

PSYCHOLOGICAL DIFFERENTIATION
AND ACHIEVEMENT IN MATHEMATICS

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CURRICULUM STUDIORUM

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TABLE OF CONTENTS

Chapter	page
INTRODUCTION	vii
I.- REVIEW OF THE LITERATURE	1
1. Spatial Orientation and Field Dependence	2
2. Characteristics Related to Field Dependence and Field Independence	6
3. Psychological Differentiation	8
4. Socialization and Genetic Factors of Differentiation	12
5. Criticisms of Witkin	15
6. Ability in Mathematics	21
7. Relationship of Field-Dependence-Independence to Mathematical Performance	28
8. Summary and Research Problem	34
II.- EXPERIMENTAL DESIGN	37
1. The Subjects	37
2. The Psychometric Instruments	37
3. The Procedure	41
4. The Statistical Technique	43
III.- PRESENTATION OF RESULTS	45
1. Test Results and Reliabilities	45
2. The Null Hypotheses	47
3. Results of Testing the Null Hypotheses	47
4. Results of Post Hoc Procedures	50
IV.- DISCUSSION OF RESULTS	51
1. Discussion of the Psychological Differentiation Hypothesis	51
2. Discussion of Sex Differences Hypothesis	52
3. Limitations and Suggestions for Further Research	53
4. Implications of the Study	54
SUMMARY AND CONCLUSIONS	56
BIBLIOGRAPHY	58
 Appendix	
1. SCORES FOR CONCEALED FIGURES, ACHIEVEMENT IN ALGEBRA AND ACHIEVEMENT IN GEOMETRY TESTS	60
2. GEOMETRY ITEM ANALYSIS	63
3. ALGEBRA ITEM ANALYSIS	64

TABLE OF CONTENTS

v

Appendix	page
4. INFORMATION FOR SUBJECTS REGARDING THE RESEARCH PROJECT	65
5. THE CLOSURE FLEXIBILITY TEST (CONCEALED FIGURES TEST) ...	67
6. GEOMETRY TEST	76
7. ALGEBRA TEST	82
8. ABSTRACT OF <u>Psychological Differentiation and Achievement in Mathematics</u>	88

LIST OF TABLES

Table	page
I.- Means, Standard Deviations and Reliability Coefficients for Algebra, Geometry and Concealed Figures Tests	46
II.- Statistical Design	48
III.- Results of Multivariate Analysis of Variance	49

INTRODUCTION

For the past twenty-five years, Witkin and his colleagues have been doing extensive research in the area of perceptual and cognitive functioning. Initially they investigated spatial orientation and discovered that there existed wide individual differences among subjects regarding whether visual or postural cues were the predominant factor. Their findings suggested that individuals were self-consistent in the various tasks. This led to the definition of the extreme modes of perception as field dependence and field independence. In the perceptual sphere, field dependency relates to the extent to which an item or an object is influenced by its position in the sometimes confusing influence of its surroundings.

Since the original discoveries, Witkin and other investigators amassed data which related field-dependence-independence to a number of personality characteristics. To explain these relationships, Witkin adopted Werner's principle of differentiation. Under this principle, the extent of differentiation was reflected in the degree of field-dependency-independency.

An analytical mode of thinking was found to be a characteristic of field independent individuals. This in turn was considered to be of great importance in mathematical achievement. Robert Gagné, in analyzing achievement of a mathematical task, identified three major abilities required - identification of a stimulus, recollection of relevant learning sets, and integration of these learning sets into a solution.

M. K. Barakat conducted extensive research in the field of mathematical ability. His psychological and statistical analysis presented solid evidence of the existence of a unique mathematical ability independent of intelligence. This mathematical factor was subdivided into two sub-factors, one for mechanical arithmetic (related to memory), and another for mathematical work (related to manipulation of schemes and relations). He also identified a division distinguishing geometry from problem arithmetic and algebra. Geometry appeared to possess a saturation with the spatial factor in the form of visual imagery, while arithmetic and algebra depended on facility in dealing with formal variables.

Barakat also concluded that ability in mathematics appeared to be influenced by school teaching (perhaps more than any other subject), and in a smaller degree, by other environmental conditions. In addition, genetic tendencies, temperamental as well as cognitive, contributed to mathematical ability. Other researchers tended to concur that this ability was related to a complex interaction of many variables.

The ability to concentrate on one aspect of a mathematical task while ignoring others, and then relating these to other pertinent aspects in the solution of a problem required an analytic approach and the establishment of a field of awareness. Such an observation suggested that the extent of psychological differentiation was related to achievement in mathematics. This experiment investigates the problem.

The first chapter contains a review of the research relating to

psychological differentiation, mathematical achievement and a statement of the research hypotheses. The second chapter includes a discussion of the psychometric instruments to be used and the statistical design employed in the experiment. A presentation of the data and the results of the analyses are provided in chapter III. The last chapter consists of a discussion of the results and their implications, along with suggestions for further research. A summary of the results and conclusions is also provided.

CHAPTER I

REVIEW OF THE LITERATURE

Witkin's early studies of spatial orientation which led to the eventual formulation of his psychological differentiation theory has been the subject of much research for the past several years. This research linked the field-dependence-independence dimension to a wide variety of psychological functions. An analytical mode of thinking has been associated with field independent individuals and this in turn was considered to be a factor in mathematics achievement.

Various sections relating to the studies are presented in this chapter. In the first section, Witkin's spatial orientation tests which led to the definition of field dependence and field independence, are introduced. Some of the personality characteristics associated with field dependence are considered in the second section. This is followed by Witkin's psychological differentiation theory as it has been derived from Werner's theory. Socialization and genetic factors relating to the origins of differentiation are considered in the fourth section. Criticisms of Witkin's work in several aspects are considered next. The sixth section assesses studies of achievement in mathematics, especially the work of Barakat and Gagné. The relationship of field-dependence-independence to mathematics is considered in the next section and this is followed by a statement of the research problem.

1. Spatial Orientation and Field Dependence

Usually, with little effort or thought, an individual is quickly able to establish vertical and horizontal directions. Objects in space are perceived as upright, tilted, or horizontal. This orientation is independent of an individual's body position. Since orientation in space involves a frame of reference in a literal sense, its investigation furnishes opportunities to establish the role of varied and changing frames of reference. Witkin and Asch, influenced by the work of German Gestalt psychologists, investigated various aspects of spatial orientation.¹ By varying the visual frame of reference and the postural position of the body, Witkin and Asch were able to study the constancy of perceiving the upright within individuals and the variance among individuals. Their task was to study the relative importance of the principal sources of orientation, visual and postural, and to determine the manner in which they interact. By separating the sources experimentally, Witkin and Asch were able to determine the effects on a subject's perception of the upright.

The major tests developed to measure a person's spatial orientation included the Rod and Frame Test (RFT), and the Tilting Room - Tilting Chair Test (TRTC). The latter consisted of two subtests, the Room-Adjustment Test (RAT) and the Body Adjustment Test (BAT).² The RFT

1 H. A. Witkin, Psychological Differentiation, New York, Wiley, 1962, p. i - xii.

2 H. A. Witkin, op. cit. p. 37.

was conducted in a completely darkened room. The apparatus consisted of a square frame with a rod pivoted at the centre of the frame. A coating of luminous paint on the rod and frame enabled the subject to see them. The rod and frame were then placed in various tilted positions independent of one another, and the subject was instructed to adjust the position of the rod to the upright, leaving the frame unchanged. In one series of trials, the subject sat upright, while in another series, the subject was tilted.

Witkin found that subjects differed markedly from each other in their ability to perform this task. At one extreme of the performance range, people adjusted the rod to align with the tilted frame. For example if the frame was tilted 28° to the left, the subject tilted the rod 28° to the left, and thereby perceived it as upright. At the opposite extreme, some individuals regardless of either the frame's position or their body orientation, were able to adjust the rod to the true upright.

The TRTC experiment did not require a rod adjustment.³ In this procedure, a subject was placed on a chair in a room where both the chair and the room could be tilted independently. In the BAT, the subject's chair and the room were brought to prepared tilted positions, and the subject's task was to adjust his body to the upright position. Here again a marked difference was observed among individuals. Some individuals performed the task successfully, while some aligned their bodies with the

3 H. A. Witkin, op. cit., p. 38.

room (which was tilted as much as 30⁰) and thereby perceived themselves upright. In the RAT subtest, a room adjustment was required instead of body adjustment. Results were again similar in the degree to which subjects were successful.

Both the RFT and TRTC tests involved the manner of perceiving an object in relation to the surroundings, and the results indicated that for some people, perception of the object was strongly affected by the surrounding field, while others were able to deal with the object as an independent unit. In order to extend the kinds of perceptual areas covered, it was desirable to find situations that did not involve perception of the upright. Witkin used material originally developed by Kurt Gottschaldt.⁴ In this experiment the subject had to locate a simple figure within a larger complex figure. The test was called the Embedded Figures Test (EFT). While this situation did not involve space orientation, what was at issue was the extent to which the surrounding visual framework dominated perception of an item within it. The simple figure was incorporated in the complex one in such a way as to be obscured perceptually. Its outlines might form boundaries of several prominent subpatterns in the complex figure, making its detection difficult. Differences in performance by subjects in this task were very marked and similar in nature to those of the other tests.

In the first two situations described (RFT and TRTC) the subject

4 H. A. Witkin, "Individual Difference in Ease of Perception of Embedded Figures", in Journal of Personality, Vol. 19, No. 1, issue of September 1950, p. 1-15.

was scored by measuring the total of the absolute tilt of the rod in degrees from the true vertical. In the EFT, the score was the time taken to locate the simple figure in the complex design. The scores provided a quantitative indicator of the extent to which an individual perceived an item influenced by an organized surrounding field.

The importance of these experiments rested with the fact that subjects tended to be self-consistent in performance across these tasks. When the same subjects were tested in these situations, it was found that a person who moved the rod towards the tilted frame also adjusted his body towards the tilted room, and was likely to take a long time to find the simple figure in a complex design.⁵ The extent to which a surrounding visual framework dominates perception of an item in it was referred to by Witkin and his associates as the field-dependence-independence dimension.⁶ A field dependent person was defined as being strongly dominated by the prevailing field, and a field independent person was one who was able to deal with an item independently of the surrounding field. Witkin pointed out however, that he did not imply that individuals were either field dependent or field independent. Scores on his tests formed a continuous distribution.

Having examined spatial orientation and developed tests to differentiate between field dependent and field independent individuals

5 H. A. Witkin, Psychological Differentiation, p. 22.

6 _____, "The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations", Princeton, Educational Testing Service, 1973, p. 5.

Witkin and his associates then turned their attention to determining whether these groups differed on other psychological and personality characteristics.

2. Characteristics Related to Field Dependence and Field Independence

The study of field-dependence-independence led Witkin and his associates to state:

The way in which a person orients himself in space is an expression of more general preferred mode of perceiving which in turn, is linked to a broad and varied array of personal characteristics involving a great many areas of psychological functioning.⁷

Originally, Witkin and his associates were interested only in general results and conclusions derived from studies of spatial orientation. The fact that individuals varied considerably in their perceptual style led Witkin to investigate the possibility that this style extended into other psychological domains. As a result, subjects who had been given the basic orientation tests described earlier were given an additional series of perceptual tests and intensive personality analyses. These included a Rorschach Test, A Thematic Apperception Test, a clinical interview, a Draw-a-Person Test, a word-association test, a sentence completion test, and an autobiography.⁸ The samples included university students, patients at a Psychiatric Hospital, and children.

7 H. A. Witkin, Psychological Differentiation, p. 1.

8 H. A. Witkin, "The Nature and Importance of Individual Differences in Perception", in Journal of Personality, Vol. 18, No. 2, issue of December 1949, p. 169.

On the basis of interpretations, Witkin et al summarized the personality correlates of field independence under two major headings: (a) activity-passivity, and (b) orientation toward inner life.⁹ The field dependent person was conventional, submissive to authority, blandly unaware of his inner feeling life, got into comfortable ruts, and was said to fear, deny, and have poor control over such impulses as sex and aggression. The field independent person, on the other hand, actively attempted to master and reorganize his environment. He strove for independence, leadership, special skills, and competencies. He was concerned with his inner life, motives of behaviour, and was able to express hostility with directness and control.

Witkin referred to field independent subjects as having an analytical field approach, while a global field approach was associated with field dependence.¹⁰ This resulted from the observation that field dependent people required considerable time to locate familiar figures in complex designs, and were less likely to decipher ambiguous stimuli which appeared vague and indefinite. They also experienced more difficulty in picture completion and in the analytical sections of intelligence tests. Field independent individuals on the other hand exhibited the opposite characteristics.

9 H. A. Witkin, et al, Personality Through Perception, New York, Harper, 1954, quoted by Rogers Elliott, "Interrelationships Among Measures of Field Dependence Ability and Personality Traits" in the Journal of Abnormal and Social Psychology, Vol. 63, No. 1, 1961, p. 27.

10 H. A. Witkin, Psychological Differentiation, p. 35-36.

Cognitive style is a person's characteristic approach to a wide range of situations which encompass both perceptual and intellectual activities. Since Witkin and his associates found that individuals tend to be self-consistent across many perceptual and intellectual activities related to his dimension of field-dependence, he referred to this dimension as a cognitive style.¹¹ As Coop and Sigel pointed out, there has been a great deal of confusion and lack of agreement in the definition of cognitive styles and thus the term has become investigator specific.¹²

Witkin's studies of cognitive style eventually led to his formulations of the psychological differentiation hypothesis.

3. Psychological Differentiation

While considering the developmental aspects of field-dependence-independence, Witkin observed that children tended to function in a field dependent manner, and that with age they tended to function more field independently.¹³ In a later study, Witkin and his co-workers studied two groups longitudinally.¹⁴ One group ranged in age from 8 to 13 years, and

11 H. A. Witkin, "The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations", p. 7.

12 R. H. Coop, and I. E. Sigel, "Cognitive Style: Implications for Learning and Instruction", in Psychology in the Schools, Vol. 8, No. 2, 1971, p. 152-161.

13 H. A. Witkin, Psychological Differentiation, p. 377.

14 _____, "Stability of Cognitive Style From Childhood to Young Adulthood", in Journal of Personality and Social Psychology, Vol. 7, No. 3, 1967, p. 291-300.

the other group 10 to 24 years. The results showed a progressive increase in the extent of field independence up to 17 years of age, with no further change to 24. However if a child at an early age was lower on the field-dependence-independence continuum than his peers, he continued to occupy a similar relative position on the continuum as a young adult. Hence they exhibited marked relative stability over the period from age 8 to age 24.

The trend to increasing field-independence with age of children closely paralleled development in general personality characteristics. The interrelationship of these characteristics led Witkin to his concept of differentiation.

Differentiation refers to the complexity of a system's structure. A less differentiated system is in a relatively homogeneous structured state: a more differentiated system in a relatively heterogeneous state.¹⁵

A highly differentiated system is manifested in degrees of specialization and integration. Specialization relates to the degree of separation of psychological areas such as feeling from perceiving, and thinking from acting. Integration is the working together of the system's components. Complex modes of integration and a high degree of specialization are characteristic of high differentiation.

The preceding concept of differentiation was based on the work of

15 H. A. Witkin, Psychological Differentiation, p. 9.

Heinz Werner. Werner in his organic approach to psychological development concluded that:

The development of biological form is expressed in an increasing differentiation of parts and an increasing subordination or hierarchization. (---) Such a process of hierarchization means for any organic structure the organization of the differentiated parts for a closed totality, an ordering and grouping of parts in terms of the whole organism.¹⁶

Later in his book, Werner concluded that the fundamental laws of development might logically be applied to the mental functions per se. "An increasing differentiation and refinement of mental phenomena and functions, and a progressive hierarchization may be accepted as a basic principle."¹⁷

Within Werner's framework, Witkin stated his hypothesis as follows, "Individuals are likely to function at a more differentiated or less differentiated level in many areas of psychological activity, making for self-consistency in behaviour."¹⁸ From this hypothesis, Witkin predicted that **at** a high level of differentiation there would be an association among characteristics such as an articulated way of experiencing the world, a differentiated self, and the use of specialized defences.

