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NEW FRINGE-FIELD CAPACITIVE SENSORS

by

Karunanayake P.A.P. Esselle

A thesis
presented to the School of Graduate Studies and Research
of the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Master in Applied Science
in
Electrical Engineering

Ottawa, Ontario, Canada, 1986

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ABSTRACT

A modified open-ended coaxial line sensor with high fringing-field capacitance is described in this thesis. The sensor called "banyan-tree" sensor allows accurate in-vivo measurements of biological tissues in the 10 kHz-40 MHz frequency range. Several 7-mm and 14-mm sensor configurations were analyzed using the Method of Moments. Two dimensional subsectional bases and Dirac testing functions were used. Numerical results are presented in tabular and graphical forms. Throughout the work, the sensor is represented by a linear two-capacitance (C_0 and C_f) model. An experimental 14-mm banyan-tree sensor is also described. The capacitances, C_0 and C_f of the experimental sensor were measured in a range of frequencies. They were found to be constant up to 40 MHz. Differences of 13% and 3% were observed between the calculated and measured values of C_0 and C_f , respectively.

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Chapter 1

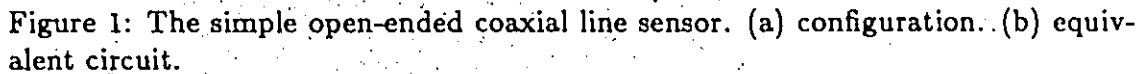
INTRODUCTION

The way electromagnetic waves interact with dielectric materials is determined by the dielectric properties of the latter. With the increasing number of radio, micro-, and millimeter wave applications in many disciplines, many electromagnetic engineers have to deal with such material-wave interaction problems today. In order to understand these problems, and sometimes to make predictions, it is essential to know the dielectric properties of the materials involved in the interactions.

It is the permittivity that plays a major role in many problems. Medical applications of microwaves such as hyperthermia treatment of cancer [1] and diagnosis of pulmonary diseases [2] require the knowledge of the permittivity of biological materials. In theoretical electromagnetic dosimetry, for the determination of local and average specific absorption rates, the permittivity distribution of the body is necessary [3]. Some industrial applications of microwaves also depend on permittivity. Hence, the need for accurate methods of permittivity measurements is evident.

The method should allow in-vivo measurements of biological materials. When the spacial distribution of permittivity is desired, sample size should be made as small as possible. It was found that, the method which uses an open-ended coaxial line as a sensor satisfies these requirements. This method also offers a wide frequency band as compared with methods using other sensor structures.

The use of open-ended coaxial line as a sensor in dielectric measurements has been extensively studied by many authors [4,5]. Figure 1 shows such a sensor,



1.1 Limitations and Modifications of Open-ended Coaxial Sensors

The major limitation of the simple, fully-open-ended coaxial sensor shown in Figure 1 is that it is not usually suitable for low frequency measurements (Practically, below 100 MHz). The small fringing field capacitance of this sensor (0.02-0.20 pF, in air) results in a very high impedance of the sensor as compared with the characteristic impedance of the line, at low frequencies. As shown by Stuchly et al.

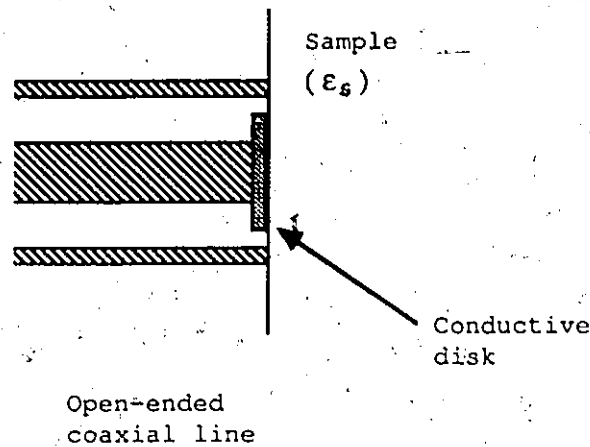


Figure 2: Coaxial line sensor with the disc-loaded inner conductor.

[6], this leads to large uncertainties in permittivity measurements. Therefore, for accurate measurements at low frequencies, the capacitance of the sensor has to be increased. Several attempts have been made to achieve this, basically by modifying the open end of the sensor.

The configuration shown in Figure 2 uses a concentric cylindrical conducting disc placed over the end of the inner conductor of the coaxial line [4]. This reduces the gap between the two coaxial conductors at the end and increases the energy stored in the fringing field. However, the achieved improvement in this way was not significant.

The multi-ring coaxial sensor described by Stuchly et al. [5] is shown in Figure 3. This modification allows an increase in the fringing field capacitance by an order of magnitude as compared with the fully-open-end sensor. This sensor, when fabricated in a standard 14-mm coaxial line with GR 900 connector, extended the range of in-vivo measurements of biological tissues down to a frequency of approximately 10 kHz.

Despite of attractive capacitance values, this sensor creates two problems. First,

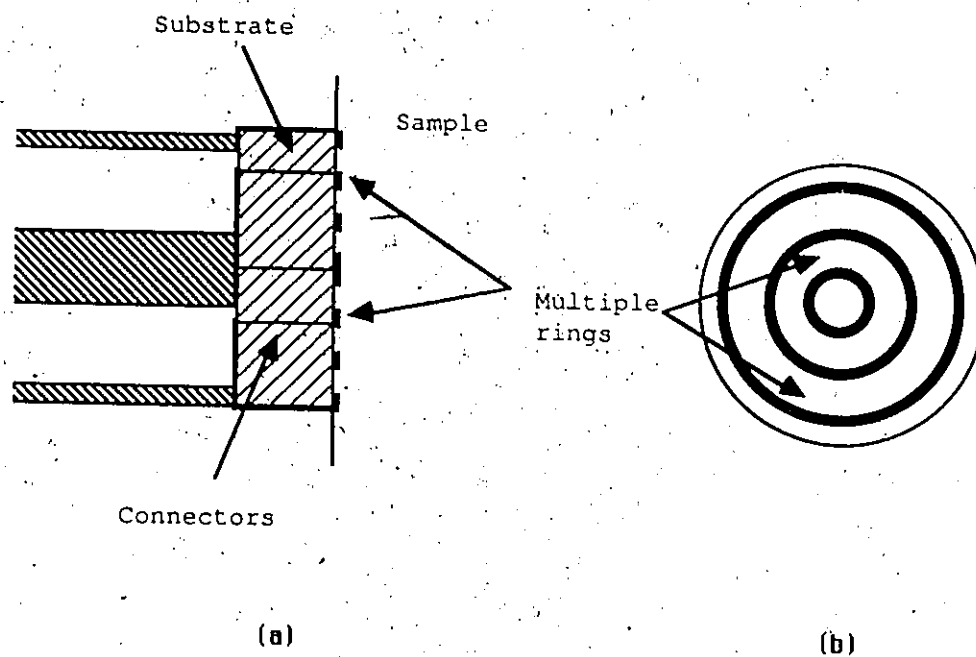


Figure 3: Multi-ring coaxial sensor. (a) sectional view. (b) gap pattern on the front face.

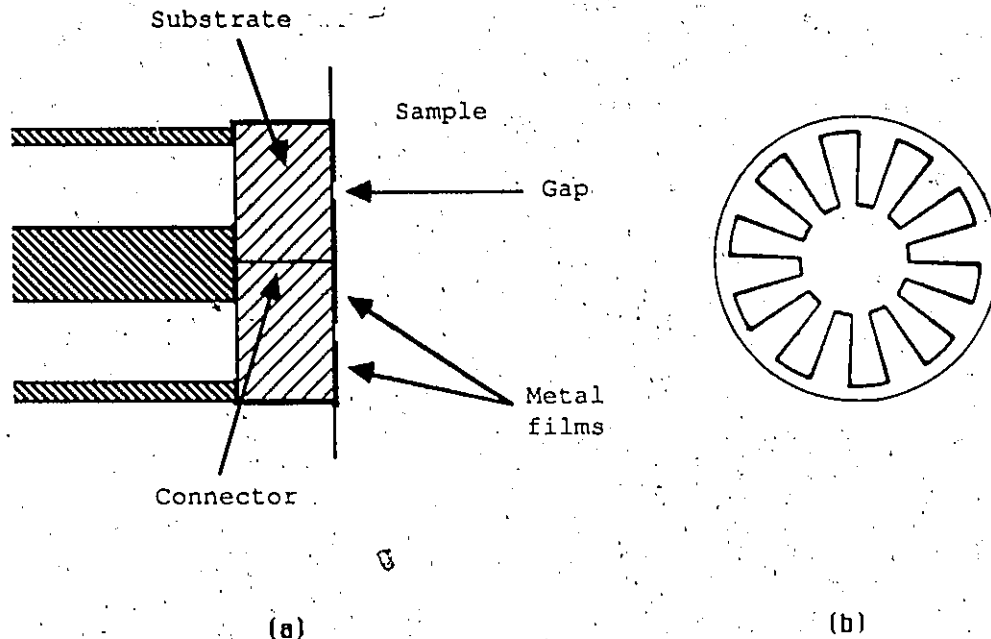


Figure 4: Rising-sun coaxial sensor. (a) sectional view. (b) gap pattern on the front face.

it is very difficult to fabricate a 7-mm sensor and therefore making measurements over small sample areas very difficult. Secondly, the conductive straps used to connect the rings of the sensor to the inner and outer conductors of the line have introduced unaccountable stray capacitances. Apparently, this has caused considerable errors in the theoretical evaluation of the sensor.

After this brief review, a need for a new design of a simple coaxial line sensor having high fringing-field capacitance should be apparent. The "Rising sun" coaxial sensor shown in Figure 4 was analyzed by the author, theoretically. The gap between the conductive films which are fabricated over a dielectric substrate resembles a curved interdigital line. The films are oppositely charged by connecting to the inner and outer coaxial conductors, respectively. Although this configuration satisfied the previously mentioned requirements, it was found that the contribution to the fringing-field made by straight-radial segments of the gap was relatively small. This

led to a conclusion that it is more advantageous to increase the number of arc-like segments of the gap rather than the number of straight-radial segments. Hence the idea of the "Banyan-tree" coaxial sensor, which is thoroughly studied in this work, has emerged.

1.2 The "Banyan-tree" Sensor

The banyan-tree sensor shown in Figure 5 comprises of two metal films which are deposited on a substrate and separated by a narrow gap. The gap which traces a large number of straight lines and arcs is responsible for the high fringing-field capacitance. The outer metal film is connected to the outer conductor. A conductive wire which passes through the substrate connects the inner film to the inner conductor. The banyan-tree pattern formed by the gap on the front surface of the sensor is shown in Figure 5(b).

1.2.1 Sensor Parameters

The important design parameters of the banyan-tree sensor are defined below. The values of some parameters pertaining to the pattern shown in Figure 5 are given in curly braces.

- (1) Number of "trees" (M) : Number of banyan-tree-like units in the pattern {3}.
- (2) Number of "branches" (N) : Number of branches on each side of each tree {2}.
- (3) Gap width (g) : Average width of the gap.
- (4) Inner-gap radius (a) : Smallest radius of the gap.
- (5) Outer gap radius (b) : Largest radius of the gap.
- (6) Outer radius (c) : Radius of the metal pattern.
- (7) Tree width (α) : Spread angle of each tree.
- (8) Substrate dielectric constant (ϵ_{sub})
- (9) Substrate thickness (l)

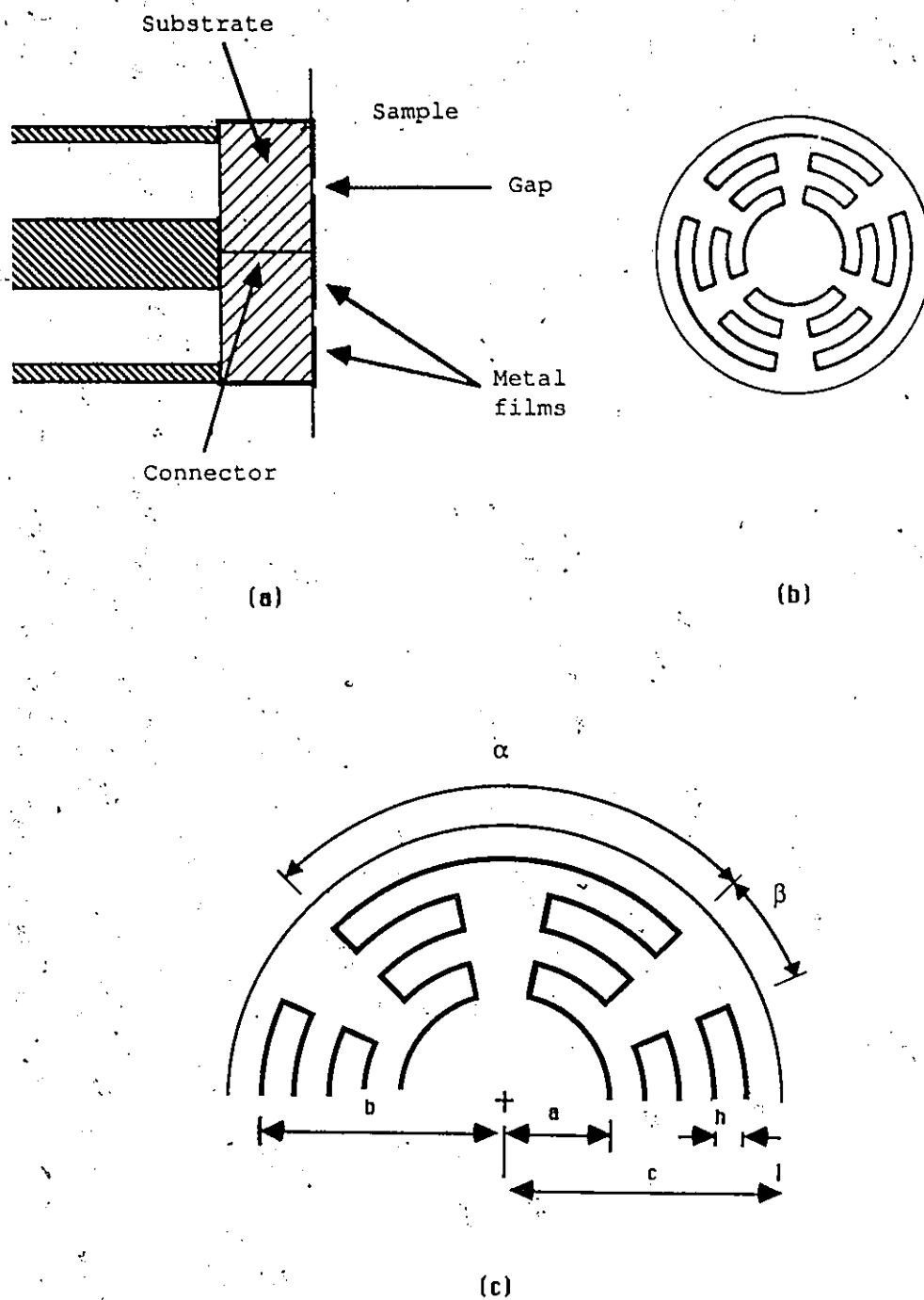


Figure 5: Banyan-tree coaxial sensor. (a) sectional view. (b) gap pattern on the front face. (c) pattern parameters.

(10) Metal film thickness (d)

1.2.2 Equivalent Circuit

If radiation by the narrow gap is neglected, the sensor can be represented by a lumped capacitance terminating a coaxial line. The value of this complex capacitance C is dependent on the complex permittivities of the substrate, the sample and the medium filling the coaxial line (assumed to be air) and frequency.

In general, the relationship between the capacitance C and the sample complex relative permittivity ϵ_s is so complex that it is difficult to determine ϵ_s by simply measuring C . However, it will be shown in the next chapter that with a few reasonable approximations, the capacitance C in this specific problem can be expressed as a linear function of ϵ_s . That is,

$$C = C_f + \epsilon_s C_0 \quad (1)$$

where C_0 and C_f are constants for a given sensor and are independent of the sample permittivity. The capacitance C_0 here may be interpreted as the external capacitance whose value, when multiplied by the sample permittivity, represents the storage of energy and losses in the dielectric sample. The internal capacitance C_f represents the storage of energy in fringing field outside the dielectric sample. Losses other than the dielectric losses in the sample are neglected thus C_0 and C_f both are real quantities. The equivalent circuit of the sensor now reduces to the form shown in Figure 6.

1.3 Method of Measurement

For the measurement of the permittivity ϵ_s of a sample, it is kept in contact with the front face of the sensor. The input reflection coefficient of the sensor is then measured by the Automatic Network Analyser (ANA) and is given by :

$$\rho^* = \rho e^{j\psi} = \frac{Y_0 - Y}{Y_0 + Y} \quad (2)$$

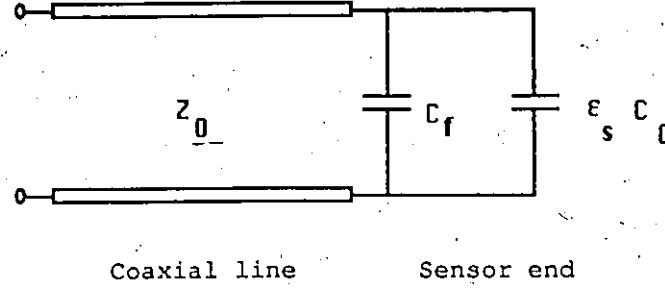


Figure 6: Simplified equivalent circuit of the banyan-tree sensor.

where Y_0 is the characteristic admittance of the coaxial line and Y is the input admittance of the sensor given by :

$$Y = j\omega C = j\omega(C_f + \epsilon_s C_0) \quad (3)$$

Here, ρ represents the magnitude of the reflection coefficient and ψ represents the phase of the reflection coefficient referenced to the plane of the open end of the sensor. When expression (3) is substituted in (2), the sample dielectric constant ϵ' and loss factor ϵ'' can be expressed as :

$$\epsilon' = -\frac{2\rho \sin \psi}{\omega C_0 Z_0 [1 + 2\rho \cos \psi + \rho^2]} - \frac{C_f}{C_0} \quad (4)$$

$$\epsilon'' = \frac{1 - \rho^2}{\omega C_0 Z_0 [1 + 2\rho \cos \psi + \rho^2]} \quad (5)$$

where $Z_0 = \frac{1}{Y_0}$ is the characteristic impedance of the coaxial line.

The use of the linear model for the capacitance results in simple closed-form expressions for the real and imaginary parts of the permittivity in terms of the reflection coefficient of the sensor. The ANA controller can be programmed to read values of ω , Z_0 , C_0 and C_f , measure ρ and ψ using the ANA, calculate ϵ' & ϵ'' by (4) & (5) and print results.

For the determination of the dielectric constant and loss factor of a sample using equations (4) and (5), it is necessary to know the two capacitances associated with

the sensor, namely C_0 and C_f . These capacitances can be obtained experimentally. For that, a sample with known ϵ' and ϵ'' is used and the input reflection coefficient of the sensor is measured. Then (4) and (5) are solved for C_0 and C_f . This can be repeated with a number of samples to improve the accuracy.

1.4 Objectives of This Work

The main objective of this work is to design a sensor with high fringing-field capacitance, for low frequency dielectric measurements.

This was accomplished in two steps. First, the proposed banyan-tree sensor was numerically analyzed to estimate its capacitance. Then, the capacitance of an experimental banyan-tree sensor was measured to verify numerical results.

Another objective is to provide design data for banyan-tree sensors. The results obtained from the numerical analysis of several banyan-tree sensors are presented in tables and graphs. These will be useful for one to find the parameters of banyan-tree sensors to meet given specifications.

1.5 Summary, Chapter 1

The banyan-tree coaxial sensor proposed in this work provides a high fringing field capacitance and hence allows accurate in-vivo measurements at low frequencies. Further, it preserves the simplicity of the coaxial sensors, making the fabrication of small sensors easy.

