Analyzing Dynamic Football Passing Network

by

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Abstract

In this thesis we are concerned with the analysis of football matches represented as passing graphs, where players are nodes and directed edges indicate the passing of the ball.

As opposed to previous work, we label the edges of the graph with the time instants when the ball is passed between two players, thus constructing a time-varying graph. We then employ techniques from social network analysis to study centrality roles and other indicators, keeping into account the evolution of the game in time.

More precisely, we focus on degree centrality, closeness, betweenness, pagerank, eigenvector centrality, and clustering coefficient, and we compute these measures dividing the overall play time into time windows following two different models. The results are to be compared with a static analysis performed on a unique time window. Our study provides observations that are different and possibly more accurate than the ones that can be obtained by a static analysis, opening the door to a dynamic study of football matches.

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Dedication

This is dedicated to

My wife *Elham* whose scarifies provided the foundation for achieving of my academic goals.

My daughter *Tina* and my son *Rayan* whose patients, which were realized by our loss of precious time together, were for me the most painful and humbling of all.

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Chapter 1

Introduction

In this chapter, we introduce the main motivation behind our study and we describe goals and contributions of the thesis.

1.1 Football Passing Networks and their Limitations

A football passing network is a graph where nodes are football players and a directed weighted edge between two players represents the number of passes that have occurred between them during a game. Although the nature of football passing networks is highly dynamic, in the literature they have been studied only using static models (e.g., see [39, 27], discussed in Chapter 2). In fact, in the existing work, the performances of football teams and players have been studied by representing a game with a passing network and graph parameters have been used to obtain an indication of key individuals, to highlights potential weaknesses, and to evaluate the general performances of a team. For example, high scores of out-degree and in-degree would be indication of an active player who passes the ball frequently during the game.

This static model can provide some interesting results, but it clearly fails to incorporate essential factors. For example, the model is not designed to capture mistakes made by the players, either critical or non-critical, and it also fails to incorporate different factors representing the team's performance other than the final match result. Most noticeably, it does not consider the time when passes are done and thus fails to provide a temporal account of the players' performances. It would be then desirable to develop the model into a more sophisticated system in which some of the missing factors are incorporated.

1.2 Motivation and Goals

In this thesis, we give a new look at football passing networks trying to overcome some of the drawbacks indicated above. In particular, the static football passing network model does not allow us to perform a time-dependent analysis of football matches, and a natural question is how the model could be modified so to possibly achieve more accurate results. Since the network under study is dynamic and is changing over time, an immediate proposal would be to keep track of the passing information in a time-dependent manner.

The goal of the thesis is to devise a time-dependent method to study the dynamics of players' passes during football games. To achieve this goal, we propose to use the notion of time varying graph with dynamic nodes and edges. More precisely, We consider football passing network as a directed weighted evolving graph. In our model, players are nodes (which are entering and exiting the system in different time periods), and at an arbitrary time t there is an edge from node A to node B if player A has passed the ball to player B in a certain interval time $[0, t)$. In addition, in this model, we associate a non-negative weight k to an edge at time t indicating the number of passes from the initial point to the end point of that edge in the considered interval of time. We call such a network a *dynamic passing network*.

The dynamic passing network could be used to determine players in a team who are either successful or insignificant, depending on whether the team discovers how to use or abuse passes, and whether a player is not involved enough in a team. Furthermore, it could be used by a team to indicate under-performing players, determine weak spots and discover potential problems between teammates who are not passing the ball as often as their position dictates. We should mention that the above characteristics can be obtained by observing specific time intervals and so the dynamic passing network over time might give us more accurate results than the static network.

The dynamic passing networks employed in the thesis are based on two models explained in Chapter 5: the aggregated model, and the non-overlapping model. For the aggregated model we consider growing time windows of different sizes and we study the corresponding growing evolving graphs; for the non-overlapping model, we partition the overall time into consecutive time windows and we consider the corresponding sequences of evolving graphs. For each model, we compute several social network measures. All these measures have been calculated both for the static network consisting of a single window (one snapshot), as well as with the time-dependent method consisting on several windows of variable size. We then compare the average of each measure for all players in the networks created with

the two models, highlighting the differences with the static approach.

1.3 Thesis Contributions

The contribution of the thesis is twofold. On one hand, we introduce a new general method to analyze football matches considering time-dependent passing information; this method could be also exploited in ways not covered in the thesis. On the other hand, we performed an analysis of football matches based on the introduced technique by studying classical centrality parameters over growing and consecutive time windows. More precisely:

- Our first contribution is to propose a temporal model that includes in the football passing network the information on the time when each pass occurs. We then propose to divide the overall time in windows of different sizes (either growing or consecutive) and to analyze the passing network over time.

- We then consider a specific match: Spain-Netherlands, the final match of the 2010 World cup, and we focus on: degree centrality, betweenness, closeness, pagerank, eigenvector centrality and clustering coefficient. We compute these values for each player in the static aggregated graph, as well as in the time-window models with windows of different sizes. For a given size, we average the values over the windows covering the overall time and we compare the results with the ones obtained by a single window.

  We observe that our methods generally produce a different ranking of the players' scores. Among the specific observations that we can make, particularly interesting is the fact that, in both windows models, Spain has more evenly distributed betweenness scores than the Netherlands, which may indicate a better global tactic by the Spain team. On the other hand, the average betweenness scores are fluctuating among different window sizes for the Netherlads team, indicating a more uneven involvement of the players.

  From the analysis of the pagerank scores, we can see that Spain has a more aggressive behaviour, keeping the ball consistently closer to the Netherlands' goalkeeper. This is particularly evident from the non-overlapping windows model, but confirmed also by the aggregated model. In both, in fact, the Netherlands' pagerank scores present more fluctuations within window sizes with a high score consistently kept by the goalkeeper (ranked first in the average of the non-overlapping windows, and 3rd in the aggregated ones).

Other interesting observations concern the identification of players who start to perform poorly during the game. This might be the case, for example, of player 11 (Robben) in the Netherlads team, whose betweenness, in the aggregated model, decreases when increasing the window size, thus indicating that the ball flow does not depend much on this player over time.

- The time-windows technique could be used to focus on a particular player and observe his/her performance over time thus discovering special characteristics of the player and anomalies. For example, a useful application could be the online monitoring of the players by the coach to determine which player to substitute or which strategies to suggest. Another interesting application would be to record the number of shots towards the opponent goalkeeper instead of just recording passes among the players. These applications are beyond the scope of the thesis, but would constitute an interesting continuation.

1.4 Thesis organization

The rest of the thesis is organized as follows: In Chapter 2, we present a brief survey of football passing networks. As we will see, very little has been done. Some drawbacks around the recent work about analyzing football passing network are discussed. In particular, we notice that all studies are based on a static model, and thus contain limitations. Being all the data collected at the end of match, with no attention to the time when the passes occurred, the existing studies are inherently inaccurate to calculate local and global measures of the network.

In Chapter 3, we first give an overview of fundamental graph theory concepts that will be needed throughout the thesis. We then define the social networks indicators that will be employed, adapting them for our models: degree centrality, betweenness, closeness, pagerank, eigenvector centrality and clustering coefficient.

In Chapter 4, we introduce the concept of dynamic graphs and the general definition of time varying graph, we give some examples and show different representations. We then focus on a class of TVG, evolving graphs, that better represent our purposes, and we define our *dynamic passing network*.

In Chapter 5, we introduce two models: aggregated and non-overlapping windows. These models represent two different ways of dividing the overall time in time intervals.

In the first case. by windows of growing size ultimately covering the whole time, in the second case, by windows of equal size partitioning the overall time.

In Chapters 6 and 7, we specifically focus on the final game of the 2010 World Cup in South Africa (Netherlands - Spain). For each team, we create dynamic passing networks corresponding to each window size, based on each model explained in previous chapter. Then we calculate degree centrality, betweenness, closeness, pagerank, eigenvector centrality and clustering coefficient for each dynamic passing network, by taking the average over all windows. Then we obtain the average centralities for each player. We calculate the same measures for the static representation of the network and we analyze the data obtained by our methods.

In Chapter 8, we close the thesis by summarizing our results and indicating open problems and research directions.

Chapter 2

Literature Review

Football is one of most popular and exciting sports around the world and recently has been considered as a research area not only in health science, but also in mathematics and computer science. The combination of graph theory and concepts of social science, known as *social network*, is a powerful tool and is used to study some areas in applied mathematics such as algorithms, social commerce and marketing. For example, social graph is used in the Internet context that refers to a graph that depicts personal relations of Internet users. In this chapter we give a review of the work that has been done to analyze football matches, applying tools from graph theory and concepts of social networks.

2.1 Graphs and Social Networks

A graph as an object of mathematics can be applied to represent any system consisting of many single units interacting through a certain kind of relationship. In this graph representation, a node stands for one of the elementary units of the system and edges are defined as interactions between different units. Some examples in social networks are as follows: friendship graphs, where nodes are people and edges join two people who are friends; communication graphs, where nodes are terminals of a communication system, such as mobile phones or email boxes, and edges are defined between two terminals if and only if there is an exchange of a message between them.

In the above examples, all graphs or networks are dynamic, i.e., the activity corresponding to an edge depends on time. More precisely, node adjacency or the relationships among units of a networked system, are rarely persistent over time. In the study of social networks the crucial information typically considered to analyze its static structure is often

related to centrality measures. In network analysis, centrality deals with measures, at local or global scale, which determine the most important vertices within a network. There are many applications in social network such as identifying the most influential person(s), key infrastructure nodes in the Internet or urban networks, and super spreaders of disease. Centrality concepts were first developed in social network analysis, and many of the terms used to measure centrality reflect their sociological origin, see [33].

As mentioned above, an important question in network analysis is *What factors or indices can characterize a significant node?* The answer to this question is given by centrality indices, for example, by defining a real-valued function on the nodes of a network, where the values produced are expected to provide a ranking which identifies the most important nodes, see [3, 4] Clearly, there are many meanings we can associate to the word "importance", depending on different definitions of centrality. There are mainly two approaches for this issue: in the first, the concept of "importance" can be understood in relation to a type of flow or transfer across the network. By taking this concept of importance, centralities are classified by the type of flow, see [4]. In the second, "importance" can alternately be considered as involvement in the cohesiveness of the network. Using this concept of importance, centralities can be classified based on how they measure cohesiveness, see [5]. These two approaches divide centralities in distinct categories.

One of the most significant centrality measures in network is *eigenvector centrality*, which is defined as a measure of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. For instance, Google's PageRank is a variant of the eigenvector centrality measure. We will talk in details about centralities of both static and dynamic networks in Chapter 3. In Section 4.2, we will provide several indicators of static networks and generalize them for dynamic networks.

2.2 Football Passing Network

Many team sports involve passing between players, and Football [1] (known as soccer in North of America) has traditionally lagged behind other sports such as Baseball, Hockey, Basketball, in terms of statistical information made available after the games. Without

[1] England invented a game of running around kicking a ball in the mid-19th century (although the Chinese claim to have played a version centuries earlier). They called it football, not because the ball is played with the feet, but because the game is played on foot rather on horseback.[keepingscore.blogs.time.com]

doubt, Football nature is unique in terms of the continuous ball movement, and the comparatively low scores obtained compared to other sports. These characteristic makes it more difficult to study it using simple statistics such as assists of goals, which are insufficient as measures of team and players performance. Recently, starting with 2008 Euro Cup, an unrivalled amount of data has been made public after the games. The issuance of considerably vast quantity of data opens up the way for creating new and more detailed analyses of football. Towards this direction one can find more details in [20].

Let us look closely at football passing networks viewed as social networks.

In [14], the authors define the passing network of a football team as the network with the team players as nodes and directed edge between two players, let say player A and player B, weighted by the successful number of passes completed from A to B. In fact, the authors use the passing network as a tool for visualizing a team's tactics by fixing its nodes in positions neglectfully corresponding to the player's formation on the field. Some local and global network measures are studied by the authors.

In network service, analyzing a football match can be an important role for coaches, talent scouts, players and even media, and with developed technologies such as image processing, more and more match data is captured. There are companies that provide data based on the position of the players and the ball with high accuracy and resolution. Moreover, such companies also present software with basic analysis tools, for instance straightforward statistics about speed, distance run and number of passes. It is, however, a non-trivial task to perform more advanced analysis. In [27], the authors develop a collection of tools specifically for analyzing the performance of football players and teams.

As we can see from references such as [30, 23], so far football passing networks have been studied by considering static graphs, which means that edges are not affected over time. More precisely, time is not involved in analyzing and getting data in such a network. In fact, only a single aggregated graph has been considered disregarding the fact that edges keep changing over time. On the other hand, in realistic passing networks, the graph structure is changing every second, just by looking at one snapshot, namely at the end of match, we can not provide enough information in order to analyze football passing network, and such data captured with a single static snapshot is not accurate enough to calculate the local and global measures of the network.

By the above reasoning, in order to get a more accurate, efficient and complete data from football passing networks, we need to define a new network in which time is considered. The concept of time varying graphs (TVG) along with visualization are presented in Chapter 4.

2.3 Related Work

There are various approaches to analyze football passing networks based on which source of data is available and which kind of parameters are most significant for the researchers. In this section, we summarize some passing networks which have been studied so far; as we will see, very little has been done.

2.3.1 Passing network analysis

In football passing networks, players are nodes and links are passes through players Looking at football networks in this way leads to a method to assess passing through a team that has been increasingly used in football. For each player, the number of passes played and received is considered according to the player they passed to and who they received from respectively. By this information, you can check who passes to a particular player and who receives from whom, along with how often they do this.

The following example[2] illustrates an analysis from the Liverpool vs Gomel match in the UEFA Europa League, has been done the 9th of August 2012. The data is for Liverpool and shows completed passes only. As we can see, it is a directed graph and the arrow with larger and darker features indicates greater number of passes executed by one player towards another. This is a static graph and its structure (positions of players) is based on the rough formation of the team.

The above passing network shows only completed passes. The number of passes between two players are indicated by thicker or narrow arrows corresponding to large or small number of passes between them respectively. The position of each marker is obtained based on the approximate formation of the team and the size of each marker is related to their closeness centrality.

By looking at the graph, one can see that there are clearly four players who are interchanging passes in different areas of the pitch. For instance, Reina is the most distributer of the ball to his centre-backs, and he often drops deeper and wider to get the ball. Further up the field, the centre-backs players Lucas, Johnson and Shelvey make triangles all together and, for instance, Luca makes other triangle with their full-back and nearest midfielder. Such a situation happens on the left-hand-side where Enrique, Borini and Gerrard linked up.

[2]Access at http://2plus2equals11.wordpress.com

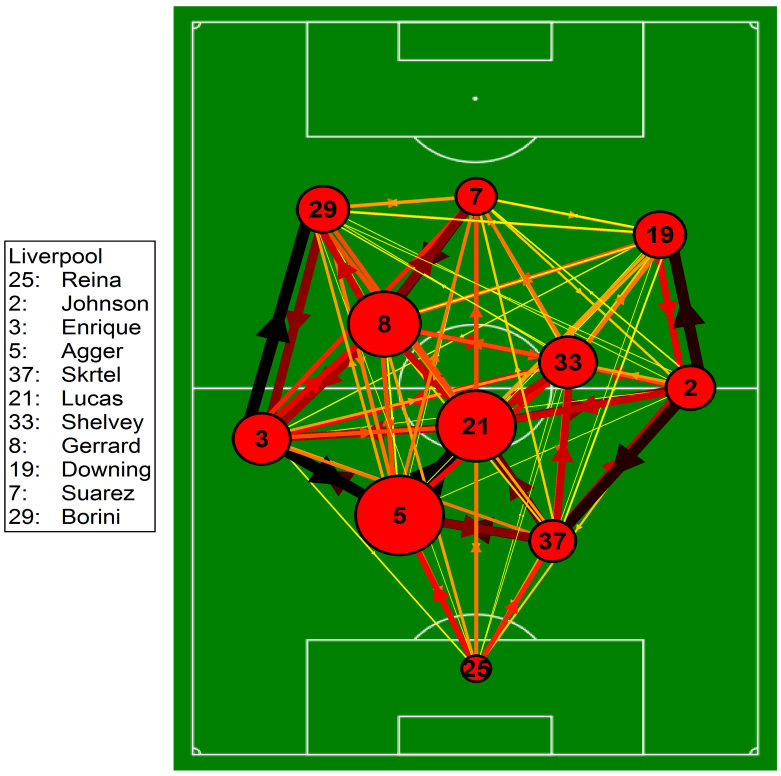


Figure 2.1: Passing Network- Liverpool

Now, let us look at the nodes with reciprocal arrows, mainly Enrique-Agger, Borini-Enrique, Johnson-Downing and Skrtel-Johnson. These node pairs have large interaction between themselves and it shows that the number of passes played back and forth between each node pair has high significance in comparison with other node pairs. By looking at player Borini we find that he often receives the ball and then passes it back to whoever passed it to him.