16 Heinz Werner, Comparative Psychology of Mental Development, (revised ed.), Chicago, Follett, 1948, p. 41.

17 H. Werner, op. cit., p. 51.

18 H. A. Witkin, et al, "Witkin Et Al On Zigler On Witkin Et Al", Contemporary Psychology, Vol. 8, No. 7, 1963, p. 363.

An articulated way of experiencing the world included the ability to distinguish objects as discrete from a structured field. A differentiated self was reflected in an articulated body concept (reflected by sophistication of human figure drawings), and a well developed sense of separate identity (as evaluated by observations of the attention to faces of others, adherence to instructions in performing routine tasks, etc.) Specialized defences included intellectualization and isolation as observed in maze solving situations and other situations.¹⁹

The conclusion drawn from the preceding research was that a high degree of differentiation is related to a high degree of field independence. It was observed that the measures of perceptual style developed by Witkin and his associates effectively defined operationally the extent to which a person is differentiated. As Witkin himself stated:

With evidence like this it is now possible to say that field independence is a manifestation in the perceptual sphere of a broad dimension of personal functioning which extends into the sphere of social behaviour and into the sphere of what may be called personality as well.²⁰

Since this broad dimension of personal functioning which Witkin referred to as an individual's cognitive style showed itself in perception, it offered an objective route to the study of individual differences in personal functioning.

19 H. A. Witkin, et al, "Witkin et al On Zigler On Witkin Et Al", p. 364.

20 H. A. Witkin, The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, p. 9.

The degree of differentiation or field independence was found to vary from person to person. The underlying reasons for the observed broad differences became an interesting research problem to which attention was next directed.

4. Socialization and Genetic Factors of Differentiation.

Another major area of research which received a great deal of attention by Witkin and associates was directed at the origin of the observed broad individual differences in differentiation. This problem was approached by examining effects of child rearing and socialization, and by considering the role of genetic factors.

Effects of child rearing were investigated both from the child's and the mother's viewpoint. The characteristic of child rearing that seemed most closely associated with the development of a more independent style was the early encouragement of autonomous functioning.²¹ This result was arrived at by interviewing mother and child separately, and by observing the child's fantasies in response to Thematic Apperception Test pictures regarding the role of parents.

The socialization aspect was investigated through cross-cultural

21 H. A. Witkin, Psychological Differentiation, p. 341-367.

studies. For example, Berry studied child rearing among the Temne of Sierra Leone and the Eskimo of Baffin Bay.²² These studies concluded that socialization experience can contribute significantly to the development of individual differences in field dependence. Children in these cultures appeared to have highly developed spatial skills.

The study of genetic factors as relating to the development of cognitive style resulted from the many findings of sex differences in various perceptual and cognitive tasks. Sex differences first arose when Witkin was conducting tests of spatial orientation. He found that women were likely to tilt the rod farther toward the tilted frame and their bodies farther towards the tilted room than men, and they take significantly longer to find the simple figure in the complex design.²³ Witkin also referred to a number of studies which confirmed this finding for a variety of groups with different educational and socio-economic backgrounds as well as in groups with several different cultural backgrounds. These studies showed that both in perceptual and intellectual situations, men tend to be relatively more analytical than women who tend toward a global field approach.²⁴

22 J. W. Berry, "Temne and Eskimo Perceptual Skills", International Journal of Psychology, No. 1, 1966, p. 207-229.

23 H. A. Witkin, Psychological Differentiation, p. 214-221.

24 Idem, ibid.

If genetic factors are involved, the sex chromosomes are especially likely to be implicated, although not necessarily to the exclusion of autosomal chromosomes. Witkin and his associates were recently studying this possibility in three different types of studies.²⁵ In one study, a recessive gene on the X chromosome was hypothesized as playing a major role in the development of individual differences in field dependence. In a second study in Denmark, men having an extra X or Y chromosome in addition to the usual XY sex chromosome complement were examined for differences in cognitive style. In a third study in Italy, 3-son families have been identified where two sons share one of the mother's two X chromosomes, while a third has received her other X chromosome. If a gene on the X chromosome contributed to development of individual differences in field-dependence-independence, the brothers with the same X chromosome should be more similar on this dimension than either is to the third brother. Results of these studies have not yet been reported.

Hartlage, in a study involving twenty-five families with male and female children found some evidence to support the hypothesis that the recessive gene on the X chromosome might be involved in the transmission of spatial ability.²⁶

25 H. A. Witkin, The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, p. 11-12.

26 L.C. Hartlage, "Sex-Linked Inheritance of Spatial Ability", in Perceptual and Motor Skills, Vol. 31, 1970, p. 610.

In a recent study, Fennema claimed that in the area of field independence tasks, when sex differences were found, the differences were often of relatively small magnitude and in many cases the variation within sexes was greater than the variation between sexes.²⁷ Also, sex differences don't appear in all cultures. In the study by Berry among the Eskimos, children had highly developed spatial skills, but no significant sex differences were found in spatial ability of male and female children.²⁸

As Witkin concluded, it appeared that socialization factors were of overwhelming importance in the development of individual differences in field-dependence-independence. Genetic factors might be implicated as well, but probably to a much smaller degree.²⁹

Sex differences are not the only doubtful area in Witkin's studies. Several researchers have questioned Witkin's techniques and findings, and some consideration must be given to these problems.

5. Criticisms of Witkin

The work by Witkin and his associates has come under attack from many sources. Common to most critics was Witkin's apparent lack of control

27 E. Fennema, Mathematics, Spatial Ability and the Sexes, paper presented at the American Educational Research Association Annual Meeting, Chicago, 1974, p. 1-2.

28 J. W. Berry, op. cit., p. 229.

29 H. A. Witkin, The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, p. 12.

for intelligence in many of his experiments. Zigler felt that the empirical relationships found between scores of field dependency and other scores employed by Witkin were due to the common relationship between all these scores and general intelligence as defined by standard intelligence tests.³⁰ Wachtel claimed that traditional measures of field dependence may be viewed as tests of ability and correlate as highly with some intelligence test subscales as they do with each other.³¹ Spotts and Mackler, in support of this criticism found a significant correlation of .28 between the Otis Intelligence Test and the Hidden Figures Test scores, and $r = .20$ between Otis and the Embedded Figures Test (N=135).³² Dickstein found a significant correlation of .24 (N=96) between Thurstone's Concealed Figures Test, and the Otis test scores.³³

Witkin however, did examine the hypothesis that field independence might be the result of superior general intelligence, since he had found significant correlations himself between intelligence and measures for perceptual tests, especially EFT.³⁴ He considered two alternative

30 E. Zigler, "A Measure in Search of a Theory?" in Contemporary Psychology, Vol. 8, No. 4, 1963, p. 133-135.

31 P. L. Wachtel, "Field Dependence and Psychological Differentiation: Reexamination", in Perceptual and Motor Skills, Vol. 35, No. 1, 1972, p. 179-189.

32 J. Spotts and S. Mackler, "Relationship of Field-Dependent and Field-Independent Cognitive Styles to Creative Test Performance", in Perceptual and Motor Skills, Vol. 24, 1967, p. 239-268.

33 L. Dickstein, "Field Independence in Concept Attainment", in Perceptual and Motor Skills, Vol. 27, 1968, p. 641.

34 H. Witkin, Psychological Differentiation, p. 59-71.

hypotheses: (1) the relation with total intelligence was a function of general intelligence, and (2) the ability to separate item from context expressed itself in intellectual abilities as well as in perception, and the relation with total intelligence was carried primarily by subtests of standard intelligence tests featuring this ability. In one study, The Wechsler (WISC) scales were employed. The three major factors isolated in these scales were the verbal comprehension factor, the attention-concentration (memory) factor, and the closure or spatial-perceptual factor (analytical). The analytical factor was represented by the block design, picture-completion, and object-assembly subtests. Using composite perceptual scores, Witkin found significant correlation with the analytical factor of .66 ($p < .01$). Correlations with the attention concentration and verbal comprehension factors were not significant ($r = .18$ and $.23$) respectively. Evidence such as this suggested that field-dependence-independence was tapped by particular subtests of the Wechsler scales, and it was the performance on these subtests which was primarily responsible for the correlation found with total intelligence. Hence Witkin rejected the general intelligence hypothesis.

However, Dubois and Cohen seem to have confounded Witkin's findings on intelligence.³⁵ They found significant correlations between measures of field independence and various intelligence tests which had

35 T. E. Dubois and W. Cohen, "Relationship Between Measures of Psychological Differentiation and Intellectual Ability", in Perceptual and Motor Skills, Vol. 31, 1970, p. 411-416.

no requirement for overcoming embedded contexts. Correlations ranged from $-.30$ to $-.56$ ($N=143$) between EFT and ability measures of verbal, quantitative, total aptitude, English, Social Studies, Art and Music, Science, Mathematics, and total achievement.

Yamada, in his thesis, also used factored tests of intelligence to determine whether there would be a difference in performance between field dependent and field independent subjects on tests not weighted with analytic ability. His results did support Witkin, but he also found evidence to support the contention that field independence was related to general intellectual abilities.³⁶

Thus, the relationship between intelligence and field independence is still an unresolved issue.

Another area of criticism lies in Witkin's scoring and measuring techniques, and his use of statistics in general.

Gruen questioned the validity with which the data presented by Witkin's scoring reflected performance in the space orientation situations.³⁷ For example, in using an interview technique with the subject to evaluate self-assurance, the subject received a positive score if anxiety is present, or he received a negative score for self-assurance. This process was

36 K. T. Yamada, Extent of Psychological Differentiation as Related to Intelligence, unpublished doctoral dissertation, University of Ottawa, 1968, viii- 100 p.

37 A. Gruen, "A Critique and Re-Evaluation of Witkin's Perception and Perception-Personality Work", in Journal of General Psychology, Vol. 56, 1957, p. 73-93.

repeated for five other interview variables, and then the scores were added together to produce a total interview score. A positive score was said to reflect personality associated with field dependence. Gruen felt such a score did not indicate anything along a continuum and questioned whether it was logically correct to correlate that score obtained from discontinuous variables with scores from a continuous function. But such results pointed out, as Gruen admitted, that perception-personality relationships of Witkin's told us that more of similar kinds of people are more likely to be found at one end of the perceptual scale than at the other. Cause and relationship could not be concluded.

Zigler concluded that many relationships between perceptual style and personality were suspect due to failure to deal with the validity of personality measures and interview scores.³⁸ He felt that many relationships were highly inflated due to the contamination of certain scores by the experimenter's familiarity with relationships previously discovered.

Young also was aware of these methodological weaknesses.³⁹ He pointed out further that there was no statement given by Witkin to estimate interscorer agreement, nor were there any data reported concerning intra-test or test-retest reliability. Young proceeded to retest Witkin's conclusions

38 E. Zigler, "A Measure in Search of a Theory?" in Contemporary Psychology, Vol. 8, No. 4, 1963, p. 133-135.

39 H. Young, "A Test of Witkin's Field Dependence Hypothesis", Journal of Abnormal and Social Psychology, Vol. 59, 1959, p. 118-192.

in order to eliminate these methodological weaknesses, and his results concurred with Witkin's.

Witkin's use of differentiation has not gone without criticism. Gardner stated that "the term psychological differentiation seems to imply more generality than is warranted even by the notable consistencies described".⁴⁰ He based this on studies which indicated that certain problem solving and verbal skills which clearly required a high level of differentiation were not found to relate to Witkin's measures of differentiation. Witkin was aware of this when he concluded that many questions were left unanswered and that there was evidence that some kinds of verbal skills might develop in a different way.⁴¹ Zigler disputed this conclusion and felt that evidence like this should force Witkin to abandon his psychological differentiation hypothesis.⁴² Hellkamp in his thesis stated that the differentiation hypothesis should be retained until further evidence was accumulated.⁴³ He observed that the results might be due to the fact that the tests might be measuring the process of differentiation rather than the extent of differentiation.

Yet despite the criticisms, the extensive findings by Witkin and his associates in the area of perceptual style and psychological differentiation

40 R. W. Gardner, "Book Reviews", American Journal of Psychology Vol. 76, 1962, p. 710.

41 H. Witkin, Psychological Differentiation, p. 198.

42 E. Zigler, "Zigler Stands Firm" Contemporary Psychology, Vol. 8, No. 11, 1963, p. 459-461.

43 D. Hellkamp, Extent of Psychological Differentiation Among Hospitalized Male Schizophrenics, unpublished doctoral dissertation, University of Ottawa, 1967, p. 25-26.

continued to receive considerable support. Werner, who wrote the Foreward to Witkin's Psychological Differentiation text, and on whose model Witkin based his hypothesis was very impressed. He concluded that the differentiation hypothesis

(---) becomes capable of channeling the inquiry toward significant particular questions, as well as recruiting powerful and relevant experimental methods. (---) the hypothesis is sufficiently incisive to be validated and tested against the findings without undue ambiguity.⁴⁴

Factors affecting ability in mathematics will be examined next. This will lead to an examination of the link between mathematical ability and field-dependence-independence.

6. Ability in Mathematics.

The process of learning a mathematical concept has been analyzed by Gagné in terms of an hierarchy of subordinate learning sets which mediate transfer of learning. The attainment of each particular learning set within the hierarchy was considered to be affected by the following conceptual variables: (a) the identifying stimulus features of the new task; (b) the ability to recall relevant subordinate learning sets; (c) the effective integration of the subordinate learning sets into the solution of the new tasks.⁴⁵ Each of these intervening variables was conceived as

44 H. Witkin, Psychological Differentiation, p. vi.

45 R. M. Gagné et al, "Factors in Acquiring Knowledge of a Mathematics Task", in Psychological Monographs, Vol. 76, No. 7, 1962, p. 1-21.

being a function of particular conditions established by the learning program.

Most research in mathematics has been related to the problem of manipulating these particular conditions to observe the effect on achievement. These studies basically ignore the psychological factors involved in how an individual learns. In 1967, Patrick Suppes stated:

Theories of learning have little to offer in providing insight into how one learns to think mathematically. The nature of abstraction or the process of imagery and association that are surely essential to thinking in the domain of mathematics have as yet scarcely been studied from a scientific viewpoint.⁴⁶

There exists, however, one extensive study of mathematical abilities by M. K. Barakat.⁴⁷ In his study, Barakat investigated the nature of the main cognitive factors, and the influence of affective traits and environmental circumstances on a student's achievement in mathematics.

Barakat began his study with a review of relevant theories. The first evidence of a group factor underlying work in mathematics appeared in a study by Burt in 1917.⁴⁸ He found that among elementary school subjects

46 J. M. Scandura, editor, Research in Mathematics Education, National Council of Teachers of Mathematics, Washington, 1967, p.5.

47 M. K. Barakat, "A Factorial Study of Mathematical Abilities", in British Journal of Psychology Statistical Section, Vol. 4, Nov. 1951, p. 137-156.

48 C. Burt, Distribution and Relations of Educational Abilities, 1917, as quoted by M. K. Barakat in "A Factorial Study of Mathematical Abilities", p. 137.

problem work in arithmetic furnished one of the highest saturations for general ability or intelligence, but when this factor was partialled out, there remained sufficient evidence of a number factor. This factor was thought to involve a variety of sub-factors including the ability to remember and work with numerical symbols and an ability to perceive and work with numerical relations.

