Barriers To Entry and Market Coverage in Vertically Differentiated Markets

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Αρνάκι άσπρο και παχύ
της μάνας του καμάρι...
(Demotic Greek Poetry)

Στους γονείς μου
Παναγιώτη και Αρχοντία.

Acknowledgment

Γηράσκω αεί διδασκόμενος

These words of Socrates can be interpreted as 'education never ends'. However, a Ph.D. thesis is the last requirement to one's *formal* education and for this reason I would like to express through this page my gratitude to all those who have helped me to reach and complete this final stage. There are too numerous for all to be named in this short space, I wish however to mention some of them who were particularly important for my academic development, starting with professors G. Kottis and P. Korliras of the Athens School of Economics and Business, whose influence and encouragement induced me to pursue graduate studies.

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Abstract

The literature on product heterogeneity distinguishes two kinds of differentiation: *quality differences* when there is unanimity in consumer preferences with respect to the different types of a product, and *variety* when such unanimity is absent. The concept of *vertical differentiation* is used in this thesis to denote cases characterised by quality differences and in which the unanimity of consumer preferences translates to a unanimous choice when all the products are priced at average cost.

In this context, a number of authors have shown that there is a limited number of single-product firms that can have a positive market share in a Bertrand-Nash equilibrium (finiteness property). This upper bound to the number of firms is independent of the size of the market, depending only on the width of the consumer income distribution. Once the maximum number of firms has entered into the industry, then a new firm can enter and obtain a positive market share only by displacing one of the old firms—the one producing the lowest quality—which will find itself with no sales. The terms *natural monopoly, duopoly etc.* characterise markets where the upper bound to the number of firms is one, two etc.

The situations examined in this work are all vertically differentiated markets that can sustain two or more firms in equilibrium, and our interest is focussed successively on market coverage, conditions of entry, and the interaction between the two.

We first derive the optimal price-quality choice of a protected multi-product monopolist operating in a market that could otherwise sustain two or more firms. The main result of this analysis is that in many instances the monopolist will choose not to serve the entire market even when the fixed entry cost is very low and the corresponding number of qualities marketed by the monopolist is very large. Whether this will happen depends on the relation between the range of available qualities and the width of the income distribution.

Next, we turn our attention to natural duopolies with single-product firms and we examine the implications of entry threats when entry is sequential. Using a three-stage perfect equilibrium concept, we find that neither *maximal differentiation* is a general outcome of such competition in the absence of entry threats, nor *minimal differentiation* is the necessary outcome when entry is contested, two results that had been suggested in the literature.

When the incumbent firms are facing entry threats, we show that the nature of the fixed cost can confer first mover advantages to the incumbent firms. This cannot be done in the same way as in the homogeneous product cases, i.e. by an incumbent's commitment to higher output, because the incumbent's share will be reduced to zero by the finiteness property if an entrant introduces a quality higher than that of the incumbent. This logic has induced some researchers to think that the only way for the lower quality incumbent[1] to protect its mere presence in the market is by marketing a product of quality as close to the high quality as the nonnegative profit condition would allow.

However, if exit is a costly activity, the incumbent can credibly threat to leave its product potentially available at a price equal to its variable cost, even if it does not sell anything. This will imply a lower consumer's willingness-to-pay for the entrant's product. In many instances, this reduction is sufficient to forestall entry at any quality between the top one and that of the lower quality incumbent. If entry is not blockaded this way, the incumbent can increase its quality but not as much as to reduce his/her profits to zero.

Next we combine our previous results to examine whether an entry threat will induce a multiproduct monopolist to cover any parts of the market he/she would choose to leave unserved in the absence of such threat. We find that there are many cases where the uncovered market result is robust to the threat of entry. This is so because, if entry is feasible, then it will take place between the top two qualities of the incumbent. If it is blockaded there, the incumbent's optimal product line fully protects him from entry. Note that the number of incumbent qualities is endogenously

[1] The high quality's position is garanteed by the Bertrand assumption and the presence of even a slight entry cost.

determined by the level of the fixed cost, which is assumed to be common for both the incumbent and the entrant.

Our next concern is whether a strategic quality choice can protect the monopolist from entry without an increase in the number of qualities as well as whether such a choice can be superior, in terms of profits, to product proliferation. The answer to both questions is positive and the last issue of this thesis is how market coverage is affected by such a strategy. Surprisingly, we find that a larger part of the market will remain uncovered in the presence of an entry threat unless the monopolist is forced to increase the number of his/her qualities.

Although a full welfare treatment is beyond the scope of this research, we note that the question of market coverage has an importance *per se* in terms of public policy since, if the good is normal, it is the low-income part of the population that most likely will be hurt by a monopolist's uncovered market strategy.

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1 Introduction

Product differentiation has always been recognised as a variable of paramount importance in studying situations involving market power. Indeed, the very notion of market power is inseparable from the existence of some product heterogeneity, for even a monopoly firmly protected by a patent or any other device, confers little advantage to its owner if it sells a product for which close substitutes exist.

Differentiation is itself a differentiated concept with two types of heterogeneity being easily identified within the relevant literature. The first describes cases in which, when price is not an issue,[1] consumers are not unanimous in their ranking of different products, and substantial differences in assessing the merits of each product type may exist among them. According to the terminology introduced by Lancaster in [25], this kind of differentiation is refered to as *variety*, the term *quality* being reserved by the same author to describe the second case of product differentiation; here, when equal prices apply to all the product types, consumers' rankings of the different products are unanimous.

Two main models can be said to dominate the literature focusing on situations where *variety* is the main feature of differentiation. The first, originating in the pioneering work of Chamberlin [3], considers that the set of products in an industry is finite[2] and consumers, although they each one have their favoured product type, are all indifferent as far as their second choice is concerned. Hence, the set of consumers can be viewed as a collection of representative individuals, who like equally well all the goods and have a taste for variety. This approach implies that competition, rather than being focused between 'close substitutes', is generalised among all the goods; hence, each product faces equally intense competition by all its competitors. This symmetry in competition implies that new entry is of equal concern to all the incumbent firms in the industry, since it will reduce their market shares to the same extent. The resulting market structure could be loosely described as a bunch of monopolies whose products are closely related, and is usually refered to as *monopolistic competition*.

[1] Or, equivalently, when all the products are sold at the same price.

[2] As opposed to a continuum of possible goods, see [45], p. 2.

The alternative to monopolistic competition is the so-called *address* or *locational* approach, where there is a *continuum* of possible product specifications; every product competes directly only with its *neighbours*. As the terms *address*, *location* and *neighbours* suggest, this approach bears a spatial interpretation under which it was first presented in the pathbreaking work of Harold Hotelling.[3] Although the use of circular models to describe such markets has been said to avoid certain analytical problems,[4] the most widespread spatial depiction is the one proposed by Hotelling himself in [12]. According to this, consumers are located along a linear horizontal segment on which firms must choose their own location (address) according to profit maximisation criteria. The equivalent of this space metaphor in terms of variety is obtained if one assumes that product differentiation is due to a single characteristic, with product types differing among each other by the amount of the characteristic each contains. Thus, at the two endpoints of the line segment we find the product types containing the minimum and the maximum amounts of that characteristic, while in between we find all intermediate cases. The location of consumers along the line segment indicates their most prefered type of product. However, instead of assuming that consumers are indifferent among products which do not correspond to their first choice,[5] location models assume a complete ranking of the product set. If, additionally, consumer preferences among locations are assumed to be single-peaked, then this ranking is equivalent to ranking the different products according to their distance from a consumer's location. Thus, having consumers located at different points along the product line introduces diversity in their preferences, and no 'representative individual' can be said to exist.

On the other hand, the location of firms indicates the product types actually available in the market. Each consumer purchases—at equal prices—the product that is 'closer' to his/her ideal specification. Having firms located among consumers implies that no product can be the closest to all the consumer locations; this is equivalent to saying that there is no single product occupying the top position in all consumers' preferences.

The straight line analogy has coined the term *horizontal differentiation*

[3]See [12].

[4]On this issue see the work of Salop in [33].

[5]As in the models of monopolistic competition.

to the address model dealing with variety; as this analogy so well illustrates, the main features of horizontal differentiation are *a*) the absence of unanimity among consumers with respect to the ranking of different product types and, *b*) the fact that only neighbouring products are in direct competition, so that the system of demand curves is *chain linked*.

Comparing the *address* models to those of *monopolistic competition* one finds that, although they resemble each other in that in both models every consumer has a product type that he/she favours, they differ with respect to the consumer's reaction over the remaining elements of the product set: in address models the consumer ranks them according to their distance from his/her top product, although in 'Chamberlinian' models he/she is indifferent among them.

Equally important, the way variety is developed in the two models differs significantly. In the former there is an infinity of possible products and tastes, although the latter can be analysed in terms of a *typical* consumer who faces a specific set of products equally well liked, but who also has a taste for variety itself.[6]

Since *variety* is not the subject of this thesis, and given the fact that the interested reader can find numerous good references on this topic,[7] we will not explore the many interesting issues arising within the context of the two models mentioned above. Instead, we turn now our attention to the study of cases where quality can be unambiguously assessed.

As mentioned earlier, *quality* is the main feature of differentiation when, in the absence of any price considerations, all the consumers agree on the ranking of the elements of the product set. Considering again the case of a single characteristic, we note that now, the entire consumer population favours the presence—or absence—of it; hence, the product containing the maximum—or minimum—amount of the characteristic can now be defined as the highest quality, and all the other products are ranked according to the amount of that attribute that each contains. Examples of technical characteristics unanimously valued by consumers are speed, durability etc., although attributes such as noise or the physical volume of computers or sound equipment provide examples of characteristics that all consumers

[6]See [45], p. 14.

[7]See for instance [14], [45] or [10], to mention but a few.

would like to avoid.

The presence of a unidirectional ranking makes clear that the preferences of all the consumers are single-peaked, with the peak occuring at one endpoint of the quality spectrum. Hence, direct competition must take place between neighbouring qualities and the demand curves must be chain-linked, properties that 'quality' models share with the ones referring to variety. What really differentiates the former from the latter is that firms operating in a market with products distinguished by their quality level have a clear advantage or disadvantage over each of their neighbours.

The reader must probably have already noticed that in our discussion price considerations have been excluded; in other words, the similarity or not among consumer rankings which is at the basis of the distinction between quality differences and variety, refers to *preferences* and not to *choices*. We want to emphasise this point adequately, because, the price considerations necessarily entering in any decision made by the consumers may reverse the picture: strong differences in price may turn all the consumers unanimously towards a single product type, even in a context characterised by variety; at the same time, those differences can divert consumers' purchases from the 'top' quality to some lower ones, provided they are available at more 'reasonable' prices. Thus, the price structure in differentiated markets has a decisive influence on whether consumers will buy one or many products.

The above observation is of particular importance for quality models, because quality improvements can usually be obtained at a higher average cost. It is, therefore, of rather limited interest to discuss consumers' choices at equal prices, since often high quality products cannot be available at a price equal or slightly above the average cost of lower qualities. Moreover, if competition prevails among the producers of every single quality, the price schedule in the industry will reflect the evolution of average cost with quality increases. Hence, consumer reaction towards the different qualities when the price structure reflects average cost must be of particular importance in the study of address models of differentiation.

We define[8] as *vertical product differentiation*—hereafter **VPD** —the situation where, if all qualities are priced at average cost, consumers will

[8]This definition is similar to the one provided by Shaked and Sutton in [36].

be unanimous in their ranking of these qualities.

The essence of the above definition lies in that it rules out the following case: a market characterised by quality differences which becomes horizontally differentiated, with cheap/low-quality products occupying one end of the straight line segment, and expensive/high-quality ones located at the other endpoint of the product spectrum. By contrast, in a market that satisfies the above definition of VPD, the advantage of a higher quality product over its lower qualities does not vanish when production cost becomes an issue; this in turn implies that it is at the discretion of a high quality producer whether his/her lower quality competitors are going to have any business at all. It should not be surprising therefore, that models like the one of Mussa and Rosen[9] do not constitute cases of VPD according to our definition: when fixed costs are zero and competition in every quality brings the price of all the qualities down to average cost consumer unanimity breaks down and an infinite number of products are sold, even if consumers were to agree in ranking the different qualities at equal prices.

Saying that consumers rank the different qualities unanimously when all products are priced at average cost does not, however, mean that these qualities must be *actually* sold at average cost for VPD to exist. In fact, and unless competition takes place within the same quality, higher qualities will never be priced at average cost since they can always find entry-limiting prices, somewhat above their average cost. Hence, in the present notion of vertical differentiation, entry deterrence of low qualities is a strategy always available to a higher quality firm, a feature that obviously affects market structure and generates the *finiteness property*.[10] This feature, however, should not be taken to imply that high quality firms will necessarily apply entry limit pricing, nor that they will keep the largest market share for themselves: accommodation of lower qualities can sometimes be more profitable, since it allows the high quality producers to concentrate on a smaller segment of the market, to which they can now sell at a higher price. As Sutton observes in [44],

"...Rolex watches, like IBM computers, may be an enviable

[9] See [28]; a more detailed discussion of this paper is given in sections 2.1.2 and 2.4.1 of the present work.

[10] For a full presentation and references on the finiteness property, see subsection 2.2.2 of this thesis.

product, but they aren't noted for their share in the watch market. Having a "better" product presumably means that the demand schedule faced by the firm is shifted further outwards, than would otherwise be the case. But the firm, so advantaged, *might find it optimal to take its profit (largely) in the form of a higher price rather than in the form of an increased volume of sales.*"[11]

Before closing this rather short introduction to vertical product differentiation we must remind the reader of a well known difficulty that differentiation introduces into the industry analysis. This difficulty concerns nothing less than the definition of the industry itself: how "much" differentiation between two products would justify them being considered as part of the same industry? Although this problem was identified some sixty years ago with the work of Chamberlin,[12] no satisfactory answer has been given to it; theoretical researchers tend to usually assume it away, although in empirical studies common sense and rules of thumb are used to define the boundaries of the industry under examination. Since a rigorous answer to this problem lies beyond the scope of this thesis, we accept this caveat and follow the tradition of considering that the industry is somehow well defined. This assumption is a *sine qua non* for any further analysis, and also seems to fit a bit more naturally in the context of quality differences—as compared to that of variety—where the concept of 'product' is more clearly defined.

The plan for the rest of this thesis is as follows: hoping to have placed in this introduction the concept of VPD within the more general concept of differentiation, we devote the next chapter to a more extensive treatment of the literature focusing on this topic. The target of this review will be two-fold: on one hand we want to find the major questions that have been posed on the subject and analyse the answers proposed by various researchers; at the same time, we want to introduce some key analytical concepts that will be used extensively in the rest of the text.

In the last part of chapter 2 we determine some important questions which have not yet been addressed or adequately answered in the litera-

[11] Emphasis added.
[12] See [3].

ture. These questions refer to the choice of quality and the extent of market coverage, as well as to the relation between these two issues and the conditions of entry in a vertically differentiated market; they will be the subject of our research, which is presented in chapters 3 to 5.

Finally, in the last chapter we present the conclusions from our research, we discuss its limitations and point out some questions which constitute our agenda for further research.

2 Review of the Literature

2.1 The Model

The literature which will be reviewed in this section is rather technical in nature, and revolves around the same basic model. We start therefore by introducing this model, since it would allow all subsequent discussion to be more concise while gaining in precision; moreover, this part introduces the main framework that serves as basis for the results of our research

Rather than preserving the notation as it appears in the original works, we try to keep it uniform throughout this research in order to facilitate comparisons between different approaches.

2.1.1 The Income-dispersion Model

It is intuitively obvious that in order to have a market with differentiated products sold simultaneously consumers must differ with respect to some characteristic; otherwise they would all buy the same product. In models of horizontal product differentiation they are assumed to be located at different points, so that the distance of each consumer from the different selling points varies among consumers. This can be captured either by the direct presence of transportation costs, or by the introduction of distance as a 'bad' in the utility function.[13]

In the case of vertical differentiation, the spatial interpretations have not been very popular in the literature and, therefore, the transportation cost approach has not received much attention.[14] Taste differences are therefore the predominant approach in VPD where almost all the results have been derived using two main models: the **income-dispersion** model and the **taste dispersion** model.

The first was introduced by Jaskold Gabszewicz and Thisse in 1979[15] and it assumes a) that consumers differ by their level of income and b) that quality interacts with income remaining after purchase in a multiplicative

[13]See for instance the original article of Hotelling [12] for the first approach, or the works of Shaked and Sutton [38] and Champsaur and Rochet [4] for the second.

[14]Although it has not been entirely neglected either. See Jaskold Gabszewicz and Thisse [18].

[15]See [19].

way.[16] Thus, consumers endowed with different incomes have different intensities of preference for quality. Further, it is assumed that quality is a normal good; hence, as income increases consumers are willing to pay more for a given increment in quality.[17] An individual's demand for the product is totally price inelastic so either he/she purchases one unit of a certain quality or nothing from this market; thus different qualities are strict substitutes. The most common form of the utility function is:

$$U_i^j = u_i \cdot z_j \tag{1}$$

where U_i^j is the total utility experienced by individual j after the consumption of one unit of the good in question and z_j units of the numéraire good. The index u is an increasing index of quality that characterises every unit of the good with $u_i \in [\underline{u}, \bar{u}]$, If we define t^j as individual j's income endowment, and p_i the price for quality u_i, the utility function becomes:

$$U_i^j \equiv U^j(u_i, p_i) = u_i \cdot (t^j - p_i) \tag{2}$$

It is normal to assume that consumers participate *voluntarily* in this market so they can purchase none of the qualities offered if they wish. We define:

$$U_0^j \equiv u_0 \cdot t^j \tag{3}$$

as being the level of utility from non-purchase. The term u_0 is the *reservation quality*, i.e., a level of quality that is available to the consumers at a zero price, so none of them would ever consider a quality lower than u_0 even if it is offered for free. The presence of u_0 corresponds to the existence of an *outside good* in Salop's terminology.[18]

Consumers are distributed uniformly with respect to their income according to the density function:

$$h(t) = \begin{cases} g & \text{if } a \leq t \leq b \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

[16] An interpretation of this interaction is that quality enhances the consumption of the *numéraire* good.

[17] The consideration of quality as a normal good is a desirable feature rather than a restrictive assumption of this model.

[18] See [33].

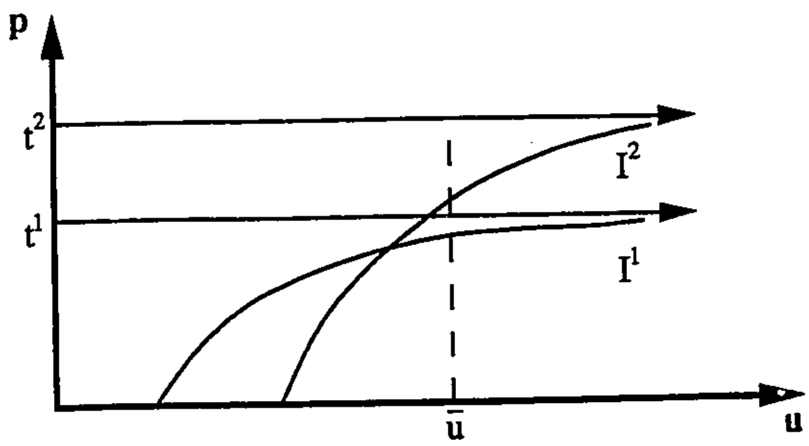

Figure 1: **The Indifference Curves of the Income Dispersion Model**
I^1 and I^2 are typical indifference curves taken from the indifference maps of two individuals with incomes t^1 and t^2 respectively. They both are asymptotic to their corresponding income level and they cross only once. The point $\bar{u}$ represents the highest level of quality that is technologically available.

The set of preferences described by the utility function in (2) can also be represented by indifference curves in the (p, u) space that are upwardly sloping rectangular hyperbolas according to the expression:

$$p = t^j - \frac{\bar{U}}{u} \tag{5}$$

As $u \longrightarrow \infty$, $p \longrightarrow t^j$, hence, the indifference curves of any individual are asymptotic to his/her level of income as in Figure 1. The marginal rate of substitution is $(t^j - p)/u$, clearly decreasing in u; the resulting concavity of the indifference curves indicates that the willingness to pay for increments in quality decreases as quality increases. This happens because as quality improves the marginal utility of income increases.

Notice that for equal (p, u) combinations, the marginal rate of substitution depends positively on t, so at any intersection of two indifference curves taken from the preference maps of two different consumers, the indifference curve of the consumer with the higher income is steeper. Moreover, any such pair of indifference curves can have at most one intersection, and hence the so-called "single-crossing" property holds for the preferences expressed in (2).[19] Therefore, assuming that many discrete qualities are offered at dif-

[19]This is the result of the marginal rate of substitution being monotonically increasing

ferent prices, if any two of them, say u_h and u_l with $u_h > u_l$, have positive market shares, then:

$$\begin{aligned} u_h(t^j - p_h) &\geq (\leq) \; u_l(t^j - p_l) \Longleftrightarrow \\ u_h(t^m - p_h) &> (<) \; u_l(t^m - p_l). \qquad \text{for all } m > (<) j. \end{aligned} \tag{6}$$

In this framework, we introduce a price schedule $p = p(u)$ and let the consumer choose a quality u_i^* such as to maximise (2) subject to this price schedule. The introduction of such a schedule constitutes from the point of view of the monopolist a *self-selection problem*[20] in that every consumer must be induced to choose a certain quality assigned to him/her by the seller. The result will be a tangency point between an indifference curve in the (p, u) space and the above price schedule; at that point we have

$$p' = (t^j - p)/u, \tag{7}$$

the marginal rate of substitution. Unless a corner solution prevails,[21] the above equality determines the optimal quality-price pair (u_j^*, p_j^*) for consumer j, provided that $U^j(u_j^*, p_j^*) \geq U_0^j$.

According to Lemma 2 in [6], the single crossing property guarantees that both u_j^* and p_j^* are increasing in t. Thus, if a consumer purchases a quality higher than that purchased by another consumer, his/her income must also be greater. Hence, if by t_{j+1} we denote the income of the consumer just indifferent between u_j and u_{j+1}, we note that all the consumers with income $t_\lambda > t_{j+1}$ prefer product u_{j+1} at price p_{j+1} rather than the pair (u_j, p_j). Similarly, all the consumers with income lower than t_j prefer the

in t; see Cooper [6].

[20]See subsection 2.4.1.

[21]As long as the number of products is finite (not a 'quality interval') while the consumers are a continuum, there will be some consumers unable to obtain their first best; therefore in the case where the distribution of consumers can be approximated by a continuous function, the non-satisfaction of the first order condition for all consumers is equivalent to having a finite number of products and bunching many consumers into the same quality. A sufficient condition for this result is of course the existence of a fixed cost associated with the entry of a new quality, and the resulting economies of scale. However, the presence of fixed cost is not a necessary condition for the violation of (7); as shown later, the relation between income distribution and the way variable cost changes with quality can also produce corner solutions, bunching, and a finite number of products even in the absence of any fixed costs (see the discussion on the finiteness condition in subsection 2.2.2).

pair (u_{j-1}, p_{j-1}) over the pair (u_j, p_j). Hence, the single crossing property implies that if a set of n qualities $u_1 \dots u_n$ is offered at prices $p_1 \dots p_n$, the market shares of the qualities with positive sales will be ***chain-linked*** rather than overlapping; as a result, the whole population of consumers will be divided in terms of product choice in such a way that if $t^j > t^m$, then $u_j^* \geq u_m^*$, where $(*)$ denotes optimal choices. Therefore the model at hand is an *address* model of differentiation since every product is in direct competition only with its immediate neighbours, as is evident in the following expressions for the volume of sales M^m of any product m:

$$M^n = (b - t_n)g \tag{8a}$$

$$M^m = (t_{m+1} - t_m)g \qquad \text{for } m = 2, \dots, n-1 \tag{8b}$$

$$M^1 = \begin{cases} (t_2 - t_1)g & \text{for } m = 1 \text{ and } t_1 \geq a \\ (t_2 - a)g & \text{for } m = 1 \text{ and } t_1 \leq a \end{cases} \tag{8c}$$

whereby t_m $(m = 1, \dots, n)$ denotes the level of income of the consumer just indifferent between qualities u_m and u_{m-1}. The determination of t_m is straightforward:

$$\begin{aligned} u_m(t_m - p_m) &= u_{m-1}(t_m - p_{m-1}) \iff \\ t_m &= p_m r_m - p_{m-1}(r_m - 1) \qquad \text{where } m = 2, \dots, n. \end{aligned} \tag{9}$$

Also for t_1 we have

$$t_1 = p_1 r_1 \qquad \text{if } p_1 r_1 \geq a \tag{10a}$$

$$t_1 = a \qquad \text{if } p_1 r_1 \leq a \tag{10b}$$

where

$$r_m \equiv \frac{u_m}{u_m - u_{m-1}} \qquad \text{for } m = 1, \dots, n. \tag{11}$$

2.1.2 The Taste-dispersion Model

The taste dispersion model has been used first by Mussa and Rosen in 1978,[22] and it assumes that the intensity of the desire for quality improvements differs among consumers, who are distributed according to a taste

[22] See [28].

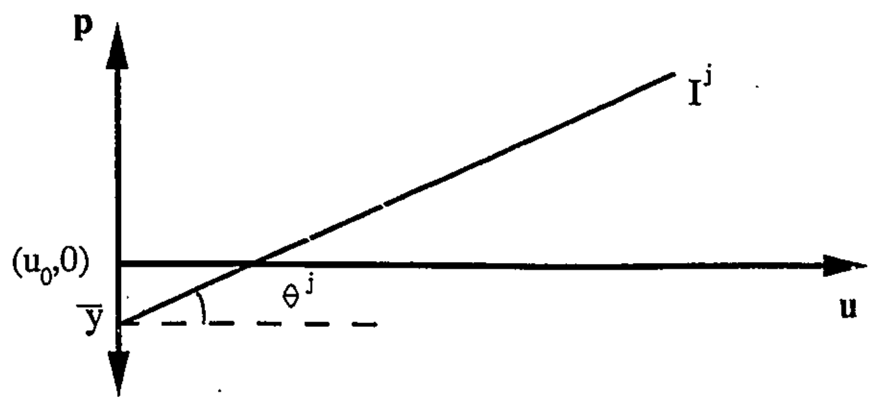

Figure 2: **Indifference Curves in the Taste-Dispersion Model.**

parameter. More specifically, the utility function of the individual j when consuming product i has the form:

$$U_i^j = u_i \cdot \theta^j + z_i^j \tag{12}$$

where θ^j is the value of the taste parameter for the j-th individual and everything else is as in page 8. Since $z_i^j = t^j - p_i$, equation (12) can also be written as:

$$U_i^j = u_i \cdot \theta^j + t^j - p_i \tag{13}$$

In the taste-dispersion approach, income enters additively in the utility function so marginal utility of income is constant (and equal to one) and the income effect is zero. Since $\partial U^j/\partial u = \theta^j > 0$, all the consumers unanimously prefer a higher u in the absence of price considerations but those with high θ's are willing to pay more for a given quality improvement; moreover, a consumer's willingness to pay for improvements in quality is independent of the level of quality he/she actually enjoys, so the indifference curves from (13) in the (p, u) space are *linear* with slope equal to θ^j; such a curve is depicted in Figure 2.[23] The origin in this figure corresponds to the pair $(u_0, 0)$, i.e., the no-purchase option, although the vertical intercept of any indifference curve—the distance $0\overline{y}$ for the (I^j) curve—represents the surplus consumer j derives from any combination (u, p) along that curve.[24]

[23] Adapted from Mussa and Rosen's Fig. 1 (see [28]).

[24] This is so since the consumer is indifferent between any point along (I^j) and the

The model is completed by a specification of the distribution of consumers, which is again uniform but refers now to the taste parameter, rather than income, according to the function:

$$h(\theta) = \begin{cases} g & \text{if } \theta_a \le \theta \le \theta_b \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

A careful examination of the income and taste dispersion models reveals that the latter is a special case of the former, with constant marginal utility of income as its special feature. This can be seen if we translate the sacrifice for additional quality in terms of **utility** (rather than money), so that $up(u)$ is measured on the vertical axis of Figure 1 and Figure 2; in this case the indifference curves will be linear for both models. Also, it is not difficult to show that the Bertrand-Nash equilibrium prices and market shares in the taste-dispersion model can be obtained if we substitute θ_a, θ_b for a, b and $u_i - u_{i-1}$ for r_i, $i = 1, 2$ into the corresponding expressions of the income-dispersion model.[25] Notice, however, that $u_i - u_{i-1} = r_i u_i^{-1}$, with u_{i-1}^{-1} being the ratio of the marginal utilities of income which correspond to the utility functions (2) and (13). Thus, the price-maximising rules for the two firms, as well as their Bertrand-Nash equilibrium prices and market shares, are similar in the two models after the appropriate adjustment to the marginal utilities of income has taken place.

The separability of the utility function in (12) allows for a further simplification, namely setting $u_0 = 0$. This simplification can be used in the context of (1) only if we treat t^j as the maximum amount consumer j is ready to spend on any quality from this market, so U_i^j now represents *additional* utility from the purchase of quality u_i rather than total utility,[26]

combination $(u_0, -0\bar{y})$. Notice however, that, since the no-purchase option corresponds to having the quality u_0 at price 0, the above combination where u_0 is combined with a 'negative price' such as $0\bar{y}$, implies a positive surplus over $(u_0, 0)$. Of course, points along an indifference curve that crosses the vertical axis at a positive point are equivalent to having u_0 at a positive price, hence they compare unfavourably to the no-purchase option, i.e., the corresponding surplus is negative. Obviously, if market participation is free, no consumer will ever accept to buy any combination along such a curve.

[25]See the expressions (26a)-(26d) in subsection 2.2.4, page 29, for the Bertrand-Nash equilibrium prices of the inome-dispersion model; substituting these prices into (8a)-(8c) we can obtain the Bertrand-Nash equilibrium market shares of the firms in the Bertrand-Nash equilibrium of the income-dispersion model.

[26]Ireland uses this interpretation (see [14] p.71).

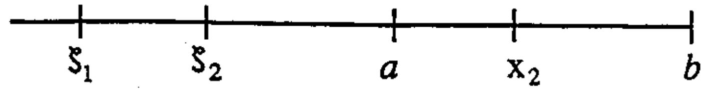

Figure 3: **Spatial Models of Vertical Differentiation**
Consumers are located between a and b although the firms' locations are ζ_1 and ζ_2. The point x_2 represents the boundary of the market shares of the two firms: with convex transportation cost consumers between a and x_2 purchase from seller ζ_1 while those between x_2 and b from seller ζ_2. The exact location of x_2 depends on the cost of transportation.

since otherwise $u_0 = 0$ together with (1) would imply that opting out of the market is always an inferior strategy.