From the above local formation, we get some insights into how the passing network fits together as a whole. But of course, this cannot be an accurate analysis as we need to get many snapshots to obtain a more accurate time-dependent study.

Let us consider one important measure, closeness centrality, that can assess the number of passes played and received by a given player. This measure would be greater if the number of passes received and played by a player are distributed more evenly across the team. Let us illustrate it by an example in the network. The closeness centrality for a player in the team would be greater if he passes the ball 70 times and received it almost 70 times from teammate compared to if they simply passed the ball back and forth to just 5

teammate. So, players with a large closeness centrality measure have more impact on the passing network by means of the movement of the ball among the teammate.

In Figure 2.1, the size of each node is shown based on its closeness centrality score. According to this scores, Daniel Agger, Lucas Leiva and Steven Gerrard are Liverpool high performers. Agger outpoints Lucas partially due to their differing passing accuracy, according to Anfield-Index this score is 98.8 vs 90.8 as they got passes from players with almost a similar amount but Lucas misplaced more passes. On the other hand, Gerrard is obviously the play-maker in the attacking third with good link up between players Suarez and Borini along with spreading the play to Enrique and Johnson as they overlapped from full-back. This shows that there is a good distribution throughout the spine of the team and it represents a potentially beneficial division of responsibilities. Finally, if a team has a singular player with a high centrality comparing with other teammate, this could have negative consequences as a team can become overly reliant on a single player.

2.3.2 Passing Network: Spain vs Netherlands

In [39], the authors have analyzed the performance of football teams and players using network theory. They mention that their approach produces a quantifiable representation of a team style, identifying key individuals and highlighting potential weaknesses. For each team, they consider a graph in which nodes are players and edges are passes between players. This network is the same as networks considered in the previous model. Figure 2.2 shows the resulting networks for the Netherlands (right) and Spain using data from the knockout stages of the 2010 World Cup in South Africa. This match was in the final contest and Spain won.

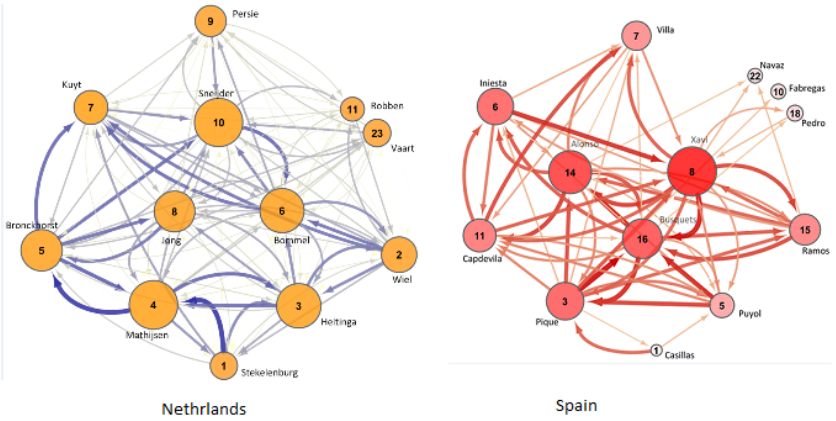


Figure 2.2: The Netherlands vs Spain

An immediate result from a visual inspection of these network implies that the thickness of the arrows indicates the number of passes between players and clearly the Spanish team pass more often. This image captures 417 passes by the Spanish team versus 266 for the Netherlands, data captured from the knockout stages of the 2010 World Cup. By this data, most significant players who played and received the ball are players number 16 Sergio Busquets and number 8 Xavi, as they stand by the number of passes they make and receive.

Let us look at some important measures of the network. In graph theory, for a given node, roughly speaking, the closeness centrality measures how easy it is to reach the node in the graph. In passing network terminology, it measures how well connected a player is in the team. In the Spain team, Busquets and Xavi have the highest scores. In fact, if we compare their scores with the rest of the players in both teams, we see that both are better connected than the best connected Dutch player, player number 1 Steckelenberg, the goal keeper. From the data, we see that the goal keeper is the Netherland best connected player itself is quite revealing.

Betweenness centrality is another important indicator in analyzing networks. In graph theory, this measures the extent to which a node lies on a path to other nodes. If we want to interpret this measure in football passing network, betweenness centrality measures how the ball flow between players depends on another player. So, by this definition, we can conclude that if some players have a high betweenness centrality score, then they have a significant impact on the structure of the network and in fact such players are fundamental keys for keeping the momentum of the game going. In fact, these players are playing important roles in the pitch, because removing them from the network can change the team from high performance to low performance. Hence, if a team has a single player with a high betweenness centrality, it would be a weakness, since it is most likely such unique player receives a red card or is injured during entire match. In the Spain team, player number 11 Joan Capdevilla is the one with the highest betweenness centrality in this match. Again, by looking at the figure, one can realize that many players pass the ball to Capdevilla who feeds mainly to player number 14 Xabi Alonso.

The other important measures in a social network is PageRank. Roughly speaking, this invariant measures popularity of a player in terms of the number of passes he receives from other popular players. Using data from FIFA 2010, it shows Xavi has highest PageRank in this match. It means that after a reasonable large number of passes among all players in both teams, Xavi is most likely to end up with the ball.

2.3.3 Limitations of Passing Network

As we saw in this section, many studies have been done for analyzing passing network and all of them are based on static network. As such a network has static nature, some data is missed, is not available or is not accurate enough in order to get high proficiency measures of all invariants of the network. In addition, the position of nodes and the structure of the network is built based on the a-priori ideal formation, which is not preserved during the match. In this direction, in order to get more accurate measures from the network, there are some suggestion such as counting the number of shots toward opponents goal by adding an extra node for each team. Furthermore, using a similar approach to measure the accuracy of passes by taking into account the probability of pass from one player to another being successful, see [39, 41] for more details.

2.4 Conclusions

In this chapter we reviewed the few available studies on football passing networks done so far. All these studies are based on a static representation of the match. In the subsequent Chapters we will indicate the parameters and measure that we intend to study and we will take a more dynamic approach by sliding the time of a match into windows of different sizes and performing a more accurate time-dependent study.

Chapter 3

Social Network Analysis

Analyzing social networks relies on concepts of graph theory. In order to study a network and its measures, one needs to know fundamental topics in graph theory. In this chapter, some significant measures of social networks will be presented. We first provide some fundamental terminologies and definitions in graph theory based on [18, 44] which will be used frequently throughout the thesis. Some examples of graphs in social networks are illustrated.

3.1 Some Graph Concepts

In the following section, the main terminology and the basic definitions are given for directed and undirected graphs.

3.1.1 Simple and directed graphs

A *simple graph* $G = (V, E)$ consists of two finite sets V and E of vertices (nodes) and edges, respectively, where E is set of 2-element subsets of V. So, each edge is a 2-subset $e = \{u, v\}$ of vertices joining one vertex to another. The vertices u and v are called *endpoints* of edge e. In a simple graph $G = (V, E)$, if E is a subset of $E \subseteq V \times V$ instead of 2-element subsets of V, then G is called a *directed graph*. In this case, the elements of E are sometimes called *arcs* or *arrows*. An arc $e = (u, v)$ consists of initial endpoint u and *terminal endpoint*. An arc (u, u) is called a *loop* of the graph. There are two significant differences between simple and directed graphs. In simple graphs, there are no loops and edges have no direction. The *degree* of graph G is the number of vertices $|V|$. If G is a directed graph, for a given vertex

u, $\Gamma_{in}(u)$ ($\Gamma_{out}(u)$, resp.) is defined as the set of nodes v in which (v,u) ((u,v), resp.) is an arc (edge)in G. The number $|\Gamma_{in}(u)|$ ($|\Gamma_{out}(u)|$) is called the *in-degree* (*out-degree*, resp.) of u. Note that for simple graphs, $\Gamma_{in}(u) = \Gamma_{out}(u)$ and so $|\Gamma_{in}(u)| = |\Gamma_{out}(u)|$. We denote this unique number by $|\Gamma(u)|$ and we call it the *degree* of u.

One important and fundamental concept in graph theory is the one of *isomorphism*. Let $G_i = (V_i, E_i), i = 1, 2$, be two graphs. A bijection map $f : V_1 \longmapsto V_2$ is called an isomorphism from G_1 to G_2 if it satisfies the property: any two vertices u and v are adjacent in G_1 if and only if $f(u)$ and $f(v)$ are adjacent in G_2. If there is an isomorphism between G_1 and G_2, then we say that G_1 is *isomorphic to* G_2, denoted by $G_1 \simeq G_2$.

All measure measures in two graphs which are isomorphic would be preserved under an isomorphism. For instance, the degrees of u and $f(u)$ are the same, where f is an isomorphism. In the following section, we will see that the characteristic polynomials of two isomorphic graphs are the same. This is one significant observation from algebraic graph theory point of view which enable us to explore the indicators of networks.

3.1.2 Adjacency matrix and characteristic polynomial of graphs

For a given graph $G = (V, G)$, the *adjacency matrix* $A = A(G)$ of G is defined as a square matrix $n \times n$, where n is the number of vertices of G, with (u, v)-entry equal to 1 if the vertex u is adjacent to v, and equal to 0 otherwise. Note that if G and G' are isomorphic, then the connection between their adjacency matrices are as follows: there is an $n \times n$ permutation matrix P such that

$$P^{-1}A(G)P = A(G').$$

Note that if we apply a trace function on both sides of the above equality, then $\mathrm{tr}A(G) = \mathrm{tr}A(G')$. The characteristic polynomial $\chi_G(x)$ of G is defined by:

$$\chi_G(x) = \det(xI - A(G))$$

A *walk* in a graph is an alternating sequence of vertices and arcs:

$$u_0, e_1, u_1, e_2, u_2, \ldots, e_m, u_m$$

where e_t is the arc (u_{i-1}, u_i). A *path* is a walk with no repeated vertices or edges. The length of the latter walk is m and starts at u_0 and ends at u_m. Now we describe the

relationship between characteristic polynomial and walks in a graph. If $u_0 = u_m$, then the walk is called a *closed* walk. Walks of length zero are allowed, for each vertex u there is only one such walk starting at u. In the following lemma, by using the adjacency matrix of a graph we can find the number of walks between any two given vertices.

Lemma 3.1.1 *[18, 2.1 Lemma] Let G be a graph with adjacency matrix A and let u and v be vertices in G. Then the number of walks in G from u to v with length m is equal to $(A^m)_{uv}$. The number of closed walks of length m in G is equal to $\mathrm{tr}A^m$, where tr is the trace function.*

A simple graph G is *connected* if for every pair of vertices u and v, there exits a path from u to v, otherwise, it is called *disconnected*. A largest connected subgraph of G is called a *connected component* of G. If G is a directed graph, it is called *strongly connected*, if for every pair of vertices u and v, there is a directed path from u to v and a directed path from v to u. The strong connected component of G is a largest strongly directed subgraph of G.

3.1.3 Eigenvectors of graphs

In this section we explain a very useful way of studying eigenvectors of a graph. Suppose that $G = (V, E)$ is a graph and $A = A(G)$ is its adjacency matrix. Let X be an eigenvector of A with eigenvalue λ. Since A is a $(0, 1)$ matrix, the equation $AX = \lambda X$ is equivalent to the following system of linear equations:

$$\lambda x_v = \sum_{u \in \Gamma(v)} x_u, \qquad v \in V \tag{3.1}$$

From the above equations, we can get a useful interpretation for eigenvector as follows: look at X as a vector from V to the real numbers, i.e., $X : V \mapsto \mathbb{R}$, then these equations imply that λ times of the function X at u-th component is nothing but the sum of components of X on the neighbors of u. Conversely, any real value function on vertex set V of G which satisfies equalities 3.1 can be seen to be an eigenvector. In fact, by looking at eigenvectors in this way, we find that such a real map provides a weighted map on the vertices of the graph. Suppose that λ is an eigenvalue of the adjacency matrix A of G with multiplicity k. So, the dimension of the eigenspace W corresponding to λ is equal to k. Now, let B be an $n \times k$ matrix with its columns forming a basis for W. Then it is easy to see that $AB = \lambda B$. This implies that the rows of B give rise to a vector valued function b, on the

vertex set of G such that $\lambda b(u)$ is equal to the sum of the values of b on the neighbors of u. Conversely, if a vector valued function satisfies the latter conditions, it determines an eigenspace of A.

We illustrate the above argument for the cube in $\mathbb{R}^3$, seen Figure 3.1. Each vertex of the graph corresponds to the vector with all entries ± 1. Two vertices are adjacent if and only if the corresponding vectors differ in only one position. As we see, if we sum the vectors adjacent to a given vector X, the result is exactly X. Hence, by looking at the vectors adjacent to $(-1, 1, 1)$, we get

$$(1, 1, 1) + (-1, -1, 1) + (-1, 1, -1) = (-1, 1, 1)$$

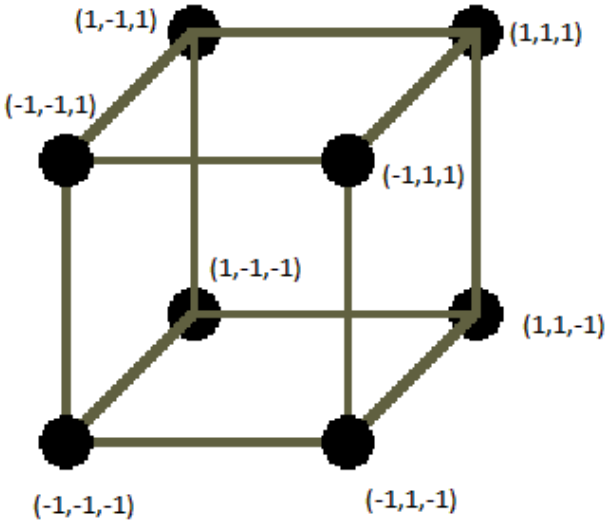


Figure 3.1: The cube in $\mathbb{R}^3$

3.1.4 Union graphs and its characteristic polynomials

Suppose that $G_i = (V_i, E_i), i = 1, 2$ is a graph. The union of G_1 and G_2 is defined as the graph $G = (V, E)$, where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. Just by definition of characteristic polynomial of graph, it is easy to see that

$$\chi_G(x) = \chi_{G_i}(x)\chi_{G_2}(x). \tag{3.2}$$

3.1.5 Perron-Frobenius theorem

As we know, if C is an arbitrary $n \times n$ matrix then its eigenvalues can be complex numbers. The *spectral radius* of C, denoted by $\rho(C)$, is defined by the non-negative real number $\max\{|\lambda| : \lambda \text{ is an eigenvalue}\}$. Suppose that D is a matrix. We write $C \geq D$ where D is a matrix such that $C - D$ exists and is non-negative. For a given arbitrary square matrix C, we may define the *underlying directed graph* corresponding to C as a directed graph whose adjacency matrix is obtained by replacing each nonzero entry of C with 1.

Theorem 3.1.2 *(Perron-Frobenius theorem) Suppose that C is a non-negative $n \times n$ matrix such that its underlying directed graph is strongly connected. Then the following statements hold:*

(a) The spectral radius $\rho(G)$ is a simple, non-zero, eigenvalue of C, and the corresponding eigenvector can be taken to be positive.

(b) Let $\lambda_1, \ldots, \lambda_k$ be all the eigenvalues of C with absolute value equal to λ. Then $k > 1$ if and only if all closed walks in G have length divisible by k. For all i, λ_i/λ is a k-th root of unity.

(c) If D is an $n \times n$ matrix with $|D| \leq C$, then $\rho(D) \leq \rho(C)$, with equality if and only if $D = \pm C$.

We will use the Perron-Frobenius theorem in order to measure a significant indicator in social network, namely eigenvector centrality.

3.2 Social Network Analysis

The study of the relationships between social entities can be done by means of social networks. In the sport world, any football match establishes a social network called *football passing network* in which relationships are passes among players as entities of a team. Social network analysis deals with the structural analysis of networks. For instance, analyzing the football passing network gives us information about how a player involves with the others (local level) or how a team has performed (global level).

We give a brief overview of important measures of networks which will be used in analyzing the passing networks.

3.2.1 Node centralities

One of most important indicators in analyzing network is degree which is used as a first step when studying a network, see [16, 32, 45]. To mathematically describe this indicator, let X be the adjacency matrix of a network $G = (V, E)$ under study and $u \in V$ be a node. Then the degree of u is given as follows:

$$k_u = C_D(u) = \sum_{v \in V} X_{u,v}. \tag{3.3}$$

Note that this is true only for an undirected and binary network, it means that $X_{u,v} = X_{v,u}$, for all $u, v \in V$ and $X_{u,v} = 1$ if u and v are neighbors otherwise $X_{u,v} = 0$.