Spearman strongly denied the existence of any special aptitude.⁴⁹ Thorndike also denied the existence of any general factors and claimed that skill in arithmetic consisted simply of forming and strengthening an immense number of specific bonds or connections between, for example, the sequence of numbers in counting, items of addition and multiplication tables, and rules of algebra.⁵⁰ Thurstone, however, supported Burt's views. His studies led him to accept a number factor that "almost certainly represents a unique ability."⁵¹ While Burt considered the factor as a unitary or primary ability, Thurstone cited evidence that number facility might be an inherited trait. Barakat felt that the main reason for these contradictory statements lay in their failure to combine statistical with other psychological evidence.

Traditionally, the associationists (including Hume, Mill, Spearman

49 C. Spearman, The Abilities of Man, London, Macmillan, 1927, p. 230-234

50 E. L. Thorndike, The Psychology of Arithmetic, 1922, as quoted by M. K. Barakat, op. cit., p. 137-138.

51 L. L. Thurstone, Primary Mental Abilities, 1938, as quoted by M. K. Barakat, op. cit., p. 138.

Thorndike) define mathematics as the science of quantity or magnitude. As such it was considered to have two main branches - one concerned with discrete quantity and abstract or numerical magnitude (arithmetic, algebra), and the other with continuous quantity and concrete or extensional magnitude (geometry). The former deals with temporal relations, as in counting, the latter with spatial relations as in length and shape. These relations were considered to be affected by educating relations and correlates, and hence did not require a new factor.⁵²

These views of the associationists were considered by Barakat to be at fault on several counts. Their concept of mathematical thinking was too analytic and relational. Philosophy of mathematics and the study of pre-school children support a synthetic and schematic interpretation. Apperception rather than association is the central process. In addition, modern views (Russell, Whitehead) describe mathematics as a branch of logic concerned primarily with formal processes, not as a science distinguished by special subject matter.⁵³

In a case study of students with high and low mathematical ability, Burt found several characteristics whose differences were found to be statistically significant. Listed in order of significance as they

52 M. K. Barakat, op. cit., p. 139.

53 M. K. Barakat, op. cit., p. 141.

characterized the students of high ability they were: good long-distance mechanical memory; high intelligence; emotional stability; introverted tendencies; ease in apperceiving formal schemes, patterns and designs; an interest in numbers at an early age; ability to reason deductively; ability to reason inductively; ability to detect and apply implicit relations; audito-motor imagery; ease in using substitute symbols according to arbitrary schemes; and a readiness for an abstract, formal, symbolic mode of thinking.⁵⁴ Thus mathematical ability appears to depend on a complex combination or conjunction of several comparatively elementary cognitive processes.

Barakat chose nine tests of mental abilities related to the above characteristics, and developed four mathematics achievement tests - arithmetical computation, arithmetical problems, algebra and geometry. The thirteen tests, administered and factor analyzed, yielded three significant factors (sexes were separated). The first was a general common factor roughly identifiable with general intelligence. The second factor divided the tests into two main classes, the mathematical and the non-mathematical while the third factor subdivided the classes into two subclasses. The non-mathematical tests subdivided into verbal and non-verbal (spatial) and the mathematical into mechanical (arithmetic) and logical (mathematics).⁵⁵

54 C. Burt, L.C.C. Reports on Backwardness in Special Subjects, 1928, as quoted by M.K. Barakat, op. cit., p. 142-143.

55 M. K. Barakat, op. cit., p. 146-150.

Of major interest was the mathematical factor, since it supported the assumption of a broad factor common to the mathematical subjects as contrasted with the non-mathematical, for both sexes (independent of intelligence). Barakat attributed this factor to differences either (a) in the learning program, (b) in the formal nature of the mental processes concerned, or (c) in the subject-matter.⁵⁶ He rejected the first possibility on the grounds that a child's way of learning was far less dependent on the methods of teaching than is commonly supposed. The second possibility was also rejected, since many researchers agreed that mathematical work involved no specific mode of mental activity found only in mathematical work. Thus Barakat concluded that the difference between the non-mathematical and the mathematical factor was due to the subject matter of the mathematical tests.

In subdividing the mathematical factor the results confirmed that the factor underlying arithmetical or mathematical tasks was highly complex. Burt agreed with Barakat by stating:

There was a broad distinction between an arithmetical factor depending on mechanical use of memorized numerical associations, and a mathematical factor involving the ability to perceive, attend to and combine numerical relations.⁵⁷

Moreover, the rotated factors suggested that proficiency in geometry

56 Ibid., p. 149.

57 C. Burt, "The Structure of the Mind: A Review of the Results of Factor Analysis", British Journal of Educational Psychology, Vol. 19, 1949, p. 186-187.

depends on the spatial factor far more than arithmetic or algebra.⁵⁸

Also in the study by Barakat, the influence of character qualities, environment, and sex differences in mathematical ability were investigated.⁵⁹ Emotional stability, integrated character and industry appeared to have negligible influence, compared to cognitive ability, on mathematical performance. Significant differences were found however, on performance of children from highest and lowest occupational classes; but otherwise social differences had a small influence. Significant differences were also found among several schools. Students' performance at schools in good neighbourhoods was better than at schools located in poorer neighbourhoods.

There was also a marked difference by boys and girls on average mathematical attainment, the boys being superior. In general intelligence there were no significant sex differences.⁶⁰

In Barakat's study of mathematical ability, the nature of the main cognitive factors and the influence of personality and environment were investigated. Of major interest was the mathematical factor which separated into mechanical and logical subclasses. The mechanical subclass was related to arithmetic and numerical association, while the logical subclass was related to algebra and geometry. Algebra and geometry appear

58 M. K. Barakat, op. cit., p. 150.

59 Ibid., p. 151-155

60 Ibid., p. 153.

to require different abilities to some extent, in that geometry relied on the spatial factor more than algebra. However both required the basic abilities to perceive, attend to, and combine numerical relations. The relationship of this mathematical factor to the field-dependence-independence dimension will be considered next.

7. Relationship of Field-Dependence-Independence
to Mathematical Performance.

In his studies, Witkin referred to field independent subjects as having an analytic field approach, while a global field approach was associated with field dependent subjects. Gardner identified an analytic field approach as a capacity to differentiate complex stimulus fields and to deploy attention selectively towards those aspects of the field that were task relevant while ignoring those aspects that were task irrelevant.⁶¹ Witkin concluded that:

The individual who, in perception cannot keep an item separate from the surrounding field is also likely to have difficulty with the kind of problem that requires taking some critical element out of the context in which it is presented and restructuring the problem material so that the element is now used in a different way. This is often the requirement in problems of mathematical reasoning.⁶²

61 R. W. Gardner, et al, "Personality Organization in Cognitive Controls and Intellectual Abilities", in Psychological Issues, Vol. 2, No. 4, 1960, p. 19.

62 H. A. Witkin, The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, p.6.

Andrew Baggaley found a high correlation between an embedded figures test and a level of concept formation which required analytic ability.⁶³ He concluded that the analytic thinker evolves and tests hypotheses by concentrating on one dimension and ignoring others. The common process involved in concept formation and location of an embedded figure seemed to be a concentration on one aspect of a complex stimulus situation.

Elkind found that field independent subjects scored significantly higher than field dependent subjects on a test requiring the formation of concepts derived from perceptual items, specifically the Abstraction Test of the Shipley Hartford Scale (SHA).⁶⁴ On the SHA a subject must abstract novel relations from the arrangement of the elements where verbal meanings were not involved. He concluded that the result was consistent with the hypothesis that an analytic approach to perception, characteristic of field independent people, was an asset on tests that required the abstraction of elements and relations **from** things as opposed to words.

The general characteristics of analytic ability are usually recognized as essential for mathematical performance. In his book Teaching High-School Mathematics, Fehr referred to geometry as a generalization

63 A. R. Baggaley, "Concept Formation and its Relation to Cognitive Variables", in Journal of General Psychology, Vol. 52, April 1955, p.297-306.

64 D. Elkind, et al., "Field Independence and Concept Formation", in Perceptual and Motor Skills, Vol. 17, 1963, p. 383-386.

of space concepts, with logic as the structural binder.⁶⁵ He felt that the study of proof in plane geometry was aided by analytical thinking. In addition he pointed out that one of the most frequently mentioned outcomes of geometric studies has been the development of space perception. Such an outcome implied that space perception was an important attribute of success in geometry.

Fehr also considered algebra as a logical extension of arithmetic to generalized relationships between numbers. Basically the problems in learning mathematics rested in the relationship between mastery of fundamental concepts and computational skills.⁶⁶ Thorndike defined algebraic skill as the ability to deal with symbols, quantitative relations, generalizations, and with selection and organization of ideas and habits to analyze algebraic situations.⁶⁷ Thus algebra, as well as geometry, required an analytic approach.

There have also been many studies which related field independence to concept attainment. Dickstein demonstrated that field independence was related to performance on concept attainment tasks which involved several perceptual attributes.⁶⁸ Bieri et al also reported a significant correlation

65 H. F. Fehr, Teaching High-School Mathematics, Washington, National Education Association, No. 9, 1955, p. 16.

66 Ibid., p. 17.

67 E. L. Thorndike, The Psychology of Algebra, New York, MacMillan, 1923, p. 198.

68 T. S. Dickstein, "Field Independence in Concept Attainment", Perceptual and Motor Skills, Vol. 27, 1968, p. 635-642.

between scores on an embedded figures test and mathematical aptitude as measured by the mathematics subtest of the Scholastic Aptitude Test, with college undergraduates.⁶⁹

Rapaport related the field-dependence-independence dimension to performance on a mathematics aptitude test which was used in a factor analytic study.⁷⁰ The test included some relatively difficult algebraic problems. Rapaport felt that field-independent persons developed a field of awareness in which distinctions between the elements of a problem were maintained and in which attention could be directed to different combinations of the elements.

Spatial ability is another frequently cited requirement for achievement in mathematics. Fennema believed that the relationship between mathematics and spatial ability was particularly evident with geometry.⁷¹ However empirical data supporting this relationship has been contradictory. Tim, in a thorough review of literature relevant to geometry and spatial ability concluded that evidence of a relationship remains inconsistent and unreliable.⁷²

69 J. Bieri, et al., "Sex Differences in Perceptual Behaviour", Journal of Personality, Vol. 26, No. 1, p. 1-12.

70 D. Rapaport et al., Diagnostic Psychological Testing, 1945, as quoted by R. W. Gardner et al., "Personality Organization in Cognitive Controls and Intellectual Abilities", in Psychological Issues, Vol. 2, No. 4, 1960, p. 62.

71 E. Fennema, Mathematics, Spatial Ability and the Sexes, p. 4.

72 H. Tim, Geometry and the Space Factors, School Mathematics Study Group, 1963, as quoted by E. Fennema, op cit., p. 6.

The two most popular measures of field dependence, the Embedded Figures Test and the Rod-and-Frame Test, were claimed by Sherman to involve visual space perception.⁷³ Thurstone reported significant correlations of .411 and .429 between scores on two different forms of the Gottschald Figures Test (embedded figures) and the Primary Mental Abilities Space Test.⁷⁴ Therefore, subjects who performed well on these tests and thereby classified as field independent might be aided by spatial ability.

Perney hypothesized a greater functional relationship between a field independent cognitive style and achievement, when the cognitive activities increased in complexity.⁷⁵ Specifically, the tasks were more complex if they included skills of comprehension and application, rather than knowledge alone. He tested a sample of elementary school students using six Stanford Achievement subtests. Field independence was ascertained by means of three different embedded figures tests. Included in the achievement battery were Arithmetic Computation and Arithmetic Application classified at the level of comprehension and application respectively. The hypothesized relationship was supported completely and without exception by males and only partially by females.

73 J. A. Sherman, "Problem of Sex Differences in Space Perception and Aspects of Intellectual Functioning", in Psychological Review, Vol. 74, No. 4, 1967, p. 290-299.

74 L. L. Thurstone, A Factorial Study of Perception, Chicago, University of Chicago Press, 1944.

75 L. R. Perney, The Relationship of Field-Dependence-Field Independence with Academic Achievement, doctoral dissertation, Case Western Reserve University, 1971, VIII-61 p.

In a survey of literature, Spitler examined the relationship of field-dependence-independence to mathematics.⁷⁶ By examining mathematical topics from several school textbooks, she concluded that both mathematical ability and achievement were related to field independence. No empirical study was performed to verify her conclusions. Spitler concluded her review by recommending that curriculum designers develop separate courses for field independent and field dependent subjects, and that perhaps field dependent students could begin studying mathematics later than field independent students.

Considerable evidence has also been accumulated implying that cognitive style was an important variable in the preferences expressed by students in the choice of electives and majors in educational institutions. A consistent finding of several studies was that field independence was predictive of educational - vocational interest in analytical and abstract areas such as the sciences, mathematics, engineering, technical and mechanical activities.⁷⁷ Stein for example, found that in a sample of fourth year university students registered in courses such as science and mathematics, those students who were found to be field independent, as determined by an embedded figures test, were significantly higher achievers than field dependent students and were also more interested in scientific areas.⁷⁸

76 G. Spitler, An Investigation of Various Cognitive Styles and the Implications for Mathematics Education, doctoral dissertation, Wayne State University, 1970, V-221 p.

77 H. A. Witkin, The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, p. 13-25.

78 F. Stein, Consistency of Cognitive Interest and Personality Variables with Academic Mastery: A study of Field-Dependence-Independence, Verbal Comprehension, Self-Perception, and Vocational Interest in Relation to Academic Performance Among Male Juniors Attending an Urban University, doctoral dissertation, New York University, 1967, VIII-120 p.

On the basis of evidence discussed, the learning of mathematics would seem to make frequent demands for a field independent cognitive style. A brief summary of the evidence will be made leading to the research problem.

8. Summary and Research Problem.

In the review of the literature, several key points were identified which related psychological differentiation to achievement in mathematics.

(1) Witkin in his studies of perceptual style and psychological differentiation concluded that the ability to disembed pertinent patterns in a complex stimulus situation was characteristic of a field independent cognitive style. Students who exhibited such a style were considered to possess a high degree of analytic ability.

(2) Many researchers agree that mathematics in general required an analytic ability on the part of subjects necessary for successful mathematical performance. Specifically, both algebra and geometry have been identified as subjects in which achievement was dependent upon an analytic approach.

(3) Barakat, in a factorial analysis of mathematical ability determined that there was sufficient evidence of a unique mathematical ability independent of intelligence. He also found a distinction between an arithmetic factor involving mechanistic or memorized relations (not requiring analytic ability) and a mathematics factor requiring an ability to perceive, attend to, and combine numerical relations.

(4) Many personality characteristics found by Witkin in studies of differentiation related to a field independent cognitive style were found to be similar to the characteristics of students who were successful in mathematics. In addition to analytic ability mentioned previously, other characteristics included good long-distance memory, emotional stability, introverted tendencies and interest in numbers and abstract thought.

(5) Sex differences have been reported in many studies of spatial ability, mathematical achievement and degree of psychological differentiation. Barakat found boys superior to girls on mathematical attainment. Witkin found males were more field independent. Other studies showed that in both perceptual and intellectual situations, men tended to be relatively more analytical than women who tended toward a global field approach.

(6) Spatial ability was another frequently cited requirement for mathematical achievement. Since measures of field independence involve spatial ability, field independent individuals might be expected to have spatial ability.

(7) A field independent cognitive style has been found to be important in many studies of concept attainment and mathematical aptitude. The particular aptitude tests employed were general in nature, and the samples usually involved elementary and university students. Perney found evidence that the more complex the task, the greater functional relationships existed between a field independent style and achievement.

(8) Several studies have shown that field independence was

predictive of educational - vocational interest in analytical and abstract areas such as mathematics. Students in these areas have been found to be more field independent than students in non scientific areas.

