FREE VIBRATION ANALYSIS OF RECTANGULAR CANTILEVER PLATES WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

by
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Thesis presented to the School of Graduate Studies
as partial fulfillment of the requirements
for the degree of M.A.Sc. in
Mechanical Engineering

UNIVERSITY OF OTTAWA
OTTAWA, CANADA, 1982

ABSTRACT

The objective of the present thesis is to provide an analytical "exact" solution to the free vibration problem of the cantilever plate with symmetrically distributed point supports using the method of superposition.

Starting with a brief review of the underlying theory in Chapter 1, Chapter 2 looks into the free vibration of the cantilever plate establishing the general procedure of the superposition method. Chapters 3 and 4 deal specifically with the application of this method to the free vibration problem of the cantilever plate with edge and lateral point supports respectively. Eigenvalues obtained are tabulated throughout this work. Mode shapes are illustrated in Appendix 2, while Appendix 3 shows a complete list of computer programs used to generate both eigenvalues and mode shapes. The following is a review of the historical progress of plate analysis.

HISTORICAL

According to Todhunter, and Pearson(1), the development of structural mechanics began with the investigation of static problems. However the first analytical and experimental studies on plates were almost exclusively devoted to the problem of free vibration(2). In 1766, Euler was the first to formulate a mathematical approach to the membrane theory of plates, and to use the analogy of two systems of stretched strings perpendicular to each other to solve the problem of rectangular and circular elastic membranes free vibrations(3). In 1789, by introducing the gridwork analogy, Euler's student, J. Bernoulli extended Euler's analogy to plates(4). However, Bernoulli found only resemblance between theory and experiments but no general agreement. That is to be expected since the torsional resistance of plates was not included in the differential equation of motion.

As to the various modes of free vibrations they were first discovered by the german physicist Chlandi(1). On a horizontal plate, he used evenly distributed powder which, after induction of vibration, accumulated along the nodal lines.

Later, using the calculus of variations, Sophie Germain, a French mathematician, obtained a differential equation for the vibration of plates. Although, the Parisian Academy presented a prize award to Germain for her work in 1816, the work done by warping of the middle surface was not represented in her expression for the strain energy. However, one of Germain's judges, Lagrange, corrected the strain energy expression by adding the missing term, and he is believed to be the first to use the correct differential equation of the free vibration of plates.

Although, the correct differential equation of the free vibration of plates was found in Lagrange's notes in 1813, its derivation is attributed to Navier(1785-1836). A great engineer and bridge designer, Navier is known to be the real originator of the modern theory of elasticity. The solution of various plate problems is one of his numerous scientific activities. He derived the correct differential equation of rectangular plates with flexural resistance, and using the trigonometric series introduced by Fourier in the same decade, he provided an exact method for the solution of certain boundary value problems. This method yields mathematically exact solutions for the plate with Navier type boundary conditions, and that is the simply supported plate. Then in 1829, Poisson extended Navier's solution to the lateral vibration problem of thick circular plates.

Later, Kirchhoff came along(1824-1887), to go a step further in introducing the extended plate theory which takes into account the combined bending and stretching(5). He found that in analyzing large deflections of plates, the non linear terms could no longer be neglected. He also introduced the method of virtual displacement and the equation of frequency of plates.

Clebsch, in his translation to Kirchhoff's book(6), provided numerous valuable comments by Saint-Venant, among which, the extension of Kirchhoff's differential equation of thin plates, is considered to be the most important. In this extended equation the combined action of bending and streching were correctly considered.

Russian scientists made a significant contribution to naval architecture by being the first to replace the ancient trade traditions by solid mathematical theories. However, due to the language barrier, western scientists could not make use of these achievements until Timoshenko came along. It is through his work that western scientists started to direct their attention toward the Russian research in the field of theory of elasticity. Among these Russian scientists are Krylov(1863-1945)(7), and his student Boobnov(8)(9)(10). As for Timoshenko's numerous important books, and contributions we list(11)(12)(13).

Among the numerous scientists that have worked on plate related problems, we name Foppl, who in his book on engineering mechanics(14) treated the nonlinear theory of plates, and Karman(15), who developed the final form of the differential equation of the large deflection theory.

For the period between the two world wars, we list only Bleich, Federhofer, Wagner, and Lévy. Then from the Soviet Union, Oniashvili and Gontkevitsh investigated the free and forced vibration of plates. Then came Johansen with the yield-line analysis(16) to introduce the first important deviation from the classic theory of elasticity in the solution of plate problems.

It is only recently that high speed electronic computers influenced the static and dynamic analysis of plates. Although, Hrennikoff had already developed an equivalent gridwork system for the static analysis of complex plate problems in 1941(17), his technique could not be fully utilized due to the lack of high speed computers. Then with the introduction of high speed computers(1950's) Turner Clough, Martin, and Topp in 1956, introduced the finite element method, which permits the numerical solution of complex plate and shell problems in an economical way(18). However, the use of this method anticipates the availability of high speed computers with considerable storage capacity. One should also name here Argyris and Zienkiewicz for their numerous contributions in this field. Another numerical method for the static and dynamic analysis of plates of arbitrary shape subjected to arbitrary loads is based on improved finite difference techniques and was developed by Stüssi and Collatz.

Although, considerable amounts of time and effort were put into the solution of the problem of free vibration of plates by numerous scientists and researchers for the past several centuries, up to the year 1969, it had been designers' experience that there is no single reference to which they can turn for a comprehensive list of availa.le data and results on the subject of plate free vibration. It is thanks to Leissa of the Ohio State University, Columbus, Ohio, that such a reference came into existence. Writing of the manuscript began in the summer of 1965, and its publication came about in 1969(19). In his book Leissa provides a comprehensive list of available results for the frequencies and mode shapes of free vibration of plates. He also lists an exhaustive list of references on related research and publications. Although, Leissa in his book claims a "reasonable completeness of results published through the end of the year 1965," he acknowledges that some significant publications were not included in his book for the reason of unavailability.

With the publication of Leissa's book it became even clearer that most of the available solutions of plate vibration problems do not satisfy exactly the governing differential equation, the prescribed boundary conditions, or even both. Furthermore, none of the references that Leissa lists in his book treats the subject of rectangular plate free vibration in a clear and orderly manner.

Most recently(1982) it is to D.J. Gorman's credit that such an orderly book came into existence(20). In this book, Gorman not only provides ~~the~~ graduate students and the design engineers with a reference "to which they can turn for a clear and orderly exposition of the general subject of rectangular plate free vibration" as he sees it, but he added a new dimension to an old concept, the superposition concept(21), and by so doing, he provided an exact analytic method for the solution of the plate free vibration problem. The method of superposition ~~itself~~ is not new, in fact it has been in use for solving static plate problems for decades. However, its introduction to the dynamic plate problems is credited to Gorman. In his book and numerous other publications, Gorman treated the problem of rectangular plate free vibration with numerous different types of boundary conditions and point supports. One case that Gorman did not approach in his book, or any of his publications is the case of the rectangular cantilever plate with edge or lateral point supports. Furthermore, to the author's knowledge, this particular problem has not been published or approached by any other researcher. Therefore, this thesis will concern itself with the study of this particular problem, providing a solution using two different methods. First, the method of superposition will be used as it was introduced by Gorman, to provide an exact analytical solution to the problem of the free vibration of the rectangular cantilever plate with or without point supports. Then the method of improved finite element will be used to generate eigenvalues for comparison purposes.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the excellent cooperation and help given by members of the Mechanical Engineering Department in particular and all of the University of Ottawa in general.

It is the author's wishes to pay particular thanks to Dr. D.J. Gorman, who was the advisor for this thesis and who provided countless thoughtful suggestions and ideas throughout this work that enabled the end product to be reached. The interest, patience, encouragement, and dedication offered by Dr. Gorman are highly valued.

Thanks are also due to the staff of the University Computing Centre for their understanding and cooperation.

Thank You All

The Author

LIST OF SYMBOLS

Symbol	Explanation
a	Plate lateral dimension in the x direction
b	Plate lateral dimension in the y direction
D	Plate flexural rigidity = $Eh^3/\{12(1-\nu^2)\}$
E	Young's modulus of the plate material
f	Plate vibration frequency in Hertz
h	Plate thickness
t	Time in seconds
u	distance between the concentrated force and the η axis divided by side length a
u^*	$= 1 - u$
v	distance between the concentrated force and the ξ axis divided by the side length b
v^*	$= 1 - v$
ζ	Distance between the concentrated edge force and the plate central axis divided by the plate edge dimension
$W(\xi,\eta,t)$	Plate lateral displacement
$W(\xi,\eta)$	Amplitude of the plate lateral displacement
x ; y	Plate spatial coordinates
λ^2	$= \omega a^2\sqrt{\rho/D}$, plate eigenvalue
ρ	Mass of the plate per unit area
ω	Circular frequency of plate vibration
ϕ	$= b/a$, plate aspect ratio
ϕ_I	$= a/b$, inverse of the plate aspect ratio

Symbol	Explanation
ν	Poisson's ratio for the plate material
ν^*	$= 2 - \nu$
ξ	$= x/a$, dimensionless plate spatial coordinate
η	$= y/b$, dimensionless plate spatial coordinate
β	Plate solution parameter, as defined in the thesis
γ	Plate solution parameter, as defined in the thesis

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Chapter 1

THE THEORY OF PLATES FREE VIBRATION

The objective of this thesis is to provide an analytical "exact" solution to the free vibration problem of the cantilever plate with edge and lateral point supports. However, it seems appropriate to start with a brief discussion on the underlying theory of the free vibration of plates in general and the method of superposition as introduced by Gorman(20) in particular. Although the author appreciates the importance of providing the reader with the general theory and the procedure that are followed throughout this thesis, and form the basis of the final results, it is left up to the reader to consult Gorman's book for any detailed derivations or explanation.

The Differential Equation

The correct differential equation governing the pure bending of plates subjected to lateral static loading is presented with detailed derivation by Timoshenko(11), and is written as

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{q(x,y)}{D} \qquad (1.1)$$

where q(x,y) is the applied static loading.

The governing differential equation for the free vibration of rectangular plates is obtained by replacing the lateral force q(x,y) of Equation (1.1) by the inertial force and by introducing the time variable parameter t(20).

$$\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2\frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y,t)}{\partial y^4} + \frac{\rho}{D}\frac{\partial^2 W(x,y,t)}{\partial t^2} = 0 \quad \ldots(1.2)$$

by replacing W(x,y,t) by its equivalent W(x,y)T(t), it can be shown that $T(t) = A\sin(\omega t+\alpha)$. And Equation (1.2) can be written as

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} - \frac{\omega^2 \rho}{D} W(x,y) = 0 \quad \ldots\ldots\ldots\ldots(1.3)$$

in its dimensionless form Equation (1.3) can be written as

$$\frac{\partial^4 W(\xi,\eta)}{\partial \xi^4} + 2\frac{\partial^4 W(\xi,\eta)}{\phi^2 \partial \xi^2 \partial \eta^2} + \frac{\partial^4 W(\xi,\eta)}{\phi^4 \partial \eta^4} - \lambda^4 W(\xi,\eta) = 0 \quad \ldots\ldots\ldots\ldots(1.4)$$

or

$$\frac{\partial^4 W(\xi,\eta)}{\partial \eta^4} + 2\phi^2 \frac{\partial^4 W(\xi,\eta)}{\partial \eta^2 \partial \xi^2} + \phi^4 \frac{\partial^4 W(\xi,\eta)}{\partial \xi^4} - \phi^4 \lambda^4 W(\xi,\eta) = 0 \quad \ldots(1.5)$$

The Lévy Type Solution

In 1820, Navier presented a paper to the French Academy of Sciences on the solution of bending of simply supported rectangular plates by double trigonometric series. This solution is sometimes called the forced solution. At the turn of the century, in 1899, a solution by single Fourier series was introduced by Levy(22). This powerful method obtains the solution of Equation (1.5) in the form(20)

$$W(\xi,\eta) = \lim_{\kappa\to\infty} \sum_{m=1}^{\kappa} Y_m(\eta)\sin(m\pi\xi) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.6)$$

by substituting this solution in Equation (1.5) and rearranging we have

$$\frac{d^4Y_m(\eta)}{d\eta^4} - 2\phi^2(m\pi)^2\frac{d^2Y_m(\eta)}{d\eta^2} + \phi^4\{(m\pi)^4 - \lambda^4\}Y_m(\eta) = 0 \quad \ldots\ldots\ldots(1.7)$$

the solution of Equation (1.7) is well known and it depends on whether $\lambda^2 - (m\pi)^2$ is negative or positive.

If $\lambda^2 > (m\pi)^2$ then

$$Y_m(\eta) = A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sin\gamma_m\eta + D_m\cos\gamma_m\eta \quad \ldots\ldots\ldots(1.8)$$

If $\lambda^2 < (m\pi)^2$ then

$$Y_m(\eta) = A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sinh\gamma_m\eta + D_m\cosh\gamma_m\eta \quad \ldots\ldots(1.9)$$

where $\beta_m = \phi\sqrt{\lambda^2+(m\pi)^2}$

and $\gamma_m = \phi\sqrt{\lambda^2-(m\pi)^2}$ or $\phi\sqrt{(m\pi)^2-\lambda^2}$ $\ldots\ldots\ldots(1.10)$

whichever is real. And where A_m, B_m, C_m, and D_m are constants to be determined by means of prescribed boundary conditions[1]

Limitations

Although, Lévy's method, which uses a single trigonometric series is more general than Navier's solution, it does not have an entirely general character since in its original form it can only be applied to rectangular plate vibration problems where at least two opposite edges have simple supports. However, it has been shown that by making use of the superposition method(20)(21), this Lévy-type solution is readily employed to solve not only rectangular plate vibration problems of all possible combinations of classical boundary conditions, but also to analyze numerous rectangular plates with nonclassical type boundary conditions. Next, a word about the classical boundary conditions and their mathematical formulation.

Classical Boundary Conditions

Whenever classical boundary conditions are mentioned, our attention is directed toward the three types of boundary conditions that have been studied so thoroughly in the classical literature, and they are, the clamped, the simply supported, and the

1.- unless stated otherwise, the coordinate system used will allways be that of Figure 1.1 throughout this work free edge conditions. Although, we are dealing with the problem of free vibration, the formulation of these boundary conditions is identical to that found in Timoshenko's work(11) on static plate analysis. This formulation will be presented here in both conventional and dimensionless coordinate systems. Reference is made to figures 1.1 and 1.2.

Clamped Edges:

Conventional coordinates

$$W(x,y) = \frac{\partial W(x,y)}{\partial x} = 0 \qquad \text{.................}(1.11)$$

Dimensionless coordinates

$$W(\xi,\eta) = \frac{W(\xi,\eta)}{\partial \xi} = 0 \qquad \text{.................}(1.12)$$

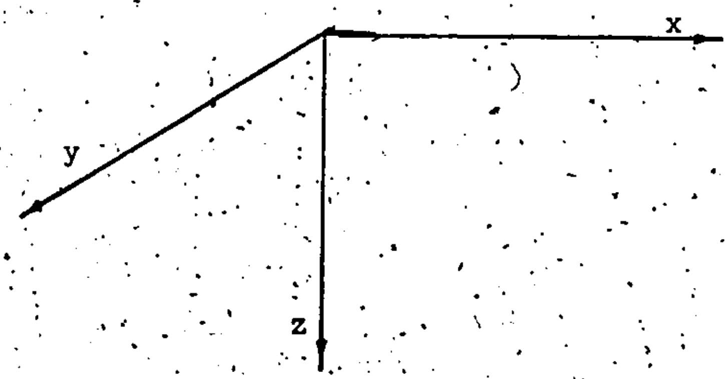


Figure 1.1. Conventional rectangular coordinates system

Simply Supported Edges:

Conventional coordinates

$$W(x,y) = \frac{\partial^2 W(x,y)}{\partial x^2} = 0 \qquad \text{.................(1.13)}$$

Dimensionless coordinates

$$W(\xi,\eta) = \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} = 0 \qquad \text{.................(1.14)}$$

Free Edges:

Conventional coordinates

$$\frac{\partial^2 W(x,y)}{\partial x^2} + \nu\frac{\partial^2 W(x,y)}{\partial y^2} = 0 \;\Big|_{x=a} \qquad \text{.................(1.15)}$$

$$\frac{\partial^3 W(x,y)}{\partial x^3} + \nu^*\frac{\partial^3 W(x,y)}{\partial x \partial y^2} = 0 \;\Big|_{x=a}$$

$$\frac{\partial^2 W(x,y)}{\partial y^2} + \nu\frac{\partial^2 W(x,y)}{\partial x^2} = 0 \;\Big|_{y=b}$$

$$\frac{\partial^3 W(x,y)}{\partial y^3} + \nu^*\frac{\partial^3 W(x,y)}{\partial y \partial x^2} = 0 \;\Big|_{y=b}$$

Dimensionless coordinates

$$\frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} = 0 \;\Big|_{\xi=1} \qquad \text{.................(1.16)}$$

$$\frac{\partial^3 W(\xi,\eta)}{\partial \xi^3} + \frac{\nu^*}{\phi^2}\frac{\partial^3 W(\xi,\eta)}{\partial \xi \partial \eta^2} = 0 \;\Big|_{\xi=1}$$

$$\frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} + \nu\phi^2 \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} = 0 \;\Big|_{\eta=1}$$

$$\frac{\partial^3 W(\xi,\eta)}{\partial \eta^3} + \nu^*\phi^2 \frac{\partial^3 W(\xi,\eta)}{\partial \eta \partial \xi^2} = 0 \;\Big|_{\eta=1}$$

where the displacement and coordinate x are divided by plate dimension a, while the coordinate y is divided by dimension b. And where $\phi=b/a$.

As we progress further ahead, it will become apparent that mathematical formulations for bending moments and vertical edge reactions are also required, as presented in the next paragraph.

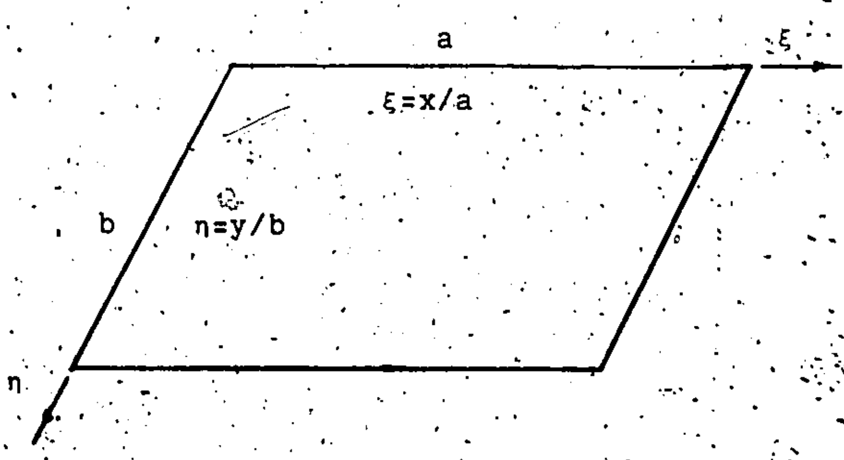


Figure 1.2. Rectangular plate dimensionless coordinate system.

Edge Bending Moments and Vertical Reactions

The mathematical expressions for bending moments and vertical edge reactions distributed along the edges of a rectangular plate have been developed in the conventional coordinates by Timoshenko(11), and in the dimensionless coordinates by Gorman(20). These expressions are as follows.

Distributed Bending Moments:

Conventional coordinates

$$M_x = -D\left(\frac{\partial^2 W(x,y)}{\partial x^2} + \nu \frac{\partial^2 W(x,y)}{\partial y^2} \right)$$
$$M_y = -D\left(\frac{\partial^2 W(x,y)}{\partial y^2} + \nu \frac{\partial^2 W(x,y)}{\partial x^2} \right) \quad \ldots\ldots\ldots\ldots\ldots (1.17)$$

Dimensionless coordinates

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} + \frac{\nu^*}{\phi^2} \frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} \right)$$
$$\frac{M_\eta \phi b}{D} = -\left(\frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} + \nu^* \phi^2 \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} \right) \quad \ldots\ldots\ldots\ldots\ldots (1.18)$$

reference is made to Figure 1.3.

Vertical Edge Reactions:

Conventional coordinates

$$V_x = -D\left(\frac{\partial^3 W(x,y)}{\partial x^3} + \nu^* \frac{\partial^3 W(x,y)}{\partial x \partial y^2} \right)$$

$$V_y = -D\left(\frac{\partial^3 W(x,y)}{\partial y^3} + \nu^* \frac{\partial^3 W(x,y)}{\partial y \partial x^2} \right) \qquad \text{............(1.19)}$$

Dimensionless coordinates

$$\frac{V_\xi a^2}{D} = -\left(\frac{\partial^3 W(\xi,\eta)}{\partial \xi^3} + \frac{\nu^*}{\phi^2} \frac{\partial^3 W(\xi,\eta)}{\partial \xi \partial \eta^2} \right)$$

$$\frac{V_\eta \phi b^2}{D} = -\left(\frac{\partial^3 W(\xi,\eta)}{\partial \eta^3} + \nu^* \phi^2 \frac{\partial^3 W(\xi,\eta)}{\partial \eta \partial \xi^2} \right) \qquad \text{............(1.20)}$$

where $\nu^* = 2 - \nu$ and where reference is made to Figure 1.4

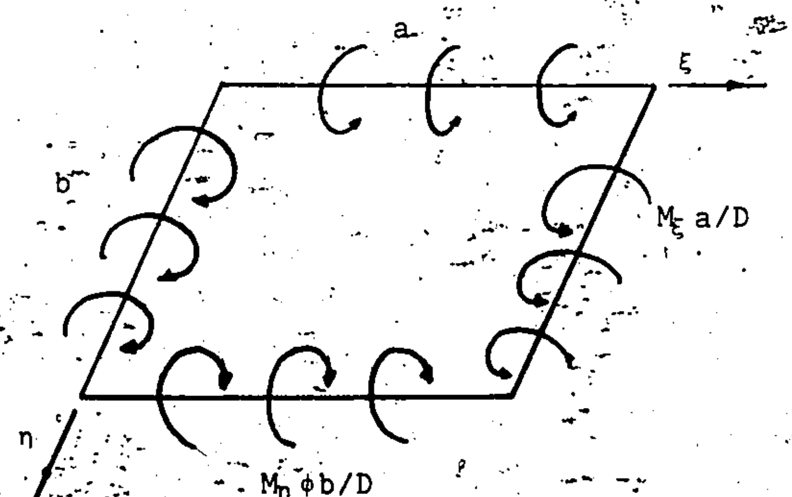


Figure 1.3. Bending moments acting along the edges of a rectangular plate.

The vertical concentrated force that will act at each corner of a rectangular plate, as explained by Timoshenko(11) is represented by the following expressions.

Corner Vertical Reaction:

Conventional coordinates

$$R = 2D(1-\nu)\frac{\partial^2 W(x,y)}{\partial x \partial y} \qquad \text{................(1.21)}$$

Dimensionless coordinates

$$R = \frac{2D}{b^3}(1-\nu)\frac{\partial^2 W(\xi,\eta)}{\partial \xi \partial \eta} \qquad \text{................(1.22)}$$

So far we have looked at all the basic equations that will be used throughout this work. Next, a word about the superposition method as it applies to rectangular plate free vibration problems(21).

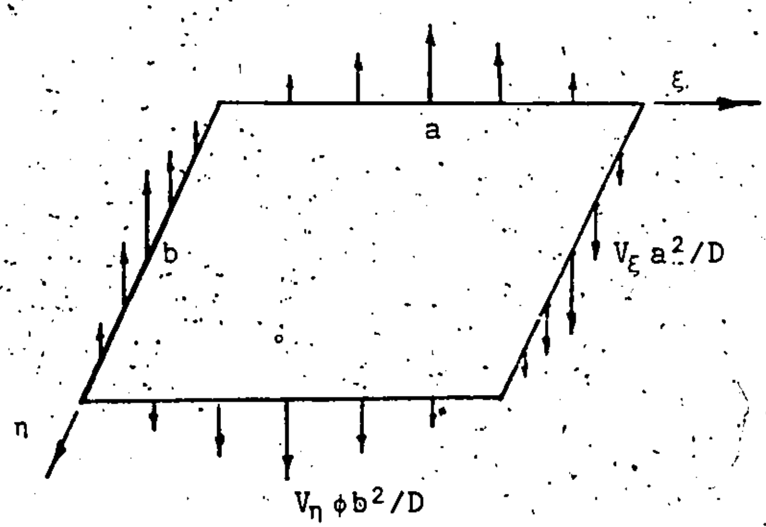


Figure 1.4. Vertical edge reactions acting along the edges of a rectangular plate.

The Method of Superposition

It has been mentioned that Lévy-type solutions are easily found, and well established for the family of rectangular plates with at least two opposite edges simply supported(20). A second family of rectangular plates is that of plates for which no two opposite edges are simply supported. Most of these plates were analyzed by Gorman using the method of superposition. In this method, two or more appropriate plate problems whose Lévy-type solutions can be obtained are superimposed, and the constants, appearing in their boundary condition formulation, are adjusted in such a way so that their combination provides boundary conditions similar to those specified in the original problem. The plate problems that are superimposed are often called building blocks. Another example of the application of this powerful method is demonstrated in the forthcoming chapters.

We now conclude this chapter with few words on limitations and assumptions that one should keep in mind when using the above theory.

Discussion and Conclusion

We have looked so far at the rectangular plate free vibration underlying theory, and we have also stated all the basic equations for the exact analytical solution of any rectangular plate free vibration problem for which the following assumptions apply.

- Plate thickness is small compared to its lateral dimensions.

- For higher vibration modes, plate thickness is small compared to the distance between nodal lines.

- Lateral displacement (W) is small compared to the thickness of the plate.

- Negligible rotary inertia effects.

- No significant in plate forces.

Fortunately, most rectangular plate vibration problems satisfy the above assumptions well enough for most practical purposes. In the next chapter we will be dealing with the application of the superposition techniques to solve the problem of the rectangular cantilever plate.

Chapter 2

THE CANTILEVER PLATE

In chapter one we discussed the method of superposition and its applications to the rectangular plate free vibration problems in general. The application of this method will be demonstrated in the present chapter by solving the problem of a plate that has traditionally presented the analyst with serious difficulties when trying to satisfy its free edge conditions, namely the cantilever plate. Solution for this particular problem already exists(20). However, the present solution is being presented here for three important reasons.

1.- To demonstrate first hand the steps involved in the application of the superposition techniques. Once that is done repetition will be avoided as much as possible.

2.- By using different building blocks than those used by Gorman to solve this same problem, it is hoped to demonstrate the freedom in choosing the appropriate building blocks without altering the final results, eventhough, the formulation of the problem might look somewhat different. Therefore, one would be inclined to select building blocks that are as simple, and as easy to solve as possible.

3.- The last and most important reason for the choice of the cantilever plate problem is the usefulness of its solution in the analysis of the cantilever plate with symmetricaly distributed point supports. This usefulness will become apparent to the reader in the forthcoming chapters.

The plate under consideration is shown in Figure 2.1. The length of the clamped edge is designated 2b, and the other dimension is a. The ξ axis is chosen to lie along the center line normal to the clamped edge of the plate. As discussed by Gorman, all free vibration modes of the cantilever plate will be either symmetric or antisymmetric with respect to the ξ axis. Dealing with each of these two possible families of modes , separately will help greatly simplify the analysis.

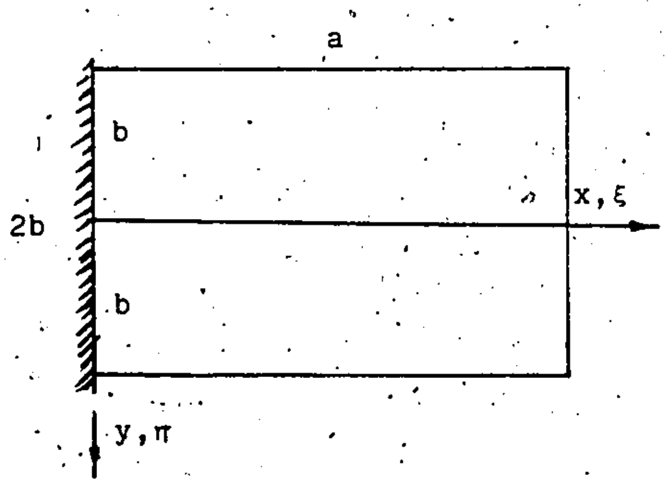


Figure 2.1. The cantilever plate.

Symmetric Modes

In order to determine its symmetric modes, only half of the cantilever plate need to be analyzed. This half plate with the three building blocks used in its solution are shown in Figure 2.2. Dotted lines along the edges of a building block refer to the slope normal to that edge. A small pair of circles attached to a building block's edge indicates that along that edge the plate has zero vertical edge reaction and that the slope of the plate taken normal to that edge is equal to zero. These boundary conditions are referred to as slip shear conditions(20). Extended edges indicate simple support.

The First Building Block:

Superimposing solutions of the three building blocks of Figure 2.2, a solution for the symmetric modes of the cantilever plate can be obtained. We begin by formulating the Lévy-type solution for the first of these building blocks.

Slip shear conditions are to be imposed along the edges $\eta=0$, and $\xi=1$, and simple support conditions on the edge $\xi=0$. The remaining edge $\eta=1$ is to have zero vertical edge reaction and a prescribed slope that varies cyclicly with time with the same frequency as the plate vibration. This last condition is an important deviation from Gorman's solution where bending moment is prescribed. This deviation eliminates rejection modes as explained later.

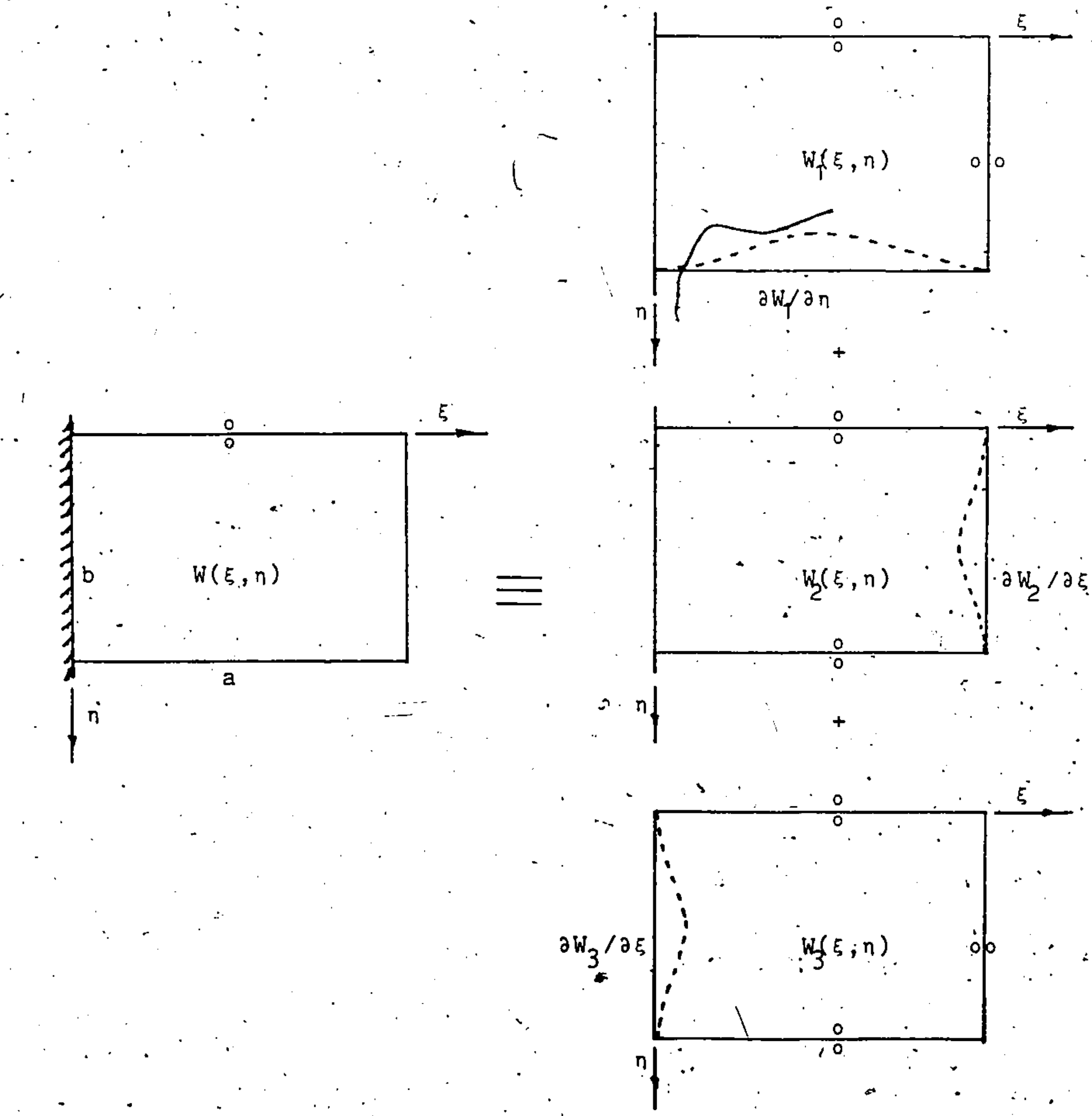


Figure 2.2. Building blocks used in the solution of the free vibration symmetric modes of the cantilever plate.

From Equation 1.6, 1.8, and 1.9 the Lévy-type solution of this block may be written as (20)

$$W_1(\xi,\eta) = \sum_{m=1,3,5}^{k^*} (A_m \cosh\beta_m\eta + D_m \cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} (A_m \cosh\beta_m\eta + D_m \cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \quad \ldots\ldots(2.1)$$

where $\beta_m = \phi\sqrt{\lambda^2+(m\pi/2)^2}$

and $\gamma_m = \phi\sqrt{\lambda^2-(m\pi/2)^2}$ or $\phi\sqrt{(m\pi/2)^2-\lambda^2}$

whichever is real, and where the first summation applies to terms for which $\lambda^2 > (m\pi/2)^2$. Also note that since we are interested in symmetric modes, antisymmetric terms are deleted from Eqn. 2.1 and therefore, boundary conditions at $\eta=0$ are automatically satisfied. Furthermore, Equation 2.1 satisfies exactly the prescribed boundary conditions at the edges $\xi=0$ and $\xi=1$, as required of Levy solutions. The two boundary conditions along the edge $\eta=1$ remain to be satisfied. We begin by enforcing zero vertical edge reaction. Equation 1.20 and the first summation of Equation 2.1 permit us to write for each value of m

$$\frac{\partial^3}{\partial\eta^3}\left\{(A_m\cosh\beta_m\eta + D_m\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}\right\}$$

$$+ \nu^*\phi^2\frac{\partial^3}{\partial\eta\partial\xi^2}\left\{(A_m\cosh\beta_m\eta + D_m\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}\right\}_{\eta=1} = 0 \quad \ldots\ldots\ldots(2.2)$$

or

$$A_m\{\beta_m(\beta_m^2 - (m\pi/2)^2\nu^*\phi^2)\}\sinh\beta_m$$

$$+D_m\{\gamma_m(\gamma_m^2 + (m\pi/2)^2\nu^*\phi^2)\}\sin\gamma_m = 0 \qquad \ldots\ldots\ldots\ldots(2.3)$$

solving for D_m in terms of A_m and substituting into Equation 2.1, the first summation of Equation 2.1 takes the following form

$$\sum_{m=1,3,5}^{k^*} A_m(\cosh\beta_m\eta + \theta_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \ldots\ldots\ldots\ldots(2.4)$$

A similar expression is obtained for terms in which we have $\lambda^2 < (m\pi/2)^2$ by using the second summation of Equation 2.1

$$\sum_{m=k^*+2}^{\infty} A_m(\cosh\beta_m\eta + \theta_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \ldots\ldots\ldots\ldots(2.5)$$

and $W_1(\xi,\eta)$ = Equation 2.4 + Equation 2.5 $\qquad \ldots\ldots\ldots\ldots(2.6)$

where

$$\theta_{1m} = \frac{\beta_m\{\nu^*\phi^2(m\pi/2)^2 - \beta_m^2\}\sinh\beta_m}{\gamma_m\{\nu^*\phi^2(m\pi/2)^2 + \gamma_m^2\}\sin\gamma_m}$$

and

$$\theta_{2m} = \frac{\beta_m\{\nu^*\phi^2(m\pi/2)^2 - \beta_m^2\}\sinh\beta_m}{\gamma_m\{\gamma_m^2 - (m\pi/2)^2\nu^*\phi^2\}\sinh\gamma_m}$$

Next, in order to satisfy the last prescribed boundary condition along the edge $\eta=1$ the slope along lines normal to this edge is expanded in series form

$$\frac{\partial W_1(\xi,\eta)}{\partial \eta} = \sum_{m=1,3}^{\infty} E_m \sin\frac{m\pi\xi}{2} \Big|_{\eta=1} \qquad \text{..........(2.7)}$$

and in keeping with Equation 2.6 we have[1].

$$\frac{\partial W_1(\xi,\eta)}{\partial \eta} = \sum_{m=1,3,5}^{k^*} A_m(\beta_m \sinh\beta_m\eta - \theta_{1m}\gamma_m \sin\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} A_m(\beta_m \sinh\beta_m\eta + \theta_{2m}\gamma_m \sinh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \text{..........(2.8)}$$

equating the right hand sides of Equations 2.7, and 2.8, and setting $\eta=1$ we obtain

$$E_m = A_m(\beta_m \sinh\beta_m - \theta_{1m}\gamma_m \sin\gamma_m) \qquad \text{..........(2.10)}$$

or if $\lambda^2 < (m\pi/2)^2$

$$E_m = A_m(\beta_m \sinh\beta_m + \theta_{2m}\gamma_m \sinh\gamma_m) \qquad \text{..........(2.11)}$$

solving for A_m and substitutng into Equation 2.6 we obtain

1.- It was proven by Gorman(20) that such derivations are legitimate.

$$W_1(\xi,\eta) = \sum_{m=1,3,5}^{k^*} \frac{E_m}{\theta_{11m}}(\cosh\beta_m\eta + \theta_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} \frac{E_m}{\theta_{22m}}(\cosh\beta_m\eta + \theta_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \quad \ldots\ldots(2.12)$$

where

$$\theta_{11m} = \beta_m\sinh\beta_m - \theta_{1m}\gamma_m\sin\gamma_m$$

and

$$\theta_{22m} = \beta_m\sinh\beta_m + \theta_{2m}\gamma_m\sinh\gamma_m$$

Equation 2.12 represent the solution of the first building block. We next, turn to the second building block.

The Second Building Block:

In finding a Lévy-type solution to the second building block of Figure 2.2, the reader will agree that in light of the boundary conditions the trigonometric part of this solution has to run in the direction of the η axis. In order to take greater advantage of equilibrium equations and forms of solutions already developed(20), a solution for the building block of Figure 2.3 will first be found. Then, a simple interchange of coordinates ξ and η will lead to the solution of the second block of interest.

The lévy-type solution of the building block of Figure 2.3 may be written as(20)

$$W(\xi,\eta) = \sum_{m=0,1,2}^{\infty} Y_m(\eta)\cos m\pi\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.13)$$

it can easily be verified that Equation 2.13 fully satisfies the boundary conditions along the edges $\xi=0$ and $\xi=1$. Then substituting Equation 2.13 into Equation 1.5 we obtain Equation 1.7, for which a solution is expressed by Equations 1.8, and 1.9.

Next, the simple support conditions along the edge $\eta=0$ is enforced by deleting the symmetric terms from Equations 1.8 and 1.9. The expression for $W(\xi,\eta)$ of Equation 2.13 becomes

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} (B_m \sinh\beta_m \eta + C_m \sin\gamma_m \eta)\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} (B_m \sinh\beta_m \eta + C_m \sinh\gamma_m \eta)\cos m\pi\xi \quad \ldots\ldots (2.14)$$

where the first summation pertains to terms for which $\lambda^2>(m\pi)^2$ and where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi)^2}$$

and

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi)^2} \qquad \text{or} \qquad \phi\sqrt{(m\pi)^2-\lambda^2}$$

whichever is real.

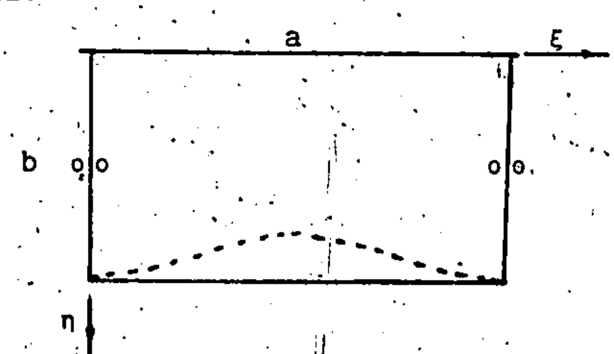


Figure 2.3. Intermediate block used for the solution of the second block of Figure 2.2

The next step is to enforce the prescribed boundary conditions along the edge $\eta=1$ and in the process eliminate the two unknowns of Equation 2.14 B_m and C_m. Note that these boundary conditions are the same as those pertaining along the same edge of the first building block of Figure 2.2. Therefore the steps followed here would be the same as those followed when the first building block was under study with the exeption of the prescribed slope that should be expanded in a cosine series form as

$$\frac{\partial W}{\partial \eta} = \sum_{m=0,1}^{\infty} E_m \cos m\pi\xi \qquad \text{.................(2.15)}$$

and in order to avoid repetition, only the final results will be shown.

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} \frac{E_m}{\theta_{11m}}(\sinh\beta_m\eta + \theta_{1m}\sin\gamma_m\eta)\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} \frac{E_m}{\theta_{22m}}(\sinh\beta_m\eta + \theta_{2m}\sinh\gamma_m\eta)\cos m\pi\xi \qquad \text{...(2.16)}$$

where

$$\theta_{1m} = \frac{\beta_m\{\beta_m^2 - \nu^*\phi^2(m\pi)^2\}\cosh\beta_m}{\gamma_m\{\gamma_m^2 + \nu^*\phi^2(m\pi)^2\}\cos{}_m}$$

$$\theta_{2m} = \frac{\beta_m\{\beta_m^2 - \nu^*\phi^2(m\pi)^2\}\cosh\beta_m}{\gamma_m\{\nu^*\phi^2(m\pi)^2 - \gamma_m^2\}\cosh\gamma_m}$$

$$\theta_{11m} = \beta_m\cosh\beta_m + \theta_{1m}\gamma_m\cos\gamma_m$$

$$\theta_{22m} = \beta_m\cosh\beta_m + \theta_{2m}\gamma_m\cosh\gamma_m$$

Now, the solution of the second building block of Figure 2.2 is extracted from the above solution, by interchanging the coordinates ξ and η. However, because we are using side length a to allow a dimensionless eigenvalue λ^2, in addition to the interchange of coordinates, it is necessary to replace the aspect ratio by its inverse, and to multiply the eigenvalue λ^2 by the square of the aspect ratio. The solution of the second block is then given by Equation 2.17. Note that the subscript m is replaced by n to distinguish this solution from that of the first building block.

$$W_2(\xi,\eta) = \sum_{n=0,1,2}^{k^*} \frac{E_n}{\theta_{11n}}(\sinh\beta_n\xi + \theta_{1n}\sin\gamma_n\xi)\cos n\pi\eta$$

$$+ \sum_{n=k^*+1}^{\infty} \frac{E_n}{\theta_{22n}}(\sinh\beta_n\xi + \theta_{2n}\sinh\gamma_n\xi)\cos n\pi\eta \qquad (2.17)$$

where the first summation includes terms for which $\lambda^2\phi^2 > (n\pi)^2$ only. And where

$$\beta_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(n\pi)^2}$$

$$\gamma_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(n\pi)^2} \quad \text{or} \quad \frac{1}{\phi}\sqrt{(n\pi)^2-\lambda^2\phi^2}$$

whichever is real. And

$$\theta_{1n} = \frac{\beta_n\{\beta_n^2 - \nu^*\phi_I(n\pi)^2\}\cosh\beta_n}{\gamma_n\{\nu^*\phi_I(n\pi)^2 + \gamma_n^2\}\cos\gamma_n}$$

$$\theta_{2n} = \frac{\beta_n\{\beta_n^2 - \nu^*\phi_I^2(n\pi)^2\}\cosh\beta_n}{\gamma_n\{\nu^*\phi_I^2(n\pi)^2 - \gamma_n^2\}\cosh\gamma_n}$$

$$\theta_{11n} = \beta_n\cosh\beta_n + \theta_{1n}\gamma_n\cos\gamma_n$$

$$\theta_{22n} = \beta_n\cosh\beta_n + \theta_{2n}\gamma_n\cosh\gamma_n$$

and the slope of lines normal to the edge $\xi=1$ is expressed as

$$\frac{\partial W_2(\xi,\eta)}{\partial\xi} = \sum_{n=0,1}^{\infty} E_n \cos n\pi\eta$$

Next, we look at a solution for the third building block.

The Third Building Block:

Following the same reasonning as before, the solution for this third and final building block of Figure 2.2 will be found by first finding a solution to the block of Figure 2.4 then transforming to that of Figure 2.5 and then, following the same steps we followed for the solution of the second building block, the expression for $W_3(\xi,\eta)$ is found.

We now focus our attention on the building block of Figure 2.4, the Lévy-type solution may be written as

$$W(\xi,\eta) = \sum_{m=0,1,2}^{\infty} Y(\eta)\cos m\pi\xi \qquad (2.18)$$

The reader should appreciate by now how the Lévy-type solution is selected to satisfy the boundary conditions along the edges $\xi=0$ and $\xi=1$.

Substituting Equation 2.18 back into the equilibrium equation we obtain Equation 1.7 that has as a solution Equations 1.8 and 1.9. Deleting the antisymmetric terms from this solution will enforce the slip shear conditions prescribed at the edge $\eta=0$. And $W(\xi,\eta)$ will assume the form

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} (A_m \cosh\beta_m\eta + D_m \cos\gamma_m\eta)\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} (A_m \cosh\beta_m\eta + D_m \cosh\gamma_m\eta)\cos m\pi\xi \qquad \text{......(2.19)}$$

enforcing zero displacement at the edge $\eta=1$ we have

$$W(\xi,\eta) = 0 \qquad \text{at } \eta=1$$

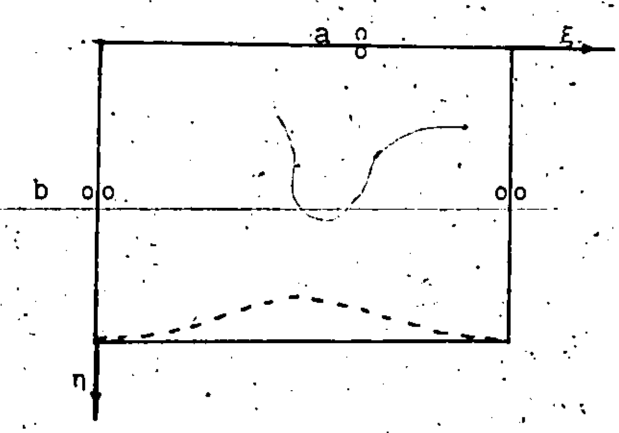


Figure 2.4. Intermediate building block used in arriving at a solution for the building block of Figure 2.5

or $$A_m \cosh\beta_m + D_m \cos\gamma_m = 0 \qquad \ldots\ldots\ldots(2.20)$$

or if $\lambda^2 < (m\pi)^2$

$$A_m \cosh\beta_m + D_m \cosh\gamma_m = 0 \qquad \ldots\ldots\ldots(2.21)$$

then

$$D_m = -A_m \frac{\cosh\beta_m}{\cos\gamma_m} = -A_m \theta_{1m}$$

or

$$D_m = -A_m \frac{\cosh\beta_m}{\cosh\gamma_m} = -A_m \theta_{2m}$$

where

$$\theta_{1m} = \frac{\cosh\beta_m}{\cos\gamma_m}$$

and

$$\theta_{2m} = \frac{\cosh\beta_m}{\cosh\gamma_m}$$

therfore,

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} A_m(\cosh\beta_m\eta - \theta_{1m}\cos\gamma_m\eta)\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} A_m(\cosh\beta_m\eta - \theta_{2m}\cosh\gamma_m\eta)\cos m\pi\xi \qquad \ldots\ldots(2.22)$$

we now express the prescribed slope $\partial W/\partial\eta$ at $\eta=1$ in series form

$$\frac{\partial W(\xi,\eta)}{\partial\eta} = \sum_{m=0,1,2}^{\infty} E_m \cos m\pi\xi \qquad \text{at } \eta=1 \qquad \ldots\ldots(2.23)$$

but from Equation 2.22 $\partial W(\xi,\eta)/\partial\eta$ is

$$\frac{\partial W(\xi,\eta)}{\partial\eta} = A_m(\beta_m \sinh\beta_m + \gamma_m\theta_{1m}\sin\gamma_m)\cos m\pi\xi$$

or if $\lambda^2<(m\pi)^2$ ....(2.24)

$$\frac{\partial W(\xi,\eta)}{\partial\eta} = A_m(\beta_m \sinh\beta_m - \gamma_m\theta_{2m}\sinh\gamma_m)\cos m\pi\xi$$

equating the right hand sides of Equations 2.24 and 2.23, and solving for A_m we obtain

$$A_m = \frac{E_m}{\theta_{11m}} \quad \text{or} \quad A_m = \frac{E_m}{\theta_{22m}}$$

where

$$\theta_{11m} = \beta_m \sinh\beta_m + \gamma_m\theta_{1m}\sin\gamma_m$$

$$\theta_{22m} = \beta_m \sinh\beta_m - \gamma_m\theta_{2m}\sinh\gamma_m$$

the expression for $W(\xi,\eta)$ then becomes

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} \frac{E_m}{\theta_{11m}}(\cosh\beta_m\eta - \theta_{1m}\cos\gamma_m\eta)\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} \frac{E_m}{\theta_{22m}}(\cosh\beta_m\eta - \theta_{2m}\cosh\gamma_m\eta)\cos m\pi\xi \quad \ldots(2.25)$$

where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi)^2}$$

and

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi)^2} \quad \text{Or} \quad \phi\sqrt{(m\pi)^2-\lambda^2}$$

whichever is real, and where $\partial W(\xi,\eta)/\partial\eta$ at $\eta=1$ is assumed to be

$$\frac{\partial W(\xi,\eta)}{\partial\eta} = \sum_{m=0,1,2}^{\infty} E_m \cos m\pi\xi$$

The solution for the building block of Figure 2.5 is now obtained from Equation 2.25 simply by replacing η by $1-\eta$. Note that the slope will change sign and Equation 2.23 becomes

$$\frac{\partial W(\xi,\eta)}{\partial \eta} = \sum_{m=0,1,2}^{\infty} -E_m \cos m\pi\xi \qquad \text{at} \quad \eta=0 \qquad \ldots\ldots(2.26)$$

and the solution of the building block of Figure 2.5 is

$$W(\xi,\eta) = \sum_{m=0,1,2}^{k^*} \frac{E_m}{\theta_{11m}}\{\cosh\beta_m(1-\eta) - \theta_{1m}\cos\gamma_m(1-\eta)\}\cos m\pi\xi$$

$$+ \sum_{m=k^*+1}^{\infty} \frac{E_m}{\theta_{22m}}\{\cosh\beta_m(1-\eta) - \theta_{2m}\cosh{}_m(1-\eta)\}\cos m\pi\xi \qquad \ldots\ldots(2.27)$$

Furthermore, by interchanging the variables ξ and η of Equation 2.27 we arrive at a solution for the third building block of Figure 2.2. Following the same transformation rules established earlier, the aspect ratio is replaced by its inverse and λ^2 is multiplied by ϕ^2. This final solution is

$$W_3(\xi,\eta) = \sum_{p=0,1,2}^{k^*} \frac{E_p}{\theta_{11p}}\{\cosh\beta_p(1-\xi) - \theta_{1p}\cos\gamma_p(1-\xi)\}\cos p\pi\eta$$

$$+ \sum_{p=k^*+1}^{\infty} \frac{E_p}{\theta_{22p}}\{\cosh\beta_p(1-\xi) - \theta_{2p}\cosh\gamma_p(1-\xi)\}\cos p\pi\eta \qquad \ldots\ldots(2.28)$$

where

$$\beta_p = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(p\pi)^2}$$

$$\gamma_p = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(p\pi)^2} \qquad \text{or} \qquad \frac{1}{\phi}\sqrt{(p\pi)^2-\lambda^2\phi^2}$$

whichever is real, and

$$\theta_{1p} = \cosh\beta_p/\cos\gamma_p$$

$$\theta_{2p} = \cosh\beta_p/\cosh\gamma_p$$

$$\theta_{11p} = \beta_p\sinh\beta_p + \gamma_p\,\theta_{1p}\sin\gamma_p$$

$$\theta_{22p} = \beta_p\sinh\beta_p - \gamma_p\,\theta_{2p}\sinh\gamma_p$$

and where the slope at $\xi=0$ is expressed as

$$\frac{\partial W(\xi,\eta)}{\partial\xi} = \sum_{p=0,1,2}^{\infty} -E_p\cos p\pi\eta \qquad \text{at} \quad \xi=0$$

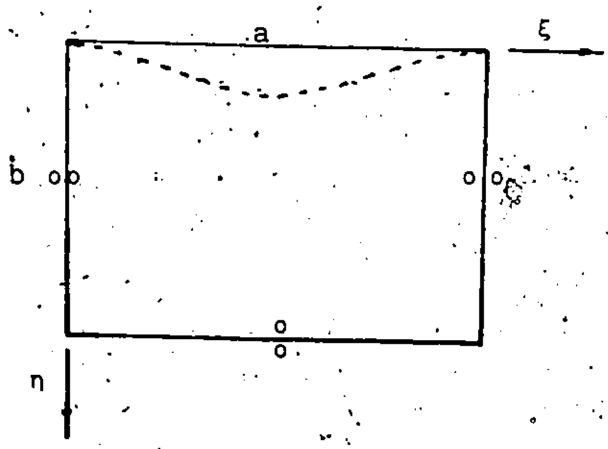


Figure 2.5. Intermediate building block used in arriving at a solution for the third building block of Figure 2.2.

Equations 2.12, 2.17, and 2.28 provide Lévy-type solutions for the first, second, and third building blocks of Figure 2.2 respectively. The next step is to superimpose these solutions constraining the Fourier coefficients that are present in them so that the net effect is to satisfy the boundary conditions of the plate of the left hand side of Figure 2.2. These conditions are, first the slope of lines normal to the edge $\xi=0$ must be set to zero, second the bending moments along the edges $\xi=1$ and $\eta=1$ must also be set to zero. Since these solutions, $W_1(\xi,\eta)$, $W_2(\xi,\eta)$, and $W_3(\xi,\eta)$ satisfy exactly the plate vibration governing differential equation, their summation, $W(\xi,\eta)$ will also satisfy this equation.

We begin by examining the contribution to bending moment along the edge $\xi=1$ of each term of the solution $W_1(\xi,\eta)$. From Equations 1.18, and 2.12 we obtain

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_1}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_1}{\partial \eta^2}\right) \qquad \text{at} \quad \xi=1$$

$$= E_m \frac{\sin m\pi/2}{\theta_{11m}}\{((m\pi/2)^2 - \nu\beta_m^2/\phi^2)\cosh\beta_m \eta$$

$$+\theta_{1m}((m\pi/2)^2 + \nu\gamma_m^2/\phi^2)\cos\gamma_m \eta\}$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.29)

$$= E_m \frac{\sin(m\pi/2)}{22m}\{((m\pi/2)^2 - \nu\beta_m^2/\phi^2)\cosh\beta_m \eta$$

$$+\theta_{2m}((m\pi/2)^2 - \nu\gamma_m/\phi^2)\cosh\gamma_m \eta\}$$

The contribution to the slope along the edge $\xi=0$ of each term of the solution $W_1(\xi,\eta)$ is

$$\frac{\partial W_1}{\partial \xi} = \frac{E_m(m\pi/2)}{\theta_{11m}}(\cosh\beta_m\eta + \theta_{1m}\cos\gamma_m\eta)$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.30)

$$= \frac{E_m(m\pi/2)}{\theta_{22m}}(\cosh\beta_m\eta + \theta_{2m}\cosh\gamma_m\eta)$$

The contribution to bending moment along the edge $\eta=1$ of each term of the solution $W_1(\xi,\eta)$ is

$$\frac{M_\eta\phi b}{D} = -\left(\frac{\partial^2 W_1}{\partial\eta^2} + \nu\phi^2\frac{\partial^2 W_1}{\partial\xi^2}\right) \qquad \text{at} \quad \eta = 1$$

$$= \frac{E_m}{\theta_{11m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\cosh\beta_m$$
$$+ \theta_{1m}(\nu\phi^2(m\pi/2)^2 + \gamma_m^2)\cos\gamma_m\}\sin\frac{m\pi\xi}{2}$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.31)

$$= \frac{E_m}{\theta_{22m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\cosh\beta_m$$
$$+ \theta_{2m}(\nu\phi^2(m\pi/2)^2 - \gamma_m^2)\cosh\gamma_m\}\sin\frac{m\pi\xi}{2}$$

Similar derivations are required for solutions $W_2(\xi,\eta)$ and $W_3(\xi,\eta)$.

The contribution to bending moment at the edge $\xi=1$ of each term of the solution $W_2(\xi,\eta)$ is

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_2}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_2}{\partial \eta^2}\right) \qquad \text{at} \quad \xi=1 \quad .$$

$$= \frac{E_n}{\theta_{11n}}\left\{\left(\frac{\nu}{\phi^2}(n\pi)^2 - \beta_n^2\right)\sinh\beta_n\right.$$

$$\left. + \theta_{1n}\left(\frac{\nu}{\phi^2}(n\pi)^2 + \gamma_n^2\right)\sin\gamma_n\right\}\cos n\pi\eta$$

or if $\lambda^2\phi^2 < (n\pi)^2$ ....(2.32)

$$= \frac{E_n}{\theta_{22n}}\left\{\left(\frac{\nu}{\phi^2}(n\pi)^2 - \beta_n^2\right)\sinh\beta_n\right.$$

$$\left. + \theta_{2n}\left(\frac{\nu}{\phi^2}(n\pi)^2 - \gamma_n^2\right)\sinh\gamma_n\right\}\cos n\pi\eta$$

The contribution to the slope at the edge $\xi=0$ of each term of the solution $W_2(\xi,\eta)$ is

$$\frac{\partial W_2}{\partial \xi} = \frac{E_n}{\theta_{11n}}(\beta_n + \theta_{1n}\gamma_n)\cos n\pi\eta$$

or if $\lambda^2\phi^2 < (n\pi)^2$ ....(2.33)

$$= \frac{E_n}{\theta_{22n}}(\beta_n + \theta_{2n}\gamma_n)\cos n\pi\eta$$

The contribution of each term of the solution $W_2(\xi,\eta)$ to bending moment along the edge $\eta=1$ is

$$\frac{M_\eta \phi b}{D} = -\left(\frac{\partial^2 W_2}{\partial \eta^2} + \nu\phi^2\frac{\partial^2 W_2}{\partial \xi^2}\right) \qquad \text{at} \quad \eta=1$$

$$= \frac{E_n \cos n\pi}{\theta_{11n}}\{((n\pi)^2 - \nu\phi^2\beta_n^2)\sinh\beta_n\xi$$

$$+ \theta_{1n}((n\pi)^2 + \nu\phi^2\gamma_n^2)\sin\gamma_n\xi\}$$

or if $\lambda^2\phi^2 < (n\pi)^2$ ....(2.34)

$$= \frac{E_n \cos n\pi}{\theta_{22n}}\{((n\pi)^2 - \nu\phi^2\beta_n^2)\sinh\beta_n\xi$$

$$+ \theta_{2n}((n\pi)^2 - \nu\phi^2\gamma_n^2)\sinh\gamma_n\xi\}$$

The contribution of each term of the solution $W_3(\xi,\eta)$ to bending moment along the edge $\xi=1$ is

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_3}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_3}{\partial \eta^2}\right) \qquad \text{at} \quad \xi=1$$

$$= \frac{E_p}{\theta_{11p}}\{\frac{\nu}{\phi^2}(p\pi)^2(1-\theta_{1p}) - \beta_p^2 - \gamma_p^2\theta_{1p}\}\cos p\pi\eta$$

or if $\lambda^2\phi^2 < (p\pi)^2$ ....(2.35)

$$= \frac{E_p}{\theta_{22p}}\{\frac{\nu}{\phi^2}(p\pi)^2(1-\theta_{2p}) - \beta_p^2 + \gamma_p^2\theta_{2p}\}\cos p\pi\eta$$

The contribution to the slope along the edge $\xi=0$ of each term of the solutin $W_3(\xi,\eta)$ is

$$\partial W_3/\partial\xi\Big|_{\xi=0} = -E_p\cos p\pi\eta \qquad \text{................(2.36)}$$

The contribution to bending moment along the edge $\eta=1$ of each term of the solution $W_3(\xi,\eta)$ is

$$\frac{M_\eta \phi b}{D} = -\left(\frac{\partial^2 W_3}{\partial \eta^2} + \nu\phi^2\frac{\partial^2 W_3}{\partial \xi^2}\right) \qquad \text{at} \quad \eta=1$$

$$= \frac{E_p \cos p\pi}{\theta_{11p}}\{((p\pi)^2 - \nu\phi^2\beta_p^2)\cosh\beta_p(1-\xi)$$

$$- \theta_{1p}((p\pi)^2 + \nu\phi^2\gamma_p^2)\cos\gamma_p(1-\xi)\}$$

or if $\lambda^2\phi^2 < (p\pi)^2$ ....(2.37)

$$= \frac{E_p \cos p\pi}{\theta_{22p}}\{((p\pi)^2 - \nu\phi^2\beta_p^2)\cosh\beta_p(1-\xi)$$

$$- \theta_{2p}((p\pi)^2 - \nu\phi^2\gamma_p^2)\cosh\gamma_p(1-\xi)\}$$

Focusing our attention on the edge $\xi=1$, where the net effect of all contributions to bending moment must be zero we write

Equation 2.29 + Equation 2.32 + Equation 2.35 = 0 ......(2.38)

It is seen that the second and third terms in Equation 2.38 which are Equations 2.32 and 2.35 respectively are in the form of Fourier series involving the trigonometric functions $\cos(n\pi\eta)$ and $\cos(p\pi\eta)$. However, since n and p vary in the same order, n = p = 0,1,2,....., both functions are one and the same. The coefficients of these series involve the quantities E_n and E_p. It is easily seen that if one could expand the first term on the left hand side of Equation 2.38 in the same type of series, one could impose the constraint that the sum of the coefficients before each of the Fourier trigonometric functions must equal zero. Expanding Equation 2.29 in a Fourier series of the type mentioned above we obtain

$$f(\eta) = \sum_{n=0,1}^{\infty} A_n \cos n\pi\eta \qquad \text{............(2.39)}$$

where

$$A_n = \begin{cases} \int_0^1 f(\eta)\cos n\pi\eta d\eta & \text{if } n=0 \\ 2\int_0^1 f(\eta)\cos n\pi\eta d\eta & \text{if } n\neq 0 \end{cases} \qquad \text{............(2.40)}$$

Appendix 1 provides a list of the type of integrals needed for the determination of all such Fourier coefficients that will be required throughout this work. From Equation 2.40 it follows that these coefficients are obtained by multiplying Equation 2.29 by $2\cos(n\pi\eta)$ and integrating between zero and one where n takes on values 0,1,.....,k-1., and dividing by 2 if n=0. The results will be shown later in this chapter.

We next turn to the net contribution of the three building block's solutions to the slope at $\xi=0$, and set it to zero. We obtain

Equation 2.30 + Equation 2.33 + Equation 2.36 = 0 ......(2.41)

It is clearly seen that Equations 2.33, and 2.36 are in the form of a Fourier series involving the trigonometric function $\cos n\pi\eta$. Therfore, following the same reasoning as before, Equation 2.30 is expanded in a Fourier series of the same type. Its coefficients are easily obtained by multiplying the appropriate expressions in Equation 2.30 by $2\cos n\pi\eta$, and integrating between zero and one, and dividing by 2 when n=0.

Finally, we turn to the net contribution of the three building block solutions of Figure 2.2 to the bending moment along the edge $\eta=1$ and set it to zero. We write

Equation 2.31 + Equation 2.34 + Equation 2.37 = 0 .....(2.42)

The first term on the left hand side of Equation 2.42 is in the form of a Fourier series involving the trigonometric function $\sin(m\pi\xi/2)$, where $m = 1,3,5.....,2k-1,(k\to\infty)$. Following the same reasoning as before, it is clear that the second and third term of the left hand side of Equation 2.42 must be expanded in a Fourier series of the same type.

The coefficients for a function $f(\xi)$ expanded in a series of the type $\sin(m\pi\xi/2)$ are known to be given by

$$A_m = 2\int_0^1 f(\xi)\sin(m\pi\xi/2)d\xi$$

It follows that the coefficients of Equation 2.34 expanded in a Fourier series of this type are obtained simply by multiplying the appropriate expressions of Equation 2.34 by $2\sin(m\pi\xi/2)$ and integrating between zero and one. The coefficients of the expanded Equation 2.37 are obtained in the same way.

In their proper expanded form, and if one choose to have k term in each of the involved series, Equations 2.38, 2.41, and 2.42 provide 3k homogeneous algebraic equations relating 3k unknown coefficients E_m's, E_n's, and E_p's. By matrix techniques these equations are handled with relative ease. Figure 2.6 is a schematie representation of such matrix showing only the first three terms of the relevent series. By trial and error, a value of λ^2 that causes the coefficient matrix determinant to vanish is established. Since a vanishing determinant indicates that a nontrivial solution for E_m's, E_n's, and E_p's exists, the associated λ^2 is in effect an eigenvalue.

Since we are more interested in the shape of the vibration function rather than its magnitude, one could arbitrarily set one of the unknown coefficients to unity, and by doing so transforms the 3k Homogeneous equations into a set of nonhomogeneous equations with 3k-1 unknowns. Using any 3k-1 equations of the set, we can easily solve for these 3k-1 unkowns using well known standard computer techniques.

In generating the matrix of Figure 2.6 it must be noted that the first quadrant, referred to as (1,1), represents the first term on the left hand side of Equation 2.42. The second quadrant (1,2) represents the second term. And the third (1,3) quadrant represents the third term on the left hand side of this equation. And in the same order Equation 2.38 is represented by Quadrants (2,1), (2,2), and (2,3). Also Equation 2.41 is represented by Quadrants (3,1), (3,2), and (3,3). These quadrants are

m=1	3	5	n=0	1	2	p=0	1	2	
-	0	0	-	-	-	-	-	-	
	(1,1)			(1,2)			(1,3)		
0	-	0	-	-	-	-	-	-	Eqn. 2.42
0	0	-	-	-	-	-	-	-	
-	-	-	-	0	0	-	0	0	
	(2,1)			(2,2)			(2,3)		
-	-	-	0	-	0	0	-	0	Eqn. 2.38
-	-	-	0	0	-	0	0	-	
-	-	-	-	0	0	-1	0	0	
	(3,1)			(3,2)			(3,3)		
-	-	-	0	-	0	0	-1	0	Eqn. 2.41
-	-	-	0	0	-	0	0	-1	
	E_m			E_n			E_p		

Figure 2.6. Matrix schematic representation for the symmetric mode problem of the cantilever plate.

Quadrant (1,1):

For $m = 1,3,5,\ldots\ldots,2k-1$

$$A_{(m+1)/2,(m+1)/2} = \{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\cosh\beta_m + \theta_{1m}(\nu\phi^2(m\pi/2)^2 + \gamma_m^2)\cos\gamma_m\}/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= \{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\cosh\beta_m + \theta_{2m}(\nu\phi^2(m\pi/2)^2 - \gamma_m^2)\cosh\gamma_m\}/\theta_{22m}$$

Quadrant (1,2):

For $m = 1,3,5,\ldots\ldots,2k-1$; $n = 0,1,2,\ldots\ldots,k-1$

$$A_{(m+1)/2,k+n+1} = 2\cos(n\pi)\sin(m\pi/2)(x_1-x_2)/\theta_{11n}$$

where

$$x_1 = \{(n\pi)^2 - \nu\phi^2\beta_n^2\}\beta_n\cosh\beta_n/\{\beta_n^2 + (m\pi/2)^2\}$$

$$x_2 = \{(n\pi)^2 + \nu\phi^2\gamma_n^2\}\gamma_n\theta_{1n}\cos\gamma_n/\{\gamma_n^2 - (m\pi/2)^2\}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= 2\cos(n\pi)\sin(m\pi/2)(x_1+x_2)/\theta_{22n}$$

where

$$x_1 = \{(n\pi)^2 - \nu\phi^2\beta_n^2\}\beta_n\cosh\beta_n/\{\beta_n^2 + (m\pi/2)^2\}$$

$$x_2 = \{(n\pi)^2 - \nu\phi^2\gamma_n^2\}\gamma_n\theta_{2n}\cosh\gamma_n/\{\gamma_n^2 + (m\pi/2)^2\}$$

Quadrant (1,3):

For $m = 1,3,5,\ldots\ldots\ldots,2k-1$; $p = 0,1,2,\ldots\ldots\ldots,k-1$

$$A_{(m+1)/2,2k+p+1} = 2\cos(p\pi)(m\pi/2)(x_1+x_2)/\theta_{11p}$$

where

$$x_1 = \{(p\pi)^2 - \nu\phi^2\beta_p^2\}\cosh\beta_p/\{\beta_p^2 + (m\pi/2)^2\}$$

$$x_2 = \{(p\pi)^2 + \nu\phi^2\gamma_p^2\}\theta_{1p}\cos\gamma_p/\{\gamma_p^2 - (m\pi/2)^2\}$$

or if $\lambda^2\phi^2 < (p\pi)^2$

$$= 2\cos(p\pi)(m\pi/2)(x_1-x_2)/\theta_{22p}$$

where

$$x_1 = \{(p\pi)^2 - \nu\phi^2\beta_p^2\}\cosh\beta_p/\{\beta_p^2 + (m\pi/2)^2\}$$

$$x_2 = \{(p\pi)^2 - \nu\phi^2\gamma_p^2\}\theta_{2p}\cosh\gamma_p/\{\gamma_p^2 + (m\pi/2)^2\}$$

Quadrant (2,1):

For $n = 0,1,2,\ldots\ldots\ldots,k-1$; $m = 1,3,5,\ldots\ldots\ldots,2k-1$

$$A_{(k+n+1),(m+1)/2} = 2\sin(m\pi/2)\cos(n\pi)(x_1+x_2)/\theta_{11m}$$

where

$$x_1 = \{(m\pi/2)^2 - \nu\phi_I^2\beta_m^2\}\beta_m\sinh\beta_m/\{\beta_m^2 + (n\pi)^2\}$$

$$x_2 = \{(m\pi/2)^2 + \nu\phi_I^2\gamma_m^2\}\gamma_m\theta_{1m}\sin\gamma_m/\{\gamma_m^2 - (n\pi)^2\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$A_{k+n+1,(m+1)/2} = 2\sin(m\pi/2)\cos(n\pi)(x_1+x_2)/\theta_{22m}$$

where

$$x_1 = \{(m\pi/2)^2 - \nu\phi_I^2\beta_m^2\}\beta_m \sinh\beta_m/\{\beta_m^2 + (n\pi)^2\}$$

$$x_2 = \{(m\pi/2)^2 - \nu\phi_I^2\beta_m^2\}\theta_{2m}\gamma_m \sinh\gamma_m/\{\gamma_m^2 + (n\pi)^2\}$$

the above results must be divided by 2 if $n=0$

Quadrant (2,2):

For $n = 0,1,2,\ldots\ldots\ldots,k-1$

$$A_{k+n+1,k+n+1} = \{(\nu\phi_I^2(n\pi)^2-\beta_n^2)\sinh\beta_n + \theta_{1n}(\nu\phi_I^2(n\pi)^2+\gamma_n^2)\sin\gamma_n\}/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$=\{(\nu\phi_I^2(n\pi)^2-\beta_n^2)\sinh\beta_n + \theta_{2n}(\nu\phi_I^2(n\pi)^2-\gamma_n^2)\sinh\gamma_n\}/\theta_{22n}$$

Quadrant (2,3):

For $p = 0,1,2,\ldots\ldots\ldots,k-1$

$$A_{k+p+1,2k+p+1} = \{\nu\phi_I^2(p\pi)^2(1-\theta_{1p})-\beta_p^2 - \gamma_p^2\theta_{1p}\}/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi)^2$

$$= \{\nu\phi_I^2(p\pi)^2(1-\theta_{2p}) - \beta_p^2 + \gamma_p^2\theta_{2p}\}/\theta_{22p}$$

Quadrant (3,1):

For $n = 0,1,2,\ldots\ldots\ldots,k-1$; $m = 1,3,5,\ldots\ldots\ldots,2k-1$

$A_{2k+n+1,(m+1)/2} = 2\cos(n\pi)(m\pi/2)(x_1+x_2)/\theta_{11m}$

where

$x_1 = \beta_m \sinh\beta_m/\{\beta_m^2 + (n\pi)^2\}$

$x_2 = \gamma_m \theta_{1m} \sin\gamma_m/\{\gamma_m^2 - (n\pi)^2\}$

or if $\lambda^2 < (m\pi/2)^2$

$= 2\cos(n\pi)(m\pi/2)(x_1+x_2)/\theta_{22m}$

where

$x_1 = \beta_m \sinh\beta_m/\{\beta_m^2 + (n\pi)^2\}$

$x_2 = \gamma_m \theta_{2m} \sinh\gamma_m/\{\gamma_m^2 + (n\pi)^2\}$

above results must be divided by 2 if n=0

Quadrant (3,2):

For n = 0,1,2,........k-1

$A_{2k+n+1,k+n+1} = (\beta_n + \theta_{1n}\gamma_n)/\theta_{11n}$

or if $\lambda^2\phi^2 < (n\pi)^2$

$= (\beta_n + \theta_{2n}\gamma_n)/\theta_{22n}$

Quadrant (3,3):

For p = 0,1,2,........,k-1

$A_{2k+p+1,2k+p+1} = -1$

Note that all of the off-diagonal elements in Quadrants (1,1), (2,2), (2,3), (3,2), and (3,3) are zero

Now, we are ready to feed the above information to a digital computer, and solve for eigenvalues, and mode shapes. Program 1 in Appendix 3 is specially designed for eigenvalue searches. While program 2 will solve for the unknown coefficients and shape data of this symmetric modes problem. In order to determine the number of terms k, that should be used in each of the Fourier series involved in the solution of this problem, a convergence test like the one shown in Figure 2.7, proved to be very effective, specially in the case of point supports.

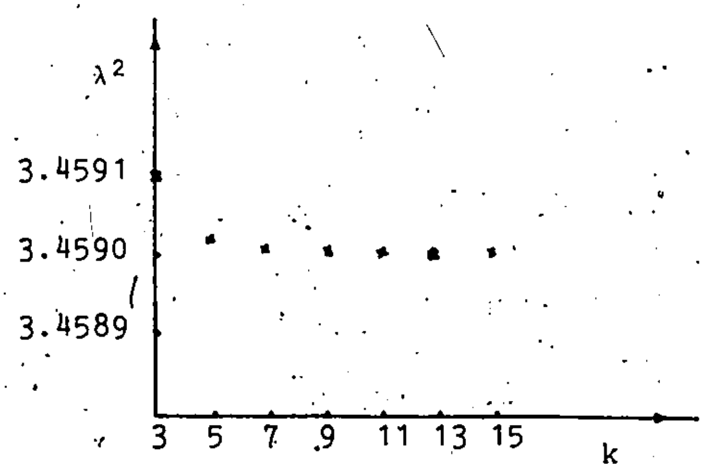


Figure 2.7. Eigenvalue λ^2 versus the number of terms k for the first symmetric mode of a squar plate, $\phi=1$.

this conclude our discussion on the free vibration analysis of the symmetric modes of the rectangular cantilever plate. Eigenvalues for this family of modes will be tabulated and discussed later in this chapter. Next, we turn to the analysis of the antisymmetric modes.

Antisymmetric Modes

The steps followed in this analysis are identical to those followed in the case of symmetric modes. We only need to analyze half of the full plate as shown in Figure 2.8, where the right hand side illustrates the three building blocks used in this solution with their boundary conditions.

The First Building Block:

We begin by finding a Lévy-type solution for the first building block of Figure 2.8. In light of the boundary conditions along the edges $\xi=0$ and $\xi=1$ this solution may be written as

$$W_1(\xi,\eta) = \sum_{m=1,3,5}^{\infty} Y_m(\eta)\sin\frac{m\pi\xi}{2} \qquad \text{.............(2.43)}$$

where $Y_m(\eta)$ is given by Equations 1.8 and 1.9. In order to enforce the simple support conditions along the edge $\eta=0$, symmetric terms must be deleted from the expression of $Y_m(\eta)$. Therfore, we may write

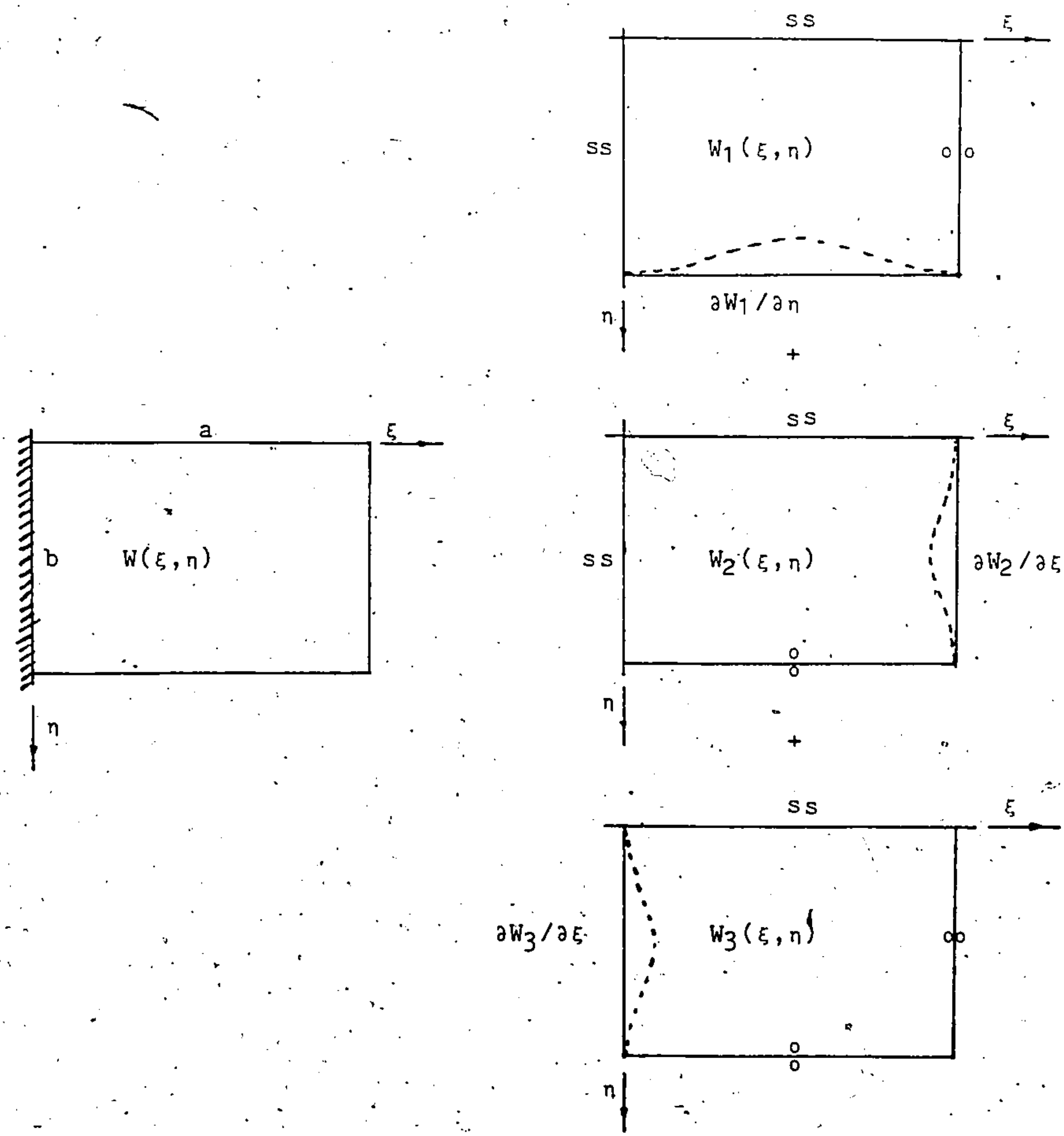


Figure 2.8. Building blocks used in the solution of the free vibration antisymmetric modes of the cantilever plate.

$$Y_m(\eta) = B_m \sinh\beta_m\eta + C_m \sin\gamma_m\eta$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.44)

$$= B_m \sinh\beta_m\eta + C_m \sinh\gamma_m\eta$$

where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi/2)^2}$$

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi/2)^2} \quad \text{or} \quad \phi\sqrt{(m\pi/2)^2-\lambda^2}$$

whichever is real.

It remains to satisfy the boundary condition at the edge $\eta=1$. Enforcing zero vertical edge reaction at $\eta=1$ we write

$$\frac{\partial^3 W_1(\xi,\eta)}{\partial\eta^3} + \nu^*\phi^2 \frac{\partial^3 W_1(\xi,\eta)}{\partial\eta\partial\xi^2}\Big|_{\eta=1} = 0 \quad \text{...........(2.45)}$$

performing the necessary derivations and solving for B_m we have

$$B_m = \theta_{1m} C_m$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.46)

$$= \theta_{2m} C_m$$

where

$$\theta_{1m} = \frac{\gamma_m\{\gamma_m^2 + (m\pi/2)^2\nu^*\phi^2\}\cos\gamma_m}{\beta_m\{\beta_m^2 - (m\pi/2)^2\nu^*\phi^2\}\cosh\beta_m}$$

$$\theta_{2m} = \frac{\gamma_m\{\nu^*\phi^2(m\pi/2)^2 - \gamma_m^2\}\cosh\gamma_m}{\beta_m\{\beta_m^2 - \nu^*\phi^2(m\pi/2)^2\}\cosh\beta_m}$$

We now represent the slope at $\eta=1$ by a series of the same type as $W_1(\xi,\eta)$, or

$$\partial W_1/\partial\eta = \sum_{m=1,3,5}^{\infty} E_m \sin\frac{m\pi\xi}{2} \qquad \text{.............(2.47)}$$

substituting Equation 2.46 into 2.44, then Equation 2.44 into 2.43, and differentiating with respect to η at $\eta=1$ and equating the results to the right hand side of Equation 2.47 we have

$$E_m = C_m(\beta_m\theta_{1m}\cosh\beta_m + \gamma_m\cos\gamma_m)$$

or if $\lambda^2 < (m\pi/2)^2$

$$= C_m(\beta_m\theta_{2m}\cosh\beta_m + \gamma_m\cosh\gamma_m)$$

Solving for C_m, and substituting into the expression of $W_1(\xi,\eta)$ we write

$$W_1(\xi,\eta) = \sum_{m=1,3,5}^{k^*} \frac{E_m}{\theta_{11m}}(\theta_{1m}\sinh\beta_m\eta + \sin\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \text{.............(2.48)}$$

$$+ \sum_{m=k^*+2}^{\infty} \frac{E_m}{\theta_{22m}}(\theta_{2m}\sinh\beta_m\eta + \sinh\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

where

$$\theta_{11m} = \beta_m\theta_{1m}\cosh\beta_m + \gamma_m\cos\gamma_m$$

$$\theta_{22m} = \theta_{2m}\beta_m\cosh\beta_m + \gamma_m\cosh\gamma_m$$

With the solution of the first building block being completed, we next turn to the solution of the second block.

The Second Building Block:

A look at the second building block of Figure 2.8 reveals that the solution for this block can easily be extracted from the solution of the first block of the same figure by exchanging the variables ξ and η. Also, in keeping with transformation rules discussed earlier λ^2 must be multiplied by ϕ^2 and ϕ must be changed into $1/\phi$. Therfore, we may write

$$W_2(\xi,\eta) = \sum_{n=1,3,5}^{k^*} \frac{E_n}{\theta_{11n}}(\theta_{1n}\sinh\beta_n\xi + \sin\gamma_n\xi)\sin\frac{n\pi\eta}{2}$$

$$+ \sum_{n=k^*+2}^{\infty} \frac{E_n}{\theta_{22n}}(\theta_{2n}\sinh\beta_n\xi + \sinh\gamma_n\xi)\sin\frac{n\pi\eta}{2} \qquad \text{...........(2.49)}$$

where

$$\theta_{1n} = \frac{\gamma_n\{\gamma_n^2 + (n\pi/2)^2\nu^*\phi_I^2\}\cos\gamma_n}{\beta_n\{\beta_n^2 - (n\pi/2)^2\nu^*\phi_I^2\}\cosh\beta_n}$$

$$\theta_{2n} = \frac{\gamma_n\{\nu^*\phi_I^2(n\pi/2)^2 - \gamma_n^2\}\cosh\gamma_n}{\beta_n\{\beta_n^2 - \nu^*\phi_I^2(n\pi/2)^2\}\cosh\beta_n}$$

$$\beta_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(n\pi/2)^2}$$

$$\gamma_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(n\pi/2)^2} \quad \text{or} \quad \frac{1}{\phi}\sqrt{(n\pi/2)^2-\lambda^2\phi^2}$$

whichever is real, and

$$\theta_{11n} = \beta_n\theta_{1n}\cosh\beta_n + \gamma_n\cos\gamma_n$$

$$\theta_{22n} = \theta_{2n}\beta_n\cosh\beta_n + \gamma_n\cosh\gamma_n$$

$$\partial W_2/\partial\xi = \sum_{n=1,3,5}^{\infty} E_n\sin\frac{n\pi\eta}{2} \qquad \text{at} \quad \xi=1$$

The Third Building Block:

Here, consideration is given to the solution of the third and last building block of Figure 2.8. A careful examination of the building block under study reveals that the task of finding a solution is made easier by first solving the building block shown in Figure 2.9. The solution of the building block of figure 2.10 is then extracted from that of Figure 2.9 simply by changing the variable η into $(1-\eta)$. Furthermore, this solution will in turn lead to the required solution of the last building block of Figure 2.8 as it is shown below.

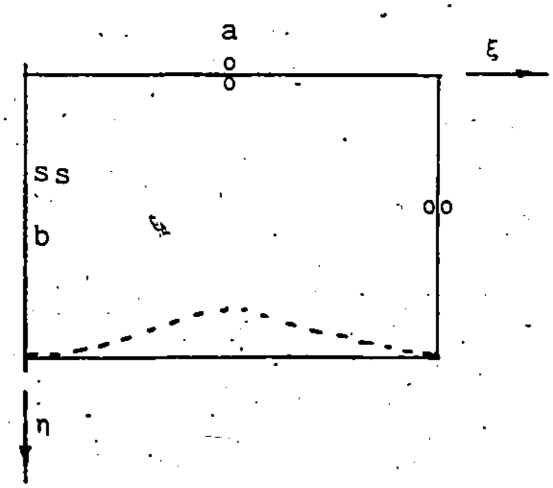


Figure 2.9. Intermediate building block used in the solution of the building block of Figure 2.10.

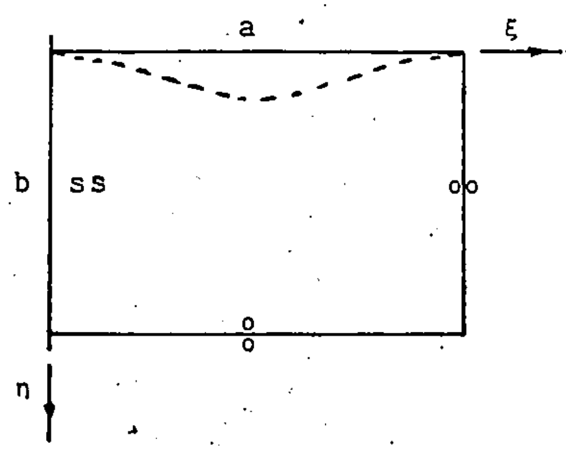


Figure 2.10. Intermediate building block used in the solution of the third building block of Figure 2.8.

Focusing our attention on the building block of Figure 2.9, we write its Lévy-type solution as

$$W(\xi,\eta) = \sum_{m=1,3,5}^{\infty} Y_m(\eta)\sin\frac{m\pi\xi}{2} \qquad \text{.........(2.50)}$$

where

$$Y_m(\eta) = A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sin\gamma_m\eta + D_m\cos\gamma_m\eta$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.51)

$$= A_m\cosh\beta_m\eta + B_m\sinh\beta_m\eta + C_m\sinh\gamma_m\eta + D_m\cosh\gamma_m\eta$$

In light of the slip shear conditions along the edge $\eta=0$, anti-symmetric terms must be deleted from Equation 2.51

$$Y_m(\eta) = A_m\cosh\beta_m\eta + D_m\cos\gamma_m\eta$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.52)

$$= A_m\cosh\beta_m\eta + D_m\cosh\gamma_m\eta$$

Enforcing zero displacement along the edge $\eta=1$ we write

$$A_m\cosh\beta_m + D_m\cos\gamma_m = 0$$

or $$A_m\cosh\beta_m + D_m\cosh\gamma_m = 0$$

and $$D_m = -A_m\theta_{1m}$$

or $$D_m = -A_m\theta_{2m}$$

where $\theta_{1m} = \cosh\beta_m/\cos\gamma_m$; $\theta_{2m} = \cosh\beta_m/\cosh\gamma_m$

Therfore,

$$W(\xi,\eta) = \sum_{m=1,3,5}^{k^*} A_m(\cosh\beta_m\eta - \theta_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} A_m(\cosh\beta_m\eta - \theta_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \quad \ldots\ldots(2.53)$$

Introducing,

$$\partial W(\xi,\eta)/\partial\eta = \sum_{m=1,3,5}^{\infty} E_m\sin\frac{m\pi\xi}{2} \qquad \text{at} \quad \eta=1$$

and equating to the first derivative of Equation 2.53 with respect to η at $\eta=1$, we have

$$E_m = A_m(\beta_m\sinh\beta_m + \theta_{1m}\gamma_m\sin\gamma_m)$$

or if $\lambda^2 < (m\pi/2)^2$

$$= A_m(\beta_m\sinh\beta_m - \theta_{2m}\gamma_m\sinh\gamma_m)$$

Solving for A_m we write

$$A_m = E_m/\theta_{11m} \qquad \text{or} \qquad A_m = E_m/\theta_{22m}$$

where

$$\theta_{11m} = \beta_m\sinh\beta_m + \theta_{1m}\gamma_m\sin\gamma_m$$

$$\theta_{22m} = \beta_m\sinh\beta_m - \theta_{2m}\gamma_m\sinh\gamma_m$$

Substituting A_m by its value in Equation 2.53 we obtain

$$W(\xi,\eta) = \sum_{m=1,3,5}^{k^*} \frac{E_m}{\theta_{11m}}(\cosh\beta_m\eta - \theta_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} \frac{E_m}{\theta_{22m}}(\cosh\beta_m\eta - \theta_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \quad \text{..........(2.54)}$$

Now, the solution of the building block of Figure 2.10 is extracted from Equation 2.54, simply by changing the variable η into $(1-\eta)$

$$W(\xi,\eta) = \sum_{m=1,3,5}^{k^*} \frac{E_m}{\theta_{11m}}\{\cosh\beta_m(1-\eta) - \theta_{1m}\cos\gamma_m(1-\eta)\}\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} \frac{E_m}{\theta_{22m}}\{\cosh\beta_m(1-\eta) - \theta_{2m}\cosh\gamma_m(1-\eta)\}\sin\frac{m\pi\xi}{2} \quad \text{.........(2.55a)}$$

Finally, the solution of the third building block of Figure 2.8 is obtained from Equation 2.55a by exchanging the variables ξ and η, multiplying λ^2 by ϕ^2, and replacing ϕ by its inverse. Therfore,

$$W_3(\xi,\eta) = \sum_{p=1,3,5}^{k^*} \frac{E_p}{\theta_{11p}}\{\cosh\beta_p(1-\xi) - \theta_{1p}\cos\gamma_p(1-\xi)\}\sin\frac{p\pi\eta}{2}$$

$$+ \sum_{p=k^*+2}^{\infty} \frac{E_p}{\theta_{22p}}\{\cosh\beta_p(1-\xi) - \theta_{2p}\cosh\gamma_p(1-\xi)\}\sin\frac{p\pi\eta}{2} \quad \text{..........(2.55)}$$

where

$$\theta_{1p} = \cosh\beta_p/\cos\gamma_p$$

$$\theta_{2p} = \cosh\beta_p/\cosh\gamma_p$$

$$\theta_{11p} = \beta_p\sinh\beta_p + \gamma_p\theta_{1p}\sin\gamma_p$$

$$\theta_{22p} = \beta_p \sinh\beta_p - \gamma_p \theta_{2p} \sinh\gamma_p$$

and

$$\partial W_3(\xi,\eta)/\partial\xi = \sum_{p=1,3,5}^{\infty} -E_p \sin\frac{p\pi\eta}{2} \qquad \text{.........(2.56)}$$

$$\beta_p = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(p\pi/2)^2}$$

$$\gamma_p = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(p\pi/2)^2} \qquad \text{or} \qquad \frac{1}{\phi}\sqrt{(p\pi/2)^2-\lambda^2\phi^2}$$

whichever is real. Note that when changing a variable ξ into $1-\xi$, the first derivative with respect to that variable changes sign as shown in Equation 2.56.

We now have solutions for all three building blocks of Figure 2.8. The next step is to superimpose these solutions, adjusting their unknown coefficients to satisfy the prescribed boundary conditions of the left hand side plate of Figure 2.8. We begin by examining the contribution of each of the building blocks to these boundary conditions.

The contribution of each term of the solution $W_1(\xi,\eta)$ to the bending moment along the edge $\xi=1$ is

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_1}{\partial\xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_1}{\partial\eta^2}\right)\Big|_{\xi=1}$$

$$= \frac{E_m}{\theta_{11m}}\sin\frac{m\pi}{2}\{((m\pi/2)^2 - \nu\phi_I^2\beta_m^2)\theta_{1m}\sinh\beta_m\eta$$

$$+ ((m\pi/2)^2 + \nu\phi_I^2\gamma_m^2)\sin\gamma_m\eta\}$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.57)

$$\frac{M_\xi a}{D} = \frac{E_m}{\theta_{22m}}\sin\frac{m\pi}{2}\{((m\pi/2)^2 - \nu\phi_I^2\beta_m^2)\theta_{2m}\sinh\beta_m\eta$$

$$+ ((m\pi/2)^2 - \nu\phi_I^2\gamma_m^2)\sinh\gamma_m\eta\}$$

The contribution to the slope at $\xi=0$ of each term of $W_1(\xi,\eta)$ is

$$\partial W_1(\xi,\eta)/\partial\xi = \frac{E_m}{\theta_{11m}}(m\pi/2)(\theta_{1m}\sinh\beta_m\eta + \sin\gamma_m\eta)$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.58)

$$= \frac{E_m}{\theta_{22m}}(m\pi/2)(\theta_{2m}\sinh\beta_m\eta + \sinh\gamma_m\eta)$$

The contribution to bending moment along the edge $\eta=1$ of each term of $W_1(\xi,\eta)$ is

$$\frac{M_\eta\phi b}{D} = -\left(\frac{\partial^2 W_1}{\partial\eta^2} + \nu\phi^2\frac{\partial^2 W_1}{\partial\xi^2}\right)\Big|_{\eta=1}$$

$$= \frac{E_m}{\theta_{11m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\theta_{1m}\sinh\beta_m$$

$$+ (\nu\phi^2(m\pi/2)^2 + \gamma_m^2)\sin\gamma_m\}\sin\frac{m\pi\xi}{2}$$

or if $\lambda^2 < (m\pi/2)^2$ ....(2.59)

$$= \frac{E_m}{\theta_{22m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\theta_{2m}\sinh\beta_m$$

$$+ (\nu\phi^2(m\pi/2)^2 - \gamma_m^2)\sinh\gamma_m\}\sin\frac{m\pi\xi}{2}$$

The contribution of each term of $W_2(\xi,\eta)$ to bending moment along the edge $\xi=1$ is

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_2}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_2}{\partial \eta^2}\right)\Big|_{\xi=1}$$

$$= \frac{E_n}{\theta_{11n}}\{(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2)\theta_{1n}\sinh\beta_n$$

$$+ (\nu\phi_I^2(n\pi/2)^2 + \gamma_n^2)\sin\gamma_n\}\sin\frac{n\pi\eta}{2}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$ ....(2.60)

$$= \frac{E_n}{\theta_{22n}}\{(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2)\theta_{2n}\sinh\beta_n$$

$$+ (\nu\phi_I^2(n\pi/2)^2 - \gamma_n^2)\sinh\gamma_n\}\sin\frac{n\pi\eta}{2}$$

Contribution to the slope at $\xi=0$ of each term of W_2 is

$$\partial W_2(\xi,\eta)/\partial\xi = \frac{E_n}{\theta_{11n}}(\theta_{1n}\beta_n + \gamma_n)\sin\frac{n\pi\eta}{2}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$ ....(2.61)

$$= \frac{E_n}{\theta_{22n}}(\theta_{2n}\beta_n + \gamma_n)\sin\frac{n\pi\eta}{2}$$

Contribution to bending moment along the edge $\eta=1$ of each term of $W_2(\xi,\eta)$

$$\frac{M_\eta\phi b}{D} = -\left(\frac{\partial^2 W_2}{\partial\eta^2} + \nu\phi^2\frac{\partial^2 W_2}{\partial\xi^2}\right)\Big|_{\eta=1}$$

$$\frac{M_n \phi b}{D} = \frac{E_n}{\theta_{11n}} \sin\frac{n\pi}{2}\{((n\pi/2)^2 - \nu\phi^2\beta_n^2)\theta_{1n}\sinh\beta_n\xi$$

$$+((n\pi/2)^2 + \nu\phi^2\gamma_n^2)\sin\gamma_n\xi\}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$ ....(2.62)

$$= \frac{E_n}{\theta_{22n}} \sin\frac{n\pi}{2}\{((n\pi/2)^2 - \nu\phi^2\beta_n^2)\theta_{2n}\sinh\beta_n\xi$$

$$+ ((n\pi/2)^2 - \nu\phi^2\gamma_n^2)\sinh\gamma_n\xi\}$$

The contribution of each term of $W_3(\xi,\eta)$ to bending moment along the edge $\xi=1$ is

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_3}{\partial\xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_3}{\partial\eta^2}\right)\Big|_{\xi=1}$$

$$= \frac{E_p}{\theta_{11p}}\{\nu\phi_I^2(p\pi/2)^2(1-\theta_{1p}) - \beta_p^2 - \theta_{1p}\gamma_p^2\}\sin\frac{p\pi\eta}{2}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$ ....(2.63)

$$= \frac{E_p}{\theta_{22p}}\{\nu\phi_I^2(p\pi/2)^2(1-\theta_{2p}) - \beta_p^2 + \theta_{2p}\gamma_p^2\}\sin\frac{p\pi\eta}{2}$$

Contribution to the slope at $\xi=0$ of each term of W_3 is

$\partial W_3(\xi,\eta)/\partial\xi = -E_p\sin\frac{p\pi\eta}{2}$ ....(2.64)

The contribution to bending moment along the edge $\eta=1$ of each term of $W_3(\xi,\eta)$ is

$$\frac{M_\eta \phi b}{D} = -\left(\frac{\partial^2 W_3}{\partial \eta^2} + \nu\phi^2\frac{\partial^2 W_3}{\partial \xi^2}\right)\Big|_{\eta=1}$$

$$= \frac{E_p}{\theta_{11p}}\sin\frac{p\pi}{2}\{((p\pi/2)^2 - \nu\phi^2\beta_p^2)\cosh\beta_p(1-\xi)$$

$$- ((p\pi/2)^2 + \nu\phi^2\gamma_p^2)\theta_{1p}\cos\gamma_p(1-\xi)\}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$ ....(2.65)

$$= \frac{E_p}{\theta_{22p}}\sin\frac{p\pi}{2}\{((p\pi/2)^2 - \nu\phi^2\beta_p^2)\cosh\beta_p(1-\xi)$$

$$- ((p\pi/2)^2 - \nu\phi^2\gamma_p^2)\theta_{2p}\cosh\gamma_p(1-\xi)\}$$

We now adjust and add relevent contributions to satisfy the boundary conditions of the plate under study. First, for zero slope at $\xi=0$ we write

Equation 2.58 + Equation 2.61 + Equation 2.64 = 0 ....(2.66)

Second, for zero bending moment along the edge $\xi=1$ we have

Equation 2.57 + Equation 2.60 + Equation 2.63 = 0 ....(2.67)

Finally, for zero bending moment along the edge $\eta=1$ we write

Equation 2.59 + Equation 2.62 + Equation 2.65 = 0 ....(2.68)

The final step is to generate the cefficients matrix by expanding the necessary terms of Equations 2.66, 2.67, and 2.68 in appropriate Fourier series as it was don for the symmetric case. The generated matrix represented schematically by Figure 2.11 is as follow

Quadrant (1,1):

For m=1,3,5,.......,2k-1

$$A_{(m+1)/2,(m+1)/2} = \frac{1}{\theta_{11m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\theta_{1m}\sinh\beta_m$$

$$+ (\nu\phi^2(m\pi/2)^2+\gamma_m^2)\sin\gamma_m\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= \frac{1}{\theta_{22m}}\{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)\theta_{2m}\sinh\beta_m$$

$$+ (\nu\phi^2(m\pi/2)^2-\gamma_m^2)\sinh\gamma_m\}$$

Quadrant (1,2):

For m=1,3,5,.......,2k-1 ; n=1,3,5,.......,2k-1

$$A_{(m+1)/2,(n+1)/2+k} = 2\sin(m\pi/2)\sin(n\pi/2)(x_1-x_2)/\theta_{11n}$$

where

$$x_1 = \{(n\pi/2)^2 - \nu\phi^2\beta_n^2\}\theta_{1n}\beta_n\cosh\beta_n/\{\beta_n^2 + (m\pi/2)^2\} -$$

$$x_2 = \{(n\pi/2)^2 + \nu\phi^2\gamma_n^2\}\gamma_n\cos\gamma_n/\{\gamma_n^2 - (m\pi/2)^2\}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$A_{(m+1)/2,(n+1)/2+k} = 2\sin(m\pi/2)\sin(n\pi/2)(x_1+x_2)/\theta_{22n}$$

where

$$x_1 = \{(n\pi/2)^2 - \nu\phi^2\beta_n^2\}\theta_{2n}\beta_n\cosh\beta_n/\{\beta_n^2 + (m\pi/2)^2\}$$

$$x_2 = \{(n\pi/2)^2 - \nu\phi^2\gamma_n^2\}\gamma_n\cosh\gamma_n/\{\gamma_n^2 + (m\pi/2)^2\}$$

m=1	3	5	n=1	3	5	p=1	3	5	
–	0	0	–	–	–	–	–	–	
0	(1,1) –	0	–	(1,2) –	–	–	(1,3) –	–	Eqn. 2.68
0	0	–	–	–	–	–	–	–	
–	–	–	–	0	0	–	0	0	
–	(2,1) –	–	0	(2,2) –	0	0	(2,3) –	0	Eqn. 2.67
–	–	–	0	0	–	0	0	–	
–	–	–	–	0	0	-1	0	0	
–	(3,1) –	–	0	(3,2) –	0	0	(3,3) -1	0	Eqn. 2.66
–	–	–	0	0	–	0	0	-1	
	E_m			E_n			E_p		

Figure 2.11. Matrix schematic representation for the antisymmetric mode problem.

Quadrant (1,3):

For m=1,3,5,.......,2k-1 ; p=1,3,5,.......,2k-1

$$A_{(m+1)/2,(p+1)/2+2k} = 2\sin(p\pi/2)(m\pi/2)(x_1+x_2)/\theta_{11p}$$

where

$$x_1 = \{(p\pi/2)^2 - \nu\phi^2\beta_p^2\}\cosh\beta_p/\{\beta_p^2 + (m\pi/2)^2\}$$

$$x_2 = \{(p\pi/2)^2 + \nu\phi^2\gamma_p^2\}\theta_{1p}\cos\gamma_p/\{\gamma_p^2 - (m\pi/2)^2\}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$

$$= 2\sin(p\pi/2)(m\pi/2)(x_1-x_2)/\theta_{22p}$$

where

$$x_1 = \{(p\pi/2)^2 - \nu\phi^2\beta_p^2\}\cosh\beta_p/\{\beta_p^2 + (m\pi/2)^2\}$$

$$x_2 = \{(p\pi/2)^2 - \nu\phi^2\gamma_p^2\}\theta_{2p}\cosh\gamma_p/\{\gamma_p^2 + (m\pi/2)^2\}$$

Quadrant (2,1):

For m=1,3,5,.......,2k-1 ; n=1,3,5,.......,2k-1

$$A_{(n+1)/2+k,(m+1)/2} = 2\sin(m\pi/2)\sin(n\pi/2)(x_1-x_2)/\theta_{11m}$$

where

$$X_1 = \{(m\pi/2)^2 - \nu\phi_I^2\beta_m^2\}\theta_{1m}\beta_m\cosh\beta_m/\{\beta_m^2 + (n\pi/2)^2\}$$

$$x_2 = \{(m\pi/2)^2 + \nu\phi_I^2\gamma_m^2\}\gamma_m\cos\gamma_m/\{\gamma_m^2 - (n\pi/2)^2\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$A_{(n+1)/2+k,(m+1)/2} = 2\sin(m\pi/2)\sin(n\pi/2)(x_1+x_2)/\theta_{22m}$$

where

$$x_1 = \{(m\pi/2)^2 - \nu\phi_I^2\beta_m^2\}\theta_{2m}\beta_m\cosh\beta_m/\{\beta_m^2 + (n\pi/2)^2\}$$

$$x_2 = \{(m\pi/2)^2 - \nu\phi_I^2\gamma_m^2\}\gamma_m\cosh\gamma_m/\{\gamma_m^2 + (n\pi/2)^2\}$$

Quadrant (2,2):

For n=1,3,5,.......,2k-1

$$A_{(n+1)/2+k,(n+1)/2+k} = \{(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2)\theta_{1n}\sinh\beta_n$$

$$+ (\nu\phi_I^2(n\pi/2)^2 + \gamma_n^2)\sin\gamma_n\}/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= \{(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2)\theta_{2n}\sinh\beta_n$$

$$+ (\nu\phi_I^2(n\pi/2)^2 - \gamma_n^2)\sinh\gamma_n\}/\theta_{22n}$$

Quadrant (2,3):

For p=1,3,5,.......,2k-1

$$A_{(p+1)/2+k,(p+1)/2+2k} = \{\nu\phi_I^2(p\pi/2)^2(1-\theta_{1p}) - \beta_p^2 - \theta_{1p}\gamma_p^2\}/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$

$$A_{(p+1)/2+k,(p+1)/2+2k} = \{\nu\phi_I^2(p\pi/2)^2(1-\theta_{2p}) - \beta_p^2 + \theta_{2p}\gamma_p^2\}/\theta_{22p}$$

Quadrant (3,1):

For $m=1,3,5,\ldots\ldots.,2k-1$; $n=1,3,5,\ldots\ldots.2k-1$

$$A_{(n+1)+2k,(m+1)/2} = 2(m\pi/2)\sin(n\pi/2)(x_1-x_2)/\theta_{11m}$$

where

$$x_1 = \theta_{1m}\beta_m\cosh\beta_m/\{\beta_m^2 + (n\pi/2)^2\}$$

$$x_2 = \gamma_m\cos\gamma_m/\{\gamma_m^2 - (n\pi/2)^2\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= 2(m\pi/2)\sin(n\pi/2)(x_1+x_2)/\theta_{22m}$$

where

$$x_1 = \theta_{2m}\beta_m\cosh\beta_m/\{\beta_m^2 + (n\pi/2)^2\}$$

$$x_2 = \gamma_m\cosh\gamma_m/\{\gamma_m^2 + (n\pi/2)^2\}$$

Quadrant (3,2):

For $n=1,3,5,\ldots\ldots.,2k-1$

$$A_{(n+1)/2+2k,(n+1)/2+k} = (\theta_{1n}\beta_n + \gamma_n)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$A_{(n+1)/2+2k,(n+1)/2+k} = (\theta_{2n}\beta_n + \gamma_n)/\theta_{22n}$$

Quadrant (3,3):

For $p=1,3,5,\ldots\ldots,2k-1$

$$A_{(p+1)/2+2k,(p+1)/2+2k} = -1$$

We now feed the above information to a digital computer through specially designed program 3 of Appendix 2 to generate appropriate eigenvalues, and through program 4 of the same appendix to generate mode shapes. Results obtained will be tabulated and discussed in the following paragraph.

<u>Eigenvalues for the Cantilever Plate</u>

An eigenvalue search was conducted for the cantilever plate problem using programs 1 and 3 of Appendix 2. Results are tabulated in Tables 2.1 and 2.2. Table 2.1 shows eigenvalues for the first five symmetric vibration modes. It must be recalled that analyses were made for half the plate where aspect ratio is denoted ϕ' and equal b/a. The full plate has dimensions 2b by a as indicated in Figure 2.1, and its aspect ratio is denoted ϕ where $\phi=2\phi'=2b/a$.

Earlier in this chapter we have said that this particular problem, the problem of the free vibration of the rectangular cantilever plate, has been discussed and analyzed by Gorman in earlier publications using a set of building blocks different than the set used in the present analysis. We have also said that our intention is not to repeat the work that has already been done but to compare our results with those of Gorman's, and to try to show that one has the complete freedom in choosing any appropriate set of building blocks as long as their superposition can lead to the prescribed boundary conditions of the plate under study, and as long as no nonlinearity is introduced in any of the boundary conditions. Results found by Gorman are shown in Tables 2.3 and 2.4, where it is seen that the above are satisfied.

Tables 2.1 through 2.4 show a very good agreement between the two sets of results. In fact, both sets are identical as expected. The insignificant difference in some of the higher modes is attributed to computational errors and/or to a difference in the number of terms used in the series involved. However, some building blocks have the advantage of simplicity and ease in arriving at their individual solutions over the others. As one becomes more involved, his ability in choosing those blocks improves. Simplicity and ease of solution are not the only differences one should look for. Another rather significant advantage is the fact that no rejection modes are possible when using the set of building blocks discussed in the present analysis. Rejection modes are modes of zero net displacement and yet nontrivial solutions for the unknown coefficients are indicated. For a full understanding of these modes, one is referred to Gorman's work(20).

Mode shape data for the symmetric and antisymmetric vibration modes for the rectangular cantilever plate may easily be obtained using programs 3 and 4 of Appendix 2. Also in Appendix 3, a full set of mode shape graphics corresponding to the set of eigenvalues of Tables 2.1 and 2.2 is found. All of these mode shapes enable the analyst to locate in advance those areas of the plate under consideration that are likely to undergo high displacement when vibrating in a specific mode, not to mention their usefulness in helping to establish, in advance, those areas where the highest bending stresses are likely to be encountered.

The task of determining the vibration circular frequencies for a given plate from known eigenvalues is easily accomplished as illustrated in the following example.

Example 2.1- Let us consider the plate of Figure 2.12 where the following are given; $E=10.0 \times 10^6$ psi. , $\rho=9.75 \times 10^{-2}$ lb.mass/in^3

and ν=0.333 , h=0.125 , and a=b=10in. Determine the second lowest symmetric and antisymmetric vibration mode frequencies.

Solution:

we have $\rho = (9.75\times10^{-2}\times0.125)/386 = 3.16\times10^{-5}\frac{lb}{in^2}\cdot\frac{sec^2}{in}$

where the gravitational constant g is taken to be 386 in./sec^2

we also have $D = \frac{Eh^3}{12(1-\nu^2)} = \frac{10.0\times10^6\times0.125^3}{12(1-0.333^2)} = 1690$ in.lb.

from Tables 2.1 and 2.2 we have for the second lowest symmetric mode λ^2 = 21.09. Therfore, from the relation

$$\omega = \frac{\lambda^2}{a^2}\sqrt{D/\rho} \qquad \text{.........(2.70)}$$

we have $\omega = \frac{21.09}{10\times10}\sqrt{(1690/3.16\times10^{-5})}$ =1542.3 rad/sec.

and $f = \omega/2\pi$ = 245.5 Hz.

for the second lowest antisymmetric mode λ^2 = 30.55, and

$$\omega = (1542.3\times30.55)/21.09 = 2234.1 \text{ rad/sec.}$$

and $f = \omega/2\pi$ = 355.6 Hz.

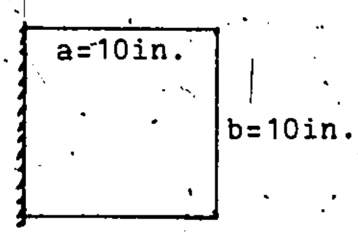


Figure 2.12. Square cantilever plate for Example 2.1

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ ($\nu = 0.333$)

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	3.395	3.406	3.402	3.437	3.448	3.459	3.470	3.477	3.487	3.492	3.496
2	21.20	21.24	21.28	21.32	21.31	21.09	18.18	14.26	10.03	7.497	6.738
3	59.60	59.73	59.75	53.06	38.93	27.07	22.21	21.88	21.78	21.36	16.73
4	117.3	117.1	92.70	61.55	60.63	53.54	43.81	37.96	31.07	22.96	21.93
5	191.9	143.4	117.7	84.45	68.25	61.12	60.95	56.03	33.88	28.52	26.53

Table 2.1. First five eigenvalues for the symmetric vibration modes of the cantilever plate of Figure 2.1.

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	20.68	17.58	14.50	11.43	9.897	8.356	7.115	6.288	5.278	4.711	4.368
2	65.07	56.14	47.33	38.71	34.53	30.55	27.58	25.59	18.84	13.63	10.73
3	118.0	104.3	91.25	79.29	73.87	63.62	42.76	31.30	24.55	23.45	22.69
4	183.7	166.7	151.1	135.3	99.03	70.64	66.07	56.14	42.66	34.37	25.43
5	265.2	246.2	228.1	143.7	134.1	92.21	69.97	65.17	52.43	36.44	32.10

Table 2.2. First five eigenvalues for the antisymmetric vibration modes of the cantilever plate of Figure 2.1.

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ $(\nu = 0.333)$

ϕ=2b/a	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
ϕ'=b/a	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	3.395	3.406	3.420	3.438	3.448	3.459	3.470	3.477	3.487	3.493	3.497
2	21.20	21.24	21.28	21.32	21.31	21.09	18.18	14.26	10.03	7.947	6.738
3	59.60	59.72	59.76	53.06	38.94	27.06	22.21	21.88	21.78	21.36	16.73
4	117.3	117.1	92.68	61.54	61.01	53.53	43.80	37.95	31.07	22.96	21.93
5	191.9	143.4	117.7	84.44	68.25	61.12	60.94	56.03	33.88	28.53	26.53

Table 2.3. First five eigenvalues for the symmetric vibration modes of the cantilever plate as found by Gorman(20).

ϕ=2b/a	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
ϕ'=b/a	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	20.68	17.58	14.50	11.43	9.897	8.356	7.115	6.288	5.278	4.711	4.368
2	65.07	56.14	47.32	38.70	34.53	30.55	27.58	25.59	18.84	13.66	10.73
3	118.0	104.3	91.24	79.29	73.86	63.62	42.76	31.30	24.55	23.45	22.69
4	183.7	166.7	151.2	135.3	99.03	70.64	66.07	56.14	42.66	34.37	25.43
5	265.2	245.5	228.1	143.7	134.1	92.21	69.97	65.17	52.43	36.44	32.10

Table 2.4. First five eigenvalues for the antisymmetric vibration modes of the cantilever plate as found by Gorman(20).

In this chapter we have discussed the method of superposition as it applies to the problem of free vibration of rectangular plates in general. We have also shown in great detail its application in the analysis of the free vibration of rectangular cantilever plates. Major steps followed in solving this problem are summarized below.

1- The introduction of the building blocks to be employed in the analysis, making sure that the governing differential equation is satisfied exactly by each of these building blocks, and that their superposition may lead to a solution that will satisfy the prescribed boundary conditions of the plate under study to any desired degree of accuracy by increasing the number of terms in the series involved. One must be careful not to introduce any nonlinearity.

2- Establish a Lévy-type solution for each of the building blocks involved.

3- Establish the contribution of each of these solution to the boundary conditions of the plate under study.

4- Expand the relevent expressions in one type of Fourier series. Adjust their coefficients to satisfy the prescribed boundary conditions they represent, and in so doing we develop a set of nk homogeneous algebraic equations relating the nk coefficients. Where n is the number of blocks involved, and k is the number of terms used in each of the series involved. k is dictated by the degree of accuracy required.

5- Handle these nk equations by matrix techniques to establish the required number of eigenvalues.

6- solve for the unknown coefficients by arbitrarily setting one of them to unity, and using well known computer techniques to solve for the remaining nk-1, using any nk-1 non homogenious equations formed from the previous n homogeneous equations we had.

7- Using the computed values of these coefficients, generate the various mode shapes.

We have come now to the end of the second chapter of this thesis. In the following chapter we will look at the problem of the rectangular cantilever plate with symmetric point supports along the edges. Although, the above seven steps will be followed in analysing this family of plates, a new function, known as the Dirac Delta Function will have to be introduced to deal with the problem of the point supports.

Chapter 3

THE CANTILEVER PLATE
WITH EDGE POINT SUPPORTS

In Chapter one we discussed the general theory of plates, we also discussed the introduction of the method of superposition to dynamic plate problems. In Chapter two we went a step further to show the application of the concept of superposition in analyzing the free vibration problem of the cantilever plate. In the present Chapter an even further step will be taken by introducing point supports on the boundary of the cantilever plate of Chapter two. These point supports are assumed to be symmetric with respect to the ξ axis as shown in Figure 3.1 and Figure 3.2.

Cantilever plates with point supports will be divided into three distinct families. The first family being that of Figure 3.1 where point supports are situated on edges parallel to the ξ axis. The second family is that of Figure 3.2 where point supports are located on the edge parallel to the η axis. And the third family is that of the cantilever plate with lateral point supports. The first two families will be analyzed fully in the present chapter, while the third will be the subject of the next chapter.

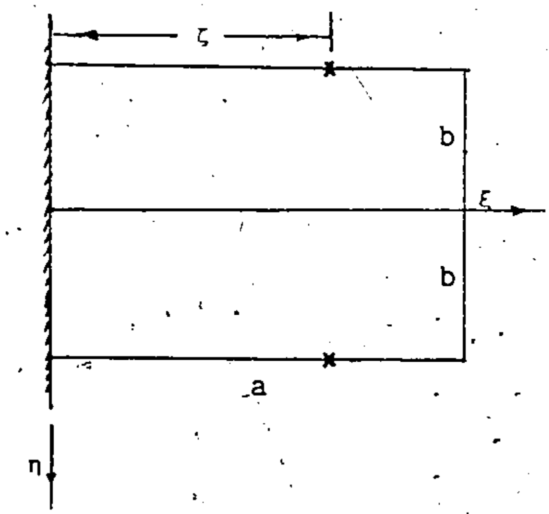


Figure 3.1. The cantilever plate with point supports along edges parallel to the ξ axis. x indicates a point support.

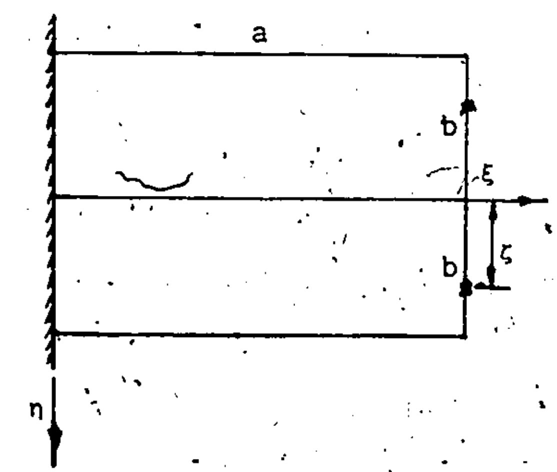


Figure 3.2. The cantilever plate with point supports along the edge parallel to the η axis. x indicates a point support.

Analysis of The First Family

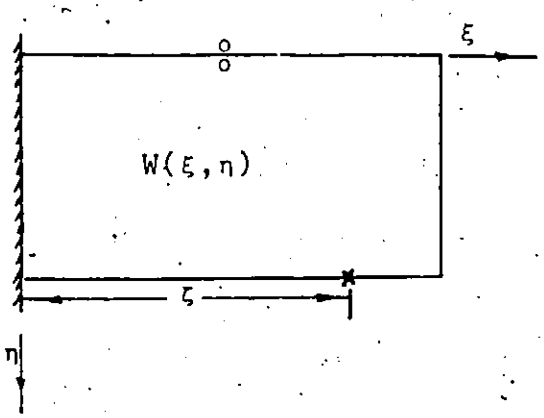
Due to the symmetry of the point supports with respect to the ξ axis of Figure 3.1, physical reasoning tells us that all free vibrations of this plate will either be symmetric or antisymmetric with respect to this axis. Each of these two families of vibration modes will be analyzed separately.

Symmetric Modes

Due to the symmetry that exist about the ξ axis, only half the plate of Figure 3.1 need to be analyzed, as shown in Figures 3.3 and 3.5. It will be noted that building blocks of Figure 3.3 are exactly the same as those of Figure 2.2 with the exception of the fourth building block of Figure 3.3 which is added here to account for the effect of the point support. Consequently, solutions for the first three building blocks of Figure 3.3 are given by Equations 2.12, 2.17, and 2.28. Therefore, our attention is focused on the solution of the fourth building block.

The Fourth Building Block:

Concentrating on the last building block of Figure 3.3, we note that the solid dots indicate that slope taken



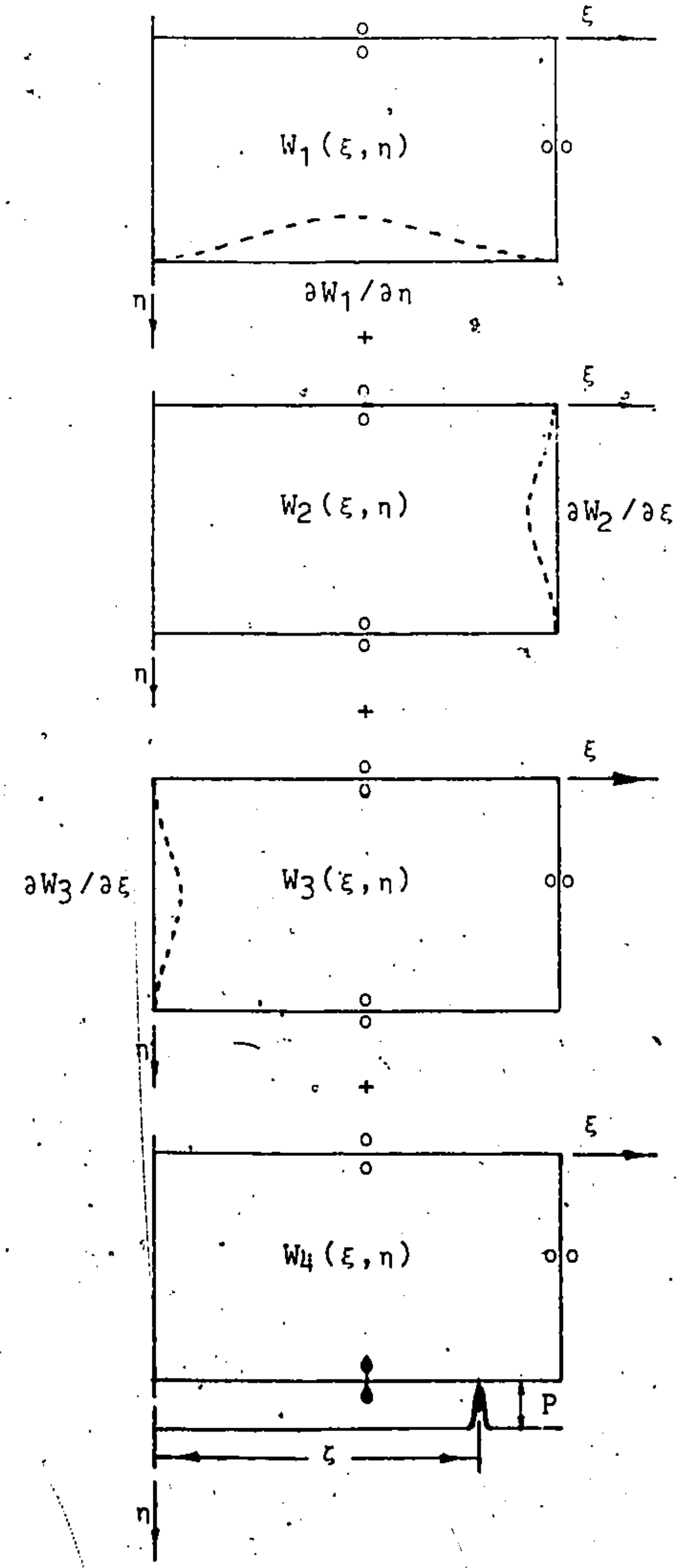


Figure 3.3. Building blocks for analyzing the symmetric modes of the cantilever plate with point supports symmetrically located on opposite edges.

normal to the edge is forbidden. We also note that the force exerted on the plate by the point support is represented schematically as a Dirac Function just below this fourth block. The Lévy-type solution of this building block may be written as[1].

$$W_4(\xi,\eta) = \sum_{l=1,3,5}^{\infty} Y_l(\eta)\sin\frac{l\pi\xi}{2} \qquad ....(3.1)$$

It can easily be shown that each term of the above solution satisfies exactly the boundary conditions along the edges $\xi=0$ and $\xi=1$. Due to the symmetry which $Y(\eta)$ must have with respect to the edge $\eta=0$, we may write $Y_l(\eta)$ as

$$Y_l(\eta) = A_l\cosh\beta_l\eta + D_l\cos\gamma_l\eta$$

or if $\lambda^2 < (l\pi/2)^2$ $\qquad ....(3.2)$

$$= A_l\cosh\beta_l\eta + D_l\cosh\gamma_l\eta$$

where

$$\beta_l = \phi\sqrt{\lambda^2+(l\pi/2)^2}$$

and

$$\gamma_l = \phi\sqrt{\lambda^2-(l\pi/2)^2} \quad \text{or} \quad \phi\sqrt{(l\pi/2)^2-\lambda^2}$$

which ever is real.

