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Effects of Spatial Correlation and Channel Estimation Errors on the Performance of Space-Time Block Coded Systems

by

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Abstract

Today's wireless communication systems require more efficiency in terms of data rate. One of the ways to improve channel capacity of such systems is to utilize diversity. Space-time block codes provide transmit diversity gain without any significant increase in complexity of systems. However in some cases, the gain by utilizing diversity decreases due to some factors. In this work we studied two of these factors and their effects to several space-time block coded systems. The first factor is the spatial correlation among channels and the second factor is channel estimation errors. Both of these factors are experienced in real life situations. The systems under consideration are 2, 3 and 4 antenna orthogonal and quasi-orthogonal space-time block coded systems. We have also compared the results with the maximum ratio combining schemes with 2 and 4 receiver antennas.

Table of Contents

List of Symbols

In order of use

h	Channel coefficient or channel impulse response
α	Amplitude of channel coefficient or amplitude of one of the multipath components in the channel response
θ	The phase of the channel coefficient or the carrier phase shift of one of the multipath component carriers in the channel impulse response
t	Time index
τ	The delay of one of the multipath components in channel impulse response
$\delta(t)$	Delta function
L	Number of multipath components or number of branches for a diversity system
f_d	Doppler shift
R	Data rate (in bits/second)
B	Bandwidth of the system (in Hz)
M_0	Modulation order
E_b	Energy per received information bit
N_0	Noise power spectral density
γ_b	Instantaneous signal to noise ratio per information bit
$\bar{\gamma}_b$	Average signal to noise ratio per information bit
$\overline{\alpha^2}$	Average gain of the channel
γ_s	Instantaneous signal to noise ratio per symbol
E_s	Energy per received information symbol
r	Received signal
n	Noise coefficient
γ	Random variable representing signal to noise ratio
$p(\gamma)$	The probability density function of γ
$p_e(\gamma)$	Probability of error for a specific γ value

$Q(\ )$	Generalized Marcum Q function
S	Transmitted signal or symbol
G	Gain introduced at the l^{th} branch in transmit diversity with feedback or receive diversity systems
$d^2(x,y)$	The squared Euclidean distance between signals x and y
$\tilde{S}$	Decision variable for a maximum likelihood decision
N	Number of transmit antennas
M	Number of receive antennas
c_t^i	The codeword at i^{th} transmit antenna at time t
$\boldsymbol{c}$	Code vector
$\boldsymbol{e}$	Error vector
$\mathbf{B}$	The error matrix
λ	Eigen value
$\mathbf{I}$	Identity matrix
ρ	Correlation coefficient
σ	Standard deviation
E[ ]	The expected value
e_{ce}	Channel estimation error
$\hat{h}$	Estimated channel coefficient

List of Abbreviations

AOA	Angle of arrival
AWGN	Additive white gaussian noise
BER	Bit error rate
BPSK	Binary phase shift keying
EGC	Equal gain combining
MLSE	Maximum likelihood sequence estimation
MRC	Maximum ratio Combining
PAM	pulse amplitude modulation
PSK	Phase shift keying
QAM	Quadrature amplitude modulation
QPSK	Quadrature phase shift keying
SC	Selection combining
SNR	Signal to noise ratio
SNR_{ce}	Channel Estimation SNR
STD	Simple transmit diversity
8PSK	8ary phase shift keying
16QAM	16ary quadratue amplitude modulation

List of Figures

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Finally I want to dedicate this thesis to my grandfathers Fehmi Baykaş and Sadık Tezel.

CHAPTER 1
Introduction

1.1 Background

Space-time codes are channel codes that have been specifically designed to efficiently utilize multiple antenna systems. There are mainly two types of space-time codes, space-time block codes and space-time trellis codes. These codes differ by their coding method, performance and complexity. The main goal of this thesis is to study the effects of spatial correlation in the performance of space-time block coded systems in multipath fading cellular wireless channels. Furthermore we investigate the effect of channel estimation errors on their performance.

Before studying those effects let us briefly review some concepts about the cellular wireless channels that are relevant to our work.

1.2 Cellular Wireless Channels

In a communication system, channel is the medium that is used to transfer the information [Pro01 p.2]. The characteristics of a channel could limit the performance of the system and most of the time they cannot be changed by the system designer. This statement is also valid for the cellular wireless communication systems. In Figure 1.1, an example of such a system is illustrated. Another definition of communication channel is, that it provides the connection between transmitters and receivers. Therefore not only free space or atmosphere between mobile users and base stations but every object that affects communication signals is a part of the channel. Examples of these objects are mountains, large buildings, homes and cars as seen in Figure 1.1.

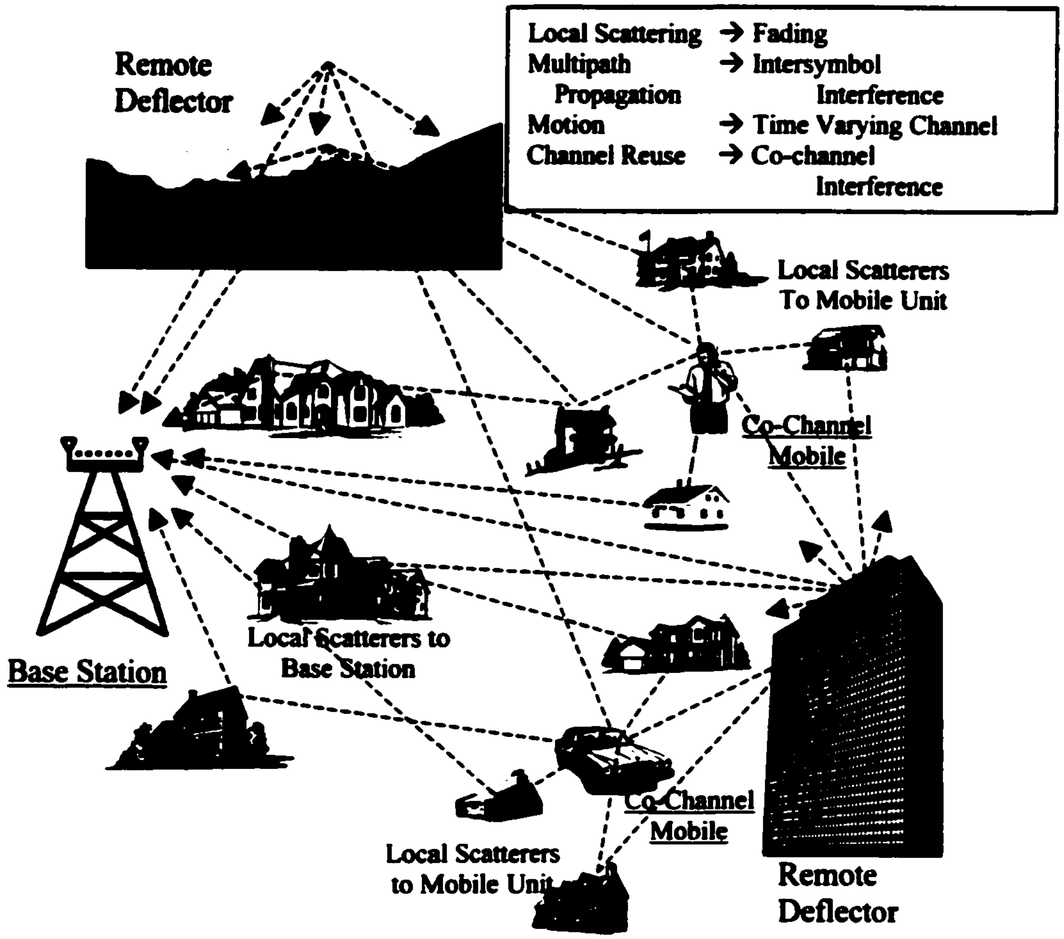


Figure 1.1 Time varying multipath fading channel for cellular wireless systems.

In terms of their effect to the signals, these objects can be divided into two groups as deflectors and scatterers.

The deflectors are large far away objects from the base stations and mobile units like mountains or skyscrapers, which reflect the signals like a mirror. Due to these reflections the signals follow different paths to reach the antennas of transceivers (multipath propagation). Intersymbol interference occurs in wireless channels, if the delay between signals from different paths (multipath delay) is significant (e.g. more than half a symbol duration). In cellular wireless systems, the same frequency band can be used by different pairs of transceivers (channel reuse). The interference between these pairs is known as co-channel interference. Deflectors could increase this type of interference as well, since the reflected signals could reach other antennas rather then the aimed ones.

The scatterers, the second group of objects, are small nearby objects to the transceivers up to a few wavelengths in dimension such as houses or vehicles. They scatter the signal to its replicas with different amplitudes and phases. Due to the short distance between the scatterers and transceivers, these replicas arrive at target antennas with small multipath delay with respect to the symbol duration. As a result, those replicas are added together at the receiver. This addition can be destructive or constructive, depending on the phases of the replicas. The destructive superposition of various components of a signal is called fading. Another effect of small multipath delay is frequency non-selective channels. This means all frequencies of interest experience fading simultaneously.

The effect of motion is important in wireless channels. The relative motion between a base station and mobile antennas such as the motion of a user or the motion of scatterers changes the channel coefficients in time. This motion causes Doppler shift. In essence, a Doppler shift is the increase or decrease in the frequency of a signal due to the relative motion between the transmitter and receiver. The amount of the shift depends on the velocity and the direction of the motion and the frequency of the signal. The coherence time of a channel is the time duration that the channel coefficients are considered constant. If the absolute value of Doppler shift is larger than the bandwidth of the channel, then the coherence time of the channel is less than the signal duration. Such channels are called fast fading channels. Otherwise the channel is a slow fading channel.

Considering all of the above conditions, the channel in a cellular wireless communication system is defined as time varying multipath fading channel.

Now we focus on the effects of these channel characteristics to the received signal power. In Figure 1.2, an example of signal power versus distance curve is illustrated. The power of radio signals decreases with distance in free space, which is called free space path loss. The mean of signal power in a wireless channel decreases as the distance increases due to the free space path loss and general characteristics of the environment as seen in Figure

1.2. This phenomenon is known as mean path loss. In most cellular networks, the signal decreases exponentially with an order between 2.5 and 5 [Rap96 p.104]. On top of mean path loss, the shadowing from large buildings or the effect of being in or out of a building is added. This effect is defined as large scale fading. Small scale fading is due to scatterers and motion. It determines the final form of the curve.

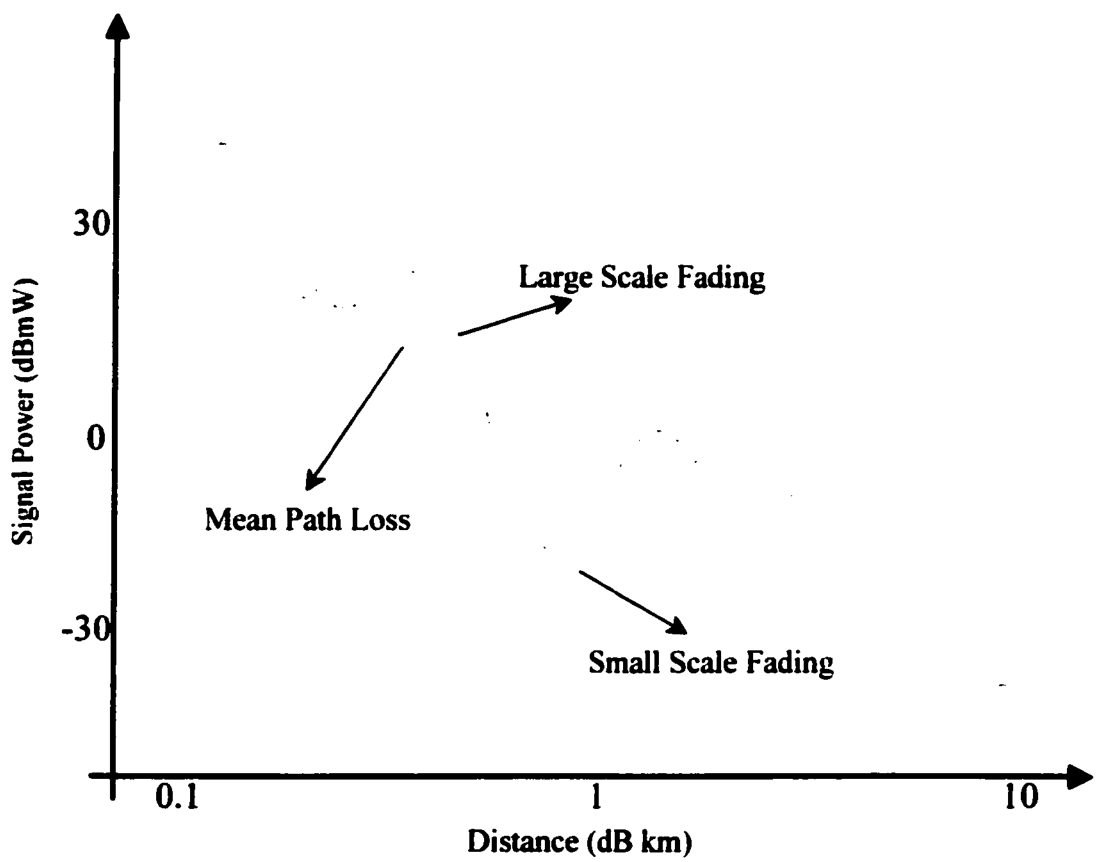


Figure 1.2 An example of signal power at the receiver vs. distance for multipath fading channel with path loss exponent 3.

In general the baseband complex representation of the channel impulse response for these communication systems is [Ert98]:

$$h(t)=\sum_{l=0}^{L(t)-1}\alpha_l(t)e^{j\theta_l(t)}\delta(t-\tau_l(t)) \quad (1.1)$$

where $L(t)$ is the number of multipath components at time t, $a_l(t)$ is the amplitude, θ_l is the carrier phase shift at time t, τ_l is the delay of the l^{th} multipath component. Angle of arrival (AOA) and the Doppler shift f_d of these components are other important parameters that affect the performance of the system.

Fading is affected by the relative position of the antennas and the environment. Therefore several methods are proposed to simulate different types of fading channels at different environments [Jak74] [Rap96, p.139] [Ert98]. For example, if there is a line of sight between antennas then the effect of fading is less. Being in a high position or low with respect to scatterers changes the effect of fading too. Especially for the case of no line-of-sight between the base station and mobile, the amplitude of the components α_k is modeled by Rayleigh distributed random variables, while phase shift θ_k is modeled by uniformly distributed random variables between 0 and 2π. Figure 1.3 is an example of amplitude variation in Rayleigh fading channel for 900Hz around its mean value [Skl97]. Observe that in this example the average signal power level is 0 dB, while the instantaneous power can be up to 40 dB below the average signal level due to deep fades and up to 7 dB above due to constructive superpositioning of the multipath components.

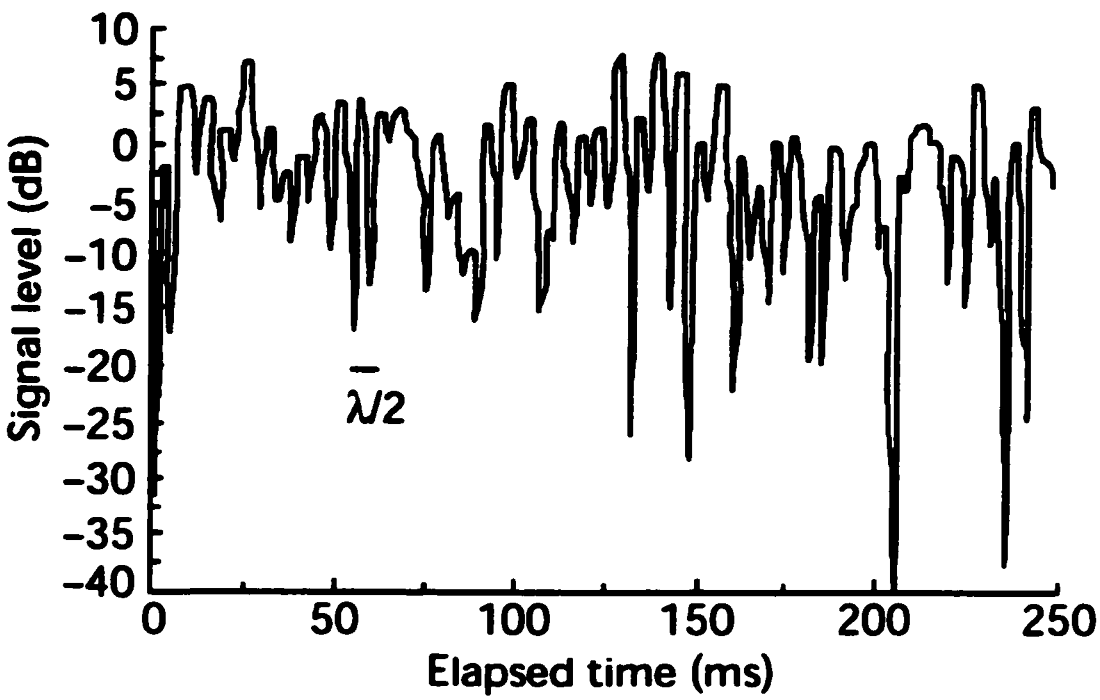


Figure 1.3 Simulation of a Rayleigh fading channel at 900 Mhz with carrier receiver speed 120 km/hr [Skl97].

The performance of a communication system is usually compared by its bit error rate versus average signal power curve. For a fading channel even if the average signal power is high, at deep fade instants bit error rate could have the value up to 0.5. As a result of this the bit error rate of the system raises. This effect can be seen in Figure 1.4 The bit error rate performances of additive white Gaussian noise (AWGN) channel and Rayleigh fading channel are shown for binary phase shift keying (BPSK) modulation. Rayleigh fading channel requires much higher signal to noise ratio (SNR) than the AWGN channel to have the same bit error rate. For example, for a 10^{-3} bit error rate, 6.5 dB SNR is enough for AWGN channel but Rayleigh channel requires an SNR around 24 dB.

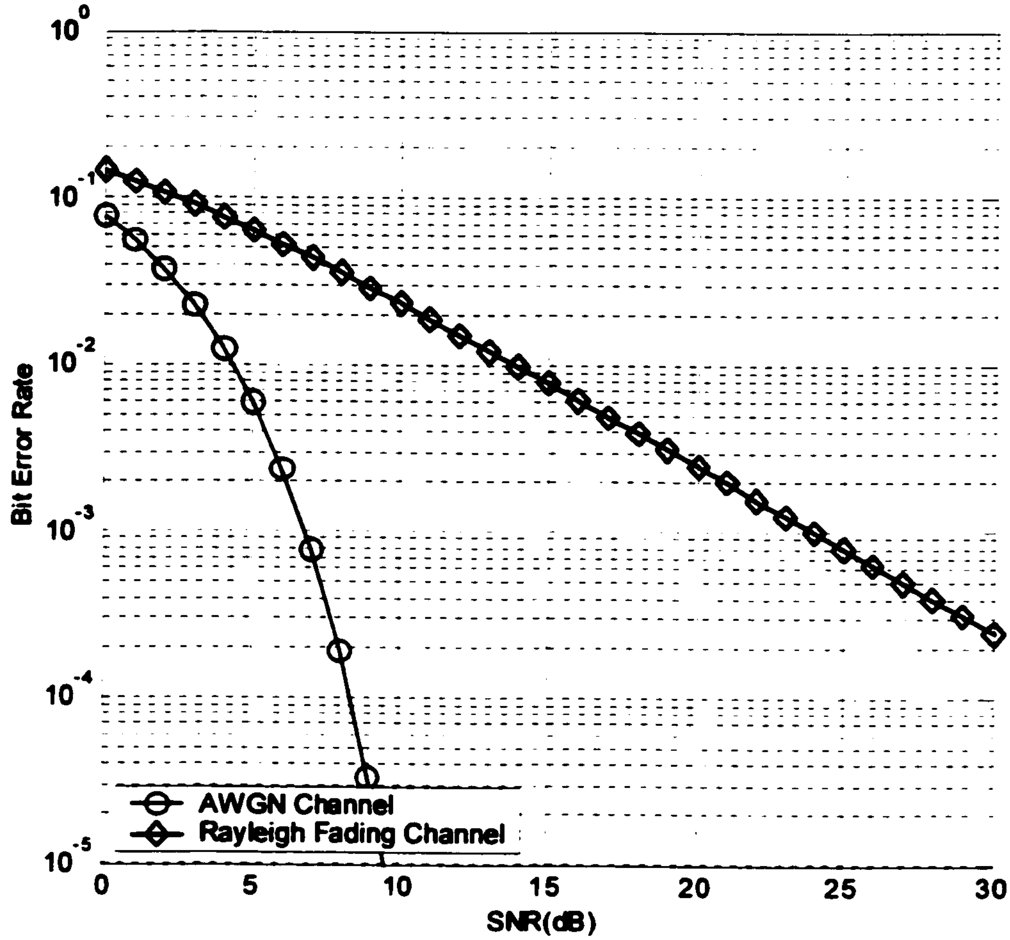


Figure 1.4 Bit error rate performance of BPSK modulation in AWGN channel and Rayleigh fading channel.

Another observation from Figure 1.4 is the inverse linear relationship between BER and SNR in Rayleigh fading channel at low bit error rates, whereas in AWGN channel an inverse exponential relationship is experienced. Due to these relationships the improvement of the BER requires more increase in signal power in Rayleigh fading. For example to decrease the bit error rate from 10^{-2} to 10^{-3} in an AWGN channel a 3 dB additional signal power is enough, whereas in Rayleigh fading channel for the same BER improvement a 10 dB increase in the signal power is required.

Since deep fades are mainly responsible for the poor error performance of the system, the most efficient way to combat fading is power adjustment with respect to the fading level of the channel. The power of the signal at the transmitter is increased or decreased according to the signal power at the receiver. Since the power level at the transmitter is kept down, when there is no fading, the average power consumption stays constant. This method can be impractical due to the power range limitations of the amplifiers. For example the amplifier cannot provide the necessary power level at a deep fade instant or utilizing high power amplifiers could increase the size and the cost of the transmitter unit beyond limits.

Naturally the transmitter should have the knowledge of channel gains (channel state information) to adjust the power. The transmitter can obtain the channel state information in two ways. In the first case, the receiver sends the channel state information to the transmitter. The transmitter uses this information to adjust the power of the signal. The second case is applicable if the transmitter and receiver use the same channel by time division duplexing. The transmitter uses the acquired channel state information for controlling the power level. These systems have some disadvantages. First of all if the transmission is simplex (one way), they are useless. They increase the complexity of the transmitter. The first approach increases the complexity of receiver as well. That approach requires the exchange of additional information, which decreases the total throughput of the system.

Utilizing diversity is another widely used approach to cope with fading. The main idea of diversity is: "to increase the performance of the system by using the random nature of radio propagation by finding independent or at least sufficiently uncorrelated signal channels for communication" [Rap96 p.325]. Basically if the signal level has a deep fade in one channel, another channel might have a high signal level. Diversity uses different channels to increase the performance of the system. In Figure 1.5 two independent channels with Rayleigh fading are in part 1 and part 2. Their mean value is 0dB. Part 3 of the figure is their average. Observe that the resulting path has less deep fades than the first two parts and the average of deep fade levels is lower than deep fade levels of the first two channels.

The elimination of deep fades will increase the performance of the system, since the instants with high BER are less. In Figure 1.5 we demonstrated the basic idea of diversity, but we did not elaborate on how this diversity is achieved. Nor did we discuss what happens when the channels are correlated or if there are channel estimation errors at the receiver. These topics will be covered in details in the following chapters of this thesis.

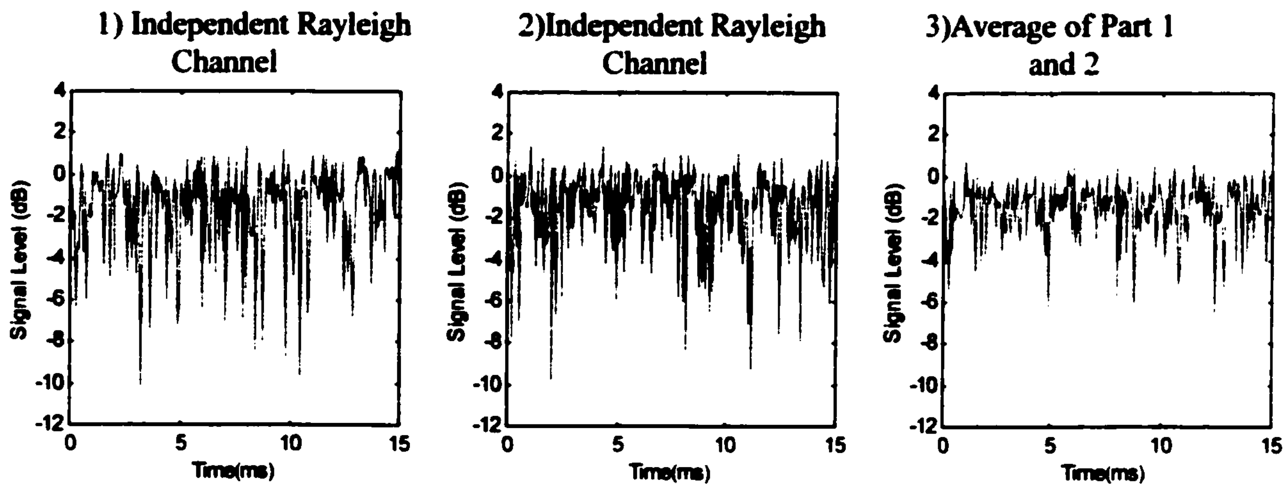


Figure 1.5 Effect of combining two independent Rayleigh fading channels.

1.3 Thesis Outline

In Chapter 2 of this thesis we will review the parameters used to evaluate the performance of communication systems and different types of diversity techniques and the applicable combination methods for diversity systems.

The focus of this thesis is transmit diversity, where the diversity gain is obtained by transmitting the signals from more than one antenna using precoding. In Chapter 3, the necessity of precoding and different types of transmit diversity techniques will be investigated.

The independence of different signal paths is the basic assumption of most diversity schemes. However in practice independent and uncorrelated paths are difficult to obtain. Existence of correlation among channels can decrease the system performance. The effects of correlation to the performance of space-time block coding systems will be investigated in Chapter 4.

Channel estimation errors at the receiver side could decrease the performance of a wireless communication system. The topic of Chapter 5 is the effect of channel estimation errors in the performance of space-time block coded systems.

The last chapter discusses the conclusions of this thesis and suggests some future work.

1.4 Thesis Contributions

One major contribution of this thesis is to bring several transmit diversity techniques proposed by several different authors and present them using a consistent and uniform notation.

After defining the transmit diversity techniques of interest, this work provides the effect of correlation at different levels to space-time block coded systems. Correlation among channels and channel estimation errors are among factors, that can decrease the performance of a wireless communication system utilizing diversity. We have shown that the robustness of different systems is not the same. We made suggestions to system designer how to decrease the effect of correlation in quasi-orthogonal systems. Channel estimation errors are another source of performance degradation. In this work their effect to space-time block coded systems are also studied.

CHAPTER 2
Background for Diversity Systems

2.1 Introduction

In this chapter we provide the background information on the parameters we used to evaluate the performance of the communication systems and review the basic principles of diversity systems.

First we discuss three important performance parameters of bandwidth efficiency, signal to noise ratio and bit error rate. Then different types of diversity systems are explained, such as temporal and frequency diversity. The last topic of the chapter is the combining schemes used in the diversity systems.

2.2 Performance Parameters for Communication Systems

In evaluating a communication system, several performance parameters are necessary, for example cost, complexity, delay, data rate, error rate. However the importance of a parameter varies from one system to another. For example the delay caused by a system is an important parameter for interactive communication systems but not important for simplex data transmission.

In this section three of these parameters, which we frequently use in the following chapters, are explained.

2.2.1 Bandwidth Efficiency

The bandwidth efficiency of a system is basically the maximum data rate R (in bits/second) achievable over the available bandwidth B (in Hz). For linear modulation techniques such as quadrature amplitude modulation (QAM) and phase shift keying (PSK), the maximum bandwidth efficiency is approximately equal to:

$$\frac{R}{B} = \log_2(M_0) \; bits/sec/Hz \tag{2.1}$$

where M_0 is the number of possible symbols the transmitter can send during a symbol interval. For binary phase shift keying (BPSK) modulation, since the number of signal waveforms is two, the bandwidth efficiency is 1 bits/sec/Hz, for quadrature phase shift keying (QPSK) modulation bandwidth efficiency is 2 bits/sec/Hz due to 4 different waveforms being used in the modulation.

