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Experimental Investigation of the Dynamic Behaviour of Inclined Sagged Cables

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EXPERIMENTAL INVESTIGATION OF THE DYNAMIC BEHAVIOUR OF INCLINED SAGGED CABLES

by

Michael Hoftyzer

A thesis submitted in partial fulfillment of
the requirements for the degree of
Master of Applied Science in Civil Engineering

to

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This thesis is dedicated to my parents

For their support and patience during my studies at university

This thesis is dedicated to the memory of my grandparents

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ABSTRACT

The nonlinear characteristics of two-dimensional in-plane free vibration of elastic cables with arbitrary sags over a range of inclinations were analyzed numerically and some typical results were experimentally verified. The analytical investigation was accomplished by using a multi-degree-of-freedom cable model, which was discretized by the use of finite elements. Qualitative and quantitative results obtained for various physical parameters of the elasto-geometrical cable parameter are discussed in detail, investigating the modal transition phenomena occurring in planer nonlinear dynamics. Close attention is placed towards investigating the frequency avoidance (or veering) regions of the internal resonances. This resulted in the analysis of the influence of cable inclination, by looking at the overall nonlinear behaviours against those previously outlined for the horizontal cable. The thesis concludes with a summary of results and some conclusions derived from it.

Limited experimental analysis was preformed for specific cases by considering, for example, the angle of inclination and cable properties. The experimental results are used to verify the results from FEM analysis. Particular attention is paid on the phenomena of frequency crossover of flat sagged cables and to resolve issues on the avoidance points of inclined sagged cables.

The existence of modal cross-over points for flat sagged cables was clearly demonstrated both analytically and experimentally. The first and third natural frequencies approach to the natural frequencies of the second and fourth modes, coinciding at the modal cross-over point, and increases past this point, when the cable tension is reduced or the sag is increased. Trials were performed on a wide range of angles of chord inclination, including 0 degrees as a reference and for an inclination of 15, 30, 45 and 60 degrees.

For an inclined sagged cable, the frequency of the first symmetric mode is less than the frequency of the first asymmetric mode up to the first avoidance point. This is similar to the case of the flat sagged cable. As the first avoidance point is approached, the first symmetric mode becomes a combination of the first symmetric and first asymmetric mode, or a hybrid mode. The frequencies of these mode shapes do not actually cross as they approach, but diverge and switch from symmetric to asymmetric and asymmetric to symmetric. As above for the flat sagged cable case, the third and fourth frequency lines (second symmetric and second

asymmetric modes) show a similar phenomenon, but at a different set of geometric and physical properties. These phenomena were analytically predicted before but this may have been the first time to have them verified experimentally.

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NOMECLATURE

A	cross-sectional area of the cable, mm^2 , $[\text{L}^2]$
c	wave speed, velocity of wave propagation along the string, m/s , $[\text{L T}^{-1}]$
E	Young's modulus of elasticity, MPa
f_n	natural frequency, s^{-1} , $[\text{T}^{-1}]$
g	acceleration due to gravity, m s^{-2} , $[\text{L T}^{-2}]$
H	horizontal component of cable tension, N
l	span length of cable, $[\text{L}]$
L	total cable length, $[\text{L}]$
t	time, s^{-1} , $[\text{T}^{-1}]$
T	static cable tension, N
w	unit weight of cable, N m^{-1}
x	local horizontal rectangular coordinates, $[\text{L}]$
y	local vertical rectangular coordinates, $[\text{L}]$
δ	sag of the cable, $[\text{L}]$, defined as the vertical distance from the chordline between two supports to the deflected cable, taken at the centre of the span.
λ^2	cable parameter to account both geometric and elastic effects, $\left(\frac{EA}{H}\right)\left(\frac{wl}{H}\right)^2$, $[\text{0}]$
ρ	mass density per unit length, kg/m
ω	natural circular frequency of vibration definition, rad s^{-1}
Ω	non-dimensional natural frequency of the cable, $[\text{0}]$

CHAPTER 1

INTRODUCTION

Cables encounter dynamic problems in a wide variety of engineering applications. Included in this is large amplitude nonlinear vibration, which has the potential to become a serious problem and also a great challenge for engineers. As the tension applied to the structural cable is lowered, the cable begins to deflect under its own weight; the linear vibration theory no longer applies to the natural frequencies and mode shapes resulting from that vibration. For this reason, there is need to study nonlinear cable dynamics.

In dealing with the vibration of sagged cables, most research studies have concentrated on a suspended cable with fixed supports at the same vertical level. These studies have focused on nonlinear free and forced vibrations both theoretically and experimentally. The internal resonance of the sagged cable systems can cause strong modal coupling effects and result in responses with multiple modes and multiple frequencies. Also, when the cable system vibrates, a mode transition phenomenon develops. This phenomenon occurs in purely free planar dynamics; with two neighbouring modes interacting and combining with each other to create a hybrid mode.

Inclined sagged cables are encountered in several practical applications; such as when they are used to span large distances. When the cable is subjected to a force either due to wind or hydrodynamic drag excitations, it may start to vibrate three-dimensionally.

The linear vibrations of inclined cables demonstrate interesting dynamic features. They include frequency avoidance (or veering) and the associated hybrid mode shapes, which distinguish them, as well as other asymmetric structural features, from horizontal symmetric sagged cables. Past studies have included the investigation of parametric and external resonances in an inclined elastic cable for application to a stack/wire system. A nonlinear dynamic model of an inclined cable in the framework of a cable-structure system was also investigated. The coupling between in-plane and forced out-of-plane vibrations of a two-degree-of-freedom inclined cable model has also been studied when subject to input disturbance. In all these cases, very small values of cable sag-to-span ratios were considered. Takahashi and

Konishi [16] examined nonlinear free vibrations of inclined sagged cables and included a brief discussion on the geometrically nonlinear effects as cable inclination, cable extensibility, and internal resonances. Narakorn et al [7] developed a model formulation for large amplitude three-dimensional free vibrations of inclined cables, which is not limited to cables having small sag-to-span ratios and also takes into account the axial deformation effects. Therefore, a relevant systemic investigation seems to be worthwhile for both theoretical and practical view points.

In the present study, the nonlinear characteristics of two-dimensional in-plane free vibration of elastic cables with arbitrary sags over a range of inclinations are analyzed numerically. This is accomplished by using a multi-degree-of-freedom cable model, which is discretized through the use of finite elements and subsequently solved. Qualitative and quantitative results obtained for different values of the elasto-geometrical cable parameter are discussed in detail, investigating the modal transition phenomena occurring in planer nonlinear dynamics. Close attention will be placed towards investigating the frequency avoidance (or veering) regions of the internal resonances. This will result in the analysis of the influence of cable inclination, by looking at the overall nonlinear behaviours against those previously outlined for the horizontal cable. The thesis concludes with a summary of results and some conclusions derived from it.

The development of the knowledge of cable dynamics over the past number of centuries is summarized in the following chapter. This is followed by a summary of the physical model used in the experimental portion of this research, including some of the results produced from this work. In the fourth chapter, the computational model used in the finite element analysis software is reviewed and results are presented along with the experimental results obtained in the previous chapter. In Chapter 5, the results from both the previous chapters on experimental and computational models are reviewed and discussed in greater detail. In the sixth and final chapter, findings are summarized, conclusions are drawn and recommendations made.

CHAPTER 2

DYNAMICS OF SUSPENDED CABLES

2.1 HISTORY OF CABLE DYNAMICS

The knowledge of cable vibration has a history dating back many centuries. An early example of this vibration is the Aeolian harp by the Greeks, which produced musical sounds when exposed to the wind, due to the strumming effect of the shed vortices. String vibration interested Pythagoras and his disciples, who had a qualitative understanding of the relationship between the pitch of a note and the tension, length and mass of a vibrating string.

However, it was not until 1636, when Mersenne experimentally discovered the basic laws governing the vibrations of taut string. Galileo was also interested in the subject and, around the same time, determined the formula for the period of a pendulum. This would be given a mathematical formulation later by Huygens. In 1676, Noble and Pigott established the idea of different modes of vibration and the associated concept of internal nodes [3]. The vibration of a taut string was one of the first physical systems to which modern mechanics and mathematics were applied. This problem was investigated originally by Taylor in 1713 and continued by D'Alembert, Euler, and Daniel Bernoulli in later part of the eighteenth century [6][7][14]. In 1755, Bernoulli proposed that a general compound vibration could be broken down into independent constituent modes [3]. This is the foundation on which the cable dynamics are now based.

By 1788, Lagrange and others had reached solutions of various degrees of completeness for the vibrations of an inextensible, mass-less string, fixed at each end, from which numerous weights were hung [14].

In 1820, Poisson proposed the general Cartesian partial differential equations of motion of a cable element under the action of a general force system [14]. This provided a natural complement to the string equation first proposed by D'Alembert in 1750 [3]. The equations proposed by Poisson were used to improve the solution previously obtained for the vertical free-hanging cable and the taut string [3].

By this time, the correct solution had been proposed for the linear vibrations of uniform cables, the geometries of which were the extreme forms of the catenary, the taut string and the pendulum. No results had been proposed for cables, where the ratio of sag to span was between these extremes.

Rohrs [7] obtained an approximate solution of the equation of motion for the symmetrical modes of a uniform chain with small sag-to-span ratio [19]. A form of Poisson's general equations correct to the first order was used, together with another equation termed the continuity of a chain. This continuity equation related only to the geometric compatibility, as the assumption that the chain was inextensible, was used [3].

In 1868, Routh was able to come up with the exact solution for the symmetric vertical vibrations, along with the associated longitudinal motion, of a heterogeneous cable assuming that it was suspended in the form of a cycloid [14]. Like Rohrs approximation above, Routh used the assumption that the cable was inextensible. His results reduced to Rohrs' solution when the sag-to-span ratio was small. Routh was also able to obtain an exact solution for the antisymmetric modes [3].

More work was prompted in this area in 1940 with the aerodynamic failure of the Tacoma Narrows suspension bridge. In was 1941 when Rannie and von Kármán were able to independently derive the results for both the symmetric and antisymmetric in-plane modes of an inextensible three span cable [7]. Cable elasticity was considered for the first time by Klöppel and Lie in 1942 [14]. Vincent, in 1945, was able to extend Rannie and von Kármán analysis to include the effects of cable elasticity in the symmetric modes [3]. The lack of emphasis on cable stretch in the above analyses is hardly surprising, as cables with relatively deep profiles are not affected very much by stretching, at least in the lower modes.

Pugsley resumed the study in 1948. He sought a simple approximate theory that would lead to explicit expressions, empirical if necessary, for the natural frequencies of suspension chains. Guided by the results of some elementary experiments, Pugsley presented a simple approximate theory for the oscillations and compared the results with those obtained by Routh for the cycloidal chain. Pugsley proposed semi-empirical formulas for the first three in-plane natural frequencies and compared the resulted frequencies with experimental results [19]. Experiments were conducted on cables for sag-to-span ratios ranging from 1:10 up to 1:4, to demonstrate the applicability of the above results [3][14].

In 1953, Saxon and Cahn obtained an asymptotic solution to the linearized equation of motion for the chain vibrating with small amplitude in the plane of a catenary, which formed the original equilibrium configuration [19]. The solution was obtained by assuming that the cable was inextensible. It reduced to previously known results for inextensible cables of small sag-to-span ratios, and for which asymptotic solutions gave extremely good results for large ratios of sag-to-span [3][14]. Their theory was demonstrated to be accurate by comparison to experimental results for sag ratios of 1:10 or greater. Later in 1961, Goodey addressed the same problem for deep-sag cables as Saxon and Cahn did 1952, using a different mathematical formulation [3][19]. Smith and Thompson extended this analysis further by showing how the analysis can be extended to include inclined cables [3].

The linear theory was assumed in the above work, limiting the investigation to small displacements. A systematic exploration of nonlinear cable dynamics was developed by Tonis [14], which can be considered representative of this new period.

For small sag-to-span ratios, the theories based on the imposition of inextensibility show that the first symmetric in-plane mode, primarily involving vertical motion, occurs at a frequency contained in the first non-zero root of

$$\tan \frac{\omega}{2} = \frac{\omega}{2} \quad \text{or} \quad \omega_1 = 2.86\pi \quad (2.1)$$

The frequency of the first symmetric mode of the transverse vibration of a taut string is contained in the first root of

$$\cos \frac{\omega}{2} = 0 \quad \text{or} \quad \omega_1 = \pi \quad (2.2)$$

The discrepancy of almost 300% in the first symmetric mode of transverse vibration cannot be resolved with the analysis restricted by the assumption of inextensibility. Several analytical studies, which were all performed independently of one another, demonstrated that the inclusion of cable stretch resolves this discrepancy.

Irvine and Caughey studied the free vibrations of a rigidly supported cable, with both supports at the same elevation. As part of their work, they used the linear theory with a ratio of sag-to-span from approximately 1:8 to zero. These ratios for sag are small enough for the static geometry to be adequately described by parabolic profiles; this theory provides good results and explains previous discrepancies caused by the inextensibility assumptions [3]. They assumed

that the elastic deformation of the cable due to vibration is quasi-static; the dynamic cable tension is a function of time. Irvine and Caughey were able to come up with one geometric-elastic system parameter for the dynamic behaviour of a cable [14]. The natural frequencies of symmetric in-plane modes and the respective antisymmetric in-plane modes coincide for certain values of this parameter, the so-called 'cross-over'.

West, Geschwindner, and Suhoski developed a numerical technique that indicated modal transition for small sag-to-span ratios, as well as explain the discrepancies in earlier theories caused by the assumed cable inextensibility. They treated the cable as a linkage of finite number of straight bars connected by smooth pins. Their work is limited to in-plane motion, as well as the assumption of equal height supports are required [3].

2.2 DYNAMICS OF INCLINED CABLES

In the studies outlined above, the dynamic motion of a taut cable is accompanied by an inelastic change in its configuration or by stretching of the cable, or by a combination of the two. The dynamic solution is affected by this change in the configuration and stretching of the cable. The change in the configuration of the cable and the stretching affect the dynamic solution differently. These effects are particularly interesting in the case of an inclined cable, due to different parts of the cable having different curvature and static tension. This results in hybrid modes, which is a mixture of taut wire and inelastic-chain dynamics.

In the numerical formulations (mathematical framework for studying such dynamics) outlined above for flat sagged cables, assumes the axes are directed horizontally and vertically. This becomes a concern as the horizontal inertia of the vibrating cable assumes an increasing importance as the angle of inclination (chord inclination) increases.

Irvine attempted to extend the theory of flat sagged cables for use with inclined cables, by neglecting the weight component parallel to the cable chord. This was accomplished by transforming the axis to align them with the chord of the inclined cable [6]. Irvine, along with Griffin, performed an analysis of a cable response due to dynamic loading as a result of support acceleration due to earthquakes [14].

Triantafyllou [18] derived an asymptotic solution for the dynamics of a shallow-sag, inclined, elastic cables. He was able to include dynamic tension (quasi-statically) and weight component variability parallel to chord of the inclined cable. Through this, it was demonstrated

that inclined cables have different properties than horizontal cable, which cannot be obtained by simply extending horizontal cable results.

Triantafyllou was able to obtain a solution where the cross-over, as defined by Irvine and Caughey [7], does not occur for inclined sagged cables. Instead, he was able to obtain two natural frequencies at values of $\lambda = 2n\pi$, but the two values would never coincide with one another unless the cable was taut or a horizontal case. Triantafyllou argued that if the horizontal cable is considered as the limiting case, it could be claimed that the cross-over does not occur. Instead, the two curves would touch each other in a horizontal sagged cable case; otherwise there would be an avoidance point.

The noncrossing curves phenomenon, or ‘avoided crossing,’ is important in determining the mode shapes for the inclined cable, which are totally different from those for horizontal cables. In the region of $\lambda = 2n\pi$ and over a region that becomes wider with increasing sag and inclination, the modes become hybrid modes. These hybrid modes are a mixture of symmetric and asymmetric shapes. It is known that symmetric modes cause most of the dynamic tension in a vibrating sagged cable system, in the case of an inclined cable forming hybrid modes, both the symmetric and asymmetric modes are equally important.

It has been shown that the three components of cable motion are coupled in nonlinear terms but that the out-of-plane and in-plane motions become decoupled [21] in the linear theory, by assuming infinitesimally small amplitudes. This was done by formulating a flexible and extensible cable as a continuum and deriving the nonlinear governing equations by representing them in terms of the dynamic displacement. Therefore, by using the linear theory of cable dynamics, the in-plane motion of the cable is totally independent of the out-of-plane motion.

Two conclusions were drawn by Yamaguchi [21]. The first conclusion is the existence of modal cross-overs for in-plane dynamic motion of horizontal cables. The other is, for very large sag, some natural frequencies for in-plane motion converge to the natural frequencies of a vertically suspended cable supported at only one end.

Obtaining an analytical solution to this nonlinear equation is very difficult. For this reason, a linearized eigenvalue model can be derived by discretizing the cable system into a finite degree-of-freedom system using the finite element method. By solving this eigenvalue problem, the natural frequencies and mode shapes of the inclined deep cable can be numerically obtained.

Forghani-Arani [4] and Oh [9] developed a computational approach to the solution of the natural vibration of sagged cables hanging under the influence of their own weight using linear finite element analysis. This approach was used in an attempt to solve cable problems not covered by the linear theory, including deep sagged cables (sag-to-span ratio larger than 1/8) and included cable. This was completed once the program had been validated by comparison with results of the linear theory for shallow cables.

Continued work is required on the study of sagged cables using finite element analysis. To complement this work, and to verify the finite element method as a method of analysis, experiments are required to verify these results. These experiments would include the study of frequencies and mode shapes in the region of the modal/frequency cross-over for flat sagged cables and in the region of the avoidance points for the inclined sagged cables. The experiments could also be expanded to investigate the limiting case of sagged cable dynamics, the deep sagged cable, where the natural frequency approaches the natural frequency of a vertically suspended cable supported at only one end.

2.3 EXPERIMENTAL STUDIES

Experimental analysis plays a fundamental role in the verification of theoretical models used in the prediction of nonlinear dynamic behaviour (relaxing the assumption of small displacements) of structural elements. Also, experimental analysis can be used in the detection of new phenomena associated with the actual nonlinearities existing in a given system.

Rega et al [10] worked with an experimental model of elastic cable made by a nylon wire carrying eight equally spaced concentrated masses and hanging at supports at the same level which are given either in-phase or out-of-phase vertical harmonic motions. Different ranges in excitation frequencies were used in the analysis. A 0.45 mm diameter nylon wire and 8 equidistant steel masses of 3.45 g were used. The values of the cable span and sag were assumed equal to 58.60 and 2.96 cm respectively. The elasticity of this model is supplied by the wire, and the mass per unit length of the equivalent continuous system is given by the distributed concentrated masses.

The dynamic behaviour of a discrete cable model is expected to be close to that of a sagged cable/mass suspend system, which can exhibit further peculiar dynamic phenomena depending on the distribution of masses and the modes considered. In the case of a symmetric array of equal masses and considering only lower modes, the main difference with respect to the

continuous cable system consists in a shift of the crossover points towards lower λ^2 values, while the sequence of the first natural frequencies and mode shapes remains practically unchanged.

The natural frequencies of the cable/mass suspended system were also obtained using a finite element code. The frequencies were calculated by performing an eigenvalue analysis on a nonlinear static equilibrium configuration of the system. It was reported that some difficulties were encountered in calibrating some parameter values in the numerical model to reliably reproduce the unknown undeflected configuration of the experimental system.

The experimental model was suspended between two points and vertical harmonic motions were applied in-phase and out-of-phase to the system by means of two shakers driven by a function generator.

Rega et al concentrated on investigating the modal amplitudes as a function of the applied frequency. Some of the relevant features are outlined below. It was noted how corresponding in-plane and out-of plane modes could couple with each other in mixed responses as identified as the first (V2H2) and second (V4H4) asymmetric ballooning. Besides those involving mainly asymmetric modes, couplings of the first in-plane and out-of-plane modes in a symmetric ballooning (V1H1), along with others have been observed in various frequency ranges. Also, near perfect crossovers were identified.

Koh and Rong [8] used an experimental verification for validating a numerical model and solution scheme. The experimental study was performed by means of a shake table. Two C-channel columns were used to support the two ends of a single span cable. One of the columns was bolted to the shake table and the other to the strong floor. The cable length between the two supports is 1.2 m; with the at-rest horizontal distance is 1.160 m. Each end of the cable was supported by a threaded rod. The dimensions and properties of the rubber cable used in the experimental study are the same as those used in the numerical study, with a cable length of 1.2 m as stated above, diameter of 25 mm, density of 1430 kg/m³, and modulus of elasticity of 59 MPa. The modulus of elasticity was determined by a material tensile test in a small range of strain.

Limited experimental validation was performed due to the constraints of the limited locations for mounting the support columns. The experimental results of these limited tests were in good agreement with the numerical results. The problem of cable motion due to support excitation was used to illustrate the numerical scheme and was verified experimentally. The goal of this study was to predict approximately the cable profile and the dynamic tensions with a

numerical scheme and validated the accuracy of the numerical results obtained by the scheme by experimental results. The numerical results from this study show that cable velocities do not differ significantly for slightly different sag ratio, but the cable dynamic tensions do. It was discovered in this study that it is important to study the cable tension, in addition to the cable displacement and velocity in selecting an appropriate numerical scheme.

Campbell et al [2] studied a 294 cm uncoated galvanized steel wire rope excited harmonically at the base. They measured the response of the cable both in the plane of the sag and out-of-plane and were compared to simulation results. Also, the mount at the upper support included a hard-rubber damper to study the effect of boundary condition modification. Results for the in-plane and out-of-plane response were presented for a single angle of inclination and cable property.

Berlioz and Lamarque [1] used a cable composed of a steel wire surrounded by copper wire to compare with numerical for the theory presented by Triantafyllou and Grinfogel [18]. The cable length as tested is 1.905 m with a mass per unit length of 0.177 kg/m. The angle of inclination was fixed at 27.5 degrees, with the ability to vary the initial tension applied to the cable, the driven amplitude and frequency. A base excitation was applied to the cable, matching the imposed displacement of the roadway of the bridge deck. Displacements along the vertical and horizontal directions in the middle of the cable were measured using non-contact type sensors.

2.4 SUMMARY OF CABLE DYNAMICS

The experimental results exhibited good agreement between the theory and the experiment, with several nonlinear motions observed and identified. Again, results for the in-plane and out-of-plane response were presented for a single angle of inclination and cable property, under different initial cable tension and base excitation amplitude.

Limited experimental analysis has been done, and when it has, it has been preformed for a specific case, for example angle of inclination and cable properties. To build on this work, experimental analysis will be used to verify FEM analysis and attempt to study the frequency crossover of flat sagged cables and to resolve issues on the avoidance points of inclined sagged cables.

The study of cable vibrations have continued for a number of centuries. This started with the study of the vibrations of a taut string, and has developed to the study of sagged cables.

The work in the area of cable dynamics and vibrations was renewed with the incident of the Tacoma Narrows suspension bridge in 1940. Many numerical, computational and experimental studies have tried to resolve many issues involved in the dynamics of sagged cables. This includes work into the effect of elastic stretching effect and inelastic change in the cable configuration on the cable profile during crossover for flat sagged cables, as reviewed by Irvine [6].

Work has continued into the study of inclined cables. Development of numerical formulation in the area of inclined cable dynamics started with the extension of the theory of flat cable dynamics. Triantafyllou developed a theory and discovered a non-crossing phenomenon, or avoided crossing in the study of the vibrations of inclined cables.

Limited experimental analysis has been performed to verify and complement numerical and computational analysis. What experimental activities have been performed has been limited in its scope. For example, experiments involving a specific angle of inclination and cable properties. To build on this pool of information, experimental analysis will be used to verify finite element analysis and complement work already performed by Oh [9]. In the following chapters, the frequency crossover of flat sagged cables and the avoidance of inclined sagged cables will be investigated.