The next step is to eliminate A_l and D_l. Enforcing the condition of zero slope along the edge $\eta=1$ we have

1.- Note the difference between the EL (l) and the ONE (1).

$$A_1\beta_1\sinh\beta_1 - D_1\gamma_1\sin\gamma_1 = 0$$

or if $\lambda^2 < (1\pi/2)^2$ ....(3.3)

$$A_1\beta_1\sinh\beta_1 + D_1\gamma_1\sinh\gamma_1 = 0$$

Eliminating the unknown D_1 from Equation 3.2 we write :

$$Y_1(\eta) = A_1(\cosh\beta_1\eta + \theta_{11}\cos\gamma_1\eta)$$

or if $\lambda^2 < (1\pi/2)^2$ ....(3.4)

$$= A_1(\cosh\beta_1\eta + \theta_{21}\cosh\gamma_1\eta)$$

where

$$\theta_{11} = \beta_1\sinh\beta_1/(\gamma_1\sin\gamma_1)$$

$$\theta_{21} = \beta_1\sinh\beta_1/(\gamma_1\sinh\gamma_1)$$

The other edge condition is the application of a time varying concentrated force of amplitude P at a distance ζ from the η axis. This load varies sinusoidally with time with the same frequency as the vibrating plate, and represents the force that the point support exerts on the plate. Expanding this concentrated edge reaction in a Dirac sine series we have (20)

$$V(\xi) = 2P\sum_{m=1,3,5}^{\infty}\sin\frac{1\pi\zeta}{2}\sin\frac{1\pi\xi}{2} \qquad ....(3.5)$$

Introducing the dimensionless force amplitude $P^* = Pb^3/Da$, Equation 3.5 may be written as

$$\frac{V(\xi)b^3}{Da} = 2P^*\sum_{m=1,3,5}^{\infty} \sin\frac{l\pi\zeta}{2}\sin\frac{l\pi\xi}{2} \qquad \text{........(3.6)}$$

From Equation 1.20 we have

$$\frac{V(\xi)b^3}{Da} = -\left(\frac{\partial^3 W_l}{\partial\eta^3} + \nu^*\phi^2\frac{\partial^3 W_l}{\partial\eta\partial\xi^2}\right)\Bigg|_{\eta=1} \qquad \text{........(3.7)}$$

where ϕ of the left hand side of Equation 1.20 has been replaced by b/a. Equating the right hand side of Equations 3.6 and 3.7, performing the necessary derivations, and solving, it is easily shown that

$$Y_l(\eta) = P^*\phi_{111}(\cosh\beta_1\eta + \theta_{11}\cos\gamma_1\eta)$$

or if $\lambda^2 < (l\pi/2)^2$ ....(3.8)

$$= P^*\phi_{221}(\cosh\beta_1\eta + \theta_{21}\cosh\gamma_1\eta)$$

where

$$\theta_{111} = \frac{-2\sin(l\pi\zeta/2)}{\beta_1\{\beta_1^2-\nu^*\phi^2(l\pi/2)^2\}\sinh\beta_1+\theta_{11}\gamma_1\{\gamma_1^2+\nu^*\phi^2(l\pi/2)^2\}\sin\gamma_1}$$

and

$$\theta_{221} = \frac{-2\sin(l\pi\zeta/2)}{\beta_1\{\beta_1^2-\nu^*\phi^2(l\pi/2)^2\}\sinh\beta_1+\theta_{21}\gamma_1\{\gamma_1^2-\nu^*\phi^2(l\pi/2)^2\}\sinh\gamma_1}$$

Therefore, ..

$$W_4(\xi,\eta) = \sum_{l=1,3,5}^{k^*} P^*\theta_{11l}(\cosh\beta_l\eta + \theta_{1l}\cos\gamma_l\eta)\sin\frac{l\pi\xi}{2}$$

$$+ \sum_{l=k^*+2}^{\infty} P^*\theta_{22l}(\cosh\beta_l\eta + \theta_{2l}\cosh\gamma_l\eta)\sin\frac{l\pi\xi}{2} \qquad \text{.........(3.9)}$$

We next turn to the contribution of each of the building blocks on the right hand side of Figure 3.3 to the prescribed boundary conditions of the plate on the left hand side of this figure. These boundary conditions are exactly the same as those of the plate on the left hand side of Figure 2.2 with the exception of the point support where displacement is forbidden. Hence Equations 2.29 through 2.37 provide contributions of the first three building blocks to bending moments along the edges $\xi=1$ and $\eta=1$ as well as their contribution to the slope at $\xi=0$. We now evaluate their contribution to displacement at the point support.

The contribution of each term of $W_1(\xi,\eta)$ to displacement at $\eta=1$ and $\xi=\zeta$ is

$$\frac{E_m}{\theta_{11m}}(\cosh\beta_m + \theta_{1m}\cos\gamma_m)\sin\frac{m\pi\zeta}{2}$$

or if $\lambda^2 < (m\pi/2)^2$ ....(3.10)

$$\frac{E_m}{\theta_{22m}}(\cosh\beta_m + \theta_{2m}\cosh\gamma_m)\sin\frac{m\pi\zeta}{2}$$

The contribution of each term of $W_2(\xi,\eta)$ to displacement at $\eta=1$ and $\xi=\zeta$ is

$$\frac{E_n}{\theta_{11n}}(\sinh\beta_n\zeta + \theta_{1n}\sin\gamma_n\zeta)\cos(n\pi)$$

or if $\lambda^2\phi^2 < (n\pi)^2$ ....(3.11)

$$\frac{E_n}{\theta_{22n}}(\sinh\beta_n\zeta + \theta_{2n}\sinh\gamma_n\zeta)\cos(n\pi)$$

The contribution of each term of $W_3(\xi,\eta)$ to displacement at $\eta=1$ and $\xi=\zeta$ is

$$\frac{E_p}{\theta_{11p}}\{\cosh\beta_p(1-\zeta) - \theta_{1p}\cos\gamma_p(1-\zeta)\}\cos(p\pi)$$

or if $\lambda^2\phi^2 < (p\pi)^2$ ....(3.12)

$$\frac{E_p}{\theta_{22p}}\{\cosh\beta_p(1-\zeta) - \theta_{2p}\cosh\gamma_p(1-\zeta)\}\cos(p\pi)$$

We remain with the task of determining the contribution of the fourth building block to the prescribed boundary conditions. Starting with the contribution of each term of $W_4(\xi,\eta)$ to bending moments along the edge $\xi=1$ we write

$$\frac{M_\xi a}{D} = -\left(\frac{\partial^2 W_4}{\partial \xi^2} + \frac{\nu}{\phi^2}\frac{\partial^2 W_1}{\partial \eta^2}\right)\bigg|_{\xi=1}$$

$$= P^*\theta_{11l}\sin\frac{l\pi}{2}\{((l\pi/2)^2 - \nu\phi_I^2\beta_l^2)\cosh\beta_l\eta$$

$$+ \theta_{1l}((l\pi/2)^2 + \nu\phi_I^2\gamma_l^2)\cos\gamma_l\eta\}$$

or if $\lambda^2 < (l\pi/2)^2$ ....(3.13)

$$= P^*\theta_{22l}\sin\frac{l\pi}{2}\{((l\pi/2)^2 - \nu\phi_I^2\beta_l^2)\cosh\beta_l\eta$$

$$+ \theta_{2l}((l\pi/2)^2 - \nu\phi_I^2\gamma_l^2)\cosh\gamma_l\eta\}$$

The contribution to slope at $\xi=0$ of each term of $W_4(\xi,\eta)$ is

$$W_4(\xi,\eta)/\partial\xi = P^*\theta_{11l}(l\pi/2)(\cosh\beta_l\eta + \theta_{1l}\cos\gamma_l\eta)$$

or if $\lambda^2 < (l\pi/2)^2$ ....(3.14)

$$= P^*\theta_{22l}(l\pi/2)(\cosh\beta_l\eta + \theta_{2l}\cosh\gamma_l\eta)$$

The contribution to bending moment along the edge $\eta=1$ of each term of $W_4(\xi,\eta)$ is

$$\frac{M_\eta\phi b}{D} = -\left(\frac{\partial^2 W_4}{\partial \eta^2} + \nu\phi^2\frac{\partial^2 W_4}{\partial \xi^2}\right)\bigg|_{\eta=1}$$

$$= P^*\theta_{11l}\{(\nu\phi^2(l\pi/2)^2 - \beta_l^2)\cosh\beta_l$$

$$+ \theta_{1l}(\nu\phi^2(l\pi/2)^2 + \gamma_l^2)\cos\gamma_l\}\sin\frac{l\pi\xi}{2}$$

or if $\lambda^2 < (l\pi/2)^2$ ....(3.15)

$$\frac{M_\eta \phi b}{D} = P^*\theta_{22l}\{(\nu\phi^2(l\pi/2)^2 - \beta_l^2)\cosh\beta_l + \theta_{2l}(\nu\phi^2(l\pi/2)^2 - \gamma_l^2)\cosh\gamma_l\}\sin\frac{l\pi\xi}{2}$$

The contribution to displacement at $\eta=1$ and $\xi=\zeta$ of $W_4(\xi,\eta)$ is

$$W_4(\zeta,1) = \sum_{l=1,3,5}^{k^*} P^*\theta_{11l}(\cosh\beta_l + \theta_{1l}\cos\gamma_l)\sin\frac{l\pi\zeta}{2} + \sum_{l=k^*+2}^{\infty} P^*\theta_{22l}(\cosh\beta_l + \theta_{2l}\cosh\gamma_l)\sin\frac{l\pi\zeta}{2} \qquad (3.16)$$

Next, in order to satisfy the prescribed boundary conditions of the plate under study we write

1.- The net bending moment along the edge $\xi=1$ is zero.

Equation 2.29 + Equation 2.32 + Equation 2.35 + Equation 3.13 = 0 ....(3.17)

2.- The net slope at =0 is zero. Or

Equation 2.30 + Equation 2.33 + Equation 2.36 + Equation 3.14 = 0 ....(3.18)

3.- The net bending moment along the edge $\eta=1$ is zero.

Or

Equation 2.31 + Equation 2.34 + Equation 2.37 + Equation 3.15 = 0 ....(3.19)

4.- The net displacement at $\eta=1$ and $\xi=\zeta$ is also zero.

Or

Equation 3.10 + Equation 3.11 + Equation 3.12 + Equation 3.16 = 0 ....(3.20)

We now focus our attention on solving Equations 3.17 through 3.20 using the same matrix techniques we have used in chapter two. Note that the required matrix is generated simply by adding one column and one row to the matrix of Figure 2.6. The additional column represents the coefficients of P* and the additional row represents the displacement at the point support. The resulting matrix is shown schematically by Figure 3.4. Elements of the last column and row are as follow.

Quadrant (1):

For $l=1,3,5,\ldots\ldots.2k-1$

$$A_{(l+1)/2,3k+1} = \theta_{1l}\{(\nu\phi^2(l\pi/2)^2 - \beta_l^2)\cosh\beta_l + \theta_{1l}(\nu\phi^2(l\pi/2)^2 + \gamma_l^2)\cos\gamma_l\}$$

or if $\lambda^2 < (l\pi/2)^2$

$$A_{(l+1)/2,3k+1} = \theta_{22l}\{(\nu\phi^2(l\pi/2)^2 - \beta_l^2)\cosh\beta_l + \theta_{2l}(\nu\phi^2(l\pi/2)^2 - \gamma_l^2)\cosh\gamma_l\}$$

Quadrant (2):

For $l=1,3,5,\ldots\ldots.2k-1$; $n=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{k+n+1,3k+1} = \sum_{l=1,3,5}^{k^*} 2\theta_{11l}\sin(l\pi/2)(x_1+x_{11}) + \sum_{l=k^*+2}^{2k-1} 2\theta_{22l}\sin(l\pi/2)(x_2+x_{22})$$

where

$$x_1 = \{(l\pi/2)^2 - \nu\phi_I^2\beta_l^2\}\beta_l\cos(n\pi)\sinh\beta_l/\{\beta_l^2 + (n\pi)^2\}$$

$$x_{11} = \{(l\pi/2)^2 + \nu\phi_I^2\gamma_l^2\}\gamma_l\theta_{1l}\sin\gamma_l\cos(n\pi)/\{\gamma_l^2 - (n\pi)^2\}$$

$$x_2 = \{(l\pi/2)^2 - \nu\phi_I^2\beta_l^2\}\beta_l\cos(n\pi)\sinh\beta_l/\{\beta_l^2 + (n\pi)^2\}$$

$$x_{22} = \{(l\pi/2)^2 - \nu\phi_I^2\gamma_l^2\}\theta_{2l}\gamma_l\cos(n\pi)\sinh\gamma_l/\{\gamma_l^2 + (n\pi)^2\}$$

the above must be divided by 2 if n=0.

Quadrant (3):

For $l=1,3,5,\ldots\ldots.2k-1$; $n=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{2k+n+1,3k+1} = \sum_{l=1,3,5}^{k^*} 2\theta_{11l}\cos(n\pi)(l\pi/2)(x_1+x_{11})$$

$$+\sum_{l=k^*+2}^{2k-1} 2\theta_{22l}\cos(n\pi)(l\pi/2)(x_2+x_{22})$$

where

$$x_1 = \beta_l \sinh\beta_l / \{\beta_l^2 + (n\pi)^2\}$$

$$x_{11} = \gamma_l\,\theta_{11}\sin\gamma_l / \{\gamma_l^2 - (n\pi)^2\}$$

$$x_2 = \beta_l \sinh\beta_l / \{\beta_l^2 + (n\pi)^2\}$$

$$x_{22} = \gamma_l\,\theta_{21}\sinh\gamma_l / \{\gamma_l^2 + (n\pi)^2\}$$

the above must be divided by 2 if $n=0$.

Quadrant (4):

For $m=1,3,5,\ldots\ldots.2k-1$

$$A_{3k+1,(m+1)/2} = (\cosh\beta_m + \theta_{1m}\cos\gamma_m)\sin(m\pi\zeta/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= (\cosh\beta_m + \theta_{2m}\cosh\gamma_m)\sin(m\pi\zeta/2)/\theta_{22m}$$

Quadrant (5):

For n=0,1,2,........k-1

$$A_{3k+1,k+n+1} = (\sinh\beta_n\zeta + \theta_{1n}\sin\gamma_n\zeta)\cos(n\pi)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= (\sinh\beta_n\zeta + \theta_{2n}\sinh\gamma_n\zeta)\cos(n\pi)/\theta_{22n}$$

Quadrant (6)

For p=0,1,2,........k-1

$$A_{3k+1,2k+p+1} = \{\cosh\beta_p(1-\zeta) - \theta_{1p}\cos\gamma_p(1-\zeta)\}\cos(p\pi)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi)^2$

$$= \{\cosh\beta_p(1-\zeta) - \theta_{2p}\cosh\gamma_p(1-\zeta)\}\cos(p\pi)/\theta_{22p}$$

Quadrant (7):

For l=1,3,5,........2k-1

$$A_{3k+1,3k+1} = \sum_{l=1,3,5}^{k^*} \theta_{11l}(\cosh\beta_l + \theta_{1l}\cos\gamma_l)\sin\frac{l\pi\zeta}{2}$$

$$+ \sum_{l=k^*+2}^{2k-1} \theta_{22l}(\cosh\beta_l + \theta_{2l}\cosh\gamma_l)\sin\frac{l\pi\zeta}{2}$$

The above matrix is fed to a digital computer through programs 5 and 6 of Appendix 3 in order to generate eigenvalues that are listed in Table 3.1, and to generate mode shapes shown in Appendix 2. Program 5 may be used to generate eigenvalues for any combination of plate aspect ratio, point support position, and vibration mode. Also, program 6 can be used

m=1	3	5	n=0	1	2	p=0	1	2	l=1,3,5	
-	0	0	-	-	-	-	-	-	-	
0	-	0	-	-	-	-	-	-	-	Eqn. 3.19
0	0	-	-	-	-	-	-	-	-	
-	-	-	-	0	0	-	0	0	-	
-	-	-	0	-	0	0	-	0	-	Eqn. 3.17
-	-	-	0	0	-	0	0	-	-	
-	-	-	-	0	0	-1	0	0	-	
-	-	-	0	-	0	0	-1	0	-	Eqn. 3.18
-	-	-	0	0	-	0	0	-1	-	
-	-	-	-	-	-	-	-	-	-	Eqn. 3.20
	E_m			E_n			E_p		P^*	

Figure 3.4. Matrix schematic representation for the symmetric mode problem. Point support at $\eta=1$ and $\xi=\zeta$.

to generate corresponding mode shape data.

We turn next to the antisymmetric vibration modes of the present plate problem.

Antisymmetric Modes

Building blocks used in this analysis are shown in Figure 3.5, and are exactly the same as those of Figure 2.8 with the exception of the fourth block which is added to account for the point support condition. Therfore, solutions for the first three building blocks of Figure 3.5 are given by Equations 2.48, 2.49, and 2.55. Hence, we only need to solve for the fourth building block and the necessary contributions to the prescribed boundary conditions. Steps followed here are exactly the same as those followed in the analysis of the symmetric case of Figure 3.3.

Fourth Building Block:

The Lévy-type solution can be written as

$$W_4(\xi,\eta) = \sum_{l=1,3,5}^{\infty} Y_l(\eta)\sin\frac{l\pi\xi}{2}$$

It is relatively easy to show that each term of this solution

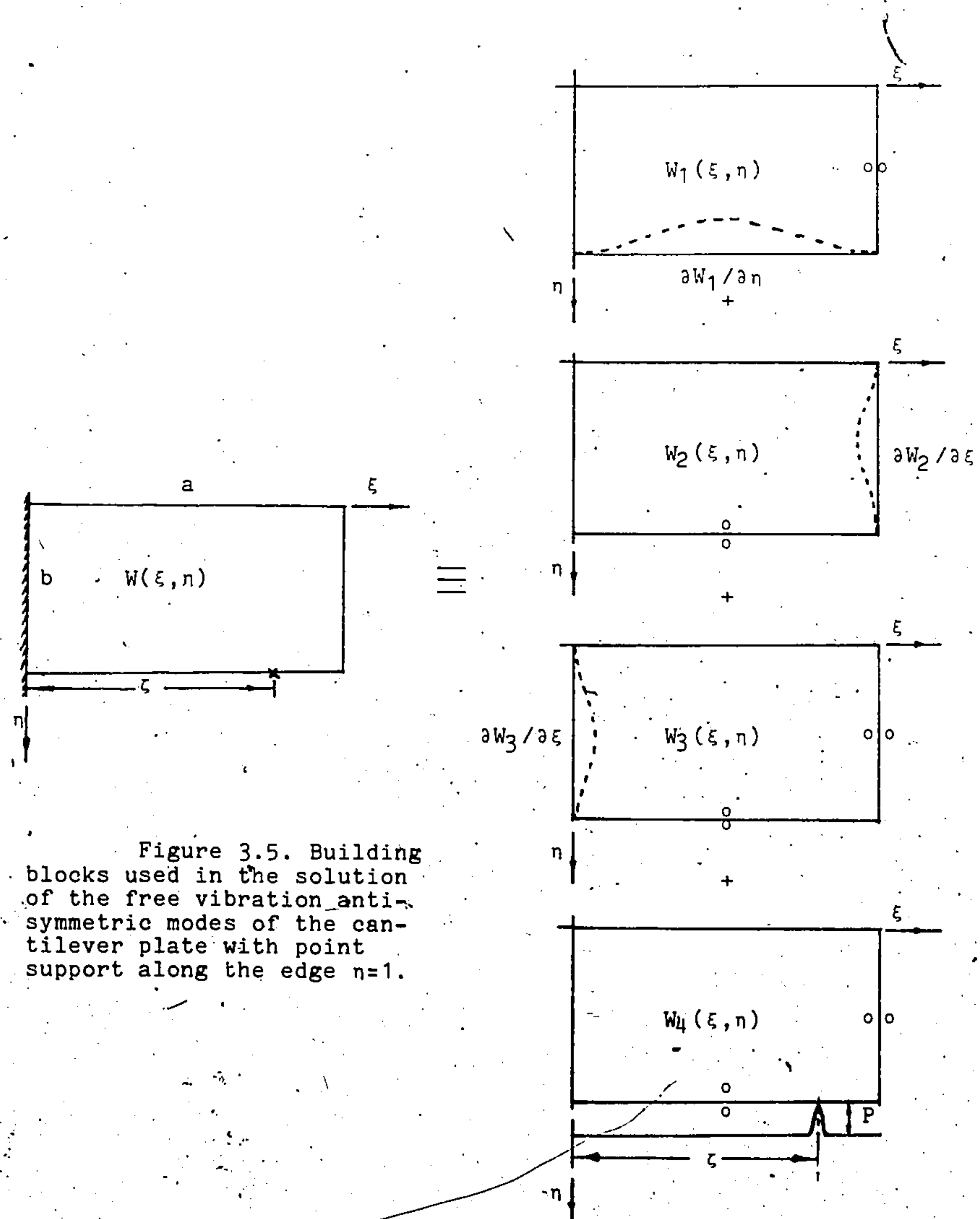


Figure 3.5. Building blocks used in the solution of the free vibration anti-symmetric modes of the cantilever plate with point support along the edge $\eta=1$.

satisfies exactly the boundary conditions for $\xi=0$ and $\xi=1$. In view of the antisymmetric nature of this block in the η direction we may write

$$Y_1(\eta) = B_1 \sinh\beta_1\eta + C_1 \sinh\gamma_1\eta$$

or if $\lambda^2 < (1\pi/2)^2$ ....(3.22)

$$= B_1 \sinh\beta_1\eta + C_1 \sinh\gamma_1\eta$$

where

$$\beta_1 = \phi\sqrt{\lambda^2+(1\pi/2)^2}$$

$$\gamma_1 = \phi\sqrt{\lambda^2-(1\pi/2)^2} \qquad \text{or} \qquad \phi\sqrt{(1\pi/2)^2-\lambda^2}$$

which ever is real.

The zero slope condition for $\eta=1$ leads to

$$B_1\beta_1\cosh\beta_1 + C_1\gamma_1\cos\gamma_1 = 0$$

or

$$B_1\beta_1\cosh\beta_1 + C_1\gamma_1\cosh\gamma_1 = 0$$

and

$$Y_1(\eta) = B_1(\sinh\beta_1\eta + \theta_{11}\sin\gamma_1\eta)$$

or ....(3.23)

$$Y_1(\eta) = B_1(\sinh\beta_1\eta + \theta_{21}\sinh\gamma_1\eta)$$

where

$$\theta_{11} = -\beta_1\cosh\beta_1/\gamma_1\cos\gamma_1$$

and

$$\theta_{21} = -\beta_l \cosh\beta_l / \gamma_l \cosh\gamma_l$$

Next, representing the vertical edge reaction along the edge $\eta=1$ by a Dirac function of the sin series type we have

$$\frac{V_\xi b^3}{Da} = 2P^* \sum_{l=1,3,5}^{\infty} \sin\frac{l\pi\zeta}{2} \sin\frac{l\pi\xi}{2} \quad \text{.........(3.24)}$$

where $\quad P^* = Pb^3/Da$

Equating the right hand side of Equation 3.24 to the right hand side of Equation 1.18 and solving we have

$$Y_l(\eta) = P^*\theta_{11l}(\sinh\beta_l\eta + \theta_{1l}\sin\gamma_l\eta)$$

or if $\quad \lambda^2 < (l\pi/2)^2$ ....(3.25)

$$Y_l(\eta) = P^*\theta_{22l}(\sinh\beta_l\eta + \theta_{2l}\sinh\gamma_l\eta)$$

where

$$\theta_{11l} = \frac{-2\sin(l\pi\zeta/2)}{\beta_l\{\beta_l^2-\nu^*\phi^2(l\pi/2)^2\}\cosh\beta_l - \theta_{1l}\gamma_l\{\gamma_l^2+\nu^*\phi^2(l\pi/2)^2\}\cos\gamma_l}$$

$$\theta_{22l} = \frac{-2\sin(l\pi\zeta/2)}{\beta_l\{\beta_l^2-\nu^*\phi^2(l\pi/2)^2\}\cosh\beta_l + \theta_{2l}\gamma_l\{\gamma_l^2-\nu^*\phi^2(l\pi/2)^2\}\cosh\gamma_l}$$

The contribution of this fourth building block to the prescribed boundary conditions are as follow.

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at the edge $\eta=1$ is

$$\frac{M_\eta \phi b}{D} = P^*\theta_{111}\{((1\pi/2)^2\nu\phi^2 - \beta_1^2)\sinh\beta_1 + \theta_{11}((1\pi/2)^2\nu\phi^2 + \gamma_1^2)\sin\gamma_1\}\sin\frac{1\pi\xi}{2}$$

or if $\lambda^2 < (1\pi/2)^2$

$$= P^*\theta_{221}\{((1\pi/2)^2\nu\phi^2 - \beta_1^2)\sinh\beta_1 + \theta_{21}((1\pi/2)^2\nu\phi^2 - \gamma_1^2)\sinh\gamma_1\}\sin\frac{1\pi\xi}{2}$$

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at $\xi=1$ is

$$\frac{M_\xi a}{D} = P^*\theta_{111}\sin\frac{1\pi}{2}\{((1\pi/2)^2 - \nu\phi_I^2\beta_1^2)\sinh\beta_1\eta + \theta_{11}((1\pi/2)^2 + \nu\phi_I^2\gamma_1^2)\sin\gamma_1\eta\}$$

or if $\lambda^2 < (1\pi/2)^2$

$$= P^*\theta_{221}\sin\frac{1\pi}{2}\{((1\pi/2)^2 - \nu\phi_I^2\beta_1^2)\sinh\beta_1\eta + \theta_{21}((1\pi/2)^2 - \nu\phi_I^2\gamma_1^2)\sinh\gamma_1\eta\}$$

The contribution of each term of $W_4(\xi,\eta)$ to slope at the edge $\xi=0$ is

$$\partial W_4/\partial\xi = P^*\theta_{11l}(l\pi/2)(\sinh\beta_l\eta + \theta_{1l}\sin\gamma_l\eta)$$

or if $\quad \lambda^2 < (l\pi/2)^2$

$$= P^*\theta_{22l}(l\pi/2)(\sinh\beta_l\eta + \theta_{2l}\sinh\gamma_l\eta)$$

The contribution of $W_4(\xi,\eta)$ to displacement at $\eta=1$ and $\xi=\zeta$ is

$$W_4(\zeta,1) = \sum_{l=1,3,5}^{k^*} P^*\theta_{11l}(\sinh\beta_l + \theta_{1l}\sin\gamma_l)\sin\frac{l\pi\zeta}{2}$$

$$+ \sum_{l=k^*+2}^{2k-1} P^*\theta_{22l}(\sinh\beta_l + \theta_{2l}\sinh\gamma_l)\sin\frac{l\pi\zeta}{2}$$

The necessary matrix is generated by adding an extra column and an extra row to that of Figure 2.11. The additional column is as follow.

Quadrant (1):

For $l=1,3,5,\ldots\ldots,2k-1$

$$A_{(l+1)/2,3k+1} = \theta_{11l}\{((l\pi/2)^2\nu\phi^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{1l}((l\pi/2)^2\nu\phi^2 + \gamma_l^2)\sin\gamma_l\}$$

or if $\lambda^2 < (l\pi/2)^2$

$$A_{(l+1)/2,3k+1} = \theta_{22l}\{((l\pi/2)^2\nu\phi^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{2l}((l\pi/2)^2\nu\phi^2 - \gamma_l^2)\sinh\gamma_l\}$$

Quadrant (2)

For $l=1,3,5,\ldots\ldots,2k-1$; $n=1,3,5,\ldots\ldots 2k-1$

$$A_{k+(n+1)/2,3k+1} = \sum_{l=1,3,5}^{k^*} 2\theta_{11l}\sin(l\pi/2)\sin(n\pi/2)(x_1-x_{11})$$

$$+ \sum_{l=k^*+2}^{2k-1} 2\theta_{22l}\sin(l\pi/2)\sin(n\pi/2)(x_2+x_{22})$$

where

$$x_1 = \{(l\pi/2)^2 - \nu\phi_I^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (n\pi/2)^2\}$$

$$x_{11} = \{(l\pi/2)^2 + \nu\phi_I^2\gamma_l^2\}\theta_{11}\gamma_I\cos\gamma_l/\{\gamma_l^2 - (n\pi/2)^2\}$$

$$x2 = \{(l\pi/2)^2 - \nu\phi_I^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (n\pi/2)^2\}$$

$$x_{22} = \{(l\pi/2)^2 - \nu\phi_I^2\gamma_l^2\}\theta_{21}\gamma_l\cosh\gamma_l/\{\gamma_l^2 + (n\pi/2)^2\}$$

Quadrant (3):

For $l=1,3,5,\ldots\ldots,2k-1$; $n=1,3,5,\ldots\ldots,2k-1$

$$A_{2k+(n+1)/2,3k+1} = \sum_{l=1,3,5}^{k^*} 2\theta_{11l}(l\pi/2)\sin(n\pi/2)(x_1-x_{11})$$

$$+ \sum_{l=k^*+2}^{2k-1} 2\theta_{22l}(l\pi/2)\sin(n\pi/2)(x_2+x_{22})$$

where

$$x_1 = \beta_l \cosh\beta_l/\{\beta_l^2 + (n\pi/2)^2\}$$

$$x_{11} = \theta_{1l}\gamma_l\cos\gamma_l/\{\gamma_l^2 - (n\pi/2)^2\}$$

$$x_2 = \beta_l\cosh\beta_l/\{\beta_l^2 + (n\pi/2)^2\}$$

$$x_{22} = \theta_{2l}\gamma_l\cosh\gamma_l/\{\gamma_l^2 + (n\pi/2)^2\}$$

The last row in the coefficient matrix is

Quadrant (4):

For m=1,3,5,.......,2k-1

$$A_{3k+1,(m+1)/2} = (\theta_{1m}\sinh\beta_m + \sin\gamma_m)\sin(m\pi\zeta/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= (\theta_{2m}\sinh\beta_m + \sinh\gamma_m)\sin(m\pi\zeta/2)/\theta_{22m}$$

Quadrant (5):

For $n=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,k+(n+1)/2} = (\theta_{1n}\sinh\beta_n\zeta + \sin\gamma_n\zeta)\sin(n\pi/2)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= (\theta_{2n}\sinh\beta_n\zeta + \sinh\gamma_n\zeta)\sin(n\pi/2)/\theta_{22n}$$

Quadrant (6):

For $p=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,2k+(p+1)/2} = \{\cosh\beta_p(1-\zeta) - \theta_{1p}\cos\gamma_p(1-\zeta)\}\sin(p\pi/2)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$

$$= \{\cosh\beta_p(1-\zeta) - \theta_{2p}\cosh\gamma_p(1-\zeta)\}\sin(p\pi/2)/\theta_{22p}$$

Quadrant (7):

For $l=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,3k+1} = \sum_{l=1,3,5}^{k^*} \theta_{11l}(\sinh\beta_l + \theta_{1l}\sin\gamma_l)\sin\frac{l\pi\zeta}{2}$$

$$+ \sum_{l=k^*+2}^{2k-1} \theta_{22l}(\sinh\beta_l + \theta_{2l}\sinh\gamma_l)\sin\frac{l\pi\zeta}{2}$$

We now have all the necessary information to generate the coefficient matrix and start our eigenvalue search. Using program 7, eigenvalues listed in Table 3.2 were found. Also mode shapes were generated using program 8 of Appendix 3. Results are shown in Appendix 2. In the remainder of this chapter we will look at the free vibration problem of the rectangular cantilever plate with point supports symmetrically located on the edge $\xi=1$ with respect to the ξ axis. This type of plate is referred to here as the second family.

Analysis of The Second Family

Once again only half the plate need to be examined. Also vibration modes will either be symmetric or antisymmetric with respect to the ξ axis. We choose to start with the symmetric family.

Symmetric Modes

Building blocks used in analyzing this family of vibration modes are shown in Figure 3.6, where it is seen that solutions W_1, W_2, and W_3 are readily available from Equations 2.12, 2.17, and 2.28 respectively. Therfore, we need only analyze the fourth building block in order to establish its solution W_4.

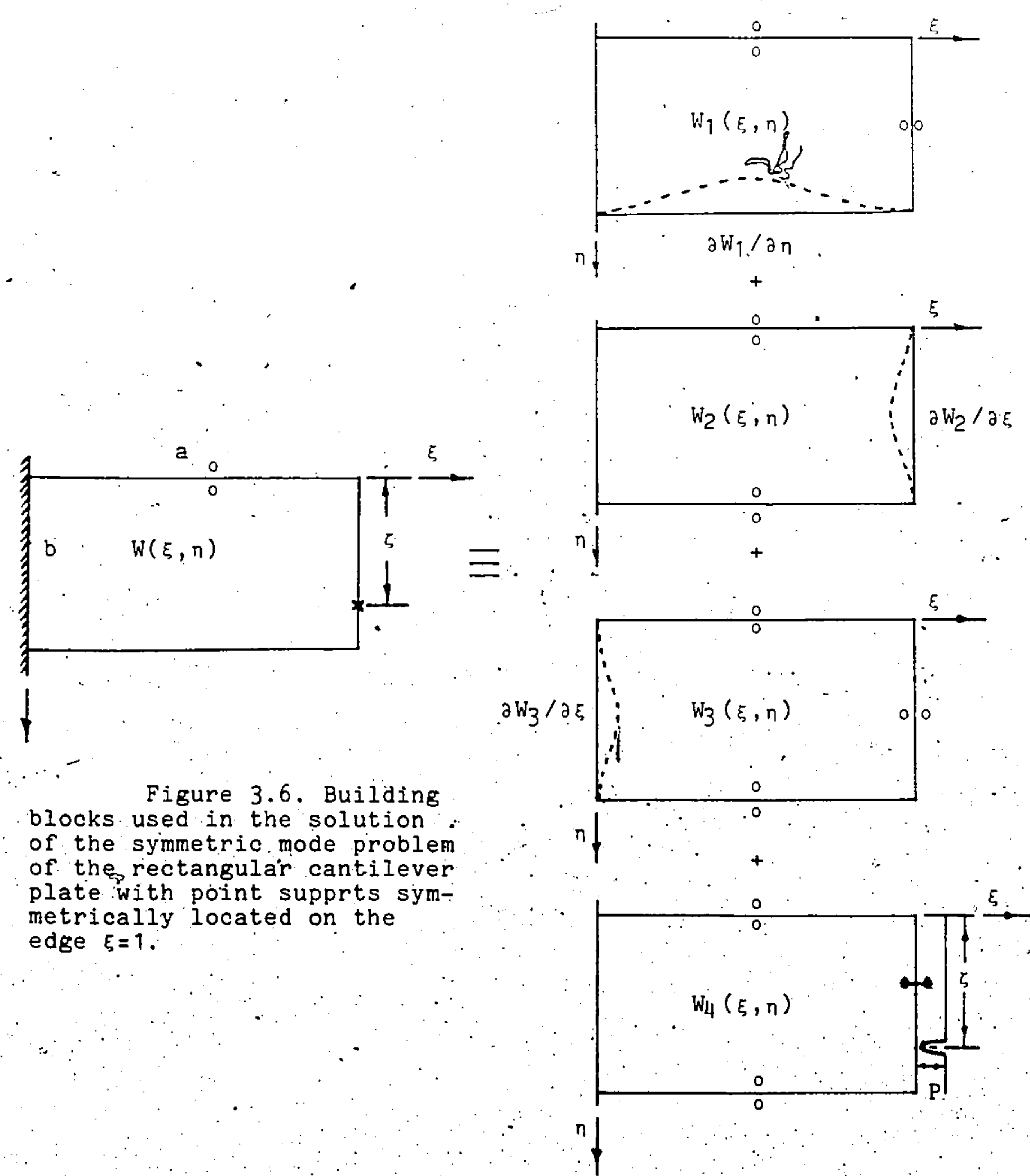


Figure 3.6. Building blocks used in the solution of the symmetric mode problem of the rectangular cantilever plate with point supprts symmetrically located on the edge $\xi=1$.

The Fourth Building Block:

Concentrating on the fourth block of Figure 3.6, and comparing it closely with the fourth building block of Figure 3.5, it becomes clear that the solution of the present block can be extracted from the readily existing solution of that of Figure 3.5. To obtain the thought solution, the following changes must be made.

1.- Change the sine function in the Lévy-type solution to a cosine function and change the quantities $(m\pi/2)$ to $(l\pi)$, and divide the first term by two.

2.- Interchange the variables ξ and η, replace the aspect ratio ϕ by its inverse $1/\phi$, andmultiply λ^2 by ϕ^2.

Therefore, $W_4(\xi,\eta)$ is

$$W_4(\xi,\eta) = \sum_{l=0,1,2}^{k^*} P^*\theta_{11l}(\sinh\beta_l\xi + \theta_{1l}\sin\gamma_l\xi)\cos(l\pi\eta)$$

$$+ \sum_{l=k^*+2}^{\infty} P^*\theta_{22l}(\sinh\beta_l\xi + \theta_{2l}\sinh\gamma_l\xi)\cos(l\pi\eta) \qquad \text{.............(3.26)}$$

where

$$\beta_l = \frac{1}{\phi}\sqrt{\phi^2\lambda^2+(l\pi)^2}$$

and

$$\gamma_l = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(l\pi)^2} \quad \text{or} \quad \frac{1}{\phi}\sqrt{(l\pi)^2-\lambda^2\phi^2}.$$

whichever is real, and

$$\theta_{1l} = -\beta_l \cosh\beta_l / \gamma_l \cos\gamma_l$$

$$\theta_{2l} = -\beta_l \cosh\beta_l / \gamma_l \cosh\gamma_l$$

$$\theta_{11l} = \frac{-2\cos(l\pi\zeta)}{\delta_l\{\beta_l(\beta_l^2-\nu^*\phi_I^2(l\pi)^2)\cosh\beta_l + \theta_{1l}\gamma_l(\gamma_l^2+\nu^*\phi_I^2(l\pi)^2)\cos\gamma_l\}}$$

$$\theta_{22l} = \frac{-2\cos(l\pi\zeta)}{\delta_l\{\beta_l(\beta_l^2-\nu^*\phi_I^2(l\pi)^2)\cosh\beta_l+\theta_{2l}\gamma_l(\gamma_l^2-\nu^*\phi_I^2(l\pi)^2)\cosh\gamma_l\}}$$

and where the first summation pertaining to terms for which $\lambda^2\phi^2 > (l\pi)^2$. And where $\delta_l=2$ if $l=0$, and $\delta_l=1$ if $l\neq0$.

Turning now to the contribution of $W_4(\xi,\eta)$ to the prescribed boundary conditions we write

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at the edge $\xi=1$ is

$$\frac{M_\xi a}{D} = P^*\theta_{11l}\{(\nu\phi_I^2(l\pi)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{1l}(\nu\phi_I^2(l\pi)^2 + \gamma_l^2)\sin\gamma_l\}\cos(l\pi\eta)$$

or if $\lambda^2\phi^2 < (l\pi)^2$ ....(3.27)

$$\frac{M_\xi a}{D} = P^*\theta_{221}\{(\nu\phi_I^2(l\pi)^2 - \beta_l^2)\sinh\beta_l + \theta_{21}(\nu\phi_I^2(l\pi)^2 - \gamma_l^2)\sinh\gamma_l\}\cos(l\pi\eta)$$

The contribution of each term of $W_4(\xi,\eta)$ to the slope at the edge $\xi=0$ is

$$\partial W_4/\partial\xi = P^*\theta_{111}(\beta_l + \theta_{11}\gamma_l)\cos(l\pi\eta)$$

or if $\lambda^2\phi^2 < (l\pi)^2$ ....(3.28)

$$= P^*\theta_{221}(\beta_l + \theta_{21}\gamma_l)\cos(l\pi\eta)$$

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at the edge $\eta=1$ is

$$\frac{M_\eta\phi b}{D} = P^*\theta_{111}\cos(l\pi)\{((l\pi)^2 - \nu\phi^2\beta_l^2)\sinh\beta_l\xi + \theta_{11}((l\pi)^2 + \nu\phi^2\gamma_l^2)\sin\gamma_l\xi\}$$

or if $\lambda^2\phi^2 < (l\pi)^2$ ....(3.29)

$$= P^*\theta_{221}\cos(l\pi)\{((l\pi)^2 - \nu\phi^2\beta_l^2)\sinh\beta_l\xi + \theta_{21}((l\pi)^2 - \nu\phi^2\gamma_l^2)\sinh\gamma_l\xi\}$$

The coefficient matrix is next generated simply by adding the following column and row to the matrix of Figure 2.6.

Quadrant (1):

For $l=0,1,2,\ldots\ldots\ldots,k-1$; $m=1,3,5,\ldots\ldots.2k-1$

$$A_{(m+1)/2,3k+1} = \sum_{l=0,1,2}^{k^*} 2\theta_{11l}\cos(l\pi)\sin(m\pi/2)(x_1-x_{11})$$
$$+\sum_{l=k^*+1}^{k-1} 2\theta_{22l}\cos(l\pi)\sin(m\pi/2)(x_2+x_{22})$$

where

$$x_1 = \{(l\pi)^2 - \nu\phi^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (m\pi/2)^2\}$$

$$x_{11} = \{(l\pi)^2 + \nu\phi^2\gamma_l^2\}\gamma_l\,\theta_{11}\cos\gamma_l/\{\gamma_l^2 - (m\pi/2)^2\}$$

$$x_2 = \{(l\pi)^2 - \nu\phi^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (m\pi/2)^2\}$$

$$x_{22} = \{(l\pi)^2 - \nu\phi^2\gamma_l^2\}\gamma_l\,\theta_{21}\cosh\gamma_l/\{\gamma_l^2 + (m\pi/2)^2\}$$

Quadrant (2):

Fore $l=0,1,2,\ldots\ldots\ldots k-1$

$$A_{k+l+1,3k+1} = \theta_{11l}\{(\nu\phi_I^2(l\pi)^2 - \beta_l^2)\sinh\beta_l$$
$$+ \theta_{11}(\nu\phi_I^2(l\pi)^2 - \gamma_l^2)\sin\gamma_l\}.$$

or if $\lambda^2\phi^2 < (l\pi)^2$

$$A_{k+l+1,3k+1} = \theta_{22l}\{(\nu\phi_I^2(l\pi)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{2l}(\nu\phi_I^2(l\pi)^2 - \gamma_l^2)\sinh\gamma_l\}$$

Quadrant (3):

For $l=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{2k+l+1,3k+1} = \theta_{11l}(\beta_l + \theta_{1l}\gamma_l)$$

or if $\lambda^2\phi^2 < (l\pi)^2$

$$= \theta_{22l}(\beta_l + \theta_{2l}\gamma_l)$$

Quadrant (4):

For $m=1,3,5,\ldots\ldots\ldots,2k-1$

$$A_{3k+1,(m+1)/2} = (\cosh\beta_m\zeta + \theta_{1m}\cos\gamma_m\zeta)\sin(m\pi/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= (\cosh\beta_m\zeta + \theta_{2m}\cosh\gamma_m\zeta)\sin(m\pi/2)/\theta_{22m}$$

Quadrant (5)

For $n=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{3k+1,k+n+1} = (\sinh\beta_n + \theta_{1n}\sin\gamma_n)\cos(n\pi\zeta)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= (\sinh\beta_n + \theta_{2n}\sinh\gamma_n)\cos(n\pi\zeta)/\theta_{22n}$$

Quadrant (6):

For p=0,1,2,.......,k-1

$$A_{3k+1,2k+p+1} = (1-\theta_{1p})\cos(p\pi\zeta)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi)^2$

$$= (1-\theta_{2p})\cos(p\pi\zeta)/\theta_{22p}$$

Quadrant (7):

For l=0,1,2,.......,k-1

$$A_{3k+1,3k+1} = \sum_{l=0,1,2}^{k^*} \theta_{11l}(\sinh\beta_l + \theta_{1l}\sin\gamma_l)\cos(l\pi\zeta)$$

$$+ \sum_{l=k^*+1}^{k-1} \theta_{22l}(\sinh\beta_l + \theta_{2l}\sinh\gamma_l)\cos(l\pi\zeta)$$

Finally program 9 of Appendix 2 is designed to generate eigenvalues for this family of vibration modes as listed in Table 3.3. The corresponding mode shape data are generated using program 10 of Appendix 3. Generated shapes are also illustrated in appendix 2. We now turn to the analysis of the antysimmetric modes.

Antisymmetric Modes

This family of vibration modes is analyzed using building blocks illustrated in Figure 3.7. Solutions for the first three of these blocks are available from those of Figure 2.8, and are given by Equations 2.48, 2.49, and 2.55 respectively. The solution of the fourth building block is easily obtained from the readily available solution of the fourth building block of Figure 3.5 by interchanging the variables ξ and η. As it was discussed earlier, such interchange implies the replacement of the aspect ratio ϕ by its inverse and the multiplication of λ^2 by ϕ^2. Hence $W_4(\xi,\eta)$ is

$$W_4(\xi,\eta) = \sum_{l=1,3,5}^{k^*} P^*\theta_{11l}(\sinh\beta_l\xi + \theta_{1l}\sin\gamma_l\xi)\sin\frac{l\pi\eta}{2}$$

$$+ \sum_{l=k^*+2}^{\infty} P^*\theta_{22l}(\sinh\beta_l\xi + \theta_{2l}\sinh\gamma_l\xi)\sin\frac{l\pi\eta}{2} \qquad \ldots\ldots\ldots\ldots(3.30)$$

where

$$\beta_l = \frac{1}{\phi}\sqrt{\phi^2\lambda^2+(l\pi/2)^2}$$

$$\gamma_1 = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(1\pi/2)^2} \quad \text{or} \quad \frac{1}{\phi}\sqrt{(1\pi/2)^2-\lambda^2\phi^2}$$

whichever is real, and

$$\theta_{11} = -{}_1\cosh\beta_1/\gamma_1\cos\gamma_1$$

$$\theta_{21} = -\beta_1\cosh\beta_1/\gamma_1\cosh\gamma_1$$

$$\theta_{111} = \frac{-2\sin(1\pi\zeta/2)}{\beta_1\{\beta_1^2-\nu^*\phi_1^2(1\pi/2)^2\}\cosh\beta_1 - \theta_{11}\gamma_1\{\gamma_1^2+\nu^*\phi_1^2(1\pi/2)^2\}\cos\gamma_1}$$

$$\theta_{221} = \frac{-2\sin(1\pi\zeta/2)}{\beta_1\{\beta_1^2-\nu^*\phi_1^2(1\pi/2)^2\}\cosh\beta_1+\theta_{21}\gamma_1\{\gamma_1^2-\nu^*\phi_1^2(1\pi/2)^2\}\cosh\gamma_1}$$

The contribution of $W_4(\xi,\eta)$ to the prescribed boundary conditions are found to be

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at $\eta=1$ is

$$\frac{M_\eta\phi b}{D} = P^*\theta_{111}\sin\frac{1\pi}{2}\{((1\pi/2)^2 - \nu\phi^2\beta_1^2)\sinh\beta_1\xi$$

$$+ \theta_{11}((1\pi/2)^2 + \nu\phi^2\gamma_1^2)\sin\gamma_1\xi\}$$

or if $\lambda^2\phi^2 < (1\pi/2)^2$,

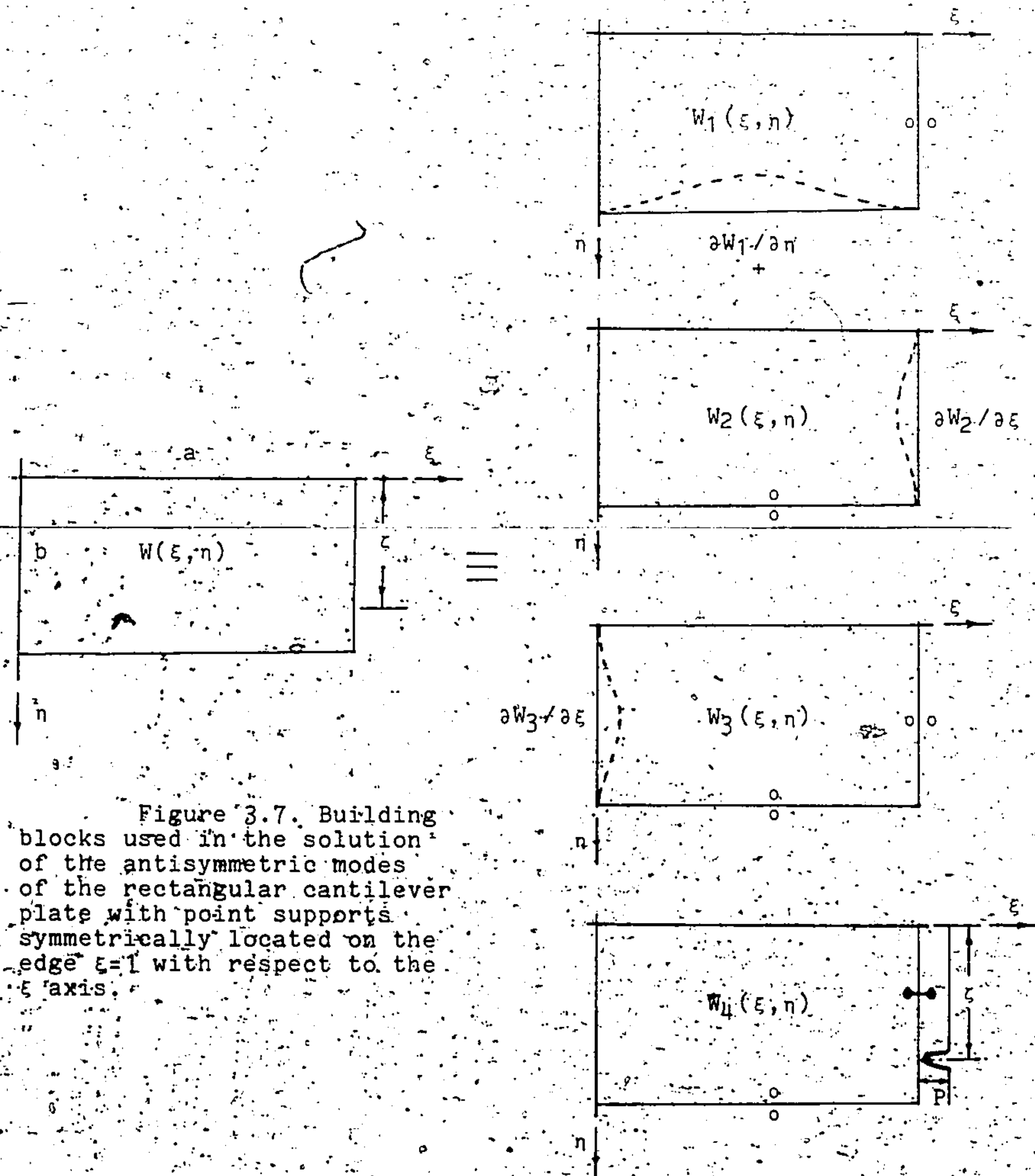


Figure 3.7. Building blocks used in the solution of the antisymmetric modes of the rectangular cantilever plate with point supports symmetrically located on the edge $\xi=1$ with respect to the ξ axis.

$$\frac{M_\eta \phi b}{D} = P^*\theta_{221}\sin\frac{l\pi}{2}\{((l\pi/2)^2 - \nu\phi^2\beta_l^2)\sinh\beta_l\xi$$

$$+ \theta_{21}((l\pi/2)^2 - \nu\phi^2\gamma_l^2)\sinh\gamma_l\xi\}$$

The contribution of each term of $W_4(\xi,\eta)$ to the slope at $\xi=0$ is

$$\partial W_4/\partial\xi = P^*\theta_{111}(\beta_l + \theta_{11}\gamma_l)\sin\frac{l\pi\eta}{2}$$

or if $\lambda^2\phi^2 < (l\pi/2)^2$

$$= P^*\theta_{221}(\beta_l + \theta_{21}\gamma_l)\sin\frac{l\pi\eta}{2}$$

The contribution of each term of $W_4(\xi,\eta)$ to bending moment at $\xi=1$ is

$$\frac{M_\xi a}{D} = P^*\theta_{111}\{(\nu\phi_I^2(l\pi/2)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{11}(\nu\phi_I^2(l\pi/2)^2 + \gamma_l^2)\sin\gamma_l\}\sin\frac{l\pi\eta}{2}$$

or if $\lambda^2\phi^2 < (l\pi/2)$

$$= P^*\theta_{221}\{(\nu\phi_I^2(l\pi/2)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{21}(\nu\phi_I^2(l\pi/2)^2 - \gamma_l^2)\sinh\gamma_l\}\sin\frac{l\pi\eta}{2}$$

The coefficient matrix is now generated by adding the following column and row to the matrix of Figure 2.11. Note that in these matricies the additional column is always multiplied by P*.

Quadrant (1):

For $l=1,3,5,.......,2k-1$; $m=1,3,5,.......2k-1$

$$A_{(m+1)/2,3k+1} = \sum_{l=1,3,5}^{k^*} 2\theta_{11l}\sin(l\pi/2)\sin(m\pi/2)(x_1-x_{11})$$
$$+ \sum_{l=k^*+2}^{2k-1} 2\theta_{22l}\sin(l\pi/2)\sin(m\pi/2)(x_2+x_{22})$$

where

$$x_1 = \{(l\pi/2)^2 - \nu\phi^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (m\pi/2)^2\}$$

$$x_{11} = \{(l\pi/2)^2 + \nu\phi^2\gamma_l^2\}\theta_{11}\gamma_l\cos\gamma_l/\{\gamma_l^2 - (m\pi/2)^2\}$$

$$x_2 = \{(l\pi/2)^2 - \nu\phi^2\beta_l^2\}\beta_l\cosh\beta_l/\{\beta_l^2 + (m\pi/2)^2\}$$

$$x_{22} = \{(l\pi/2)^2 - \nu\phi^2\gamma_l^2\}\theta_{21}\gamma_l\cosh\gamma_l/\{\gamma_l^2 + (m\pi/2)^2\}$$

Quadrant (2):

For $l=1,3,5,.......2k-1$

$$A_{k+(l+1)/2,3k+1} = \theta_{11l}\{(\nu\phi_I^2(l\pi/2)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{1l}(\nu\phi_I^2(l\pi/2)^2 + \gamma_l^2)\sin\gamma_l\}$$

or if $\lambda^2\phi^2 < (l\pi/2)^2$

$$= \theta_{22l}\{(\nu\phi_I^2(l\pi/2)^2 - \beta_l^2)\sinh\beta_l$$

$$+ \theta_{2l}(\nu\phi_I^2(l\pi/2)^2 - \gamma_l^2)\sinh\gamma_l\}$$

Quadrant (3):

For $l=1,3,5,\ldots\ldots,2k-1$

$$A_{2k+(l+1)/2,3k+1} = \theta_{11l}(\beta_l + \theta_{1l}\gamma_l)$$

or if $\lambda^2\phi^2 < (l\pi/2)^2$

$$= \theta_{22l}(\beta_l + \theta_{2l}\gamma_l)$$

Quadrant (4):

For $m=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,(m+1)/2} = (\theta_{1m}\sinh\beta_m\zeta + \sin\gamma_m\zeta)\sin(m\pi/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$A_{3k+1,(m+1)/2} = (\theta_{2m}\sinh\beta_m\zeta + \sinh\gamma_m\zeta)\sin(m\pi/2)/\theta_{22m}$$

Quadrant (5):

For n=1,3,5,......,2k-1

$$A_{3k+1,k+(n+1)/2} = (\theta_{1n}\sinh\beta_n + \sin\gamma_n)\sin(n\pi\zeta/2)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= (\theta_{2n}\sinh\beta_n + \sinh\gamma_n)\sin(n\pi\zeta/2)/\theta_{22n}$$

Quadrant (6):

For p=1,3,5,......,2k-1

$$A_{3k+1,2k+(p+1)/2} = (1-\theta_{1p})\sin(p\pi\zeta/2)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$

$$= (1-\theta_{2p})\sin(p\pi\zeta/2)/\theta_{22p}$$

Quadrant (7):

For l=1,3,5,......,2k-1

$$A_{(3k+1),(3k+1)} = \sum_{l=1,3,5}^{k^*} \theta_{11l}(\sinh\beta_l + \theta_{1l}\sin\gamma_l)\sin\frac{l\pi\zeta}{2}$$

$$+ \sum_{l=k^*+2}^{2k-1} \theta_{22l}(\sinh\beta_l + \theta_{2l}\sinh\gamma_l)\sin\frac{l\pi\zeta}{2}$$

Eigenvalues for this family of vibration modes are generated using program 11 of Appendix 3 and listed in Table 3.4. Corresponding mode shapes are also illustrated in Appendix 2 using data generated by specially designed program 12 of Appendix 3.

In Tables 3.1 through 3.4 eigenvalues are obtained with four digits accuracy. While this practice is followed throughout this work, greater accuracy is easily obtained by increasing the number (k) of terms used in the appropriate series involved. k may be decided upon following a convergence test similar to the one shown in Figure 2.7. Since point supports may assume arbitrarily their symmetric position s with respect to the ξ axis along the edges of the cantilever plate, it is virtually impossible to list all possible combinations of plate aspect ratios and point support positions along the edges. However, programs 5 through 12 are provided to generate eigenvalues and mode shape data for any possible combination one might encounter. As for Tables 3.1

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ $(\nu = 0.333)$ $\zeta = 0.5$

ϕ=2b/a	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
ϕ'=b/a	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	8.821	8.600	8.209	7.465	6.866	6.082	5.361	4.884	4.353	4.089	3.939
2	59.50	58.37	47.72	35.91	30.40	25.42	21.58	18.73	14.43	11.45	9.406
3	66.75	59.87	59.76	58.55	49.81	38.64	31.40	27.74	24.57	22.77	19.31

Table 3.1. First three eigenvalues for the symmetric vibration modes of the cantilever plate of Figure 3.1. ζ=0.5.

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ $(\nu = 0.333)$ $\zeta = 0.5$

ϕ=2b/a	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
ϕ'=b/a	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	41.53	35.31	29.05	22.70	19.43	16.03	13.14	11.08	8.384	6.808	5.857
2	107.5	96.81	86.33	75.22	65.62	50.76	38.30	30.59	21.96	17.17	13.98
3	142.4	124.8	106.2	84.62	75.04	68.80	53.63	43.31	33.16	28.40	24.99

Table 3.2. First three eigenvalues for the antisymmetric vibration modes of the cantilever plate of Figure 3.1. ζ=0.5

$\lambda^2 = \omega a^2 \sqrt{\rho/D}$ $(\nu = 0.333)$ $\zeta = 0.5$

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	14.81	14.83	14.82	14.77	14.68	14.44	13.90	12.80	9.775	7.852	6.693
2	47.99	47.95	47.72	46.17	38.51	27.02	19.54	15.94	13.24	11.20	9.369
3	100.2	99.61	91.30	56.09	47.96	45.17	40.94	36.19	27.62	22.85	20.34

Table 3.3. First three eigenvalues for the symmetric vibration modes of the plate shown in Figure 3.2. $\zeta=0.5$.

$\lambda^2 = \omega a^2 \sqrt{\rho/D}$ $(\nu = 0.333)$ $\zeta = 0.5$

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	40.18	34.19	28.33	22.70	20.00	17.33	15.10	13.40	10.73	8.791	7.455
2	87.75	76.88	66.43	56.14	50.51	43.45	35.78	29.88	23.06	19.62	17.27
3	147.3	132.3	117.3	99.42	86.75	69.42	53.60	44.17	34.23	29.25	26.71

Table 3.4. First three eigenvalues for the antisymmetric vibration modes of the plate shown in Figure 3.2. $\zeta=0.5$.

through 3.4, we have chosen the point support position to be at a distance of 1/2 from the axis perpendicular to the edge on which it is located. Further discussions on eigenvalues listed above and others will be forthcoming in later chapters.

We now come to the end of the present chapter where we have analyzed rectangular cantilever plates with edge point supports. The next chapter will concern itself with the study of the free vibration problem of the rectangular cantilever plate with lateral point supports. Also in the next chapter a discussion and a comparison of some of the eigenvalues found so far and others with existing eigenvalues will be carried out whenever is possible.

Chapter 4

THE RECTANGULAR CANTILEVER PLATE WITH LATERAL POINT SUPPORTS

Consider the cantilever plate of Figure 4.1. The free vibration problem of this family of plates was fully discussed in Chapter 2. Then in Chapter 3 we introduced the condition of point supports on the boundary of the cantilever plate as illustrated by Figures 3.1 and 3.2 where lateral movement of the plate is forbidden at these points. In this chapter internal point supports are being introduced, where lateral movement of the plate is forbidden where the coordinates ξ and η of the lower portion of the plate shown in Figure 4.2 assume the values u and v. Here also point supports are considered to be symmetric with respect to the ξ axis. Therefore, only the lower half of the plate of Figure 4.2 will be analyzed. If we assume that the plate is vibrating in one of its vibration modes with a circular frequency ~~ω, then lateral motion~~ at the fixed point will have to be prevented by a time varying force that may be represented as $P\sin(\omega t)$. Solution for this type of problems was described by Nowaki(23), and then discussed by

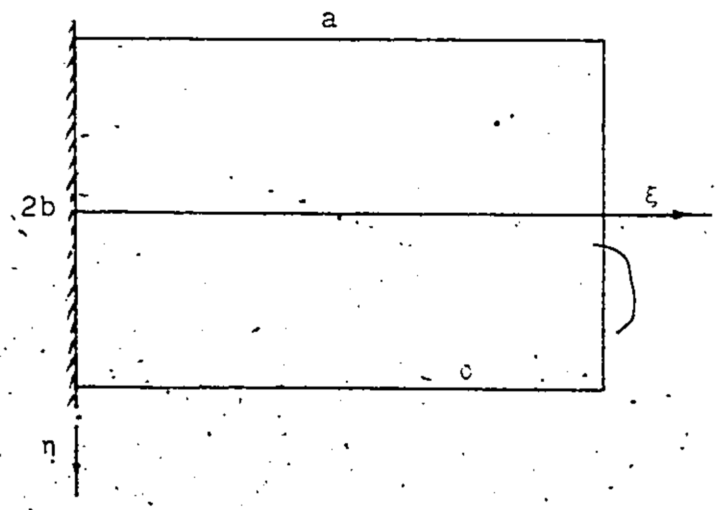


Figure 4.1. Cantilever plate discussed in Chapter 2.

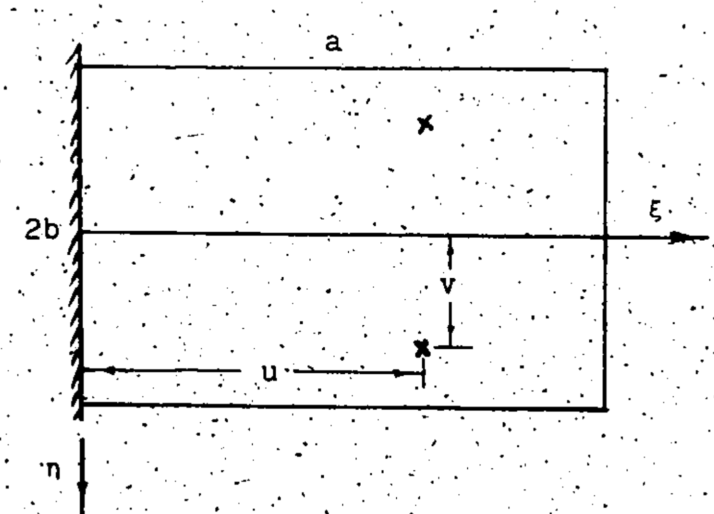


Figure 4.2. Cantilever plate with lateral point supports.

Gorman in his "Free Vibration Analysis of Rectangular Plates". Following our practice of dividing the free vibration modes of our plate into two families, we will first focus our attention on the symmetric mode family.

Symmetric Modes

Building blocks used in the solution of this family of free vibration modes consist of four blocks the first three of which are the same as those used in analyzing the same family of free vibration modes of the cantilever plate of Figure 3.1 and for which solutions are readily developed in Chapter 2. Figure 4.3 illustrates all four building blocks needed for this analysis. Focussing our attention on the last building block of Figure 4.3, we divide this block into two segments I and II. Each segment has its own coordinate system with the point support lying on the common line between the two segments as shown in Figure 4.4.

The Fourth Building Block:

This fourth building block was used by Gorman in his analysis of modes symmetric with respect to the ξ axis and antisymmetric with respect to the η axis for the rectangular plate with symmetrically distributed point supports(20).

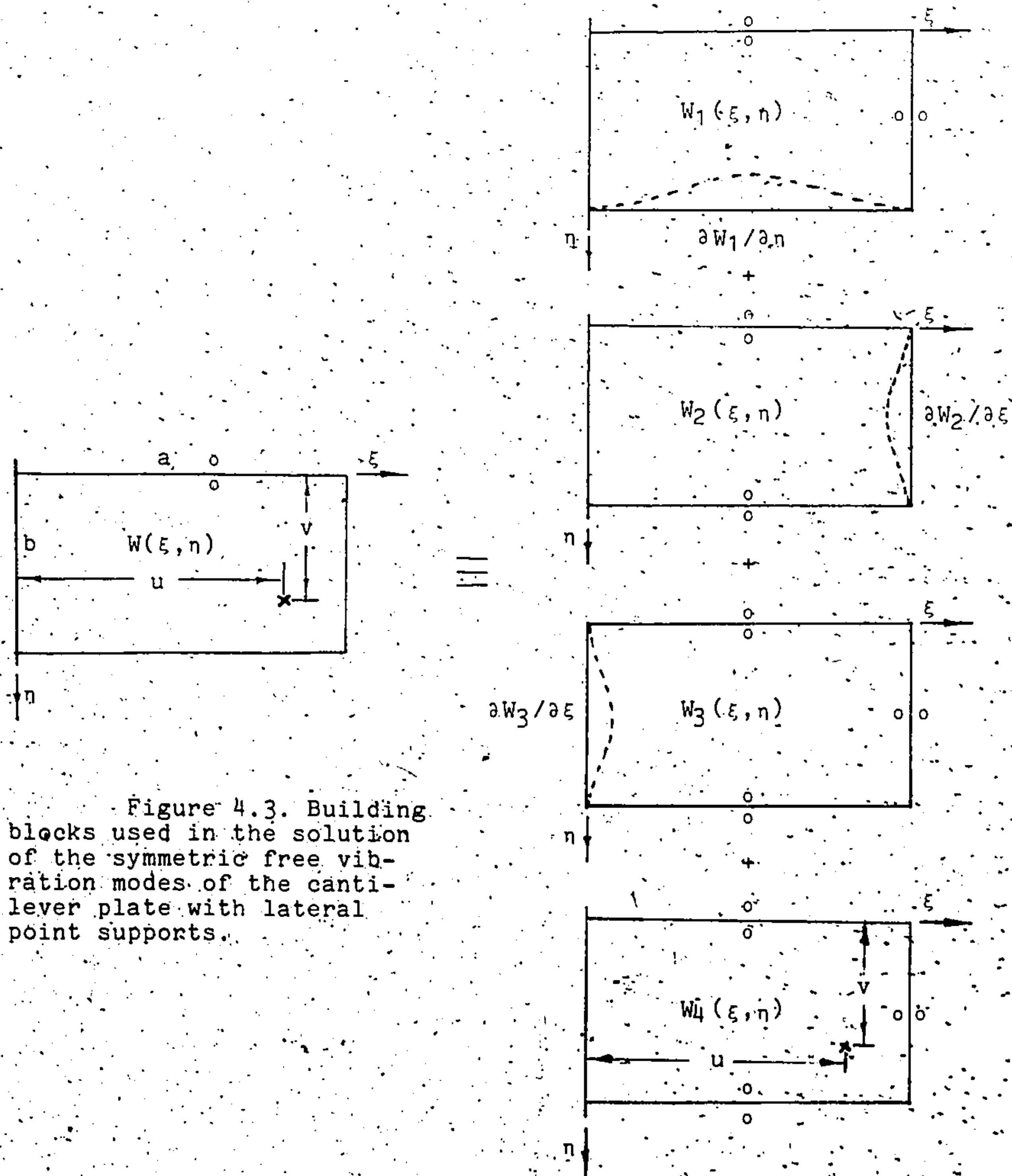


Figure 4.3. Building blocks used in the solution of the symmetric free vibration modes of the cantilever plate with lateral point supports.

Although this solution is readily available, it will be reproduced here for convenient reference and to show briefly the various steps involved.

In view of the boundary conditions, Levy-type solution may be written for each segment as follow

$$W_{41}(\xi,\eta) = \sum_{m=1,3,5}^{k^*} (A_{1m}\cosh\beta_m\eta + B_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} (A_{2m}\cosh\beta_m\eta + B_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \ldots\ldots\ldots\ldots(4.1)$$

and

$$W_{42}(\xi,\eta) = \sum_{m=1,3,5}^{k^*} (C_{1m}\cosh\beta_m\eta + D_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} (C_{2m}\cosh\beta_m\eta + D_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \ldots\ldots\ldots\ldots(4.2)$$

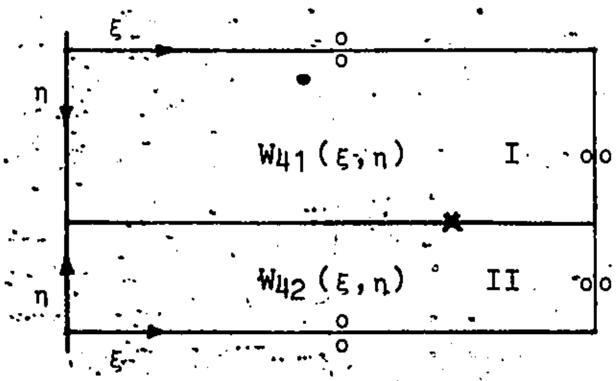


Figure. 4.4. Fourth building block of Figure 4.3 divided into two segments by a line parallel to the ξ axis and passing through the lateral point support.

where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi/2)^2}$$

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi/2)^2} \quad \text{or} \quad = \phi\sqrt{(m\pi/2)^2-\lambda^2}$$

whichever is real. Then the concentrated force is represented as

$$P(\xi) = \sum_{m=1,3,5}^{\infty} P^{*}\sin\frac{m\pi u}{2}\sin\frac{m\pi\xi}{2} \quad \ldots\ldots\ldots(4.3)$$

The next step is to enforce the conditions of continuity and force equilibrium along the common edge. These conditions are:

1.- The plate displacement must be continuous across the boundary between the two segments.

2.- The slope of the plate taken in a direction normal to the boundary must also be continuous across the boundary.

3.- Bending moments must be continuous across the boundary between the two segments.

4.- There must be an equilibrium force balance between the applied force and the vertical edge reactions taken along the intersegment boundary.

By enforcing the above conditions we have

$$A_{1m}\cosh\beta_m v + B_{1m}\cos\gamma_m v - C_{1m}\cosh\beta_m v^* - D_{1m}\cos\gamma_m v^* = 0$$

$$A_{1m}\beta_m\sinh\beta_m v - B_{1m}\gamma_m\sin\gamma_m v + C_{1m}\beta_m\sinh\beta_m v^* - D_{1m}\gamma_m\sin\gamma_m v^* = 0$$

$$A_{1m}\beta_m^2\cosh\beta_m v - B_{1m}\gamma_m^2\cos\gamma_m v - C_{1m}\beta_m^2\cosh\beta_m v^* + D_{1m}\gamma_m^2\cos\gamma_m v^* = 0$$

$$A_{1m}\beta_m^3\sinh\beta_m v + B_{1m}\gamma_m^3\sin\gamma_m v + C_{1m}\beta_m^3\sinh\beta_m v^* + D_{1m}\gamma_m^3\sin\gamma_m v^* = P^*\sin\frac{m\pi u}{2}$$

and $\lambda^2 < (m\pi/2)^2$

$$A_{2m}\cosh\beta_m v + B_{2m}\cosh\gamma_m v - C_{2m}\cosh\beta_m v^* - D_{2m}\cosh\gamma_m v^* = 0$$

$$A_{2m}\beta_m\sinh\beta_m v + B_{2m}\gamma_m\sinh\gamma_m v + C_{2m}\beta_m\sinh\beta_m v^* + D_{2m}\gamma_m\sinh\gamma_m v^* = 0$$

$$A_{2m}\beta_m^2\cosh\beta_m v + B_{2m}\gamma_m^2\cosh\gamma_m v - C_{2m}\beta_m^2\cosh\beta_m v^* - D_{2m}\gamma_m^2\cosh\gamma_m v^* = 0$$

$$A_{2m}\beta_m^3\sinh\beta_m v + B_{2m}\gamma_m^3\sinh\gamma_m v + C_{2m}\beta_m^3\sinh\beta_m v^* + D_{2m}\gamma_m^3\sinh\gamma_m v^* = P^*\sin\frac{m\pi u}{2}$$

where $v^* = 1-v$

Solving the above for $A_{1m}, \ldots, D_{2m}$ we have

$$A_{1m} = P^*\sin(m\pi u/2)\cosh\beta_m v^*/(\beta_m^2 + \gamma_m^2)\beta_m\sinh\beta_m$$

$$B_{1m} = P^*\sin(m\pi u/2)\cos\gamma_m v^*/(\beta_m^2 + \gamma_m^2)\gamma_m\sin\gamma_m$$

$$C_{1m} = P^*\sin(m\pi u/2)\cosh\beta_m v/(\beta_m^2 + \gamma_m^2)\beta_m\sinh\beta_m$$

$$D_{1m} = P^*\sin(m\pi u/2)\cos\gamma_m v/(\beta_m^2 + \gamma_m^2)\gamma_m \sin\gamma_m$$

$$A_{2m} = P^*\sin(m\pi u/2)\cosh\beta_m v^*/(\beta_m^2 - \gamma_m^2)\beta_m \sinh\beta_m$$

$$B_{2m} = -P^*\sin(m\pi u/2)\cosh\gamma_m v^*/(\beta_m^2 - \gamma_m^2)\gamma_m \sinh\gamma_m$$

$$C_{2m} = P^*\sin(m\pi u/2)\cosh\beta_m v/(\beta_m^2 - \gamma_m^2)\beta_m \sinh\beta_m$$

$$D_{2m} = -P^*\sin(m\pi u/2)\cosh\gamma_m v/(\beta_m^2 - \gamma_m^2)\gamma_m \sinh\gamma_m$$

We now have a solution for the fourth building block of Figure 4.3 with an intersegment line running in the direction of the ξ axis. However, since we require the bending moment contribution of this building block along the edge $\xi=1$, and the slope at the edge $\xi=0$, although not necessary, it would be helpful to have a solution with intersegment line running parallel to these edges, as demonstrated by Figure 4.5, to eliminate any irregularity along these edges.

Solution of the block of Figure 4.5 may easily be obtained from that of the block of Figure 4.6 by interchanging the variables ξ and η and following transformation rules already discussed. Focussing our attention on Figure 4.5, in view of the boundary conditions along the edges $\xi=0$ and $\eta=0$ we may write

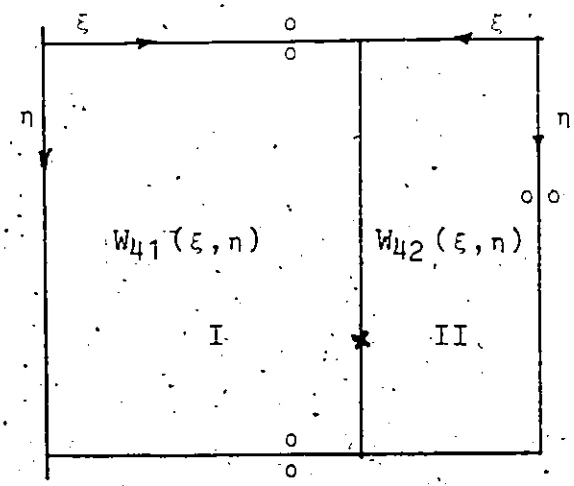


Figure 4.5. Same as Figure 4.4 with intersegment line running in the η direction.

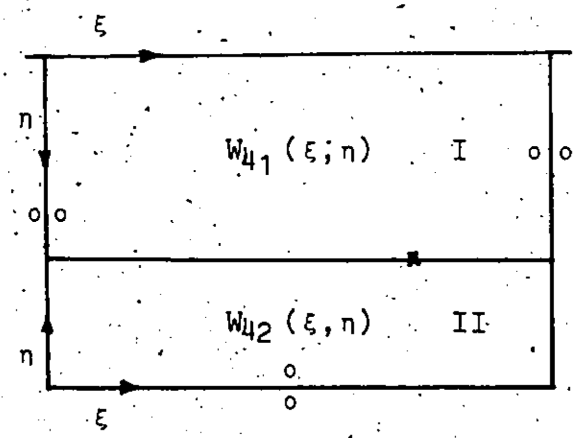


Figure 4.6. Intermediate block used in the solution of the block of Figure 4.4.

$$W_{41}(\xi,\eta) = \sum_{m=0,1,2}^{k^*} (A_{1m}\sinh\beta_m\eta + B_{1m}\sin\gamma_m\eta)\cos(m\pi\xi)$$

$$+ \sum_{m=k^*+1}^{\infty} (A_{2m}\sinh\beta_m\eta + B_{2m}\sinh\gamma_m\eta)\cos(m\pi\xi) \qquad \ldots\ldots\ldots\ldots(4.4)$$

and

$$W_{42}(\xi,\eta) = \sum_{m=0,1,2}^{k^*} (C_{1m}\cosh\beta_m\eta + D_{1m}\cos\gamma_m\eta)\cos(m\pi\xi)$$

$$+ \sum_{m=k^*+1}^{\infty} (C_{2m}\cosh\beta_m\eta + D_{2m}\cosh\gamma_m\eta)\cos(m\pi\xi). \qquad \ldots\ldots\ldots\ldots(4.5)$$

where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi)^2}$$

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi)^2} \qquad \text{or} \qquad = \phi\sqrt{(m\pi)^2-\lambda^2}$$

whichever is real.

By enforcing the four conditions of continuity and force equilibrium across the boundary of the intersegment line and solving we have

$$A_{1m} = P^*\cos(m\pi u)\cosh\beta_m v^*/\delta_m(\beta_m^2 + \gamma_m^2)\beta_m\cosh\beta_m$$

$$B_{1m} = -P^*\cos(m\pi u)\cos\gamma_m v^*/\delta_m(\beta_m^2 + \gamma_m^2)\gamma_m\cos\gamma_m$$

$$C_{1m} = P^*\cos(m\pi u)\sinh\beta_m v/\delta_m(\beta_m^2 + \gamma_m^2)\beta_m\cosh\beta_m$$

$$D_{1m} = -P^*\cos(m\pi u)\sin\gamma_m v/\delta_m(\beta_m^2 + \gamma_m^2)\gamma_m\cos\gamma_m$$

and

$$A_{2m} = P^*\cos(m\pi u)\cosh\beta_m v^*/\delta_m(\beta_m^2 - \gamma_m^2)\beta_m\cosh\beta_m$$

$$B_{2m} = -P^*\cos(m\pi u)\cosh\gamma_m v^*/\delta_m(\beta_m^2 - \gamma_m^2)\gamma_m\cosh\gamma_m$$

$$C_{2m} = P^*\cos(m\pi u)\sinh\beta_m v/\delta_m(\beta_m^2 - \gamma_m^2)\beta_m\cosh\beta_m$$

$$D_{2m} = -P^*\cos(m\pi u)\sinh\gamma_m v/\delta_m(\beta_m^2 - \gamma_m^2)\gamma_m\cosh\gamma_m$$

Where the concentrated force has ben expanded as a Dirac function using the cosine series

$$P(\xi) = \sum_{m=0,1,2}^{\infty} P^*\frac{1}{\delta_m}\cos(m\pi u)\cos(m\pi\xi) \qquad \text{......(4.6)}$$

where $\delta_m = \begin{cases} 2 & \text{if } m=0 \\ 1 & \text{if } m\neq 0 \end{cases}$

and $P^* = -2Pb^3/Da^2$

We now interchange the variables ξ and η in Equations 4.4 and 4.5 to obtain solutions for the two segments of Figure 4.5. Note that m is also replaced by n.

$$W_{41}(\xi,\eta) = \sum_{n=0,1,2}^{k^*} (A_{1n}\sinh\beta_n\xi + B_{1n}\sin\gamma_n\xi)\cos(n\pi\eta) \qquad \text{.........(4.7)}$$

$$+ \sum_{n=k^*+1}^{\infty} (A_{2n}\sinh\beta_n\xi + B_{2n}\sinh\gamma_n\xi)\cos(n\pi\eta)$$

and

$$W_{42}(\xi,\eta) = \sum_{n=0,1,2}^{k^*} (C_{1n}\cosh\beta_n\xi + D_{1n}\cos\gamma_n\xi)\cos(n\pi\eta)$$

$$+ \sum_{n=k^*+1}^{\infty} (C_{2n}\cosh\beta_n\xi + D_{2n}\cosh\gamma_n\xi)\cos(n\pi\eta) \qquad \ldots\ldots\ldots(4.8)$$

where

$$\beta_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(n\pi)^2}$$

$$\gamma_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(n\pi)^2} \qquad \text{or} \qquad = \frac{1}{\phi}\sqrt{(n\pi)^2-\lambda^2\phi^2}$$

We may also deduce the coefficients if we replace subscript m by n, and interchange the quantities u and u* with v and v* respectively. Furthermore, we must divide the new coefficients by ϕ^4 to leave unchanged our definition for P*. Displacement will still be dimensionless with respect to side length a.

$$A_{1n} = P^*\cos(n\pi v)\cosh\beta_n u^*/\delta_n\phi^4(\beta_n^2 + \gamma_n^2)\beta_n\cosh\beta_n$$

$$B_{1n} = -P^*\cos(n\pi v)\cos\gamma_n u^*/\delta_n\phi^4(\beta_n^2 + \gamma_n^2)\gamma_n\cos\gamma_n$$

$$C_{1n} = P^*\cos(n\pi v)\sinh\beta_n u/\delta_n\phi^4(\beta_n^2 + \gamma_n^2)\beta_n\cosh\beta_n$$

$$D_{1n} = -P^*\cos(n\pi v)\sin\gamma_n u/\delta_n\phi^4(\beta_n^2 + \gamma_n^2)\gamma_n\cos\gamma_n$$

and

$$A_{2n} = P^*\cos(n\pi v)\cosh\beta_n u^*/\delta_n\phi^4(\beta_n^2 - \gamma_n^2)\beta_n\cosh\beta_n$$

$$B_{2n} = -P^*\cos(n\pi v)\cosh\gamma_n u^*/\delta_n\phi^4(\beta_n^2 - \gamma_n^2)\gamma_n\cosh\gamma_n$$

With the solution of the final building block developed we next turn to derive expressions to its contribution to the prescribed boundary conditions. Contributions of the first three building blocks are readily available from Chapter 2, and they are given by Equations 2.29 to 2.37. Focussing our attention on the contribution of the last building block we write

The contribution to bending moment at the edge $\xi=1$ of each term of Equation 4.8 is

$$\frac{M_\xi a}{D} = \{(\nu\phi_I^2(n\pi)^2 - \beta_n^2)C_{1n} + (\nu\phi_I^2(n\pi)^2 + \gamma_n^2)D_{1n}\}\cos(n\pi\eta)$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= \{(\nu\phi_I^2(n\pi)^2 - \beta_n^2)C_{2n} + (\nu\phi_I^2(n\pi)^2 - \gamma_n^2)D_{2n}\}\cos(n\pi\eta)$$

The contribution to slope at $\xi=0$ of each term of Equation 4.7 is

$$\partial W_{41}/\partial\xi = (A_{1n}\beta_n + B_{1n}\gamma_n)\cos(n\pi\eta)$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= (A_{2n}\beta_n + B_{2n}\gamma_n)\cos(n\pi\eta)$$

The contribution to bending moment at the edge $\eta=1$ of each term of Equation 4.2 is

$$\frac{M_n\phi b}{D} = \{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)C_{1m} + (\nu\phi^2(m\pi/2)^2 + \gamma_m^2)D_{1m}\}\sin\frac{m\pi\xi}{2}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= \{(\nu\phi^2(m\pi/2)^2 - \beta_m^2)C_{2m} + (\nu\phi^2(m\pi/2)^2 - \gamma_m^2)D_{2m}\}\sin\frac{m\pi\xi}{2}$$

The contribution of Equation 4.1 to displacement at $\eta=v$ and $\xi=u$ is

$$W_{41}(u,v) = \sum_{m=1,3,5}^{k^*}(A_{1m}\cosh\beta_m v + B_{1m}\cos\gamma_m v)\sin\frac{m\pi u}{2}$$

$$+ \sum_{m=k^*+2}^{\infty}(A_{2m}\cosh\beta_m v + B_{2m}\cosh\gamma_m v)\sin\frac{m\pi u}{2}$$

The coefficient matrix is generated by adding the following column and row to the matrix of Figure 2.6.

Quadrant (1):

For $m=1,3,5,\ldots\ldots,2k-1$

$$A_{(m+1)/2,3k+1} = \{(\nu\phi^2(m\pi/2)^2-\beta_m^2)C_{1m} + (\nu\phi^2(m\pi/2)^2+\gamma_m^2)D_{1m}\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= \{\nu\phi^2(m\pi/2)^2-\beta_m^2\}C_{2m} + \{\nu\phi^2(m\pi/2)^2-\gamma_m^2\}D_{2m}$$

Quadrant (2):

For n=0,1,2,........,k-1

$$A_{k+n+1,3k+1} = \{\nu\phi_I^2(n\pi)^2 - \beta_n^2\}C_{1n} + \{\nu\phi_I^2(n\pi)^2 + \gamma_n^2\}D_{1n}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= \{\nu\phi_I^2(n\pi)^2 - \beta_n^2\}C_{2n} + \{\nu\phi_I^2(n\pi)^2 - \gamma_n^2\}D_{2n}$$

Quadrant (3):

For n=0,1,2,........,k-1

$$A_{2k+n+1,3k+1} = A_{1n}\beta_n + B_{1n}\gamma_n$$

or if $\phi^2\lambda^2 < (n\pi)^2$

$$= A_{2n}\beta_n + B_{2n}\gamma_n$$

Quadrant (4):

For m=1,3,5,.......,2k-1

$$A_{(3k+1),(m+1)/2} = (\cosh\beta_m v + \theta_{1m}\cos\gamma_m v)\sin(m\pi u/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$A_{3k+1,(m+1)/2} = (\cosh\beta_m v + \theta_{2m}\cosh\gamma_m v)\sin(m\pi u/2)/\theta_{22m}$$

Quadrant (5):

For $n=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{3k+1,k+n+1} = (\sinh\beta_n u + \theta_{1n}\sin\gamma_n u)\cos(n\pi v)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi)^2$

$$= (\sinh\beta_n u + \theta_{2n}\sinh\gamma_n u)\cos(n\pi v)/\theta_{22n}$$

Quadrant (6):

For $p=0,1,2,\ldots\ldots\ldots,k-1$

$$A_{3k+1,2k+p+1} = \{\cosh\beta_p(1-u) - \theta_{1p}\cos\gamma_p(1-u)\}\cos(p\pi v)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi)^2$

$$= \{\cosh\beta_p(1-u) - \theta_{2p}\cosh\gamma_p(1-u)\}\cos(p\pi v)/\theta_{22p}$$

Quadrant (7):

For $m=1,3,5,\ldots\ldots\ldots,2k-1$

$$A_{3k+1,3k+1} = \sum_{m=1,3,5}^{k^*} (A_{1m}\cosh\beta_m v + B_{1m}\cos\gamma_m v)\sin\frac{m\pi u}{2}$$

$$+ \sum_{m=k^*+2}^{2k-1} (A_{2m}\cosh\beta_m v + B_{2m}\cosh\gamma_m v)\sin\frac{m\pi u}{2}$$

Next, using specially designed Program 13 of Appendix 3 we generate the coefficient matrix to establish those values of λ^2 which cause the determinant of the matrix to vanish. A vanishing determinant indicates that the associated value of λ^2 is an eigenvalue. Due to the fact that the coordinates of the point support of Figure 4.3 may assume any combination of real numbers between zero and one, it is physically impossible to cover eigenvalues for this infinit number of possibilities. However, Appendix 3 provides Program 13 to handle any possibility one may encounter, and Program 14 will generate the corresponding mode shape data. Meanwhile, Table 4.1 lists the first three eigenvalues for eleven different plate aspect ratio where u and v assume the value 1/2. Corresponding mode shapes are also illustrated in Appendix 2.

We next turn to the antisymmetric free vibration modes of the cantilever plate with lateral point supports.

Antisymmetric Modes

We start by studying the four building blocks of Figure 4.7. It soon becomes apparent that solutions for the first three building blocks are readily available from the analysis of the antisymmetric free vibration modes of the cantilever plate of Chapter 2. Hence, we start with the analysis of the fourth building block by dividing it into two segments as shown in Figure 4.8.

The Fourth Building Block:

The block of Figure 4.8 was also used by Gorman in his analysis of the fully antisymmetric modes of the rectangular plate with symetrically distributed point supports on the lateral surface. Hence, its solution is readily available and is repeated here for convenient reference. Using a second subscript to refer to the relevent segment of the building block we have

$$W_{41}(\xi,\eta) = \sum_{m=1,3,5}^{k^*} (A_{1m}\sinh\beta_m\eta + B_{1m}\sin\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} (A_{2m}\sinh\beta_m\eta + B_{2m}\sinh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \quad \ldots\ldots\ldots(4.9)$$

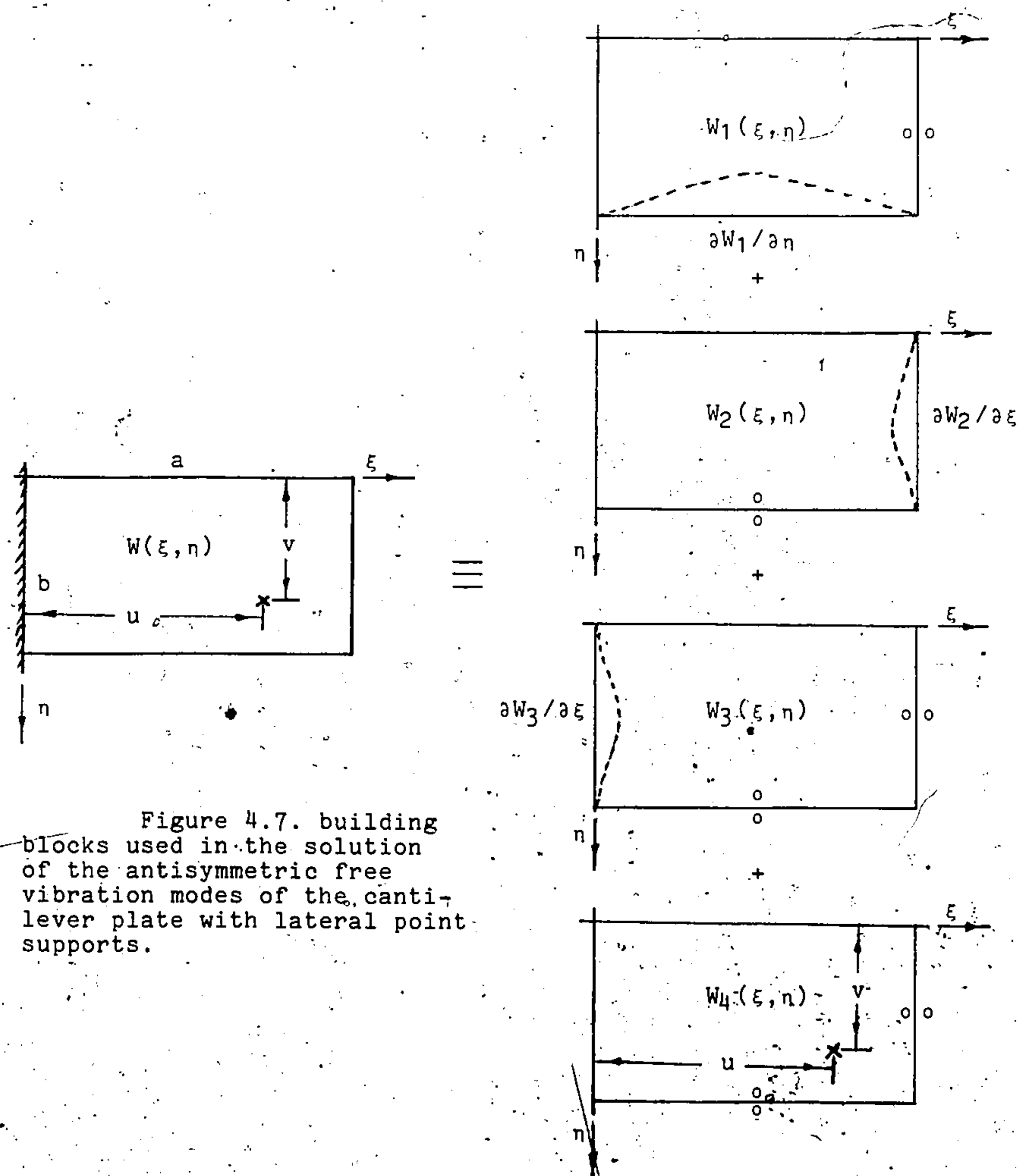


Figure 4.7. building blocks used in the solution of the antisymmetric free vibration modes of the cantilever plate with lateral point supports.

and

$$W_{42}(\xi,\eta) = \sum_{m=1,3,5}^{k^*} (C_{1m}\cosh\beta_m\eta + D_{1m}\cos\gamma_m\eta)\sin\frac{m\pi\xi}{2}$$

$$+ \sum_{m=k^*+2}^{\infty} (C_{2m}\cosh\beta_m\eta + D_{2m}\cosh\gamma_m\eta)\sin\frac{m\pi\xi}{2} \qquad \text{.......(4.10)}$$

where

$$\beta_m = \phi\sqrt{\lambda^2+(m\pi/2)^2}$$

$$\gamma_m = \phi\sqrt{\lambda^2-(m\pi/2)^2} \qquad \text{or} \qquad = \phi\sqrt{(m\pi/2)^2-\lambda^2}$$

whichever is real, following our established procedure, the first summation pertains to terms for which $\lambda^2 > (m\pi/2)^2$.

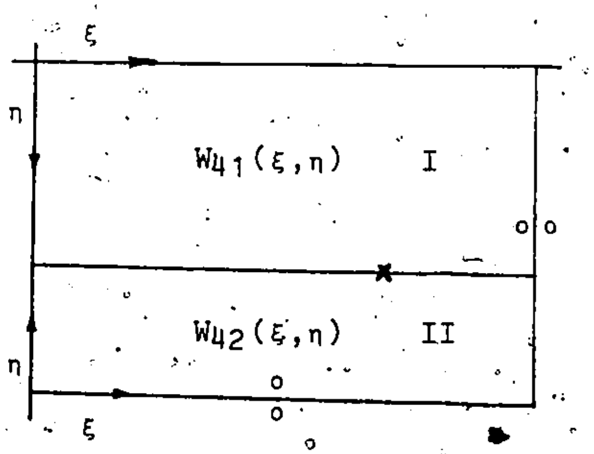


Figure 4.8. Fourth building block of Figure 4.7 with intersegment line parallel to the ξ axis.

Two of the original constants in Equations 4.9 and 4.10 have ben eliminated by means of the boundary conditions prescribed at the edge $\eta=0$. If we represent the concentrated force by a sine Dirac function,

$$P(\xi) = \sin\frac{m\pi u}{2}\sin\frac{m\pi\xi}{2} \qquad \text{.........(4.11)}$$

the continuity and force equilibrium equations become

$$A_{1m}\sinh\beta_m v + B_{1m}\sin\gamma_m v - C_{1m}\cosh\beta_m v^* - D_{1m}\cos\gamma_m v^* = 0$$

$$A_{1m}\beta_m\cosh\beta_m v + B_{1m}\gamma_m\cos\gamma_m v + C_{1m}\beta_m\sinh\beta_m v^* - D_{1m}\gamma_m\sin\gamma_m v^* = 0$$

$$A_{1m}\beta_m^2\sinh\beta_m v - B_{1m}\gamma_m^2\sin\gamma_m v - C_{1m}\beta_m^2\cosh\beta_m v^* + D_{1m}\gamma_m^2\cos\gamma_m v^* = 0$$

$$A_{1m}\beta_m^3\cosh\beta_m v - B_{1m}\gamma_m^3\cos\gamma_m v + C_{1m}\beta_m^3\sinh\beta_m v^* + D_{1m}\gamma_m^3\sin\gamma_m v^* = P^*\sin\frac{m\pi u}{2}$$

and

$$A_{2m}\sinh\beta_m v + B_{2m}\sinh\gamma_m v - C_{2m}\cosh\beta_m v^* - D_{2m}\cosh\gamma_m v^* = 0$$

$$A_{2m}\beta_m\cosh\beta_m v + B_{2m}\gamma_m\cosh\gamma_m v + C_{2m}\beta_m\sinh\beta_m v^* + D_{2m}\gamma_m\sinh\gamma_m v^* = 0$$

$$A_{2m}\beta_m^2\sinh\beta_m v + B_{2m}\gamma_m^2\sinh\gamma_m v - C_{2m}\beta_m^2\cosh\beta_m v^* - D_{2m}\gamma_m^2\cosh\gamma_m v^* = 0$$

$$A_{2m}\beta_m^3\cosh\beta_m v + B_{2m}\gamma_m^3\cosh\gamma_m v + C_{2m}\beta_m^3\sinh\beta_m v^* + D_{2m}\gamma_m^3\sinh\gamma_m v^* = P^*\sin\frac{m\pi u}{2}$$

Solving the above equations we have

$$A_{1m} = P^*\sin(m\pi u/2)\cosh(\beta_m v^*)/(\beta_m^2 + \gamma_m^2)\beta_m \cosh\beta_m$$

$$B_{1m} = -P^*\cos\gamma_m v^*\sin(m\pi u/2)/(\beta_m^2 + \gamma_m^2)\gamma_m \cos\gamma_m$$

$$C_{1m} = P^*\sin(m\pi u/2)\sinh\beta_m v/(\beta_m^2 + \gamma_m^2)\beta_m \cosh\beta_m$$

$$D_{1m} = -P^*\sin\gamma_m v\sin(m\pi u/2)/(\beta_m^2 + \gamma_m^2)\gamma_m \cos\gamma_m$$

and

$$A_{2m} = P^*\sin(m\pi u/2)\cosh\beta_m v^*/(\beta_m^2 - \gamma_m^2)\beta_m \cosh\beta_m$$

$$B_{2m} = -P^*\sin(m\pi u/2)\cosh\gamma_m v^*/(\beta_m^2 - \gamma_m^2)\gamma_m \cosh\gamma_m$$

$$C_{2m} = P^*\sin(m\pi u/2)\sinh\beta_m v/(\beta_m^2 - \gamma_m^2)\beta_m \cosh\beta_m$$

$$D_{2m} = -P^*\sin(m\pi u/2)\sinh\gamma_m v/(\beta_m^2 - \gamma_m^2)\gamma_m \cosh\gamma_m$$

With the solution for the building block of Figure 4.8 available, it will be recalled from the solution technique for the symmetric modes that we also require a solution for this fourth building block with the segments divided by a line parallel to the η axis as shown in Figure 4.9. This solution may easily be inferred from the solution of the block of Figure 4.8 which is given by Equations 4.9 and 4.10 by a simple interchange of the variables ξ and η, at the same time replacing subscript m by n.

$$W_{41}(\xi,\eta) = \sum_{n=1,3,5}^{k^*} (A_{1n}\sinh\beta_n\xi + B_{1n}\sin\gamma_n\xi)\sin\frac{n\pi\eta}{2}$$

$$+ \sum_{n=k^*+2}^{\infty} (A_{2n}\sinh\beta_n\xi + B_{2n}\sinh\gamma_n\xi)\sin\frac{n\pi\xi}{2} \qquad \text{........(4.11)}$$

and

$$W_{42}(\xi,\eta) = \sum_{n=1,3,5}^{k^*} (C_{1n}\cosh\beta_n\xi + D_{1n}\cos\gamma_n\xi)\sin\frac{n\pi\eta}{2}$$

$$+ \sum_{n=k^*+2}^{\infty} (C_{2n}\cosh\beta_n\xi + D_{2n}\cosh\gamma_n\xi)\sin\frac{n\pi\eta}{2} \qquad \text{........(4.12)}$$

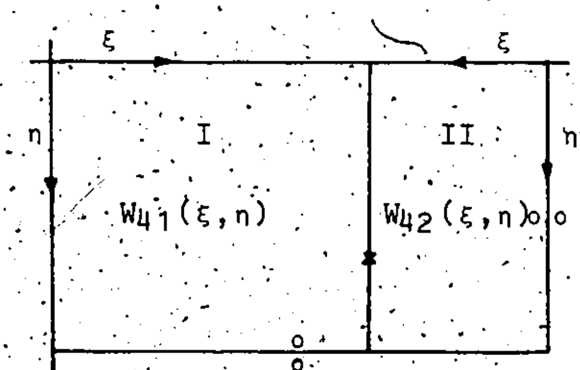


Figure 4.9. Fourth building block of Figure 4.7 with intersegment line running parallel to the η axis.

where

$$\beta_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2+(n\pi/2)^2}$$

$$\gamma_n = \frac{1}{\phi}\sqrt{\lambda^2\phi^2-(n\pi/2)^2} \qquad \text{or} \qquad = \frac{1}{\phi}\sqrt{(n\pi/2)^2-\lambda^2\phi^2}$$

whichever is real, and

$$A_{1n} = P^*\sin(n\pi v/2)\cosh\beta_n u^*/(\beta_n^2 + \gamma_n^2)\beta_n\phi^4\cosh\beta_n$$

$$B_{1n} = -P^*\cos\gamma_n u^*\sin(n\pi v/2)/(\beta_n^2 + \gamma_n^2)\gamma_n\phi^4\cos\gamma_n$$

$$C_{1n} = P^*\sin(n\pi v/2)\sinh\beta_n u/(\beta_n^2 + \gamma_n^2)\beta_n\phi^4\cosh\beta_n$$

$$D_{1n} = -P^*\sin\gamma_n u\sin(n\pi v/2)/(\beta_n^2 + \gamma_n^2)\gamma_n\phi^4\cos\gamma_n$$

and

$$A_{2n} = P^*\sin(n\pi v/2)\cosh\beta_n u^*/(\beta_n^2 - \gamma_n^2)\beta_n\phi^4\cosh\beta_n$$

$$B_{2n} = -P^*\sin(n\pi v/2)\cosh\gamma_n u^*/(\beta_n^2 - \gamma_n^2)\gamma_n\phi^4\cosh\gamma_n$$

$$C_{2n} = P^*\sin(n\pi v/2)\sinh\beta_n u/(\beta_n^2 - \gamma_n^2)\beta_n\phi^4\cosh\beta_n$$

$$D_{2n} = -P^*\sin(n\pi v/2)\sinh\gamma_n u/(\beta_n^2 - \gamma_n^2)\gamma_n\phi^4\cosh\gamma_n$$

Contributions of the fourth building block to the prescribed boundary conditions are:

The contribution of each term of Equation 4.12 to bending moment at $\xi=1$ is

$$\frac{M_\xi a}{D} = \{C_{1n}(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2) + D_{1n}(\nu\phi_I^2(n\pi/2)^2 + \gamma_n^2)\}\sin\frac{n\pi\eta}{2},$$

or if $\quad \phi^2\lambda^2 < (n\pi/2)^2$

$$= \{C_{2n}(\nu\phi_I^2(n\pi/2)^2 - \beta_n^2) + D_{2n}(\nu\phi_I^2(n\pi/2)^2 - \gamma_n^2)\}\sin\frac{n\pi\eta}{2}$$

The contribution of each element of Equation 4.11 to slope at $\xi=0$ is

$$W_{41}(\xi,\eta) = (A_{1n}\beta_n + B_{1n}\gamma_n)\sin\frac{n\pi\eta}{2}$$

or if $\quad \lambda^2\phi^2 < (n\pi/2)^2$

$$= (A_{2n}\beta_n + B_{2n}\gamma_n)\sin\frac{n\pi\eta}{2}$$

The contribution of each term of Equation 4.10 to bending moment along the edge $\eta=1$ is

$$\frac{M_\eta \phi b}{D} = \{C_{1m}(\nu\phi^2(m\pi/2)^2 - \beta_m^2) + D_{1m}(\nu\phi^2(m\pi/2)^2 + \gamma_m^2)\}\sin\frac{m\pi\xi}{2}$$

or if $\lambda^2 < (m\pi/2)^2$

$$\frac{M_n \phi b}{D} = \{C_{2m}(\nu\phi^2(m\pi/2)^2 - \beta_m^2) + D_{2m}(\nu\phi^2(m\pi/2)^2 - \gamma_m^2)\}\sin\frac{m\pi\xi}{2}$$

The contribution of Equation 4.11 to displacement at the point support(u,v) is

$$W_{41}(u,v) = \sum_{n=1,3,5}^{k^*} (A_{1n}\sinh\beta_n u + B_{1n}\sin\gamma_n u)\sin\frac{n\pi v}{2}$$

$$+ \sum_{n=k^*+2}^{\infty} (A_{2n}\sinh\beta_n u + B_{2n}\sinh\gamma_n u)\sin\frac{n\pi v}{2}$$

The coefficient matrix may now be generated by adding the following column and row to the matrix of Figure 2.11.

Quadrant (1):

For $m=1,3,5,\ldots\ldots,2k-1$

$$A_{(m+1)/2,3k+1} = C_{1m}\{\nu\phi^2(m\pi/2)^2 - \beta_m^2\} + D_{1m}\{\nu\phi^2(m\pi/2)^2 + \gamma_m^2\}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= C_{2m}\{\nu\phi^2(m\pi/2)^2 - \beta_m^2\} + D_{2m}\{\nu\phi^2(m\pi/2)^2 - \gamma_m^2\}$$

Quadrant (2):

For n=1,3,5,.......,2k-1

$$A_{k+(n+1)/2,3k+1} = C_{1n}\{\nu\phi_I^2(n\pi/2)^2 - \beta_n^2\} + D_{1n}\{\nu\phi_I^2(n\pi/2)^2 + \gamma_n^2\}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= C_{2n}\{\nu\phi_I^2(n\pi/2)^2 - \beta_n^2\} + D_{2n}\{\nu\phi_I^2(n\pi/2)^2 - \gamma_n^2\}$$

Quadrant (3):

For n=1,3,5,.......2k-1

$$A_{2k+(n+1)/2,3k+1} = A_{1n}\beta_n + B_{1n}\gamma_n$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= A_{2n}\beta_n + B_{2n}\gamma_n$$

Quadrant (4):

For m=1,3,5,........,2k-1

$$A_{3k+1,(m+1)/2} = (\theta_{1m}\sinh\beta_m v + \sin\gamma_m v)\sin(m\pi u/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$A_{3k+1,(m+1)/2} = (\theta_{2m}\sinh\beta_m v + \sinh\gamma_m v)\sin(m\pi u/2)/\theta_{22m}$$

Quadrant (5):

For $n=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,k+(n+1)/2} = (\theta_{1n}\sinh\beta_n u + \sin\gamma_n u)\sin(n\pi v/2)/\theta_{11n}$$

or if $\lambda^2\phi^2 < (n\pi/2)^2$

$$= (\theta_{2n}\sinh\beta_n u + \sinh\gamma_n u)\sin(n\pi v/2)/\theta_{22n}$$

Quadrant (6):

For $p=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,2k+(p+1)/2} = \{\cosh\beta_p(1-u) - \theta_{1p}\cos\gamma_p(1-u)\}\sin(p\pi v/2)/\theta_{11p}$$

or if $\lambda^2\phi^2 < (p\pi/2)^2$

$$= \{\cosh\beta_p(1-u) - \theta_{2p}\cosh\gamma_p(1-u)\}\sin(p\pi v/2)/\theta_{22p}$$

Quadrant (7)

For $n=1,3,5,\ldots\ldots,2k-1$

$$A_{3k+1,3k+1} = \sum_{n=1,3,5}^{k^*} (A_{1n}\sinh\beta_n u + B_{1n}\sin\gamma_n u)\sin\frac{n\pi v}{2}$$

$$+ \sum_{n=k^*+2}^{2k-1} (A_{2n}\sinh\beta_n u + B_{2n}\sinh\gamma_n u)\sin\frac{n\pi v}{2}$$

Using the above information, Program 15 of Appendix 3 was designed to generate eigenvalues of any cantilever plate with lateral point supports symmetrically located with respect to the ξ axis as shown in Figure 4.2. These eigenvalues are obtained simply by feeding Program 15 to a digital computer with the plate aspect ratio and its point support coordinate u and v. As an example, eigenvalues for the first three antisymmetric modes of the cantilever plate of different aspect ratios, and point support coordinate of (1/2,1/2) are listed in Table 4.2. Corresponding mode shape data are obtained using Program 16 of Appendix 3. Mode shapes corresponding to eigenvalues of Table 4.2 are illustrated in Appendix 2.

Results and Discussion

At the begining of this work we mentioned that most of the published data and research work relating to the problem of plate vibrations up to the year 1965 were grouped

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ $\nu = 0.333$ $u=v=0.5$

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	9.467	9.501	9.547	9.602	9.622	9.602	9.484	9.255	8.494	7.483	6.555
2	59.52	59.64	59.62	51.66	37.20	26.07	18.43	14.26	10.11	8.207	7.114
3	86.10	85.37	81.05	80.24	60.60	52.58	42.89	36.74	27.42	21.51	18.00

Table 4.1. First three symmetric mode eigenvalues for the cantilever plate with symmetrically located lateral point supports.

$\lambda^2 = \omega a^2\sqrt{\rho/D}$ $\nu = 0.333$ $u=v=0.5$

$\phi=2b/a$	1/3	1/2.5	1/2	1/1.5	1/1.25	1	1.25	1.5	2	2.5	3
$\phi'=b/a$	1/6	1/5	1/4	1/3	1/2.5	1/2	5/8	3/4	1	1.25	1.5
Mode 1	41.26	35.02	28.76	22.41	19.19	15.93	13.27	11.45	9.065	7.579	6.599
2	106.9	96.15	85.58	73.45	61.93	46.73	34.62	27.37	19.81	16.08	13.76
3	141.0	122.9	103.7	82.02	73.98	68.21	56.09	45.54	34.57	28.90	23.96

Table 4.2. Eigenvalues for the first three antisymmetric free vibration modes of the cantilever plate with lateral point supports.

together and presented in a single reference thanks to Leissa(19). Leissa's book lists a good number of references relating to the analysis of the rectangular cantilever plate. He also lists their published data most of which agrees favorably with the results of Gorman's work(20) relating to the cantilever plate problem. Gorman's results were listed in Tables 2.3 and 2.4 for convenient reference. In Chapter two we have compared these results with our own of Tables 3.1 and 3.2 and found a near total agreement. Leissa's book also lists a good number of references relating to the analysis of point supported plates, none of which dealt with the point supported cantilever plate. Although, a more recent literature survey shows that researchers and scientists are still devoting time and effort to the analysis of the point supported plates including Gorman(24) and G. Venkateswara Rao, I.S. Raju and C.L. Amba-Rao(25), none of them has dealt with the cantilever type plates. Therefore there is no apparent published data with which one may compare in order to test the validity of one's results. However, one can think of numerous options open to test these results, some of which will be explored in the following.

Focussing our attention on Figure 4.3, it is readily seen that by removing the third building block we would end up with three building blocks that could be used to analyze modes symmetric with respect to the ξ axis and antisymmetric with respect to the η axis of the rectangular plate with symmetrically distributed point supports, readily analyzed by Gorman(20). With this in mind, the following test was conducted. The effects of the third building block of Figure 4.3 was eliminated by dropping rows and columns representing these effecis from the appropriate coefficient matrix. Eigenvalues for plates similar to those analyzed by Gorman were generated and general agreement was found.

We next turn to Figure 4.7, where by dropping the third building block we end up with a set of building blocks that can be used to analyze the fully antisymmetric modes of the rectangular plate with symmetrically distributed point supports on the lateral surface, also analyzed by Gorman. Therefore, effects of the third building block of Figure 4.7 were removed by dropping out appropriate rows and columns from the coefficient matrix. Relevent eigenvalues were generated, and when compared with existing results, general agreement was also found.

In the above two tests we have checked and compared our derivations and programming of the present chapter. We now turn to check for those of Chapter 3 as well. A close look at Figures 4.3 and 4.7 shows that by decreasing the value of the dimensionless quantity u, and as u becomes closer and closer to zero, at the limit we should end up with the same eigenvalues as those of the cantilever plate, and that proved to be the case. The same was found for the plate of Figure 3.1 as ζ became close to zero. Furthermore, it is clearly seen that if the dimensionless quantity u of Figure 4.3 and 4.7 was to be increased so as to become very close to unity, at the limit we expect the same eigenvalues as those of Figure 3.2 That was also proven to be the case.

CONCLUSION

One of the most commonly used structures in the industrialized world is the rectangular plate. In many design problems, specifications merely ensuring that plates will withstand applied static loads prove to be inadequate. It is for this reason that analysis of the free vibration of plates has been and still is gaining momentum. The cantilever plate with symmetrically distributed point supports constitutes one of the classical plate free vibration problems. It has many industrial applications, as well as applications in civil engineering structures. Analysis of this type of plate has traditionally presented the analyst with serious difficulties because of the formidable problem of trying to satisfy the free edge conditions and the condition of zero displacement at the point supports.

In this paper, such difficulties were obviated, and a highly accurate analytical type solution was provided for the first time for this particular engineering problem. Through exploitation of the method of superposition, and techniques developed by Gorman(21), a solution for the above problem was obtained by superimposing a number of solutions that were obtained by classical methods. Unlike numerical and other approximate solutions, the analytical solution satisfies exactly the differential equation throughout the plate. It also satisfies all boundary conditions to any desired degree of accuracy.

Highly accurate eigenvalues and mode shapes have been tabulated for both the symmetric and antisymmetric modes of a wide selection of plates with a value of Poisson's ratio equal 0.333. All of the results reported here were computed with a view to providing accuracy to four significant digits in the eigenvalues. Greater accuracy is easily obtained simply by increasing the number of terms used in the appropriate series involved. After conducting numerous convergence tests of the type described in Chapter 2, it was recognized that thirty term expansions would exceed the number of terms required for many plate configurations and modes. But in order to take care of the most demanding cases, it was decided to go with thirty terms throughout, in the case of point supported plates. However, seven term expansions proved to be adequate for most plate configurations and modes of the cantilever plate. It was decided to go with fifteen terms in this case to satisfy the most demanding cases.

A study of the literature reveals that exact or highly accurate solutions have been obtained for many rectangular thin-plate free vibration problems including the rectangular cantilever plate. However, to the author's knowledge, this proves not to be the case for the problem at hand. Consequently, no comparison of results is possible. But, eigenvalues generated were carefully examined using various techniques such as those discussed at the end of Chapter 4. Mode shape data were also generated and carefully analyzed using specially designed computer programs of Appendix 3.

The analytical solution presented herein may be considered as an improvement and an extension to the solution of the cantilever plate of Reference (20). In this paper, rejection modes introduced by the solution of the above reference were eliminated. Furthermore, point supports were introduced. A highly accurate analytical solution was provided using a technique already described in the literature. This mathematical technique is not limited in application to problems with symmetrically distributed point supports. Its application could be extended to any combination of point supports along the edges or the lateral surface by prudently choosing a sufficent number of building blocks to satisfy the boundary conditions. The only requirement is that these boundary conditions and the differential equation do not introduce any non-linearities. In fact, a solution for the free vibration problem of the cantilever plate with four symmetrically distributed point supports, two along the edges parallel to the ξ axis and two along the edge opposite to the clamped base, can easily be obtained by superimposing appropriate building blocks already discussed in this thesis. As indicated earlier, a search of the literature has failed to provide eigenvalue and mode shape information for this important industrial problem. A main goal of this paper is to fill this serious gap in the technical literature.

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Appendix 1

In this Appendix a list of integrals is reproduced from Reference (20) for convenient reference. This list, although brief, it is sufficient for all the integrations required in this thesis.

$$\int_0^1 \cosh\beta\xi\cos m\pi\xi\, d\xi = \frac{\beta\cos m\pi\sinh\beta}{\beta^2+(m\pi)^2} \tag{A1.1}$$

$$\int_0^1 \sinh\beta\xi\cos m\pi\xi\, d\xi = \frac{\beta}{\beta^2+(m\pi)^2}(\cosh\beta\cos m\pi-1) \tag{A1.2}$$

$$\int_0^1 \cos\beta\xi\cos m\pi\xi\, d\xi = \frac{\beta\sin\beta\cos m\pi}{\beta^2-(m\pi)^2} \tag{A1.3}$$

$$\int_0^1 \sin\beta\xi\cos m\pi\xi\, d\xi = \frac{\beta}{\beta^2-(m\pi)^2}(1-\cos\beta\cos m\pi) \tag{A1.4}$$

$$\int_0^1 \cosh\beta\xi\sin\frac{m\pi\xi}{2}\, d\xi = \frac{m\pi/2+\beta\sinh\beta\sin m\pi/2}{\beta^2+(m\pi/2)^2} \tag{A1.5}$$

$$\int_0^1 \sinh\beta\xi\sin\frac{m\pi\xi}{2}\, d\xi = \frac{\beta\cosh\beta\sin m\pi/2}{\beta^2+(m\pi/2)^2} \tag{A1.6}$$

$$\int_0^1 \cos\beta\xi\sin\frac{m\pi\xi}{2}\, d\xi = \frac{m\pi/2-\beta\sin(m\pi/2)\sin\beta}{(m\pi/2)^2-\beta^2} \tag{A1.7}$$

$$\int_0^1 \sin\beta\xi\sin\frac{m\pi\xi}{2}\, d\xi = \frac{-\beta\cos\beta\sin m\pi/2}{\beta^2-(m\pi/2)^2} \tag{A1.8}$$

Equation (A1.6) is valid for m=1,3,5,... only.

$$\int_0^1 \cosh(\beta(1-\xi))\sin\frac{m\pi\xi}{2}\, d\xi = \frac{(m\pi/2)\cosh\beta}{\beta^2+(m\pi/2)^2} \tag{A1.9}$$

$$\int_0^1 \sinh(\beta(1-\xi))\sin\frac{m\pi\xi}{2}d\xi = \frac{(m\pi/2)\sinh\beta-\beta\sin(m\pi/2)}{\beta^2+(m\pi/2)^2} \quad (A1.10)$$

$$\int_0^1 \cos(\beta(1-\xi))\sin\frac{m\pi\xi}{2}d\xi = \frac{-(m\pi/2)\cos\beta}{\beta^2-(m\pi/2)^2} \quad (A1.11)$$

$$\int_0^1 \sin(\beta(1-\xi))\sin\frac{m\pi\xi}{2}d\xi = \frac{\beta\sin(m\pi/2)-(m\pi/2)\sin\beta}{\beta^2-(m\pi/2)^2} \quad (A1.12)$$

Equqtions (A1.9) and (A1.11) are valid for m=1,3,5,... only.

$$\int_0^1 \sinh\beta\xi\sin m\pi\xi d\xi = \frac{-m\pi\cos m\pi\sinh\beta}{\beta^2+(m\pi)^2} \quad (A1.13)$$

$$\int_0^1 \cosh\beta\xi\sin m\pi\xi d\xi = \frac{m\pi(1-\cos m\pi\cosh\beta)}{\beta^2+(m\pi)^2} \quad (A1.14)$$

$$\int_0^1 \sin\beta\xi\sin m\pi\xi d\xi = \frac{m\pi\cos m\pi\sin\beta}{\beta^2-(m\pi)^2} \quad (A1.15)$$

$$\int_0^1 \cos\beta\xi\sin m\pi\xi d\xi = \frac{m\pi(1-\cos m\pi\cos\beta)}{(m\pi)^2-\beta^2} \quad (A1.16)$$

$$\int_0^1 \sinh(\beta(1-\xi))\sin m\pi\xi d\xi = \frac{m\pi\sinh\beta}{(m\pi)^2+\beta^2} \quad (A1.17)$$

$$\int_0^1 \cosh(\beta(1-\xi))\sin m\pi\xi d\xi = \frac{m\pi(\cosh\beta-\cos m\pi)}{\beta^2+(m\pi)^2} \quad (A1.18)$$

$$\int_0^1 \sin(\beta(1-\xi))\sin m\pi\xi d\xi = \frac{m\pi\sin\beta}{(m\pi)^2-\beta^2} \quad (A1.19)$$

$$\int_0^1 \cos(\beta(1-\xi))\sin m\pi\xi d\xi = \frac{m\pi(\cos\beta-\cos m\pi)}{(m\pi)^2-\beta^2} \quad (A1.20)$$

$$\int_0^1 \cosh(\beta(1-\xi))\cos m\pi\xi d\xi = \frac{\beta\sinh\beta}{\beta^2+(m\pi)^2} \quad (A1.21)$$

$$\int_0^1 \sinh(\beta(1-\xi))\cos m\pi\xi d\xi = \frac{\beta(\cosh\beta-\cos m\pi)}{\beta^2+(m\pi)^2} \quad (A1.22)$$

$$\int_0^1 \cos(\beta(1-\xi))\cos m\pi\xi \, d\xi = \frac{\beta\sin\beta}{\beta^2-(m\pi)^2} \tag{A1.23}$$

$$\int_0^1 \sin(\beta(1-\xi))\cos m\pi\xi \, d\xi = \frac{\beta(\cos m\pi-\cos\beta)}{\beta^2-(m\pi)^2} \tag{A1.24}$$

FREE VIBRATION ANALYSIS
OF RECTANGULAR CANTILEVER PLATES
WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

VOLUME II

Appendix 2

MODAL SHAPES

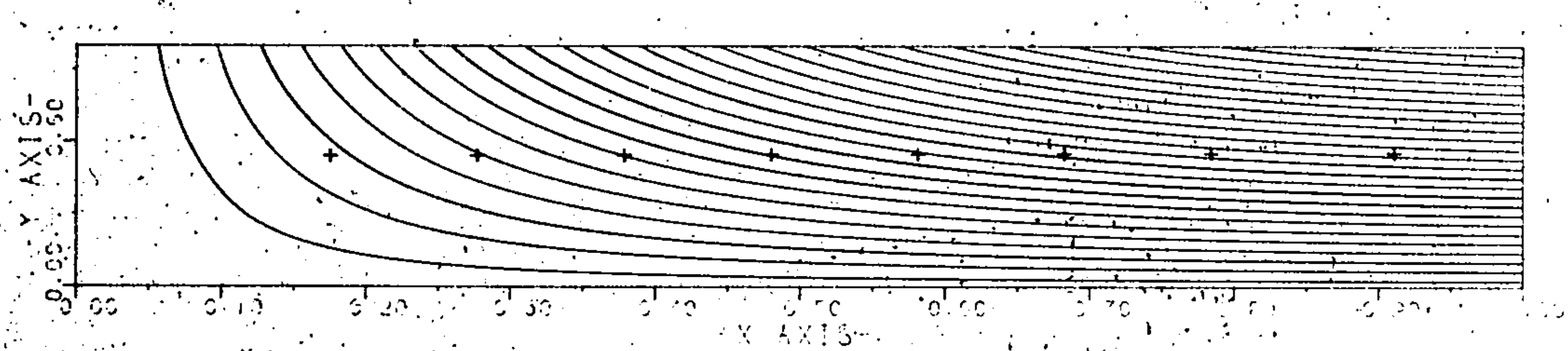
This Appendix contains a comprehensive computer mode shape printouts illustrating both symmetric and antisymmetric mode shapes for the cantilever plates discussed in Chapters 2, 3, and 4. In these illustrations (PHI) indicates the full plate aspect ratio divided by two. U and V indicate the point support coordinates on the lower half of the cantilever plate as explained in the list of symbols. It also must be noted that in these illustrations only the lower half of the plate is shown.

The original computer printouts show three different colors, black, blue, and red. Where black indicates positive displacements, blue indicates zero displacements, and red indicates negative displacements. In future copies these displacements are indicated by (+), (0), and (-) respectively.

WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

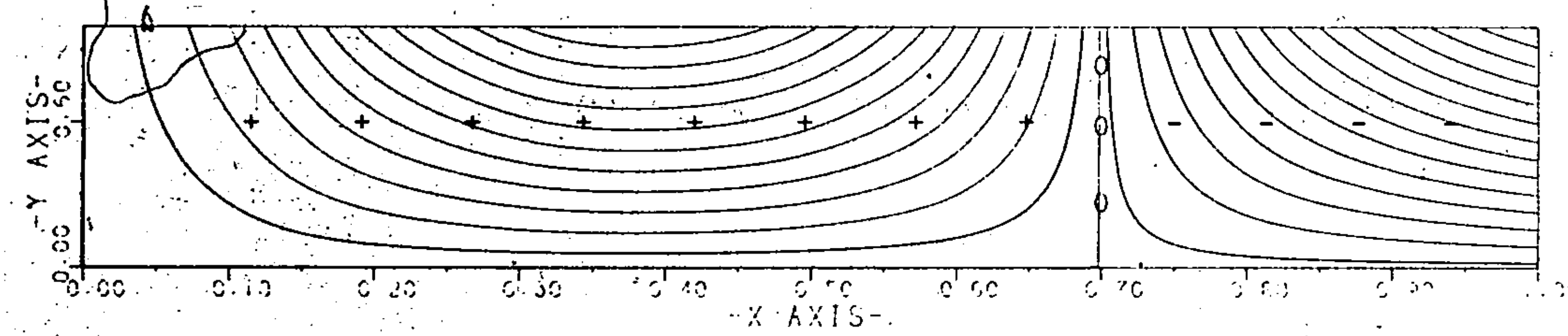
••••••• FIRST ANTISYMMETRIC MODE WITH PHI = .76 •••••••



WAVE PLATE GRAPHIC

*********** THE RECTANGULAR CANTILEVER PLATE ***********

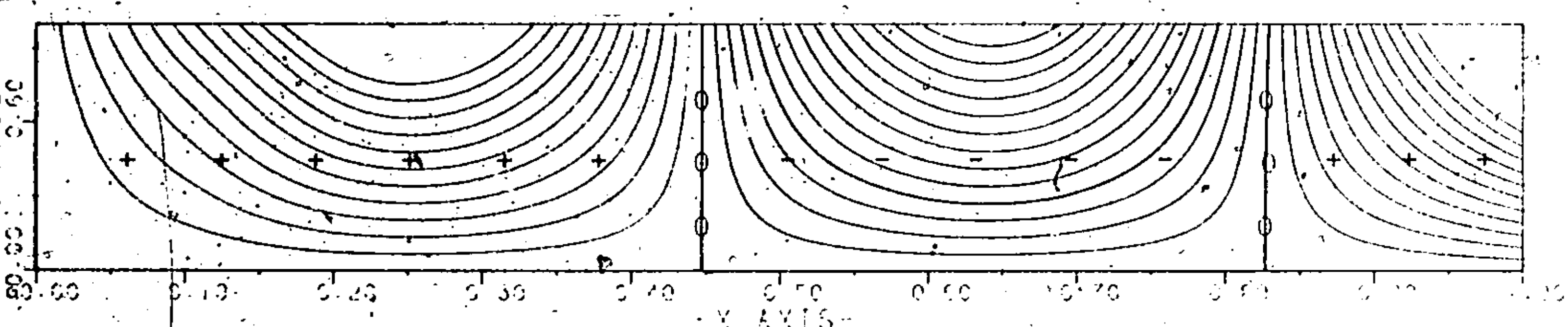
******* SECOND ANTISYMMETRIC MODE WITH PHI = π/6 *******



WAVE PLATE GRAPHIC

••••••••••• THE RECTANGULAR CANTILEVER PLATE •••••••••••

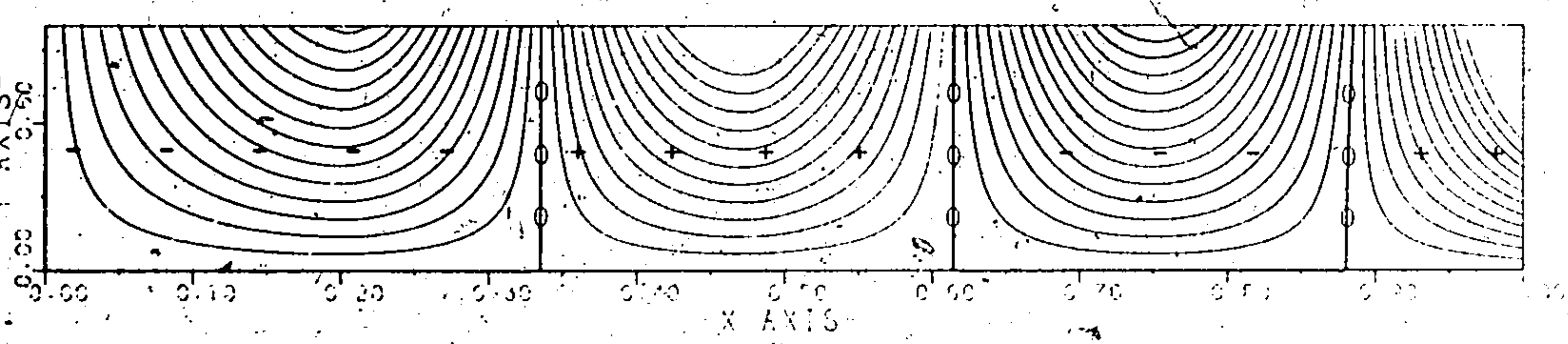
••••••• THIRD ANTISYMMETRIC MODE WITH PHI = 1/6 •••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

•••••••• FOURTH ANTISYMMETRIC MODE WITH PHI = [illegible] ••••••••



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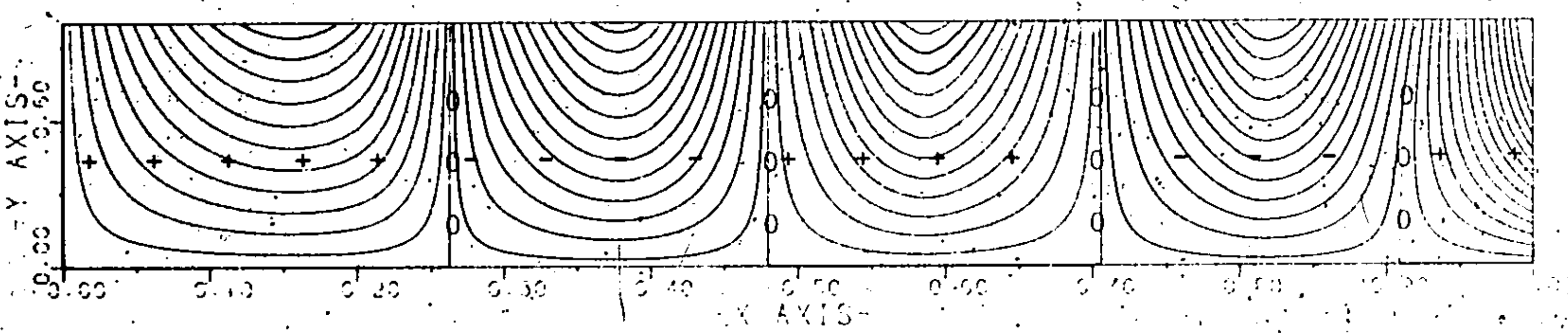
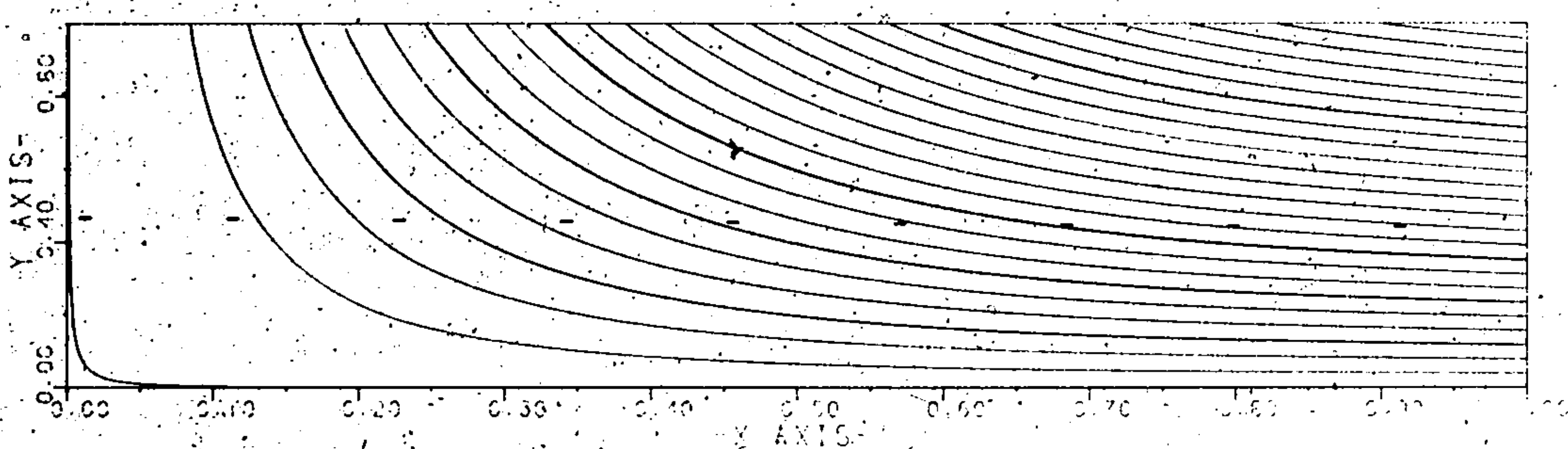
••••••• FIFTH ANTISYMMETRIC MODE WITH PHI = 1/6 •••••••



WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

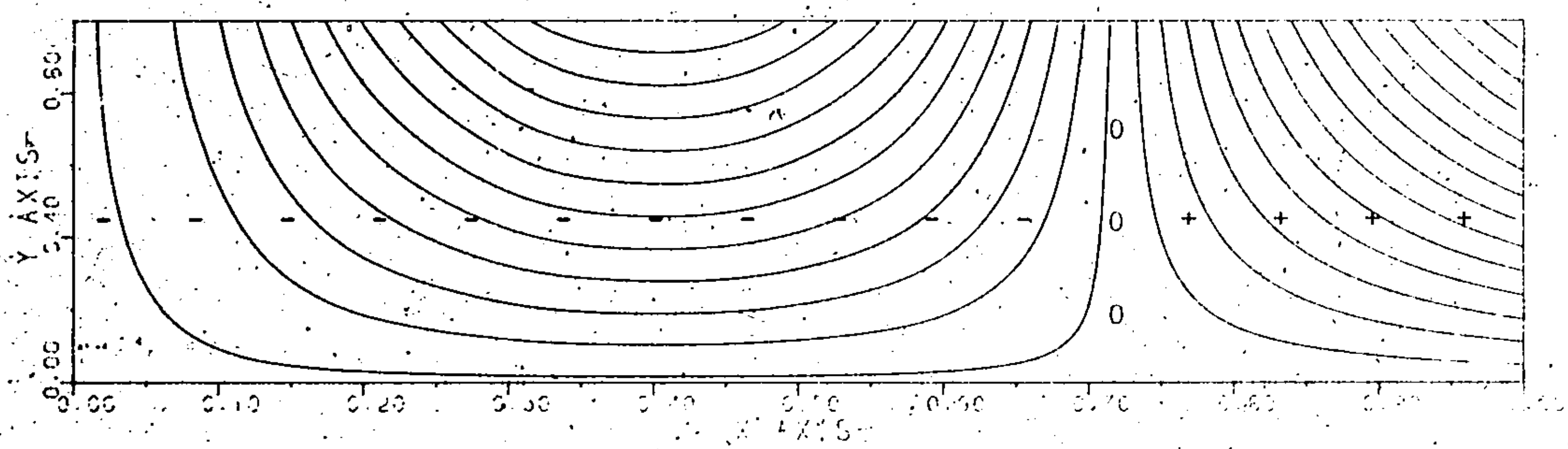
FIRST ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

••••••• SECOND ANTISYMMETRIC MODE WITH PHI ... •••••••



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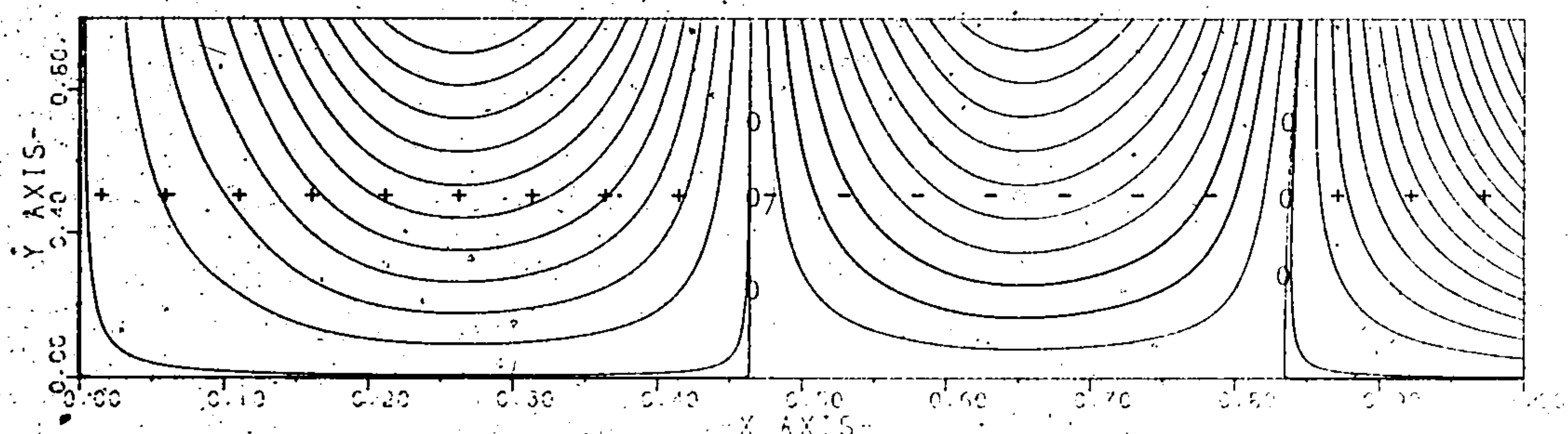
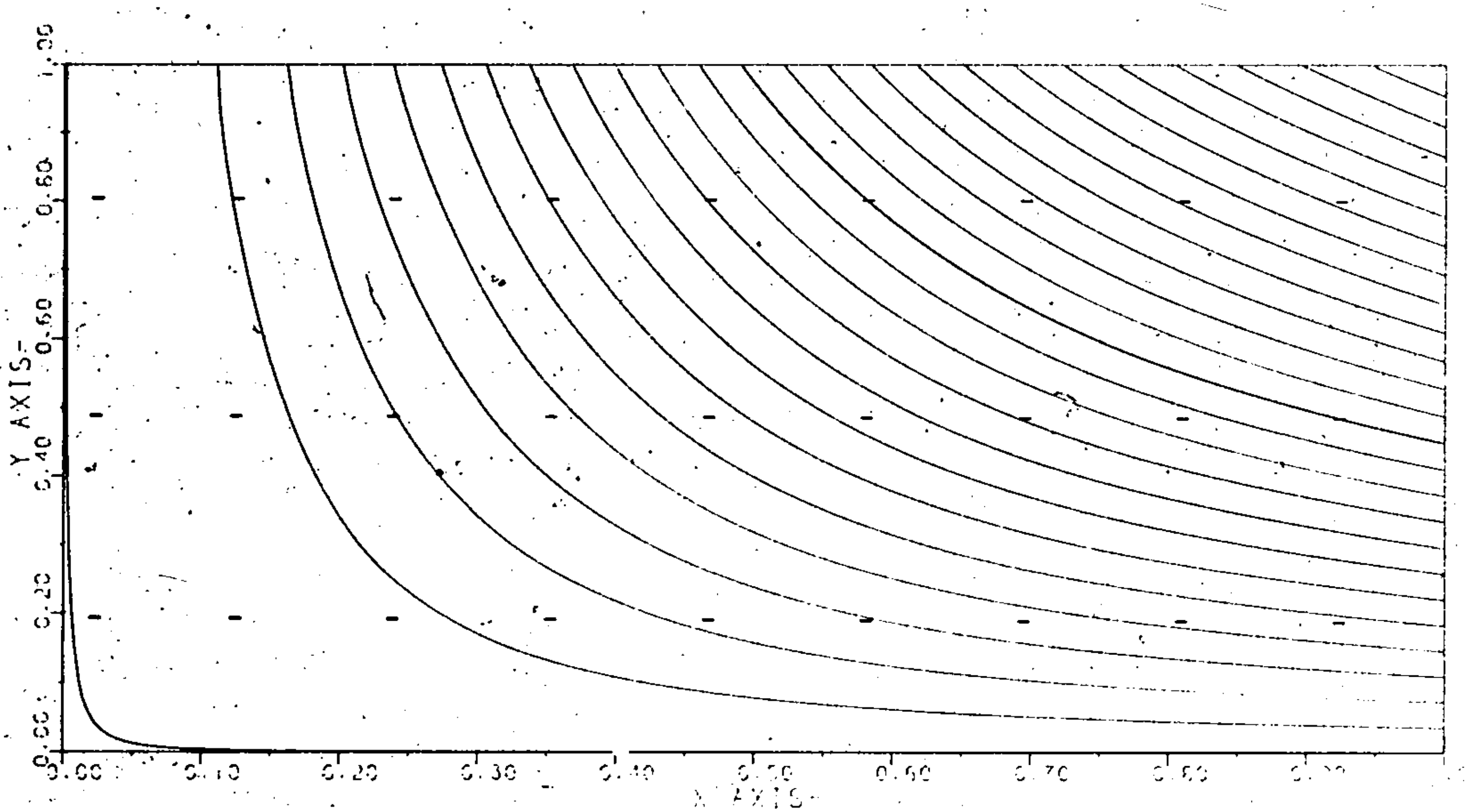
••••••• THIRD ANTISYMMETRIC MODE WITH PHI ••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

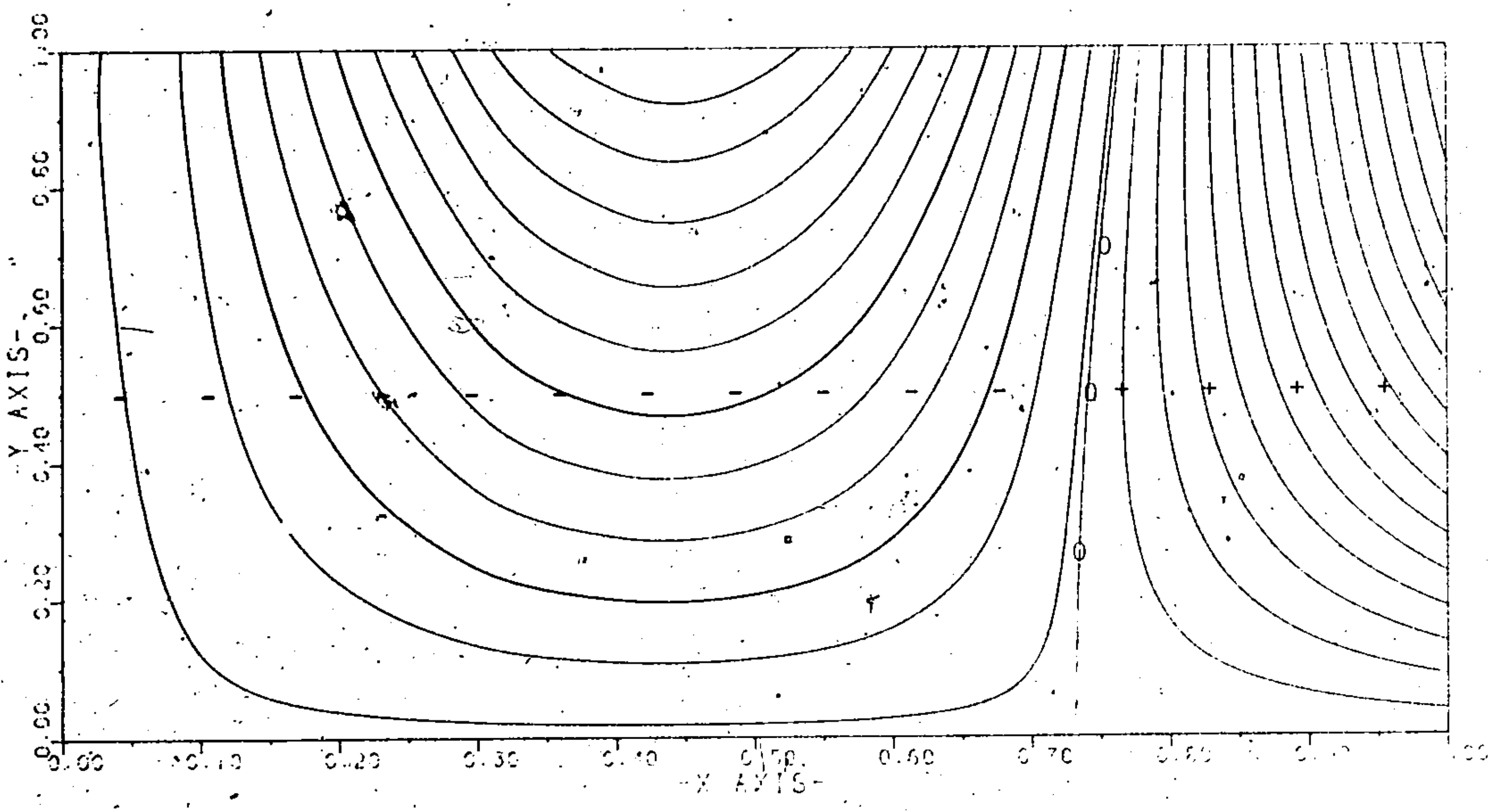
••••••• FIRST ANTISYMMETRIC MODE WITH/PHI = [illegible]2 ••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

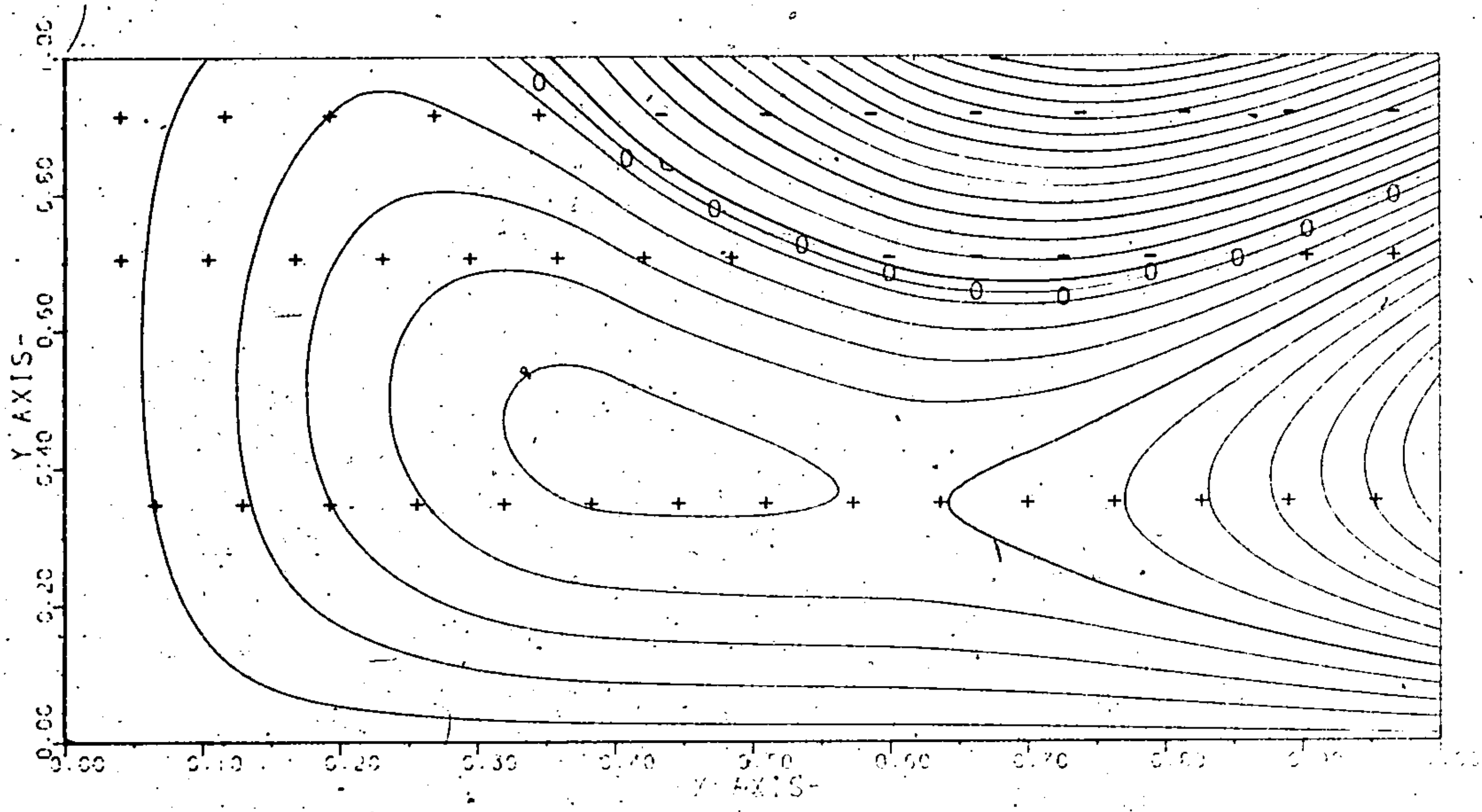
••••••• SECOND ANTISYMMETRIC MODE WITH PHI = [illegible] •••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

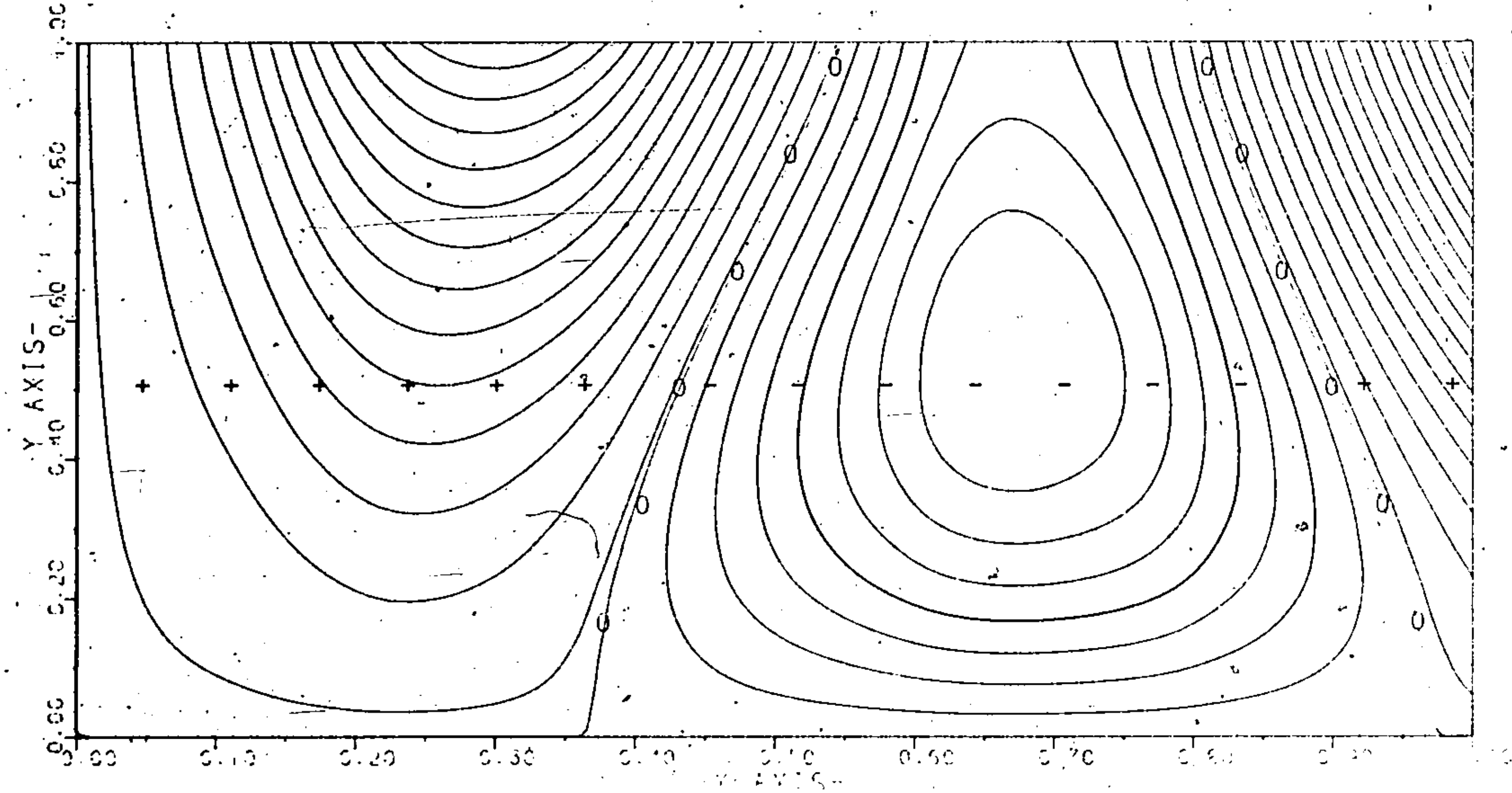
••••••• THIRD ANTISYMMETRIC MODE WITH PHI - .../2 •••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

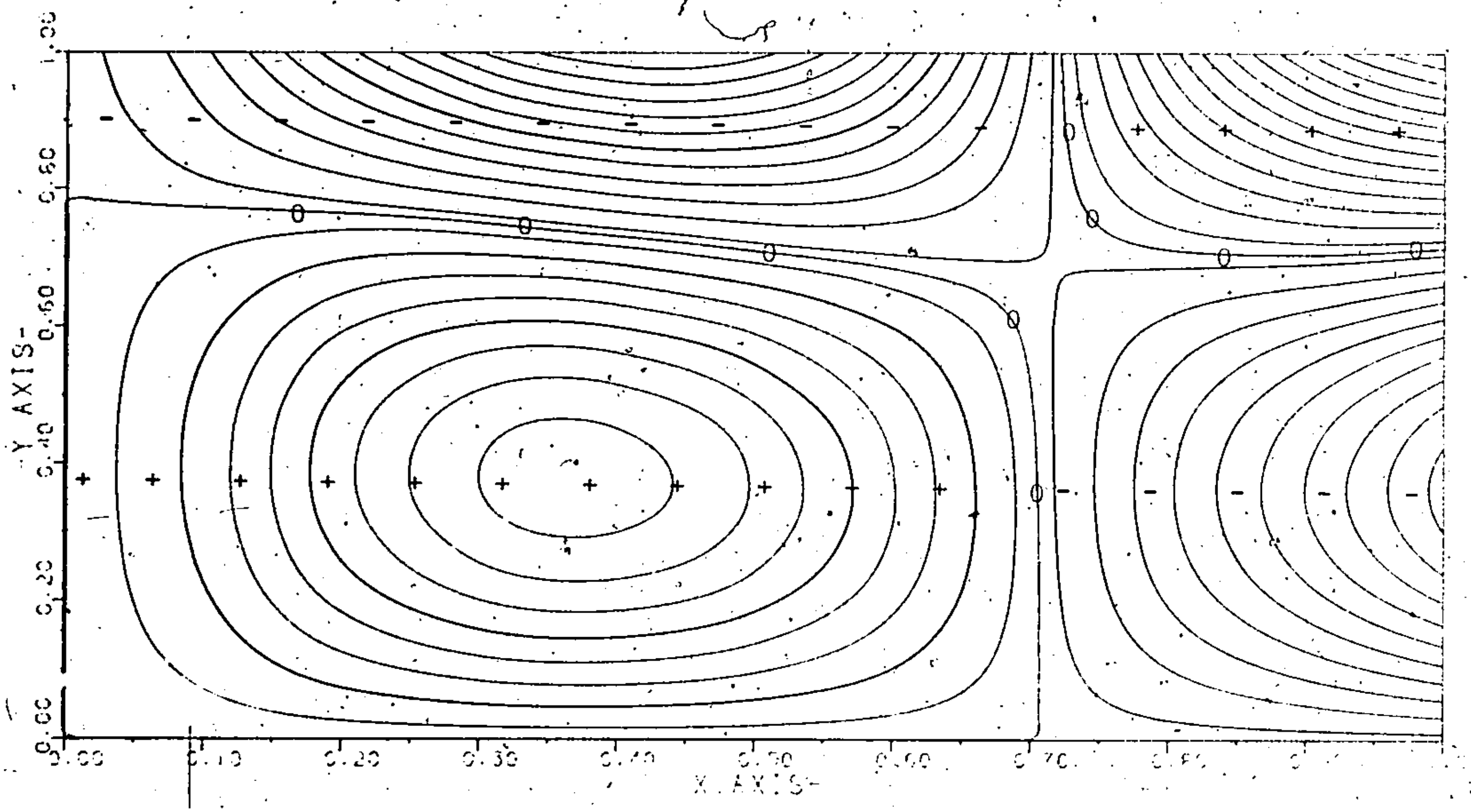
••••••• FOURTH ANTISYMMETRIC MODE WITH PHI = [illegible]2 •••••••



WAVE PLATE GRAPHIC

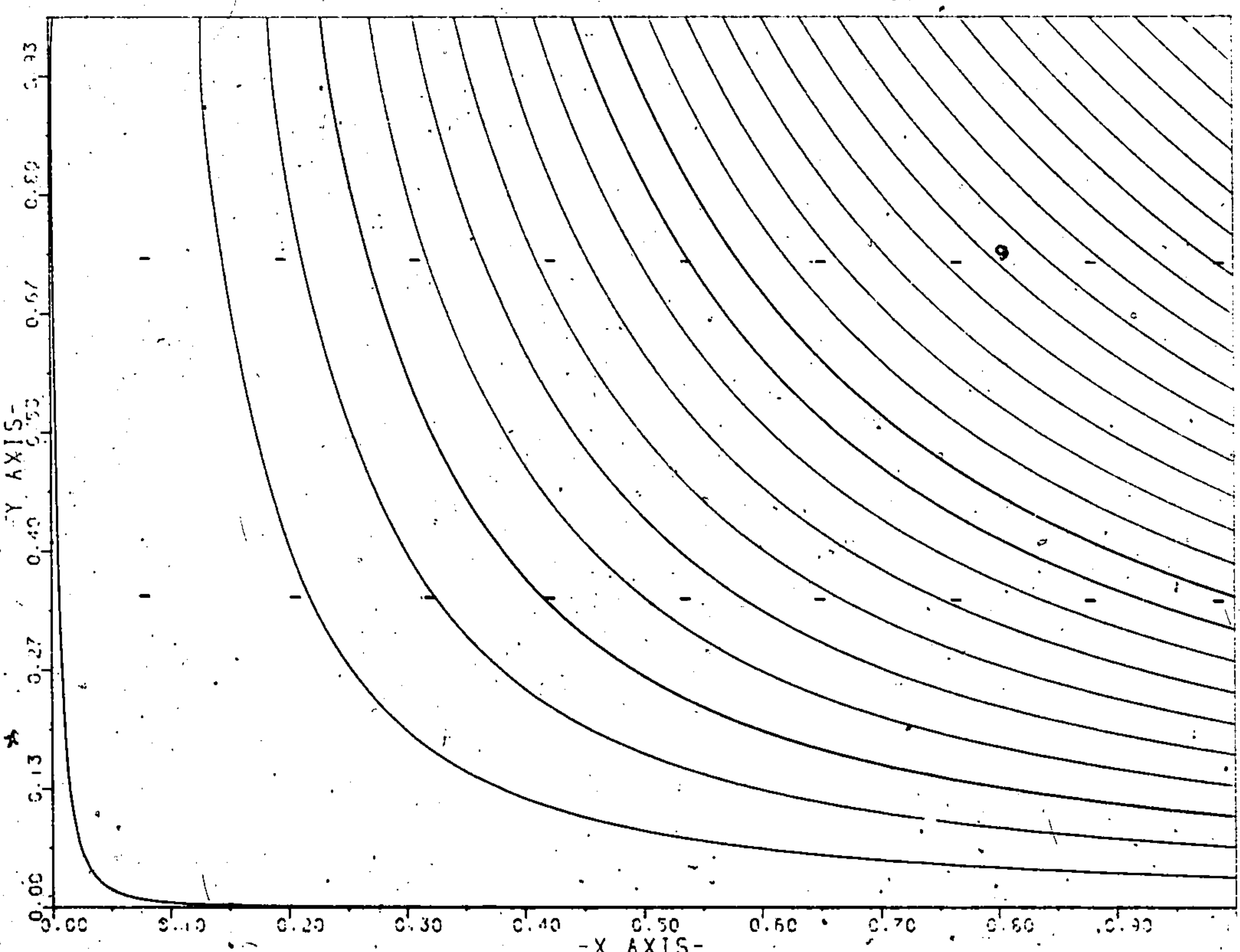
•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

••••••• FIFTH ANTISYMMETRIC MODE WITH PHI - .42 •••••••



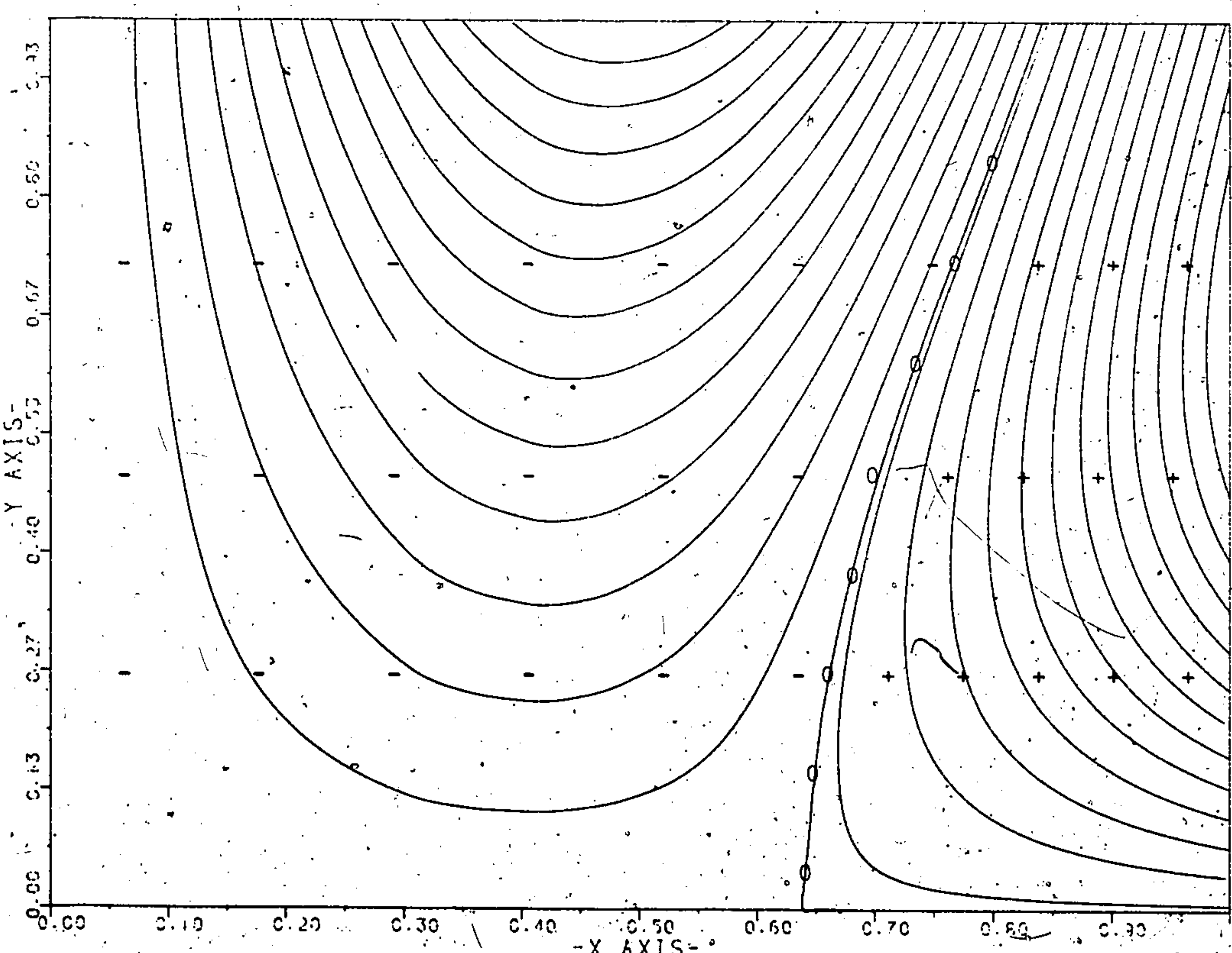
••••••••••• THE RECTANGULAR CANTILEVER PLATE •••••••••••

••••••• FIRST ANTISYMMETRIC MODE WITH PHI = 0.4 •••••••



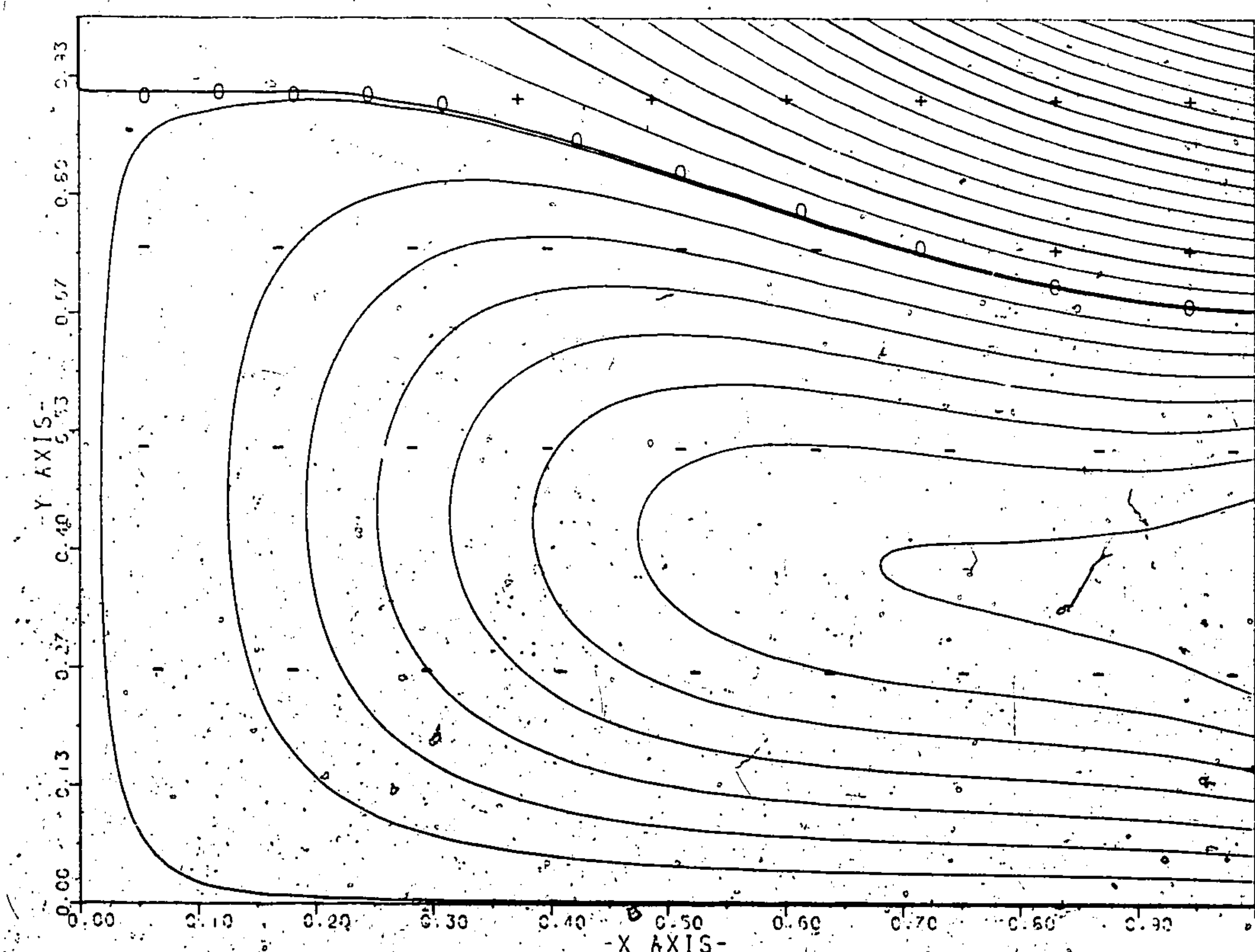
•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

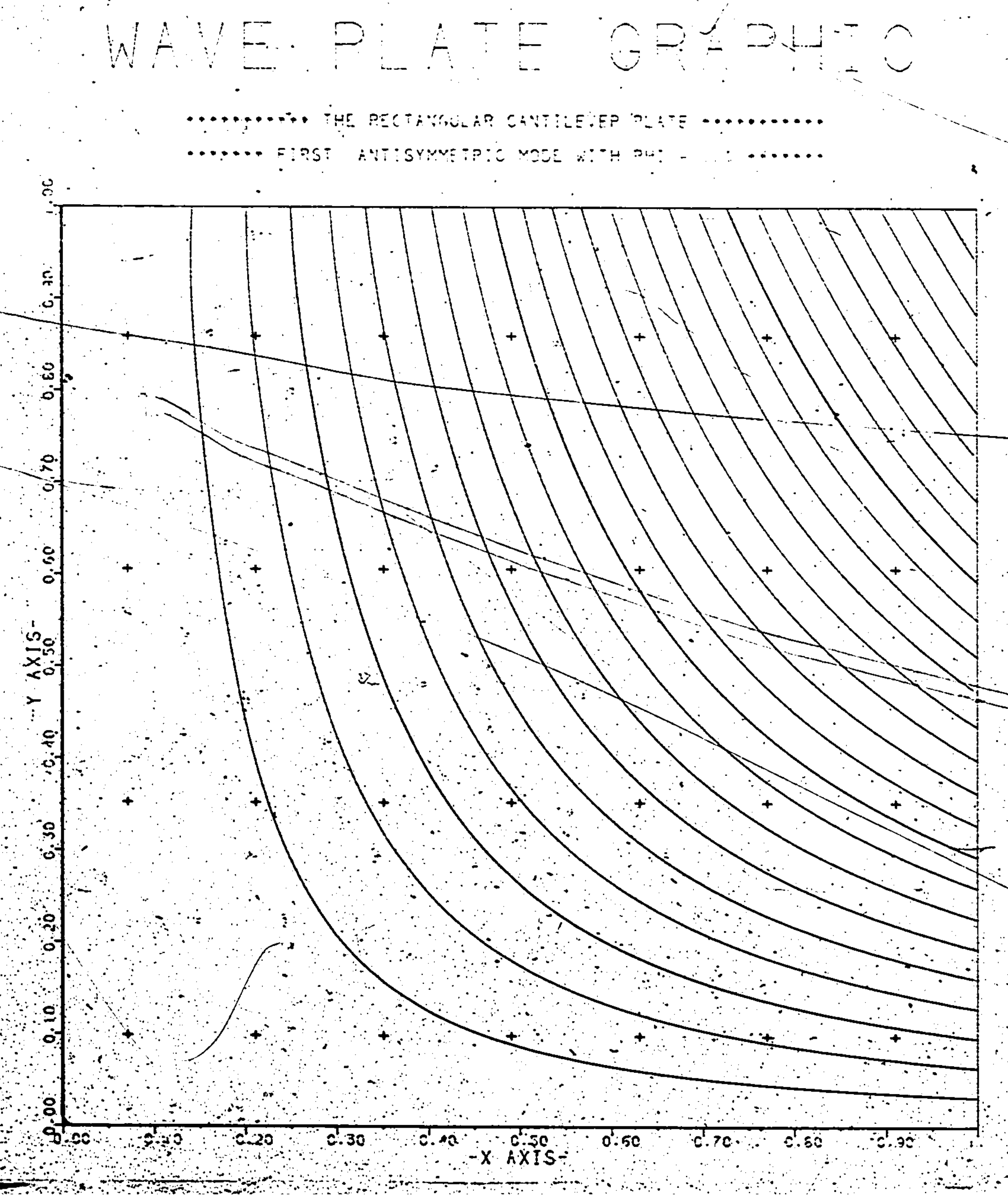
••••••• SECOND ANTISYMMETRIC MODE WITH PHI = [illegible] •••••••



•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

••••••• THIRD ANTISYMMETRIC MODE WITH PHI = 3.4 •••••••

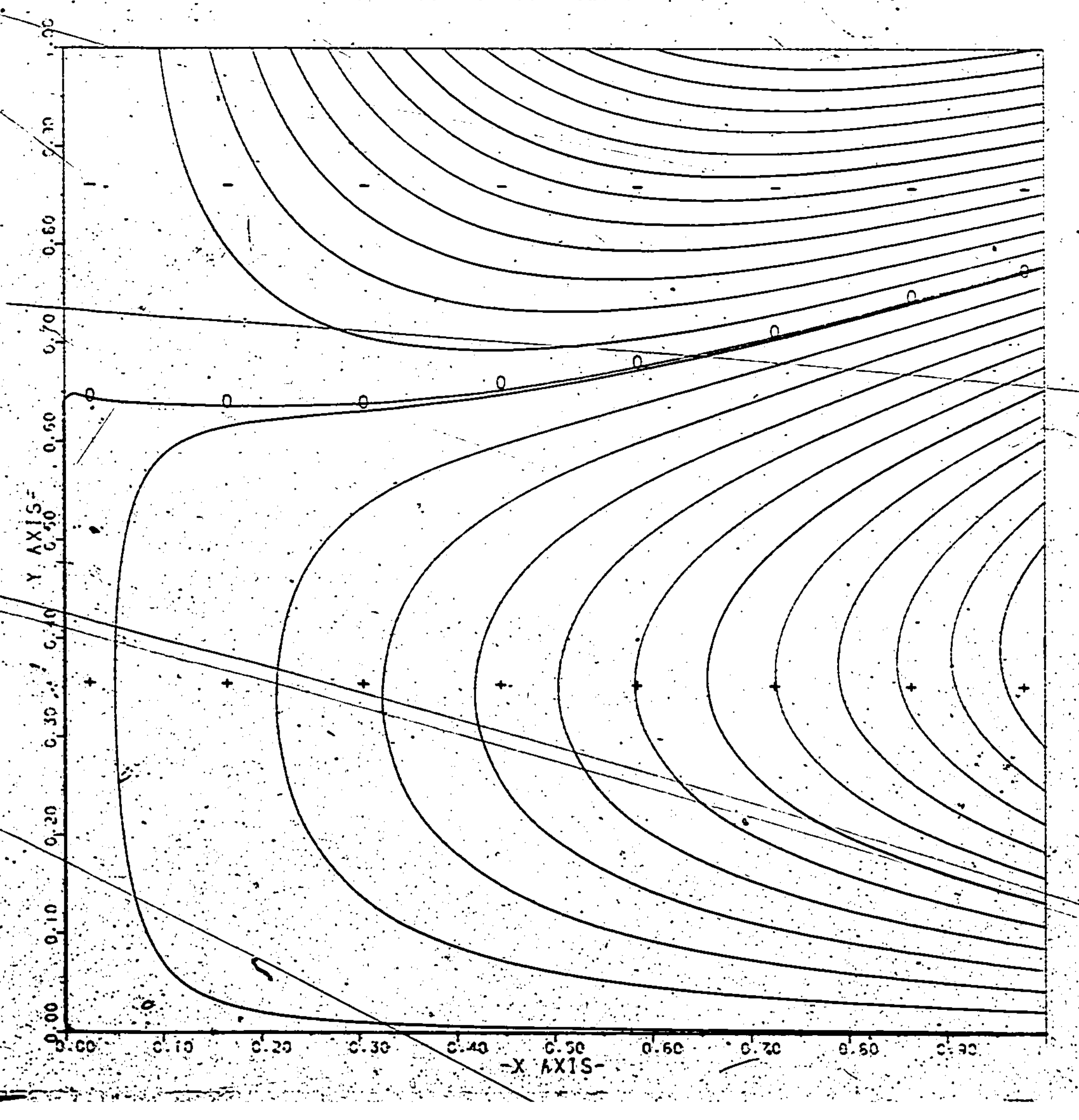



WAVE PLATE GRAPHIC
THE RECTANGULAR CANTILEVER PLATE
FIRST ANTISYMMETRIC MODE WITH PHI
-Y AXIS-
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
-X AXIS-

WAVE PLATE GRAPHIC

************ THE RECTANGULAR CANTILEVER PLATE ************

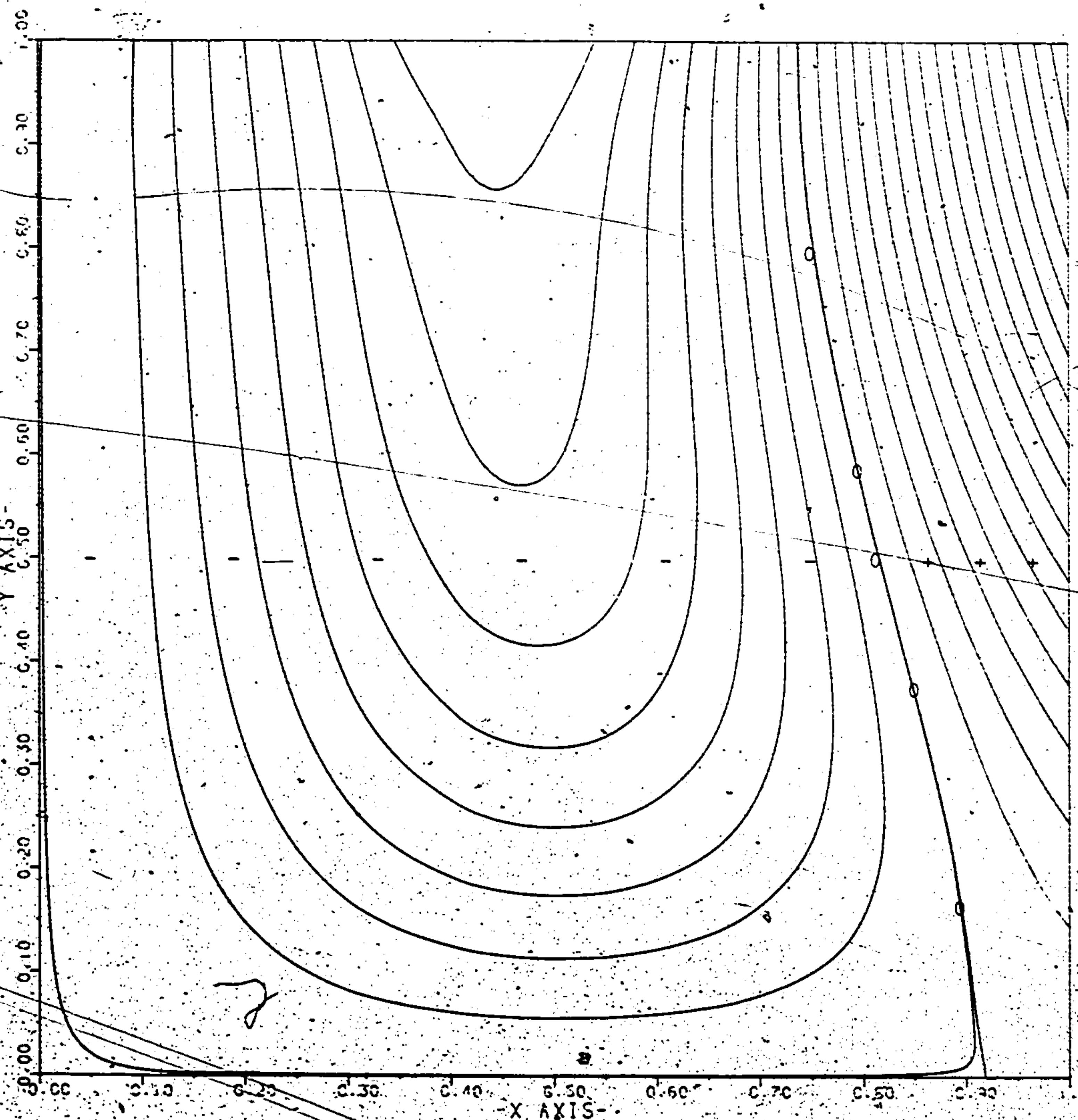
******* SECOND ANTISYMMETRIC MODE WITH PHI = ... *******

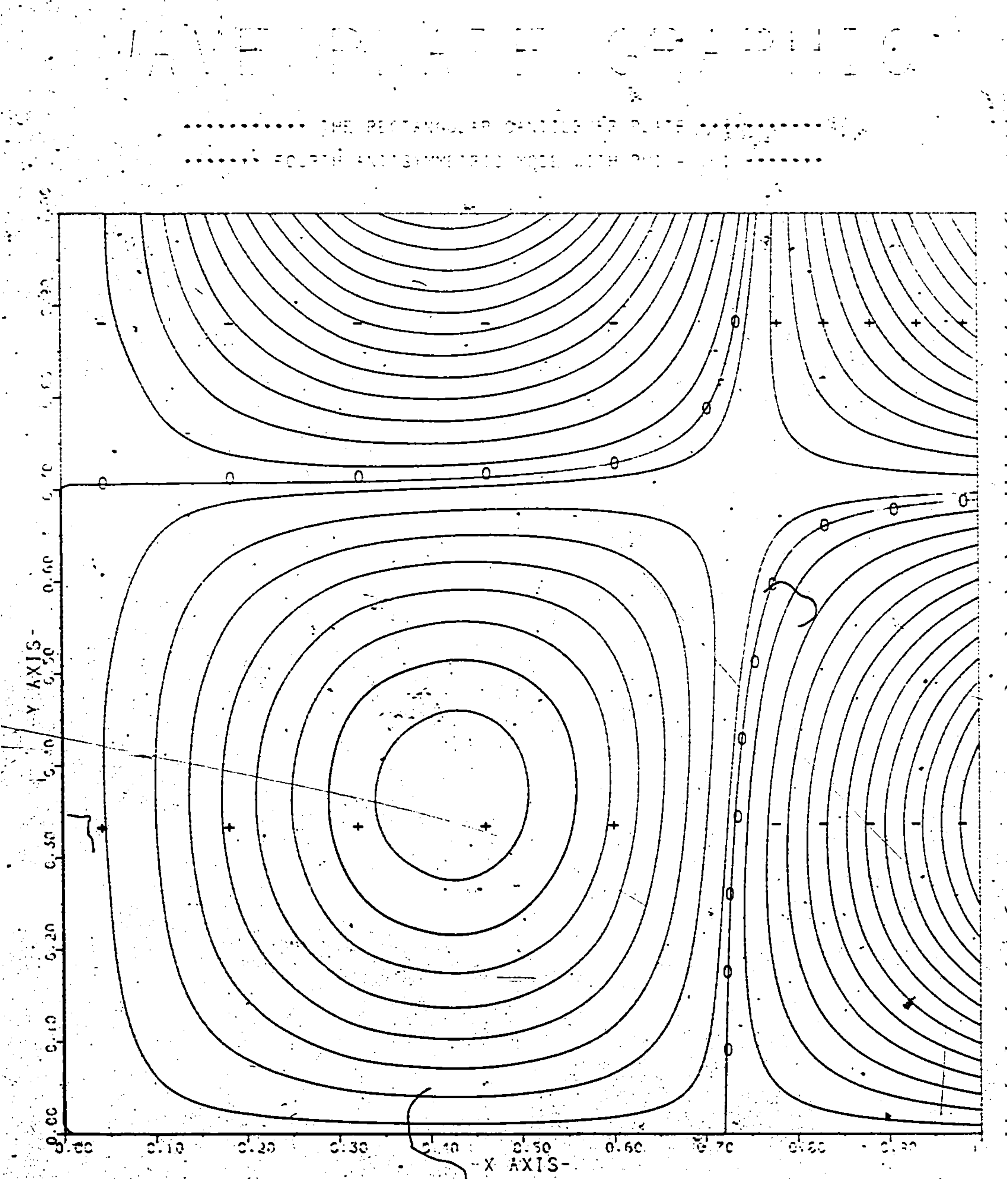


WAVE PLATE GRAPHICS

THE RECTANGULAR CANTILEVER PLATE

THIRD ANTISYMMETRIC MODE WITH PHI = [illegible]

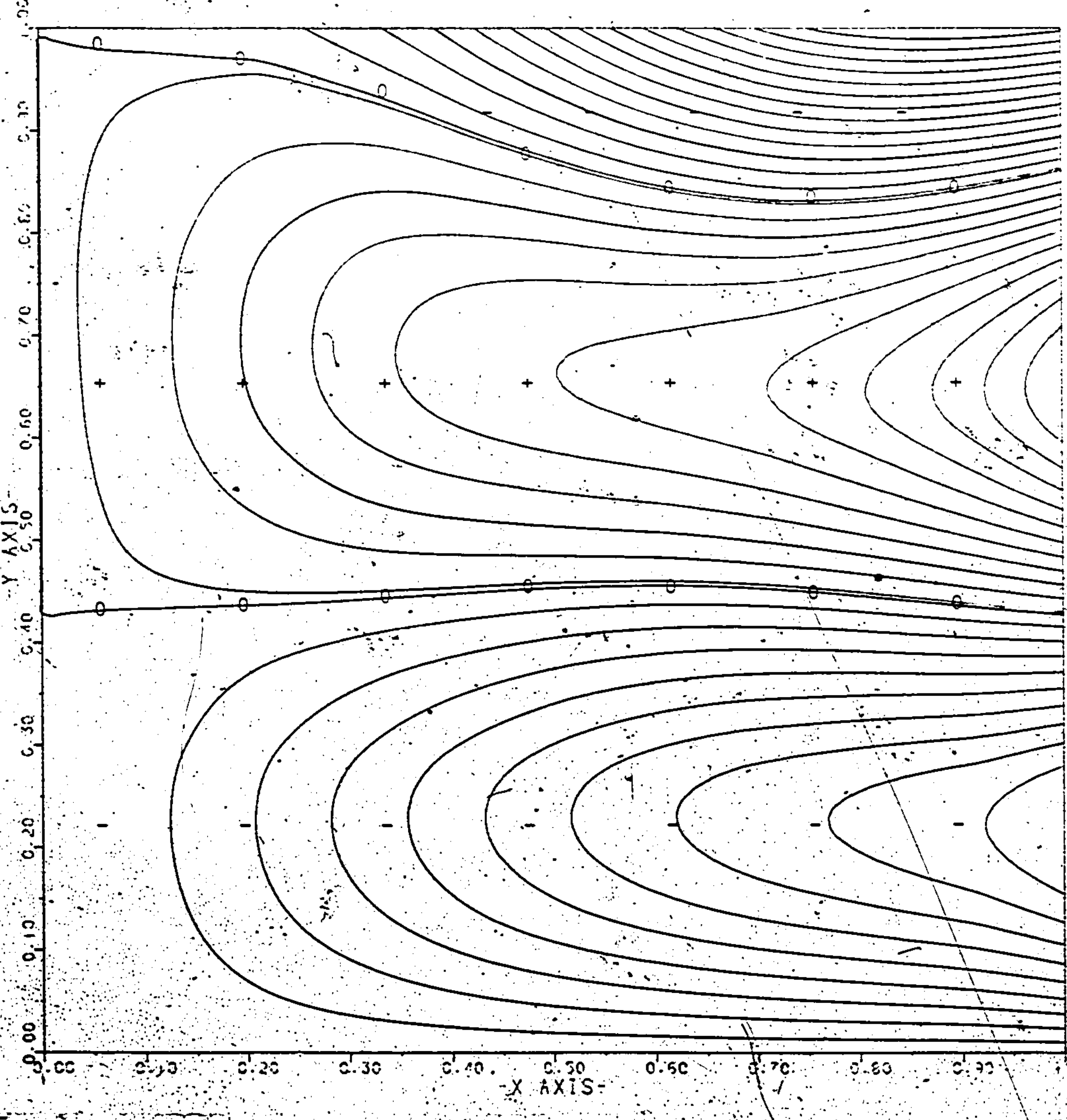



-X AXIS-
-Y AXIS-

WAVE PLATE GRAPHICS

************ THE RECTANGULAR CANTILEVER PLATE ************

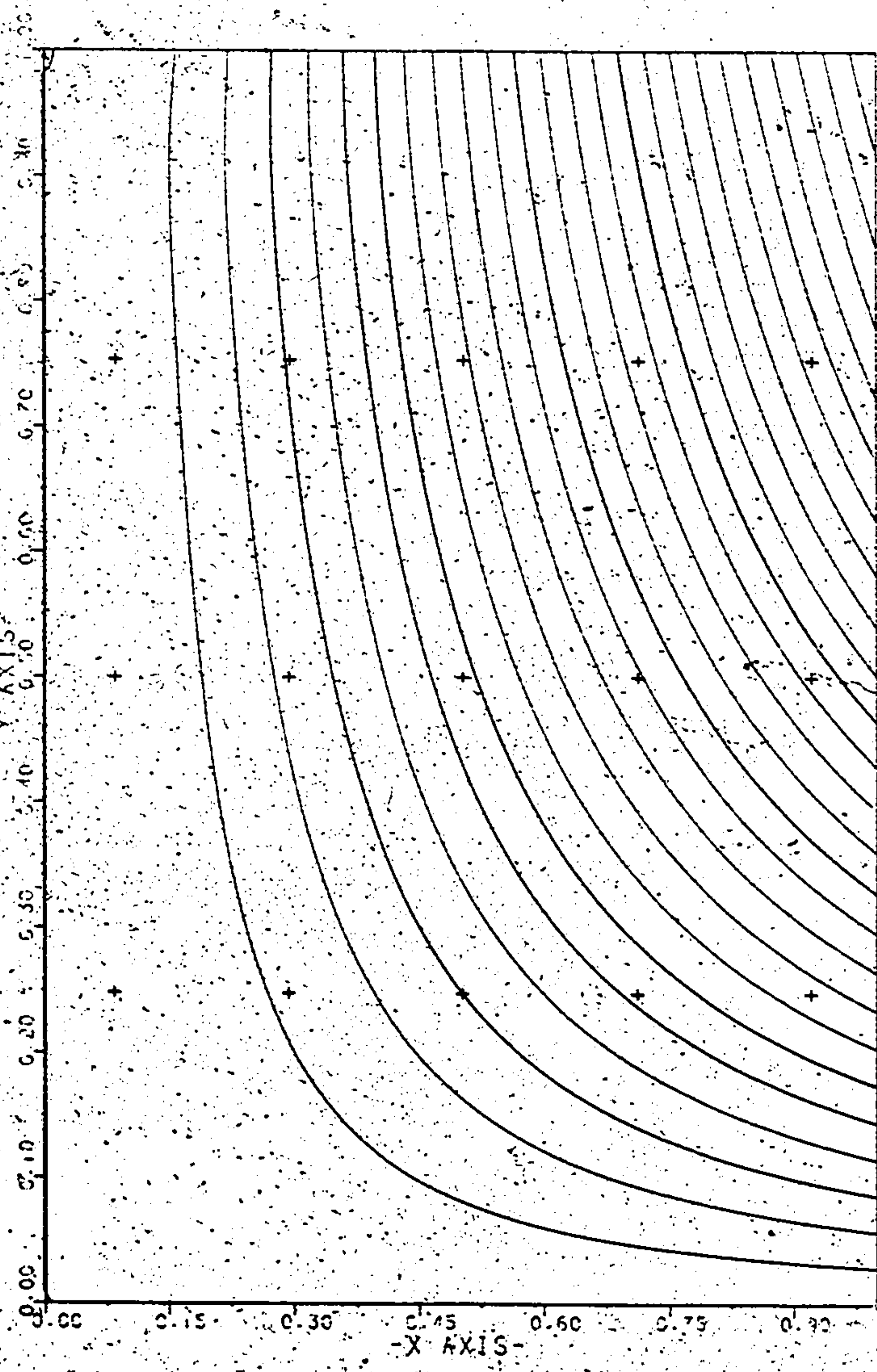
******** FIFTH ANTISYMMETRIC MODE WITH [illegible] ********



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

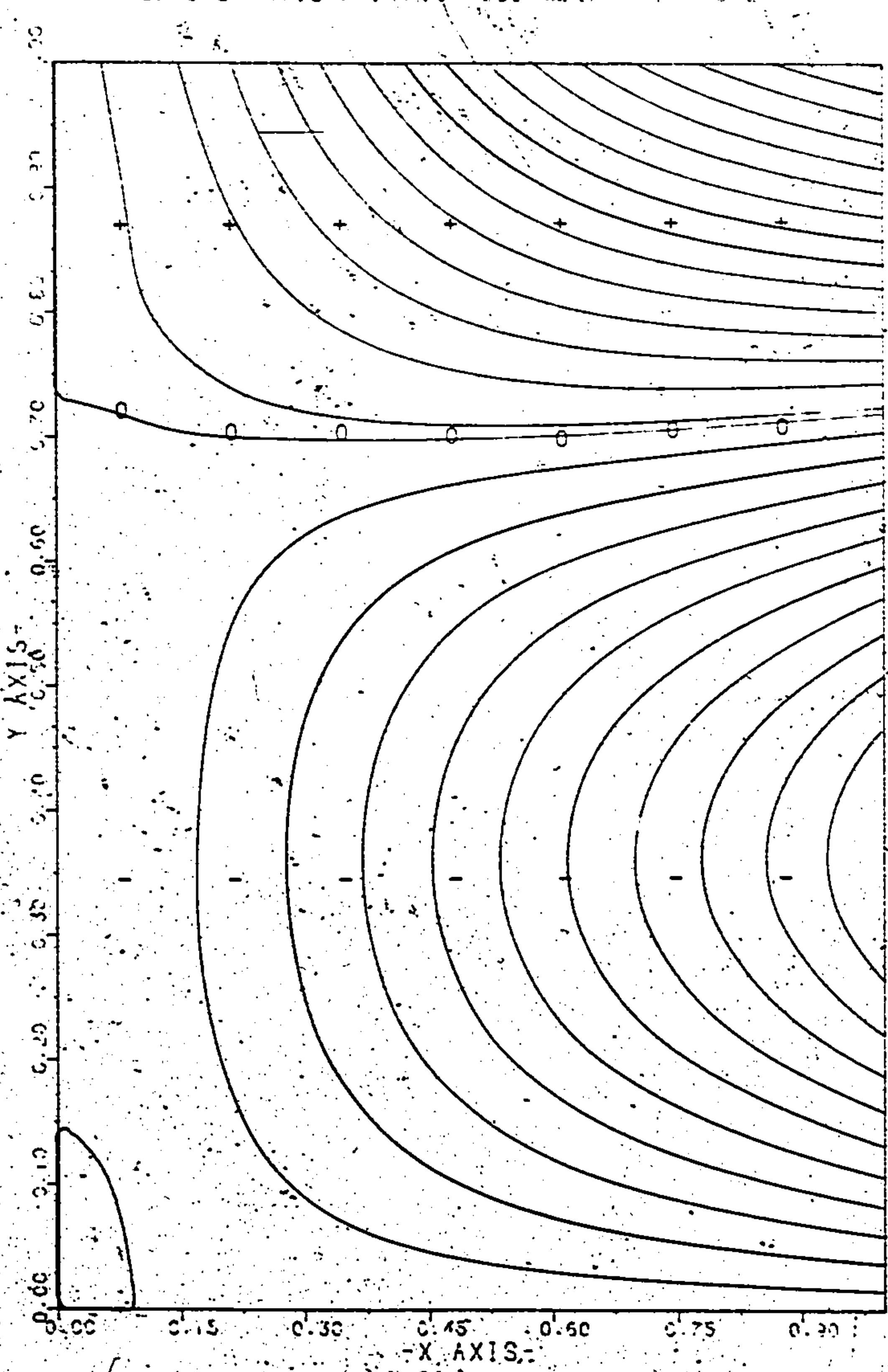
••••••• FIRST ANTISYMMETRIC MODE WITH PHI = 3/2 •••••••



WAVE PLATE GRAPHIC

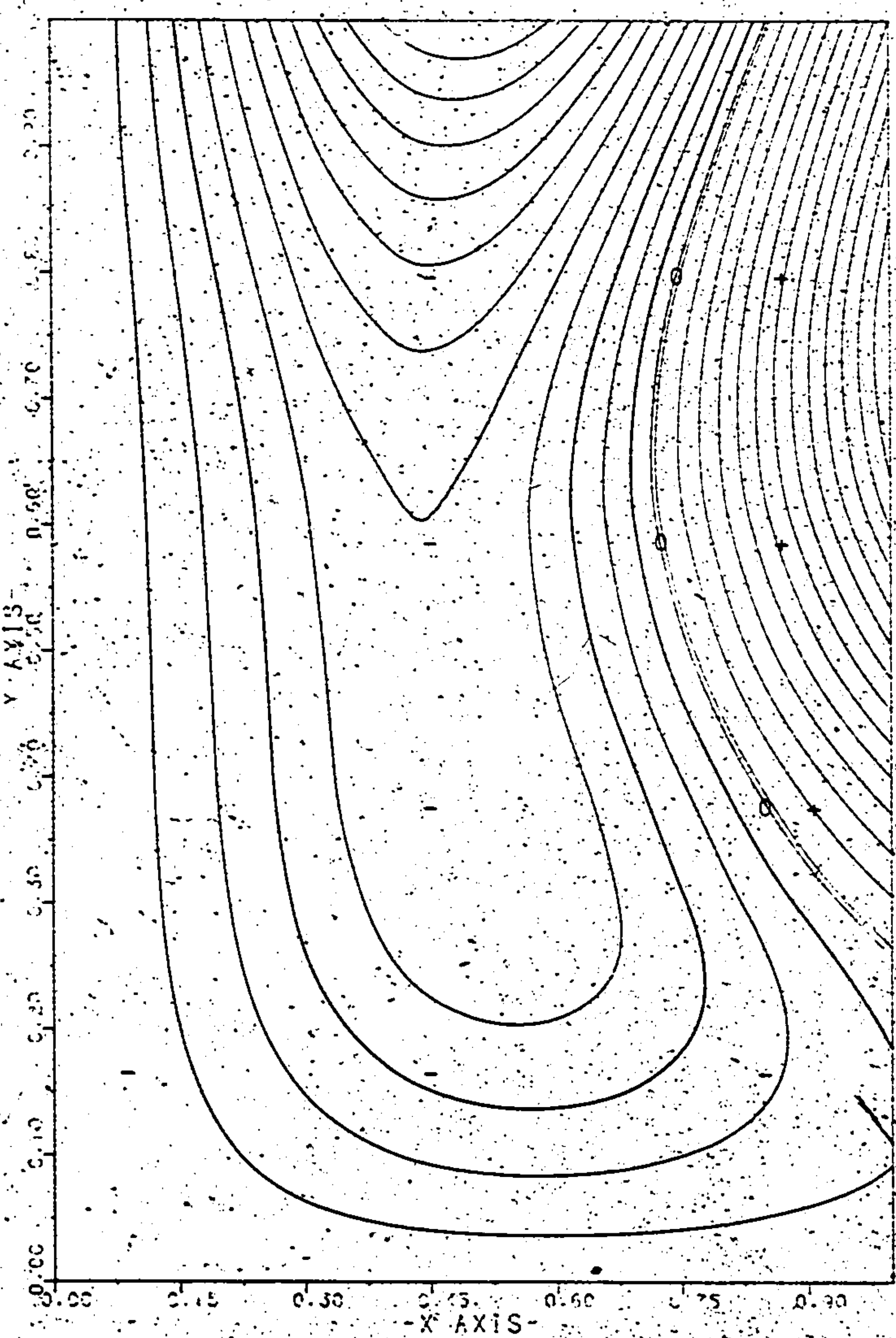
************ THE RECTANGULAR CANTILEVER PLATE ************

******* SECOND ANTISYMMETRIC MODE WITH



THE RECTANGULAR CANTILEVER PLATE

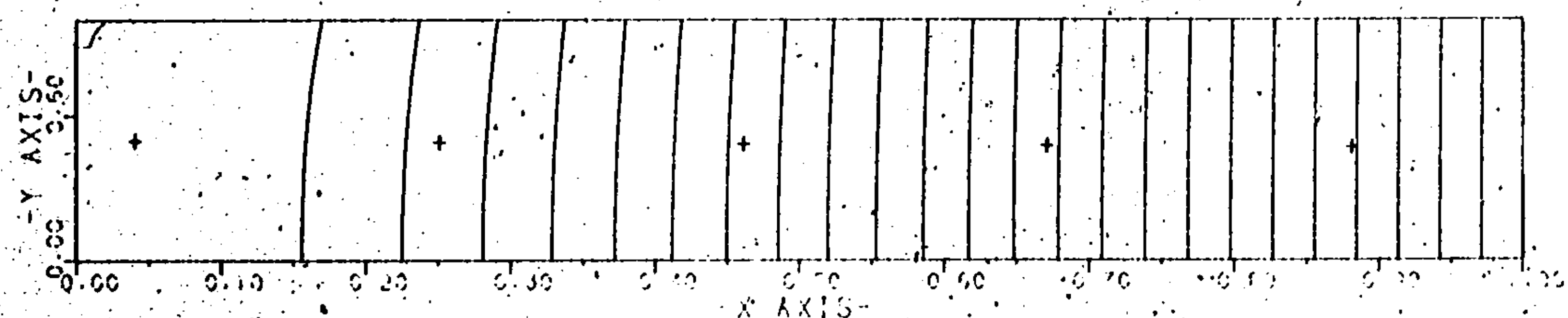
THIRD ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

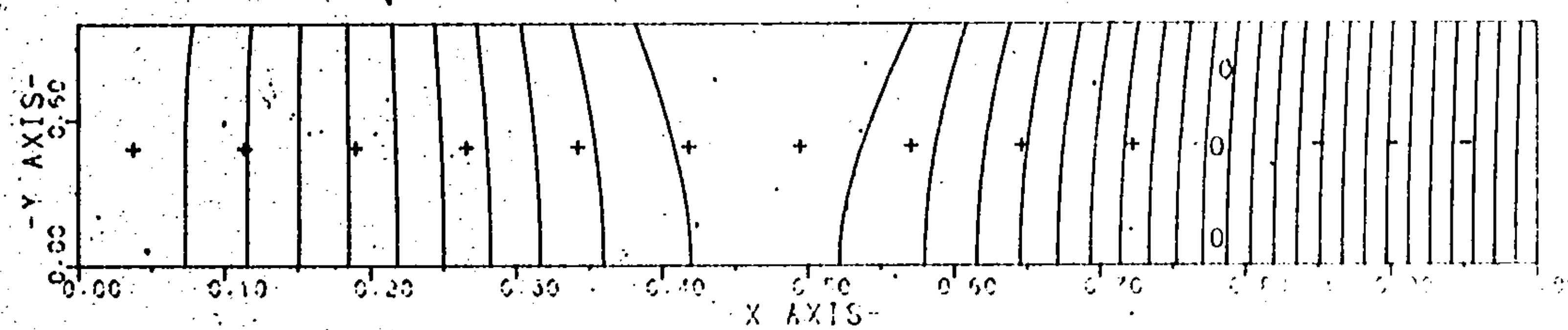
••••••••• FIRST SYMMETRIC MODE WITH PHI = 1/6 •••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

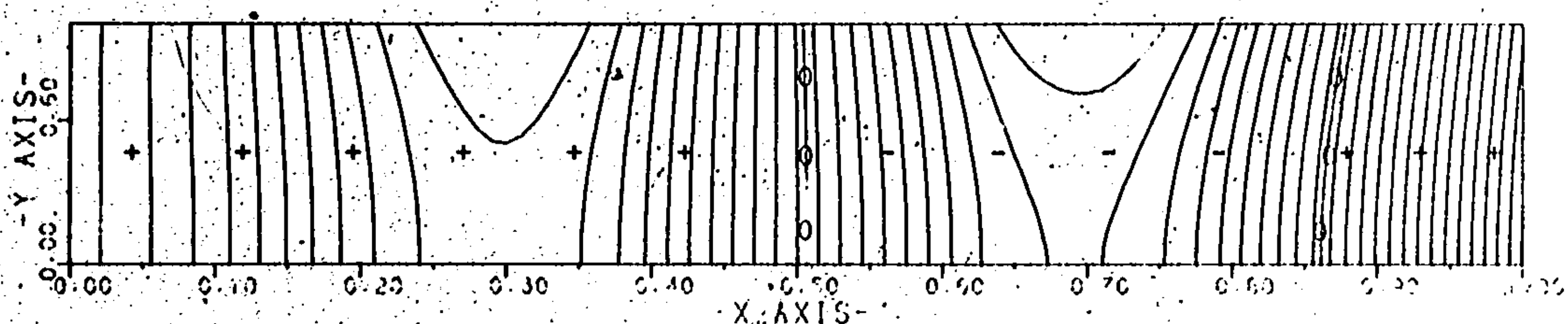
••••••••• SECOND SYMMETRIC MODE WITH PHI = 1/6 •••••••••



WAVE PLATE GRAPHIC

•••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••

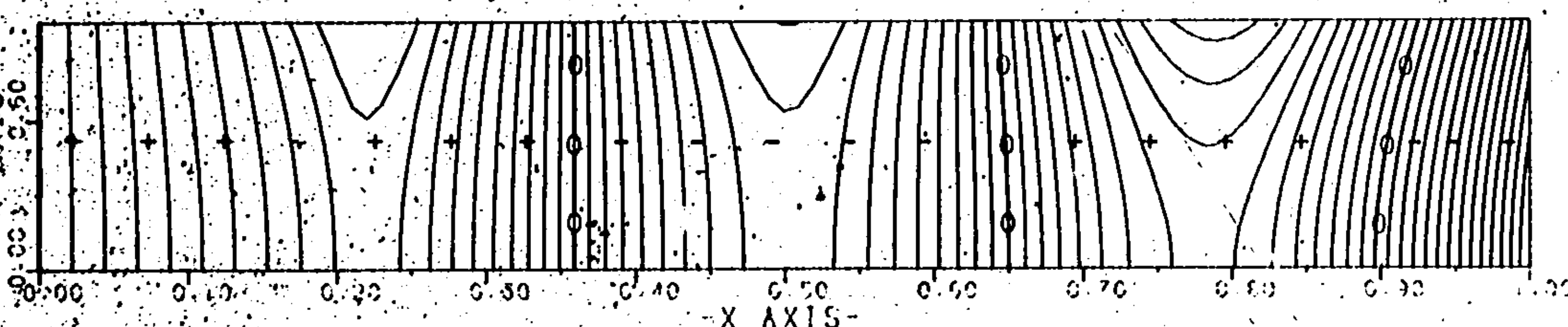
••••••••• THIRD SYMMETRIC MODE WITH PHI = 1/6 •••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

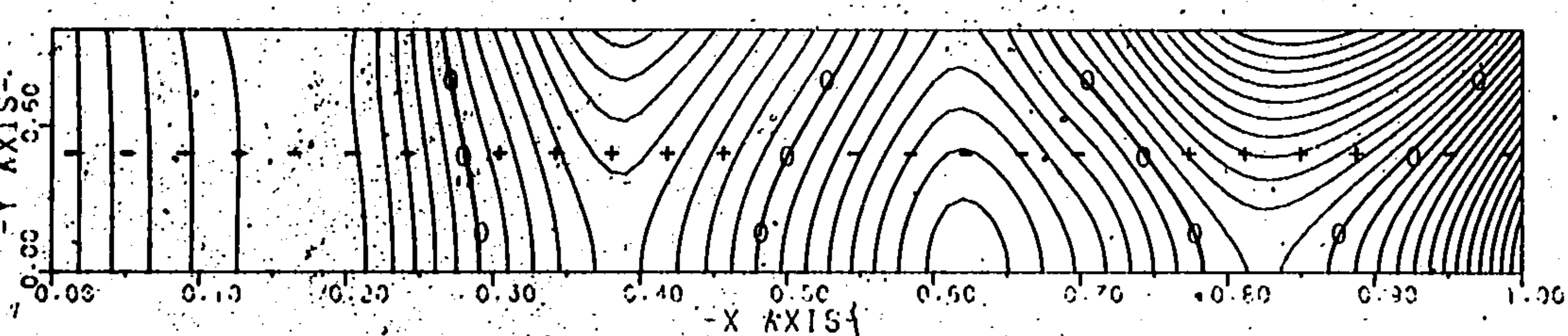
•••••••••• FOURTH SYMMETRIC MODE WITH PHI = 1/6 ••••••••••



WAVE PLATE GRAPHIC

●●●●●●●●●●● THE RECTANGULAR CANTILEVER PLATE ●●●●●●●●●●●

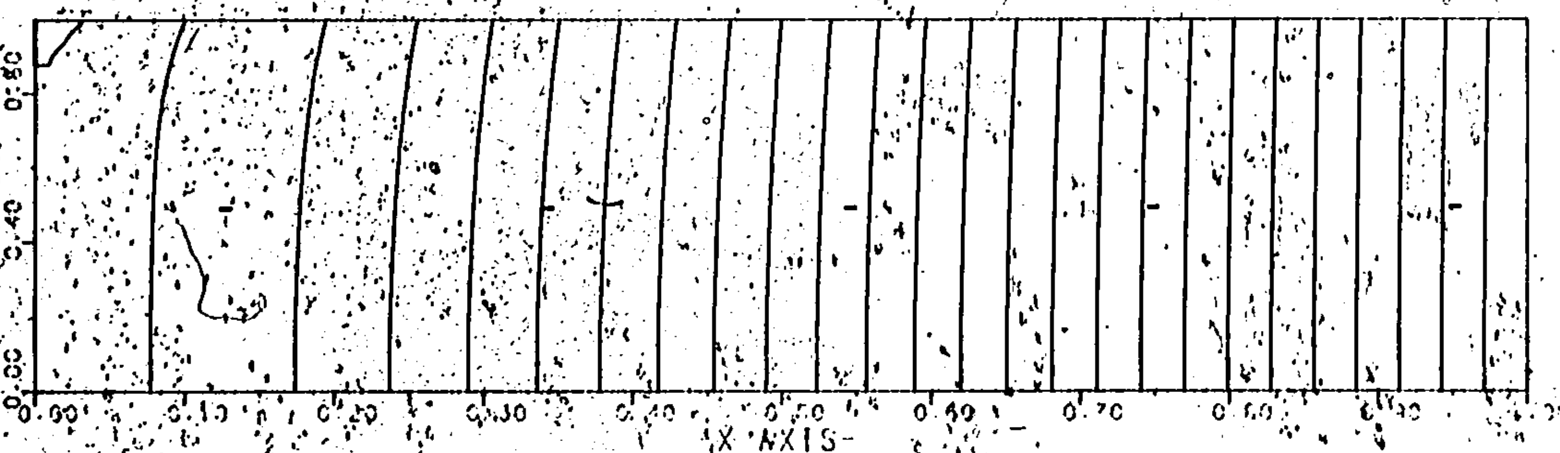
●●●●●●●●● FIFTH SYMMETRIC MODE WITH PHI = 1/6 ●●●●●●●●●



WAVE PLATE GRAPHIC

••••••••••••• THE RECTANGULAR CANTILEVER PLATE •••••••••••••

••••••••• FIRST SYMMETRIC MODE WITH PHI = 1/4 •••••••••



WAVE PLATE GRAPHIC

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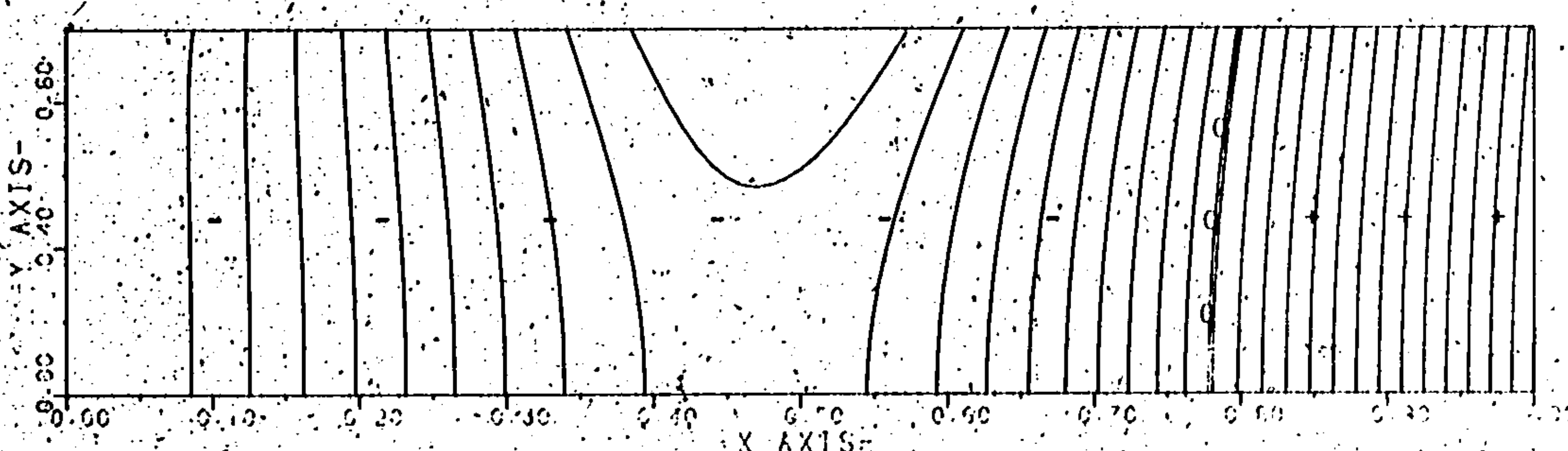

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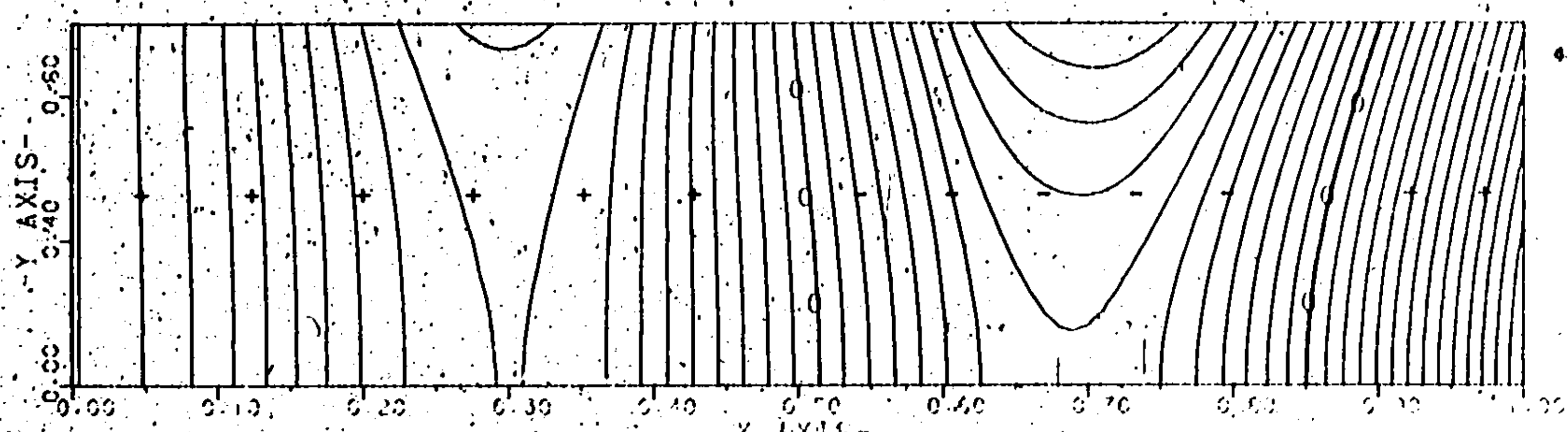
•••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••

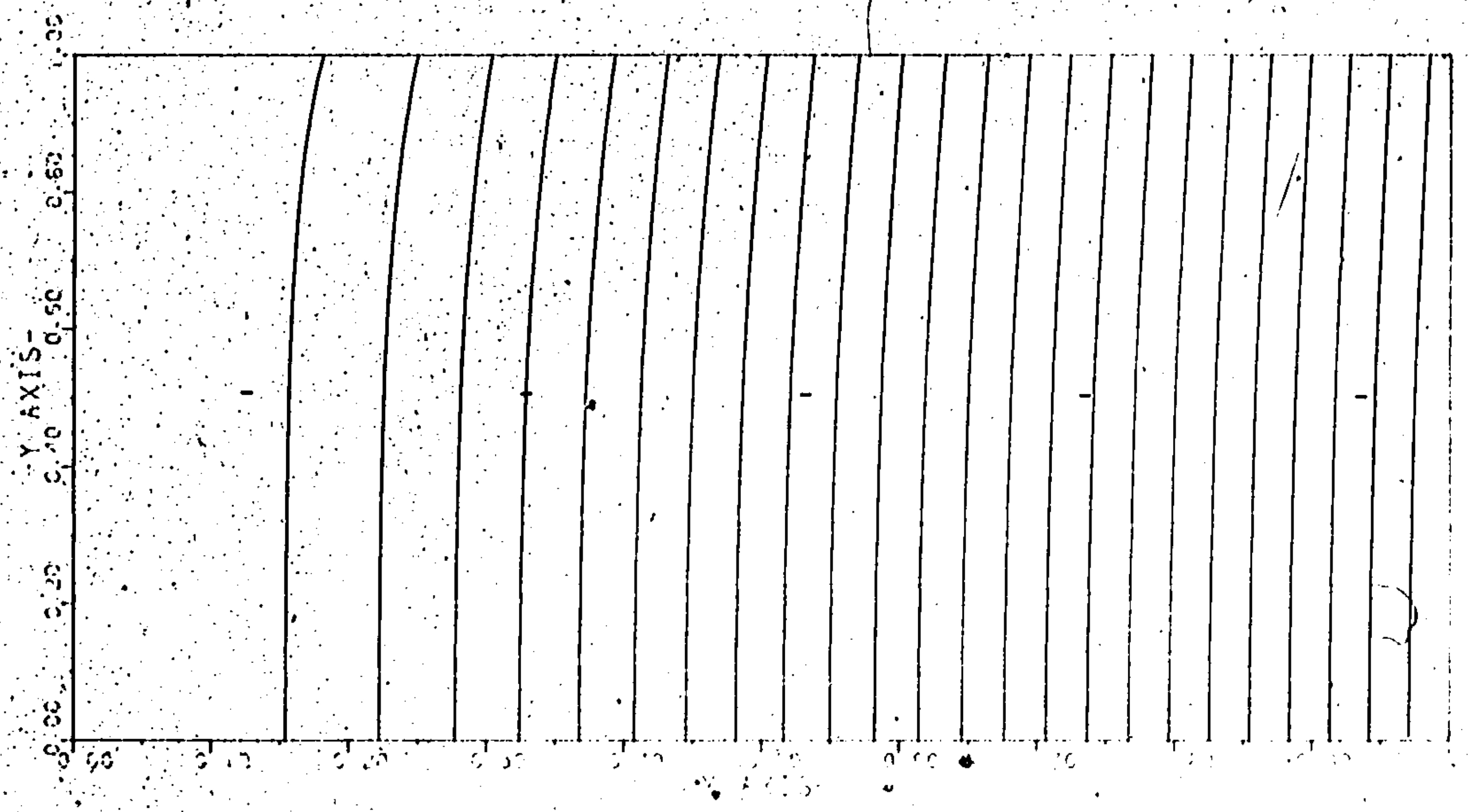
•••••••••• THIRD SYMMETRIC MODE WITH PHI = 1/4 ••••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

••••••••• FIRST SYMMETRIC MODE WITH PHI =1/2 •••••••••



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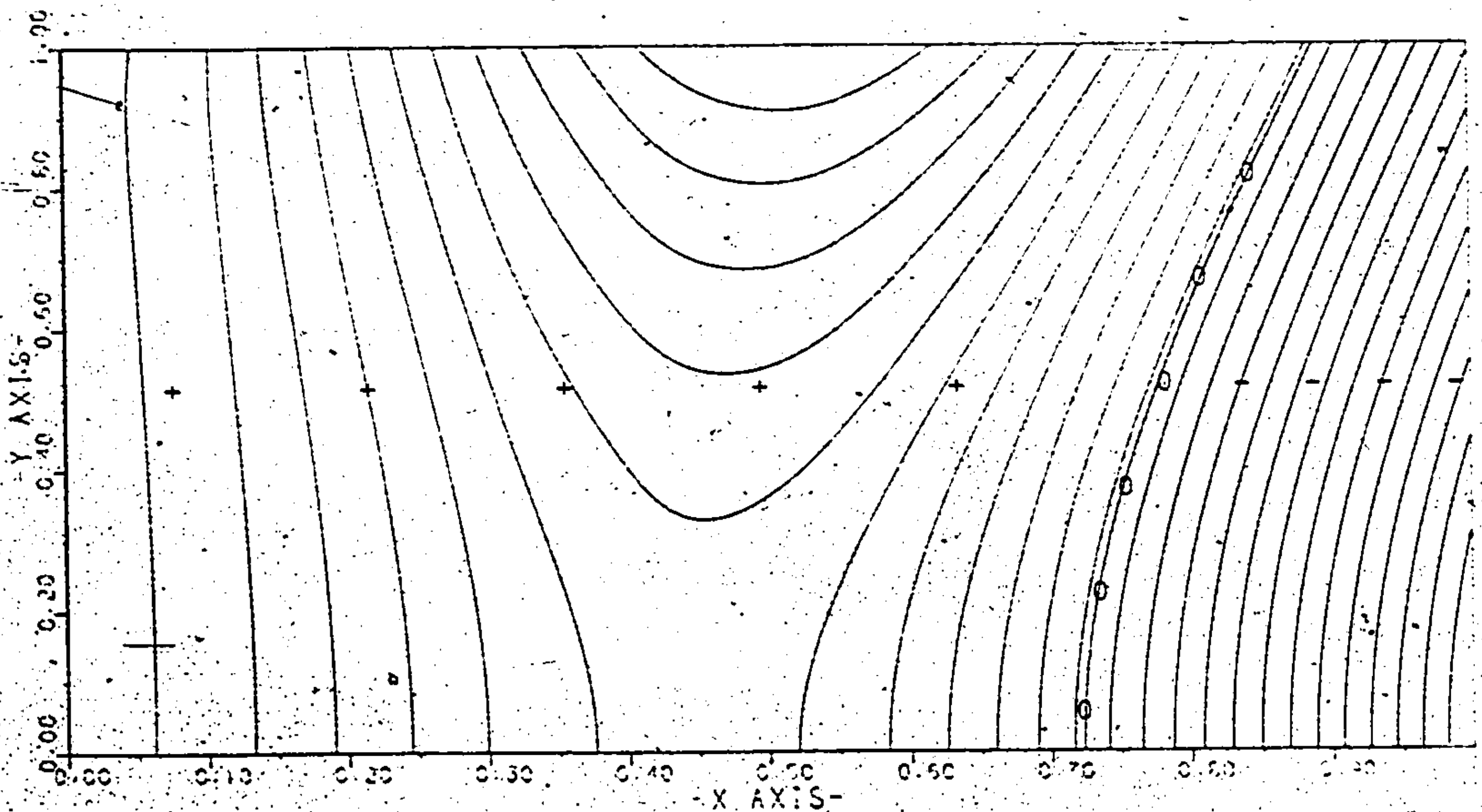
************ THE RECTANGULAR CANTILEVER PLATE ************

********** SECOND SYMMETRIC MODE WITH PHI=PI/2 **********



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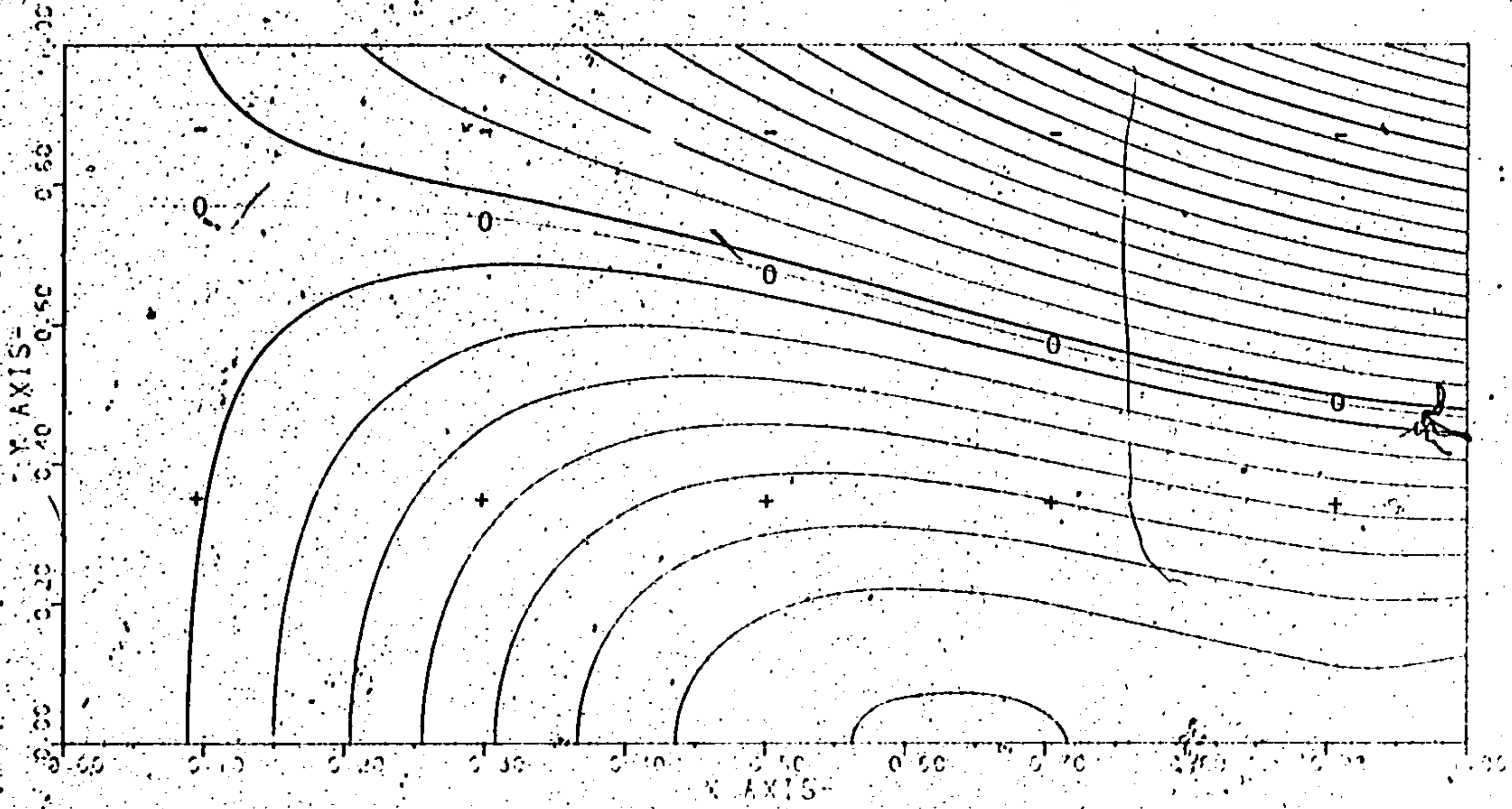
THE RECTANGULAR CANTILEVER PLATE

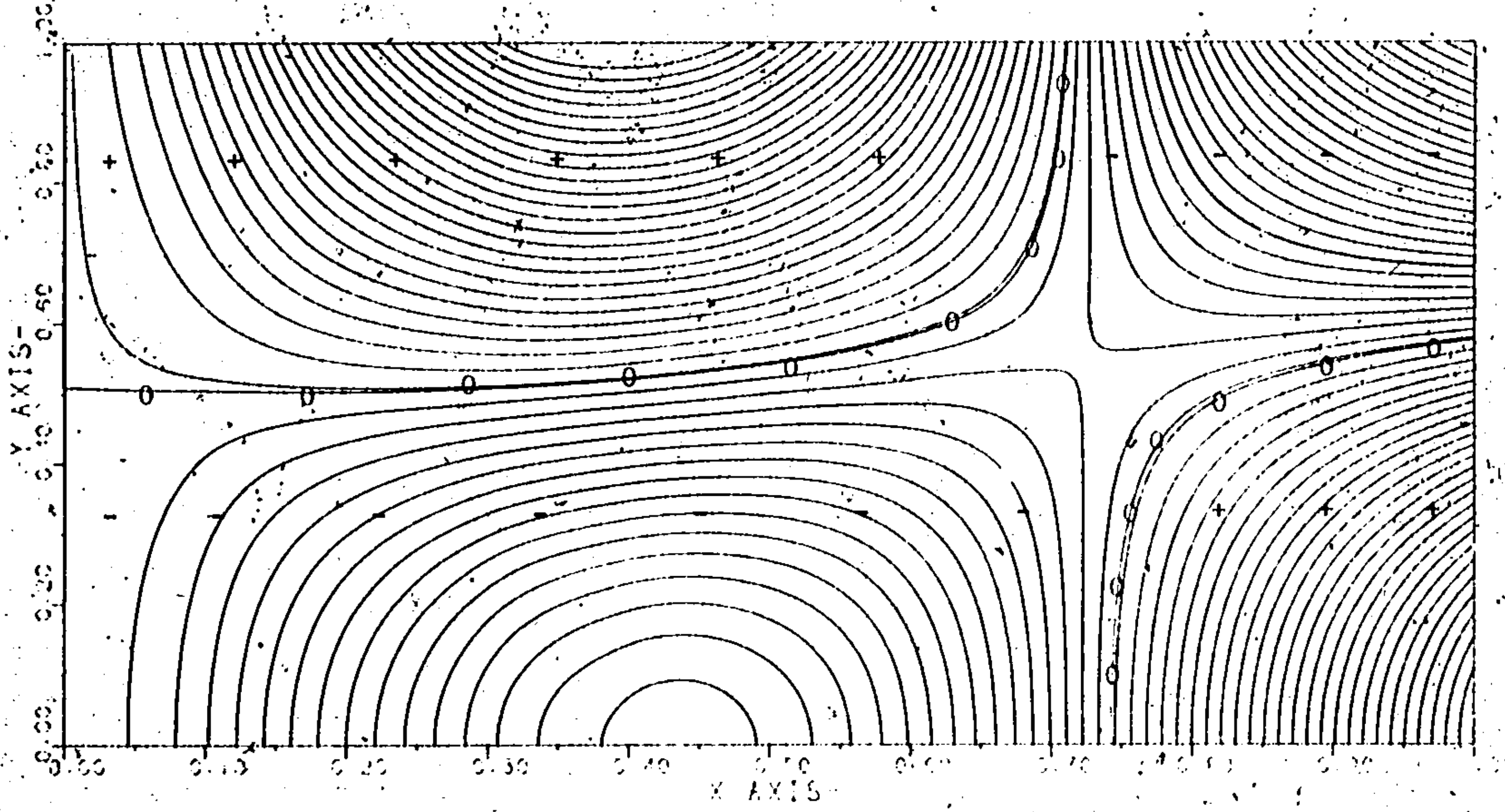
THIRD SYMMETRIC MODE WITH PHI = 1/2



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

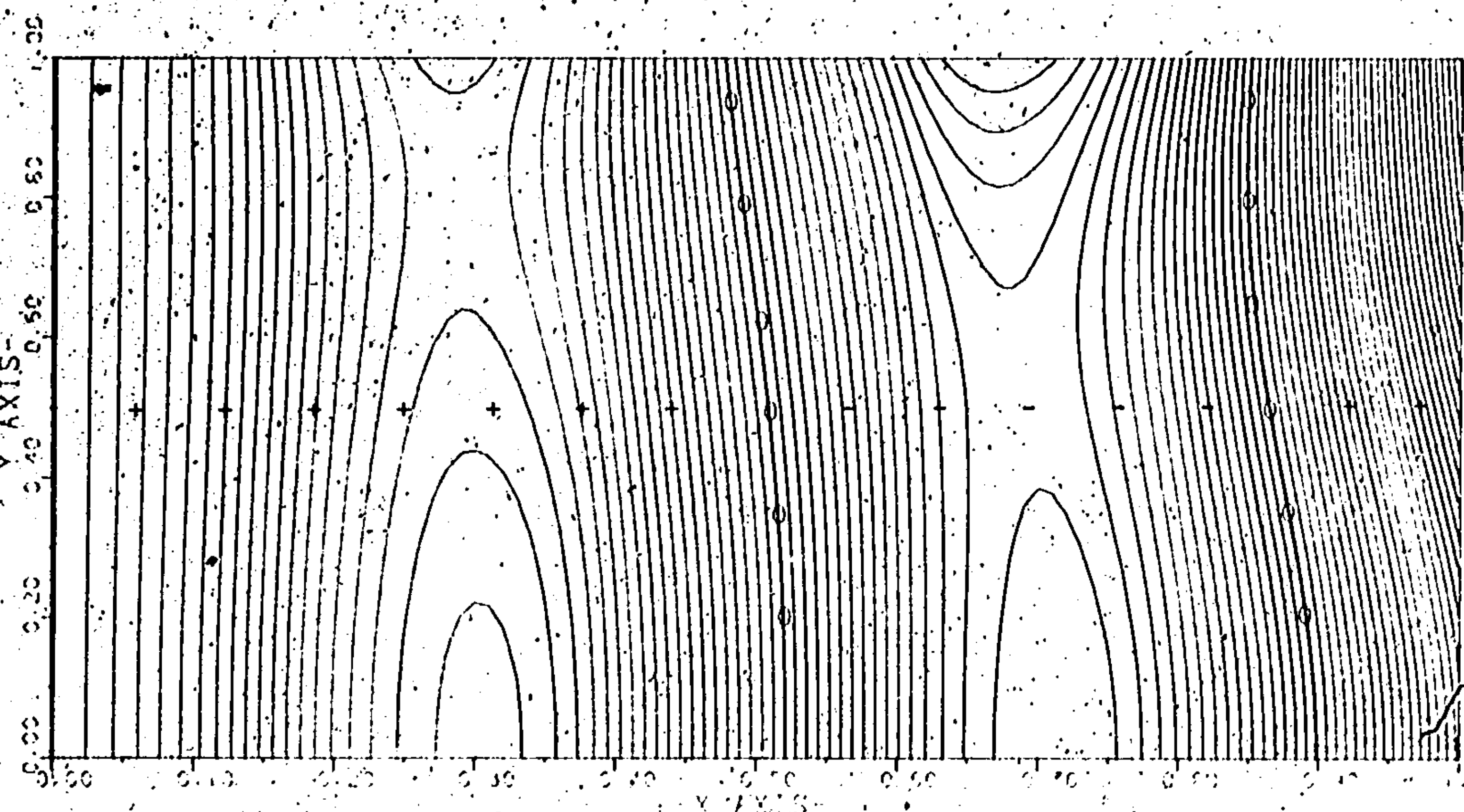
••••••••• FOURTH SYMMETRIC MODE WITH PHI = [illegible]/2 •••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

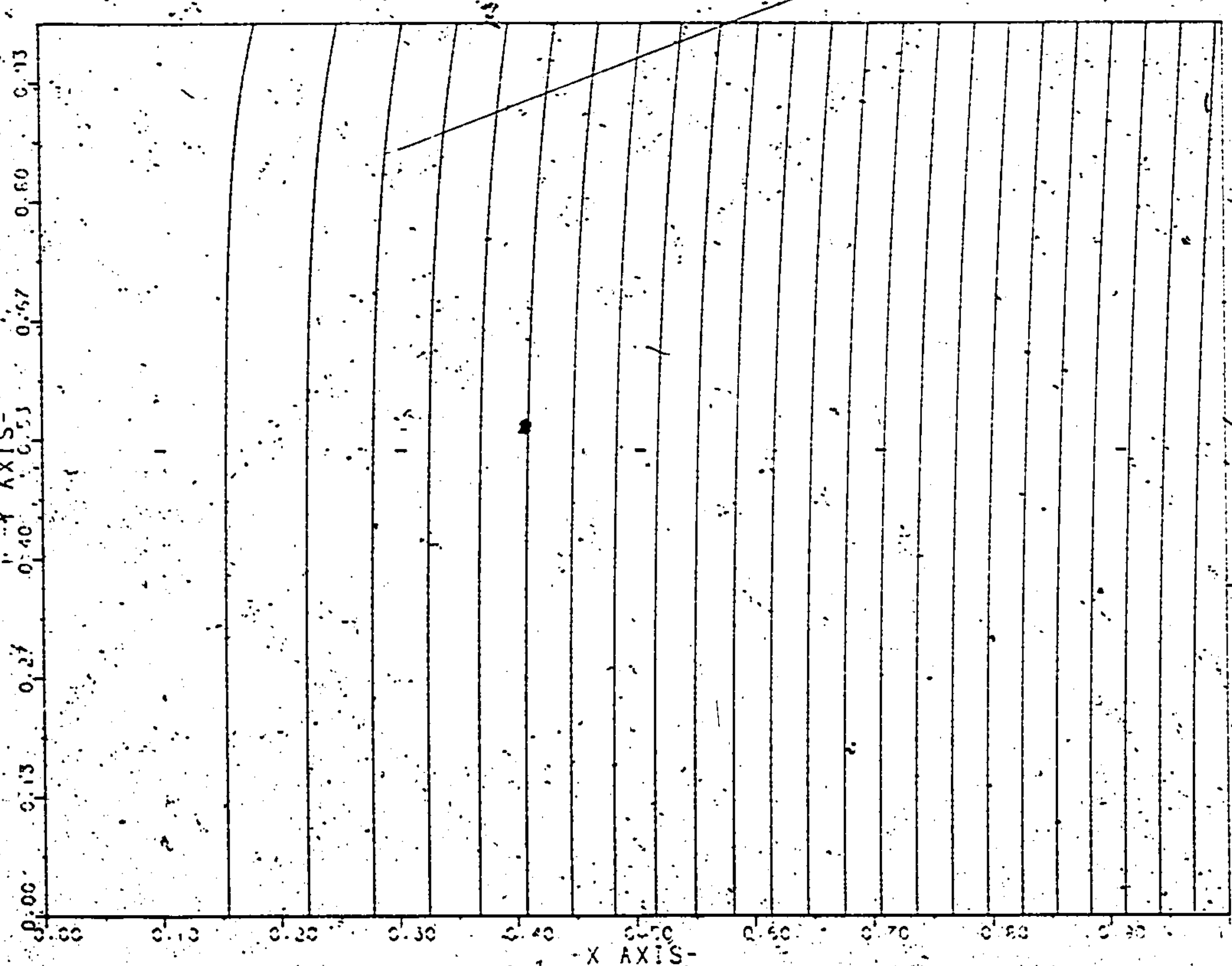
••••••••• FIFTH SYMMETRIC MODE WITH PHI = 1/2 •••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

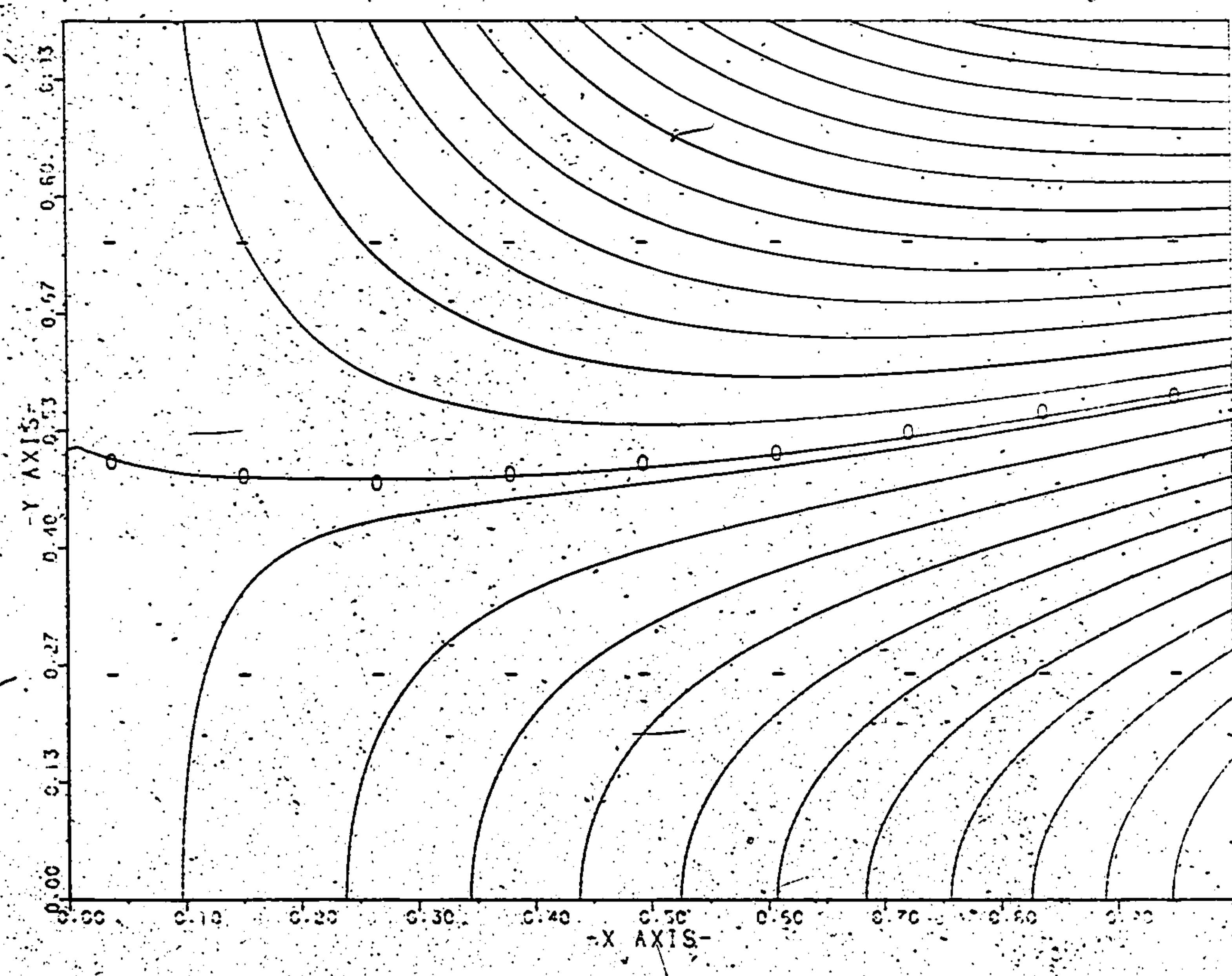
•••••••••• FIRST SYMMETRIC MODE WITH PHI = 3.4 ••••••••••



WAVE PLATE GRAPHIC

•••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••

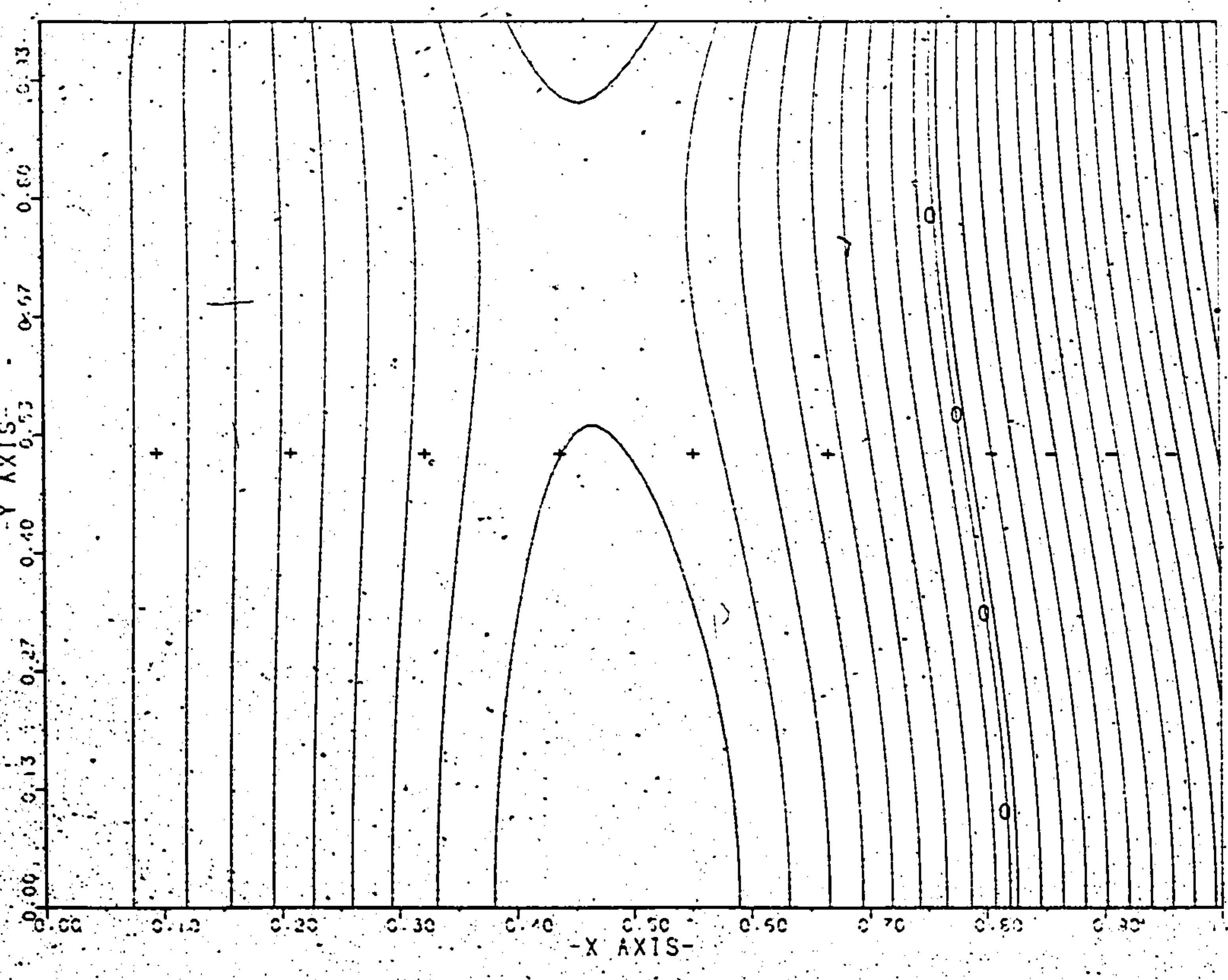
•••••••••• SECOND SYMMETRIC MODE WITH PHI = 3/4 ••••••••••



WAVE PLATE GRAPHIC

•••••••••••• THE RECTANGULAR CANTILEVER PLATE ••••••••••••

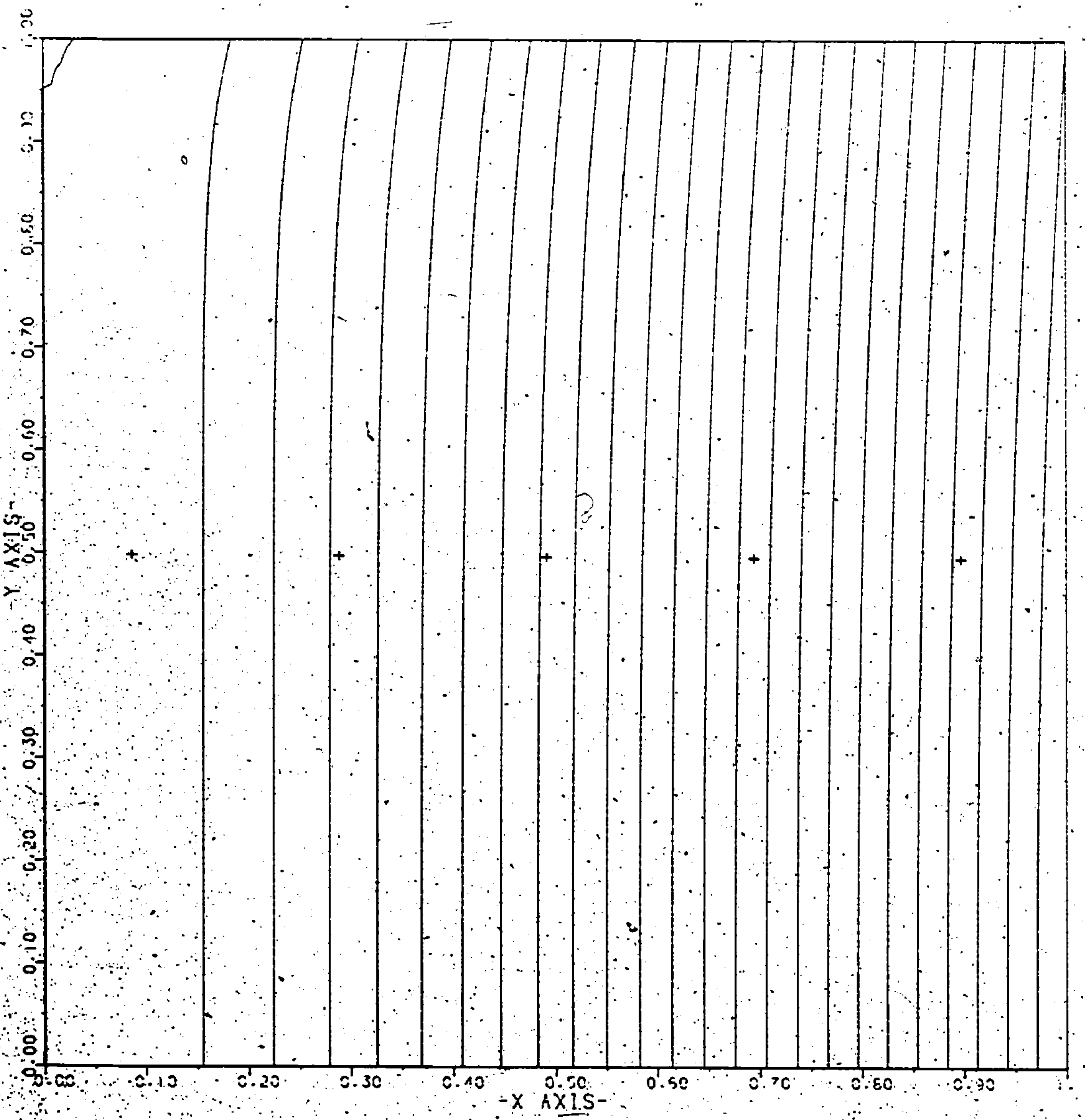
•••••••••• THIRD SYMMETRIC MODE WITH PHI = 3/4 ••••••••••



