WITTGENSTEIN'S RECEPTION OF GÖDEL'S INCOMPLETENESS THEOREMS

LINA AZERKI

A thesis submitted to the University of Ottawa in partial fulfillment of the requirements for the Master's degree in Philosophy

Department of Philosophy
Faculty of Arts
University of Ottawa

Abstract

Wittgenstein's remarks on Gödel's (first) incompleteness theorem were initially dismissed by scholars who thought Wittgenstein misunderstood the syntactic nature of Gödel's proof. Following the release of the 1998 Bergen Electronic Edition of Wittgenstein's *Nachlass*, remarks on Gödel that were excluded in the two editions of the *Remarks on the Foundations of Mathematics* were brought to light, reviving an interest in the topic. As part of the same efforts of reinterpretation, this thesis delineates an alternative approach to Wittgenstein's reception of Gödel. Rather than simply comparing Wittgenstein's comments on Gödel to Gödel's seminal paper on Undecidability, the present work does not assume that "Gödel's theorem," as an object of philosophical investigation, is identical to "Gödel's theorem" as conceived within the discipline of mathematics. Hence, the object of Wittgenstein's discussions must be determined first. To do this, I give an account of late Wittgenstein's philosophical methods in general, and interpret his remarks on the aims of his discussions of the foundations of mathematics. Wittgenstein's philosophy is a dialectical exposition of our tendencies in thinking which avoids issuing any apodictic claims. He proposes a kind of philosophical investigation which describes rather than explains. Thus, his intention is not to advance competing theories or interpretations of mathematical theories, but to describe the use of words which appear in mathematics but originate in ordinary language. Given these strictures, a more charitable reading of Wittgenstein's reception of Gödel is available. For Wittgenstein, Gödel's theorem is a case study of our tendencies to imagine metaphysical theories, being misled by pictures associated with words that occur in the prose which accompanies the proof. Wittgenstein shows us how we are tempted to interpret words like "truth" and "proof" and thus to impose unwarranted expectations on Gödel's proof. This is further demonstrated by appealing to passages from the *Nachlass* (MS 163, 43r-47v) that are for the first time shown to concern Gödel's paper, passages in which Wittgenstein problematizes the notion of "interpreting content." As a result, this thesis shows that by a close study of Wittgenstein's method and stated intentions, the common conclusion that Wittgenstein misunderstood Gödel can be avoided.

Table of Contents

List of Abbreviations

C&V	*Culture and Value*
LFM	*Lectures on the Foundations of Mathematics, Cambridge 1939*
PI	*Philosophical Investigations*
RFM	*Remarks on the Foundations of Mathematics*
TLP	*Tractatus Logico-Philosophicus*
TS / MS	Typescript and Manuscript Items from *Wittgenstein's Nachlass Bergen Electronic Edition.*

1. Introduction

1.1. The Philosophical Problem of Dealing with Mathematics Philosophically (Or What Can the Philosopher Tell Us About Mathematics?)

> Our problems are totally different from those which a mathematician tackles. The difficulty is to get into the new dimension in which these problems can be solved.
>
> –Wittgenstein, **MS 161, 1r (1939)** [written in English].

How can a philosopher who is not a mathematician, or a philosopher *qua* philosopher say something about mathematics which is relevant to the mathematician? For how can two different disciplines talk about the same object when being 'different disciplines' by definition makes them consider the seemingly identical object from different points of view or a different set of properties? If a philosopher is addressing an issue in mathematics, are they addressing the mathematician if the philosopher is talking about something other than that which is relevant to the mathematician? Or is the *way* the philosopher speaks about mathematics, that is philosophical? In which case, how can the philosophical perspective enlighten the mathematician? Wittgenstein distinguishes the philosopher and mathematician as well as their disciplines in a multitude of his late remarks[1]. His reflections on mathematics were developing during a time when the foundation of mathematics was thought to be shaken, a time when philosophy and mathematics were deeply intertwined[2]. It is at the heart of his mature philosophy to argue that mathematics *cannot* solve philosophical questions. However, a multitude of scholars do not entirely take this into consideration when reading Wittgenstein's remarks on mathematics, for instance, when it comes to Wittgenstein's remarks on Gödel. Gödel's Incompleteness Theorems (GIT) are a key development during this critical period of mathematical history. It is the object of this monograph to present an interpretation of these remarks.

[1] For instance: MS157b, 30v,31r; MS 121, 77v; MS 123, 7; MS 161, 1r; MS 124, 115r; RFM.V.52, 53; RFM.VI.12,13; PI 125; LFM 111; MS117, 172.

[2] José Ferreirós, "The Crisis in the Foundations of Mathematics", 2008, 143.

What can we know of Wittgenstein's project in his remarks on GIT? By his project, I mean the *purpose* of the Gödelian remarks, as well as the *means* used to this end. The Gödelian remarks are a subset of his philosophy of the foundations of mathematics, which in turn is a subset of the philosophy of mathematics remarks, which are, finally, subsets of his philosophy in general. As such, metaphilosophical remarks on any of these levels are helpful in determining the purpose of his remarks on GIT. As we will see, several points made about GIT are also made about other theorems and proofs that share the same features. This provides further reason for the claim that the Gödelian remarks do not have a purpose that is separate from the larger project of his philosophy of (the foundations of) mathematics. I surmise that a reading that is strongly informed by his own conception of his project (and even from observing the way he tends to carry it out) would lead to a better understanding of Wittgenstein's Gödelian remarks, and that this account would be distinct from the usual "mathematical" one which many scholars default to[3]. That their approach differs in this significant way can be seen for instance in their claim that the Gödelian remarks are a specific topic Wittgenstein deals with, and it can be separated from the rest of his philosophy. I will argue the opposite[4]. As for the means to arrive at what his project is, it is less a matter of being attentive to his metaphilosophical remarks and more so a matter of observing his method at play. Taking into account both of these facets of his project supports the interpretation of each.

Seeing that they are not an isolated event in Wittgenstein's philosophy, the Gödelian remarks are thus worth revisiting for a few reasons. They provide a conveniently well-defined topic which acts as a case study or, otherwise put, a frame of focus from which to see his metaphilosophy and his approach at play. This strategy of approaching the remarks (taking the metaphilosophy as a guide) is itself that which the commentators neglect, thereby misreading Wittgenstein and making claims such as that he misunderstood or that he was criticizing Gödel. The misstep in the commentaries tends to manifest itself as the apparent expectation that Wittgenstein should deal with the mathematical moves in Gödel's

[3] *E.g.*, Timothy Bays' (2004), Victor Rodych's (1999), (2002), (2003) ; Graham Priest's (2004); even Putnam to a certain extent (Floyd's solo papers are not like this, they are more philosophical) in Floyd, Juliet, and Hilary Putnam (2000); Alan Ross Anderson (1958), Paul Bernays' (1964) to name a few.

[4] Juliet Floyd argues this point too in (1995, 375).

proof, rather than looking at Wittgenstein's own way of defining what the problem is from his own philosophical perspective. The focus on the methodological comments and commitments of Wittgenstein should assist in providing a reading of the remarks against most commentators[5], and more interestingly, present a somewhat systematic flaw in their interpretation. This reading should show the intelligibility and centrality of these traditionally detested "gnomic utterances"[6] (as Graham Priest once characterized them) on Gödel, in Wittgenstein's philosophy of mathematics. I also claim that the distinction between a philosophical and mathematical perspective is not merely necessary from external considerations, but if we pay more attention to the context of the Gödelian remarks, we notice that they are frequently accompanied by reflections on this distinction. In other words, this topic itself is central to understanding what Wittgenstein uses GIT as an example for.

1.2. Exegetical Considerations and Method

The aim of this work is to discern how Wittgenstein deals with Gödel's theorems philosophically. We must, first, briefly speak of the texts used here and what we are considering to be the Gödel remarks, the reliability of posthumous publications and lecture notes, and whether it is necessary to consider the chronology of writing.

Wittgenstein's remarks on Gödel are typically considered to be those remarks in which Wittgenstein explicitly addresses Gödel's proof. I will first add under the heading "Gödelian remarks" *any* that are considered in the secondary literature to be remarks on Gödel's proof (I will aim to be as inclusive as possible). Most of these are either those explicitly mentioning Gödel's proof or key distinctive features, or the other seemingly non-related remarks which occur in the third appendix of RFM (AppIII). The proof and the theorem, however, are important results in the field of the foundation of mathematics and they involve notions common in this discourse which Wittgenstein treats. Thus his remarks pertaining to such notions as proof, comparison of proofs in different systems,

[5] Although due to time and space limitations, I will not be able to address many of them specifically.
[6] Priest 2004, 208.

decidability, provability, diagonalization should in theory be relevant. Should we count them as part of Wittgenstein's Gödelian remarks? Many of these remarks are surely pertinent, but, besides the impracticality of such a measure, given the approach to Wittgenstein's philosophy of mathematics I am defending here, it is not strictly necessary to consider all these facets of the proof. This is because Wittgenstein's point about Gödel is rather an application or an articulation of a larger point/purpose. It is not really the sum of his treatments of the parts (mathematical notions) that constitute Gödel's proof. I consider that his remarks on many specific issues are for him at the service of a general goal. These will be cited in case they *add* something or pose a problem. There are many other remarks that I consider to be very relevant and that are absent in the secondary literature. These are in the same manuscripts as a few previously cited remarks (*e.g.* MS 163), but are not too overtly on topic. Moreover, I suggest including them in the set of Gödelian remarks not just as loose remarks, but as part of entire passages. In the secondary literature, the only passage taken as a whole is RFM.AppIII (perhaps also RFM.VII, 22-3, given the lengths of these remarks). Wittgenstein's philosophical style is quintessentially dialectical, thus taking whole passages into account offers more than just additional remarks, but also gives a clue as to the problems Wittgenstein is concerned with. This is especially important given that Wittgenstein is not a philosopher who has doctrines to contribute, but a way of investigation. I will now give an overview of the primary sources.

❖ *Remarks in Print.* The first group of remarks that is canonically considered to be Wittgenstein's remarks on Gödel is the set of remarks grouped under the title Appendix III (App.III), absent in the first edition of RFM but appearing in the revised one. It is grafted from MS 118, 105v-114r (September 22-24, 1937), MS162a and the beginning of MS 162b (April 1938-Jan 1939)[7], §20 is from MS 159, 24r-24v (1938). Wittgenstein made a typescript (TS221, 1938) of what is now the first part of RFM (RFM.I), which was intended as the second part of the *Philosophical Investigations*. These were cut into clippings and subject to revisions. The three appendices that follow are also in typescript form, but they were not clipped and were separated from the rest of the collection of clippings. The third (as well as second) appendix are part of the same typescript (in the *Nachlass*, *Item 221 Typoskript der zweiten Hälfte der*

[7] RFM, Editors' Preface to the Revised Edition; Rodych 2003, 294.

Vorkriegsfassung der Untersuchungen [Typescript of the second half of the pre-war version of the Investigations]). Wittgenstein also added some handwritten changes, yielding the typescript TS 223[8]. We should thus date the App.III remarks to the year of their final revision by Wittgenstein, that is 1938. Given that App.III is grouped by Wittgenstein, the first few remarks, although apparently having nothing to do with GIT, are widely considered part of the Gödelian remarks. The typescripts are missing some symbols in the manuscripts and have not all been restored in RFM[9]. The editors mention App.II as well for Gödelian remarks, but it is not deployed as much in the scholarly debates in the late nineties and early two thousands.

In terms of published remarks, RFM.VII.19,21-22 (MS 124, 87-94: July 2-3, 1941), considered by Rodych (2003) and Floyd (2001), the latter warning us to look at the manuscript's order of remarks and handwritten changes that were not reflected in print[10]. I would also add these remarks from RFM, III.5 (possibly a joke though) about the response to the philosophical question about the unprovability of P in Russell's system; RFM.III.46, about the creation of a new calculus from Russell's system, and translation between calculi, 81, 82, 84-90 are about proofs of consistency and contradictions, recursive proofs, impossibility proofs, metamathematics, as well as some methodological remarks.

❖ *Remarks not published in print but available in the Nachlass*. Including but not limited to: Some remarks in MS 117 and MS 118 around those that have been selected for publication, MS 159, 13r, 14r, 21r-v, 22v, 23r; MS 117, 147-8, 151-2 (1938 and 1940); MS 121, 32r, 69v (RFM.II.52), 76r-84r (1938-9); MS 122 (1939); MS 163, 22v-23v, 31r-63r (1941) and MS 124 (1944)[11]. Not all of these are discussed in the secondary literature. Some of these remarks Rodych labels as "ruminations."[12] These are remarks where Wittgenstein is simply trying to figure out something, something like a draft. Wittgenstein is indeed working things out in his writing, but I argue that there is no such clear line between ruminating and asserting. Rodych uses this classification to try to make a case for Wittgenstein's understanding of

[8] Rodych 2003, 294.

[9] See Rodych (2003, ft. 23) for a detailed inventory of these discrepancies.

[10] Floyd, 2001, ft. 7.

[11] A lot of these are mentioned by Rodych in (2002) and (2003) taken together, but not all (not MS117, MS 118, many in MS 163). Rodych considers the *Nachlass* remarks more in isolation than as passages.

[12] Rodych 2003, 296; 2002, ft.4.

Gödel's proof over time, I disagree. I would add, however, that some remarks are clearly emotionally charged exclamations, *e.g.* MS 121 81r-81v[13] wherein he calls "the Gödelian reason [...] a stupid one". This is not just a cheap way to discard a damning remark. The reason why I would give less attention to such remarks (contrary to Rodych, who repeatedly references them to make the case that Wittgenstein was very critical of Gödel), is that Wittgenstein has largely made the same point elsewhere, without the aggressive tone. The language here is noticeably more emotionally charged than usual[14]. He also had enough self-control to start the sentence where he calls the Gödelian reason stupid by "I would almost like to say".

❖ *Lecture notes.* LFM is a result of compiling notes from four manuscripts belonging to R. G. Bosanquet, Rush Rhees, Norman Malcolm, and Yorick Smithies, none of them present in all the lectures. Cora Diamond, the editor herself in the preface, warns us against taking the text as "based on clear and conclusive evidence" for sometimes, notes are hard to parse, sometimes they simply disagree. She admits that "[m]uch of the text given here *is* accurate; that is Wittgenstein did say the words in the text of something very close."[15] But given that he did not read these notes and approve of them, she says we must be cautious in asserting novel theses about Wittgenstein from LFM, or even providing evidence in support of an interpretation[16]. I think this level of caution is rather unrealistic. Scholars hardly worry that Aristotle's works are lecture notes. Besides, one can also be suspicious of the widely studied PI.

---

[13] "But damn it, it is either self-evident or not self-evident!" – The truth is that you would not normally call something like this 'self-evident!', nor assert it or its opposite. Most importantly, you do not have the slightest use for such a proposition. And if, nonetheless, I were to press you to make up your mind whether you are willing to accept it, etc., you will see that here there are not the usual reasons for a decision. [*Alt transl. Using Witt's variations*: And if, nonetheless, I were to press you to make up your mind whether you are willing to accept it, etc., you have to see that it is of no consequence how you make up your mind, that therefore in this case the usual <decision|choice> is <not available|absent>.] I would almost like to say: whatever you opt for do not make up your mind for the Gödelian reason, as this is a stupid one. I would prefer your having the courage here to say something obviously nonsensical to <your shying away from such a consequence | your saving the external form>." The '<_|_>' notation expresses is to include the versions Wittgenstein used without choosing.

[14] We are dealing with private notes after all. Wittgenstein is not trying to stay "objective" in his own notes for himself. Emotional outbursts of frustration, hopelessness or despair are not unusual. The notebooks which contain a lot of the relevant remarks contain a lot of diary entries. Some very intimate ones even *immediately* preceding or following remarks discussed here.

[15] LFM, 9.

[16] LFM, 9.

LFM is in fact unique in what it can offer. We have here a record of what Wittgenstein is saying *to an audience*, not to himself. He should make himself clearer. Perhaps, the fact that people understood such-and-such from Wittgenstein speaking, and that such-and-such can be independently read from the written remarks, gives even more credibility to an interpretation.

As far as separating times of writing, I think a developmental hypothesis can be discarded. It seems to me that Wittgenstein's purpose with Gödel remains the same. I will not argue for this explicitly, but will include the dates. Wittgenstein's approach is pluralistic in terms of the questions he is asking, and he is examining Gödel's theorem under different possible assumptions and reasoning. By this I mean that he does *not* give *one* interpretation of Gödel's theorem, he does not claim to address *the* Gödelian proof. He gives several ones one might have under the influence of certain philosophical pictures, even in remarks written the same day. So such variations are not due to his changing his mind about what he thinks Gödel's theorem is saying. His goal with discussing this topic being the same from 1937 (or even earlier) until 1944, and the similarity of considerations about it across time, led me to conclude that it would be appropriate to treat Wittgenstein on the matter in a more unified manner, contra what some commentators like Rodych were arguing (that Wittgenstein's treatment was ill informed up until the point where he speaks of technical notions from Gödel's proof).

1.3. The Project

The goal is to present an interpretation of Wittgenstein's purpose with the topic of Gödel's incompleteness theorem from what we can parse out to be Wittgenstein's own perspective rather than comparing Wittgenstein's supposed claims about Gödel to an independent understanding of Gödel's own proof. A comparison of Wittgenstein's remarks to the content of the proof is a characteristic of a "mathematical" approach to the remarks, not a philosophical one. The program is therefore to expose (1) Wittgenstein's philosophical disposition, especially in dealing with mathematical matters; (2) the distinction between dealing with mathematics "mathematically" in contrast to philosophically from

observing Wittgenstein's method at play, as well as from his remarks on this distinction, a lot of which, it turns out, occur within/around passages on Gödel; (3) to read him on Gödel's theorems (mostly his first incompleteness theorem (GFIT)), while showcasing the relevance of (1) and (2). (1) and (2) are not merely aids to see Wittgenstein's angle of approach, but they are also relevant to understanding the content.

This monograph is to provide a reading of a *sufficient amount* of Wittgenstein's remarks on Gödel while framing his perspective as an approach in the philosophy of mathematics which can be constructed from his methodological remarks. These are remarks about what philosophy is, about what it means for a philosopher to speak of mathematics, about the contrast between the philosopher and the mathematician, about his purpose in speaking about mathematics, and lastly about his aim with Gödel's Theorem/proof and related mathematical devices.

As a result, this leads to a different reading from that of several commentators, particularly those that claim that Wittgenstein misunderstood Gödel or was criticizing him. Wittgenstein's understanding of GIT or lack thereof is irrelevant. What matters is the method he exhibits when dealing with such matters. Wittgenstein sets up a situation, a set of hypothetical conditions, and we must take them as they are and analyze what kind of thing he is trying to tell us, what he is warning us against, what picture he is trying to paint or trying to erase. Comparing what he seems to be talking about to the mathematics of Gödel's theorem is inadequate, and I hope to show this.

2. Wittgenstein's Philosophy of Mathematics (Theory)

2.1. The Temptation of Philosophy

> **RFM VI 12.** Philosophy has to work things out in face of the temptations to misunderstand on *this* level of knowledge. (On another level there are again new temptations.) But that doesn't make philosophising any easier!
>
> **MS 121 76v-77r (Dec. 30, 1938):** Philosophy owes an enormous amount to set theory– for now we have gained *experience* of traps posed by phraseology of which we could not otherwise have dreamt.[17]

The inaugural lecture of LFM is immensely useful to discern Wittgenstein's subject matter (after all, he owes clarity to his students). In it, he specifies that he will not talk about "foundations of mathematics", the branch of mathematics to which *Principia Mathematica* (PM) belongs. Rather, he is concerned with "the word 'foundation' in the phrase 'foundation of mathematics'. This is a most important word and will be one of the chief words we will deal with."[18] This is also partly an announcement of his separation of the treatment of mathematical matters as a philosopher from that of the mathematician. How can he legitimately talk about matters that are exogenous to his discipline? As a philosopher, he can speak about mathematics despite his lack of expertise (and external approach), because, Wittgenstein claims, he "will only deal with puzzles which arise from the words of our ordinary everyday language, such as 'proof', 'number' , 'series' , 'order', etc."[19] He will not provide "new calculations", neither will he provide, "a new interpretation of these calculations"[20]. Instead, he is "going to talk about the interpretation of mathematical symbols, but [he] will not give new interpretations."[21]

[17] Translation from Rodych (2002, ft. 33).
[18] LFM, 14.
[19] *Ibid*.
[20] LFM, 13. This already gives us an idea about what Wittgenstein thinks mathematics is.
[21] *Ibid.*

Should he happen to produce novel interpretations, these will not be intended to be the true ones falsifying the older ones, but merely to "show that the old interpretation and the new are equally *arbitrary*."[22]

The material upon which philosophy 'acts' is language[23]. For Wittgenstein, philosophy is (best) used[24] to clarify our language, especially to reveal our misunderstandings of it[25]. Wittgenstein's philosophy is thus meant to intervene in mathematics because the latter discipline employs words from ordinary language, and this is what Wittgenstein called "prose." Prose refers to the words that accompany the proof, words that we use to explain or talk about the proof[26], as opposed to the *calculus* which is the rather algorithmic aspect[27]. We have a 'natural' *inclination* to import the 'meanings' of these words from their ordinary usage[28], which is the context in which these words took their meaning, and then, to some extent, informed mathematical reasoning.

It is worth noting that one must bear in mind Wittgenstein's criticism of the philosopher's (metaphysical) conception of the notion of "meaning". When Wittgenstein himself employs the term *meaning*, it means something rather informal: it is simply the ordinary use of the term, not something technical as it is the norm in (analytic) philosophy. Wittgenstein, in fact, rejects any theory of meaning, any "general conception of meaning"[29]. But in order to do so he appeals to a description and a "counter-picture"[30] that inevitably comes with the said description. This description is essentially human

---

[22] My emphasis. LFM, 13-4.

[23] In MS 159 2v he says that philosophy originates from the feeling of "the need to be familiar with our language".

[24] As in the way he advocates to use it.

[25] PI 90. Rupert Read claims that "Wittgenstein is setting out here for the first time really the heartland [*sic*] of his method." (2021, 44) I believe it encapsulates that too. This will be shown over the course of my thesis.

[26] This is a problem Wittgenstein is concerned with: the articulation of the difference "between philosophical prose about mathematics and mathematics". (Floyd, "Prose Versus Proof:", 2001: 289). That prose tends to mislead (Floyd, 2001, 299). More on this later.

[27] WVC, 129, 149; Mathieu Marion 2008, 5.

[28] A neutral example of the way imagination and pictures play a role:

- "We extend our ideas from calculations with small numbers to ones with large numbers in the same kind of way as we imagine that, if the distance from here to the sun could be measured with a footrule, then we should get the very result that, as it is, we get in quite different way. That is to say, we are inclined to take the measurement of length with a footrule as a model even for the measurement of the distance between two stars." (RFM.III.4)

[29] Diamond (2004, Criss-Cross Philosophy, 217); Read 2021, 63.

[30] This is Read's term (2021, 63).

behaviour, construed as an 'indubitable fact,' and the observation that language functions in the language-game model. For example, when we say that someone has understood what such-and-such utterance *means*, it is because they employ or respond to the such-and-such utterance in a way that we deem to be the appropriate response. But these norms depend on context, and one cannot define all the contextual conditions that are proper to a concept. Therefore, while we can see that what we mean by "meaning" reflects the role/interest/purpose of the employment of the concept in the form of life that uses it (leading to construction of the norm), we cannot fully determine the range of application (or even "truth-conditions" of the term).

The application of words is not exactly determinate (we may speak of them by means of "rule following" but this is a mere metaphor). Yet, in the case of some 'basic' words that lend themselves to ostension, it is straightforward. This simplicity is unfortunately very limited. Ambiguity begins already in words like chair (when does an armchair become a sofa?)[31]. Furthermore, the use of words spans in a spectrum between literal and figurative. Wittgenstein illustrates this with a few examples where the context slowly shifts from what we would deem *fair* use of the word, to one where we would deem it undeniably misleading, and the difficulty–if not the practical impossibility–of drawing that line. Consider the utterance of the phrase "everyone in Cambridge has a telephone" in each of these contexts[32]: a) when everyone has a telephone receiver, some are connected, some are not; b) when everyone has a telephone case, some are full and some are empty; c) in a situation where only some appear to have a telephone, but someone says "no, everyone has a telephone, some are just invisible".

> We learn our ordinary everyday language; certain words are taught us by showing us things, etc.–and in connexion with them we conjure up certain pictures. We can then change the use of words gradually; and the more we change it, the less appropriate the picture becomes, until finally it becomes quite ridiculous. In the earlier cases we should say Smith was exaggerating or using high-flown language; finally we should say that he was simply using sophistry to cheat us.
>
> To think this difference is irrelevant because it is a difference of degree is stupid.[33]

[31] See LFM, 20: the example of Lewy recognizing sigmas.

[32] LFM, 18. The example is not very well presented. The cases presented are disorganized and a bit muddled, but the point of the example is clear from when he clearly states it, and from the other examples he uses to illustrate the same concept.

[33] LFM, 18.

Several rhetorical devices can be examples of shifts of meaning that are not immediately obvious: exaggeration, manipulation, metaphor, *etc*. In a way, this obscurity is due to the words' tempting us towards certain conceptions (perhaps the most normal ones). This temptation, Wittgenstein also calls tendency (*e.g.* MS 162a 95), inclination (*e.g.* PI 252) or even bewitchment (PI 109). Wittgenstein indicates in a few places that the temptation of 'extrapolating' meanings takes a hold of us on account of the fact that it stimulates (or "interests" is the word Wittgenstein uses the most) us in some way[34]: we find the result "surprising," beautiful, mind-boggling, or "charming."[35] Hence our tendency to make those interpretations. There is a danger when we are not aware of this slippery slope. For example, when we are told about some result in a discipline in which we are not experts, we falsely believe that we have understood it because we take for granted the meaning of the word from previous contexts and believe that it applies in this case too. Our inclination to see things in this way becomes ground for finding things convincing or evident, eventually leading to the construction of metaphysical theses we take to be true. Wittgenstein also means to warn us not to take these examples as evidence of something mysterious, only that some expressions are just incomprehensible without context[36]. Another common inclination is to avoid the inconvenient fact that there is no foolproof way of defining, determining, and categorizing things accurately and without any outlying cases, that is, the avoidance of the fact that we group things under a class not because of a unique shared genus or essence, but merely through something like family resemblance[37].

In the first LFM lecture, he appeals to several other examples to the same end (showcasing the slight shifts in meaning and the way in which they can be more or less tricky)[38] which we find are also alluded to in his own writings as well in MS 162a (*Taschennotizbuch*), pp91-98. These remarks are in fact sandwiched between reflections on Gödel, constituting a further piece of evidence in support of the

[34] *Cf.* RFM.I.167.
[35] RFM, AppII.
[36] MS 162a, 97.
[37] Read 2021, 190.
[38] The examples in the MS that are also in Lec. I of LFM are about the ambiguities of "seeing in the dark" and "he has found a cure for unemployment". In MS 162a, the example of "he has discovered the East Pole of the Earth" can correspond to the LFM example of "Mr. Smith flew to the North Pole and found tulips all around".

relevance of these remarks to our topic, to understanding Wittgenstein's angle of approach to GIT[39]. Within those pages[40], Wittgenstein links this phenomenon to the general problem he has with the discourse of the foundations of mathematics: Why do we think that mathematics needs a foundation? For, isn't this conception just the fruit of temptation? (We shall investigate this more attentively in 3.1.)