In weighted network, the degree of a node u has generally been extended to the sum of weights of edges e in which e meets u, and labeled node strength, see [37, 35]. Node strength has been formalized as follows:

$$s_u = C_D^W(u) = \sum_{v \in V} W_{u,v} \tag{3.4}$$

where W is the weighted adjacency matrix of the network. If the network is binary, each edge has weight 1, then this definition is equal to the definition of degree. On the other hand, in weighted networks, the outcomes of two measures given in (3.3) and (3.4) are different. In weighted networks, in order to analyze the network, the node strength has been preferred as a measure since it takes into consideration the weighted edges, see [36, 37]. In fact, node strength is a straight measure as it calculates the node's total degree of involvement in the network and not the number of nodes that connected to it. In Figure 3.2, node **2** and node **3** have the same strength, but node **2** has twice as many neighbors as node **3**. It implies that node **2** is participated in more parts of the networks.

We note that degree and strength are both measures of the level of involvement of a node in the network. So, when analyzing the centrality of a node, it is important to identify both these measures.

In order to combine both degree and strength, we use an adjustment parameter α which indicates the relative significant of the number of edges compared to edge weights. Precisely, a degree centrality measure is proposed by the product of the number of nodes that a focal node is connected to and the average weight to these nodes adjusted by the adjustment parameter, see [36]. The following formula gives a formal definition for a degree centrality measure:

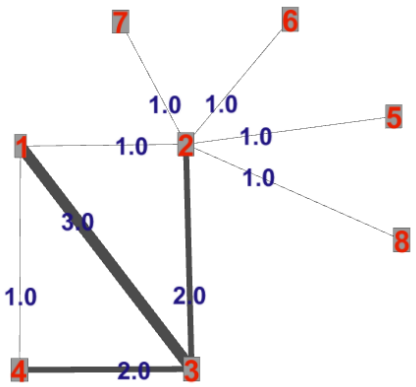


Figure 3.2: A weighted network with 8 nodes. The thickness of edges correspond to their weights

$$C_D^{w\alpha}(i) = k_i \times \left(\frac{s_i}{k_i}\right)^{\alpha} = k_i^{(1-\alpha)} \times s_i^{\alpha} \tag{3.5}$$

where α is a positive real number that can be set according to the setting and the data. If $0 \leq \alpha \leq 1$, then high degree would be preferable , whereas if $1 < \alpha$, a low degree is favourable.

So far, we discussed undirected network, i.e., links without direction. Since, we would analyze directed and weighted network, let us consider the degree centrality measure in a directed and weighted network. In this case, two additional aspects of a node's involvement are added to identify the degree measure. The activity of a node or its social role can be quantified by two measures as follows: the number of edges that are directed towards the node, denoted by k^{in}, is a representative of its popularity, the number of edges that originate from the node, denoted by k^{out}. Since not all edges are not reciprocated, in general k^{out} is not equal to k^{in}. The strength of a node is involved with two measures: s_u^{in} (resp. s_u^{out}) is the total weight attached to incoming edges (resp. outgoing edges). These two measures have the same limitation as s, i.e., the number of edges are not considered. The following two measures are proposed to access a node's activity and popularity, respectively

$$C_{D_{out}}^{w\alpha}(u) = k_u^{out} \times \left(\frac{s_u^{out}}{k_u^{out}}\right)^{\alpha} \tag{3.6}$$

$$C_{D_{in}}^{w\alpha}(u) = k_u^{in} \times \left(\frac{s_u^{in}}{k_u^{in}}\right)^{\alpha} \tag{3.7}$$

The adjustment parameter α is the same as the one in Equation 3.4. If two nodes have the same s^{out} and different k^{out}, the measure would assign a higher score to the node with the

highest k^{out} if $\alpha < 1$, whereas if $\alpha \geq 1$ the measure would assign a highest score to node with lowest k^{out}. For more information we refer the reader to [36].

3.2.2 Closeness and betweenness centralities

The closeness and betweenness measures depend on recognition and length of the shortest paths among nodes in the network. In order to generalize these measures for weighted networks, we need to know how to generalize the shortest distances and their length from binary to weighted networks. In [24, 34, 40, 45, 48] there have been results about the shortest distances among nodes in binary networks. In a binary network, the shortest path between two nodes A and B is defined as the minimum of the length of all path between A and B. Clearly, the length of a path is the number of edges connecting any two consecutive nodes in the path. If there are many intermediary nodes between A and B on a path that connects A to B, the cost of interaction between these two nodes increases. Moreover, the intermediary nodes can slant information or delay interaction between nodes, see [7, 17]. Clearly, in binary networks the shortest path between A and B is the smallest number of intermediary nodes.

There are different aspects of the shortest distances among nodes in a network such as the all-pairs shortest- path and the single-source shortest-path problem. Closeness centrality relies on the single-source shortest-path, i.e., the length of the shortest path from one node to all other nodes, whereas betweenness relies on the identification of the all-pairs shortest-path, i.e., the lengths of shortest paths between all possible source-destinations pairs.

The definition of closeness and betweenness in an undirected and unweighted network is given in [15] as follows, respectively:

$$C_C(u) = \left[\sum_{v \in V} d(u,v)\right]^{-1} \tag{3.8}$$

$$C_B(u) = \sum_{v \neq w \in V \setminus \{u\}} \frac{P_{vw}(u)}{P_{vw}} \tag{3.9}$$

where d is the distance function, so $d(u,v)$ is the length of the shortest path between node u and node v, P_{vw} is the number of shortest paths from v to w and $P_{vw}(u)$ is the number of those paths that go through node u.

It is more complicated if we want to calculate these two measures for weighted networks. For instance, data can be transmitted though a longer path of strong link more quickly, and diseases have higher probability to carry on through a sequence containing individuals through strong links than through a weak direct connection.

This situation is illustrated in Figure 3.3. In this example, we have a weighted network with three paths between two nodes, node **1** and **3** which are connected through three different paths with different number of intermediary nodes with different weights. If our network in Figure 3.3 was binary, then the shortest path would be the direct connection $1-3$. However, in a weighted network, we would like to know if this path is the quickest path for flow. Although path $1, 5, 4, 3$ goes through two intermediary nodes, it could be quicker since the path consists of stronger links.

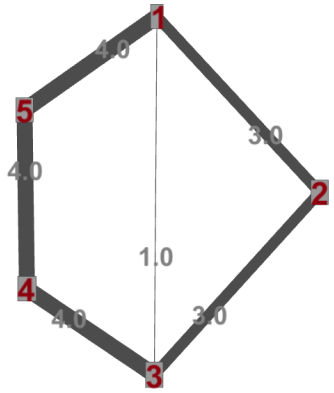


Figure 3.3: A weighted network with three paths between two nodes: **1** and **3**, directly path 13; through one intermediary node 123; through two intermediary nodes 1543. The thickness of edges correspond to their weights

Finding shortest paths in weighted networks has been studied in [10, 24, 40, 48]. For instance, in [10] the author proposed an algorithm that finds the shortest path for networks in which the weights are identified as cost of transmitting, e.g., time to route Internet traffic or distances in GPS devices. In reality, in most cases weights are considered based on edge strength not on their cost, in which case the edge weights must be reversed before directly applying Dijkstra's algorithm to find the shortest paths. In [34] the author has proposed to invert the weights in order to generalize closeness and betweenness centralities. To do so, weights on each edge are considered as costs since large value on each edge represented weak or costly edge, whereas small value on each edge identifies strong or cheap edge. For example, if edge $< A, B >$ has weight 2 and edge $< C, D >$ has weight 1, then the distance between A and B is half of the distance between C and D. If there is no path or edge between two nodes, then the distance would be considered as infinite number. More

precisely, the distance function for a weighted network for implementation of Dijkstra's algorithm is defined as the following:

$$D_w(u,v) = \min\left(\frac{1}{W_{uu_1}} + \frac{1}{W_{u_1u_2}} + \ldots + \frac{1}{W_{u_kv}}\right) \tag{3.10}$$

where minimum is taken over all path $< u, u_1 >< u_1, u_2 > \ldots < u_k, v >$ from u to v. In this model, the number of intermediary nodes are not considered, so the distance between two nodes is not affected by the number of nodes that lie on the path connecting them.

Opsahl and et al., by following ideas from [10, 34], have extended the shortest path algorithm by taking into consideration the number of intermediary nodes. They transform the inverted weights by a similar adjustment parameter used in the formula for degree measure. Equation 3.5 is used to find the least costly path before applying Dijkstra's algorithm. Using an adjustment parameter grantees that both weight and the number of intermediary nodes affect the determination of shortest paths. So, from the above argument, the length of the shortest path between two nodes is defined as follows:

$$D_{w\alpha}(u,v) = \min\left(\frac{1}{(W_{uu_1})^\alpha} + \frac{1}{(W_{u_1u_2})^\alpha} + \ldots + \frac{1}{(W_{u_kv})^\alpha}\right) \tag{3.11}$$

where α is a positive real number which is called tuning parameter (see [36]).

By combining Equations 3.11 and 3.8 the centrality measure in weighted networks is provided in [36] as follows:

$$C_C^{w\alpha}(u) = \left[\sum_{v\in V} D_{w\alpha}(u,v)\right]^{-1}. \tag{3.12}$$

Furthermore, it is possible to extend betweenness centrality by taking advantage of the defined shortest path algorithm. To do so, by combining the weights on edges and the number of intermediary nodes, betweenness centrality is given as follows:

$$C_B^{w\alpha}(u) = \sum_{v\neq w\in V\setminus\{u\}} \frac{P_{vw}^{w\alpha}(u)}{P_{vw}^{w\alpha}}. \tag{3.13}$$

Now, we have a generalization of betweenness and closeness centralities from simple networks to weighted networks. Since we would like to analyze directed and weighted networks, we need to generalize the betweenness and closeness centrality to directed and weighted networks. In fact, the calculation of the shortest paths and their length in directed networks can be done in the same way as in undirected networks with a constraint. In order

to find the length of a path from one node to another, we should follow the direction of the corresponding edges. For example, in road networks, the car can move from one direction to reach location B from location A. This implies that the distance from node A to B is not necessarily equal to the distance from node B to node A.

3.2.3 Pagerank

The pagerank centrality is one of the fundamental measures in element-level analysis of a network element, i.e. a node or a link, which indicates how significant is this node in the network. This concept is introduced by Brin and Page in the area of Web search, see 3.4. We can consider the Web as a graph whose vertex set is the set of web pages and the edges are hyperlinks. A fundamental idea of pagerank is that links from an "important" node must weight more than links from nodes with less "important" ones. So, in the following a node stands for a web page (or simply page).

To identify the pagerank measure, we need to have an algorithm to calculate it. So, the pagerank algorithm states that if node B pointed by an important node A, i.e., the node has important link to it, when it points to other node D, this would be an important node, i.e., the link from B to D becomes important. Thus, pagerank considers an incoming hyperlink from one web page to another website (backlinks) and distributes the ranking through links: if a the sum of the ranks of backlinks for node B is high, then B has a high pagerank.

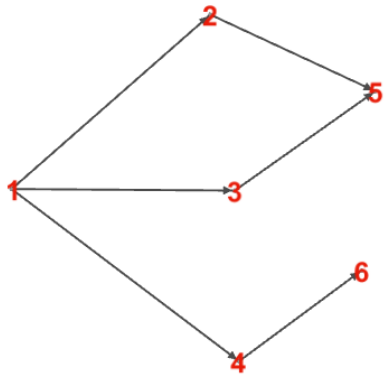


Figure 3.4: Example of backlinks

In Figure 3.4, it shows that node **1** is a blacklink of nodes **2, 3** and **4**; node **2** and **3** are blacklinks of node **5** and finally node **4** is a blacklink of node **6**.

In [38] a slightly simplified version of pagerank is given as follows:

$$\mathtt{Pr}(u) = \alpha \sum_{v \in \mathtt{R_{in}}(u)} \frac{\mathtt{Pr}(v)}{|\mathtt{R_{out}}(v)|} \tag{3.14}$$

where u represents a web page and α is a factor used for a normalization.

From 3.14, we see that the rank score of a page u is obtained recursively based on score of all pages that point to u. In fact, this rank can be calculated iteratively starting from any webpage and it does not depend on any particular page. It might be possible within a webpage to have two or more pages connected to each other to make a loop. If these pages are referred by other pages outside the pages of the loop, and they did not refer to any page outside the loop, they would never propagate any rank but they would accumulate rank. This process is called *rank sink*, see [38] for more information.

The original pagerank is given in 3.14 based on the rank sink problem as follows:

$$\mathtt{Pr}(u) = (1-d) + d \sum_{v \in \mathtt{R_{in}}(u)} \frac{\mathtt{Pr}(v)}{|\mathtt{R_{out}}(v)|} \tag{3.15}$$

where d is a dampening factor and normally is set to 0.85.

Mathematically, we can define the pagerank vector $\mathtt{pr}(\alpha, s)$ of a network G as the unique solution of the following system of linear equations:

$$\mathtt{pr}(\alpha, s) = \alpha s + (1-\alpha)\mathtt{pr}(\alpha, s)W \tag{3.16}$$

where $\alpha \in (0, 1]$ is a jumping constant; $s = \frac{1}{n}\mathbf{1}$ is a starting vector and $W = (w_{ij})$ as follows $w_{ij} = \frac{1}{d_i}$, if node i adjacent node j, otherwise $w_{ij} = 0$.

In [47], pagerank is generalized to weighted networks as follows:

$$\mathtt{Pr}(u) = (1-d) + d \sum_{v \in \mathtt{R_{in}}(u)} \mathtt{Pr}(v) W^{in}_{(v,u)} W^{out}_{(v,u)} \tag{3.17}$$

where $W^{in}_{(v,u)}$ and $W^{out}_{(v,u)}$ are the popularity from the number of in-links and out-links, respectively. More precisely, $W^{in}_{(v,u)}$ is the number of in-links of page u out of the number of in-links of all reference pages of page v:

$$W^{in}_{(v,u)} = \frac{|\mathtt{R_{in}(u)}|}{\sum_{p \in R(v)} |\mathtt{R_{in}(p)}|}$$

where $R(v)$ denotes the reference page list of page v. Similarly, $W^{out}_{(v,u)}$ is the number of out-links of page u out of the number of out-links of all reference pages of page v:

$$W^{out}_{(v,u)} = \frac{|\mathtt{R_{out}(u)}|}{\sum_{p \in R(v)} |\mathtt{R_{out}(p)}|}$$

Interpretation of Pagerank in Football Network

As we mentioned in previous section, pagerank centrality is a recursive notion of popularity when a node is referred by other popular node in the network. Following 3.16, we extract an exact formula for pagerank centrality x_i of each node i as follows: Let $\mathtt{pr}(\alpha, s) = (x_1, x_2, \ldots, x_n)$ be the pagerank vector associated with search ranking vector $s = \frac{1}{n}\mathbf{1}$. From 3.16 we have

$$\mathtt{pr}(\alpha, s) = (x_1, x_2, \ldots, x_n) \quad = \quad \alpha \frac{1}{n}\mathbf{1} + (1-\alpha)(x_1, x_2, \ldots, x_n)W.$$

This implies that

$$x_j = \frac{\alpha}{n} + (1-\alpha) \sum_{t:\ t \to j} \frac{1}{\mathtt{d_{out}}(t)} x_t \tag{3.18}$$

where $t \to j$ means that there is a directed link from t to j.

From 3.18, we see that PageRank is an eigenvector algorithm which assigns scores to each node. The score for a given node may be considered as the fraction of time spent "visiting" that node (measured over all time) in a random walk over the nodes (following outgoing edges from each node). Pagerank modifies this random walk by adding to the model a probability, indicating by a parameter α in 3.18 of jumping to any node. From 3.18, it is easy to see that if $\alpha = 0$, this is equivalent to the eigenvector centrality algorithm. On the other hand, if $\alpha = 1$, all vertices will receive the same score $1/|V|$. Therefore, parameter α acts as a sort of score smoothing parameter.

Let $\alpha = 1 - p$ and $q = \frac{1-p}{n}$, then from 3.18 we conclude the following formula x_j for each node j

$$x_j = q + p \sum_{t:\ t \to j} \frac{1}{\mathtt{d_{out}}(t)} x_t. \tag{3.19}$$

In the case of Football passing networks, the interpretation of pagerank formula 3.19 is as follows: $\mathtt{d_{out}}(t)$ is the total number of passes made by player t; p is heuristic parameter indicating the probability that a player will pass the ball away rather than keep it and go for a shot by himself. Finally, q is a parameter proportional to the probability that

a player will shoot the ball away rather than pass it to other player. According to the recursive formula given in 3.19, the pagerank score of a player depends on the scores of all his teammates. This implies that all pagerank scores in a team should be calculated simultaneously.

Roughly speaking, the pagerank centrality assigns to each player the probability that he will get the ball after a reasonable number of passes have been made by all players in the team. The value of probability p is not created by the team alone, as it can be different from one team to another, and that is why p must be determined by heuristics. As a proof of concept, in our analysis we might apply a uniform value $p = 0.6$, then $q = \frac{1-0.6}{11} = 0.036$. for all the teams studied.