2.2.2 Signal to Noise Ratio

Signal to noise ratio (SNR) is one of the most important parameters used in digital communication systems. In the general context, it refers to the ratio of signal power to the noise power for a given bandwidth. The most convenient way of expressing SNR is in terms of the average energy per information bit (E_b) over the noise power spectral density N_0 at the receiver. When the channel is varying, like Rayleigh fading channel, the gain due to the channel should be also considered. In Eq. (2.2) the instantaneous SNR per bit at the input of the system is given:

$$\text{SNR}_{(bit)} = \gamma_b = \frac{E_b}{N_0}\alpha^2 \tag{2.2}$$

where α^2 is instantaneous gain from the channel. The average SNR per bit will be denoted by:

$$\text{SNR} = \bar{\gamma}_b = \frac{E_b}{N_0}\overline{\alpha^2} \tag{2.3}$$

and $\overline{\alpha^2}$ represents the average gain of the channel.

For some equations it is necessary to use SNR as average energy per symbol (E_s) over the noise power spectral density. Then the equation becomes

$$\text{SNR}_{(sym)} = \gamma_s = \frac{E_s}{N_0}\alpha^2 \qquad (2.4)$$

In the receiver, during the process of decoding the SNR value could change due to several factors. Diversity systems could increase the SNR via combining of different signals. Whereas channel estimation errors at the receiver or intersymbol interference could act like noise in the system. Therefore they should be considered in the SNR calculations.

2.2.3 Error Probability

Error probability is another important criterion for the communication systems. For digital communication systems, the definition of error is basically, the mismatch between the sent data from the transmitter and the decoded data at the receiver. For example if the bit sent as 1 is decoded as 0, it is a bit error. An incorrectly decoded symbol is a symbol error and if there is one or more bit errors in a frame then there exists a frame error.

The importance of bit error probability or bit error rate (BER) comes from the fact that it permits the performance comparison of different modulation techniques.

In general to compare different systems one of these three parameters is kept fixed and performance curves are obtained. In this work comparisons will be made using input SNR versus BER curves among the systems with the same bandwidth efficiency.

2.2.3.1 Error Probabilities in Rayleigh Fading Channels with BPSK and QPSK Modulation

In this work only slow and flat fading cases are investigated so that the attenuation and phase shift of the signal are considered constant over at least one or more symbol intervals. Let the channel coefficient be:

$$h = \alpha e^{j\theta} \tag{2.5}$$

where α represents the channel gain and θ represents the channel phase at time t. In Eq. (2.5) to simplify the notation time index has been omitted.

Suppose the signal S is transmitted and the multipath components arrive at the receiver without any significant delay differences, (e.g. less than a quarter of a symbol period). Then the received signal r is denoted by:

$$r = hS + n = \alpha e^{j\theta} S + n \tag{2.6}$$

where n is the additive white Gaussian noise. For fading channels, channel gains vary in time as mentioned in Chapter 1. As a result, SNR at the receiver is not constant and every SNR level contributes to the BER in proportion of its probability.

Let γ be the random variable representing the SNR of the received signal and let $p(\gamma)$ be the probability density function of γ. For each value of γ there is a value of the probability of error $p_e(\gamma)$, which depends on the modulation and the detection technique used. To find the probability of error for a signal having mean SNR $\bar{\gamma}$, we should find the average of the error probabilities of the different signal powers due to fading, i.e.,

$$P_e(\bar{\gamma}) = \int p_e(\gamma) p(\gamma) d\gamma \tag{2.7}$$

The SNR per bit in the signal of Rayleigh fading channels has the chi-square distribution, and has the probability density function given by [Pro01, p.817]:

$$p(\gamma_b) = \frac{e^{-\gamma_b/\bar{\gamma}_b}}{\bar{\gamma}_b} \tag{2.8}$$

For BPSK and QPSK modulation with coherent detection, probability of bit error (BER) in terms of γ_b is given by:

$$p_e(\gamma_b) = Q(\sqrt{2\gamma_b}) \tag{2.9}$$

In Eq. (2.9) $Q(\ )$ represents the generalized Marcum Q function. If equations (2.8) and (2.9) are substituted in Eq. (2.7), the result is:

$$P_e(\bar{\gamma}_b) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right] \tag{2.10}$$

and for large values of SNR this can be approximated by:

$$P_e(\bar{\gamma}_b) = \frac{1}{4\bar{\gamma}_b} \tag{2.11}$$

The inverse linear relationship between the signal power and probability of error is obvious in Eq. (2.11).

2.3 Diversity

In this section different diversity methods and different combining techniques are reviewed. As explained in Chapter 1, the main goal of diversity is to improve the BER performance by finding independent or weakly correlated fading channels. By sending replicas of the data from these channels and using some combining techniques, diversity systems reduce the bit error rate of the system for the same SNR values. The way of obtaining independent channels determines the type of the diversity, such as temporal diversity, frequency diversity and spatial diversity.

2.3.1 Temporal Diversity

The idea behind the temporal (time) diversity is the random nature of noise and channel attenuation with respect to time. Even if the channel is constant, since the noise amplitudes at two different moments of time are different, sending the same data more than once can increase the system performance. This is the starting point of the error control coding systems. In wireless systems temporal diversity is not only used to combat noise but also against fading. As explained in Chapter 1, there can be a relative motion between transmitters and receivers in a wireless channel, which results in time variant channels. For this reason the replicas of data are sent to obtain temporal diversity with time interleaving. Naturally the time difference between two replicas should exceed the time that channel is considered as constant (i.e. coherence time) [Rap96 p.338].

Another form of temporal diversity is obtained by combination of channel coding and interleaving. Most of the channel coding methods can correct non-bursty errors in a data frame, hence the system first encodes the data bits and interleave coded bits, due to severe attenuation; a part of the frame could be received erroneously. After deinterlaving, these errors are spread in the data bits and channel coding can correct these errors [Rap96 p.339].

Note that temporal diversity is not applicable if the communication system is very sensitive to delay and the channel has a long coherence time.

2.3.2 Frequency Diversity

In frequency diversity, the signals are transmitted at different carrier frequencies, which are separated by more than the coherence bandwidth of the channel. Hence if there is fading at one frequency, the information can be gathered from the other frequencies. Such a system is employed in microwave line-of-sight links [Rap96 p.335]. Basically there is a backup frequency, which is idle and if there is fading in one frequency in the system then the traffic is switched to the backup frequency.

Another way of obtaining the frequency diversity is by using spreading codes to exceed several times the coherence bandwidth of the channel like in CDMA systems. Therefore the fading will occur only in one portion of the frequency band while the information can still be acquired from the other parts. In outdoor environments if the channel is not flat i.e. system bandwidth is larger than the coherence bandwidth of the channel, there can be significant time difference between the multipath components and since those components are bearing the same information, they can be combined using a RAKE receiver.

2.3.3 Space Diversity

The diversity is obtained from the channels that are independent in spatial domain due to spatially separated antennas [Tar98]. The importance of space diversity is the fact that it does not introduce any delay or bandwidth expansion. The major disadvantages of spatial diversity are the increase in the complexity and the cost of the system.

Receive diversity and transmit diversity are the main types of the space diversity used in cellular wireless communication systems.

2.3.3.1 Receive Diversity

In receive diversity, sufficiently spaced apart multiple antennas are deployed at the receiver side to obtain independent channels. For example, in cellular communication this can be easily done at the base stations. The purpose is to increase link margin and co-channel interference suppression and to compensate low power transmission from mobile systems [Tar98, Win94]. The received data is combined according to one of the combining techniques which will be explained in the Section 2.4.

2.3.3.2 Transmit Diversity

In transmit diversity systems, data are sent from several antennas, which are deployed far apart to generate independent channels in the system. Unlike receive diversity, transmit diversity requires either feedback information or proper coding of the transmitted signals. Wittneben proved [Wit93] that without precoding no diversity gain could be obtained for schemes without feedback. The proof and detailed analysis of transmit diversity systems with precoding will be given in the next chapter, where an analysis of space-time coding systems is performed in addition.

2.4 Combining Techniques for Diversity Systems

In the previous section the types of diversity and the ways of obtaining independent channels have been explained. The aim of this section is to explain different ways of combining the received signals from those channels so as to improve the system performance. Without taking the delay into consideration, in Figure 2.1 the basic lowpass equivalent structure for diversity systems excluding transmit diversity is shown.

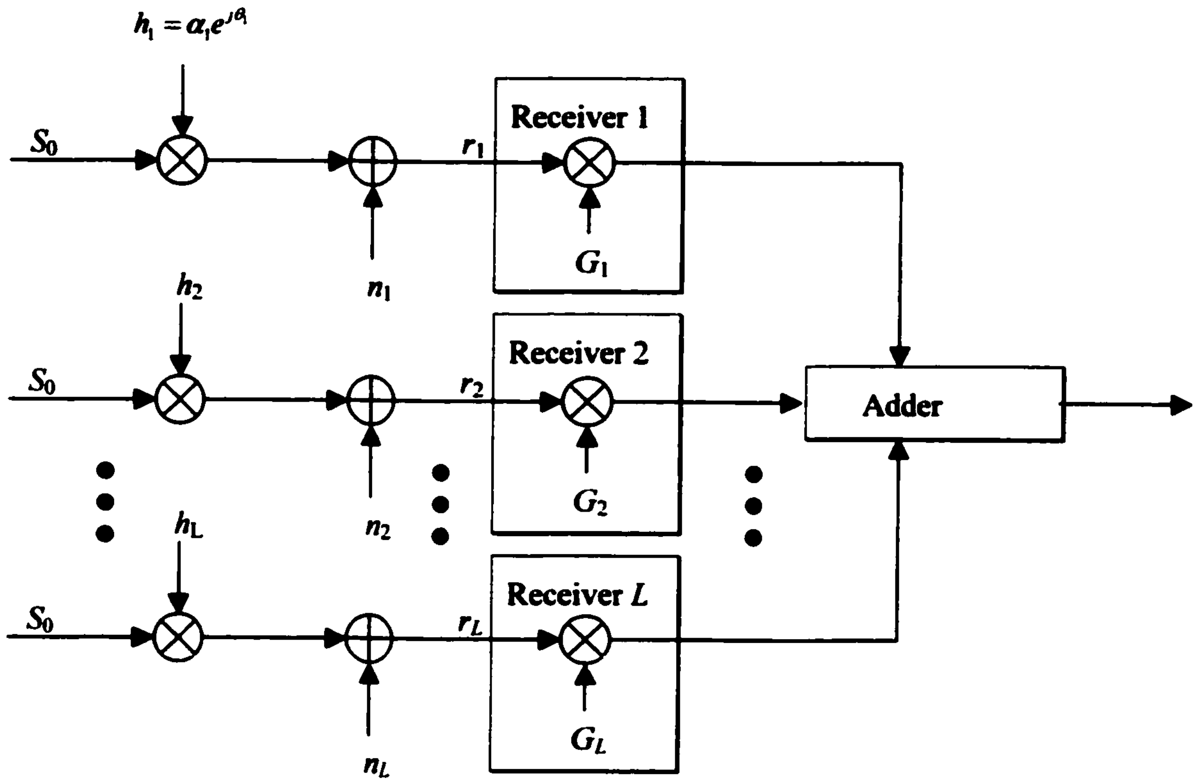


Figure 2.1 Basic scheme of diversity systems excluding transmit diversity.

In Figure 2.1 S_0 is the transmitted symbol, h_i is the complex channel coefficient of the i^{th} channel with amplitude α_i and phase θ_i. The AWGN coefficient n_i has a variance $N_{0,i}$ at the i^{th} receiver. The received signal at branch i is:

$$r_i = h_i S_0 + n_i = \alpha_i e^{j\theta_i} S_0 + n_i, \qquad i = 1,2.....L \qquad (2.12)$$

Suppose the complex gain introduced from i^{th} path is G_i, then the total received signal is

$$r_T = \sum_{i=1}^{L} G_i r_i \tag{2.13}$$

The way of choosing the gain coefficients determines the type of combining used and the performance of the system. Three combining schemes, which are studied in this work are selection combining (SC), equal gain combining (EGC) and maximum ratio combining (MRC) [Pro01] [Stu96] [Rap96].

Selection combining is the most basic combining technique. Only the channel which has the highest SNR value is fed to the system and the rest is neglected. In a practical system this is done by switching circuits. In Figure 2.1 selection combining corresponds to the situation with only one of the complex gains G_i being equal to 1 and the rest are zeros.

In equal gain combining, unlike selection combining, all of the branches are taken into consideration and the channels are co-phased and combined. In Eq. (2.12) every channel has a phase denoted by θ_i. For equal gain combining the gain coefficients G_i are chosen to be $e^{-j\theta_i}$. As a result the combined signal is:

$$r_T = S_0 \sum_{i=1}^{L} \alpha_i + \sum_{i=1}^{L} e^{-j\theta_i} n_i \tag{2.14}$$

For this combining scheme, even if all the branches are lower then a certain threshold, it is probable to obtain an SNR value higher than the threshold at the end of the combiner, which is impossible for selection combining.

The optimum combining scheme in terms of the increase in the SNR is maximal ratio combining, where the gain coefficients are chosen proportional to $G_i = h_i^* / N_{0,i}$. Since the transmit diversity systems that we are going to study aim to obtain the performance of MRC systems, we study the SNR improvement MRC system in more detail.

The combined signal for this system is:

$$r_T = S_0 \sum_{i=1}^{L} \frac{\alpha_i^2}{N_{0,i}} + \sum_{i=1}^{L} \frac{h_i^*}{N_{0,i}} n_i \qquad (2.15)$$

Let the instantaneous SNR per bit value at branch i be

$$SNR = \gamma_i = \frac{E_b}{N_{0,i}} \alpha_i^2 \qquad (2.16)$$

Considering L independent branches, the combined signal has the resulting instantaneous SNR:

$$\gamma' = \frac{\left| \sum_{i=1}^{L} \frac{\alpha_i^2}{N_{0,i}} \right|^2 E_b}{\sum_{i=1}^{L} \frac{\alpha_i^2}{N_{0,i}^2} N_{0,i}} = \sum_{i=1}^{L} \frac{\alpha_i^2 E_b}{N_{0,i}} = \sum_{i=1}^{L} \gamma_i \qquad (2.17)$$

Eq. (2.17) shows that the instantaneous SNR of the combined signal is the sum of SNR's at every branch. As a result, the average SNR of the MRC system (SNR_{MRC}) is total of the average SNR's at every branch.

$$SNR_{MRC} = \sum_{i=1}^{L} \bar{\gamma}_i \qquad (2.18)$$

If the branches have the same average SNR value $\bar{\gamma}$ then,

$$SNR_{MRC} = L\bar{\gamma} \qquad (2.19)$$

Maximum likelihood decision is the optimum decision method for equally likely symbols. In the next section we study, how maximum ratio combining and maximum likelihood decisions are combined in receive diversity systems.

2.4.1 Maximum Ratio Combining with Maximum Likelihood Decisions

Suppose the symbol S_0, among all possible symbols, is sent in a system with L diversity branches, then the resulting received signal at the l^{th} branch is:

$$r_l = h_l S_0 + n_l, \qquad l = 1,2.....L \tag{2.20}$$

where n_l denotes the additive noise. Assuming the additive noise is Gaussian distributed and has equal power at every branch, the symbol S_i is decoded for the transmitted symbol, if it minimizes the decision metric (2.21) of the maximum likelihood decision for the case of equal probability symbols.

$$\sum_{l=0}^{L} d^2(r_l, h_l S_i) \tag{2.21}$$

where $d^2(x,y)$ is the squared Euclidean distance between signals x and y, which is equal to:

$$d^2(x,y) = (x-y)(x^* - y^*) \tag{2.22}$$

In the receiver, the decision variable $\widetilde{S}$ is constructed from the equation below:

$$\widetilde{S} = \sum_{l=0}^{L} h_l^* r_l = \sum_{l=0}^{L} (h_l\ ^2 S_0 + h_l^* n_l) \tag{2.23}$$

Combination of metric in (2.21) and equations (2.22), (2.23) results in the decision metric below,

$$\sum_{l=0}^{L} h_l\ ^2\ S_i\ ^2 - \widetilde{S} S_i^* - \widetilde{S}^* S_i + r_l\ ^2 \tag{2.24}$$

Since the last term is constant for all S_i values, it can be ignored in the minimization process and the rest of the metric is equal to:

$$(\sum_{l=0}^{L} h_l^{\,2} S_l^{\,2}) - S_l^{\,2} - \tilde{S}^2 + d^2(\tilde{S}, S_l) \tag{2.25}$$

If the energy of all possible symbols is the same as, in the case of PSK modulation, the first three terms in the metric (2.25) are constants. Therefore if $d^2(\tilde{S}, S_l)$ is minimized, the decision metric (2.21) is minimized too.

Since the system uses a linear decoding algorithm, i.e. just multiplications and additions of the received symbols and comparing one variable with the symbols in the constellation, it is easy to implement. In Figure 2.2 the structure of a Maximum Ratio Combining for receive diversity is illustrated. If all diversity branches are independent and have the same average SNR per bit $\bar{\gamma}_b$ then bit error probability of BPSK and QPSK modulations in Rayleigh fading is [Pro01 p.825]:

$$P_e(\bar{\gamma}_b) = \left[\frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right)\right]^L \sum_k^{L-1}\left(\binom{L-1+k}{k}\left[\frac{1}{2}\left(1+\sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right)\right]^k\right) \tag{2.26}$$

where L denotes the number of branches. BER performance of those modulations for 2 and 4 branches are shown in Figure 2.3.

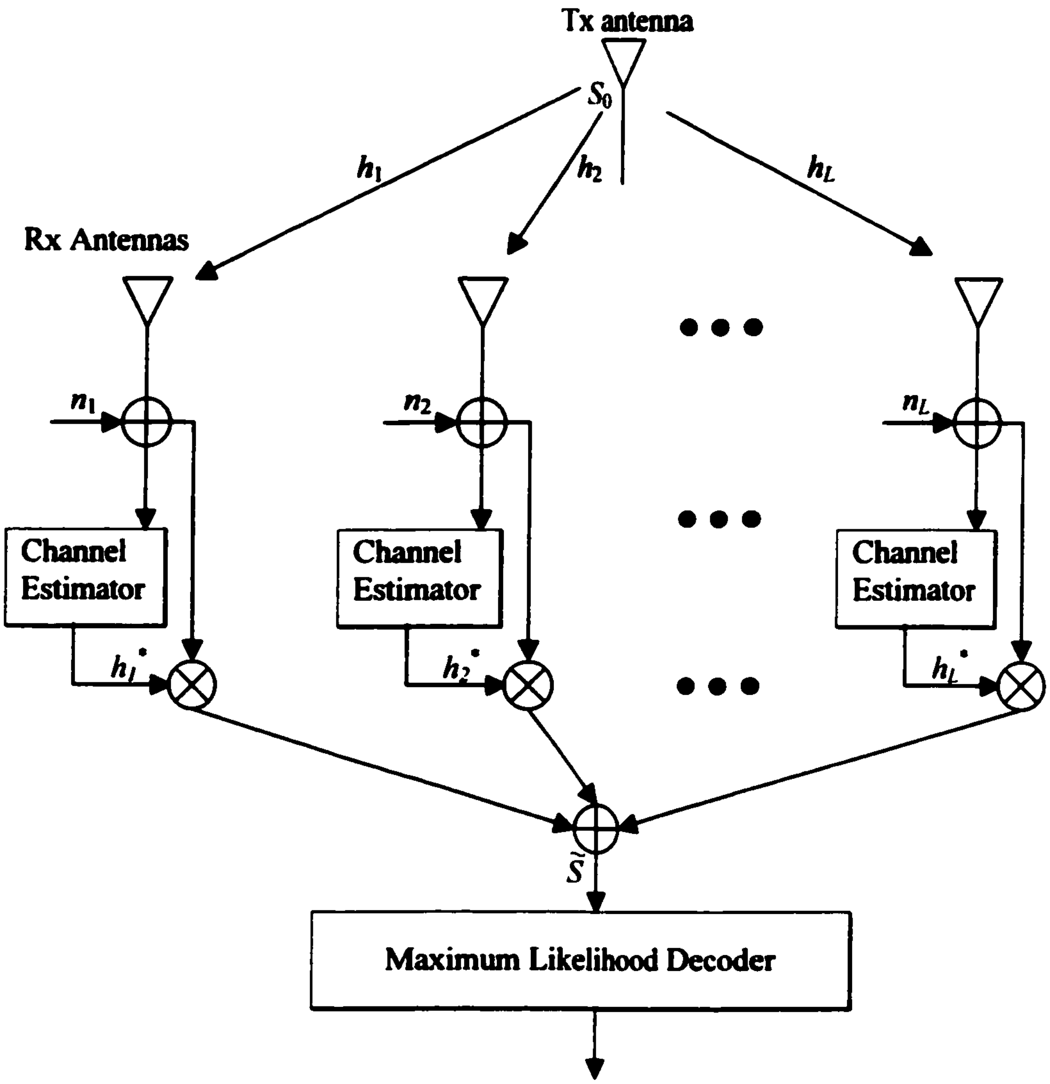


Figure 2.2 Maximum ratio combining structure for same noise power spectral density at every branch.

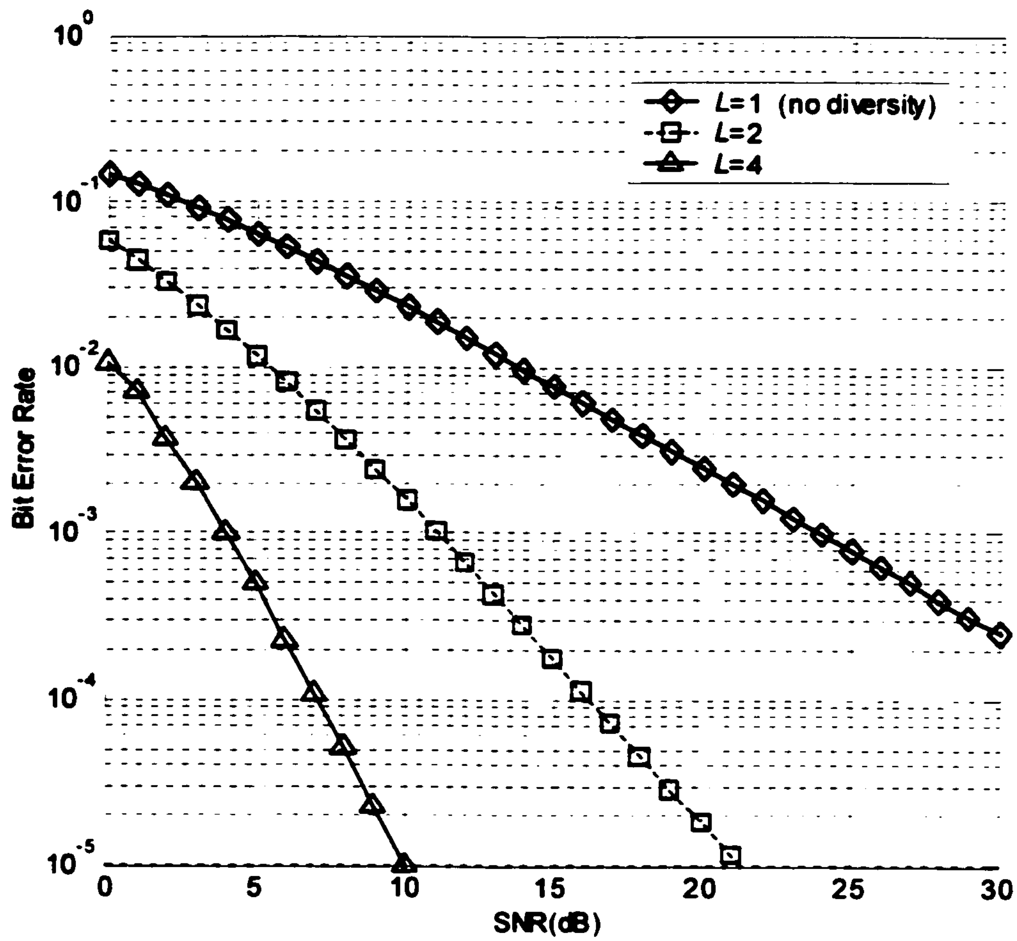


Figure 2.3 Bit error rate performance of BPSK and QPSK modulations in Rayleigh fading channel utilizing L branches.

2.5 Summary

This chapter provided the necessary information to evaluate the performance of diversity systems and reviewed different types of diversity systems. Among the combining techniques explained in this chapter MRC is the most important one since the aim of space-time block codes is to obtain the performance of this combining scheme.

CHAPTER 3

Transmit Diversity

3.1 Introduction

In Chapter 2 different types of diversity techniques and combining methods were investigated. In this chapter we focus only on transmit diversity schemes. First transmit diversity without precoding is studied. Afterwards we focus on feedback in transmit diversity. Finally the schemes without feedback, especially space-time coding are investigated.

3.2 Transmit Diversity without Precoding

For temporal, frequency and receive diversity, it is sufficient to sent replicas of data at the transmitter side from independent channels to obtain diversity gain. In this section we check the validity of this statement for transmit diversity systems.

Suppose the same symbol is sent from N transmit antennas to one receive antenna at the same time interval, then the total received signal at the receiver is:

$$r_T = (\sum_{i=1}^{N} h_i S_0) + n \tag{3.1}$$

At the transmitter side the energy per symbol E_s is the sum of energy per symbols at each antenna. If the energy per symbol is divided equally to transmit antennas then the instantaneous SNR at the receiver is:

$$SNR_{Tr} = \frac{E_s / N}{N_{0,i}} \sum_{i=1}^{N} |h_i|^2 = \frac{E_s / N}{N_{0,i}} \sum_{i=1}^{N} \alpha^2 \tag{3.2}$$

In Eq. (3.2), if the channel coefficients are independent and Rayleigh distributed, then gain from the channels is the sum of each gain of a channel. The SNR for channels with equal gain α^2 becomes:

$$SNR_{Tr} = \frac{E_s / N}{N_{0,i}} N\alpha^2 = \frac{E_s}{N_{0,i}} \alpha^2 \tag{3.3}$$

Eq. (3.3) shows that the SNR of the system is not increased. Therefore it offers no diversity gain. Sending the replicas is not sufficient but necessary for transmit diversity systems. A more generalized proof of this statement can be found in [Wit93].

3.3 Transmit Diversity with Feedback

For transmit diversity systems, the signal at the receiver is the superposition of all signals sent from the transmit antennas. The gain coefficients mentioned in the previous chapter cannot be applied at the receiver side for every branch, but they can be applied at the transmitter side as shown in Figure 3.1 to obtain diversity. Such a scheme is termed "maximum ratio transmission" in [Lo99].

The aim of the feedback system is again to maximize the signal to noise ratio at the receiver [Cal02]. For receive diversity systems the maximum diversity gain is obtained by choosing the gain coefficients G_i proportional to $h_i^*/N_{0,i}$. Since the same noise affects all the channels in transmit diversity systems, it is sufficient to choose gain coefficients proportional to h_i^*. As a result, if the amplitude and phase of the channels are known at the transmitter side, the diversity order of maximal ratio combining can be obtained. If only amplitudes or only phases are known, then the maximum diversity gain is not attainable, but still the other combining methods can be applied. For the case of known amplitudes, the diversity gain of selection combining is achieved by sending the data from the transmitter antenna, which has the highest SNR among the channels.

For the case of known phases, by predistorting the phase of the symbol at the ith antenna with the gain coefficient G_i equals to $e^{-j\theta_i}$, equal gain combining could be achieved.

The main drawbacks of the feedback systems are decreasing the total throughput or making the system more complex. As well they are unfeasible for some systems as explained in Chapter 1. Cavers shows also in [Cav00] that their performance is not good in multiuser systems. Due to this drawbacks, transmit diversity systems without feedback are considered to obtain diversity as well.