CHAPTER 3

PHYSICAL MODEL EXPERIMENT

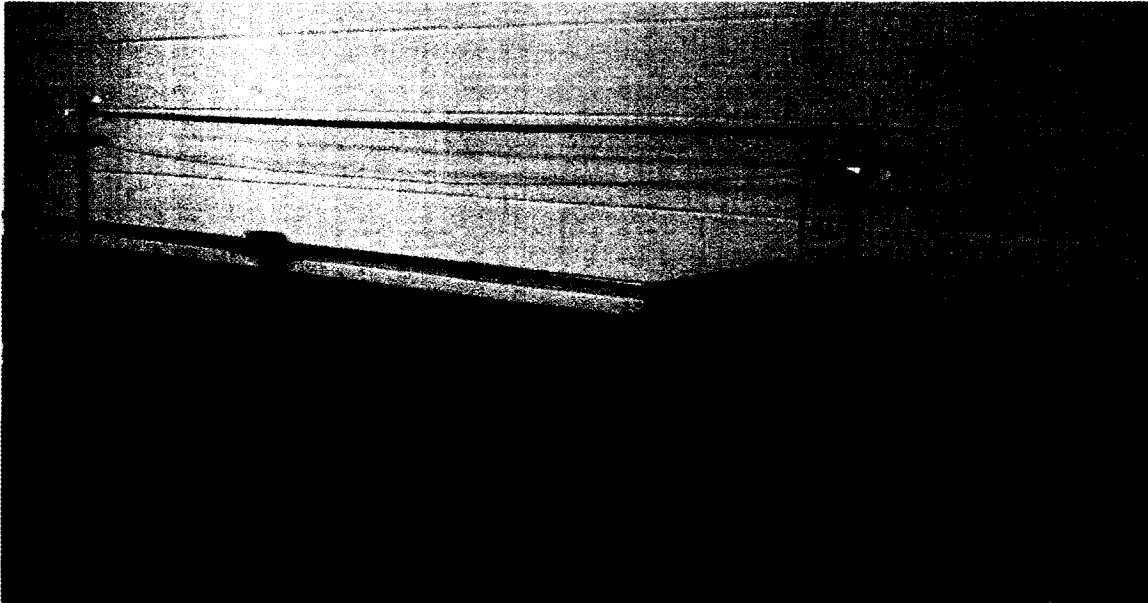
3.1 DESCRIPTION OF EXPERIMENT

An experiment was performed to verify a relationship between physical and geometrical properties of a cable and its natural frequencies. To do this, various types and styles of materials were investigated for their performance as a model cable.

The material explored for use in the experiment included a bungee cord, wires and cables of different sizes. The plastic nature of a bungee cord made it impractical for the wide range of tensions required for the tests. A 1.59 mm diameter galvanized aircraft cable was used in the experiments. The choice of this cable allowed for a wide range of elasto-geometric parameters to be investigated, covering the crossover region of a flat sag cable, and avoidance region of inclined cables to deep sagged cables. The cable has an approximate metallic cross sectional area of 1.187 mm^2 , (detailed calculations are found in Appendix B) consisting of 7×7 construction right regular lay, IWRC (independent wire rope core). The mass per unit length of the cable was calculated to be $9.39 \times 10^{-6} \text{ kg/mm}$. This was calculated by assuming the density of steel to be 7850 kg/m^3 , and multiplied by the metallic cross sectional area. The modulus of elasticity of the cable was assumed to be 93,080 MPa, following the published [20] modulus value by the manufacturer for all cables of similar construction, in this case, 7×7 construction.

The distance between fixed supports was set to be 2.50 m, even when the model was inclined. The size and span of cable was decided in order to obtain a wide range of property parameters, while maintaining a manageable size for setting-up and performing the experiments. The setup was supported using experiment (retort) stands, and span maintained a constant distance using steel rods clamped to the experimental stands, as shown in the pictures below. The test took place in the Environmental and Geotechnical Teaching Laboratory at the University of Ottawa during the fall of 2004 and winter of 2005.

Figure 3.1: Experimental Setup for Flat Sagged Cable



The amount of sag was changed from 10 mm up to the maximum of 2000 mm over the 2.50 m span. The 10 mm sag was the smallest sag possible before the inextensibility of the cable inhibited the vibration of the model; 2000 mm was the maximum sag used for detecting the differences between the natural frequencies was still possible.

Trials were performed on a wide range of angles of support inclination, including 0, 15, 30, 45, and 60 degrees, with 60 degrees being the maximum applicable angle used.

The length of the cable was calculated using the hyperbolic-sine function, given the tension of the cable, which is calculated by measuring the sag [15]:

$$L = \frac{2H}{w} \sinh \frac{wl}{2H} \quad (3.1)$$

where L is the total cable length,

H is the horizontal tension,

w is the weight per unit length,

l is the span length.

Figure 3.2: Experimental Setup for an Inclined Cable, 15 Degrees



The cable was excited by using a function generator connected to an audio speaker, and the frequencies were adjusted until maximum deflection was reached due to resonance. The input frequency value was then recorded. The speaker was connected loosely to the cable using a light rubber band. This allows the cable to vibrate at the excitation frequency and yet practically independent of the excitation force.

3.2 EXPERIMENTAL RESULTS

Below are the results of the physical cable experiments for each of the inclination angles, flat sag of 0 degrees, and the inclination angles of 15, 30, 45, and 60 degrees with respect to the horizontal.

The experimental results with the small sag confirmed the location of the first two cross-over points at approximately $\lambda = 2\pi^2$ and $4\pi^2$ for the flat sagged cable case. For the angles of inclination of 15 and 30 degrees, the magnitude of the avoidance points appear to be near that of the flat sagged cable case, if not slightly higher. It is not until the angle of inclination is increased to 45 degrees that a noticeable change in the avoidance points occurs. At this angle, the avoidance point was observed at λ value of 62.1 and 241.7. These points increase to 136.9 and 531.9, when the angle of inclination increases to 60 degrees. A summary of the cross-over points for flat sagged cables and the avoidance points for inclined cables are presented in the table below.

Table 3.1: Experimental Crossover/Avoidance λ -Values for Various Angles of Inclination

Crossover or Avoidance	Theoretical	Numerical	Crossover	Avoidance				
			0	15	30	45	60	
First	$(2\pi)^2$	39.5	35.1	34.4	42.7	62.1	136.9	
Second	$(4\pi)^2$	157.9	156.2	149.1	175.1	241.7	531.9	

The results of the experiments are plotted in comparison to the dimensionless frequencies as defined by the following relationship:

$$\Omega = \omega l \sqrt{\frac{w}{Hg}} \quad (3.2)$$

where ω is the circular frequency,

l is the span length,

w is the weight per unit length,

H is the horizontal tension,

g is the gravitational acceleration;

against the elasto-geometric parameter λ^2 :

$$\lambda^2 = \frac{EA}{H} \left(\frac{wl}{H} \right)^2 \quad (3.3)$$

where E is the modulus of elasticity,

A is the cross-sectional area.

Figure 3.3: Results of Experimental Modelling, Flat Sag

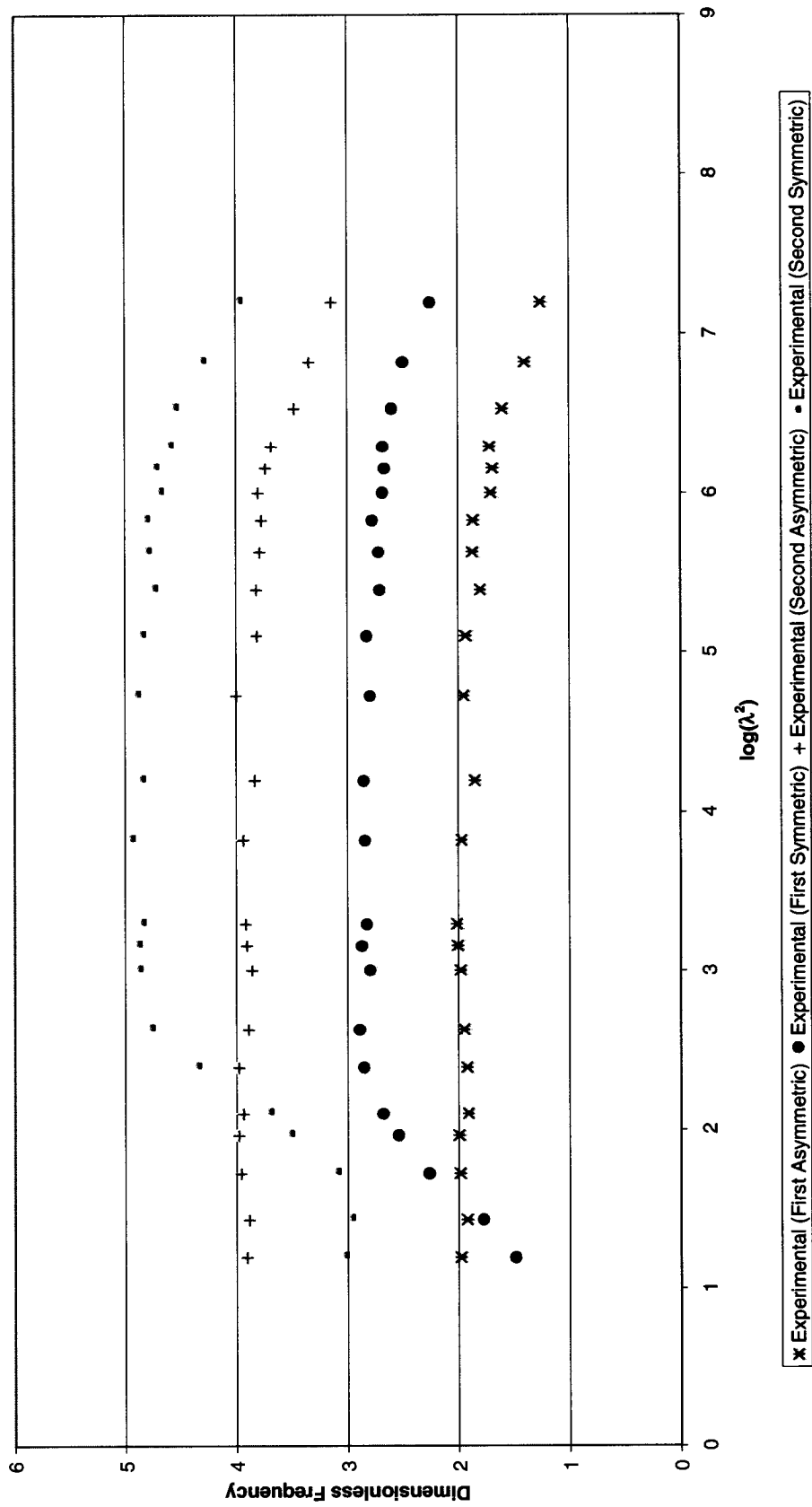


Figure 3.4: Results of Experimental Modelling: 15 Degrees

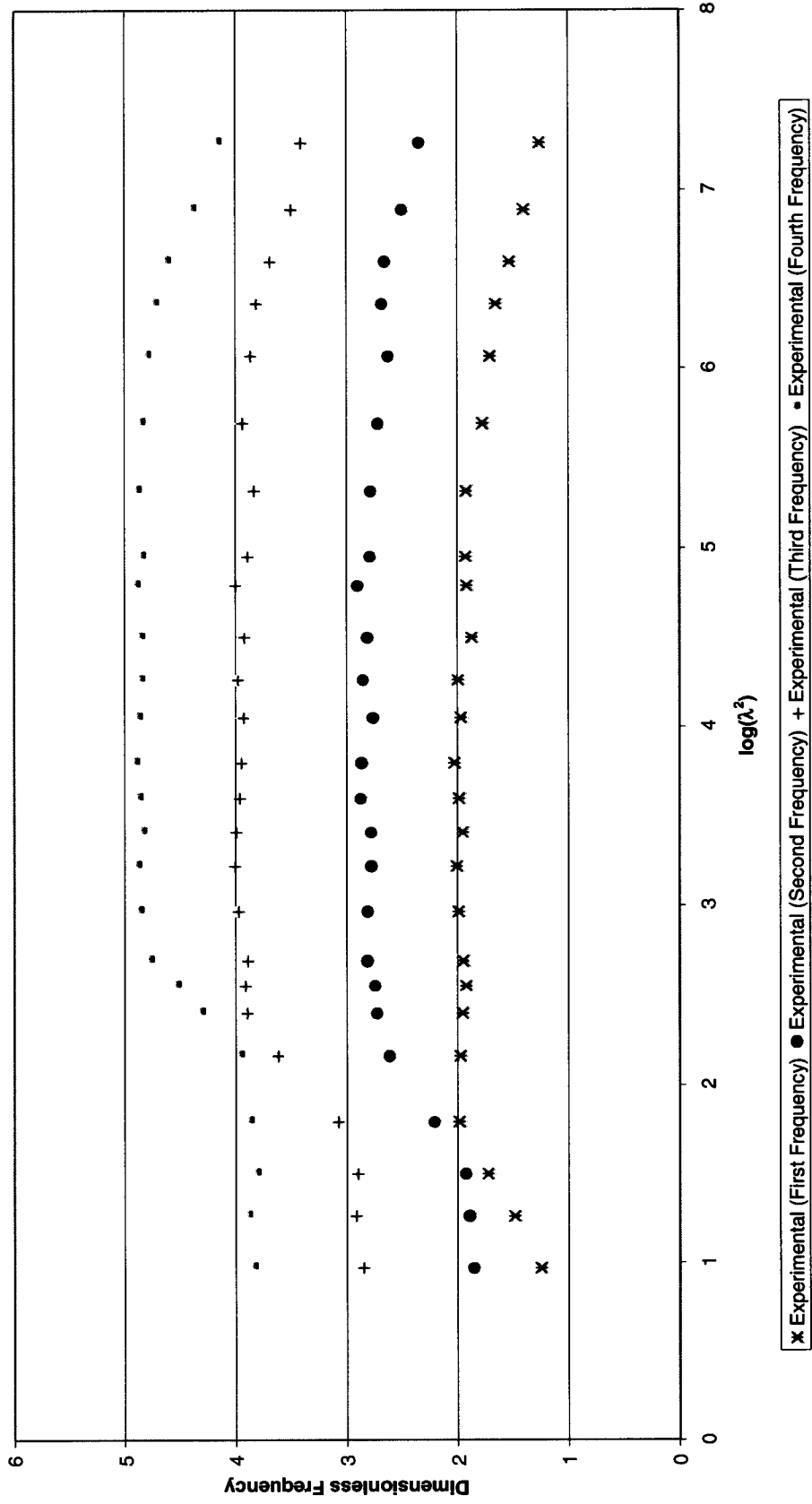


Figure 3.5: Results of Experimental Modelling: 30 Degrees

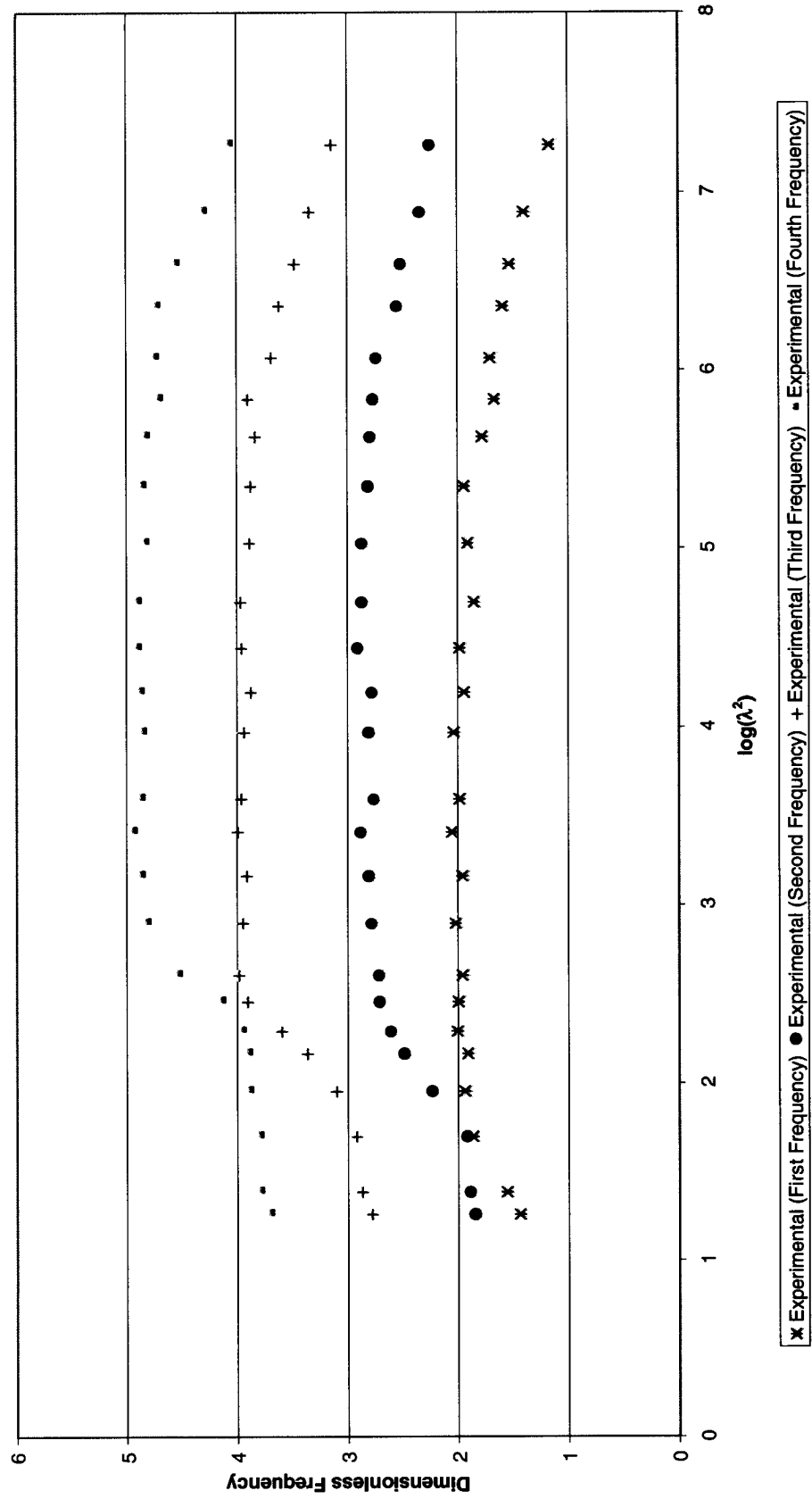


Figure 3.6: Results of Experimental Modelling: 45 Degrees

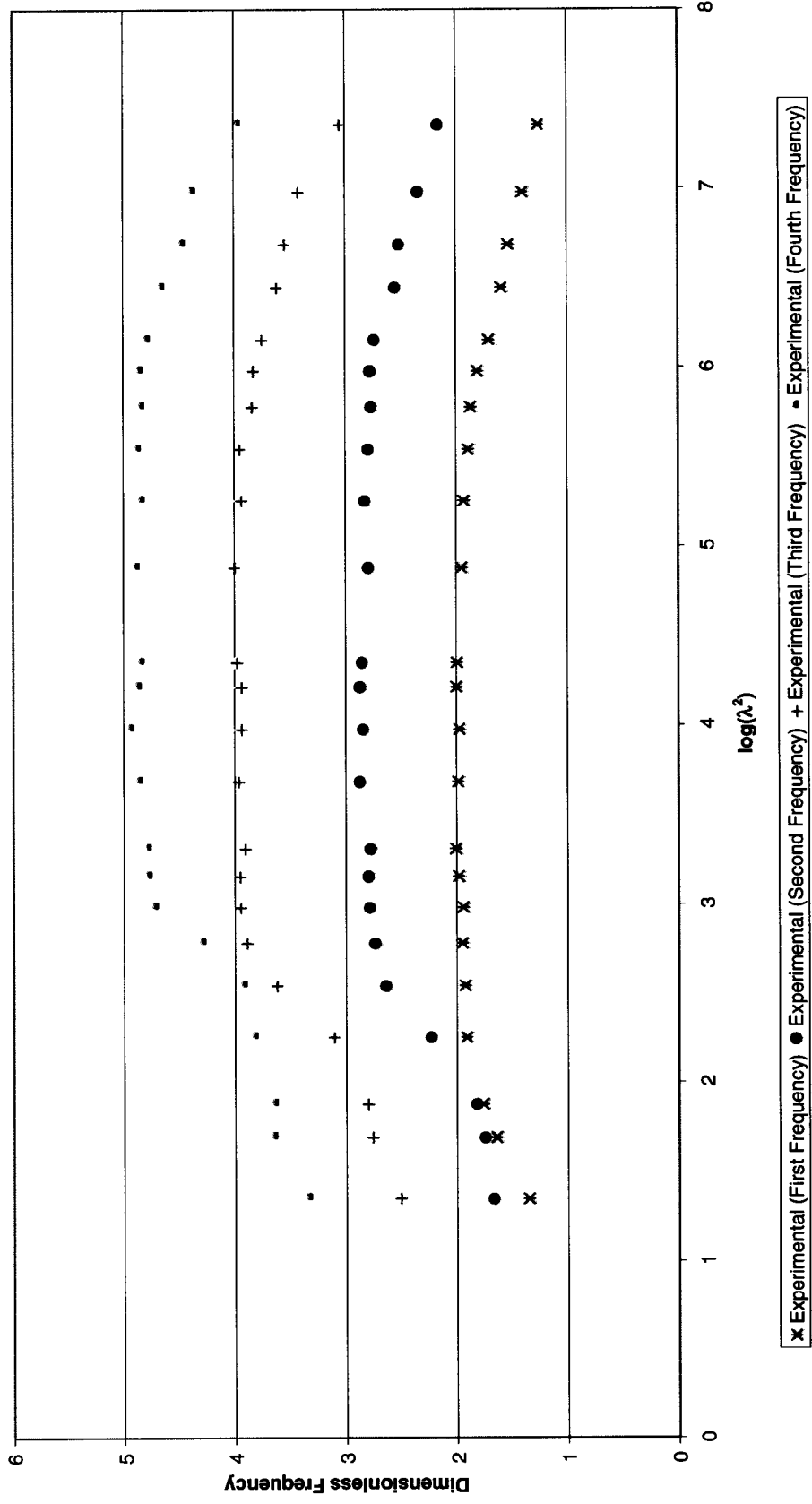
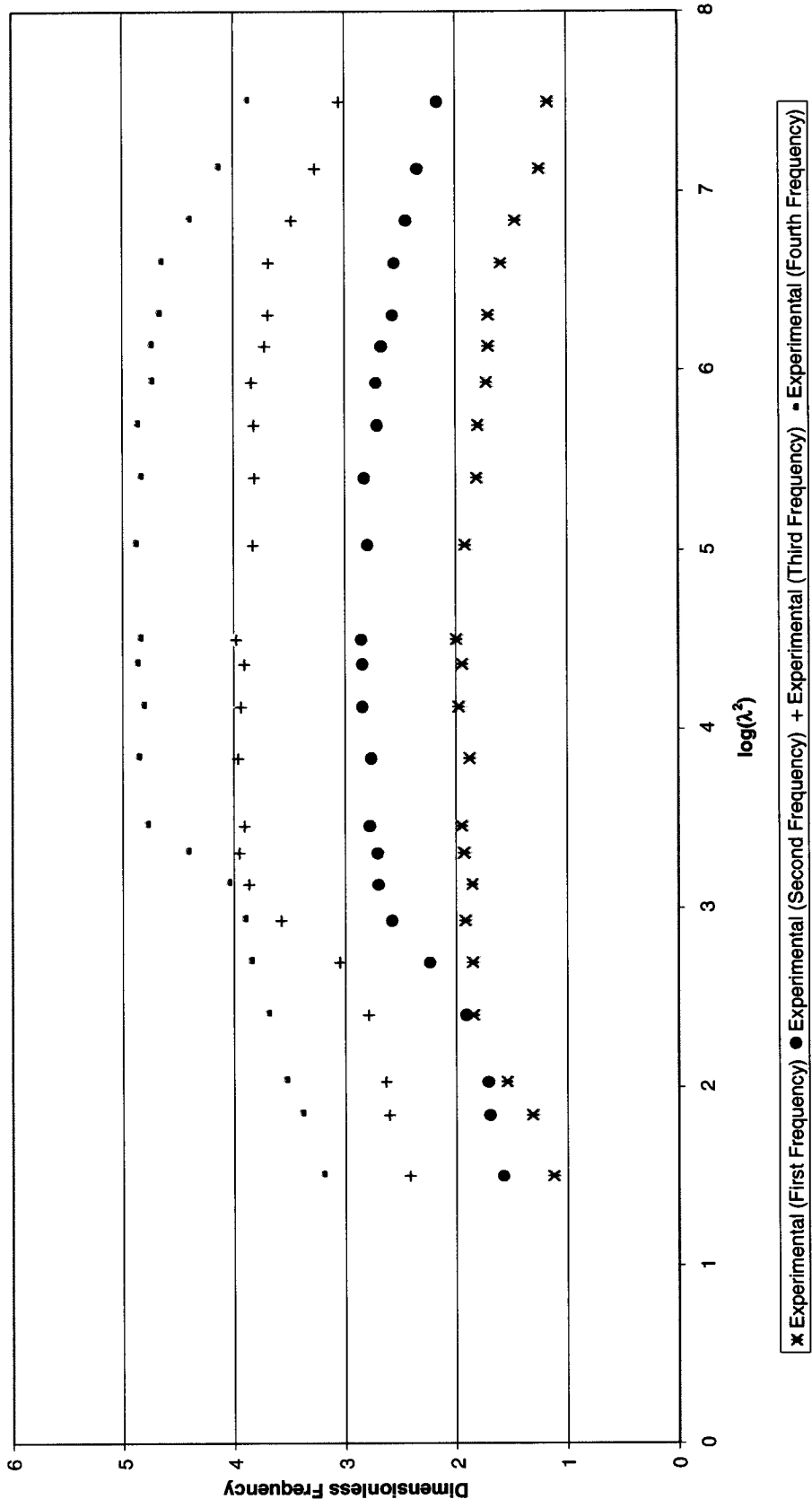


Figure 3.7: Results of Experimental Modelling: 60 Degrees



CHAPTER 4

COMPUTATIONAL SIMULATIONS

4.1 COMPUTER SIMULATIONS

In this chapter, the finite element program was used to calculate the in-plane natural frequencies and vibration mode shapes of the experimental cable with different geometric properties, e.g. inclination and sag-to-span ratio. This will allow us to investigate the differences in the geometric properties outside the experimental range described in the previous chapter. We will also be able to investigate the phenomenon of cable frequency and modal avoidance for the case of inclined cables. The results of the frequency calculations are given below.