As a result of these observations, the present study will examine the relationship between a field independent cognitive style and achievement in mathematics, especially in the more complex areas. Since both algebra and geometry require a high degree of analytic ability, a field independent cognitive style would apparently be an asset to students attempting problems in these areas. Also, since sex differences are frequently reported, sex must be a consideration in any study involving these variables.

Therefore the following hypotheses will be investigated:

(1) Students who exhibit a field independent cognitive style are more successful on mathematical achievement tests as measured by separate tests of algebra and geometry than students who are field dependent.

(2) Boys are more successful on mathematical achievement tests as measured by separate tests of algebra and geometry than girls.

CHAPTER II

EXPERIMENTAL DESIGN

The first chapter contained a review of the literature which led to a problem and the statement of the hypotheses. The method by which the hypotheses were investigated will be presented in this chapter. Consideration will be given to (1) The Subjects, (2) The Psychometric Instruments, (3) The Procedure, and (4) The Statistical Techniques for Analyzing the Data.

1. The Subjects.

The subjects involved in the study consisted of 107 students, 62 male and 45 female, who represented all students registered in the third year of mathematics in a large urban secondary school in Ontario. The mean age of the students was 16.5 years. These students were chosen for the study because they would be familiar with algebraic and geometric concepts and would have developed adequate skills and familiarity with general symbols.

2. The Psychometric Instruments .

For each subject, scores were obtained on tests of field-dependence-independence, achievement in algebra, and achievement in geometry.

The achievement tests for both algebra and geometry were constructed by the experimenter. Content validity was achieved by selecting topics for

the tests from the courses of study employed in the subject's school. The subjects' teachers assisted in the selection and also recommended some changes on a preliminary draft of the tests.

Since it is analytic ability which is related to cognitive style, test items based simply on memorization were avoided. Topics included in the algebra test were equations (one or several unknowns), word problems, simplifying algebraic expressions, and factoring. The geometry test included congruency, similarity, parallelism, and angle sum of polygons. All the items on the geometry test included figures, since it is perception in the use of figures that makes geometry basically different from algebra. All items were of the multiple choice type, with 25 items included in the algebra test and 20 items in the geometry test. Copies of the tests are included in appendices 5, 6 and 7.

An initial draft of both the geometry and algebra tests was administered in a different school. The purpose was to check on content validity, internal consistency, and the length of time required by most students to complete the test.

A Kuder Richardson 20 reliability coefficient was calculated for each test. The results were .74 for algebra and .75 for geometry ($N=29$). An item analysis was made for each test, and as a result of this, several items were revised or replaced before the final draft was constructed.

The test chosen for measuring the degree of field dependence was Thurstone's test for Closure Flexibility , more commonly called the Concealed Figures Test (CFT). This test is easily administered in group form. As with Witkin's Embedded Figures Test, Thurstone's test was developed from material originally developed by Kurt Gottschaldt. Each item of CFT consists of a simple figure and four complex figures in which the simple figure may or may not be embedded. The subjects' task is to identify those figures which contain the design and those which do not. The subject's score is obtained by subtracting the number of incorrect responses from the correct responses.

In the test manual, Thurstone reported a split-half reliability coefficient of .78 on an original test, while Pemberton reported a corrected split-half reliability coefficient of .94 on the present form of the test.¹ In a recent study, Bowles reported a KR-21 reliability coefficient of .93.² These results are acceptable measures of internal consistency.

The validity of this test as a measure of psychological differentiation can be defended from previous research. In particular, Witkin

1 L. L. Thurstone, and T. E. Jeffrey, Closure Flexibility (Concealed Figures) - Test Administration Manual , Industrial Relations Centre, University of Chicago, 1965, 1-18 p.

2 A. Bowles, Extent of Psychological Differentiation as Related to Achievement in Science and Attitude in Science, unpublished Master's thesis, University of Ottawa, Ottawa 1973, p. 38.

substantiated the use of the test in this context. He stated that:

Flexibility of closure, spatial decontextualization, and field dependence may be different names for the same dimension. In view of this evidence we may appropriately consider results obtained with tests loaded on the flexibility-of-closure and spatial decontextualization factors as bearing upon the field-dependence dimension.³

In discussing measures of field-dependence-independence, namely the Rod-and-Frame Test (RFT), the Embedded Figures Test (EFT) and Thurstone's CFT, Elliot found that they were all significantly intercorrelated.⁴ In particular, EFT and CFT had a correlation of .55, CFT and RFT were correlated .30, and RFT and EFT were correlated .42.

Other researchers have used Thurstone's CFT as measures of psychological differentiation. Bowles, using the CFT, supported her hypothesis that field independent students were higher achievers in science and exhibited a more positive attitude towards science than field dependent students.⁵

Rudin and Stagner, using both CFT and RFT as measures of physical object perception, found a significant positive correlation between perception of physical objects and social perception. They

3 H. A. Witkin, Psychological Differentiation, p. 52.

4 R. Elliot, "Interrelationships Among Measures of Field Dependence, Ability, and Personality Traits", Journal of Abnormal and Social Psychology, Vol. 63, No. 1, p. 35.

5 A. Bowles, op. cit., vii 71 p.

also reported a correlation of .55 (N =34) between RFT and CFT measures.⁶

Podell and Phillips, on a cluster analysis of a wide variety of tests obtained a significant correlation of .77 (N =32) between Thurstone's CFT and Witkin's EFT.⁷

Thus construct validity for the CFT has been demonstrated by the fact that researchers have used this test in furthering Witkin's theory of psychological differentiation. Although correlations between Witkin's measure of field-dependence-independence and Thurstone's CFT are not as high as might be desirable, most studies using a combination of these measures have found them significantly correlated and found them all effective in supporting their hypotheses.

3. The Procedure.

All tests were administered to the subjects in their four regular classroom groups when scheduled for mathematics. This procedure was followed to minimize disruptions in normal school routine. All subjects were first given the Concealed Figures Test by the researcher in one day. At the time of this administration, the nature of study was outlined to the students. A copy of this outline may be found in the appendix 4.

6 S. A. Rudin and R. Stagner, "Figure-Ground Phenomena in the Perception of Physical and Social Stimuli", in Journal of Psychology, Vol. 45, 1958, p. 225.

7 J. E. Podell and T. Phillips, "A Developmental Analysis of Cognition as Observed in Dimensions of Rorschach and Objective Test Performance", in Journal of Personality, Vol. 27, 1959, p. 439-463.

Four weeks later, the geometry test was administered by the regular classroom teacher. The four classes received the test the same day, and they were allowed 40 minutes to complete it. Two weeks later the algebra test was administered in a similar manner, but for this test, students were allowed 35 minutes for its completion. At most seven students failed to complete the algebra test, since this is the number who omitted the last item on the test. On the geometry test, only one student omitted the last question.

Prior to writing the tests, no advance preparations were made by either classroom teachers or students. The study of geometry was completed by the students approximately six weeks before they wrote the geometry achievement test, and no review was given before the test. At the time of writing the algebra test, students were studying algebra. However with the exception of one or two items, most topics covered by the algebra test had been studied by the students earlier in the year or in preceding years. An item analysis showed that all test items were of moderate difficulty and provided good discrimination. A copy of the item analyses for each test is included in appendices 2 and 3.

The tests were spaced over several weeks to minimize class disruption and to make them appear as part of the normal routine. Teachers were able to choose a day for testing which suited their planning.

After all testing was completed, extreme groups based on the

Concealed Figures Test scores were chosen. The top one third of the students formed the field independent group, with the lowest one third forming the field dependent group. The middle group was not used. Appendix 1 contains all the scores obtained by all students who wrote the three tests.

4. The Statistical Technique

To check for internal consistency, Kuder-Richardson 20 (KR-20) reliability coefficients were calculated for both the algebra and geometry achievement tests. A Kuder-Richardson 21 coefficient (KR-21) was calculated for the Concealed Figures Test. Since all items of the CFT are of the same level of difficulty, the KR-21 gives a very accurate estimate of the KR-20. When the items vary in difficulty, as is the case with achievement tests, the KR-21 underestimates the KR-20. The KR-21 is much easier to calculate, and since the CFT is not machine scored, the 196 items on the test make it extremely difficult to calculate a KR-20 coefficient which requires the proportion of the subjects who answered each item correctly. Also, the KR-20 coefficient is not recommended for use with speeded tests such as the Concealed Figures Test.

The research hypotheses were tested in their null form by means of a multivariate analysis of variance. The independent variables were psychological differentiation and sex. The dependent variables were performance in algebra and geometry. The level of significance was set at .05.

A multivariate analysis of variance was employed since research indicates that the dependent variables, achievement scores in algebra and geometry, are interdependent. Multivariate tests make allowance for the correlation of dependent measures and thereby provide more meaningful results.

Where significant differences were found, Scheffé type post hoc procedures were employed to determine which of the dependent variables contributed to the differences.

CHAPTER III

PRESENTATION OF RESULTS

In this chapter, the results of the study are presented under the following headings: (1) Test Results and Reliabilities, (2) The Null Hypotheses, (3) Results of Testing the Null Hypotheses and (4) Results of Post Hoc Procedures.

1. Test Results and Reliabilities

Of the 107 students registered in the classes used for the study, 103 students wrote the algebra test. The scores ranged from a low of 7 to a high of 25 (maximum possible mark 25) with a mean of 15.4. Similarly, 103 students wrote the geometry test, and the marks ranged from 4 to 19 (maximum 20) with a mean of 13.6. The Kuder-Richardson 20 reliability coefficients for algebra and geometry were .77 and .62 respectively. Table I provides a summary of the results.

For the Concealed Figures Test, 94 students were present when the test was administered. The scores ranged from 12 to 128 with a mean of 72.4. The Kuder-Richardson 21 reliability coefficient was .91.

Although the reliability coefficients of the algebra and geometry tests are not as high as might be desirable, they are quite satisfactory for teacher made tests. An item analysis of the test results showed that the test items did provide good discrimination. The item analyses are contained in appendices 2 and 3.

Table I.-

Means, Standard Deviations and Reliability Coefficients
for Algebra, Geometry and Concealed Figures Tests

	N	Number of items	Mean	Standard Deviation	Reliability Coefficient
Algebra	103	25	15.37	4.57	.77 (KR20)
Geometry	103	20	13.55	3.11	.62 (KR20)
Concealed Figures	94	196	72.38	22.25	.91 (KR21)

2. The Null Hypotheses.

The two research hypotheses were tested in the following null form:

(1) There is no significant difference on results of mathematical achievement as measured by tests of algebra and geometry between students who exhibit a field independent cognitive style and students who exhibit a field dependent style.

(2) There is no significant difference between boys and girls on results of mathematical achievement as measured by tests of algebra and geometry.

3. Results of Testing the Null Hypotheses.

In Table II the statistical design is presented, showing for each cell the means and standard deviations for each dependent variable and the number of subjects.

The results of the multivariate analysis of variance are provided in Table III. Since the F ratio of 6.03 is significant for the first hypothesis, the null hypothesis was rejected. There is a significant difference between field independent subjects and field dependent subjects on tests of mathematical achievement in algebra and geometry. The field independent subjects scored significantly higher.

For the second hypothesis the F ratio of 2.70 is not significant.

Table II.-
Statistical Design

	Psychological Differentiation			
	Field Independent		Field Dependent	
	Algebra	Geometry	Algebra	Geometry
Girls	m = 17.07 s.d. = 3.68	m = 14.92 s.d. = 2.50	m = 12.50 s.d. = 3.90	m = 11.43 s.d. = 3.16
	n = 13		n = 14	
Boys	m = 18.24 s.d. = 3.51	m = 14.94 s.d. = 2.75	m = 16.06 s.d. = 4.99	m = 13.50 s.d. = 3.44
	n = 17		n = 16	
Total	N = 60			

Table III.-
Results of Multivariate Analysis of Variance

Hypothesis	F ratio	Degrees of Freedom	P less than
Psychological Differentiation	6.03	2,55	.0043 significant
Sex	2.70	2,55	.0764
Interaction	0.54	2,55	.5861

From tables, $F(2,55) = 3.15$, level of significance .05

Therefore the null hypothesis can not be rejected. There are no significant sex differences on the tests of mathematical achievement employed in this study.

The third variance source indicated in the analysis would be the result of an interaction hypothesis between sex and psychological differentiation. Since the theory did not imply that there would be an interaction, such an hypothesis was not stated. The non significant F ratio of .54 supports this conclusion of no interaction.

4. Results of Post Hoc Procedures.

Post hoc procedures were followed to determine whether it was geometry, algebra or both which contributed to the significant difference between the field independent and field dependent groups on mathematical achievement. Confidence intervals from a Scheffé type post hoc procedure were established for the difference between the means for each of the dependent variables (algebra and geometry).

The confidence interval for the geometry mean was 2.47 ± 1.97 and similarly the interval for algebra was 3.37 ± 2.68 . Since neither of these intervals spans zero, it may be concluded that both algebra and geometry contributed significantly to the difference in achievement between the field independent and the field dependent groups.

CHAPTER IV

DISCUSSION OF RESULTS

In this chapter the results of testing the research hypotheses will be discussed. The limitations of the study and suggestions for future research will also be considered. The last section will contain a discussion of the implications of this study.

1. Discussion of the Psychological Differentiation Hypothesis.

Students who exhibited a field independent cognitive style were found to score significantly higher on tests of mathematical achievement than field dependent students. This result provides additional support for Witkin's theory of psychological differentiation. In the review of the literature most researchers had suggested such a relationship but failed to support their contention with empirical data related to specific mathematical topics. In several studies, mathematical aptitude tests were employed which by their nature are very general in character. Also, most researchers tended to rely on samples drawn from elementary or post secondary students. Therefore the results of this research extend and strengthen Witkin's hypothesis that a field independent cognitive style facilitates mathematical achievement.

The post hoc procedures also support the theory that both algebra and plane geometry require analytic ability and that achievement in both

these particular subjects is dependent on a field independent cognitive style. Evidence provided by this study cannot lead to any conclusion as to whether algebra or geometry contributes most to the finding of the significant differences among subjects exhibiting the extreme cognitive styles.

2. Discussion of the Sex Differences Hypothesis.

Although there was some indication that boys perform better than girls on mathematical achievement tests, this study failed to confirm such a finding. An inspection of the results of this study shows that boys tend to score higher than girls on the average, but the differences were not large enough to be significant.

There are several reasons why significant differences, if they should exist, did not occur in this study. First of all in the review of the literature evidence was cited that field independent students were more likely to be found in analytical areas such as mathematics. In the Ontario school system mathematics is not a compulsory subject. Therefore by their third year, a large number of students may have dropped the study of mathematics. These students who are no longer enrolled in mathematics may be mainly field dependent as compared to those involved in this study. Thus the students classified as field dependent may have been relatively higher on the field-dependence-independence continuum than many students in the population consisting of all students registered in the third year of secondary school.

Since girls have been found to be more field dependent than boys in many studies, girls were probably more likely to drop mathematics in high school. **This** is suggested by the fact that of the students available for this study, there were 62 boys and 45 girls. Generally, approximately equal numbers of each sex would be enrolled in high school.

Another possible reason for failure to observe sex differences is the relatively small number of subjects available. The small numbers involved in each cell make it very difficult to show statistically that sex differences in achievement are significant.

3. Limitations and Suggestions for Further Research.

There are several limitations in this study that indicate generalization of these results should be made with caution. For example, the mathematical achievement tests included multiple choice items exclusively. Although such tests are highly desirable due to their ease and accuracy of scoring and analysis, further experiments could employ tests consisting of more detailed problems. Many topics in mathematics require the solution of problems consisting of several steps and requiring a high degree of analytic ability.

In the review of the literature there is evidence that the Rod-and-Frame Test may measure slightly different properties than an embedded figures test. Therefore it would be desirable to employ both types of tests as a measure of field independence to obtain more definite results. Perhaps

the failure to find sex differences may be attributable to the exclusive use of the Concealed Figures Test.