In dielectric measurements, sensors can be represented by the two capacitances, C_0 and C_f . The usual problem is to design a sensor having specific capacitances. In this work, several banyan-tree sensor configurations are theoretically analyzed to produce some useful design charts. Using them, one can find the values of sensor parameters required to obtain the required capacitance values.

Chapter 2

APPROACH TO THE ANALYSIS

2.1 Assumptions

In this work, following assumptions were made to simplify the numerical analysis of the banyan-tree sensor.

1. **Quasi-Static Conditions** : It is assumed that the quasi-static conditions are valid in the frequency range of interest. Hence, the radiation of energy at the end and the higher-order mode propagation in the line are not accounted for. Fringing field capacitance is determined by solving an electrostatic problem.
2. **Infinitely-thin Metal Film** : The finite thickness of the metal film is neglected. This avoids the difficulty of modelling the shape of the metal film edges. It also reduces the real 3D-charge distribution over metal films to a 2D-charge distribution.
3. **No Effect of Near-by Conductors** : The effect of the near-by conductors (other than metal films) such as electrodes, connecting wires and inner and outer conductors of the coaxial line on the fringing field is neglected.

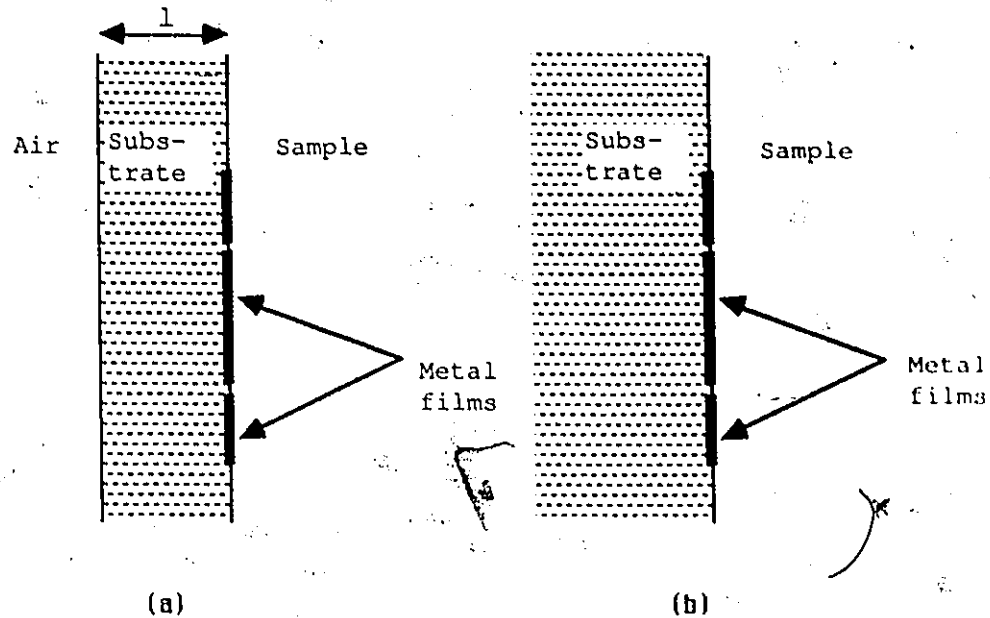


Figure 7: Simplified sensor for analysis. (a) After assumptions (1) to (4). (b) After final simplification.

4. **Infinitely Large Substrate Radius :** Although the radius of the dielectric substrate is actually equal to the radius of the outer conductor of the coaxial line, it is assumed here that the substrate extends radially to infinity.

2.2 Simplification of the Problem

With above mentioned assumptions, the structure to be analyzed is reduced to the one shown in Figure 7(a). It has two infinitely thin metal films deposited on a substrate and separated by the narrow gap forming the banyan tree pattern. The substrate which extends laterally to infinity has a finite thickness l .

For the analysis of the simplified structure shown in Figure 7(a), Green's function can be derived using the method of images adopted by Stuchly *et al.* in the analysis of the multi-ring sensor [5]. The presence of the air/substrate boundary is accounted

for by introducing an infinite series of image charges. Since the strength of images decreases with the distance from the metal film, it is possible to truncate this series at a finite number of terms. The number of terms in the truncated series for Green's function is determined by the required accuracy of the analysis.

When one tries to follow the method described above for the analysis of the structure in Figure 7(a), the following difficulties are usually encountered :

1. The presence of the air/substrate interface in this structure creates, a weak normal component of the electric field at the substrate/sample interface (gap). As described by Gajda and Stuchly [7], this has to be represented by a more complex non-linear capacitance model. Hence the simple two-shunt-capacitance model shown in Figure 6 and described by expression (1) fails and the fringing field capacitance is not a linear function of sample permittivity anymore. Expressions (3) and (4) have to be modified according to the new model.
2. With the introduction of the image-terms into the Green's function, computational effort increases significantly. The introduction of the first image term almost doubles the computer time required for the analysis.

If one neglects the presence of the air/substrate boundary and assumes that the substrate has infinite thickness (semi-infinite solid), these difficulties will disappear. After this assumption, the structure to be analyzed is reduced to the one shown in Figure 7(b).

In the analysis of the rising-sun sensor shown in Figure 4, simplified forms corresponding to those shown in Figure 7(a) and Figure 7(b) were considered. In the analysis of the first one, only the strongest image was taken into account. When practical values such as $l = 3.2$ mm and $\epsilon_{sub} = 2.35$ were assumed for sensor parameters, the difference between the values obtained for the fringing field capacitance was less than 1% for sample permittivities greater than 10. The higher the sample permittivity, the lesser was the difference between the two results.

Similar results could be expected for the banyan-tree sensor. Due to the two

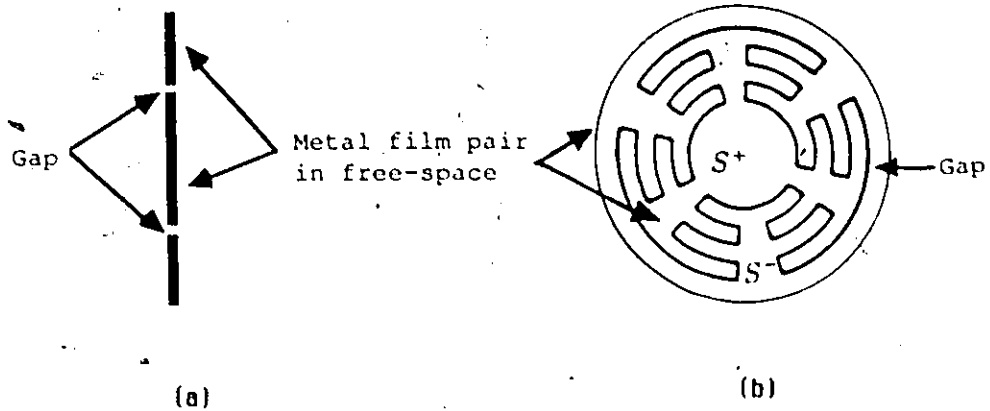


Figure 8: Hypothetical structure to be analyzed. (a) sectional view, (b) front view (S^+ - inner metal film, S^- - outer metal film).

facts that the main application of this sensor is for low frequency dielectric measurements of biological materials and most biological materials exhibit high permittivities at low frequencies, it is reasonable to neglect this difference without further investigation. Hence, the presence of air/substrate boundary is hereafter neglected and the structure shown in Figure 7(b) is analyzed instead of that in Figure 7(a).

2.3 The Simplified Problem

After above simplifications, the structure shown in Figure 7(b) is analyzed. Let us imagine the case where the same infinitely thin metal film pair lies in space without any supporting or nearby materials. This hypothetical structure is shown in Figure 8. Let C_{fs} be the corresponding (free-space) fringing-field capacitance between the metal films. Identifying the fact that the only difference between the structure shown in Figure 7(b) and this is that in the former, one half of the space is filled by the substrate and the other half by the sample, we can express the total fringing-field capacitance C in terms of C_{fs} and relative permittivities ϵ_s and ϵ_{sub} .

That is :

$$C = \frac{C_{fs}}{2} \epsilon_{rub} + \frac{C_{fs}}{2} \epsilon_s \quad (6)$$

Comparing expressions (1) and (6), the two capacitances in the banyan-tree sensor model can be identified as :

$$C_0 = \frac{C_{fs}}{2} \quad (7)$$

$$C_f = \frac{C_{fs}}{2} \epsilon_{rub} \quad (8)$$

Hence, for the determination of capacitances C_0 and C_f , it is enough to analyze the free-space structure shown in Figure (8). Once C_{fs} is obtained, C_0 and C_f can be calculated from (7) and (8), respectively.

2.4 Integral Equation Formulation

For the determination of capacitance C_{fs} of the hypothetical structure shown in Figure (8), let us assume that an electrostatic differential voltage of 1 V is applied across the gap. The resulting surface charge density distribution on metal films is denoted by $\rho(s)$. Then, the total charge on the inner metal film is given by,

$$Q = \int_{S^+} \rho(s) ds \quad (9)$$

where S^+ represents the surface of the positively charged (inner) metal film. Due to the unit voltage difference, the capacitance is simply given by,

$$C_{fs} = Q \quad (10)$$

According to the electrostatic theory, Poisson's equation has to be satisfied everywhere in space. That is,

$$\nabla^2 \phi(v) = -\frac{\rho(v)}{\epsilon} \quad (11)$$

where

$\phi(v)$ = electrostatic potential

$\rho(v)$ = volume charge density

ϵ = absolute permittivity

and ∇^2 — is the 3D-Laplacian operator.

The solution to this partial differential equation has been identified as [8]:

$$\phi(v) = \int_V \frac{\rho(v') dv'}{4\pi\epsilon R} \quad (12)$$

where V is the volume over which the charge is distributed. Primed coordinates represent the source point and unprimed coordinates represent the observation point. R is the distance between the two points in space.

When dealing with surface charge distributions instead of volume charge distributions, equation (12) has to be modified as follows :

$$\phi(v) = \int_S \frac{\rho(s') ds'}{4\pi\epsilon R} \quad (13)$$

The factor $\frac{1}{4\pi\epsilon R}$ which represents the voltage produced by an unit charge at a distance R could be identified as the Green's function of the problem.

Coming back to the problem on hand, the domain of integration is $S = S^+ \cup S^-$, where S^+ is the inner metal film surface and S^- is the outer metal film surface. According to the geometry, the polar coordinate system is preferred. If (r', θ') is the source point and (r, θ) is the observation point,

$$ds' = r' dr' d\theta' \quad (14)$$

$$R = \sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')} \quad (15)$$

$\phi(r, \theta) = 1$ if the observation point lies on the inner metal film (S^+) and $\phi(r, \theta) = 0$ if it lies on the outer metal film (S^-). For this free-space problem, $\epsilon = \epsilon_0$. All this knowledge can be combined to re-write the equation (13) as:

$$\iint_{S^+ \cup S^-} \frac{\rho(r', \theta') r' dr' d\theta'}{\sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')}} = \begin{cases} 4\pi\epsilon_0 & \text{if } (r, \theta) \in S^+ \\ 0 & \text{if } (r, \theta) \in S^- \end{cases} \quad (16)$$

This is the integral equation which is to be solved for $\rho(r, \theta)$. Once the surface charge density is calculated, C_{fs} can be determined from (9) and (10) as :

$$C_{fs} = \iint_{S^+} \rho(r, \theta) r dr d\theta \quad (17)$$

Unfortunately, the nature of equation (16) does not lead to an analytical solution. Therefore, a numerical technique called Method of Moments (MOM) is used to obtain an approximate solution to this equation numerically.

2.5 Summary, Chapter 2

The complex three-dimensional geometry of the practical banyan-tree sensor does not permit an exact and efficient analysis method to be used. Thus some approximations are made to reduce the complexity. The problem is further simplified by assuming infinite substrate thickness, mainly to save computer time required for the analysis. According to past experiences, this should not introduce any significant inaccuracies in the final results.

The hypothetical structure shown in Figure (8) is analyzed to determine the corresponding fringing field capacitance C_f . Then, the required sensor equivalent circuit parameters, namely C_0 and C_f , are calculated using expressions (7) and (8).

The determination of C_f requires an integral equation to be solved. The available analytical tools fail here, and therefore Method of Moments is adopted to obtain an approximate solution.

Chapter 3

THEORY OF THE METHOD OF MOMENTS

3.1 General Method

The method of moments has been widely used in solving both deterministic and eigen-value problems [9,10]. Theory relevant to solving deterministic problems is presented here. Let us assume that,

$$\mathcal{L}f = y \quad (18)$$

be the linear functional equation to be solved. Here,

$\mathcal{L}$: a linear operator
 f : unknown to be determined
and y : input function.

The method involves two basic steps; expansion and testing. In the expansion, the unknown function (f) is expressed in terms of known functions called expansion (or basis) functions and unknown coefficients. The expansion functions are selected so as to satisfy boundary conditions of the problem and to best approximate the exact solution. Let

$$f = \sum_{j=1}^N \alpha_j f_j \quad (19)$$

where α_j : unknown coefficients
and f_j : expansion functions.

Substitution of (19) in (18) and the use of linear properties of the operator lead to :

$$\sum_{j=1}^N \alpha_j (\mathcal{L} f_j) = y \quad (20)$$

For testing, a set of functions called testing (or weighting) functions is used to minimize the error introduced by the approximate expansion process, in a weighted sense. Selecting N testing functions w_i , and multiplying (20) by each, we get

$$\sum_{j=1}^N \alpha_j w_i (\mathcal{L} f_j) = w_i y \quad (21)$$

Let us define the inner product as :

$$\langle u, v \rangle = \int_{\Omega} uv \, d\Omega. \quad (22)$$

Applying this to (21), we get,

$$\sum_{j=1}^N \alpha_j \langle w_i, \mathcal{L} f_j \rangle = \langle w_i, y \rangle \quad (23)$$

which is a set of N linear simultaneous equations. It can also be written in the matrix form as,

$$[A] \vec{\alpha} = \vec{b} \quad (24)$$

where

$$A_{ij} = \langle w_i, \mathcal{L} f_j \rangle \quad (25)$$

$$b_i = \langle w_i, y \rangle \quad (26)$$

and

$$\vec{\alpha} = [\alpha_1, \dots, \alpha_N]^T \quad (27)$$

The matrix equation (24) is solved for the unknown vector $\vec{\alpha}$. If $[A]^{-1}$ exists,

$$\vec{\alpha} = [A]^{-1} \vec{b}. \quad (28)$$

The substitution of coefficients α_j in (19) gives the moment method solution to the problem.

3.2 Point Matching and Subsection Methods

Selection of Dirac's functions as testing functions simplifies the evaluation of inner products (25) and (26). This specialization of the moment method which is most suitable to solve integral equations is called the point matching method. Let,

$$w_i = \delta(\vec{r} - \vec{r}_i) \quad (29)$$

where $\vec{r}_i$ represents the testing points at which exact satisfaction of equation (18) is guaranteed. Then, expressions (25) and (26) reduce to :

$$\begin{aligned} A_{ij} &= \mathcal{L}f_j |_{\vec{r}=\vec{r}_i} \\ b_i &= y |_{\vec{r}=\vec{r}_i} \end{aligned} \quad (30)$$

As mentioned earlier, the expansion functions have to satisfy the boundary conditions of the problem. In many cases, it is difficult to find such functions defined over the entire domain of the solution. Hence, sometimes they are defined only over specific subsections.

The simplest expansion functions defined over subsections are the pulse functions. A pulse function assumes the value of unity over its domain of existence but zero elsewhere. In solving integral equations with point matching, the use of pulse expansion functions greatly simplifies the evaluation of the matrix coefficients.

Chapter 4

NUMERICAL ANALYSIS

4.1 Implementation of MOM

In this section, the moment method with pulse expansion functions and Dirac's delta testing functions is used to find an approximate solution to the integral equation (16). The domain of solution $S = S^+ \cup S^-$ is shown in Figure 9(a).

Expansion : 2-dimensional pulse functions defined over sub-areas are used for expansion. The j^{th} sub-area (S_j) is defined by the four parameters -

r_{1j} = smallest radius

r_{2j} = largest radius

θ_{1j} = smallest angle

θ_{2j} = largest angle

as shown in Figure 9(b). Then the j^{th} expansion function f_j is defined as

$$f_j(r, \theta) = \begin{cases} 1 & \text{if } (r, \theta) \in S_j \\ 0 & \text{otherwise.} \end{cases} \quad (31)$$

The entire domain of solution (S) is divided into large number of sub-areas. Let, S^+ area is divided into L_1 number of sub-areas and S^- area into L_2 number of

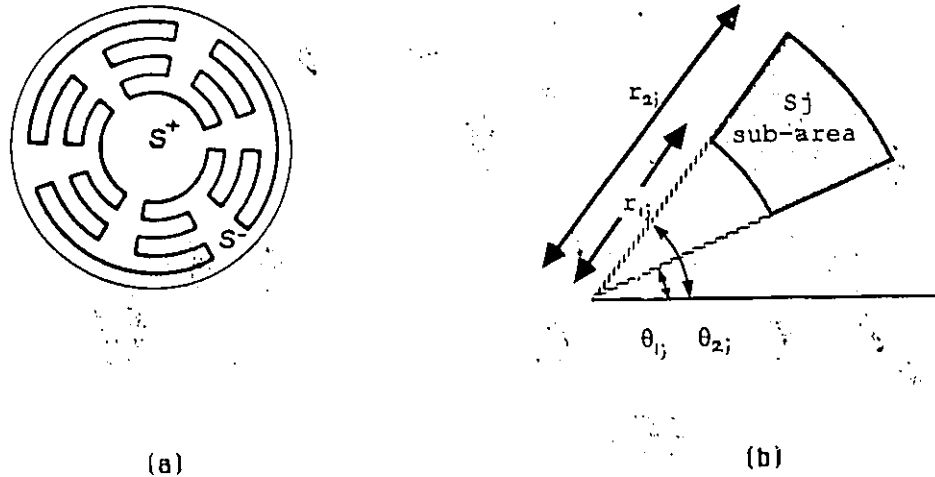


Figure 9: (a) The entire domain of solution. (b) definition of j^{th} sub-area.

sub-areas. Within each sub-area, surface charge density is assumed to be constant. Hence, the solution to equation (16) is approximated as

$$\rho(r, \theta) = \sum_{j=1}^{L_1+L_2} \rho_j f_j(r, \theta) \quad (32)$$

where ρ_j is the surface charge density over j^{th} sub-area. It is evident that the use of pulse functions for expansion has resulted in a two dimensional step approximation to the surface charge distribution.

Testing : 2-dimensional Dirac's delta functions are used for testing. Selecting the centre of the i^{th} sub-area as the i^{th} testing point, we have the i^{th} testing function as -

$$w_i(r, \theta) = \delta(r - r_{mi}, \theta - \theta_{mi}) \quad (33)$$

where

$$\begin{aligned} r_{mi} &= \frac{r_{1i} + r_{2i}}{2} \\ \theta_{mi} &= \frac{\theta_{1i} + \theta_{2i}}{2} \end{aligned} \quad (34)$$

With this testing, the approximate solution (32) and the exact solution both should produce the same voltage at least at all the testing points.