A string of 100 link elements was setup in the ADINA 7.5.2 commercial software package. The stressed cable was analysed for natural frequencies and their corresponding mode shapes. A convergence analysis was performed to find the appropriate number of elements to be used to obtain a converged solution. This was done using the well known case of the vibration of a taut wire, and 100 elements for the length of the cable provided adequate results (see appendix A). For comparison purposes of the numerical results from this study with similar studies [4][9], all numerical simulations were performed with 100 elements per length of cable. This comparison will be done later in this report.

The parameters for the computer analysis were the same as the measured properties of the physical model. These parameters are summarized below:

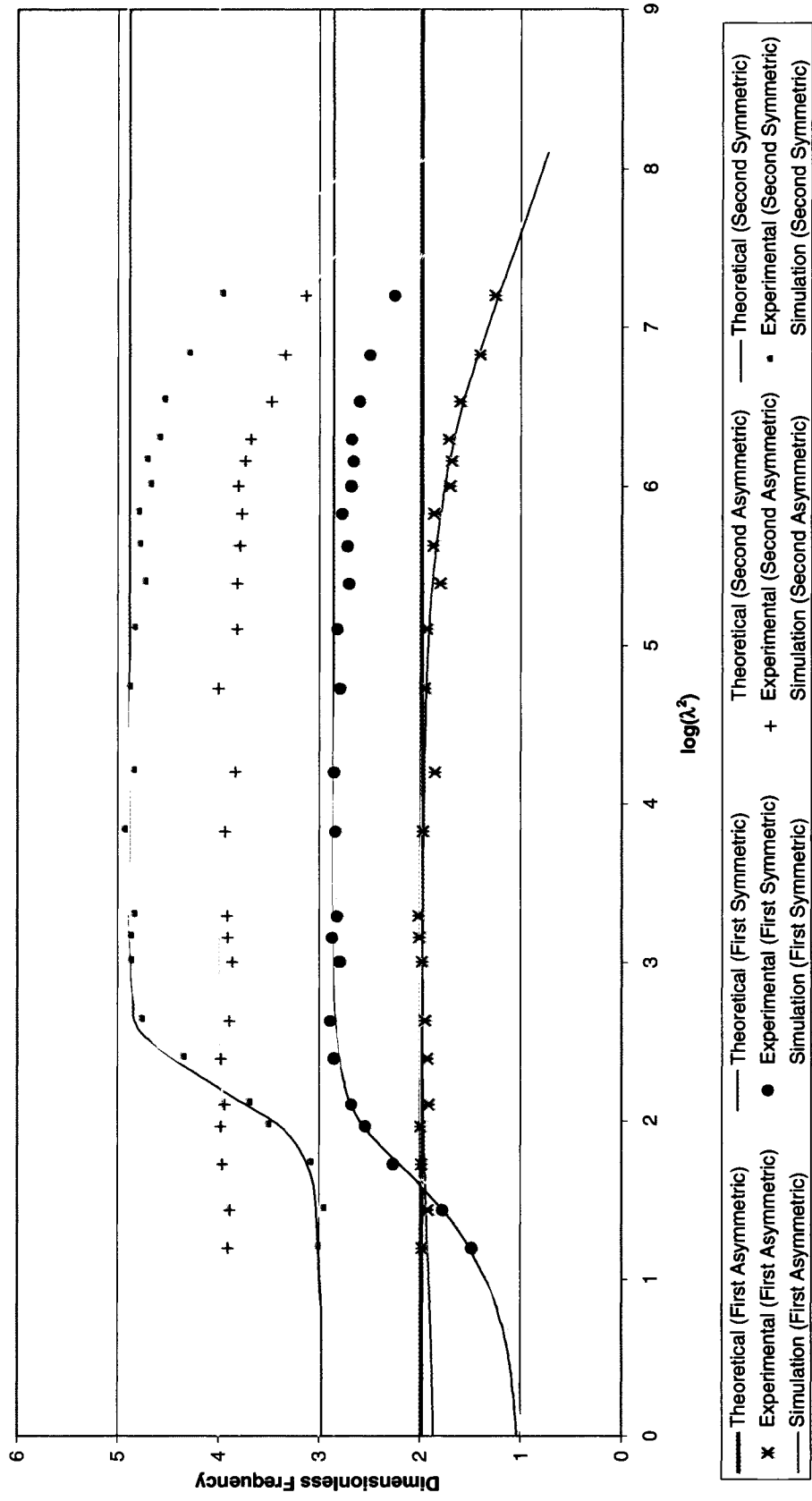
- metallic cross-sectional area: 1.187 mm^2
- modulus of elasticity: $93\ 080 \text{ MPa}$
- density: 7850 kg/m^3
- span: 2.5 m between supports, even for inclined cases.

The sag for the models was varied from 5 mm up to the maximum of 4400 mm. The length of the cable was modified to maintain a span length of 2500 mm with the changes in sag. Angles of inclination of 0, 15, 30, 45, and 60 were analysed.

Due to the accuracy of the experimental analysis for this study, the avoidance points for inclined cables were unable to be developed. To assist in this, computational analysis was used and concentrated in the area in and around the avoidance points. It is noted that as the angle of inclination increased, the width of the region around the avoidance point also increases, known as the transition region. The phenomenon is in agreement with numerical results by Triantafyllou and Grinfogel [20]. The results from this study indicate that the location of the avoidance points increase in value along the λ^2 axis with the increase in angle of inclination.

The difference of the modal crossover from flat sagged cables to inclined cables could be caused by non-symmetric dynamic effects of the cable mass. In an attempt to deal with inclination of the cables, Irvine had rotated the coordinates of the horizontal cables but was unable to recognize the crossover phenomenon.

Figure 4.1: Results of Computer Simulations, Flag Sag



- theoretical values in the above graph are results obtained by Irvine and are included in this graph for comparison with results from this study
- simulations are the numerical (finite element analysis) results from this study

Figure 4.2: First Symmetric and Asymmetric Frequency Crossover, Flat Sag

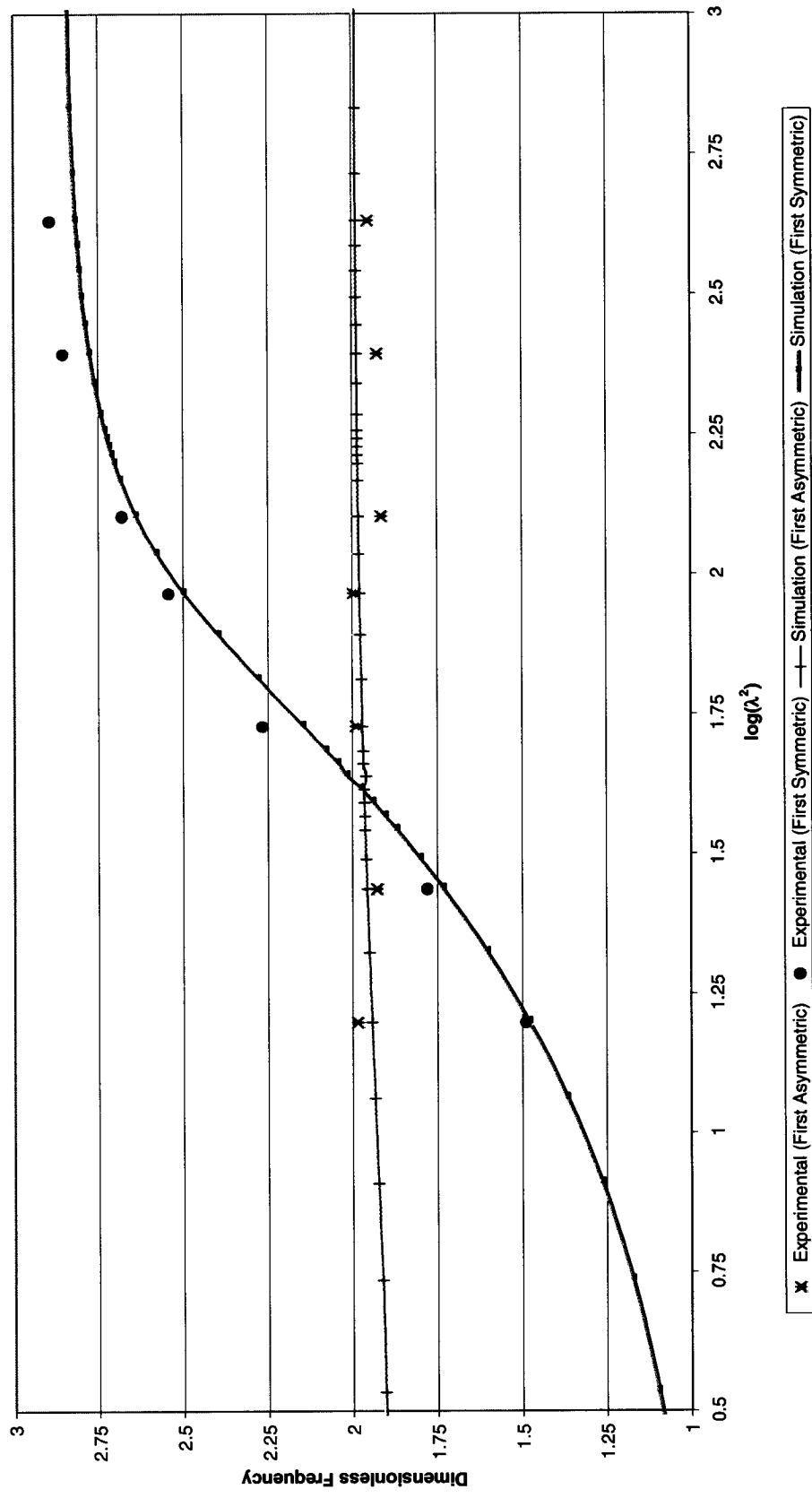


Figure 4.3: Second Symmetric and Asymmetric Frequency Crossover, Flat Sag

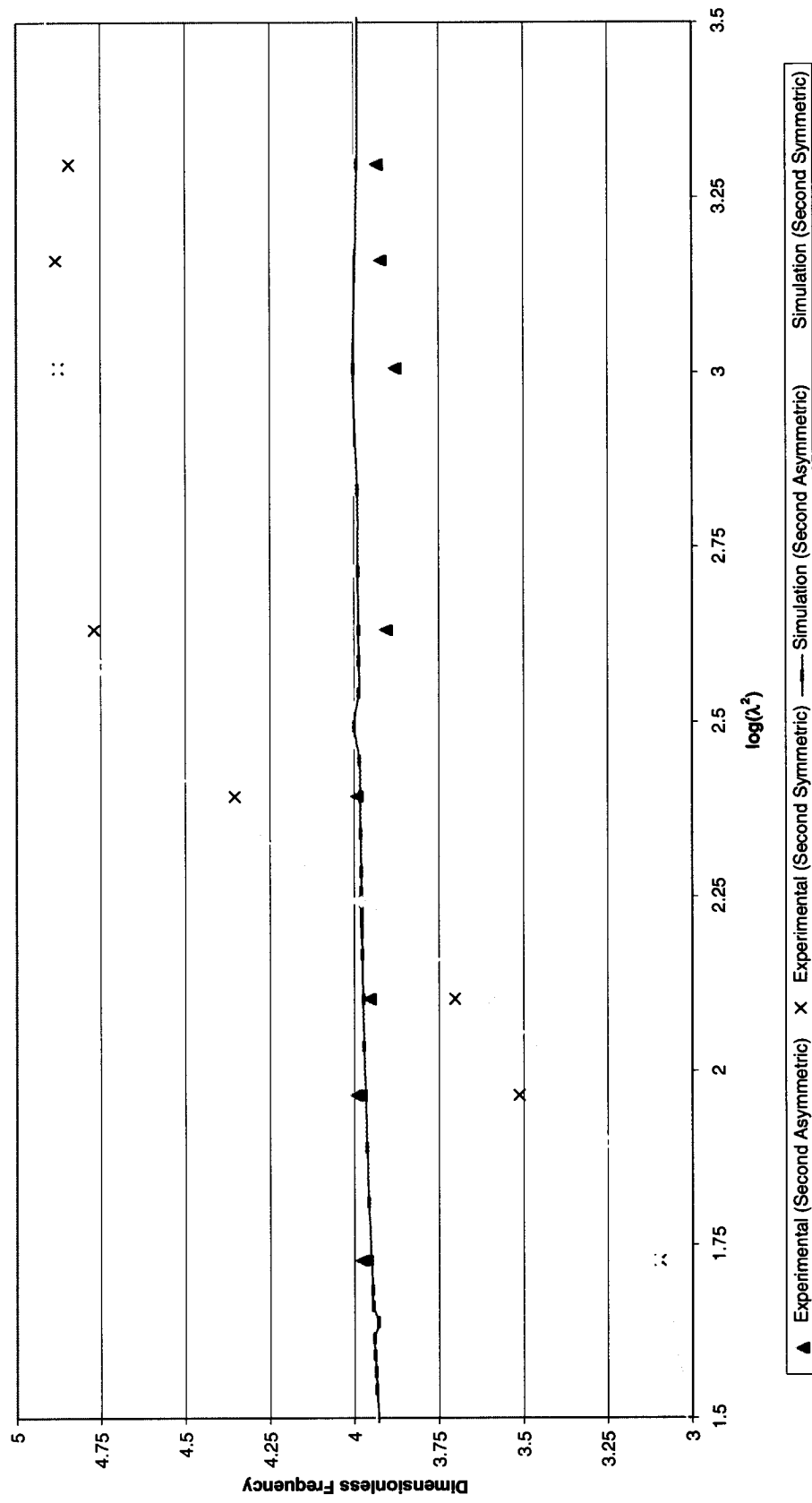


Figure 4.4: Results of Computer Simulations, 15 Degrees

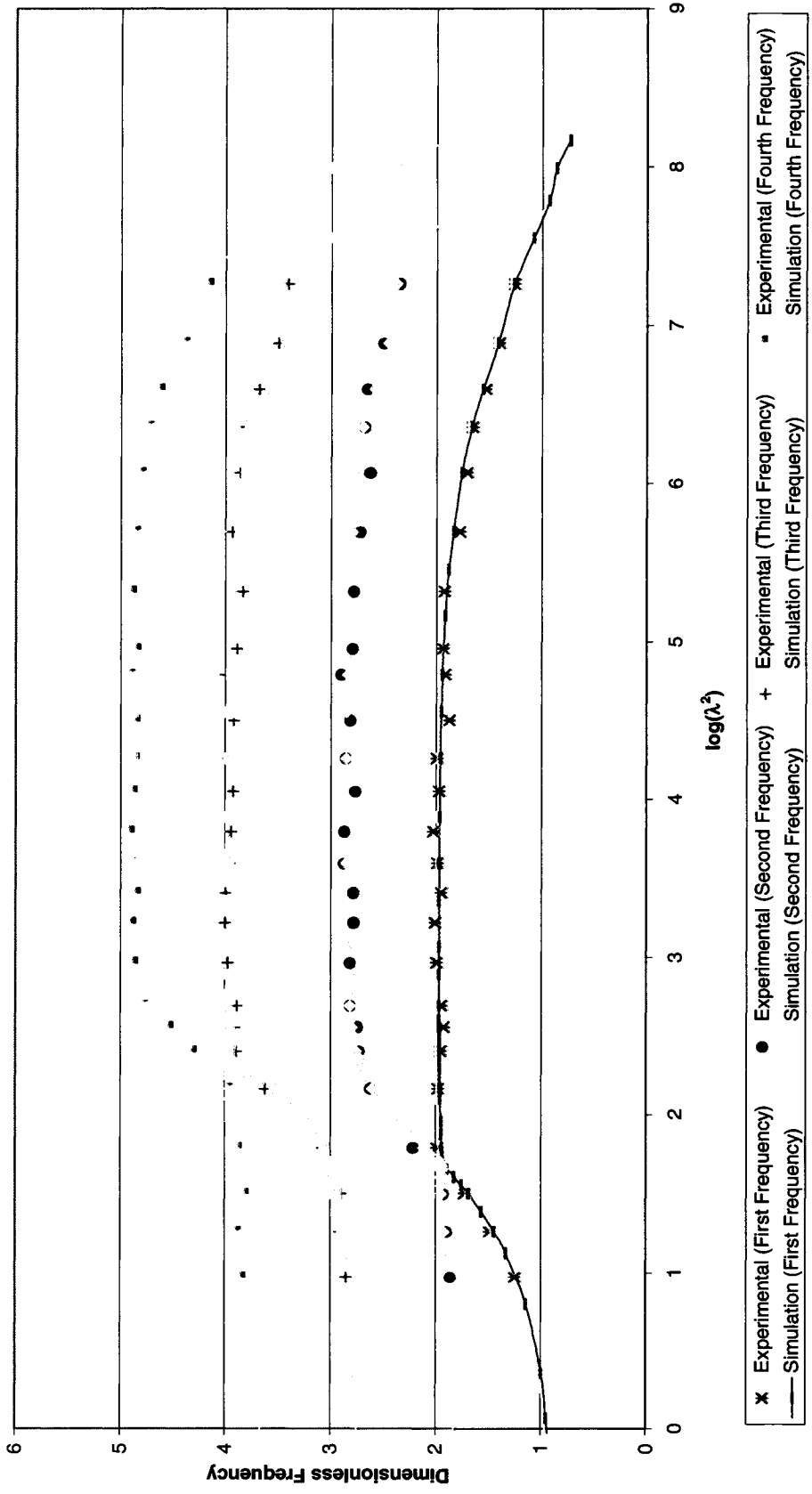


Figure 4.5: First Symmetric and Asymmetric Frequency Avoidance, 15 Degree Inclination

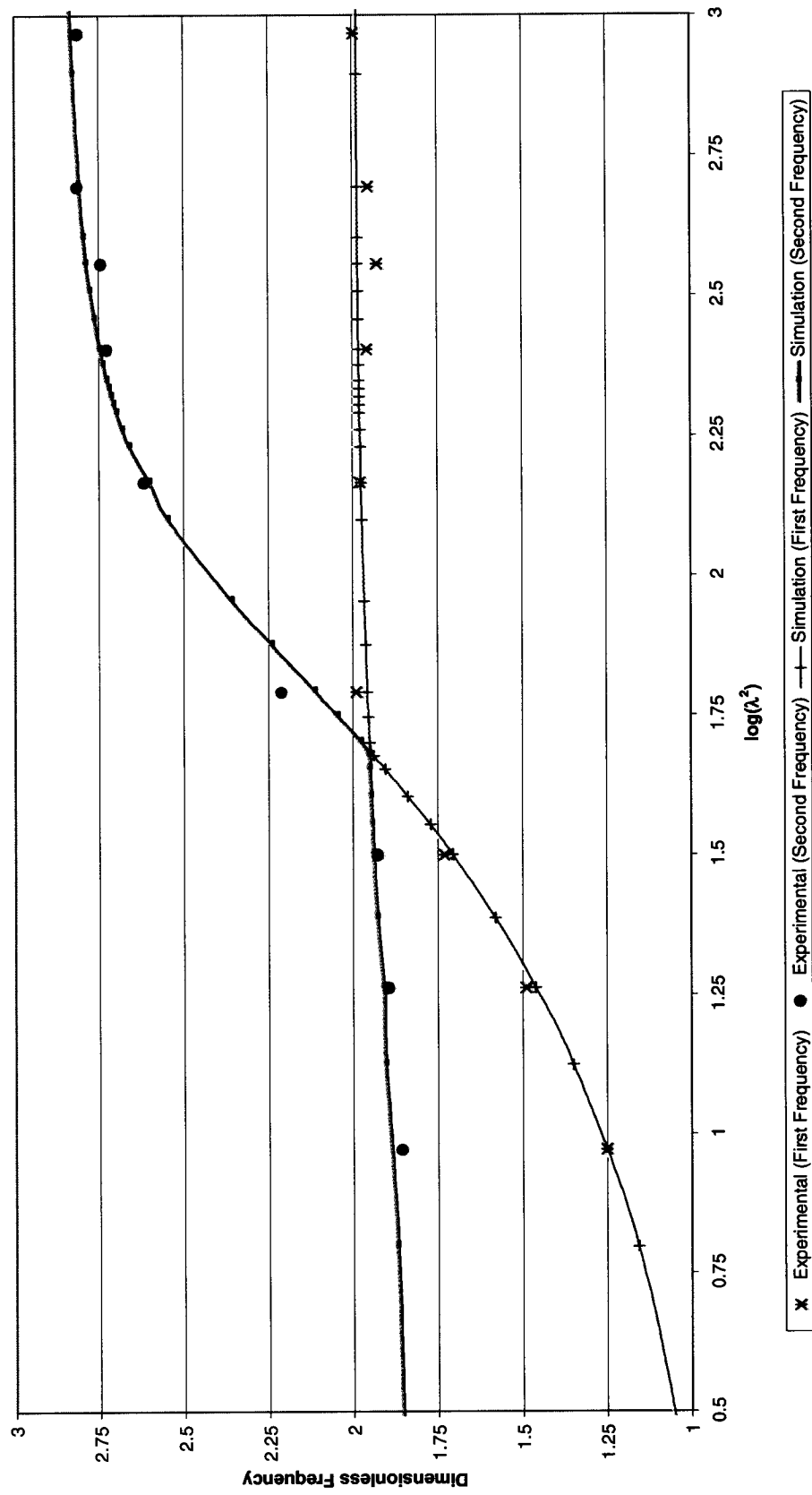


Figure 4.6: Second Symmetric and Asymmetric Frequency Avoidance, 15 Degree Inclination