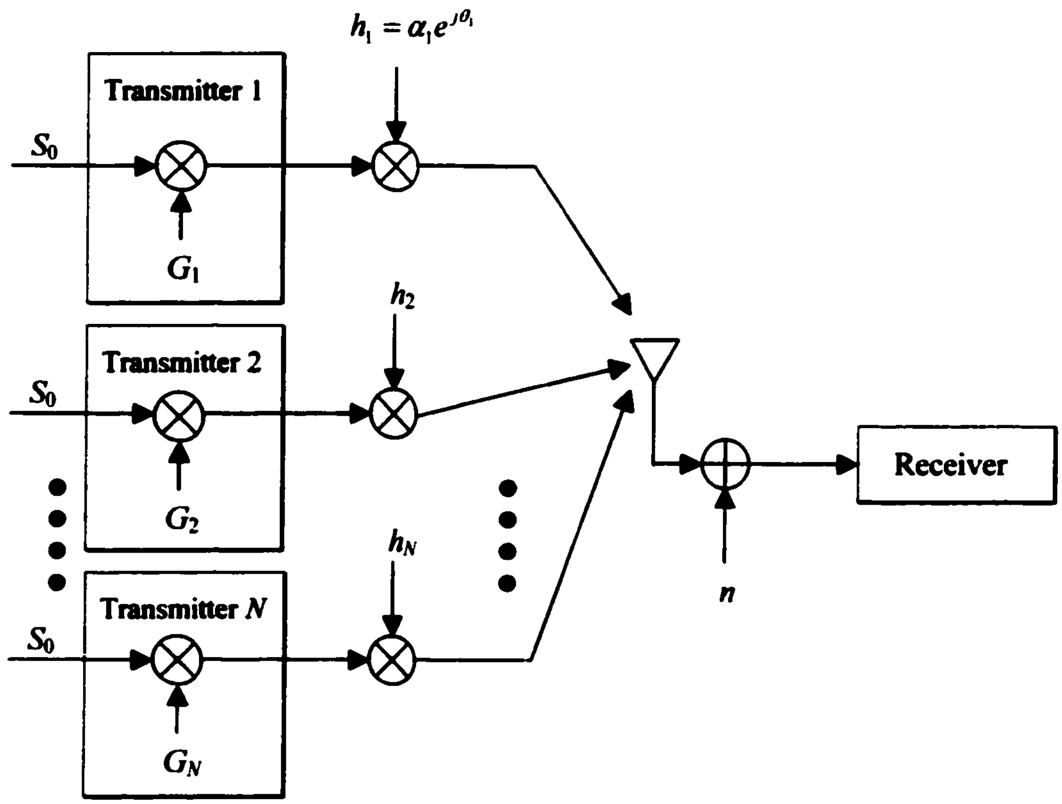


Figure 3.1 Basic receiving scheme for a transmit diversity system with feedback, where G coefficients are sent by receiver.

3.4 Transmit Diversity without Feedback

In systems without feedback the channel state information is not known at the transmitter side. Therefore intelligent precoding schemes should be applied at the transmitter side to obtain diversity gain.

There are two subcategories for transmit diversity systems without feedback. In the first category feed forward information is necessary to obtain the diversity gain. This information is used to estimate the channel between the receiver and transmitter. The examples of such systems are delay diversity of Seshadri and Winters [Ses94], space-time trellis codes which were introduced first by Tarokh et al, [Tar98] and space-time block codes which was introduced by Alamouti [Ala98]. The second category does not require any feedback or feed forward information to obtain diversity gain. One of the schemes in this category uses channel encoding to send symbols from different antennas in orthogonal manner, using frequency multiplexing [Cim96], time multiplexing [Ses94] or using orthogonal spreading sequences. The disadvantage of this category over the previous category is the bandwidth expansion of the system, which decreases total throughput. Most wireless systems are bandwidth limited, to serve a maximum number of users and they use feed forward information for detection of symbols. For these reasons the first category will be studied more deeply in this work.

3.4.1 Diversity via Delay Diversity

In this system, the same information symbol is transmitted with delay differences between the transmit antennas as shown in Figure 3.2. At the receiver side maximum likelihood sequence estimation is used to decode the symbols.

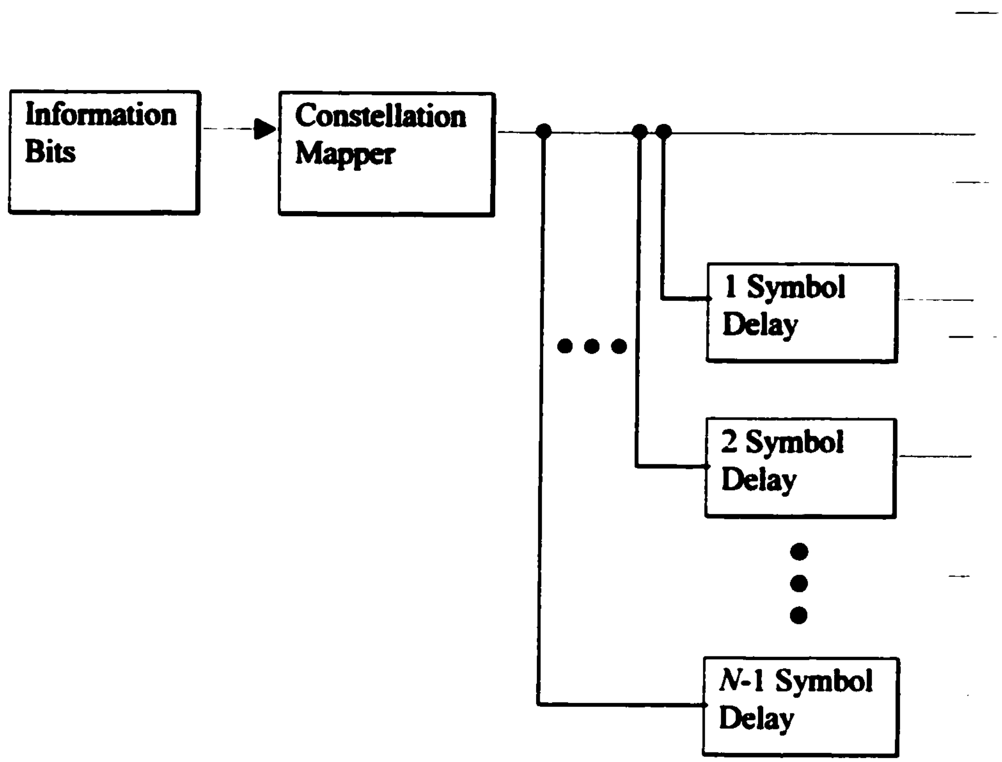


Figure 3.2 Delay diversity structure.

From the coding point of view, the system uses a repetition code for channel coding with the rate $1/N$, where N is the number of transmit antennas in the system. Tarokh et al, studied the channel codes for the best performance curves in [Tar98] and came up with the notion of space-time trellis codes. Before explaining the trellis coded systems, first the parameters for space-time trellis codes will be introduced.

3.4.2 Parameters for Space-Time Coded Systems

In evaluating the system performance, error probability, SNR and bandwidth efficiency are among the parameters, which are valid both for coded and uncoded systems. There are some parameters which are applicable only to coded systems, such as minimum distance for block coded systems and free distance for trellis coded systems. Tarokh et al, [Tar98] suggested some performance measures for space-time coded systems in fading channels. Suppose data are sent in frames. Assuming that the system consists of M receive and N transmit antennas, the received signal at the decoder of the receiver antenna j at time t can be given under perfect channel knowledge by:

$$r_j(t) = \sum_{i=1}^{N} h_{i,j}(t) c_t^i \sqrt{E_s} + n_j(t) \quad ,1 \le j \le M \tag{3.4}$$

where, $h_{i,j}(t)$ is the channel coeffiecient from the i^{th} transmit antenna to the j^{th} receive antenna at time t. c_t^i is the codeword at i^{th} transmit antenna at time t, E_s is the average energy per coded symbol and $n_j(t)$ is additive white Gaussian noise with variance $N_0/2$ per dimension.

We assume that the frames consist of l symbols and the channel path gains are constant over the frame length. Let

$$c = c_1^1 c_1^2 \cdots c_1^N c_2^1 \cdots c_l^1 c_l^2 \cdots c_l^N \tag{3.5}$$

be the code vector transmitted and the decoder decides erroneously for

$$e = e_1^1 e_1^2 \cdots e_1^N e_2^1 \cdots e_l^1 e_l^2 \cdots e_l^N \tag{3.6}$$

then the average pair wise error probability is approximated by [Tar98]

$$P\left(c \to e \middle| \alpha_{i,j}, i = 1,2,..N, j = 1,2,..M\right) \le \exp(-d^2(c,e)E_s / 4N_0) \tag{3.7}$$

For equal energy systems, the distance function $d^2(c,e)$ is equal to

$$d^2(c,e) = \sum_{j=1}^{M} \sum_{t=1}^{l} \left| \sum_{i=1}^{N} h_{i,j}(c_t^i - e_t^i) \right|^2 \tag{3.8}$$

By setting $\mathbf{\Omega}_j = (h_{1,j}, \cdots, h_{n,j})$ and its Hermitian (transpose conjugate) as $\mathbf{\Omega}_j^*$ and

$$\mathbf{A}(c,e) = \mathbf{B}(c,e)\mathbf{B}^*(c,e) \tag{3.9}$$

where **B** is the error matrix between the transmitted *c* and received *e* code sequences such that

$$\mathbf{B}(c,e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_l^2 - c_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & e_l^n - c_l^n \end{bmatrix} \tag{3.10}$$

then Tarokh et al, [Tar98] showed that the distance function is equal to:

$$d^2(c,e) = \sum_{j=1}^{m} \mathbf{\Omega}_j \mathbf{A}(c,e) \mathbf{\Omega}_j^* \tag{3.11}$$

Since $\mathbf{A}(c,e)$ is a Hermitian matrix, due to Eq. (3.9) $\mathbf{A}(c,e)$ can be written as $\mathbf{U \Lambda U}^*$ where Λ is the diagonal matrix consisting of the eigen values of $\mathbf{A}(c,e)$.

$$\Lambda = diag(\lambda_1, \lambda_1, \cdots \lambda_N) \tag{3.12}$$

U is the orthonormal matrix whose columns are the eigen vectors of $\mathbf{A}(c,e)$. Let $\beta_j = \mathbf{U}^* \mathbf{\Omega}_j^*$, then,

$$d^2(c,e) = \sum_{j=1}^{M} \mathbf{\Omega}_j \mathbf{A}(c,e) \mathbf{\Omega}_j^* = \sum_{j=1}^{M} \beta_j^* \Lambda \beta_j = \sum_{j=1}^{M} \sum_{j=1}^{N} \lambda_j \left| \beta_{ij} \right|^2 \tag{3.13}$$

For Rayleigh fading channels $\left| \beta_{ij} \right|^2$ has Rayleigh distribution and from equations (3.7) and (3.13) the probability of error can be approximated by:

$$P(\mathbf{c} \to e) \le \left(\frac{1}{\prod_{i=1}^{N} (1 + \lambda_i E_s / 4N_0)} \right)^M \tag{3.14}$$

If the rank of matrix **A** is r then **A** has r nonzero eigen values and the approximation becomes:

$$P(\mathbf{c} \rightarrow \boldsymbol{e}) \leq \left(\prod_{i=1}^{r} \lambda_i\right)^{-M} \left(\frac{1}{E_s/4N_0}\right)^{rM} \tag{3.15}$$

3.4.2.1 The Rank Criterion

From Eq. (3.15), two criterions for space-time coding systems can be seen.

$$\left(\frac{1}{E_s/4N_0}\right)^{rM} \tag{3.16}$$

is the diversity gain part of the Eq. (3.15) and is related to the rank of matrix **B**(***c***,***e***), which is also the rank of matrix **A**(***c***,***e***). To have the maximum available diversity order *NM*. **B**(***c***,***e***) has to be full rank for any pair of distinct code vectors ***c*** and ***e***. Otherwise the minimum rank r of **B**(***c***,***e***) over all set of pairs will determine the diversity order as rM.

3.4.2.2 The Determinant Criterion

The coding gain of the system is determined by

$$\left(\prod_{i=1}^{r} \lambda_i\right)^{-M} \tag{3.17}$$

which is equal to the absolute value of the sum of determinants of all principal rxr cofactors of **A**(***c***,***e***). Therefore the minimum of the r^{th} roots of the sum of determinants of all principal rxr cofactors of **A**(***c***,***e***) over all pairs of distinct code vectors ***c*** and ***e*** corresponds to coding gain [Cal02]. These are the criterions for quasi-static Rayleigh distributed fading channels. For fast fading channel other measures are obtained in [Tar98].

3.4.3 Space-Time Trellis Codes

Using the parameters explained in Section 3.3.2, Tarokh et al, [Tar98] constructed several space-time trellis codes. In Figure 3.3, a space-time trellis code for 4-PSK modulation and 2 transmit antennas is given. This code has 4 states and the symbols are transmitted according to the trellis diagram depending on the current state and next state. The information bits determine state values. An example of information bits and corresponding symbols are given in Figure 3.3. The trellis is set to 00 state at the beginning. The first bits are 10, therefore there is a transition from the state 00 to state 10 and signals s_0 and s_2 are sent from the transmitter antennas 0 and 1 respectively. The second set of bits are 11, which makes the next state 11 and the transmitter antenna 0 sends the symbol s_2, while the transmitter antenna 1 sends the symbol s_3. At the end of the data frame the trellis ended again in zero state.

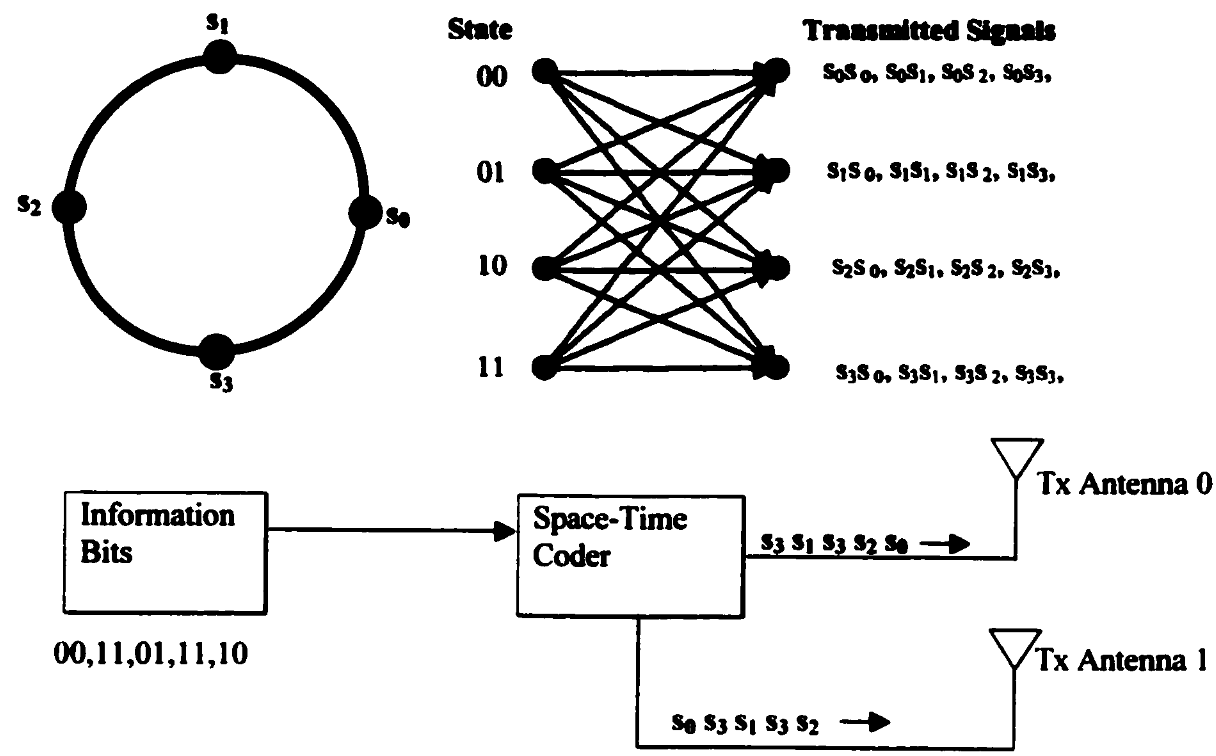


Figure 3.3 A space-time trellis code for two transmit antennas.

We have explained the encoding process of the space-time trellis codes for N transmit and M receive antennas. Let us look at the encoding process of these codes in the presence of quasistatic flat fading channel and assuming ideal channel state information, the channel coefficients $h_{i,j}$ $i=1,2...N$, $j=1,2...M$, are known at the receiver side. The received signal at time t at the j^{th} receive antenna could be written as:

$$r_j(t) = \sum_{i=1}^{N} h_{i,j} s^i(t) + n_j(t) \tag{3.18}$$

where $s^i(t)$ is the symbol sent from the i^{th} transmit antenna at time t. $n_j(t)$ is the noise coefficient at the j^{th} receive at time t.

The optimum decoding scheme of such a system is maximum likelihood sequence estimation (MLSE), which searches the path trough the trellis that minimizes the total Euclidean distance between the received data frame and all possible transmitted symbols. The total Euclidian distance between a received data frame length l and a set of possible transmitted symbols can be written as:

$$\sum_{t=1}^{l} \sum_{j=1}^{M} \left| r_j(t) - \sum_{i=1}^{N} h_{i,j}(t) \tilde{s}^i(t) \right|^2 \tag{3.19}$$

where $\tilde{s}^i(t)$ is a possible transmitted symbol from ith antenna at time t. The set of symbols, which minimizes the above metric, is chosen by the decoder.

Since their introduction the space-time trellis codes have been studied extensively and codes with better performances are introduced, for example in [Che01] [Yan00] [Ion01] [Tao01]. The codes of better performance are obtained either by computer search or changing the determinant criterion for different SNR regions or the number of transmit and receive antennas. Research shows that coding gain is important when the SNR is low or the number of receive antennas is large. Nevertheless rank criterion still determines the diversity gain introduced from the transmitter side and space-time block coded systems are mainly based on the rank criterion.

3.4.4 Space-Time Block Codes

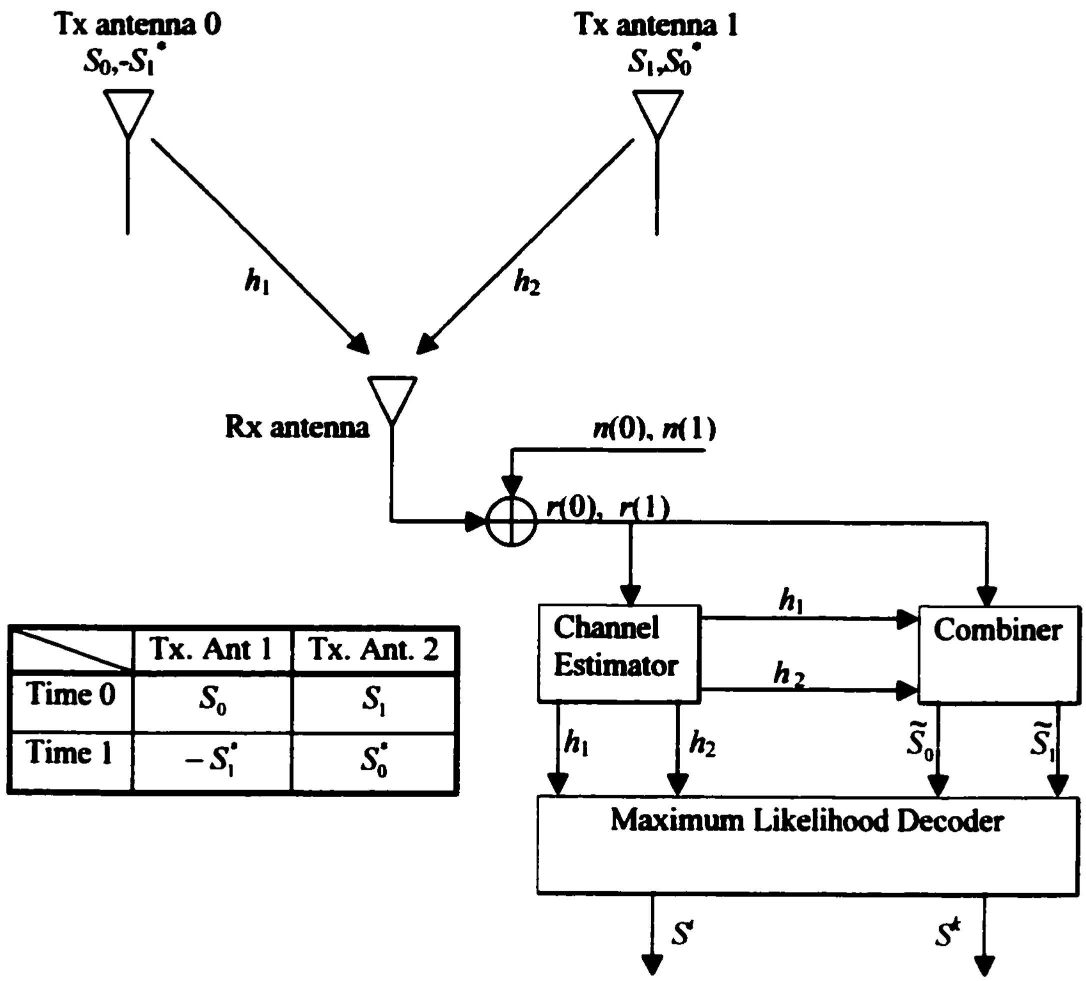
In [Tar98] Tarokh et al, show that no block code (that admits a trellis representation)can outperform their space-time trellis codes in terms of the tradeoff between diversity gain, rate, and trellis complexity. Despite this drawback, space-time block codes are much less complex than the space-time trellis codes and they provide full diversity gain.

To explain the properties and the coding scheme of transmit diversity systems Alamouti's simple transmit diversity scheme will be explained in the next section.

3.4.4.1 Simple Transmit Diversity

The simple transmit diversity (STD), is shown in Figure 3.1. The system employs two transmit antennas and there is no restriction for the number of receive antennas.

To show the effect of transmit diversity, the system for only one receive antenna is introduced first. The extension of the system to more than one receive antenna will be explained later. It is assumed that the channels between the transmit antennas and the receive antenna are quasi-static, i.e. constant for the time slot of a block and independent from one block to another one. Time 0 and 1 correspond to two consecutive symbol periods. The complex channel from the k^{th} transmit antenna to the receive antenna at times 0 and 1 is denoted by h_k. Let S_0 and S_1 denote two symbols to be transmitted over two signaling intervals, and let S_0^* denote the complex conjugate of S_0. At times 0 and 1 the symbols are sent according to the antenna-time slot correspondence table shown in Figure 3.4 The noise samples at times 0 and 1 are denoted by $n(0)$ and $n(1)$ respectively and they are independent samples of a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension.



	Tx. Ant 1	Tx. Ant. 2
Time 0	S_0	S_1
Time 1	$-S_1^*$	S_0^*

Figure 3.4 Structure of simple transmit diversity.

The received signals for time 0 and time 1 are:

$$r(0) = h_1 S_0 + h_2 S_1 + n(0) \tag{3.20}$$

$$r(1) = -h_1 S_1^* + h_2 S_0^* + n(1) \tag{3.21}$$

respectively. In section 2.4.1 the maximum likelihood decision for maximum ratio combining was explained. For this transmit diversity system the maximum likelihood decision metric has the form below:

$$[d^2(r(0),(h_1S^i + h_2S^k)) + d^2(r(1),(-h_1S^{k*} + h_2S^{i*}))] \tag{3.22}$$

The symbols S^i, S^k are decoded as the transmitted symbols for S_0 and S_1 among all possible symbols from the given constellation, if they minimize the decision metric in (3.22). That metric is also equal to:

$$\begin{aligned}&\Big[|r(0)|^2 - r(0)(h_1S^i + h_2S^k)^* - r(0)^*(h_1S^i + h_2S^k) + |h_{1,m}S^i + h_{2,m}S^k|^2 \\ &+ |r(1)|^2 - r(1)(-h_1S^{k*} + h_2S^{i*})^* - r(1)^*(-h_1S^{k*} + h_2S^{i*}) + |-h_1S^{k*} + h_2S^{i*}|^2\Big]\end{aligned} \tag{3.23}$$

If the metric is expanded and the terms independent from the codewords are deleted, the following metric is obtained:

$$\begin{aligned}&\Big[-r(0)h_1^*S^{i*} - r(0)^*h_1S^i + |h_1|^2|S^i|^2 - r(1)h_2^*S^i - r(1)^*h_2S^{i*} + |h_2|^2|S^i|^2\Big] \\ &+\Big[-r(0)h_2^*S^{k*} - r(0)^*h_2S^k + |h_2|^2|S^k|^2 + r(1)h_1^*S^k + r(1)^*h_1S^{k*} + |h_1|^2|S^k|^2\Big]\end{aligned} \tag{3.24}$$

In (3.24) it can be seen that the metric consists of two parts. One part is a function of S^k and the other one is a function of S^i. Therefore the minimization of (3.22) is equivalent to the minimization of these two parts separately. Using mathematical techniques the metric in (3.25) is constructed for S^i:

$$d^2\left(r(0)h_1^* + r(1)^*h_2, S^i\right) + \left(|h_1|^2 + |h_2|^2 - 1\right)|S^i|^2 \tag{3.25}$$

The metric (3.26) for S^k is

$$d^2\left(r(0)h_2^* - r(1)^*h_1, S^k\right) + \left(|h_1|^2 + |h_2|^2 - 1\right)|S^k|^2 \tag{3.26}$$

In case of equal energy symbols (like PSK modulation), the metrics will reduce:

$$d^2\left(r(0)h_1^* + r(1)^* h_2, S^i\right) \tag{3.27}$$

$$d^2\left(r(0)h_2^* - r(1)^* h_1, S^k\right) \tag{3.28}$$

for S^i and S^k respectively.

From metrics (3.27), (3.28) it can be seen that two decision variables should be created in the receiver part, which are:

$$\widetilde{S}_0 = r(0)h_1^* + r(1)^* h_2 \tag{3.29}$$

$$\widetilde{S}_1 = r(0)h_2^* - r(1)^* h_1 \tag{3.30}$$

Then these variables are fed into a maximum likelihood decision decoder for the final part of the process. The expansions of the decision variables in equations (3.29) and (3.30) lead to:

$$\begin{aligned}\widetilde{S}_0 &= r(0)h_1^* + r(1)^* h_2 \\ &= \left(h_1 S_0 + h_2 S_1 + n(0)\right)h_1^* + \left(-h_1 S_1^* + h_2 S_0^* + n(1)\right)^* h_2 \\ &= |h_1|^2 S_0 + h_1^* h_2 S_1 + h_1^* n(0) - h_2 h_1^* S_1 + |h_2|^2 S_0 + h_2 n(1) \\ &= |h_1|^2 S_0 + |h_2|^2 S_0 + h_1^* n(0) + h_2 n(1)\end{aligned} \tag{3.31}$$

$$\begin{aligned}\widetilde{S}_1 &= r(0)h_2^* - r(1)^* h_1 \\ &= \left(h_1 S_0 + h_2 S_1 + n(0)\right)h_2^* - \left(-h_1 S_1^* + h_2 S_0^* + n(1)\right)^* h_1 \\ &= h_2^* h_1 S_0 + |h_2|^2 S_1 + h_2^* n(0) + |h_1|^2 S_1 - h_1 h_2^* S_1^* - h_1 n(1) \\ &= |h_1|^2 S_1 + |h_2|^2 S_1 + h_2^* n(0) - h_1 n(1)\end{aligned} \tag{3.32}$$

These equations show that, the system provides full diversity order, since both variables have the combination from two independent channels for their symbols.

From Eq. (3.31), we can calculate the SNR for the decision variable $\widetilde{S}_0$. Due to the symmetry in the system the other decision variable also has the same SNR. If the energy per information bit is E_b then for a fair comparison between STD and receive diversity systems, every symbol in STD should have half energy per bit, since a symbol is sent twice in a block. The signal to noise ratio of STD system ($\mathrm{SNR}_{\mathrm{STD}}$) is:

$$\mathrm{SNR}_{\mathrm{STD}} = \frac{\frac{E_b}{2}\left(\sum_{l=0}^{2}\overline{\alpha^2}\right)^2}{\sum_{l=0}^{2}\overline{\alpha^2}N_0} = \frac{E_b\sum_{l=0}^{2}\overline{\alpha^2}}{2N_0} \tag{3.33}$$

Eq. (3.33) shows that STD system with one receive antenna provides 3 dB less diversity gain than MRC system with 2 receive antennas.

