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Meaning and Interchangeability

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Meaning and Interchangeability

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Thesis submitted to the Faculty of Graduate and Postdoctoral Studies
In partial fulfillment of the requirements
For the PhD degree in Philosophy

Department of Philosophy
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Abstract

This dissertation focuses on the notion of synonymy, or identity of meaning, applied to monadic predicates. Beginning with an examination of W. V. Quine's remarks about synonymy in "Two Dogmas of Empiricism," it then considers interchangeability *salva veritate*, interpreted in various different formal languages, as a possible criterion of synonymy. The languages considered have the structure of (i) first order logic, (ii) quantified modal logic, in particular S5, (iii) a language obtained by supplementing first order logic with doxastic operators of the form "Speaker *s* believes that ..." for each speaker of the language, and (iv) a quantified polymodal logic which contains both modal and doxastic operators. A positive result is achieved in that it is argued that synonymy can be identified with interchangeability *salva veritate* in language (iv). Furthermore, ISV in (iv) is shown to correspond to a proposition in that language affirming that two predicates cannot possibly be the predicates of divergent beliefs.

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Introduction

This dissertation focuses on the problem of identity as applied to linguistic meanings. It does so by considering the conditions under which the meanings of two distinct expressions are the same, and when they are different, restricting the problem to that of one-place predicates.

Much of the inspiration derives from Quine's well known essay "Two Dogmas of Empiricism" (hereafter Two Dogmas). Like Quine, I see the notion of synonymy, or identity of meaning to be of central importance in the larger theory of meaning. I will focus on the prospect of using interchangeability *salva veritate*, that is, interchangeability in any sentential context without disturbance to truth-value, as a possible criterion for synonymy.

Two Dogmas was not written in isolation: it is important to understand that Quine was responding primarily to Carnap's philosophy, and the logical positivism associated with the Vienna Circle. Carnap understood the rise of modern logic to be something that would change philosophy for the better; traditional metaphysics would be replaced with a theory of syntax, so as to provide an *a priori* structure in which to situate the data of experience. The *a priori* content of such a philosophical system would reduce to mere conventions regarding the use of signs.

Quine and Carnap agreed that there was no substantial fact of the matter as to which statements of a language are analytic and which are synthetic, however their interpretations of this absence of fact were very different. For Quine, the distinction was to be utterly rejected, because it is hopelessly vague at its so-called boundary:

I do not know whether the statement ‘Everything green is extended’ is analytic. Now does my indecision over this example really betray an incomplete understanding, and incomplete grasp of the “meanings” of ‘green’ and ‘extended’? I think not. The trouble is not with ‘green’ or ‘extended’ but with ‘analytic’.¹

Carnap on the other hand allowed for the distinction, but in a deflationary way. For Carnap, languages come with rules. One and the same system of symbols can give rise to entirely different languages; a language has not been specified until, along with its symbols and grammar, a list of semantic rules has been determined. Which rules may we adopt under this conception of language? Any rule that suits our purpose, for it is a matter of choice and convention. The analytic truths of a language are the result of arbitrary choices, such as the chemist’s decision to abbreviate ‘calcium’ with ‘Ca,’ or ‘lead’ with ‘Pb.’ Had our chemist chosen ‘Le’ for lead, the resulting theory of chemistry would not differ from our actual one in any interesting way. There is no fact of the matter for Carnap about which statements are the analytic in some final and absolute sense, in the same way that there is no fact of the matter that Ca is *the* correct abbreviation for calcium, beyond the historical contingency that chemists agreed to use this abbreviation.

In *Meaning and Necessity*² Carnap developed a system of modal logic which captured this conception of analyticity grounded in semantic rules. Carnap’s theory unified names, descriptions, predicates and other linguistic forms under the general concept of a

¹ Quine, W. V. 1980. *From a Logical Point of View*. 2nd Edition. Cambridge, MA: Harvard University Press. p32.

² Carnap, R. 1964. *Meaning and Necessity*. Chicago: University of Chicago Press.

designator: each designator would have both an extension and intension. The former corresponds roughly to Frege's concept of reference, and the latter to sense. A key aspect of Carnap's approach is that the intension of a linguistic form be determined purely by the semantic rules of the language. For our considerations here, it is important to note that *Meaning and Necessity* was written decades before Kripke's *Naming and Necessity*, and the popular acknowledgement of *a posteriori* necessities associated with the reception of Kripke's work. One could only speculate how Carnap might have modified his project if Kripke's ideas had occurred before *Meaning and Necessity*. My departure from Carnap's overall position is ultimately one towards metaphysics: I advocate the Aristotelian essentialism which Quine insisted would be the consequence of taking quantified modal logic seriously.

In *Meaning and Necessity*, Carnap effectively doubles each of the basic notions used in the semantics of first order logic: (1) truth, (2) (material) equivalence and (3) extension. They are related in that any two predicates are equivalent exactly on the condition that they have the same extension. The truth value of a sentence depends solely on the extension of the terms which occur in it, hence any two equivalent predicates can be interchanged in any true first order sentence, and the resulting sentence is true.

Carnap offers six such concepts: in addition to the list above, he adds (1') L-truth:

“The concept of *L-truth* is what philosophers call logical or necessary or analytic truth.”³

(2') L-equivalence:

³ *Meaning and Necessity* p7

“Two designators are said to be *equivalent* if the \equiv -sentence connecting them is true; they are said to be *L-equivalent* if this sentence is L-true.”⁴

and (3') intension:

“...we shall say of two designators of any kind that they have the same *extension* if they are equivalent, and that they have the same *intension* if they are L-equivalent.”⁵

(Carnap's uses ' \equiv ' above for the material biconditional, which is represented with a double arrow ' \leftrightarrow ' in this dissertation.)

Carnap's analysis is on the level of designators, which includes not only predicates, but also names, and other linguistic forms. However, we are interested primarily in what he said about predicates. Carnap explains the distinctions of (1'), (2'), and (3') from their first order counterparts by reference to semantic rules: a sentence is L-true when its truth follows from the semantic rules alone. Two predicates F and G are said to be equivalent when the following statement is true:

$$(x) (Fx \leftrightarrow Gx)$$

⁴ *Meaning and Necessity* p13

⁵ *Meaning and Necessity* p23

Predicates are said to be L-equivalent when their equivalence follows from the semantic rules alone: that is, when the statement expressing their equivalence is an L-truth. Intensions are associated with predicates in a one-to-many way: two predicates are said to have the same intension when they are L-equivalent.⁶

My approach involves making finer distinctions among kinds of truth: as indicated in the quote above, under the blanket concept of L-truth, Carnap collects together the theorems of logic, those statements that are true in virtue of meaning, and those that are true come what may. He did not in any way distinguish logical, necessary and analytic truth from one another. Where Carnap sees one concept, I see three.

I would distinguish the logical truths from the analytic along the same lines as Quine:

Statements which are analytic ... fall into two classes. Those of the first class, which may be called logically true are typified by:

(1) No unmarried man is married.

The relevant feature of this example is that it not merely is true as it stands, but remains true under any and all reinterpretations of 'man' and 'married'... But there is also a second class of analytic statements typified by (2) No bachelor is married.⁷

So the logical rules must answer to a criterion to which, presumably, the analytic truths do not. For example, to interpret 'bachelor' as 'mammal,' and 'married' as 'carnivorous' in

⁶ This definition of 'intension' is roughly compatible with the use of this term in Chapter 3 of this dissertation.

⁷ Quine, W. V. 1980. *From a Logical Point of View*. 2nd Edition. Cambridge, MA: Harvard University Press.

No bachelor is married

the resulting statement is false. The distinction Quine refers to in this quote relates to Tarski's semantic conception of logicity as truth in every model, insofar as the reinterpretation of predicates can be construed as allowing their extensions to vary. For example:

“John is a married bachelor”

is forbidden by the semantic rules of English, yet obviously has a model: interpret “John” with zero, “married” with the set of even numbers, and “bachelor” with the set of numbers less than two. In contrast,

“John is a bachelor and John is not a bachelor”

has no model. So there is a *prima facie* reason to distinguish logical truth from analyticity, if the latter is to be couched in terms of semantic *rules* (as opposed to *semantics* proper).

Since *Meaning and Necessity* was written decades before *Naming and Necessity*, it unclear how the Carnap of *Meaning and Necessity* would have reacted to Kripke's contention that some necessary truths are *a posteriori*. Consider, for instance, our quote above:

“The concept of *L-truth* is what philosophers call logical or necessary or analytic truth.”

To the traditionalist, it would seem obvious that a key term is missing here: surely anyone who would run together the concepts of necessity, logicity and analyticity must have the *a priori* in mind, mustn't he? From my reading of *Meaning and Necessity*, it would seem that Carnap understood the *a prioricity* of necessary truth to result from our knowledge of the semantic rules of the language we speak, and conversely, the necessity of *a priori* truth would be understood in a deflationary way: necessary truths would be the result of mere convention, or abbreviation. For example,

(L) The natural logarithm of *e* squared is two.

is indeed necessarily true, but in a particularly uninteresting way for anyone who understands the usual definitions of the terms which occur in (L). Someone who sincerely doubted the truth of (L) could not be considered a competent speaker of the language in which (L) is stated, hence the *a prioricity*. Consider the following:

(1) 'Fido is black or Fido is not black.'

(2) 'If Jack is a bachelor, then he is not married'

In either case it is sufficient to understand the statement in order to establish its truth; knowledge of (extra-linguistic) facts is not involved.⁸

This sounds something like the *a priori*, yet Carnap stops short of actually using the term, for whatever reason.

⁸ *Meaning and Necessity* p222

The concept of necessary truth need not be confined to the *a priori* in the way that analyticity must be. My reasons for making this claim are in part motivated by Kripke's arguments about natural kinds, but are also grounded in a metaphysical position abstracted from logic. Quine's repeated failures to formally establish the illegitimacy or incoherence of quantified modal logic, coupled with the relative success we have seen in recent decades in the semantics of quantified modal logic move me to adopt the position that Quine's rejection of *de re* modality is misfounded.

Unlike Carnap, I allow for a metaphysical conception of necessity, and with this, I am an advocate quantified modal logic. By 'metaphysical,' I mean that there can be necessary particular propositions. For example, I believe that there are truths of the following form:

(FD) Fido is necessarily a dog.

Names and free variables share a common syntax: given any well-formed expression in which a name occurs, the expression resulting from substituting a variable in place of the name is also well formed, and consequently so is the expression obtained by prefixing a quantifier to this result:

(XD) it is necessarily a dog

(ED) Something is such that it is necessarily a dog.

I agree with Quine that as a consequence of this view, I am committed to some form of Aristotelian essentialism just by allowing for *de re* modality, as quantified modal logic does. Because of this, I can offer no good reason why some necessary truths should not be *a posteriori*, in fact I have every reason to suspect that some of them are, simply by condoning quantified modal logic. Consider:

$$\Box Fx$$

the content of this symbol is that whatever *x* is refers to, it is necessarily an *F*. Whether names have Fregean senses is a matter over which there is considerable disagreement. Supposing that names do have meanings, however, a true statement of the form

$$Fa$$

could be analytic, in that the meaning of the predicate could be conceptually contained in the meaning of the name. This could therefore be the basis for asserting

$$\Box Fa$$

So, whether or not this could only be genuine *de re* modality depends on the extent to which Frege's duality of sense and reference applies to proper names.

However, free variables are devoid of descriptive content. A variable shows no preference towards one object or another, it can refer to one object just as easily as it could

to any other. A variable could not merely be a description in disguise, as Russell and Quine both suggested to be the case with proper names. Consider the use of the variable x in $(x)Fx$: the very generality we intend in such an expression hinges on the absence of any preference toward one object or another on the part of the variable x . Any deviation from this would disable our very mechanism of expressing generality. Insofar as the sense of an expression is the manner in which its referent is designated, variables have no senses, because to have a particular sense is to show preference toward designating some things as opposed to others. Hence, if $\Box Fx$ is true, the necessity attaches to the object directly, rather than via an overlap of senses or meanings, which is the mark of an analytic truth. So unless the predicate F abbreviated a compound expression which, if negated, led to a contradiction (e.g. letting Fx abbreviate $[Gx \vee \sim Gx]$), then Fx could not be true in virtue of meaning alone, and hence $\Box Fx$ would not be an analytic truth. Since we can rarely, if ever, know an external object completely, there is no good reason to assume that all such necessities would be knowable by mere cognitive reflection on the object. At the very least, I don't have a counter-argument to Kripke, hence I doubt the claim that all necessary truths are *a priori*.

On the other hand *a prioricity* is an essential constituent of the concept of analyticity. A truth cannot be analytic if it is *a posteriori*, because our grasp of meanings would appear not to vary with the truth of factual statements (assuming that factual statements presuppose meanings), and an analytic truth is one that can be ascertained by mere reflection on meaning, and hence independently of any particular observation or matter of fact. In contrast, an *a posteriori* truth cannot be known independently of empirical means. A theory of analyticity that would deviate from this condition would, in my view, do so much violence to Kant's original intentions, that it would simply be misleading to use the word

'analytic' at all, and that, whatever we were pursuing, we should use a new term instead. I am not saying that it is altogether incoherent to sacrifice this condition, but rather that we do so only at the peril of abandoning much of what Frege intended by sense, or meaning. Perhaps Kripke himself may lean in that direction, and in Putnam's essay "The Meaning of Meaning,"⁹ we see an even more committed form of semantic externalism, such that the connection of meaning with the limitations of a single speaker's cognition, and hence with *a priori* knowledge no longer holds.

This is where I would mark my point of departure from those theories of meaning and analyticity that have hitherto occurred in the literature. It is my intention to offer a compositionalist, non-externalist theory of meaning which would (i) maintain the Kantian condition that analytic truths be *a priori*, yet (ii) allow for Kripkean *a posteriori*, or metaphysical necessities - criteria which Carnap's method of intension and extension cannot simultaneously satisfy. For this reason, I believe it is important to reconsider the use of interchangeability as a criterion of synonymy.

Interchangeability *salva veritate* can be thought of as an interpretation of Leibniz's principle of the identity of indiscernables. Predicates should primarily be thought of in terms of the constitutive role they play in complete sentences. If two expressions differ in their meaning, then there should be some sentence that, upon substituting one expression for the other, disturbs the truth value of this sentence. That is, there should be some sentence containing the expressions by which we can discern the meaning of one from that of the other. In this sense, truth will be taken as primitive over meaning.

⁹ Putnam, H. 'The Meaning of Meaning' in Language, Mind and Knowledge. *Minnesota Studies in the Philosophy of Science* vol. VII. 1975. University of Minnesota Press: Minneapolis.

Throughout this dissertation, a compositionalist view of meaning will be assumed. That is, the meaning of a complex symbol should be determined by the meanings of its constituents coupled with the order in which they occur. Alternatively, this can be stated as follows: if the meanings of the constituent symbols are fixed, then so is the meaning of any complex symbol in which these constituents are arranged in a fixed order. A consequence of this view is that the meaning of a complete sentence, i.e., an expression that is capable of having a truth value, is determined by the meanings of the words which make up the sentence, and the order in which they occur.

Similarly, it will be assumed that the truth value of a complete sentence depends on its meaning, in combination with the relevant facts of the matter in the world. That is, provided the meanings of all its constituent expressions are determined, and their order is fixed, the truth value of a sentence is determined by the presence or absence of the fact that it asserts to be in the world. It is the converse of this dependence, however, that underlies the hope of using interchangeability *salva veritate* as an instrument to measure distinctions among meanings. That is, provided that two expressions can be interchanged with one another in any possible sentence (without the use of quotation), without disturbing its truth-value, then we should say that the meanings of the expressions are identical. Thus the outcome of this dissertation is a 'two-way,' or symmetric compositionism with respect to meaning. By this I mean that the meaning of a complex expression depends on the meanings of its constituents, and the meaning of a constituent expression can be reconstructed from the meanings of the complex symbols in which it occurs, in particular from the meanings of the complete sentences in which it occurs.

My main concern is not, however, the relation of the meaning of a symbol to the meanings of the sentences in which it occurs, but rather to their truth values. Hence this dissertation is experimental in spirit. It can be viewed as the first step in the attempt to construct a bridge between the meanings of words and truth. For it is assumed that the truth value of a sentence should be determined by the relevant presence or absence of a fact in the world together with the meanings of the symbols occurring in the sentence, and the order in which they occur. What we ask is: Does the reverse relation hold? Can the meaning of a symbol be uniquely associated with a distribution of truth values attached not to sentences but to sentence schemata with a free predicate variable? If synonymy can be identified with interchangeability *salva veritate*, then these questions are answered in the affirmative.

As Quine has pointed out, interchangeability *salva veritate* in an extensional language L is a necessary, but not a sufficient condition for the identity of meaning. (By extensional, I mean a language which has the syntactic and semantic structure of first order logic.) First order logic, however, is hardly the only formal language that has arisen in the literature. In particular, the logic of non-truth functional operators such as those of modal logic has enjoyed a large degree of formal success in the more recent years since *Two Dogmas* was written. Aside from an outright rejection of modal approaches, there is no good reason not to examine the nature of interchangeability *salva veritate* in such languages. I will therefore consider the properties of interchangeability *salva veritate* in various strengthened languages. By this I mean that the logical connectives and quantification will be maintained, while the language is supplemented with various non-truth functional operators. The sequence of the argument is as follows.

In Chapter 1, I will discuss Quine's Two Dogmas and the connection of meaning with the problem of analyticity. This will lead into the next chapter by concluding that synonymy, or the identity of meaning is the main problem to be addressed. I will also discuss the failures of interchangeability *salva veritate* to provide us with sufficient criteria for the identity of meaning in an extensional language. Thus, I will define interchangeability *salva veritate* in formal terms which can be adapted to any particular language. Chapter 1 will include a proof of the fact that coextensive predicates are interchangeable *salva veritate*, using this formal definition. The proof will proceed by mathematical induction on the complexity of formulas, and will serve as a platform on which subsequent proofs will be based, in particular that occurring in Chapter 3.

In the Chapter 2, I will discuss the motivating factors involved in this dissertation, and will lay out the basic relations connecting the theory of meaning with the problem of synonymy, or identity of meaning. It will be suggested that the problem of meaning raised by Quine is a manifestation of the traditional debate on universals between the schools of nominalism and realism. I will propose that, provided all parties can agree on a criterion for synonymy, then we can use the theory of sets to establish a common currency of discourse among nominalist and realist alike. Synonymy is an equivalence relation: it is reflexive, symmetric and transitive. Taking inspiration from the fundamental homomorphism theorem of group theory, I will propose that the system of meanings of the expressions of a language can be thought of as the quotient structure (i.e., set of equivalence classes) resulting from the equivalence relation of synonymy. In particular, the meaning of a predicate is to be identified with its equivalence class of predicates under the relation of synonymy. In this sense, when the realist speaks of abstract meanings in the robust sense, the nominalist can

simply translate such discourse into talk of inductively definable equivalence classes of expressions.

