Analysis of the Telecommunications Industry using Smooth Transition Autoregressive ($STAR$) Models

by

Michael J. Sloboda

Major Paper presented to the Department of Economics of the University of Ottawa in partial fulfillment of the requirements of the M.A. Degree

Supervisor

Professor Gabriel Rodríguez

ECO 7997

Ottawa, Canada
August 2002

Abstract

This paper examines eleven quarterly revenue time series from the Telecommunications Industry using Smooth Transition Autoregressive (STAR) models. We use the modeling cycle methodology proposed by Terasvirta (1994). After estimation of linear models, we find strong evidence of nonlinearities in the data with the exception of one company. Selection of the delay variable and the kind of the transition function is done. Estimation of the corresponding nonlinear model shows interesting dynamics of adjustment in all time series. Also, the nonlinear models improves the linear models in term of the estimated variance. Other indicators such as the AIC favor selection of the nonlinear model. All ten time series were evaluated for non remaining autocorrelation, non remaining nonlinearity and parameter constancy. Only for one time series do we reject the respective null hypothesis. For all other estimations, we are noy able to reject the corresponding null hypothesis, indicating that the estimated models are adequate.

Keywords: Telecommunications Industry, Smooth Autoregressive Models, Transition Function, Structural Change.

JEL Classification: C2, C5.

Table of Contents

1 Introduction

This paper examines the impact that re-regulation of the U.S. telecommunications industry has had on the dynamics of AT&T, the seven Regional Operating Companies (ROCs): Ameritech (AIT), Bellsouth (BLS), Bell Atlantic (BEL), Southwestern Bell (SBC), NYNEX (NYN), Pacific Telesis (PAC), U.S. West (USW), two independent operating telecommunications companies: GTE, a major independent, SNG, a small local bell operating company, and a large telecommunications equipment supplier Nortel (NT). The Telecommunications Act of 1996 opened a pandora's box of new entrants into the telecommunications marketplace, new markets for incumbent players to enter, and large build-outs of new long distance fiber optic networks and high speed metropolitan access networks. However, the euphoria and spending did not last long and within four years many of the new entrants couldn't attract enough customers to pay for their network builds and started to fail. This has lead to the present unprecedented round of bankruptcies and layoffs. With it has also come network over capacity and consolidation in the industry. In addition, the stock markets, especially the NASDAQ, have fallen to historical lows affecting millions of individuals. Therefore an examination of the impact of re-regulation of this sort is warranted to better understand its ramifications on the dynamics of an industry and perhaps better design them in order to prevent a similar meltdown in other industries.

To model all these observed structural changes, we use Smooth Transition Autoregressive (STAR) models. This kind of model allows us to consider two regimes (named 0 and 1), where the way to pass from one to other is done either in abrupt or smooth form. In fact, this kind of model allows one to nest for a wide variety of other models.

We follow the methodology proposed by Terasvirta (1990) and others, which is known as the STAR modelling cycle. This methodology consists in first estimating a linear model and then verifying the hypothesis of linearity for different lags and for different possible variables. In an autoregressive model, the explanatory variables are lagged dependent variables. It is one (or more) of these variables that may determine the transition from one regime to another regime. As a consequence, this variable is named the transition variable. With this variable in hand, the methodology then applies a statistic to decide what is the form of the transition process. Two classes of functions are widely used for this purpose. The first is the logistic function, the second is the exponential function. When a function is selected, the estimations are realized using nonlinear least squares which under some conditions, is equivalent to maximum of likelihood. Estimation produces two important parameters. The first is the smoothing parameter and it indicates the degree of smoothness of the adjustment from one regime to the other regime. The second parameter is the threshold, which is the point where the curve changes from one regime to the other regime. In the paper, we consider models where we can allow up to three thresholds.

After the model is estimated, the nonlinear model has to be evaluated. Essentially this is done using three kinds of tests. The first test is a statistic to verify the null hypothesis that there is no remaining autocorrelation in the residuals from the nonlinear model. The second test is a statistic to evaluate the null hypothesis that there is no remaining nonlinearity in the model. Finally, the third statistic deals with the null hypothesis that the parameters are constant. Once the model passes all these tests, normally it is evaluated for forecasting. We do not follow this step. We stop at the evaluation step and we compare with the variance of the linear models.

We applied the methodology mentioned above to eleven time series from the telecommunications industry. The series are quarterly revenues that span from 1984:1 until 2002:1. Our results show the important presence of nonlinearities for almost all time series analyzed. We have only one time series for which the null hypothesis of linearity cannot be rejected. Consequently, this variable is not used in the rest of the estimations. The other ten variables are submitted to the modeling cycle methodology. Different LSTAR models are estimated and evaluated. Results show that these kind of models are preferred to the estimated linear models. There are important improvements in the variances estimated from the regressions, compared to those from the linear models. Using AIC as an indicator allows us to select again the nonlinear estimated models over the linear models. The graphical results allow us to observe important dynamics in our time series.

This paper is organized as follows. Section 2 presents a background about the telecommunications industry. We think that this section will allow to the reader to understand some characteristics of this industry over the last two decades . Section 3 presents a revision of the STAR models. In this section, these models are defined and analyzed in the context of the modeling cycle methodology. Section 4 presents the empirical results. The text presents only the more essential results but all the tables containing detailed results are presented at the end of the document. Section 5 concludes. Figures and Tables are presented at the end of the document.