> Has there always been a problem with the fundamentals of mathematics? —
>
> The problem arises most easily where there is a strong tendency to assimilate expressions with very different applications.[41]

Temptation yields what he terms "misunderstandings" in LFM. These are the main object of his 1939 seminar, but they are not any kind of misunderstanding. One in particular stands out: the kind "without which the calculus would never have been invented, being of no other use, where the interest is centered entirely on the words which accompany the piece of mathematics you make."[42] As a result of *temptation*, these words influence the way we conceptualize and imagine mathematics. At best they lead to the creation of new calculi (like Dedekind or Gödel). It is important to note that the issue is not *that* this happens at all, especially in mathematics wherein, he says, some of these "misunderstandings" drive the creation of new calculi. I argue that Wittgenstein is not deducing that this calculus is therefore wrong, that it should not have happened, or is not real mathematics. For once it becomes recognized as a calculus, applied like the rest of calculi, plays that same function, it is a calculus, we can rightfully call it "mathematics." Consequently, although mathematicians were misled by the term "foundations," the calculus they invented to found mathematics is legitimate mathematics. Hence, what Wittgenstein deems intellectual straying does not concern mathematics *qua* mathematics.

When the Austrian philosopher says in Lecture I "I will be concerned with cases where having a picture is no guarantee whatever for going on in the normal way"[43], he is in fact indicating that which

---

[39] This refers to the theorem as well as the proof.

[40] MS 162a, 94-95.

[41] The German original: Gab es immer das Problem der Grundlagen der Mathematik? —

> Das Problem erwächst dort am leichtesten, wo starke Tendenzen der Assimilation von Ausdrucksweisen mit ganz verschiedener Anwendung vorhanden sind.

[42] LFM, 16-17.

[43] LFM, 21.

interests philosophy in general, not just his. These are the cases where our application of some words gets dodgy (e.g. "It's weighing *on* my mind"). The opposite of this (in the extreme) is the case where we *can* tell by having "a picture": this refers to the trivial day-to-day practical things (like when I presently utter, "I am sitting *on* my chair"). Philosophy precisely likes to deal with ambiguous cases. The ambiguous cases are, in turn, precisely these instances where an ordinary word deviates from its ordinary usage. And this is where the philosopher, or indeed *anyone*, including "*the man in the street*," tends to be tempted by the preexisting pictures, *i.e.* the *conditioned* perspectives inherited from ordinary usage[44]. Temptations inform our *attitude,* and attitude effectively becomes a bias. The result is simply narrow-mindedness, a clinging to the views that feel more convincing, which stands in the way of "seeing *more*"[45], or, one might say, compels us to "hide the things one does not *want* to see"[46]. Wittgenstein illustrates this graphically too (*fig. 1*) in a digression in the midst of a passage on Gödel. The initially mysterious graph seems to be perfectly explained in words a few pages later:

> What we teach is the diversity of concepts, as expressed neither in the surface grammar of our language nor in the pictures we associate with our expressions, but in the structure of the use we make of them. This structure[47] shows us, as it were, new dimensions of the concept. When viewed from above, everything seems to lie on a single plane. The connectivity of the concept is different from what it appears to be when viewed from the usual standpoint.

The philosophical problem Wittgenstein in particular is trying to draw our attention to by means of the aforementioned examples is that there are no sharp lines between the cases where it is appropriate to say "I don't yet understand what you mean" and "oh, really?"[48] or "this is nonsense!" Wittgenstein is a philosopher of *degrees of difference*. (This (partly) explains his refusal to make theses or theories. Theories rest upon making rigid differences, which Wittgenstein is almost always preoccupied with showing that it is practically unfeasible.) In a way this is one of the central things he wants to convey, or one of the defining characteristics of his philosophical attitude: the thought that

---

[44] *Cf.* Read 2021, 60.

[45] RFM.III.85: "Philosophical dissatisfaction disappears by our seeing *more*".This seems to be a claim that on the other side of what Wittgenstein is inviting us to partake in is liberation from philosophical dissatisfaction.

[46] Read 2021, 56.

[47] We must be careful though not to take this "structure" literally!

[48] LFM, 17-18.

considering differences of degree as irrelevant as a different is "stupid"[49]. This also indicates that Wittgenstein is not here to tell us what makes sense and what doesn't. To think that one can make this distinction fairly is itself a temptation we might fight, and a reason why Wittgenstein is so critical (perhaps too much) of the author of the *Tractatus*. "Don't for heaven's sake be afraid of talking nonsense!" He exclaims, "[b]ut you must pay attention to your nonsense.—"[50]

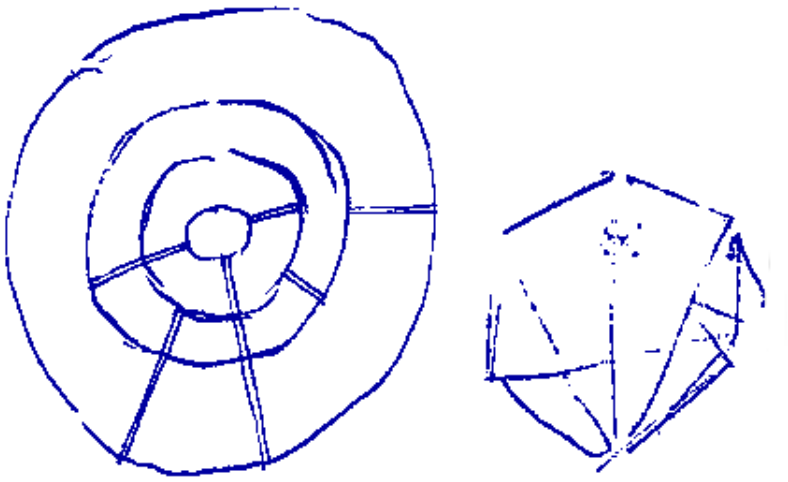

Figure 1. *From MS162a p79.*
The figure on the left is analogous to our automatic way of seeing. This given unquestioned perspective misleads us into conceptualizing something in two-dimensions. But as it turns out, we are as though lacking in imagination, because we could actually conceptualize something richer, as though we had been flattening a 3D object (on the right) because we were stuck looking at it from the top.

2.2. Paying Attention to Our Nonsense: Wittgenstein's Remedy Against Temptation (Description Over Explanation)

> "We shall see contradiction in a quite different light if we look at its occurrence and its consequences as it were anthropologically—and when we look at it with a mathematician's exasperation. That is to

[49] LFM, 18.
[50] C&V, 64.

> say, we shall look at it differently, if we try merely to *describe* how the contradiction influences language-games, and *if* we look at it from the point of view of the mathematical law-giver." (Wittgenstein, RFM III 87)

Temptation, inclination, or tendency, is a recurring theme. It is a description of symptoms that Wittgenstein often employs not only in metaphilosophical passages as seen in 2.1, but it can also be noticed in the procedure of his philosophy. He is often referring to the ways in which (or the 'fact' that) we are tempted or inclined to say such-and-such (in such-and-such context). This is a cornerstone of Wittgenstein's late view of philosophy as "philosophy as therapy", or we might even say, "philosophy as liberation." The telos of Wittgenstein's philosophy is to lead the fly out of the fly trap[51]. Wittgenstein wants to help philosophers (including himself!) to cure themselves[52]. What sort of thing imprisons the philosopher? What kind of illness befalls them? It is *pictures* that they tend to have ("A *picture* held us captive"[53]), pictures that inform *how* they see philosophical problems, pictures that *tempt* them towards certain conceptualizations, certain forms of philosophizing. "The problem is: getting stuck in one metaphor, which one mistakes for the concept itself, rather than retaining and indeed expanding a space of autonomy *vis-à-vis* one's metaphors."[54] These are pictures we take for granted, but not as statements or premises that we do not question, but as *perspectives* which inform our interpretations. (An analogy could be drawn with Necker's cube[55], the duck-rabbit image, and the like.) As such, it is very difficult to escape them because they become the background against which we think, the space wherein we form alternate views (thus we hardly get beyond it, even in our skepticism). Wittgenstein endeavours to shake

[51] PI 309; PI 255 also fits here: "The philosopher's treatment of a question is like the treatment of an illness."(more informative) Alternative translation: "The philosopher treats a question; like an illness." (more accurate translation) But note that strictly speaking, it is not clear whether he is speaking here prescriptively (what the philosopher must do), which is presumably what he should be doing, and we indeed can observe that he does that, or if it is a manifestation of a misled philosopher. I would say that this is an instance of the self-reflection he prescribes, which is also a manifestation of his "description over explanation" approach. He is indeed describing what he does. But also in a way it also makes sense that this is about philosophers in general. This remark, I would say, is just a description, not a judgement over whether this should be done or not.

[52] RFM.V.53.

[53] PI 115.

[54] Read 2021, 46.

[55] Already in TLP 5.5423.

the underlying framework which informs such distinctions such that we become able to see something else *too*. These frameworks are the perspectives through which we see problems or are inclined to make sense of things, which then influences the philosophical theories we created. These are pernicious pictures that are very difficult to call into question, which is why when Wittgenstein did it, he was (and still is) so prone to being misread. Wittgenstein himself knew that he was not immune to temptation. He was not completely freed either. "I am inclined/tempted to say__" is often something he utters. This is his confession, his introspection, as one stray among us. This is why Rupert Read prefers to qualify Wittgenstein's philosophy as "liberatory" instead of "therapy." The idea of therapy involves a model where there is a "therapist" and a "therapee," while the more appropriate dynamic for (late) Wittgensteinian philosophy is not really hierarchical. It is a mutual effort between equals, a dialogue, a confessional on both sides[56]. This attitude can also be observed in LFM. "Temptation" is not inherently bad. We are always tempted to say things and Wittgenstein constantly admits this: "I am tempted to say__" or "are we not tempted to say__" and "how come__". Read proposes that the problem is merely "that tyranny, which comes from the metaphysical urge to define"[57]. The idea of therapy misleads people into thinking the temptation of metaphysics is bad and must be purged, but nay, "the fact that we are inveterately tempted by metaphysical theses is part of what makes philosophy possible, necessary, worthwhile. Moreover, [...] any given metaphysical thesis itself is almost certainly not without value as it can become part of the practice of comparison or perspicuous presentation which leads us past the problem/confusion/trap."[58]

Wittgenstein tells us how he would respond to Hilbert when the latter says: "No one is going to turn us out of the paradise which Cantor has created."

> "I wouldn't dream of trying to drive anyone out of this paradise." I would try to do something quite different: I would try to show you that it is not a paradise–so that you'll leave of your own accord. I would say, "You're welcome to this; just look about you."[59]

[56] Read 2021, 7, 52.

[57] *Ibid.*, 46. The underlining here is Read's emphasis.

[58] Read 2021, 8.

[59] LFM, 103.

Wittgenstein's remedy to counter these dominant strains of thinking is not a counter-theory. So how does he defeat a view, without having another view? It is through somehow making us *aware* of how we are misled. How can this be done? By presenting examples that trigger our temptations to think a certain way, contexts that reveal that our intuitions are questionable. In the context of mathematics specifically, a few pictures that get a hold of us have a large and influential reach (both in terms of one's own ideas, and the number of people that fall under their spell): that mathematics *needs* a foundation, that mathematics are discovered[60], mathematical Platonism *etc*. These are the objects of his investigation.

> **MS 179, 22v-23r**: You say what you are inclined to say. And it has interest only because we too feel the same temptation to say it. However, now it is not yet true nor merely probable, but the object of our investigation.
>
> **PI 254.** [...] What we 'are tempted to say' in such a case is, of course, not philosophy; but it is its raw material. Thus, for example, what a mathematician is inclined to say about the objectivity and reality of mathematical facts, is not a philosophy of mathematics, but something for philosophical treatment.

This is an application of a metaphilosophical principle that Wittgenstein both explicitly advocates for and applies: philosophy must switch over from explanation to description[61]. Philosophers are motivated by the desire to understand which becomes the desire to explain things. One might say, driven by wishful thinking, philosophers assume that they can indeed explain things. Wittgenstein intervenes to say not that explanation is *impossible*, for such a claim would itself be a thesis and would require 'extraordinary' evidence. Wittgenstein is instead merely saying we seem to misunderstand what can and cannot be known. We have unrealistic expectations so to speak. Note that this does not imply he is asserting the existence of something we cannot say or do, nor that there is an ideal we pursue yet can never attain. But how else can we conceptualize this? It is a *helpful* picture that we might keep, but we must be *aware* of that. We must in fact study this very 'phenomenon' that is the application of this

[60] MS 162a,66 : In mathematics, we don't discover the properties of concepts, rather we make definitions of them. [In der Mathematik werden Begriffsbestimmungen gemacht, nicht die Eigenschaften von Begriffen gefunden.]; RFM.I.168: "The mathematician is an inventor, not a discoverer." *Cf*. RFM.AppII.2.

[61] "All *explanation* must disappear, and description alone must take its place." (PI 109) Also said about mathematics in particular: "Our task is, not to discover calculi, but to describe the *present* situation." (RFM.III.81 (1939-1940))

picture[62]. The object of philosophy is language, and "[l]anguage, I should like to say, relates to a *way* of living. // In order to describe the phenomenon of language, one must describe a practice, not something that happens once".[63] In other words, things we *tend* to do.

This is a kind of investigation Wittgenstein wants "to draw [our] attention to"[64]. Because in fact, it is the kind of investigation, even standpoint, to which we tend to be *resistant*. He tells his students, a dozen lectures later:

> One of the greatest difficulties I find in explaining what I mean is this: You are inclined to put our difference in one way, as a difference of *opinion*. But I am not trying to persuade you to change your opinion. I am only trying to recommend a certain sort of investigation. If there is an opinion involved, my only opinion is that this sort of investigation is immensely important, and very much *against the grain* of some of you. If in these lectures I express any other opinion, I am making a fool of myself.[65]

Notice also here the renunciation of any doctrine or thesis. His non-revisionism might seem paradoxical in his would-be reply to Hilbert, but his intention is clearer here.

This investigation, Wittgenstein says, might lead to "trivial facts"[66]. Indeed, he explicitly commits himself to dropping any point of contention precisely to avoid advancing anything that is subject to dispute[67][68]. If this were the whole story, however, why do the philosophically minded resist it? The answer is that such readers expect an *explanatory* account, not merely a descriptive one. The "trivial facts" Wittgenstein uncovers are simultaneously trivial and philosophically counterintuitive precisely because of this shift from explanation to description. What Wittgenstein offers is not an explanation of meaning, necessity, or truth, but a description of our own attempts to explain—one that

---

[62] "The great difficulty here is not to present the matter as if there were something one *couldn't* do. As if there really were an object, from which I extract its description, which I am not in a position to show anyone.—And the best that I can propose is that we yield to the temptation to use this picture, but then investigate what the *application* of the picture looks like." (PI 374)

[63] RFM.VI.34.

[64] LFM, 22.

[65] LFM, 103.

[66] LFM, 22; This must be partly the reason why Wittgenstein's philosophy tends to be unsatisfactory to most readers (and why my thesis feels trivial to me).

[67] LFM, 22.

[68] This stance helps explain why some read him as an ordinary language philosopher. He gives the impression of attempting to return to some pre-theoretical understanding of our terms, as though their meaning were exhausted by their ordinary usage.

turns philosophical reflection back upon itself and exposes the limitations of explanatory ambition. The typical target of this investigation is a characteristic philosophical posture: the conviction that one is entitled to make categorical claims such as "this is how things really are" or "these are the most fundamental concepts upon which reality, or our knowledge of it, is built." Such claims are deluded pretenses to the discovery of the necessary and certain truths about reality. Yet despite the triviality of its results, the descriptive endeavour is no easy task. It requires sustained resistance to deeply entrenched habits of thought and to the philosophical urge to theorize.

> How does it come about that we are at all tempted (or at any rate come near it) to divide through by (3 - 3) in (3 - 3) × 2 = (3 - 3) ×5? How does it come about that by the rules this step looks plausible, and that even so it is still unusable?
>
> When one tries to describe this situation it is enormously easy to make a mistake in the description. (So it is very difficult to describe.) The descriptions which immediately suggest themselves are all misleading--that is how our language in this field is arranged.
>
> And there will be constant lapses from description into explanation here.[69]

The philosophical motivation expressed thus far is exactly the one that Rupert Read was able to excavate from the first remarks of PI, while explaining the significance of opening with a quote from St. Augustine's *Confessions*, which Wittgenstein deemed "the most serious book ever written"[70]. Rupert Read takes PI as a confession, and the *Confessions* as a work wherein the spirit is that of self-doubt and self-reflection rather than self-satisfaction with some found answer[71]. Wittgenstein is modelling his work in the same spirit of honesty, self-reflection, and the goal of eliminating 'natural' and hubristic tendencies (the delusion that we can *know*). What is a sin for Augustine is for Wittgenstein "compulsive [philosophical] commitments"[72]. But there is another reason for the latter to address the former: it is to show that despite Augustine's intention, he is still "vulnerable to dogmatism just where he wasn't aware of making any claim at all"[73]. R. Read, contra Hacker, takes PI.1 not as presenting the view against which Wittgenstein attempts to position himself, but to present an instance of a 'natural' and 'automatic' way

[69] RFM.III.85.
[70] Rhees 1984, 90 ; Read 2021, 42.
[71] Read 2021, 54.
[72] *Ibid*., 55,57.
[73] *Ibid.*

philosophers think[74]. Wittgenstein is not offering a counter-theory[75]. Wittgenstein's commitment in philosophy is *ethical*: he wants to say that it is possible to liberate ourselves from 'automatic' and 'natural' tendencies of thinking since these are born out of inclinations to *avoid* a more honest look at what confronts us, but we must actively choose to do so: "what is in play is already the incorrectness and indeed, more crucially, the moral dubiety, the tacit failure of intellectual honesty, of the assumption, the presumption, that 'I know how language works/how people work.'"[76] This is the kind of hubris that we must purge[77].

To summarize, in this chapter, a few characteristics/aspects of Wittgenstein's philosophical approach have been noted: i) its reliance on particular instances, particular thought experiments as a starting point or drive for ii) the dialectical process that it is, which manifests as a constant self-undermining, or arguments against other views, real or fabricated. iii) This serves to expose the object he wishes to treat and reflect upon (his and our tendencies of thinking), hence the abundance of such expressions as "we are tempted to think…", "aren't we inclined to say…", "what are we to say now?", "I want to say…", "when does one have the thought…?"[78] that which he wants to *describe* instead of explain. He does not put forth theories. The description is not empirical[79]. iv) Humility and freedom in thinking. This is almost unusual to say about Wittgenstein due to the multiple anecdotes of his attitude and personality (especially in his early period), but his later writings (and reports from his 1939 lectures)

---

[74] *Ibid.*, 56.

[75] *Ibid.*, 64.

[76] Read 2021, 57.

[77] *Ibid.*, 58.

[78] RFM.I.125.

[79] "It was true [to say] that our considerations must [could] not be scientific ones. It was not of any possible interest to us to find out empirically 'that it is possible, contrary to our preconceived ideas, to think this or that'—whatever that may mean. (The pneumatic conception of thinking [thought as a gaseous medium].) And we may not advance any kind of theory. There must not be anything hypothetical in our considerations. We must do away with all *explanation*, and description alone must take its place. And this description gets its light, that is to say its purpose, from the philosophical problems. These are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way that these workings are recognized– *despite* an urge to misunderstand them. The problems are solved, not by coming up with new discoveries [giving new information], but by assembling what we have long been familiar with[always known]. Philosophy is a struggle against the bewitchment of our understanding [intelligence] by the resources [means] of our language." (PI 109). The square brackets indicate an alternative translation form an anterior edition.

demonstrate an openness to being challenged, acknowledgement of his limits, and a drive to push (our collective) reasoning as far as it can go, demonstrating the freedom which is obscured by some pictures. To go beyond the pictures that usually inform our thinking is to free oneself to think thoughts that are normally thought of as 'irrational', or more accurately, thoughts that are normally unlikely to even occur.

3. Wittgenstein on Gödel (Application)

3.1. Altering the Attitude and By-passing Gödel's Proof (Wittgenstein's Aim with Gödel)

Gödel's incompleteness theorems present a particularly fertile ground for philosophical temptation: they invoke the appearance of paradox, the unexpected contradiction, the possibility of undermining foundational ambitions, the notion of unprovability, the formalization of metamathematical concepts, the treatment of provability as a predicate, and consequences for the concepts of mathematical truth and proof themselves. Wittgenstein seeks to engage in dialogues centering these themes for the sake of enlightening us (including himself) about our unjustified and conditioned dispositions towards these concepts. Chief among these are our attitudes towards contradiction and toward proofs of consistency[80]. Contradiction is commonly regarded with a kind of

[80] RFM.III.82-5 (around March 1940); MS 117, 230 (3 March 1940).

"dread"[81]: it is assumed that it is harmful, capable of ruining the calculus[82]. Consistency proofs are often taken to *establish* consistency thereby conferring reliability on a formal system, and even to be a precondition for our trust in the calculus. Gödel's theorems come as a surprising discovery about the foundations of mathematics and the limits of systems like Russell's.

***

> **RFM.VII.19**. My task is, not to talk about (e.g.) Gödel's proof, but to by-pass it.
>
> **RFM.VII.22. (MS 124, 90-98, 3-4 July 1941)** It might justly be asked what importance Gödel's proof has for our work. For a piece of mathematics cannot solve problems of the sort that trouble *us*. –The answer is that the *situation*, into which such a proof brings us, is of interest to us. 'What are we to say now?' –That is our theme.
>
> However queer it sounds, my task as far as concerns Gödel's proof seems merely to consist in making clear what such a proposition as: "Suppose this could be proved" means in mathematics.

One cannot attempt to decipher Wittgenstein's viewpoint on Gödel without taking these remarks seriously, for Wittgenstein is clearly stating what he takes his task to be. It is in order, thus, to understand what is meant here by "by-pass". According to what has been said thus far: rather than engaging with mathematics mathematically, *i.e.* doing calculus, or saying things that are somehow intended to affect the calculus, Wittgenstein is going to simply attempt to *describe* the *situation*. He asks : "What are we to say now?" Can Gödel's proof establish that systems like PM cannot be at mathematics' foundation? Does it reveal something about what mathematical truth is? What are mathematical propositions about? Does it tell us something about the essence of mathematics? The situation is the reception of Gödel's theorem by the mathematical and philosophical communities (as they were intricately related at the time). But he is engaging with the problem philosophically[83]. We must by-pass

[81] RFM.AppIII.17.

[82] RFM.III.80-2; RFM.IV.60; MS 117, 230.

[83] MS 161, 1r (1939): " Our problems are totally different from those which a mathematician tackles. The difficulty is to get into the new dimension in which these problems can be solved." ; RFM.V.52: "The philosopher must twist and turn about so as to pass by the mathematical problems, and not run up against one, –which would have to be solved before he could go further. // His labour in philosophy is as it were an idleness in mathematics. // It is not that a new building has to be erected, or that a new bridge has to be built, but that the geography, as it now is, has to be described. // We certainly see

the proof in order to pass through the 'conceptual prison' we find ourselves in when we allow ourselves to think according to that which we find most 'intuitive.' This is because opinion is relevant in philosophy and not in mathematics according to Wittgenstein, because doing mathematics hinges not on interpretation but technique *i.e.*, a correct application of the rules, that is, a manipulation of signs such as the correct result is obtained.

To begin undoing the ways in which we were conditioned which keep us philosophically confused, we must first disengage from our traditional manner of engaging with mathematics within philosophy. This tradition assumes, for instance, that philosophical work will *solve* mathematical problems, that philosophical problems pose genuine threats to mathematics, or conversely that mathematics—or logic—can resolve philosophical problems. Closely related to this is the picture of logic or mathematics is the most general science, for instance[84], or of logic as the foundation of mathematics[85]. One helpful way to characterize Wittgenstein's departure from this entire framework—his "by-passing"—is by contrasting this "by-passing" with "talking about." To say that we are talking *about* Gödel's proof is misleading since "a technique is '*about*' nothing" or "a technique 'deals' with nothing"[86]. Thus, we cannot discuss what Gödel's proof is *really about* given that the proof—a mathematical device—is a technique. (I will return to this point in later sections.)

The preceding remainder of RFM.VII.19 is telling:

> **RFM.VII.19.** But can't we imagine a human society in which calculating quite in our sense does not exist, any more than measuring quite in our sense?—Yes.—But then why do I want to take the trouble to work out what mathematics is?
>
> Because we have a mathematics, and a special conception of it, as it were an ideal of its position and function,–and this needs to be clearly worked out.
>
> Don't demand too much, and don't be afraid that your just demand will dwindle into nothing.
>
> It is my task, not to attack Russell's logic from within, but from without.

bits of the concepts, but we don't clearly see the declivities by which one passes into others. // This is why it is of no use in the philosophy of mathematics to recast proofs in new forms. Although there is a strong temptation here."

[84] MS 162a, 16-7.

[85] RFM.V.24.

[86] MS 162a 67.

> That is to say: not to attack it mathematically –otherwise I should be doing mathematics –but its position, its office.
>
> My task is, not to talk about (e.g.) Gödel's proof, but to by-pass it.[87]

Attacking "the position" of mathematics is one way of by-passing the proof. The "office" under attack is one that is bestowed upon mathematics in general due to philosophical confusion. This is not a confusion unique to philosophers: such a conception can be inherited or assumed by anyone (even laymen). Mathematicians, too, are susceptible to philosophical misunderstandings, which can affect how they conceptualize their own work as well as the mathematical discourse itself. They are indeed criticized by Wittgenstein for their lack of philosophical training or practice that could have enabled them to correct their confusions[88]. One of such passages is one directed towards Gödel from 1944, one of Wittgenstein's last remarks on him.

> **MS 124, 115r (March 5, 1944)** What's unphilosophical in Gödel's essay is that he doesn't understand the relationship between mathematics and its application. In this, he maintains the murky notions of most mathematicians.

In his supposed conclusive blow to Floyd and Putnam's defence of Wittgenstein's understanding of the proof, Rodych—assuming that "Gödel's essay" refers to (1931)—takes the above passage as further evidence of Wittgenstein's "mocking"[89] of Gödel's proof due to the former's ignorance of it and its misapprehension as a direct self-referential argument[90]. Rodych connects this to RFM.AppIII.19 in which Wittgenstein asks, "how could you make the truth of the assertion plausible to me, since you can make no use of it except to do these bits of legerdemain?" He takes the penultimate remark of Appendix III as concluding a series of remarks wherein Wittgenstein aimed to provide increasing evidence in an effort to showcase errors in the proof[91].

However, it is crucial to note that Wittgenstein is concerned here with what is "*unphilosophical* in Gödel's essay", not what is mathematically wrong. Does a philosophical shortcoming necessarily

---

[87] *Cf*. PI 124.

[88] MS 157b, 30v (February 1937); MS 121, 77v (December 1938); MS 123, 7 (October 1940); MS 124, 115r (March, 1944).

