An Empirical Investigation of the Random Walk Hypothesis in the US and UK Stock Markets

Do Well Developed Economies with High Market Capitalization Yield the Same Conclusions?

Leila Al-Mqbali
Student Number: 5600405

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Supervisor: Professor FrancescaRondina
ECO6999

Ottawa, Ontario
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Leila Al-Mqbali
5600405

Abstract

This paper assesses the validity of random walk in the US and UK stock markets, in order to assess whether controlling for the size of market capital and the extent of economic development impacts results. We find that results are not robust to the empirical techniques employed, nor are they robust to the choice of horizon length. Therefore, the lack of consensus in professional opinion regarding the validity of the random walk hypothesis cannot be wholly attributed to differences in market capital and economic development. Future research should attempt to control for other factors such as trading frequency and volume, with the aim of identifying key economic dynamics which influence the results.

Leila Al-Mqbali
5600405

I. Introduction

In recent years, there has been increased interest in the behaviour of stock market returns, and as yet there is no clear consensus among experts with regards to their predictability. This paper will assess whether stock prices in the US and UK markets tend to follow a random walk.

The random walk hypothesis (RWH) relies on two premises: (1) stock price changes conform to some probability distribution, and (2) stock price changes are independent of each other, which implies that previous movements in stock prices cannot be used to forecast future changes. Investors react instantaneously to any new information they are presented with, and this competition between investors eliminates all prospects of profiting from such information. Advocates of RWH thus maintain that stocks take an unpredictable and random path, and as a consequence it is impossible to outperform the market as stock prices reflect all information available at the time of exchange. Conversely, opponents of RWH argue that stock price changes are, at least to some extent, predictable.

Previous results from testing the validity of RWH appear to be sensitive to the empirical tests conducted as well as the particular market which is studied. However, as yet, research has focusedon using a variety of empirical methods to assess RWH in a particular country, rather than exploring how their results may differ *across* countries when each index is subjected to the same empirical methods. This project will therefore determine whether stock indices in two markets which are very similar in terms of size of market capital and the extent of economic development yield the same conclusions when subjected to the same empirical tests for RWH.

More specifically, we will focus on the UK and US economies, two well developed markets with high market capitalization.Our empirical methods include a subset of various parametric and non-parametric tests frequently employedby researchers testing for RWH. If our results differ between indices, then we must acknowledge that the difference must be due not todissimilarities in market capitalization and economic development, but rather to some unknown market characteristic which has not been accounted for. If, on the other hand, we obtain similar results for each index, then future research can attempt to extend this analysis to demonstrate that, in general, well developed markets with high market capitalization will yield the same conclusions with regards to RWH.

This research will be a valuable contributor to the existing literature on the RWH in several ways. First, no previous papers have focussed on a comparative analysis of the RWH for indices in two well developed countries with similar market characteristics using such an extensive array of empirical methods. Second, Section V of this paper presents a thorough and comprehensive description of the primary parametric and non-parametric tests used to assess the validity of RWH, which researchers may find to be a useful reference for future studies in this area.

The next section will present a review of previous literature on the RWH.Early theoretical contributions in this area were followed by empirical studies that addressed the behaviour of stock market returns through a variety of empirical techniques. As we will see, these studies provide evidence both for and against RWH, as results appear to be sensitive to the tests employed as well as the market which is studied.

II. A Review of Previous Literature

An Early Theoretical Argument for RWH

The doctoral thesis of French mathematician Louis Bachelier(2006, originally published in 1900) documents an early discussion of the random walk hypothesis in financial markets. In *Théorie de la Spéculation*, Bachelier investigates the market for French government bonds and notes that “the influences that determine the movements of the exchange are innumerable; past, current and even anticipated events that often have no obvious connection with its changes ... it is thus impossible to hope for mathematical predictability” (Bachelier, 2006, p. 15). Consequently, Bachelier infers a fundamental hypothesis that predates much of the existing literature on RWH: since it is not possible to predict future movements in the market, the mathematical expectation of the economic agent is equivalent to zero (p. 18).

However, Bachelier’s work did not receive widespread notice outside the field of mathematics until the 1950s, when economist Paul Samuelson connected the theory of random walk in stock markets with economic fundamentals. Samuelson (1965) proposes a theoretical argument that the market achieves randomness through the competitive activities of a large number of investors all seeking to increase their wealth. Creating a general stochastic model of price change, he demonstrates that the price differences in any one period are uncorrelated with the price differences in the previous period; in other words, there is zero expected capital gain. However, Samuelson did emphasize that his theory does not suggest that it would be impossible for one investor who has superior information to make a profit – only that it is unlikely they can outperform the market average. Throughout his career, Samuelson continued to propose theoretical frameworks in support of RWH; yet he did not conduct econometric and empirical research to test his hypotheses. The following section will address empirical findings on the subject.

Empirical Evidence

Following Samuelson's research, Fama (1965) found that there was strong evidence to support RWH. Using daily stock prices as data, Fama employed statistical techniques to show both the independence of stock price movements and their conformity to stable Paretian distributions with characteristic exponents less than 2. Moreover, Osborne (1959) supports Fama's results. Using the theory of Brownian motion applied to financial markets, Osborne considers prices evolving continuously in time and concludes in favour of RWH. It is important to note, however, that Brownian models assume a frictionless market and perfectly divisible assets. This assumption is unrealistic as all financial markets are, to some extent, subject to frictions such as transaction costs and barriers to trade.

As the field of econometrics advanced, so too did the empirical methods which could be used to test RWH. Lo and MacKinlay (1988) use a simple specification test based on variance estimators to refute RWH over the whole of the sample period (1962-1965) for the New York Stock Exchange (NYSE). Their empirical results imply that RWH is not consistent with the behaviour of weekly stock returns, as they find shorter horizon return lengths to exhibit negative serial correlation[1], and longer horizon returns to exhibit positive serial correlation.Lo and

[1]Serial correlation refers to the relationship between a specified variable and lagged values of itself over time, and indicates that future levels of the given variable are, to some extent, dependent on its past levels.

MacKinlay also developed a version of the specification test of RWH which is robust to fluctuations in variance, and conclude that even when heteroscedasticity is present, the null hypothesis of RWH is still rejected at all conventional levels of significance (Lo and MacKinlay, p. 19). Fama and French (1988) also find RWH to be invalid; however, in contrast to Lo and MacKinlay (1988), they demonstrate that introducing a mean-reverting component of stock prices traded on the NYSE generates a strong *negative* serial correlation for longer horizon returns. Likely, these conflicting results are due to the small sample sizes for long horizon returns, and as such it is difficult to infer reliable conclusions from such small samples (Fama and French, 1988, p. 4).

Supporting Lo and MacKinlay's findings, Paul Cootner (1962) also rejects RWH at standard significance levels. Cootner observes the distribution of changes in weekly stock prices, and notes that the mean of these changes is much smaller than the standard deviation. As the time period lengthens, however, he finds that the mean becomes relatively larger than the standard deviation and that the mean of each of the component trends becomes more discernible from the group mean (Cootner, p. 5). Put differently, Cootner also finds positive autocorrelation for longer horizon returns, and concludes that stock prices are therefore not free to wander completely randomly.

Directly opposing this position, Malkiel (2007) argues that short run movements in stock prices cannot possibly be predicted. Contrary to many econometric approaches undertaken to test RWH, Malkiel closely analyzes existing methods used in critical analyses in order to cast doubt on their soundness of argument. For example, he generates a stock chart of a fictitious asset whose price changes are determined by a coin toss. Using conventional empirical methods, Malkiel demonstrates that although the stock prices may seem to follow a predictable cycle, this is clearly not the case in reality.

Further arguments in favour of RWH demonstrate the absence of profitability when technical trading strategies are followed over the long term (Fama, 1965). Moreover, Malkiel (2003) uses statistical methods to show that in general, traders do not outperform the benchmark indices. In cases where traders *do* perform better than the benchmark, Malkiel finds that this success is not replicated or sustained in the long run.

Other research has yielded results which imply conclusions are dependent upon the empirical methods employed. Gourishankar Hiremath (2014) studies the behaviour of stock returns in India for 14 different indicestraded on the National Stock Exchange (NSE) and the Bombay Stock Exchange (BSE) using a number of parametric and non-parametric methods. Applying the same parametric tests to both exchange markets, Hiremath finds that RWH is supported for well-capitalized indices traded on the BSE, but refuted for those indices traded on the NSE. Yet, when nonparametric methods were implemented, Hiremath rejects RWH for all indices, thereby suggesting that conclusions are not robust to the statistical techniques used to test the hypothesis.

Existing literature also suggests that results may be dependent upon which market is tested, due to factors such as differences in policies and in trading frequency. For example, Solnik (1973) rejects random walk in European markets, and suggests the deviation from random walk is due to a combination of inadequate disclosure norms and insider trading. Furthermore, Mustafa and Nishat (2007) assert that thin trading is a primary cause of significant correlation in stock returns.

Criticisms of Existing Literature

New methods of empirical testing are still being developed as criticisms of existing techniques continue to emerge. Chow and Denning (1993) criticised Lo and MacKinlay's variance ratio test, citing the fact that the test has a large probability of Type I error (rejecting the null hypothesis when it is true). The Lo-MacKinlay variance ratio test assesses whether the variance ratio is equal to one for a particular holding period, whereas RWH necessitates that the test should be implemented simultaneously over a number of holding periods, and the variance ratios for *all* holding periods should equal one. Noting the sequential nature of the Lo-MacKinlay test, Chow and Denning proposed a multiple variance ratio test to control the test size and thereby reduce Type I errors. It is possible that similar issues with other test techniques are partly responsible for the lack of consensus among experts on the behaviour of stock market returns.

Another possible explanation for the discord regarding the validity of RWH is that, often, scholars appear to confuse the random walk hypothesis with the efficient markets hypothesis (EMH). Though the two theories are related, they are by no means equivalent: i.e. random movements in stock prices do not necessarily imply an efficient stock market where all investors are rational, and vice versa (Lo and MacKinlay, 2002).

A New Approach Moving Forward

Though the research and existing literature exploring the validity of RWH is vast, there is still no clear agreement among scholars regarding the behaviour of stock market returns. There are as many published studies in support of RWH as there are studies which refute it, and many others maintain that the legitimacy of RWH depends on the indices used and the particular market economy studied in the sample. Furthermore, the existing literature has shown that results are not robust to empirical methods, and as such further research is warranted to address the issue. As econometric techniques continue to advance, it is possible that experts will establish new methods to better test the behaviour of stock market prices.

In this paper, we assess the validity of RWH for indices in two well developed economieswith high market capital, in order to see how results compare when both indices are subjected to the same empirical tests.To assess the robustness of our results, both parametric and non-parametric methods will be used, and the tests will be applied to different horizon length returns as well as to various sub-periods and additional market indices.

III. Data Collection

To test for random walk in stock market prices, we use data from the NASDAQ and from the London Stock Exchange Group (LSE). Comparing indices from markets in the United States and the United Kingdom allows us to examinemarkets with similar market capitalization in countries with comparable economic development. In choosing well developed markets, we hope to avoid more serious market inefficiencies which could prohibit random fluctuations. Indeed, Samuels (1981) suggests that since emerging markets are inherently more prone to inefficiencies such as a high degree of imperfection with regards to competition, they cannot possibly fluctuate randomly. With regards to market capitalization, the World Federation of Exchanges estimates that as of January 31st 2015, the LSE and NASDAQ had $6187 billion USD and $6831 billion USD respectively in market capital. Moreover, the two exchanges are comparable with respect to the number of companies traded on each market, with LSE listing 2475 companies as of June 2014, and NASDAQ listing 2472 (Statistica, 2015). Data pertaining to the volume of trade per index was not available for both markets; however it is sufficient to say that two well developed markets with high capitalization will very likely trade at a volume above some critical threshold that minimizes any potential threats from thin and/orinsider trading.

For the London Stock Exchange, we select the Financial Times Stock Exchange (FTSE) 100 index to test the validity of RWH. The FTSE 100 is the LSE's primary stock index, and the UK's most widely recognized stock market indicator. The FTSE 100 is a capitalization weighted index, and includes the 100most highly capitalized companies traded on the LSE, both financial and non-financial. For the NASDAQ market we consider the NASDAQ 100 index, which is also a capitalization weighted index. Comprised of 100 of the largest non-financial companies traded on the NASDAQ in terms of market capital, the NASDAQ 100 is one of the major stock indices in the United States.

The sample period consists of weekly closing price data from October 1985 through April 2015, the longest time period over which data are available for both indices. Retrieved from NASDAQ and LSE via the Google Finance website[2], this presents us with 1517 observations for each index denominated in local currency. Summary statistics for each index's closing price data are presented in Table 1. The choice of weekly observations is determined by several factors. First, since sampling theory is contingent upon asymptotic properties, we require a large number of observations in order to infer accurate results from our analysis and so we discard monthly observations as a potential candidate for our study. However, we also discard daily observations despite the obvious fact that daily observations would provide us with a much larger dataset. As noted previously by Lo and MacKinlay (1988), "the biases associated with non-trading, the bid-ask spread, asynchronous prices, etc. may become statistically significant [with daily observation intervals]" (p.50). Therefore, choosing weekly observations affords us both a large number of

[2]Further details regarding the data sources can be found in Appendix A.

data points whilst minimizing these types of inherent biases. Furthermore, Dickinson and Muragu observe that "infrequent trading of particular shares can introduce serious biases into the results" (1994, p.137), and thus the use of longer time intervals (such as weekly price series) increases the power of the statistical tests for random walk[3] (Taylor, 1986).

IV. A Simple Model of Random Walk

A sequence of random variables X_t ($t = 1, 2, ...$) is said to be a random walk if the increments $u_t = X_t - X_{t-1}$ are independently identically distributed (i.i.d.) for all $t, t-1$ (Praetz, 1980, p. 123), and if they conform to some probability distribution. In the context of financial markets, the random walk hypothesis asserts that:

$$p_t = p_0 + \sum_{j=1}^{t} u_j \ , \qquad (1)$$

where p_t is the stock price (or some transformation of the stock price, for instance its logarithm) at time t and p_0 is the initial stock price. We examine the distribution of price changes over the entire sample period in order to assess whether the data are stationary. Inferring meaningful results is rendered impossible when a clear trend is present in the data, as many statistics such as correlations with other variables cannot be deduced with any degree of reliability.Figure 1 graphs weekly closing prices from October 10th 1985 – April 13th 2015 for the NASDAQ 100 and FTSE 100 respectively. It is evident that both indices exhibit a clear upwards trend, which is a reasonable result since prices in general economic conditions exhibit an upward tendency due to inflation.

Confirming this result, a Dickey-Fuller Generalized Least Squares (DF-GLS) test is conducted on the level price data. Dickey Fuller tests are employed to test the null that the series contains a unit root against the alternative hypothesis that it is stationary, and are particularly useful when analyzing models of unknown orders.Stochastic processes contain unit roots if 1 is a

[3]Note that most of the existing literature also employs weekly observation intervals for these same reasons.

root of their characteristic equation, and are modelled as ARIMA(p,1,q) where p≥0, q≥0. Random walks are special cases of processes containing unit roots and are modelled as ARIMA(0,1,0).

Therefore, it is important to note that failure to reject the null hypothesis does not necessarily mean that the data represent a random walk – only that there is no strong evidence to indicate that the data are stationary. Quite possibly the data is ARIMA(p,1,q) with p>0, q>0, and consequently does not conform to a random walk distribution.The DF-GLS test is used to detect stationarity, not randomness.

Elliott, Rothenberg, and Stock (1996) modified the Dickey Fuller statistic using GLS and found that the performance of the modified statistic far surpassed the original in terms of power (Elliot et. al, 1996, p.813). Furthermore, the DF-GLS test automatically accounts for the distinct trend visible in the primary data, whereas the original Dickey Fuller test does not. Presented in Table 2 are the results from the DF-GLS tests for NASDAQ 100 and FTSE 100 weekly price data at various lag lengths, including both a constant and a trend in the analysis.[4]

Clearly, the null hypothesis cannot be rejected for either index at any of the conventional levels of significance, and therefore we cannot conclude that the price data comes from a stationary process. However, research has shown that conducting many statistical tests – including the tests described in the following section – using non-stationary data generally yields unreliable results. In many instances, results are spurious in that they imply relationships between variables where none exist in reality. Thus, it follows that stationarity of our time series is a necessary prerequisite in order to derive clear and consistent results from our empirical tests.

Wetransform the data using logarithms and differencing in order to create a series of returns which may be stationary. In fact, previous research has shown that taking the first difference of a series which follows a random walk results in a stationary process (Mbululu, D., Auret, C.J., and Chiliba L., 2014, p. 61). Taking the logarithm of prices, we define returns as:

$$r_{i,t} = \log(p_{i,t}) - \log(p_{i,t-1}) \tag{2}$$

[4]The lag lengths presented for each index were chosen so as to be consistent with the lengths selected for use in variance ratio tests later on in the paper. Also presented is the maximum lag selected by the Schwert Criterion, and the optimal lag lengths consistent with the Ng-Perron criterion, the Schwarz (SC) criterion, and the Modified Akaike Information Criterion (MAIC).

where *i* indicates the particular index and *p* denotes price. Theoretically, the logarithmic transformation of prices is warranted since absolute price changes exhibit some degree of dependency on the price level. Moreover, the change in the natural logarithms of the stock price from time *t-1* to time *t* represents the yield with continuous compounding from holding the stock over two consecutive intervals (Fama, 1965, p.43).

Graphing the time series of index weekly returns in Figure 2 reveals that the data no longer exhibits the general upward trend clearly visible in the primary data. Summary statistics for index weekly returns are depicted in Table 3. In order to confirm that the returns are stationary, the DF-GLS test is employed once more to each index, this time including a constant but no trend, and the results are displayed in Table 4. The null hypothesis can now be rejected for both indices at all conventional levels of significance, and the returns series is therefore concluded to be stationary. We also plot the return residuals for each index in Figure 3, where residuals are a measure of statistical error defined as the difference between the actual observation and the predicted observation. The residual series of the index returns also both appear to be stationary, although it is perhaps worth noting that there appears to be a much larger disturbance around the year 2000 in the NASDAQ 100 than in the FTSE 100.[5]Furthermore, we can observe a significant outlier in the NASDAQ 100 series around the year 1994 in Figures 2 and 3 that does not appear to be present in the FTSE 100 series. However, the DF-GLS test results have verified that this outlier is not statistically significant enough to conclude that the data is not stationary. Thus, having now confirmed the stationarity of the data, the following section will discuss the empirical methodology employed to assess randomness.

