

**PRELIMINARY INVESTIGATIONS
OF (n, α) REACTIONS**

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ABSTRACT

It is proposed that the (n, ∞) direct interaction will provide a sensitive test of whether or not 'alpha clusters' exist in the nuclear surface. Appropriate theoretical analyses and specific reactions for future study are given. The apparatus developed to study the direct (n, ∞) reaction is described and the alpha particle spectrum obtained by irradiating a natural nickel foil with 14.7 Mev neutrons is presented. A new method for comparing the (n, d) and (n, ∞) ground state transition cross sections on Silicon, as well as two new techniques for pulse multiplication are introduced.

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PRELIMINARY INVESTIGATIONS OF (n, α) REACTIONS

SECTION I: INTRODUCTION

It is hoped that the (n, α) reaction at 14 Mev will give insight into conditions at the nuclear surface. Our lack of knowledge about the surface, regardless of the particular model we choose, is only surpassed by our ignorance of the nuclear forces. It is probable that information gained in either will aid in the understanding of both.

In recent years there have been many discussions about alpha particle clusters at the nuclear surface. Calculations by Brueckner¹ and others indicate that at the normal density of nuclear matter such clustering does not occur. At lower densities, however, it seems that some degree of clustering is possible. On the basis of collision phenomena, alpha decay, and non mesonic K^- -meson absorption, Wilkinson² suggests that in the nuclear surface the probability of a nucleon belonging to an alpha particle may be as high as 0.3 to 0.5.

It is proposed that the direct (n, α) reaction at 14 Mev will provide a sensitive test for the validity of this theory. Since a direct interaction occurs at the nuclear surface, it is expected that if alpha clusters exist the direct interaction will proceed by way of a knock on reaction. If such clusters do not exist, the reaction must proceed through a pick up

mechanism, and variations in ground state cross sections should follow the predictions of the shell model.

Bayman, Brady and Sherr³ have analyzed the direct (p, α) reaction at 18 Mev in a number of medium weight nuclei by assuming that the reaction occurs through single nucleon pick up. The agreement with theory is only fair. The object of this, and of future experiments to be performed at this laboratory, is to determine whether or not their interpretation is correct when extended to (n, α) reactions.

With the availability of solid state radiation detectors it has become relatively easy to study alpha particles emitted in nuclear reactions. They have been used to study (\bar{p}, α) , (d, α) and other reactions induced by charged particles since such particles can be shielded easily from the silicon of the detector. It is more difficult to shield against neutrons and there are, therefore, many technical difficulties which must be overcome before the (n, α) reaction can be observed. One of the contributions of this report is to present the method which was used.

Briefly, the experimental approach was the following. An alpha particle produced in the reaction was identified twice as it passed through two proportional counters before stopping in the solid state detector. If the three pulses occurred in coincidence, the pulse from the silicon detector was allowed to enter a 256 channel pulse-height analyzer. By this technique it has been possible to observe, with good resolution, the alpha particles of energy greater than 10 Mev obtained from irradiating

a natural nickel foil with 14.7 Mev neutrons. From the observed spectrum, an estimate of the integrated cross section for the ground state, $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ transition is $1 \pm .3$ millibarns.

Since further work at this laboratory on the direct (n, α) reaction is contemplated, a number of proposals are made. Specific reactions to study are suggested and analyzed in Section III(c), and improvements in the apparatus are presented in Section VI.

SECTION II: PREVIOUS EXPERIMENTS

There have been virtually no direct, (n, α) reactions studied. The reason for this is the difficulty of shielding neutrons from the alpha particle energy detector. The use of the energy detector as the target has been reported for the (n, α) reaction at 14 Mev on silicon⁴ and on sodium³.

Two other techniques of studying the (n, α) reaction have been reported. Nuclear emulsions⁶ have been used to observe the compound nucleus contributions to the (n, α) cross sections bombarding 14 Mev neutrons on Al^{27} , S^{32} , V^{51} , Mn^{55} , Co^{59} and As^{75} . Secondly, the total (n, α) cross sections in many nuclei have been obtained by activation techniques.⁷

There is an abundance of published (p, α) , (d, α) and (α, p) studies. The most interesting as far as this project is concerned, is the (p, α) reactions reported by Bayman, Brady and Sherr. Their results are shown in Figures 1 and 2.* The three features to notice are that the cross sections for the ground state transition increase with A for even-even target nuclei, decrease for odd-even targets, and that the ratio of the two is between 20 and 30 to one.

The analyses of past experimental results have been conflicting. The majority of the (p, α) and (d, α) reactions

*Private communication, from E. P. Bayman.

reported, like those of Bayman et al., have been compared to theoretical predictions whose basis is the shell model. Notable exceptions, for which clustering in the target nucleus is proposed are: the (p, α) reaction at 45 Mev on light and heavy emulsion nuclei of Heigson⁸; the (α, p) reaction at 19 Mev on several light nuclei of Eleuer et al.⁹; and the $C^{12}(\alpha, 2\alpha)$ reaction at 915 Mev reported by Gooding and Igo¹⁰.

Clarke et al.¹¹ concluded from their (p, α) reactions on P^{31} , Cl^{35} , Cl^{37} , and K^{41} that the appearances at the nuclear surface of an alpha particle or of a single nucleon separated from the residual nucleus have about equal probabilities. They also found that the reduced width for alpha particle emission decreased by a factor of about 8 in going from mass 31 to mass 41. A similar decrease was observed in the integrated cross sections for the ground state transitions in the (d, α) reactions¹² on C^{12} , O^{16} , Ne^{20} , Mg^{24} and S^{32} . It is interesting to notice that the ground state (d, α) cross sections in the odd-odd and odd-even nuclei, N^{14} , P^{19} , Al^{27} and P^{31} , while they decreased at a similar rate are about a factor of 10 smaller than for the even-even target nuclei.

There have been a number of interesting (n, d) reactions reported¹³. Colli and her collaborators have investigated the seniority number in the $17/2$ proton shell by determining the magnitudes of the (n, d) ground state cross sections bombarding 14 Mev neutrons on Sc^{45} , V^{51} , Cr^{52} , Fe^{54} , Fe^{56} , Mn^{55} , Co^{59} , Ni^{58} and Ni^{60} . They conclude that the agreement between the experimental results and that predicted by the shell model is rather satisfactory.

FIGURE 1

(p, α) GROUND STATE TRANSITIONS
EVEN Z TARGETS

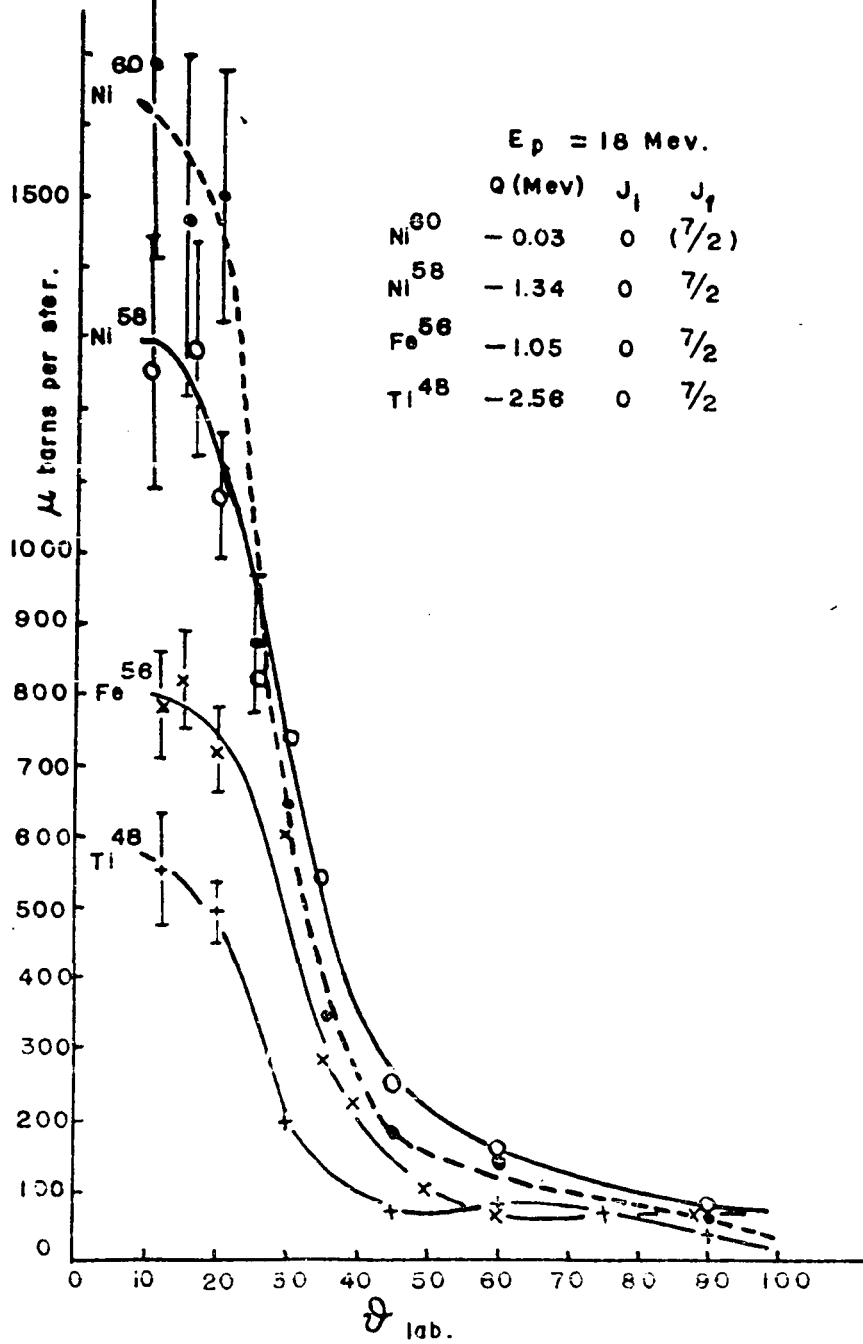
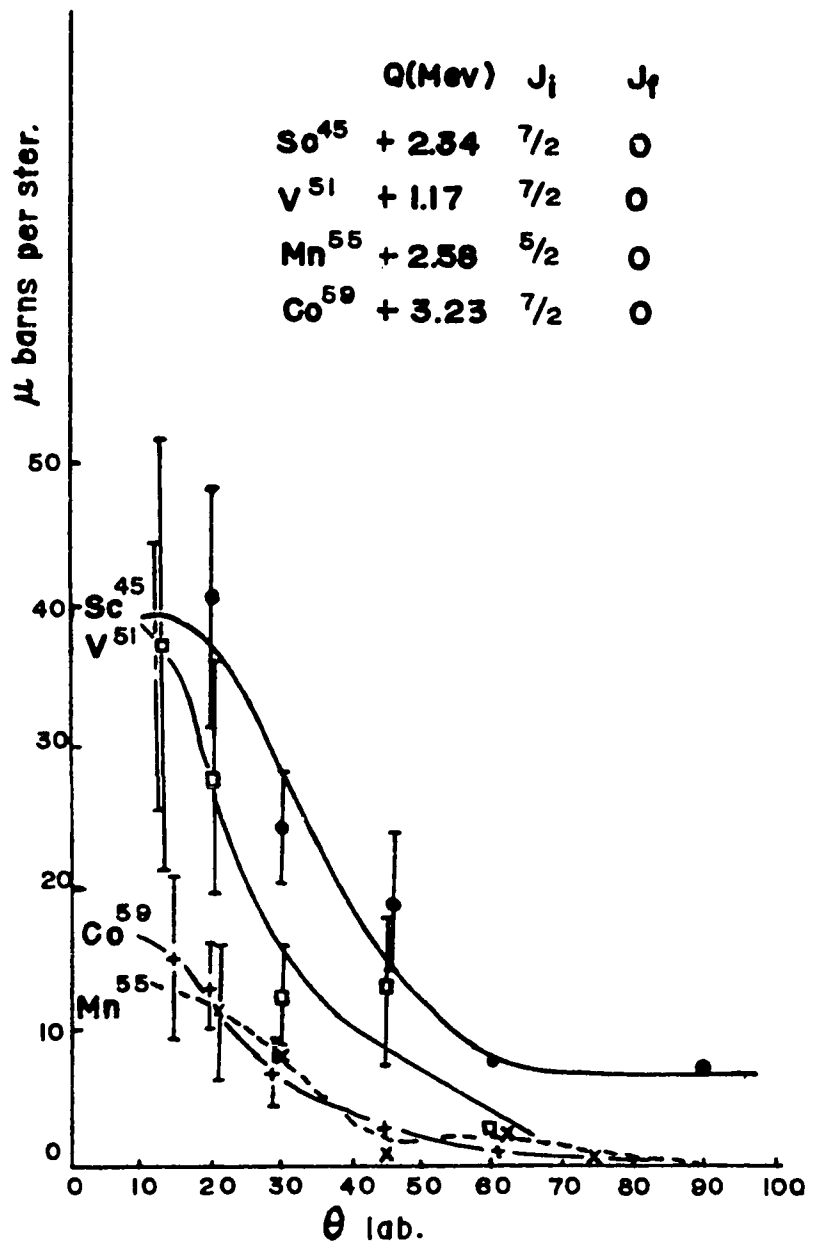


FIGURE 2

(p, α) GROUND STATE TRANSITIONS — $E_p = 18.0$ Mev

ODD Z TARGETS



SECTION III: THEORY

As was mentioned in the introduction, if alpha particle clusters exist in the nuclear surface, then the majority of the direct (n, α) or (p, α) reactions should proceed via a knock on process. If there are no such correlations between nucleons it should be possible, using theory developed from the independent particle model, to predict variations in the magnitudes of, for example, the ground state cross sections reported by Bayman et al. In Section III (a) the (p, α) results of Bayman et al. are analyzed with the pick up theory. The sum rules that are used are derived in Appendix I. Next, a simple cluster model is proposed and with it an analysis of the same (p, α) cross section variations is made. Finally, in Section III (c) a number of interesting (n, α) reactions are suggested for future study.

At the outset, the (p, α) work of Bayman et al. shows us that the process of pick up or knock on is not simple. Butler¹⁹ states:

"In an (α, p) reaction, for example, the absolute magnitude of the direct cross section is proportional to the probability of finding an α particle in the final nucleus at the radius r . In the case of light nuclei, correlations between groups of nucleons is probably quite strong, so that the required probability for finding an α particle at the nuclear 'radius' may be appreciable. For heavier nuclei, however, where the independent particle model becomes more accurate, such strong correlations should decrease in importance. Thus one might expect that the absolute magnitude for the (α, p) or (p, α) reactions, for example, should decrease strongly as the mass of the initial nucleus increases."

It can be argued that the work of Bayman et al. is over a too-limited range (from Tl^{48} to Ni^{60}) to invalidate Butler's remarks. The (α, ∞) work mentioned in Section II did follow such a prediction. However, since an increase, rather than a decrease, was found in the (p, ∞) cross sections for even-even targets, it indicates that this simple picture is inadequate.

We can infer from Wilkinson's remarks² that the opposite effect to that proposed by Butler should occur. He states that where the nuclear density, ρ , is very much less than the density at the center of the nucleus, $\rho(0)$: "effectively all nucleons are in 'alpha particles' ". For $0.5\rho(0) > \rho > 0.15\rho(0)$, he suggests that there is perhaps a 0.3 to 0.5 probability for a nucleon to belong to an 'alpha particle'. He writes: 'alpha particle' to signify that the process is a highly dynamical one with the clusters continually forming and dissolving. If we neglect the decreasing penetrability of an alpha particle as the Coulomb barrier increases with increasing A , an increase in the direct cross section for (p, ∞) or (n, ∞) (or the inverse) reactions should occur.