[89] Rodych 2003, 308.

[90] Rodych 2003, 308-9.

[91] Rodych 2003, 311.

undermine a mathematical result? As we have already seen, Wittgenstein sustains a continuous effort throughout the years to articulate a difference between the two disciplines. Even in his 1939 Lectures on the Foundations of Mathematics, he put forth disclaimers from day one specifying his non-interference with the mathematics (the calculus[92])—his non-revisionism. Misunderstandings, he holds, may accompany or even motivate the development of a calculus, but they do not thereby invalidate it: a calculus remains a calculus. What makes something a calculus, for Wittgenstein—is not the content of our conception or interpretation of it, nor of our understanding that may accompany, facilitate, or complicate the calculations and application of the rules, but that it is that kind of activity where one answer is correct, where the manipulation of the signs is as predetermined as possible, as though mechanical[93]. It is moreover, the kind of technique that can be applied across domains and integrated into other sciences. In short, it occupies what Wittgenstein calls "the special position that we assign to the activity of calculating".[94] Wittgenstein does in fact make this point immediately before the above remark MS 124, 115r—an element omitted by the editors when extracting surrounding material to form RFM.VII.31. Restoring it to its original context supports the present interpretation (especially the second paragraph):

> We condition a man in such-and-such ways; then bring a question to bear on him; and get a number-sign. We go on to use this for our purposes and it proves practical. That is calculating. –No, it isn't enough! It might be an eminently *sensible* procedure[95] – but need not be what we call 'calculating'. As one could imagine sounds being emitted for purposes now served by language, which sounds yet did not form a language.
>
> It is essential to calculating that everyone who calculates right produces the same pattern of calculation. And 'calculating right' does not mean calculating with a clear understanding or smoothly; it means calculating *like this*.
>
> What's unphilosophical in Gödel's essay is that he doesn't understand the relationship between mathematics and its application. In this, he maintains the murky notions of most mathematicians.

[92] I often use "mathematics" to mean "calculus" in the context where I am contrasting the object of mathematics and the object of philosophy.

[93] The image of logic or mathematics as being "mechanical" is examined extensively by Wittgenstein throughout his work. *E.g* RFM.III.81ff.

[94] RFM.VII.24; *Cf.* RFM.III.47's last two paragraphs.

[95] This is the voice of an interlocutor who thinks that our imagination and interpretation, a mental or experiential thing, of mathematics is important and essential to (doing) mathematics.

Rodych only mentions this editorial choice and only gives it weight in his (2002)[96], though his account remains lacking. He claims that Wittgenstein's view is rather that 'real mathematics' is that which has an "extra-systemic application". As a result, on this account, Wittgenstein is reproaching Gödel for being unphilosophical due to the latter's neglecting to consider the *usability* of the Gödel sentence within and/or without PA[97]. In other words, pure mathematics is not real mathematics, only applied mathematics is legitimate. Wittgenstein indeed often toys with this idea[98] and is often *inclined* to agree. But he also questions it[99]. As far as his dialectic goes, he has not supplied stronger reasons to defend the view of the illegitimacy of pure mathematics. The emphasis on applicability is not a point against Gödel's proof (we do not even know for sure that by "Gödel's essay" he was talking about the (1931)), but any conception that Gödel would have had of the proof, and of the "office" of mathematics. And it is known that Gödel, being a Platonist, would have a philosophical conception that is often criticized by Wittgenstein. However, the problem for Wittgenstein is not necessarily *that* Gödel is a Platonist about mathematics (in case he knew that about him[100]). What he would be accusing Gödel of is of not reflecting enough or well enough about the matter, "he maintains murky notions of most mathematicians", he thinks that he is *discovering* something about mathematics in-itself, *i.e.*, that he is discovering properties of objects that exist independently of humans, that he is unveiling the essence of mathematical truth—a "philosophical naïveté"[101]. Even if Gödel is named, this attack does not concern him exclusively, it concerns "most mathematicians." And as far as this effect of temptation is at play, Wittgenstein is "attacking" almost everyone (again, including himself). Against the mainstream view, Wittgenstein wants to say that developments in mathematics such as GIT do "not [...] improve bad

[96] Rodych 2002, 387.

[97] *Ibid.*

[98] In funny ways sometimes: "For one person [(Hilbert)] can see it [(set theory)] as a paradise of mathematicians, why should not another see it as a joke?" (RFM.V.7)

[99] *E.g.* RFM.V.7 (questioning why we think pure mathematics would be mathematics using a thought experiment of mathematics being invented for satirical reasons); V.8 (thought experiment of math done for ceremonial reasons, still is an application); III.85.

[100] He seemed not to know much about Gödel. He refers to nothing other than GIT.

[101] Floyd and Putnam 2000, 632.

mathematics, but [...] create a new bit of mathematics"[102]. GIT does not show PM to be deficient, for instance, but provides a new technique—a new game as it were.

Gödel is accused of failing to "understand the relationship between mathematics and its application." Reflecting on this relationship would reveal something about mathematics (and what this reflection itself can yield), according to Wittgenstein, namely that what makes mathematics itself is not the correspondence to a mathematical reality—*i.e.,* truth about independent mathematical entities—but its applicability, its role in our form of life. This applicability/function hinges more on "method" and "technique" than "interpretation" and "understanding."[103] After all one can somehow partake in a mathematical activity 'mechanically'—without understanding—but the converse does not hold (given that for him the criterion for assessing someone's understanding of a rule is to see them succeed in its application, in multiple instances preferably). Wittgenstein "should like to say: mathematics is a MOTLEY [*Gemish*][104] of techniques of proof.—And upon this is based its manifold applicability and its importance"[105]. "I want to give an account of the motley of mathematics",[106] he says. Thus despite sometimes phrasing things as above: "It is essential to calculating...", he does not mean to say that the *essence* of mathematics is technique (after all, many things are techniques but are not mathematical). 'Essence' is not meant literally. He flags this in RFM.VII.33, which, although beginning as a remark about the essence of mathematics being the formation of concepts, it continues to reject any literal talk of *essence*. That is to say, concept formation is a feature of much "of [w]hat is *called* 'mathematics'", "and yet [...] it plays no part in other regions" of mathematics [107]. "Mathematics is, then, a <u>family</u>; but that is not to say that we shall not mind what is incorporated into it"[108]

[102] RFM.VII.12.

[103] RFM.III.25,34; RFM.VII.34.

[104] Floyd disagrees with this translation by Anscombe (so does Mülholzer) because of the negative connotations. Instead she and Mülholzer prefer "colorful mix." She also says the French "*bigarrée*" is unnecessarily laden with negative connotations. (Floyd, "Das Überraschende: Wittgenstein on the Surprising in Mathematics", ft. 1)

[105] RFM.III.46.

[106] RFM.III.48.

[107] RFM.VII.33.

[108] *Ibid.*

Wittgenstein is *describing* mathematics as "an anthropological phenomenon," as a human activity[109], and as such, he asks about its application, its *usage*. Is pure mathematics without application because it lacks "extra-systemic" application? The application (or use) of pure mathematics (*e.g.*, foundations of mathematics in general, and GIT in particular) is indeed not immediately obvious, as evident in Wittgenstein's abundant questioning of it[110]. However, his conclusion is not that it is useless, but merely *that the use has not been clarified or is misunderstood*. This clarification is exactly the work Wittgenstein is intending to accomplish. (A task that will be further examined in 3.2.3.2 and 3.2.4.) That Wittgenstein does not exclude mathematics of the pure kind or of foundations of mathematics can be seen in his treatment of PM, particularly in response to claims that it is defective mathematics as a consequence of Gödel's result[111]. Wittgenstein argues instead that what is at stake is the creation of a new calculus, rather than a revision of our representation of some independent mathematical reality[112]. Such is the office of this calculus that Wittgenstein seeks both to understand and to challenge: attitudes such as the conviction that "[o]nly the proof of consistency shews me that I can rely on this calculus."[113][114] The authority which mathematics is taken to possess rests on a cluster of presumptions which manifest as an attitude, hence:

> **RFM III 82 (1940)** My aim is to alter the *attitude* to contradiction and to consistency proofs. (Not to shew that this proof shews something unimportant. How *could* that be so?)

How could Wittgenstein be understood as contesting Gödel's *proof*, an established piece of mathematics? This is not what he finds problematic. What is at issue instead is the kind of *status* that tends to be accorded[115] to GIT by "[m]ost people"— a status that reflects a *philosophical stance*. The

---

[109] *Ibid.*

[110] RFM.VII.32.

[111] RFM.III.85.

[112] RFM.III.27: "I am trying to say something like this: even if the proved mathematical proposition seems to point to a reality outside itself, still it is only the expression of acceptance of a new measure (of reality)", *i.e.,* of a new *paradigm* (he is not making an ontological claim).

[113] RFM.III.84, a remark that summarizes the "situation."

[114] A view that has in fact largely been abandoned following Gödel, or at least that "the sort of consistency proof to which we would have been inclined to attribute [the] effect [of security of a system] is in key cases probably unavailable." (Crispin Wright, *Wittgenstein on the Foundations of Mathematics*, 1980, 297.)

[115] MS 122, 28v; November 1939; *Cf.* Rodych 2002, 388-9.

problem is in large part the conflation of mathematical and philosophical matters. This muddling manifests as an "*attitude*": a (quasi-)systematic way of responding to certain notions under the influence of the pictures that accompany them. It is precisely here where Wittgenstein seeks to intervene in order to soften this attitude and the rigidity of thought it sustains. "The motto here is always: Take a *wider* look around."[116]

3.2. Wittgenstein's Remarks on Gödel

3.2.1. Wittgenstein's Angle (RFM.AppIII.1-7)

The first textual instance of Wittgenstein's addressing of GIT is in a letter to Schlick dated July 31, 1935 (see the appendix below). It is the only mention up until the 1937 manuscripts. What is notable is the extent to which the way he dealt with it here resembles the subsequent treatments:

- Your surprise shows your misunderstanding and ignorance. Once you understand it, the surprise will dissipate. Therefore, you should take surprise merely as an indication that there is something you do not understand;
- You won't know anything until you examine the proof carefully, for the summary is obviously ambiguous;
- Philosophy cannot help you understand the proof;
- Philosophy cannot tell you about the possibilities of the calculus;
- Do not make up your mind about the proof before you read it.

For instance, the gist of the message is echoed four years later in MS 121, 84r (January 2, 1939):

> Gödel confronts us with a new situation. What shall we say about this? One mustn't of course be hasty in deciding what to say. In particular, one mustn't be hasty in wanting to say what sounds most sensational. The situation is more difficult to assess than it seems. [117]

Following the 1935 letter, Wittgenstein's next substantial treatment of Gödel appears in Appendix III in RFM. Its opening remarks **§§1-4** (TS 221, pp. 246-7) stand out in that they are not,

[116] RFM.II.6.

[117] Translation from Rodych 2002.

strictly speaking, 'about' Gödel. Their function, however, is to set the terrain for Wittgenstein's ways of questioning, the kind of thing which is put into question, and the kind of desired response. So how is Wittgenstein (philosophically) approaching Gödel's (mathematical) theorem?

Remark **§1**[118] invites us to do a simple thought experiment in which there is no special form for questions and commands. Instead of the subject-verb inversion and the question mark, or the employment of the imperative mood, they are instead phrased as declarative sentences. By appealing to the ease of imagining such a scenario, Wittgenstein is warning us of the possibility that our language conceals some other function, role, or purpose among declarative statements. We take declarative sentences to be propositions and thereby to have a truth-value, or to be truth-apt. What if some of these sentences are actually not what they seem to be? The question that is in the form of a declarative sentence[119] ("I should like to know if…") is actually not a proposition, but one could think on account of its grammatical structure that it is a proposition and thus inappropriately wonder about or attribute a truth-value to it[120].

The explicit link to truth-values is made in the following remark (**§2**[121]) which continues by noting that most sentences in our language are these "statement sentences. And—you say[122]—these sentences are true or false". But the declarative form is not sufficient to make a sentence a proposition. The utterance of a proposition plays a different *role* than that of a question or command, even when they share the same declarative form. As Wittgenstein puts it, "For assertion is not something that gets

[118] Penned first in MS 118, 105v (November 22 1937).

[119] Or a command that is in the form of a sentence "will you do this?" (PI.21)

[120] Floyd also read this (1995, 396), and Rodych (1999, 177). They both already talk about truth at this point, even if it is not explicitly mentioned in RFM.AppIII.1. Since Floyd's exegetical paper (1995) begins by considerations on Wittgenstein's discussions on the trisection of the angle and mathematical conjectures (which guide her reading of Wittgenstein on Gödel), she also connects this remark to conjectures: mathematical conjectures are like such disguised commands and questions. They seem to be assertoric but they do not fulfill this propositional role, rather, conjectures have the effect of something like a command to make a mathematical search for a proof, or a wish to know what it means (more on this below). (Floyd, *ibid.*)

[121] September 23, 1937 all the way to the rest of the appendix (except §20 which comes from MS 159, 24r-24v, it's added in TS 221, 255) . These are the MS dates, not TS (which are not specifically dated).

[122] "And–<u>you say</u>– these sentences are true or false". Note that Wittgenstein here makes it rather clear that this is some interlocutor, probably representing the opinion of many. The interesting thing about this remark is that he seems to translate this opinion he criticizes into his own terminology, one that is nascent in the TLP, "Or, <u>as I might also say</u>, the game of truth-functions is played with them".

added to the proposition, but an essential feature of the game we play with it." A proposition seems to be defined by *the fact that it asserts something*. "My wish is that you close the window" plays not the role of a proposition because the expected response is not assent or dissent but the action of closing the window. Whereas, propositions *play the role of assertion*. Hence we can say that assertion is "an essential feature" of the proposition, in that it is "an essential feature of the game we play with it." Just as winning and losing is an essential part of the game of chess, and if we play chess without this feature, we are no longer playing chess but another game that merely resembles it[123].

Floyd's comment explains the larger implication of such a remark:

> Wittgenstein tried to bring out that (and how) our grammatical taxonomies, such as our divisions of utterances into 'kinds of sentences', are not ordered according to a pre-given structure or concept, but are only appropriate for particular (i.e., restricted) given purposes.[124]

Rodych[125] maintains that this remark communicates that within "the game of truth-functions," propositions have two characteristics: they are in the present or past tense, and "the logic of truth functions applies to them"[126]. On this view, the truth of meaningful contingent propositions refers to the correspondence to fact and thus their truth is time-stamped, that it "is true or false the *instant* it is asserted" (the truth-value of sentence changes if the same sentence is uttered at a different time)[127]. Rodych further argues that "the <u>real</u> point of this passage" is to contrast these propositions to "meaningful mathematical propositions" whose truth is not relative to the time of assertion but it is rather linked to the fact of there being a proof[128]. In other words, a "meaningful mathematical proposition"'s truth is not issued, so to speak, at the time of its utterance (since it does not depend on a temporal fact), but it receives a truth-value once it is algorithmically decided. It can be "written down or considered and not *yet* be true or false because it has yet to be proved or refuted"[129]. On Rodych's

[123] RFM.App.III.2.
[124] Floyd 1995, 397.
[125] Rodych1999, 178.
[126] *Ibid*.
[127] *Ibid*.
[128] *Ibid.*
[129] *Ibid.*

interpretation, this is how the LEM applies to meaningful mathematical propositions: their truth is grounded in "that we knowingly have in hand an applicable and effective DP"[130].

Between 1942 and 1944, several years after composing the remarks under discussion, Wittgenstein connects these considerations on declarative sentences and their tendency to mislead us into assuming them to be assertoric with a variant of the Liar paradox. Gödel mentions this paradox in his (1931) introduction, as does Wittgenstein later in the third appendix of RFM. "I always lie", Wittgenstein notes, is not really an assertion but an exclamation. And if we see it as such, it's no longer paradoxical[131], it no longer appears to us as a problem, as a malfunction, a "logical collision", as something impossible to make sense of [132], or a limit of making sense.

§3 consists of the following:

> Imagine it were said: A command consists of a proposal ('assumption' [*Annahme*]) and the commanding of the thing proposed.

This ironic formulation has the effect of inciting skepticism over the Fregean categories of content and the "(utterly independent) willing of that content" that are supposedly fundamental logical categories in a proposition[133]. Why, Wittgenstein asks implicitly, should such categories not apply equally to commands? It is not entirely incoherent to have such a theory, to fabricate these categories in a command, but it is not something we are inclined to think. Wittgenstein's point is that the corresponding division in the case of propositions is no less artificial. What, after all, grants the analysis of a proposition more legitimacy? Wittgenstein is drawing attention to the tendency produced by the grammar of our language which gives us "the possibility [...] of writing every assertoric sentence in the form 'it is asserted that such-and-such is the case'."[134] The availability of this paraphrase encourages a picture of propositions that may not be philosophically warranted.

---

[130] *Ibid.*
[131] RFM.IV.58.
[132] LFM, 184.
[133] *Cf.* PI.22; Floyd 1995, 397.
[134] PI.22.

In **§4** Wittgenstein finally turns more explicitly to the topic of metamathematics. This remark initiates the turn towards the topic of arithmetic specifically. He examines the notion of 'proposition' in arithmetic in order to elucidate its "illusory"[135] character and to challenge the *foundational* role it is often assumed to play. This examination serves to unsettle our intuitive understanding of the notions of truth and provability in mathematics[136]. What we deem to be 'clear and intuitive', Wittgenstein suggests, is only so under the influence of an unexamined picture, one that may *legitimately* be called into question once we are able to adopt a perspective that is external (to these pictures) . In this context, Wittgenstein *entertains* the idea that "arithmetical propositions" is not essential to the practice of arithmetic itself, *i.e.*, to calculating[137]. Wittgenstein casts doubt on the common tendency[138] to regard a statement such as "2 × 2 = 4" as a proposition, for, he claims, this resemblance is only due to the verb "is" (read: "2 times 2 is 4"), which is rather superficial. Wittgenstein's aim is not to offer a causal explanation of why we are inclined toward such metaphysical conclusions—an explanatory endeavour he explicitly rejects—but rather to draw attention to grammatical features that lead us to jump to metaphysical conclusions. The resemblance between arithmetical equations and assertoric sentences, he seems to suggest, is no more significant than the resemblance between shaking one's head at a miscalculation, correcting someone saying it is raining when it is not, or reacting to a dog's misbehaviour. Wittgenstein merely notes this "point of connection". He is drawing attention to the fact that the "is" resemblance is no more legitimate than the latter (where 'true' and 'false' are construed pragmatically). And is it legitimate? Are we prepared to say that? If it is not a superficial relation, what would it tell us?

One way of synthesizing the three points raised in §4 is to see Wittgenstein as issuing a warning against the philosophical tendency to accord the notion of a *proposition* the status of a fundamental epistemological unit, one thought to stand in a privileged or essential relation to the special concept of

[135] Floyd 1995, 395.

[136] Floyd 1995, 389.

[137] *Cf.* RFM.I.144: "If somebody calculates like this must he utter any 'arithmetical *proposition*'? Of course, we teach children the multiplication tables in the form of little *sentences*, but is that essential? Why shouldn't they simply: *learn to calculate*? And when they can do so haven't they learnt arithmetic?"

[138] "We are used to saying" etc. (RFM.AppIII.4)

Truth. This tendency is characteristic of many metaphysical frameworks. Wittgenstein's suggestion, by contrast, is that the concept of "proposition"—and, more generally, various metamathematical notions—may not be essential to arithmetic. More precisely, these concepts may lack the foundational role or explanatory priority they are often assumed to have with respect to other arithmetical concepts. On this view, arithmetic is to be understood as a language-game. What is essential to such a game are not abstract entities or theoretical posits, but rather the practices, rules, and forms of use that constitute it. The resemblance between two activities/games such as describing an empirical fact and carrying out a multiplication, does not warrant the inference that there must be some abstract entity—the proposition—underlying both. At most, these activities share some features. The term "proposition," however, need not relate to them in the same way, and in some cases (*e.g.*, arithmetic[139],) it may even be dispensable for the purposes of the practice itself.

Rodych is correct in noting that the relationship at issue is merely a superficial one, amounting to what he describes as a "dichotomy of '+' vs. '–', 'right' or 'good' (doggy) vs. 'bad' (doggy)" [140]. Wittgenstein's point is that we are misled into thinking that the correspondence schema of truth in contingent propositions is replicated in the case of mathematical equations. This, Wittgenstein argues, is a confusion born of surface grammar, it does not license the postulation of something independent—something 'real'—to which mathematical equations, construed as propositions, might refer. Rodych explains this supposed independence by suggesting that we are inclined to think of a mathematical equation such as "3245 × 9983 = 2286725" as being "true or false *right now*, even before we have decided its 'truth-value'"[141]. The introduction of a temporal dimension, however, is not required by Wittgenstein's remarks at this stage, nor in the sense Rodych proposes. Temporality becomes relevant only once the question concerns the meaning of a mathematical proposition before and after it has been proved[142]. To say that a mathematical proposition is "true or false *right now*, even before we have decided its 'truth-value'" adds little beyond the claim that its truth is independent of our methods for

[139] RFM.I.144.
[140] Rodych 1999, 179.
[141] *Ibid.*
[142] As Floyd noted in (1995).

establishing it, *i.e.*, that it is somehow already determined. If the truth-value of a mathematical proposition is independent of our having found a way to determine its truth-value (a proof), then the proposition is either always true or always false, not as possessing a truth-value *now* in any temporally indexed sense. Even within this (misguided) scenario, therefore, the appeal to "right now" does no explanatory work. Rodych further maintains that it is already clear by now that Wittgenstein is trivializing the notion of truth insofar as saying ''p' is true' amounts to no more than asserting 'p' itself (citing LFM, 188), but this is clear by the 6th remark of this appendix, where the redundancy of the truth predicate in mathematical contexts is explicitly stated.

Graham Priest briefly comments on the four remarks together in his paper about Wittgenstein's remarks on Gödel. According to Priest, these four remarks lay the groundwork for the ensuing discussion by reviving a broadly Tractarian idea, namely that mathematical and arithmetical propositions lack content and are, in the Tractarian sense, *unsinnig*[143]. That "sentences such as 'I want to know whether it is raining' do not really have propositional content. [...] This is the context which triggers his ruminations on Gödel's theorem"[144] Priest reinforces this reading by appealing to the last remark of the appendix (§20, found only in TS 223), which he takes to advance the same point. This interpretation is not implausible. However, the notion of "propositional content" is not very clear in Priest's paper, and insofar as it functions as a technical term in his analysis, it is even more precarious to attribute this to late Wittgenstein. §20 states that "one needs to remember that the propositions of logic are so constructed as to have *no* application as *information* in practice. So it could very well be said that they were not *propositions* at all;" and that "the mere *ring of a sentence* is not enough to give these connexions of signs any meaning [*Bedeutung*]." Here, Wittgenstein is not claiming that such sentences lack some intrinsic property (propositional content), but rather drawing attention to their use, their practical function. The point is not about a deficiency in the sentences themselves, but about the fact that their *application* does not play the role of conveying information.

[143] Priest 2004, 209. On this note, at the end of Lecture IX, Cunningham asks Wittgenstein "Isn't it queer [to that someone can believe that '25 x 25 = 624' (which is false) or even that '25 x 25 = 625' (true)] because the expression '25 x 25 = 624' is meaningless?" To which Wittgenstein responds: "Well, this 'meaningless' road has now been trodden so often that it has become muddy and one cannot see one's way clearly; it needs rolling." (LFM, 92)

[144] *Ibid.*

In this way, Wittgenstein prepares the terrain for his engagement with Gödel by calling into question our tendency to confer propositional status on mathematical sentences—a tendency that typically brings with it a particular conception of truth and reality. This preparatory discussion will then transition into a reflection on the Gödel sentence itself, which Wittgenstein treats as differing from other mathematical sentences. His concern is not to characterize this difference in terms of content, but rather in terms of the distinctive role the Gödel sentence is taken to play within mathematical and philosophical discourse.

***

Now we turn to the Remarks on Gödel proper, which begin with the brief and straightforward question:

> **§5**[145] Are there true propositions in Russell's system, which cannot be proved in his system? — What is called a true proposition in Russell's system, then?

Rodych reads the connective "then" as indicating a conditional progression. "*If* there *are* true but unprovable propositions in Russell's system, what *then* is a *true* proposition in Russell's system?"[146] This much is rather evident. But the remark may also be read as both a set-up of what will follow and a clarification of the concerns raised in §2. On this reading, the "then" also suggests that the second question is a rhetorical answer to the first. For what else could true propositions of Russell's system mean, if not propositions that are proved—or more precisely, asserted in Russell's system? In Russell's system, assertion is possible only for axioms or propositions derived from axioms. The underlying motivation of these questions, as Floyd emphasizes, is the vagueness of "P cannot be proved" or "P is unprovable", that is, the vagueness of the notion of provability[147].

The connection between Wittgenstein's inquiries on truth and on assertion becomes clear in **§6**[148]. Truth is here treated as redundant, reducible to assertion (assertion being contrasted with mere

[145] TS 221, 248 // MS 118, 108r.
[146] Rodych 1999, 179.
[147] Floyd 1995, 400-1.
[148] MS 118, 108r.

utterance)[149]. Wittgenstein writes: "For what does a proposition's '*being true*' mean? '*p*' is *true* = *p*. (That is the answer)." The question 'What does it mean for a proposition to be true?' receives the answer: it means that the proposition is *asserted* under certain *circumstances*. What, then, are the circumstances under which a proposition is asserted in "Russell's game"? At the end of a proof, or when the proposition is one of the axioms (a "fundamental law", or, as it is called in PM, "primitive proposition (Pp.)"[150]). As Wittgenstein puts it, "There is no other way in this system of employing asserted propositions in Russell's symbolism." This need not be read as Wittgenstein's general theory of truth, but it clearly characterizes what "truth" amounts to *within Russell's system*[151].

Against this background, the idea of a proposition that is both true and unprovable sounds like a contradiction in terms[152]. This, according to Wittgenstein, is precisely why Gödel's theorem sounds *surprising*, and our persistent *inclination* to dwell on it. Already in a letter to Schlick in 1935, Wittgenstein cautions that "if you hear that someone has proven that there must be unprovable statements in mathematics, *there is nothing surprising about that at first*, because you have no idea what this seemingly clear prose statement means." This reaction strikingly exemplified the methodological stance Wittgenstein articulates in the inaugural lecture of his 1939 seminar[153], wherein he says: "Suppose Professor Hardy came to me and said, 'Wittgenstein, I've made a great discovery. I've found that...' I would say 'I am not a mathematician, and therefore I won't be surprised at what you say. For I cannot know what you mean until I know how you've found it."[154] Or even better is his analogy of the scientist who comes to you saying "I have at last discovered how to see what people look like in the dark"[155]. In such cases, we illegitimately carry over ordinary or preestablished understandings of terms into a new situation wherein they mean something else for they have acquired a different role. This perspective is echoed in RFM.VII.22: "It might justly be asked what importance Gödel's proof has for our work.