Although innocuous for the determination of prices when qualities are given, the assumption $u_0 = 0$ entails a significant loss of generality and can be proven misleading when the choice of qualities is considered.[27]

2.1.3 Spatial Models

Although most of the results in VPD have been derived on the basis of the "utility approach" some studies have used a "space equivalent" approach with the appropriate specification of a cost function. Jaskold Gabszewicz and Thisse considered in [18] the familiar straight line metaphor from horizontal differentiation and required that the two firms be located at two points that lie outside the consumers' segment $[a, b]$, like the selling points ζ_1 and ζ_2 in Figure 3. The fact that ζ_2 is closer than ζ_1 to all the customers is equivalent to saying that ζ_2's product is of higher quality since, everybody prefers it at equal prices. With a linear cost specification it can be seen that if $p_2 < p_1 + (\zeta_1 - \zeta_2)c$, the sales of ζ_1 are zero, with the opposite holding if the inequality is reversed.[28] To avoid this type of bang-bang switch, a non-linear cost function must be introduced. Gabszewicz and

[27]See section 2.3 and chapter 4.

[28]In case of equality all consumers are indifferent between the two firms.

Thisse [18,21] use the following quadratic cost function in order to discuss the stability of the Bertrand-Nash equilibrium in a duopoly game:

$$c(\zeta, x) = d|\zeta - x| + l(\zeta - x)^2 \qquad d, l > 0 \tag{15}$$

where $x \in [a, b]$ denotes consumer location and $\zeta = \zeta_1, \zeta_2$ is the selling point. It can easily be seen that with a cost function like the one above, which is convex in distance, and with an appropriate parameter specification, there exists a critical consumer x_2 such that consumers in $[a, x_2]$ buy from ζ_1 and those in $[x_2, b]$ buy from ζ_2. The position of x_2 can be obtained by solving the equation in x_2:

$$p_1 + d|\zeta_1 - x_2| + l(\zeta_1 - x_2)^2 = p_2 + d|\zeta_2 - x_2| + l(\zeta_2 - x_2)^2 \tag{16}$$

As is obvious, for a given location of (ζ_1,ζ_2) the position of x_2 depends on the relative prices of ζ_1 and ζ_2. In order to show the equivalence between the utility models and those with transportation costs, Champsaur and Rochet[29] show that the surplus generated by the taste dispersion model can be approximated by using a distance function and a quadratic specification for the transportation cost.

2.2 Equilibrium Prices and Market Structure

In this, as well as in the following section, we assume that each firm produces only one quality unless it is stated otherwise; the literature on multiproduct firms will be reviewed in section 2.4.

2.2.1 Pricing Game

We consider the oligopoly game as a three stage game, with firms deciding in the first stage whether to enter while in the second stage they select the quality of their products; finally at the third stage they compete in prices *à la* Bertrand.[30]

[29] See [4].

[30] The order of the stages is determined so the short run decisions of the firms—i.e., pricing—be taken condititional on the medium and long run decisions which correspond to choice of quality and entry decision respectively.

The *subgame perfect* concept is used to describe the equilibrium in this game, requiring that "after any stage, that part of the firm's strategies pertaining to the game consisting of [the remaining stages], form a Nash Equilibrium in that game." [31]

In order to find this equilibrium one must proceed backwards solving first for the equilibrium in the price game with qualities considered as given; substitute next the expression for the optimal prices as functions of qualities in the revenue function and find the choice of qualities assuming entry has been decided; and finally, compare the optimised revenue to the entry cost in order to determine whether entry can be profitable.

Assuming for simplicity zero variable costs, the revenue function represents also operating profits; hence, the objective of the m-th firm is to choose the price that maximises its revenue function given by one of the equations below:

For $m = n$:

$$R_n = p_n(b - t_n)g \tag{17}$$

For $1 < m < n$:

$$R_m = p_m(t_{m+1} - t_m)g \tag{18}$$

For $m = 1$ and $t_1 \leq a$:

$$R_1 = p_1(t_2 - t_1)g \tag{19a}$$

For $m = 1$ and $t_1 \geq a$:

$$R_1 = p_1(t_2 - a)g \tag{19b}$$

The presence of the expression (19a) indicates the possibility of a corner solution for the price of the lowest surviving quality. The first order conditions for the system are:

For $m = n$:

$$(1/g)\partial R_n/\partial p_n = b - t_n - p_n r_n = 0 \tag{20}$$

For $1 < m < n$:

$$(1/g)\partial R_m/\partial p_m = t_{m+1} - t_m - p_m(r_m + r_{m+1} - 1) = 0 \tag{21}$$

[31] See [39], page 3.

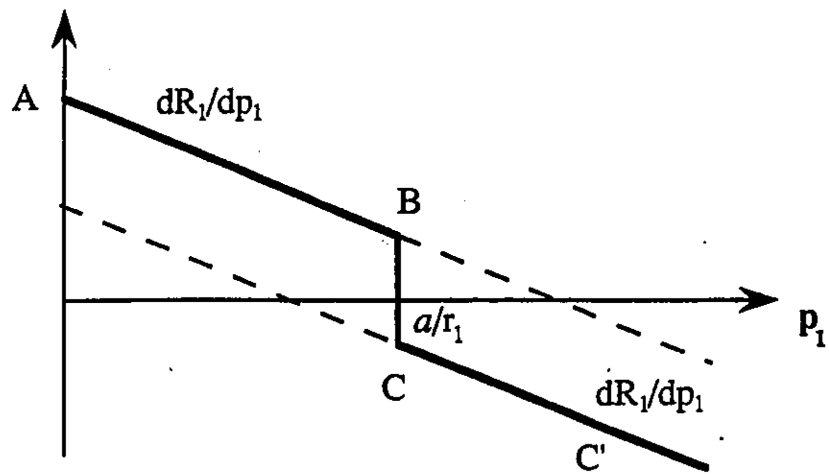

Figure 4: **Choice of p_1 and Market Coverage in a Bertrand-Nash equilibrium**

The higher of the two lines holds for $p_1 \leq a/r_1$ ($t_1 \leq a$), and the lower for $p_1 \geq a/r_1$. Thus, the relevant expression for $\partial R_1/\partial p_1$ is represented by the solid line $ABCC'$; if the p_1 axis crosses this line along the segment AB, $t_1 < a$ and consumer a has some surplus; if $ABCC' \cap 0p_1 \in CC'$ firm 1 leaves some market uncovered and if $0p_1$ crosses $ABCC'$ along AB the market is just covered and consumer a gets no surplus.

For $m = 1$ and $t_1 \leq a$:

$$(1/g)\partial R_1/\partial p_1 = t_2 - a - p_1(r_2 - 1) = 0 \tag{22a}$$

For $m = 1$ and $t_1 \geq a$:

$$(1/g)\partial R_1/\partial p_1 = t_2 - t_1 - p_1(r_1 + r_2 - 1) = 0 \tag{22b}$$

The form of the $\partial R_1/\partial p_1$ function is given in Figure 4, taken from Ireland.[32]

Because we have assumed the price of u_0 to be zero, $t_1 = p_1 r_1$. Therefore when $p_1 = a/r_1$ the market is 'just covered,' i.e., the poorest consumer is just indifferent between the lowest quality in the market at price p_1 and the outside good at price zero. If $p_1 > a/r_1$, some consumers are left out

[32]See [14].

of the market and in the absence of a fixed cost there is always room for a lower quality. Finally, if $p_1 < a/r_1$ the poorest consumer is having some positive surplus.[33] Depending on which segment of the $\partial R_1/\partial p_1$ function crosses the horizontal axis, the appropriate solution for p_1 applies; if the intersection occurs within the vertical segment, then $p_1 = a/r_1$.

Before we derive explicit solutions for the prices in the case where $n = 2$, let us discuss some important implications stemming from the system (20)-(22).

2.2.2 The Finiteness Property

By the very definition of VPD it is obvious that a higher quality enjoys an absolute advantage in demand over a lower quality. Thus, as Bain suggested,[34] if cost differences do not eliminate this advantage there will be no easy entry and the potential for blockaded or effectively impeded entry[35] suggests itself together with the possibility of a small number of firms in the market. The *finiteness property*—first developed by Gabszewicz and Thisse[36] and further elaborated by Shaked and Sutton[37]—states indeed that "...there will exist an upper bound independent of product qualities, to the number of firms which can coexist with positive market shares and prices exceeding unit variable costs, at a Nash Equilibrium in prices."[38]

The following example adapted from Shaked and Sutton [37][39] illustrates the case where the upper bound to the number of firms is one (natural monopoly). Consider the utility function given by (2) and a uniform distribution of consumer incomes as in (4) with density $g = 1$; assuming that there are two firms in the market, the demand for the higher quality

[33] It is interesting to notice that setting $p_1 < a/r_1$ allows even the poorest customer to obtain the product at a price lower than what he is willing to pay, despite the fact that there are no customers between t_1 and a to be attracted by this low price. The answer to this little paradox lies of course in the competition between product 1 with product 2: the positive surplus of consumer a is a side effect of the competition for consumers with incomes around t_2; this situation arises when qualities are too close.

[34] See [1], page 12.

[35] See [1], pp. 21-22.

[36] See [19] [17].

[37] See [39] [36] [44] [38].

[38] See [36], page 1472.

[39] See also Ireland [14] and Waterson [46].

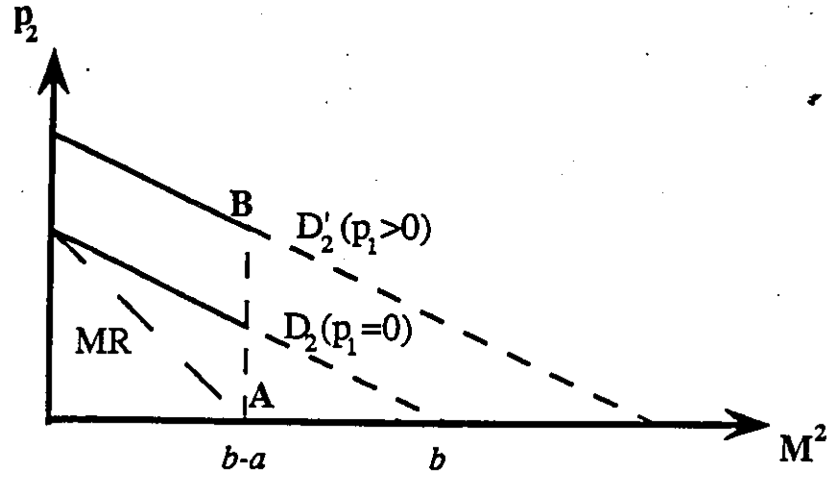

Figure 5: **The Demand for the High Quality Product**
The position of the demand curve depends on the price of the lower quality: the curve D_2 is drawn for $p_1 = 0$; an increase in p_1 will shift the D_2 curve to a position like D_2'. Since the width of the income distribution is $b - a$, there will be no consumers between $b - a$ and a; hence, the demand curve is vertical at $b - a$. If the marginal revenue MR crosses the axis to the right of $b - a$, there will be no room for a second firm; thus, for $b \leq 2a$ there is no $p_1 \geq 0$ leaving any market share to firm 1.

product will be:

$$M^2 = min(b - t_2, b - a) \tag{23}$$

Given the choice of qualities, t_2 must depend directly on p_2 and inversely on p_1.

In Figure 5 we consider the relation between p_2 and M^2; this relation depends of course on the price of product 1: the higher p_1 is, the higher the position of the demand curve for product 2. Assume that quality u_1 is priced at average cost c_1 and consider the discontinuity in the demand for product 2 at $b - a$. The price $\bar{p}_2$, which is the highest price that confers all the market to product 2, cannot be lower than the average cost of this quality since by our definition of VPD there is no $p_1 \geq c_1$ such as to make $\bar{p}_2 \leq c_2$, the average cost of product 2. The question therefore is whether the intersection of marginal revenue[40] with marginal cost of product 2 lies

[40]Marginal revenue is defined here in the traditional way, i.e. the derivative of total

to the left or right of the segment AB. Since we have assumed zero marginal cost, it is the intersection of marginal revenue with the horizontal axis that determines the optimal p_2 and M^2: if it occurs to the left of AB there will be entry accommodation, although in the case where marginal revenue and cost meet on or to the right of AB the market will be monopolised.

It is now obvious that the width of the income distribution, $b - a$, is of paramount importance for the determination of the number of firms in the market. To see this, start from a situation where monopoly prevails and keep lowering a: the segment AB will move to the right till eventually the situation will turn to a duopoly.

Notice that for given qualities the demand for product 2 shifts downwards as p_1 decreases. Thus, even when the lower quality ends up being out of business, its (potential) presence is not without consequences for the high quality: it forces p_2 to be as low as it is necessary to keep the low quality out of the market, which in Modigliani's terminology[41] is the *entry limiting price*. "Hence, the limit price is viewed here as an equilibrium strategy which endows it with a game theoretic stability content."[42]

Assuming zero costs[43] in a model similar to the above, Jaskold Gabszewicz and Thisse in [19] determined the width of income distribution that could sustain one, two or more firms. They considered two firms in the market and they found that if $b < 2a$ only one firm serves the whole market, if $2a < b < 4a$ the whole market is served by the two firms (with no room for a third one) and if $b > 4a$ the two firms will not serve the entire market, and thus, with zero fixed cost, there is room for at least a third firm. These results have also been presented in Shaked and Sutton in [39]; the presentation below follows this last reference.

To determine an upper bound to the width of the income distribution so that it can sustain at most $\bar{m}$ firms we simply solve the system of first order conditions for the top $\bar{m}$ firms and we require that $t_{n-\bar{m}+1} \leq a$.

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revenue with respect to quantity. Since consumer distribution is assumed to be uniform with density equal to one, the marginal revenue is $\partial R_2/\partial M^2 = -\partial R_2/\partial t_2$.

[41]See [27].

[42]See [19].

[43]The assumption of no fixed entry costs is allowed in [19] because entry in the industry is assumed to be simultaneous and firms are single-product. In the present work, we relax the zero-entry-cost assumption and allow for sequential entry and multiproduct firms.

Subtracting t_m from (20) and (21) and adding its equivalent from the RHS of (9), we obtain the expressions:

For $m = n$

$$b - 2t_n - (r_n - 1)p_{n-1} = 0 \tag{24a}$$

For $1 < m < n$

$$t_{m+1} - 2t_m - (r_{m+1} + r_m - 1)p_{m-1} = 0 \tag{24b}$$

Notice now that for any of these conditions to hold, we must have $t_m \geq a$.[44]

Assuming all costs being equal to zero, a necessary and sufficient condition for any quality other than the top one to have a positive market share is that the lower boundary of the market share of its higher quality neighbour must be greater than a in a Bertrand-Nash equilibrium. Thus, in order to get $\bar{m} \geq 2$ qualities in the market, it must be that $t_{n-\bar{m}+2} > a$, where $\bar{m} = 2, \ldots, n$. From (24b) we obtain that $t_{m+1} > 2t_m$ for all m and from (24a) that $b > 2t_n$. Proceeding recursively we can obtain the condition $b > 2^{\bar{m}-1}a$ which guarantees the existence of at least $\bar{m}$ firms in the market. In order to guarantee that $\bar{m}$ is also the *maximum* number of products, it suffices to ensure that $t_{n-\bar{m}+1} \leq a$. Since $2^{\bar{m}}t_{n-\bar{m}+1} < b$, the above condition holds whenever $b < 2^{\bar{m}}a$. Thus we will have exactly $\bar{m}$ qualities in the market when $2^{\bar{m}-1}a < b < 2^{\bar{m}}a$.[45]

As Shaked and Sutton state, "the mechanism that brings this result about is price competition among the 'surviving' products which drives their prices down to levels sufficiently low to make every consumer who chooses to purchase the product to prefer one of these qualities at its equilibrium price."[46] Notice that for many of the surviving products these "sufficiently low" prices may involve substantial price-cost margins and, consequently, positive profits.

A most striking feature of the finiteness property is that, although it depends on the width of the income distribution, it is independent of its density: as can be seen from (22) optimal prices and market boundaries

[44]If for some $m = \bar{m}$, $t_{\bar{m}} < a$, condition (22) applies for that $\bar{m}$ (with $p_{\bar{m}}$ and $t_{\bar{m}+1}$ replacing p_1 and t_2 respectively) and all the qualities below $\bar{m}$ are left out of the market.

[45]This expression generalises as $2^{\bar{m}-1}a < b + c < 2^{\bar{m}}$ for the case where there is a per unit cost $c > 0$; see [22].

[46]See [36].

of all the qualities are independent of y. In other words, if the size of the market increases uniformly, there will be no tendency towards a larger number of firms! Contrast this with the paradigm of horizontal differentiation where the number of firms depends on the relation between the size of the market and the level of fixed cost: if the former tends to infinity (or the latter to zero) the number of firms will tend to infinity leading to a *fragmented* market structure and an infinitesimal market share for each firm.[47] By contrast, in models of VPD once the upper bound to the number of firms is reached, the market shares of the different products remain invariable as the size of the market tends to infinity or the fixed costs tend to zero.

We have shown that the upper bound to the number of firms in the market imposed by the finiteness property is independent of the relation between market size and fixed costs, being determined by the pattern of tastes and the income distribution. The terms *natural monopoly*, *natural duopoly* and *natural oligopoly* have been used in the context of VPD to indicate situations where the upper bound to the number of firms allows for at most one, two or a few firms respectively. This is not to say, however, that the relation between fixed cost and market size has no role to play in the determination of the actual number of firms; large increases in fixed cost can transform a natural duopoly, for example, to a monopoly through the usual profitability condition for entry, but this result is not symmetric: substantial reductions in fixed cost cannot make room for a second firm in a natural monopoly.

The question that naturally arises at this point concerns the generality and robustness of the finiteness property.

General Conditions for Finiteness In order to examine this question we keep the initial assumptions of the model, i.e. a uniform income distribution and a utility function of the kind described in (2) and ask which are the necessary and sufficient conditions for finiteness. Shaked and Sutton treated this subject in their 1983 paper.[48] In order to present their work we assume the variable cost function $c(u)$ to be a nondecreasing, continu-

[47]See for instance [14], [10] or [45].
[48]See [36].

ous and differentiable function of quality for $u \in [\underline{u}, \bar{u}]$ with $\underline{u} \geq u_0$, and we define as $t(u, v)$ the income of the consumer who is indifferent between qualities v and u ($v > u$) when both are priced at average cost. Therefore, t must be given by the solution of

$$u(t - c(u)) = v(t - c(v))$$

so

$$t(u, v) = \frac{vc(v) - uc(u)}{v - u} = c(v)r_{uv} + c(u)(1 - r_{uv})$$

where $r_{uv} \equiv v/(v - u)$; next, let us define the function $t(u, u)$ as:[49]

$$\lim_{v \to u} t(v, u) = c(u) + uc'(u)$$

Since we have assumed $c(u)$ to be differentiable, $t(u, u)$ is well defined. Let us now consider the utility maximisation problem of a consumer with income Y:

$$\max_u u(Y - c(u))$$

The first order condition for this problem is:

$$Y = uc'(u) + c(u) = t(u, u)$$

In other words, when all the qualities are priced at average cost, it is the consumer of income $t(u, u)$ who has his first order conditions for utility maximisation satisfied when quality u is chosen. This implies that the consumer $t(u, u)$ attains either a maximum or a minimum of utility by choosing quality u. It is precisely this type of consumer that must not be present in the population for the finiteness property to hold. For this, it is required that "either (a) $t(u, u) \notin (a, b)$ for $[\forall u \in [\underline{u}, \bar{u}]]$, so that all such consumers lie outside our range of incomes, or (b) $t(u, u) < t(u_0, u)$, so that any such consumer prefers to make no purchase, rather than buy u."[50]. The following condition combines the two cases:

$$Condition(F): \quad t(u, u) \notin [max(a, t(u_0, u)), b] \quad \forall u \in [\underline{u}, \bar{u}]$$

[49]Since $c(v)$ is differentiable this is derived easily from de l'Hospital's rule.
[50]See [36].

Shaked and Sutton prove indeed that the fulfilment of the condition (F) is necessary and sufficient for the finiteness property to hold. In what follows, we will not try to reproduce their rigorous proof but rather to discuss the intuition lying behind the condition (F).

As already said, finiteness is absent whenever, by reducing the fixed cost or by increasing the size of the market, we can make room for an arbitrarily large number of firms. This can be done in two ways. First, allow the existence of an arbitrarily small range of consumers $(t', t'') \subset (a, b)$ for whom condition (F) is violated, i.e. $\forall t(u_j, u_j) \in (t', t'')$, $t(u_0, u_j) < a < t(u_j, u_j) < b$ for some $u_j \in [\underline{u}, \bar{u}]$. Then, for any such consumer, quality u_j can be dominated only if some other quality is sold below average cost, which is of course unsustainable. Therefore we can say that the consumer is "located" on his/her preferred price-quality package. Combinations to the left or right of it are considered inferior and are ranked according to their distance from the optimum one. This setting falls into the horizontal differentiation context where, if fixed cost is zero, an infinite number of firms can be introduced, each one serving a particular type of consumer in the (t', t'') interval. Therefore, the first case that condition (F) rules out is the transformation of vertical to horizontal differentiation through cost of quality considerations; the way we defined VPD in page 4 aimed to rule out this possibility.

The second way in which an unbounded number of firms can be entered is "by introducing new products of successively lower quality at the end of the existing range."[51] Assume that there is a consumer, with income zero, and assume that the average cost is zero as well; from the utility function in (2) it is obvious that he/she is indifferent between any qualities priced at average cost, hence, if any firm wants to attract this consumer, it must price its product at average cost and make zero profits. Assume now that after the entry of firm m, $m \leq n$, there is still some part of the market not served by the incumbent firms, and consider the decision of the prospective entrant: he/she has either to cover all the remaining market in which case his/her profits will be zero, or to price above average cost and accept that some low income customers do not purchase its product. In the latter case, the assumptions of vertical differentiation will guarantee a positive market

[51]See [36].

share to the firm, so that its profits—no matter how small—will be positive. Hence firm $m+1$ will always leave some market uncovered. In the absence of fixed cost a new firm can enter and obtain positive profits by excluding some consumers in the neighbourhood of the zero income consumer thus leaving room for another firm, and so on. Since a positive market share at a price above average cost can be guaranteed to any quality as long as no higher qualities are sold at or below average cost, and since the top quality makes positive profits, it follows that this process will entail the entry of an infinite number of firms in the absence of any positive fixed cost. What destroys the finiteness property in this case is the presence of a consumer who is locally indifferent between a certain range of goods—in our example the consumer of income zero.

In the more general case where variable costs are not zero, the above example indicates that in order to preserve the finiteness, we must exclude the consumer who is locally indifferent between two qualities priced at average cost. The indifference curve of that troublesome consumer must be tangent in some section to the average variable cost schedule, since his/her willingness to pay for additional quality increases at the same rate (locally) as the average variable cost necessary to obtain the quality improvement. Condition (F) eliminates this case as well.

2.2.3 Robustness of the Finiteness Property

Until now, all our discussion of the finiteness property has been based on the very specific assumptions of: (i) pure VPD, (ii) a two-stage perfect equilibrium with *Bertrand-Nash* equilibrium in the second stage, (iii) the utility function of the *income-dispersion* model and a uniform income distribution. It arises therefore naturally to ask at this point about the robustness of the finiteness property under alternative specifications.

Finiteness in the Taste Dispersion Model Donsimoni and Hamilton [9] examined the finiteness property in a *taste* (rather than income) dispersion model and found that a necessary and sufficient condition for the absence of finiteness is that

$$c''(\hat{u}(\theta)) - U''(\theta, \hat{u}(\theta)) > 0 \qquad (25)$$

where $U(\theta, u)$ is the utility function, c is the cost per unit of output, primes denote derivatives with respect to u and $\hat{u}(\theta)$ represents the ***optimal-u-for-type-θ*** function as defined by the maximisation of (13) for different values of θ.

Condition (25) implies that cost must be more convex than utility with respect to quality. If the marginal utility of quality ($U'' \leq 0$) is assumed to be non-increasing, then the finiteness property cannot hold when marginal cost increases with quality ($c'' > 0$). Thus, in the taste-dispersion model as exemplified by (12) a necessary but not sufficient condition for the finiteness to be present is that the average cost be a concave function of quality.

Notice that the violation of condition (25) is close in spirit to the requirement that average variable cost does not rise "too fast with quality."

Finiteness in the Presence of Horizontal Differentiation In their 1987 paper, Shaked and Sutton[52] carry the generalisation a few steps further by combining horizontal and vertical differentiation. In such a context, however, one must be less ambitious, for it is obvious that the finiteness condition as defined earlier cannot be maintained in the presence of horizontal differentiation. Nevertheless, a modified version of the finiteness property (MFP from now on) still holds. In this version, the number of firms is no longer bounded from above, yet, as the size of the market increases, the relation between technology and tastes can prevent the market from becoming extremely fragmented: the market share of some firms will remain positive, even if the size of the market tends to infinity. Sufficient for this result is a set of two conditions on tastes and technology.

Let us assume that products differ according to a horizontal attribute h and a vertical one, u; hence, $U = U(u, d, t)$. Utility is strictly increasing in u and strictly decreasing in $d \equiv |h - \alpha|$, the distance of the horizontal attribute h from the consumer's favoured location α. In addition, it is assumed that the marginal rate of substitution between u and d is positive and finite, so that if a firm from its horizontal location jumps up "vertically" to a higher quality it can capture some of the consumers located rather closely to its rivals. The first of the two sufficient conditions for the MFP to hold is that at any location, there exists an increase in quality, sufficient

[52]See [38].

to allow the firm operating at that location to obtain any market share at a price above average cost. For this to be possible average cost must not rise too sharply with quality, for if it does, the advantage in terms of market share that a higher quality could obtain would be dissipated by the higher price necessary to cover the new (increased) average variable cost. Restated in terms of utility and cost, this condition implies that: $\partial U/\partial u$ is bounded away from zero; $\partial U/\partial y, |\partial U/\partial d|$ are bounded from above, and $c(u)$ is bounded from above by some income level which is less than b—the maximum value of consumer income.[53] This condition is analogous to the familiar one of finiteness property in the pure VPD.

The second condition admits that quality improvement always involves an increase in fixed costs but it stipulates that the elasticity of fixed cost with respect to quality changes is bounded from above. The role of this condition become clear in the following situation: as the size of the economy increases, given the condition on average variable cost there will be a tendency for the maximum quality to increase. However, this tendency may be stopped if the increase in fixed cost due to better quality is prohibitively large. The condition bounding the elasticity of fixed cost with respect to quality helps to prevent this from happening.

The above mentioned conditions can be combined in a single sufficient condition for the market never to become fragmented: *"there is some factor k such that, by incurring k times more fixed cost than any other participant, a firm can guarantee sales of μS at a mark-up p over unit variable cost, irrespective of the prices set by rival firms"* where S is the size of the market and μ is market share.[54] What really rules out the horizontal differentiation paradigm and the resulting fragmented market is the *"tendency for firms to jump to higher levels of u in order to escape crowding by competitors at lower levels."*[55]

In [38] is also discussed the case of a *Cournot* (rather than *Bertrand*) equilibrium in the final stage as well as sequential entry and multiproduct firms, and it is shown that the MFP is robust to all of these generalisations.

[53]See [38].

[54]See [38].

[55]See Sutton in [44] p. 390.

2.2.4 Comparative Statics

Having dealt with the finiteness property we proceed now to the complete solution, followed by a short analysis of comparative statics, of the pricing stage of a Bertrand-Nash equilibrium with single-product firms; we assume that only two firms enter the market. The analysis presented below is taken from Gabszewicz and Thisse [19]. We define

$$\tilde{V}(u_1, u_2) \equiv \frac{u_2 - u_0}{u_2 - u_1} \tag{25}$$

where $\tilde{V}$ is a measure of the relative distance of u_0 and u_1 from u_2. By straightforward solution of (20) and (22) for the case where $n = 2$, we obtain:

If $\tilde{V} \leq (b-a)/3a$ *(region III)*:

$$p_1 = \frac{b}{4r_1 + 3(r_2 - 1)} \qquad p_2 = \frac{2b(r_1 + r_2 - 1)}{r_2(4r_1 + 3(r_2 - 1))} \tag{26a}$$

If $(b-a)/3a \leq \tilde{V} \leq (b+a)/3a$ *(region II)*:

$$p_1 = \frac{a}{r_1} \qquad p_2 = \frac{b + a(r_2 - 1)/r_1}{2r_2} \tag{26b}$$

If $(b+a)/3a \leq \tilde{V}$ *(region I)*:

$$p_1 = \frac{b - 2a}{3(r_2 - 1)} \qquad p_2 = \frac{2b - a}{3r_2} \tag{26c}$$

If $b \leq 2a$ *(monopoly)*:

$$p_1 = 0 \qquad p_2 = a(u_1^{-1} - u_2^{-1}) \tag{26d}$$

The first thing to notice in the above expressions is again the absence of g: prices are not affected by the size of the market. Thus, the short-run gains to consumers from joining two **similar** (though possibly differently sized) economies through international trade are nil.[56] Based on the expressions (26a)-(26d), Gabszewicz and Thisse examine through comparative

[56] See [37].

statics the impact on prices of the parameters of the income distribution and the qualities.[57] A simple calculation shows that the mean of the income distribution is $\bar{x} = (b+a)/2$ and its standard deviation is $\sigma = (b-a)/2\sqrt{3}$. We first notice that p_1^* and p_2^* are piecewise linear functions of $\bar{x}$ .Then, keeping qualities constant we have:

If $(b-a)/3a \leq \tilde{V} \leq (b+a)/3a$:

$$\left.\frac{\partial p_1^*}{\partial \bar{x}}\right|_{\sigma=cst} > 0 \qquad \left.\frac{\partial p_2^*}{\partial \bar{x}}\right|_{\sigma=cst} > 0$$

If $(b+a)/3a \leq \tilde{V}$ and $b > 2a$

$$\left.\frac{\partial p_1^*}{\partial \bar{x}}\right|_{\sigma=cst} < 0 \qquad \left.\frac{\partial p_2^*}{\partial \bar{x}}\right|_{\sigma=cst} > 0$$

If $b \leq 2a$:

$$\left.\frac{\partial p_2^*}{\partial \bar{x}}\right|_{\sigma=cst} > 0$$

This shows that at low income levels both duopolists benefit from a policy that increases average income; beyond some level the low quality firm will be forced to decrease its price and eventually exit the market since the consumers are now rich enough to switch to the high quality product.