SECTION III (a): SHELL MODEL ANALYSIS

Bayman et al. have analysed their results using theory developed from the shell model with the added assumption that the seniority is a good quantum number. The seniority number is simply the number of non saturated (that is, unpaired) particles in a given state. The statement that, for example the seniority of the ground state of ${}_{23}^{51}\text{V}$ is 1, means there is only one unpaired nucleon, the odd proton in the ${}_{7/2}^+$ shell. This approximation simplifies the theory considerably and, even for fairly large departures from the good seniority scheme, the theoretical predictions are not expected to be affected, appreciably. ¹⁵

With this simplification, the result of a Born approximation calculation, for the differential cross section describing a (p, α) ground state transition, can then be written as ¹⁶

$$\frac{d\sigma}{d\Omega}(\theta) \stackrel{\text{ground state}}{=} \frac{V_f}{V_i} \mathcal{S} \left| \int_{\lambda=R}^{\infty} u_n^2(r) u_p(r) j_1(qr) r^2 dr \right|^2 \quad (1)$$

Here V_i and V_f are initial and final relative velocities, R is the interaction radius, $u_n(r)$ and $u_p(r)$ the radial wave functions of the picked up neutrons and proton, j_1 is a spherical Bessel function, $q = \left| \vec{k}_2 - \frac{A-2}{A} \vec{k}_1 \right|$ is the momentum transfer vector and \mathcal{S} is a spectroscopic factor. If we wish to compare the magnitudes of ground state (p, α) cross sections for two target nuclei, A_1 and A_2 , then obviously:

$$\frac{d\sigma}{d\Omega} \Big|_{A_1} / \frac{d\sigma}{d\Omega} \Big|_{A_2} = \frac{V_f}{V_i} \mathcal{S} \Big|_{A_1} / \frac{V_f}{V_i} \mathcal{S} \Big|_{A_2} \quad (1a)$$

If we assume that the integral in equation (1) may be neglected,

\mathcal{S} , the spectroscopic factor or relative reduced width, is essentially the square of the overlap integral between the relevant states. For the (p, α) reactions studied by Bayman et al., the 'active' i.e., picked up protons are all in the $f_{7/2}$ shell. It can be shown (see Appendix I) that for a given shell, if the states concerned are unique (that is, of lowest seniority) for the chain:

$$(n, J) = (0, 0) \leftarrow (1, j) \leftarrow (2, 0) \leftarrow \dots,$$

where J is the nuclear spin and n is the number of protons in the shell with angular momentum j ,

$$\mathcal{S}(n, 0 \rightarrow n-1, j) = n, \text{ if } n \text{ is even, and} \quad (2)$$

$$\mathcal{S}(2j+1-n+1, j \rightarrow 2j+1-n, 0) = 1 - \frac{n-1}{2j+1}, \quad (3)$$

if n is odd.

These expressions hold for each of the picked up neutrons as well.

It is helpful to develop a simple explanation for these expressions although the interpretation is not exact. We wish to find the probability of removing a single nucleon from a given shell subject to conditions imposed on the spins of the initial and final nuclei. There are two cases. If the initial nuclear spin is 0 , we demand that the spin of the final nucleus be j , the total angular momentum quantum number specifying the 'active shell' of nucleons. If the initial spin is j , the final spin must be 0 .

If the number of protons (say) is even, we can visualize this condition in the following way. The n protons in the active shell are paired, with their spins antiparallel. The nuclear spin is then 0. If we pick up any one of the n protons we will leave an unpaired proton. This one proton will give the spin of the final nucleus (the good seniority assumption). Thus the probability, $\mathcal{S}(n, 0 \rightarrow n-1, j) = n$.

If the number of protons is odd, it is more difficult to satisfy $(n, j \rightarrow n-1, 0)$. There is only one of the n protons that is unpaired, and it is this one that, if removed, will leave the nucleus with spin 0. If there were only one nucleon in the shell, then obviously $\mathcal{S} = 1$. When there are n , the probability for hitting the correct one is reduced to $\mathcal{S}(n, j \rightarrow n-1, 0) = 1 - \frac{n-1}{2j+1}$.

Bayman et al. have analysed their results differently from ^{the method} that which will be presented here. We shall explain the distinction below. The reason why we choose an alternate formalism is to point out two improvements which appear in the comparison with the experimental results. In addition, it is not certain whether the additional factor that they introduced is exact. *

From equations (2) and (3), the spectroscopic factor corresponding to the removal of a proton and two neutrons from an even-even target nucleus is:

* private communication with P. West

$$\mathcal{S}_{EE} = 2 \frac{N}{2} \left(1 - \frac{N-2}{2j_n+1} \right) \quad (4).$$

For the odd-even targets,

$$\mathcal{S}_{OE} = \left(1 - \frac{Z-1}{2j_p+1} \right) \frac{N}{2} \left(1 - \frac{N-2}{2j_n+1} \right) \quad (5).$$

Here Z and N refer to the number of protons and neutrons, respectively, in the active shell. The factor, 2 , can be thought of as expressing the number of neutron pairs available for pick up.

Bayman et al. write these as:

$$\mathcal{S}_{EE} = C Z N (2j_n + 3 - N) \quad (6)$$

$$\text{and } \mathcal{S}_{OE} = C \left(\frac{2j_p + 2 - Z}{2j_p + 1} \right) N (2j_n + 3 - N) \quad (7)$$

Where C is a constant. Notice that they have removed the (common) denominator of the last term in both expressions. This was done in an attempt to correct for the probability that the two neutrons exist in space near enough to the incoming proton for an interaction to occur. Effectively, they assumed that the interaction can be mathematically expressed by a delta function. This probability is proportional to the total, possible number of neutrons in the active shell; that is, it is proportional to $(2j_n + 1)$.

In Table I, our interpretation of their results is displayed. The consistency of the experimental results with the theoretical predictions using equations (4) and (5), is noticeably better for odd-target cross sections compared with that predicted by Bayman et al., is not much different for even-targets, but leaves an odd-even discrepancy in the theory of a factor of 6 compared with the 3-8 reported by Bayman et al.

TABLE I: COMPARISON OF (p, α) CROSS SECTIONS AT 10°

(1) Target	(2) Protons	(3) Neutrons	(4) $\frac{V_1}{V_f} \frac{d\sigma}{d\Omega}(10^\circ)$	(5) \mathcal{S}	(4) / (5) [See equation 1(a)]
Sc ⁴⁵	$(r_{7/2})^1$	$(r_{7/2})^4$	74 ± 15	$3/2$	49 ± 10
V ⁵¹	$(\text{"})^3$	$(\text{"})^8$	66 ± 25	$3/4$	88 ± 33
Mn ⁵⁵	$(\text{"})^5$	$(\text{"})^8 (r_{5/2})^2$	32 ± 10	$1/2$	64 ± 20
Co ⁵⁹	$(\text{"})^7$	$(\text{"})^8 (\text{"})^4$	32 ± 10	$1/3$	96 ± 30
Ti ⁴⁸	$(\text{"})^2$	$(\text{"})^6$	1160 ± 150	3	390 ± 50
Fe ⁵⁶	$(\text{"})^6$	$(\text{"})^8 (\text{"})^2$	1620 ± 150	6	270 ± 25
Ni ⁵⁸	$(\text{"})^8$	$(\text{"})^8 (\text{"})^2$	2650 ± 400	8	330 ± 50
Ni ⁶⁰	$(\text{"})^8$	$(\text{"})^8 (\text{"})^4$	3150 ± 600	$32/3$	295 ± 55

Because of their impulse approximation, Bayman et al. state that the consistency for both even- and odd-targets is poorer if the ν neutrons in excess of eight are assumed to populate the $f_{7/2}$ shell. Macfarlane et al.^{15(a)} have found that the occupation numbers for the neutrons outside the $f_{7/2}$ shell in Ni^{58} and Ni^{60} satisfy:

$$\begin{array}{rcl} 0.3 << \langle \text{neutrons} \rangle_{f_{5/2}} < 0.6 \\ 1.7 >> \langle \text{neutrons} \rangle_{2p_{3/2}} > 1.4 & \text{in } \text{Ni}^{58} \\ \text{and } 0.8 << \langle \text{neutrons} \rangle_{f_{5/2}} < 1.6 \\ 3.2 >> \langle \text{neutrons} \rangle_{2p_{3/2}} > 2.4 & \text{in } \text{Ni}^{60}. \end{array}$$

Here $\langle \text{neutrons} \rangle_{l,j}$ is the average number of neutrons in the (l, j) orbit, and $3/2$ is bracketed because the contributions from $2p_{3/2}$ and $2p_{1/2}$ were not separated.

With these occupation numbers and our statistical factors we find that for Ni^{58} , the quantity listed in the last column of Table I does not change (as would be the case for Ni^{55} also). For Ni^{60} , the quantity becomes 365 ± 70 , which is not significantly different from the listed value.

Finally, it is seen that the shell model pick up theory presented is not adequate to explain the (p, α) results. The increase and decrease for even-even and odd-even targets, respectively, is consistent with the theory, but there remains a residual factor of 6 between the two. The justification for cancelling the integral in equation 1, when comparing the cross sections for different nuclei, is found in the consistency of the even-even and odd-even cross sections separately.

SECTION III (b): A SIMPLE CLUSTER MODEL

As yet, it is not known how to perform quantitative comparisons of direct interaction experimental results with a cluster model¹⁶ or with the assumption that alpha particles 'cluster' in the nuclear surface. However, let us analyze the (p, α) cross sections with the aid of the following four assumptions:

- (1) The shell model does not require that the nucleons fill the shells as rigorously as is usually assumed. The most striking success of the model has been the prediction of the magic numbers and we assume that nuclei prefer closed shells to filling them in the 'correct' order.
- (2) The pairing force between two identical nucleons is strong.
- (3) Direct interactions occur at the nuclear surface and normally do not disturb closed shells.
- (4) If convenient, nucleons cluster at the surface to form 'alpha particles'.

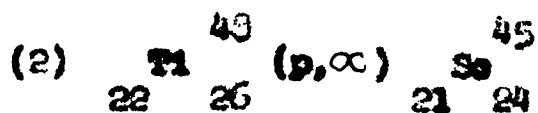
We now proceed to the analysis.* From the experimental results of Bayman et al. we obtain $V_i / V_f \cdot d\sigma / d\Omega$ evaluated at a specific angle (10°) for each ground state transition. Again, $V_i / V_f \cdot d\sigma / d\Omega$ is proportional to the relative reduced width δ . A table indicating the results of this section is given on page 21.

* It should be pointed out that this analysis makes no attempt to predict the ground-state cross section, but only attempts to account for clustering; the experimental results and the previous pick-up analysis both have to do with the ground-state transition only.



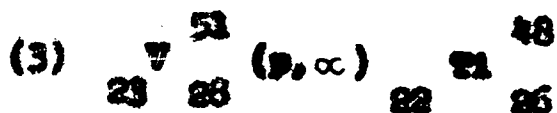
Se^{45} has 1 proton and 4 neutrons in the $f_{7/2}$ shells - outside the 'core' of closed shells. If the (p, α) reaction does not disturb the core, then the reaction proceeds by picking up the one proton and two of the four neutrons. It is possible that the three nucleons already exist as a He^3 nucleus in Se^{45} but since He^3 is not bound very tightly, the cluster probably dissolves quickly. The major contribution to the direct (p, α) reaction is then expected to be from pick up of single nucleons. Bayman et al. found for this reaction,

$$V_1 / V_F \cdot d\sigma / d\Omega (10^\circ) \approx 75 \text{ microbarns/steradian.}$$



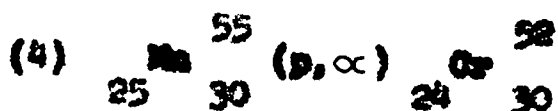
Ti^{48} has 2 protons and 6 neutrons outside the core. We expect that the nucleus would like to fill its $f_{7/2}$ neutron shell even by accepting for short periods the 2 protons, thereby forming a fairly stable 'alpha cluster'. Thus the (p, α) reaction on Ti^{48} could proceed in the main via a knock on process. Bayman et al. found for this reaction,

$$V_1 / V_F \cdot d\sigma / d\Omega (10^\circ) \approx 1200 \text{ microb./sterad.}$$



V^{51} has 3 protons in the $f_{7/2}$ shell and the neutron shell is closed. The neutron shell can not be broken easily, and on this simple cluster model the active nucleons are expected to correlate little. Therefore, both the pick up and knock on contributions to the cross section are expected to be small. For this reaction, Bayman et al. found,

$$V_1 / V_T \cdot d\sigma / d\Omega (\text{at } 10^\circ) \simeq 65 \text{ microb. / sterad.}$$



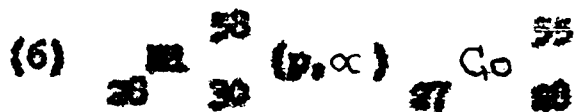
Mn^{55} has 5 protons in the $f_{7/2}$ shell, a filled $f_{7/2}$ shell of neutrons plus two more which probably occupy both the $2p$ and $1f_{5/2}$ shells. Because the extra 2 neutrons are in a different orbit from the protons, they would have little tendency to cluster with any of the paired protons. In addition, as is assumed for V^{51} , a closed neutron shell can not be broken easily. Thus the reaction is expected to proceed via a difficult pick up process. Bayman et al. found

$$V_1 / V_T \cdot d\sigma / d\Omega (\text{at } 10^\circ) \simeq 30 \text{ microb. / sterad.}$$



Fe^{56} has 6 protons and 8 neutrons in the $f_{7/2}$ shells and 2 neutrons outside. Even though, as in the case of Mn^{55} , the extra two neutrons are not in the same orbits as are the protons, we suspect considerably more clustering here for this would complete the proton shell. Since, in addition, neutrons can not 'come in' to the $f_{7/2}$ shell by the Pauli principle, the alpha cluster may drift further out into the nuclear surface than in the case of Ti^{48} . Again, from Bayman et al.,

$$V_1 / V_T \cdot d\sigma / d\Omega (\text{at } 10^\circ) \approx 1500 \text{ microb./st.}$$



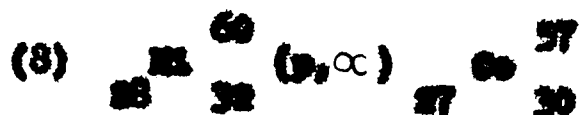
Ni^{58} has filled both the proton and neutron $f_{7/2}$ shells. The occupation numbers for the last two neutrons have been given in Section III (a). We expect that this reaction will proceed in the main through a knock on process. Since the spatial orbits of the two closed $f_{7/2}$ shells are the same, alpha clusters could form easily without 'breaking' either shell. For this reaction, Bayman et al. found

$$V_1 / V_T \cdot d\sigma / d\Omega (\text{at } 10^\circ) \approx 2500 \text{ microb./st.}$$



${}_{27}^{60}\text{Co}$ has only 7 $f_{7/2}$ protons with a closed $f_{7/2}$ shell of neutrons and 4 more in higher orbits. As in the case of Ni^{58} , the reaction is expected to proceed through a difficult pick up mechanism. Reason $\frac{d\sigma}{d\Omega}$ found for this reaction,

$$V_1 / V_2 = d\sigma / d\Omega (\text{at } 10^\circ) \approx 30 \text{ microbarns / sterad.}$$



Finally, Ni^{60} has two more neutrons than Ni^{58} . It is possible that the $f_{7/2}$ protons are influenced by the Coulomb repulsion of the inner protons, thereby drifting further into the surface, and occasionally clustering with the outer neutrons. However, we expect that the majority of the cross section arises from knock on reactions with alpha clusters formed from $f_{7/2}$ shell nucleons. For this reaction, Reason $\frac{d\sigma}{d\Omega}$ found

$$V_1 / V_2 = d\sigma / d\Omega (\text{at } 10^\circ) \approx 3000 \text{ microb. / sterad.}$$

In Table II, the results of this analysis are summarized. From it we conclude that the reactions studied by Ryzan et al., proceed both by pick up and knock on processes, depending on the particular nucleus investigated. The total pick up cross section in the $1/2$ shell, for the ground state transitions is small, about 50 microns per steradian at 10° to the incident beam, while the knock on cross section is of the order of 1000 microns per steradian per cluster.