2 U. S. Telecommunications Industry Background

2.1 The Modified Final Judgement (MFJ)

On January 8, 1982 AT&T and the U.S. Justice Department reached a settlement in their eight year anti-trust suit. The agreement would split up the largest corporation in U.S. history and divide up $155 billion in assets and one million employees. The structure of AT&T before this momentous event is shown in Figure 1, from Simon (1985).

The main divisions were the 22 *Bell Operating Companies* (BOCs) which handled most of the telephone business, AT&T Long Lines which was the interstate long-distance division carrying all Bell System interstate call traffic and Western Electric, the equipment manufacturing arm of AT&T. There were many changes to the original agreement but as of August 24, 1982 all modifications were in place and Judge Harold Greene approved what would become known in the industry as the *Modified Final Judgement* (MFJ). The structure and organizations it outlined are illustrated in Figure 2, from Simon (1985).

The final Plan of Reorganization encompassed the number of *Regional Operating Companies* (ROCs), the formation of geographical areas, known as *Local Access and Transport Areas* (LATA's), to distinguish local service from long-distance service and Equal Access, that is, steps to be put in place to allow competitors to be on an equal footing with AT&T Long-Lines in the long-distance telephone market. In the end seven ROC's were formed composed of the original BOC's with each starting off with approximately the same amount of assets as shown in the Table 1.

The other key ingredient of the Plan of reorganization, the LATA, was defined as the new regional exchange area. Roughly speaking it divided local service from long-distance service in the geographic service area defined by the LATA. In addition, LATA had to be correspond more or less to *Standard Metropolitan Statistical Areas* (SMSA) with some consideration given to common socioeconomic characteristics of an area. In the end, Judge Greene approved the LATA structure illustrated in Figure 2.

The MFJ prohibited the ROCs from providing long-distance telecommunications services or information services, manufacturing telecommunications equipment, or providing any product or service, except exchange telecommunications and exchange access service, that is not a natural monopoly service actually regulated by tariff. The MFJ allowed the ROCs to provide printed directory advertising and to provide, but not manufacture, customer premises equipment, see U.S. Securities and Exchange Commission (2002).

2.2 The Telecommunications Act of 1996

On February 8, 1996, the Telecommunications Act of 1996 (the Act) was enacted into law. The Act is intended to address various aspects of competition within, and regulation of, the telecommunications industry. The Act provides that all post-enactment conduct or activities which were subject to the MFJ are now subject to the provisions of the Act. Among other things,the Act also defines conditions ROC's must comply with before being permitted to offer interLATA long-distance service and establishes certain terms and conditions intended to promote competition for the Telephone Company's local exchange services. The MFJ, as originally approved by the District Court in 1982, had placed restrictions, known as the "line of business" restrictions, on the types of businesses in which ROC'S could engage. ROC'S could obtain relief from these restrictions upon a showing that there was no substantial possibility that it could use its monopoly power to impede competition in the specific market it sought to enter (the Waiver Standard). Over time, the Court granted waivers to the ROCs to engage in otherwise prohibited lines of business upon a showing to the Court that there was no substantial possibility that the company could use its monopoly power to impede competition in the market it sought to enter.

As a result of waiver proceedings before the District Court since divestiture, the MFJ's initial line of business restrictions against engaging in non telecommunications businesses, providing intraLATA information services, and providing telecommunications products had all been removed. ROC's were also authorized to engage in the restricted lines of business outside the United States, subject to certain conditions designed to prevent an impact on United States markets. However, ROC's were prohibited from providing interexchange telecommunications services and manufacturing telecommunications products and customer premises equipment (CPE).

Over time, the Court granted waivers to the ROCs to engage in otherwise prohibited lines of business upon a showing to the Court that there was no substantial possibility that the company could use its monopoly power to impede competition in the market it sought to enter. In decisions handed down in September 1987 and March 1988, the Court continued prohibitions relating to equipment manufacture and long-distance services. The rulings allowed limited provision of information services by transmission of information and provision of information gateways, but excluded generation or manipulation of information content. In addition, the rulings eliminated the need for a waiver for entry into non-telephone related businesses. In July 1991, the Court lifted the information services ban, but stayed the effect of the decision pending outcome of the appeals process. Soon after, the stay was lifted on appeal and in July 1993, the U.S. Court of Appeals unanimously upheld the Court's order allowing the ROCs to produce and package information for sale across business and home phone lines. In November 1993, the U.S. Supreme Court declined to review the lower court ruling„ see U.S. Securities and Exchange Commission (2002).

In July 1994, four of the ROCs (Ameritech was not a participant) filed a motion in the Court to vacate the entire MFJ. The filing was supported by numerous affidavits from consultants to the companies which largely suggested that ROC entry into restricted markets would not impede competition in those markets, but actually spur competition and result in lower prices for consumers. After a brief review by the Court, the matter was referred to the DOJ which has taken comments from interested parties as part of an extensive fact finding effort. The DOJ's recommendation was expected in late 1995 or early 1996. The Figure 3 illustrates the consolidation that has occurred post Telecommunications ACT 1996.