To extend the knowledge of the relationship between a field independent cognitive style and mathematics, other mathematical subjects might be examined. Trigonometry, other geometries, calculus, and vector analysis for example, might be examined to determine whether a field independent cognitive style is conducive to achievement in these areas.

4. Implications of the Study.

In addition to providing support for Witkin's theories of psychological differentiation, several other implications of the study might be observed.

The identification of a student's cognitive style can provide teachers with an important cue for relating more effectively teaching methods and material to individual differences. At least the knowledge that students differ in cognitive style should be brought to the attention of teachers so that when difficulties arise, corrective action might be undertaken.

Since the Concealed Figures Test is so easily administered, it might be used as screening device to aid in the formation of class groupings for regular or tutorial purposes. A different mode of presentation might then be employed to cope with the cognitive styles identified. Spitler

as cited in the review of the literature recommended that curriculum designers develop separate courses for field independent and field dependent subjects. Such an outcome would be highly desirable but undoubtedly quite difficult to achieve.

A final implication involves school counsellors. An emphasis on the causative factors of academic failure may require changes in counselling goals. An understanding of the causes of academic failure might lead counsellors to provide their subjects with information regarding methods of learning suited to their individual cognitive style.

A summary of the results and conclusions of this study will be found in the next section.

SUMMARY AND CONCLUSIONS

Witkin's theory of psychological differentiation has stimulated extensive research. A field independent cognitive style has been found to be an important factor for success in analytical tasks. The learning of mathematics appears to make frequent demands for a field independent cognitive style in more complex areas.

In this study, 107 students enrolled in third year secondary school mathematics were given the Concealed Figures Test as a measure of field-dependence-independence and achievement tests in algebra and plane geometry. Based on the results of the Concealed Figures Test, extreme groups were identified as field dependent or field independent and these groups were then separated with respect to sex.

As a result of a multivariate analysis of variance, support was found for the hypothesis that subjects who are field independent would score significantly higher on tests of mathematical achievement than subjects who are field dependent. An hypothesis that boys would perform better than girls on the tests of mathematical achievement was not supported.

Post hoc procedures showed that for the first hypothesis both algebra and geometry contributed significantly to the difference found between the two extreme groups.

Knowledge of these results might be beneficial to teachers of

mathematics and school counsellors. A knowledge of a student's cognitive style might be used as a guide for class groupings, teaching styles and materials. Counsellors might provide information regarding methods of learning suited to an individual's cognitive style.

To extend and strengthen Witkin's theory of psychological differentiation, researchers might employ other measures of field-dependence-independence and examine other areas of mathematics for possible relationships.

BIBLIOGRAPHY

Barakat, M. K., "A Factorial Study of Mathematical Abilities", in British Journal of Psychology Statistical Section, Vol. 4, issue of November 1951, p. 137-156.

The author summarizes studies relating to ability in mathematics and personality characteristics of high achievers in mathematics. By means of a factor analysis, strong evidence of a mathematical factor independent of general intelligence was found. A distinction was found between mechanical arithmetic and mathematics involving manipulation of schemes and relations. Boys were more successful than girls.

Burt, C., "The Structure of the Mind: A Review of the Results of Factor Analysis", in British Journal of Educational Psychology, Vol. 19, 1949, p. 176-199.

Support was found for Barakat's findings of a distinction between an arithmetic factor relating to memory and a mathematical factor involving the ability to perceive, attend to and combine numerical relations.

Elliot, Rogers, "Interrelationships Among Measures of Field Dependence, Ability, and Personality Traits", in the Journal of Abnormal and Social Psychology, Vol. 63, No. 1, 1961, p. 27-36.

Significant intercorrelations exist between Witkin's measures of psychological differentiation and Thurstone's Concealed Figures Test.

Perney, L. R., The Relationship of Field Dependence Field Independence With Academic Achievement, unpublished doctoral dissertation, Case Western Reserve University, 1971, VIII-61 p.

As part of this study, it was found that a greater functional relationship between a field independent cognitive style and achievement existed when the cognitive activities increased in complexity.

Spitler, Gail, J., An Investigation of Various Cognitive Styles and the Implications for Mathematics Education, unpublished doctoral dissertation, Wayne State University, 1970, V-221 p.

A review of related research and an examination of specific mathematical topics enabled the author to conclude that a field independent cognitive style was important for mathematics education. It was recommended that special courses catering to cognitive styles be developed.

Witkin, Herman A., et al., Personality Through Perception, New York, Harper, 1954, XXVI-571 p.

Witkin presents results of the extensive research from the origins of the field-dependence-independence construct to its relationship to personality characteristics and patterns of adaptation.

Witkin, H. A. et al., Psychological Differentiation, New York, Wiley, 1962, XII-418 p.

In this second major publication, the authors bring their research up to date and introduce the psychological differentiation hypothesis as a framework for their findings.

Witkin, H. A., The Role of Cognitive Style in Academic Performance and in Teacher-Student Relations, Princeton, Educational Testing Service, 1973, 58 p.

Witkin discusses applications of the concepts, techniques and findings from cognitive style research to a wide range of educational problems.

Young, Harl H., "A Test of Witkin's Field Dependence Hypothesis", in Journal of Abnormal and Social Psychology, Vol. 59, 1959, p. 188-192.

Due to criticisms of Witkin's findings with respect to validity and reliability, Young retested Witkin's hypotheses and provided experimental support.

Zigler, Edward, "A Measure in Search of a Theory?", in Contemporary Psychology, Vol. 8, No. 4, 1963, p. 133-135.

In this review of Witkin's 1962 text, the author questioned the reliability and validity of many conclusions and suggested that field independence was an extension of general intelligence.

APPENDIX 1

Scores for Concealed Figures, Achievement in
Algebra and Achievement in Geometry Tests

Lowest Group on Psychological Differentiation

Sex	Concealed Figures	Algebra	Geometry	Age
M	12	10	14	17.1
F	15	12	11	17.5
M	16	23	17	16.2
F	29	12	9	16.8
M	32	17	15	15.8
F	38	13	14	16.3
F	39	18	10	16.0
M	40	17	14	15.6
M	46	21	13	16.2
F	46	8	7	17.1
F	50	10	7	16.3
M	50	7	4	16.5
M	52	13	14	16.8
M	53	19	12	15.7
F	57	7	13	16.6
F	57	9	12	15.7
M	58	19	14	16.3
F	58	11	12	16.8
M	59	19	14	16.7
F	60	10	8	19.1
F	60	12	10	16.8
F	61	16	14	16.9
M	61	9	12	17.0
F	62	17	17	16.9
M	62	13	10	16.0
M	62	19	17	16.8
M	62	20	16	15.1
M	63	21	19	15.8
F	63	20	16	16.1
M	63	10	11	16.4

APPENDIX 1 (continued)

Middle Group on Psychological Differentiation

Sex	Concealed Figures	Algebra	Geometry	Age
M	64	15	9	16.3
F	64	15	10	16.0
M	64	9	8	16.6
M	64	9	12	16.8
M	64	9	13	16.4
F	65	18	13	16.2
M	65	20	16	17.1
F	65	16	12	16.2
F	65	22	13	16.8
M	67	13	18	17.2
M	68	20	13	16.9
M	68	7	13	16.3
M	68	24	18	17.4
F	68	9	13	16.7
M	70	22	14	15.8
F	72	19	18	16.5
M	72	17	11	16.5
F	72	14	9	16.3
M	72	13	16	16.2
M	74	15	16	16.6
M	74	9	12	17.3
F	74	7	11	16.3
M	76	10	12	18.8
M	76	9	12	16.9
F	76	21	17	16.9
F	78	11	11	16.8
F	78	15	13	17.6
M	78	18	16	16.3
M	78	13	12	15.9
F	79	11	9	16.5

APPENDIX 1 (continued)

Highest Group on Psychological Differentiation

Sex	Concealed Figures	Algebra	Geometry	Age
M	80	19	13	16.2
M	80	16	16	17.0
F	80	21	17	16.8
F	81	22	14	16.7
M	82	11	11	16.6
M	82	18	13	17.8
F	82	15	14	15.9
M	82	25	18	15.6
M	83	21	16	16.3
M	85	18	13	17.8
F	85	14	12	16.4
F	86	23	15	16.5
M	88	17	15	15.4
M	89	20	17	16.3
F	89	16	16	16.5
F	91	16	15	16.2
F	92	16	12	16.3
M	92	21	16	16.8
F	92	15	19	15.0
M	94	16	16	16.5
F	97	17	19	16.3
M	99	13	8	16.5
F	102	13	16	16.9
M	113	23	18	16.3
M	114	18	14	16.3
M	115	15	16	15.8
F	119	12	11	16.0
M	120	18	15	16.2
F	123	22	14	16.6
M	128	21	19	16.1

APPENDIX 2

Geometry Item Analysis

N = 103

Item	Response					Omit	Above ¹ Median	Below ² Median	Item ³ Difficulty	Discrimination ⁴ Index
	A	B	C	D	E					
1	1	63*	0	5	34	0	41	22	.61	.37
2	16	5	49*	24	5	4	34	15	.48	.37
3	3	15	5	8	71*	1	42	29	.69	.25
4	3	6	20	63*	8	3	43	20	.61	.45
5	10	83*	4	3	3	0	48	35	.81	.25
6	9	61*	11	13	9	0	40	21	.59	.37
7	2	1	1	91*	7	1	50	41	.88	.18
8	73*	6	5	2	15	2	45	28	.71	.33
9	13	2	0	3	85*	0	47	38	.83	.18
10	10	1	4	86*	2	0	49	37	.83	.24
11	3	4	95*	1	0	0	51	44	.92	.14
12	1	76*	3	12	5	6	46	30	.74	.31
13	5	8	10	3	77*	0	45	32	.46	.25
14	47*	21	10	14	8	3	34	13	.66	.41
15	8	5	68*	2	20	0	39	29	.75	.20
16	2	7	13	77*	1	3	47	30	.46	.33
17	47*	6	7	25	14	4	30	17	.53	.25
18	13	55*	6	28	1	0	38	17	.72	.41
19	14	8	74*	4	2	1	42	32	.50	.20
20	32	10	2	6	52*	1	39	13	.75	.51

* = correct response

1 Above Median = Number of students above the median who answered item correctly.

2 Below Median = Number of students below the median who answered item correctly.

3 Item Difficulty = Number of students answering item correctly ÷ number of students.

4 Discrimination Index = Difference in proportions of students above and below the median who answered the item correctly

For example: Item 1: Discrimination Index = $\frac{41}{51} - \frac{22}{51} = .37$

APPENDIX 3

Algebra Item Analysis

N = 103

Item	A	Response				Omit	Above Median	Below Median	Item Difficulty	Discrimination Index
		B	C	D	E					
1	2	90*	5	1	3	2	51	39	.87	.24
2	12	77*	5	5	3	1	48	29	.75	.37
3	10	62*	17	12	1	1	39	23	.60	.31
4	1	0	21	6	75*	0	45	30	.73	.29
5	6	7	10	77*	3	0	48	29	.75	.37
6	10	9	3	45*	31	5	31	14	.44	.33
7	3	7	75*	10	8	0	46	29	.73	.33
8	18	65*	3	0	15	2	47	18	.63	.57
9	4	13	19	10	44*	13	35	9	.43	.51
10	5	27	15	51*	4	1	33	18	.50	.29
11	1	9	10	79*	3	1	51	28	.77	.45
12	12	20	8	46*	10	7	35	11	.45	.47
13	8	36*	22	33	3	1	28	8	.35	.39
14	11	79*	8	2	2	1	47	32	.77	.29
15	6	9	5	30	49*	4	33	16	.48	.33
16	15	71*	4	8	5	0	43	28	.69	.29
17	3	4	4	10	80*	2	45	35	.78	.20
18	66*	25	2	5	2	3	42	24	.64	.35
19	5	20	48*	6	21	3	35	13	.47	.43
20	14	5	81*	2	0	1	47	34	.79	.25
21	65*	5	11	9	8	5	42	23	.63	.37
22	10	6	31*	40	8	8	23	8	.30	.29
23	8	23	52*	4	6	9	38	14	.50	.47
24	4	5	4	65*	20	5	46	19	.63	.53
25	67*	10	5	11	3	7	47	20	.65	.53

* = correct response

APPENDIX 4

Information for Subjects Regarding the Research Project

Having been introduced to the class by the regular classroom teacher, the experimenter provided the students with the following information:

"As most of you know, this year I am back at school to find out what it is like to be a student again. I am doing graduate work at the University of Ottawa. As part of my requirements, I am conducting a research project to attempt to learn why some students are more successful in mathematics than others. As you know, general intelligence is not the only reason since some of you who find mathematics difficult might do very well in other subjects. Recently research has shown that the extent of psychological differentiation might be related to achievement in mathematics. This term, psychological differentiation, is related to the way in which a person sees an object in relation to its surroundings. Research has shown that individuals vary widely in their ability to see these objects.

Today I am going to give you a short ten minute test called the Concealed Figures Test. In the next few weeks you will be given tests in algebra and geometry. I want to emphasize that the results of these tests are completely confidential. No one else will be given these results and therefore they will not be used in determining your term mark for mathematics. You are not expected to do any studying for these tests

since they are very general in nature. When I return in September, if you are interested in knowing how well you did, you may see me for your results. If you do not wish to participate in this research just leave your answer sheets blank. Are there any questions?"

Several students wanted further information regarding psychological differentiation and the content of the achievement tests. After the questions were answered, the Concealed Figures Test was administered according to the instructions provided in the manual.

APPENDIX 5

The Closure Flexibility Test
(Concealed Figures Test)

CLOSURE FLEXIBILITY

(Concealed Figures)

(Form A)

Please fill in:

Name _____

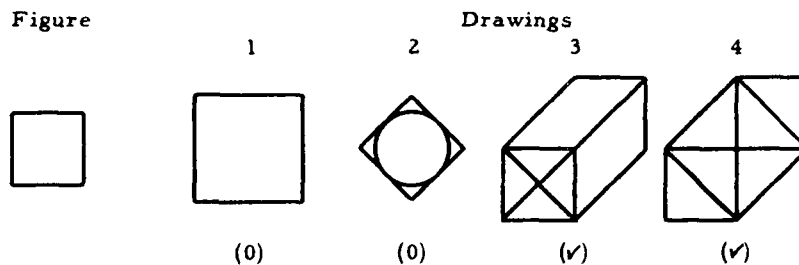
Age _____ Sex _____ Date _____

Occupation _____

Developed by: L.L. Thurstone, Ph.D. and T.E. Jeffrey, Ph.D. - The Psychometric Laboratory - The University of North Carolina

Directions:

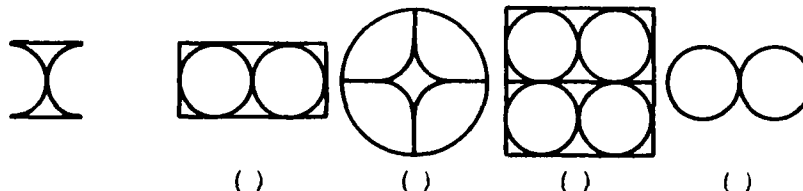
The row of designs below is a sample item of this test. The parts have been labeled to make description easier. These labels do not appear in the test items. The left hand design in each row is the figure. You are to decide whether or not the figure is concealed in each of the four drawings to the right. Put a check mark (✓) in the parentheses under a drawing, if it contains the figure. Put a zero (0) in the parentheses under a drawing, if it does not contain the figure. Look at the row of designs below.



In the row above a zero (0) has been written in the parentheses under drawing 1. The first drawing is a square but it is larger than the figure. A zero (0) has been written under drawing 2. Although the second drawing contains a square of exactly the same size as the figure, it has been turned. Check marks (✓) have been written under the third and fourth drawings since they each contain a square of exactly the same size as the figure and have not been turned. It does not matter that the figure contained in drawings three and four is on a different level from the figure at the left.

