} \tag{1.18}$$

$$EP_{i,g} = \omega + \xi(g) - \phi.g^2, \omega, \xi, \phi \geq 0 \tag{1.19}$$

where, Y^l is labour income, r_i is the real interest rate, $\tau's$ are tax rates on labour income, capital income and consumption, Pen is the pension benefit, P is index price of unit aggregate consumption and A is individual's asset holdings. LS is exogenous supply of physical units of labour and EP is defined as a quadratic function of age. Since the model incorporates individual heterogeneity, the labour income differs across regions due to the wage rate per unit of effective labour and demographic structure. Because of wage differences by age, the income of an individual is described by a hump-shaped income profile [16].

Lifetime budget constraint is derived by solving the dynamic budget constraint for k equal 0 to 6 and is as follow:

$$\sum_{k=0}^{k=6}\left[\prod_{k=k+1}\left(\frac{SR_{t+k,g+k}}{(1+(1-\tau_i^k)r_{i,t+k})}\right)(1+\tau_i^c)C_{i,t+k,g+k}PC_{i,t+k}\right]=\sum_{k=0}^{k=6}\left[\prod_{k=k+1}\left(\frac{SR_{t+k,g+k}}{(1+(1-\tau_i^k)r_{i,t+k})}\right)Y^*_{i,t+k,g+k}\right] \quad (1.20)$$

$$Y^*_{i,t+k,g+k}=Y^l_{i,t+k,g+k}+Pen_{i,t+k,g+k} \quad (1.21)$$

where, Y^* represents lifetime earning, Pen_i is zero for the working-age cohorts and it will differ from zero once the individual chooses to retire. When individuals are young they have no assets and they do not receive pensions, so that their saving and pension benefit is zero. When old, individuals have no labour income and stop accumulating assets; therefore, saving and labour income is equal to zero. Taking derivatives from utility function, with

[16] With quadratic income profile, the individuals reach their maximum income between middle age and retirement. We assume that the parameters of the quadratic age function are the same across regions. ω=1, ξ=0.25, ϕ=0.285.

respect to the lifetime budget constraint, we have the first order conditions:

$$(\frac{1}{1+\varphi_i})^{t+1}(C_{i,t+1,g+1})^{-\gamma_i} = \lambda\left(\frac{(1+\tau^c_{i,t+1})PC_{i,t+1}}{\prod_{k=0}^{k=6}\left(1+(1-\tau^k_{i,t+1})r_{i,t+1}\right)}\right) \tag{1.22}$$

$$(\frac{1}{1+\varphi_i})^{t}(C_{i,t,g})^{-\gamma_i} = \lambda\left(\frac{(1+\tau^c_{i,t})PC_{i,t}}{\prod_{k=0}^{k=6}\left(1+(1-\tau^k_{i,t})r_{i,t}\right)}\right) \tag{1.23}$$

where, λ represents the shadow price of the lifetime budget constraint and is the utility value of an additional unit of income. Combining the two first order conditions yields the following equation:

$$\frac{Con_{i,t+1,g+1}}{Con_{i,t,g}} = \left[\left(\frac{\left(1+(1-\tau^k_{i,t+1})\right)r_{i,t+1}}{1+\varphi_i}\right)\left(\frac{(1+\tau^c_{i,t})PC_{i,t}}{(1+\tau^c_{i,t+1})PC_{i,t+1}}\right)\right]^{\frac{1}{\gamma_i}} \tag{1.24}$$

Second Household Optimization

As is mentioned above, the household optimization consists of two steps. Once the households determine the aggregate consumption path over lifetime, they allocate the consumption expenditure among differentiated regional goods[17] using a CES function.

$$min \quad C_{i,t,g}PC_{i,t} = \sum_j (1+\tau_{j,i,t})P_{j,t}x_{j,i,t,g} \tag{1.25}$$

Subject to:

$$C_{i,t,g} = \left[\sum_j \alpha_{C,j,i} x_{j,i,t,g}^{\frac{(\sigma^c_i-1)}{\sigma^c_i}}\right]^{\frac{\sigma^c_i}{\sigma^c_i-1}} \tag{1.26}$$

[17]Home produced goods and imported goods from other regions

where, $x_{j,i,t,g}$ represents the household-g of region-i's consumption demand for region-j's good, $\alpha_{C,j,i}$ is region-j share of region-i consumption good and σ_i^c , the substitution elasticities in consumption goods and $\tau_{j,i,t}$ represents the tariff [18]. The first order conditions for consumption expenditure minimization is derived by taking the derivative with respect to $x_{k,i,t,g}$:

$$(1+\tau_{k,i,t})P_{k,t} = \mu \left[\sum_j \alpha_{C,j,i} x_{j,i,t}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \right]^{\frac{\sigma_i^c}{\sigma_i^c-1}-1} \alpha_{C,k,i} x_{k,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}-1} \tag{1.27}$$

where, μ is the lagrangian multiplier. Multiplying both sides by $x_{k,i,t,g}$:

$$(1+\tau_{k,i,t})x_{k,i,t,g}P_{k,t} = \mu \left[\sum_j \alpha_{C,j,i} C_{j,i,t}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \right]^{\frac{\sigma_i^c}{\sigma_i^c-1}-1} \alpha_{C,k,i} x_{k,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \tag{1.28}$$

Taking the sum over all goods gives:

$$\sum_k (1+\tau_{k,i,t})x_{k,i,t,g}P_{k,t} = \mu \left[\sum_j \alpha_{C,j,i} x_{j,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \right]^{\frac{1}{\sigma_i^c-1}-1} \left[\sum_k \alpha_{C,k,i} x_{k,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \right] \tag{1.29}$$

Substituting equation (1.25) in the above equation yields the following form for the Lagrangian multiplier:

$$C_{i,t,g}PC_{i,t} = \mu C_{i,t,g} \Rightarrow \mu = PC_{i,t} \tag{1.30}$$

Replacing μ by its value into equation (1.28) gives the final consumption demand of

[18] Since there are no trade barriers across regions, we assume the tariff is equal to zero.

the representative household of region-i for good produced in region-k at period t:

$$(1+\tau_{k,i,t})x_{k,i,t,g}P_{k,t} = PC_{i,t}\left[\sum_j \alpha_{C,j,i}x_{j,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}}\right]^{\frac{\sigma_j^c}{\sigma_i^c-1}-1}\alpha_{C,k,i}x_{k,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \tag{1.31}$$

$$(1+\tau_{k,i,t})x_{k,i,t,g}P_{k,t} = PC_{i,t}(C_{i,t})^{\frac{1}{\sigma_i^c}}\alpha_{C,k,i}x_{k,i,t,g}^{\frac{(\sigma_i^c-1)}{\sigma_i^c}} \tag{1.32}$$

$$x_{k,i,t,g} = \alpha_{C,k,i}^{\sigma_i^c}\left(\frac{1}{1+\tau_{k,i,t}}\right)^{\sigma_i^c}\left(\frac{PC_{i,t}}{P_{k,t}}\right)^{\sigma_i^c}C_{i,t} \tag{1.33}$$

Then using the constraint function, equation (1.26), and substituting $x_{k,i,t,g}$, I derive the composite consumption price index:

$$C_{i,t,g} = \left[\sum_j \alpha_{C,j,i}\left(\alpha_{C,j,i}^{\sigma_i^c}\left(\frac{1}{1+\tau_{j,i,t}}\right)^{\sigma_i^c}\left(\frac{PC_{i,t}}{P_{j,t}}\right)^{\sigma_i^c}C_{i,t}\right)^{\frac{\sigma_i^c-1}{\sigma_c}}\right]^{\frac{\sigma_i^c}{\sigma_i^c-1}} \tag{1.34}$$

$$PC_{i,t}^{\sigma_i^c} = \left[\sum_j \alpha_{C,j,i}^{\sigma_i^c}\left((1+\tau_{j,i,t})P_{j,t}\right)^{1-\sigma_i^c}\right]^{\frac{\sigma_i^c}{1-\sigma_i^c}} \tag{1.35}$$

$$PC_{i,t} = \left[\sum_j \alpha_{C,j,i}^{\sigma_i^c}\left((1+\tau_{j,i,t})P_{j,t}\right)^{1-\sigma_i^c}\right]^{\frac{1}{1-\sigma_i^c}} \tag{1.36}$$

The composite consumption price index is a non-linear weighted average of regional prices.

1.5.3 Pension

The main pillar of the income security system in Canada consists of two pension plans: CPP and QPP. The residents of Quebec contribute to the QPP, while Atlantic and ROC regions contribute to the Canada Pension Plan. We assume that both pension plans are PAYG (Pay as you go) systems. Therefore, for each pension plan, the total pension benefits should be equal to the total contributions.

For household optimization, we assume that the earning profiles of all individuals are similar; however, labour incomes are different since each region has its specific wage rate per unit of effective labour. Retirees receive pension benefits that are proportional to their lifetime earning. The retiree pension benefit is determined by the public pension replacement, *Pen*, that is identical across all regions. The elderly in ROC and the Atlantic are under the same pension plan, CPP, and their pension benefits are thus equal to:

$$Pen_{i,t+5,g+5} = Pen_{i,t+6,g+6} = PenR_{i,t,g}\left((\frac{1}{5})\sum_{k=0}^{4} Y^{L}_{i,t+k,g+k}\right), i \in [ROC, Atlantic] \quad (1.37)$$

For CPP, the total pension benefits should be equal to the total contributions. Five working generations contribute to the plan and two retired generations receive pension benefits that are proportional to their lifetime income[19]:

$$\sum_{gw} N_{i,t,gw} Pen_{i,t,gw} = CTR_{i,t} \sum_{gr} N_{i,t,gr} Y_{i,t,gr}, i \in [ROC, Atlantic] \quad (1.38)$$
$$gw = \{g+k; k = 0, ..., 4\}, gr = \{g+5, g+6\}$$

As the above equations indicate, the size of the population of the retirees and the working generations is crucial in sustainability of the pension plan. While the residents of both ROC and Atlantic contribute to the CPP, only the residents of Quebec contribute to QPP and

[19]The contribution rates across all regions are equal.

the retired generation receive their benefits from QPP. There is the same set of equations for QPP:

$$Pen_{QC,t+5,g+5} = Pen_{QC,t+6,g+6} = PenR_{QC,t,g}\left((\frac{1}{5}) \sum_{k=0}^{4} Y^{l}_{QC,t+k,g+k} \right) \tag{1.39}$$

$$\sum_{gw} N_{Qc,t,gw} Pen_{Qc,t,gw} = CTR_{Qc,t} \sum_{gr} N_{Qc,t,gr} Y_{Qc,t,gr} \tag{1.40}$$

The fiscal sustainability of the pension plan depends on the aging of the population. As shown in section 1.3, demographic transition will intensify over coming decades and the ratio of individuals 65 years of age and over to the population 15 to 64 years of age (old age dependency ratio) will rise sharply. With the aging of the population and withdrawal of Canadians out of their "prime working-age" years into their retirement years, the labour force shrinks. The reduced number of working population will contribute to provide for the larger retiree population.

1.5.4 Government

Government earns revenues from taxation and spends its income on government consumption and on interest payments on borrowing. I assume that social security has a separate system. Social security has its own contribution instrument, the payroll tax, and is self-financing. Also, the representative government in each region purchases goods from all regions and minimizes its spending.

The Government Budget Constraint

We assume government spending per capita remains constant over time; however, as the population dynamics change the government issues bonds to keep the budget balanced. The government targets a constant debt-to-GDP ratio. The government budget constraint is as follow:

$$
\begin{aligned}
& PG_{i,t}B_{i,t+1} - PG_{i,t-1}B_{i,t} + \sum_{g} N_{i,t,g}[\tau_{i,t}^{L}(Y_{i,t,g}^{L}) + \tau_{i,t}^{c}(C_{i,t,g}PC_{i,t}) + \tau_{i,t}^{K}(re_{i,t}A_{i,t,g})] \\
& = G_{i,t}PG_{i,t} + \left(rb_{i,t-1} + \frac{PG_{i,t} - PG_{i,t-1}}{PG_{i,t-1}} \right) PG_{i,t-1}B_{i,t}
\end{aligned}
\tag{1.41}
$$

where, $G_{i,t}$ represents the government spending, $PG_{i,t}$ is composite government good price index for region-i, re is expected rate of return on physical capital, $N_{i,t,g}$ is population size for generation g in region i and rb represents interest rate on government bonds. Different taxes are assumed to be exogenous.

Government Optimization

The representative government in each region purchases goods from all regions and minimizes its total spending. Government spending is a composite good of the three regional final goods. The allocation of government spending across the regions is determined as follow:

$$min \quad G_{i,t}PG_{i,t} = \sum_j P_{j,t}g_{j,i,t} \tag{1.42}$$

$$subject\ to \quad G_{i,t} = \left[\sum_j \alpha_{G,j,i} g_{j,i,t}^{\frac{\sigma_i^g - 1}{\sigma_i^g}}\right]^{\frac{\sigma_i^g}{\sigma_i^g - 1}} \tag{1.43}$$

where, $g_{j,i,t}$ represents public spending of government-i on goods from region-j. $alpha_{G,j,i}$ is region-j share of region-i government consumption demand and σ_i^g is the substitution elasticities in government consumption. The first order conditions for government expenditure minimization are derived by taking derivatives with respect to $g_{k,i,t}$:

$$P_{k,t} = \nu \left[\sum_j \alpha_{G,j,i} g_{j,i,t}^{\frac{\sigma_j^g - 1}{\sigma_j^g}}\right]^{\frac{\sigma_j^g}{\sigma_j^g - 1} - 1} \alpha_{G,k,i} g_{k,i,t}^{\frac{\sigma_j^g - 1}{\sigma_g^g} - 1} \tag{1.44}$$

where, ν is the lagrangian multiplier. Multiplying both sides by $g_{k,i,t}$:

$$g_{k,i,t}P_{k,t} = \nu \left[\sum_j \alpha_{G,j,i} g_{j,i,t}^{\frac{\sigma_j^g - 1}{\sigma_j^g}}\right]^{\frac{\sigma_j^g}{\sigma_j^g - 1} - 1} \alpha_{G,k,i} g_{k,i,t}^{\frac{\sigma_j^g - 1}{\sigma_g^g}} \tag{1.45}$$

Taking the sum over all goods gives:

$$\sum_k G_{k,i,t}P_{k,i,t} = \nu \left[\sum_j \alpha_{G,j,i} G_{j,i,t}^{\frac{\sigma_j^g - 1}{\sigma_j^g}}\right]^{\frac{\sigma_j^g}{\sigma_j^g - 1} - 1} \left[\sum_k \alpha_{G,k,i} G_{k,i,t}^{\frac{\sigma_j^g - 1}{\sigma_g^g}}\right] \tag{1.46}$$

Substituting equation (1.43) into the above equation yields the following form for the Lagrangian multiplier:

$$G_{i,t,g}PG_{i,t} = \nu G_{i,t,g} \Rightarrow \nu = PG_{i,t} \tag{1.47}$$

Replacing ν by its value into equation (1.45) gives the final government consumption demand of region-i for good produced in region-k at period t, $g_{k,i,t}$:

$$g_{k,i,t}P_{k,t} = PG_{i,t}G_{i,t}^{\frac{1}{\sigma_i^g}}(\alpha_{G,k,i})g_{k,i,t}^{\frac{\sigma_i^g-1}{\sigma_i^g}} \tag{1.48}$$

$$g_{k,i,t} = \alpha_{G,j,i}^{\sigma_i^g}\left(\frac{PG_{i,t}}{P_{k,t}}\right)^{\sigma_i^g} G_{i,t} \tag{1.49}$$

Then using the constraint function, equation (1.43), and substituting $g_{k,i,t}$, I derive the composite government price index:

$$G_{i,t} = \left[\sum_j \alpha_{G,j,i}\left(G_{i,t}(\alpha_{G,j,i})^{\sigma_i^g}PG_{i,t}^{\sigma_i^g}P_{j,i}^{-\sigma_i^g}\right)^{\frac{\sigma_i^g-1}{\sigma_i^g}}\right]^{\frac{\sigma_i^g}{\sigma_i^g-1}} \tag{1.50}$$

$$PG_{i,t} = \left[\sum_j (\alpha_{G,j,i})^{\sigma_i^g}P_{j,t}^{1-\sigma_i^g}\right]^{\frac{1}{1-\sigma_i^g}} \tag{1.51}$$

The composite government consumption price index is a non-linear weighted average of regional prices.

1.5.5 Market Clearing Condition

Aggregate demand is composed of consumption demand, investment demands and government consumption demand. Equation (1.52) shows the market equilibrium condition for commodity market in region i. The total aggregate demand for each good is equal to total output.

$$Q_{i,t} = (\sum_{g} N_{i,t,g}Con_{i,t,g}) + I_{i,t} + G_{i,t} + X_{i,t} - M_{i,t} \tag{1.52}$$

$X_{i,t}$ and $M_{i,t}$ represent respectively region i's export and import with other regions. Also the market clearing condition for commodity market holds for Canada, in which the GDP is equal to the sum of aggregate consumption, investment and net export.

The labour and capital are immobile across regions and input market clears for these factors in each region. Equation (1.53) provides the equilibrium condition for labour market. Capital market must be in equilibrium. This means that the total household saving must be equal to stock of government bonds plus stock of capital,(equation (1.55)).

$$L_{i,t} = \sum_{g} N_{i,t,g}LS_{i,g} \tag{1.53}$$

$$K_{i,t} = Kstock_{i,t} \tag{1.54}$$

$$\sum_{i}\sum_{g} N_{i,t+1,g}A_{i,t+1,g} = \sum_{i} PG_{i,t}B_{i,t+1} + \sum_{i} PI_{i,t}Kstock_{i,t+1} \tag{1.55}$$

where, $Kstock_{i,t}$ is the exogenous supply of capital stock in region-i.

It is assumed that Canada is a closed economy. Obviously, other options were available. First, I could assume a global model that incorporates the rest of the world regions. This would be an ideal option but the model will become very large and hard to handle and also very demanding in data in order to calibrate the model. Second, Canada could be a small open economy that consequently it would be a price taker. Therefore it wouldn't be possible to study the impact of the demographic changes on domestic prices since they would not alter. This would be hard to defend as all large economies of the world are now aging. The third option, which is used in this chapter, is to assume a closed economy. The advantage of making this assumption is that I can analyse the impact of demographic changes on wage and interest rate. The disadvantage is that we implicitly assume that the demographic changes in the Canadian economy are the same as the rest of the world. This might not be far from the reality as demographic changes in Canada are similar to other OECD (Organisation for Economic Co-operation and Development) countries.

1.6 Data and Calibration

The data for the calibration are obtained from Statistics Canada for the year 2007. We use the symmetric Input-Output (I-O) tables for provinces and territories. The I-O tables provide detailed statistics on all the transactions in an economy and involve production activity and intermediate and final consumption of goods and services. We use the data provided in the I-O tables jointly with other relevant data, such as government revenue and expenditure [20], to create the Social Accounting Matrix (SAM) for Canada's provinces. The social Accounting Matrix provides a framework to analyse and build general equilibrium models. Harberger [65] and Johansen [78] pioneered the application of SAM for providing a data base to build an applied general equilibrium (Kehoe [81]). Since then, many research studies have been developing general equilibrium (Scarf [120], Scarf [119], Shoven and Whalley [121], Ballard et al. [13], Mercenier and Srinivasan [94], Dixon [41]).

The symmetric industry-by-industry I-O tables for Canada's provinces represent the most detailed accounting of the Canadian economy. They include data on labour and capital value added, transactions among the 25 sectors, household spending, investment, inter-provincial trade and international trade for each province. I use the I-O tables to construct the SAM for each province and then I aggregate the provinces to create the regional SAM [21]. In this chapter, I assume each region has only one sector and therefore, I aggregate across the 25 sectors. As an accounting principle, the total value of each column

[20] Government data are obtained from CANSIM tables, available at: `http://www.statcan.gc.ca/tables-tableaux/sum-som/l01/cst01/govt02a-eng.htm`

[21] We aggregate the SAM for Newfoundland and Labrador, New Brunswick, Nova Scotia and Prince Edward Island to create the Atlantic region and aggregate Ontario, Manitoba, Saskatchewan, Alberta, British Columbia, Yukon, Nunavut and Northwest Territories to create the ROC and Quebec remains as a single province.

(expenditure) and its corresponding row (income) should be equivalent.

1.6.1 Demographic Structure

As mentioned in section 1.5, the population is divided into seven generations (i.e., 15-24, 25-34, 35-44, 45-54, 55-64, 65-74, 75-84). The population data for the baseline year are obtained from Statistics Canada, "Population Projections for Canada, Provinces and Territories" [22]. Statistics Canada provides different population scenarios and we choose the projection with medium growth in population; however, the population projection data for Canadian provinces are only available until 2031. In order to have a longer horizon for the model to allow it to converge to the steady state, I also use the population projections from the United Nations [23]. The United Nations Statistics Division provides national population projection datasets by age groups until the year 2100.

The aging of the Canadian population based on United Nations data [24] is shown in figure 1.6. The figure shows the proportion of children aged 14 years and less and persons 65 and over.