V. Description of Empirical Methods

In our analysis, we subject both indexes to both parametric and non-parametric tests, in order to ensure that our results are robust to alternative empirical methods. Devised over many years, there exist a wide variety of techniques commonly employed to assess the validity of

[5]This may be explained by the fact that the US experienced a recession in the early 2000s due to the dot com bubble, whereas the UK ultimately managed to avoid recession during this period.

RWH in stock markets. This paper will make use of some of the most recognized methods in order to evaluate the hypothesis of random walk across the sample period.

Parametric Methods

Autocorrelation Test

The autocorrelation test is a parametric method used to test the hypothesis that returns are not correlated over time. Put differently, it assesses the probability that the process generating the observations is a series of *i.i.d.* random variables. If the series follows a random walk, the autocorrelation function should not be statistically different from zero for all lags. Conversely, if the series is not random, not all autocorrelations will be statistically different from zero. The following test for correlation closely adheres to the methods implemented by Fama (1965), Solnik (1973), and Cooper (1982).

We compute the autocorrelation coefficients for each index across a number of lagschosen as k=min(T/2 – 2, 40), where T denotes the sample size, as is customary in the existing literature.The autocorrelation coefficients are generated using the following formula:

$$\gamma_{ik} = \frac{\text{cov}(\varepsilon_{i,t}, \varepsilon_{i,t-k})}{\text{var}(\varepsilon_{i,t})} \qquad (3)$$

where γ_{ik} is the autocorrelation coefficient of index i at lag k, and $\varepsilon_{i,t}$ is the residual of the returnsdefined as:

$$\varepsilon_{i,t} = r_{i,t} - \hat{r}_{i,t} \qquad (4)$$

where $r_{i,t}$ is the observed value of returns as defined in (2), and $\hat{r}_{i,t}$ is the predicted value of returns of index i at time t given by the simple linear regression line.The null hypothesis states that residual returns are not correlated over time (i.e., γ_{ik} = 0). The alternative hypothesis holds that residuals do exhibit some degree of correlation over time ($\gamma_{ik} \neq 0$).

In order to determine whether all autocorrelation coefficients are *simultaneously* equal to zero, we employ the portmanteau Q-statistic developed by Ljung and Box (1978). Under the null hypothesis the test statistic asymptotically follows a chi-square distribution, and is defined as follows:

$$Q = T(T+2)\sum_{k=1}^{L}\left(\frac{(\gamma_k)^2}{(T-k)}\right) \tag{5}$$

where T again represents the sample size, γ_k is as defined in (3), and L represents the number of lags of autocorrelation. If the associated p-value is larger than our chosen level of significance, then we cannot reject the null hypothesis that the data obtained are random.

Lo and MacKinlay (1988) Variance Ratio Test

Variance ratio methodology generally involves testing the hypothesis of random walk against several alternative stationary processes (Charles, A. and Darné, O., 2009).The Lo and MacKinlay variance ratio test is frequently employed by academics assessing the validity of RWH in stock markets, and is founded on the principle that, for all sampling intervals, the variance of increments of a random walk process X_t is linear (Chen, 2008). Put differently, a stochastic process is identified as being a random walk if the sample variance of $X_t - X_{t-k}$ is comparable to k times the sample variance of $X_t - X_{t-1}$, where k denotes the investment return horizon. Thus, Lo and MacKinlay (1988) define the variance ratio at lag k as:

$$VR(k) = \frac{\sigma^2(k)}{\sigma^2(1)} \tag{6}$$

where

$$\sigma^2(k) = \frac{\mathrm{var}(x_t - x_{t-k})}{k} \tag{7}$$

Following the procedures of Lo and MacKinlay (1988), Chen (2008), and Hiremath (2014), we set $X_t = \ln P_t$, where P_t denotes the closing price of the stock. $X_t - X_{t-k}$ can subsequently be interpreted as the return over a horizon period of length k.

The null hypothesis stipulates that returns are serially uncorrelated; the variance ratio must not be statistically different from unity in order for the process to be identified as a random walk (Campbell, J.Y., Lo, A.W., and MacKinlay, A.C., 1997). A variance ratio larger than unity at a given horizon length *k* implies positive autocorrelation, whilst a variance ratio less than one indicates negative autocorrelation (Hiremath, p.26). Since we are dealing with sample data, the estimators of $\hat{\sigma}(k)$ will be used to calculate the variance ratios in place of $\sigma(k)$.Letting the number of observations be $nk+1$, $X_0, X_1, ..., X_{nk}$, where *k* denotes the horizon length and *n*represents the number of periods of length *k*in 1,…,T,the equationsused to calculate the variance ratio are as follows:

$$\hat{\sigma}^2(1) = \frac{1}{nk-1}\sum_{t=1}^{nk}(X_t - X_{t-1} - \hat{\mu})^2 \tag{8}$$

$$\hat{\sigma}^2(k) = \frac{1}{m}\sum_{t=k}^{nk}(X_t - X_{t-k} - k\hat{\mu})^2 \tag{9}$$

where

$$\hat{\mu} = \frac{1}{nk}\sum_{t=1}^{nk}(X_t - X_{t-1}) = \frac{1}{nk}(X_{nk} - X_0) \tag{10}$$

and

$$m = k(nk - k + 1)\left(1 - \frac{k}{nk}\right) \tag{11}$$

Assuming that *k* is fixed as the sample size extends to infinity, Lo and MacKinlay (1988) employ standard approximations to determine the asymptotic distribution of variance ratios. Under the null hypothesis of homoscedastic increments (i.e. X_t is i.i.d.) and a variance ratio equal to unity, the asymptotically standard normal test statistic is calculated as:

$$Z(k) = \frac{[VR(k) - 1]}{\sqrt{\theta(k)}} \sim N(0,1) \tag{12}$$

where

$$\theta(k) = \frac{2(2k-1)(k-1)}{3k(nk)} \tag{13}$$

where $\theta(k)$ denotes the asymptotic variance of the distribution. Furthermore, on account of the fact that financial time series are often prone to conditional heteroscedasticity, Lo and MacKinlay (1988) propose a second test statistic $Z*(k)$ which is asymptotically standard normal and also robust to heteroscedasticity:

$$Z*(k) = \frac{[VR(k) - 1]}{\sqrt{\theta*(k)}} \sim N(0,1) \tag{14}$$

wherethe asymptotic variance is defined as:

$$\theta*(k) = \sum_{j=1}^{k-1} \left[\frac{2(k-j)}{k} \right]^2 \hat{\partial}(j) \tag{15}$$

and

$$\hat{\partial}(j) = \frac{\sum_{t=j+1}^{nk} (X_t - X_{t-1} - \hat{\mu})^2 (X_{t-j} - X_{t-j-1} - \hat{\mu})^2}{\left[\sum_{t=1}^{nk} (X_t - X_{t-1} - \hat{\mu})^2 \right]^2} \tag{16}$$

An important consideration when employing variance ratio tests is that limiting distributions are used to approximate sampling distributions of the variance ratio test statistics, and as such the empirical application of such tests is impeded by poor approximations of the sampling distributions when the sample size is small. Fundamentally, the ability of the test to reliably infer the sampling distribution is contingent upon the horizon length k (Charles and

Darné, 2009, p.7). In particular, large *k* relative to the sample size has been shown to result in biased and right skewed statistics (Lo and MacKinlay, 1989) which can weaken the power of the test, and consequently Lo and MacKinlay recommend only employing the variance ratio test for $k \leq 16$. Noting that our primary analysis employs 1516 returns observations, we can therefore deduce that any negative effects from this issue are minimal as 16/1516 yields a ratio of 0.0106.

Moreover, the Lo-MacKinlay variance ratio test investigates whether the variance ratio is equal to unity for a particular horizon length, thereby rendering the method sequential in nature. According to Chow and Denning (1993), assessing RWH for an individual value of *k*increases the probability of Type I error, since true randomness would require that the variance ratios should equal one for all horizon lengths *simultaneously*. As a result, the Lo-MacKinlay variance ratio test is often employed in conjunction with a multiple variance ratio test, in order to ascertain the validity of RWH when various *k* are jointly subjected to the testing procedure.

Chow and Denning (1993) Multiple Variance Ratio Test

Extending the work on variance ratio tests favoured by Lo and MacKinlay, Chow and Denning (1993) proposed a multiple variance ratio test which addressed the theoretical challenges imposed by the sequential nature of the Lo-MacKinlay test. Following Hochberg's (1974) method for comparing a group of variance ratios for several distinct horizon lengths with unity, Chow and Denning propose an empirical method for assessing RWH which controls for the overall test size.

The null hypothesis under this test necessitates that each variance ratio in a set of *m* estimates is jointly equal to one, and the null is rejected if any of the *m* estimated statistics is statistically different from unity. More specifically, whereas the individual hypothesis test of Lo and MacKinlay has null hypothesis $M_r(k) = VR(k) - 1 = 0$, the joint hypothesis test of Chow and Denning considers a set of *m* tests $\{M_r(k_i) \mid i = 1, 2, ..., m\}$ associated with a set of predetermined lag lengths $\{ k_i \mid i = 1, 2, ..., m \}$,where the null and alternative hypotheses are as follows:

$$H_{0i} : VR(k_i) = 1 \text{ for all } i = 1, 2, ..., m \quad (17)$$

(18)

$H_{1i} : VR(k_i) \neq 1$ for at least one $i = 1, 2, ..., m$

In order to assess the validity of RWH, Chow and Denning (1993) propose the following test statistic for homoscedastic increments:

$$CD_1 = \sqrt{T} \max_{1 \leq i \leq m} | Z(k_i) | \quad (19)$$

where T denotes sample size and $Z(k_i)$ is as defined in (11).Derived from the notion that our decision with regards to the rejection of the null hypothesis can be made conditional on the maximum absolute value of individual variance ratio test statistics (Chen, 2008, p.100), this test statistic follows the Studentized Maximum Modulus (SMM) distribution with m and T degrees of freedom.[6]For any given level of significance, the null hypothesis is rejected where the test statistic CD_1 exceeds the $[1-(\alpha^*/2)]^{\text{th}}$ percentile of the standard normal distribution, with α* defined as $\alpha^* = 1-(1-\alpha)^{1/m}$ (Charles and Darné, 2009).

Furthermore, Chow and Denning (1993) also propose a heteroscedasticity robust test statistic defined as:

$$CD_2 = \sqrt{T} \max_{1 \leq i \leq m} | Z^*(k_i) | \quad (20)$$

where $Z^*(k_i)$ is as defined in (14) and the critical values are equivalent to those derived for the statistic under homoscedasticity. Moreover, ensuring that the ratiok/T is small is a necessary prerequisite to using the multiple variance ratio test, just as it waswith the Lo-MacKinlay test (Charles and Darné, 2009).

Non-Parametric Methods

Whereas the ability of parametric tests to infer reliable conclusions is contingent upon the assumptions they impose on the data distribution, non-parametric tests exhibit no such sensitivity. Financial time series data are often non-normal and non-linear, and so conducting

[6]Critical values for the test can be found in Stoline and Ury (1979). Moreover, following standard asymptotic theory, when T is large, the limiting distribution can be used to estimate the critical values.

empirical tests which do not impose restrictions on the returns distribution is warranted. Furthermore, as discussed previously in SectionII, results from earlier studies have been shown to be dependent upon the empirical methods employed by the researchers, and so conducting parametric tests in conjunction with non-parametric tests increases the robustness of our results.

Runs Test

The runs test (Bradley, 1968) is one of the primary non-parametric methods used by researchers to determine the validity of RWH. Siegel (1956) defines a run as “a succession of identical symbols which are followed or preceded by different symbols or no symbol at all” (p.52). In order to determine the validity of RWH, we can therefore characterize a run as the succession of consecutive changes in the series of returns, and identify it as positive (negative) wherever a sequence is positive (negative). Similarly, the run is zero if there are no changes in the sequence (Hiremath, 2014, p. 27). The length of a run is delineated by the number of consecutive positive, or negative, values.

In a truly random dataset, the probability of a run – that is, the likelihood that the $(j+1)^{th}$ value will differ from the j^{th} value – follows a binomial distribution (NIST/SEMATECH, par.2). Therefore, the total number of runs in the series serves as an indicator of the extent of randomness, since an excessive tendency towards one particular type of run is itself indicative of a trend (Muthotya, 2013, p. 50). If the number of runs is less than the expected number, this reflects the overreaction of the market to new information (Poshokwale, 1996, p.89). Conversely, a higher than expected number of runs suggests a delayed response to new information (Muthotya, 2013, p.54). In order to test the null hypothesis that our data is generated by a random process, we assess RWH by testing whether the total number of runs in our series is statistically different from the number of runs expected in a random series containing the same number of observations.

We calculate the expected number of runs by:

$$\overline{R} = \frac{2n_0 n_1}{T} + 1 \qquad (21)$$

where n_0 denotes the number of negative runs, n_1 is the number of positive runs, and T is the total number of observations. The Z-statistic is then computed by calculating:

$$Z = \frac{R - \overline{R}}{s_R} \tag{22}$$

$$s_R^2 = \frac{2n_0 n_1 (2n_0 n_1 - n_0 - n_1)}{(n_0 + n_1)^2 (n_0 + n_1 - 1)} \tag{23}$$

where R is the number of actual runs and S_R^2 is defined as the standard deviation of the returns series. We then reject the null hypothesis of random returns if $|Z| > Z_{1-\alpha/2}$, where α represents the chosen level of significance. Small sample runs tests make use of critical values described in Mendenall, Wackerly, and Scheaffer (2008). However, when the sample is large (i.e. when the number of positive runs and negative runs each exceed ten) the test statistic can be approximated as following a standard normal distribution.

<u>The Brock, Dechert and Scheinkman (BDS) Test</u>

Brock, Dechert, and Scheinkman (1987) developed a non-parametric method, commonly known as the BDS test, to assess randomness in a dataset by detecting possible deviations from independence. It is important to note that unlike the autocorrelation test which only has the ability to detect linear dependence, the BDS test can detect various types of dependence, including linear and non-linear dependence and chaos.[7] Thus, failing to reject our null hypothesis for the autocorrelation test does not necessarily mean that our data follows a random walk – only that it does not exhibit linear dependence. In order to rule out other types of time-based dependencies, the BDS test must also be conducted.

After anypossible linear structure has been removed from the time series by first differencing or some other method of de-trending, the BDS test null hypothesis is that the underlying data generating process is i.i.d. The alternative hypothesis allows for linear dependence, non-linear dependence, and chaos. The theory behind the BDS test is rooted in the

[7]Characterized by intermittent periods of high volatility, chaotic processes are deterministic and non-linear processes which can bear a similar resemblance to financial time series (Hsieh, 1991).

work on correlation integrals in time series pioneered by Grassberger and Procaccia (1983). The discussion of the BDS test presented here closely follows Taylor (2005); the reader is encouraged to consult this reference for further details and clarification.

For a sample of T observations $\{x_1,...,x_T\}$, the correlation integral $C_m(T,\varepsilon)$ must be computed before the BDS test can be conducted on the returns observations. Denoting the embedding dimension and the distance m and ε respectively, the equations are as follows:

$$I(x_s,x_t,\varepsilon)=\begin{cases}1 & \text{if } |x_s-x_t|<\varepsilon \\ 0 & \text{otherwise}\end{cases} \tag{24}$$

$$I_m(x_s,x_t,\varepsilon)=\prod_{k=0}^{m-1} I(x_{s+k},x_{t+k},\varepsilon) \tag{25}$$

$$C_m(T,\varepsilon)=\frac{2}{(T-m)(T-m+1)}\sum_{s=1}^{T-m}\sum_{t=s+1}^{T-m+1} I_m(x_s,x_t,\varepsilon) \tag{26}$$

where $x_1 \le x_s \le x_T$ and $x_1 \le x_t \le x_T$. The function $I(x_s,x_t,\varepsilon)$ identifies whether the observations at times s and t are close to each other at a given distance ε, and the product $I_m(x_s,x_t,\varepsilon)$ is equal to unity if and only if the two m-period histories $(x_s,x_{s+1},...,x_{s+m-1})$ and $(x_t,x_{t+1},...,x_{t+m-1})$ are sufficiently close to each other such that each x_{s+k} is near x_{t+k}. Therefore, we can also define our approximation of the correlation integral as the fraction of pairs of m-period histories that are close to one another (Taylor, 2005, p.136).

As the sample size extends to infinity, observations from many processes define the limit of correlation integrals as:

$$C_m(\varepsilon)=\lim_{T\to\infty} C_m(T,\varepsilon) \tag{27}$$

If the data are indeed generated by an i.i.d. process, then the probability that a pair of observations are near one another is the same for all possible combinations of data points, and consequently the probability that there exists *m* consecutive near pairs is simply the product of the equal probabilities:

$$C_m(\varepsilon) = C_1(\varepsilon)^m \qquad (28)$$

However, when the data generating process is chaotic, the conditional probability that x_{s+k} is near to x_{t+k}, given that there has already been a near pair at some earlier time period *j*, is greater than the unconditional probability and thus:

$$C_m(\varepsilon) > C_1(\varepsilon)^m \qquad (29)$$

Considering these properties of correlation integrals led Brock, Dechert, and Scheinkman to study a random variable which, as the sample size increases, converges to a normal distribution with mean zero and variance V_m for i.i.d. processes. The BDS test statistic is calculated as:

$$W_m(\varepsilon) = \sqrt{\frac{T}{\hat{V}_m}}[C_m(T,\varepsilon) - C_1(T,\varepsilon)^m] \qquad (30)$$

which has been demonstrated to be asymptotically standard normal (Brock, Hsieh, and Le Baron, 1991). $\hat{V}_m$ represents a consistent estimator of the variance of the distribution and is given by:

$$\hat{V}_m = 4\left(K^m + (m-1)^2 C^{2m} - m^2 K C^{2m-2} + 2\sum_{j=1}^{m-1} K^{m-j} C^{2j}\right) \qquad (31)$$

as discussed in Brock, Dechert, Scheinkman, and LeBaron (1996). Here, $C = C_1(T,\varepsilon)$ and K is calculated as:

$$K = \frac{6}{(T-m-1)(T-m)(T-m+1)}\left\{\sum_{s=2}^{T-m}\left[\sum_{r=1}^{s-1} I_m(x_r, x_s, \varepsilon)\right]\left[\sum_{t=s+1}^{T-m+1} I_m(x_s, x_t, \varepsilon)\right]\right\}, \qquad (32)$$

where $I_m(x_s, x_t, \varepsilon)$ and $I_m(x_r, x_t, \varepsilon)$ are as defined in (23) and (24). As demonstrated by Hsieh (1991), with regards to detecting deviations from i.i.d. processes and alternative specifications, the BDS test exhibits considerable power.Conventionally, the BDS test is conducted on the sample of returns observations at several distinct *m* and ε, where ε assumes values between one half and two times the standard deviation. Furthermore, it is generally accepted that a minimum of 500 observations is required in order to infer reliable results from the BDS test (Abbas, 2014, p.321) and that the accuracy of results diminishes for $m \geq 5$ (Brock et. al, 1991).