Sample:

Here is another example for practice. Try it.



You should have placed check marks (✓) in the parentheses under the first and third drawings and zeros (0) in the parentheses under the second and fourth drawings.

WHEN YOU GET THE SIGNAL TO BEGIN, turn the page and mark more problems of the same kind. Work as fast and as accurately as you can, but do not guess. Wrong answers will count against you. You are not expected to finish in the time allowed. You will have exactly ten minutes to do as much as you can.

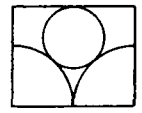
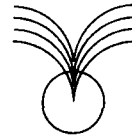
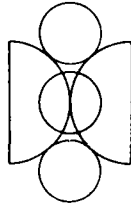
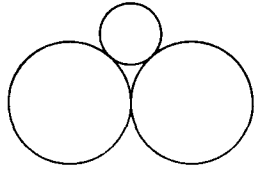
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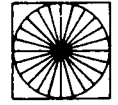
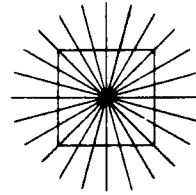
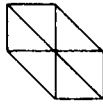
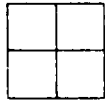
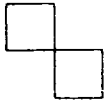


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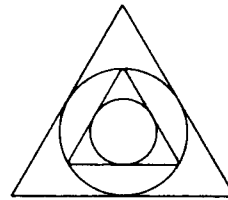
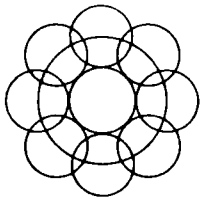
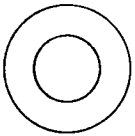


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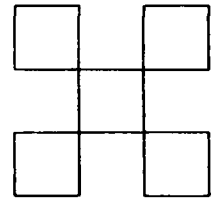
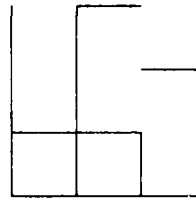
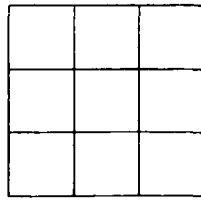
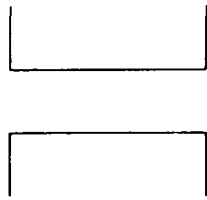
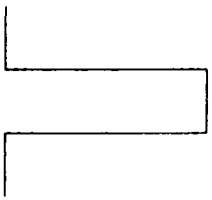


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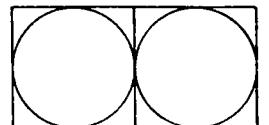
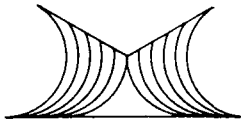
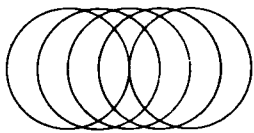


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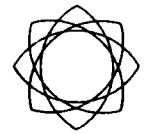
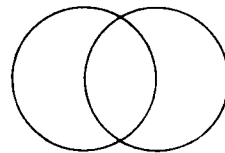
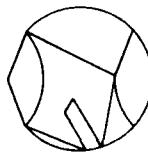
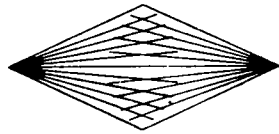
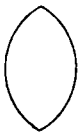


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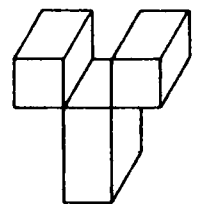
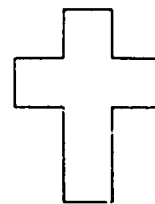
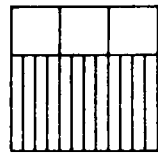
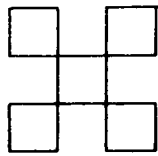
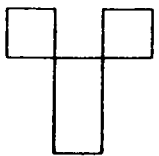


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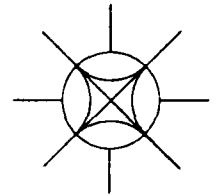
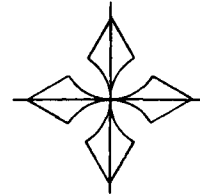
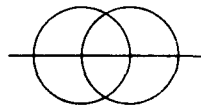
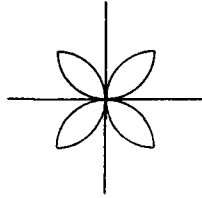
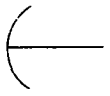
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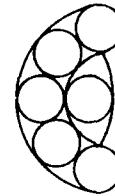
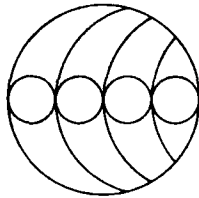


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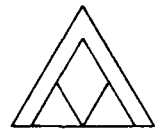
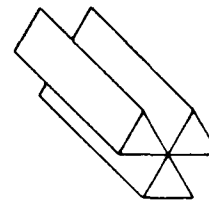
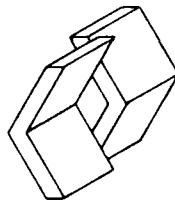
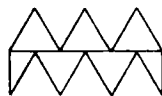


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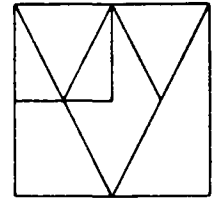
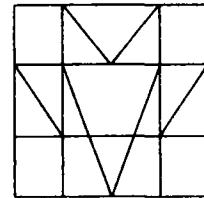
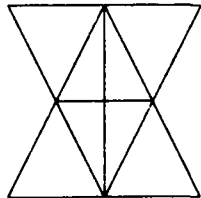
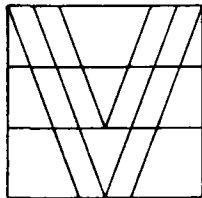
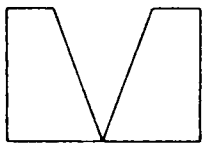


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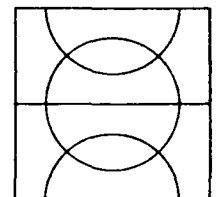
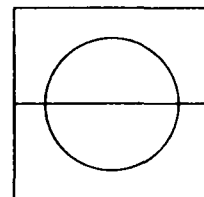
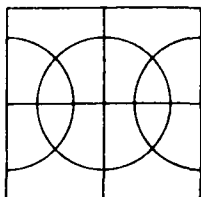
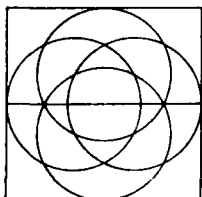
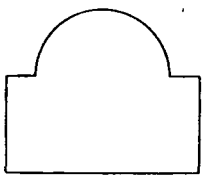


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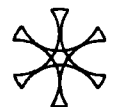
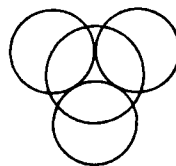
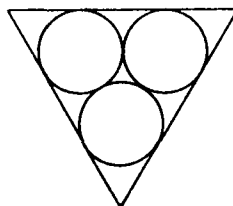


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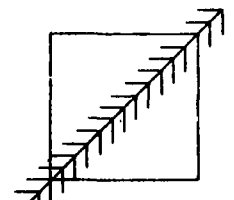
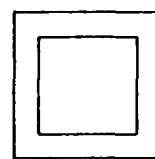
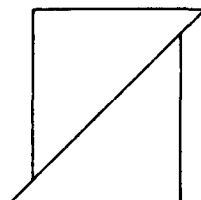
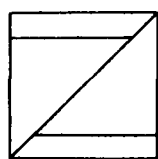
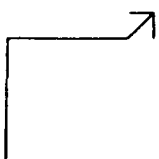


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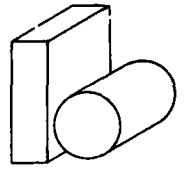
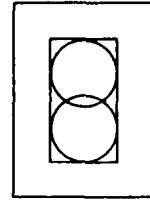
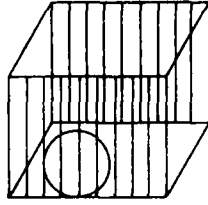
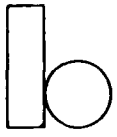
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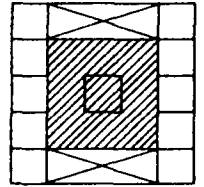
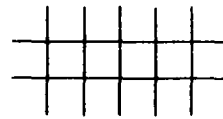
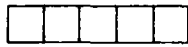
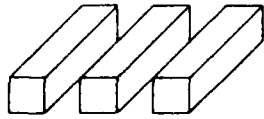


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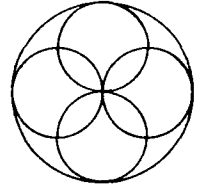
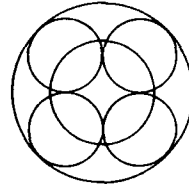
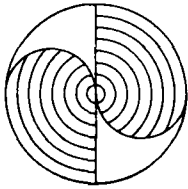
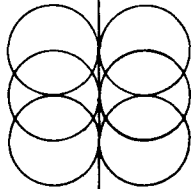
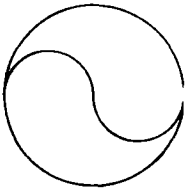


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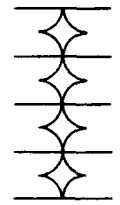
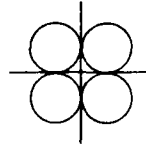
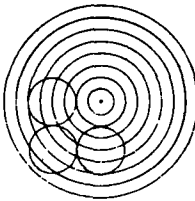
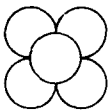


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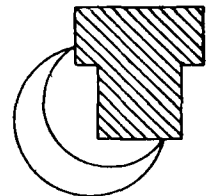
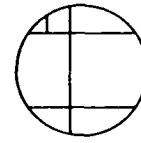
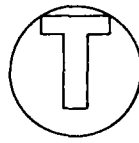
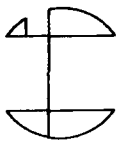
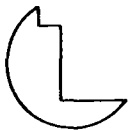


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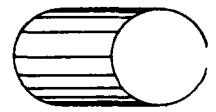
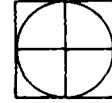
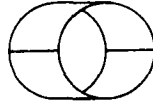
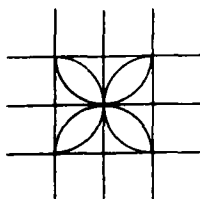
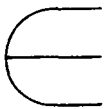


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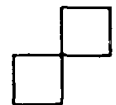
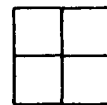
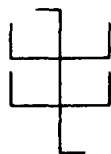
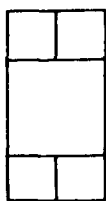


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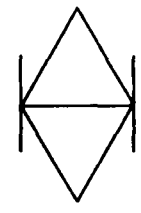
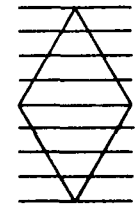
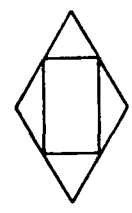
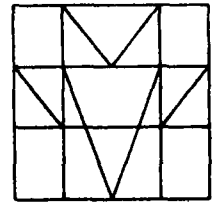
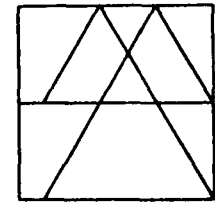
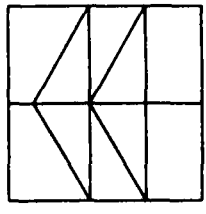
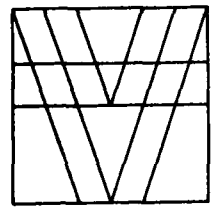
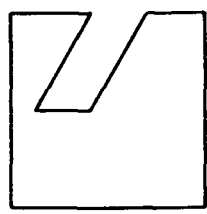
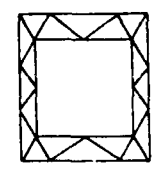
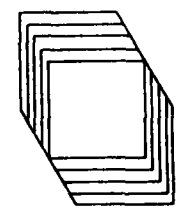
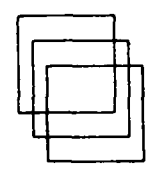
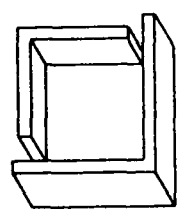
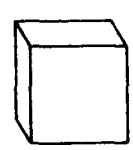
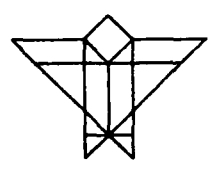
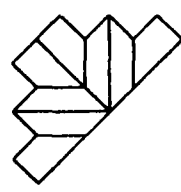
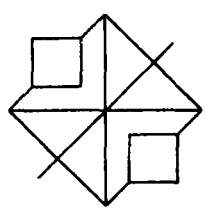
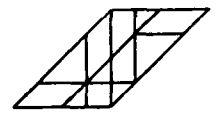
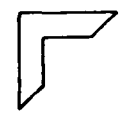
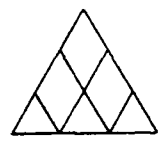
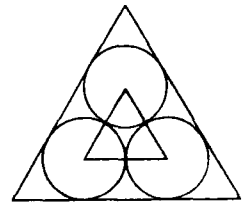
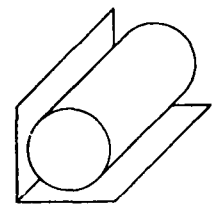
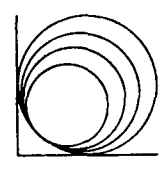
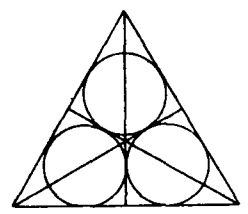
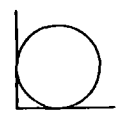
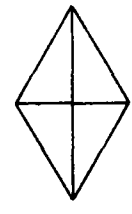
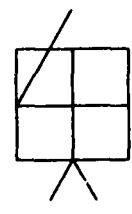
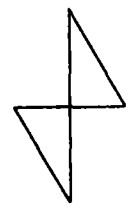
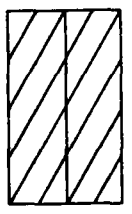


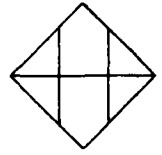
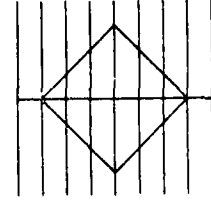
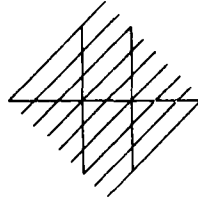
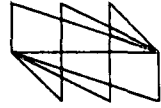
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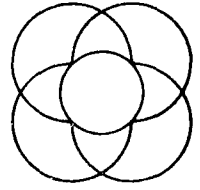
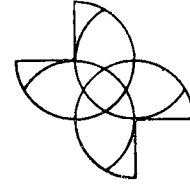
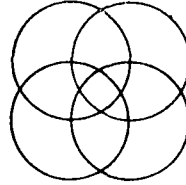
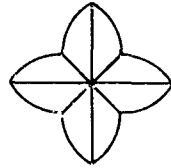
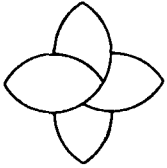


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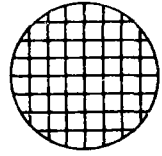
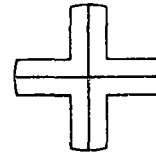
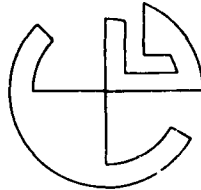
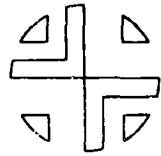