TABLE II: COMPARISON OF (σ, ∞) CROSS SECTIONS AT 10°

TARGET	NUCLEI	# OF ALPHA CLUSTERS	PICKUP CROSS SECTION	KNOCK ON CROSS SECTION
^{23}Na	^{23}Na	—	pick up	75 ± 15
	only one active proton			
^{27}Al	^{27}Al	1	knock on	1160 ± 150
	One 'closed' neutron shell			
^{27}V	^{27}V	—	pick up	66 ± 25
	Closed neutron shell			
^{29}Mn	^{29}Mn	—	pick up	32 ± 10
	" "			
^{56}Fe	^{56}Fe	$\sim 1\frac{1}{2}$	knock on	1680 ± 150
	One 'closed' proton shell			
^{58}Ni	^{58}Ni	~ 2	knock on	2650 ± 400
	Closed neutron and proton shells			
^{59}Co	^{59}Co	—	pick up	32 ± 10
	Closed neutron shell			
^{58}Ni	^{58}Ni	~ 2	knock on	3150 ± 600
	As for ^{58}Ni			

SECTION III(a): (n, ∞) TRANSITION FOR EVEN TARGET

Two of the most interesting (n, ∞) reactions that can be studied using the apparatus that has been developed, are $Ca^{42}(n, \infty) Ti^{40}$ and $Ti^{48}(n, \infty) Ca^{46}$. These target nuclei are the only stable isotopes filling the $f_{7/2}$ shell, for which the (n, ∞) ground state transitions can be analyzed easily on the good similarity scheme. The spectroscopic factor for the removal of two protons and a neutron from an even-even target is:

$$S_{HE} = \frac{1}{2} \cdot \left(1 - \frac{1}{2j_p + 1} \right) \cdot N \quad (8)$$

and from an even-odd nucleus,

$$S_{HO} = \frac{1}{2} \cdot \left(1 - \frac{1}{2j_p + 1} \right) \left(1 - \frac{1}{2j_n + 1} \right) \quad (9)$$

If the delta force or impulse approximation is used, these expressions should each be multiplied by $(2j_p + 1)$. In this case, the approximation would not affect a comparison of the two cross sections.

From the pick up theory we expect for the ratio of the ground state cross sections:

$$\frac{\sigma_{\alpha}}{\sigma_{f}} \text{ (Cr}^{52}\text{)} \quad ; \quad \frac{\sigma_{\alpha}}{\sigma_{f}} \text{ (Ti}^{49}\text{)} = 48$$

From the simple cluster model proposed the ratio could be as much as an order of magnitude smaller. Ti^{49} has 2 protons and 7 neutrons in its $f_{7/2}$ shells. The neutron shell can be closed if it accepts, for brief periods, the 2 protons to form an alpha cluster. The seventh neutron would then be relatively free. The results of Berman¹⁷ that 50 per cent of the ground state eigenfunction of Ti^{49} has seniority 1 and that the remaining 42 per cent is distributed between the $J = 7/2$ states at about 2.5 Mev excitation suggests that such a phenomenon is occurring. In addition, ${}_{24}\text{Cr}^{52}$ has a closed $f_{7/2}$ shell of neutrons. It is expected that alpha clusters will form in Cr^{52} , but not so readily as in the case of ${}_{26}\text{Fe}^{56}$, for example.

A second, series of experiments that can be performed is to study the (n, ∞) reactions in the same target nuclei as were studied by Berman et al. It is considerably more difficult to predict the ground state (n, ∞) cross sections than the corresponding (p, ∞) cross sections because of the nuclear spins involved. However, if the (n, ∞) cross

sections show variations similar to those found by Figure 21. We expect that some of the reactions are proceeding through a knock on mechanism.

Leaving the $f_{7/2}$ shell, two direct (n, α) reactions in the $d_{5/2}$ shell which can be analyzed easily are $^{21}\text{Si}^{28} (n, \alpha) ^{18}\text{Ne}^{25}$ and $^{22}\text{Ne}^{25} (n, \alpha) ^{19}\text{Ne}^{22}$. From the pick up theory we expect for the ratio of the ground state cross sections:

$$\frac{\frac{V}{V_0} \frac{d\sigma}{d\Omega} (^{21}\text{Si}^{28})}{\frac{V}{V_0} \frac{d\sigma}{d\Omega} (^{22}\text{Ne}^{25})} = \frac{10}{1}$$

From the simple cluster model proposed, the ratio may be less, if the 4 protons and 3 neutrons in the $d_{5/2}$ shells of $^{21}\text{Si}^{28}$ prefer to cluster, temporarily at least, one of the shells. In addition, an alpha cluster would leave 2 protons and a neutron relatively free for a pick up reaction.

SECTION IV: APPARATUS

To be able to measure the direct interaction contribution to the (n, α) cross section it is necessary to have reasonably good resolution. In particular, to compare ground state transitions, it must be possible to separate clearly the ground and first excited states of the final nucleus. To perform such experiments, therefore, the energy detector must have good resolving power and the targets must be thin to avoid straggling of the α particles at their source. Fortunately, direct interactions are known to be peaked forward strongly (for example, see Figure 1) and the energy resolution is not expected to be deteriorated seriously even if the angular resolution of the system is poor.

The new solid state energy detectors satisfy the first requirement admirably. Whereas in a typical proportional counter the ionizing particle, whose energy is to be determined, loses 35 ev per 'collision' with an ^{atom,} electron, in a solid state detector it loses only 1/10 this amount. Consequently many more electrons are produced before the incoming particle is stopped and better resolutions can be obtained. Unlike proportional counters and photo-multiplier tubes, solid state detectors have no internal amplification and it is therefore imperative that the detector's preamplifier have a low noise figure.

The use of the solid state detectors to study the (α, α) reaction has not been reported in the literature, except in special circumstances to be explained below. More than half the time devoted to this project has been involved in developing the techniques of measurement. In Appendix III, the problems encountered are listed.

The method used was to identify the alpha particle emitted from the target in two proportional counters as it travelled the 5 cm. from the target to the solid state detector. If the energy pulse was in coincidence with pulses from both proportional counters, it was allowed to enter a 200 channel pulse-height analyzer.

There are two modifications to the apparatus shown in Figure 3. The second proportional counter was changed to one similar to the design of the first, with a wire mesh isolating the two. On this mesh were placed two small ^{241}Am , 5.5 Mev alpha particle sources. Also, the R.C.A. Victor junction detector developed considerable back leakage current with a corresponding deterioration in resolution. This may have been caused by neutron irradiation. It was replaced by an OMMO surface-barrier type which, though it also developed some leakage, did not suffer a serious deterioration in its resolving power.

The block electronics are shown in Figure 4. The solid state detector's preamplifier, using 7586 Muristons,

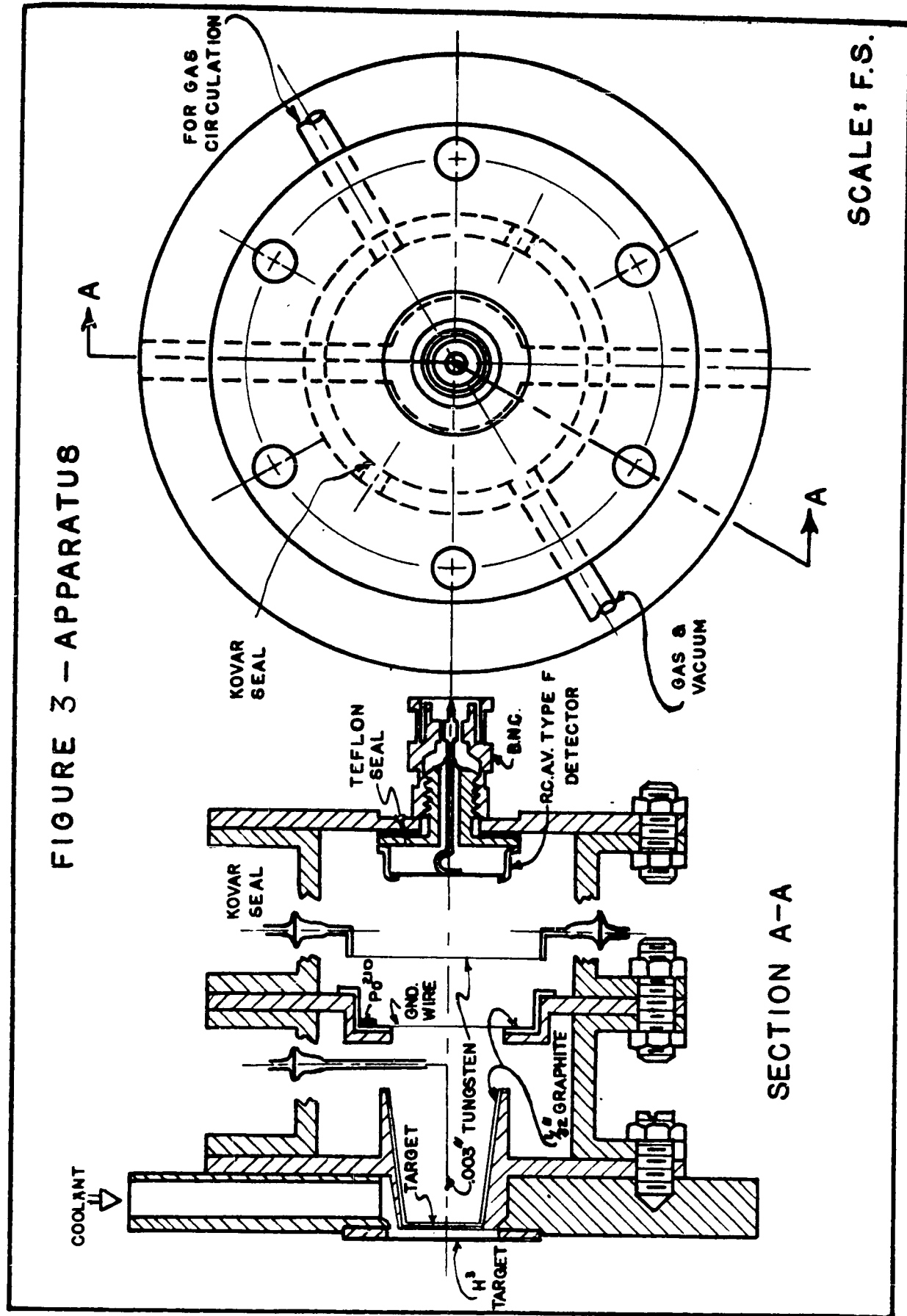
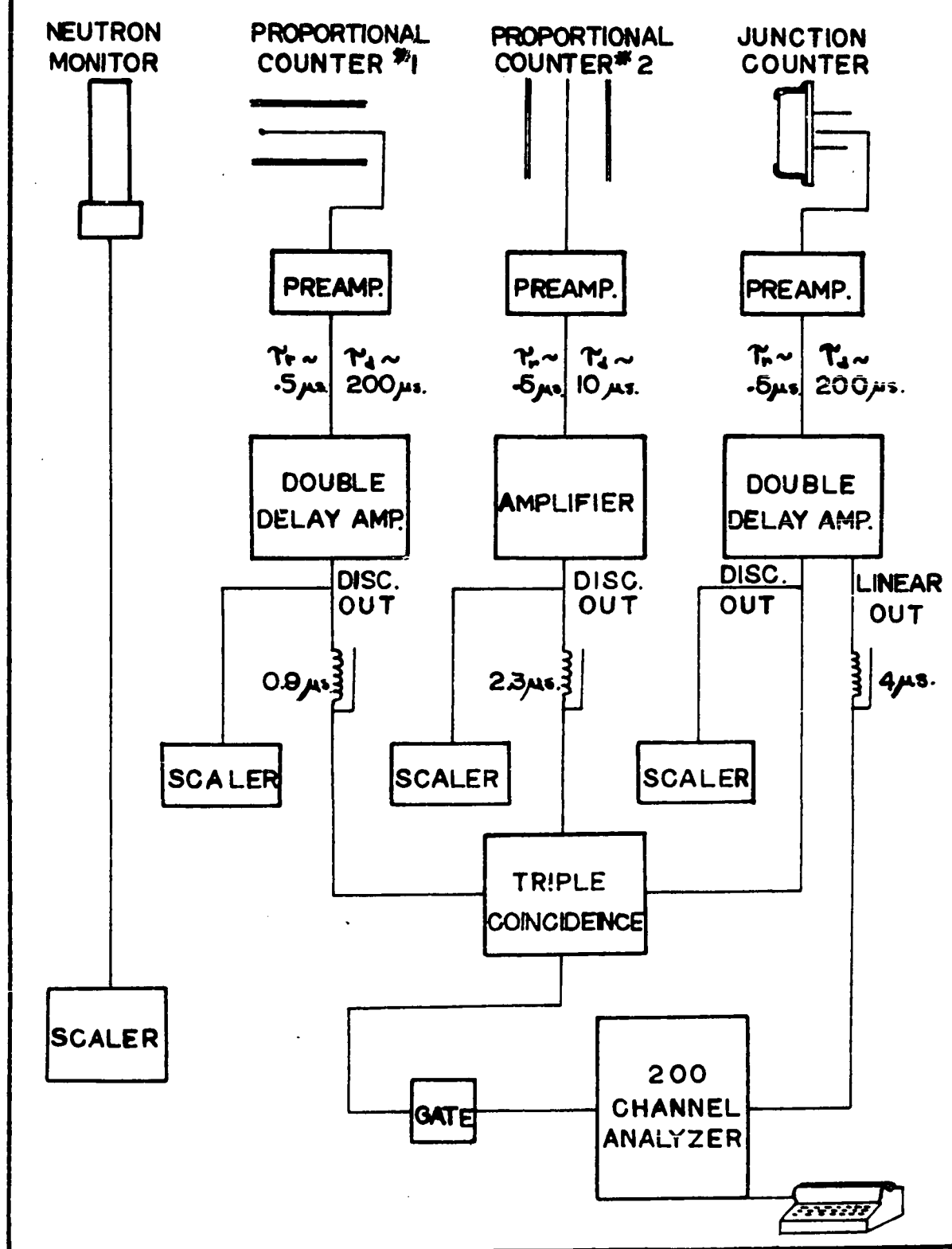


FIGURE 4
BLOCK ELECTRONICS



is described by Glass et al.¹⁸ The preamplifiers for the proportional counters were similar in design but have 12X7A's for the cascade stage and 12X17A's for two long tail pairs. The proportional counters' preamplifiers had open loop gains in excess of 1000, closed loop gains of about 200, and rise times, with capacitive feedback, 0.4μ sec.

The proportional counters' gas was a mixture of 60 per cent helium and 40 per cent methane, at a total pressure of 150 mm. Hg. Tungsten wire is suitable for the central wire of a proportional counter and was specifically chosen because of its low (n, α) cross section. The inside walls of the proportional counters were lined with $1/32$ inch thick graphite cylinders because the maximum energy of an alpha particle is 8.5 Mev for the $C^{12} (n, \alpha) Li^9$ reaction, at 14.7 Mev.