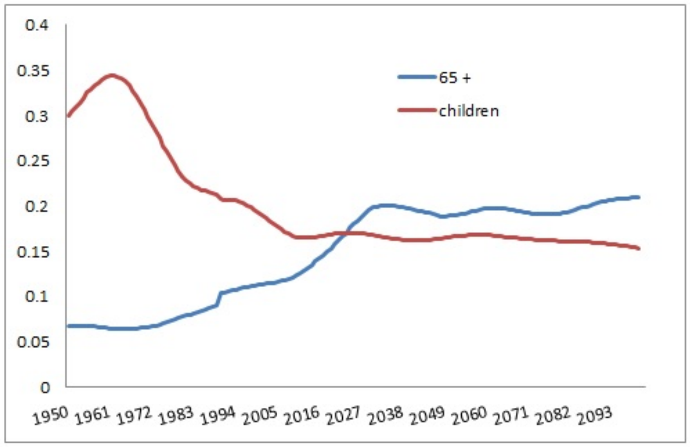
The share of older age groups continues to increase after 2031 with lower speed. As demonstrated in Figures 1.6, the share of elderly population in Canada will increase from 13% in 2011 to 22% in 2100. The share of people aged 65 years and above continues to be lower than those aged 14 years and less until 2022 and then it surpasses and remains

[22] The data are available at the following link: `http://www5.statcan.gc.ca/bsolc/olc-cel/olc-cel?catno=91-520-XWE&lang=eng`

[23] United Nations Population Censuses Datasets, available at: `http://unstats.un.org/unsd/demographic/products/dyb/dybcensusdata.htm`

[24] United Nations, World Population Prospects, 2012.

Figure 1.6: Proportion of Persons Aged 65 Years and Over and Children Aged 14 Years and Less in Canada, 1950-2100



Source: United Nations, World Population Prospects, 2012

above this group until the year 2100. While the senior population is continuously growing, the proportion of children is decreasing. During the past two decades the share of children aged 14 years and less has dropped significantly, and it was approximately 16% in 2011. It continues to decreases with a lower pace until 2100.

As the United Nations population projections reveal, Canada will experience significant demographic changes in the coming decade.

In order to obtain demographic variables such as survival rate and fertility rate for extended periods until the year 2100, I combine the data provided by the UN and Statistics Canada. First, I calculate these rates for each cohort based on population projection data available by Statistics Canada until the year 2031. Then, I use the UN population projection and calculate the cohorts survival rate for Canada from 2031 to 2100. Subsequently, I obtain the trend in UN demographic variables and extrapolate the survival rates for each age group from 2032 to 2100. This procedure will allow us to have demographic variables for a longer horizon and improve the simulation results.

To simplify the model, I assume that demographic variables, fertility rate and survival rate, are exogenous. Each generation has the following law of motions:

$$N_{i,t,g+k} = \begin{cases} N_{i,t-1,g+k} * FR_{i,t-1} & k = 0 \\ N_{i,t-1,g+k-1} * SR_{i,t-1,g+k+1} & k = 1, 2, 3, 4, 5, 6 \end{cases}$$

where, $FR_{i,t-1}$ represents the fertility rate, $SR_{i,t-1,g+k+1}$ is the survival rate and $N_{i,g+k}$ is the population size for generation $g + k$ in region i. The first law of motion indicates that in region i at time t the number of the young generation (i.e, age group 15-24) is equal to the size of the same cohort at time t-1 times the fertility rate at time t-1. The second law of motion specifies the size of any generation (except the first generation) at time t equal to the size of the same cohort at time t-1, times the survival rate of that generation at time t-1. The survival rates are different for each generation at any period. The survival rate for the last generation is equal to zero. Table 1.2 represents the projected old age dependency ratio across the regions.

Table 1.1: Projected Old-age Dependency Ratio

Region	2007	2020	2030
ROC	18.57	25.85	35.00
Atlantic	20.56	33.25	47.18
Quebec	20.85	30.23	40.81

Source: Statistic Canada, Population Projection for Canada Provinces and Territories, Table 052-0005.

1.6.2 Calibration

I use SAM, first-order conditions, regions-specific exogenous parameters and market clearing conditions to calibrate the model and obtain other parameters of the model and values of the non-observed variables at the baseline year. The calibration is performed in three steps to estimate the parameters of the model. In the first step, I employ the regional data on output production, labour and capital demand and the first order conditions from the production optimization to calibrate wage, rental rate of capital and the scaling parameter (TFP) of production function.

The second step includes household optimization, market clearing conditions in goods and assets and government budget constraints to derive consumption profile and government expenditure. Jointly, rate of time preferences, interest rate and intertemporal elasticity of substitution determine the consumption profile. Also, government spending adjusts to population dynamics. I assume that debt-to-GDP remains constant.

The third step calculates a matrix and estimates the distribution of wealth across generations and regions. Individuals hold their wealth in the form of physical capital and government bonds but the distribution of the wealth is unknown. Although assets are perfectly substitutable and mobile across regions, it is important to know the initial portfolio distribution in case of unanticipated shocks. Each individual's wealth consists of the local physical capital and bonds issued by local and other regional governments.

After the three steps, I use the calibration results to examine the model and verify whether the model could replicate the observed data on baseline year. Then the calibration results are used to simulate the impact of population dynamics on pension plan.

Table 1.2: Calibration Results

Region	α	A	δ	γ	r
ROC	0.43	1.72	0.42	0.25	0.67
Atlantic	0.48	1.45	0.21	0.15	0.47
Quebec	0.39	1.64	0.34	0.17	0.59

Notes:
α= share of physical capital
A= shift parameter in production
δ= depreciation rate over 10 year period
γ= inverse of intertemporal elasticity of substitution over 10 year period
r= real rental rate of capital

1.7 Simulation

As mentioned in section 1.2, the main pillar of the income security system in Canada consists of two pension plans. The residents of Quebec contribute to the Quebec Pension Plan (QPP), while the rest of the provinces contribute to the Canada Pension Plan (CPP). In order to investigate the impact of population aging on the pension plans I use the model under steady state to calculate the contribution rate for each region, in three different scenarios for pension programs and in two different years, 2007 and 2030.

For the first scenario, I assume that all the regions together have one single pension plan. The working-age population from all three regions contributes to one pension system that provides benefits for all the retiree population in all three regions. The pension equation is equal to:

$$\sum_{i}\sum_{gw} N_{i,t,gw} Pen_{i,t,gw} = CTR_t \sum_{i}\sum_{gr} N_{i,t,gr} Y_{i,t,gr} \quad i \in (ROC, Atlantic, Qc) \qquad (1.56)$$

where, gw represents the working generation and gr is the retired generation. As the above equation shows, in a PAYG system the working-age population contributes to the system to provide for the benefits of the elderly. According to the above equation, the pension plan has a single contribution rate that is the same for all regions.

For the second scenario, I take into consideration the current situation of the income security system in Canada in which Quebec has its own pension plan (QPP) and Atlantic and ROC regions are under CPP. Therefore, there are two pension equations, as follow:

$$\sum_{gw} N_{i,t,gw} Pen_{i,t,gw} = CTR_{i,t} \sum_{gr} N_{i,t,gr} Y_{i,t,gr} \quad i \in (ROC, Atlantic) \qquad (1.57)$$

$$\sum_{gw} N_{Qc,t,gw} Pen_{Qc,t,gw} = CTR_{Qc,t} \sum_{gr} N_{Qc,t,gr} Y_{Qc,t,gr} \qquad (1.58)$$

According to equations (1.57) and (1.58), there exist two contributions rates. The elderly in Quebec receive benefits from QPP while benefits to elderly in the ROC and Atlantic are paid under the CPP.

For the third scenario, I assume each region has its own pension plan. Therefore, there are three contribution rates and the pension equation is equal to:

$$\sum_{gw} N_{i,t,gw} Pen_{i,t,gw} = CTR_{i,t} \sum_{gr} N_{i,t,gr} Y_{i,t,gr} \quad i \in (ROC, Atlantic, Quebec) \qquad (1.59)$$

Table 1.3: Contribution Rates for Three Scenarios

Contribution Rate 2007	ROC	Atlantic	Quebec
Scenario 1	10		
Scenario 2	9.74		11.00
Scenario 3	9.68	10.62	11.00

Contribution Rate 2030	ROC	Atlantic	Quebec
Scenario 1	16.66		
Scenario 2	15.35		21.74
Scenario 3	15.02	19.88	21.74

The above three scenarios are calculated for the model and the results are reported in Table 1.3. To do a comparative analysis among the three scenarios and across years, I choose scenario one in 2007 as a benchmark and compare the results of scenarios two and three with the benchmark. According to scenario one at baseline year 2007, the contribution rate is 10 percent. If the economy switches to scenario two in which there are two pension systems, one for Quebec and one for Atlantic and ROC, the contribution rate for ROC and Atlantic drops by about 3 percent while the contribution rate for Quebec increases by 10 percent in order to maintain its pension system. Quebec is better off under a joint pension plan with other regions. The difference in the contribution rate is due to the demographic structure of the working population.

As explained in section 1.3, the population aging is heterogenous across Canadian regions. Under scenario one, the share of the retired population in the total population is 12 percent; however, once the regions are separated and Quebec has its own pension system, the share of the retired population in Quebec becomes 15 percent. This significant increase in the share of the retiree population puts pressure on the provision of pension benefits and leads to a rise in contribution rate.

If the economy chooses scenario three, the contribution rates for Atlantic region should increase by 6 percent in order to maintain the pension system. The contribution rate in ROC drops by 3 percent indicating that ROC is better off when it has its own pension plan. The share of the working age population in ROC is significantly larger than the share of the working age population in Atlantic and Quebec (by 6 times in Atlantic and 3 times in Quebec). Therefore, under a joint pension system, the larger and young working population of ROC contribute to the provision of benefits of elderly in all three regions.

The retiree population is continuously growing in all regions by 2030. It increases by 60% in Quebec, 80% in the Atlantic and 46% in the ROC. These significant changes in the structure of the population show that population aging is faster in Quebec and Atlantic region.

The results for the three scenarios in year 2030 confirm that the aging of the population imposes higher contribution rates to the pension plan. Comparing the results in 2030 with scenario one in 2007 reveals that the contribution rate increases in all three scenarios and the increase is higher in Quebec, if it keep its own pension system. In 2030, according to scenario one, the contribution rate should increase by 66 percent from the benchmark, in order to maintain the balance in the pension system. Under the scenario two the contribution rate should increase by 53 percent for ROC and Atlantic; however, the contribution rate should increase more than 100 percent for Quebec in order to maintains its pension system.

The main benefits of having a joint pension system are for the Atlantic. Under the scenario three, the Atlantic region should increase its contribution rate by 100 percent, but by joining to ROC under the scenario two, it pays lower contributions. Regions with faster aging of the population benefit more from a joint pension system. The Atlantic region and Quebec should double their contribution rates by 2030, if they choose to have a separate pension plan. But, by joining to a joint pension system their contribution rates increase by less.

A shrinking labour force has to provide for the growing retiree population. The share of working age population declines for all regions from 2007 to 2030. It decreases by 9 percent in Quebec, 8 percent in the Atlantic and 4 percent in the ROC.

1.7.1 Implicit Transfers Across Regions

To calculate the implicit transfer of funds across regions and to get a good sense of the amount of implicit transfers, I perform the following experiment, using the data on total contributions to CPP and QPP for year 2013-2014 from Canada's Pension Plan report [25]. I assume the economy's pension system is scenario two. The total contributions paid to CPP in 2013-2014 are $43.3 billion. This amount is contributed by the working age population in both ROC and Atlantic region. I use the share of the working-age population in 2007 and estimate the share of each region in total contributions. The working-age population in ROC is seven times larger than the working-age population in Atlantic. Therefore, the amount of pension contribution paid by the ROC is seven times more than the total

[25] https://www.canada.ca/en/employment-social-development/programs/pensions/pension/statistics/2015-quarterly-january-march.html

Table 1.4: Total Contribution for Three Scenarios, Using 2013-2014 Data, $Billions

2007	ROC	Atlantic	Quebec
Scenario 1	34.63	5.00	15.87
Scenario 2	37.73	5.45	12.32
Scenario 3	20.94	22.24	12.32

2030	ROC	Atlantic	Quebec
Scenario 1	37.00	5.62	12.89
Scenario 2	37.49	5.69	12.32
Scenario 3	19.41	23.77	12.32

contribution paid by Atlantic. Thus, the shares of ROC and Atlantic from the total contributions of $43.2 billion are respectively, $37.7 and $5.4 billion. The results of this experiment are reported in Table 1.4.

When the economy moves from scenario two to scenario three, in which each region has its own independent plan, the working-age population in each region should pay for the benefits of the retiree population in the respective region. The retiree population of the Atlantic region is 1.6 percent larger than in the ROC, therefore the amount of benefits paid to retirees of Atlantic is 1.6 percent larger than the amount paid to retirees in ROC. Consequently, the total benefits paid to the retiree population are equal to $20.9 billion and $22.2 billion in ROC and Atlantic, respectively.

As a result, under a PAYG pension system, the total contribution in the Atlantic region should increase from $5.4 billion to $22.2 billion, while it declines from $37.7 to $20.9 billion in the ROC. The difference between the two levels of contributions, which is equal to $16.8 billion, is the implicit transfer of funds (contributions) from the ROC to Atlantic region when the economy chooses scenario two and ROC and Atlantic have a joint pension system.

The Atlantic region under scenario two pays $16.8 billion less in contributions, since it has a smaller share of working population. Therefore, this amount is paid by the working population of the ROC under a joint pension plan.

Quebec cannot benefit from the same type of implicit transfers if it chooses to have a joint pension system with ROC and Atlantic. The contribution rate declines if Quebec joins the other two regions, but since it has higher share of working population than the Atlantic, it compensates for part of the benefits paid to retiree in Atlantic. In fact, if Quebec chooses to join the other two regions to have a single plan (scenario one), it will be worse off than having its own separate plan. This is because by having a shared plan with other regions it will take away some of the pressure on the ROC in providing for the Atlantic region. In other words, there is a small implicit transfer of funds from Quebec to Atlantic. According to the pension plan report for 2013-2014, the total contribution for the QPP is equal to $12.3 billion. Therefore, the total pension benefits in the economy equal to $55.5 billion. Using the same justification as above, the amounts of pension contributions in ROC and Quebec are, respectively, seven and three times larger than the total contributions paid by Atlantic because the share of the working-age population is larger, by the same ratio, in ROC and Quebec compared to Atlantic. Thus, under scenario one, the shares of ROC, Atlantic and Quebec from the total contributions of $55.5 billion are, respectively, $34.6, $5 and $15.8 billion. Comparing the results of scenarios one and two shows the transfer of implicit funds from ROC and Quebec to Atlantic.

The share of the retiree population in Atlantic is larger than the retiree population in ROC by 1.6 percent and the same share is 0.9 percent smaller in Quebec compared to ROC. Therefore, if each region decides to have its own pension plan (scenario 3), the amounts of benefits paid to the retiree population in ROC, Atlantic and Quebec are equal to \$20.94, \$22.2 and \$12.3 billion. By comparing scenario three with scenario two, the implicit transfer of funds (contributions) from the ROC to Atlantic region is equal to \$17 billion.

I repeat the same experiment for year 2030 to further explore the impact of aging and the amount of implicit transfers. I use the same data on total contributions for year 2007. The lower panel of Table 1.4 reports on the total contributions by region for year 2030. In 2030, the share of the working-age population in ROC is 6.6 times larger than in Atlantic, therefore, under scenario two, the working age population in ROC will be paying for the larger share of total contributions. Comparing the results of scenarios two and three shows that the implicit funds transferred from ROC to Atlantic equal to \$18 billion.

Moving from scenario two to scenario one in 2030, in which Quebec joins the other two regions, benefits Atlantic region significantly, but it also reduces the pressure on the ROC region by one percent. Also, in 2007 Quebec pays a higher amount of contributions than in 2030 under scenario one. In other words, for Quebec to have its own program is beneficial in 2007 but that benefit is disappearing slowly and by 2030 the benefit is almost zero. The big winner of the current system (scenario two) is obviously the Atlantic region, both in 2007 and in 2030.

1.8 Conclusion

This chapter provides, from a theoretical and quantitative point of view, a methodology to investigate the impact of an aging population on the pension system from a regional perspective. I developed, calibrated and simulated an OLG model to investigate the impact of demographic changes on public pension and measure the transfer of implicit funds across regions. Three scenarios are suggested for the pension system. The results show that as a result of aging of the population, all regions are affected negatively if they choose to have an independent pension system. Under a joint pension system most of the pressure of the provision of the pension system is on the ROC, since it has the highest share of working population. Atlantic region benefits the most from a joint pension plan, as the aging of the population intensifies significantly in this region over the coming years.

ROC bears the brunt of a joint system. The pressure of providing pension benefits for the elderly in Atlantic is mainly on the ROC, since it has a much larger working population. If Quebec joins the pension plan with the other two regions, it takes away some of the pressure on the ROC, by paying larger contributions. Therefore it is beneficial for Quebec to have its own pension system; however, over time, as the share of working population drops in Quebec, engaging in a joint pension system with other regions benefits Quebec. This is because the larger share of working population in ROC is contributing to the provision of pension benefits in Quebec. A joint pension system dramatically burdens the future working population in ROC. Nonetheless, when reduction in GDP per capita and cohort welfare are taken into account, all regions experience contractions in response to aging of the population.

The model and the results contained in this chapter provide valuable and useful insights on the consequences of a joint pension system in Canada. In particular, it shows that there is an argument for Quebec to have its own pension program and not join the other regions under a joint pension program and that the implicit transfers of resources from the west part of Canada to the Atlantic provinces is substantial. The results of the chapter suggest that it would be relevant in future research to investigate further the question by going beyond the pay-as-you-go specification for the pension plan and incorporating the three pillars of the Canada pension system.

1.9 Appendix for Chapter 1

1.9.1 List of variables

Subscript i indicates region:

$Q_{i,t}$ =region-i output

$P_{i,t}$= output price in region-i

$K_{i,t}$=physical capital

$Kstock_{i,t}$=stock of capital in region-i

$L_{i,t}$=effective unit of labour

$r_{i,t}$=real rental rate of capital

$W_{i,t}$=wage for effective unit of labour

$C_{i,t,g}$=consumption demand of household of generation g at time t

xj, i, t, g= household-g of region-i's consumption demand for region-j's good

$PC_{i,t}$=composite consumption price index

$A_{i,t,g}$=household-g stock of wealth at end of period t-1

$Y^l_{i,t,g}$=labour income of household-g

$LS_{i,g}$=labour supply by household-g

$EP_{i,g}$=household-g age-dependent earning profile

$N_{i,t,gw}$=population size of working age cohorts

$N_{i,t,gr}$=population size of retired cohorts

$Pen_{i,t,gr}$=pension benefit to retired generation gr

$I_{i,t}$=investment in region-i

$i_{j,i,t}$=region-j's investment demand for region-i

PIi,t= composite investment good price

$re_{i,t}$= expected rate of return on physical capital

$rb_{i,t}$= rate of interest on region-i government bond issued at end of t-1

$G_{i,t}$= region-i public expenditure

$G_{j,i,t}$= government-i public expenditure on region-j good

$PG_{i,t}$= composite government spending price index (price of bond issued by region-i government)

1.9.2 List of parameters

Subscript i indicates region:

$A_{i,t}$= total factor productivity

α_i= share of physical capital in Cobb-Douglas function

φ_i= rate of time preferences

γ_i= inverse of the constant inter-temporal elasticity of substitution

τ_i^c= consumption tax rate

τ_i^k= tax rate on capital income

τ_i^l= tax rate on labour income

$\tau_{j,i,t}$= tariffs on goods

$CTR_{i,t}$= contribution to the pension plan

$PenR_{i,t,g}$= pension replacement rate

$Pen_{i,t,g}$= pension benefit

$SR_{i,t,g}$= survival rate of cohort g at timet

δ_i= depreciation rate of capital

$\alpha_{C,j,i}$= region-j share of region-i consumption good demand

$\alpha_{I,j,i}$= region-j share of region-i investment good demand

$\alpha_{G,j,i}$= region-j share of region-i government good demand

σ_i^c= substitution elasticities in consumption goods

σ_i^I= substitution elasticities in investment goods

σ_i^g= substitution elasticities in government goods

ω= constant term in quadratic function of age

ξ= coefficient on age term in quadratic function of age

ϕ= coefficient on the quadratic term in age function

Chapter 2

Age Variable Rate of Time Preference in OLG-CGE Model

2.1 Introduction

Intertemporal choices require decision-making that involves trade-off between current and future outcomes. Intertemporal choices and their outcomes are important for economists. Economic models construct assumptions about preferences and sometimes these assumptions have led us to solutions that are general and abstract. Olson and Bailey [103] state that "without time preference, models of intertemporal choice do not yield predictions consistent with observed behaviour". The early studies on the importance of intertemporal choices started in the early nineteen century with Scottish economist John Rae and Austrian Economist Eugen von Bohm-Bawerk who examined the sociological and psychological determinants of intertemporal choice. The perspective on intertemporal choices changed in 1937 when Paul Samuelson proposed the discounted utility model as a "descriptively accurate" representation of individual behaviour. In his model, he assumes that all the underlying motives for intertemporal choices are condensed into one single parameter, the discount rate. Many studies began to employ innovative approaches to better understand and highlight the origins and stability of the preference parameter over individuals life cycles (Borghan et al. [25], Posner [107], Trostel and Tyler [125]).

