# THREE ESSAYS ON HUMAN CAPITAL, CHILD CARE AND GROWTH, AND ON MOBILITY

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**Thesis Abstract**

This thesis contributes to the fields of Public Economics and Development Economics by studying human capital formation under three scenarios. Each scenario is represented in an individual paper between Chapters 2 to 4 of this thesis.

Chapter 2 examines the effect of child care financing, through human capital formation, on growth and welfare. There is an extensive literature on the benefits of child care affordability on labour market participation. The overall inference that can be drawn is that the availability and affordability of appropriate child care may enhance parental time spent outside the home in furthering their economic opportunities. In another front, the endogenous growth literature exemplifies the merits of subsidizing human capital in generating growth. Again, other contributions demonstrate the negative implications of taxes on the returns from human capital on long run growth and welfare. This paper assesses the long run welfare implications of child care subsidies financed by proportional income taxes when human capital serves as the engine of growth. More specifically, using an overlapping-generations framework (OLG) with endogenous labour choice, we study the implications of a distortionary wage income tax on growth and welfare. When the revenues from proportional income taxes are channelled towards improving economic opportunities for both work and schooling investments in the form of child care subsidies, long run physical and human capital stock may increase. A higher level of growth may ensue leading to higher welfare.

Chapter 3 answers the question of how child care subsidization works in the interest of skill formation, and specifically, whether child care subsidization policies can work to the effect of human capital subsidies. Ample studies have highlighted the significance of early childhood learning through child care in determining the child's longer-term outcomes. The general conclusion has been that the quality of life for a child, higher earnings during later life, as well as the contributions the child makes to society as an adult can be traced back to exposures during the first few years of life. Early childhood education obtained through child care has been found to play a pivotal role in the human capital base amongst children that can benefit them in the long run. Based on this premise, the paper develops a

simple Overlapping Generations Model (OLG) to find out the implications of early learning on future investments in human capital. It is shown that higher costs of child care will reduce skill investments of parents. Also, for some positive child care cost, higher human capital obtained through early childhood education can induce further skill investments amongst individuals with a higher willingness to substitute consumption intertemporally. Finally, intervention that can internalize the intra-generational human capital externalities arising from parental time spent outside the home - for which care/early learning is required to be purchased for the child - can unambiguously lead to higher skill investments by all individuals. Chapter 3 therefore proposes policy intervention, such as child care subsidization, as the effect of such will be akin to a human capital subsidy.

The objective of Chapter 4 is to understand the implications of inter-regional mobility on higher educational investments of individuals and to study in detail the impact of mobility on government spending for education under two particular scenarios – one in which human capital externalities are non-localized and spill over to other regions (e.g. in the form of R&D), and another in which the externalities are localized and remain within the region. It is shown that mobility enhances private investments in education, and all else equal, welfare should be higher with increased migration. The impacts on government educational expenditures are studied and some policy implications are drawn. In general, with non-localized externalities, all public expenditures decline under full-migration. Finally under localized externalities, the paper finds that governments will increase their financing of education to increasingly mobile individuals only when agglomeration benefits outweigh congestion costs from increases in regional population.

*To my parents,*

*my husband and*

*my lovely little boy.*

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# CHAPTER 1

## Introduction to the Thesis

This dissertation is motivated by the notion that an understanding of why investments in higher education matter - as well as how they are influenced - deserves attention at the theoretical level. Much of today's policy debates revolve around factors influencing - and influenced by - investments in higher education, child care, and skill migration. This dissertation therefore explores human capital investments under three scenarios. The first scenario uses human capital as the engine of growth to justify public financing of child care. The second scenario substantiates these findings by linking the merits of child care financing to human capital investments later in life. The third scenario studies the private and public financing mix for human capital when skilled individuals are mobile across regions in a federation. The overall goal of this dissertation is to advance theoretical understanding of the benefits and constraints for human capital formation, and to inform policy towards enhancements in higher education.

Chapters 2 and 3 particularly address the link between child care financing and human capital formation, the implications on growth, and the bearing of child care financing on human capital subsidization policies. Chapter 3 brings sole focus on human capital mobility and its implications on skill investments at the private and public level. In other words, it describes the role and behaviour of regional and federal governments in a framework that examines skill mobility.

The availability and affordability of child care is a common theme for policy debate. The support for child care provision and the importance of its affordability is backed by a vast amount of literature emphasizing benefits arriving from two main sources. The first source has been well documented to arrive from children's exposure to formal child care that in turn can foster their long term development (Garces, Thomas and Currie 2000, Currie 2001, Heckman and Carneiro 2002, Nelson 2000, Shonkoff and Phillips 2000 and Karoly 2000). These benefits from "early learning opportunities" will be the subject matter of Chapter 3. The second source of benefits channels towards parents by furthering their economic opportunities outside from spending time with their children at home. Assessing the latter, Heckman (1974), Powell (1997), Michalopoulos and Robins (2000), Lefebvre and Merrigan (2008) and Baker, Gruber and

Milligan (2008) are some examples of contributions that highlight the importance of child care in enhancing labour market participation.

Considering the implication of affordable child care on labor market participation, Chapter 2 illustrates the opportunity for allocating time towards human capital investments when child care is increasingly affordable. Thus, in a framework motivated by the endogenous growth literature that exemplifies the merits of human capital formation in generating long run growth (Lucas 1988, Romer 1990, Glomm and Ravikumar 1992), the implications of using distortionary taxes to financing child care subsidies, and the long run implications are drawn. This is interesting since the endogenous growth literature has also implied that labour income taxes may reduce long-run growth through lower productivity (e.g., King and Rebelo 1990, Rebelo 1991 and Stokey and Rebelo 1995). In the context of child care, even if an increase in child care subsidies may have a positive effect on human capital accumulation and hence growth, opposing implications may appear when these subsidies are financed by distortionary income taxes.

Chapter 2 therefore assesses the long run welfare implications of child care subsidies financed by proportional income taxes when human capital serves as the engine of growth. The revenues from distortionary income taxes are assumed to be used to finance child care subsidies. Using an overlapping-generations framework (OLG) with endogenous labour choice, it explores the implications of a distortionary wage income tax on growth and welfare. The chapter contributes to the literature in several ways. First, it draws an unexplored link between the child care literature and human capital as an engine of growth. Second, it builds on OLG models by allowing endogenously determined time allocation decisions, and hence labour supply. Finally, it addresses concerns about the negative implications of distortionary income tax on growth to demonstrate conditions under which child care financing, through human capital accumulation, may lead to growth and welfare.

A pertinent query that can evolve from Chapter 2 is the choice of child care subsidies over a human capital subsidy which could also serve directly as the engine of growth – and whether the two are comparable instruments. Chapter 3 aims to answer this by focusing on the first set of benefits from child care discussed earlier.

Long-term benefits to children from early-learning opportunities could be dependent on parental requirements for child care as determined by the amount of time they spend outside the home seeking economic opportunities. In other words, parental choices of times spent outside the home – whether at work or at school - often requires them to send their children to child care. Less (leisure) time spent at home with the child, and more "outside opportunities" sought by a parent could directly translate into higher "early learning opportunities" from formal child care services.

With exposure to child care leading to higher human capital endowments from which children may reap benefits in the future, parental decisions on times spent outside the home can therefore have a positive inter-generational externality on their children. Based on this additional feature, Chapter 3 assesses the effects of child care exposure on human capital investments later in life, and whether internalizing this externality will at all be welfare improving. In short, the aim of Chapter 3 is to clearly motivate the use of child care subsidies, and to determine whether child care subsidies can encourage skill formation akin to human capital subsidies.

Impending from the literature on human capital enhancements are the implications of skill migration through what is commonly known as either a brain drain or a brain gain. It is with more ease today than before that human capital endowed labour move between regions after investing in human capital. The incentives of governing authorities in financing the educational expenses of individuals may be affected by the potential migration of the same skill endowed individuals. Although mobility within a federation such as Canada, where migration is within the same country, may seem to pose less concern than mobility amongst different countries in an economic union such as the European Union, the essence of the problems remains since one regional government may have spent on the human capital of a skilled worker located in another province. Under different inter-regional skill-spillover scenarios, Chapter 4 addresses two main issues pertaining to human capital mobility – (i) how does human capital mobility affect human capital investments and (ii) how does human capital mobility affect government financing of education.

# CHAPTER 2

## Child care, Taxes, Human Capital and Growth

### 2.1 Introduction:

Parental allocation of time between caring for their children at home and seeking economic opportunities outside the home may critically depend on the availability of adequate child care arrangements. As such, the availability and affordability of child care is often a common theme for policy debate. The support for child care provision and the importance of its affordability is backed by a vast amount of literature that emphasizes resultant benefits arriving from two main sources. The first source of benefits has been well documented to arrive from children's exposure to formal child care that in turn can foster their long term development (Garces, Thomas and Currie 2000, Currie 2001, Heckman and Carneiro 2002, Nelson 2000, Shonkoff and Phillips 2000 and Karoly 2000). The second source of benefits channels towards parents by furthering their economic opportunities outside from spending time with their children at home. Assessing the latter, Heckman (1974), Powell (1997), Michalopoulos and Robins (2000), Lefebvre and Merrigan (2008) and Baker, Gruber and Milligan (2008) are some examples of contributions that highlight the importance of child care in enhancing labour market participation.

In his paper, Powell (1997) shows that child care costs have a negative impact on the hours of work and labor-force-participation of married mothers. Incorporating government aid towards child care costs, Michalopoulos and Robins (2000) find significant effects of child care subsidies, child care prices, and wage rates on employment and child care choices for parents. In particular, they show that child care subsidies positively affect parental labour supply. Taking this further, and discussing the welfare implications of child care financing, Baker, Gruber and Milligan (2008) show that while increased access to early child care through publicly financed systems can lead to a higher labour supply among secondary income earners, public expenditures will require

extensive revenue collection that may come at the cost of higher taxes and reduced economic efficiency.

While the relationships between child care availability, child care costs and labour supply have been extensively explored in the literature, most are empirical studies that investigate their significance and magnitude. Unfortunately, only few theoretical studies back up the role of child care in determining labour market participation. One noteworthy study amongst them is Bergstrom and Blomquist (1996) who present a theoretical model of political support for child care subsidization. They show that in an economy with high taxes on wage income, selfish taxpayers with no children may still favour substantial public financing of child care. The reason for this is that subsidies induce mothers to join the labour force and hence to pay their share of income taxes.

In an intertemporal framework where individuals need to take care of their children in all periods of their lives, Minagawa and Upmann (2006) study the impact of various public child care fees on parental time allocation. They arrive at a general conclusion that public child care policies work to encourage labor supply.

An interesting paper by Brink and Nordblom (2005) analyzes the effects of child care subsidies on parental time allocation. They show that parental time allocation depends on whether child care fees are subsidized. These subsidies are shown to have different effects depending on the fee structure on which they are administered. They also show that while a subsidy may increase the use of formal child care, the effect on labour supply may be unclear. This is due to the possibility that parents may be using child care even when they are not working, such as when they attend school. This is an interesting result as it could indicate the possible use of child care for the purposes of parental human capital investments.

This link between child care and human capital has been well addressed in Joshie and Davies (1993). They show that support for child care, like a human capital subsidy, is an aid to skill formation for individuals choosing to do so, and the resources generated from such subsidies exceed the resource costs of child care. They generate their results using simulations of the lifetime earnings of a typical British mother under various child care regimes. Their study, however, misses to capture general equilibrium effects. It also does not fully capture the distortionary effects of child care subsidies when financed by proportional income taxes. What is

interesting is the indication that child care subsidies may be akin to human capital subsidies. However, long term growth implications of these subsidies have not been investigated in their paper and would be worth studying.

While child care subsidies increase labour market participation through encouraging time spent outside of the home, this time could be split between working and schooling. In an environment where the individual receives a subsidy that provides the opportunity to invest in schooling, this can enhance human capital accumulation. Building on the indication in Joshie and Davis (1993) that child care subsidies may be akin to human capital subsidies, an assessment of the growth and welfare implications of child care financing, through human capital accumulation, deserves to be explored[1].

The importance of human capital accumulation on economic growth has been emphasized by the endogenous growth literature through key contributions among which are Uzawa 1965, Lucas 1988, Romer 1990, Glomm and Ravikumar 1992. The same literature has also shown that labour (and capital) income taxes may reduce long-run growth through lower productivity (e.g, King and Rebelo 1990, Rebelo 1991 and Stokey and Rebelo 1995). Thus, when effective labour is an accumulable factor, a tax on labour income has been shown to act as a tax on human capital earnings and discourage skill accumulation. Besides, in models where leisure is modelled as "quality time" spent at home, labour income taxes and human capital subsidies provided at the same rate may have equal and opposing income and substitution effects. Thus subsidies and taxes will have no implications on long run growth (Milessi-Ferretti and Roubini 1998).

In the context of child care, even if an increase in child care subsidies may have a positive effect on human capital accumulation and hence growth, opposing implications can be expected when these subsidies are financed by distortionary income taxes. This link between the child care subsidies, their financing by distortionary income taxes as well as the overall implications on human capital formation and growth deserves to be assessed properly.

A minimal setting for analyzing the impact of child care subsidies on human accumulation and growth must make it possible to assess several conditions. First, the requirement to allocate time

[1] A theoretical framework identifying the merits of child care in enhancing human capital formation is detailed in Alamgir-Arif (2011). The paper provides an understanding of how encouraging child care use can enhance human capital formation by all individuals across generations, and can therefore generate the effects of a human capital subsidy. This is detailed in the next chapter.

between child care, work and human capital accumulation may change over the course of an individual's lifetime. Second, while the benefits of child care subsidies may accrue to the relatively young participants in the labour market, a share of the underlying costs arising from income taxes may be borne by other labour market participants for whom child care is no longer a requirement. Overlapping-generations (OLG) models can provide an excellent setup to assess lifetime implications of government policies, especially when needs, characteristics, and policy requirements may differ during each stage of an individual's life.

Several papers have been developed to assess the impact of taxation policies on growth particularly in an OLG framework with human capital accumulation. Yakita (2003), Valente (2008) are examples amongst several others. As an illustration, Yakita (2003) develops an OLG model with human capital as the engine of growth to examine the effects of taxes on growth and welfare. In contrast to most previous findings in the literature with exogenous labour supply, the paper finds that a flat-rate wage income tax can have a positive effect on growth. The reason for this is that that there is a period in the course of the lifetime when the individual does not invest in human capital. Increased savings from a lump sum subsidy from the richer old to the poorer young generation generates lower the interest rate, inducing individuals to invest in human capital accumulation. Unfortunately, like several other papers using OLG models, labour supply is exogenous in Yakita (2003). Exogenous labour supply models such as the one in Yakita (2003) are not able to capture the key mechanisms through which child care subsidies financed through distortionary taxes may affect time allocation decisions between work, school and home, therefore missing important implications for human accumulation.

The reasons for not considering endogenous labour supply are possibly related to complications in the model dynamics brought about by the variations in labour supply over time. Detailed explanations are available in Nourry (2001) and Nourry and Venditti (2006). Both papers address the indeterminacies related to multiple steady states that are governed by a system of two functional equations – a dynamic path of labour supply in addition to that of physical capital. Nourry (2001) is one of the pioneers who introduce endogenous labour supply in an OLG framework. Unfortunately, however, neither of these two papers model second period labour supply. As such, they do not incorporate human capital accumulation incentives. Human capital transferred through generations could potentially add an interesting perspective to the OLG literature with endogenous labour-supply. Gaumont and Leonard (2010) model human

capital in an OLG framework with endogenous labour supply in both periods. However, human capital is formed in their model through "the intensity of (their) efforts, and the interaction on the factory floor" by workers producing physical goods. They do not model schooling efforts or time allocation towards schooling as a key endogenous factor determining human capital formation.

This paper expands on these discussions by allowing schooling investments in human capital, as well as labour to be supplied in both periods together with endogenous labour supply in the first period. A strand of the growth literature emerges in this paper that also acknowledges the long run merits of early childhood learning for children and the importance of child care affordability. We therefore model child care financing in an OLG model with human capital accumulation, endogenous labour supply as well as child care in order to study the implications of the latter on growth and welfare. In order to do so, we build upon interesting contributions in the literature of endogenous growth with human capital and endogenous labour supply and child care like Yakita(2003), Nourry (2001), Milessi-Ferretti and Roubini (1998) and Brink and Nordblom (2005). Specifically, we assess the implications of increasing child care subsidies that are funded by distortionary taxes on labour income in a framework that models human capital accumulation and labour supply in an OLG framework. We derive conditions under which the overall growth implications are positive, indicating elevated welfare in the long-run.

The remainder of this paper will present the model in Section 2. Section 3 provides introduces taxes and the child care subsidy into the framework. We conclude in Section 4.

### 2.2 Basic Model:

#### 2.2.1 Overview

We model a closed-economy populated by overlapping generations, g, who are economically active over two periods within which they provide labour services. Individuals within each generation are assumed to be identical. Within each period, an individual has a fixed time endowment that is normalize to unity.

At any time t, two economically active generations coexist– the young and the old. Each young individual is also a young parent to a child. Without loss of generality, as in Yakita (2003), we restrict our attention to the case of one child per individual[2]. It is assumed that the young adult raises the child by the end of the first period of her life. It follows that the population growth rate is equal to zero. The total number of individuals within each generation has been normalized to unity for simplicity. Therefore, the total economically active population coexisting in any period t is two.

We assume that each child requires full-time care throughout his childhood. The young individual derives utility from devoting time to take care of her child at home. In the second period of the adult's life, the child is assumed to have grown up, no longer requiring supervision and care.

As an alternative to spending time at home with the child, the adult can also spend time outside the home, seeking economic opportunities through working or schooling. The increase human capital arising from schooling can make it possible for the individual to enjoy a higher wage in the second period.

When the parent leaves the home to seek economic opportunities, adequate supervision of her child is acquired through the purchase of formal child care services. We assume that formal child care services are a perfect substitute for the care provided by the parent.

Young adults save for their old-age by investing in physical capital. Older adults reap the benefits of their investments made when young, whether through schooling or savings. As such, the benefits received as older adults come from wage and interest earnings, respectively. The older generation need not choose between schooling and work as they supply labour inelastically.

[2] We do not allow for heterogeneity amongst households by allowing some households to have children while others do not. Heterogeneity in this form could imply that some households would require child care due to having children, while other households could potentially have little interest in child care due to having no children. Our paper retains focus on the general outcome of child care financing on growth and welfare by assuming identical individuals and households. Studying the potential implications on our results from adding heterogeneity amongst households may be an avenue for future research.

The government can play the appropriate role of facilitating economic participation by reducing child care procurement costs, using income taxes to help finance child care subsidies.

The production sector consists of one representative firm. Aggregate production is a constant-returns-to-scale function of physical capital and effective labour that is supplied by the young and the old. Output produced at any time t is fully exhausted through consumption by the young and the old as well as through investments.

### 2.2.2 Individual decisions

The young generation $g$ at time $t$ will consist of individuals choosing to leave the home to become participate in the labour force or invest in schooling. The old generation supplies labour inelastically to earn their human-capital endowed income.