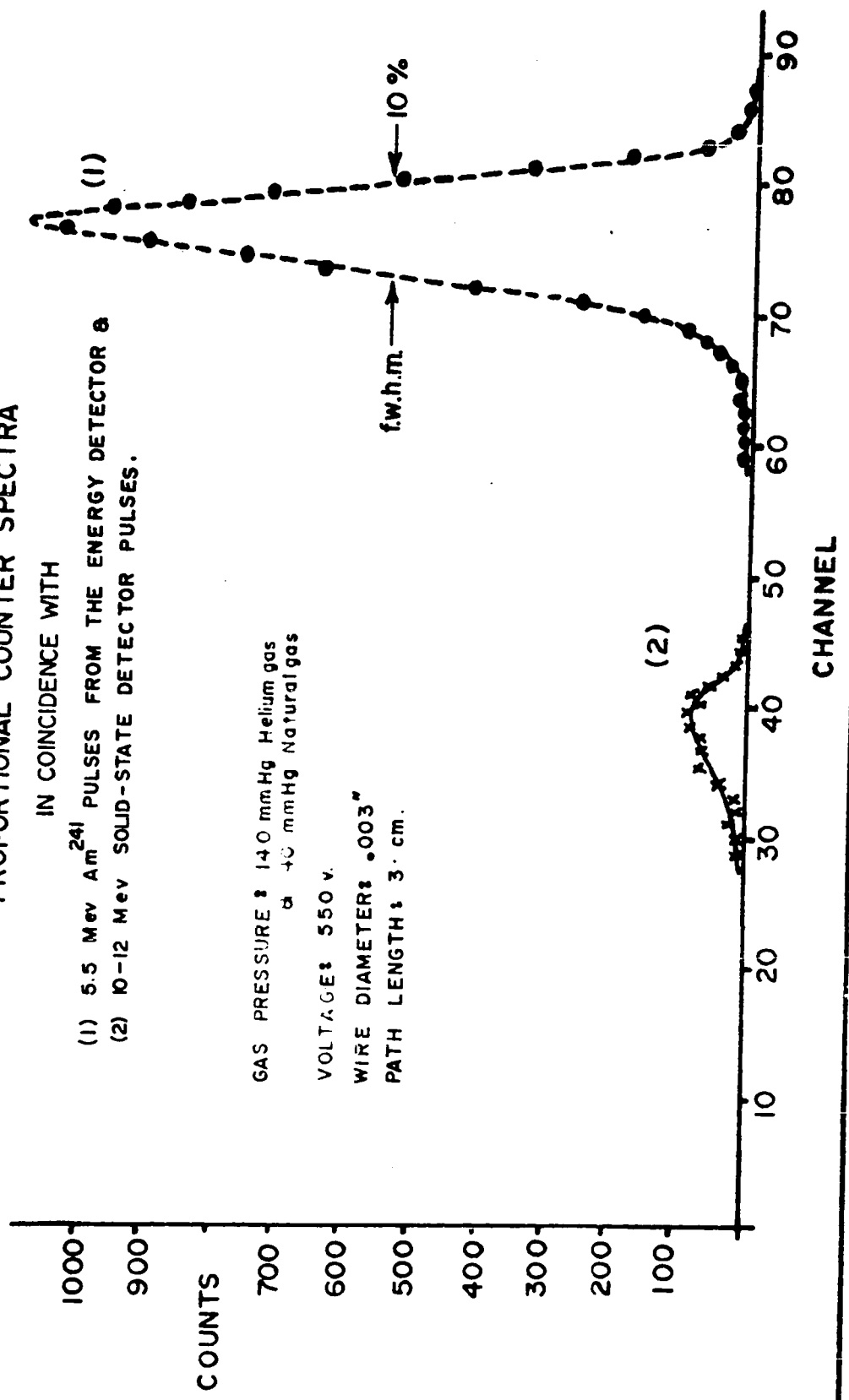
In Figure 5, the resolution of the second proportional counter is demonstrated. The two spectra were obtained by analyzing only those events in the proportional counter which were in coincidence with pulses corresponding to 5.5 Mev and 10 to 12 Mev alpha particles stopped in the solid state radiation detector. The two peaks correspond to about 600 and 300 Kev, respectively, lost by the alpha particles as they traversed 3 cm. of proportional counter gas.

This technique of identifying the alpha particles before they reach the energy detector is a version of the standard 'telescope' apparatus¹⁹. There are other methods which could be used. As reported by Facchini et al.²⁰, the detector alone

FIGURE 5
PROPORTIONAL COUNTER SPECTRA
IN COINCIDENCE WITH

- (1) 5.5 Mev A^{241} PULSES FROM THE ENERGY DETECTOR B
- (2) 10-12 Mev SOLID-STATE DETECTOR PULSES.

GAS PRESSURE: 140 mmHg Helium gas
 40 mmHg Natural gas
VOLTAGE: 550 v.
WIRE DIAMETER: .003"
PATH LENGTH: 3 cm.



can be used to study the alpha spectrum ≈ 15 Mev. Below this energy, the $\text{Si}^{28}(n, \alpha) \text{Mg}^{25}$ and $\text{Si}^{29}(n, \alpha) \text{Mg}^{25}$ reactions in the detector itself, produce too many background counts. Also, there are many $\text{Fe}^{56}(n, \alpha) \text{Cr}^{53}$ reactions in the case supporting the detector which would be observed.

A second method that could be used is to eliminate the (n, α) reactions in the detector by putting it in a 'cone' of neutrons defined by the associated alpha particle from the $\text{T}(d, n) \text{He}^4$ reaction. If alpha particles were detected in both detectors, an anticoincidence circuit would not allow the pulse from the silicon detector to proceed to the analyzer. The limitation in this method would be the rate at which the associated alpha detector could operate ($\sim 10^6$ counts per second) and the efficiency of the anticoincidence circuit. The low neutron flux could be compensated by placing a ring of the target material about the 'eliminated cone' of neutrons.

SYSTEM V: RESULTS AND DISCUSSION

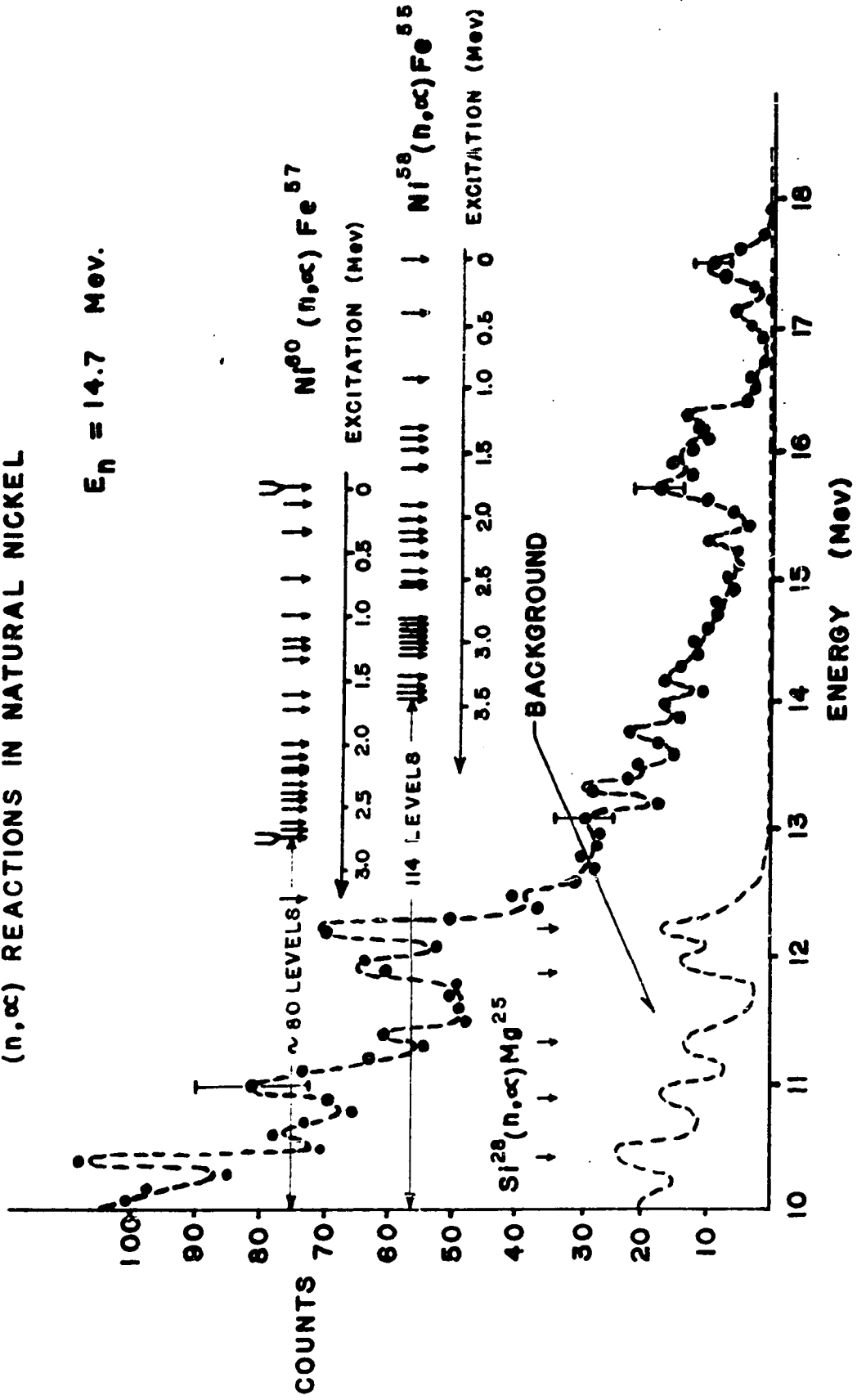
Figure 6 shows the energy spectrum of alpha particles emitted from a 1 mg/cm^2 natural nickel foil. It was bombarded for approximately 100 hours by 5×10^6 neutrons per second. The foil thickness corresponds to a 100 Nev straggle for a 15 Nev alpha particle. Also shown is the expected background which would be observed in an equivalent period. It is extrapolated from data accumulated for about 20 hours.

The electronics were checked every two hours for drifts and every two days for linearity. Drifting did not exceed 100 Nev per day and the linearity was better than 1 per cent over the range of interest. The drift calibrations were easily obtained by determining the position of the ground state transition of the $\text{Si}^{23}(n, \alpha)\text{Mg}^{25}$ reaction (see Figure 7). on page 34.

The second proportional counter was continuously displayed on an oscilloscope and was calibrated by observing the pulse height corresponding to Am^{241} alpha particles that traversed the proportional counter and stopped in the solid state detector. The first proportional counter was set in analogy, allowing a considerable factor for uncertainty. As an independent check on the proportional counters, a Po^{210} source was placed in the target's position and the behaviour of the two compared. Finally, the spectra of both proportional counters, with the neutron generator in operation, were observed on the pulse height analyser, gated by the silicon detector.

FIGURE 6
(n, α) REACTIONS IN NATURAL NICKEL

$E_n = 14.7$ Mev.



The statistics of the results are poor and because natural nickel is 68 per cent Ni^{58} and 26 per cent Ni^{60} , the usefulness of the spectra is decreased. It should be noticed, however, that the resolution of the ground state $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ transition is better than 2 per cent. We can not compare the $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ with the $\text{Ni}^{60}(n, \alpha)\text{Fe}^{57}$ ground state cross section because it is impossible to determine the contributions to the peak at 15.7 Mev from the three states in Fe^{57} (at 0, 14, and 136 Kev) and from the excited states in Fe^{55} at 1.93 and 2.06 Mev. *

The apparatus was designed to determine relative direct interaction cross sections for different target nuclei. The comparison could be made by bombarding different targets with the same number of neutrons and simply counting the number of events appearing in the ground state peak of each spectrum. It will be some time before such a comparison can be made. However, since an observation of the $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ ground state transition has not been reported previously, it is worthwhile to try to obtain its absolute cross section.

The difficulty is in determining the solid angle subtended by the nickel target at the neutron source. The areas of the nickel and tritium targets are about the same (1 cm^2) and they were $1/8$ inch apart. To hit the solid state detector an alpha particle must travel nearly parallel to the center line of the apparatus. Immediately we can see that the 25 ± 5 counts that appear within ± 200 Kev of the ground state peak, represent an integrated cross section ($\int \sigma(\Omega) d\Omega$) over at most, 2π steradians.

* The Q-values for the competing reactions from Ni^{62} (9%) & Ni^{64} (1%) are about 0 Mev.

The integration limits can be determined more accurately by observing that these 25 ± 3 counts appear within a 400 Nev interval. From the kinematics of the $T(d, n)Ni^4$ and $Ni^{58}(n, \alpha)Fe^{55}$ reactions, we find that more appropriate integration limits are ^{for θ_L} -15° to $+45^\circ$. They are not symmetrical because the center line of the apparatus was at 45° to the direction of the incident deuteron beam.

From the definition of the differential cross section as the number of alpha particles emitted into a solid angle per unit time per nucleus divided by the incident flux of neutrons, we have

$$\int_{-90^\circ}^{+90^\circ} \sigma(\Omega) d\Omega \geq \left\{ \frac{4\pi SA}{N N_A \rho_s \omega_D} \right\} .$$

Here, C is the number of alpha particles detected,

A is the atomic weight of Ni^{58} ,

N is the total number of neutrons emitted into 4π steradians,

N_A is Avogadro's number,

ρ_s is the per cent of the total surface density that corresponds to Ni^{58} ,

and, ω_D is the solid angle subtended by the detector at the nickel target.

Substituting the experimental values, a lower limit to the integrated (from -90° to $+90^\circ$), $Ni^{58}(n, \alpha)Fe^{55}$ ground state transition cross section is $1.0 \pm .3$ mb. That is, for the more realistic integration limits of -15° to $+45^\circ$, the cross section is approximately $1 \pm .3$ mb.

This experimental result will now be compared with theoretical predictions. From the $Mi^{58}(p, \alpha)Co^{55}$ ground state angular distribution, Figure 1, we obtain:

$$2\pi \int_{-90^\circ}^{+90^\circ} \sigma(\vartheta_L) \sin \vartheta_L d\vartheta_L = \frac{2.3 \pm .5}{\cancel{11.1 \pm 1.1}} \text{ mb.}$$

and

$$2\pi \int_0^{+45^\circ} \sigma(\vartheta_L) \sin \vartheta_L d\vartheta_L = 1.6 \pm .5 \text{ mb.}$$

The proposed cluster model predicts that, when we correct for the different penetrabilities of an 18 Mev proton and a 14.7 Mev neutron, the (p, α) and (n, α) cross sections should be identical. The energies, and therefore the penetrabilities, of the two alpha particles are almost the same. From Blatt and Weisskopf²¹ and from Shapiro²², using $r_0 = 1.5 \text{ fm}$, a 14.7 Mev neutron, because it does not experience Coulomb repulsion, penetrates 'into' Mi^{58} 1.5 times more easily than an 18.0 Mev proton. Consequently the predicted (n, α) cross sections are: $6 \pm .6$ mb. for the integration limits of -90° to $+90^\circ$, and $2.4 \pm .3$ mb. for the more realistic limits of -15° to $+45^\circ$. This is larger, by a factor of 2, than the $\approx 1 \pm .3$ mb. experimentally determined.

To be able to compare the (n, α) and (p, α) ground state cross sections for Mi^{58} with the prediction of the shell model theory that has been presented, simplifying assumptions must be made. The spin of the ground state of Co^{55} is $3/2$. We assume that it can be attributed to an unpaired $p_{3/2}$ neutron which, in Mi^{58} , was paired with the picked up neutron. Secondly, we assume

that the angular distributions for the (n, α) and (p, α) reactions are the same. Finally, when taking the ratio of the two predicted cross sections, we drop the integrals that appear in equation 1, Section III(a). That is, we assume that the integrated product of the wave functions for the picked up neutron and the protons in the (n, α) case, is identical to that for the two neutrons and one proton in the (p, α) case.

Some justification for this last approximation is found in the consistency of the (p, α) results for even-even or odd-even targets separately. For example, the two neutrons picked up in the $Tl^{48}(p, \alpha)$ reaction are in the $f_{7/2}$ shell whereas those picked up from Mi^{58} are not. Nevertheless, the ratio of the two cross sections, when the contributions from the two integrals are ignored, are consistent with the theory within experimental error.