[149] Cf. Priest 2004, 210.
[150] Hoffman 2024, 78.
[151] Floyd 1995, 401.
[152] "[O]n [Wittgenstein's] own terms" according to Rodych (1999, 180).
[153] LFM 17-18.
[154] LFM 17; *Cf.* MS 162a, 93-4.
[155] *Ibid.*

[...]The answer is that the *situation*, into which such a proof brings us, is of interest to us." There is a clear before and after, here. Prior to Gödel's proof, the idea of "true and unprovable" propositions functioned more as a conjecture (for someone like Hilbert who thought it necessary to have a proof of negation-completeness[156]). Its meaning was vague, if it had any[157], and it seemed contradictory given prevailing conceptions of truth and provability. Gödel's proof, however, effects a conceptual transformation, as it were. It introduces a new way of using these terms, thereby creating new concepts[158]. Gödel's proof gave these names ("true", "provable") a new meaning. In particular, "provable" in PM could only be ascribed previously once a proof *had actually been* constructed, while Gödel's novel technique of arithmetization allows for making statements about provability without a proof being found.

Wittgenstein continues to press the interlocutor to clarify "what is being asked *for*" really[159]. In **§7**, he establishes a distinction between propositions of *Russell's system*, and propositions merely written in *Russell's symbolism*[160]. The set-up of the remark involves the imaginary interlocutor asking, as a follow up to the previous remark which gives the impression that Russell's system cannot contain a true but unprovable proposition. §7 specifies that the question must be understood *in Russell's system's own terms*. Wittgenstein suggests that a proposition which is true in another system, "i.e. can rightly be asserted in another game" may be unprovable *in Russell's system*. This does not imply that to assert a proposition in that other system amounts to its being *proved* there; it only indicates that truth and assertion are governed by different criteria across different language-games. Likewise, a proposition belonging to another system—physics, for instance—may be expressed in Russell's *symbolism* (without being part of his system) can be true while being 'uttered' in Russell's system. Conversely, propositions provable in Russell's system or Euclidean geometry may be false in some other system[161]. Wittgenstein

[156] Hoffman 2024, 58.

[157] Frege *e.g.* thought a proof of consistency was unnecessary because following from the axioms which are true meant you have truth and can never derive a contradiction (See Hintikka 1988, "On the Development of the Model-Theoretic Viewpoint in Logical Theory", 8f.)

[158] RFM.III.24,31, 39-41, 46; RFM.IV.47; RFM.V.5,42,45; RFM.VII.44,45,70,72.

[159] Floyd 1995, 403.

[160] *Cf*. Floyd 1995, 402.

[161] RFM.AppIII.7.

illustrates this with the familiar sum of the angles of a triangle equals two right angles is provable in Euclidean geometry, but false in hyperbolic geometry. But aren't these triangles in a different sense? "Of course", Wittgenstein replies. Similarly, "a proposition which cannot be proved Russell's system is 'true' or 'false' in a different sense from a proposition of *Principia Mathematica.*" In other words, if truth and provability are decoupled, then we are facing a situation in which "true" and "provable" cannot be both understood in according to Russell's system[162]. This observation is central to Wittgenstein's philosophical diagnosis.

Priest offers an alternative reading. For him, Wittgenstein wants to argue that mathematical propositions have no propositional content. The redundancy account appealed to in §6 is compatible with this view and becomes more apparent in §7. Since mathematical propositions have "no propositional or informational content", their truth can amount only to being "generated by certain rules"[163]. The difference between propositions with content (paradigmatically empirical/contingent propositions) and those which lack it is in the application, in how an asserted proposition in each of these language-games is applied. Priest appeals to §20, the final remark of the Appendix III (which was added later in TS 223) where Wittgenstein states that "the propositions of logic are so constructed as to have *no* application as *information* in practice." On this basis, Priest concludes that Wittgenstein differentiates logical (but we can also say mathematical) "propositions" from ordinary propositions in that the latter have a '*Bedeutung*'[164] that the former lacks. What makes a proposition have a *Bedeutung*? That it has an *application as information*. This much is plausible. It is more explanatory to frame the matter in this way than in Tractarian terms (lack of propositional content) which Priest employs. Priest, however, takes §7 in a strange direction. While acknowledging the difference in meaning of "provability" in different axiomatic systems, an uncontroversial opinion, he somehow infers that the Gödel sentence "is not provable in the theory itself, but is provable (or can be proved to be true) in a metalanguage/metatheory."[165] I do not *yet* see evidence to justify this claim. The ambiguity that

[162] This is what Rodych (1999, 180-1) reads as well.
[163] Priest 2004, 210.
[164] This is the term used in the remark.
[165] *Ibid.*, 211.

Wittgenstein refers to here, according to Priest, is that of *provability*. It is that which is analogous to the term whose meaning changes between Euclidean geometry and a non-Euclidean one[166][167]. I do not see, however, why this must concern provability alone, and not also the notion of truth, to say the least. Juliet Floyd also sees that the concern here is with both[168]. On her account, the belief that Wittgenstein is trying to undo here is that of provability and truth being independent notions; he is trying to show that they are rather relative to a system or language-game, or more precisely, relative to what Floyd calls "an ongoing technique of use" and not "a formalism itself"[169]. Floyd reads Wittgenstein as invoking skepticism towards a general intuitive notion of "truth" or "provability". She explains though a historical analogy to how Euclidean geometry seems to have been seen as providing "knowledge of (the form of) space"[170], but then the advent of other geometries led to the decrease in popularity of this view, for instance, helping give reason to form the formalist position, for these geometries while they all work, their conceptions of 'point' and 'line' are not the same[171]. As Floyd puts it, "that which is taken to be mathematicized [should] not be separated from the mathematics (i.e., techniques) employed in discussing it."[172] The meaning or sense of space is relative to the geometric system (which, strictly

---

[166] *Ibid.*

[167] Wittgenstein's remark, I find, is quite ambiguous about what is similar from one geometric system and another. Priest simply talks about some proposition in the abstract. Wittgenstein talks about propositions, sure, though he also mentions triangles. "May not triangles be–in another system–similar (*very* similar) which do not have equal angles?" I presume that the analogy concerns triangles having the property that their angles are equal to a certain number of degrees. But what is similar here? It is not the whole sentence "the sum of the angles of a triangle is ___". It must be the predicate, which is, here, (somewhat unusually) being a triangle. We call this thing a triangle and that thing in the other system also a triangle. They are very similar in some aspects of triangularity, yet an essential property differs, and we can recognize that. But I do not see how this is necessarily about the property of being provable and not about the property of being true. To make things more problematic for us, Wittgenstein says another intriguing thing, that "in that case they are not 'similar' in the same sense!" I am inclined to say that this cannot be about the sum of the angles. Is it that similarity differs in sense from one system to another, or is it that our comparing something cross-system and calling them similar is different from comparing things within a system and calling them similar. In geometry, the property of similarity of triangles is different in Euclidean and non-Euclidean geometries. The latter ones require triangles not only to have equal angles and proportional sides, but equal sides. Perhaps Wittgenstein is supposing some (non-existent) geometry wherein equality of angles is not a measure of similarity. This is plausible. But it is still strange that Wittgenstein says "*very* similar". What is this degree adverb supposed to mean? Floyd (1995, 403) thinks it is similarity within different geometric systems.

[168] Floyd 1995, 403.

[169] *Ibid.*

[170] Floyd 1995, 404.

[171] Michael Potter, *Set Theory and its Philosophy*, 2004, 8.

[172] Floyd 1995, 404.

speaking for Wittgenstein, being a language-game, is defined by what we consider to be the correct application/manipulation of the signs). This analogy ties together Euclidean geometry and mathematical logic[173].

In both Floyd and Priest, we see a Wittgenstein who mistrusts a semantic conception of truth in mathematics. Truth for a proper proposition (one with content), has to do with this content. Truth with regards to content is another to way to say semantic correctness/truth[174]. Because this conception is underlain by a correspondence theory of truth—a correspondence to some intuitive notion that does not exist according to Wittgenstein—to speak about the truth of contentless sentences is to speak of truth in a different sense. Some scholars have taken this to mean that truth is always linked to derivability/provability. Priest's reading tends in this direction, identifying truth with conditions of assertion that ultimately reduce to those of derivability. On the other hand, by emphasizing the language-game aspect, Floyd is able to avoid the conclusion that the conditions of assertion are those of derivation. The rules characterizing each language-game or "system" are defined by what we consider to be a correct way of carrying out the rule, in other words, a correct human behaviour is the application of the rule. This conception of rule and the former formal one may coincide in terms of what is considered a proper outcome of following the rule, but the Floydean one is more true to Wittgenstein's (own brand of) skepticism about the conception of logic as something *mechanical*[175] and independent of humans, logic that "takes care of itself", and more importantly, it accords with his own emphasis on *Praxis* (in §20 for instance). To construe the relevant 'systems' here simply as formal systems risks obscuring Wittgenstein's broader philosophical method and purpose.

This divergence is understandable given Priest's general exegetical approach, which aims to assess Wittgenstein's remarks through the lens of contemporary logic. Priest's reading foregrounds logical elements more than other ones, thereby highlighting what may be taken as Wittgenstein's

[173] Floyd 1995, 404.

[174] This is what Gödel calls semantic concept of truth, *inhaltlich richtig* (1931, 176), the term he employs in the introduction to say that is not the notion he will employ in his proof (Prince 2022, 14). This is to be contrasted with truth as in some relation holding or predicate being applicable, which is a syntactic notion that Gödel does employ in his proof, that of "correctness" [*Richtigkeit*] (1931, 194).

[175] The last two paragraphs in RFM.III.81, first in 82, penultimate in 87; echoed in LFM, Lecture XX.

contribution to (or anticipation of) later logical conceptions. On such a reading, Wittgenstein can be seen as pioneering some logical conceptions (namely, paraconsistency). From Wittgenstein's own perspective, however, if any such anticipation took place, it would be *accidental*, an accidental product of his philosophizing, for his investigation is *philosophical* in nature[176]. Philosophy reflects and describes its subject in a way proper to it (it is not part of the object language of logic, nor some standardized meta-logic).

"Here I can only say: depart as quickly as possible from this picture, and see the interest of this calculation in its application. (It is as if we were at a masked ball at which every calculation appears in a queer guise.)"[177] The "picture" in question is that in "the logical calculus of Frege and Russell [...] every word that is spoken in mathematics has exact significance, is an element of the calculus. Thus in every calculus we can really prove that 'multiplying is possible'."[178] This strongly corroborates Floyd's reading, according to which Wittgenstein is resisting a conception of logical calculi as self-contained structures whose expressions are already endowed with determinate meanings that are prior to their use.

Priest's framing in terms of "object language" and "meta-language" is, in some respect, defensible. The problem is the connotations of this vocabulary. (*This* is an opportunity to apply the Wittgensteinian method!) These terms mislead us into imagining a formal distinction thereby reintroducing the very picture Wittgenstein is trying to dismantle. However, it is correct in the sense that Wittgenstein does sometimes distinguish a level of calculation (syntax) and a level at which that calculation/syntax is interpreted (loosely speaking, an 'aboutness'.)[179] Wittgenstein's concern is not to

---

[176] Priest's paper starts with this amazing quote from Wittgenstein's middle period (in 1930) in which he prophetically says: "Indeed, even at this stage, I predict a time when there will be mathematical investigations of calculi containing contradictions, and people will actually be proud of having emancipated themselves from consistency." (PR, 3)

[177] RFM.II.57. *Cf.* MS 121, 69v (27.Dec.1938).

[178] RFM.II.56. (MS 121, 69r)

[179] For Wittgenstein, math is a technique, and "Eine Technik '*handelt*' von nichts. Man mag sie aber mit Hinblick auf die und die Anwendung lehren. Es wäre seltsam, eine Technik des Schleuderns, z.B., eine allgemeine Technik zu nennen, weil sie allerlei verschiedene Verwendungen hat." (MS 162a, 67r) A technique '*deals*' with or '*is about*' nothing. All we can 'mean' by it is an application. See also MS 162a, 36-8: " 'Der Kalkül gibt dem Zeichen seine Bedeutung' heißt : Der *Kalkül* offenbart Dir den Zweck des Zeichens; wozu es denn überhaupt dient — nicht seine Schreibweise allein, nicht ein Bild, das sich mit ihm verbindet, nicht irgend ein Ausschnitt bloß des Kalküls. (So wie man sagen kann: wenn Du den inneren Bau eines Hauses verstehen willst schau nicht nur auf die Fassade — obwohl diese *manchmal* über den Bau Aufschluß geben kann.)" [The calculation gives the sign its meaning" means: The *calculation* reveals to you the purpose of the sign; what it is actually used for — not just its spelling, not an image associated with it, not just some part of the calculation. (Just as one

posit a formal hierarchy of language. Priest's reading here is not entirely out of peril. For this distinction is also not the object of §7. The more plausible interpretation is that Wittgenstein is comparing two systems ('object languages', as it were)—two language-games. But does this not imply that we are talking at the metalinguistic level, one that encompasses these two systems? Sure, but this level is not the object of the comparison in §7.

One might propose another defence for Priest. The relation between truth and provability is a relation between the object language (where proofs take place) and a metalanguage (in which truth is ascribed), and truth is the process of acknowledging that a proposition has been proved, recognizing (from outside) the assertability of the last line in the proof. But this is not entirely compatible with a redundancy conception of truth (which Priest attributes to Wittgenstein). Such a two-level account is closer to a Tarskian framework, precisely insofar as it relies on a principled distinction between levels. By contrast, in the context in which Wittgenstein couples truth and provability (and recall that for Priest this is always the case for Wittgenstein), there is no room for such a stratification. Truth is entirely reducible to the fact of being proved. Commenting on this provability does not generate a metalanguage in any substantive sense.

3.2.2. Disambiguating "P is unprovable in Russell's system" (App.III.8)

RFM.III.8, dubbed "the Notorious Paragraph" by Floyd and Putnam, deserves its own part given the controversy surrounding its interpretation. This polemic is arguably less a function of the paragraph *per se* than of the influential reading proposed by Floyd and Putnam in (2000), which had elicited a lot of responses[180]. §8 is composed of two paragraphs, the first of which is the voice of an interlocutor asking Wittgenstein for "advice". The interlocutor says:

> "I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says: 'P is not

might say: if you want to understand the internal structure of a house, don't just look at the façade — although this can *sometimes* provide information about the structure.)]

[180] Rodych wonders why this is the only remark they chose to qualify so dramatically. He says the same could be said of RFM.App.III.5,10,17 (2003, 282).

> provable in Russell's system'. Must I not say that this proposition on the one hand is true, and on the other hand is unprovable? For suppose it were false; then it is true that it is provable. And that surely cannot be! And if it is proved, then it is proved that it is not provable. Thus it can only be true, but unprovable."

On the one hand, there is some attempt at a representation of the mathematics going on in Gödel, and on the other, it is a set up for a philosophical question. Aiming to describe the present (actual) situation does not (necessarily) preclude Wittgenstein from presenting particular hypothetical scenarios. These scenarios are meant to change our view of the present and actual situation in that they serve to bring to light and to challenge our preconceptions of the notions involved. In other words, all Wittgenstein needs to do is to present a point of view (an interpretation) of the particular mathematical situation that is Gödel's theorem, a "possibility,"[181] a situation presenting symptoms of the attitude Wittgenstein aims to alter[182], an attitude that is the fruit of a picture that (may) cloud our judgment. He does not *need* to present a real point of view—a point of view that somebody holds, because the picture that informs it is not particular to Gödel's theorem. It can be found in any similar enough situation involving the same notions or forms of reasoning. Hence, insofar as it is irrelevant, I will not try to trace the historical origin (if any) of such points of view. Some scholars (Rodych, Steiner and the early reception) infer Wittgenstein's misunderstanding of Gödel's theorem from §8 when they see it as an intended portrayal of Gödel's proof. To oppose this attribution of misunderstanding in §8, it suffices to argue that this passage is not intended by Wittgenstein to represent Gödel's own perspective. Two considerations support this denial. First, the context of the remark: Nothing in the text indicates that he is addressing Gödel's argument as such; he does not attribute the reasoning to Gödel, nor does he frame it as an exposition of Gödel's proof. Second, the long-running thread in this thesis showcasing Wittgenstein's methodology: he consistently distinguished between mathematics itself and philosophical reflection on its use and interpretation. Floyd and Putnam supply some anecdotal material to leverage a defence of Wittgenstein. They reopen the exegetical case to argue that he understood

[181] "What interests me is not Gödel's theorem but the possibilities to which G. draws our attention through his discussion." (MS163, 37v-38r)
[182] RFM.III.82.

Gödel's theorem while surmising that the Austrian philosopher even had a mathematically significant insight. Anecdotal evidence might help support the case, but it should only be secondary, and not what initiates an interpretation of the remarks. Floyd and Putnam's generous interpretation is not incredibly evident, but it is by no means impossible. Setting aside the low level of reliability of the evidence presented, even if it were granted, given Wittgenstein's methods, it would remain accidental that the insight is relevant mathematically, and thus, I will not address this claim here.

Returning to §8 itself, it is worth asking why Wittgenstein introduces the passage by "imagin[ing] someone asking [his] advice" rather than by something like "Gödel says", invoking Gödel explicitly. To insist that the interlocutor represents Gödel is to introduce an extra assumption (which a hermeneutic 'Occam's razor' can cut away). Besides, Wittgenstein recognizes the importance of Gödel's proof[183], and it is known that Wittgenstein maintains a non-revisionist stance toward mathematics: one can only change mathematics by presenting mathematical arguments, "[philosophy] leaves everything as it is."[184] It is neither his goal to change the calculus nor is it characteristic of his method to be of the mathematical sort. Wittgenstein is just trying to describe the situation into which Gödel brings us[185].

> **MS 121. 84r (January 2, 1939).** Gödel confronts us with a new situation. What shall we say about this? One mustn't of course be hasty in deciding what to say. In particular, one mustn't be hasty in wanting to say what sounds most sensational. The situation is more difficult to assess than it seems.

The other major charge leveled against Wittgenstein is that he (mistakenly) interprets Gödel's argument as resting on drawing a contradiction or paradox between a mathematical fact (that P has been proved) and the natural language self-referential interpretation (that P says of itself that it is unprovable). But, I am arguing, there is no sufficient evidence to justify that that is what he thinks Gödel is saying.

---

[183] This is uncontroversial, it is acknowledged by: Steiner (in that at least that's officially what Wittgenstein claims to do, though for Steiner, not the case for the Gödelian remarks) (2001, 258); Lampert (2017,325-6); Floyd and Putnam (*ibid*.), among others.

[184] PI 124.

[185] "It might justly be asked what importance Gödel's proof has for our work. For a piece of mathematics cannot solve problems of the sort that trouble *us*.–The answer is that the *situation*, into which such a proof brings us, is of interest to us. 'What are we to say now?' –That is our theme." (RFM.VII.22 / MS 124, 98, 4.Jul.1941)

Even if he is rehearsing a self-referential interpretation and criticizing it, he need not be attributing it to Gödel. He cannot be critical of mathematics, but an interpretation of it[186]. This is more coherent with his method and broader aim to describe how our understanding of mathematical concepts changes. Wittgenstein is not even offering *one* way to see GIT, but several possible ones.

Wittgenstein's reply—the "advice" given in §8—begins with yet another attempt to clarify the meaning of 'true', but here the emphasis is the *relativity to a system*. "Just as we ask '"provable" in what system?'" he writes, "so we must also ask: '"true" in what system?'" Since systems are essentially defined by their rules and proofs tend to be an application of those rules, it is natural to think of provability relative to a system. Wittgensteinstein's point is that the same holds for truth. What "true" means in Russell's system has been established in §6; in §7, the relativity to the system of truth and provability, as well as their relationship, was made explicit. Now, in §8, he considers an argument for there being a proposition in Russell's system that is true and unprovable, and examines it in light of the preceding.

As Wittgenstein puts it: "'True in Russell's system' means, as was said [in §7]: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system."[187] So 'true' in (or *according to*) Russell's system—or in "*the Russell sense*" as he says later— refers to the assertion of a proposition (*i.e.*, is said of a proposition which we have rightfully asserted), and this is *identical*[188] with the proposition's being at the end of a proof, that is, with the proposition's being proved. Similarly, a proposition being 'false' is *identical* to the opposite having been proved. In Russell's system, there is no "unproof" that could establish non-truth/falsehood.

3.2.2.1. Derivability VS. Provability: A Way to Distinguish the Philosophical and the Mathematical

Having identified reasons to doubt the standard interpretations of §8 in general, I will now attempt to articulate an alternative reading that avoids attributing mathematical confusion to Wittgenstein while addressing some specific exegetical moves in the secondary literature. I claim that

[186] Interestingly, Lampert (2017) agrees with this, but he also thinks Wittgenstein is legitimately attacking Gödel because Gödel's proof *does* involve a meta-mathematical interpretation which "is a valid object of philosophical critique." (325)

[187] RFM.AppIII.8.

[188] In §6, we have an identity/equality sign: " '*p*' is *true* = *p*".

Wittgenstein's aim here is to demonstrate a risk of an equivocation. He thus endeavours to disambiguate possible meanings of *"P is not provable in Russell's system"*.

(1) "Provable in Russell's system" can mean 'provable *in Russell's system/sense*', I will call this one *derivability* in RS (Russell's sense). It refers to the definition just stated above, that is we say of a proposition that it is derivable (or not) *only* when there *is* a proof (or there is a proof of its negation). Deriv*ability* does *not* refer to the *potential* of a proposition to be proved. Strictly speaking, it is retrospective: it tracks what has been proved. I nonetheless keep the suffix "-ability" because in the context of Gödel, Wittgenstein speaks of prov*ability*, whether it is in Russell's sense or not. Wittgenstein does not disambiguate by systematically contrasting "proved" or "derived" with "provable" or "derivable". His point is precisely that we can indeed talk about 'derivability/provability' while specifying that it is in Russell's sense, or hear about 'derivability/provability' and assume it is the one belonging to Russell's system, but, as Wittgenstein aims to stress, we have to be careful to identify what we really mean, lest confusions remain unchecked. Once clarified, we understand that this "ability" (possibility) is not asserted in itself, but only inferred from the established fact that a proof is known.

(2) The other possible reading of '*provable in Russell's system*', as the italics suggest, emphasizes that this expression is taken as a single technical expression. It is, for instance and in this case, the provability predicate "$Bew_F(x)$" (x is provable in the system F[189]) in *Gödel's formal system*. The provability predicate is in Gödel's system, while it concerns proofs carried in RS according to RS's rules of inference. Here, provability does concern the *potential* of there being a proof, for, in Gödel's system, $Bew_F(x)$ is contrasted with xB_Fy (x is a proof sequence for y in the formal system F), a formula expressing that the sequence of formulas x is a proof of this formula y. In xB_Fy the proof is presented, it is actual. To disambiguate, I will call this one *Provability in PM*. I say 'PM' and not 'RS' because there is no operation to determine whether something is prov*able* in Russell's sense, independently of a proof being *found*, that provability predicate does not exist. Even if Gödel's formal system includes (arithmetical) predicates and relations that can, roughly speaking, be taken as 'about' Russell's system, Gödel introduces new ways of understanding *Principia Mathematica*—ways previously foreign to Whitehead

[189] More specifically, x is *the Gödel number of a formula* that is provable in the formal system F.

and Russell's own framework—where *Principia Mathematica* itself becomes subject to (indirect, syntactic) representation, *i.e.* by making it possible for arithmetic to represent and reason about PM itself. In this way, PM acquires a different sense than RS.

Rodych (1999) also notices "an apparent vacillation" between "proved" and "provable". He takes this to be a wavering between "Wittgenstein's own view (e.g. §6 and §8), and [...] Wittgenstein considering GIT on its own terms (e.g., §7 and §17)"[190]. These apparent shifts are precisely what motivates the present section. However, I have chosen to frame the relevant distinction in terms of *derivability* versus *provability*, rather than *proved* versus *provable* because the latter were already employed by Wittgenstein (in a, strictly speaking, inconsistent way), risking confusion when citing his remarks. There are a few problems with Rodych's reading. Wittgenstein's use of "proved" and "provable" does not reliably track a shift between Wittgenstein's view and Gödel's because, as I already argued, Wittgenstein is not addressing Gödel himself, and Wittgenstein does not present a "view" but approaches the topic from several angles and suppositions. Instead, what is worth distinguishing in his discussion is the conceptual distinction between considering the fact of proof in actuality or potentiality. There is no textual clue, no difference in way of expression that warrants interpreting §7 as Wittgenstein considering GIT on its own terms and §6 as his own. As has been argued, it is plausible that Wittgenstein is laying out what the meaning of "provability" *would be* in RS in §6 according to RS's own terms, then, in §7, that he is expanding his scope to talk about other systems to insist that truth *and* provability are relative to a system/language-game. Had Wittgenstein intended §6 to articulate a general theory of truth he holds, he would have been making the sort of claim he is well known to rebuke. If, instead, he meant to express his own understanding of truth in RS in particular, then it would still make sense that §7 introduces other systems in order to show how easily we are misled when we ignore this relativity when, for example, we speak of truth and provability in different systems in the course of the same sentence.

[190] Rodych 1999, 177.

The distinction between provability and derivability also helps mark the difference alluded to earlier between *Gödel's system* and Russell's system, which seems to underlie Wittgenstein's remarks. Gödel's system is constructed on the basis of the axioms of *Principia*, but it is, in addition to that, that formal system in which syntax is arithmetized thereby rendering metamathematical statements mathematical. Gödel's proof proceeds using the axioms of PM, supplemented by Peano Arithmetic and other technical devices introduced for convenience, so strictly speaking, they could be dropped since the proof could in principle be carried out using PM alone, albeit at the cost of considerable complexity. Given this, from a *mathematical* point of view—*i.e.* the informal way mathematicians commonly describe the situation[191]—we can consider that Gödel's result has been derived within PM, demonstrating the existence or the construction of an undecidable sentence *within PM*.

From Wittgenstein's philosophical perspective, however, given that *Principia* as Russell and Whitehead devised it (RS) and Gödel's proof lay a multitude of additional *techniques*. Having different techniques at our disposal is, for Wittgenstein, *sufficient* ground for distinguishing Gödel's system from Russell's system (or at least, a sufficient ground to warrant skepticism towards the coincidence of the system that Russell and Whitehead created and the one Gödel used[192]). For this reason, when I speak of "PM", I mean PM as it is seen by mathematicians (according to Wittgenstein); while when I say "RS" (Russell's system) I mean the same system but as seen philosophically by Wittgenstein.

Consequently, this is another point at which an investigation of mathematics within philosophy can be distinguished from a mathematical one in Wittgenstein's Gödelian context. This example constitutes an ostensible difference in this particular case, it does not reveal the *essential* difference between these two kinds of investigations. In the investigation of the philosophical kind, it is sufficient to differentiate between Gödel's system and Russell's system on the grounds that a new concept has been created (through a new technique). What may appear to be a change in a concept—or more misleadingly, according to Wittgenstein, what appears to be a clarification of the mathematical concepts

[191] For instance, Prince 2022, 14-15.

[192] If we want to be careful about attributing theses to Wittgenstein. His point is not to present a thesis, but to propose alternative views we could not or did not imagine or consider.

of "truth" and "provability" by Gödel's proof, is in fact the creation of a new concept[193], the establishment of a new language-game, which is equivalent to having a new application, and in this case, it is the effect of the introduction of a new technique. There is no 'real' concept of "truth" or "provability" or even of "the mathematical" which we are to *discover*.