Over the past three decades, due to the anomalies and inconsistencies in the discounted utility model, a series of theoretical and empirical studies attempted to explain and measure discount rate. The theoretical studies attempted to make more realistic assumptions such as relaxing the assumption of constant discounting, or incorporating more considerations into utility function such as the "utility from anticipation", "habit formation function" and "reference point model" (Laibson [89], Strotz [122]). The empirical studies derive estimates of discount rate by observing "real-world" behaviour (Rutherman et al. [114], Kirby [83], Burks et al. [33]). There is a striking variation in the estimated discount rate among empirical studies, which is mostly due to the elicitation procedure.

In this chapter, I review the existing theoretical and empirical research on intertemporal choice. Recent empirical studies have demonstrated that the age profile of consumption is bell shaped (National Transfer Account, United Nations 2013[1]). This allows for a new calibration procedure in which the rate of time preference varies by age. The advantage of the new procedure is that it respects consumption behaviour observed in data. The consumption profile derived from NTA allows the development of a new approach that measures the rate of time preference more accurately. Contrary to the mainstream studies in the area of intertemporal optimization that assume a constant rate of time preference over individuals' life cycles, I describe a new approach in which the rate of time preference

[1] `http://www.ntaccounts.org/web/nta/show/Methodology`

evolves by age. I construct an OLG model and assume that people are not characterized by a constant rate of time preference and allow the rate of time preference to vary with age. Age-variable rate of time preference would permit us to capture many other elements that affect the life cycle profile of consumption observed in the data. Better-calibrated life cycle profiles should improve simulation results. I also derive the asset holding by age and compare it at the individual level with the total asset holding for the economy. Furthermore, I investigate the sensitivity of the calibrated rate of time preference to the other parameters of the model.

The rest of the chapter is organized as follows. In section 2.2, I review the various studies that have attempted to explain the intertemporal choice and discounting behaviour and highlight the assumptions underlying those studies. Section 2.3 describes the OLG model developed to measure the rate of time preference. Section 2.4 describes the calibration process and sections 2.5 and 2.6 explain two calibration approaches. Section 2.7 reports the calibration results and section 2.8 concludes.

2.2 Review of Relevant Literature

In this section, I review the existing literature and highlight the striking variations across studies that attempt to explain and measure the rate of time preference. I stress out the importance of non-economic factors affecting individual time preferences and therefore discount rate.

Investigating the determinants of intertemporal choices goes back to the early nineteenth century. Rae [108] invented the topic of intertemporal choice and recognized the non-economic factors, sociological and psychological, that determine individuals' intertemporal choices and therefore the level of saving and investment.

Rae [108] strives to explain why wealth differs among nations. Earlier, Adam Smith argues that national wealth is determined by labour allocation toward production, ;however, Rae [108] believes that this explanation is incomplete and the missing element is "the effective desire of accumulation". In his view, a psychological factor that is different among countries determines individuals' preferences toward saving and investment. He lists two main elements that determines the accumulation: bequest motive and "propensity to exercise self-restraint"[2]. For the first time, Rae [108] defined intertemporal choice and its underlying motives for time preferences. He develops two views. In the first one, he claims that individuals care about current consumption and postpone consumption only if they anticipate higher utility in the future. The second view treats present and future outcomes equally. In other words, individuals have no intrinsic preferences toward either present or future consumption. Based on these two views, "intertemporal choice behaviour" shapes

[2]" The extent of intellectual powers and prudence in the minds of the members of society".

differently, but in both views people's mental ability in imagining future situations affects their intertemporal choice.

The next major figure in development of intertemporal choices is Eugen von Bohm-Bawerk [23]. He added a new factor to the list proposed by Rae, arguing that people's intertemporal choices are affected by "systematic tendency to underestimate future wants". He begins modeling the intertemporal choice by allocating one's resources to different points in time. Bohm-Bawerk concurs with Rae that humans' decisions are heavily dependent on psychological traits. The early theoretical framework developed by these authors became the dominant theoretical framework for studying intertemporal choice.

A decade later Irving Fisher formalized Bohm-Bawerk's approach and developed the intertemporal consumption decision on a two-good indifference curve with consumption in the current year and following year. Fisher [52], as for Bohm-Bawerk, differentiates between "objective factors" and "personal factors" that affect individuals' decisions over time. In their views, time preference is an amalgamation of diverse intertemporal motives that change over lifetime. In Fisher's depiction, consumption depends on two factors: time preferences and diminishing marginal utility. Fisher, like Rae and Bohm-Bawerk, considers psychological factors in the determination of time preferences.

In 1937, Paul Samuelson introduced the discounted utility model. He condenses all the analysis of time preferences mentioned by his predecessor into one single factor: discount rate. Samuelson [117] developed a "generalized" model of intertemporal choices with multiple time periods. An underlying assumption for Rae [108], Bohm-Bawerk [23], Fisher [52] and Samuelson [117] is the constant discounting, which means that individuals have a single, constant discount rate over life cycle.

Over the past four decades a body of research, both theoretical and empirical, has been developed on intertemporal choices and these studies suggest that the discount rate is not necessarily constant over time.

Many studies attempted to improve the theoretical model developed by Samuelson. These studies formulate more realistic assumptions by incorporating more considerations into utility function, such as relaxing the assumption of constant discounting (Thaler [124], Chapman and Elstein [35], MacKeigan et al. [92], O'Donoghue and Rabin [101], ODonoghue and Rabin [102]), including "habit formation function"[3] (Pollak [106], Becker and Murphy [17]) and using "reference point concept" [4] (Tversky and Kahneman [126], Lowenstein and Prelec [91], Laibson [89], Stortz [122]).

The empirical studies derive estimates of discount rate by observing "real-world" behaviour (Rutherman et al. [114], Kirby [83], Burks et al. [33]). There is a striking variation in the estimated discount rate among empirical studies, which is mostly due to the elicitation procedure. The estimated discount rates range from -6 to infinity (Frederick et al.[55]).

In the following sections I review the literature on time preferences and divide the studies into three main categories: natural selection, psychological traits and experimental studies.

[3]Habit formation means that individuals' current preferences depend on their past consumption patterns.

[4]The reference point concept demonstrates that when people choose between immediate and delayed consumption, the reference point used to evaluate alternatives can significantly influence choice. Preferences formation may differ using reference point concept.

2.2.1 Time Preferences and Natural Selection

Some of the studies on theory of time preferences propose that preferences originated from "natural selection". In other words, individuals' preferences are affected by their biological characteristics such as size, shape and pattern of fertility. These models link discount rate with age, because fertility capacity varies with age. Reproduction requires successful propagation of one's genes, which necessitates spending resources. These studies claim that the time preferences are affected by theory of the evolutionary genetics[5]. They argue that "if genes affect preferences, then an evolutionary model may succeed in predicting them" (Becker [16], Hirshleifer [69], Rubin and Paul [112], Ursprung [127]).

Roger [111] discusses the hypothesis that human time preferences are in evolutionary equilibrium [6]. This hypothesis defines "Marginal Rate of Substitution (MRS) in fitness as the marginal rate at which two goods can be substituted without changing Darwinian fitness"[7]. By assuming that in evolutionary equilibrium the MRS in fitness must equal MRS of preferences, Roger [111] estimates the evolutionary discount function. His model shows that younger adults "should discount at a higher rate than their elders" because "young adults seem to live as if there were no tomorrow". Roger [111] identifies three factors that encourage individuals to choose immediate over delayed benefits: "expectation of rising consumption", "declining reproductive value" and the possibility of future benefits

[5]Human evolutionary genetics studies how one human genome differs from another human genome, the evolutionary past that gave rise to it and its current effects.

[6]"An equilibrium that no mutation changing time preferences could be favoured by natural selection."

[7]" In models with discrete generations, Darwinian fitness is the conditionally expected number of an individual's offspring, given its genotype. In models with overlapping generations, fitness is measured by "Malthusian parameter", which measures the asymptotic rate of exponential increase in the numbers of one's descendants."

to be received by descendants rather than the individual himself. For the second factor, reproduction declines because fertility declines with age. This element underlies the age effect in his model. As individuals age the reproductive value declines and encourages individuals to choose present benefits rather than delayed benefits.

Posner [107] asserts that discounting exists because individuals have "multiple selves", that is "people are weighting their present consumption far more heavily than their future consumption...[because] the present self and the future self are, in some meaningful sense, separate persons". This argument favours separable utility functions (Koszegi [85]). Becker and Mulligan [18] contend that the reason for discounting is "rational defective recognition of future utilities". They show that elements such as wealth, addiction, mortality and uncertainty affect the "degree of time preferences" across cohorts. For example, wealth can improve patience and the "patience endowment" can be transferred from one cohort to another.

Trostel and Taylor [125] argue that individuals do not have an "intrinsic preference" for present consumption but with aging they increase discounting (or have a lower rate of time preference) because their expected utility falls as their physical and mental health declines. The two authors construct a new theory of time preferences with this hypothesis that "the instantaneous utility function is expected to vary with age, just as people's physical and mental abilities vary with age". In other words, the capacity to enjoy consumption is expected to rise while young and therefore discount rate may be negative. Trostel and Taylor [125] use household consumption data from the Panel Study of Income Dynamics (PSID)[8] and test their theory by estimating discount rate as a function of age, income,

[8]PSID contains 14 years of consumption data on income and demographic data, from 1974 to 1987.

education and race. Their estimation results confirm their theory on increasing discount rate due to the decline in physical and mental health.

Robson and Samuelson [109] also assume that people's preferences are affected by biological evolution. They show theoretically that individuals discount future reproduction, because an "intervening death" may prevent individuals from realizing future outcomes. They derive an equation for discount rate, which is a product of population growth rate and death rate. When there is an idiosyncratic uncertainty, the discount rate is equal to the product of population growth rate and death rate; however when there is aggregate uncertainty [9] the discount rate surpasses the product of population growth rate and death rate. Aggregate uncertainty breaks the link between discounting and the sum of the growth and death rates. They show that aggregate uncertainty lowers the growth rate "below that arising with comparable idiosyncratic uncertainty". Aggregate uncertainty slows down population growth but it does not distort the trade-off between current and future outcomes and hence the discount rate. Therefore, the product of the growth rate and the death rate falls behind discounting.

Robson and Szentes [110] use Rogers' [111] biological approach and reexamine the functional form for the discount rate. They show that the functional form of the utility function does not produce an equilibrium evolutionary behaviour. Roger [111] assumes that utility function of an individual is derived from "reproductive value" and all offspring are identical; however Robson and Szentes [110] allow for different types of offspring, based on their level of transfers that vary with age. They show that offspring with larger (intergenerational)

[9] Aggregate uncertainty, such as fluctuations in the weather, epidemics and failures of food sources are all bound to have a common effect on death rates.

transfers have higher reproductive value.

Fisher [51] argues that future rewards are discounted because "an intervening death may prevent us from realizing such rewards". The possibility of death plays an important role in individuals' discounting behaviour. When there is a risk affecting individuals' survival rates, the constant discounting may not hold. Uncertainty affects intertemporal choice and its effect differs across ages. In the presence of uncertainty, discount rate varies with age (Robson and Szentes [110]).

Curry [37] supports the "evolution of time preferences under natural selection" by assuming that individuals care about the number of offspring they produce. Individuals make decisions under uncertainty and choose from sets of risky alternatives (lotteries). These lotteries are choices over commodities that may involve uncertainty. Commodity choices at present time are transformed into expected surviving offspring in the next period. To choose the optimal selection of lotteries, Curry [37] uses the Robson [109] evolution criterion that gives the preferences to lotteries that maximize reproductive value. Curry [37] shows that a lottery should be selected only if it maximizes "expected number of offspring relative to the average number of offspring in the population". When all agents are maximizing reproductive value, a game is constructed based on the choice among lotteries (expected number of offspring). By allowing for this kind of preference, the reproductive value is maximized and a unique Nash equilibrium is reached.

Blackburn and Cipriani [20] show that an increase in the probability of survival increases the future returns on human capital accumulation and therefore causes the opportunity cost of current work and reproduction to increase. Individuals increase their human capital in exchange for a smaller number of children. The paper endogenises the survival rate by allowing it to change as a function of the economy. They develop an overlapping generation model with three generations and endogenous survival rate. Individuals choose between consumption and number of children. They show that time preferences are affected by changes in survival rate. When probability of survival increases, individuals expect to live longer and opt for more education versus child bearing.

Rutstrom et al. [115] show that reasonable estimates of discount rate depend on joint estimation of risk and time preferences rather than separate estimation. In their experiment to jointly elicit time preference and risk, they use a utility function for income and consumption, and present individuals with one task that indicates "risk attitude" and another task that indicates "discount rate". They show that "joint elicitation of risk and discount rate results in significantly lower discount rates than separate elicitation of discount rate". They performed a field experiment[10] to collect data on a Danish adult population.

2.2.2 Personality Traits and Time Preferences

There is a huge literature in psychology on the importance of personality traits on economic outcomes, such as job performance, health and academic achievement (Barrick and Mount [14]; Chamorro-Premuzic and Furnham [34]; Hogan and Holland [70]; Hampson et al. [64]). Individuals' personality traits affect the choices they make at any age. These traits are not stable over life cycle and consequently, individuals' preferences vary over time. Heckman and Honore [68] discuss the relevance and importance of personality traits in

[10]Rutstrom et al. [115], Chapter 4, page 18.

economics and show that learning ability affects consumption, labour supply and earning profiles. They incorporate the measurement of learning ability in a life cycle model and show that "more able people are more likely to attend more years of school" and individuals with higher levels of education have lower discount rates. Rubinstein and Heckman [113] use the data from the General Education Development[11] (GED) program to show the "quantitative importance" of personality traits. They show that GED recipients have the same cognitive ability as high school graduates but they differ in personality traits. GED recipients perform poorly in the labour market compared to high school graduates.

Most studies in the field of psychology are dependent on questionnaires that capture personality traits. Many economic studies strive to measure standard economic preference parameters such as time preferences. Economic studies are now beginning to incorporate psychological measurements of personality traits into economic models and filling the gap between personality psychology and conventional economic preferences. Personality traits affect individuals' choices and basically their time preferences over the life cycle. In other words, psychological characters appear as constraints into individuals' optimization.

Personality traits are defined as "patterns of thought, feelings and behaviour" (Borghans et al. [25]). One of the most prominent personality trait measurements is the "Big Five"[12], which economists have begun to use.

Considering the impact of personality traits on life outcomes, it is important to understand how much and to what extent they can change over time and how environments and investments affect development of personality traits. Environments and investments can

[11] An exam-certified alternative high school degree.

[12] Conscientiousness, openness, emotional stability, extraversion, agreeableness.

have an immediate or short-term impact as well as long-term impact on behaviour. For example, individuals will most probably invest more efforts if they are compensated to do so and will be "more sociable" if there are incentives to do so (Borghans et al. [25]). Consequently, the individual's behaviour will vary in different settings. Therefore, taking the "malleability" of personality traits into analysis is essential because "certain personality traits are more susceptible to change over individuals' life cycles" (Borghans et al. [25]).

Studies in psychology use various methods[13] such as *Mean-level change* to capture and measure the "malleability" of personality and its impact on economic performance. *Mean-level change* considers "changes over time in absolute levels of a trait and is measured by changes in personality scores over time."[14]

Many studies in the field of psychology and economics estimate time preferences using "observational data" (Heckman and Honore [68]; Borghans and Golsteyn [26]; Hausman [67]; Ruderman et al. [114]); however, this type of study suffers from imperfect information and consequently the interpretation of discount rate estimated this way will be complicated. For example, Borghans and Golsteyn [26] explore the relationship between body mass index (BMI) and proxies for discount rates; however, the relationship is not only dependent on the choice of the proxy but also on the other parameters that determine the intertemporal decisions. These empirical studies are discussed in section 2.3.

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[13]Some other methods to measure changes in personality traits are: *Rank order change, normative change and non formative change*. *Rank order change* refers to changes in the ordinal ranking of a trait in a population and is measured by test-retest rank correlations. *Normative changes* are "changes that are typical of average individuals in a given population, and caused either by biological programming or by predictable changes in social roles". *Non normative change* "encompasses both intentional change, caused by deliberate, self-directed efforts, deliberately chosen changes in social roles and atypical life events".

[14]Psychologists believe that it's impossible to see dramatic change in mean-level change in personality after adulthood.

To avoid issues concerning observational data, some other studies use an experimental approach to estimate discount rates. They use a series of monetary rewards for people choices. Discount rate changes with the size of the reward (Green et al. [60], Kirby [83]) and with type of the reward (Chapman et al. [36], Bonato and Bolan [24]); however, these monetary choices are usually hypothetical and therefore individuals may choose not to maintain their preferences when faced with real life situations (Fuch [56], Green and Myerson [60], Kirby and Marakovic [84]).

Discount rate estimates are affected by the context of study and they may not successfully demonstrate separable economic preferences. Kahneman et al. [79] argue that individuals' choice in the valuation of environmental public goods does not comply with economic preferences. They introduce the concept of "attitude" that determines the size and the strength of an individual's stated willingness to pay for public goods. By including "personality attributes" into the utility function and outlining "dynamic constraints connected with skill and trait formation", utility optimization becomes consistent with more "general preference specifications" (Phelps and Pollak [105], Ainslie [3], Laibson [89]).

Frederick et al. [55] propose a tri-dimensional rate of time preference, consisting of three motives: "Impulsivity" [15], "compulsivity" [16] and "inhibition" [17], and claim that these dimensions can be measured accurately. They do not argue a precise link between rate of time preference and these personality traits and state that more analysis is required to clearly estimate a rate of time preference that is a composite of these dimensions.

[15] Tendency to act spontaneously and without planning.
[16] Tendency to stick with plans.
[17] Ability to override automatic responses to urgencies or emotions.

Borghans et al. [27] use a variety of personality trait measurements to predict social and economic outcomes and show to what degree the traits are stable over the life cycle. They create a framework for choice model that imposes psychological variables as a constraint and preference parameter is derived as a consequence of these constraints. They conclude that certain economic concepts require modification in order to fully capture the impact of psychological traits into the choice model.

Borghans et al. [25] show the importance of personality traits by comparing the impact of Big Five personality traits and IQ on time preferences and economic performances such as job performance, college grades and years of education. They show that conscientiousness is the best personality trait in predicting college grades and years of education. Chamorro-Premuzic and Furnham [34] investigate other aspect of conscientiousness such as self-control and perseverance on individuals' success in school and in life. Nyhus [100] shows that emotional stability leads to higher wages. Salgado [116] and Hogan and Holland [70] show the importance of emotional stability in predicting job performance across professions.

2.2.3 Experimental Studies

There is a group of studies that use observational data to observe and measure time preference and display its variability. Although the scope of these studies is limited to specific subject matter, they provide insight on choice preferences (Hausman [67], Ruderman et al. [114], Gately [57], Borghans et al. [27]). The estimated time preferences by various studies, depending on the characteristics of the population, psychological state of individuals and particular procedure or situation, range from -6 to infinity (Frederick et al [55])[18]. A positive rate of time preference means that individuals are willing to sacrifice one unit of consumption today for more than one unit of consumption later. A negative rate of time preference would require substantial reductions in present consumption in favour of the future.

Borghans et al. [27] investigate individuals' emotional stability and conscientiousness in performing IQ tests. By providing financial incentives, they measure to what extent people are responsive to these incentives. They show that individuals spend more time in answering questions if they are offered financial reward. Multiple financial incentives were offered to the participants and the amount of time spent on the test was measured, along with their performance. The discount rate varies between 0 and 0.6 and its mean is 0.247. They chose 128 students from Maastricht University for the experiment and concluded that people with motivation and self-discipline are more successful in the test and have lower discount rate.

Hausman [67] studies individuals' behaviour in the market for durable goods. He investigates the trade-off between "capital costs" and "operation cost" in the market for energy efficient air conditioners and measures discount rate in purchasing air conditioners. He uses household data from a Midwest Research Institute (MRI, 1978) survey for energy consumption and electricity appliances. The MRI survey includes a random sample of 1985 households in sixteen cities across United States. This survey also collects socioeconomic data for each household in the sample. Individuals consider "initial capital cost" and "operating cost" when making decisions toward purchasing energy efficient appliances.

[18]Fredrick et al. [55] survey methods for measuring discount rate.

Individuals trade off "capital costs" and expected "operating cost" and therefore the rate of substitution among energy appliances depends on the above two factors.