Following Lucas (1988) and Redding (1996), each young adult is assumed to inherit a portion $\chi$ of the stock of human capital from the preceding generation. This idea of intergenerational human capital transfers is widely accepted with supporting studies such as Ermisch and Francesconi (2001), Oreopoulos, Page and Stevens (2006) and Chevalier (2004) and Becker and Tomes (1986) that empirically support the transmission of human capital from previous generations to offspring[3]. Taking this intergenerational human capital externality into account, the period $t$ human capital of a representative worker from generation g is given by:

$$h_{t,g} = \chi H_t = \chi h_{t,g-1}, \ \chi \leq 1 \qquad (1)$$

where $H_t$ is the stock of knowledge accumulated by generation $g-1$ that will be available for generation $g$ at time $t$. For simplicity, we assume full transfer of the stock of knowledge, i.e.,

[3] The intergenerational transfer of human capital is a positive spillover across generations that must be internalized. This can beckon government intervention to encourage skill formation. Following the notion that a child care subsidy could encourage skill formation just like a human capital subsidy (Joshie and Davis 1993, Alamgir-Arif 2011), the former can then therefore be of similar interest to policy makers in order to internalize this human capital externality. This is important to note in order to justify our use of child care subsidies instead of human capital subsidies at the outset.

$\chi = 1$. Thus, $h_{t,g} = H_t = h_{t,g-1}$ at time $t$. Forwarding a period, $h_{t+1,g} = H_{t+1}$ will be transferred to the young at time $t+1$.

Workers may augment their second period stock of human capital by devoting a fraction of their time towards schooling. The timing of their decision is as follows. At period $t$, young adults decide on the fraction of time $x_t$ they wish to allocate towards investing in education as well as the time $l_t$ they wish to devote to working.

The human capital production technology in our paper is assumed to follow the formulation of Lucas (1988) and Redding (1996). The general notion of Lucas (1988) and the specific structure of Redding (1996) assumes that an individual who devotes $x_t$ units of time to schooling will acquire the following level of human capital the next period

$$h_{t+1,g} = \left(1 + \gamma_h \, \phi\left(x_t\right)\right) h_{t,g}, \text{ where } \phi' > 0, \; \phi'' < 0 \tag{2}$$

where $h_{t,g}$ is the human capital stock that the individual inherits from the previous generation. $\phi\left(x_t\right)$ is an increasing and concave function that determines the efficiency with which time invested in schooling $x_t$ generates higher human capital in the next period, and $\gamma_h$ parameterises the productivity of the human capital formation technology.

Lucas (1988), however, assumes the Uzawa (1965) and Rosen (1976) formulation that human capital to accumulated is in direct proportion to time spent in schooling. In other words, no time spent in schooling is equivalent to no human capital accumulated for the future period. We take this simpler notion and generalize with human capital accumulation to be strictly increasing and strictly concave in schooling time. Thus, instead of restricting ourselves to constant returns to schooling efforts, we allow for diminishing returns to schooling. Human capital accumulation is therefore assumed to follow (3):

$$h_{t+1,g} = \phi\left(x_t\right) h_{t,g}, \text{ where } \phi' > 0, \; \phi'' < 0, \; \phi(0) > 0 \tag{3}$$

Given the notion of inter-generational human capital spillover discussed above, (3) establishes the vital concept of human capital as the engine of growth that can easily be seen as:

$$\frac{h_{t+1,g}}{h_{t,g}} = \frac{H_{t+1}}{H_t} = \phi(x_t) \tag{3´}$$

This highlights the significance of schooling investments in determining growth in our model.

Total effective labour supplied at time $t$ is therefore:

$$L_t = h_{t.g-1} + l_{t.g} h_{t.g} \tag{4}$$

where the first component on the right hand side denotes the effective labour supply of the old generation in period $t$, while the second component denotes the effective labour supplied by the young adult in period $t$. Note that the first component $h_{t.g-1}$ is in effective labour terms as it is multiplied by the total labour supplied by the old which is inelastic and equal to one.

In aggregate human capital terms effective labour supply at time t can be re-written as:

$$L_t = H_t[1 + l_{t.g}] \tag{4´}$$

As mentioned earlier, higher education through increased schooling efforts in the first period will lead to a higher human-capital embodied wage earning of $w_{t+1} h_{t+1.g}$ in the second period, while choosing to work in the first period allows them an effective wage of $w_t h_{t.g}$.

We assume that a parent may find a job suitably meeting her labour supply decision at any time, and may also enrol in schooling if desired. While spending time outside the home can yield an individual a mix of current period earnings coupled with higher future earnings, spending quality time at home in the form of leisurely activities with the child increases a young adult's utility

benefits. Leisure time spent by the parent at home can be assumed to be equivalent to parental child care, $d^P$. This, as discussed, comes at the expense of effective time spent outside the home either in the form of first period work $l_{t.g}$ or first period schooling investments $x_{t,g}$ by generation *g*.

Effective leisure time, or "quality time" spent with the child at home, is assumed to be boosted by the amount of human capital a parent is endowed with and is denoted by $\left(1-l_{t.g}-x_t\right)h_{t.g}$ [4]. Total effective time spent outside the home by a young adult is thus $\left(l_{t.g}+x_t\right)h_{t.g}$. Since the second economically active period of their lives is entirely spent in working, $l_{t+1.g}=1$. The young adult's time allocation between home and outside-home possibilities will therefore determine how a child's need for care is fulfilled.

When the parent chooses to leave the home, the child will require adequate formal care services, $d^F$ [5]. It is assumed that $d^P$ and $d^F$ are perfectly substitutable care options for the child. The total time constraint for the parent can be summarized as:

$$d^P + l_{t,g} + x_t = 1 \tag{5}$$

The entire care time for the child must equal the total amount of disposable time:

$$d^P + d^F = 1 \tag{5.1}$$

(5) and (5.1) together imply that the total time spent outside the home by the parent should be equivalent to the amount of formal child care necessary to be procured.

[4] We follow the underlying principles of Milessi-Ferretti and Roubini (1998) in defining leisure as quality time that can be boosted by the amount of human capital embodied in the person. In this paper, quality time at home with human capital embodiment can represent "home production" of child care.

[5] In order to retain focus on the effects of child care subsidies and income taxes on individual choices and the resultant impact on welfare and growth, we abstract from modelling the child care sector as another market and assume that child care is outsourced (and when individuals are assumed to live for three periods, with economically inactive old age in the third period t+2 ,child care can be imagined to be provided by grandparents). We expect a general equilibrium model with an explicit child care producing sector to generate the same results as the ones we obtain, but the opportunity exists for trial in future research.

We assume the price of a unit of child care time to be $q$. Therefore, for a total of $(l_t + x_t)h_t$ effective units of time spent outside the home in period t, the individual will be required to pay a total of $q(l_t + x_t)h_t$ for child care[6]. We denote the total expenditure on child care in the economy as $E^F$. Thus,

$$E^F = q(l_t + x_t)h_t \tag{5.2}$$

Individuals from each generation $g$ will maximize their lifetime utility by choosing consumption in each period, $c_{t,g}$ and $c_{t+1,g}$, and by deciding on how much time to allocate between schooling and work in order to determine their utility enhancing amount of leisure. We assume without loss of generality that the consumption choice of a child is made by the parent. The child's consumption is therefore incorporated in $c_{t,g}$.

### 2.2.3 Household utility maximization

We assume the following log-linear lifetime utility function for any generation *g* that is increasing in first and second period consumption as well as first period leisurely time spent with the child[7].

$$U\left(c_{t,g}, c_{t+1,g}, (1-l_t-x_t)\right) = \log c_{t,g} + \log(1-l_{t,g}-x_{t,g}) + \beta \log c_{t+1,g}$$

The individual chooses lifetime consumption as well as the proportion of time spent between school and work during the first period of life in order to maximize lifetime utility.
The young adult's utility is not altruistic in terms of caring for the child. In other words, the parent does not take into account the child's utility in her own utility.

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[6] A child care subsidy a offered by the government will reduce the price of child care by $(q-a)(l_t + x_t)h_t$ Explicit modelling of the income-tax financed child care subsidy will be provided in section 3. For now, we abstract from modelling the government's financing of child care.

[7] We choose a log linear utility function for simplicity. Using other forms of utility, such as a CRRA function may add complexities to the intuition arising from the curvature of the utility function. We intentionally avoid these complexities, but the option remains for further exploration in future research.

Households are responsible for savings in the economy. Each household saves $s_t$ during the first period of the adult's economically active life. Using (5), the first period budget constraint of an individual is written as[8]:

$$c_{t,g} + q_t(l_{t,g} + x_{t,g})h_{t,g} + s_t = w_t l_{t,g} h_{t,g} \tag{6.1}$$

where $w_t$ is the first period wage rate.

The second period budget constraint, given inelastic second period labour supply is given as:

$$c_{t+1,g} = w_{t+1} h_{t+1}^g + R_{t+1} s_t \tag{6.2}$$

where again $w_{t+1}$ is the second period wage rate and $R_{t+1}$ is the gross second period return from savings.

Using (6.1) and (6.2), the household's inter-temporal budget constraint is solved as:

$$c_{t,g} + q_t(l_{t,g} + x_{t,g})h_{t,g} + \frac{c_{t+1,g}}{R_{t+1}} = w_t l_{t,g} h_{t,g} + \frac{w_{t+1} h_{t+1,g}}{R_{t+1}} \tag{7}$$

Rearranging the intertemporal budget constraint to account for net-of care earnings gives:

$$c_{t,g} + \frac{c_{t+1,g}}{R_{t+1}} = (w_t - q_t) l_{t,g} h_{t,g} + \frac{w_{t+1} h_{t+1,g}}{R_{t+1}} - q_t x_{t,g} h_{t,g} \tag{7.1}$$

The individual's problem is to solve the following maximization problem:

$$\max_{\{C_{t,g}, C_{t+1,g}, l_{t,g}, x_{t,g}\}} U = U\left(c_{t,g}, (1 - l_{t,g} - x_{t,g})\right) + \beta\, U(c_{t+1,g}) = \log c_{t,g} + \log(1 - l_{t,g} - x_{t,g}) + \beta \log c_{t+1,g}$$

[8] We normalize the price of consumption to unity, and assume no inflation between periods in order to maintain focus on the effects of child care subsidies and income taxes on growth and welfare, ceteris paribus.

subject to (1), (3) and (7)

The first order conditions for this utility maximization problem are laid out in Appendix 1.

Simplification of the first order conditions give (8.1) – (8.6) listed below. The description of (8.1) – (8.6) will follow shortly.

$$c_{t+1,g} = \beta R_{t+1} c_{t,g} \tag{8.1}$$

$$c_t = (w_t - q_t)(1 - l_{t,g} - x_{t,g}) h_{t,g} \tag{8.2}$$

The optimal choice of schooling $x_{t,g}$ can be determined from the following condition:

$$w_t = \phi'(x_t) \frac{w_{t+1}}{R_{t+1}} \tag{8.3}$$

An implicit form for labour choice made by the young in period $t$ can be derived as:

$$l_{t,g} = \frac{1}{2+\beta}\left[(1-\beta)(1-x_t) - \frac{1}{(w_t - q_t)}\left(\frac{w_{t+1}\phi(x_t)}{R_{t+1}} - q_t x_t\right)\right] \tag{8.4}$$

It is worth noting that a closed form solution to the decision on labour time spent cannot be obtained since (8.4) has the optimal choice of schooling as an argument. We will call this the implicit labour supply decision.

Finally, the consumption plan by the young in period $t$ and the old in period $t+1$ is

$$c_{t,g} = \frac{h_{t,g}}{(2+\beta)}\left[w_t(1 - x^*_{t,g}) - q + \frac{w_{t+1,g}\phi(x^*_{t,g})}{R_{t+1}}\right] \tag{8.5}$$

Likewise,

$$c_{t+1.g} = \frac{\beta R_{t+1} h_{t,g}}{(2+\beta)} \left[ w_t (1 - x^*_{t,g}) - q + \frac{w_{t+1,g} \phi(x^*_{t,g})}{R_{t+1}} \right] \tag{8.6}$$

Condition (8.1) follows from simplifying the first order conditions with respect to first and second period consumption. It is the Euler equation giving the time-path for consumption smoothing choices that an individual can make throughout the lifetime. At the margin, a higher return on savings will increase the individual's second period consumption[9].

Condition (8.2) gives the intratemporal decision between consumption and leisure and can be obtained from the first period consumption and leisure choices. A higher cost of child care will reduce consumption spending, while a higher first period wage rate wt will increase first period consumption spending.

Condition (8.3) is obtained from solving the first order conditions with respect to consumption and labour supply simultaneously. Dividing the first order condition related to labour choice by that of schooling gives

$$w_t - q_t = \phi'(x_t) \frac{w_{t+1}}{R_{t+1}} - q_t \tag{8.3'}$$

The left hand side describes the opportunity cost in terms of wages lost when spending an additional unit of time in school. The right hand side describes the earnings net of child care costs arising from an additional unit of time invested in schooling. The intuition from this is clear – optimal skill choice arrives from equating the net marginal opportunity cost with the marginal benefit from schooling net child care costs. Equation (8.3) therefore depicts an arbitrage condition between the first period opportunity cost of schooling and the second period earnings from schooling.

As $\phi'$ is monotonically decreasing, the expression for the optimal level of schooling time $x_t$ can be found as

[9] The effects of a higher return on savings on the *levels* of consumption in both periods, human capital investments, labour supplied are more complicated and will depend on the Euler equation, the inter-temporal budget constraint as well as the properties of the utility function and can be seen within the analysis provided later.

$$x_t = \phi'^{-1}\left( w_t \left( \frac{1}{w_{t+1}/R_{t+1}} \right) \right) \tag{8.3''}$$

Clearly, higher current wages will deter time spent in school, while potentially higher future wages will encourage the same[10].

Finally, the optimal implicit labour supply decision from (8.4) depicts ambiguity on how current wages $w_t$ affect implicit labour supply. Rearranging further gives

$$l_{t,g} = \frac{1}{2+\beta}\left( (1-\beta)(1-x_t) + \frac{q_t x_t}{(w_t - q_t)} - \frac{w_{t+1}\phi(x_t)}{(w_t - q_t)R_{t+1}} \right) \tag{8.4'}$$

The first component in square brackets is simply the fixed component arising from logarithmic utility with the cancelling of income and substitution effects. The second term gives the relative impact of the cost of schooling described as the direct cost of education $q_t x_t$ as a ratio of the opportunity cost of schooling through net wage lost. The third term in (8.4´) describes the relative benefit of schooling as a ratio of the opportunity cost from net wages lost.

In order for the implicit labour supply decision to be positive, the following condition must hold:

$$\frac{w_{t+1}\phi(x_t)}{(w_t - q_t)R_{t+1}} < \left[ (1-\beta)(1-x_t) + \frac{q_t x_t}{(w_t - q_t)} \right]$$

In other words, the relative cost of schooling described as the direct cost of education as a ratio of the opportunity cost from schooling $\frac{q_t x_t}{(w_t - q_t)}$, together with the fixed benefit

[10] It is worth noting from the individual's intertemporal budget constraint that the cost of child care affects first period earnings (on the right hand side) and first period expenditures (on the left hand side) equally. This results in a human capital choice that is not directly impacted by child care costs. This is due to the linear entry of the child care cost into the intertemporal budget constraint.

$(1-\beta)(1-x_t)$ from spending time away from school, must exceed the relative benefit from schooling, $\frac{w_{t+1}\phi(x_t)}{(w_t-q_t)R_{t+1}}$.

The effect of current wage rate wt on labour supply will be ambiguous as can be seen below:

$$\frac{\partial l_{t,g}}{\partial w_t}=\frac{1}{2+\beta}\left[-(1-\beta)\frac{\partial x_t}{\partial w_t}+\frac{q_t}{(w_t-q_t)}\frac{\partial x_t}{\partial w_t}-\frac{w_{t+1}\phi'(x_t)}{(w_t-q_t)R_{t+1}}\frac{\partial x_t}{\partial w_t}-\frac{q_t x_t}{(w_t-q_t)^2}+\frac{w_{t+1}\phi(x_t)}{(w_t-q_t)^2R_{t+1}}\right]$$

In order for current wages to positively affect the labour supply decision:

$$\left[-\frac{w_{t+1}\phi'(x_t)}{(w_t-q_t)R_{t+1}}\frac{\partial x_t}{\partial w_t}+\frac{w_{t+1}\phi(x_t)}{(w_t-q_t)^2R_{t+1}}-(1-\beta)\frac{\partial x_t}{\partial w_t}>\frac{q_t x_t}{(w_t-q_t)^2}-\frac{q_t}{(w_t-q_t)}\frac{\partial x_t}{\partial w_t}\right]$$

Note that the first and third term on the left hand side are positive since $\frac{\partial x_t}{\partial w_t}<0$. Likewise, the second term on the right hand side is also positive. The first two components on the left hand side describe the impact of higher first period wages on second period returns from schooling. The second component on the left hand side describes the direct effect of higher wages through the resultant decline in schooling efforts on the labour supply decision. The right hand side describes the impact of higher wages on the first period relative cost of schooling. Thus, a higher first period wage rate will induce higher labour efforts as long as its direct effect through declining schooling efforts $-(1-\beta)\frac{\partial x_t}{\partial w_t}$ coupled with its impact on the relative future returns from education $\frac{w_{t+1}\phi(x_t)}{(w_t-q_t)^2R_{t+1}}-\frac{w_{t+1}\phi'(x_t)}{(w_t-q_t)R_{t+1}}\frac{\partial x_t}{\partial w_t}$ exceed its impact on the relative cost of schooling.

The cost of child care has two opposing effects on first period labour supply.

$$\frac{\partial l_{t,g}}{\partial q_t} = \frac{1}{2+\beta}\left[\frac{x_t}{(w_t - q_t)} + \frac{q_t x_t}{(w_t - q_t)^2} - \frac{w_{t+1}\phi(x_t)}{(w_t - q_t)^2 R_{t+1}}\right]$$

On one hand, child care costs will increase the direct cost of investing in education relative to the opportunity cost of schooling from the income forgone $\frac{x_t}{(w_t - q_t)}$ and the child care cost of education relative to the opportunity cost of schooling by $\frac{q_t x_t}{(w_t - q_t)^2}$. Both effects should induce higher work effort. We will call this the inducement effect. On the other hand, child care costs will increase the benefit from schooling relative to the opportunity cost of attending school $\frac{w_{t+1}\phi(x_t)}{(w_t - q_t)^2 R_{t+1}}$, which in turn will induce lower time spent at work. We will call this the disincentive effect.

The actual implication on the labour supply decision will depend on the forces between the two:

$$\frac{\partial l_{t,g}}{\partial q_t} > 0 \ \textit{iff} \ \left[\frac{x_t}{(w_t - q_t)} + \frac{q_t x_t}{(w_t - q_t)^2} > \frac{w_{t+1}\phi(x_t)}{(w_t - q_t)^2 R_{t+1}}\right] \tag{9}$$

In other words, as long as the inducement effects outweigh the disincentive effect, higher child care costs will induce higher labour effort. Further analysis of (9) at the steady state will be provided in the next section.

Using (4), total effective labour supply in the economy which comprises of the labour supply by both young and old generations is given by (10):

$$L_t = H_t\left[1 + \frac{1}{2+\beta}\left((1-\beta)(1-x_t^*) + \frac{1}{w_t - q_t}\left(q_t x_t^* - \frac{w_{t+1}\phi(x_t^*)}{R_{t+1}}\right)\right)\right] \tag{10}$$

This endogenously determined labour supply will be key in determining the long-run dynamic plan that we will soon address.

Condition (8.5) and (8.6) give the consumption plan during the individual's lifetime. Taking (8.5) taking (8.3″) into account for the optimal schooling decision, a higher first period wage rate will lead to higher first period consumption due to a higher labour earnings. It will also lead to a higher second period consumption ceteris paribus, as a higher first period income will increase savings for the second period.

There are two effects from an increase in the second period wage rate on first and second period consumption. First, higher second period wage earnings will lead to higher second period consumption. First period consumption also increases, and this may be due to the lower need for savings for the future, ceteris paribus. However, higher second period wage earnings also lead to a higher investment in education from (8.3´´). This takes time off from work, resulting in lower first period income and consumption, as well as lower savings for future consumption, all else equal.