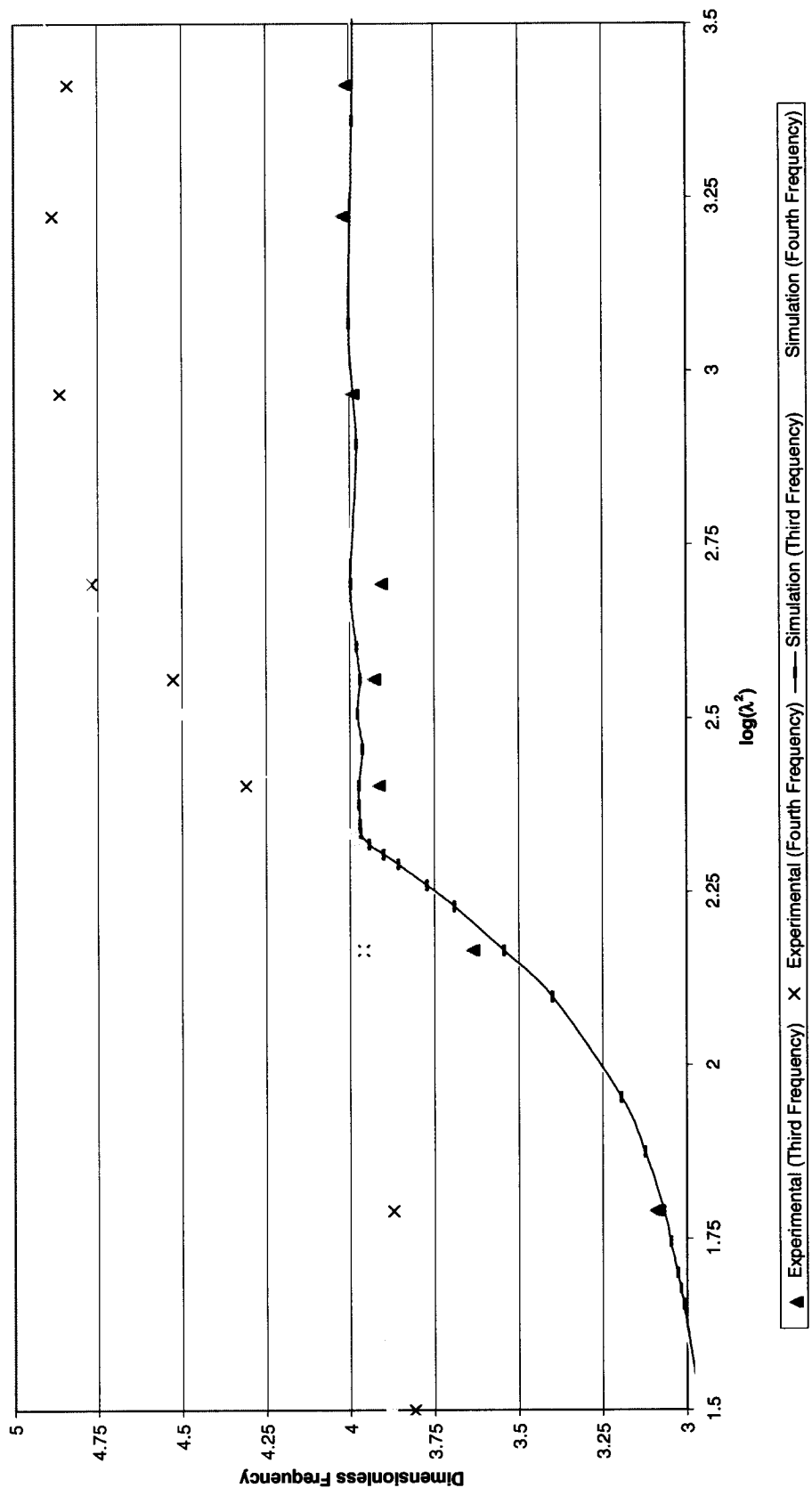


Figure 4.7: Results of Computer Simulations, 30 Degrees

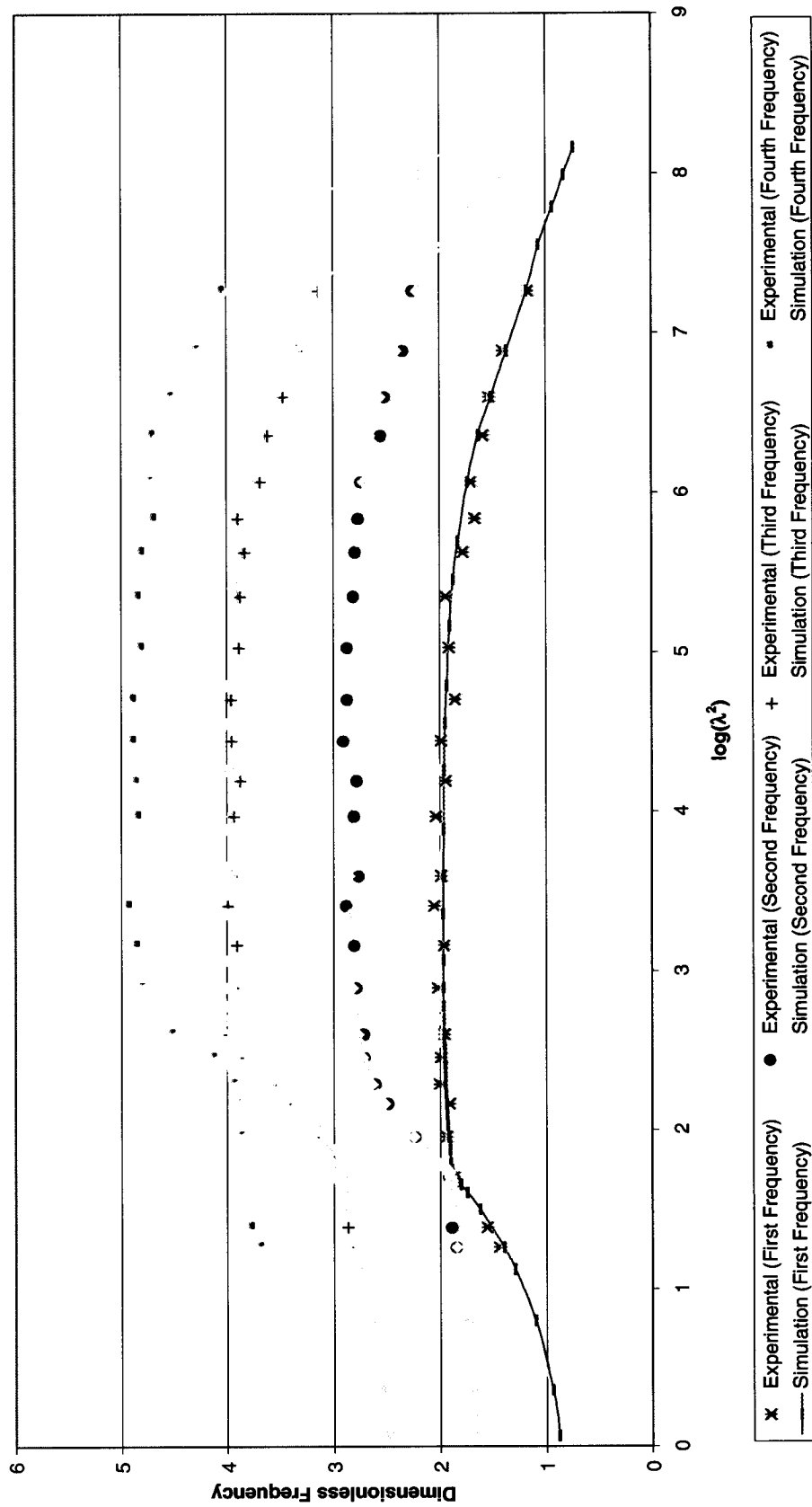


Figure 4.8: First Symmetric and Asymmetric Frequency Avoidance, 30 Degree Inclination

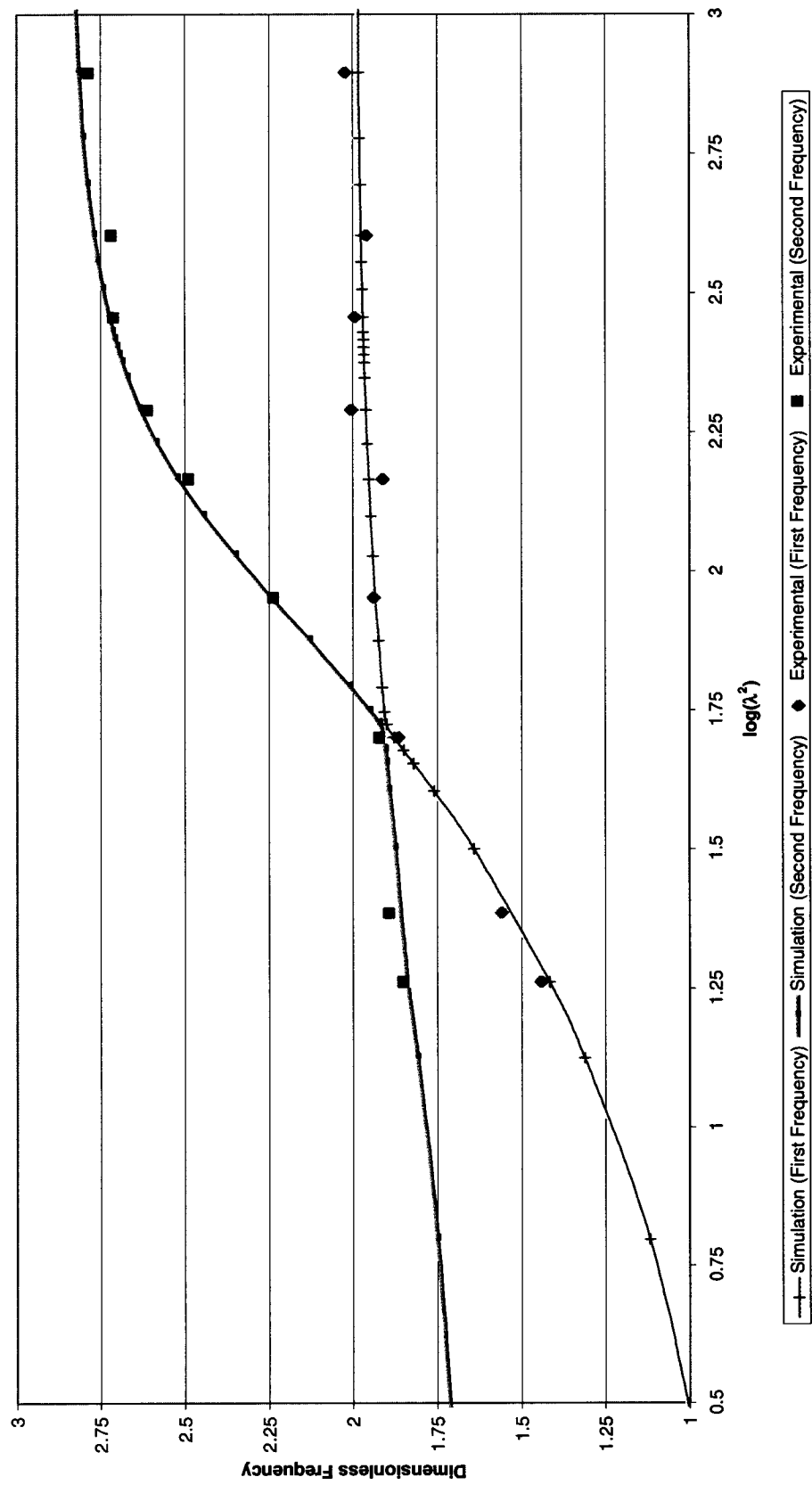


Figure 4.9: Second Symmetric and Asymmetric Frequency Avoidance, 30 Degree Inclination

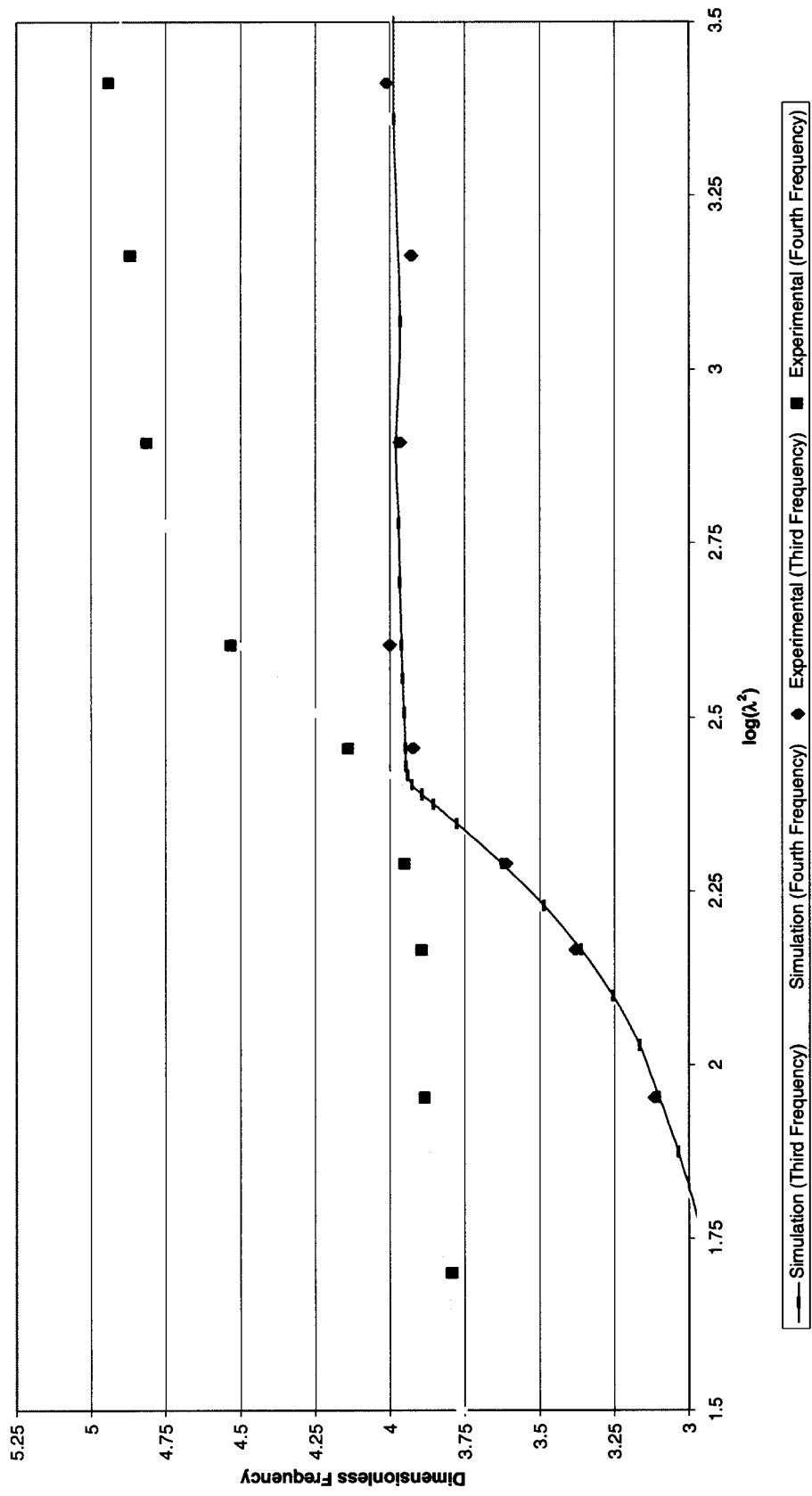


Figure 4.10: Results of Computer Simulations, 45 Degrees

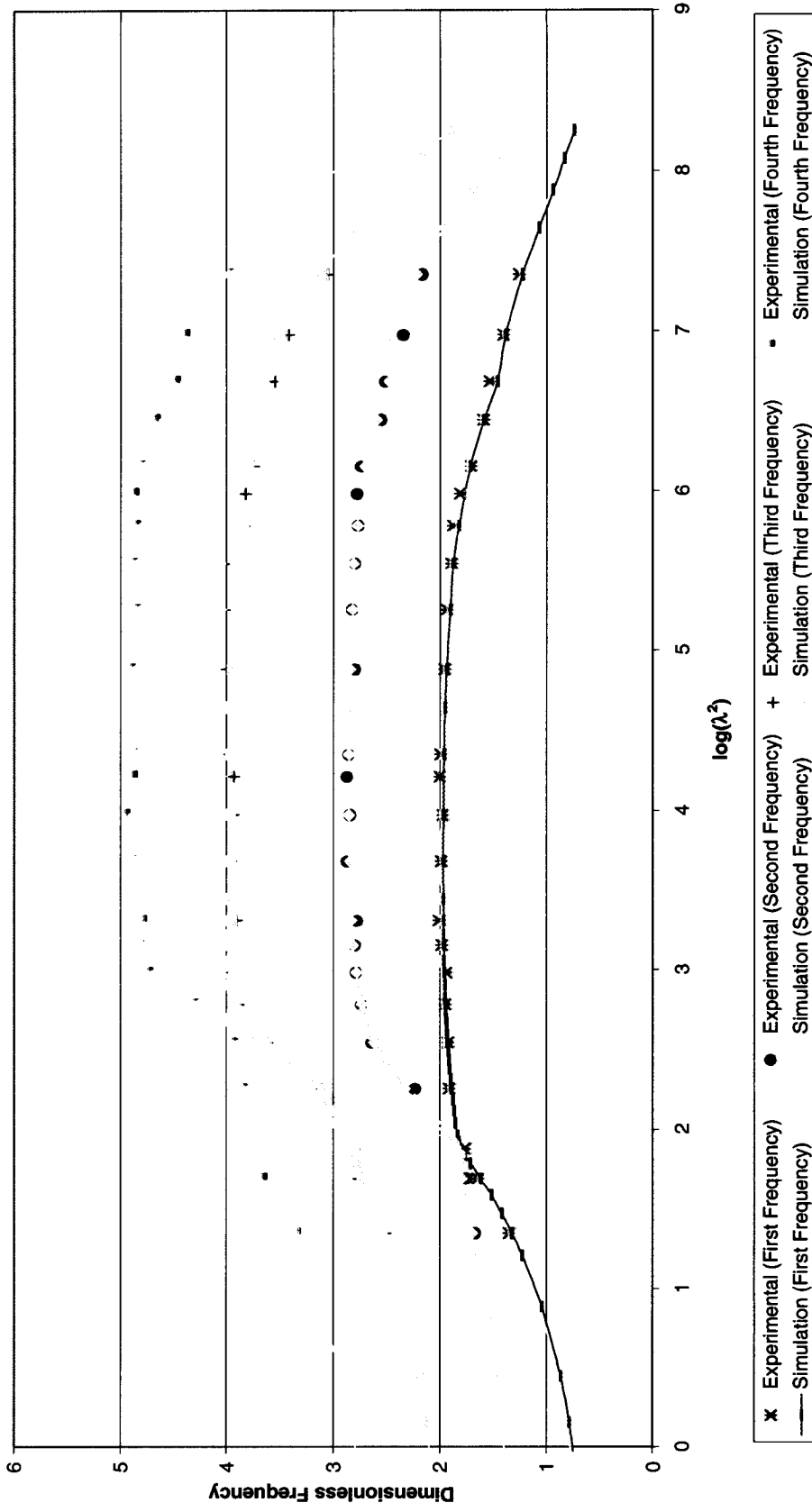


Figure 4.11: First Symmetric and Asymmetric Frequency Avoidance, 45 Degree Inclination

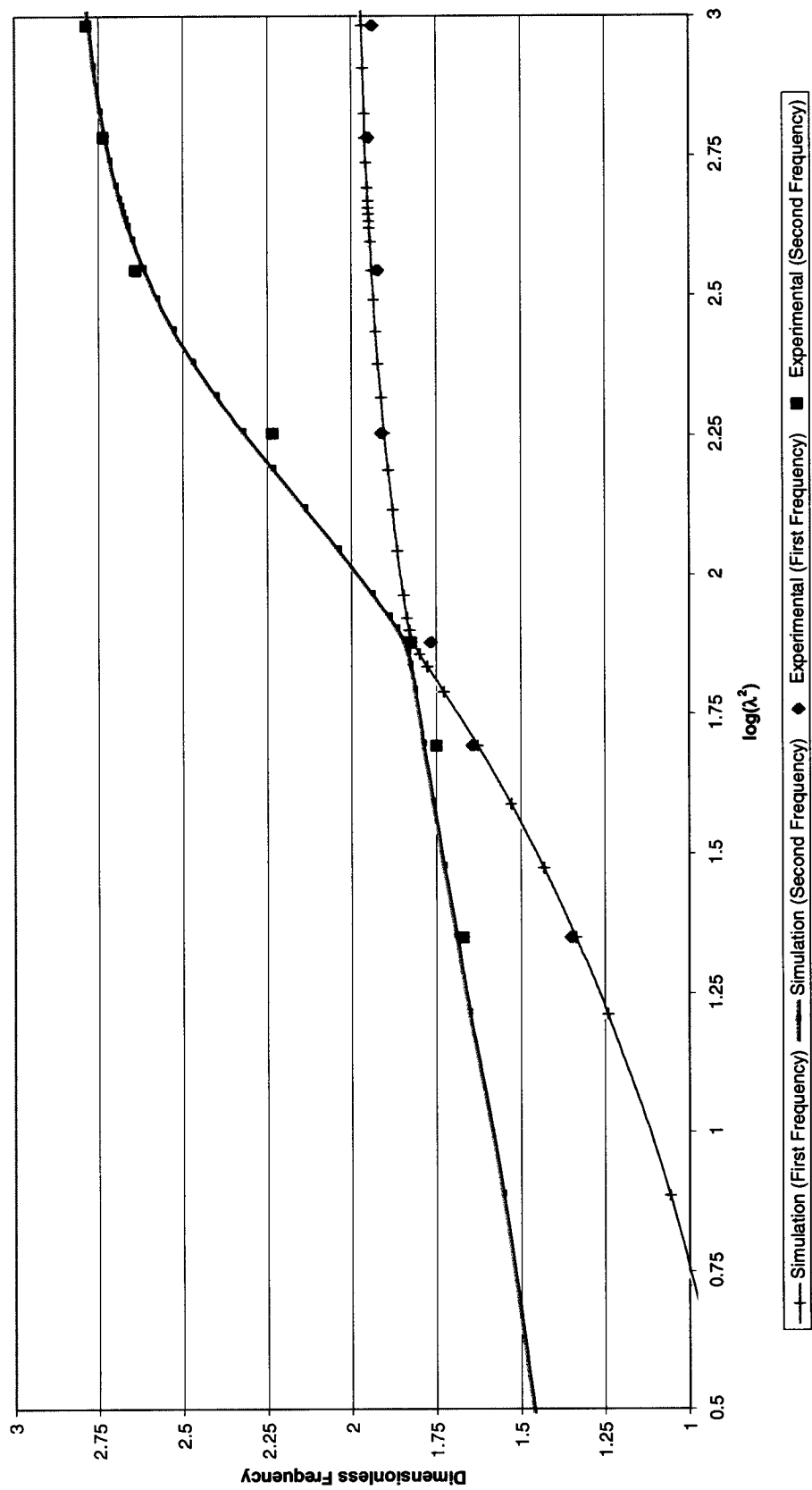


Figure 4.12: Second Symmetric and Asymmetric Frequency Avoidance, 45 Degree Inclination