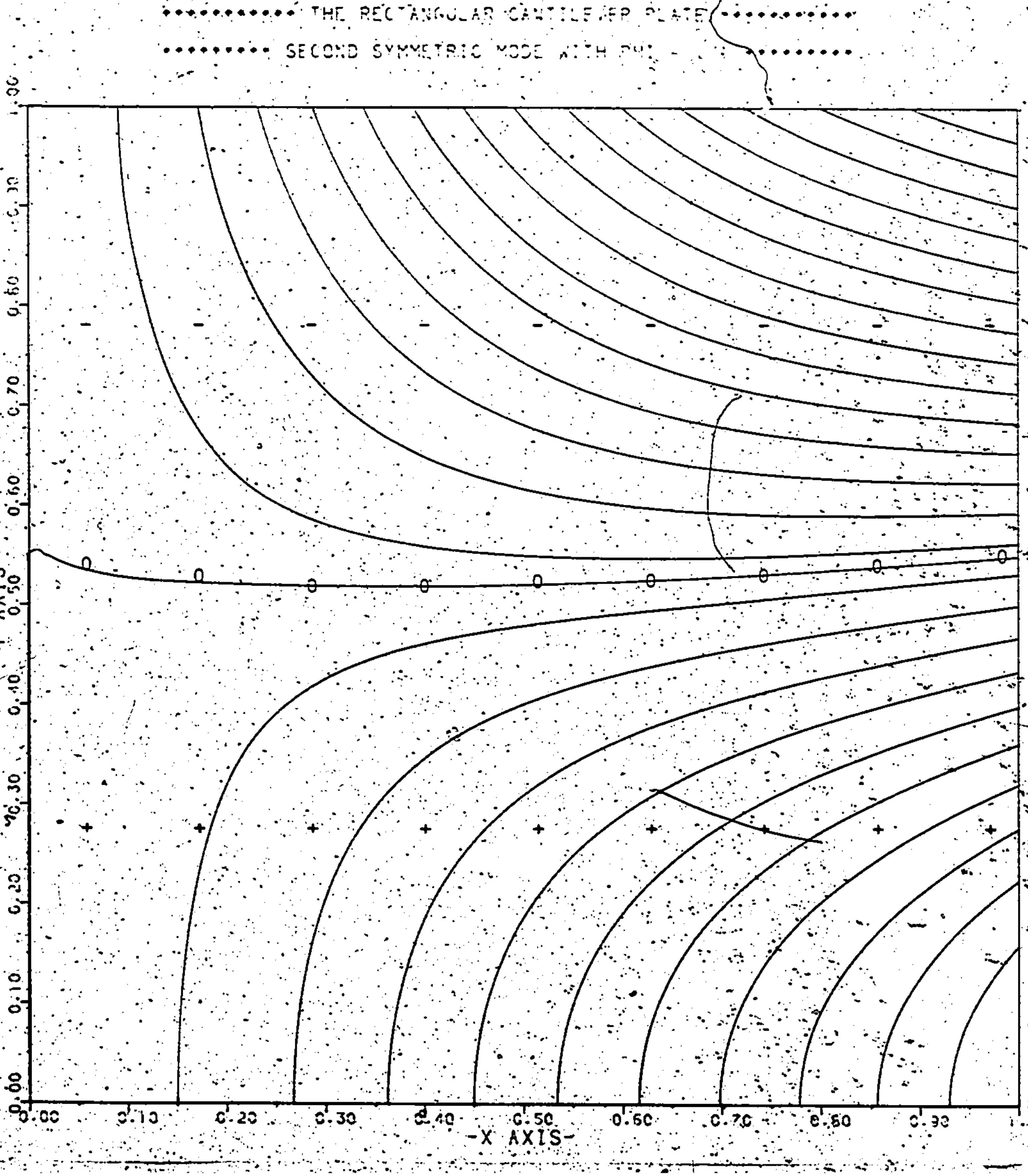
WAVE PLATE GRAPHIC

••••••••••••• THE RECTANGULAR CANTILEVER PLATE •••••••••••••

•••••••••• FIRST SYMMETRIC MODE WITH PHI = [illegible] ••••••••••



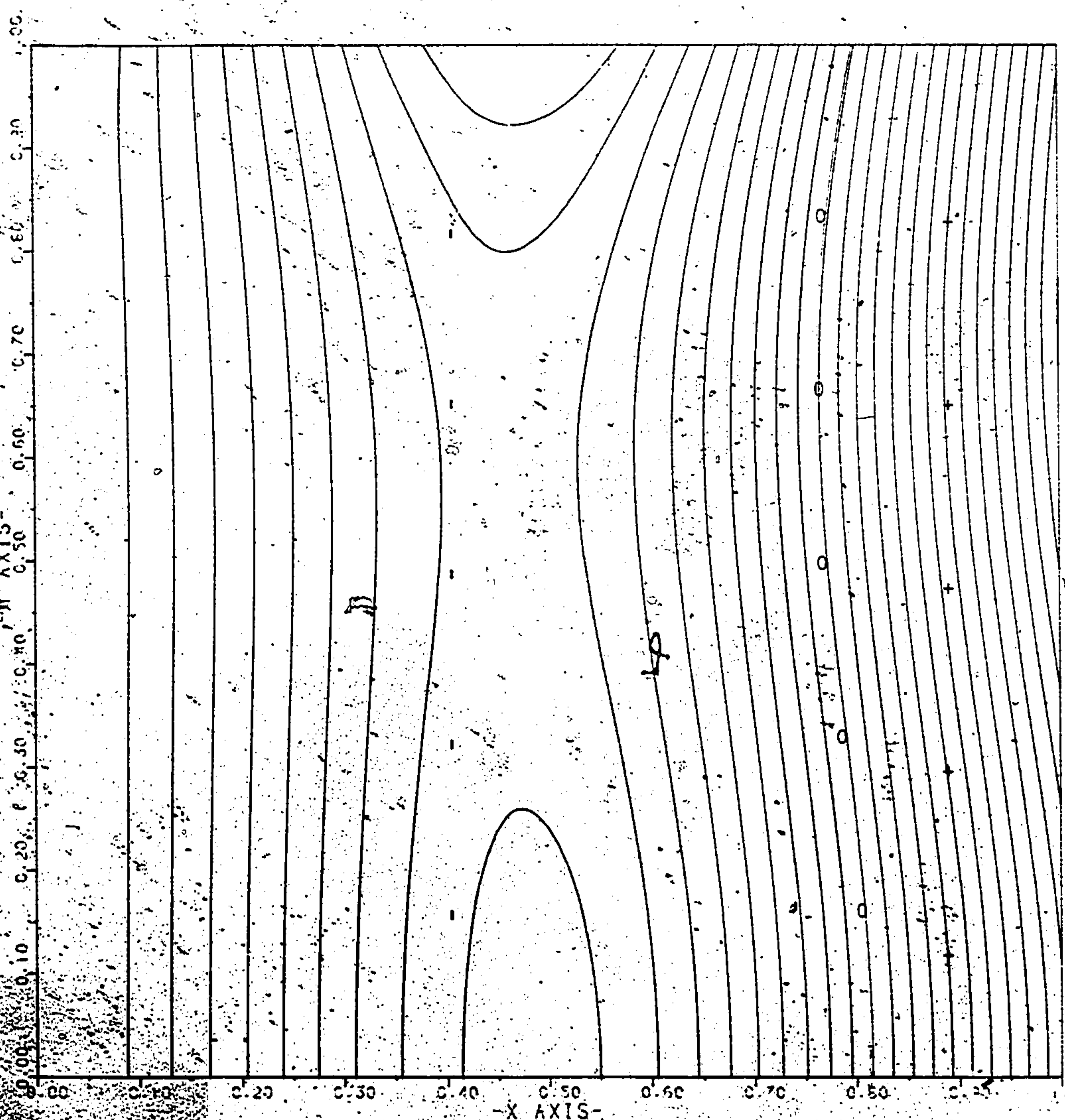
WAVE PLATE GRAPHIC



WAVE/PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

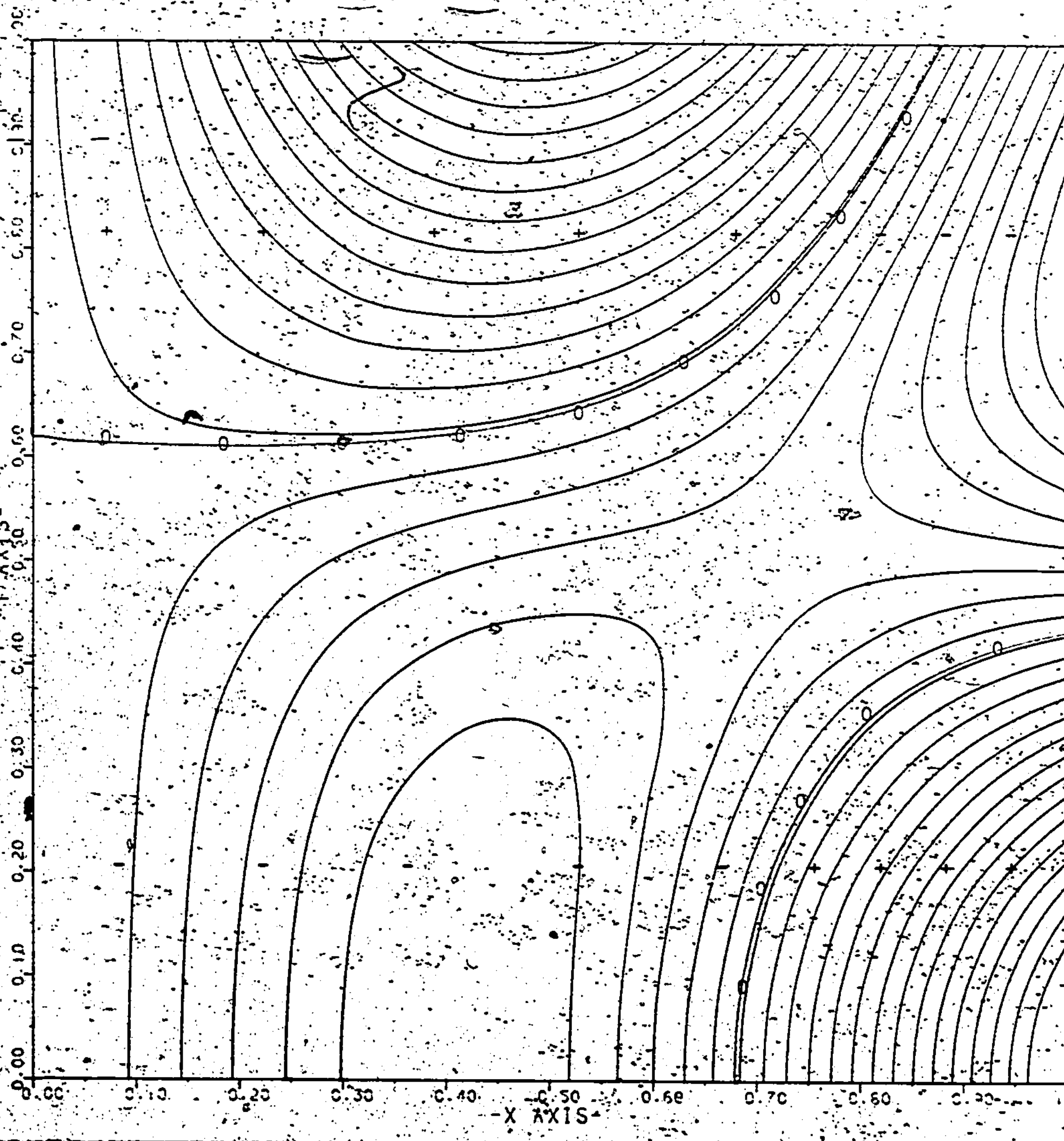
THIRD SYMMETRIC MODE WITH PHI



WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

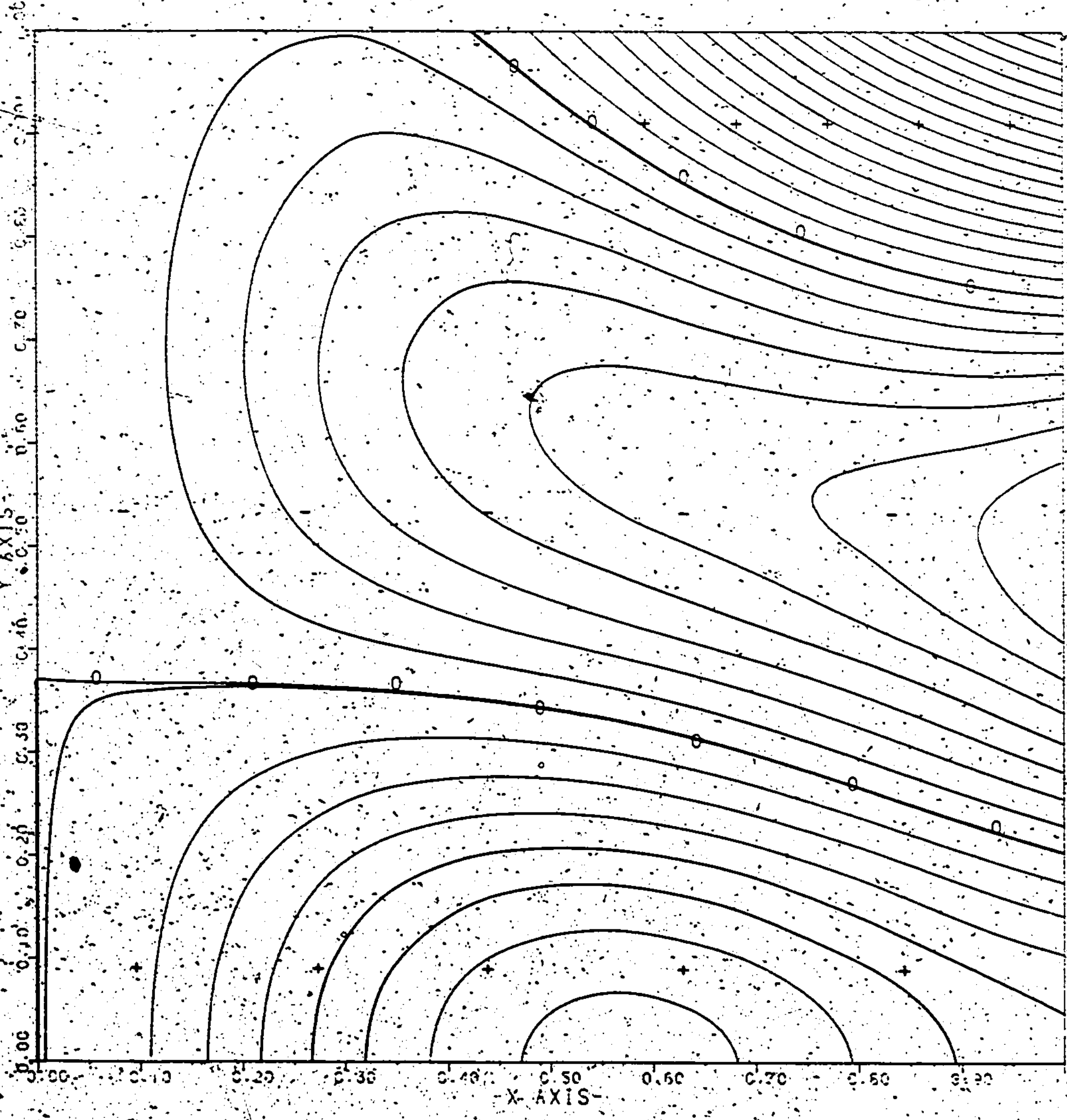
FOURTH SYMMETRIC MODE WITH PHI



WAVE PLATE GRAPHIC

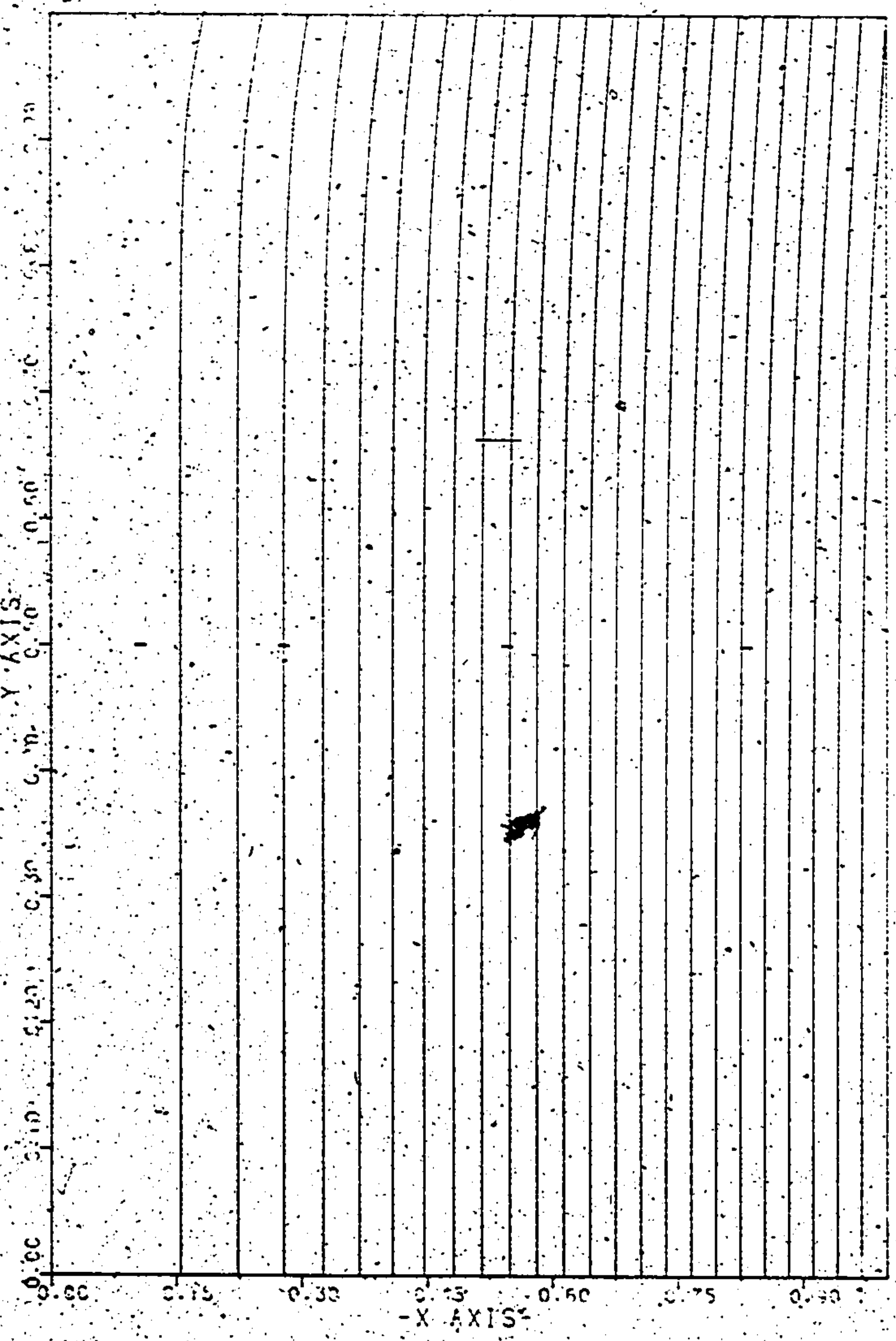
THE RECTANGULAR CANTILEVER PLATE

FIFTH SYMMETRIC MODE WITH



THE RECTANGULAR CANTILEVER PLATE

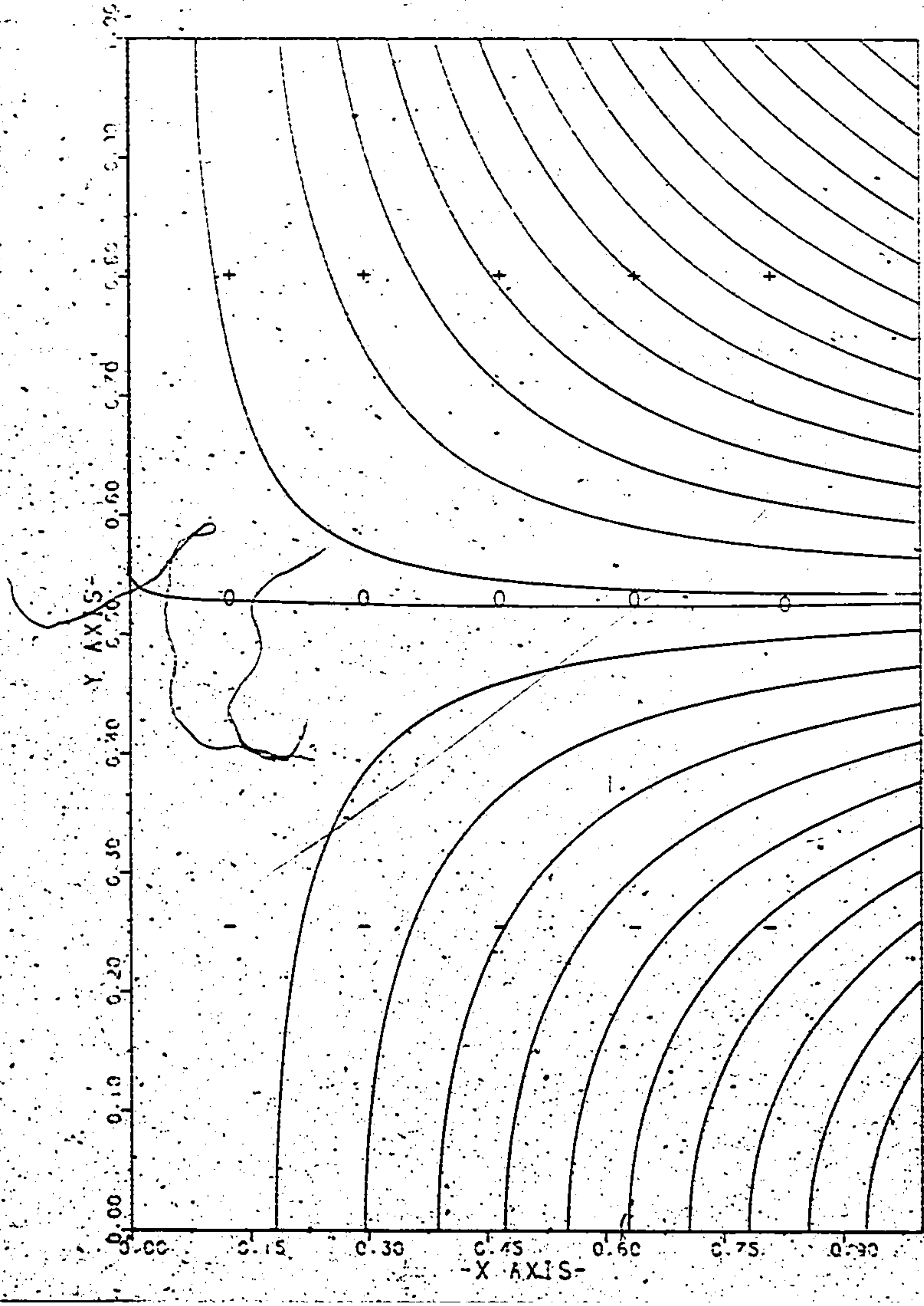
FIRST SYMMETRIC MODE WITH [illegible]



WAVE PLATE GRAPHIC

************ THE RECTANGULAR CANTILEVER PLATE ************

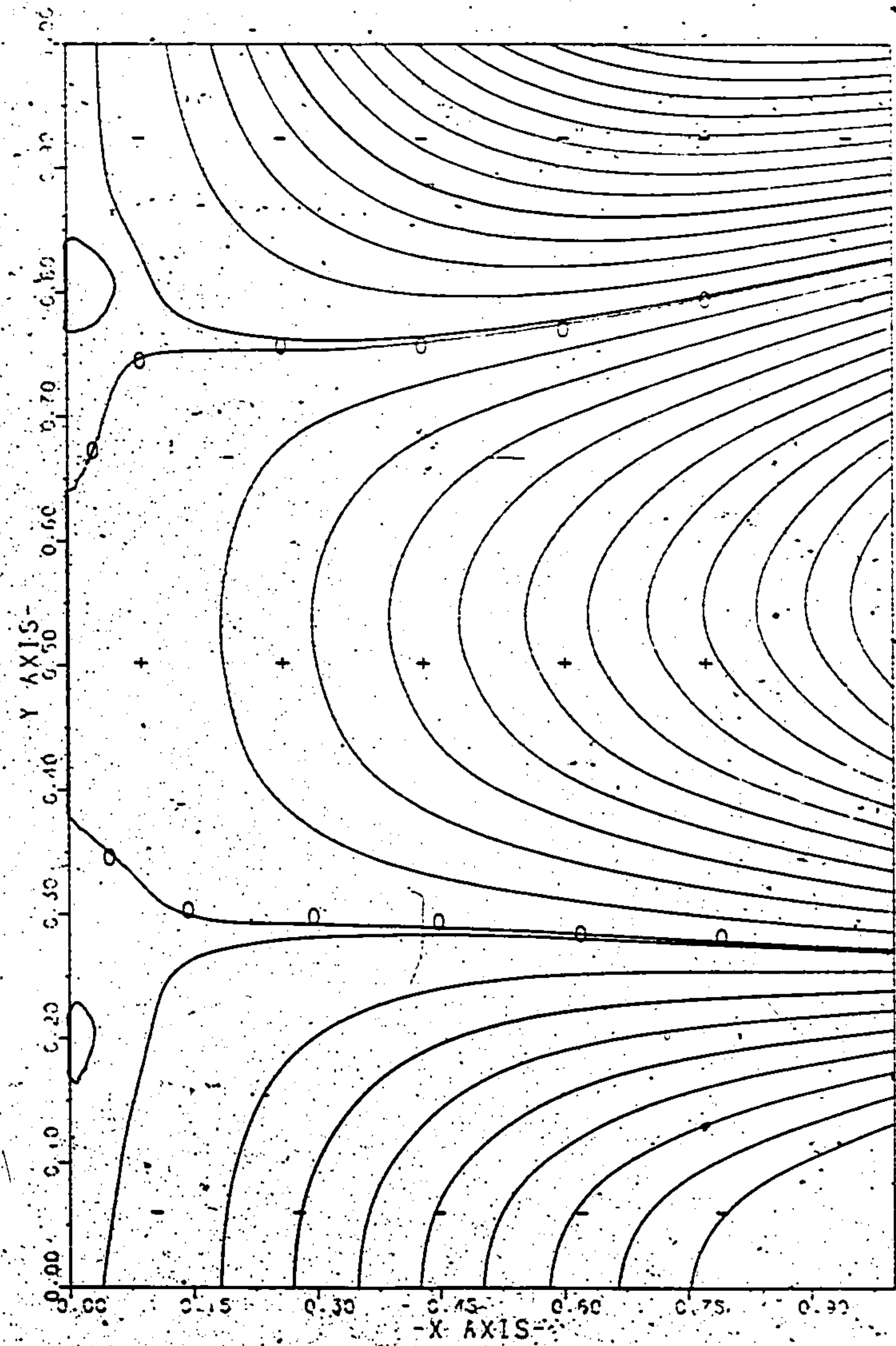
********** SECOND SYMMETRIC MODE WITH PHI = 3/2 **********



WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

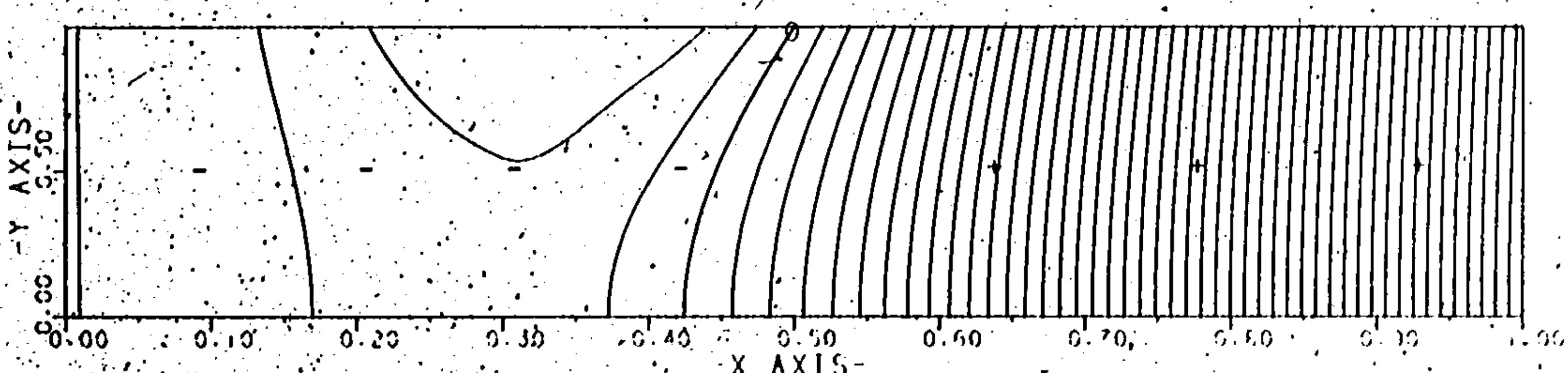
THIRD SYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

•••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ••••

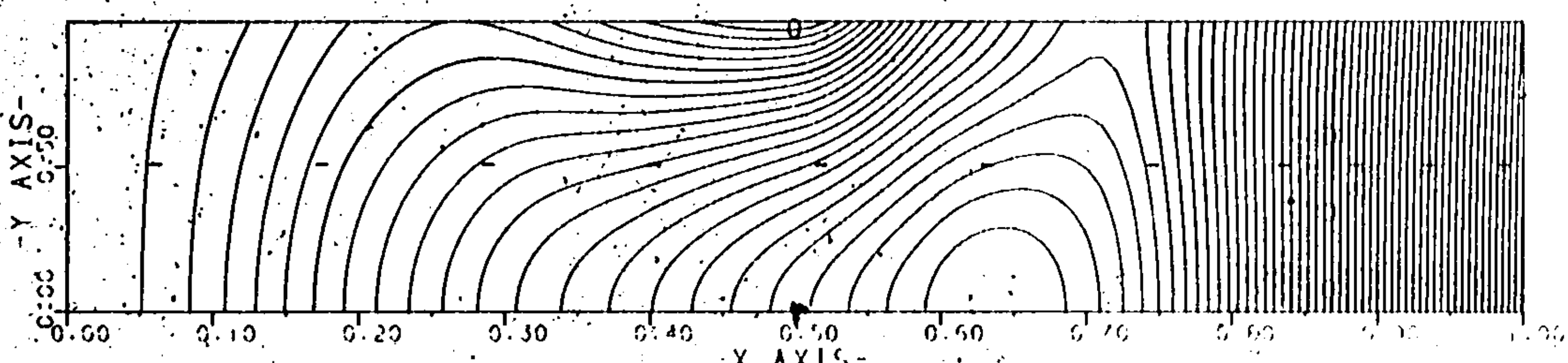
FIRST SYMMETRIC MODE WITH PHI = 1/5, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

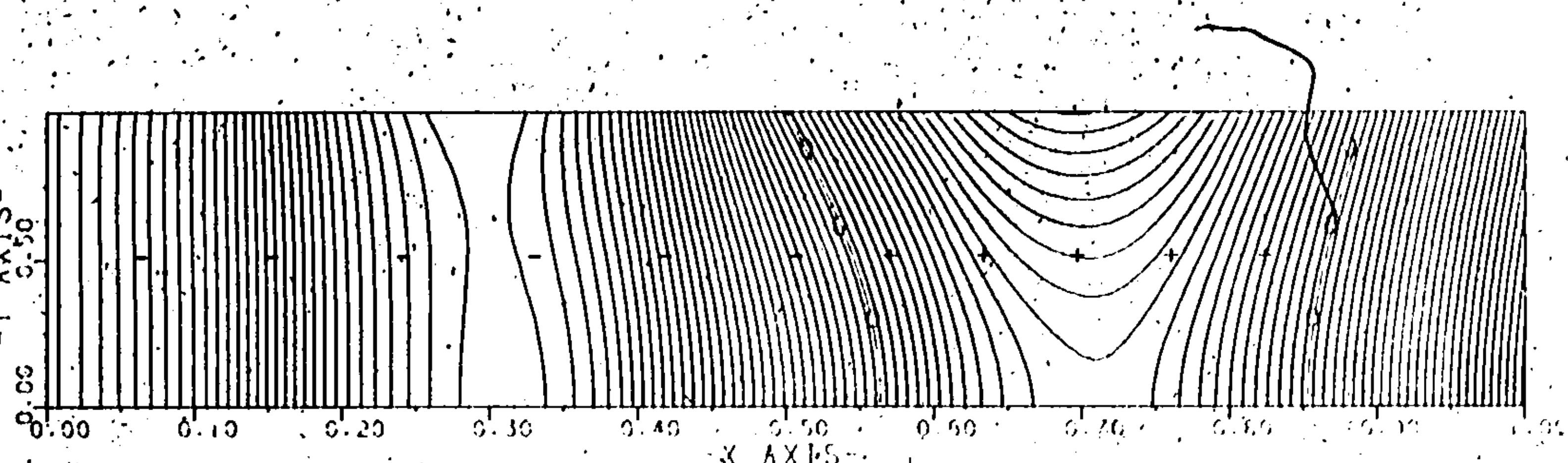
SECOND SYMMETRIC MODE WITH PHI = 1/5, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

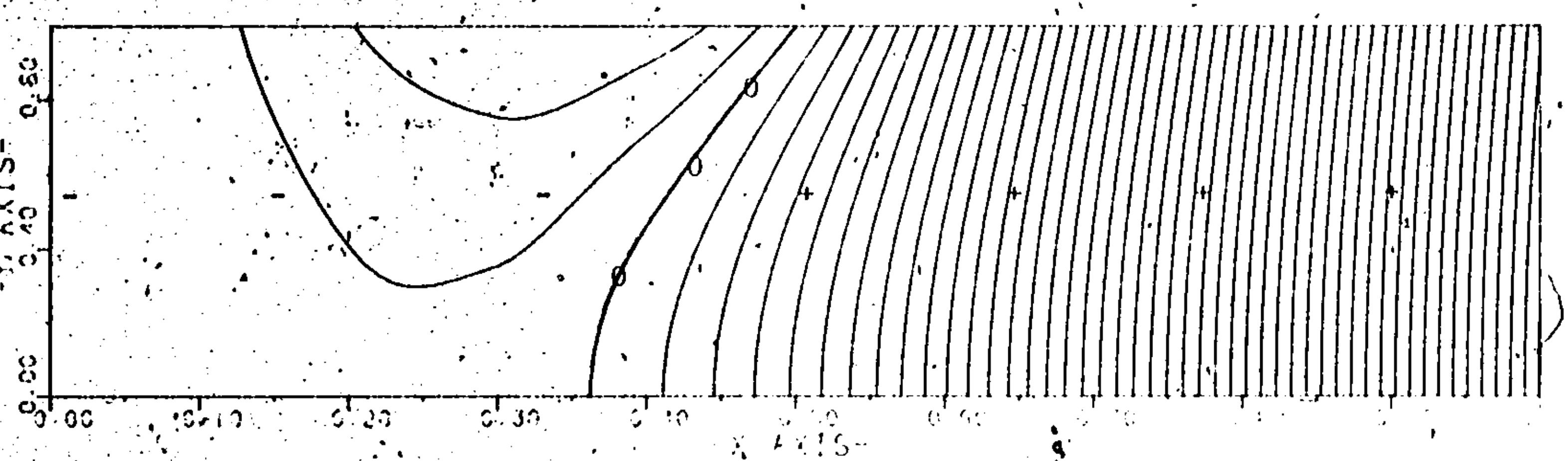
THIRD SYMMETRIC MODE WITH PHI = 1/5, U = 1/2, V = .171



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

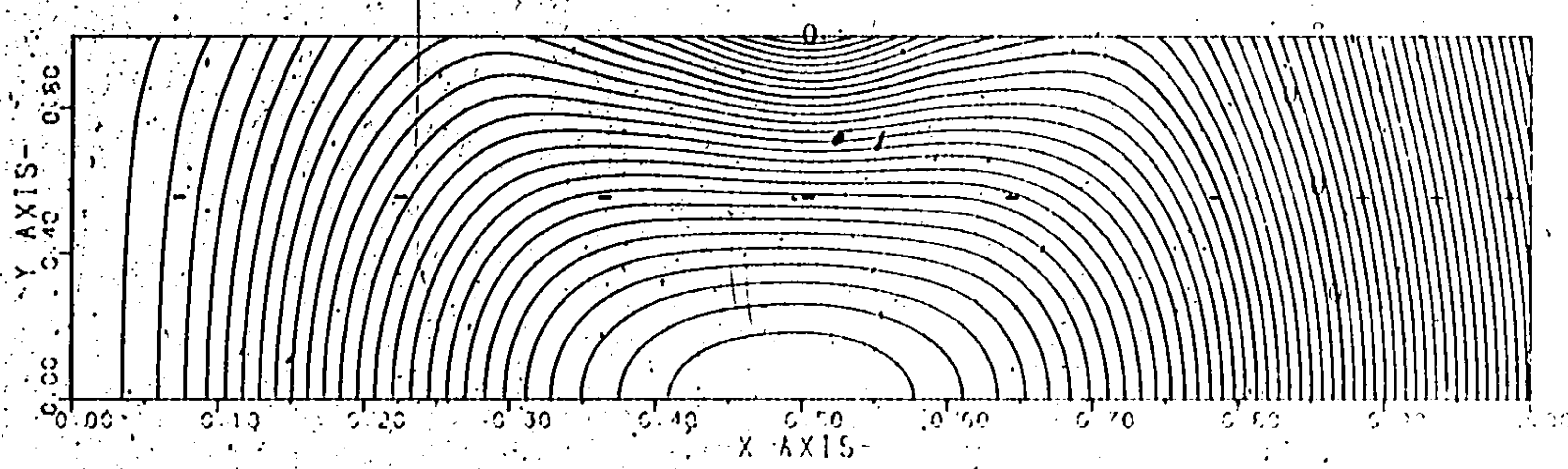
FIRST SYMMETRIC MODE WITHOUT – 1/4, U = 1X2, ...



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT-SUPPORT ***

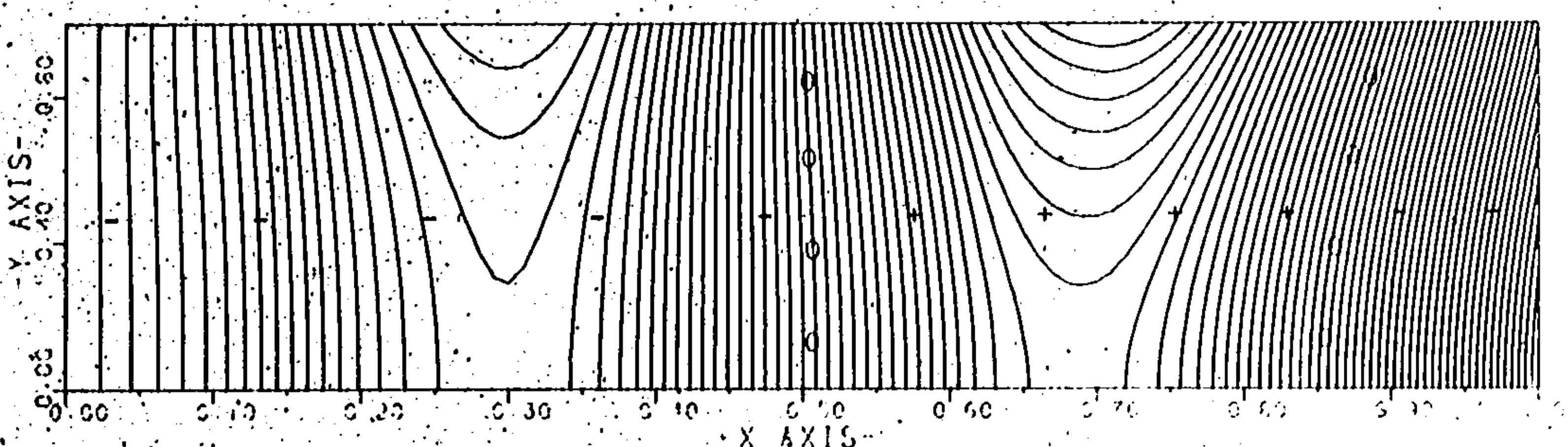
SECOND SYMMETRIC MODE WITH PHI - 1/4, U - 3/2, V - 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

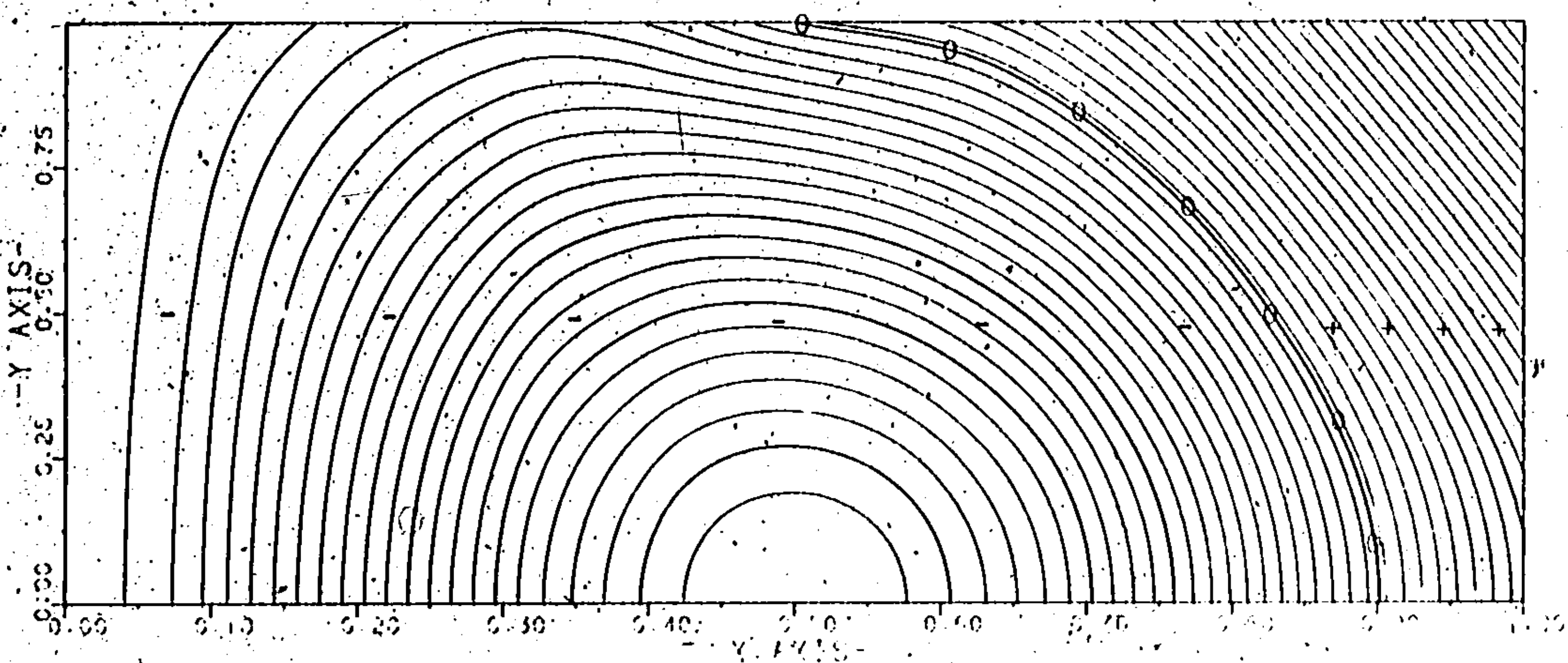
THIRD SYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

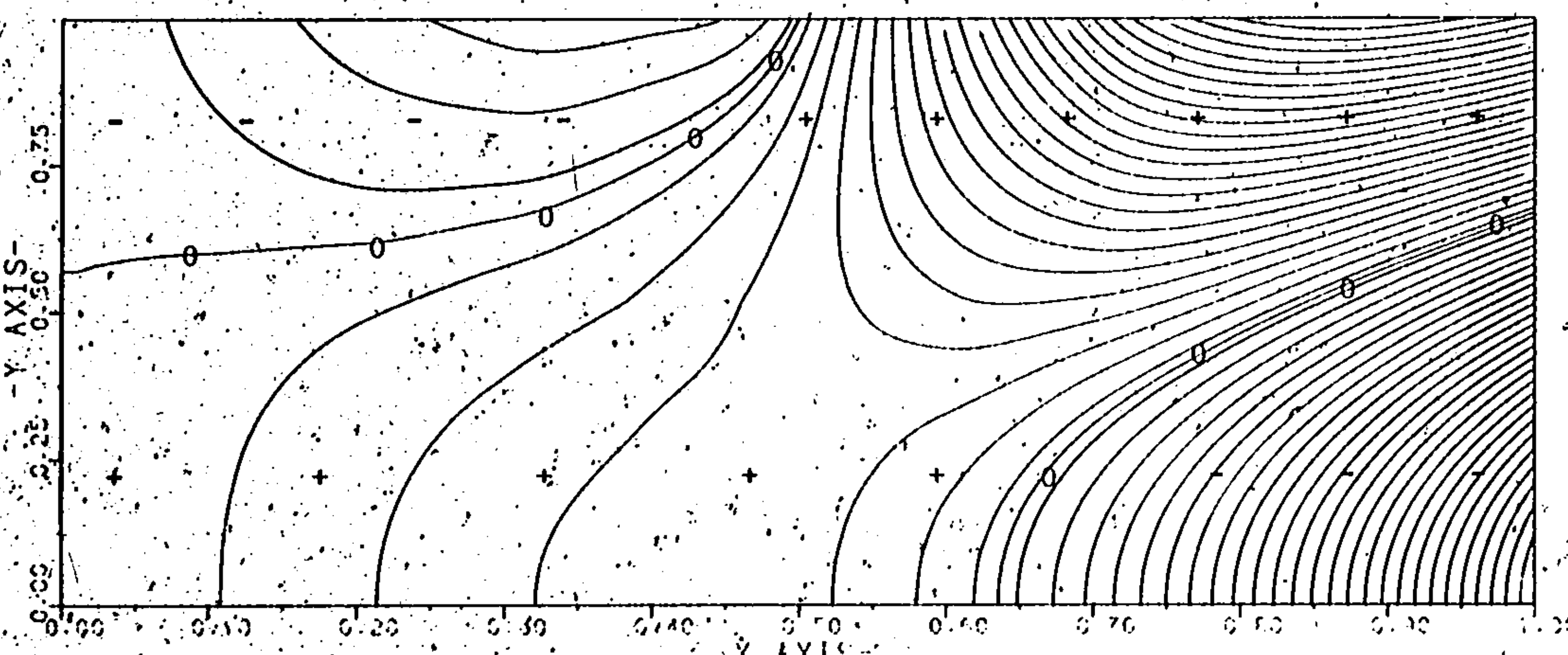
SECOND SYMMETRIC MODE WITH PHI=1/2.5, U = 1/2, V = [illegible]



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

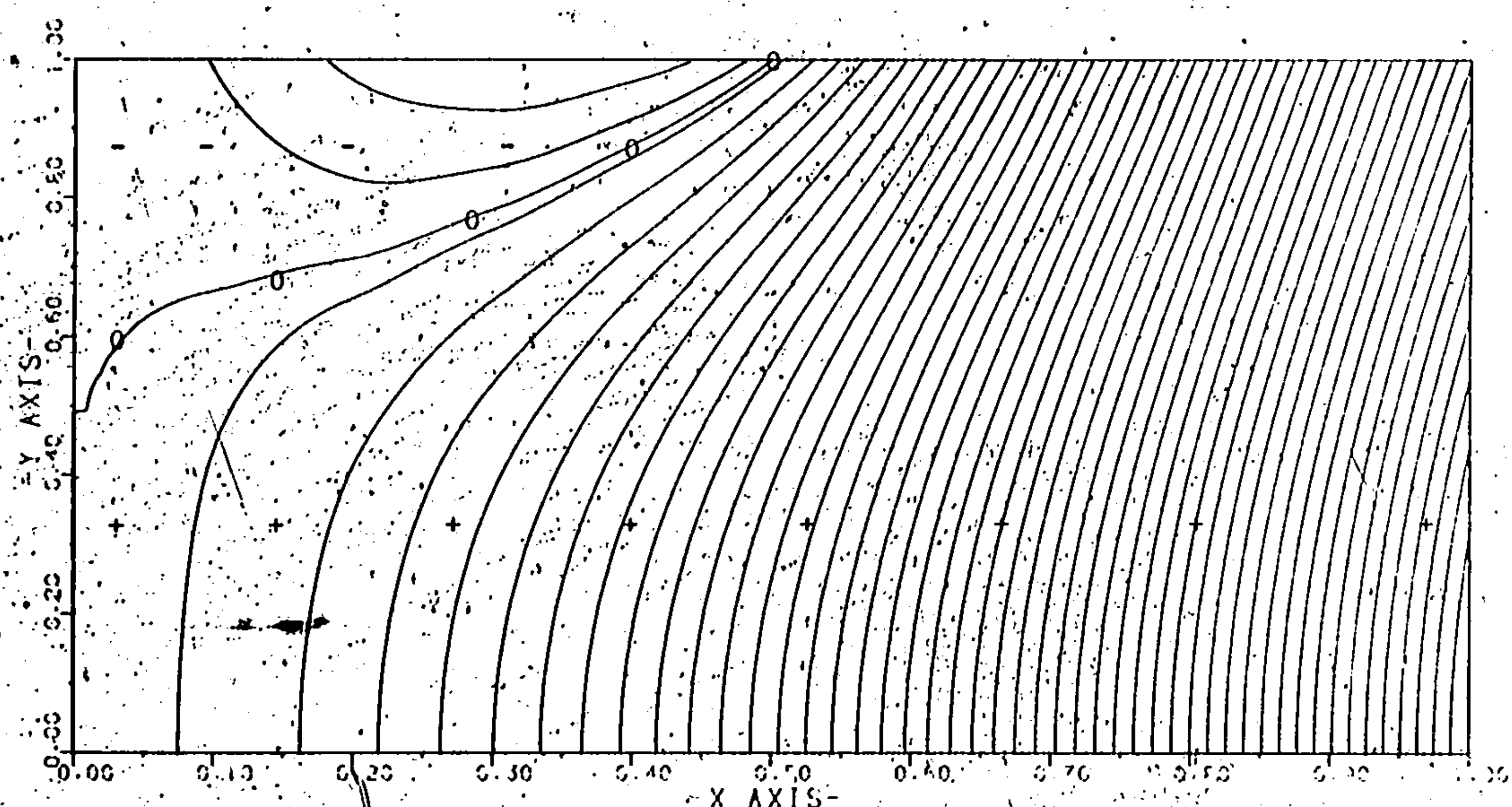
THIRD SYMMETRIC MODE WITH PHI=1/2.5, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

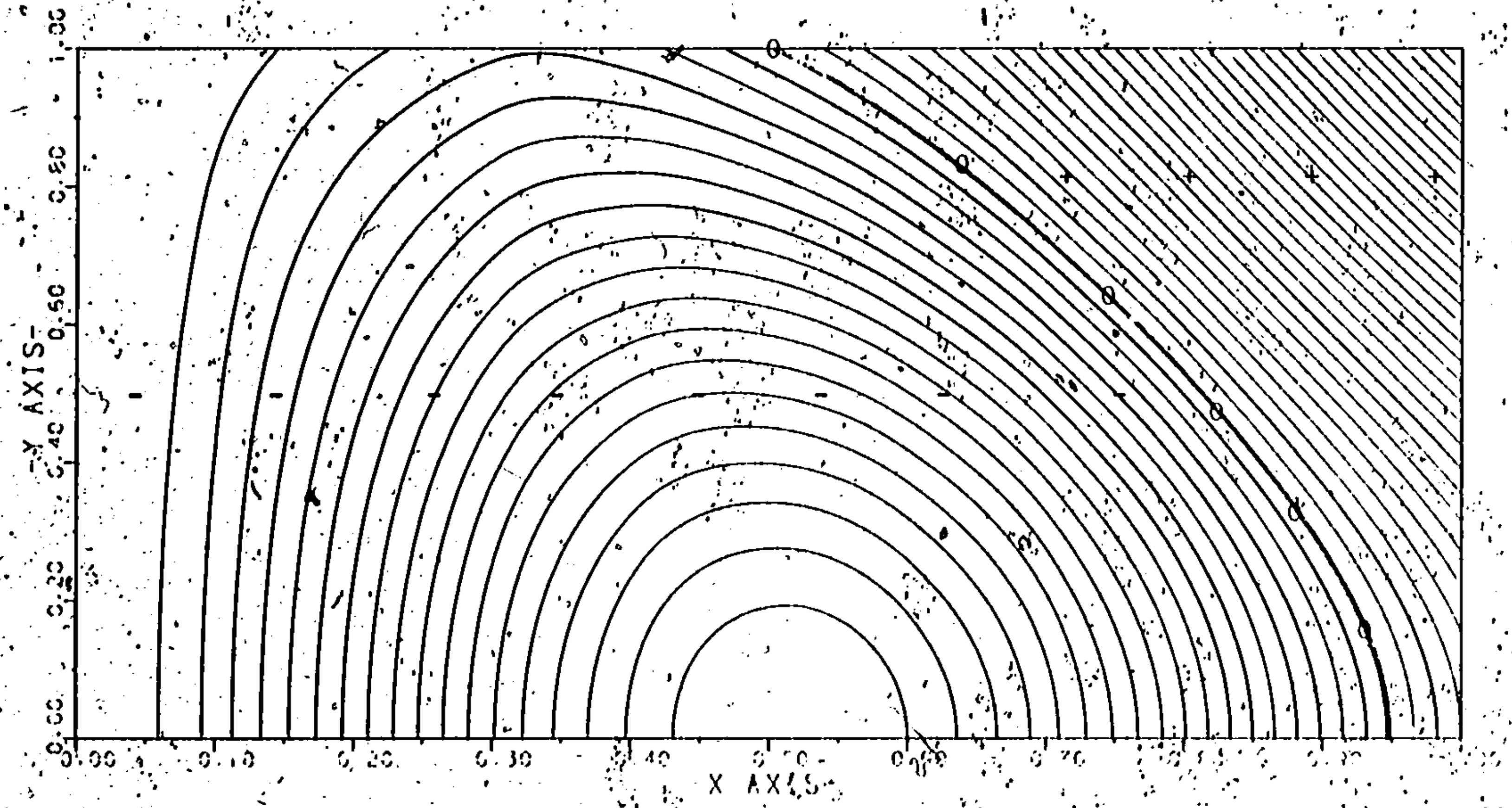
FIRST SYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

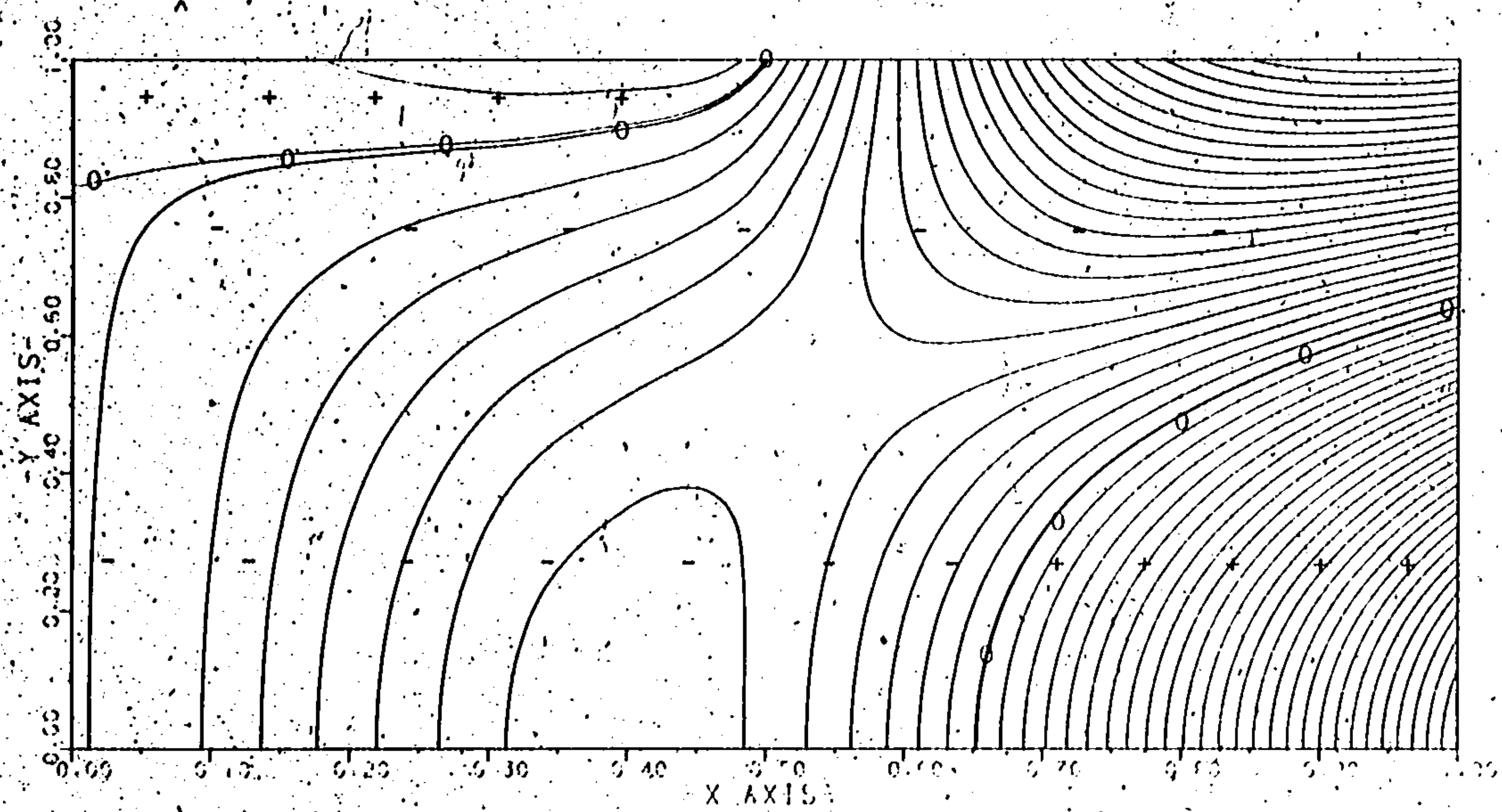
SECOND SYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

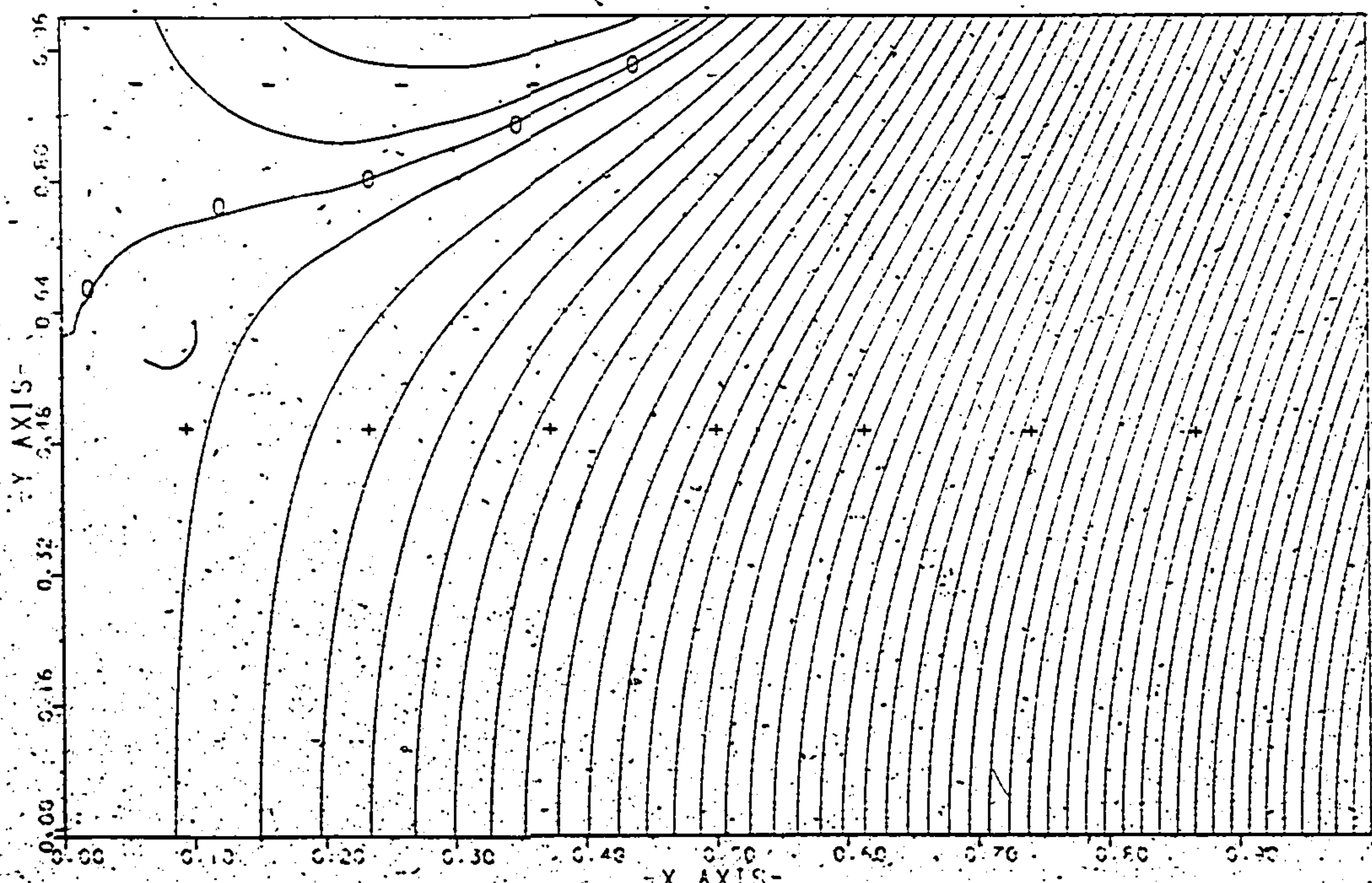
THIRD SYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

**** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ****

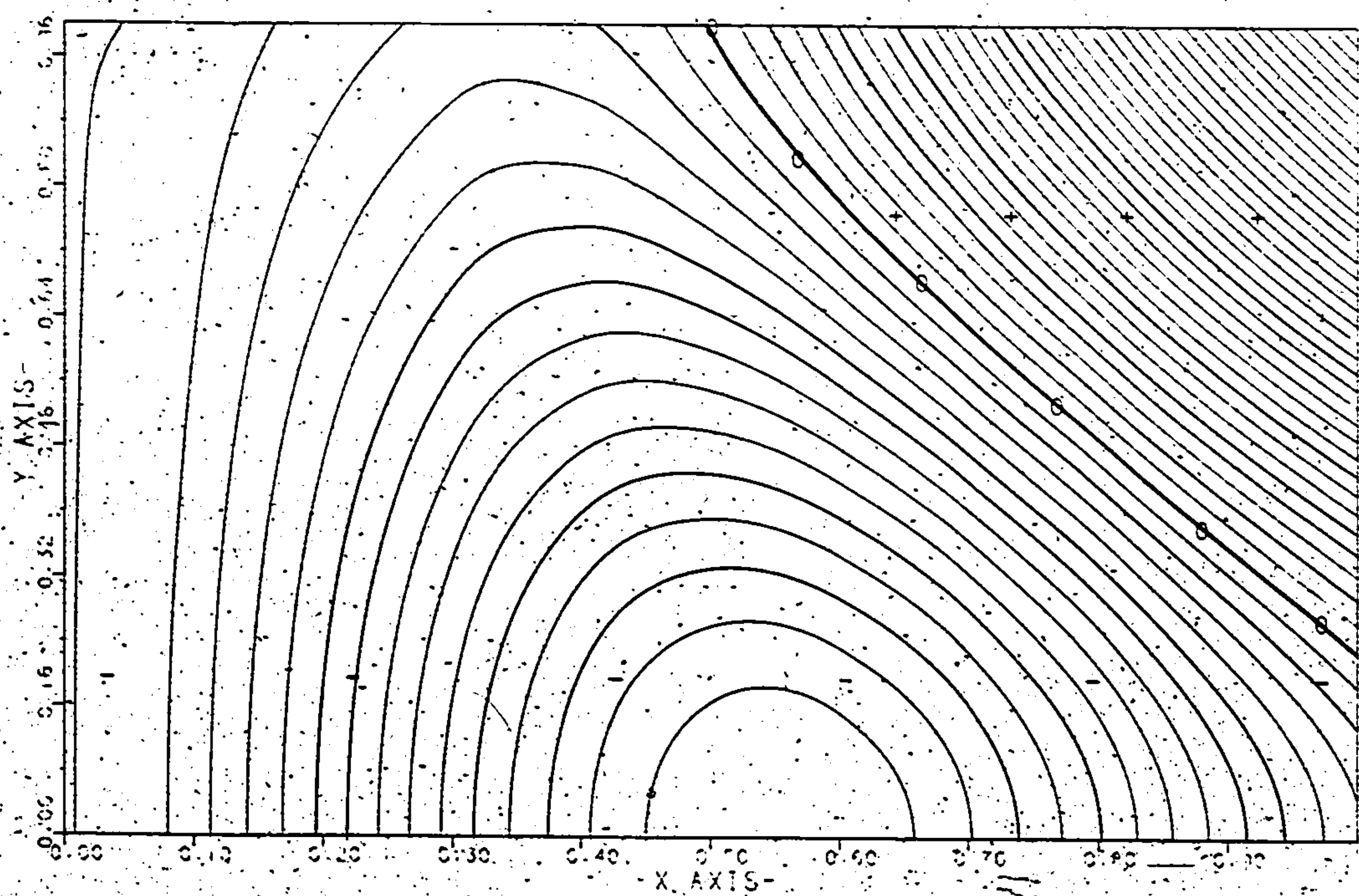
FIRST SYMMETRIC MODE WITH PHI = 5/6 [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

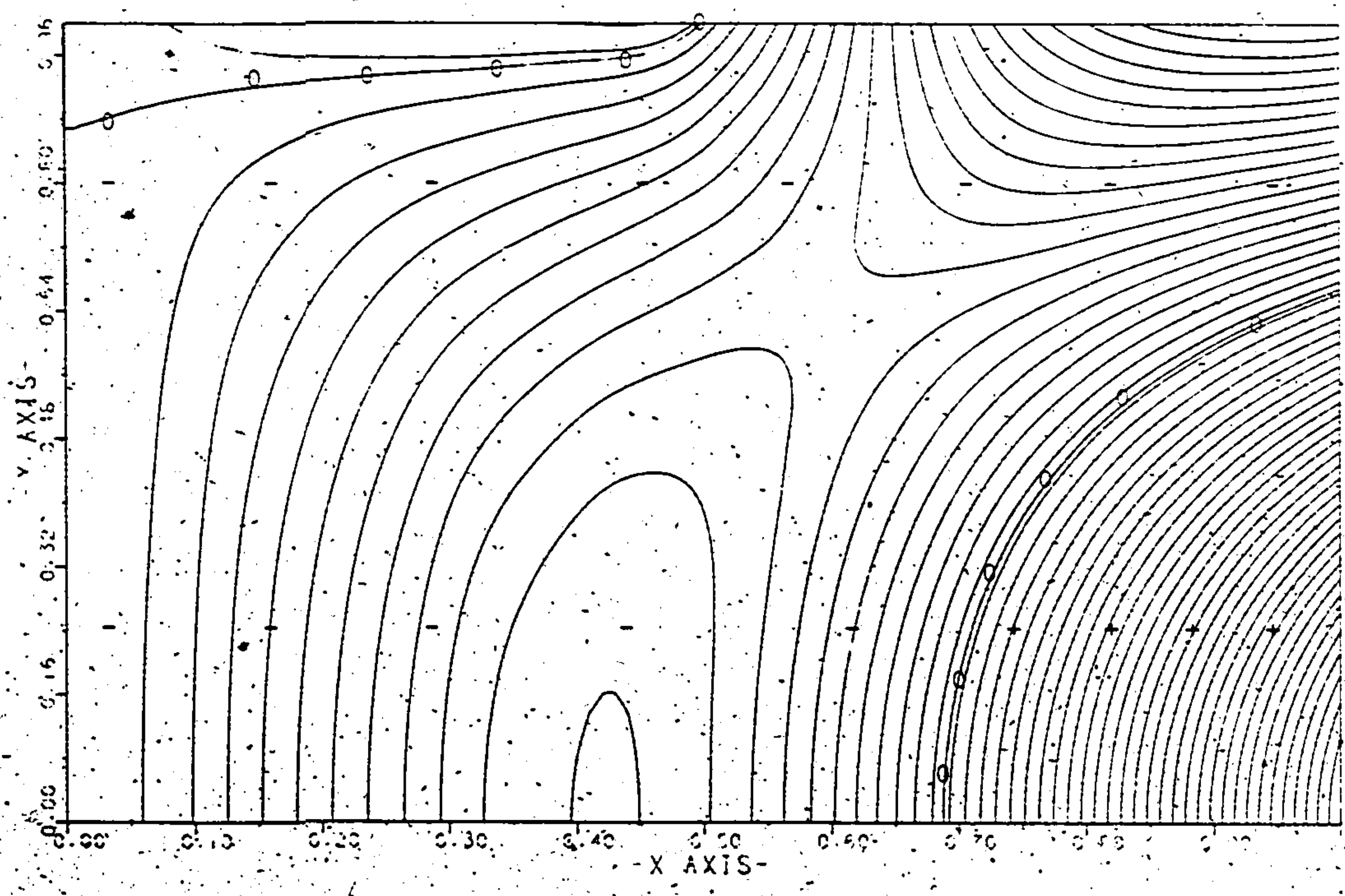
SECOND SYMMETRIC MODE WITH PHI = S/S. [illegible]



WAVE PLATE GRAPHIC

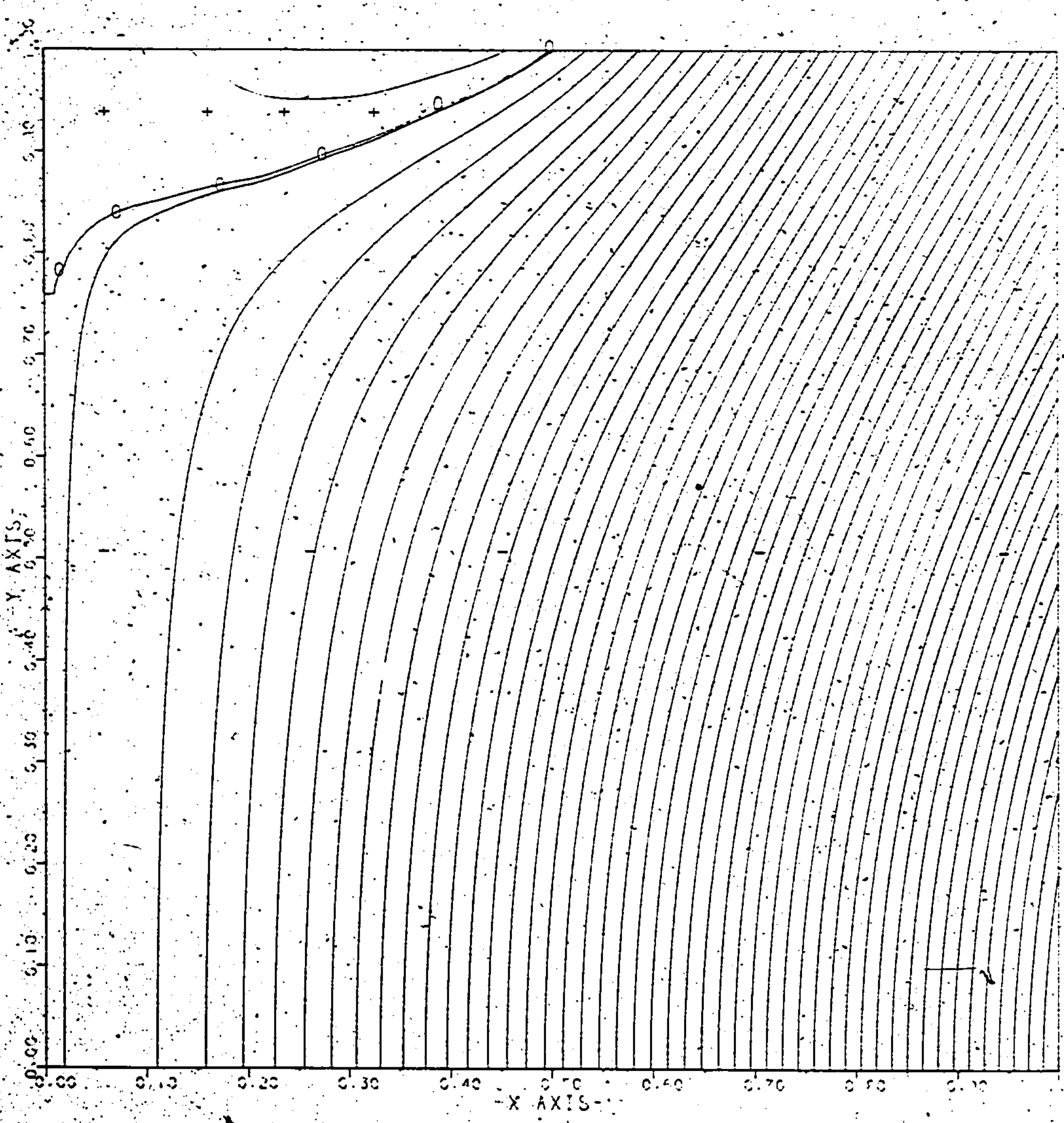
*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

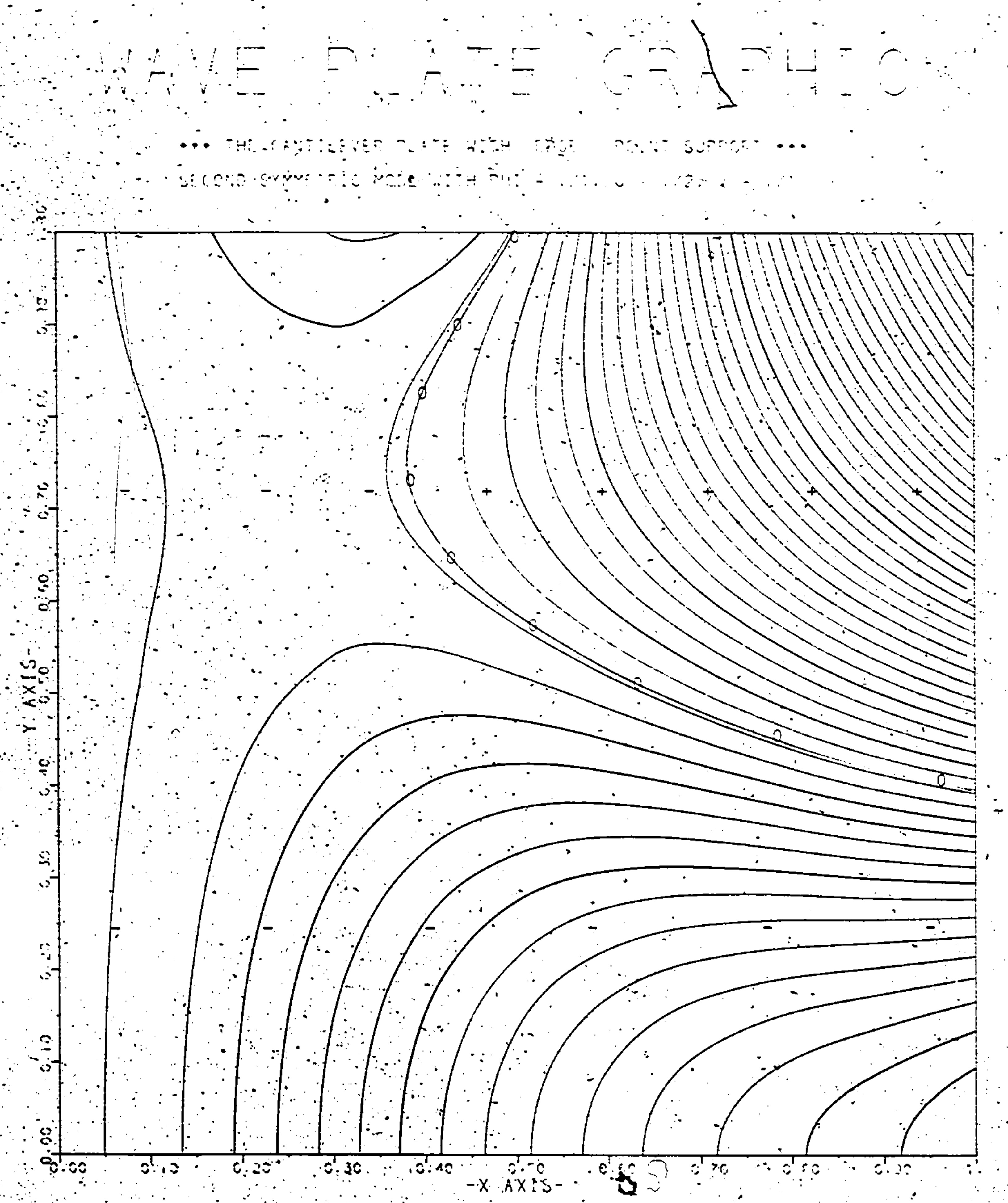
THIRD SYMMETRIC MODE WITH PHI = 5/6, U = 1/2, V = 1/1



*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST SYMMETRIC MODE WITH

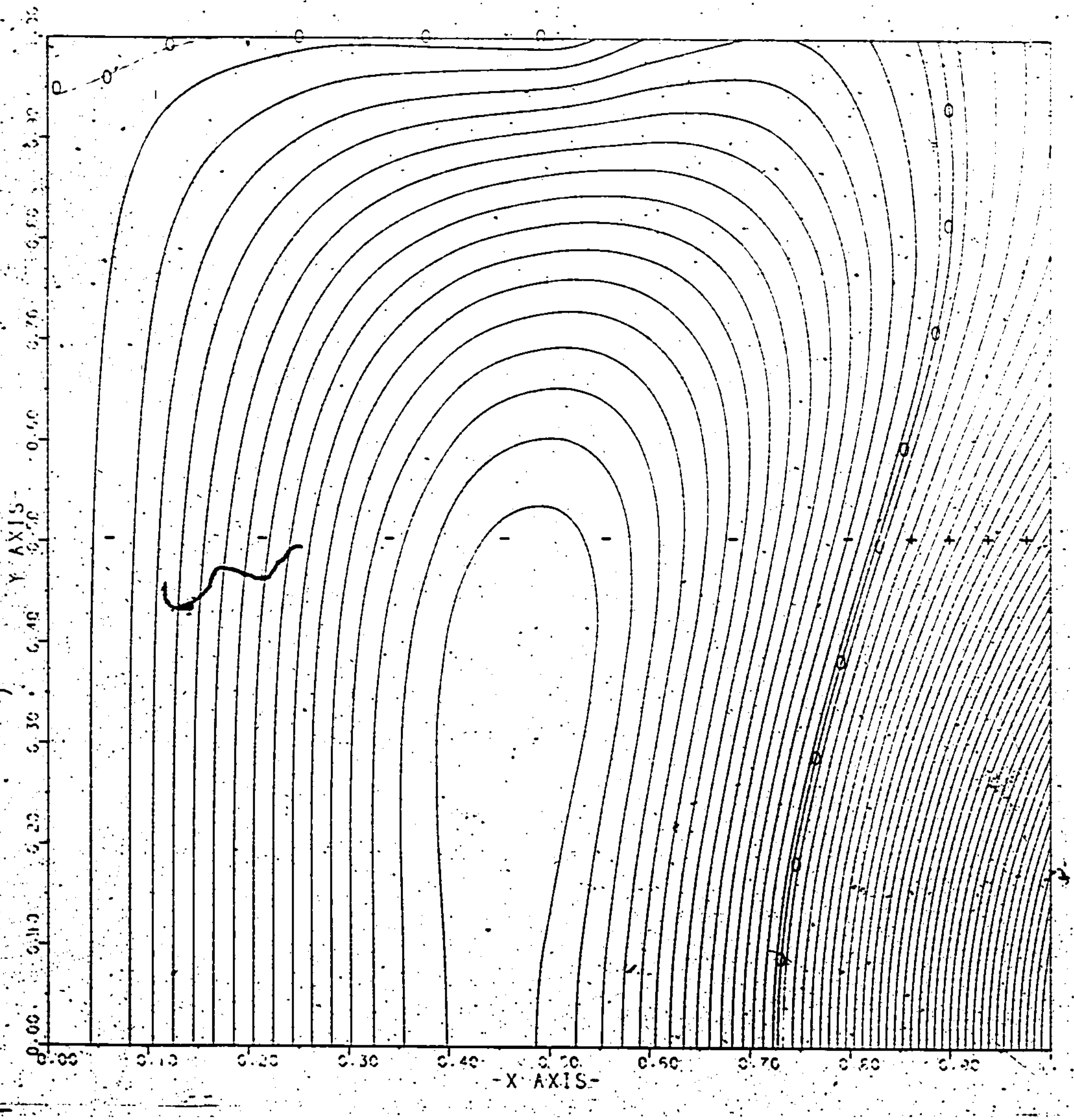



-X AXIS-
-Y AXIS-
0.00
0.10
0.20
0.30
0.40

WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH [illegible] POINT SUPPORT ***

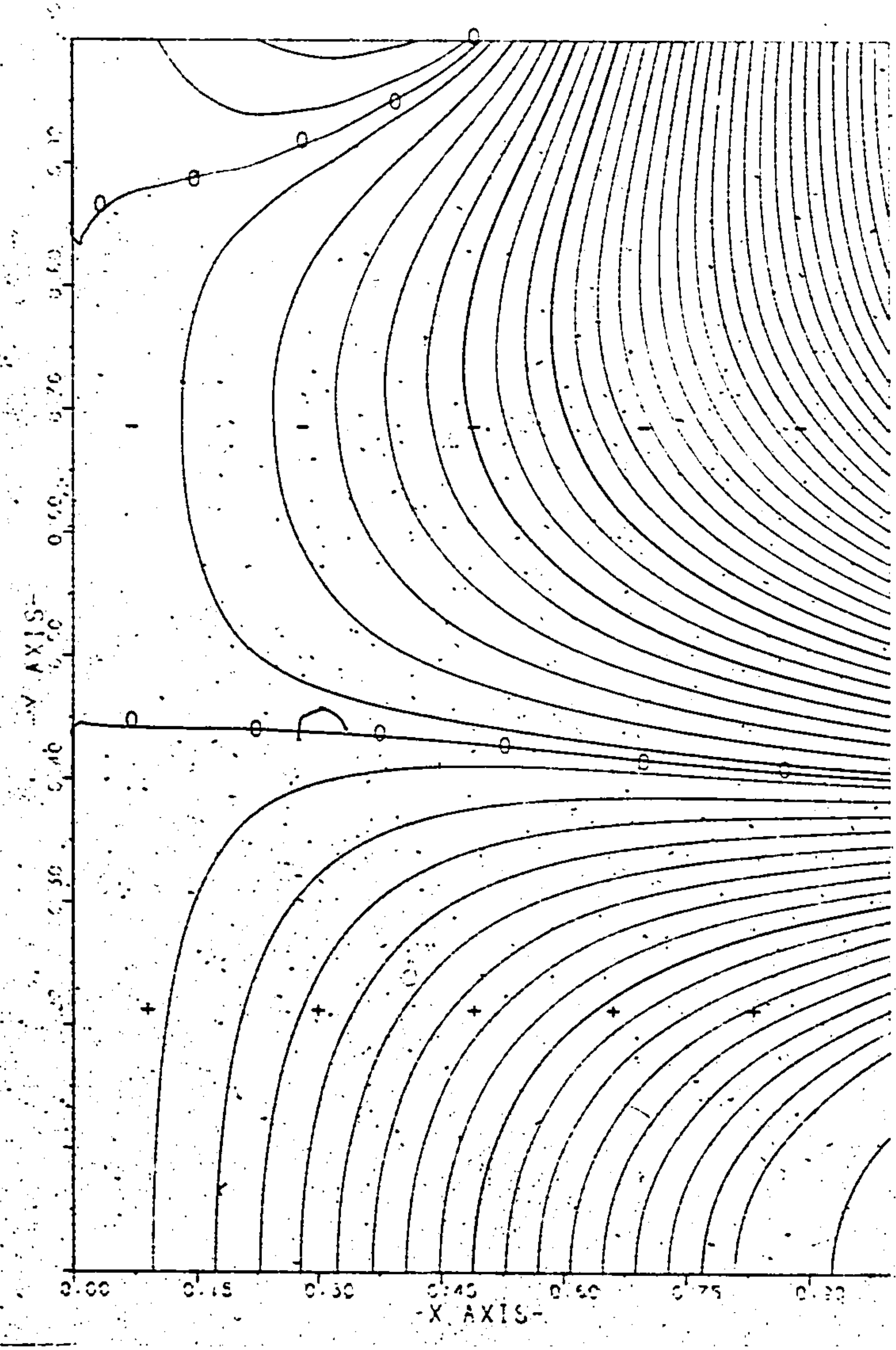
THIRD SYMMETRIC MODE WITH [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH. EDGE POINT SUPPORT ***

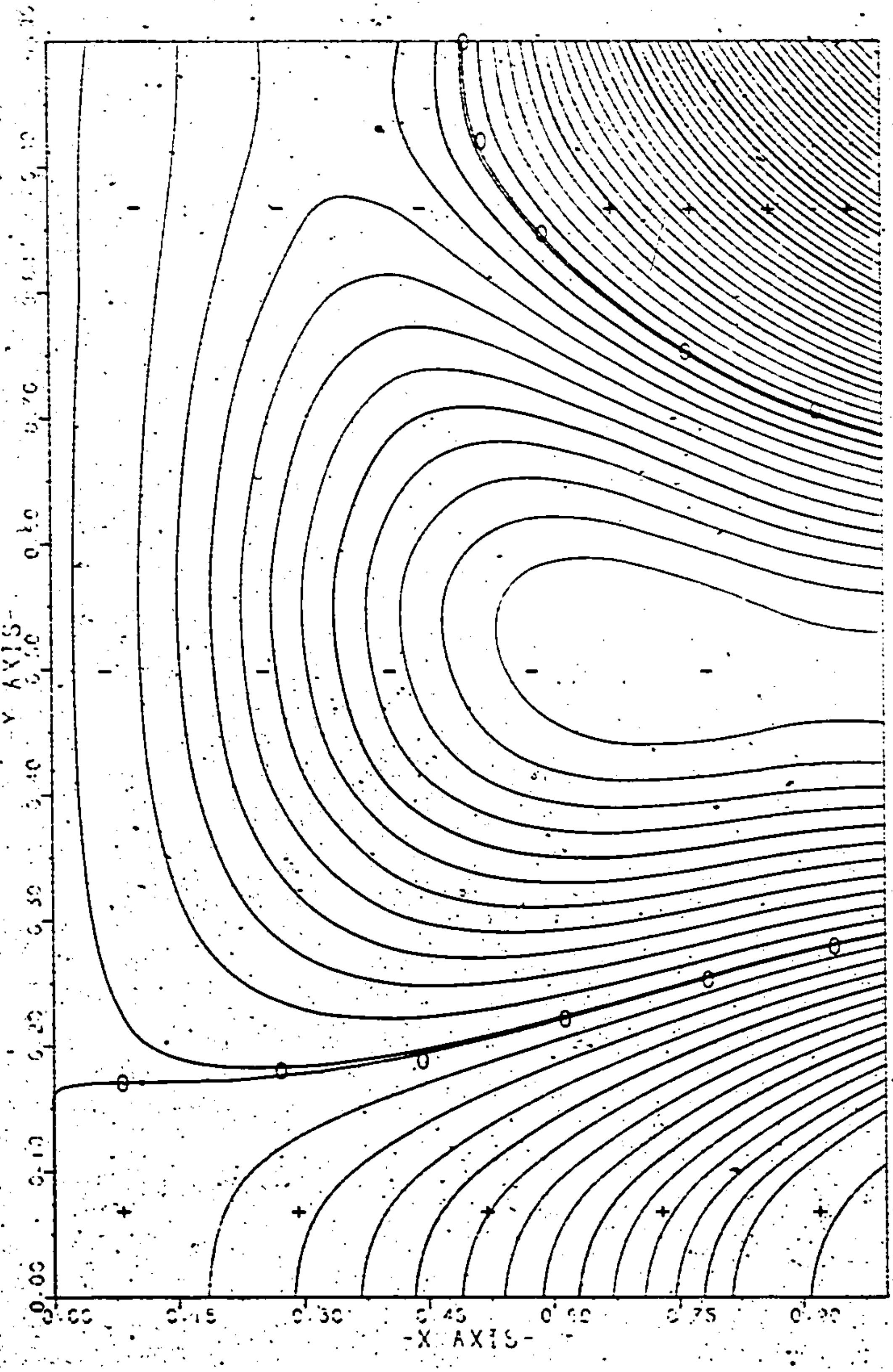
SECOND SYMMETRIC MODE WITH PHI = 3/2, U = [illegible], V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

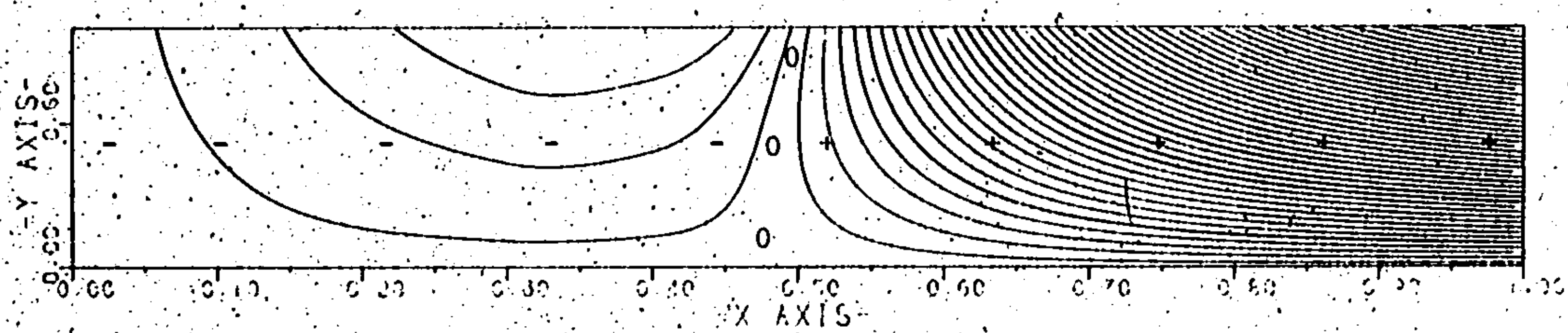
THIRD SYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE (POINT) SUPPORT ***

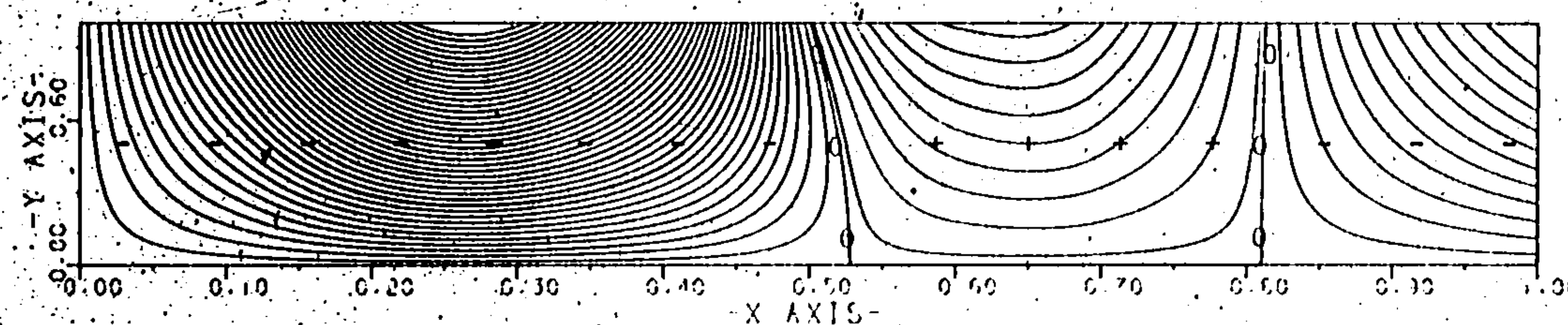
FIRST ANTISYMMETRIC MODE WITH PHI = 1/6; U = 1/2; V = [illegible]



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

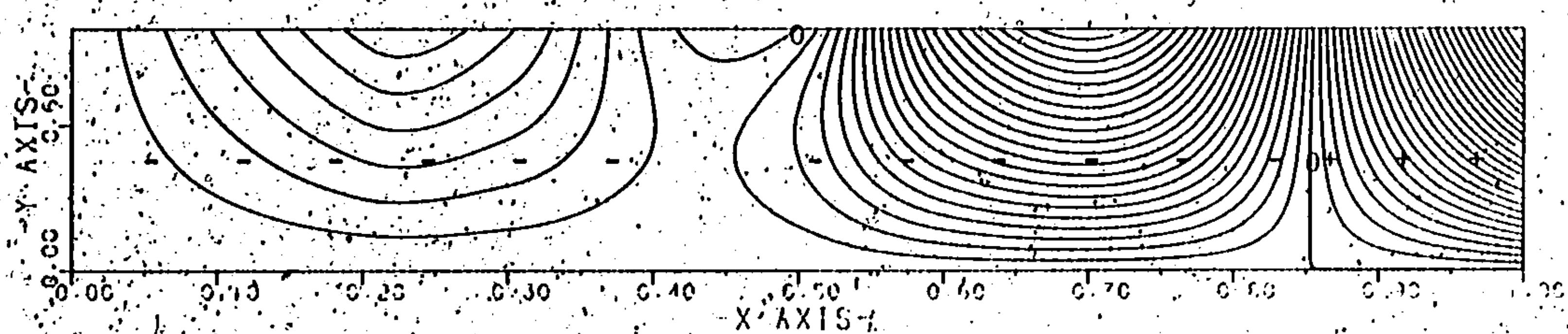
SECOND ANTISYMMETRIC MODE WITH PHI = 1/6, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

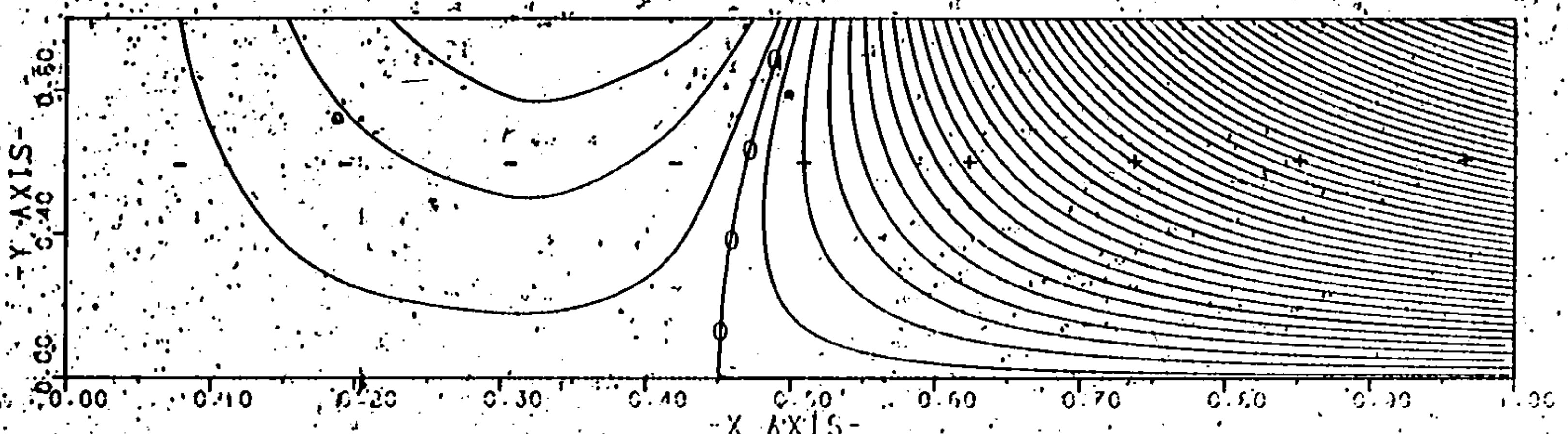
THIRD ANTISYMMETRIC MODE WITH PHI = 1/6, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

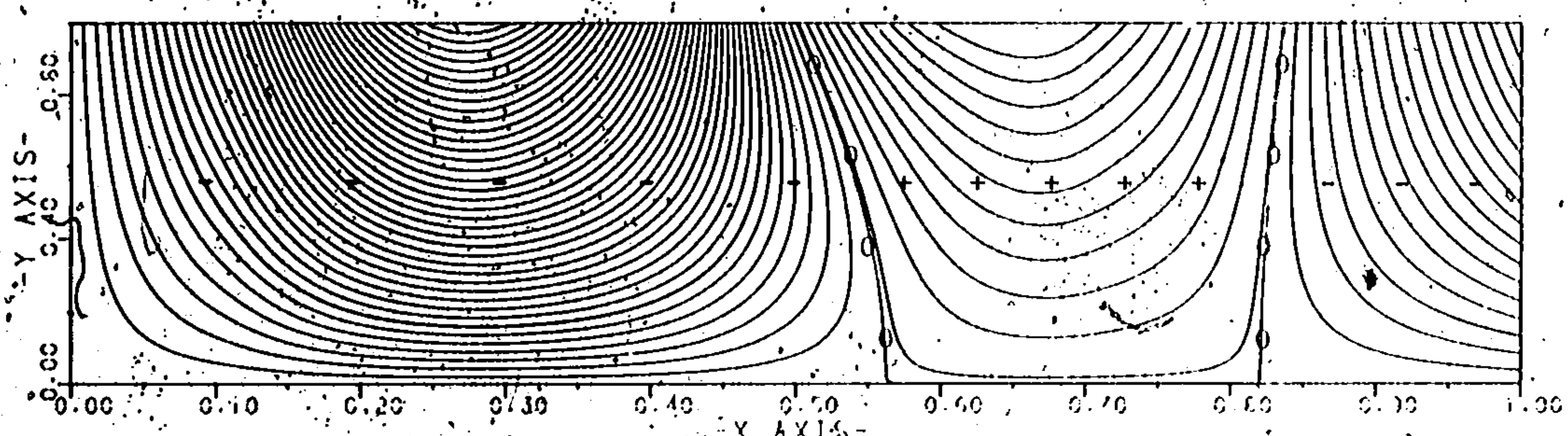
FIRST ANTISYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE · POINT SUPPORT ***

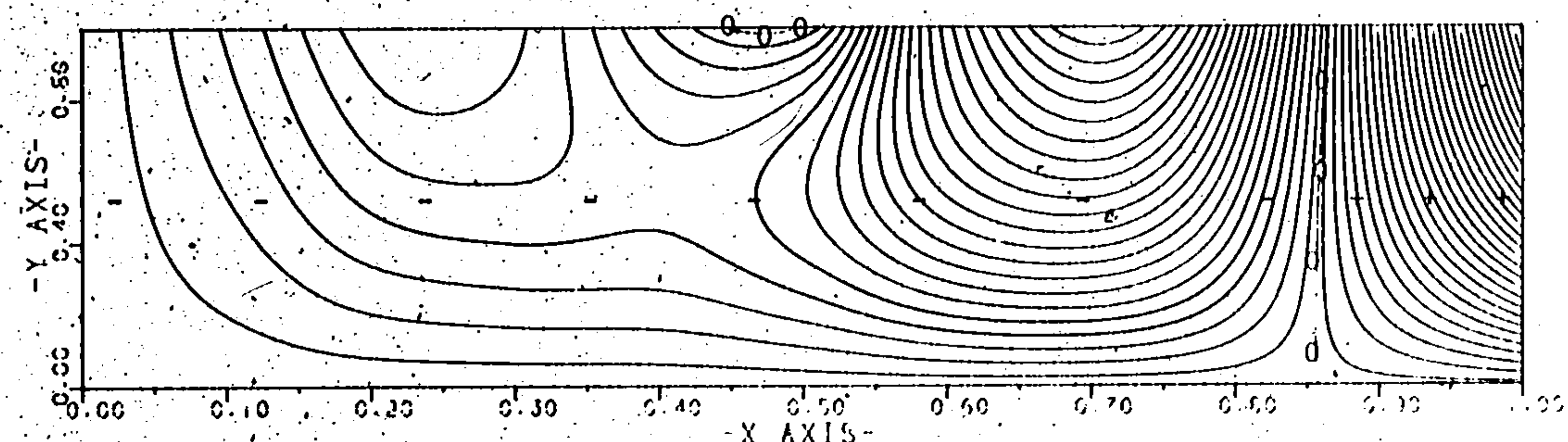
SECOND ANTISYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

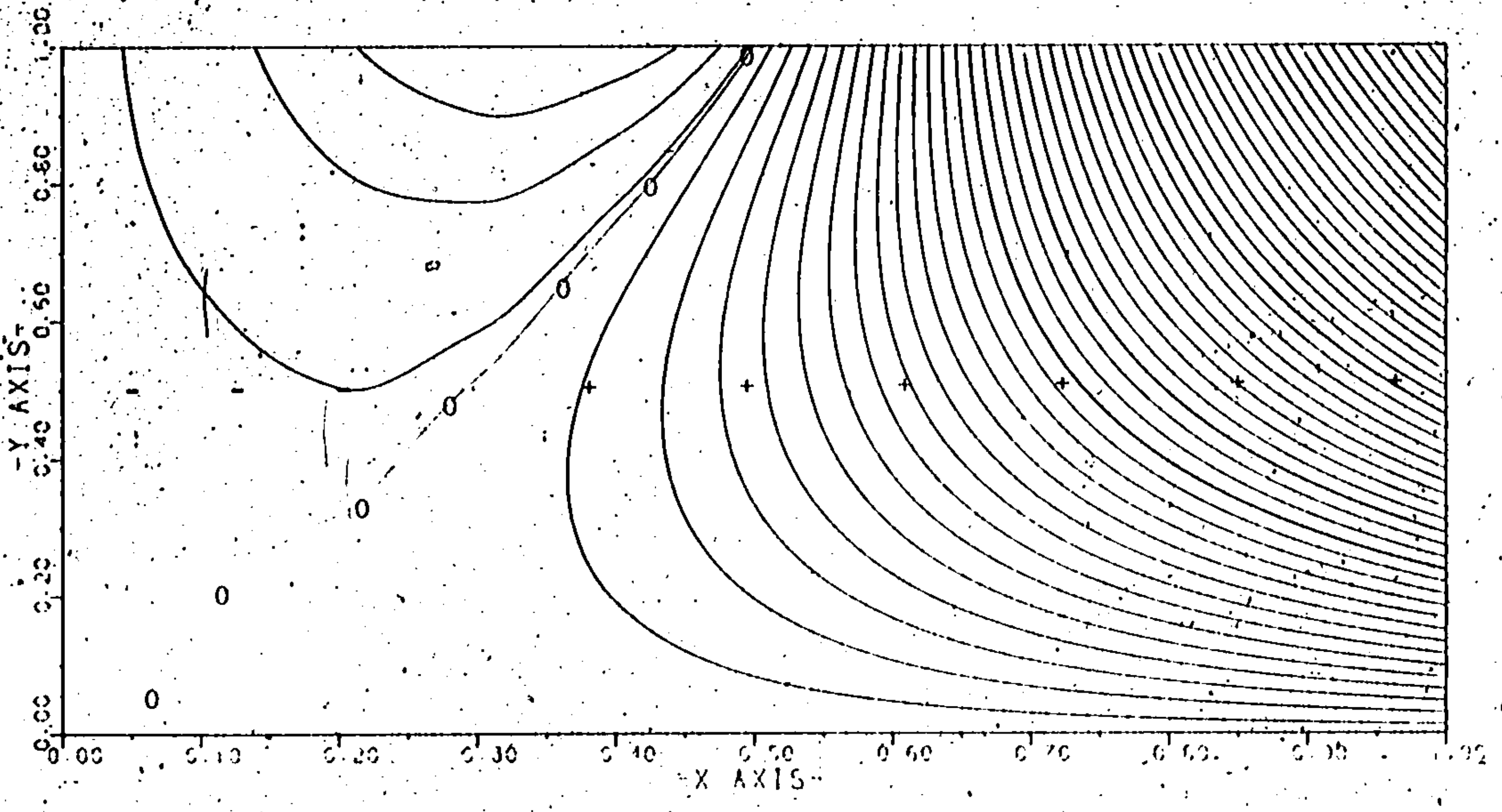
THIRD ANTISYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

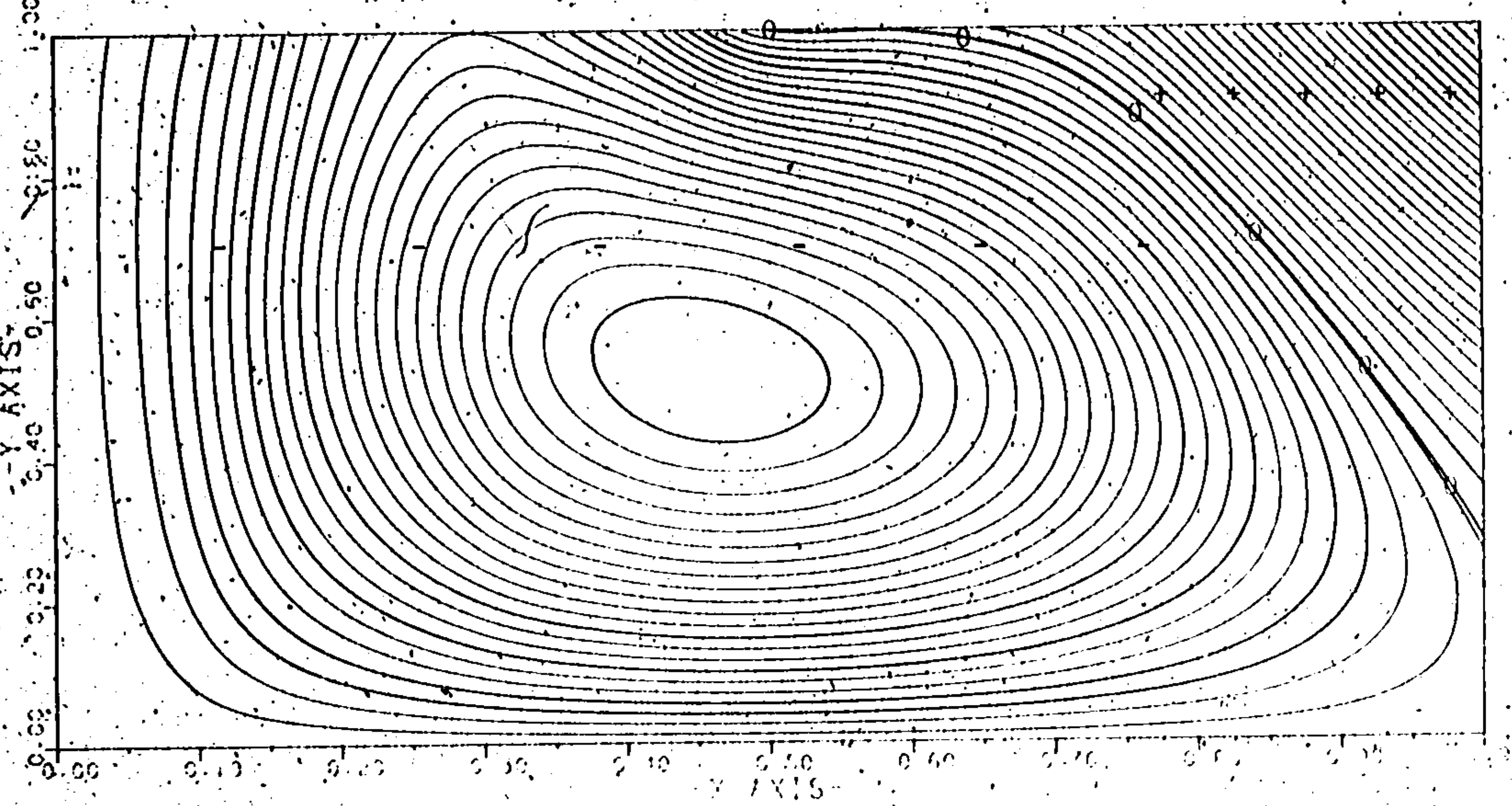
FIRST ANTISYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

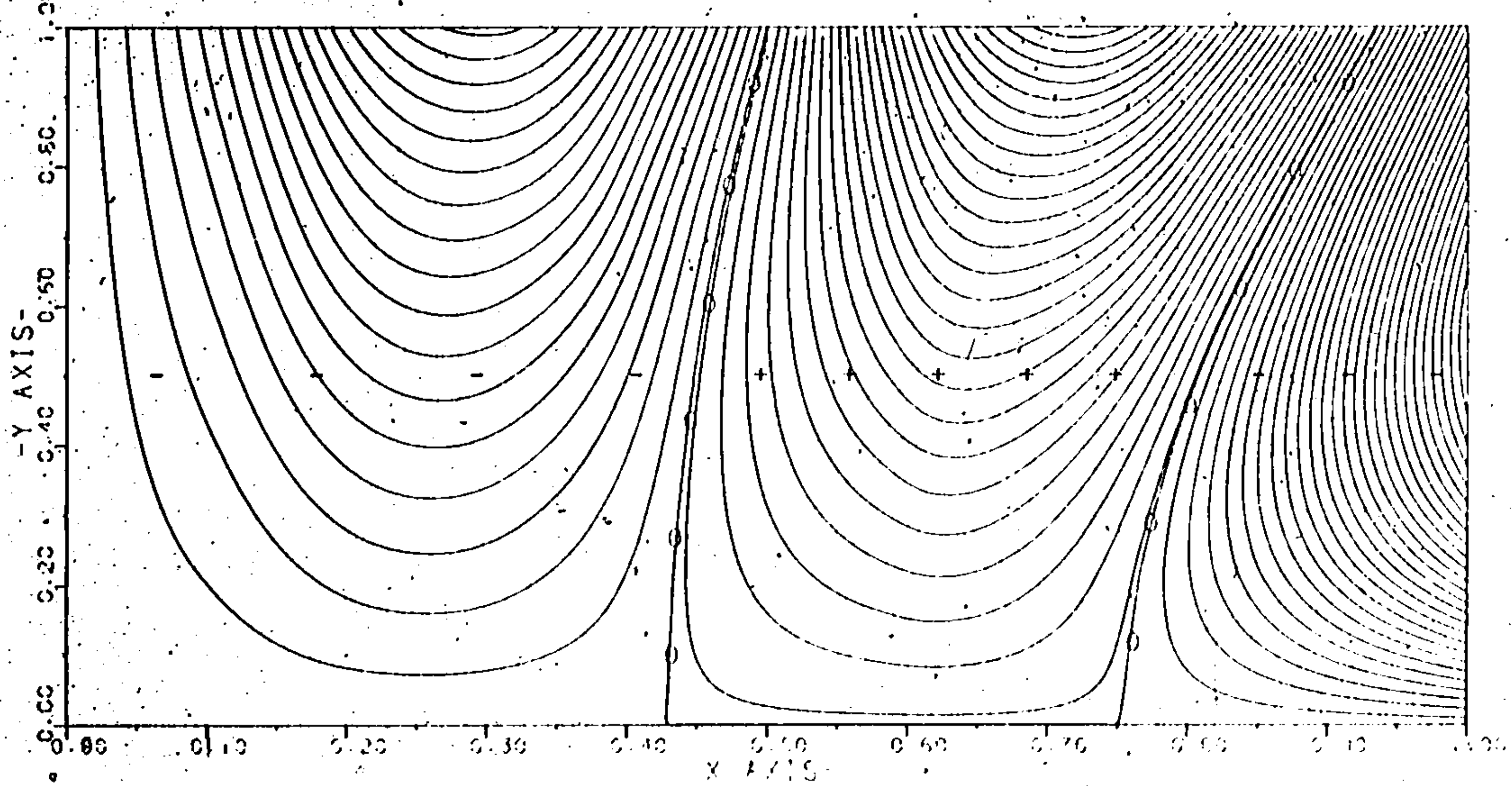
SECOND ANTISYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

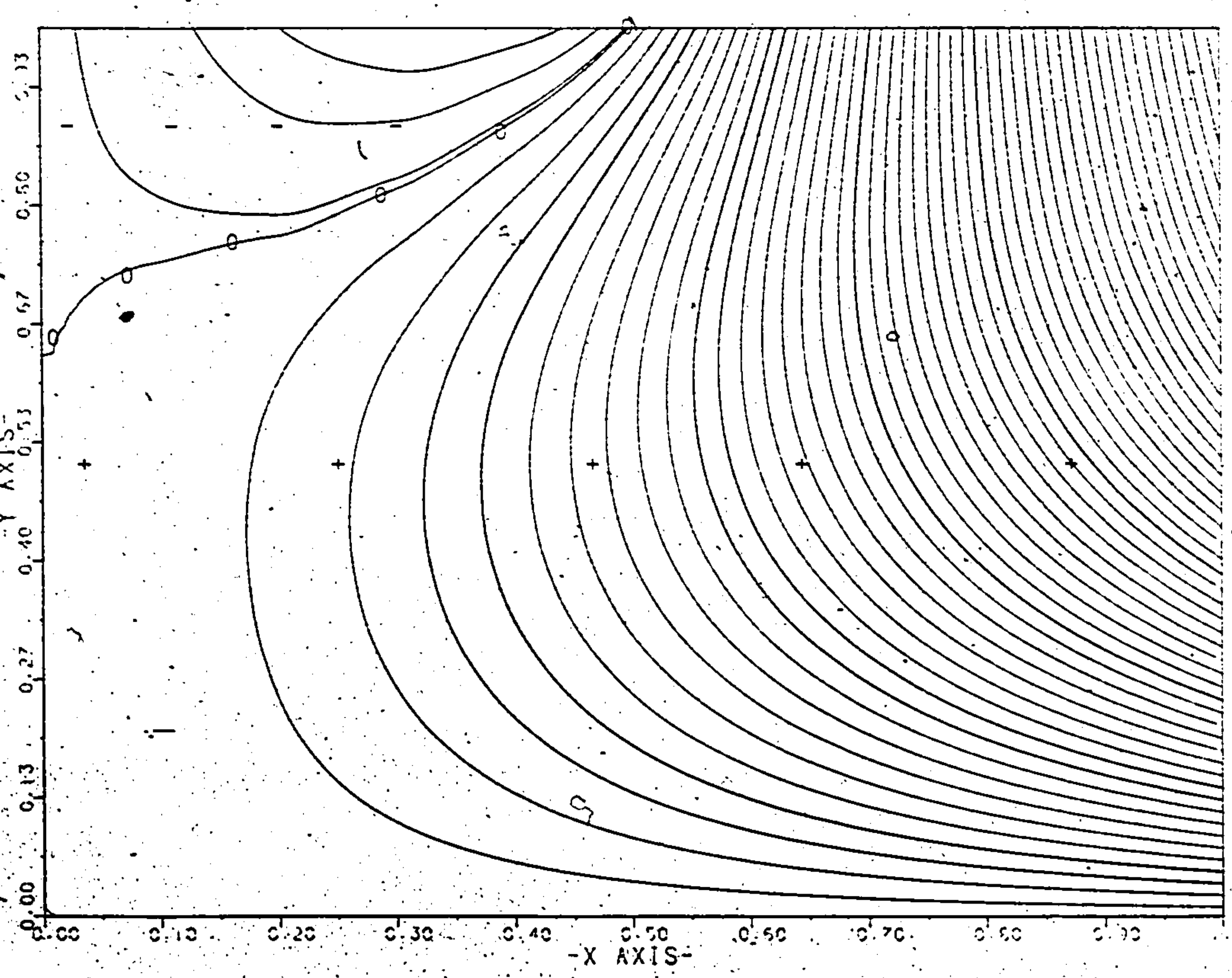
THIRD ANTISYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/4



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

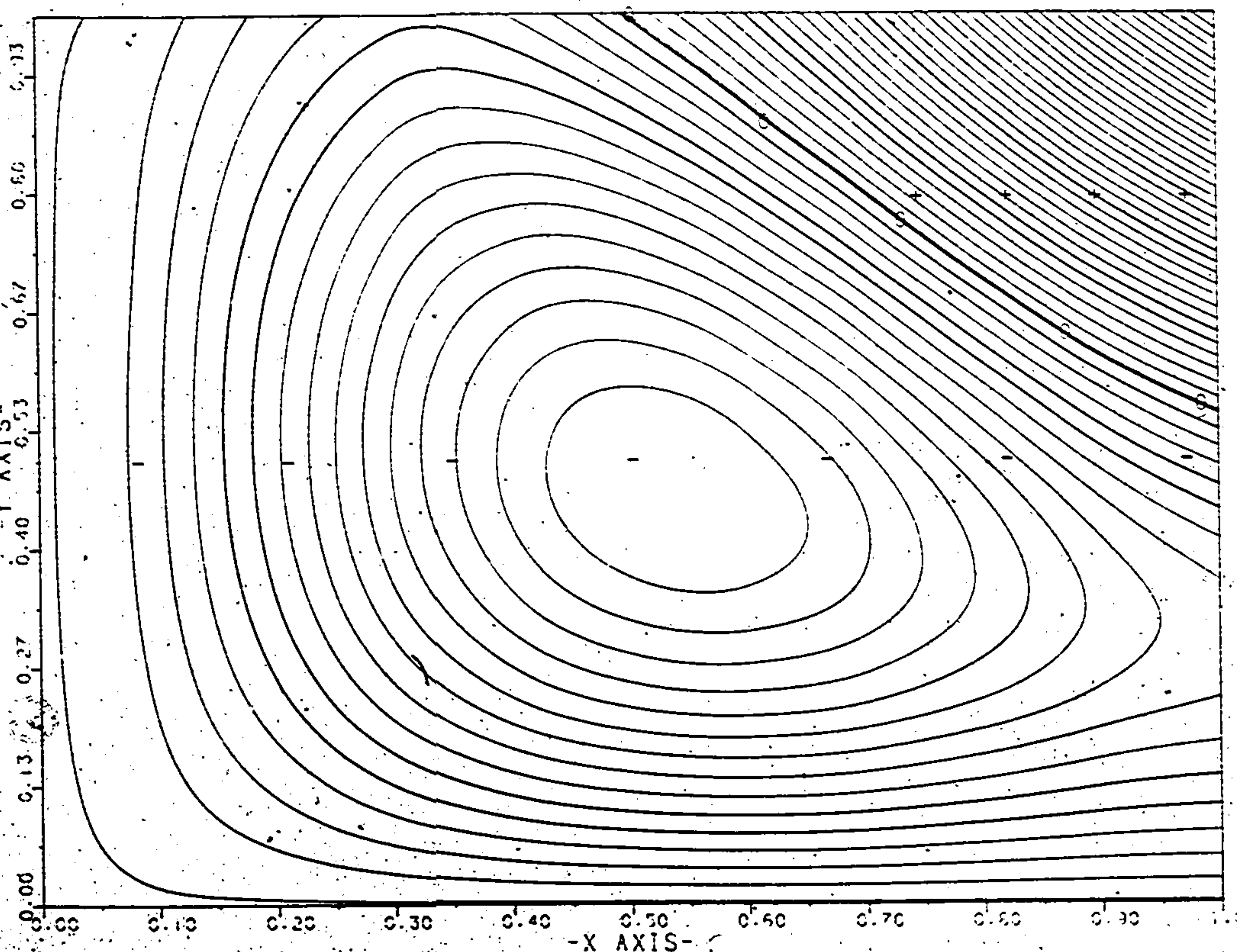
FIRST ANTISYMMETRIC MODE WITH PHI = .5/4. [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

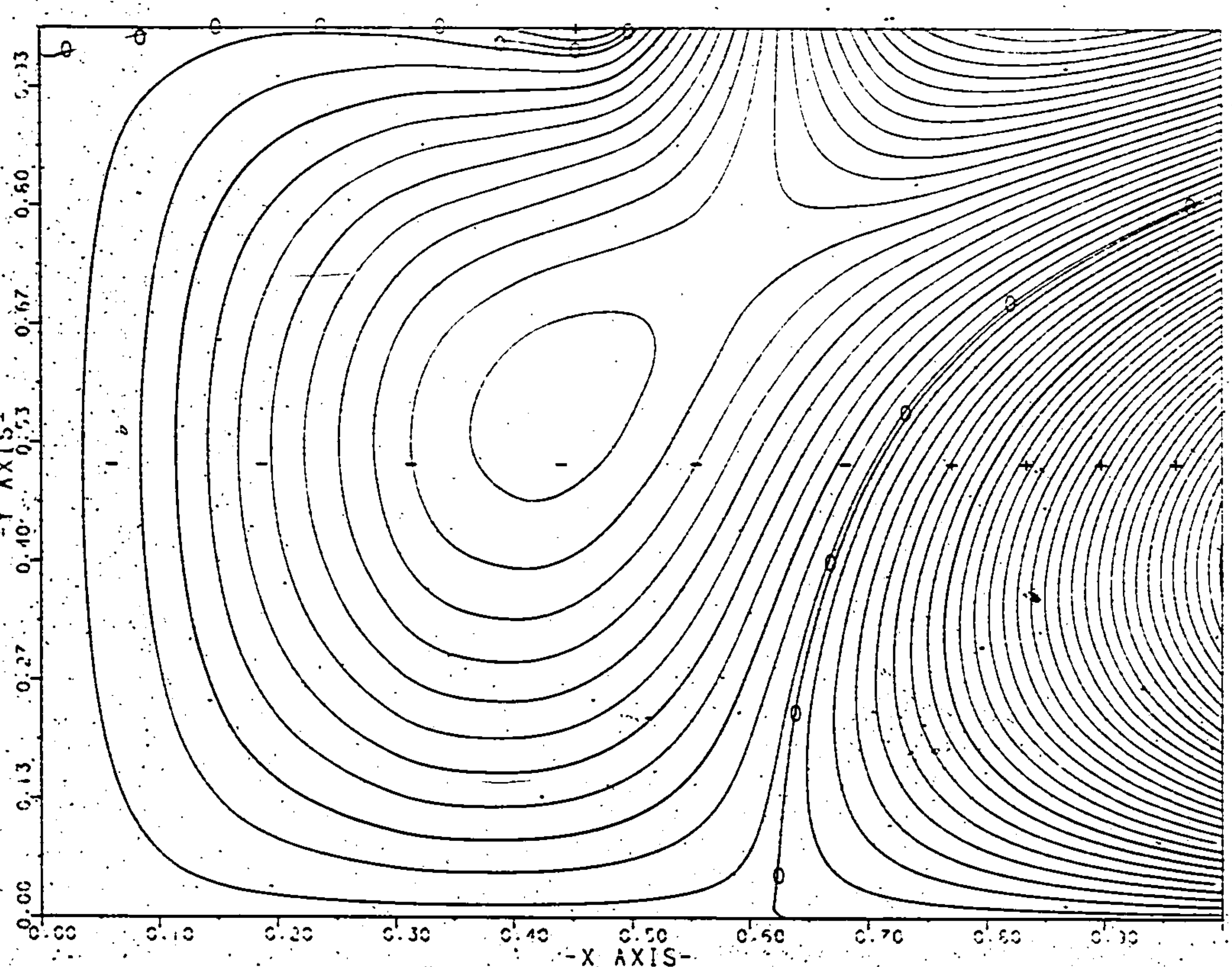
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

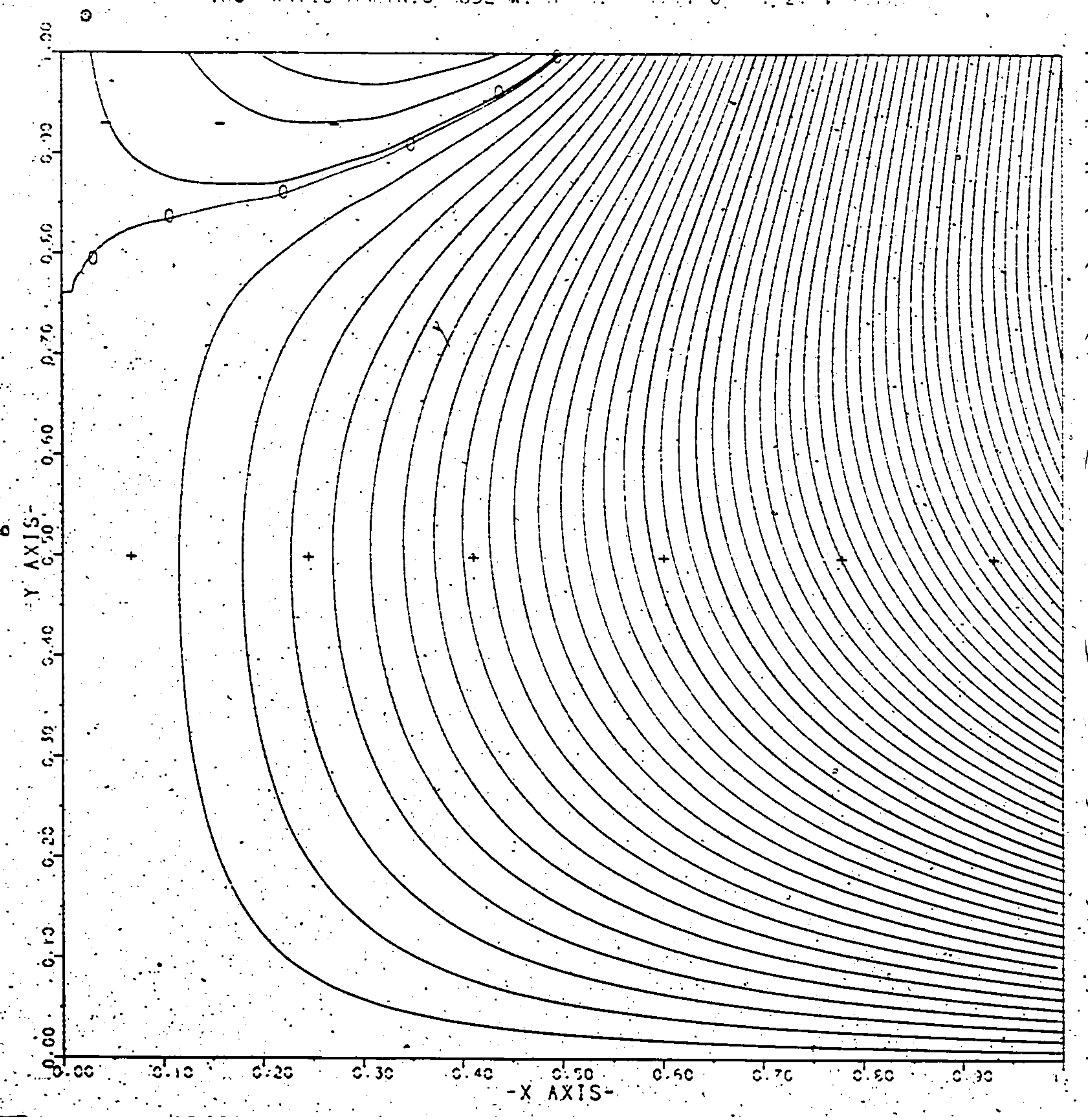
THIRD ANTISYMMETRIC MODE WITH PHI = 3/4. U = 1/2. V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

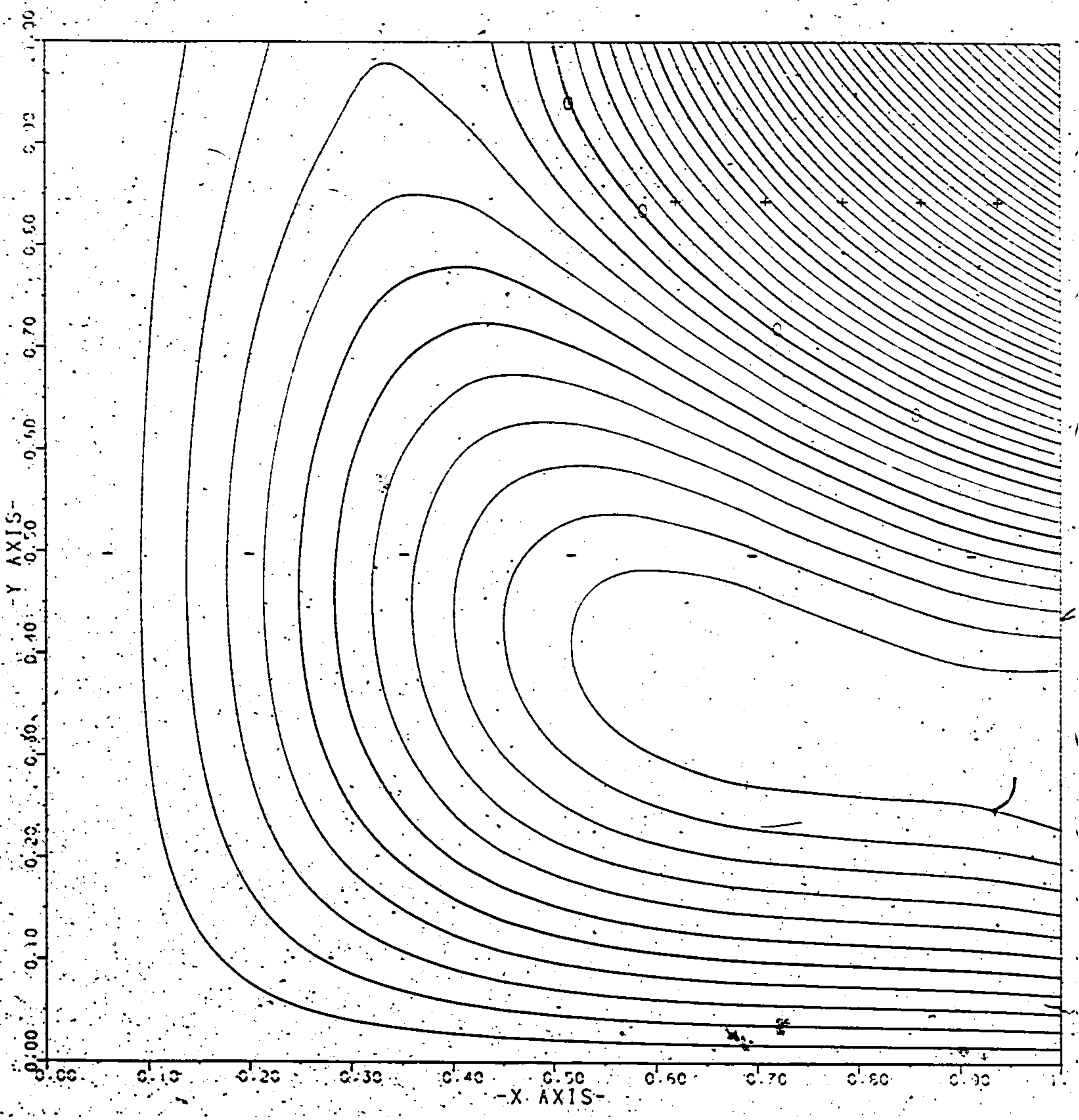
FIRST ANTISYMMETRIC MODE WITH PHI =



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

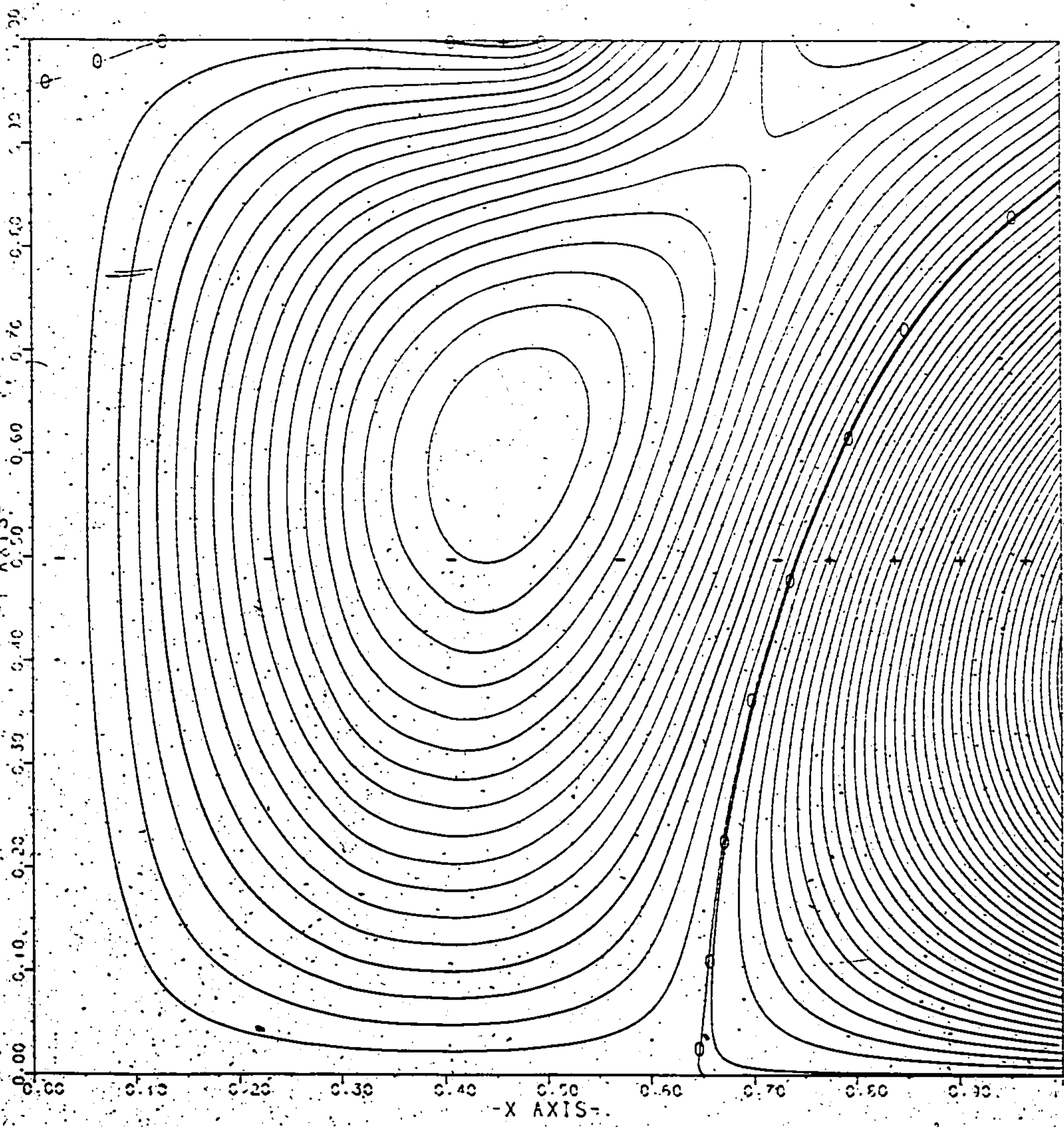
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible], U = [illegible]2, V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

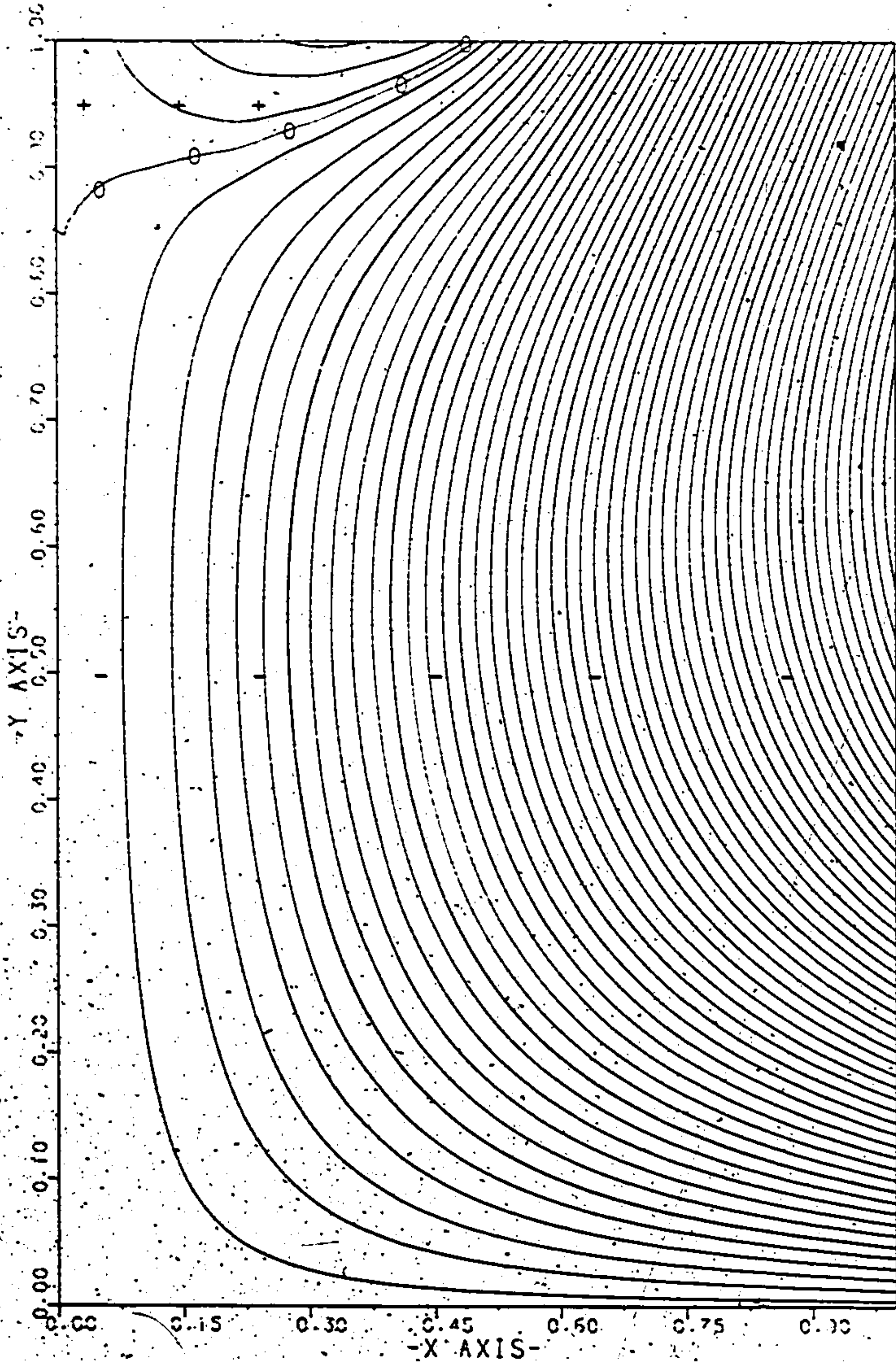
THIRD ANTISYMMETRIC MODE WITH PHI = [illegible], U = [illegible]2, V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

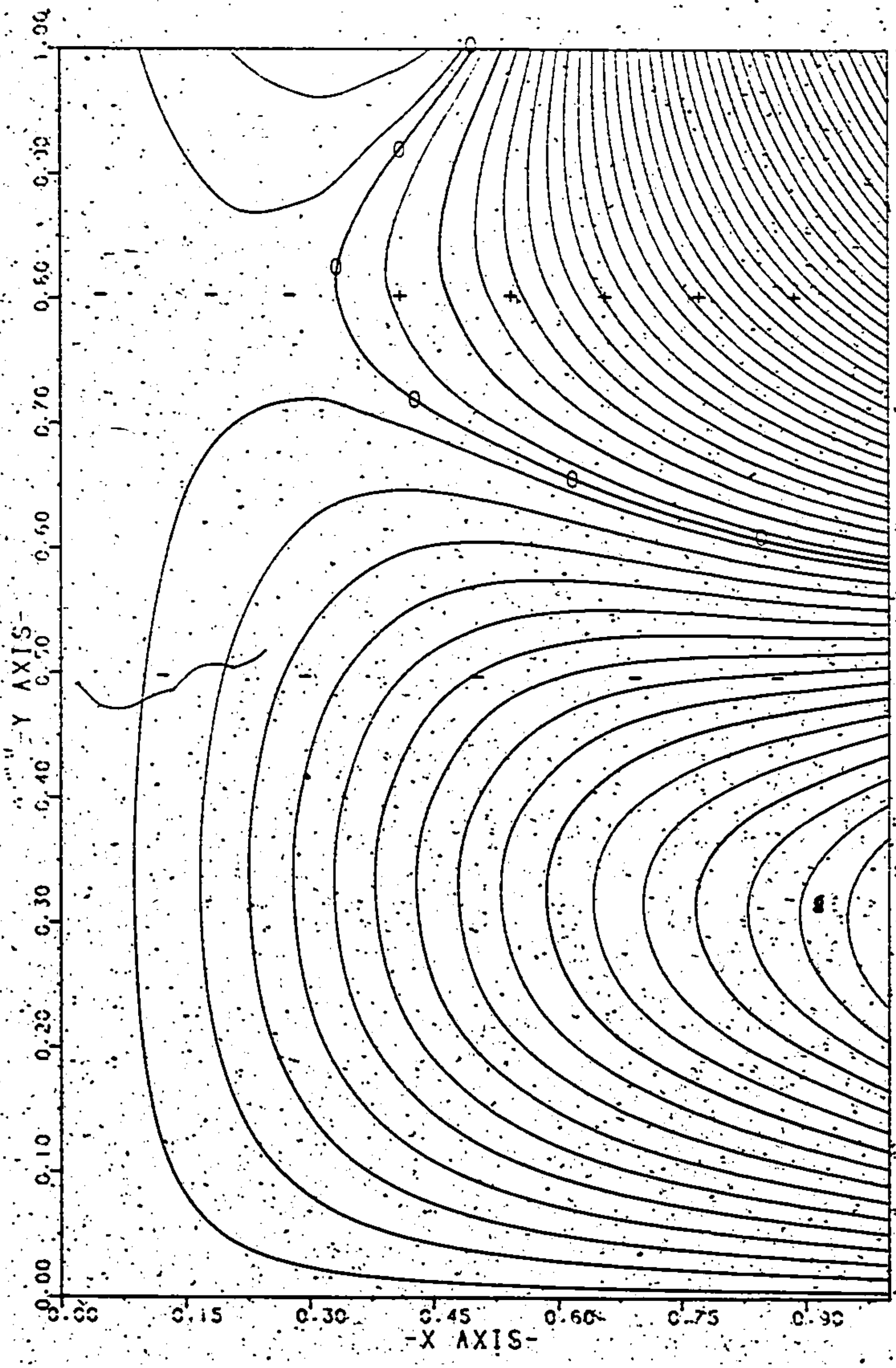
FIRST ANTISYMMETRIC MODE WITH PHI = 3/2. U = 1/2.