3 STAR Models

According to van Dijk, Franses, and Teräsvirta (2000), the smooth transition autoregressive (*STAR*) model for the independent time series variable observed at $t = 1-p, 1-(p-1), ..., -1, 0, 1..., T-1, T$, is given by

$$\begin{aligned} y_t &= (\phi_{1,0} + \phi_{1,1}y_{t-1} + ... + \phi_{1,p}y_{t-p})(1 - F(s_t;\gamma, c) + \\ &\quad (\phi_{2,0} + \phi_{2,1}y_{t-1} + ... + \phi_{2,p}y_{t-p})(F(s_t;\gamma, c)) + \varepsilon_t \end{aligned} \quad (1)$$

or

$$y_t = \phi_1' x_t(1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c) + \varepsilon_t \tag{2}$$

where $x_t = (1, \tilde{x}_t')$ with $\tilde{x}_t = (y_{t-1}, ..., y_{t-p})'$ and $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, $i = 1, 2$ and $F(.)$ is named the transition function. The model can easily be extend to include exogenous variables $z_{1t}, ..., z_{kt}$ as additional regressors and this is discussed in detail in Teräsvirta (1998). In the $STAR$ model, we assume that the ε_t is a martingale difference sequence with respect to the time series history up to and including time $t-1$, it is denoted as $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, ..., y_{1-(p-1)}, y_{1-p}\}$, or $E[\varepsilon_t|\Omega_{t-1}] = 0$. In addition, we assume that the conditional variance of ε_t is constant, $E[\varepsilon_t^2|\Omega_{t-1}] = \sigma^2$. As well, Lundbergh and Teräsvirta (1998) discuss an extension of this model that accounts for autoregressive conditional heteroscedasticity ($ARCH$).

The transition function is a continuous function that is bounded between 0 and 1. The most popular choices for $F(s_t; \gamma, c)$ include the logistic and exponential functions (see below). In the $STAR$ model (2), the transition variable can be a lagged endogenous variable, $s_t = y_{t-d}$ for $d > 0$. It can also be an exogenous variable $s_t = z_t$ or a function of lagged endogenous variables $s_t = g(\tilde{x}_t; \alpha)$ for some function g, which depends on the $q \times 1$ parameter vector α . Lastly, the transition variable can be a linear time trend $s_t = t$ giving a model with smoothly changing parameters as discussed in Lin and Teräsvirta (1994).

There are two ways to interpret the $STAR$ model (2). First, the $STAR$ model can be interpreted as a regime-switching model that switches between two regimes according to the limiting values of the transition function $F(s_t; \gamma, c) = 0$, and $F(s_t; \gamma, c) = 1$, where the transition from one regime to another is smooth. Secondly, the $STAR$ model (2) can be interpreted as a continuum of regimes each associated with a different value of $F(s_t;\gamma,c)$ between 0 and 1. In this discussion we will only be concerned with the two-regime viewpoint.

The regime that occurs at time t is determined by the observable variable s_t and the associated values of $F(s_t;\gamma,c)$. Different choices for the transition function $F(s_t;\gamma,c)$ give rise to different types of regime-switching behaviour. A popular choice for $F(s_t;\gamma,c)$ is the first-order logistic function (see Figure 5)

$$F(s_t;\gamma,c) = (1+\exp\{-\gamma\{s_t-c)\})^{-1}, \qquad \gamma > 0 \tag{3}$$

and the resultant model is called the logistic $STAR$ (L$STAR$) model. The parameter c in (2) or (3) can be interpreted as the threshold between the two regimes, in the sense that the logistic function changes monotonically between 0 and 1 as s_t increases and $F(s_t;\gamma,c) = 0.5$. The parameter γ determines the smoothness of the change in the value of the logistic function and thus, the smoothness of the transition from one regime to the other. As γ becomes very large, the logistic function $F(s_t;\gamma,c)$ approaches the indicator function $I[s_t > c]$, defined as $I[A] = 1$ if A is true and $I[A] = 0$ otherwise, and, consequently, the change of $F(s_t;\gamma,c)$ from 0 to 1 becomes almost instantaneous at $s_t = c$. Hence, the $LSTAR$ model (2) with (3) nests a two-regime threshold autoregressive (TAR) model as a special case. In the case $s_t = y_{t-d}$, this model is called a self exciting TAR (SETAR) model. An extensive discussion of the SE(TAR) models can be found in Tong (1990). When $\gamma \to 0$, the logistic function becomes equal to a constant (equal to 0.5) and when $\gamma = 0$, the L$STAR$ model (2) reduces to a linear AR model with parameters $\phi_j = (\phi_{1,j} + \phi_{2,j})/2$, $j = 0, 1, ..., p$.

In the $LSTAR$ model, the two regimes are associated with small and large values of the transition variable s_t relative to c. This type of regime-switching can be convenient for modelling, for example, business cycle asymmetry to distinguish expansions and recessions. If y_t is the growth rate of an output variable, and if $c \approx 0$, the model distinguished between periods of positive and negative growth, that is, between expansions and recessions. The $LSTAR$ model has been successfully applied by Teräsvirta and Anderson (1992) and Teräsvirta, Tjøstheim and Granger (1994) to characterize the different dynamics of industrial production indexes in a number of OECD countries during expansions and recessions.