VI. Discussion of Empirical Results

Results from Autocorrelation Tests

Having previously confirmed the stationarity of the residual series, we plot the autocorrelation functions using STATA computing software with a 5% significance level. As shown in Figure 4, all lags except two (approximately lag 26 and lag 38)for the NASDAQ 100 are within the confidence bands, suggesting that the NASDAQ residual series do indeed follow a white noise process. However, the ACF for the FTSE 100 has a number oflags significantly outside the confidence bands, which could indicate that there is some degree of non-randomness in the data.

In order to assess the overall randomness present in the time series, we apply the Ljung-Box test. Previous studies investigating the RWH have found the significance of autocorrelations to be dependent upon the chosen horizon length of the returns; conducting the Ljung-Box test serves to investigate the joint null hypothesis that all autocorrelations are *simultaneously* equal to zero.It is evident from the results in Table 5 that we cannot reject the null hypothesis for the NASDAQ 100index at the five percent level of significance, as we obtain a p-value larger than 0.05. On the contrary, the null is rejected at the five percent level of significance (and indeed at all conventional significance levels) for the FTSE 100, and thus we conclude from the Ljung-Box test that all autocorrelations are not simultaneously equal to zero for the UK index. Put differently, there appears to be some degree of serial correlation in the FTSE 100 residuals series. Therefore, the results from the autocorrelations test appear to indicate that the RWH is validated for the US index, but not the UK index.

Results from Lo-MacKinlay Variance Ratio Tests

The variance ratios VR(k) for both indices are computed by entering the raw data into Microsoft *Excel* and inputting the necessary formulae. Recorded in the primary rows of Table 6, the variance ratio for each index at each horizon level is approximately unity, although it is readily apparent that the variance ratios for the NASDAQ 100 are marginally higher than those reported for the FTSE 100 at each respective investment horizon. The horizon levels reported are 2, 4, 8, and 16, as is standard procedure when conducting Lo-MacKinlay tests.

Retrieved using R computing technology, the variance ratio homoscedastic test statistics and heteroscedasticity robust test statistics for both indices are reported in the second and third row parentheses of Table 6. The Lo-MacKinlay test dictates that the null hypothesis of random fluctuations in returns should be rejected at the 5% level of significance wherever the absolute value of the test statistic exceeds 1.96. Notably, we do not reject the null hypothesis for either index at any of the standard investment horizons, supporting RWH in both the U.S. and U.K. markets.

Results from Chow and Denning Multiple Variance Ratio Tests

In order to ensure the reliability of Lo-MacKinlay test results, the Chow and Denning multiple variance ratio test is conducted. The objective of this exercise is to guarantee that the null of RWH cannot be rejected even when we test the joint null hypothesis that the variance ratios for all investment horizons simultaneously are not statistically different from unity. Recall that the Chow and Denning multiple ratio test is only reliable when k/T is small, where k denotes the investment horizon and T represents sample size. Since the longest horizon length tested is 16 and the returns series contains 1540 observations, their ratio is approximately 0.01, and thus we can proceed with the test.

Conducted once again using R computing software, the Chow and Denning homoscedastic test statistics and heteroscedasticity robust test statistics are reported in Table 7. Recall that the test statistics for the Chow and Denning test follow an SMM distribution. R software generates these critical values electronically, and presents us with a critical value of 2.490915 at the 5% significance level. The test statistics for the FTSE 100 are markedly higher than those reported for the NASDAQ 100 index; however, none of the statistics exceeds the critical value of 2.490915, and so we cannot reject the null hypothesis for either index at the 5% level of significance. Therefore, the results from the Chow and Denning multiple variance ratio tests also support RWH in both markets.

Results from Runs Tests

In order to assess the robustness of the results to alternative empirical methods, we next use STATA to perform the runs test; a non-parametric approach which tests whether the residuals are mutually independent. Recall that if the data does indeed follow a random walk, then the observations should be independent and identically distributed.

The results of the runs tests depicted in Table 8 reveal that the actual number of runs for both indexes is very close to the number of runs that would be expected if the data were truly generated by a random process. The negative Z-score for the FTSE 100 indicates that the observed number of runs is less than the expected number, while the positive Z-score of the NASDAQ 100 reflects that the observed number of runs is greater than the expected number. However, since the Z-score for the UK index is greater than -1.96 and the Z-score for the US index is less than 1.96, we cannot reject the null hypothesis that the residuals are independent at the five percent level of significance for either index.

Results from BDS Tests

Finally, we conduct the BDS test for both indices in order to determine whether the underlying data is generated by an i.i.d. process.In accordance with general procedure, we test distances between one half and two times the standard deviation of the returnsseries, and we use embedding dimensions between 2 and 5 (recall that conducting the test with dimensions greater than five diminishes the reliability of the results).

Performed on the returns data series, R software reports the results for the NASDAQ 100 and the FTSE 100 BDS tests, which are summarized in Table 9 and Table 10 respectively. The primary rows report the test statistics, while the values in parentheses denote the corresponding p-values. The BDS test statistics are asymptotically standard normal, and once again we assess the validity of the null hypothesis using a 5% significance level to maintain consistency with the other empirical methods. From Table 9 and Table 10, each p-value is approximately zero and is therefore less than the critical value of 0.05. Indeed, the BDS results are significant even at the 1% significance level for both indices. Consequently, the BDS test is the only test for which RWH is strongly refuted for both the NASDAQ 100 and the FTSE 100.

VII. Summary and Discussion of the Empirical Results

The primary focus of this study was to investigate the validity of RWH for markets in two countries with comparable economic development, market capitalization, and a similar number of trading companies, in order to test how controlling for these factors impacted our results.

Analyzing weekly closing price data from October 1985 through April 2015 we find that results broadly seem to support RWH in both markets. When subjected to the same three parametric tests, we fail to reject the null hypothesis of randomness in all tests for the NASDAQ 100, and for two of the three tests for the FTSE 100. The only discrepancy in the results for the parametric tests is that the FTSE 100 exhibits a statistically significant degree of correlation in the returns series when subjected to the autocorrelation test.

With regards to non-parametric tests, we fail to reject the null hypothesis of random returns for both indices when the Runs test is conducted. Conversely, when the BDS test is performed, RWH is strongly refuted for both indices at all conventional levels of significance.

It is perhaps important to note that, typically, previous studies have strongly rejected RWH when the data is subjected to the BDS test (Hsieh 1989, De Grauwe, Dewachter and Embrechts

1993, Steurer 1995, Brooks 1996). Some experts believe that this tendency may be due to the fact that the BDS test is capable of detecting general dependence, whereas most statistical tests are only capable of detecting linear dependence. Indeed, Lim, Azali, and Lee (2003)suggest that it is the breakthroughs in non-linear dynamics and chaos theory which have prompted the discovery of “dependencies in the underlying financial time series that often appear completely random to standard linear statistical tests, such as serial correlation tests, non-parametric runs tests, variance ratio tests and unit root tests” (p. 42).

Rejection of the null of an independent and identically distributed process implies that there is some pattern in the behaviour of the stock prices that appears more frequently than one would expect if the data were indeed truly generated by an i.i.d. process. However, at present, existing empirical methods are not sophisticated enough to determine the exact cause of rejection or indicate *what* pattern exists, if any. Further diagnostic tests, in particular non-parametric tests which focus on non-linear dynamics, are warranted in order to infer more consistent and reliable results.

Empirical analysis has shown that RWH is broadly supported for both the U.K. index and the U.S. index, although improved testing procedures are needed to further investigate possible nonlinear types of dependence. The following section will ascertain the robustness of our results when various sub-periods of the sample data are used, and when alternative market indices are employed in place of the FTSE 100 and the NASDAQ 100.

VIII. Robustness Analysis

In order to confirm that our results are consistent and reliable, we perform the parametric and non-parametric tests once more with different stock market indices, using an alternative observation horizon length, and using sub-samples of our original time series. If our results from these further tests agree with our original findings in that they also broadly support RWH in both markets, then we can conclude that our results appear robust to the choice of market indices, horizon length,and the elected sample period.

Analyzing Alternative Indices

Data Selection and Stationarity

To ensure that the results from our empirical analysis are robust to the market indices, we select another index from each market and repeat the testing procedures using weekly observations across the same sample period. From the NASDAQ stock exchange market, we select the NASDAQ Composite index. Also a capitalization weighted index, the NASDAQ Composite contains data from several thousand companies and is weighted towards technology companies. From the London Stock Exchange, we select the FTSE All-Share index, which is again capitalization weighted and includes the majority of companies listed on the London Stock Exchange.Summary statistics for the weekly closing prices of each alternative index are illustrated in Table 11.

As expected, the raw price data from both indices exhibits a clear upwards trend, as is evidenced from Figure 5.Performing the DF-GLS test confirms the non-stationarity of the U.S. data, as we cannot reject the null hypothesis of a unit root at any conventional significance level for any lag length of the NASDAQ Composite index. Furthermore, the results from the DF-GLS tests also generally confirm the non-stationarity of the U.K. data, as we fail to reject the null hypothesis for any lag length at the 1% and 5% levels of significance(although, at the 10% level of significance we do reject the null at lags 2 and 23). The results are reportedin Table 12.

Having confirmed the non-stationarity of the price data, we once again compute the returns and repeat the DF-GLS procedure. Summary statistics of the index weekly returns are given in Table 13. From Figure 6 we can see that the returns data does appear to be stationary, and in Table 14 the DF-GLS results confirm this, as we now reject the null hypothesis at all significance levels and at all lag lengths for both indices.

Results from Parametric Tests

In order to determine whether the residual series of our alternative indices are correlated over time, we once again perform the autocorrelations test. Figures 7 and 8 illustrate the residual series and the autocorrelation functions of each index respectively. In contrast to our earlier findings, it appears that there are more observations which lie outside the 95% confidence bands for the NASDAQ Composite index than there were for the NASDAQ 100 index, and there are also a significance number of data points outside the confidence bands for the FTSE All-Share index. Performing the autocorrelation test yields p-values which are less than 0.05 for both indices, indicating thatRWH is rejected at the 5% significance level. Conveyed in Table 15, there appears to be some degree of serial correlation in the residual series for both of our alternative indices, and therefore the autocorrelations testsdo not support RWH in either market.

Conducting the Lo-MacKinlay variance ratio test once morepresents us with results similar to those obtained when the NASDAQ 100 and FTSE 100 indices were tested. Table 16 reports the variance ratios for the NASDAQ Composite and the FTSE All-Share in the primary rows, while the values in the second and third row parentheses denote the homoscedastic test statistic and the heteroscedasticity robust test statistic respectively. With regards to the FTSE All-Share, we cannot reject the null of random returns at the 5% significance level for any investment horizon as we obtain test statistics whose absolute value is less than 1.96. Furthermore, the only statistic to be rejected for the NASDAQ Composite index is the statistic for homoscedastic increments at a horizon length of 8, and consequently the results obtained from the Lo-MacKinlay tests on alternative indices closely resemble the results acquired from our original tests.

We next perform the Chow andDenning multiple variance ratio test in order to assess the joint null hypothesis that all variance ratios simultaneously are not statistically different from unity. Once again, our critical value is 2.490915 at the 5% level of significance as our test statistic follows a SMM distribution.Table 17 presents the results from the Chow and Denning multiple variance ratio test, and it is clear that we cannot reject the null hypothesis for either index since the test statistics for homoscedastic increments and the heteroscedasticity robust test statistics all have an absolute value less than the critical value. Consequently, the results from the Chow and Denning test also support RWH for both of our alternative indices.

Results from Non-Parametric Tests

Proceeding now to non-parametric methods, Table 18 reports the results from the runs tests. For both the NASDAQ Composite and the FTSE All-Share, it appears that the observed number of runs is less than the expected number if the data were truly random, as is evidenced by the negative Z-scores of each index. However, we cannot reject the null hypothesis of random returns at the 5% significance level since both Z-scores obtained are less than 1.96 in terms of absolute value. Therefore, the results from the runs tests also defend the validity of RWH in both financial markets.

We perform the BDS test once more for our alternative indices using the same range of values for the distance and embedding dimensions as previously employed in this paper. The results are documented in Tables 19 and 20. The primary rows report the test statistics whilst the values in parentheses denote the corresponding p-values. Yet again, the p-values are approximately zero for both of our alternative indices and therefore the BDS results reject the null of randomness at all conventional significance levels in both the UK and US markets.

Conclusions from Alternative Indices

Clearly, with the exception of the results from the autocorrelations tests, the results from testing alternative indices broadly agree with the results from analyzing our initial data. With our original indices we find that results from autocorrelations tests support RWH in the NASDAQ but not in the FTSE; however when we subject our alternative indices to the autocorrelations tests we find that there is no support for RWH in either market.

Interestingly, the NASDAQ 100 does not include financial securities such as those from investment companies in its calculations, whilst the NASDAQ Composite index does not discriminate against such securities. Therefore, one possible reason for the discrepancy in the results from the autocorrelations tests is that the presence of large financial and investment companies in the NASDAQ Composite index are driving the behaviour of the index, thereby causing the autocorrelations to be more pronounced over time. Furthermore, the NASDAQ Composite index has a strong focus on technology firms, and it is possible that the stock prices of these large technology companies may be more likely to exhibit common patterns that can cause the index to become more autocorrelated during some of the phases of the business cycle.

The results from the rest of the parametric tests and from the non-parametric tests on our alternative indices all closely follow the results obtained from our original indices. RWH is once again supported by the Lo-MacKinlay test, the Chow and Denning test, and the runs test, while strongly rejected by the BDS test. As discussed earlier, it is possible that this rejection may be due to some underlying non-linear deterministic behaviourwarranting further investigation. Overall, it appears that our results seem to be largely unaffected by the choice of index, and that RWH is generally valid for our chosen markets with similar economic development and market capitalization.

Adjusting the Horizon Length of Observations

<u>Data and Stationarity</u>

As a second robustness check, we adjust the horizon length of the returns data for our original indices. Our tests thus far have employed a sample of weekly observations for each index. Data are also available for monthly and daily time intervals. Since using monthly data would significantly reduce the number of observations in our sample, we collect daily data which will serve to increase our sample size dramatically. However, as discussed previously, we must consider that potential biases may become statistically significant with the use of daily observation intervals.

Conveyed in Table 21 are summary statistics for the daily price data for the NASDAQ 100 and FTSE 100indices. Despite the fact that our horizon length is now much shorter than before, the mean and standard deviations of the raw price data are close to those retrieved from our original weekly observations. The DF-GLS test results are given in Table 22, and once again we cannot reject the null hypothesis of a unit root for either index at any significance level. Computing daily returns in an attempt to retrieve a time series that is stationary yields summary statistics presented in Table 23, and after performing the DF-GLS test once more for both indices we can now reject the null at all conventional levels of significance. These results are illustrated in Table 24.

<u>Results from Parametric Tests</u>

Now that our data is confirmed to be stationary we can conduct the autocorrelation tests on the return residuals for both indices. Displayed in Table 25, the analysis generates p-values that are once again less than 0.05 for each index, and so we reject the RWH for both indices at the 5% level of significance using daily horizon returns. Indeed, the results are significant even at the1% significance level.

Results from the Lo-MacKinlay tests for each index are depicted in Table 26. The null of random returns cannot be rejected at the 5% significance level for the FTSE 100 index at any investment horizon, which agrees with our results using weekly observation intervals. However, for the NASDAQ 100 index we now reject the null hypothesis at the 5% significance level at horizons 4 and 8 (for both the statistic for homoscedastic increments and the heteroscedasticity robust statistic) and also at horizon16 (for the homoscedastic statistic). This is in contrast to our earlier results using weekly data where we could not reject the null hypothesis at the 5% level for any investment horizon. Therefore, the analysis from the Lo-MacKinlay tests yield mixed results regarding the validity of RWH: random returns seem to be supported in the UK market while broadly refuted in the US market for all investment horizons greater than 2.

Performing the Chow and Denning tests we find further evidence of mixed empirical results. Presented in Table 27, we find that we cannot reject the null hypothesis at the 5% level for the FTSE 100 index, which again agrees with our results from weekly observation intervals. Conversely, the null for the homoscedastic NASDAQ 100 test statistic is rejected at the 5% level of significance, whereas when weekly observations were employed earlier in the paper we could not reject the null at the 5% level.

Results from Nonparametric Tests

The runs test results and BDS test results for daily observation intervals do not support the RWH for either index. Conducting the runs tests reveals that the number of runs for each index was less than the expected number, as is evidenced by the negative Z-scores in Table 28. Moreover, the Z-scores for both indices were significantly greater than 1.96 in terms of absolute value and thus we strongly reject the null hypothesis of random returns at the 5% significance level. In addition, the results from the BDS tests given in Table 29 and Table 30 reveal that – as with weekly observations and the use of alternative indices – we strongly reject the null hypothesis of random returns at all conventional significance levels for both indices.

Conclusions from Daily Observation Intervals

After close analysis of the data using daily observation intervals, we observe that our results do not appear robust to the modified horizon length. Although conclusions from the autocorrelations tests, Chow and Denning tests, and BDS tests agree with our previous analysis, results from the Lo-MacKinlay and runs tests diverge from those derived from our original investigation.Using daily horizon intervals the runs test now refutes RWH in both markets, whereas when weekly observation intervals were employed, the results supported random returns for each index. With regards to the Lo-MacKinlay tests, whereas our original analysis found support for RWH in both markets, employing daily observation intervals provides mixed results. Specifically, the results from the Lo-MacKinlay test on the FTSE 100 index is unaffected by the modified horizon length, however we now reject the null for the NASDAQ 100 at all investment horizons larger than 2.

Therefore, employing daily observations as opposed to weekly observations reveals ambiguous results with regards to the validity of random walk in the stock market. Conclusions drawn from the autocorrelation tests, runs tests, and BDS tests refute random walk for both markets, results from the Chow and Denning tests support random walk for both markets, and results from the Lo-MacKinlay tests are mixed. However, itis quite possible that this ambiguity is due to latent biases becoming statistically significant with the shorter observation interval, as discussed previously.

In particular, consider that the shortest investment horizon for the Lo-MacKinlay test with weekly observation intervals was 14 days, while the longest horizon was 112 days. In contrast, using daily observations implies that our longest horizon is just 16 days – approximately the same length as the shortest horizon employed in our primary analysis. Therefore, all empirical tests which involve the use of investment horizons cover a significantly different length of time relative to the baseline scenario. It follows that a difference in results is not entirely unexpected.

Sub-sampling Original Data

Lastly, we draw subsamples of our original datasets for the NASDAQ 100 and the FTSE 100 in order to assess whether our findings are dependent on the choice of sample period. We do this by eliminating all observations from the original datasets which are related to periods of economic crises, and create subsamples setting the break to be the beginning of a recessionary period.