For an STD system utilizing M receive antennas, the received signal for time 0 and 1 at the m^{th} receive antenna is denoted by:

$$r_m(0) = h_{1,m}S_0 + h_{2,m}S_1 + n_m(0) \tag{3.34}$$

$$r_m(1) = -h_{1,m}S_1^* + h_{2,m}S_0^* + n_m(1) \tag{3.35}$$

For maximum likelihood decision, the decision metric is now:

$$\sum_{m=0}^{M}\left[d^2(r_m(0),(h_{1,m}S^i + h_{2,m}S^k)) + d^2(r_m(1),(-h_{1,m}S^{k*} + h_{2,m}S^{i*}))\right] \tag{3.36}$$

After some mathematical manipulations the metrics for S^i and S^k are decoupled again and the metric (3.37) for S^i is:

$$d^2\left(\left[\sum_{m=1}^{M} r_m(0)h_{1,m}^* + r_m(1)^* h_{2,m}\right], S^i\right) + \left(\sum_{m=1}^{M}\left[|h_{1,m}|^2 + |h_{2,m}|^2\right] - 1\right)|S^i|^2 \tag{3.37}$$

Similarly the metric (3.38) for S_k is:

$$d^2\left(\left[\sum_{m=1}^{M} r_m(0)h_{2,m}^* - r_m(1)^* h_{1,m}\right], S^k\right) + \left(\sum_{m=1}^{M}\left[h_{1,m}^{\ 2} + h_{2,m}^{\ 2}\right] - 1\right) S^{k\ 2} \tag{3.38}$$

In case of equal energy signals, these metrics will reduce to:

$$d\left(\left[\sum_{m=1}^{M} r_m(0)h_{1,m}^* + r_m(1)^* h_{2,m}\right], S^i\right)^2 \tag{3.39}$$

$$d\left(\left[\sum_{m=1}^{M} r_m(0)h_{2,m}^* - r_m(1)^* h_{1,m}\right], S^k\right)^2 \tag{3.40}$$

for S_i and S_k respectively. Thus at the m^{th} antenna, the variables formed are:

$$\widetilde{S}_{0,m} = r_m(0)h_{1,m}^* + r_m(1)^* h_{2,m} \tag{3.41}$$

$$\widetilde{S}_{1,m} = r_m(0)h_{2,m}^* - r_m(1)^* h_{1,m} \tag{3.42}$$

Every decision variable provides the diversity order of two and there are M different decision variables for every symbol. Therefore the total diversity order of the system is $2M$ and the system uses M+2 antennas utilizing a very simple decoding algorithm without the channel information at the transmitter.

From equations (3.41) and (3.42) the SNR of STD system utilizing M receive antenna is equal to:

$$\mathrm{SNR_{STD}} = \frac{\frac{E_b}{2}\left(\sum_{l=0}^{2M}\overline{\alpha^2}\right)^2}{\sum_{l=0}^{2M}\overline{\alpha^2}N_0} = \frac{E_b\sum_{l=0}^{2M}\overline{\alpha^2}}{2N_0} \tag{3.43}$$

Eq (3.43) shows again that STD system provides diversity of $2M$ and its performance is 3dB less than MRC system.

The rate of a space-time block code is defined as the ratio of total number of information symbols to the number of time slots in a block. STD sends 2 symbols in 2 time slots. Therefore the rate of the system is 1. Rate 1 systems are also known as full rate systems.

3.4.4.2 General Theory of Space-Time Block Codes and Orthogonal Designs

In the previous section the antenna-time slot correspondence table is used to define the space-time block codes and block coding is done in the symbol level. In this section instead of tables, matrices are used to define the codes and the coding is applied to signals, which can be symbols or codewords. For example simple transmit diversity has the matrix:

$$G_1 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \tag{3.44}$$

This system sends in the first time slot the signals x_1 and x_2 from the antenna 1 and 2 respectively. A better system design has the following matrix:

$$G_1' = \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} \tag{3.45}$$

In this system the first antenna sends the signals x_1 and x_2 at time slots 1 and 2 respectively. If the receiver couldn't construct the decision variables, then the second antenna can be turned off and the system can still operate. In other words above system is exactly like a no diversity system, if it is utilized only with one transmit antenna.

The theory of orthogonal designs is a branch of mathematics and was studied by theorists including Radon and Hurwitz [Ger79]. A real orthogonal design of size n is an nxn orthogonal matrix with entries $\pm x_1, \pm x_2, \ldots, \pm x_n$ and has the property;

$$GG^T = G^T G = (x_1^2 + x_2^2 + \ldots + x_n^2)I \tag{3.46}$$

where G^T denotes the transpose of G and I is the identity matrix of size n. Real orthogonal designs can be used with any one dimensional modulation such as pulse amplitude modulation (PAM).

A complex orthogonal design of size n can have the entries $\pm x_1, \pm x_2, \dots, \pm x_n$ and their conjugates $\pm x_1^*, \pm x_2^*, \dots, \pm x_n^*$ and has the property of

$$GG^* = G^*G = (x_1^2 + x_2^2 + \dots + x_n^2)I \tag{3.47}$$

where G^* denotes the hermitian conjugate of G. Complex designs can be applied to both one and two dimensional modulation techniques such as PAM, PSK or QAM. For real designs there exist orthogonal ones if and only if the size n=2, 4 or 8. Below are the examples of such designs.

$$G_1^R = \begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix}, \quad n = 2 \tag{3.48}$$

$$G_2^R = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}, \quad n = 4 \tag{3.49}$$

$$G_3^R = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}, \quad n = 8 \tag{3.50}$$

For complex designs, Tarokh et al, [Tar99] proved that only size 2 orthogonal designs exist. The examples of that code are given in the beginning of this section and in the previous section in (3.44) and (3.45).

We have shown that the complex orthogonal design has full transmission rate and full diversity order with a simple decoding algorithm. Let us check the validity of this statement for real orthogonal designs. A real orthogonal design of size n sends n information symbols in n time slots in a block. Therefore the rate of the orthogonal real designs is $n/n=1$, and they provide full rate. To check the diversity order of the space-time codes, we can use the rank criterion given in section 3.4.2.1. Rank criterion states that if all possible error matrices of a code are nonsingular, then the code provides full diversity gain.

The error matrix of code sequences $\boldsymbol{c}=(c_1, c_2, \ldots, c_n)$ for a real orthogonal design is

$$G(\boldsymbol{c},\boldsymbol{e}) = G(\boldsymbol{c}) - G(\boldsymbol{e}) \tag{3.51}$$

where, $\boldsymbol{c}$ and $\boldsymbol{e}$ are distinct code sequences and $\boldsymbol{c}=(c_1, c_2, \ldots, c_n) \neq \boldsymbol{e}=(e_1, e_2, \ldots, e_n)$. A matrix is nonsingular if its determinant is nonzero. The determinant of G is:

$$\det(G) = \det(GG^T)^{1/2} \tag{3.52}$$

If we substitute (3.47) in (3.52), we obtain:

$$\det(GG^T)^{1/2} = \det((x_1^2 + x_2^2 + \ldots + x_n^2)I)^{1/2} = \left[\sum_{i=1}^{n} x_i^2\right]^{n/2} \tag{3.53}$$

Therefore the determinant of an error matrix is:

$$\det(G(\boldsymbol{c},\boldsymbol{e})) = \left[\sum_{i=1}^{n} |c_i - e_i|^2\right]^{n/2} \tag{3.54}$$

Since $\boldsymbol{c}$ and $\boldsymbol{e}$ are different for at least one element for any error matrix, all error matrices are nonsingular. Hence real orthogonal designs have also full diversity order. Their decoding is also simple due to the orthogonal columns; every signal is decoded independently from the others, like Simple Transmit Diversity.

To obtain diversity gain for arbitrary number of transmit antennas, Tarokh et al, defined generalized real and complex orthogonal designs in [Tar99].

A generalized real orthogonal design G for n transmit antennas and p time slots is a pxn matrix with entries $\pm x_1, \pm x_2, \ldots, \pm x_k$. It has the property $G^T G = D$, where D is a diagonal matrix of size nxn with elements:

$$(l_1^i x_1^2 + l_2^i x_2^2 + \ldots + l_k^i x_k^2) \quad i = 1,..n \tag{3.55}$$

In (3.55) l' coefficients are strictly positive to obtain diversity gain. Property in (3.55) provides sufficient condition to all error matrices be nonsingular. Hence codes have full diversity order. The number of entries (the number of symbols transmitted) is k in one block. Therefore the rate of the system is k/p. Using the theorem of Radon-Hurwitz, Tarokh et al, [Tar99] show that. there exists a generalized orthogonal design for any number of transmit antennas. In (3.56) an example of such codes for 3 transmit antennas is provided.

$$G_4^R = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_1 & -x_3 & x_2 \end{pmatrix}, \quad n = 3 \tag{3.56}$$

A generalized complex orthogonal design G for n transmit antennas and p time slots is a pxn matrix with entries $\pm x_1, \pm x_2, \ldots, \pm x_k, \pm x_1^*, \pm x_2^*, \ldots, \pm x_n^*$ or their multiplication by complex number i. It has the property $G^* G = D$, where D is a diagonal matrix of size nxn with elements:

$$(l_1^i |x_1|^2 + l_2^i |x_2|^2 + \ldots + l_k^i |x_k|^2) \quad i = 1,..n \tag{3.57}$$

where l' coefficients are strictly positive. These codes have also full diversity order, due to matrix D. Their rate is again k/p. For complex codes there is no rate 1 code other than codes for 2 transmit antennas. In [Tar99] and [Gan01] rate 3/4 codes are given for n=3,4 such as

$$G_{3/4} = \begin{pmatrix} x_1 & 0 & x_2 & -x_3 \\ 0 & x_1 & x_3^* & x_2^* \\ -x_2^* & -x_3 & x_1^* & 0 \\ x_3^* & -x_2 & 0 & x_1^* \end{pmatrix}, \quad n = 4 \tag{3.58}$$

Tarokh et al, showed also in [Tar99], for any number of transmit antennas, there exist a code at least with rate 1/2 such as

$$G_{1/2} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{pmatrix}, \quad n = 4 \tag{3.59}$$

For the orthogonal designs, the aim is to obtain the full diversity gain with the maximum transmission rate. In the next section, codes with full transmission rate and maximum available diversity rate will be considered.

3.4.4.3 Quasi-Orthogonal Designs in Space-Time Block Codes

Quasi-orthogonal space-time block codes are designed to obtain full transmission rate and maximum diversity order. Yongacoglu [Yon00] and Jafarkhani [Jaf01] independently proposed quasi-orthogonal transmit diversity schemes for four transmit antennas. Since both of them have the same properties, in this work only Yongacoglu's scheme STD4 will be investigated. The coding scheme has the following coding matrix:

$$Q_1 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2 & x_1 & x_4 & x_3 \\ x_3^* & x_4^* & -x_1^* & -x_2^* \\ x_4^* & x_3^* & -x_2^* & -x_1^* \end{pmatrix}, \quad n = 4 \tag{3.60}$$

Then for the symbols S_0, S_1, S_2, S_3 the system has the following antenna time slot correspondance table.

Table 3-1 Antenna time slot correspondance table for STD4 system

	Tx. Ant 1	Tx. Ant. 2	Tx. Ant 3	Tx. Ant. 4
Time 0	S_0	S_1	S_2	S_3
Time 1	S_1	S_0	S_3	S_2
Time 2	S_2^*	S_3^*	$-S_0^*$	$-S_1^*$
Time 3	S_3^*	S_2^*	$-S_1^*$	$-S_0^*$

For the sake of simplicity, at the receiver side only one antenna case is considered. The received signals became:

$$\begin{bmatrix} r(0) \\ r(1) \\ r(2)^* \\ r(3)^* \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & h_2 & h_3 \\ h_1 & h_0 & h_3 & h_2 \\ -h_2^* & -h_3^* & h_0^* & h_1^* \\ -h_3^* & -h_2^* & h_1^* & h_0^* \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} n(0) \\ n(1) \\ n(2)^* \\ n(3)^* \end{bmatrix} \tag{3.61}$$

In the combiner, the four decision variables are obtained below:

$$\begin{bmatrix} \tilde{S}_0 \\ \tilde{S}_1 \\ \tilde{S}_2^* \\ \tilde{S}_3^* \end{bmatrix} = \begin{bmatrix} h_0^* & h_1^* & -h_2 & -h_3 \\ h_1^* & h_0^* & -h_3 & -h_2 \\ h_2^* & h_3^* & h_0 & h_1 \\ h_3^* & h_2^* & h_1 & h_0 \end{bmatrix} \begin{bmatrix} r(0) \\ r(1) \\ r(2)^* \\ r(3)^* \end{bmatrix} \tag{3.62}$$

This is equivalent to:

$$\begin{bmatrix}\tilde{S}_0\\ \tilde{S}_1\\ \tilde{S}_2\\ \tilde{S}_3\end{bmatrix}=\begin{bmatrix}A & B & 0 & 0\\ B & A & 0 & 0\\ 0 & 0 & A & B\\ 0 & 0 & B & A\end{bmatrix}\begin{bmatrix}S_0\\ S_1\\ S_2\\ S_3\end{bmatrix}+\begin{bmatrix}h_0^* & h_1^* & -h_2 & -h_3\\ h_1^* & h_0^* & -h_3 & -h_2\\ h_2^* & h_3^* & h_0 & h_1\\ h_3^* & h_2^* & h_1 & h_0\end{bmatrix}\begin{bmatrix}n(0)\\ n(1)\\ n(2)^*\\ n(3)^*\end{bmatrix} \tag{3.63}$$

where,

$$A = h_0^{\ 2} + h_1^{\ 2} + h_2^{\ 2} + h_3^{\ 2} \tag{3.64}$$

$$B = h_0 h_1^* + h_1 h_0^* + h_2 h_3^* + h_3 h_2^* \tag{3.65}$$

Both A and B are real quantities. A is always positive and B can be positive or negative, There is orthogonality between pairs $(\tilde{S}_0, \tilde{S}_1)$ and $(\tilde{S}_2, \tilde{S}_3)$. Because of this orthogonality, the decisions are made pair wise in the maximum likelihood receiver. Symbols, S^i and S^j are chosen for S_0 and S_1 respectively if

$$\begin{aligned} &d^2(\tilde{S}_0,(AS^i + BS^j)) + d^2(\tilde{S}_1,(AS^j + BS^i)) \quad < \\ &\quad d^2(\tilde{S}_0,(AS^k + BS^l)) + d^2(\tilde{S}_1,(AS^l + BS^k)) \quad \forall k,l \end{aligned} \tag{3.66}$$

Symbols, S^m and S^n are chosen for S_2 and S_3 respectively if

$$\begin{aligned} &d^2(\tilde{S}_2,(AS^m + BS^n)) + d^2(\tilde{S}_3,(AS^n + BS^m)) \quad < \\ &\quad d^2(\tilde{S}_2,(AS^k + BS^l)) + d^2(\tilde{S}_3,(AS^l + BS^k)) \quad \forall k,l \end{aligned} \tag{3.67}$$

Although A combines 4 independent channels, there is degradation in the performance due to B. Besides the minimum rank of the coding matrix $Q(c,e)$ is equal to 2 [Jaf01]. Therefore the diversity gain with M receive antennas is only $2M$, and the decoding system is much more complicated. Another quasi-orthogonal code is:

$$Q_2 = \begin{pmatrix} x_1 & x_2 & x_3 & 0 & x_4 & x_5 & x_6 & 0 \\ -x_2^* & x_1^* & 0 & -x_3 & x_5^* & -x_4^* & 0 & x_6 \\ x_3^* & 0 & -x_1^* & -x_2 & -x_6^* & 0 & x_4^* & x_5 \\ 0 & -x_3^* & x_2^* & -x_1 & 0 & x_6^* & -x_5^* & x_4 \\ -x_4 & -x_5 & -x_6 & 0 & x_1 & x_2 & x_3 & 0 \\ -x_5^* & x_4^* & 0 & x_6 & -x_2^* & x_1^* & 0 & x_3 \\ x_6^* & 0 & -x_4^* & x_5 & x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_6^* & -x_5^* & -x_4 & 0 & x_3^* & -x_2^* & -x_1 \end{pmatrix} \qquad (3.68)$$

This code is constructed from two rate 3/4 transmission matrices and that's why this matrix has the coding rate 3/4 with diversity order 4. In the following sections the performance of STD4 will be compared with the orthogonal codes, since the code in (3.68) is not practical.

3.4.4.4 Performance of Space-Time Block Codes

In this section the performance of several space-time block coded systems will be studied. Monte Carlo simulation method and Matlab 6.1 is used during the simulations. Minimum number of errors count is 100 for any studied system. One of the functions used in simulations can be found in Appendix A. The first system under consideration is Alamouti's STD.

In Figure 3.5 the BER performance versus energy per bit of the system with 1 receive antenna is compared with a MRC receive diversity system utilizing 2 receive antennas. The modulation is coherent BPSK and the channels are assumed to be uncorrelated Rayleigh fading channels. Another assumption is that the receiver has the perfect channel state information. If the no diversity system curve were extended to the bit error rate of 10^{-3}, the coding gain of STD is found to be about 10 dB, which is 3 dB less than a MRC system. The reason for the 3 dB penalty is keeping the total transmitted energy per bit fixed for both systems. STD has two transmit antennas and due to this the energy is halved at each transmit antenna, whereas the receive diversity system combines the signals emitted from one transmit antenna through two separate paths.

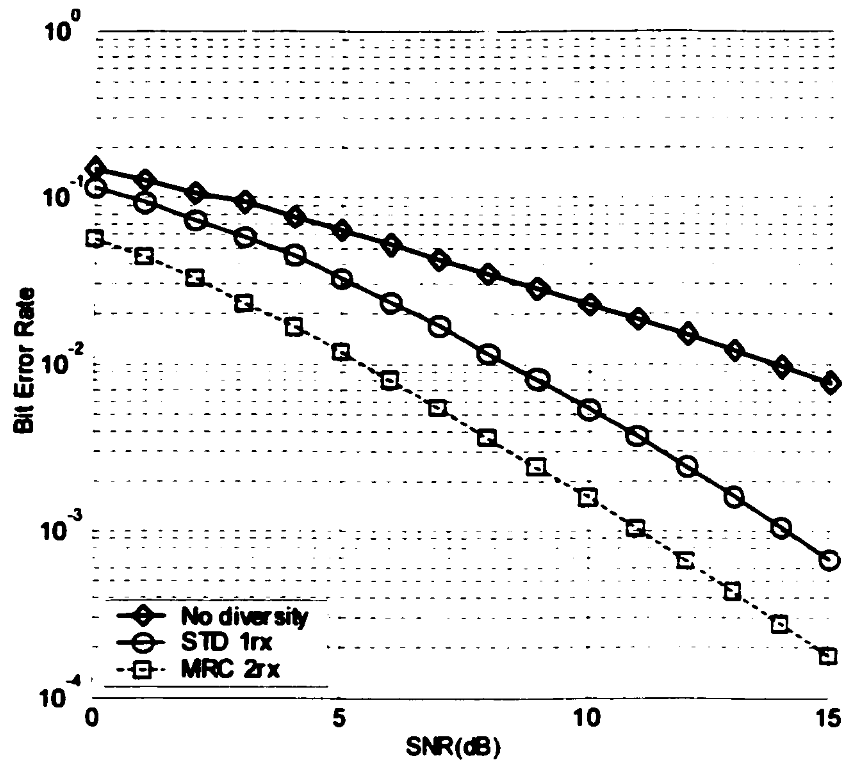


Figure 3.5 BER comparison of coherent BPSK systems (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) receive diversity with 1 transmitter and 2 receivers (MRC) in Rayleigh fading.

In section 3.4.4.1 we showed that STD provides $2M$ diversity order, if there are M antennas at the receiver side. This effect can be seen in Figure 3.6 where STD system with two receive antennas is compared with a receive diversity system utilizing MRC. STD again provides 3 dB less diversity gain than MRC.

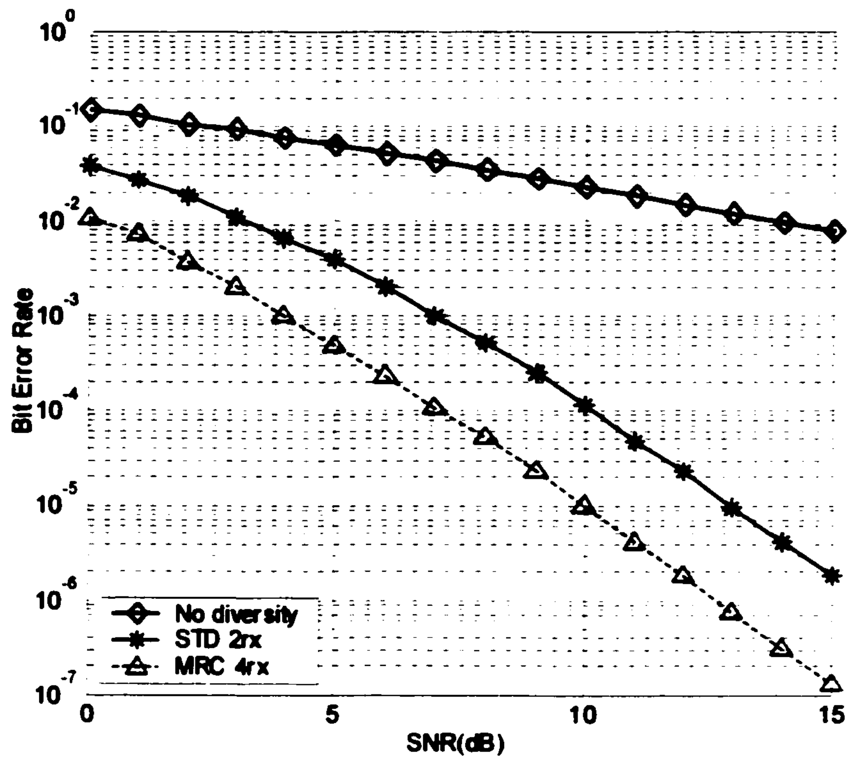


Figure 3.6 BER comparison of coherent BPSK systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 2 receivers (iii) receive diversity with 1 transmitter and 4 receivers (MRC).

For a fair comparison between orthogonal codes, all the systems should have the same bandwidth efficiencies. In Figure 3.7 a no diversity system and STD are modulated using BPSK for 1 bit/sec/Hz, whereas the following code for 3 transmit antennas and code $G_{1/2}$ in (3.59) for 4 transmit antennas have QPSK modulation since their rate is 1/2.

$$H_{1/2} = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_1 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_1^* & -x_3^* & x_2^* \end{pmatrix} \quad n = 3 \qquad (3.69)$$

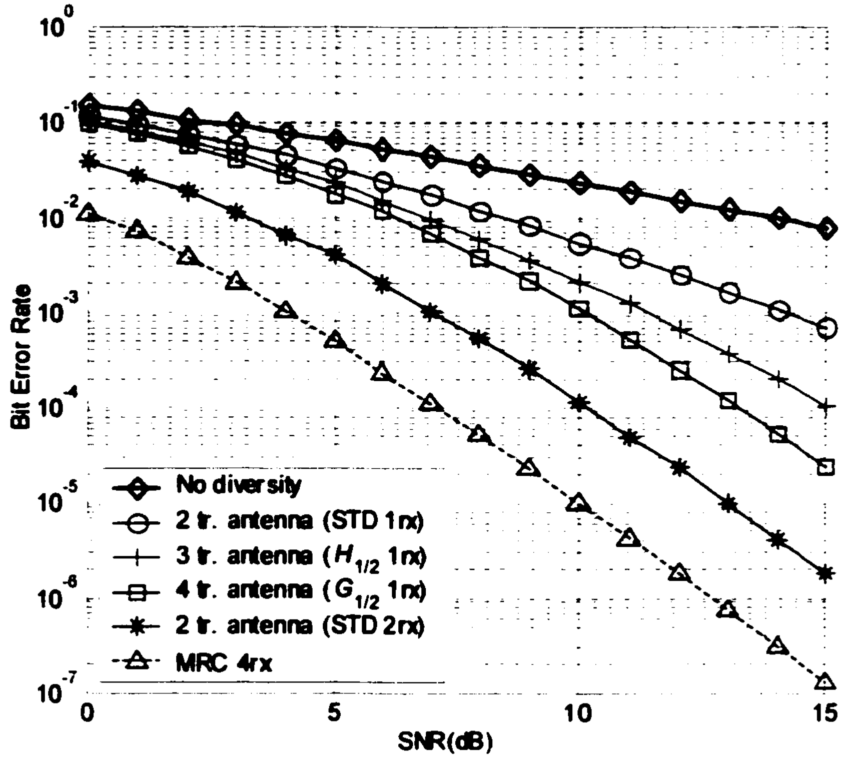


Figure 3.7 BER comparison of 1 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{1/2}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{1/2}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC).

The 3 transmit antenna system add additional gain around 2.7 dB at 10^{-3} BER. Whereas 4 antenna systems gain is around 4 dB at the same BER. In Figure 3.7, a STD system with 2 receive antennas and a MRC system with 4 receive antennas are shown for comparison. 4 transmit antenna systems performance is 3 and 6 dB less than STD and MRC systems respectively at all bit error rates. This loss is due to power considerations, however it provides the same diversity order of MRC system.

For 2 bit/sec/Hz, no diversity system and STD should use QPSK. The code $H_{1/2}$ in (3.69) for 3 transmit antennas and code $G_{1/2}$ in (3.59) for 4 transmit antennas utilize 16 QAM since their rate is 1/2. In Figure 3.8 the BER performance of these systems are shown.

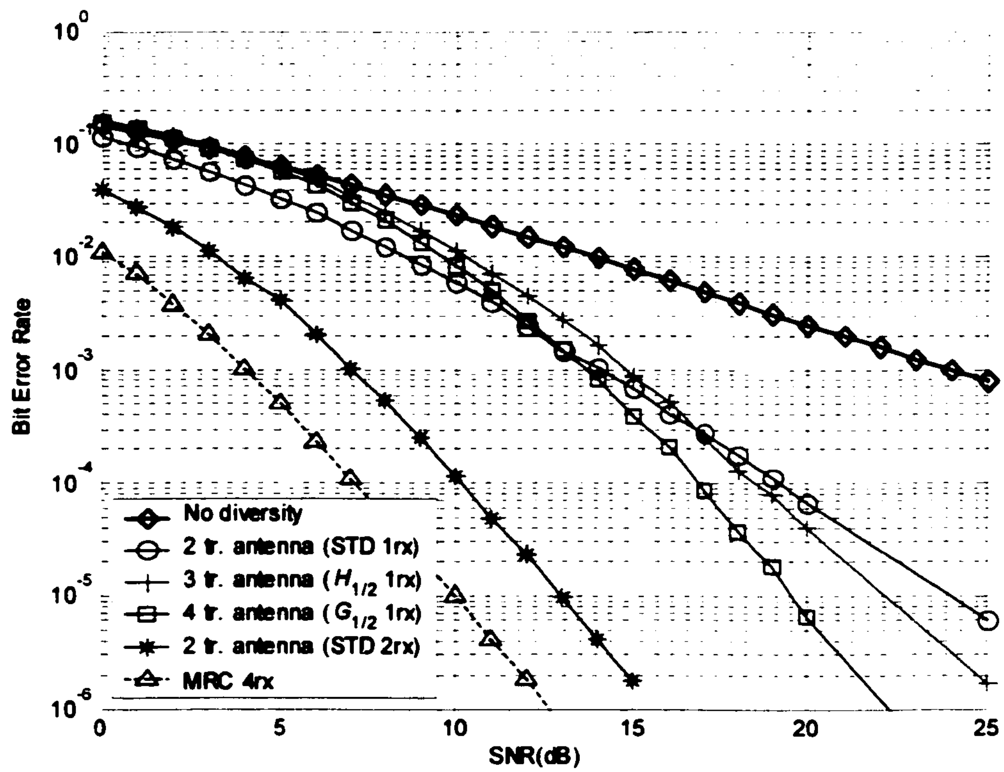


Figure 3.8 BER comparison of 2 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{1/2}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{1/2}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC).