In certain applications another type of regime-switching behaviour might be more appropriate. For example, it can be argued that the behaviour of the real exchange rate depends on the size of the deviation from purchasing power parity (PPP). In particular, the presence of transaction costs, such as costs of transportation and storage of goods, leads to the notion of different regimes in real exchange rates. The profits from commodity arbitrage do not make up for the costs involved in the necessary transactions for small deviations from the equilibrium real exchange rate, which implies the existence of a band around the equilibrium rate in which there is no tendency of the real exchange rate to revert to its equilibrium value. Outside this band, commodity arbitrage becomes profitable, which forces the real exchange rate back towards the band. See Taylor, Peel and Sarno (2001) for a review and discussion of theoretical models that incorporate effects of transaction costs as described above. If regime switching of this form is to be captured by a $STAR$ model with y_t denoting the real exchange rate and $s_t = y_{t-d}$, it appears more appropriate to specify the transition function such that the regimes are associated with small and large absolute values of s_t . This can be achieved by using, for example, the exponential function (see Figure 6)

$$F(s_t;\gamma,c)=1-\exp[-\gamma(s_t-c)^2], \qquad \gamma>0 \tag{4}$$

The exponential function has the property that $F(s_t;\gamma,c)\rightarrow 1$, both as $s_t\rightarrow-\infty$ and as $s_t\rightarrow\infty$ whereas $F(s_t;\gamma,c)=0$ for $s_t=c$. The resultant exponential $STAR$ (E$STAR$) model has been applied to real exchange rates by Michael, Nobay and Peel (1997) and Taylor, Peel and Sarno (2001) and to real effective exchange rates by Sarantis (1999). A drawback of the exponential function (4) is that for either $\gamma\rightarrow 0$ or $\gamma\rightarrow\infty$, the function collapses to a constant (equal to 0 or 1, respectively). Hence, the model becomes linear in both cases and the $ESTAR$ model does not nest a SETAR model as a special case. If this is thought to be desirable, one can instead use the second-order logistic function (see Figure 7)

$$F(s_t;\gamma,c)=\{1+\exp[-\gamma(s_t-c_1)(s_t-c_2)]\}^{-1}, \tag{5}$$

where $\gamma>0, c_1\leqslant c_2$. Now $c=(c_1,c_2)'$, as proposed by Jansen and Tersävirta (1996). In this case, if $\gamma\rightarrow 0$, the model becomes linear, whereas if $\gamma\rightarrow\infty$ and $c_1\neq c_2$, the function $F(s_t;\gamma,c)$ is equal to 1 for $s_t<c_1$ and $s_t>c_2$ and equal to 0 in between. Hence, the $STAR$ model with this particular transition function nests a restricted three-regime SETAR model, where the restriction is that the other regimes are identical.

Note that for moderate values of γ, the minimum value of the second-order logistic function attained for $s_t = (c_1 + c_2)/2$, remains between 0 and 1/2, unless $\gamma \to \infty$. In the latter case, the minimum value does equal zero. This has to be kept in mind when interpreting estimates from the models with this particular transition function. Finally, the transition functions (3) and (4) are special cases of the general *nth*-order logistic function

$$F(s_t; \gamma, c) = \{1 + \exp[-\gamma \prod_{i=1}^{n} (s_t - c_i)]\}^{-1}, \tag{6}$$

where $\gamma > 0, c_1 \leqslant c_2 \leqslant ... \leqslant c_n$. This function can be used to obtain multiple switches between the two regimes.

4 The *STAR* Modeling Cycle

The approach suggested by Teräsvirta (1994) follows a specific-to-general strategy for building nonlinear time series models. This approach allows the construction of a suitable model starting from a simple model and proceeding to a more complicated model by performing diagnostic testing of the model's adequacy, also this approach allows us to work around the nuisance parameter issues mention earlier.

Teräsvirta's (1994) method consisted of the following six steps: (i) specify a linear model of order p for the time series under investigation; (ii) test the null hypothesis of linearity against the alternative *STAR* nonlinearity. If linearity is rejected, choose the appropriate transition variable s_t and the form of the transition function $F(s_t; \gamma, c)$; (iii) estimate the parameters in the chosen $STAR$ model; (iv) evaluate the model, using both diagnostic testing and impulse response analysis; (v) modify the model if warranted; (vi) use the model to forecast or describe situations. In the following lines, steps 1-4 will be described.

4.1 Specification of a linear model

The main element involved in the first step is the choice of the correct lag order p in the $AR(p)$ model (7) for y_t,

$$y_t = \phi_0 + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t \tag{7}$$

where the lag order should be chosen so that the residuals are as close to be white noise as possible because the nonlinearity tests in step two are sensitive to remaining linear residual autocorrelation. One can then use the usual methods to select the order of the AR model, that is, the Akaike Information Criterion (AIC), the Schwarz Information Criterion (BIC) or the Ljung-Box statistic. Even if linearity is rejected in step two, the lag order of the AR model can still be used as an initial starting point for the $STAR$ model order.