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

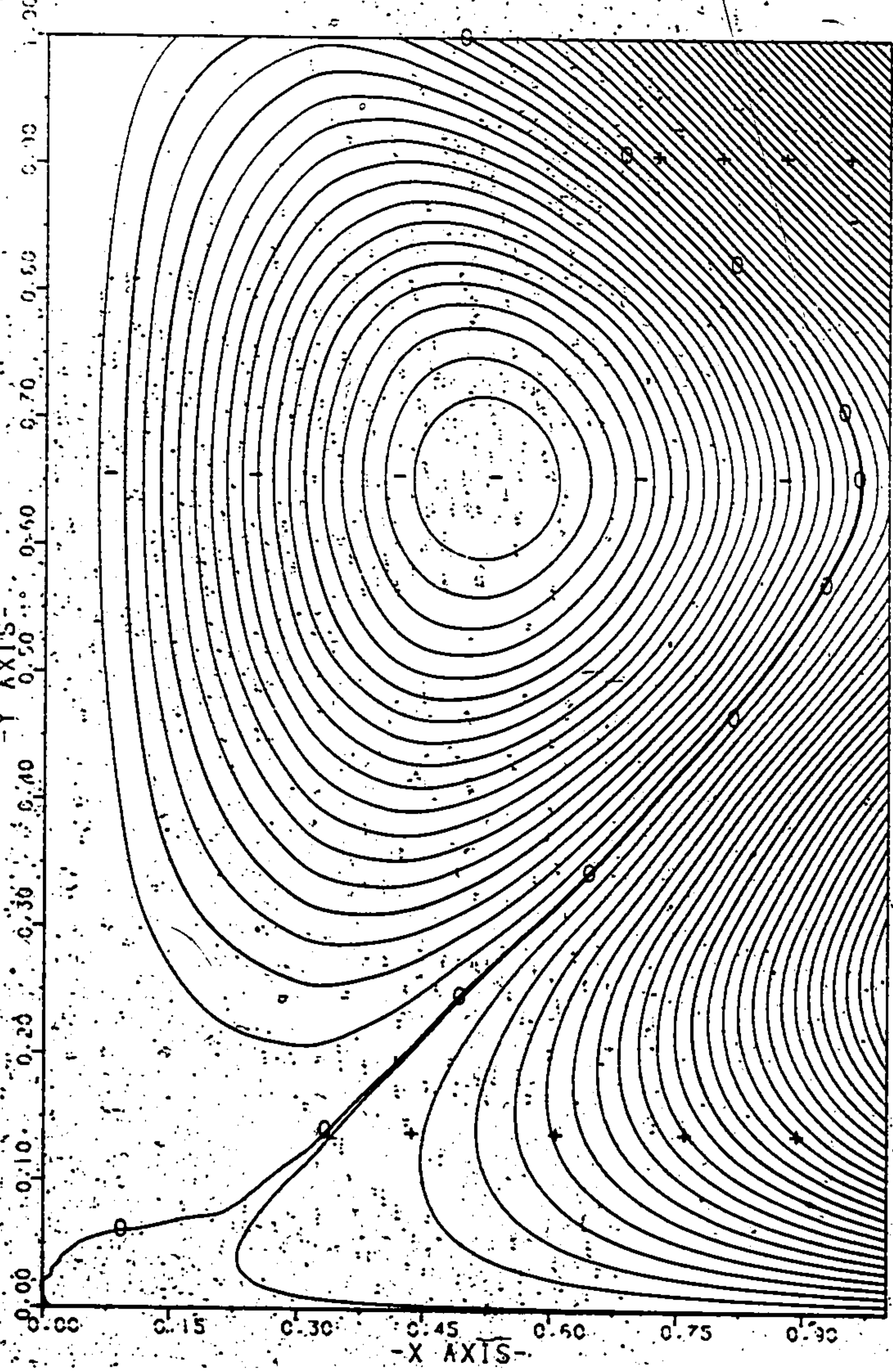
SECOND ANTISYMMETRIC MODE WITH PHI = 3/2, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

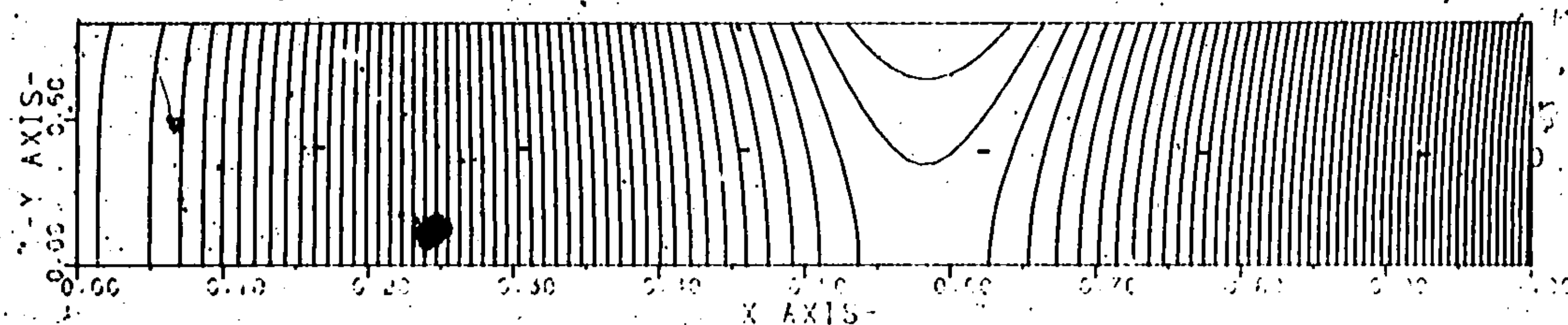
THIRD ANTISYMMETRIC MODE WITH PHI = 3/2, U = 1/2, V = 1/1



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

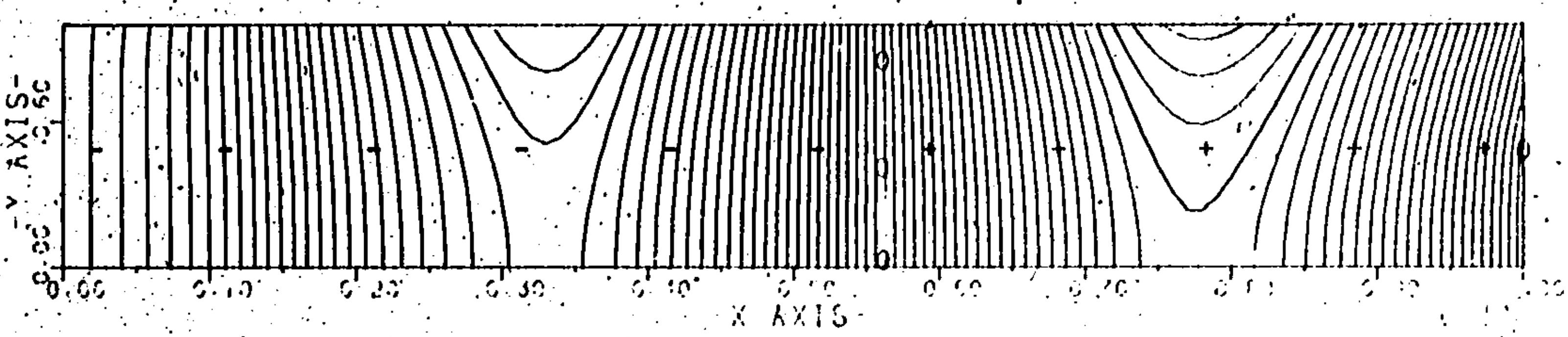
FIRST SYMMETRIC MODE WITH PHI = 1/6, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

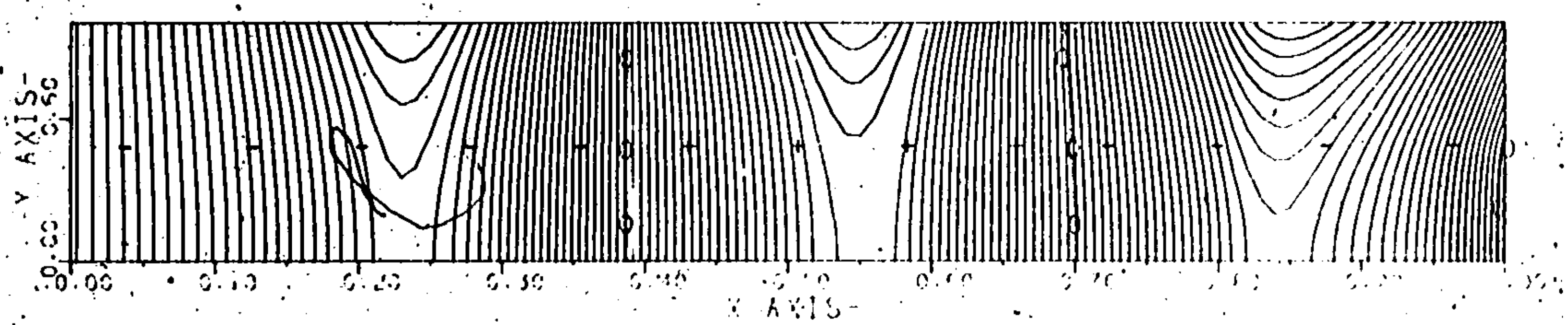
SECOND SYMMETRIC MODE WITH PHI = 1/6, U = ./1, V = ./2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ****

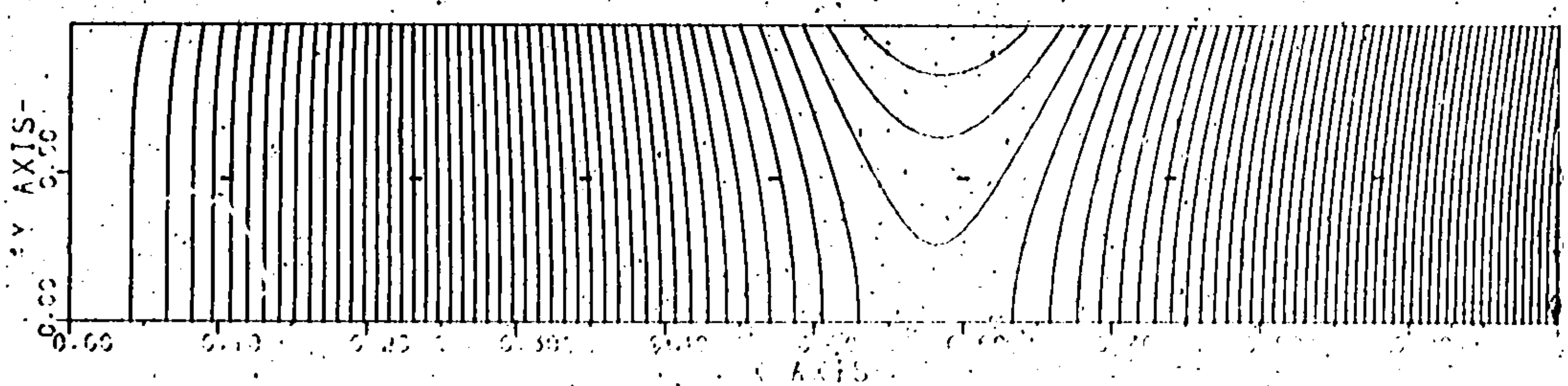
THIRD SYMMETRIC MODE WITH PHI = .1/6, U = .71, V = .72



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

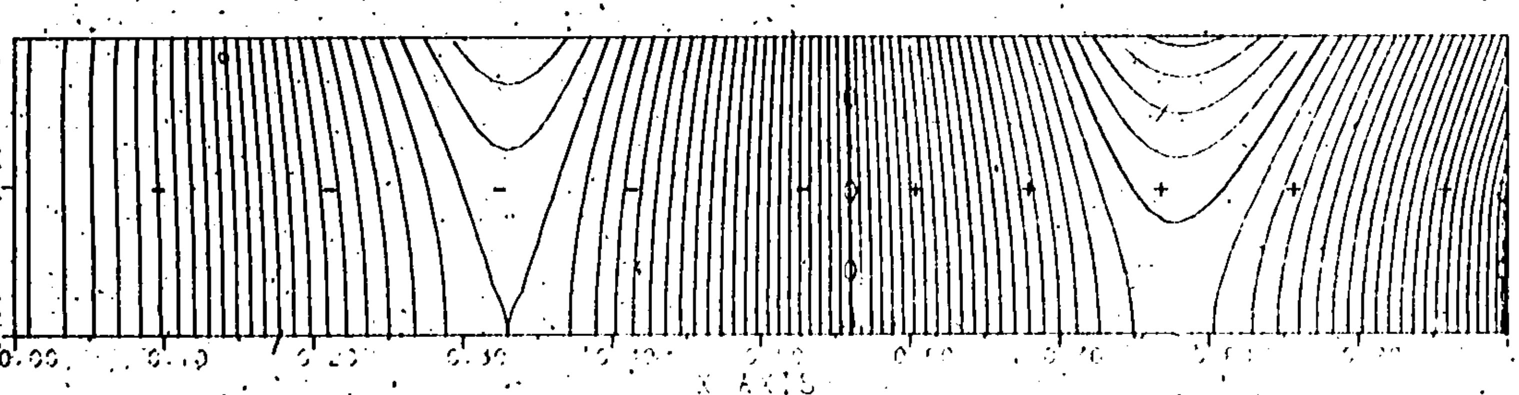
FIRST SYMMETRIC MODE WITH PHI = 1/5, U = 1/1, V = .7?



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

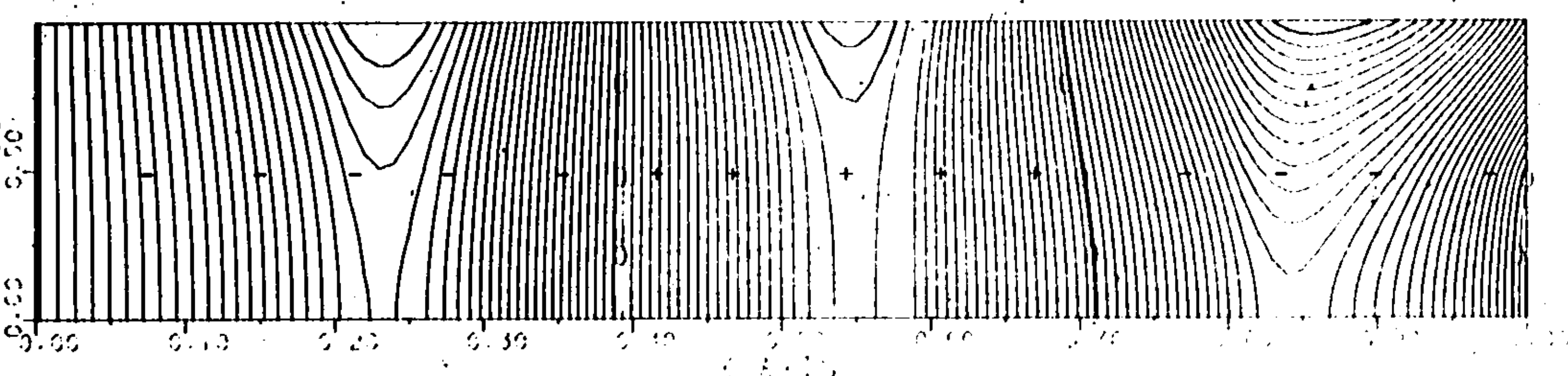
SECOND SYMMETRIC MODE WITH PHI = 1/5, U = 5/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE - POINT SUPPORT ***

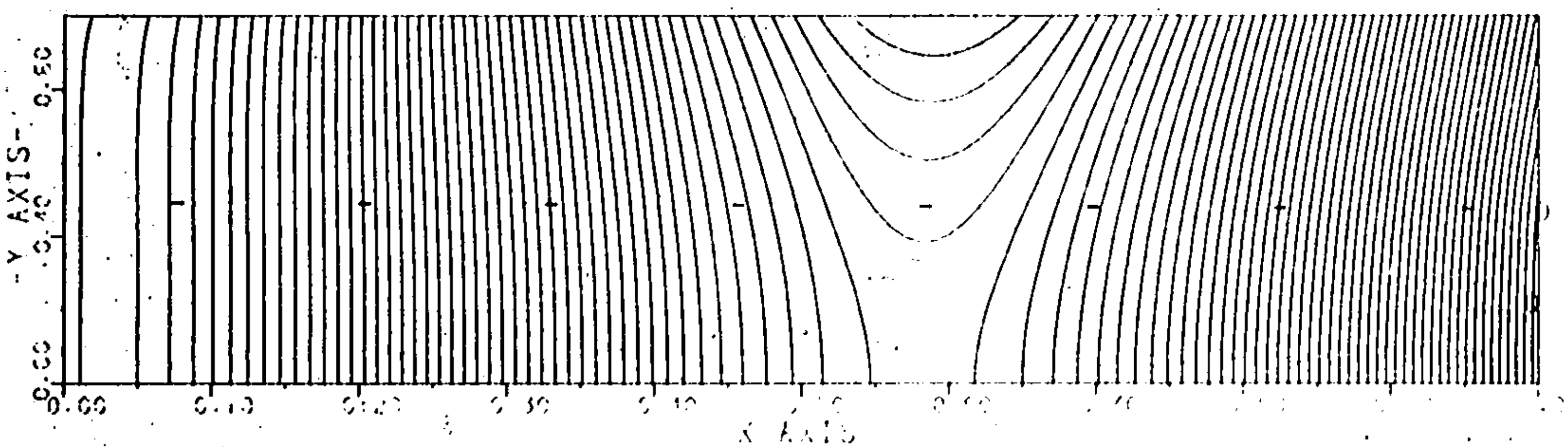
THIRD SYMMETRIC MODE WITH PHI = 1/5, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

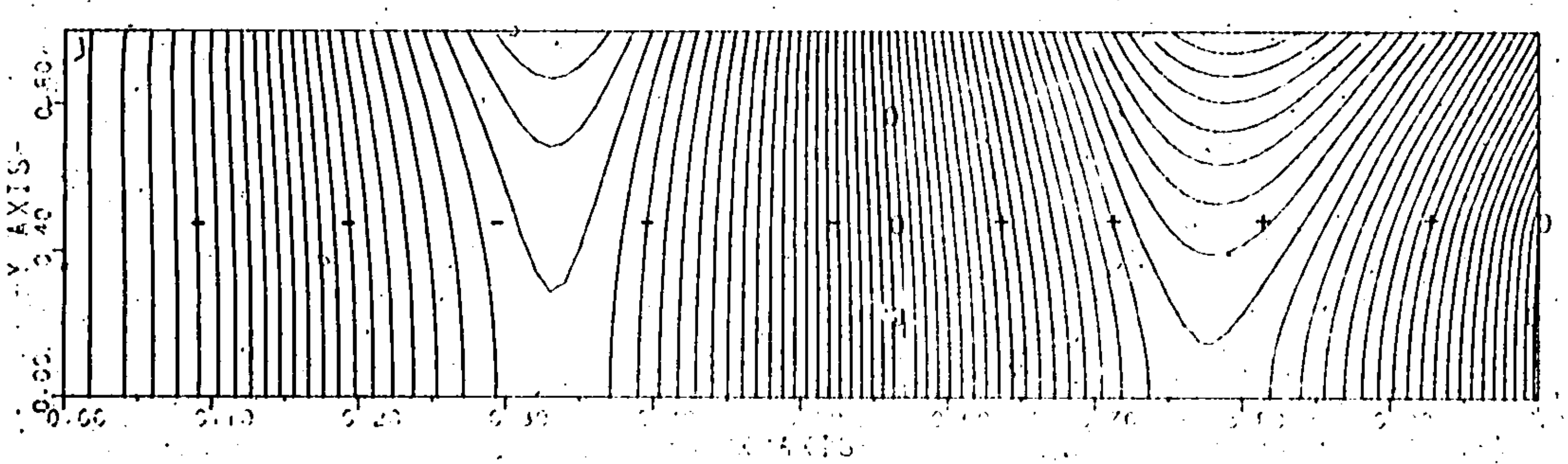
FIRST SYMMETRIC MODE WITH PHI = 1/4, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

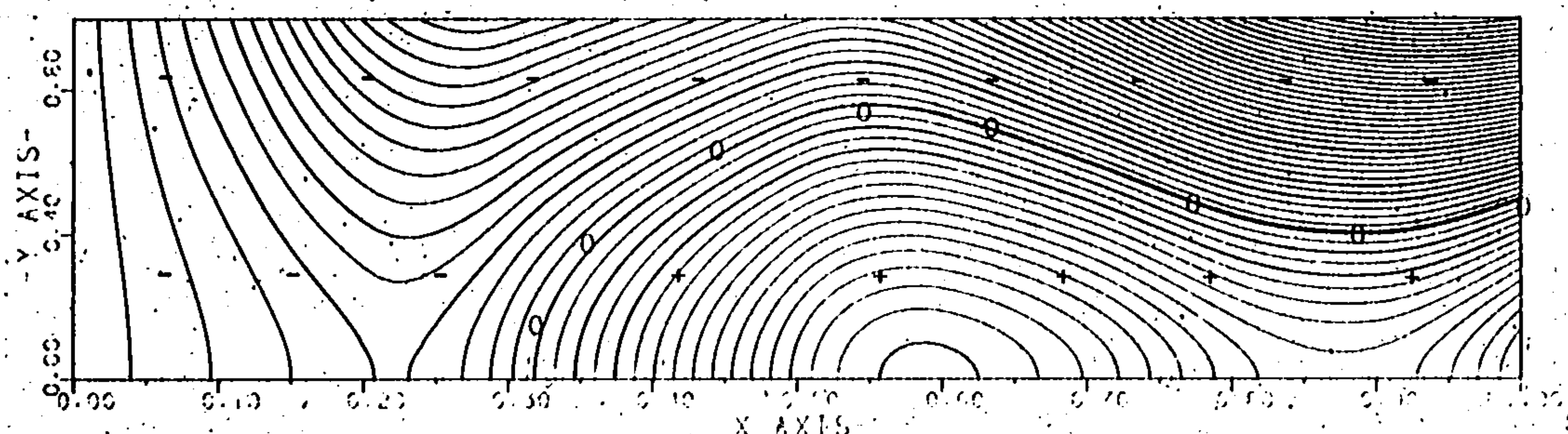
SECOND SYMMETRIC MODE WITH PHI = 1/4, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

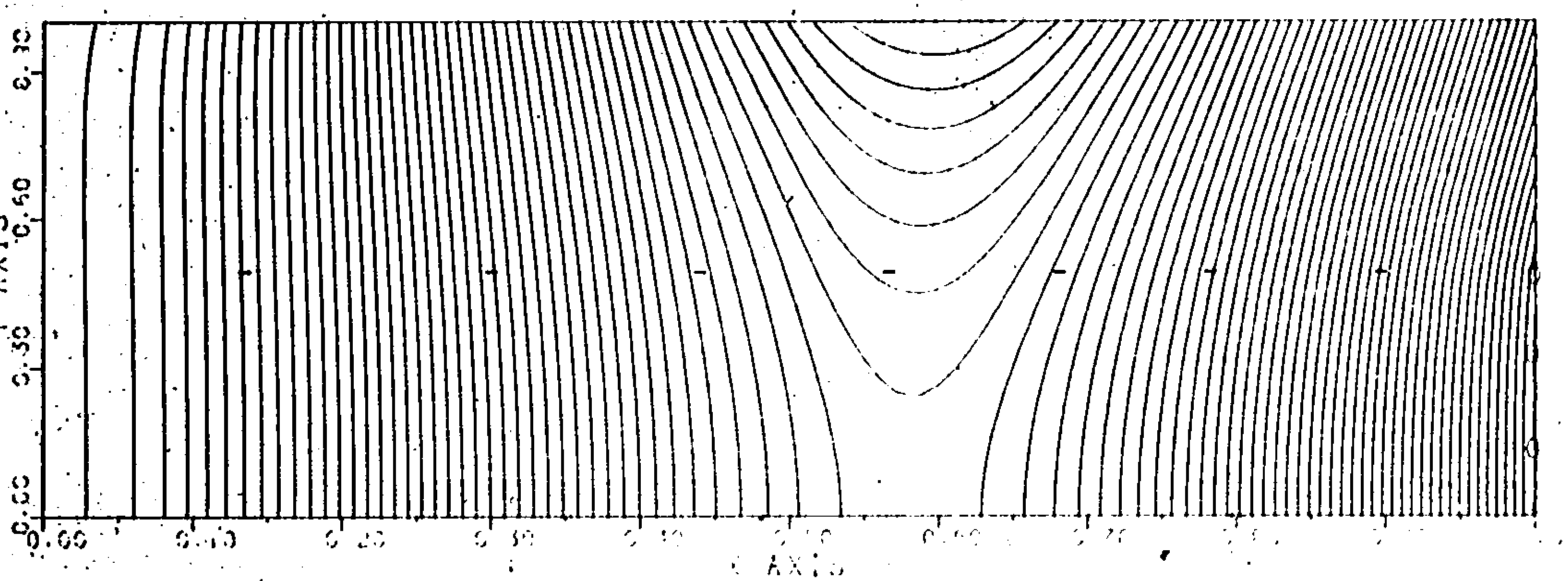
THIRD SYMMETRIC MODE WITH PHI = 1/4, U = [illegible], V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE [illegible] POINT SUPPORT ***

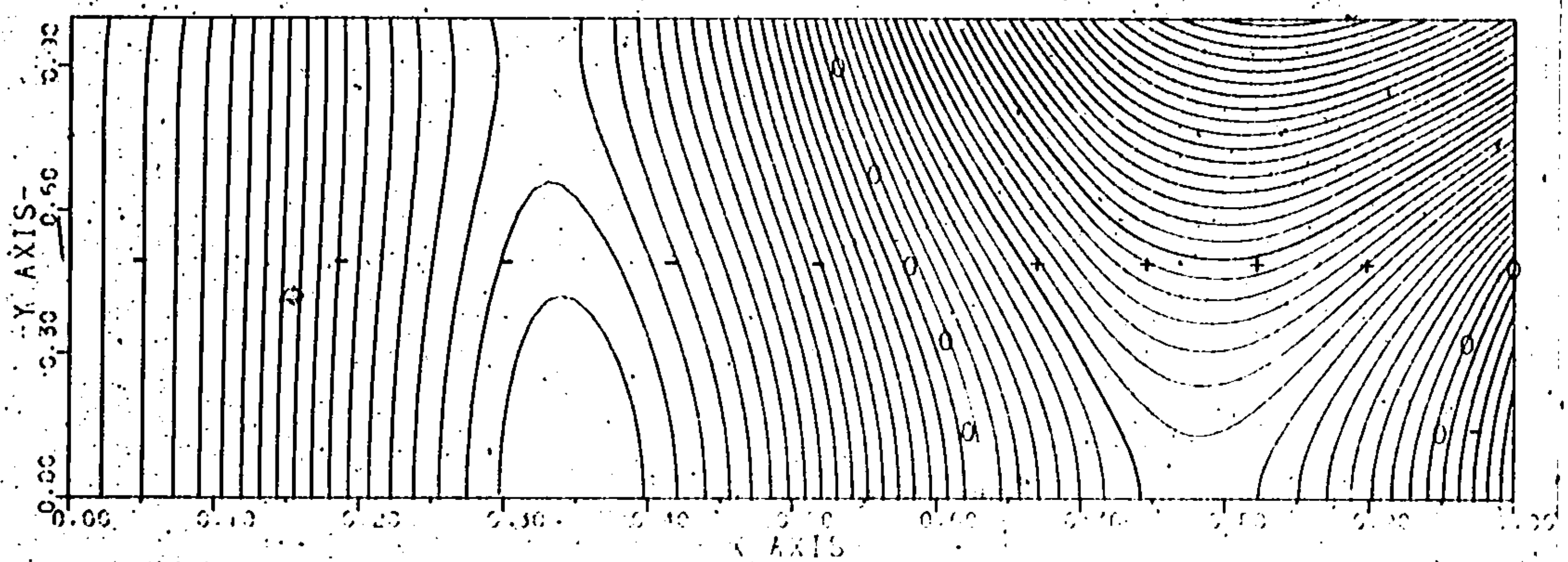
FIRST SYMMETRIC MODE WITH PHI = 1/3, [illegible] V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

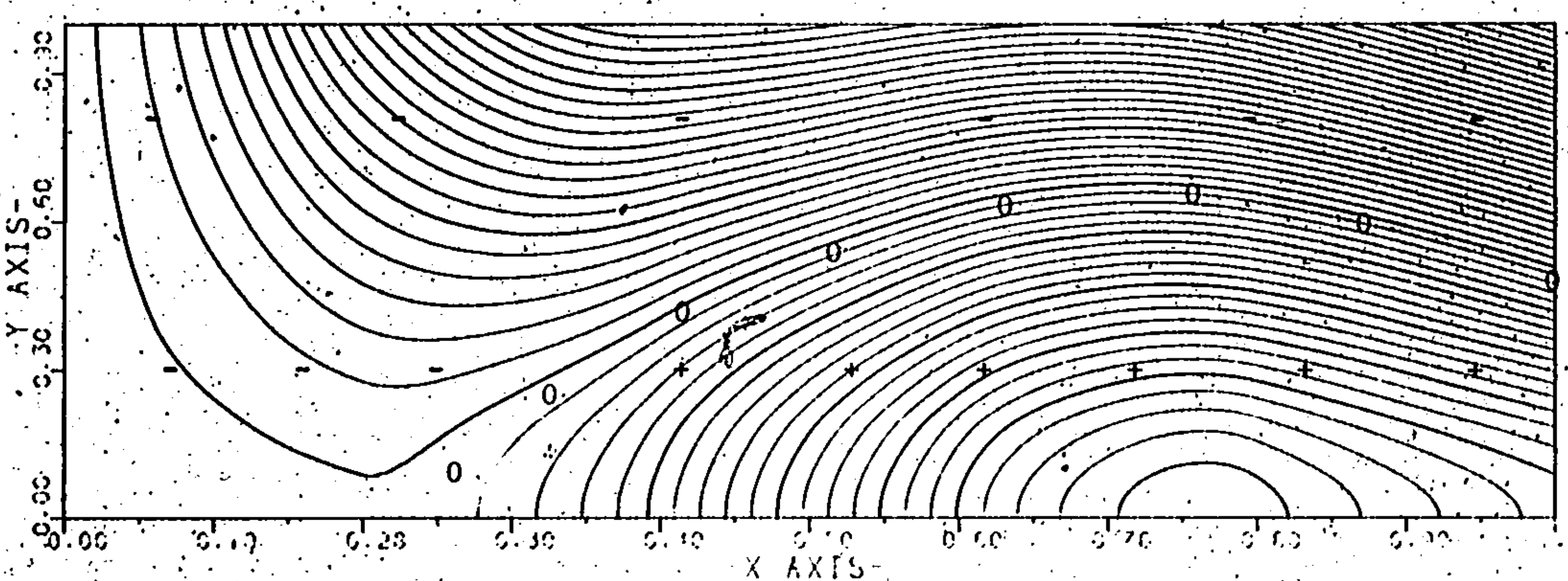
SECOND SYMMETRIC MODE WITH PHI = 1/3 U = 1/1, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

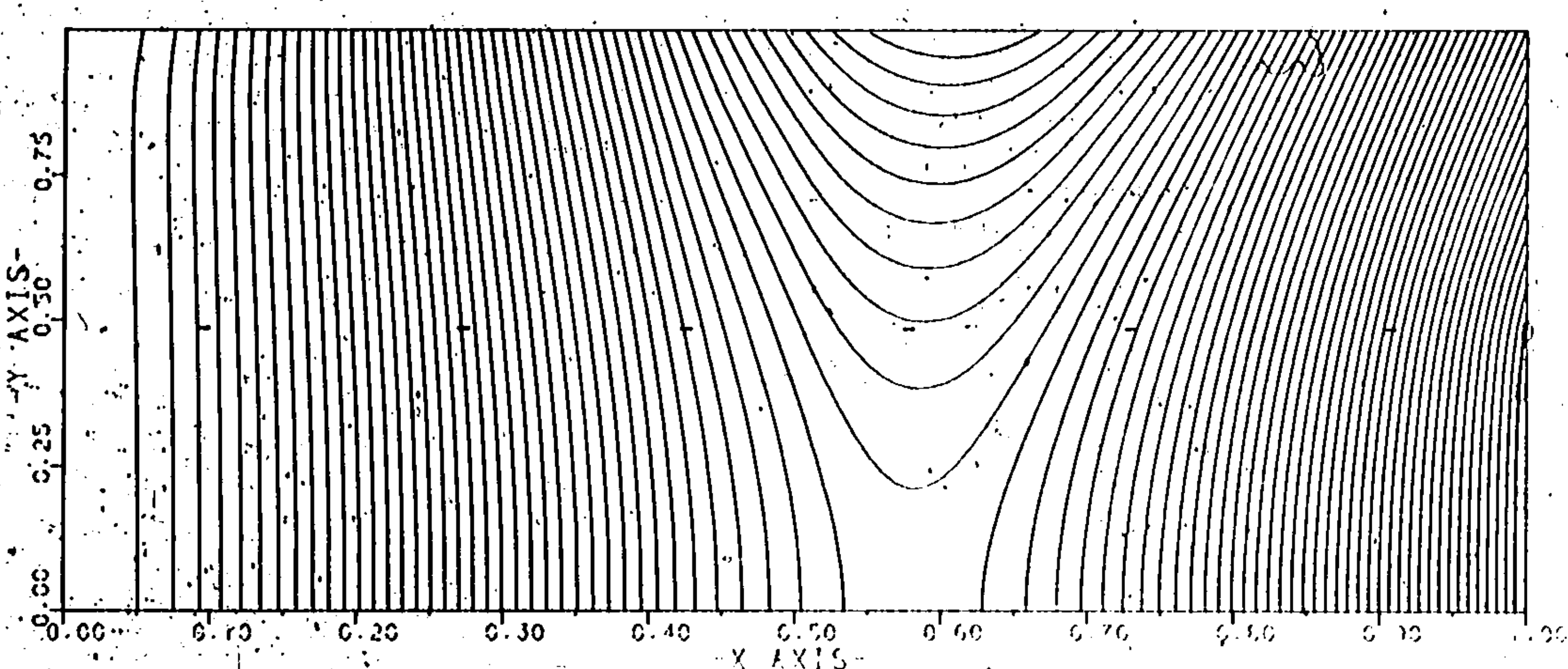
THIRD SYMMETRIC MODE WITH PHI = 1/3, C = .71, W = .72



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

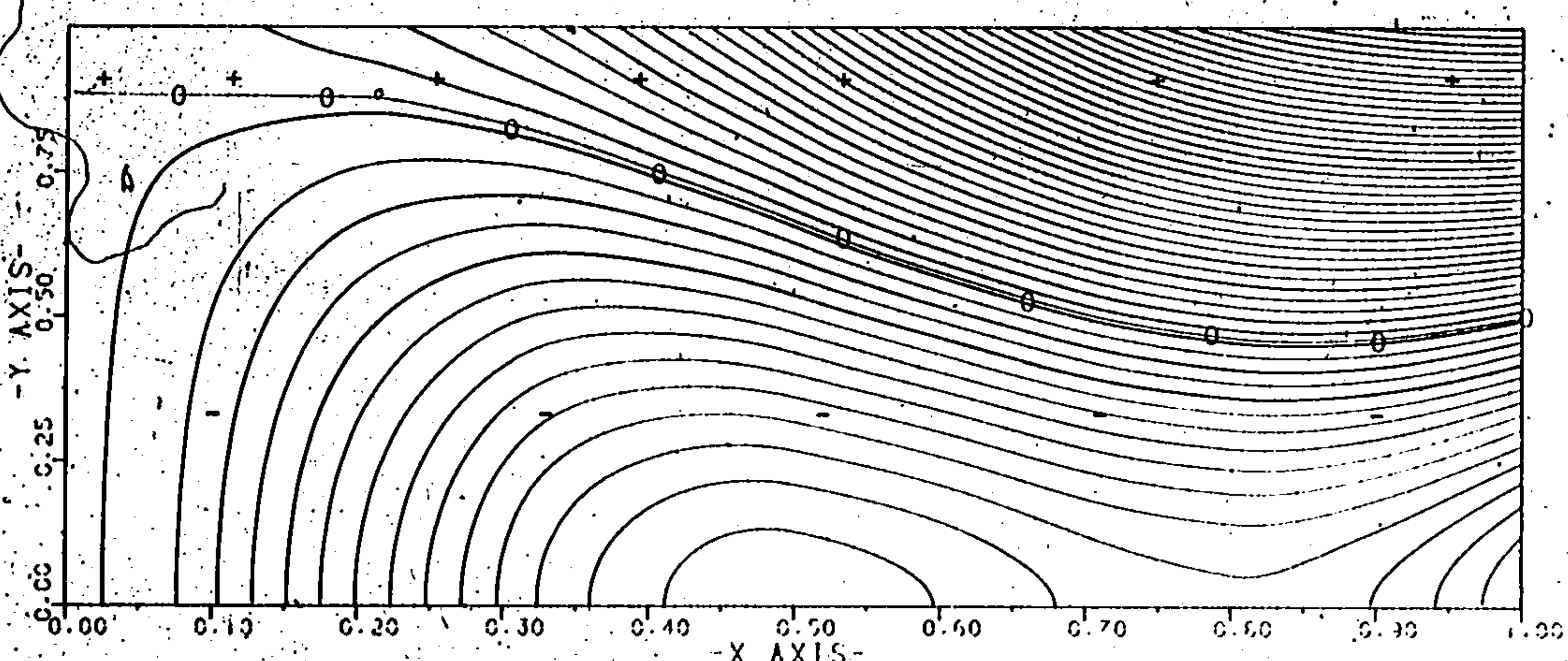
FIRST SYMMETRIC MODE WITH PHI=1/2.5 U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

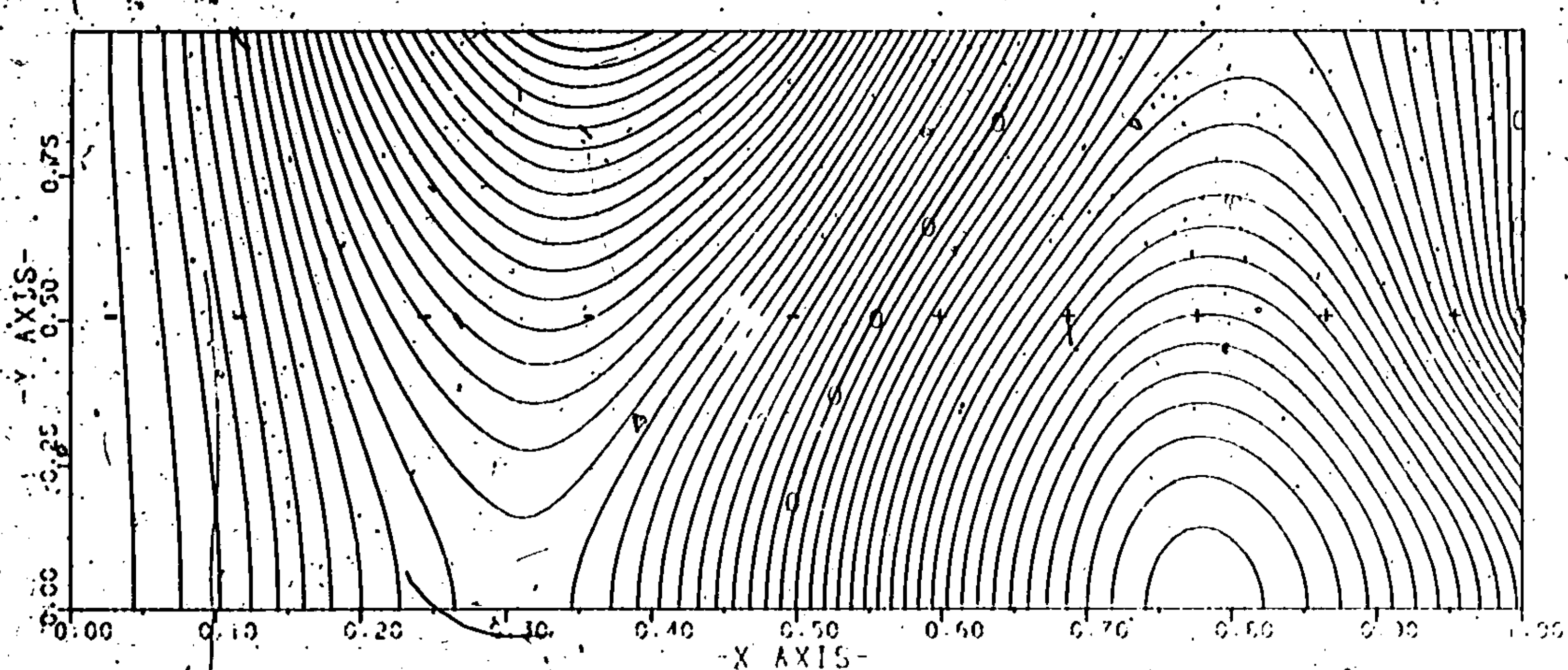
SECOND SYMMETRIC MODE WITH PHI-1/2.5, U - 1/1, V - 1/2



WAVE PLATE GRAPHIC

••• THE CANTIDEVER PLATE WITH EDGE POINT SUPPORT •••

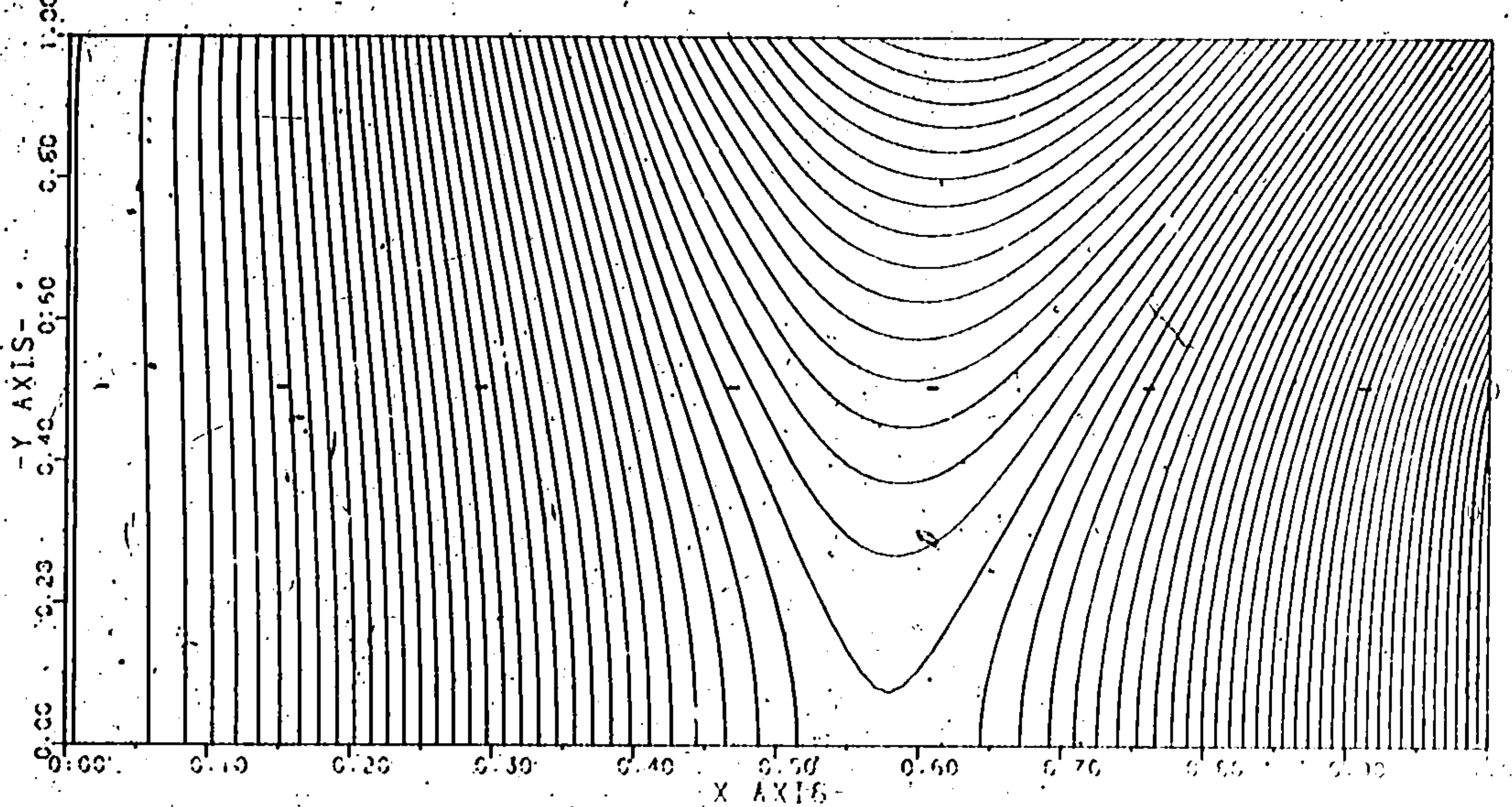
THIRD SYMMETRIC MODE WITH PHI=1/2.5, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

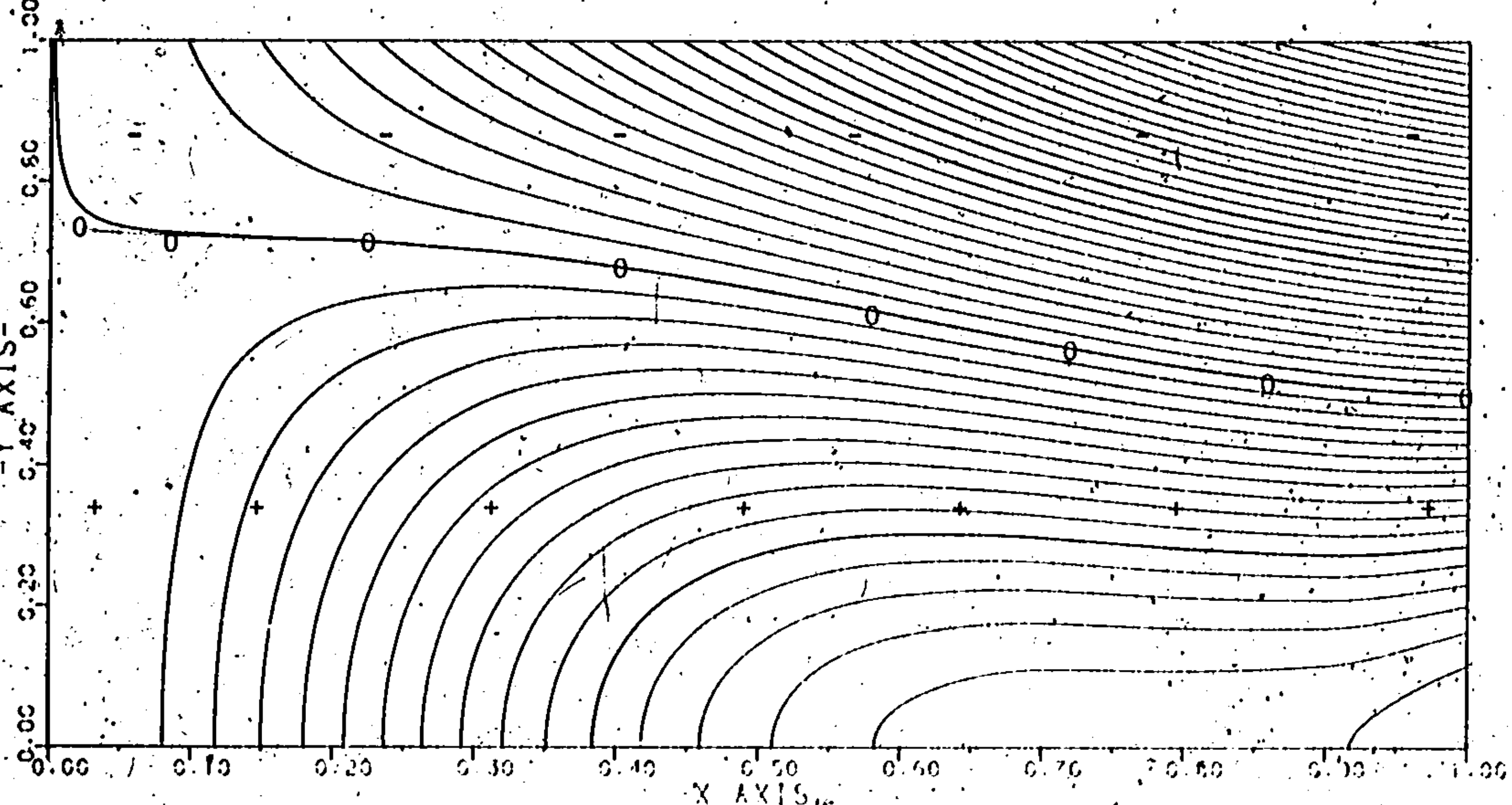
FIRST SYMMETRIC MODE WITH PHI = 1/2, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

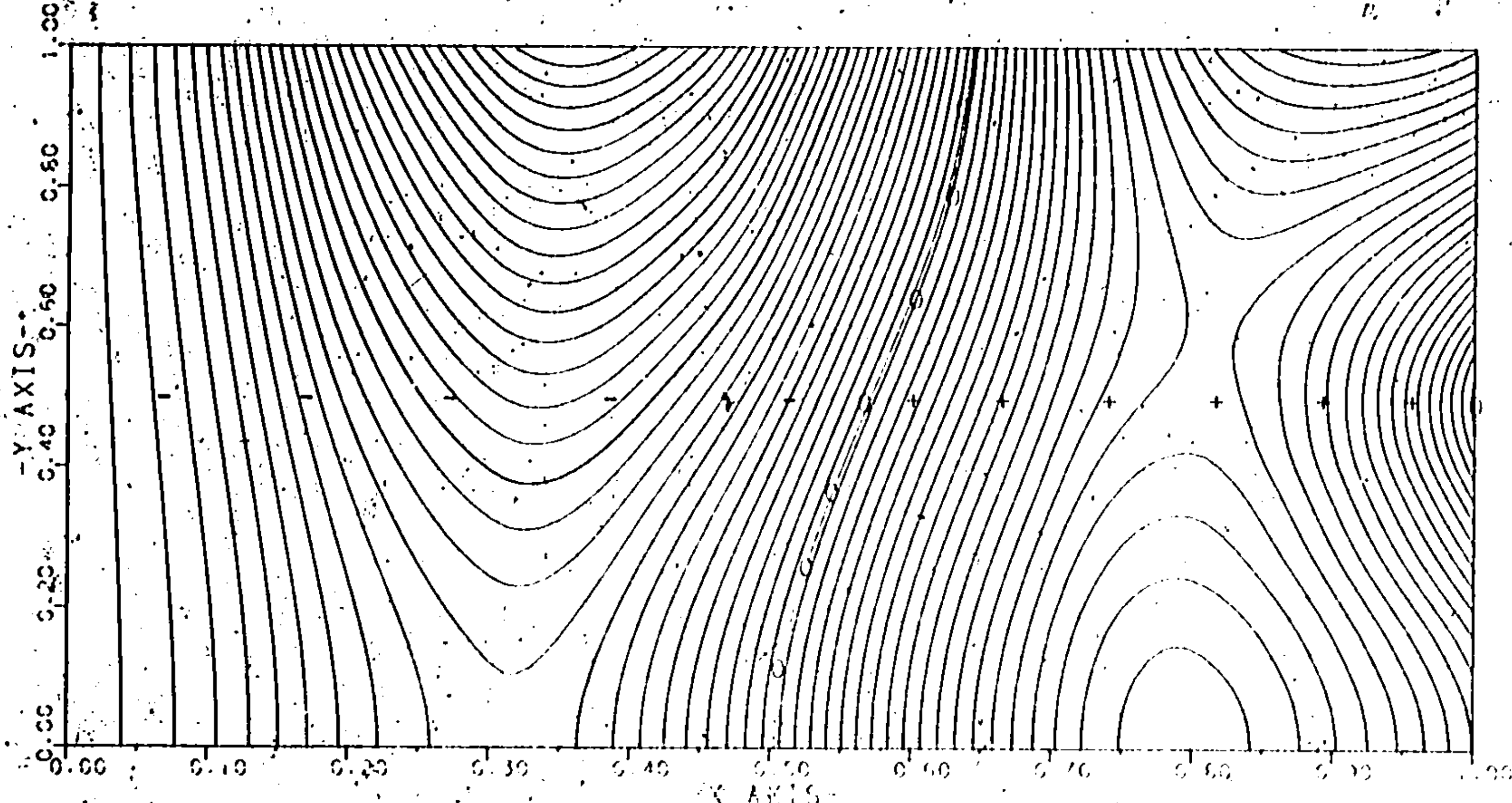
SECOND SYMMETRIC MODE WITH PHI = 1/2, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

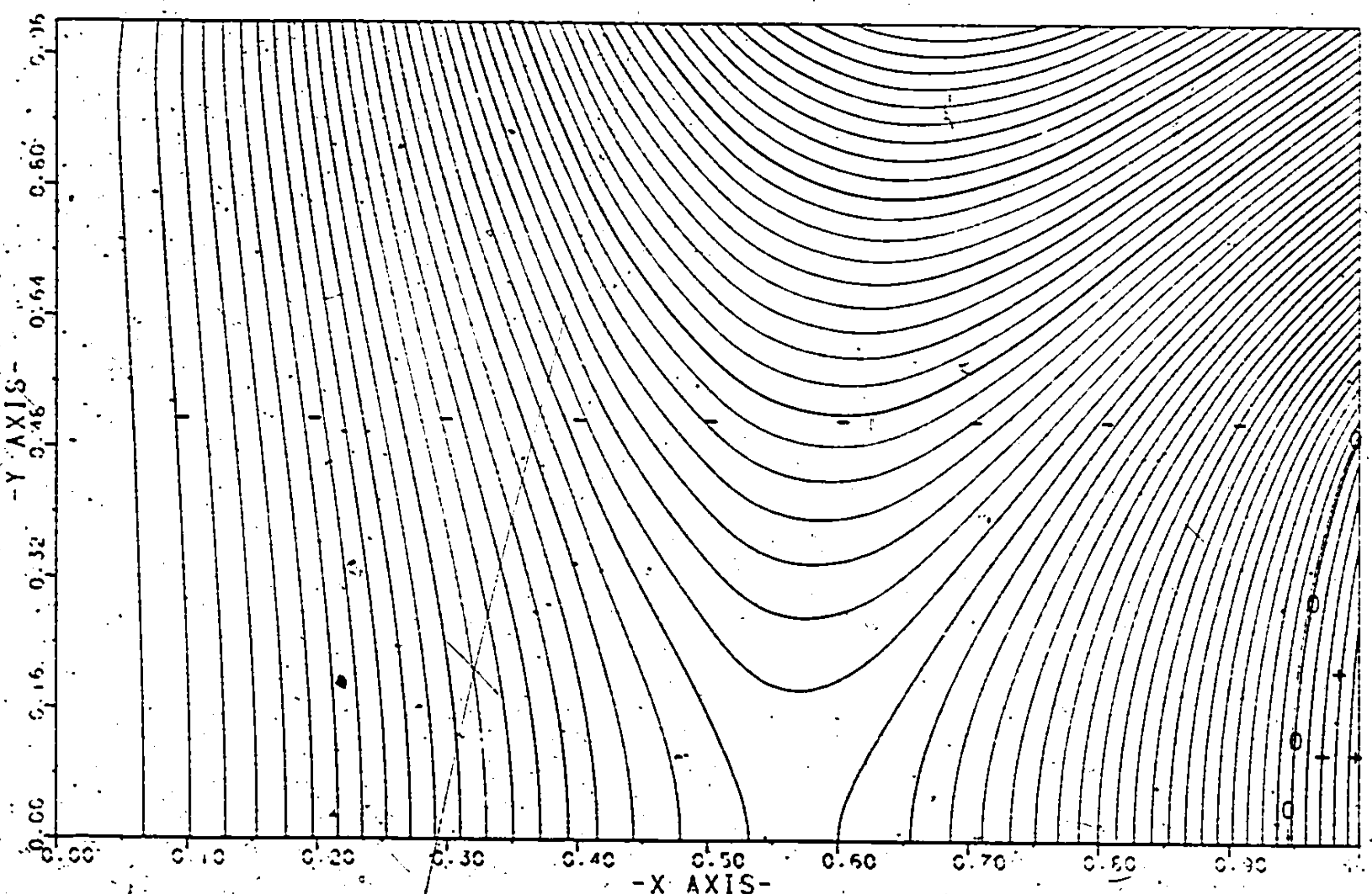
THIRD SYMMETRIC MODE WITH PHI = 1/2, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

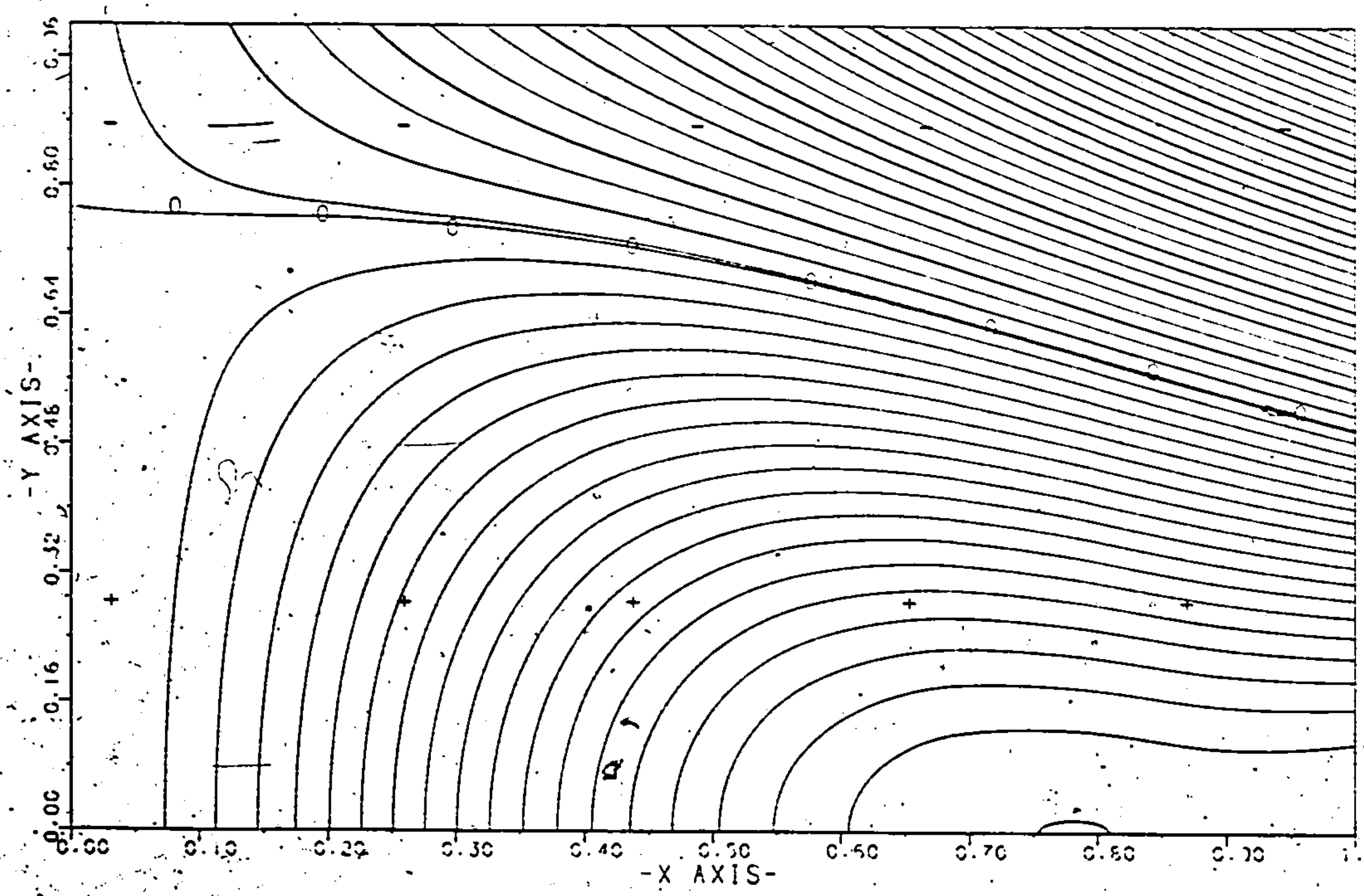
FIRST SYMMETRIC MODE WITH PHI - 5/8. U - 1/1. V - [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

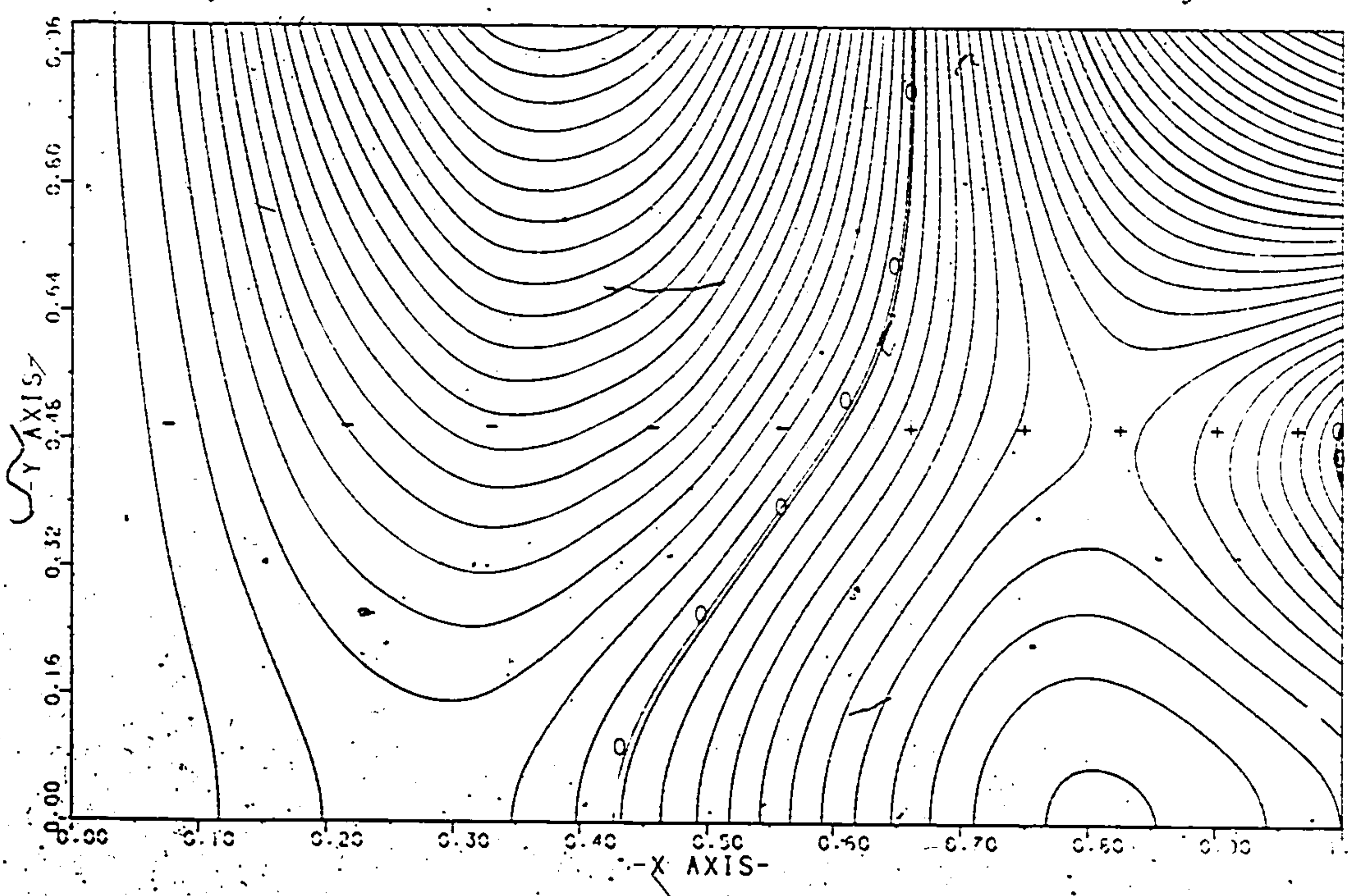
SECOND SYMMETRIC MODE WITH PHI - 5/8, U - ..., V - ./2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

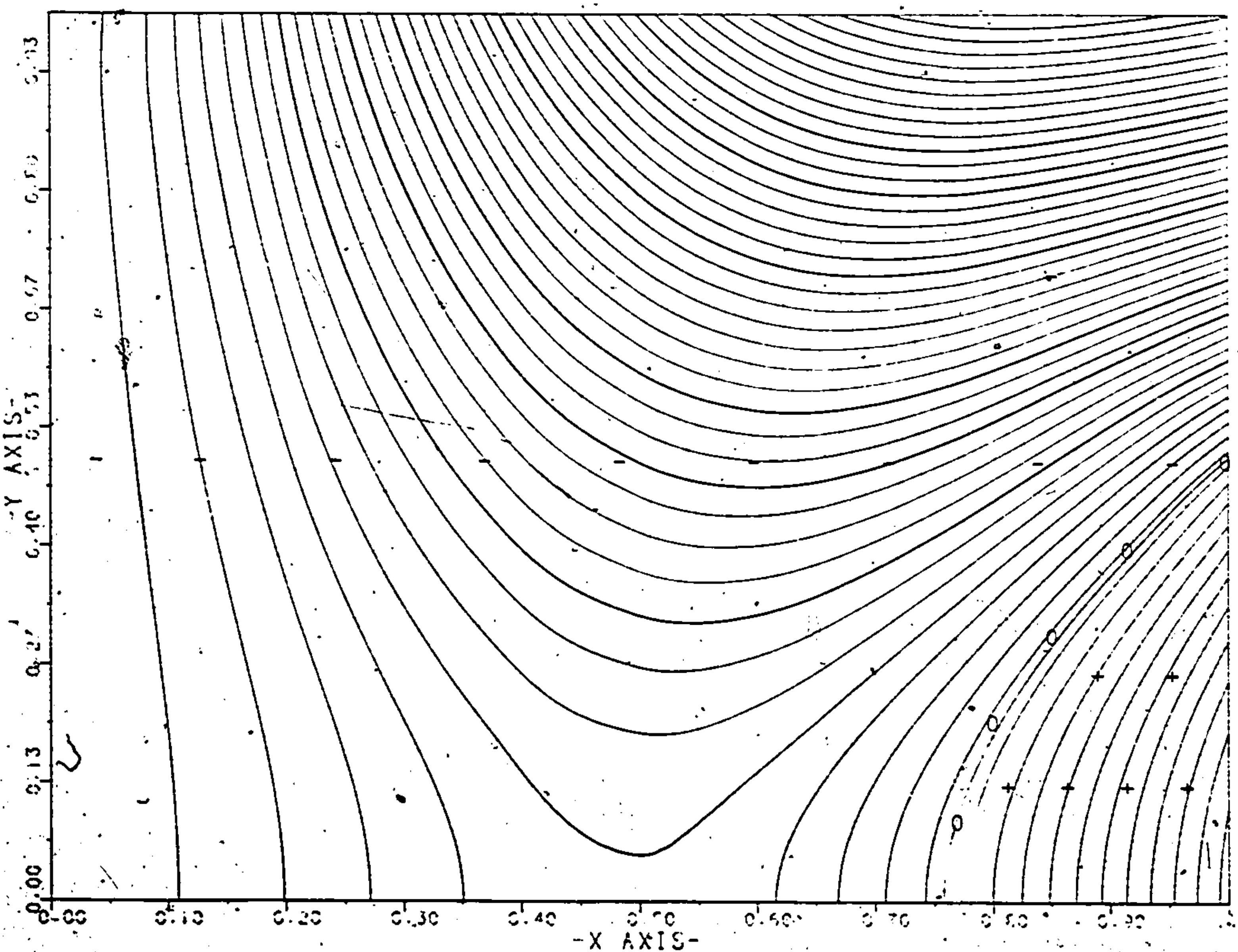
THIRD SYMMETRIC MODE WITH PHI = 5/8, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

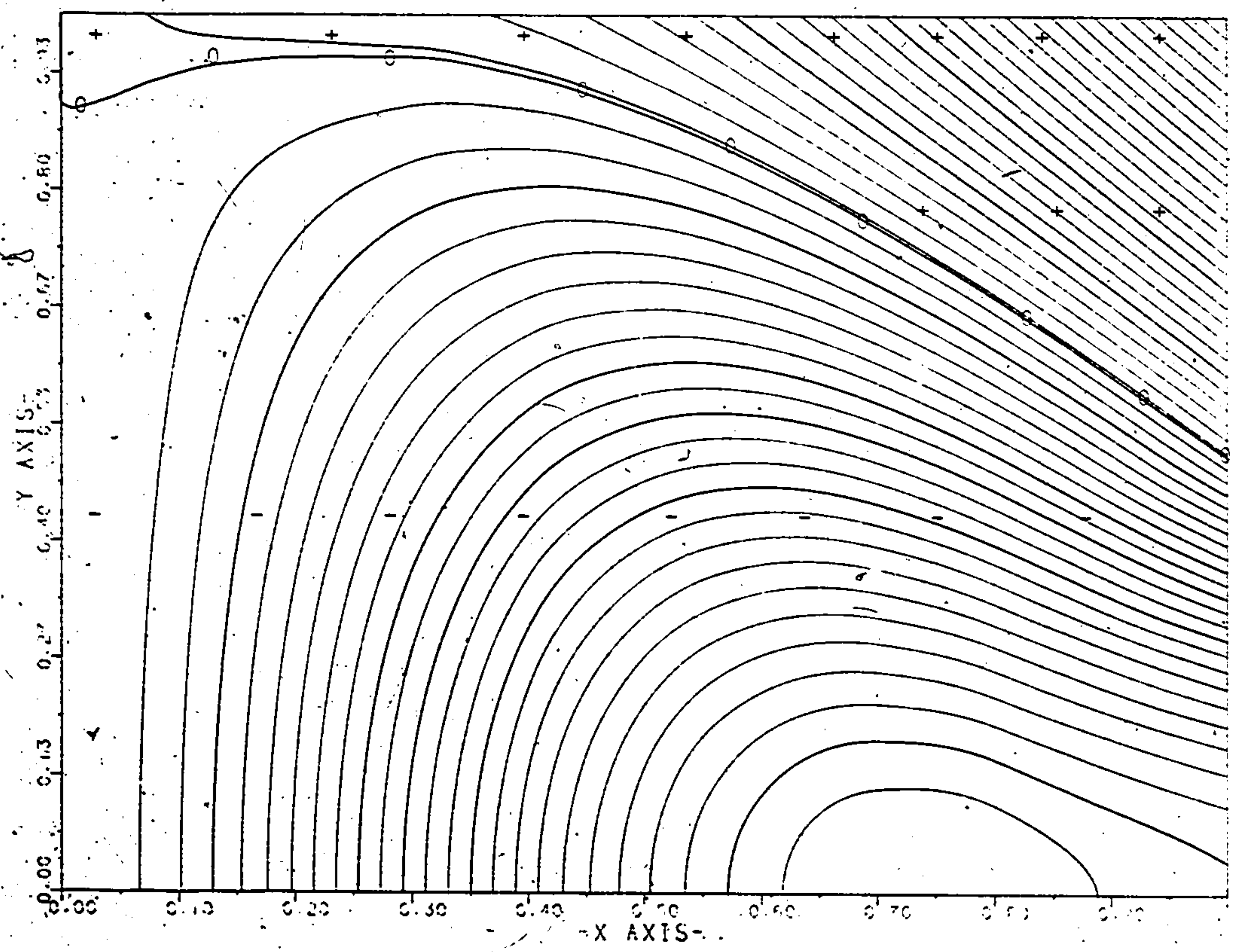
FIRST SYMMETRIC MODE WITH PHI = 3.4 ...



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE- POINT SUPPORT ***

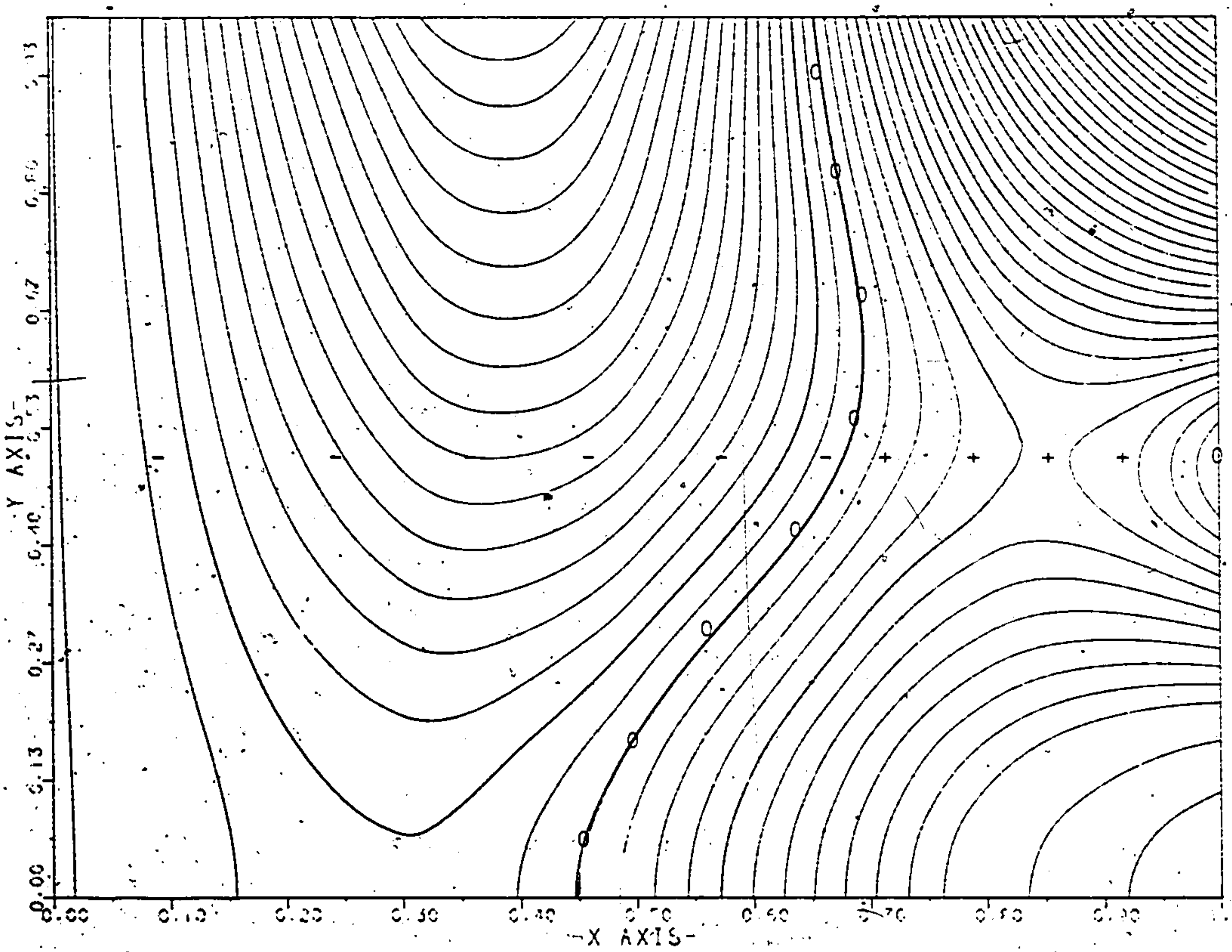
SECOND SYMMETRIC MODE WITH PHI = 3/4, U = [illegible], V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

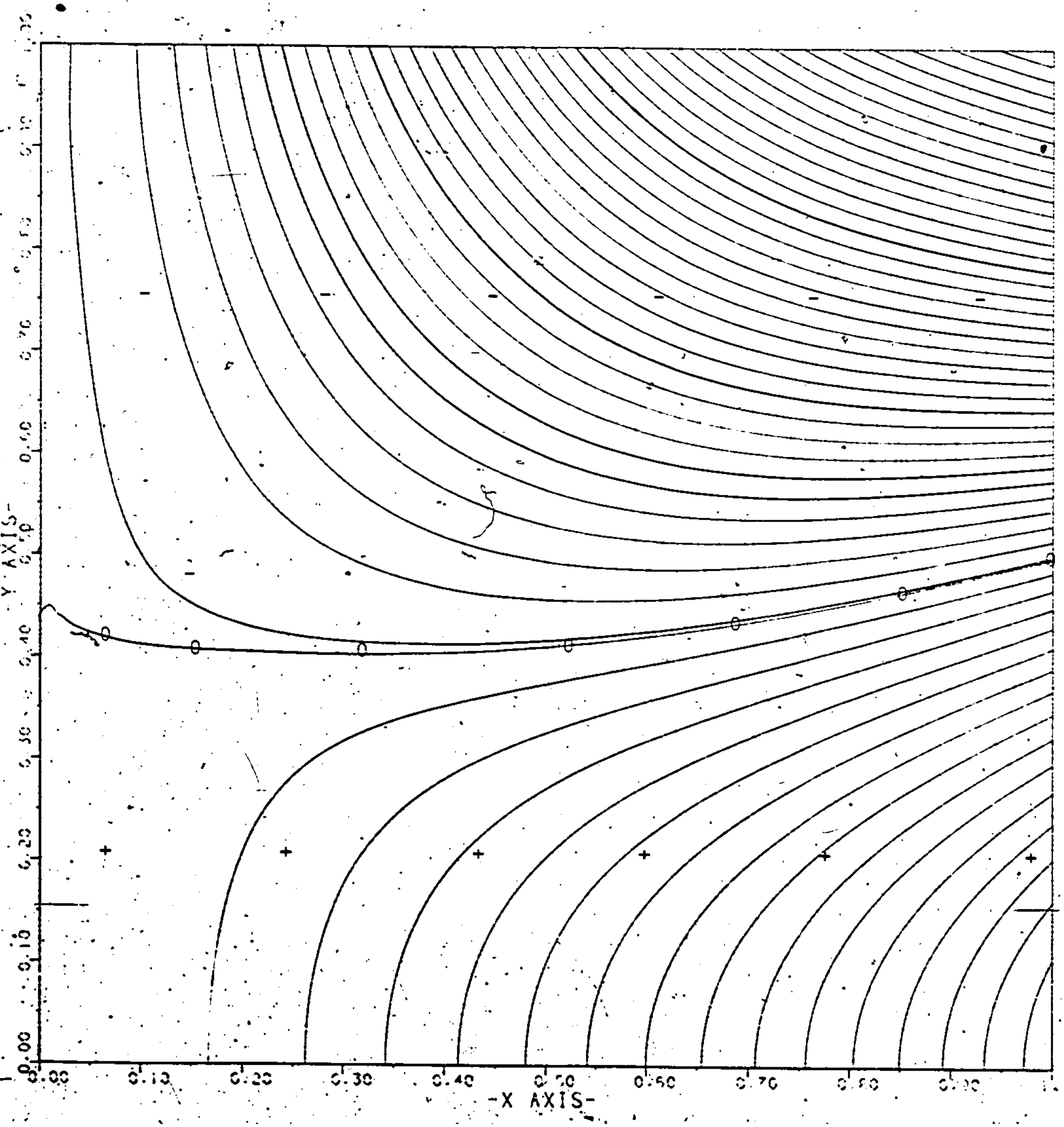
THIRD SYMMETRIC MODE WITH PHI = 3/4, [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH [illegible] POINT SUPPORT ***

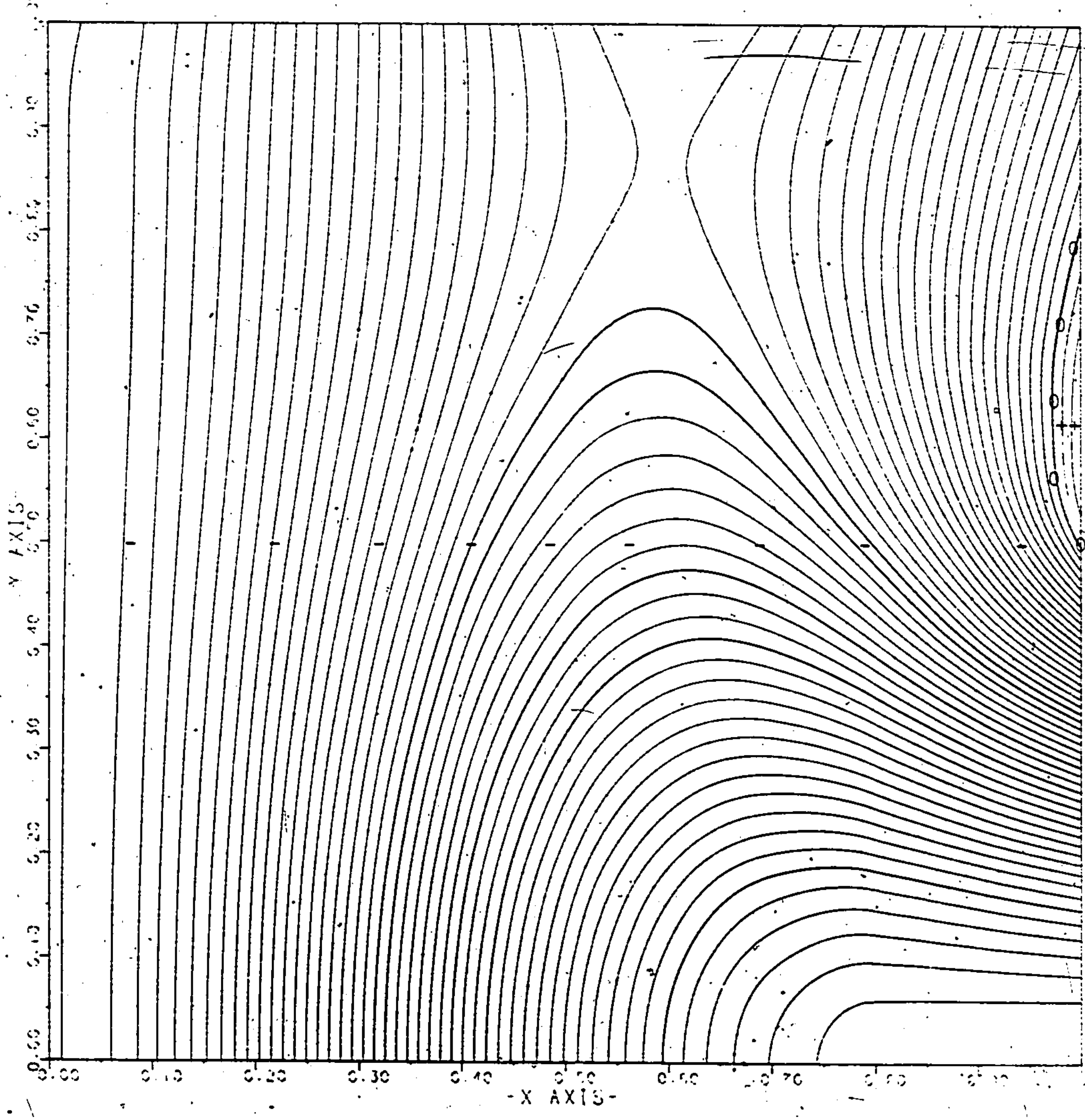
FIRST SYMMETRIC MODE WITH PHI - [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

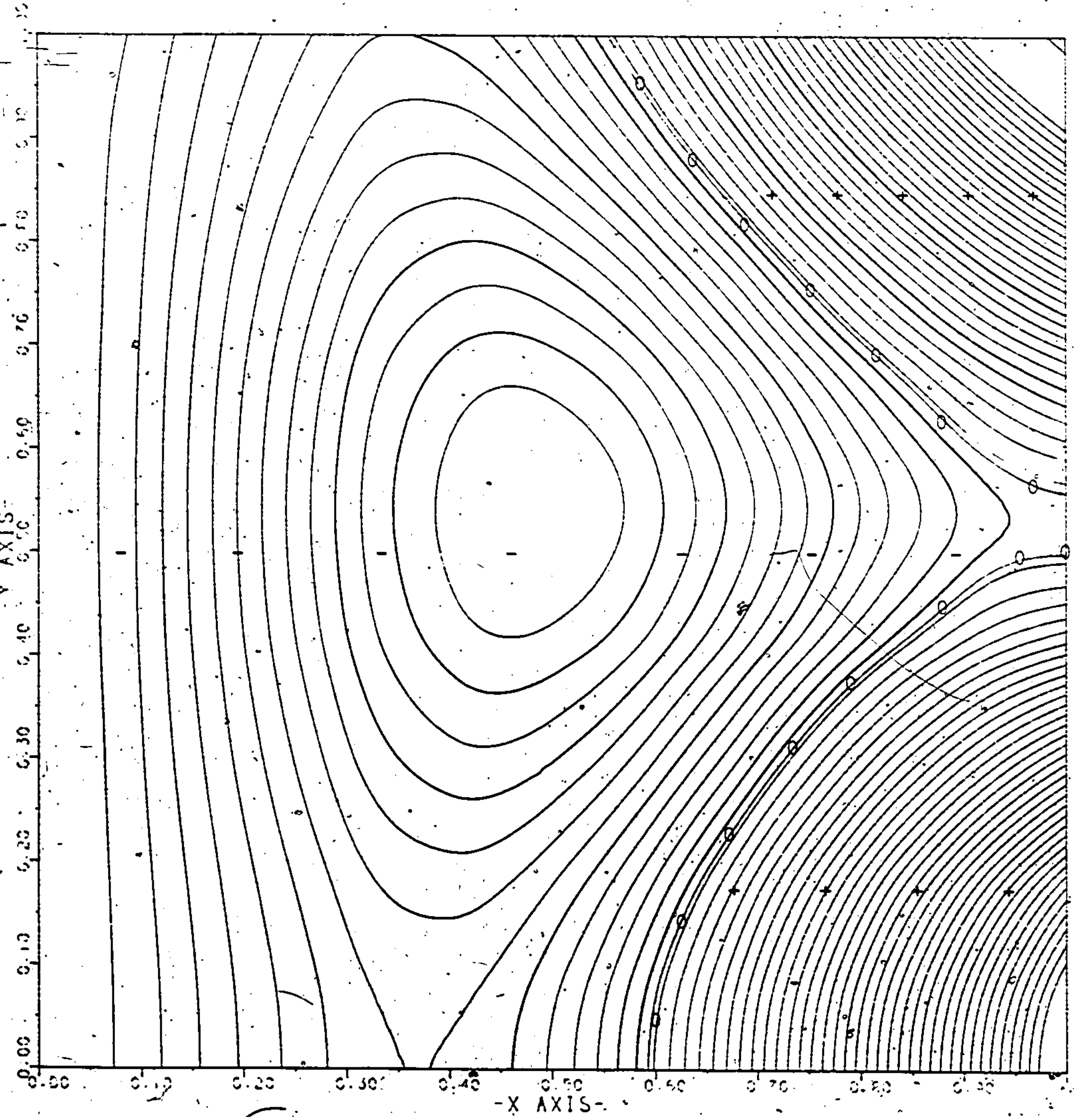
SECOND SYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

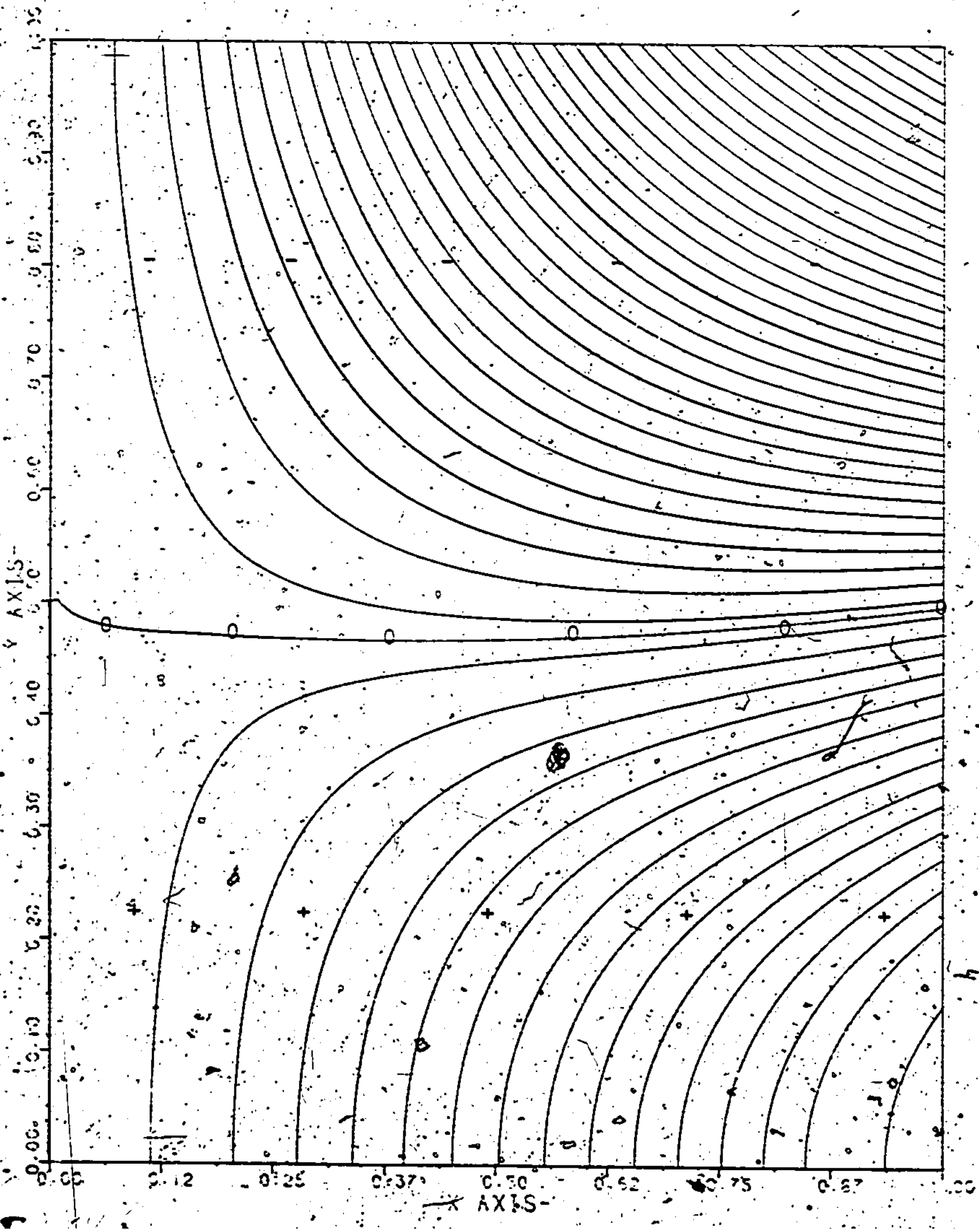
*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH PHI = [illegible]



*** THE CANTILEVER PLATE WITH ... POINT SUPPORT ***

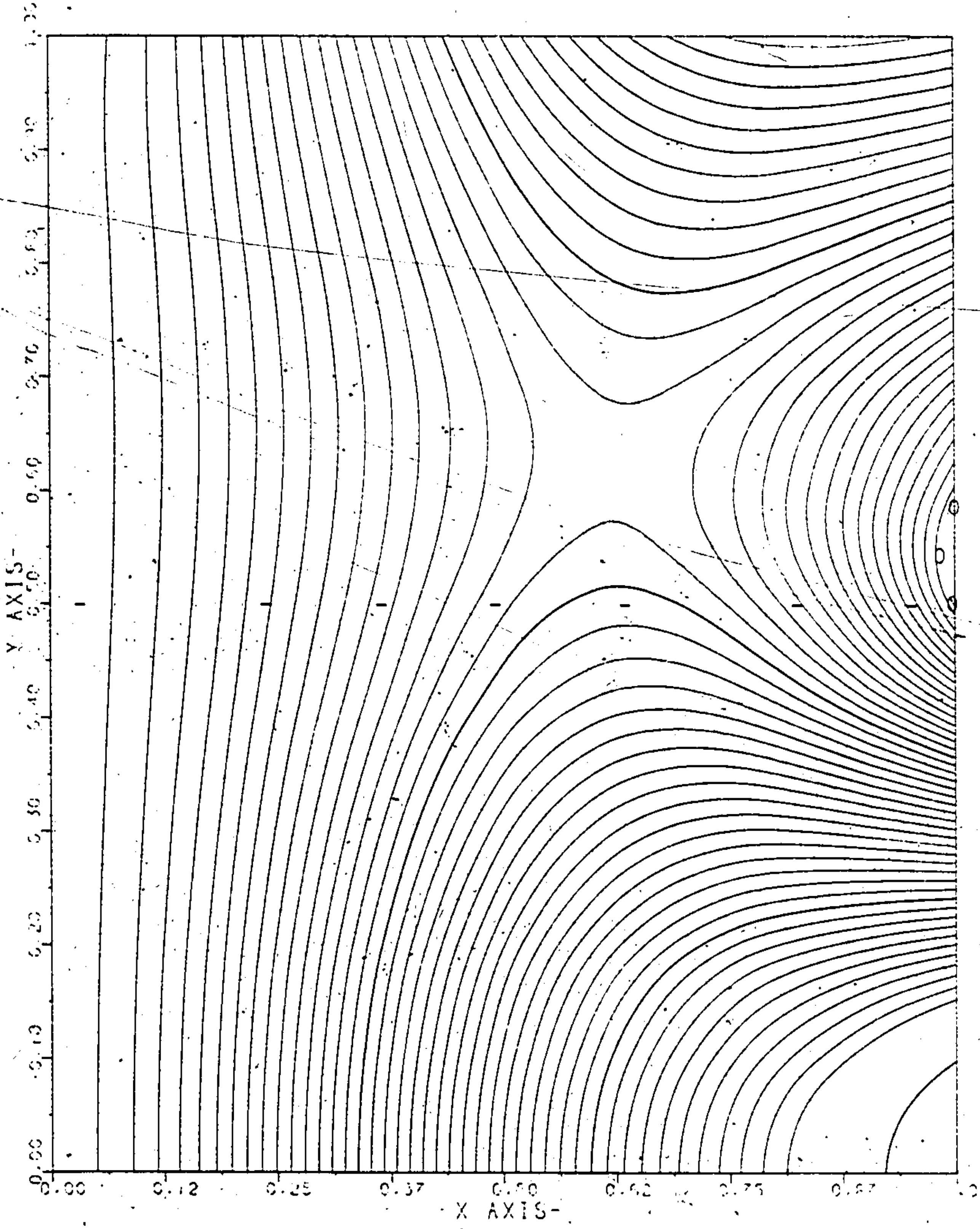
FIRST SYMMETRIC MODE WITH PHI = .25 ...



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

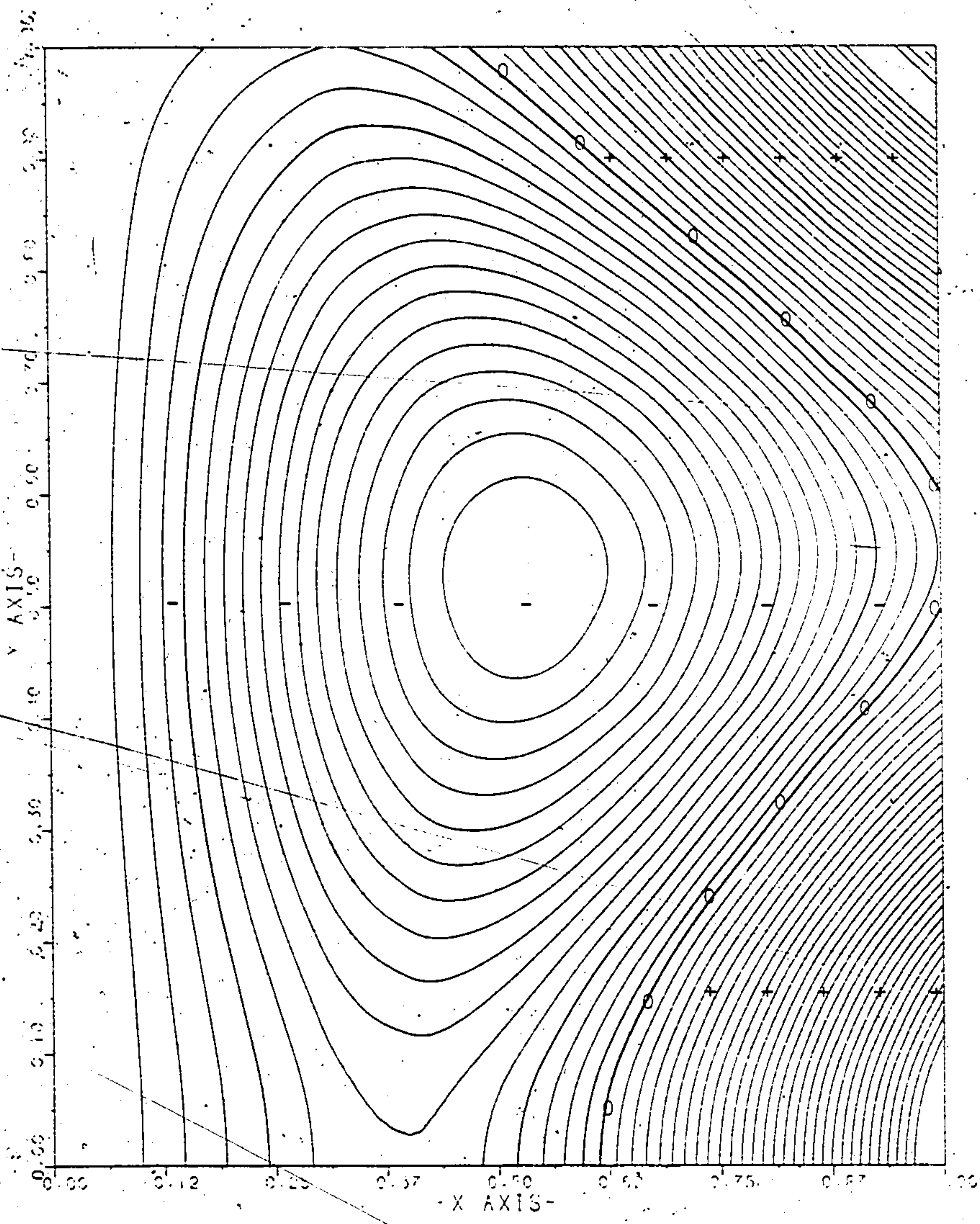
SECOND SYMMETRIC MODE WITH PHI= 0.25 [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

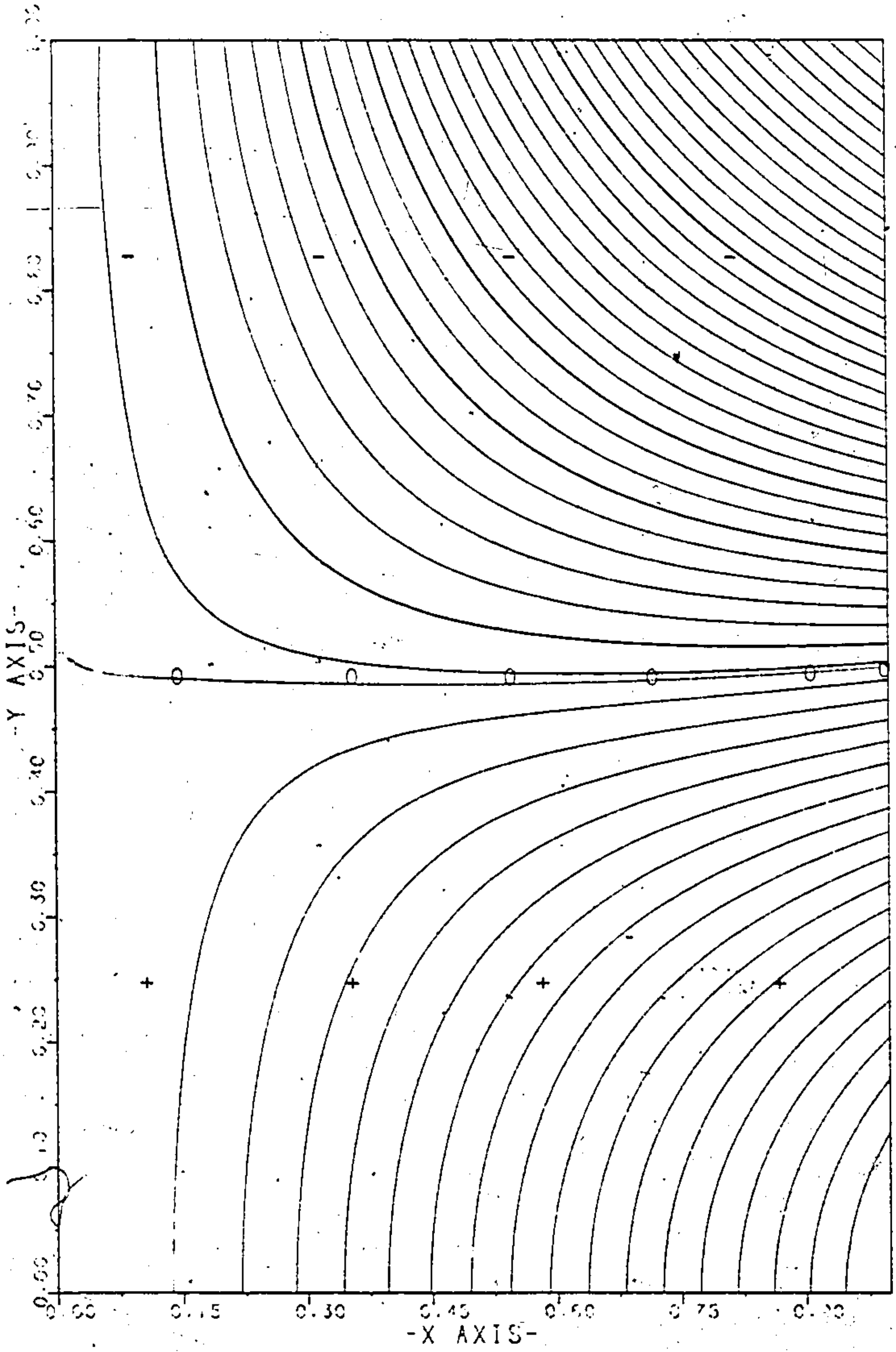
THIRD SYMMETRIC MODE WITH PHI= [illegible].25 [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

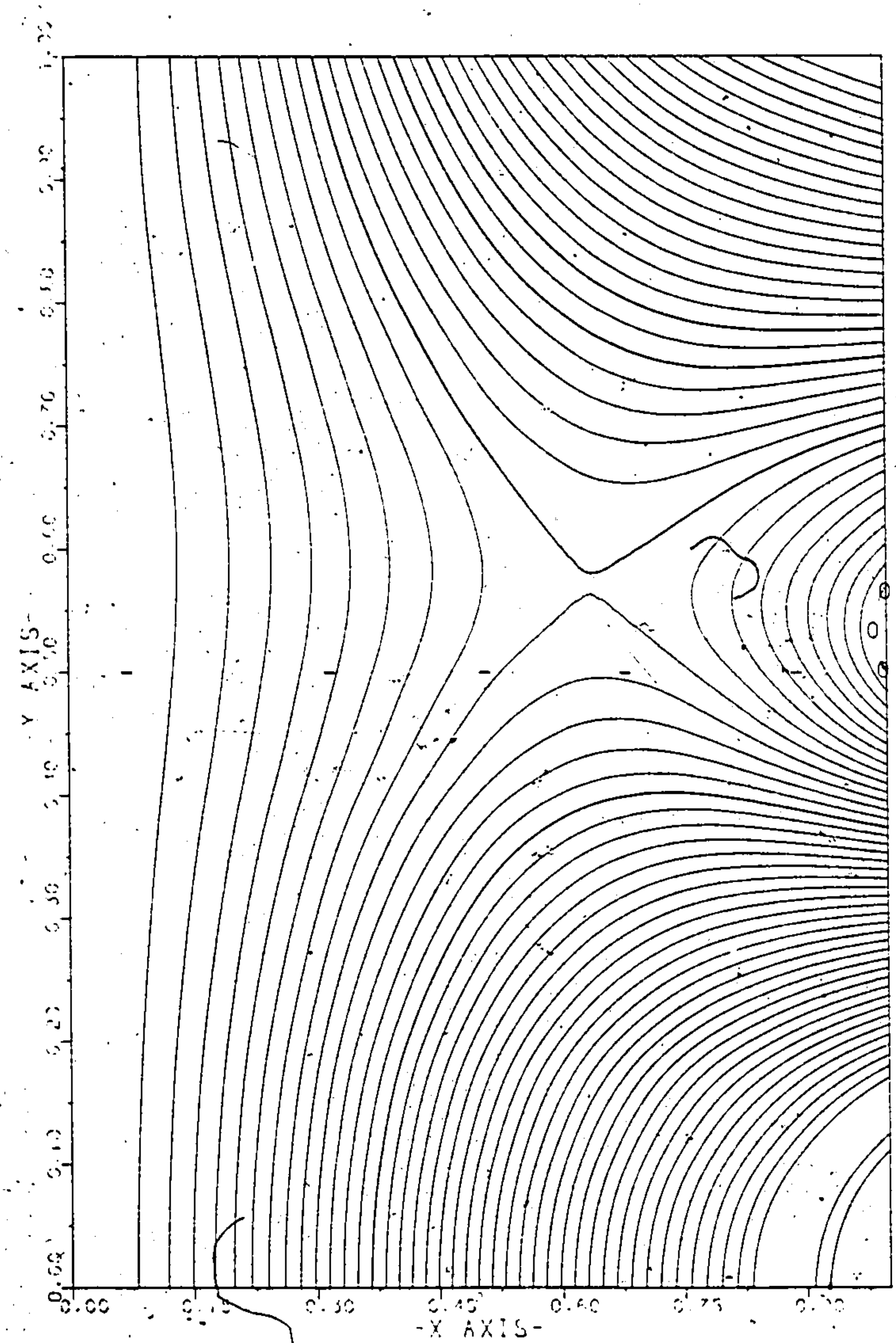
FIRST SYMMETRIC MODE WITH PHI = 3/2, [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH FOUR POINT SUPPORT ***

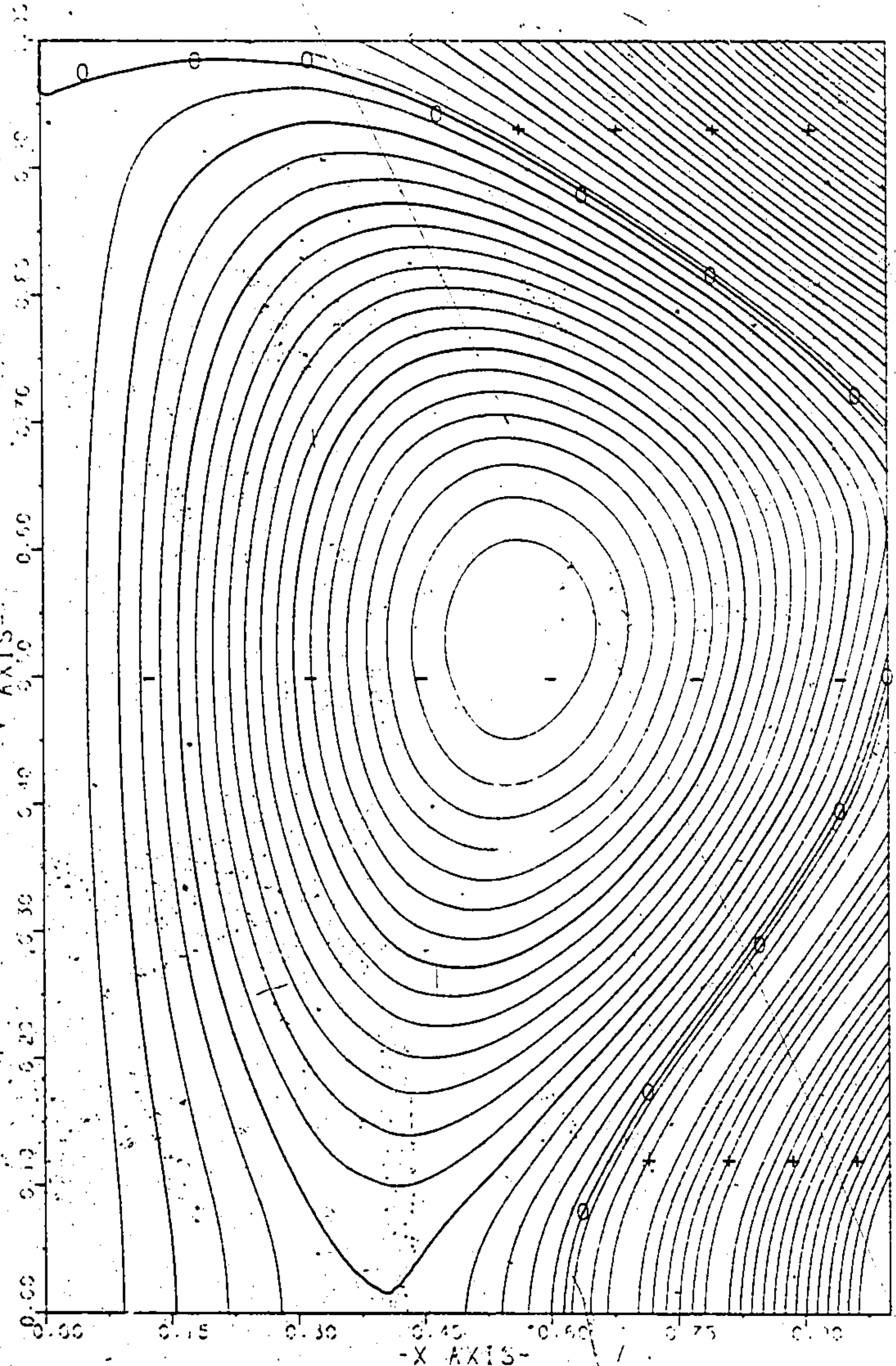
SECOND SYMMETRIC MODE WITH PHI = 3/2, ...



WAVE ... GRAPHIC

*** THE CANTILEVER PLATE WITH ... POINT SUPPORT ***

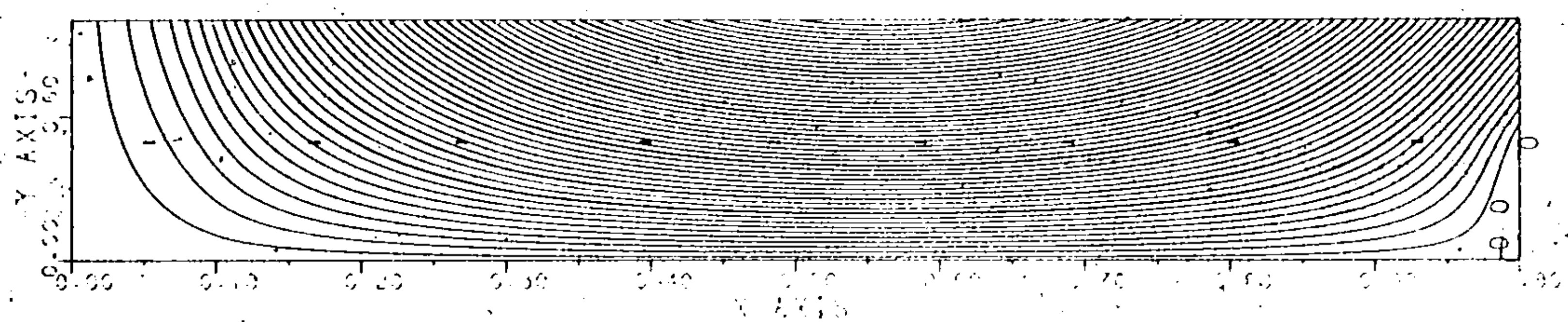
THIRD SYMMETRIC MODE WITH PHI = 3.2 ...