Justification

Constructing such modified samples where we remove all observations related to recessionary periods is justified for several reasons. Periods of crises are associated with greater insecurity in a number of markets, and this increased market instability is generally reflected by an increase in the volatility of stock market indices. The intuition behind this is simple: once an economy enters recession, firms either profit less from their investments or suffer losses, resulting in lower dividends paid to investors. This decrease in dividend payments consequently renders stocks less attractive assets.

It is possible that the behaviour of stock prices during recessionary periods is responding to increased uncertainty in the financial and economic markets, and therefore stock returns cannot possibly be fluctuating completely at random. For this reason, we eliminate all observations pertaining to major economic crises and instead evaluate whether, under general economic conditions, RWH is valid for the U.S. and U.K. markets.

Identification of Financial Crises

During the course of the original sample period (October 1st 1985-April 13th 2015) there were three major recessionary periods in the United States: the early 1990s recession, the early 2000s recession, and the Great Recession of 2007-2008. The United Kingdom also experienced recessions in the early 1990s and in 2007-2008; however the UK managed to avoid the recession in the early 2000s. As we noted earlier in section IV, from Figure 3 there appears to be a large disturbance around the year 2000 in the NASDAQ 100 residuals that is not readily apparent in the residuals for the FTSE 100.[8] It is likely that this inconsistency in residual pattern is, at least in part, due to the fact that the recession of 2000-2001 was not felt in the United Kingdom. In addition, it is worth noting that the fact that two of these three major economic events were common to both countries serves to underscore the mounting significance of globalization and also highlights the increasing importance of financial markets.

Lasting from July 1990 through March 1991 in the United States (Walsh, C.E., 1993) and from July 1990 through September 1991 in the United Kingdom (Chamberlin, G., 2010), the early 1990s recession was largely due to decreased consumer confidence resulting from rising oil prices following Iraq's invasion of Kuwait, the savings and loans crisis, and the stock market crash of 1987. Following this economic downturn, the U.S. economy then experienced further crisis from March 2001 – November 2001 (NBER, 2003), largely due to the dot-com crash and the 9/11 attacks. Finally, due to the subprime mortgage crisis and subsequent global financial meltdown, the U.S. encountered recession from December 2007 – June 2009 (NBER, 2010) and the U.K. was in recession from June 2008 – September 2009 (Chamberlin, G., 2010).

Data and Stationarity

Removing these dates from our original sample periods, we then create subsamples of weekly observations for the US and UK markets. After eliminating all data points related to the recessionary periods delineated above, we set the break between subsamples to be the start of a recession. Specifically, we obtain 4 subsamples for the NASDAQ 100, and 3subsamples for the FTSE 100 index. Summary statistics detailing these sub-periods of raw price data are presented in Table 31. The DF-GLS test results for the price data of subsamples of each index are portrayed in Tables 33 and 35. Once again, the null of a unit root in the price data cannot be rejected at any conventional significance level for the NASDAQ 100 index. For the FTSE 100, we cannot reject the null in two of the three subsamples, the exception being the third sub-period where the results are significant at least at the 10% level for all lag lengths. It may be that the small sample size of 289 observations is responsible for this unexpected result.

[8] We also noted earlier that there is a significant outlier in the NASDAQ 100 series around the year 1994; however, this does not appear to coincide with any period of major financial crisis.

In order to conduct our empirical analysis, the data must be stationary and so we compute the returns for each index subsample, and report the summary statistics in Table 32. Performing the DF-GLS test once more, as is evidenced from Tables 34 and Table 36, we can now reject the null of a unit root at the 1% level for most lags of each subsample of the NASDAQ 100 and FTSE 100 indices. The only exceptions are the 3rd subsample of the NASDAQ 100 index (where we can only reject at the 1% significance level for lags 1,2, and 4) and the 4th subsample of the NASDAQ 100 index(where we can only reject at the 1% significance level for lags 1 and 2).Therefore, we can conclude thatour data generally appears to be stationary, and we can proceed with our empirical tests.

Results from Parametric Tests

Table 37 presents the results from conducting the autocorrelations test on the residuals of each index subsample.For the NASDAQ 100 index, the only statistically significant result at the 5% level is the third sub-period containing 308 observations. The p-values for all other subsamples are much larger than 0.05, indicating that for these sub-periods there does not appear to be a high degree of serial correlation in the residuals over time. Noting the small sample size of the third sub-period and that the results for all other subsamples are not significant, we conclude that our results from the autocorrelations tests on the NASDAQ 100 generally appear to support RWH in contrast to our earlier findings.

For subsamples of the FTSE 100 index, we find ambiguous results. The first subsample, consisting of 248 observations, is significant at the 5% level which indicates a high degree of correlation in the residuals. Conversely, the second and third sub-periods, containing 868 and 289 observations respectively, are not significant at the 5% level.It is possible that these conflicting results are due to the relatively small sample size of some of the sub-periods, or else due to some underlying effect caused by the removal of observations relating to periods of financial crises. Furthermore, it is important to note that the number of observations employed in the tests will actually be smaller than the number of data points in our original subsample. This is due to the fact that several of the tests involve using lags of the variable, thereby reducing the number of observations.

The Lo-MacKinlay test results for subsamples seem to broadly agree with our original findings for both indices, as is evidenced from Table 38. Generally, the null hypothesis cannot be rejected at the 5% level: the only exceptions are the third sub-period for the NASDAQ 100 where we reject the null of randomness at investment horizons 8 and 16, the first sub-period for the FTSE 100 where we reject the null at investment horizon 4, and the third sub-period for the FTSE 100 where we reject the null at horizons 8 and 16. Otherwise, our analysis closely resembles the results retrieved using the original samples for each index.

Employing the Chow and Denning multiple variance ratio tests also yields results similar to our original findings, as is evidenced in Table 39. The null hypothesis of random returns cannot be rejected for any subsample of the NASDAQ 100 index, and is rejected only for the first subsample of the FTSE 100 index. Again, it is likely that this rejection is due to the small sample size.

<u>Results from Non-Parametric Tests</u>

Table 40 reports the results from the runs tests performed on each subsample of index returns. For both indices, we find that we cannot reject the null of randomness at the 5% level for any subsample, as the absolute value of the Z-scores is less than 1.96 for each period.

Results from the BDS tests for subsamples of the NASDAQ 100 and FTSE 100 are conveyed in Table 41 and Table 42, respectively. Once more, the BDS test proves robust to alternative specifications of the data, and again refutes RWH for both markets. The only cases for which randomness cannot be rejected are presented in red, and pertain to the sub-periods containing the smallest number of observations.

<u>Conclusions from Sub-sampling Original Data</u>

Evidently, our original analysis appears to be generally robust to the choice of sample period, as testing sub-periods of our original data yields similar conclusions. Results from the Lo-MacKinlay tests, Chow and Denning tests, and runs tests on subsamples broadly agree with the conclusions from testing the entire sample period, in that RWH is supported for both markets. Furthermore, the BDS test once again refutes random walk for both indices, and the autocorrelations tests offers mixed conclusions, as was the case in our original investigation.

IX. Conclusions and Recommendations for Future Research

The purpose of this paper was to assess whether stock prices in two well developed markets with high market capitalization follow a random walk, in order to determine whether controlling for these dynamics produces consistent results with regards to the validity of RWH. Using a consistent number of observations across the same sample period, we subjected the NASDAQ 100 and FTSE 100 indices to a variety of empirical tests in an attempt to assess the validity of RWH in each market. Both parametric and non-parametric techniques were employed in order to gauge the robustness of the results to empirical methods, particularly since previous literature has found results to be dependent upon the empirical approach.

Our primary results from testing weekly observations from each index suggested that randomness was broadly supported for both markets. However, the autocorrelation test results were mixed, as RWH was supported for the NASDAQ 100 while refuted for the FTSE. Notably, the BDS test was the only method to strongly reject RWH for both indices.

Indeed, when subjecting our analysis to a number of robustness checks, the BDS test continued to strongly reject random returns for all alternative data specifications. The robustness checks included using different stock market indices for each market, using daily returns as opposed to weekly returns, and using subsamples of our original dataset.

Employing alternative market indices and sub-sampling our original sample period generated results which closely resembled our original findings; however, the autocorrelations tests for the alternative indices rejected random walk in both marketsas opposed to providing mixed evidence. Conversely, our original findings were not robust to the use of daily observation intervals. Using daily horizons, results from the Lo-MacKinlay tests became mixed, and the runs tests refuted RWH for both markets.

Thus, when our original dataset is modified to assess the robustness of our empirical results,the only tests to offer consistent and reliable conclusions are the BDS and Chow and Denning tests. However, since the results from these tests directly oppose each other, this fact provides little insight. Since the use of alternative indices and subsamples leaves our original findings largely unchanged, it is possible that the discrepancies in results when assessing daily horizon returns are due to underlying biases becoming statistically significant with shorter observation intervals. Advanced research and statistical methods are needed to further investigate these inconsistencies. In particular, additional means of testing for non-linear dependence in time series are warranted. Future research must work to develop new means of testing for randomness in datasets, as at present, empirical investigations of random walk are not robust to the techniques employed.

Furthermore, the fact that our results for each index are not robust to the horizon interval suggests that the diversity in professional opinion regarding the validity of RWH cannot be wholly attributed to differences in market capital and economic development. Further research should attempt to control for other factors when testing the legitimacy of RWH in stock prices, such as trading frequency and volume, with the aim of identifying key economic dynamics which influence the results.

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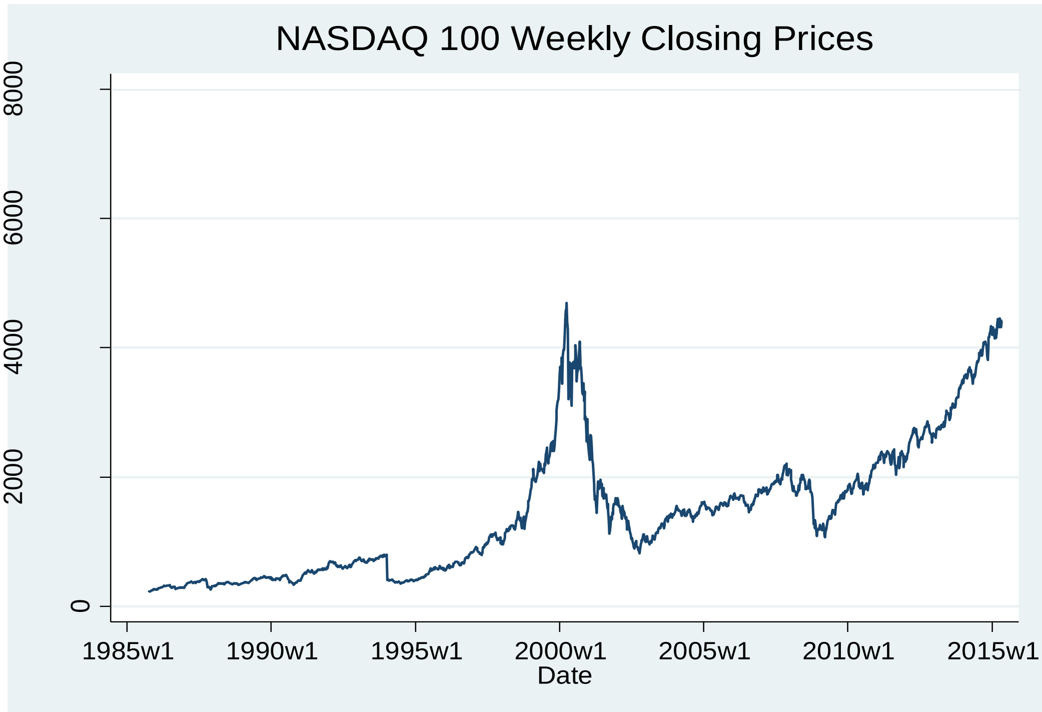
Appendix A: Data Sources

Data	Source
Weekly Closing Price Data for the NASDAQ 100	NASDAQ 100 Historical Prices. Obtained from the NASDAQ via: https://www.google.ca/finance/historical?q=INDEXNASDAQ%3ANDXandei=Dp6BVfmiEYGH2Aa_4YKoDw
Weekly Closing Price Data for the FTSE 100	FTSE 100 Historical Prices. Obtained from the LSE via: https://www.google.ca/finance/historical?q=INDEXFTSE%3AUKXandei=rp6BVZiYH8O72Aa7h4DwBw
Weekly Closing Price Data for the NASDAQ Composite	NASDAQ Composite Historical Prices. Obtained from the NASDAQ via: https://www.google.ca/finance/historical?q=INDEXNASDAQ%3A.IXICandhl=enandei=YX69VfnpI5TwjAHxxqf4Dg
Weekly Closing Price Data for the FTSE All-Share	FTSE All-ShareHistorical Prices. Obtained from the LSE via: https://www.google.ca/finance/historical?q=INDEXFTSE%3AASXandhl=enandei=tH69VejLE5bLjAHQ7IbAAw
Daily Closing Price Data for the NASDAQ 100	NASDAQ 100 Historical Prices. Obtained from the NASDAQ via: https://www.google.ca/finance/historical?q=INDEXNASDAQ%3ANDXandei=vH29VaHUGMStjAHclbnoDg
Daily Closing Price Data for the FTSE 100	FTSE 100 Historical Prices. Obtained from the LSE via: https://www.google.ca/finance/historical?q=INDEXFTSE%3AUKXandhl=enandei=IX69VdmUNYjHjAGpkoeADw

Appendix B: Plots and Figures

Figure 1: Index Closing Prices



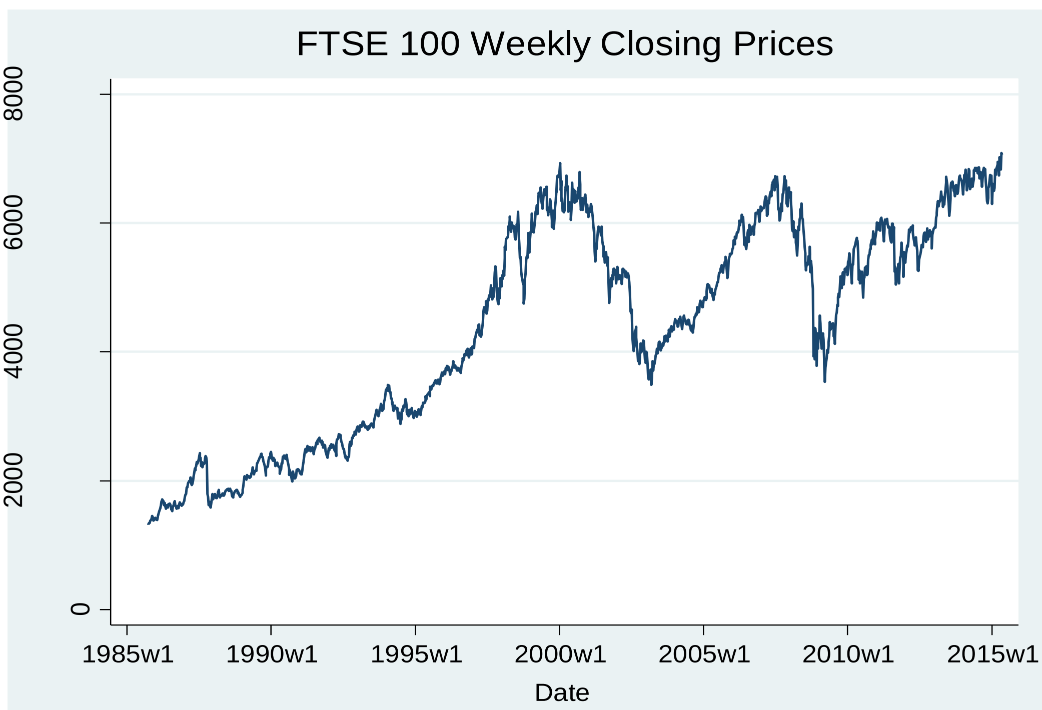
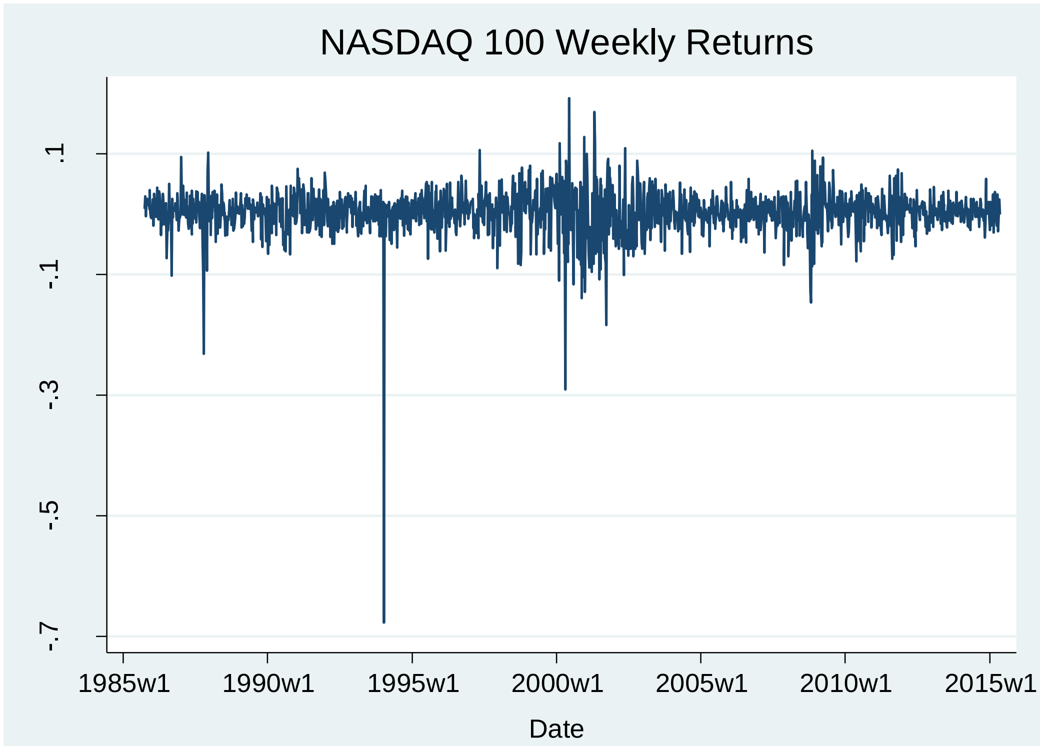