4.2 Inference and Hypothesis testing

Testing linearity against $STAR$ is our first step in assembling a $STAR$ model. However, testing for linearity against $STAR$ alternatives is complicated by the presence of nuisance parameters. This can be illustrated in the following manner, one way to formulate the null hypothesis of linearity is to equate the AR parameters in the two regimes giving, $H_0 : \phi_1 = \phi_2$, where

$$y_t = \phi_1' x_t (1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c) \tag{8}$$

but an alternative hypothesis, $H_0 : \gamma = 0$, also determines a linear model. Thus if our transition function is the logistic function (3) with $\gamma = 0$, then $F(s_t; \gamma, c) = 0.5$ for all values of s_t, and the $STAR$ model above reduces to the AR model with parameters $(\phi_1 + \phi_2)/2$. So if we use the hypothesis $H_0 : \gamma = 0$, the location parameter c and the parameters ϕ_1 and ϕ_2 are unidentified. The presence of these nuisance parameters disallows the use of conventional asymptotic distribution theory to derive test statistics such as, the classical likelihood ratio, Lagrange-Multiplier, and Wald statistics and would force us to perform extensive simulation in order to calculate critical values. To get around this conundrum, Luukkonen, Saikkonen and Teräsvirta (1988) proposed replacing the transition function $F(s_t; \gamma, c)$ by a third-order Taylor series approximation and using it to test linearity. This re-formulization removes the identification problem and allows us to test linearity using an $LM - type$ statistic with a standard asymptotic χ^2 distribution under the null hypothesis. This approach also enables us use standard asymptotic theory to calculate critical values for the test statistics and frees us from having to estimate the model under the alternative hypothesis.

4.2.1 Testing Linearity versus LSTAR

Let's rewrite the $LSTAR$ model (2) as

$$y_t = \phi_1' x_t + (\phi_2 - \phi_1)' x_t F(s_t; \gamma, c) + e_t \tag{9}$$

and assume $\{e_t\} \sim n.i.d.(0, \sigma^2)$. Following Luukkonen, Saikkonen and Teräsvirta (1988), it is possible to approximate the logistic function (3) with a first-order Taylor approximation around $\gamma = 0$ which results in the following auxiliary regression

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + e_t \tag{10}$$

where $\beta_i = (\beta_{i,0}, \beta_{i,1}, ..., \beta_{i,p})'$, $i = 0, 1$, and $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_1(s_t; \gamma, c)$, with $R_1(s_t; \gamma, c)$ the remainder term from the first-order Taylor approximation. This remainder term under our null hypothesis of linearity is defined as equal to zero leaving $e_t = \varepsilon_t$. This allows us to use asymptotic theory to calculate test statistics. Now, given this auxiliary regression we note that the parameters β_i, $i = 0, 1$, are functions of the parameters in (9). This implies that if $\gamma = 0$ in equation (2), then $\beta_{0,j} \neq 0$ and $\beta_{1,j} = 0$ for $j = 0, ..., p$ in the auxiliary regression. Thus testing the null hypothesis $H_0 : \gamma = 0$ in the original $LSTAR$ is the same as testing the null hypothesis $H_0 : \beta_1 = 0$ in the auxiliary regression. The $LM - type$ statistic calculated using this approach, denoted LM_1, is asymptotically χ^2 distributed with $p + 1$ degrees of freedom under the null hypothesis of linearity. Hansen (1997, 1999) also use a very similar test statistic to estimate a SETAR model.

In the case where the transition variable $s_t = y_{t-d}$, for $1 < d \leqslant p$, $\beta_{1,0} s_t$ must be dropped due to multicollinearity. Luukkonen *et al.* (1988) noted that when $\phi_{1,0} \neq \phi_{2,0}$ but $\phi_{1,j} = \phi_{2,j}$ for $j = 1, ..., p$, that is, only the intercept differs across regimes, the LM_1 test statistic lacks power. To circumvent this problem they proposed approximating the transition function $F(s_t;\gamma,c)$ by a third-order Taylor approximation which gives us the following auxiliary regression equation

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + e_t \tag{11}$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_3(s_t;\gamma,c)$, and $\beta_{0,0}$ and the β_i, $i = 1,2,3$, are functions of the parameters ϕ_1, ϕ_2, γ and c . In this case, the null hypothesis $H_0 : \gamma = 0$ in model (2) corresponds to the null hypothesis $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ in the above auxiliary regression. A new $LM - type$ test statistic, LM_3, is used to test the auxiliary regression null hypothesis and is asymptotically χ^2 distributed with $3(p+1)$ degrees of freedom. Similar to that mentioned above, if $s_t = y_{t-d}$ for $d \leqslant p$, $\beta_{1,0} s_t$ should be dropped from the auxiliary regression as well.

A more economical version of the test statistic LM_3 can be derived by noting that when s_t is not included in $\tilde{x}_t$ where $\tilde{x}_t$ was defined as $\tilde{x}_t = (y_{t-1}, ..., y_{t-p})'$ in the basic $LSTAR$ model, the only parameters that depend on $\phi_{1,0}, \phi_{2,0}$ are $\beta_{1,0}, \beta_{2,0}$ and $\beta_{3,0}$. Therefore adding the regressors, s_t^2 and s_t^3 to the auxiliary regression obtained from the third-order Taylor approximation above we get the following modified auxiliary regression

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_{2,0}' s_t^2 + \beta_{3,0}' s_t^3 + e_t \tag{12}$$

where the new null hypothesis to be tested is $H_0 : \beta_1 = 0$ and $\beta_{2,0} = \beta_{3,0} = 0$. The $LM - type$ test statistic obtained in this situation is denoted LM_3^e

and is asymptotically χ^2 distributed with only $p+3$ degrees of freedom. Again as above, when if $s_t = y_{t-d}$ for some $d \leqslant p$, then only $\beta_{2,d}$ and $\beta_{3,d}$ provide information about $\phi_{1,0}$ and $\phi_{1,0}$ therefore by augmenting the auxiliary regression

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + e_t \tag{13}$$

with y_{t-d}^3 and y_{t-d}^4 we can calculate the $LM-type$ test statistic LM_3^e.