3.2.4 Eigenvector centrality

There is another centrality measure in networks, proposed by Peter Gould [19], that is more sophisticated than the degree centrality, called eigenvector centrality. It is a measure of the influence of a node in a network. Roughly speaking, the eigenvector centrality of a node is the sum of its connections to other node in network, weighted by their centrality.

By using the adjacency matrix of a network, we can identify eigenvector centrality of all nodes in the network. Suppose that $G = (V, E)$ is a graph and $A = (a_{uv})$ is its adjacency matrix, so $a_{uv} = 1$ if u is a neighbor of v, otherwise $a_{uv} = 0$. Define a new matrix $B = A+I$, simply by replacing the diagonal zeros with ones. This has the effect of giving an eigenvalue λ of B, then $\lambda - 1$ would be an eigenvalue of A with the same corresponding eigenvector. Since A is a symmetric matrix, B is so and it can be diagonalized by an orthogonal matrix. This implies that the eigenvalues of B are real and we can select the largest one. Now we check the requirement of Perren-Frobenius Theorem 3.1.2. We remind the reader that a matrix $D = (d_(u, v)$ us is called non-negative if $d_{(u,v)} \geq 0$ for all u, v. A non-negative matrix D is called primitive if there exits an integer $N > 0$ such that every entry in D^N is strictly positive.

To check the hypothesis of Perron-Frobenius Theorem 3.1.2, we calculate B^k, where k is a positive integer number. Let us see what is the (u, v) entry of matrix B^k. In fact, $(B^k)_{uv}$ is the number of ways traversing from node u to node v by walks of length k including stopover. Since the diameter $\texttt{diam}(G)$ of a connected graph is the smallest integer k such that any two nodes u, v can be connected by a walk no longer than k, if we chose $k \geq \texttt{diam}(G)$ then B^k would be positive. Therefore, B is primitive and all requirements in 3.1.2 are satisfied.

The above argument ensures that for a connected graph, we can find a principle vector v_0 whose entries are all positive. Let $\lambda_0, \lambda_1, \ldots, \lambda_n$ be all eigenvalues of B with eigenvectors $\mathbf{v_0}, \mathbf{v_1}, \ldots, \mathbf{v_n}$, respectively. Take the following linear combination of eigenvectors which is clearly not orthogonal to $\mathbf{v_0}$:

$$\mathbf{v} = \alpha_0\mathbf{v_0} + \alpha_1\mathbf{v_1} + \cdots + \alpha_n\mathbf{v_n}, \quad (\alpha_0 \neq 0).$$

Since each λ_i is an eigenvalue with corresponding eigenvector $\mathbf{v_i}$, we get

$$B^k\mathbf{v} = \lambda_0^k\alpha_0\mathbf{v_0} + \lambda_1^k\alpha_1\mathbf{v_1} + \cdots + \lambda^n\alpha_n\mathbf{v_n}.$$

This implies that

$$\frac{B^k\mathbf{v}}{\lambda_0^k} = \alpha_0\mathbf{v_0} + \frac{\lambda_1^k\alpha_1\mathbf{v_1}}{\lambda_0^k} + \cdots + \frac{\lambda_n^k\alpha_n\mathbf{v_n}}{\lambda_0^k}.$$

By taking limit from both sides of the above equation, when $k \to \infty$, we get $\frac{B^k\mathbf{v}}{\lambda_0^k} \to \alpha_0\mathbf{v_0}$, it holds because $\lambda_0 > \lambda_i, i \neq 0$. We conclude that when k is increasing, the ratio of all components of $B^k\mathbf{v}$ tends to corresponding components of $\mathbf{v_0}$.

By the above argument, entry v in row u determines all k length walks from node u to node v. It shows that the row sum is the total number of k-length paths to all other nodes from node u. If $\mathbf{v} = \mathbf{1}$, the vector with entries are 1, the row sums of B^k are exactly equal to $B^k\mathbf{v}$. As we saw in the above, counting longer and longer paths would lead us to bring $B^k\mathbf{v}$ to the same ratio as eigenvector centrality.

Therefore, in oder to formalize the eigenvector centrality of nodes we use the above discussion and we can define the centrality score as an eigenvector $\mathbf{v}$ corresponding to the largest eigenvalue λ as follows: $A\mathbf{v} = \lambda\mathbf{v}$. We can find exact value of scores for every node. Let $\mathbf{X} = (x_v)_{v\in V}$ be the eigenvector corresponding to the largest eigenvalue λ mentioned above. Then the solution of the system of linear equation $A\mathbf{X} = \lambda\mathbf{X}$ is as follows:

$$x_v = \frac{1}{\lambda}\sum_{u\in R(v)} x_u = \frac{1}{\lambda}\sum_{u\in V} A_{vu}x_u. \tag{3.20}$$

In equation 3.20, λ is the largest eigenvalue and all entries of X are positive. The u^{th} component of the related eigenvector gives the centrality score of the node u in the network.

In the above we explained how to achieve the centrality scores by power iteration. This process is one of many eigenvalue algorithms that can be used to obtain this dominant scores.

3.2.5 Clustering coefficient

To analyze a social network there is another measure called *clustering coefficient*. In undirected networks, it is a measure of the number of triangles in a graph. In [46], the concept of clustering coefficient have been proposed for unweighted and undirected networks and Fagiolo in [11] has generalized it for binary directed and also weighted directed networks.

Let $G = (V, E)$ represents a weighted and directed graph. Let A be adjacency matrix of G whose entries are 0 and 1 and let W be adjacency matrix of G whose entries represent weights on edges. Define $d_u^{\mathtt{in}} = |\mathtt{R}_{in}(u)|$, $d_u^{\mathtt{out}} = |\mathtt{R}_{out}(u)|$ as in-degree of node u and out-degree of node u. Let $d_u = d_u^{\mathtt{in}} + d_u^{\mathtt{out}}$ be the total degree of node u and let

$$d^{\leftrightarrow} = \sum_{u \neq v \in V} A_{uv} A_{vu}.$$

If $W = A$, i.e., G is a binary network, then the clustering coefficient of node u is defined as the ratio between all the possible triangles formed by node u and the total number of triangles that could be formed:

$$C_u^D = \frac{(A + A^T)^3_{uu}}{2[d_u(d_u - 1) - 2d^{\leftrightarrow}_u]}. \tag{3.21}$$

The above formula can be easily extended to the wighted directed graph as follow:

$$C_u^W = \frac{(\hat{W} + \hat{W}^T)^3_{uu}}{2[d_u(d_u - 1) - 2d^{\leftrightarrow}]} \tag{3.22}$$

where $\hat{W} = (w_{uv}^{\frac{1}{3}})$. In [11] it is mentioned that the above definitions 3.21 and 3.22 are not characterizing the richness of patterns that take place in a complex directed network. The reason is that Equations 3.21 and 3.22 consider all possible triangles without their directions. However, in directed graph we should consider the direction edges in underline triangles. So, in order to remove this problem, four more definitions are treated as shown in the following:

Case (a): cycle, consider a directed cycle started at node A. In this case, the associated clustering coefficient is given as follows:

$$C_u^{cyc} = \frac{(\hat{W})^3_{uu}}{d_u^{\mathtt{in}} d_u^{\mathtt{out}} - d^{\leftrightarrow}_u}. \tag{3.23}$$

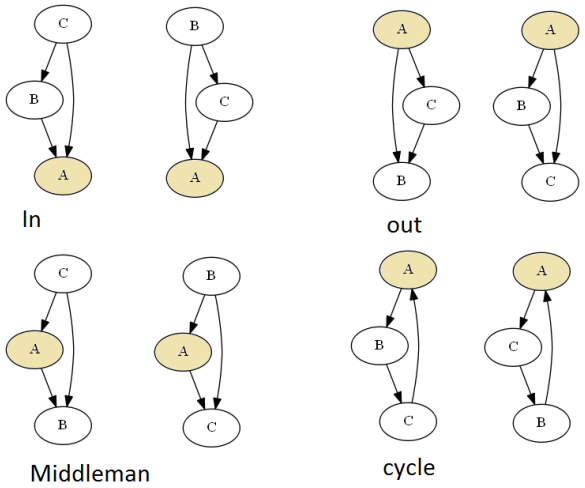


Figure 3.5: classification of the triangles based on directed networks

Case (b): Middleman, one of the adjacent node of A has out-degree of 2 and other has in-degree of 2. In this case, the associated clustering coefficient is given by the following:

$$C_u^{mid} = \frac{(\hat{W}\hat{W}^T\hat{W})_{uu}}{d_u^{\mathtt{in}} d_u^{\mathtt{out}} - d_{\leftrightarrows}}. \tag{3.24}$$

Case (c): node A has in-degree of 2. The clustering coefficient is given by the following:

$$C_u^{in} = \frac{(\hat{W}^T\hat{W}^2)_{uu}}{d_u^{\mathtt{in}}(d_u^{\mathtt{in}} - 1)}. \tag{3.25}$$

Case (d): out, the out-degree of node A is equal to 2. The clustering coefficient is as follows:

$$C_u^{out} = \frac{(\hat{W}^2\hat{W}^T)_{uu}}{d_u^{\mathtt{out}}(d_u^{\mathtt{out}} - 1)}. \tag{3.26}$$

3.3 Conclusions

In this Chapter we discussed all the measures that we intend to use for the analysis of a specific dynamic passing network corresponding to a World cup match between the Netherlands and Spain: degree centrality, closeness, betweenness, page rank, and clustering coefficient.

Chapter 4

Time-Varying Graphs

A vast majority of real-world networks are dynamic (or time-varying or evolving) as opposed to static. In the past few years, intensive research effort has been devoted to study dynamic graphs. Since the study of dynamic graphs is a relatively new research area, the concepts and definitions are not well-developed yet, and the same concept, sometimes, is referred to by different terms. For instance, there is not even a common terminology on naming the concept of graphs that change over time. For example, Kempe et al. [25] uses the term temporal network, while Leskovec et al. [29] refer to them as graphs over time; Flocchini et al. [13] employ the term time-varying graphs (TVG). In this Section we introduce the notion of time-varying graphs and the main definitions around the concept of graphs that change over time.

4.1 Definitions

The need for dynamics-related concepts emerged from a range of very different investigations. In fact, dynamic networks play an important and central role in a variety of research areas. For instance, delay-tolerant networks (DTNs), highly-dynamic, infrastructure-less networks whose essential characteristic is a possible absence of an end-to-end communication route at any instant, are an exact mapping for dynamic networks [22, 21, 31, 42, 43, 49]. While DTNs mostly focused on fixed network objects, researches on moving network objects continued with studying opportunistic-mobility networks [50, 1, 6, 9]. Dynamic networks have also been an interest in complex systems [28, 26]. Our focus is the application of TVGs in dynamic football passing networks.

Time varying graphs have been defined in [8] and they constitute a general framework

to describe dynamic networks. We give their formal definition below.

Definition Let $\mathcal{G}$ be a system consisting of a set of units V and set of pairs E of units interacts with each other, over a time interval $\mathbf{T}$, called lifetime of system $\mathcal{G}$. If the system $\mathcal{G}$ under study runs over discrete (continuous, resp.) time, then $\mathbf{T}$ is s subset of $\mathbb{N}$ ($\mathbb{R}^+$, resp.). Then the quintuple $\mathcal{G} = (V, E, \mathbf{T}, \rho, \zeta)$ is called *time varying graph*, abbreviated by TVG, where $E \subseteq V \times V$ is the set of edges in such a way that $(u, v) \in E$ if and only if u has at least one interaction with v at time $t \in \mathbf{T}$, and

$$\rho : E \times \mathbf{T} \longrightarrow \{0, 1\}$$

is a map indicating whether a given edge exists at a given time; the latency function $\zeta : E \times \mathbf{T} \longrightarrow \mathbf{T}$ indicates the time it takes to cross a given edge if starting at a given date.

In this thesis, we assume that the temporal domain $\mathbf{T}$ is discrete (the unit of time is given by a second) and finite. It means that we focus on systems with finite lifetime, namely $\mathbf{T} = [t_0, t_k]$, where $t_0 < t_k$ are positive real numbers. When the interaction between nodes are directed, it means that our binary relation E on V gives us ordered pairs (u, v); pair (u, v) indicates an interaction from node u to node v.

Example Let us provide several examples of both directed and undirected TVGs as follows:

- Friendship network. In social networks, the friendship relation between people generates a time varying graph in which the underlying graph is simple. In this network, in the most general sense of the term, the set of nodes is a group of people, and edges are defined by social interactions or ties of some kind. This is an example of undirected TVG.

- Transportation networks. In this system nodes are cities, and directed edges are, for example, flights whose departure dates are given by punctual presences. This is a directed TVG.

- Wireless mobile networks. This is a well known example of communication networks, where an edge is present whenever its two endpoints are within range. This gives us an undirected TVG.

- Scientific networks. In this social network, nodes are researchers in a specific branch of science, and edges are coauthors. Clearly, this network provides an undirected TVG.

The above examples demonstrate aspects of modules over which the TVG definition can extend. As we see, some contents are simpler than others and call for limitations, such as directed vs undirected edges or single vs multiple edges. Additional restrictions might be considered, for instance the latency function could be determined constant over time, over the edges, over both, or actually ignored. In fact, a vast majority of work in social networks does not require such information (e.g., the propagation time of an email is of little interest to the understanding of a community behavior). Since the scope of this thesis is about analyzing a realistic social network, we will intentionally omit the latency function.

In order to analyze the network measures during the lifetime of a time varying graph, two types of indicators are described: *atemporal* and *temporal* ones. Atemporal parameters are defined based on static networks and their evolution over time can be obtained by measuring them over sequences of static graphs, where each of the sequence corresponds to the aggregation of interactions that occur in a given interval of time, any elements of such a sequence is called *footprints* of a TVG. Whereas, temporal indicators are defined on time varying graphs in terms of their temporal nature. In this model, a sequence of non-aggregated time varying graphs, each of which corresponds to a temporal subgraph of the original one for the considered interval, is required for the evolution of temporal measures. In this thesis we use atemporal indicators.

4.2 Football Passing Networks as Evolving Graphs

The notion of evolving graph introduced in [12] is the one that closely fits our description of the dynamic football passing network. Consider the situation, in discrete time systems, where each footprint interval corresponds to the unit of time, or to the time between any two consecutive modification of the graph. In these cases, every footprint corresponds to an instant snapshot of the network, and the sequence corresponds exactly to the evolving graph model. In our setting, instead of considering a new snapshot for each network change, we construct sequences of footprints by aggregating the time information in intervals of time. The extreme situation would be when a single interval is considered, covering the whole lifetime of the system, resulting in a single static footprint which is the aggregation

of all interactions over the network lifetime. More precisely, we will use the following, slightly changed, definition of evolving graphs.

Definition Let $\mathbf{T} = [t_0, t_k]$ be a time interval, where $t_0 < t_k$. Partition $\mathbf{T}$ into subintervals $[t_0, t_1), [t_1, t_2), \ldots, [t_{k-2}, t_{k-1}), [t_{k-1}, t_k]$. For each $i = 0, 1, \ldots, m$, define atemporal time varying graph $G_i = (V_i, E_i)$ where the set of vertices and edges E_i are considered during time subinterval $[t_i, t_{i+1})$. Then the triple $\mathcal{E} = (G, S_G, S_T)$ is called an evolving graph with respect to $\mathbf{T}$, where S_T is the sequence $t_0, t_1, \ldots, t_k$ of time instance, S_G is the sequence $G_1, G_2, \ldots, G_k$ of the graphs G_i and $G = \bigcup_{0=1}^{k-1} G_i$. Denote E be the edge set of G.

For a given time varying graph, when considering a sequence of footprints SF that correspond to a sequence of time intervals, we have a sequence of static graphs, and any classical network measure, such as pagerank, centrality, betweenness, etc. can be directly applied to each. When we look at the evolution of an measure over a sequence SF, we might capture diverse values of granularity by modifying the size of the footprint intervals. According to measures and applications, different choices of granularity might be better to capture a significant behavior.

Suppose that there is a football match between two teams $\mathcal{A}$ and $\mathcal{B}$. For teams $\mathcal{A}$ and $\mathcal{B}$, we define football passing network $G(\mathcal{A})$ and $G(\mathcal{B})$, respectively as follows: for team $\mathcal{A}$ network $G(\mathcal{A})$ is a graph in which the set of vertices is all 11 players from team $\mathcal{A}$ and the set of edges is the set of links between any two players of team $\mathcal{A}$. Here, a link between player A to player B means that player A passes the ball to player B. This graph is a directed weighted graph, with the weight being the number of passes from player A to player B. Moreover, each edge is labeled with the time when the corresponding passes occurred.