Figure 2: Index Returns



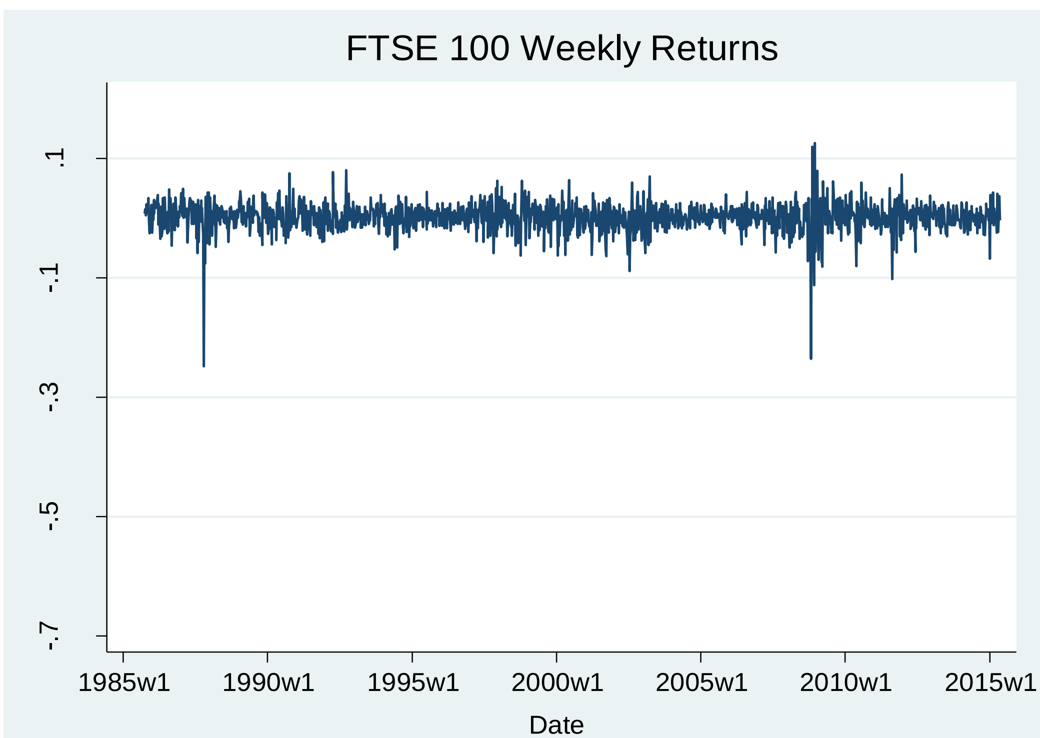
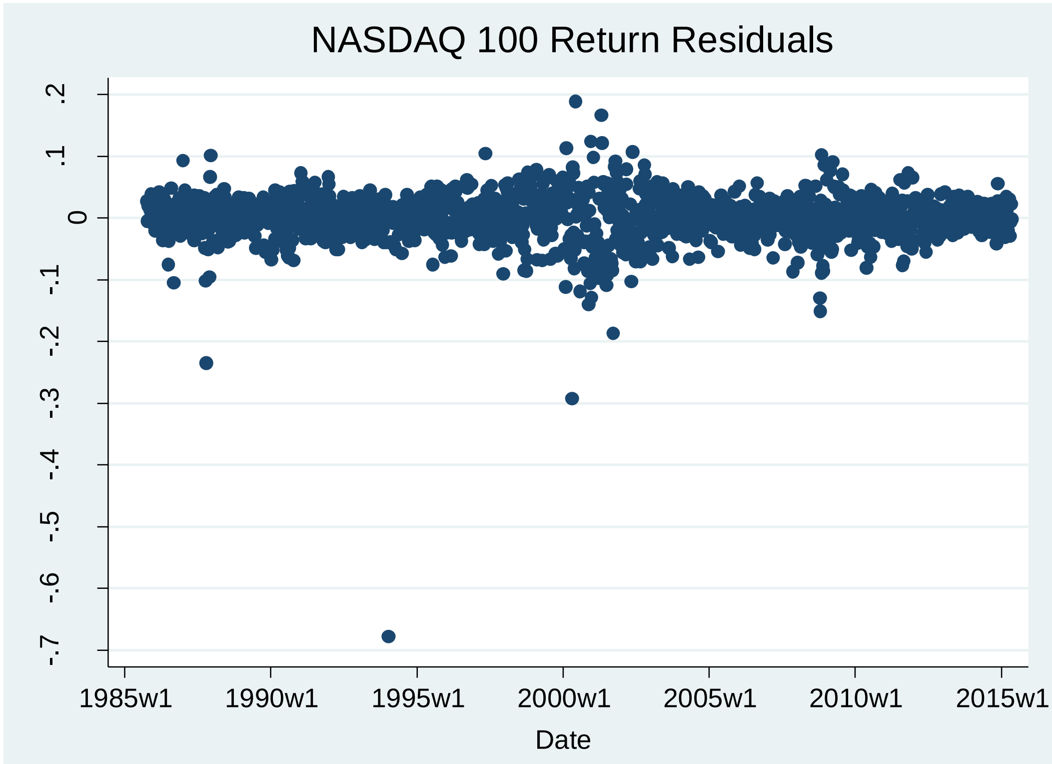


Figure 3: Residuals of Index Returns



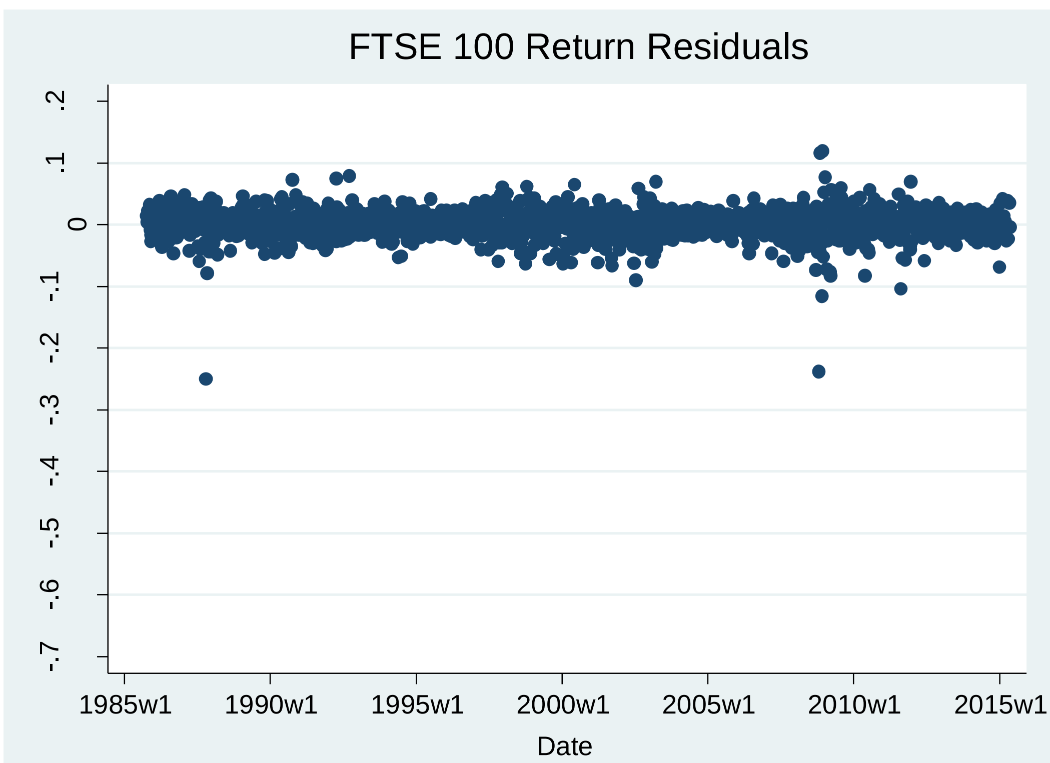
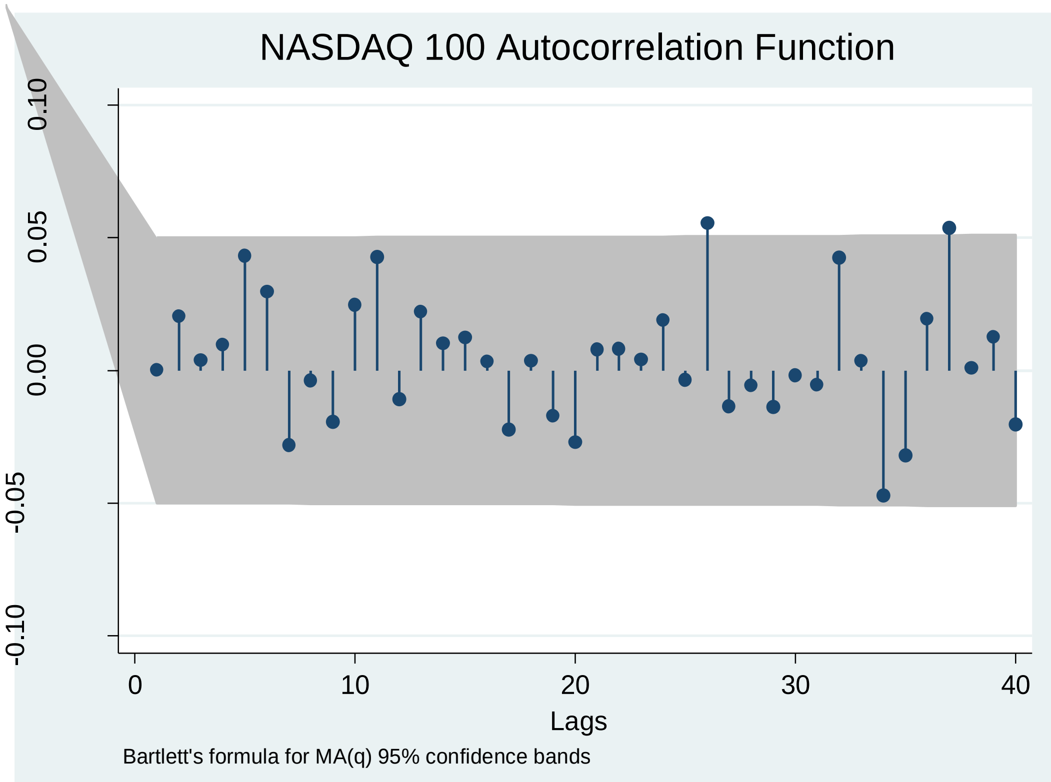


Figure 4: Index Autocorrelation Functions



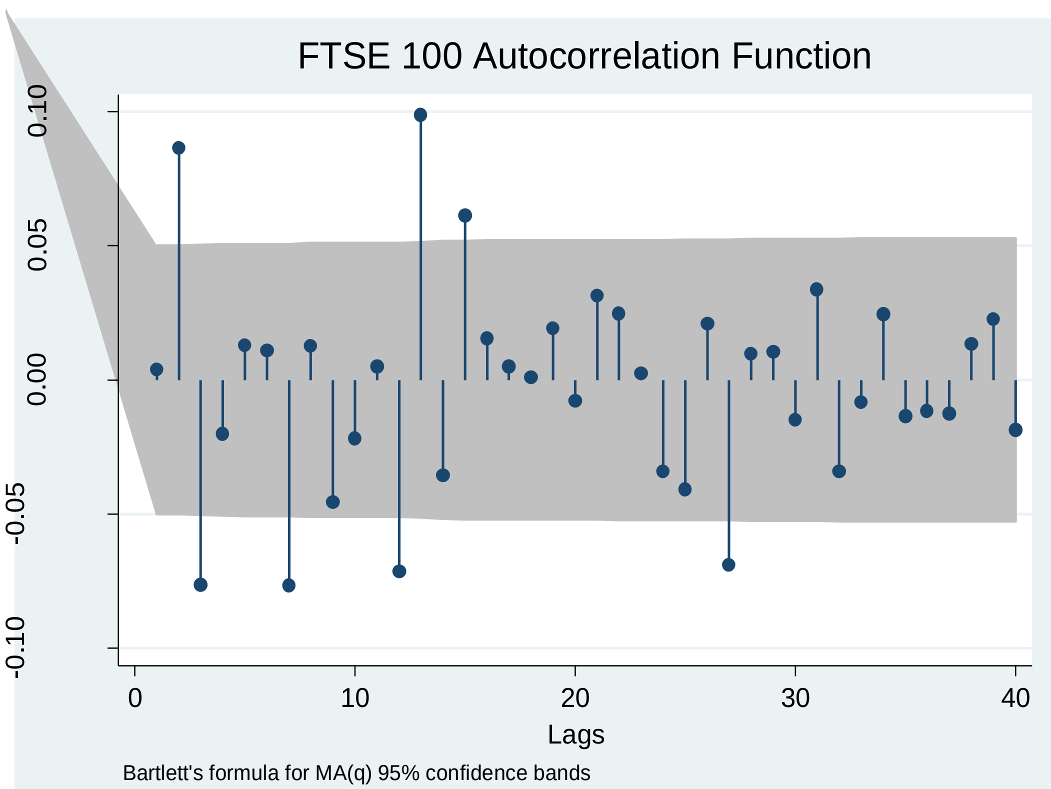
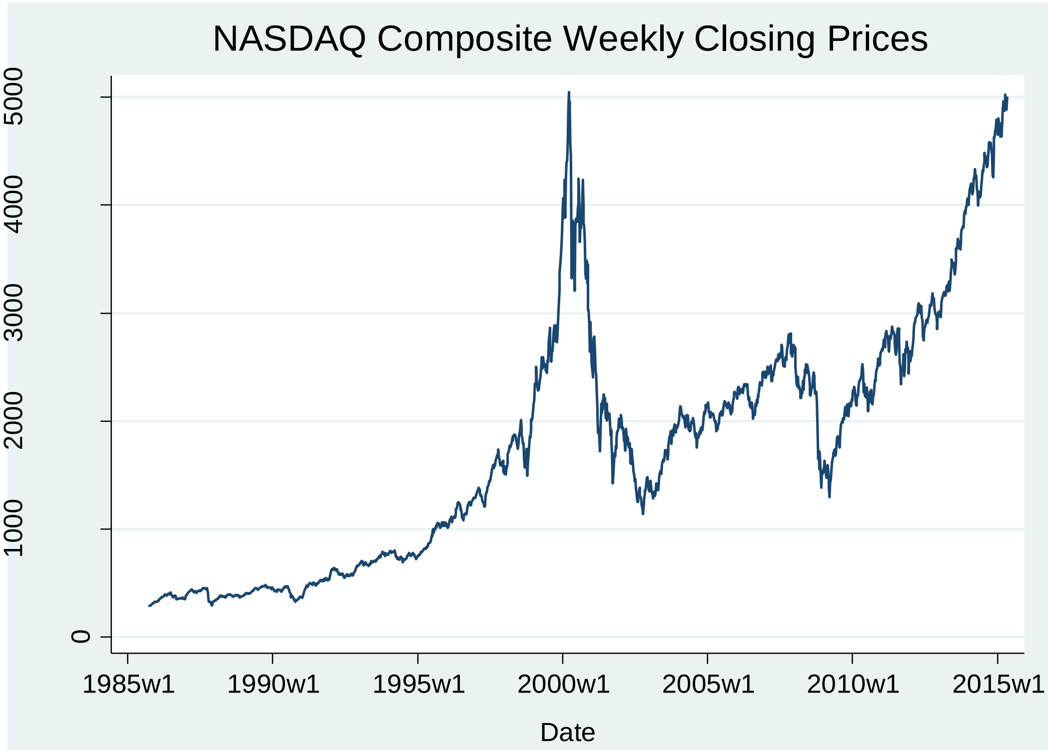


Figure 5: Alternative Index Closing Prices



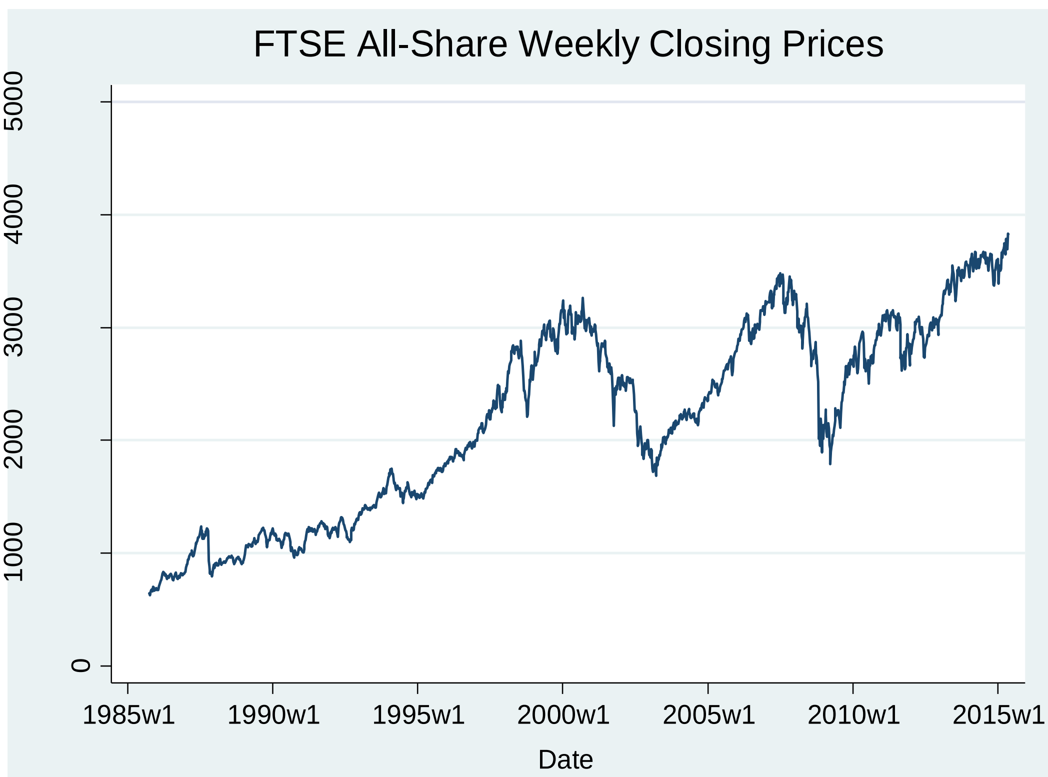
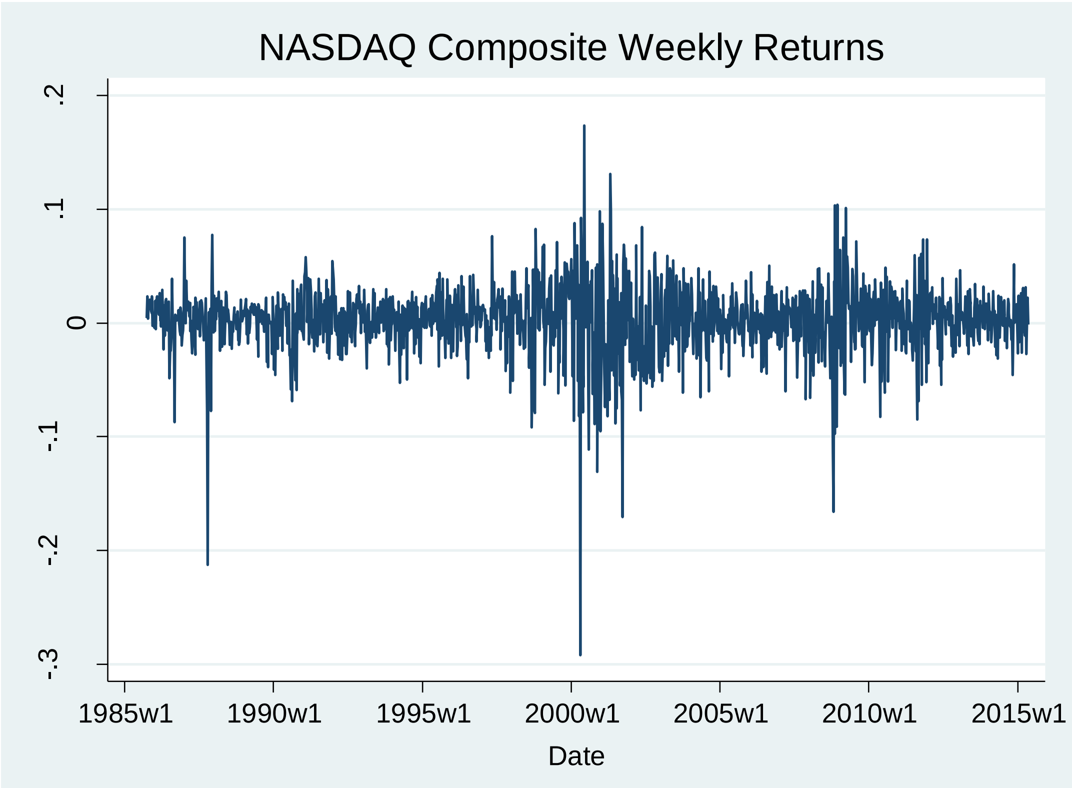