***

Now let us return to §8 and examine how the concepts derivability and provability are at work there. Wittgenstein follows the definition he provided of *being true* and *being false* with a question about the meaning of *supposing*[194] the falsehood of *P*. His answer:

> In the Russell sense it means 'suppose the opposite is proved in Russell's system'; if that is your assumption, you will now presumably give up the interpretation [*Deutung*] that it is unprovable. And by 'this interpretation' I understand the translation into this English sentence.

The conditional "*if that is your assumption*" means 'if you assume that *P* is false in RS'[195]. Its consequence is that "you will now presumably give up the interpretation that it is unprovable. And by 'this interpretation' I understand the translation into this English sentence." If *P* is false in (or according to) RS, this means that ¬*P* has been derived in RS. Now suppose that P had initially been interpreted as "*P* is unprovable," and thus ¬*P* as "*P* is provable." The existence of a proof of ¬P, which is described in English as ¬*P* being proved, then comes into conflict with that interpretation, *i.e.* it contradicts another English sentence purporting to describe the same fact, namely "*P* is provable". This would be a contradiction between two *English sentences* that claim to describe the fact of the proof. In §11, the same idea reappears, though in a slightly different form. Instead of talking about it in terms of a description of the fact of the proof of *P*, he rather speaks of *P*'s belonging to Russell's system, which is equivalent to *P* being a theorem, and hence equivalent to the fact that *P* has been proved.

---

[193] "The idea that proof creates a new concept might also be roughly put as follows: a proof is not its foundations plus the rules of inference, but a *new* building—although it is an example of such and such a style. A proof is a *new* paradigm." (RFM.III.41)

[194] Recall RFM.VII.22 :"However queer it sounds, my task as far as concerns Gödel's proof seems merely to consist in making clear what such a proposition as: 'Suppose this could be proved' means in mathematics."

[195] One might wonder if by "that assumption" he could be referring to the fact that true in RS means proved and false meaning the opposite is proved. But given the possibilities listed in this remark, and the end of the remark, it only makes sense that the supposition of being false in RS is listed at first.

Faced with such a contradiction, the interpretation must be abandoned. Why? Because the proof itself cannot be rejected (supposing it is correct or at least that we take it to be so[196]) given what we think a mathematical proof is. If what stands against the mathematical proof is the interpretation, wouldn't we sooner consider altering our interpretation? Aren't interpretations more malleable than proofs? This is why, for Wittgenstein, we must conclude that our initial interpretation was incorrect, that the signs of RS cannot be so interpreted (*if* proof and truth are understood according to RS). This is the question he asks in §10: If someone somehow insists on holding on to the interpretation, what reason could they have for doing so? "[W]hy should I not let the proof stand and say I must withdraw my interpretation 'unprovable'?" What would be a compelling reason to deviate from our usual practice here?[197]

3.2.2.2. The Logical Possibilities in §8

The remainder of §8 considers the other logical possibilities. Having dealt with the supposition that *P* is false in RS, Wittgenstein turns to the consequences of assuming that *P* is provable in RS (*i.e.* derived), then that *P* is true in RS, and finally that *P* is false in another sense than RS's. The first three are logically exhaustive of the possibilities within Russell's system, because the supposition that *P* is unprovable, or "unproved" in RS is not, at least at this stage, meaningful in RS[198].

> **§8.** [...]If you assume that the proposition is provable in Russell's system, that means it is true *in the Russell sense*, and the interpretation "*P* is not provable" again has to be given up. If you assume that the proposition is true in the Russell sense, *the same* thing follows. Further: if the proposition is supposed to be false in some other than the Russell sense, then it does not contradict this for it to be proved in Russell's system. (What is called "losing" in chess may constitute winning in another game.)

The second conditional states that if we assume that *P* is *derivable* (in RS), it means we have found a proof in RS. As a result, the English translation/description "*P* is not provable" has to be abandoned, since as a matter of fact, the proof exists, and this fact is *identical* to the one denoted by "*P* is provable in Russell's sense". Describing what is the case in the calculus with the English sentence "*P*

[196] A consideration that Wittgenstein raises in §10: "if this were now proved, or if I believed–perhaps through an error–that I had proved it". Because what matters is that we consider it proved, and it is in this case that we wonder what we should do next. Wittgenstein is avoiding any appeal to some independent objectivity.
[197] *Cf*. Lampert 2017, 335.
[198] RFM.App.III.16.

is *not* provable in RS" is therefore unwarranted, we would have to drop the negation on account of the fact that there is a proof. For in RS, '*P*' is true = *P* (*i.e.*, to say that *P* is true is just to write $\vdash P$), so '*P* is provable' is true = *P* being proved (*i.e.*, $\vdash P$). As the next remark (§9)[199] indicates, we employ English (natural language) expressions to designate parts of the calculus, the occurrence of a string of signs in its notation. Sometimes, we have two expressions that denote the same thing: "*P*"[200] and "*P* is unprovable" denote the same formula of the calculus.

The third possibility consists in the assumption that *P* is true in RS, which implies that a proof in RS has been found (because that is what it means to assert *P* in that system), and thus the same consequences follow as in the second case. Finally, if we assume that *P* is false but in another sense than that in RS, then whether it is proved or unproved in RS would not yield a contradiction. A contradiction is intrasystemic. Two claims cannot contradict one another if they are incommensurable, that is if they belong to different language-games. The other sense of false is not specified here, so whatever it may be, it bears no relation to provability in RS. This is the point of the parenthetical chess analogy with which the remark concludes.

3.2.3. The Problem of Semantics

3.2.3.1. The SRNLI (The Self-Referential Natural Language Interpretation)

> **MS 163 39v-40r** (July 8, 1941) Gödel's Theorem develops a problem that must appear in a much more elementary way. (And herein, it seems to me, lies both Gödel's great service to the philosophy of mathematics and, at the same time, the reason why it is not his particular theorem that interests us.)

---

[199] This is §9:

> For what does it mean to say that P and "P is unprovable" are the same proposition? It means that these *two* English sentences have a *single* expression in such-and-such a notation.

[200] Wittgenstein does not put this one in quotes. If Wittgenstein is rigorous with his quote usage, I presume, that this is because Wittgenstein thinks of the use of "P" as consisting in denoting a sentence, since it is an abbreviation. He does not put it in quotes in "P is unprovable" which might be taken against his understanding of the indirect reference. But this does not hold because he is aware of the kind of reference that it is, and says so explicitly in MS 121, 82v (Dec, 31, 1938), and 83v (Jan 1, 1939) which Rodych (2002, 380) cites as evidence to Wittgenstein's understanding of the number-theoretic nature of the Gödel sentence. Priest (2004, 213) notes "a small slip" here: that P is not a sentence of English but of *Principiaese*. I would say that it is not a sentence of Principiaese, but an abbreviation of a sentence.

> **MS 163 40r** (July 11, 1941) I could say: Gödel's theorem gives us the stimulus to change the perspective from which we used to see mathematics. *What* he proves doesn't concern us, but we have to come to grips with his mathematical *method of proof.*

Wittgenstein often employs the expression "true and unprovable" to refer to the Gödel sentence. The use of the expression "true", "true and unprovable" is considered to involve semantics, but Gödel's proof is notoriously syntactic in nature (and not semantic), something that Gödel continuously draws attention to in his (1931) paper[201]. Gödel, however, does provide a semantic equivalent of his theorem as well as a proof sketch for it in the introduction as it is more intuitive to the reader. He speaks of "a proposition that says about itself that it is not provable", "interpreted according to the meaning of the terms of PM"[202]. At the end of this introduction, Gödel states that his proof "will replace [the assumption that every provable formula is true in the interpretation considered] by a purely formal and much weaker one". This gives reason for many scholars (the early reception, Steiner, Rodych) to conclude that Wittgenstein's treatment of Gödel is solely or mainly on the semantic level, that Wittgenstein takes the semantic proof to be Gödel's proof, thus demonstrating the inadequacy of the remarks. Many take Wittgenstein's critical stance to the semantic arguments to be a critique of Gödel. Some argue that Wittgenstein is unfair to Gödel, while some argue that he is unfair to the semantic proof (on account of it being a legitimate piece of mathematics too).

In this section and the following, three questions are of interest to us: 1) Does Wittgenstein read Gödel's first incompleteness theorem (GFIT) (only) in the semantic way? 2) Does he think that that is where Gödel went wrong? 3) Is it accurate at all to talk about "true but unprovable" in the context of GFIT? To what extent are semantics relevant to Gödel's theorem? As I have already been arguing so far, to conclude (or assume) that Wittgenstein is addressing Gödel is a leap. I will now address their arguments and showcase this interpretive jump. I will also show that Wittgenstein does not exclusively make such a reading of GFIT. Finally, I will also address the extent to which semantics are relevant to GFIT, as this is more relevant in general.

[201] A few occurrences will be cited below.

[202] Gödel (1967) 598.

What semantic proof in Wittgenstein?

The semantic interpretation is suggested both in the first paragraph of RFM.AppIII.8 and in Wittgenstein's mentions of "true and unprovable". Steiner, for instance, highlights the paradoxical aspect presented by Wittgenstein, claiming that he treats the emergence of a contradiction between *P*'s falsehood and its provability as pivotal in the argument he attributes to Gödel[203]. The argument may be summarized as follows: '*P*' expresses the proposition '*P* is not provable'; if *P* is false, then ¬*P* is true and can be asserted, which means that ¬(*P* is not provable) can be asserted, and this is equivalent to '*P* is provable'. Therefore, we can assert *P* is provable when *P* is false[204]. Given the assumption of soundness (or correctness) of the system, namely that only true sentences are provable, the simultaneity of the *P*'s falsehood and its provability constitutes a contradiction[205]. Steiner charges Wittgenstein with failing to acknowledge this crucial assumption. In Steiner's eyes, Wittgenstein not only misses his mark against Gödel, but also unduly rejects a valid informal semantic version of the proof[206]. Moreover, Steiner maintains that the interlocutor's reasoning in §8 relies on a conception of "truth in Russell's system" that turns out to be Tarskian, specifically, "truth as satisfaction by all sequences of natural numbers."[207] In order to make the case that Wittgenstein is wrong about criticizing this notion of truth, Steiner formalizes a set theoretic semantic proof based on Tarski's definition after presuming to have established that all its properties underlie the conception of truth in the interlocutor's reasoning[208]. Steiner, however, needed to add one more property to this conception of truth for his proof, and several other assumptions[209]. "What is fallacious here is that we have a *mathematics theorem* that 'losing' in Russell's system implies losing in Tarski's! So if 'false in some other than the Russell sense' includes the 'Tarski sense' Wittgenstein is simply wrong", Steiner concludes[210]. Steiner's argument is methodologically

[203] Steiner 2001, 263.
[204] Steiner 2001, 261.
[205] *Ibid*., 262.
[206] Steiner 2001, 263.
[207] Steiner 2001, 264.
[208] *Ibid.*
[209] See Steiner 2001, 265f.
[210] *Ibid.*, 267.

suspect. Steiner himself acknowledges that there is no evidence to establish that Wittgenstein was familiar with Tarski's work[211].Moreover, if one were to judge Wittgenstein's response to an argument, mustn't one look at the argument itself, rather than reconstruct another that is based on the same concept (or similar rather, for one concept has a property that the other lacks)? Should Wittgenstein be faulted for failing to anticipate Tarski[212]?

Rodych likewise argues that Wittgenstein commits a mistake in §8 and the surrounding, and that this error was to blame for the unforgiving early reception of his reflections[213]. The mistake, Rodych claims, lies in Wittgenstein's purported assumption that GFIT requires the natural language interpretation 'P is not provable in Russell's system' in order "to obtain the threatened contradiction", when in fact, the contradiction arises purely from syntactic manipulations, *i.e.*, without appeal to such an interpretation[214]. Although Rodych acknowledges Wittgenstein's conditional phrase "*if that is your assumption*" and concedes that this is not Gödel's own assumption, he nevertheless concludes that Wittgenstein takes himself to be addressing Gödel. In doing so, Rodych maintains that Wittgenstein presupposed that Gödel's argument relies on the conception of provability in Russell's sense[215] —which I have called *derivability*. Rodych does not distinguish this one from provability according to Gödel's system. In re-endorsing his (1999) reading of §8 in (2003), Rodych writes that "[i]t is virtually impossible to interpret the interlocutor's Gödelian reasoning in (§8, par. 1) any other way."[216]

But should the issue not be what the interlocutor is actually saying? After all, the interlocutor may well be presenting a defective SRNLI, if Wittgenstein's aim is precisely to critique such a misunderstanding. What ultimately matters, therefore, is Wittgenstein's response to the claim as presented. Rodych contends that the response in §8 is inadequate because Wittgenstein urges the withdrawal of an interpretation that is irrelevant to Gödel's actual proof, whereas the only interpretation

[211] *Ibid.*, ft.22.

[212] Tarski did publish the relevant work a few years before Wittgenstein's remarks, but it seems "Wittgenstein never once mentioned Tarski's work on truth;" so it is often assumed that he was not familiar with them. (Floyd 1995, 303)

[213] Rodych 1999, 182.

[214] Rodych 1999, 182.

[215] Rodych 2003, 307.

[216] Rodych 2003, 287.

that bears on the proof "is the *standard number-theoretic* interpretation of 'P', which Wittgenstein does not (here) challenge."[217] Yet Rodych's charge of inadequacy presupposes that Wittgenstein intends to address Gödel's genuine proof, and thus evaluates Wittgenstein's remarks by comparing the interlocutor's claim to the structure of Gödel's argument itself.

In other passages of the *Nachlass*, Wittgenstein does display a number-theoretic understanding of the Gödel sentence—one that does not rely on the SRNL interpretation[218]. Interestingly, Rodych himself acknowledges this understanding of Wittgenstein's as well as that concerning the indirect reference in his 2002[219] and 2003[220] papers. In the latter, Rodych moderates his earlier assessment, speaking of Wittgenstein's "partial understanding" rather than claiming outright that these passages exhibit genuine understanding[221]. In (2002), Rodych examines "the newly published remarks" from the *Nachlass*, arguing that they support the same position he had defended in 1999, when his access was limited to the RFM material. Across these discussions, Rodych engages both the early reception and several of Floyd's arguments in her 1995 paper. In (2003), Rodych's primary aim is to respond to Floyd and Putnam who had advanced a more robust defense of the philosophical value of Wittgenstein's remarks on Gödel[222]. Consequently, the shifts in the strength of Rodych's claims about Wittgenstein's understanding appear to stem not from the availability of additional *Nachlass* material, but rather from his reading of AppIII *in the attempt* to respond to Floyd and Putnam's interpretation of the third appendix, in particular that of §8, the notorious paragraph. Rodych's revisiting of the *Nachlass* remarks is less charitable, because he examines those remarks in light of Floyd and Putnam's thesis in order to undermine their argument that Wittgenstein possessed a deeper grasp of Gödel's result. For instance,

---

[217] Rodych 1999, 182.

[218] MS 121, 82v (Dec 31, 1939), 83v (Jan 1, 1939).

[219] Rodych 2002, 380.

[220] Rodych 2003, 303.

[221] For instance, about MS 121, 83v (Jan 1, 1939), Rodych says in (2002, 380) that this remark "is further evidence that Wittgenstein correctly understood the number-theoretic nature of Gödel's proposition." Whereas in (2003, 303), he claims that it only shows "partial understanding". Concerning MS 163, 41v (July 11, 1941), Rodych describes it in (2002, 381) as "still further indication that Wittgenstein understood how Gödel's proof was intended to work". While in (2003, 303), it's still a matter of partial understanding.

[222] Rodych 2003, 282.

> **MS 121 83v (January 1, 1939): '**This proposition is one that cannot be obtained by the operations…' If one takes here the "this" as reflexive, then this could simply be a shorthand for : "This proposition: 'This proposition…' is not obtainable…", & here 'this' would not be used reflexively. The proposition would be a mathematical proposition but written as a proposition about its own form (a special way of writing).

Rodych recognizes that this remarks shows "Wittgenstein's genuine *understanding* of the mathematical nature of Gödel's proposition"[223], and that "despite repeated mistaken construals of the Gödelian proposition [in RFM.AppIII.7-10], Wittgenstein does appear to understand, even in 1939, that a natural-language, self-referential interpretation of the Gödelian proposition is unnecessary."[224] Yet in his next article, Rodych simply says of MS 121, 83v that it shows "perhaps partial, understanding" without offering much justification other than pointing to its similarity to MS 121, 76r, which states: "Gödel shows us an unclarity in the concept of 'mathematics', which is indicated by the fact that mathematics was taken to be a *system*."[225] But while both of these remarks pertain to Gödel's theorem, they concern widely different aspects of it. The two remarks are concerned with different modes of reflection, one seems to be a reflection on the elusive (indirect)"self-reference", the other one is a general reflection on Gödel's theorem's insightful contribution. Thus to compare them cannot be informative about Wittgenstein's understanding of GIT in either. It does not provide any justification for the attribution of partial understanding to Wittgenstein from the first remark. Moreover, passing over such remarks while emphasizing select passages that appear extreme or dismissive—such as the one calling the Gödelian reason "a stupid one"—is difficult to regard as even-handed treatment[226]. This is the case for a few passages from MS 121, MS 163 and MS159 to which Rodych does not do justice. Another example is MS159, 22v, composed in August 1938, prior to the writing of Appendix III. Rodych reads this remark as mere "rumination", *i.e.*, part of Wittgenstein's process of working out Gödel's reasoning.[227] In this remark, Wittgenstein speaks of the sentences: "No. 27 is not provable" and "(No.*x*)No.*y* is not

[223] Rodych 2002, 380.
[224] Rodych 2002, 381.
[225] Rodych 2003, 303.
[226] Rodych 2003, 302.
[227] Rodych 2003, 296.

provable"[228]. These can easily be read as corresponding, respectively, to 'the sentence with the Gödel number 27 is not provable' and 'there is no sentence with the Gödel number *x* such that it is a proof for the sentence with the Gödel number *y*'[229]. These are the ways one would read Gödel's crucial arithmetical functions involved in GFIT, namely the provability predicate *Bew(x)* (so Wittgenstein's sentence can be formalized as: ¬*Bew(27)*), and the proof predicate *xBy* (here: *(x)*¬*xBy*). Gödel intends these predicates to express purely number-theoretic relations, i.e. to be functions calculating relations between numbers, though he does not always present them in fully formalized notation. At times, he writes propositions of the form: such-and-such number-theoretic function is not provable (a predicate that is also a number-theoretic function) rather than explicitly referring to formulae via their Gödel number. [230] Indeed, Gödel even writes "17 Gen *r* is not K-PROVABLE"[231] when demonstrating the undecidability of 17 Gen *r*, instead of writing ¬Bew$_K$(17 Gen *r*). Since, 17 Gen *r*, the Gödel sentence, may be replaced by *P*, what is being calculated can be legitimately paraphrased as"P is not K-provable" or "*P* is not provable" or even "I am not (K-)provable"[232], provided we know these are such paraphrases, or, as Wittgenstein calls them, "English translations." These are one of many hints that Wittgenstein does not conceive of GIT as necessarily involving a SRNLI. As a consequence, this makes it all the more suspicious to read those passages in which Wittgenstein appears to invoke an SRNLI as attempts to represent Gödel's actual argument.

3.2.3.2. *"To Interpret in Terms of Content" and the Application of Mathematical Propositions*

> **MS 163, 35r-36v** (July 8, 1941): The difficulty here is to gain the appropriate perspective for this investigation, one that does not show the insignificant, but rather everything essential. Our perspective should show us the parts of mathematics that seem so important and promising to those investigating the fundamentals

[228] MS 159, 22v; "(No.x)" is to be read as "for all numbers x".

[229] For the definitions of these functions, see Gödel's paper (1967) , 606; For Gödel stating that he will abbreviate sentences expressing 'so and so is the Gödel number of a formula' as 'so and so is a FORMULA.' see (*ibid.*, 601). Some translations use italics rather than capital letters to indicate a number theoretic formula.

[230] Gödel (1967), 601.

[231] 17 is the Gödel code of a variable of type 1: x_1.

[232] Wittgenstein talks about formulating it with the K in MS 163, 32r.

> in the most condensed form, while showing those aspects of mathematics that seem uninteresting to them in their full extent.

According to Dirk W. Hoffman, Kurt Gödel deliberately avoided the notion of truth (unlike Tarski[233]) in his proof because it was controversial at the time, owing to the prevalence of semantic paradoxes that threatened the developing axiomatic systems that were meant to be at the foundation of mathematics[234]. John W. Dawson reports Gödel to have said that "a concept of objective mathematical truth [*ein Konzept der objektiven mathematischen Wahrheit*] [...] was viewed with [the] greatest suspicion and [was] widely rejected as meaningless"[235]. Moreover, the syntactic character of the proof of GFIT had the additional advantages of addressing Hilbert's program and of providing grounds for deriving GSIT (the second incompleteness theorem) as a corollary[236].

In a certain respect, this is highly compatible with Wittgenstein's perspective, according to which mathematics is, broadly speaking, akin to a 'mechanical'[237] technique and 'interpretation' or 'understanding' are irrelevant to it—or, as Priest puts it, he rejects the idea of "propositional content" in mathematical propositions. This bias against semantics, for which Steiner criticized Wittgenstein, has in fact also been directed at Gödel by Jaakko Hintikka[238]. Hintikka claims that the influence of Gödel's bias towards syntactic or proof-theoretical methods was responsible for delaying the development of model theory to "some twenty-five years later", "even though Gödel's incompleteness results strikingly demonstrated the need of a serious model theory"[239]. Today, however, GIT is often discussed and taught in model theoretic terms[240]. Just like Gödel's initiating the reader with a semantic rendition, GIT is often taught by semantic arguments, at least at an initial stage, or by replacing some subproofs by their semantic counterparts, on account of the complexity of the fully formal method[241]. While GIT is

[233] Dawson 1997, 59.
[234] Hoffman 2024, 135.
[235] *Ibid*, 58-9.
[236] Bays 2004, ft. 28.
[237] For Wittgenstein's suspicions of the *picture* of logical *machinery*, see Lecture XX of LFM, in particular pp194-199.
[238] Jaakko Hintikka, "Kurt Gödel: An Introduction", 2005, 453.
[239] *Ibid.*
[240] Priest 2004, 224.
[241] See Raymond Smullyan's *Gödel's Incompleteness Theorems* (1992) ; Dirk Hoffman's (2024) ; Peter Smith's (2020).

recognized as a syntactic proof, and this has its own strengths in that it relies on fewer assumptions and is thus much stronger, semantics are not wholly irrelevant nor entirely absent. For starters, the interpretability of Gödel's formal language F is what makes his result significant[242]. As Peter Smith writes:

> we are normally interested in *interpreted* languages—i.e., we are concerned with formal expressions which have some intended significance, which can be true or false. After all, our formalized proofs are ideally supposed to be just that, i.e., *proofs* with content, which show things to be true.
> Agreed, we'll often be very interested in certain features of proofs that can be assessed independently of their significance (for example, we will want to know whether a putative proof does obey the formal syntactic rules of a given deductive system). But it is one thing to set aside their semantics for some purposes; it is another thing entirely to drain formal proofs of all semantic significance.[243]

The distinction between semantics and syntax may be easy to draw today on account of the legacy of model theory, but judging this distinction in the historical proof of Gödel may be met with some difficulties. One may, as Hoffman did, situate Gödel's variant of the first incompleteness theorem between a semantic variant (assuming the correctness/soundness of the formal system), and a syntactic one (assuming the consistency, Barkley Rosser's proof) which Gödel sketched in his introduction, for Gödel assumes ω-consistency, a condition stronger than ordinary consistency[244]. Meanwhile, in his introduction to a print of Gödel's paper, Braithwaite judges that

> Gödel intermingles semantical with syntactical considerations in sketching a proof of the undecidability of [the Gödel sentence]. The distinction between what is syntactical and what is semantical was not made explicitly until a year or two later (by Tarski, whose work included rigorously establishing unprovability theorems that were semantical)"[245]

The typical assumption, however, is that Gödel's introduction sketches a semantic proof since it relies on the assumption of correctness/soundness, and that his main proof is purely syntactic in that it excludes that assumption. Nevertheless, Gödel does not entirely preclude references to "content"

[242] *Cf.* Hintikka 2005, 453.
[243] Smith 2020, 27.
[244] Hoffman 2024, 135-6.
[245] R. B. Braithwaite, *Introduction to On Formally Undecidable Propositions of Principia Mathematica and Related Systems* by Kurt Gödel, 1992, 29.

throughout his proof, nor to "truth" or "satisfiability". The English translations make matters more tricky. These references are either intended to be helpful comments, or can be an actual part of the proof. "True" for example, is sometimes used to render "*richtig*", while "gelte" is sometimes translated as "true", sometimes (of a relation) that it "holds". However, even when Gödel meant "true" semantically in his introduction, he used the term "*richtig*" instead of the more standard "*wahr*"[246]. Gödel also involves satisfiability [*Erfüllbarkeit*] in a subproof in order to call upon his *completeness theorem* in theorems [*Sätze*] IX and X. For the term "*inhalt(lich)*", the discrepancy among translations is even more unfortunate. "Content" is hardly ever translated[247]. For instance, in ft. 29 concerning the use of Hilbert's symbol '∞', we observe the following:

- Van Heijenoort's translation: "Whenever formulas are used to express a meaning (in particular, in all formulas expressing metamathematical propositions or notions), Hilbert's symbolism is employed"
- Mendelson: "In all informal (in particular, metamathematical) considerations Hilbert's symbolism is employed."
- Prince: "observations about meaning"
- Original: "*Für alle inhaltlichen (insbes. Auch die metamathematischen) Überlegungen wird die Hilbertsche Symbolik verwendet.*"

The pivotal Theorem V then links the formal system F and the primitive recursive functions. It is the theorem which decisively sets up the one that states the main result of the GFIT (Theorem VI). It bridges primitive recursive (PR) functions with the system F: "stat[ing] that the computation of a primitive-recursive function is replicable within the formal system [F]"[248]. The formal system can conduct PR operations. The PR functions, including those that can calculate the metamathematical properties of formulas via calculation on the Gödel numbers associated with the said formulas, can be represented and used in the formal system F. Using more (anachronistic) technical terminology, Theorem V states that every PR relation is *syntactically representable.* Gödel introduces it "vaguely" at

[246] "Denn angenommen der Satz [R(q); q] wäre er auch richtig" (1931, 175).
[247] Except B. Meltzer's translation (1992). Meltzer does translate inhalt as "content".
[248] Hoffman 2024, 279.

first, calling it "definable[249] [*definienbar*] within the system [F] (under its intuitive interpretation [*dieses inhaltlich gedeutet*])"[250] (Mendelson's translation)[251]. Prince translates it as "(interpreted semantically)"[252], while Van Heijenoort: "(if the usual meaning is given to the formulas of this system)"[253]. Gödel formulates it without the semantic aspect, without interpreting the content ("*ohne auf eine inhaltliche Deutung*") immediately after[254]. But in the proof, which he merely sketches, he uses a semantic argument, specifying only in a footnote (ft. 41) that "[w]hen this proof is carried out in detail, *r*, of course, is not defined indirectly with the help of its meaning but in terms of its purely formal structure." (Van Heijenoort). [*Bei der genauen Durchführung dieses Beweises wird natürlich r nicht auf dem Umweg über die inhaltliche Deutung, sondern durch seine rein formale Beschaffenheit definiert.*] Compare to Mendelson's translation: "*r* will not be defined by this shortcut through the intuitive interpretation"; and to Prince's: "*r* is naturally not defined indirectly, through its meaning" (interesting use of 'indirect'). In Ft. 42 as well: "Which, therefore, in the usual interpretation expresses the fact that this relation holds" (Van Heijenoort) [*Welches also, inhaltlich gedeutet das Bestehen dieser Relation ausdrückt.*] Note the variation in Van Heijenoort with himself. Mendelssohn here: "expresses intuitively that this relation holds". Also varies. Prince "interpreted semantically, says that this relation holds." Translated literally, *inhaltlich gedeutet* is "interpreted in terms of content".