In Chapter 3, I will take up a point raised in section 3 of Quine's Two Dogmas, namely that interchangeability *salva veritate* is a sufficient condition for identity of meaning provided that we supplement the extensional language with intensional adverbs such as "necessarily" and "possibly." Quine warns us that to do so would invoke a vicious circle, for he equates necessity with analyticity, or truth in virtue of meaning, the very concept which is to be analysed. In Chapter 1, I will give reasons for departing from the view that necessity and analyticity name the same concept, and in Chapter 3 I will go on to consider the nature of interchangeability *salva veritate* in quantified modal logic. In the main I will focus on languages with the semantic structure of S5. The approach will involve the application of the concept of a 'possible world,' in the usual fashion. It will be demonstrated that interchangeability *salva veritate* in such a language can be equated with necessary coextension. The proof of this fact will resemble that given in Chapter 1, and will similarly proceed by mathematical induction on the complexity of formulas. I will then consider various arguments suggesting that heteronymous predicates are sometimes necessarily coextensive, drawing largely on the theory of the modal properties of natural kinds laid out by Saul Kripke. Interchangeability in a modal language does reflect certain key aspects of synonymy, in particular the essential connections which synonyms must have, however it does not reflect the epistemic, or doxastic elements of meaning and analyticity which are normally associated with *a priori* knowledge.

In Chapter 4, I will construct a language by supplementing the extensional language with another set of intensional operators of the form "Speaker S believes that," for every

speaker of the language. I will then go on to examine the implications of interchangeability *salva veritate* in such a doxastic language. I will define a function which represents the attitudes that a speaker has towards the sentences of the language, and it will be demonstrated that two predicates are interchangeable *salva veritate* in this doxastic language provided that they are coextensive, and that the output of this function remains stable upon their mutual substitution. It will be proposed that human speakers are capable of having determinate beliefs about sentences of a limited degree of complexity, and based on this proposal, a second proof will be given which shows how interchangeability *salva veritate* can be identified with a statement of the doxastic object language. The inclusion of doxastic operators ensures that the connection of meaning and analyticity with *a priori* knowledge is better represented by interchangeability in this language than in that of Chapter 3: however the main problem in Chapter 4 will involve what Quine called 'collateral information.' By collateral information, I mean widespread beliefs regarding accidental, or contingent facts. Through the application of the concept of collateral information, counterexamples will be generated which suggest that interchangeability *salva veritate* in this doxastic language is not a sufficient condition for identity of meaning.

In Chapter 5, I will go on to consider the combination of the ideas of Chapters 3 and 4. Firstly, I will consider as a prospective criterion of synonymy the condition that predicates be interchangeable both in the modal language of Chapter 3 and also in the doxastic language of Chapter 4. This consideration will be rejected based on counterexamples constructed from Kripkean natural kinds. Thus, in Chapter 5, I will construct a language by supplementing the extensional language of Chapter 1 with both kinds of intensional operators examined in Chapters 3 and 4. In this polymodal language,

doxastic operators can occur within the scope of modal operators, and conversely, modal operators can occur within the scope of doxastic operators. By closely examining certain specific forms of sentence in this polymodal language, it will be argued that interchangeability *salva veritate* in this language is a sufficient condition for the synonymy of predicates. This conclusion will be reflective of the Leibnizian idea that *a priori* knowledge of the analytic sort involves compulsory beliefs.

This thesis will therefore conclude with a positive result, namely that synonymy can be understood as interchangeability *salva veritate* in the appropriate language, however this language must include several kinds of non-truth functional operators, and suitable axioms for their interpretation must be chosen. Modal and doxastic contexts, when combined together in the right way, provide enough of a space of possible facts that meanings can be directly associated with truth values.

Chapter 1

It can be argued that the birth of the analytic tradition was marked by Frege's departure from the Kantian view that the truths of arithmetic are synthetic *a priori*. Kant offered at least two ways of conceiving of the analytic: (1) analytic truths are those in which the subject concept is contained in the predicate concept, and (2) analytic truths are those truths that follow from the principle of non-contradiction alone. Appealing to the latter definition, and using the formal machinery pioneered in the *Begriffsschrift*, Frege set out to give a proof that the elementary truths of number theory could be established using the logical canon derived solely from the principle of non-contradiction with no further assumptions. Borrowing from Cantor's concept of cardinality¹, Frege offered a non-circular definition of number. This machinery evolved into what we now call first order predicate logic.

One suggestion that emerged from Frege's writing was to make the idea of analyticity precise by defining it in terms of logicality and meaning. An analytic truth is one that is synonymous with a logical theorem. Frege was also a compositionalist. He believed that the meaning of a compound symbol was determined by the meanings and arrangement of its constituent symbols, in the way that the image of a function is determined by the value of its argument. An important consequence of this view is that the substitution of synonyms for synonyms in any one part of a compound symbol results in a synonymous compound symbol. With this, a criterion of analyticity emerges, namely that a statement is analytically true when it can be converted to a logical theorem by the substitution of synonyms for synonyms on component parts of the statement. In this way, the broader category of analytic

¹ Some say that the origin of this characterization of number is attributable to Hume, before Cantor.

truth could be viewed as the expansion of the category of logical truth by way of the relation of synonymy.

The exposition of this canon of logical truth has enjoyed a prosperous history, and has been clarified well beyond just about any other comparable concept in philosophy. Unfortunately, the same cannot be said for the ideas of meaning and synonymy. A particularly effective assault that has been waged on the above conception of analyticity, and the related concepts of meaning and synonymy is to be found in the writings of Quine. In "Two Dogmas of Empiricism"² (hereafter Two Dogmas) Quine explicitly focuses on this way of defining analyticity.

The first problem that Quine points out is that the idea of synonymy suffers from the same lack of clarity as the idea of 'conceptual containment,' used in Kant's definition above. Invoking a Leibnizian theme, Quine suggests that we examine the prospect of using interchangeability *salva veritate* (hereafter ISV) as a criterion of synonymy to remedy this lack of clarity. Two terms are ISV when the substitution of one term for the other in any true statement results in a true statement.

Note that to quantify as we do when we say 'any true statement,' we must first specify a language (viewed as a set of sentences or expressions) over which we are quantifying. Hence ISV is always relative to a particular language. From a logical point of view, the simplest relevant language would be an extensional language; i.e., a language whose sentences readily admit translation into the symbolism of first order predicate logic. The problem that Quine indicates here is then that two predicates in such a language can satisfy ISV while differing in their meaning. As it happens, any two predicates that are merely coextensive (that is, true of exactly the same entities) are ISV. To the best of my

² Quine, W. V. 1980. *From a Logical Point of View*. 2nd Edition. Cambridge: Harvard University Press.

knowledge, Quine does not give a formal proof of this fact. From one point of view, it is evident from the model theoretic semantics that is standard for first order languages. The semantic datum with which a predicate symbol is associated is an extension, so the truth value of a closed sentence is determined by the extensions of the names and predicates which occur in it. Substituting one coextensive predicate for another therefore should not change the truth value of the sentence.

For our purposes, however, we will step back from the semantic point of view, and think of ISV in inferential terms. To say that ISV is equivalent to $(x) (Fx \leftrightarrow Gx)$ is to say that from a single statement of the language L , infinitely many consequences, also statements of L , can be inferred. On the face of it, it seems as though ISV must be stated in a metalanguage in that it requires variables which range over arbitrary sentences of L to be stated. From the perspective of the object language, ISV can be thought of as an infinite conjunction of biconditionals.

A proof that $(x) (Fx \leftrightarrow Gx)$ is equivalent to ISV is given here for two reasons. Firstly, it demonstrates how, through exploiting the recursive structure of the language, infinitely many consequences can be shown to be derivable from a single premise. This is to be done in terms of the symbols alone, without reference to semantic concepts such as extensions. Secondly, and more importantly, it will trace out a method for proving similar results about ISV in Chapters 3 and 4 when we consider languages richer in structure than first order logic, which contain non-truth functional operators. The proof presented below will be a sort of template for the proofs that follows in subsequent chapters.

We can distinguish between two strengths of ISV. We define $\phi[F/G : i]$ to be the result of rewriting ϕ replacing the i^{th} occurrence of G in ϕ with F . If the number of G 's in ϕ

is less than i , then $\varphi[F/G : i] = \varphi$. We define $\varphi[F/G] =$ the result of rewriting φ replacing every occurrence of G in φ with F .

(Defⁿ) Weak $ISV^L(F,G) =$ For all $\varphi \in L$, $\varphi \leftrightarrow \varphi[F/G]$

(Defⁿ) Strong $ISV^L(F,G) =$ For all $\varphi \in L$, For all $i \in \mathbf{N}$, $\varphi \leftrightarrow \varphi[F/G : i]$

In the ‘closed curve’ argument of section 3 of Two Dogmas, Quine makes use of strong ISV.

Proposition 1.01 Strong ISV entails Weak ISV

Proof:

Assume Strong ISV. Suppose that there are n occurrences of F in φ .

Let $\varphi^0 = \varphi$ and let $\varphi^{i+1} = \varphi^i[F/G : 1]$. Therefore $\varphi^n = \varphi[F/G]$

And by our assumption

$$\begin{array}{l} \varphi = \varphi^0 \leftrightarrow \varphi^1 \\ \dots \\ \varphi^i \leftrightarrow \varphi^{i+1} \\ \dots \\ \varphi^{n-1} \leftrightarrow \varphi^n = \varphi[F/G] \end{array}$$

hence by the transitivity of ‘if and only if,’ $\varphi \leftrightarrow \varphi[F/G]$ QED.

Strong ISV is defined in terms of substitution on one occurrence at a time, however any number of substitutions can be made if strong ISV holds. For example, suppose F and G are ISV, and consider the following sentence which contains three occurrences of G :

$$\begin{aligned}
& (w) (x) (y) (Gw \wedge Gx \rightarrow (Hy \leftrightarrow (z) (Gz \wedge Jz)))[F/G : 1] \\
& = (w) (x) (y) (Fw \wedge Gx \rightarrow (Hy \leftrightarrow (z) (Gz \wedge Jz)))
\end{aligned}$$

And

$$\begin{aligned}
& (w) (x) (y) (Fw \wedge Gx \rightarrow (Hy \leftrightarrow (z) (Gz \wedge Jz)))[F/G : 2] \\
& = (w) (x) (y) (Fw \wedge Gx \rightarrow (Hy \leftrightarrow (z) (Fz \wedge Jz)))
\end{aligned}$$

So with repeated applications of operators of the form $[F/G : i]$, we can choose exactly which Gs to convert to Fs.

We restrict our attention to those first order formulas that involve only the universal quantifier, conjunction and negation. Disjunction, implication, the biconditional and existential quantification are defined in the usual way:

$$\begin{aligned}
\phi \vee \psi & \stackrel{\text{def}}{=} \sim(\sim\phi \wedge \sim\psi) \\
\phi \rightarrow \psi & \stackrel{\text{def}}{=} \sim(\phi \wedge \sim\psi) \\
\phi \leftrightarrow \psi & \stackrel{\text{def}}{=} (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi) \\
(\text{Ex}) \phi & \stackrel{\text{def}}{=} \sim(x) \sim\phi
\end{aligned}$$

We define the *degree* of a formula in terms of the recursive structure of L as follows:

$\text{deg}(\varphi) = 1$ if φ is atomic, i.e., $\varphi = Fx$ or $\varphi = Fa$ (where 'F' is a predicate,

'x' is a variable and 'a' is a name)

$\text{deg}(\varphi \wedge \psi) = 1 + \max\{\text{deg}(\varphi), \text{deg}(\psi)\}$

$\text{deg}(\sim\varphi) = 1 + \text{deg}(\varphi)$

$\text{deg}((x)\varphi) = 1 + \text{deg}(\varphi)$

In other words, $\text{deg}(\varphi)$ = the number of nodes on the longest branch of the parsing tree of φ . Note that if ψx contains the free variable x , then $\text{deg}(\psi x) = \text{deg}(\psi u)$ for any variable or name u .

Proposition 1.02 $(x)(Fx \leftrightarrow Gx)$ if and only if Strong $\text{ISV}^L(F, G)$

Proof: To show that $\text{ISV}^L(F, G)$ implies $(x)(Fx \leftrightarrow Gx)$, we first note that $(x)(Gx \leftrightarrow Gx)$ is a theorem, and therefore is true. Hence by strong ISV,

$$((x)(Gx \leftrightarrow Gx))[F/G : 1] = (x)(Fx \leftrightarrow Gx)$$

is true. To show that $(x)(Fx \leftrightarrow Gx)$ implies Strong $\text{ISV}^L(F, G)$ we proceed by induction on $\text{deg}(\varphi)$. We assume $(x)(Fx \leftrightarrow Gx)$

Base case: let $\text{deg}(\varphi) = 1$. Therefore φ consists of a predicate followed by a variable or name; either the predicate is G or it is not G ; hence we consider 4 cases: Gx, Ga, Hx, Ha

Now for every i , $Hx[F/G : i] = Hx$, and $Ha = Ha[F/G : i]$, and if $i > 1$, then $Gx[F/G : i] = Gx$ and $Ga[F/G : i] = Ga$, so the desired result follows trivially, since $\varphi \leftrightarrow \varphi$ is true for any formula φ . Hence, we consider Gx and Ga where $i = 1$. By $(x) (Fx \leftrightarrow Gx)$, universal instantiation gives $Fx \leftrightarrow Gx$ and $Fa \leftrightarrow Ga$, hence $\varphi \leftrightarrow \varphi[F/G : i]$ for every positive integer i .

Inductive Hypothesis: suppose that for every φ , $\text{deg}(\varphi) \leq n$ implies that for every positive integer k ,

$$\varphi \leftrightarrow \varphi[F/G : k]$$

Consider φ such that $\text{deg}(\varphi) = n + 1$, and choose any particular $i \in \mathbf{N}$.

Case 1: $\varphi = \sim\psi$

First, we note that for every formula χ , since the number of occurrences of G in χ equals the number of occurrences of G in $\sim\chi$,

$$[\text{SUB1}] \quad \sim(\chi[F/G : i]) = (\sim\chi)[F/G : i]$$

To show that $\varphi \rightarrow \varphi[F/G : i]$, assume φ , i.e., assume $\sim\psi$. Since $\text{deg}(\psi) = n$, our inductive hypothesis entails that

$$\text{For every } k \in \mathbf{N}, \psi \leftrightarrow \psi[F/G : k]$$

In particular, $\psi \leftrightarrow \psi[F/G : i]$. Hence, by modus tollens we get $\sim(\psi[F/G : i])$ and by SUB1 we get $(\sim\psi)[F/G : i] = \phi[F/G : i]$, therefore $\phi \rightarrow \phi[F/G : i]$.

To show the converse, assume $\phi[F/G : i]$, i.e., assume that $(\sim\psi)[F/G : i] = \sim(\psi[F/G : i])$. Again by our inductive hypothesis, $\psi \leftrightarrow \psi[F/G : i]$, and by modus tollens, we get $\sim\psi = \phi$, hence $\phi[F/G : i] \rightarrow \phi$. Therefore $\phi \leftrightarrow \phi[F/G : i]$.

Case 2: $\phi = \psi \wedge \chi$

Let j be the number of occurrences of G in ψ . To show that $\phi \rightarrow \phi[F/G : i]$, assume ϕ , that is, assume $\psi \wedge \chi$. By simplification, we have ψ , and similarly we have χ . Notice $\deg(\psi) \leq n$ and $\deg(\chi) \leq n$. If $i \leq j$ then $\phi[F/G : i] = \psi[F/G : i] \wedge \chi$. Since $\deg(\psi) \leq n$, by modus ponens and our inductive hypothesis we have $\psi[F/G : i]$, and therefore we have $\psi[F/G : i] \wedge \chi = \phi[F/G : i]$. If $j < i$, then $\phi[F/G : i] = \psi \wedge \chi[F/G : i - j]$. Similarly, since $\deg(\chi) \leq n$, by modus ponens and our inductive hypothesis we have $\chi[F/G : i - j]$, hence we have $\psi \wedge \chi[F/G : i - j] = \phi[F/G : i]$. Therefore $\phi \rightarrow \phi[F/G : i]$. To show the converse, assume $\phi[F/G : i] = (\psi \wedge \chi)[F/G : i]$. If $i \leq j$, then $(\psi \wedge \chi)[F/G : i] = \psi[F/G : i] \wedge \chi$. Therefore by simplification, we have $\psi[F/G : i]$, and similarly we have χ . Since $\deg(\psi) \leq n$, by modus ponens and our inductive hypothesis we have ψ , and therefore we have $\psi \wedge \chi = \phi$. Hence $\phi[F/G : i] \rightarrow \phi$.

If $j < i$ then $(\psi \wedge \chi)[F/G : i] = \psi \wedge \chi[F/G : i - j]$, hence we have ψ and $\chi[F/G : i - j]$ by simplification. Since $\deg(\chi) \leq n$, by modus ponens and our inductive hypothesis, we have χ , and therefore we have $\psi \wedge \chi = \phi$. Hence $\phi[F/G : i] \rightarrow \phi$. Therefore $\phi \leftrightarrow \phi[F/G : i]$.

Case 3: $\phi = (\forall x) \psi x$ (where the variable x occurs free in ψx)

First we note that since the number of occurrences of G in $(\forall x) \psi x$ equals the number of occurrences of G in ψx ,

$$[\text{SUB2}] \quad ((\forall x) \psi x)[F/G : i] = (\forall x) \psi x[F/G : i]$$

To show that $\phi \rightarrow \phi[F/G : i]$, assume ϕ , that is, assume $(\forall x) \psi x$. We use a reductio approach; assume $\sim\phi[F/G : i]$, i.e., assume that $\sim((\forall x) \psi x)[F/G : i] = \sim(\forall x) \psi x[F/G : i]$ by SUB1 and SUB2. Therefore, by quantifier negation and existential instantiation, there exists u such that $\sim\psi u[F/G : i]$. Since $\text{deg}(\psi u) = n$, by our inductive hypothesis and modus tollens, we have $\sim\psi u$, however, by our conditional assumption, ψu . Since the assumption that $\sim\phi[F/G : i]$ leads to a contradiction, we have $\phi[F/G : i]$. To show the converse, assume that $\phi[F/G : i] = (\forall x) \psi x[F/G : i]$ by SUB2. Suppose $\sim\phi$, i.e., suppose that $\sim(\forall x) \psi x$. Therefore, there exists u such that $\sim\psi u$. Since $\text{deg}(\psi u) = n$, by our inductive hypothesis and modus tollens, $\sim\psi u[F/G : i]$, however by our conditional assumption, $\psi u[F/G : i]$. Hence, the assumption that $\sim\phi$ leads to a contradiction, and we conclude that ϕ . Therefore $\phi \leftrightarrow \phi[F/G : i]$. QED.

So to say that two predicates are ISV in an extensional language is to say no more and no less than to say that they are coextensive. If we were to turn the tables, and hold to ISV, insisting that it defines something *like* meaning here, then the class of possible predicate meanings would be identified with the powerset of the universe of individuals. But, as Quine argues, this will hardly do. Using the natural language example of the clearly

heteronymous predicates 'creature with a heart' and 'creature with a kidney,' he illustrates that mere coextensiveness in no way ensures the desired synonymy of predicates, provided the language over which ISV is quantified has the structure of first order logic.