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

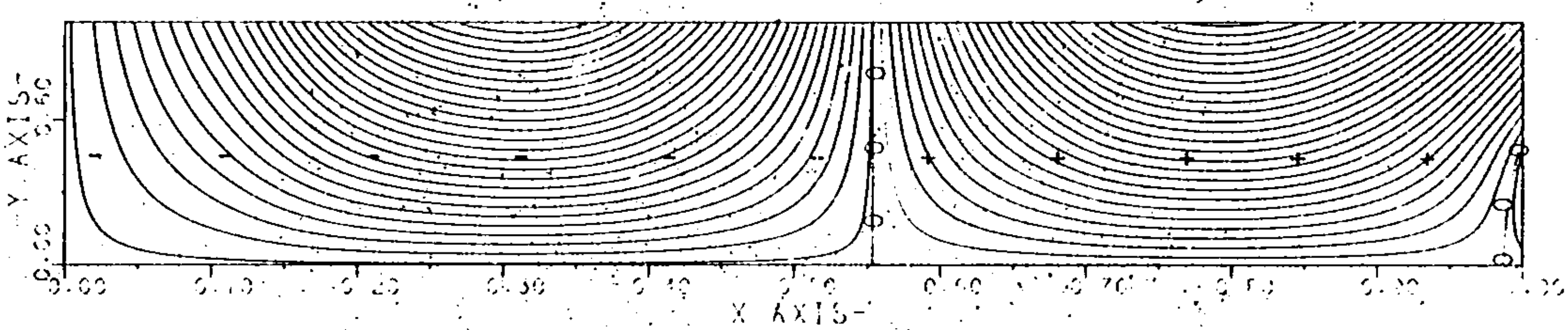
FIRST ANTISYMMETRIC MODE WITH PHI - 1/6, U - 1/1, V - 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

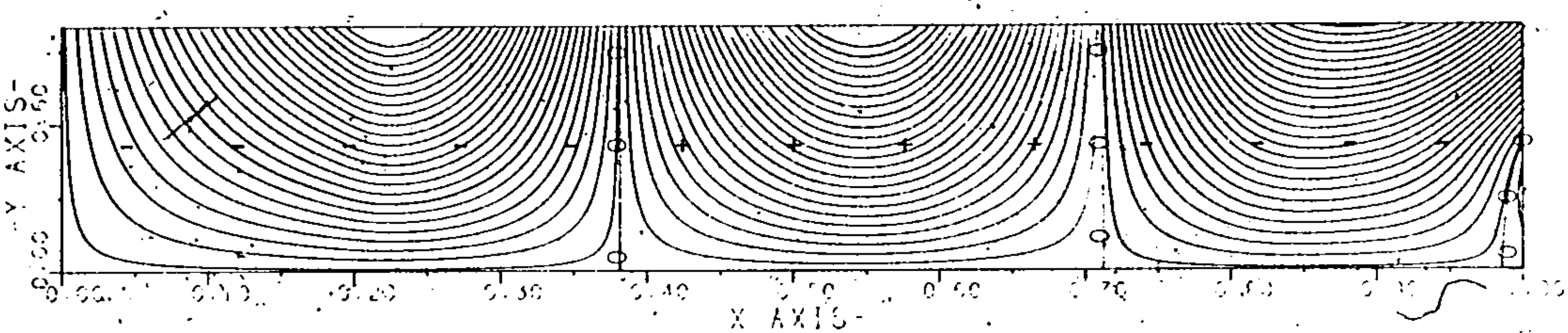
SECOND ANTISYMMETRIC MODE WITH PHI = 1/6, U = [illegible]/1, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

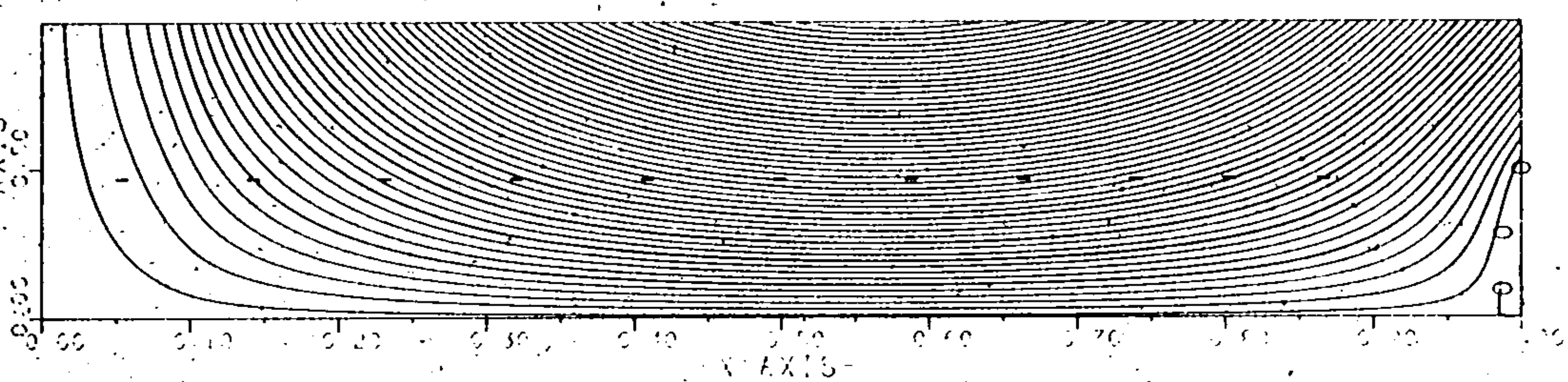
THIRD ANTISYMMETRIC MODE WITH PHI = 1/6, U = [illegible], V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

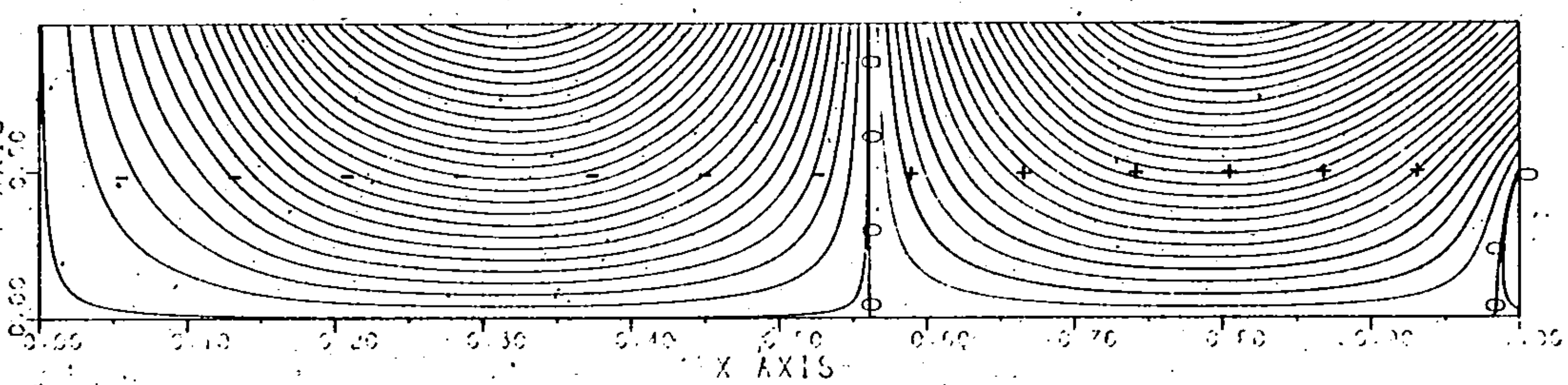
FIRST ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

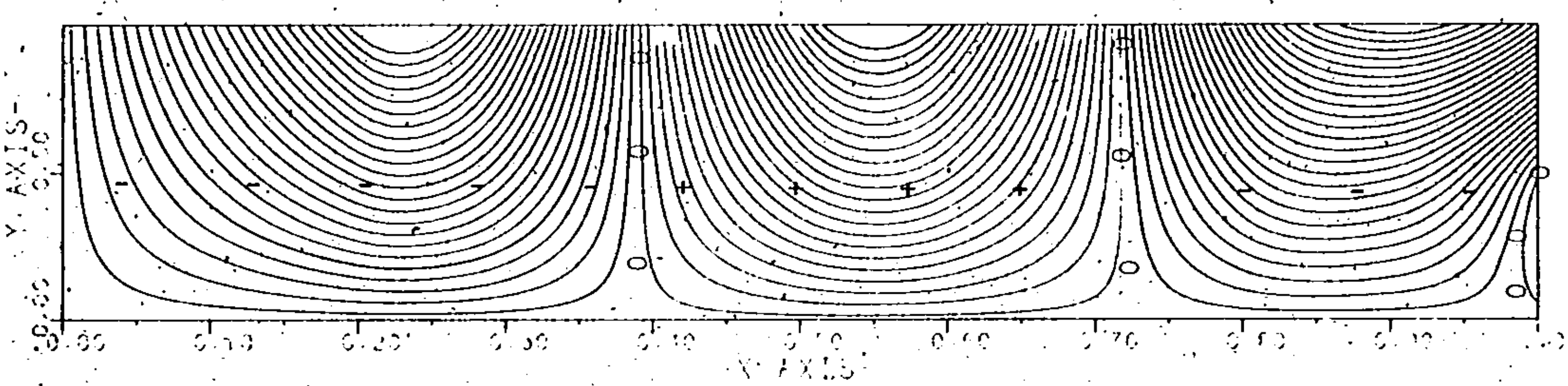
SECOND ANTISYMMETRIC MODE WITH PHI = 1/5 U = [illegible] V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

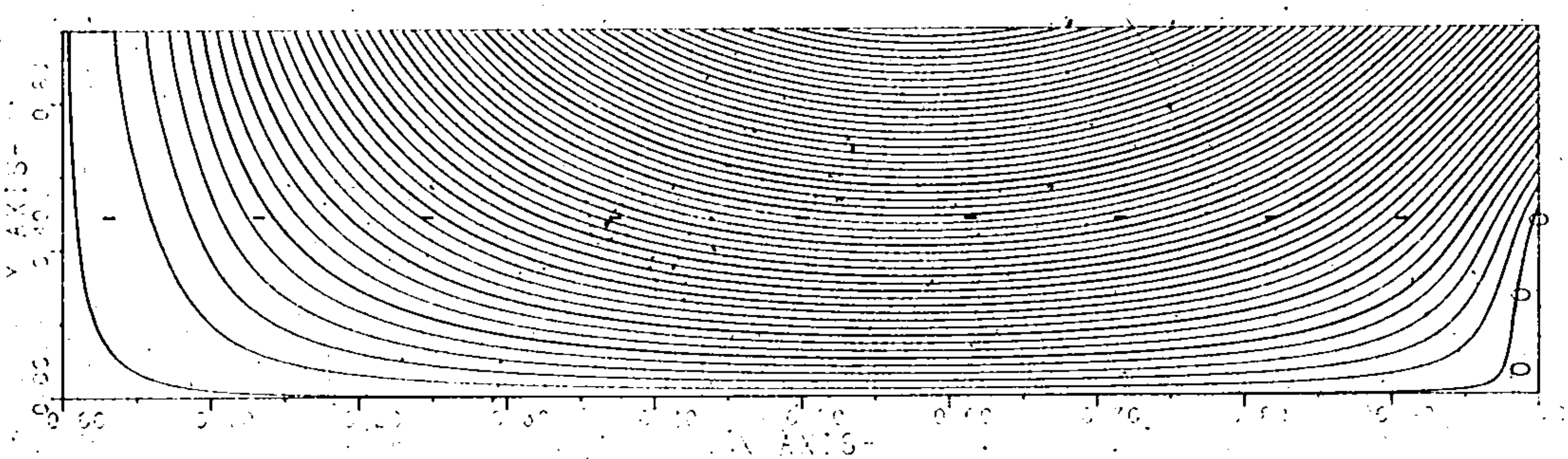
THIRD ANTISYMMETRIC MODE WITH PHI = .75. U = .[illegible]. V = .72



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

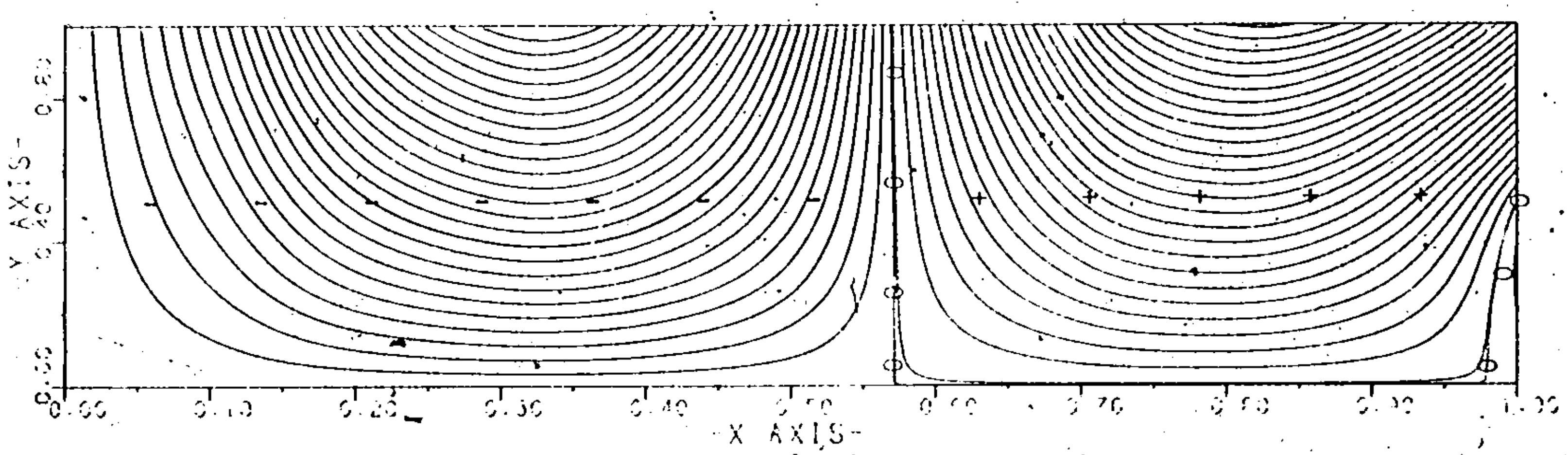
FIRST ANTISYMMETRIC MODE WITH PHI - [illegible]



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

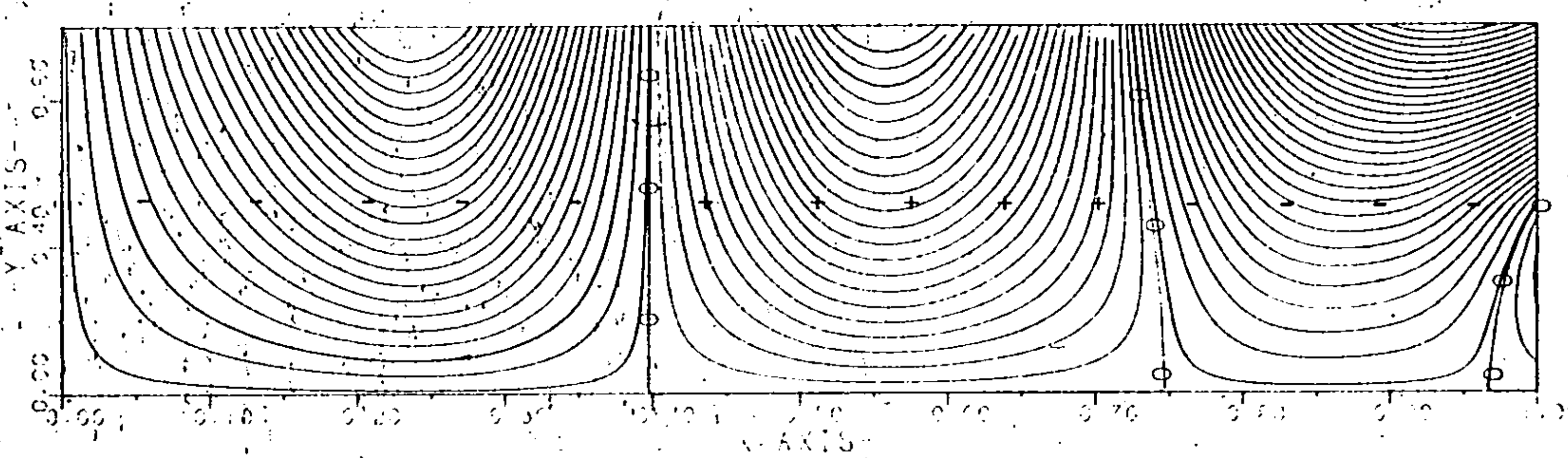
SECOND ANTISYMMETRIC MODE WITH PHI = 1/4, U = 1/1, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

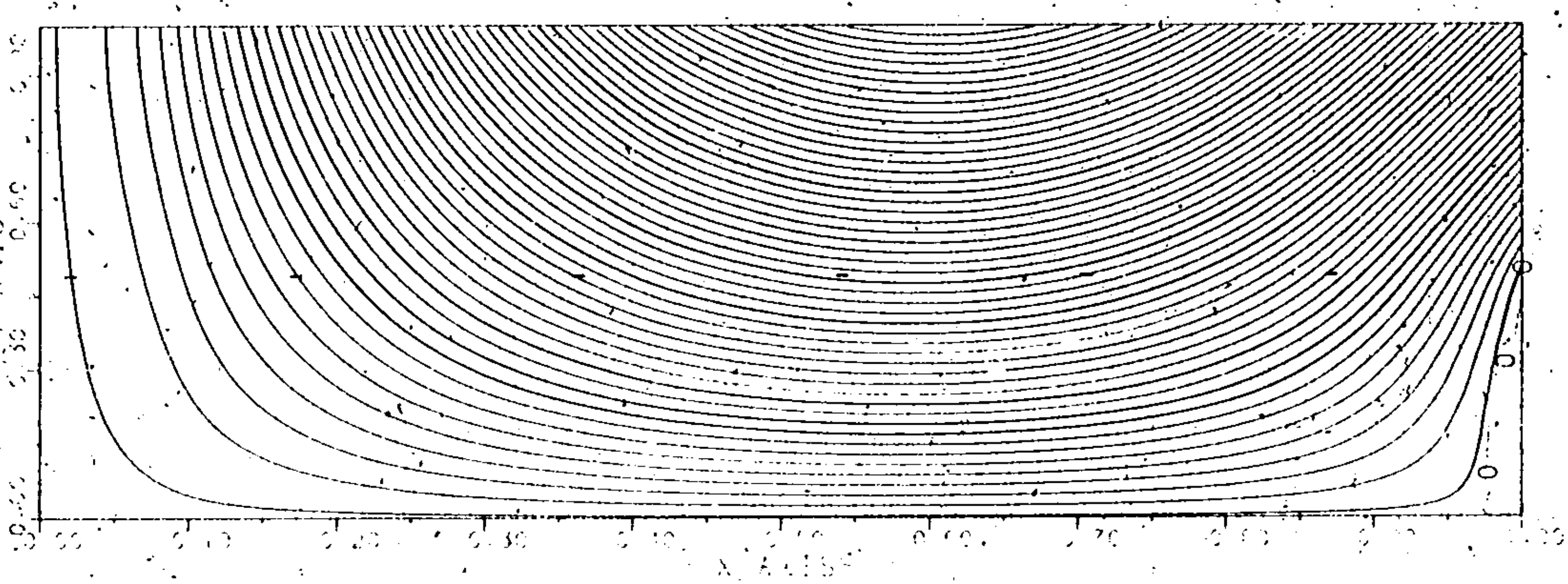
THIRD ANTISYMMETRIC MODE WITH PHI = 1/4, U = [illegible]/1, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

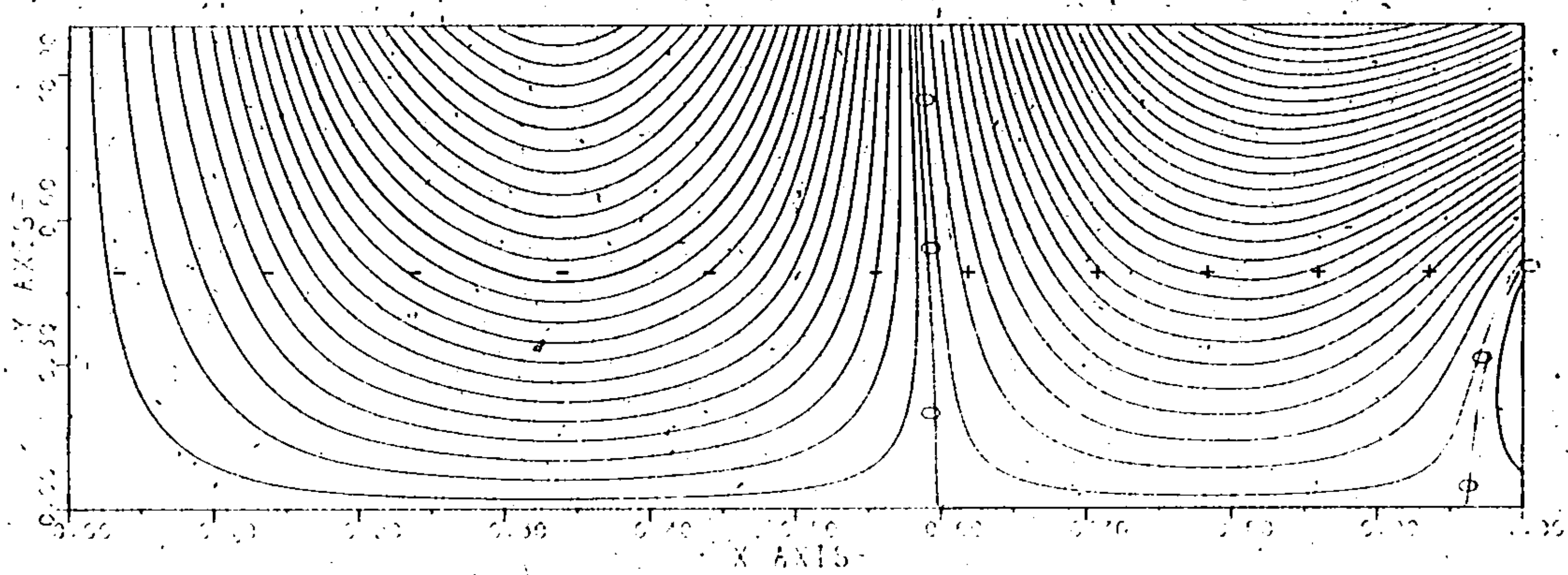
FIRST ANTISYMMETRIC MODE WITH PHI = 1/3, C = [illegible], V = 1/2



WAVE PLATE GRAPHIC

**** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ****

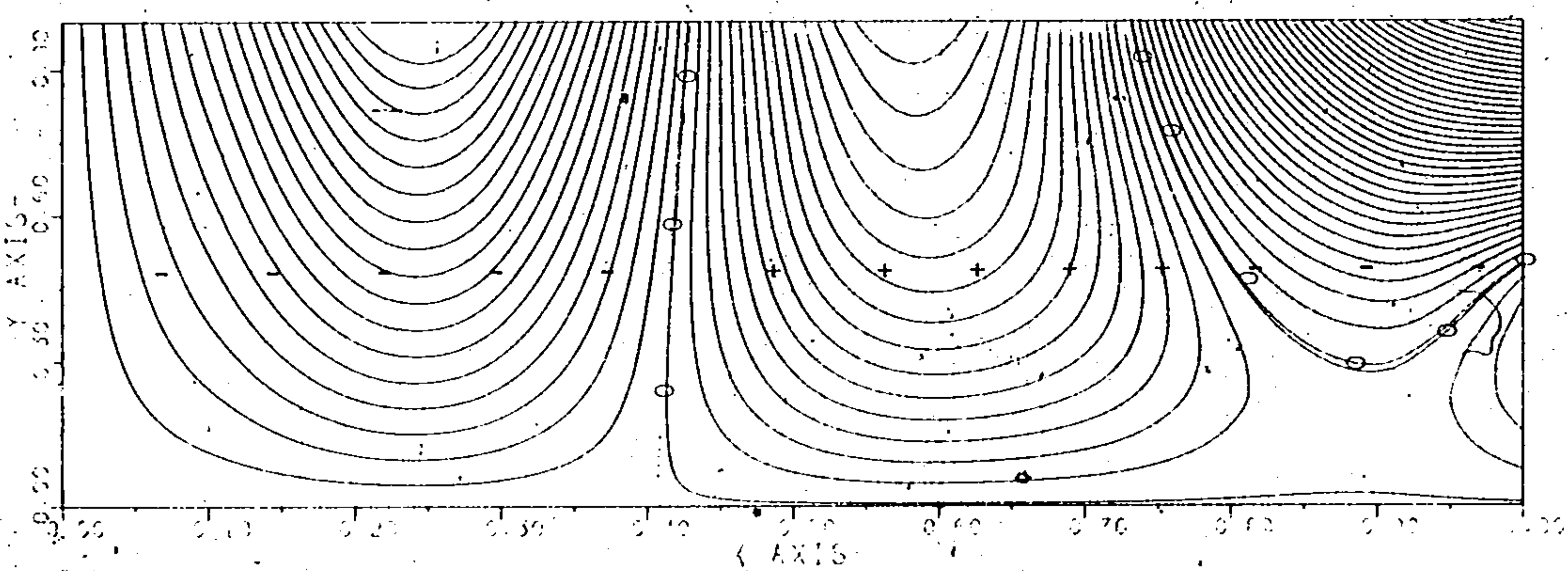
SECOND ANTISYMMETRIC MODE WITH PHI = 1/3, C = [illegible], V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

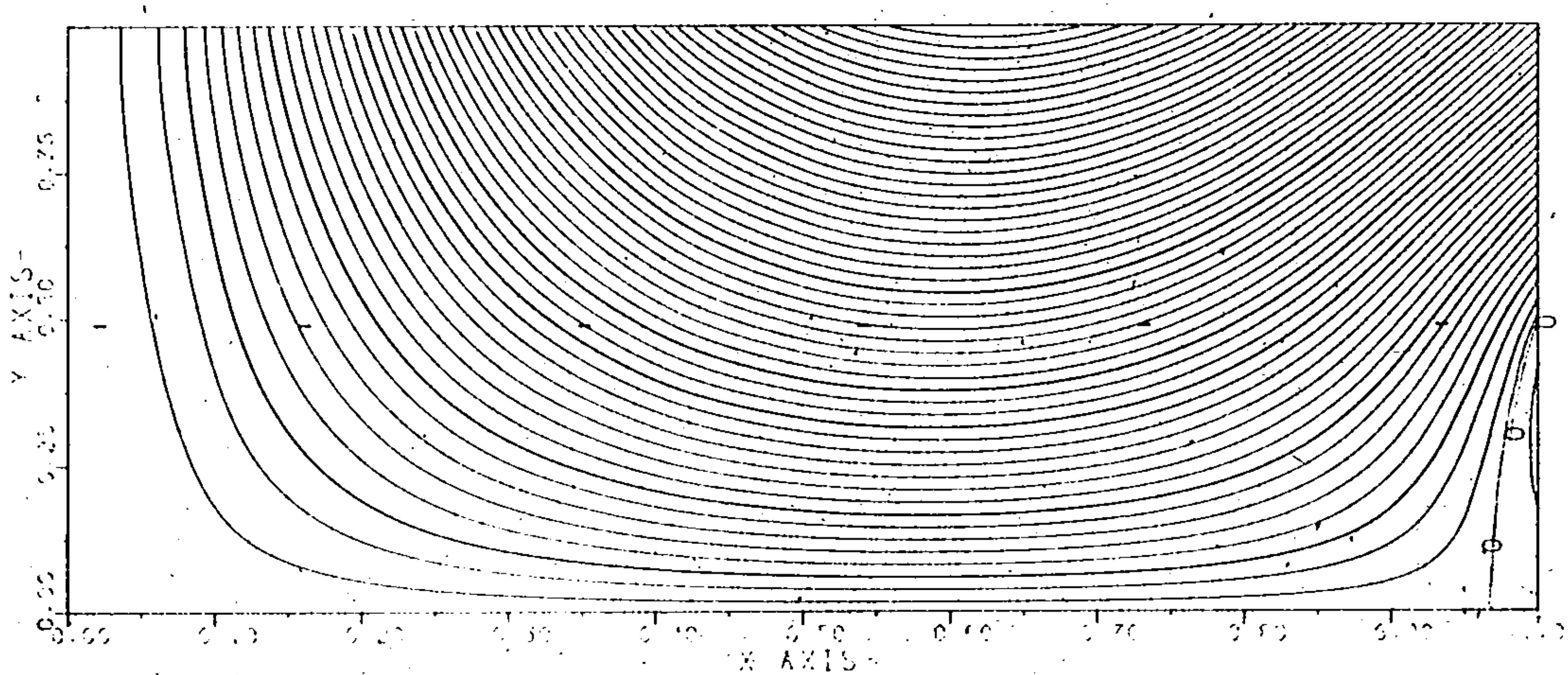
THIRD ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI=1/2.5, [illegible]



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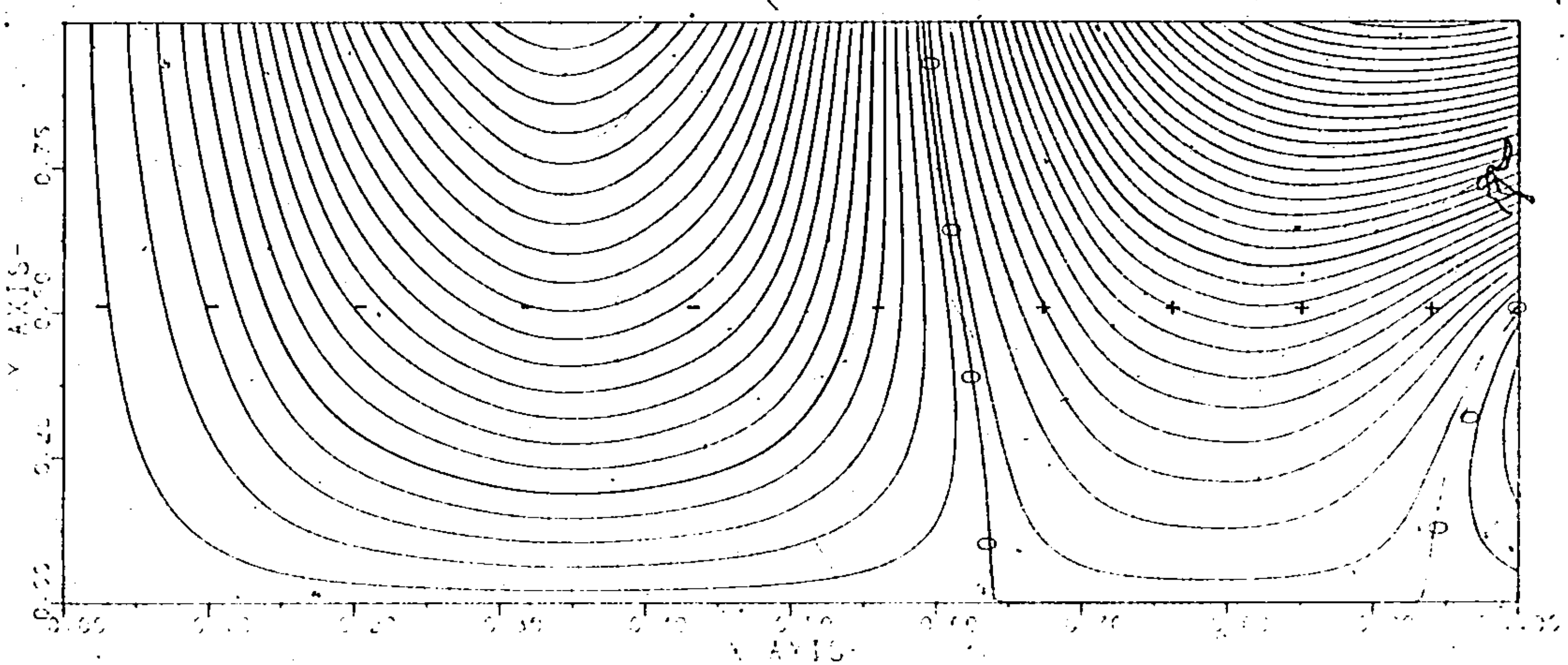
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

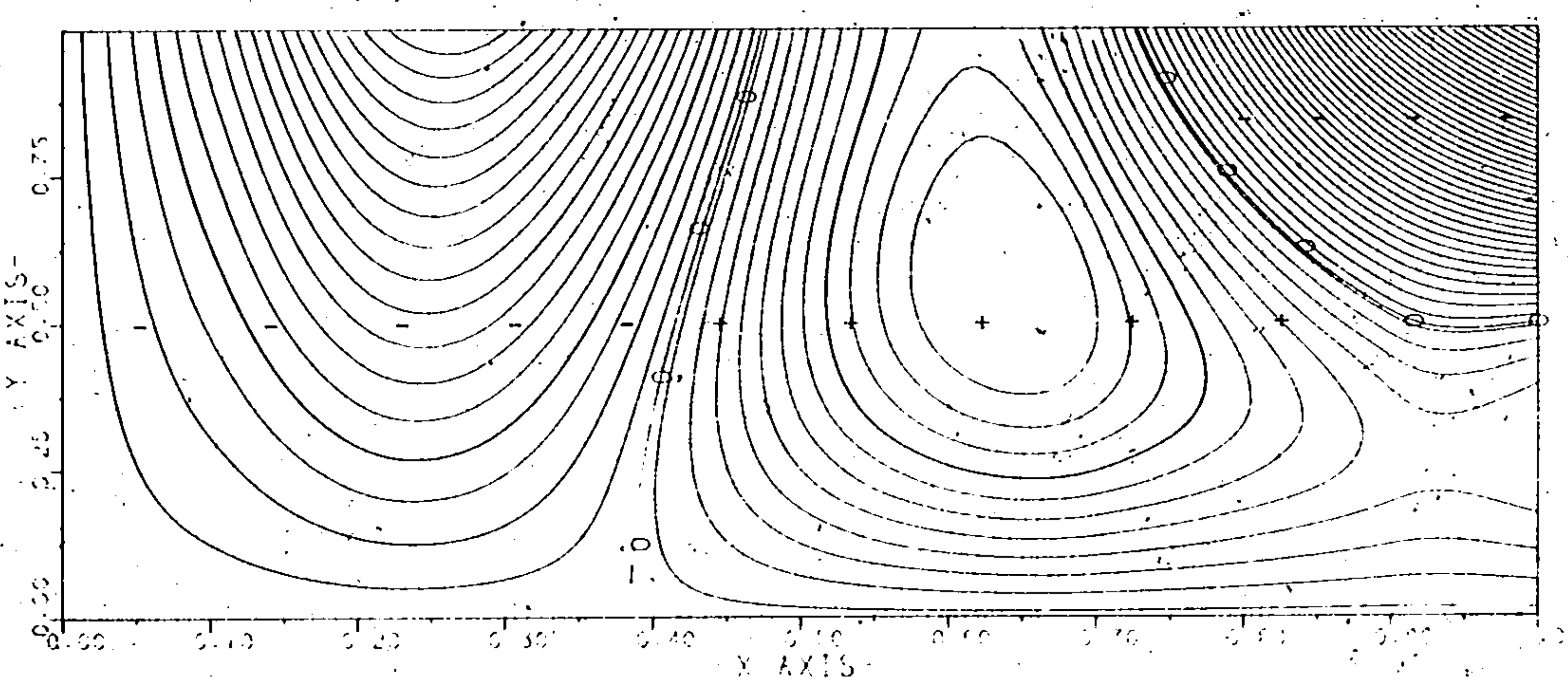
SECOND ANTISYMMETRIC MODE WITH PHI=172.5, U = [illegible], V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

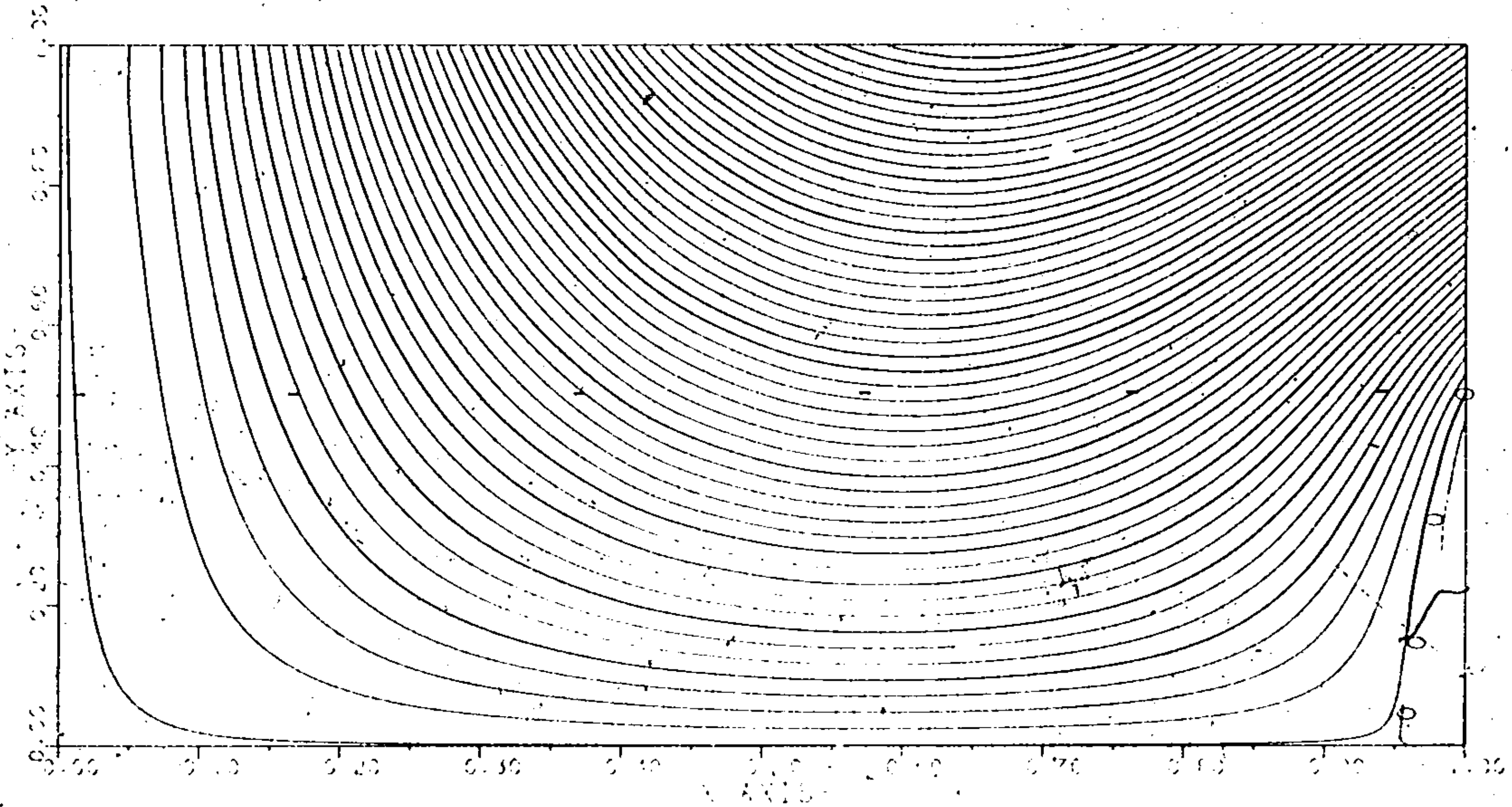
THIRD ANTISYMMETRIC MODE WITH PHI=172.5 U = [illegible] V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

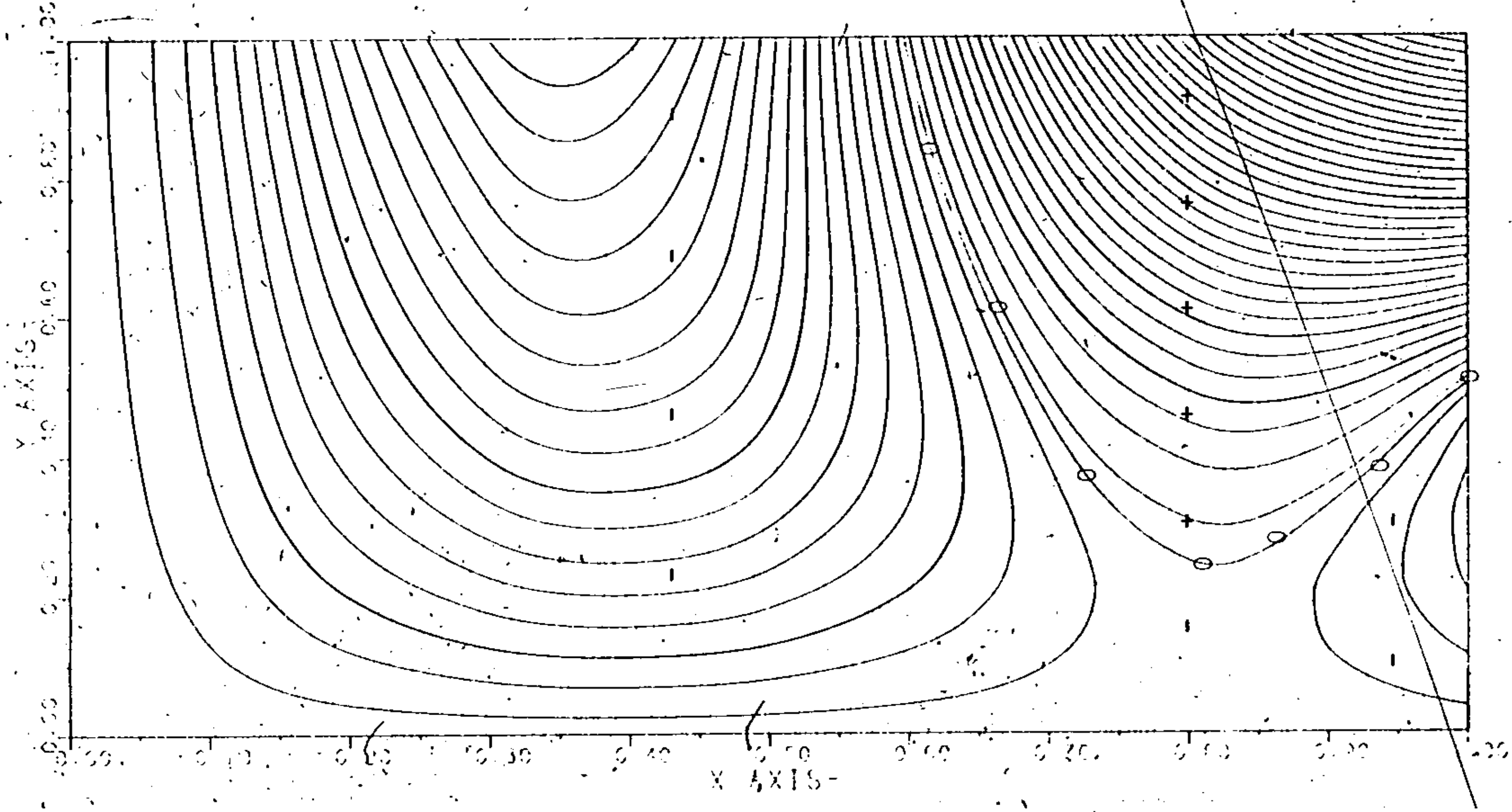
FIRST ANTISYMMETRIC MODE WITH PHI = 1/2, U = [illegible], V = [illegible]/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH EDGE POINT SUPPORT •••

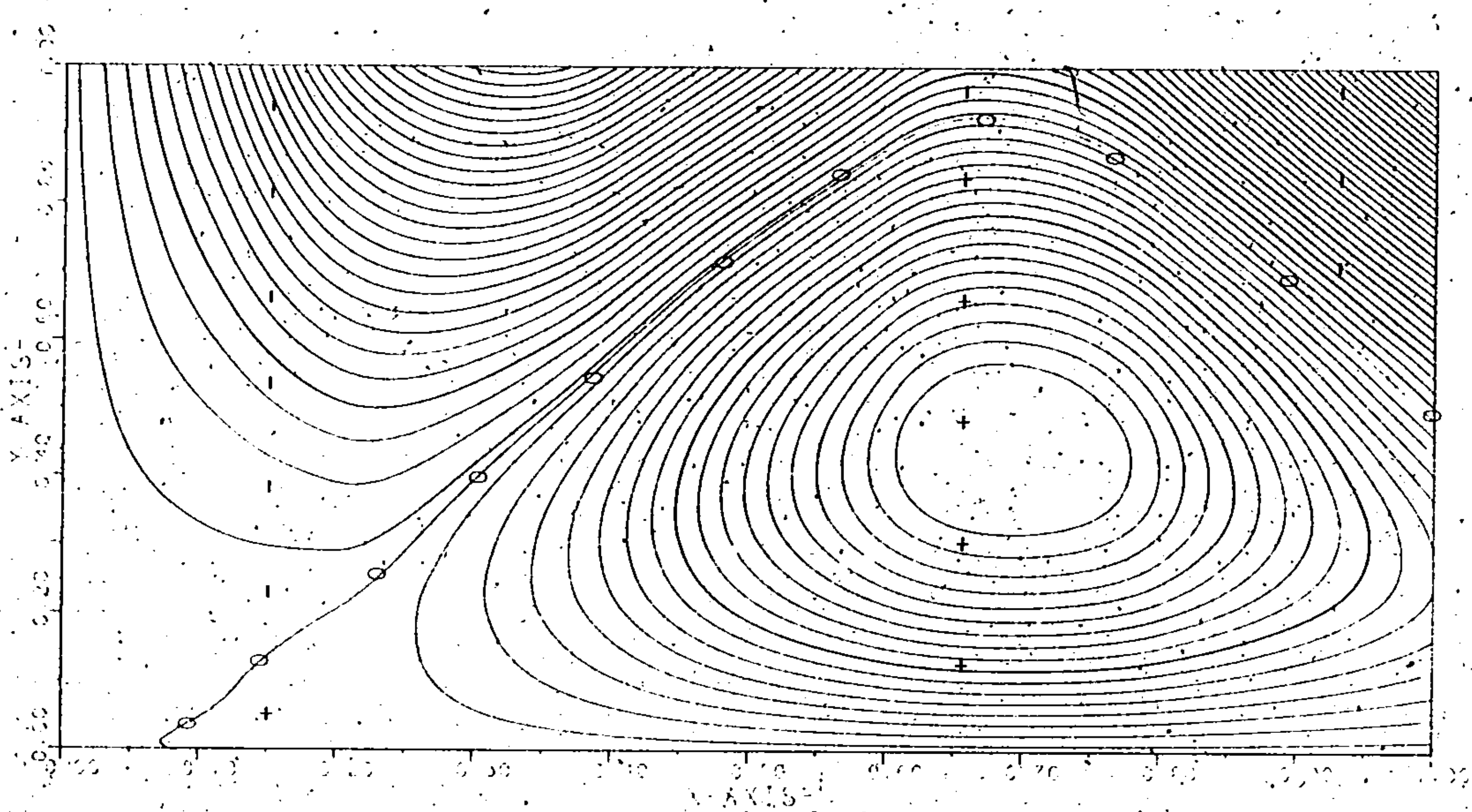
SECOND ANTISYMMETRIC MODE WITH PHI = 1/2, U = [illegible]/1, V = [illegible]/2



WAVE PLATE GRAPHIC

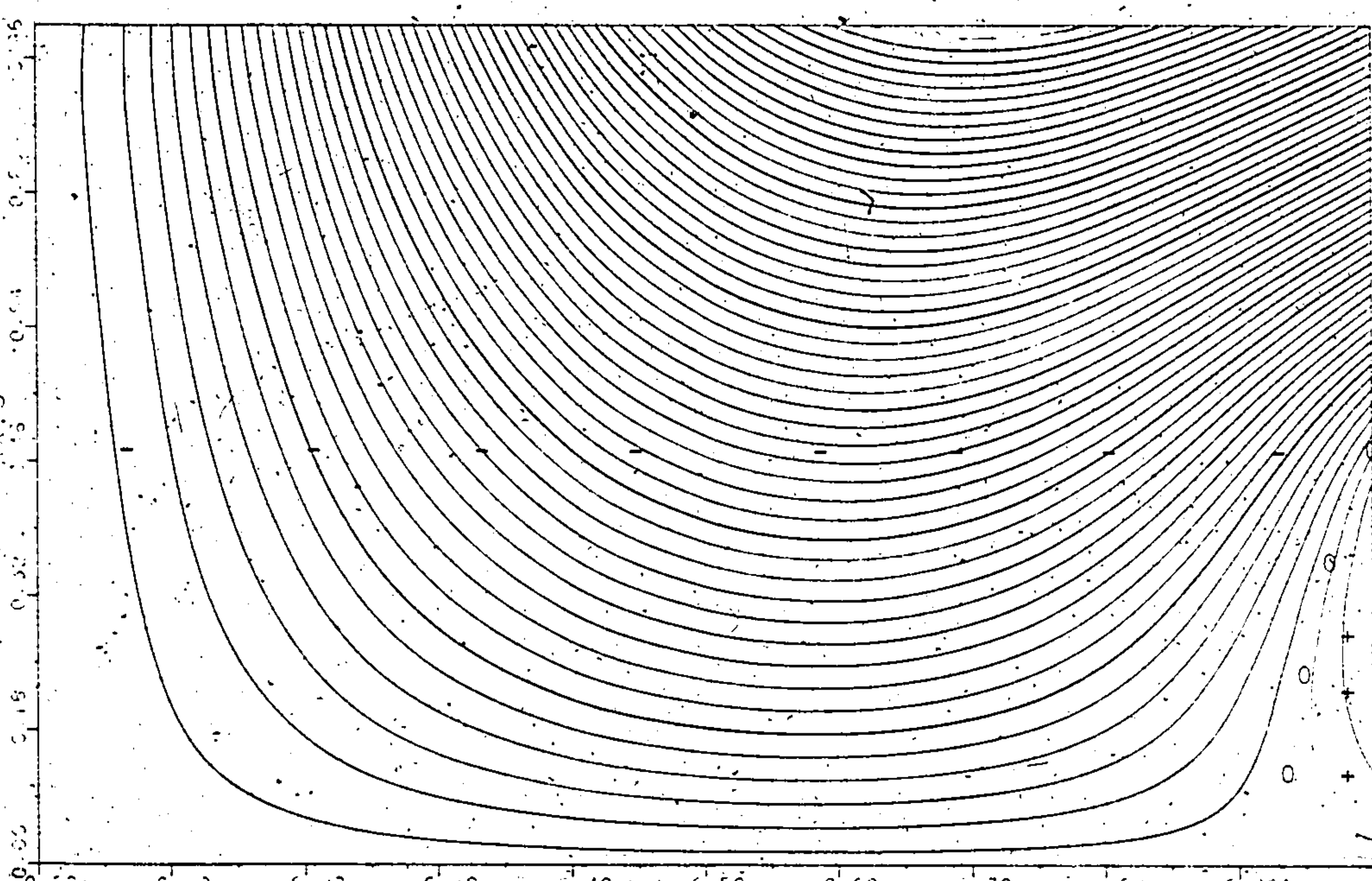
*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD ANTISYMMETRIC MODE WITH PHI = 1/2, U = [illegible]



THE CANTILEVER PLATE WITH [illegible] POINT SUPPORT

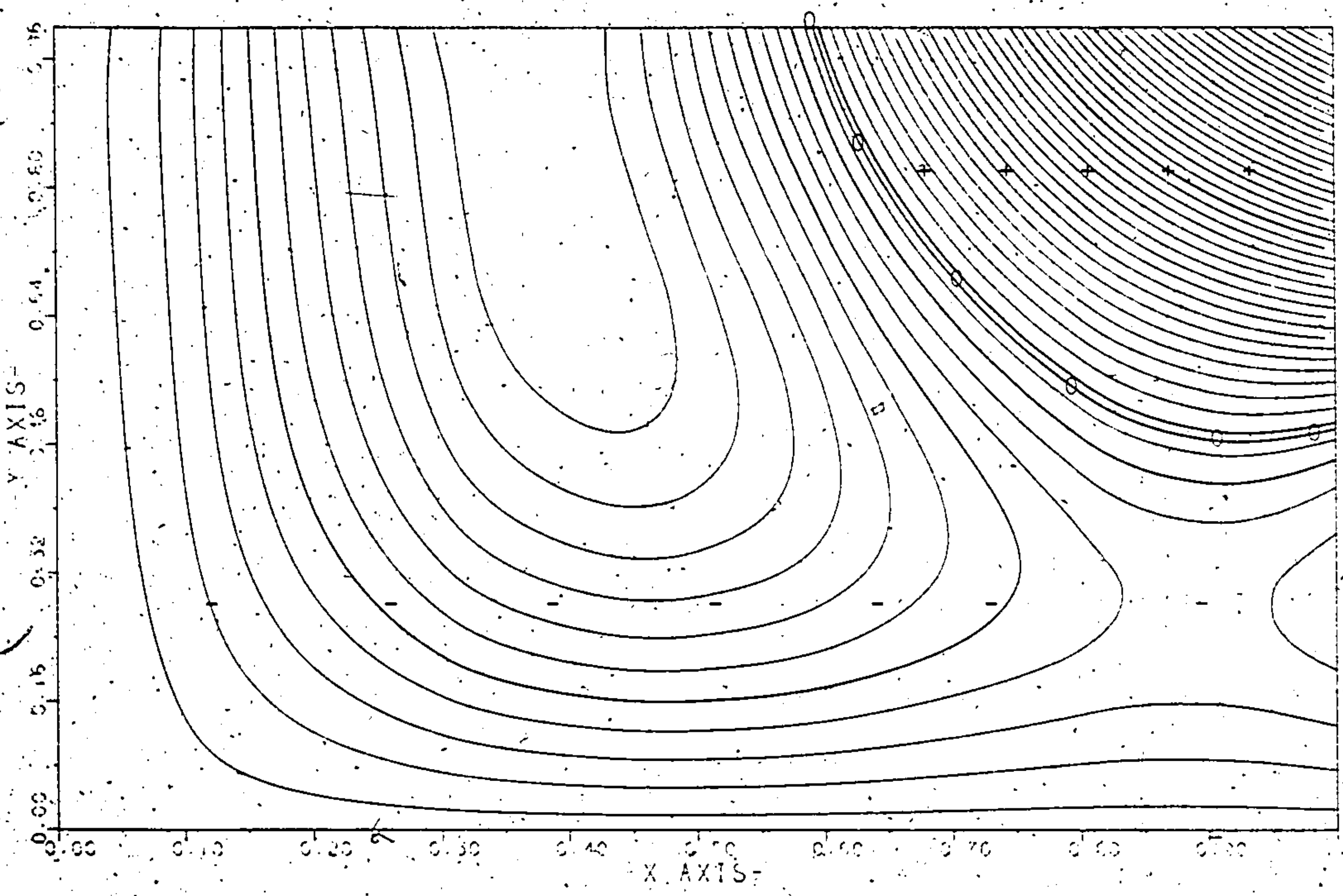
FIRST ANTISYMMETRIC MODE WITH PHI [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH 'EDGE' POINT SUPPORT ***

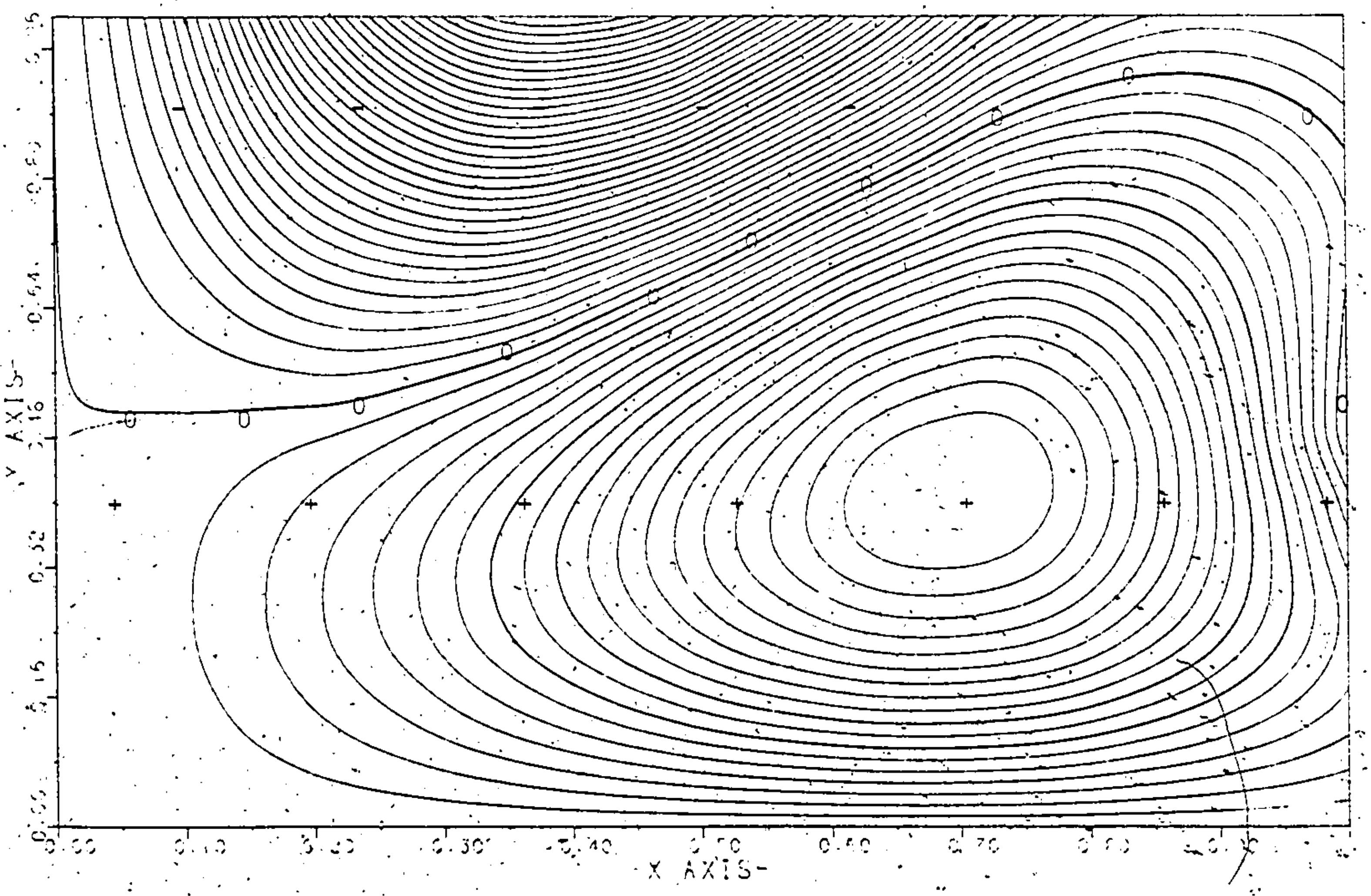
SECOND ANTISYMMETRIC MODE WITH PHI = 5/6, [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH FOUR POINT SUPPORT ***

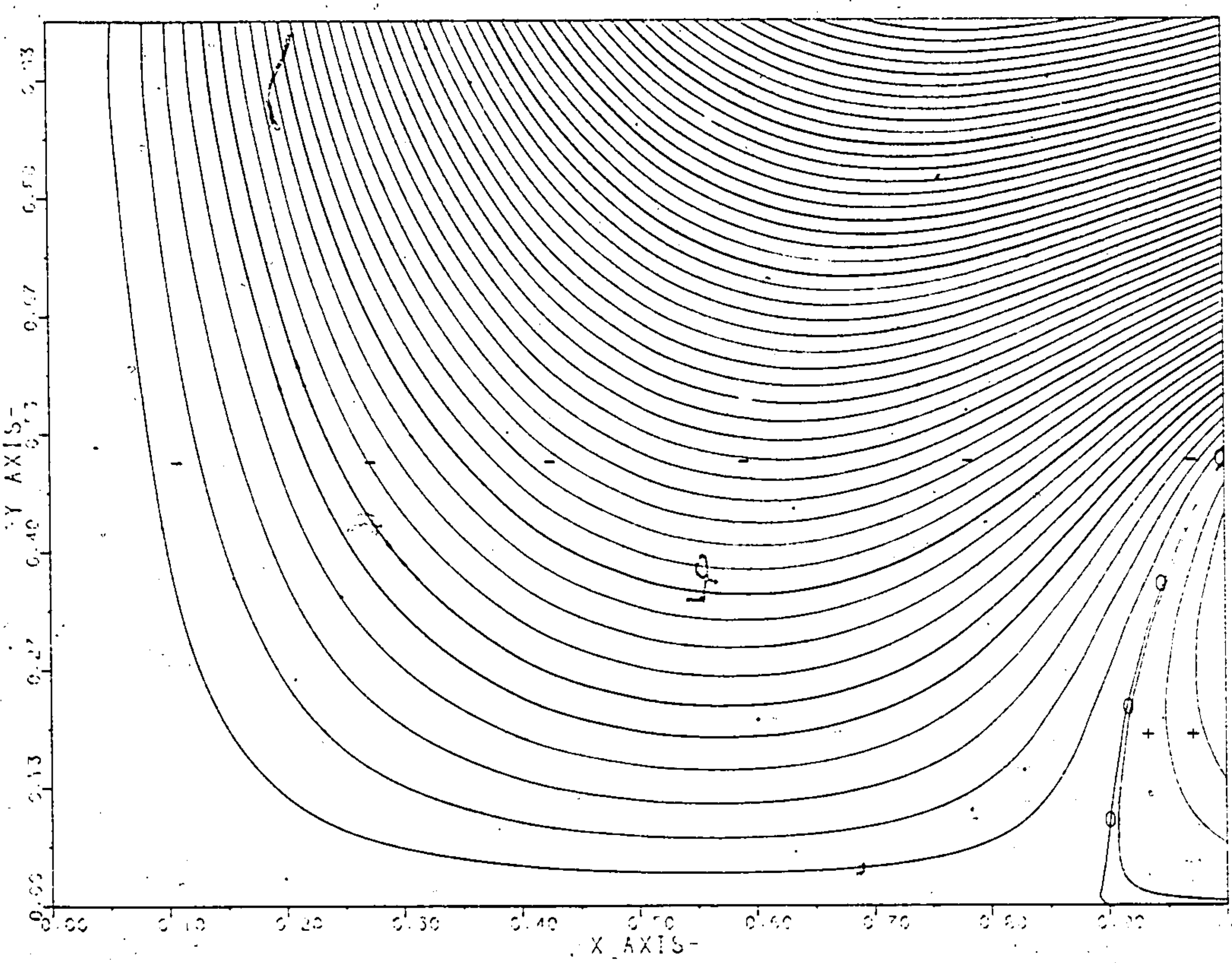
THIRD ANTISYMMETRIC MODE WITH PHI [illegible]

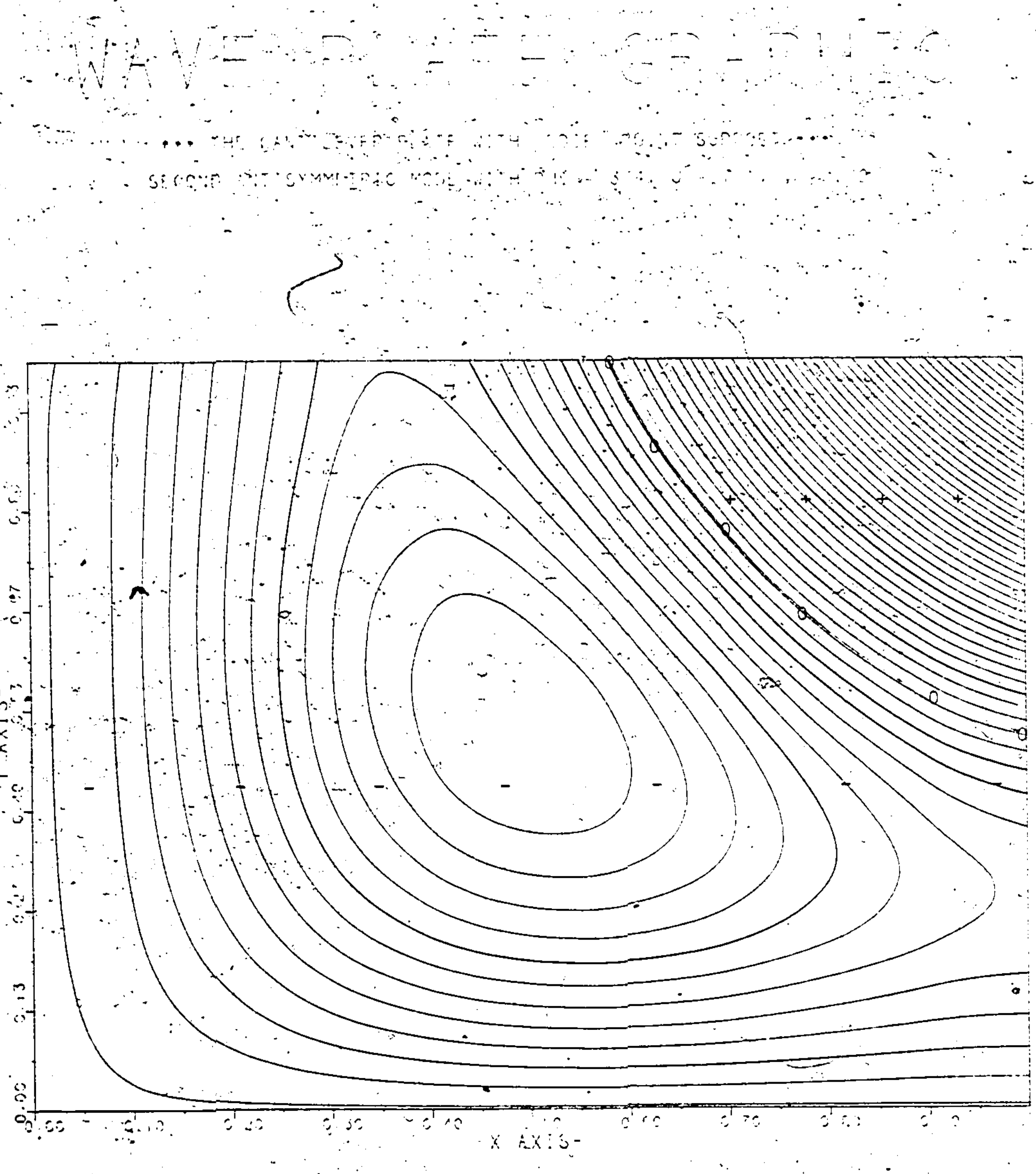


WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI = 3.4, [illegible]

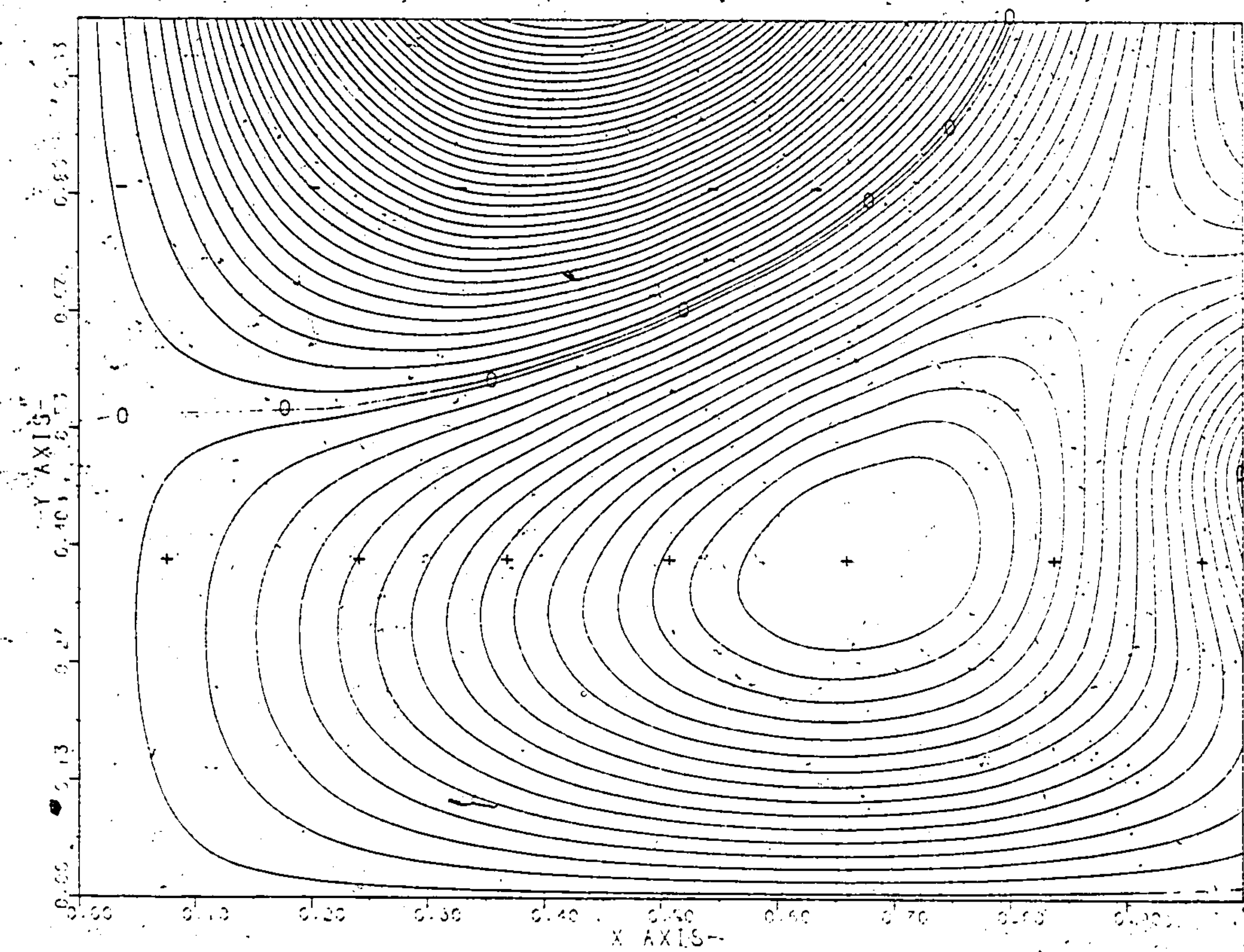



WAVE
THE CANTILEVER PLATE WITH
SECOND ANTISYMMETRIC MODE WITH
X AXIS
Y AXIS
0.00
0.13
0.40
0.67
0.10
0.20
0.30
0.40
0.60
0.70
0.80

WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

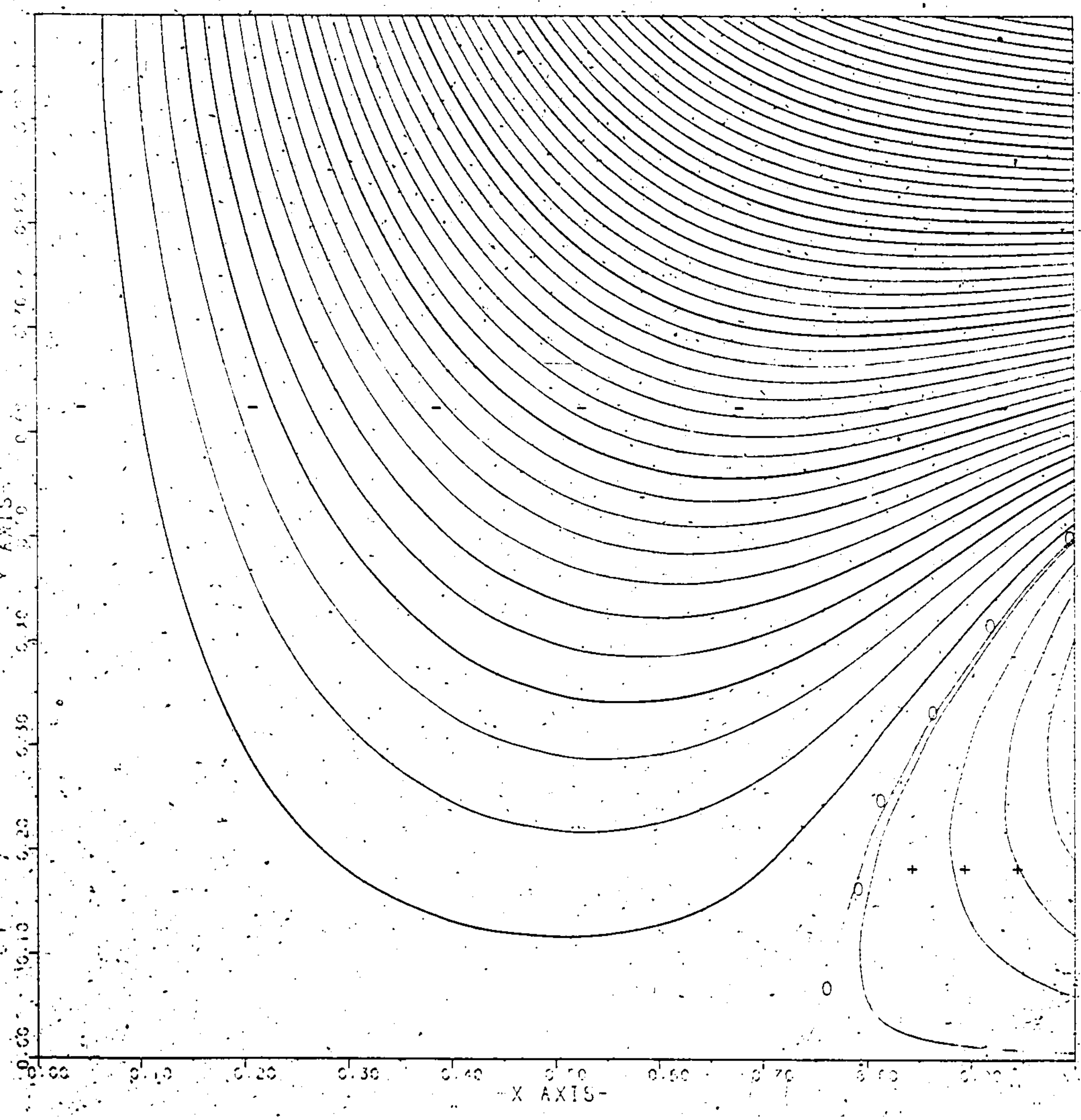
THIRD ANTISYMMETRIC MODE WITH



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH FOUR POINT SUPPORT ***

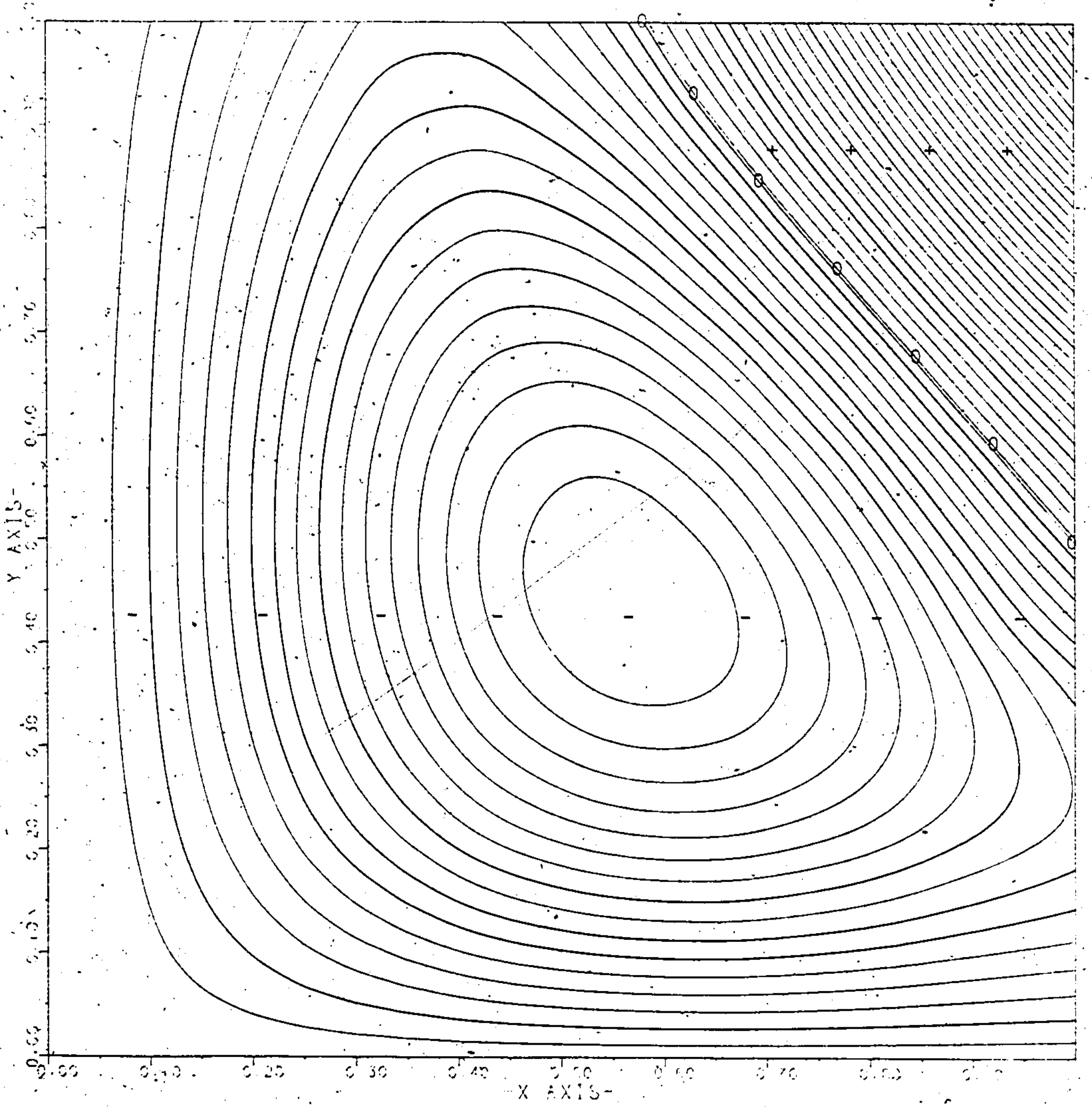
FIRST ANTISYMMETRIC MODE WITH PHI= [illegible] V = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH FOUR POINT SUPPORT ***

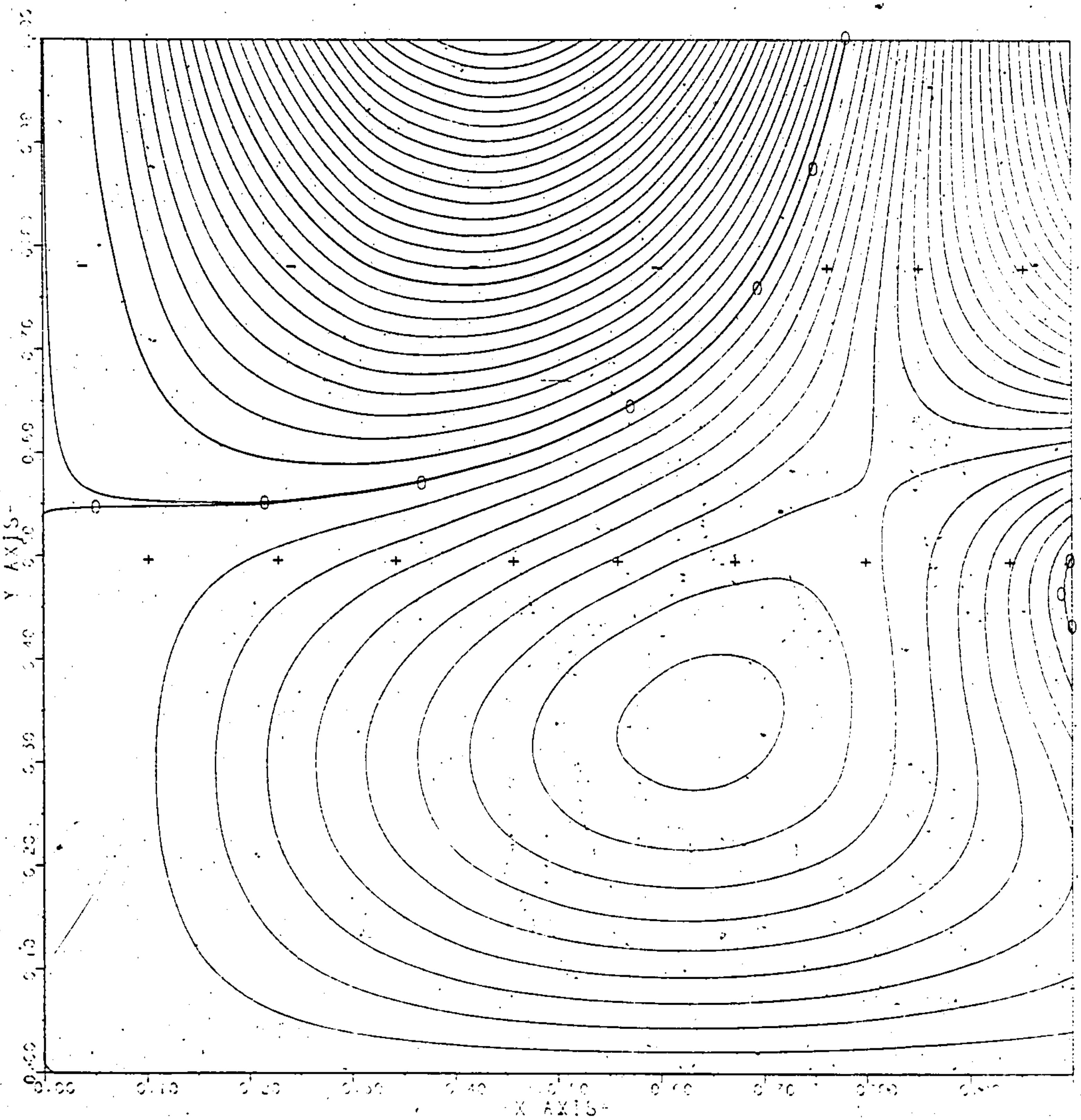
SECOND ANTISYMMETRIC MODE WITH PHI [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

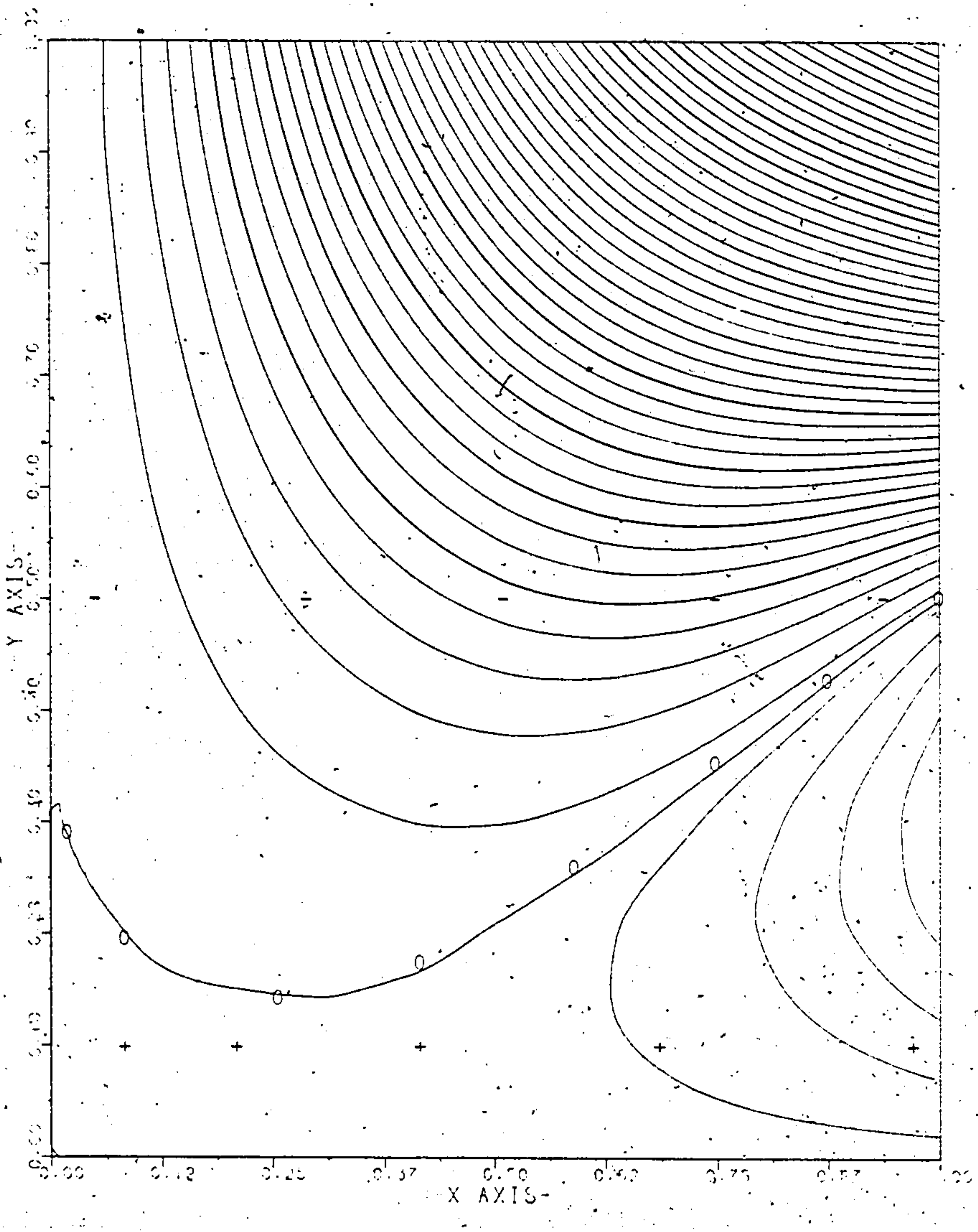
THIRD ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

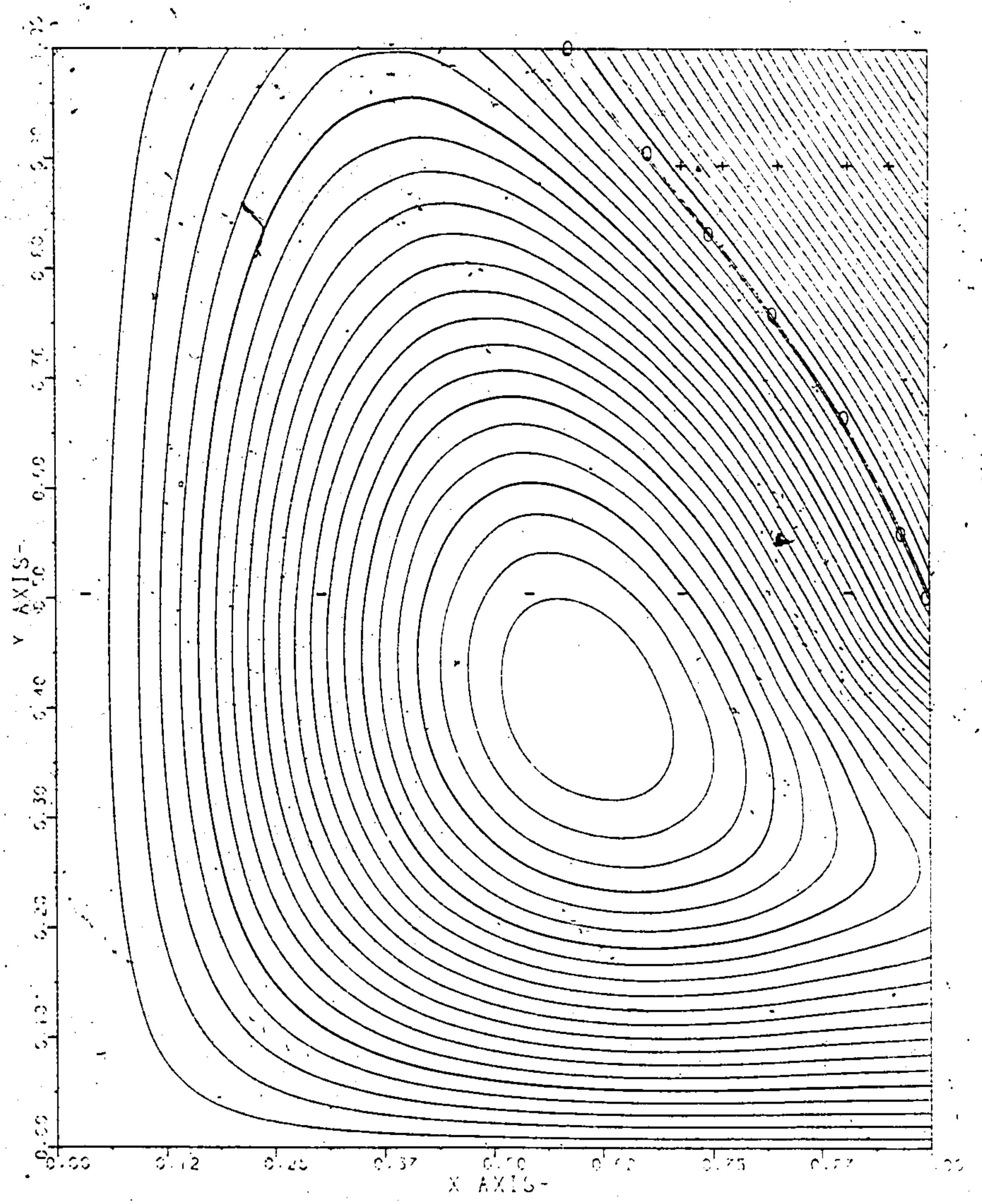
FIRST ANTISYMMETRIC MODE WITH PHI= 1.25, ...



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

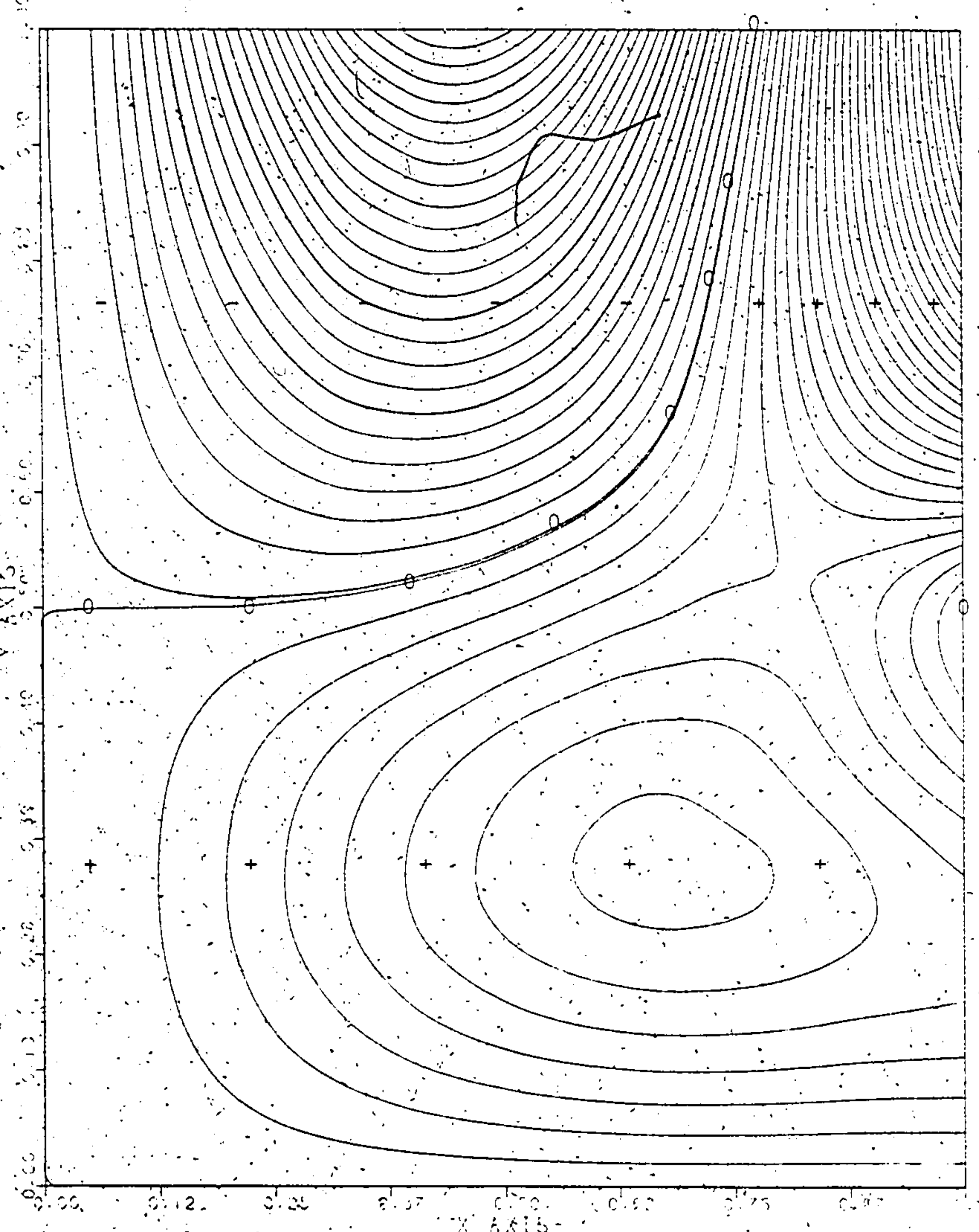
SECOND ANTISYMMETRIC MODE WITH PHI= 0.25,



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORTS ***

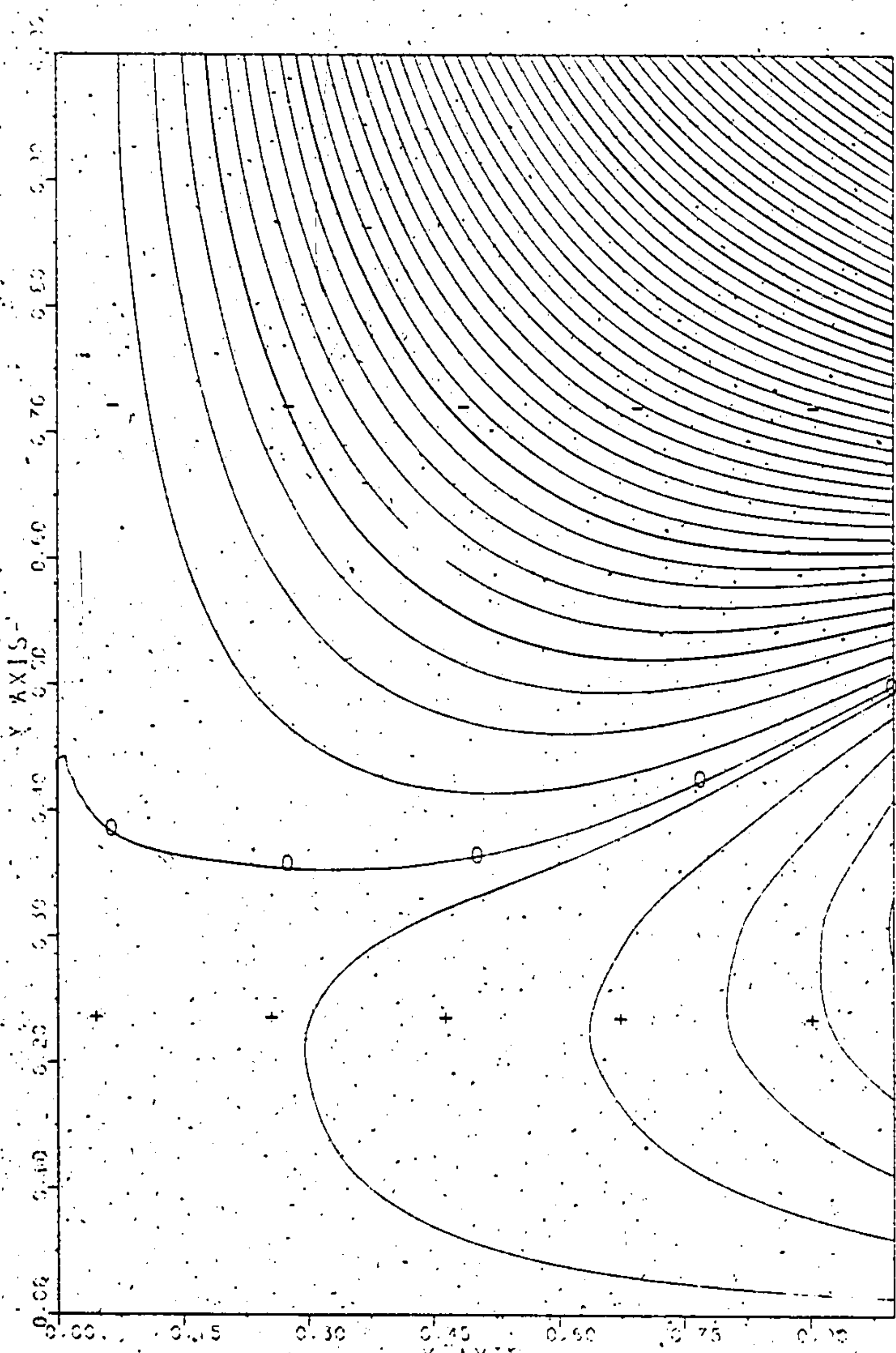
THIRD ANTISYMMETRIC MODE WITH PHI= [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LEDGE POINT SUPPORT ***

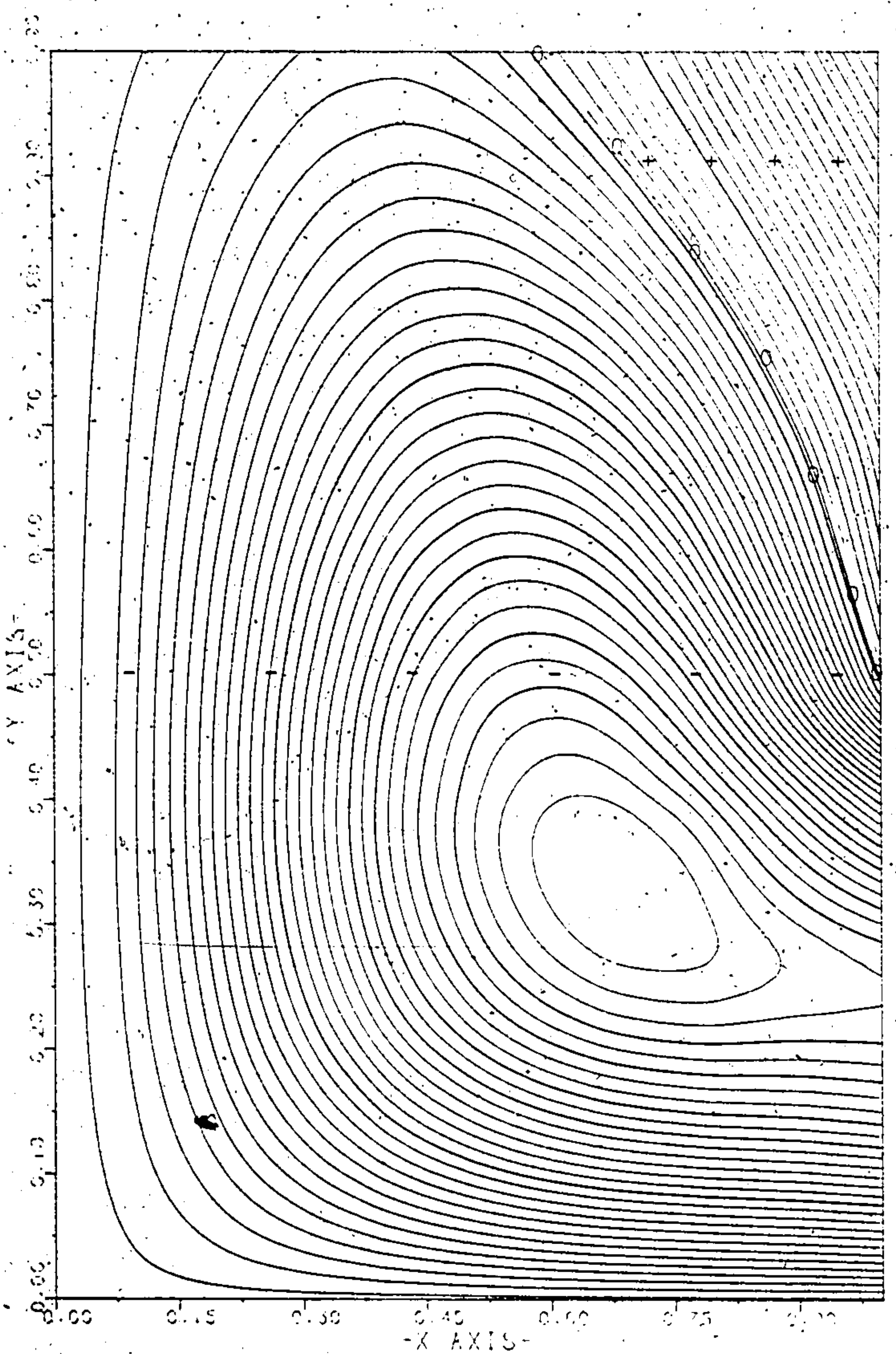
FIRST ANTISYMMETRIC MODE WITH PHI = [illegible]

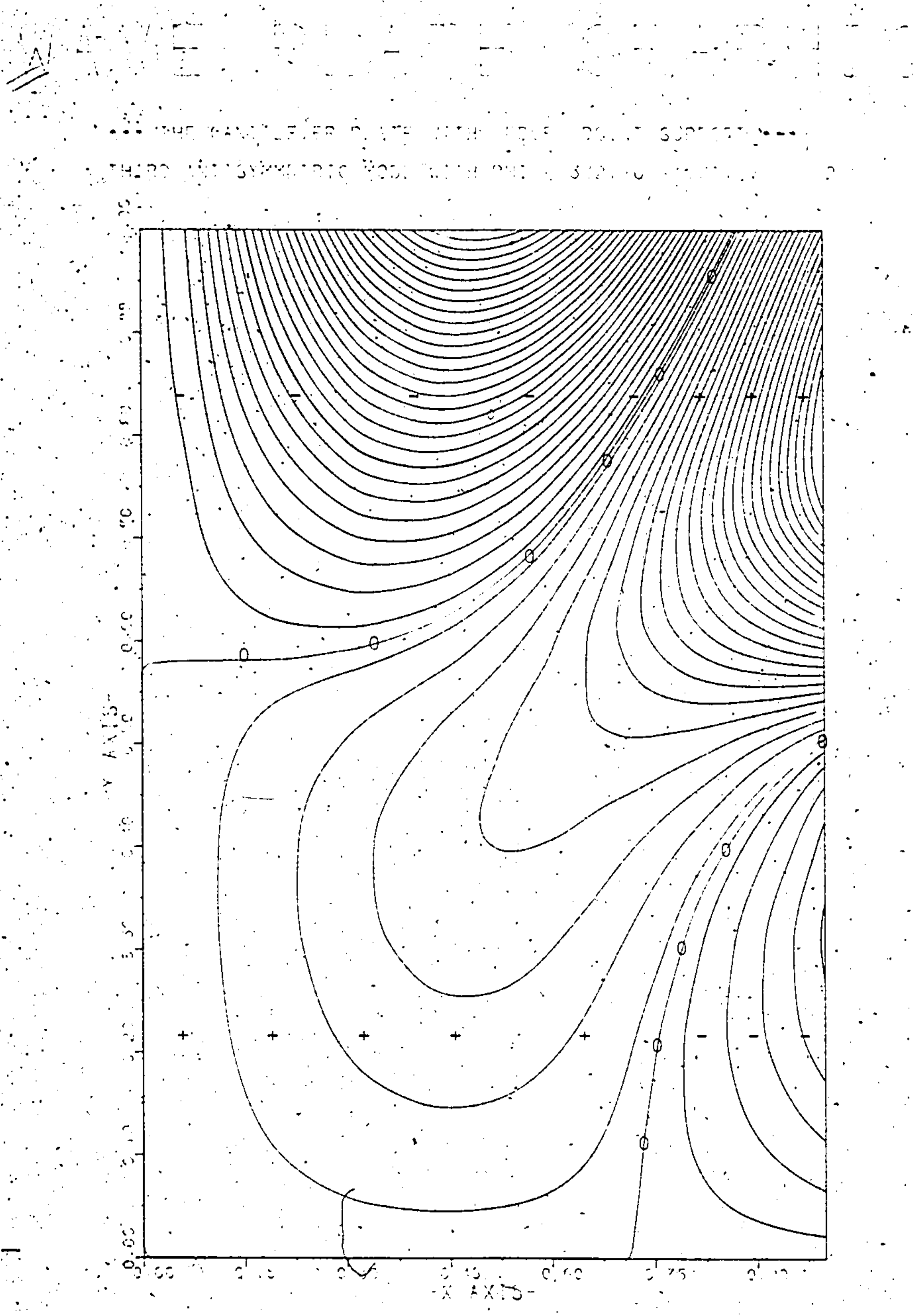


WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

SECOND ANTISYMMETRIC MODE WITH PHI = [illegible]

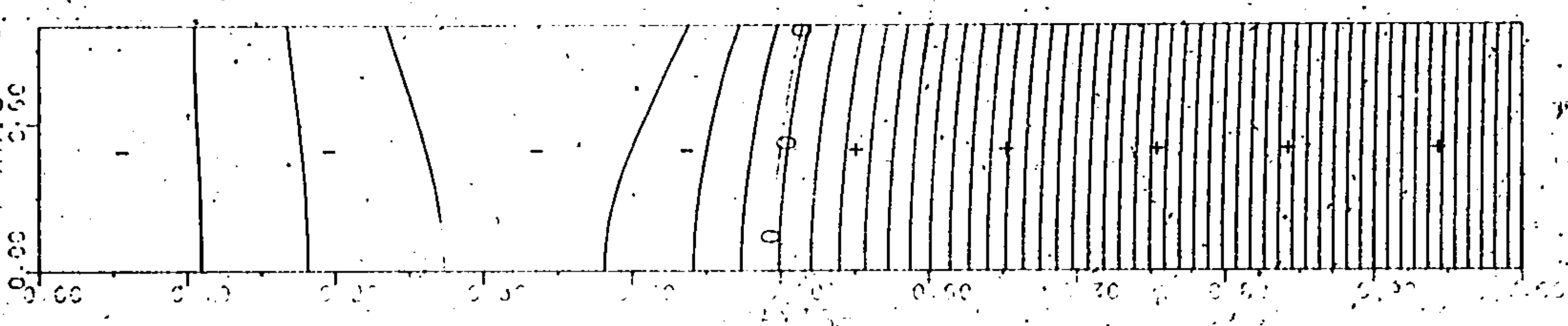



Y AXIS
X AXIS

WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

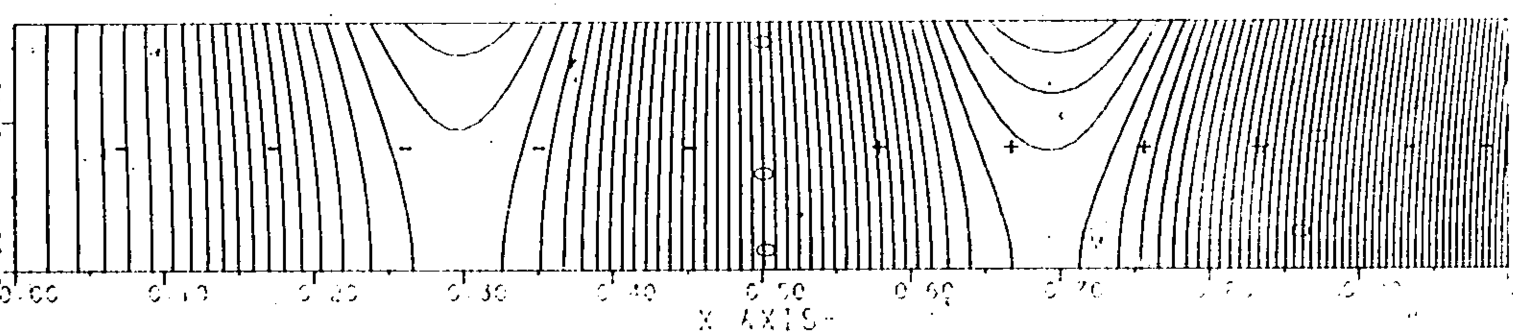
FIRST SYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

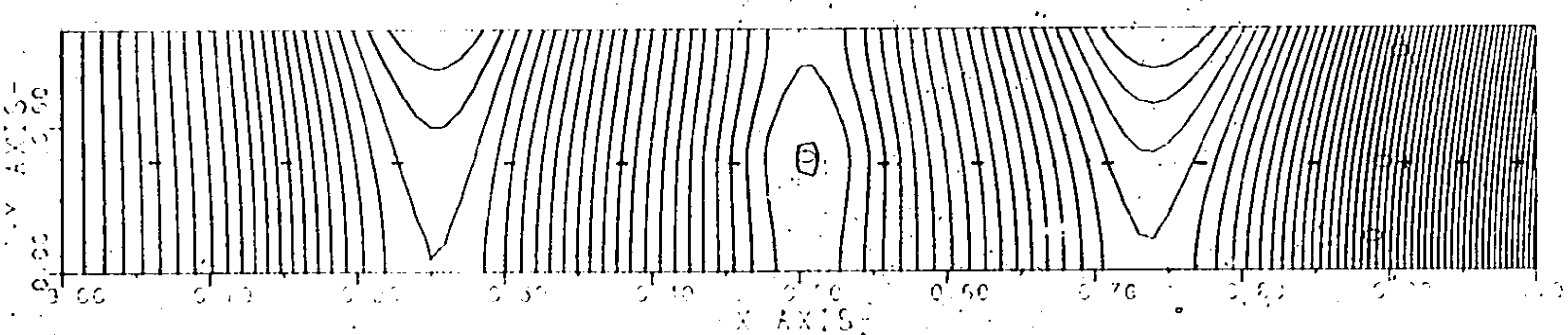
SECOND SYMMETRIC MODE WITH PHI = 1/6, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

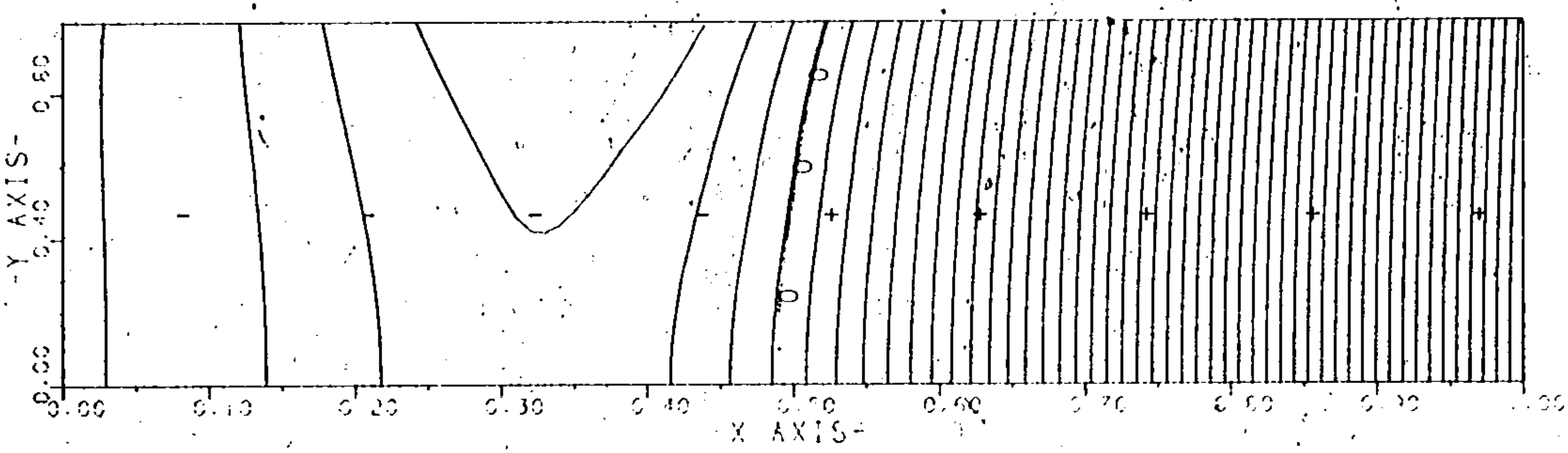
THIRD SYMMETRIC MODE WITH PHI - 1/6, U - 1/2, V - 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

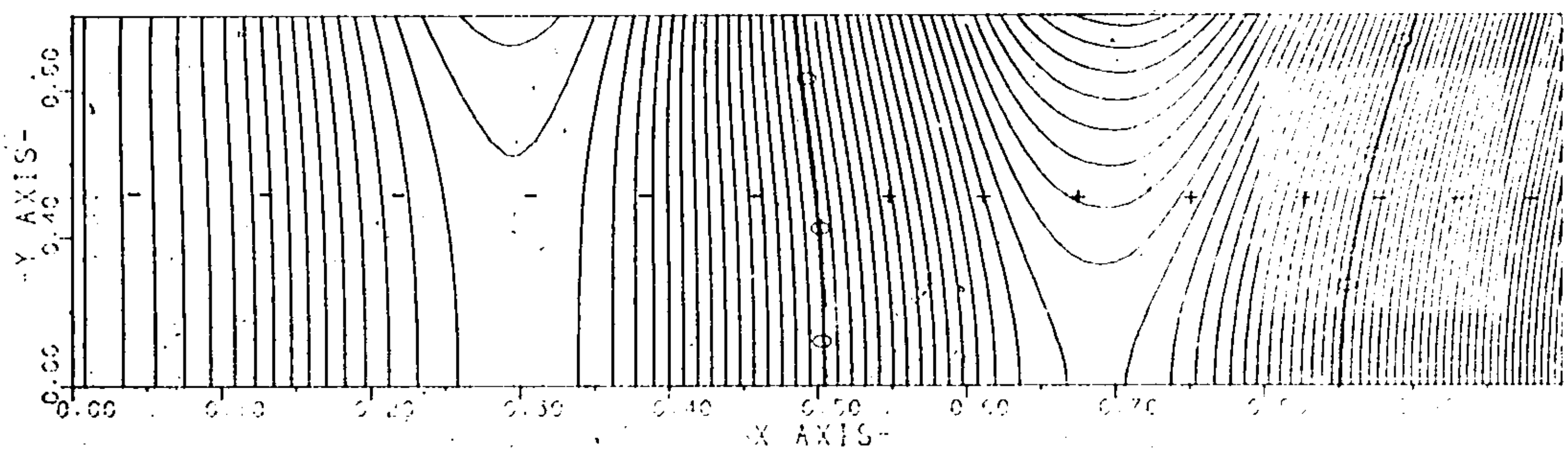
FIRST SYMMETRIC MODE WITH PHI = [illegible]/4, [illegible] = [illegible]/2, [illegible] = [illegible]/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

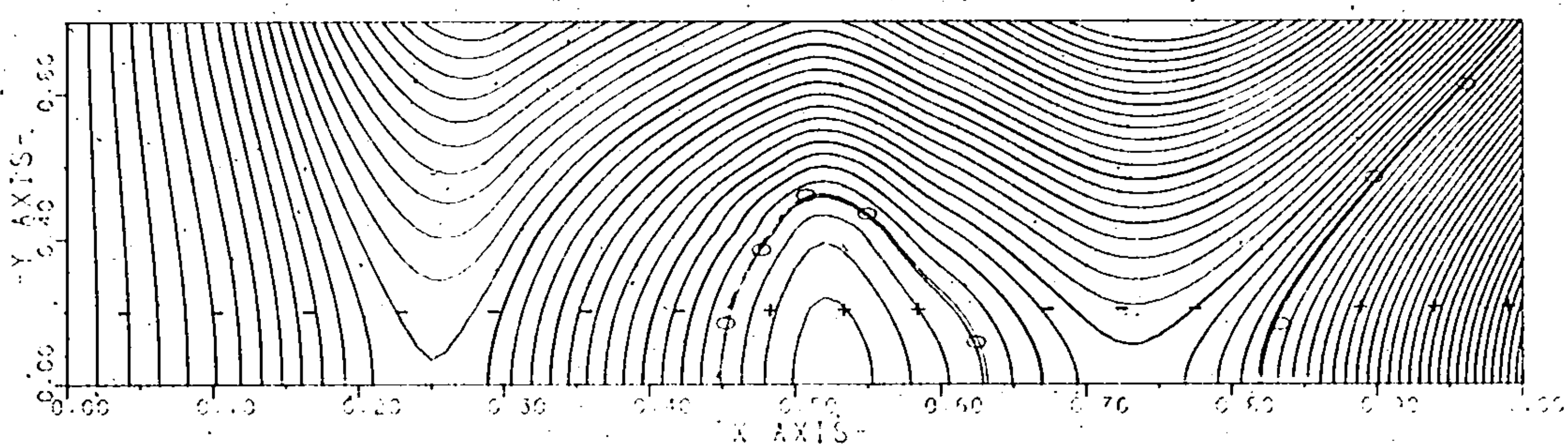
SECOND SYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

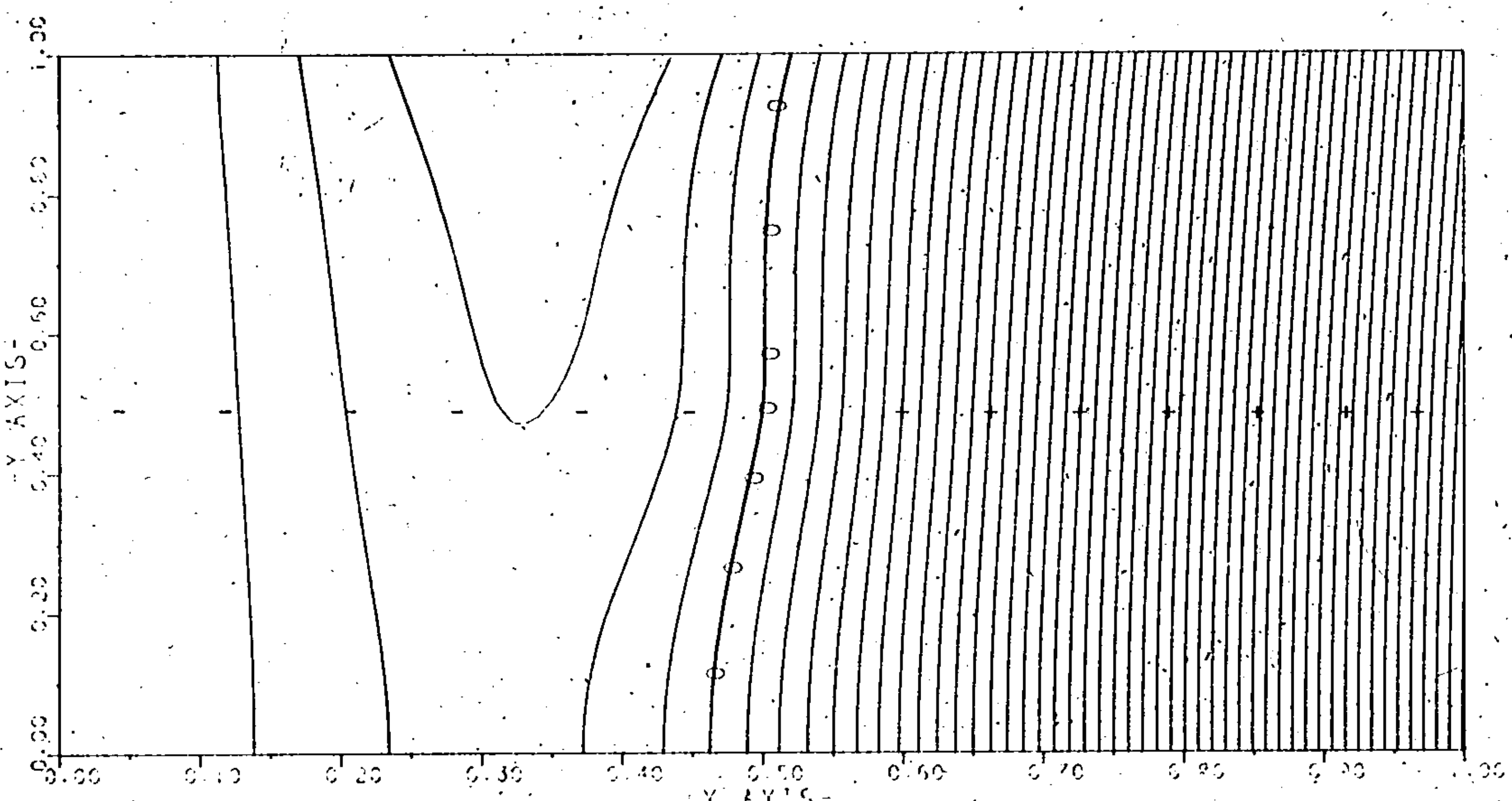
THIRD SYMMETRIC MODE WITH PHI = 1/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

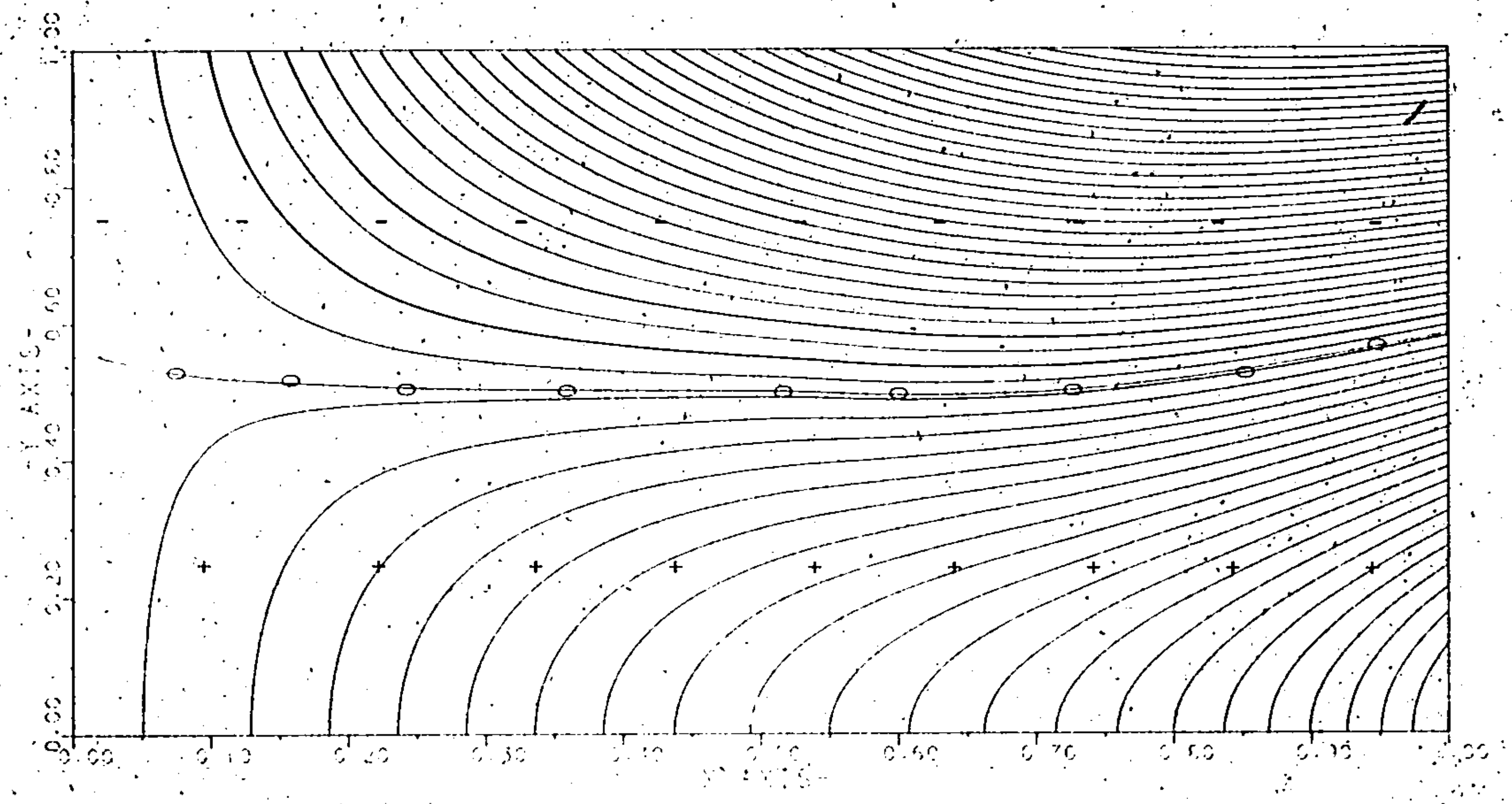
FIRST SYMMETRIC MODE WITH PHI = [illegible]2, θ = [illegible]2, V = [illegible]2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

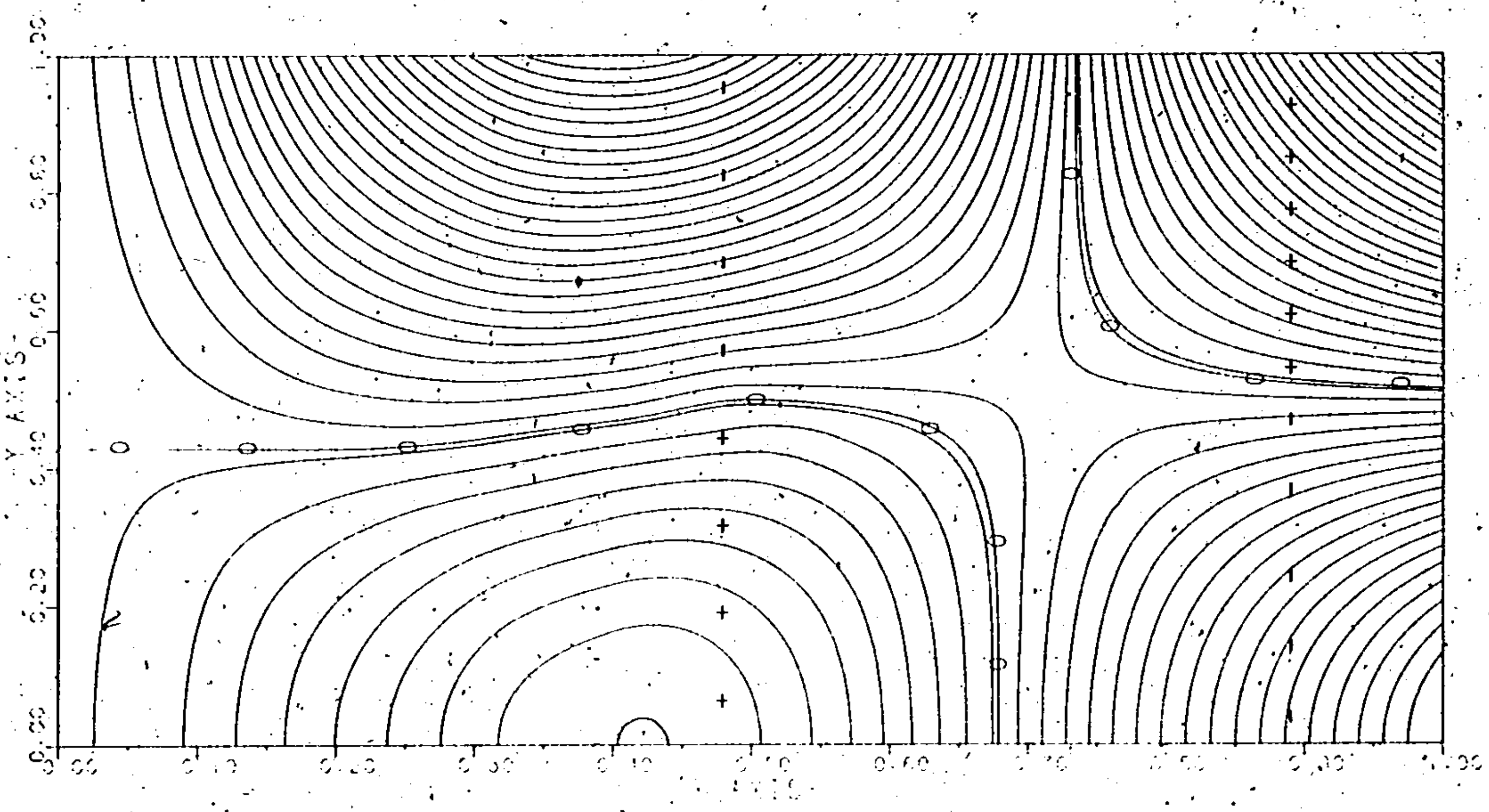
SECOND SYMMETRIC MODE WITH PHI = 1/2, U = [illegible]/2, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

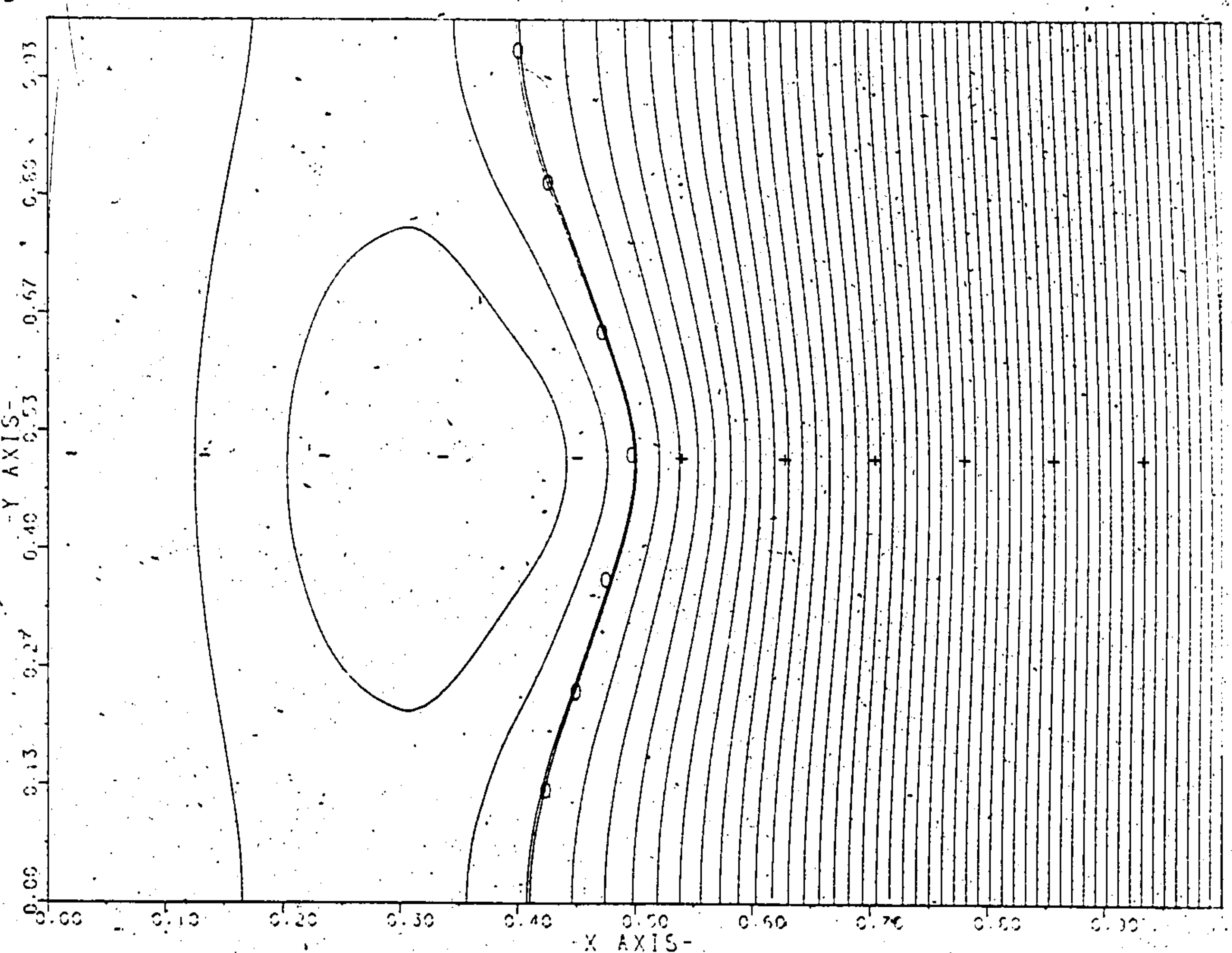
THIRD SYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL-POINT SUPPORT ***