4.2.2 Testing Linearity versus ESTAR

Saikkonen and Luukkonen (1988) use the following auxiliary regression to test linearity against the $ESTAR$ alternative

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + e_t \tag{14}$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_2(s_t; \gamma, c)$. This equation is based on the first-order expansion of (2) where $x_t = (1, \tilde{x}_t')$ with $\tilde{x}_t = (y_{t-1}, ..., y_{t-p})'$ and $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, $i = 1, 2$. Then we have either

$$F(s_t; \gamma, c) = 1 - \exp[-\gamma(s_t - c)^2], \qquad \gamma > 0 \tag{15}$$

or

$$F(s_t; \gamma, c) = \{1 + \exp[-\gamma(s_t - c_1)(s_t - c_2)]\}^{-1}, \tag{16}$$

where $\gamma > 0, c_1 \leqslant c_2$. Note that the restriction $\gamma = 0$ is equivalent to $\beta_1 = \beta_2 = 0$ in (14). The LM_2 statistic that tests the null hypothesis H_0 :

$\beta_1 = \beta_2 = 0$ is asymptotically χ^2 distributed with $2(p+1)$ degrees of freedom.

Elsewhere in the literature, Escribano and Jordá (1999) show that the first-order Taylor approximation of the exponential function is not sufficient enough to capture all its characteristic features. Instead, they suggest that a second-order Taylor approximation is needed especially to capture the two inflection points in the original exponential function. They suggest the following auxiliary regression

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + \beta_4' x_t s_t^4 + e_t \qquad (17)$$

where the null hypothesis to be tested in this case is $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Under this null hypothesis, the LM-type statistic derived from this test and denoted, LM_4, is asymptotically χ^2 distributed with $4(p+1)$ degrees of freedom. The trade-off with using this test is between the increase in the dimension of the null hypothesis and the number of variables in the auxiliary regression (17).

4.2.3 Computing the Test Statistics

We usually compute the F-versions of the $LM - type$ test statistics because of the superior size properties versus their χ^2 counterparts especially when sample size is small. Note that both F and χ^2 versions can be computed using the auxiliary regressions outlined in the proceeding discussions. The LM_3 test statistic computation is outlined below as per van Dijk, Franses and Teräsvirta (2000) as an illustration of the procedure:

- Regress y_t on x_t to estimate the model under the null hypothesis of linearity and compute $\hat{\varepsilon}_t$ and the sum of squared residuals. That is

$$SSR_0 = \sum_{t=12}^{T} \hat{\varepsilon}_t^2, \tag{18}$$

- Estimate the auxiliary regression of y_t on x_t and $x_t s_t^i$, with $i = 1, 2, 3$ and compute $\hat{e}_t$ and the sum of squared residuals. That is

$$SSR_1 = \sum_{t=12}^{T} \hat{e}_t^2, \tag{19}$$

- Compute the χ^2 version of the LM_3 test statistic as

$$LM_3 = \frac{T(SSR_0 - SSR_1)}{SSR_1/(T - 4(p+1)} \tag{20}$$

or calculate the F version

$$LM_3 = \frac{(SSR_0 - SSR_1)/3(p+1)}{SSR_1/(T - 4(p+1)} \tag{21}$$

where under the null hypothesis the F version of the test is F distributed with $3(p+1)$ and $T - 4(p+1)$ degrees of freedom and T is the total number of observations of y_t the dependent variable.

4.3 Specification

Specification is the second phase in the STAR modeling cycle and its main goals, in addition to testing for linearity are, choosing the transition variable and the form of the transition function.

4.3.1 Choosing the transition variable s_t

By noting that all the auxiliary regressors in the first-order Taylor approximation to the $ESTAR$ model are contained in the third-order Taylor approximation to the $LSTAR$ model derived earlier, we see that the LM_3 statistic developed earlier has power against both the $ESTAR$ and $LSTAR$ models. This implies that we can pick our transition variable before selecting the transition function by computing the LM_3 statistic for different choices of transition variable and picking the transition variable which gives the lowest $p-value$ for the LM_3 test statistic. One thing to note about this test is that the significance level is not under our control, however, because we are using this test to help in model specification and not strict linearity testing, it will not be an issue.

4.3.2 Choosing the transition function $F(s_t; \gamma, c)$

At this stage of the modeling cycle we have rejected linearity in favor of $STAR$ nonlinearity and chosen the transition variable. The last step in this phase is to pick a transition function. Usually the form of the transition function is limited to the types mentioned before.