If Leibniz's criterion is to help us identify the conditions of synonymy, then we must consider ISV in a richer language. Notice that both strong and weak ISV become more selective as we increase the number and types of sentences over which they are quantified. After observing that coextensive predicates are ISV in a first order language, Quine's next suggestion was to consider supplementing our first order language with the modal adverb 'necessarily.' Quine does indicate that ISV in such a language would ensure synonymy. This suggestion was not to be taken seriously, however, as it is merely the first step in a *reductio* argument. To take this approach, according to Quine, would involve a sort of vicious circularity, for Quine did not distinguish necessity from analyticity. On this view, the attempt to clarify synonymy in terms of ISV in a modal language would be to take as primitive the very concept which is to be analyzed, for 'Necessarily p ' and ' p is analytic' effectively say the same thing. Let us examine Quine's argument in section 3 of Two Dogmas.

The 'closed curve' argument in section 3 of Two Dogmas is intended to demonstrate the circularity of using ISV in a modal language to define synonymy and analyticity. The argument involves an interplay of three sentences:

- (3) All and only bachelors are unmarried men.
- (4) Necessarily all and only bachelors are bachelors.
- (5) Necessarily all and only bachelors are unmarried men.

Now, Quine notes that to say

the meaning of 'bachelor' = the meaning of 'unmarried man'

is to say 'no more nor less than' that (3) is analytic, for

(B) All and only bachelors are bachelors.

is a logical truth, and is therefore analytic. So if we suppose that

(I) the meaning of 'bachelor' = the meaning of 'unmarried man'

then we can substitute 'unmarried man' for the second occurrence of 'bachelor' in (B) to obtain (3), hence (3) is analytic on this supposition. (Recall that ISV is a necessary, although not a sufficient condition for synonymy). Therefore to say that

(I) the meaning of 'bachelor' = the meaning of 'unmarried man'

is to say no *less* than

“(3) All and only bachelors are unmarried men” is analytic,

in the sense that (3) is a consequence of the meaning identity. On the other hand, 'bachelor' and 'unmarried man' are the only predicates in (3); the only logical truths that (3) could be converted into through interchange of predicates³, one at a time, would be either (B)

(B) All and only bachelors are bachelors.

or

(M) All and only unmarried men are unmarried men.

If analyticity is to be logicality expanded along the lines of synonymy, then we can only allow that (3) be analytic if we are prepared to link (3) with either (B) or (M). Therefore, we would have to accept (I), that the meaning of "bachelor" is the meaning of "unmarried man," since (B) and (M) are the only logical truths that

(3) All and only bachelors are unmarried men.

could be converted to by interchange of one predicate at a time. So to assert that

(I) the meaning of 'bachelor' = the meaning of 'unmarried man'

is to say no *more* than that (3) is analytic, in the sense that the meaning identity is a consequence of (3). Hence in general, we can assert that

the meaning of F = the meaning of G

³ For the sake of argument, we treat 'unmarried man' as a simple predicate.

is equivalent to

"All and only *F*s are *G*s." is analytic.

So there is a close connection between the meaning of a predicate and the analytic statements that ascribe conditions to its extension.

Unfortunately, for the present purpose this insight is of little help. It would only be useful in a context in which we had some independent characterization of analyticity from which we wished to derive the relation of synonymy. Following Quine's argument, however, we are considering synonymy to be primitive, and analyticity as something to be derived:

(P) We treat synonymy (i.e. the identity of meaning) as primitive over analyticity, in the sense that it is to be characterized in terms independent of (i.e., that do not presuppose, or make reference to) analyticity.

Quine suggests that we consider the intensional adverb 'Necessarily' to be so narrowly construed so as only to apply to analytic statements:

(*) For any sentence *p* if 'Necessarily *p*' is true, then *p* is analytic.

According to (*),

(4) Necessarily all and only bachelors are bachelors.

is true since

(B) All and only bachelors are bachelors.

is a logical, and hence analytic truth. So if ISV holds of 'bachelor' and 'unmarried man,' (5) must like (4) be true. Hence

the meaning of 'bachelor' = the meaning of 'unmarried man'

entails the truth of

(5) Necessarily all and only bachelors are unmarried men.

via the evident truth of

(4) Necessarily all and only bachelors are bachelors.

Quine concludes by pointing out that, due to (*), (5) is equivalent to saying that

(3) All and only bachelors are unmarried men.

is analytic, and hence (P), the assumption that synonymy is primitive over analyticity must be rejected, on grounds of circularity.

At the beginning of this chapter, a reference was made to Frege's view that the truths of arithmetic are analytic. Mathematics, in particular arithmetic is usually considered the example of necessity *par excellence*. Recall Descartes' eternal truths, Hume's relations of ideas, and Kant's synthetic *a priori* - all use mathematics and in particular arithmetic when the subject of necessity is to be raised. In this sense, Frege can be thought of as a Kantian revisionist, in that he set out to show that arithmetic is analytic, not synthetic. So there seems to be an ongoing connection between necessity and arithmetic. A natural development to be expected in the Fregean tradition would be the generalization that the only logically coherent form of necessity is analyticity, and perhaps this explains Quine's adoption of the premise (*). The view that analyticity is the only kind of necessity was dominant among Quine's generation. In particular, Carnap⁴ did much to clarify and formalize this position.

There are, however, other senses in which the term 'necessity' can be understood. Recall Hume's analysis of causation, which is to be explained as necessary succession. Necessity is not the only choice of a modal primitive, for 'necessary' can be defined as 'not possibly not,' and hence be dealt with as a quantifier over possible worlds. David Lewis offers a realistic theory of possible worlds, and has done much to clarify the semantics of counterfactual conditionals, whose truth conditions are presumably grounded in what the world is like as opposed to mere meanings. Consider

⁴ Carnap, R. 1964. *Meaning and Necessity*. Chicago: University of Chicago Press.

(BG) If Bill Gates had never gone into software then he would be richer than he is now.

The problem is, our intuition dictates that a sentence like (BG) should be false, however it is true according to the definition of implication above, by virtue of the falsehood of its antecedent. To me, these so-called paradoxes of material implication indicate a serious defect in first order logic with respect to the representation of human language. To the best of my knowledge, possible worlds provide the best method for explaining the conditions of truth for such false counterfactuals with false antecedents. This is reason to consider languages with a richer structure than that of first order logic, in particular, with non-truth functional operators.

When Quine wrote *Two Dogmas*, little was known about the formalization of modal logic. The explanatory power of possible worlds, combined with the degree of success witnessed in the last half century in the project of formalizing the logic of modality are reasons enough to question Quine's identification of necessity with analyticity. Modal languages have existed in the literature for some time. There is no good reason not to examine the nature of ISV in a modal language, just as there is reason to question the view that analyticity is the only kind of necessity, on which Quine's 'closed curve' argument rests. Proposition (*) seems to support his argument only by making the conclusion an assumption at the outset.

In the ensuing chapters, we will investigate the properties of ISV in various languages. These languages will have a richer structure than that of first order logic, and will include the intensional, or non-truth functional operators. In particular, we will consider

two types of non-truth functional operators, namely the alethic modalities, and contexts of belief.

Chapter 2

We have seen that ISV in a language with the semantic structure of first order predicate logic falls short of the intuitive idea of synonymy. When two predicates in an extensional language are coextensive they are ISV, but clearly there is an abundance of examples taken from the languages of science, and from ordinary life, of intuitive heteronyms that happen to be actually coextensive. That Quine takes this as his point of departure from the doctrine of meaning indicates a nominalist spirit. Given a predicate F, the class of Fs can be thought of as really nothing more than the individual Fs taken plurally. There is nothing more to the class of Fs than the particular individual Fs. For the nominalist, the meaning of a predicate F is nothing more than the individual Fs taken plurally, and to posit something over and above the individual Fs, a meaning or metaphysical property, would be to take up a realist line of inquiry.

I would agree with Quine on this point if the only formal languages available in the literature all had the semantics of first order logic. However, this is not the case. For example, simple declarative sentences of natural language such as

"Some of your soldiers have surrounded the building"

cannot be translated into first order logic without the introduction of plural quantification along the lines suggested by Boolos (otherwise such a sentence requires second order quantifiers). In *Two Dogmas*, Quine himself suggests that the language of ISV could be boosted with the introduction of modal adverbs, but then warns us not to do so for other

reasons. Even if we did heed his warning, considering ISV in a modal language suggests looking at ISV in other intensional contexts, the most salient of which would be belief, or perhaps knowledge. Quine didn't warn us against considering ISV in doxastic or epistemic languages, however he did warn against quantifying into beliefs. Perhaps, in light of his naturalized epistemology and physicalistic outlook on the human mind, he thought that such languages are in principle reducible to a first order language because the correct analysis of belief and knowledge should be in terms of physiology and physics. So we should not give up on ISV yet until we have examined it in languages with a semantics richer than that of first order logic.

Before examining the prospect of defining meaning in terms of ISV, I should say a few words about the question of why philosophers should care about the conditions of identity for predicate meanings in the first place. One reason is the Kantian idea of analyticity, which has been among the focal subjects of interest throughout the Analytic tradition. If we are to speak of an analytic/synthetic distinction, we must be prepared to say of a given true sentence whether it is true in virtue of its meaning. If there is vagueness in the identity and distinction of the meanings of terms, then the analytic/synthetic distinction is vague too. To say that one true sentence is synthetic, and that another is analytic minimally presupposes that we can say whether their meanings are distinct or identical. Hence, by clarifying the problem of identity of meaning, one clarifies the category of analytic truths. If we adopt a compositionalist view of sentence meaning, then the problem of analytic sentences reduces to the synonymy of words and other sub-sentential units.

But there are other issues concerning the problem of meaning identity, whether we use meaning to clarify the analytic/synthetic distinction or not. The philosophical traditions

of realism and nominalism are reflected within this debate. Predicates combine with singular terms to form the units of assertion, those which assert that a specific individual possesses a certain property. Hence, predicates are closely associated with properties.

Certain privileged, or natural properties have traditionally been called universals. One can think of the meaning of a predicate which attributes such a property to its instance as the property, or universal itself. Hence, Quine's meaning skepticism can be viewed as a variation of nominalism. It is a compromised nominalism, however Quine retains one minimal stock of all purpose universals, the standard constructions of the theory of sets, applied to the individuals of the world. Given a predicate F , the set of individual things that are F , combined with the usual operations of set theory provide us with enough structure to replace the meaning of the predicate, or the universal which they instantiate. For Quine, coextensiveness is the only respectable condition of identity for predicates, apart from the trivial relation of orthographic identity, i.e., when the identity relation is on strings of symbols.

There are grounds for thinking that identity is a metaphysical primitive, and as such it cannot be properly analysed. Definitions of identity invariably involve a circularity. Take for instance the traditional definition of identity as the relation which every object bears to itself and nothing else. This statement may have the appearance of an analysis, however it is not because it can only be stated using the word 'itself,' which is simply another way of saying 'identity.' Similarly, one might seek to analyse identity at the level of the symbolism, where it can be defined by means of the concept of co-designation, i.e., when two names or variables designate one and the same object. Any statement of the form

$$A = B$$

is true provided that the names, or variables A and B are co-designative. However the concept of co-designation depends on that of identity, since terms co-designate if and only if they designate *the same thing*. The concept of co-designation cannot be used to define identity because it presupposes identity. The metaphysical relation of identity therefore cannot be reduced to the linguistic property of co-designation. I suspect that a similar problem will arise with any attempted reduction of identity to other concepts; no matter how one tries to define identity, one ends up invoking it⁵.

This primitive character of identity is coupled to the notion of essential characteristics, a connection reflected for instance in Leibniz's principle of the identity of indiscernibles: two things that share all the same characteristics are identical; if they differ in a characteristic, then they are not identical. So the conditions of identity are simultaneously the defining characteristics of a thing: given a particular object O, the conditions under which one can say that something is or is not O reveals the essential characteristics of *what it is to be* O. So identity is connected with being, another metaphysical primitive. Statements expressing necessary and sufficient conditions of identity with a particular object reflect the essential features of that object.

These considerations are important when we consider "definitions" of identity in various domains. To take an example from natural science, in the classical theory of physical objects the conditions of identity and distinction of objects are clear cut; to be

⁵ It is possible to define identity in a non-circular way, provided we avail ourselves of second order quantifiers: $x = y$.if and only if. $(X) (x \in X$ if and only if $y \in X)$. Such a definition is unavailable however unless one is prepared to make the metaphysically expensive move of counting along with the individuals in one's ontology, all sets of such individuals.

different is to have distinct locations at the same time; to be identical is to have the same position at every time, possibly augmented with the assumption that all motion is continuous. This reveals the essence of the classical idea of corporeal body. Some of the more counterintuitive aspects of the Copenhagen school of quantum physics, such as non-locality, or twin particles are the direct outcome of the challenge this new physics presents to the old criterion of corporeal identity, thereby challenging the essential features of this traditional idea of body. *Location* is the essence of the classical, or Cartesian conception of *body*, and this is displayed in the conditions for identity of classical bodies. Notice, that the nature of identity is not qualified here, but rather the concept of body itself.

Another example of a criterion of identity is the principle of extensionality in the theory of sets.

(EXT) $A = B$.if and only if. For all x , $x \in A$ if and only if $x \in B$ ⁶

When we have a biconditional statement, flanked on the left with an identity statement involving variables limited to a certain type of object, and some criterion on the right, what we have is *not* a definition of identity. The identity relation is preordained and fixed, and by virtue of the rigidity of its meaning is incapable of definition. Such biconditional statements instead show something essential about the *type* of object the variables A and B are to range over.

To the best of my knowledge, (EXT) is a universally accepted principle in any formalization of set theory. Through the rigidity of the meaning of the identity sign, (EXT) qualifies not identity, but *set-hood*. The essential nature of set-hood is clarified with (EXT)

⁶ It is enough for the variable x to range over $A \cup B$.

in terms of the most fundamental and elementary property that sets are involved in; membership. We can say exactly where one set begins, and another ends. For example, (EXT) shows us that ordering of elements is to be disregarded when it comes to set-hood:

$$\{1, 4, 5\} = \{5, 4, 1\}$$

In this way, sets are distinguished from other objects which have members, such as lists, or sequences. The theory of sets has enjoyed such a rich history of development, I speculate, partly because of the clarity and applicability of (EXT). The virtue of (EXT) is that it gives us a logically workable condition which makes it possible to demonstrate that two sets are distinct or identical.

The principle of extensionality is not an optional axiom; dispense with it and you have dispensed with sets altogether. On the other hand, if we have a domain of objects D , a subset S of D and a relation R on D such that for all a, b in S

$$(x) (Rxa \leftrightarrow Rxb) \text{ implies } a = b,$$

then we can immediately raise a set theoretic representation of this, associating each member s of S with $\{x : x \text{ in } D \wedge Rxs\}$ in a one to one fashion. So anything that obeys extensionality can be represented by a set theoretic construction.

To make this more concrete, let us borrow an example from Quine. Let B be the set of citizens of Canada, and C be the set of possible incomes, and let D be the union of B and C . The relation R is defined as follows: Rbc if and only if the income of b is c . The point

is, we could do without the members of C; incomes can be eliminated in favour of the set of individuals who earn this income. The relation 'has the same income as' is an equivalence relation, and particular incomes give way to the equivalence classes of individuals who have the same income.

If we ignore savings, there is nothing that can be said about incomes which could not easily be translated into the purchasing habits of the class of individuals which earn that income. A complete description of the situation can be given in terms of sets of citizens and the transmission of goods and services without ever mentioning numerical income.

The previous examples are meant to illustrate how membership is the essential characteristic of sets, and it is the only characteristic mentioned in the definition of identity for sets, (EXT). Synonymy is also an identity relation, the identity of meaning, and could therefore provide us with the essential characteristics of what it is to be a meaning. What is desired is a characterization of meaning identity that resembles (EXT), in that it makes use of similarly primitive and essential characteristics of linguistic terms. In light of what has been said regarding the rigidity of the concept of identity, and the role of type-specific identity criteria, what I hope to achieve by doing this is to expose some of the essential features that characterize what it is to be a meaning.

The main thesis of this chapter is to propose a deflationary theory of synonymy that makes use of only two primitive concepts: truth, and the relation of occurrence of one symbol within another. Like the membership relation is to sets, the occurrence of a symbol within a larger string is about as fundamental a property of linguistic terms as one could find. The same is not obviously so in the case of truth. Indeed there are those who would argue that one arrives at truth only through the meaning of a term, and that what I propose is

putting the cart before the horse. I choose to take truth as primitive for two reasons. Firstly, the concept of truth is better understood than that of meaning. Secondly, I hold that the clearest, and best defined instances of meaning are not individual terms, but rather complete sentences, i.e., those symbols which have a truth value. The meaning of a sub-sentential symbol is best characterized in terms of the meanings of the sentences in which it occurs.

Who needs meanings when we have synonymy? The "meaning" of a predicate can simply be identified with the class of symbols which are its synonyms. Synonymy is reflexive, symmetric and transitive. Every predicate has the same meaning as itself, and if the meaning of F is the meaning of G then the meaning of G is the meaning of F. If the meaning of F is the meaning of G and the meaning of G is the meaning of H, then the meaning of F is the meaning of H. Synonymy, whatever it turns out to be, will be an equivalence relation. I propose that the meaning of a predicate be conceived as its equivalence class under the relation of synonymy.

If such an approach were successful, then everything that one could say about meanings could be translated into talk of classes of symbols. Meanings could enjoy a place in the system by courtesy; this solution is meant to invite both Nominalists and Realists alike to the table. The Realist could happily maintain his belief in a world of abstract meanings, and provided they agree on the extension of the synonymy relation, the Nominalist can translate the Realist's words into talk of equivalence classes of expressions, so that they could successfully communicate with each other without compromising their metaphysical positions. The point is that with a little bit of set theory, one can get by with the symbols alone. Meanings in the robust sense are unnecessary for the analysis of a language for which the synonymy relation is completely understood.

The essential nature of set-hood is given by (EXT) in terms of the most fundamental and essential property that sets are involved in; membership. What is desired is a similar characterization of meaning in terms of things everyone can agree upon, namely the symbol of which it is the meaning, and the truth and falsehood of the sentences in which this symbol occurs. Like Davidson, we take truth as our primitive, and attempt to construct meaning from it.⁷

If we are to exhaust the meaning of a symbol by the truth and falsehood of the sentences in which that symbol occurs, then two symbols must be synonymous if they are ISV. If truth and occurrence are all we allow ourselves, then when two predicates are ISV, there is nothing left at our disposal for distinguishing further. On the one hand, it has been demonstrated that in the language of first order logic, ISV merely means coextensive; at the other extreme, in full-blown natural language ISV reduces to the identity relation on strings of symbols, since no distinct symbols are ISV wherever quotation is possible. Consider:

'Bachelor' begins with a 'B'	is true
------------------------------	---------

'Unmarried man' begins with a 'B'	is false
-----------------------------------	----------

A similar example can be constructed for any pair of alleged synonyms provided that they are symbolized with distinct strings of letters.