Comparing expressions (18) and (16), following analogies can be identified :

$$\mathcal{L} \rightarrow \iint_S \frac{r' dr' d\theta'}{\sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')}} \quad (35)$$

$$f \rightarrow \rho(r', \theta') \quad (36)$$

$$y \rightarrow \begin{cases} 4\pi\epsilon_0 & \text{if } (r, \theta) \in S^+ \\ 0 & \text{if } (r, \theta) \in S^- \end{cases}$$

$$\text{Also } \alpha_j \rightarrow \rho_j \quad (37)$$

Hence, the expression (30) for matrix coefficients can be re-written as:

$$\begin{aligned} A_{ij} &= \iint_S \frac{f_j r' dr' d\theta'}{\sqrt{r^2 + r'^2 - 2rr' \cos(\theta - \theta')}} \Big|_{r=r_{mi}, \theta=\theta_{mi}} \\ &= \iint_S \frac{f_j r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \end{aligned} \quad (38)$$

With the definition of f_j in (31), above expression reduces to :

$$\begin{aligned} A_{ij} &= \iint_{S_j} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \\ &= \int_{\theta_{1j}}^{\theta_{2j}} \int_{r_{1j}}^{r_{2j}} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \end{aligned} \quad (39)$$

Also

$$b_i = \begin{cases} 4\pi\epsilon_0 & \text{if } (r_{mi}, \theta_{mi}) \in S^+ \\ 0 & \text{if } (r_{mi}, \theta_{mi}) \in S^- \end{cases} \quad (40)$$

The matrix coefficient A_{ij} can be interpreted as the voltage produced at the i^{th} testing point by a unit surface charge density distributed over the j^{th} sub-area.

The matrix $[A]$ and the vector $\vec{b}$ are evaluated using above expressions. Then the unknown charge coefficients ρ_j are found by the expression corresponding to (28), ie.

$$\vec{\rho} = [A]^{-1} \vec{b} \quad (41)$$

where

$$\vec{\rho} = [\rho_1, \dots, \rho_{L_1+L_2}]^T \quad (42)$$

Finally, these coefficients are substituted in (32) in order to obtain the approximate solution to (16).

4.2 Evaluation of Matrix Coefficients

It can be seen that the evaluation of A_{ij} using (39) involves double integration. To best of the author's knowledge, there is no closed-form analytical solution to this. However, fortunately, the first integration with respect to r' can be as shown in Appendix A done analytically.

$$\begin{aligned} F_{ij}(\theta') &= \int_{r_{1j}}^{r_{2j}} \frac{r' dr'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \\ &= \begin{cases} F_{1ij}(\theta') & \text{if } \theta' \neq \theta_{mi} \text{ or } (\theta' = \theta_{mi} \text{ and } r_{mi} < r_{1j}) \\ F_{2ij}(\theta') & \text{if } \theta' = \theta_{mi} \text{ and } r_{mi} > r_{2j} \end{cases} \end{aligned} \quad (43)$$

where

$$\begin{aligned} F_{1ij}(\theta') &= r_{mi} \cos(\theta_{mi} - \theta') \ln |r_{2j} - r_{mi} \cos(\theta_{mi} - \theta') + F_{3ij}(\theta')| \\ &\quad - r_{mi} \cos(\theta_{mi} - \theta') \ln |r_{1j} - r_{mi} \cos(\theta_{mi} - \theta') + F_{4ij}(\theta')| \\ &\quad + F_{3ij}(\theta') - F_{4ij}(\theta') \end{aligned} \quad (44)$$

$$F_{2ij}(\theta') = (r_{1j} - r_{2j}) + r_{mi} \ln \left| \frac{r_{mi} - r_{1j}}{r_{mi} - r_{2j}} \right| \quad (45)$$

$$F_{3ij}(\theta') = \sqrt{r_{mi}^2 + r_{2j}^2 - 2r_{mi}r_{2j} \cos(\theta_{mi} - \theta')} \quad (46)$$

$$F_{4ij}(\theta') = \sqrt{r_{mi}^2 + r_{1j}^2 - 2r_{mi}r_{1j} \cos(\theta_{mi} - \theta')} \quad (47)$$

Here $F_{ij}(\theta')$ represents the voltage produced at a testing point (r_{mi}, θ_{mi}) by a radial line charge extending from r_{1j} to r_{2j} and kept at $\theta = \theta'$. It can be evaluated using expressions (43) to (47). The only exception is when $\theta' = \theta_{mi}$ and $r_{1j} < r_{mi} < r_{2j}$. In this case, the testing point falls on the line charge itself thus making the resultant voltage $F_{ij}(\theta')$ infinite. It can be shown that this only occurs when $i = j$. This is because the testing point is always in the middle of the i^{th} sub-area whereas the

line charge source should be within j^{th} sub-area. Their coincidence hence implies that $i = j$.

Substituting (43) in (39), we have,

$$A_{ij} = \int_{\theta_{1j}}^{\theta_{2j}} F_{ij}(\theta') d\theta' \quad (48)$$

As long as $i \neq j$, the function $F_{ij}(\theta')$ does not produce singularities, due to reasons mentioned before. This allows one to perform the integration in (48) numerically. The function $F_{ij}(\theta')$ is evaluated at sufficiently large number $(K_j + 1)$ of equally spaced sample points falling within the interval of integration $(\theta_{2j} - \theta_{1j})$. Let the k^{th} sample point be :

$$\begin{aligned} \theta_k &= \theta_{1j} + k\Delta\theta_j \\ \text{where } \Delta\theta_j &= \frac{\theta_{2j} - \theta_{1j}}{K_j} \end{aligned} \quad (49)$$

If the trapezoidal rule is used for numerical integration, expression (48) becomes :

$$A_{ij} = \Delta\theta_j \left[\frac{F_{ij}(\theta_{1j})}{2} + \frac{F_{ij}(\theta_{2j})}{2} + \sum_{k=1}^{K_j-1} F_{ij}(\theta_k) \right] \quad (50)$$

All the off-diagonal coefficients of the matrix $[A]$ can be calculated using this formula.

However, the determination of diagonal coefficients of the matrix $[A]$ is not so simple. In this case, $i = j$ hence expressions (34) can be re-written as :

$$\theta_{mi} = \theta_{mj} = \frac{\theta_{1j} + \theta_{2j}}{2}$$

and

$$r_{mi} = r_{mj} = \frac{r_{1j} + r_{2j}}{2} \quad (51)$$

As described before, the function $F_{ij}(\theta')$ produces a singularity at θ_{mj} . This has to be considered in performing the integration (48) numerically. In this work, the problem of numerically integrating this singular function was attacked in two different ways.

In the first approach, usual numerical integration was performed on $F_{ij}(\theta')$, according to the formula (50). The singularity was deliberately eliminated by not selecting θ_{mj} as a sample point. For this, an odd number is chosen for each K_j . Then always $\theta' \neq \theta_{mi}$, therefore according to (43), $F_{ij}(\theta') = F_{1ij}(\theta')$ which could be evaluated using (44). This approach is outlined in Figure 10(a).

The algorithm of this approach looks simple. Nevertheless, one has to select sufficiently large numbers for K_j in order to reduce the error introduced by neglecting the singularity. Still there are limitations. More sample points means more calculations and more computer time. Apart from this, with higher values of K_j , the two sample points closest to the singularity get more closer to it. At certain point, they become so close that the values of the function $F_{ij}(\theta')$ at them become too large to be handled by the computer.

The second approach divides the range of integration into two parts : the neighbourhood of the singularity and the rest. In the singularity neighbourhood, the function $F_{ij}(\theta')$ is approximated by a suitable simple function and it is integrated analytically. In the rest of the range, usual numerical integration is performed according to (50). The introduction of analytical integration makes the algorithm more complex. The advantages of this approach over the first are (1) higher accuracy (2) K_j is only limited by computer time. The approach is outlined in Figure 10(b).

Since $i = j$, the function $F_{ij}(\theta')$ is now symmetrical about the singularity. This property is exploited to express the diagonal coefficients as :

$$A_{jj} = \int_{\theta_{1j}}^{\theta_{2j}} F_{jj}(\theta') d\theta' = 2 \int_{\theta_{mj}}^{\theta_{2j}} F_{jj}(\theta') d\theta' \quad (52)$$

where $F_{jj}(\theta')$ is still given by,

$$F_{jj}(\theta') = F_{1jj}(\theta') \quad (53)$$

Here $F_{1jj}(\theta')$ is obtained by substituting $i = j$ in (44). It can be seen that the second term in the expression (44) is responsible for the singularity of $F_{jj}(\theta')$ at θ_{mj} . This is because when $i = j$, as $\theta' \rightarrow \theta_{mj}$,

$$\begin{aligned} F_{1jj}(\theta') &\rightarrow r_{mj} - r_{1j} \\ \cos(\theta_{mi} - \theta') &\rightarrow 1 \end{aligned}$$

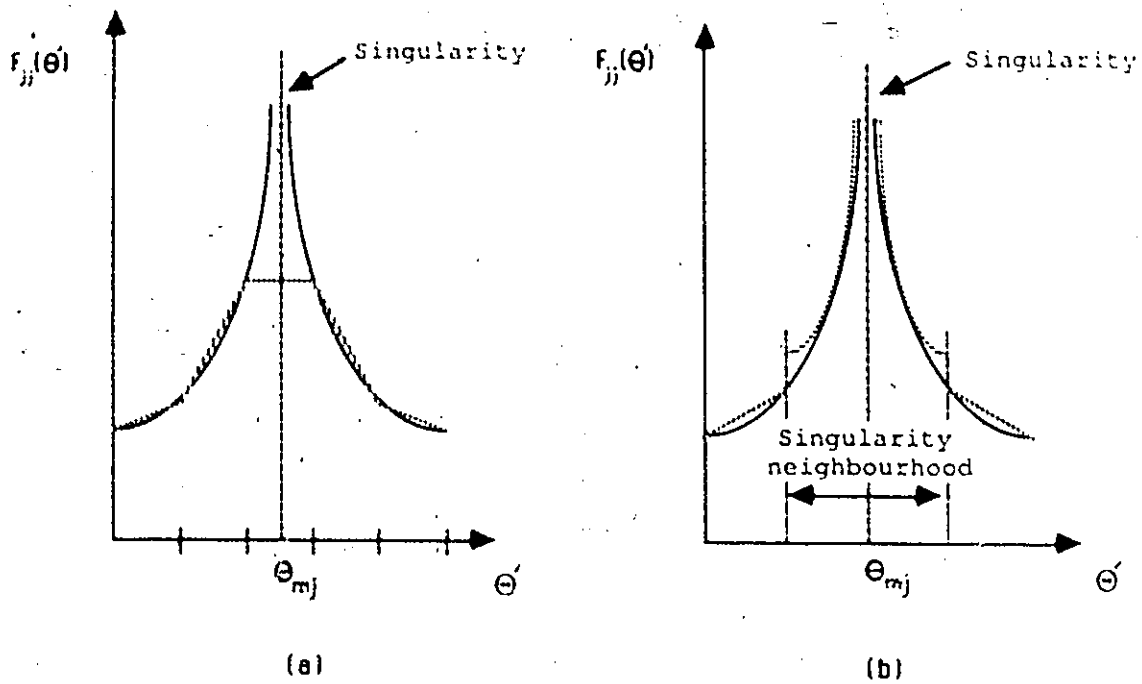


Figure 10: Two approaches to handle the singularity at θ_{mj} . (a) usual numerical integration, neglecting singularity. (b) analytical integration of approximate function at singularity. (— exact function, approximation for numerical integration, ... approximation for analytical integration.)

therefore, the argument of the natural logarithm approaches zero, making its logarithm very large. Hence it is advisable to separate this term from the rest. Let,

$$F_{1jj}(\theta') = F'_{1jj}(\theta') - F''_{1jj}(\theta') \quad (54)$$

where

$$F'_{1jj}(\theta') = r_{mj} \cos(\theta_{mj} - \theta') \ln | r_{2j} - r_{mj} \cos(\theta_{mj} - \theta') + F_{3jj}(\theta') | \\ + F_{3jj}(\theta') - F_{4jj}(\theta') \quad (55)$$

$$F''_{1jj}(\theta') = r_{mj} \cos(\theta_{mj} - \theta') \ln | r_{1j} - r_{mj} \cos(\theta_{mj} - \theta') + F_{4jj}(\theta') | \quad (56)$$

Expressions (52) to (54) can be combined as :

$$A_{jj} = 2 \int_{\theta_{mj}}^{\theta_{2j}} F'_{1jj}(\theta') d\theta' - 2 \int_{\theta_{mj}}^{\theta_{2j}} F''_{1jj}(\theta') d\theta' \quad (57)$$

As the function $F'_{1jj}(\theta')$ does not produce singularities, the first integral can be simply evaluated numerically. Nevertheless, the second integral is to be dealt with differently.

In the Appendix B, $F''_{1jj}(\theta')$ is approximated by a simple function in the neighbourhood of the singularity. A problem arises here in defining this "neighbourhood" quantitatively. In this work, the upper limit of that (θ_{mj}^+) is defined as follows. θ' is increased from θ_{mj} in steps of $\Delta\theta'_j$ given by,

$$\Delta\theta'_j = \frac{\theta_{2j} - \theta_{1j}}{K'_j} \quad (58)$$

where K'_j is a large even integer defined for each sub-area. The smallest value of θ' at which $F''_{1jj}(\theta')$ becomes small enough to be handled by the computer is taken as θ_{mj}^+ . Let,

$$\theta_{mj}^+ = \theta_{mj} + l_j \Delta\theta'_j \quad (59)$$

By symmetry, the lower limit of the neighbourhood is automatically defined as,

$$\theta_{mj}^- = \theta_{mj} - l_j \Delta\theta'_j \quad (60)$$

As shown in the Appendix B, the analytical integration of approximate function over the neighbourhood results :

$$\int_{\theta_{mj}}^{\theta_{mj}^+} F_{1jj}''(\theta') d\theta' = r_{mj}\gamma_j \ln \left| \frac{2r_{mj}r_{1j}}{r_{mj} - r_{1j}} \right| - 2r_{mj} \left[\gamma_j \ln(2) + \sum_n \frac{\sin(n\gamma_j)}{n^2} \right] \quad (61)$$

where

$$\gamma_j = \theta_{mj}^+ - \theta_{mj} = l_j \Delta\theta'_j$$

when $r_{1j} \neq 0$. Otherwise, the argument of the logarithm, ie. $(2r_{mj}r_{1j})/(r_{mj} - r_{1j})$ should be replaced by $(2r_{mj})$. The infinite series can be truncated according to the accuracy required.

Outside of this neighbourhood, $F_{1jj}''(\theta')$ is numerically integrated using the trapezoidal rule. However, in order to follow the steep variation of the function, it is necessary to sample more frequently. Hence a small sample spacing $\Delta\theta'_j$ is used. Then,

$$\int_{\theta_{mj}^+}^{\theta_{2j}} F_{1jj}''(\theta') d\theta' = \Delta\theta'_j \left[\frac{F_{1jj}''(\theta_{mj}^+) + F_{1jj}''(\theta_{2j})}{2} + \sum_{k=l_j+1}^{K'_j/2-1} F_{1jj}''(\theta_{mj} + k\Delta\theta'_j) \right] \quad (62)$$

Combining all these expressions, A_{jj} in (57) can be expressed as¹:

$$\begin{aligned} A_{jj} = & \Delta\theta_j \left[\frac{F_{1jj}'(\theta_{mj}) + F_{1jj}'(\theta_{2j})}{2} + \sum_{k=1}^{K_j-1} F_{1jj}'(\theta_{mj} + \frac{k\Delta\theta_j}{2}) \right] \\ & - 2\Delta\theta'_j \left[\frac{F_{1jj}''(\theta_{mj}^+) + F_{1jj}''(\theta_{2j})}{2} + \sum_{k=l_j+1}^{K'_j/2-1} F_{1jj}''(\theta_{mj} + k\Delta\theta'_j) \right] \\ & - 2r_{mj}\gamma_j \ln \left| \frac{2r_{mj}r_{1j}}{r_{mj} - r_{1j}} \right| \\ & + 4r_{mj} \left[\gamma_j \ln(2) + \sum_n \frac{\sin(n\gamma_j)}{n^2} \right] \end{aligned} \quad (63)$$

¹If $r_{1j} = 0$, the term $\ln \left| (2r_{mj}r_{1j})/(r_{mj} - r_{1j}) \right|$ should be replaced by $\ln(2r_{mj})$.

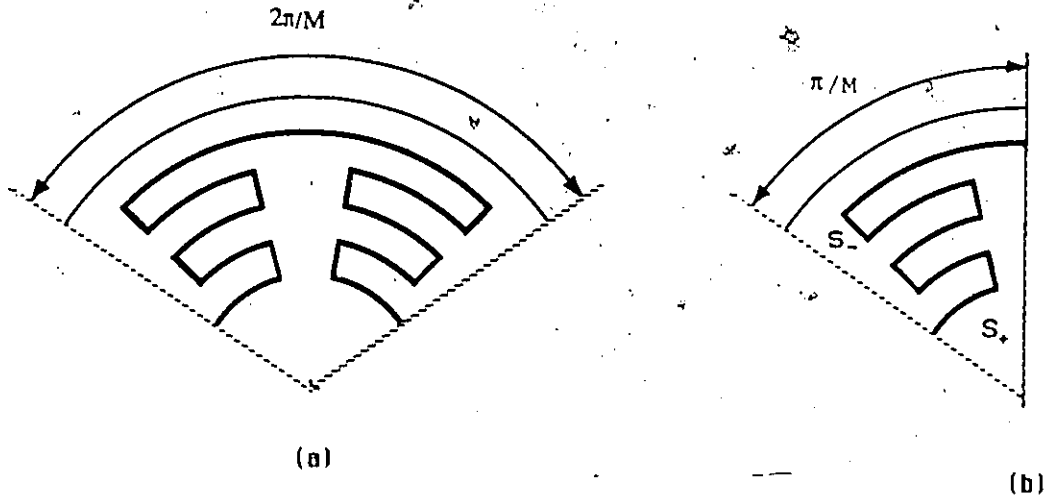


Figure 11: The 'period' and 'unit' of the pattern. (a) One tree forms one period. (b) Half-period is one unit.

4.3 Use of Circular Periodicity

A brief look at the banyan-tree pattern on the sensor face reveals the circular periodicity of the pattern [Figure 5(b)]. One period has the angular spread of $2\pi/M$ radians, where M is the number of "trees" in the pattern [Figure 11(a)]. Since two points on the metal film separated by an integral multiple of this angle should have the same surface charge density, it is sufficient to determine the charge distribution only over one period, in order to calculate the capacitance.

Further investigation shows that the charge distribution over one period should be symmetric; allowing one to concentrate only on one half of the period - the 'unit' shown in Figure 11(b). In fact, only this unit is divided into sub-areas. Once the charge distribution over this is determined, the overall distribution can be found simply by considering the symmetry and periodicity of the problem.

So far we assumed that S^+ and S^- represents the entire positively and negatively charged surfaces, respectively. Since our target is now only one unit, a correction is made. Let S^+ be the positively charged area of the unit shown in Figure 11(b). It is divided into L_1 number of sub-areas. S^- , the negatively charged area of the same

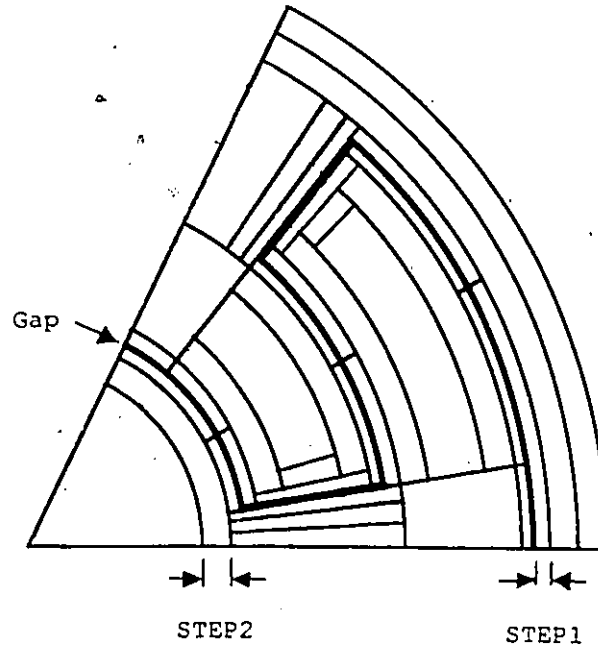


Figure 12: Division of the main unit into sub-areas. The sub-areas closest to the gap have the average width STEP1. The next row has the average width STEP2. (Assumed: $N = 1$, $L_1 = L_2 = 18$)

unit, is divided into L_2 number of sub-areas. This division is shown in Figure 12.