"Interpretation", "meaning", "content", "intuitive", "informal", these terms suggest vastly different ideas when received in a philosophical register. This discrepancy in translation may confuse the English reader and lead them to conclude that the notion of "*inhatlich gedeutet*" lacks specificity and/or clarity, but it seems that Wittgenstein, a German speaker himself, agrees to this assessment as he spends a few pages in MS163 expressing. The discussion of "*inhaltlich gedeutet*" occurs within reflections on Gödel, and begins with the remark

[249] Definition from Prince (2025, 15): "A relation R(x) on the natural numbers is said to be definable in a formal system if there is a formula *r* in the language, with one free variable *x*, such that for any natural number *n*, *R(n)* holds if and only if *r* is true when the expression for *n* is substituted for the free variable *x* in *r*."

[250] Hoffman 2024, 283.

[251] Used by Hoffman in (2024, 283).

[252] Prince 2022, 83.

[253] Gödel 1967, 606.

[254] Prince 2022, 83.

> The phrase "interpreted in terms of content" ["*inhaltlich gedeutet*"] is a miserable fabrication. This expression gives us an incorrect idea [*Begriff*]of the application of mathematics.[255]

The expression "*inhaltlich gedeutet*" is here said to be "a miserable fabrication" presumably because it is a formulation of the sort that tends to be confusing, especially in philosophy. It may be for this reason that Wittgenstein said, towards the end of this section on "*inhaltlich gedeutet*", that "[t]he language of philosophers is already deformed, as if by shoes that are too tight."[256] Outside of philosophy, such linguistic imprecision might seem relatively harmless; but couldn't this vagueness also be problematic for mathematicians in that it produces misunderstandings of which they may not be fully aware?[257] The variability in translation indicates that, insofar as the phrase occurs in the context of a mathematical proof, it may not function as a technical term at all, but rather as an informal expression whose role is not unambiguous[258]. At the same time, its apparent dispensability is not obvious, particularly in a proof where the distinction between syntactic and semantic considerations is central. Nor is the degree of informality clear if, as Gödel suggests in the introduction, a formal proof might rely on the semantic considerations.

In philosophy, the question of what constitutes the "content" of a mathematical proposition is itself a substantive problem. To illustrate the controversial nature of such a notion in philosophy and the kind of questions it may elicit, one need look no further than G.E. Moore, an important influence on Wittgenstein. G.E. Moore, as part of the early development of analytic philosophy in opposition to

---

[255] MS 163, 43r; it occurs immediately after the sentence: "A mathematical theorem is undecidable in *the proof system for which it claims its own unprovability*." ; *Cf.* MS 127, 169-70.

[256] MS 163, 47v.

[257] See also MS 121, 77v-78r (Dec. 1938). "One of the <most pernicious | embarrassing> unclarity of mathematicians as to what they – *now* almost derogatorily – call the '*interpretation*' [*interpretation*] of their signs. By 'interpretation' [*interpretation*]or 'conception' [*Auffassung*] one imagines some kind of psychological processes of no interest to us, which accompany the <words | signs>, whereas the interpretation of a sign lies in its application. //Except in rare cases, the meaning of a sign [*die Bedeutung eines Zeichens*] does not lie in the mental processes that accompany its pronunciation, writing, etc., but in the complicated, yet familiar, practice [*Praxis*] of its use. " This remark also occurs in the midst of reflections on Gödel. Here, however, Wittgenstein seems to reproach mathematicians of discarding any mention of "*interpretation*" because they falsely conceive of it as something mental. Interpretation seems to come together with "*die Bedeutung eines Zeichens*", interpretation as the particular application perhaps, and the meaning as the result of the repeated applications among the members of the community.

[258] Some translations suggest that it (rather) *refers* to something informal.

English Idealism, grappled with F.H. Bradley's account of the content of a judgment (who in turn was reacting to the empiricists before him.) Moore writes in "The Nature of Judgment":

> "Truth and falsehood," says Mr. Bradley (Logic, p.2), "depend on the relation of our ideas to reality." And he immediately goes on to explain that, in this statement, "ideas" must not be understood to mean mere "states of my mind". The ideas, he says, on the relation of which to reality truth depends, are "*mere* ideas, signs of an existence other than themselves," and this aspect of them must not be confused either with their existence in my mind or with their particular character as so existent, which may be called their content. "For logic, at least," he says, "all ideas are signs" (p. 5); and "A sign is any fact that has a meaning," while "meaning consists of a part of the content (original or acquired) cut off, fixed by the mind, and considered apart from the existence of the sign" (p. 4).
>
> But Mr. Bradley himself does not remain true to this conception of the logical idea as the idea *of* something. As such, indeed it *is* only the psychological idea, related, indeed, to that which it signifies, but only related to it. Hence he finds it necessary, later, to use "idea," not of the symbol, but of the symbolised. Ideas, as *meanings*, not as "facts, which have a meaning," "are," he says (p. 8), "the ideas we spoke of, when we said 'Without ideas no judgment'". And he proceeds to show that "in predication we do not *use* the mental fact, but only the meaning"; [...] "we cannot judge until we use ideas *as* ideas. We must be aware that they are *mere* ideas, signs of an existence other than themselves." [...] "But it is better to say the idea *is* the meaning". The question is surely not which is "better to say," but which is true.[259]

Moore accuses Bradley of unduly psychologizing *universal meaning*, which Moore replaces by "concept", a supposedly more objective alternative[260]. However, Bradley is explicit in opposing "the psychological attitude" of considering ideas as phenomena and therefore as psychical facts[261]. Moore finds the notion of an idealist substratum, an idea of objectivity that is not independent of ideality, as completely incoherent. Frege's notion of "thought" [*Gedanke*] may be seen as emerging from engagement with similar philosophical problems. Frege explicitly characterizes thought as "not the subjective performance of thinking but its objective content [*objektiven Inhalt*], which is capable of being the common property of several thinkers."[262] Underlying all these debates and investigations is a shared philosophical project: the attempt to account for something that is grasped in understanding and

[259] G.E. Moore, 'The Nature of Judgment', 1899, 176.

[260] Moore 1899, 177.

[261] F. H. Bradley, *The Principles of logic*, 1992, 2.

[262] Frege 1948, 'On Sense and Reference', ft.5; Frege. 'Über Sinn und Bedeutung' 1892, ft. 5.

yet appears to be independent of individual minds by virtue of the universality of its truth, *i.e.* something objective that flirts with subjectivity.

If this is the intellectual atmosphere in which Wittgenstein was immersed in Cambridge, it is no surprise that an expression such as "*inhaltlich gedeutet*" would have caught his attention. The general problem of the meaning and truth of mathematical propositions indeed always interested Wittgenstein, but here, his questioning is taking the form of an incredulity toward the very intelligibility of the notion of "*inhaltlich gedeutet*". In very broad terms, Wittgenstein's 'descriptive' method leads him to construe meaning and understanding in terms of application, since use is all that can be perspicuously described. In the case of mathematics, such application consists largely in the manipulation of signs. As a result, Wittgenstein blurs the distinction—or, as he would see it, reveals the inherent blurriness of—the distinction between meaning, content, interpretation, or semantic content on the one hand, and application or syntactic/formal manipulation on the other. From this standpoint, to express the content of a proposition amounts to equating (or "translating", as he puts it sometimes) one expression into another. For Wittgenstein, this is not sufficiently different from asserting that $v(x) = y$ or that $P = Q$. Put differently, the association of one set of signs or formulae with another set by means of a rule of "translation" (*e.g.*, the association of signs/formulas of PM and Gödel numbers) is not, in itself, fundamentally distinct from the association of a sign or formula with what is typically called its semantic content or an interpretation[263]. In both cases, an expression is used to *refer* to another one, and no further explanatory work is accomplished by invoking a notion of content over and above such rule-governed use.

In the present context (MS 163), it can be ascertained that Wittgenstein's comments are directed at the (1931) undecidability paper. In addition to being situated amongst explicit references to Gödel's proof, this can be seen in the specific concerns Wittgenstein raises and the fact that they closely parallel questions he raises elsewhere about the proof, notably in AppIII[264]. Wittgenstein's frustration with the

[263] Indeed, this is the case for formal or informal semantic association/reference. It is more evident in the formal case. In the informal one, according Wittgenstein, to express the "semantic content" or "meaning" or "interpretation" of X consists in the answer to the question of what does X mean? And the answer is none other than a linguistic expression.

[264] This doesn't exclude the possibility—and indeed the fact—that Wittgenstein's resolution of the issue is also applicable in the case of Moore and the others.

use of "*inhaltlich gedeutet*" here concerns its ambiguity in a proof where syntactic considerations are *technically* (as in through technique) distinguished from semantics ones, without philosophically clarifying this distinction while nevertheless liable to giving the impression that such a philosophical distinction is unproblematic. In other words, the *prose* is prone to mislead when it contains expressions that have already proven philosophically treacherous.

Accordingly, in MS 163, Wittgenstein explores a few ways in which "*inhaltich gedeutet*" fails to make sense. What does it mean, for example, to ignore the content of a mathematical proposition, as Gödel's proof seems to demand? And in what sense does "this expression [give] us an incorrect idea of the application of mathematics"? Wittgenstein follows by giving an example:

> This concept could be described as follows: Let us imagine playing a game with any set of German sentences, such as "Hans is a stupid boy," "his hat is dusty," etc., in which understanding these sentences is not important. We could also play this game if the strings of words were sentences in a language unknown to us, or even if they were not sentences at all. But let us assume that they are actually German sentences. In that case, this fact plays a role in the game, as it does in written chess, in that we use real letters and numbers for notation.—But let us now assume that the game proves to be useful in that, under certain circumstances, it generates German sentences that turn out to be true [*wahr*]. So that if, under certain circumstances, the result of certain transformations is the sentence "Hans is stupid," this sentence, *interpreted in terms of its content* [*inhaltlich gedeutet*], usually applies [*für gewöhnlich zutrifft*].—But haven't you described here—albeit in its rawest form—the nature of mathematics and its application?
>
> What does it mean to interpret a sequence of 'signs' in terms of content? Does it mean anything other than understanding them as a sentence or expression of a language familiar to us and thus mastering their conventional application, or, if they are not the expression of a language *familiar* to us, having some kind of fixed application in mind?[265]

The use of *inhaltlich deuten* in Gödel's (1931) paper suggests that a mathematical sentence may be said to possess a kind of "content" independently of our interpreting it: we do not need to interpret or even acknowledge this content in order to operate with the sentence within the formal system, yet when we do attend to it, the sentence typically turns out to be true. This raises an immediate question: what, precisely, is meant by such an interpretation? Isn't it simply our understanding of the meaning of the word, a grasp that can only be seen in the correct (usual) application (use) of it? Or, if we are not

[265] MS 163, 43v-45r.

familiar with the language, *i.e.* we have not learned this meaning through *training*, then isn't interpreting the idea one has in the mind of how the sign is to be applied? If so, Wittgenstein asks, then

> *When* do we interpret? That is, when do we carry out the interpretation?

The psychologistic or mentalist conception proves unhelpful in explaining the meaning of mathematical propositions[266]. Wittgenstein's questions are rhetorical, intended to prompt us to abandon this conception. Wittgenstein tests alternative conceptions.

> 'Interpreting the content' should mean: *applying*; and, more specifically, applying in the manner suggested by these words.
>
> 'Interpreted in terms of content, this formula means ...' therefore means: "this formula can be expressed in words as: ..."

That "it should mean", given Wittgenstein's method, is to say "I am/we are inclined to think this". Here in particular, it is Wittgenstein's own tendency, as he is the one always considering meaning in terms of use and application. But this answer is not sufficient to cleanly separate between syntactic and semantic representation. One can easily say that "*P* says '*P* is unprovable'" is an instance of giving an interpretation in terms of content, a semantic interpretation of *P*. *P* refers to the meaning "*P* is unprovable." But is it so different to say that "*P* says that there is no sequence of formulas whose Gödel number is the Gödel number of the proof of the formula whose Gödel number is ⌜*P*⌝[267]"? In both cases we can say that "*P* refers to itself", but in different senses. The second 'syntactic' one is "*P* expresses a sentence containing a symbol which refers to P by means of Gödel coding"[268]. In both cases, it *is* a translation into English.

> "The proposition says that this number cannot be got from these numbers in this way" — But are you certain that you have translated it correctly into English?
> Certainly it looks as if you had. —But isn't it possible to go wrong here?[269]

---

[266] And Wittgenstein says so explicitly in the beginning of MS 163 and a bit after his discussion of *inhaltlich gedeutet*.

[267] ⌜*P*⌝ refers to the Gödel number of *P*.

[268] *Cf*. RFM.VII.22

[269] Occurs many pages before, in the same notebook: MS 163, 19r-19v. They are also remarks that have been selected for publication in RFM.VII.21.

Wittgenstein's suspicion extends not only to what substantial meaning (description of a tangible application) can be given to "*inhaltlich gedeutet*", but also to the very idea of reference/representation operative in both semantic and syntactic cases. The reference between sign and meaning is nebulous, and so is the "purely formal" one between the Gödel number and the sequence of signs/formulas of PM. We are under the impression of a *picture* when we think that formalization *can represent* proofs[270], or even more ambitiously, language itself (Frege's *Begriffsschrift*). What is that picture? Whence does it originate? It appears to come from the picture of propositions of physics, or empirical propositions, which are taken to represent or correspond to physical facts, a model that itself traces back to the picture of the practice of ostension. This picture of propositions as carriers of *information* was explicitly problematized in RFM.AppIII. There, the highlighted 'cause' of the problem is the superficial similarity of syntactic form. More fundamentally, however, the basis upon which Wittgenstein judges the sentences of mathematics to be different from empirical propositions lies in their differing modes of application. Sentences of mathematics are not applied as or used to convey information.

> **RFM.AppIII.20 (1939)**Here one needs to remember that the propositions of logic are so constructed as to have no application as *information* in practice. So it could very well be said that they were not *propositions* at all; and one's writing them down at all stands in need of justification. Now if we append to these 'propositions' a further sentence-like structure of another kind, then we are all the more in the dark about what kind of application this system of sign-combinations is supposed to have; for the mere *ring of a sentence* is not enough to give these connexions of signs any meaning.

> **MS 163, 46r-48r (1941)** [...] The whole idea of interpreting content is based on the view of mathematics as the physics of 'mathematical objects'.
> [...]
> "The mathematical proposition, as we usually understand it, does have content!" — That is, we understand it as *a proposition*, not as *an empty* group of figures! — Now, this obviously comes from the fact that the signs of the mathematical proposition are signs (words) of our *language*.

Another remark Wittgenstein makes about the "content" of mathematical sentences is that whatever content comes to be ascribed to them does not originate from mathematics. It is not inherently mathematical, nor does it constitute an additional aspect of the mathematical object, the counterpart of *form*. Any meaning that is bestowed upon the mathematical signs comes from *outside of mathematics*.

[270] *Cf*. Floyd 2001, 290.

Whatever content these words may be said to have, it comes from their ordinary use which is outside of the technical practice of the discipline. (Recall that this is the justification behind Wittgenstein's treatment of mathematics as an object of philosophical study, namely that a lot of the words used in mathematics are drawn from ordinary usage.) About the application of mathematics, he adds:

> **MS 163, 61v-62r.** Never forget that the *application* of mathematics does not lie in the mathematics.
> Or: When we believe we are obtaining information *in* mathematics, it is only apparent information; the actual information lies *outside* of mathematics. In other words: never be tempted to view mathematics as the natural history of numbers, operations, etc.![271]

This provides a further objection to Rodych's conception of Wittgenstein's notion of application, particularly what Rodych named "extra-mathematical" application. Rodych appears to understand this notion in much the same way as contemporary "applied mathematics", and on this basis he takes Wittgenstein to reject pure mathematics and to regard Gödel's proof with suspicion on the grounds that foundational mathematics lacks practical application[272]. But this simple and intuitive conception of "practical," "application," and "use" is not the one Wittgenstein employs, and, in a way, to think otherwise is to be misled by the vocabulary. Wittgenstein's notion of application is, in a way, more abstract.

> **MS 163, 41v-42r.** Gödel demonstrates beyond question that the proposition he has constructed occupies an exceptional position in the system of propositions. (I.e.,) However one describes this exceptional position, *it still remains such*.
> Gödel's discovery is a mathematical discovery. Now if such a discovery can be regarded as an extension of grammar, what is the grammatical meaning of the construction?
> Could this also be expressed as follows: Which extra-mathematical application can we give to Gödel's theorem?

---

[271] This command appears to enter in contradiction with what Wittgenstein is doing when he says that Gödel's theorem introduces a new notion of truth and provability. But Wittgenstein is not viewing mathematics itself as the history of numbers *etc*.. Wittgenstein does not think his account is mathematical but philosophical. To see mathematics as a history of numbers *etc.* would be something Wittgenstein himself would be tempted to entertain given his "descriptions". In this remark, it is plausible that he is reminding himself (or someone who is coming to adopt his perspective) not to equate this kind of description (of what provability used to mean compared to after Gödel, for instance) with mathematics itself. Mathematics is simply another kind of practice. The considerations that matter to the mathematician are not at all historical and thus this is not what mathematics is in practice (or, we could say, what it 'really' is). Despite Wittgenstein appearing to be speaking *about* "mathematics", the "mathematics" characterized by his account is distinct from the mathematics *in practice*, or as seen inside the practice for practical reasons.

[272] Rodych 2002, 387.

It is from a reformulation of this last question that Wittgenstein's discussion of content proceeds (starting at MS 163, 43r). Rodych is mistaken in deeming Wittgenstein's diagnosis of Gödel's supposed 'unphilosophical naïveté' to be the failure to consider the "application in the real world"[273] of the Gödel sentence. The misunderstanding of the relationship between mathematics and its application[274] which Wittgenstein calls 'unphilosophical naïveté' is rather that, for him, Gödel does not appear to have to have considered the way "meaning" or "content" is nothing but the way the words are applied. And this is a non-mathematical consideration. Had Gödel paid attention to that, he would have clarified his employment of the term "*inhaltlich gedeutet*", for instance. Gödel could have clarified how he differentiated the semantic self-reference and the indirect reference, as this distinction is in a way at the heart of the confusion between 'fact of the proof' and 'a proposition asserting unprovability'. This is the limit of Wittgenstein's 'reproach'. As Wittgenstein himself remarks, "what [Gödel] proves doesn't concern us"[275], "it is not his particular theorem that interests us"[276]. Wittgenstein cannot be seriously accusing Gödel *qua* mathematician (there is no evidence that Wittgenstein saw him as anything other) of failing to address the question of the nature of mathematics, an issue Wittgenstein consistently treats as philosophical and not mathematical[277].

3.2.4. The Usability of the Gödel Sentence

> **MS 163 39r-39v** So if I prove that a certain number cannot be produced in such and such a way, then this must be a proof that is valid for the geometry of the symbols. It must be *physically* trustworthy. But doesn't that just mean that if we cannot trust it, we are misinterpreting the theorem? Viewing it as an instrument for something it is not?
> Gödel's proof develops a difficulty that must also appear in a much more elementary way. (And herein lies, it seems to me, both Gödel's great contribution to the philosophy of mathematics and the reason why his particular proof is not what interests us.)

[273] Rodych 2002, 387.
[274] MS 124, 115r (1944).
[275] MS 163, 40r.
[276] MS 163, 39v-40r.
[277] Many threads of Wittgenstein on this, see for example "but can it be a mathematical problem to make mathematics into mathematics?" (RFM.VII.34)

> **MS 163, 40r-40v (July 11, 1941)**. I could say: Gödel's proof gives us an incentive to change the perspective from which we viewed mathematics. *What* he proves is not our concern, but we must deal with this type of mathematical *proof*.
> **MS 163, 41r-42v (July 11, 1941)** This brings us back to the idea that spelling the word "spell" is not a higher degree of spelling.
>
> If the two ω-contradictory pieces of evidence really are present, then it becomes problematic what we can do with what has been proven and refuted in this way.
>
> Gödel shows *clearly* that the theorem he constructed occupies an *exceptional position* in the system of theorems. (That is to say,) however one describes this exceptional position, *it remains such.*
>
> G's discovery is a mathematical discovery. If such a discovery can be understood as an extension of grammar, what is the grammatical meaning of the construction?
> Could one also express it this way: What *non-mathematical* use can we give to G's theorem?

The *situation* into which Gödel places us is one in which we have to re-examine mathematics. For Wittgenstein, this is because Gödel's results challenge the ways in which we had regarded the notions of proof, provability, mathematical truth—concepts that are central to our conception of mathematics and the mathematical. Wittgenstein nevertheless thinks that it is misguided to conceive of this as precipitating a foundational crisis (a notion he generally resists, given his skepticism toward any attempt to establish "foundations" in an epistemological or (onto-)logical sense). The question for him is instead the *application* of a kind of mathematical proof that appears both to represent and, in a certain sense, to undermine mathematical proof as such.

The notion of application of mathematics is the philosophical 'concept' which Wittgenstein deems mathematicians to struggle with. Application concerns the use of a mathematical statement or device, it concerns its role within the practice of mathematics. When Wittgenstein "attacks" the "position" of a theorem such as Gödel's incompleteness theorems[278], he is not disputing the mathematical result nor the mechanics of the proof, but expressing skepticism over philosophical views that underlie the typical interpretation of the significance—or role—of the theorem. Otherwise put, for Wittgenstein, these are views purporting to describe, explain, and justify the applicability—or

[278] RFM, VII, 19.

"exceptional position" of the theorem, and on this ground he finds them inadequate. Thus Wittgenstein's issues regarding the "position" of the theorem are articulated through the notions of application and applicability (or, equivalently, use, usability, or utility).

The aim of this section is to present Wittgenstein's accounts on application which he suggests in AppIII, the most well-known and most disputed locus of his engagement with Gödel's work. In particular, his account can be divided into a negative and positive component. Wittgenstein rejects the implicit assumption that a contradiction would render Russell's system unusable, or that contradictions can rob anything of its usability. Instead, he argues that contradictions themselves may be said to have a use, and thus our instinctive abhorrence towards them is unwarranted. His positive account of the usability of Gödel's proof emerges through a comparison with proofs of impossibility. Such proofs offer a cogent reason for a prediction. And this prediction is construed in a non-empirical sense: it is more like establishing a new measure of what it means to do mathematics, *i.e.*, one cannot derive the Gödel sentence within a consistent PM or similarly expressive axiomatic system without thereby ceasing to engage in what we recognize as mathematical practice.

3.2.4.1. The Illusory Harm of Contradictions

> There is a particular mathematical method, the method of *reductio ad absurdum*, which we might call "avoiding the contradiction". In this method one shows a contradiction and then shows the way from it. But this doesn't mean that a contradiction is a sort of devil. **–LFM, 209.**
>
> My aim is to alter the *attitude* to contradiction and to consistency proofs. **–RFM.III.82.**

To challenge the "office"[279] of Gödel's theorem is not to deny its importance—indeed, one could easily say that it is of "exceptional" importance—but rather to question the status that is conferred upon it. Wittgenstein seems to be addressing *tendencies* which underlie the adoption of some views about the

[279] RFM.VII.19.

foundations of mathematics[280]. The status commonly attributed to Gödel which Wittgenstein is resisting is—at least partly—informed by attitudes on contradiction and (proofs of) consistency, chief among which is that our natural/intuitive/ordinary avoidance of contradiction is not a "constraint [that] is *imposed* on us."[281] (For a detailed list of these attitudes, see Crispin Wright (1980, 296-7).)

Gödel's first incompleteness theorem shows that for any sufficiently expressive axiomatic formal system, assuming its (ω-)consistency, there exists an undecidable sentence, *i.e.*, a sentence for which neither it nor its negation is provable. Otherwise put, either the system is (ω-)consistent and undecidable, or it is (ω-)inconsistent and decidable. Since an inconsistent system is generally regarded as uninteresting or useless, mathematicians ordinarily take Gödel's result to concern undecidability, while consistency (or ω-consistency) is simply assumed.

The worry about inconsistency and desire to establish consistency (in particular, the ability to anticipate that we *won't* cross a contradiction in the calculus) is one of Wittgenstein's central themes. Occupying three out of the famous twenty Gödelian remarks of Appendix III and many other places (notably the last dozen remarks of RFM.III), his reflection on the harm of contradictions—or lack thereof—as well as their application, understandably touches on the applicability of indirect proofs and use as prediction (which is the use of the Gödel sentence that will be discussed in the next section). Wittgenstein's Gödelian remarks in Appendix III about contradictions suggest a particular interest in the two indirect proofs concluding the demonstration of Theorem VI[282]. At this junction of the argument, the assumption of consistency is invoked. Wittgenstein seems to consider the hypothetical situation in which consistency is not assumed, (a consideration that is perhaps further justified by the fact that some regard consistency itself as something requiring proof)[283]. Thus, at the moment where we

[280] It is worth insisting on the fact that Wittgenstein is concerned more with temptations in thought which can be shared among different philosophical positions than about full-fledged views specifically, which explains his own seemingly dismissive habits of neglecting to consider other parts of a position or of providing a satisfying portrayal.

[281] Wright 1980, 296.

[282] Theorem VI states that "for every ω-consistent [primitive] recursive class K of FORMULAS, there are [primitive] recursive CLASS SIGNS *r* such that neither *v Gen r* nor *Neg(v Gen r)* belongs to *Flg(x)* (where *v* is the FREE VARIABLE of *r*)." (Gödel 1967, 607)

[283] To be clear, Wittgenstein is not critiquing the assumption of consistency as a mathematical device (nor ignoring it altogether as Kreisel and Bernays' early reception presumed). He does not *explicitly* say that he is concerned with it either,

are confronted with a contradiction, or with what he calls an "ω-contradict[ion]"[284] in the case of ω-inconsistency (where each individual instance *P(n)* holds, yet the universal generalization $\forall x P(x)$ fails), and before the assumption of (ω-)consistency is invoked to complete the *reductio*, Wittgenstein pauses to examine our attitude toward contradiction itself in general. His aim is to undermine the picture that contradictions are not inherently bad, or even, evil[285], and that we have an imperative to fix to do something about their appearance. The question largely is: If we indeed find a contradiction, does it really affect the entire language, does it affect its usability? Wittgenstein encourages us to see contradictions as not inherently harmful. It is rather a (practical) choice to adopt the principle of explosion, he seems to presume[286]. One of the ways he explores this line of inquiry is considering if a "contradiction is harmless if it can be sealed off"[287], for that should be what the harm is for the language, if contradictions were a sickness affecting language as a whole. But if that is the only way the language is harmed, then it not the contradiction that is the problematic element; the real problem is this "*inability*" to use the language/calculus the way one wants[288]. Contradictions in fact are useful, he proposes, they have a use as an indication, a signpost, a warning indicating that "only the opposite of what you do not want to abandon is combinable with [the assumption heading the indirect proof]."[289]

Wittgenstein suggests an attitude shift through viewing the usability of contradictions. The previous attitude we reason from unawares being a "superstitious dread and veneration by mathematicians in the face of contradiction"[290], or the view of contradictions as though they are a disease that would infect the whole body[291] (system)."But it is vitally important to see that a contradiction is

---

but it appears implicitly. It could be said that this is also an instance of his—as some scholars named it—*voluntarism* at play.