So on one end of the spectrum, we have the structure of first order logic, and on the other hand, the full suite of natural language. As we increase the structure of the language of

⁷ Davidson, D. 2001. *Inquiries into Truth and Interpretation*. Oxford, UK: Clarendon Press.

first order logic step-by-step towards natural language, the entailments of ISV gradually change. Note that in any language, ISV is always an equivalence relation. When considering such families of languages, we have not so much a linear spectrum, but rather a 'convergent' tree with one root (first order logic) and one leaf (natural language). We can frame this idea as follows:

(TM)

If synonymy, or the identity of the meanings of two distinct symbols can be given wholly in terms of the truth and falsehood of the sentences in which these symbols occur, then synonymy must be definable as ISV in some language that occurs as a node on the convergent tree (i.e., some sub-quotational fragment of natural language).

The justification of this conditional is best thought through contraposition: if ISV in every language on the convergent tree allows pairs of heteronymous equivalents, then meaning cannot be defined in terms of truth and falsehood alone. That is not to say that there is no such thing as meaning, but rather that something beyond the notions of sentence and truth is required to define it.

On the other hand, supposing we are successful in discovering a language L^* on the convergent tree in which ISV does satisfactorily capture our intuitive notion of synonymy, then we can declare peace between Nominalist and Realist alike. For when the Realist speaks of meanings, in however robust a sense, the Nominalist is free to substitute the idea

of an inductively definable equivalence class of expressions of natural language in its stead. Which equivalence class? The equivalence class of the predicate under ISV in L^* .

The debate whether abstract meanings really exist dissolves into idle speculation insofar as Realist and Nominalist can agree on the conditions of synonymy. Synonymy is the identity of meaning, and thus provides a definition of meaning. The equivalence classes become the place of reconciliation of the nominalist and realist points of view. Our approach is to give the conditions of synonymy in terms of truth and symbol occurrence, both of which are taken as primitive. Given this approach, the most natural candidate for synonymy would be ISV. In the next chapters, we proceed to examine the properties of ISV in various nodes on our convergent tree of languages. We will focus on languages in which quotation is not possible, and which allow the construction of the so-called intensional contexts, namely those involving possibility, belief and finally their combination.

Chapter 3

We have seen in Chapter 1 that ISV in the plain language of ordinary predicate logic is too weak a criterion to be identified with synonymy. However, let us not give up on Leibniz's idea, let us now consider ISV in a more powerful language. As has been stated in Chapter 1, we have departed from Quine's position that necessity and analyticity name one and the same idea. In doing so, Quine's 'closed curve' argument no longer applies to our consideration when we go on to consider the properties of ISV in a modal language.

The problem we immediately face in doing so is the spectrum of choice before us with respect to the concept of a 'modal language.' It will be assumed throughout this chapter that by 'modal,' we mean a language that is capable of expressing necessity and possibility, as opposed to belief, knowledge, obligation, or other modalities understood in the broad sense. Since our investigation is primarily directed toward the meaning of predicates, we may bypass the various propositional systems for modal logic. Unlike first order predicate logic, there is a variety of competing systems that could represent quantified modal reasoning. For the purposes of this chapter, we will focus on quantified S5. Our system will be treated using the ideas of possible world semantics.

3.1

Recall that the language L of Chapter 1 was to be a standard first order language equipped with predicates, names, variables and quantifiers. The variables are intended to range over the entities that exist in the actual world. From L we construct the language M as follows:

(3I) If ϕ is a formula of L (with or without free variables) then

ϕ is a formula of M

(3II) If ϕ and ψ are formulas of M (with or without free variables) then so

are $\Box\phi$, $\phi \wedge \psi$, $\sim\phi$ and $(\alpha)\phi$ for any variable α of L

(3III) Nothing else is a formula of M; M is the smallest possible language

that satisfies (I) and (II)

We define the possibility operator in the usual way:

$\Diamond\phi$ if and only if $\sim\Box\sim\phi$

By allowing ϕ to contain free variables in clause (3II), we have provided for the expression of both *de re* and *de dicto* modality. An expression to which a modal operator is prefixed is said to be *de dicto* if no free variables or names occur within the scope of the operator, and *de re* otherwise. By defining M in this way, each formula of M has an unique recursive structure, which can be represented by a parsing tree, the nodes of which are all formulas of M, e.g.

$$\begin{array}{c}
(x) (y) (Fx \wedge \Box Gy) \wedge \Box(x) \sim Hx \\
/ \quad \backslash \\
(x) (y) (Fx \wedge \Box Gy) \quad \Box(x) \sim Hx \\
| \quad | \\
(y) (Fx \wedge \Box Gy) \quad (x) \sim Hx \\
| \quad | \\
Fx \wedge \Box Gy \quad \sim Hx \\
/ \quad \backslash \quad | \\
Fx \quad \Box Gy \quad Hx \\
| \\
Gy
\end{array}$$

We interpret the operator \Box with the usual structure: a domain of all possible worlds Ω , and an accessibility relation R on the worlds of Ω . By this, we mean that if wRv for $w, v \in \Omega$, then world v is possible relative to world w ; i.e., that everything that is true in v is possible in w . In this sense, we can consider the deductive system associated with M to depend on R ; different conditions on the accessibility relation will result in differing modal logics, corresponding to which the entailments of ISV may vary.

Since we are concerned primarily with quantified modal logic, M must also be equipped with a function $ONT: \Omega \rightarrow \Sigma$, where Σ is the set of all possible individuals. $ONT(w)$ is the 'ontology' of w ; i.e., $ONT(w) = \{x : x \text{ exists in } w\}$. It may seem that the invocation of the class of all possibilia is metaphysically extravagant, however, ONT is an onto function; we could simply say that each world w is associated with a domain of individuals $ONT(w)$, and then define

$$\Sigma = U\{ONT(w) : w \in \Omega\}$$

Some assumption more or less like this is required any time one mingles modal operators with quantifiers. Therefore in M, predicates do not simply have an extension as they would in L, but rather an extension that varies from world to world. Borrowing from David Lewis's notation, we will let @ designate the actual world in which we live, and we denote the set of actual existing individuals with $ONT@$. If ϕ expresses what is for us a necessary truth, then ϕ is true in every world w such that $@Rw$. To avoid unnecessary complications, we have imposed the restriction that the only proper names occurring in M are those that occur in L. In other words, each proper name that occurs in M designates a member of $ONT@$, i.e., an object that actually exists. In distinction from Lewis's Counterpart Theory, we allow worlds to overlap:

$$ONT(w) \cap ONT(v) \text{ may be non-empty where } w \neq v$$

That is, we invoke the relation of transworld identity of individuals; individuals will typically exist in more than one possible world. With this in mind, we adopt Kripke's convention that proper names be rigid designators, i.e., that proper names designate the same individual in every world. We write $w \Vdash \phi$ to say that ' ϕ is true in w.' Where a modal operator is prefixed to a formula containing a free variable such as $\Box Fx$, we interpret this along the essentialist line; in any accessible world in which the referent of the variable exists, F is true of it.

Much of the structure we have defined so far is highly simplified by the choice S5. For instance, in S5, the accessibility relation R must be an equivalence relation. Because of this, we will naturally be concerned with the equivalence class of the actual world under R.

$$\{w : @Rw\}$$

Also, by choosing S5, the set of individuals existing in any world accessible to the actual world must remain constant:

$$@Rw \text{ implies } \text{ONT}(w) = \text{ONT}@$$

Similarly, the problem of interpreting de re modality is highly simplified by the choice of S5, for in this system, the domain of individuals must be constant from world to world:

$$\text{For all } w, \text{ if } @Rw \text{ then } \text{ONT}(w) = \text{ONT}@$$

The formula $\Box Fx$ is understood to be true if the sub-formula to which the modal operator is prefixed, Fx , is true in every accessible world. This can present a problem when the individual in this world designated by the variable x does not exist in some accessible world. However, in our system the domain of individuals remains constant, so we will not face this problem. This means that we can simply understand $\Box\phi$ as asserting that ϕ is true in every accessible world, even when ϕ contains free variables.

The set of propositions that are true in a world should be closed under logical consequence. Hence, a rule of inference we adopt is

(IR1) If $w \Vdash \varphi^1, \dots, w \Vdash \varphi^n$ where each φ^i is a formula of M
and from $\varphi^1 \dots \varphi^n$, ψ can be validly inferred, then $w \Vdash \psi$

Similarly, we want the set of propositions true in a world to be maximal; if a formula is not true in a world, then it ought to be false:

(IR2) For all $\chi \in M$ $\sim(w \Vdash \chi)$ if and only if $w \Vdash \sim\chi$

We introduce some new notation. Since L is a proper subset of M , and by the construction of M , we have introduced no new predicate letters, a predicate F occurs in some sentence of L if and only if it occurs in some sentence of M . We define:

$ISV^L(F, G)$ to mean that the predicates F and G are ISV in the language L
according to the usual rules of first order logic, interpreting the
variables as ranging over $ONT@$.

$ISV^{M(w)}(F, G)$ to mean that the predicates F and G are ISV in the
language M interpreted with domain of possible worlds Ω and
accessibility relation R , with respect to world w . That is, for every
 $\varphi \in M$ and every positive integer i $w \Vdash \varphi \leftrightarrow \varphi[F/G : i]$

In particular, we are interested in $ISV^{M@}$.

3.2

In Chapter 1, we saw that the meta-statement $ISV^L(F, G)$ is true if and only if a certain first order statement was true, namely that F and G are coextensive. The question that naturally occurs here is whether $ISV^{M@}(F, G)$ has a similar correlate in M . The most obvious candidate would be the condition that F and G be necessarily coextensive. We have three apparently distinguishable options as to how to interpret the phrase 'necessarily coextensive'

$$(1) \quad \Box(x) (Fx \leftrightarrow Gx)$$

$$(2) \quad (x) \Box(Fx \leftrightarrow Gx)$$

$$(3) \quad \Box(x) \Box(Fx \leftrightarrow Gx)$$

We should note that (1) implies (2):

Proof: Suppose $w \Vdash (1)$. Therefore, (i) for all $u \in \Omega$, if wRu then $u \Vdash (x) (Fx \leftrightarrow Gx)$. Now consider the content of (2). It says that (ii) for all $y \in \text{ONT}(w)$, if wRu and $y \in \text{ONT}(u)$ then $u \Vdash Fy \leftrightarrow Gy$, which follows by (i) and (IR1) since wRu .

This proof can be generalized for any formula ϕx containing one free variable. That is, the Converse Barcan Formula,

$$(CBR) \quad \Box(x) \phi x \rightarrow (x) \Box \phi x$$

holds in our system. This choice is adopted in the main to accommodate our understanding of *de re* modal statements. If a certain condition holds for all x , not just in this world, but in every world accessible to ours, then it is certain that this condition holds of a thing in any accessible world in which it exists. (CBR) can be defended further on more intuitive grounds. Consider

$$(CBR^*) \quad (Ex) \diamond Fx \rightarrow \diamond(Ex) Fx$$

which states that if there exists a thing in this world which could possibly be an F , then it is possible in this world that something be an F . But doesn't that follow from our common sense understanding of modality? Suppose it is possible for Adam Smith to jump ten feet in the air. Then by existential generalization, a statement having the form of the antecedent of (CBR*) is true. It seems an obvious and unavoidable conclusion then that it is possible that someone jump ten feet in the air. Note that (CBR*) implies (CBR):

$$(Ex) \diamond \sim Fx \rightarrow \diamond(Ex) \sim Fx$$

$$\sim(x) \sim \diamond \sim Fx \rightarrow \diamond \sim(x) Fx$$

$$\sim(x) \square Fx \rightarrow \sim \square(x) Fx$$

$$\square(x) Fx \rightarrow (x) \square Fx$$

Because we are working within the S5 structure, the Barcan Formula

$$(BAR) \quad (x) \square \phi x \rightarrow \square(x) \phi x$$

also holds. Hence, (1) and (2) are equivalent. Since we only consider R that are reflexive, axiom T holds

$$(T) \quad \Box\phi \rightarrow \phi$$

and since R is transitive,

$$(IT) \quad \Box\phi \rightarrow \Box\Box\phi$$

also holds, hence (1), (2) and (3) above are mutually equivalent.

The problem with option (2) in modal logics where (BAR) does not hold, such as some S4 systems is that the quantifier ranges only over actual things (provided we are dealing with a statement of the form $@ \Vdash \dots$), and fails to rule out the case that some possible, non-actual F is not a G. (2) could be true because of some essential *de re* property shared by all of the objects that are actual Fs, and there could be a possible world in which a non-actual possible thing exists that is both an F and by virtue of lacking this property is not a G. More precisely, suppose that (2) is true. I.e. suppose that

$$@ \Vdash (x) \Box(Fx \leftrightarrow Gx)$$

(2) allows us to also suppose that there is a possible world w such that

@Rw

and

$$w \Vdash \sim(x) \Box(Fx \leftrightarrow Gx)$$

In other words, (2) does nothing to prevent the case that, for some z in $\text{ONT}(w)$, z is not a member of $\text{ONT}@$, and for some accessible world $u \in \Omega$ such that $z \in \text{ONT}(u)$:

$$u \Vdash (Fz \wedge \sim Gz) \vee (\sim Fz \wedge Gz)$$

(2) only ensures that among the individuals that actually exist, in every other world in which they exist, every F is a G and every G is an F .

To give an example of this problem, suppose that there are exactly six billion people on earth, and let

$P_1, P_2, \dots, P_n, \dots, P_{6,000,000,000}$

be a list of their names. Assuming that identities are necessary, the n^{th} person satisfies the formula

$$\Box(x = P_n)$$

Define $V(x)$ to be the disjunction of open identity statements ranging over all living people's names:

$$V(x) = (x = P1) \vee (x = P2) \vee \dots \vee (x = P6,000,000,000)$$

Now, for every n , it is a theorem of logic that,

$$(x = Pn) \rightarrow V(x),$$

Hence for every n ,

$$\Box((x = Pn) \rightarrow V(x))$$

In most modal logics, the axiom K holds, which states that the necessity operator distributes over the conditional connective,

$$(K) \quad \text{For all } \phi, \psi \in M \quad \Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$$

hence, for every n

$$\Box(x = Pn) \rightarrow \Box V(x)$$

Hence, $\Box V(x)$ is satisfied precisely by those people who are alive in the world today. It is an essential property possessed by all and only those human beings who now exist in the actual world.

It is intuitively plausible that Wittgenstein could have had a son. Suppose that in world u , such that $@Ru$, Wittgenstein had a son named Karl. Once again with Kripke, assuming the necessity of origin, we can say that $V(\text{Karl})$ is false in u , and therefore $\Box V(\text{Karl})$ is false in both u and $@$; hence Karl fails to have a necessary property possessed by all humans that exist in $@$, and

$$u \Vdash \text{Karl is human} \wedge \sim V(\text{Karl})$$

Hence (2) is not a satisfactory expression of 'necessary coextension' in any system in which (BAR) does not hold.

Notice how our example requires a world to be accessible to ours in which a man exists who does not exist in our world. Recall that in S5, both (BAR) and (CBR) hold. (BAR) prevents such an accessibility relation; the domain of individuals in a world accessible to ours in such a system must be a subset of the domain of the actual, $\text{ONT}@$. Conversely, (CBR) prevents such an accessibility relation in which the domain of individuals existing in a world accessible to ours is a proper subset of the actual domain. This is a consequence of adopting an S5 structure. Admittedly this may seem a shortcoming, however it does much to simplify the interpretation of de re modal statements, as discussed above. Our decision to maintain a fixed domain from world to world has had its supporters in the literature, most notably, Ruth Barcan Marcus.

3.3

At the beginning of the previous section, we said we were searching for an object language expression that would encode ISV in M. In this section, we will consider the views on interchangeability in modal logics found in the writings of the some of the original figures in the development of modal logic, namely C. I. Lewis, C. H. Langford and Ruth Barcan Marcus.

In 1946 Ruth Barcan Marcus published "A Functional Calculus of First Order Based on Strict Implication" in which she presented a quantified extension of C. I. Lewis' S2. Note that any rule of inference that holds in S2 also holds in S4 and S5. In both propositional S2, and in Marcus' first order extension, the following rule of inference holds:

(SUBST) If A, B and Γ are such that E results from Γ by the substitution of B for one or more occurrences of A in Γ , and if $\Box(A \leftrightarrow B)$, then infer Γ from E and E from Γ ⁸

(SUBST) is directly adopted from C. I. Lewis's⁹ substitution rule in (Lewis, C. I., 1959). From this rule, and the principle of universal instantiation, Marcus extended (SUBST) to a rule applied to quantified formulas:

⁸ Marcus, R. B. 1946. A Functional Calculus of First Order Based on Strict Implication. *The Journal of Symbolic Logic*, Vol. 11. No. 1. pp1-16.

⁹ Lewis, C.I., Langford, C.H. 1959. *Symbolic Logic*. New York: Dover.

(XVI) If the well formed formula Γ , E and B are such that B results from A by the substitution of E for one or more occurrences of Γ in A and if $(\alpha^1)(\alpha^2)\dots(\alpha^n)\Box(\Gamma \leftrightarrow E)$ where $\alpha^1, \alpha^2, \dots, \alpha^n$ is a complete list of the free variables occurring in Γ and E then we may infer B from A and A from B .

In particular,

$$@ \Vdash (x) \Box(Fx \leftrightarrow Gx) \text{ .implies. } ISV^{M@}(F, G)$$

follows from this rule. $ISV^{M@}(F, G)$ states that F and G are interchangeable *salva veritate* in any sentence $\phi \in M$ such that $@ \Vdash \phi$. Recall that in our system

$$(x) \Box(Fx \leftrightarrow Gx)$$

is equivalent to

$$\Box(x) (Fx \leftrightarrow Gx)$$

which intuitively states that F and G are necessarily coextensive, or more precisely, that it is necessary that F and G be coextensive.

(SUBST) is not a proven meta-theorem, but rather is a rule that is stated at the outset for both Lewis's propositional system as well as Marcus's quantified extension. Notice the

location of the modal operator in (XVI), the quantified version of (SUBST): it is attached to the open sub-formula. A rule of inference is a syntactic entity however, and we may ask for its semantic justification.

We must remember the namesake of the Barcan and Converse Barcan formulas, (BAR) and (CBR). Marcus includes an equivalent formulation of (BAR) as an axiom, and later proves (CBR), which as stated above has the semantic consequence of assuming a fixed domain:

$$@Rw \text{ .implies. } \text{ONT}@ = \text{ONT}(w)$$

Hence there is no substantial distinction between $(x) \Box(Fx \leftrightarrow Gx)$ and $\Box(x) (Fx \leftrightarrow Gx)$ in her system. The example of Wittgenstein's son does not hold in our system.