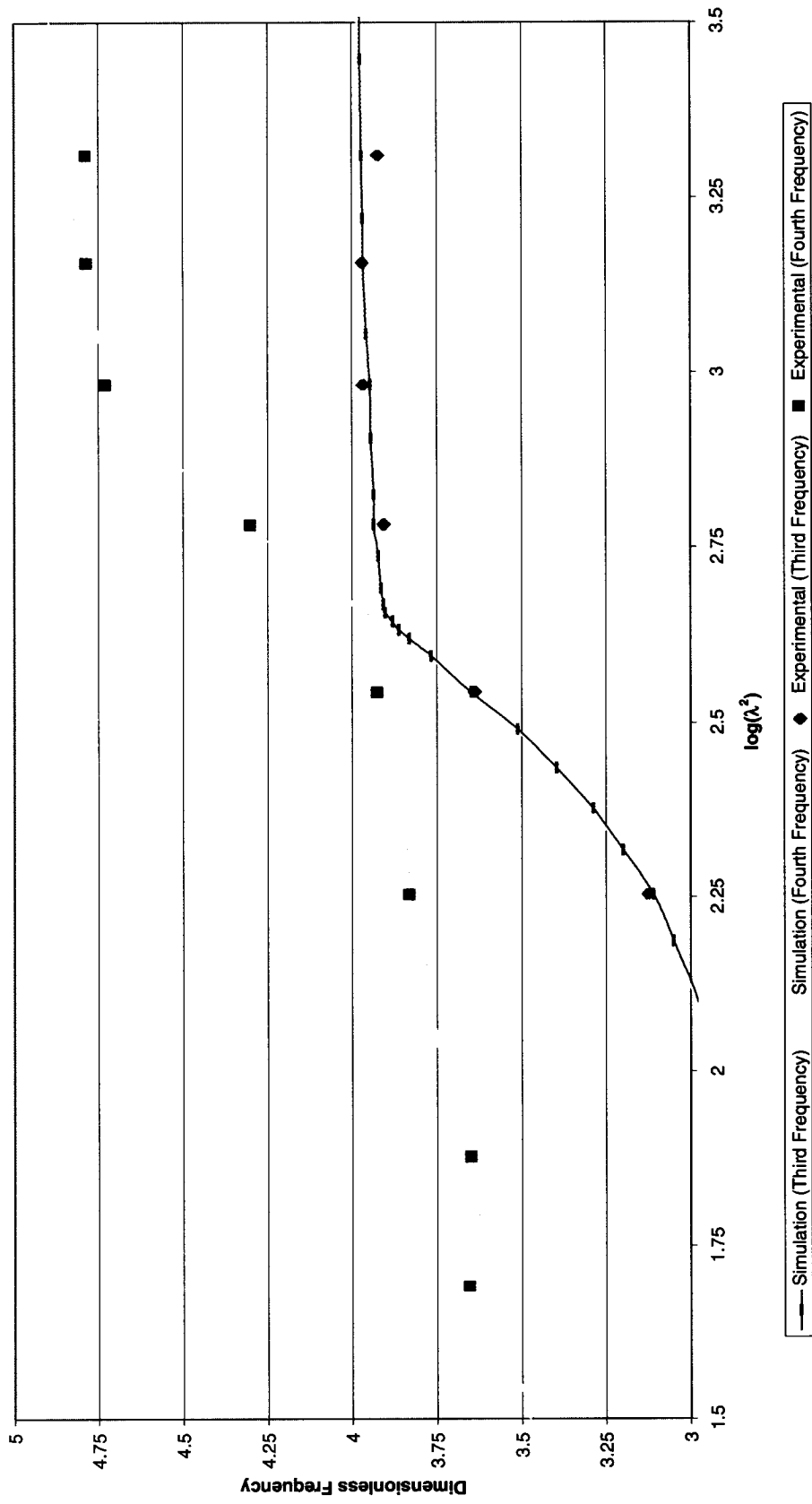


Figure 4.13: Results of Computer Simulations, 60 Degrees

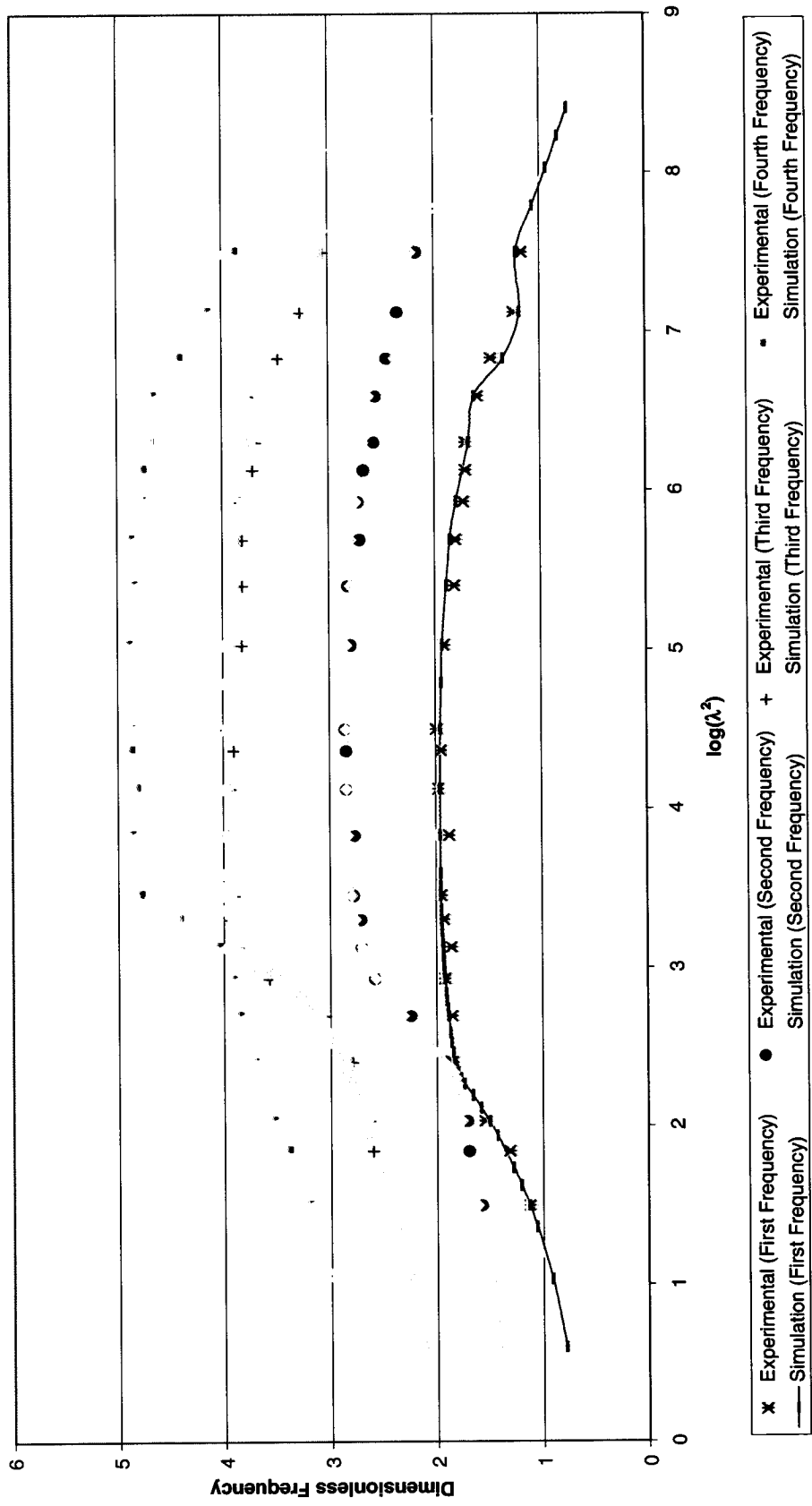


Figure 4.14: First Symmetric and Asymmetric Frequency Avoidance, 60 Degree Inclination

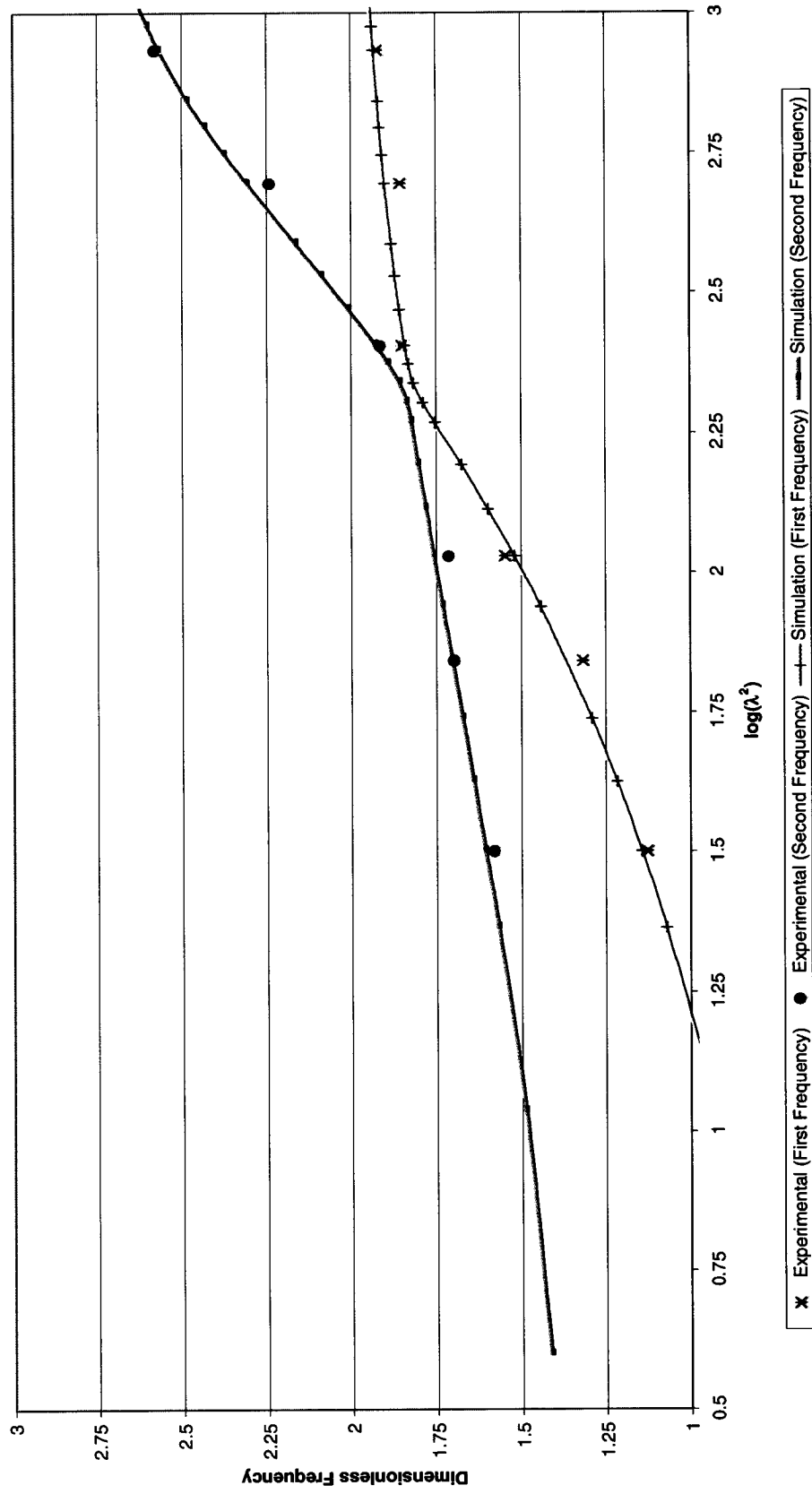
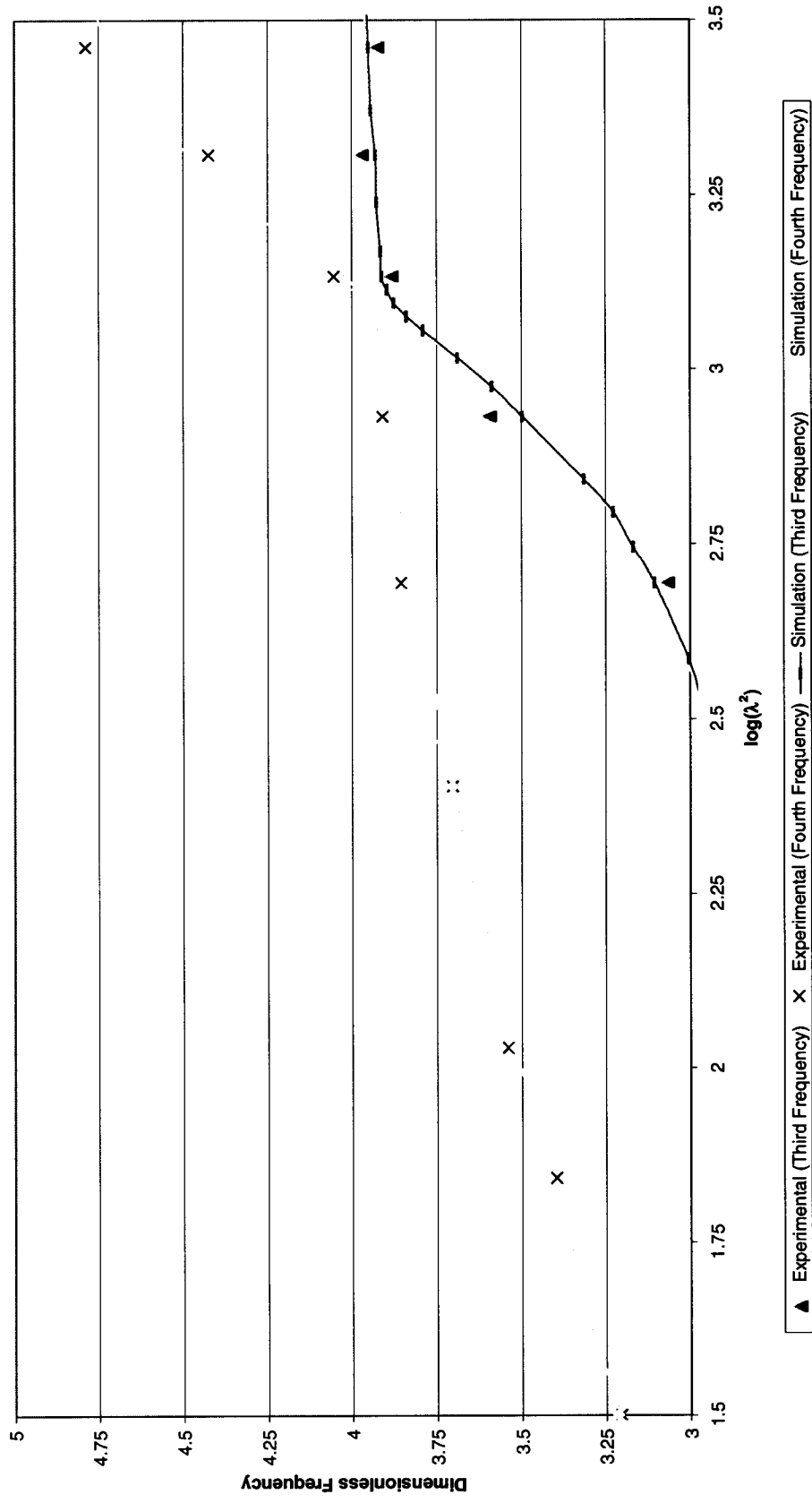


Figure 4.15: Second Symmetric and Asymmetric Frequency Avoidance, 60 Degree Inclination



4.2 RESULTS OF COMPUTATIONAL MODELLING

The results of the computer simulations of the cable model are presented above, one for each of the 5 angles of inclination: 0, 15, 30, 45, and 60 degrees.

The theoretical location of the first two crossover points for a flat sag cable are approximately $\lambda = 2\pi^2$ and $4\pi^2$. The λ value of the avoidance points increases minimally as the angle of inclination is increased to 15 and 30 degrees. It is not until the angle of inclination is increased to 45 degrees that that a noticeable change in the cross-over point occurs. At this angle, the cross over point occurs at $\lambda = 59.1$ and 325.9. These points increase to 105.9 and 616.4 when the angle of inclination increases to 60 degrees. A summary of the cross over points are given in the table below.

Table 4.1: Computed λ -Values for Various Angles of Inclination

Crossover or Avoidance	Theoretical	Numerical	Crossover	Avoidance				
			0	15	30	45	60	
First	$(2\pi)^2$	39.5	40.1	41.0	44.3	59.1	105.9	
Second	$(4\pi)^2$	157.9	175.8	183.1	218.6	325.9	616.4	

For a flat sagged cable, the natural frequencies of the experimental and computer simulations begin to deviate when they are compared to those predicted by the theoretical linear theory for flat sagged cable of the study at approximately a sag-to-span ratio (δ/l) exceeds 1/8. The linear theory for horizontal shallow cable is in good agreement with the numerical and experimental results of this study when the sag to span ratio is small, as defined by the ratio of ($\delta/l < 1/8$). This confirms the conclusion drawn by Irvine [6] when reviewing the deep cable profile by Saxon and Cahn [12]. As the sag-to-span ratio increases, the difference between the frequencies obtained by the linear theory and the frequencies obtained from the experiment and computation analysis also increases. This shows a similar trend to the results presented by Saxon and Cahn [12] for deep sagged cables, via a numerical formulation and confirmed experimentally.

For the case of inclined cables, similar conclusions can be drawn. As the λ^2 value increases past the avoidance points and after plateauing at the crossover dimensionless

frequency, these frequencies begin to decrease in a similar pattern as for the case of a flat sagged cable. Through inspection of the results, this occurs at approximately the sag-to-span ratio (δ/l) of 1/8. The difference in the dimensionless frequencies continues to increase for the maximum value reached following the avoidance point as the λ^2 value increases, in a similar pattern to the flat sagged cable.

Also, as the angle of inclination increases, the width of the region around the avoidance point increases, known as the transition region. The phenomenon is in agreement with numerical results by Triantafyllou and Grinfogel. The results from this study do not agree in the location of the avoidance points outlined by Triantafyllou and Grinfogel. As described above, Table 5.1, the λ^2 values increase in value with the increase in angle of inclination. Triantafyllou and Grinfogel predicted the avoidance points for all angles of inclination would occur in the region of $\lambda = 2\pi m$.

CHAPTER 5

DISCUSSION ON THE RESULTS

5.1 EXPERIMENTAL AND COMPUTATIONAL RESULTS

Both the experimental and computational results show, for a flat sagged cable, the frequency of the first symmetric mode is less than the frequency of the first asymmetric mode up to the first cross-over point, where the first symmetric mode frequency becomes greater than the first asymmetric mode frequency. A similar phenomenon is observed between the third and fourth frequency lines (second symmetric and second asymmetric modes) only at a different set of geometric and physical properties as illustrated in Figure 5.2 below. This confirms the cross-over phenomenon of a shallow cable similar to what is found analytically.

For an inclined sagged cable, the frequency of the first symmetric mode is less than the frequency of the first asymmetric mode up to the first avoidance point. This is similar to the case of the flat sagged cable. As the first avoidance point is approached, the first symmetric mode becomes a combination of the first symmetric and first asymmetric mode, or hybrid mode. The frequencies of these mode shapes do not actually cross as they approach, but diverge and switch from symmetric to asymmetric and asymmetric to symmetric. As above for the flat sagged cable case, the third and fourth frequency lines (second symmetric and second asymmetric modes) show a similar phenomenon, but at a different set of geometric and physical properties. This is illustrated in Figure 5.3 to Figure 5.6 below.

As stated above, the cable vibrates in a combination of symmetric and asymmetric modes, or a hybrid mode, in an area of the avoidance point. This is of significant practical consequence, since asymmetric modes are not considered to be the most important for fatigue and strength studies [17], as they have less significant engineering influence on fatigue and strength. This is because when a cable is vibrating in an asymmetric mode it does not involve elastic elongation due to structural dynamics. Therefore, due to the hybrid mode shapes, both symmetric and asymmetric modes near the avoidance points need to be studied in further detail for fatigue and strength studies for inclined cables.

For a flat sagged cable, the natural frequencies of the experimental and computer simulations begin to deviate when they are compared to those predicted by the theoretical linear

theory for flat sagged cable of the study at approximately a sag-to-span ratio (δ/l) exceeds 1/8. The linear theory for horizontal shallow cable is in good agreement with the numerical and experimental results of this study when the sag to span ratio is small, as defined by the ratio of ($\delta/l < 1/8$). This confirms the conclusion drawn by Irvine [6] when reviewing the deep cable profile by Saxon and Cahn [12]. As the sag-to-span ratio increases, the difference between the frequencies obtained by the linear theory and the frequencies obtained from the experiment and computation analysis also increases. This shows a similar trend to the results presented by Saxon and Cahn [12] for deep sagged cables, via a numerical formulation and confirmed experimentally.

For the case of inclined cables, similar conclusions can be drawn. As the λ^2 value increases past the avoidance points and after plateauing at the crossover dimensionless frequency, these frequencies begin to decrease in a similar pattern as for the case of a flat sagged cable. Through inspection of the results, this occurs at approximately the sag-to-span ratio (δ/l) of 1/8 for the angle of inclination of 30 degrees. The same cannot be said for the angle of inclination of 60 degrees. The difference in the dimensionless frequencies continues to increase for the maximum value reached following the avoidance point as the λ^2 value increases, in a similar pattern to the flat sagged cable.

The difference in the flat sagged and inclined cable cases is the location of the avoidance points for the inclined cables to that of the crossover point for the flat sagged cable. As the angle of inclination increase, the λ^2 values of the avoidance point also increases. The greater the angle of inclination of the supports, the greater the increase for the λ^2 value at the avoidance point compared with smaller angles of inclination. This is illustrated in the following table for the experimental and computational (computer simulations) of this study.

Table 5.1: Comparison between Experimental and Computational λ^2 -Values

Angle of Inclination	Crossover	Avoidance			
	0	15	30	45	60
First Experimental	35.1	34.4	42.7	62.1	136.9
First Computational	40.1	41	44.3	59.1	105.9
Second Experimental	156.2	149.1	175.1	241.7	531.9
Second Computational	175.8	183.1	218.6	325.9	616.4

Also, as the angle of inclination increases, the width of the region around the avoidance point increases, known as the transition region. The phenomenon is in agreement with numerical results by Triantafyllou and Grinfogel. The results from this study do not agree in the location of the avoidance points outlined by Triantafyllou and Grinfogel. As described above, Table 5.1, the λ^2 values increase in value with the increase in angle of inclination. Triantafyllou and Grinfogel predicted the avoidance points for all angles of inclination would occur in the region of $\lambda = 2\pi$.