With these assumptions and the sum rules of Appendix I, the prediction for the ratio of the (n, α) to (p, α) ground state cross sections, correcting for the different penetrabilities of the bombarding particles, is:

$$1.13 (1.5) \cdot \frac{\frac{Z}{2} \left(1 - \frac{Z-2}{2j_p+1} \right) \cdot N}{Z \cdot \frac{N}{2} \left(1 - \frac{N-2}{2j_n+1} \right)} = .4 \quad (10)$$

That is, we expect for the $Mi^{58}(n, \alpha)Fe^{55}$ ground state cross section, $0.9 \pm .2$ mb. from the (p, α) result integrated from -0° to $+90^\circ$, and $.64 \pm .2$ mb. if we integrate from -0° to $+45^\circ$.

This is to be compared with the experimentally determined cross section of $\approx 1 \pm .3$ mb.

Instead of using the spectroscopic factors that were found to 'improve' the comparison of the (p, α) results with the pick up theory, let us introduce the impulse approximation here. We have seen that \mathcal{S} deals with the number of nucleons that exist in the active shell; this new factor identifies the shell. Speaking loosely, four pairs of nucleons in a $f_{7/2}$ shell, for example, are more 'crowded' than three pairs in a $f_{5/2}$ shell. Hence it will be easier to pick up a pair from the $f_{7/2}$ shell or, mathematically, we multiply \mathcal{S} by $2j_n + 1$.

Multiplying the numerator of expression (10) by $2j_p + 1 = 8$ and the denominator by $2j_n + 1 = 4$, we obtain for the predicted cross sections: $3.2 \pm .4$ mb. integrating from -90° to $+90^\circ$, and $1.3 \pm .15$ mb., using the more realistic integration limits. This is the best agreement with the experimental result.

We conclude this section with Table III. It is not reasonable to pursue the exact determination of the solid angle subtended by the target at the neutron source because of the nonuniformity of the data and because of the theoretical approximations made. No firm conclusion can be drawn, but one interesting result that should be noticed is how the prediction for the (n, α) cross section improved when the impulse assumption was introduced. It did not improve the predictions for the (p, α) results. Compounding the uncertainty

in the explanation for the direct interaction (n, ∞) and (p, ∞) experiments, Sherr* reports that the (p, ∞) cross sections at 12 Mev on the same nuclei as were studied at 18 Mev, show entirely different characteristics.

TABLE III: The $^{58}\text{Ni}(\alpha, \infty)^{55}\text{Fe}$ Cross Section

Integration Limits	Cross Section Predictions From the $^{58}\text{Ni}(\alpha, \infty)^{55}\text{Fe}$ Reaction			Experiment
	Cluster Model	Pick up	Pick up & Transfer	
0 to 90°	3.5 ± .7 mb.	.9 ± .2 mb.	1.8 ± .4 mb.	> 2 ± .3 mb.
0 to 45°	2.4 ± .7 mb.	.6 ± .2 mb.	1.3 ± .4 mb.	≈ 2 ± .3 mb.

* private communication, to be published.

SECTION VI: IMPROVEMENTS

There are a number of useful modifications to the apparatus that should be made before work is continued on the study of the direct (n, α) reactions. The most obvious one is to obtain a larger detector. The one used to study the $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ reactions had a surface area of $1\frac{1}{2}$ cm².

To determine absolute values for the direct interaction cross sections, it would be helpful if the center line of the apparatus were collinear with that of the neutron generator. This is one of the reasons why the comparison between the $\text{Ni}^{58}(n, \alpha)\text{Fe}^{55}$ ground state cross section and the $\text{Ni}^{58}(p, \alpha)\text{Co}^{55}$ cross section is only tentative. At 0° to the deuteron beam, neutrons from the $\text{F}(d, n)\text{He}^4$ reaction have an energy of 14.8 Mev; where the center line of the apparatus is now situated, the neutron energy is 14.65 Mev; at 60°, it is 14.5 Mev. If the center line of the apparatus were placed along the neutron generator's center line, the neutrons incident on the (nickel) target would produce alpha particles whose energy was symmetric with the laboratory angle.

A third improvement would be to design a movable target holder. It could be simply a 1/32 inch thick piece of graphite with three holes. The first hole to be covered with the target material, the second with an aluminum foil with a small Am^{241} alpha particle source, and the third left blank for background runs. This graphite slab should be movable from the outside of the apparatus, possibly with the aid of magnets. With this

alpha particle source, both proportional counters could be calibrated frequently as well as the silicon detector. Consequently, the energy loss in the proportional counters of the alpha particles of interest would be known more exactly and a considerably higher neutron flux could be used.

In conjunction with this improvement in the calibration of the proportional counters, improvements in the electronics could be made. The proportional counters' preamplifiers have a signal to noise ratio of about 100:1. The pulses of interest are about .1 to .5 volts. These pulses should be sent through a differential discriminator with a variable slit width so that both upper and lower discriminations can be realized. For the results shown, an integral discriminator was used. At the same time a double delay line amplifier should be used to amplify the energy pulses. With this modification the neutron flux could be increased still further.

There exists a simple method by which the ^{production rate} ~~tritium content~~ of the ^{of 14 Mev neutrons} ~~neutron generator's target~~ can be determined while an experiment is in progress. It is known that after about 50 hours of continuous operation at about 10 μ amps of deuteron beam current, many $D(d, n)He^3$ reactions occur. These 2.5 Mev neutrons do not induce the reactions under study but they do register on the slow neutron detector that is used to monitor the neutron flux. To compare the direct (n, α) cross sections, it is necessary to know what fraction of those monitored are 14 Mev neutrons. The method proposed is to place an additional

solid state detector, in a permanent location, to count the number of $Si^{28}(n, \alpha)He^{25}$ ground state (or ground and first few excited states) transitions. Obtaining equal numbers of such transitions will insure that the comparisons between the direct (n, α) reactions for different nuclei will be exact.

CONCLUSION

In summary, this survey of (n, α) reactions represents an introduction to an interesting field of study. The original project to compare the direct (n, α) cross sections at 14 Mev in middle weight nuclei with the results obtained by Dayman, Brady and Sherr, has not been completed. However, the major experimental difficulties have been overcome, and the $Mi^{58}(n, \alpha)Fe^{55}$ direct interaction has been observed. After another year or two of experimentation, a decision on whether or not 'alpha clusters' exist may be reached.

APPENDIX I: THE PICK UP THEORY

The sum rules that have been used extensively in this report will now be derived. In this appendix, no original work is presented. Complete (and rather complicated) derivations of the sum rules have been given by French and Macfarlane,²³ but it was felt that a review of a number of more easily understood papers²⁴ would be useful for future reference. The underlying assumption of the theory to be presented is that the shell model is an accurate representation of the nucleus.

The two sum rules that have been used in the text can be written (considering neutrons only for the moment) :

$$\sum_i G = \langle \text{neutrons} \rangle_{l,j} \quad (11)$$

$$\sum_r G = \langle \text{neutron holes} \rangle_{l,j} \quad (12)$$

Here G is the 'strength' of the neutron transition

$$\psi_r \rightleftharpoons \psi_{l+1} + (l,j)$$

and $\langle \text{neutrons} \rangle_{l,j}$ is the average number of target neutrons in the (l,j) orbit. We shall derive these two expressions and then, introducing the seniority number, arrive at equations (2) and (3) of Section III (a).

The 'strength' of the transition is defined as the reduced width, multiplied by the statistical factors which enter into the cross section. The reduced width, in turn, can be thought of as a product of two factors. One factor measures the probability

that in a given compound state the nucleons will arrange themselves in a configuration corresponding to the final state; the other factor measures the intrinsic probability that when this happens the two components will actually separate. More will be said about the statistical factors later, although at this point it is worthwhile to note that they do not enter in the pick up case.

The definition of the reduced width can be written formally as: $\mathcal{V}^2 = \mathcal{S} \mathcal{V}_0^2$, where \mathcal{S} is the relative reduced width and \mathcal{V}_0^2 is the single particle reduced width. We shall be more interested in \mathcal{S} than in \mathcal{V}_0^2 . The latter gives the probability that the two components actually separate. A reasonable measure of \mathcal{V}_0^2 can be obtained by considering the problem of a single nucleon moving in a potential well produced by the other nucleons. ²⁵

\mathcal{S} , the relative reduced width or spectroscopic factor, a measure of the probability that the nucleons arrange themselves in a configuration corresponding to the final state is determined from the coefficients of fractional parentage. Consider a single nucleon pick up reaction for example. We wish to describe a collection of n nucleons so that one is separated from the others. Let us write the wave function of this one nucleon as $\psi(jn)$ - we discuss the problem here in jj coupling for definiteness although any coupling scheme will do. We now ask how simultaneously to describe the other $n - 1$ equivalent nucleons.

Since we are hypothesizing independent particle motion in some potential well it is clear that the behaviour of the other $n - 1$ equivalent nucleons in the nucleus of mass A ,

related to what it must be ~~just the same as~~ would be in the nucleus of mass $A-1$ that would result if we actually removed our last nucleon from nucleus A , instead of only mathematically setting it to one side by writing a separate wave function for it. However, if the $n-1$ nucleons were really making a state of mass $A-1$ by themselves this state must be an eigenstate; whereas in mass A they can be simultaneously represented by any or all of these same eigenfunctions of $A-1$ whose spins, parities, and isotopic spins can couple to the spin, parity and isotopic spin of the last nucleon to give the correct spin, parity and isotopic spin of the state we described in $A-1$. These various possible states of $A-1$, that we find underlying the state of A when we single out one nucleon and then describe the remainder, are called the parent states of the particular state of A . They are not, as a rule, equally represented.

More formally, we write

(13)

$$\Psi \left(\begin{matrix} J_A & M_A \\ J & T \end{matrix} \alpha \right) = \sum_P \langle \alpha_P | \alpha \rangle \psi \left(\begin{matrix} J_P & M_P \\ J_P & T_P \end{matrix} \alpha_P \right) \varphi \left(\begin{matrix} J_n \\ J_n \end{matrix} \right) C_{\begin{matrix} J_P & J_n \\ J_A & T_A \end{matrix}}^{J_P & T_P} C_{\begin{matrix} J_P & J_n \\ J_A & T_A \end{matrix}}^{J_P & T_P}$$

Here J , M_J , T , M_T and α refer to, respectively: the angular momentum, the observable component of angular momentum, the isotopic spin, its observable component, and α - to any other quantum numbers that are necessary to specify the particular state in A . The subscript P refers to the parent states. The upper case symbols without subscripts refer to the state to be described in nucleus A ; the lower case symbols to the last nucleon. The C 's are the coefficients of vector addition.

In equation (13) the $\langle \alpha_p | \alpha \rangle$ are the respective amplitudes in which the various parent states are represented. They are fractional percentage coefficients. They are effectively the coefficients that are demanded in order that this expansion of $\Psi (J^2 \alpha)$ should have over all antisymmetry, namely that we should not pretend to know which of the n equivalent nucleons we are singling out for this special treatment. We normalize

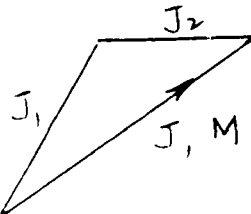
$$\sum_p \langle \alpha_p | \alpha \rangle^2 = 1 \quad (14)$$

It is useful to introduce the simpler notation used by French and Macfarlane.²⁵ The coupling of two angular momenta, $J_1 M_1$ and $J_2 M_2$ to give $J M$ is usually written as²⁶

$$| J_1 J_2 J M \rangle = \sum_{M_1 M_2} | J_1 J_2 M_1 M_2 \rangle \langle J_1 J_2 M_1 M_2 | J_1 J_2 J M \rangle \quad (15)$$

where $\langle J_1 J_2 M_1 M_2 | J_1 J_2 J M \rangle = \begin{matrix} J_1 & J_2 & J \\ M_1 & M_2 & M \end{matrix}$

are the Clebsch-Gordan or vector addition coefficients. French denotes this same relationship, graphically, by



J_1 and J_2 are understood. To

describe an antisymmetric state of n identical nucleons, each with angular momentum j coupled to a resultant J , we write:



, in which the circular arc represents antisymmetrization. Finally the three basic recoupling rules²⁷

can be written as:

$$\begin{array}{c} a \\ \diagup \\ \triangle \\ \diagdown \\ b \\ \hline c \end{array} = (-1)^{a+b-c} \begin{array}{c} b \\ \diagup \\ \triangle \\ \diagdown \\ a \\ \hline c \end{array} \quad (16)$$

$$\begin{array}{c} b \\ \diagup \\ a \quad e \quad d \\ \diagdown \\ c \end{array} = \sum_s U(\text{abcd : ef}) \begin{array}{c} b \\ \diagup \\ a \quad f \quad d \\ \diagdown \\ c \end{array} \quad (17)$$

$$\begin{array}{c} b \\ \diagup \\ a \quad f \quad d \\ \diagdown \\ c \end{array} = \sum_s U(\text{abcd : ef}) \begin{array}{c} b \\ \diagup \\ a \quad e \quad d \\ \diagdown \\ c \end{array} \quad (18)$$

The first of these defines the phase convention for the Clebsch-Gordan coefficients²⁸, the second may be regarded as the definition of the normalized Racah coefficient²⁹, and the third equation is the inverse of the second.

Using this notation, equation (13) becomes (without introducing the jj coupling scheme)

$$\begin{array}{c} \curvearrowright \\ J \end{array} \cdot \begin{array}{c} \curvearrowright \\ T \end{array} = \sum_{\substack{C, J, T \\ l, j}} \langle C J T | C_P J_P T_P \rangle l_{ij} \cdot \begin{array}{c} l(n) \\ \diagup \\ C_P \quad J_P \\ \diagdown \\ j \end{array} \cdot \begin{array}{c} t(n) \\ \diagup \\ T_P \\ \diagdown \\ T \end{array} \quad (19)$$

where C represents all the quantum numbers, apart from J, T (and their s components) necessary to define the state. l(n) and s(n) are, of course, the orbital angular momentum and spin of the separated particle, and we write (following French) $\langle C J T | C_P J_P T_P \rangle l_{ij}$ for the coefficient of fractional percentage (c.f.p.) connecting the states (CJT) and (C_PJ_PT_P).

If we now introduce the channel spin, s , using the recoupling rules (15) to (18), equation (19) becomes:

$$\begin{aligned}
 \langle C_J \cdot T \rangle = \sum_{\substack{C_P J_P \\ l, j, z}} (-1)^{z+s} z^{s-j-J} U(J_P \begin{matrix} S \\ J l : z^j \end{matrix}) \cdot \\
 \cdot \langle C_J T | C_P J_P T_P \rangle_{l, j} \cdot \left[\begin{array}{c} \text{Diagram 1: Triangle with vertices } C_P, J_P, z \text{ and side } J \\ \text{Diagram 2: Triangle with vertices } S(n), T_P, z \text{ and side } t(n) \end{array} \right] \quad (20)
 \end{aligned}$$

The relative reduced width for orbital angular momentum l and channel spin s , corresponding to $(CJT) \rightarrow \left[\begin{matrix} (S J T) + 1 \\ P P P \end{matrix} \right]_z$

now follows from (20) and is given by

$$\mathcal{J}(l, s) = n \sum_j (-1)^{s-j-J} z^{s-j-J} U(J_P \begin{matrix} S \\ J l : z^j \end{matrix}) \langle CJT | C_P J_P T_P \rangle_{l, j} \quad (21)$$

The usefulness of introducing the channel spin now becomes apparent because, upon summing over channel spins and using the unitary property of the U functions^{*}, we see that the interference between different j values vanishes and

$$\mathcal{J}(l) = n \sum_j \langle CJT | C_P J_P T_P \rangle_{l, j}^2 \quad (22)$$

The factor n occurs because the s.f.p. describes the separation of particle number n whereas the reaction allows every particle to be emitted on the same footing.