3.4

In each world, a predicate has an extension, but this extension typically varies from world to world. Following Montague's usage of the term,¹⁰ we define the intension of a predicate F , to be a function $\sigma^F: \Omega \rightarrow P(\Sigma)$, where P is the powerset operator, such that $\sigma^F(w)$ is a subset of $\text{ONT}(w)$. The intension of F maps a world w to the extension of F in w , or to the set consisting of those members of $\text{ONT}(w)$ of which F is true. To make this work for all predicates, we need the following conditions:

$$\begin{aligned} (\text{INT}\wedge) \quad & \text{For all } w, \sigma^{F\wedge G}(w) = \sigma^F(w) \cap \sigma^G(w) \\ (\text{INT}\sim) \quad & \text{For all } w, \sigma^{\sim F}(w) = \text{ONT}(w) \setminus \sigma^F(w) \end{aligned}$$

It may seem strange to label such formal objects as 'intensions,' however to do so is not to stray far from the traditional view of meaning. The meaning of a predicate, according to Frege's view, should determine its extension. If the world was different than it actually is, we would expect the meaning of a predicate to remain stable, although its extension may vary. The meaning of a predicate should be rich enough not just to determine the actual extension of a predicate, but the extension of a predicate in any counterfactual state of affairs. Hence, the function which assigns to a world the extension of F in that world should be closely linked with the meaning of a predicate.

The intension of a sentence ϕ is a function $\sigma^\phi: \Omega \rightarrow \{0, 1\}$ such that $\sigma^\phi(w) = 1$ if and only if $w \Vdash \phi$, i.e., ϕ is true in w . We let $\Omega^w = \{u \in \Omega : wRu\}$, and we let σ^{F^w} be the

¹⁰ Montague, R. 1974. *Formal Philosophy*. Edited by R. Thomason. London: Yale University Press.

restriction of σ^F to Ω^w , and call it the intension of F local to w, or simply the local intension of F when the context makes the world unambiguous. It is clear then that two predicates are coextensive in every world accessible to ours if and only if $\sigma^{F@} = \sigma^{G@}$.

$$@ \Vdash \Box(x) (Fx \leftrightarrow Gx) \text{ .if and only if. } \sigma^{F@} = \sigma^{G@}$$

and that if $@Rw$

$$w \Vdash (x) (Fx \leftrightarrow Gx) \text{ .if and only if. } \sigma^{F@}(w) = \sigma^{G@}(w)$$

The question of whether $ISV^{M@}(F, G)$ holds if $\sigma^{F@} = \sigma^{G@}$, i.e. if ISV holds provided that F and G are coextensive in every world accessible to ours, becomes the question of whether the @-local intension of a sentence, and hence its truth value in any world accessible to @, is functionally dependent on $\sigma^{F@}$ in the way that the truth value of a molecular compound in the propositional calculus is functionally dependent on the truth values of its atomic constituents, or in the way that the truth value a sentence of first order logic is functionally dependent on the extensions of the predicates and names that occur in it. That is, we ask whether σ^ϕ is fixed if σ^F is fixed for all F occurring in ϕ . It turns out that the answer is yes.

In what follows, we will consider a proof of the fact that if the predicates F and G share a common intension, i.e. $\sigma^F = \sigma^G$, then F and G are ISV in M. This proof will closely resemble the chapter 1 proof that coextensive predicates are ISV in L. The strategy for this proof will be to proceed by induction on the complexity of the formulas of M, defined in terms of the recursive structure of M. We define $Mdeg(\phi)$ as follows:

$Mdeg(\varphi) = 1$ if φ is atomic

$Mdeg(\varphi \wedge \psi) = 1 + \max\{Mdeg(\varphi), Mdeg(\psi)\}$

$Mdeg(\sim\varphi) = 1 + Mdeg(\varphi)$

$Mdeg((x)\varphi) = 1 + Mdeg(\varphi)$

$Mdeg(\Box\varphi) = deg(\varphi) + 1$

Proof: (To simplify the issue, we ignore sentences which include proper names)

Suppose $\sigma^F = \sigma^G$

Let $Mdeg(\varphi) = 1$, and suppose G occurs in φ

Therefore $\varphi = Gx$

Suppose $w \Vdash \varphi$ i.e., $w \Vdash Gx$

It makes no difference what particular world w is, because $\sigma^F = \sigma^G$. Therefore for some $t \in \text{ONT}(w)$, x designates t , and $t \in \sigma^F(w)$

Since $\sigma^F = \sigma^G$, in particular $\sigma^F(w) = \sigma^G(w)$. Hence, $t \in \sigma^F(w)$, i.e., $w \Vdash Fx$

Without loss of generality, $w \Vdash Fx$.if and only if. $w \Vdash Gx$, i.e., for every world w ,

$w \Vdash \varphi$.if and only if. $w \Vdash \varphi[F/G : 1]$

Since $\varphi[F/G : i] = \varphi$ for all $i > 1$, it follows that for every world w , for every positive integer i ,

$w \Vdash \varphi$.if and only if. $w \Vdash \varphi[F/G : i]$

IH Suppose for all $\psi \in M$, for all $w \in \Omega$ for all $n \leq k$, $Mdeg(\psi) = n$ implies that

for all $i \in \mathbb{N}$,

$w \Vdash \psi$.if and only if. $w \Vdash \psi[F/G : i]$

Suppose that $Mdeg(\varphi) = k + 1$

Case 1: $\varphi = \sim\psi$

Since $\text{deg}(\psi) = k$, by IH we have

$$\begin{aligned} w \Vdash \psi \text{ .if and only if. } w \Vdash \psi[F/G : i] \\ \sim(w \Vdash \psi) \text{ .if and only if. } \sim(w \Vdash \psi[F/G : i]) \\ w \Vdash \sim\psi \text{ .if and only if. } w \Vdash \sim(\psi[F/G : i]) \quad \text{by (IR2)} \\ w \Vdash \sim\psi \text{ .if and only if. } w \Vdash (\sim\psi)[F/G : i] \quad \text{by (SUB1)} \\ w \Vdash \phi \text{ .if and only if. } w \Vdash \phi[F/G : i] \quad \text{Since } (\phi = \sim\psi) \end{aligned}$$

Case 2: $\phi = \psi \wedge \chi$

Without loss of generality, suppose $\text{Mdeg}(\chi) \leq \text{Mdeg}(\psi)$. Therefore $\text{Mdeg}(\psi) = k$

Note that by (IR1) for all sentences α, β , $w \Vdash \alpha \wedge \beta$.if and only if. $w \Vdash \alpha$ and $w \Vdash \beta$

To show that $w \Vdash \phi$.implies. $w \Vdash \phi[F/G : i]$, suppose (*) $w \Vdash \phi$

Let j = the number of occurrences of G in ψ , and g = the number of occurrences of G in χ

Subcase1 $i \leq j$, note $\phi[F/G : i] = \psi[F/G : i] \wedge \chi$

By our supposition (*), $w \Vdash \psi$ and $w \Vdash \chi$. Since $\text{Mdeg}(\psi) = k$, by IH we have

$$w \Vdash \psi[F/G : i]$$

Since $w \Vdash \psi[F/G : i]$ and $w \Vdash \chi$, we have $w \Vdash \psi[F/G : i] \wedge \chi$, hence $w \Vdash \phi[F/G : i]$

Subcase2 $j < i \leq j + g$ (note if $j + g < i$, then ϕ is a fixed point for $[F/G : i]$)

Note $\phi[F/G : i] = \psi \wedge \chi[F/G : i - j]$

Since $\text{Mdeg}(\chi) \leq k$, and $w \Vdash \chi$, by IH we have $w \Vdash \chi[F/G : i - j]$

Since we have $w \Vdash \psi$ and $w \Vdash \chi[F/G : i - j]$, by IR1, we have $w \Vdash \psi \wedge \chi[F/G : i - j]$, hence

$$w \Vdash \phi[F/G : i]$$

Therefore without loss of generality, $w \Vdash \phi$.if and only if. $w \Vdash \phi[F/G : i]$

Case 3: $\phi = (x) \psi x$

To show $w \Vdash \phi$.implies. $w \Vdash \phi[F/G : i]$, suppose (**) $w \Vdash \phi$.

And suppose that for some i , $\sim(w \Vdash \varphi[F/G : i])$

By (IR2) $w \Vdash \sim\varphi[F/G : i]$, i.e., $w \Vdash (Ex) \sim\psi x[F/G : i]$

Let y witness this. I.e., by (IR1) suppose $w \Vdash \sim\psi y[F/G : i]$

By (IR1) and (**), we have $w \Vdash \psi y$ by universal instantiation, and since $\text{deg}(\psi y) = k$, by IH

we have $w \Vdash \psi y[F/G : i]$, a contradiction. Therefore the supposition that

$\sim(w \Vdash \varphi[F/G : i])$ must be false, therefore

$$w \Vdash \varphi[F/G : i]$$

Therefore without loss of generality, $w \Vdash \varphi$.if and only if. $w \Vdash \varphi[F/G : i]$.

Case 4: $\varphi = \Box\psi$

Suppose $w \Vdash \varphi$, i.e., $w \Vdash \Box\psi$. Therefore for all u such that wRu , $u \Vdash \psi$. Note that

φ and ψ have exactly the same free variables. Again, since (CBR) holds, the designations of the free variables occurring in ψ exist in $\text{ONT}(u)$ if they exist in $\text{ONT}(w)$,

so we do not need to consider the case that ψ is false in u because the object designated by x in w does not exist in u .

Since $M\text{deg}(\psi) = k$ by IH we have

$$u \Vdash \psi \text{ .if and only if. } u \Vdash \psi[F/G : i]$$

Therefore for all u such that wRu , $u \Vdash \psi[F/G : i]$, therefore $w \Vdash \Box(\psi[F/G : i])$

i.e., $w \Vdash (\Box\psi)[F/G : i]$, i.e., $w \Vdash \varphi[F/G : i]$.

Without loss of generality, $w \Vdash \varphi$.if and only if. $w \Vdash \varphi[F/G : i]$

QED

This proof reveals that the structure of our modal language M fits the Fregean compositionalist view on the nature of meaning and semantics. That is, the semantic determinations of a complex symbol depend functionally on its constituent inputs; truth

values in propositional logic, extensions in quantified logic, and now intensions for modal logic. If we fix σ^F , the intension of F , for every atomic F , we have thereby fixed the truth value for every sentence of M relative to every world w . Notice, our proof did not require the assumption that R be transitive, hence no special stance on the topic of iterated modality has been assumed. Notice also, that the preceding proof did make use of the assumption (CBR), which is not true in all modal logics, however it does hold for us in S5. This is not a major loss however, because (CBR) is a consequence of (CBR*), which has been argued for above, independently of the choice of S5.

If synonymy is to be identified with $ISV^{M@}$ then meanings are associated with local intension functions in a one to one way. Two predicates would have the same meaning if and only if they have the same local intension function, or as stated in our object language, F and G are necessarily coextensive, in the sense of (1). The range of all possible predicate meanings would then be associated with the set of all functions σ from Ω to the powerset of Σ such that $\sigma^F(w)$ is a subset of $ONT(w)$, and both (INT \wedge) and (INT \sim) hold for molecular predicates.

The question is, do intensions, so defined, provide enough semantic input to determine the meaning of a predicate? There seem to be reasons to answer no. For example, any two self-contradictory predicates, say 'round square' and 'least positive real number' have the same intension function, namely that which assigns the empty set to each world. Similarly, any two tautologically valid predicates, such as 'human or non-human' and 'self identical' have the same intension function, namely that which assigns $ONT(w)$ to each world w . These results should at least raise the suspicion that there is more to the meaning of a predicate than its intension.

3.5

In the preceding section, we asked whether the intension of a sentence of M functionally depends on the intension of the predicates, that is, we asked whether σ^ϕ is fixed if σ^F is fixed for all F occurring in ϕ , and the answer was yes. The section concluded with counterexamples such as the empty predicate and the universal predicate. In this section we will consider other weaknesses of modal ISV as a candidate for synonymy by considering counterexamples arising from the work of Kripke.¹¹ Let us return to expression (1):

$$(1) \quad \Box(x) (Fx \leftrightarrow Gx)$$

Stated in English, (1) says

'It is necessary that every F is a G and every G is an F'

or in the language we have adopted,

'It is necessary that F and G are coextensive'

In possible worlds talk, this says that F and G are coextensive not only in this world but in every world that is accessible to ours.

¹¹ Kripke, S. 1980. *Naming and Necessity*. Cambridge: Harvard University Press.

Now, since it turns out that (1) entails $ISV^{M@}(F, G)$, then we face some trouble in our pursuit of synonymy, as indicated in the concluding remarks of the previous section. If Kripke is right when he alleges that natural kind equivalences are necessary, then it turns out that predicates like 'heat' and 'molecular motion' are coextensive in every possible world accessible to ours. However, from an intuitive standpoint, there is nothing analytic about the statement 'Heat is molecular motion.' The discovery of this identification was surely an *a posteriori* affair, and not a matter of mere synonymy of predicates. The same holds for 'table salt' and 'sodium chloride,' 'water' and 'dihydrogen oxide' and all the familiar examples of natural kind equivalences. Empirical science is full of such non-analytic, discovered equivalences, hence ISV is still too blunt an instrument to separate distinct meanings.

Assuming that Kripke is right, I suppose we would have the option of forcing this definition of synonymy, although this would do so much violence to the intuitive concepts of meaning and analyticity and their connection to *a priori* knowledge, that it would seem appropriate to admit that $ISV^{M@}$ does not sufficiently capture synonymy in any intuitively compelling sense. So if Kripke is right, since (1) entails $ISV^{M@}(F, G)$, then we need to somehow modify the language over which we consider ISV. 'Heat' and 'molecular motion' stand in the same counter example relation to $ISV^{M@}$ as do 'creature with a heart' and 'creature with a kidney' to ISV^L .

Let us not, however, disregard the things that modal ISV contributes towards synonymy. Modal ISV does rule out Quine's 'heart and kidney' example, while validating the 'bachelor' example. What Modal ISV guarantees is that two predicates are not accidentally coextensive. This is one ingredient of meaning, namely that if two predicates

have the same meaning, they share an essential connection, and must enjoy all the same modal properties. For example,

$$(SELF1) \quad \Box(x) (Fx \leftrightarrow Fx)$$

is true for every predicate F on any conception of necessity, since the sub-formula to which the modal operator is attached is a logical truth. Hence, if ISV(F, G), then

$$(SELF1)[G/F : 2] = \Box(x) (Fx \leftrightarrow Gx) = (1)$$

also holds, i.e., F and G stand in the same necessary relation to their common extension. ISV ensures that every modal property possessed by F is also possessed by G. This is a considerable advancement towards synonymy and meaning considering the limits of ISV in Chapter 1. An interesting distinction arises here. Extending the *de re* view from objects to states of affairs, that is, supposing that necessity attaches directly to states of affairs, as opposed to the symbols which denote an existing state of affairs, then not only the meaning, but the hidden essential content of the class of objects forming the extension of F must also stand in the same essential relation to G, whether we recognize it or not. However, if one maintains the *de dicto* interpretation of sentences where no variable occurs free within the scope of a modal operator, then we would be more inclined to focus on those modal properties that reflect the cognitive meaning of a symbol, and not such hidden yet necessary facts. Perhaps here we have a reflection of the *a priori/a posteriori* distinction among modalities.

What modal ISV lacks, in light of Kripke's natural kinds, is a guarantee of the epistemic and doxastic properties shared by synonyms. It is neither obvious nor compelling to believe that hydrogen is (in some sense) a constituent of water. If we assume that at least one of the primary destinations of meaning is to be the individual human mind, then synonyms should be recognizable as such. Given a compositionalist view of meaning, if sentence meanings are to be the objects of belief, then if a person believes in the proposition symbolized by a sentence, she should also believe the proposition symbolized by a sentence which differs from the first by the interchange of two synonymous predicates. That is, synonyms should be interchangeable in all belief contexts. With this in mind, in the next chapter we will consider ISV in a language which is capable of expressing belief contexts.

Chapter 4

4.1

In Chapter 3 we investigated ISV in a modal language. This investigation was primarily motivated by Quine's remarks in *Two Dogmas*, where Quine suggested that ISV under modal operators would ensure identity of meaning, while concluding that the 'closed curve' equivalence of analyticity with necessity ruined this criterion. This is reflective of his reading of analyticity primarily in terms of the work of Rudolph Carnap, as is evident in *Two Dogmas*. Carnap's idea of a possible world was a state description; that is to say, a possible world was to be understood as function that would assign a truth value to all the atomic propositions of a language. Doing so would thereby produce a truth value for every compound sentence through the familiar rules governing the logical connectives and quantifiers. Of course, the latter must involve the notion of a variable assignment, and hence a domain of quantification must be presupposed. The crucial issue occurs at the base level. Consider the propositions

(JR) John earns more than Rick

(RJ) John earns less than Rick

or

(MJ) For every x , x earns more than John

(LJ) There exists x , x earns less than John

Now as we saw in the previous chapter, if synonymy is mere modal ISV, then meanings can be identified with intension functions. But the question is how can we explain the inconsistency of (JR) with (RJ) or of (MJ) with (LJ)? Necessity is to be understood in terms of quantification over possible worlds, however if these possible worlds are mere state descriptions, then the totality of all possible worlds should be the totality of all possible assignments of truth values to the atomic sentences. So can we justify ignoring those assignments which make both (JR) and (RJ) true?

To the best of my knowledge, Carnap was not very worried about this question. He spoke of 'meaning postulates' i.e., certain statements, apparently adopted without further justification, which would rule out married bachelors and the like. But consider the fate of other postulates, such as the parallel line postulate of Euclidean Geometry. The parallel line postulate is intuitively compelling, however history showed that it is logically independent of the other axioms of Euclid's geometry, as demonstrated by the models of non-Euclidean space. Furthermore, physics eventually came to suggest that actual space is non-Euclidean. So the question that we can abstract from this story is: How do we know that a bachelor couldn't be married? Given what happened in the physics of spacetime, how do we know that there may not one day be a married bachelor, or that bachelors might not turn out to be married after all?

If one is to explain synonymy in terms of modality, modality is to be explained in terms of possible worlds, and if possible worlds are to be essentially linguistic entities then trouble occurs. By essentially linguistic, I mean that such possible worlds are conceived as maximally consistent sets of sentences, and that necessity is to be determined by the totality of such sets. The very circularity that Quine identifies in Section III of *Two Dogmas* recurs

as the question of which assignments of truth values to the sentences of a language we ought to rule out as analytically false. David Lewis identifies a similar problem, in that we are not sure which descriptions of possible worlds actually refer, because of the problem of analyticity. If meanings are intension functions, then the incompatibility of bachelorhood with marriage is a property of these functions, however the explanation stops there; it gives us no reason why these functions ought to have such a property. Carnap's meaning postulates stand in need of justification. Furthermore, given Kripke's arguments about natural kinds, it seems that modal ISV may not be fine-grained enough for synonymy in the first place. Modal ISV is nothing more than necessary coextension. It does give us part of what we expect from synonymy, namely necessary connection. However, what we overlooked in Chapter 3 are the doxastic and epistemic properties of meaning and analyticity, and these are what we will attempt to take account of in this chapter.