In other words, when the "capital cost" is high, individuals opt for purchasing energy efficient appliances only if "operating cost" is low. His estimation results show individuals use a discount rate of 20 percent in making the trade-off. He comments that a high level of discount rate can be reduced if the choice of other energy-using appliances is added to the analysis. If consumers have more options available the estimated discount rate might be lower.

Ruderman et al. [114] analyse the market for energy appliances and estimate the discount rate. They use historical data from surveys of appliance manufactures. The data consist of energy cost, purchase price and efficiency factors and are captured by the U.S Department of Energy for 1972-80. They measure the aggregate discount rate for eight products by assuming that individuals minimize the life cycle cost over these products. Their results show that higher discount rate is associated with lower sensitivity between energy consumption and appliance efficiency. Also, discount rates for various energy appliances show the same trends. They justify high discount rates by unavailability of full information for consumers.

Hardisty et al. [66] extract time preferences for financial gains and losses for delays up to 50 years. They employed various methods such as matching, fixed-sequence choice iteration and a dynamic "staircase" choice method. They recruited five hundred sixteen participants from the virtual lab participant pool of the Center for Decision Sciences at

Columbia University and randomly assigned an experiment [19]. Unlike other studies that use short-range delays of gains and losses, Hardisty et al. [66] use long timescale (50 years), to model "real-world" intertemporal choices. For all methods used, they find that gains are being discounted more than losses, which means that participants' wish to have gains immediately is stronger than their wish to postpone losses.

Anderoni et al.[4] use two methods, convex time budgets (CTB)[20] and double multiple price lists (DMPL), [21] to elicit the discount rate. They show that the discounting behaviour is different with and without accounting for curvature of utility function. Estimated discount rate is lower for linear utility function. They evaluate the predictive power of the estimated parameter and show that both methods make predictions close to average behaviour.

Burks et al. [33] calculate measures of time preference and test the measures to evaluate how well these measures predict important health outcomes such as smoking and Body Mass Index (BMI), credit scores and subsequent job related outcomes. They use a large (1069 participants) sample of "non student subjects" (trainee truckers) who participated in an extensive set of economic field experiments between December 2005 and August 2006 and gather data on preference and outcome. Their results show that all measures of time preferences predict individual behaviour to some extent.

[19] Hardisty et al. [66], page 239.

[20] "Given a set of functional form assumptions about discounting and curvature of the utility function, preference parameters are estimated at either the group or individual level."

[21] "This method compares safe and risky gambles with different payoffs and different odds, including some gambles that payout with certainty. The response to risky choices identifies attitude towards risk by the point at which individuals switch from a safe payout to a risky gamble. This information identifies the degree of utility function curvature".

This section reviewed the existing literature in estimating and measuring time preference. Individual differences in personality and cognition shape the constraints of individuals and therefore their choices over the life cycle. Many studies argue that constant time preference for choice behaviour is an unrealistic assumption. Individuals' preferences change through the life cycle due to various elements such as age, health, personality traits, etc. If these elements are considered in shaping the individuals' preferences, the rate of time preference will not be constant over life cycle. This chapter contributes to this literature by assuming that the rate of time preference is not constant and is dependent on age. In the next section we describe an OLG model with age dependent rate of time preference.

2.3 Model Description

The model is a modified version of Auerbach and Kotlikoff [7] and Auerbach et al. [8]. It is an OLG model with three sectors, 21 generations and exogenous labour supply. Two of the sectors are public goods (education and health) and the third is the unique private good sector. In this section, I give an overview of the model. In the next section, I discuss in detail the calibration challenge of such a model.

2.3.1 Firm Behaviour

The representative firm maximizes its profits by employing labour and capital. It is assumed that capital is homogeneous and depreciating. Labour supply differs in its efficiency because individuals of different ages provide different amounts of labour input. The production function is assumed to be of the Cobb-Douglas form with constant return to scale. Perfectly competitive firms maximize profits. The demand for factors of production is derived accordingly.

2.3.2 Household Behaviour

There are 21 overlapping generations [22] that live side by side. Each generation lives 21 periods of 5 years. Individuals in an age-cohort are identical and they have identical tastes. An individual is born at the age zero, enters the labour market at age 20, retires at age

[22] The 21 age cohorts are: 0-4, 5-9, 10-14, 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-65, 65-69, 70-74, 75-79, 80-84, 85-89, 90-94, 95-99 and 100+.

65 and dies at age 100. The first four generations (children) do not contribute to the economy and they do not have active roles in the model. Individuals are assumed to be forward-looking with perfect foresight. Individuals make lifetime decisions on consumption based on life cycle behaviour and they have bequest motives. The oldest generation leaves a bequest for the first working generation [23]. Also, as households encounter uncertain lifetimes, they might leave "unintentional bequests" [24] (Barro and Friedman [15]). Assuming a "perfect annuity market", the unintentional bequests are evenly distributed across the active generations [25]. The household preferences are represented by a Constant Elasticity of Substitution (CES) function with current and future consumption. Households maximize the inter-temporal utility function with respect to the budget constraint to derive the consumption demand. The rate of time preference denotes the degree to which the households prefer current consumption over future consumption during their lifetimes.

Future utilities are discounted by an exogenous survival rate. This form of utility function imposes some constraints on preferences. At any point in time the individuals' decisions depend only on current and future consumption. Past levels of consumption affect households' current wealth.

[23]This assumption is made in order to ensure the model solves. In calibrating the Euler Equation, (equation (2.17)), due to the ratio of C(g+1)/(C(g), the number of equations is one less than the number of variables. By assuming that the oldest generation leaves a bequest for the first working generation, one equation is added to the model and guarantees feasibility of the solution.

[24]Unintentional bequest occurs when the date of death differs from what the individual might forecast.

[25]The unintentional bequest approach was first developed by Yaari [130], for a continuous time model. Application of this theory to the overlapping generation model was implemented by Borsch-Supan et al. [28].

Budget Constraint

In each period, the household earns income from labour and capital and decides how much to spend on consumption. The remainder is saved and added to its lifetime stock of assets. The present value of lifetime consumption should be equal to the present value of lifetime earnings. Individuals contribute to the pension plan when young and receive pension benefits when retired. Individuals have no labour income and stop accumulating assets when retired.

2.3.3 Pension

I assume that the pension plan is a PAYG (pay as you go) system. Therefore, the total pension benefits should be equal to the total contributions. Contribution rates of the pension program are dependent on the age distribution of the population. Retirees receive pension benefits that are proportional to their lifetime earnings.

2.3.4 Government

Government earns revenues from taxation and spends its income on government consumption, education, health and interest payments on borrowing. The government targets a constant debt-to-GDP ratio. We assume government consumption per capita remains constant over time; however, public spending on education and health is sensitive to the age structure of the population. Consequently, as the population dynamics change the government will adjust the income tax to keep the budget balanced.

2.3.5 Market Clearing Condition

The model is a closed economy and all markets are competitive. Labour supply equals to labour demands. With population aging, labour supply becomes scarce hence wage rate will rise. On the contrary, capital demand will decline for the same reason and capital return will drop. Aggregate demand is composed of consumption demand, investment demand and government consumption demand. The market equilibrium condition for commodity market ensures that the total aggregate demand is equal to total output.

2.4 Calibration

2.4.1 Data

The model is calibrated using United Kingdom (UK)data for the year 2010[26]. The population projection data are from the Office for National Statistics (ONS)[27] and are used to derive fertility rate, survival rate and migration rate. The formula is provided in section 4.2. Data on GDP, public revenue and expenditure are extracted from ONS. Data on pension are taken from the Government Actuarial Department (GAD)[28]. Labour income tax, consumption tax and capital tax are calculated using the government revenue data [29]. Labour force data are obtained from the Labour Force Survey in ONS. The data obtained are used to derive the Social Accounting Matrix (SAM). Table 2.1 provides the SAM for the base year 2010.

SAM is a particular representation of the macro and intermediate economic accounts of a socio-economic system, that captures the transactions and transfers between all economic agents. As we can see in Table 2.1, the value added of production is captured in rows 5 and 6 (capital, labour, with the sum of them constituting GDP). Final consumption expenditure by households is shown in column 7, government spending in columns 8-10 and investment in column 10.

[26]The UK data are used in this chapter due to the availability of long term population projection data for all age groups. Canadian population projection data per province is merely available until 2031. This limitation hinders the calibration of a large model such as the one deployed in this chapter.

[27]`http://www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationprojections/bulletins/nationalpopulationprojections/2011-10-26`

[28]`https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/220568/pesa_2010_complete.pdf`

[29]`http://budgetresponsibility.org.uk/download/public-finances-databank`

Table 2.1: Social Accounting Matrix, United Kingdom, £mil, 2010

	FIRM	HEALTH	EDUCATION	CAPITAL	LABOUR	CONSUMPTION	GOV	GOVH	GOVE	INVESTMENT	TOTAL
FIRM						953264	124515	65458	12979	203778	1359994
HEALTH								55847			55847
EDUCATION									77782		77782
CAPITAL	431578										431578
LABOUR	928416	55847	77782								1062045
HH				157100	798845		148336				1104281
TAX				70700	263200	151017					484917
SAVING				203778							203778
TOTAL	1359994	55847	77782	431578	1062045	1104281	484917	121305	90761	203778	

In our new calibration procedure, I also use life cycle data on labour income and consumption. These data will be presented in the following sections.

2.4.2 Demographic Structure

The population data for the baseline year (2010) is obtained from the Office of National Statistics (ONS), which provides projections up to the year 2100. In order to obtain demographic variables such as survival rate, migration rate and fertility rate for extended periods until year 2100, we calculate these rates for each cohort using population projection data.

To simplify the model, I assume that demographic variables, fertility rate, migration rate and survival rate, are exogenous. Each generation has the following law of motions:

$$N_{t,g+k} = \begin{cases} N_{t-1,g+k+5} * fr_{t-1} & k = 0 \\ N_{t-1,g+k-1} * (sr_{t-1,g+k+1} + mr_{t-1,g+k+1}) & k = 1, 2, ...19, \end{cases}$$

where, N represents population, fr_{t-1} the fertility rate, $sr_{t-1,g+k+1}$ is the survival rate and mr_{t-1} represents migration rate. The first law of motion indicates that the number of children (i.e., age group 0-4) at time t is equal to the size of the first adult generation times the fertility rate at time t-1. The second law of motion specifies the size of any generation (except the first generation) at time, t, equal to the size of the same cohort at time t-1, times the sum of survival rate and net migration rate of that generation at time t-1. The survival rates are different for each generation in any period. The survival rate for the last generation is equal to zero.

2.4.3 Calibration Procedure

In this section I review the fundamental aspects of the calibration procedure that underlies most overlapping generations computable general equilibrium (OLG-CGE) models and introduce a new approach for calibrating the model described in section 3. The calibration procedure is sequential and there are two sets of parameters: *predetermined* and *free* parameters. *Predetermined* parameters are estimates from other studies and are regarded as exogenous in this model.

The model described in section 2.3 is solved to derive the first order conditions. Data obtained from SAM, first-order conditions, exogenous parameters and market clearing conditions are used to obtain the other parameters of the model and values of the non-observed variables at the baseline year. The calibration procedure allows one to determine the scaling parameter of production, the depreciation rate, the age-specific consumption and the rate of time preference, assuming that all markets are in steady state equilibrium.

Production

I calibrate the supply side of the economy by using data from SAM. Deploying the data on labour demand, capital demand and total output from SAM and assuming a Cobb-Douglas production function, I calibrate the scale parameter for the production function. The production function is:

$$Q = A(K)^{\alpha}(L)^{1-\alpha} \tag{2.1}$$

where, Q represents the output, K is the capital, L is labour, A represents the scaling parameter and α measures the capital intensity in production. Rearranging equation (2.1) to derive A results in:

$$A = \frac{Q}{K^{\alpha}L^{1-\alpha}} = \frac{0.910534}{0.651944} = 1.39 \tag{2.2}$$

Households

The Constant Elasticity of Substitution (CES) utility function is maximized with respect to the dynamic budget constraint to derive the consumption path. The utility function for a representative cohort at time t is:

$$Max \quad U = \frac{1}{1-\theta}\sum_{k=5}^{k=21}\left[\left(\frac{1}{1+\rho}\right)^{k} q_{g+k}(C_{g+k})^{1-\theta}\right], 0<\theta<1, \tag{2.3}$$

where, θ is the inverse of inter-temporal elasticity of substitution between consumption in different years. C is aggregate consumption and ρ is the rate of time preference that denotes the degree to which the households prefers current consumption over future consumption. A large ρ indicates that households spend more of their resources early in lifetime and save less. q_{g+k} is the probability of survival up to age $g+k$ and denotes as follow:

$$q_{g+k} = \prod_{n=0}^{n=k} sr_{g+n} \tag{2.4}$$

where, sr_{g+n} indicates exogenous survival rate. The household budget constraint is:

$$\begin{aligned} A_{g+k+1} = \frac{1}{sr_{g+k}}[(1-\tau^{l}-CTR)Y^{l}_{g+k} + (1+(1-\tau^{K})r)A_{g+k} \\ +Pen_{g+k} - (1-\tau^{c})C_{g+k}] \end{aligned} \tag{2.5}$$

$$Y^{l}_{g} = w.EP_{g}.LS_{g} \tag{2.6}$$

$$EP_{g} = LFPR_{g} * hc_{g} \tag{2.7}$$

where, CTR is the contribution to the pension system and Pen represents pension benefits, which is zero for the working age cohorts and it will differ from zero once the individual chooses to retire. r is the real interest rate, τ^l, τ^K and τ^c are respectively tax rates on labour income, capital income and consumption and A represents individual's asset holdings. Y^l is labour income, LS is the exogenous supply of physical units of labour and EP is individual's earning profile that depends on age-specific productivity. $LFPR$ is the labour force participation rate, hc represents the age-specific productivity profile.

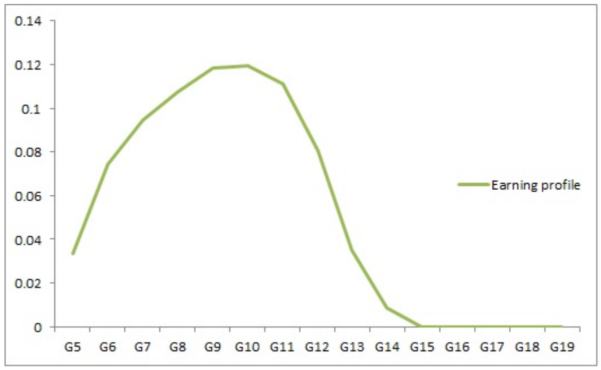
Table 2.2: Labour Force Participation and Productivity Profile

Age	$LFPR_g$	hc_g
G5	0.045	0.74
G6	0.082	0.91
G7	0.103	0.92
G8	0.117	0.92
G9	0.13	0.91
G10	0.131	0.91
G11	0.125	0.89
G12	0.103	0.78
G13	0.07	0.5
G14	0.031	0.28
G15	0.022	0
G16	0.01	0
G17	0.015	0
G18	0.006	0
G19	0.001	0

In equation (2.6), labour supply and wages are exogenously known but the earning profile, EP, is derived using equation (2.7). The data on age-specific productivity profile are obtained from NTA. Using the age-specific productivity profile and labour force participation rate, presented in Table 2.2, I derive the age-specific earning profile.

Since productivity differs across ages, the income of an individual will vary by age and

Figure 2.1: Earning Profile



is described by a hump-shaped [30] profile. The earning profile of individuals is depicted in Figure 2.1.

Lifetime budget constraint is derived by solving the dynamic budget constraint for k from 5 to 21. The present value of lifetime consumption should be equal to the present value of lifetime earnings.

$$\sum_{k=5}^{k=21}\left[\prod_{n=5}^{n=k}\left(\frac{SR_{g+n}}{(1+(1-\tau^k)r_k)}\right)(1+\tau^c-CTR)C_{g+k}\right]=\sum_{k=5}^{k=21}\left[\prod_{n=5}^{n=k}\left(\frac{SR_{g+n}}{(1+(1-\tau^k)r_k)}\right)Y^*_{g+k}\right] \tag{2.8}$$

$$Y^*_{g+k}=Y^l_{g+k}+Pen_{g+k} \tag{2.9}$$

Taking derivatives from utility function, with respect to the lifetime budget constraint,

[30]Individuals reach their maximum income between middle age and retirement.

using lagrangian function, the first-order condition is derived:

$$(\frac{1}{1+\rho})(Cg+1)^{-\theta} = \lambda \left(\frac{(1+\tau^c)}{\prod (1+(1-\tau^k)r)} \right) \tag{2.10}$$

$$(C_g)^{-\theta} = \lambda \left(\frac{(1+\tau^c)}{\prod (1+(1-\tau^k)r)} \right) \tag{2.11}$$

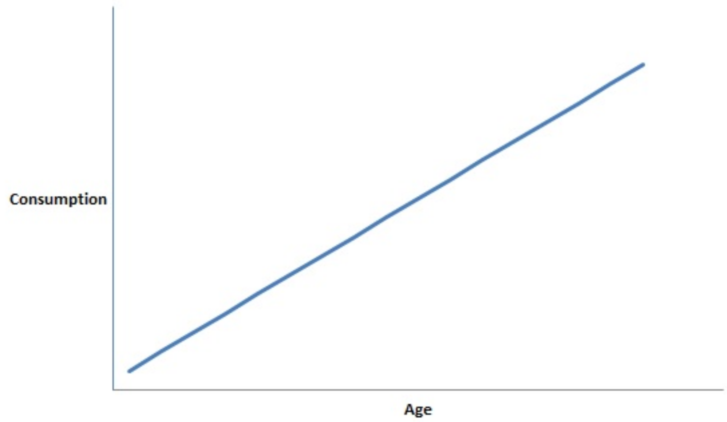
where, λ represents the shadow price of the lifetime budget constraint and is the utility value of an additional unit of income. Combining the first order conditions yields the Euler equation:

$$\frac{Con_{g+1}}{Con_g} = \left[\frac{\left(1+(1-\tau^k)r\right)}{1+\rho} \right]^{\frac{1}{\theta}} \tag{2.12}$$

In the above equations the consumption profile by age is derived endogenously. As θ, ρ, r and τ are given, the conventional approach to calibrate the Euler equation is to generate the consumption profile using the parameters on the right hand side of the above equation. Assuming that the rate of time preference is constant and the inverse of intertemporal elasticity of substitution is between zero and one, the above equation suggests that depending on the value of the interest rate and rate of time preference, the consumption takes three plausible paths, which differ distinctly. As shown in Figure 2.2, if interest rate, net of tax, exceeds the rate of time preference, the consumption increases monotonically and consumption profile over life cycle will follow an upward path. The slope of the graph depends on the value of θ.

Unlike the above case, if the rate of time preference exceeds the interest rate, net of tax, the consumption profile will take a decreasing path. The consumption profile graphed in Figure 2.3 shows that for higher value of the rate of time preference, individuals prefer

Figure 2.2: Consumption Path, $r > \rho$



present consumption over future consumption and therefore the consumption path will be decreasing.

When the rate of time preference equals the interest rate, net of tax, the path of consumption is constant over time. Figure 2.4 shows that individuals consume a constant level of goods over their life cycle.

There are many empirical evidences against the above consumption profiles (Krueger and Perri [87], Parker [104], Kazianga and Udry [80], Vettese [128]) that show consumption changes dramatically with age and over time. Krueger and Perri [87] show that individuals are not inclined to smooth their consumption over their life cycle. Individuals may face uncertainty regarding their income and consumption. Uncertain situations at different ages prevent individuals from consuming freely over the life cycle.

Vettese [128] argues that individuals' spending, excluding mortgage payments, declines with age later in life. He claims that individuals' consumption increases with age when they

Figure 2.3: Consumption Path, $r < \rho$

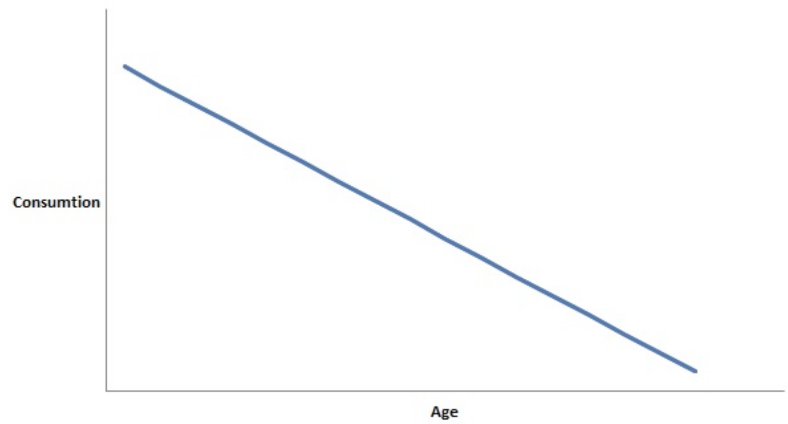


Figure 2.4: Consumption Path, $r = \rho$

are young, but after a certain age it starts declining. His results are not consistent with the consumption path provided in Figures 2.2, 2.3 and 2.4. The consumption profiles obtained in many countries have a hump-shaped form over age. Indeed, Vettese [128] provides evidence based on data from four developed countries (United States, Canada, Germany and United Kingdom), and shows that contrary to "theoretical expectation", household assets rise and spending declines after retirement. For example, in Germany seniors save 10 percent of their income, which is much higher than the saving rate among the working German population. His results for the United Kingdom reveal that as people grow older they spend less on consumption regardless of their income. A household headed by an 80-year-old consumes 43 percent less than a household headed by a 50-year-old. Vettese [128] also provides evidence for the United States that consumption declines in old age, although the rate of decline depends on the individual's marital status and education. The rate of decline in consumption is lower for college-educated people than less-educated cohorts. He also shows that the average Canadian household headed by a 77-year-old consumes 40 percent less than a household headed by a 54-year-old. He claims that the fall in spending after retirement is not because of a lack of financial resources. One suggested explanation by Vettese [128] for lower consumption in old age is that older individuals may have "physical limitations" that prevent them from spending money.