* $\sum_f U(abcd : ef) U(abed : e'f) = \delta_{ee'}$

We now explicitly introduce the shell model. The jj coupling s.f.p. are defined by

$$\left(\begin{array}{c} j^n \\ J \end{array} \right) \cdot \left(\begin{array}{c} t^n \\ T \end{array} \right) = \sum_{J_P T_P} \langle j^n : JT | j^{n-1} : J_P T_P \rangle \cdot \quad (23)$$

Since this is a special case of equation (19), we have for

$$\left(\begin{array}{c} j^N \\ JT \end{array} \right) \longrightarrow \left(\begin{array}{c} j^{N-1} \\ J_P T_P \end{array} \right) + j$$

$$\mathcal{S} = n \langle j^N : JT | j^{N-1} : J_P T_P \rangle^2 \quad (24)$$

The strength of a transition was defined as the reduced width, multiplied by the statistical factors which enter into the cross section. From equation (24), we have that for the removal of the single nucleon in a shell, $\mathcal{S} = 1$. At the other end of the shell, if we fill the last hole, we have $\mathcal{S} = 2j + 1$. This does not mean, however, that the latter event is more probable; we have neglected the fact that there are $2j + 1$ positions in the shell, only one of which can be filled. Thus for this case (letting $N = 2j + 1$), the cross section for a (d,p) reaction, for example, becomes:

$$\sigma^{dp} \left(\begin{array}{c} N-1 \\ j + j \rightarrow j^N \end{array} \right) \propto \frac{1}{2j+1} \mathcal{S} (N \rightarrow N-1) = 1$$

Generalizing this, French has shown that the relationship between holes and particles in a shell is:

$$(2J+1) (2T+1) \mathcal{S} (n, J, T \rightarrow n-1, J_P, T_P) \quad (25)$$

$$= (2J_P+1) (2T_P+1) \mathcal{S} (N-n+1, J_P, T_P \rightarrow N-n, J, T)$$

Finally, the sum rules (26) and (27) follow. From the normalization of the wave function (13) or (19),

$$\sum_{J_p, T_p} \delta(n, J, T \rightarrow n-1, J_p, T_p) = n \quad (26)$$

or, considering neutrons only, $\sum_1^1 \theta = \langle \text{neutrons} \rangle_{1, J} \quad (27),$

and combining equations (25) and (26),

$$\sum_{J_p, T_p} \frac{N+1}{2J_p+1} \cdot \frac{N+1}{2T_p+1} \delta(N-n+1, J, T \rightarrow N-n, J_p, T_p) = N-n+1 \quad (28)$$

or, $\sum_p^1 \theta = \langle \text{neutron holes} \rangle_{1, J} \quad (29).$

To obtain the sum rules used by Elyman et al., we must introduce the seniority quantum number. If the seniority is a good quantum number (or, in other words, if the states involved are unique) we will be able to drop the summations in the sum rules.

The usefulness of the seniority quantum number can be shown using, for the moment, V^{51} as an example. In this nucleus there are three protons (of the possible eight) in the $f_{7/2}$ shell. This is written $(f_{7/2})^3$. If there were instead, three different, uncoupled particles in the state, it could be specified with $j_1, j_2, j_3, m_1, m_2, m_3$. If there were three different particles, jj coupled, the individual m 's would not be good quantum numbers and we could use instead the five quantum numbers: j_1, j_2, j_3, J, M . One other quantum number is necessary. If particles 1 and 2 are coupled more strongly than are 1 and 3 or 2 and 3, then it may be possible to use the six quantum numbers: $j_1, j_2, j_3, J_{12}, J, M$. However, if, as in the case of the $(f_{7/2})^3$ protons in V^{51} , the particles

are identical, then they are indistinguishable and it is impossible to choose a particular pair such as particle number 1 and particle number 2. J_{12} therefore has no meaning, and the need arises for a new quantum number.

A possible way of classifying the states of such a system of identical particles, is to introduce an operator which measures the number of pairs of particles coupled together with antiparallel angular momenta, i.e. to a total angular momentum zero. Notice that it is not necessary to designate any particular pairs of particles in order to consider the number of pairs of particles coupled to zero angular momentum. The number is therefore an observable consistent with the Pauli principle.

It is known that a number of 'saturated' pairs can be added to any given system of particles without changing the properties of the original system very much. Thus we can consider a state as consisting of a number, v , of non-saturated particles which determines most of the properties of the state, plus a number of —saturated pairs. We can construct a series of states for systems containing $v, v+2, v+4$ etc. particles, each differing from the others only by the number of additional saturated pairs and all having very similar properties. The number, v , of non-saturated particles is called the "seniority number", because it gives the smallest number of particles needed for building a state with a given set of properties and therefore specifies the simplest (oldest) configuration which contains such a state.

After these remarks, the expressions used by Bayman et al. follow easily. From equations (25) and (26), considering identical nucleons only (without the isotopic spin formalism) and assuming that the seniority is a good quantum number, we have for the chain

$$(n\bar{j}) = (00) \leftarrow (1j) \leftarrow (20) \leftarrow (3j) \dots,$$

$$\mathcal{S}(n \rightarrow n-1) = n, \quad \text{if } n \text{ is even}$$

$$\text{and} \quad \mathcal{S}(n \rightarrow n-1) = 1 - \frac{n-1}{2j+1}, \quad \text{if } n \text{ is odd.}$$

For $j = 7/2$ the chain of \mathcal{S} -values is then:

$$\mathcal{S} = 1, 2, 3/4, 4, 1/2, 6, 1/4, 8.$$

APPENDIX XI: THE (n, α) AND (n, d) REACTIONS IN SILICON

In this appendix we introduce a new method for determiningⁱⁿ the ratio of the ground state $Si^{28}(n, \alpha)He^{25}$ to $Si^{28}(n, d)Si^{27}$ cross sections. We will also show that the tentative assignment of the $Si^{28}(n, d)Si^{27}$ ground state transition reported by Deushere and Lawrence³⁰ is incorrect.

The depletion depth or sensitive region of a solid state detector varies as the square root of the applied reverse voltage. The fractional energy loss of a charged particle as it travels through the silicon of the detector is proportional to $\frac{z^2}{mE}$, where m , z , and E are respectively the mass, charge and E energy of the particle. A monogram prepared by Weiss and Whitley,³¹ relates the reverse bias to the depletion depth and to the maximum energy of ^{various} particles which can be stopped in these depths. Table IV shows some pertinent data obtained from the monogram for a 3000 ohm-cm N-type surface barrier diode.

TABLE IV: CHARACTERISTICS OF THE SILICON DETECTOR

Reverse Bias (volts)	Depletion Depth (microns)	Maximum energy in Mev that the appropriate particle can lose.		
		proton	deuteron	alpha particle
100	85	3	4	12
150	100	3.1	4.1	14
200	125	3.6	4.7	15
250	150	4	5.3	17
300	175	4.3	6	18
400	200	4.5	6.2	19
500	250	5.3	7.2	25
700	400	7	10	30
1000	900	12	15	45

The number of hole-electron pairs created in the silicon detector by a nuclear event which occurs within it, is proportional to the total energy released in the transition. For example, the detector stops the Mg^{25} nucleus as well as the alpha particle in a $\text{Si}^{28}(n, \alpha)\text{Mg}^{25}$ reaction. The three reactions in the silicon detector which can be observed when bombarded by 14.7 Mev neutrons are:

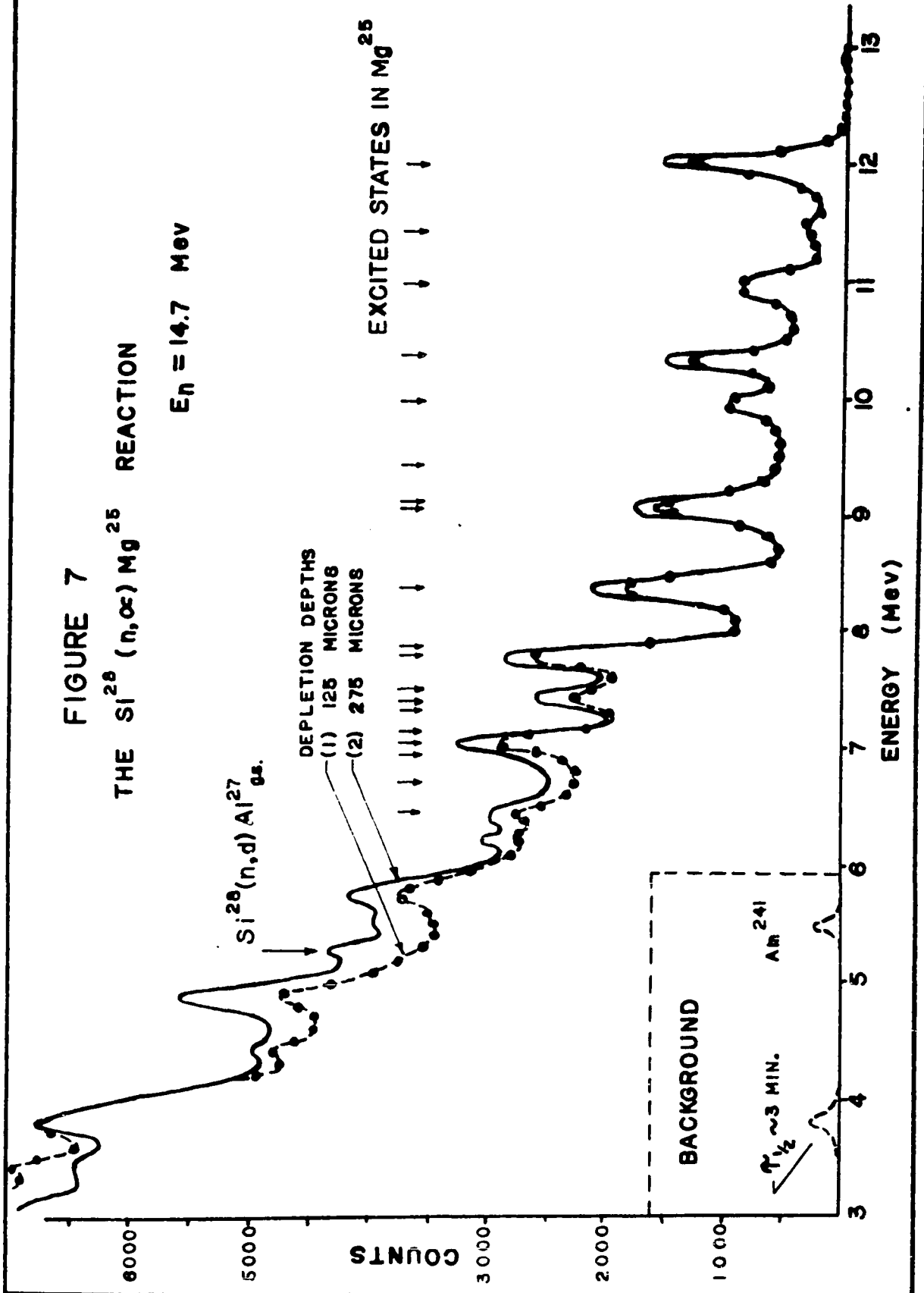
1. $\text{Si}^{28}(n, \alpha)\text{Mg}^{25}$, for which the ground state transition energy which is detected is 12 Mev.
2. $\text{Si}^{28}(n, d)\text{Al}^{27}$, " " 5.3 Mev.
3. $\text{Si}^{28}(n, p)\text{Al}^{28}$ " " 10.8 Mev.

From Table IV, it can be seen that as the depletion depth is increased from 100, through 150 to 250 microns, the 5.3 Mev deuterons representing the $\text{Si}^{28}(n, d)\text{Al}^{27}$ ground state transitions, should not exist, appear, and remain, respectively. This is the basis of the proposed method to compare the (n, d) and (n, α) reactions in Silicon. Notice that the depletion depth must be at least 250 microns before protons will lose 5.3 Mev. Figure 7 shows typical changes which occur in the observed spectrum as the depletion depth is varied. To emphasize these changes, different numbers of neutrons were incident on the detector at the two depths.

Besides appreciating the capability of the detector to stop different particles of varying energies, there is another effect to consider before a comparison of the (n, d) and (n, α) cross sections can be made. First, we assume that the angular distributions for the deuterons and alpha particles are the same. Direct interactions are peaked forward strongly and,

FIGURE 7
THE $Si^{28}(n,\alpha)Mg^{25}$ REACTION

$E_n = 14.7$ Mev



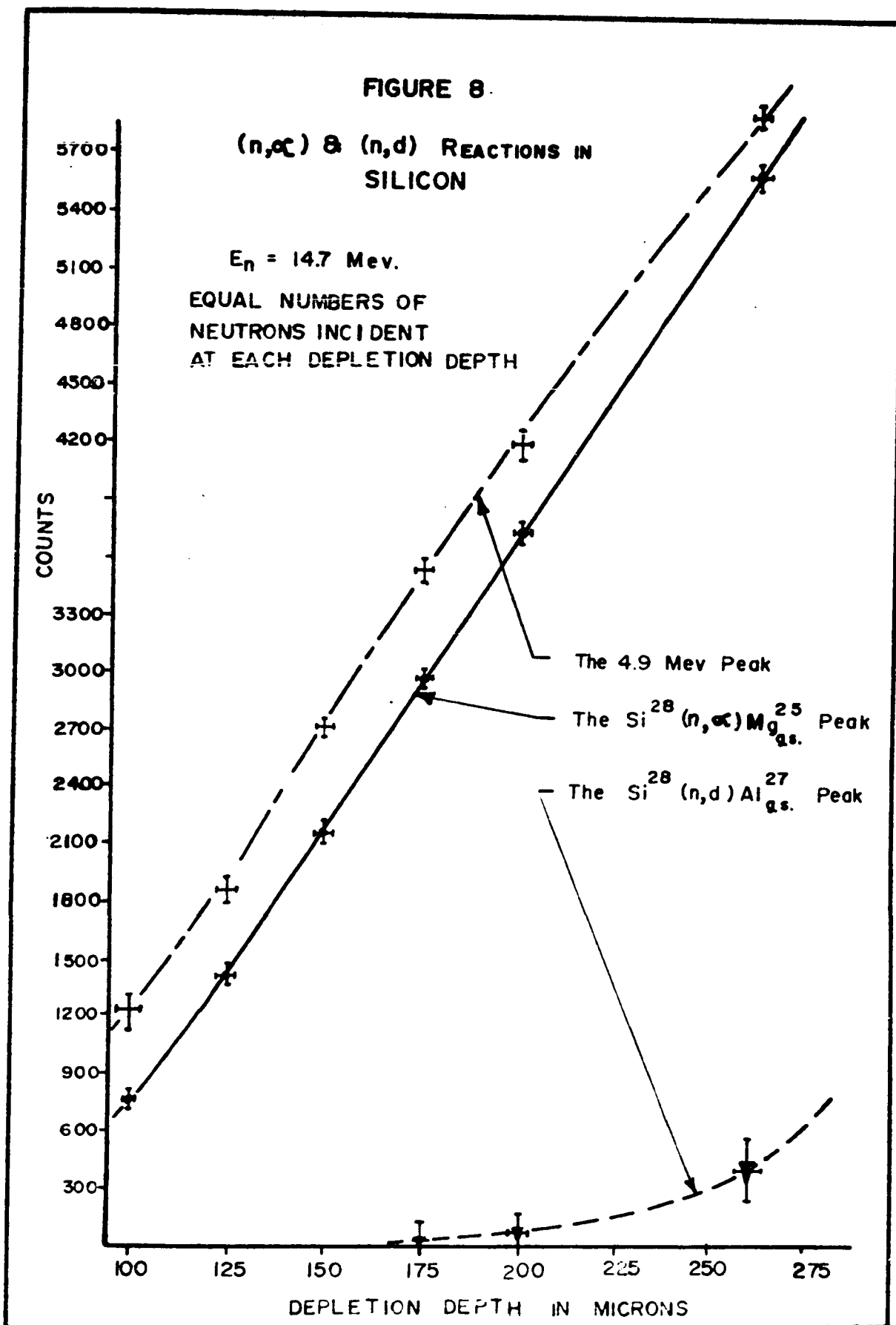
in addition, Mg^{25} and Al^{27} have the same nuclear spin. However, at 150 microns, although the detector is capable of stopping a 5.3 Mev deuteron, there is no 'target', that is, depletion depth in front of the 150 microns which will act as a deuteron source, when bombarded by neutrons. At the same depletion depth, there is about 65 microns acting as a source of 12 Mev ground state transition alpha particles. To be able to compare the magnitudes of the two cross sections, it is necessary to have equal target depths.