In this case the degradation effect of low rates to BER performance is seen for 3 and 4 antennas. Due to their low rate higher order constellation 16 QAM is used, and up to 17 dB the performance of 3 transmit antenna system is inferior to STD system in terms of bit error rate. For 4 antenna system the gain is around 0.5 dB at 10^{-3} BER. However at high

SNR values the performances of both systems are better than STD system, since their slopes are steeper. The loss due to power is around 10 dB between MRC and 4 transmit antenna case at 10^{-3}.

For 3 bit/sec/Hz bandwidth efficiency, no diversity system and STD are utilizing 8PSK. The following code for 3 transmit antennas and code with coding matrix $G_{3/4}$ in (3.58) for 4 transmit antennas utilize 16QAM since their rate is 3/4.

$$H_{3/4} = \begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{(-x_1 + x_2 - x_1^* - x_2^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{(x_1 + x_2 - x_1^* + x_2^*)}{2} \end{pmatrix}, \quad n = 3 \tag{3.70}$$

In Figure 3.9 the BER performance of the systems are shown.

In this case both 3 and 4 transmit antenna systems have better performances than STD. At 10^{-4} bit error rate, it is observed that their extra gains of $H_{3/4}$ and $G_{3/4}$ are around 2 and 3 dB with respect to STD respectively. The difference between 4 antenna system and STD system utilizing 2 receive antennas is around 3.2 dB at 10^{-3} BER.

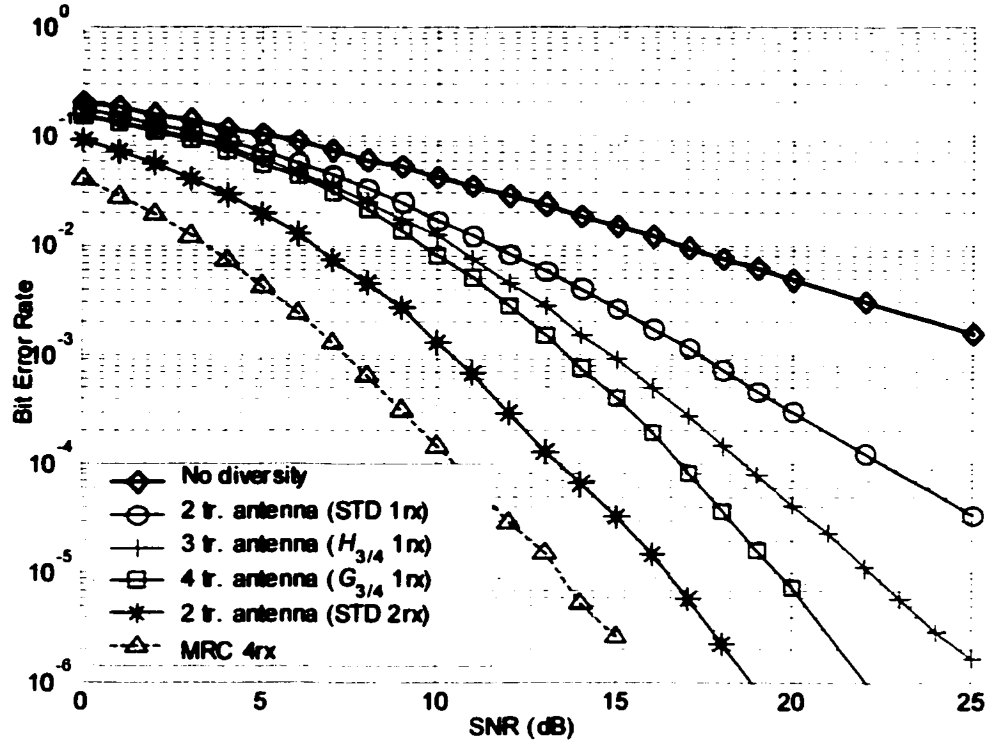


Figure 3.9 BER comparison of 3 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{3/4}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{3/4}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC).

To compare Yongacoglu's quasi-orthogonal system with the orthogonal codes the same methodology is used. At 1 bit/sec/Hz the performance of the system is shown in Figure 3.10

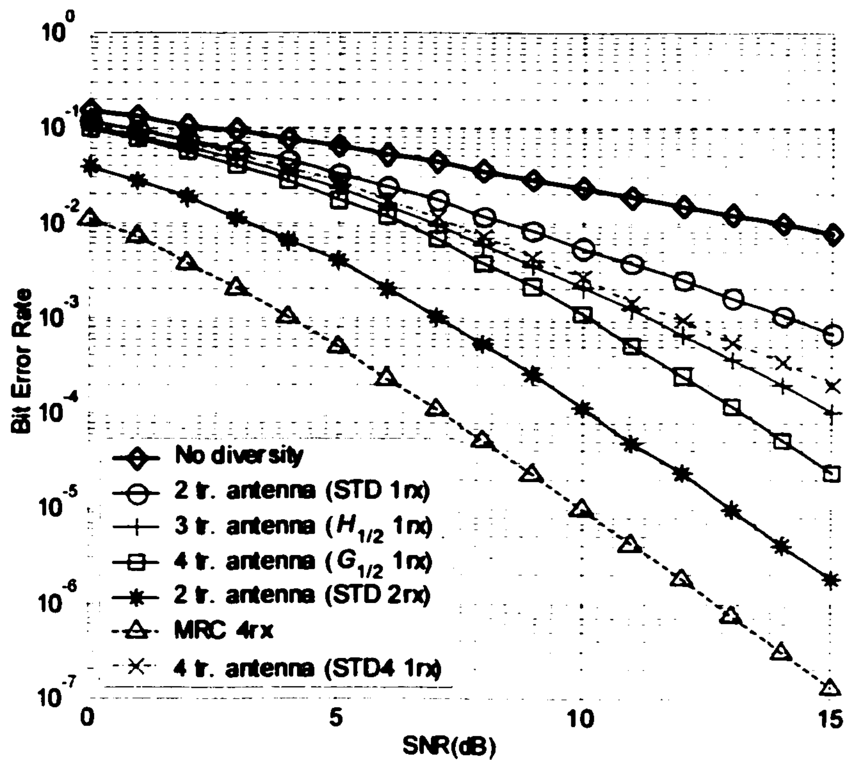


Figure 3.10 BER comparison of 1 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{1/2}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{1/2}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC) (vi) transmit diversity STD4 with 4 transmitters and 1 receiver.

Since STD4 system does not provide full diversity gain and there is no modulation penalty between BPSK and QPSK, 4 transmit antenna orthogonal system has better performance at every signal to noise ratio. Figure 3.11 shows the performance of STD4 for 2 bit/sec/Hz bandwidth efficiency.

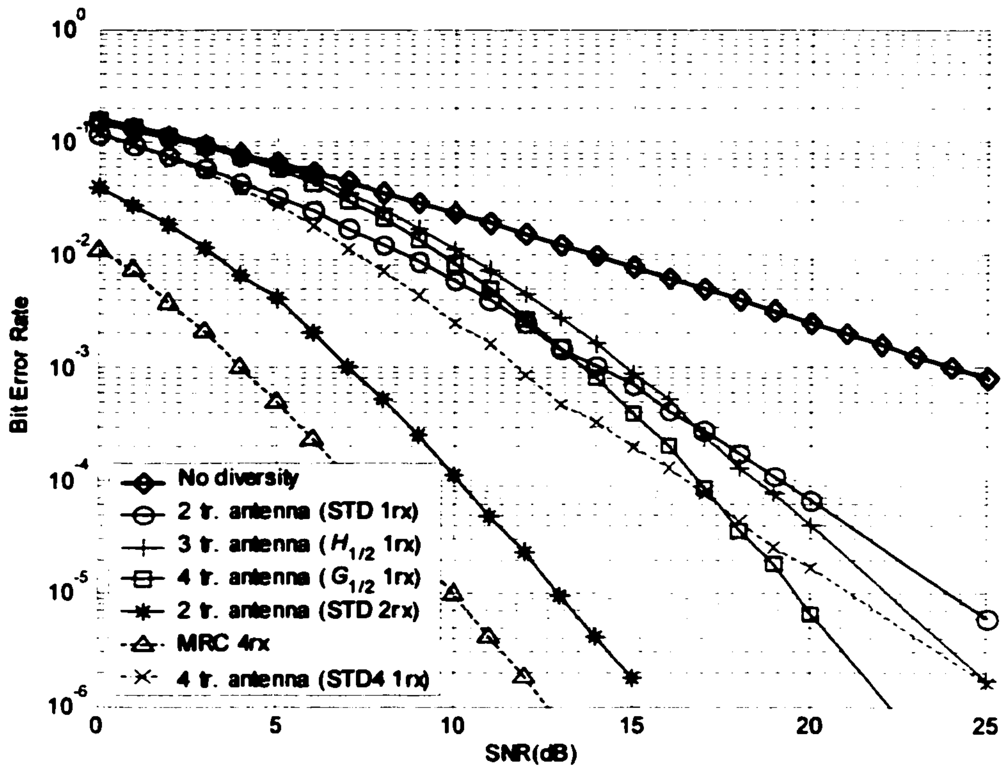


Figure 3.11 BER comparison of 2 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{1/2}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{1/2}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC) (vi) transmit diversicy STD4 with 4 transmitters and 1 receiver.

In this case despite its low diversity order the performance of STD4 is better than the 4 transmit antenna orthogonal system up to 17.5 dB. However if the SNR is increased then the improvement in the performance of the orthogonal system is more than STD4, since the slope of the orthogonal system is steeper than the quasi-orthogonal one. 3 antenna system has also better performance than the quasi orthogonal code but only after 25 dB SNR.

If 8PSK is utilized for the STD4 system, the bandwidth efficiency is 3 bit/sec/Hz. In this case the systems have the performance curves shown in Figure 3.12

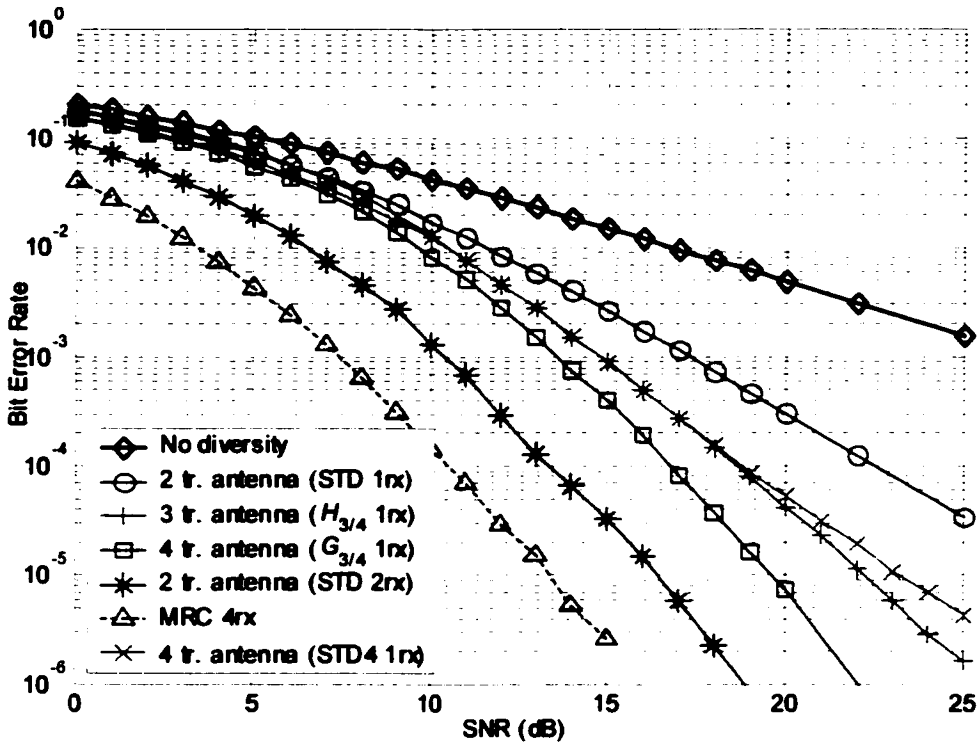


Figure 3.12 BER comparison of 3 bit/sec/Hz systems in Rayleigh fading (i) no diversity (ii) transmit diversity STD with 2 transmitters and 1 receiver (iii) transmit diversity $H_{3/4}$ with 3 transmitters and 1 receiver (iii) transmit diversity $G_{3/4}$ with 4 transmitters and 1 receiver (iv) transmit diversity STD with 2 transmitters and 2 receivers (v) receive diversity with 1 transmitter and 4 receivers (MRC) (vi) transmit diversity STD4 with 4 transmitters and 1 receiver.

In this case the performance of 3 transmit antenna system is more or less the same with the quasi-orthogonal system between 10 and 19 dB signal to noise ratios. Outside this region, 3 transmit antenna has better performance than the quasi-orthogonal code. For example at 10^{-5} the system has 1 dB gain.

3.5 Summary

The necessity of precoding for transmit diversity systems is demonstrated in the first part of this chapter. Afterwards different transmit diversity techniques with and without feedback are introduced. Among systems without feedback the space-time block codes are studied more deeply. The necessary condition to provide full diversity gain is shown. Rate of a space-time code is defined and codes with different rates are given. Quasi-orthogonal codes, which are specifically designed to obtain high rate rather than diversity order is studied. At the end of the chapter simulation results of space-time block code performances are given. It is observed that at 1 bit/sec/Hz, diversity order determines, which code is better in terms of the performance. However for 2 bit/sec/Hz the rate of the system is more important than the diversity order at low SNR values.

CHAPTER 4
Effects of Correlation to Space-Time Block Coded Systems

4.1 Introduction

The main idea behind the diversity concept was increasing the system performance via employing independent or at least sufficiently uncorrelated signal paths. However, for some real life situations the assumption of highly uncorrelated channels is not valid [Sim00, p.316]. For example insufficient antenna spacing in small mobile units is one of the sources of correlation among wireless channels. In this work we will study the effect of spatial correlation to the BER performance of space-time coded systems and compare it to MRC systems having the same diversity order. In the literature there are papers about orthogonal codes [Dam01, Abd01], however they are for specific correlation models and do not consider quasi-orthogonal codes. Before these effects, we will define spatial correlation among channels and describe how to model the correlation among wireless channels.

4.2 Correlation Models for Wireless Channels

Correlation is basically a measure of the degree of linear relationship between two variables. For example, if two channels are usually experiencing deep fades together, then we can claim that there is a high correlation between those two channels.

The correlation coefficient between two random variables X and Y is defined as

$$\rho = \frac{E[XY] - E[X]E[Y]}{\sigma_X \sigma_Y} \tag{4.1}$$

where E[Z] is the expected value of Z, σ_Z is the standard deviation of Z. The correlation coefficient may take any value between -1 and 1 and if it is equal to zero, the variables are uncorrelated.

In this work correlation in time is not considered, therefore the channel coefficients at a given time instant can be considered as random variables. The correlation among several random variables can be shown by using a covariance matrix. The following covariance matrix is for 4 random variables with variance 1, such as channel coefficients from 4 different transmit antennas to 1 receive antenna.

$$\begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} \\ \rho_{1,2} & 1 & \rho_{2,3} & \rho_{2,4} \\ \rho_{1,3} & \rho_{2,3} & 1 & \rho_{3,4} \\ \rho_{1,4} & \rho_{2,4} & \rho_{3,4} & 1 \end{bmatrix} \tag{4.2}$$

By definition the correlation coefficient between a random variable and itself is 1, therefore the first diagonal of the matrix is 1. $\rho_{i,j}$ denotes the correlation coefficient between i^{th} and j^{th} random variables and from Eq. (4.1) $\rho_{i,j} = \rho_{j,i}$.

The first model that we are going to consider is Aalo's [Sim00 p. 323] [Aal95] constant correlation model. This model assumes that correlation coefficient ρ is the same between all channel pairs. Then the matrix in (4.2) has the form:

$$\begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix} \tag{4.3}$$

This model may correspond to a scenario of very closely spaced antennas. In addition to that, if the maximum correlation coefficient among the channels is known, this model provides the minimum BER performance achievable from those channels.

The second model considers situations when correlation exists only between specific pairs of channels. For example, at the transmitter side the antennas are spaced closely whereas at the receiver side far enough to be uncorrelated or four transmit antennas are placed in two distant groups of two. The first and second channels are from the first transmit antenna. The third and fourth channels are from the second transmit antenna. Consequently it should be some correlation between first and second and between third and fourth channels. If the value of correlation coefficient is the same for those pairs, then the matrix has the form:

$$\begin{bmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho \\ 0 & 0 & \rho & 1 \end{bmatrix} \tag{4.4}$$

In the following sections effect of spatial correlation to space-time block coded systems is studied with comparison to their effects to MRC receive diversity systems. Monte Carlo simulation method and Matlab 6.1 is used during the simulations. Again minimum numbers of error count is 100. How correlated channels are obtained, and a sample function can be found in Appendix B.

4.3 Effect of Correlation to Orthogonal Space-Time Block Coded Systems

The first orthogonal system under consideration is STD. In Figure 4.1 the effect of correlation is obtained using simulations for STD 1 receive antenna and MRC 2 receive antennas. Since there are only 2 channels, a correlation coefficient value is enough to describe the situation. The simulations show that the effect of correlation among channels in the decrease of the performance of the systems is identical in terms of the loss in dB, which means the 3 dB difference stays constant between MRC and STD. In high correlation situations, even when the correlation coefficient is 0.95, there is a significant coding gain for STD system. At 0.95 correlation, the gain is around 2 dB at 10^{-2}. The performance of the STD system drops to a no diversity system if the channels are identical (ρ=1).

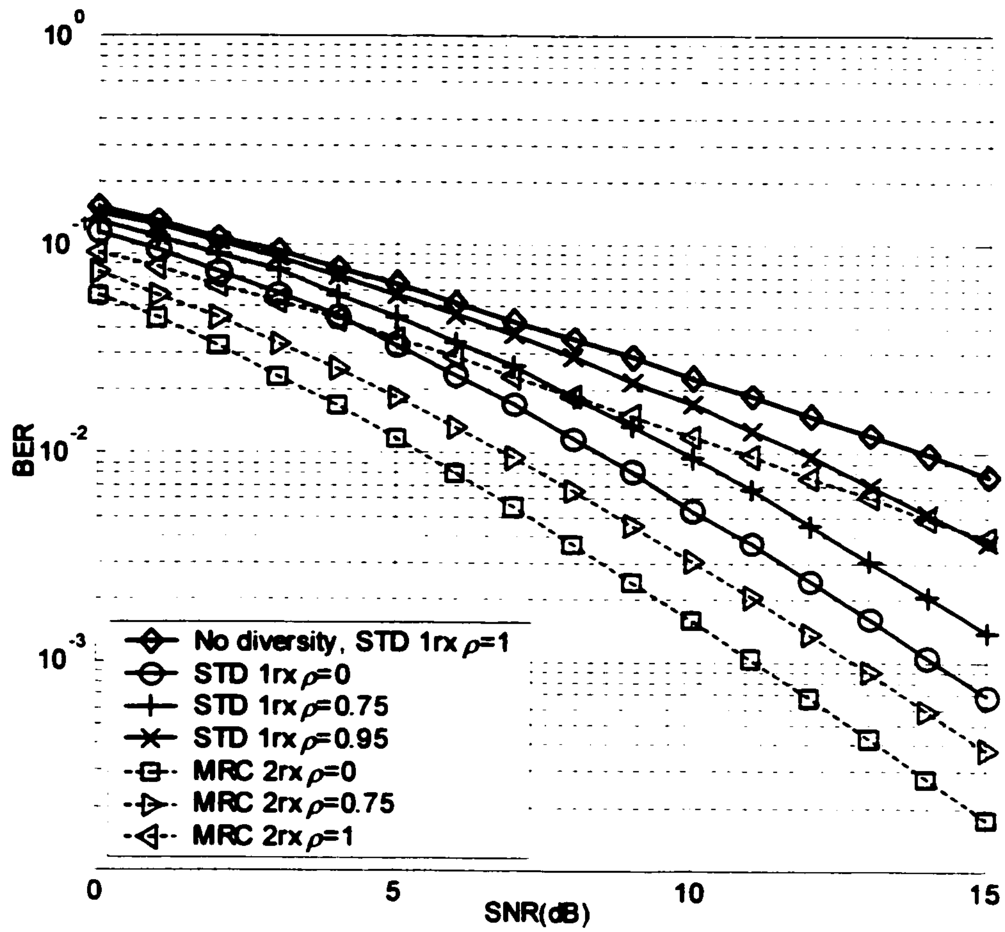


Figure 4.1 Effect of spatial correlation for different correlation coefficients to STD system with 2 transmitters and 1 receiver and a MRC system with 1 transmitter and 2 receivers at 1 bit/sec/Hz.

For STD system we have simulated 2 receive antennas case for Aalo's constant correlation model. The first observation is that the system is robust against fading at least up to 0.5 correlation. In Figure 4.2 it is observed that at 10^{-4} bit error rate the loss is around 1.1 dB for 0.5 correlation coefficient. Although it is not shown in the figure, at ρ=1, the performance of the STD system with 2 receive antennas is 3dB better than a no diversity system. Although correlation decreases the effect of diversity, 2 receive antennas provide a combining gain by the doubling total received power. Another observation is that even if the correlation is very high ρ=0.9, the performance of the system is better than a STD system with 1 receive antenna and no correlation.

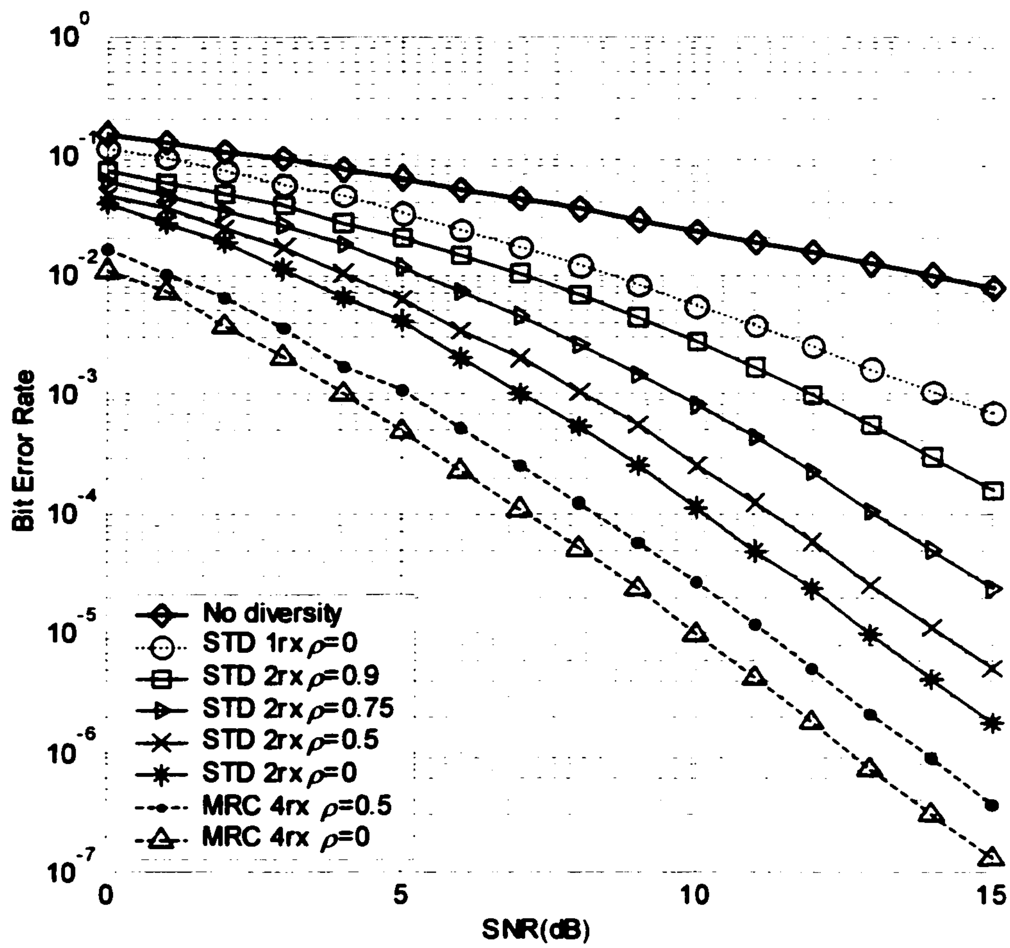


Figure 4.2 Effect of spatial correlation among all channels to STD system with 2 transmitters and 2 receivers and a MRC system with 1 transmitter 4 receivers at 1 bit/sec/Hz.

From the three transmit antenna systems, the one with rate 1/2 denoted by the matrix $H_{1/2}$ in (3.72) has the following performance curves for identically distributed channels with constant correlation. 0.5 correlation among channels results in a 1 dB loss at 10^{-3} BER.

This system has a gain with respect to, a no diversity system around 3 dB at 10^{-2} even if the correlation coefficient among channels is 0.95. At full correlation the system performance drops to performance of a no diversity system.

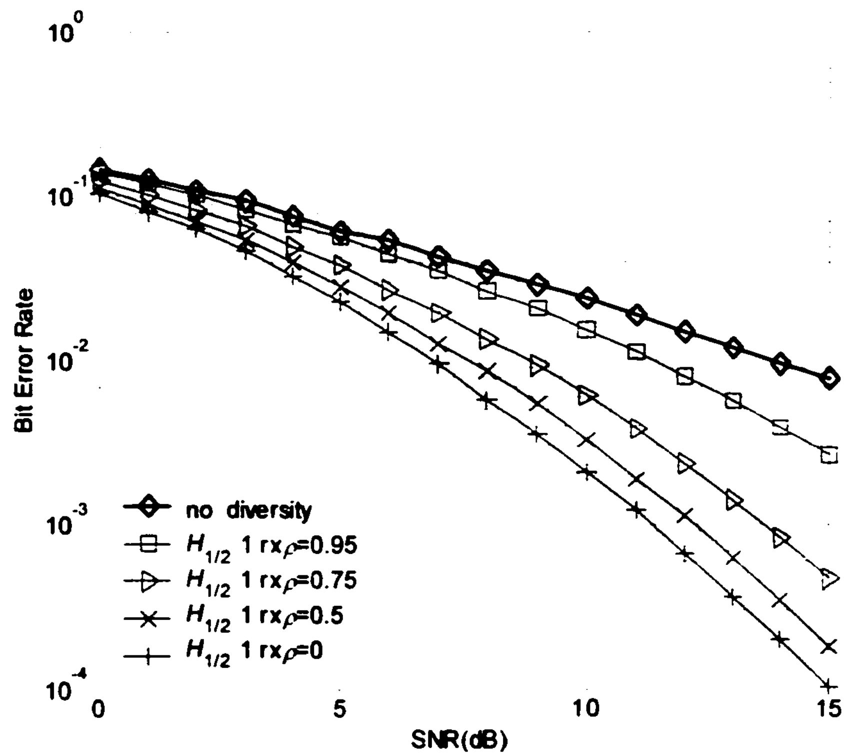


Figure 4.3 Effect of spatial correlation among all channels to $H_{1/2}$ system utilizing 1 receiver at 1 bit/sec/Hz.

At 2 bit/sec/Hz the same system has the performance curves shown in Figure 4.4 Once more 0.5 correlation among channels results in a 1 dB loss at 10^{-3} BER for this system although it is not shown in the figure. At full correlation the performance of the system is worse than a no diversity system because of the modulation penalty between 16QAM and 4PSK. At full correlation the performance of the system equals to a no diversity system utilizing 16QAM. For 0.95 correlation the system performance is better than a no diversity system only after 15 dB SNR.