Similarily, inserting the value of standard deviation in (20) and (22) and again keeping qualities constant and noting that p_1^* and p_2^* are piecewise linear functions of σ, we get:

If $(b-a)/3a \leq \tilde{V} \leq (b+a)/3a$:

$$\left.\frac{\partial p_1^*}{\partial \sigma}\right|_{\bar{x}=cst} < 0 \qquad \left.\frac{\partial p_2^*}{\partial \sigma}\right|_{\bar{x}=cst} \text{ indeterminate}$$

If $\tilde{V} \leq (b-a)$ or $(b+a)/3a \leq \tilde{V}$ and $b > 2a$

$$\left.\frac{\partial p_1^*}{\partial \sigma}\right|_{\bar{x}=cst} > 0 \qquad \left.\frac{\partial p_2^*}{\partial \sigma}\right|_{\bar{x}=cst} > 0$$

If $b \leq 2a$:

$$\left.\frac{\partial p_2^*}{\partial \sigma}\right|_{\bar{x}=cst} < 0$$

[57] See [19].

As expected from the finiteness property, a substantial reduction of income inequality drives the lower quality out of the market; if the distribution of income continues to shrink beyond that point, the price of the surviving product rises since consumers now become more homogeneous. Hence, the ideal income dispersion from the point of view of consumers is the one that just eliminates the low quality from the market.

If we keep now the parameters of the distribution constant we note with respect to the impact of quality choices on prices[58] that all the derivatives with respect to u_2 are nonnegative; however, with respect to u_1, things are as follows:

If $\tilde{V} \leq (b-a)$

$$\left.\frac{\partial p_1^*}{\partial u_1}\right|_{u_2=cst} \text{ indeterminate} \qquad \left.\frac{\partial p_2^*}{\partial u_1}\right|_{u_2=cst} < 0$$

If $(b-a)/3a \leq \tilde{V} \leq (b+a)/3a$:

$$\left.\frac{\partial p_1^*}{\partial u_1}\right|_{u_2=cst} > 0 \qquad \left.\frac{\partial p_2^*}{\partial u_1}\right|_{u_2=cst} < 0$$

If $(b+a)/3a \leq \tilde{V}$ and $b > 2a$

$$\left.\frac{\partial p_1^*}{\partial u_1}\right|_{u_2=cst} < 0 \qquad \left.\frac{\partial p_2^*}{\partial u_1}\right|_{u_2=cst} < 0$$

If $b \leq 2a$:

$$\left.\frac{\partial p_2^*}{\partial u_1}\right|_{u_2=cst} < 0$$

Surprisingly, when $(b+a)/3a \leq V$ holds, an improvement in the quality of product 1 will result in a reduction in its price. On the other hand, in all the above cases $\partial p_2^*/\partial u_1^*$ is negative implying, as expected, that the optimal price of the high quality decreases as the quality of product 1 increases. The lesson from this analysis is that being too close in terms of quality can hurt both firms, and *"relaxing price competition through product price differentiation"* may be a profitable strategy for both duopolists.[59]

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[58] Of course qualities are ultimately determined endogenously in the previous stage of the game, but here we consider only the comparative statics of their effect on prices.

[59] See Shaked and Sutton in [39].

Although suggestive, the above statement is not yet rigorously substantiated since revenues depend not only on prices but also on sales; for this reason we now turn our attention to the optimal choice of qualities.

2.3 Product Selection

The examination of the literature on the pricing stage of the duopoly game has revealed that, in situations described by the model of VPD as defined in page 4, there will be only a *limited* number of firms, each one producing a single quality.[60] We turn to the discussion of the attempts in the literature to identify the specific qualities that will be chosen in the context of VPD. We start this examination by assuming the absence of any fixed costs, and also that variable costs do not depend on quality (it actually makes no difference if we assume them equal to zero as well). Further, it will be assumed that qualities can only be chosen from the set $[\underline{u}, \bar{u}]$ with $\underline{u} \geq u_0$; hence we exclude any technical progress. The case where $\bar{u}$ is allowed to increase will be discussed later. Unless otherwise stated we also assume that the market sustains only two firms, i.e., $2a \leq b \leq 4a$.

Since a higher quality has always an advantage over a lower one, it may appear that both firms have an incentive to choose the highest quality available, so that the market is characterised by *minimal differentiation.* This is indeed the conclusion in [20].

However, a quite different conclusion can emerge upon closer examination of the revenue functions after prices have been chosen optimally. Inserting the expressions for optimal prices from (26b)-(26c) in (17)-(19) we obtain:

If $t_1 \leq a$:

$$R_1 = \frac{1}{r_2 - 1} \cdot \left(\frac{b - 2a}{3}\right)^2 \qquad (28a)$$

$$R_2 = \left(\frac{2b - a}{3}\right)^2 \frac{1}{r_2} \qquad (28b)$$

[60] By assumption.

If $t_1 = a$:

$$R_1 = \frac{a[b - a(V+1)]}{2r_1} \quad (29a)$$

$$R_2 = \frac{[b + a(V-1)]^2}{4r_2} \quad (29b)$$

Before focussing our attention on the revenue functions let us examine the conditions determining whether the first or the second set of the above expressions hold. First notice that both the expressions (28) and (29) are increasing in u_2; this implies that the optimal choice of u_2 is $\bar{u}$, which also conforms to our intuition. We define $V(u_1) \equiv \tilde{V}(u_1, \bar{u})$ so that the terms *region I (region II)*[61] refer to the set of values of u_1 for which $V(u_1) \geq (\leq)(b+a)/3a$; hence, the expressions (28a)-(28b) apply in region I.

The function $V(u_1)$ is continuous and monotonically increasing in u_1 and as $u_1 \longrightarrow \bar{u}$, $V \longrightarrow +\infty$, so that the values of u_1 in the neighbourhood of $\bar{u}$ belong to region I. From (28a) and (28b) it is obvious that the revenues of both firms are decreasing in r_2; since r_2 is increasing in u_1,[62] both revenues must also be decreasing in u_1. Thus, **both** firms can do better by *"relaxing price competition through product differentiation"* as Shaked and Sutton suggest in [39].

The intuition behind this argument indicates that as u_1 gets further away from u_2, the two products become less homogeneous, thus conferring market power to the firms producing them. The question at this point is how far this process can go on. In other words, can one suggest as Hung and Schmitt [13] and Ireland [14] do that VPD markets are characterised by *maximal differentiation*? The answer to this question is generally negative because firm 1 trades off the additional heterogeneity at the expense of lower quality premia for its product.[63] Thus, although the Bertrand conjectures that characterise the pricing stage of the game require some heterogeneity for profits to be nonzero, too much differentiation may harm the revenue of the lower quality product. To make the point more rigorously, assume that $\underline{u} = u_0$, so that maximal differentiation implies $u_1 = u_0$.

[61] See page 29.

[62] See the definition of r_m in (11).

[63] Recall that we have assumed zero costs so there are no cost savings by choosing a lower quality.

Then, p_1 must equal 0 and $R_1 = 0$. Moreover, since $V(u_1)$ is continuous with $\lim_{u_1 \to u_0} V(u_1) = 1 < (b+a)/3a$[64], the expressions (28b) and (28a) must hold for low values of u_1. Also the continuity and monotonicity of $V(u_1)$ guarantee the existence of a unique value of u_1, name it $\hat{u}_1$, that belongs to both sets and is determined by the relation $V(\hat{u}_1) \equiv (2b+a)/3a$. Therefore, maximal differentiation will be observed only when $\underline{u}$ is close enough to $\bar{u}$, as is the case in [13] where Hung and Schmitt consider $\underline{u} \geq \hat{u}_1$, although they never state this assumption.[65] In the case of Ireland,[66] the maximal differentiation is due to his special assumptions that $u_0 = 0$[67] and that the set of feasible qualities is bounded away from zero.

Thus, even if the principle of maximal differentiation does not represent the general case in VPD, as some authors believe, it remains true that some differentation is always profitable for both firms. However, this is not to say that both firms benefit equally from product heterogeneity: as Shaked and Sutton show[68] the revenue of the higher quality always exceeds that of a lower quality[69]. Thus, when entry is simultaneous, both producers may be seeking the high quality although differentiation would be mutually beneficial. This problem can be easily assumed away by requiring that the firms enter sequentially, which is probably a more realistic assumption.

Shaked and Sutton have examined the equilibrium configuration of products in the context of natural duopoly in the presence of a fixed cost function that is increasing and convex in quality.[70] This situation is illustrated in Figure 6 taken from [37], where $R_i(u; u_j)$ represents the revenue of firm i as a function of its quality u for a given choice of quality u_j by the other firm. Thus, for instance, if firm 2 sets its quality equal to u_1, its revenue will be zero from the Bertrand assumption. Notice also that the position of the lower quality's revenue function depends on the *reservation quality* although both functions R_1 and R_2 depend on the quality level of each firm's *rival product*. Every firm maximises profit for a given quality

[64] By the assumption $b > 2a$.

[65] For more discussion on this point, see chapter 4.

[66] See [14] pp. 71,72.

[67] This assumption is allowed by his interpretation of (2) as being additional rather than total utility from the purchase of quality u_i.

[68] See [39].

[69] This result may not hold if the income distribution is very skewed on the right side.

[70] See [37].

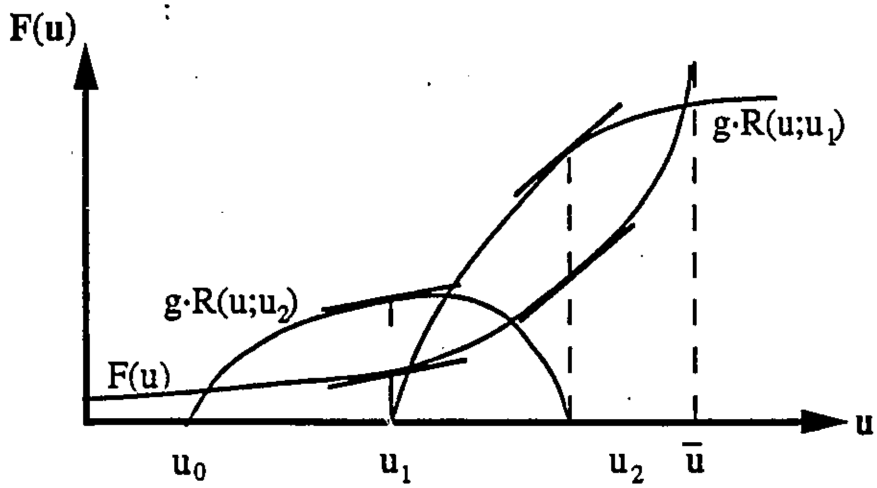

Figure 6: **Equilibrium Product Configuration when the Fixed Cost Increases with Quality.**
The curves $gR_1(u; u_2)$ and $gR_2(u; u_1)$ represent each firm's revenue as function of its quality for a given quality choice of its rival. The curve $F(u)$ represents the fixed cost as function of the quality levels.

of the rival product by setting $\partial R_i/\partial u_i = F' \equiv \partial F(u)/\partial u$, where $F(u)$ represents the fixed cost function. As can be seen from Figure 6, the combination (u_1,u_2) is an equilibrium configuration; Shaked and Sutton [37] claim to have found that such an equilibrium exists.

Two things are of importance in this diagram: the first is that the highest quality may no longer be the most profitable choice for firm 2, because now any increases in revenue from chosing a higher quality must be considered against increases in costs. The second is that, although the incumbent (firm 2) has not chosen the highest feasible quality, for the depicted solution to be a Nash equilibrium firm 1 must not have any incentive to 'jump above' and become the high quality producer. This may happen since the revenue of any 'high' quality depends on the position of its rival; it may be better for firm 1 to be the low quality producer with sufficient differentiation, rather than producing the higher quality in a market where qualities are too close.[71]

Extending the above discussion Shaked and Sutton conclude that, in the presence of fixed costs that depend on quality, increasing g, the size of the market, results in higher qualities being produced.[72] They use this result to assess the welfare impact of joining two similar economies through trade.[73]

2.4 Multiproduct Firms

2.4.1 Multiproduct Monopoly and Price Discrimination

All the works we have examined up to this point refer—with the exception of [28]—to single product firms. We now turn our attention to the more realistic case of multiproduct firms. By 'multiproduct firms' we mean firms producing a number of qualities of the *same product* and not firms selling in many different markets.[74] We start by assuming the whole market being supplied by a **protected** monopolist, so that entry threat is ruled out; in a situation like this, why would the monopolist introduce many qualities

[71]This point is also discussed in [13].

[72]Provided that the fixed cost function does not have kinks at the present quality levels.

[73]See [37].

[74]The old question of defining the boundaries of the market lies outside the scope of this paper.

competing with each other? To answer this question, consider the indifference map resulting from (13);[75] the indifference curves are linear with slope θ^j, the taste parameter of individual j. Define $\bar{I}^j$ as the indifference curve in (u, p) space with slope θ^j such that $(0,0) \in \bar{I}^j$,[76] and $\bar{p}^j(u)$ such that $(u_i, \bar{p}^j(u_i)) \in \bar{I}^j$, with $\bar{p}^j(u_i)$ representing the maximum amount consumer j is willing to pay for quality u_i. Since we are discussing quality differences rather than variety, $\bar{p}^j$ is monotonically increasing $\forall\ j$, so that any consumer's willingness to pay is at a maximum at $\bar{u}$. Thus, if average cost $c(u)$ rises slowly with quality,[77] the monopolist would produce only $\bar{u}$ in the absence of any price discrimination. On the other hand, assuming that the monopolist could identify consumers by their taste parameter and could also monitor purchases, would again imply the availability of only $\bar{u}$ except that now it would be offered at an interval of prices indicated by points A and B in Figure 7, where $A \in \bar{I}^a$, $B \in \bar{I}^b$, and the superscripts a and b represent the consumers with taste parameter θ_a and θ_b.[78] This solution represents the maximum maximorum for the monopolist's profit.

However, if the monopolist, although aware of the position of $\bar{I}^a$ and $\bar{I}^b$ as well as of the distribution of the taste parameter, is unable to identify consumers or monitor their purchases, the perfectly discriminating solution becomes infeasible; nevertheless, the monopolist can still practise (imperfect) price discrimination if he/she offers the consumers a price-quality schedule based on the distribution of willingness-to-pay rather than on cost conditions. To see how this can be done, consider only two types of consumers characterised by $\theta_a < \theta_b$[79] and assume initially that only combination B is available. Consumers with θ_a will not purchase the good since

[75]In order to simplify the diagrams, we develop the intuition related to this topic using the taste-dispersion model since the resulting indifference curves are linear; this is a purely simplifying assumption since the main idea of this section does not depend on the choice of model.

[76]The (u, p) combinations along $\bar{I}^j$ yield zero surplus to consumer j. Since the marginal utility of income is one in this model, the utility level corresponding to $\bar{I}^j$ is equal to consumer income.

[77]More rigorously, it is required that $\partial c/\partial u$ be bounded from above, at a level below θ_a.

[78]Recall that θ_a and θ_b are the low and high boundaries of the distribution of the taste parameter.

[79]The intuition presented here applies for any two values of θ, say θ_h and θ_l, for which $\theta_a \leq \theta_l < \theta_h \leq \theta_b$. We are using though θ_a and θ_b to avoid complicating Figure 7 too much.

Figure 7: **Multiproduct Monopoly and Price Discrimination**
This Figure superimposes two indifference maps similar to the one illustrated in Figure 2, representing two consumers with different θ's. The superscripts to the indifference curves (I) indicate consumers.

they derive a negative surplus, equal to OE.[80] If the monopolist tries to tempt them with a combination like A, they will accept the offer but, at the same time, the θ_b customers will purchase A as well since it yields a positive surplus equal to OF although the surplus from combination B is zero.

Consider now the alternative of offering a pair of combinations like C and D.[81] In this case, the θ_b consumers are indifferent between the two, and an infinitessimally small reduction in p_C would induce them to buy C. On the other hand, the θ_a consumers will find D more advantageous than both C and the non-purchase option. Therefore, quality differences are necessary in order to induce consumers to reveal their willingness-to-pay for quality.

How large ought this difference to be from the monopolist's point of view? To answer this question notice first that, if the two groups are

[80] See page 14 and in particular footnote 24 on that page.
[81] See again Figure 7.

considered separately, the monopolist revenue from each group increases monotonically with u.[82] Thus the monopolist would like to offer to each group a level of quality as high as possible. However, *if C is already supplied*, the monopolist cannot combine it with any quality $u_G > u_D$, because if u_G is offered at a price above p_G[83] nobody will buy it, while, if the price of u_G is lower than p_G, nobody will buy C anymore. Nonetheless, if the combination offered to high taste consumers is allowed to vary, a lower combination along I^a opens the possibility to extract more surplus from the high taste types, since the price of $\bar{u}$ can now be raised without inducing them to buy the lower quality. There is, therefore, a trade-off between revenues from the two groups, and the relative populations of the two groups (consumers' distribution) is obviously important for the determination of the profit-maximising price-quality combinations. If, for instance, the θ_l group is too small in size, or the range of technologically feasible qualities $[\underline{u}, \bar{u}]$, with $u_0 \leq \underline{u} < \bar{u}$, is too narrow, it may be optimal for the monopolist not to serve the low-taste customers. We examine this point in greater detail later in this work.

As Phlips points out the above idea is not new: this practice was observed already in the nineteenth century by Dupuit. The following quotation from Dupuit[84] "... introduces the idea of a *reduction* in quality (of the *lower-quality* goods) as a market segmentation technique":[85]

> It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriages or to upholster the third-class seats that some company or other has open carriages with wooden benches...What the company is trying to do is to prevent the passengers who can pay the second-class

[82]Notice that if differentiation is *vertical* in the sense we defined the concept on page 4, not only revenue, but *profit* will also be increasing with u. This is so because assuming a unanimous ranking among consumers of the qualities *priced at average cost* implies that the willingness-to-pay for quality improvements always exceeds the cost increments necessary to bring about those improvements. This point does not hold in the model of Mussa and Rosen presented below, because the cost structure together with the utility function employed in that paper do not imply VPD and the presence of the finiteness property.

[83]See Figure 7.

[84]As cited in Phlips, see [31] p. 216.

[85]See [31] p. 216.

> fare from traveling third-class; it hits the poor, not because it wants to hurt them, but to frighten the rich...And it is again for the same reason that the companies, having proved almost cruel to third-class passengers and mean to second-class ones, become lavish in dealing with first-class passengers. Having refused the poor what is necessary, they give the rich what is superfluous.

Three general characteristics of the solution to the above self-selection problem must be noted: a) quality $\bar{u}$ is always offered when cost does not rise with quality, b) the surplus of the high-taste consumers is always positive (imperfect discrimination), and c) the surplus of the low-taste buyers is always zero.

Despite the fact that the use of many qualities for purposes of discrimination has long been noticed by economists, it received a rigorous analytic treatment only in 1978, in the work of Mussa and Rosen [28]. Their assumptions include, together with a utility function as in (12) and a distribution as in (14), a cost function that contains no fixed part, so that a continuum of qualities is usually produced, and a per unit part that is constant in quantity and increasing and convex in quality.[86]

Using the above assumptions, Mussa and Rosen conclude that, if such an industry is monopolised, a larger interval of qualities would be produced than in the competitive case, with every consumer other than the one characterised by the highest taste parameter purchasing a lower quality than under competition; hence, the extension of the quality interval occurs at the lower end of the quality scale. Further, it is shown that some consumers may be bunched on the same quality.

The first of these results constitutes a rigorous proof of the Dupuit observation. In order to see how the 'bunching' result arises, let us consider the 'true' marginal revenue from extending sales to lower θ customers, which is shown to be:[87]

$$MR(\theta_\lambda) \equiv \theta_\lambda - \left[\frac{1 - F(\theta_\lambda)}{f(\theta_\lambda)}\right] \tag{30}$$

where $f(\theta)$ is the density function of the taste parameter among consumers,

[86] Recall that according to the condition (25) even under free entry we would observe an infinite number of qualities—the finiteness condition does not hold in this model.

[87] See [28].

$F(\theta) \equiv \int_{\underline{\theta}}^{\theta} f(s)ds$ is the cumulative distribution, and θ_λ is the taste parameter of the marginal consumer. Label the negative term in (30) $H(\theta)$; the term $H(\theta_\lambda)$ "represents the ratio of the number of consumers whose θ are larger than θ_λ to the density of the marginal consumer[s] whose θ... [are] equal to θ_λ."[88] The presence of $H(\theta)$ in the expression of MR indicates the loss in revenue from accepting to lower the price of all the qualities designed for higher taste customers, according to the condition for self-selection.

In this context, it would seem that the natural solution to the monopolist's problem is to determine the quality offered to any consumer by equating $MR(\theta)$ to $C'(u(\theta))$ for that consumer, where $u(\theta)$ is the assignment of customer type to product type. However, $MR'(\theta) = 2 + [1 - F(\theta)] \cdot f'(\theta)/f(\theta)$, so the $MR(\theta)$ function may have downward-sloping parts. This means that lower qualities would be offered to individuals with a higher taste for quality, a solution that is obviously not feasible since it violates the self-selection constraints. Moreover, the monopolist cannot ignore the consumers in the segment where $MR(\theta)$ is decreasing, unless he/she decides to exclude all those with lower θ as well, a strategy that may well be unprofitable. As it turns out, the optimal solution involves bunching of a range of consumers into an intermediate quality.

Using assumptions similar to those of Mussa and Rosen, Itoh [15] considers discrete qualities and investigates the impact of making the product differentiation finer or coarser on the price schedule of the multiproduct monopolist. The first result that he derives is that the introduction of a new product affects the prices of only products of higher quality; hence, the welfare impact of introducing relatively low qualities may be substantial. Note though that in this model the introduction of a new quality does not imply the optimal relocation of the already existing ones, as if the decision to increase the number of products is taken *after* some cost has been commited.[89] Because of this assumption, the direction of any effect on welfare depends only on whether the new product raises or lowers the prices of its higher qualities; this in its turn depends on whether the function $H(\theta)$ is respectively convex or concave at the θ of the potential demander of the new good.

[88] See [15] p. 93-94.

[89] In the example that Itoh provides the new quality is introduced after a *change* in its average cost.

Unfortunately, the otherwise interesting results of Itoh depend crucially upon the assumption of constant marginal rate of substitution between quality and price, and they do not hold for the utility specification of the income-dispersion model.[90]

2.4.2 Diffusion of New Products

When we examined the optimal quality configuration in a duopoly context, we saw that qualities may change over time due to advances in technology. In the case in which a monopolist controls the whole market he/she may find it optimal to introduce a new high quality only gradually in the market by lowering its price in consecutive periods. Stokey [41] considered this case as a possibility for price discrimination since the monopolist can sell initially to individuals with high valuation of the good and then lower the price for those with a less urgent need for the good. In a sense, the date of purchase becomes the attribute according to which 'quality' is assessed and the 'the sooner the better' principle applies.

Surprisingly, Stokey's analysis suggests that it is never optimal for the monopolist to discriminate intertemporally! A striking result indeed, since, as Salant [32] points out, it contradicts the analysis of Mussa and Rosen.[91] However, the comparative analysis of the two papers contained in [32] reveals that the result in [41] is due to the assumption of average cost being independent of quality that Stokey uses together with the taste-dispersion model[92] and that once this assumption is removed, discrimination becomes optimal in the intertemporal context as well.

On the other hand, Stokey's cost assumption can support intertemporal discrimination when combined with a non-additive utility function, like (2) of the income-dispersion model. This is the case with the analysis contained

[90]In that model, all the prices except p_1 and the price differences depend on the number and location of **all** the qualities (and not only on the lower ones), as can be seen by equations (33a) - (33c) below.

[91]Recall that the main idea in Mussa and Rosen is that the monopolist finds it optimal to use many qualities as a means of price discrimination. Assuming that availability at an earlier date is considered as better quality by all the consumers, Stokey's analysis seems to imply that the monopolist will never use the above device.

[92]Recall condition (25) together with the linearity of indifference curves in the taste-dispersion model.

in section 6.3 of Ireland's book[93] where consumers' expectations and the monopolist's ability to credibly *pre-commit* himself on the announced price path[94] are pointed as factors of fundamental importance for the determination of the price path and the speed of quality diffusion in equilibrium.

In a recent paper, Stoneman [42] introduces vertical differentiation in a model of diffusion so, as time goes on, changes in technology improve the quality of the product. Using the income-dispersion model with *myopic* buyers, he argues that the diffusion path may involve a period of rising product quality followed by a period of fixed quality and falling price.

2.4.3 Multiproduct Natural Monopoly

The price-quality schedule of the multiproduct monopolist has also been analyzed by Gabszewicz, Shaked, Sutton and Thisse in [16]. In this work, the demand side is described by the expressions (1) and (4) so it fits into the income-dispersion strand of the literature. On the cost side, it is assumed that the variable cost is zero; this is a simplifying assumption in order to preserve the finiteness property. Fixed costs are assumed to be independent of quality, and their presence transforms the continuum of qualities in [28] to a discrete set; this implies that one must determine where the qualities are located in the $[\underline{u},\bar{u}]$ segment.

The fact that the cost structure implies the presence of the finiteness condition, together with an assumption limiting the width of the income distribution to the interval $[a, b]$, where $a > 0$ and $b \leq 2a$, guarantees that the whole market will be supplied by a single firm even in the absence of legal protection or any other type of entry barrier. Since the work of Gabszewicz *et al.* is closely related to the analysis and the results of this thesis it is worth devoting a little more space to its presentation.

Supposing that the monopolist has already decided to sell n products of

[93]See [14] pp. 81-91.

[94]When consumers have perfect foresight, the monopolist's optimal strategy is *time inconsistent* which means that profit maximisation at any future date will dictate to the monopolist a price path different from the solution of the same problem at the present period. This renders the optimal path infeasible since the consumers will anticipate the future temptations of the seller to deviate from his/her announced path and they will react accordingly. The problem of time consistency was first introduced in the economic analysis by R.H. Strotz [43].

quality u_i, with $i = 1, 2, \ldots, n$ and $u_i \in [u_0, \bar{u}]$, his/her problem is to choose $p_1, \ldots, p_n$ so as to maximise the revenue function:

$$R = p_1(t_2 - a) + p_2(t_3 - t_2) + \cdots + p_n(b - t_n) \tag{31}$$

on $\Im = \{(p_1, \ldots, p_n) : p_\kappa \geq 0 \, \text{for} \, \kappa = 1, \ldots, n-1 \, \text{and} \, t_n \leq b\}$. The natural monopoly assumption guarantees that, even with only one quality introduced, the seller will always find it optimal to 'cover' the entire market so no consumer opts for a non-purchase. In this situation, the optimal p_1 must be such as to leave no surplus to the consumer of income a, for otherwise, additional revenue could be raised by charging a higher p_1 since no customer would drop out or choose a quality lower than before. Thus:[95, 96]

$$p_1^* = \frac{a}{r_1}. \tag{32}$$

Once p_1^* is given, the prices of the other qualities can be calculated as functions of p_1:[97]

$$p_\kappa^* = p_1^* + s_2 \left[\frac{1}{s_2} + \frac{1}{s_3} + \cdots + \frac{1}{s_\kappa}\right] \cdot (p_2^* - p_1^*), \tag{33a}$$

$$p_2^* = p_1^* + \frac{b - p_1^*}{s_2 B}, \tag{33b}$$

$$p_n^* = b - \frac{b - p_1^*}{B}, \tag{33c}$$

where

$$s_i \equiv \frac{u_i + u_{i-1}}{u_i - u_{i-1}}, \qquad \text{for } i = 1, \ldots, n \tag{34}$$

from which we obtain by using the definition of r_i from (11)[98] and the fact that $u_{i-1}/(u_i - u_{i-1}) = r_i - 1$:

$$s_i = 2r_i - 1 \text{for } i = 1, \ldots, n; \tag{35}$$

[95]The notation continues as in pages 12 and 9.
[96]Equation (3) in [16].
[97]Equations (6),(7),(8) in [16].
[98]See page 12.

similarly,

$$B \equiv 1 + \frac{1}{s_2} + \frac{1}{s_3} + \cdots + \frac{1}{s_n} = 1 + \sum_{i=2}^{n} \left\{ \frac{u_i - u_{i-1}}{u_i + u_{i-1}} \right\}. \tag{36}$$

Inserting the above expressions into (31), we obtain the price-maximised revenue function:[99]

$$2\,R = (b - p_1^*)p_n^* + p_1^*(b - 2a + p_1^*) \tag{37}$$

which obviously depends on qualities through p_1^*, p_2^* and p_n^*, as given by (33a), (33b), and (33c).

The monopolist's optimal choice of qualities can be obtained by maximising (37) with respect to $u_1, \ldots, u_n$. The satisfaction of the first order conditions requires that:

$$u_n^* = \bar{u}, \tag{38a}$$

$$u_\kappa^* = \sqrt{u_{\kappa-1}^* \cdot u_{\kappa+1}^*} \qquad \text{for } \kappa = 2, \ldots, n-1 \tag{38b}$$

The expression (38b) indicates that the monopolist constrained to produce at most n goods will produce exactly n goods[100] since in no case can $u_\kappa^* = u_{\kappa-1}^*$ or $u_\kappa^* = u_{\kappa+1}^*$. Treating u_1 as a parameter, the optimal expressions for all $u < \bar{u}$ are:[101]

$$u_\kappa^* = u_1 \cdot q^{\kappa-1} \tag{39}$$

where

$$q \equiv \left(\frac{\bar{u}}{u_1}\right)^{\frac{1}{n-1}}. \tag{40}$$

Inserting the optimal values of u_κ in the definition of s_κ and B, we obtain:[102]

$$s_\kappa = \frac{q+1}{q-1} \tag{41}$$

and

$$B^* = 1 + \frac{(n-1)(q-1)}{q+1}. \tag{42}$$

[99] Equation (10) in [16].

[100] We ignore for the moment the constraint for the set of optimal prices to be in $\Im$.

[101] Equation (12) in [16].

[102] Equations (13) and (14) in [16].

It is easy to check that if qualities have been optimally chosen, $s_i = s_j, \forall i, j = 2, \ldots, n$, hence prices form an arithmetic progression, *viz.*,

$$p_\kappa^* = p_1^* + (\kappa - 1)(p_2^* - p_1^*), \qquad \kappa = 3, \ldots, n. \tag{43}$$

Once all the optimal qualities and prices have been expressed in terms of u_1, it remains to examine whether there exists some $u_1 \in [u_0, \bar{u}]$ such that the corresponding vector of prices belongs to $\Im$, and to determine the optimal policy of the monopolist in both cases—existence or non-existence—of such a u_1.