Finally, a higher price of child care will imply lower consumption in both periods as lower disposable earnings will be available for consumption for the first period and savings for the next period, all else equal. It follows that the consumption levels of the two economically active generations g and g-1 coexisting at any period t are given by

$$C_t = \{c_{t,g}, c_{t,g-1}\}$$
$$= \left\{ \frac{h_{t,g}}{(2+\beta)} \left[ w_t(1-x^*_{t,g}) - q_t + \frac{w_{t+1,g}\phi(x^*_{t,g})}{R_{t+1}} \right], \frac{\beta R_t h_{t,g-1}}{(2+\beta)} \left[ w_{t-1}(1-x^*_{t-1,g-1}) - q_{t-1} + \frac{w_{t,g-1}\phi(x^*_{t-1,g-1})}{R_t} \right] \right\} \quad (11)$$

**2.2.4 Savings in the economy**

Aggregate savings is used for investment in physical capital. Therefore,

$$S_t = I_t \quad (12)$$

The motion equation for capital is given by

$$K_{t+1} = (1-\delta)K_t + I_t$$

where $I_t$ is the portion of output invested for the generation of future physical capital and $\delta$ is the rate of depreciation of physical capital.

We assume capital to fully depreciate over one period, i.e, $\delta = 1$. Therefore, the motion for physical capital accumulation reduces to $K_{t+1} = I_t$. With population in each generation being normalized to unity, (12) reduces to

$$s_t = K_{t+1} \tag{12.1}$$

Using the definition of savings from (6.2), i.e., $s_t = \dfrac{c_{t+1,g} - w_{t+1,g}\phi(x^*_{t,g})h_{t,g}}{R_{t+1}}$, and using (8.6) gives

$$s_t = \frac{h_{t,g}}{2+\beta}\left[\beta(w_t(1-x_t) - q\ ) - \frac{2w_{t+1}\phi(x_t)}{R_{t+1}}\right] \tag{12.2}$$

The effect of a higher first period wage rate on the savings of individuals will depend on the following forces

$$\frac{\partial s_t}{\partial w_t} > 0 \ \textit{iff}\ \left[\beta(1-x_t) > \beta w_t \frac{\partial x_t}{\partial w_t} + \frac{2w_{t+1}\phi'(x_t)}{R_{t+1}}\frac{\partial x_t}{\partial w_t}\right] \tag{12.3}$$

As long as the direct positive effect on savings from time spent away from schooling exceeds the negative effect of higher first period wages on schooling effort, a higher current wage rate will induce individuals to save more.

We can also generate the following proposition:

**Proposition 1:** *All else equal, higher child care costs will reduce the savings of individuals in the economy.*

**Proof:**

Equation (12) shows that

$$\frac{\partial s_t}{\partial q_t} = -\frac{\beta\ h_{t,g}}{2+\beta} < 0$$

Thus, higher costs of child care borne by young adults when leaving the home, all else equal, will reduce the funds available to save for the future. This implies that the benefit from child care subsidies that can potentially reverse the decline in individual savings behaviour.

### 2.2.5 Production

A representative firm produces output in each period using a constant-returns-to-scale production function $Y_t = F(K_t, L_t)$ with $F' > 0$ and $F'' < 0$ that respects the Inada conditions. The firm therefore uses the total labour supplied in the economy, $L_t$ together with physical capital stock $K_t$, in order to produce its output.

As mentioned earlier, physical capital stock $K_t$ is supplied by the households through their savings. $K_t$ is assumed to fully depreciate in one period upon being used in production. In any period $t$, the firm's profit maximization problem is given by:

$$\max_{\{L_t, K_t\}} F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t$$

where $r_t$ is the real interest rate and $\delta$ is the depreciation rate of capital. As assumed earlier, $\delta = 1$ implying that the rental rate of capital becomes $(1 + r_t)$ which is simply the gross returns to savings, $R_t$. The firm's profit maximization problem is then:

$$\max_{\{L_t, K_t\}} F(K_t, L_t) - w_t L_t - R_t K_t$$

which yields:

$$R_t = F_1(K_t, L_t) = f'(k_t) \tag{13}$$

and

$$w_t = F_2(K_t, L_t) = [f(k_t) - k_t f'(k_t)] \tag{14}$$

where $k_t = \frac{K_t}{L_t}$ is the capital labour ratio. (13) and (14) together with (11), (12.1) and (1) will generate the following resource constraint for the economy

$$Y_t = C_t + K_{t+1} + CC^d \tag{15}$$

where $CC^d$ is simply the aggregate demand for child care in the economy, the price of which is normalized to unity. In other words, $CC^d$ coupled with its unitary price will equal the total expenditure on child care in the economy $E^F$ that was defined earlier in (5.2).

**2.2.6 Competitive equilibrium**

A competitive equilibrium in this economy is characterized by a sequence of price variables $\{w_t, w_{t+1}, R_{t+1,} q_t\}_0^\infty$ and a sequence of quantity variables $\{L_t, K_t, C_t, CC^d\}_0^\infty$ such that

1) at time $t$ individuals of generation $g$ maximize their inter-temporal lifetime utility to determine work and schooling investments $l_{t,g}$ and $x_{t,g}$ as well as first and second period consumption $c_{t,g}$ and $c_{t+1,g}$.

2) firms maximize profits by using inputs $L_t$ and $K_t$ at prices $w_t$ and $R_t$

3) the factor market clears, i.e, the demand for effective labour $L_t$ equals the supply of effective labour $H_t\left(l_{t,g-1} + l_{t,g}\right)$ at time $t$, and the supply of capital through savings $S_t$ equals the demand for capital $K_t$.

4) the goods market clears, i.e, $Y_t = C_t + K_{t+1} + CC^d$

### 2.2.7 Dynamics

Using the intensive form for capital, (4) gives the capital market equilibrium as

$$s_t = k_{t+1} L_{t+1} \tag{12´´}$$

where, forwarding the definition of effective labour supply from (10) to period $t+1$ gives:

$$L_{t+1} = \phi(x_t) H_t \left[ 1 + \frac{1}{2+\beta} \left( (1-\beta)(1-x_t^*) + \frac{1}{w_t - q} \left( q\ x_t^* - \frac{w_{t+1}\phi(x_t^*)}{R_{t+1}} \right) \right) \right] \tag{16}$$

We will study the competitive equilibrium directly, focusing on the conditions that are relevant to determine the long run growth rate in order to be more informative about the economic mechanisms at work in this model. The conditions under which a long-run steady state equilibrium exists will soon be demonstrated. Before that, however, we describe the dynamics dictating our economy in the next sub-section.

The dynamic system for our economy is obtained from the analogues of equations (12´), (12´´) and (13) plus equation (8.3) that determines investment in human capital. The entire system of relevant equations comprise of:

[i] $k_{t+1} L_{t+1} = s_t$

[ii] $s_t = \frac{h_{t,g}}{2+\beta} \left[ \beta(w_t\ (1-x_t^*) - q) - \frac{2 w_{t+1} \phi(x_t)}{R_{t+1}} \right]$

[iii] $w_t = f(k_t) - k_t f'(k_t)$

[iv] $R_t = f'(k_t)$

[v] $L_{t+1} = \varphi(x_t) H_t\ (1 + l_{t,g})$

[vi] $l_{t,g} = \frac{1}{2+\beta}\left[(1-\beta)(1-x_t^*) + \frac{1}{w_t - q_t}\left(q_t x_t^* - \frac{w_{t+1}\phi(x_t^*)}{R_{t+1}}\right)\right]$

[vii] $R_{t+1} = \left[\frac{\phi'(x_t)w_{t+1}}{w_t}\right]$

[viii] $H_{t+1} = \chi\phi(x_t)H_t$

Equations [i] and [viii] are the basic dynamic equations that, together with the other relevant equations described above, will have bearing on the long run behaviour of our economy. Particularly, [vii] and [viii] describe the role of human capital in the growth process while [i] balances investment with savings[11]. [v] and [vi] describe the path taken by the implicit labour supply decision as it affects the savings plan through [i].

A steady state that evolves according to [i]-[viii] is a competitive equilibrium in which the intensive variables $x$ and $k$ are constant over time while extensive variables such as $Y_t$, $C_{t,g}$, $C_{t,g-1}$, $H_t$, $K_t$ and $L_t$ may either be constant or grow geometrically.

The only inter-generational dynamics in this model arrives from human capital accumulation that is transferred through generations. That is, the only impetus that drives changes in consumption, output and hence the capital stock between periods is fed in through the intergenerational knowledge transfer along with schooling investments.

Using the firm's problem to determine $\frac{Y_{t+1}}{Y_t}$, keeping in mind that capital stock is generated through savings that in turn depends on the consumption path, we obtain $\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} = \phi(x_t)$.

In other words, the long run growth rate of the economy depends crucially on the time spent at school. If a steady state equilibrium exists, then we will be interested in studying the intermediate and transitory impacts of wage income taxes and child care subsidies through dynamic paths [i]

[11] Note that (vii) arrives from rearranging (8.3).

to [viii] on long run physical and human capital stock. The conditions under which a steady state will exist for our overlapping generations economy is demonstrated in the next sub-section.

### 2.2.8 Existence of a steady-state equilibrium

At the steady state, all state variables stabilize to their long-run value. In other words, $k_t = k_{t+1} = k$ and $x_t = x_{t+1} = x$. A steady state will exist in our economy if the dynamic paths described by the savings-investment decision and the human capital investment decision intersect to determine an equilibrium. A more detailed explanation is provided below.

Evaluated at the steady state, the two dynamic paths described by equations [i]-[vi] and [vii] that reflect the savings-investment behaviour as well as the path for human capital investment are simplified to (D1) and (D2), respectively:

$$\left[\beta(w\ (1-x\ )-q)-\frac{2w\ \phi(x\ )}{R}\right]=\phi\ k\left[2+\beta+\left((1+\beta)(1-x)+\frac{1}{w\ -q}\left(\frac{Rq\ x-w\ \phi}{R}\right)\right)\right] \qquad \text{(D1)}$$

and

$$R\ =\phi'(x) \qquad \text{(D2)}$$

Describing the balanced growth path as a time path on which schooling time and the capital-labour ratio are constant, the (D1) describes the balance between investment and savings. The left hand side describes the optimal savings plan as determined from the individual's utility maximization decision. The right hand side gives the total investment in physical capital stock for period $t+1$ as a product of total labour supplied and physical capital per worker at time $t+1$.

The second dynamic equation (D2) follows from [vii] of Section 2.8 when evaluated at the steady state. This condition describes the arbitrage between returns to physical and human capital that will in turn determine an equilibrium choice for each at the steady state.

Using equation [iv], this becomes:

$$f'(k)=\phi'(x) \qquad \text{(D2´)}$$

Fully differentiating (D2´) gives $\frac{dx}{dk} = \frac{f''}{\phi''} > 0$ by the concavity of the production function for output as well as human capital. Thus, plotting $x$ against $k$, (D2´) would give a positive sloped line that we will call HC.

The existence of a steady-state equilibrium will then require that the relationship between savings and investment derived from (17) evaluated in a $(k, x)$ space be downward sloping. Plotted in a $(k, x)$ space, we will call this relationship between savings and investment SI. We now derive the conditions under which this will occur.

**Proposition 2:** *The overlapping generations economy experiences a steady state equilibrium if the following sufficient conditions hold:*

1) *the marginal impact of an additional unit of capital on savings through its impact on (discounted) second period wage income* $\frac{\phi}{R}$ *should exceed the marginal impact of an additional unit of capital on savings through its impact on first period income* $\frac{\beta(1-x)}{2}$. *In short,* $\frac{\phi}{R} > \frac{\beta(1-x)}{2}$.

2) *the elasticity of substitution between capital and labour* $\sigma_{k,l}$ *must exceed capital's share in output* $\frac{kf'}{f}$. *In short,* $\sigma_{k,l} > \frac{kf'}{f}$.

3) *production technologies of output and human capital respectively must be continuous in physical and human capital.*

**Proof:**

For simplicity, we will conduct the proof of condition (a) from proposition 2 under the assumption that the price of child care $q_t$ is zero. Homotheticity of the utility function implies

that savings is linear homogenous in the wage income of both periods. The left hand side of (D1) is therefore homothetic in wage rate w. Following the method of Azariadis and Drazen (1990) and simplifying gives:

$$\frac{k}{w} = \frac{m(k,x)}{v(k,x)} \qquad \text{(D1´)}$$

where $m(k,x) = \left[ \beta(1-x\ ) - \frac{2\ \phi}{R} \right]$ and $v(k,x) = \phi \left[ 2 + \beta + \left( (1+\beta)(1-x) + \left( \frac{-\phi}{R} \right) \right) \right]$.

This relationship given by (D1´) gives the curve SI when plotted in a $(k,x)$ space. We will be interested in finding sufficient conditions that will generate a negative slope for SI.

Considering the right hand side of (D1´),

$$\frac{\partial \dfrac{m(k,x)}{v(k,x)}}{\partial k} = \frac{v(k,x)\left( \dfrac{2\phi}{R^2} f'' \right) - m(k,x)\left( \dfrac{\partial v(k,x)}{\partial k} \right)}{v(k,x)^2} \qquad \text{(D1´´)}$$

The sign of (D1´´) will depend on the sign of $v(k,x)$ and $m(k,x)$.

Working with the dynamic plan equations (v) and (vi) evaluated at the steady state, and considering $q = 0$, $v(k,x)$ is simply $\frac{L}{wH}$. This ratio of total labour supplied to the effective wage rate is unambiguously positive. The first term in the numerator is clearly negative. Also, from the definition of $v(k,x)$, $\frac{\partial v(k,x)}{\partial k} = \frac{\phi^2}{R^2} f'' < 0$. The sign of (D1´´) will therefore rest on the sign of $m(k,x)$.

If $m(k,x) > 0$, then $\beta(1-x\ ) > \frac{2\ \phi}{R}$. This requires that the marginal impact of an additional unit of capital on first period wage earnings should be twice as much as the impact of an

additional unit of capital on (discounted) second period wage income. However, the dynamic plan from equation (ii) points to unbound increases in savings when $\beta(1-x) > \frac{2\phi}{R}$ holds. Therefore, a steady-state equilibrium will not exist when this holds.

A steady-state [equilibrium will only exist if (D1") has a negative sign and the following condition holds:

$$\frac{\phi}{R} > \frac{\beta(1-x)}{2} \tag{17}$$

Thus $m(k,x) > 0$ is exactly condition (a) in proposition 2. This requires that the marginal impact of an additional unit of capital on savings through its effect on (discounted) second period wage income should be larger than one half of the marginal impact of capital on savings through the effect on first period wage earnings. This ensures that savings does not increase unbound from changes in the steady state capital stock through a larger impact on first period earnings than second period earnings. Clearly then, $\frac{\partial \frac{m(k,x)}{v(k,x)}}{\partial k} < 0$

In order for SI to be downward sloping, given $\frac{\partial \frac{m(k,c)}{v(k,x)}}{\partial k} < 0$, the left hand side of (D1´) should increase in $k$ as in Azariadis and Drazen (1990). It is easy to show that the necessary condition for $\frac{k}{w}$ to be increasing in $k$ is that $\frac{kf'}{f}(1-\frac{kf''}{f'}) < 1$. Azariadis and Drazen (1990) demonstrate this to further imply that the elasticity of substitution between capital and labour services must exceed capital's share in output – i.e., $\sigma_{k,l} = -\left(\frac{f'}{kf''}\right)(f - kf') > \frac{kf'}{f}$. This proves condition (b).

An upward sloping HC curve, together with a downward sloping SI curve will, under the sufficient conditions stated above, ensure that at least one solution to the dynamic system

comprising of (D1) and (D2) will exist. Another sufficient condition would require that the production function for output and human capital be continuous. This is because (D2) implies that as $k \to \infty$, $x \to 1$. This in turn implies negative savings. Again, $k \to 0$, $x \to 0$ implies positive savings. Therefore, savings and capital stock will only be zero for some $x < 1$. Continuity ensures an interior solution to equations (D1) and (D2). Since both the technologies of output and human capital production are continuous, we satisfy this requirement in our model and ensure condition (c) holds.

**2.3 Income Taxes and Child care Subsidy:**

We now introduce a government that helps to ease the young adults' monetary cost of leaving the home when seeking outside economic opportunities. The role of the government is to provide subsidies for the time a child requires formal care, thereby reducing the cost of child care necessary to be procured for the entire time spent by the parent outside the home.

The government finances its expenditure on child care through proportional income taxes. The young adult's first period budget constraint now becomes:

$$C_{t,g} + (q_t - a)(l_{t,g} + x_{t,g})h_{t,g} + s_t = (1-\tau)w_t l_{t,g} h_{t,g}$$

where $a$ is the subsidy portion that reduces the cost of effective time $(l_{t,g} + x_{t,g})h_{t,g}$ spent outside the home and $\tau$ is the tax rate on the wage earnings of the entire working population.

It follows from Proposition 1 that a child care subsidy that decreases the cost of child care will raise savings. The intuition is clear – a lower portion of income goes out to finance child care for each unit of effective time spent outside the home. This enables increased savings for the future by young adults. However, with a tax on wage earnings, savings can decline. The overall impact on savings and the dynamic path from growth is therefore not lucid and requires further inquiry. We proceed with such in the following sub-sections.

### 2.3.1 Government budget

The government budget equation in period t is given by

$$a(l_{t,g} + x_{t,g})h_{t,g} = \tau w_t L_t \tag{18}$$

where again $a$ is the subsidy rate and $\tau$ is the tax rate. The left hand side of (18) is the subsidy expenditure per unit of effective child care time procured. The right hand side of (18) depicts the tax revenue collected from the earnings of the entire workforce. It takes account of the fact that taxes are collected from both economically active generations who participate in the labour force $L_t$. At any time t, the earnings of the young and old generations will be taxed so that the tax revenue from the young will be $\tau w_t l_{t,g} h_{t,g}$ and from the old will be $\tau w_t l_{t,g-1} h_{t,g-1}$.

An important point to note is that a unit increment in the tax rate will generate a more than unit increase in the subsidy since taxes are collected from both economically active generations but are channelled to only the young amongst them. In other words, $\frac{\partial a}{\partial \tau} > 1$.

### 2.3.2 Tax and subsidy induced dynamics

With taxes and child care subsidies introduced, the savings-investment relationship from [D1] becomes:

$$\left[\beta\left((1-\tau)w\ (1-x\ ) - q + a\right) - \frac{2(1-\tau)w\ \phi(x\ )}{R}\right] = \phi\ k\left[1 + \frac{1}{2+\beta}\left((1+\beta)(1-x) + \frac{1}{(1-\tau)w\ - q + a}\left((q\ - a)x - \frac{(1-\tau)w\ \phi}{R}\right)\right)\right]$$

Using the definitions of $L_t$, $l_{t,g}$ and $x_{t,g}$ from [v], [vi] and [vii] of Section 2.8 and the government budget constraint (18) and then differentiating the system with respect to the tax rate evaluated at $\tau = 0$ (and hence $a = 0$) gives:

$$\left.\frac{dx}{d\tau}\right|_{\tau=0} = \frac{A + B - C}{D} \tag{19}$$

where

$$A = w\left(\frac{2\phi}{R} - \beta(1-x)\right),\ B = \frac{k\phi}{2+\beta}\left[\frac{\left(\frac{(1-\tau)w\phi}{R} - (q-a)x\right)}{\left((1-\tau)w\ -q+a\right)^2}\left(w - \frac{\partial a}{\partial \tau}\right)\right],$$

$$C = \frac{k\phi}{2+\beta}\left(\frac{w\phi}{R\left((1-\tau)w\ -q+a\right)}\right) \text{ and } D = \frac{w(1-\tau)}{2+\beta}\left(\beta + \frac{2\phi'}{R}\right) + \frac{\partial}{\partial x}\left(k\phi(x)(1+l)\right).$$

$A > 0$ by condition (17) that is required for the existence of a steady state[12]. A close study of savings plan [ii] from Section 2..2.7 describes $A$ as the impact of additional wage earnings on the savings behaviour of individuals. Similar scrutiny to study total labour supply from equations [v] and [vi] will describe $B$ as the effect of a wage tax on labour supply, and therefore savings, through its impact on first period wages. Likewise, $C$ describes the impact of a wage tax on the implicit labour supply decision, and hence savings, through second period earnings. The first term in $D$ is positive. We will discuss the second component and the full sign of $D$ shortly.