It seems that, in the calculation of charge distribution, one has to consider only the 'main unit' shown in Figure 11(b); and the rest $(2M-1)$ number of 'image units' can be neglected. This is really not true. That is because not only the charges on main unit but also those on image units contribute to the voltage at testing points. Hence a new interpretation to the matrix coefficient A_{ij} is necessary. It is now the voltage produced at the i^{th} testing point, by a unit surface charge density distributed over the j^{th} sub-area (S_j) in the main unit and the other corresponding 'image sub-areas' located in image units. Correspondingly, the expressions for A_{ij} [(50) and (63)] have to be modified in order to take into account the contributions

from image sub-areas.

If we consider a source point (r', θ') on S_j , the corresponding image points on the $(2M - 1)$ image units can be identified as:

$$\begin{aligned} (r', \frac{2\pi l}{M} + \theta') & ; \quad l = 1, \dots, M-1 \\ (r', \frac{2\pi l}{M} - \theta') & ; \quad l = 1, \dots, M \end{aligned} \quad (64)$$

In this case, the expression (39) for A_{ij} is modified as:

$$A_{ij} = \int \int_{S_j \text{ sub-area}} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} + \int \int_{\text{image sub-areas}} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \quad (65)$$

After combining (64) and (65) and rearranging, we have

$$\begin{aligned} A_{ij} = & \int \int_{S_j} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta')}} \\ & + \sum_{l=1}^{M-1} \int \int_{S_j} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} - \theta' - 2\pi l/M)}} \\ & + \sum_{l=1}^M \int \int_{S_j} \frac{r' dr' d\theta'}{\sqrt{r_{mi}^2 + r'^2 - 2r_{mi}r' \cos(\theta_{mi} + \theta' - 2\pi l/M)}} \end{aligned} \quad (66)$$

Previous sections dealt with the integration of the first term in detail. The other two terms can be integrated following the same procedure, namely analytical integration with respect to r' and numerical integration with respect to θ' . Since all the testing points are confined to the main unit and the image sub-areas are away from that, singularities are impossible. Thus numerical integration can be safely performed for integration with respect to θ' . With these contributions, expression (50) for off-diagonal matrix coefficients is modified as:

$$A_{ij} = \Delta \theta_j \left[\frac{F'_{ij}(\theta_{1j}) + F'_{ij}(\theta_{2j})}{2} + \sum_{k=1}^{K_j-1} F'_{ij}(\theta_k) \right] \quad (67)$$

where

$$F'_{ij}(\theta') = F_{ij}(\theta') + \sum_{l=1}^{M-1} F_{1ij}(\frac{2\pi l}{M} + \theta') + \sum_{l=1}^M F_{1ij}(\frac{2\pi l}{M} - \theta') \quad (68)$$

with $F_{1ij}(\theta')$ defined by expression (44). Similarly, the expression for diagonal coefficients now becomes²:

$$\begin{aligned}
 A_{jj} = & \Delta\theta_j \left[\frac{F''_{jj}(\theta_{1j}) + F''_{jj}(\theta_{2j})}{2} + \sum_{k=1}^{K_j-1} F''_{jj}(\theta_k) \right] \\
 & + \Delta\theta_j \left[\frac{F'_{1jj}(\theta_{mj}) + F'_{1jj}(\theta_{2j})}{2} + \sum_{k=1}^{K_j-1} F'_{1jj}(\theta_{mj} + \frac{k\Delta\theta_j}{2}) \right] \\
 & - 2r_{mj}\gamma_j \ln \left| \frac{2r_{mj}r_{1j}}{r_{mj} - r_{1j}} \right| \\
 & + 4r_{mj} \left[\gamma_j \ln(2) + \sum_n \frac{\sin(n\gamma_j)}{n^2} \right] \\
 & - 2\Delta\theta'_j \left[\frac{F''_{1jj}(\theta_{mj}^+) + F''_{1jj}(\theta_{2j})}{2} + \sum_{k=l_j+1}^{K'_j/2-1} F''_{1jj}(\theta_{mj} + k\Delta\theta'_j) \right] \quad (69)
 \end{aligned}$$

where

$$F''_{jj}(\theta') = \sum_{l=1}^{M-1} F_{1jj}(\frac{2\pi l}{M} + \theta') + \sum_{l=1}^M F_{1jj}(\frac{2\pi l}{M} - \theta') \quad (70)$$

4.4 Calculation of Capacitance

We are now in a position to determine the surface charge density distribution over metal films, following the procedure outlined in section 4.1. It is given by the expression (32). Then, the total charge on j^{th} sub-area and its $(2M - 1)$ image sub-areas can be calculated as:

$$\begin{aligned}
 Q_j &= \rho_j \sigma_j \\
 \text{where } \sigma_j &= M(r_{2j}^2 - r_{1j}^2)(\theta_{2j} - \theta_{1j}) \quad (71)
 \end{aligned}$$

The total charge on the positively charged metal film is given by:

$$Q = \sum_{j=1}^{L_1} Q_j \quad (72)$$

According to the expression (10), the capacitance of the hypothetical free-space structure shown in Figure 8 is given by,

$$C_{fs} = Q \quad (73)$$

²If $r_{1j} = 0$, the term $\ln \left| \frac{2r_{mj}r_{1j}}{r_{mj} - r_{1j}} \right|$ should be replaced by $\ln(2r_{mj})$.

4.5 Summary, Chapter 4

Moment method is used with Dirac's testing functions and 2-dimensional pulse expansion functions to translate the integral equation (16) into a system of linear equations. The coefficients of the matrix $[A]$ associated with the linear system are determined by a combination of analytical and numerical integration techniques. The circular periodicity and symmetry of the problem are exploited to reduce the order of the matrix effectively. The approximate charge distribution obtained by solving the linear system is used to find the capacitance of the structure shown in Figure 8.

Chapter 5

NUMERICAL RESULTS

In this chapter, several banyan-tree sensor designs are analyzed following the procedure described before and numerical results are presented. The analysis involves following steps:

1. Division of main unit into sub-areas (Figure 12).
2. Formulation of the linear system of equations.
3. Solving the linear system.
4. Calculation of free-space capacitance (C_{fs}).
5. Calculation of sensor capacitances (C_0, C_f and C).

A FORTRAN program called DIVI is used in the first step. The next three steps are handled by another FORTRAN program named CAPA which calculates the value of C_{fs} . The actual sensor capacitances, namely internal capacitance (C_f), external capacitance (C_0) and total fringing-field capacitance (C), are then evaluated using expressions (7) and (8).

For sensors used with 14mm lines, linear dimensions are chosen according to the following criteria.

- a = line inner conductor radius = 3.102 mm.
- b = line outer conductor inner radius = 7.144 mm.
- c = line outer conductor outer radius = 10.20 mm.

For convenience, all the linear dimensions are normalized such that the largest radius is unity. With the scale factor (for 14mm sensors) of 10.2×10^{-3} , normalized dimensions are: $a_n = 0.3041$, $b_n = 0.7003$ and $c_n = 1.0000$.

Always, the radial separation between any two successive arc-like gap segments is kept constant (h in Figure 5). Tree's spread angle (α) is assumed to be $5\pi/3M$ radians. Since $\alpha + \beta$ is one period (Figure 5), β is automatically set to $\pi/3M$ radians. Calculated free-space capacitance is denormalized by CAPA itself. All the computer programs are listed in the Appendix C.

5.1 Program Descriptions

5.1.1 Program 'DIVI'

This program divides the main unit of the banyan-tree pattern into sub-areas and labels them. Each sub-area takes the general shape shown in Figure 9(b) and is defined by the four parameters r_{1j} , r_{2j} , θ_{1j} and θ_{2j} . The positively charged area (S^+) is divided into

$$L_1 = 13N + 5$$

number of sub-areas. S^- , the negatively charged area is divided into another L_2 sub-areas, where

$$L_2 = L_1$$

A divided and labelled main unit for the case $N = 1$ is shown in Figure 12, as an example. For higher values of N , the same method of division is repeated by DIVI.

The sub-areas closest to the gap have the average width of STEP1 and the next row has the average width of STEP2 (Fig. 12). The values of M, N, g (GAP, in millimeters), STEP1 and STEP2 (normalized values) are the inputs to the program. Normalized dimensions a_n (ANORM), b_n (BNORM) and c_n are preset according to the sensor dimensions.

The program calculates the coordinate parameters of all sub-areas. It also determines the value of K_j for each sub-area, which is used in expressions (67) and (69). Recall that $K_j + 1$ is the number of sample points used for the numerical

integration of expression (48). Sample frequency of 2/degree is used. However, it is ensured that at least five samples are selected for each sub-area.

Finally, this program stores the coordinate parameters $(r_{1j}, r_{2j}, \theta_{1j}, \theta_{2j})$ and K_j of all the sub-areas in the data file called 'COORD', in a tabular format. Also written into the same file the inputs to DIVI, namely M, N, GAP, STEP1 and STEP2.

5.1.2 Program 'CAPA'

This program calculates the approximate charge distribution and capacitance of the free-space structure shown in Figure 8. first, it reads $M, N, g, \text{STEP1}, \text{STEP2}$ and sub-area data from COORD file. Then the linear system is formed as follows:

- (a) Testing points are selected according to expression (34).
- (b) Input vector to the linear system, $\vec{b}$ is formed according to expression (40).
- (c) Off-diagonal terms of the system matrix $[A]$ are evaluated by expression (67). The subroutine EBOTH calculates $F'_{ij}(\theta')$ according to (68), when the input parameter K is set to 1.
- (d) Diagonal terms are evaluated from expression (69). The first term of this expression, which represents the contribution from image sub-areas is calculated first. $F''_{jj}(\theta')$ defined in expression (70) is calculated by subroutine EBOTH, setting K to 0.

Then, the contribution from second term of (69) is calculated and added. The function subprogram EXP01 calculates the value of $F'_{1jj}(\theta')$ from expression (55).

For the evaluation of the last three terms of expression (69), K'_j is defined as:

$$K'_j = 100K_j \quad (74)$$

Next, the 'singularity neighbourhood' half-width γ_j is found by calling the subroutine HWIDTH. This subroutine uses the criterion described in section 4.2 to determine θ_{mj}^+ and l_j .

Then the third and fourth terms of expression (69) are calculated and added. The infinite series in fourth term is truncated with an error of less than 0.1%.

Finally, the contribution from last term is added. To evaluate that, the function subprogram EXP02 calculates $F''_{1j}(\theta')$ from expression (56).

The linear system so formed has the charge density vector $\vec{\rho}$ as the unknown. However, it is desirable to select the charge vector $\vec{Q}$ defined by,

$$\vec{Q} = [Q_1, \dots, Q_{L_1+L_2}]^T$$

as the unknown. This ensures a well-behaving system matrix. According to expression (71),

$$Q_j = \rho_j \sigma_j$$

hence A_{ij} is divided by σ_j to form the new system matrix. The linear system is solved using library subroutines. Finally, free-space capacitance C_{fs} is calculated using expressions (72) and (73). All calculations are done using double precision arithmetic.

5.2 Results

5.2.1 14-mm Sensors

Results obtained by the numerical analysis of few 14-mm banyan-tree sensor designs are given below. Following sensor parameters are common for all designs.

Inner gap radius	(a)	=	3.102 mm
Outer gap radius	(b)	=	7.144 mm
Outer radius	(c)	=	10.20 mm
Tree width	(α)	=	$5\pi/3M$ rad

Table 1 and Table 2 give the calculated values of free-space capacitance (C_{fs}) for several sensor designs. STEP1 and STEP2 for these calculations were selected as $0.003/N$ and $0.027/N$, respectively. Plots of C_{fs} versus average gap width for three-tree sensors are shown in Figure 13.

Table 1: Calculated C_{fs} for 14-mm three-tree sensors ($M = 3, \alpha = 100^\circ$).

N	Gap width g (mm)	C_{fs} (pF)
1	.10	2.086
1	.15	1.881
1	.20	1.725
1	.35	1.408
2	.10	2.597
2	.15	2.280
2	.20	2.049
2	.35	1.576
3	.10	3.037
3	.15	2.612
3	.20	2.286
3	.35	1.652

The calculation of the equivalent circuit parameters and the total fringing-field capacitance is straightforward. From expressions (6) to (8),

$$\begin{aligned} C_f &= \epsilon_{sub} \frac{C_{fs}}{2} \\ C_0 &= \frac{C_{fs}}{2} \\ \text{and } C &= C_f + \epsilon_s C_0 \end{aligned} \quad (75)$$

No doubt, the most useful design charts are the plots of total capacitance. It is plotted against sample permittivity for different sensor designs in Figures 14 to 16. The graphs given are for lossless samples thus both ϵ_s and C are real quantities. In the case of lossy samples, ϵ' replaces ϵ_s and $\text{Real}(C)$ replaces C . Substrate permittivity of 2.35 is assumed, which corresponds to RT-Duroid. For other substrates, similar plots can be easily generated using expressions (75).

5.2.2 7-mm Sensors

Linear dimensions for 7-mm sensors are chosen as follows:

Inner gap radius	(a)	=	1.52 mm
Outer gap radius	(b)	=	3.50 mm
Outer radius	(c)	=	5.00 mm

This selection is made according to the following criteria. Inner conductor radius of the 7-mm coaxial line is chosen as a and the inner radius of its outer conductor is chosen as b . To achieve same normalized dimensions a_n and b_n as 14-mm sensors, a scale factor of 5.0×10^{-3} is necessary. Then, denormalization of outer radius c_n ($=1.0$) gives the value of c as 5.0 mm.

It is clear that this 7-mm configuration is a scaled down version of the 14-mm configuration. Therefore, corresponding capacitances can also be obtained by properly scaling down the corresponding values of 14-mm sensors. The correct scaling down factor is the ratio between the two scale factors, ie. $(10.2 \times 10^{-3}) / (5.0 \times 10^{-3}) = 2.04$. The results of scaling are given in Table 3 and Table 4. It should be

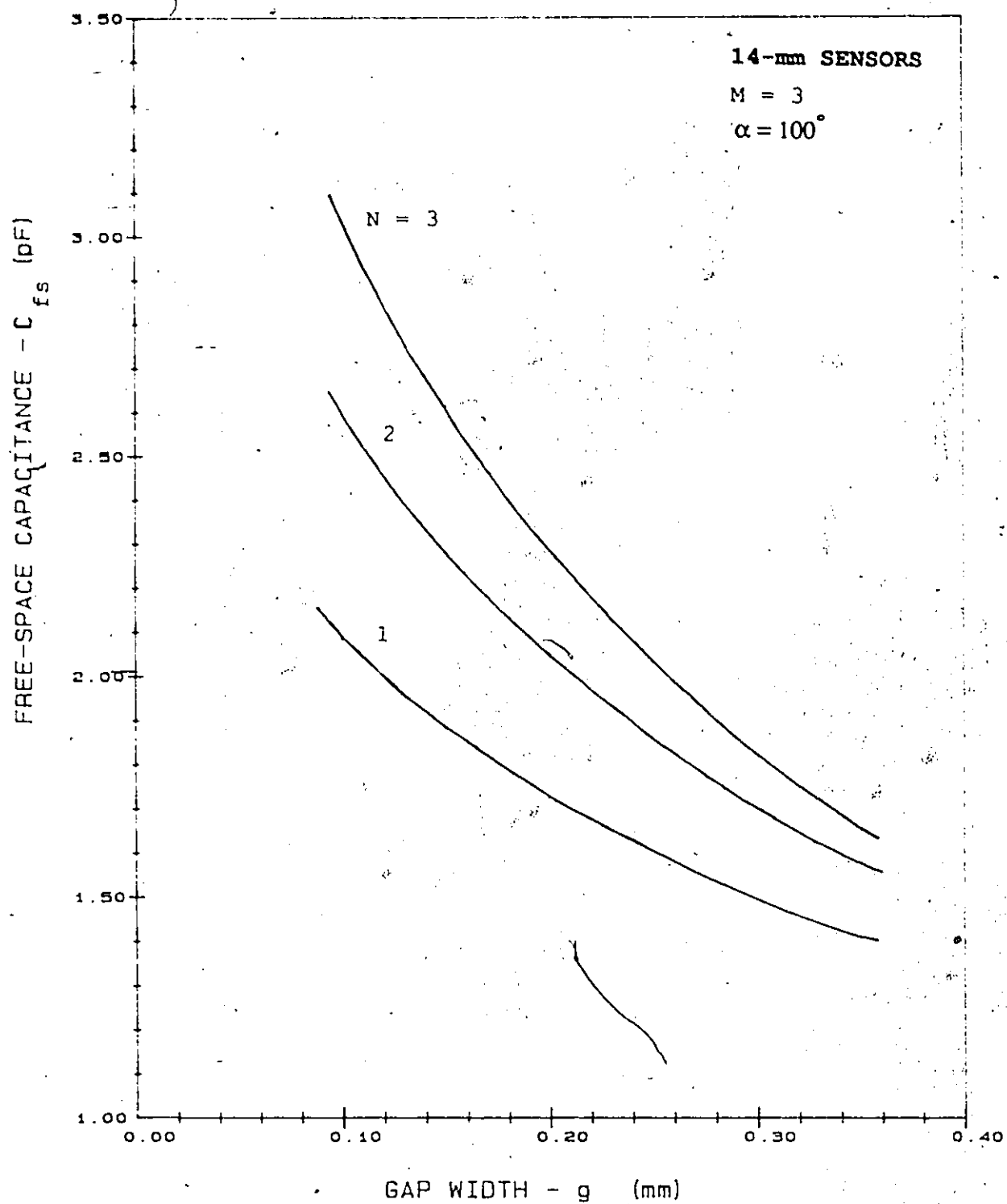


Figure 13. The graph of free-space capacitance versus gap width for 14-mm three-tree sensors.

Table 2: Calculated C_f , for 14-mm four-tree sensors ($M \equiv 4, \alpha = 75^\circ$).