The answer to the existence question comes from the condition that $t_2 \geq a$. Substituting s_κ and B^* in (32)-(33b) and using the definition, of t_2, this can be rewritten as

$$\frac{b-a}{a} \geq \frac{u_0}{u_1}\left[B\left(1+\frac{1}{q}\right)-1\right]. \tag{44}$$

This condition is satisfied if[103]

$$\frac{b-a}{a} > \frac{u_0}{\bar{u}}; \tag{*}$$

in which case there exists a u_1 capable to generate (together with (32), (33b) and (43)) a price vector that belongs to $\Im$. In this case the (unique) solution for u_1 is given implicitly in[104]

$$\frac{b-a}{a} = \frac{u_0}{u_1}[(n-1)(q-1)+q] \tag{45}$$

and the optimal strategy for the monopolist is to introduce as many qualities as he/she is allowed to.[105] On the other hand, if condition (*) does not hold, it is shown in [16] that the profit-maximising policy involves the bunching of all consumers into the highest quality. The intuition behind this 'paradoxical' result[106] is that when the market is too narrow (the LHS

[103] Condition (*) in [16].

[104] Equation (18) in [16].

[105] If fixed costs are assumed positive they will indicate the optimal n. Of course if the fixed cost is zero, it is optimal to produce a whole interval of qualities.

[106] In fact not so paradoxical after the analysis of Mussa and Rosen.

of (*) is too small), and/or the available possibilities for differentiation rather restricted (the RHS of (*) is too large), the loss in revenue due to self-selection requirements can be more important than the one resulting from a slight drop in the price of $\bar{u}$ sufficient to attract everybody into that quality.

Chander and Leruth [5] have presented an interesting extension to the problem of the multiproduct monopolist, where the quality of a type of product depends inversely on the number of this type's purchasers. Congestion problems come immediately to mind as an example of this situation and, indeed, the example used in [5] is that of the subway in Paris, where seats on special coaches are sold at higher prices only because they confer to their purchaser the privilege of being less crowded.[107]

In such a situation it is shown that there exists a profit maximising policy and that this policy always involves differentiation; hence, the possibility of introducing only one quality, as suggested by Gabszewicz *et al.*[16], does not exist in this context.

2.4.4 Multiproduct Duopolists

Price discrimination by means of product differentiation and the possibility of bunching are also examined in [4], where Champsaur and Rochet study these questions assuming *two* multiproduct sellers in the market. Their analysis uses the taste dispersion model, but the utility function is more general, assuming the form

$$U = v(\theta, u) + t - p, \tag{46}$$

where $v(\cdot)$ is increasing in both its arguments, three times differentiable and satisfies the single-crossing property.[108] It is easy to see that the utility in (13) is a special case of (46) with linear $v(\cdot)$, and that in this more generalised form indifference curves need no longer be linear. Nevertheless, income effects are again zero since income enters additively in the utility function, and it implies no further loss of generality to ignore the term t in (46).

[107] In all other aspects they are identical to the regular seats.
[108] See (6), page 11.

Although, as explained earlier, the taste dispersion model allows one to assume that the outside good is the numéraire, Champsaur and Rochet consider the existence of a reservation quality u_0 at a price that can be other than zero; this allows them to introduce the idea that the reservation quality can be a **higher** quality than those offered by the monopolist.

On the cost side, the assumptions in [4] are similar to the ones in [28], but the unanimous choice of a single quality in case all qualities are priced at unit variable cost is ruled out even as a special case;[109] hence, although their model can encompass many cases of 'vertical differentiation' in the definition of Lancaster[110] it does not fit into our definition of the concept.[111]

The analysis in [4] starts by examining the case of the discriminating monopolist in the light of (46) and the aforementioned considerations of the reservation quality. Conditions—in a spirit similar to that in [28]—for bunching some consumers in a single quality are deduced for both cases, that is, in which the reservation quality is either above or below the qualities in the market.

Assuming that the bunching conditions do not hold, and using the concept of a two stage perfect equilibrium as in [39], Champsaur and Rochet investigate the duopoly case and show that bunching may be present even under conditions that would exclude it under a monopolistic market structure.

More specifically, it is shown that when both firms are making positive profits, the one producing the high qualities will bunch some consumers at the lower quality end of its product line although the lower quality producer will bunch some of his/her customers to the upper end of his/her product line. This occurs because it is in the interest of both firms to relax price competition through **product line** differentiation. In other words, the firms avoid not only overlapping each other's product line, but even having them touch with each other. As a consequence, they bunch their customers whose θ would require some of the middleground qualities to the nearest quality they produce.

[109] It is not surprising then that, as shown in [4], the horizontal and 'vertical' differentiation cases are analytically identical.

[110] See [25].

[111] See chapter 1, page 4.

2.4.5 Effects of Public Policy

Although no work in the literature has formulated explicitly a call for government intervention, the impact of certain common policy measures has been examined in the context of a multiproduct monopoly.

Kluger [23] studied the implications of a minimum quality standard regulation for multiproduct monopoly pricing. As he observes, "[i]n response to a minimum quality standard, the monopolist will no longer be able to sell a full range of products. Because the profits from the sale of each quality depend on the prices of other qualities in the product line, a quality standard will, in many circumstances, affect the entire continuum of consumers."[112] Thus, one must expect changes in the whole pricing schedule as well as some consumers altering their purchase decision. With respect to the latter, Kluger finds that "there will always be a flat zone,[113] or a bunching of consumers who purchase the minimum quality."[114]

With respect to consumers' participation and their choice of quality it is shown that, in general, some consumers will drop out of the market and others will purchase higher qualities than before.[115]

In what concerns price changes, it is shown that they can occur in either direction, since the monopolist may decide either to lower the price schedule in order to reduce the dropouts, or to increase it, so he/she can extract more profits from the remaining buyers.

In the work of Krishna [24] the effect of *specific* and *ad valorem* taxes is examined, with attention to welfare consequences of such policies when the monopolist is a domestic or a foreign one.

It is shown that both forms of taxes remove consumers with low valuations who purchase the low quality products from the market; this tends to raise average quality. Thus in the case of a specific tax or a quota,[116] average quality rises, although in the case of an *ad valorem* measure the result is ambiguous.

The welfare effect of quotas is shown to depend on the distribution of

[112]See [23] p. 61.

[113]In the $u(\theta)$ function, see page 41.

[114]See [23] p. 67.

[115]This result may be reversed with the use of some decreasing distribution function.

[116]Quotas here correspond to the sale of licences for specific quantities, hence they can be considered equivalent to *specific tariffs*.

θ; if $H'(\theta_\lambda) < 0$ [117] prices rise by less than the tax, and if the monopolist's welfare is not taken into account (foreign firm) total surplus rises.[118] The welfare effect of *ad valorem* taxes is ambiguous.

2.5 Conclusions from the Literature Review

As a conclusion to the review of research on vertically differentiated markets we may say that most of the results in the literature have been derived from specific models, the most popular being the income and taste dispersion models. It can also be said that the two models have 'exclusive territories' in the domain of research interests, the first focusing on market structure and the second on the problem of a multiproduct monopolist.[119]

Surprisingly, little attention has been paid to questions of entry, and this constitutes a problem common to both strands in the literature.

Although the finiteness property implies an entry barrier, careful examination reveals that, except for the case of natural monopoly, the entry of new firms is not excluded by it when entry is *sequential* rather than simultaneous. On the contrary, potential entry in a vertically differentiated market may represent a more serious threat for the incumbent firms compared to the usual case of entry considered in the literature; if a new firm finds a place in the market, one of the incumbent firms will see its market share disappearing once the upper bound to the number of firms has been reached.[120].

Moreover, as already mentioned, in a vertically differentiated market a higher quality can always eliminate the market share of a lower one using a price **above average cost**. Thus, when firm 1—which from now on we assume to be the second firm to enter into the market—decides on how much to 'relax competition through product differentiation,'[121] **vulnera-**

[117] Recall from page 41 that $H(\theta) = [1 - F(\theta_\lambda)/f(\theta_\lambda]$ and that θ_λ represents the taste parameter of the marginal consumer.

[118] Recall that a) there is no deadweight loss as long as consumers do not change their quality choice because the demand is perfectly inelastic for participants in the market, and b) while marginal consumers are removed by the tax, their welfare is not affected since their surplus from the product was zero anyway.

[119] With the exceptions of [16] et [9].

[120] See [17].

[121] See page 33.

bility must be added to the list of factors affecting its quality decision; consequently, the presence or not of an entry threat can have a substantial impact on the degree of differentiation.

The only attempt to explore the possibilities that can arise from such a threat is the work of Hung and Schmitt.[122] Unfortunately, their analysis covers only a small part of the possibilities because their result of reduced differentiation rests on very specific assumptions.[123] We defer the discussion of their paper to chapter 4 so we can give it a more extensive treatment.

On the other hand the analysis of the multiproduct firm has always assumed the absence of an entry threat, except for the cases of natural monopoly. In this light the robustness of some of the results might be questionable, particularly the one relating to the uncovered market. As it turns out, however, the problem of market coverage not only persists despite the presence of entry threats but it can also be worsened by the incumbent's actions to impede entry.

In the present work we intend to give a more elaborate discussion to the impact of an entry threat in a vertically differentiated market. In particular, we focus our attention on two cases of natural duopoly: a) both firms produce a single product and b) the incumbent is a multiproduct firm. In the first case we study the impact that the entry threat can have on the choice of qualities in a more general setting than the one of [13]; in the second we focus on the question of market coverage, by first asking what are the precise conditions under which the monopolist would decide to serve only part of the market in the absence of a threat, and then determining whether under those conditions entry can be naturally blockaded.[124] This turns out to be a possibility, implying that, insofar as economic policy is concerned, one must be aware of the fact that the presence of an unserved portion of the market is not necessarily a temporary phenomenon indicating possibilities of further entry in the industry.

Further, when entry is not blockaded we examine the possible strategic

[122]See [13].

[123]For more details, see page 74.

[124]Unlike the usual case, the existence of blockaded entry in a non-natural monopoly context with a multiproduct incumbent is not a trivial question, since an increase in fixed cost reduces the incumbent's number of products and makes entry easier. For a more detailed discussion see chapter 5.

actions of the firm; Schmalensee's paper in 1978 [35] showed in the case of horizontal differentiation how the incumbent can forestall entry by increasing the number of his/her brands; however, the monopolist can also deter entry by changing his/her product mix. While this research was in progress, we found that Bonanno [2] showed that this strategy can be superior to product proliferation for the monopolist's profit in a horizontally differentiated market. As we show, this result holds when quality differences rather than variety characterise the market, and we examine the consequences of such an entry-deterring strategy on market coverage. As we already suggested, it turns out that the uncovered part of the market **increases** when the monopolist decides to impede entry by relocating his/her qualities; this counterintuitive result implies that, compared to the situation of a protected monopoly, more consumers will be now deprived from the surplus they could enjoy from purchasing the product. Although we do not address general welfare questions, it is obvious that market coverage is strictly related to the welfare of low income people; thus, especially if the product is a basic one (like housing), the participation of this part of the population can be a very sensitive issue of public policy.

The plan of the remaining chapters is as follows: in chapter 3 we analyse the optimal price-quality rules for a multiproduct monopolist, operating in a market which could sustain more than one firm but legally protected from entry and the related question of market coverage. Then, in chapter 4, we analyse the effects of *sequential entry* on the qualities produced by two single-product firms in a natural duopoly and the role of fixed cost in conferring first-mover advantages to the low quality incumbent. In chapter 5, we combine the results from the previous chapters to examine entry conditions and their impact on market coverage in a natural duopoly where the first firm is allowed to introduce many qualities. Finally, in chapter 6, together with the conclusions of this thesis, we discuss some remaining questions which constitute plans for future research.

3 Protected Monopoly and Uncovered Market

In this chapter we will analyse the case of a multiproduct monopoly which is legally protected from entry. More specifically, we will address questions concerning the monopolist's optimal choice of product line and of the price vector associated with it. As the answer to the above questions depends crucially on whether the incumbent firm decides to serve the whole range of consumers, the determination of the conditions that induce the monopolist to do so occupies a central place in this chapter's discussion. Further, the partial coverage of the market has welfare implications analogous to those arising from a single product monopoly since, as will be argued later, some consumers are deprived of the surplus they could obtain by purchasing some quality of the product. The modification of the monopolist's optimal strategies in the presence of entry threat will be treated in chapter 5 of this research.

As we saw previously, an analysis along similar lines has been conducted by Gabszewicz, Shaked, Sutton and Thisse in [16], where a monopolist facing no entry threat is allowed to introduce a finite number of qualities. In that study the protection of the monopolist is due to the fact that the income distribution is assumed to be narrow enough to result in a monopoly situation even in a free-entry Bertrand-Nash equilibrium. Our analysis differs from theirs in that it relaxes the narrowness assumption and determines the monopolist's pricing and quality rules for the whole set of values of the width of the income distribution. The important new element that surfaces from our generalisation is the possibility that a part of the market may remain unserved by the multiproduct monopolist. This is so because in a case of natural monopoly, by definition, a single product firm will always choose a price such that all the potential buyers will be served and no room will be left for a lower quality competitor; obviously the above result holds *a fortiori* for a multiproduct incumbent. However, when the market is wide enough to support more than one firm, it is not clear whether the protected monopolist will be willing to adopt a price-quality configuration that would induce even the poorest consumer to buy something. We must therefore start our research by trying to answer the above question and to determine the exact relations between the parameters that induce the monopolist to cover the entire market.

The possibility of an uncovered market has also been noted by Mussa and Rosen in [28]; however, a number of different assumptions, and the fact that Mussa and Rosen do not determine **the exact conditions** under which the uncovered market case occurs, make the analysis contained in this chapter necessary for any further examination of the case of multiproduct monopoly. The assumptions that differentiate this part of our study from the work of Mussa and Rosen are:

a) The use of the *income-dispersion* model, whereas Mussa and Rosen base their analysis on the taste-dispersion model using the separable utility function described in (2); as has been shown in the review of the literature, the taste-dispersion model can be considered as a special case of the income-dispersion model, so our results are more general than those suggested by the analysis in [28].

b) The product line is not restricted to be continuous but may be represented by a finite number of qualities, the continuous case being examined as the limit of the product line when the number of products allowed is infinite. This approach, besides being more realistic, allows for the determination of the different qualities to be produced and for the *endogenous* specification of the **number** of qualities when a cost structure is specified.

c) Our cost structure[125] implies *vertical differentiation* as defined on page 4. Thus, if all prices are equal to average cost, all the consumers will favour the high quality product, whereas in [28], the unanimity of preferences disappears at average cost pricing. Moreover, the cost function employed in this work implies the presence of the finiteness property: in a Bertrand-Nash equilibrium among single product firms there is an upper bound to the number of products that can find a positive market share and this bound is independent of the level of fixed cost. The cost structure of Mussa and Rosen does not possess the finiteness property; therefore, in a Bertrand-Nash equilibrium among single product firms an infinite number of products can coexist if the fixed costs is assumed to be zero. Hence, the model of Mussa and Rosen is one of *horizontal differentiation* even if it refers to quality differences.

[125]See below.

It must be noted from the outset that the presence of an uncovered market segment has important implications, both positive and normative. Concerning the former, there is a kink in the revenue function at the point where the market is just covered, so the monopolist's rules about pricing and choice of product line are substantially modified according to whether he/she decides to serve the entire market or not.

With respect to the normative aspect we must note that almost all the potential buyers excluded by the monopolist's policy are strictly worse off even if the less-than-perfect situation of oligopoly is considered as a standard of comparison[126]; this is so because the voluntary participation assumption guarantees that almost all the consumers[127] get a positive surplus from the purchase of the product in question.

Of course, this does not constitute a full welfare analysis since the effects of the monopolist's policies on the surplus of those who still purchase the product must also be considered. Although we undertake some discussion on this point later on in this research, we believe that in a welfare discussion market coverage is of interest *per se* since, when quality is a normal good, those that tend to be excluded from the market are the poorest customers. When distribution matters, this fact could be a problem on its own, calling for public intervention.

We begin by stating the assumptions pertaining to the demand side of our model. The taste dispersion model as expressed by the utility function in (1) will be the basis of our analysis. Following the tradition in the literature of product differentiation we will assume demand to be completely inelastic so that each consumer either buys one unit of the good or otherwise makes no purchase from the market in question.

The distribution of consumers according to their income is assumed uniform over a support $[a, b]$ with $b \geq 2a$. The density of the distribution will be assumed equal to one without any loss of generality; the $b \geq 2a$ assumption is meant to rule out the case of natural monopoly. Obviously more complicated distributions would be more realistic but since it is possible to create uncovered market situations just by skewing the lower end

[126]With a cost stucture that supports the finiteness property, the first best solution implies no differentiation whatsoever since all the consumers prefer unanimously the highest quality at a price vector that would allow the firm to break even on every quality.

[127]Except possibly for the poorest, see footnote 33 on page 19.

of the distribution and eliminate them by skewing the upper end, the use of any distribution other than the uniform would tend to obscure the point of market coverage without adding much to the intuition of the results.

The cost structure of the model consists of a fixed part that is independent of quality but must be paid every time the monopolist introduces a new quality, like a set-up cost; all variable costs are assumed zero.

To make the fixed cost dependent on quality would unnecessarily complicate the analysis to a substantial degree since the highest quality would not always be part of the monopolist's product line. On the other hand, if we accept the presence of a variable unitary cost that is independent of the quantity and quality produced, even the detailed expressions for the monopolist's rules would not be altered significantly. Again, making the variable cost per unit a function of quality would not destroy the results provided that average cost would not rise "too fast" with quality, as explained in pp. 23-26.

We consider the monopolist taking his/her decisions sequentially: first he/she decides on the qualities to be introduced and then the corresponding prices are determined. The reason for preferring this approach, instead of assuming a simultaneous choice of prices and qualities, is that in most relevant instances, changes in quality are less frequent than changes in price. This is due to the presence of some irrecoverable costs associated with a particular product type (design, advertising, selling expenses etc.), and to the fact that switching the production process from one type of product to another is a costly activity. The sunk nature of these costs forces the firm to commit itself to the production of a specific product line for a certain period, within which it finds it easier to change its price vector several times if necessary.[128]

3.1 Optimal Policies when the Market *is not* Covered

3.1.1 Pricing Policy

The revenue function of the monopolist is:

$$R = p_1(t_2 - t_1) + p_2(t_3 - t_2) + \ldots + p_{n-1}(t_n - t_{n-1}) + p_n(b - t_n) \qquad (47)$$

[128]For more discussion on this point see chapter 4.

with $t_1 \geq a$.

Noting that each price affects not only the revenue from the quality to which it is related but also the revenue from the two neighbouring qualities, the set of derivatives of (47) with respect to $p_1 \dots p_n$ is:

$$\begin{aligned} \frac{\partial R}{\partial p_1} &= t_2 - t_1 - p_1(r_2 - 1) - p_1 r_1 + p_2(r_2 - 1) \\ &= 2t_2 - t_1 - p_1 r_1 - p_2 && (48a) \\ \frac{\partial R}{\partial p_\kappa} &= t_{\kappa+1} - t_\kappa - p_\kappa r_\kappa - p_\kappa(r_{\kappa+1} - 1) + p_{\kappa-1} r_\kappa + p_{\kappa+1}(r_{\kappa+1} - 1) \\ &= 2t_{\kappa+1} - t_\kappa - p_\kappa r_\kappa + p_{\kappa-1} r_\kappa - p_{\kappa+1} && (48b) \\ \frac{\partial R}{\partial p_n} &= b - t_n - p_n r_n + p_{n-1} r_n && (48c) \end{aligned}$$

Adding and subtracting $p_{\kappa-1}$ from (48b), b and p_{n-1} from (48c) and using the definition of t_κ from (9), the above derivatives can be written as

$$\begin{aligned} \frac{\partial R}{\partial p_1} &= 2(t_2 - t_1) - p_2 && (49a) \\ \frac{\partial R}{\partial p_\kappa} &= 2(t_{\kappa+1} - t_\kappa) + p_{\kappa-1} - p_{\kappa+1} && (49b) \\ \frac{\partial R}{\partial p_n} &= 2(b - t_n) - b + p_{n-1} && (49c) \end{aligned}$$

Equating the above expressions to zero we obtain the following system of n equations the solution of which will determine the optimal price vector:

$$\begin{aligned} 2M^1 &= p_2^* && (50a) \\ 2M^\kappa &= p_{\kappa+1}^* - p_{\kappa-1}^* \qquad \text{for } \kappa = 2, \dots, n-1 && (50b) \\ 2M^n &= b - p_{n-1}^* && (50c) \end{aligned}$$

where[129] $M^i \equiv t_{i+1} - t_i$ for $i = 1, \dots, n-1$, $M^n \equiv b - t_n$, and (*) indicates optimal values. Obviously, M^i represents the width of the market for quality u_i[130] and as (50a) and (50b) indicate, for any quality except u_n, M^i is one half the difference of the prices of its neighbouring qualities.

[129] As defined on page 12.

[130] So the volume of sales of u_i is $M^i \cdot g$ where g is the density of the income distribution.

From the definitions of M^i and t_i the system (50) can be written as

$$\begin{aligned} 2p_2r_2 - 2p_1(r_2-1) - 2p_1r_1 - p_2 &= 0 \\ 2p_{\kappa+1}r_{\kappa+1} - 2p_\kappa(r_{\kappa+1}-1) - 2p_\kappa r_\kappa + 2p_{\kappa-1}(r_\kappa - 1) - p_{\kappa+1} + p_{\kappa-1} &= 0 \\ b - 2p_nr_n + 2p_{n-1}(r_{n-1}-1) + p_{n-1} &= 0 \end{aligned}$$

and using the definition of s_i,[131] we can rewrite the above equations as:

$$p_1^*(s_2+s_1) = p_2^*s_2 \tag{51a}$$
$$p_\kappa^*(s_{\kappa+1}+s_\kappa) = s_\kappa p_{\kappa-1}^* + s_{\kappa+1}p_{\kappa+1}^* \tag{51b}$$
$$p_n^*(1+s_n) = s_np_{n-1}^* + b. \tag{51c}$$

From (51a) and (51b) we have

$$p_2^* - p_1^* = \frac{s_1}{s_2}p_1^* \tag{52a}$$
$$p_{\kappa+1}^* - p_\kappa^* = \frac{s_\kappa}{s_{\kappa+1}}(p_\kappa^* - p_{\kappa-1}^*) \qquad \text{for } \kappa = 2,\ldots,n-1 \tag{52b}$$

Thus

$$p_{\kappa+1}^* - p_\kappa^* = \prod_{i=1}^{\kappa}\frac{s_i}{s_{i-1}}p_1^* = \frac{s_1}{s_{\kappa+1}}p_1^* \tag{53}$$

and the system of $n-1$ equations (52a), (52b) can be expressed as

$$p_{\kappa+1}^* - p_\kappa^* = \frac{s_1}{s_{\kappa+1}}p_1^* \qquad \text{for } \kappa = 1,\ldots,n-1$$

and since $p_{\kappa+1}^* = p_1^* + \sum_{i=2}^{\kappa+1}(p_i^* - p_{i-1}^*)$ for $\kappa = 2,\ldots,n-1$,

$$p_{\kappa+1}^* = p_1^*\left(1 + \sum_{i=2}^{\kappa+1}\frac{s_1}{s_i}\right) = p_1^*s_1\sum_{i=1}^{\kappa+1}\frac{1}{s_i}. \tag{54}$$

Obtaining from (54) the expressions for p_n^* and p_{n-1}^* and substituting them into (51c) we get

$$p_1^*s_1(1+s_n)\sum_{i=1}^{n}\frac{1}{s_i} = p_1^*s_1s_n\sum_{i=1}^{n-1}\frac{1}{s_i} + b \iff$$

[131] See the expressions (34) and (35) on page 44.

$$p_1^* = \frac{b}{\hat{B} \cdot s_1}, \qquad \hat{B} \equiv 1 + \frac{1}{s_1} + \frac{1}{s_2} + \ldots + \frac{1}{s_n}. \tag{55}$$

Using (54) and (55) we obtain, for $\kappa = 1, \ldots, n$:

$$p_\kappa^* = \frac{b}{\hat{B}} \left(\frac{1}{s_1} + \cdots + \frac{1}{s_\kappa} \right) \tag{56}$$

which fully characterises the optimal price vector as a function of qualities.

Notice that the optimal prices depend on the number of qualities and the relative distance between **all** the qualities. This is in sharp contrast with the analysis of Itoh in [15] where the additivity of the utility function implied that the introduction of a new quality would affect only the prices of higher qualities;[132] according to equation (56) the introduction of a new quality would affect **all** the prices since it will add a term in the parenthesis and it will modify at least the two s_i^{-1} terms, immediately below and above the new quality. Notice also that no p_κ^* depends on a since the market is uncovered

Multiplying each equation of the system (50a)–(50c) by the corresponding price and adding up all the equations, we have:

$$2R^* = bp_n^* = b^2 \left(1 - \frac{1}{\hat{B}} \right) \Longleftrightarrow R^* = \frac{b^2}{2} \left(1 - \frac{1}{\hat{B}} \right). \tag{57}$$

Again we see that the revenue, after being optimised w.r.t. prices, is a function of the n qualities through $\hat{B}$. Notice that revenue is measured in terms of the *numéraire* despite the presence of the b^2 term, since the whole expression in (57) is multiplied by the density g of the income distribution which measures 'number of consumers' per level of income; of course g does not appear in the above expression because it has been assumed constant and equal to one.

3.1.2 Choice of Quality

Differentiating (57) with respect to qualities and setting the derivatives equal to zero, we obtain a system of n equations described by:

$$\frac{\partial R}{\partial u_\kappa} = \frac{b^2}{2\hat{B}^2} \cdot \frac{\partial \hat{B}}{\partial u_\kappa} \qquad \text{for } \kappa = 1, \ldots, n \tag{58}$$

[132] See also the discussion in subsection 2.4.1, page 42.

with:

$$\frac{\partial \hat{B}}{\partial u_\kappa} = \frac{2u_{\kappa-1}}{(u_\kappa + u_{\kappa-1})^2} - \frac{2u_{\kappa+1}}{(u_{\kappa+1} + u_\kappa)^2} \qquad \text{for } \kappa = 1, \ldots, n-1 \quad (59a)$$

and

$$\frac{\partial \hat{B}}{\partial u_n} = \frac{2u_{n-1}}{(u_n + u_{n-1})^2} \qquad (59b)$$

Since $b^3/2\hat{B}^2 \geq 0$, the sign of $\partial R/\partial u_\kappa$in (58) depends on $\partial \hat{B}/\partial u_\kappa$. We set the latter equal to zero in the first n-1 equations, and from straightforward calculation we obtain:

$$u_\kappa^* = (u_{\kappa-1}^* \cdot u_{\kappa+1}^*)^{\frac{1}{2}} \qquad \text{for } \kappa = 2, \ldots, n-1 \quad (60a)$$

$$u_1 = (u_2^* \cdot u_0)^{\frac{1}{2}} \qquad (60b)$$

Therefore each quality is the geometric mean of its two neighbours. Also, since $\partial \hat{B}/\partial u_n$ is obviously strictly positive, we conclude that $u_n^*=\bar{u}$ as expected.

This set of optimal qualities is similar to the one obtained by Gabszewicz *et al.* in [16] for the natural monopoly but with one important difference: here, u_1 too is the geometric mean of the adjacent qualities u_2^* and u_0 just like the other qualities. In Gabszewicz *et al.*, by constrast, u_1^* was given by a corner solution.

Solving the system of first order conditions we obtain

$$u_\kappa = u_0 h^{\kappa/n} \qquad \text{for } \kappa = 1, \ldots, n-1 \qquad (61)$$

where

$$h \equiv \bar{u}/u_0 \qquad \text{for } \kappa = 1, \ldots, n-1. \qquad (62)$$

Define w as the ratio of any adjacent qualities; obviously,

$$w \equiv h^{1/n}. \qquad (63)$$

By straightforward calculation we obtain

$$s_\kappa^* = \frac{w+1}{w-1}\,, \qquad (64a)$$

$$\hat{B}^* = 1 + n \cdot \frac{w-1}{w+1}\,, \tag{64b}$$

$$r_\kappa^* = \frac{w}{w-1} \qquad \text{for } \kappa = 1, \ldots, n \tag{64c}$$

Using (56) and (64) we get

$$p_\kappa^* = b \cdot \frac{w-1}{w(n+1)-(n-1)} \tag{65}$$

which describes the optimal price vector. Equations (61) - (65) are valid as long as the t_1 implied by them is greater than a. Substituting the optimised expressions for p_1 and r_1 from (65) and (64) in the definition of t_1^* we get

$$t_1 = b \cdot \frac{w}{w(n+1)-(n-1)}\,, \tag{66}$$

so the total sales volume will be

$$\sum_{\kappa=1}^{n} M^\kappa = b \cdot \frac{n(w-1)+1}{w(n+1)-(n-1)}\,. \tag{67}$$

Therefore, the above rules hold and the market is uncovered as long as $\sum_{\kappa=1}^{n} M^\kappa > b - a$, or

$$t_1 \geq a \Longleftrightarrow \frac{kw-1}{w-1} \geq n \tag{68}$$

where

$$k \equiv (b-a)/a \tag{69}$$

is a measure of the relative width of the income distribution. This condition can be rewritten as:

$$t_1 \geq a \Longleftrightarrow w(n-k) < n-1. \tag{70}$$

Equation (70) is the necessary and sufficient condition guaranteeing that the multiproduct protected monopolist will not cover the entire market with the n available qualities. This implies that some consumers—the poorer ones—will be left out of the market, preferring not to buy anything rather than to buy any of the qualities available at their corresponding optimally-chosen prices. On the other hand, if (70) does not hold we have the case of the covered market presented in the next section.

The key issue is under what sets of values of the parameters h, k and n does (70) hold. First, note that (70) is always satisfied when $n - k < 0$, so the set of triples (h, k, n) that leaves the market uncovered ($t_1 > a$) is not empty. Also, it is immediately obvious that higher values for k make the presence of an unserved part in the market more probable. This is in accordance with intuition, since large differences in the willingness to pay among consumers imply that the introduction of a low quality, priced adequately to attract those with a low taste for quality, would require sharp reductions in the prices of higher qualities in accordance with the self selection constraints. The ensuing income reduction from those qualities may not be compensated by the extra revenue from the new quality. Since the LHS of (70) is increasing in w and thus in h as well, larger values of h point towards reversal of the above inequality. An increase in h implies that the range of qualities available to the monopolist will be wider and so will be the distance between any two of the optimally chosen qualities. This in turn implies that the effect of any quality upon the prices of the qualities above it will be weakened, thus making the option of offering to customers at the lower end of the distribution a cheaper product more appealing to the monopolist.