$B$ and $C$ arrive from incorporating (vi) in (v) from Section 2..2.7 and differentiating it with respect to the tax rate $\tau$. Clearly, the first term in brackets in the numerator of $B$ is positive since the net of tax earnings must exceed the cost of child care $(q-a)x$, and represents the change in the return from schooling relative to the return from working when first period income is taxed. A higher tax reduces the return from first period work, and therefore increases the relative benefit of attending school. Similarly, $C$ denotes the change in the return from schooling relative to the return from first period work when the second period earnings are taxed, creating a disincentive to attend school.

Keeping the sign of $D$ constant for now, the numerator of (19) will unambiguously have a positive sign as long as the effect of a wage income tax on the (net-of-child care-cost) first

[12] Note that $x$ is simply the optimal choice of skill investments. The asterisk is eliminated for simplicity of formulation.

period income $B$ exceeds the effect of the same on second period earnings $C$, leading to higher investments in schooling.

Intuitively, a decline in first period implicit labour supply due to the lower wage earnings from a tax will free up time for the young adult to either stay home or invest in schooling as the opportunity cost of availing each of these will decline. A labour income tax can therefore cause a shift to human capital accumulation and away from work. On the other hand, a lower net-of tax second period income can induce a young adult to work more in the first period in order to save more for the future. This will reduce the time available to invest in schooling. Thus, if the sign of $D$ is positive, long-run welfare will unambiguously increase through a higher stock of knowledge as long as the former exceeds the latter, and vice versa when $D$ is a negative.

The second component of $D$ given by $\frac{\partial}{\partial x}\left(k\phi(x)(1+l)\right)$ will now be studied to determine conditions under which welfare will unambiguously improve with wage income taxes and child care subsidies.

**Proposition 3:** *A tax-induced shift from labour time to human capital accumulation will increase savings in the economy as long as the effect on total labour supplied from increases in time spent in schooling is positive.*

**Proof:**

A close inspection of equations (i), (v) and (vi) will reveal that $\frac{\partial}{\partial x}\left(k\phi(x)(1+l)\right)$ is simply the impact of human capital investment on the aggregate capital stock, or savings, evaluated at the steady state. Simplifying this second component of $D$ in order to yield a positive sign gives:

$$\left[\phi'(1+l)+\phi\frac{\partial l}{\partial x}\right]>0 \Rightarrow \frac{\partial L}{\partial x}>0 \qquad (20)$$

where the response of the optimal implicit labour decision with schooling choices is taken into account.[13]:

Individuals choose to spend time in human capital accumulation in order to avail higher earnings in the future. When labour income is taxed, the opportunity cost of schooling declines. This encourages schooling in the first period and lower labour supplied. All else equal, less time spent at work will decrease labour earnings that could potentially have been saved for the second period. On the other hand, higher schooling efforts can in turn generate higher effective labour earnings in the second period. However, due to higher effective labour earnings in the second period, the total tax payment amount is higher for the future period than for the first period. As a result, there is a need for investment in physical capital through savings to smooth lifetime income. A decline in the returns to human capital will therefore trigger a shift from human to physical capital accumulation. As long as the latter effect dominates, i.e., as long as the decline in savings due to a lower first period labour supply is less than the increase in savings from a substitution from human to physical capital, more schooling should increase savings in the economy.

According to condition (20), higher savings in the economy will rest on the response of first period work to changes in second period human capital. While on the one hand, a higher human capital stock will imply lower labour supplied and hence lower savings, a higher human capital stock may also trigger a shift from human to physical capital investments due to a higher return from the latter. The left hand side of (20) shows the change in total implicit labour supply from a larger human capital stock. As long as the negative impact on implicit labour supplied from a higher human capital is not too high, and higher stocks of human capital relative to physical

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[13] Re-arranging this condition gives $\frac{\partial l}{\partial x} > -\phi'(1+l)$ or $-\frac{\partial l}{\partial \phi}\frac{\phi}{l} < 1+\frac{1}{l}$. For simplicity of notation, we denote the responsiveness of implicit labour supply to second period human capital stock on the left hand side by $\varepsilon_{l,\phi} = -\frac{\partial l}{\partial \phi}\frac{\phi}{l}$. The elasticity is determined using the expression $x = \phi'^{-1}\left(\frac{w_t R_{t+1}}{w_{t+1}}\right)$, which is derived from (8). The condition therefore requires that $\varepsilon_{l,\phi} < \kappa$ where $\kappa = 1+\frac{1}{l} > 1$. This therefore requires that the percentage change in implicit labour supply from higher second period human capital stock be less than a number exceeding a hundred percent. Therefore, this condition is not a binding restriction.

capital triggers the need for more time spent in first period work, the total effect on national savings will increase with improvements in human capital.

**Proposition 4:** *The introduction of a child care subsidy to young parents that is financed by a wage tax on both economically active generations will lead to higher long-run stocks of physical and human capital, and hence higher welfare, as long as the following sufficient conditions hold:*

1) a wage tax on incomes from both periods generates a larger incentive to invest in schooling due to a lower impact of the tax on second period earnings relative to first period earnings.

2) $\frac{\partial L}{\partial x} > 0$

Individuals are required to take time out from work when making skill investments. This will increase the opportunity cost of forming human capital. An income tax that reduces the opportunity cost of skill formation should positively affect long run welfare as long as: i. the effect of a wage tax on the optimal implicit labour supply decision is through a larger reduction in first period earnings relative to the reduction in second period earnings. This leads to greater incentives for skill investments, and ii. higher investments in schooling that generate higher returns to physical capital investments relative to human capital investments lead to a shift from human to physical capital accumulation, and therefore higher long run stocks of physical capital.

Both effects work to generate a higher physical and human capital stock in the economy, and hence higher growth (as seen from equation (3)) and welfare. This can be clearly depicted in the figure below. The new tax and subsidy incorporated SI and HC curves plotted on an $(k, x)$ space, keeping in mind that HC is unaffected by taxes and subsidies at the steady state, will illustrate Proposition 4.

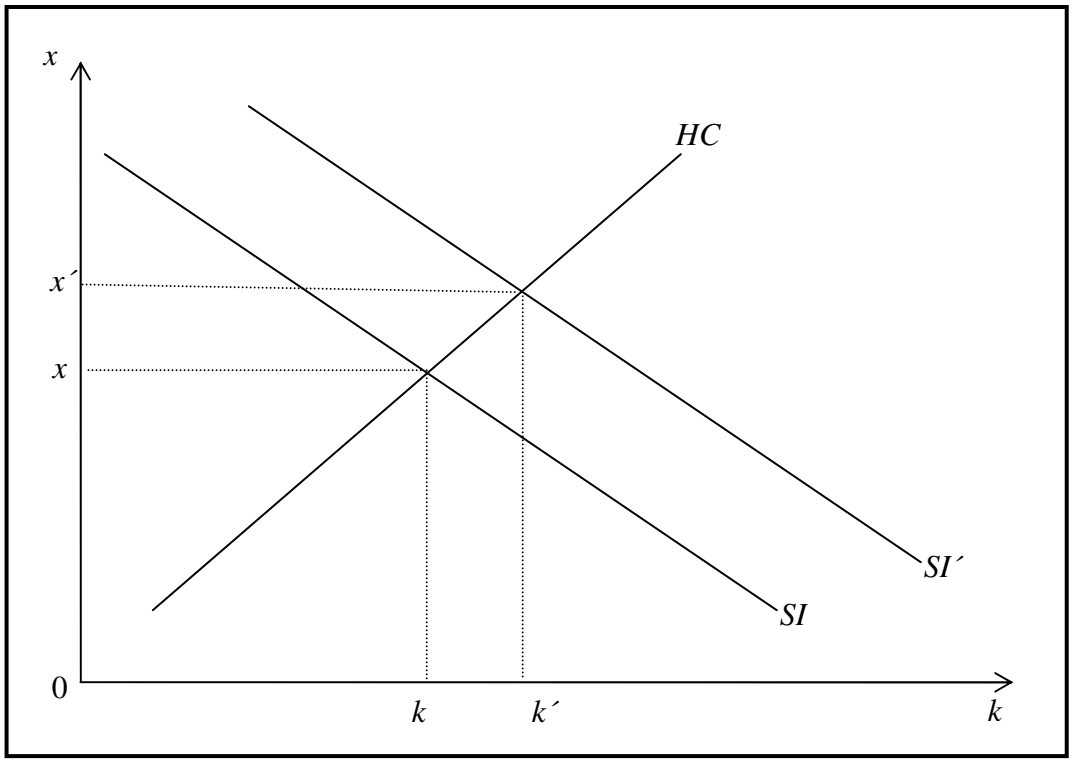


**Figure: Effects of wage-income taxes**

Contrary most findings, the above figure demonstrates that when taxes are used to finance the cost of child care required by parents who decide to work or invest in schooling, they can actually improve long run growth and welfare.

This result contrasts with the findings in the endogenous growth literature that postulate that while human capital will have positive implications on growth, income taxes will negate such effects such that growth should be lower in high income countries and vice versa (e.g., Rebelo 1991). This also contrasts with the notion that, in an economy where leisure choice improves welfare, income taxes can reduce growth and welfare since the opportunity cost of leisure declines with higher income taxes.

The intuition behind the results from the tax-subsidy proposal simply arrives from two angles. First, the time spent away from the home is subsidized, and this may be used either through schooling – that can enhance human capital and growth, or working - that can enhance savings in the economy. Both have direct implications on growth and welfare. The reason for this is that, while the tax has distortionary impacts on labour-leisure-skill choices, the tax revenue is

used not only to finance human capital but also time spent at work. Thus, while investments in human capital can enhance the long run growth rate directly, higher time spent at work can generate increases in physical capital stock through higher potential savings. This in turn would depress the returns from investing in physical capital relative to human capital, once again making skill formation a lucrative choice. The mechanism of subsidizing child care translates as a subsidy to non-leisure time and can therefore generate a higher physical and human capital stock in the long run, despite the use of distortionary income taxes to finance this public expenditure.

Second, the results also arrive in the way of internalizing the transmission externality present in the model. Each child values the parental human capital that they will inherit when they grow to become young adults. As such, children would like their parents to accumulate as much human capital as possible. However, having no way of contracting with their parents, these children cannot ensure this for themselves. The current decision to undertake schooling by a young parent takes into account only the private return that the individual receives from education and not the return that will also be received by the next generation. Hence, human capital is under-accumulated in a competitive equilibrium. Therefore, a subsidy to education financed by a tax on income is a way of ensuring the internalization of that externality. While the very first old generation do incur the cost, every generation there after will gain, leading to long-run growth and welfare in our OLG economy.

**2.4 Conclusion:**

The endogenous growth literature postulates the positive implications of human capital formation on growth. We adopt this notion in our framework where human capital is the engine of growth. Again, in a separate strand of literature, the merits of child care financing in enabling the active economic participation of parents has been highlighted. This economic participation is enabled through lower costs for parents to leave their home to avail such opportunities. However, little has ever been explored regarding the potential use of parental time outside the home in the form of own human capital investments. When child care subsidies are provided to parents, both working and attending school can become more affordable for them in terms of the time they would require to procure from formal care providers for their children. While the

decision to work enhances parental earnings that can potentially be saved for the future, the decision to invest in schooling can build human capital that can generate higher earnings in the future. Keeping in mind the merits of human capital in determining endogenous growth, our model allows this parental division of time between caring for their children, going to school and working. In other words, we merge the endogenous growth literature that highlights the long run benefits of human capital formation with the literature on child care that provides evidence on the positive implications of child care affordability on labour market participation in order to study the long run welfare implications of child care subsidies.

Government expenditures on child care, however, need to be financed through tax revenues. The endogenous growth literature shows that proportional taxes on the earnings from human capital investments may lead to negative growth effects. In fact, income taxes will negate the positive growth effects of human capital formation such that growth should be lower in high income countries and vice versa (e.g., Rebelo 1991). Our paper addresses the different forces of child care subsidies as they encourage skill formation and distortionary income taxes as they discourage the same.

A useful way to take account of inter-generational spillovers from human capital when studying the long run benefits of child care is to use an OLG framework. This captures intergenerational externalities as well as lifetime heterogeneity of child care needs. A realistic framework will have some young and economically active population requiring child care while others who have raised their children will no longer require the same. Also, while the benefits from child care subsidies will directly accrue to some, the costs of providing the subsidies may be financed by taxes on all labour market participants – old and young. Using a setup that incorporates these features, we show that when subsidies are provided for work and human capital formation simultaneously, the benefits from the tax-financed subsidy can offset its negative implications on growth. Placed within the endogenous growth models with OLG, this is a stark and interesting result. The essential reasoning behind this result is the idea that individuals have different needs for child care and human capital investments over their lifetime. While the earnings of all labour market participants can be taxed, the benefits accrue to only those for whom child care matters. The overall benefits from child care financing remain lucrative despite the distortionary taxes due to two main reasons. Higher taxes on the earnings of human capital endowed individuals

reflect lower returns to human capital investments. Individuals switch to physical capital investments in order to save for the future that is taxed. This improved savings plan sets the path to growth. Also, as long as the marginal effect of the income tax on first period earnings outweighs the effect of the same on second period wage earnings, first period labour supply will be less lucrative, while human capital investments will be increasingly rewarding. Long run welfare can therefore be ensured similar conditions.

This paper therefore derives conditions based on which distortionary income taxes can still have positive long-run welfare implications when financing child care. Real life facts testify our findings of high taxes rates in developed economies. Countries such as Canada, Sweden and the UK have high tax rates that, in fact, support their growth and welfare. Contrary to the literature, therefore, we find that when the revenues from taxes are channelled towards improving the economic opportunities amongst those for whom leisure may have become the fall-back option with higher taxes, endogenizing labour supply and taxing income can still be welfare improving.

This paper provides a framework that allows numerous possibilities for extension. One of the limitations of this paper is the non-normative focus on taxes and subsidies as they affect long run income and growth. In other words, the current paper does not evaluate taxes and subsidies from an optimality point of view. However, the presence of the inter-generational human capital externality allows for two possibilities. One possibility is the option for allowing altruism in parental preferences such that a parent's utility would increase in the child's utility. This would allow a normative evaluation of the optimal child care subsidy and wage income tax that would internalize the intergeneration human capital externality. The second possibility relates to endogenizing fertility decisions within the model. This would help justify the case of why parents would choose to have children at the outset, and also further justify the need for child care subsidization. This paper also offers possibility for incorporating interest-income taxes in addition to labour-income taxes in assessing long run income and growth implications. Further complexities introduced in the process could potentially be addressed with the help of numerical simulations. This could also help to address another limitation of this paper – the complexities involved in the stability analysis of an OLG model with endogenous labour supplied. Numerical analysis could help to demonstrate the dynamics brought about by the model in the transition from one steady state to another through stability analysis.

**Appendix to Chapter 1**

**Appendix 1:**

The Lagrangian maximization problem is given by:

$$\max_{\{C_{t,g},C_{t+1,g},l_{t,g},x_{t,g}\}} L = LogC_t + Log(1-l_{t,g}-x_{t,g}) + \frac{1}{1+\rho} LogC_{t+1,g} + \lambda\left[ w_t l_{t,g} h_{t,g} + \frac{w_{t+1}\phi(x_{t,g})h_{t,g}}{R_{t+1}} - C_{t,g} - q(l_{t,g}+x_{t,g})h_{t,g} - \frac{C_{t+1,g}}{R_{t+1}} \right]$$

where $\lambda$ is the Lagrange multiplier to the constrained maximization problem.

The first-order conditions are given by:

$$C_{t,g}: \frac{1}{C_{t,g}} = \lambda \tag{A.1}$$

$$C_{t+1,g}: \frac{\beta}{C_{t+1,g}} = \frac{\lambda}{R_{t+1}} \tag{A.2}$$

$$l_{t,g}: \frac{1}{(1-l_{t,g}-x_{t,g})} = \lambda(w_t - q)h_{t,g} \tag{A.3}$$

$$x_{t,g}: \frac{1}{(1-l_{t,g}-x_{t,g})} = \lambda\left[\frac{w_{t+1}\phi'(x_{t,g})}{R_{t+1}} - q\right]h_{t,g} \tag{A.4}$$

$$\lambda: C_{t,g} + \frac{C_{t+1,g}}{R_{t+1}} = (w_t - q)l_{t,g}h_{t,g} + \frac{w_{t+1,g}\phi(x_{t,g})h_{t,g}}{R_{t+1}} - qx_{t,g}h_{t,g} \tag{A.5}$$

The first order conditions with respect to $C_{t,g}$ and $C_{t+1,g}$ yield the following relationship:

$$\frac{\beta C_{t+1}^g}{C_t^g} = R_{t+1} \tag{A.6}$$

This describes the inter-temporal choice between current and future consumption. Future consumption will increase with higher returns to savings and likewise, current consumption will decline.

(A.6) together with (A.3) to (A.5) will give (8.1) to (8.6) in Section 1.

**Appendix 2:**

As long as some positive wage rate exists in the current and future periods, the amount of time spent in schooling should be positive. Condition (23) for a steady state gives

$$\frac{\phi}{R} > \frac{\beta(1-x)}{2}$$

If $\left(x - \frac{\phi}{R}\right) > 0$ then $\frac{\phi}{R} < x$. Together with the condition for the steady state, this means that $x > \frac{\beta(1-x)}{2}$ which simplifies to $x > \frac{\beta}{2+\beta}$ which is a very small fraction. Therefore, in order for some positive amount of time spent in schooling, we have $\left(x - \frac{\phi}{R}\right) > 0$ as a satisfying condition.

# CHAPTER 3

## Child care Exposure and Human Capital Formation

### 3.1 Introduction:

Increased policy attention to early childhood has been warranted by new evidence towards the life-long implications of brain development during the early years, as well as the efficacy of early education programs (Nelson 2000, Shonkoff and Phillips 2000; Karoly 2000). Neuroscience research has further documented how complex cognitive capacities depend on earlier foundational skills, and that many cognitive skills are sensitive to early life experiences (Knudsen, E.I et al. 2006).

The quality of life for a child and the contributions the child makes to society as an adult can be traced back to the first few years of life. From birth until about 5 years of age, a child undergoes tremendous growth and change. If this period of life includes support for growth in cognition, language, motor skills, adaptive skills and social-emotional functioning, the child is more likely to succeed in school and later contribute to society. In other words, preschool and early learning interventions may improve lifetime outcomes in part through the possibility that "learning begets learning"—that mastery by young children of a range of cognitive and social competencies may improve their ability to learn when they are older (Garces, Thomas and Currie 2002, Currie 2001 and Heckman and Carneiro 2002.)