Figure 6: Alternative Index Returns



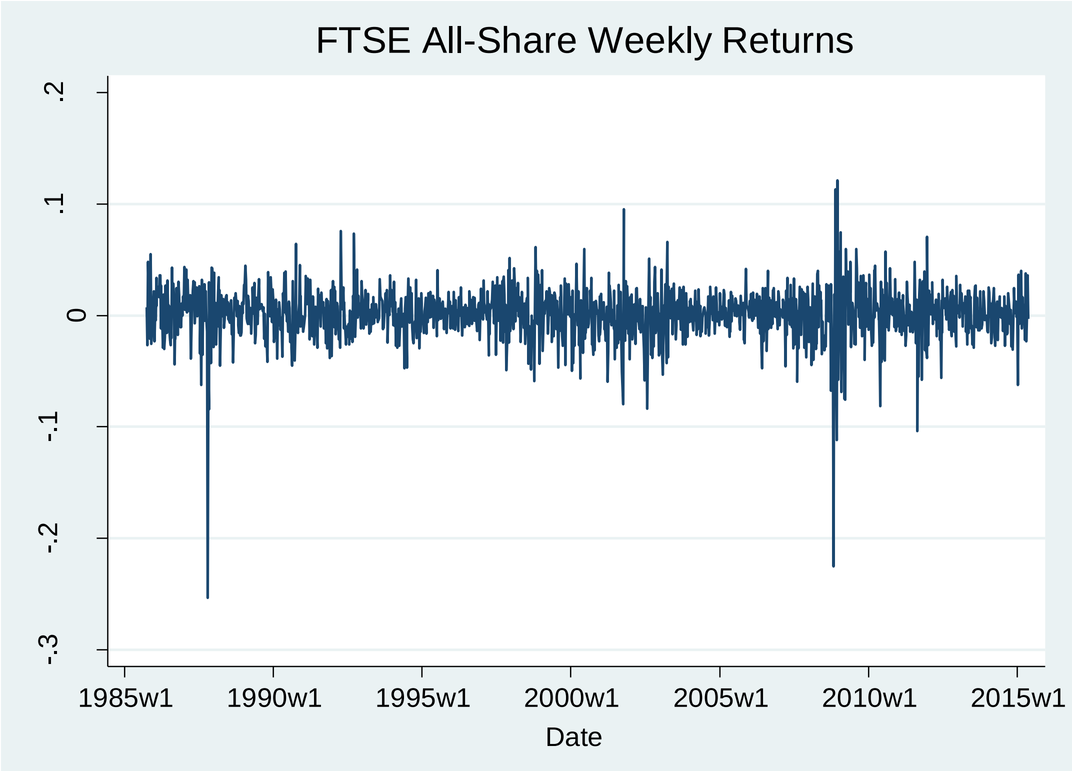
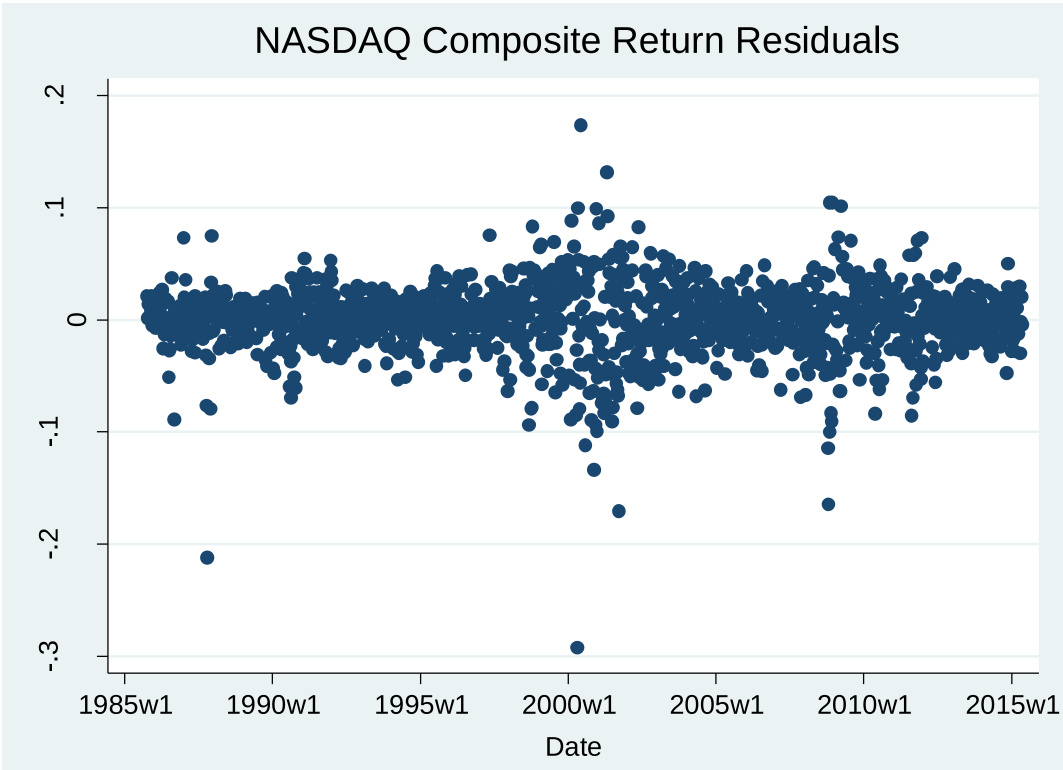


Figure 7: Residuals of Alternative Index Returns



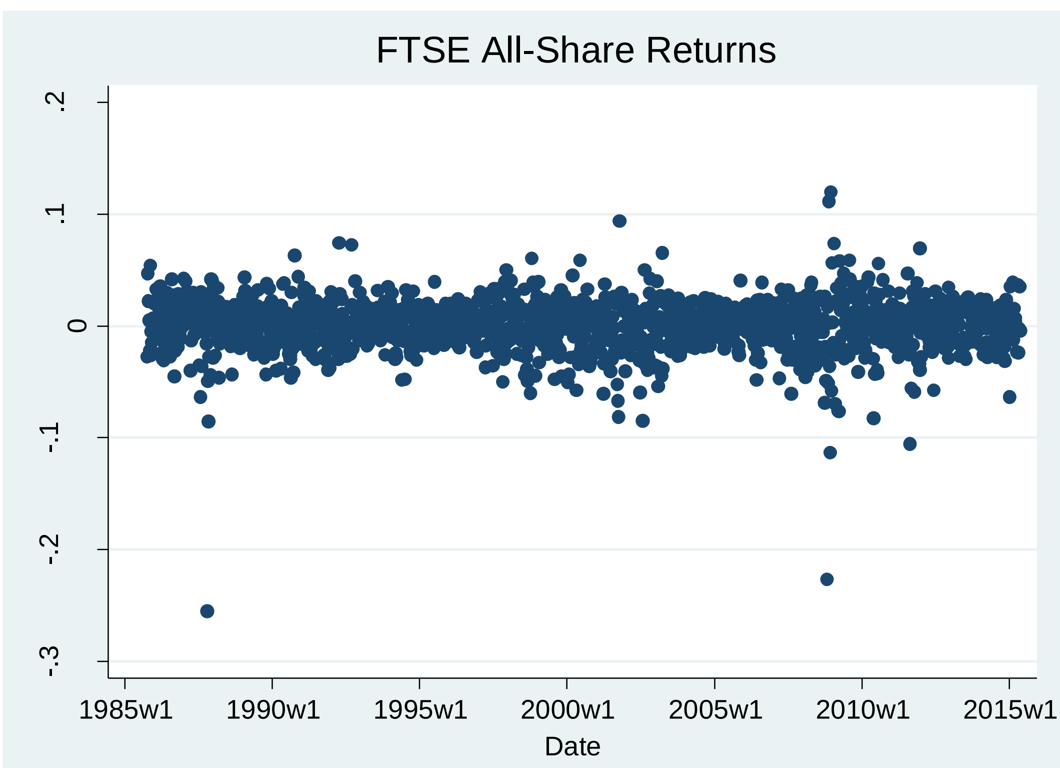
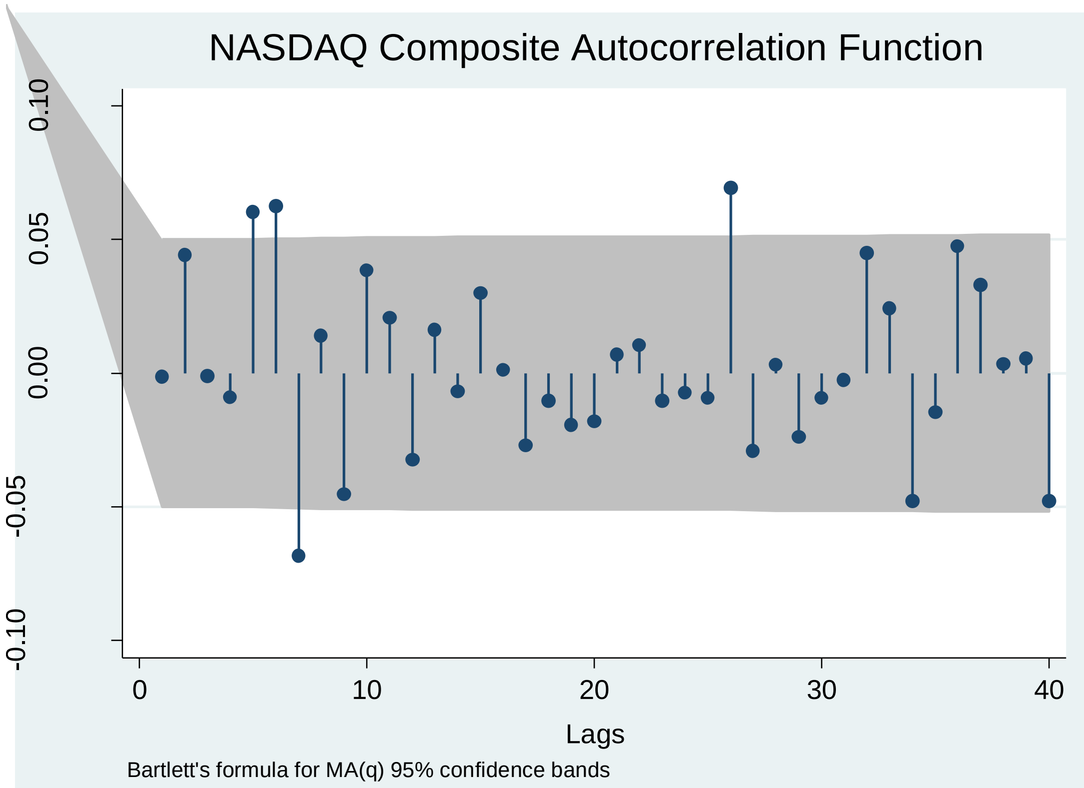
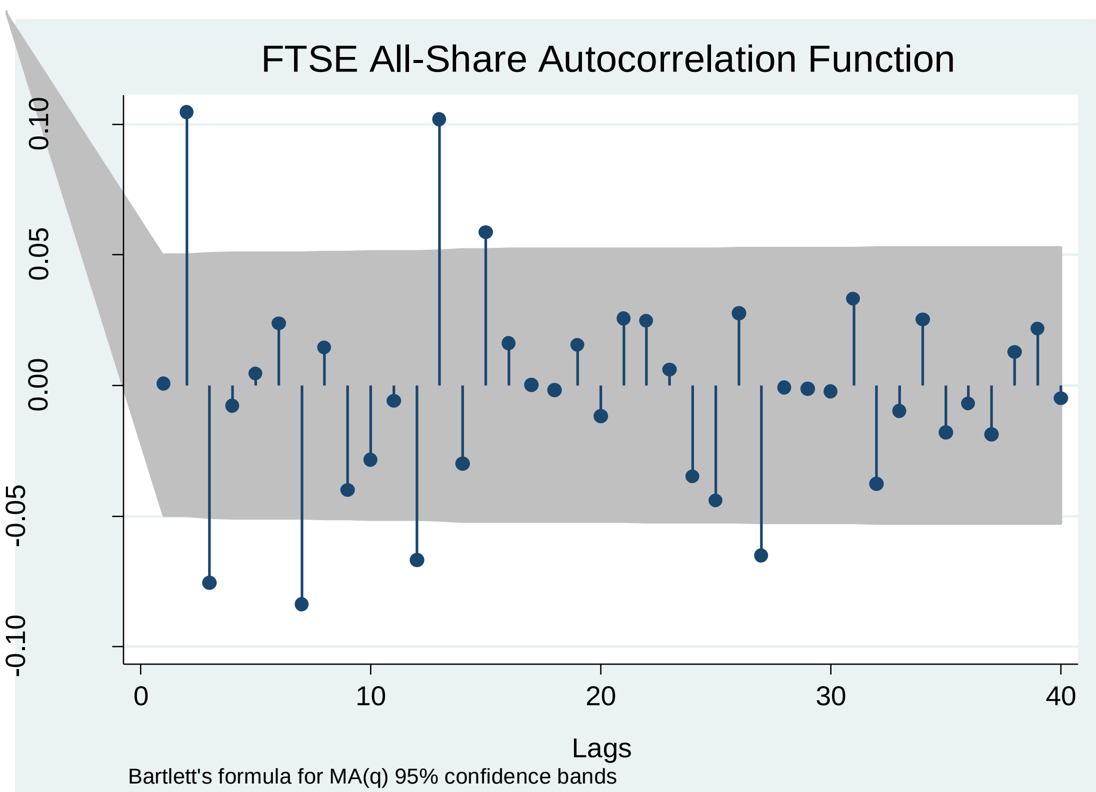


Figure 8: Alternative Index Autocorrelation Functions





Appendix C: Tables

Table 1: Summary Statistics for Index Weekly Closing Prices

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ 100	1541	1444.5530	1031.1950	220.1500	4691.6100
FTSE 100	1541	4390.6910	1656.8070	1313.0000	7089.8000

Table 2: DF-GLS Tests for Index Weekly Closing Prices

Index	Number of Lags	DF-GLS Test Statistic for Index Weekly Prices	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ 100	23	–1.939	–3.480	–2.830	–2.544
	20	–1.813	–3.480	–2.833	–2.546
	16	–1.972	–3.480	–2.837	–2.550
	15	–2.018	–3.480	–2.838	–2.551
	8	–1.480	–3.480	–2.844	–2.557
	4	–1.413	–3.480	–2.848	–2.560
	2	–1.431	–3.480	–2.850	–2.562
	1	–1.413	–3.480	–2.851	–2.562
FTSE 100	23	–2.196	–3.480	–2.830	–2.544
	22	–2.018	–3.480	–2.831	–2.545
	16	–2.018	–3.480	–2.837	–2.550
	13	–1.904	–3.480	–2.840	–2.553
	8	–1.975	–3.480	–2.844	–2.557
	4	–2.033	–3.480	–2.848	–2.560
	2	–2.266	–3.480	–2.850	–2.562
	1	–2.153	–3.480	–2.851	–2.562

NASDAQ 100: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 20 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 15 respectively.
FTSE 100: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 22 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 13 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 3: Summary Statistics for Index Weekly Returns

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ 100	1540	0.0019	0.0387	– 0.6770	0.1914
FTSE 100	1540	0.0011	0.0239	– 0.2486	0.1258

Table 4: DF-GLS Tests for Index Weekly Returns

Index	Number of Lags	DF-GLS Test Statistic for Index Weekly Prices	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ 100	23	–7.070 ***	–3.480	–2.830	–2.544
	16	–8.510 ***	–3.480	–2.837	–2.550
	10	–10.531 ***	–3.480	–2.843	–2.555
	8	–12.469 ***	–3.480	–2.844	–2.557
	4	–16.104 ***	–3.480	–2.848	–2.560
	2	–21.883 ***	–3.480	–2.850	–2.562
	1	–26.890 ***	–3.480	–2.851	–2.562
FTSE 100	23	–6.751 ***	–3.480	–2.830	–2.544
	22	–6.630 ***	–3.480	–2.831	–2.545
	21	–6.832 ***	–3.480	–2.832	–2.546
	16	–8.567 ***	–3.480	–2.837	–2.550
	8	–13.377 ***	–3.480	–2.844	–2.557
	4	–7.229 ***	–3.480	–2.848	–2.560
	2	–22.682 ***	–3.480	–2.850	–2.562
	1	–25.418 ***	–3.480	–2.851	–2.562

NASDAQ 100: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 10 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 23 respectively.
FTSE 100: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 21 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 22 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 5: Autocorrelation Test Results for Index Weekly Returns

Index Returns	Number of Lags	Ljung Box Q-Statistic	P-Value
NASDAQ 100	40	34.8051	0.7028
FTSE 100	40	89.6936	0.0000

The number of lags is chosen conventionally as k=min(T/2 – 2, 40) where T denotes the number of observations. The null hypothesis is rejected at the 5% level of significance wherever the p-value ≤ 0.05.