[284] MS163, 41r-41v (July 11, 1941): "If both ω-contradicting proofs are really present, then it becomes problematic what we are able to do with the thusly proven and refuted propositions."

[285] LFM, 209.

[286] Priest also invokes Wittgenstein's voluntarism to refrain from applying the principle of explosion (2004, ft. 17). This is not an explicit acknowledgement by Wittgenstein, but again, it appears to underlie much of his inquiry.

[287] RFM.III.80.

[288] RFM.III.80.

[289] RFM.V.28.

[290] RFM.AppIII.17

[291] *E.g.* LFM, 209.

not a germ which shows general illness", Wittgenstein declares[292]. *If* it were shown that there was a contradiction in RS, the contradiction merely indicates that this is a road we do not want to pursue, and that we will (as a matter of a new rule) not try assuming or deriving that which yields this, but the other parts of RS should still be safe, given that they were supposedly in use without problems prior to the finding of this contradiction.

This overview concerns Wittgenstein's treatment of contradiction in relation to Gödel's theorem, drawing on his Gödelian remarks and his remarks on contradiction more generally. When it comes to those in RFM.AppIII in particular, whether he is treating hypotheticals based on the *reductio* in Gödel's demonstration of theorem VI in his 1931 proof of undecidability, or instead addressing the semantic contradiction[293] is not decisive for present purposes. Either way, both readings are compatible with Wittgenstein's central aim, which is what matters here. The reading involving consistency (whose relevance to Gödel is undeniable both within Wittgensteinian concerns and without) comes mainly from RFM.III, and not RFM.App.III, but the discussion in the latter is organically supplemented with remarks in the former (as well as other parts of RFM and LFM).

***

"Is Russell's logic vitiated by a contradiction?"[294]Wittgenstein asks in his 1939 lectures. In the Gödelian context, he means: (setting aside the assumption of consistency) would PM be harmed if a ω-inconsistency were displayed in it? Would the Gödel sentence ruin PM (by rendering it useless)? There is a significant amount of occurrences of this question in Wittgenstein. In RFM.AppIII for example, are "remarks which have [perhaps] drawn the ire of commentators more than any other"[295]:

> **§11**. Let us suppose I prove the unprovability (in Russell's system) of *P*; then by this proof I have proved *P*. Now if this proof were one in Russell's system—I should in that case have proved at once that it belonged and did not belong to Russell's system.—That is what comes of making up such sentences.—But there is a contradiction here!—Well, then there is a contradiction here. Does it do any harm here?

[292] LFM, 211.

[293] For a more formal exposition of a semantic contradiction see Priest, *In Contradiction*, 2006, 39-50.

[294] LFM, 209.

[295] Priest 2004, 214.

> **§12.** Is there harm in the contradiction that arises when someone says: "I am lying.—So I am not lying.—So I am lying.—etc."? I mean: does it make our language less usable if in this case, according to the ordinary rules, a proposition yields its contradictory, and vice versa?—the proposition *itself* is unusable, and these inferences equally; but why should they not be made?—It is a profitless performance!—It is a language-game with some similarity to the game of thumb-catching.
>
> **§13.** Such a contradiction is of interest only because it has tormented people, and because this shews both how tormenting problems can grow out of language, and what kind of things can torment us.

Recall that, as has already been established, there is no such thing as a proof of unprovability *within* Russell's system, since proofs in RS are exhibited rather than propositions asserting provability or unprovability, *i.e.*, asserting that a proof is possible or not. Accordingly, in the eleventh remark of RFM Appendix III, Wittgenstein suggests that *if* we suppose that *P*'s unprovability in RS is itself proved *in* RS (as opposed to Gödel's system), then all that has been proved is a proposition asserting unprovability—namely, the proposition called *P*. But this would amount to *P* being derived according to the rules of RS. The result is a contradiction between *P*'s asserted unprovability and the fact that *P* has been derived: *P* "belonged and did not belong to Russell's system"; *P* would be both a theorem and not a theorem.

"That is what comes of making up such sentences". The meaning of "making up such sentences" is ambiguous. Does Wittgenstein mean constructing the Gödel sentence itself, or merely formulating the English sentence that asserts unprovability (the "translation in English")? Before resolving this ambiguity, it would be helpful to examine the nature of the contradiction that, in Wittgenstein's opinion, does not cause any harm.

In the following remark, Wittgenstein explains that for a contradiction to "cause harm" would mean that it renders "our language less usable". But is this the case for PM? As an example, he offers: " I am lying.—so I am not lying.—So I am lying.—etc." The problem here is not that the language itself becomes unusable, but rather that the proposition "I am lying," together with this chain of inferences, lacks any practical point. They are unusable because there is nothing to be gained from uttering it in this

way[296]. One may therefore ask: why would anyone say this? What purpose would it serve? One can give a use to such an expression in ordinary contexts such as an exclamation[297], or in statements such as "He is 34—I am lying, he's 32."[298] In this sentence, "I am lying" is unproblematic; it simply means "I am mistaken," "I am joking," or "I am misleading you," *etc*.. No one can or would *assert* the Liar sentence in the manner required by the paradox, nor utter it in a context to be treated the way we ordinarily treat assertion[299]. If someone did, we would assume that they meant something else by "lying". One might object, however, that logicians themselves construct and employ such sentences! In this case, Wittgenstein's response is to invite us to wonder if the interest of "making up" such sentences is nothing but "the pleasant [or dreadful] feeling of paradox"[300], whereas his own interest lies in the fact that such sentences have exerted a peculiar intellectual "torment".

During one of the 1939 lectures, Turing remarks: "What puzzles one is that one usually uses a contradiction as a criterion for having done something wrong. But in this case one cannot find anything wrong." Wittgenstein responds: "Yes—and more: nothing has been done wrong. One may say, 'This can only be explained by a theory of types.' But what is there which needs to be explained?"[301] The problem of the Liar becomes a problem in logic only under the assumption that the mere possibility of generating such a sentence reveals a defect in the language itself. This assumption reflects a broader picture according to which possible inferences are somehow independent of the human practice of deriving them—as if they belonged to the language itself. This picture underlies the theory of types, Hilbert's programme, the search for consistency proofs, and Gödel's own efforts to find a proof of completeness. What these projects seek to assuage is the fear that a *hidden* contradiction might be lurking within the system[302]. This worry is expressed in the idea that "[o]nly the proof of consistency shews me that I can rely on the calculus."[303] Wittgenstein examines this (as an) anthropological

[296] *Cf.* LFM 206-7.
[297] RFM.IV.58: "It might also be said: his 'I always lie' was not really an *assertion*. It was rather an exclamation."
[298] LFM, 208.
[299] RFM.IV.58.
[300] LFM, 16.
[301] LFM, 207.
[302] LFM, 210.
[303] RFM.III.84.

phenomenon in the lengthy remarks closing RFM.III, attempting to *describe* the "practical purpose" served by proofs of consistency and by the avoidance of contradictions[304]. Although a detailed reconstruction of his path to this conclusion would be illuminating, practical constraints require moving directly to the result.

Wittgenstein aims to expose the absurdity of fearing a hidden contradiction: such fear, he argues, is unfounded. What remains to be asked, then, is what utility the construction of Gödelian sentences actually has, what role it plays within mathematical practice. Its function, to be discussed in the next section, is not merely to provoke excitement through the appearance of paradox, but rather to operate analogously to a prediction. It establishes a new rule, a new measure, or a new criterion of what it means to do mathematics with sufficiently expressive axiomatic formal systems.

3.2.4.2. The Application of the Gödel Sentence: The Comparison with Impossibility Proofs

> **RFM.III.86.** [...] The proof of consistency must give us reasons for a prediction; and that is its *practical purpose*. [...]
> I wanted to say: the consistency-proof can only set our minds at rest, if it is a cogent reason for this prediction.
> **RFM.III.87.** Where it is enough for me to get a proof that a contradiction or a trisection of the angle cannot be constructed in *this* way, the recursive proof achieves what is required of it.
>
> **RFM.AppIII.14.** A proof of unprovability is as it were a geometrical proof; a proof concerning the geometry of proofs. Quite analogous e.g. to a proof that such-and-such a construction is impossible with ruler and compass. Now such a proof contains an element of prediction, a physical element. For in consequence of such a proof we say to a man: "Don't exert yourself to find a construction (of the trisection of an angle, say)—it can be proved that it can't be done". That is to say: it is essential that the proof of unprovability should be capable of being applied in this way. It must–we might say–be a *forcible reason* for giving up the search for a proof (i.e. for a construction of such-and-such a kind).
> A contradiction is unusable as such a prediction.

[304] "We shall see contradiction in a quite different light if we look at its occurrence and its consequences as it were anthropologically—and when we look at it with a mathematician's exasperation. That is to say, we shall look at it differently, if we try merely to describe how the contradiction influences language-games, and if we look at it from the point of view of the mathematical law-giver." (RFM.III.87)

At the end of RFM.III, Wittgenstein concludes that the *practical purpose* of seeking a proof of consistency is to provide a *forcible* or *cogent reason for a prediction*[305]. This conclusion emerges from his *descriptive* account of the "anthropological" phenomenon of why mathematicians are compelled to look for consistency proofs in the first place. It contrasts sharply with an *explanatory* account according to which a proof, such as Gödel's completeness theorem, could establish the trust in (or legitimacy of) the calculus, or confirm that a given axiomatic system genuinely lies at the foundation of mathematics. As can be seen in the quotes above, being a forcible reason for a prediction characterizes the application of Gödel's proof and of proofs of impossibility. The comparison to this geometric example serves to elucidate the applicability of the Gödel sentence/GIT, the role it plays in practice, one that is not the revelation of something about the foundations of mathematics nor of defeating the purpose of PM.

A proof of impossibility—say, that of the construction of a regular heptagon with a straightedge and compass (a common example in LFM), or that of the trisection of the angle with the same tools ("a result he discussed in almost every one of his lecture courses for which we have records and in every major volume of his posthumously published writings"[306])—demonstrates that carrying out such constructions is impossible. But what does it mean to speak about the heptagon, then? Did two millennia of speculation about it amount to mere nonsense? And if such constructions are nonsensical, what are we denying?[307] The answer, according to Juliet Floyd, is that before the impossibility is proven, we are in fact dealing with a conjecture[308]. Since such a construction never took place, when we talk about the heptagon or the trisected angle, the concept is not clear, not fully determined. Terms such as "heptagon" and "trisected angle" may occur in declarative-looking sentences—much like the question that is disguised as an assertion[309]—but these sentences do not (yet) have a truth-value. They do not function as information. Rather, a sentence like

> 'It is (im)possible to give a general Euclidean method of trisecting the angle', uttered before one accepts the proof, can be construed, if one wishes, to be either true or false; but saying this, i.e., working with the truth-functions on such a 'statement' (or is it a question?) amounts in effect to a demand for clarification,

[305] RFM.III.86.
[306] Floyd 2001, 287.
[307] A problem Wittgenstein poses in RFM.VI.13.
[308] Floyd 1995,384.
[309] RFM.AppIII.1-4.

> the announcement that one is going to *try* to prove something, to change the circumstances of 'the statements''s utterance. It says (like a command), 'Go out and make a mathematical search!'[310]

Note Floyd's own hesitation. She's not affirming that such an utterance is a question, a command, or something else. The point is this: the purpose that such conjectural propositions serve in our language-game is different from that of ordinary or "real" propositions, *i.e.*, they are not used to make truth-claims. It could be a use we do not have a name for, which we are trying to describe, as Floyd does, using names we already have like "question" and "command". This suits the Wittgensteinian spirit of aiming to show that there is room to think or question something we previously hadn't thought to look into, over giving us answers.

Moreover, "trisecting an angle" is not absolutely impossible either. It is only impossible (and thus can be qualified as meaningless) if we mean trisecting in the Euclidean sense, *i.e.*, relative to a specific technique, Euclidean construction using only a compass and a straightedge[311]. If other instruments are allowed, such as a protractor, then we can in fact divide an angle into three equal ones. Two things are at play here. First, sentences involving such notions do not play the informative role characteristic of declarative propositions. Second, the meaning of said notions is relative to a language-game/system. These two points converge in Wittgenstein's claim that whatever we mean by the 'meaning' of a mathematical notion is none other than a (vague) way to denote the applicability/application of said notion[312], its *usability* within a practice. This should not be read as advancing a theory of meaning, the 'meaning is use' thesis. "Wittgenstein's emphasis on use is a paradigm, it is a paradigm to end all theoretical paradigms; [...] it aims to break the spell of philosophy as theory."[313]

Applying Floyd's schema of the conjecture to the Gödelian case, we obtain the following. Prior to reading Gödel's proof, "true but unprovable" is not yet clear. We wonder about its meaning, and given our prior conceptions of truth and provability, it may even appear contradictory. Once the proof is read and accepted, we are forced to recognize that these notions are not functioning in the way we

[310] Floyd, 1995, 396.
[311] Floyd 1995, 388 ff.
[312] MS 162a, 67.
[313] Mulhall, *Inheritance and Originality: Wittgenstein, Heidegger, Kierkegaard*, 2001, 43. (cited in Read 2021, 64).

previously assumed, and that the sentence "true but unprovable" plays a different role from that of theorems in PM. The task, then, is to inquire into its use. In this sense, the proof does not uncover the (preexisting) meanings of 'true' and 'provable'—concepts we intuitively understand that always underlay mathematics—but contributes to the formation of new ones (new criteria of application of the notions).

A proof of unprovability functions much like a proof of the impossibility of a geometric construction. Both effectively tell us: do not bother trying to construct the heptagon or to prove *P,* you will not succeed. A proof of unprovability, like that of impossibility, plays the role of providing (or being) "a *forcible reason* for giving up the search for a proof (i.e. for a construction of such-and-such a kind)"[314]. They contain this element of prediction; they predict that you cannot obtain such-and-such a result, and if you do, you *must* have done it in a different sense, one that is irrelevant to us. To be more specific and by appealing to other remarks of RFM, this predictive element is the mathematical *must* whose bind lies not in that it predicts a physical fact (like an oracle)[315], nor that it binds us by a (psychological) "compulsion"[316], but that it establishes a new rule of what may be done with the symbols, a "new measure" of what we accept as a *mathematical* construction[317].

The final line of §14 is "A contradiction is unusable as such a prediction." This line does not exist in MS 118 nor TS 221, but was added in the second typescript TS 223, p253. It is intriguing initially since *reductio* argument often establish impossibility by deriving contradictions[318]. Wittgenstein's point,

---

[314] RFM.AppIII.14.

[315]In RFM.VI.15, Wittgenstein specified a difference between this and a prediction of an empirical fact, which is more aptly called 'prediction'. The mathematical 'prediction' is not in the form of 'you will ___', but of 'you must ___'.

[316] *Cf.* RFM.I.113-118ff.

[317] RFM.III.26-27:

> §26. Let us remember that in mathematics we are convinced of *grammatical* propositions; so the expression, the result, of our being convinced is that we *accept a rule*.
>
> Nothing is more likely than that the verbal expression of the result of a mathematical proof is calculated to delude us with a myth.
>
> §27. I am trying to say something like this: even if the proved mathematical proposition seems to point to a reality outside itself, still it is only the expression of acceptance of a new measure (of reality).
>
> Thus we take the constructability (provability) of this symbol (that is, of the mathematical proposition) as a sign that we are to transform symbols in such and such a way.

[318] This is in fact one of the things that Bernays (1959,523) reproached Wittgenstein with. He retorts: "Such proofs of impossibility in fact always proceed by the deduction of a contradiction."

however, is that contradiction as a purely formal structure does not by itself predict anything[319]. It is only within a practice, where certain routes are abandoned and others retained, that a contradiction acquires this role. The *necessity* to calculate or proceed in one way rather than another does not arise at the purely formal level, but the level of use.

3.3. Wittgenstein Wrestling with Himself

The *situation* into which Gödel places us prompted Wittgenstein to ask a variety of philosophical questions: questions about change in mathematics; about changes in our conception of truth and provability, which, it turns out, is not a change *per se* but the creation of new concepts following new proofs and invention of new language-games. This raises further questions. What makes us accept, or be convinced by, a change in the notion of "proof," given that proof itself is typically the means by which we are convinced? What distinguishes the creation of a new grammar from making a mistake within an existing game? And what, precisely, is Gödel's contribution to our practice of mathematics? (A question whose answer Wittgenstein would call "extra-mathematical" application of Gödel's theorem.) Related questions proliferate. For example, there are questions about the effect of importing words from ordinary language into mathematical contexts; about the difference between an experiment and a mathematical proof construed as a human activity; about the grammatical form of sentences; and about the difference between sentences that function as information and those that do not. One could notice a trend in Wittgenstein's investigations. For instance, he repeatedly looks to the application of such and such a notion to find its 'meaning'; he speaks of tendencies, convictions, and of what it is to accept the proof—psychological concepts. Wittgenstein seems to hold that mathematics is a motley of techniques rather than a unified theoretical edifice, and that concepts are applied on the basis of family resemblance rather than strict definition.

At this point, one might object: is Wittgenstein not advancing a thesis of his own? This worry can be addressed by recalling the methodological point already emphasized. Wittgenstein does not aim

[319] Floyd 2001, 293.

to explain mathematical or abstract concepts or to give their causal origin. Instead, he attempts to describe an anthropological—or "ethnological"—phenomenon. And this is no easy task, especially given our entrenched tendencies in thinking:

> **MS 123, 11r-11v (7 Oct. 1940).** The *phenomenon*, I mean the ethnological phenomenon, of mathematics, and which features can be regarded as characteristics of this phenomenon, is very difficult to describe, especially the transitions (gradients) from characteristic mathematical actions to those of a different kind.[320]

There is a certain irony in the fact that Wittgenstein himself appears at times to become entangled in questions on this notion of 'interpreted in terms of content' in Gödel's paper. In a way, one could say he fell in the very philosophical trap he diagnoses. But he is not oblivious to this. This awareness surfaces both in general remarks about his exasperation with the limiting (even tormenting) tendencies of thinking of the philosopher[321], but also in more specific remarks pertaining to the terms he employs and the kinds of claims he might seem to be making. As though he anticipated accusations of defending doctrines :

> **MS 163, 55v**. <u>One must not be dogmatic</u> here: one will be <u>inclined</u> to say that some new proofs change our concept, while others—trivial ones, so to speak—do not. But for us, it is precisely the transition between the inclination to say one thing and the inclination to say the other that is important.[322]

---

[320] *Cf.* RFM.III.84:

> When one tries to describe this situation it is enormously easy to make a mistake in the description. (So it is very difficult to describe.) The descriptions which immediately suggest themselves are all misleading–that is how our language in this field is arranged.
> And there will be constant lapses from description into explanation here.

[321] "The language of philosophers is already deformed, as if by shoes that are too tight." (MS 163, 47v (July 11, 1941)); "A philosophical problem is like a serious illness from which I must free myself and others."(MS 163, 62v (July 11, 1941)). From the occasional introspection and diary entries found in these manuscripts, Wittgenstein's affect seemed very depressed and in pain. *E.g.* MS 163, 58v (July 11): "Something terrible seems to be happening to me—" and the next date entry was more than a month later MS 163, 63r-v (Aug. 23, 1941) "My soul has suffered so much in recent months that it is completely ill, and I cannot seriously think about my work without feeling nauseous. — A great injustice is taking its revenge here. I was deeply hurt, and perhaps I deserved to be hurt so badly, if that is the fate of those who cannot restrain themselves and therefore impose themselves on others." This is also the case in 1938 in his MS 118, the source of the AppIII remarks.

[322] Immediately preceding is: "Here, however, we must distinguish between the concept in mathematics and outside mathematics. Only *in the latter case* can we say that it has changed. [Terribly unclear!]" This is his own acknowledgement of his unclarity. But keeping in mind his remarks elsewhere, one can see that what he means here is that when he speaks

It is a recurring theme of Wittgenstein's that new concepts in mathematics are created through proofs[323]. As we have seen, this idea is also at the heart of his description of the transition between Russell's sense of 'truth' and 'provability' and Gödel's. But this naturally invites further questions. Does *every* new proof create a new concept? What about two proofs that show the same thing? Where to draw the line? Wittgenstein's response is not to supply a criterion, but to diagnose the very urge to draw such a boundary. We are naturally inclined to think that there *must* be a line to draw, or at least that one *can* be drawn. Wittgenstein's philosophy—liberatory and confessional in character—takes as its object precisely these inclinations. He is inviting us to become self-aware so as to free ourselves from illusory imperatives, forms of determinacy imposed not by the demands of the concepts themselves, but by 'culturally' inherited pictures of what explanation and rigor must look like. And to do this he must attend to degrees of difference as well as draw attention to the importance of noticing them, against our automatic dismissal of them in pursuit of fixed classifications.

Wittgenstein's sensitivity to gradations helps explain the unclarity of his use of terms such as 'our concept of __' or 'our conception', 'understanding'. We have an 'innate' desire to force everything into rigid categories (like Frege who says everything is either concept or object); to capture everything and leave nothing out (like Frege's attempt to capture natural language in a formal system). But if we are really honest, Wittgenstein seems to suggest, we will see that this cannot be achieved.

> **MS 163, 60r.** If one said that every new *method* of calculating changes the concepts, one would have the same vagueness in the concept of 'new method of calculation' as in that of the change of concept.
> Think of it as talking about concepts [*Begriffen*] and conceptual pathways [*Begriffsbahnen*]. Of course, this is vague and should be vague.

> **MS 163, 57r-58v.** But here, what I understand by the *concept* [*Begriff*] is still quite unclear [vague]. Of course, I am thinking of the technique of our use of an expression. It is like the railway network that we have built for it.
> [...]

about the concept of "unprovability" and "truth" changing, he does not mean that the mathematical concept itself changes, but he speaks at a level that is outside of mathematics, the philosophical discourse.

[323] See in RFM *e.g.,* RFM.III.24,31, 39-41, 46; RFM.IV.47; RFM.V.5,42,45; RFM.VII.44,45,70,72.

> For us, it is precisely the gradual or steep fall [*steilere Abfallen*] of concepts towards others that is interesting.
> For in this fall lies our justification [*Berechtigung*] for calling something one thing or another. It is often quite sufficient for us to show that one *need* not call something *that*, that one can call it *that* way. For *that* alone changes the face of things.
>
> In this sense, my dogmatic statements were incorrect. But they could be corrected if, where I said, "this is how it should be viewed," one said, "it can also be viewed this way."[324] And it would be wrong to believe that this would take away the actual wit of the sentence.

4. Conclusion

The aim of this work was to present a reading of Wittgenstein on Gödel's incompleteness theorems. I first argued for a method to approach Wittgenstein's controversial remarks, one centering Wittgenstein's metaphilosophy instead of comparing the remarks to GIT. In clarifying Wittgenstein's dialectical mode of philosophizing and its emancipatory end, it becomes necessary to take seriously the distinction between approaching a mathematical topic philosophically and approaching it mathematically, especially taking care to determine what this distinction amounts to for Wittgenstein himself. Once it is recognized that Wittgenstein is approaching GIT as a philosopher of the Wittgensteinian brand, it is no longer appropriate to attempt to read his remarks by comparing their content to that of Gödel's paper. Such an approach presupposes, without justification, the level at which Wittgenstein's investigation of GIT is conducted. It amounts to assuming in advance what his object and the import of his reflections must be, rather than allowing these to be inferred from the texts. This

[324] Recall it is also something he made explicit to his students "I am not trying to persuade you to change your opinion. I am only trying to recommend a certain sort of investigation."(LFM, 103).

methodological error gives rise to exegetically misguided questions, such as whether Wittgenstein 'really understood Gödel.' The considerations developed in the first part of this work (2.) therefore also constitute the basis of my disagreement with a large body of commentaries.

In the main part of the dissertation (3.), I connected these methodological considerations to Wittgenstein's Gödelian remarks in view of demonstrating that he is not addressing GIT itself, nor GIT as a mathematical theorem, but rather a series of suppositions of what we could say about it given the misleading pictures that shape our thinking. I began by assembling textual evidence of Wittgenstein's own characterization of his interest in Gödel, showing that these remarks fit squarely within his metaphilosophical framework. Given that Wittgenstein did not arrange his writings in a systematically expository manner, it is sensible to first collect the passages wherein he explicitly signals his aims. Appendix III constitutes something of an exception in this respect, insofar as Wittgenstein at one point deliberately gathered remarks from several sources in a typescript (TS 223) arranging them in a determinate order. For this reason, I took this order into account, and I treated the opening remarks as introducing the topic, and the final one as concluding it.

Appendix III is the most discussed set of Gödelian remarks in the secondary literature. It is thought that the "remarks on Gödel's (first incompleteness) theorem are contained almost entirely within an appendix [...] to Part I." [325] I disagree. Even under a conservative criterion of inclusion, I think there is a wealth of Gödelian remarks that have yet to have the attention they deserve. Given this multiplicity, selectivity was unavoidable, and I therefore did not comment on every potentially relevant remark, including several within AppIII. This is because I prioritized fleshing out Wittgenstein's own thoughts with his peculiar method and questions over engaging exhaustively with the secondary literature. I submit that what has been presented is sufficient to address, or at least to provide the essential resources for addressing, many popular and often antagonistic interpretations of Wittgenstein. The Appendix III remarks that remain undiscussed do not, in my view, introduce substantially new considerations beyond those developed elsewhere. Furthermore, I judged it more important to draw attention to remarks hitherto ignored by the secondary literature, especially when they provide new line

[325] Priest 2004, 207.

inquiry from Wittgenstein. I chose not to engage with any of the early reception of Wittgenstein's remarks on Gödel (Kreisel, Bernays, Anderson) who were not even convinced of the philosophical value of these remarks[326], for the next generation of commentaries already dealt with them and developed beyond them.

In the exegetical section (3.2.), I examined how Wittgenstein frames his discussion of GIT in AppIII before offering an interpretation of the eighth and most notorious remark. My reading rests on a distinction between *derivability* and *provability* that I argue Wittgenstein is making implicitly, two conceptions pertaining to the determination of provability that correspond to Russell's system and to Gödel's system. I then reflected on this distinction itself, arguing that to devise it is itself an interpretive move that is proper to the discipline of philosophy and not mathematics. From a strictly mathematical standpoint[327], one may simply say that there is a single system that is the object of the proof's inquiry—*Principia Mathematica*. From a philosophical standpoint, however, the question arises whether Gödel's extension of PM, with its novel techniques and arithmetical resources, ought to be regarded as the same system in a relevant sense. This contrast provides a concrete illustration of Wittgenstein's philosophical mode of engagement with mathematics, as opposed to an account that is more mathematical[328]. Roughly speaking, to say that Gödel made PM yield an undecidable sentence would be a more mathematical way to view the theorem; while to ask whether Gödel's methods transform the object of inquiry (PM) into something conceptually different is a philosophical one.