Examples of successful early childhood intervention programs with positive long term effects include the HeadStart program, the Perry School Program and the Abecedarian Project in the United States, each of which provided children full-time, high-quality educational experience from infancy through age 5. Academic achievement in both reading and math was higher for program participants relative to nonparticipants into young adulthood (Farran, 2000). The

differences in such achievements vary upon quality and funding across programs (Barnett 1995, Currie, Garces and Thomas 2000).

A study on child care provided by the High/Scope Perry School Program shows the total estimated return on a dollar invested in the program was $8.74 in 1992 dollars estimating a real internal rate of return of 16 percent. Garces, Thomas, and Currie (2002) also show that "… participation in Head Start is associated with a significantly increased probability of completing high school and attending college, and we find some evidence of elevated earnings in one's early twenties. The evidence also suggests that there are positive spillovers from older children who attended Head Start to their younger siblings." In other words, early learning opportunities may bolster the human capital endowments amongst children.

Higher human capital endowments obtained by children exposed to child care can benefit them when they enter the job market. In other words, a higher "early learning" human capital obtained from child care could supplement the earnings of these children when they grow up and enter the labour market. This is supported by the finding that the earnings and employee benefits for adults who had been exposed to formal child care when young can rise by a total of $30,331 in 1992 dollars discounted at 3 percent. (Schweinhart, 1993).

Acknowledging the merits of child care in building the human capital endowments that these children will bring along into the labour market during their adulthood, a relevant question still remains. Can higher "endowments" of human capital potentially affect own individual human capital investment decisions in later periods of their lives? In other words, when these children grow up and have the option to choose between working and schooling, will their exposure to early learning/child care have any effect on their individual choices of how much to invest in own human capital? This question is crucial since investments in schooling or human capital have been directly linked with economic growth in the endogenous growth literature (Uzawa 1965, Lucas 1988, Romer 1990, Glomm and Ravikumar 1992). Our objective is not to study growth per se, but to draw a link between one's exposure to child care and future investments in human capital that could render potential implications for economic growth. In one paper, Teles and Andrade (2008) provide a simple Overlapping Generations (OLG) framework with exogenous labor supply that examines the effects of higher public education (e.g., elementary or

basic education) on individual human capital decisions later in life. Our concern, however is regarding the effects of exposure to child care on skills formation later in life.

Parental choices of spending time outside the home – whether at work or at school- often requires them to send their children to child care. In other words, less leisure spent at home with the child, and more "outside opportunities" sought by a parent could directly translate into higher "early learning opportunities" from formal child care services. With exposure to child care leading to higher human capital endowments from which children may reap benefits in the future, parental decisions on times spent outside the home can therefore have a positive inter-generational externality on their children. With this additional feature linking the child care literature with the literature on human capital, we aim to find out the effects of child care exposure on human capital investments later in life, and whether internalizing this externality will at all be welfare improving.

Early child care education can have interesting implications for individuals later in their lifetime when all time allocation decisions can be made endogenously. While the human capital stock can improve from early learning opportunities arriving from child care, this higher endowment can have income and substitution effects later in the children's lives when they grow into adults and are ready to decide on their time allocation between leisure, schooling and work. Without modeling the specific choice of how much leisure time to enjoy, and how much residual time to spend outside the home either at work or school, the income and substitution effects can be simplified. What needs to be well demonstrated in order to advocate the merits of child care - and potentially, the government financing of child care - is how an individual's time allocation decisions during later stages of life are affected by their exposure to child care or early learning opportunities. Can positive benefits be theoretically justified?

The fundamental goal of this study is to demonstrate this relationship between early learning through child care and the human capital accumulation process, specifically observing the impacts of child care on individual investments in higher education. In summary, we attempt to answer the following set of questions: (i) under what conditions will increased exposure to child care lead to increases in individual choices of human capital investments later in life, and (ii) will

internalizing the intergenerational externality from parent to child unambiguously lead to higher human capital investments?

The remainder of this paper will include a description of the model in Section 2. This will include a private individual's choice in Section 2.1, and a planner's choice in Section 2.2. We conclude in Section 3.

## 3.2 The Model

### 3.2.1 The individual's choice

An overlapping generation model is designed in its simplest form in order to demonstrate the model characteristics clearly. The economy comprises of the young, the old, and children. Therefore, agents live for three periods.

We assume that each child requires full-time care throughout his childhood. The young individual derives utility from devoting time to take care of her child at home in the form of leisurely activities. In the final period of the adult's life, her child is assumed to have grown up, no longer requiring supervision and care.

As an alternative to spending time at home with the child, the adult can also spend time outside the home, seeking economic opportunities through working or schooling. The increase human capital arising from schooling can make it possible for the individual to enjoy higher second period consumption.

When the parent leaves the home to seek economic opportunities, adequate supervision of her child is acquired through the purchase of formal child care services. We assume that formal, program oriented child care services are the only form of child care available outside the home[14].

[14] We do not model the market for child care services. All care providers are lumped into one category of services that offer a formal learning environment. These services are exogenously provided.

During the initial period or childhood, agents receive an endowment of human capital stock $\bar{H}$ that arrives from their exposure to early learning through child care. The first economically active period begins as soon as the agent grows into a young adult. The young adult allocates all her non-leisure time in choosing between work and investment in human capital. Investment in human capital is the vehicle for the household to smooth consumption over time. The second economically active period is one in which the agent only works. We will call these economically active periods the first and the second periods respectively. The agent's consumption during the first and the second periods is determined, respectively, by the equations below[15]:

$$c_{t,g} = w_t(T^o_{t,g} - h_{t,g})\bar{H} - qT^o_{t,g} \quad (1)$$

$$c_{t+1,g} = w_{t+1}h_{t+1,g} \quad (2)$$

where $h_{t+1,g} = h_{t,g} + \bar{H}$ (2.1)

$T^o_{t,g}$ in equation (1) is the total time spent outside the home at time t by a young adult of generation g. This time can be split between schooling $h_{t,g}$ or working $T^o_{t,g} - h_{t,g}$. Wage rates in periods $t$ and $t+1$ are assumed to be exogenously given as $w_t$ and $w_{t+1}$. The price per unit of child care purchased is assumed to be positive and given by $q > 0$. The expenditure on child care, $qT^o_{t,g}$, is the price of child care coupled with the total raw time an individual chooses to spend outside the home for which care is required for the child. $\bar{H}$ is the human capital obtained from child care exposure that will lead to higher effective earnings in both periods. The technology for human capital accumulation depicted in equation (2) is simple. The term in the brackets describes second period human capital. This comprises of first period time spent in schooling $h_{t,g}$ together with the human capital endowment $\bar{H}$ that also enhances their effective earnings in the second period.

[15] Notational differences in the notations for leisure and labour times between Chapters 2 and 3, respectively, are solely for ease of mathematical exposition and algebra.

The human capital endowment received by an individual from her exposure to child care is assumed to depend directly on the time her parent spends outside the home seeking economic opportunities in the form of schooling or work. This time spent outside the home requires the child to be sent to formal care. Thus, an intergenerational spillover occurs from generation $g-1$ to generation $g$ as described in the following equation:

$$\bar{H} = f(T^{o}_{t-1,g-1}),\ \ f' \geq 0,\ \ f'' \geq 0,\ \ f(0) = 0 \tag{3}$$

where $T^{o}_{t-1,g-1}$ is the time spent outside the home at time $t-1$ by the previous generation, $g-1$. This human capital transfer occurs indirectly through child care services available to the child when the parent seeks economic opportunities outside the home. During this time the child is exposed to such "early learning" opportunities. The benefit from this human capital transfer coming from "early learning" is reaped when the child grows into an adult and makes earnings in period $t$ and $t+1$, as seen in equations (1) and (2) below.

For ease of mathematical exposition, equation (3) is assumed to be of the simple form given below[16]:

$$\bar{H} = T^{o}_{t-1,g-1} \tag{3'}$$

Individuals within each generation are assumed to be identical. Within each period, an individual has a fixed time endowment that is normalized to unity. Upon choosing the amount of time to be spent outside the home, the remaining time may be spent enjoying leisure $l_{t,g}$. Therefore,

$$T^{o}_{t,g} = (1 - l_{t,g}) \tag{4}$$

Decision making for children is assumed to be made by their parents with their own utility maximizing choices. Therefore, an individual's economically active life will include the second

[16] Note that this simplified assumption implies that $\bar{H} < 1$.

and third periods during which they will make lifetime choices to maximize their own utility. Consumption choices during the second period of an adult's life are therefore imagined to include the first period consumption decisions for their children.

The individual's second period utility obtains from consumption and leisure as follows:

$$u_t = \frac{c_{t,g}^{1-\theta}}{1-\theta} + l_{t,g}$$

This form of quasi-linear utility agrees with the type of utilities described in Milessi-Ferretti and Roubini 1998 that is consistent with growth models. The first part is a constant relative risk aversion (CRRA) function of consumption, where $\theta$ is the inverse of the intertemporal elasticity of substitution, and $0 < \theta < 1$. As mentioned earlier, the entire time in the third period is spent working. Therefore, third period utility only arrives from third period consumption, i.e.

$$u_{t+1} = \frac{c_{t+1,g}^{1-\theta}}{1-\theta}.$$

The individual maximizes her lifetime utility to choose consumption and leisure from the following problem[17]:

$$\max_{\{c_t, l\}} U = u_t + \beta\, u_{t+1} = \frac{c_{t,g}^{1-\theta}}{1-\theta} + l_t + \frac{\beta\, c_{t+1,g}^{1-\theta}}{1-\theta}$$

subject to (1), (2), (3′) and (4).

The linearity of the utility function in leisure has been assumed for simplicity of method. This implies that the non-linear component of utility, i.e.., consumption, will not have a direct income effect. This simplifying assumption is not expected to restrict our model results or intuition.

[17] It is worth mentioning the underlying assumption that the parent's consumption choice includes the child's consumption as well. Being economically dependent, the child's utility is solely the satisfaction from the consumption bundle chosen by the parent. The parent therefore maximizes her utility that includes the child's utility-generating consumption.

Also, this utility function takes into account the fact that a parent chooses consumption and leisure to maximize her utility, where the former is a bundle that includes the child's consumption. Altruistic parental utility functions that explicitly consider the child's utility is not modeled, nor are endogenous fertility choices for the optimal number of children. Normative questions of optimal childcare subsidies and income taxes could well be answered through such techniques, which is not of primary concern in this positive modeling framework.

Incorporating constraints (1), (2) and (4), the individual's problem may be re-written as:

$$\max_{\{l_{t,g},h_{t,g}\}} U = \frac{\left((w_t\bar{H}-q)(1-l_{t,g})-w_t h_{t,g}\bar{H}\right)^{1-\theta}}{1-\theta} + l_{t,g} + \frac{\beta\ \left(w_{t+1}(h_{t,g}+\bar{H})\right)^{1-\theta}}{1-\theta}$$

The individual's choice does not incorporate the intra-generational human capital spillover. First order conditions with respect to $l_{t.g}$ and $h_{t.g}$ are the following respectively:

$$-\left[\left(1-l_{t.g}\right)(w_t\bar{H}-q)-w_t h_{t.g}\bar{H}\right]^{-\theta}\left(w_t\bar{H}-q\right)+1=0 \tag{5}$$

$$-\left[\left(1-l_{t.g}\right)\left(w_t\bar{H}-q\right)-w_t h_{t.g}\bar{H}\right]^{-\theta}\left(w_t\bar{H}\right)+\beta\left(w_{t+1}(h_{t.g}+\bar{H})\right)^{-\theta}=0 \tag{6}$$

Simultaneous solving of (5) and (6) to isolate $h_{t.g}$ gives

$$h_{t,g} = \frac{\beta^{\frac{1}{\theta}}}{w_{t+1}}\left(\frac{w_t\bar{H}-q}{w_t\bar{H}}\right)^{\frac{1}{\theta}} - \bar{H} \tag{7}$$

We assume that the amount of time spent in schooling $h_{t.g}$ is non-negative. The condition to fulfill this can easily be derived as

$$q \le w_t\bar{H}\left(\frac{\beta-\left(w_{t+1}\bar{H}\right)^{\frac{1}{\theta}}}{\beta}\right) \tag{7'}$$

Since wage rates are assumed to be set exogenously, the condition in (7') can be evaluated for $w_t = 1$ and $w_{t+1} = 1$ that gives the following full restriction on the price of child care[18]:

$$0 < q \leq \bar{H}\left(\frac{\beta - \bar{H}^{\frac{1}{\theta}}}{\beta}\right) \tag{7"}$$

Note that since $\bar{H} < 1$, the right hand side of (7") is positive.

Equation (7) clearly shows that $\frac{\partial h_{t,g}}{\partial q} < 0$. This leads us to proposition 1:

**Proposition 1:** *Human capital investments will unambiguously decline in the cost of child care, all else equal. Any subsidy to reduce the cost of child care can therefore have positive effects on skill formation.*

This proposition bears significance for potential growth implications arising from the analysis. Any policy aiming to reduce the cost of child care can encourage the skill formation decisions of parents. Higher human capital, it is well known, may work as an engine of growth.

Analysis of equation (7) further leads to the next proposition:

**Proposition 2:** *Higher stocks of human capital transferred to children through parental choices of seeking economic opportunities outside the home, that in turn requires their children to attend formal child care services offering "early learning opportunities," can generate higher human capital investments when these children grow up as long as (i) there is a positive net cost of child care borne by parents; and (ii) individuals are willing to substitute consumption intertemporally.*

**Proof:**

$$\frac{\partial h_{t,g}}{\partial \bar{H}} = \frac{q\beta^{\frac{1}{\theta}}\left(w_t\bar{H} - q\right)^{\frac{1-\theta}{\theta}}}{\theta w_{t+1}\bar{H}\left(w_t\bar{H}\right)^{\frac{1}{\theta}}} - 1$$

[18] For the remainder of the paper, however, and particularly for clarity of analysis of the anticipated income and substitution effects, we will continue to use the notation of wage rates rather than using their normalized values.

A positive substitution effect from the first component, and a negative income effect from the second component will together determine the sign of $\frac{\partial h_{t,g}}{\partial \bar{H}}$. Note that the second component comes from the additive functional form of human capital technology described in equation (2).

Thus, $\frac{\partial h_{t,g}}{\partial \bar{H}} > 0$ as long as $\frac{q\beta^{\frac{1}{\theta}}\left(w_t\bar{H}-q\right)^{\frac{1-\theta}{\theta}}}{\theta w_{t+1}\bar{H}\left(w_t\bar{H}\right)^{\frac{1}{\theta}}} > 1.$

*Condition (i)*: A full reduction or waiver in the cost of child care (e.g. through a subsidy) imparts the first term on the right hand side as zero as the cost of subsidy would be fully offset. A necessary condition required for a higher stock of human capital endowments $\bar{H}$ to induce increased investments in schooling is the existence of some positive net cost of child care necessary to be paid in order for a parent to seek economic opportunities outside the home.

*Condition (ii.)*: The above relationship will be positive for certain values of the intertemporal elasticity of substitution parameter. The critical level of $\theta$ below which child care-induced early learning human capital will generate higher skill investments is given by the following condition:

$$\frac{\partial h_{t,g}}{\partial \bar{H}} > 0 \ \textit{iff} \quad \theta < \bar{\theta} \ , \ \bar{\theta} = \frac{q\beta^{\frac{1}{\theta}}\left(w_t\bar{H}-q\right)^{\frac{1-\theta}{\theta}}}{w_{t+1}\bar{H}\left(w_t\bar{H}\right)^{\frac{1}{\theta}}}$$

Therefore, as long as the inverse of the intertemporal elasticity of substitution is not too high (i.e., individuals are increasingly willing to substitute consumption intertemporally), and condition (i) holds, a higher human capital stock inherited through early learning can influence an individual into investing more in education.

The intuition arrives from the first and second period budget constraints (1) and (2). When the individual leaves the home either to attend school or work, the individual must pay for child care for the entire amount of time spent outside the home. The gain from this expense comes from two sources - the first period earnings from working, and the higher second period consumption

from attending school in the first period. A higher $\bar{H}$ would imply higher first and second period consumption, all else equal. This could be in the form of various effects.

An income effect would imply that a rise in second period income arising from a higher $\bar{H}$ would in turn summon a lower need for human capital investments $h_t$ [19]. The substitution effect would involve comparisons of the relative wage incomes of periods $t$ and $t+1$. A higher second period return from human capital investments would encourage skill investments through a substitution of work time with schooling time in the first period. Furthermore, a higher income in the first period from increases in $\bar{H}$ could also encourage an individual to spend more time in schooling $h_t$ (and less time at work) without affecting her first period consumption.

An individual with a lower willingness to substitute consumption intertemporally would seek to reduce skill investments and simply gain from first period work. Given that consumption in the second period exceeds consumption in the first period for a higher $\bar{H}$, this individual would seek to smooth lifetime consumption by increasing work hours to improve first period earnings. However, an individual who is more willing to substitute consumption intertemporally would be willing to pay child care costs to attend school only to gain from an even higher second period consumption. We can therefore conclude that the substitution effect would dominate. An individual with higher willingness to substitute intertemporally would therefore be willing to increase own investments in education the more they are exposed to early learning opportunities.

With endogenously determined leisure as a fall back option for increasing utility, individuals who are less willing to substitute consumption intertemporally would most likely choose to stay home with their children and enjoy less consumption in both periods. Therefore, only individuals who are increasingly willing to substitute consumption intertemporally (with $\theta$ lower than a critical level $\bar{\theta}$) would be willing to leave the home to invest in human capital for higher future consumption.

[19] Note that the income effect will not lead to an increase in leisure due to utility being linear in leisure.

Further algebra from the first order conditions gives the optimal choice of leisure as follows:

$$l_{t,g} = 1 - (w_t \bar{H} - q)^{\frac{1}{\theta}-1} \left[ 1 + \left( \frac{\beta}{w_t \bar{H}} \right)^{\frac{1}{\theta}} \right] + \left( \frac{w_t \bar{H}^2}{w_t \bar{H} - q} \right) \tag{9}$$

Further analysis of (9) to check for the response of leisure choice to the cost of child care will lead to the following result:

**Proposition 3:** *An increase in the cost of child care unambiguously leads to an increase in time spent for leisure.*

**Proof:**

The first derivative of (9) with respect to the price of child care q gives

$$\frac{\partial l_{t,g}}{\partial q} = \left[ \left( \frac{1-\theta}{\theta} \right) + \left( \frac{\beta}{w_t \bar{H}} \right)^{\frac{1}{\theta}} \right] \left( w_t \bar{H} - q \right)^{\frac{1}{\theta}-1} + \left( \frac{w_t \bar{H}^2}{\left( w_t \bar{H} - q \right)^2} \right) > 0$$

These findings also generally support Proposition 1 that higher costs of child care will reduce the amount of time spent in schooling unambiguously. Reduction in schooling investments arising from higher costs of child care can unequivocally translate into increases in leisure time spent at home with the child in order to minimize on higher child care expenditures.