N	Gap width g (mm)	C_f , (pF)
1	.10	2.183
1	.15	1.959
1	.20	1.787
1	.35	1.443
2	.10	2.675
2	.15	2.335
2	.20	2.087
2	.35	1.581
3	.10	3.084
3	.15	2.652
3	.20	2.309
3	.35	1.632

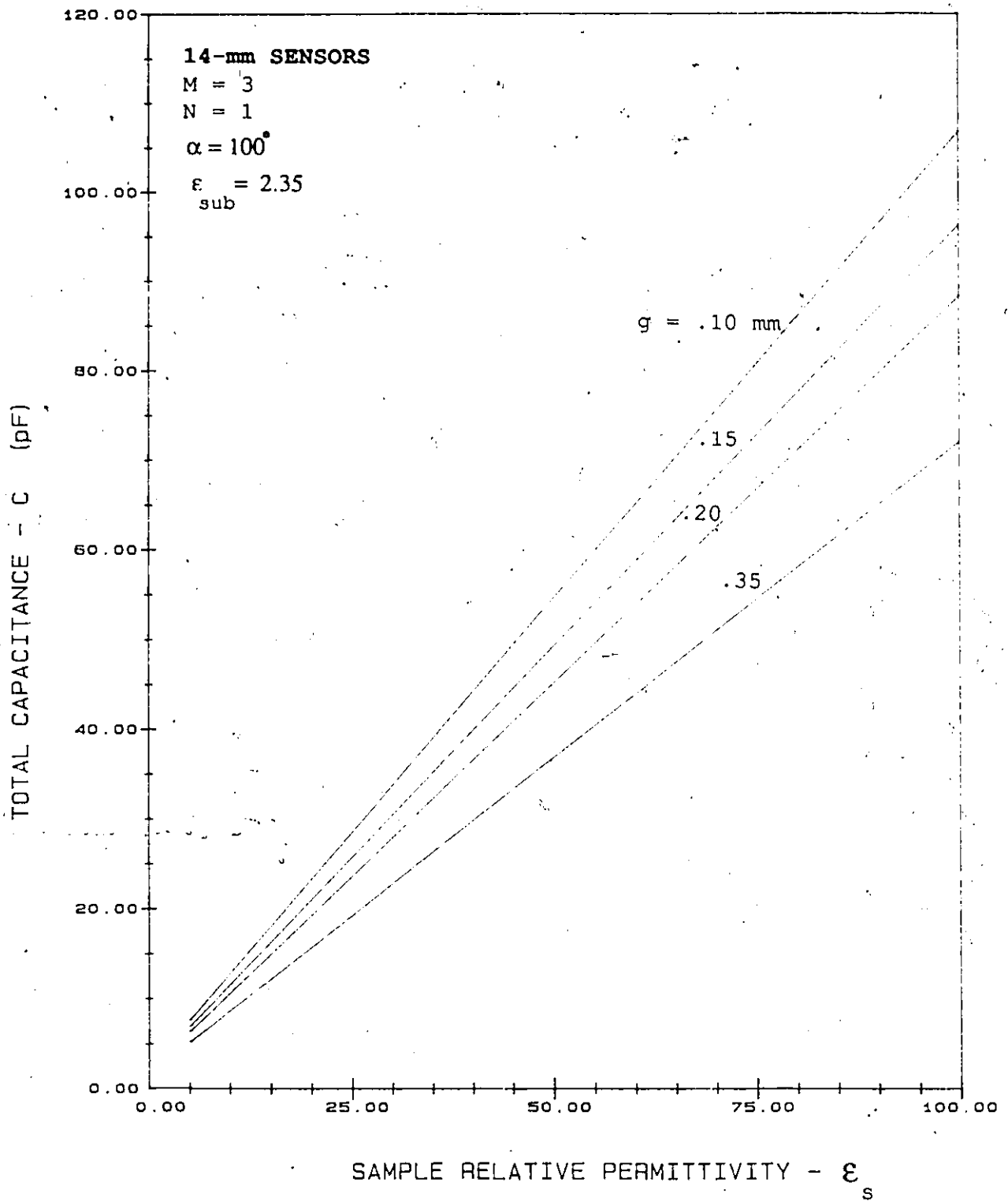


Figure 14. The graph of total capacitance versus sample relative permittivity

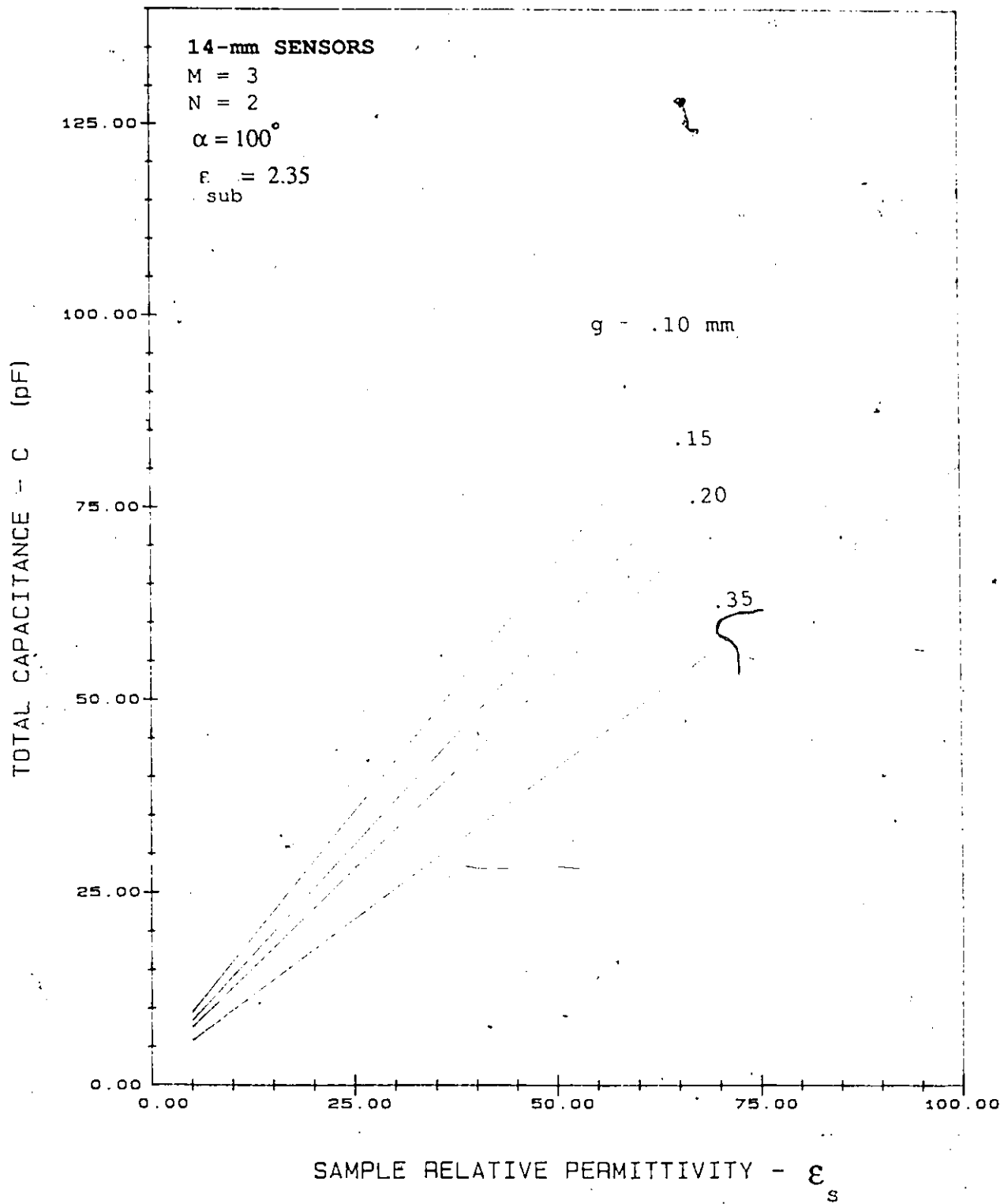


Figure 15. The graph of total capacitance versus sample relative permittivity

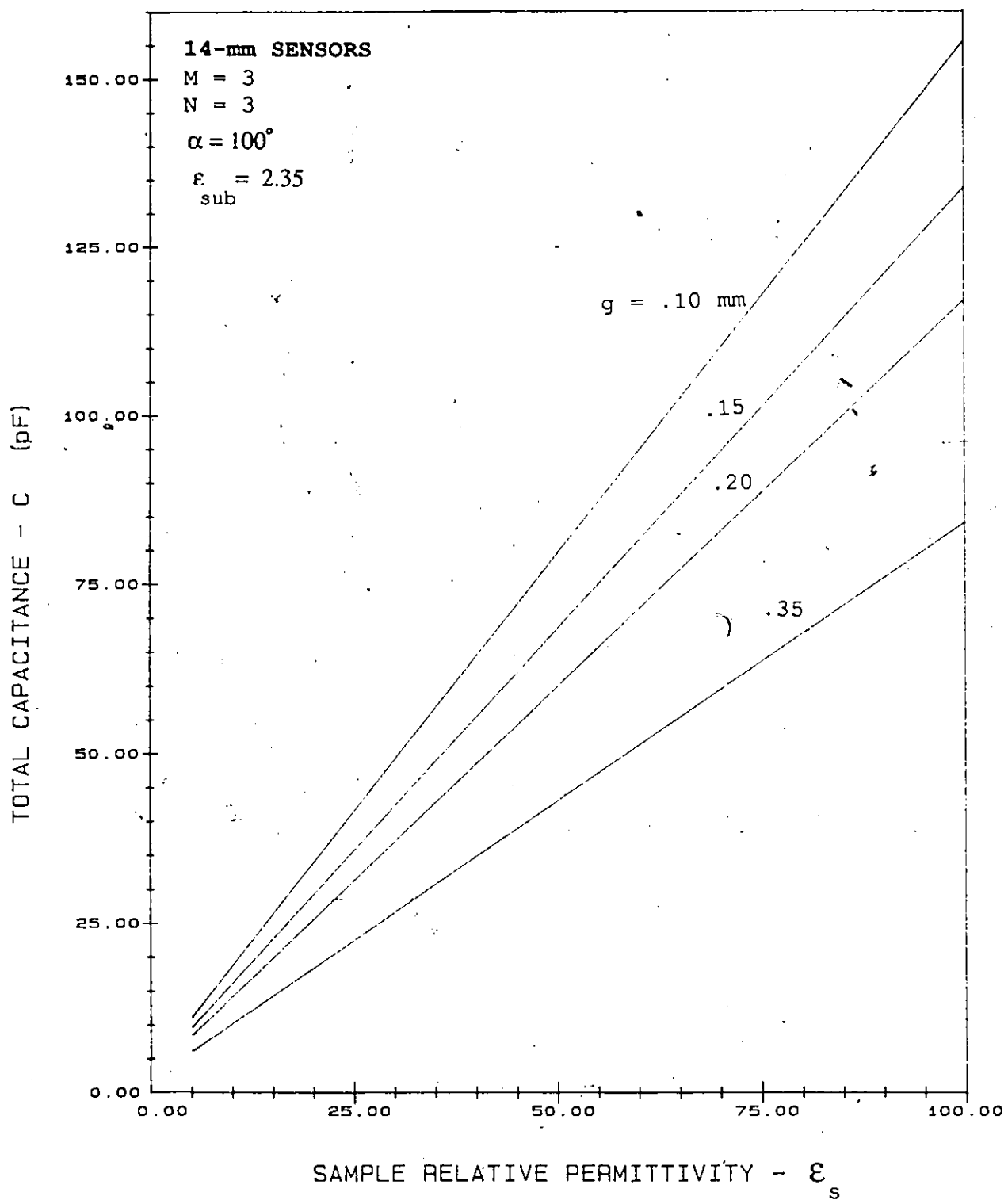


Figure 16. The graph of total capacitance versus sample relative permittivity

Table 3: Calculated C_{fs} for 7-mm three-tree sensors ($M = 3, \alpha = 100^\circ$).

N	Gap width g (mm)	C_{fs} (pF)
1	.05	1.023
1	.07	0.992
1	.10	0.846
1	.17	0.690
2	.05	1.273
2	.07	1.118
2	.10	1.004
2	.17	0.773
3	.05	1.489
3	.07	1.280
3	.10	1.121
3	.17	0.810

Table 4: Calculated C_{fs} for 7-mm four-tree sensors ($M = 4, \alpha = 75^\circ$).

N	Gap width g (mm)	C_{fs} (pF)
1	.05	1.070
1	.07	0.960
1	.10	0.876
1	.17	0.707
2	.05	1.311
2	.07	1.145
2	.10	1.023
2	.17	0.775
3	.05	1.512
3	.07	1.300
3	.10	1.132
3	.17	0.800

noted that the average gap width is also scaled down by the same factor. Plots of total capacitance versus sample permittivity are shown in Figures 17 to 19. Again, substrate permittivity of 2.35 is assumed.

5.3 Discussion

In this analysis, division into sub-areas could be done in numerous ways. Many of the possibilities were tried and finally the criterion used in the program DIVI was selected. It generates only $26N + 10$ sub-areas; a reasonably small number as far as practical sensors are concerned (N up to 3). It should not be forgotten that the required computer time is approximately proportional to the square of the number of sub-areas. It was seen that the result of the analysis (C_{fs}) is rather insensitive

to the changes in values of STEP1 and STEP2.

As expected, the computed results in Tables 1 to 4 show that the increase of number of trees does not contribute much to the capacitance. In fact, increasing M does not increase the total length of arc-like segments of the gap. Instead, it adds few more radial segments into the pattern. As observed in the analysis of rising-sun sensors, this cannot enhance the capacitance appreciably. Hence, the graphs were produced only for three-tree sensors.