In the subsequent section, I brought up instances of a prevalent interpretation according to which Wittgenstein is taken to be addressing Gödel himself and criticizing him on the basis of an erroneous understanding of the proof as being semantic rather than syntactic. My goal was to argue against this, particularly as developed by Steiner and Rodych. I first identified weaknesses in their hermeneutic assumptions and then showed that AppIII admits of a coherent interpretation without recourse to this reading. Rodych, who has written several papers on the topic and with whom I had a lot

[326] Floyd 2001, 286.

[327] A point of view itself formulated within (Wittgensteinian) philosophy.

[328] And of course these two kinds of accounts are not really categorical. Like Wittgenstein, I think it is an interesting question, and worthwhile to attempt to draw a line between the two as precisely as possible, but there will always be ambiguous cases.

of disagreements, received the most sustained attention. He engages with the *Nachlass* remarks more than any other commentator and highlights a range of remarks where Wittgenstein refers to specific elements of Gödel's proof, as well as supposedly incriminating passages. Throughout this work, I offer alternative interpretations of these remarks. In this section, it was a matter of showing that the problematic self-referential natural language interpretation (SRNLI) is not the only conception of the proof that Wittgenstein considers. I tried to show that Wittgenstein is in fact very sensitive to these issues of reference, treating them with greater nuance than his critics acknowledge. What Rodych and Steiner interpret as direct criticism of Gödel is better understood as criticism of philosophical opinions one might be tempted to adopt when misled by certain terminological formulations. Finally, by appealing to a passage from MS 163 that I argue is Gödelian, I identified a part where Wittgenstein might be reproaching Gödel with something, namely with the potentially misleading use of the phrase *inhaltlich gedeutet*, a use lacking clarification. I argue that the discussion on "*inhaltlich gedeutet*" concerns the use of this expression in Gödel's proof despite the fact that he is not named, because it is indeed an unclear piece of prose that is pertinent to key concepts involved in the proof, in addition to the fact that it occurs between explicit references to Gödel. The reproach concerns the philosophical implications of Gödel's prose rather than the validity of the proof itself, and it is directed especially at philosophers who reflect on questions of content and meaning. Wittgenstein may be taking issue with this unclarity, but it is mostly the fact that philosophers' attempts to give this concept of *Inhaltlich deutung* are far from satisfying.

In the rest of this section, I returned to Appendix III to elaborate on two further themes: the applicability of the Gödel sentence as a prediction, and the idea that contradictions are not inherently harmful. On the first point, I aligned my interpretation with Juliet Floyd here, and rebuked Rodych's account of applicability. On the second, Wittgenstein challenged the widespread tendency to regard contradictions as symptoms of systemic collapse, as though a single inconsistency inevitably renders an entire calculus unusable. Against this picture, Wittgenstein argues that a contradiction need not destroy anything if the calculus can function in other parts, as it has been priori to the contradiction. Wittgenstein invites us to see that contradictions do not need to be construed as 'unmathematical' in

the sense that they are part of the practice of mathematics because they have a use within it (*e.g.* in indirect proofs). Wittgenstein infers that if a contradiction appears, like the sentence threatening PM with (ω-)inconsistency, it would not ruin the edifice that was PM. This is underscored by Wittgenstein's proposal that PM (or any formalism) was not an edifice in the first place, it is not an entity independent from our practices. In his emancipatory lens, a contradiction can be seen to merely indicate that certain derivational paths are ones we do not wish to pursue.

I then added a final section, "Wittgenstein wrestling with himself"[329], devoted to the remaining portions of the MS 163 passage introduced earlier. This part of the text closes the discussion on *Inhaltlich deutung* in which Wittgenstein explains the vagueness of certain notions he uses (and by extension, that I used) such as 'concept,' 'conception,' and 'understanding,' as well as gives further clarification on how his statements that appear assertoric are to be understood.

When one follows Wittgenstein's own way of determining the general interest of his philosophy, it becomes clear that his primary target is the unreflective *opinions we* are prone to form. His reputation as "*anti-philosophe*"[330] is therefore understandable, given his sustained attention on the 'bias' in the opinions of philosophers, down to their conviction that philosophy (must) consist(s) in the construction of a theory or theories. The intention of his philosophy is to expose such conditioned patterns of thinking and to reveal their shortcomings and groundlessness. Wittgenstein's philosophy treats the meaning of concepts as an 'anthropological' or 'ethnological' phenomenon, though not in an empirical sense, since his object is not people but language as an expression of our form of life. To speak of 'meaning' or significance of a word or 'concept,' on this view, is to describe how it seems to be applied, or the *interest* we have in such an application. This is not a theoretical enterprise. We can say that when he is discussing theories, he does not engage with them or refute them at a theoretical level. Wittgenstein is not conducting this level of analysis because it yields the truth. Instead, his commitment is to lucidity, to self-awareness. It is ethical. His aim is to free us—himself foremost—from automatic ways of thinking which entrap us and could end up being harmful in a variety of ways. The philosopher, on this view,

[329] An expression taken from Priest (2004, 208).

[330] Badiou, *L'antiphilosophie de Wittgenstein*, 2009.

needs healing when s/he is tormented by problems s/he is under a spell to think are legitimate or solvable, and is aiming for something truly 'out there' that is not. The appropriate remedy must therefore be dialectical, precisely in order to avoid one doctrine with another. Otherwise, this endeavour "can degenerate into presentations that do little more than make their creator (or beholder) happy."[331] As Rupert Read suggests, Wittgenstein's philosophy may be understood as a post-Enlightenment philosophy, the 'post' due to the call for "*overcom*[*ing*] the individualistic, rationalistic and thus limited—in a way, *insufficiently* ambitious"[332], yet still a philosophy of 'enlightenment' in a broader, even Buddhist, sense. This affinity is particularly striking when compared to Madhyamaka thought: in its emphasis on self-knowledge, its rejection of theoretical reification, its paraconsistent tendencies, and its repeated efforts to show—through dialectical analyses reducing a concept into emptiness—that concepts lack inherent sense [333]. Emphasizing these aspects is essential in order not to leave Wittgenstein vulnerable to accusations of criticizing Gödel out of ignorance, or of contradicting himself, or Gödel on the basis of presuming that he had his own view[334]. I maintain that it is hardly possible to understand what Wittgenstein is saying in his Gödelian remarks outside of this perspective.

[331] *Ibid*, 171.

[332] Read 2021, 61.

[333] Read 2021, 35, ft.6; see also *e.g*. Jay L. Garfield and Priest, 'Nagarjuna and the Limit of Thought', 2003.

[334] For instance, he is often called a finitist (*E.g.*, Rodych (2006, 56), Berto (2009, 217). But take a look at his explicit renunciation of such a view in LFM, 141:

> If you say that mathematical propositions are about a mathematical reality–although this is quite vague, it has very definite consequences. And if you deny it, there are also queer consequences—for example, one may be led to finitism. Both would be quite wrong. There is a *muddle* at present, an unclarity. But this doesn't mean that certain mathematical propositions are *wrong*, but that we think their interest lies in something in which it does not lie. I am *not* saying transfinite propositions are *false*, but that the wrong pictures go with them. And when you see this the result may be that you lose your interest. It may have enormous consequences but not mathematical consequences, not the consequences which finitists expect.

Appendix

Note that the emphases here are all Wittgenstein's own.

Letter to Schlick 31.7.1935.

Über die Anwendung des von mir gesagten auf Fälle wie den von Ihnen zitierten will ich nur eines sagen: Wenn Sie hören jemand habe bewiesen, es müsse unbeweisbare Sätze in der Mathematik geben, so ist daran vorerst gar nichts Erstaunliches, weil sie ja noch gar keine Ahnung haben, was dieser scheinbar so klare Prosasatz sagt. Sie haben also den Beweis von A bis Z. durchzugehen um zu sehen, was er beweist. D.h.: ehe Sie diesen speziellen Beweis bis in sein letztes Detail durchgegangen sind, wissen Sie noch gar nichts. Z.B. wissen Sie nicht, was in der Auffassung dieses Beweises ein "mathematischer Satz" ist. Denn anderseits ist ja eine Mathematik abgrenzbar, in der es nicht unbewiesene Sätze giebt, z.B die elementare Arithmetik. Daß der Prosasatz, der als Resultat des Beweises gilt, erstaunlich klingt, sagt gar nichts. Es ist klar, daß auch der gegenteilige Prosasatz bewiesen werden kann. D. h.: ich zweifle nicht, daß ein geschickter Mensch die Anwendung der Worte so wenden kann, daß wir geneigt sein werden den Gegenteiligen Satz als das Resultat eines Beweises anzuerkennen; wie man ja beweisen kann, daß jede Gerade einen Kreis schneidet & auch, daß nicht jede Gerade einen Kreis schneidet. Wer sich nun wundern würde, daß beide entgegengesetzten Sätze beweisbar sind, dem würde ich sagen: Schau Dir die Beweise an dann wirst Du sehen "in welchem Sinn" das eine & "in welchen Sinn" das andere bewiesen ist. Und ehe Du die Beweise genau durchstudiert hast ist gar kein Grund zur Verwunderung. Alles was Sie aus "meinen Anweisungen" lernen können ist, daß über so einen Beweis & sein Resultat nichts gesagt werden kann ehe sie nicht diesen bestimmten Beweis untersucht haben. D.h.: der Philosoph ist immer im Unrecht, der quasi etwas in der Mathematik prophezeien will & sagt "das ist unmöglich", "das kann nie bewiesen werden". Warum nicht? das was bewiesen wurde ist ja nur ein Wortausdruck & der Beweis gibt ihm seinen besonderen Sinn; & mit wieviel Berechtigung wir dann diesen Beweis den Beweis dieses Prosasatzes nennen ist teils Geschmacksache; d. h. es ist Sache unseres Ermessens & unserer Neigung ob wir das in diesen Prosasatz ausgedrückte Bild hier anwenden wollen oder nicht. Wie es Sache unserer Neigung ist ob wir von imaginären Punkten reden wollen oder nicht; oder von unsichtbarem Licht, oder nicht. – Die genaue Untersuchung eines komplizierten Beweises ist außerordentlich schwer. D. h., es ist außerordentlich schwer den Beweis ganz durchsichtig zu gestalten & volle Klarheit über seine Beziehung zu andern Beweisen, seine Stellung in gewissen Systemen etc zu gewinnen. Sie brauchen nur einmal zu versuchen einen Beweis wie den des Satzes, daß √2 irrational ist, genau zu untersuchen & Sie werden sich davon überzeugen. Das bedeutet aber nicht, daß in diesem Beweis von dieser Untersuchung etwas Geheimnisvolles ist, sondern nur daß wir ihn, & besonders seine Stellung zu anderen Beweisen & Sätzen, noch nicht klar überschauen. – Sie sind auf einem falschen Weg, wenn Sie sagen Sie fühlten sich trotz meiner Anweisungen noch immer hilflos den & den Beweisen gegenüber. Der Witz ist eben daß man gar nichts von vornherein über einen Beweis sagen kann, den man nicht tatsächlich untersucht hat, außer das, daß man ihn selbst untersuchen muß um zu sehen, wie sehr oder wie wenig angemessen es ist ihn

einen Beweis dieses Prosasatzes zu nennen. Philosophie kann Ihnen nichts über einen Beweis sagen, & versucht sie es, so müssen Sie allerdings immer zittern, ob sie nicht wie ein falscher Prophet durch die Wirklichkeit lügengestraft werde. Sie denken etwa: "Aber die Philosophie soll mir doch sagen, ob so etwas bewiesen werden kann". Aber es ist ja hier ganz unbestimmt gelassen, was unter einem Beweis eines Wortlautes zu verstehen ist. Erst wenn ich den Beweis kenne, oder das System von Beweisen deren einer unser Beweis ist sehe ich was hier Beweis genannt wird, wieviel Ähnlichkeit dieses mit andern Konstruktionen hat die wir Beweise nennen & mit welchem Teil der ungeheuren Familie die wir mathematische Beweise nennen es die meiste Ähnlichkeit hat. Und erst dann kann man überhaupt über den Beweis & den Satz reden. Wenn also ein Mathematiker auf sie zukommt & sagt: "Nun, was sagen sie zu dem neuen Beweis, daß .....", so haben Sie zu antworten: "Gar nichts", – aber nicht darum weil sie dem Beweis solange nicht trauen können, sondern weil sie auch den Wortlaut des Bewiesenen noch nicht verstehen solange sie den Beweis nicht kennen. Ebenso: Wenn Prof N. N. auf sie zukommt & ihnen sagt, man habe experimentell festgestellt daß es Schmerzen gibt die man nicht fühlt, so ist die Antwort nicht: "Ehe ich das glaube, müßte ich die Experimente kennen", sondern: "Ehe ich das verstehe muß ich die Experimente kennen". Ich will damit nicht sagen, daß es nicht sehr interessant ist diese Experimente zu studieren, & jene Beweise. Aber Sie sind in der falschen Auffassung begriffen, wenn Sie glauben die Ohren spitzen zu müssen, wenn Ihnen von einem neuen Beweis erzählt wird.

English translation:

Regarding the application of what I have said to cases such as the one you cited, I would like to say only one thing: if you hear that someone has proven that there must be unprovable statements in mathematics, *there is nothing surprising about that at first*, because you have no idea what this seemingly clear prose statement means. So you have to go through the proof from A to Z to see what it proves. In other words, until you have gone through this particular proof in every detail, you know nothing at all. For example, you do not know what a “mathematical theorem” is in the context of this proof. On the other hand, it is possible to define a branch of mathematics in which there are *no* unproven theorems, e.g., elementary arithmetic. The fact that the prose statement, which is considered the result of the proof, sounds surprising does not mean anything. It is clear that the opposite prose statement can also be proven. In other words, I have no doubt that a skilled person can use words in such a way that we will be inclined to accept the opposite statement as the result of a proof; just as it can be proven that every straight line intersects a circle and also that not every straight line intersects a circle. To anyone who would be surprised that both opposing statements are provable, I would say: Look at the proofs and you will see “in what sense” the one is proven and “in what sense” the other is proven. And before you have studied the evidence thoroughly, there is no reason to be surprised. All you can learn from “my instructions” is that *nothing* can be said about such evidence and its results until you have examined this particular evidence. In other words, philosophers are always wrong when they try to predict something in mathematics and say “*that*'s impossible” or “*that* can never be proven.” Why not? What has been proven is only an expression in words, and the proof gives it its special meaning; and *how* justified we are

in calling this proof the proof of this prose sentence is partly a matter of taste; i.e., it is a matter of our discretion and our inclination whether we want to apply the image expressed in this prose sentence here or not. Just as it is a matter of our inclination whether we want to talk about imaginary *points* or not; or about invisible light, or not.—It is extremely difficult to examine a complicated proof in detail. In other words, it is extremely difficult to make the proof completely *transparent* and to gain full clarity about its relationship to other proofs, its position in certain systems, etc. You only need to try to examine a proof such as the theorem that √2 is irrational in detail, and you will see this for yourself. However, this does not mean that there is anything mysterious about this proof *from* this investigation, but only that we do not yet have a clear overview of it, especially its position in relation to other proofs and theorems. You are on the wrong track when you say that, despite my instructions, you still feel helpless when faced with the proofs. The point is that you cannot say anything in advance about a proof that you have not actually examined, except that you must examine *it yourself* to see *how appropriate or inappropriate* it is to call it a proof of *this* prose statement [*Prosasatzes*]. Philosophy cannot tell you anything about proof, and if it tries to do so, you must always fear that it will be exposed as a false prophet by reality. You might think, "But philosophy should tell me whether something like this *can* be proven." But here it is left completely undefined what is meant by proof of a statement. Only when I know the proof, or the system of proofs of which ours is one, do I see what is called proof here, how much similarity it has with other constructions that we call proofs, and with which part of the enormous family that we call mathematical proofs it has the most similarity. And only then can one talk about the proof and the theorem at all. So if a mathematician approaches you and says, "Well, what do you say to the new proof that …", you should reply, "Nothing" – not because you cannot trust the proof yet, but because you do not understand the wording of the proof until you know the proof itself. Similarly, if Prof. N. N. approaches you and tells you that experiments have shown that there is pain that cannot be felt, the answer is not, "Before I *believe* that, I would have to know the experiments," but rather, "Before I *understand* that, I would have to know the experiments." I am not saying that it is not very interesting to study these experiments and that evidence. But you are mistaken if you believe that you have to prick up your ears when you hear about new evidence.—

Bibliography

Wittgenstein – primary sources

Wittgenstein, Ludwig, Friedrich Waismann, Moritz Schlick, Gordon P Baker, Brian McGuiness, et Antonia Soulez. *Dictées de Wittgenstein à Friedrich Waismann et pour Moritz Schlick*. Paris: PUF, 1997.

Wittgenstein, Ludwig, R. G Bosanquet, and Cora Diamond. *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge, 1939 : From the Notes of R.G. Bosanquet, Norman Malcolm, Rush Rhees, and Yorick Smythies*. University of Chicago Press ed. Chicago, IL: University of Chicago Press, 1989.

Wittgenstein, Ludwig, and Brian McGuiness. *Ludwig Wittgenstein, Letters, Lectures, Conversations, Memoirs : Wittgenstein and the Vienna Circle*. Charlottesville, Va: InteLex Corporation, 2001.

Wittgenstein, Ludwig, Claus Huitfeldt, The Wittgenstein Archive at the University of Bergen., and G. H. von Wright. *Wittgenstein's Nachlass, the Bergen electronic edition*. Charlottesville, Va: InteLex Corporation, 1998.

Wittgenstein, Ludwig, G. E. M Anscombe, G. H. von Wright, and Heikki Nyman. *The Collected Works of Ludwig Wittgenstein. Notebooks: 1914-1916*. Charlottesville, Va: InteLex Corporation, 1998.

Wittgenstein, Ludwig, G. E. M Anscombe, P. M. S Hacker, and Joachim Schulte. *Philosophical investigations*. 2nd ed. / by P.M.S. Hacker and Joachim Schulte. Oxford: Wiley-Blackwell, 1958.

Wittgenstein, Ludwig, G. E. M Anscombe, P. M. S Hacker, and Joachim Schulte. *Philosophische Untersuchungen = Philosophical investigations*. Rev. 4th ed. / by P.M.S. Hacker and Joachim Schulte. Chichester, West Sussex, U.K. ; Wiley-Blackwell, 2009.

Wittgenstein, Ludwig. *Remarks on the foundations of mathematics.* Edited by G. H. von Wright, R. Rhees, and G.E.M. Anscombe. Translated by G.E.M. Anscombe. Revised 4th ed. MIT Press, 1978.

Wittgenstein, Ludwig, G. E. M Anscombe, G. H. von Wright, Heikki Nyman, and Rush Rhees. *The published works of Ludwig Wittgenstein : Some Remarks on Logical Form*. Charlottesville, Va: InteLex Corporation, 1993.

Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus*. Translated by: C.K. Ogden. Trowbridge, Wiltshire, G.B.; Routledge & Kagan Pau, 1983.

The Rest

Anderson, Alan Ross. "Mathematics and the 'Language Game.'" *The Review of Metaphysics* 11, no. 3 (1958): 446–58.

Badiou, Alain. *L'antiphilosophie de Wittgenstein*. Caen: Nous, 2009.

Bays, Timothy. 'On Floyd and Putnam on Wittgenstein on Gödel': *Journal of Philosophy* 101, no. 4 (2004): 197–210.

Bernays, Paul. "Comments on Ludwig Wittgenstein's Remarks on the Foundations of Mathematics." In *Philosophy of Mathematics: Selected Readings*, edited by Paul Benacerraf, and Hilary Putnam, 510– 528. Englewood Cliffs, NJ: Prentice-Hall, 1964.

Berto, Francesco. 'The Gödel Paradox and Wittgenstein's Reasons'. *Philosophia Mathematica* 17, no. 2 (1 June 2009): 208–19.

Berto, Francesco. 'Wittgenstein on Incompleteness Makes Paraconsistent Sense'. In *Paraconsistency: Logic and Applications*, edited by Koji Tanaka, Francesco Berto, Edwin Mares, and Francesco Paoli, 257–76. Dordrecht: Springer Netherlands, 2013.

Bradley, F. H. *The Principles of Logic, Vol. I*. London: Oxford University Press, 1922.

Dawson, John W. *Logical Dilemmas : The Life and Work of Kurt Gödel*. Wellesley, Mass: A K Peters, 1997.

Dawson Jr., John W., "IV. The reception of Gödel's Incompleteness Theorem." *Godel's Theorem in Focus*, Taylor & Francis Group, 1990.

Diamond, Cora. "Criss-Cross Philosophy". In *Wittgenstein at Work: Method in the Philosophical Investigations* by E. Ammereller & E. Fischer (Eds.). London: Routledge (2004): 201–220.

Ferreirós, José."The Crisis in the Foundations of Mathematics". *The Princeton Companion to Mathematics*. Princeton university press (2008) 142–156.

Floyd, Juliet, Jonathan Ellis, and Daniel Guevara. "Das Überraschende: Wittgenstein on the Surprising in Mathematics." In *Wittgenstein and the Philosophy of Mind*. United States: Oxford University Press, 2012.

Floyd, Juliet. "Prose Versus Proof: Wittgenstein on Godel, Tarski and Truth." *Philosophia Mathematica* 9, no. 3 (October 1, 2001): 280–307. https://doi.org/10.1093/philmat/9.3.280.

Floyd, Juliet. 'On Saying What You Really Want to Say: Wittgenstein, Gödel and the Trisection of the Angle.' In Jaakko Hintikka, *From Dedekind to Gödel: The Foundations of Mathematics in the Early Twentieth Century*, Synthese Library Vol. 251.Kluwer Academic Publishers (1995): 373-426.

Floyd, Juliet, and Hilary Putnam. 'Bays, Steiner, and Wittgenstein's "Notorious" Paragraph about the Gödel Theorem'. *The Journal of Philosophy* 103, no. 2 (2006): 101–10.

Floyd, Juliet, and Hilary Putnam. 'A Note on Wittgenstein's "Notorious Paragraph" about the Gödel Theorem'. *The Journal of Philosophy* 97, no. 11 (2000): 624–32.

Frege, Gottlob. 'Sense and Reference' In *The Philosophical Review,* Vol. 57, No. 3 (May, 1948), 209–230.

Frege, Gottlob. 'Über Sinn und Bedeutung'. In *Zeitschrift für Philosophie und philosophische Kritik*, NF 100, (1892): 25–50.

Garfield, Jay L. and Priest, Graham "Nagarjuna and the Limit of Thought" (2003). *Philosophy: Faculty Publications*, Smith College, Northampton, MA.

Gödel, Kurt. *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*. Translated from by B. Meltzer. New York: Dover Publications, 1992.

Gödel, Kurt. "On Formally Undecidable Propositions of Principia Mathematica and Related Systems". Translated from by Jean van Heijenoort In *From Frege To Gödel: A Source Book in Mathematical Logic*. Harvard University Press: Cambridge, Massachusetts, 1967.

Gödel, Kurt. "Über Formal unentscheidbare Sätze der 'Principia Mathematica' und Verwandten Systeme I" In *Monatshefte für Mathematik und Physik*, 38 (1931), 173–198.

Hintikka, Jaakko. 'Kurt Gödel: An Introduction'. *Revue Internationale de Philosophie* 59, no. 234 (4) (2005): 451–57.

Hintikka, Jaakko. 'On the Development of the Model-Theoretic Viewpoint in Logical Theory'. *Synthese* 77 (1988): 1–36.

Hoffman, *Dirk W. Gödel's Incompleteness Theorems: A Guided Tour Through Kurt Gödel's Historic Proof*. Translated from German by Dirk W. Hoffman. Springer, 2024.

Kreisel, G. 'Wittgenstein's Remarks on the Foundations of Mathematics'. Edited by L. Wittgenstein. *The British Journal for the Philosophy of Science* 9, no. 34 (1958): 135–58.

Lampert, Timm. 'Wittgenstein and Gödel: An Attempt to Make "Wittgenstein's Objection" Reasonable'. *Philosophia Mathematica* 26, no. 3 (1 October 2018): 324–45. https://doi.org/10.1093/philmat/nkx017.

Marion, Mathieu. *Wittgenstein, Finitism, and the Foundations of Mathematics*. Oxford: Clarendon, 1998.

Moore, G.E. 'The Nature of Judgment'. In *Mind*, New Series, Vol. 8, No. 30 (Apr., 1899): 176–193.

Mulhall, S. *Inheritance and Originality: Wittgenstein, Heidegger, Kierkegaard*. Oxford Clarendon Press, 2001.

Potter, Michael. *Set Theory and its Philosophy*. Oxford University Press, 2004.

Priest, Graham. *In Contradiction*. Oxford University Press, 2006.

Priest, Graham. 'Wittgenstein's Remarks on Gödel's Theorem'. In *Wittgenstein's Lasting Significance*. Routledge, 2004.

Prince, Hal. *The Annotated Gödel: A Reader's Guide to His Classic Paper On Logic and Incompleteness*. Homebred Press: 2022.

Read, Rupert J. *Wittgenstein's Liberatory Philosophy : Thinking through His Philosophical Investigations*. Abingdon ; Routledge, 2021.

Rhees, R. *Recollections of Wittgenstein.* Oxford University Press. 1984

Rodych, Victor. 'Misunderstanding Gödel: New Arguments about Wittgenstein and New Remarks by Wittgenstein'. *Dialectica* 57, no. 3 (2003): 279–313. https://doi.org/10.1111/j.1746-8361.2003.tb00272.x.

Rodych, Victor. 'Who Is Wittgenstein's Worst Enemy?: Steiner on Wittgenstein on Godel'. *Logique Et Analyse* 49, no. 193 (2006): 55–84.

Rodych, Victor. "Wittgenstein's Inversion of Gödel's Theorem." *Erkenntnis (1975-)* 51, no. 2/3 (1999): 173–206. http://www.jstor.org/stable/20012949.

Rodych, Victor. 'Wittgenstein on Gödel: The Newly Published Remarks'. *Erkenntnis* 56, no. 3 (1 June 2002): 379–97. https://doi.org/10.1023/A:1016379820371.

Shanker, S.G. "Wittgenstein's Remarks on the Significance of Gödel's Theorem 1" in *Godel's Theorem in Focus* (1st ed.). Routledge, 1988: 155–256.

Smith, Peter. *An Introduction to Gödel's Theorems*. Second edition. Cambridge: Cambridge University Press, 2013.

Smullyan, Raymond M. *Gödel's Incompleteness Theorems.* Oxford University Press: 1992.

Steiner, Mark. 'Wittgenstein as His Own Worst Enemy: The Case of Gödel's Theorem'. *Philosophia Mathematica* 9, no. 3 (2001): 257–79. https://doi.org/10.1093/philmat/9.3.257.

Wright, Crispin. *Wittgenstein on the Foundations of Mathematics*. London: Duckworth, 1980.