Further analysis of (9) makes it easy to show the following

$$\frac{\partial l_{t,g}}{\partial \bar{H}} = -w_t \left( \frac{1-\theta}{\theta} \right) (w_t \bar{H} - q)^{\frac{1}{\theta}-2} \left( 1 + \left( \frac{\beta}{w_t \bar{H}} \right)^{\frac{1}{\theta}} \right) + \left( \frac{w_t \bar{H}}{w_t \bar{H} - q} \right) \left( 2 - \frac{w_t \bar{H}}{w_t \bar{H} - q} \right) + \left( \frac{\beta}{w_t} \right)^{\frac{1}{\theta}} \left( w_t \bar{H} - q \right)^{\frac{1}{\theta}-1} < 0$$

for $\theta < \hat{\theta}$, where $\hat{\theta} = \dfrac{w_t^{\frac{1}{\theta}+1}(w_t\bar{H}-q)^{\frac{1}{\theta}-1}\left(1+\left(\dfrac{\beta}{w_t\bar{H}}\right)^{\frac{1}{\theta}}\right)}{w_t^{\frac{1}{\theta}+1}\bar{H}+\beta^{\frac{1}{\theta}}}$

A comparison of $\bar{\theta}$ from Proposition 2 and $\hat{\theta}$ from above reveals $\hat{\theta} > \bar{\theta}$. This leads us to the following result:

**Proposition 4:** *Lower choices of leisure for $\theta < \hat{\theta}$, and higher human capital investments (as in Proposition 2) for $\theta < \bar{\theta}$ together imply that higher stocks of human capital from early learning may also increase work hours or labour market participation amongst some individuals.*

The explanation is illustrated in the figure below.

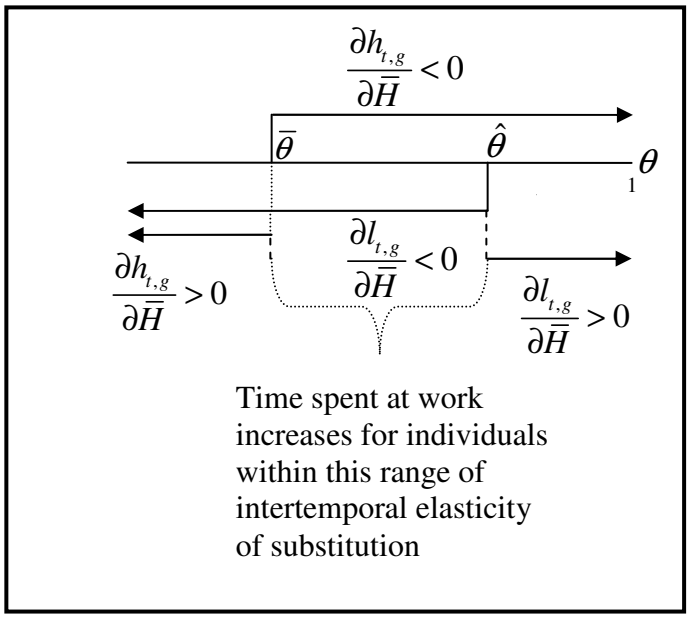


**Figure: Critical values of intertemporal elasticity of substitution**

Proposition 2 conditions higher human capital investments arising from increased human capital endowments (from child care exposure) only to individuals who are willing to substitute consumption intertemporally. Again, leisure will decline with increments in human capital endowments for individuals with intertemporal elasticity of substitution for consumption higher than another threshold level. Given that this threshold exceeds the threshold required for

individuals to increase their own skill investments, there must be some individuals with $\bar{\theta} < \theta < \hat{\theta}$ who will both reduce their skills investments, but will also reduce their enjoyment of leisure. Since time is perfectly substitutable, these individuals must therefore be spending an increasing amount of time at work during the second period of their lives. Under the conditions discussed so far, higher stocks of human capital may not only lead to higher stocks of human capital for individuals who are increasingly willing to substitute consumption, but may also lead to higher labour market participation in the workforce by others having a lower willingness to substitute consumption intertemporally.

This makes it possible to conclude that time spent outside the home may increase with increases in child care exposure $\bar{H}$ under risk aversion parameters $\theta < \hat{\theta}$. A pertinent question that arises now is, if the decision on time spent outside the home by parents were taken considering the beneficial implications from child care on their children, could these results be improved? In other words, would investments in schooling improve unconditionally when the intergenerational externality is internalized? We address this aspect in the following section.

### 3.2.2 The planned optimum

Individuals do not account for the intra-generational benefit spillovers from parents to their offspring when choosing times spent outside the home for seeking economic opportunities. An optimum will take this externality into account. The planned optimum will involve maximizing the sum of the utilities of both coexisting generations at any time t. In other words, the utilitarian optimum will involve

$$\max U^P = u_{t,g} + u_{t,g-1}$$

where $u_{t,g}$ is the utility of the representative young adult in period t and $u_{t,g-1}$ is the utility of the representative old adult in period $t$. The planned optimum would consider the time allocation choices by any individual to be invariant across generations. Therefore, time spent outside the home by an individual from generation *g* would be identical to that of her previous generation.

Thus, $T^{o,p} = 1 - l^p_{t,g} = 1 - l^p_{t-1,g-1} = 1 - l^p$. Likewise, the time subscript on schooling investments will be eliminated and replaced in the following problem by $h^p$.

Therefore the optimum would require solving:

$$\max_{l^p, h_{t,g}} U^P \frac{\left((1-l^p-h^p)w_t(1-l^p) - q(1-l^p)\right)^{1-\theta}}{1-\theta} + l_{t,g} + \frac{\left[w_t\left(h^p + (1-l^p)\right)\right]^{1-\theta}}{1-\theta}$$

The first-order conditions for $l^p$ and $h^p$ give the following respectively

$$\left[\left(1-l^p-h^p\right)w_t(1-l^p) - q(1-l^p)\right]^{-\theta}\left(-w_t(1-l^p) + q - w_t\left(1-l^p-h^p\right)\right)$$
$$+1 - w_t^{1-\theta}\left(h^p + H\ (1-l^p)\right)^{-\theta} = 0 \qquad (11)$$

$$\left[\left(1-l^p-h^p\right)w_t(1-l^p) - q(1-l^p)\right]^{-\theta}\left(-w_t(1-l^p)\right)$$
$$+w_t^{1-\theta}(h^p + (1-l^p))^{-\theta} = 0 \qquad (12)$$

Keeping in mind that $T^o = (1-l^p)$, a comparison of (11) with (5) reveals that the choice of leisure time at home with the child is lower than the private choice at the optimum. In other words, the optimum would involve more time being spent outside the home either at work or at school. Similarly, a comparison of (12) and (6) shows that the optimal skill choice is higher than the private skill choice, irrespective of the value of the risk aversion parameter.

**Proposition 5:** *An optimum will be one that internalizes inter-generational externalities from human capital formation through "early learning opportunities" so that higher times spent seeking economic opportunities outside the home by any individual at any time will result in higher skill investment choices irrespective of the cost of child care or individual preferences for intertemporal consumption substitutability.*

Private choices on the allocation of time between work, schooling and leisure critically depend on the intertemporal elasticity parameters of individuals, and any increase in child care exposure would affect these decisions based on these individual preferences parameters. However, when the positive intergenerational externality is considered in determining an optimal choice for the allocation of time, schooling investments would unambiguously increase with increases in child care exposure for all intertemporal elasticity values.

A government can play a role in ensuring higher skill investments by private individuals by providing child care subsidies that reduce the cost of time spent outside the home (reducing the cost of one unit of care below $q$), leading to their children's exposure to early learning opportunities. Based on Proposition 5, this in turn can lead to higher skill investments. This result is described in Proposition 6.

**Proposition 6:** *A child care subsidy that can reduce the cost of time spent outside the home may induce more skill formation and therefore can be akin to a human capital subsidy*[20].

This justifies the role of government intervention through reducing child care costs. Since higher times spent either at work or school by parents can have positive inter-generational effects in terms of inducing further skill investments by their offspring, policy intervention that can reduce the costs of purchasing child care may lead to growth enhancing human capital formation. This is an interesting result that supports the financing of child care when human capital financing can necessarily be another policy alternative, and bolsters the results in Chapter 2 that demonstrate economic growth from the child care subsidies.

### 3.3 Conclusion:

This paper contributes to the literature on human capital and early childhood education by building on the literature in an OLG framework with endogenous leisure choice and an inter-generational child care-induced human capital externality.

[20] When the subsidy is financed by proportional income taxes, the resultant distortions on work, leisure and human capital formation decisions can oppose the result in proposition 5. The net effect of both the subsidy and a distortionary income tax on growth and welfare has been dealt with in a related working paper (Alamgir-Arif and Dissou 2011).

We show that for some positive cost of child care, individuals with a higher willingness to substitute consumption intertemporally will increase their investments in human capital the more they are exposed to higher human-capital generating child care services. This result is interesting due to the addition of endogenous leisure choices as a feature in our model that is absent in the literature. With leisure as a fall back option for increasing utility, individuals with a low willingness to intertemporally substitute consumption would most likely choose to stay home with their children and enjoy less consumption in both periods. In other words, only individuals who are willing to intertemporally substitute consumption more than a threshold level would be willing to leave to home to invest more in human capital for higher consumption in the last period of their lives. The split between work and leisure decisions therefore critically relies on the curvature of individual preferences.

When the inter-generational externality from parental choices of times spent either at work or school, for which their children require child care is internalized, increased exposure to learning through child care can unambiguously lead to increased investments in human capital.

While policy intervention that can encourage human capital investments has been generally accepted to be growth and welfare improving, it is necessary to understand where the intervention will work best, given limited public resources. In other words, it is necessary for policymakers to identify specific educational investments that yield the highest public returns. Here our results are clear. Policy intervention in early childhood learning can yield desirable public returns over time through higher individual skill investment choices by all individuals.

Possible extensions of this paper may provide useful insight towards policy intervention in child care. One interesting question that demands attention is the implication of the intergenerational externality in an OLG model of the type discussed in the previous chapter, and its implications for growth. While the application will be straightforward, both the normative question of an optimal tax and subsidy policy for childcare financing, as well as the positive question of how economic growth can be affected, could well be addressed. Again, numerical analysis could be useful in order to obtain tractability of results from that set up. This could also help eliminate the limitations of this current paper in terms of eliminating the need for simplifying assumptions

such as the utility function being linear in leisure choice or the intertemporal human capital externality being directly and positively proportional to the time spent by a parent outside the home while also abstracting, as in this current model, from any further human capital endowment (direct parent to child knowledge transfer) component.

# CHAPTER 4

## The Implications of Human Capital Mobility on Private and Public Skill Investments

### 4.1 Introduction:

In a world of rapidly advancing technology and integration, the consequences of factor mobility are of crucial concern to economic unions and federations. Not only are capital markets integrating, but so are labour markets. It is with more ease today than before that human capital endowed labour move between regions after investing in human capital. This may affect the incentives of the governing authorities to finance, either partially or fully, educational expenses. Although mobility within a federation such as Canada, where migration is within the same country, may seem to pose less concern than mobility amongst different countries in an economic union such as the European Union, the essence of the problems remains since one provincial government may have spent on the human capital of a skilled worker located in another province. This paper addresses two issues pertaining to human capital mobility – (i) how does human capital mobility affect human capital investments, (ii) how does human capital mobility affect government financing of education. We analyze the effects of mobility on private investments in skills, on the need for government subsidies, and on the decisions in invest in education by sub-national governments.

It has long been understood that redistributive goals of governments are constrained by, what has emerged in the literature as, fiscal competition for mobile resources among jurisdictions (Oates 1972, Flatters, Henderson and Mieszkowski 1974, Boadway and Flatters 1982, Wildasin 1991, Cremer, Fourgeaud, M. Leite-Monteiro, M. Marchand and P. Pestieau 1996,). Such would ultimately lead to a race to the bottom in the choice of social benefits. Most of the literature in this area deals with taxation of mobile capital for the financing of public expenditures, and the subsequent competition amongst regions for acquiring capital. In a world of rapidly advancing

technology and ease of communication, the mobility of capital as well as goods and services, and to an extent, labour, has intensified.

Within the fiscal federalism literature, there are theoretical work on the impacts of tax competition on redistribution and welfare when skilled labour is mobile (Andersson and Konrad 2003, Poutvaara 2001). Further within this domain there are those that incorporate factor mobility and skill investments in the same framework with particular interest in tax competition for mobile skilled labor (Wildasin 2000, Poutvaara 2000). In another domain, some authors abstract from detailing government financing of education, and solely focus on how private human capital investments are impacted when individuals are allowed to migrate (Grubel and Scott 1966, Helmenstein and Prskawetz 1998).

Within the realm of public finance, Poutvaara (2000) shows that a policy that aims to eliminate tax competition can be welfare improving when educated individuals are mobile across regions. Wildasin (2000) uses a general equilibrium framework to compare the welfare aspects on private and public financing of education for mobile individuals. He shows that private financing of human capital investment is possible through the capital market, freeing up migration amongst regions raises welfare and reduces any ex post earnings inequalities. Again, when skill formation is publicly financed from local taxes, free migration can lead to inter-jurisdictional fiscal competition that shifts the tax burden to low-skilled immobile workers. His paper is an improvement over prior fiscal competition literature (e.g., Epple and Romer 1991, Wildasin 1994, Keen and Marchand 1997), in that he endogenizes skill investments of labor in a model of tax and expenditure policies. Yet, since the model focuses on the cases of either public or private provision, it does not examine the effect of mobility on the public/private expenditure mix. An interesting exercise would be one that explores the effect of skill mobility on government incentives to finance education when knowledge accumulation involves positive spillovers within and across regions.

In their simple and succinct model, Justman and Thisse (1997) show that inter-jurisdictional mobility can lead to lower public expenditures in education. The education financing public authority in their framework is, however, a rent-maximizing one, and the intuition for reductions

in educational expenditures with increased probabilities of migration is clear – rents are maximized by retaining individuals within the region.

In our endogenous migration framework, we incorporate a welfare maximizing government. Our aim is to study the implications of mobility on public and private human capital financing when skilled individuals are mobile, and when the accumulated human capital in a region has positive externalities on the productivities of others. To keep this focus, we abstract from tax competition issues by assuming centralized tax collection for all regions. We abstract from redistribution by considering homogenous individuals, and this is also where our paper differs from most of the existing fiscal competition literature, where tax collection is mostly for redistributive purposes (e.g.,Wildasin 1998, Epple and Romer 1991).

As mentioned earlier, our paper justifies the need for public financing of education, in addition to private educational investments made by individuals, by modeling a positive externality from human capital. A similar path was also undertaken by Stark and Wang (2002), who show that as the probability of migration increases, welfare will increase but will decline at the limit for those remaining if the income in the recipient region is sufficiently high. A probability of migration higher than a critical level can then lead to a brain drain. This is more so if the income difference between the two regions is low such that the initial benefit from migration is low. They therefore identify a brain gain, in the form of increased welfare for all, within restrictive conditions. Results of a brain gain are supported empirically in models that highlight the gains from high-income remittances from rich to poor regions (see Beine, Docquier and Rapoport 2001).

The intuition in Stark and Wang is similar to that of our basic model in that individuals have a possibility of migrating to a higher income region, and this inducement effect leads individuals to form more human capital. However, rather than assuming migration to a higher income region as exogenous, we endogenously derive the result that income is higher when migration is allowed. In other words, we prove, contrary to assuming, that income of individuals will increase if they are allowed to migrate, all else equal. This result is key to our finding that human capital formation increases when migration is possible. Further, Stark and Wang (2002) do not study the effects of migration on the incentives of regional and federal governments to finance

education, whereas our key focus is to study these precise effects. We characterize the effect of increased mobility on human capital formation - not only from private skill investments but also from government support (the effect of which, together, we can refer to as a brain gain).

Finally, we study the strategic incentives of governments in their financing of skills when there are localized human capital externalities and migration costs. Incorporating the Mansoorian and Myers (1993) attachment to home notion of migration costs (later referred to as M-M), we show that governments would reduce their subsidies to education as mobility costs decrease when the negative effect of congestion outweighs the positive externality from having a larger number of people in their region. Again, when the reverse is true and individuals are increasingly mobile, governments engage in expenditure competition by increasing their subsidy offers to attract more individuals to their region.

We continue by defining our basic model in Section 2. In Section 3, we introduce non-localized externalities into the model sans strategic behaviour amongst regions. Section 4 extends the model to one with localized externalities and allows for migration to be taken into account by governments in their choices of educational subsidies. Finally, we conclude and provide a brief discussion on possible avenues for extension in Section 5.

### 4.2 Model Definition:

A federation comprises of two regions, *A* and *B* that are identical ex-ante. Productivity in these regions will vary upon the realization of an adverse shock. The federation has $N$ individuals indexed by $n$, $n \in [o, N]$, equally divided within each region initially. Individuals are identical in all aspects but their attachment to home. Following Mansoorian and Myers (1993), the parameter $n$ determines the psychic benefit to individuals from living in each region. The attachment to home of individual $n$ if residing in region A is $\alpha(N-n)$ and if living in region B is $\alpha n$. Households with a relatively small *n* have region A as their home and those with a larger $n$ have their home in region B. $\alpha$ determines the cost of migration. Therefore $\alpha = 0$ implies no attachment to home (or perfect mobility) and the degree of attachment to home increases as $\alpha$ increases. Attachment to home can also be thought of as the "recognition of (educational)

qualifications[21]." Often it is the case, especially in the EU, that educational qualifications are better recognized in the home region than in any other region. Skilled individuals' choices of migrating will, thus, be encouraged or discouraged by the ease with which their skills are recognized in the region to which they would like to migrate. Note that the parameter $n$ will also denote the individual who is just indifferent between residing in either region in the migration equilibrium. This implies that $n$ will also denote the population of region $A$ in equilibrium.

Each individual supplies one unit of labour that is augmented by human capital. Individuals decide on higher education or human capital investments $h$, in order to maximize their expected utilities. An individual's choice of $h$ can be thought of as her efficiency units of labour. For notational simplicity, the human capital formed in region A is denoted by $h$ and that formed in region B is determined by $\bar{h}$. The generalized production function in region A is given by:

$$Y = F(n,h) + n\ g(n\ h,(N-n)\ \bar{h})$$

where $F_n > 0, F_{nn} < 0, F_h > 0, F_{hh} = 0,\ g'(\cdot) > 0, g''(\cdot) < 0,$ and the populations of regions A and B are given by $n$ and $(N-n)$, respectively.

Regional output not only increases in skilled labour, albeit at a decreasing rate due to diminishing returns to the amount of labour in a region (a *productivity effect*), but also in the positive externality function, $n\ g(\cdot)$, from the total skills within the federation (an *indirect effect*). Thus, the positive externality from education received by each individual in the region, $g(\cdot)$ comprises of a localized externality and a non-localized externality. A localized externality can be thought of as agglomeration generated from being among, and interacting with skilled individuals. The non-localized externality can be thought of as larger scale impacts of skill formation on, e.g., Research and Development (R&D) that has a spillover effect across regions. In a federation such as Canada, a rise in the level of skills in one or more provinces can be seen to bump up the overall income structure of the economy. For example, it is reasonable to think that research and technological knowledge advances in British Columbia will benefit Alberta as well. Therefore,

[21] $\alpha$ can also be imagined to be language barriers across regions (e.g., dominance of English or French in any region)

benefits from the stock of knowledge in a federation may not be limited to the region that produces them, but can spillover to other regions. Since both regions in our model are identical, they benefit equally from the total stock of human capital formed in the federation. It is therefore worth noting that the amount of non-local externality generated from skill formation will not vary with migration or the number of people in each region after migration since it is generated during the skill formation stage and, irrespective, will be equally benefiting residents of the federation whether they migrate or not. In our paper we will focus on the special cases of fully non-localized externalities and fully localized externalities.

For the specific reason of obtaining the migration equilibrium later in this paper, the productivity effect is simplified to be separable in $h$. Thus, output is simplified to:

$$Y = F(n)h \ + n \ g(n \ h,(N-n)\bar{h})$$

The specific production function for region A will be $Y = F(n,h) \ + n \ g(n \ h+(N-n)\bar{h})$ in the case of non-localized externalities and $Y = F(n,h) + n \ g(nh)$ in the case of localized externalities. However, for the general function (for either localized or non-localized human capital externalities, output per worker is given by

$$Y/n = y = \left[\frac{F(n)}{n}\right]h + g(n \ h,(N-n)\bar{h}) \tag{1}$$

We re-write the above as

$$y = f(n) \ h + g(n \ h,(N-n)\bar{h}) \tag{1.1}$$

Close inspection of (1) shows that the first derivative of the productivity component of output per worker with respect to n gives $f' = \left(\frac{F'(n)}{n} - \frac{F(n)}{n^2}\right)h$. Since $F'(n) > 0$ and $F''(n) < 0$, the

term within the brackets describing $f'$ is negative. Following the same logic, $f''(n) < 0$.