As we know, the duration of a football game is 90 minutes (5400 seconds), which is divided into two 45-minute (2700 seconds) halves. Each half can be extended as decided by the referee. This extension of the game length is called injury period or stoppage time. We assume that a match between two teams over time interval $[0, 2700]$ for the first half time. Note that number 2700 is taken as 45 minutes in terms of seconds and can be extended to 3000, for instance if there is 5 minutes extra time. Let us define the football passing network of team $\mathcal{A}$ as the following:

Definition Let $\mathbf{T} = [0, t]$, where t is the match duration. The football passing network $G(\mathcal{A})$ of team $\mathcal{A}$ is an evolving graph $\mathcal{E} = (G, S_G, S_T)$ with respect to $\mathbf{T}$, where S_T is the sequence $0, t_1, \ldots, t_k = t$ of time instance, S_G is the sequence $G_1, G_2, \ldots, G_k$ of directed

graph G_i with a weight attached to each edge in which the set of vertices is represented as a subset of players among all of 18 players and there is an edge from A to B with weight w if w number of passes has been made from player A to player B and $G = \bigcup_{t=1}^{k} G_i$.

In the above definition, each component G_i is called *aggregated football passing network* over time interval $[t_i, t_{i-1}]$.

Remark Suppose that we would like to study the evolution of a social network over a time interval $\mathbf{T} = [t_0, t_k]$. Depending on the topology of social network under study, we partition $\mathbf{T}$ into subintervals $[t_0, t_1), [t_1, t_2), \ldots, [t_{k-2}, t_{k-1}), [t_{k-1}, t_k]$. Now, the aggregated graph $\mathcal{A}(G)$ over total time interval $[t_0, t_k]$ is exactly the evolving graph $\mathcal{E} = (G, S_G, S_T)$.

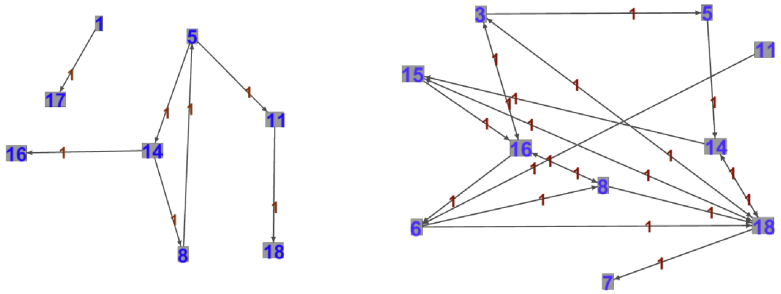


Figure 4.1: letf: Aggregated football passing graph over time interval [0,88]; right: Aggregated football passing graph over time interval [88,176]

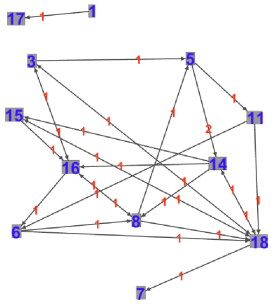


Figure 4.2: Aggregated football passing graph over time interval [0,176]

Figures 4.1 and 4.1 illustrate the above remark showing examples of aggregated football passing networks corresponding to a match. In figure 4.1 there are two graphs: on the left, let G_0 is the aggregated football passing network over time interval $[0, 88)$ and on the right, G_1 is the aggregated football passing network over $[88, 176)$. Moreover, in Figure 4.2, we can see the aggregated football passing network G over interval $[0, 176)$.

Chapter 5

Models for analyzing dynamic passing network

In this section, we introduce the two models on which we base our study of a dynamic football passing network: the Aggregated Model, and the Non-Overlapping Sliding Windows Model.

5.1 The Data

In the next Chapter we will analyze the measures described in Chapter 3 to study the structure of a dynamic football passing network that has been extracted from the final match between the Netherlands and Spain during World Cup 2010. The 2010 FIFA World Cup Final was a football match that took place in Johannesburg, South Africa, to determine the winner of the 2010 FIFA World Cup. Spain defeated the Netherlands 1-0 with a goal from Andres Iniesta four minutes from the end of extra time. The data correspond to the passes among players in the first half of the game, and it is tagged by the time when each pass occurred. More precisely, for each team, we created a table with three columns called **From, To** and **Time**. Let N, M and t be numbers corresponding to **From**, **To** and **Time** respectively; this means that player number N passed the ball to player number M at time t, the time is in terms of seconds. The data has been collected based on the first half time period of the match plus extra time considered by the referee, for a total of 2772 seconds.

5.2 The Aggregated Model

By nature, a football passing network is dynamic, both in terms of passes and in terms of players involved in them throughout the match. Although the number of players in the football team is fixed (11), this number and the interactions between the players change over time, i.e., some players may appear in a specific time period and some of them may not appear in that period. In such a dynamic setting, an aggregation of all data would lead to a loss of all temporal information. An intuitive way to capture the dynamics of football passing network is to aggregate edge weights, which represent the number of passes between two players, only over a certain time period and extend this period to develop a discrete notion from aggregated data. In our network where players interact through passing the ball, the aggregation could be to sum of the number of passes over a specific time period. So, the network is growing over time in terms of number of players and number of passes between them. We expect that for each football team, the network under study tends to a complete graph after a certain time. With this approach, the aggregation of the passes over time would influence the old players in the network over players who just joint to this subgraph.

Following this idea the aggregated model proceed as follows. The total time span of our data is $T = 2772$ seconds. This time is covered in an aggregated way by using windows of different sizes: 198, 308, 924, 1386, 2772. The "static" case correspond to a single window of size 2772; for window size 1386 we obtain 2 periods of time [0, 1386] and [0,2772]; for window size 924 we obtain 3 periods of time [0, 924], [0,1848], and [0, 2772] and so on for smaller windows.

Windows Size (second)	Windows Number
198	14
308	9
924	3
1386	2
2772	1

Table 5.1: Aggregating Periods Model

We construct a directed weighted graph for each team, G for Spain and H for the Netherlands respectively, by associating nodes to players with arcs between nodes representing passes between them. Arcs are weighted by the number of passes in the time

period and labeled by the times when those passes have occurred. We create two aggregated graphs G_i and H_i for each period $[0, i \times t]$, where t is the window size, thus obtaining a sequence of graphs for each team in correspondence of each window size.

Note that aggregating windows can create anomalous situations where a time period might contain more than 11 players. This is the case when a player is substituted. Consider, for example, the situation of player number 17, who is replaced by player 20, 40 seconds after the start of the game; his inactive presence is recorded throughout the game and impacts the network measures.

5.3 The Non-Overlapping Sliding Windows Model

In the aggregated model, while we are analyzing the football passing network there is a drawback that we cannot prevent: the prevalence of players who have performed many passes at the beginning of the match over players who have a chance to pass the ball to other players only later during of the match. This can happen, for example, when a new player appears in the network towards the end of a window under consideration. If we look at this new player who was added to the network at the last moment, all his scores will be quite low in comparison with the other players. In order to reduce this disadvantage, we consider another approach called *sliding windows*. In this model, we partition the viewing period in equidistant time windows, each window starting when the last window finished.

Windows Size (second)	Windows Number
99	28
396	7
1386	2

Table 5.2: Non-Overlapping Sliding Windows Model

In our analysis, we consider 4 different window sizes as is illustrated in Table 5.2: 2772, 1386, 396, 99 corresponding to a single time period of the total time of the match (2772 sec.), 2 consecutive periods of 1386 sec. each, 7 consecutive periods of 396 sec. each, 28 consecutive periods of 99 sec. each.

After specifying the interval for each time window, we create a directed weighted graph G_t based on the observations made during that period, where t shows the time interval over which the information is captured. For example, for window size = 99 there are 28 windows. According to the first window, two aggregated graphs G_0 for Spain and H_0 for

Spain over time interval $[0, 99]$ are created. For the second window, an aggregated graph G_1 for Spain and H_1 for the Netherlands are established over time interval $[99, 198]$. For the third window, two aggregated graph G_2 and H_2 are built over time interval $[198, 396]$, and so on. Finally, for the last window, aggregated graphs G_{27} and H_{27} are built over time interval $[2673, 2771]$.

5.4 The Study

In the next chapter, we focus on degree, closeness, betweenness, pagerank, eigenvector centrality and clustering coefficient in two models described above. More precisely, we will compute these measures for each player and for each set of time windows corresponding to the various different ways of dividing the overall time of play. For example, in the non-overlapping windows model with window size 396, we have 7 windows corresponding to 7 consecutive periods and 7 passing networks. For each player, we compute the six measures indicated above for each of 7 time periods and we average them. We then obtain a table for each team indicating the average score of each player during the match for that particular choice of window size. Finally, the measures of a single snapshot network is also computed and compared with the average scores obtained over window times.

The data has been collected based on a close inspection of the match. The data has then been adapted to be imported into Gephi [2] a tool for the analysis of social networks that has a pre-defined set of network parameters already available. The various parameters investigated in the thesis have been computed using this tool.

5.5 Conclusions

In this chapter, we defined the two models on which we base our study. In the next two chapters we perform the analysis on the aggregated and on the non-overlapping window model, respectively.

Chapter 6

Dynamic Passing Networks in the Aggregated Model

In this chapter, we analyze some measures for dynamic passing networks, such as degree centrality, betweenness, closeness, pagerank, eigenvector centrality and finally clustering coefficient in the aggregated model.

We remind that in the aggregated model, the overall time is divided in growing windows of different sizes: 198, 308, 924, 1386, 2772. The "static" case correspond to a single window of size 2772; for window size 1386 we have 2 time intervals: [0, 1386] and [0,2772]; for window size 924 we have 3 time intervals: [0, 924], [0,1848], and [0, 2772], finally, for window size 198, we have 14 growing time intervals.

6.1 Degree centrality in the Aggregated Model

One of the most important scores in networks is degree centrality. This score indicates how many balls a specific player received or passed from or to other players. If a player has a high degree it means that the player has high impact in the team in terms of the number of his neighbors regardless of whether the other players are passing the ball to him (in-degree) or are receiving the ball from him (out-degree). To consider the total number of passes done by each player, we therefore use formulas 3.6 and 3.7 defined in Chapter 3 with adjustment parameter $\alpha = 1$ and with strength equal to the degree.

Table 6.1 and 6.2 contain the result of the experiment for the two teams, each column containing the average degree for a given window size.

Id	Aggregated Model WS = 198 Weighted Degree	WS = 308 Weighted Degree	WS = 924 Weighted Degree	WS = 1386 Weighted Degree
1	8.93	9.22	11.00	13.00
3	22.21	22.89	27.00	29.00
5	26.71	27.44	32.33	35.50
6	22.79	23.33	28.00	30.50
7	8.14	7.22	9.33	10.50
8	36.64	40.67	49.67	52.00
11	18.29	17.33	20.67	23.00
14	34.43	35.56	43.33	45.00
15	18.86	20.22	23.33	25.00
16	27.07	28.89	34.00	37.50
17	3.71	1.00	1.00	1.00
18	29.36	30.00	36.33	38.00

Table 6.1: Average of weighted degrees for window sizes 198, 308, 924 and 1386 for Spain

We notice that the degree of each player is increasing when its windows size is growing. This is to be expected since the number of passes that a player performs grows in time. There are a few exceptions in the Spain team; in fact, the degrees of players number 17 (Arbeloa) and number 7 (Villa) are decreasing while the window size is growing. The reason is that Arbeloa played for just 5 minutes and then was replaced by Villa.

Id	Aggregated Model: WS = 198 Weighted Degree	Aggregated Model WS =308 Weighted Degree	Aggregated Model WS = 924 Weighted Degree	Aggregated Model WS = 1386 Weighted Degree
1	17.57	18.67	21.33	24.00
2	14.86	15.56	18.33	20.50
3	13.50	14.22	16.67	20.00
4	17.43	18.33	21.33	23.00
5	8.00	8.44	9.67	11.00
6	11.79	12.56	14.67	17.50
7	14.50	14.67	16.67	19.00
8	12.29	12.44	14.67	15.50
9	8.50	8.78	10.67	11.00
10	15.86	16.00	18.33	20.00
11	6.00	6.33	7.67	8.50

Table 6.2: Average of weighted degrees for window sizes 198, 308, 924 and 1386 for the Netherlands

An observation that can be made by looking at the values reported in the Tables is that the Spain team has generally highest degree centrality scores, indicating a more active play. In particular, in the Spain team, player number 8 (Xavi) has consistently the highest average degree centrality, for any window size. In fact we have $C_D = 36.64$ for $WS = 198$, $C_D = 40.67$ for $WS = 308$, $C_D = 49.67$ for $WS = 924$, and finally $C_D = 52$ for $WS = 1386$. In the Netherland's team, on the other hand, player number 1 (Stekelenburg) has the highest average degree centrality respectively of 17.57, 18.67, 21.33,24 for the

various window sizes, indicating difficulties in defense.

Player	Average of Weighted Degree	One snapshot Weighted Degree
Casillas(1)	10.54	18
Pique(3)	25.28	40
Puyol(5)	30.50	48
Iniesta(6)	26.15	38
Villa(7)	8.80	15
Xavi(8)	44.74	72
Capdevila(11)	19.82	35
Alonso(14)	39.58	62
Ramos(15)	21.85	31
Busquets(16)	31.87	53
Arbeloa(17)	1.68	1
Pedro(18)	33.42	49

Table 6.3: Average of weighted degrees over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time periods $[0, 2772]$ Spain

Tables 6.4 and 6.3 show the average degree centrality over all window sizes in the first column, and the degree based on one snapshot over the total time period $[0, 2772]$ (i.e., based on a static representation of the network) for the two teams.

Player	Average of Weighted Degree	One Snapshot Weighted Degree
Stekelenburg(1)	20.39	30
Van der Wiel(2)	17.31	27
Heitinga(3)	16.10	27
Mathijsen(4)	20.02	31
Van Bronkhorst(5)	9.28	14
Van Bommel(6)	14.13	24
Kuyt(7)	16.21	22
De Jong(8)	13.72	19
Van Persie(9)	9.74	15
Sneijder(10)	17.55	24
Robben(11)	7.13	13

Table 6.4: Average of weighted degrees over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time periods $[0, 2772]$ for the Netherlands

Although the players with the lowest and highest degree centrality scores are preserved with the aggregated and snapshot methods, some differences can be noticed if we observe the rankings of the two team members. In fact, in the aggregated model we can detect fluctuations in players' degree in time by observing different window sizes, even if they have

the same total degree in the single snapshot model. For example, in the Netherlands team both players 6 and 10 are ranked 4th in the single snapshot analysis, while the aggregated windows method place them in different ranks giving player number 10 a higher score.

6.2 Closeness in the Aggregated Model

Since the dynamic passing network under study is a directed weighted graph, closeness centrality of a player can be calculated by the formula given in 3.12, where the length of a shortest path between two nodes is given by formula 3.11.

The interpretation of closeness centrality in dynamic passing networks can be explained as a direct measurement on how easy it is to reach a particular player within a team. Formula 3.12 shows that a large closeness centrality corresponds to a small average distance, indicating a well-connected player within the team.

	Aggregated Mode WS = 198	WS = 308	WS = 924	WS = 1386
Id	Closeness Centrality	Closeness Centrality	Closeness Centrality	Closeness Centrality
1	0.61	0.60	0.59	0.61
2	0.69	0.69	0.74	0.77
3	0.65	0.64	0.66	0.75
4	0.67	0.68	0.72	0.74
5	0.46	0.47	0.48	0.55
6	0.72	0.73	0.78	0.84
7	0.59	0.59	0.59	0.65
8	0.58	0.58	0.64	0.67
9	0.49	0.51	0.54	0.57
10	0.62	0.63	0.67	0.69
11	0.27	0.30	0.38	0.29

Table 6.5: Average of Closeness centrality over window sizes 198, 308, 924 and 1386 for the Netherlands

By looking at Table 6.5, we see that player number 6 (Van Bommel) in the Netherlands team has highest closeness centrality among all players in each window size. In Table 6.6, we see that player number 16 (Busquets) in the Spain team has highest closeness centrality among all players in each window size.

Similarly to the degree centrality, Tables 6.5 and 6.6 show that closeness centrality is increasing with the increase of the window size, with a few exceptions.

By looking at Table 6.7 and 6.8, we can see that the average closeness centrality of players and the single snapshot present almost the same ranking of players, with a few exceptions.

	Aggregated Model			
	WS = 198	WS = 308	WS = 924	WS = 1386
Id	Closeness Centrality	Closeness Centrality	Closeness Centrality	Closeness Centrality
1	0.73	0.72	0.63	0.69
3	0.74	0.74	0.77	0.77
5	0.70	0.64	0.67	0.69
6	0.68	0.66	0.64	0.69
7	0.48	0.34	0.43	0.46
8	0.69	0.73	0.75	0.74
11	0.62	0.59	0.59	0.62
14	0.76	0.74	0.77	0.76
15	0.67	0.63	0.65	0.65
16	0.75	0.79	0.80	0.85
17	0.05	0.00	0.00	0.00
18	0.71	0.67	0.70	0.71

Table 6.6: Average of Closeness centrality over window sizes 198, 308, 924 and 1386 for Spain

	Average of	One Snapshot
Player	Closeness Centrality	Closeness Centrality
Stekelenburg(1)	0.60	0.63
Van der Wiel(2)	0.72	0.77
Heitinga(3)	0.67	0.83
Mathijsen(4)	0.70	0.77
Van Bronkhorst(5)	0.49	0.63
Van Bommel(6)	0.77	0.91
Kuyt(7)	0.60	0.67
De Jong(8)	0.62	0.71
Van Persie(9)	0.53	0.67
Sneijder(10)	0.65	0.71
Robben(11)	0.31	0.59

Table 6.7: Average of Closeness centrality over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time periods $[0, 2772]$ for the Netherlands

6.3 Betweenness in the Aggregated Model

Betweenness centrality is a very different notion of centrality, which measures the extent to which a node lies on the paths between other nodes in a network. Since our network is a directed weighted graph, this measure has to be calculated by the formula given in 3.13.