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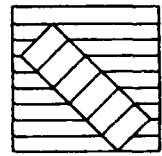
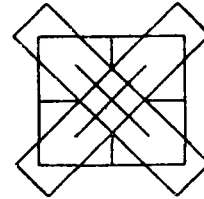
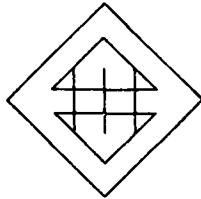
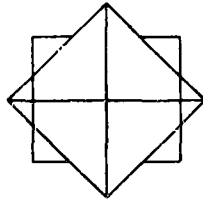
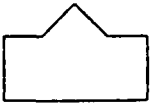


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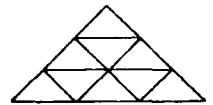
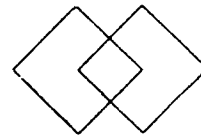
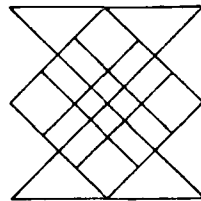
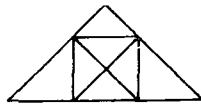


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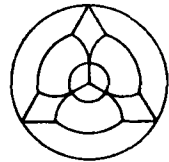
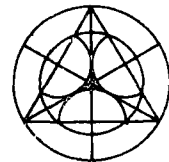
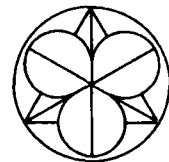
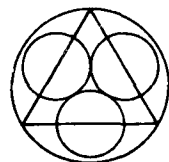
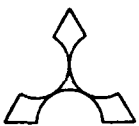


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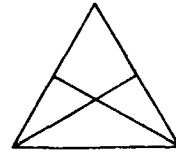
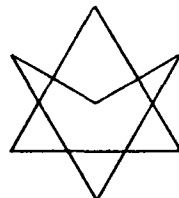
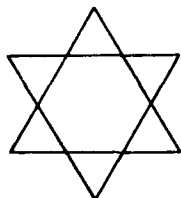
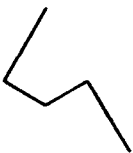


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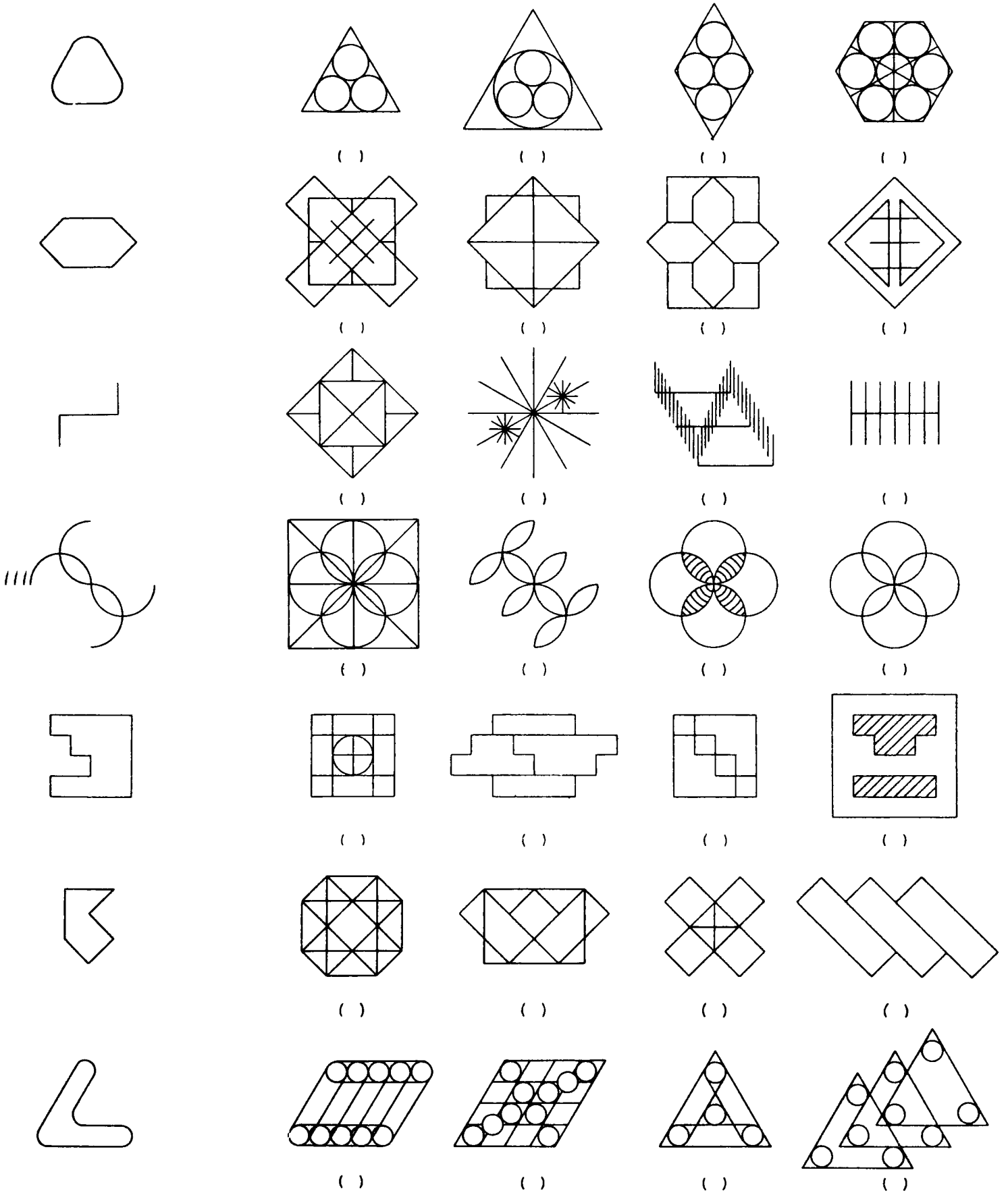
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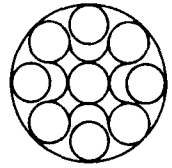
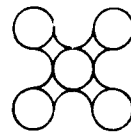
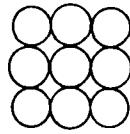
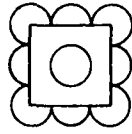
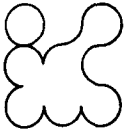
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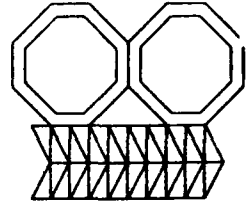
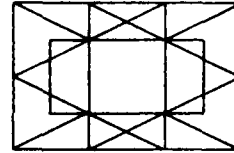
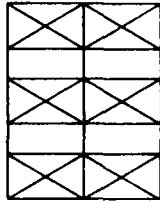
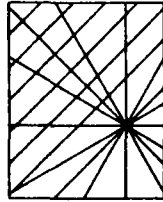


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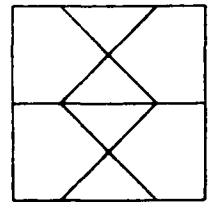
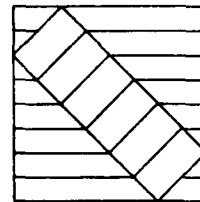
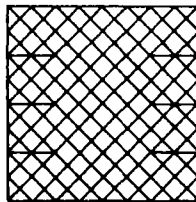
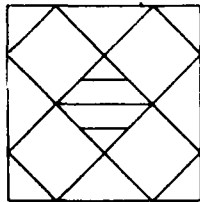
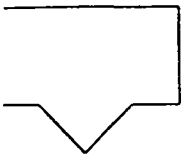


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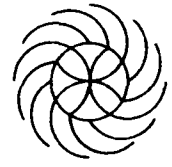
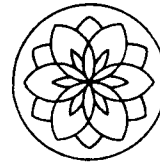
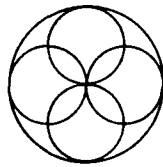


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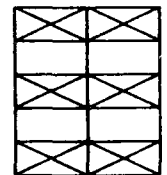
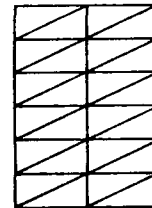
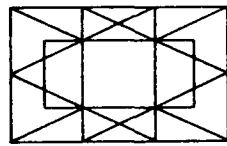
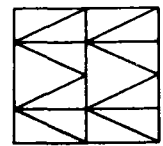
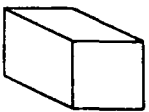


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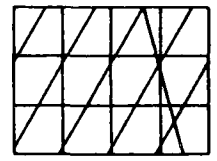
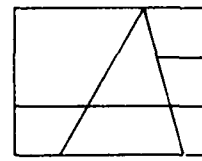
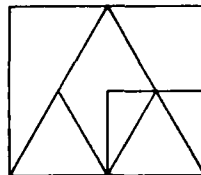
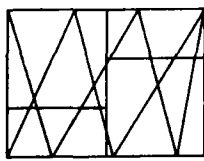
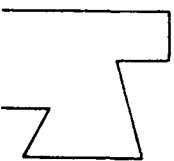


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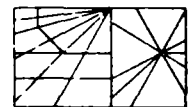
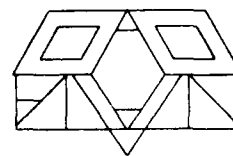
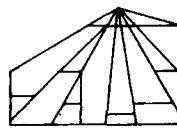
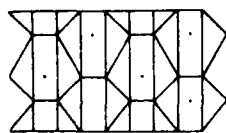


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APPENDIX 6

Geometry Test

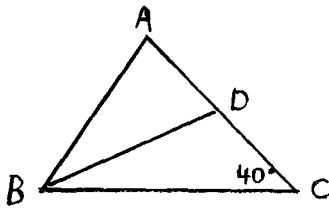
GEOMETRY TEST

Instructions:

1. On the answer sheet provided, record your answer by drawing an X over the appropriate letter.
e.g. 21. A B ~~X~~ D E
2. If you change your mind about an answer, circle the original answer and then draw an X through your revised answer.
e.g. 21. A B (X) ~~X~~ E
3. Note that there is only one correct answer per question.
4. There is no penalty for guessing.
5. The diagrams are representative only. They are not drawn to scale, and therefore not intended to be measured.
6. Do any rough work on scrap paper.
7. Do not spend too much time on any one question. You can go back and try questions you missed if you complete the tests in the time allowed.
8. You will be allowed exactly 40 minutes for this test.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO.

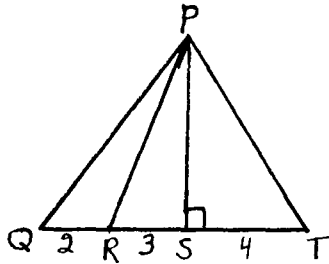
1.



In $\triangle ABC$, $AB=AC$, $\angle C=40^\circ$, and BD bisects angle ABC . The size of angle DBC is:

- (A) 10° (B) 20° (C) 25° (D) 40° (E) 50°

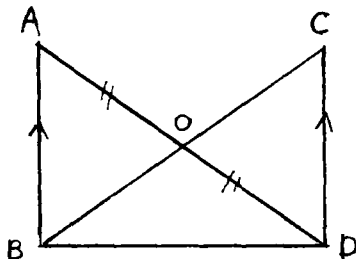
2.



In the given figure, $PS \perp QT$, $QR=2$, $RS=3$, and $ST=4$. Arrange PQ , PR , and PT in order of size beginning with the shortest.

- (A) PQ, PT, PR (B) PT, PQ, PR (C) PR, PT, PQ
(D) PR, PQ, PT (E) PT, PR, PQ

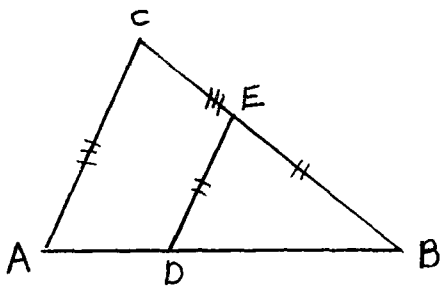
3.



In the diagram, $AB \parallel CD$ and $AO=OD$. It follows that:

- (A) $\angle A = \angle C$ (B) $\angle B = \angle D$
(C) $AO=OC$ (D) $BO=OD$
(E) $BO=OC$

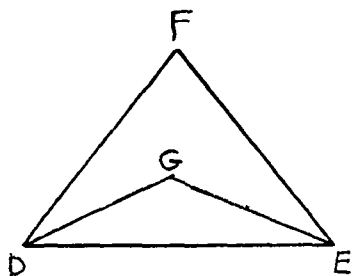
4.



In the given figure, if $CA=CB$ and $ED=EB$, then which of the following conclusions can be made?

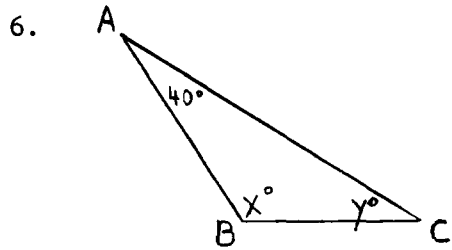
- (A) $\triangle ABC$ is equilateral
(B) $\triangle BDE$ is equilateral
(C) $AD=CE$
(D) $CA \parallel ED$
(E) $AD=DE$

5.



In the diagram, the bisectors of angles EDF and DEF intersect at G . If the number of degrees in $\angle F=60^\circ$, then the number of degrees in $\angle G$ is

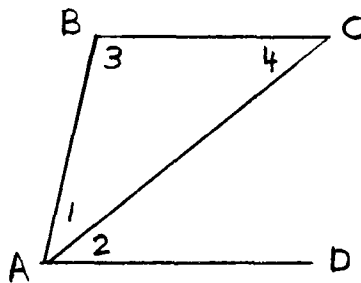
- (A) 30° (B) 120° (C) 135° (D) 150° (E) 90°



In $\triangle ABC$, $\angle A = 40^\circ$. If $60^\circ \leq y^\circ \leq 100^\circ$ then

- (A) $0^\circ < x^\circ < 60^\circ$ (B) $40^\circ \leq x^\circ \leq 80^\circ$
 (C) $60^\circ < x^\circ < 100^\circ$ (D) $40^\circ < x^\circ < 80^\circ$
 (E) $80^\circ < x^\circ < 120^\circ$

NOTE: Questions 7 and 8 refer to the following diagrams and statements.



Given: $\angle 1 = \angle 2$ and $BC \parallel AD$

Prove: $AB = BC$

Proof: 1) $BC \parallel AD$ given

2) \therefore

3) $\angle 1 = \angle 2$ given

4) $\therefore \angle 1 = \angle 4$ transitive

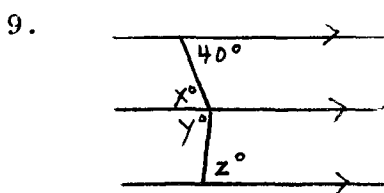
5) \therefore

7. The statement and authority for line 2 is

- (A) $\angle 1 = \angle 3$ corresponding angles (PLT)
 (B) $\angle 1 = \angle 4$ supplements of equal angles (SAT)
 (C) $\angle 1 = \angle 4$ isosceles triangle theorem (ITT)
 (D) $\angle 2 = \angle 4$ alternate angles (PLT)
 (E) none of these

8. The statement and authority for line 5 is

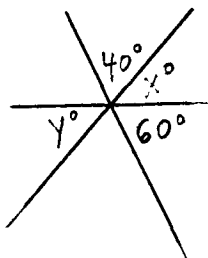
- (A) $AB = BC$ isosceles triangle theorem (ITT)
 (B) $AB \parallel CD$ equal alternate angles (PLT)
 (C) $AB = BC$ congruent triangles (SAS)
 (D) $AB \parallel BC$ corresponding angles (PLT)
 (E) none of these.