Figure 5.1: The First Two Natural Frequencies of a Catenary in Air versus λ^2 [17]

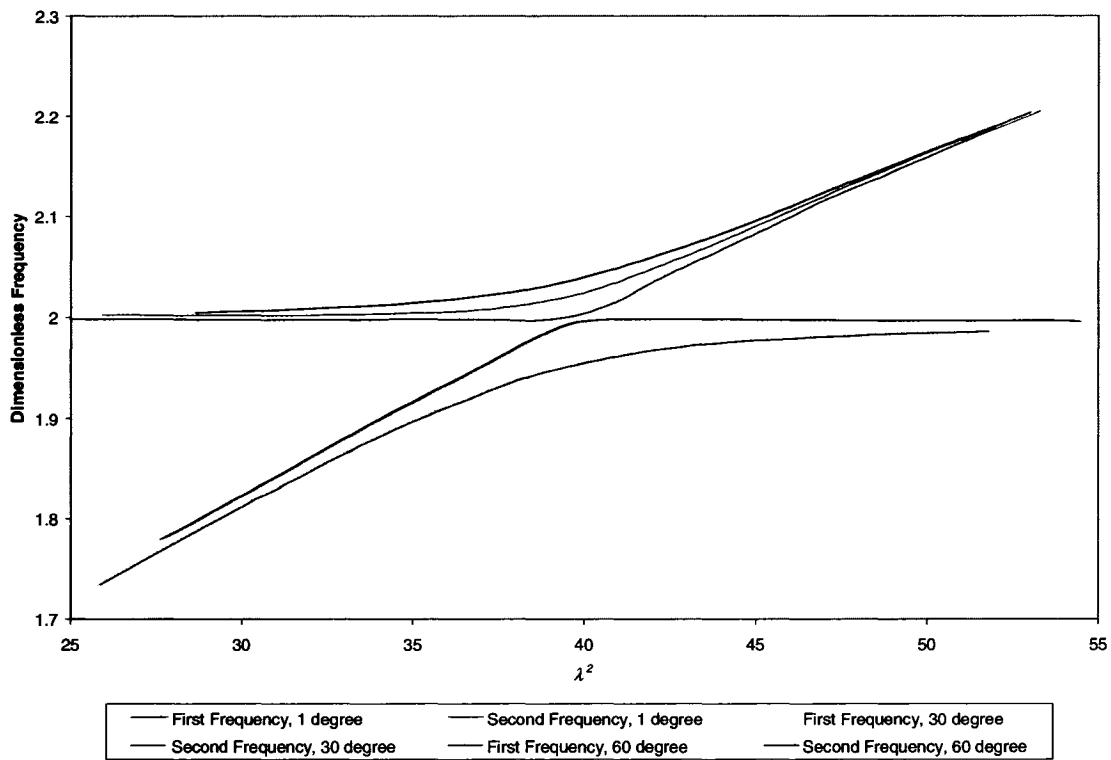


Figure 5.2: Comparison between Experimental and Computational Results, Flat Sag

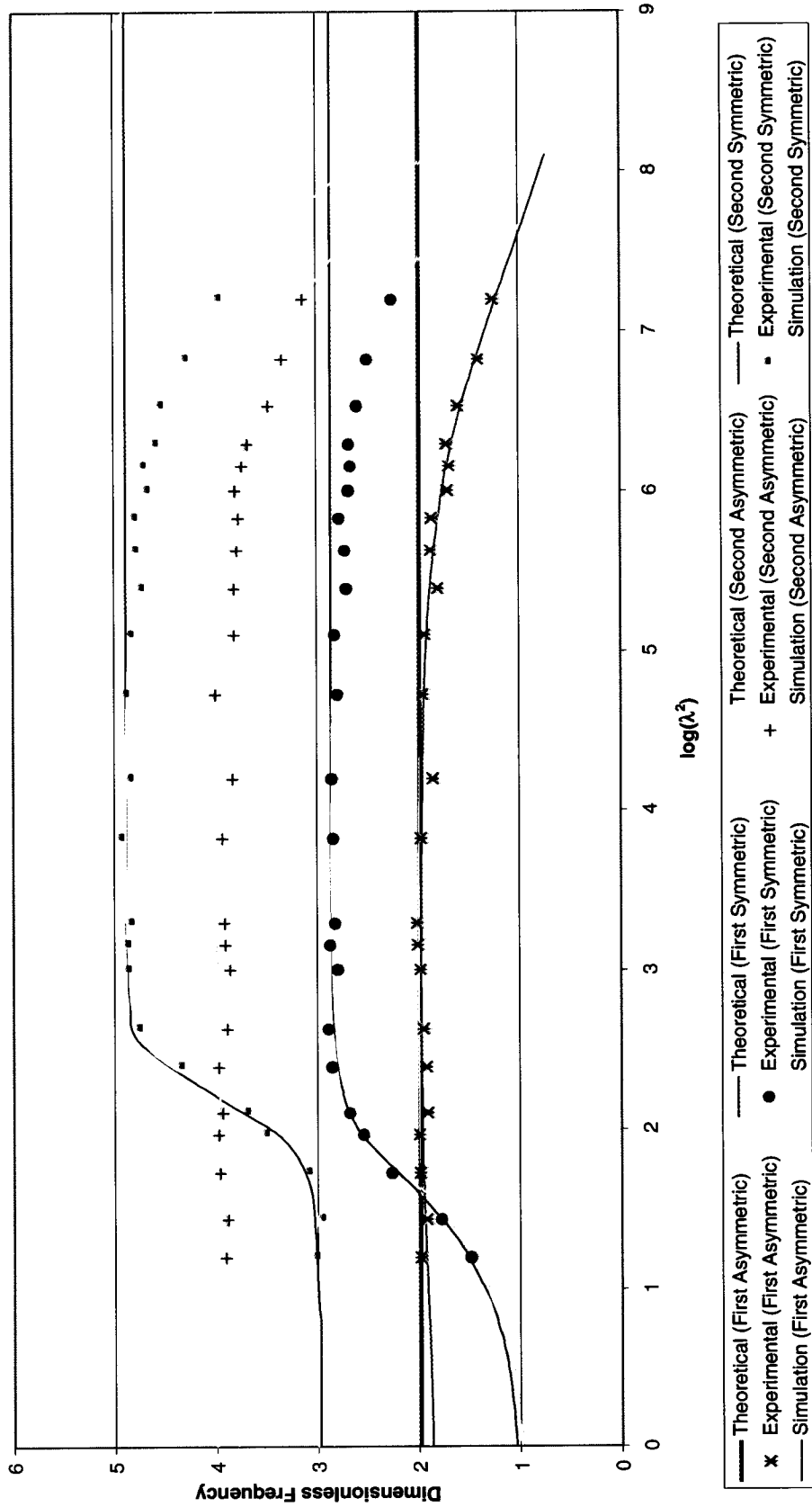


Figure 5.3: Comparison between Experimental and Computational Results, 15 Degrees

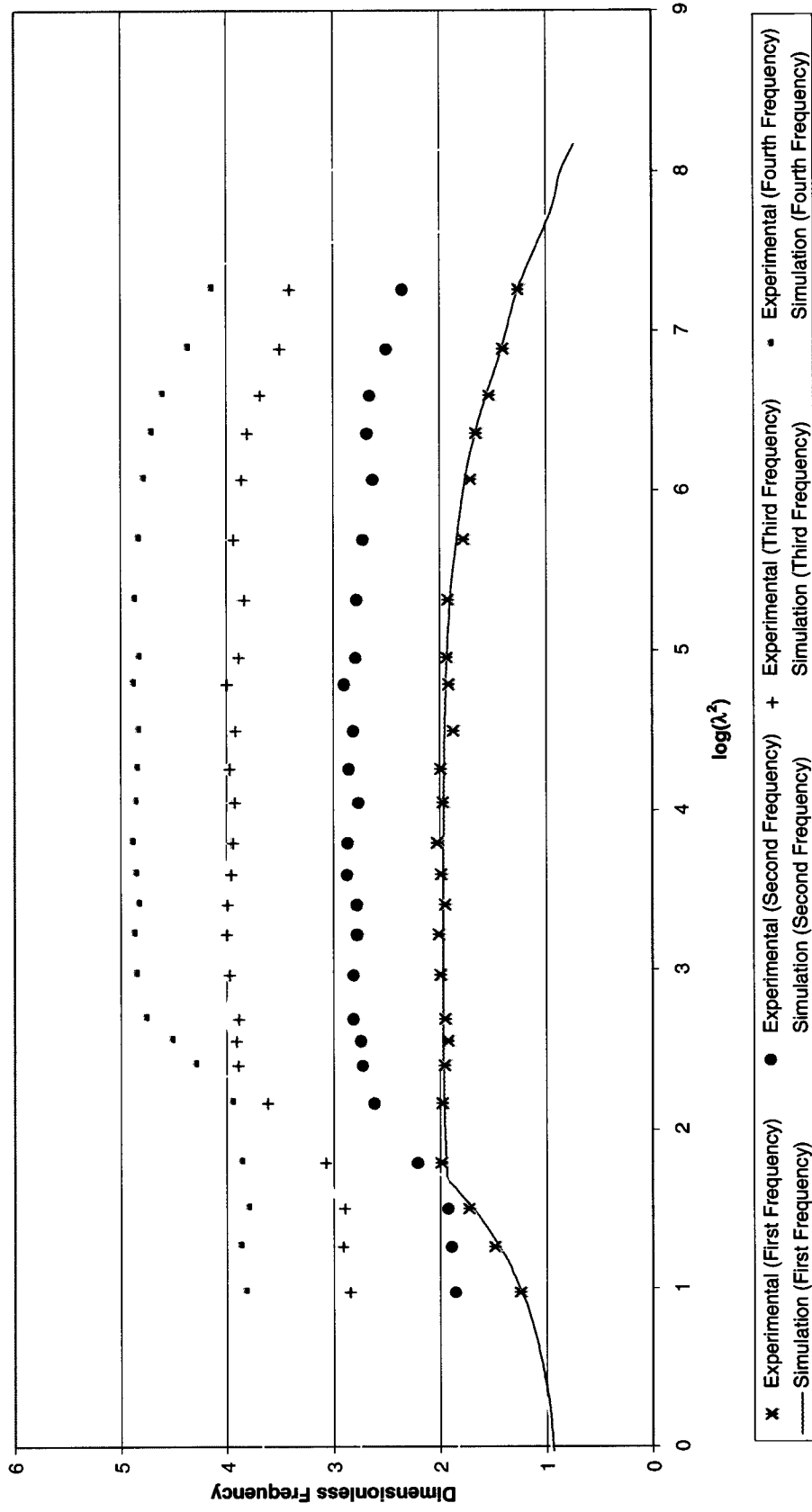


Figure 5.4: Comparison between Experimental and Computational Results, 30 Degrees

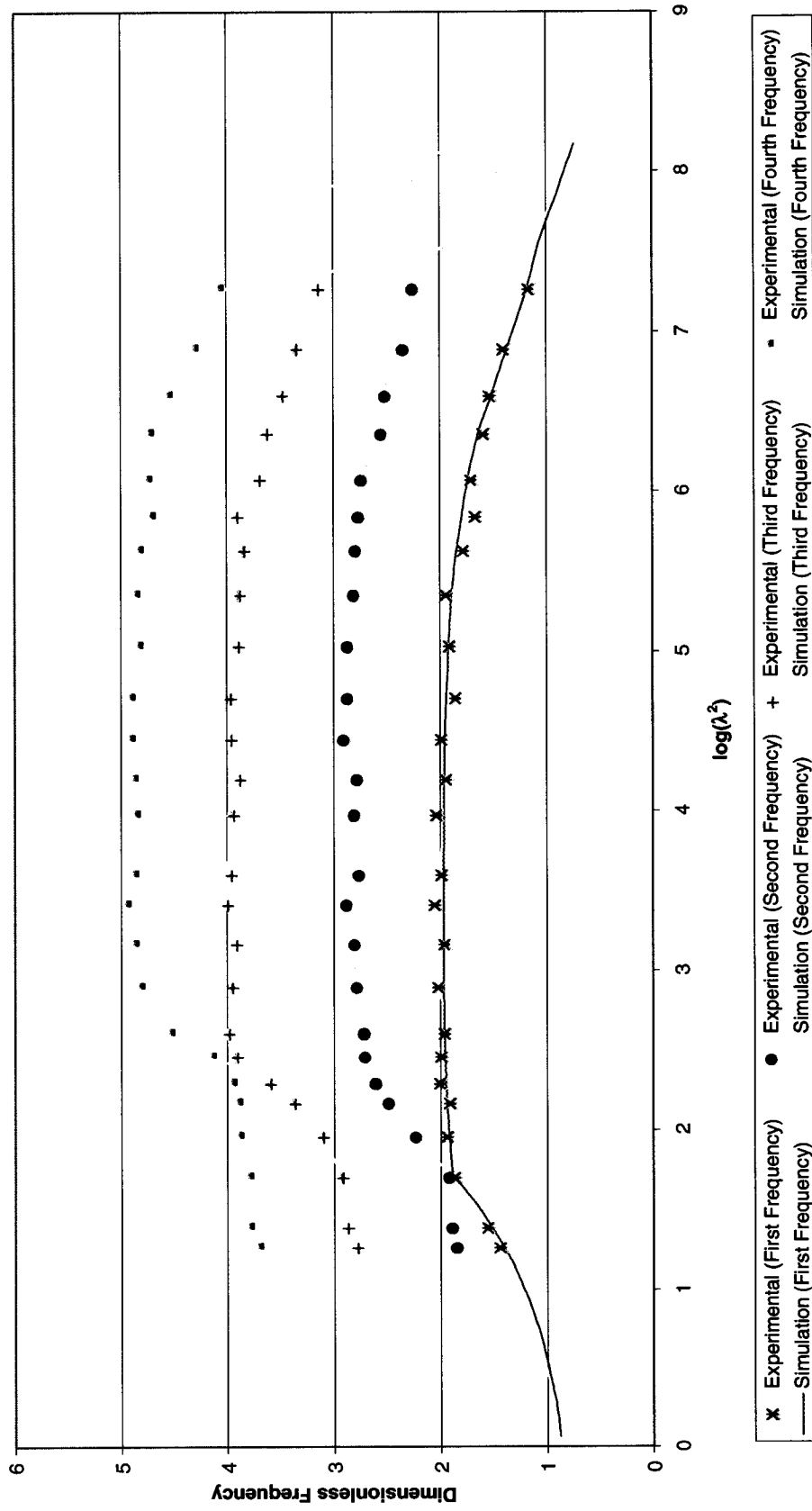


Figure 5.5: Comparison between Experimental and Computational Results, 45 Degrees

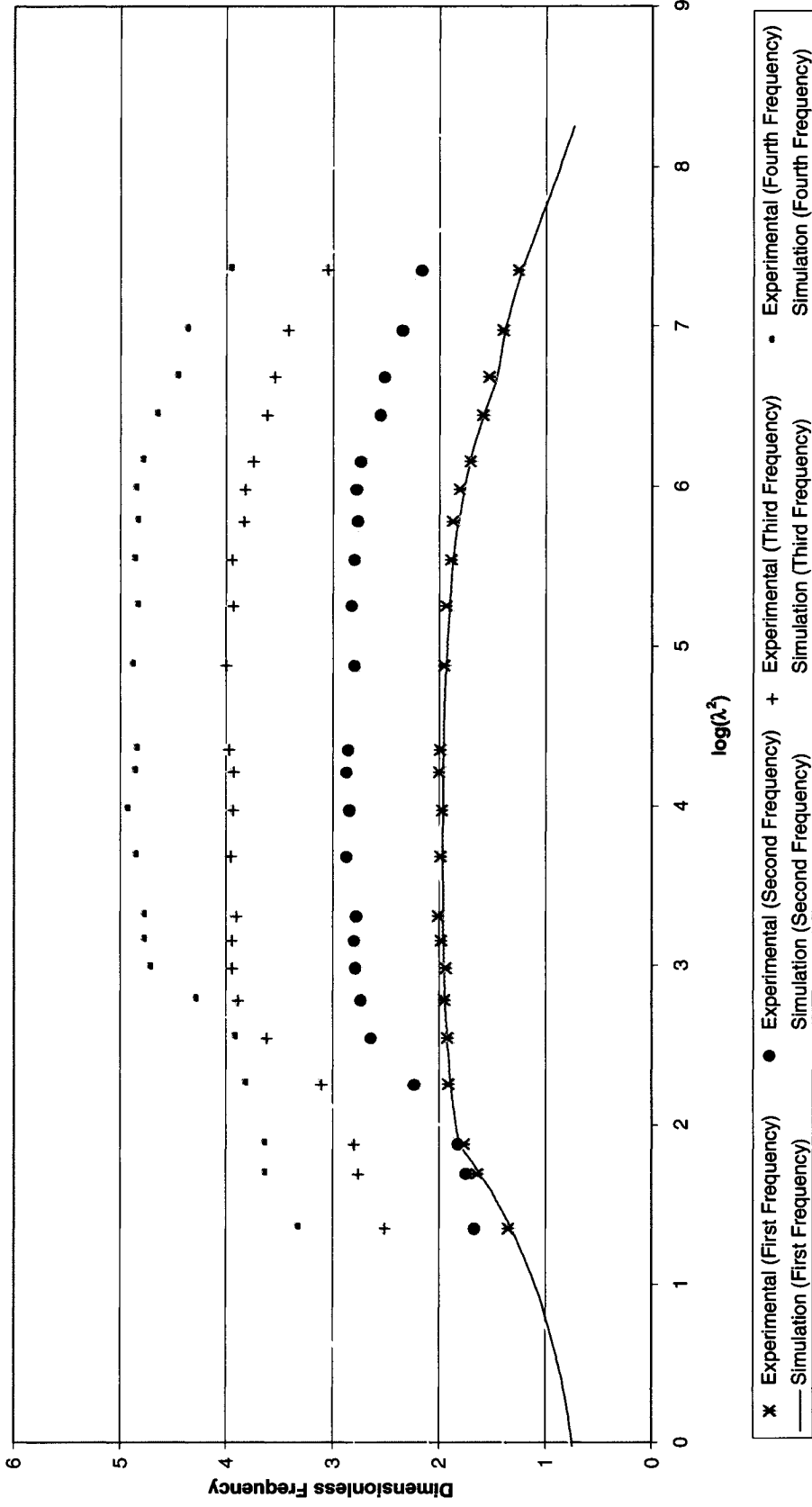
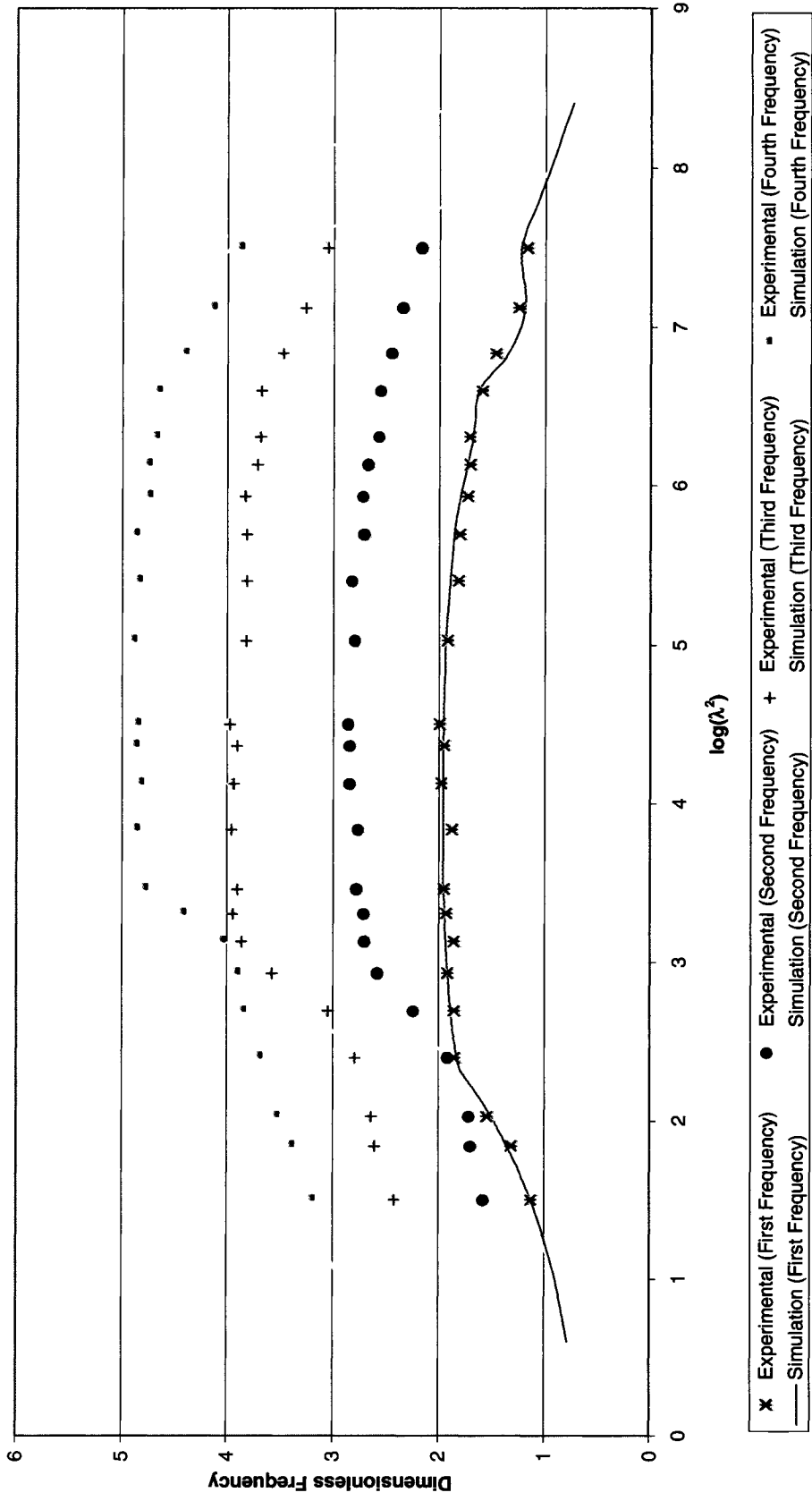


Figure 5.6: Comparison between Experimental and Computational Results, 60 Degrees



5.2 COMPARISON OF RESULTS WITH PREVIOUS STUDY

The results of this study are in agreement with previous studies by Oh [9] and Forghani-Arani [4] when compared using the elasto-geometric parameter λ^2 . The results from this study are presented in the next three figures as plots of dimensionless frequencies versus the logarithm of λ^2 . The agreement occurs in the following aspects.

For flat sagged cables, the existence of frequency and modal crossover points, and the change of the mode shape corresponding with the crossover points. Also, as the sag of the cable increases, the convergence of the frequencies and mode shapes to the frequencies and mode shapes of a vertically suspended cable. With the increase of inclination of the cable, the modal avoidance occurs at a larger value of λ^2 as illustrated by the movement of the avoidance point to the right when compared with the crossover point of the flat sagged cable case and with smaller angles of inclination.

As seen in the following three graphs, the results of this study are compared with that of Oh [9] for the angles of inclination which are common in both studies. These include 0 degrees, or flat sag, 30 degrees and 60 degrees. The two test cases from [9] are finite element calculations composed of two different elastic moduli, with all other cable parameters remaining the same. This change in modulus of elasticity is a decrease in the magnitude of a value of 10. The cross sectional area and the elastic moduli for the present study are both different than the parameters used by Oh in the previous study. When all three cases are plotted on the same graph, the crossover points for the flat sagged model and the avoidance points of the inclined models coincide for the respective models.

The dimensionless frequencies results from the previous study for the inclined cable results were divided by the square root of the cosine of the angle of inclination to match the frequency calculations of the study. This inclusion of the angle of inclination to the dimensionless frequency calculations brings the previous study results in line with theory by Triantfyllou [17] and calculations of the dimensionless frequency for this study.

The transition regions around the avoidance points for inclined cables increase in width as the angle of inclination increases. This is illustrated better with the results from Oh's computer modeling than with the results of this study. This is in agreement with previous studies by Triantfyllou [17] described in the previous section. Unlike Triantfyllou results, as the angle of inclination increases, the avoidance point moves to the right, to higher λ^2 values.