Again consider the third-order Taylor approximation to the $LSTAR$ model derived earlier. If we now consider the following null hypothesis:

$$H_{03} \; : \; \beta_3 = 0, \tag{22}$$

$$H_{02} \; : \; \beta_2 = 0 \mid \beta_3 = 0 \tag{23}$$

$$H_{01} \quad : \quad \beta_1 = 0 \mid \beta_3 = \beta_2 = 0 \tag{24}$$

all of which can be tested with the LM-type tests derived earlier, we have the following possible results:

- $\beta_3 \neq 0$ only if the model is a $LSTAR$ model,
- $\beta_2 = 0$ if the model is an $LSTAR$ model with $\phi_{1,0} = \phi_{2,0}$ and $c = 0$ but is always nonzero if the model is an $ESTAR$ model, and
- $\beta_1 = 0$ if the model is an $ESTAR$ model with $\phi_{1,0} = \phi_{2,0}$ and $c = 0$ but is always nonzero if the model is an $LSTAR$ model.

4.4 Estimation

In this stage of the modeling cycle we are ready to estimated the parameters of the $STAR$ model. The estimation procedure outlined below is with respect to the two-regime model expressed in (2). The $STAR$ model parameters can be estimated using non-linear least squares (NLS) where the parameters $\theta = (\phi_1', \phi_2', \gamma, c)'$ are estimated as

$$\hat{\theta} = \arg\min_{\theta} Q(\theta) = \arg\min_{\theta} \sum_{t=1}^{T} (y_t - F(x_t; \theta))^2 \tag{25}$$

where $F(x_t; \theta)$ is the skeleton of the $STAR$ model,

$$F(x_t; \theta) = \phi_1'(1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c). \tag{26}$$

Adding the assumption that the residual errors ε_t are normally distributed, the NLS estimation is equivalent to maximum likelihood. As outlined in Pötscher and Prucha (1997), given some regularity conditions, the NLS estimate are consistent and asymptotically normal, which can be represented as

$$\sqrt{T}(\hat{\theta} - \theta_0) \longrightarrow N(0, C), \tag{27}$$

where θ_0 represents the true parameter values and $\hat{A}_T^{-1}\hat{B}_T\hat{A}_T^{-1}$ is a consistent estimate of the asymptotic covariance-matrix C of $\hat{\theta}$, with $\hat{A}_T$ as the Hessian evaluated at $\hat{\theta}$, that is,

$$\hat{A}_T = -\frac{1}{T}\sum_{t=1}^{T} \nabla^2 q_t(\hat{\theta}) = \frac{1}{T}\sum_{t=1}^{T}(\nabla F(x_t;\hat{\theta})\nabla F(x_t;\hat{\theta})' - \nabla^2 F(x_t;\hat{\theta})\hat{\varepsilon}_t) \tag{28}$$

with $q_t(\hat{\theta}) = (y_t - F(x_t;\hat{\theta}))^2$, and $\hat{B}_T$ is the outer product of the gradient

$$\hat{B}_T = \frac{1}{T}\sum_{t=1}^{T} \nabla q_t(\hat{\theta})\nabla q_t(\hat{\theta})' = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^2 \nabla F(x_t;\hat{\theta})\nabla F(x_t;\hat{\theta})' \tag{29}$$

Performing estimation by NLS is possible but some attention must be paid to the following issues: (i) concentrating the sum of squares function; (ii) the choice of starting values for the parameters; and (iii) the estimation of γ, the smoothness parameter in $F(s_t;\gamma,c)$ the transition function.

4.5 Estimating γ

Obtaining accurate estimates of the smoothness parameter γ in the transition function becomes difficult as $\gamma \to \infty$. When γ is large, the transition function approaches a step function and as a consequence in order to accurately determine value of γ, one needs a large number of observations around c. Large changes in γ have little effect of the transition function in general therefore an accurate estimate of γ is usually unnecessary. Even though the t-statistic may indicate that γ is insignificant we must remember that under the null hypothesis of $\gamma = 0$ it does not have the usual asymptotic $t-$ distribution. Therefore the bottom-line is that we do not need high accuracy when estimating γ.

4.6 Evaluation

The third step in our $STAR$ modeling cycle is to evaluate the soundness of the nonlinear model. We will follow the method of Eitrheim and Teräsvirta (1996) using their $LM - type$ test statistics to examine the hypotheses of (i) no residual autocorrelation; (ii) no remaining nonlinearity; and (iii) parameter constancy. We explain briefly each statistic.

4.6.1 No error autocorrelation

Let's consider the basic two-regime $STAR$ model given in expression (2) and represent the skeleton of the model given by (26). The $LM - test$ for no serial correlation in the residuals $\widehat{\varepsilon}_t$ is calculated using TR^2, where R^2 is the coefficient of determination from the regression of $\hat{\varepsilon}_t$ on $\nabla F(x_t; \hat{\theta}) = \partial \nabla F(x_t; \hat{\theta})/\partial\theta$, with $\theta = (\phi_1, \phi_2, \gamma, c)'$, and q lagged residuals $\hat{\varepsilon}_{t-1}, ..., \hat{\varepsilon}_{t-q}$. The test statistic, $LM_{SI(q)}$, is asymptotically χ^2 distributed with q degrees of freedom.