Table 6.9 shows that player number 2 (Van der Weil) has the highest betweenness score among all players within each window sizes in the Netherlands team. Therefore, Van der Weil's score is consistent throughout the windows.

The scores of betweenness are evenly distributed among players in the Spain team; when this occurs, then the performance of the team is considered to be at high level:

Player	Average of Closeness Centrality	One snapshot Closeness Centrality
Casillas(1)	0.69	0.73
Pique(3)	0.76	0.85
Puyol(5)	0.68	0.69
Iniesta(6)	0.67	0.73
Villa(7)	0.43	0.5
Xavi(8)	0.73	0.79
Capdevila(11)	0.60	0.73
Alonso(14)	0.75	0.79
Ramos(15)	0.65	0.65
Busquets(16)	0.80	0.92
Arbeloa(17)	0.01	0
Pedro(18)	0.70	0.73

Table 6.8: Average of Closeness centrality over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time periods $[0, 2772]$ Spain

Id	Aggregated Mode WS = 198 Betweenness Centrality	WS = 308 Betweenness Centrality	WS = 924 Betweenness Centrality	WS = 1386 Betweenness Centrality
1	0.12	0.12	0.11	0.05
2	0.12	0.12	0.13	0.08
3	0.08	0.08	0.08	0.08
4	0.09	0.09	0.08	0.03
5	0.02	0.02	0.02	0.02
6	0.09	0.09	0.09	0.09
7	0.08	0.08	0.06	0.08
8	0.08	0.08	0.08	0.06
9	0.04	0.04	0.02	0.02
10	0.11	0.11	0.08	0.08
11	0.01	0.01	0.02	0.02

Table 6.9: Average of Betweenness over window sizes 198, 308, 924 and 1386 for the Netherlands

in fact, having high betweenness scores among some players indicates a high dependence on those players, whereas well distributed low betweenness scores are an indication of a well-balanced performance technique. This result might then indicate that the Spain team applies a better play strategy than the Netherlands. We also notice that player number 1, the goalkeeper (Casillas) has the highest betweenness centrality over all players within the Spain team regardless of the window sizes. This, together with the homogeneity of the values seems to indicate the entire team played well, with a strong role by the goalkeeper.

We notice that if a player's betweenness score vanishes, which is the case of players number 7 and 17 in the Spain team and player 11 in the Netherlands team, then a player is not getting involved in the match and so his removal from the team would not have much impact.

	Aggregated Model			
	WS = 198	WS = 308	WS = 924	WS = 1386
Id	Betweenness Centrality	Betweenness Centrality	Betweenness Centrality	Betweenness Centrality
1	0.88	0.08	0.10	0.10
3	0.32	0.05	0.07	0.04
5	0.47	0.07	0.07	0.07
6	0.51	0.08	0.08	0.08
7	0.03	0.00	0.00	0.00
8	0.22	0.04	0.04	0.02
11	0.25	0.02	0.01	0.02
14	0.35	0.05	0.06	0.05
15	0.40	0.03	0.04	0.03
16	0.47	0.06	0.04	0.06
17	0.00	0.00	0.00	0.00
18	0.26	0.08	0.07	0.07

Table 6.10: Average of betweenness over window sizes 198, 308, 924 and 1386 for Spain

	Average of	One Snapshot
Player	Betweenness Centrality	Betweenness Centrality
Stekelenburg(1)	0.10	0.03
Van der Wiel(2)	0.11	0.06
Heitinga(3)	0.08	0.06
Mathijsen(4)	0.07	0.04
Van Bronkhorst(5)	0.02	0.03
Van Bommel(6)	0.09	0.08
Kuyt(7)	0.08	0.05
De Jong(8)	0.08	0.05
Van Persie(9)	0.03	0.03
Sneijder(10)	0.09	0.05
Robben(11)	0.01	0.03

Table 6.11: Average of Betweenness centrality over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time periods $[0, 2772]$ for the Netherlands

	Average of	One snapshot
Player	Betweenness Centrality	Betweenness Centrality
Casillas(1)	0.29	0.1
Pique(3)	0.12	0.03
Puyol(5)	0.17	0.04
Iniesta(6)	0.19	0.03
Villa(7)	0.01	0
Xavi(8)	0.08	0.02
Capdevila(11)	0.07	0.03
Alonso(14)	0.13	0.03
Ramos(15)	0.12	0.02
Busquets(16)	0.16	0.06
Arbeloa(17)	0.00	0
Pedro(18)	0.12	0.05

Table 6.12: Average of Betweenness centrality over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

A closer look at Tables 6.9 and 6.10 gives us an indication of how betweenness centrality changes over different window sizes. We notice that betweenness centrality is not increasing when window sizes are increasing. In fact, in both tables, there is a fluctuation in betweenness centrality that can be caused by these changes. Table 6.12 shows that the betweenness scores of most players in the single snapshot model is significantly smaller than the average. When a player obtains the betweenness score in the single snapshot network much bigger than in the aggregated model, such a player is getting good score even in small window sizes. Therefore, he can be a good candidate to keep high score in betweenness. For example, player number 1 (Casillas), not only is placed in the first rank in aggregated model, but also the difference of his scores between the two models is very high. On the other hand, he gets first rank in one snapshot model. The same phenomenon occurs for the Netherlands; this is the case, for example, of player number 2 (Van der Wiel) in Table 6.11

Some differences in the players' rankings are noticeable between the static analysis and the one based on aggregating windows. For example, the Spanish player number 8 (Pedro) is ranked 6th by averaging the different size windows, while he is 3rd in the static analysis.

6.4 Pagerank in the Aggregated Model

In passing networks, the pagerank centrality of a player evaluates the probability that the player will get the ball after a reasonable number of passes have been exchanged between players in the team.

Tables 6.13 and 6.14 show that pagerank of each player for the two teams.

From Table 6.13 as we see, player number 1, the goalkeeper (Stekelenburg) has maximum pagerank score for each window sizes among all players in the Netherland team, and from Table 6.14 we observe that two players: number 8 (Xavi) and number 18 (Pedro) have maximum average pagerank scores among all players in the Spain team.

As we see, the average of pagerank for each players within a team over window sizes is very close to the one that takes on one snapshot network and, in this case, we cannot really observe any significant difference.

Aggregated Model WS = 198		WS = 308	WS = 924	WS = 1386
Id	PageRank	PageRank	PageRank	PageRank
1	0.16	0.16	0.14	0.13
2	0.07	0.07	0.08	0.08
3	0.09	0.09	0.09	0.09
4	0.13	0.13	0.13	0.11
5	0.06	0.06	0.07	0.07
6	0.06	0.06	0.07	0.08
7	0.13	0.13	0.12	0.13
8	0.09	0.09	0.09	0.09
9	0.07	0.07	0.06	0.06
10	0.10	0.09	0.09	0.10
11	0.05	0.06	0.06	0.06

Table 6.13: Average of pagerank over window sizes 198, 308, 924 and 1386 for the Netherlands

Aggregated Model WS = 198		WS = 308	WS = 924	WS = 1386
Id	PageRank	PageRank	PageRank	PageRank
1	0.03	0.03	0.03	0.03
3	0.08	0.08	0.08	0.08
5	0.09	0.09	0.09	0.09
6	0.11	0.11	0.12	0.12
7	0.06	0.05	0.05	0.06
8	0.12	0.13	0.14	0.13
11	0.06	0.05	0.05	0.06
14	0.12	0.12	0.12	0.11
15	0.08	0.09	0.08	0.08
16	0.10	0.10	0.09	0.09
17	0.03	0.02	0.02	0.02
18	0.13	0.13	0.13	0.12

Table 6.14: Average of pagerank over window sizes 198, 308, 924 and 1386 for Spain

6.5 Clustering Coefficient in the Aggregated Model

The clustering coefficient gives us the degree to which players in a network tend to cluster to each other. In our analysis we use clustering coefficient of each node (player) given in 3.22. More precisely, for transitivity of the dynamic passing network we count the percentage of all possible triangles consisting of the player. Suppose that player A wants to pass the ball to player B, but since this line is blocked by a competitor he has to pass the ball to C to reach player B. In this case, player A is acting as middle-man. For all other kind of cycles, we are able to compute the clustering coefficient as explained in section 3.2.5.

Player	Average of PageRank	One snapshot PageRank
Stekelenburg(1)	0.15	0.12
Van der Wiel(2)	0.07	0.09
Heitinga(3)	0.09	0.1
Mathijsen(4)	0.12	0.12
Van Bronkhorst(5)	0.07	0.07
Van Bommel(6)	0.06	0.09
Kuyt(7)	0.13	0.11
De Jong(8)	0.09	0.08
Van Persie(9)	0.07	0.06
Sneijder(10)	0.10	0.09
Robben(11)	0.06	0.07

Table 6.15: Average of Pagerank scores over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

Player	Average of PageRank	One snapshot PageRank
Casillas(1)	0.03	0.03
Pique(3)	0.08	0.08
Puyol(5)	0.09	0.09
Iniesta(6)	0.11	0.1
Villa(7)	0.06	0.06
Xavi(8)	0.13	0.13
Capdevila(11)	0.05	0.07
Alonso(14)	0.12	0.11
Ramos(15)	0.09	0.08
Busquets(16)	0.10	0.1
Arbeloa(17)	0.02	0.02
Pedro(18)	0.13	0.13

Table 6.16: Average of Pagerank scores over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

In Table 6.17, we see that the clustering coefficient of most players is increasing with the increase of the window sizes. Players number 6 and number 7 are the only players whose measure is decreasing with the increase of the window size. In order to have a uniform results for all players we need to take average over all window sizes, this would also give

Aggregated Mode WS = 198		WS = 308	WS = 924	WS = 1386
Id	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient
1	0.15	0.15	0.17	0.19
2	0.19	0.19	0.21	0.24
3	0.18	0.19	0.20	0.22
4	0.17	0.17	0.17	0.19
5	0.22	0.22	0.21	0.33
6	0.17	0.16	0.18	0.18
7	0.24	0.23	0.23	0.14
8	0.31	0.33	0.25	0.26
9	0.18	0.18	0.20	0.22
10	0.16	0.16	0.18	0.19
11	0.52	0.54	0.61	0.56

Table 6.17: Average of Clustering coefficient over window sizes 198, 308, 924 and 1386 for the Netherlands

more informative results. By taking average over all window sizes, we see that player number 11 (Robben) has highest score equal to 0.56 among all players in the Netherlands team.

Aggregated Model WS = 198		WS = 308	WS = 924	WS = 1386
Id	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient
1	0.14	0.12	0.14	0.17
3	0.26	0.27	0.29	0.31
5	0.28	0.30	0.32	0.29
6	0.25	0.22	0.23	0.22
7	0.47	0.43	0.68	0.46
8	0.19	0.20	0.19	0.20
11	0.35	0.36	0.39	0.41
14	0.21	0.21	0.20	0.21
15	0.28	0.25	0.27	0.29
16	0.21	0.20	0.23	0.23
17	0.01	0.00	0.00	0.00
18	0.15	0.15	0.16	0.16

Table 6.18: Average of clustering coefficient over window sizes 198, 308, 924 and 1386 for Spain

Table 6.18 shows the clustering coefficient of all players with different window sizes within the Spain team. As we see, player number 7 (Villa) has the highest average clustering coefficient score (0.47). In addition, Villa has maximum value of clustering coefficient in each window size. This means that Villa has a largest balance between the amount of passes involved in the team.

Table 6.19 shows that the maximum value of average clustering coefficient of players in the Netherland team belongs to player number 11 (Robben). From the same table we see that Robben has also maximum score in the single snapshot network.

As for the other parameters studied so far, there is generally not a big difference between

Player	Average of Weighted Clustering Coefficient	One snapshot Weighted Clustering Coefficient
Stekelenburg(1)	0.16	0.25
Van der Wiel(2)	0.21	0.25
Heitinga(3)	0.20	0.18
Mathijsen(4)	0.18	0.2
Van Bronkhorst(5)	0.24	0.33
Van Bommel(6)	0.17	0.21
Kuyt(7)	0.21	0.18
De Jong(8)	0.29	0.3
Van Persie(9)	0.20	0.23
Sneijder(10)	0.18	0.22
Robben(11)	0.56	0.38

Table 6.19: Average of Clustering coefficient over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

Player	Average of Weighted Clustering Coefficient	One snapshot Weighted Clustering Coefficient
Casillas(1)	0.15	0.18
Pique(3)	0.28	0.24
Puyol(5)	0.30	0.31
Iniesta(6)	0.23	0.22
Villa(7)	0.51	0.37
Xavi(8)	0.20	0.2
Capdevila(11)	0.38	0.28
Alonso(14)	0.20	0.21
Ramos(15)	0.27	0.31
Busquets(16)	0.22	0.2
Arbeloa(17)	0.00	0
Pedro(18)	0.16	0.15

Table 6.20: Average of Clustering coefficient over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

the static analysis and the one based on aggregated windows, but some players are placed in rather different ranks. This is the case, for example, of the Netherlands goalkeeper, who is ranked 3rd in the static analysis, while he is ranked last with the aggregated windows method.

6.6 Eigenvector Centrality in the Aggregated Model

The eigenvector centrality evaluates the most central players i.e. those with the smallest "farness" from others, in terms of the global structure of the network. In fact, this score pays less attention to patterns that are more local. As we explained in section 3.2.4, this measure identifies dimensions of the distances among players. The location of each player with respect to each dimension is called an *eigenvalue*, and the collection of such values is called the *eigenvector*. Two highest eigenvector centralities in the Netherlands team are

	Aggregated Mode WS = 198	WS = 308	WS = 924	WS = 1386
Id	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality
1	0.65	0.66	0.67	0.63
2	0.60	0.63	0.75	0.68
3	0.61	0.64	0.70	0.72
4	0.53	0.51	0.52	0.50
5	0.48	0.51	0.54	0.67
6	0.61	0.62	0.67	0.71
7	0.82	0.83	0.87	0.91
8	0.89	0.89	0.91	0.88
9	0.83	0.84	0.86	0.81
10	0.94	0.94	0.96	1.00
11	0.65	0.68	0.70	0.76

Table 6.21: Average of eigenvector centrality over window sizes 198, 308, 924 and 1386 for the Netherlands

player number 10 (Sneijder), and player number 8 (De Jong). This means that Sneijder and Jong have smallest "farness" from others in the team, and their removal from the match would have a great impact on the performance of the team. In the Spain team,

	Aggregated Model WS = 198	WS = 308	WS = 924	WS = 1386
Id	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality
1	0.34	0.34	0.38	0.44
3	0.68	0.70	0.68	0.72
5	0.76	0.77	0.80	0.87
6	0.80	0.80	0.94	0.92
7	0.50	0.48	0.51	0.53
8	0.83	0.90	0.93	0.89
11	0.54	0.49	0.56	0.56
14	0.89	0.90	0.96	0.94
15	0.80	0.89	0.88	0.88
16	0.79	0.81	0.80	0.85
17	0.11	0.05	0.06	0.06
18	0.97	0.99	1.00	1.00

Table 6.22: Average of eigenvector centrality over window sizes 198, 308, 924 and 1386 for Spain

Table 6.22 shows that the two highest eigenvector centrality scores are for player number

18 (Pedro) and player number 14 (Alonso).

Another observation from Tables 6.22 and 6.21 is that a fluctuation in eigenvector centralities occurs by increasing window sizes. In other words, there is no correlation between eigenvector measures and the size of the windows. In order to get a stable result, we take the average of eigenvector centrality of players over all window sizes and compare them with eigenvector centrality captured from one snapshot, see Table 6.23 and 6.24.