In the given diagram, lines are parallel as marked. What is the value of Z ?

- (A) 40° (B) 45° (C) 50° (D) 60° (E) cannot be determined.

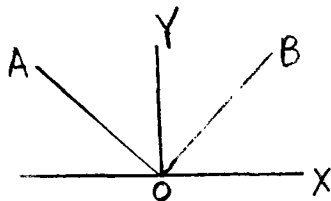
10.



The diagram shows three lines intersecting at one point. Calculate the value of $X^\circ + Y^\circ$.

- (A) 80° (B) 90° (C) 120° (D) 160°
 (E) cannot be determined

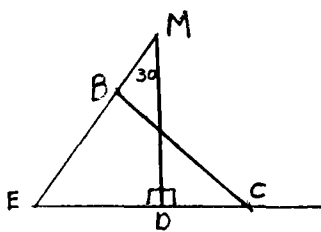
11.



Given $OX \perp OY$, $OA \perp OB$, and OB bisects $\angle YOX$. The degree measure of $\angle AOX$ is

- (A) 120° (B) 130° (C) 135° (D) 45° (E) 90°

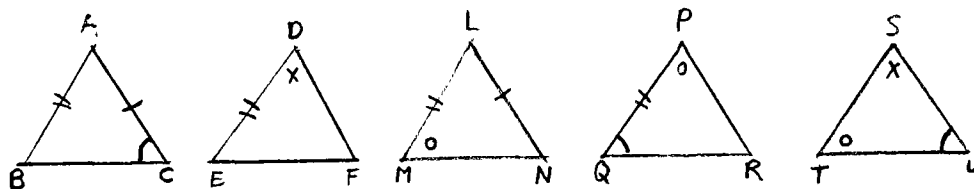
12.



In the diagram, $MD \perp EC$, and $\angle M = 30^\circ$. Also $BE = BC$. The degree measure of $\angle MBC$ is

- (A) 140° (B) 120° (C) 110° (D) 60° (E) 100°

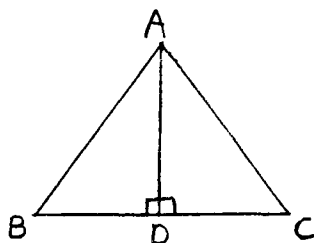
13.



Which of the following statements is correct for the diagrams shown above? (Angles and sides marked similarly are equal)

- (A) $\triangle ABC \cong \triangle LMN$ (B) $\triangle DEF \cong \triangle STU$
 (C) $\triangle DEF \cong \triangle PQR$ (D) $\triangle PQR \cong \triangle STU$
 (E) No two triangles are congruent.

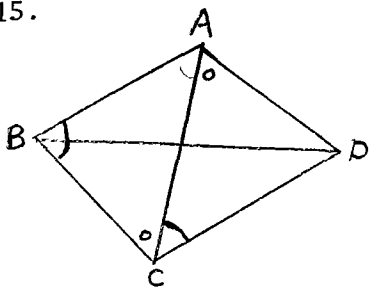
14.



In the diagram, $\triangle ABC$ is equilateral, with each side 2 units in length. The length of the altitude AD is

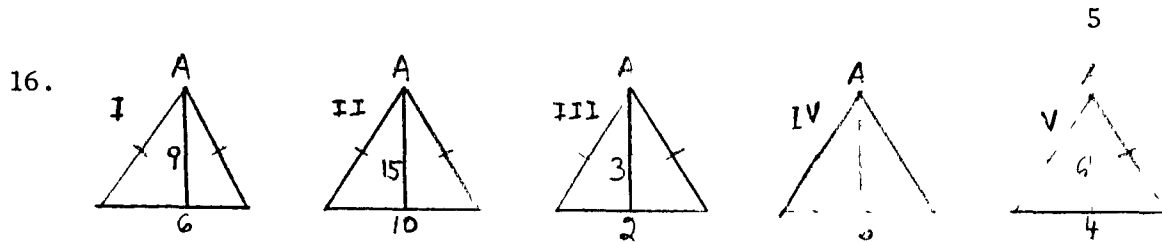
- (A) $\sqrt{3}$ (B) $\sqrt{5}$ (C) 3 (D) 2 (E) 1.9

15.

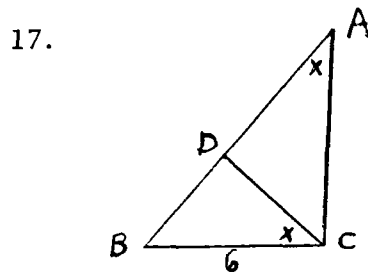


In the given diagram, $AC = BC$, $\angle DAC = \angle ACB$, $\angle ABC = \angle ACD$. Therefore $\triangle ABC \cong \triangle ADC$ for the following reason:

- (A) AAA (B) SAS (C) ASA (D) SSS (E) ASA and PLT

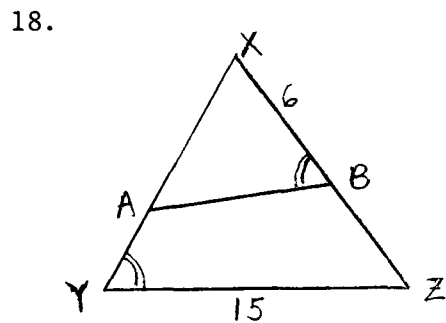


In each of the isosceles triangles above, the base and the altitude are given. The vertex angles A in all triangles are equal except in
 (A) I (B) II (C) III (D) IV (E) V



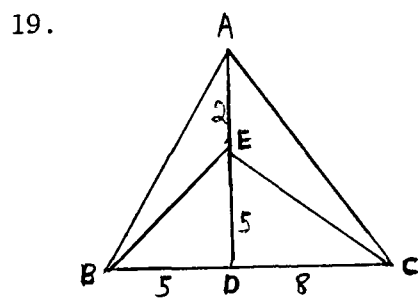
In the given figure, $\angle A = \angle BCD$, and BC has a length of 6 units. The ratio of the area of $\triangle ABC$ to $\triangle DBC$ is 9:4. The length of BD is

- (A) 4 (B) 9 (C) 8 (D) $2\frac{2}{3}$ (E) $13\frac{1}{2}$



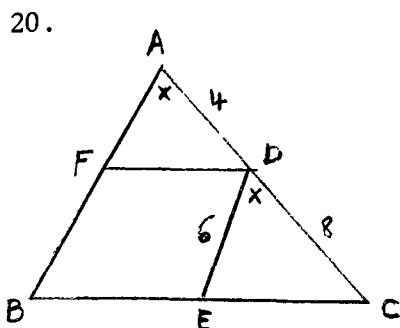
In the diagram, $\triangle XAB$ is similar to $\triangle XYZ$. Which of the following proportions is correct?

- (A) $\frac{XB}{XY} = \frac{YZ}{AB}$ (B) $\frac{XB}{XY} = \frac{AX}{XZ}$
 (C) $\frac{AB}{XY} = \frac{XB}{XZ}$ (D) $\frac{XA}{AY} = \frac{XB}{BZ}$ (E) $\frac{XA}{XB} = \frac{YZ}{XY}$



In $\triangle ABC$, D is a point in BC and E is a point in AD such that AE = 2 units, ED = 5 units, BD = 5 units, and DC = 8 units. The ratio of the area of $\triangle BED$ to $\triangle BEA$ is

- (A) $\frac{2}{5}$ (B) $\frac{25}{4}$ (C) $\frac{5}{2}$ (D) $\frac{8}{5}$ (E) $\frac{4}{25}$



In $\triangle ABC$, $\angle EDC = \angle A$. If AD = 4 units, DC = 8 units, and DE = 6 units, the length of AB is

- (A) 12 (B) 10 (C) 4 (D) 16 (E) 9

APPENDIX 7

Algebra Test

ALGEBRA TEST

Instructions:

1. This is a multiple choice test with five possible answers for each question. There is only one correct answer for each question.
2. Do any necessary calculations or rough work on a separate sheet of paper.
3. Record your answers on the answer sheet provided, by drawing an X through the correct letter of the answer.

e.g. A ~~B~~ C D E

In this example, B is the correct answer.

4. After choosing an answer, if you change your mind, circle the incorrect choice and then place an X over the new answer.

e.g. A ~~(B)~~ C ~~D~~ E

In this example, D would be marked as the correct answer, since B is circled.

5. Do not spend too much time on any one question. You can go back and try questions you missed if you complete the test in the time allowed.
6. You are allowed exactly 35 minutes for this test.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO.

1. $\frac{5a^2b^2}{4} \div \frac{10b^2}{3a^2}$ is equal to ($a \neq 0, b \neq 0$)
- (A) $\frac{3b^4}{8}$ (B) $\frac{3a^4}{8}$ (C) $\frac{25b^4}{6}$ (D) $\frac{8}{3a^4}$ (E) $\frac{3}{8a^2b^2}$
2. If $y = \frac{1}{x}$ and x is greater than zero, which of the following statements is true?
- (A) as x increases, y increases (B) as x increases, y decreases
 (C) as x decreases, y decreases (D) when x is greater than 1, y is greater than one
 (E) when x is less than 1, y is less than 1.
3. An equivalent expression for $x > -1$ is:
- (A) $-x < -1$ (B) $-x < +1$ (C) $-x > +1$ (D) $-x > -1$ (E) $x < -1$
4. When expanded, $(2m-n)^2$ is equal to:
- (A) $4m^2 - n^2$ (B) $2m^2 - n^2$ (C) $4m^2 - 4mn - n^2$ (D) $4m^2 + n^2$
 (E) $4m^2 - 4mn + n^2$
5. When $3x^2 - 4x + 2$ is subtracted from zero, the result is:
- (A) 0 (B) $-3x^2 - 4x + 2$ (C) $3x^2 - 4x + 2$ (D) $-3x^2 + 4x - 2$
 (E) $3x^2 + 4x + 2$
6. For what value(s) of y is $\frac{y-3}{9-y^2}$ undefined?
- (A) $Y=0$ (B) $y=9$ or -9 (C) $y=9$ only (D) $y=+3$ or -3
 (E) $y=3$ only
7. After simplifying, $2\sqrt{x}(2\sqrt{x} - 5\sqrt{xy})$ is equal to
- (A) $2x - 10x\sqrt{y}$ (B) $2x - 10\sqrt{x^2y}$ (C) $4x - 10x\sqrt{y}$
 (D) $4x^2 - 10\sqrt{x^2y}$ (E) $4x^2 - 10x\sqrt{y}$

8. If $2x^2 - bx^2 - 2 = 0$ when $x = -2$, what is the value of b ?
- (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $3\frac{1}{2}$ (D) $-3\frac{1}{2}$ (E) cannot be determined
9. If $\frac{x}{3} - 1 = \frac{x}{5} + 2$, then x equals
- (A) 15 (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{15}{2}$ (E) $\frac{45}{2}$
10. The statement, "A certain number X exceeds twice another number n by 30" can be written as
- (A) $x + 2n = 30$ (B) $x > 2n + 30$ (C) $x = 2n - 30$ (D) $x - 2n = 30$ (E) $2x - n = 30$
11. If $x + y = 4$ and $x - y = 2$, then x equals
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 6
12. If $3R = \frac{2K}{\pi d}$, then d equals
- (A) $3R - \frac{2K}{\pi}$ (B) $\frac{6KR}{\pi}$ (C) $\frac{2K}{3R}$ (D) $\frac{2K}{3\pi R}$ (E) $2K - 3\pi R$
13. An automobile is moving at " a " miles per hour and an airplane is moving three times as fast. How many hours will the plane require for a flight of " b " miles?
- (A) $\frac{3b}{a}$ (B) $\frac{b}{3a}$ (C) $3ab$ (D) $\frac{3a}{b}$ (E) $b - 3a$
14. Given the equation $\frac{x+4}{3} = \frac{2x-5}{2}$. What is the first incorrect line in the solution below?
- (A) $6\left(\frac{x+4}{3}\right) = 6\left(\frac{2x-5}{2}\right)$ $2x+4 = 6x-5$
- (B) $2x+4 = 6x-5$
- (C) $-4x = 9$ $x = -\frac{9}{4}$
- (D) $x = \frac{9}{4}$
- (E) There are no errors in the solution.

15. Given $y = A+2B-3C$. If $A=3C$ and $B=2A$, express y in terms of C .
(A) $y=2C$ (B) $y=3C$ (C) $y=4C$ (D) $y=6C$ (E) $y=12C$
16. $\frac{5m-2}{15} - \frac{3m+5}{15}$ is equal to
(A) $\frac{2m+3}{15}$ (B) $\frac{2m-7}{15}$ (C) $2m-3$ (D) $2m-7$ (E) $2m+3$
17. When $2x^2-3x+5$ is divided by $x-3$, the remainder is
(A) 4 (B) 32 (C) -22 (D) -4 (E) 14
18. $3(2x+1)-2(x+3)(2x-1)$ is equal to
(A) $-4x^2-4x+9$ (B) $-4x^2+16x-3$ (C) $-8x^2-14x+15$
(D) $-4x^2-4x-9$ (E) $-4x^2+11x-3$
19. If $\sqrt{x^2+5} - 3=0$, then the value(s) of x is (are)
(A) $x=4$ or $x=-4$ (B) $x=2$ only (C) $x=2$ or $x=-2$
(D) $x=\sqrt{14}$ or $x=-\sqrt{14}$ (E) there are no real roots
20. A boy has q quarters and d dimes. He buys p pencils at 5 cents each. How many cents does he have left?
(A) $q+d-5p$ (B) $25q+10d-p$ (C) $25q+10d-5p$
(D) $q+d-p$ (E) $\frac{1}{4}q+10d-5p$
21. If a varies directly as b , and $b=3$ when $a=5$, the value of b when $a=9$ is
(A) $\frac{27}{5}$ (B) $\frac{45}{3}$ (C) 7 (D) $\frac{15}{9}$ (E) 15

22. $\frac{x^3+x^2}{a-b} \times \frac{a^2-b^2}{x+1}$ equals ($a \neq b, x \neq -1$)
- (A) $(x^3+x)(a-b)$ (B) $2x^2(a-b)$ (C) x^2a+x^2b
(D) $\frac{(x^3+x^2)(a-b)}{x+1}$ (E) $(x^2+x)(a+b)$
23. One factor of $x^3-3x^2-3x+10$ is
- (A) $x+2$ (B) x^2-x+5 (C) $x-2$ (D) $x-5$ (E) $x+1$
24. If $p = \frac{2}{x-3}$ and $q = \frac{2}{x+3}$, then $p+q$ equals
- (A) 0 (B) $\frac{2}{x}$ (C) $\frac{4}{x}$ (D) $\frac{4x}{(x-3)(x+3)}$ (E) $\frac{4}{(x-3)(x+3)}$
25. What is the quotient when $x^3-x^2-17x+20$ is divided by x^2+3x-5 ?
- (A) $x-4$ (B) $x-2$ (C) $x-1$ (D) $x+2$ (E) $x+4$

APPENDIX 8

ABSTRACT OF

Psychological Differentiation and Achievement in Mathematics

Based on research with respect to psychological differentiation and achievement in mathematics there is evidence that high achievement in more complex areas of mathematics is related to a field independent cognitive style. Also, males appear to be more successful than females on mathematical tasks.

A group of 107 secondary school students were given the Concealed Figures Test to identify field independent and field dependent extreme groups. Tests of achievement in algebra and plane geometry were also administered.

Results of a multivariate analysis of variance provided support for the hypothesis that field independent subjects would score significantly higher on tests of mathematical achievement than field dependent subjects. Post hoc procedures showed that both algebra and geometry contributed significantly to this difference between the groups. There was no significant difference between the sexes on mathematical achievement.

Teachers of mathematics and counsellors might find the results beneficial to aid in class groupings and the choice of teaching methods.