The following proposition, which constitutes the main result of this chapter, establishes the exact relation between k, h and n that determines whether the market is covered or not.

Proposition 1 *a) If $k \geq 1+\ln h$ the market will remain uncovered ($t_1 > a$) for any given finite n. b) If $k < 1+\ln h$ then there exists a maximum integer n_0 satisfying (70) strictly. The monopolist will cover the market if $n > n_0$, and will leave it uncovered for $n \leq n_0$.*

Proof: We first show that t_1 is always decreasing in n, i.e. the introduction of a new quality followed by a re-arrangement of the already existing qualities according to (61) increases market coverage. From (66), the derivative of t_1 with respect to n is

$$\frac{\partial t_1}{\partial n} = \frac{w'[w(n+1)-(n-1)] - w[w'(n+1)+w-1]}{[w(n+1)-(n-1)]^2} \tag{71}$$

where

$$w' \equiv \frac{\partial w}{\partial n} = -\frac{1}{n^2} h^{1/n} \ln h = -\frac{1}{n} w \ln w \tag{72}$$

and since the denominator is positive, (71) is ≤ 0 if and only if its numerator is negative, which is shown to be the case in what follows. First notice that

$$\begin{aligned} w'[w(n+1)-(n-1)] - w[w'(n+1)+w-1] &= \\ -w'(n-1) - w(w-1) &= \\ w\left(\frac{n-1}{n}\ln w - w + 1\right) &\leq 0 \qquad (73) \end{aligned}$$

It can easily be seen that the LHS of (73) is monotonically increasing since

$$\frac{\partial LHS(73)}{\partial n} = -\frac{(n-2)\ln w}{n^2} + \frac{1}{n}w\ln w = \frac{\ln w}{n}\left(w - \frac{n-2}{n}\right) > 0,\ \forall n > 1 \qquad (74)$$

On the other hand, as $n \longrightarrow +\infty$, $w \longrightarrow 1$ and the LHS of (74) goes to zero. Thus, the LHS of (73) can never be positive which proves that t_1 is a monotonically decreasing function of n; it also is obviously continuous. In order to see whether the market will be covered for some n it remains to check the limit of t_1^* as n approaches infinity; for this purpose, we rewrite (66) as

$$t_1^* = b \cdot \frac{\mathcal{N}}{\mathcal{D}}$$

where

$$\mathcal{N} \equiv \frac{w}{n-1}, \quad \text{and } \mathcal{D} \equiv w \cdot \frac{n+1}{n-1} - 1.$$

From the above expressions it is easy to see that as $n \longrightarrow \infty$, both $\mathcal{N}$ and $\mathcal{D}$ tend to zero so, by applying *l'Hospital's* rule, we can obtain

$$\begin{aligned} \lim_{n\to+\infty} t_1^* &= b \cdot \lim_{n\to+\infty} \frac{\partial \mathcal{N}}{\partial n} \Big/ \frac{\partial \mathcal{D}}{\partial n} \\ &= b \cdot \lim_{n\to+\infty} \frac{\dfrac{w'(n-1)-w}{(n-1)^2}}{\dfrac{[w'(n+1)+w](n-1) - w(n+1)}{(n-1)^2}} \Longleftrightarrow \end{aligned}$$

$$\lim_{n\to+\infty} t_1^* = b \cdot \frac{\lim_{n\to+\infty}[W(n-1)-1]}{\lim_{n\to+\infty}[W(n+1)(n-1)-2]} \qquad (75)$$

where $W \equiv w'/w = -n^{-1}\ln w = -n^{-2}\ln h$. Therefore, as $n \longrightarrow +\infty$,

$$W(n-1) - 1 = -\frac{1}{n}\left(1 - \frac{1}{n}\right)\ln h - 1 \longrightarrow -1$$

and

$$W(n+1)(n-1) - 2 = -\left(1 - \frac{1}{n}\right)\left(1 + \frac{1}{n}\right)\ln h - 2 \longrightarrow -(2 + \ln h)$$

hence

$$\lim_{n \to +\infty} t_1^* = b \cdot \frac{1}{2 + \ln h} = a \cdot \frac{k+1}{2 + \ln h} \tag{76}$$

since $b = a(k+1)$ from the definition of k. Obviously $\lim t_1 \geq a$ as long as $(k+1)/(2+\ln h) > 1$ and t_1 will be greater than a for any n, Q.E.D. If on the other hand, $k < 1 + \ln h$, there will be some finite value of n, name it $\hat{n}$, for which $t_1^* = a$ and the market will be covered for any number of qualities $\geq \hat{n}$, while for $n < \hat{n}$, the market remains uncovered. Q.E.D.

Proposition 1 implies the surprising result that if $k \geq 1 + \ln h$ then there is no finite number of qualities that will cover the entire market. The monopolist will introduce a number of qualities that depends on the fixed costs, but it will always leave some consumers (the poorest ones), who will prefer not to purchase any product at the optimally-chosen price-quality combination. When $k < 1 + \ln h$, on the other hand, the coverage of the market in its entirety will depend on the number of qualities that the monopolist may introduce. This number is given as a parameter here, but its ultimate choice will be a function of the fixed costs of introducing new qualities; it is determined in a subsequent section. In either case the results are considerably at variance with those of the natural monopoly in [16] where it is never optimal to leave consumers outside the market.

3.2 Optimal Policies when Market is Covered

This case will be outlined briefly since it is essentially similar to Gabszewicz *et al.*, [16].

3.2.1 Pricing Policy

It is easy to see here that the monopolist will never choose a price-quality configuration that yields $t_1 < a$, the reason being that, if $t_1 < a$, the monopolist can increase the price of u_1 and increase his/her revenue without losing any customers, since there are none between t_1 and a. If the market is covered we have, therefore, for every choice of u_1:

$$p_1^* = \frac{a}{r_1} \tag{77}$$

and

$$R = p_1(t_2 - a) + p_2(t_3 - t_2) + \ldots + p_n(b - t_n). \tag{78}$$

Differentiating R with respect to prices we get a system of $n-1$ equations. The pricing rules in this case are similar to the ones for the natural monopolist as determined in [16], namely:

$$p_\kappa^* = p_1^* + \frac{b - p_1^*}{B}\left(\frac{1}{s_2} + \frac{1}{s_3} + \cdots + \frac{1}{s_\kappa}\right) \tag{79a}$$

$$p_n^* = b - \frac{b - p_1^*}{B} \tag{79b}$$

where p_1^* is given by (77) and $B = \hat{B} - 1/s_1$ as in (36).

The maximised revenue can be written as

$$2R^* = (b - p_1^*)p_n^* + p_1^*(b - 2a + p_1^*) \tag{80}$$

which is equation (10) in [16].

3.2.2 Choice of Quality

Maximising (80) with respect to $u_1, \ldots, u_n$ we get the optimal quality choice of the multiproduct monopolist. Since this case is similar to the one presented by Gabszewicz *et al.* we will only give the results, i.e., the solutions for $u_1^*, \ldots, u_n^*$.

$$u_\kappa^* = u_1^* q^{\kappa-1} \quad \text{for } \kappa = 2, \ldots, n-1 \tag{80a}$$

$$u_n = \bar{u}, \quad \text{(equation [12] in Gabszewicz } et\ al.) \tag{80b}$$

because R is monotonically increasing in u_n. The solution for u_1^* is given implicitly in

$$k = \frac{u_0}{u_1}\left[(n-1)(q-1) + q\right] \quad \text{(equation [12] in Gabszewicz } et\ al.) \tag{80c}$$

where

$$q \equiv \left(\frac{\bar{u}}{u_1}\right)^{\frac{1}{n-1}}.$$

Furthermore, for this system to be valid we must have $t_2 \geq a$, implying:

$$k \geq \frac{u_0}{u_1} \cdot \left[B\left(1+\frac{1}{q}\right)-1\right] \tag{81}$$

which is equation (16) in Gabszewicz *et al.* We will discuss this condition in the next section.

3.3 The Optimal Number of Qualities

The previous analysis was conducted under the assumption that n was given. We now turn to the determination of the optimal number of qualities. Evidently, the crucial parameter in this analysis will be the value of the fixed cost. Nevertheless, in order to bridge this discussion with the previous sections, as well as with the work of Gabszewicz, Shaked, Sutton and Thisse, we will start from the case of a zero fixed cost of introducing an additional quality. In the Gabszewicz *et al.* analysis for $k < 1$, ($b < 2a$) it was shown that the monopolist will introduce an infinite number of qualities as long as (81) is satisfied. Further, they show that (81) will be satisfied if and only if

$$\frac{b-a}{a} > \frac{u_0}{\bar{u}} \qquad \text{(Condition (*) in Gabszewicz } et\ al.\text{)} \tag{82}$$

This condition (82) is independent of n. Therefore, if it holds, it does so for any number of qualities, and as Gabszewicz, Shaked, Sutton and Thisse show, the optimal n will be infinite. On the other hand, if (82) does not hold, only $\bar{u}$ will have a positive market share. This means that when the market is quite narrow and so is the available quality variation, it may be profitable for the monopolist to bunch all the consumers in the highest quality. The reason for this is that, when the monopolist introduces a lower quality, if this quality is to have some positive market share it will decrease the revenue of the immediately higher quality; moreover, if the market is too narrow and the available qualities are too close, this reduction of the revenue from $\bar{u}$ may not be compensated from the extra revenue any lower quality would bring. In our case, since $k > 1 > h^{-1}$, condition

(82) is always fulfilled but, of course, it is relevant only when the market is covered. The examination of the uncovered market case, leads to the following proposition:

Proposition 2 *If k is larger than 1 and the fixed cost of introducing an additional quality is zero then the optimal number of qualities that the monopolist would want to introduce is always infinite.*

Proof: If $k \geq 1 + \ln h$, Proposition 1 implies that condition (70) will be satisfied for any finite n, and the maximised revenue function will always be determined by (57) if we substitute in it the optimal value for $\hat{B}$ from (64b). Taking the derivative of the resulting expression with respect to n, we obtain:

$$\frac{\partial R^*}{\partial n} = b^2 \cdot \frac{A}{2[(n+1)w - (n-1)]^2} \tag{83}$$

where $A \equiv w^2 - 2w \ln w - 1$. Obviously, the sign of $\partial R^*/\partial n$ depends on the sign of A. It is easy to prove that $A > 0$ if $w > 1$ independently of the value of n. We have

$$\frac{\partial A}{\partial w} = 2w - 2\ln w - 2 = 2(w - \ln w - 1) \tag{84}$$

$$\frac{\partial^2 A}{\partial w^2} = 2\left(1 - \frac{1}{w}\right) > 0 \qquad \text{since } w > 1. \tag{85}$$

Since it is easily verified that $\left.\frac{\partial A}{\partial w}\right|_{w=1} = 0$, it follows that $\partial A/\partial w \geq 0$ for $w \geq 1$. Also, $A = 0$ for $w = 1$. Consequently, A starts from 0 and rises as w increases, therefore it can never be negative. Q.E.D.

Hence, if $k \geq 1 + \ln h$ and fixed costs are zero the optimal number of qualities is infinite. Similarly, with $k < 1 + \ln h$ and as long as $n < n_0$ the market is uncovered and the analysis is the same as when $k \geq 1 + \ln h$, while for $n > n_0$ the market is covered and the fulfilment of (82) guarantees a monotonically increasing revenue function, Q.E.D.

Thus, when $k > 1$ and there are no fixed costs the possibility of a finite number of qualities disappears, unlike the case in [16]. This is perhaps not surprising. If the natural monopoly of Gabszewicz *et al.* finds it optimal to introduce an infinite number of qualities when the income distribution is wide enough one expects that a similar result would obtain when that distribution gets even wider.

We now turn our attention to the calculation of the optimal number of qualities when fixed costs are present. We will assume for simplicity that there is a fixed fee F for the introduction of any new quality and that F is like a licence, i.e. independent of the level of quality; in such a case the total cost is equal to nF, which means that the cost function is a linear function of total output units.

The first order condition for profit maximisation requires that the derivative of the revenue function[133] as expressed in (83) must be set equal to the marginal cost of introducing a new quality, which in our case is F. In order to establish that the thus determined value for n, call it n^*, represents indeed the optimal number of products, we must also show that the revenue function is concave around n^* and that $\nexists \ddot{n} > n_0$ such that $R^*(\ddot{n}) > R^*(n^*)$. The concavity of $R^*(n)$ is shown to hold in Lemma 1 although the fact that n^* is a global optimum is established as part of the proof of Proposition 3 below.

It is easy to see that, by Proposition 1, the optimal number n^* of qualities when $F > 0$ will always leave the market uncovered when $k \geq 1+\ln h$. Similarly, the market may also remain uncovered when $k < 1+\ln h$, provided F is sufficiently large to produce an optimal choice of qualities $n^* \leq n_0$. In what follows we shall derive conditions that determine under what circumstances this happens. We first prove the following auxiliary results.

Lemma 1 *When the market is uncovered the monopolist's optimal revenue $R^*(n)$ is a concave function of n.*

Proof: We treat n as a continuous variable and examine the second derivative $\partial^2 R^*/\partial n^2$. From (83) we know that $\partial R^*/\partial n = b^2 A/2D^2$ where $D \equiv (n+1)w - (n-1) > 0$, and A was defined in the proof of Proposition 2. Hence, for the concavity of R^*(n) we need to show that

$$D\frac{\partial A}{\partial n} - 2A\frac{\partial D}{\partial n} \leq 0 \tag{86}$$

[133] Already maximised for prices and qualities.

Substituting and simplifying, we find that the left-hand side (LHS) of (86) is proportional to the expression $4(w-1)w\ln w - w\ln^2 w - w^2\ln^2 w - wln^2w(w-1)/n$. Since the last term is obviously negative, it suffices to show that the expression

$$L(w) \equiv 4(w-1)w\ln w - wln^2w(w+1)$$

is negative for $w \in [1,h]$. We note that $L(1)=0$ and

$$L'(w) = 3(w-1)[\ln w - (w-1)] + \ln w[3(w-1) - 2w\ln w - \ln w]$$

For $w>1$, both expressions in brackets in the above equation can be shown to be negative, implying that $L'(w)<0$ for $w>1$ and $L(w)<0$, which in turn implies that (86) holds, Q.E.D.

The next auxiliary result is somewhat obvious, but will be used extensively in the proof of Proposition 3 that follows.

Lemma 2 *Suppose that the number of qualities n is fixed exogenously, and that the upper limit b of consumer income is fixed. Then the optimal revenue $R^*(n)$ is a non-decreasing function of the width of the income distribution, i.e. a non-increasing function of the lower limit a of consumer income.*

Proof: If the optimal $R^*(n)$ leaves the market uncovered then decreasing a would clearly leave the optimal revenue unaffected.[134] If the market is covered then the previous optimal price and quality choices would still be available when a decreases, implying that the new optimal revenue is at least as large, Q.E.D.

The two lemmas allow us now to prove the last result of this section.

Proposition 3 *If $F>0$ and $k<1+\ln h$ the monopolist will leave the market uncovered if the following relation holds:*

$$\frac{2F}{b^2} \geq \frac{{w_0}^2 - 2w_0\ln w_0 - 1}{[(n_0+1)w_0-1)]^2} \tag{87}$$

where $w_0 \equiv h^{1/n_0}$ provided F is greater than[135] $(b^2/4)[(h-1)/h]$. *On the*

[134]Notice that, since the density of consumer distribution is unaffected, lowering a implies *increasing* the size of the market. However, if the market is initially uncovered consumer a is not purchasing anything and therefore no new consumer with income $I<a$ will do so either. If on the other hand, instead of expanding the market size by lowering a we do so by increasing the density of consumer distribution, the firm's revenue will increase proportionately.

[135]Since $R^*(1) = \frac{b^2}{4}\cdot\frac{h-1}{h}$.

other hand, if (87) is violated the market is either covered, or the optimal number of qualities is $n^* = n_0$.

Proof: Define as $\check{R}^*(n,a)$ the expression of the revenue of the monopolist after prices and qualities have been chosen optimally and the whole market is covered, and let $\hat{R}^*(n)$ be the same function in the presence of an uncovered market. Notice that the RHS of (87) corresponds to $(2/b^2)\partial\hat{R}^*(n)/\partial n$ evaluated at n_0. This, in conjunction with the fact that $\partial\hat{R}^*(n)/\partial n$ is decreasing in n, implies that if condition (87) holds, there exists an $n^* \leq n_0$ which equates the marginal revenue from introducing an extra quality to the marginal cost of that quality. The concavity of $\hat{R}^*(n)$ in n implies that the profit function with uncovered market has a unique maximum at n^*.[136] For Proposition 3 to hold we must show that there is no $n^* \geq n_0 + 1$ such as $\check{R}^*(n^*,a) \geq \hat{R}^*(n^*,a)$.

Define as $\Gamma(b,h,n) \equiv t_1(b,h,n) - a$ the uncovered part of the market. For the market to be 'just' covered Γ must be equal to 0. For every value of n, we can define as $\hat{a}(n)$ that value of a that guarantees $\Gamma \equiv 0$.

Notice that $\hat{R}^*(\cdot)$ is independent of a, as indicated by the expression (57), although $\check{R}^*(\cdot)$ is decreasing in a[137] as indicated by Lemma 2. Hence, the $\check{R}^*(n,a)$ reaches a maximum when $a = \hat{a}$ so $\check{R}^*(n,a) \leq \check{R}^*(n,\hat{a}(n))$ for all n. However, $\check{R}^*(n,\hat{a}(n)) \equiv \hat{R}^*(n)$ for any n. Hence, $\check{R}^*(n,a) \leq \hat{R}^*(n)$ for any n. Also, since n^* represents the unique maximum of $\hat{R}^*(\cdot)$, $\hat{R}^*(n) - nF \leq \hat{R}^*(n^*) - n^*F$. Combining the above inequalities, we obtain

$$\check{R}^*(n,a) - nF \leq \hat{R}^*(n^*) - n^*F, \qquad \forall n,$$

which implies that if $a \leq \hat{a}(n^*)$, the uncovered market strategy with n^* qualities dominates the whole set of strategies with covered market. Q.E.D.

Proposition 3 provides therefore a sufficient condition for the market to remain uncovered even when it is sufficiently "narrow" to allow coverage given an adequate number of qualities. This condition guarantees that the monopolist will leave the poorest customers unsatisfied, in the sense that the optimal prices even for the cheapest produced qualities would be high enough to make them prefer not to purchase the product.

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[136]For convenience we treat n as continuous. If n^* happens not to be an integer the optimal value of n will be represented by one of its neighbouring integers.

[137]Recall that when a increases the market becomes more narrow.

3.4 Conclusions from Chapter 3

The analysis presented in this chapter showed that, once we relax the assumption of natural monopoly market, a single firm may decide not to serve the entire spectrum of consumers even if the fixed cost is so low as to allow a very large number of qualities.

The intuition behind the appearance of an uncovered market is linked to the presence of self-selection constraints in the choices of the monopolist. Since the seller does not know the exact amount each consumer is willing to pay for quality, an incentive must be provided for high taste consumers to reveal their identities. Improving market coverage by reducing the low quality prices interferes with this incentive, unless the high quality prices are lowered as well; this, however, reduces profits. Note also that optimal market coverage tends to be lower when the range of available qualities is narrow (h is small). In such a case the optimal distance $h^{1/n}$ between qualities is also small, implying a high degree of interdependence between quality prices.

Obviously, in order to motivate the analysis of a monopoly firm in a nonnatural monopoly setting, one must assume that the monopolist is protected against entry by a patent, licence or any other type of entry barrier; this of course is far from being an unrealistic possibility. However, although the results of Proposition 2 and Proposition 3 hold only if the monopolist is protected, they are very important for the analysis of entry because they constitute the unconstrained optimal decisions of the firm which will be used as a benchmark in discussing how the behaviour of the incumbent monopolist is affected by the presence of an entry threat. This discussion will be resumed in chapter 5 after some additional insights into the nature of competition between two firms are presented in the next chapter.

4 Entry Threat and Quality Determination in a Natural Duopoly

Having completed the analysis of the *protected* monopoly we move now to the examination of situations in which more than one firms are present, either as actual or as potential competitors. Our first step in this direction will be to examine the competition between single product firms. Also, in order to keep things manageable we will deal with *natural duopolies*, i.e., markets that can sustain only two firms with positive market shares in a Bertrand-Nash equilibrium.

As is discussed in chapter 2, many aspects of the natural duopoly case have already been studied in [39]. More specifically, Shaked and Sutton have examined the optimal pricing policies of the two firms and implicitly considered their quality choices. Concerning the latter point their interest was focussed on showing that one firm in the market will choose the highest quality ($\bar{u}$) and that its rival will prefer to choose a lower quality rather than face stiff competition on the top quality product. In order to prove this point, they provided expressions that contain the solution for the optimal u_1, name it u_1^*, implicitly.

However, Shaked and Sutton did not provide any further analysis of the aforementioned solution since this was neither part of their objectives nor a necessary step for their conclusions.

Most important, their analysis considers entry to be *simultaneous* rather than *sequential*. After two firms enter, the finiteness property indicates to all the other potential competitors that, if they enter, they would loose their entry cost, which in Shaked and Sutton is assumed to be sunk.

This approach can be interpreted as assuming the fixed cost to be *firm specific* rather than *quality specific*. In other words, the firm cannot recover its entry cost in the case of exit, but, while in the market, it can change its quality at no penalty.

The problem with this line of argument is that it deprives the choice of quality from any potential *strategic importance*: the first two firms to enter will get the whole market **no matter which quality they decide to produce**. This is so since, any further entrant knows that, if it enters with a quality between the two incumbent qualities and tries to seize the market share of the lower quality incumbent, the latter will react by increasing the quality of its product. The final equilibrium will involve all three firms producing the highest quality and making a loss equal to their entry cost, according to the Bertrand-Nash assumption. However, although such an outcome can be acceptable for the incumbent firms since they have already sunk their entry cost, it will completely discourage any prospective entrants. Therefore, once in, the first two firms can decide their quality levels on the basis of profit maximisation, without considering any constraints related to protection from further entry.

The assumption of a sunk fixed cost related to entry rather than to the choice of quality is in many instances a reasonable one. However, it is also reasonable to assume that changes in quality are not costless. This is especially true when the postulated concept of equilibrium is that of *multistage perfect*, with entry, qualities and prices decided successively rather than all three of them being determined at once. For, all the arguments justifying the use of a multistage perfect equilibrium—namely the presence of design costs, costs related to switching production from one type to the other or advertising expenditures to introduce a new quality—represent quality specific costs that are sunk, at least in the short run.

Once the fixed cost—or just a large part of it—is considered quality-specific, the first two firms in the market, and more specifically the one producing the lower quality, can no longer ignore the threat coming from potential entrants. This is so because, a large percentage of the sunk cost being quality-specific, the lower-quality incumbent's threat of increasing his/her quality becomes non-credible. Thus, even if the number of firms with positive market shares is to be no greater than two, the lower quality producer may find itself in serious trouble if some entrant decides to market a new quality that lies between the two already existing ones. In this case, entry will not simply reduce the market share of the lower quality incumbent, but will bring it down to zero.

Hung and Schmitt[138] correctly noticed this point and modified the analysis of Shaked and Sutton by considering entry to be sequential rather than simultaneous and assuming the presence of a large number of potential entrants ready to enter and displace the lower quality incumbent, if this can

[138]See [13].

be a profitable move.

In such a setting Hung and Schmitt concluded a) that potential entry would have the effect of *narrowing product differentiation* below the levels identified by Shaked and Sutton, and b) that the ratio of high to low qualities produced by the two firms in the market would be constant, depending only on cost parameters and the width of the income distribution.

However, the analysis in [13] relies heavily on two specific assumptions that are both arbitrary and unstated. It is therefore necessary to remove them and re-examine the optimal choice of quality of the duopolists that arises under general conditions; the present chapter of this thesis will be devoted to this end.

The first of these assumptions states that the lower boundary of the technologically-feasible range of qualities is sufficiently high to induce a corner solution to the problem of quality choice of the lower-quality firm; here, this assumption is removed and we determine this choice without any restrictions. The immediate implications of relaxing the restrictions on the choice set are that the principle of *maximal differentiation*[139] is not implied by the analysis of Shaked and Sutton, as Hung and Schmitt seem to believe, and that the ratio of high to low qualities becomes now a function of consumer utility parameters, especially of the *utility of the "reservation quality"*, the lowest quality potentially available in the market. It turns out that this more general solution also has crucial implications for the subsequent entry game.

The second assumption relates to the logic that governs the rules of that game. Hung and Schmitt adopt a zero-profit condition for the lower quality firm, arguing that if the lower quality is making a positive profit, then a subsequent entrant can choose a slightly higher quality level and capture all of its market share. In their words,[140]

> [the entrant] ... can safely adopt this price strategy because ... firm [1—the lower quality] is forced to exit whatever its (rational) price strategy. Hence, in order to stay in the market, firm [1] must choose ...[its quality level] from its zero-profit condition. This condition deters any further entry.

[139] See the discussion on page 33.

[140] See [13], p. 290.

This reasoning seems to imply that *in a vertically differentiated oligopoly, the irrecoverability of the fixed cost does not confer any first-mover advantages.* Indeed, if anything, it creates a *problem* to the incumbent, since it forbids the leap-frogging in qualities, a necessary element of defence against new entry.

As we hope to be able to show here, this conclusion is not robust: although a subsequent entrant may capture all the market share of firm 1, the latter's decision to stay or exit will depend on the *nature of the fixed cost.* If this cost is recoverable upon exit, firm 1 will quit the market; if it is sunk into the production of a specific quality, the optimal decision concerning the exit of firm 1 is indeterminate without any further cost considerations. If there are some pure exit costs, like cleaning up the site,[141] firm 1 may choose not to exit even if its sales are zero; this decision may have serious consequences for the revenue of any prospective entrant thus creating an entry barrier.

In our analysis we examine the implications of the entry cost being *quality specific* and *totally irrecoverable*, so that the lower quality entrant has no incentive to liquidate its fixed investment. Its product may then remain in the market as potentially available to consumers, even though its market share may fall to zero. In such a case, *the lower quality incumbent product would provide the new reservation quality, which will affect the profits of any subsequent entrant.* The ultimate result then is that this new (higher) reservation quality will have a depressing effect on these profits. It may then happen that such an effect would be sufficient to drive these profits down to zero, thus ensuring that no entry would take place, even if firm 1 realises positive profits. Potential entry in such cases would not reduce product differentiation, in contrast to the Hung and Schmitt results. At the end of this chapter we provide a numerical example that illustrates this point.

[141]Another, and potentially more interesting, example of exit cost can be given when there is uncertainty over future realisations of k, the relative width of the income distribution. In that case, a possible large realisation of k could give again some market share to the lower quality incumbent, turning the market to a natural triopoly for at least some time. No matter how small the *expected* revenues can be, they make exit an inferior strategy for the displaced incumbent, since, by staying in, he/she can now expect to recover part of the otherwise lost fixed cost.

It is important to notice that the entry deterrence mechanism described just above *differs significantly* from the one through which entry can be impeded in the case of homogeneous products,[142] despite the importance of cost irrecoverability being common to both cases. When products are homogeneous, it is the threat of producing a sufficiently large quantity that convinces the entrant to stay out. In the case of vertically differentiated products the post-entry level of output of the incumbent is of no importance, being always equal to zero. The entrant's profit is now curtailed through the reduction of the price that consumers are willing to pay for the entrant's product. This reduction occurs because even the *potential* availability of an alternative better than opting out of the market reduces the distance between any available quality and the consumer's reservation quality.

Because of its implications on entry deterrence and product selection, the above discussion constitutes a serious warning that the simplifying assumption which equates the reservation quality to zero[143] may not be always innocuous, the case depending on the specific purposes of each study.

4.1 The Duopolists' Choice of Qualities

We continue, in line with the previous parts of this research, to consider that the demand side is represented by the income dispersion model as described in subsection 2.1.2 while the cost structure is as in the previous chapter.[144] Entry is sequential so firm 2, the incumbent, enters first followed by the first entrant, firm 1. We also assume that $k \in [1,3]$[145] which, as said earlier, implies that at any moment at most two firms will have positive sales in a Bertrand-Nash equilibrium and that the whole market will be covered as long as the fixed cost allows the entry of two firms. At this point we ignore any threats of further entry, assuming that the duopoly equilibrium takes place under the belief that no other firm will enter the market; hence we are back to the model of Shaked and Sutton. We limit the scope of this section

[142] See for instance the work of Dixit in [7,8] as well as that of Schmalensee [35] and Perrakis and Warskett [29,30].

[143] See the discussion in page 14.

[144] See page 56.

[145] Or that $2a \leq b \leq 4a$.

to the discussion of the expressions (28) and (29)[146] originally derived by them in [39]. The equilibrium concept adopted is that of *subgame perfect* which is described on page 17. Thus, the duopoly game consists of three sequential stages: entry, choice of quality and choice of price; decision-making at each stage is conditional on the decisions made at the previous stages. Both entry and quality choices are assumed irreversible. Qualities are chosen from within a technologically-feasible interval $[\underline{u}, \bar{u}]$, where $\underline{u}$ is some lower quality level.

After prices have been chosen optimally, the revenue functions of the two firms are thus given by the expressions (28) and (29); in their analysis Hung and Schmitt consider only one of these expressions, assuming that the parameters will determine an optimal pair (u_2,u_1) that justifies this choice. It will be shown that their assumption is in general incorrect. While this has no effect on the derivation of the optimal quality u_2, which turns out to be $\bar{u}$ in either case, it changes fundamentally the optimal u_1 which we name u_1^*.

As mentioned on page 33, the appropriate expressions to use in firm revenues depend on the quality choices of the two firms according to whether

$$\frac{b-a}{3a} \leq V \leq \frac{a+b}{3a} \tag{88a}$$

in which case the price p_1^* is always determined in such a way as to set $t_1 = a$ and the expressions (29) represent the price-maximised revenue functions for the two firms, or

$$\frac{a+b}{3a} \leq V \tag{88b}$$

with t_1^*, the optimal t_1, being $\leq a$ and the revenue functions given by (28); V is as defined on page 33.[147]

Hung and Schmitt assumed that (88b) holds always when they considered the expression for firm revenues. This is justifiable only if either the optimally-chosen u_1 and u_2[148] happen to satisfy (88b), or if this optimal u_1 is not achievable because it lies below the lowest available quality $\underline{u}$. It will be shown here that, in the absence of this last technological constraint, the

[146] See page 32.