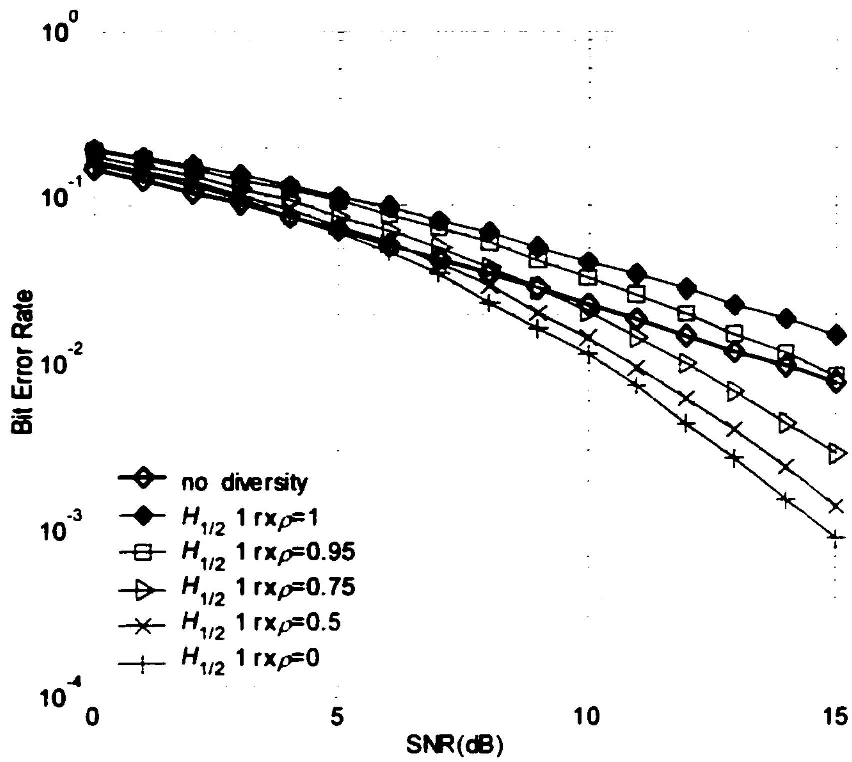


Figure 4.4 Effect of spatial correlation among all channels to $H_{1/2}$ system utilizing 1 receiver at 2 bit/sec/Hz.

At 3 bit/sec/Hz, rate 3/4 system for 3 transmit antennas $H_{3/4}$ in (3.70) has the same performance of $H_{1/2}$ at 2 bit/sec/Hz.

Among the 4 antenna systems at 1 bit/sec/Hz, the system utilizing the coding matrix $G_{1/2}$ in (3.59), has the following performance curves for Aalo's constant correlation model in Figure 4.5 0.5 correlation among channels outcomes a loss around 1 dB at 10^{-3} BER and the system has a gain with respect to a no diversity system around 3 dB at 10^{-2} even if the correlation coefficient among channels is 0.95, like the previous $H_{1/2}$ system. If Figure 4.2 and Figure 4.5 are compared, it can be seen that the 3 dB performance difference between $G_{1/2}$ and STD with 2 receivers is constant for all correlation coefficients.

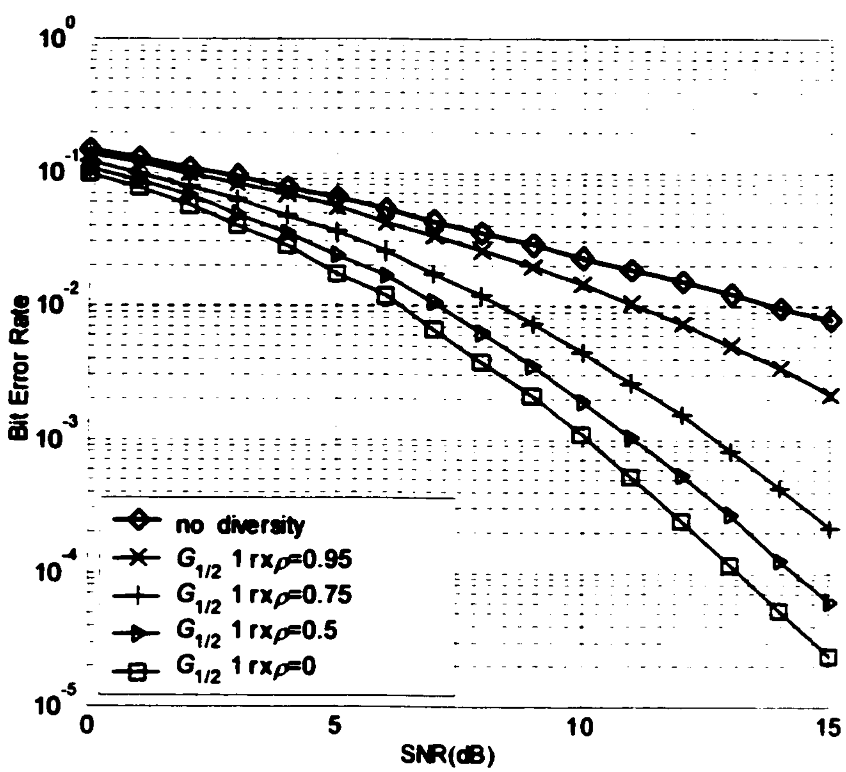


Figure 4.5 Effect of spatial correlation among all channels to $G_{1/2}$ system utilizing 1 receiver at 1 bit/sec/Hz.

The performance curves for different correlation coefficients at 2 bit/sec/Hz for the same system are shown in Figure 4.6. Similar to the $H_{1/2}$ system, due to the modulation penalty between 16QAM and QPSK, at full correlation the performance of the system is worse than no diversity system. Although the diversity order of this system is higher than $H_{1/2}$ at high correlation levels like 0.95, both systems have almost the same performance up to 15 dB SNR. Similar to the other codes, at 0.5 correlation the degradation in the performance is around 1 dB at 10^{-3} BER. At 3 bit/sec/Hz, rate 3/4 system for 4 antennas ($G_{3/4}$ in (3.58)) has the same performance of $G_{1/2}$ at 2 bit/sec/Hz. In this section we have investigated constant correlation model for orthogonal codes. In the following section special cases for correlation among channels will be studied as well to observe the effects of correlation to quasi-orthogonal codes.

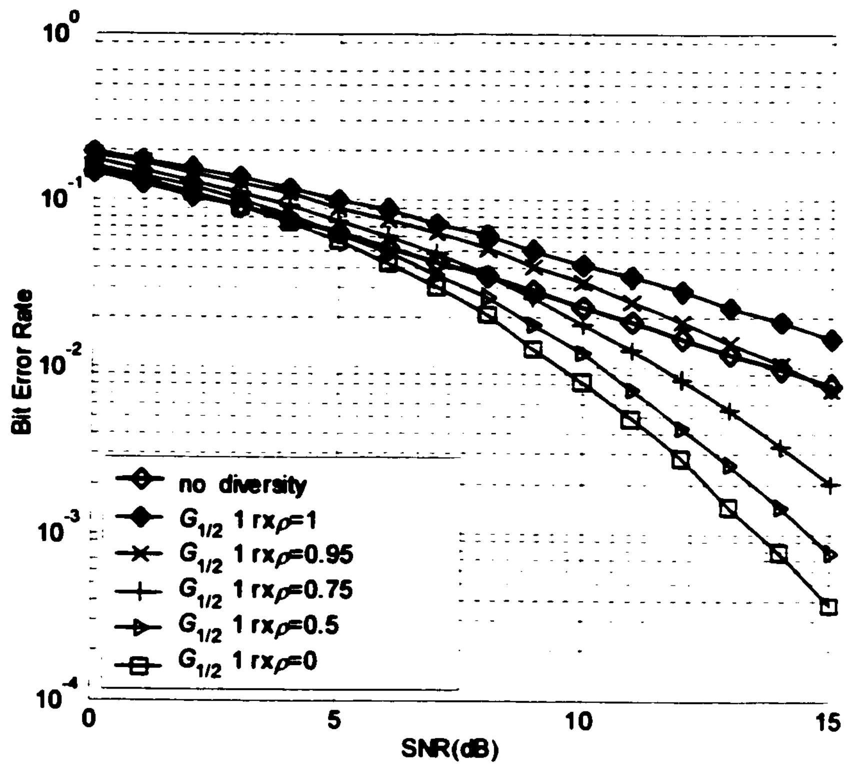


Figure 4.6 Effect of spatial correlation among all channels to $G_{1/2}$ system utilizing 1 receiver at 2 bit/sec/Hz.

4.4 Effect of Correlation to Quasi-orthogonal Space-Time Block Coded Systems

Aalo's constant correlation model is first studied for STD4 system fro 4 transmit antennas and 1 receive antenna. The system has the performance curves for different values of correlation coefficient in Figure 4.7 It can be seen that the STD4 system is vulnerable to high correlation. If the channels are identical, the bit error rate rises to 0.25, which makes the system useless. Furthermore at 0.75 correlation the performance is worse than a no diversity system up to 5 dB. If correlation coefficient is 0.5, the loss in STD4 system is around 1.5 dB at 10^{-3} bit error rate. This value is higher than the losses of orthogonal systems.

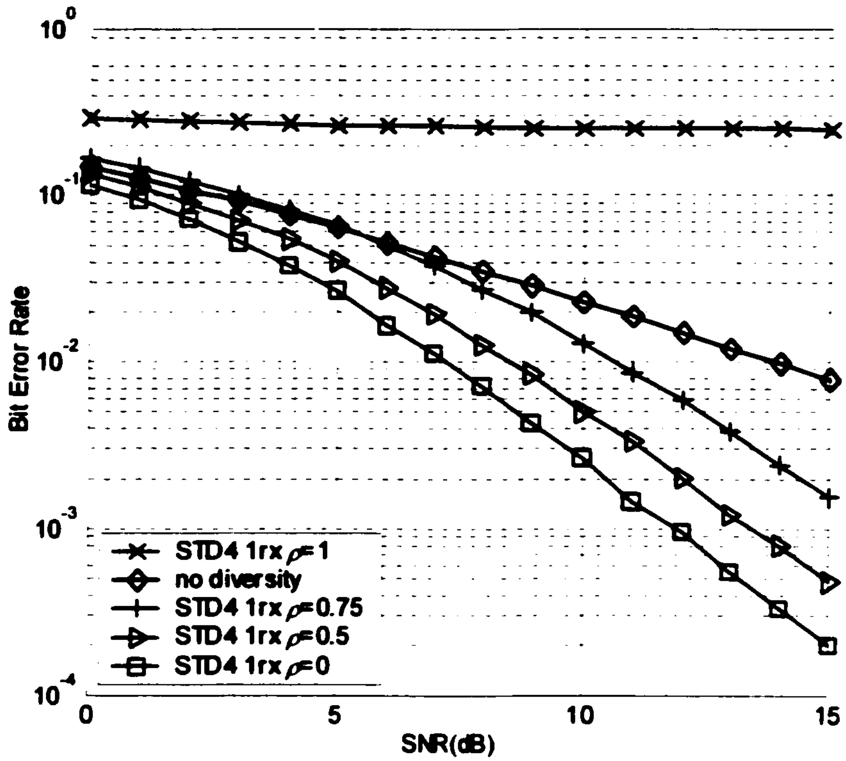


Figure 4.7 Effect of spatial correlation among all channels to STD4 system utilizing 1 receiver at 1 bit/sec/Hz.

Since the correlation has greater impact on the quasi-orthogonal system, we extend our research to different correlation models. These models include correlation among channels that are decoded together, correlation among channels that are decoded separately, correlation among three channels and two cases of correlation between two channels.

In the next case, correlation exists between the first and second and between the third and fourth channels, hence the channels are pair wise independent and the covariance matrix has the form:

$$\begin{bmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho \\ 0 & 0 & \rho & 1 \end{bmatrix} \tag{4.5}$$

Subsequently following performance curves are obtained in Figure 4.8 via simulations. Again if ρ is 1, the bit error rate raises to 0.25. The proof of this phenomenon is given in appendix C. The performance is slightly better at other values of correlation coefficient with respect to the first situation. If we compare this system with the orthogonal $G_{1/2}$ system, we observe that for orthogonal system maximum performance degradation is about 4 dB at 10^{-4} bit error rate. However for a relatively high correlation like 0.75 the loss is only 1.2 dB at the same BER.

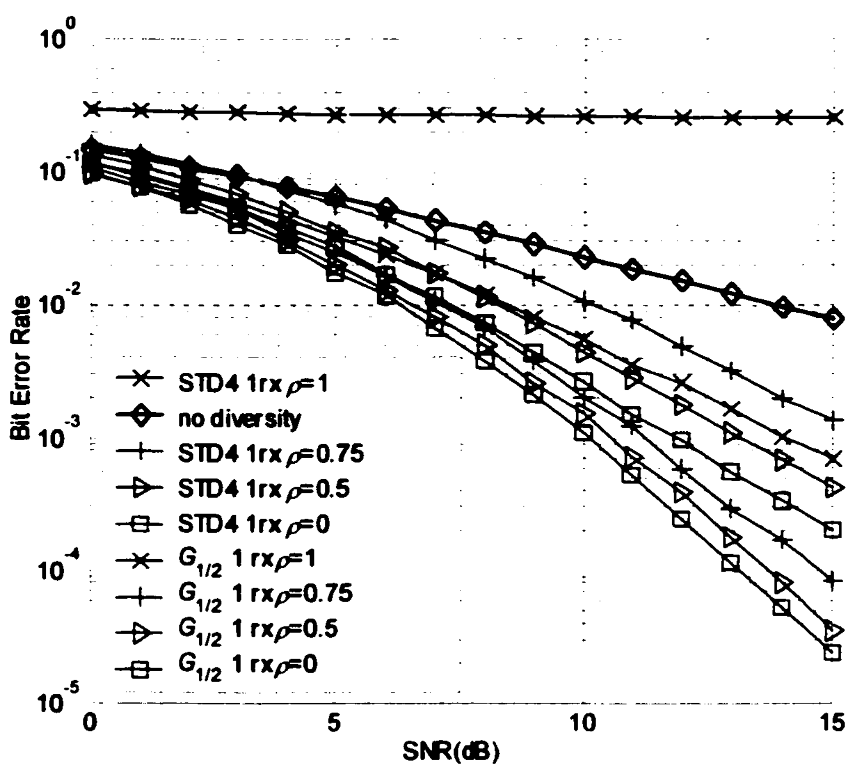


Figure 4.8 Effect of spatial correlation between 1st and 2nd and between 3rd and 4th channels to STD4 system with 4 transmitters and 1 receiver and $G_{1/2}$ system with 4 transmitters and 1 receiver at 1bit/sec/Hz.

If correlation exists between the first and third and between second the and fourth channels with a covariance matrix,

$$\begin{bmatrix} 1 & 0 & \rho & 0 \\ 0 & 1 & 0 & \rho \\ \rho & 0 & 1 & 0 \\ 0 & \rho & 0 & 1 \end{bmatrix} \tag{4.6}$$

the performance of the STD4 system is better than a no diversity system (Figure 4.9), even if there exists full correlation among channels. Although it is not shown in the figure for SNR up to 15dB and ρ=0.75 bit error rate is less than a STD2 system with independent channels (no correlation). Although the difference between the effects of correlation to STD4 and $G_{1/2}$ is not so drastic in this case, still the degradation effect is more in STD4.

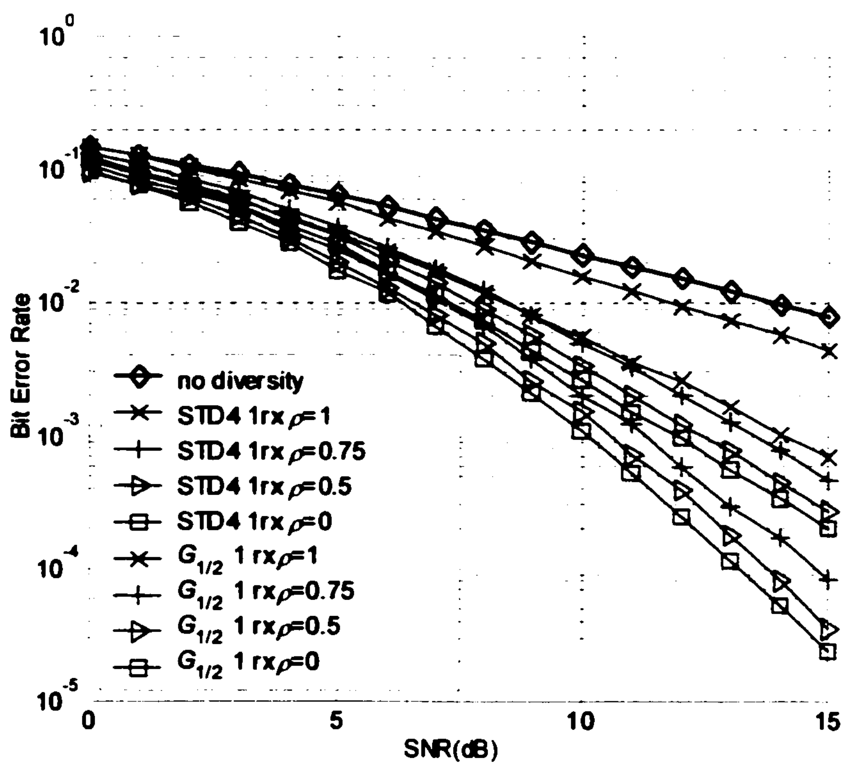


Figure 4.9 Effect of spatial correlation between 1st and 3rd and between 2nd and 4th channels to STD4 system with 4 transmitters and 1 receiver and $G_{1/2}$ system with 4 transmitters and 1 receiver at 1bit/sec/Hz.

In the fourth situation, there exists correlation between the first, second and third channels. The performance of the system is slightly worse than the previous situation as seen in Figure 4.10. For orthogonal $G_{1/2}$ code we obtained a slightly worse performance with respect to the previous situation too.

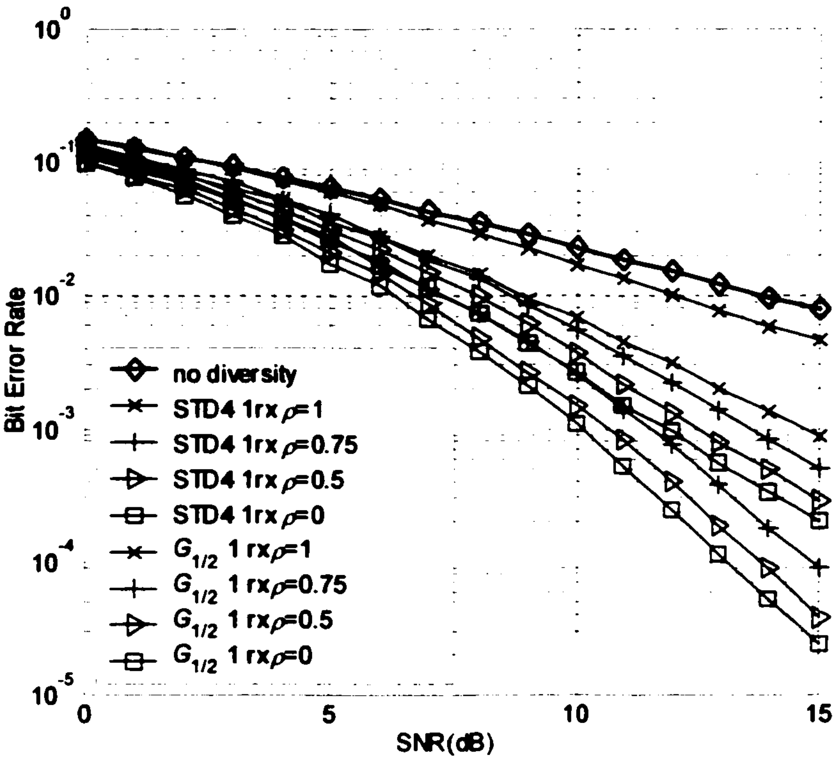


Figure 4.10 Effect of spatial correlation between 1st, 2nd and 3rd channels to STD4 system with 4 transmitters and 1 receiver and $G_{1/2}$ system with 4 transmitters and 1 receiver at 1bit/sec/Hz.

In the last two situations, correlation exists only between two channels. In the fifth one due to correlation between the first and second channels, for STD4 system bit error performance is slightly better than the third and fourth situations as seen in Figure 4.11. Whereas for the orthogonal system, the degradation effect of the correlation is much less in this case. Only 1.5 dB loss occurs at 10^{-3} BER, for a full correlation scenario.

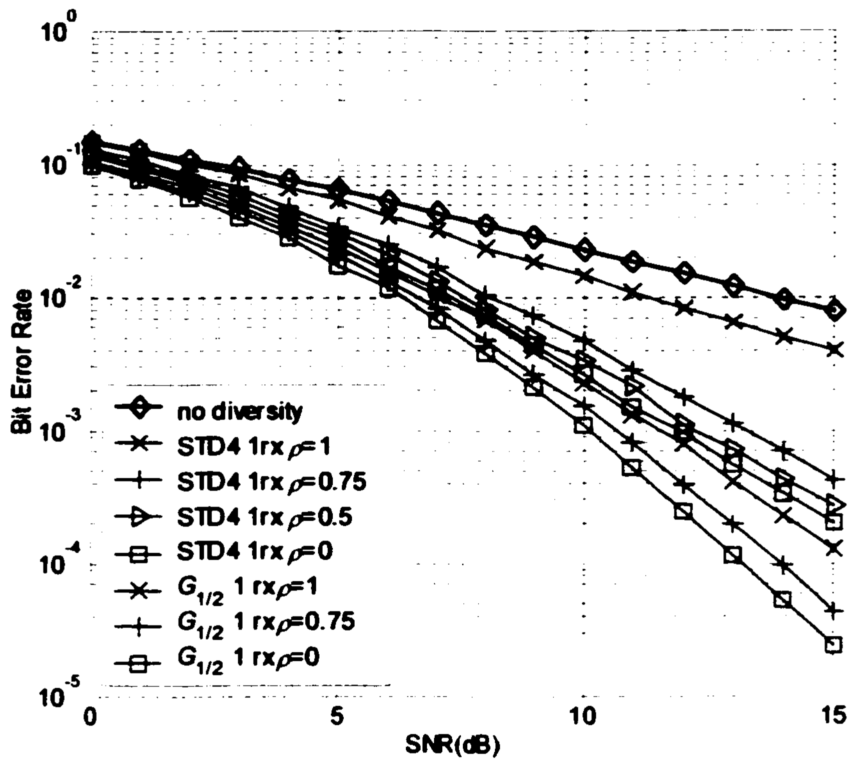


Figure 4.11 Effect of spatial correlation between 1st and 2nd channels to STD4 system with 4 transmitters and 1 receiver and $G_{1/2}$ system with 4 transmitters and 1 receiver at 1bit/sec/Hz.

In the last situation, correlation exists between the first and third channels. We observed the best performance curves in all values of correlation coefficient. Albeit $\rho=1$, the loss in the performance of the system is only around 1 dB at 10^{-3} bit error rate. This is the only case that the losses in SNR due to correlation at the STD4 system are less than $G_{1/2}$ orthogonal system. For example, at 10^{-3} bit error rate the loss of $G_{1/2}$ is around 1.2 dB for full correlation. Moreover the performance at $\rho=0.75$ is better than all other correlation situations at $\rho=0.5$ for STD4.

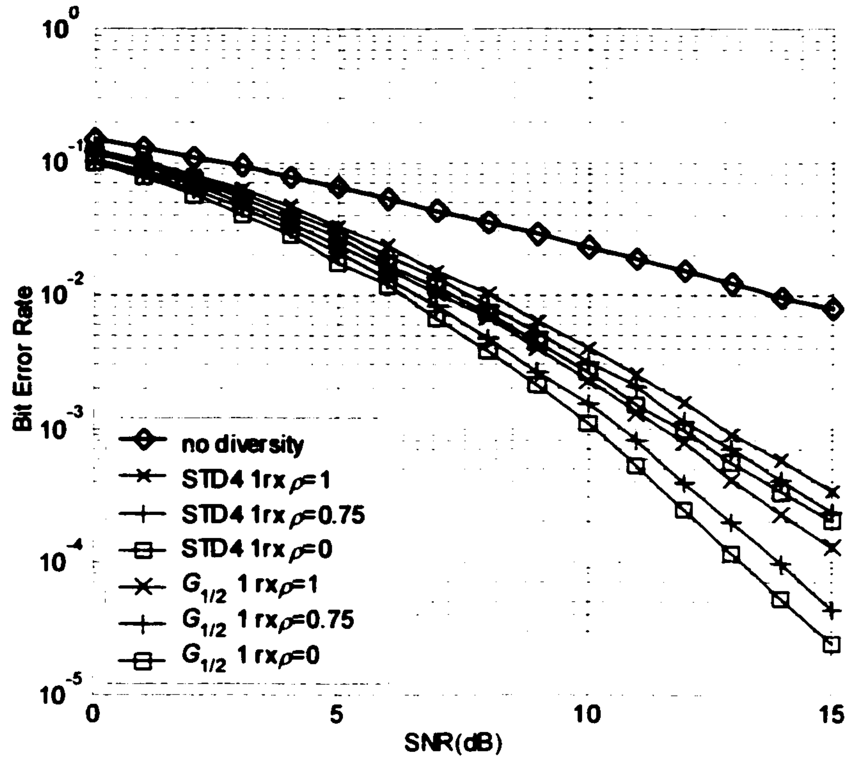


Figure 4.12 Effect of spatial correlation between 1st and 3rd channels to STD4 system with 4 transmitters and 1 receiver and $G_{1/2}$ system with 4 transmitters and 1 receiver at 1bit/sec/Hz.

4.5 Summary

In this chapter the effects of correlation among channels on the system BER performance has been studied. First the definition of spatial correlation has been made and different models for correlation has been given. Aalo's constant correlation model among all channels is studied for orthogonal channels. We observed that orthogonal space-time block codes are robust against correlation and the effect of correlation is the same with respect to MRC systems utilizing the same modulation.

The quasi-orthogonal code is sensitive to the correlation especially between its first and second and between third and fourth channels. If a high correlation exists between those channels then there are important losses in the system. Simulations show that the BER could have the values up to 0.25. Therefore system designers should avoid the correlation among the channels that are decoded together.

CHAPTER 5

Effects of Channel Estimation Errors to Space-Time Block Coded Systems

5.1 Introduction

Like correlation between channels, channel estimation errors (imperfect channel state information) could decrease the performance of diversity systems. Among the combining types, maximum ratio combining requires both the amplitude and phase information of the channel to achieve the diversity gain sought. If there are errors on this information, then the performance drops. In this chapter we will study the effect of channel estimation errors on the performance of space-time block coded systems and compare them with MRC systems.

5.2 Model for Channel Estimation Errors

In this work we model channel estimation errors e_{ce} as additive complex Gaussian random variablesas don in [Tar992]. The estimated channel coefficient $\hat{h}$ has the value:

$$\hat{h} = h + e_{ce} \tag{5.1}$$

To define the variance of the channel, we will use the term channel estimation SNR (SNR_{ce}) and the variance of the errors σ_{ce} is equal to:

$$\sigma_{ce} = \frac{1}{\mathrm{SNR}_{ce}} \tag{5.2}$$

In section 2.4.1 we have studied the maximum likelihood decoding for MRC systems. If the symbol S_0 is sent from a transmitter, then the resulting received signal at the l^{th} branch for a L branch system is:

$$r_l = h_l S_0 + n_l, \qquad l = 1,2.....L \tag{5.3}$$

where n_l denotes the additive white Gaussian noise sample with a variance N_0 at every branch. Consequently, in the presence of channel estimation errors the decision variable has the value from equations (5.1) and (2.23):

$$\widetilde{S} = \sum_{l=0}^{L} \hat{h}_l^* r_l = \sum_{l=0}^{L} (|h_l|^2 S_0 + e_{ce}^* h_l S_0 + h_l^* n_l + e_{ce}^* n_l) \tag{5.4}$$

Suppose energy per bit is equal to E_b and AWGN noise has the variance N_0 and every channel has equal average gain $\overline{\alpha^2}$, then the SNR at the input of the system for every branch is:

$$\overline{\gamma}_b = \frac{E_b}{N_0}\overline{\alpha^2} \tag{5.5}$$

Then from Eq. (5.4), the system has the SNR_{dv} after the construction of the decision variable below:

$$\begin{aligned}\mathrm{SNR}_{dv} &= \frac{E_b\left(\sum_{l=0}^{L}\overline{\alpha^2}\right)^2}{\sum_{l=0}^{L}\overline{\alpha^2}\sigma_{ce}E_b + \overline{\alpha^2}N_0 + \sigma_{ce}N_0} \\ &= \frac{E_b\sum_{l=0}^{L}\overline{\alpha^2}}{N_0 + \sigma_{ce}E_b + \frac{\sum_{l=0}^{L}\sigma_{ce}N_0}{\sum_{l=0}^{L}\overline{\alpha^2}}} = \frac{LE_b\overline{\alpha^2}}{N_0 + \sigma_{ce}E_b + \frac{\sigma_{ce}N_0}{\overline{\alpha^2}}}\end{aligned} \tag{5.6}$$

If σ_{ce} is equal to 0 (i.e. no channel estimation error), then the performance of the system is equal to the ideal system as expected, however at high signal level where the noise is negligible, SNR is equal to:

$$\lim_{N_0 \to 0} \mathrm{SNR}_{dv} = \lim_{N_0 \to 0} \frac{LE_b\overline{\alpha^2}}{N_0 + \sigma_{ce}E_b + \frac{\sigma_{ce}N_0}{\overline{\alpha^2}}} = \frac{L\overline{\alpha^2}}{\sigma_{ce}} \tag{5.7}$$

Eq. (5.7) shows that for channel estimation errors, in our model there is a maximum SNR achievable for the system and it depends on the gain from the channels and the SNR_{ce}. In Chapter 2 we have shown that if SNR at a branch is $\bar{\gamma}_b$, then the output SNR has the value $L\bar{\gamma}_b$. Therefore from Eq. (5.7) the performance limit of a system with channel estimation errors utilizing BPSK and QPSK modulation, is equal to the ideal system with SNR at every branch,

$$\bar{\gamma}_b = \frac{\overline{\alpha^2}}{\sigma_{ce}} \tag{5.8}$$

For higher order constellations the effect of channel estimation errors are more severe.