Table 6: Lo and MacKinlay Variance Ratio Test Statistics for Index Weekly Returns

Index Returns	Lo and MacKinlay Variance Ratios for Various Investment Horizons			
	2	4	8	16
NASDAQ 100	0.9856	1.0016	1.0610	1.1275
	(–0.5894)	(–0.0098)	(0.7446)	(1.0387)
	(–0.4461)	(–0.0077)	(0.5979)	(0.8394)
FTSE 100	0.9545	0.9848	0.9406	0.8399
	(–1.7402)	(–0.3216)	(–0.8218)	(–1.4863)
	(–1.2868)	(–0.2215)	(–0.5472)	(–0.9867)

The variance ratios VR(k) are reported in the primary rows, while the test statistic for homoscedastic increments Z(k) and the heteroscedasticity robust test statistic Z*(k) are given in the second and third row parentheses respectively. Asterisked values indicate rejection of the random walk hypothesis at the 5% level of significance, i.e. wherever the |Z-statistic| ≥ 1.96.

Table 7: Chow and Denning Multiple Variance Ratio Test Statistics for Index Returns

Index Returns	Homoscedastic Statistic	Heteroscedastic Statistic
NASDAQ 100	1.038719	0.839373
FTSE 100	1.740174	1.286797

The 5% critical value is 2.490915, and so we reject the null hypothesis if |Z-statistic| ≥ 2.490915. Asterisked values indicate rejection of random walk hypothesis at the 5% level of significance.

Table 8: Runs Test Statistics for Index Returns

Index Returns	Residuals ≤ 0	Residuals > 0	Expected Runs	Actual Runs	Z-Score	Prob>|z|
NASDAQ 100	713	826	766	788	1.11	0.27
FTSE 100	721	818	767	752	–0.79	0.43

The null hypothesis rejected at the 5% level of significance if | Z-score | ≥ 1.96.

Table 9: BDS Test Statistics for NASDAQ 100 Index Weekly Returns

Distance (ε)	Dimension (m)			
	2	3	4	5
0.5S	10.4805 (0.0000)	12.6378 (0.0000)	15.3339 (0.0000)	18.1596 (0.0000)
1.0S	11.7339 (0.0000)	14.3461 (0.0000)	16.4868 (0.0000)	18.3048 (0.0000)
1.5S	12.2136 (0.0000)	14.9902 (0.0000)	16.8608 (0.0000)	18.0126 (0.0000)
2.0S	12.1134 (0.0000)	14.3052 (0.0000)	15.9673 (0.0000)	16.8336 (0.0000)

ε,*m*, and *S* denote the distance, dimension, and standard deviation respectively. ***ε*** is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the NASDAQ index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 10: BDS Test Statistics for FTSE 100 Index Weekly Returns

Distance (ε)	Dimension (m)			
	2	3	4	5
0.5S	6.1624 (0.0000)	9.2907 (0.0000)	11.1658 (0.0000)	13.1169 (0.0000)
1.0S	6.4578 (0.0000)	9.4122 (0.0000)	10.7497 (0.0000)	12.2003 (0.0000)
1.5S	6.1119 (0.0000)	8.9963 (0.0000)	9.9877 (0.0000)	10.9656 (0.0000)
2.0S	5.4647 (0.0000)	8.2492 (0.0000)	9.0695 (0.0000)	9.7359 (0.0000)

ε,*m*, and *S* denote the distance, dimension, and standard deviation respectively. ***ε*** is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the London Stock Exchange index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 11: Summary Statistics for Alternative Index Weekly Closing Prices

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ Composite	1541	1760.7790	1153.2670	280.5000	5048.6200
FTSE All-Share	1541	2196.5700	854.9778	625.7500	3830.8700

Table 12: DF-GLS Tests for Alternative Index Weekly Closing Prices

Index	Number of Lags	DF-GLS Test Statistic for Index Weekly Prices	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ Comp.	23	–1.832	–3.480	–2.830	–2.544
	16	–1.960	–3.480	–2.837	–2.550
	15	–1.986	–3.480	–2.838	–2.551
	8	–1.692	–3.480	–2.844	–2.557
	4	–1.741	–3.480	–2.848	–2.560
	2	–1.748	–3.480	–2.850	–2.562
	1	–1.641	–3.480	–2.851	–2.562
FTSE All-Share	23	–2.575 *	–3.480	–2.830	–2.544
	22	–2.541	–3.480	–2.831	–2.545
	16	–2.409	–3.480	–2.837	–2.550
	13	–2.233	–3.480	–2.840	–2.553
	8	–2.294	–3.480	–2.844	–2.557
	4	–2.319	–3.480	–2.848	–2.560
	2	–2.583 *	–3.480	–2.850	–2.562
	1	–2.421	–3.480	–2.851	–2.562

NASDAQ Comp.: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 15 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 15 respectively.
FTSE All-Share: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 22 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 13 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 13: Summary Statistics for Alternative Index Weekly Returns

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ Composite	1540	0.0019	0.0304	– 0.2918	0.1738
FTSE All-Share	1540	0.0012	0.0232	– 0.2536	0.1211

Table 14: DF-GLS Tests for Alternative Index Weekly Returns

Index	Number of Lags	DF-GLS Test Statistic for Index Weekly Returns	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ Comp.	23	–7.973 ***	–3.480	–2.830	–2.544
	16	–9.211 ***	–3.480	–2.837	–2.550
	8	–12.911 ***	–3.480	–2.844	–2.557
	6	–14.149 ***	–3.480	–2.846	–2.559
	5	–13.957 ***	–3.480	–2.847	–2.559
	4	–16.054 ***	–3.480	–2.848	–2.560
	2	–21.617 ***	–3.480	–2.850	–2.562
	1	–25.930 ***	–3.480	–2.851	–2.562
FTSE All-Share	23	– 6.726 ***	–3.480	–2.830	–2.544
	22	–6.583 ***	–3.480	–2.831	–2.545
	21	–6.811 ***	–3.480	–2.832	–2.546
	16	–8.435 ***	–3.480	–2.837	–2.550
	8	–13.014 ***	–3.480	–2.844	–2.557
	4	–16.829 ***	–3.480	–2.848	–2.560
	2	–22.083 ***	–3.480	–2.850	–2.562
	1	–24.385 ***	–3.480	–2.851	–2.562

NASDAQ Comp.: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 6 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 5 respectively.
FTSE All-Share: Maximum lag for the index is 23, selected by the Schwert Criterion. The Ng-Perron criterion selects 21 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 22 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 15: Autocorrelation Test Results for Alternative Index Returns

Index Returns	Number of Lags	Ljung Box Q-Statistic	P-Value
NASDAQ Composite	40	61.9059	0.0147
FTSE All-Share	40	94.5642	0.0000

The number of lags is chosen conventionally as k=min(T/2 – 2, 40) where T denotes the number of observations. The null hypothesis is rejected at the 5% level of significance wherever the p-value ≤ 0.05.

Table 16: Lo and MacKinlay Variance Ratio Test Statistics for Alternative Index Weekly Returns

Index Returns	**Lo and MacKinlay Variance Ratios for Various Investment Horizons**			
	2	**4**	**8**	**16**
NASDAQ Comp.	1.0293 (1.1254) (0.6520)	1.0898 (1.8397) (1.0980)	1.1750 (2.2507) ** (1.3991)	1.2220 (1.8736) (1.1931)
FTSE All-Share	0.9949 (–0.5894) (–0.4461)	1.0602 (–0.0098) (–0.0077)	1.0427 (0.7446) (0.5979)	0.9461 (1.0387) (0.8394)

The variance ratios VR(k) are reported in the primary rows, while the test statistic for homoscedastic increments Z(k) and the heteroscedasticity robust test statistic Z*(k) are given in the second and third row parentheses respectively. Asterisked values indicate rejection of the random walk hypothesis at the 5% level of significance, i.e. wherever the |Z-statistic| ≥ 1.96.

Table 17: Chow and Denning Multiple Variance Ratio Test Statistics for Alternative Index WeeklyReturns

Index Returns	**Homoscedastic Statistic**	**Heteroscedastic Statistic**
NASDAQ Composite	2.250702	1.399123
FTSE All-Share	1.038719	0.839373

The 5% critical value is 2.490915, and so we reject the null hypothesis if |Z-statistic| ≥ 2.490915. Asterisked values indicate rejection of random walk hypothesis at the 5% level of significance.

Table 18: Runs Test Statistics for Alternative Index Returns

Index Returns	**Residuals ≤ 0**	**Residuals > 0**	**Expected Runs**	**Actual Runs**	**Z-Score**	**Prob>|z|**
NASDAQ Comp.	714	825	766	752	–0.74	0.46
FTSE All-Share	725	815	768	753	–0.79	0.43

The null hypothesis rejected at the 5% level of significance if | Z-score | ≥ 1.96.

Table 19: BDS Test Statistics for NASDAQ Composite Index Weekly Returns

Distance (ε)	**Dimension (m)**			
	2	**3**	**4**	**5**
0.5S	11.9948 (0.0000)	15.2696 (0.0000)	19.6497 (0.0000)	24.4294 (0.0000)
1.0S	13.5724 (0.0000)	17.0270 (0.0000)	19.9602 (0.0000)	22.7019 (0.0000)
1.5S	13.6790 (0.0000)	16.9039 (0.0000)	19.1239 (0.0000)	20.6393 (0.0000)
2.0S	13.2537 (0.0000)	15.8836 (0.0000)	17.6276 (0.0000)	18.6589 (0.0000)

ε,m, and S denote the distance, dimension, and standard deviation respectively. ε is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the NASDAQ index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 20: BDS Test Statistics for FTSE All-Share Index Weekly Returns

Distance (ε)	Dimension (m)			
	2	3	4	5
0.5S	10.4805 (0.0000)	12.6378 (0.0000)	15.3339 (0.0000)	18.1596 (0.0000)
1.0S	11.7339 (0.0000)	14.3461 (0.0000)	16.4868 (0.0000)	18.3048 (0.0000)
1.5S	12.2136 (0.0000)	14.9902 (0.0000)	16.8608 (0.0000)	18.0126 (0.0000)
2.0S	12.1134 (0.0000)	14.3052 (0.0000)	15.9673 (0.0000)	16.8336 (0.0000)

ε,*m*, and *S* denote the distance, dimension, and standard deviation respectively. ***ε*** is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the NASDAQ index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 21: Summary Statistics for Index Daily Closing Prices

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ 100	10787	1441.7400	1026.6390	214.3200	4704.7300
FTSE 100	10787	4384.4450	1654.0890	1296.0000	7089.8000

Table 22: DF-GLS Tests for Index Daily Closing Prices

Index	Number of Lags	DF-GLS Test Statistic for Index Daily Prices	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ 100	38	–1.642	–3.480	–2.835	–2.548
	8	–1.266	–3.480	–2.839	–2.551
	4	–1.419	–3.480	–2.839	–2.551
	2	–1.575	–3.480	–2.840	–2.552
	1	–1.584	–3.480	–2.840	–2.552
FTSE 100	38	–2.041	–3.480	–2.835	–2.548
	35	–2.044	–3.480	–2.836	–2.548
	8	–2.137	–3.480	–2.844	–2.557
	4	–2.250	–3.480	–2.848	–2.560
	2	–2.425	–3.480	–2.850	–2.562
	1	–2.403	–3.480	–2.851	–2.562

NASDAQ 100: Maximum lag for the index is 38, selected by the Schwert Criterion. The Ng-Perron criterion, the SC criterion, and the MAIC criterion also select 38 as the optimal lag, while the SC criterion chooses 1 lag as optimal.
FTSE 100: Maximum lag for the index is 38, selected by the Schwert Criterion. The Ng-Perron criterion selects 35 as the optimal lag, while the SC criterion and MAIC criterion choose 4 and 35 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 23: Summary Statistics for Index Daily Returns

Index	Observations	Mean	Standard Deviation	Min	Max
NASDAQ 100	10786	0.0003	0.0135	– 0.2912	0.1720
FTSE 100	10786	0.0002	0.0085	– 0.1303	0.0847

Table 24: DF-GLS Tests for Index Daily Returns

Index	Number of Lags	DF-GLS Test Statistic for Index Weekly Returns	1% Critical Value	5% Critical Value	10% Critical Value
NASDAQ 100	38	–5.787 ***	–3.480	–2.835	–2.548
	23	–8.841 ***	–3.480	–2.837	–2.549
	8	–21.238 ***	–3.480	–2.839	–2.551
	4	–33.067 ***	–3.480	–2.848	–2.560
	2	–44.530 ***	–3.480	–2.850	–2.562
	1	–55.523 ***	–3.480	–2.851	–2.562
FTSE 100	38	–6.351***	–3.480	–2.835	–2.548
	35	–6.739 ***	–3.480	–2.836	–2.545
	8	–22.428 ***	–3.480	–2.844	–2.557
	4	–34.883 ***	–3.480	–2.848	–2.560
	2	–46.694 ***	–3.480	–2.850	–2.562
	1	–57.713 ***	–3.480	–2.851	–2.562

NASDAQ 100: Maximum lag for the index is 38, selected by the Schwert Criterion. The Ng-Perron criterion also selects 38 as the optimal lag, while the SC criterion and MAIC criterion choose 23 and 38 respectively.
FTSE 100: Maximum lag for the index is 38, selected by the Schwert Criterion. The Ng-Perron criterion also selects 38 as the optimal lag, while the SC criterion and MAIC criterion choose 35 and 38 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 25: Autocorrelation Test Results for Index Daily Returns

Index Returns	Number of Lags	Ljung Box Q-Statistic	P-Value
NASDAQ 100	40	70.8975	0.0019
FTSE 100	40	149.2809	0.0000

The number of lags is chosen conventionally as k=min(T/2 – 2, 40) where T denotes the number of observations. The null hypothesis is rejected at the 5% level of significance wherever the p-value ≤ 0.05.

Table 26: Lo and MacKinlay Variance Ratio Test Statistics for Index Daily Returns

Index Returns	Lo and MacKinlay Variance Ratios for Various Investment Horizons 2	4	8	16
NASDAQ 100	0.9846 (–1.3386) (–0.8844)	0.9249 (–3.4812) ** (–2.2780) **	0.8857 (–3.3572) ** (–2.2262) **	0.8630 (–2.7191) ** (–1.8454)
FTSE 100	1.000164 (–0.5894) (–0.4461)	0.9396 (–0.0100) (–0.0077)	0.9051 (–0.7446) (–0.5979)	0.9170 (–1.0387) (–0.8394)

The variance ratios VR(k) are reported in the primary rows, while the test statistic for homoscedastic increments Z(k) and the heteroscedasticity robust test statistic Z*(k) are given in the second and third row parentheses respectively. Asterisked values indicate rejection of the random walk hypothesis at the 5% level of significance, i.e. wherever the |Z-statistic| ≥ 1.96.

Table 27: Chow and Denning Multiple Variance Ratio Test Statistics for Index Daily Returns

Index Returns	Homoscedastic Statistic	Heteroscedastic Statistic
NASDAQ 100	3.481191 **	2.278031
FTSE 100	1.038719	0.8393733

The 5% critical value is 2.490915, and so we reject the null hypothesis if |Z-statistic| ≥ 2.490915. Asterisked values indicate rejection of random walk hypothesis at the 5% level of significance.

Table 28: Runs Test Statistics for Index Daily Returns

Index Returns	Residuals ≤ 0	Residuals > 0	Expected Runs	Actual Runs	Z-Score	Prob>|z|
NASDAQ 100	5224	5561	5388	4024	–26.3	0.00
FTSE 100	5554	5231	5389	4185	–23.2	0.00

The null hypothesis rejected at the 5% level of significance if \| Z-score \| ≥ 1.96.

Table 29: BDS Test Statistics for NASDAQ 100 Index Daily Returns

Distance (ε)	Dimension (m) 2	3	4	5
0.5S	18.3725 (0.0000)	26.7738 (0.0000)	33.6127 (0.0000)	42.3939 (0.0000)
1.0S	21.9768 (0.0000)	30.7533 (0.0000)	36.1855 (0.0000)	41.9352 (0.0000)
1.5S	24.2540 (0.0000)	32.8442 (0.0000)	37.3409 (0.0000)	41.3468 (0.0000)
2.0S	24.4371 (0.0000)	32.5397 (0.0000)	36.4054 (0.0000)	39.4711 (0.0000)

ε, m, and S denote the distance, dimension, and standard deviation respectively. ε is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the NASDAQ index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 30: BDS Test Statistics for FTSE 100 Index Daily Returns

Distance (ε)	**Dimension (m)**			
	2	**3**	**4**	**5**
0.5S	10.4805 (0.0000)	12.6378 (0.0000)	15.3339 (0.0000)	18.1596 (0.0000)
1.0S	11.7339 (0.0000)	14.3461 (0.0000)	16.4868 (0.0000)	18.3048 (0.0000)
1.5S	12.2136 (0.0000)	14.9902 (0.0000)	16.8608 (0.0000)	18.0126 (0.0000)
2.0S	12.1134 (0.0000)	14.3052 (0.0000)	15.9673 (0.0000)	16.8336 (0.0000)

ε,*m*, and *S* denote the distance, dimension, and standard deviation respectively. ε is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the London Stock Exchange index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05.