In the next session I explain the two approaches used in this chapter to calibrate the model.

2.5 Two Calibration Approaches

I develop two procedures to calibrate consumer optimization. In the first procedure, consumption is endogenous and calibrated such that the sum of individuals' consumption is equal to the amount provided by National Accounts reported in SAM. This is the traditional procedure used in the literature. Using the Euler equation, and assuming that the interest rate, tax rate on capital, intertemporal elasticity of substitution and the rate of time preference are determined exogenously, consumption profile is calibrated endogenously.

For the second procedure, which is the procedure promoted in this chapter, I assume that consumption is given exogenously and instead the rate of time preference is calibrated endogenously.

Indeed, a long standing challenge with calibrating the CGE-OLG models is the difficulty of obtaining microdata reflecting life cycle behaviour with respect to consumption, income and saving, and yet being consistent with national accounting data. The National Transfer Accounts (NTA) Project [31](United Nations, 2013) is a serious attempt to fill this gap.

NTA for Selected Countries : The NTA project disaggregates National Accounts by age. The NTA project provides a "systematic, comprehensive and coherent" method to disaggregate the major components of national accounts by age groups. NTA describes economic flows from one age group to another, during a particular time period. National Transfer Accounts shed light on measuring how people at different ages consume and share

[31] `http://www.ntaccounts.org/web/nta/show/Methodology`

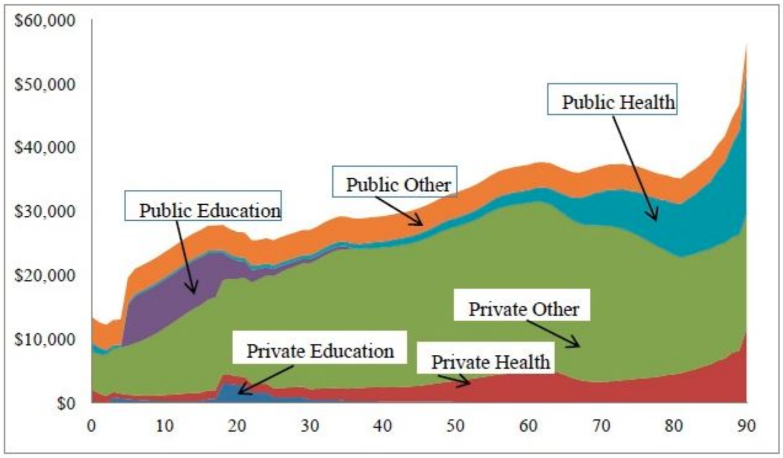
resources. More than 40 countries have joined the NTA project and are constructing and measuring accounts for consumption, production, resources and saving for each group. The purpose of the NTA is to complement the UN System of National Accounts, population data and other important economic and demographic indicators. The NTA project is available for a range of both developed and developing countries and allows the estimation of the life cycle patterns for income and consumption, which are consistent with national accounts. The following section reviews the consumption profile for a selected number of countries that utilize the NTA methodology. A general result obtained by NTA studies in many countries is that the age profile of consumption over individuals' life cycles is not an upward or downward path, but instead a bell- shaped profile.

The consumption profile for the United States in 2003 is shown in Figure 2.5 (UN, 2013, page 139). The age pattern of consumption in the U.S. consists of a combination of public and private components. As illustrated in Figure 2.5 [32], if other components of aggregate consumptions such as education and health are deducted, the remainder that represents private consumption depicts a bell-shape over the life cycle, which is not reproducible using the conventional calibration procedure. Private consumption is large for adults in their prime age but decreases when individuals approach retirement.

At the bottom of the graph the horizontal band depicts the equal allocation of public expenditures such as military spending, government-funded research, roads, public buildings, and the operating costs of government, that are not age dependent. "Private other" consumption represents all private spending except for health, education and owned housing. Private health expenditures increase until age 65, when "government-provided Medicare

[32]United Nations, National Transfer Accounts: Population Aging and Generational Economy, 2011.

Figure 2.5: Per Capita Consumption, Private and Public by Sector, United States, 2003



insurance" becomes available but declines sharply due to the decrease in private medical spending. The "private other" expenditure rises later in life again to cover "out-of-pocket" costs not chargeable to Medicare. The last component of private consumption is private education which is large over two periods: 1) the pre-school years until age 5, for elementary school, and 2) for the years of higher education after age 17.

A similar consumption distribution is derived for Canada for the year 2006 (Zhang et al. [131]). Figure 2.6 shows that individuals consume less when young. As they age their consumption of private goods increases and eventually declines as they approach retirement. Figure 2.7 shows the age profiles of aggregate consumption in Germany [33]. The adult consumption in Germany increases significantly with age. As shown in Figure 2.7, private consumption, excluding education and health, does not have a linear path, instead depicting a hump-shaped pattern.

[33]United Nation, National Transfer Accounts: Population Aging and Generational Economy, 2011.

Figure 2.6: Per Capita Consumption, Private and Public by Sector, Canada, CAD $, 2006

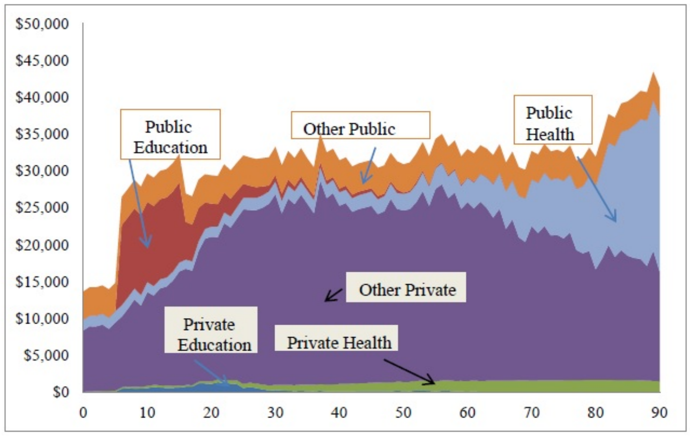


Figure 2.7: Per Capita Consumption, Private and Public by Sector, Germany, 2003

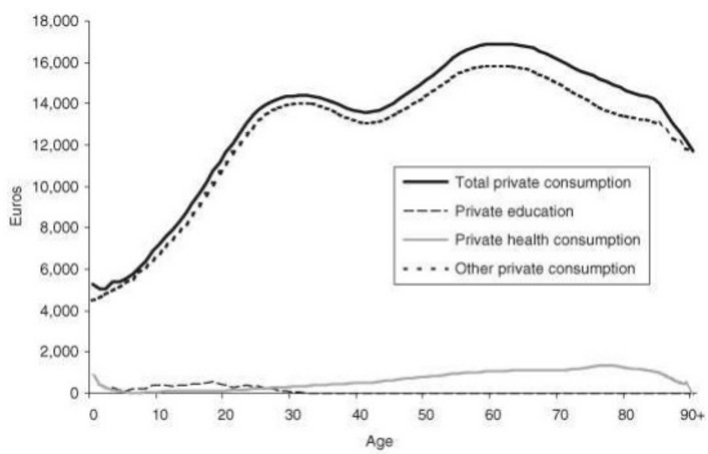


Figure 2.8: Per Capita Private Consumption, Slovenia, 2014

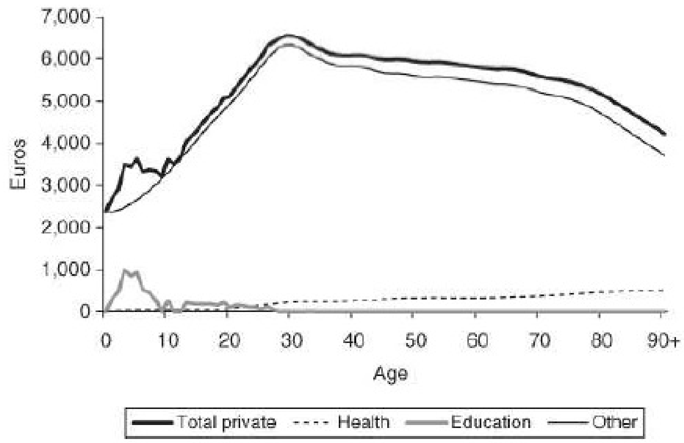


Figure 2.8 shows the consumption profile for Slovenia (UN, 2013) [34]. The private expenditures represent the majority of private consumption expenditures. Consumption is low when individuals are young (until age 20) and also when individuals are retired. The spike during the late 20s represents an increase in one of the private consumption subcategories (expenditure on vehicle and apartment).

2.6 Calibration Procedure

In this section I explain the two calibration procedures introduced in section 2.5.

[34]United Nation, National Transfer Accounts: Population Aging and Generational Economy, 2011.

2.6.1 First Calibration Procedure

For this approach, I use equation (2.12), the Euler equation, to calibrate the consumption profile. The right hand side of equation (2.12) is exogenous while the left hand side is endogenous. In equation (2.12), we have in fact 16 equations and 17 endogenous variables (C_g). To find a unique solution, equation (2.13) is required so that household consumption times population for each age-group is equal to the aggregate consumption found in the data. The calibration results, depicted in section 2.7, show that consumption profile has a smooth upward path, which means that consumption is increasing as individuals age.

$$C_{ag} = (\sum_g N_g C_g), \tag{2.13}$$

where, C_{ag} represents aggregate consumption. Once the consumption is calibrated, I impose the dynamic budget constraint, equation (2.5), to ensure that this equation is satisfied for all generations. To make certain that the bequest from the last generation is distributed, I assume that any bequest left by the last generation is transferred to the first working generation. Therefore, I add a new component to the dynamic budget constraint of the last generation, equation (2.14), in order to satisfy this condition. Thus, I satisfy two dynamic budget constraints: one equation for all generations except the last generation (equation (2.5)) and one equation for the last generation (equation (2.14)).

$$(1 + (1 - \tau^k)r) * A_{gl} + Pen_{gl} + (1 - \tau^l - CTR)Y^l_{gl} = (1 + \tau^c) * C_{gl} + \left(\frac{A_{g5} * N(g5)}{N_{gl}}\right) \tag{2.14}$$

where, A_{gl}, C_{gl} and Y_{gl} respectively represent the asset holding, consumption and labour income for the last generation. A_{g5} and N_{g5} are the asset holding and population for the first working generation. Since the last generation is retired, Y_{gl} is equal to zero. By imposing the dynamic budget constraint I derive individuals' asset holdings.

2.6.2 Second Calibration Procedure

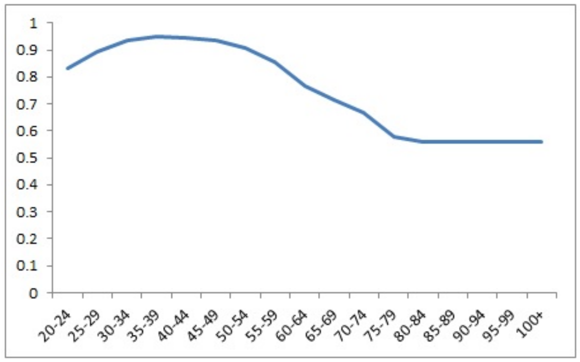
The calibration approach promoted in this chapter is to fully respect the consumption profile derived in the NTA exercises. In other words, consumption profiles are taken as given (exogenous) rather than generated by the calibration procedure.

As mentioned before, the aggregate consumption data are obtained from ONS, and expenses on health and education are excluded from the consumption expenditure in order to derive the private consumption and to be consistent with the theoretical model. I differentiate between the adult and child consumption. Age Specific Fertility Rates (ASFR) from NTA are used to calculate the children's consumption profile. Here it is assumed that a 5-9 year old consumes half as much as a 25-29 year old adult.

The calibration procedure selects values of model parameters in a manner to ensure that the model's reference equilibrium is consistent with given data. The NTA data complement the SAM by offering the life cycle profiles. I assume that the economy is in an initial steady-state equilibrium and calibrate the model to a given base year's social accounting data.

The solution to the consumer intertemporal optimization is still the Euler equation. In calibrating the Euler equation, this time the consumption profile is taken from NTA. NTA data provide the distribution of consumption by age. To make NTA data consistent with

Figure 2.9: Distribution of Consumption from NTA



the SAM, we conduct the following exercise:

$$C_{ag} = (\sum_{g} N_g C_g) \tag{2.15}$$

$$C_{g+1} = \left[\frac{DC_{G+1}}{DC_g} \right] C_g, \tag{2.16}$$

where, C_{ag} is aggregate consumption and DC_g is distribution of consumption by age obtained from NTA. The distribution of DC_g is provided in Figure 2.9. With C_{ag}, N_g and DC_g known, we can build consumption per generation, C_g using equations (2.15) and (2.16).

Once the consumption by generation is derived, we impose the Euler equation (2.12) to calibrate the rate of time preference. Relaxing the assumption of constant rate of time preference adds 18 unknown parameters to the model that needs to be calibrated. Replacing consumption distribution obtained from NTA into equation (2.12) adds an equal number of constraints and therefore we reach a feasible solution. In equation (2.17) the left hand side is given and on the righthand side, only ρ_g is endogenous and all the other parameters are given.

$$\frac{Cg+1}{Cg} = \left[\frac{\big(1+(1-\tau^k)r\big)}{1+\rho_g}\right]^{\frac{1}{\theta}} \tag{2.17}$$

Similar to the first approach, I impose the dynamic budget constraint to derive individual asset holdings.

It is important to distinguish the two approaches. In the traditional approach, the rate of time preference will be constant over the entire life cycle. Hence, a rate of time preference indicates essentially to what extent the individual discounts the future. A rate of time preference larger (smaller) than the rate of interest rate, net of tax, implies that the individual is more impatient (patient) than the market. In such a case, we expect dissavings (savings).

In the new approach, I take consumption profile over the life cycle as given. Since consumption choices can be influenced by a large numbers of factors that interfere in the lifetime of a person (such as survival and reproduction risks, personality traits and physical and mental health capacity), I will interpret the rate of time preference as the parameter capturing these less observable factors. With this new approach, the rate of time preference evolves with age. The dynamic evolution of the rate of time preference will tell us if consumption is increasing or decreasing.

2.7 Calibration Results

In this section I report on the results of the calibration for both approaches explained in section 2.6. I show the consumption profile for the two approaches and depict the calibrated age-variable rate of time preference, described in approach 2. Finally I investigate to see how sensitive the rate of time preference is to the changes in other parameters of the model, such as interest rate and tax rate on capital. I also report the results on households assets.

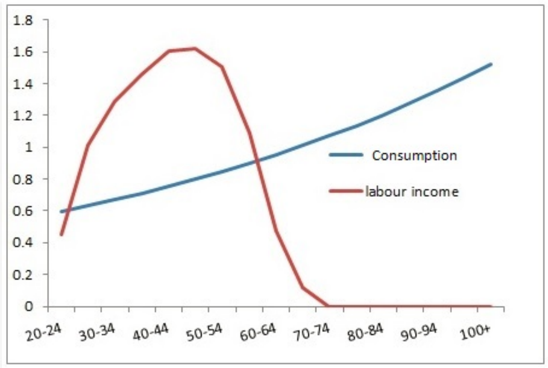
2.7.1 Calibration Results for Approach 1

I use equation (2.12) to calibrate consumption profile endogenously, by assuming that all the parameters on the right hand side of equation (2.12), including rate of time preference, are constant and exogenous. Assuming an interest rate of 0.18, an intertemporal elasticity of substitution of 1.25, a rate of time preference equal to 0.045 and tax rate on capital equal to 0.24, I derive the consumption profile [35]. Labour income curve and consumption path are depicted in Figure 2.10. The calibrated consumption profile has a smooth upward path and is increasing constantly over an individual's life cycle. Individuals have different levels of income during their life cycles. When young, they receive labour income (wages) until age 64, and when they retire receive pension payments to support their consumption. They also receive interest payments on asset holdings during their lives.

When consumers are making consumption-saving choices, they respond to the differences between their own rate of time preference (degree of impatience) and market interest

[35] The interest rate of 18 percent is for a 5-year period. This is equivalent to a 0.036 annual interest rate. The 1.25 value for the intertemporal elasticity of substitution is equivalent to a 0.25 annual value. The same for capital tax, the 0.24 is equivalent to a 0.048 annual value.

Figure 2.10: Approach 1: Consumption and Labour Income Age Profile

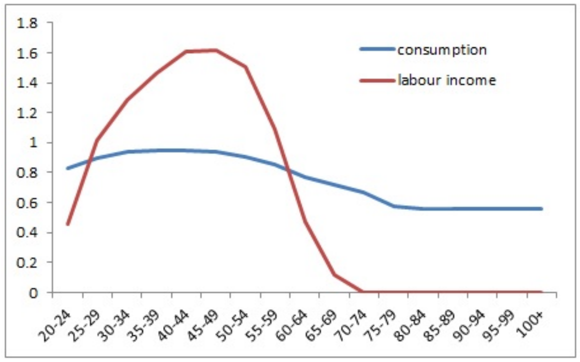


rate. The prices of goods are normalized to one, therefore one plus the interest rate shows the relative price between future and present consumption. In this approach, interest rate is greater than rate of time preference for all age groups. This means that the market rate of trade-off, given by interest rate, is greater than the rate of trade-off (between current consumption and future consumption), given by rate of time preference. Therefore, individuals are willing to postpone current consumption in favour of future consumption. Consequently, the consumption over an individual life cycle has an increasing path as a result of this calibration approach.

2.7.2 Calibration Results for Approach 2

Figure 2.11 shows the bell-shaped consumption profile derived from NTA data. When comparing the bell-shaped consumption profile in Figure 2.11 with the upward consumption profile in Figure 2.10, it is clear that the upward consumption profile does not reflect the consumption of younger and older age groups. It underrates consumption of young age groups while overrating the consumption of older age groups.

Figure 2.11: Approach 2: Consumption and Labour Income Age Profile

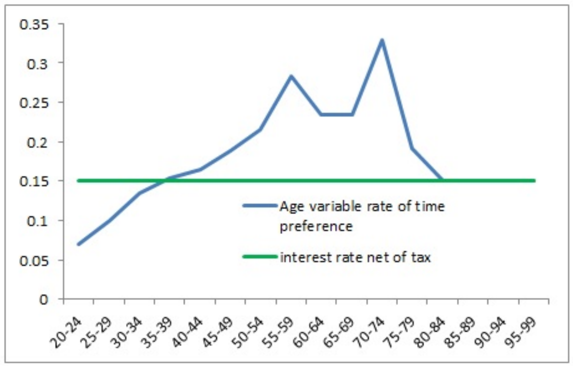


Equation (2.17) is used to derive the age-variable rate of time preference. The left hand side of equation (2.17) is replaced by the consumption profile derived from NTA data. For the right hand side of equation (2.17), I assume all the parameters, except for the rate of time preferences are given. Assuming an interest rate of 0.18, an intertemporal elasticity of substitution of 1.25, a capital tax rate equal to 0.24 and replacing $C(g+1)/C(g)$ by the solution to equations (2.15) and (2.16), I derive the rate of time preferences for each generation [36]. Figure 2.12 depicts the interest rate and the calibrated rate of time preference.

In this approach the rate of time preference varies by age. Three phases for the rate of time preference are observed in Figure 2.12. In the first phase, at young age, individuals' rate of time preference is positive and increasing with age. In the second phase, at middle age, the rate of time preference is unstable but eventually declines. In the third phase, at very old age, the rate of time preference remains constant.

[36] Same as in approach one, the interest rate, the intertemporal elasticity of substitution and capital tax are for a 5-year period.

Figure 2.12: Interest Rate and Calibrated Rate of Time Preference



From the traditional approach, a low rate of time preference, as in the young age, implies a low discount factor of future consumption; however, a lower rate than the interest rate, net of tax, implies positive savings.