In Section 4.2, we will first construct a doxastic language and assert three axioms regarding inferences involving belief contexts. We will then identify a function which indicates the distribution of beliefs over the speakers of this language, and prove a result connecting this function with ISV, drawing inferences on this connection pertaining to the larger problem of synonymy. Then we will move on to consider a condition under which ISV would be definable within the doxastic language constructed.

In section 4.3, we will closely consider certain specific examples of expressions which inform us about the consequences of ISV in the doxastic language, labeled W , W' , P , and P' respectively. These examples will shed light on the consequences of ISV that lead us closer to capturing the relation of synonymy. They will also help to illuminate the main

problem uncovered in this chapter, namely that of collateral information that is universally possessed by every speaker of the language..

4.2

An analytic statement should be one which, if a person understands completely, they will believe to be true without further information. This suggests that perhaps synonymy should be linked with the property of ISV under doxastic rather than modal operators. Ideally, synonyms should be ISV under both; however let us first consider ISV in a doxastic language without modal operators.

To build a doxastic language from L, we first specify a subset Γ of $\text{ONT}@$, namely the set of all actual speakers. We define the language B as follows:

(BI) If φ is a formula of L then φ is a formula of B

(BII) If $s \in \Gamma$ and φ and ψ are formulas of B then so are

$$\varphi \wedge \psi, \sim\varphi, (\text{x}) \varphi, \text{ and } \beta^s\varphi$$

(BIII) Nothing else is a formula of B; B is the smallest possible language that satisfies (I) and (II)

Each sentence of B has a unique recursive structure as is the case with L and M.

For $\varphi \in B$, we define $\text{Bdeg}(\varphi)$ as the length of the longest branch in the parsing tree of φ .

More rigorously,

$$\text{Bdeg}(\varphi) = 1 \text{ if and only if } \varphi \text{ is an atomic formula of L}$$

$$\text{Bdeg}(\sim\varphi) = 1 + \text{Bdeg}(\varphi)$$

$$\text{Bdeg}(\varphi \wedge \psi) = 1 + \max\{\text{Bdeg}(\varphi), \text{Bdeg}(\psi)\}$$

$$\text{Bdeg}((x) \varphi) = 1 + \text{Bdeg}(\varphi)$$

$$\text{Bdeg}(\beta^s \varphi) = 1 + \text{Bdeg}(\varphi)$$

We define the operators of B as follows: for each speaker $s \in \Gamma$,

$$\beta^s \varphi = \text{Speaker } s \text{ believes that } \varphi$$

We adopt the following axioms concerning the interrelations of a speaker's beliefs:

$$\text{(DOX1)} \quad \text{For all speakers } s, \beta^s \varphi \rightarrow \sim \beta^s \sim \varphi$$

$$\text{(DOX2)} \quad \text{For all speakers } s, \beta^s \sim \varphi \rightarrow \sim \beta^s \varphi$$

$$\text{(DOX3)} \quad \text{For all speakers } s, \beta^s(\varphi \wedge \psi) \rightarrow \beta^s \varphi \wedge \beta^s \psi$$

For the sake of logical simplicity, it would be wonderful if we could make the assumption that for every speaker s , for every formula φ

$$\beta^s \varphi \text{ or } \beta^s \sim \varphi$$

which is equivalent to the permutation assumption that

$$\beta^s \sim \varphi \text{ if and only if } \sim \beta^s \varphi$$

however such an assumption would be unwarranted. One reason is that to do so would be to presuppose that for every speaker s , for every sentence of B (including sentences which express beliefs about the beliefs of others, etc.), s has an opinion about the truth of that sentence, which is clearly not the case with any actual speaker. For example, few people would boast knowledge of exactly how many water molecules made up the Pacific Ocean.

Hence, for each explicitly consistent speaker s , the language B is partitioned into three mutually disjoint subsets:

$$s^+ = \{\varphi \in B : \beta^s \varphi\}$$

$$s^- = \{\varphi \in B : \beta^s \sim \varphi\}$$

$$s^? = \{\varphi \in B : \sim \beta^s \varphi \wedge \sim \beta^s \sim \varphi\}$$

These are, respectively, the set of sentences s believes true, the set of sentences s believes false, and the set of sentences on which s has no opinion.

We can associate with each speaker $s \in \Gamma$ a function $\pi^s : B \rightarrow \{0, 1, 2\}$ such that

$$\pi^s(\varphi) = 0 \text{ .if and only if. } \varphi \in s^+$$

$$\pi^s(\varphi) = 1 \text{ .if and only if. } \varphi \in s^-$$

$$\pi^s(\varphi) = 2 \text{ .if and only if. } \varphi \in s^?$$

Using belief alone, we cannot define knowledge, however we can express a necessary condition for knowledge

(EP1) For every $\varphi \in B$ for every $s \in \Gamma$ If s knows that φ then $\varphi \wedge \beta^s \varphi$

One useful aspect of Γ being a subset of $\text{ONT}@$ is that we can quantify over speakers: that is, we allow a quantifier to bind the symbol s in $\beta^s \varphi$. We define

$$\text{ISV}^B(F, G) = \text{For every } \varphi \in B, \text{ For every positive integer } i, \varphi \leftrightarrow \varphi[F/G : i]$$

Conjecture 4.01: $\text{ISV}^B(F, G)$.if and only if. $\text{ISV}^L(F, G)$ and For all $\varphi \in B$, For all $s \in \Gamma$,

$$\text{For every positive integer } i, \pi^s(\varphi) = \pi^s(\varphi[F/G : i])$$

Proof:

(\rightarrow) Suppose (*) $\text{ISV}^B(F, G)$. Since L is a subset of B , $\text{ISV}^L(F, G)$.

First we note:

$$\text{(SUB3)} (\beta^s \varphi)[F/G : i] = \beta^s(\varphi[F/G : i])$$

Case 1 : Suppose $\pi^s(\varphi) = 0$

Therefore $\beta^s \varphi$. By (*), $\beta^s \varphi \leftrightarrow (\beta^s \varphi)[F/G : i]$, therefore $(\beta^s \varphi)[F/G : i]$, and by (SUB3) $\beta^s(\varphi [F/G : i])$, therefore $\pi^s(\varphi[F/G : i]) = 0$

Case 2 Suppose $\pi^s(\varphi) = 1$

Therefore $\sim \beta^s \varphi$ and $\beta^s \sim \varphi$ By (*) and (SUB3) we have $\beta^s \varphi \leftrightarrow \beta^s(\varphi[F/G : i])$, therefore $\sim \beta^s(\varphi[F/G : i])$. Similarly by (*) and (SUB3) we have $\beta^s \sim \varphi \leftrightarrow \beta^s(\sim \varphi[F/G : i])$, therefore $\beta^s(\sim \varphi[F/G : i])$, therefore since $\sim \beta^s(\varphi[F/G : i])$ and $\beta^s(\sim \varphi[F/G : i])$, $\pi^s(\varphi[F/G : i]) = 1$

Case 3 Suppose $\pi^s(\varphi) = 2$

Therefore $\sim\beta^s\varphi$ and $\sim\beta^s\sim\varphi$. By (*) and (SUB3) we have both $\beta^s\varphi \leftrightarrow \beta^s(\varphi[F/G : i])$ and $\beta^s\sim\varphi \leftrightarrow \beta^s(\sim\varphi[F/G : i])$, therefore we have $\sim\beta^s(\varphi[F/G : i])$ and $\sim\beta^s(\sim\varphi[F/G : i])$, therefore $\pi^s(\varphi[F/G : i]) = 2$

(\leftarrow) (**)**Suppose** $ISV^L(F, G)$ and For all $\varphi \in B$, For all $s \in \Gamma$, For every positive integer i ,

$$\pi^s(\varphi) = \pi^s(\varphi[F/G : i])$$

Let $\varphi \in B$. Suppose $Bdeg(\varphi) = 1$ Therefore φ is an atomic formula of L .

Since $ISV^L(F, G)$,

$$\text{For every } i, \varphi \leftrightarrow \varphi[F/G : i]$$

(IH) Suppose for all $\psi \in B$ if $Bdeg(\psi) \leq n$ then for all positive integers i , $\psi \leftrightarrow \psi[F/G : i]$

Let $Bdeg(\varphi) = n + 1$

Case 1 $\varphi = \sim\psi$

Since $Bdeg(\psi) = n$, by (IH) $\psi \leftrightarrow \psi[F/G : i]$

$$\sim\psi \leftrightarrow \sim(\psi[F/G : i]) \text{ By contraposition}$$

$$\sim\psi \leftrightarrow (\sim\psi)[F/G : i] \text{ By (SUB)}$$

$$\varphi \leftrightarrow \varphi[F/G : i] \quad \text{Since } \varphi = \sim\psi$$

Case 2 $\varphi = \psi \wedge \chi$ (without loss of generality let $Bdeg(\chi) \leq Bdeg(\psi)$)

Let $j =$ the number of occurrences of G in ψ , and $g =$ the number of occurrences of G in χ

Subcase 1 $i \leq j$

Note $\varphi[F/G : i] = \psi[F/G : i] \wedge \chi$

Since $Bdeg(\psi) = n$, by (IH) $\psi \leftrightarrow \psi[F/G : i]$

Therefore $\psi \wedge \chi \leftrightarrow \psi[F/G : i] \wedge \chi$, therefore $\phi \leftrightarrow \phi[F/G : i]$

Subcase2 $j < i \leq j + g$ (note if $j + g < i$, then ϕ is a fixed point for $[F/G : i]$)

Note $\phi[F/G : i] = \psi \wedge \chi[F/G : i - j]$

Since $Bdeg(\chi) \leq n$, by IH we have $\chi \leftrightarrow \chi[F/G : i - j]$

Therefore $\psi \wedge \chi \leftrightarrow \psi \wedge \chi[F/G : i - j]$, therefore $\phi \leftrightarrow \phi[F/G : i]$

Case 3 $\phi = \beta^s \psi$

By supposition (**) $\pi^s(\psi) = \pi^s(\psi[F/G : i])$ for every positive integer i

If $\beta^s \psi$ is true then $\pi^s(\psi) = 0$, therefore $\pi^s(\psi[F/G : i]) = 0$ therefore $\beta^s(\psi[F/G : i])$ is true

if $\beta^s \psi$ is false, then $\pi^s(\psi) = 1$ or $\pi^s(\psi) = 2$, therefore $\pi^s(\psi[F/G : i]) = 1$ or

$\pi^s(\psi[F/G : i]) = 2$, therefore $\beta^s(\psi[F/G : i])$ is false.

Therefore $\beta^s \psi \leftrightarrow \beta^s(\psi[F/G : i])$, i.e., $\phi \leftrightarrow \phi[F/G : i]$ QED.

This allows us to specify what ISV^B achieves. Firstly, to be ISV in B , predicates must be actually coextensive. This much can be inferred from the fact that L is a subset of B . Secondly, the value of every π function must remain undisturbed whenever one predicate is substituted for the other. That is to say, for every speaker, the predicates can be interchanged with one another without disturbing the attitude which this speaker has towards a sentence.

4.3

Most people would agree that for each speaker there is an upper bound on the degree of complexity a proposition can have before we can neither believe it true or false. We consider the following finitude restriction:

(FIN) There exists $n \in \mathbb{N}$ such that for all $s \in \Gamma$ for all $\varphi \in B$,
 if $n < \text{Bdeg}(\varphi)$ then $\pi^s(\varphi) = 2$

This condition was suggested by Bertrand Russell¹² and it would ensure that each person only has finitely many active beliefs.

Conjecture: If (FIN) then $\text{ISV}^B(F, G)$ is definable in B

Proof:

Suppose (FIN), and let n be such an integer. Enumerate the formulas of B $\varphi^0, \varphi^1, \varphi^2, \dots, \varphi^m$ such that $\text{Bdeg}(\varphi^i) \leq n + 2$. Hence if there exists a speaker s that has an opinion on the truth value of φ , then for some $i \leq m$, $\varphi = \varphi^i$. Consider:

(FIN1)

(x) $(Fx \leftrightarrow Gx) \wedge (\varphi^0 \leftrightarrow \varphi^0[F/G : 1]) \wedge \dots \wedge (\varphi^0 \leftrightarrow \varphi^0[F/G : k^0]) \wedge \dots \wedge (\varphi^m \leftrightarrow \varphi^m[F/G : k^m])$

¹² Russell, B. 1992. *The Principles of Mathematics*. 2nd Edition. London: Routledge. Section 141.

Where k^i = the number of occurrences of G in ϕ^i

Now for every $\psi \in B$ such that $n < \text{Bdeg}(\psi)$, $\pi^s(\psi) = 2$, hence for every positive integer j,

$$\text{If } n < \text{deg}(\psi) \text{ then } \pi^s(\psi) = \pi^s(\psi[F/G : j])$$

Suppose $\text{Bdeg}(\psi) \leq n$. Since for every formula $\psi \in B$, $\psi \leftrightarrow \psi$, if

$$\phi^i \leftrightarrow \phi^i[F/G : 1] \wedge \dots \wedge \phi^i \leftrightarrow \phi^i[F/G : k^i]$$

then for every positive integer j,

$$\phi^i \leftrightarrow \phi^i[F/G : j]$$

Since $\text{Bdeg}(\beta^s \psi) \leq n + 1$, by (FIN1) for every integer j

$$\beta^s \psi \leftrightarrow \beta^s \psi[F/G : j]$$

Similarly, since $\text{Bdeg}(\beta^s \sim \psi) \leq n + 2$, by (FIN1) for every integer j

$$\beta^s \sim \psi \leftrightarrow (\beta^s \sim \psi) [F/G : j]$$

and by (SUB3)

$$\beta^s \sim \psi \leftrightarrow \beta^s (\sim \psi [F/G : j])$$

hence for every positive integer j , $\pi^s(\psi) = \pi^s(\psi[F/G : j])$, hence $ISV^B(F, G)$. QED.

So it turns out that if (FIN) is true, as Russell has argued for, then doxastic ISV is definable in the object language as modal ISV was in Chapter 3, and first order ISV in Chapter 1.

4.4

Doxastic ISV has the virtue of reflecting speaker sensitive properties of knowledge and belief. Doxastic ISV preserves the equivalence of "bachelor" and "unmarried man," in the sense that any competent English speaker will not have a belief about bachelors that does not apply to unmarried men. This is a welcome condition in our pursuit of synonymy. Doxastic ISV preserves the distinction between 'water' and 'H2O' provided Γ includes all speakers throughout history, because at least one of them would have had different beliefs concerning the two expressions.

Connections of synonymy should be brought by considering speakers beliefs about sentences in which two predicates occur, in particular, statements which are generalized over the entire class of speakers. To examine the consequences of ISV, we will consider particular statements of B. These statements will be chosen because they reflect essential relations between a predicate and its negation in the context of belief. We will then apply substitution operators to these sentences to derive propositions which ought to reflect essential relations between meanings in the context of belief. Consider the following

example which is intended to reflect a speaker's ability to recognize the negative implications of the principle of non-contradiction, that is to recognize that certain states of affairs can be said not to exist without any auxiliary information.

$$W(F, G) = \text{For every speaker } s, s \text{ does not believe that } (Ex) (Fx \wedge \sim Gx)$$

And consider the case where $G = F$

$$W(F, F) = \text{For every speaker } s, s \text{ does not believe that } (Ex) (Fx \wedge \sim Fx)$$

Something like $W(F, F)$ is hinted at by our doxastic axioms, however, it does not follow from them, since we can give the belief context a *de dicto* reading. Suppose for some speaker s

$$s \text{ believes that } (Ex) (Fx \wedge \sim Fx)$$

Now, our doxastic axioms rule out the case that this belief was derived from a belief about particular a particular object x via existential generalization, for if s believed that

$$Fx \wedge \sim Fx$$

then DOX3 would yield

$$\beta^s Fx$$

and also

$$\beta^s \sim Fx$$

But with $\beta^s Fx$, DOX1 would yield

$$\sim \beta^s \sim Fx$$

which is a contradiction. However, existential beliefs needn't be derived from existential generalization on *de re* beliefs, even if this is the more common mechanism by which existential beliefs are formed. For example, consider the belief

There is a student in my class who got 100% on the quiz.

This belief is typically derivative of another belief, namely one of the form

Sally, a student in my class, got 100% on the quiz.

with some name in place of 'Sally.' That is to say the belief in the existential statement followed from a *de re* belief, that is a belief about a specific person, through the mechanism of existential generalization. But this is not the only way an existential belief may come about. Consider for instance Quine's example

There are spies.

a statement which most of us believe, however relatively few of us believe a sentence of the form

Orcutt is a spy.

with some name in the place of 'Orcutt.' What is typified here is the case in which a speaker possesses an existential belief, even though this belief was not derived from a *de re* belief about a particular object. The former example of the student is of the form

$\beta^s Fa$ Therefore $\beta^s (Ex) Fx$

whereas the latter example does not depend on the prior possession of a *de re* belief about a particular such as $\beta^s Fa$. One might believe $(Ex) \phi$ because one has an argument in mind against the claim that $(x) \sim\phi$. This is often the case in classical set theory, and marks a point of departure of the intuitionist school of logic, in which existential proofs must proceed from a particular construction by way of existential generalization. So nothing in our doxastic axioms ensures that $W(F, F)$ must hold. To examine what we can infer about speakers who do conform to this condition, consider the open formula

$W(F, F) = s$ does not believe that $(Ex) (Fx \wedge \sim Fx)$

and let the class of speakers that satisfy $W'(F, F)$ for every predicate F of L be called the class of sensible speakers. A sensible speaker is a person who recognizes the fact that no object can exist that is both an F and a non-F, i.e., a person who grasps the principle of non-contradiction in a sense that is indifferent to the *de dicto/de re* distinction. Our doxastic axiom allow speakers to be insensible, but an interesting correlation takes place when we assume that $ISV^B(F, G)$. First note that

$$W'(F, G) = W'(F, F)[G/F : 2]$$

and

$$W'(F, F) = W'(F, G)[F/G : 1]$$

Hence, if ISV holds, s satisfies $W'(F, F)$ if and only if s satisfies $W'(F, G)$. That is to say, that the class of sensible speakers - those who acknowledge the existential consequences of the principle of non-contradiction - coincide exactly with the class of speakers who recognize the incompatibility of the predicates F and G, on the same existential level.