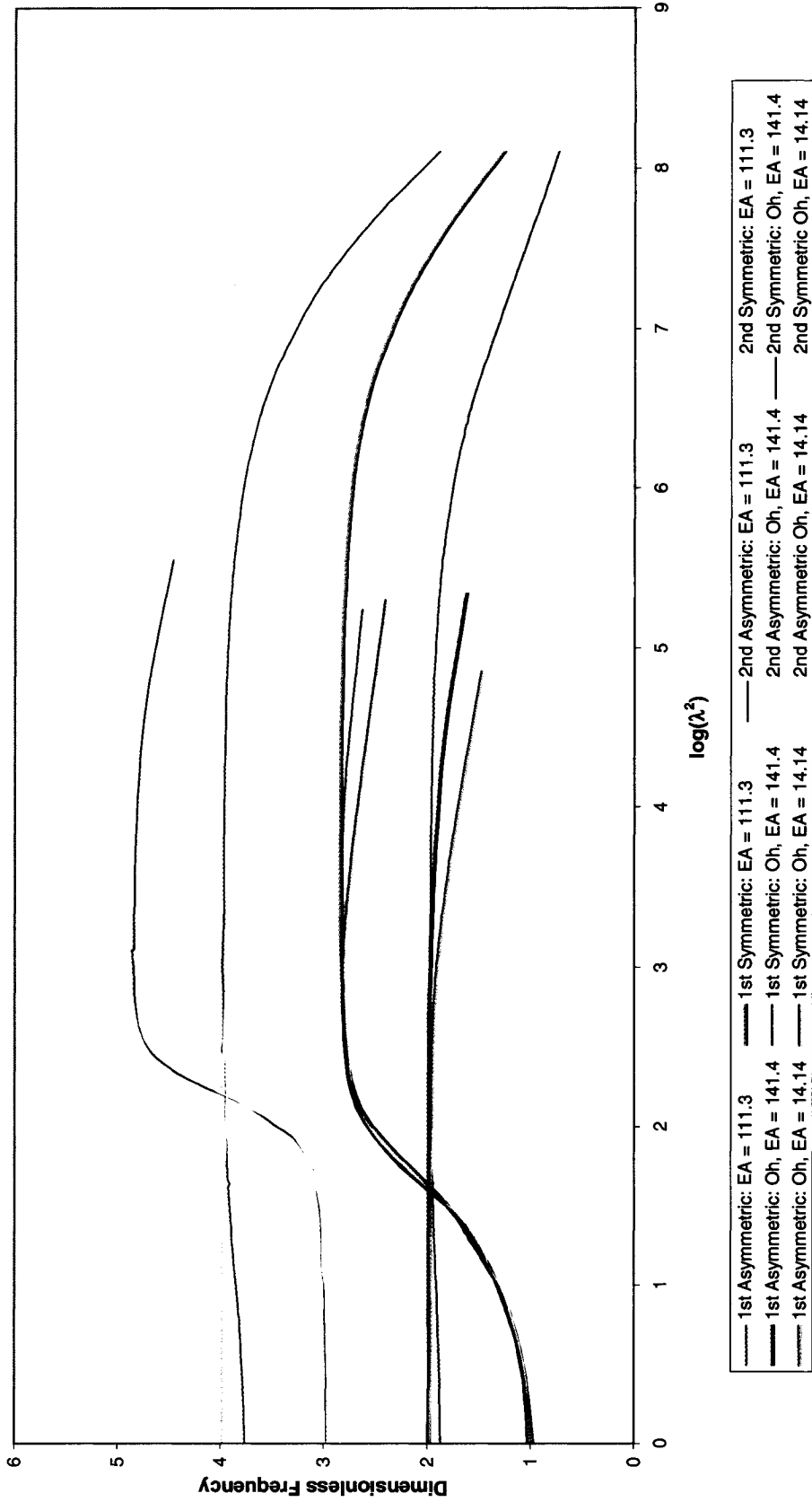
The results for the three models are not in agreement for the higher λ^2 . As the λ^2 value increases, starting approximately at the sag-to-span ratio of 1/8 for the flat sagged models, the dimensionless frequencies predicted by each model differed from one another. This occurs at approximately the sag-to-span ratio of 1/8 as predicted by Irvine [6] for the end of the linear theory for flat sagged cables. For the inclined cable cases, this tendency of a lack of agreement of the dimensionless frequencies for λ^2 still exist, but the value of 1/8 used for the flat sagged cable does not appear applicable. This can be seen when the results from both studies are plotted using the sag-to-span ratio, as illustrated in the graphs below.

For the flat sagged cable case, as the sag-to-span ratio of 1/8 is reached (at approximately the value of -2.7 in the $\log((\delta/l)^3)$ graph) the dimensionless frequencies begin to decrease in value. This could also be taken to be true of the angle of inclination of 30 degrees, but it is not true of the angle of inclination of 60 degrees. It can be seen from these graphs that as the sag-to-span ratio continues to increase, the dimensionless frequency continues to decrease. As the sag-to-span ratio approaches its maximum, the frequencies of the sagged cable approach that of a cable suspended by one end.

In the previous study by Oh [9] the cable length was held constant and the sag of the cable was changed by varying the span along the chord of the cable. For this study, the span or chord length of the model remained constant and the total cable length was modified to obtain the sag-to-total cable lengths for the study. This was to facilitate the experimental model testing and compare the contrast between the two approaches to studying sag-to-total cable length.

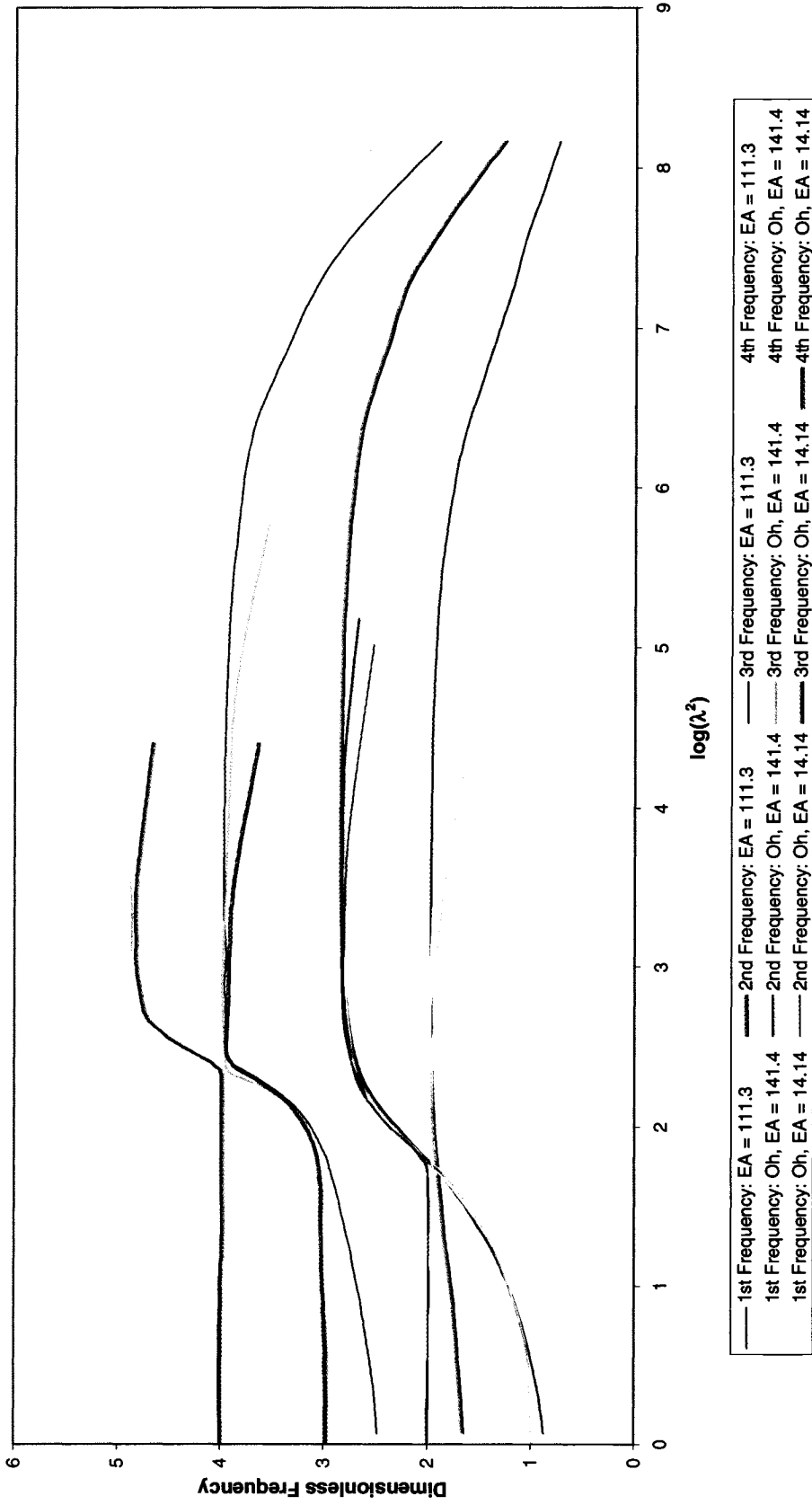
For the smaller sag-to-span ratios (approaching a case of a taut wire) the dimensionless frequencies obtained for the numerical simulations of this study are lower than that of the previous study's results. As the angle of inclination increases, the dimensionless frequencies continue to decrease. The dimensionless frequencies increase up to the crossover point for the flat sagged case and the avoidance points for the inclined cable cases. At these points, the dimensionless frequencies are close to that predicted by Triantyllou and Irvine of 2 and 4.

Figure 5.7: Comparison of Numerical Results with Previous Study, Flat Sag



- difference in the two trails from Oh's [9] previous study in the modulus of elasticity by a magnitude of 10
- EA = 111.3 kN for present study, EA = 141.4 MN for the first test of Oh's work, EA = 14.14 MN for the second test of Oh's work

Figure 5.8: Comparison of Numerical Results with Previous Study, 30 Degrees



- results from Oh [9] for inclined sagged cables, the dimensionless frequency has been divided by the square root of the cosine of the angle of inclination to match the frequency calculations from this study. This is to match theory by Triantyllou [17] in using the horizontal cable tension instead of the cable tension along the chord.

Figure 5.9: Comparison of Numerical Results with Previous Study, 60 Degrees

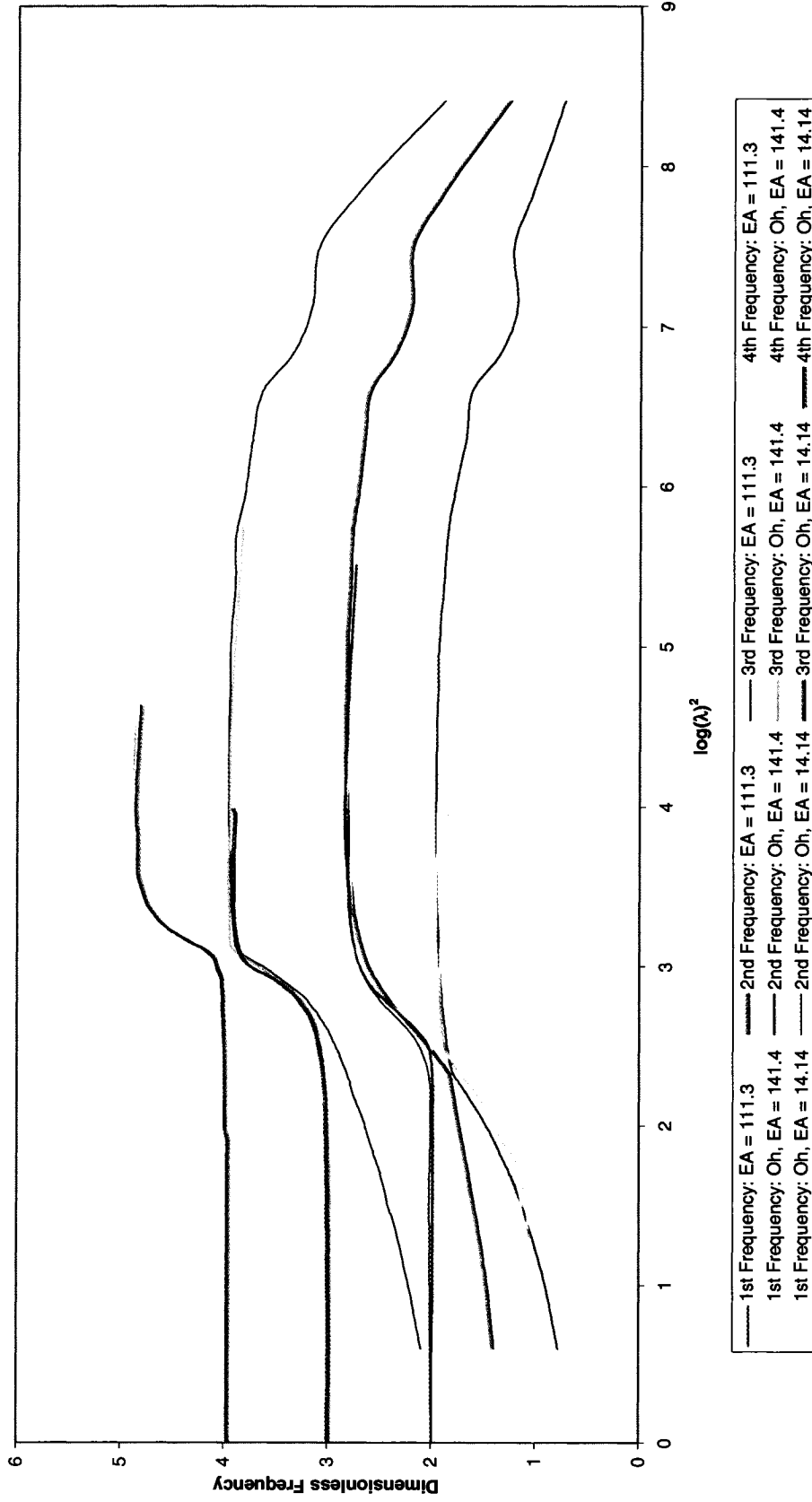


Figure 5.10: Comparison of Numerical Results with Previous Study, Flat Sag

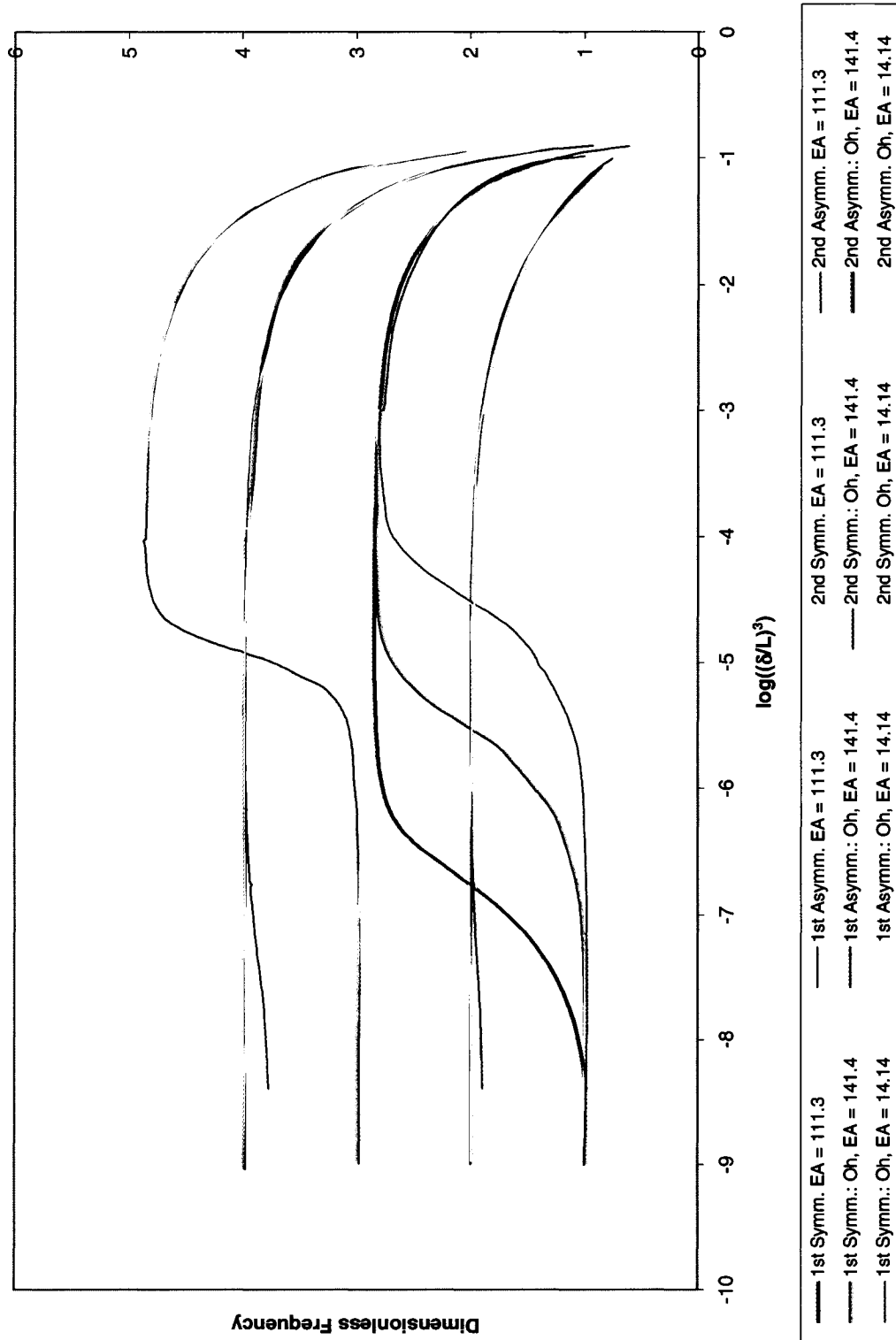


Figure 5.11: Comparison of Numerical Results with Previous Study, 30 Degrees

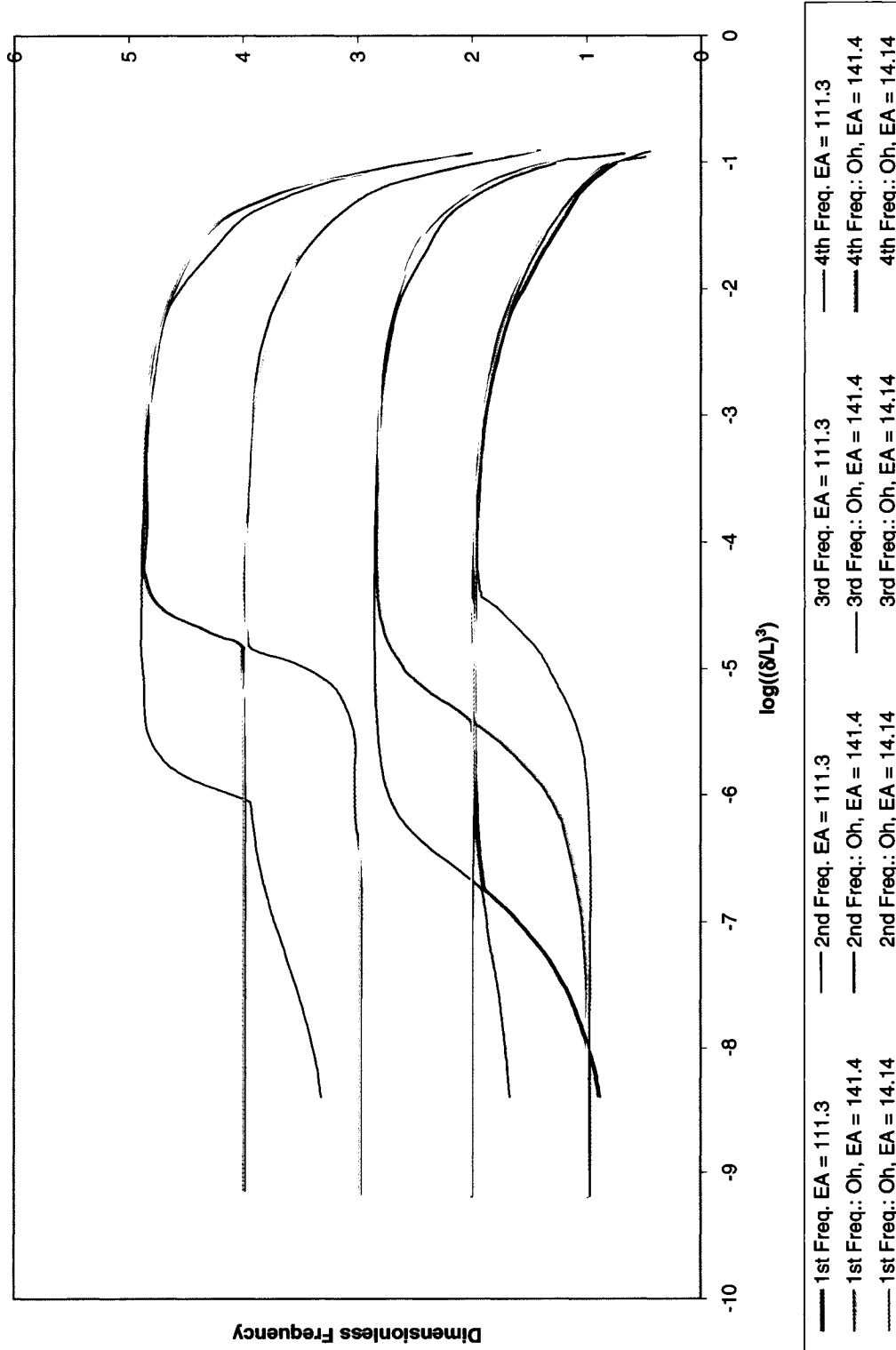
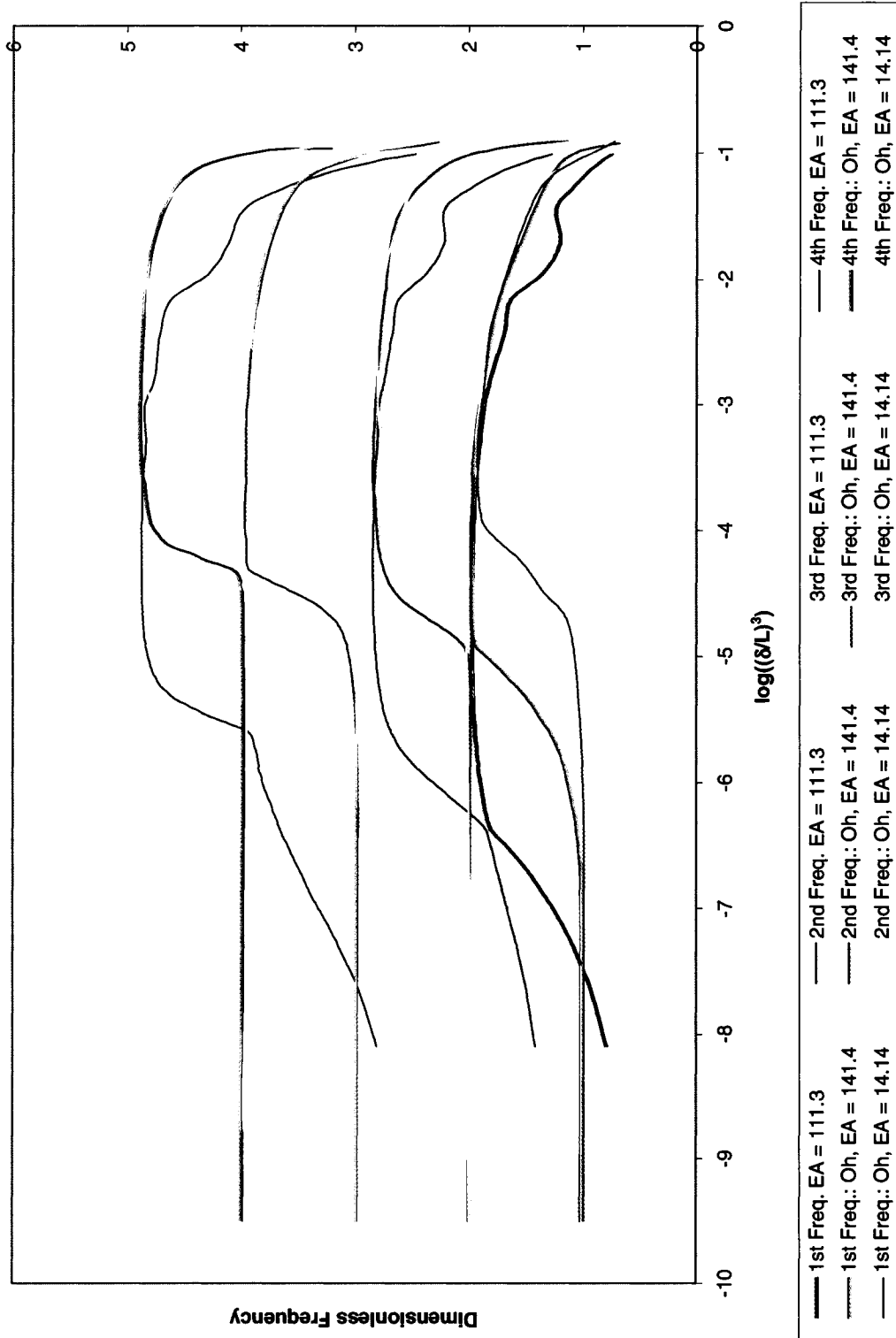


Figure 5.12: Comparison of Numerical Results with Previous Study, 60 Degrees



CHAPTER 6

SUMMARY, CONCLUSIONS AND FURTHER RESEARCH

6.1 SUMMARY AND CONCLUSIONS

The study of natural frequencies and corresponding mode shapes of vibration of suspended cables has been an area of research interest for a number of years. The current study compares the results from a physical model test with a computational model, which is a chain of extensible links of cable elements connected together by frictionless pins and concentrated masses at the connection joints. The greater discretization creates a closer representation to a real cable with uniformly distributed mass. The finite element method formulation is applied using a commercial software package ADINA.