[147] $V \equiv (\bar{u} - u_0)/(\bar{u} - u_1)$.

[148] Which is of course equal to $\bar{u}$ when fixed cost is independent of quality.

optimal u_1 will always satisfy (88a). Hence, the Hung and Schmitt results are only tenable if and only if the lowest available quality $\underline{u}$ is sufficiently high in order to force (88b) to hold. While this is possible, the imposition of such a structure to the solution constitutes, in our opinion, an arbitrary assumption whose relaxation should be investigated. Hereafter, therefore, we shall assume that $\underline{u}$ is at (or slightly above) the reservation quality u_0.

In order to simplify the exposition we introduce some definitions and notation: first, we define the parameter u_c as the critical value of u_1 that satisfies the relation

$$V(u_c) = (a + b)/3a; \tag{89}$$

in other words, the solution $(\bar{u}, u_c)$ is the borderline case between the expressions (88a) and (88b). Also, we define as $h_1(u_1) \equiv u_1/u_0$ the relative 'distance' of the lower quality product from the reservation quality, and we denote as h_c the value of h_1 when $u_1 = u_c$; obviously h_1 is a monotonically increasing function of u_1 for a given u_0, so we can carry our analysis in terms of h_1 rather than u_1, and we find convenient to rewrite the expressions (29a) and (28a) as functions of h, h_1 and k[149]

$$R_1^* = \frac{a^2(k - V)}{2} \cdot \frac{h_1 - 1}{h_1} \tag{90a}$$

and

$$R_1^* = \frac{h - h_1}{h_1} \cdot \left(\frac{a(k-1)}{3}\right)^2 \tag{90b}$$

Dividing the numerator and the denominator in the definition of V[150] by u_0 ,we obtain that $V = (h - 1)/(h - h_1)$; on the other hand, the use of the definition of k allows us to write the RHS of (89) as $(k + 2)/3$. Since the optimal u_2 will always be equal to $\bar{u}$, the expression (89) can be written as

$$\frac{h - 1}{h - h_1} = \frac{k + 2}{3}$$

from which we can solve to obtain the value of h_c as

$$h_c = \frac{h(k - 1) + 3}{k + 2} \tag{91}$$

[149] $k \equiv (b - a)/a$ as defined in (69); see page 61.

[150] See footnote 147.

Equation (91) shows that u_c, the critical value of u_1, is a weighted average between $\bar{u}$ and u_0, the weights being $(k-1)/(k+2)$ and $3/(k+2)$ respectively;[151] thus, u_c is completely independent of the choice of the optimal u_1.

Since the critical value for h_c is a parameter, it is important to determine the position of h_1^* relative to h_c, where as h_1^* we define the ratio u_1^*/u_0. The following lemma deals with this issue.

Lemma 3 *The unconstrained optimal u_1^* is always less than or equal to h_c.*

Proof: It is easy to see that for $u_1 = u_c$ the expressions (90a) and (90b) coincide. Also, the RHS of (90b) is clearly a decreasing function of h_1, implying that R_1^* is maximised at u_c. On the other hand, in (90a) the derivative of R_1^* with respect to h_1 is either negative or nonnegative at $h_1 = h_c$ (i.e. $u_1 = u_c$). If it is nonnegative then u_c is the optimal quality for firm 1, while if it is negative, $u_1 \leq u_c$, Q.E.D.

Thus, if there is no constraint on the width of the set of technologically feasible qualities, it suffices to maximise (90a) with respect to u_1. Proposition 4 describes the *unconstrained* optimal choice of quality for firm 1.

Proposition 4 *If u_1^* denotes the unconstrained optimal value of u_1, then we have:*

(a) *If $hk \leq h+3$ then $u_1^* = u_c$.*

(b) *If $hk > h+3$ then $u_1^* < u_c$; in such a case, if $k \geq (\leq) h-1$ then $h_1^* \equiv u_1^*/u_0$ is the smallest (largest) root of the following equation in h_1:*

$$H(h_1) \equiv k(h-h_1)^2 - (h-1)(h+{h_1}^2 - 2h_1) = 0. \qquad (92)$$

Proof: Replacing V by the ratio $(h-1)/(h-h_1)$ in (90a) we obtain

$$R_1^* = \frac{a^2}{2}\left(k - \frac{h-1}{h-h_1}\right)\left(1 - \frac{1}{h_1}\right)$$

[151] Note that the two weights add up to one.

the derivative of which with respect to h_1 is

$$\frac{\partial R_1^*}{\partial h_1} = \frac{a^2}{2}\left[-\frac{h-1}{(h-h_1)^2}\frac{h_1-1}{h_1} + \frac{1}{{h_1}^2}\left(k - \frac{h-1}{h-h_1}\right)\right].$$

Setting the above expression equal to zero, we obtain

$$\frac{1}{h_1}(kh - kh_1 - h + 1) - \frac{(h_1-1)(h-1)}{h-h_1} = 0 \iff$$

$$h^2(k-1) - 2khh_1 + {h_1}^2(k+1) + h - 2h_1 - h{h_1}^2 + 2hh_1 = 0 \iff$$

$$k(h-h_1)^2 - (h-1)(h + {h_1}^2 - 2h_1) = 0$$

where this last expression is equation (92). For $u_1 = u_0$ ($h_1 = 1$) we get

$$H(1) = (h-1)^2(k-1) > 0.$$

Hence, $u_1^* = u_c$ if $H(h_1)$ is positive for all $h_1 \leq h_c$; otherwise, the optimal u_1 is found at the roots of (92). It is easy to see that H is convex (concave) if $k \geq (\leq) h-1$ and that $\partial H/\partial h_1$ is proportional to $k(h_1-h)-(h-1)(h_1-1)$, which is always negative for $h_1 \in (1,h)$. Obviously, $H(h_1)$ will be positive for all $h_1 \leq h_c$ if and only if $H(h_c) > 0$. Substituting h_c from (91) into the LHS of (92) we find

$$\begin{aligned} H(h_c) &= k(h-h_c)^2 - (h-1)[h - h_c + h_c(h_c-1)] \\ &= \left[\frac{(h-1)}{k+2}\right]^2 (k-1)[3 - h(k-1)]; \end{aligned} \quad (93)$$

recalling that h is strictly greater than one, it is clear that $H(h_c) \geq 0$ is strictly equivalent to $kh \leq h+3$, thus proving (a).

If on the other hand $H(h_c) < 0$ then (92) has a unique root in the interval $(1,h)$. This is the largest (smallest) root of (92) depending on $H(h_c)$ being concave (convex) and this proves part (b), QED.

Thus, Proposition 4 substantiates the intuition that *maximal differentiation* is not a general characteristic of vertically differentiated markets.[152]

[152]See the discussion on page 33

Most important though, it implies that the choice of the lower quality product is more complex than Hung and Schmitt have assumed, being a function of the *reservation quality*.[153] This has very important consequences for the entry game, as argued in the next section, and it shows that simplifying assumptions on the reservation quality,[154] far from being innocuous, affect directly the determination of the lower quality product.

4.2 Sunk Costs and Entry Deterrence

According to Proposition 4, the optimal revenue of firm 1 in the absence of an entry threat will be given by (90a) with u_1^* replacing u_1, provided that $\underline{u} \leq u_c$. It is easy to see that this optimal revenue, denoted by R_1^*, is a function of the reservation quality u_0. In fact, we can prove the following result:

Proposition 5 *If $\underline{u} \leq u_c$, the optimal revenue of firm 1 is a decreasing function of the reservation quality u_0.*

Proof: We need to show that $\partial R_1^*/\partial u_0 < 0$ and for this, it suffices to show that $\partial R_1/\partial u_0 < 0$, for all u_1.

Differentiating (90a) with respect to u_0, we find that

$$\frac{\partial R_1}{\partial u_0} = \frac{a^2}{2u_1(\bar{u} - u_1)} A(u_1) \tag{94}$$

where $A(u_1) \equiv (k+1)u_1 - 2u_0 - (k-1)\bar{u}$. Obviously, the sign of $\partial R_1/\partial u_0$ depends on the sign of $A(\cdot)$, which is clearly an increasing function of u_1.

However, we know that the optimal u_1 is at most equal to u_c. Substituting the value of u_c from (91), we find that $A(u_c)$ is proportional to the quantity $(k-1)(u_0 - u_1)$, which is negative. Thus, $A(u_1)$ is always negative, implying $\partial R_1/\partial u_0 < 0$, QED.

Suppose now that there is a number of potential entrants ready to enter the market. Recall that the presence of a fixed cost, combined with Bertrand conjectures, rules out the possibility that a new firm enters producing an already existing quality. On the other hand, the natural duopoly

[153]Except of course in case of a corner solution, i.e. if the lowest feasible quality $\bar{u}$ is greater than u_c.

[154]See reference to Ireland on page 33.

assumption makes it impossible to have more than two firms with positive market shares in a Bertrand-Nash equilibrium. Hence, the entry of a new firm must be accompanied by the exit of the lower quality incumbent, thus resulting in an increase in the lower quality.

In such a situation, the assumptions about the fixed costs incurred upon entry become crucial in determining the nature of the pre-entry game. Hung and Schmitt assume that firm 2, the first to enter the market and the one who produces the high quality, has no fixed entry costs, while all subsequent entrants must pay the same fixed fee, denoted by F. They then conclude that firm 1, the first entrant, will deter subsequent entry simply by increasing its quality level above u_1^*, to the point where the optimal revenue is barely sufficient to cover F.

The cost structure we have adopted in this work is similar to that of Hung and Schmitt except for the fact that we consider F being sunk after entry and paid by *all* the firms. Notice that the difference in our conclusions does not come from the fact that we require firm 2 to pay the fixed cost,[155] but from that we assume any other firm's cost to be sunk after entry as well, even if that firm may eventually end up having no market share.

We then show that an increase in the quality level u_1 above u_1^* may not be necessary in order to deter entry. The mere fact that firm 1 is located at u_1^* and realises positive profits does not necessarily imply that a subsequent entrant can choose a higher quality and force it out of business. Given that the entry fee F is sunk, and that no other costs are incurred by firm 1, there is nothing to be gained by that firm in withdrawing from the market.

Thus, the decision of the firm whether to exit or not is indeterminate without further assumptions on the nature of the fixed cost. If, for instance, F is sunk and there is a positive exit cost (disposal of inventories, clearing of the site, etc.) then the firm will stay in the market with zero market share. This will affect the revenue of the firm that displaced it, as expressed by the following result:

Proposition 6 *If a displaced lower quality incumbent (firm 1) decides to stay in the market and dispose of its product at zero price then it will have*

[155]The fact that Hung and Schmitt do not consider the first firm in the market paying the fixed cost can be interpreted as assuming that firm 2 has already paid its sunk entry cost.

a negative impact on the revenue of the firm that displaced it even if its market share is zero.

Proof: If the displaced firm, with quality level u_1, disposes its product at zero price then no consumer will accept a quality lower than u_1 even for free. Hence, u_1 will now take the place of the reservation quality u_0. Also, since $u_1 \geq \underline{u}$, all qualities from (the new) reservation quality to $\bar{u}$ are now feasible. It follows that with the new reservation quality, the price-maximised revenue of the new entrant (firm ε) is given by (90a), with the subscript 1 replaced by ε. Since this revenue is, by Proposition 5, a decreasing function of u_0, $R_\varepsilon(u_\kappa) < R_1(u_\kappa), \forall u_\kappa \in [u_1, u_c(u_0 = u_1)]$, Q.E.D.

The importance of Proposition 6 lies in the fact that, whenever the nature of costs makes exit an inferior strategy, it also confers to firm 1 a first mover advantage. At any quality level, subsequent entrants obtain lower revenues than firm 1 would have obtained at that same quality, as can be seen in Figure 8.

The entry-deterring choice of quality can now be expressed as follows.

Proposition 7 *Let $u_1^*(u_0)$ denote the optimal quality choice of firm 1, which is either $\underline{u}$ or is given by Proposition 4 as a function of the reservation quality u_0. Let also $R_1^*(u_1^*(u_0), u_0)$ be the revenue R_1, given by (90a) or (90b) when u_1 is replaced by $u_1^*(u_0)$. Then, if the nature of costs makes exit an inferior strategy at zero market share, either further entry is blockaded at $u_1^*(u_0)$, or the entry-deterring choice of u_1 will be given by the unique solution of the equation in y:*

$$R_1^*(u_1^*(y), y) = F \tag{95}$$

At that quality level firm 1 will enjoy positive profits if it did so when the duopoly faced no entry threat.

Proof: By Proposition 5 the function R_1^* is decreasing in u_0 and $\lim_{u_0 \to 0} R_1^* \geq F$, for otherwise the fixed cost would be prohibitively high for a second firm to enter; hence (95) has a unique solution y^*. Clearly, if $y^* \leq u_1^*(u_0)$ the profits of any further entrant will be negative or zero if $u_1(u_0)$ becomes the new reservation quality, and entry is blockaded, Q.E.D. A situation of blockaded entry is depicted in Figure 8.

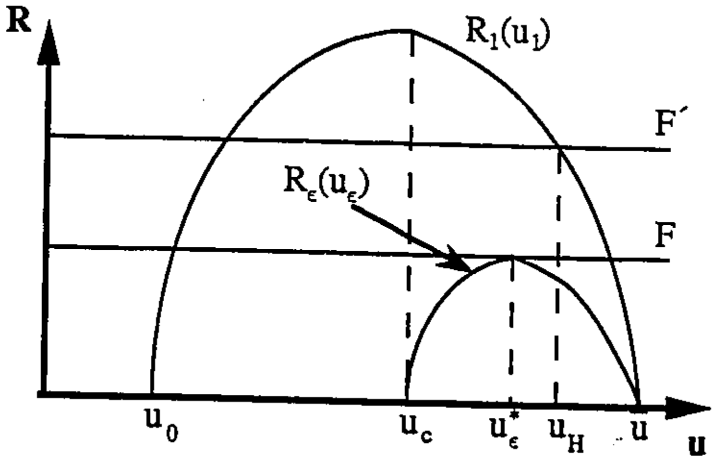

Figure 8: **The First Mover Advantages of Firm 2**
This figure shows the revenue of the lower quality firm as a function of the quality of its product. The curve R_1 corresponds to the revenue of the incumbent; it takes the value of 0 for $u_1 = u_0$ and $u_1 = \bar{u}$, and is maximised at $u_1 = u_c$ (it is assumed that $hk \leq h+3$). The curve R_ε corresponds to the revenue of the entrant if the incumbent leaves the quality u_c potentially available; it is equal to 0 for $u_\varepsilon = u_c$ and $u_\varepsilon = \bar{u}$, it is maximised when $u_\varepsilon = u_\varepsilon^*$ and it lies below R_1 for any $u_\varepsilon < \bar{u}$. If the fixed cost is F', entry is blockaded with $u_1 = u_c$; compare this solution to the one adopted by Hung and Schmitt where $u_1 = u_H$, in terms of product differentiation and the profits of the incumbent. Fixed cost must be below F for entry to be feasible when $u_1 = u_c$.

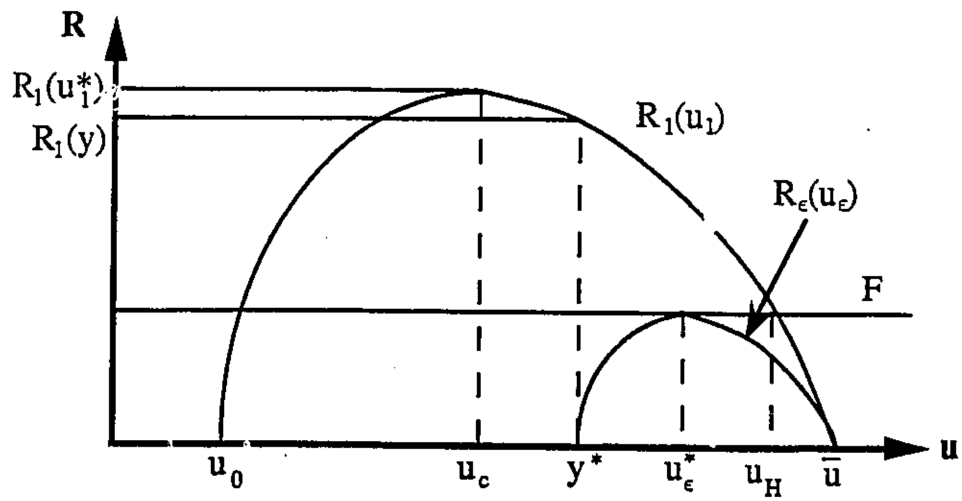

Figure 9: **Entry Deterrence in a Natural Duopoly**
This figure is identical to Figure 8 except for the fact that the incumbent produces the quality y^* instead of u_c, where y^* is the solution of equation (95). The distance $R_1(u_1^*) - R_1(y^*)$indicates the reduction in incumbent's profit necessary for entry to be deterred. With the reservation quality at y^*, the entrant cannot find a quality level that makes positive profits. Notice again the difference between $u_1 = y^*$ and the solution of Hung and Schmitt where $u_1 = u_H$, in terms of differentiation and the profit of the incumbent firm.

If y^* exceeds $u_1^*(u_0)$ then firm 2 must choose a quality level corresponding to y^* to deter entry; with such a choice, if exit is an inferior strategy, then y^* becomes the new reservation quality. The best any subsequent entrant can do is to choose $u_1^*(y^*)$ which yields zero profits by (95), Q.E.D. A situation of entry deterrence is depicted in Figure 9.

By assumption firm 1 has positive profits in the absence of entry, equal to $R_1^*(u_1^*(u_0), u_0) - F$. It remains to show that $R_1(y^*, u_0) - F > 0$, given that y^* solves (95) and that $u_1^*(u_0) < y^*$. From the proof of Proposition 7 we know that $u_1^*(u_0)$ is always greater than u_0. Hence, $u_1^*(y^*) > y^*$. Similarly, $R_1^*(u_1^*(u_0), u_0) - F > 0$ implies, together with (95) and the fact that R_1^* is decreasing in u_0, that $y^* > u_0$. Recall, however, that $R_1^*(u_1, u_0)$ was shown in Proposition 5 to be decreasing in u_0, $\forall u_1$. Set $u_1 = u_1^*(y^*)$ and consider the function $R_1^*(u_1^*(y^*), u_0) - F$, which becomes 0 by (95) for $u_0 = y^*$. It follows that $R_1^*(u_1^*(y^*), u_0) - F > 0$, for $u_0 < y^*$.

We have, therefore, established that if entry is not blockaded then $u_0 < u_1^* < y^* < u_1^*(y^*)$. Recall, however, that in Proposition 4 we showed that $\partial R_1^*(u_1, u_0)/\partial u_1 < 0$, $\forall u_1 > u_1^*(u_0)$.[156] Hence, $R_1^*(y^*, u_0) > R_1(u_1^*(y^*), u_0)$; since the RHS of this inequality is greater than F, we also have that $R_1(y^*, u_0) - F > 0$, Q.E.D.

From this last expression it is deduced that for $R_1(y^*, u_0)$ to be positive it is sufficient to show that $\partial R_1(u_1, u_0)/\partial u_1 < 0$, $\forall u_1 \geq y^*$. Since however we have excluded the case of blockaded entry[157] and because the function $R_1(u_1, u_0)$ is single peaked,[158] the above derivative is always negative, Q.E.D.

The significance of Proposition 7 is that it shows the nature of fixed cost as being very important even in the context of *vertical product differentiation*, whereas one would be inclined to think that it is not. Its impact may be felt on the revenue of subsequent entrants, not through its effect on the quantities that the incumbents will produce, but through the price the consumers will be willing to pay for the different qualities. This of course implies that the zero profit result in Hung and Schmitt must be qualified by some further assumptions on cost recoverability.

An illustration The dramatic impact of Proposition 7 upon the extent of vertical differentiation in a duopoly can be illustrated by the following numerical example. Suppose that in a market the wealthiest consumers have three times the income of the poorest ones, or $b = 3a$ and $k = 2$, and that the ratio of the highest available to the reservation quality $h \equiv \bar{u}/u_0 = 3$. Then, we have by Proposition 4 that in the absence of an entry threat $u_1^* = u_c$ and that $h_c = 1.5$.[159] This yields from (90a) a profit for firm 2 equal to $(a^2/9) - F$.

Suppose, however, that F is totally sunk and firm 1's product remains potentially available even if its market share goes to zero. Then, any quality

[156] Since a unique maximum always exists.

[157] I.e. we assume $y^* > u_1^*(u_0)$.

[158] Remember that for $u_1 \in [u_0, u_c]$, $R_1(\cdot)$ is single-peaked although it decreases monotonically for $u_1 > u_c$.

[159] By (91).

level u_1 introduced by this firm forms the new reservation quality, implying that for any subsequent entry the ratio h will become $\breve{h}(u_1) \equiv \bar{u}/u_1$; since the post-entry value of $u_c \equiv \breve{u}_c$ is a function of u_1 rather than u_0, we can define as $\breve{h}_c(u_1) \equiv \breve{u}_c(u_1)/u_1$ where $\breve{}$ represents the values of the quality-related parameters after entry, in the duopoly game between the entrant and firm 2.

Let us first assume that firm 1 sets $u_1 = u_1^* = u_c$ and determine the values of F for which entry is blockaded. According to the above definitions, $\breve{h}(u_c) = h/h_c = 2$ and $\breve{h}_c(u_c) = 1.25$ from (91). It is easy to confirm that if the condition $hk \leq h + 3$ holds for some value of h, it also holds for any h smaller than that value. Thus, $\breve{h}k < \breve{h} + 3$ and the profit maximising quality for the entrant is $u_\varepsilon^* = \breve{u}_c$ while his/her maximised revenue is given by $R_\varepsilon^* = a^2/15$, where the subscript ε denotes the entrant. Hence, if $F \in \left(\frac{a^2}{15}, \frac{a^2}{9}\right)$ there will be no possibility of further entry and the initial amount of product differentiation is unchanged.

If on the other hand $F < \frac{a^2}{15}$, then entry deterrence is achieved by an increase in the quality level of firm 1, from u_c to y, sufficient to set the entrant's profits equal to zero, $\forall u_\varepsilon$. Since in the event of entry $u_\varepsilon^* = \breve{u}_c(y)$ (the new u_c), we may use expression (90b) and the fact that $\breve{h}_c = (\breve{h}+3)/4$, according to (91), to find the value for $\breve{h}$ which makes the entrant's profits zero. This entry deterring value, $\breve{h}(y)$, can be found by solving the following equation in $\breve{h}$

$$\frac{a^2}{9}\left[\frac{4\breve{h}}{3+\breve{h}} - 1\right] = F$$

which yields

$$\breve{h}(y) = \frac{9F + a^2}{a^2 - 3F}.$$

This is the ratio of high to low qualities that will deter any further entry. It can be shown that this ratio is less than 2 if $F < \frac{a^2}{15}$, thus reducing differentiation. Nonetheless, it can be easily seen that firm 1's profits are unequivocally positive.

As hinted earlier, the analysis of Hung and Schmitt must be qualified by further assumptions on the nature of the fixed cost before their zero-profit-under-entry-threat conclusion is accepted. This does not mean that their analysis is wrong, but rather that they have examined only a special case. If for instance a sufficient portion of the fixed cost is recoverable to offset any possible exit costs and if $\underline{u}$ is sufficiently high to impose a corner solution as the choice of u_1, then the results of Hung and Schmitt will continue to hold.

Robustness Let us now discuss how Proposition 7 is affected if we assume *a*) the presence of *variable cost*, *b*) the existence of a *restart cost* necessary for the firm in order to produce something after shutting down its operation for a while and *c*) the possibility of a *transfer payment* between the lower quality incumbent and the entrant.

With respect to variable cost one might reason as follows: since they are always recoverable, it will be in the interest of firm 1, whether exiting the market or not, to shut down its plant so its product will no longer be available and it will have no effect on the reservation quality.

However, upon second reflection one realises that, even in the absence of any variable cost, the low quality product would never actually be produced in a post-entry Bertrand-Nash equilibrium. Even more, not even the initial reservation quality need ever be produced since no consumer purchases it at a Bertrand-Nash equilibrium. The effect of any reservation quality—u_0 or the displaced lower quality product—owes to the **potential** rather than the actual availability of that quality.

The above observation sheds a different light on the case where part of the production cost is variable; for simplicity, we assume average variable cost to be constant. Since it is potential availability of the displaced product that matters in determining whether that product becomes the new reservation quality, the entry deterrence mechanism described in this chapter will be robust in the presence of variable costs because consumers are aware that the displaced quality may become available at any moment at price equal to variable cost. The potential availability of quality u_1 depends of course on reentry conditions and the length of the depreciation period of the sunk cost. Although the harmful consequences of the new reservation quality on the entrant's profit will be reduced as the price at which this quality is offered—actually or potentially—increases towards covering variable costs, our conclusions with respect to the asymetry between the second firm and potential entrants need not be altered. First notice that since there is a fixed component in the total cost, entry at zero profit implies that the price of the new firm is above variable cost. This implies that the incumbent—who cannot recover the fixed cost—can always find prices strictly lower than the price of the entrant that allow him/her to cover the variable cost, thus keeping the product potentially available at such prices. On the other hand, all that matters for the asymmetry between the two firms to exist is that the entry-blocking quality $u_1 = y$, available at a positive price no less than average variable cost, is preferred to u_0 offered for free; after considering the consumers' choice at the pre-entry game there should be no doubt that this is indeed the case.

When a restart cost is required for the firm to produce after an interruption in its production process, then the product can be potentially available to consumers provided that variable plus restart cost can be covered by the price. This case can be considered as a variant of the previous one, with the difference that now the fixed cost must be spread over a number of consumers. Thus every single consumer must consider the product potentially available at average variable plus average fixed cost. The question that arises from the individual consumer's point of view is the following: how many other consumers would eventually be willing to buy the product at any given situation and share the restart cost with him/her. The answer to this question is simply that, since consumers with the same income level are identical in all meaningful aspects and exhibit identical behaviour, whenever one of them is willing to purchase a unit of u_1, everybody will do so. Hence the density of the income distribution will indicate to each individual consumer at what minimum price the displaced quality could become again available. Thus, knowledge of the density of the income distribution, together with information on the nature of fixed cost, is sufficient to allow consumers an evaluation of the potential availability of the displaced product. This discussion applies also to cases where some part of the fixed cost is recoverable and must be paid again if the firm decides to restart. Finally, a similar case exists when, instead of a restart cost, one considers that average variable cost falls with the quantity produced; again, the density of the consumer income distribution will be of crucial importance for the validity of Proposition 7.

The other way that the first mover advantage of the firm 1 could be reduced or eliminated is the possibility of a post-entry transfer payment. More specifically, after entry has taken place the entrant can propose to buy and destroy the plant and any inventories of the displaced firm. In this way, it will be made clear to his/her customers that product u_1 will no longer be available. If costs are only sunk without any pure exit cost element and this option is available, it will be indeed in the interest of firm 1 to accept any small amount of money and leave the market;[160] this in turn will dissipate all the credibility of the incumbent's threat through the reservation quality.

However, if exit entails some extra costs, the transfer payment must equal the amount of these costs or, alternatively, the entrant must assume them after they purchase the incumbent's plant. Thus, although the possibility of ex-post transfer payments may eliminate the asymmetry between incumbent and entrant on the *revenue* side, it creates a new one on the cost side. This means that the incumbent can enjoy profits up to the amount of exit costs without fear of further entry.

4.3 Conclusions from Chapter 4

This part of the research dealt with the choice of quality in a natural duopoly situation. This question was implicitly considered in the work of Shaked and Sutton,[161] but the framework of these two authors did not allow for the threat of further entry—accompanied by the replacement of an incumbent—to be present. Hung and Schmitt improved this situation by using sequential entry and a large number of potential entrants; based on these assumptions they concluded that (i) the ratio of qualities will depend only on cost parameters; (ii) differentiation will be reduced relative to the case of no entry threat.

Unfortunately, although these results have an intuitive appeal, this part of our research has shown that they were the product of two very specific assumptions: the lowest technologically available quality $\underline{u}$ exceeds the critical quality u_c; a firm must leave the market if its market share is captured by a higher quality.

[160] Note, however, that the incumbent can inflict losses to the entrant by refusing this option. This implies that there is an amount of transfer payments up to which the entrant is willing to negotiate. Hence, the transfer payment can be far from trivial, depending on the relative bargaining strength.

[161] See [39].

The first assumption cannot obviously be substantiated on any grounds of intuition or empirical relevance; section 4.1 revealed its constraining nature and extended the discussion on the choice of the lower quality to cover all the possibilities.

As argued in section 4.2, the validity of the second assumption depends on the nature of costs. There are empirically relevant cases in which the product remains potentially available to the consumers even if the firm's market share goes down to zero.[162] In such cases the lower quality firm in the duopoly enjoys first mover advantages relative to subsequent entrants; these advantages come through a change in the reservation quality. The interesting feature of this situation is the relation between the nature of cost and the reservation quality.

The role of the irrecoverability of the fixed entry costs in deterring competitors' entry in an oligopoly is well-known in markets with homogeneous commodities.[163] In this chapter we hope to have shown the role that this irrecoverability plays in vertically differentiated markets, in which it can also create entry barriers and/or reduce product differentiation while maintaining positive incumbent profits. It is important to realise the difference between our case and the role of sunk costs in homogeneous markets. In the latter, it is the threat of the incumbent that he/she will produce

[162] The case of oil transporting tankers remaining unused for long periods illustrates this point. The oil tankers are vertically differentiated since more modern versions of the product are faster and better equipped. On the other hand, their cost is sunk since they cannot be used for anything else but oil transportation. If the market share of such a tanker becomes zero, its owner will keep it available because of exit costs taking two forms: the first is high cost of disposal, the second is related to future opportunities; the latter cannot be understood unless we introduce time and uncertainty considerations into our model. More specifically, business cycles affect the willingness to pay of the users of oil tankers, thus creating variability in a and b and ultimately in k. Now consider the following situation: with a value of $k \in [1,3]$ the market is a natural duopoly and entry with a quality higher than u_1 reduces the market share of the incumbent low quality to zero. However, if the value of k increases later on beyond 3 at least for a certain time some room will be created again for u_1. Although the full analysis of the effects of uncertainty in a multi-period game rests beyond the scope of this work, it is intuitively obvious that a prospect of some positive market share in the future—which in turn implies a positive expected revenue, will act as a strong barrier to exit for the low quality firm, thus reducing the profit opportunities of the higher qualities according to the analysis of this chapter.