We assume that a worker's gross earnings is also the worker's output. We also assume that each region may receive either a positive or a negative shock after skill formation with probability 1/2. For simplicity, shocks are assumed to be perfectly negatively correlated, so that a positive shock in one region implies a negative shock in the other[22]. The productivity shock is denoted by $z$ and after-shock output per worker in region $A$ is:

$y = (1+z) f(n)\ h + g\left(n\ h, (N-n)\bar{h}\right)$ for a good shock

and

$y = (1-z) f(n)\ h + g\left(n\ h, (N-n)\bar{h}\right)$ for a bad shock.

Due to the interplay of private as well as external benefits from $h$ within each region, there is a role for public intervention to induce skill formation through subsidization. By a parallel token, the existence of positive external benefits across regions implicates the need for central coordination of some form by the federal government. The stages of the game are as follows: the regional governments first announce subsidies towards skill formation that maximize expected utility of the natives. Based on the subsidy offered, individuals take private decisions on how much to invest in skills in order to maximize expected utility. In the third stage, nature randomly provides either a positive or negative shock to the region. When migration is possible, utility maximizing individuals can relocate from one region to another in stage four. Migration continues until net-of-shock utilities equate across regions.

Upon investing in human capital, the utility of an individual residing in region A for a good shock situation will be given by:

[22] Using different probabilities for shocks in each region would add complications in terms of asymmetric government behaviour on expectation. We use equal probabilities so that the regional governments behave identically on expectation. This allows simplicity in achieving our goal of studying the implications of skill mobility on human capital financing under the different knowledge spillover scenarios. However, while our simple symmetric region case provides the benchmark for public financing implications, unequal probabilities leading to heterogeneous regions (on expectation) could be worth exploring, e.g., to fit a model with remittances.

$$U = (1+z) f(n) h + g(n\ h, (N-n)\ \bar{h}) + s\ h - k\ h^2 + \alpha(N-n) - T$$

and for a bad shock:

$$U = (1-z) f(n) h + g(n\ h + (N-n)\ \bar{h}) + s\ h - k\ h^2 + \alpha(N-n) - T$$

where $k$ is a cost parameter for skill investments, $s$ is the subsidy rate offered on each unit of investment in skills and $T$ is a lump-sum tax on each individual in the region[23]. Note that when $\alpha = \infty$ there is absolutely no mobility, and individuals remain in the region despite a negative shock. The marginal productivity from human capital for individuals residing in region A, given by $f(n)$ will be denoted by $W^n$ [24].

No other public expenditures, but educational spending, are modeled in order to restrict our focus to human capital financing. Lump-sum taxes are used for simplicity. It is assumed that individuals pay taxes to their region of origin when they obtain education (or before they leave the region if they need to migrate). We assume that individuals can borrow freely from a capital market exogenous to our model, or that they are born with an income endowment that can be exhausted to pay taxes that help finance their education. This notion of tax will work similar to the concept of a graduate tax in which individuals graduating from a region will pay back to the same region with portions taken from their income, except that, for simplicity we assume lump-sum taxes. We begin our analysis in the section below with the case of non-localized human capital externalities.

### 4.3 Equilibrium with Non-Localized Human Capital Externalities:

The objective of this basic case is to see how, in the absence of strategic competition, the possibility of migration affects individuals' decisions to invest in human capital as well as

[23] Note that the subsidy shifts the marginal cost of education curve down. Alternatively, the subsidy could affect the slope itself, e.g. $(k-s)h$ in an otherwise formulated utility function, but this essentially would not affect the results of our model.

[24] Likewise, $W^{(N-n)}$ for region B in our set up.

government financing of education. The absence of strategic behaviour obtains with our special case of $g(n\ h+(N-n)\bar{h})$. This is the case because the externality benefits individuals of both regions identically irrespective of location. Any incentive by governments to induce individuals to their region would only be possible if regions benefit from agglomeration. We first consider the extreme case of no-migration $(\alpha=\infty)$, then that of perfect mobility ($\alpha=0$).[25]

### 4.3.1 The no-mobility case

#### 4.3.1.1 Private skill choice under no-migration

When migration possibilities are not present, individuals in a bad shock region cannot avoid experiencing the negativity from the shock on their rate of return from skill investments. The expected utility maximizing problem for a representative individual under the "no-migration $(nm)$" case is given by:

$$\max_{h}\left\{\left(\frac{(1+z+1-z)}{2}\right)W^{n}h+g(n\ h+(N-n)\bar{h})-k\ (h)^{2}+s\ h-T\right\};$$

This yields:

$$\max_{h}\left\{W^{n}h+g(n\ h+(N-n)\bar{h})-k\ (h)^{2}+s\ h-T\right\}$$

The solution is:

$$h^{*}_{nm}=\frac{W^{n}+s}{2k} \qquad (2)$$

[25] With migration costs between the extremes, i.e, with some costly migration, the analysis would become complicated (when coupled with the shocks) without adding much insight. The primary effects of mobility on individuals' incentives to invest in education would become difficult to decipher.

We make use of our assumption that $n$ is large when obtaining the above and other similar results throughout this paper. [26]Two points come out of (2). First, if government funding is not available, the skill choice would be lower (since $h^* = \frac{W^n + s}{2k} > h^* = \frac{W^n}{2k}$). Second, individuals do not take into account the positive externality from skill formation when making their decision. A regional social optimum would involve taking the externality into consideration in determining the choice of skills. This implies that government intervention is potentially desirable.

#### 4.3.1.2 The regional government problem

We assume identical individuals, and therefore our utilitarian welfare optimization problem is that of a representative individual. Internalizing the positive externality that private individuals do not generally take into account, the optimum skill formation, denoted by $O,R$ below, is characterized by:

$$\max_h \left\{ W^n h + g(n\ h + (N-n)\bar{h}) - k(h)^2 \right\}$$

and the regional optimum will yield:

$$h^{O,R}_{nm} = \frac{W^n + n\ g'(n\ h + (N-n)\bar{h})}{2k} \tag{3}$$

The regional optimum takes account of the positive externality that benefits residents of the region. Clearly, (3) exceeds an individual's choice of skills had there been no government subsidies, $h^*_{nm} = \frac{W^n}{2k}$. The regional government would induce the social optimum by choosing the appropriate level of subsidy.

[26] This large $n$ is assumed for mathematical pithiness so that each individual's impact on the knowledge externality is meagre. This effect can then be omitted from the algebra (Dahlby, Mintz and Wilson 2003, Acemoglu 1993). This is also realistic – an individual's choice of skills in a region (e.g. Ontario) will have a nominal effect on the knowledge spillover within and across the region, but collectively, the effect of total human capital on the externality will be non-trivial.

This is clear from the regional government's problem below:

$$\max_{s,T} \; n\left\{W^n h_{nm}^{\;*} + g(n\; h_{nm}^{\;*} + (N-n)\bar{h}_{nm}^{\;*}) - k(h_{nm}^{\;*})^2 + s\; h_{nm}^{\;*} - T\right\}$$

s.t. $\quad n\; s\; h_{nm}^{*} = n\; T$

Using the optimal value of $h_{nm}^{\;*}$ from (2) into the problem above results in

$$s_{nm}* = n\; g'(n\; h_{nm}^{\;*} + (N-n)\bar{h}_{nm}^{\;*}) \tag{4}$$

and

$$T_{nm}^{*} = n\; g'(n\; h_{nm}^{\;*} + (N-n)\bar{h}_{nm}^{\;*})\left[\frac{\left(W^n + n\; g'(n\; h_{nm}^{\;*} + (N-n)\bar{h}_{nm}^{\;*})\right)}{2k}\right] \tag{5}$$

The regional government can thus induce the socially optimal level of skill formation by choosing a subsidy rate of $n\; g'(n\; h_{nm}^{\;*} + (N-n)\bar{h}_{nm}^{\;*})$.

### 4.3.2 The perfect-mobility case

#### 4.3.2.1 Migration equilibrium

It is assumed that regions are symmetric apart from the regional shock that is realized. It is also assumed that there are no migration costs, and migration decisions are made as soon as a shock is realized. The existence of private efforts to form human capital and public subsidies to partially cover their costs enables the framework to incorporate both private and public investments in skills[27]. It can be noted that, since taxes and subsidies are announced in the first stage of the game, and regions are ex-ante symmetric, both regional governments behave identically in their choices. Individuals migrate from a negative shock region to a positive shock region until their utility is equal irrespective of where they choose to reside. The migration equilibrium is characterized by the population of region $A$, $\tilde{n}$, for whom utility is equalized across regions.

[27] While Wildasin (2000) compares private and public financing of skill formation, Poutvaara (2000) incorporates both public and private financing in the same framework to compare the effects of tax competition and the absence of it.

Assuming a positive shock in region A, and equating the utilities from being in each region while eliminating identical terms from each side, the productivity component of income is determined as follows[28]:

$$\{(1+z)f(n)=(1-z)f(N-n)\}=\{(1+z)W^{n}=(1-z)W^{(N-n)}\}=W^{M} \quad (6)$$

With migration from an adverse shock region to a positive shock region, the number of people n changes for each region and the return to human capital adjusts through changes in $f(n)$. In equilibrium, the return from skill formation in one region equals that in the other and there is no further migration between regions. The productivity component in income adjusts to a fixed level, $W^{M}$, as determined from (6).

A comparison of the equilibrating level of $W^{M}$ must be made with the no-migration productivities, $(1-z)W^{I}$ or $(1+z)W^{I}$, where $I=n$ or $(N-n)$. Prior to doing so, we identify the private choice of skills with migration.

#### 4.3.2.2 Private skill choice under migration

Prior to the shock occurring and migration taking place, individuals choose the skill level that maximizes their utility keeping in mind that migration will occur amongst regions once s shocks occur. Denoting the migration case by m and solving by backward induction, they take the migration income, $W^{M}$, as given and choose skill level $h$ to maximize:

$$\max_{h}\left\{W^{M}h+g(n\ h+(N-n)\bar{h})-k(h)^{2}+s\ \ h-T\right\}$$

and the resulting individual skill choice is given by:

$$h^{*}_{m}=\frac{W^{M}+s}{2k} \quad (7)$$

[28] Symmetry implies that the results will not change if region B receives a positive shock and A receives the negative shock.

A comparison of (7) and (2) will determine the desirability of migration as a means to encourage private skills investments in education.

**Proposition 1:** *Migration possibilities augment investments in skill formation and, all else equal, result in a higher level of welfare.*

One can easily imagine the benefit from migration. Although an individual is risk-neutral, migration increases expected income because it leads to more people living in the relatively productive region.

**Proof:**

We reiterate our simplifying assumption of ex-ante identical regions with equal population. Mathematically, the equilibrium prior to shocks occurring is:

$$f(n) = f(N-n)$$

Assuming a negative shock to region A, the equilibrium initially regresses to

$$(1-z)f(n) < (1+z)f(N-n)$$

This triggers migration from region A to B until the productivity component of income through a migration of $x$ number of people from a bad shock to a good shock region gives:

$$(1-z)f(n-x) = (1+z)f(n+x) \qquad \text{(A)}$$

Given the concavity of output production with additional individuals migrating to a region, the decline in the productivity component in region B would be lower than the rise in the productivity component from individuals leaving region A. As a result, the equilibrium will be at a level that results from a slower decline of post-shock income in region B and a faster increment in post-shock income in region A. In other words, migration from region A to B leads income to equilibrate at a level that is higher than the original, pre-shock level.[29]

[29] The magnitude of this will depend on the curvature of $f'$ and on the magnitude of the shock, $z$.

It is worth recalling also that the pre-shock productivity component of income is identical to that of the no-migration case. That is, the productivity component of income, based on expectation, is given by:

$$\frac{(1-z)f(n)}{2}+\frac{(1+z)f(n)}{2}=f(n) \qquad \text{(B)}$$

which is the pre-shock productivity component of income.

Given that the productivity component of income settles at a higher level with migration (equation A) than without (equation B), individuals obtain a higher income component, $W^m$ in (7) than in (2). This is depicted in the figure below.

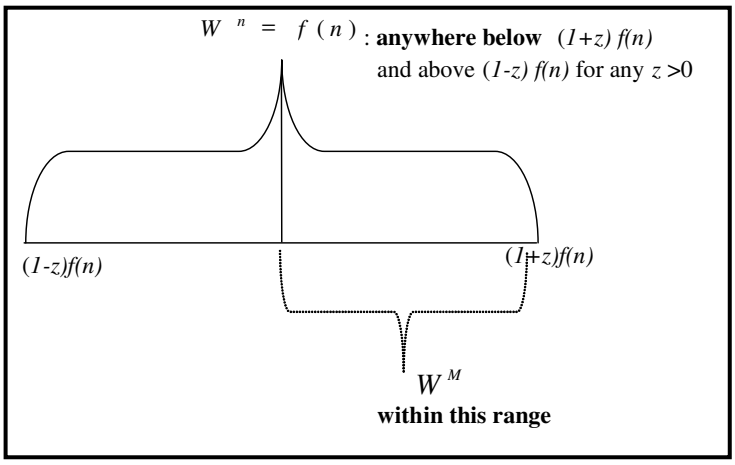


**Figure: Post migration income**

Therefore, all else equal, increased possibilities of migrating encourages skill investments while improving welfare.

#### 4.3.2.3 The regional government problem

The optimum solution to a planner problem, denoted by O,R below, would take the externality into consideration, while choosing individual skills from the optimization problem

$$\max_{h_m}\left\{W^M h_m + g\left(n\ h_m + (N-n)\bar{h}_m\right) - k(h_m)^2\right\}$$

yielding[30]:

$$h_m^{O,R} = \frac{W^M + n\ g'\left(n\ h_m + (N-n)\bar{h}_m\right)}{2k} \tag{8}$$

Clearly, (8) exceeds an individual's choice of skills had there been no government subsidies, $h^*_m = \frac{W^M}{2k}$. The regional government will use educational subsidies to induce the social optimum from the private choice of skills.

The government's problem that would achieve this is:

$$\max_{s,T} n\left\{W^M h_m^* + g\left(n\ h^*_m + (N-n)\bar{h}^*_m\right) - k(h_m^*)^2 + s\ h_m^* - T\right\}$$

s.t. $\quad n\ sh^*_m = n\ T$

Using the optimal value of $h^*$ from (8), where $h^*$ is a function of $s$, into the problem above results in

$$s^*_m = n\ g'\left(n\ h_m + (N-n)\bar{h}_m\right) \tag{9}$$

$$T^*_m = n\ g'\left(n\ h_m + (N-n)\bar{h}_m\right)\left[\frac{\left(W^M + g'\left(n\ h_m + (N-n)\bar{h}_m\right)\right)}{2k}\right] \tag{10}$$

**Proposition 2:** *When the marginal externality decreases as the total level of skills increases, a socially optimal level of skill formation can be induced with lower subsidies when migration possibilities are allowed. Therefore, mobility increases the share of private expenditures and reduces the share of public expenditures within the total expenditure mix.*

[30] As mentioned earlier, the non-localized externality will not vary with the number of people locating in each region and hence, will not vary with migration. Thus, $g(nh^A + (N-\tilde{n})h^B) = g(nh^A + (N-n)h^B)$. We will continue our current analysis using the latter.

**Proof:**

Imagining a move from a case of no migration to one of migration, the productivity component of income *W*, all else equal, is expected to increase.

Total differentiation of the first order condition that leads to (8) in order to find $\frac{dh^{o,R}}{dW^{M}}$ gives:

$$\frac{dh^{o,R}}{dW} = \frac{1}{2k - g''\left(nh + (N-n)\bar{h}\right)n^2} \tag{11}$$

Note that generalization is drawn here between the migration and no migration cases by dropping the (super/) subscripts that distinguished them. The same exercise for the private individual's choice, i.e, the first order condition that derives (7) gives:

$$\frac{dh^{*}}{dW} = \frac{1}{2k} \tag{12}$$

Let

$$H = \frac{dh^{o,R}}{dW} - \frac{dh^{*}}{dW} \tag{13}$$

It is clear that, when the marginal externality diminishes in the total level of skills, the second component in the denominator of (11) is positive together with the negative sign before it. As a result, the overall value of (11) is lower than (12) implying that $H < 0$.

Therefore, when migration possibilities are enhanced leading to an increase in the productivity component of income, *W*, the gap between the socially desirable level of skills and the individually chosen level of skills declines. This result also implies that although the socially desirable level of human capital would also increase when migration is allowed (following Proposition 1), it would not increase as much as the private choice of skills would increase.

The optimal regional subsidy required under migration would therefore be lower than that required under the absence of migration because the private skill formation is higher under the

former (from Proposition 1). Migration would reduce the fiscal pressure on the government in its effort to internalize the externality. Migration possibilities therefore work towards achieving a desirable outcome. This is of significant policy relevance since by increasing mobility, the mix of private to public expenditures on education is affected. While the private share of human capital investments can increase with enhanced mobility amongst regions, the public share of educational financing can decline.

#### 4.3.2.4 Federal optimum

The federal optimum takes the positive externalities across regions from skill choices into account since it considers the welfare of both regions in the federation. The social optimum, denoted by $O,C$ below, would consider the effect of skill choices by region A on region B from the following problem:

$$\max_{h} \quad n\left\{W^M h + g(n\, h_m + (N-n)\bar{h}_m) - k\ (h)^2\right\} + (N-n)\left\{W^M \bar{h} + g(n\, h_m + (N-n)\bar{h}_m) - k\ (\bar{h}_m)^2\right\}$$

and obtain:

$$h^{o.c} = \frac{W^M + N\ g'(n\, h_m + (N-n)\bar{h}_m)}{2k} \tag{14}$$

When the federal government actively chooses the subsidy to be offered for skill formation in the first stage of our game, it centrally chooses the amount of financial support to provide to the residents of both regions. The problem for the central government solves:

$$\max_{s,T} \quad N\left\{W^M h_m^* + g(n\, h_m^* + (N-n)\bar{h}_m^*) - k(h_m^*)^2 + s\ h^*_m - T\right\}$$

$$\text{s.t.} \qquad N\ s\ h_m^* = N\ T \tag{15}$$

The central government will take both inter-regional and intra-regional positive externalities into consideration when centrally deciding on the skill subsidization. The subsidy choice obtained must satisfy the first-order condition:

$$\left(\frac{W^M}{2k}\right)+g(n\ h_m^*+(N-n)\bar{h}_m^*)\left(\frac{n}{2k}+\frac{(N-n)}{2k}\right)-\left(\frac{2k\ h_m^*}{2k}\right)=0 \tag{16}$$

Using (7), the central government's subsidy choice is:

$$s_m^{c*}=N\ g'(n\ h_m^*+(N-n)\bar{h}_m^*) \tag{17}$$

Thus, with non-localized externalities, subsidy amount that induces the federal social optimum (14) from individual choices (7) is given by (17).