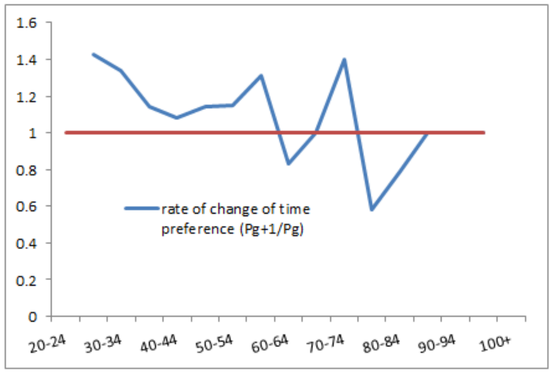
In the new approach, the low but increasing rate of time preference means that consumption opportunities are improving and hence young individuals in this case are somewhat optimistic. This is in part consistent with the work of Rogers [111] since he claims that young people expect rising consumption and hence discount the future differently than old people. A rate of time preference higher than the rate of interest, net of tax, as in the middle age, implies dissavings, according to the traditional approach. In the new approach, however, the declining rate of time preference during middle age implies declining opportunities for consumption and hence more pessimistic views with respect to the future. This is also consistent somewhat with Rogers [111]. In the very old age phase the rate of time preference equals the rate of interest rate, net of tax. In both approaches this means that the consumption path is constant during old age; however, in the traditional approach, this also means no savings and dissavings. In my approach, a low but constant rate of time preference means a lower capacity to enjoy consumption because of physical and health issues. This is consistent with the work of Trostel and Taylor [125]. Moreover, low and constant consumption levels at very old age may imply positive savings, as is the case in my model. This low level of consumption and positive savings at very old age is consistent with the results found by Vettese [128].

In the traditional approach, a positive rate of time preference means that individuals are willing to sacrifice one unit of consumption today for more than one unit of consumption later. An increasing rate of time preference implies that there are more opportunities to consume in future. Thus, the consumption profile should be increasing. In the new approach, in phase one the rate of time preference is increasing but as the difference between rate of time preference and interest rate decreases, and individuals reach to the end of phase one, the increase in consumption slows down.

At age 40, phase two starts and lasts until age 80. The rate of time preference has two sub stages during phase two. At the beginning of phase two, the rate of time preference has an increasing trend until age 59. During this stage, the rate of time preference overpasses interest rate and the difference between these two rates rises until age 59, when the rate of time preference starts to decline. The dynamics of the rate of time preference reflect the consumption profile. Indeed, at the beginning of phase two, the consumption profile is decreasing but the decline in consumption speeds up as the difference between the two rates rises.

During phase three, which starts at age 80 and lasts until the end of the life cycle, the rate of time preferences does not change. Individuals' consumption drops to its lowest level

Figure 2.13: Rate of Change of Time Preference



during this phase, even though interest rate equals the rate of time preference.

In the traditional approach, the rate of time preference fully reflects the discounting behaviour of individuals with respect to the future; however, in the new approach the rate of time preference captures all the other elements such as physical and mental health and personality traits. Therefore, people's will, ability and capacity to enjoy current consumption will affect their intertemporal decision-making.

An age-variable rate of time preference reflects the fact that old individuals have a different discounting behaviour than young. Rogers [111] and Becker and Mulligan [18] contend that discount rate varies with age. They assert that elements such as wealth, mortality and uncertainty affect the degree of discounting. Posner [107] also asserts that individuals have "multiple selves" and they weight their present consumption differently than their future consumption. He argues that the "present self" and the "future self" are separate persons, therefore the discounting behaviour is expected to be different.

Figure 2.13 shows the rate of change of time preference. In the traditional approach, the rate of change of time preference, as shown in Figure 2.13, is equal to one, and reflects a constant rate of time preference over time. In the traditional approach, when the rate of time preference is constant and smaller than the interest rate, net of tax, individuals are more patient and willing to postpone consumption. Therefore the consumption path is constantly increasing. In our approach, the rate of time preference is changing with age.

As is observed in Figure 2.13, until age 59, the rate of change for rate of time preference is greater than one, reflecting an increasing rate of time preference; however, the slope of this rate is negative until age 40 and then it becomes positive. These changes reflect the consumption path. As shown in Figure 2.10, consumption is constantly increasing until age 40, but with a lower rate as it reaches age 40. After age 40, the rate of time preference is still rising but this time with an increasing rate. After age 59, the rate of change of time preference is below one, except for a sudden rise at age 69. A rate of change below one, reflects a decreasing level of consumption. After age 40, consumption starts to decrease but with declining rate. The decline in consumption speeds up after age 59 and continues to decrease until age 79. From age 80 until the end of the life cycle consumption remains constant and individuals reach their lowest level of consumption.

The change in consumption is reflected in the individuals' asset holdings. The asset holding is derived as follow:

$$IHA_g = HA_g * sr_g \tag{2.18}$$

where, HA_g represents asset holding for generation g. The total asset holding for the economy, THA, is equal to:

$$THA = \sum_g N_g * IHA_g \tag{2.19}$$

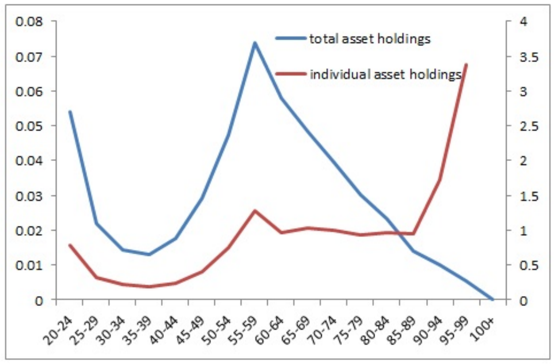
The saving profile from the traditional approach follows the life cycle theory, which means that individuals accumulate assets when they earn and dissave when they are retired. Saving by individuals during their earning years would be offset by the dissaving of the retired. When individuals are very young they have little wealth but at middle age they have more, and they reach the peak of asset holding just before retirement.

The saving profile, derived from the new approach, is different. As shown in Figure 2.14 [37], at early stages of life cycle when individuals are young they dissave but they start saving once they reach middle age and continue saving until the end of the life cycle. In other words, after middle age they never stop saving. When young, individuals dissave until age 40, and then start saving actively until the end of their life cycle. They reach their highest level of saving at the end of their life cycle, when individuals' consumption drops dramatically. This result is consistent with Vettese [128] that provides evidence on reduced consumption when individuals are old and shows that consumption spending decreases after retirement. The reduced consumption level at the end of the life cycle leads to accumulation of assets among older generations.

In the new approach, the asset holding increases smoothly and climaxes at old age, due to reduced consumption. Furthermore, in the new approach the asset holding at the economy level does not show the same trend as the individual asset holding. As presented in Figure 2.14, the total asset holding is calculated using equation (2.19). Figure 2.14 shows that the individuals' assets increase at old age, but taking into consideration the lower population size and survival rate for the older age groups, the total asset holding declines by age. In the new approach, by transferring the assets left from the last generation to the

[37] The data for this graph are presented in the appendix for Chapter 2.

Figure 2.14: Asset Holdings

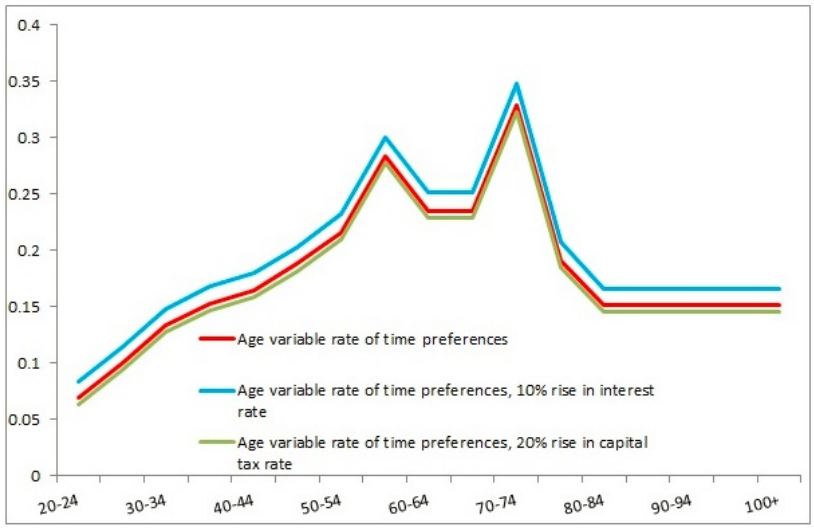


first working generation, the saving profile is flattened. It is noted that the asset holding for the economy is consistent with life cycle theory for the entire population.

Figure 2.15 shows the changes in calibrated rate of time preference in response to changes in interest rate and tax rate on capital. As Figure 2.15 shows, an exogenous increase in interest rate shifts the calibrated rate of time preference upward, while an increase in the tax rate of capital moves the rate of time preference downward, but the magnitude of the shift is very small. The exogenous shocks shift the rate of time preference slightly but they do not affect the shape of the rate of time preference. This fact signifies the importance of all the elements affecting the consumption behaviour that are reflected in the rate of time preferences. Notwithstanding a relatively large change in interest rate and the tax rate, the rate of time preference does not change as much. There are other elements that shape the consumption profile derived from NTA data and these elements are not affected by exogenous changes.

The results confirm the significance of other elements such as personality traits and physical and mental abilities in determination of consumption profile. Individuals at dif-

Figure 2.15: Calibrated Rate of Time Preference Response to Exogenous Shocks



ferent ages are endowed with distinct personality traits and physical and mental capabilities, and their expectations of future outcome vary, therefore their discounting behaviours differ.

2.7.3 Sensitivity Analysis

In this chapter, I assume that individuals have a finite time horizon and also no risk is associated with labour income; however, one can validly argue that in reality individuals may not foresee the future perfectly and therefore are not fully aware of risks associated with income and consumption. If so, the changes in consumption profile could also result from individuals' relative risk aversion, or in other words, the inverse of intertemporal elasticity of substitution. Therefore, individuals' risk tolerance can change with age. To perform a sensitivity analysis, I calibrate the model assuming that the intertemporal elasticity of substitution is changing by age, capturing the notion that individuals' decision-making is affected by risk. I use the Euler equation (equation (2.17)), but this time I assume that rate of time preference is constant but the intertemporal elasticity of substitution changes with age. Table 2.3 shows the calibrated intertemporal elasticity of substitution.

Table 2.3: Intertemporal Elasticity of Substitution

Age	$Intertemporal Elasticity of Substitution$
20-24	0.8
25-29	1.32
30-34	4
35-39	25
40-44	4.76
45-49	1.85
50-54	1.05
55-59	0.54
60-64	0.83
65-69	0.83
70-74	0.4
75-80	1.67

As the results show, the intertemporal elasticity of substitution is first increasing with age and then declines. Assuming interest rate and rate of time preference are constant, the dynamics of intertemporal elasticity of substitution is determined by the rate of change in consumption profile. The dynamics between the interest rate and the rate of time preference and the growth rate of consumption controls the intertemporal elasticity of substitution.

Using the consumption profile from the NTA and assuming that interest rate is larger than the rate of time preference, intertemporal elasticity of substitution is calibrated using

Euler Equation (equation (2.17)). When people are young and have an increasing growth rate of consumption, the intertemporal elasticity of substitution is increasing. Once the growth rate of consumption starts to decrease the intertemporal elasticity of substitutions declines.

One caveat is that we cannot separate the impact of changes in consumption profile on the rate of time preferences and intertemporal elasticity of substitution. Further study and experiments are required to disentangle these two effects. Some studies discuss that using a different type of preferences allow separating these two effects (Epstein et al. [43], Epstein and Zein [42]).

Consumption profile is capturing many elements that affect individuals decision-making. Due to data limitation, one of the elements that this chapter does not discuss is the existence of uncertainty in the individual life cycle. It is possible that the calibrated rate of time preference is capturing some aspect of individual decision-making while encountering uncertainty. Incorporating uncertainty into the model may result in a different pattern for the rate of time preference.

2.8 Conclusion

This chapter re-examines the importance of intertemporal rate of time preference by developing a calibration procedure that allows for age-variable rate of time preference. I construct an OLG model in which "the instantaneous utility function is expected to vary with age". In this new calibration procedure the rate of time preference captures the consumption behaviour observed in data. Since Samuelson's discounted utility model, many studies have taken rate of time preference as given or exogenous with little discussion of what determines the rate of time preference. The process of anticipating future outcomes is not merely one image formation. Numerous factors such as mental and physical health and personality traits affect intertemporal decision-making and future outcomes. These factors are affected by age and therefore carry through individuals' life cycles. By developing a new approach, I calibrate and show how the rate of time preference is affected by age. Using NTA consumption data in the calibration procedure shows that rate of time preference is different for young versus old individuals.

The rate of time preference exhibits three phases. During phase one, individuals' rate of time preference is positive and increasing with age. Increasing rate of time preference means that consumption opportunities are improving and individuals are optimistic. In phase two, the rate of time preference is unstable but it eventually declines. A declining rate of time preference may reflect pessimistic views with respect to future opportunities for consumption. During phase three, at a very old age, the rate of time preference does not change significantly and consumption profile is constant during this phase.

One can validly claim that the rate of time preference calibrated in this chapter does not represent discounting behavior; instead it reflects many other factors such as physical and mental health and personality traits that are captured by the consumption profile during the life cycle. People's capacity to enjoy consumption affects their decision-making. The results on saving profiles show that at early stages of the life cycle when individuals are young, they dissave but they start saving when they reach middle age and continue saving until the end of the life cycle. The individuals' asset holdings increase smoothly and climax at old age, due to reduced consumption, but the total asset holding declines at very old age. Exogenous changes in interest rate and tax rate on capital, shift the calibrated rate of time preference but the magnitude of the shift is very small and the shape of the rate of time preference does not change.

It is appropriate to raise certain caveats here. The calibrated rate of time preference is based on the consumption age profile derived by NTA. This consumption profile only includes private consumption and does not incorporate public consumption (such as health and education). It is possible that incorporating public consumption changes the consumption profile and consequently the calibrated rate of time preference; but one can argue that since public goods are determined by collective bargaining and not by private decision, they should not be included in the determination of the rate of time preference.

2.9 Appendix for Chapter 2

Table 2.4: Asset Holdings for Approach 2

Age	N_g	SR^*HA_g	THA_g
20-24	0.0692	0.77760	0.0538
25-29	0.0683	0.3220	0.0220
30-34	0.0625	0.2275	0.0142
35-39	0.0675	0.1931	0.0130
40-44	0.0744	0.2357	0.0175
45-49	0.0733	0.3969	0.0291
50-54	0.0639	0.7423	0.0475
55-59	0.0575	1.2861	0.0739
60-64	0.0604	0.9604	0.0581
65-69	0.0471	1.0247	0.0482
70-74	0.0396	0.9982	0.0396
75-79	0.0322	0.9369	0.0301
80-84	0.0240	0.9680	0.0232
85-89	0.0150	0.9413	0.0141
90-94	0.0059	1.7293	0.0101
95-99	0.0016	3.3806	0.0053

Chapter 3

Computing Demographic Change Simulation under Constant and Age-variable Rate of Time Preference

3.1 Introduction

Economic consequences of an aging population on current and future generations increase the importance of its long-run impacts. Various studies have been carried out to simulate long-run impact of aging in an OLG model (Auerbach and Kotlikoff [6], Fehr, Jokisch and Kotlikoff ([46], [48], [47])). These studies have simulated the impact of demographic changes assuming younger and older individual have same rate of time preference. This chapter simulates the economic impact and welfare effects of population aging using a computable general equilibrium overlapping generations (CGE-OLG) and assumes that the rate of time preference varies by age.

The model presented in this chapter demonstrates a broad array of demographic, economic and policy factors such as tax spending, age-specific fertility and earnings and bequests that can be simultaneously incorporated into the analysis. Including these elements is a significant programming challenge but it is a unique way to measure the impacts of aging and welfare effects of different generations and the efficiency of the economy.

In this chapter I use the calibration results produced in Chapter 2 and simulate the model until a new steady state is reached, using an iterative solution technique. The OLG model used in this chapter is the descendant of Auerbach and Kotlikoff [6].

Indeed, various studies have been carried out with the original model of Auerbach and Kotlikoff [6]. Auerbach and Kotlikoff [6] develop an OLG model based on Samuelson's [118] discounted utility model and simulate the impact of aging in an OLG model with 55 overlapping generations. These studies focus on many aspect of an OLG model and strive to explain the implication of aging on policy reforms. These models differ depending on the assumptions they make in developing the model. Here I briefly review some of these studies.

Auerbach and Kotlikoff [7] and Auerbach et al. [8] further extend the original OLG model of Auerbach and Kotlikoff [6] and quantify the impact of aging population on the economy of four countries: U.S., Japan, Sweden and Germany. They simulate the impact of aging population within the structure of a worldwide model. Kotlikoff et al. [86] present a model with more detailed population dynamics for the U.S. De Nardi, Imrohoroglu and Sargent [98] incorporate desired and undesired bequests in the model.

Broer and Lassila [30] incorporate uncertainty and more detailed demographic scenario into the model and investigate the impact of aging population on the economy. They introduce age-specific survival probabilities, uncertain life span and perfect annuity markets into the model and assume that the remaining assets of those who have died are distributed to the surviving members of the respective cohort. Fehr et al. [46] simulate a dynamic OLG model for the world economy. They divide the world economy into three regions: the U.S., the EU and Japan and incorporate immigration, age-specific fertility, life span uncertainty, bequests and intra-cohort heterogeneity.

More recent studies explore the effects of aging within a CGE-OLG model. The French INGENUE Team's [123] OLG model divides the world into six large regions and shows that aging has a small negative impact on interest rate. Boersch-Supan, et al. [31] develop a three-region OLG model and examine the international consequences of alternative pension reforms. They find small changes in capital-labour ratios in response to pension reforms.

Fehr, Jokisch and Kotlikoff ([46], [48], [47]) develop a multi-regional OLG model for the U.S., Europe and Asian countries that are interlinked through the capital market. They also incorporate immigration and different skill levels into the model.

Recent studies have incorporated labour leisure choice into the model and optimize the retirement age, when individuals quit working and start to receive pension benefits. For example, Fehr, Strekeby and Thogersen [50] simulate an OLG model for the Norwegian economy, in which the pension system "distinguishes five classes of income within a generation". Fehr, Kallweit and Kindermann [49] also simulate an OLG model with endogenous retirement age and evaluate the impact of alternative policy options and explore various policy reforms for improving the sustainability of the German pension system. They show that increasing the retirement age is an important policy instrument that leads to reducing contribution rate.

Some other studies simulate OLG models that incorporate idiosyncratic economic risk such as income risk. Imrohoroglu et al. [77] were the first to develop an OLG model with stochastic employment opportunities and examine the welfare effects of alternative social security arrangements. Fehr et al. [45] develop and simulate an OLG model that incorporates disability risk, a pension system with old-age pension and health insurance, and endogenous labour supply.

There are studies that incorporate the social security system into the model. Huang et al. [75] simulate an OLG model assessing two social security systems: unfunded and mandatory state-run funded systems. De Nardi et al. [98] extend the Huang et al. [75] model and incorporate U.S. demographic trends and variable labour supply into the model. Nishiyama and Smetters [99] simulate an OLG model for the U.S. economy with 50 percent privatization of the social security system. By introducing a Lump-Sum Redistribution Authority (LSRA) that "compensates initial agents and distributes the accumulated asset or debt to newborn and future agents, they show that there are efficiency gains from privatization per household. Fehr and Haberman [44] reach a similar conclusion for the German social security system. Fehr et al. [45] extend the model and assume that the social security consists of an old age pension and disability insurance.

Kudrna and Woodland [88] simulate a dynamic OLG model for the Austrian economy, in which households make decisions on both consumption and labour supply and also determine the age they retire.

All the above studies have assumed that younger and older individuals have the same rate of time preference. In this chapter I address the importance of time preference in quantifying the long-run effect of population aging, by assuming that rate of time preference varies by age. I show that the conventional methodology, specification with constant rate of time preference, does not accurately measure the effects on GDP per capita and on welfare.

In this chapter, I specify and simulate an OLG model and incorporate age-specific fertility rates and age-variable rate of time preference. I investigate the impact of aging on labour, capital, wage rate, interest rate and welfare. In particular, I demonstrate that the age-variable rate of time preference version of the model generates very different implications for welfare. The rest of the chapter is as follow. Section 3.2 presents the model. Section 3.3 provides the simulation results and section 3.4 concludes.

3.2 Model Structure

The OLG model described in this section is the descendant of the model developed by [7] and [8] with exogenous labour supply.

3.2.1 Firm Behaviour

The Production Function

The production technology is represented by a Cobb-Douglas production function. It is assumed that capital is homogeneous and depreciating. Labour differs in its efficiency because individuals of different ages provide different amounts of labour input.

$$Q_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha} \tag{3.1}$$

where, Q_t represents the output at time t, K_t is the capital stock, L_t, labour, A denotes the scaling constant and α measures the share of capital in production. Firms maximize profit renting capital and hiring labour from households.

Labour Demand

I assume firms employ labour without any costs. As mentioned above, the competitive behaviour assumption leads to marginal product of labour equal to wage, w_t. Given the

Cobb-Douglas production function, we have:

$$w_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{3.2}$$

Capital Demand

I also assume that capital is adjusted costlessly and the firm sets the marginal product of capital equal to the interest rate, r_t:

$$r_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{3.3}$$

The above two equations give the wage rate and interest rate as a function of capital stock and labour.