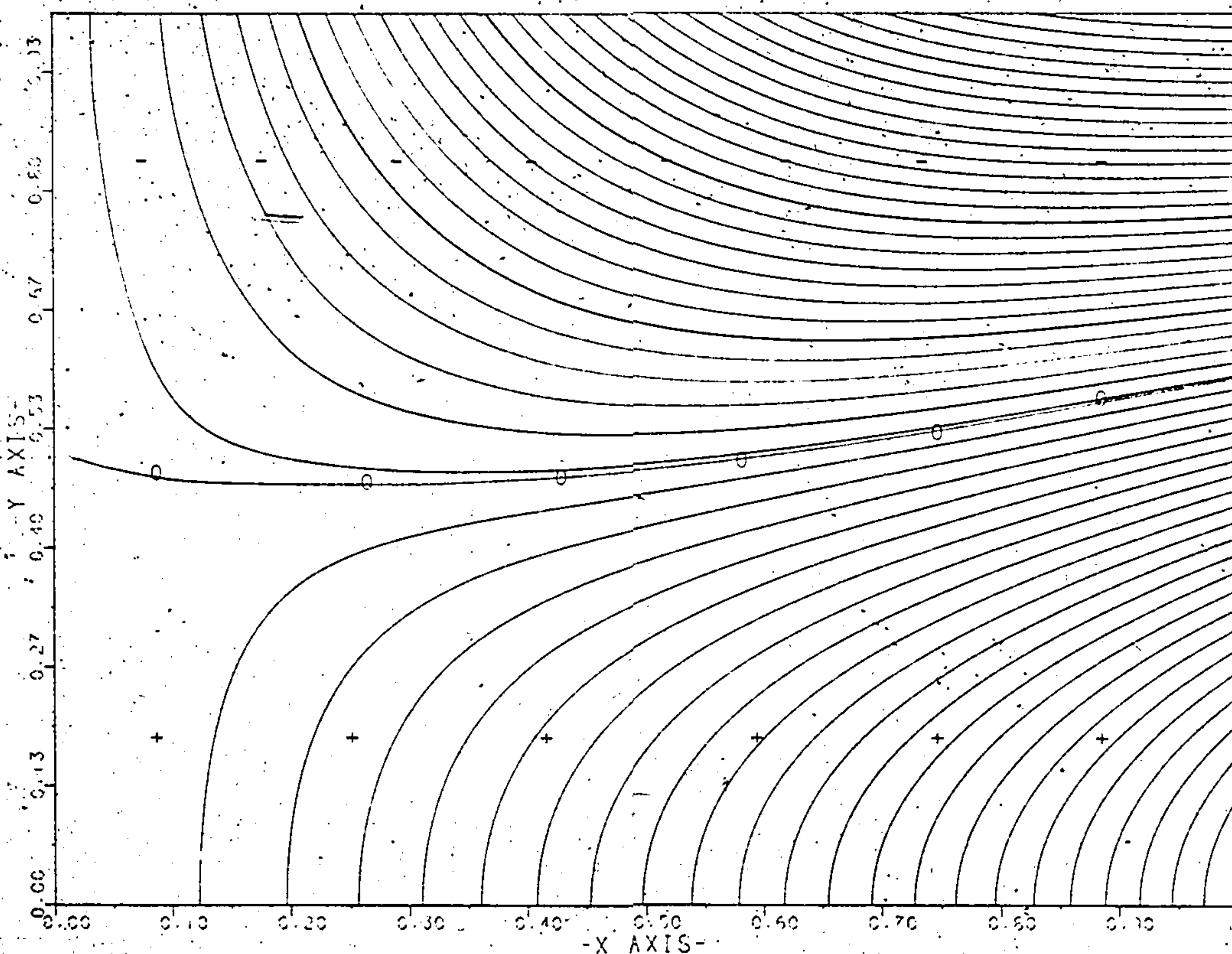
FIRST SYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

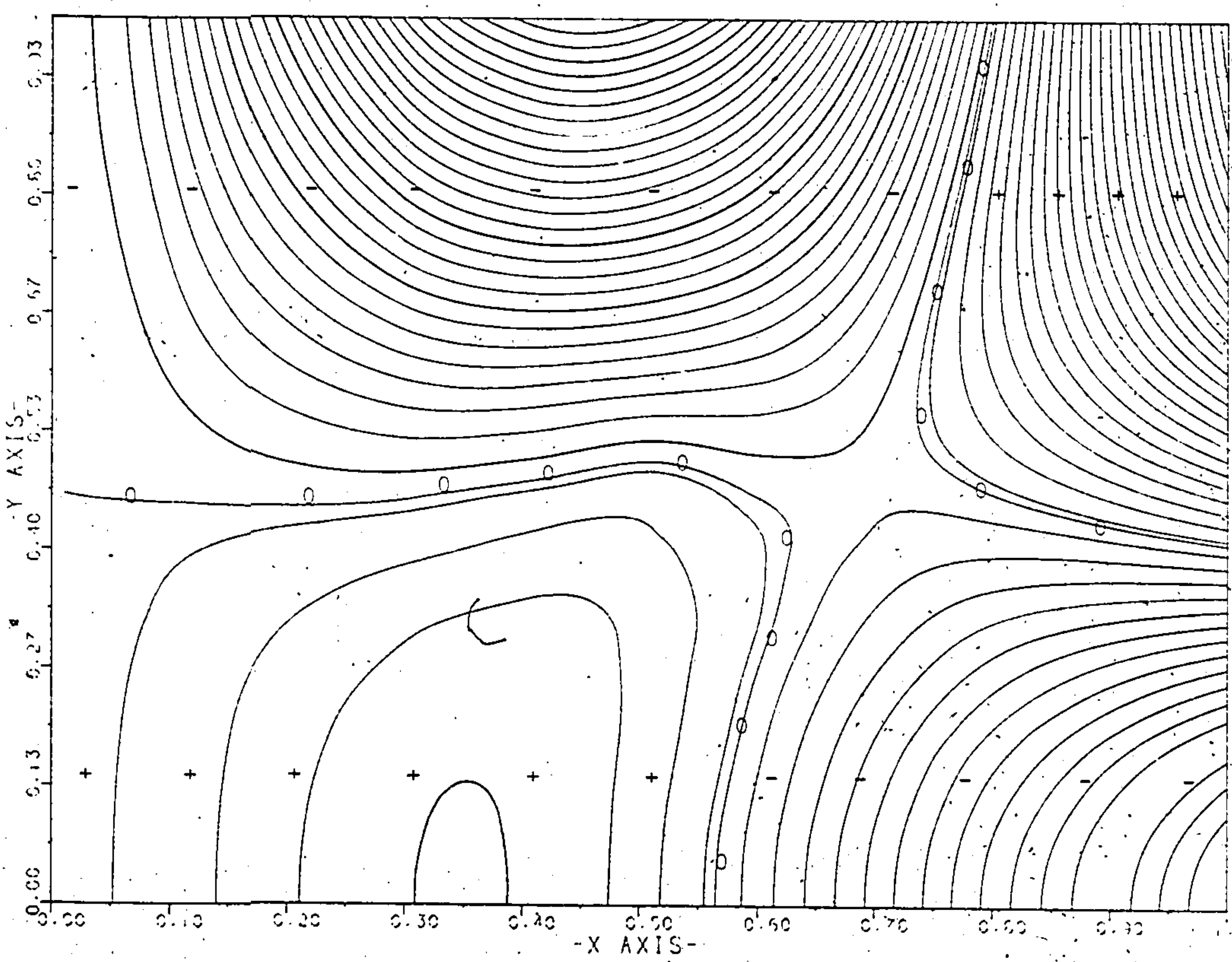
SECOND SYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

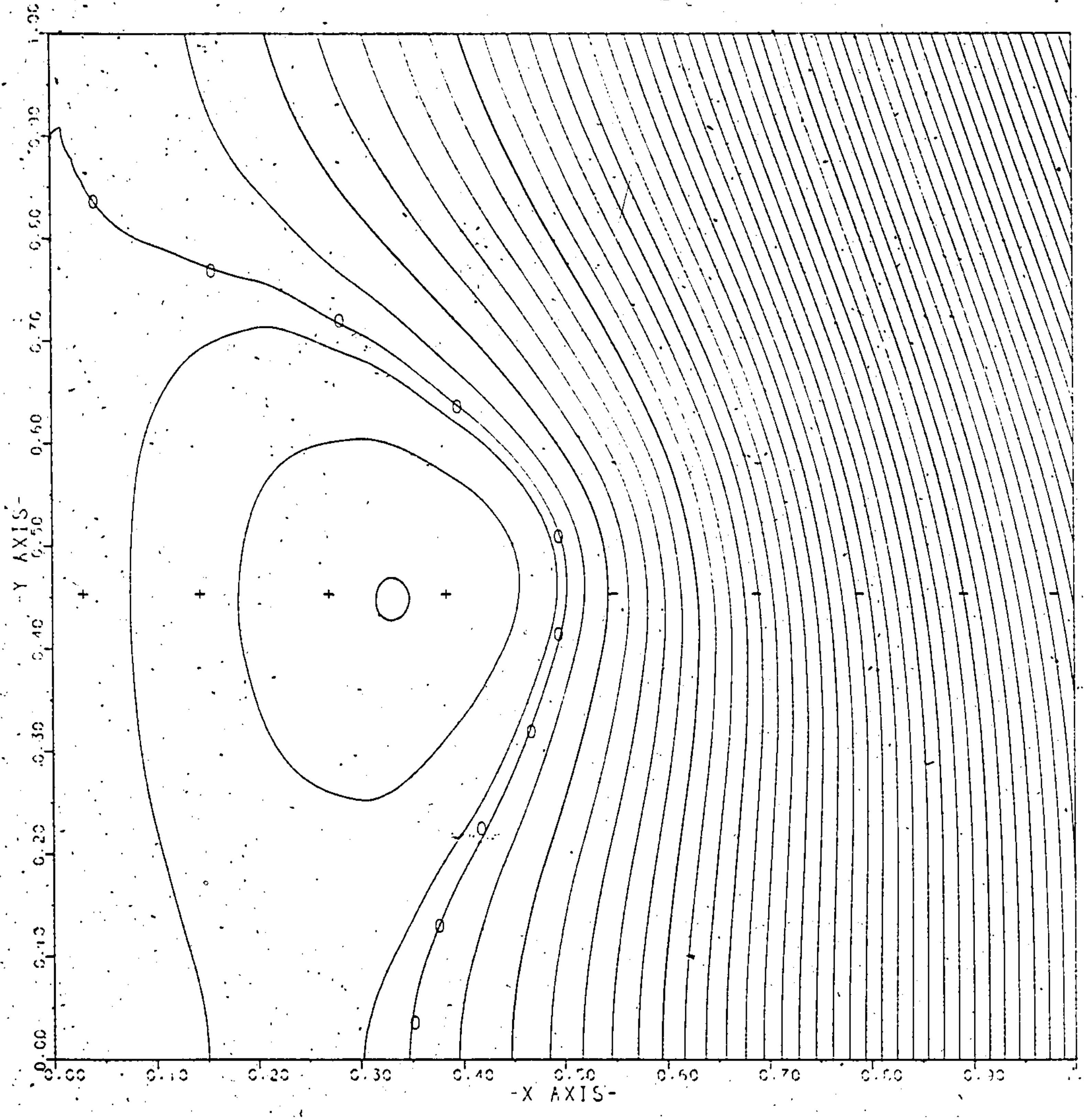
THIRD SYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = .72



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

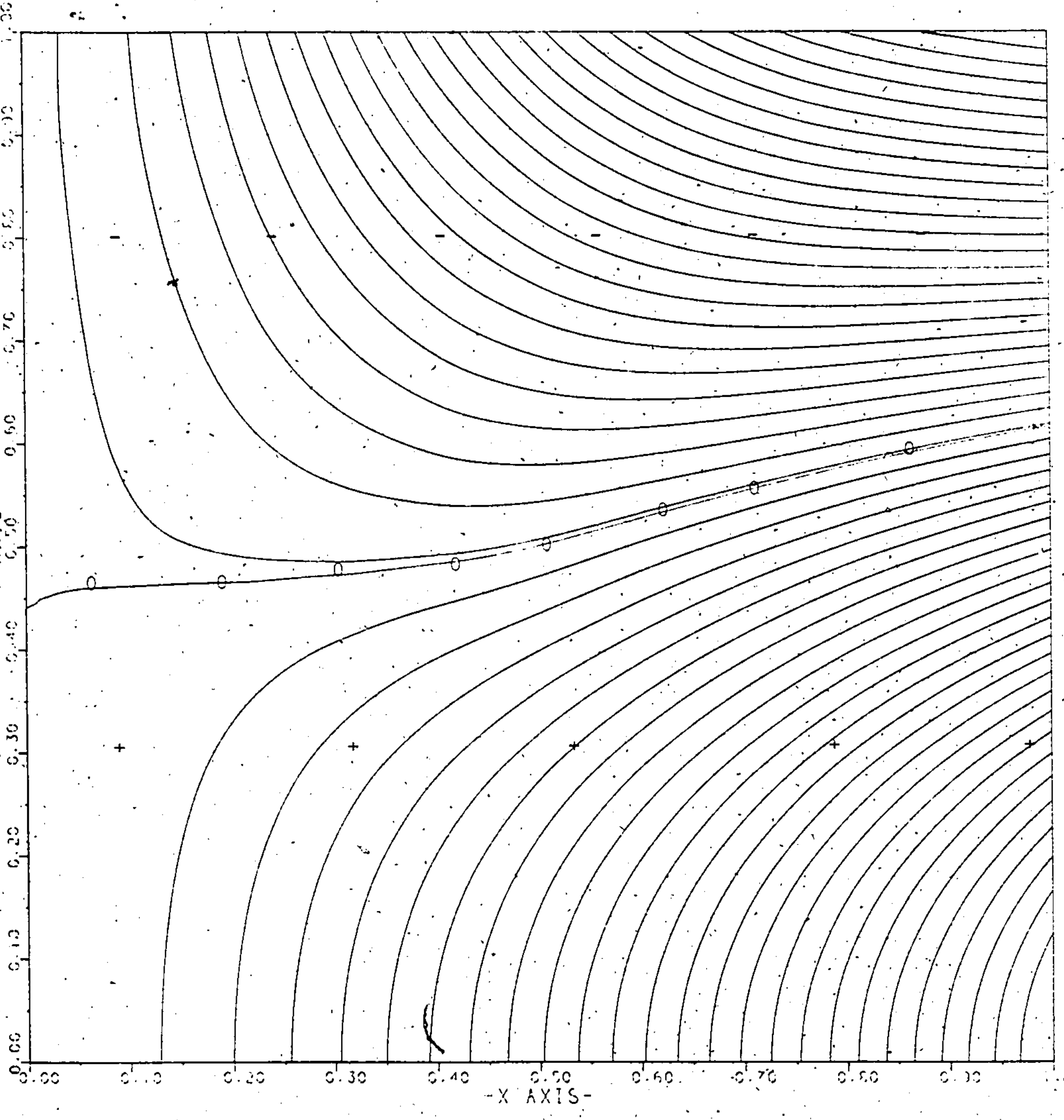
FIRST SYMMETRIC MODE WITH PHI = 1/1, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

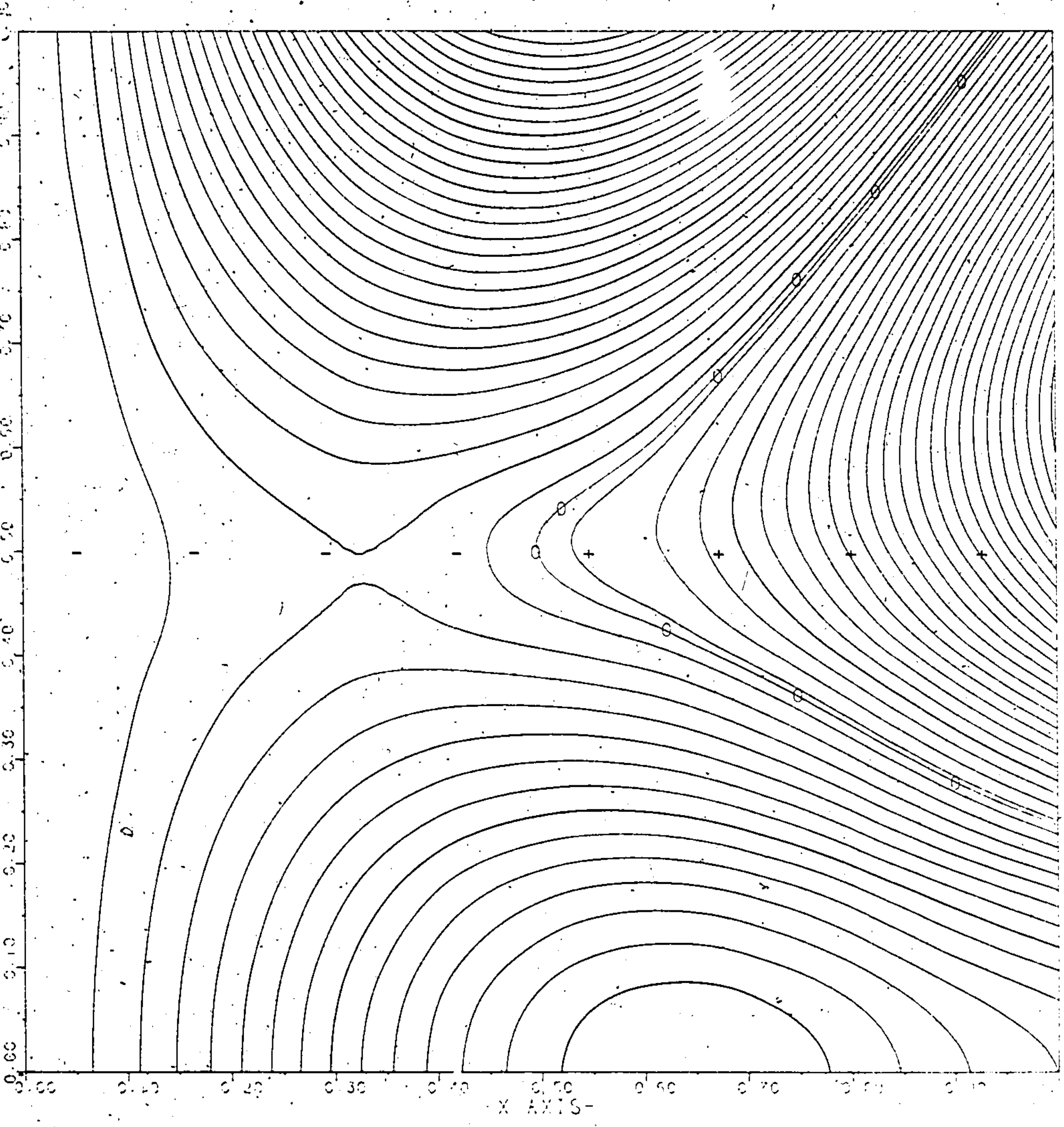
SECOND SYMMETRIC MODE WITH PHI = [illegible], U = [illegible]/2, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

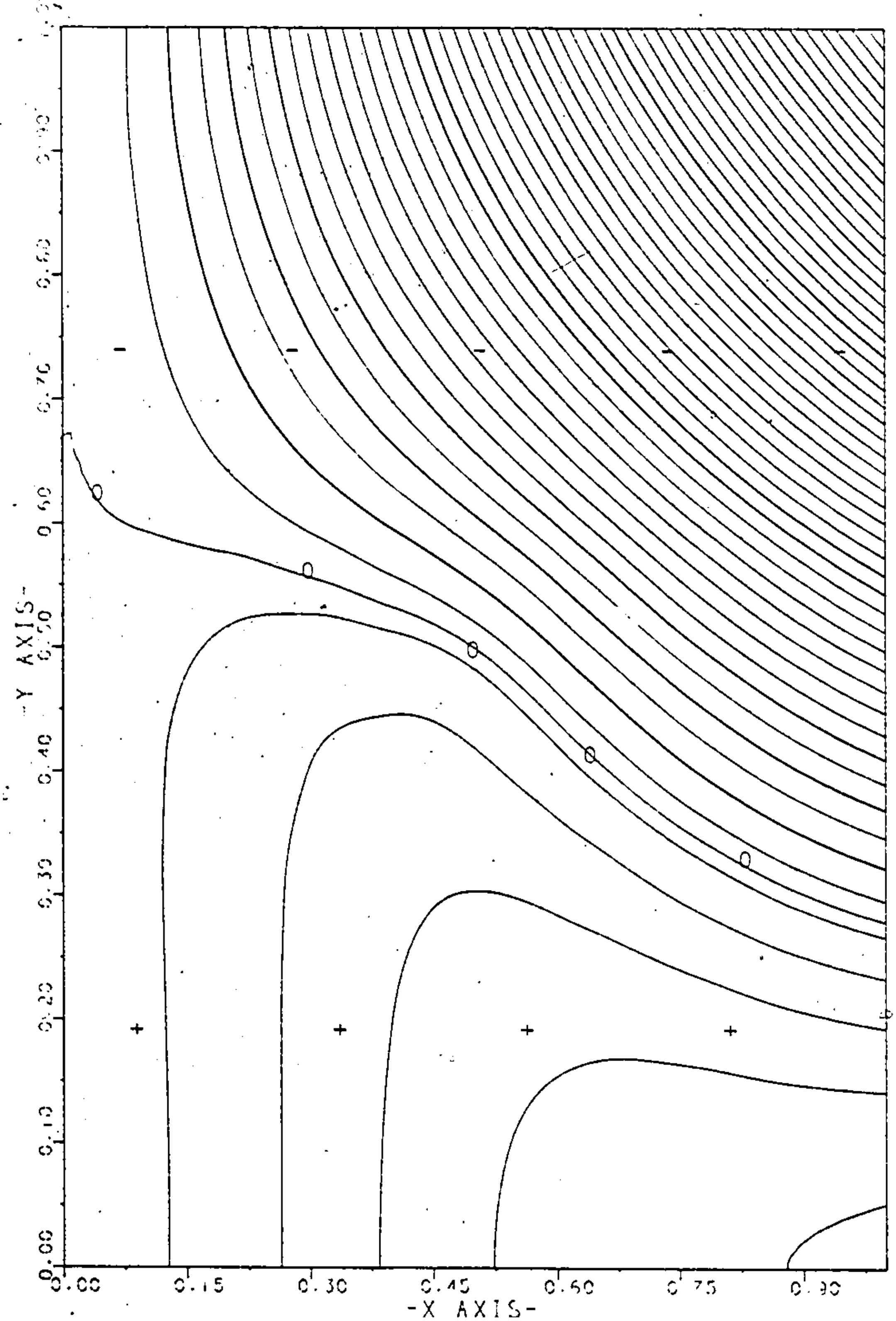
THIRD SYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

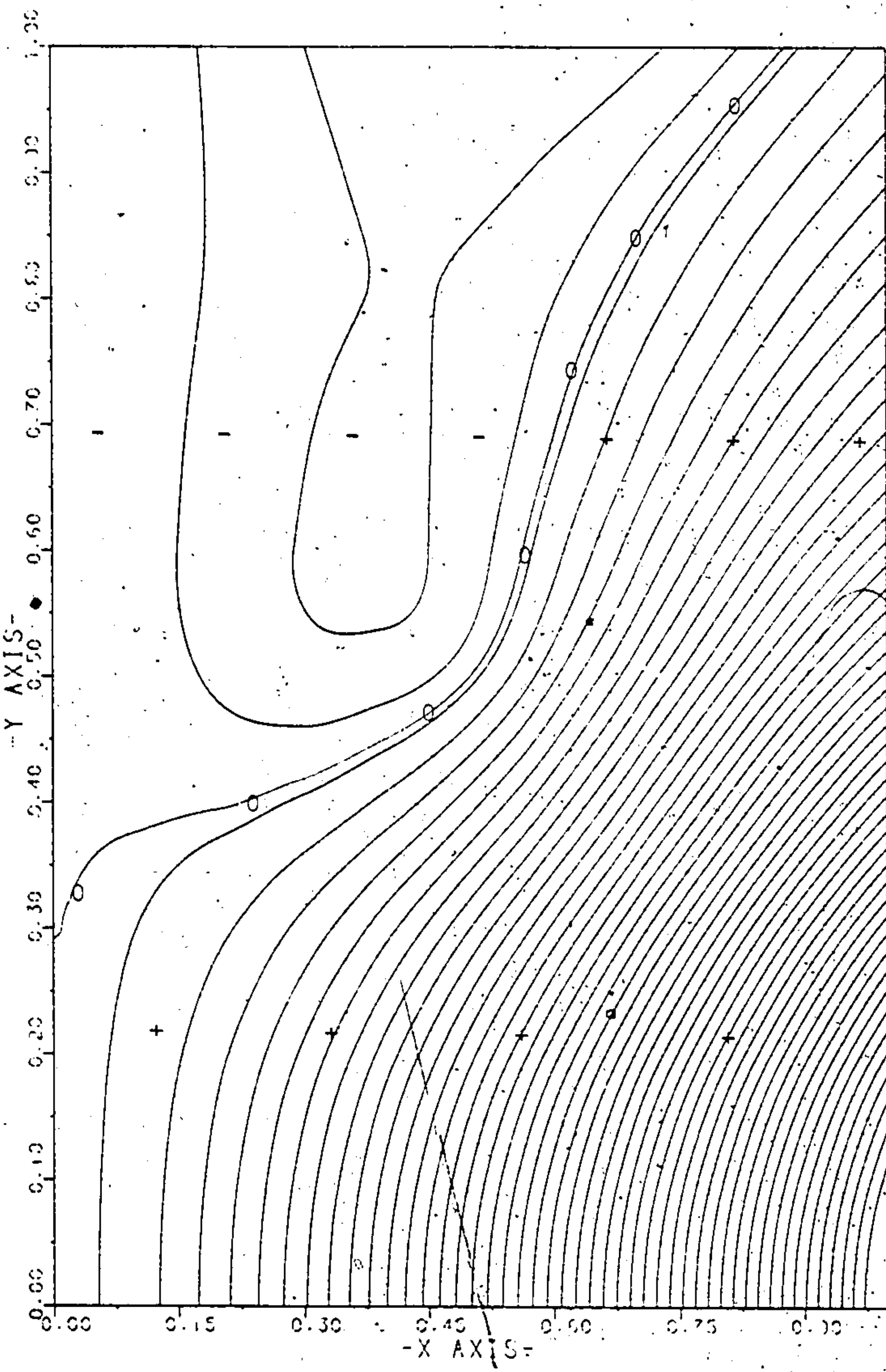
FIRST SYMMETRIC MODE WITH PHI - 3/2, U - [illegible]/2, V - [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

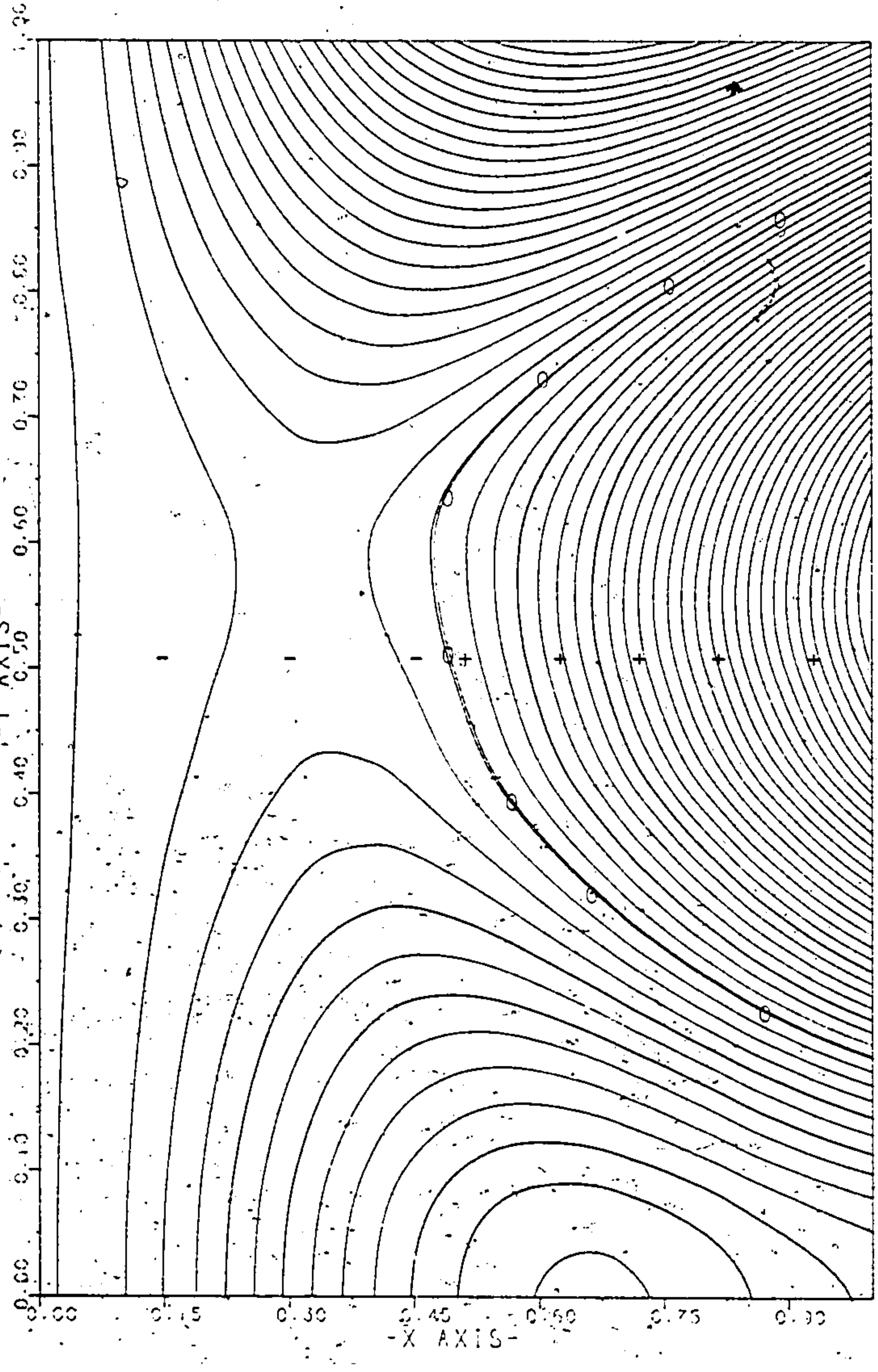
SECOND SYMMETRIC MODE WITH PHI - 3/2, U - 1/2, V1 - .32



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

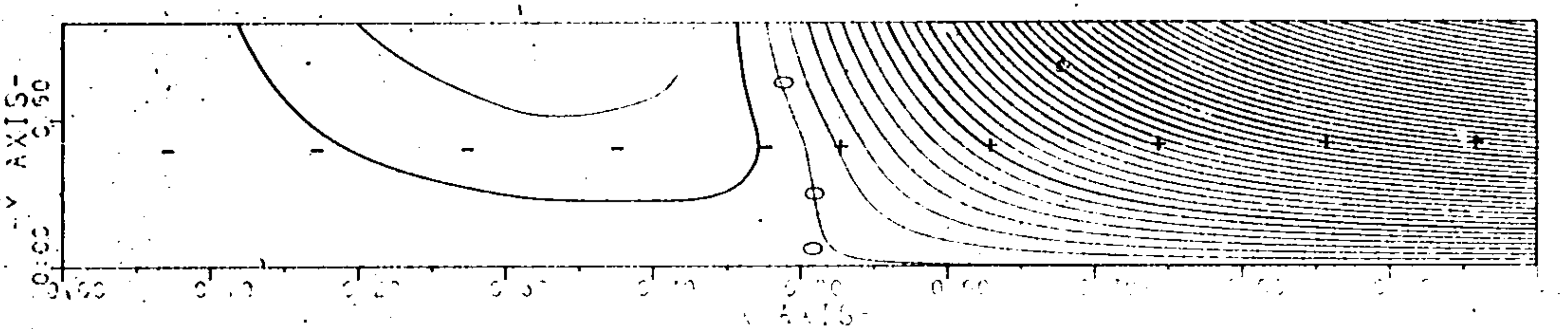
THIRD SYMMETRIC MODE WITH PHI - 3/2: U - 9/2, V - ./2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

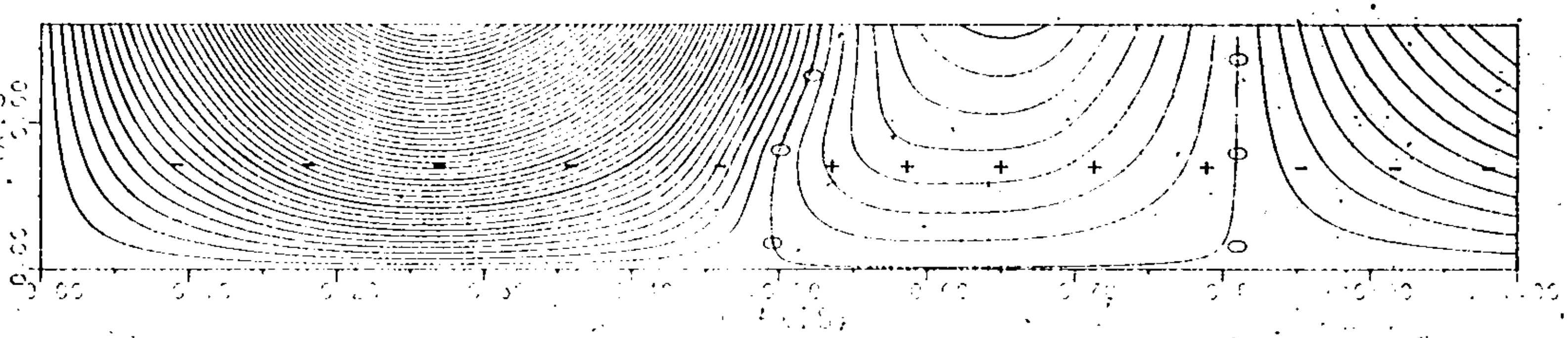
FIRST ANTISYMMETRIC MODE WITH PHI = 1/6, U = ?/2, V = ./2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

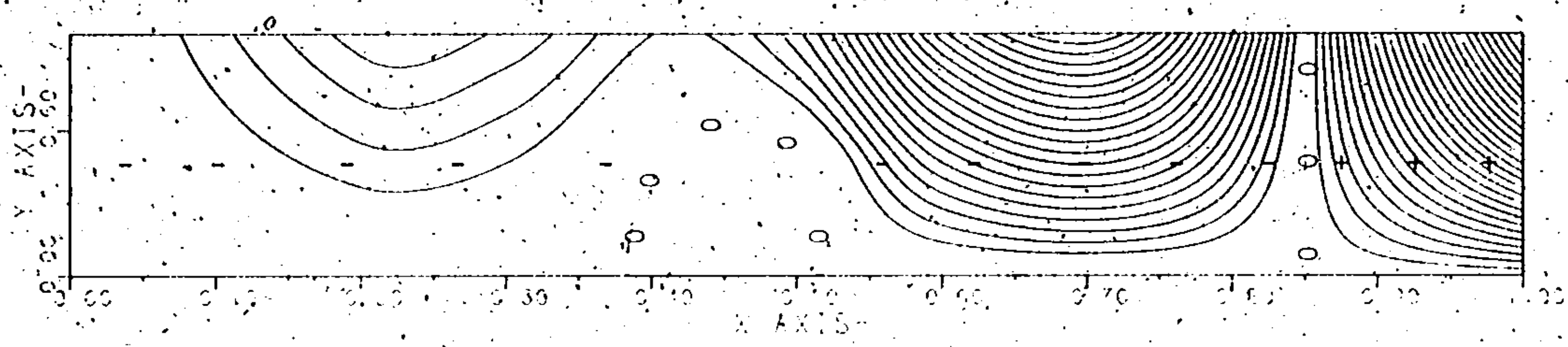
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible]/6, U = [illegible]/2, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

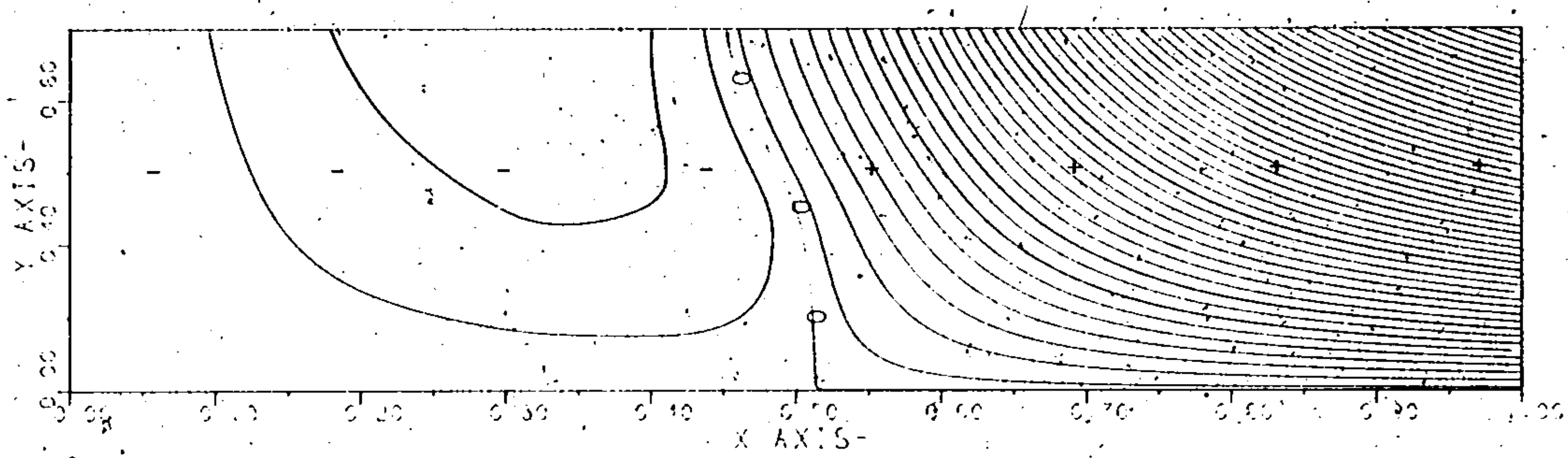
THIRD ANTISYMMETRIC MODE WITH PHI = 1.6, U = [illegible]2, V = [illegible]2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

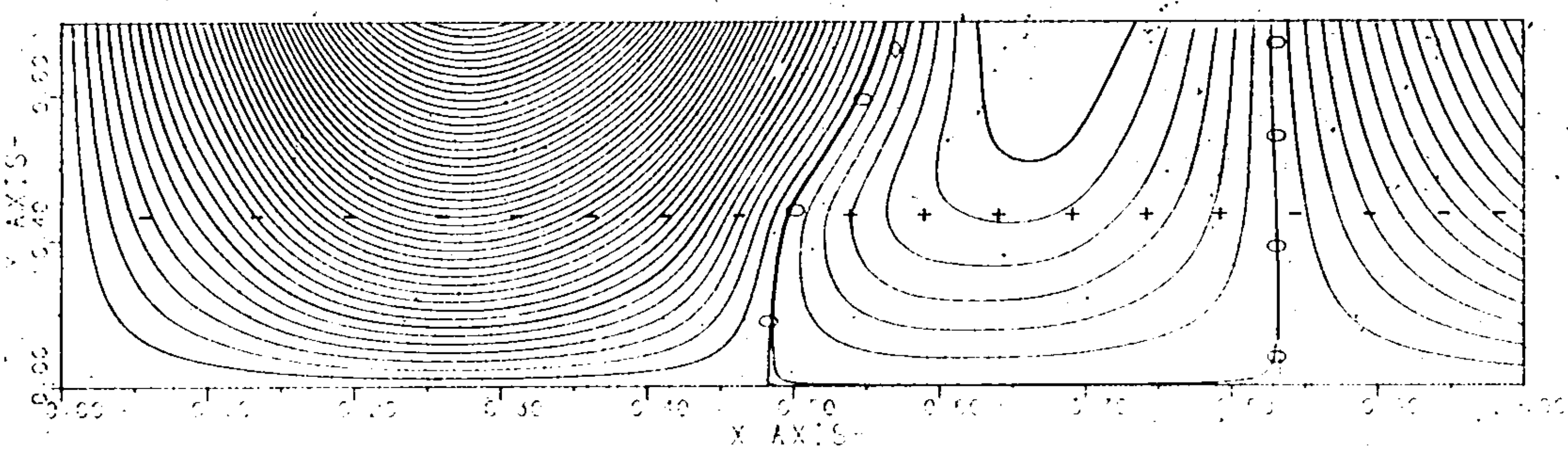
FIRST ANTISYMMETRIC MODE WITH PHI = π/4, U = π/2, V = π/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

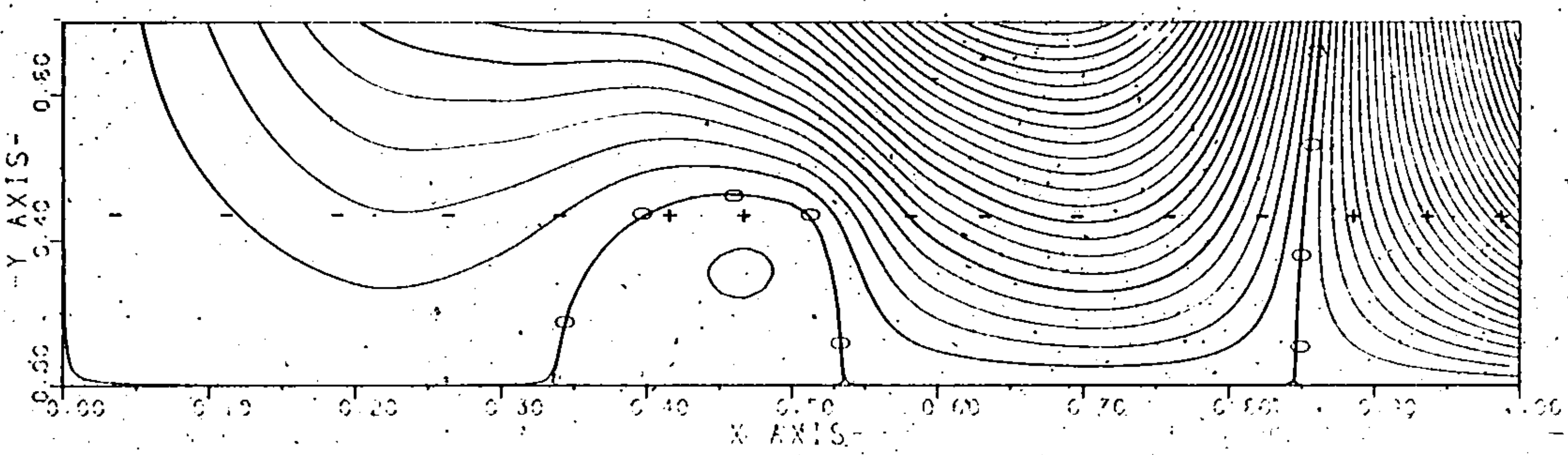
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible]4, U = [illegible]2, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

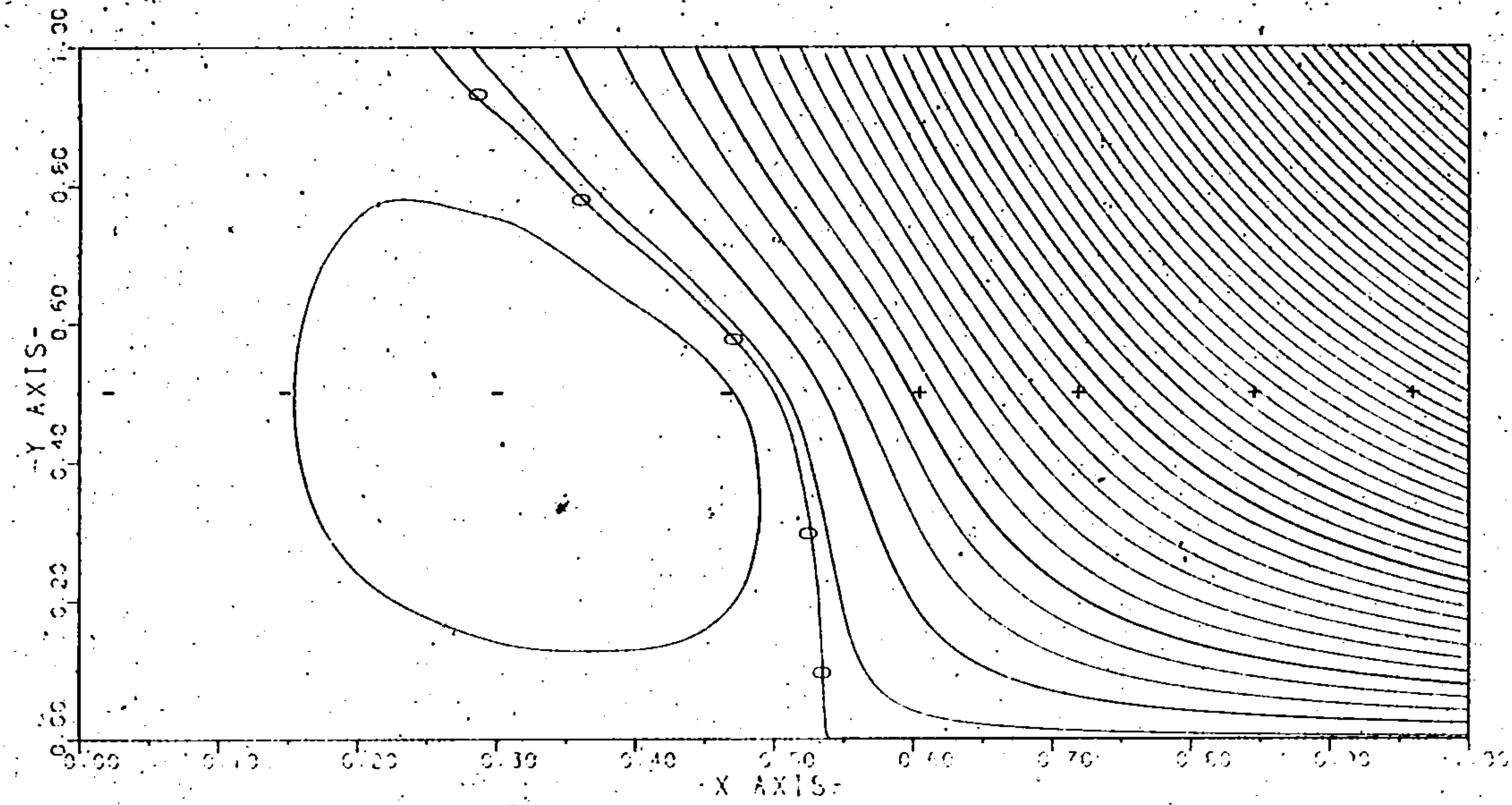
THIRD ANTISYMMETRIC MODE WITH PHI - [illegible]/4, [illegible] - [illegible]/2, [illegible] [illegible]/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

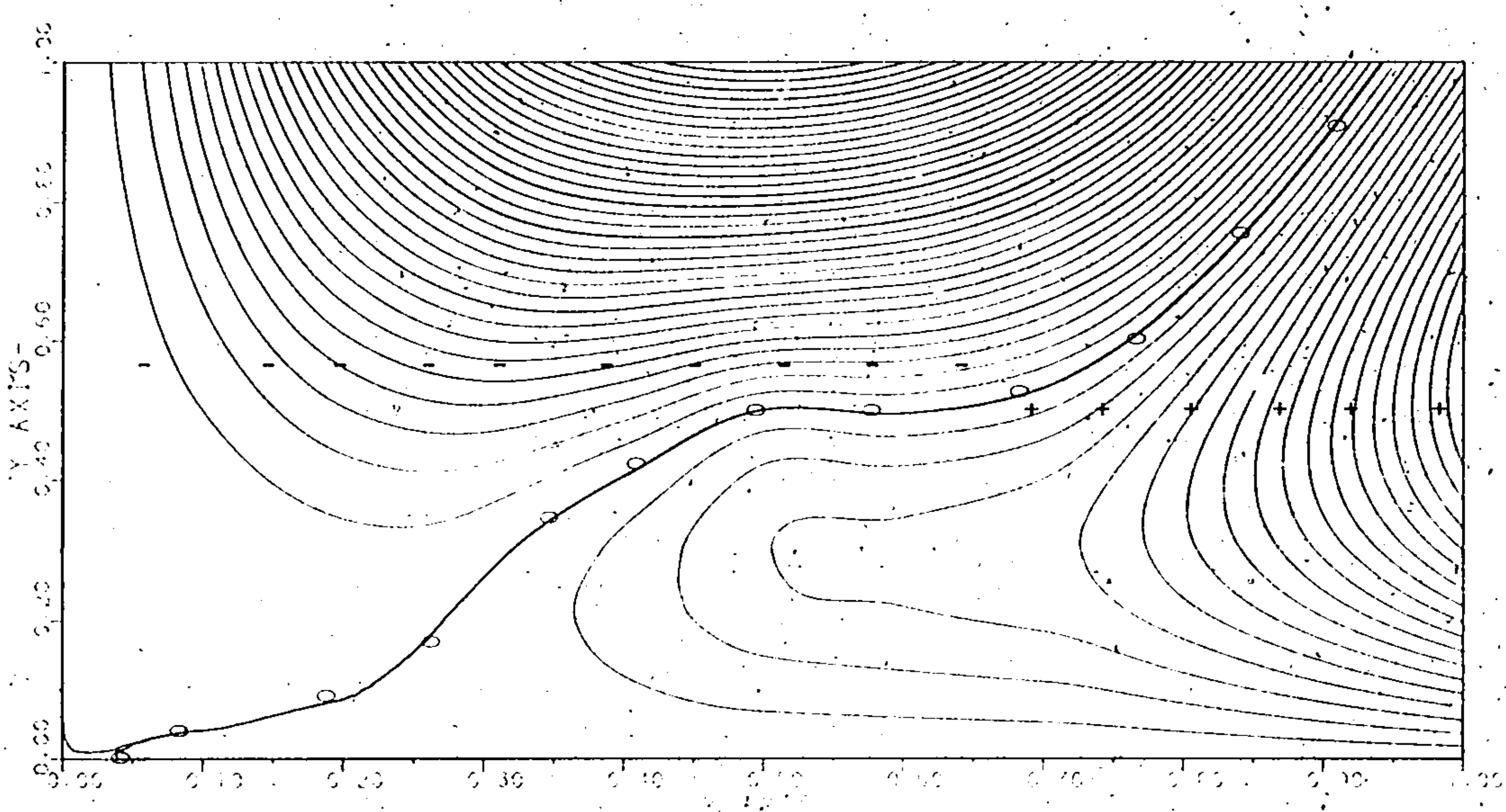
FIRST ANTISYMMETRIC MODE WITH PHI = [illegible]2, U = [illegible]/2, V = [illegible]/2



WAVE PLATE GRAPHIC

***. THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

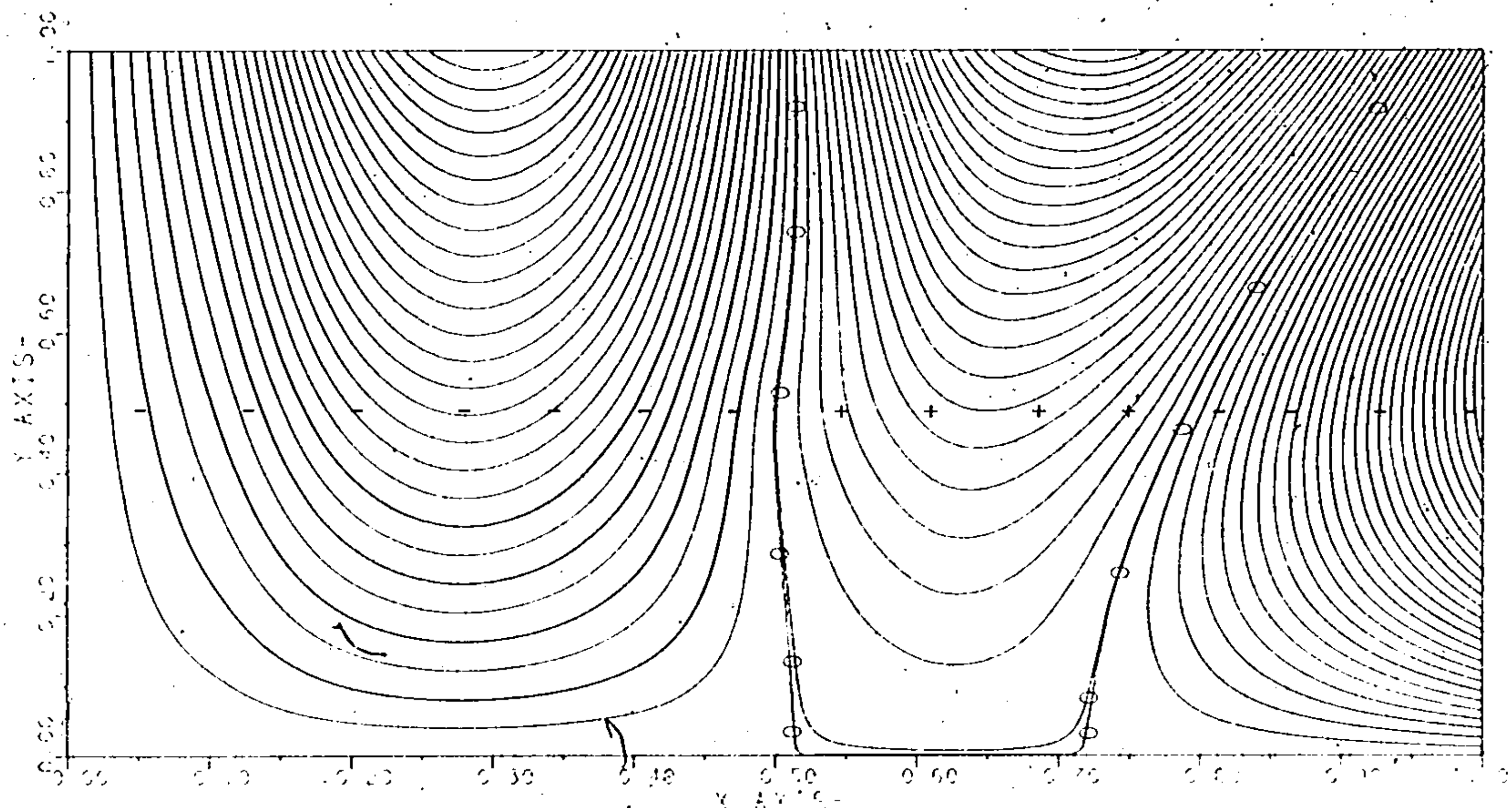
SECOND ANTISYMMETRIC MODE WITH PHI - 1/2, U - 1/2, V = 1/2



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

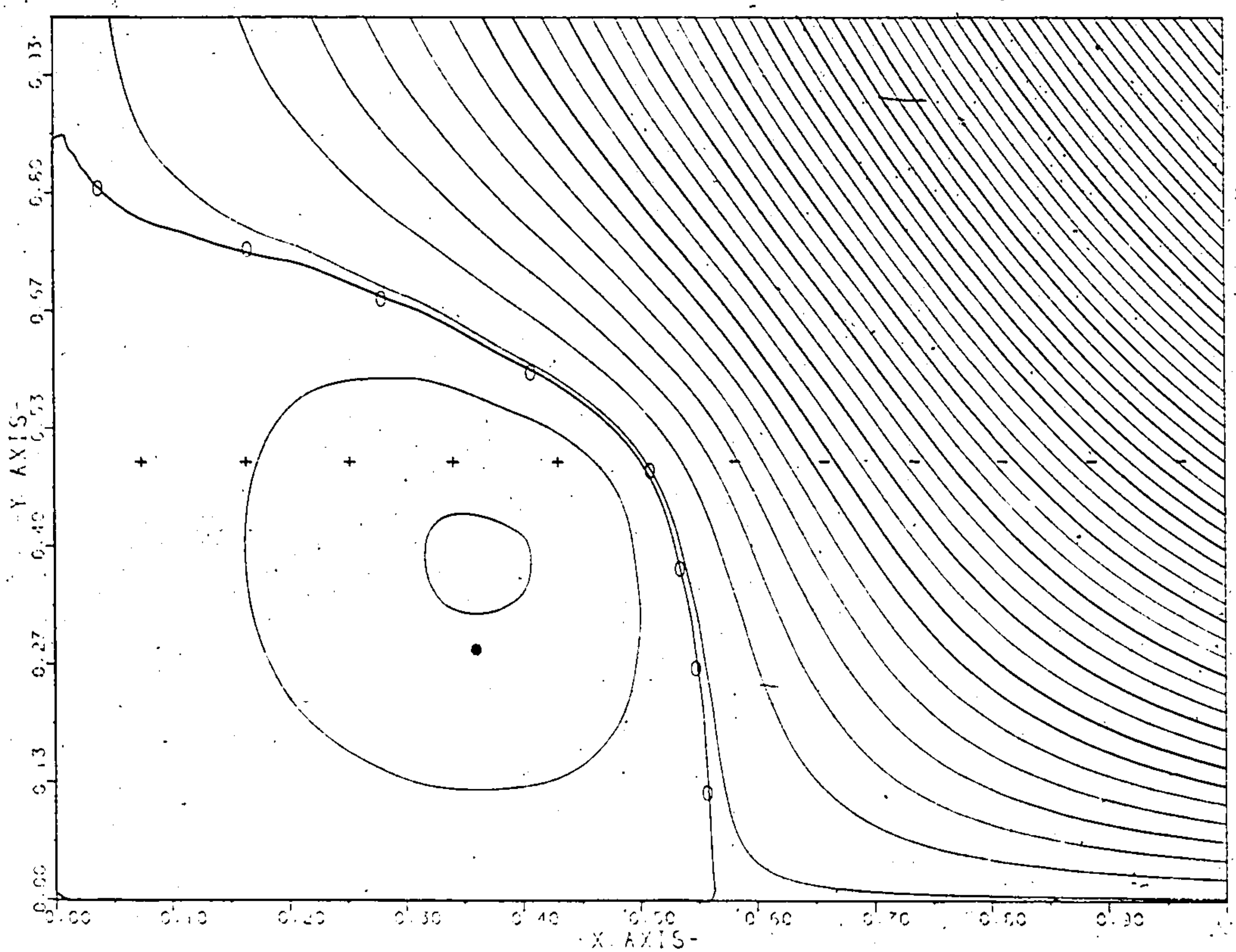
THIRD ANTISYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

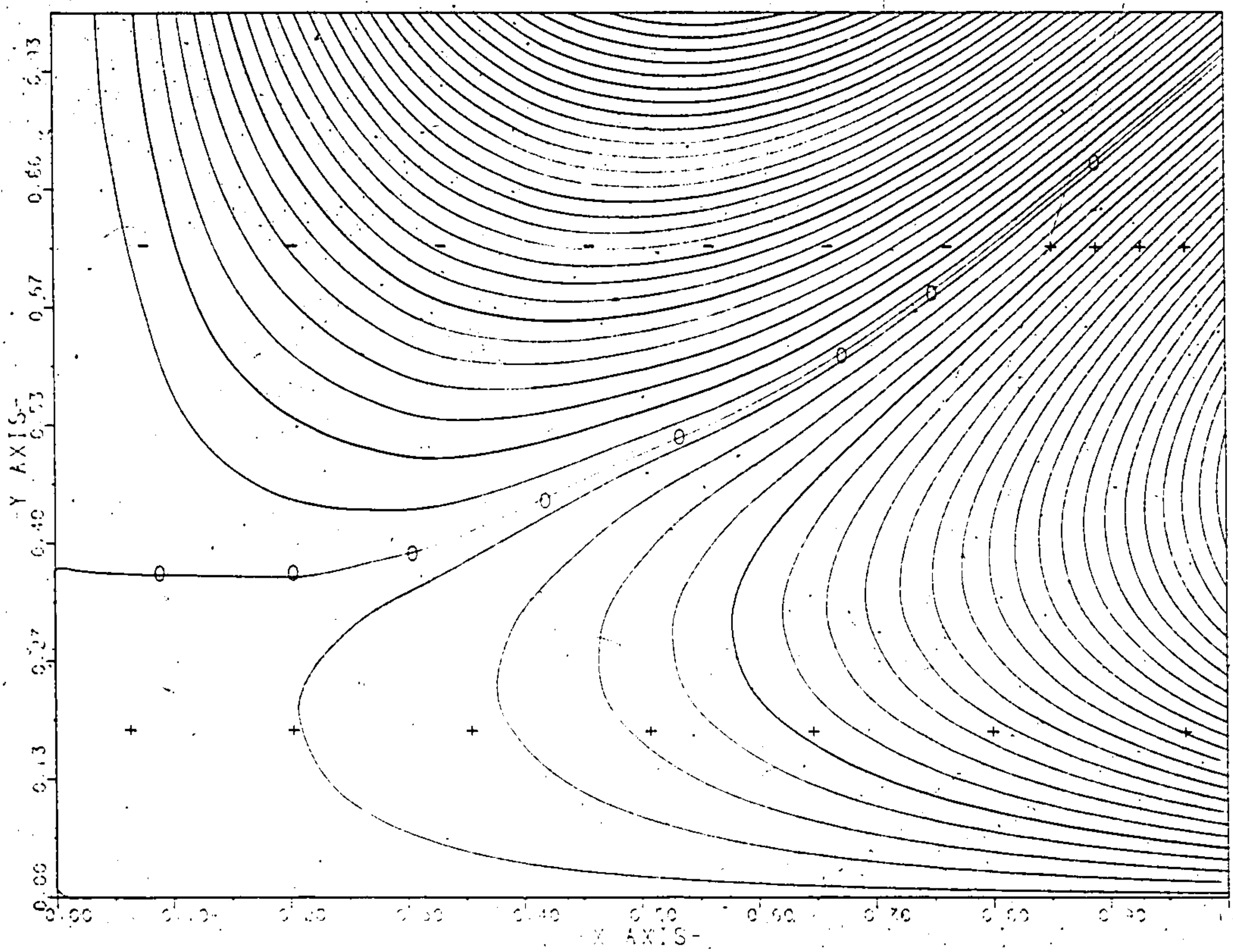
FIRST ANTISYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

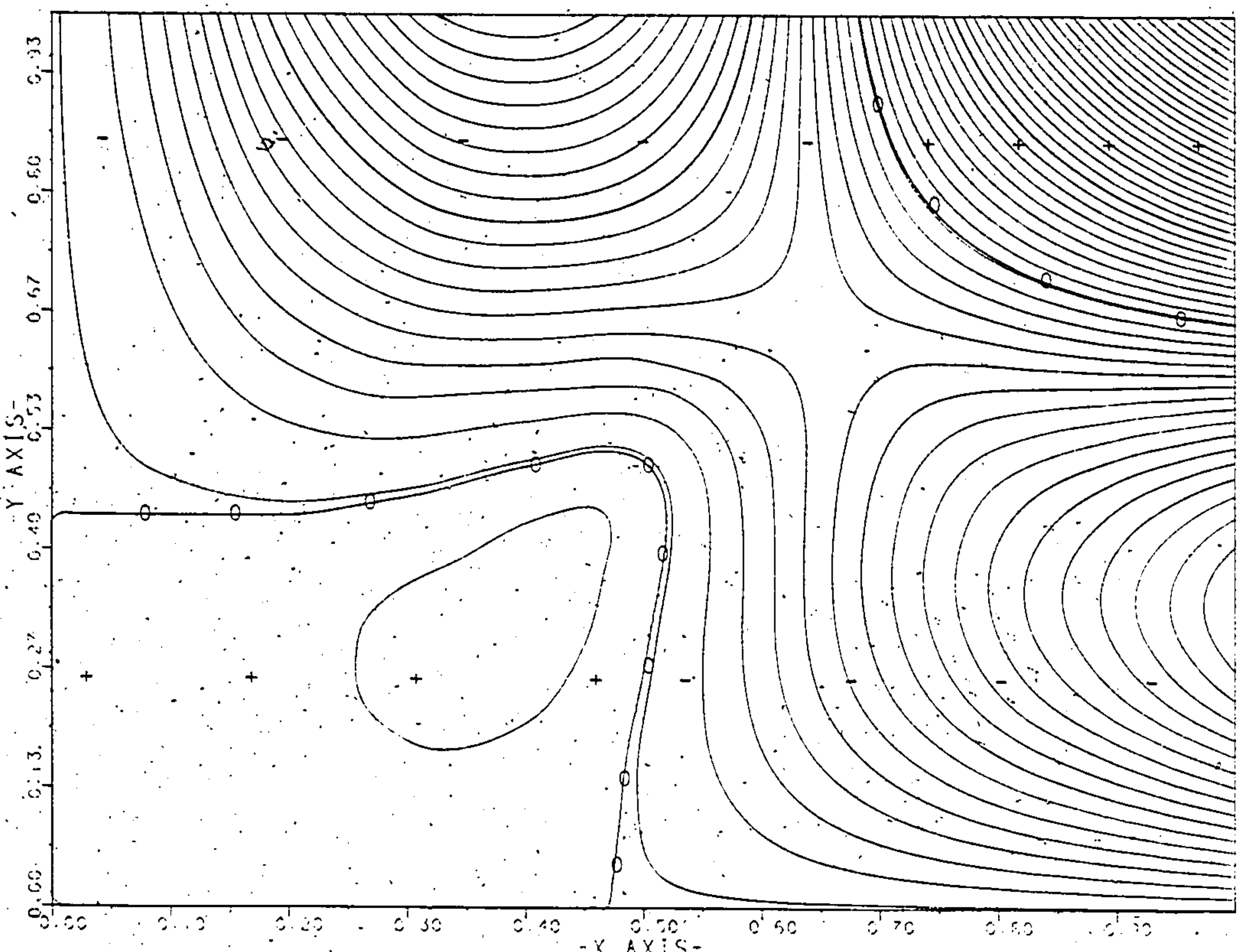
SECOND ANTISYMMETRIC MODE WITH PHI = 3/4. U = [illegible]/2. V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

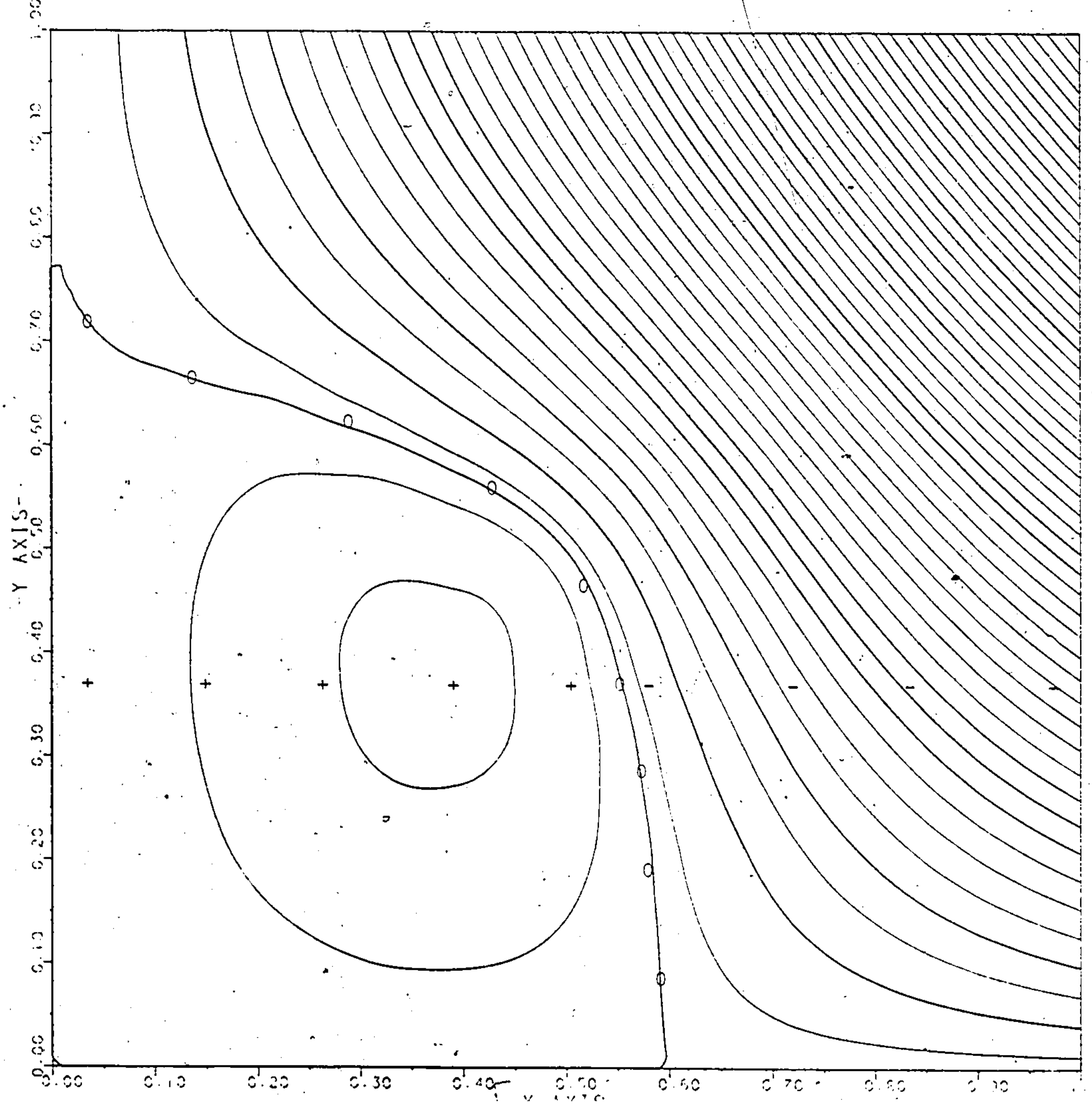
THIRD ANTISYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

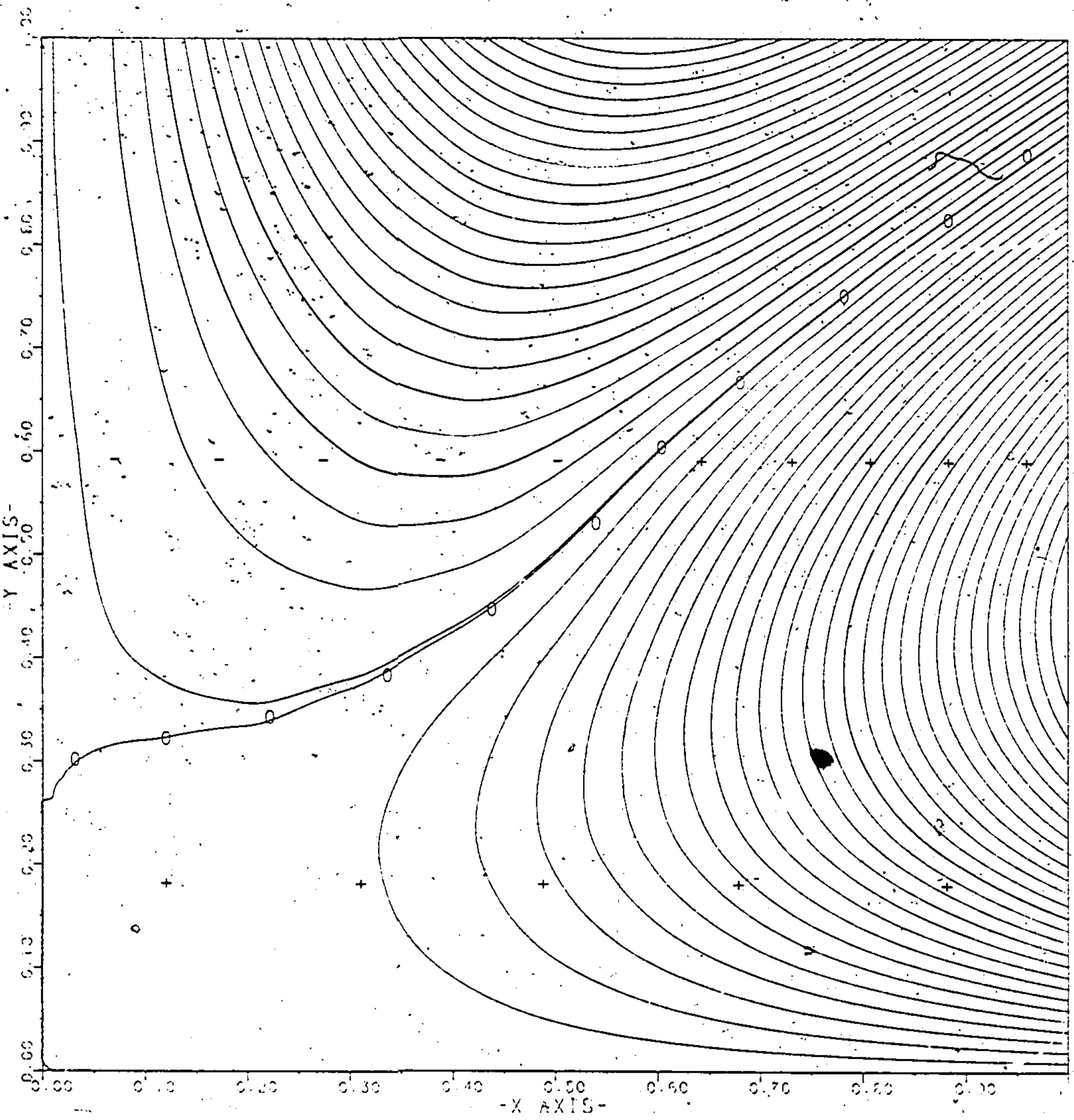
FIRST ANTISYMMETRIC MODE WITH PHI = [illegible], U = 1/2, V = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

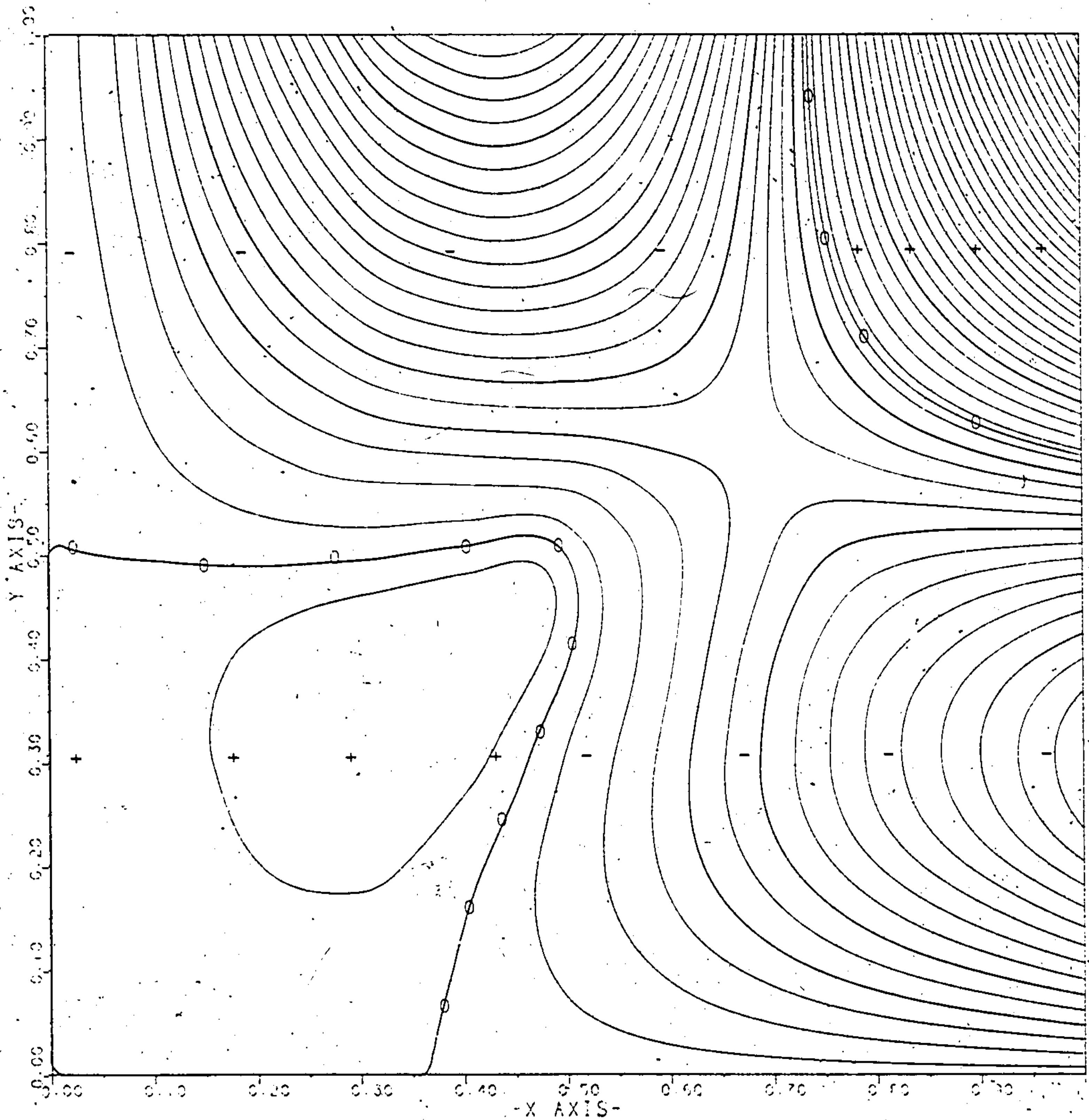
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible] Q = 1/2, [illegible] = 1/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

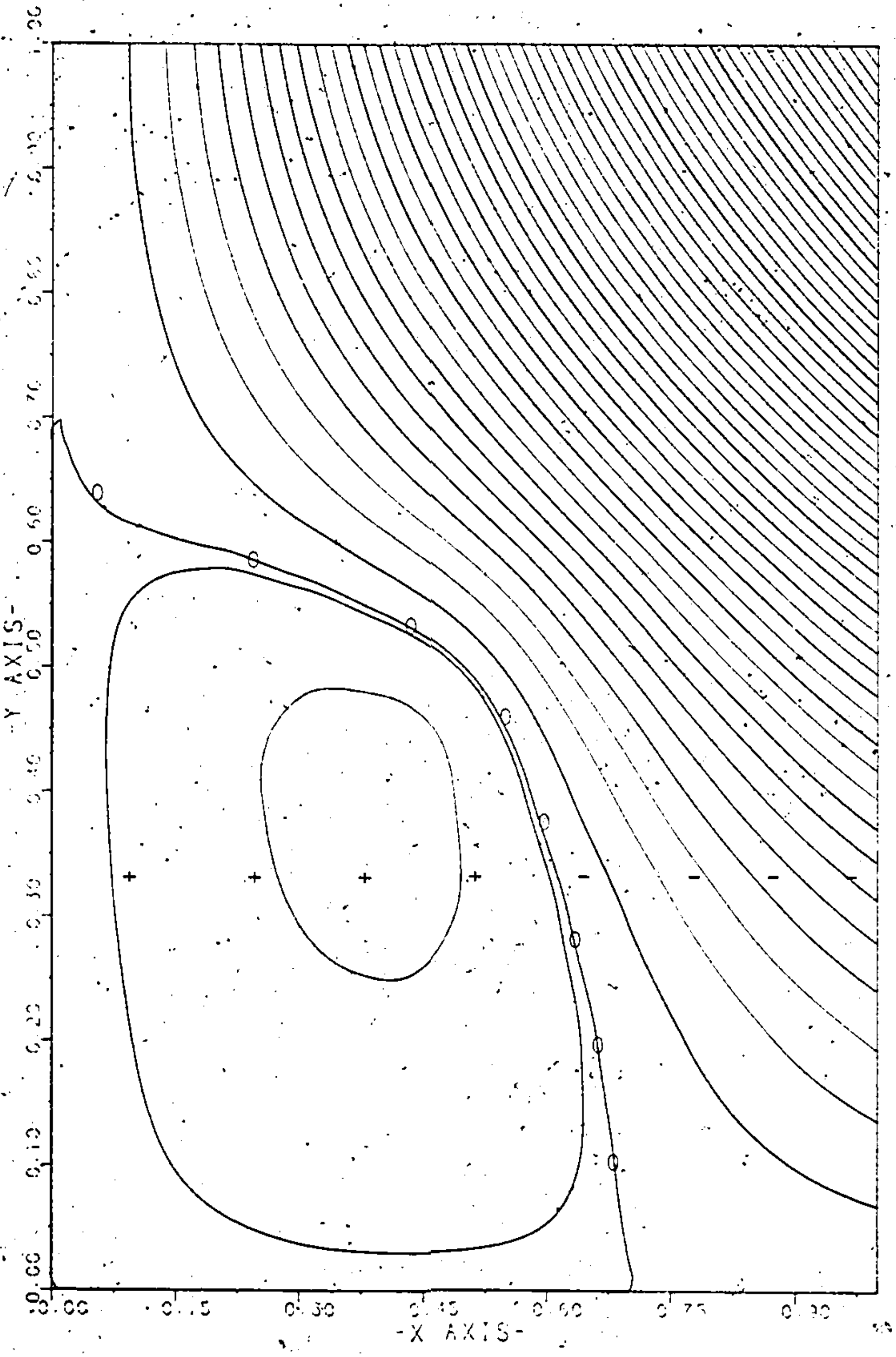
THIRD ANTISYMMETRIC MODE WITH PHI = [illegible], U = [illegible]/2, V = [illegible]/2



WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

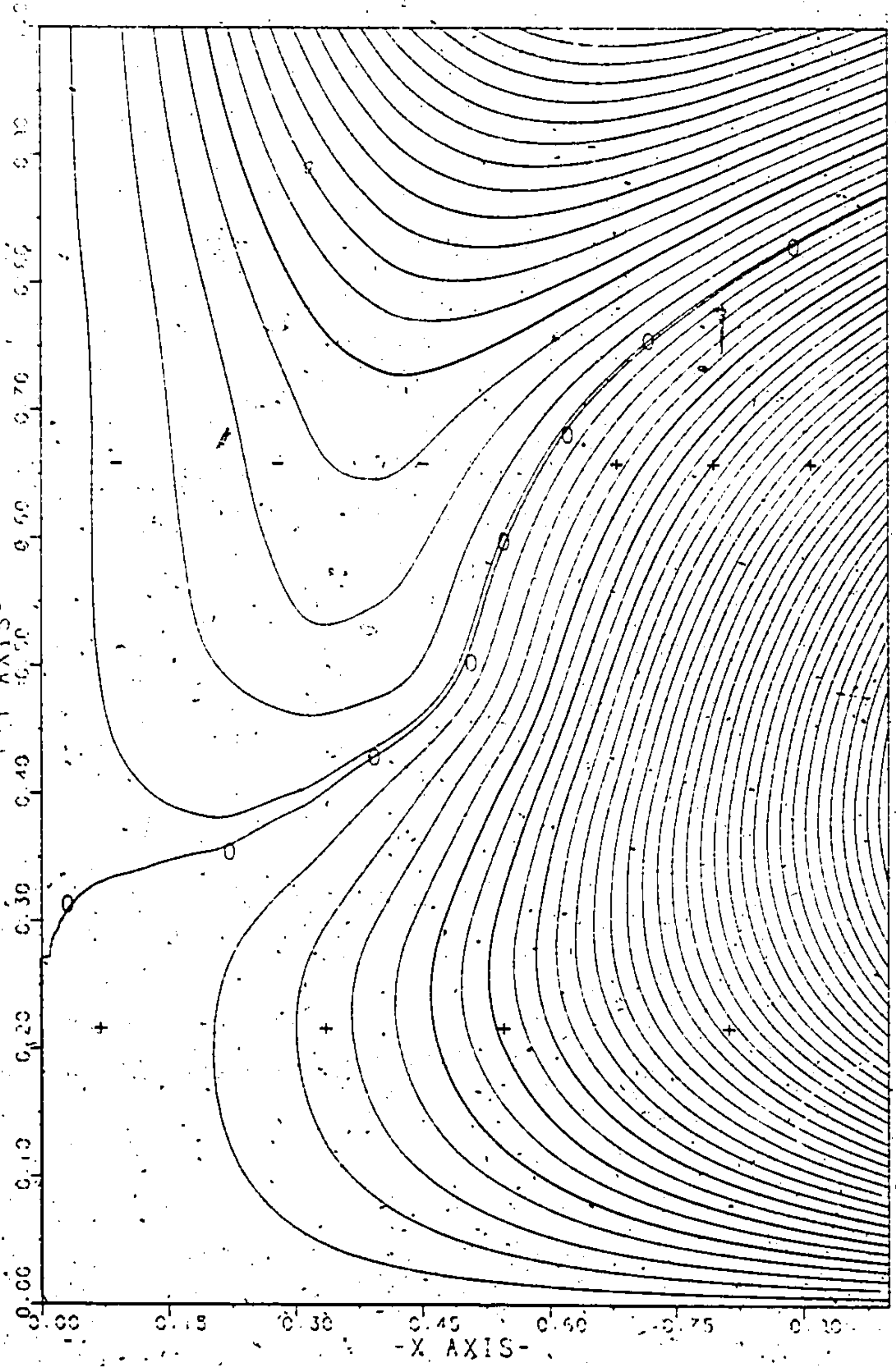
FIRST ANTISYMMETRIC MODE WITH PHI = 3?2. ...



WAVE PLATE GRAPHIC

••• THE CANTILEVERED PLATE WITH LATERAL POINT SUPPORT •••

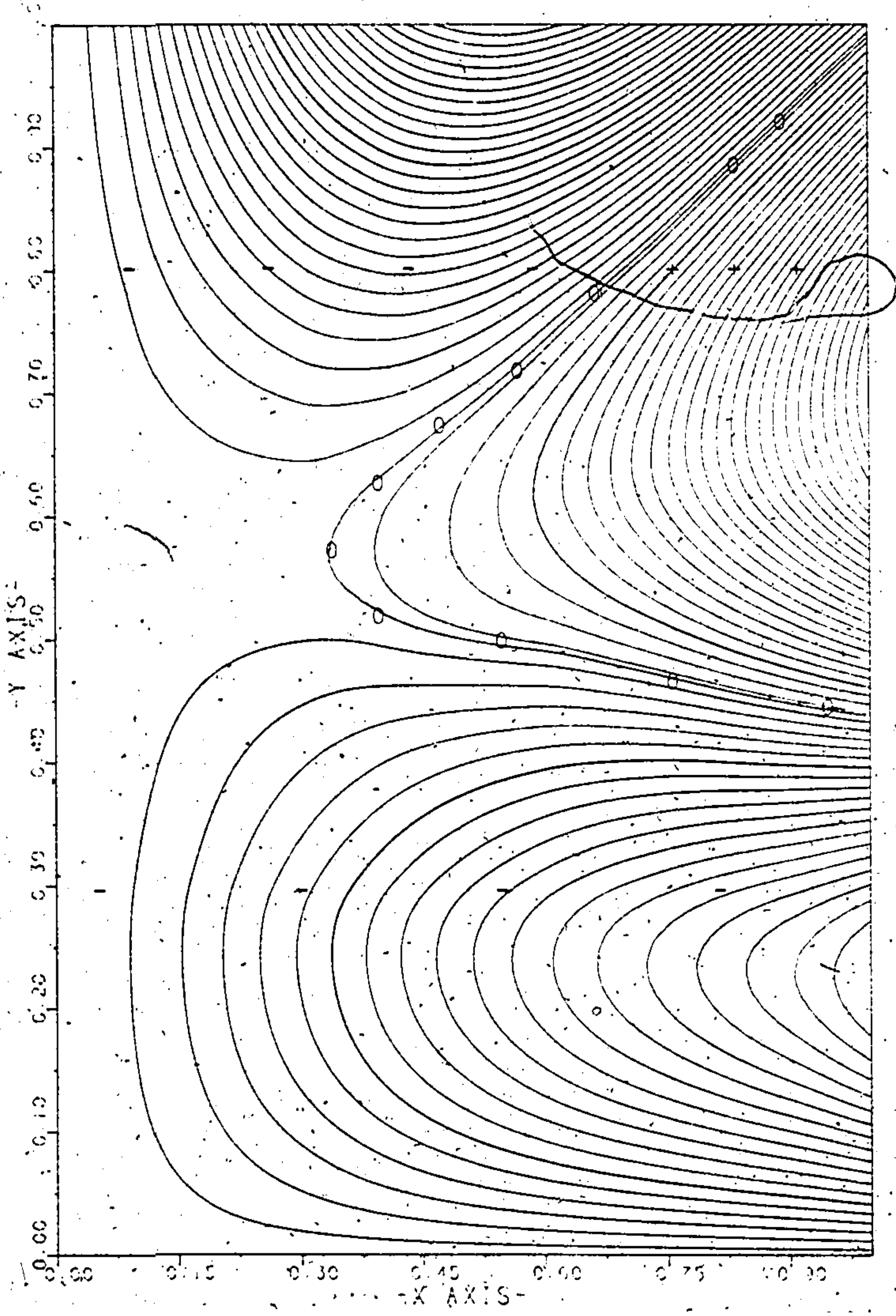
SECOND ANTISYMMETRIC MODE WITH PHI = [illegible]



WAVE PLATE GRAPHIC

••• THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT •••

THIRD ANTISYMMETRIC MODE WITH PHI = 3/2, U = 1/2, ... = 1/2



FREE VIBRATION ANALYSIS
OF RECTANGULAR CANTILEVER PLATES
WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

VOLUME III

Appendix 3

COMPUTER PROGRAMS
USED IN THE FREE VIBRATION ANALYSIS
OF THE RECTANGULAR CANTILEVER PLATE

This appendix presents a complete list of computer program listings to determine the eigenvalues and mode shapes for the symmetric and antisymmetric modes of the cantilever plate of Chapters 2, 3, and 4. While Programs one through 16 are based on the analytical solutions of Chapters 2, 3, and 4, Programs 17 and 18 use the finite element techniques. To generate desired eigenvalues or mode shapes, a full understanding of these programs is not required for enough explanatory comments have been entered at the begining of each program, and no further description should be required here.

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C                              PROGRAM 1                                  C
C                                                                         C
C            THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM           C
C     FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.        C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                   C
C                                                                         C
C             1.- K     = THE NUMBER OF TERMS TO BE USED IN THE           C
C                         SERIES INVOLVED. FOR 5 DIGITS ACCURACY          C
C                         USE K = 10.                                     C
C                                                                         C
C             2.- PHIR  = 2B/A, IS THE FULL PLATE ASPECT RATIO.           C
C                                                                         C
C             3.- ALMDS = AN INITIAL STARTING VALUE FOR THE EIGEN-        C
C                         VALUE SEARCH.                                   C
C                                                                         C
C             4.- DLIM  = A FINISHING OR EIGENVALUE SEARCH ENDING         C
C                         LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO        C
C                         HALT EXECUTION.                                 C
C                                                                         C
C             5.- DEL   = EIGENVALUE INCREMENT.                           C
C                                                                         C
C             6.- POI   = POISSON'S RATIO.                                C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45)
      K=????????????????????
      Z2=3600
      PHIR=????????????????????
      PHI=PHIR/2
      PRINT50,PHIR,K
   50 FORMAT('1','PHIR=',F8.4,10X,'K=',I5)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
      ALMDS=????????????????????
      DLIM=????????????????????
      DEL=????????????????????
    1 CONTINUE
      ALMDS=ALMDS+DEL
```

```
C
C     INITIALIZE THE MATRIX A
C
      DO 2 M=1,I
      DO 2 N=1,I
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.LT.Z2) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      IF (M.GT.1.0) GO TO 7
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
```

```
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (M.GT.1.0) GO TO 8
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN          *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(ENS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSINH(GM)
      XX            =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      IF (JN.GT.0.0) GO TC 12
      XX=XX/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
   12 A(N+K,(M+1)/2)=XX
      GO TC 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z22) GO TO 45
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
```

```
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
14    A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
15    TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
20    CONTINUE
      CALL DETERM (A,I,DET)
      PRINT110,ALMDS,DET
110   FORMAT(' ','ALMDS=',F10.4,10X,'DET=',D20.5)
      IF (ALMDS.LT.DLIM) GO TO 1
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
30    BIG=TERM
      L=K
50    CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
```

```
C         L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C         I IS NOT EQUAL TO L,SWITCH ROWS I AND L
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C         NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C         FOR EACH OF THE ROWS AFTER THE I-TH
      DO 200 J=NEXTR,N
C
C         MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C         NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C         END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                              PROGRAM 2                               C
C                                                                      C
C              THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE REC-     C
C      TANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.               C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C      USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;               C
C                                                                      C
C              1.- K       = AS DEFINED IN PROGRAM 1.                  C
C                                                                      C
C              2.- PHIR    = 2B/A FULL PLATE ASPECT RATIO.             C
C                                                                      C
C              3.- ALMDS = EIGENVALUE.                                 C
C                                                                      C
C              4.- KS      = NUMBER OF POINTS AT WHICH THE DISPLACE-   C
C                            MENT W IS REQUIRED.                       C
C                                                                      C
C              5.- POI     = POISSON'S RATIO.                          C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,K,K1,KS
      DIMENSION A(45,45)
      K=?????????????????????
      KS=????????????????????
      Z2=3600
      PHIR=??????????????????
      ALMDS=?????????????????
      PHI=PHIR/2
      PRINT50,PHIR,K
   50 FORMAT('1','PHIR=',F8.4,10X,'K=',I5)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
    1 CONTINUE
C
C     INITIALIZE THE MATRIX A
C
      DO 2 M=1,I
      DO 2 N=1,I
      A(M,N)=0.0
    2 CONTINUE
```

```
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      EN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (ENS.LT.Z2) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      IF (M.GT.1.0) GO TO 7
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (M.GT.1.0) GO TO 8
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
```

```
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN              *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSINH(GM)
      XX                        =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
   12 A(N+K,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
```

```
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
   20 CONTINUE
      CALL DETERM (A,I,DET)
      PRINT25
   25 FORMAT('1','END')
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45),X(45),EM(45),EN(45),EP(45)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
```

```
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C     FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C     FIND AND PRINT EM,EN,AND EP
C
      L=N/3
      DO 550 I=1,L
      J=2*I-1
      K=I-1
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
```

```
      GO TO 550
540   EP(J)=1.
550   PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J)
600   FORMAT('-','M=',I2,5X,'N=',I2,5X,'P=',I2,10X,'EM(',I2,') =',D14.5
     1,10X,'EN(',I2,') =',D14.5,10X,'EP(',I2,') =',D14.5)
      DO 601 I=1,L
      J=2*I-1
      EN(I)=EN(J)
      EP(I)=EP(J)
601   CONTINUE
      PRINT 650
650   FORMAT('1','THE SYMMETRIC MODE SHAPE DATA ARE:'////)
      CALL SHAPL (EM,EN,EP)
 60   RETURN
      END
      SUBROUTINE SHAPE (EM,EN,EP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,K,K1,KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W(21,21),EM(45),EN(45),
     1EP(45)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      DO 620 N=1,K
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 606
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (ETA.EQ.0.0) GO TO 601
      IF (ENP.EQ.0.0) GO TO 601
      XX=DCOS(ENP*ETA)
      GO TO 602
601   XX=1.0
602   IF (PSI.EQ.0.0) GO TO 604
      XW2=EN(N)*(DSINH(BN*PSI)+TD1*DSIN(GN*PSI))*XX/TD11N
      IF (PSI.EQ.1.0) GO TO 603
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD1P*DCOS(GN*(1-PSI)))*XX/TD11P
      GO TO 617
603   XW3=EP(N)*(1-TD1P)*XX/TD11P
      GO TO 617
604   XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD1P*DCOS(GN))*XX/TD11P
      GO TO 617
606   X1=-X1
```

```
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.GT.Z2) GO TO 612
      TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (ETA.EQ.0.0) GO TO 607
      IF (ENP.EQ.0.0) GO TO 607
      XX=DCOS(ENP*ETA)
      GO TO 608
  607 XX=1.0
  608 IF (PSI.EQ.0.0) GO TO 610
      XW2=EN(N)*(DSINH(BN*PSI)+TD2*DSINH(GN*PSI))*XX/TD22N
      IF (PSI.EQ.1.0) GO TO 609
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD2P*DCOSH(GN*(1-PSI)))*XX/TD22P
      GO TO 617
  609 XW3=EP(N)*(1-TD2P)*XX/TD22P
      GO TO 617
  610 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*XX/TD22P
      GO TO 617
  612 TD2N=BN*(BNS-POIS*PHINS*ENPS)/(GN*(POIS*PHINS*ENPS-GNS))
      IF(ETA.EQ.0.0) GO TO 613
      IF (ENP.EQ.0.0) GO TO 613
      XX=DCOS(ENP*ETA)
      GO TO 614
  613 XX=1.0
  614 IF (PSI.EQ.0.0) GO TO 616
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.) B=0.0
      XW2=(EN(N)*(DEXP((BN*PSI-BN)*B)+TD2N*DEXP((GN*PSI-GN)*B))*XX/
     1(BN+TD2N*GN))*B
      IF (PSI.EQ.1.0) GO TO 615
      TEST=BN*PSI
      IF (TEST.GT.60.) GO TO 615
      XW3=EP(N)*(DEXP(-BN*PSI)-DEXP(-GN*PSI))*XX/(BN-GN)
      GO TO 617
  615 XW3=0.0
      GO TO 617
  616 XW2=0.0
      XW3=0.0
  617 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 623
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
```

```
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSINH(GM)
      IF (PSI.EQ.0.0) GO TO 622
      IF(ETA.EQ.0.0) GO TO 621
      XW1=EM(M)*(DCOSH(BM*ETA)+TD1*DCOS(GM*ETA))*DSIN(EMP2*PSI)/TD11
      GO TO 629
621   XW1=EM(M)*(1+TD1)*DSIN(EMP2*PSI)/TD11
      GO TO 629
622   XW1=0.0
      GO TO 629
623   X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 625
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2S)
     1 *DSINH(GM))
      TD2=TD2*(-1)
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      IF (PSI.EQ.0.0) GO TO 622
      IF (ETA.EQ.0.0) GO TO 624
      XW1=EM(M)*(DCOSH(BM*ETA)+TD2*DCOSH(GM*ETA))*DSIN(EMP2*PSI)/TD22
      GO TO 629
624   XW1=  EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
      GO TO 629
625   TD2M=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
      IF (PSI.EQ.0.0) GO TO 627
      IF (ETA.EQ.0.0) GO TO 627
      TEST=BM-BM*ETA
      IF (TEST.GT.60.) GO TO 627
      XW1=EM(M)*(DEXP(BM*ETA-BM)+TD2M*DEXP(GM*ETA-GM))*DSIN(EMP2*PSI)/
     1(BM+TD2M*GM)
      GO TO 629
627   XW1=0.0
629   W1(I,J)=XW1+W11
      W11=W1(I,J)
630   CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)
      PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W(I,J)
635   FORMAT('-','PSI =',F5.3,4X,'ETA =',F5.3,4X,'W1(',I2,',',I2,') =',
     1D12.5,4X,'W2(',I2,',',I2,') =',D12.5,4X,'W3(',I2,',',I2,') =',
     1D12.5,4X,'W(',I2,',',I2,') =',D12.5)
      PSI=PSI+1.0/DFLOAT(KS)
640   CONTINUE
      ETA=ETA+1.0/DFLOAT(KS)
650   CONTINUE
      RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                        C
C                               PROGRAM 3                                C
C                                                                        C
C            THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH             C
C     PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION        C
C     PROBLEM.                                                           C
C                                                                        C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                        C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                  C
C                                                                        C
C             1.- K      = THE NUMBER OF TERMS TO BE USED IN THE         C
C                          SERIES INVOLVED. FOR 5 DIGITS ACCURACY        C
C                          USE K = 10.                                   C
C                                                                        C
C             2.- PHIR   = 2B/A, IS THE FULL PLATE ASPECT RATIO.         C
C                                                                        C
C             3.- ALMDS  = AN INITIAL STARTING VALUE FOR THE EIGEN-      C
C                          VALUE SEARCH.                                 C
C                                                                        C
C             4.- DLIM   = A FINISHING OR EIGENVALUE SEARCH ENDING       C
C                          LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO      C
C                          HALT EXECUTION.                               C
C                                                                        C
C             5.- DEL    = EIGENVALUE INCREMENT.                         C
C                                                                        C
C             6.- POI    = POISSON'S RATIO.                              C
C                                                                        C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45)
      K=????????????????????
      K2=2*K
      K1=2*K-1
      Z2=2600
      PHIR=???????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,K
   50 FORMAT('1','PHIR=',F8.4,10X,'K=',I5)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
      ALMDS=??????????????????
      DEL=????????????????????
      DLIM=??????????????????
```

```
    1   CONTINUE
C
C       INITIALIZE THE MATRIX TO 0.0
C
        DO 2 M=1,I
        DO 2 N=1,I
    2   A(M,N)=0.0
        DO 10 M=1,K1,2
        DO 10 N=1,K1,2
        EMP2=M*PI/2.
        EMP2S=EMP2*EMP2
        ENP2=N*PI/2.
        ENP2S=ENP2*ENP2
        BNS=PHINS*(ENP2S+ALMDS*PHIS)
        BN=DSQRT(BNS)
        X1=ALMDS*PHIS-ENP2S
        IF (X1.LT.0.0) GO TO 4
        GNS=PHINS*X1
        GN=DSQRT(GNS)
        TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
       1POIS*PHINS))
        TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
        TD1P=DCOSH(BN)/DCOS(GN)
        TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
        IF (M.GT.1.) GO TO 3
        A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
        A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
       1*ENP2S+GNS)*DSIN(GN))/TD11N
        A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
    3   X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
        X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
        XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
        A((M+1)/2,(N+1)/2+K)=XX
        X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
        X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
        XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
        A((M+1)/2,(N+1)/2+K2)=XX
        GO TO 10
    4   X1=-X1
        GNS=PHINS*X1
        GN=DSQRT(GNS)
        IF (BNS.LT.Z2) GO TO 6
        TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
        TD22N=TD2N*BN+GN
        TD2P=1.
        TD22P=BN-GN*TD2P
        IF (M.GT.1) GO TO 5
        A((N+1)/2+K2,(N+1)/2+K)=0.0
        A((N+1)/2+K,(N+1)/2+K2)=0.0
        A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
       1-GNS)/TD22N
    5   X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
        X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
        XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
        A((M+1)/2,(N+1)/2+K)=XX
        X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
        X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
        XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
        A((M+1)/2,(N+1)/2+K2)=XX
        GO TO 10
```

```
    6  TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
      1ENP2S)*DCOSH(BN))
       TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
       TD2P=DCOSH(BN)/DCOSH(GN)
       TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
       IF (M.GT.1) GO TO 7
       A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
       A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
      1ENP2S-GNS)*DSINH(GN))/TD22N
       A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
    7  X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
       X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
       XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
       A((M+1)/2,(N+1)/2+K)=XX
       X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
       X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
       XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
       A((M+1)/2,(N+1)/2+K2)=XX
   10  CONTINUE
       DO 11 N=1,K
   11  A(N+K2,N+K2)=-1.
       DO 20 N=1,K1,2
       DO 20 M=1,K1,2
       ENP2=N*PI/2.
       ENP2S=ENP2*ENP2
       EMP2=M*PI/2.
       EMP2S=EMP2*EMP2
       BMS=PHIS*(ALMDS+EMP2S)
       BM=DSQRT(BMS)
       X1=ALMDS-EMP2S
       IF (X1.LT.0.0) GO TO 13
       GMS=PHIS*X1
       GM=DSQRT(GMS)
       TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
      1POIS*PHIS)*DCOSH(BM))
       TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
       IF (N.GT.1) GO TO 12
       A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
      1EMP2S+GMS)*DSIN(GM))/TD11M
   12  X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
       X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
       XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
       A((N+1)/2+K,(M+1)/2)=XX
       X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
       X2=GM*DCOS(GM)/(GMS-ENP2S)
       XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
       A((N+1)/2+K2,(M+1)/2)=XX
       GO TO 20
   13  X1=-X1
       GMS=PHIS*X1
       GM=DSQRT(GMS)
       IF (BMS.LT.Z2) GO TO 15
       TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
       TD22M=TD2M*BM+GM
       IF (N.GT.1) GO TO 14
       A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14  X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
       X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
       XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
       A((N+1)/2+K,(M+1)/2)=XX
```

```
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      IF (N.GT.1) GO TO 16
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
     1GMS)*DSINH(GM))/TD22M
   16 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
   20 CONTINUE
      CALL DETERM (A,I,DET)
      PRINT 110,ALMDS,DET
  110 FORMAT(' ','ALMDS=',F10.4,10X,'DET=',D20.5)
      ALMDS=ALMDS+DEL
      IF (ALMDS.LE.DLIM) GO TO 1
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
```

```
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C                               PROGRAM 4                                 C
C                                                                         C
C              THIS IS AN ANTISYMMETRIC MODE SHAPE PROGRAM FOR            C
C      THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.           C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                   C
C                                                                         C
C              1.- K     = AS DEFINED IN PROGRAM 3.                       C
C                                                                         C
C              2.- PHIR  = 2B/A FULL PLATE ASPECT RATIO.                  C
C                                                                         C
C              3.- ALMDS = EIGENVALUE.                                    C
C                                                                         C
C              4.- KS    = NUMBER OF POINTS AT WHICH THE DISPLACE-        C
C                          MENT W IS REQUIRED.                            C
C                                                                         C
C              5.- POI   = POISSON'S RATIO.                               C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(45,45)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,ALMDS,K,K1,KS
      K=??????????????????????
      KS=?????????????????????
      K2=2*K
      K1=2*K-1
      Z2=2600
      PHIR=???????????????????
      ALMDS=??????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,PHI,K,ALMDS
   50 FORMAT('1','PHIR =',F8.4,10X,'PHI =',F8.4,10X,'K =',I4,
     110X,'ALMDS =',F9.4)
      PRINT60
   60 FORMAT('-','THE VALUES OF EM,EN,AND EP ARE',////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UPS=POI*PHIS
      UPIS=POI*PHINS
      I=3*K
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I
```

```
      DO 2 N=1,I
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*PUIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1PUIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      IF (M.GT.1.) GO TO 3
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-PUIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      IF (M.GT.1) GO TO 5
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-PUIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
```

```
      IF (M.GT.1) GO TO 7
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (N.GT.1) GO TO 12
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      IF (N.GT.1) GO TO 14
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
```

```
 15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
   1EMP2S)*DCOSH(BM))
    TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
    IF (N.GT.1) GO TO 16
    A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
   1GMS)*DSINH(GM))/TD22M
 16 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
    X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
    XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
    A((N+1)/2+K,(M+1)/2)=XX
    X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
    X2=GM*DCOSH(GM)/(GMS+ENP2S)
    XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
    A((N+1)/2+K2,(M+1)/2)=XX
 20 CONTINUE
    CALL DETERM (A,I,DET)
    PRINT25
 25 FORMAT('1','END')
    STOP
    END
    SUBROUTINE DETERM (A,N,DET)
    IMPLICIT REAL*8(A-H,O-Z)
    DIMENSION A(45,45),X(45),EM(45),EN(45),EP(45)
    SIGN=1.
    M=N-1
    LAST=M-1
C
C   START OVERALL LOOP FOR(N-1) PIVOTS
C
    DO 200 I=1,LAST
C
C   FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
    BIG=0.
    DO 50 K=I,M
    TERM=DABS(A(K,I))
    IF (TERM-BIG)50,50,30
 30 BIG=TERM
    L=K
 50 CONTINUE
C
C   CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
    IF (BIG)80,60,80
C
C   L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
 80 IF (I-L)90,120,90
C
C   I IS NOT EQUAL TO L.SWITCH ROWS I AND L
C
 90 DO 100 J=1,N
    TEMP=A(I,J)
    A(I,J)=A(L,J)
100 A(L,J)=TEMP
C
C   NOW START PIVOTAL REDUCTION
C
120 PIVOT=A(I,I)
    NEXTR=I+1
```

```
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C     FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C     FIND AND PRINT EM,EN,AND EP
C
      L=N/3
      DO 550 I=1,L
      J=2*I-1
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      GO TO 550
  540 EP(J)=1.
  550 PRINT600,J,J,EM(J),J,EN(J),J,EP(J)
  600 FORMAT('-','J=',I2,10X,'EM(',I2,')=',D15.5,10X,'EN(',I2,')='
     1,D15.5,10X,'EP(',I2,')=' ,D15.5)
      PRINT 650
  650 FORMAT('1','THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:',////)
      CALL SHAPE(EM,EN,EP)
   60 RETURN
```

```
      END
      SUBROUTINE SHAPE(EM,EN,EP)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,ALMDS,K,K1,KS
      DIMENSION W1(51,51),W2(51,51),W3(51,51),W(51,51),EM(45),EN
     1(45),EP(45)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      DO 620 N=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 605
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      IF (ETA.EQ.0.0) GO TO 602
      IF (PSI.EQ.0.0) GO TO 603
      XW2=EN(N)*(TD1N*DSINH(BN*PSI)+DSIN(GN*PSI))*DSIN(ENP2*ETA)/TD11N
      IF (PSI.EQ.1.0) GO TO 601
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD1P*DCOS((1-PSI)*GN))*DSIN
     1(ENP2*ETA)/TD11P
      GO TO 604
  601 XW3=EP(N)*(1-TD1P)*DSIN(ENP2*ETA)/TD11P
      GO TO 604
  602 XW2=0.0
      XW3=0.0
      GO TO 604
  603 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD1P*DCOS(GN))*DSIN(ENP2*ETA)/TD11P
  604 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
      GO TO 620
  605 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.GT.Z2) GO TO 610
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      IF (ETA.EQ.0.0) GO TO 607
      IF (PSI.EQ.0.0) GO TO 608
      XW2=EN(N)*(TD2N*DSINH(BN*PSI)+DSINH(GN*PSI))*DSIN(ENP2*ETA)/TD22N
      IF (PSI.EQ.1.0) GO TO 606
```

```
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD2P*DCOSH((1-PSI)*GN))*DSIN(ENP2*
     1ETA)/TD22P
      GO TO 609
  606 XW3=EP(N)*(1-TD2P)*DSIN(ENP2*ETA)/TD22P
      GO TO 609
  607 XW2=0.0
      XW3=0.0
      GO TO 609
  608 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*DSIN(ENP2*ETA)/TD22P
  609 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
      GO TO 620
  610 TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      IF (ETA.EQ.0.0) GO TO 612
      IF (PSI.EQ.0.0) GO TO 612
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.) B=0.0
      XW2=B*EN(N)*(TD2N*DEXP((BN*PSI-BN)*B)+DEXP((GN*PSI-GN)*B))*
     1DSIN(ENP2*ETA)/(BN*TD2N+GN)
      TEST=BN*PSI
      IF (TEST.GT.60.) GO TO 611
      XW3=EP(N)*(DEXP(-PSI*BN)-DEXP(-PSI*GN))*DSIN(ENP2*ETA)/(BN-GN)
      GO TO 614
  611 XW3=0.0
      GO TO 614
  612 XW2=0.0
      XW3=0.0
  614 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 625
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (ETA.EQ.0.0) GO TO 622
      IF (PSI.EQ.0.0) GO TO 622
      XW1=EM(M)*(TD1M*DSINH(BM*ETA)+DSIN(GM*ETA))*DSIN(EMP2*PSI)/TD11M
      GO TO 624
  622 XW1=0.0
  624 W1(I,J)=W11+XW1
      W11=W1(I,J)
      GO TO 630
  625 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 627
```

```
 15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
   1EMP2S)*DCOSH(BM))
    TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
    IF (ETA.EQ.0.0) GO TO 626
    IF (PSI.EQ.0.0) GO TO 626
    XW1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD22M
    GO TO 629
626 XW1=0.0
    GO TO 629
627 TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
    IF (ETA.EQ.0.0) GO TO 628
    IF(PSI.EQ.0.0) GO TO 628
    TEST=BM-BM*ETA
    IF (TEST.GT.60.) GO TO 628
    XW1=EM(M)*(TD2M*DEXP(BM*ETA-BM)+DEXP(GM*ETA-GM))*DSIN(EMP2*PSI)/
   1(BM*TD2M+GM)
    GO TO 629
628 XW1=0.0
629 W1(I,J)=W11+XW1
    W11=W1(I,J)
630 CONTINUE
    W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)
    PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W(I,J)
635 FORMAT('-','PSI =',F5.3,4X,'ETA =',F5.3,4X,'W1(',I2,',',I2,') =',
   1D12.5,4X,'W2(',I2,',',I2,') =',D12.5,4X,'W3(',I2,',',I2,') =',
   1D12.5,4X,'W(',I2,',',I2,') =',D12.5)
    PSI=PSI+1.0/DFLOAT(KS)
640 CONTINUE
    ETA=ETA+1.0/DFLOAT(KS)
650 CONTINUE
    RETURN
    END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                              PROGRAM 5                               C
C                                                                      C
C           THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM         C
C     FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.     C
C     WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE EDGES NORM-     C
C     AL TO THE CLAMPED BASE.(ONE POINT SUPPORT ON EACH EDGE.)         C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                C
C                                                                      C
C           1.- K = THE NUMBER OF TERMS TO BE USED IN THE SERIES       C
C                   INVOLVED. FOR 5 DIGITS ACCURACY USE K = 30.        C
C                                                                      C
C           2.- CSI = 0 TO 1, PROVIDING THE DIMENSIONLESS DISTANCE     C
C                     OF THE POINT SUPPORT TO THE X OR PSI AXIS.       C
C                     SEE FIGURE 3.1.                                  C
C                                                                      C
C           3.- PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.           C
C                                                                      C
C           4.- ALMDS = AN INITIAL STARTING VALUE FOR THE EIGEN-       C
C                       VALUE SEARCH.                                  C
C                                                                      C
C           5.- DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING         C
C                      LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO        C
C                      HALT EXECUTION.                                 C
C                                                                      C
C           6.- DEL = EIGENVALUE INCREMENT.                            C
C                                                                      C
C           7.- POI = POISSON'S RATIO.                                 C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      K=??????????????????????
      Z2=3600
      CSI=????????????????????
      PHIR=???????????????????
      PHI=PHIR/2
      PRINT50,PHIR,K,CSI
   50 FORMAT('1',10X,'PHIR =',F8.4,10X,'K =',I4,10X,'CSI =',F8.4,////)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
```

```
      I=3*K
      ALMDS=??????????????????
      DLIM=???????????????????
      DEL=????????????????????
    1 CONTINUE
      ALMDS=ALMDS+DEL
C
C  INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(I+1,N+2*K)=(DCOSH(BN*(1-CSI))-TD1P*DCOS(GN*(1-CSI)))*DCOS(ENP)/
     1TD11P
      A(I+1,N+K)=(DSINH(BN*CSI)+TD1*DSIN(GN*CSI))*DCOS(ENP)/TD11N
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.LT.Z2) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      X=UFIS*ENPS
      TD22P=BN-GN
```

```
      TD22N=BN+TD2*GN
      IF (M.GT.1.0) GO TO 7
      B=1.
      TEST=BN*CSI
      IF (TEST.GT.60) B=0.0
      A(L,N+2*K)=DCOS(ENP)*(DEXP((-BN*CSI)*B)-DEXP((-GN*CSI)*B))*B/
     1(BN-GN)
      B=1.0
      TEST=BN-BN*CSI
      IF (TEST.GT.60) B=0.0
      A(L,N+K)=DCOS(ENP)*(DEXP((-BN+BN*CSI)*B)+TD2*DEXP((-GN+GN*CSI)*B)
     1)*B/(BN+TD2*GN)
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (M.GT.1.0) GO TO 8
      A(I+1,N+2*K)=(DCOSH(BN*(1-CSI))-TD2P*DCOSH(GN*(1-CSI)))*DCOS(ENP)
     1/TD22P
      A(I+1,N+K)=(DSINH(BN*CSI)+TD2*DSINH(GN*CSI))*DCOS(ENP)/TD22N
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN          *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
```

```
      TD1L=BM*DSINH(BM)/(GM*DSIN(GM))
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)
     1+TD1L*GM*(GMS+POIS*PHIS*EMP2S)*DSIN(GM))
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX                =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      X1=BM*DCOS(ENP)*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD1L*DCOS(ENP)*DSIN(GM)/(GMS-ENPS)
      XX1=2*TD11L*EMP2*(X1+X2)
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
      XX1=XX1/2
11    A(N+2*K,(M+1)/2)=XX
      A(N+2*K,I+1)=A(N+2*K,I+1)+XX1
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TDI
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1L
      XX1=2*TD11L*DSIN(EMP2)*(X1+X2)
      XX2=TD11L*(DCOSH(BM)+TD1L*DCOS(GM))*DSIN(EMP2*CSI)
      XX3=TD11L*((UFS*EMP2S-BMS)*DCOSH(BM)+TD1L*(UFS*EMP2S+GMS)*
     1DCOS(GM))
      XX4=(DCOSH(BM)+TD1*DCOS(GM))*DSIN(EMP2*CSI)/TD11
      IF (JN.GT.C.0) GO TO 12
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A((M+1)/2,I+1)=XX3
      A(I+1,(M+1)/2)=XX4
      A(I+1,I+1)=A(I+1,I+1)+XX2
12    A(N+K,(M+1)/2)=XX
      A(N+K,I+1)=A(N+K,I+1)+XX1
      GO TO 20
13    X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      TD2L=-BM*DSINH(BM)/(GM*DSINH(GM))
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)+
     1TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DSINH(GM))
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2L*DSINH(GM)/(GMS+ENPS)
      XX2=2*TD22L*EMP2*(X1+X2)*DCOS(ENP)
      X11=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM*DSINH(GM)/(GMS+ENPS)
      XX3=2*TD22L*DSIN(EMP2)*(X11+X22)*DCOS(ENP)
      IF (JN.GT.0.0) GO TO 14
```

```
      XX4=TD22L*((UFS*EMP2S-BMS)*DCOSH(BM)+TD2L*(UFS*EMP2S-GMS)
     1*DCOSH(GM))
      XX5=TD22L*(DCOSH(BM)+TD2L*DCOSH(GM))*DSIN(EMP2*CSI)
      XX6=(DCOSH(BM)+TD2*DCOSH(GM))*DSIN(EMP2*CSI)/TD22
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,I+1)=XX4
      A(I+1,I+1)=A(I+1,I+1)+XX5
      XX=XX/2
      XX1=XX1/2
      XX2=XX2/2
      XX3=XX3/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      A(N+2*K,I+1)=A(N+2*K,I+1)+XX2
      A(N+K,I+1)=A(N+K,I+1)+XX3
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      TD2L=-BM/GM
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*(GMS-
     1POIS*PHIS*EMP2S))
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      X1=BM/(BMS+ENPS)
      X2=GM*TD2L/(GMS+ENPS)
      XX2=2*TD22L*EMP2*(X1+X2)*DCOS(ENP)
      X11=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM/(GMS+ENPS)
      XX3=2*TD22L*DSIN(EMP2)*(X11+X22)*DCOS(ENP)
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      XX2=XX2/2
      XX3=XX3/2
      XX4=TD22L*((UFS*EMP2S-BMS)+TD2L*(UFS*EMP2S-GMS))
      XX5=TD22L*(1+TD2L)*DSIN(EMP2*CSI)
      XX6=(1+TD2)*DSIN(EMP2*CSI)/TD22
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,I+1)=XX4
      A(I+1,I+1)=A(I+1,I+1)+XX5
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
   20 CONTINUE
      CALL DETERM (A,L,DET)
      PRINT110,ALMDS,DET
  110 FORMAT(' ','ALMDS=',F12.6,10X,'DET=',D20.5)
      IF (ALMDS.LT.DLIM) GO TO 1
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      SIGN=1.
      LAST=N-1
```

```
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C                          PROGRAM 6                                    C
C                                                                       C
C             THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE REC-       C
C     TANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH POINT      C
C     SUPPORTS SYMMETRICALY LOCATED ON THE EDGES NORMAL TO THE          C
C     CLAMPED BASE.(ONE POINT SUPPORT ON EACH EDGE.)                    C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                 C
C                                                                       C
C             1.- K     = AS DEFINED IN PROGRAM 5.                      C
C                                                                       C
C             2.- CSI   = AS DEFINED IN PROGRAM 5.                      C
C                                                                       C
C             3.- PHIR  = 2B/A FULL PLATE ASPECT RATIO.                 C
C                                                                       C
C             4.- ALMDS = EIGENVALUE.                                   C
C                                                                       C
C             5.- KS    = NUMBER OF POINTS AT WHICH THE DISPLACE-       C
C                         MENT W IS REQUIRED.                           C
C                                                                       C
C             6.- POI   = POISSON'S RATIO.                              C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      DIMENSION A(99,99)
      K=?????????????????????
      KS=????????????????????
      Z2=3600
      CSI=???????????????????
      PHIR=??????????????????
      ALMDS=?????????????????
      PHI=PHIR/2
      PRINT50,K,KS,CSI,PHIR,ALMDS
   50 FORMAT('1',10X,'K =',I3,10X,'KS =',I3,10X,'CSI =',F7.4,10X,
     1'PHIR =',F7.4,10X,'ALMDS =',F8.4,////)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
    1 CONTINUE
```

```
C
C     INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=N*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=PUIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(I+1,N+2*K)=(DCOSH(BN*(1-CSI))-TD1P*DCOS(GN*(1-CSI)))*DCOS(ENP)/
     1TD11P
      A(I+1,N+K)=(DSINH(BN*CSI)+TD1*DSIN(GN*CSI))*DCOS(ENP)/TD11N
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=PUIS*PHINS*ENPS
      IF (BNS.LT.Z2) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      IF (M.GT.1.0) GO TO 7
      B=1.
      TEST=BN*CSI
      IF (TEST.GT.60) B=0.
      A(L,N+2*K)=DCOS(ENP)*(DEXP((-BN*CSI)*B)-DEXP((-GN*CSI)*B))*B/
```

```
      1(BN-GN)
      B=1.
      TEST=BN-BN*CSI
      IF (TEST.GT.60) B=0.
      A(L,N+K)=DCOS(ENP)*(DEXP((-BN+BN*CSI)*B)+TD2*DEXP((-GN+GN*CSI)*B)
     1)*B/(BN+TD2*GN)
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (M.GT.1.0) GO TO 8
      A(I+1,N+2*K)=(DCOSH(BN*(1-CSI))-TD2P*DCOSH(GN*(1-CSI)))*DCOS(ENP)
     1/TD22P
      A(I+1,N+K)=(DSINH(BN*CSI)+TD2*DSINH(GN*CSI))*DCOS(ENP)/TD22N
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN              *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1L=BM*DSINH(BM)/(GM*DSIN(GM))
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BNS-POIS*PHIS*EMP2S)*DSINH(BM)
     1+TD1L*GM*(GMS+POIS*PHIS*EMP2S)*DSIN(GM))
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
```

```
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX            =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      X1=BM*DCOS(ENP)*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD1L*DCOS(ENP)*DSIN(GM)/(GMS-ENPS)
      XX1=2*TD11L*EMP2*(X1+X2)
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
      XX1=XX1/2
   11 A(N+2*K,(M+1)/2)=XX
      A(N+2*K,I+1)=A(N+2*K,I+1)+XX1
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1L
      XX1=2*TD11L*DSIN(EMP2)*(X1+X2)
      XX2=TD11L*(DCOSH(BM)+TD1L*DCOS(GM))*DSIN(EMP2*CSI)
      XX3=TD11L*((UFS*EMP2S-BMS)*DCOSH(BM)+TD1L*(UFS*EMP2S+GMS)*
     1DCOS(GM))
      XX4=(DCOSH(BM)+TD1*DCOS(GM))*DSIN(EMP2*CSI)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A((M+1)/2,I+1)=XX3
      A(I+1,(M+1)/2)=XX4
      A(I+1,I+1)=A(I+1,I+1)+XX2
   12 A(N+K,(M+1)/2)=XX
      A(N+K,I+1)=A(N+K,I+1)+XX1
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (EMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      TD2L=-BM*DSINH(BM)/(GM*DSINH(GM))
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)+
     1TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DSINH(GM))
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2L*DSINH(GM)/(GMS+ENPS)
      XX2=2*TD22L*EMP2*(X1+X2)*DCOS(ENP)
      X11=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM*DSINH(GM)/(GMS+ENPS)
      XX3=2*TD22L*DSIN(EMP2)*(X11+X22)*DCOS(ENP)
      IF (JN.GT.0.0) GO TO 14
      XX4=TD22L*((UFS*EMP2S-BMS)*DCOSH(BM)+TD2L*(UFS*EMP2S-GMS)
     1*DCOSH(GM)).
      XX5=TD22L*(DCOSH(BM)+TD2L*DCOSH(GM))*DSIN(EMP2*CSI)
      XX6=(DCOSH(BM)+TD2*DCOSH(GM))*DSIN(EMP2*CSI)/TD22
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,I+1)=XX4
```

```
      A(I+1,I+1)=A(I+1,I+1)+XX5
      XX=XX/2
      XX1=XX1/2
      XX2=XX2/2
      XX3=XX3/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      A(N+2*K,I+1)=A(N+2*K,I+1)+XX2
      A(N+K,I+1)=A(N+K,I+1)+XX3
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      TD2L=-BM/GM
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*(GMS-
     1POIS*PHIS*EMP2S))
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      X1=BM/(BMS+ENPS)
      X2=GM*TD2L/(GMS+ENPS)
      XX2=2*TD22L*EMP2*(X1+X2)*DCOS(ENP)
      X11=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X22=(EMP2S-UFIS*GMS9*TD2L*GM/(GMS+ENPS)
      XX3=2*TD22L*DSIN(EMP2)*(X11+X22)*DCOS(ENP)
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      XX2=XX2/2
      XX3=XX3/2
      XX4=TD22L*((UFS*EMP2S-BMS)+TD2L*(UFS*EMP2S-GMS))
      XX5=TD22L*(1+TD2L)*DSIN(EMP2*CSI)
      XX6=(1+TD2)*DSIN(EMP2*CSI)/TD22
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,I+1)=XX4
      A(I+1,I+1)=A(I+1,I+1)+XX5
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
   20 CONTINUE
      DO 30 M=1,L
      TEMP=A(M,I)
      A(M,I)=A(M,I+1)
      A(M,I+1)=TEMP
   30 CONTINUE
      CALL DETERM (A,L,DET)
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
```

```
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=CABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
```

```
C
  400 DO 450 J=2,I
C
C      WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C      FOUND VALUES
C
       K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C      FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C      FIND AND PRINT EM,EN,EP,AND P*
C
       L=(N-1)/3
       DO 550 I=1,L
       J=2*I-1
       K=I-1
       EM(J)=X(I)
       EN(J)=X(I+L)
       IF(I.EQ.L) GO TO 540
       EP(J)=X(I+2*L)
       P=X(3*L)
       GO TO 550
  540 EP(J)=-1.0
  550 PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J),P
  600 FORMAT('-','M=',I2,3X,'N=',I2,3X,'P=',I2,5X,'EM(',I2,') =',
     1D13.5,5X,'EN(',I2,') =',D13.5,5X,'EP(',I2,') =',D13.5,5X,
     1'P* =',D13.5)
       DO 601 I=1,L
       J=2*I-1
       EN(I)=EN(J)
       EP(I)=EP(J)
  601 CONTINUE
       PRINT 650
  650 FORMAT('1','THE SYMMETRIC MODE SHAPE DATA ARE:'////)
       CALL SHAPE (EM,EN,EP,P)
   60 RETURN
       END
       SUBROUTINE SHAPE (EM,EN,EP,P)
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
      1KS
       DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
      1,EM(75),EN(75),EP(75)
       KS1=KS+1
       ETA=0.0
       DO 650 I=1,KS1
       PSI=0.0
       DO 640 J=1,KS1
       W11=0.0
       W22=0.0
       W33=0.0
       W44=0.0
       DO 620 N=1,K
       JN=N-1
       ENP=JN*PI
       ENPS=ENP*ENP
       BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
```

```
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 606
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (ETA.EQ.0.0) GO TO 601
      IF (ENP.EQ.0.0) GO TO 601
      XX=DCOS(ENP*ETA)
      GO TO 602
  601 XX=1.0
  602 IF (PSI.EQ.0.0) GO TO 604
      XW2=EN(N)*(DSINH(BN*PSI)+TD1*DSIN(GN*PSI))*XX/TD11N
      IF (PSI.EQ.1.0) GO TO 603
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD1P*DCOS(GN*(1-PSI)))*XX/TD11P
      GO TO 617
  603 XW3=EP(N)*(1-TD1P)*XX/TD11P
      GO TO 617
  604 XW2=0.0
      XW3=0.0
      GO TO 617
  606 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.GT.Z2) GO TO 612
      TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      IF (ETA.EQ.0.0) GO TO 607
      IF (ENP.EQ.0.0) GO TO 607
      XX=DCOS(ENP*ETA)
      GO TO 608
  607 XX=1.0
  608 IF (PSI.EQ.0.0) GO TO 610
      XW2=EN(N)*(DSINH(BN*PSI)+TD2*DSINH(GN*PSI))*XX/TD22N
      IF (PSI.EQ.1.0) GO TO 609
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD2P*DCOSH(GN*(1-PSI)))*XX/TD22P
      GO TO 617
  609 XW3=EP(N)*(1-TD2P)*XX/TD22P
      GO TO 617
  610 XW2=0.0
      XW3=0.0
      GO TO 617
  612 TD2=BN*(BNS-POIS*PHINS*ENPS)/(GN*(POIS*PHINS*ENPS-GNS))
      IF(ETA.EQ.0.0) GO TO 613
      IF (ENP.EQ.0.0) GO TO 613
      XX=DCOS(ENP*ETA)
      GO TO 614
  613 XX=1.0
  614 IF (PSI.EQ.0.0) GO TO 616
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.0) B=0.
```