W and W' reflect a speakers ability to recognize the negative implications of the principle of non-contradiction, that is to recognize that certain states of affairs can be said not to exist without any auxiliary information. On the positive side of this principle, we consider

$$P(F, G) = \text{For all speakers } s, s \text{ believes that } (x) (Fx \leftrightarrow Gx)$$

This statement could very well be true, however like W, it involves a *de dicto* belief, and does not follow from our doxastic axioms. If we consider the open expression

$$P'(F, G) = s \text{ believes that } (x) (Fx \leftrightarrow Gx)$$

a similarly interesting correlation occurs if we suppose F and G are interchangeable. Since

$$P'(F, G) = P'(F, F)[G/F : 2]$$

and

$$P'(F, F) = P'(F, G)[F/G : 1]$$

ISV^B(F, G) means that

$$P'(F, G) \text{ if and only if } P'(F, F)$$

and that the class of speakers who satisfy $P'(F, G)$ is precisely the class of speakers who satisfy $P'(F, F)$. Hence, the only kind of person who does not believe that

$$(x) (Fx \leftrightarrow Gx)$$

also doesn't believe that

$$(x) (Fx \leftrightarrow Fx)$$

or that all and only Fs are Fs. Again, as with W this intuitively seems to indicate synonymy, since if F and G are heteronymous, there should be nothing to prevent someone from believing that

$$\sim(x) (Fx \leftrightarrow Gx)$$

while still being able to recognize that every F is an F. Of course, there is still the possibility that widespread collateral information account for this correlation, even though F and G are heteronymous. This is only possible however if such collateral information is universally known by all speakers. If two predicates are doxastically ISV, then no speaker both has a belief involving one predicate and does not have the corresponding belief involving the other.

What we have gained from the perspective of Chapter 3 is sensitivity to the conditions of belief formation; this reflects the intuition that linguistic synonyms should correspond to pairs of beliefs in each speaker. The principle drawback of doxastic ISV, however, is the vulnerability of this criterion to what Quine called 'collateral information,' or mundane, yet non-trivial knowledge that is universally possessed among a community of speakers. Quine argued that the role of collateral information in belief formation is

hopelessly intermingled with our web of belief, and that so-called analyticity could never be cleanly separated from its influence. So long as Γ includes certain historically relevant speakers then water and H₂O are not ISV. If we were to restrict Γ to only those English speakers who are alive in the year 2009, then it is possible that every mentally competent adult is aware of the equivalence of water and H₂O; it is actually easy to imagine an English speaking culture in which every adult knows this. Another example that would likely hold in our time would be the predicates 'country of which Barack Obama is president' and 'country of which George Bush was recently president.'

These considerations lead to the idea that combining doxastic and modal operators might lead us to a positive result. The reason for suspecting this is connected to the difficulty we ran into with examples of W and P above. Doxastic ISV does provide us with interesting generalizations over all speaker's habits of belief which do bring us closer to synonymy. However, there is nothing preventing the presence of universally possessed collateral information from being the basis for the interchangeability of two heteronymous predicates. However, the belief connections that are held together by contingent collateral information are likely to be recognized as beliefs about contingencies, hence such connections may not be manifest when the speakers are asked about the possibilities concerning Fs and Gs as opposed to the factual situation. Also, the universal distribution of such collateral information can itself be a contingent, or accidental fact, in which case there would be possible worlds in which such information is not universally possessed, hence disrupting ISV. We will explore these points in the next chapter.

Chapter 5

5.1

In Chapters 3 and 4 we examined the properties of ISV in languages that are richer in structure than standard predicate logic, in the attempt to bring the formal language of investigation closer to the structure of natural language, without the possibility of quotation, spelling or any other equivalent procedure that collapses ISV to the identity relation on strings of symbols. ISV in the modal language of Chapter 3 brought us closer to synonymy, in that it reflected the element of necessary connection which synonymous predicates ought to have. ISV in the doxastic language of Chapter 4 lacked this element of necessary connection, but reflected the element of epistemic or doxastic equivalence that synonyms should have with respect to one another, a property of synonymy which was lacking in the modal language of Chapter 3. Both may be said to reflect essential ingredients of meaning and synonymy. Synonyms should enjoy all the same modal properties, and they should be constitutively equivalent with respect to belief formation. It is desirable to combine these two ingredients together.

In the natural language of everyday life, such restrictions are missing; we can speak both of possibility and of belief. We do not use separate languages for either task, both modal and doxastic terms occur together in our ordinary speech. In this chapter, we investigate the nature of interchangeability in languages that contain fragments isomorphic to M and B from Chapters 3 and 4.

There are at least two ways in which this combination can be carried out. The simplest way would be to consider as a criterion for synonymy, conjunctions of the form

$$ISV^M(F, G) \wedge ISV^B(F, G)$$

where F and G are predicates of L.

Unfortunately, however, there are counterexamples that threaten the idea that ISV in both B and M can be identified with synonymy of predicates. Before focusing on specific examples, we lay out the conditions in which two heteronymous terms could be ISV in both B and M. Firstly, we need a pair of necessarily coextensive, yet heteronymous predicates. Secondly, an empirical fact must be the case; provided that the culture of speakers is suitably educated, it may be a common belief that the predicates are coextensive, and a common belief that it is a common belief that the predicates are coextensive, etc., thereby making them ISV.

For example, suppose the group of speakers of this language consists wholly of an isolated culture living above the Arctic Circle, and suppose the predicates translate into English as "water" and "melted snow." In such a case it would be critical to the survival of the group to recognize that drinking water can be obtained by warming up snow, hence it is plausible to assume that every speaker, at least every speaker of a certain minimal level of maturity is educated on this fact; it would be a mundane artifact of common knowledge that every speaker had visual contact with from the earliest age. Since both predicates refer to natural kinds, it is necessary that they are coextensive. However it surely is not an analytic truth that melting snow results in drinking water. The empirical content of this truth is non-

trivial, and hence is not a matter of mere synonymy of terms. This is reflective of the traditional assumption that analytic truths must be devoid of genuine empirical content.

The interference of common education, and common knowledge makes ISV, even in both B and M, vulnerable to counter examples like the one above. This is what Quine referred to as collateral information. The presence of collateral information, with genuine empirical content, when applied to natural kinds can result in the interchangeability of heteronymous terms under the scope of both modal and doxastic operators. If the meaning of a predicate is to be isolated or distilled in terms of the symbols with which it is interchangeable, then we must take some measure to guard against this interference of empirically significant information distributed in the form of commonly possessed collateral information.

It should be noted here that the interference of collateral information when trying to cleanly separate the analytic from the synthetic played a major role in Quine's overall assault on the concepts of analyticity and meaning, as discussed¹³ in (Quine, 1960). There is no way, Quine alleged, to separate the so-called analytic truths from those obvious truths of common knowledge such as "There have been black dogs." From Quine's perspective this obstacle is insurmountable, however this is because he denied the admission, and hence the logical utility of modal concepts, of which we have availed ourselves fully here. It is this key ingredient that hopefully will make the difference for us.

In the example above, we have a community in which widespread knowledge, or universally possessed collateral information concerns the connection of two natural kind terms, i.e., concerns a necessary truth. The knowledge of this connection however can not

¹³ Quine, W. V. 1960. *Word and Object*. Cambridge, MA: MIT Press.

be obtained by mere reflection on the synonyms of the language. The connection itself is necessary, but the fact that these speakers have knowledge of this connection is an accident, or a contingent matter. Regarding this distinction, it seems what we need is some way of speaking about accidents of belief. With this in mind, we go on to construct a language in which doxastic operators can occur *within* the scope of modal operators, and modal operators can occur within the scope of doxastic operators. We define the language H as follows:

(HI) If ϕ is a formula of L (with or without free variables) then
 ϕ is a formula of H

(HII) If ϕ and ψ are formulas of H (with or without free variables) then so
are $\Box\phi$, $\phi \wedge \psi$, $\sim\phi$, $\beta^s\phi$ and $(x)\phi$ for any variable x of L

(HIII) Nothing else is a formula of H; H is the smallest possible language that
satisfies (HI) and (HII)

By this construction, clearly both M and D are subsets of H. We now add on the axioms for belief:

(DOX1) For all speakers s , in every world w , $\beta^s\phi \rightarrow \sim\beta^s\sim\phi$

(DOX2) For all speakers s , in every world w , $\beta^s\sim\phi \rightarrow \sim\beta^s\phi$

(DOX3) For all speakers s , in every world w , $\beta^s(\phi \wedge \psi) \rightarrow \beta^s\phi \wedge \beta^s\psi$

5.2

In this section, an argument will be presented that synonymy can be associated with ISV in H, via a consequence Γ of ISV^H that can be stated in the language H itself. It will be argued that this consequence is a sufficient criterion for the identity of meaning. To identify Γ , we consider sentences with two or more occurrences of predicate symbols, such that the substitution of the same predicate for each occurrence always leads to a true statement, no matter which predicate we choose. This method is inspired by an example mentioned in Chapter 1 from Quine's *Two Dogmas*:

(4) Necessarily all and only bachelors are bachelors
is evidently true, even supposing 'necessarily' so narrowly
construed as to be applicable only to analytic statements¹⁴

The sentence Quine uses is of the form

Necessarily all and only Fs are Gs

A key feature of this sentence schema is that upon substitution of the same predicate for both F and G, the schema results not only in a true sentence, but one that is true *a priori*.

Necessarily all and only Fs are Fs

¹⁴ Quine, W. V. 1980. *From a Logical Point of View*. 2nd Edition. Cambridge, MA: Harvard University Press.

This result holds not only for bachelors, but for any other predicate. Notice that

"Necessarily all and only Fs are Fs"[G/F : 2]

= Necessarily all and only Fs are Gs

Hence

Necessarily all and only Fs are Gs

is a consequence of $ISV(F, G)$. According to Quine, this consequence of ISV would be a sufficient condition to assert the synonymy of F and G , however for reasons argued in Chapter 3, we do not recognize this as a sufficient condition.

Our reason for adopting a strategy based on this example is this: to examine whether ISV^H should be a sufficient condition for synonymy of predicates, we need to look at the consequences on the relationship of two predicates, F and G , when $ISV^H(F, G)$ holds. If G is to be substituted for F , we need to start with a sentence containing F , which is assumed to be true. This approach is specifically modeled after Quine's example above. Let us suppose that $\Gamma(F, G)$ is some formula of H in which the predicates F and G each occur at least once. There are two key features relevant to the selection of $\Gamma(F, G)$: (i) that $\Gamma(F, F)$ be true for every predicate F , preferably true *a priori*, and (ii) that $\Gamma(F, G)$ be a sufficient condition for the synonymy of F and G . What sorts of sentences are good candidates for $\Gamma(F, G)$?

Recall that above, a subtle distinction was made between accidental and non-accidental knowledge, connected with the example of collateral information of a truth that is

necessary, where the speakers possession of such collateral information is contingent. To cleave synonymy away from the influence of collateral information requires this distinction because the possession of a belief that constitutes collateral information can be an accidental state of affairs. To pursue this notion, we must consider possible states of belief. Synonymy of distinct terms will generate pairs of necessarily connected beliefs, if beliefs are to be individuated on the basis of the sentence which asserts their content. That is to say, synonymy of terms will result in connections between distinct belief statements that are forced upon the speaker. Later in this chapter we will return to this idea negatively when arguing that heteronymy is marked by the lack of such forced connections between belief.

The distinction we are looking for, which involves possible states of belief, should be reflected in statements containing the various permutations of modal and doxastic operators. To discuss possible states of belief, we should consider sentences in which a doxastic operator occurs within the scope of a modal operator:

$$\diamond\beta^s\phi$$

where a quantifier may or may not occur to the left of the diamond. On the other hand, to discuss peoples attitudes about possibility, we would consider sentences where a modal operator lies within the scope of a doxastic operator:

$$\beta^s\diamond\phi$$

again, where a quantifier may or may not occur to the left of the beta. Hence, the choices for Γ that we will consider will range over statements containing the elementary permutations of the modal operators, doxastic operators and quantifiers.

The argument of this section is intended to demonstrate that ISV^H is a sufficient condition for synonymy, and it will have the following form:

A1. If the meaning of $F =$ the meaning of G , then $ISV^H(F, G)$

A2. For every predicate F of L , $\Gamma(F, F)$

A3. If $ISV^H(F, G)$ then $\Gamma(F, F)$ implies $\Gamma(F, G)$

A4. Therefore by A3 and A2, $ISV^H(F, G)$ implies $\Gamma(F, G)$

A5. If $\Gamma(F, G)$ then the meaning of $F =$ the meaning of G

A6. Therefore by A4 and A5, if $ISV^H(F, G)$

then the meaning of $F =$ the meaning of G

A7. Therefore by A1 and A6, $ISV^H(F, G)$ if and only if

the meaning of $F =$ the meaning of G

A1 is a consequence of the compositionalist view of meaning adopted in Chapter 1. A2 indicates a key feature determining the selection of Γ ; we are specifically interested in Γ such that $\Gamma(F, F)$ is true for all predicates F . Our reason for this should be evident in the logical structure of the schematic argument A1 ... A7. The strength of the assumption A2 will rely on an argument that $\Gamma(F, F)$ is true, and hence will depend on the choice of Γ . Ideally we would prefer Γ such that for every predicate F of L , $\Gamma(F, F)$ follows from our doxastic axioms and assumptions about the possible worlds structure underlying the modal

operators. A3 is a demonstrable inference, since $\Gamma(F, G) = \Gamma(F, F)[G/F :2]$. A5 indicates another key feature in the selection of Γ , namely that Γ must be a sufficient condition for synonymy. As with A2, the argument for A5 will depend on the choice of Γ . A3, A4 and A7 follow from their predecessors by familiar logical rules.

In light of the criteria we have given for the selection of Γ , let us consider the following alternatives. One candidate for assumption $\Gamma(F, G)$ would be

$$T(F, G) = \text{For every speaker } S, \text{ it is not possible that } S \text{ believe that} \\ (Ex) (Fx \wedge \sim Gx)$$

Given universal knowledge among speakers of the most elementary aspects of predication and negation, it seems reasonable to assume that $T(F, F)$ is true for every predicate F :

$$(PB1) \text{ For every speaker } S, \text{ it is not possible that } S \text{ believe} \\ \text{that } (Ex) (Fx \wedge \sim Fx)$$

(PB1) is a reasonable hypothesis, provided we insist that each speaker conform to certain basic, minimal conditions of consistent thought, and its falsehood does seem to clash with our doxastic axioms. It does seem reasonable to assume that $T(F, G)$ is false wherever F and G stand for heteronymous predicates, such as the case of above of water and melted snow. A competent speaker who is confused could easily believe that melted snow is gasoline, or vodka (perhaps she was tricked by a friend to think this), provided she had not been educated on this piece of collateral information.

Another permutation of modal and doxastic operators that could evade the interference of collateral information, and hence be a condition towards synonymy, would be $U(F, G)$ as follows:

$$U(F, G) = \text{For every speaker } s, s \text{ does not believe that it is possible} \\ \text{that } (Ex) (Fx \wedge \sim Gx)$$

Consider

$$U(F, F) = \text{For every speaker } s, s \text{ does not believe that it is possible that} \\ (Ex) (Fx \wedge \sim Fx)$$

If the world were made up of perfect speakers, then for every predicate F , $U(F, F)$ ought to be true. However, this may not be the case. Consider the open statement schema

$$U'(F, G) = s \text{ does not believe that it is possible that } (Ex) (Fx \wedge \sim Gx)$$

Let us say that s is a lawful thinker if s satisfies every substitution of a predicate of L for F in $U'(F, F)$. That is to say, s is a lawful thinker provided that s respects the principle of non-contradiction, and moreover understands the modal ingredient of this principle, that is that contradictions are not merely false, but are necessarily false. A lawful thinker is not simply a speaker who believes in no contradiction (which is guaranteed by our doxastic axioms), but one who actively believes that contradictions are impossible.

In chapter 4, we considered the statement

$$W(F, G) = \text{For every speaker } s, s \text{ does not believe that } (Ex) (Fx \wedge \sim Gx)$$

The problem being that, although on the supposition that $W(F, F)$ is true for all predicates F , $ISV^B(F, G)$ gives us $W(F, G)$, there is no way of distinguishing whether the speakers possession of this belief is a consequence of universally held collateral information on the coextensiveness of F and G . Quine's 'creature with a heart,' 'creature with a kidney' example comes to mind here, particularly in connection with the example of a hunter gatherer society in which all people take part in the preparation of harvested game into food. However, the inclusion of the modal operator in $U(F, F)$ evades such possible collateral influence. The lawful speaker who satisfies every formula of the form $U(F, F)$ could easily satisfy

$$W(F, G) = s \text{ does not believe that } (Ex) (Fx \wedge \sim Gx)$$

but not satisfy $U(F, G)$. Such differences would indicate precisely the points at which the influence of collateral information occurs.

What is interesting about the lawful/unlawful distinction that has been raised is this: provided $ISV^H(F, G)$ holds, it is precisely the set of lawful thinkers, i.e., those speakers who comprehend the modal force of the principle of non-contradiction that are the very speakers who recognize the truth of statement

$$\sim \diamond (Ex) (Fx \wedge \sim Gx)$$

since

$$U(F, G) = U(F, F)[G/F : 2]$$

Hence by ISV, if s satisfies $U(F, F)$ then s satisfies $U(F, G)$. That is, on the hypothesis that ISV holds for F and G , it is precisely those speakers who respect the modal force of the principle of non-contradiction who also affirm the impossibility of something being an F and not a G , something we should not expect them to do in the case of heteronymous predicates. It is possible, and moreover very likely that at least one lawful speaker believe

$$\diamond(\exists x) (Fx \wedge \sim Gx)$$

when F and G are heteronymous, regardless of their possession of collateral information that F and G are contingently coextensive. The only kind of collateral information that could easily by pass this sieve would be knowledge of a necessary connection, and moreover knowledge that the connection is necessary.

A third arrangement of modal and doxastic operators would be

$$V(F, G) = \text{For every speaker } S, \text{ it is not possible that } S \text{ believe that it is} \\ \text{possible that } (\exists x) (Fx \wedge \sim Gx)$$

As with $T(F, F)$, it could be reasonable to assume that $V(F, F)$ is true for every predicate of the language, provided that each speaker is acquainted with the minimal conditions of

consistent thought and speech, i.e., that every speaker is necessarily lawful. Little can be achieved in any case if we do not assert such minimal conditions of consistency of thought. It is not the universal knowledge that contradictions are simply always false that makes $V(F, G)$ true, as is the case with $T(F, G)$, but rather the universal knowledge that contradictions are always necessarily false.

(PB2) For every speaker S, it is not possible that S believe that it is possible that $(\exists x) (Fx \wedge \sim Fx)$

(PB1) and (PB2) are merely hypotheses, suggesting a link between the predicate and itself that may illuminate the subject of synonymy. However, they do not follow directly from our doxastic axioms. T , U , and V also share the common trait of being expressions of *de dicto* belief (or possibility). In other words, no variables occur free within the scope of a modal or belief operator.

As opposed to this, we now take a look at the *de re* case. Let's consider the class of sentences of H that are of the following form:

$$Q(F, G) = (\forall x) (\forall s) (\text{Speaker}(s) \rightarrow \sim \diamond \beta^s (Fx \wedge \sim Gx))$$

Where F and G are predicates of L. In words, we have

$Q^{\text{Eng}}(F, G) =$ "For all things x, For all speakers s it is not possible that s believe that x is both an F, and not a G"

Now, it follows from our doxastic axioms that $Q(F, F)$ is true for every predicate F .