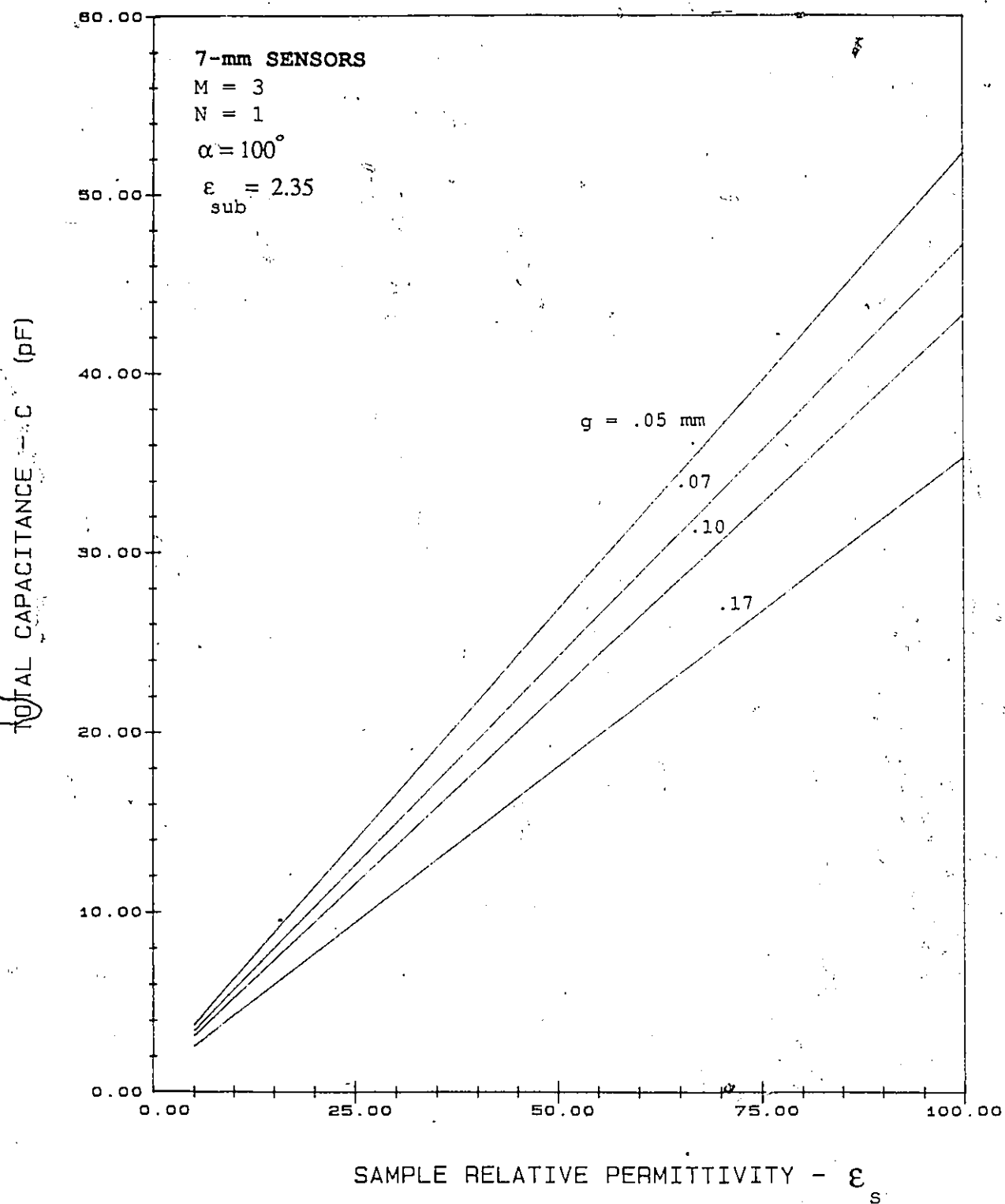


Figure 17. The graph of total capacitance versus sample relative permittivity

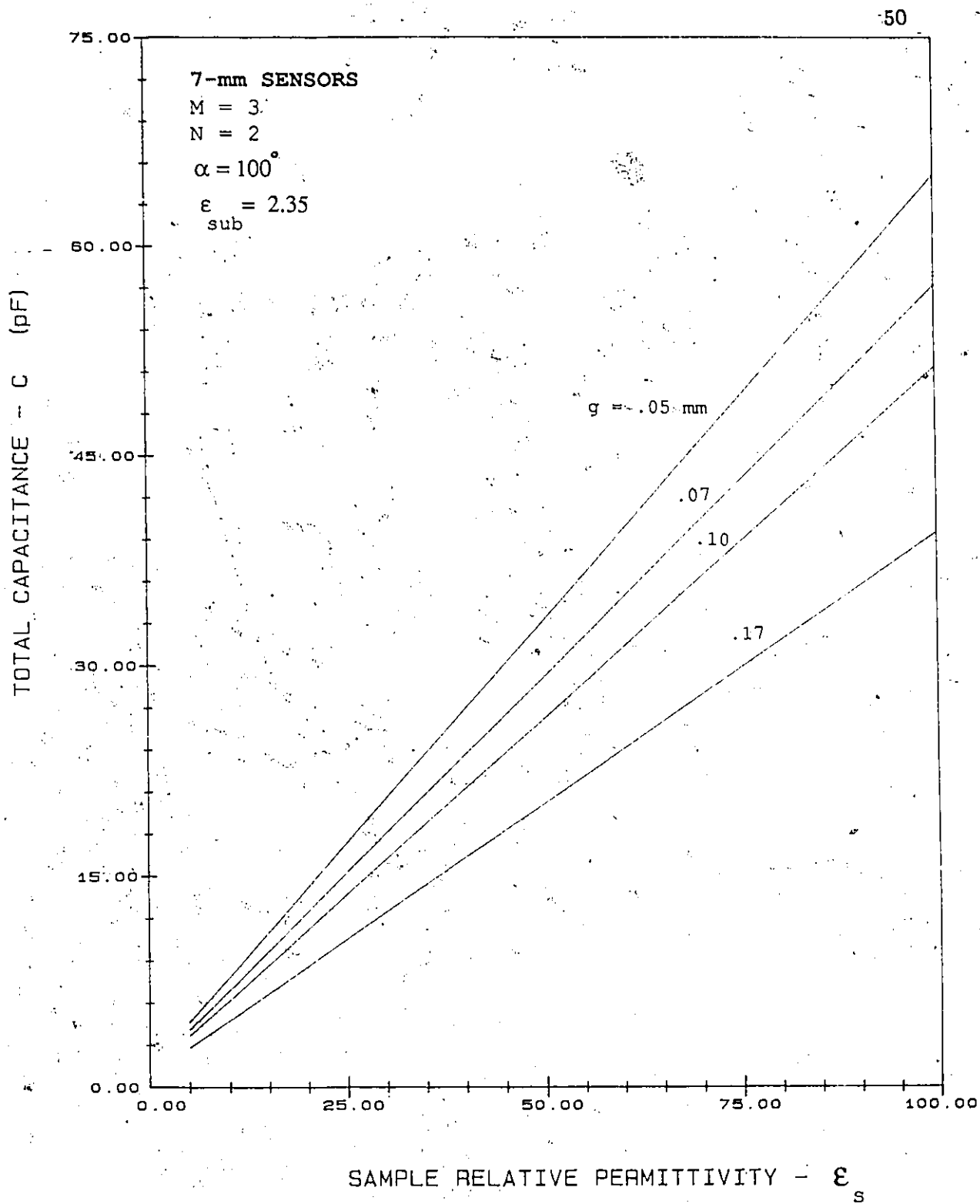


Figure 18. The graph of total capacitance versus sample relative permittivity

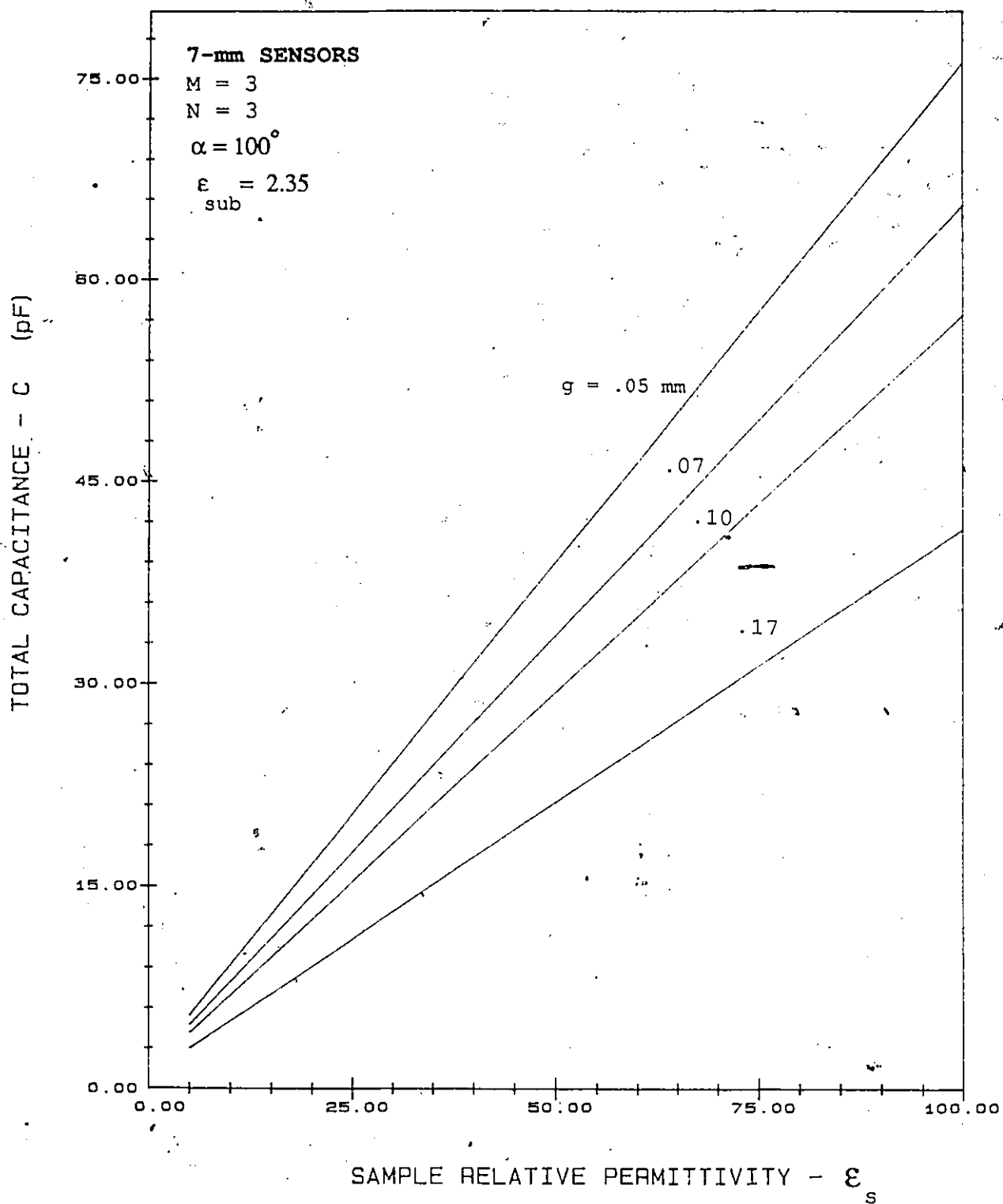


Figure 19. The graph of total capacitance versus sample relative permittivity

Chapter 6

EXPERIMENTAL RESULTS

6.1 Experimental Sensor

A banyan-tree sensor was fabricated to verify the results of the numerical analysis. Figure 20 shows a sectional view of the experimental sensor. Photograph of the front and rear faces is given in Figure 21. Very thin metal films were realized over an RT-Duroid substrate disk, by selective etching. Copper electrodes on the back side were also made by etching. Outer metal film was electrically connected to the outer electrode by a layer of conductive paint. A thin wire passing through the substrate connects the inner film to inner electrode. Sensor dimensions were chosen for use with the GR-900 connector.

The parameters of the experimental sensor (as defined in section 1.2.1) are: $M=3$, $N=2$, gap width (g) = 0.1 mm, inner gap radius (a) = 3.0 mm, outer gap radius (b) = 7.1 mm, outer radius (c) = 13.0 mm, tree width (α) = 100° , substrate relative permittivity (ϵ_{sub}) = 2.35, substrate thickness (l) = 3.2 mm and metal film thickness (estimated) (d) = $5\mu\text{m}$.

The sensor was installed in a GR-900 connector fitted to a 14-mm coaxial line. It was attached by a nut screwed to the connector. A step in the sensor front face (Figure 20) was used to accomodate the nut. (This metal nut increased the value of c up to 13.0 mm.)

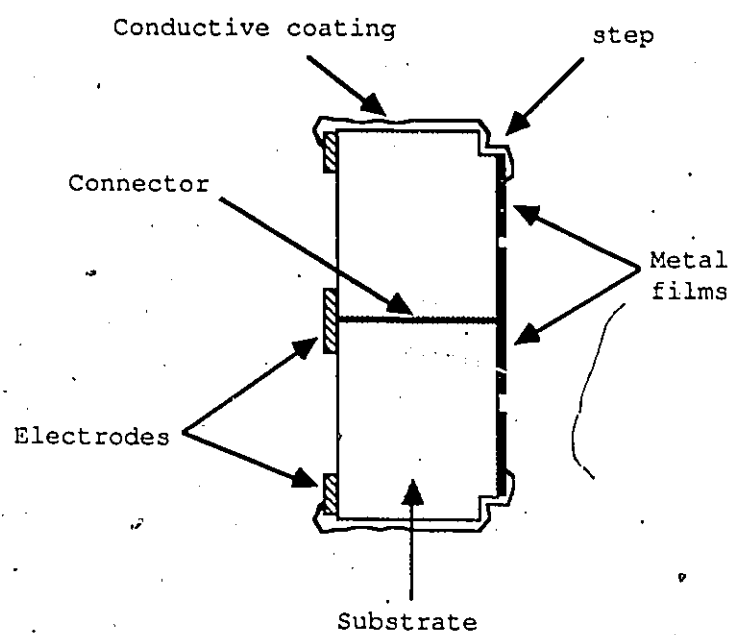


Figure 20: The experimental banyan-tree sensor, sectional view.



Figure 21. Photograph of the front and rear sides of experimental sensors.
(left: banyan-tree. right: rising-sun.)

6.2 Measurement of Sensor Capacitance

The external and internal capacitances (C_0 and C_f) of the experimental sensor were measured in the frequency range 1 MHz - 100 MHz; as described below.

6.2.1 Theory

The sensor is kept in contact with several dielectrics whose electrical properties are known and the input reflection coefficients are measured at a particular frequency. If the reflection coefficient measured with the k^{th} sample having the dielectric constant ϵ'_k (at the measurement frequency) is $\rho_k e^{j\psi_k}$, the total (real) capacitance of the sensor can be expressed as:

$$C_k = \epsilon'_k C_0 + C_f = -\frac{2\rho_k \sin \psi_k}{\omega Z_0 [1 + 2\rho_k \cos \psi_k + \rho_k^2]} \quad (76)$$

by re-arranging expression (4). Using this, C_k corresponding to each sample is calculated at that frequency. For N samples, the following system of N equations can be written with two unknowns, C_0 and C_f .

$$\epsilon'_k C_0 + C_f = C_k; \quad k = 1, \dots, N \quad (77)$$

for $N > 2$, the system has the redundancy of $(N - 2)$. Hence, C_0 and C_f can be determined according to the least square method as:

$$\begin{aligned} C_0 &= \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}^2} \\ C_f &= \frac{a_{11}b_2 - a_{12}b_1}{a_{11}a_{22} - a_{12}^2} \end{aligned} \quad (78)$$

where

$$\begin{aligned} a_{11} &= \sum_{k=1}^N \epsilon_k'^2, \quad a_{12} = \sum_{k=1}^N \epsilon'_k, \quad a_{22} = N \\ b_1 &= \sum_{k=1}^N \epsilon'_k C_k \quad \text{and} \quad b_2 = \sum_{k=1}^N C_k \end{aligned} \quad (79)$$

6.2.2 Procedure

Measurements were carried out with the following five pure liquids whose dielectric properties are available in the literature: 1-propanol, ethanol, methanol, water and formamide. For each liquid, the dielectric constant at the measurement frequency was calculated from the Cole-Cole dispersion equation [14]:

$$\epsilon' - j\epsilon'' = \frac{\epsilon_s - \epsilon_0}{1 + (2\pi f\tau)^{1-\alpha}} + \epsilon_0 \quad (80)$$

where

- ϵ_s - static value of dielectric constant
- ϵ_0 - optical value of dielectric constant
- τ - characteristic relaxation time
- α - distribution parameter
- and f - measurement frequency.

For methanol and formamide, ϵ_s , ϵ_0 , τ and α corresponding to the measurement temperature were obtained by interpolating data published by Jordan et.al. [15]. The formulae given in References [16] and [17] were used to calculate ϵ_s and τ of water, respectively. Temperature independent values of 4.6 and 0.014 were selected for ϵ_0 and α of water, according to Ref. [18]. Parameters of ethanol were found in Reference [19]. Values of 20.5, 3.3 and 385 ps were chosen for ϵ_s , ϵ_0 and τ of propanol at 25°C [6]. In the temperature interval 20-25°C, ϵ_0 and τ of propanol were assumed to be constant, but ϵ_s was calculated by interpolating the data given in Reference [20]. α of propanol was always assumed to be zero.

The input reflection coefficient of the sensor was measured by an HP 3577A automatic network analyzer. It was seen that propanol, the liquid with low dielectric constant, gave very small phase angles of reflection coefficient ($< 1^\circ$) at frequencies below 1 MHz. The resulting high percentage uncertainty of the phase angle could cause large errors in the final results. Hence measurements were made above 1 MHz. Thirty measurement frequencies, uniformly distributed in the log scale over the 1 MHz-100 MHz frequency interval were selected. At each frequency, ϵ'_k for each sample was calculated using the Cole-Cole equation (80). The measured values of

Table 5: Comparison of the calculated and measured values of C_0 and C_f of the 14-mm experimental banyan-tree sensor.

Parameter	Measured value (pF)	Calculated value (pF)	Difference
C_0	1.49	1.30	12.8%
C_f	3.15	3.05	3.2%

ρ_k and ψ_k were used to calculate C_k for each sample, according to (76). Then, C_0 and C_f at that frequency were calculated using (78) and (79). In Figure 22, the sample dielectric constants, corresponding C_k values and the least square fit are graphically presented at 20.5 MHz, as an example.

6.3 Results and Comparison

The external capacitance (C_0) of the experimental banyan-tree sensor, determined as described just before, is plotted against frequency in Figure 23. This graph clearly shows that C_0 is almost constant up to 40 MHz and increases with frequency thereafter. The average value of C_0 over 1-40 MHz interval is found as 1.49 pF. The standard deviation of C_0 over this interval is 0.015 pF. The internal capacitance C_f has the average value of 3.15 pF. The large variation of C_f about this value shows a standard deviation of 0.473 pF.

From Table 1, the theoretically estimated values of C_0 and C_f of a banyan-tree sensor with parameters, $M=3$, $N=2$, $g=1$, $a=3.102$ mm, $b=7.144$ mm, $c=10.20$ mm, $\alpha=100^\circ$ and $\epsilon_{ub}=2.35$ are found as 1.30 pF and 3.05 pF respectively. These values are compared with the measured values of the experimental sensor, in Table 5.

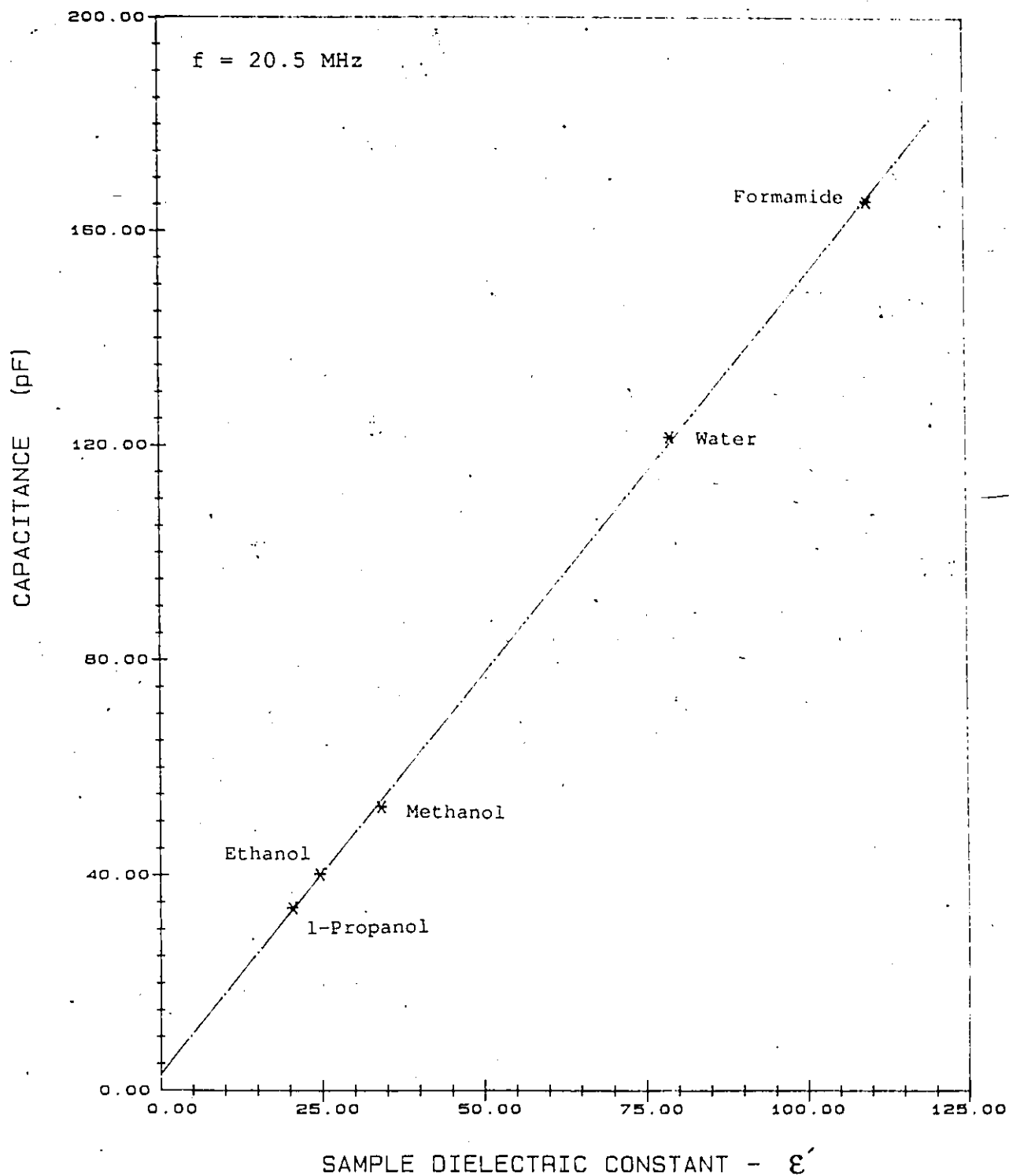


Figure 22. Measured capacitances with five reference liquids and the least-square-fit line at 20.5 MHz.

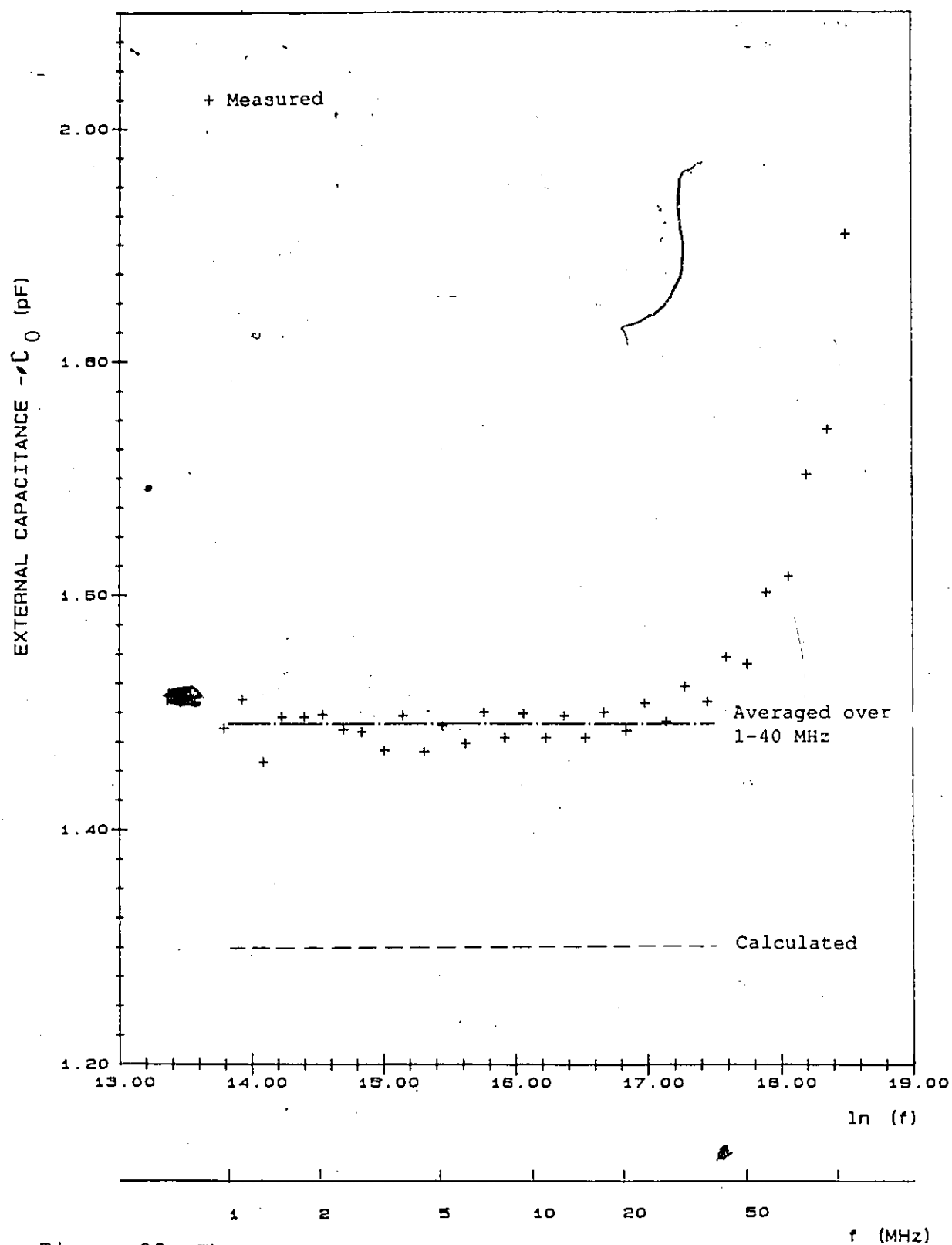


Figure 23. The measured, averaged and calculated values of external capacitance versus frequency.