```
      XW2=(EN(N)*(DEXP((BN*PSI-BN)*B)+TD2*DEXP((GN*PSI-GN)*B))*XX/
     1(BN+TD2*GN))*B
      IF (PSI.EQ.1.0) GO TO 615
      B=1.
      TEST=BN*PSI
      IF (TEST.GT.60.0) B=0.
      XW3=(EP(N)*(DEXP((-BN*PSI)*B)-DEXP((-GN*PSI)*B))*XX/(BN-GN))*B
      GO TO 617
  615 XW3=0.0
      GO TC 617
  616 XW2=0.0
      XW3=0.0
  617 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 623
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD1L=BM*DSINH(BM)/(GM*DSIN(GM))
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)
     1+TD1L*GM*(GMS+POIS*PHIS*EMP2S)*DSIN(GM))
      IF (PSI.EQ.0.0) GO TO 622
      IF(ETA.EQ.0.0) GO TC 621
      XW1=EM(M)*(DCOSH(BM*ETA)+TD1*DCOS(GM*ETA))*DSIN(EMP2*PSI)/TD11
      XW4=P*TD11L*(DCOSH(BM*ETA)+TD1L*DCOS(GM*ETA))*DSIN(EMP2*PSI)
      GO TO 629
  621 XW1=EM(M)*(1+TD1)*DSIN(EMP2*PSI)/TD11
      XW4=P*TD11L*(1+TD1L)*DSIN(EMP2*PSI)
      GO TO 629
  622 XW1=0.0
      XW4=0.0
      GO TO 629
  623 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 625
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2S)
     1 *DSINH(GM))
      TD2=TD2*(-1)
      TD2L=-BM*DSINH(BM)/(GM*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)
     1+TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DSINH(GM))
      IF (PSI.EQ.0.0) GO TO 622
      IF (ETA.EQ.0.0) GO TO 624
      XW1=EM(M)*(DCOSH(BM*ETA)+TD2*DCOSH(GM*ETA))*DSIN(EMP2*PSI)/TD22
      XW4=P*TD22L*(DCOSH(BM*ETA)+TD2L*DCOSH(GM*ETA))*DSIN(EMP2*PSI)
      GO TO 629
```

```
624  XW1=  EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
     XW4=P*TD22L*(1+TD2L)*DSIN(EMP2*PSI)
     GO TO 629
625  TD2M=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
     IF (PSI.EQ.0.0) GO TO 627
     IF (ETA.EQ.0.0) GO TO 627
     B=1.
     TEST=BM-BM*ETA
     IF (TEST.GT.60.) B=0.
     XW1=EM(M)*(DEXP((BM*ETA-BM)*B)+TD2M*DEXP((GM*ETA-GM)*B))*DSIN
    1(EMP2*PSI)*B/(BM+TD2M*GM)
     TD2L=-BM/GM
     TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*(GMS-
    1POIS*PHIS*EMP2S))
     XW4=P*TD22L*(DEXP((BM*ETA-BM)*B)+TD2L*DEXP((GM*ETA-GM)*B))*B*DSIN
    1(EMP2*PSI)
     GO TO 629
627  XW1=0.0
     XW4=0.0
629  W1(I,J)=XW1+W11
     W11=W1(I,J)
     W4(I,J)=XW4+W44
     W44=W4(I,J)
630  CONTINUE
     W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
     PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
635  FORMAT('-','PSI =',F5.3,3X,'ETA =',F5.3,3X,'W1(',I2,',',I2,') =',
    1D12.5,3X,'W2(',I2,',',I2,') =',D12.5,3X,'W3(',I2,',',I2,') =',
    1D12.5,3X,'W4(',I2,',',I2,') =',D12.5,/)
     PRINT636,PSI,ETA,I,J,W(I,J)
636  FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'
    1W(',I2,',',I2,') =',D12.5)
     PSI=PSI+1./DFLOAT(KS)
640  CONTINUE
     ETA=ETA+1.0/DFLOAT(KS)
650  CONTINUE
     RETURN
     END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C                            PROGRAM 7                               C
C                                                                    C
C          THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH           C
C     PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION    C
C     PROBLEM. WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE      C
C     EDGES NORMAL TO THE CLAMPED BASE.(ONE SUPPORT ON EACH EDGE)    C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED:              C
C                                                                    C
C              1.- K     = THE NUMBER OF TERMS TO BE USED IN THE     C
C                          SERIES INVOLVED. FOR 5 DIGITS ACCURACY    C
C                          USE K = 30.                               C
C                                                                    C
C              2.- CSI   = 0 TO 1. PROVIDING THE DISTANCE BETWEEN    C
C                          THE CONCENTRATED EDGE FORCE(POINT SUP-    C
C                          PORT) AND THE PLATE CENTRAL AXIS DIVIDED  C
C                          BY THE PLATE EDGE DIMENSION.              C
C                                                                    C
C              3.- PHIR  = 2B/A. IS THE FULL PLATE ASPECT RATIO.     C
C                                                                    C
C              4.- ALMOS = AN INITIAL STARTING VALUE FOR THE EIGEN-  C
C                          VALUE SEARCH.                             C
C                                                                    C
C              5.- DLIM  = A FINISHING OR EIGENVALUE SEARCH ENDING   C
C                          LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO  C
C                          HALT EXECUTION.                           C
C                                                                    C
C              6.- DEL   = EIGENVALUE INCREMENT.                     C
C                                                                    C
C              7.- POI   = POISSON'S RATIO.                          C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      K=?????????????????????
      CSI=???????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600.
      PHIR=??????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,K,CSI
   50 FORMAT('1','PHIR =',F7.4,10X,'K =',I5,10X,'CSI =',F7.4,////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????
      POIS=2-POI
      PHIN=1./PHI
```

```
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K+1
      ALMDS=??????????????????
      DLIM=???????????????????
      DEL=????????????????????
    1 CONTINUE
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I
      DO 2 N=1,I
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      IF (M.GT.1.) GO TO 3
      A(I,(N+1)/2+K)=(TD1N*DSINH(BN*CSI)+DSIN(GN*CSI))*DSIN(ENP2)/TD11N
      A(I,(N+1)/2+K2)=(DCOSH(BN*(1-CSI))-TD1P*DCOS(GN*(1-CSI)))*DSIN(
     1ENP2)/TD11P
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      IF (M.GT.1) GO TO 5
      TEST=BN-BN*CSI
      B=1.
      IF (TEST.GT.60.) B=0.
```

```
      A(I,(N+1)/2+K)=B*(TD2N*DEXP((BN*CSI-BN)*B)+DEXP((GN*CSI-GN)*B))
     1*(DSIN(ENP2)/(TD2N*BN+GN))
      TEST=BN*CSI
      B=1.0
      IF (TEST.GT.60.) B=0.0
      A(I,(N+1)/2+K2)=B*(DSIN(ENP2)/(BN-GN))*(DEXP((-BN*CSI)*B)
     1-DEXP((-GN*CSI)*B))
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    6 TD2N=GN*(PUIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-PUIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      IF (M.GT.1) GO TO 7
      A(I,(N+1)/2+K)=(TD2N*DSINH(BN*CSI)+DSINH(GN*CSI))*DSIN(ENP2)/TD22N
      A(I,(N+1)/2+K2)=(DCOSH(BN*(1-CSI))-TD2P*DCOSH(GN*(1-CSI)))*
     1DSIN(ENP2)/TD22P
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*PUIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1PUIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      TD1L=-BM*DCOSH(BM)/(GM*DCOS(GM))
```

```
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)-
     1TD1L*GM*(GMS+POIS*PHIS*EMP2S)*DCOS(GM))
      X1=(EMP2S-UFIS*BMS)*BM*DCOSH(BM)/(BMS+ENP2S)
      X11=(EMP2S+UFIS*GMS)*TD1L*GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*TD11L*DSIN(EMP2)*DSIN(ENP2)*(X1-X11)
      A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
      X1=BM*DCOSH(BM)/(BMS+ENP2S)
      X11=TD1L*GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*TD11L*EMP2*DSIN(ENP2)*(X1-X11)
      A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
      IF (N.GT.1) GO TO 12
      XX=TD11L*(DSINH(BM)+TD1L*DSIN(GM))*DSIN(EMP2*CSI)
      A(I,I)=A(I,I)+XX
      A(I,(M+1)/2)=(TD1M*DSINH(BM)+DSIN(GM))*DSIN(EMP2*CSI)/TD11M
      A((M+1)/2,I)=TD11L*((EMP2S*UFS-BMS)*DSINH(BM)+TD1L*(EMP2S*UFS+GMS)
     1*DSIN(GM))
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      TD2L=-BM/GM
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*
     1(GMS-POIS*PHIS*EMP2S))
      X2=(EMP2S-UFIS*BMS)*BM/(BMS+ENP2S)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM/(GMS+ENP2S)
      XX=2*TD22L*DSIN(EMP2)*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
      X2=BM/(BMS+ENP2S)
      X22=TD2L*GM/(GMS+ENP2S)
      XX=2*TD22L*EMP2*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
      IF (N.GT.1) GO TO 14
      XX=TD22L*(1+TD2L)*DSIN(EMP2*CSI)
      A(I,I)=A(I,I)+XX
      A(I,(M+1)/2)=(TD2M+1)*DSIN(EMP2*CSI)/TD22M
      A((M+1)/2,I)=TD22L*((EMP2S*UFS-BMS)+TD2L*(EMP2S*UFS-GMS))
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
```

```
      1EMP2S)*DCOSH(BM))
       TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
       TD2L=-BM*DCOSH(BM)/(GM*DCOSH(GM))
       TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)+
      1TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DCOSH(GM))
       X2=(EMP2S-UFIS*BMS)*BM*DCOSH(BM)/(BMS+ENP2S)
       X22=(EMP2S-UFIS*GMS)*TD2L*GM*DCOSH(GM)/(GMS+ENP2S)
       XX=2*TD22L*DSIN(EMP2)*DSIN(ENP2)*(X2+X22)
       A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
       X2=BM*DCOSH(BM)/(BMS+ENP2S)
       X22=TD2L*GM*DCOSH(GM)/(GMS+ENP2S)
       XX=2*TD22L*EMP2*DSIN(ENP2)*(X2+X22)
       A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
       IF (N.GT.1) GO TO 16
       XX=TD22L*(DSINH(BM)+TD2L*DSINH(GM))*DSIN(EMP2*CSI)
       A(I,I)=A(I,I)+XX
       A(I,(M+1)/2)=(TD2M*DSINH(BM)+DSINH(GM))*DSIN(EMP2*CSI)/TD22M
       A((M+1)/2,I)=TD22L*((EMP2S*UFS-BMS)*DSINH(BM)+TD2L*(
      1EMP2S*UFS-GMS)*DSINH(GM))
       A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
      1GMS)*DSINH(GM))/TD22M
 16    X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
       X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
       XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
       A((N+1)/2+K,(M+1)/2)=XX
       X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
       X2=GM*DCOSH(GM)/(GMS+ENP2S)
       XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
       A((N+1)/2+K2,(M+1)/2)=XX
 20    CONTINUE
       CALL DETERM (A,I,DET)
       PRINT 110,ALMDS,DET
 110   FORMAT(' ','ALMDS=',F10.4,10X,'DET=',D20.5)
       ALMDS=ALMDS+DEL
       IF (ALMDS.LE.DLIM) GO TO 1
       STOP
       END
       SUBROUTINE DETERM (A,N,DET)
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION A(99,99)
       SIGN=1.
       LAST=N-1
C
C      START OVERALL LOOP FOR(N-1) PIVOTS
C
       DO 200 I=1,LAST
C
C      FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
       BIG=0.
       DO 50 K=I,N
       TERM=DABS(A(K,I))
       IF (TERM-BIG)50,50,30
 30    BIG=TERM
       L=K
 50    CONTINUE
C
C      CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
       IF (BIG)80,60,80
```

```
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C                            PROGRAM 8                                  C
C                                                                       C
C             THIS IS AN ANTISYMMETRIC MODE SHAPE PROGRAM FOR THE       C
C      RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH        C
C      POINT SUPPORTS SYMMETRICALY LOCATED ON THE EDGES NORMAL TO       C
C      THE CLAMPED BASE.(ONE POINT SUPPORT ON EACH EDGE)                C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C      USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                 C
C                                                                       C
C              1.- K      = AS DEFINED IN PROGRAM 7.                    C
C                                                                       C
C              2.- CSI    = AS DEFINED IN PROGRAM 7.                    C
C                                                                       C
C              3.- PHIR   = 2B/A FULL PLATE ASPECT RATIO.               C
C                                                                       C
C              4.- ALMDS  = EIGENVALUE.                                 C
C                                                                       C
C              5.- KS     = NUMBER OF POINTS AT WHICH THE DISPLACE-     C
C                           MENT W IS REQUIRED.                         C
C                                                                       C
C              6.- POI    = POISSON'S RATIO.                            C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      K=????????????????????
      KS=????????????????????
      CSI=???????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600.
      PHIR=??????????????????
      PHI=PHIR/2.
      ALMDS=?????????????????
      PRINT50,PHIR,K,CSI,ALMDS
   50 FORMAT('1','PHIR =',F7.4,10X,'K =',I5,10X,'CSI =',F7.4,10X,'ALMDS
     1 =',F7.4,////)
      PRINT51
   51 FORMAT('-',30X,'THE VALUES OF EM,EN,EP,AND P*, ARE:',////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
```

```
      UFIS=POI*PHINS
      I=3*K+1
    1 CONTINUE
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I
      DO 2 N=1,I
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      IF (M.GT.1.) GO TO 3
      A(I,(N+1)/2+K)=(TD1N*DSINH(BN*CSI)+DSIN(GN*CSI))*DSIN(ENP2)/TD11N
      A(I,(N+1)/2+K2)=(DCOSH(BN*(1-CSI))-TD1P*DCOS(GN*(1-CSI)))*DSIN(
     1ENP2)/TD11P
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      IF (M.GT.1) GO TO 5
      TEST=BN-BN*CSI
      B=1.
      IF (TEST.GT.60.) B=0.
      A(I,(N+1)/2+K)=B*(TD2N*DEXP((BN*CSI-BN)*B)+DEXP((GN*CSI-GN)*B))
     1*(DSIN(ENP2)/(TD2N*BN+GN))
      TEST=BN*CSI
      B=1.0
      IF (TEST.GT.60.0) B=0.0
```

```
      A(I,(N+1)/2+K2)=B*(DSIN(ENP2)/(BN-GN))*(DEXP((-BN*CSI)*B)
     1-DEXP((-GN*CSI)*B))
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22F=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      IF (M.GT.1) GO TO 7
      A(I,(N+1)/2+K)=(TD2N*DSINH(BN*CSI)+DSINH(GN*CSI))*DSIN(ENP2)/TD22N
      A(I,(N+1)/2+K2)=(DCOSH(BN*(1-CSI))-TD2P*DCOSH(GN*(1-CSI)))*
     1DSIN(ENP2)/TD22P
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      TD1L=-BM*DCOSH(BM)/(GM*DCOS(GM))
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)-
     1TD1L*GM*(GMS+POIS*PHIS*EMP2S)*DCOS(GM))
      X1=(EMP2S-UFIS*BMS)*BM*DCOSH(BM)/(BMS+ENP2S)
      X11=(EMP2S+UFIS*GMS)*TD1L*GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*TD11L*DSIN(EMP2)*DSIN(ENP2)*(X1-X11)
```

```
      A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
      X1=BM*DCOSH(BM)/(BMS+ENP2S)
      X11=TD1L*GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*TD11L*EMP2*DSIN(ENP2)*(X1-X11)
      A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
      IF (N.GT.1) GO TO 12
      XX=TD11L*(DSINH(BM)+TD1L*DSIN(GM))*DSIN(EMP2*CSI)
      A(I,I)=A(I,I)+XX
      A(I,(M+1)/2)=(TD1M*DSINH(BM)+DSIN(GM))*DSIN(EMP2*CSI)/TD11M
      A((M+1)/2,I)=TD11L*((EMP2S*UFS-BMS)*DSINH(BM)+TD1L*(EMP2S*UFS+GMS)
     1*DSIN(GM))
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (EMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      TD2L=-BM/GM
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*
     1(GMS-POIS*PHIS*EMP2S))
      X2=(EMP2S-UFIS*BMS)*BM/(BMS+ENP2S)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM/(GMS+ENP2S)
      XX=2*TD22L*DSIN(EMP2)*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
      X2=BM/(BMS+ENP2S)
      X22=TD2L*GM/(GMS+ENP2S)
      XX=2*TD22L*EMP2*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
      IF (N.GT.1) GO TO 14
      XX=TD22L*(1+TD2L)*DSIN(EMP2*CSI)
      A(I,I)=A(I,I)+XX
      A(I,(M+1)/2)=(TD2M+1)*DSIN(EMP2*CSI)/TD22M
      A((M+1)/2,I)=TD22L*((EMP2S*UFS-BMS)+TD2L*(EMP2S*UFS-GMS))
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      TD2L=-BM*DCOSH(BM)/(GM*DCOSH(GM))
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)+
     1TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DCOSH(GM))
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```
      X2=(EMP2S-UFIS*BMS)*BM*DCOSH(BM)/(BMS+ENP2S)
      X22=(EMP2S-UFIS*GMS)*TD2L*GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*TD22L*DSIN(EMP2)*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K,I)=A((N+1)/2+K,I)+XX
      X2=BM*DCOSH(BM)/(BMS+ENP2S)
      X22=TD2L*GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*TD22L*EMP2*DSIN(ENP2)*(X2+X22)
      A((N+1)/2+K2,I)=A((N+1)/2+K2,I)+XX
      IF (N.GT.1) GO TO 16
      XX=TD22L*(DSINH(BM)+TD2L*DSINH(GM))*DSIN(EMP2*CSI)
      A(I,I)=A(I,I)+XX
      A(I,(M+1)/2)=(TD2M*DSINH(BM)+DSINH(GM))*DSIN(EMP2*CSI)/TD22M
      A((M+1)/2,I)=TD22L*((EMP2S*UFS-BMS)*DSINH(BM)+TD2L*(
     1EMP2S*UFS-GMS)*DSINH(GM))
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
     1GMS)*DSINH(GM))/TD22M
16    X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
20    CONTINUE
      DO 30 M=1,I
      TEMP=A(M,I-1)
      A(M,I-1)=A(M,I)
      A(M,I)=TEMP
30    CONTINUE
      CALL DETERM (A,I,DET)
      PRINT31
31    FORMAT('1',10X,'XXXXXXXXXXXXXXXXXXXX',10X,'END',10X,
     1'XXXXXXXXXXXXXXXXXXXX')
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
30    BIG=TERM
      L=K
50    CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
```

```
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C     FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C     FIND AND PRINT EM,EN,EP,AND P*
C
```

```
      L=(N-1)/3
      DO 550 I=1,L
      J=2*I-1
      K=J
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      P=X(3*L)
      GO TO 550
  540 EP(J)=-1.
  550 PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J),P
  600 FORMAT('-','M=',I2,3X,'N=',I2,3X,'P=',I2,5X,'EM(',I2,') =',
     1D13.5,5X,'EN(',I2,') =',D13.5,5X,'EP(',I2,') =',D13.5,5X,
     1'P* =',D13.5)
      PRINT601
  601 FORMAT('1',30X,'THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:',////)
      CALL SHAPE (EM,EN,EP,P)
   60 RETURN
      END
      SUBROUTINE SHAPE (EM,EN,EP,P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON ZZ,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
     1,EM(75),EN(75),EP(75)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      W44=0.0
      DO 620 N=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 605
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      IF (ETA.EQ.0.0) GO TO 602
      IF (PSI.EQ.0.0) GO TO 603
      XW2=EN(N)*(TD1N*DSINH(BN*PSI)+DSIN(GN*PSI))*DSIN(ENP2*ETA)/TD11N
      IF (PSI.EQ.1.0) GO TO 601
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD1P*DCOS((1-PSI)*GN))*DSIN
     1(ENP2*ETA)/TD11P
      GO TO 604
  601 XW3=EP(N)*(1-TD1P)*DSIN(ENP2*ETA)/TD11P
      GO TO 604
  602 XW2=0.0
      XW3=0.0
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      GO TO 604
  603 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD1P*DCOS(GN))*DSIN(ENP2*ETA)/TD11P
  604 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
      GO TO 620
  605 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.GT.Z2) GO TO 610
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      IF (ETA.EQ.0.0) GO TO 607
      IF (PSI.EQ.0.0) GO TO 608
      XW2=EN(N)*(TD2N*DSINH(BN*PSI)+DSINH(GN*PSI))*DSIN(ENP2*ETA)/TD22N
      IF (PSI.EQ.1.0) GO TO 606
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD2P*DCOSH((1-PSI)*GN))*DSIN(ENP2*
     1ETA)/TD22P
      GO TO 609
  606 XW3=EP(N)*(1-TD2P)*DSIN(ENP2*ETA)/TD22P
      GO TO 609
  607 XW2=0.0
      XW3=0.0
      GO TO 609
  608 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*DSIN(ENP2*ETA)/TD22P
  609 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
      GO TO 620
  610 TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      IF (ETA.EQ.0.0) GO TO 612
      IF (PSI.EQ.0.0) GO TO 612
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.0) B=0.0
      XW2=B*EN(N)*(TD2N*DEXP((BN*PSI-BN)*B)+DEXP((GN*PSI-GN)*B))
     1*DSIN(ENP2*ETA)/(BN*TD2N+GN)
      TEST=BN*PSI
      B=1.
      IF (TEST.GT.60.0) B=0.
      XW3=EP(N)*(DEXP((-BN*PSI)*B)-DEXP((-GN*PSI)*B))*DSIN(ENP2*ETA)*B
     1/(BN-GN)
      GO TO 614
  612 XW2=0.0
      XW3=0.0
  614 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W22=W2(I,J)
      W33=W3(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
```

```
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 625
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      TD1L=-BM*DCOSH(BM)/(GM*DCOS(GM))
      TD11L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)-TD1L*
     1GM*(GMS+POIS*PHIS*EMP2S)*DCOS(GM))
      IF (ETA.EQ.0.0) GO TO 622
      IF (PSI.EQ.0.0) GO TO 622
      XW1=EM(M)*(TD1M*DSINH(BM*ETA)+DSIN(GM*ETA))*DSIN(EMP2*PSI)/TD11M
      XW4=P*TD11L*(DSINH(BM*ETA)+TD1L*DSIN(GM*ETA))*DSIN(EMP2*PSI)
      GO TO 624
  622 XW1=0.0
      XW4=0.0
  624 W1(I,J)=W11+XW1
      W4(I,J)=W44+XW4
      W11=W1(I,J)
      W44=W4(I,J)
      GO TO 630
  625 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.22) GO TO 627
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      TD2L=-BM*DCOSH(BM)/(GM*DCOSH(GM))
      TD22L=-2*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)+
     1TD2L*GM*(GMS-POIS*PHIS*EMP2S)*DCOSH(GM))
      IF (ETA.EQ.0.0) GO TO 626
      IF (PSI.EQ.0.0) GO TO 626
      XW1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD22M
      XW4=P*TD22L*(DSINH(BM*ETA)+TD2L*DSINH(GM*ETA))*DSIN(EMP2*PSI)
      GO TO 629
  626 XW1=0.0
      XW4=0.0
      GO TO 629
  627 TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD2L=-BM/GM
      TD22L=-2.*DSIN(EMP2*CSI)/(BM*(BMS-POIS*PHIS*EMP2S)+TD2L*GM*(GMS-
     1POIS*PHIS*EMP2S))
      IF (ETA.EQ.0.0) GO TO 628
      IF(PSI.EQ.0.0) GO TO 628
      B=1.
      TEST=BM-BM*ETA
      IF (TEST.GT.60.) B=0.
      XW1=EM(M)*(TD2M*DEXP((BM*ETA-BM)*B)+DEXP((GM*ETA-GM)*B))*B*DSIN(
     1EMP2*PSI)/(TD2M*BM+GM)
      XW4=B*P*TD22L*(DEXP((BM*ETA-BM)*B)+TD2L*DEXP((GM*ETA-GM)*B))*DSIN
     1(EMP2*PSI)
      GO TO 629
  628 XW1=0.0
      XW4=0.0
  629 W1(I,J)=W11+XW1
      W4(I,J)=W44+XW4
```

```
      W11=W1(I,J)
      W44=W4(I,J)
630   CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
      PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
635   FORMAT('-','PSI =',F5.3,3X,'ETA =',F5.3,3X,'W1(',I2,',',I2,') =',
     1D12.5,3X,'W2(',I2,',',I2,') =',D12.5,3X,'W3(',I2,',',I2,') =',
     1D12.5,3X,'W4(',I2,',',I2,') =',D12.5,/)
      PRINT636,PSI,ETA,I,J,W(I,J)
636   FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'
     1W(',I2,',',I2,') =',D12.5)
      PSI=PSI+1.0/DFLOAT(KS)
640   CONTINUE
      ETA=ETA+1.0/DFLOAT(KS)
650   CONTINUE
      RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                               PROGRAM 5                              C
C                                                                      C
C          THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM          C
C     FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.     C
C     WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE EDGE PARAL-     C
C     LEL TO THE CLAMPED BASE.                                         C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED:                C
C                                                                      C
C             1.- K = THE NUMBER OF TERMS TO BE USED IN THE SERIES     C
C                     INVOLVED. FOR 5 DIGITS ACCURACY USE K = 30.      C
C                                                                      C
C             2.- CSI = 0 TO 1. PROVIDING THE DIMENSIONLESS DISTANCE   C
C                       OF THE POINT SUPPORT TO THE X OR PSI AXIS.     C
C                       SEE FIGURE 3.2.                                C
C                                                                      C
C             3.- PHIR = 2B/A. IS THE FULL PLATE ASPECT RATIO.         C
C                                                                      C
C             4.- ALMDS = AN INITIAL STARTING VALUE FOR THE EIGEN-     C
C                         VALUE SEARCH.                                C
C                                                                      C
C             5.- CLIM = A FINISHING OR EIGENVALUE SEARCH ENDING       C
C                        LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO      C
C                        HALT EXECUTION.                               C
C                                                                      C
C             6.- DEL = EIGENVALUE INCREMENT.                          C
C                                                                      C
C             7.- POI = POISSON'S RATIO.                               C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      K=????????????????????????????
      ZZ=3600
      CSI=???????????????????????????
      PHIR=??????????????????????????
      PHI=PHIR/2
      PRINT50,PHIR,K,CSI
   50 FORMAT('1',10X,'PHIR =',F8.4,10X,'K =',I4,10X,'CSI =',F8.4,////)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UPS=POI*PHIS
      UPIS=POI*PHINS
```

```
      I=3*K
      ALMDS=??????????????????????????
      DLIM=??????????????????????????
      DEL=???????????????????????????
    1 CONTINUE
C
C   INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=PCIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD11L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-X)*DCOSH(BN)-TD1L*GN*(GNS+X)
     1*DCOS(GN)))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(L,N+K)=(DSINH(BN)+TD1*DSIN(GN))*DCOS(ENP*CSI)/TD11N
      A(L,N+2*K)=(1-TD1P)*DCOS(ENP*CSI)/TD11P
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
      A(N+K,L)=TD11L*((X-BNS)*DSINH(BN)+TD1L*(X+GNS)*DSIN(GN))
      A(N+2*K,L)=TD11L*(BN+TD1L*GN)
      XX=TD11L*(DSINH(BN)+TD1L*DSIN(GN))*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X11=((ENPS+UFS*GNS)/(GNS-EMP2S))*GN*TD1L*DCOS(GN)
```

```
      XX=2*TD11L*DCOS(ENP)*DSIN(EMP2)*(X1-X11)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
      GO TO 10
 5    X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=PCIS*PHINS*ENPS
      IF (ENS.LT.Z2) GO TO 6
      TD2=EN*(BNS-X)/(GN*(X-GNS))
      TD2L=-BN/GN
      TD2P=1.0
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      CL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD22L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-PCIS*PHINS*ENPS)+TD2L*GN
     1*(GNS-PCIS*PHINS*ENPS)))
      IF (M.GT.1.0) GO TO 7
      A(L,N+K)=(1+TD2)*DCOS(ENP*CSI)/TD22N
      A(L,N+2*K)=0.0
      A(N+2*K,L)=0.0
      A(N+K,L)=TD22L*((X-ENS)+TD2L*(X-GNS))
      XX=TD22L*(1+TD2L)*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
 6    TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      CL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD22L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-PCIS*PHINS*ENPS)*DCOSH(BN)+
     1TD2L*GN*(GNS-POIS*PHINS*ENPS)*DCOSH(GN)))
      IF (M.GT.1.0) GO TO 8
      A(L,N+K)=(DSINH(BN)+TD2*DSINH(GN))*DCOS(ENP*CSI)/TD22N
      A(L,N+2*K)=(1-TD2P)*DCOS(ENP*CSI)/TD22P
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      A(N+K,L)=TD22L*((X-BNS)*DSINH(BN)+TD2L*(X-GNS)*DSINH(GN))
      A(N+2*K,L)=TD22L*(BN+TD2L*GN)
      XX=TD22L*(DSINH(BN)+TD2L*DSINH(GN))*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
      GO TO 8
 7    X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN                 *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      X2=(ENPS-UFS*BNS)*BN/(BNS+EMP2S)
      X22=(ENPS-UFS*GNS)*GN*TD2L/(GNS+EMP2S)
```

```
      XX=2*TD22L*DCOS(ENP)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*EN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      X2=((ENPS-UFS*BNS)/(BNS+EMP2S))*EN*DCOSH(BN)
      X22=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*TD2L*DCOSH(GN)
      XX=2*TD22L*DCOS(ENP)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1.)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX                =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      XX4=(DCOSH(BM*CSI)+TD1*DCOS(GM*CSI))*DSIN(EMP2)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A(I+1,(M+1)/2)=XX4
   12 A(N+K,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
```

```
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX6=(DCOSH(BM*CSI)+TD2*DCOSH(GM*CSI))*DSIN(EMP2)/TD22
      A(I+1,(M+1)/2)=XX6
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      B=1.0
      TEST=BM-BM*CSI
      IF (TEST.GT.60.) B=0.0
      XX6=B*(DEXP((BM*CSI-BM)*B)+TD2*DEXP((GM*CSI-GM)*B))*DSIN(EMP2)/
     1(BM+TD2*GM)
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
   20 CONTINUE
      CALL DETERM (A,L,DET)
      PRINT110,ALMDS,DET
  110 FORMAT(' ',' ','ALMDS=',F12.8,10X,'DET=',D20.5)
      ALMDS=ALMDS+DEL
      IF (ALMDS.GT.DLIM) GO TO 21
      GO TO 1
   21 CONTINUE
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
```

```
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                             PROGRAM 10                               C
C                                                                      C
C            THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE REC-       C
C     TANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH POINT     C
C     SUPPORTS SYMMETRICALY LOCATED ON THE EDGE PARALLEL TO THE        C
C     CLAMPED BASE.                                                    C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                C
C                                                                      C
C             1.- K       = AS DEFINED IN PROGRAM 9.                   C
C                                                                      C
C             2.- CSI     = AS DEFINED IN PROGRAM 9.                   C
C                                                                      C
C             3.- PHIR    = 2B/A FULL PLATE ASPECT RATIO.              C
C                                                                      C
C             4.- ALMDS   = EIGENVALUE.                                C
C                                                                      C
C             5.- KS      = NUMBER OF POINTS AT WHICH THE DISPLACE-    C
C                           MENT W IS REQUIRED.                        C
C                                                                      C
C             6.- POI     = POISSON'S RATIO.                           C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      DIMENSION A(99,99)
      K=?????????????????????
      KS=????????????????????
      Z2=3600
      CSI=???????????????????
      PHIR=??????????????????
      ALMDS=?????????????????
      PHI=PHIR/2
      PRINT50,K,KS,CSI,PHIR,ALMDS
   50 FORMAT('1',10X,'K =',I3,10X,'KS =',I3,10X,'CSI =',F7.4,10X,
     1'PHIR =',F7.4,10X,'ALMDS =',F8.4,////)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=???????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
    1 CONTINUE
```

```
C
C     INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD11L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-X)*DCOSH(BN)-TD1L*GN*(GNS+X)
     1*DCOS(GN)))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(L,N+K)=(DSINH(BN)+TD1*DSIN(GN))*DCOS(ENP*CSI)/TD11N
      A(L,N+2*K)=(1-TD1P)*DCOS(ENP*CSI)/TD11P
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
      A(N+K,L)=TD11L*((X-BNS)*DSINH(BN)+TD1L*(X+GNS)*DSIN(GN))
      A(N+2*K,L)=TD11L*(BN+TD1L*GN)
      XX=TD11L*(DSINH(BN)+TD1L*DSIN(GN))*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X11=((ENPS+UFS*GNS)/(GNS-EMP2S))*GN*TD1L*DCOS(GN)
      XX=2*TD11L*DCOS(ENP)*DSIN(EMP2)*(X1-X11)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
```

```
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.LT.Z2) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      TD2L=-BN/GN
      TD2P=1.0
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD22L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-POIS*PHINS*ENPS)+TD2L*GN
     1*(GNS-POIS*PHINS*ENPS)))
      IF (M.GT.1.0) GO TO 7
      A(L,N+K)=(1+TD2)*DCOS(ENP*CSI)/TD22N
      A(L,N+2*K)=0.0
      A(N+2*K,L)=0.0
      A(N+K,L)=TD22L*((X-BNS)+TD2L*(X-GNS))
      XX=TD22L*(1+TD2L)*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      TD22L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-POIS*PHINS*ENPS)*DCOSH(BN)+
     1TD2L*GN*(GNS-POIS*PHINS*ENPS)*DCOSH(GN)))
      IF (M.GT.1.0) GO TO 8
      A(L,N+K)=(DSINH(BN)+TD2*DSINH(GN))*DCOS(ENP*CSI)/TD22N
      A(L,N+2*K)=(1-TD2P)*DCOS(ENP*CSI)/TD22P
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      A(N+K,L)=TD22L*((X-BNS)*DSINH(BN)+TD2L*(X-GNS)*DSINH(GN))
      A(N+2*K,L)=TD22L*(BN+TD2L*GN)
      XX=TD22L*(DSINH(BN)+TD2L*DSINH(GN))*DCOS(ENP*CSI)
      A(L,L)=A(L,L)+XX
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN                      *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      X2=(ENPS-UFS*BNS)*BN/(BNS+EMP2S)
      X22=(ENPS-UFS*GNS)*GN*TD2L/(GNS+EMP2S)
      XX=2*TD22L*DCOS(ENP)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
```

```
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      X2=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X22=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*TD2L*DCOSH(GN)
      XX=2*TD22L*DCOS(ENP)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,L)=A((M+1)/2,L)+XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX                  =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      XX4=(DCOSH(BM*CSI)+TD1*DCOS(GM*CSI))*DSIN(EMP2)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A(I+1,(M+1)/2)=XX4
   12 A(N+K,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX6=(DCOSH(BM*CSI)+TD2*DCOSH(GM*CSI))*DSIN(EMP2)/TD22
      A(I+1,(M+1)/2)=XX6
```

```
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      B=1.0
      TEST=BM-BM*CSI
      IF (TEST.GT.60.) B=0.0
      XX6=B*(DEXP((BM*CSI-BM)*B)+TD2*DEXP((GM*CSI-GM)*B))*DSIN(EMP2)
     1/(BM+TD2*GM)
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      GO TO 14
   20 CONTINUE
      DO 30 M=1,L
      TEMP=A(M,I)
      A(M,I)=A(M,I+1)
      A(M,I+1)=TEMP
   30 CONTINUE
      CALL DETERM (A,L,DET)
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=1,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
```

```
C       L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C       I IS NOT EQUAL TO L.SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C       NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C       FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C       MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C       NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C       END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C       IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C       GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C       NOT WORKING ON LAST ROW,I IS 2 OR GREATER.
C
  400 DO 450 J=2,I
C
C       WORK BACKWARD FOR X(N),X(N-1)-------SUBSTITUTING PREVIOUSLY
C       FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C       FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C       FIND AND PRINT EM,EN,EP,AND P*
C
      L=(N-1)/3
```

```
      DO 550 I=1,L
      J=2*I-1
      K=I-1
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      P=X(3*L)
      GO TO 550
540   EP(J)=-1.
550   PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J),P
600   FORMAT('-','M=',I2,3X,'N=',I2,3X,'P=',I2,5X,'EM(',I2,') =',
     1D13.5,5X,'EN(',I2,') =',D13.5,5X,'EP(',I2,') =',D13.5,5X,
     1'P* =',D13.5)
      DO 601 I=1,L
      J=2*I-1
      EN(I)=EN(J)
      EP(I)=EP(J)
601   CONTINUE
      PRINT 650
650   FORMAT('1','THE SYMMETRIC MODE SHAPE DATA ARE:'////)
      CALL SHAPE (EM,EN,EP,P)
 60   RETURN
      END
      SUBROUTINE SHAPE (EM,EN,EP,P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
     1,EM(75),EN(75),EP(75)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      W44=0.0
      DO 620 N=1,K
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 606
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      DL=1.0
      IF (N.EQ.1) DL=2.0
      TD1RL=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-X)*DCOSH(BN)-TD1L*GN*(GNS+X)*
     1DCOS(GN)))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
```

```
      IF (ETA.EQ.0.0) GO TO 601
      IF (ENP.EQ.0.0) GO TO 601
      XX=DCOS(ENP*ETA)
      GO TO 602
 601  XX=1.0
 602  IF (PSI.EQ.0.0) GO TO 604
      XW2=EN(N)*(DSINH(BN*PSI)+TD1*DSIN(GN*PSI))*XX/TD11N
      XW4=P*TD11L*(DSINH(BN*PSI)+TD1L*DSIN(GN*PSI))*XX
      IF (PSI.EQ.1.0) GO TO 603
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD1P*DCOS(GN*(1-PSI)))*XX/TD11P
      GO TO 617
 603  XW3=EP(N)*(1-TD1P)*XX/TD11P
      GO TO 617
 604  XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 617
 606  X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.GT.Z2) GO TO 612
      TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      DL=1.0
      IF (N.EQ.1) DL=2.0
      TD22L=-2*DCOS(ENP*CSI)/(DL*(BN*(BNS-X)*DCOSH(BN)+TD2L*GN*(GNS-X)*
     1DCOSH(GN)))
      IF (ETA.EQ.0.0) GO TO 607
      IF (ENP.EQ.0.0) GO TO 607
      XX=DCOS(ENP*ETA)
      GO TO 608
 607  XX=1.0
 608  IF (PSI.EQ.0.0) GO TO 610
      XW2=EN(N)*(DSINH(BN*PSI)+TD2*DSINH(GN*PSI))*XX/TD22N
      XW4=P*TD22L*(DSINH(BN*PSI)+TD2L*DSINH(GN*PSI))*XX
      IF (PSI.EQ.1.0) GO TO 609
      XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD2P*DCOSH(GN*(1-PSI)))*XX/TD22P
      GO TO 617
 609  XW3=EP(N)*(1-TD2P)*XX/TD22P
      GO TO 617
 610  XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 617
 612  TD2N=BN*(BNS-POIS*PHINS*ENPS)/(GN*(POIS*PHINS*ENPS-GNS))
      IF(ETA.EQ.0.0) GO TO 613
      IF (ENP.EQ.0.0) GO TO 613
      XX=DCOS(ENP*ETA)
      GO TO 614
 613  XX=1.0
 614  IF (PSI.EQ.0.0) GO TO 616
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.0) B=0.0
      XW2=B*(DEXP((BN*PSI-BN)*B)+TD2N*DEXP((GN*PSI-GN)*B))*EN(N)*XX/
     1(BN+TD2N*GN)
```

```
      C1=-2*DCOS(ENP*CSI)
      C2=BN*(BNS-POIS*PHINS*ENPS)
      C3=GN*(GNS-POIS*PHINS*ENPS)
      DL=1.0
      IF (N.EQ.1) DL=2.0
      TD2L=-BN/GN
      TD22L=C1/(DL*(C2+TD2L*C3))
      XW4=B*P*TD22L*XX*(DEXP((BN*PSI-BN)*B)+TD2L*DEXP((GN*PSI-GN)*B))
      IF (PSI.EQ.1.0) GO TO 615
      TD2P=1.
      TD22P=BN-GN*TD2P
      B=1.
      TEST=BN*PSI
      IF (TEST.GT.60.) B=0.
      XW3=EP(N)*B*XX*(DEXP((-BN*PSI)*B)-TD2P*DEXP((-GN*PSI)*B))/TD22P
      GO TO 617
  615 XW3=0.0
      GO TO 617
  616 XW2=0.0
      XW3=0.0
      XW4=0.0
  617 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 623
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      IF (PSI.EQ.0.0) GO TO 622
      IF(ETA.EQ.0.0) GO TO 621
      XW1=EM(M)*(DCOSH(BM*ETA)+TD1*DCOS(GM*ETA))*DSIN(EMP2*PSI)/TD11
      GO TO 629
  621 XW1=EM(M)*(1+TD1)*DSIN(EMP2*PSI)/TD11
      GO TO 629
  622 XW1=0.0
      GO TO 629
  623 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 625
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2S)
     1 *DSINH(GM))
      TD2=TD2*(-1)
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      IF (PSI.EQ.0.0) GO TO 622
      IF (ETA.EQ.0.0) GO TO 624
      XW1=EM(M)*(DCOSH(BM*ETA)+TD2*DCOSH(GM*ETA))*DSIN(EMP2*PSI)/TD22
```

```
      GO TO 629
  624 XW1=   EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
      GO TO 629
  625 A=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
      IF (PSI.EQ.0.0) GO TO 627
      IF (ETA.EQ.0.0) GO TO 627
      TEST=BM-BM*ETA
      IF (TEST.GT.60.0) GO TO 627
      XW1=EM(M)*(DEXP(BM*ETA-BM)+A*DEXP(GM*ETA-GM))*DSIN(EMP2*PSI)/
     1(BM+A*GM)
      GO TO 629
  627 XW1=0.0
  629 W1(I,J)=XW1+W11
      W11=W1(I,J)
  630 CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
      PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
  635 FORMAT('-','PSI =',F5.3,3X,'ETA =',F5.3,3X,'W1(',I2,',',I2,') =',
     1D12.5,3X,'W2(',I2,',',I2,') =',D12.5,3X,'W3(',I2,',',I2,') =',
     1D12.5,3X,'W4(',I2,',',I2,') =',D12.5,/)
      PRINT636,PSI,ETA,I,J,W(I,J)
  636 FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'
     1W(',I2,',',I2,') =',D12.5)
      PSI=PSI+1.0/DFLOAT(KS)
  640 CONTINUE
      ETA=ETA+1.0/DFLOAT(KS)
  650 CONTINUE
      RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                            C
C                            PROGRAM 11                                      C
C                                                                            C
C              THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH              C
C       PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION         C
C       PROBLEM. WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE           C
C       EDGE PARALLEL TO THE CLAMPED BASE.                                  C
C                                                                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                            C
C       USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                    C
C                                                                            C
C               1.- K       = THE NUMBER OF TERMS TO BE USED IN THE          C
C                             SERIES INVOLVED. FOR 5 DIGITS ACCURACY         C
C                             USE K = 30.                                    C
C                                                                            C
C               2.- CSI     = 0 TO 1. PROVIDING THE DISTANCE BETWEEN         C
C                             THE CONCENTRATED EDGE FORCE(POINT SUP.)        C
C                             AND THE PLATE CENTRAL AXIS DIVIDED BY THE      C
C                             PLATE EDGE DIMENSION.                          C
C                                                                            C
C               3.- PHIR    = 2B/A. IS THE FULL PLATE ASPECT RATIO.          C
C                                                                            C
C               4.- ALMDS   = EIGENVALUE.                                    C
C                                                                            C
C               5.- DLIM    = EIGENVALUE SEARCH ENDING LIMIT. IT             C
C                             INSTRUCTS THE COMPUTER WHEN TO HALT            C
C                             EXECUTION.                                     C
C                                                                            C
C               6.- DEL     = EIGENVALUE INCREMENT.                          C
C                                                                            C
C               7.- PUI     = POISSON'S RATIO.                               C
C                                                                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      K=????????????????????
      CSI=??????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600.
      PHIR=?????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,K,CSI
   50 FORMAT('1','PHIR =',F7.4,10X,'K =',I5,10X,'CSI =',F7.4,/////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      PUI=??????????????????
      POIS=2-PUI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
```

```
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K+1
      ALMDS=??????????????????
      DLIM=??????????????????
      DEL=???????????????????
    1 CONTINUE
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I
      DO 2 N=1,I
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      TD11L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)
     1-TD1L*GN*(GNS+POIS*PHINS*ENP2S)*DCOS(GN))
      IF (M.GT.1.) GO TO 3
      A(I,(N+1)/2+K)=(TD1N*DSINH(BN)+DSIN(GN))*DSIN(ENP2*CSI)/TD11N
      A(I,(N+1)/2+K2)=(1-TD1P)*DSIN(ENP2*CSI)/TD11P
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
      XX=TD11L*(DSINH(BN)+TD1L*DSIN(GN))*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,I)=TD11L*(BN+TD1L*GN)
      A((N+1)/2+K,I)=TD11L*((UFIS*ENP2S-BNS)*DSINH(BN)+TD1L*(UFIS*
     1ENP2S+GNS)*DSIN(GN))
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X11=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1L*GN*DCOS(GN)
      XX=2*TD11L*DSIN(ENP2)*DSIN(EMP2)*(X1-X11)
      A((M+1)/2,I)=A((M+1)/2,I)+XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
```

```
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      TD2L=-BN/GN
      TD22L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)+TD2L*GN*
     1(GNS-POIS*PHINS*ENP2S))
      IF (M.GT.1) GO TO 5
      A(I,(N+1)/2+K)=(TD2N+1)*DSIN(ENP2*CSI)/TD22N
      A(I,(N+1)/2+K2)=0.0
      A((N+1)/2+K2,I)=0.0
      A((N+1)/2+K,I)=TD22L*(UFIS*ENP2S-BNS+TD2L*(UFIS*ENP2S-GNS))
      XX=TD22L*(1+TD2L)*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      X2=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN
      X22=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2L*GN
      XX=2*TD22L*DSIN(ENP2)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,I)=A((M+1)/2,I)+XX
      GO TO 10
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      TD22L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)+
     1TD2L*GN*(GNS-POIS*PHINS*ENP2S)*DCOSH(GN))
      IF (M.GT.1) GO TO 7
      A(I,(N+1)/2+K)=(TD2N*DSINH(BN)+DSINH(GN))*DSIN(ENP2*CSI)/TD22N
      A(I,(N+1)/2+K2)=(1-TD2P)*DSIN(ENP2*CSI)/TD22P
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
      XX=TD22L*(DSINH(BN)+TD2L*DSINH(GN))*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,I)=TD22L*(BN+TD2L*GN)
      A((N+1)/2+K,I)=TD22L*((UFIS*ENP2S-BNS)*DSINH(BN)+TD2L*
     1(UFIS*ENP2S-GNS)*DSINH(GN))
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
```

```
      A((M+1)/2,(N+1)/2+K2)=XX
      X2=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X22=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2L*GN*DCOSH(GN)
      XX=2*TD22L*DSIN(ENP2)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,I)=A((M+1)/2,I)+XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (N.GT.1) GO TO 12
      A(I,(M+1)/2)=(TD1M*DSINH(BM*CSI)+DSIN(GM*CSI))*DSIN(EMP2)/TD11M
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      IF (N.GT.1) GO TO 14
      TEST=BM-BM*CSI
      B=1.0
      IF (TEST.GT.60.) B=0.0
      A(I,(M+1)/2)=B*DSIN(EMP2)*(TD2M*DEXP((BM*CSI-BM)*B)+DEXP((GM*CSI
     1-GM)*B))/(TD2M*BM+GM)
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
```

```
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      IF (N.GT.1) GO TO 16
      A(L,(M+1)/2)=(TD2M*DSINH(BM*CSI)+DSINH(GM*CSI))*DSIN(EMP2)/TD22M
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
     1GMS)*DSINH(GM))/TD22M
   16 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
   20 CONTINUE
      CALL DETERM (A,I,DET)
      PRINT 110,ALMDS,DET
  110 FORMAT(' ','ALMDS=',F13.7,10X,'DET=',D20.5)
      IF (ALMDS.GT.DLIM) GO TO 21
      ALMDS=ALMDS+DEL
      GO TO 1
   21 CONTINUE
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
```

```
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C                          PROGRAM 12                                 C
C                                                                     C
C            THIS IS AN ANTISYMMETRIC MODE SHAPE PROGRAM FOR THE      C
C     RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH       C
C     POINT SUPPORTS SYMMETRICALY LOCATED ON THE EDGE PARALLEL TO     C
C     THE CLAMPED BASE.                                               C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;               C
C                                                                     C
C             1.- K     = AS DEFINED IN PROGRAM 11.                   C
C                                                                     C
C             2.- CSI   = AS DEFINED IN PROGRAM 11.                   C
C                                                                     C
C             3.- PHIR  = 2B/A FULL PLATE ASPECT RATIO.               C
C                                                                     C
C             4.- ALMDS = EIGENVALUE.                                 C
C                                                                     C
C             5.- KS    = NUMBER OF POINTS AT WHICH THE DISPLACE-     C
C                         MENT W IS REQUIRED.                         C
C                                                                     C
C             6.- POI   = POISSON'S RATIO.                            C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      K=??????????????????????
      KS=?????????????????????
      CSI=????????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600.
      PHIR=???????????????????
      PHI=PHIR/2.
      ALMDS=??????????????????
      PRINT50,PHIR,K,CSI,ALMDS
   50 FORMAT('1','PHIR =',F7.4,10X,'K =',I5,10X,'CSI =',F7.4,10X,'ALMDS
     1 =',F7.4,////)
      PRINT51
   51 FORMAT('-',30X,'THE VALUES OF EM,EN,EP,AND P* ARE:',////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=0.333
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
```

```
      UFIS=POI*PHINS
      I=3*K+1
    1 CONTINUE
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I
      DO 2 N=1,I
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      TD11L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)
     1-TD1L*GN*(GNS+POIS*PHINS*ENP2S)*DCOS(GN))
      IF (M.GT.1.) GO TO 3
      A(I,(N+1)/2+K)=(TD1N*DSINH(BN)+DSIN(GN))*DSIN(ENP2*CSI)/TD11N
      A(I,(N+1)/2+K2)=(1-TD1P)*DSIN(ENP2*CSI)/TD11P
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
      XX=TD11L*(DSINH(BN)+TD1L*DSIN(GN))*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,I)=TD11L*(BN+TD1L*GN)
      A((N+1)/2+K,I)=TD11L*((UFIS*ENP2S-BNS)*DSINH(BN)+TD1L*(UFIS*
     1ENP2S+GNS)*DSIN(GN))
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X11=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1L*GN*DCOS(GN)
      XX=2*TD11L*DSIN(ENP2)*DSIN(EMP2)*(X1-X11)
      A((M+1)/2,I)=A((M+1)/2,I)+XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
```

```
      TD2P=1.
      TD22P=BN-GN*TD2P
      TD2L=-BN/GN
      TD22L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)+TD2L*GN*
     1(GNS-POIS*PHINS*ENP2S))
      IF (M.GT.1) GO TO 5
      A(I,(N+1)/2+K)=(TD2N+1)*DSIN(ENP2*CSI)/TD22N
      A(I,(N+1)/2+K2)=0.0
      A((N+1)/2+K2,I)=0.0
      A((N+1)/2+K,I)=TD22L*(UFIS*ENP2S-BNS+TD2L*(UFIS*ENP2S-GNS))
      XX=TD22L*(1+TD2L)*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      X2=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN
      X22=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2L*GN
      XX=2*TD22L*DSIN(ENP2)*DSIN(EMP2)*(X2+X22)
      A((M+1)/2,I)=A((M+1)/2,I)+XX
      GO TO 10
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      TD22L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)+
     1TD2L*GN*(GNS-POIS*PHINS*ENP2S)*DCOSH(GN))
      IF (M.GT.1) GO TO 7
      A(I,(N+1)/2+K)=(TD2N*DSINH(BN)+DSINH(GN))*DSIN(ENP2*CSI)/TD22N
      A(I,(N+1)/2+K2)=(1-TD2P)*DSIN(ENP2*CSI)/TD22P
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
      XX=TD22L*(DSINH(BN)+TD2L*DSINH(GN))*DSIN(ENP2*CSI)
      A(I,I)=A(I,I)+XX
      A((N+1)/2+K2,I)=TD22L*(BN+TD2L*GN)
      A((N+1)/2+K,I)=TD22L*((UFIS*ENP2S-BNS)*DSINH(BN)+TD2L*
     1(UFIS*ENP2S-GNS)*DSINH(GN))
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      X2=((ENP2S-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X22=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2L*GN*DCOSH(GN)
      XX=2*TD22L*DSIN(ENP2)*DSIN(EMP2)*(X2+X22)
```

```
      A((M+1)/2,I)=A((M+1)/2,I)+XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (N.GT.1) GO TO 12
      A(I,(M+1)/2)=(TD1M*DSINH(BM*CSI)+DSIN(GM*CSI))*DSIN(EMP2)/TD11M
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      IF (N.GT.1) GO TO 14
      TEST=BM-BM*CSI
      B=1.0
      IF (TEST.GT.60.0) B=0.0
      A(I,(M+1)/2)=B*DSIN(EMP2)*(TD2M*DEXP((BM*CSI-BM)*B)+DEXP((GM*CSI
     1-GM)*B))/(TD2M*BM+GM)
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      IF (N.GT.1) GO TO 16
      A(I,(M+1)/2)=(TD2M*DSINH(BM*CSI)+DSINH(GM*CSI))*DSIN(EMP2)/TD22M
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
```

```
     1GMS)*DSINH(GM))/TD22M
  16  X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
  20  CONTINUE
      DO 30 M=1,I
      TEMP=A(M,I-1)
      A(M,I-1)=A(M,I)
      A(M,I)=TEMP
  30  CONTINUE
      CALL DETERM (A,I,DET)
      PRINT31
  31  FORMAT('1',10X,'XXXXXXXXXXXXXXXXXXXX',10X,'END',10X,
     1'XXXXXXXXXXXXXXXXXXXX')
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
```

```
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C     FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C     FIND AND PRINT EM,EN,EP,AND P*
C
      L=(N-1)/3
      DO 550 I=1,L
      J=2*I-1
      K=J
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      P=X(3*L)
      GO TO 550
  540 EP(J)=-1.
  550 PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J),P
  600 FORMAT('-','M=',I2,3X,'N=',I2,3X,'P=',I2,5X,'EM(',I2,') =',
     1D13.5,5X,'EN(',I2,') =',D13.5,5X,'EP(',I2,') =',D13.5,5X,
     1'P* =',D13.5)
```

```
      PRINT601
  601 FORMAT('1',30X,'THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:',////)
      CALL SHAPE (EM,EN,EP,P)
   60 RETURN
      END
      SUBROUTINE SHAPE (EM,EN,EP,P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
     1KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
     1,EM(75),EN(75),EP(75)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.C
      W44=0.0
      DO 620 N=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 605
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      TD1L=-BN*DCOSH(BN)/(GN*DCOS(GN))
      TD11L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)-
     1TD1L*GN*(GNS+POIS*PHINS*ENP2S)*DCOS(GN))
      IF (ETA.EQ.0.0) GO TO 602
      IF (PSI.EQ.0.0) GO TO 603
      XW2=EN(N)*(TD1N*DSINH(BN*PSI)+DSIN(GN*PSI))*DSIN(ENP2*ETA)/TD11N
      XW4=P*TD11L*(DSINH(BN*PSI)+TD1L*DSIN(GN*PSI))*DSIN(ENP2*ETA)
      IF (PSI.EQ.1.C) GO TO 601
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD1P*DCOS((1-PSI)*GN))*DSIN
     1(ENP2*ETA)/TD11P
      GO TO 604
  601 XW3=EP(N)*(1-TD1P)*DSIN(ENP2*ETA)/TD11P
      GO TO 604
  602 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 604
  603 XW2=0.0
      XW4=0.0
      XW3=EP(N)*(DCOSH(BN)-TD1P*DCOS(GN))*DSIN(ENP2*ETA)/TD11P
  604 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
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```
      GO TO 620
  605 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.GT.Z2) GO TO 610
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD2L=-BN*DCOSH(BN)/(GN*DCOSH(GN))
      TD22L=-2*DSIN(ENP2*CSI)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)+
     1TD2L*GN*(GNS-POIS*PHINS*ENP2S)*DCOSH(GN))
      IF (ETA.EQ.0.0) GO TO 607
      IF (PSI.EQ.0.0) GO TO 608
      XW2=EN(N)*(TD2N*DSINH(BN*PSI)+DSINH(GN*PSI))*DSIN(ENP2*ETA)/TD22N
      XW4=P*TD22L*(DSINH(BN*PSI)+TD2L*DSINH(GN*PSI))*DSIN(ENP2*ETA)
      IF (PSI.EQ.1.0) GO TO 606
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD2P*DCOSH((1-PSI)*GN))*DSIN(ENP2*
     1ETA)/TD22P
      GO TO 609
  606 XW3=EP(N)*(1-TD2P)*DSIN(ENP2*ETA)/TD22P
      GO TO 609
  607 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 609
  608 XW2=0.0
      XW4=0.0
      XW3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*DSIN(ENP2*ETA)/TD22P
  609 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
      GO TO 620
  610 TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD2L=-BN/GN
      C1=-2*DSIN(ENP2*CSI)
      C2=BN*(BNS-POIS*PHINS*ENP2S)
      C3=GN*(GNS-POIS*PHINS*ENP2S)
      TD22L=C1/(C2+TD2L*C3)
      IF (ETA.EQ.0.0) GO TO 612
      IF (PSI.EQ.0.0) GO TO 612
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.0) B=0.0
      XW2=B*EN(N)*(TD2N*DEXP((BN*PSI-BN)*B)+DEXP((GN*PSI-GN)*B))*DSIN(
     1ENP2*ETA)/(BN*TD2N+GN)
      XW4=B*P*TD22L*(DEXP((BN*PSI-BN)*B)+TD2L*DEXP((GN*PSI-GN)*B))*
     1DSIN(ENP2*ETA)
      B=1.
      TEST=BN*PSI
      IF (TEST.GT.60.0) B=0.0
      XW3=EP(N)*B*(DEXP((-BN*PSI)*B)-DEXP((-GN*PSI)*B))*DSIN(ENP2*ETA
     1)/(BN-GN)
      GO TO 614
  612 XW2=0.0
      XW3=0.0
```

```
      XW4=0.0
  614 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 625
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (ETA.EQ.0.0) GO TO 622
      IF (PSI.EQ.0.0) GO TO 622
      XW1=EM(M)*(TD1M*DSINH(BM*ETA)+DSIN(GM*ETA))*DSIN(EMP2*PSI)/TD11M
      GO TO 624
  622 XW1=0.0
  624 W1(I,J)=W11+XW1
      W11=W1(I,J)
      GO TO 630
  625 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 627
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      IF (ETA.EQ.0.0) GO TO 626
      IF (PSI.EQ.0.0) GO TO 626
      XW1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD22M
      GO TO 629
  626 XW1=0.0
      GO TO 629
  627 TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      IF (ETA.EQ.0.0) GO TO 628
      IF(PSI.EQ.0.0) GO TO 628
      B=1.
      TEST=BM-BM*ETA
      IF (TEST.GT.60.0) B=0.0
      XW1=EM(M)*(TD2M*DEXP((BM*ETA-BM)*B)+DEXP((GM*ETA-GM)*B))*DSIN(
     1EMP2*PSI)*B/(BM*TD2M+GM)
      GO TO 629
  628 XW1=0.0
  629 W1(I,J)=W11+XW1
      W11=W1(I,J)
  630 CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
                 IF (PI.GT.0) GO TO 800
      PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
  635 FORMAT('-','PSI =',F5.3,3X,'ETA =',F5.3,3X,'W1(',I2,',',I2,') =',
     1D12.5,3X,'W2(',I2,',',I2,') =',D12.5,3X,'W3(',I2,',',I2,') =',
     1D12.5,3X,'W4(',I2,',',I2,') =',D12.5,/)
```

```
800   PRINT636,PSI,ETA,I,J,W(I,J)
 636 FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'
    1W(',I2,',',I2,') =',D12.5)
      PSI=PSI+1.0/DFLOAT(KS)
 640 CONTINUE
      ETA=ETA+1.0/DFLOAT(KS)
 650 CONTINUE
      RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C                             PROGRAM 13                                  C
C                                                                         C
C              THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM          C
C       FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.     C
C       WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS ON THE LATERAL      C
C       SURFACE.(ONE POINT SUPPORT ON EACH SIDE OF THE PLATE CENTRAL      C
C       AXIS.)                                                            C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C       USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                 C
C                                                                         C
C               1.- K      = THE NUMBER OF TERMS TO BE USED IN THE        C
C                            SERIES INVOLVED. FOR 5 DIGITS ACCURACY       C
C                            USE K = 30.                                  C
C                                                                         C
C               2.- U      = 0 TO 1, PROVIDING THE DISTANCE BETWEEN       C
C                            THE CONCENTRATED FORCE(POINT SUPPORT)        C
C                            AND THE PLATE CLAMPED EDGE. AXIS DIVIDED     C
C                            BY SIDE LENGTH A.                            C
C                                                                         C
C               3.- V      = 0 YO 1, PROVIDING THE DISTANCE BETWEEN       C
C                            THE CONCENTRATED FORCE(POINT SUPPORT)        C
C                            AND THE PLATE CENTRAL AXISDIVIDED BY         C
C                            SIDE LENGTH B.                               C
C                                                                         C
C               4.- PHIR   = 2B/A, IS THE FULL PLATE ASPECT RATIO.        C
C                                                                         C
C               5.- ALMDS  = AN INITIAL STARTING VALUE FOR THE EIGEN-     C
C                            VALUE SEARCH.                                C
C                                                                         C
C               6.- DLIM   = A FINISHING OR EIGENVALUE SEARCH ENDING      C
C                            LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO     C
C                            HALT EXECUTION.                              C
C                                                                         C
C               7.- DEL    = EIGENVALUE INCREMENT.                        C
C                                                                         C
C               8.- POI    = POISSON'S RATIO.                             C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      K=??????????????????????
      Z2=3600
      V=??????????????????????
      U=??????????????????????
      PHIR=???????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,K,U,V
   50 FORMAT('1',6X,'PHIR =',F8.4,6X,'K =',I4,6X,'U =',F8.4,6X,
     1'V =',F8.4,////)
```

```
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
      I=3*K
      DLIM=????????????????????
      ALMDS=???????????????????
      DEL=?????????????????????
 1    CONTINUE
C
C  INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
 2    CONTINUE
      DO 3 N=1,K
 3    A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A1N=DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*BN*
     1DCOSH(BN))
      B1N=-DCOS(ENP*V)*DCOS(GN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*GN*
     1DCOS(GN))
      C1N=DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS+GNS)*BN*DCOSH(BN))
      D1N=-DCOS(ENP*V)*DSIN(GN*U)/(DL*PHIS*PHIS*(BNS+GNS)*GN*DCOS(GN))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(L,N+K)=(DSINH(BN*U)+TD1*DSIN(GN*U))*DCOS(ENP*V)/TD11N
      A(L,N+2*K)=(DCOSH(BN*(1-U))-TD1P*DCOS(GN*(1-U)))*DCOS(ENP*V)/TD11P
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
      A(N+K,L)=(X-BNS)*C1N+(X+GNS)*D1N
```

```
      A(N+2*K,L)=A1N*BN+B1N*GN
4     X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
5     X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.LT.22) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      TD2P=1.0
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A2N=DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      B2N=-DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      C2N=DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      D2N=-DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      IF (M.GT.1.0) GO TO 7
      B=1.0
      TEST=BN-BN*U
      IF (TEST.GT.60.0) B=0.0
      A(L,N+K)=B*(DEXP((BN*U-BN)*B)+TD2*DEXP((GN*U-GN)*B))*DCOS(ENP*V)/
     1(BN+TD2*GN)
      B=1.0
      TEST=BN*U
      IF (TEST.GT.60.0) B=0.0
      A(L,N+2*K)=B*(DEXP((-BN*U)*B)-DEXP((-GN*U)*B))*DCOS(ENP*V)/(BN-GN)
      A(N+2*K,L)=B*(A2N*BN*DEXP((-BN*U)*B)+B2N*GN*DEXP((-GN*U)*B))
      B=1.0
      TEST=BN-BN*U
      IF (TEST.GT.60.0) B=0.0
      A(N+K,L)=B*((X-BNS)*C2N*DEXP((BN*U-BN)*B)+(X-GNS)*D2N*
     1DEXP((GN*U-GN)*B))
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
6     TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A2N=DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*BN*
     1DCOSH(BN))
      B2N=-DCOS(ENP*V)*DCOSH(GN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*GN*
     1DCOSH(GN))
      C2N=DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS-GNS)*BN*DCOSH(BN))
      D2N=-DCOS(ENP*V)*DSINH(GN*U)/(DL*PHIS*PHIS*(BNS-GNS)*GN*DCOSH(GN))
      IF (M.GT.1.0) GO TO 8
```

```
      A(L,N+K)=(DSINH(BN*U)+TD2*DSINH(GN*U))*DCOS(ENP*V)/TD22N
      A(L,N+2*K)=(DCOSH(BN*(1-U))-TD2P*DCOSH(GN*(1-U)))*DCOS(ENP*V)/
     1TD22P
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      A(N+K,L)=(X-BNS)*C2N+(X-GNS)*D2N
      A(N+2*K,L)=A2N*BN+B2N*GN
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      A1M=DSIN(EMP2*U)*DCOSH(BM*(1-V))/((BMS+GMS)*BM*DSINH(BM))
      B1M=DSIN(EMP2*U)*DCOS(GM*(1-V))/((BMS+GMS)*GM*DSIN(GM))
      C1M=DSIN(EMP2*U)*DCOSH(BM*V)/((BMS+GMS)*BM*DSINH(BM))
      D1M=DSIN(EMP2*U)*DCOS(GM*V)/((BMS+GMS)*GM*DSIN(GM))
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX=((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      XX4=(DCOSH(BM*V)+TD1*DCOS(GM*V))*DSIN(EMP2*U)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      A((M+1)/2,L)=(UFS*EMP2S-BMS)*C1M+(UFS*EMP2S+GMS)*D1M
```

```
      XX2=(A1M*DCOSH(BM*V)+B1M*DCOS(GM*V))*DSIN(EMP2*U)
      A(L,L)=A(L,L)+XX2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A(I+1,(M+1)/2)=XX4
   12 A(N+K,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      A2M=DSIN(EMP2*U)*DCOSH(BM*(1-V))/((BMS-GMS)*BM*DSINH(BM))
      B2M=-DSIN(EMP2*U)*DCOSH(GM*(1-V))/((BMS-GMS)*GM*DSINH(GM))
      C2M=DSIN(EMP2*U)*DCOSH(BM*V)/((BMS-GMS)*BM*DSINH(BM))
      D2M=-DSIN(EMP2*U)*DCOSH(GM*V)/((BMS-GMS)*GM*DSINH(GM))
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX6=(DCOSH(BM*V)+TD2*DCOSH(GM*V))*DSIN(EMP2*U)/TD22
      A(I+1,(M+1)/2)=XX6
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,L)=(UFS*EMP2S-BMS)*C2M+(UFS*EMP2S-GMS)*D2M
      XX2=(A2M*DCOSH(BM*V)+B2M*DCOSH(GM*V))*DSIN(EMP2*U)
      A(L,L)=A(L,L)+XX2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      D=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
      B=1.0
      TEST=BM-BM*V
      IF (TEST.GT.60.0) B=0.0
      XX6=B*(DEXP((BM*V-BM)*B)+D*DEXP((GM*V-GM)*B))*DSIN(EMP2*U)/
     1(BM+D*GM)
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      A2M=DSIN(EMP2*U)/(BM*(BMS-GMS))
      B2M=-DSIN(EMP2*U)/(GM*(BMS-GMS))
      C2M=DSIN(EMP2*U)/(BM*(BMS-GMS))
      D2M=-DSIN(EMP2*U)/(GM*(BMS-GMS))
```

```
      B=1.0
      TEST=BM-BM*V
      IF (TEST.GT.60.0) B=0.0
      A((M+1)/2,L)=B*((UFS*EMP2S-BMS)*C2M*DEXP((BM*V-BM)*B)+
     1(UFS*EMP2S-GMS)*D2M*DEXP((GM*V-GM)*B))
      TEST=BM-BM*V
      IF (TEST.LT.60.0) GO TO 16
      XX=(A2M/2)*(1+DEXP(-2*BM*V))+(B2M/2)*(1+DEXP(-2*GM*V))
      GO TO 17
   16 XX=(A2M/2)*(1+DEXP(2*(BM*V-BM)))+(B2M/2)*(1+DEXP(2*(GM*V-GM)))
   17 A(L,L)=A(L,L)+XX
      GO TO 14
   20 CONTINUE
      CALL DETERM (A,L,DET)
      PRINT110,ALMDS,DET
  110 FORMAT(' ','ALMDS=',F12.6,10X,'DET=',D20.5)
      ALMDS=ALMDS+DEL
      IF (ALMDS.GT.DLIM) GO TO 21
      GO TO 1
   21 CONTINUE
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND.
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
```

```
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTICN---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C                          PROGRAM 14                                C
C                                                                    C
C              THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE REC-   C
C      TANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH POINT  C
C      SUPPORTS SYMMETRICALLY DISTRIBUTED ON THE LATERAL SURFACE OF  C
C      THE PLATE.(ONE POINT SUPPORT ON EACH SIDE OF THE CENTRAL AXIS) C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C      USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;             C
C                                                                    C
C              1.-. K      = AS DEFINED IN PROGRAM 13.               C
C                                                                    C
C              2.- U       = AS DEFINED IN PROGRAM 13.               C
C                                                                    C
C              3.- V       = AS DEFINED IN PROGRAM 13.               C
C                                                                    C
C              4.- PHIR    = 2B/A FULL PLATE ASPECT RATIO.           C
C                                                                    C
C              5.- ALMDS   = EIGENVALUE.                             C
C                                                                    C
C              6.- KS      = NUMBER OF POINTS AT WHICH THE DISPLACE- C
C                            MENT W IS REQUIRED.                     C
C                                                                    C
C              7.- POI     = POISSON'S RATIO.                        C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,C-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,U,V,K,K1,
     1KS
      DIMENSION A(99,99)
      K=??????????????????????
      KS=?????????????????????
      Z2=3600
      V=??????????????????????
      U=?????????????????????
      PHIR=?????????????????
      ALMDS=????????????????
      PHI=PHIR/2
      PRINT50,PHIR,K,CSI
   50 FORMAT('1',10X,'PHIR =',F8.4,10X,'K =',I4,10X,'CSI =',F8.4,////)
      K1=2*K-1
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
      UFIS=POI*PHINS
```

```
      I=3*K
C
C  INITIALIZE THE MATRIX A
C
      L=I+1
      DO 2 M=1,L
      DO 2 N=1,L
      A(M,N)=0.0
    2 CONTINUE
      DO 3 N=1,K
    3 A(N+2*K,N+2*K)=-1.
      DO 10 M=1,K1,2
      DO 10 N=1,K
      JN=N-1
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 5
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A1N=DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*BN*
     1DCOSH(BN))
      B1N=-DCOS(ENP*V)*DCOS(GN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*GN*
     1DCOS(GN))
      C1N=DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS+GNS)*BN*DCOSH(BN))
      D1N=-DCOS(ENP*V)*DSIN(GN*U)/(DL*PHIS*PHIS*(BNS+GNS)*GN*DCOS(GN))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (M.GT.1.0) GO TO 4
      A(L,N+K)=(DSINH(BN*U)+TD1*DSIN(GN*U))*DCOS(ENP*V)/TD11N
      A(L,N+2*K)=(DCOSH(BN*(1-U))-TD1P*DCOS(GN*(1-U)))*DCOS(ENP*V)/TD11P
      A(N+K,N+K)=((UFIS*ENPS-BNS)*DSINH(BN)+ TD1*(UFIS*ENPS+GNS)*
     1DSIN(GN))/TD11N
      A(N+2*K,N+K)=(BN+TD1*GN)/TD11N
      A(N+K,N+2*K)=(X*(1-TD1P)-BNS-GNS*TD1P)/TD11P
      A(N+K,L)=(X-BNS)*C1N+(X+GNS)*D1N
      A(N+2*K,L)=A1N*BN+B1N*GN
    4 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=(TD1*(ENPS+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=(DCOS(ENP)*DSIN(EMP2)/TD11N)*(X1-X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD1P*(ENPS+UFS*GNS)/(GNS-EMP2S))*DCOS(GN)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    5 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
```

```
      IF (BNS.LT.22) GO TO 6
      TD2=BN*(BNS-X)/(GN*(X-GNS))
      TD2P=1.0
      X=UFIS*ENPS
      TD22P=BN-GN
      TD22N=BN+TD2*GN
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A2N=DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      B2N=-DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      C2N=DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      D2N=-DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      IF (M.GT.1.0) GO TO 7
      B=1.0
      TEST=BN-BN*U
      IF (TEST.GT.60.0) B=0.0
      A(L,N+K)=B*(DEXP(BN*U-BN)+TD2*DEXP(GN*U-GN))*DCOS(ENP*V)/
     1(BN+TD2*GN)
      B=1.0
      TEST=BN*U
      IF (TEST.GT.60.0) B=0.0
      A(L,N+2*K)=B*(DEXP(-BN*U)-DEXP(-GN*U))*DCOS(ENP*V)/(BN-GN)
      A(N+2*K,L)=B*(A2N*BN*DEXP(-BN*U)+B2N*GN*DEXP(-GN*U))
      B=1.0
      TEST=BN-BN*U
      IF (TEST.GT.60.0) B=0.0
      A(N+K,L)=B*((X-BNS)*C2N*DEXP(BN*U-BN)+(X-GNS)*D2N*DEXP(GN*U-GN))
      A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
      A(N+2*K,N+K)=0.0
      A(N+K,N+2*K)=0.0
      GO TO 7
    6 TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
      TD2P=DCOSH(BN)/DCOSH(GN)
      X=UFIS*ENPS
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      DL=1.0
      IF (N.EQ.1.0) DL=2.0
      A2N=DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*BN*
     1DCOSH(BN))
      B2N=-DCOS(ENP*V)*DCOSH(GN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*GN*
     1DCOSH(GN))
      C2N=DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS-GNS)*BN*DCOSH(BN))
      D2N=-DCOS(ENP*V)*DSINH(GN*U)/(DL*PHIS*PHIS*(BNS-GNS)*GN*DCOSH(GN))
      IF (M.GT.1.0) GO TO 8
      A(L,N+K)=(DSINH(BN*U)+TD2*DSINH(GN*U))*DCOS(ENP*V)/TD22N
      A(L,N+2*K)=(DCOSH(BN*(1-U))-TD2P*DCOSH(GN*(1-U)))*DCOS(ENP*V)/
     1TD22P
      A(N+K,N+K)=((X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
      A(N+2*K,N+K)=(BN+TD2*GN)/TD22N
      A(N+K,N+2*K)=(UFIS*ENPS*(1-TD2P)-BNS+GNS*TD2P)/TD22P
      A(N+K,L)=(X-BNS)*C2N+(X-GNS)*D2N
      A(N+2*K,L)=A2N*BN+B2N*GN
      GO TO 8
    7 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN              *TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
      X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
```

```
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
      GO TO 10
    8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
      X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
      XX=(DCOS(ENP)*DSIN(EMP2)/TD22N)*(X1+X2)
      A((M+1)/2,N+K)=XX*2.0
      X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=(TD2P*(ENPS-UFS*GNS)/(GNS+EMP2S))*DCOSH(GN)
      XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,N+2*K)=XX
   10 CONTINUE
      DO 20 N=1,K
      DO 20 M=1,K1,2
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF(X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      A1M=DSIN(EMP2*U)*DCOSH(BM*(1-V))/((BMS+GMS)*BM*DSINH(BM))
      B1M=DSIN(EMP2*U)*DCOS(GM*(1-V))/((BMS+GMS)*GM*DSIN(GM))
      C1M=DSIN(EMP2*U)*DCOSH(BM*V)/((BMS+GMS)*BM*DSINH(BM))
      D1M=DSIN(EMP2*U)*DCOS(GM*V)/((BMS+GMS)*GM*DSIN(GM))
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      XX                 =((BM*DSINH(BM))/(BMS+ENPS)+TD1*GM*DSIN(GM)/(GMS-
     1 ENPS))*2*EMP2*DCOS(ENP)/TD11
      IF (JN.GT.0.0) GO TO 11
      XX=XX/2
   11 A(N+2*K,(M+1)/2)=XX
      X1=((EMP2S-UFIS*BMS)/(BMS+ENPS))*BM*DCOS(ENP)*DSINH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENPS))*GM*DCOS(ENP)*DSIN(GM)*TD1
      XX=(X2+X1)*2*DSIN(EMP2)/TD11
      XX4=(DCOSH(BM*V)+TD1*DCOS(GM*V))*DSIN(EMP2*U)/TD11
      IF (JN.GT.0.0) GO TO 12
      XX=XX/2
      A((M+1)/2,L)=(UFS*EMP2S-BMS)*C1M+(UFS*EMP2S+GMS)*D1M
      XX2=(A1M*DCOSH(BM*V)+B1M*DCOS(GM*V))*DSIN(EMP2*U)
      A(L,L)=A(L,L)+XX2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD1*(
     1UFS*EMP2S+GMS)*DCOS(GM))/TD11
      A(I+1,(M+1)/2)=XX4
   12 A(N+K,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 15
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))*(-1)/(GM*(GMS-POIS*
     1PHIS*EMP2S)*DSINH(GM))
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      A2M=DSIN(EMP2*U)*DCOSH(BM*(1-V))/((BMS-GMS)*BM*DSINH(BM))
```

```
      B2M=-DSIN(EMP2*U)*DCOSH(GM*(1-V))/((BMS-GMS)*GM*DSINH(GM))
      C2M=DSIN(EMP2*U)*DCOSH(BM*V)/((BMS-GMS)*BM*DSINH(BM))
      D2M=-DSIN(EMP2*U)*DCOSH(GM*V)/((BMS-GMS)*GM*DSINH(GM))
      X1=BM*DSINH(BM)/(BMS+ENPS)
      X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM*DSINH(BM)/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX6=(DCOSH(BM*V)+TD2*DCOSH(GM*V))*DSIN(EMP2*U)/TD22
      A(I+1,(M+1)/2)=XX6
      XX=XX/2
      XX1=XX1/2
      A((M+1)/2,L)=(UFS*EMP2S-BMS)*C2M+(UFS*EMP2S-GMS)*D2M
      XX2=(A2M*DCOSH(BM*V)+B2M*DCOSH(GM*V))*DSIN(EMP2*U)
      A(L,L)=A(L,L)+XX2
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
     1GMS)*DCOSH(GM))/TD22
   14 A(N+2*K,(M+1)/2)=XX
      A(N+K,(M+1)/2)=XX1
      GO TO 20
   15 TD2=(BM*(BMS-POIS*PHIS*EMP2S))*(-1)/(GM*(GMS-POIS*PHIS*EMP2S))
      TD22=BM+TD2*GM
      X1=BM/(BMS+ENPS)
      X2=GM*TD2/(GMS+ENPS)
      XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
      X1=(EMP2S-UFIS*BMS)*BM/(BMS+ENPS)
      X2=(EMP2S-UFIS*GMS)*TD2*GM/(GMS+ENPS)
      XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD22
      IF (JN.GT.0.0) GO TO 14
      XX=XX/2
      XX1=XX1/2
      D=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
      B=1.0
      TEST=BM-BM*V
      IF (TEST.GT.60.0) B=0.0
      XX6=B*(DEXP(BM*V-BM)+D*DEXP(GM*V-GM))*DSIN(EMP2*U)/(BM+D*GM)
      A(I+1,(M+1)/2)=XX6
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD22
      A2M=DSIN(EMP2*U)/(BM*(BMS-GMS))
      B2M=-DSIN(EMP2*U)/(GM*(BMS-GMS))
      C2M=DSIN(EMP2*U)/(BM*(BMS-GMS))
      D2M=-DSIN(EMP2*U)/(GM*(BMS-GMS))
      B=1.0
      TEST=BM-BM*V
      IF (TEST.GT.60.0) B=0.0
      A((M+1)/2,L)=B*((UFS*EMP2S-BMS)*C2M*DEXP(BM*V-BM)+
     1(UFS*EMP2S-GMS)*D2M*DEXP(GM*V-GM))
      TEST=BM-BM*V
      IF (TEST.LT.60.0) GO TO 16
      XX=(A2M/2)*(1+DEXP(-2*BM*V))+(B2M/2)*(1+DEXP(-2*GM*V))
      GO TO 17
   16 XX=(A2M/2)*(1+DEXP(2*(BM*V-BM)))+(B2M/2)*(1+DEXP(2*(GM*V-GM)))
   17 A(L,L)=A(L,L)+XX
      GO TO 14
   20 CONTINUE
      DO 30 M=1,L
      TEMP=A(M,I)
      A(M,I)=A(M,I+1)
```

```
      A(M,I+1)=TEMP
   30 CONTINUE
      CALL DETERM (A,L,DET)
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
```

```
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
C     FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C     FIND AND PRINT EM,EN,EP,AND P*
C
      L=(N-1)/3
      DO 550 I=1,L
      J=2*I-1
      K=I-1
      EM(J)=X(I)
      EN(J)=X(I+L)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      P=X(3*L)
      GO TO 550
  540 EP(J)=-1.
  550 PRINT600,J,K,K,J,EM(J),K,EN(J),K,EP(J),P
  600 FORMAT('-','M=',I2,3X,'N=',I2,3X,'P=',I2,5X,'EM(',I2,') =',
     1D13.5,5X,'EN(',I2,') =',D13.5,5X,'EP(',I2,') =',D13.5,5X,
     1'P* =',D13.5)
      DO 601 I=1,L
      J=2*I-1
      EN(I)=EN(J)
      EP(I)=EP(J)
  601 CONTINUE
      PRINT 650
  650 FORMAT('1','THE SYMMETRIC MODE SHAPE DATA ARE:'////)
      CALL SHAPE (EM,EN,EP,P)
   60 RETURN
      END
      SUBROUTINE SHAPE (EM,EN,EP,P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,U,V,K,K1,
     1KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
     1,EM(75),EN(75),EP(75)
```

```
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      W44=0.0
      DO 620 N=1,K
      JN=N-1
      ENP=JN*PI
      ENPS=ENP*ENP
      BNS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP*ENP
      IF (X1.LT.0.0) GO TO 606
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      TD1=(BN*(BNS-X)*DCOSH(BN))/(GN*(GNS+X)*DCOS(GN))
      TD1P=DCOSH(BN)/DCOS(GN)
      DL=1.0
      IF (N.EQ.1) DL=2.0
      A1N=P*DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*
     1BN*DCOSH(BN))
      B1N=-P*DCOS(ENP*V)*DCOS(GN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*GN*
     1DCOS(GN))
      C1N=P*DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS+GNS)*BN*
     1DCOSH(BN))
      D1N=-P*DCOS(ENP*V)*DSIN(GN*U)/(DL*PHIS*PHIS*(BNS+GNS)*GN*DCOS(GN))
      X=UFIS*ENPS
      TD11N=BN*DCOSH(BN)+TD1*GN*DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
      IF (ETA.EQ.0.0) GO TO 601
      IF (ENP.EQ.0.0) GO TO 601
      XX=DCOS(ENP*ETA)
      GO TO 602
  601 XX=1.0
  602 IF (PSI.EQ.0.0) GO TO 604
      XW2=EN(N)*(DSINH(BN*PSI)+TD1*DSIN(GN*PSI))*XX/TD11N
      IF (PSI.LT.U) GO TO 60
      XW4=(C1N*DCOSH(BN*(1-PSI))+D1N*DCOS(GN*(1-PSI)))*XX
      IF (PSI.EQ.1.0) GO TO 603
   60 XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD1P*DCOS(GN*(1-PSI)))*XX/TD11P
      IF (PSI.GE.U) GO TO 617
      XW4=(A1N*DSINH(BN*PSI)+B1N*DSIN(GN*PSI))*XX
      GO TO 617
  603 XW3=EP(N)*(1-TD1P)*XX/TD11P
      GO TO 617
  604 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 617
  606 X1=-X1
      GNS=(1./PHIS)*X1
      GN=DSQRT(GNS)
      X=POIS*PHINS*ENPS
      IF (BNS.GT.22) GO TO 612
      TD2=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCOSH(GN))
```

```
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      DL=1.0
      IF (N.EQ.1) DL=2.0
      A2N=P*DCOS(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*BN*
     1DCOSH(BN))
      B2N=-P*DCOS(ENP*V)*DCOSH(GN*(1-U))/(DL*PHIS*PHIS*(BNS-GNS)*GN*
     1DCOSH(GN))
      C2N=P*DCOS(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS-GNS)*BN*
     1DCOSH(BN))
      D2N=-P*DCOS(ENP*V)*DSINH(GN*U)/(DL*PHIS*PHIS*(BNS-GNS)*GN*
     1DCOSH(GN))
      IF (ETA.EQ.0.0) GO TO 607
      IF (ENP.EQ.0.0) GO TO 607
      XX=DCOS(ENP*ETA)
      GO TO 608
  607 XX=1.0
  608 IF (PSI.EQ.0.0) GO TO 610
      XW2=EN(N)*(DSINH(BN*PSI)+TD2*DSINH(GN*PSI))*XX/TD22N
      IF (PSI.LT.U) GO TO 61
      XW4=(C2N*DCOSH(BN*(1-PSI))+D2N*DCOSH(GN*(1-PSI)))*XX
      IF (PSI.EQ.1.0) GO TO 609
   61 XW3=EP(N)*(DCOSH(BN*(1-PSI))-TD2P*DCOSH(GN*(1-PSI)))*XX/TD22P
      IF (PSI.GE.U) GO TO 617
      XW4=(A2N*DSINH(BN*PSI)+B2N*DSINH(GN*PSI))*XX
      GO TO 617
  609 XW3=EP(N)*(1-TD2P)*XX/TD22P
      GO TO 617
  610 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 617
  612 A=BN*(BNS-POIS*PHINS*ENPS)/(GN*(POIS*PHINS*ENPS-GNS))
      IF(ETA.EQ.0.0) GO TO 613
      IF (ENP.EQ.0.0) GO TO 613
      XX=DCOS(ENP*ETA)
      GO TO 614
  613 XX=1.0
  614 IF (PSI.EQ.0.0) GO TO 616
      B=1.0
      TEST=BN-BN*PSI
      IF (TEST.GT.60.0) B=0.0
      XW2=B*(DEXP(BN*PSI-BN)+A*DEXP(GN*PSI-GN))*EN(N)*XX/(BN+A*GN)
      DL=1.0
      IF (N.EQ.1) DL=2.0
      A2N=P*DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      B2N=-P*DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      C2N=P*DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*BN)
      D2N=-P*DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS)*GN)
      A1=0.0
      A2=0.0
      A3=0.0
      B1=0.0
      B2=0.0
      B3=0.0
      TEST=U+.000001
      IF (PSI.GT.TEST) GO TO 74
      IF (BN*U-BN*PSI.GT.60.0) GO TO 71
      A1=DEXP(-BN*U+BN*PSI)
```

```
      B1=DEXP(-GN*U+GN*PSI)
   71 IF (2*BN-BN*U-BN*PSI.GT.60.0) GO TO 72
      A2=DEXP(-2*BN+BN*U+BN*PSI)
      B2=DEXP(-2*GN+GN*U+GN*PSI)
   72 IF (BN*U+BN*PSI.GT.60.0) GO TO 73
      A3=DEXP(-BN*U-BN*PSI)
      B3=DEXP(-GN*U-GN*PSI)
   73 XW4=(A2N*(A1+A2-A3)+B2N*(B1+B2-B3))*XX/2.
      GO TO 65
   74 IF (BN*PSI-BN*U.GT.60.0) GO TO 75
      A1=DEXP(BN*U-BN*PSI)
      B1=DEXP(GN*U-GN*PSI)
   75 IF (BN*U+BN*PSI.GT.60.0) GO TO 76
      A2=DEXP(-BN*U-BN*PSI)
      B2=DEXP(-GN*U-GN*PSI)
   76 IF (2*BN-BN*U-BN*PSI.GT.60.0) GO TO 77
      A3=DEXP(-2*BN+BN*U+BN*PSI)
      B3=DEXP(-2*GN+GN*U+GN*PSI)
   77 XW4=(C2N*(A1-A2+A3)+D2N*(B1-B2+B3))*XX/2.
   65 IF (PSI.EQ.1.0) GO TO 615
   66 TEST=BN*(1-PSI)
      IF (TEST.LT.40.) GO TO 615
      XW3=EP(N)*(DEXP(-BN*PSI)-DEXP(-GN*PSI))*XX/(BN-GN)
      GO TO 617
  615 XW3=0.0
      GO TO 617
  616 XW2=0.0
      XW3=0.0
      XW4=0.0
  617 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
  620 CONTINUE
      DO 630 M=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 623
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS+POIS*PHIS*EMP2S)
     1 *DSIN (GM))
      TD1=TD1*(-1)
      TD11=BM*DSINH(BM)-TD1*GM*DSIN(GM)
      IF (PSI.EQ.0.0) GO TO 622
      IF(ETA.EQ.0.0) GO TO 621
      XW1=EM(M)*(DCOSH(BM*ETA)+TD1*DCOS(GM*ETA))*DSIN(EMP2*PSI)/TD11
      GO TO 629
  621 XW1=EM(M)*(1+TD1)*DSIN(EMP2*PSI)/TD11
      GO TO 629
  622 XW1=0.0
      GO TO 629
  623 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
```

```
      IF (BMS.GT.Z2) GO TO 625
      TD2=(BM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2S)
     1 *DSINH(GM))
      TD2=TD2*(-1)
      TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
      IF (PSI.EC.0.0) GO TO 622
      IF (ETA.EQ.0.0) GO TO 624
      XW1=EM(M)*(DCOSH(BM*ETA)+TD2*DCOSH(GM*ETA))*DSIN(EMP2*PSI)/TD22
      GO TO 629
  624 XW1=  EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
      GO TO 629
  625 A=BM*(POIS*PHIS*EMP2S-BMS)/(GM*(GMS-POIS*PHIS*EMP2S))
      IF (PSI.EQ.0.0) GO TO 627
      IF (ETA.EQ.0.0) GO TO 627
      TEST=BM-BM*ETA
      IF (TEST.GT.60.) GO TO 627
      XW1=EM(M)*(DEXP(BM*ETA-BM)+A*DEXP(GM*ETA-GM))*DSIN(EMP2*PSI)/
     1(BM+A*GM)
      GO TO 629
  627 XW1=0.0
  629 W1(I,J)=XW1+W11
      W11=W1(I,J)
  630 CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
      PRINT635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
  635 FORMAT('-','PSI =',F5.3,4X,'ETA =',F5.3,4X,'W1(',I2,',',I2,') =',
     1D12.5,4X,'W2(',I2,',',I2,') =',D12.5,4X,'W3(',I2,',',I2,') =',
     1D12.5,4X,'W4(',I2,',',I2,') =',D12.5)
      PRINT636,PSI,ETA,I,J,W(I,J)
  636 FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'
     1W(',I2,',',I2,') =',D12.5)
      PSI=PSI+1.0/DFLOAT(KS)
  640 CONTINUE
      ETA=ETA+1.0/DFLOAT(KS)
  650 CONTINUE
      RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C                            PROGRAM 15                                C
C                                                                      C
C            THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH           C
C      PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION     C
C      PROBLEM. WITH POINT SUPPORTS SYMMETRICALLY DISTRIBUTED ON       C
C      THE LATERAL SURFACE OF THE PLATE.(ONE SUPPORT ON EACH SIDE      C
C      OF THE CENTRAL AXIS) .                                          C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                 C
C                                                                      C
C             1.- K       = THE NUMBER OF TERMS TO BE USED IN THE      C
C                           SERIES INVOLVED. FOR 5 DIGITS ACCURACY     C
C                           USE K = 30.                                C
C                                                                      C
C             2.- U       = 0 TO 1, PROVIDING THE DISTANCE BETWEEN     C
C                           THE CONCENTRATED FORCE(POINT SUPPORT)-     C
C                           AND THE PLATE CLAMPED EDGE DIVIDED BY      C
C                           SIDE LENGTH A.                             C
C                                                                      C
C             3.- V       = DISTANCE BETWEEN THE CONCENTRATED FORCE    C
C                           AND THE PLATE CENTRAL AXISDIVIDED BY       C
C                           SIDE LENGTH B.                             C
C                                                                      C
C             4.- PHIR    = 2B/A, IS THE FULL PLATE ASPECT RATIO.      C
C                                                                      C
C             5.- ALMDS   = AN INITIAL STARTING VALUE FOR THE EIGEN-   C
C                           VALUE SEARCH.                              C
C                                                                      C
C             6.- DLIM    = A FINISHING OR EIGENVALUE SEARCH ENDING    C
C                           LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO   C
C                           HALT EXECUTION.                            C
C                                                                      C
C             7.- DEL     = EIGENVALUE INCREMENT.                      C
C                                                                      C
C             8.- POI     = POISSON'S RATIO.                           C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(150,150)
      U=??????????????????????
      V=??????????????????????
      K=??????????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600
      PHIR=????????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,K,U,V
   50 FORMAT('1','PHIR=',F8.4,10X,'K=',I5,10X,'U =',F8.4,10X,'V =',
```

```
      1F8.4,////)
       C=1.
       PI=4.*DATAN(C)
       PHIS=PHI*PHI
       POI=????????????????????
       POIS=2-POI
       PHIN=1./PHI
       PHINS=PHIN*PHIN
       UFS=POI*PHIS
       UFIS=POI*PHINS
       I=3*K
       I1=I+1
       ALMDS=??????????????????
       DEL=????????????????????
       DLIM=???????????????????
    1  CONTINUE
C
C      INITIALIZE THE MATRIX TO 0.0
C
       DO 2 M=1,I1
       DO 2 N=1,I1
    2  A(M,N)=0.0
       DO 10 M=1,K1,2
       DO 10 N=1,K1,2
       EMP2=M*PI/2.
       EMP2S=EMP2*EMP2
       ENP2=N*PI/2.
       ENP2S=ENP2*ENP2
       BNS=PHINS*(ENP2S+ALMDS*PHIS)
       BN=DSQRT(BNS)
       X1=ALMDS*PHIS-ENP2S
       IF (X1.LT.0.0) GO TO 4
       GNS=PHINS*X1
       GN=DSQRT(GNS)
       TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
      1POIS*PHINS))
       TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
       TD1P=DCOSH(BN)/DCOS(GN)
       TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
       A1N=DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS+GNS)*BN*PHIS*PHIS*DCOSH(BN)
      1)
       B1N=-DCOS(GN*(1-U))*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*DCOS(GN))
       C1N=DSIN(ENP2*V)*DSINH(BN*U)/((BNS+GNS)*BN*PHIS*PHIS*DCOSH(BN))
       D1N=-DSIN(GN*U)*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*DCOS(GN))
       IF (M.GT.1.) GO TO 3
       A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
       A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
      1*ENP2S+GNS)*DSIN(GN))/TD11N
       A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
       A((N+1)/2+K,I1)=C1N*(UFIS*ENP2S-BNS)+D1N*(UFIS*ENP2S+GNS)
       A((N+1)/2+K2,I1)=A1N*BN+B1N*GN
       A(I1,(N+1)/2+K)=(TD1N*DSINH(BN*U)+DSIN(GN*U))*DSIN(ENP2*V)/TD11N
       A(I1,(N+1)/2+K2)=(DCOSH(BN*(1-U))-TD1P*DCOS(GN*(1-U)))*DSIN(ENP2*V
      1)/TD11P
    3  X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
       X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
       XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
       A((M+1)/2,(N+1)/2+K)=XX
       X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
       X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
```

```
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
4     X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      IF (M.GT.1) GO TO 5
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TEST=BN-BN*U
      B=1.0
      IF(TEST.GT.60.) B=0.0
      A(I1,(N+1)/2+K)=((TD2N*DEXP((BN*U-BN)*B)+DEXP((GN*U-GN)*B))*
     1DSIN(ENP2*V)/(TD2N*BN+GN))*B
      TEST=BN*U
      B=1.
      IF(TEST.GT.60.) B=0.
      A(I1,(N+1)/2+K2)=((DEXP((-BN*U)*B)-DEXP((-GN*U)*B))*DSIN(ENP2*V
     1)/(BN-GN))*B
      C2N=DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
      D2N=-DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
      TEST=BN-BN*U
      B=1.0
      IF(TEST.GT.60.) B=0.
      A((N+1)/2+K,I1)=(C2N*(UFIS*ENP2S-BNS)*DEXP((BN*U-BN)*B)+D2N*(UFIS*
     1ENP2S-GNS)*DEXP((GN*U-GN)*B))*B
      A2N=DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
      B2N=-DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
      TEST=BN*U
      B=1.
      IF(TEST.GT.60.) B=0.
      A((N+1)/2+K2,I1)=(A2N*BN*DEXP((-BN*U)*B)+B2N*GN*DEXP((-GN*U)*B))*B
5     X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
6     TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      A2N=DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN)
     1)
      B2N=-DSIN(ENP2*V)*DCOSH(GN*(1-U))/((BNS-GNS)*GN*PHIS*PHIS*DCOSH(GN
     1))
      C2N=DSIN(ENP2*V)*DSINH(BN*U)/((BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN))
      D2N=-DSIN(ENP2*V)*DSINH(GN*U)/((BNS-GNS)*GN*PHIS*PHIS*DCOSH(GN))
```

```
      IF (M.GT.1) GO TO 7
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
      A((N+1)/2+K,I1)=C2N*(UFIS*ENP2S-BNS)+D2N*(UFIS*ENP2S-GNS)
      A((N+1)/2+K2,I1)=A2N*BN+B2N*GN
      A(I1,(N+1)/2+K)=(TD2N*DSINH(BN*U)+DSINH(GN*U))*DSIN(ENP2*V)/TD22N
      A(I1,(N+1)/2+K2)=(DCOSH(BN*(1-U))-TD2P*DCOSH(GN*(1-U)))*DSIN(ENP2
     1*V)/TD22P
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
   10 CONTINUE
      DO 11 N=1,K
   11 A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      C1M=DSIN(EMP2*U)*DSINH(BM*V)/((BMS+GMS)*BM*DCOSH(BM))
      D1M=-DSIN(GM*V)*DSIN(EMP2*U)/((BMS+GMS)*GM*DCOS(GM))
      IF (N.GT.1) GO TO 12
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
      A(I1,(M+1)/2)=(TD1M*DSINH(BM*V)+DSIN(GM*V))*DSIN(EMP2*U)/TD11M
      A((M+1)/2,I1)=C1M*(UFS*EMP2S-BMS)+D1M*(UFS*EMP2S+GMS)
      A(I1,I1)=A(I1,I1)+(C1M*DCOSH(BM*V)+D1M*DCOS(GM*V))*DSIN(EMP2*U)
   12 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   13 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      C2M=DSIN(EMP2*U)/((BMS-GMS)*BM)
```

```
      D2M=-DSIN(EMP2*U)/((BMS-GMS)*GM)
      IF (N.GT.1) GO TO 14
      B=1.
      TEST=BM-BM*V
      IF(TEST.GT.60.) B=0.
      A(I1,(M+1)/2)=((TD2M*DEXP((-BM+BM*V)*B)+DEXP((-GM+GM*V)*B))*
     1DSIN(EMP2*U)/(TD2M*BM+GM))*B
      A((M+1)/2,I1)=(C2M*DEXP((-BM+BM*V)*B)*(UFS*EMP2S-BMS)+D2M*DEXP
     1((-GM+GM*V)*B)*(UFS*EMP2S-GMS))*B
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
      B=1.
      TEST=BM-2*BM*V
      IF (TEST.GT.60) B=0.
      A(I1,I1)=A(I1,I1)+.5*(C2M*DEXP((2*BM*V-BM)*B)+D2M*DEXP((2*GM*V-
     1GM)*B))*B*DSIN(EMP2*U)
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      C2M=DSIN(EMP2*U)*DSINH(BM*V)/((BMS-GMS)*BM*DCOSH(BM))
      D2M=-DSIN(EMP2*U)*DSINH(GM*V)/((BMS-GMS)*GM*DCOSH(GM))
      IF (N.GT.1) GO TO 16
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
     1GMS)*DSINH(GM))/TD22M
      A(I1,(M+1)/2)=(TD2M*DSINH(BM*V)+DSINH(GM*V))*DSIN(EMP2*U)/TD22M
      A((M+1)/2,I1)=C2M*(UFS*EMP2S-BMS)+D2M*(UFS*EMP2S-GMS)
      A(I1,I1)=A(I1,I1)+(C2M*DCOSH(BM*V)+D2M*DCOSH(GM*V))*DSIN(EMP2*U)
   16 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
   20 CONTINUE
      CALL DETERM (A,I1,DET)
      PRINT 110,ALMDS,DET
  110 FORMAT(' ','ALMDS=',F13.7,10X,'DET=',D20.5)
      ALMDS=ALMDS+DEL
      IF (ALMDS.LE.DLIM) GO TO 1
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(150,150)
      SIGN=1.
      LAST=N-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
```

```
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,N
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 SIGN=-SIGN
      DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,N
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT
C
      DET=SIGN
      DO 300 I=1,N
  300 DET=DET*A(I,I)/10.
      GO TO 61
   60 DET=0.
   61 RETURN
      END
```

```
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C                             PROGRAM 16                                C
C                                                                       C
C            THIS IS AN ANTISYMMETRIC MODE SHAPE PROGRAM FOR THE        C
C      RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM, WITH        C
C      SYMMETRICALLY DISTRIBUTED POINT SUPPORTS ON THE PLATE LATERAL    C
C      SURFACE.(ONE SUPPORT ON EACH SIDE OF THE CENTRAL AXIS)           C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C      USAGE : THE FOLLOWING VARIABLES MUST BE PROVIDED;                C
C                                                                       C
C              1.- K      = AS DEFINED IN PROGRAM 15.                   C
C                                                                       C
C              2.- U      = AS DEFINED IN PROGRAM 15.                   C
C                                                                       C
C              3.- V      = AS DEFINED IN PROGRAM 15.                   C
C                                                                       C
C              4.- PHIR   = 2B/A FULL PLATE ASPECT RATIO.               C
C                                                                       C
C              5.- ALMDS  = EIGENVALUE.                                 C
C                                                                       C
C              6.- KS     = NUMBER OF POINTS AT WHICH THE DISPLACE-     C
C                           MENT W IS REQUIRED.                         C
C                                                                       C
C              7.- POI    = POISSON'S RATIO.                            C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,ALMDS,U,V,K,K1,KS
      U=????????????????????????
      V=????????????????????????
      K=????????????????????????
      KS=???????????????????????
      K2=2*K
      K1=2*K-1
      Z2=3600
      PHIR=?????????????????????
      ALMDS=????????????????????
      PHI=PHIR/2.
      PRINT50,PHIR,PHI,K,ALMDS,U,V
   50 FORMAT('1','PHIR=',F8.4,10X,'PHI =',F8.4,10X,'K =',I5,10X,'ALMDS =
     1',F8.4,10X,'U =',F8.4,10X,'V =',F8.4,////)
      C=1.
      PI=4.*DATAN(C)
      PHIS=PHI*PHI
      POI=??????????????????????
      POIS=2-POI
      PHIN=1./PHI
      PHINS=PHIN*PHIN
      UFS=POI*PHIS
```

```
      UFIS=POI*PHINS
      I=3*K
      I1=I+1
    1 CONTINUE
C
C     INITIALIZE THE MATRIX TO 0.0
C
      DO 2 M=1,I1
      DO 2 N=1,I1
    2 A(M,N)=0.0
      DO 10 M=1,K1,2
      DO 10 N=1,K1,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 4
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      A1N=DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS+GNS)*BN*PHIS*PHIS*DCOSH(BN)
     1)
      B1N=-DCOS(GN*(1-U))*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*DCOS(GN))
      C1N=DSIN(ENP2*V)*DSINH(BN*U)/((BNS+GNS)*BN*PHIS*PHIS*DCOSH(BN))
      D1N=-DSIN(GN*U)*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*DCOS(GN))
      IF (M.GT.1.) GO TO 3
      A((N+1)/2+K2,(N+1)/2+K)=(TD1N*BN+GN)/TD11N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD1N*DSINH(BN)+(UFIS
     1*ENP2S+GNS)*DSIN(GN))/TD11N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD1P)-BNS-TD1P*GNS)/TD11P
      A((N+1)/2+K,I1)=C1N*(UFIS*ENP2S-BNS)+D1N*(UFIS*ENP2S+GNS)
      A((N+1)/2+K2,I1)=A1N*BN+B1N*GN
      A(I1,(N+1)/2+K)=(TD1N*DSINH(BN*U)+DSIN(GN*U))*DSIN(ENP2*V)/TD11N
      A(I1,(N+1)/2+K2)=(DCOSH(BN*(1-U))-TD1P*DCOS(GN*(1-U)))*DSIN(ENP2*V
     1)/TD11P
    3 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD1N*BN*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*GN*DCOS(GN)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S+UFS*GNS)/(GNS-EMP2S))*TD1P*DCOS(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1+X2)/TD11P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    4 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.LT.Z2) GO TO 6
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TD22N=TD2N*BN+GN
      TD2P=1.
      TD22P=BN-GN*TD2P
      IF (M.GT.1) GO TO 5
```

```
      A((N+1)/2+K2,(N+1)/2+K)=0.0
      A((N+1)/2+K,(N+1)/2+K2)=0.0
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N+UFIS*ENP2S
     1-GNS)/TD22N
      TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
      TEST=BN-BN*U
      B=1.0
      IF(TEST.GT.60.) B=0.0
      A(I1,(N+1)/2+K)=((TD2N*DEXP((BN*U-BN)*B)+DEXP((GN*U-GN)*B))*
     1DSIN(ENP2*V)/(TD2N*BN+GN))*B
      TEST=BN*U
      B=1.
      IF(TEST.GT.60.) B=0.
      A(I1,(N+1)/2+K2)=((DEXP((-BN*U)*B)-DEXP((-GN*U)*B))*DSIN(ENP2*V
     1)/(BN-GN))*B
      C2N=DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
      D2N=-DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
      TEST=BN-BN*U
      B=1.0
      IF(TEST.GT.60.) B=0.
      A((N+1)/2+K,I1)=(C2N*(UFIS*ENP2S-BNS)*DEXP((BN*U-BN)*B)+D2N*(UFIS*
     1ENP2S-GNS)*DEXP((GN*U-GN)*B))*B
      A2N=DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
      B2N=-DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
      TEST=BN*U
      B=1.
      IF(TEST.GT.60.) B=0.
      A((N+1)/2+K2,I1)=(A2N*BN*DEXP((-BN*U)*B)+B2N*GN*DEXP((-GN*U)*B))*B
    5 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=(ENP2S-UFS*BNS)/(BNS+EMP2S)
      X2=(ENP2S-UFS*GNS)/(GNS+EMP2S)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
      GO TO 10
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      A2N=DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN)
     1)
      B2N=-DSIN(ENP2*V)*DCOSH(GN*(1-U))/((BNS-GNS)*GN*PHIS*PHIS*DCOSH(GN
     1))
      C2N=DSIN(ENP2*V)*DSINH(BN*U)/((BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN))
      D2N=-DSIN(ENP2*V)*DSINH(GN*U)/((BNS-GNS)*GN*PHIS*PHIS*DCOSH(GN))
      IF (M.GT.1) GO TO 7
      A((N+1)/2+K2,(N+1)/2+K)=(TD2N*BN+GN)/TD22N
      A((N+1)/2+K,(N+1)/2+K)=((UFIS*ENP2S-BNS)*TD2N*DSINH(BN)+(UFIS*
     1ENP2S-GNS)*DSINH(GN))/TD22N
      A((N+1)/2+K,(N+1)/2+K2)=(UFIS*ENP2S*(1-TD2P)-BNS+TD2P*GNS)/TD22P
      A((N+1)/2+K,I1)=C2N*(UFIS*ENP2S-BNS)+D2N*(UFIS*ENP2S-GNS)
      A((N+1)/2+K2,I1)=A2N*BN+B2N*GN
      A(I1,(N+1)/2+K)=(TD2N*DSINH(BN*U)+DSINH(GN*U))*DSIN(ENP2*V)/TD22N
      A(I1,(N+1)/2+K2)=(DCOSH(BN*(1-U))-TD2P*DCOSH(GN*(1-U)))*DSIN(ENP2
     1*V)/TD22P
    7 X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*TD2N*BN*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)
```

```
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22N
      A((M+1)/2,(N+1)/2+K)=XX
      X1=((ENP2S-UFS*BNS)/(BNS+EMP2S))*DCOSH(BN)
      X2=((ENP2S-UFS*GNS)/(GNS+EMP2S))*TD2P*DCOSH(GN)
      XX=2*DSIN(ENP2)*EMP2*(X1-X2)/TD22P
      A((M+1)/2,(N+1)/2+K2)=XX
10    CONTINUE
      DO 11 N=1,K
11    A(N+K2,N+K2)=-1.
      DO 20 N=1,K1,2
      DO 20 M=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 13
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      C1M=DSIN(EMP2*U)*DSINH(BM*V)/((BMS+GMS)*BM*DCOSH(BM))
      D1M=-DSIN(GM*V)*DSIN(EMP2*U)/((BMS+GMS)*GM*DCOS(GM))
      IF (N.GT.1) GO TO 12
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
     1EMP2S+GMS)*DSIN(GM))/TD11M
      A(I1,(M+1)/2)=(TD1M*DSINH(BM*V)+DSIN(GM*V))*DSIN(EMP2*U)/TD11M
      A((M+1)/2,I1)=C1M*(UFS*EMP2S-BMS)+D1M*(UFS*EMP2S+GMS)
      A(I1,I1)=A(I1,I1)+(C1M*DCOSH(BM*V)+D1M*DCOS(GM*V))*DSIN(EMP2*U)
12    X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD1M*BM*DCOSH(BM)
      X2=((EMP2S+UFIS*GMS)/(GMS-ENP2S))*GM*DCOS(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD1M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOS(GM)/(GMS-ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1-X2)/TD11M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
13    X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.LT.Z2) GO TO 15
      TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      TD22M=TD2M*BM+GM
      C2M=DSIN(EMP2*U)/((BMS-GMS)*BM)
      D2M=-DSIN(EMP2*U)/((BMS-GMS)*GM)
      IF (N.GT.1) GO TO 14
      B=1.
      TEST=BM-BM*V
      IF(TEST.GT.60.) B=0.
      A(I1,(M+1)/2)=((TD2M*DEXP((-BM+BM*V)*B)+DEXP((-GM+GM*V)*B))*
     1DSIN(EMP2*U)/(TD2M*BM+GM))*B
      A((M+1)/2,I1)=(C2M*DEXP((-BM+BM*V)*B)*(UFS*EMP2S-BMS)+D2M*DEXP
     1((-GM+GM*V)*B)*(UFS*EMP2S-GMS))*B
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
      B=1.
      TEST=BM-2*BM*V
```

```
      IF (TEST.GT.50) B=0.
      A(I1,I1)=A(I1,I1)+.5*(C2M*DEXP((2*BM*V-BM)*B)+D2M*DEXP((2*GM*V-
     1GM)*B))*B*DSIN(EMP2*U)
   14 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM/(BMS+ENP2S)
      X2=GM/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
      GO TO 20
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      C2M=DSIN(EMP2*U)*DSINH(BM*V)/((BMS-GMS)*BM*DCOSH(BM))
      D2M=-DSIN(EMP2*U)*DSINH(GM*V)/((BMS-GMS)*GM*DCOSH(GM))
      IF (N.GT.1) GO TO 16
      A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-
     1GMS)*DSINH(GM))/TD22M
      A(I1,(M+1)/2)=(TD2M*DSINH(BM*V)+DSINH(GM*V))*DSIN(EMP2*U)/TD22M
      A((M+1)/2,I1)=C2M*(UFS*EMP2S-BMS)+D2M*(UFS*EMP2S-GMS)
      A(I1,I1)=A(I1,I1)+(C2M*DCOSH(BM*V)+D2M*DCOSH(GM*V))*DSIN(EMP2*U)
   16 X1=((EMP2S-UFIS*BMS)/(BMS+ENP2S))*TD2M*BM*DCOSH(BM)
      X2=((EMP2S-UFIS*GMS)/(GMS+ENP2S))*GM*DCOSH(GM)
      XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K,(M+1)/2)=XX
      X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
      X2=GM*DCOSH(GM)/(GMS+ENP2S)
      XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
      A((N+1)/2+K2,(M+1)/2)=XX
   20 CONTINUE
      DO 22 J=1,I1
      TEMP=A(J,I)
      A(J,I)=A(J,I1)
      A(J,I1)=TEMP
   22 CONTINUE
      CALL DETERM (A,I1,DET)
      PRINT25
   25 FORMAT('1',30X,'END',/////)
      STOP
      END
      SUBROUTINE DETERM (A,N,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
      SIGN=1.
      M=N-1
      LAST=M-1
C
C     START OVERALL LOOP FOR(N-1) PIVOTS
C
      DO 200 I=1,LAST
C
C     FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
C
      BIG=0.
      DO 50 K=I,M
      TERM=DABS(A(K,I))
      IF (TERM-BIG)50,50,30
   30 BIG=TERM
```

```
      L=K
   50 CONTINUE
C
C     CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
      IF (BIG)80,60,80
C
C     L-TH ROW HAS THE BIGGEST TERM----IS I=L
C
   80 IF (I-L)90,120,90
C
C     I IS NOT EQUAL TO L,SWITCH ROWS I AND L
C
   90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
  100 A(L,J)=TEMP
C
C     NOW START PIVOTAL REDUCTION
C
  120 PIVOT=A(I,I)
      NEXTR=I+1
C
C     FOR EACH OF THE ROWS AFTER THE I-TH
C
      DO 200 J=NEXTR,M
C
C     MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C
      CONST=A(J,I)/PIVOT
C
C     NOW REDUCE EACH TERM OF THE J-TH ROW
C
      DO 200 K=I,N
  200 A(J,K)=A(J,K)-CONST*A(I,K)
C
C     END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
C
      M=N-1
      DO 500 I=1,M
C
C     IREV IS THE BACKWARD INDEX,GOING FROM M BACK TO 1
C
      IREV=M+1-I
C
C     GET Y(IREV) IN PREPARATION
C
      Y=A(IREV,N)
      IF (IREV-M) 400,500,400
C
C     NOT WORKING ON LAST ROW,I IS 2 OR GREATER
C
  400 DO 450 J=2,I
C
C     WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
C     FOUND VALUES
C
      K=N+1-J
  450 Y=Y-A(IREV,K)*X(K)
C
```

```
C      FINALLY,COMPUTE X(IREV)
C
  500 X(IREV)=Y/A(IREV,IREV)
C
C      FIND AND PRINT EM,EN,EP,AND P*
C
      L=M/3
      DO 550 I=1,L
      J=2*I-1
      EM(J)=X(I)
      EN(J)=X(I+L)
      P=X(M)
      IF(I.EQ.L) GO TO 540
      EP(J)=X(I+2*L)
      GO TO 550
  540 EP(J)=-1.
            GO TO 1
  550 PRINT600,J,J,EM(J),J,EN(J),J,EP(J),P
  600 FORMAT('-','J=',I2,10X,'EM(',I2,')=',D15.5,10X,'EN(',I2,')='
     1,D15.5,10X,'EP(',I2,')=' ,D15.5,10X,'P* =',D15.5)
      PRINT 650
  650 FORMAT('1','THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:',////)
    1 CALL SHAPE(EM,EN,EP,P)
   60 RETURN
      END
      SUBROUTINE SHAPE(EM,EN,EP,P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,ALMDS,U,V,K,K1,KS
      DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21),
     1EM(75),EN(75),EP(75)
      KS1=KS+1
      ETA=0.0
      DO 650 I=1,KS1
      PSI=0.0
      DO 640 J=1,KS1
      W11=0.0
      W22=0.0
      W33=0.0
      W44=0.0
      DO 620 N=1,K1,2
      ENP2=N*PI/2.
      ENP2S=ENP2*ENP2
      BNS=PHINS*(ENP2S+ALMDS*PHIS)
      BN=DSQRT(BNS)
      X1=ALMDS*PHIS-ENP2S
      IF (X1.LT.0.0) GO TO 605
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*
     1POIS*PHINS))
      TD11N=BN*TD1N*DCOSH(BN)+GN*DCOS(GN)
      TD1P=DCOSH(BN)/DCOS(GN)
      TD11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
      A1N=P*DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS+GNS)*BN*PHIS*PHIS*
     1DCOSH(BN))
      B1N=-P*DCOS(GN*(1-U))*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*
     1DCOS(GN))
      C1N=P*DSIN(ENP2*V)*DSINH(BN*U)/((BNS+GNS)*BN*PHIS*PHIS*DCOSH(BN))
      D1N=-P*DSIN(BN*U)*DSIN(ENP2*V)/((BNS+GNS)*GN*PHIS*PHIS*DCOS(GN))
      IF (ETA.EQ.0.0) GO TO 602
```

```
      IF (PSI.EQ.0.0) GO TO 603
      XW2=EN(N)*(TD1N*DSINH(BN*PSI)+DSIN(GN*PSI))*DSIN(ENP2*ETA)/TD11N
      IF (PSI.EQ.1.0) GO TO 601
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD1P*DCOS((1-PSI)*GN))*DSIN
     1(ENP2*ETA)/TD11P
      W41=(A1N*DSINH(BN*PSI)+B1N*DSIN(GN*PSI))*DSIN(ENP2*ETA)
      W42=(C1N*DCOSH(BN*(1-PSI))+D1N*DCOS(GN*(1-PSI)))*DSIN(ENP2*ETA)
      XW4=W41
      IF (PSI.GT.U) XW4=W42
      GO TO 604
  601 XW3=EP(N)*(1-TD1P)*DSIN(ENP2*ETA)/TD11P
      XW4=(C1N+D1N)*DSIN(ENP2*ETA)
      GO TO 604
  602 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 604
  603 XW2=0.0
      XW3=EP(N)*(DCOSH(BN)-TD1P*DCOS(GN))*DSIN(ENP2*ETA)/TD11P
      XW4=0.0
  604 W2(I,J)=W22+XW2
      W3(I,J)=W33+XW3
      W4(I,J)=W44+XW4
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
      GO TO 620
  605 X1=-X1
      GNS=PHINS*X1
      GN=DSQRT(GNS)
      IF (BNS.GT.Z2) GO TO 610
    6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*
     1ENP2S)*DCOSH(BN))
      TD22N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
      TD2P=DCOSH(BN)/DCOSH(GN)
      TD22P=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
      A2N=P*DSIN(ENP2*V)*DCOSH(BN*(1-U))/((BNS-GNS)*BN*PHIS*PHIS*
     1DCOSH(BN))
      B2N=-P*DSIN(ENP2*V)*DCOSH(GN*(1-U))/((BNS-GNS)*GN*PHIS*PHIS*
     1DCOSH(GN))
      C2N=P*DSIN(ENP2*V)*DSINH(BN*U)/((BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN))
      D2N=-P*DSIN(ENP2*V)*DSINH(GN*U)/((BNS-GNS)*GN*PHIS*PHIS*DCOSH(GN))
      IF (ETA.EQ.0.0) GO TO 607
      IF (PSI.EQ.0.0) GO TO 608
      XW2=EN(N)*(TD2N*DSINH(BN*PSI)+DSINH(GN*PSI))*DSIN(ENP2*ETA)/TD22N
      IF (PSI.EQ.1.0) GO TO 606
      XW3=EP(N)*(DCOSH((1-PSI)*BN)-TD2P*DCOSH((1-PSI)*GN))*DSIN(ENP2*
     1ETA)/TD22P
      W41=(A2N*DSINH(BN*PSI)+B2N*DSINH(GN*PSI))*DSIN(ENP2*ETA)
      W42=(C2N*DCOSH(BN*(1-PSI))+D2N*DCOSH(GN*(1-PSI)))*DSIN(ENP2*ETA)
      XW4=W41
      IF (PSI.GT.U) XW4=W42
      GO TO 609
  606 XW3=EP(N)*(1-TD2P)*DSIN(ENP2*ETA)/TD22P
      XW4=(C2N+D2N)*DSIN(ENP2*ETA)
      GO TO 609
  607 XW2=0.0
      XW3=0.0
      XW4=0.0
      GO TO 609
```

```
608 XW2=0.0
    XW3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*DSIN(ENP2*ETA)/TD22P
    XW4=0.0
609 W2(I,J)=W22+XW2
    W3(I,J)=W33+XW3
    W4(I,J)=W44+XW4
    W22=W2(I,J)
    W33=W3(I,J)
    W44=W4(I,J)
    GO TO 620
610 TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
    IF (ETA.EQ.0.0) GO TO 612
    IF (PSI.EQ.0.0) GO TO 612
    B=1.0
    TEST=BN-BN*PSI
    IF (TEST.GT.60.) B=0.0
    A1=DEXP((BN*PSI-BN)*B)*B
    A2=DEXP((GN*PSI-GN)*B)*B
    XW2=EN(N)*(TD2N*A1+A2)*DSIN(ENP2*ETA)/(TD2N*BN+GN)
    B=1.
    TEST=BN*PSI
    IF (TEST.GT.60.) B=0.0
    B1=DEXP(-BN*PSI*B)*B
    B2=DEXP(-GN*PSI*B)*B
    XW3=EP(N)*(B1-B2)*DSIN(ENP2*ETA)/(BN-GN)
    A2N=P*DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
    B2N=-P*DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
    C2N=P*DSIN(ENP2*V)/((BNS-GNS)*BN*PHIS*PHIS)
    D2N=-P*DSIN(ENP2*V)/((BNS-GNS)*GN*PHIS*PHIS)
    BB=1.
    B=1.
    C=1.
    D=1.
    TEST11=BN*PSI-BN*U
    TEST1=-TEST11
    TEST2=BN*U+BN*PSI
    TEST3=2*BN-BN*PSI-BN*U
    IF (TEST11.GT.60.) BB=0.
    IF (TEST1.GT.60.) B=0.
    IF (TEST2.GT.60.) C=0.
    IF (TEST3.GT.60.) D=0.
    IF (PSI.GT.U) GO TO 611
    D1=(DEXP((BN*PSI-BN*U)*B)*B-DEXP((-BN*U-BN*PSI)*C)*C+DEXP((BN*PSI
   1+BN*U-2*BN)*D)*D)/2.
    D2=(DEXP((GN*PSI-GN*U)*B)*B-DEXP((-GN*U-GN*PSI)*C)*C+DEXP((GN*PSI
   1+GN*U-2*GN)*D)*D)/2.
    XW4=(A2N*D1+B2N*D2)*DSIN(ENP2*ETA)
    GO TO 614
611 D1=(DEXP((BN*U-BN*PSI)*BB)*BB-DEXP((-BN*U-BN*PSI)*C)*C+DEXP((-2*
   1BN+BN*U+BN*PSI)*D)*D)/2.
    D2=(DEXP((GN*U-GN*PSI)*BB)*BB-DEXP((-GN*U-GN*PSI)*C)*C+DEXP((-2*
   1GN+GN*U+GN*PSI)*D)*D)/2.
    XW4=(C2N*D1+D2N*D2)*DSIN(ENP2*ETA)
    GO TO 614
612 XW2=0.0
    XW3=0.0
    XW4=0.0
614 W2(I,J)=W22+XW2
    W3(I,J)=W33+XW3
    W4(I,J)=W44+XW4
```

```
      W22=W2(I,J)
      W33=W3(I,J)
      W44=W4(I,J)
  620 CONTINUE
      DO 630 M=1,K7,2
      EMP2=M*PI/2.
      EMP2S=EMP2*EMP2
      BMS=PHIS*(ALMDS+EMP2S)
      BM=DSQRT(BMS)
      X1=ALMDS-EMP2S
      IF (X1.LT.0.0) GO TO 625
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BM*(BMS-EMP2S*
     1POIS*PHIS)*DCOSH(BM))
      TD11M=BM*TD1M*DCOSH(BM)+GM*DCOS(GM)
      IF (ETA.EQ.0.0) GO TO 622
      IF (PSI.EQ.0.0) GO TO 622
      XW1=EM(M)*(TD1M*DSINH(BM*ETA)+DSIN(GM*ETA))*DSIN(EMP2*PSI)/TD11M
      GO TO 624
  622 XW1=0.0
  624 W1(I,J)=W11+XW1
      W11=W1(I,J)
      GO TO 630
  625 X1=-X1
      GMS=PHIS*X1
      GM=DSQRT(GMS)
      IF (BMS.GT.Z2) GO TO 627
   15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
     1EMP2S)*DCOSH(BM))
      TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
      IF (ETA.EQ.0.0) GO TO 626
      IF (PSI.EQ.0.0) GO TO 626
      XW1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD22M
      GO TO 629
  626 XW1=0.0
      GO TO 629
  627 TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
      IF (ETA.EQ.0.0) GO TO 628
      IF(PSI.EQ.0.0) GO TO 628
      B=1.
      TEST=BM-BM*ETA
      IF (TEST.GT.60.) B=0.
      A1=DEXP((-BM+BM*ETA)*B)*B
      A2=DEXP((-GM+GM*ETA)*B)*B
      XW1=EM(M)*(TD2M*A1+A2)*DSIN(EMP2*PSI)/(TD2M*BM+GM)
      GO TO 629
  628 XW1=0.0
  629 W1(I,J)=W11+XW1
      W11=W1(I,J)
  630 CONTINUE
      W(I,J)=W1(I,J)+W2(I,J)+W3(I,J)+W4(I,J)
            PRINT631,I,J,W(I,J)
  631       FORMAT(' ',7X,'W(',I2,',',I2,')=',F7.0)
            IF(PI.GT.0.)GO TO 632
      PRINT 635,PSI,ETA,I,J,W1(I,J),I,J,W2(I,J),I,J,W3(I,J),I,J,W4(I,J)
  635 FORMAT('-','PSI =',F5.3,4X,'ETA =',F5.3,4X,'W1(',I2,',',I2,') =',
     1D12.5,4X,'W2(',I2,',',I2,') =',D12.5,4X,'W3(',I2,',',I2,') =',
     1D12.5,4X,'W4(',I2,',',I2,') =',D12.5)
      PRINT636,PSI,ETA,I,J,W(I,J)
```

δ

```
636 FORMAT('-',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'W(',I2,',',I2,
   1') =',D12.5)
632      CONTINUE
    PSI=PSI+1.0/DFLOAT(KS)
640 CONTINUE
    ETA=ETA+1.0/DFLOAT(KS)
650 CONTINUE
    RETURN
    END
```