Table 31: Summary Statistics for Subsamples of Index Weekly Closing Prices

Index	**Subsample**	**Observations**	**Mean**	**Standard Deviation**	**Min**	**Max**
NASDAQ 100	10/10/1985 – 01/07/1990	248	357.0414	61.1748	220.1500	481.5700
	01/04/1991 – 01/03/2001	518	1214.7820	1013.1510	351.7600	4691.6100
	01/01/2002 – 01/12/2007	308	1492.9040	291.7316	815.4000	2213.8600
	01/07/2009 – 13/04/2015	302	278.5830	795.4716	1419.8400	4458.5400
FTSE 100	10/10/1985 – 01/07/1990	248	1917.0010	305.8356	1313.0000	2444.5000
	01/10/1991 – 01/06/2008	868	4693.2740	1288.646	2312.6000	6930.2000
	01/10/2009 – 13/04/2015	289	6021.9590	563.4173	4838.1000	7089.8000

Table 32: Summary Statistics for Subsamples of Index Weekly Returns

Index	Subsample	Observations	Mean	Standard Deviation	Min	Max
NASDAQ 100	10/10/1985 – 01/07/1990	247	0.0031	0.0305	– 0.2314	0.1014
	01/04/1991 – 01/03/2001	517	0.00238	0.0485	– 0.6770	0.1914
	01/01/2002 – 01/12/2007	307	0.0008	0.0300	– 0.1019	0.1091
	01/07/2009 – 13/04/2015	301	0.00377	0.0235	– 0.0786	0.0740
FTSE 100	10/10/1985 – 01/07/1990	247	0.0024	0.0268	– 0.2486	0.0486
	01/10/1991 – 01/06/2008	867	0.0010	0.0205	– 0.0887	0.0795
	01/10/2009 – 13/04/2015	288	0.0011	0.0212	– 0.1028	0.0724

Table 33: DF-GLS Tests for Subsamples of NASDAQ 100 Weekly Closing Prices

Sub-Period	Number of Lags	DF-GLS Test Statistic for Weekly Prices	1% Critical Value	5% Critical Value	10% Critical Value
10/10/1985 – 01/07/1990	15	–2.522	–3.480	–2.804	–2.525
	8	–2.464	–3.480	–2.868	–2.584
	4	–2.413	–3.480	–2.898	–2.612
	2	–2.431	–3.480	–2.912	–2.624
	1	–2.321	–3.480	–2.918	–2.629
01/04/1991 – 01/03/2001	18	–2.026	–3.480	–2.820	–2.537
	16	–1.824	–3.480	–2.828	–2.544
	15	–1.946	–3.480	–2.831	–2.547
	8	–1.413	–3.480	–2.855	–2.569
	4	–1.347	–3.480	–2.867	–2.580
	2	–1.382	–3.480	–2.873	–2.585
	1	–1.370	–3.480	–2.876	–2.588
01/01/2002 – 01/12/2007	15	–1.337	–3.480	–2.816	–2.536
	10	–1.200	–3.480	–2.851	–2.568
	8	–1.045	–3.480	–2.864	–2.579
	4	–1.006	–3.480	–2.887	–2.600
	2	–1.006	–3.480	–2.897	–2.609
	1	–1.088	–3.480	–2.902	–2.613
01/07/2009 – 13/04/2015	15	–1.470	–3.480	–2.816	–2.535
	9	–1.613	–3.480	–2.858	–2.574
	8	–1.787	–3.480	–2.864	–2.580
	4	–2.076	–3.480	–2.888	–2.601
	2	–2.173	–3.480	–2.898	–2.610
	1	–2.296	–3.480	–2.903	–2.615

First Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion alsoselects 15 as the optimal lag, while the SC criterion and MAIC criterion both choose 1 lag as optimal.
Second Subsample: Maximum lag for the index is 18, selected by the Schwert Criterion. The Ng-Perron criterion selects 15 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 16 respectively.
Third Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 10 as the optimal lag, while the SC criterion and MAIC criterion both choose 1 lag as optimal.
Fourth Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 8 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 9 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 34: DF-GLS Tests for Subsamples of NASDAQ 100 Weekly Returns

Sub-Period	**Number of Lags**	**DF-GLS Test Statistic for Weekly Returns**	**1% Critical Value**	**5% Critical Value**	**10% Critical Value**
10/10/1985 – 01/07/1990	15	–3.781 ***	–3.480	–2.804	–2.525
	8	–5.161 ***	–3.480	–2.868	–2.584
	5	–5.148 ***	–3.480	–2.892	–2.605
	4	–6.465 ***	–3.480	–2.898	–2.612
	2	–8.659 ***	–3.480	–2.912	–2.624
	1	–10.028 ***	–3.480	–2.918	–2.629
01/04/1991 – 01/03/2001	18	–3.118 **	–3.480	–2.820	–2.537
	17	–3.169 **	–3.480	–2.824	–2.541
	10	–4.397 ***	–3.480	–2.849	–2.563
	8	–5.606 ***	–3.480	–2.855	–2.569
	4	–8.035 ***	–3.480	–2.867	–2.580
	2	–11.405 ***	–3.480	–2.873	–2.585
	1	–14.567 ***	–3.480	–2.876	–2.588
01/01/2002 – 01/12/2007	15	–1.197	–3.480	–2.816	–2.536
	11	–1.309	–3.480	–2.845	–2.562
	9	–1.536	–3.480	–2.858	–2.574
	8	–1.936	–3.480	–2.864	–2.579
	4	–3.563 ***	–3.480	–2.887	–2.600
	2	–5.886 ***	–3.480	–2.897	–2.609
	1	–7.685 ***	–3.480	–2.902	–2.613
01/07/2009 – 13/04/2015	15	–1.834	–3.480	–2.816	–2.535
	13	–1.848	–3.480	–2.830	–2.549
	8	–2.031	–3.480	–2.864	–2.580
	7	–2.062	–3.480	–2.871	–2.585
	4	–2.942 **	–3.480	–2.888	–2.601
	2	–4.049 ***	–3.480	–2.898	–2.610
	1	–5.586 ***	–3.480	–2.903	–2.615

First Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 5 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 5 respectively.
Second Subsample: Maximum lag for the index is 18, selected by the Schwert Criterion. The Ng-Perron criterion selects 10 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 17 respectively.
Third Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 11 as the optimal lag, while the SC criterion and MAIC criterion choose 9 and 11 respectively.
Fourth Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 13 as the optimal lag, while the SC criterion and MAIC criterion choose 7 and 13 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 35: DF-GLS Tests for Subsamples of FTSE 100 Weekly Closing Prices

Sub-Period	Number of Lags	DF-GLS Test Statistic for Weekly Prices	1% Critical Value	5% Critical Value	10% Critical Value
10/10/1985 – 01/07/1990	15	–2.054	–3.480	–2.804	–2.525
	13	–2.248	–3.480	–2.824	–2.564
	8	–1.839	–3.480	–2.868	–2.584
	4	–2.400	–3.480	–2.899	–2.612
	2	–2.619	–3.480	–2.912	–2.624
	1	–2.034	–3.480	–2.918	–2.629
01/10/1991 – 01/06/2008	20	–1.246	–3.480	–2.826	–2.542
	8	–1.300	–3.480	–2.849	–2.562
	7	–1.292	–3.480	–2.851	–2.564
	4	–1.424	–3.480	–2.856	–2.568
	2	–1.492	–3.480	–2.859	–2.571
	1	–1.466	–3.480	–2.860	–2.572
01/10/2007 – 13/04/2015	15	–2.958 **	–3.480	–2.813	–2.533
	13	–2.700 *	–3.480	–2.829	–2.548
	8	–2.909 **	–3.480	–2.865	–2.581
	6	–2.637 *	–3.480	–2.878	–2.592
	4	–3.035 **	–3.480	–2.890	–2.603
	2	–3.647 ***	–3.480	–2.901	–2.613
	1	–3.524 ***	–3.480	–2.906	–2.618

First Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 13 as the optimal lag, while the SC criterion and MAIC criterion choose 2 and 1 respectively.
Second Subsample: Maximum lag for the index is 18, selected by the Schwert Criterion. The Ng-Perron criterion selects 7 as the optimal lag, while the SC criterion and MAIC criterion both choose 1 lag as optimal.
Third Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 13 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 6 respectively.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 36: DF-GLS Tests for Subsamples of FTSE 100 Weekly Returns

Sub-Period	Number of Lags	DF-GLS Test Statistic for Weekly Returns	1% Critical Value	5% Critical Value	10% Critical Value
10/10/1985 – 01/07/1990	15	–3.607 ***	–3.480	–2.804	–2.525
	12	–3.974 ***	–3.480	–2.833	–2.552
	8	–5.728 ***	–3.480	–2.868	–2.584
	4	–6.720 ***	–3.480	–2.899	–2.612
	2	–8.133 ***	–3.480	–2.912	–2.624
	1	–7.980 ***	–3.480	–2.918	–2.629
01/10/1991 – 01/06/2008	20	–3.307 **	–3.480	–2.826	–2.542
	8	–6.581 ***	–3.480	–2.849	–2.562
	4	–10.032 ***	–3.480	–2.856	–2.568
	2	–14.154 ***	–3.480	–2.859	–2.571
	1	–17.589 ***	–3.480	–2.860	–2.572
01/10/2007 – 13/04/2015	15	–3.767 ***	–3.480	–2.813	–2.533
	9	–5.931 ***	–3.480	–2.858	–2.575
	8	–5.513 ***	–3.480	–2.865	–2.581
	4	–8.911 ***	–3.480	–2.890	–2.603
	2	–9.925 ***	–3.480	–2.901	–2.613
	1	–11.819 ***	–3.480	–2.906	–2.618

First Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 12 as the optimal lag, while the SC criterion and MAIC criterion both choose 1 as the optimal lag length.
Second Subsample: Maximum lag for the index is 20, selected by the Schwert Criterion. The Ng-Perron criterion also selects 20 as the optimal lag, while the SC criterion and MAIC criterion choose 1 and 20 respectively.
Third Subsample: Maximum lag for the index is 15, selected by the Schwert Criterion. The Ng-Perron criterion selects 9 as the optimal lag, while the SC criterion and MAIC criterion both choose 1 as the optimal lag length.
Note: The null hypothesis is rejected at the α% level of significance wherever the DF-GLS test statistic ≤ α% critical value. Values marked with *, **, and *** denote significance at the 10%, 5%, and 1% levels respectively.

Table 37: Autocorrelations Tests for Subsamples of Index Weekly Returns

Index	Subsample	Number of Lags	Ljung Box Q-Statistic	P-Value
NASDAQ 100	10/10/1985 – 01/07/1990	40	33.7882	7449
	01/04/1991 – 01/03/2001	40	15.5213	9998
	01/01/2002 – 01/12/2007	40	61.9277	0146
	01/07/2009 –13/04/2015	40	45.5941	2507
FTSE 100	10/10/1985 – 01/07/1990	40	65.6725	0.0064
	01/10/1991 – 01/06/2008	40	32.2861	0.8021
	01/10/2009 – 13/04/2015	40	54.0591	0.0680

The 5% critical value is 2.490915 for all subsamples, and so we reject the null hypothesis for a given sub-period if |Z-statistic| ≥2.490915. Asterisked values indicate rejection of random walk hypothesis at the 5% level of significance.

Table 38: Lo–MacKinlay Variance Ratio Test Results for Subsamples of Index Returns

Index Returns	Subsample	Lo and MacKinlay Variance Ratios for Various Investment			
		2	4	8	16
NASDAQ 100	10/10/1985 – 01/07/1990	1.0907 (1.3565) (0.6917)	1.1514 (1.1537) (0.6660)	1.2675 (1.2320) (0.8254)	1.1675 (0.3628) (0.2580)
	01/04/1991 – 01/03/2001	0.9368 (–1.4785) (–1.5499)	0.9285 (–0.9331) (–1.0139)	0.9413 (–0.5438) (–0.5823)	0.1041 (0.0694) (0.0720)
	01/01/2002 – 01/12/2007	1.0351 (0.5551) (0.4687)	0.9672 (–0.3954) (–0.3338)	0.9252 (–0.5613) (–0.4641)	1.0847 (0.1365) (0.1123)
	01/07/2009 – 13/04/2015	0.9294 (–1.2772) (–1.0370)	0.8282 (–1.6655) (–1.3451)	0.6400 (–2.1915) ** (–1.8176)	0.4919 (–2.0836) ** (–1.7573)
FTSE 100	10/10/1985 – 01/07/1990	1.0860 (1.2824) (1.1069)	1.3304 (2.6397) ** (1.8731)	1.3685 (1.7533) (1.3002)	1.2093 (0.4897) (0.3827)
	01/10/1991 – 01/06/2008	0.9602 (–1.2034) (–1.0822)	0.9355 (–1.0657) (–0.9449)	0.8846 (–1.2200) (–1.0805)	0.8269 (–1.2534) (–1.1142)
	01/10/2009 – 13/04/2014	0.9159 (–1.4821) (–1.3180)	0.8659 (–1.2987) (–1.0996)	0.6074 (–2.3370) ** (–2.0359) **	0.4939 (–2.0482) ** (–1.8034)

The variance ratios VR(q) are reported in the primary rows, while the test statistic for homoscedastic increments Z(q) and the heteroscedastic-robust test statistic Z*(q) are given in the second and third row parentheses respectively. Asterisked values indicate rejection of the random walk hypothesis at the 5% level of significance. The null hypothesis is rejected at the 5% level where we have |Z-statistic| ≥1.96.

Table 39: Chow and Denning Multiple Variance Ratio Test Results for Index Returns

Index Returns	Subsample	Homoscedastic Statistic	Heteroscedastic Statistic
NASDAQ 100	10/10/1985 – 01/07/1990	1.356487	0.825363
	01/04/1991 –01/03/2001	1.478549	1.549895
	01/01/2002 –01/12/2007	0.561320	0.468738
	01/07/2009 – 13/04/2015	2.191476	1.817578
FTSE 100	10/10/1985 – 01/07/1990	2.639678 **	1.873051
	01/10/1991 – 01/06/2008	1.253416	1.114228
	01/10/2009 – 13/04/2014	2.336798	2.035850

The 5% critical value is 2.490915, and so we reject the null hypothesis if |Z-statistic| ≥ 2.490915. Asterisked values indicate rejection of random walk hypothesis at the 5% level of significance.

Table 40: Runs Tests Results for Subsamples of Index Returns

Index Returns	Subsample	Residuals ≤ 0	Residuals 0	Expected Runs	Actual Runs	Z-Score	Prob>|z|
NASDAQ 100	10/10/1985 – 01/07/1990	108	138	122	123	0.11	0.91
	01/04/1991 – 01/03/2001	234	282	257	251	–0.51	0.61
	01/01/2002 – 01/12/2007	151	155	154	169	1.72	0.09
	01/07/2009 – 13/04/2015	141	159	150	150	–0.05	0.96
FTSE 100	10/10/1985 – 01/07/1990	112	134	123	130	0.9	0.37
	01/10/1991 – 01/06/2008	419	447	434	449	1.05	0.29
	01/10/2009 – 13/04/2015	127	160	143	134	–1.03	0.30

The null hypothesis rejected at the 5% level of significance if | Z-score | ≥ 1.96.

Table 41: BDS Test Results for Subsamples of NASDAQ 100 Index Returns

Index Returns	Subsample	Distance(ε)	Dimension (m) 2	3	4	5
NASDAQ 100	10/10/1985 – 01/07/1990	0.5S	0.7257 (0.4680)	1.9997 (0.0455)	2.0301 (0.0423)	2.1249 (0.0336)
		1.0S	1.7131 (0.0867)	2.0562 (0.0398)	1.9497 (0.0512)	2.1972 (0.0280)
		1.5S	2.7754 (0.0055)	3.0516 (0.0023)	2.6724 (0.0075)	2.7970 (0.0052)
		2.0S	3.7647 (0.0002)	4.1279 (0.0000)	3.7006 (0.0002)	3.6379 (0.0003)
	01/04/1991 – 01/03/2001	0.5S	4.5357 (0.0000)	5.4719 (0.0000)	6.9807 (0.0000)	8.0579 (0.0000)
		1.0S	5.3445 (0.0000)	6.0723 (0.0000)	7.3543 (0.0000)	8.1521 (0.0000)
		1.5S	5.1041 (0.0000)	5.6989 (0.0000)	6.7289 (0.0000)	7.5043 (0.0000)
		2.0S	4.1201 (0.0000)	4.3982 (0.0000)	4.9606 (0.0000)	5.4110 (0.0000)
	01/01/2002 – 01/12/2007	0.5S	2.6759 (0.0075)	2.6917 (0.0071)	3.5164 (0.0004)	4.7490 (0.0000)
		1.0S	2.3441 (0.0191)	2.7774 (0.0055)	3.6665 (0.0002)	4.6434 (0.0000)
		1.5S	2.2586 (0.0239)	3.2636 (0.0011)	4.5099 (0.0000)	5.5888 (0.0000)
		2.0S	2.4180 (0.0156)	3.1023 (0.0019)	4.3694 (0.0000)	5.2671 (0.0000)
	01/07/2009 – 13/04/2015	0.5S	3.6859 (0.0002)	5.4257 (0.0000)	6.7282 (0.0000)	7.5065 (0.0000)
		1.0S	3.9908 (0.0001)	5.7569 (0.0000)	7.1660 (0.0000)	7.9423 (0.0000)
		1.5S	4.7211 (0.0000)	6.1501 (0.0000)	7.0267 (0.0000)	7.3304 (0.0000)
		2.0S	4.9434 (0.0000)	6.2638 (0.0000)	6.9320 (0.0000)	7.1018 (0.0000)

ε,*m*, and *S* denote the distance, dimension, and standard deviation respectively. ε is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the London Stock Exchange index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05. Values in red denote the only cases where the null hypothesis cannot be rejected.

Table 42: BDS Test Results for Subsamples of FTSE 100 Index Returns

Index Returns	Subsample	Distance(ε)	Dimension (m) 2	3	4	5
FTSE 100	10/10/1985 – 01/07/1990	0.5S	1.7281 (0.0840)	2.8229 (0.0048)	2.4404 (0.0147)	2.5460 (0.0109)
		1.0S	1.9338 (0.0532)	3.1271 (0.0018)	2.9894 (0.0028)	3.1931 (0.0014)
		1.5S	1.5888 (0.1121)	3.1523 (0.0016)	3.1535 (0.0016)	3.4201 (0.0006)
		2.0S	1.2641 (1.2062)	3.2510 (0.0011)	3.4338 (0.0006)	3.6566 (0.0003)
	01/10/1991 – 01/06/2008	0.5S	2.9889 (0.0028)	5.3936 (0.0000)	6.6522 (0.0000)	8.1650 (0.0000)
		1.0S	3.5371 (0.0004)	5.9344 (0.0000)	7.0460 (0.0000)	8.4203 (0.0000)
		1.5S	3.5021 (0.0005)	5.6403 (0.0000)	6.4599 (0.0000)	7.5549 (0.0000)
		2.0S	2.8742 (0.0041)	4.8027 (0.0000)	5.3056 (0.0000)	6.1974 (0.0000)
	01/10/2009 – 13/04/2015	0.5S	1.9812 (0.0476)	3.2973 (0.0010)	5.8644 (0.0000)	6.7610 (0.0000)
		1.0S	2.4377 (0.0148)	3.9855 (0.0001)	4.8892 (0.0000)	4.8820 (0.0000)
		1.5S	1.9980 (0.0457)	3.5678 (0.0004)	4.2053 (0.0000)	4.2145 (0.0000)
		2.0S	1.8448 (0.0651)	3.2545 (0.0011)	3.7172 (0.0002)	3.7314 (0.0002)

ε, *m*, and *S* denote the distance, dimension, and standard deviation respectively. ε is calculated as various multiples (0.5, 1, 1.5, and 2) of the standard deviation of the data. The primary rows report the BDS test statistics for the London Stock Exchange index, while the values in parentheses yield the corresponding p-values. The test statistic is asymptotically standard normal, and so the null hypothesis is rejected at the 5% level of significance where the p-value ≤ 0.05. Values in red denote the only cases where the null hypothesis cannot be rejected.