6.4 Discussion

In our analysis, self inductance of the sensor has been neglected assuming that the inductive reactance is negligible compared with the capacitive reactance. Although this is true at low frequencies, with increasing frequency, the former increases and the latter decreases, making inductive effects more significant. The addition of a positive inductive reactance to the negative capacitive reactance appears as an increase in the sensor capacitance. This must be the reason for the apparent increase of C_0 above 40 MHz. These inductive effects limits the use of the 14-mm banyan-tree sensor to frequencies below approximately 40 MHz.

The external capacitance of the actual sensor can be different from that of the ideal sensor due to two major reasons; finite metal film thickness and variations in gap width. Due to non-zero thickness of the metal films, a miniature parallel plate capacitor is formed at the gap. This effect was not taken into account in our analysis. An approximate value for this parallel plate capacitance can be obtained by the formula $C = \epsilon A/d$. It is found that each micrometer thickness of the metal film can increase C_0 by 0.013 pF. The specified metal film thickness of the experimental sensor is $5\mu\text{m}$. However, it may possibly go up to $10\mu\text{m}$ due to imperfections in the etching process. An increase in C_0 by about 0.1 pF can be expected due to this thickness.

Theoretically, the width of the gap should be 0.1 mm along all the arc-like segments. Although it varies along the radial segments, it should be 0.1 mm again at the centre. However, variations in the gap width as large as $\pm 20\%$ were observed in the experimental sensor. This was due to errors in making the mask and imperfections in the manufacturing process. The sensitivity of C_0 to these variations was estimated using the graphs of C_0 versus gap width. At $g = 0.1$ mm, the value of $\partial C/\partial g$ was found as -4.2 pF/mm. The significance of this is evident from the following numerical example: a reduction of the average gap width by only 10% results in an increase of C_0 by as much as 0.04 pF.

On the other hand, the numerically calculated value of C_0 cannot be considered as the exact external capacitance of the ideal sensor, either. The approximate

numerical method which assumes a two dimensional step distribution for charge density may involve some inaccuracies. The differences of 12.8% in C_0 and 3.2% in C_f between the calculated and measured values may be due to an unfavourable combination of these effects.

Chapter 7

FINAL DISCUSSION AND CONCLUSIONS

7.1 Discussion

The method of moments (MOM) and the finite element method (FEM) are the most frequently used numerical techniques to solve electromagnetic boundary value problems. Although FEM used simple algorithms, MOM was found to be more appropriate for the analysis of fully-open-ended coaxial sensors [4]. This, together with the difficulty in dealing with three dimensional finite elements led to the selection of MOM in this work.

MOM itself takes several forms. The Galerkin method where expansion and testing functions are the same, is found to have good convergence properties. But, it results in more complicated expressions for matrix coefficients. On the other hand, point matching method gives simplest expressions; hence minimizes the computer time required to form the linear system. Previous studies show that both Galerkin and point matching methods give very close results when used to analyze fully-open end sensors [4]. Hence, point matching method was adopted in this work.

Some results were obtained using Amdahl/CMS computer system. VAX/VMS system was used for others. Linear system of equations was solved by double precision Fortran library routines (In VAX, SSP routine 'DGELG' [12] and in Amdahl,

IMSL routine 'LEQT1F' [13]). Most of the computer time was expended to form the system matrix. With $M=3$ and $N=2$, Amdhal computer took about 190 CPU seconds for the complete analysis. VAX computer needed 20 CPU minutes for the same task.

Numerical results presented in Chapter 5 show that banyan-tree sensors can be realized practically to achieve external capacitance (C_0) of approximately 1.5 pF. In this respect, it is comparable with multi-ring sensors where C_0 of 1.2 pF is possible with 0.1mm gap [5]. Nevertheless, other advantages of the banyan-tree sensor should not be overlooked. It needs only one connecting wire and no straps; hence fabrication is easier than multi-ring sensors.

Realization of 7-mm multi-ring sensor is very difficult. The simplicity of banyan-tree type allows easy fabrication of small sensors. If the sample is inhomogeneous, the sensor averages sample permittivity over certain volume. Hence, small sensors are preferred for the measurement of permittivity distribution of inhomogeneous media. They are also needed to deal with small samples. According to numerical results, 7-mm banyan-tree practical sensors can have C_0 values up to 0.7 pF. This should make a dramatic improvement in the measurement frequency range of small open-ended coaxial sensors.

It should be noted that the results presented in Chapter 5 are for sensors with specific dimensions. The air/substrate interface of the sensor was neglected in all the calculations. This is reasonable if the substrate is several millimeters thick ($>3\text{mm}$) and the sample relative permittivity is greater than 10. Otherwise, the effect of the interface has to be taken into account. It can be accommodated using the image coefficient method described in Reference [11]. A little modification of the CAPA program and a lot more computer time is needed for this task.

It is seen that, both external and internal capacitances (C_0 and C_f) of the sensor strongly depend on the gap width (g) and the number of branches in the pattern (N). They hardly vary with the number of trees (M). However, one can suspect that M can make a noticeable change in sensor self-inductance. Low values of M forms long current paths between the coaxial inner and outer conductors thus possibly would lead to high inductances. This in turn would cause errors in high

frequency measurements. Hence, values of $M = 3$ or 4 should be used in general applications.

Another factor which may limit the maximum usable frequency of the sensor is the radiation at the open end. Antenna effect of the open end was not treated in this analysis. Nevertheless, according to aperture radiation theory, one cannot expect the narrow gap of the sensor to radiate effectively in the frequency range of interest in this work, ie. up to 100 MHz.

The theoretical values of capacitances given in Chapter 5 would not be practically achieved unless special care is taken in mask preparation and sensor fabrication. If the metal film thickness exceeds $1\text{ }\mu\text{m}$, a correction for parallel plate effect is necessary. This can be easily done using the parallel plate capacitance formula. Nevertheless, a correction for the imperfections in gap width is not so easy. Hence, it is very important to make every effort to realize a high quality gap pattern.

7.2 Conclusions

The fringing-field capacitance of the banyan-tree sensor is by an order of magnitude higher than that of the standard open-ended coaxial sensor. For example, 14-mm banyan-tree sensors can be easily realized to have external capacitances (C_0) of 1.5 pF and therefore to allow accurate in-vivo measurements of biological tissues at frequencies as low as 10 kHz. Further, fabrication of the banyan-tree sensor is relatively easy as compared with the multi-ring sensor. It is hoped that a 7-mm banyan-tree sensor with C_0 close to 0.7 pF will significantly improve the low frequency limit of 7-mm sensors.

The tables and graphs given in this thesis can be used for the design of banyan-tree sensors to meet specific requirements. Nevertheless, it is important to keep in mind that these results were obtained by an approximate numerical technique. Further, ideal conditions such as zero metal thickness were assumed in the analysis. The measured capacitance of the experimental sensor shows that the quasi-static assumption is valid at least up to 40 MHz.

The apparent increase of the capacitance at high frequencies is most probably due to the neglected inductive effects. This limits the upper useful frequency of the banyan-tree sensor. Larger values of M may improve the high frequency performances, by reducing inductive effects.

An interesting extension of the studies of open-ended coaxial line sensors would be the design of a broadband sensor for dielectric measurements. More consideration should be given to the inductive effects and a dynamic analysis of the sensor should be performed.

Appendix A

INTEGRATION OF THE GREEN'S FUNCTION

The double integration in expression (34) has to be performed for each and every matrix coefficient. Here, the first integration with respect to r' is carried out analytically. Let,

$$F_{ij}(\theta') = \int_{r_{1j}}^{r_{2j}} \frac{r dr}{\sqrt{r^2 + r_{mi}^2 - 2rr_{mi} \cos(\theta' - \theta_{mi})}} \quad (81)$$

For convenience, let us make following variable changes.

$$\begin{aligned} r_{mi} &\rightarrow a \\ \cos(\theta' - \theta_{mi}) &\rightarrow \alpha \end{aligned}$$

Then,

$$\begin{aligned} &\int \frac{r dr}{\sqrt{r^2 + a^2 - 2\alpha ar}} \\ &= \int \frac{(r - a\alpha) dr}{\sqrt{(r - a\alpha)^2 + a^2(1 - \alpha^2)}} + \int \frac{a\alpha dr}{\sqrt{(r - a\alpha)^2 + a^2(1 - \alpha^2)}} \end{aligned} \quad (82)$$

Using well-known integration formulae, this is found to be equal to:

$$\sqrt{r^2 + a^2 - 2\alpha ar} + a\alpha \ln | (r - a\alpha) + \sqrt{r^2 + a^2 - 2\alpha ar} | \quad (83)$$

Therefore,

$$\begin{aligned}
 F_{ij}(\theta') = & \frac{\sqrt{r_{2j}^2 + r_{mi}^2 - 2r_{2j}r_{mi}\cos(\theta' - \theta_{mi})}}{\sqrt{r_{1j}^2 + r_{mi}^2 - 2r_{1j}r_{mi}\cos(\theta' - \theta_{mi})}} \\
 & + r_{mi}\cos(\theta' - \theta_{mi}) \\
 & \times \ln \left| \frac{r_{2j} - r_{mi}\cos(\theta' - \theta_{mi}) + \sqrt{r_{2j}^2 + r_{mi}^2 - 2r_{2j}r_{mi}\cos(\theta' - \theta_{mi})}}{r_{1j} - r_{mi}\cos(\theta' - \theta_{mi}) + \sqrt{r_{1j}^2 + r_{mi}^2 - 2r_{1j}r_{mi}\cos(\theta' - \theta_{mi})}} \right| \quad (84)
 \end{aligned}$$

Above expression is useful in calculation of $F_{ij}(\theta')$ when $\theta' \neq \theta_{mi}$ and when $\theta' = \theta_{mi}$ and $r_{mi} < r_{1j}$. In both cases, $F_{ij}(\theta')$ is finite. When $\theta' = \theta_{mi}$ and $r_{1j} < r_{mi} < r_{2j}$ the value of $F_{ij}(\theta')$ tends to infinity. If $\theta' = \theta_{mi}$ and $r_{mi} > r_{2j}$, the argument of the logarithm becomes indeterministic. For this case, $F_{ij}(\theta')$ can be determined as follows. Now, as $\theta' = \theta_{mi}$,

$$F_{ij}(\theta') = \int_{r_{1j}}^{r_{2j}} \frac{r dr}{\sqrt{r^2 + r_{mi}^2 - 2rr_{mi}}} \quad (85)$$

Since $r_{mi} > r_{2j}$, $r_{mi} > r$ in the interval of integration. Therefore,

$$\sqrt{r^2 + r_{mi}^2 - 2rr_{mi}} = |r - r_{mi}| = r_{mi} - r$$

and hence,

$$\begin{aligned}
 F_{ij}(\theta') &= \int_{r_{1j}}^{r_{2j}} \frac{r dr}{r_{mi} - r} \\
 &= (r_{1j} - r_{2j}) + r_{mi} \ln \left| \frac{r_{mi} - r_{1j}}{r_{mi} - r_{2j}} \right| \quad (86)
 \end{aligned}$$

Combining the two results, the integral can be expressed as:

$$F_{ij}(\theta') = \begin{cases} F_{1ij}(\theta') & \text{if } \theta' \neq \theta_{mi} \text{ or } (\theta' = \theta_{mi} \text{ and } r_{mi} < r_{2j}) \\ F_{2ij}(\theta') & \text{if } \theta' = \theta_{mi} \text{ and } r_{mi} > r_{2j} \end{cases}$$

where

$$F_{1ij}(\theta') = r_{mi}\cos(\theta' - \theta_{mi}) \ln \left| \frac{r_{2j} - r_{mi}\cos(\theta' - \theta_{mi}) + F_{3ij}(\theta')}{r_{1j} - r_{mi}\cos(\theta' - \theta_{mi}) + F_{4ij}(\theta')} \right|$$

$$\begin{aligned}
 & + F_{3ij}(\theta') - F_{4ij}(\theta') \\
 F_{2ij}(\theta') &= (r_{1j} - r_{2j}) + r_{mi} \ln \left| \frac{r_{mi} - r_{1j}}{r_{mi} - r_{2j}} \right| \\
 F_{3ij}(\theta') &= \sqrt{r_{2j}^2 + r_{mi}^2 - 2r_{2j}r_{mi} \cos(\theta' - \theta_{mi})} \\
 F_{4ij}(\theta') &= \sqrt{r_{1j}^2 + r_{mi}^2 - 2r_{1j}r_{mi} \cos(\theta' - \theta_{mi})}
 \end{aligned} \tag{87}$$

Appendix B

INTEGRATION AROUND SINGULARITY

In this section, the function $F''_{1jj}(\theta')$ defined in expression (56) is approximated by a simple function in the neighbourhood of its singularity at θ_{mj} . Then, it is integrated over this region. From (56), we have,

$$F''_{1jj}(\theta') = r_{mj} \cos(\theta' - \theta_{mj}) \ln | r_{1j} - r_{mj} \cos(\theta' - \theta_{mj}) + \sqrt{r_{1j}^2 + r_{mj}^2 - 2r_{1j}r_{mj} \cos(\theta' - \theta_{mj})} | \quad (88)$$

The integration to be performed is,

$$X = \int_{\theta_{mj}}^{\theta_{mj}^+} F''_{1jj}(\theta') d\theta' \quad (89)$$

In the interval of integration, $\theta' \approx \theta_{mj}$ hence

$$F''_{1jj}(\theta') \approx r_{mj} \ln | r_{1j} - r_{mj} + \sqrt{r_{1j}^2 + r_{mj}^2 - 2r_{1j}r_{mj} \cos(\theta' - \theta_{mj})} | \quad (90)$$

For convenience, following variable changes are made.

$$r_{1j} \rightarrow r$$

$$r_{mj} \rightarrow r_m$$

$$\theta' - \theta_{mj} \rightarrow x$$

Then,

$$F''_{1jj}(x) \approx r_m \ln | r - r_m + \sqrt{r^2 + r_m^2 - 2rr_m \cos x} |$$

$$\approx r_m \ln |(r - r_m) + (r_m - r) \sqrt{1 + \frac{4rr_m \sin^2(x/2)}{(r - r_m)^2}}| \quad (91)$$

since $r_m > r$. For very closer θ' and θ_{mj} , $x \approx 0$ hence,

$$\frac{4rr_m \sin^2(x/2)}{(r - r_m)^2} \ll 1$$

and then,

$$\begin{aligned} F''_{1jj}(x) &\approx r_m \ln |(r - r_m) + (r_m - r) [1 + \frac{2rr_m \sin^2(x/2)}{(r - r_m)^2}]| \\ &\approx r_m \ln |\frac{2rr_m \sin^2(x/2)}{r_m - r}| \\ &\approx r_m \ln \frac{2rr_m}{|r_m - r|} + 2r_m \ln |\sin(x/2)| \end{aligned} \quad (92)$$

If $\gamma_j = \theta'_{mj} - \theta_{mj}$, now X can be expressed as:

$$\begin{aligned} X &= \int_0^{\gamma_j} F''_{1jj}(x) dx \\ &= r_m \gamma_j \ln \frac{2rr_m}{|r_m - r|} + 2r_m \int_0^{\gamma_j} \ln |\sin(x/2)| dx \\ &= r_m \gamma_j \ln \frac{2rr_m}{|r_m - r|} + 4r_m \int_0^{\gamma_j/2} \ln |\sin(x)| dx \end{aligned} \quad (93)$$

According to Ref. [21],

$$\int \ln \sin(x) dx = -x \ln(2) - \sum_n \frac{\sin(2nx)}{2n^2} \quad (94)$$

for $0 < x < \pi$. Hence, X is found as:

$$\begin{aligned} X &= r_{mj} \gamma_j \ln \frac{2r_{1j} r_{mj}}{|r_{mj} - r_{1j}|} \\ &\quad - 2r_{mj} [\gamma_j \ln(2) + \sum_n \frac{\sin(n\gamma_j)}{n^2}] \end{aligned} \quad (95)$$

However, above expression fails when $r_{1j} = 0$. Then,

$$\begin{aligned} F''_{1jj}(\theta') &= r_{mj} \cos(\theta' - \theta_{mj}) \ln |r_{mj} [1 - \cos(\theta' - \theta_{mj})]| \\ &\approx r_{mj} \ln [r_{mj} (1 - \cos x)] \\ &\approx r_{mj} \ln [2r_{mj} \sin^2(x/2)] \\ &\approx r_{mj} \ln(2r_{mj}) + 2r_{mj} \ln |\sin(x/2)| \end{aligned} \quad (96)$$

Integrating with respect to x as before, X is found as:

$$\begin{aligned} X|_{r_{1j}=0} &= r_{mj}\gamma_j \ln(2r_{mj}) \\ &\quad - 2r_{mj} \left[\gamma_j \ln(2) + \sum_n \frac{\sin(n\gamma_j)}{n^2} \right] \end{aligned} \quad (97)$$

Appendix C

COMPUTER PROGRAM LISTINGS

The DIVI and CAPA computer programs used in the numerical analysis of the banyan-tree sensor are listed in the following pages.

.....

DIVISION INTO SUB-AREAS

.....

PROGRAM NAME : DIVI
 PURPOSE : TO DIVIDE THE MAIN-UNIT OF THE BANYAN
 TREE PATTERN INTO SUB-AREAS.

.....

C
 C
 DIMENSION A(50),B(60),R1(100),R2(100),T1(100),T2(100),KK(100)
 INTEGER P,Q
 DOUBLE PRECISION RA(100),RB(100),TA(100),TB(100)

C
 C
 C M - NUMBER OF TREES
 C N - NUMBER OF BRANCH PAIRS
 C AP - AVERAGE GAP WIDTH (MILLIMETERS)
 C STEP1 & STEP2 - SUB AREA WIDTHS (NORMALIZED)
 C
 C SCALE - NORMALIZING FACTOR
 C ANORM - INNER GAP RADIUS (NORM.)
 C BNORM - OUTER GAP RADIUS (NORM.)
 C CNORM - OUTER RADIUS (NORM.)
 C
 C H - RADIAL SEPARATION OF GAP
 C A - RADII OF ARC-TYPE BOUNDARIES (NORM.)
 C B - ANGLES OF RADIAL BOUNDARIES (DEG.)
 C ANGL - PERIOD OF THE PATTERN (DEG.)
 C RADIUS - MID POINT RADIUS OF RADIAL SEGMENTS
 C GAPANG - ANGLE OF RADIAL GAP (DEG.)
 C W1,W2 - ANGULAR WIDTH OF RADIAL SUB-AREAS
 C 2*L - TOTAL NUMBER OF SUB-AREAS
 C
 C

PRINT *, 'ENTER M,N,GAP(MM),STEP1,STEP2 (NORMALIZED)'
 READ *,M,N,GAP,STEP1,STEP2
 SCALE=10.2
 GAP=GAP/SCALE
 BNORM=.7003
 ANORM=.3041
 CNORM=1.0
 PI=4.*ATAN(1.)
 H=(BNORM-ANORM-(2*N+1)*GAP)/(2*N)

C
 C COORDINATES OF BOUNDARIES ARE CALCULATED :
 C ARC-TYPE BOUNDARIES :
 C

A(1)=ANORM-STEP2
 A(2)=ANORM-STEP1

```

      A(3)=ANORM
C
      J=4
      DO 10 I=1,2*N
      A(J)=ANORM+GAP+(I-1)*(H+GAP)
      A(J+1)=A(J)+STEP1
      A(J+2)=A(J)+STEP2
      A(J+5)=A(J)+H
      A(J+4)=A(J+5)-STEP1
      A(J+3)=A(J+5)-STEP2
      J=J+6
10  CONTINUE
C
      K=4+12*N
      A(K)=BNORM
      A(K+1)=A(K)+STEP1
      A(K+2)=A(K)+STEP2
      A(K+3)=CNORM
C
C  RADIAL BOUNDARY LINES :
C
      ANGL=180./M
      J=1
      DO 20 I=1,2*N
      B(J)=ANGL
      B(J+8)=0.
      B(J+5)=ANGL/6.
      B(J+1)=ANGL-B(J+5)
C
      RADIUS=ANORM+GAP+H/2.+(H+GAP)*(I-1)
      GAPANG=GAP*180./(RADIUS*PI)
      W1=STEP1*180./(RADIUS*PI)
      W2=STEP2*180./(RADIUS*PI)
C
      B(J+4)=B(J+5)+GAPANG
      B(J+3)=B(J+4)+W1
      B(J+2)=B(J+4)+W2
      B(J+6)=B(J+5)-W1
      B(J+7)=B(J+5)-W2
C
      II=2*(I/2)
      IF (II.EQ.1) THEN
        DO 30 L=1,9
30    B(J+L-1)=ANGL-B(J+L-1)
      END IF
      J=J+9
20  CONTINUE
C
C  COORDINATE PARAMETERS OF SUB-AREAS :
C      R1 - SMALLEST RADIUS (NORM.)
C      R2 - LARGEST RADIUS (NORM.)
C      T1 - SMALLEST ANGLE (DEG.)
C      T2 - LARGEST ANGLE (DEG.)
C

```

```

R1(1)=0.
R2(1)=A(1)
T1(1)=0.
T2(1)=ANGL
R1(2)=A(1)
R2(2)=A(2)
T1(2)=0.
T2(2)=ANGL
R1(3)=A(2)
R2(3)=A(3)
T1(3)=ANGL/2.
T2(3)=ANGL
R1(4)=A(2)
R2(4)=A(3)
T1(4)=B(6)
T2(4)=ANGL/2.