In Figure 5.1 the effect of channel estimation errors for a no diversity system are shown. In the simulations in this chapter, Matlab 6.1 ® is used and again the minimum number of errors count per SNR value is 100. An example function to create the channel estimation errors and simulating the systems can be found in Appendix D.

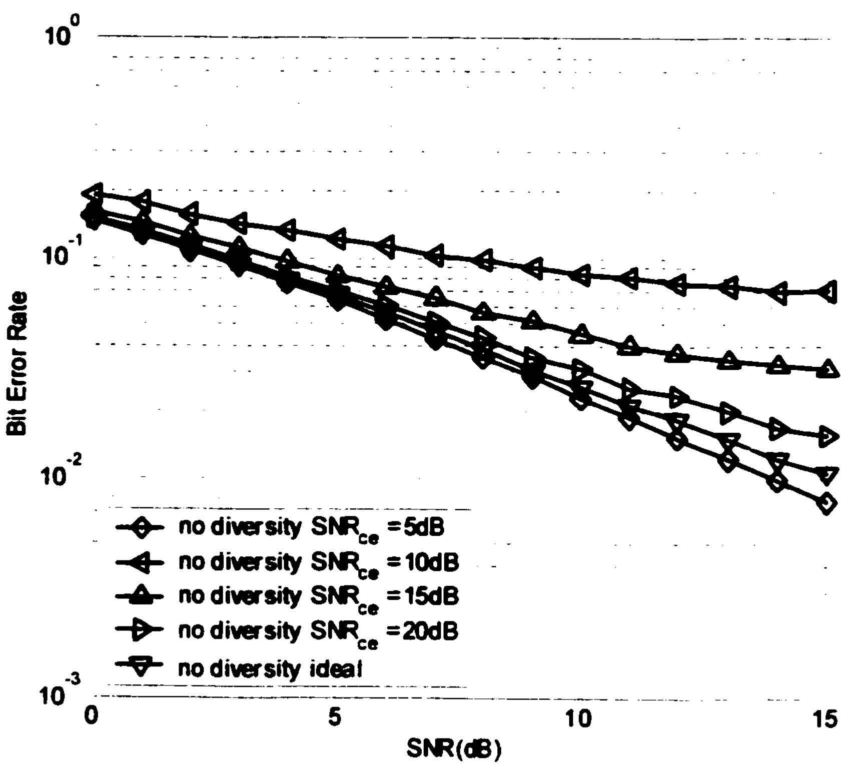


Figure 5.1 Effect of channel estimation errors to a no diversity system under Rayleigh fading at 1 bit/sec/Hz.

In this simulation we set the average channel gain to 1. Therefore the upper limit for SNR_{dv} is equal to SNR_{cc}. This effect can be seen at 5 dB SNR_{cc} the system has 0.07 bit error rate at 15 dB SNR and the performance curve is almost horizontal. We simulated the system with very high SNR values like 1000 dB and the system has the minimum BER limit of 0.0645, which is the value of ideal system at 5 dB. In Figure 5.2 the simulations results are presented for MRC system with 2 antennas. The performance lost at 10^{-3} BER for 15 dB SNR_{ce} is around 2 dB with respect to the ideal system. Again for 5 dB SNR_{ce} the system approaches the asymptotic BER rate of ideal system with branch SNR of 5 dB.

In the next section we study the effects of correlation to different space-time codes.

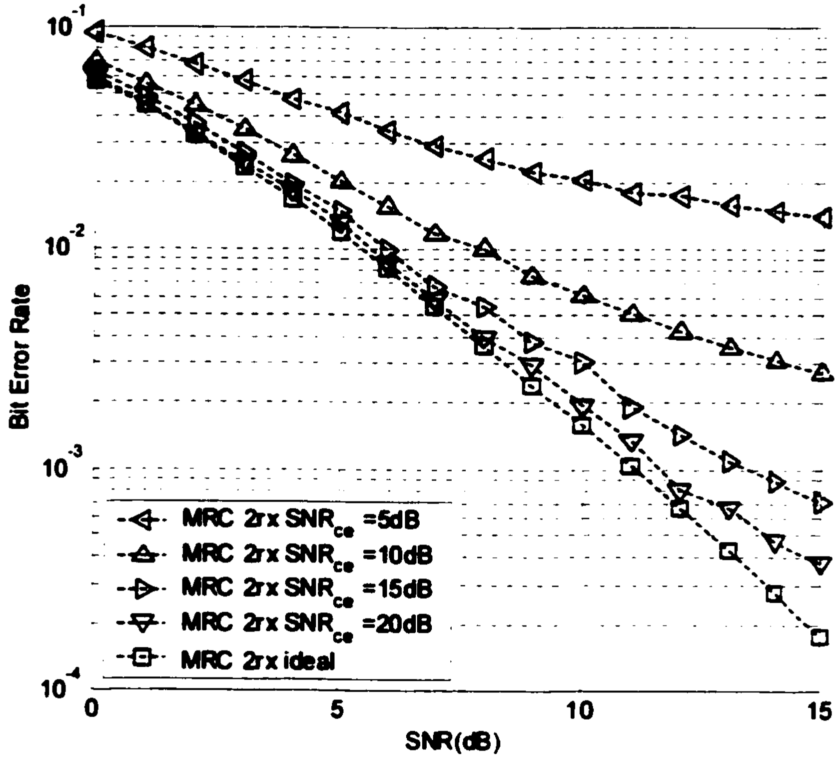


Figure 5.2 Effect of channel estimation errors to a MRC system with 2 receive antennas under Rayleigh fading at 1bit/sec/Hz.

5.3 Effects of Channel Estimation Errors to Orthogonal Space-Time Block Coded Systems

The first system under consideration is Alamouti's STD system. In section 3.4.4.1 we have shown the decision variables of STD system and calculated their SNR values. For an STD system utilizing M receiver antennas under the presence of channel estimation errors, the decision variable at the m^{th} branch has the value:

$$\begin{aligned}\widetilde{S}_{0,m} &= r_m(0)(h_{1,m}^* + e_{ec1,m}^*) + r(1)_m^*(h_{2,m} + e_{ec2,m}^*) \\ &= (|h_{1,m}|^2 + |h_{2,m}|^2)S_0 + (h_{1,m}^* + e_{ec1,m}^*)n_m(0) + (h_2, m + e_{ec2,m})n_m(1) \\ &\quad + e_{ec1,m}^*(h_{1,m}S_0 + h_{2,m}S_1) + e_{ec2,m}(h_{2,m}^*S_0 - h_{1,m}^*S_1)\end{aligned} \quad (5.9)$$

In this case SNR$_{STD}$ is equal to:

$$\mathrm{SNR}_{\mathrm{STD}} = \frac{\frac{E_b}{2}\left(\sum_{l=0}^{2M}\overline{\alpha^2}\right)^2}{\sum_{l=0}^{2M}\left(\overline{\alpha^2}\sigma_{ce}2\frac{E_b}{2}+\overline{\alpha^2}N_0+\sigma_{ce}N_0\right)}$$

$$= \frac{\frac{E_b}{2}\sum_{l=0}^{2M}\overline{\alpha^2}}{N_0+\sigma_{ce}E_b+\frac{\sum_{l=0}^{2M}\sigma_{ce}N_0}{\sum_{l=0}^{2M}\overline{\alpha^2}}} = \frac{\frac{E_b}{2}\sum_{l=0}^{2M}\overline{\alpha^2}}{N_0+\sigma_{ce}E_b+\frac{\sigma_{ce}N_0}{\overline{\alpha^2}}} \tag{5.10}$$

At very high input SNR, where noise is negligible, the system has the $\mathrm{SNR}_{\mathrm{STD}}$:

$$\lim_{N_0\to 0}\mathrm{SNR}_{\mathrm{STD}} = \lim_{N_0\to 0}\frac{\frac{E_b}{2}\sum_{l=0}^{2M}\overline{\alpha^2}}{N_0+\sigma_{ce}E_b+\frac{\sigma_{ce}N_0}{\overline{\alpha^2}}} = \frac{\sum_{l=0}^{2M}\overline{\alpha^2}}{2\sigma_{ce}} \tag{5.11}$$

In Eq. (3.43) we have shown that the STD systems performance is 3 dB less than the MRC system under no channel estimation. Eq. (5.11) shows that in the limit case for channel estimation errors, that there is still 3 dB difference in the performance but in terms of SNR_{ce}. In Chapter 3 we have shown that if SNR is

$$\overline{\gamma}_b = \frac{E_b}{N_0}\overline{\alpha^2} \tag{5.12}$$

then the output SNR has the value $M\overline{\gamma}_b$ for equal gain channels. Therefore from Eq. (5.11) the performance limit of a system with channel estimation errors utilizing BPSK or QPSK, is equal to the ideal system with SNR,

$$\overline{\gamma}_b = \frac{\overline{\alpha^2}}{\sigma_{ce}} \tag{5.13}$$

In Figure 5.3 STD system with 1 receive antenna is simulated for different SNR_{ce}. Again the channel coefficients have the average gain of 1. For 20 dB SNR_{ce} the loss is around 1 dB at 10^{-3} BER. 15 dB SNR_{ce} results in a 2 dB loss at 10^{-2} BER. 5 dB and 10 dB SNR_{ce} curves are approaching their asymptotical values in the figure. Especially for 5 dB SNR_{ce} it is easy to observe this behaviour.

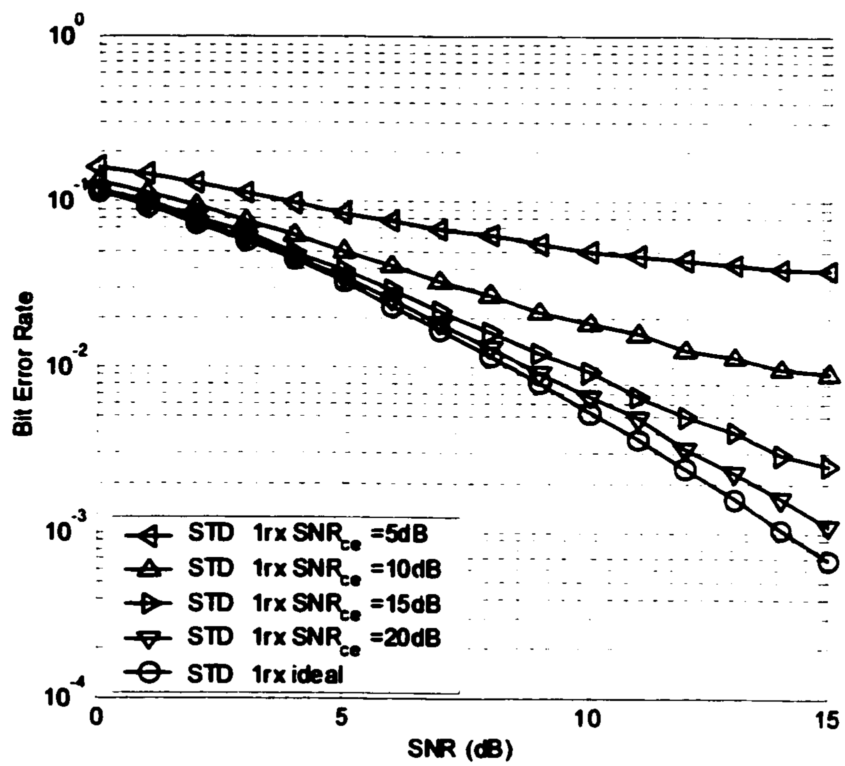


Figure 5.3 Effect of channel estimation errors to a STD system with 1 receiver under Rayleigh fading at 1bit/sec/Hz.

In Figure 5.4 STD system with 2 receive antennas is simulated for different SNR_{ce} values. At 10^{-5} BER, 20 dB SNR_{ce} results in a 1 dB loss with respect to the ideal system. Again we observe that 5 dB and 10 dB SNR_{ce} systems are close to their error floor values, which are the BER's of the ideal system with 5dB and 10 dB SNR value.

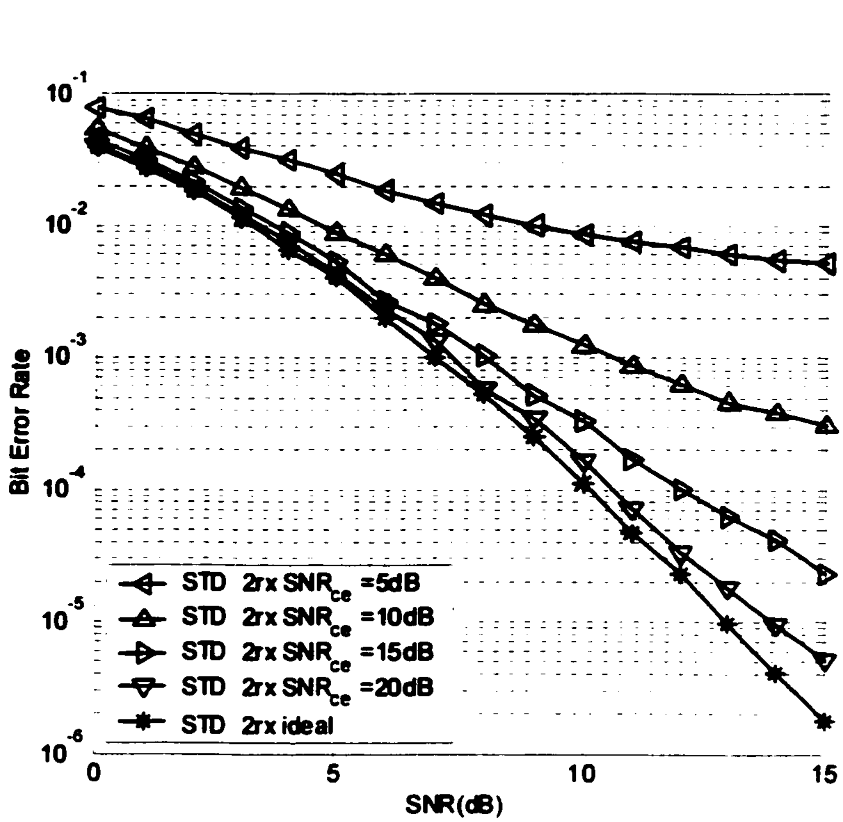


Figure 5.4 Effect of channel estimation errors to a STD system with 2 receivers under Rayleigh fading at 1 bit/sec/Hz.

For rate 1/2 systems, the first simulations are conducted at 1 bit/sec/Hz. The results show that the effect of channel estimation errors is similar to STD system.

In Figure 5.5 the performance curves of $H_{1/2}$ system with 3 transmitters under different SNR_{ce} values are presented. Up to 2×10^{-4} BER the loss of 20 dB SNR_{ce} does not exceed 1 dB. The loss of 10 dB SNR_{ce} is around 3 dB at 10^{-2} BER. 5 dB SNR_{ce} curve is almost horizontal at 15dB SNR. Its asymptotical value is 0.022.

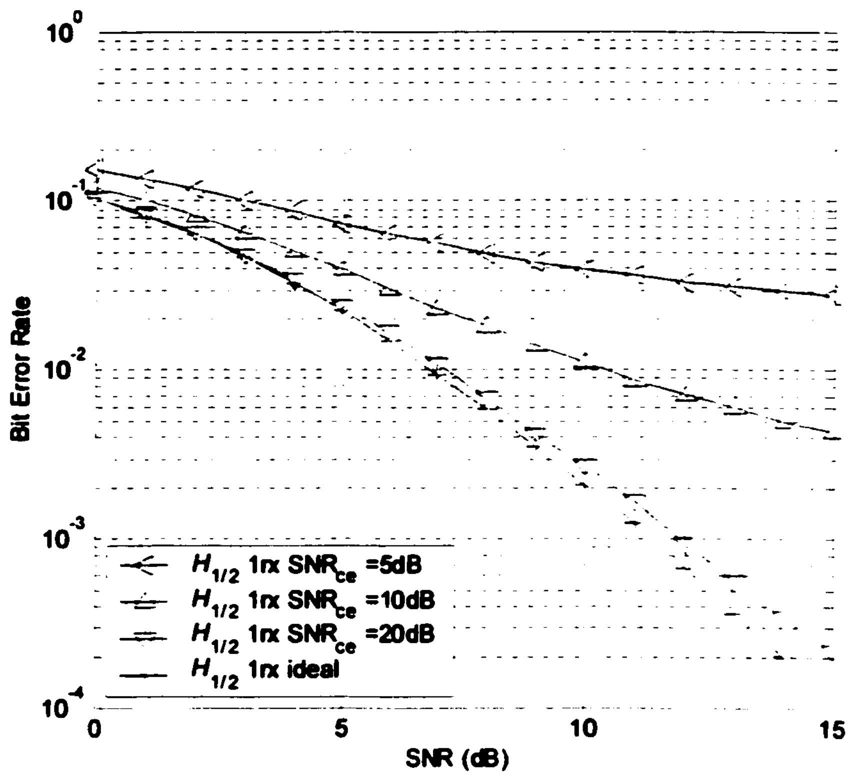


Figure 5.5 Effect of channel estimation errors to $H_{1/2}$ system with 3 transmitters and 1 receiver under Rayleigh fading at 1bit/sec/Hz.

The $G_{1/2}$ system with 4 transmitters and 1 receiver has the performance curves in Figure 5.6 under different SNR_{ce} values. We observed that the maximum loss due to 20 dB SNR_{ce} is around 1 dB in the limits of the simulation. The loss of 10 dB SNR_{ce} is around 3 dB at 10^{-2} BER, like the previous system. 5 dB SNR_{ce} curve is very close to its asymptotical limit, which is around 0.018.

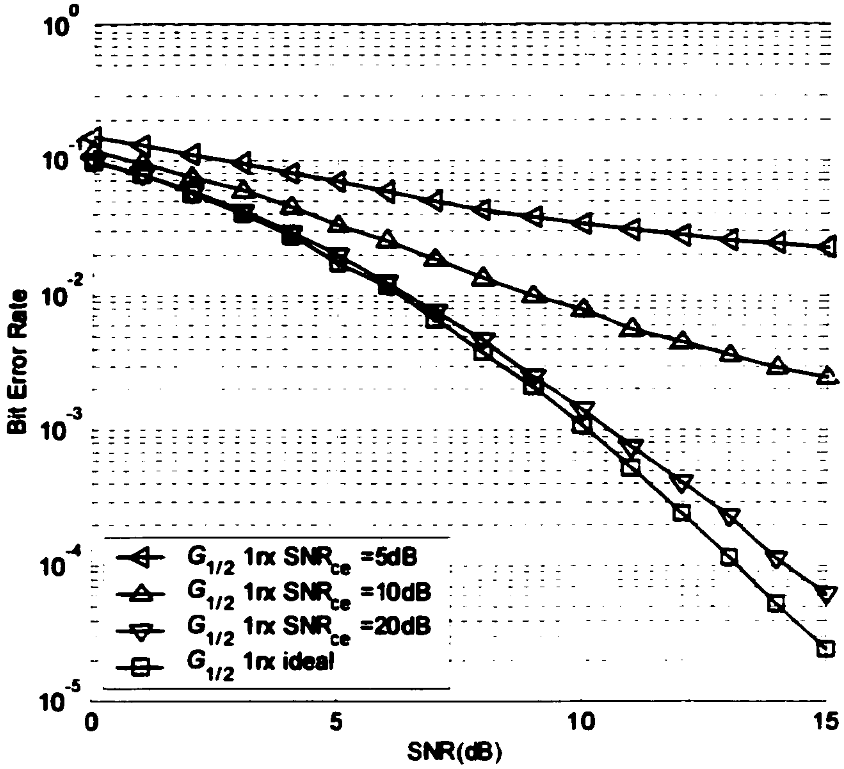


Figure 5.6 Effect of channel estimation errors to $G_{1/2}$ system with 4 transmitters and 1 receiver under Rayleigh fading at 1 bit/sec/Hz.

Rate 1/2 systems $H_{1/2}$ with 3 transmitters and $G_{1/2}$ with 4 transmitters have the performance curves in Figure 5.7 and in Figure 5.8 at 2 bit/sec/Hz. 16QAM modulation is used in the simulations. In higher order constellations the effect of channel estimation errors are much more severe. At 5 dB SNR_{ce}. the minimum BER's of $H_{1/2}$ and $G_{1/2}$ are higher than 0.1 and 0.09 respectively and the performance curves are flat. At 20 dB SNR_{ce} the loss $H_{1/2}$ system is around 0.9 dB at 0.01 BER, whereas loss of $G_{1/2}$ is around 0.6 dB. In Figure 5.9 and in Figure 5.10, the rate 3/4 orthogonal systems are at 3 bit/sec/Hz simulated. In the simulations $H_{3/4}$ system has 3 transmitters and 1 receiver. Its performance loss is around 1.5 dB at 0.01 BER for 20 dB SNR_{ce} as seen in Figure 5.9. At 5 dB SNR_{ce} the minimum BER for this system is 0.165. The $G_{3/4}$ system has 4 transmitters and 1 receiver. In our simulations the BER performance of $G_{3/4}$ at 10 dB SNR_{ce} does not exceed 0.06 as seen in Figure 5.10. For 20 dB SNR_{ce} the loss of the system is around 1.4 dB at 0.01 BER.

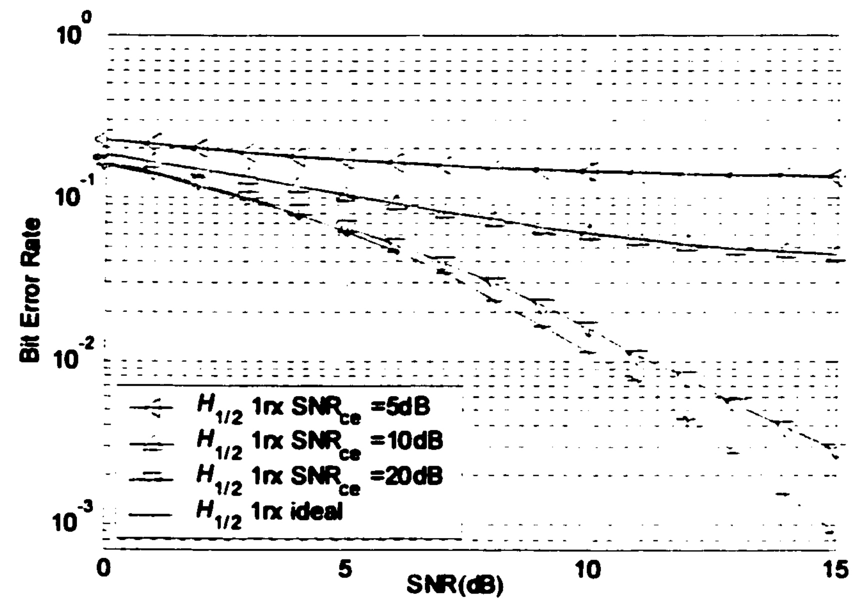


Figure 5.7 Effect of channel estimation errors to $H_{1/2}$ system with 3 transmitters and 1 receiver under Rayleigh fading at 2 bit/sec/Hz.

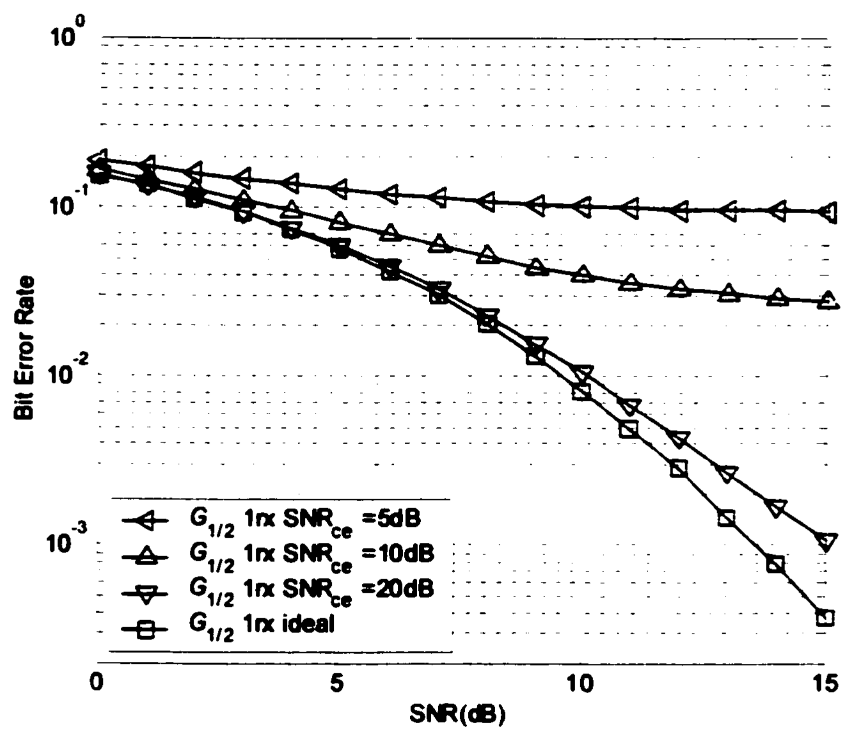


Figure 5.8 Effect of channel estimation errors to $G_{1/2}$ system with 4 transmitters and 1 receiver under Rayleigh fading at 2 bit/sec/Hz.

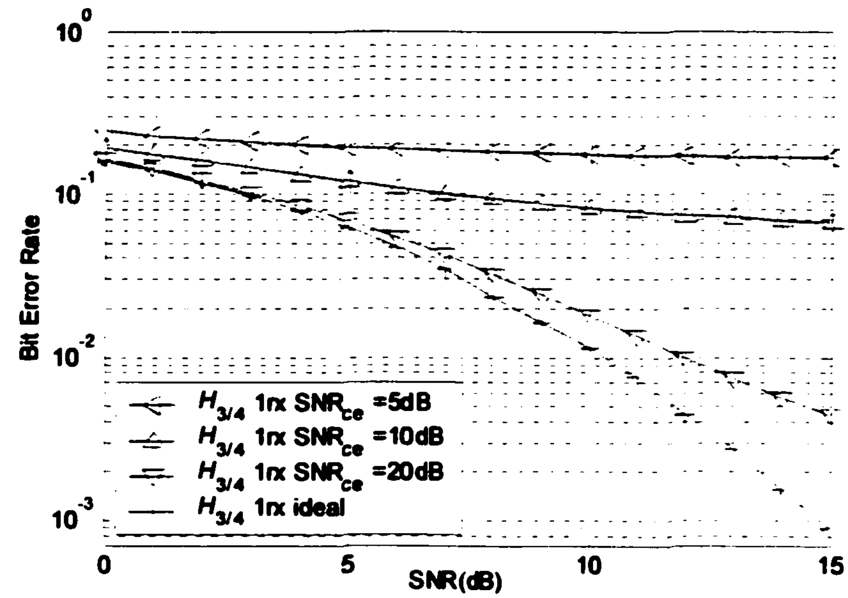


Figure 5.9 Effect of channel estimation errors to $H_{3/4}$ system with 3 transmitters and 1 receiver under Rayleigh fading at 3 bit/sec/Hz.