Capital Accumulation

Accumulation of the capital stock is given in equation (4). I assume that physical capital and financial capital are undifferentiated. Capital depreciates at a constant rate.

$$Kstock_{t+1} = I_t + (1+\delta)Ktock_t \tag{3.4}$$

$$1 + r_t = R_t + (1 - \delta_t) \tag{3.5}$$

where, I_t, represents investment and $Kstock$ and δ are respectively the capital stock and the depreciation rate of capital. R_t is the gross rate of return on capital.

3.2.2 Household Behaviour

In each period, households decide about consumption, labour supply and saving, satisfying the intertemporal budgets constraint, which ensures that the present value of lifetime earnings is equal to the present value of expenditure on consumption. Government policies affect intertemporal decision-making through the household budget constraint as well as through the household's first order condition (Euler equation). The remaining lifetime varies across living generations in a specific year, therefore tax reforms have different impacts on the budget constraints of old versus young and the future generations

There are 21 overlapping generations that live side by side. Each generation lives 21 periods of 5 years. Individuals in an age-cohort are identical and they have identical tastes. An individual is born at the age zero, enters the labour market at age 24, retires at age 65 and dies at age 100+. Individuals are assumed to be forward looking with perfect foresight. Individuals make lifetime decisions on consumption based on life cycle behaviour. The last generation leaves a bequest for the first working generation. Following Barro and Friedman [15], when households encounter uncertain lifetimes, they might leave "unintentional bequests". Assuming "perfect annuity market", the unintentional bequests are evenly distributed across the living generations [1].

[1] The unintentional bequest approach was first developed by Yaari [130] for a continuous time model. Application of this theory to an overlapping generations model was implemented by Borsch-Supan et al. [28].

Preferences

The household preferences are represented by a CES utility function. Households maximize the inter-temporal utility function with respect to the budget constraint to drive the consumption demand. At each age g during the life cycle an individual may die, and therefore each generation has a specific survival rate. The intertemporal preferences of an individual are given by:

$$Max \quad U_t = \frac{1}{1-\theta} \sum_{k=0}^{k=21} \left[\prod_{m=5}^{m=k} \left(\frac{1}{1+\rho_{g+m}} \right) q_{t+k,g+k} (C_{t+k,g+k})^{1-\theta} \right] \tag{3.6}$$

where C is aggregate consumption and ρ is the age-variable rate of time preference that denotes the degree to which the households prefer consumption during their lifetime. A large ρ indicates that a household will spend more of its resources early in lifetime and will save less. $q_{t+k,g+k}$ represents probability of survival and denotes as follow:

$$q_{t+k,g+k} = \prod_{n=0}^{n=k} SR_{t+n,g+n} \tag{3.7}$$

where, $SR_{t+n,g+n}$ indicates an exogenous survival rate between two consecutive generations and time periods. This form of utility function imposes some constraints on preferences. The intertemporal elasticity of substitution, which expresses the degree of consumption substitutability across time, is fixed and equal to one. Also, at any point in time the individual decisions depend only on future consumption. Past levels of consumption affect household current wealth.

Budget Constraint

At each period, the household earns income from labour and capital and decides how much to spend on consumption. It will save the remaining and add it to its lifetime stock of assets. Individuals have to pay social security contribution and taxes on labour income, consumption and interest income. The household dynamic budget constraint is as follow:

$$A_{t+k+1,g+k+1} = \frac{1}{SR_{t+k,g+k}}[(1-\tau_i^l - CTR)Y^l_{i,t+k,g+k} + (1+(1-\tau^K)r_{t+k})A_{t+k,g+k} + Pen_{t+k,g+k} - (1-\tau^c)C_{t+k,g+k}] \quad (3.8)$$

Where:

$$Y_g^l = w.EP_g.LS_g \quad (3.9)$$

$$EP_g = LFPR_g * hc_g \quad (3.10)$$

where, Y^l is labour income, r_t is the real interest rate, τ^l, τ^K and τ^c are respectively tax rates on labour income, capital income and consumption. A represents individual asset holdings, where CTR is the contribution to the pension system and Pen represents pension benefits, which is zero for the working-age cohorts and will differ from zero once the individual chooses to retire. LS_g is the exogenous supply of physical units of labour and EP is individuals' earning profile that depends on age-specific productivity. $LFPR$ is the labour force participation rate, hc represents age-specific productivity profile.

The present value of lifetime consumption should be equal to the present value of lifetime earnings. Lifentime budget constraint is derived by solving the dynamic budget constraint for k equal 0 to 17 and is as follow:

$$\sum_{k=0}^{k=18}\left[\prod_{m}\left(\frac{SR_{t+n,g+n}}{(1+(1-\tau^{k})r_{t+k})}\right)(1+\tau^{c}-CTR)C_{t+k,g+k}\right]=\sum_{k=0}^{k=18}\left[\prod_{m}\left(\frac{SR_{t+n,g+}}{(1+(1-\tau^{k})r_{t+k})}\right)Y^{*}_{t+k,g+k}\right] \tag{3.11}$$

$$Y^{*}_{t+k,g+k}=Y^{l}_{t+k,g+k}+Pen_{t+k,g+k} \tag{3.12}$$

where, Y^* represents lifetime earing, Pen is zero for the working age cohorts and it will differ from zero once the individual chooses to retire. When individuals are young they have no assets and they do not receive a pension, so their saving and pension benefit is zero. When old, individuals have no labour income and stop accumulating assets; therefore, saving and labour income is equal to zero. Taking derivatives from the utility function, with respect to the lifetime budget constraint, using lagrangian function, we have the first order conditions:

$$\prod_{m=0}^{m=k+1}\left(\frac{1}{1+\rho_{g+m}}\right)q_{t+k+1,g+k+1}\prod^{k+1}(C_{t+k+1,g+k+1})^{-\theta}=\lambda\left(\frac{(1+\tau^{c})}{\prod^{k+1}(1+(1-\tau^{k})r_{t+k+1})}\right) \tag{3.13}$$

$$\prod_{m=0}^{m=k}\left(\frac{1}{1+\rho_{g+m}}\right)q_{t+k,g+k}\prod^{k}(C_{t+k,g+k})^{\theta}=\lambda\left(\frac{(1+\tau^{c})}{\prod^{k}(1+(1-\tau^{k})r_{t+k})}\right) \tag{3.14}$$

where, λ represents the shadow price of the lifetime budget constraint and is the utility value of an additional unit of income.

Combining the first order conditions yields the following equation:

$$\frac{1}{1+\rho_{g+k+1}}\left(\frac{C_{t+k+1,g+k+1}}{C_{t+k,g+k}}\right)^{-\theta} = \frac{1}{(1+(1-\tau^k)r_{t+1}))} \tag{3.15}$$

$$\frac{C_{t+k+1,g+k+1}}{C_{t+k,g+k}} = \left(\frac{1+(1-\tau^k)r_{t+1}}{1+\rho_{g+k+1}}\right)^{\frac{1}{\theta}} \tag{3.16}$$

3.2.3 Pension

I assume that the pension plan is a PAYG (Pay as you go) system. Therefore, the total pension benefits should be equal to the total contributions. In each period, the pension system pays benefits and collects payroll contributions from labour income. Pension benefits in a specific year are computed from the individuals' average labour income over their lifetime earning. Pension benefits are thus equal to:

$$Pen_{t,gr} = PenR_{t,g}\left((\frac{1}{11})\sum_{k=5}^{13} Y^L_{t+k,g+k}\right) \tag{3.17}$$

where, gr represents the retired generations and Pen denotes public pension replacement. The total pension benefits should be equal to the total contributions. Eleven working generations, individuals aged between 20 and 64, contribute to the plan and eight retired generations, individuals aged 65 and over, receive pension benefits that are proportional to their lifetime income:

$$\sum_{gw} N_{t,gw} Pen_{t,gw} = CTR \sum_{gr} N_{t,gr} Y_{t,gr} \tag{3.18}$$

$$gw = \{g+k; k = 5, ..., 1\}, gr = \{g+k; k = 14, ...21\}$$

As the above equations indicate, the size of the population of the retirees and the working generations is crucial in sustainability of the pension plan. With the aging of the population and withdrawal from prime working-age years into retirement years, the labour force shrinks. The reduced number of the working population will contribute to provide for the larger retiree population.

3.2.4 Government

Government earns revenues from taxation and spends its income on government consumption, education, health and interest payments on borrowing. I assume that social security has a separate system. The social security has its own tax instrument, the payroll tax, and is self-financing.

Government Budget Constraint

I assume government spending per capita remains constant over time; however, as the population dynamics change the government issues bonds to keep the budget balanced. In each period the government issues new debt, $(1+r)B_{t-1} - B_t$, and collects taxes from households and firms as well as interest payments on its debt, in order to finance its spending, which is fixed per capita. The government budget constraint is as follow:

$$\sum_g N_{t,g}[\tau_t^l(Y_{t,g}^l) + \tau^c(C_{t,g}) + \tau^k(r_t A_{t,g})] = G_t + G_t^H + G_t^E \tag{3.19}$$

where, G_t represents the government consumption, and G_H and G_E are government spending on health and education, respectively. Different taxes are assumed to be exogenous. The left hand side of the equation is government revenue and the right hand side represents government spending.

3.2.5 Equilibrium Condition

Aggregate demand is composed of consumption demand, investment demands and government consumption demand. Equation (20) shows the market equilibrium condition for the commodity market. The total aggregate demand is equal to total output.

$$Y_t = (\sum_g N_{t,g} C_{t,g}) + I_t + G_t \tag{3.20}$$

Equations (3.21) and (3.22) provide equilibrium conditions for the labour market and the capital market. The total capital accumulated must be equal to units of capital demanded.

$$L_t = \sum_g N_{t,g} LS_g \tag{3.21}$$

$$K_t = Kstock_t \tag{3.22}$$

$$\sum_g \sum_g N_{t,g} A_{t,g} = Kstock_1 \tag{3.23}$$

where, $Kstock_t$ is the exogenous supply of capital stock.

3.3 Simulation

In this section I simulate the model to analyse the impact of aging. I present in order the macroeconomic impact and the welfare effect.

3.3.1 Baseline Simulation

The methodology for simulating the baseline year is explained in this section. The baseline simulation consists of an initial steady state that is based upon the data in 2010. Because of the changes in the population dynamics, the baseline solution moves from the initial steady state along a transition path to a new steady state. The new steady state is reached at year 2280. The baseline solution to the model (comprising the initial steady state, the final steady state and the transition path between these steady states) is compared with the solution to the demographic transition simulation.

Baseline Parametrization

Except for demographic projections, all the variables and parameters are either calibrated in Chapter 2 or obtained from the literature, for the baseline year 2010. Table 3.1 shows the values of the model parameters.

Population Projection

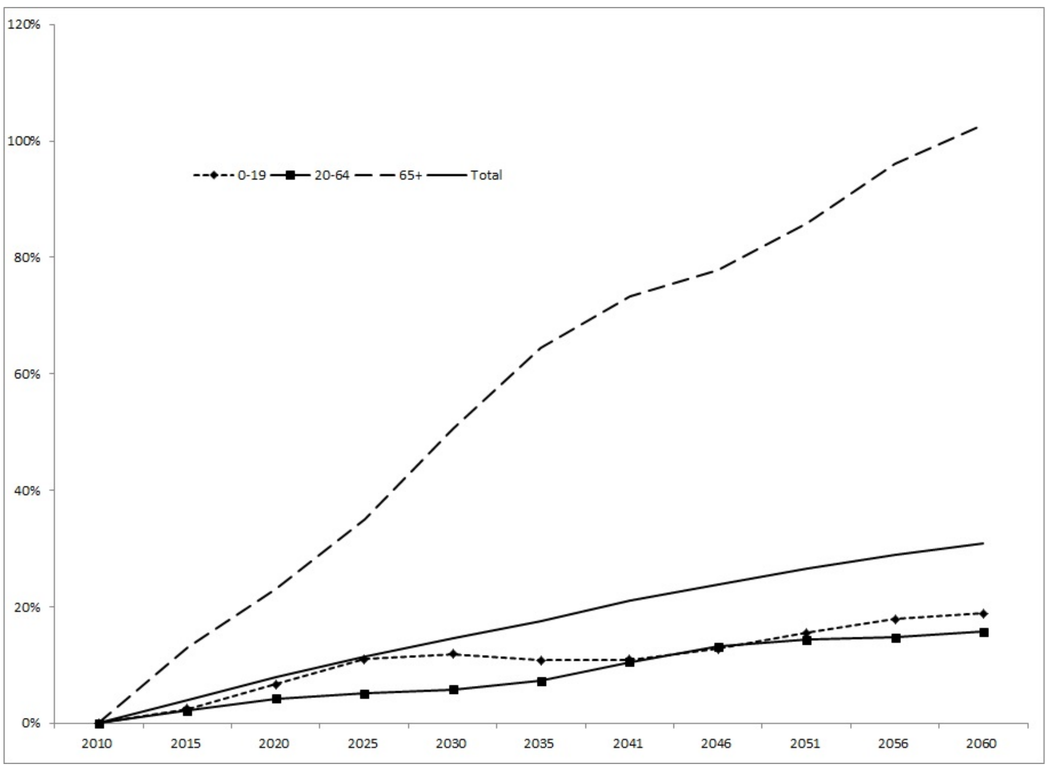
In this model, one period covers five years. I assume agents are born at age zero but start their "economically relevant life" at age 20 and live until 100+. Population projection data

Table 3.1: Value of Model Parameters

Description	Value	Source
Demographics		
Population growth rate	0.0145	Calculated from data
Survival rates	ONS(2010)	Calculated from data
Net immigration rate	ONS(2010)	Calculated from data
Preferences and Pension		
Inter-temporal elasticity of substitution	1.25	Literature
Rate of time preference	0.085, age-variable	Literature, Calibrated
Pension contribution rate	0.087461	Calculated from data
Technology		
Capital share in production	0.32	Calibrated
Technology parameter	1.33	Calibrated
Depreciation rate	0.16	Calibrated
Government		
Consumption tax rate	0.158421	Calculated from data
Capital tax rate	0.163817	Calculated from data
Income tax rate	0.160362	Calculated from data

are obtained from the United Kingdom's Office of National Statistics (ONS). To account for a reasonable demographic transition, I estimate the immigration rate and survival rate for each age group using the projection data. The average fertility rate is 0.96. The net migration rate is zero for individuals younger than 9 years old; however, it increases gradually for individuals aged between 10 and 54. For individuals older than 55, the net migration does not change, except that for individuals between 75 and 84 it declines. The survival rate declines steadily after age 60 toward zero at age 100 years old. Figure 3.1 reports the share of cohorts in total population from 2010 to 2060.

Figure 3.1: Cohort Share in Total Population



Baseline Simulation Results

The demographic transition is incorporated into the model and its impact is simulated from 2010 to a new long-run equilibrium. I present the baseline simulation for both calibrated models presented in Chapter 2. The main features of the transition path for macroeconomic variables for both models are presented in Figures 3.3 and 3.4. These figures demonstrate the transition path towards a new equilibrium for main variables such as: GDP per capita, wages, interest rate and capital-labour ratio. They provide the level values of the macroeconomic variables in the initial steady state and in the selected years of the transition path. The macroeconomic variables of the two models, along the transition path, show similar patterns in response to demographic transitions.

As a result of demographic transitions, the GDP per capita declines. The drop in GDP per capita is larger in the model with the age-variable rate of time preference. The percentage change in the decline of GDP per capita in the model with age-variable rate of time preference is, on average, 0.65 percent larger over the period of 2010 to 2080. The reason that the GDP per capita is larger in the model with constant rate of time preference is the larger capital to labour ratio. Aging raises the stock of capital compared to the supply of labour in both models and the increase in the ratio is larger in the model with constant rate of time preference. The larger capital to labour ratio is attributed to the higher level of saving in the model with constant rate of time preference.

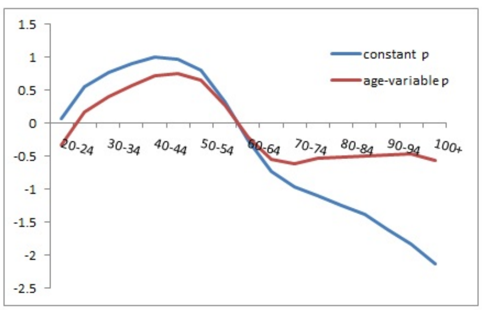
As discussed in Chapter 2, the life cycle consumption profile is very different in the two models. In the model with constant rate of time preference, the value of rate time preference is taken from the literature and the consumption profile is calibrated to match the aggregate value. The consumption profile resulting from this exercise is upward sloping. In contrast, in the model with age-variable rate of time preference, the life cycle consumption profile is taken from a reliable data source (NTA) and the rate of time preference is calibrated accordingly to satisfy the Euler equation and the aggregate value.

Figure 3.2 shows the dynamics of saving levels across individuals through the transition path for three specific years: 2010, 2030 and 2050. As shown in Figure 3.2, individual saving level is larger in the model with constant rate of time preference. As a result of aging of the population, the consumption drops in both models; however, the drop is larger in the model with age-variable rate of time preference as lower saving rates push down capital formation further, which subsequently reduces earnings by a large amount. Thus, the level of capital in the model with constant rate of time preference is higher, which leads to a higher level of capital to labour ratio and consequently higher level of GDP per capita.

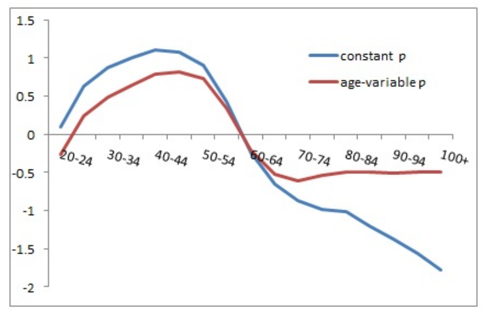
Aging raises the stock of capital compared to the supply of labour, and therefore real wages increase. One possible explanation for such capital deepening is the fact that the elderly are the primary owners and suppliers of capital, while the young are the main suppliers of labour. All else equal, more elderly relative to young means a greater supply of capital relative to labour. This fact explains the reduction in interest rate.

The hump-shaped consumption profile in the model with age-variable rate of time preference reflects more realistic consumption patterns compared to the traditional increasing consumption profile. Therefore, using the increasing consumption profile undermines the decline in the GDP per capita as a result of aging of the population. The simulation results from the model with constant rate of time preference, which uses the increasing consump-

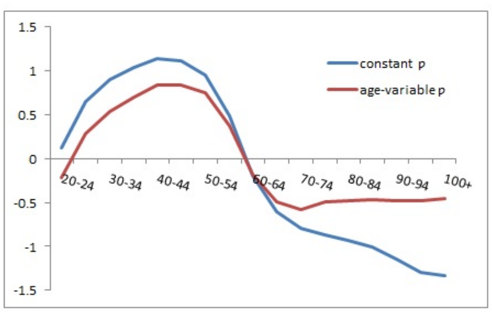
Figure 3.2: Saving Levels for Both Models for Selected Years



(a) 2010

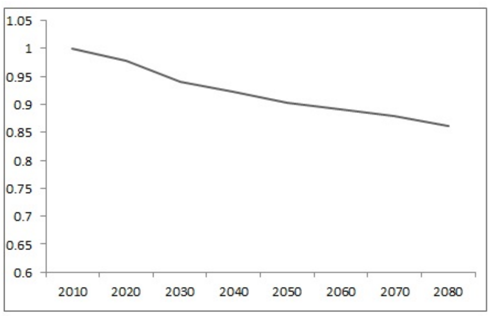


(b) 2030

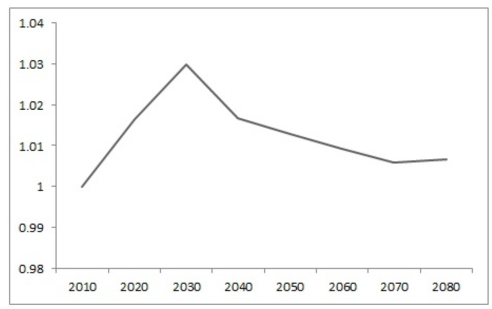


(c) 2050

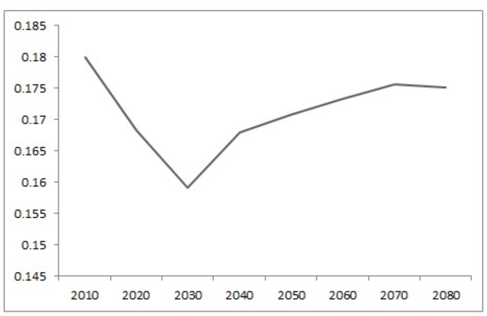
Figure 3.3: Baseline Simulation for Model with Age-variable ρ



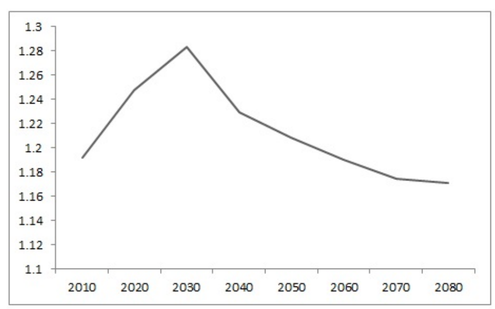
(a) GDP per capita



(b) Wage

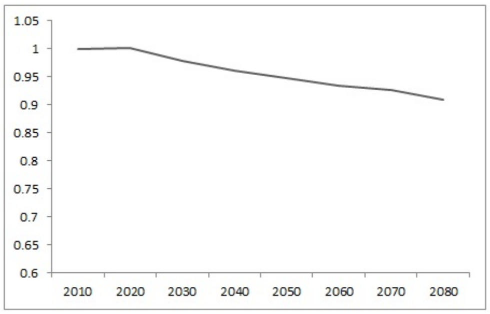


(c) Interest rate

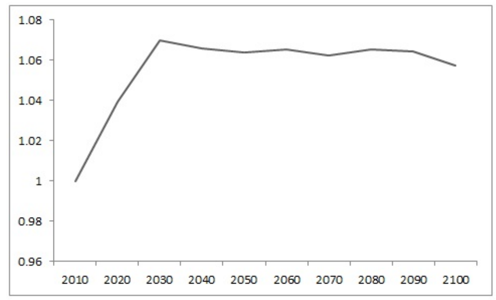


(d) Capital-labour ratio

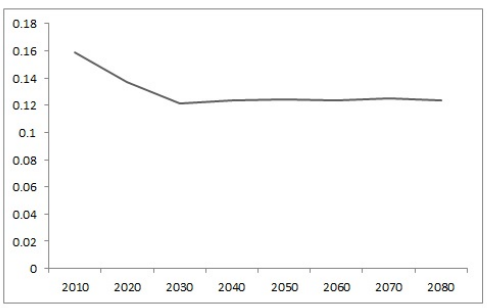
Figure 3.4: Baseline Simulation for Model with Constant ρ



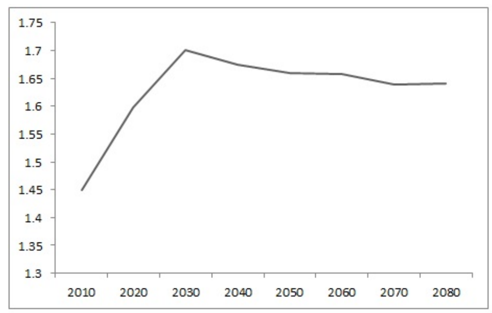
(a) GDP per capita



(b) Wage



(c) Interest rate



(d) Capital-labour ratio

tion profile, underestimate the decline in GDP per capita by 0.65 percent on average, over 2010 to 2080.