Player	Average of Eigenvector Centrality	One snapshot Eigenvector Centrality
Stekelenburg(1)	0.65	0.71
Van der Wiel(2)	0.67	0.81
Heitinga(3)	0.67	0.81
Mathijsen(4)	0.51	0.72
Van Bronkhorst(5)	0.55	0.72
Van Bommel(6)	0.65	0.83
Kuyt(7)	0.86	0.99
De Jong(8)	0.89	0.89
Van Persie(9)	0.83	0.87
Sneijder(10)	0.96	1
Robben(11)	0.70	0.85

Table 6.23: Average of Eigenvector centrality over window sizes 198, 308, 924 and 1386 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

In Table 6.23 we see that the maximum eigenvector centrality captured by taking the average of scores over the window sizes and in the single snapshot network, belong to player number 10 (Sneijder) in the Netherlands. By the same observation from Table 6.24, we see that player number 18 (Pedro) has the maximum score in both models. Also in this case, however, the rankings of the players are not always preserved. Consider, for example, player 11 (Capdevila) in the Spain team, who is ranked 9th in the aggregated windows model, while is 6th in the static model.

6.7 Conclusions

In this chapter we studied degree centrality, closeness, betweenness, pagerank, and clustering coefficient in the the aggregated model. Below are some general conclusions we could make.

From the fact that degree and pagerank of the Netherlands goalkeeper are the highest among all the players, we infer that the ball is flowing around the Netherlands net, while

Player	Average of Eigenvector Centrality	One snapshot Eigenvector Centrality
Casillas(1)	0.37	0.5
Pique(3)	0.70	0.68
Puyol(5)	0.80	0.86
Iniesta(6)	0.86	0.9
Villa(7)	0.50	0.69
Xavi(8)	0.89	0.92
Capdevila(11)	0.54	0.82
Alonso(14)	0.92	0.98
Ramos(15)	0.86	0.86
Busquets(16)	0.81	0.99
Arbeloa(17)	0.07	0.07
Pedro(18)	0.99	1

Table 6.24: Average of Eigenvector centrality over window sizes 198, 308, 924 and 1386 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

Spanish players keep it far from their net. The most important player in terms of degree and pagerank is in fact the Netherlands goalkeeper.

The average of degree centrality of each players is increasing when the window size is growing, that is, the degree centrality of each player in one snapshot network is greater than the average over all window sizes. Therefore, the player with the highest degree centrality within a team in one snapshot is also the one who has the highest degree by taking the average of all degrees over the different window sizes in the team. A similar observation holds also for closeness centrality.

We observed the fluctuation of the betweenness score in the two teams noticing that, although the betweenness scores of Spain players are lower than the ones of the Netherlands players, they are however more smoothly distributed, indicating that Spain has a well-balanced performance technique.

We have seen that there is no correlation between clustering coefficients in different window sizes. For some players, a small window size produce greater score than large window size. By taking the average of the scores over the window sizes we obtain then a more informative result than scores in the single snapshot network.

By using the window model, we easily see the presence of players that are substituted during the game; this is the case, for example, of Spanish player number 17 (Arbeloa) who just played for a small time period in the team from the start point. In the single snapshot model, this fact would have been less evident.

Chapter 7

Dynamic Passing Networks in the Non-Overlapping Model

In this chapter, we compute the same measures studied in the previous chapter based on the non-overlapping model, by partitioning the total time period into equal consecutive time windows. We remind that in the non-overlapping model we consider 4 different window sizes: 2772, 1386, 396, 99 corresponding to a single time period of the total time of the match (2772 sec.), 2 consecutive periods of 1386 sec. each, 7 consecutive periods of 396 sec. each, 28 consecutive periods of 99 sec. each.

7.1 Degree in the Non-Overlapping Model

In this section we consider degree centrality in the non-overlapping model. This data has been shown in Table 7.1 and 7.2 for the Netherlands and Spain, respectively. As expected, by decreasing the number of windows, the degree is increasing for each player within a team.

In Table 7.1 player number 7 (Kuyt) has maximum degree score within a team for window size equal to 99, whereas player number 1 (Stekelenburg) has maximum score within a team when window sizes are equal to 396 and 693. Moreover, we observe that the most active player in terms of getting and passing the ball around his co-teammate is the goalkeeper, Stekelenburg in the Netherlands team.

Tables 7.3 and 7.4 show the values calculated for the single snapshot model and for the average of the various windows in the non-overlapping windows model. No significant difference can be observed in this case.

Non-Overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Weighted Degree	Weighted Degree	Weighted Degree
1	2.31	5.00	7.50
2	1.80	3.86	6.75
3	1.69	3.86	6.75
4	1.88	4.43	7.75
5	2.00	2.33	3.50
6	1.85	4.00	6.00
7	2.44	3.67	5.50
8	2.43	2.71	4.75
9	1.88	2.50	5.00
10	1.91	4.00	6.00
11	1.22	1.86	3.25

Table 7.1: Average of weighted degrees for window sizes 99, 396 and 693 for the Netherlands

Non-overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Weighted Degree	Weighted Degree	Weighted Degree
1	1.42	3.00	4.50
3	2.92	5.71	10.00
5	2.63	6.86	12.00
6	2.56	5.43	9.50
7	1.67	2.50	3.75
8	3.19	10.29	18.00
11	2.67	5.00	8.75
14	2.95	8.86	15.50
15	2.42	5.17	7.75
16	2.42	7.57	13.25
17	1.00	1.00	1.00
18	2.93	7.00	12.25

Table 7.2: Average of weighted degrees for window sizes 99, 396 and 693 for Spain

Since player 17 has non zero degree value over $[0, 396]$, it means that he is in the field. However, his degree for the other windows vanishes. We conclude that he has been removed from the game, observation that could not be done by inspection of the single snapshot model. Another observation we can make from Graph 7.5 is that in different window times, the number of passing between players in Spain is much higher than the one of players in the Netherlands. This can be seen even better in Figure 7.5, where a window division has been fixed and the variations of degrees can be observed in time for all players through consecutive time intervals for a portion of the game.

Player	Average of Weighted Degree	One snapshot Weighted Degree
Stekelenburg(1)	4.94	30
Van der Wiel(2)	4.10	27
Heitinga(3)	4.68	27
Mathijsen(4)	4.14	31
Van Bronkhorst(5)	3.30	14
Van Bommel(6)	3.13	24
Kuyt(7)	3.87	22
De Jong(8)	2.61	19
Van Persie(9)	3.95	15
Sneijder(10)	3.97	24
Robben(11)	2.11	13

Table 7.3: Average of degree centrality over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

Player	Average of Weighted Degree	One snapshot Weighted Degree
Casillas(1)	2.97	18
Pique(3)	6.21	40
Puyol(5)	7.16	48
Iniesta(6)	5.83	38
Villa(7)	2.64	15
Xavi(8)	10.49	72
Capdevila(11)	5.47	35
Alonso(14)	9.10	62
Ramos(15)	5.11	31
Busquets(16)	7.75	53
Arbeloa(17)	1.00	1
Pedro(18)	7.39	49

Table 7.4: Average of degree centrality over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

7.2 Closeness in the Non-Overlapping Model

As explained in section 6.2, because of the character of passing network, closeness centrality of a player must be computed by formula given in 3.12, where the length of a shortest path between two nodes are given by formula 3.11.

Tables 7.6 and 7.7 show the closeness centrality of each player within the corresponding

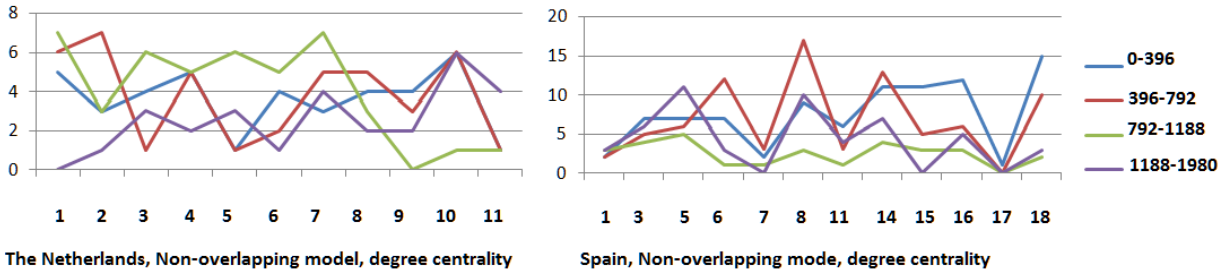


Table 7.5: Degree centrality of players over 4 consecutive windows

teams. By looking at the tables, we see that when the window size is increasing some scores are increasing while some scores are decreasing. Therefore, in order to find who is the best player in terms of how he is close to other players in the team, we take the average of closeness centrality over the window sizes and we determine the highest average score.

Non-Overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Closeness Centrality	Closeness Centrality	Closeness Centrality
1	0.65	0.50	0.50
2	0.62	0.48	0.54
3	0.55	0.45	0.52
4	0.48	0.49	0.56
5	0.57	0.48	0.32
6	0.62	0.61	0.56
7	0.43	0.38	0.41
8	0.47	0.37	0.40
9	0.44	0.50	0.40
10	0.44	0.36	0.49
11	0.16	0.18	0.21

Table 7.6: Average of closeness for window sizes 99, 396 and 693 for the Netherlands

Non-overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Closeness Centrality	Closeness Centrality	Closeness Centrality
1	0.52	0.63	0.50
3	0.49	0.51	0.62
5	0.54	0.50	0.58
6	0.47	0.35	0.53
7	0.22	0.23	0.30
8	0.59	0.60	0.72
11	0.55	0.42	0.51
14	0.61	0.58	0.65
15	0.68	0.46	0.53
16	0.48	0.55	0.64
17	0.00	0.00	0.00
18	0.49	0.45	0.53

Table 7.7: Average of closeness for window sizes 99, 396 and 693 for Spain

Player	Average of Closeness Centrality	One snapshot Closeness Centrality
Stekelenburg(1)	0.55	0.63
Van der Wiel(2)	0.55	0.77
Heitinga(3)	0.51	0.83
Mathijsen(4)	0.51	0.77
Van Bronkhorst(5	0.46	0.63
Van Bommel(6)	0.60	0.91
Kuyt(7)	0.41	0.67
De Jong(8)	0.41	0.71
Van Persie(9)	0.45	0.67
Sneijder(10)	0.43	0.71
Robben(11)	0.19	0.59

Table 7.8: Average of closeness over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

In Table 7.8, the first column of scores shows that player number 6 (Van Bommel) has maximum score in the average of window sizes. The second column of scores indicates that Van Bommel has also a maximum score in one snapshot network.

Player	Average of Closeness Centrality	One snapshot Closeness Centrality
Casillas(1)	0.55	0.73
Pique(3)	0.54	0.85
Puyol(5)	0.54	0.69
Iniesta(6)	0.45	0.73
Villa(7)	0.25	0.5
Xavi(8)	0.64	0.79
Capdevila(11)	0.49	0.73
Alonso(14)	0.61	0.79
Ramos(15)	0.56	0.65
Busquets(16)	0.56	0.92
Arbeloa(17)	0.00	0
Pedro(18)	0.49	0.73

Table 7.9: Average of closeness over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

A significant difference is shown in Table 7.9 where we can see that the highest average closeness in the windows model for the Spain team belongs to player number 14 (Alonso), whereas player number 16 (Busquets) has the maximum score in the single snapshot network. Another difference can be seen in the relative rankings of player 18 (Pedro), who is 8th in the windows model, while is ranked only 5th with a single snapshot.

7.3 Betweenness in the Non-Overlapping Model

The computation of betweenness centrality is based on 3.13, since the network under study is a directed weighted network.

From Table 7.10 one can see that the betweenness centrality of players are generally increasing with the increase of the window sizes with two exceptions: player number 7 and 8. By this observation, we can deduce that the ball flow among players does not rely on these two players, since their betweenness scores are not distributed very well in different time periods. As we see by the same Table, Casillas has maximum betweenness score between all players regardless of the window sizes. Therefore, this highest score dose not measure how Casillas is well-connected, but rather how the ball flow among the other players relays on Casillas. As we noticed before, if a player's betweenness score vanishes, which is the case of players number 8 (De Jong) in the Netherlands and players number 7 (Villa) and 17 (Arbeloa)in the Spain team, then a player is not getting involved in the match and so his removal from the team would not have much impact.

If we look at Table 6.11, we notice several discrepancies between the players rankings under the single snapshot and the non-overlapping window model. For example, player number 8 (De Jong) has the minimum scores among all players in the window model, but not in the static one. Another difference we observe with the single snapshot model, concerns player number 6 (Vam Bommel) who has the highest score within the team in the static model, but is ranked only 10th in our model, where it is instead (Stekelenburg) the one with maximum score. This seems to imply that the goalkeeper in the Netherlands team has a high impact on flowing the ball among players in the team, role that was less visible in the single snapshot model and that we already observed also in the aggregated model.

If we look at Table 7.12, we see a fluctuation of betweenness scores in different window sizes. For instance, betweenness score of player number 1 (Casillas) the goalkeeper, vanishes in window size 99. Moreover, if we compare two betweenness scores in the average and in the single snapshot model, given in Table 6.12 we observe that player number 3 (Pique) has the highest average betweenness centrality among all players. We can also see that player number 7 (Villa) has maximum betweenness score among all players regardless of the window sizes (in one snapshot network). On the on other hand, by looking at the scores of Villa from Table 7.12 we can determine that the performance of Villa in specific time periods does not show that he could hold a maximum score among the players in the team. This is one important advantage of the non-overlapping slide window model that

	Non-Overlapping Model		
	WS = 99	WS = 396	WS = 693
Id	Betweenness Centrality	Betweenness Centrality	Betweenness Centrality
1	0.11	0.18	0.21
2	0.08	0.11	0.20
3	0.09	0.09	0.14
4	0.10	0.16	0.19
5	0.02	0.03	0.05
6	0.11	0.17	0.20
7	0.10	0.11	0.08
8	0.14	0.06	0.05
9	0.02	0.04	0.09
10	0.09	0.12	0.12
11	0.01	0.05	0.11

Table 7.10: Average of Betweenness over window sizes 99, 396 and 693 for the Netherlands

can make clear the performance of players which can not be seen in the aggregated model.

As before, we notice that if a player's betweenness score vanishes, which is the case of players number 7 and 17 in the Spain team and player 11 in the Netherlands team, then a player is not getting involved in the match and so his removal from the team would not have much impact. e also note that the score of player number 17 (Arbeloa) vanishes as well, but this player was playing only for a few minutes in the team. That is why we take out his scores from our table.

By comparing betweenness centralities for players in two teams from Tables 7.11 and 7.12, we can conclude that there is more fluctuation in the Netherlands than in Spain, meaning that the scores are more evenly distributed among the players in Spain. As already observed in the analysis with the aggregated model, this may indicate that Spain has a well-balanced performance technique.

An interesting observation is about player number 6 (Vam Bommel) who is placed at first rank of betweenness scores in one snapshot network, but his score is second last in the non-overlapping model. An explanation can be given also by observing a portion of the match in time without averaging the window sizes. (see Figure 7.14) for the two teams. The graph shows that Vam Bommel has a high score in interval $[792, 1188]$, which however becomes zero in the following interval $[1188, 1980]$. Having the highest score in one of the intervals implies the maximum value in the single snapshot model.

Player	Average of Betweenness Centrality	One snapshot Betweenness Centrality
Stekelenburg(1)	0.17	0.03
Van der Wiel(2)	0.11	0.06
Heitinga(3)	0.15	0.06
Mathijsen(4)	0.13	0.04
Van Bronkhorst(5)	0.08	0.03
Van Bommel(6)	0.05	0.08
Kuyt(7)	0.10	0.05
De Jong(8)	0.03	0.05
Van Persie(9)	0.16	0.03
Sneijder(10)	0.11	0.05
Robben(11)	0.06	0.03

Table 7.11: Average of betweenness centrality over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time periods $[0, 2772]$ for the Netherlands

	Non-overlapping Model		
	WS = 99	WS = 396	WS = 693
Id	Betweenness Centrality	Betweenness Centrality	Betweenness Centrality
1	0.00	0.05	0.06
3	0.21	0.12	0.16
5	0.10	0.13	0.11
6	0.12	0.07	0.07
7	0.06	0.03	0.01
8	0.15	0.17	0.13
11	0.13	0.06	0.07
14	0.13	0.18	0.12
15	0.05	0.09	0.05
16	0.08	0.12	0.05
17	0.00	0.00	0.00
18	0.14	0.12	0.14

Table 7.12: Average of betweenness over window sizes 99, 396 and 693 for Spain

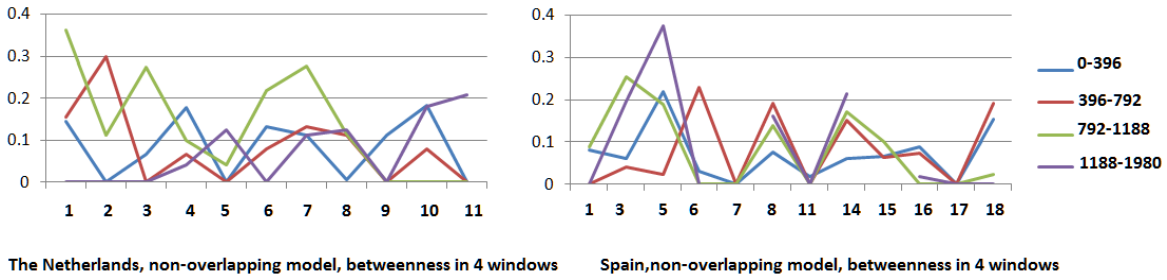
7.4 Pagerank in the Non-Overlapping Model

As mentioned in Section 6.4, roughly speaking, the pagerank centrality of a player in passing networks gives the probability that he will get the ball after a reasonable number of passes have been done between players in the team. The computation of pagerank score in our model is based on formula 3.19 described in section 6.4.