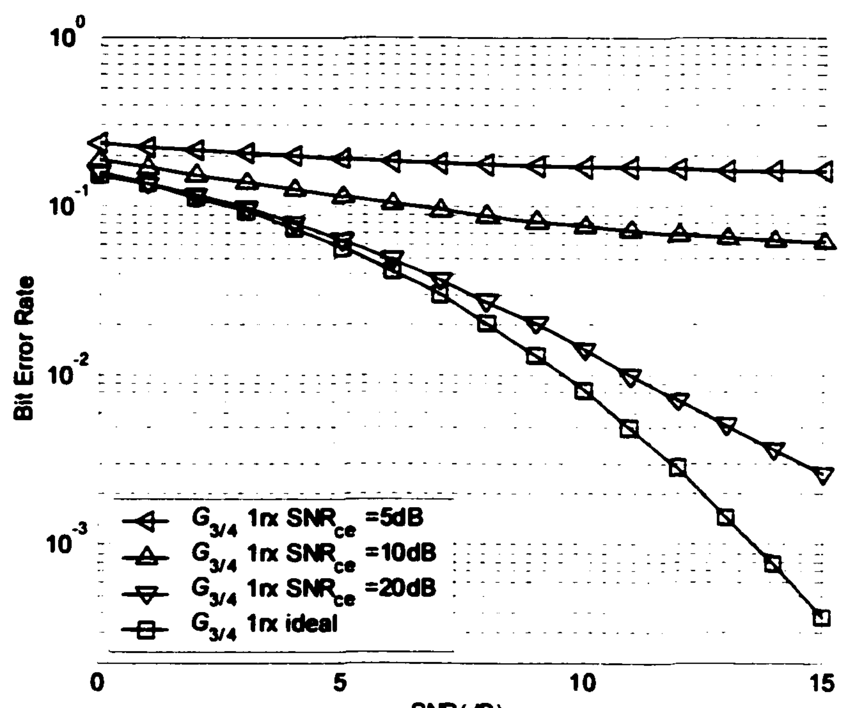


Figure 5.10 Effect of channel estimation errors to $G_{3/4}$ system with 4 transmitters and 1 receiver under Rayleigh fading at 3 bit/sec/Hz.

5.4 Effects of Channel Estimation Errors to Quasi-orthogonal Space-Time Block Coded Systems

In this section the performance of STD4 system is studied under channel estimation error at 1 bit/sec/Hz and 3 bit/sec/Hz. In Figure 5.10, the modulation scheme is BPSK and therefore the rate of the system is 1 bit/sec/Hz.

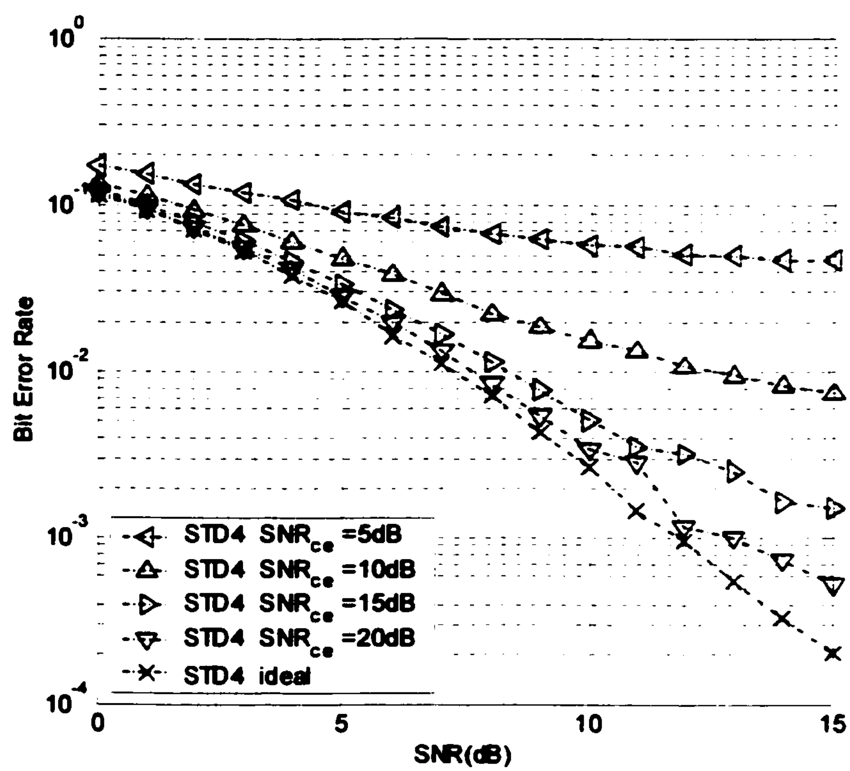


Figure 5.11 Effect of channel estimation errors to STD4 system with 4 transmitters and 1 receiver under Rayleigh fading at 1 bit/sec/Hz.

STD4 system is more sensitive to channel estimation errors with respect to the orthogonal systems at 1 bit/sec/Hz. For example at 5 dB SNR_{ce} its performance limit is 0.04 BER, which is worse than all systems excluding no diversity system. For 20 dB SNR_{ce} the loss of the system is around 1 dB at 10^{-3} BER. 5.5 dB loss is observed at 0.01 BER for the case of 10 dB SNR_{ce}. Another simulation for this system is done at 3 bit/sec/Hz by utilizing 8PSK scheme.

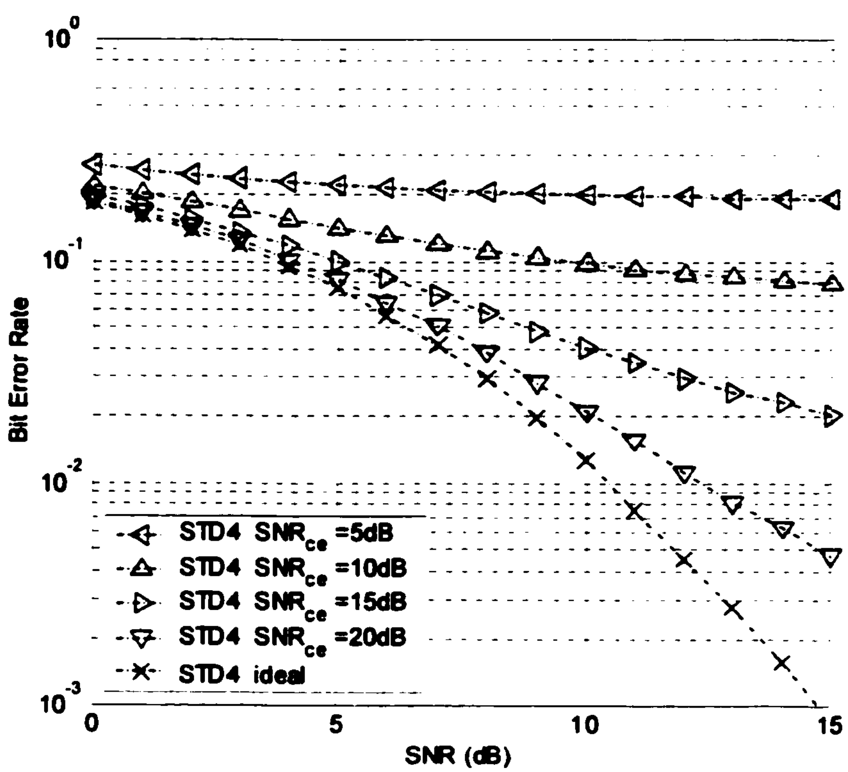


Figure 5.12 Effect of channel estimation errors to STD4 system with 4 transmitters and 1 receiver under Rayleigh fading at 3 bit/sec/Hz.

In Figure 5.12 the results are presented. For 5 dB SNR_{ce} the performance of the system is limited by 0.185 BER, which is worse than the orthogonal 4 transmit antenna code $G_{3/4}$ at 3 bit/sec/Hz. The loss of the system at 10^{-2} BER is around 1.9 dB for 20 dB SNR_{cc}.

5.5 Summary

In this chapter, effects of channel estimation errors on the performance of space-time block coded systems has been studied. A general model for channel estimation errors has been specified, where errors are modeled as complex Gaussian random variables.

If the variance of the channel estimation errors is equal to 1/100th of average channel gain (20 dB SNR_{ce}) the maximum loss of the orthogonal systems is around 1 dB at 1 bit/sec/Hz. For the STD4 system we observed a maximum 1.5 dB loss. Quasi-orthogonal system is more sensitive to channel estimation errors as was the case for correlation. At high bandwidth efficiency scenarios, the robustness of orthogonal systems drops because of utilizing higher order constellations, but again STD4 system at 3 bit/sec/HZ has the worst performance among all systems at 5 dB SNR_{ce}.

CHAPTER 6
Conclusions

6.1 Summary of the Space –Time Block Coded Systems

In this chapter we will briefly summarize the properties of systems we have studied. STD system is the only full rate full diversity order among space-time block coded systems. Therefore it achieves the diversity order and performance of its equivalent MRC system with only a 3 dB penalty. The system is robust against correlation and channel estimation errors. For example a 2 transmitter 2 receiver STD system provides 7 dB gain with respect to a no diversity system, even if the correlation among its channels is equal to 0.9. In terms of channel estimation errors the same system has only 1 dB loss at 10^{-5} BER if the SNR_{ce} is 20 dB.

Two rate 1/2 system $H_{1/2}$ with three transmit antennas and $G_{1/2}$ with four transmit antennas provide full diversity gain. At 1 bit/sec/Hz there is no penalty for these systems since they are utilizing QPSK, but at 2 bit/sec/Hz STD system has better performance than these codes up to 13 and 16 dB respectively, since their modulation scheme is 16QAM. In terms of robustness against correlation both systems are good. At 1 bit/sec/Hz they provide some diversity gain unless the channels are exactly the same, whereas at 2 bit/sec/Hz and 0.95 correlation among channels the systems start to provide some extra gain only after 15 dB SNR. If channel estimation errors are considered, their performance is robust at high SNR_{ce} values at both 1 and 2 bit/sec/Hz. At low SNR_{ce} values especially at 2 bit/sec/Hz the performance of the systems drop dramatically.

Rate 3/4 systems $H_{3/4}$ and $G_{3/4}$ are studied only at 3 bit/sec/Hz. They provide the same performance curves in correlation simulations of rate 1/2 systems at 2 bit/sec/Hz. However their robustness against channel estimation errors are less than rate 1/2 systems.

STD4 code is designed to obtain full rate at the expense of diversity order. Basically it provides diversity order of 2 with its 4 transmit antennas. Simulations show that it has only a gain advantage to 4 antenna orthogonal systems up to 17 dB SNR at 2 bit/sec/Hz. The system is very sensitive to correlation between its first and second and between third and fourth channels. In terms of channel estimation errors the system is susceptible too. Especially at 3 bits/sec/Hz, the performance of the system is inferior to the three transmit antenna $H_{3/4}$ system at every SNR_{ce} value.

6.2 Future Work

We made the simulations for the effects of channel estimation errors and correlation among channels separately. A work about their combined effect to space-time block coded systems is in progress.

Another study will be conducted about mathematical expression for the performance of STD4 system.

As mentioned in Chapter 3 there are several space-time trellis codes with better performances that the first ones introduced. But there are very few works on their performance in the presence of channel estimation errors or correlation among channels.

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Appendix A:

The following function is written in Matlab to obtain the performance of STD system with one receiving antenna. This function assumes that there is perfect channel state knowledge at the receiver.

```
function Ber=std_2(N,ebno)
%STD_2 simulates simple transmit diversity scheme for 2 transmit and
%one receive antennas for BPSK modulation.
% Ber=std_2(N,ebno)
% 'N' denotes the number of samples and 'ebno' denotes the Signal to
Noise Ratio in dB.
% 'Ber' is the output of the function, which is the bit error rate.

% Tuncer Baykas 6/4/2001

% generating the samples
% since in diversity systems total energy is conserved
% every sample is divided by square root of 2.
s1=randint(1,N);
s1=(2*s1-1);
s1=s1/sqrt(2);
s2=randint(1,N);
s2=(2*s2-1);
s2=s2/sqrt(2);

% The antenna time mapping of STD
i=1:N;
at1(2*i-1)=s1(i);
at1(2*i)=-s2(i);

at2(2*i-1)=conj(s2(i));
at2(2*i)=conj(s1(i));

% Generating channel
a=randn(1,N);
b=randn(1,N);
x=(a+b*j)/sqrt(var(a+b*j));

% Since there is no interleaving it is assumed
% the channels value is same for two time slot
channel1(2*i)=x(i);
channel1(2*i-1)=x(i);

% making the standard deviation of the channel equal to one since
% ebno will be equalized using the noise standard deviation.
channel1=channel1/std(channel1);

% generating the second channel
a=randn(1,N);
```

```
b=randn(1,N);
x=(a+b*j)/sqrt(var(a+b*j));
channel2(2*i)=x(i);
channel2(2*i-1)=x(i);
channel2=channel2/std(channel2);


% generating recieved samples
rxd1=(channel1.*at1);
rxd2=(channel2.*at2);

% generating noise...
n1=randn(1,2*N);
n2=randn(1,2*N);
% Making signal to noise equal to 'ebno'
n=sqrt(1/(10^(ebno/10)))*(n1+n2*j)/sqrt(var(n1+n2*j));

%Finding total recieved vector
rxd=rxd1+rxd2+n;

% Finding the decision vectors for the bits.
r1(i)=conj(channel1(2*i)).*rxd(2*i-1)+channel2(2*i).*conj(rxd(2*i));
r2(i)=channel2(2*i).*conj(rxd(2*i-1))-conj(channel1(2*i)).*rxd(2*i);

% The output is checked with the input and the bit error rate is found
% by dividing 2*N...
Ber=(sum((r1.*s1)<0)+sum((r2.*s2)<0))./(2*N);
```

Appendix B:

In this appendix, the way of obtaining correlated random channels will be explaind and a sample function is given.

Suppose there are n channels in a system then using Matlab , n independent channels coefficients can be obtained and since correlation in time is not considered, they can be seen as a vector of n random variables together.

$$\mathbf{X}=(X_0,X_1,X_2\ldots X_n) \tag{A.1}$$

In the simulations the gain of the channels is equal to one. therefore any correlation matrix has the form:

$$\boldsymbol{K}=\begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n} \\ \rho_{1,2} & 1 & & \vdots \\ \vdots & & \ddots & \rho_{n-1,n} \\ \rho_{1,n} & \cdots & \rho_{n-1,n} & 1 \end{bmatrix} \tag{A.2}$$

$\boldsymbol{K}$ can be expressed in the form (A.3).

$$\boldsymbol{K}=\mathrm{P}\Lambda\mathrm{P}^{T} \tag{A.3}$$

where P is a matrix whose columns consist of the eigenvalues of $\boldsymbol{K}$ and Λ is the diagonal matrix that consists of the eigenvalues of $\boldsymbol{K}$. By computing equation below,

$$\mathrm{Y}=\mathrm{P}\Lambda^{1/2}\,\mathbf{X} \tag{A.4}$$

we obtain Y, which is a vector consists of n correlated random variables. For two channels and a specified correlation coefficient ρ. $\mathrm{P}\Lambda^{1/2}$ is equal to:

$$\mathbf{P}\Lambda^{1/2} = \begin{bmatrix} \sqrt{(1-\rho)/2} & \sqrt{(1+\rho)/2} \\ -\sqrt{(1-\rho)/2} & \sqrt{(1+\rho)/2} \end{bmatrix} \qquad \textbf{(A.5)}$$

The following function in Matlab is used for STD4 system:

```
function ber=std_4_corr_8psk(N,ebno,C)
% STD_4_corr_8psk simulates simple transmit diversity scheme for 4
transmit
% and one receive antennas without interleaving using 8PSK modulation
with
% given covariance matrix among Rayleigh fading channels.

% ber=std_4_corr_8psk(N,ebno,C)
% 'N' denotes the number of samples and 'ebno' denotes the
% Signal to Noise Ratio in dB,
% 'C' is the correlation matrix among channels ,
% 'ber' is the output of the function
% which is the bit error rate.

% Tuncer Baykas 6/1/2002

% generating the samples
% since in diversity systems total energy is conserved
% each samples should halved.

psk8=sqrt(3)*[j,cos(pi/4)+sin(pi/4)*j,1,+cos(pi/4)-cos(pi/4)*j,-j,-
cos(pi/4)-sin(pi/4)*j,-1,-cos(pi/4)+sin(pi/4)*j];
ss1=floor(8*rand(1,N))+1;
ss2=floor(8*rand(1,N))+1;
ss3=floor(8*rand(1,N))+1;
ss4=floor(8*rand(1,N))+1;


s1=psk8(ss1)/2;
s2=psk8(ss2)/2;
s3=psk8(ss3)/2;
s4=psk8(ss4)/2;

i=1:N;

% Antenna-time mapping of std_4
at1(4*i-3)=s1(i);
at1(4*i-2)=s2(i);
at1(4*i-1)=s3(i);
at1(4*i)=s4(i);


at2(4*i-3)=s2(i);
at2(4*i-2)=s1(i);
at2(4*i-1)=s4(i);
at2(4*i)=s3(i);


at3(4*i-3)=conj(s3(i));
```

```
at3(4*i-2)=conj(s4(i));
at3(4*i-1)=-conj(s1(i));
at3(4*i)=-conj(s2(i));


at4(4*i-3)=conj(s4(i));
at4(4*i-2)=conj(s3(i));
at4(4*i-1)=-conj(s2(i));
at4(4*i)=-conj(s1(i));

% generating independent Rayleigh fading channels

a=randn(1,N);
b=randn(1,N);
x1=(a+b*j);
x1=x1./std(x1);
a=randn(1,N);
b=randn(1,N);
x2=(a+b*j);
x2=x2./std(x2);
a=randn(1,N);
b=randn(1,N);
x3=(a+b*j);
x3=x3./std(x3);
a=randn(1,N);
b=randn(1,N);
x4=(a+b*j);
x4=x4./std(x4);

%Finding Eigenvectors and Eigenvalues of C to correlate independent
channels
[Evec,Evar]=eig(C);
%Corralator Matrix
Cr=Evec*sqrt(Evar);

% Correlated Channels
Ch=Cr*[x1; x2; x3; x4];

% Making the values of channels for 4 time slots equal
h1(4*i)=Ch(1,i);
h1(4*i-3)=Ch(1,i);
h1(4*i-2)=Ch(1,i);
h1(4*i-1)=Ch(1,i);

h2(4*i)=Ch(2,i);
h2(4*i-1)=Ch(2,i);
h2(4*i-2)=Ch(2,i);
h2(4*i-3)=Ch(2,i);

h3(4*i)=Ch(3,i);
h3(4*i-3)=Ch(3,i);
h3(4*i-2)=Ch(3,i);
h3(4*i-1)=Ch(3,i);

h4(4*i)=Ch(4,i);
h4(4*i-1)=Ch(4,i);
h4(4*i-2)=Ch(4,i);
```

```
h4(4*i-3)=Ch(4,i);

% computing the received vectors
rxd1=(h1.*at1);
rxd2=(h2.*at2);
rxd3=(h3.*at3);
rxd4=(h4.*at4);

% generating noise and applying signal to noise ratio
n=randn(1,4*N)+j.*randn(1,4*N);
n=sqrt(1/(10^(ebno/10)))*(n)/std(n);

% calculating total recieved vector
rxd=rxd1+rxd2+rxd3+rxd4+n;

% generating the decision variables
d1(i)=conj(h1(4*i)).*rxd(4*i-3)+conj(h2(4*i)).*rxd(4*i-2)-
h3(4*i).*conj(rxd(4*i-1))-h4(4*i).*conj(rxd(4*i));
d2(i)=conj(h2(4*i)).*rxd(4*i-3)+conj(h1(4*i)).*rxd(4*i-2)-
h4(4*i).*conj(rxd(4*i-1))-h3(4*i).*conj(rxd(4*i));
d3(i)=conj(h3(4*i)).*rxd(4*i-3)+conj(h4(4*i)).*rxd(4*i-
2)+h1(4*i).*conj(rxd(4*i-1))+h2(4*i).*conj(rxd(4*i));
d4(i)=conj(h4(4*i)).*rxd(4*i-3)+conj(h3(4*i)).*rxd(4*i-
2)+h2(4*i).*conj(rxd(4*i-1))+h1(4*i).*conj(rxd(4*i));
d3=conj(d3);
d4=conj(d4);

% genrating A and B
A(i)=h1(4.*i).*conj(h1(4.*i))+h2(4.*i).*conj(h2(4.*i))+h3(4.*i).*conj(h
3(4.*i))+h4(4.*i).*conj(h4(4.*i));
B(i)=h1(4.*i).*conj(h2(4.*i))+h2(4.*i).*conj(h1(4.*i))+h3(4.*i).*conj(h
4(4.*i))+h4(4.*i).*conj(h3(4.*i));


% generating distance function and counting number of errors
y=1:64;

ber=0;

bmap=[0,0,0;0,0,1;0,1,0;0,1,1;1,1,0;1,1,1;1,0,1;1,0,0];

for i=1:N
[dd1,dind]=min(abs(d1(i)-
(A(i).*psk8(ceil(y/8))/2+B(i).*psk8(mod(y,8)+1)/2)).^2+abs(d2(i)-
(B(i).*psk8(ceil(y/8))/2+A(i).*psk8(mod(y,8)+1)/2)).^2);
ber=ber+sum(bmap(ceil(dind/8),:)~=bmap(ss1(i),:))+sum(bmap((mod(dind,8)
+1),:)~=bmap(ss2(i),:));
[dd1,dind]=min(abs(d3(i)-
(A(i).*psk8(ceil(y/8))/2+B(i).*psk8(mod(y,8)+1)/2)).^2+abs(d4(i)-
(B(i).*psk8(ceil(y/8))/2+A(i).*psk8(mod(y,8)+1)/2)).^2);
ber=ber+sum(bmap(ceil(dind/8),:)~=bmap(ss3(i),:))+sum(bmap((mod(dind,8)
+1),:)~=bmap(ss4(i),:));

end

% calculating bir error rate
ber=ber/12/N;
```

Appendix C:

In this appendix we will prove, why the BER of BPSK or QPSK modulated STD4 drops to 0.25 at high SNR, if there is full correlation between 1st and 2nd and between 3rd and 4th channels.

The antenna-time mapping of STD4 is:

	Tx. Ant 1	Tx. Ant. 2	Tx. Ant 3	Tx. Ant. 4
Time 0	S_0	S_1	S_2	S_3
Time 1	S_1	S_0	S_3	S_2
Time 2	S_2^*	S_3^*	$-S_0^*$	$-S_1^*$
Time 3	S_3^*	S_2^*	$-S_1^*$	$-S_0^*$

And the decision variables are

$$\begin{bmatrix} \tilde{S}_0 \\ \tilde{S}_1 \\ \tilde{S}_2 \\ \tilde{S}_3 \end{bmatrix} = \begin{bmatrix} A & B & 0 & 0 \\ B & A & 0 & 0 \\ 0 & 0 & A & B \\ 0 & 0 & B & A \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} h_0^* & h_1^* & -h_2 & -h_3 \\ h_1^* & h_0^* & -h_3 & -h_2 \\ h_2^* & h_3^* & h_0 & h_1 \\ h_3^* & h_2^* & h_1 & h_0 \end{bmatrix} \begin{bmatrix} n(0) \\ n(1) \\ n(2)^* \\ n(3)^* \end{bmatrix} \quad \text{(A.6)}$$

where,

$$A = h_0^2 + h_1^2 + h_2^2 + h_3^2 \quad \text{(A.7)}$$

$$B = h_0 h_1^* + h_1 h_0^* + h_2 h_3^* + h_3 h_2^* \quad \text{(A.8)}$$

We will focus on $(\tilde{S}_0, \tilde{S}_1)$ pair. But the results are valid for $(\tilde{S}_2, \tilde{S}_3)$ due to the symmetry of the system. Symbols, S^i and S^j are chosen for S_0 and S_1 respectively if

$$\begin{aligned} & d^2(\tilde{S}_0, (AS^i + BS^j)) + d^2(\tilde{S}_1, (AS^j + BS^i)) \;< \\ & d^2(\tilde{S}_0, (AS^k + BS^l)) + d^2(\tilde{S}_1, (AS^l + BS^k)) \quad \forall k, l \end{aligned} \quad \text{(A.9)}$$

If there is full correlation between 1st and 2nd and between 3rd and 4th channels. A is equal to B, and above equation becomes:

$$\begin{aligned} &d^2(\tilde{S}_0,(AS^i + AS^j)) + d^2(\tilde{S}_1,(AS^j + AS^i)) \quad < \\ &\quad d^2(\tilde{S}_0,(AS^k + AS^l)) + d^2(\tilde{S}_1,(AS^l + AS^k)) \quad \forall k,l \end{aligned} \tag{A.10}$$

If the sum of any S^i and S^j pair is equal to sum of another S^k and S^l pair, then the decision cannot be made certainly and the error probability of the decision is 0.5 at high SNR. Suppose signal S and $-S$ are used in BPSK modulation There are four possible pairs and (S,-S) and (-S,S) has the same summation value. At high SNR we can assume there are no errors in decoding of other pairs and therefore the BER is 0.25.

For QPSK assuming the symbols (S,-S,iS,-iS), there are 16 possible pairs. 4 of them has the sum 0, 2 of them has (S+iS), 2 of them has (-S+iS), 2 of them has (-S-iS) and 2 of them has (S-iS) and the rest 4 is unique. If all symbols are equally likely and gray coding is used, then if noise is ignored for the pairs of sum 0 BER is 0.5. for unique sum ones 0 and for the rest 0.25 from m the theory of probability. the total BER is equal to. 0.5*4/16+4*(0.25*2/16)+4*0*4/16= 0.5

For other modulation techniques the proof can be extended.

Appendix D:

Matlab program to simulate channel estimation errors the function below is used for MRC system with 2 receive antennas.

```
function Ber=cee_mrrc(N,ebno,c,mu)

% MRRC simulates maximum received ratio combinig scheme for 1 transmit
and 2 receive antennas
% for BPSK modulation.
% Ber=MRRC(N,ebno,c,mu)
% 'N' denotes the number of samples and 'ebno' denotes the Signal to
Noise Ratio in dB.
% 'c' denotes the correlation between the channels.
% 'mu' is the 1/varience of channel estimation errors
% 'Ber' is the output of the function which is the bit error rate.

% Tuncer Baykas 16/9/2001

% generating the samples

s1=randint(1,N);
s1=(2*s1-1);

i=1:N;

% generating the channels
a=randn(1,N);
b=randn(1,N);
x=(a+b*j)/sqrt(var(a+b*j));
channel1=x;

a=randn(1,N);
b=randn(1,N);
x=(a+b*j)/sqrt(var(a+b*j));
channel2=x;

% From Theory of Probability, we can generate from two independent
% random variables the corrolated ones with the formula below
chan1=sqrt(0.5*(1-c)).*channel1+sqrt(0.5.*(1+c)).*channel2;
chan2=-sqrt(0.5*(1-c)).*channel1+sqrt(0.5.*(1+c)).*channel2;

%generating the AWGN noise samples
n1=randn(1,N);
n2=randn(1,N);
% Making signal to noise equal to 'ebno'
n=sqrt(1/(10^(ebno/10)))*(n1+n2*j)/sqrt(var(n1+n2*j));
rxd1=chan1.*s1+n;



%generating the AWGN noise samples for the 2nd channel
n1=randn(1,N);
```

```
n2=randn(1,N);
% Making signal to noise equal to 'ebno'
n=sqrt(1/(10^(ebno/10)))*(n1+n2*j)/sqrt(var(n1+n2*j));
rxd2=chan2.*s1+n;

% Generating estimated channels...
% The std of estimation error is sqrt(1-1/mu^2)

est_er=(randn(1,2*N)+j.*randn(1,2*N));
est_er=sqrt(1/(10^(mu/10)))*est_er/std(est_er);

chan1(i)=chan1(i)+est_er(2*i);
chan2(i)=chan2(i)+est_er(2*i-1);

% The output is checked with the input and the bit error rate is found
% by dividing N...

r1=conj(chan1).*rxd1+conj(chan2).*rxd2;
Ber=(sum((r1.*s1)<0))./(N);
```