3.3.2 Welfare Effect

In this section, I report the welfare results first for each cohort, then from a social welfare function.

Cohort Welfare

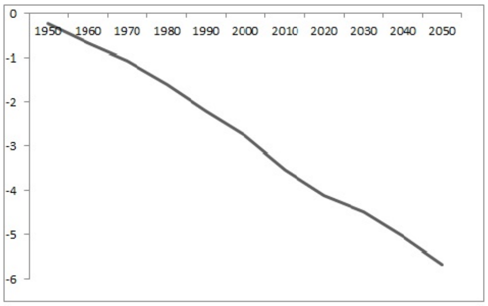
The welfare effects of demographic transition are calculated for each cohort. I assume the economy is in a steady state and then compute the new equilibrium for the economy when a demographic change occurs. I then compare the welfare effects for each cohort in the initial steady state with the welfare obtained under the new steady state.

The burden of aging of the population is mostly on the future cohorts. As the share of the elderly in the total population increases over the next decades, smaller cohorts of working-age people will support larger cohorts of elderly.

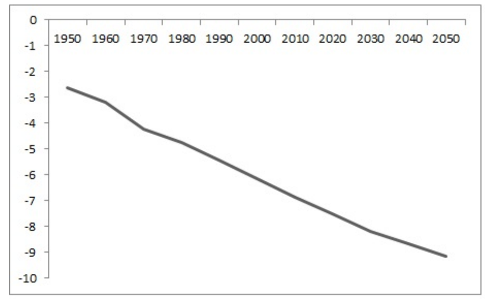
Figure 3.5 provides the percentage change in welfare for each cohort after the demographic transition for both models. The horizontal axis in Figure 3.5 identifies the year the cohort becomes active in the labour market. For instance, in Figure 3.5 (a), welfare of the cohort in 1970 declines by 1.6 percent while the drop in welfare for cohort 2020 is 4.1 percent. The results are reported from 1950 in order to incorporate the welfare effects for older cohorts still living in 2010. The 80 year old cohort alive in year 2010 became active members of the labour market in 1950. As shown in Figure 3.5, as a result of aging of the populations, each cohort's welfare decreases in both models. The decline in welfare in the model with the constant rate of time preference is larger than in the model with age-variable rate of time preference, despite the fact that GDP per capital is higher in the model with constant rate of time preference.

To disentangle the role of rate of time preference and consumption profile and to determine the impact of rate of time preference (discount rate) in the cohorts' welfare measurement, I do the following experiment for both models. First, I assume there is no discounting

Figure 3.5: Percentage Change in Cohort Welfare due to Demographic Transition



(a) Model with age-variable ρ



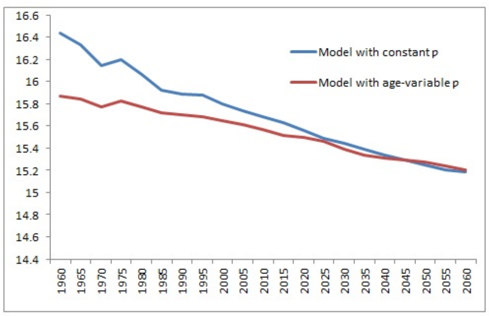
(b) Model with constant ρ

in the calculation of welfare by putting the rate of time preference equal to zero, therefore the discount rate equals to one. Then, I recalculate the welfare effects. This experiment allows me to isolate the impact of rate of time preference in analysing the cohorts' welfare.

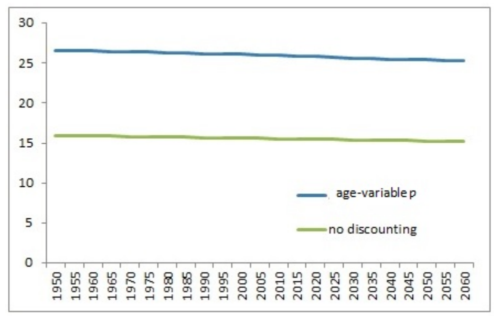
As shown in Figure 3.6, in both models the welfare declines as a result of population aging, but the level of each cohort's welfare is larger in the model with constant rate of time preference. When there is no discounting, the changes in welfare are all attributed to the consumption profiles. The two models have two different consumption profile, therefore the welfare levels varies. The differences in the consumption pattern explain the differences in welfare level, therefore welfare change depends on how fast consumption is increasing or falling over time.

In the model with constant rate of time preference, the consumption profile is increasing over the life cycle. In other words, the consumption of a succeeding year is always larger than the current year consumption during the lifetime and for all cohorts. Cohorts have higher levels of consumption at old age. In the model with age-variable rate of time preference, the dynamics of consumption profile are quite different over a lifetime. For

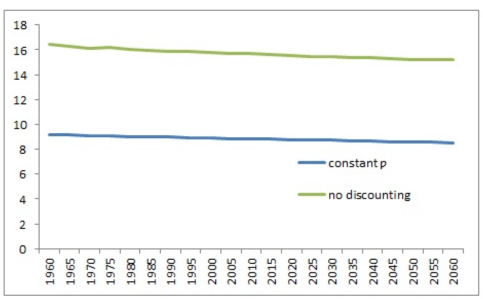
Figure 3.6: Cohort Welfare Levels due to Demographic Transition



(a) Both models: no discounting



(b) Model with age-variable ρ



(c) Model with constant ρ

each cohort, consumption increases at a young age then declines until reaching a low level after retirement. Figure 3.6 shows that when there is no discounting the welfare levels are higher for most cohorts with constant rate of time preference. This is consistent with the fact that GDP per capita is larger in the model with constant rate of time preference.

Once discounting is incorporated into the models, as shown in Figures 3.6(b) and 3.6(c), the cohorts' welfare levels increase in the model with age-variable rate of time preference but decline in the model with constant rate of time preference. An age-variable rate of time preference modifies the relationship between welfare and GDP per capita more substantially. Indeed, in the model with age-variable rate of time preference, welfare declines by less despite the fact that GDP per capita declines by more, compared to the model with constant rate of time preference.

The impact of age-variable rate of time preference on the relationship between GDP per capita and welfare can have important policy implications, as I can confirm using a social welfare function.

Social Welfare Function

The impact of demographic transition on individual welfare can be analysed from the social planner perspective. I assume that a social planner is interested in the present discounted sum of living generations consumptions.

$$W_t = \sum_g (\beta)^g N_{t,g} Con_{t,g}^{1-\theta} \tag{3.24}$$

where, W_t is social welfare evaluated at time t. This social welfare function indicates to the social planner the evolution of the degree of satisfaction of its population. Since the social planner wants to take into consideration all the living individuals, the discount rate is composed of the rate of time preference weighted by the size of the population of the corresponding age group, $N_{t,g}$. For the model with constant rate of time preference the discount rate is the same for all generations, but in the model with age-variable rate of time preference the discount rate varies by age group.

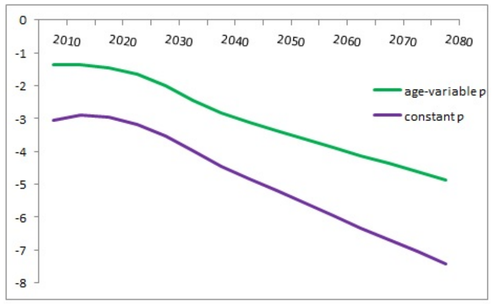
As shown in Figure 3.7, in both models the social welfare declines as a results of aging of the population. The decline in social welfare is much less dramatic in the model with age-variable rate of time preference than the model with constant rate of time preference. Consequently, policy measures that a social planner might want to implement to respond to a decline in social welfare would likely be different in the two models.

As noted before, the age-variable rate of time preference affects the relationship between the GDP per capita and welfare effect. Although the GDP per capita is larger in the model with constant rate of time preference, the social welfare is two times smaller than the model with age-variable rate of time preference.

Also, the decline in social welfare is larger than the decline in cohort welfare. As a result of aging of the population, cohort welfare and social welfare decrease in both models; however, if the social planner allows the rate of time preference to vary by age, the decline in social welfare will be smaller compared to when the rate of time preference is constant.

In the model with the age-variable rate of time preference, at any point in time the

Figure 3.7: Percentage Change in Social Welfare due to Demographic Transition



(a) Both models

social planner accommodates different discounting behaviour by allowing the rate of time preference to change by age. Therefore, when preferences change, the measurement of the welfare effects changes. In the model with constant rate of time preference, all individuals in the economy, no matter their age, are discounted equally. In both models the welfare declines as a result of aging of the population; however, when the social planner allows various discounting behaviour depending on individuals' ages, the economy augments higher welfare than when the discounting is constant for all generations, and the impact of aging population is less dramatic.

3.4 Conclusion

The simulation modeling of aging populations and welfare changes have been undertaken and discussed in this chapter. The model permits the simulation of aging and policy change so that the impacts on various macroeconomic variables as well as the cohorts' welfare can be obtained. The two approaches simulated in this chapter allow us to compare the results. The outcomes from simulations are dependent on the choice of rate of time preference as well as the structure of the model. The impact of aging on GDP per capita is larger in the model with age-variable rate of time preference, due to different saving profiles which stem from different consumption profiles. Also, in both models welfare declines as a result of population aging, but the cohort welfare is larger in the model with a constant rate of time preference. The differences in the consumption pattern explain the differences in welfare level, therefore welfare change depends on how fast consumption is increasing or falling over time. The choice of a realistic rate of time preference, which allows its variability by age, affects the cohorts' welfare noticeably.

It is appropriate to raise certain caveats here. The model has a very detailed household sector but has a relatively simple production structure. Also, the model assumes one representative agent by cohort, so our model cannot report distributional effects within each age group. There is also just one consumer good in the model. This model could be extended by assuming a multi-sector production which consequently leads to multiple consumption goods in the economy.

The results in this chapter provide a new perspective on the impact of aging population on households and macroeconomic variables. The choice of a realistic rate of time preference, which allows its variability by age, affects the results. This chapter demonstrates that the conventional methodology, specification with constant rate of time preference, underestimates the effects on GDP per capita but overestimates the negative effects on welfare.

3.5 Appendix for Chapter 3

Table 3.2: Survival Rates

	0 - 4	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85-89	90-94	95-99	100+
Survival rates	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	G13	G14	G15	G16	G17	G18	G19	G20	G21
1930	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1935	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1940	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1945	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1950	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1955	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1960	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1965	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1970	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1975	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1980	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1985	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1990	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
1995	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
2000	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
2005	0.893205	1.034979	1.096674	1.101794	0.98604	0.915668	1.079754	1.102413	0.985762	0.871882	0.898831	1.051773	0.778954	0.841756	0.811166	0.745663	0.626109	0.390527	0.269763	0.128343	0
2010	0.996041	0.999737	1.004399	1.004927	0.992638	0.992884	0.994444	0.992699	0.991373	0.986858	0.978848	0.967718	0.951469	0.924304	0.878207	0.799398	0.671174	0.51025	0.335512	0.186583	0
2015	0.996726	0.999772	1.004433	1.005117	0.992924	0.993742	0.99482	0.99288	0.991963	0.988064	0.980984	0.970409	0.954864	0.931333	0.89137	0.82331	0.711533	0.55268	0.3666	0.22485	0
2020	0.996801	0.999794	1.003936	1.005477	0.99275	0.993911	0.995059	0.993165	0.992496	0.989011	0.982503	0.97297	0.958298	0.935896	0.899662	0.839646	0.740683	0.598799	0.410803	0.254089	0
2025	0.996792	0.99982	1.003658	1.004868	0.992755	0.993865	0.995238	0.993482	0.992884	0.989876	0.983701	0.974854	0.961391	0.940315	0.905873	0.849399	0.760501	0.631834	0.459284	0.299999	0
2030	0.99676	0.999843	1.003694	1.004527	0.993427	0.993947	0.995414	0.993766	0.993282	0.990503	0.984845	0.976395	0.96372	0.94414	0.911758	0.858021	0.771784	0.65408	0.492923	0.351789	0
2035	1.00[illegible]787	0.998148	0.998321	1.000916	1.000684	1.011018	1.011104	0.980815	0.977911	1.000076	1.005908	1.001866	0.935707	0.940087	0.932728	0.895915	0.794598	0.670072	0.54893	0.414079	0
2040	0.9[illegible]695	0.999883	1.00398	1.004815	0.993947	0.99481	0.995976	0.994417	0.994164	0.991631	0.986651	0.979387	0.968458	0.950604	0.921898	0.874693	0.797709	0.686235	0.533568	0.438228	0
2045	0.997068	0.999894	1.003959	1.005	0.993931	0.994903	0.996208	0.99477	0.994542	0.992113	0.98735	0.980489	0.970074	0.953887	0.925997	0.880948	0.807842	0.701956	0.55648	0.468351	0
2050	0.997125	0.999906	1.003825	1.00499	0.993919	0.99496	0.996378	0.995072	0.994853	0.992617	0.98805	0.981545	0.97165	0.956243	0.930897	0.886964	0.816981	0.715794	0.576393	0.521912	0
2055	0.998602	0.999373	0.998125	0.996461	0.989569	0.996392	1.000944	0.998217	0.988965	0.977272	0.973614	0.995282	0.988109	0.948608	0.913529	0.869595	0.847962	0.727429	0.576707	0.514316	0
2060	0.997154	0.998477	1.004223	1.010233	1.002024	0.999313	0.995352	0.991181	0.99256	0.999672	1.005389	0.999092	0.962346	0.946239	0.947626	0.920282	0.860719	0.721408	0.611526	0.596338	0
2065	0.997192	0.999944	1.003723	1.004672	0.994496	0.99536	0.996847	0.995729	0.99573	0.993819	0.990064	0.98466	0.976449	0.963109	0.941104	0.90474	0.844821	0.756484	0.626816	0.634315	0
2070	0.997262	0.999955	1.003764	1.004742	0.994603	0.995544	0.997015	0.995948	0.995996	0.99417	0.990608	0.985537	0.977776	0.96549	0.944477	0.909649	0.852374	0.767945	0.647562	0.696432	0
2075	0.99733	0.999964	1.003754	1.004811	0.994641	0.99566	0.99717	0.996165	0.996243	0.994507	0.991124	0.986347	0.979042	0.967428	0.948025	0.9147	0.859693	0.778772	0.663237	0.721478	0
2080	0.995647	0.996357	0.999393	1.002077	0.994648	0.996914	0.996792	0.991733	0.989464	0.990667	0.992636	0.991001	0.982456	0.962296	0.934283	0.903358	0.875097	0.795473	0.663275	0.708938	0
2085	0.9974	0.999977	1.003645	1.004757	0.99474	0.99578	0.997395	0.996513	0.996642	0.995075	0.992008	0.987721	0.981164	0.970665	0.953162	0.923632	0.873919	0.79809	0.68899	0.777331	0
2090	0.999168	0.998425	0.999298	1.001604	0.996476	1.0011	1.002016	0.995658	0.988676	0.986955	0.994812	1.000149	0.993899	0.971028	0.94687	0.901159	0.870758	0.857526	0.717947	0.827708	0
2095	0.994796	0.998253	1.005169	1.008927	0.997842	0.994479	0.992554	0.99238	0.998052	1.003851	1.001337	0.986789	0.97178	0.962534	0.959554	0.940417	0.913367	0.828969	0.679813	0.896464	0
2100	0.99859	1.002659	1.005349	1.003119	0.991149	0.993427	0.999192	1.002008	1.001667	0.994853	0.985161	0.981386	0.986641	0.987152	0.97198	0.934038	0.884725	0.805219	0.726999	1.001393	0
2105	0.99852	0.998909	1.00095	1.002954	0.996571	0.999881	1.000439	0.995671	0.992408	0.991695	0.994685	0.998511	0.993655	0.974666	0.951599	0.928225	0.900291	0.845355	0.768562	0.980701	0
2110	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2115	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2120	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2125	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2130	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2135	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2140	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2145	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0
2150	0.99143	0.989535	1.004484	1.069406	1.065715	1.020419	0.995377	0.983717	0.982795	0.993394	0.996406	0.987822	0.96472	0.942644	0.931636	0.928446	0.906266	0.835593	0.723695	0.870175	0

Table 3.3: Simulation Results for Both Models

Model with age-variable rate of time	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085	2090	2095	2100	2105	2110
Output per person	1	0.993	0.979	0.958	0.94	0.929	0.922	0.914	0.904	0.895	0.891	0.886	0.88	0.871	0.86	0.853	0.846	0.839	0.833	0.824	0.815
Total Consumption	0.596	0.621	0.636	0.644	0.651	0.656	0.665	0.669	0.673	0.676	0.677	0.684	0.69	0.694	0.697	0.696	0.697	0.699	0.702	0.706	0.71
Interest rate	0.18	0.174	0.168	0.162	0.159	0.162	0.168	0.17	0.171	0.172	0.173	0.176	0.176	0.175	0.175	0.176	0.178	0.179	0.181	0.182	0.185
Wage	1	1.008	1.016	1.026	1.03	1.025	1.017	1.013	1.013	1.011	1.009	1.006	1.006	1.007	1.007	1.005	1.003	1.001	0.999	0.997	0.993
Capital labour ratio	1.192	1.22	1.248	1.275	1.283	1.262	1.229	1.214	1.208	1.199	1.19	1.177	1.175	1.175	1.17	1.163	1.152	1.142	1.131	1.121	1.106
Model with constant rate of time preference	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085	2090	2095	2100	2105	2110
Output per person	1	1.005	1.001	0.99	0.977	0.969	0.962	0.956	0.948	0.939	0.935	0.93	0.925	0.918	0.908	0.9	0.892	0.885	0.877	0.867	0.856
Total Consumption	0.486	0.515	0.539	0.557	0.57	0.579	0.59	0.596	0.6	0.604	0.604	0.611	0.616	0.621	0.626	0.625	0.628	0.63	0.634	0.639	0.645
Interest rate	0.159	0.148	0.137	0.127	0.121	0.121	0.123	0.125	0.124	0.124	0.124	0.125	0.125	0.124	0.124	0.123	0.124	0.126	0.128	0.129	0.132
Wage	1	1.019	1.039	1.058	1.07	1.071	1.066	1.063	1.064	1.065	1.065	1.063	1.062	1.064	1.065	1.066	1.064	1.061	1.058	1.054	1.048
Capital labour ratio	1.449	1.523	1.598	1.663	1.7	1.7	1.675	1.661	1.66	1.66	1.657	1.643	1.639	1.64	1.642	1.638	1.627	1.61	1.591	1.571	1.544

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