[163] See for instance Dixit [7,8], Eaton and Lipsey [11], Schmalensee [34], Perrakis and Warskett [29,30] and Spulber [40].

large enough quantities to inflict losses on entrants that deters entry (when credible). In the former, the threat consists of simply leaving the product available and thus making the consumers willing to pay less for higher qualities. In this case, the sunkness of cost guarantees only that the firm does not lose anything by staying, leaving its final decision indeterminate; therefore some explicit exit cost must be introduced in order to solve the indeterminacy of the decision.

It may appear that the existence of variable costs would provide an incentive for the firm to leave the market, but this doesn't need to be so. By producing nothing, the displaced firm can avoid all variable costs and, at the same time, be ready to produce if anybody is willing to pay a price at least equal to variable cost. On the other hand, if consumers are aware of this potential availability of the displaced firm's quality, the willingness to pay for higher qualities will be reduced at least for some consumers. Finally, we must say that the introduction of exit costs is not the only solution to the indeterminacy of the firm's decision whether to stay or exit. The entrant may also attempt to buy the existing lower quality by paying some part of its sunk cost, in which case bargaining strength will determine the final outcome. But even in that case, the part of the sunk cost that the entrant will pay to convince the incumbent to leave creates a cost asymmetry allowing the incumbent to shield some profits from subsequent entry threats.

5 Multiproduct Monopoly, Entry threat and Market Coverage

In chapter 3 we allowed one firm to introduce many qualities but we assumed that no entry was possible although in chapter 4 we allowed for the presence of entry threat while restricting the analysis to single-product firms; time has come now to combine some of the results of these two chapters in an attempt to examine the situation of an unprotected multiproduct non-natural monopoly. The focus of our attention will again be on the important question of market coverage as we try to examine how the presence of entry threat affects this issue. Two main points arise out of the possibility of having a second firm share the market.

The first is whether entry can be blockaded while the market is uncovered. In other words, does the monopolist *have to* serve the entire market in order to prevent entry once the protection is removed? If this is the case, then the importance of the results in chapter 3 will be substantially weakened because the uncovered market will be a temporary phenomenon, lasting only for the time it takes for new entry to occur. However, if for some parameter configurations the monopolist can keep the other firms out of the market while depriving the poorest consumers[164] from the surplus they could obtain from purchasing the product, then there may be a need for public action other than simply removing all the legal entry barriers. Section 5.1 is devoted to the question of the existence of situations where entry is blockaded while the monopolist leaves part of the market unserved.

The second point relates to situations in which entry is not blockaded at the monopolist's unconstrained quality-price policy described in chapter 3 and, therefore, strategic action is needed to keep other firms from entering. Provided that accommodation is not the optimal strategy, the multiproduct incumbent has two means of impeding entry: to increase the number of qualities (product proliferation) or to rearrange the spacing between them (product relocation). In this context one may ask how the issue of market coverage is affected by those strategies. Although the answer is quite straightforward in the case of product proliferation, the case of relocation

[164]See subsection 2.4.1 for an explanation of why selling to some consumers may be unprofitable even when variable production costs are zero.

requires a more careful examination and we will devote section 5.2 to its analysis.

5.1 Uncovered Market and Blockaded Entry

Suppose that a monopoly firm operating in a vertically-differentiated market has determined an optimal number n^* of qualities that leaves the market uncovered. Suppose also that the market is not a natural monopoly and the incumbent firm is not protected from competitive entry by legal entry restrictions, patents or any other typical entry barriers. Is competitive entry possible under all circumstances? Alternatively, are there sets of parameter values that result in both an uncovered market and unprofitable entry under monopolistically-chosen n^* and quality levels? If the answer to this question is yes then a monopoly can be established and maintained in a vertically-differentiated market even in the absence of protection against entry, simply by virtue of first-mover advantages.

This question is trivial when the incumbent is limited to a single quality and a fixed cost is present. For in such a case one can always find a level of fixed cost F between the maximum revenue of the entrant and that of the incumbent, thus rendering entry infeasible. However, the issue is more complex when the incumbent is allowed to introduce more than one quality. The fixed cost now has a double effect: on the one hand, increasing the fixed cost makes entry more difficult by requiring a higher post-entry revenue; on the other hand, this increased cost induces the incumbent to introduce fewer qualities and leave more space between them, thus making entry easier. Thus, it is not immediately obvious that blockaded entry is feasible in the case of a multiproduct firm.

The issue will be examined here under the assumption that the market can sustain at most two firms so $2a \leq b \leq 4a$ or $k \in (1, 3)$.[165] If, in addition, the fixed costs are sufficiently low to allow the profitable existence of two such firms under free entry, then the market would be completely covered.

As we already know, if the incumbent is limited to only one quality

[165] This assumption should not be considered as restrictive since in most instances in this chapter we are trying to prove the *existence* of certain cases; on the other hand of course it simplifies the analysis greatly since it allows us to use the results from the previous chapter.

he/she will choose the highest available one, namely $\bar{u}$. Since the numbers $1 \dots n$ as subscripts will be reserved to denote the qualities of the multiproduct incumbent, we will use the subscript ε to denote the other duopolist, the prospective entrant. The choice of u_ε is made according to the rules described in Proposition 7.[166]

Suppose now that the monopolist has already chosen the optimal number of qualities $n = n^*$ by Proposition 3, and the quality levels $u_1, \dots, u_n$ according to the expression (61), as if entry were infeasible. The entrant is now considering the choice of a quality level u_ε somewhere between the opponent's qualities. We assume that the entry cost F is sunk once a quality level is chosen and that F is the same for both incumbent and entrant, as in the previous chapter. The incumbent may, however, withdraw an existing quality from the market if it is more profitable to do so.[167]

Within such a setup the entry game can be simplified according to the following result.

Proposition 8 *In a natural duopoly if entry between the top two optimally chosen incumbent qualities is infeasible because F exceeds R_ε^* as given by (90b), it will also be infeasible in all other locations in quality space.*

Proof: Let $h_\varepsilon \equiv u_\varepsilon/u_0$ and suppose that $u_{n-1} < u_\varepsilon < u_n = \bar{u}$. After entry the incumbent withdraws all the qualities $u_j < u_\varepsilon$ from the market since on one hand their market share will be zero[168] and at the same time their potential availability will affect the reservation quality and the revenue of both the incumbent and the entrant, according to the discussion in section 4.2. Thus, if entry takes place between the top two incumbent qualities, all the qualities $u_j \leq u_{n-1}$ will be withdrawn. Then, entry between

[166]See page 79.

[167]Contrary to the case discussed in chapter 4, where the low quality incumbent might leave his/her quality available even after its market share was reduced to zero, in the present context the incumbent has an incentive to withdraw all the qualities that entry deprives from sales. This is so because, although in both cases even the potential presence of the lower qualities will affect directly or indirectly the revenue from the higher qualities, in the present analysis all the incumbent qualities are introduced by the same producer. Thus, the incumbent now has an incentive to eliminate all the qualities with no market share, since their presence will affect not only the revenue of the entrant, but that of his/her remaining higher qualities as well.

[168]Because of the natural duopoly assumption.

the top two qualities is profitable if and only if a $u_\varepsilon \in (u_{n-1}, u_n)$ exists such that $R_\varepsilon \geq F$.

Concerning the expression for R_ε, it could be given either by (90a) or (90b),[169] depending on whether u_ε is greater or smaller than u_c. Notice however that, for any $u_\varepsilon < u_c$, the RHS of (90b) is always smaller than that of (90a). Thus, to prove the Proposition it suffices to show that if F exceeds the RHS of (90b) $\forall u \in (u_{n-1}, \bar{u})$, then entry is also unprofitable $\forall u_\varepsilon \in (u_{\kappa-1}, u_\kappa)$, for $\kappa = 1, \ldots, n-1$.

It is clear that the RHS of (90b) increases when u_ε decreases; also, by virtue of (64c),[170] $(\bar{u} - u_{n-1})/u_{n-1} = w - 1$. Hence, if

$$\frac{1}{9}(b-2a)^2\frac{\bar{u} - u_{n-1}}{u_{n-1}} = \frac{1}{9}(b-2a)^2(w-1) \leq F \tag{96}$$

then R_ε is less than F for all $u_\varepsilon \in (u_{n-1}, \bar{u})$ and entry is always infeasible between the top two qualities.

Suppose now that the entrant locates between qualities $\kappa - 1$ and $\kappa < n$. Then incumbent qualities $1, \ldots, \kappa - 1$ are withdrawn, and the price game takes place between the entrant and the incumbent's lowest remaining quality u_κ. The entrant's price is p_ε, and the corresponding revenue $R_\varepsilon = p_\varepsilon(t_\kappa - a)$, given that the market is completely covered in a Bertrand-Nash equilibrium; the incumbent revenue is

$$R_I = p_n(b - t_n) + \sum_{\kappa}^{n-1} p_j(t_{j+1} - t_j).$$

Setting $r_{\kappa\varepsilon} \equiv u_\kappa/(u_\kappa - u_\varepsilon)$ we have $t_\kappa = p_\kappa r_{\kappa\varepsilon} + p_\varepsilon(1 - r_{\kappa\varepsilon})$.

Maximising R_ε with respect to p_ε yields

$$p_\varepsilon^* = (t_\kappa - a)/(r_{\kappa\varepsilon} - 1) \tag{97}$$

and

$$R_\varepsilon^* = (t_\kappa - a)^2/(r_{\kappa\varepsilon} - 1). \tag{98}$$

On the other hand, maximising R_I with respect to $p_\kappa, \ldots, p_n$ and taking into account that $t_j = p_j r_j + p_{j-1}(1 - r_j)$, for $j = \kappa + 1, \ldots, n$, we get

$$b - t_n = r_n D p_n \tag{99a}$$

[169] With the subscript ε replacing the subscript 1.
[170] See page 61.

$$t_{j+1} - t_j = r_j Dp_j - (r_{j+1} - 1)Dp_{j+1} \tag{99b}$$

$$t_{\kappa+1} - t_\kappa = r_{\kappa\epsilon} p_\kappa - (r_{\kappa+1} - 1)Dp_{\kappa+1} \tag{99c}$$

with $Dp_j \equiv p_j - p_{j-1}$, and $j = \kappa + 1, \ldots, n - 1$. Adding the relations (99a)-(99c) together, we find the total incumbent market share

$$b - t_\kappa = r_{\kappa\epsilon} p_\kappa + \sum_{j=\kappa+1}^{n} Dp_j. \tag{100}$$

However, from the definition of t_κ and the optimal p_ϵ it is easy to see that $r_{\kappa\epsilon} p_\kappa = 2t_\kappa - a$, implying that $3t_\kappa - a = b - \sum_{\kappa+1}^{n} Dp_j < b$, from which $t_\kappa - a < (b - 2a)/3$. Thus,

$$R_\epsilon^* < \frac{(b-2a)^2}{9(r_{\kappa\epsilon} - 1)} = \frac{(b-2a)^2(u_\kappa - u_\epsilon)}{9u_\epsilon}. \tag{101}$$

For $u_\epsilon \in (u_{\kappa-1}, u_\kappa)$ the RHS of (101) is no greater than $(b - 2a)^2(u_\kappa - u_{\kappa-1})/9u_{\kappa-1}$ by the same argument that establishes (96). However, as established by (64c), the ratio of the relative distances between any two optimally set qualities of the incumbent is constant; thus, $(u_\kappa - u_{\kappa-1})/u_{\kappa-1} = (\bar{u} - u_{n-1})/u_{n-1}$ and $R_\epsilon^* < (b - 2a)^2(w - 1)/9 \leq F$, Q.E.D.

We note that the presence of condition (96) for $u_\epsilon = u_{n-1}$ is necessary and sufficient for entry to be infeasible only if $u_{n-1} \geq u_c$. It is only a sufficient condition otherwise, since for $u_\epsilon \in [u_{n-1}, u_c)$ the entrant revenue R_ϵ^* is less than the LHS of (96). It turns out, however, that most observable cases of blockaded entry occur almost always for parameter values such that $u_{n-1} > u_c$, as will become clear from the next set of results.

It suffices, therefore, to consider entry between the top two qualities in order to determine if a monopolist's optimal quality choices are robust in the face of entry. Hence, we shall consider values of $k \in (1, 3)$, h and F such that the market stays uncovered at the optimal n. We shall then examine whether such values exist that render entry infeasible between the top two qualities. This is expressed by the following result.

Proposition 9 *In a natural duopoly, for all h less than a certain value approximately equal to 4.316 there exist values k, F such that the market is uncovered with at least two optimally chosen incumbent qualities in the absence of an entry threat, and entry is infeasible.*

Proof: It must be shown that for $k \in (1,3)$ there exist values of h and F such that $2 \leq n^* \leq n_0$, and for $u_\varepsilon = u_{n-1} = h^{1-1/n^*}$ the expression (96) holds. This last condition can be rewritten as $a^2(k-1)^2(w-1)/9 \leq F$, where $w \equiv h^{1/n}$.[171] The optimal n, however, can be chosen by setting $\partial R_j^*(n)/\partial n = F$ or, according to (83), by solving in w the equation below

$$\frac{a^2(k+1)^2}{2}\frac{w^2 - 2w\ln w - 1}{[(n+1)w-(n-1)]^2} = F. \tag{102}$$

Setting $n = \ln h/\ln w$ and substituting the LHS of (102) for F in the blockaded entry condition we find that entry is blockaded if there exist admissible values of h, k and $w \in (1, h^{1/2}]$ such that

$$4.5\left(\frac{k+1}{k-1}\right)^2 \frac{(w^2 - 2w\ln w - 1)\ln^2 w}{(w-1)[\ln h(w-1) + \ln w(w+1)]^2} \geq 1. \tag{103}$$

On the other hand, these same values must satisfy the uncovered market condition (70).[172] The latter can be rewritten as

$$k \geq \frac{(w-1)\ln h + \ln w}{w \ln w} \equiv k(w). \tag{104}$$

It is clear from the proof of Proposition 1 that $k(w)$ is a decreasing function, with $k(h) = 1$ and $\lim_{w\to 1} k(w) = 1 + \ln h$.[173] Setting the LHS of (103) equal to $4.5[(k+1)/(k-1)]^2\Xi(w)$ and replacing k by $k(w)$ we can rewrite (103) as

$$4.5\Xi(w) \geq \left[\frac{k(w)-1}{k(w)+1}\right]^2 \equiv k_1(w). \tag{105}$$

For $w \in (1,h)$ the RHS of (105) is a decreasing function, and $k_1(w) \in \left(0, \left[\frac{\ln h}{(2+\ln h)}\right]^2\right)$. On the other hand, it can be shown that $\lim_{w\to 1}\Xi(w) = 0$,[174]

[171]See page 60.

[172]See page 61.

[173]For $w \longrightarrow 1$, both the numerator and denominator of (104) tend to zero so, by taking the corresponding derivatives with respect to w and applying *l'Hospital's* rule, we find that $\lim_{w\to 1} k(w) = lim_{w\to 1}\frac{\ln h + (1/w)}{\ln w} = 1 + \ln h$.

[174]Define

$$N_1 \equiv w^2 - 2w\ln w - 1 \qquad N_2 \equiv \ln^2 w$$
$$D_1 \equiv (w-1) \qquad D_2 \equiv [\ln h(w-1) + \ln w(w+1)]^2.$$

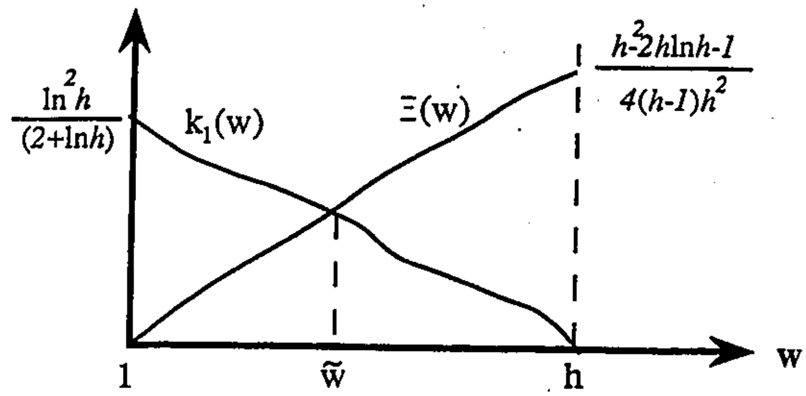

Figure 10: **Blockaded Entry in a Multiproduct Nonnatural Monopoly**
This Figure shows the existence of a solution for (105) .

while $\Xi(h) = [h^2 - 2h \ln h - 1]/4(h-1)h^2$, which is positive for all h. Hence, the expression (105), if treated as an equation, has at least one solution $\tilde{w}$ in the interval $w \in (1, h)$, as shown in Figure 10.

To prove the Proposition it suffices to show that there exist values of $h > 1$ for which $\tilde{w} \in (1, h^{.5})$. For in such cases a value of k slightly higher than $k(w)$ would satisfy (103) as a strict inequality for $w = \tilde{w}$, while from (102) values of F can be found for that k such that $n_0 \geq n^* \geq \frac{\ln h}{\ln \tilde{w}} \geq 2$. In turn, a sufficient condition for $\tilde{w} \in (1, h^{.5})$ is that $4.5\Xi(h^{.5}) > k_1(h^{.5})$ for some $h > 1$.

This last condition boils down to $4.5[h - h^{.5} \ln h - 1] > (h^{.5} - 1)^3$; the

It is clear that, as $w \to 1$, all the expressions above tend to zero and therefore we need to apply the *l' Hospital* rule. Using ′ and ″ to indicate first and second order derivatives, we have

$$
\begin{aligned}
N_1' &= 2(w - \ln w - 1) - 1 & \qquad D_1' &= 1 \\
N_2' &= (2/w) \ln w & D_2' &= \ln h + \ln w + (1 + 1/w) \\
N_1'' &= 2(1 - 1/w) & D'' &= 0 \\
N_2'' &= (2/w)(1 - 1/w) & D_2'' &= (1/w)(1 - 1/w).
\end{aligned}
$$

It is easy to verify that all the expressions above tend to zero as w tends to 1 with the only exception of $\lim_{w \to 1} D_2' = 2 + \ln h$. Thus, the first derivatives of the numerator and denominator of $\Xi(w)$ tend to zero and as far as the second derivatives are concerned, that of the numerator tends to zero although that of the denominator tends to $2D_2' > 0$ with w tending to one; hence, the corresponding limit of $\Xi(w)$ is zero.

h	k	$(F \times 10^2)/a$	t_1/a
4.0	1.501	2.845	1.0004
3.5	1.47	2.241	1.0018
3.5	1.48	2.259	1.0059
3.0	1.45	1.663	1.0134
2.5	1.40	1.061	1.0179
2.0	1.33	0.510	1.0162
1.5	1.21	0.169	1.0144
1.4	1.18	0.0688	1.0117
1.3	1.14	0.0336	1.0080
1.2	1.10	0.0117	1.0086

Table 1: **Blockaded Entry and Uncovered Market.**
This Table shows sample values of $\{h, k, F\}$ *resulting to situations of blockaded entry with uncovered market* ($t_1 > a$).

unique value of h that equates the two sides of this last inequality is approximately 4.316, implying that the inequality holds for $h < 4.316$ and is violated otherwise, Q.E.D.

Table 1 shows some examples of values of h, k and F, as well as the corresponding t_1, for which entry is blockaded and the market is uncovered (i.e. $t_1 > a$). In all entries we have taken $a = 1$ and $n^* = 2$, with F calculated exactly from (102). It is clear that, given the discrete nature of n^*, the indicated values of F define only the lower end of an open set, within which t_1 remains unchanged and entry continues to be blockaded.

It follows, therefore, that a monopoly can be established and maintained under free entry conditions in a vertically differentiated market even when that market is sufficiently wide and the fixed cost sufficiently low to accommodate two firms. Thus, product proliferation not only increases the incumbent monopolist's profit by allowing him/her to price discriminate, but it also protects these profits from the entry of new firms. Further, for some parameter values such a monopoly can coexist with an uncovered market, in which the poorest consumers would prefer not to buy a unit of product. The next section examines what happens when blockaded entry is not possible under uncovered market conditions. Surprisingly, it turns out that in many such cases the threat of entry actually worsens the extent of market coverage when compared to a protected monopolist.

5.2 Uncovered Market under Entry Deterrence

As we saw in the previous section there are many instances in which entry can be blockaded without a need for the monopolist to alter its unconstrained optimal price-quality decision, even if this decision involves forcing some consumers to leave the market. However, in many other cases the protected monopolist's optimal choices are not able to protect him/her from new entry; in these circumstances the incumbent firm has the choice between *entry deterrence* and *accommodation*. If the latter is chosen, the market will be completely covered, given that we continue assuming an income distribution leading to a natural duopoly, i.e., $k \in (1,3)$. Entry deterrence, on the other hand, can take place either by *increasing the number* of optimally-located qualities beyond n^*, or by *relocating* the n^* qualities till entry is infeasible. Thus, if entry is to be impeded by relocation, the first question we should ask is what must be the rule guiding the incumbent's decision on his/her product line? To answer this question we first prove an auxiliary result, similar to Proposition 8:

Proposition 10 *a) In a natural duopoly market with n optimally chosen incumbent qualities, if entry is possible between quality levels $\kappa - 1$ and κ, where $1 \leq \kappa \leq n-1$, then it is also possible between any pair of qualities j and $j+1$, where $j \geq \kappa$. b) The entrant's optimal market share, $t_\kappa - a$, is an increasing function of the quality index κ.*

Proof: As shown in the proof of Proposition 8, the entrant's revenue R_ε is at most equal to $(t_\kappa - a)^2(u_\kappa - u_\varepsilon)/u_\varepsilon$, which must be above F for entry to be possible. The term $(u_\kappa - u_\varepsilon)/u_\varepsilon$ is an increasing function of u_κ/u_ε; the latter can take all values less than $u_\kappa/u_{\kappa-1}$, which is equal to $h^{1/n}$, $\forall\kappa$. To prove both (a) and (b) it suffices, therefore, to show that t_κ increases with κ for any given ratio u_κ/u_ε.

In the proof of Proposition 8 we showed after maximising the incumbent and entrant's revenues with respect to their price levels $p_\kappa, \ldots, p_n$ and p_ε respectively, that (50b) and (50c) hold for $i > \kappa$, and similarly (51b) and

(51c) hold, the latter for indices greater than or equal to $k+1$. We can rewrite (51b) under the form

$$s_{i+1}Dp_{i+1} = s_i Dp_i, \qquad \text{for } i = \kappa+1,\ldots,n-1. \tag{106}$$

Similarly, we have from (97) and the definition of t_κ that

$$p_\kappa r_{\kappa\epsilon} = a + 2p_\epsilon(r_{\kappa\epsilon} - 1). \tag{107}$$

Combining (107) and (99c) we obtain[175]

$$s_{\kappa+1}Dp_{\kappa+1} + p_\kappa = (a + 3p_\kappa r_{\kappa\epsilon})/2$$

or

$$s_{\kappa+1}Dp_{\kappa+1} = p_\kappa L + a/2, \tag{108}$$

where $L \equiv (3s_\kappa - 1)/4$. Solving the system (106) recursively we find

$$p_i = p_{\kappa+1} + s_{\kappa+1}Dp_{\kappa+1}\sum_{j=\kappa+2}^{i}\frac{1}{s_j} \quad \text{for } i = \kappa+2,\ldots,n. \tag{109}$$

On the other hand, from (108) we obtain that

$$p_{\kappa+1} = p_\kappa + \frac{1}{s_{\kappa+1}}\cdot\left(p_\kappa L + \frac{a}{2}\right)$$

which substituted into (109) yields

$$p_i = p_\kappa + \left(p_\kappa L + \frac{a}{2}\right)\sum_{j=\kappa+1}^{i}\frac{1}{s_j} \quad \text{for } i = \kappa+1,\ldots,n. \tag{110}$$

---

[175] Using the expression (9)—see page 12—to replace $t_{\kappa+1}$ and t_κ, the LHS of (99c) becomes

$$t_{\kappa+1} - t_\kappa = r_{\kappa+1}Dp_{\kappa+1} - p_\kappa(r_\kappa - 1) + p_\epsilon(r_\kappa - 1)$$

which allows us to write (99c) as

$$p_\epsilon(r_\kappa - 1) = p_\kappa s_\kappa - Dp_{\kappa+1}s_{\kappa+1}.$$

Substituting the LHS of the above expression into the RHS of (107) and using the relation $s_i = 2r_i - 1$, $\forall i$, we obtain the expression which follows in the text.

Finally, substituting the value of (110) for $i = n$ into (51c), we find:

$$p_\kappa = \frac{b - a(B_1+1)/2}{1+(B_1+1)L}; \tag{111}$$

$$t_\kappa = \frac{a + r_{\kappa\varepsilon} p_\kappa}{2}, \tag{112}$$

where $B_1 \equiv \sum_{\kappa+1}^{n} 1/s_j$. It is clear that p_κ is a decreasing function of B_1 which, in turn, is a decreasing function of k. Hence, both p_κ and t_κ increase with k since s_κ (and, hence, L) depends only on u_κ/u_ε, which is constant in k, Q.E.D.

Proposition 10 implies that if strategic action is necessary to deter entry, the multiproduct monopolist, starting from an optimal quality configuration, must first reduce the space between the top two qualities till entry is infeasible there. A pure relocation strategy would, therefore, displace quality u_{n^*-1} till entry is deterred at the top, and would then choose optimally the locations of the remaining $n^* - 2$ qualities given the choice of u_{n^*-1}. If this deters entry everywhere then it is clearly the optimal relocation strategy; otherwise, quality u_{n^*-2} must also be displaced; etc. Since from (111) and (112) the entrant's market share and revenue decline with the level of the incumbent quality that the entrant faces, pure relocation may be sufficient to deter entry everywhere.

A pure *proliferation*, on the other hand, would find a number $\tilde{n} > n^*$ of qualities such that the ratio $h^{1/\tilde{n}}$ between optimally located successive quality levels be insufficient to allow profitable entry. Lastly, pure *entry accommodation* here simply implies that the incumbent has only one quality level located at $\bar{u}$, in which case the market will be completely covered. There are also many possible combinations of these pure strategies. For instance, increasing the number of qualities to $n^* + 1$ may be insufficient to deter entry, unless the top two qualities are also relocated. On the other hand, recent work by Martínez-Giralt [26] suggests that when accommodation is decided the incumbent firm will wish to withdraw *all* its qualities but one, the argument being that, in order to relax competition, the two firms will try to be as far apart as possible. However, this result has been developed using a *spatial* model of vertical differentiation *à la* Gabszewicz and Thisse[176] and for specific values of the parameters, so one must examine its

[176]See subsection 2.1.3, page 15.

robustness before transplanting it into the income dispersion model. The determination of the optimal incumbent strategy under general conditions depends on the parameters of the problem and will not be attempted here since the focus in this thesis is on market coverage.

The next two results strengthen the practical importance of an uncovered market in a natural duopoly. It is shown first that a pure relocation strategy is optimal under some conditions, and then that such a strategy always reduces market coverage.

Let $f(u_\varepsilon)$ denote the entrant's revenue R_ε as given by the LHS of (96). By Proposition 9 there exist values (h, k, F) such that entry is blockaded and the market is uncovered with $n^* \geq 2$. This implies, as shown in the proof of that proposition, that for some values of (h, k) we have

$$f(u_\varepsilon) \leq \check{F} \equiv \left.\frac{dR^*}{dn}\right|_{n=n^*}.$$

By Lemma 1[177] the concavity of $R^*(n^*)$ in an uncovered market guarantees that $\check{F} \in (F_1, F_2)$, where $F_1 \equiv R^*(n^*+1) - R^*(n^*)$, $F_2 \equiv R^*(n^*) - R^*(n^*-1)$, with (F_1, F_2) representing the range of fixed costs for which n^* is optimal. Suppose that n^* is the highest number of qualities for which entry is blockaded for the given (h, k). Then there exists an $F' \in (F_1, \check{F}]$ such that $f(u_{n^*-1}) = F'$, with entry being feasible for all $F \in (F_1, F')$.

For any such F let $\check{u}_\varepsilon(F)$ denote the unique solution of the equation $f(u_\varepsilon) = F$. A pure relocation strategy must set $u_{n-1} = \check{u}_\varepsilon$. Let $R_n(\check{u}_\varepsilon)$ denote the corresponding **incumbent** revenue, and define $g(\check{u}_\varepsilon) \equiv R^*(n^* + 1) - R_n(\check{u}_\varepsilon)$, the difference in revenue between an increase in the number of qualities by one and relocation.[178] We may now prove the following result:

Proposition 11 *a) In a natural duopoly, suppose that for some (h, k, F) entry is blockaded and the market is uncovered with $n^* \geq 2$ qualities as in Proposition 9. Then, if entry is possible for some $F \in (F_1, F_2)$, there exists an open set $(F'', F') \subset (F_1, \check{F})$ such that entry deterrence by relocation is an optimal policy for the same values of (h, k) and any $F \in (F'', F')$.*

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[177] See page 68.

[178] Note that $g(\check{u}_\varepsilon) > 0$ does not imply that proliferation is a superior strategy since introducing a new quality increases total cost by F; see also the proof of Proposition 11 below.

b) Relocation may still be an optimal strategy for some (h,k,F) even when entry is not blockaded for any F at some (h,k).

Proof: Relocation dominates proliferation if $R_n(\check{u}_\varepsilon) + F > R^*(n^* + 1)$ or $g(\check{u}_\varepsilon) < F$. It dominates accommodation if $R_n(\check{u}_\varepsilon) - \kappa F > R_I^*$, where $\kappa \geq 1$ is the number of incumbent qualities displaced by the entrant and R_I^* denotes the incumbent firm's revenue in the duopoly game that results under such an accommodation. It is clear from (59) and (61) that $R_n(\check{u}_\varepsilon) = R^*(n^*)$ as long as $\check{u}_\varepsilon \leq u_{n^*-1}$ (i.e. $F \geq F'$), and that $dR_n/du_\varepsilon = (<)0$ for $u_\varepsilon/u_0 = (>)h^{1-1/n^*}$. Similarly, (96) implies that $\check{u}_\varepsilon(F)$ is a continuously decreasing function. Hence, $g(\check{u}_\varepsilon)$ is a strictly increasing and piecewise continuous function of u_ε, with possible discontinuities arising whenever there is a need to relocate a lower quality; the function $g(\check{u}_\varepsilon)$ is depicted in Figure 11.