```

C
C

```

J=5
K=2
P=7
L=5+13*N
DO 200 JJ=1,2
DO 100 I=1,N
R1(J)=A(K)
R2(J)=A(K+9)
T1(J)=B(P)
T2(J)=B(P-1)
R1(J+1)=A(K)
R2(J+1)=A(K+9)
T1(J+1)=B(P+1)
T2(J+1)=B(P)
R1(J+2)=A(K)
R2(J+2)=A(K+9)
T1(J+2)=B(P+2)
T2(J+2)=B(P+1)
J=J+13
K=K+12
P=P+18

```

100 CONTINUE

J=L+12

K=8

P=16

200 CONTINUE

C

C

```

J=8
K=10
P=11
DO 400 II=1,2
DO 300 I=1,N
R1(J)=A(K)
R2(J)=A(K+1)
T1(J)=B(P)

```

```

T2(J)=ANGL/2.
R1(J+1)=A(K)
R2(J+1)=A(K+1)
T1(J+1)=ANGL/2.
T2(J+1)=B(P+3)
R1(J+2)=A(K+1)
R2(J+2)=A(K+4)
T1(J+2)=B(P+2)
T2(J+2)=B(P+3)
R1(J+3)=A(K+1)
R2(J+3)=A(K+4)
T1(J+3)=B(P-1)
T2(J+3)=B(P)
R1(J+4)=A(K+4)
R2(J+4)=A(K+5)
T1(J+4)=B(P)
T2(J+4)=ANGL/2.
R1(J+5)=A(K+4)
R2(J+5)=A(K+5)
T1(J+5)=ANGL/2.
T2(J+5)=B(P+3)
R1(J+6)=A(K+3)
R2(J+6)=A(K+4)
T1(J+6)=B(P)
T2(J+6)=B(P+2)
R1(J+7)=A(K+1)
R2(J+7)=A(K+2)
T1(J+7)=B(P)
T2(J+7)=B(P+2)
R1(J+8)=A(K+2)
R2(J+8)=A(K+3)
T1(J+8)=B(P+1)
T2(J+8)=B(P+2)
R1(J+9)=A(K+2)
R2(J+9)=A(K+3)
T1(J+9)=B(P)
T2(J+9)=B(P+1)
J=J+13
K=K+12
P=P+18
300  CONTINUE
      J=L+2
      K=4
      P=2
400  CONTINUE
C
C
      R1(L)=A(2+12*N)
      R2(L)=A(3+12*N)
      T1(L)=0.
      T2(L)=B(18*N-7)
      R1(L+1)=A(4)
      R2(L+1)=A(5)
      T1(L+1)=B(2)

```

```

      T2(L+1)=B(1)
      LL=2*L-3
      R1(LL)=A(4+12*N)
      R2(LL)=A(5+12*N)
      T1(LL)=ANGL/2.
      T2(LL)=B(18*N-3)
      LL=LL+1
      R1(LL)=A(4+12*N)
      R2(LL)=A(5+12*N)
      T1(LL)=0.
      T2(LL)=ANGL/2.
      LL=LL+1
      R1(LL)=A(5+12*N)
      R2(LL)=A(6+12*N)
      T1(LL)=0.
      T2(LL)=ANGL
      LL=LL+1
      R1(LL)=A(6+12*N)
      R2(LL)=A(7+12*N)
      T1(LL)=0.
      T2(LL)=ANGL
C
C
      Q=0
      DO 500 I=1,N
      DO 600 J=2,14
      T=T1(L+Q+J)
      T1(L+Q+J)=T2(L+Q+J)
      T2(L+Q+J)=T
600  CONTINUE
      Q=Q+13
500  CONTINUE
      OPEN(UNIT=10,FILE='COORD',STATUS='NEW')
      WRITE (10,1300) GAP*SCALE,M,N,STEP1,STEP2
      DO 700 I=1,2*L
      KK(I)=2*INT(T2(I)-T1(I))
      RA(I)=DBLE(R1(I))
      RB(I)=DBLE(R2(I))
      TA(I)=DBLE(T1(I))
      TB(I)=DBLE(T2(I))
      IF (KK(I).LE.4) KK(I)=4
700  WRITE(10,1200)I,RA(I),RB(I),TA(I),TB(I),KK(I)
1200  FORMAT(2X,I2,4(2X,D12.5),2X,I4)
1300  FORMAT(2X,F8.4,2(2X,I2),2(2X,F8.4))
      STOP
      END

```

NUMERICAL ANALYSIS OF BANYAN TREE END

COAXIAL SENSORS

```
* PROGRAM NAME : CAPA
* PURPOSE : TO ANALYZE "BANYAN TREE" OR SIMILAR TYPE
*           SENSORS USING THE METHOD OF MOMENTS WITH
*           POINT MATCHING.
```

C
C

```

DIMENSION K(62)
DOUBLE PRECISION R1(62),R2(62),T1(62),T2(62),T1D(62),T2D(62),
* A(62,62),B(62),AREA(62),WKAREA(3844)
DOUBLE PRECISION PI,EPS0,RR1,RR2,RM,TM,T,RES,RES1,RES2,
* SUM,SUM1,SUM2,TT1,TT2,CAP,ERR,ERROR,EPS,GAMMA,CON,DELT
COMMON RR1,RR2,RM,TM,PI,EPS0,NUM
COMMON/FIRST/TT1,TT2,KKK,I

```

c
c
c
c

CONSTANTS: EPS0 - FREE SPACE PERMITTIVITY
SCALE - SCALE FACTOR FOR NORMALIZATION

```
PI=4.D0*DATAN(1.D0)
EPS0=1.D0/(PI*36.D9)
SCALE=10.2E9
```

C
C
C
C
C
C
C

FOLLOWINGS ARE READ FROM THE FILE 'COORD'

GAP - AVERAGE GAP WIDTH:

M - NUMBER OF BANYAN TREES

N - NUMBER OF BRANCH PAIRS

STEP1,STEP2 - SUB-AREA WIDTHS

5
C
C
C
C

```
OPEN (UNIT=10,FILE='COORD',STATUS='OLD')
```

```
READ (10,5000) GAP,M,N,STEP1,STEP2
```

```
5000  FORMAT(2X,F6.4,2(2X,I2),2(2X,F6.4))
```

L1 - NUMBER OF POSITIVELY CHARGED SUB-AREAS

LT - TOTAL NUMBER OF SUB-AREAS

$$L1=13*N+5$$
$$LT=L1*2$$

C
C
C
C
C

FOLLOWINGS ARE ALSO READ FROM 'COORD':

R1,R2 - ARRAYS OF SUB-AREA RADIAL COORDINATES

T1D,T2D - ARRAYS OF SUB-AREA ANGULAR COORDINATES (DEGREES)

K - ARRAY OF THE SELECTED NUMBER OF SAMPLE POINTS

DO 2000 I-1,LT

```
2000 READ (10,3010) R1(I),R2(I),T1D(I),T2D(I),K(I)
```

```

3010  FORMAT (6X,4(D12.5,2X),I4)
      CLOSE (10)
C
C  RESULTS AND ERROR MESAGES ARE WRITTEN IN 'RESULT' FILE
      OPEN (UNIT=20,FILE='RESULT',STATUS='NEW')
C
C  A - SYSTEM MATRIX {A}
C  B - SYSTEM VECTOR {B}
C  T1,T2 - ARRAYS OF SUB-AREA ANGULAR COORDINATES (RADIAN)
C  RM - RADIAL COORDINATE OF CURRENT TESTING POINT
C  TM - ANGULAR COORDINATE OF CURRENT TESTING POINT
C
      DO 100 I=1,L1
100   B(I)=4.D0*PI*EPS0
      DO 200 I=L1+1,LT
200   B(I)=0.D0
C
      DO 1000 I=1,LT
      T1(I)=T1D(I)*PI/180.D0
      T2(I)=T2D(I)*PI/180.D0
1000  CONTINUE
C
      DO 300 I=1,LT
      RM=(R1(I)+R2(I))/2.D0
      TM=(T1(I)+T2(I))/2.D0
C
C  RR1,RR2 - RADIAL COORDINATES OF CURRENT SUB-AREA
C  TT1,TT2 - ANGULAR COORDINATES OF CURRENT SUB-AREA
C  KKK - VALUE OF K FOR CURRENT SUB-AREA
C
      DO 400 J=1,LT
      RR1=R1(J)
      RR2=R2(J)
      TT1=T1(J)
      TT2=T2(J)
      KKK=K(J)
C
      IF (I.EQ.J) GO TO 3000
C
C  EVALUATION OF OFF-DIAGONAL MATRIX COEFFICIENTS
C
      CALL EBOTH (TT1,1,RES1)
      CALL EBOTH (TT2,1,RES2)
      SUM=(RES1+RES2)/2.D0
      DELT=(TT2-TT1)/KKK
      DO 10 KK=1,KKK-1
      T=TT1+DBLE(KK)*DELT
      CALL EBOTH (T,1,RES)
10   SUM=SUM+RES
      A(I,J)=SUM*DELT
      GO TO 400
C
C  EVALUATION OF DIAGONAL COEFFICIENTS :
C  CONTRIBUTION FROM IMAGES IS CALCULATED AS SUM.

```

```

C
3000 CALL EBOTH (TT1,0,RES1)
      CALL EBOTH (TT2,0,RES2)
      SUM=(RES1+RES2)/2.D0
      DELT=(TT2-TT1)/KKK
      DO 3020 KK=1, KKK-1
      T=TT1+DBLE(KK)*DELT
      CALL EBOTH (T,0,RES)
3020 SUM=SUM+RES
      SUM=SUM*DELT
C
C      CONTRIBUTION FROM THE SECOND TERM IS ADDED.
C
      SUM1=(EXP01(TM)+EXP01(TT2))/2.D0
      DELT=DELT/2.D0
      DO 3030 KK=1, KKK-1
      T=TM+DBLE(KK)*DELT
3030 SUM1=SUM1+EXP01(T)
      SUM=SUM+SUM1*2.D0*DELT
C
C      K' IS TAKEN AS 100*K
C      KKK - VALUE OF K' OF THE CURRENT SUB-AREA
C
      KKK=KKK*100
C
C      NEXT TWO TERMS ARE CALCULATED AND ADDED.
C      INFINITE SERIES IS CALCULATED TO .1% ACCURACY OR
C      10,000 TERMS.
C      GAMMA - SINGULARITY NEIGHBOURHOOD HALF-WIDTH
C
      CALL HWIDTH(GAMMA,L)
      EPS=1.D-4
      SUM1=GAMMA*DLOG(2.D0)
      MM=1
3040 CON=DSIN(MM*GAMMA)/MM**2
      SUM1=SUM1+CON
      MM=MM+1
      ERROR=CON/SUM1
      IF (MM.LT.10001.AND.ERROR.GT.EPS) GO TO 3040
      IF (MM.GE.10001) WRITE(20,3050)
3050 FORMAT (' NO CONVERGENCE OF THE SERIES AT 10000TH TERM')
      IF (RR1.EQ.0.D0) THEN
          SUM=SUM- 2.D0*RM*GAMMA*DLOG(2.D0*RM)
          *      + 4.D0*RM*SUM1
      ELSE
          SUM=SUM- 2.D0*RM*GAMMA*DLOG(2.D0*RR1*RM/(RM-RR1))
          *      + 4.D0*RM*SUM1
      END IF
C
C      LAST TERM IS CALCULATED AND ADDED TO THE REST.
C
      SUM1=(EXP02(TM+GAMMA)+EXP02(TT2))/2.D0
      DELT=(TT2-TT1)/KKK
      DO 3060 KK=L+2, KKK/2-1

```

```

      T=TM+DBLE(KK)*DELT
3080 SUM1=SUM1+EXP02(T)
      SUM=SUM-SUM1*2.D0*RM*DELT
      A(I,J)=SUM
400  CONTINUE
300  CONTINUE
C
C   UNKNOWN IS CHANGED TO NORMALIZED CHARGE (Q)
C   AREA - ARRAY OF SUB-AREA AREAS.
C
      DO 4000 J=1,LT
      AREA(J)=DBLE(M)*(R2(J)**2-R1(J)**2)*(T2(J)-T1(J))
      DO 4000 I=1,LT
      A(I,J)=A(I,J)/AREA(J)
4000 CONTINUE
C
C   LINEAR SYSTEM IS SOLVED.
C   KEY - SHOULD BE ZERO FOR VALID RESULTS
C
      CALL LEQ1F (A,1,LT,LT,B,0,WKAREA,KEY)
      WRITE (20,1112) KEY
1112 FORMAT(// '   KEY = ',I4)
C
C   NOW, B - SOLUTION VECTOR (Q)
C   FINAL CALCULATIONS FOR FREE SPACE CAPACITANCE
C
      SUM1=0.D0
      DO 80 I=1,L1
80  SUM1=SUM1+B(I)
C
C   CAP - NORMALIZED FREE-SPACE CAPACITANCE
C
      CAP=SUM1*SCALE
C
      WRITE (20,4001)
4001 FORMAT(// '   I      R1      R2      THETA1      THETA2      CHARGE
      * DENS.   CHARGE'//)
      DO 50 I=1,LT
50  WRITE (20,20) I,R1(I),R2(I),T1D(I),T2D(I),B(I)/AREA(I),B(I)
20  FORMAT(2X,I2,2(3X,F7.4),2(3X,F8.2),2(3X,D10.3))
      WRITE (20,70) CAP,GAP,M,N,STEP1,STEP2
70  FORMAT(// '      CAPACITANCE = ',F10.4,' PF',
      *      '// '      GAP          = ',F5.2,' MM',
      *      '// '      TREES (M)     = ',I2,
      *      '// '      BRAN.PAIRS(N)= ',I2,
      *      '// ' (STEP 1 = ',F6.4,') (STEP 2 = ',F6.4,')')
      STOP
      END
C
C

```

```

.....
*
SUBROUTINE EBOTH (T,K,VAL)
*
.....
C
C   T - ANGULAR COORDINATE OF THE SOURCE
C   K - KEY TO SELECT CORRECT EXPRESSION
C   VAL - OUTPUT VALUE
C
DOUBLE PRECISION F1,F2,F3,F4
DOUBLE PRECISION R,X,Y,T,VAL,VAL1,VAL2,RR1,RR2,RM,TM,PI,EPS0
COMMON RR1,RR2,RM,TM,PI, EPS0,M
F1(R,X)= DSQRT(R**2+RM**2-2.D0*R*RM*DCOS(X-TM))
F2(R,X)= R-RM*DCOS(X-TM)+F1(R,X)
F3(X)=RM*DCOS(X-TM)*DLOG(F2(RR2,X)/F2(RR1,X))+F1(RR2,X)-F1(RR1,X)
C
VAL = F3(2.D0*PI/M-T)
DO 30 I=2,M
30 VAL = VAL + F3(I*2.D0*PI/M-T)
      + F3((I-1)*2.D0*PI/M + T)
      IF (K.EQ.0) RETURN
      VAL1= F2(RR1,T)
      VAL2= F2(RR2,T)
      IF (VAL1.GT.0.D0.AND.VAL2.GT.0.D0.AND.(T.NE.TM.OR.(T.EQ.TM.AND.
      *RM.LT.RR1))) THEN
        VAL=VAL+RM*DCOS(T-TM)*DLOG(VAL2/VAL1)+F1(RR2,T)-F1(RR1,T)
      RETURN
      ELSE IF (RM.GT.RR2) THEN
        VAL=VAL+DLOG((RM-RR1)/(RM-RR2))*RM+RR1-RR2
      RETURN
      ELSE
        WRITE (20,10)
      RETURN
    END IF
10  FORMAT ('      ERROR ')
    END
C
.....

```

```

*
SUBROUTINE HWIDTH (OUT,COUNT)
*
*   THIS ROUTINE CALCULATES SINGULARITY NEIGHBOURHOOD
*   HALF-WIDTH "GAMMA", FOR EACH SUB-AREA.
*
.....
C
C   OUT - ANGLE GAMMA IN RADIANs
C
INTEGER COUNT
DOUBLE PRECISION RR1,RR2,RM,TM,TT1,TT2,OUT,F1,R,X,ARG,LIMIT,TR
COMMON RR1,RR2,RM,TM
COMMON/FIRST/TT1,TT2,KKK,I
F1(R,X)= DSQRT(R**2+RM**2-2.D0*R*RM*DCOS(X))
LIMIT=5.D-39

```

```

COUNT=0
TR=(TT2-TT1)/KKK
OUT=0.D0
10  ARG=RR1-RM*DCOS(TR)+F1(RR1,TR)
    IF (ARG.GE.LIMIT) THEN
        OUT=TR
    ELSE
        TR=TR+(TT2-TT1)/KKK
        COUNT=COUNT+1
        IF (COUNT.LE.200) GO TO 10
        WRITE (20,20) 1,KKK/100
    END IF
20  FORMAT('  ERROR: NO ALPHA'/'  K('12,') = '13,  IS TOO
    • LARGE')
    RETURN
END
C
.....
•
FUNCTION EXP01(T)
•
.....
C  T - SAMPLE ANGULAR COORDINATE
C
COMMON RR1,RR2,RM,TM
DOUBLE PRECISION RR1,RR2,RM,TM,PVAL1,F,T,R
F(R)=DSQRT(R**2+RM**2-2.D0*R*RM*DCOS(T-TM))
EXP01=RM*DCOS(T-TM)*DLOG(DABS(RR2-RM*DCOS(T-TM)+F(RR2)))
•
+ F(RR2) - F(RR1)
END
C
.....
•
FUNCTION EXP02(T)
•
.....
C  T - SOURCE ANGULAR COORDINATE
C
COMMON RR1,RR2,RM,TM
DOUBLE PRECISION RR1,RR2,RM,TM,PVAL2,F,T,R
F(R)=DSQRT(R**2+RM**2-2.D0*R*RM*DCOS(T-TM))
EXP02=DCOS(T-TM)*DLOG(DABS(RR1-RM*DCOS(T-TM)
•
+DSQRT(RR1**2+RM**2-2.D0*RR1*RM*DCOS(T-TM))))
END

```

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