Depletion depths as large as 1000 microns can be obtained with Lithium drift solid state detectors. These would be ideal for the comparison. To be able to perform the experiment accurately, it is imperative also, to have good resolving power so that peaks in the alpha particle spectrum near 5.3 Mev can be clearly separated from the deuteron ground state peak. For the results given below, the resolution was only about 200 Kev. The detector used was purchased for its large surface area rather than for its intrinsic noise figure; smaller units can be obtained with equivalent noise figures of about 20 Kev.

The method used was the following. The detector was irradiated with an equal number of neutrons at each depletion depth. This depth, up to the maximum of 200 microns, was determined by measuring both the applied voltage and the leakage current. Then, the number of events appearing in the spectra at 12, and 4.9 Mev, and the change in the number appearing at 5.3 Mev, was plotted against the depletion depth.

The results are shown in Figure 8. The 4.9 Mev peak was tentatively assigned by Deubars and Lawrence to represent the $Si^{28}(n,d)Al^{27}$ ground state transition. It is obvious that this is incorrect. The peak appears in the spectrum at depletion depths of less than 100 microns and the number of events varies 'linearly' with the alpha particle target thickness. It should not be exactly linear because the true ground state deuteron peak has moved out from this energy to 5.3 Mev as the depletion depth was increased.

In forthcoming studies it is hoped that the ratio of the (n,d) to (n,α) ground state cross sections will be determined and compared with theory. In Figure 8 it is seen that the number of $Si^{28}(n,\alpha)Mg^{25}$ _{G.S.} events did not depend linearly on the depletion depth until an alpha particle 'target' thickness of about 25 microns was obtained. Similarly, at the maximum obtainable depletion depth, the $Si^{28}(n,d)Al^{27}$ _{G.S.} transition is just beginning to appear and the ratio of the two cross sections can not be obtained.



APPENDIX III: TWO NEW PULSE MULTIPLIERS

Many different experimental approaches were investigated attempting to observe the (n, α) reaction with the solid state detector. In Table V, we list these different methods as well as the reason for their abandonment. One technique attempted was to identify the particle that entered the system by multiplying the proportional counter pulse (dE/dx), by the solid state detector pulse (E). The two multipliers that were developed will be described below.

TABLE V: UNSUCCESSFUL EXPERIMENTAL METHODS

METHOD	ABANDONED BECAUSE:
1. The solid state detector alone, with and without an aluminum foil placed directly in front of it.	There was no significant change in the observed alpha particle spectrum.
2. The detector alone, shielded from the neutron source with 3 cm. of 'heavy met.', with and without aluminum foil.	" "
3. A single proportional counter, cathode of polyethylene covered with gold foil; demand double coincidence.	Too many random counts (many knock on n-p reactions from polyethylene.)
4. Single proportional counter, cathode of 1/32" graphite covered with gold foil; multiply dE/dx by E.	Still too many random counts, (therefore also random multiplications) Possibly statistical (n, α) reactions in Silicon causing 'backward' travelling alpha particles.
5. Two proportional counters, cathodes of graphite covered with gold foil.	Too much high energy background (> 15 Mev)
6. Two proportional counters, cathodes of 1/32" graphite; demand triple coincidence.	AS USED

The method of identifying charged particles by multiplying a dE/dx by an E pulse relies on the relationship:

$$dE/dx \cdot E \propto \frac{E \cdot E^2}{E} \cdot E = m E^2$$

Thus the product pulse is independent of the energy and is 16 times larger for an alpha particle than for a proton. A single channel analyzer then accepts only the desired product pulse, and gates a multichannel analyzer which records the E pulse. Existing techniques of multiplying (microsecond rise time) pulses either involves considerable circuitry³² or considerable expense³³, and it was hoped that a simpler circuit could be developed.

The first multiplier is shown in Figure 9. This circuit generates a rectangular pulse whose length is proportional to E and whose height is proportional to dE/dx . The 'area' of this pulse (proportional to mE^2) is then found by integration. If no pulse appears at E , V2 will short any positive pulse on its high impedance plate line to ground. If a negative pulse appears at E , the pulse appearing at the grid will decay exponentially ($T_d = R_2 C_2$) towards D^+ . The decay time from its maximum negative value to where V2 will again be in a position to conduct will be, therefore, a reasonably linear function of the negative pulse height. Consequently if pulses appear both at E and dE/dx , the pulse at the grid of V3 will be of height proportional to dE/dx , 'flat topped' because of D1, and of duration dependent on how long V2 is nonconducting. The rectangular pulse is then integrated and ^{fed to a cathode follower output.} ~~cathode followed out.~~

FIGURE 9
PULSE-HEIGHT-TO-TIME MULTIPLIER

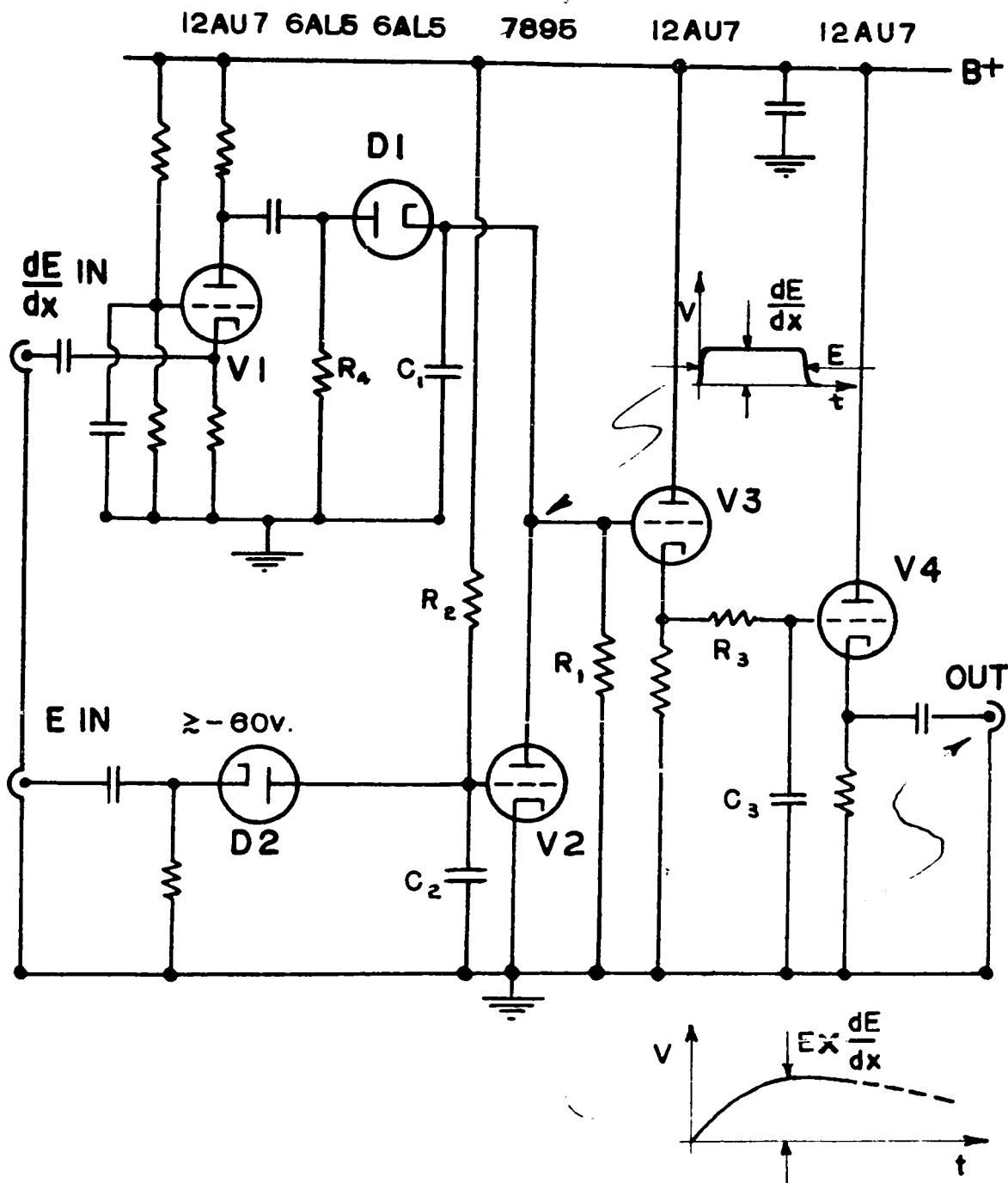
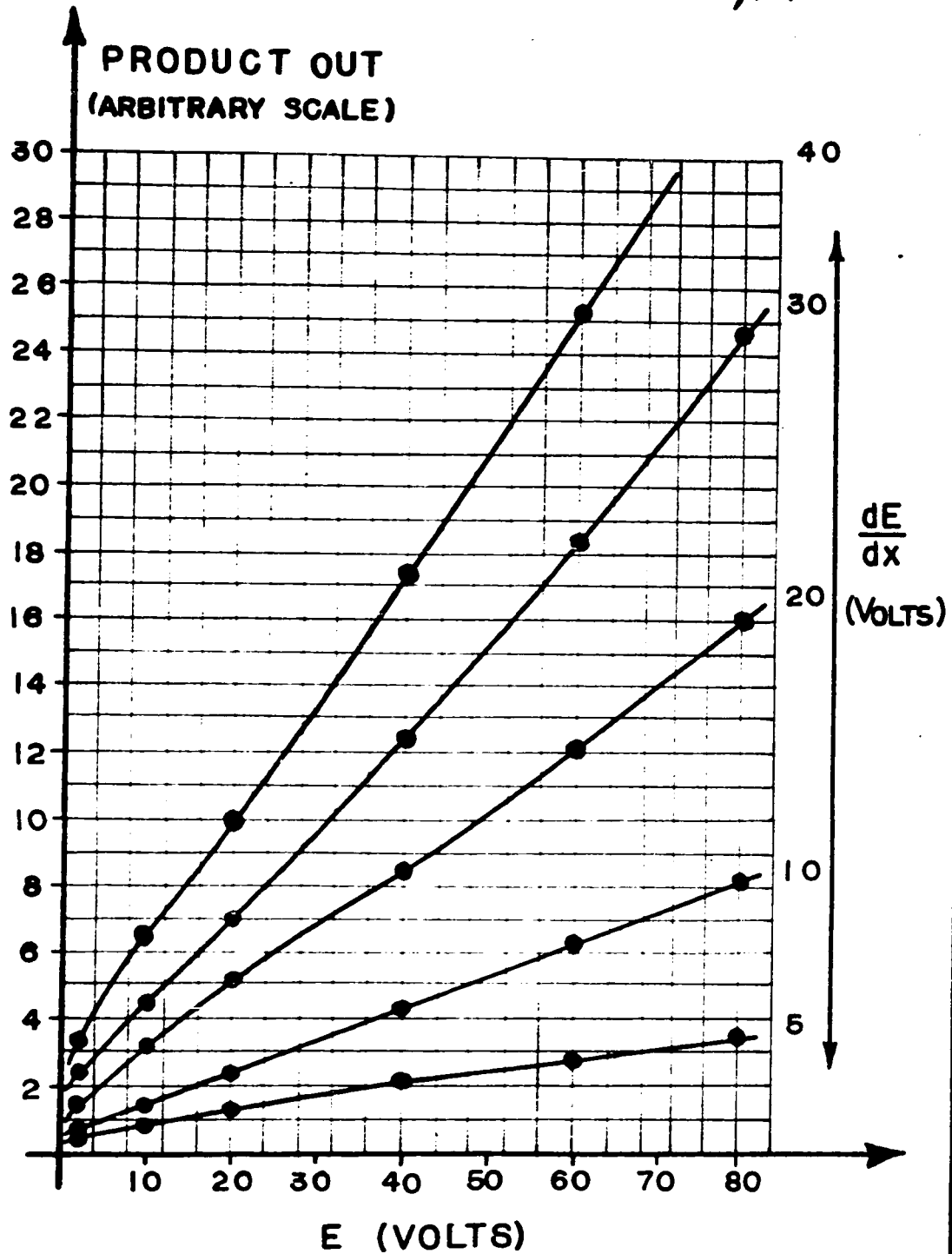


Figure 10 shows the results of multiplication with this circuit. The multiplication is only fair for the three following reasons. First, the plate to cathode resistance of V2 is not zero. Consequently an output pulse appears when there is only a dE/dx input. This was improved by using the high output impedance of the grounded-grid stage (V1) which is then loaded by the low plate to cathode resistance of V2 when it is not turned off. An upper limit to this output impedance is set, however, by the diode leak resistance R_1 . This problem would be overcome, of course, by putting the E , dE/dx and multiplied pulses in triple coincidence. Secondly, a compromise has to be made between how 'flat topped' the pulse appearing at the grid of V3 should be ($T_d = R_1 C_1$) and what dead time for the circuit is tolerable. Thirdly, V2 will not switch on immediately when its grid falls near ground and there is a resulting slow-falling edge on the 'rectangular' pulse.

However, the reason that the circuit was abandoned was because the information does not appear at the output until the integration is complete. This becomes prohibitively long, $\sim 50 \mu$ sec., if the rise time of the input pulses is $\sim 1 \mu$ sec. One could apply the principle used by Aitken³² of putting the product pulse in coincidence with the E pulse after it has been processed by a multichannel analyser. This processing is accomplished in $\approx 50 \mu$ sec. and the product pulse could then 'tell' the analyser to "add one" or "add zero" to its memory.

FIGURE 10

MULTIPLICATION BY PULSE-HEIGHT-TO-TIME MULTIPLIER
 (TESTED WITH MERCURY PULSER - $T_R \sim 1 \mu s.$)



The second multiplier is shown in Figure 11. V2, a 6L7 heptode is the 'multiplier' of this circuit. Theoretically, the gain of V2 is $k = m e_3$ where m is a constant and e_3 is the voltage applied to the third grid. The output is $e_0 = k e_1$, where e_1 is the voltage applied to the first grid. Hence the output, $e_0 = m e_1 e_3$, is proportional to the product of the two voltages e_1 and e_3 .

However in Figure 13, it is seen that although curves for a 6L7 are closer to multiplication than those for a 6AK5 pentode (Figure 12), multiplication could not be performed using a single tube. The characteristics suggest that the output is closer to $a e_1 + b e_3 + c e_1 e_3$, ignoring their nonlinearity. In Figure 11, V1 and V3 add to the output $- a e_1$ and $- b e_3$. These two tubes are biased the same as V2, but V1 receives only the dE/dx pulses and V3, only the E. The output pulses from V1 and V3 are then inverted and added to the output of V2 at its plate.

Two different bias settings are used in the resultant 'multiplication' curves shown in Figures 14 and 15. The multiplier is seen to be sufficiently accurate so that alpha particles could be separated from protons. The results could be improved by matching the pulses at the plate of V2 exactly, but are ultimately limited by the nonlinearity of the characteristics.