A physical model of a galvanized aircraft cable, 1/16" in diameter was used to investigate sagged cable dynamics. The choice of this cable allowed for a wide range of elasto-geometric parameters (λ^2) to be investigated, covering the crossover region of a flat sag cable, and avoidance region of inclined cables to deep sagged cables for both cases of flat sagged and inclined cables.

The distance between fixed supports was set to be 2.50 m, even when the model was inclined. The size and span of cable was decided in order to obtain a wide range of property parameters, while maintaining a manageable size for setting-up and performing the experiments.

The amount of sag was changed from 10 mm up to the maximum of 2000 mm over the 2.50 m span. The 10 mm sag was practically the smallest sag possible before the inextensibility of the cable inhibited the vibration of the model; 2000 mm was the maximum sag used while the detection of the differences between natural frequencies was still possible.

The existence of modal cross-over points for flat sagged cables was demonstrated. The first and third natural frequencies approach to the natural frequencies of the second and fourth modes, coinciding at the modal cross-over point, and increases past this point, when the cable tension is reduced or the sag is increased. Trials were performed on a wide range of angles of

support inclination, including 0 degrees as mentioned above, and 15, 30, 45, 60 degrees, with 60 degrees being the maximum applicable angle used in industry.

As has been shown, the experimental results and the computational results of this study are in good agreement. The deviation in the results can be attributed to the level of precision in the experimental data. The natural frequency of the cable during the tests was determined from the reading of input frequency at resonance.

When comparing the present results with the earlier studies by Forghani-Arani [4] and Oh [9], the results from this study are in agreement with previous studies regarding the location of the crossover points for a flat sagged cable and the avoidance points for inclined sagged cables. In comparing the two results from the previous study by Oh [9], one with a modulus of elasticity with the magnitude of 10 less than the other, the crossover of the model with smaller elastic modulus occurred at a higher sag ratio, but at the same elasto-geometrical cable parameter, λ^2 . A difference in dimensionless frequencies was observed when comparing the results from this study to the above mentioned previous study for the higher sag ratios (or lower tension) and higher angles of inclination (in this case, 60 degrees of inclination).

In the previous study by Oh [9], the cable length was held constant and the sag of the cable was changed by varying the span along the chord of the cable. For this study, the span or chord length of the model remained constant and the total cable length was modified to obtain the sag-to-total cable lengths for the study. This was to facilitate the experimental model testing and compare the contrast between the two approaches to studying sag-to-total cable length. In both cases, the elasto-geometrical cable parameter, or the variables incorporated in the parameter are reflected in the analysis of the results. These parameters include mass of the cable, resulting in the inertia forces, and the cable stiffness.

As the sag ratio increase, a decrease in the dimensionless frequencies occurs and the cable begins to resemble that of a vertically suspended cable. The dimensionless frequencies of all modes converge to that of a vertically suspended cable as this occurs.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

With all the work put in to the problem of cable vibration over the past centuries, work can still be done to better understand on nature of cable vibration. The case of a flat sagged cable is well understood, but the case when the cable supports are inclined and when the cable allowed

deflecting under its own weight requires a better understanding. This will be of greater importance as cables are used to span greater distances and become lighter and stronger.

Further study can be conducted in the crossover areas for λ^2 using different parameters, including stiffness (EA), span length (l), and mass per unit length (m). This would include avoidance point location for inclined sagged cables, including width of the transition region as outlined by Triantyllou [17].

The effect of cable bending stiffness can be investigated. In particular, the change in the natural frequencies and mode shapes of vibration, if any, due to the increased bending stiffness of the cable. Also, are there any changes in the location of the cross-over points for horizontal sagged cables and the location and width of the transition region around the avoidance point if the bending stiffness is changed?

The sag-to-span ratio (δ/l) of $1/8$ has been accepted as an upper limit for the use of predicting natural frequencies of flat sagged cables using the linear theory. For the results examined in this study, the dimensionless frequencies of one of the test cases did not follow this limit. Further research is required into the applicability of this ratio for inclined sagged cables before a decrease in the dimensionless frequency occurs with the increase in sag (or decrease in tension).

Improvements are possible to the experimental setup in both the method of exciting the cable and measuring the natural frequencies of the cable vibration. These include the use of non-contact methods of cable excitation and frequency measurement of the cable as outline in [1] as a possibility.

APPENDIX A

CONVERGENCE STUDY

To determine the number of elements required in the model, a convergence study was performed. To accomplish this, a taut string or cable in tension model was studied, as the theory of its vibration is well understood. The same modelling theories are used for the sagged cable. In this case, only the lateral vibration was studied.

A.1 Theory

The traverse vibration of a string or cable, is covered in most introductory books on dynamics. Consider a string with a uniform mass density ρ (kg/m) and a constant cross sectional area, fixed at both ends and under an axial tension, T . The string can vibrate laterally, in the y -direction. The lateral displacement at any point of the string must be a function of both time and the position x along the string. For free vibration, the governing equation is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} \quad (\text{A.1})$$

$$f_n = \frac{nc}{2l} = \frac{n}{2l} \sqrt{\frac{T}{\rho}} \quad \text{where} \quad c = \sqrt{\frac{T}{\rho}} \quad (\text{A.2})$$

The magnitude of c depends on the physical properties of the string, which is called the wave speed, or the velocity of wave propagation along the string. The above equation is the one-dimensional wave equation, and is subject to two initial conditions in time because of the dependence on the second derivative.

A.2 Model Description

The taut wire was modelled as a series of one-dimensional links, modelling the wire as a chain. The model is fixed in the x - and y -translations at both supports, but is allowed to rotate freely. The model is 1 meter in length. The material properties of the model were assumed to be 200 GPa for the Modulus of elasticity, and a mass density of 7920 kg/m³. These parameters were

constant for all models. The wire cross sectional area is 10 mm^2 , and the applied tension is 5000 N. The number of elements used in the model ranges from 40 to 240.

To obtain the pre-stress in the cable, a stress analysis was performed using structural analysis in static mode. The pre-stress was defined giving each element a stress value corresponding to the applied load and the material properties. In this way, both ends of the model could be fixed against displacement in both the x and y directions. The pre-stress could be applied to the model by a concentrated force on the one end, or by moving a support of the model a specified distance, but that would change the span length of the model and the corresponding natural frequency of the model.

A simple two-dimensional model was chosen due to the fact that the results could be compared for both modal shapes as well as for frequencies. An increase in the model complexity, e.g. a third dimension, would produce in more results. This would lead to greater output of frequencies and mode shapes making the resulting analysis more difficult due to the sheer volume of numerical results obtained by the solution without changing the outcome of the required task, the number of elements required to obtain a converged solution.

A.3 Results

The results of the convergence study are present in the table below. They show the number of nodes used to model the taut wire and the eigenvalues for each of the first five modes. All the results presented in Table A.1 have been represented in a Figure A.1 on the following page. The corresponding dimensionless frequencies have been calculated, as outlined in [6], for the above mentioned results, and plotted in Figure A.2. The results for the first mode and the fifth mode have also been plotted separately in Figures A.3 and A.4 to obtain a better idea of the results.

From the graphs, the results obtained were stable throughout the whole range of models, from 40 to 240 nodes. As evident in Figures A.2 and A.3, the solution obtained by increasing the number of nodes for analysis over 120 does not improve the accuracy of the solution to justify using any amount of nodes greater than 120.

Table A.1: Number of Nodes versus the Eigenvalues Obtained

Modes	40	80	120	160	200	240
First	3.9788	3.9780	3.9778	3.9778	3.9778	3.9778
Second	7.9637	7.9575	7.9564	7.9560	7.9558	7.9557
Third	11.9610	11.9400	11.9360	11.9350	11.9340	11.9340
Fourth	15.9760	15.9270	15.9180	15.9150	15.9140	15.9130
Fifth	20.0170	19.9210	19.9030	19.8970	19.8940	19.8920

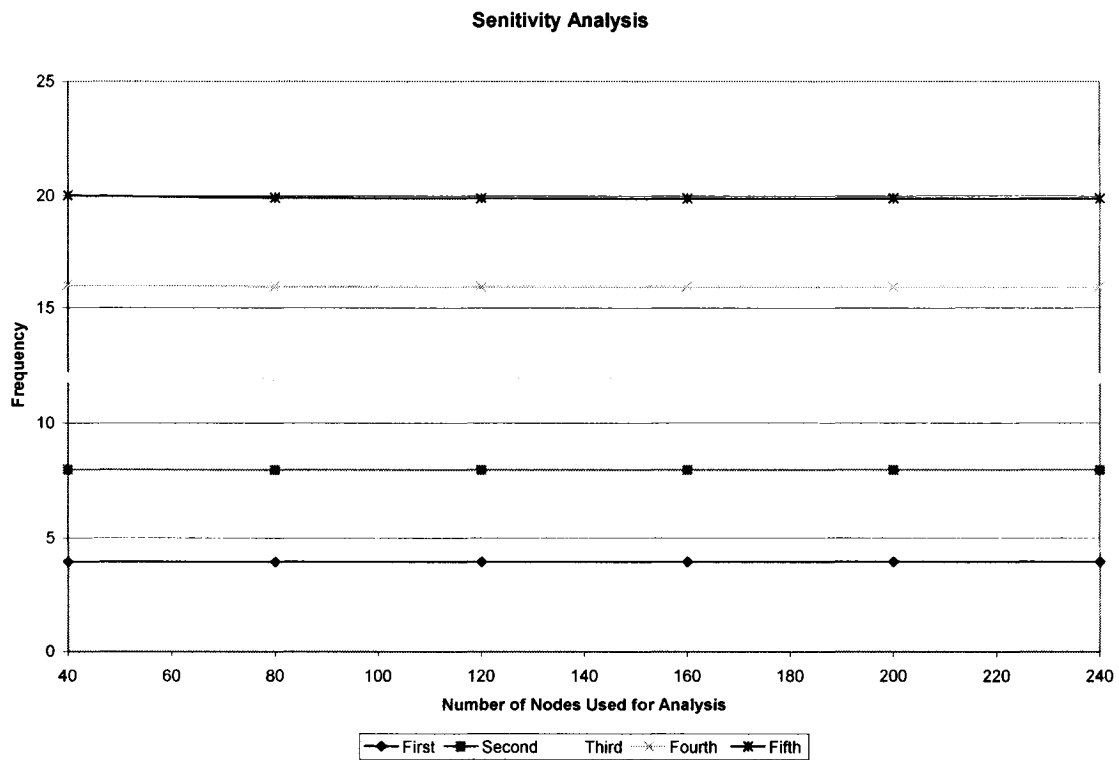


Figure A.1: Sensitivity Analysis for a Taut Wire.

Table A.2: Number of Nodes versus the Eigenvalues Obtained, Dimensionless Frequency.

Modes	40	80	120	160	200	240
First	0.504057	0.503956	0.503931	0.503931	0.503931	0.503931
Second	1.008887	1.008102	1.007962	1.007912	1.007886	1.007874
Third	1.515288	1.512628	1.512121	1.511994	1.511868	1.511868
Fourth	2.023931	2.017724	2.016584	2.016203	2.016077	2.01595
Fifth	2.535868	2.523707	2.521426	2.520666	2.520286	2.520033

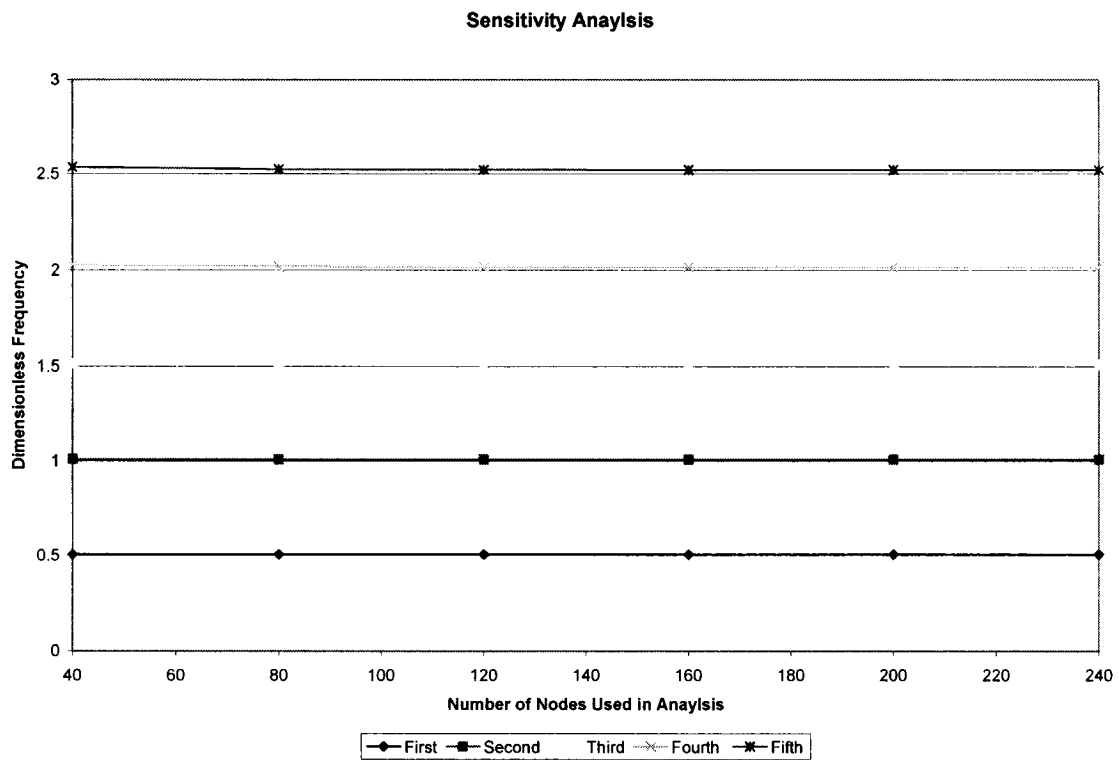


Figure A.2: Sensitivity Analysis for a Taut Wire, Dimensionless Frequency.

Sensitivity Analysis

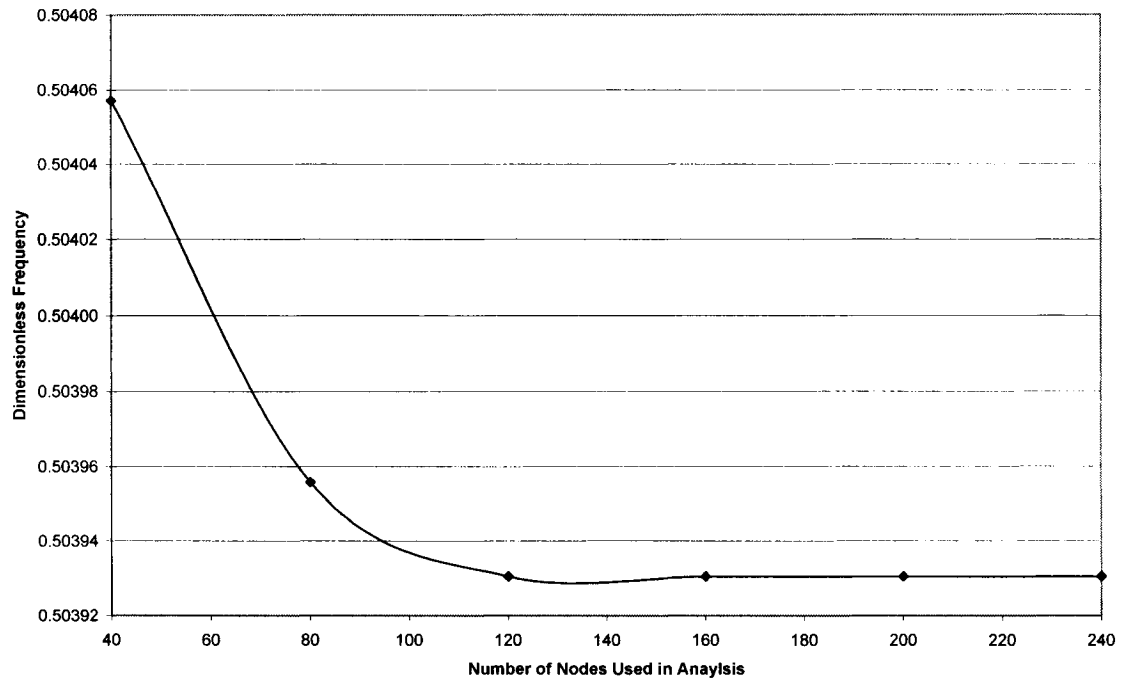


Figure A.3: Sensitivity Analysis for the First Mode of a Taut Wire, Dimensionless Frequency.

Sensitivity Analysis

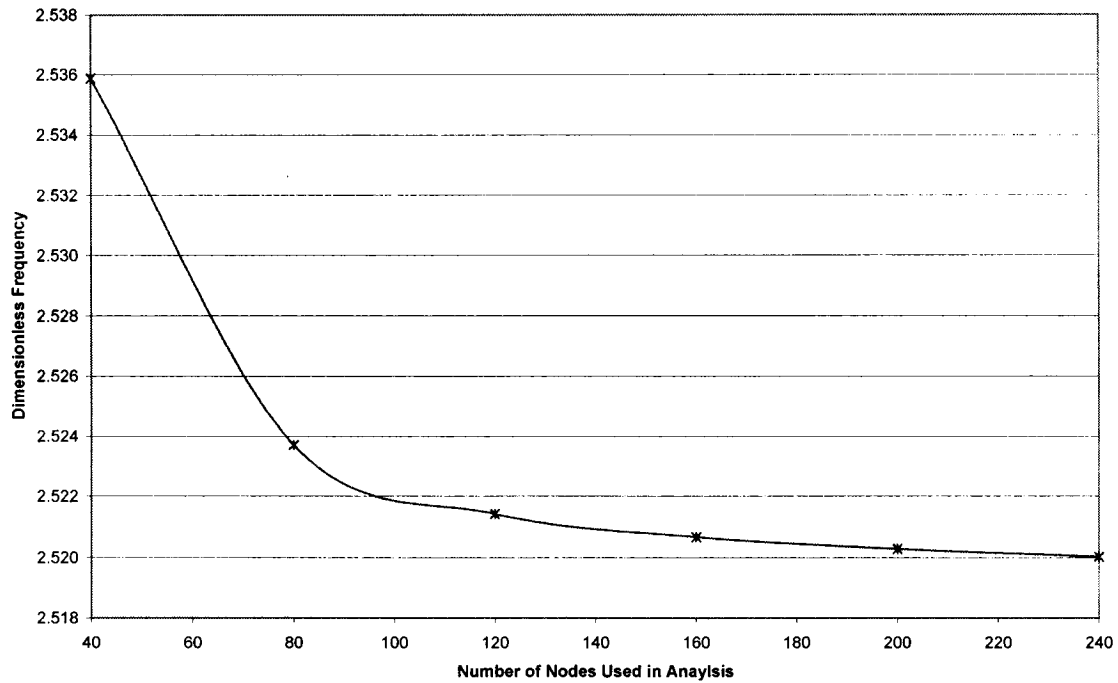


Figure 4: Sensitivity Analysis for the Fifth Mode of a Taut Wire, Dimensionless Frequency

A.4 Discussion

The convergence study in the present analysis is required to analyse the model by the finite element modelling which involves discretization. The component being studied is a continuous medium equivalent to having an infinite number of degrees of freedom. Due to the limitations of the finite element analysis process, any computer model will only be a discrete representation of the actual component, having a finite number of degrees of freedom. For this case, the continuous cable will be modelled as a chain of extensible links of cable elements connected together by frictionless pins and concentrated masses at the connection joints. The greater discretization creates a closer representation to a real cable with uniformly distributed mass.

In order to get some idea of the error developed due to modelling a continuous component as a discrete model, different number of mesh sizes was applied to the model.

As seen above in Figures A.1 through A.3, the natural frequencies for the first five modes vary with the level of discretization applied to the model. The graphs indicate that the model has

converged to a solution. The level of accuracy of the results above from the computer analysis was limited above 100 node used in the model. For this reason, and to have a consistency in the analysis process with previous studies by Forghani-Arani [4] and Oh [9], a model consisting of 100 nodes was used in the analysis.

APPENDIX B

CALCULATION OF METALLIC CROSS SECTIONAL AREA

- 1/16 of an inch (1.5875 mm) diameter galvanized aircraft cable
- 7 x 7 construction
- area of 1.187 mm², calculated using the *Wire Rope User's Manual* [20], a table for the approximate metallic areas of one inch rope of various constructions; approximate metallic area of one-inch rope of 7 x 7 construction, .471

This is a dimensionless parameter, defined by the ratio of the cross sectional area of the wire rope to the area of a solid metallic rod of the same diameter.

For diameters other than 1 inch, multiply the area given in the above referenced table by the square of the nominal rope diameter.

Find the area of 1/16" 7 x 7 IWRC

From above: 0.471

Diameter squared: $(1/16)^2 = 1/256$ or $0.0625 * 0.0625 = 0.00390625$ inches²

Multiply table value by diameter squared: Area = $0.00390625 * 0.471$

$$= 1.840 \times 10^{-3} \text{ inches}^2$$

$$\text{area} = \left(\frac{1}{16}\right)^2 (0.471) = 1.840 \text{ in}^2 = 1.187 \text{ mm}^2$$

Values given are based on 3% oversize because this is a common design target. But, this figure often varies and is not to be considered a standard. Wire sizes in specific constructions also vary, thus the given values are approximate. They are, however, within the range of accuracy of the entire method that is, in itself approximate.

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