Consider:

$$\sim Q(F, F) = \sim(x) (s) (\text{Speaker}(s) \rightarrow \sim\Diamond\beta^s(Fx \wedge \sim Fx))$$

Which is equivalent to

$$(\exists x) (\exists s) (\text{Speaker}(s) \wedge \Diamond\beta^s(Fx \wedge \sim Fx))$$

Hence, in some possible world W , for some speaker in W , say Janet, there is an object o such that Janet believes that

$$Fo \wedge \sim Fo$$

Therefore, by (DOX3), modus ponens and simplification of the conjunction, Janet believes that Fo ,

$$\beta^{\text{Janet}} Fo$$

and by (DOX1) and modus ponens, Janet does not believe that $\sim Fo$.

$$\sim\beta^{\text{Janet}} \sim Fo$$

However, it is also a consequence of (DOX3) that Janet does believe that $\sim F_0$,

$$\beta^{\text{Janet}} \sim F_0$$

a contradiction. Hence the truth of $Q(F, F)$ is secured by our doxastic axioms for every predicate F.

So if we suppose that $ISV^H(F, G)$, it follows that $Q(F, G)$ must also be true, since $Q(F, G) = Q(F, F)[G/F : 2]$. In words, we can express $Q(F, F)$ as

(PB3) For all things x, For all speakers s, it is not possible that s believe
that x is an F and not a F.

Note (PB1) and (PB2) are mere hypotheses, whereas (PB3) follows directly from our doxastic axioms.

Notice that $Q(F, G)$ can be true even if F and G have overlapping meanings, yet are not synonymous: let F = 'blue and heavy,' and let G = 'blue.' In such a case, it is demonstrable from our doxastic axioms that $Q(F, G)$ holds. Notice, however, that $Q(G, F)$ is presumably false: someone can believe that a box is blue but not blue and heavy. To remedy this asymmetry in our prospective criterion, we define:

$$Q^*(F, G) = Q(F, G) \text{ and } Q(G, F)$$

To argue that Q^* is a sufficient criterion for synonymy, we pursue the contrapositive. As indicated in the example above, heteronymy comes in degrees: sometimes the meanings of two terms overlap, sometimes they are completely distinct. If they overlap, some restrictions may apply regarding their combinatorial properties within contexts of possible belief. If the two meanings don't overlap at all, as in the case with 'blue' and 'heavy,' then they should be able to combine freely in belief contexts. This is the thesis of free combination. To state this claim more precisely, we quantify over the actual speakers in the world:

(FC) If F and G are heteronymous then

$$(Ex) (Es) (\diamond\beta^s(Fx \wedge \sim Gx) \vee \diamond\beta^s(Gx \wedge \sim Fx))$$

In other words, if F and G are not synonymous then there must be combinatorial freedom within possible belief states regarding a single object, and its predication with one, and the negation of the other.

We are now in the position to assert the contrapositive argument. Suppose F and G are heteronymous. Then by (FC),

$$(Ex) (Es) (\diamond\beta^s(Fx \wedge \sim Gx) \vee \diamond\beta^s(Gx \wedge \sim Fx))$$

By existential instantiation, there is a speaker s and an object o such that

$$(DISJ) \diamond\beta^s(Fo \wedge \sim Go) \vee \diamond\beta^s(Go \wedge \sim Fo)$$

Suppose $Q^*(F, G)$. Therefore $Q(F, G)$ and $Q(G, F)$. Therefore by universal instantiation

$$\sim\Diamond\beta^s(Fo \wedge \sim Go)$$

and

$$\sim\Diamond\beta^s(Go \wedge \sim Fo)$$

which by disjunctive syllogism with (DISJ) gives

$$\Diamond\beta^s(Fo \wedge \sim Go)$$

a contradiction. Therefore $Q^*(F, G)$ is false. Since the heteronymy of F and G implies the negation of $Q^*(F, G)$, we conclude that $Q^*(F, G)$ implies the synonymy of F and G.

(SUF1) If $Q^*(F, G)$ then the meaning of 'F' = the meaning of 'G'

An immediate corollary of this is:

(SUF2) if $ISV^H(F, G)$ then the meaning of 'F' = the meaning of 'G'

Proof: $Q^*(F, F)$ is assumed true. Suppose $ISV^H(F, G)$. Therefore,

$Q^*(F, F)[G/F : 2][G/F : 2] = Q^*(F, G)$ is also true.

Therefore by (SUF1), the meaning of 'F' = the meaning of 'G.' QED.

Recall, we have said that ISV in any language (without quotation, or an equivalent device) is a necessary condition for synonymous predicates. (SUF2) states that ISV^H is a sufficient condition for synonymy of predicates. Hence

(SYN) Predicates F and G are synonymous if and only if $ISV^H(F, G)$

Let us now sum up some important points stated so far. Firstly, in Chapter 1, it was stated that ISV is a necessary condition for synonymy, provided that quotation is not possible in the language considered. This is a direct consequence of the compositionalist view. If the meaning of a complex symbol is determined by the meanings of its constituent symbols, together with their arrangement then the substitution of synonyms for synonyms within a complex symbol cannot result in a difference in the meaning of the complex. If two complex symbols have the same meaning, then they must either both be true, or both be false, hence ISV is implied by synonymy, or equivalently, ISV is a necessary condition for synonymy.

It has been demonstrated that statements of the form $Q^*(F, F)$ are true upon the substitution for every predicate F, so we can rely on the assumption that $Q^*(F, F)$ holds for

any predicate F . Supposing that $ISV^H(F, G)$ is the case, then $Q^*(F, F)$ implies $Q^*(F, G)$. This is a direct result, and cannot be questioned, since

$$Q^*(F, G) = Q^*(F, F)[G/F : 2][G/F : 2].$$

Now, if we suppose that $Q^*(F, G)$ is a sufficient condition for the synonymy of F and G , two things follow. Firstly, $ISV^H(F, G)$ is a sufficient condition for the synonymy of F and G , and secondly that $ISV^H(F, G)$ if and only if $Q^*(F, G)$. The first follows from the sufficiency of $Q^*(F, G)$ for synonymy coupled with the truth of $Q^*(F, F)$. The second follows from left to right by the truth of $Q^*(F, F)$ for all predicates F , and from right to left by the compositionalist principle stated above that ISV is a necessary condition for synonymy. The sufficiency of ISV^H for synonymy means we have reached a positive end to our inquiry, for it turns out that synonymy can be identified with ISV in the appropriate language. This result indicates the importance of pursuing the problem of doxastic and polymodal logics, especially with respect to the theory of meaning. The equivalence of $ISV^H(F, G)$ and $Q^*(F, G)$ shows that, as in all languages considered thus far, ISV is translatable as a condition which can be asserted within the object language, hence the semantic transparency found in our earlier examples continues to hold in the polymodal language.

Conclusion

Let us summarize the argument thus far. In Chapter 1, we closely examined Quine's discussion of synonymy in Two Dogmas. Quine established that two predicates are ISV in a first order language if they are coextensive, and hence ISV in an extensional language cannot be associated with synonymy. In other words, ISV can be translated into a sentence of the first order language

$$(x) (Fx \leftrightarrow Gx)$$

We considered an inductive proof of this fact, not so much to verify Quine's claim, but to lay out the foundation of how to prove things about ISV in other languages in subsequent chapters. Quine suggested that two predicates should be synonymous if they are ISV in a modal language, however he warned that this would be a circular definition of synonymy due to his identification of necessity with analyticity. We gave reasons for doubting this latter claim, and indicated that we would construct a modal language and examine the consequences of ISV in it.

In Chapter 2, we temporarily set aside the task of examining modal ISV so as to give a justification for the project initiated in Chapter 1. In Chapter 2, it was argued that Quine was right when he asserted that synonymy is of primary concern to the theory of meaning. The importance of the synonymy relation is two-fold: (1) because a compositionalist view of meaning is assumed, the problem of analyticity can be reduced to that of synonymy, since a sentence is analytic if and only if it can be transformed into a logical theorem by substituting

synonyms for synonyms, and (2) if meanings in the abstract sense are to be introduced into one's ontology, then the conditions of identity and distinction among meanings are of primary concern, otherwise we have not clearly defined what type of thing meanings are in the first place. It was argued that the conditions of identity give the essential content of abstract postulated types, as exemplified in the principle of extensionality in the theory of sets.

It was suggested in Chapter 2 that, if all parties could agree on the extension of the synonymy relation, then a bridge could be constructed between nominalist and realist when speaking about meaning. Synonymy, like any other form of identity, is an equivalence relation. The nominalist can simply view the meaning of a predicate as its equivalence class under the synonymy relation, and thereby translate the realist's talk of abstract meanings accordingly. As long as they agree on the synonymy relation, then communication regarding meanings would be secured. In this sense, a deflationary theory of meaning is the intended outcome of this dissertation.

In Chapter 3, we returned to the last points of Chapter 1, and followed Quine's suggestion in examining ISV in a modal language. A modal language M was constructed from the first order language considered in Chapter 1, and a semantic interpretation was given in terms of possible worlds. Because of difficulties arising in the context of *de re* modality, or situations in which a variable occurs free within the scope of a modal operator, we focused our attention on quantified S5, where the domain of individuals remains constant from world to world. Because of this move, we are able to consider sentences of the form

$$(x) \Box \phi \text{ and } \Box (x) \phi$$

as equivalent and interchangeable. We noted that in both C. I. Lewis's original propositional modal logic, as well as in Ruth Barcan Marcus's quantified extension of Lewis's systems, ISV can be equated with necessary coextension. That is, ISV in a modal language is equivalent to a statement within the modal language

$$(\mathbf{x}) \Box(\mathbf{F}\mathbf{x} \leftrightarrow \mathbf{G}\mathbf{x})$$

This is a similar result to that found in Chapter 1, where ISV, a statement which at first glance requires variables that range over the entire language, and hence should be stated in a metalanguage can be translated into a condition which can be stated within the object language itself. Using the proof given in Chapter 1 as a template, a proof of this fact was given which exploited the recursive structure of the language M.

In connection with this proof, we defined the intension of a predicate to be the function which assigns to each possible world the extension of the predicate in that world, and the intension of a sentence with the function which assigns to each world the truth value of the sentence in that world. Another way of stating the result that ISV is equivalent to necessary coextension is that the intension of a sentence depends functionally on the intensions of the predicates which occur in the sentence. Hence we have a hierarchy of semantic objects: in propositional logic, the semantic content of a symbol is its truth value, in first order logic, the semantic content of a predicate is its extension, and in quantified modal logic, the semantic content of a predicate is its intension function.

In Chapter 3 it was argued that it is possible for heteronymous predicates to have the same intension function. First it was noticed that any two self-contradictory predicates have the intension which assigns the empty set to each world, and any two tautologically valid predicates have the intension which assigns $\text{ONT}(w)$ to each world w . After this, it was noted that Kripkean natural kinds also exhibit this phenomenon where apparent heteronyms share a common intension function, because natural kind terms are necessarily coextensive. Hence it was concluded that intension functions are not fine-grained enough to be identified with meanings, and that contrary to Quine's suggestion in *Two Dogmas*, ISV in a modal language cannot be identified with synonymy. At the end of Chapter 3, it was suggested that what ISV in a modal language lacks is the reflection of the *a priori* aspects of meaning and synonymy.

Since *a prioricity* is an epistemic property of propositions or judgments, we pursued these *a priori* aspects of synonymy in Chapter 4 by considering its relationship with belief states associated with speakers of the language. Here we constructed a doxastic language from the first order language given in Chapter 1 by supplementing it with operators of the form

Speaker s believes that ...

for each speaker of the language. Associated with each speaker, we defined a function which records the attitude the speaker has towards each sentence of the language, whether she believes it, denies it, or does neither. It was demonstrated that ISV holds between two predicates if and only if they are coextensive and their mutual substitution does not modify

the output of any attitude function. Differing from the results of Chapters 1 and 3, this characterization of ISV cannot be stated within the doxastic language.

Doxastic ISV can however be associated with a sentence of the doxastic language provided that for each speaker, there is an upper bound to the degree of complexity which a proposition can have in order for the speaker to have an opinion about its truth value. This idea of finite complexity was derived from Bertrand Russell. A proof was given in connection with this fact. Hence, the semantic transparency found in the languages examined in Chapters 1 and 3 is repeated in Chapter 4 in the doxastic language, however in a somewhat less elegant way. Unlike propositional, first order, and quantified modal logic, there was no clear semantic object which could be associated with a predicate such that two predicates are ISV if and only if they are associated with the same semantic object.

What was gained from Chapter 3 was sensitivity to the *a priori* aspects of synonymy and analyticity, however what was lost was the essential, or necessary connections between predicates when considering the doxastic language of Chapter 4. Because of this, ISV in a doxastic language is vulnerable to the influence of what Quine called collateral information, or non-trivial factual knowledge which is universally possessed by every speaker of the language. Two predicates can be actually coextensive, and also be interchangeable within the belief contexts of all speakers, not because they are synonymous, but because they are linked together by such widespread knowledge of non-trivial facts. Using counterexamples constructed from common knowledge in the contemporary English speaking world, it was concluded that doxastic ISV on its own is not a sufficient condition for synonymy.

In Chapter 5 we examined various ways of combining the elements of ISV from Chapters 3 and 4. When we asked whether ISV is a sufficient condition for synonymy, the

counterexamples in Chapter 3 showed that modal ISV does not reflect the *a priori* elements of synonymy and analyticity, whereas the counterexamples in Chapter 4 involved widespread knowledge of contingent facts, showing that doxastic ISV does not reflect the element of necessary connection between synonyms. So the most natural next step was to ask whether predicates can be identified as synonymous when they are ISV both in the modal and the doxastic languages. This approach was rejected on the grounds that *a posteriori* collateral information can apply to natural kinds, and hence that counterexamples similar to those occurring in Chapter 4 can be raised against the claim that ISV in both languages captures synonymy.

Such counterexamples made a distinction between two kinds of contingency: an item of knowledge may be a fact that is or is not contingent: however, the speaker's possession of such knowledge is also a fact, and we can similarly ask whether the possession of this knowledge is a contingent matter. This opened up the question of possible states of belief. Sentences which make assertions about possible states of belief are those in which a doxastic operator occurs within the scope of a modal operator. With this in mind, we constructed a new polymodal language which included both kinds of operators seen in Chapters 3 and 4.

The approach then undertaken was modeled after an aspect Quine's 'closed curve' argument in *Two Dogmas*. We examined sentence types which contain two predicate symbols:

$\Gamma(F, G)$

Specifically we searched for sentence types which, upon substitution of the same predicate in each of the two predicate places, resulted in a true statement, regardless of which predicate was chosen:

For all F, $\Gamma(F, F)$ is true

Our purpose was to find a sentence that would be a sufficient condition for synonymy. If $\Gamma(F, F)$ is true, then $ISV(F, G)$ implies $\Gamma(F, G)$. Hence, if $\Gamma(F, G)$ is a sufficient condition for synonymy, then so is $ISV(F, G)$.

We examined several alternatives for Γ , most of which involved *de dicto* modality and beliefs. They each seemed to be plausible candidates for synonymy, however the truth of $\Gamma(F, F)$, however intuitively compelling, was only derivable from our doxastic axioms when we considered a *de re* case, in which the quantifier bound a variable that occurred free within the scope both of a modal and a doxastic operator:

$$Q(F, G) = (\exists x) (\exists s) (\text{Speaker}(s) \rightarrow \sim\Diamond\Box^s(Fx \wedge \sim Gx))$$

To make this criterion symmetric, we defined

$$Q^*(F, G) = Q(F, G) \wedge Q(G, F)$$

It turned out that $Q(F, F)$ was demonstrable from our doxastic axioms, each of which was chosen based on intuitive and *a priori* considerations of the nature of belief. For example, if

a speaker believes that hot dogs are unhealthy, then she does not believe that hot dogs are healthy, and if a speaker believes that orange juice is both healthy and tastes good, then she believes that orange juice is healthy.

It was argued that $Q^*(F, G)$ is a sufficient condition for synonymy by pursuing the contrapositive: that if F and G are heteronymous, then at least one of $Q(F, G)$ and $Q(G, F)$ is false. This was called the thesis of free combination, namely that if two predicates are heteronymous then it is possible for a speaker to believe of an object that it is an instance of one predicate and not an instance of the other. With this, a positive conclusion to this dissertation was reached, in that ISV in the polymodal language is a sufficient criterion for the synonymy of predicates.

In Chapter 2, ISV was to be the tool from which we would construct a deflationary theory of meaning, of which one of the principle advantages is that it enables the nominalist to translate realist talk of meanings into her own vocabulary in a way that respects the synonymy relation. However this is not the only interpretation that is compatible with the results of Chapter 5. We now consider a realist interpretation of the argument thus far.

According to the argument presented in 5.2, $ISV^H(F, G)$ if and only if $Q^*(F, G)$. In Chapter 2 it was suggested that the meaning of a predicate F be associated with its equivalence class under ISV, thus in this case:

$$\{G : Q^*(F, G)\}$$

So if F and G are synonymous then for every object o and every speaker s, s is not free believe that Fo without believing that Go, nor is s free to believe that Go without believing that Fo. An explanation of this fact will take the form of a particular example. We say that

(P) Lewis is Quine's pupil.

(S) Lewis is Quine's student.

are distinct sentences, but they are associated with one and the same proposition. Notice,

$\beta^s(P)$

and

$\beta^s(S)$

are also distinct sentences. Propositions, not sentences are the objects of belief. The compositionalist view of meaning is compatible with an essentially mereological view of the proposition; that the meaning of a symbol occurring in a sentence is a part of the meaning of the complete sentence, i.e., the proposition associated with a sentence. Hence a speaker is not free to believe that (P) without believing that (S); to believe one is to have the relation to the proposition which constitutes believing in the other. It is literally the same cognitive state of affairs to believe that (P) as it is to believe that (S). That is

$\beta^s(P)$

and

$\beta^s(S)$

describe one and the same state of affairs, although they are different sentences. Hence it is not possible that one be true and the other false. This inference does not depend on concrete particular facts, hence it should hold in any possible worlds in which there are speakers.

Conversely, the principle of free combination is based on the idea that, given a sentence ϕ , if F and G are heteronymous, then the proposition associated with ϕ contains a part that the proposition associated with $\phi[F/G : i]$ does not have and hence they are necessarily distinct objects of belief, and there is nothing stopping a person from believing one without the believing in the other.

Regarding future developments of the ideas laid out here, the formal aspects of the doxastic language of Chapter 4, and of the polymodal language in Chapter 5 need to be further clarified. It is clear that from the doxastic axioms which have been chosen, ISV will generate pairs of distinct symbols that cannot be freely combined in possible belief contexts, however much needs to be investigated with respect to the structure and semantics of polymodal languages, in particular identifying those models and axioms sets for which $Q^*(F, G)$ is a sufficient condition for ISV(F, G). Furthermore, it is desirable that the proof given in Chapter 3 be generalized beyond the logic of S5, preferably a proof for quantified S4 that does not depend on either the Barcan or Converse Barcan formula.

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