FIGURE 12

CHARACTERISTICS OF A 6AK5

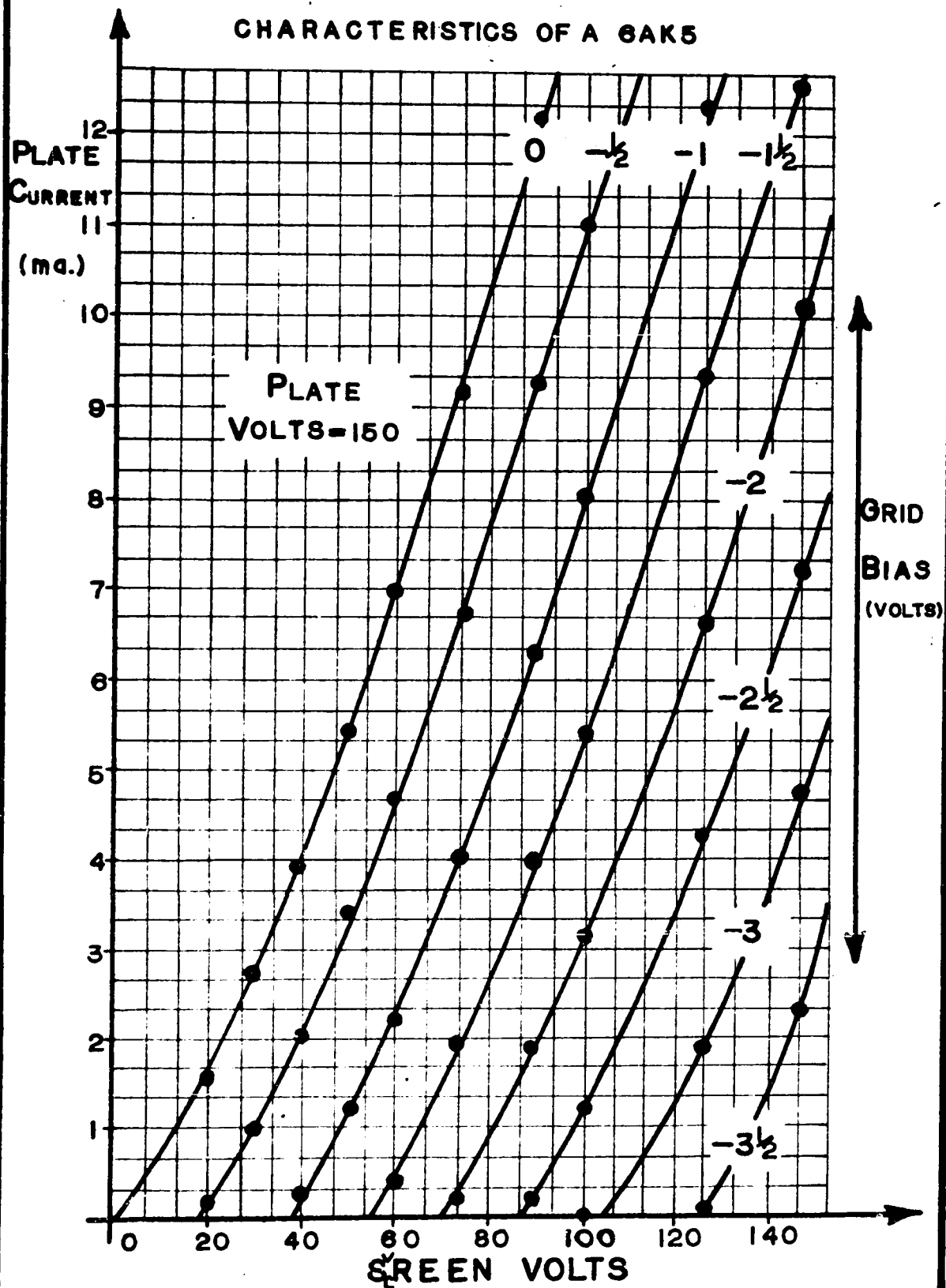


FIGURE 13
CHARACTERISTICS OF A 6L7

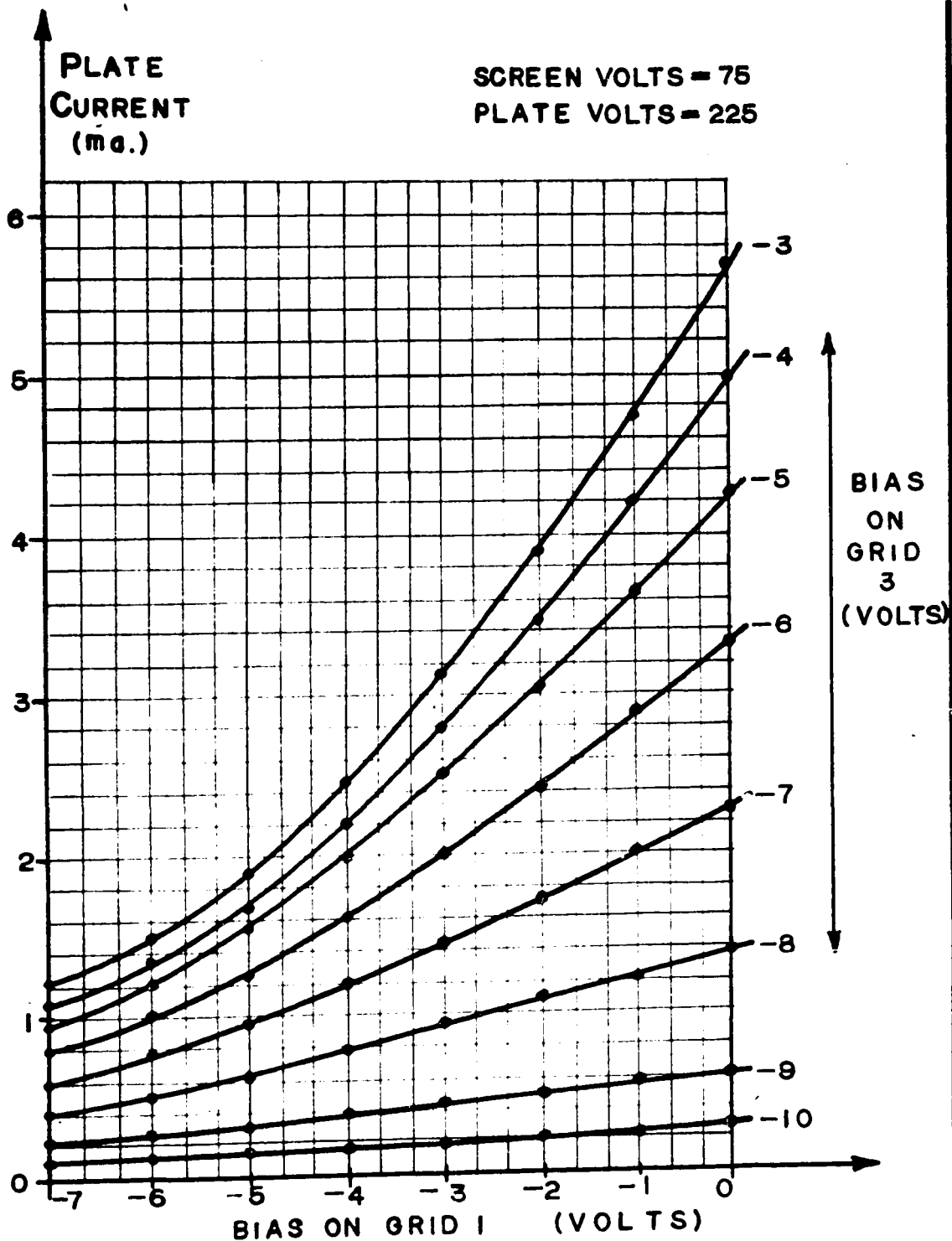


FIGURE 14

MULTIPLICATION RESULTS (I) FOR THE 6L7 MULTIPLIER (FIGURE 11)
 TESTED WITH MERCURY PULSER ($\tau_T \sim 1 \mu\text{sec.}$)

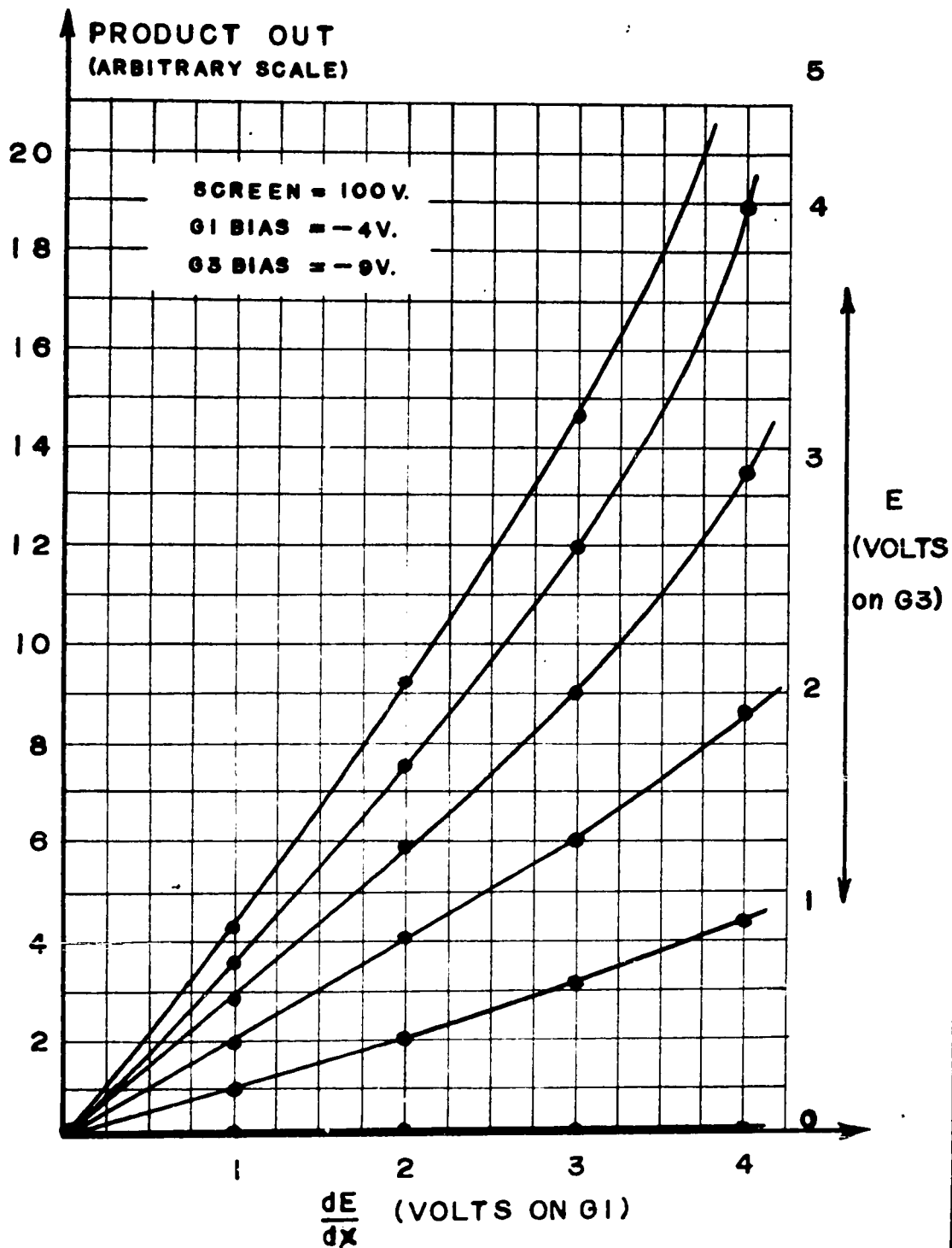
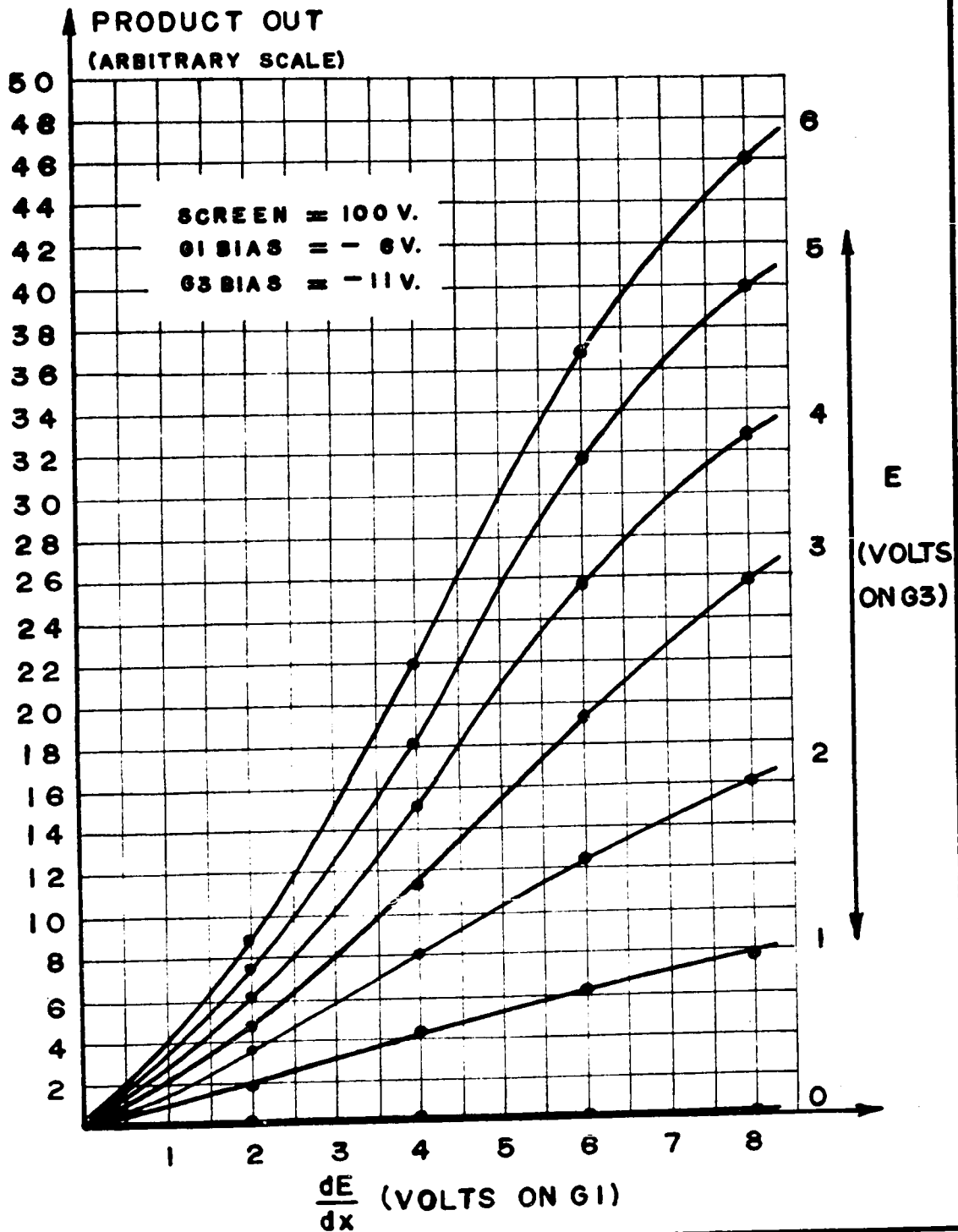


FIGURE 15

MULTIPLICATION RESULTS (2) FOR THE 6L7 MULTIPLIER
 TESTED WITH MERCURY PULSER ($T_r \sim 1 \mu s$)



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