**Proposition 3:** *All else equal, a central government that considers both regions' welfare will spend more on educational subsidies than a regional government; and the share of public expenditures in per capita total (private and public) expenditures will be greater under centralization.*

When positive externalities from skill formation are not limited to the skill-producing region, educational funds provided by regional governments will fall short of the "globally" optimal level. Since regional governments under-subsidize human capital formation from a federal welfare perspective [(9) compared to (17)], intervention by the federal government is palpably required. The fact that federal subsidies are higher than regional subsidies will be true both for the case with migration as well as the case without migration, since the need to internalize an inter-regional externality is present in both scenarios.

Interestingly, however, the analysis below (a continuation of Proposition 2) shows that increased possibilities of migration will decrease this rate of under-subsidization and will also reduce the need for government expenditures on education (federal and regional, more so for the former) as private investments in skills increase for the same level of subsidy.

Using (17) for a federal optimum, and following the same analysis as that in the proof of Proposition 2, total differentiation of the first order condition that leads to (14) gives

$$\frac{dh^{o,C}}{dW}=\frac{1}{2k-nNg''\left(nh+(N-n)\bar{h}\right)} \tag{18}$$

It is clear that, when the marginal externality diminishes in the total level of skills, the second component in the denominator of (18) is positive together with the negative sign before it. Let

$$H^C = \frac{dh^{o,C}}{dW} - \frac{dh^*}{dW}$$

This gives:

$$H^C = \frac{1}{2k - nNg''\left(nh + (N-n)\bar{h}\right)} - \frac{1}{2k} < 0 \qquad (19)$$

Thus, enhanced migration (resulting in an increase in $W$) causes the gap between the federal optimum and individual choice of skills to fall. Also, inspection of (12) and (19) shows that denominator is higher for the federal government than the regional government. Thus, the gap between the federal optimum level of human capital and the private choice of the same declines faster than the gap between the regional optimum level of human capital and the private choice of skills. This implies a declining need for federal involvement.

Allowing migration not only leads the private choice of skills to increase, but also leads the federal and regional optimum to converge even though the regional optimum is lower than the federal. Such benefits from enabling migration are explained in further detail below.

The positive externality that is not accounted for by private individuals, or regional governments when it spills across regions, implies the need for higher federal subsidies than regional subsidies. However, migration-induced enhancements in human capital investments, which in turn lead to a faster decline in the federal optimum relative to the regional optimum, would imply the need for a lower increment of federal subsidies under migration than under no migration compared to regional subsidies for the same comparison.

The explanation that follows should make the idea clearer. Based on our findings so far, as one imagines a gradual movement from an extreme no-migration scenario to a completely free-migration scenario, the subsidy required would diminish since individuals themselves would be investing more in skills for each subsidy announcement level (Proposition 1) even though the

externality that was to be to be internalized may have increased in totality with increases in $h$. With the optimally desired skill level declining too, the subsidy that would be needed to internalize the externality to the federal optimum would also decline. In other words, moving from a no-migration case to a migration case will result not only in a declining need for the subsidy (due to Proposition 1) but also a "declining" increment in the subsidy. Thus, federal intervention will imply declining fiscal efforts. With the private share of educational financing expanding with migration, the role for the federal government too, although inducing a social optimum level of education, will also decline.

Another form of federal intervention may be conditional grants. Such can be used when the economy is set on a de-centralized approach of financing skills.

#### 4.3.2.5 Conditional grants

Federal intervention to induce a socially desirable level of skill formation can either be in the form of centrally provided educational subsidies, $s^c$, provided by the federal government alone, or in the form of centrally provided grants to regional governments. When provincial or regional autonomy carries sufficient political weight, the latter provides an avenue to achieve the socially desirable result. The federal government may provide a grant to a regional government that is conditional on the subsidy it offers to individuals for their educational investments.

The budget constraint of the regional government becomes:

$$n\ s\ h_m - n\ s\ h_m x = nT \tag{19a}$$

Here $x$ (> 0) is the federal grant offered for provincial expenditures on education and this federal grant is financed by lump-sum taxes on each province. The regional government can easily transfer this tax on to its natives. Since this involves no distortion, it is not made explicit in the budget constraint above and is included in $T$ in the above equation without introducing new notation. The above budget constraint simplifies to:

$$s\ h_m(1-x) = T \tag{19b}$$

The regional government's problem becomes:

$$\max_{s,T} \quad W^m h_m^* + g(n\ h_m^* + (N-n)\bar{h}_m^*) - k(h_m^*)^2 + s\ h_m^* - T$$

s.t. $$T = s\ h_m(1-x)$$

The first order condition gives the following solution for subsidy choice:

$$s_m^* = [ng'(n\ h_m^* + (N-n)\bar{h}_m^*) + W^M x\ ]/[1-2x] \tag{20}$$

The federal government chooses $x$ from (20) so that $s^{c}{}_{m}^{*} = Ng'(n\ h_m^* + (N-n)\bar{h}_m^*)$ is induced. The required level of the conditional grant is therefore:

$$x = (N-n)g'(n\ h_m^* + (N-n)\bar{h}_m^*)/[W^M + 2Ng'(n\ h_m^* + (N-n)\bar{h}_m^*)] \tag{21}$$

This is the optimal federal grant offered to provincial governments.

**4.4 Equilibrium with Localized Human Capital Externalities:**

The objective in this section is to see the impact of mobility on private and public investments in skills when two conditions hold – individuals have positive mobility costs and - the externalities are localized implying welfare benefits from attracting more individuals to a region[31]. To study the impact of mobility on financing behaviour amongst governments when they take into account the effects of their choices on individual's migration decision, the focus in this section will be on regional governments only. Towards that end, we hold $\alpha > 0$ to see the effect of migration costs, or ease of mobility, on the subsidies offered by governments. In order for governments to take account of their subsidy choice on migration decisions of individuals, the externality function from the basic model will be restricted to a regional human capital externality that will differ based on the number of people attracted to each region. With no

[31] We assume that there are no productivity shocks.

spillovers from skills generated in another region, the externality in our setting becomes $g(n\ h)$ for region A and $g((N-n)h)$ for region B[32].

The migration equilibrium is characterized by the marginal individual, $\tilde{n}$, who is indifferent between residing in either region. Since educational decisions are made and taxes are paid prior to migration, the only components in utility determining migration decisions are the human capital externalities in each region, incomes earned in each region, and the psychic attachments to living in each region. We assume that the initial populations of regions A and B are $n_0$ and $N-n_0$ respectively, and that migration goes from region A to region B, so that the marginal migrant obtains education in region A. The migration equilibrium is then given by:

$$f(\tilde{n})\,h^A + g(\tilde{n}\ h^A) + \alpha(N-\tilde{n}) = f(N-\tilde{n})h^A + g\left((N-n_o)h^B + (n_o-\tilde{n})h^A\right) + \alpha\ \tilde{n} \qquad (24)$$

The marginal individual can earn $f(\tilde{n})h^A$ from own productivity and $g(\tilde{n}\,h^A)$ of income from educational externalities when residing in region A and $f(N-\tilde{n})h^A$ from own productivity and $g\left((N-n_o)h^B + (n_o-\tilde{n})h^A\right)$ from educational externalities if choosing to reside in region B. In equilibrium, the marginal individual's choice of mobility is determined from (24).

### 4.4.1 Private skill choice

When making their education decision, we assume that individuals are myopic with respect to the migration equilibrium. They take their region of residence as well as the population of each region as given. An individual in region A maximizes:

$$\max_h \quad \left\{f(n_0)h + g(n_0\ h) - k(h)^2 + s\ h - T + \alpha\ (N-n_0)\right\}$$

and given that the externality is ignored in the process, chooses:

$$h^* = \frac{f(n_0)+s}{2k} \qquad (25)$$

[32] As before, we will keep our focus on region A as results are not specific to the region of analysis.

An individual's choice of skills increases, as in the basic model, directly in the subsidy offered by the own region.

### 4.4.2 Government problem

The government of each region anticipates individuals' choices of human capital as well as their migration decisions in order to form its choice of subsidy, $s$. Although the government cannot directly control the migration decisions of individuals, it factors the effect of $s$ on the region's positive externality through the formation of human capital. We assume that the government maximizes the utility of individuals that are initially residing in the region and that are not expected to migrate. As the psychic benefit to each individual from residing in the region is a parameter, regional governments essentially maximize the remainder of utility for each household in the region. Taking into account the impact of $s$ in determining locational choices of individuals, the problem for the government of region A, when considering the utility of the marginal individual, is[33]:

$$\max_{s,T} \quad n_0 \left\{ f(\tilde{n})h^* + g(\tilde{n}\ h^*) - k(h^*)^2 + s\ h^* - T \right\}$$

s.t. $s\ h^* = T$, (24), (25)

Considering the budget constraint, the problem reduces to:

$$\max_{s} \quad \left\{ f(\tilde{n})h^* + g(\tilde{n}\ h^*) - k(h^*)^2 \right\}$$

s.t. (24) and (25).

The first order condition to the above problem in region A can be written as:

$$\left[ f'(\tilde{n}) + g'(\tilde{n}\,h^*) \right] h^* \frac{\delta \tilde{n}}{\delta h^*} + f(\tilde{n}) - 2k\ h^* + \tilde{n}\, g'(\tilde{n}\,h^*) = 0 \qquad (26)$$

where the following obtains from totally differentiating (24) with respect to $\tilde{n}$ and $h^*$:

$$\frac{\delta \tilde{n}}{\delta h^*} = \frac{-\left[ f(\tilde{n}) + \tilde{n}\, g'(\tilde{n}\ h^*) - f(N-\tilde{n}) - (n_0 - \tilde{n}) g' \left( (N-n_0)\, h^B + (n_0 - \tilde{n}) h^* \right) \right]}{f'(\tilde{n})h^* + h^* g'(\tilde{n}\ h^*) + f'(N-\tilde{n})h^* + g' \left( (N-n_0)\, h^B + (n_0 - \tilde{n}) h^* \right) h^* - 2\alpha} \qquad (27)$$

[33] The problem for the government in B will be likewise.

This condition derives the relationship between increases in human capital, potentially arising from increases in the subsidy, and the number of people in region *A* after migration. Clearly, the cost of migration $\alpha$ will impact this relationship. The sign of this impact will depend on the overall sign of (27). This, as well as the effect of (27) on the optimal subsidy decision by the government, will now be demonstrated.

Implicitly solving (26) for $h^*$ gives:

$$h^* = \frac{-\left[ f'(\tilde{n}) + \tilde{n}\, g'(\tilde{n} h^*) \right]}{\left( f'(\tilde{n}) + g'(\tilde{n} h^*) \right) \frac{\delta \tilde{n}}{\delta h^*} - 2k} \tag{28}$$

Considering that $h^*$ is linear in $s$ from (25) and using this relationship in (28) gives

$$s^* = \frac{-\left[ f'(\tilde{n}) + \tilde{n}\, g'(\tilde{n} h^*) \right]}{\left( f'(\tilde{n}) + g'(\tilde{n} h^*) \right) \frac{\delta \tilde{n}}{\delta s} - 1} - f\left(n_0\right) \tag{28'}$$

Equation (28') gives the implicit decision for the optimal subsidy. A relationship between equation (27) and the optimal subsidy decision is visible from the denominator of (28'). The sign of this relationship requires attention.

Differentiating (28') with respect to $\frac{\delta \tilde{n}}{\delta s}$ gives:

$$\frac{ds}{d\left( \frac{\delta \tilde{n}}{\delta s} \right)} = \frac{\left[ f'(\tilde{n}) + \tilde{n}\, g'(\tilde{n} h^*) \right]}{\left\{ \left( f'(\tilde{n}) + g'(\tilde{n} h^*) \right) \frac{\delta \tilde{n}}{\delta s} - 1 \right\}^2} \tag{29}$$

Whether (29) will be positive or negative will depend on the forces of $f'(\tilde{n})$ and $\tilde{n}\,g'(\tilde{n}\,h^*)$. The first is the negative effect from additional individuals in the region, and the second latter is the positive externality effect from increases in the level of human capital in a region. Which force outweighs the other will determine whether (29) will be increasing or decreasing with $\left(\frac{\delta \tilde{n}}{\delta s}\right)$.

The question remains about how the cost of migration will affect the subsidy choice by the government. The cost of migration, $\alpha$ affects the level of human capital, and therefore subsidy choice through equation (27).

Assuming for simplicity that the regions initially have equal population, so that the equilibrium will be symmetric, then $n_0 = \frac{N}{2}$ , $\tilde{n} = (N - \tilde{n}) = \frac{N}{2}$, and $h^A = h^B$. Equation (27) simplifies to:

$$\frac{\delta \tilde{n}}{\delta h^*} = \frac{-\tilde{n}\,g'(\tilde{n}\ h^*)}{2\left[\,f'(\tilde{n})h^* + h^* g'(\tilde{n}\ h^*)^* - \alpha\right]} \tag{30}$$

Equation (30) can be re-written using the chain rule that gives $\frac{\delta \tilde{n}}{\delta s} = \frac{\delta \tilde{n}}{\delta h^*}\frac{\delta h^*}{\delta s}$. However, equation (25) gives $\frac{\delta h^*}{\delta s} = \frac{1}{2k}$. Therefore, $\frac{\delta \tilde{n}}{\delta h^*} = 2k\frac{\delta \tilde{n}}{\delta s}$. This can be replaced in (30) to give:

$$\frac{\delta \tilde{n}}{\delta s} = \frac{-\tilde{n}\,g'(\tilde{n}\ h^*)}{4k\left[\,f'(\tilde{n})h^* + h^* g'(\tilde{n}\ h^*)^* - \alpha\right]} \tag{30'}$$

Then, any changes in the migration cost $\alpha$ can affect (30') with the following relationship:

$$\frac{d\left(\frac{\delta \tilde{n}}{\delta s}\right)}{d\alpha} = \frac{-\tilde{n}\, g'(\tilde{n}\ h^*)}{4k\left[\, f'(\tilde{n})h^* + h^* g'(\tilde{n}\ h^*)^* - \alpha \right]^2} < 0 \qquad (30'')$$

A reduction in migration costs will increase the effectiveness with which a government's subsidy choice affects the number of people in the region.

The effect of migration costs on the government's choice of subsidy using (30'') and (29) will rest on the forces of $f'(\tilde{n}) < 0$ and $\tilde{n}\, g'(\tilde{n} h^*) > 0$. The following result obtains from our analysis thus far.

**Proposition 4:** *When governments take into account the effect of their policies on the population choosing to reside in their region, an increase in the degree of mobility can lead to an increase in their subsidy offers as long as the benefits from agglomeration outweigh the costs from congestion, and vice versa.*

This result therefore depends on the following conditions:

(i) $\tilde{n}\, g'(\tilde{n} h) > \left| f'(\tilde{n}) \right|$: increases in mobility will lead to an increase in the subsidy offered as long as the agglomeration benefit outweighs the negative congestion effect from a higher population.

(ii.) $\left| f'(\tilde{n}) \right| > \tilde{n}\, g'(\tilde{n} h)$ - increases in mobility will lead to a decrease in the subsidy offered as long as the effect of a change in population on productivity exceeds the effect of the agglomeration externality.

Conditions (i) and (ii) are intuitive. When $g'(\tilde{n} h) > \left| f'(\tilde{n}) \right|$, i.e., when the positive externality effect from higher human capital exceeds the negative productivity or *congestion* effect of a higher population in the region, governments will seek to attract mobile individuals with a higher

subsidy in order to potentially gain from the agglomeration externality. In other words, when educational subsidies can be used as a tool to attract human capital, governments will compete in expenditures by increasing their educational spending. We therefore obtain a result of *expenditure competition*.

Again, when $\left|f'(\tilde{n})\right| > g'(\tilde{n}h)$, i.e., when the negative effect from additional individuals in a region outweigh the positive effect through the externality, governments would prefer less "congestion" in the region. Each government will reduce the subsidy offered, taking the policies of the other region as given. In other words, governments' optimal responses are to allow the "other region" to become a more rewarding destination for migration as a lower subsidy would induce individuals to leave the region. Therefore, when mobility is enhanced through reduced costs of migration, governments would seek to reduce subsidy offers under condition (ii).

This is an interesting result. When a localized externality poses as a tool to attract individuals to the region and when migration is less costly, we obtain a result of increased educational spending through "expenditure competition" as long as agglomeration benefits outweigh congestion costs from increases in population. When congestion costs from a larger population outweigh the agglomeration benefits, governments reduce subsidy offers in order to attract less individuals from settling into the region. Contrary to the basic model, this section develops a condition that enables higher government financing of education despite increases in mobility - one that will require agglomeration benefits from human capital accumulation to be powerful. The implication of increases in human capital financing is a resultant increase in income and welfare. The policy implications are also noteworthy and point to regional investments in agglomeration infrastructure (such as R&D/ research centers) in order to increase the human capital accumulation, income and welfare of individuals residing in those regions.

**4.5 Conclusion and Possible Extensions:**

In the above analyses we coin a simple framework that incorporates regional governments and the federal government in a model of human capital investments and migration. Our approach in this simple model answers the basic question of whether, really, migration possibilities of

individuals induce them to invest more in education, and if so, how migration affects welfare. Towards that end, we discern the consequent impact of migration possibilities on regional and federal involvements when the benefits of human capital investments cannot be restricted by individuals either to themselves or to their native regions.

While the literature explicitly claims either a brain-drain or brain-gain from migration, we are unable to support either of these results conclusively. While a movement from a no-migration scenario to a migration scenario leads individuals to invest more in human capital based on given subsidy announcements under each scenario, governments will take that into account in their announcement, and will need to offer lower subsidies to induce the optimum. Our results from the basic framework of non-localized externalities provide the intuition that as migration is increasingly allowed, individuals (for given the subsidy announcements) will earn higher incomes and increase their human capital investments while governments, taking that into account, will need to offer lower subsidies. The overall impact on welfare is difficult to ascertain. If a brain-gain is contemplated as a welfare improvement for all individuals in a region arising from increased migration by some, our analyses makes it difficult to determine whether migration indeed results in such. While migration possibilities increase individual investments in skills that will have a positive effect on welfare, subsidies offered by governments may decline with increases in mobility. In other words, enabling migration will result in a brain gain from increased individual investments and improved welfare in the short term. The implications, however, will not be clear-cut for the longer term when government subsidies are adjusted downwards despite increased private investments. We conclude in this context that a welfare gain or brain gain from migration is more difficult to achieve in the long run.

Finally, we clear our model of the indirect induction effects on government expenditures arising from mobility-induced changes in private skill investments. Rather, we focus on finding a direct relationship between mobility and government financing of education as a special case. Towards that end, we conclude that governments will increase their funding of human capital formation as long as the positive spillovers or agglomeration benefits to the region from such are sufficiently strong compared to congestion costs of population accumulation.

Another conclusion we can draw from ranking the case of non-localized externalities with that of localized externalities is that a federal government can achieve a more efficient outcome when externalities are non-localized. Welfare would be higher with non-localized externalities as everyone in the federation would benefit from the human capital investments of some individuals/regions.

There are several avenues for extending the literature on human capital migration and government financing of education. The simplistic approach adopted in this paper eliminates forces arising from migration between heterogenous regions. Migration from a poor to a richer region may have interesting implications on public financing of education when migrants that move away from the poorer region can also remit to their region of origin. This notion points to an avenue for further exploration. Furthermore, the simple model provided in this paper eliminates an equally important dimension to public financing of education – taxes on mobile factors. An interesting study could also incorporate the revenue collection dimension from, e.g., proportional income taxes on post-migration residents to draw tax and expenditure aspects of educational financing within one comprehensive framework. It would also be interesting to see the effects on expenditures towards education when income taxes are on non-residents who had benefitted from public educational financing and who, post migration, send remittances to their region of birth. Another aspect worth exploring is human capital financing by public authorities when some individuals are heterogeneous in their abilities to assimilate human capital and when a certain ability group is more mobile than the other.

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