At $F = F'$ we have by definition $g(\check{u}_\varepsilon) = F_1 < F' = f(\check{u}_\varepsilon)$, and g is continuous[179] for F in an open neighbourhood to the right of F'. At $F = F_1$, $g(\check{u}_\varepsilon) > F_1 = f(\check{u}_\varepsilon)$. Hence, as shown in Figure 11, there exists some $F'' \in (F_1, F')$ such that, depending on the continuity of g at $\check{u}_\varepsilon(F'')$, either $g(\check{u}_\varepsilon(F'')) = F''$ or $g(\check{u}_\varepsilon(F'' - e)) < F'' < g(\check{u}_\varepsilon(F'' + e))$ $\forall e > 0$. Hence, for any $F \in (F'', F')$ we have $g(\check{u}_\varepsilon) < F$, implying that relocation dominates proliferation, Q.E.D.

On the other hand, let $R^*(n^* - \kappa)$ denote the optimal incumbent revenue with $n^* - \kappa$ qualities. This revenue obviously exceeds the incumbent's total revenue R_I^* under accommodation with the same total number of qualities. However, $R^*(n^*) - \kappa F > R^*(n^* - \kappa)$ by the optimality of n^* for all $F \in (F_1, F_2)$. It follows that $R_n(\check{u}_\varepsilon) - \kappa f(\check{u}_\varepsilon) > R_I^*$ for some $F < F'$, thus proving that relocation dominates accommodation, Q.E.D.

For part (b) we first note that the proof in part (a) can still be applied even if conditions (103)-(105) in the proof of Proposition 9 are violated: entry may be feasible at $F = \check{F}$ but infeasible for some $F \in (F_1, F_2]$. The question is whether entry-deterring relocation can still be the optimal incumbent policy when entry between the top two qualities is feasible for $F \geq F_2$. In such a case $\check{u}_\varepsilon(F_2) \geq u_{n^*-1}$ and correspondingly $g(\check{u}_\varepsilon) > F_1$. However, relocation would still dominate proliferation for some set of values of $F \leq F_2$ if at $\check{u}_\varepsilon(F_2) < f(\check{u}_\varepsilon) = F_2$, i.e., if the vertical intercept of the

[179]This is a direct consequence of Proposition 10.

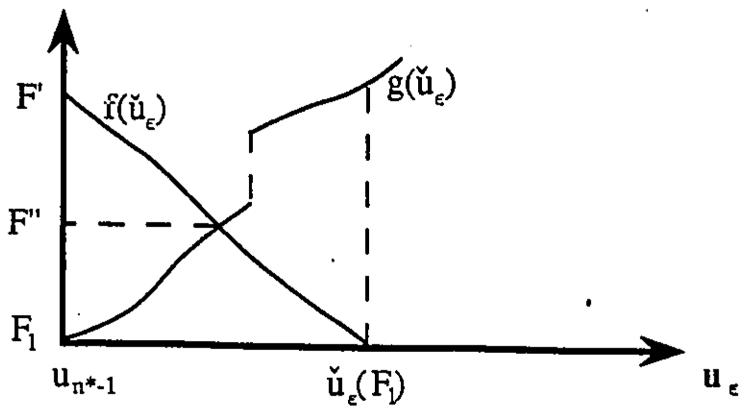

Figure 11: **Entry Deterrence with a Multiproduct Incumbent.**
This Figure illustrates the proof of Proposition 11.

function f lies above that of g in Figure 11. Similarly, relocation would dominate accommodation if at $\check{u}_\varepsilon(F_2)$, $R_n(\check{u}_\varepsilon) - \kappa F_2 > R_l^*$. The numerical examples at the end of this section show several such instances of optimal relocation even when blockaded entry is infeasible, Q.E.D.

Since several instances of blockaded entry for $n^* = 2$ were found in the previous section, the optimality of relocation when the fixed cost decreases sufficiently to allow profitable entry has been established for at least these cases.

The last result of this paper underlines the importance of relocation with respect to market coverage. We first prove an auxiliary result.

Lemma 4 *Suppose that in a pure relocation strategy with n^* qualities the last $n^* - \kappa$ qualities have been relocated to deter entry, while the first κ qualities have been aligned optimally on the $(\kappa+1)$st quality. Then for any quality index $j \geq 2$ we have $u_j^2 \geq u_{j+1}u_{j-1}$.*

Proof: From (60a) the result obviously holds with equality for the optimally aligned qualities $1, .., \kappa$. Any quality $j \in [\kappa+1, n^* - 1]$ has been determined from the relation $F = (t_j - a)^2[(u_{j+1}/u_j) - 1]$, where t_j is given by (111). By Proposition 10 t_j is an increasing function of j, implying that for a fixed F the ratio u_{j+1}/u_j must be decreasing in j, Q.E.D.

Since the last $n^* - \kappa$ qualities have been relocated successsively by equating the entrant's revenue to the fixed cost F, it follows that for any given F

each quality level becomes an increasing function of its immediately higher one. Hence, $u_{\kappa+1}$ is ultimately a function of u_{n^*-1}, while the optimally aligned qualities $1, \ldots, \kappa$ are functions of $u_{\kappa+1}$ through (60a). Of particular interest to us is the increase in u_1 and its effect on the market coverage parameter t_1. From (10)[180] we have $t_1 = p_1 r_1$, and we examine the effect on t_1 of the relocation strategy. Given the monotonicity of the functions determining the quality levels, we can consider all such levels as functions of u_1. We can then prove the following result.

Proposition 12 *If the market is uncovered at the optimally-chosen locations and number of qualities, and if pure relocation is the optimal entry deterrence strategy, then such relocation would always* decrease *the extent of market coverage.*

Proof: It suffices to show that t_1 increases. From (10) and (50) it is easy to see that

$$\frac{t_1}{a} = [\frac{k+1}{2}] \cdot \frac{1+s_1^{-1}}{1+s_1^{-1}+\ldots+s_n^{-1}}.$$

A pure relocation strategy would increase u_1. Differentiating t_1 with respect to u_1 we get

$$\frac{dt_1}{du_1} = \frac{k+1}{2} \cdot \left[-\frac{s_1'}{s_1^2}\hat{B} + \left(1+\frac{1}{s_1}\right)\sum_{i=1}^{n}\left(\frac{s_i'}{s_i^2}\right)\right] \Big/ \hat{B}^2 \tag{113}$$

and noting that s_1 decreases with u_1, we find that a sufficient condition for $\partial t_1/\partial u_1$ to be positive is

$$\sum_{j=1}^{n} \frac{s_j'}{s_j^2} \geq 0, \tag{114}$$

where $s_j' \equiv \partial s_j/\partial u_1$. Using the definitions of the s_j's and noting that $\partial u_j/\partial u_1 > 0$ for all j (since all qualities have been displaced upwards), we find that the above sum is positive if $u_{j+1}/(u_j+u_{j+1})^2 \geq u_{j-1}/(u_j+u_{j-1})^2$. This, however, boils down to $u_j^2 \geq u_{j+1}u_{j-1}$, which holds by Lemma 4, Q.E.D.

Hence, a pure relocation strategy, which is the optimal strategy for at least some parameter configurations, would always reduce market coverage.

[180]See page 12.

h	k	$F \times 10^2$	t_1
4.0	1.501	2.00	1.0521
3.5	1.48	1.60	1.0559
2.5	1.40	0.80	1.0479
1.5	1.21	0.09	1.0277
1.3	1.14	0.024	1.0206
5.0	1.60	3.20	1.0885
4.5	1.60	2.90	1.0980
2.0	1.70	1.80	1.2074
1.5	1.45	0.30	1.1613
1.4	1.40	0.19	1.1479

Table 2: **Entry Deterrence and Uncovered Market.**
This Table shows values of t_1 for selected $\{h, k, F\}$ triples, after entry deterrence.

Such a reduction can be very substantial. Table 2 shows some values of t_1 corresponding to the indicated values of h, k and F; in all cases we have $n^* = 2$ and $a \mp 1$, with relocation being the optimal strategy.

Several of the (h, k) values in Table 2 are the same as in Table 1 but the reduced value of F makes blockaded entry infeasible and relocation necessary in order to deter entry. In other cases, though, the (h, k) values do not satisfy the blockaded entry condition (103) for any $w \in (1, h^{1/2}]$, even though relocation continues to be optimal. In all instances the extent of market coverage has been significantly reduced as a result of relocation.

It should be noted that a reduced market coverage as a result of relocation stemming from an entry threat has certain implications for consumer welfare. For $n^* = 2$ this welfare, compared to the one existing under a protected monopolist, has been reduced for all but the wealthiest consumers (the ones purchasing the top quality). Indeed, not only are those consumers who now don't make any purchase worse off, but the price p_1 of the lowest quality level has risen,[181] reducing the utility of some of those who pur-

[181]Notice that after relocation, the level of the lowest quality is increased, and yet, some consumers refuse to buy this higher quality. It is therefore obvious that the price of the

chased it before and continue to purchase it now. On the other hand, this surprising welfare effect of an entry threat is not necessarily true when relocation is not the optimal strategy. Consider, for instance, the (h, k) values corresponding to the first entry of Table 2, but suppose that $F = .019$. This value is still larger than $F_1 = .01608$, implying that $n^* = 2$. Here, however, relocation sufficient to deter entry must set $u_1 = 2.3792$, which results in an incumbent revenue $R_2 = 1.24598$. It can be easily verified that $R_2 + F < R^*(3) = 1.26708$, implying that relocation is inferior to proliferation if the latter can deter entry; this turns out to be the case, since for u_ε located at $h^{2/3}$ we have $R_\varepsilon = .01638 < F$. Hence, the optimal strategy here is to introduce three optimally located qualities. It can be easily verified that for $n = 3$ (70) is violated and the market will be completely covered, thus improving welfare when compared to the protected monopolist.

5.3 Conclusions from Chapter 5

The main purpose of this chapter was to substantiate the conclusions of chapter 3 by showing that uncovered market can be a persistent phenomenon even in situations where there is no legal entry barrier. Our first question was to examine whether the monopolist's multiproduct policy can be entry-proof without need for any modification. As we showed in section 5.1, there exist cases where entry is naturally blockaded by the incumbent firm's unconstrained optimal decisions. This enhances the importance of the optimal decisions of the monopolist concerning his/her price and quality schedules as derived in section 3.1, since it shows that they can be robust in the presence of an entry threat.

What protects the monopolist in this case is not the sunkness of cost as in the case of homogeneous product markets but the fact that there is not enough room for entry between any two incumbent qualities. This result is in the spirit of Schmalensee [35]; however, two points substantially differentiate our analysis from that in [35]. The first is that the product proliferation in our case results from the unconstrained optimal choice of the monopolist rather than from being a deliberate entry impediment policy; this relates to the endogeneity of the number of products as discussed in the

lowest quality must be higher than what it was before relocation took place.

next paragraph. The second point is that we deal with vertical rather than horizontal differentiation; this is important because in our context every firm—including the entrant—has an advantage over any lower qualities even if these qualities belong to the incumbent.

We cannot insist too much on the fact that our blockaded-entry result, altough rather trivial when the incumbent is limited to introducing only one quality, is not obvious at all in the case of multiproduct firms. With only one quality, and since there is an asymmetry in revenue between high and low qualities, it is always possible to find values for the fixed cost that will make entry of a second firm unprofitable. This result is due to the fact that the number of qualities of the incumbent is in this case treated as exogenous, whereas in our analysis it was determined by the parameters (h, k, F) of the model. Having the number of incumbent products endogenous implies that the level of fixed cost imposes—for given values of (h, k)—a restriction on the space between qualities that is available for the entrant, thus affecting to a large extent the profitability of entry. Hence, varying the level of fixed cost has two effects of opposite direction on the conditions for entry. For instance, if fixed cost is increased, it is not *a priori* obvious that entry has become more difficult since the incumbent's optimal choice now results in more space between qualities, thus giving the entrant the opportunity to increase his/her revenue by further relaxing price competition through product differentiation.

The second question examined in this chapter concerns the persistence of an uncovered market when entry cannot be blockaded but the incumbent chooses the optimal strategy in anticipation of such entry. Three possible strategies were examined: *a)* to allow entry, *b)* to prevent entry by product proliferation in the spirit of Schmalensee,[182] c) to prevent entry by product relocation, as Bonanno[183] has shown to be possible in the context of horizontal differentiation. It was shown that this last strategy may emerge as the optimal response and that, if it does so, it will always reduce market coverage and diminish welfare of all but the wealthiest consumers as compared to the protected monopolist. On the other hand, a change in fixed costs may render product proliferation optimal, in which case the uncovered

[182]See [35].
[183]See [2].

market may disappear and consumer welfare increase unambiguously.

6 Conclusions

The study of differentiation has been based on two different approaches, one represented by the model of monopolistic competition and the other by the address model. Although these models assume in common that many types of a product are available in the market and that each consumer favours only one type, they differ by their assumptions on how consumers view the rest of the product set: in the former, consumers are indifferent when facing a choice that does not involve their favoured type, while in the latter they establish a non-trivial ranking on the whole product set. The differentiation of the product set is characterised as variety when there is no unanimity in consumers' rankings of the different types of the product; in the opposite case one can meaningfully talk about quality differences, since the unanimity in consumers' ranking can unambiguously determine between any two products which one is of higher quality.

In many instances, to produce a higher quality requires a higher average variable cost; therefore better product types can be available at substantially higher prices, even if there is sufficient competition among firms to bring the prices of all the qualities down to their average variable cost. In those cases, there may not be a set of feasible prices—i.e. prices at which some firms will accept to produce the various qualities—resulting in a unanimous choice of quality among consumers, despite the absence of disagreement in assessing the merits of each product; some consumers will decide to buy the lower qualities because they are not ready to pay the extra cost that a higher quality entails. Cases like the above can be handled very well by the address models of variety and for this reason we will consider them as horizontal differentiation even if the market is characterised by quality differences rather than variety.

There are, however, cases where increases in quality do not require substantial increments in variable cost in order to be made available to consumers. This is particularly true in situations where the burden of increasing quality falls primarily on fixed cost. When this is the case, an average-cost pricing structure will translate the unanimity of preferences to an identical choice by all the consumers, which in turn implies that only the top quality product will be produced. Under these circumstances, the high quality product can deviate from average cost pricing while maintaining a significant market share. Thus, in situations of *vertical differentiation*, the unanimity in consumers' choices at average cost implies that, if a set of qualities is produced, all but possibly the last quality are making positive profits.

The literature studying this type of situation is focussed on two issues: market structure, and the possibility of a single firm using many qualities in order to price discriminate when no perfect information on consumers' willingness to pay is available. Surprisingly, little has been done to bring these two issues together. The works focussing on market structure almost always assume single-product firms, while those on price discrimination assume that the monopolistic structure is determined either exogenously or by the characteristics of the income distribution, i.e. totally independently from the introduction and positioning of many qualities.

In this thesis we have attempted to bring these two issues closer together by studying the implications of the nature of the fixed cost on the natural duopoly solution, and then by investigating the implications that the number of products can have on entry conditions. At the same time, we examined the extent of market coverage in situations of protected and unprotected monopoly in order to detect the impact that potential entry may have upon this question. In what follows, we summarise the major findings of our work and place them into the context of the relevant literature.

Market structure and choice of quality The main result in the literature on market structure in VPD is the *finiteness property*; it states that in a Bertrand-Nash equilibrium there will be an *upper bound* to the number of firms present in the market, this bound being independent of the fixed cost and the choice of qualities of the different firms. This surprising result comes from the fact that, once a certain number of single-product firms has entered into the market, competition will drive the prices of all the qualities so low as to make every consumer prefer some of the available products to any lower quality even if that quality is offered at a zero price. Thus, considering a three-stage perfect equilibrium where firms decide entry, quality, and price of their product in three successive rounds, the finiteness property is an outcome of the *pricing* stage and as such it is only affected by the width of the willingness-to-pay for quality.

When the finiteness property protects up to two incumbents from further entry, Shaked and Sutton showed that, if the fixed cost allows the profitable existence of exactly two firms, these firms will decide to differentiate their products rather than both produce the top quality; they also implicitly determined the optimal quality level for the low quality product in the market. The analysis of Shaked and Sutton is based on entry being simultaneous, which implies that entry, rather than choice of quality, causes the irrecoverability of the fixed cost. If, on the contrary, the fixed cost is not only entry specific but also quality specific, then the industry must be analysed assuming that the entry of firms is sequential. The important implication of the assumption of sequential entry is that the finiteness property is no longer sufficient to protect the lower quality firm from a new entrant who decides to produce a higher quality. This consideration induced Hung and Schmitt to claim that the lower quality incumbent would try to choose a quality level sufficiently high in order to protect itself from entry and replacement. As explained in chapter 4, the result of Hung and Schmitt rests on two specific assumptions: that the range of qualities is too narrow and that the nature of the fixed cost cannot affect the decision of a firm on whether to stay in business or not. The arbitrariness of the first assumption is rather obvious; this assumption results in the revenue of the lower quality firm being independent of the level of the reservation quality. In other words, the consumers' willingness to pay for the low quality is independent of the desirability of the non-purchase option, which constitutes a rather unattractive feature of the model.

The second assumption of Hung and Schmitt constitutes a more subtle case, since a low quality product may indeed find itself with no sales in a Bertrand-Nash equilibrium. However, to say that a firm, once its sales are reduced to zero, will quit the market implies that the fixed cost is sunk with respect to changes in quality but not with respect to exit. If, on the contrary, the fixed cost is totally sunk in relation to exit, the decision of the firm concerning its continued presence in the market is indeterminate. To cope with this indeterminacy, one needs more information on the nature of the fixed cost; if there is a cost component purely associated with exit, like cleaning up the site or the expectation of future revenues, the firm may decide to stay in the market. On the other hand, if a major part of the fixed cost can be salvageable upon exit and this part exceeds any pure exit cost component, then the firm will decide to go, as Hung and Schmitt have assumed.

Can the decision of the displaced incumbent to stay or not be important? Our conclusion from the analysis contained in chapter 4 is that if the displaced quality remains potentially available, then it will become the new reservation quality and through this it will affect the willingness of consumers to pay for the other qualities. This is reflected by the fact that, in general, the revenue of the lower quality firm depends negatively on the level of the reservation quality. Thus, a major result from our analysis is that a costly exit can become a crucial weapon in the hands of the lower quality incumbent firm against new entry. This in turn implies that, in a vertically differentiated natural duopoly, the two products will not be as close as possible[184] unless the nature of the fixed cost makes permanent exit a superior strategy for the displaced incumbent. This is in sharp contrast with the analysis of Hung and Schmitt where the choice of the lower quality must always be determined by the zero profit condition for the lower quality and, as a consequence, the market is characterised by minimal differentiation. As this thesis showed, the extent of product differentiation in a natural duopoly can vary from the level suggested by the simultaneous entry model to that dictated by the zero-profit condition, depending on the *nature* and the amount of the fixed cost.

Multiproduct firms, market coverage and market structure Turning to the question of multiproduct firms, we found that there may be an upper bound to the extent of market coverage, this bound being independent of the level of the fixed cost. In other words, there are instances where even the elimination of the cost of introducing a new quality is not sufficient to induce the monopolist to cover the market. In chapter 3, we derived the monopolist's optimal rules concerning prices, qualities and number of products in general instances where the market is uncovered either because of high fixed costs or because of pure demand conditions. Surprisingly, as the analysis of chapter 5 showed, these rules can be robust in the face of an entry threat despite the presence of a number of consumers ignored by the monopolist. This is so because entry must be disputed at high quality

[184]Recall that the two qualities cannot be so close so that the lower quality sustains losses.

levels rather than at the level which could bring the excluded consumers back to the market; at the same time, the introduction of many qualities by the incumbent leaves little space for entry between the high qualities. Hence, the presence of an uncovered market is not an indication of easy entry; neither can it be considered as a short run phenomenon. This conclusion is directly related to the welfare of the poorest consumers and it can be important for policy purposes in situations where distribution is an issue.

On the other hand, the possibility of entry being blockaded by the monopolist's optimal number of products has severe implications for market structure: there are instances in a natural duopoly in which the fixed cost would allow the presence of two single-product firms completely covering the market; yet the upper bound to the number of firms may not be reached if the first firm in the market introduces many qualities. This result is not trivial since the number of qualities is endogenously determined by the level of the fixed cost. Thus, increasing the level of fixed cost may, to some extent, help the entrant, since it reduces the number of and therefore the spacing between qualities.

When entry is not blockaded at the monopolist's unconstrained choice of product line, the incumbent firm can impede entry either by product proliferation or by relocation of its qualities. As we showed, the latter can dominate the former from the monopolist's point of view in a number of instances. Surprisingly, when the monopolist relocates his/her qualities in order to avoid entry, the segment of unserved consumers will become larger. This is a consequence of the fact that entry between high qualities is easier because the distance between any two qualities increases with the quality index, while at the same time the entrant's revenue increases with the quality he/she introduces. Thus, the incumbent must first reduce the space between the top two qualities and align all the lower qualities with respect to u_{n-1}. If entry is now feasible between u_{n-1} and u_{n-2} he/she must upgrade the quality u_{n-2} until entry is blockaded and align the $n-3$ lower qualities according to u_{n-2} and so on. Thus, entry deterrence causes an upgrading of the lower qualities as well, and it is this fact that decreases market coverage.

Remaining questions As it happens, few—if any—theoretical works give a definite answer to all the interesting questions related to their topic; time limitations and increasing complexity leading to a lack of tractability, leave almost always a number of issues unexamined and the present work constitutes no exception in this respect. We would like therefore to devote this last part to the description of some issues which did not receive an adequate treatment in this thesis and which will constitute, as we hope, the agenda for our future research.

The first such question concerns changes in the overall welfare, as measured by total consumer surplus,[185] when one moves from one of the situations examined in this work to another. Since most of the results of this thesis have been rigorously proven for cases where $k \in (1, 3)$, in the discussion that follows we will consider that the width of the income distribution determines a natural duopoly.

Before proceeding any further we would like to mention the fact that in VPD models, differentiation *is not* to the advantage of the consumers. Indeed, by our cost assumptions, the optimal situation involves only the top quality produced and priced at average cost.[186,187] If more than one qualities are introduced, their purpose will be either to relax competition among firms or to allow the monopolist to extract consumer surplus, rather than better accommodate the differentiated needs of individual consumers, as is

[185]Strictly speaking, the Hicks-Kaldor criterion requires us to include firms' profits in the measurement of welfare. This however would complicate the analysis to a considerable extent. Moreover, consumer protection often constitutes on its own a goal for public policy. Thus, welfare comparisons based only on consumer surplus can be of substantial value for policy decisions.

[186]Since each individual consumer's demand is completely price inelastic, if profits are taken into account when making welfare assessments then the price of the top quality in the optimal situation can be anything between average cost and the price which would just induce the poorest consumer to stay in the market.

[187]Notice that, ideally, the determination of the optimal price of the only quality to be produced should be based on marginal rather than on average cost. However, the presence of a fixed cost together with constant average variable cost—equal to zero—create economics of scale at any level of output of a given product type. Hence, the market of any single quality taken separately is a natural monopoly; therefore, as is well known, marginal cost pricing results to losses for the firm. Ignoring the possibility of a tax subsidy covering any such losses, we consider as optimal a situation where only one firm produces the high quality product and prices it at average cost.

the case in horizontal differentiation. Thus, unless a surprising outcome results from the detailed study of the situation, the conclusion concerning the number of products should be unambiguous. What therefore is of interest in the context of VPD is to compare consumer welfare between different situations involving the same nunber of products but different price-quality decision rules or between situations involving the same amount of fixed cost but where the number of products can be different according to whether entry is impeded or not. Thus, the welfare analysis should not be conducted in terms of comparisons of different situations against an ideal, but instead as a comparison between imperfect alternatives.

One such comparison should seek to assess the impact on consumer welfare of entry being sequential rather than simultaneous. As the analysis in chapter 4 made clear, if entry is sequential in a natural duopoly with single product firms, the choice of the lower quality will depend on the nature and the amount of the fixed cost, ranging between the solution of Hung and Schmitt at the one extreme where exit is the superior strategy, and the blockaded entry solution in which the incumbent lower-quality firm can safely adhere to the rules for quality choice determined in chapter 4[188] for the absence of entry threat.[189] Thus, the situation reduces to the examination of how consumer surplus changes as the lower quality increases. The first thing one must observe is how consumers' choices are affected by an upgrading of the lower quality, which corresponds to studying the impact of the choice of u_1 on t_2.[190] Substituting the optimal pricing rules as given by (26c) and (26b) into the definition of t_2[191] one can easily obtain the expressions for t_2 in a Bertrand-Nash equilibrium as a function of the choice of qualities, which are

$$t_2 = \frac{b+a}{3}, \tag{115a}$$

[188]See Proposition 4, page 79.

[189]Note that this solution is similar to the one assuming simultaneous entry.

[190]The value of t_1 is not important as far as the consumers' choice of quality is concerned since the market is always covered. However, as already said in footnote 33 (page 19), the surplus of the consumer with income t_1 depends on whether t_1 is < 0 or $= 0$.

[191]See (9), page 12.

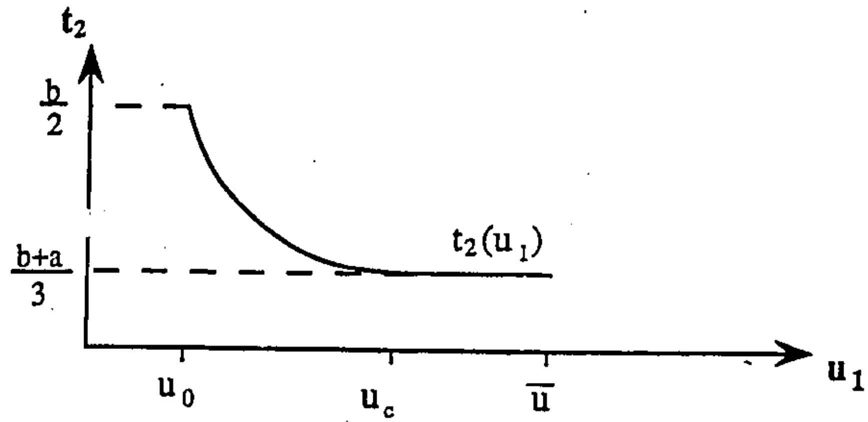

Figure 12: **The lower boundary of the market of $\bar{u}$ as function of u_1.**
This figure shows t_2 in a Bertrand-Nash equilibrium as function of the choice of the lower quality, u_1. For $u_1 \leq u_c$, improvements in the lower quality product actually reduce its market share because they are accompanied by increases in p_1 and reductions in p_2. For $u_1 \geq u_c$, as u_1 rises, both p_1 and p_2 fall leaving the market shares of the two products unaffected.

$$t_2 = \frac{1}{2}[b - a(V-1)], \tag{115b}$$

the first expression corresponding to $t_1 < a$ and therefore $u_1 \geq u_c$ and the second to $t_1 = a$ and thus $u_1 \leq u_c$. Figure 12 shows t_2 as function of u_1

As can be seen from Figure 12, if $hk \leq h+3$ so $u_1^* = u_c$, any increase in the lower quality to protect the incumbent firm from entry, leaves the consumers' choices unaffected. Moreover, as the expressions (26c) in page 29 indicate, both prices are decreasing in u_1, $\forall u_1 \in [u_c, \bar{u}]$ (*region I*). Thus, when an entry threat forces the low quality incumbent firm to increase its quality beyond u_c, all the consumers will be better off, since those with incomes between t_2 and b will purchase the same quality as before at a lower price, although those with income between a and t_2 will get a higher quality at a lower price. Unfortunately, things are not so obvious when $u_1^* < u_c$, which implies also that the price of the lower quality is always adjusted as to set $t_1 = a$. Suppose again that entry deterrence forces the lower quality incumbent firm to increase its quality but say that the entry deterring level of u_1 remains below u_c. As shown in Figure 12, the new value of t_2, call it $\ddot{t}_2$, will be lower. We can then divide the consumers into three groups:

a) those with income between t_2 and b who always buy the highest quality, b) those with income between a and $\ddot{t}_2$ who always buy the lower quality, and c) those with income between $\ddot{t}_2$ and t_2, who in the absence of entry threat were buying the low quality but after the increase of u_1 switch to $\bar{u}$. Straightforward calculations, along with intuition, show that the price of $\bar{u}$ falls as u_1 increases, and this implies that the presence of an entry threat that forces the lower quality firm to increase its quality level is beneficial for the first group. But this is all that we can say with any degree of certainty at this point. The change in the welfare of the group with incomes between t_2 and $\ddot{t}_2$ cannot be easily assessed. To see this, consider the following relations revealed by the choices of those consumers, where ¨ is used to denote the values of different variables after entry and the symbol $\succ$ reads as *'preferred to'*:

$$\begin{aligned}(u_1, p_1) &\succ (\bar{u}, p_2)\\ (\bar{u}, \ddot{p}_2) &\succ (\ddot{u}_1, \ddot{p}_1)\end{aligned}$$

Since $(\bar{u}, \ddot{p}_2) \succ (\bar{u}, p_2)$, no conclusion can be obtained from the above conditions. Similarly, the poorest group of consumers are paying a higher price after entry deterrence takes place but they also enjoy a higher quality. Is the increase in quality sufficient to compensate for the higher price? Nothing can be said without further research, except of course for the fact that those with income a get no surplus in any situation.

Another interesting welfare comparison would involve two situations with similar fixed cost and parameter values determining natural duopoly, but where a multiproduct incumbent could blockade entry by producing his/her optimal number of qualities. Although the intuition suggests that welfare would be probably reduced under the monopoly regime, it will be interesting to observe the redistributional aspects of disallowing multiproduct firms, i.e., if there are consumers who may benefit from that change. Equally important, in situations where a multiproduct incumbent relocates his/her qualities to deter entry, are consumers any better off in the presence of an entry threat? Here we know already that all the consumers left out of the market[192] and the new marginal consumer will be strictly worse off, since the entry threat will reduce their otherwise positive surplus to zero.

[192]Except for a.

Besides welfare questions, an important direction for future research would be the introduction of uncertainty in vertically differentiated markets. Uncertainty may concern the cost conditions, the density or the width of consumer income distribution. Out of these three cases we think that the most interesting is the one in which, in a multiperiod game, the future realisations of the width of the income distribution cannot be known with certainty when a firm must decide on entry, exit and choice of quality (qualities). The importance of this situation stems from the fact that changes in the width of the income distribution may reflect business cycles which is an important element of a firm's planning. Moreover, if k becomes temporarily narrow, some qualities may be totally deprived from sales for a certain time, which can possibly explain why idle capacity in times of recession is a phenomenon usually associated with the production of lower quality products.

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