As we see from Tables 7.15 and 7.16, in both teams the scores are for the most part decreasing with the increase of the window size. This means that in most of the cases, a player gets smaller pagerank than he is expected to get over a large window size. By

Player	Average of Betweenness centrality	One snapshot Betweenness Centrality
Casillas(1)	0.03	0.1
Pique(3)	0.16	0.03
Puyol(5)	0.11	0.04
Iniesta(6)	0.08	0.03
Villa(7)	0.03	0.5
Xavi(8)	0.15	0.02
Capdevila(11)	0.08	0.03
Alonso(14)	0.14	0.03
Ramos(15)	0.06	0.02
Busquets(16)	0.08	0.06
Arbeloa(17)	0.00	0
Pedro(18)	0.13	0.05

Table 7.13: Average of betweenness centrality over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain



The Netherlands, non-overlapping model, betweenness in 4 windows

Spain,non-overlapping model, betweenness in 4 windows

Table 7.14: Betweenness centrality of players in 4 consecutive windows

Id	Non-Overlapping Model WS = 99 PageRank	WS = 396 PageRank	WS = 693 PageRank
1	0.23	0.15	0.13
2	0.15	0.07	0.08
3	0.19	0.10	0.10
4	0.22	0.16	0.14
5	0.15	0.08	0.07
6	0.17	0.10	0.09
7	0.20	0.10	0.12
8	0.19	0.08	0.07
9	0.10	0.06	0.07
10	0.18	0.08	0.07
11	0.21	0.09	0.07

Table 7.15: Average of pagerank over window sizes 99, 396 and 693 for the Netherlands

taking the average over all window sizes, we will obtain small score compared with taking the average of scores over small window sizes.

Non-overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	PageRank	PageRank	PageRank
1	0.09	0.05	0.03
3	0.15	0.10	0.09
5	0.15	0.09	0.10
6	0.16	0.09	0.10
7	0.13	0.08	0.06
8	0.17	0.13	0.14
11	0.13	0.06	0.07
14	0.16	0.12	0.11
15	0.13	0.10	0.08
16	0.12	0.10	0.09
17	0.10	0.05	0.02
18	0.18	0.11	0.13

Table 7.16: Average of pagerank over window sizes 99, 396 and 693 for Spain

Player	Average of PageRank	One snapshot PageRank
Casillas(1)	0.06	0.03
Pique(3)	0.11	0.08
Puyol(5)	0.11	0.09
Iniesta(6)	0.11	0.1
Villa(7)	0.09	0.06
Xavi(8)	0.14	0.13
Capdevila(11)	0.09	0.07
Alonso(14)	0.13	0.11
Ramos(15)	0.10	0.08
Busquets(16)	0.10	0.1
Arbeloa(17)	0.06	0.02
Pedro(18)	0.14	0.13

Table 7.17: Average of pagerank centrality over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

We conclude that the most important players in the Spain team seem to be players number 8 (Xavi) and number 18 (Pedro), while player number 1 (Stekelenburg) the goalkeeper is most important player in the Netherlands.

We notice that in terms of pagerank score, the performance of Spain is higher than the one of Netherlands. The reason is twofold: on one hand, the fact that the pagerank score of the goalkeeper is high implies that Spain players keep the ball near to the Netherlands net, or shoot the ball into the Netherlands net. Second, the fluctuation of pagerank scores

Player	Average of PageRank	One snapshot PageRank
Stekelenburg(1)	0.17	0.12
Van der Wiel(2)	0.13	0.09
Heitinga(3)	0.17	0.1
Mathijsen(4)	0.10	0.12
Van Bronkhorst(5)	0.11	0.07
Van Bommel(6)	0.08	0.09
Kuyt(7)	0.14	0.11
De Jong(8)	0.10	0.08
Van Persie(9)	0.12	0.06
Sneijder(10)	0.11	0.09
Robben(11)	0.12	0.07

Table 7.18: Average of pagerank centrality over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

in Spain is much lower than the fluctuation for the Netherlands. This is again an indication of the defensive nature of the Netherlans' play versus the more aggressive play of Spain.

This behaviour is evident also by inspecting the variations of pagerank for a portion of the game, and for a given window size (see Figure 7.19).

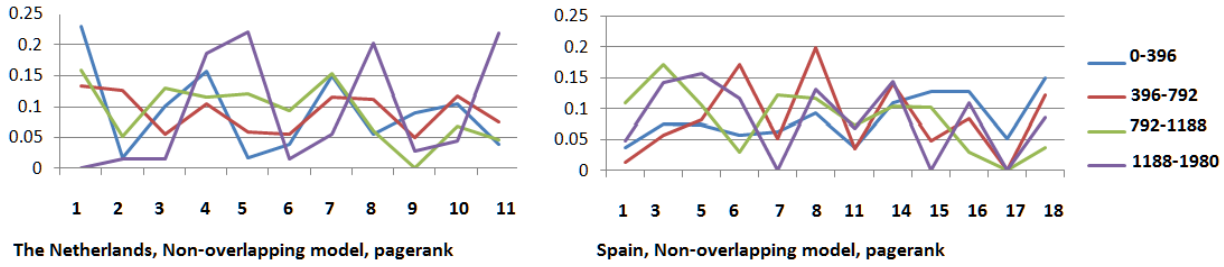


Table 7.19: Pagerank score of players for 4 consecutive windows

7.5 Clustering Coefficient in the Non-Overlapping Model

The clustering coefficient is calculated by the formula given in 3.22. As we saw in section 3.2.5, there are four models of directed triangles consisting a fix node, In, Out, Cycle and Middle-man. For each player, as a node of the network, we count the percentage of all possible triangles consisting of the node.

Non-Overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient
1	0.01	0.07	0.11
2	0.02	0.13	0.15
3	0.03	0.09	0.10
4	0.06	0.16	0.12
5	0.02	0.02	0.07
6	0.05	0.03	0.16
7	0.23	0.32	0.37
8	0.05	0.14	0.25
9	0.00	0.11	0.15
10	0.01	0.05	0.06
11	0.00	0.01	0.31

Table 7.20: Average of Clustering coefficient over window sizes 99, 396 and 693 for the Netherlands

Non-overlapping Model			
	WS = 99	WS = 396	WS = 693
Id	Weighted Clustering Coefficient	Weighted Clustering Coefficient	Weighted Clustering Coefficient
1	0.03	0.04	0.16
3	0.14	0.24	0.37
5	0.14	0.13	0.27
6	0.11	0.14	0.29
7	0.00	0.19	0.31
8	0.17	0.12	0.15
11	0.13	0.18	0.22
14	0.19	0.15	0.22
15	0.14	0.14	0.38
16	0.17	0.16	0.25
17	0.00	0.00	0.00
18	0.06	0.09	0.20

Table 7.21: Average of clustering coefficient over window sizes 99, 396 and 693 for Spain

Tables 7.20 and 7.21 shows the clustering coefficient of players in each team is increasing by growing the window sizes.

From Tables 7.22 and 7.23, we find that player number 7 (Kuyt) in the Netherlands team and player number 3 (Pique) in the Spain team have highest average scores among the players of their own team. Whereas player number 11 (Robben) in the Netherlands and player number 7 (Villa) in Spain have the highest score.

By inspecting Tables 7.20 and 7.21 we notice that Stekelemburg, the Netherlands goalkeeper has the same anomalous behaviour already remarked in the aggregated window model: he is ranked 3rd in the static analysis, while he is ranked much lower int the

Player	Average of Weighted Clustering Coefficient	One snapshot Weighted Clustering Coefficient
Stekelenburg(1)	0.06	0.25
Van der Wiel(2)	0.08	0.25
Heitinga(3)	0.11	0.18
Mathijsen(4)	0.10	0.2
Van Bronkhorst(5)	0.15	0.33
Van Bommel(6)	0.09	0.21
Kuyt(7)	0.30	0.18
De Jong(8)	0.03	0.3
Van Persie(9)	0.08	0.23
Sneijder(10)	0.04	0.22
Robben(11)	0.11	0.38

Table 7.22: Average of Clustering coefficient over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

Player	Average of Weighted Clustering Coefficient	One snapshot Weighted Clustering Coefficient
Casillas(1)	0.08	0.18
Pique(3)	0.25	0.24
Puyol(5)	0.18	0.31
Iniesta(6)	0.18	0.22
Villa(7)	0.17	0.37
Xavi(8)	0.15	0.2
Capdevila(11)	0.18	0.28
Alonso(14)	0.19	0.21
Ramos(15)	0.22	0.31
Busquets(16)	0.19	0.2
Arbeloa(17)	0.00	0
Pedro(18)	0.12	0.15

Table 7.23: Average of Clustering coefficient over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

windows method.

Another interesting observation regards player number 5 (Van Bronkhorst) in the Netherlands team, who appears to be one of best players within the team, with a large balance in the amount of passes. In fact, his score is increasing smoothly from window size 99 to window size 693 with a very small difference among the clustering coefficients within the various window sizes. This small fluctuation indicates that Bronkhorst has kept a constant high level performance during the match. Note that such an observation emerges from the non-overlapping model, but could not be seen in the single snapshot network.

7.6 Eigenvector Centrality in the Non-Overlapping Model

In Tables 7.24 and 7.25, the eigenvector centrality of players for the Netherlands and Spain have been shown for different window sizes. This computation has been done based on technique explained in section 3.2.4. These tables show fluctuation of scores among window sizes.

	Non-Overlapping Model		
	WS = 99	WS = 396	WS = 693
Id	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality
1	0.66	0.70	0.72
2	0.29	0.33	0.52
3	0.42	0.55	0.67
4	0.69	0.64	0.63
5	0.42	0.32	0.40
6	0.40	0.60	0.58
7	0.75	0.35	0.67
8	0.66	0.47	0.55
9	0.32	0.32	0.50
10	0.50	0.42	0.57
11	0.56	0.43	0.43

Table 7.24: Average of eigenvector centrality over window sizes 99, 396 and 693 for the Netherlands

	Non-overlapping Model		
	WS = 99	WS = 396	WS = 693
Id	Eigenvector Centrality	Eigenvector Centrality	Eigenvector Centrality
1	0.24	0.23	0.19
3	0.46	0.63	0.63
5	0.48	0.55	0.70
6	0.54	0.55	0.74
7	0.51	0.49	0.35
8	0.61	0.83	0.91
11	0.47	0.48	0.56
14	0.58	0.73	0.76
15	0.38	0.52	0.50
16	0.37	0.64	0.68
17	0.01	0.05	0.02
18	0.56	0.66	0.84

Table 7.25: Average of eigenvector over window sizes 99, 396 and 693 for Spain

From Table 7.24, we see that the average eigenvector centrality for player number 1 (Stekelenburg) is the highest, whereas the maximum score based on the single snapshot model belongs to player number 10 (Sneijder) which is equal to 1. This means that Sneijder

has smallest "farness" from the others in the single snapshot network, whereas he has no high score if we look at his activity over different time periods, where he is actually ranked quite low. The same phenomenon occurs in the Spain team, where player number 8 (Xavi) has the maximum average score, whereas in the single snapshot model player number 18 (Pedro) has the maximum score (see Table 7.27). Again, if we focus on Pedro's scores in different window sizes, we see that he is not obtaining maximum score in each window sizes (Table 7.25) indicating an uneven performance.

Player	Average of Eigenvector Centrality	One snapshot Eigenvector Centrality
Stekelenburg(1)	0.69	0.71
Van der Wiel(2)	0.55	0.81
Heitinga(3)	0.65	0.81
Mathijsen(4)	0.38	0.72
Van Bronkhorst(5)	0.56	0.72
Van Bommel(6)	0.38	0.83
Kuyt(7)	0.59	0.99
De Jong(8)	0.38	0.89
Van Persie(9)	0.53	0.87
Sneijder(10)	0.50	1
Robben(11)	0.48	0.85

Table 7.26: Average of eigenvector centrality over window sizes 99, 396 and 693 for the Netherlands; Right: One snapshot over time period $[0, 2772]$ for the Netherlands

Player	Average of Eigenvector Centrality	One snapshot Eigenvector Centrality
Casillas(1)	0.22	0.5
Pique(3)	0.57	0.68
Puyol(5)	0.57	0.86
Iniesta(6)	0.61	0.9
Villa(7)	0.45	0.69
Xavi(8)	0.79	0.92
Capdevila(11)	0.50	0.82
Alonso(14)	0.69	0.98
Ramos(15)	0.47	0.86
Busquets(16)	0.57	0.99
Arbeloa(17)	0.03	0.07
Pedro(18)	0.69	1

Table 7.27: Average of eigenvector centrality over window sizes 99, 396 and 693 for Spain; Right: One snapshot over time period $[0, 2772]$ for Spain

7.7 Conclusions

In this chapter, we studied degree centrality, closeness, betweenness, pagerank, and clustering coefficient in the the non-overlapping window model. Below are some general conclusions we could make, most confirming what already observed with the aggregated model.

We have noticed that the most important players in the Spain team seem to be players number 8 (Xavi) and number 18 (Pedro), while player number 1 (Stekelenburg), the goalkeeper, being the most important player in the Netherlands.

We have seen that in terms of pagerank score, the performance of Spain is higher than the one of Netherlands. Indeed, since the pagerank score of the goalkeeper is high, we conclude that Spain players keep the ball near to the Netherlands net, or shoot the ball into the Netherlands net. Moreover, we observed that, the fluctuation of pagerank scores in Spain is much lower than the fluctuation for the Netherlands. This is again an indication of the defensive nature of the Netherlans' play versus the more aggressive play of Spain.

Another consequence of our case study concerns player number 5 (Van Bronkhorst) in the Netherlands team, who appears to be one of best players within the team, with a large balance in the amount of passes. We have seen that his score (degree centrality) is increasing evenly from window size 99 to window size 693 with a very small difference among the clustering coefficients within the various window sizes.

Different results have been captured using non-overlapping model in comparison with the static analysis; for example, by this, possibly more accurate, analysis, we see that the highest closeness score belongs to player number 14 (Alonso), whereas in the single snapshot model the maximum closeness is taken by player number 16 (Busquets).

Finally, player number 17 (Arbeloa) in the Spain team has extremely low scores of degree, betweenness and closeness over different widow size, as well as in the single snapshot model. From the non-overlapping window times, we can deduce that he has been substituted very soon after the start of the game.

Chapter 8

Conclusions

In the current literature, football passing networks have been analyzed on the basis of static representations of football matches. In the thesis we propose to include temporal information in the match description and to take advantage of it to obtain possibly more accurate information about the weaknesses, strengths and connections among the players, and about the general behaviours of the team.

To do so, we adapted to our setting the centrality measures previously studied in the context of football passing networks: degree centrality, closeness, betweenness, pagerank, clustering coefficient and eigenvector centrality. These measures have been studied for static football passing networks representations, while we investigated them in two temporal models: one where time is partitioned in consecutive periods (non-overlapping windows) and another where the global play time is observed in growing time intervals (aggregated windows). The observations of those centrality parameters in the windows models has allowed us to detect interesting information about the players and about the teams, several of which could not be detected by the inspection of the static representation of the passing network.

To the best of our knowledge this is the first time that a football match has been studied focusing on the passes among players in time, and the results of the thesis constitutes a very first step towards a more complete analysis. The techniques and results of the thesis, in fact, contain several limitations and they open more problems than they close. For example:

- Some temporal measures based on the dynamic nature of the time varying graph that represents the football passing network could be employed. This is the case, for example, of foremost and fastest betweenness, where the "static" notion of shortest

path is substituted by the temporal notions of "fastest" or "earliest" path. The investigation of such measures could bring a totally different perspective to the temporal study of football matches.

- Different types of windows could be considered; for example, another option would be to perform the analysis with overlapping windows where the corresponding time intervals overlap of variable amounts.

- A study that should be performed is to focus on individual players and analyze their scores in time, after fixing a certain window size. This study would allow the analysis of the scores fluctuations in time and would identify specific characteristics of the players behaviour during the match. For lack of time, we observed only partial data in this fashion (e.g., see Figures 7.1) but a full analysis is left for future study.

- Another very interesting direction would be the study of passing patterns of successful teams, so to learn information about play strategies, and to understand patterns associated to weak or strong players.

- A study that could be of interest is to incorporate the information about shots towards the opposite goalkeeper and use social networks tools to analyze the resulting network.

- Finally, techniques similar to the ones of the thesis could be employed to focus on successful and incomplete passes within players in a team.

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