4.6.2 No remaining nonlinearity

Eitrheim and Teräsvirta (1996) have developed an LM statistic to test the two-regime $LSTAR$ model against the alternative of an additive $STAR$ model as in

$$
\begin{aligned}
y_t &= \phi_1' x_t + (\phi_2 - \phi_1)' x_t F_1(s_t; \gamma_1, c_1) \\
&\quad + (\phi_3 - \phi_2)' x_t F_2(s_t; \gamma_2, c_2) + \varepsilon_t
\end{aligned} \tag{30}
$$

where we assume $c_1 < c_2$, and the autoregressive parameters of this model change smoothly from ϕ_1 by means of ϕ_2 and ϕ_3 as s_t increases in value and as F_1 varies from 0 to 1 followed by the same type of change in F_2. A third-order Taylor approximation to model becomes

$$
\begin{aligned}
y_t &= \beta_0' x_t + (\phi_2 - \phi_1)' + x_t F_1(s_t; \gamma, c) + \beta_1' x_t s_t \\
&\quad + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + e_t
\end{aligned} \tag{31}
$$

where the parameters β_i, for $i = 0, 1, 2, 3$ are functions of the parameters $\phi_1, \phi_2, \phi_3, \gamma_2$ and c_2. The null hypothesis $H_0 : \gamma_2 = 0$ becomes instead $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ in the Taylor series approximation. The test statistic $LM_{AMR,3}$ has an asymptotic χ^2 distribution with $3(p+1)$ degrees of freedom, where AMR indicates additive multiple regime.

4.6.3 Parameter Constancy

By testing the hypothesis $H_0 : \gamma_2 = 0$, we can test for parameter constancy in the two regime $STAR$ model against the alternative of smoothly changing parameters. The $LM - type$ statistic is based on a third-order Taylor approximation of $F_2(t; \gamma_2, c_2)$, and is represented as $LM_{C,3}$.

5 Empirical Analysis

In this section the empirical results are presented. Firstly, we present the results from the linear estimations. The second step, as detailed before, is to apply linearity tests to verify the presence of nonlinearity. After choosing the transition variable, estimations are performed and the models are evaluated.

Before estimating the linear models, we tested for the presence of a unit root in the original time series. Application of a standard unit root test such as the Augmented Dickey-Fuller (ADF) (see Dickey and Fuller, 1979; Said and Dickey, 1984) indicated that the null hypothesis of a unit root can not be rejected. Because these kinds of tests have low power if there is a misspecification of the deterministic components (Perron, 1989) we applied unit root tests with structural change using the method of Zivot and Andrews (1992). The statistic allowed to reject the null hypothesis of a unit root for some time series in favor of the alternative hypothesis with a break in the intercept and/or slope. However, these kind of tests are valid when the alternative allows for only one break point. Observing the original time series, it is possible to say that more than one break point is possible to be considered. Given these reasons, we consider all our time series as nonstationary time series; consequently, the estimations are performed using data in first differences.

Because the data is quarterly, and as visual inspection suggests, there is seasonal behavior, we consider also seasonal differenced data. In consequence, our variable can be denoted as $\Delta\Delta_4 y_t$.

5.1 Linear Models and Linearity Testing

The next step in the analysis was the estimation of linear $AR(p)$ models for all the time series. These models are represented by

$$\Delta\Delta_4 y_t = \hat{\beta}_1 \Delta\Delta_4 y_{t-1} + ... + \hat{\beta}_8 \Delta\Delta_4 y_{t-p} + \hat{\varepsilon}_t \tag{32}$$

where $p = 8$. Tables 3.1-3.3 present the results obtained from the estimation of linear models. Note that we present the "best" linear model in the sense that non significant parameters have been eliminated using significance levels.

The next stage is to test linearity against $STAR$ nonlinearity using the $LM-type$ statistics discussed earlier. We consider the following variables as the transition variable, $s_t = \Delta\Delta_4 y_{t-d}$, $d = 1, ..., d_{\max}$. We set the maximum value of the delay parameter $d_{\max}$ equal to 8 .

Tables 4.1-4.11 contain the $p-values$ of the standard LM_3 tests with $\Delta\Delta_4 y_{t-d}$ as the transition variable. The results for each company are summarized in Table A. In the second column there appears the transition variable selected by the statistic according to the lowest p-value. However, we have included in most of cases, more than only one possible delay for the transition function, which allows us to find a best result. In the third column there appears the level at which the null hypothesis of linearity is rejected.

Table A. Summary of Linearity Testing Results[a]

Time Series $\{\Delta\Delta_4 y_t\}$	Transition variable(s), s_t	Linearity null Hypothesis
Ameritech (AIT)	$\Delta\Delta_4 y_{t-d}, d = 2, 8$	***
Pacific Telesis (PAC)	N/A	
U.S. West (USW)	$\Delta\Delta_4 y_{t-d}, d = 1, 4$	*
Southern New England Bell (SNG)	$\Delta\Delta_4 y_{t-d}, d = 1$	***
NYNEX (NYN)	$\Delta\Delta_4 y_{t-d}, d = 2, 5$	*
Bellsouth (BLS)	$\Delta\Delta_4 y_{t-d}, d = 2, 3, 4, 6$	*
GTE	$\Delta\Delta_4 y_{t-d}, d = 1, ..., 8$	*
Bell Atlantic (BEL)	$\Delta\Delta_4 y_{t-d}, d = 1, 2, 3, 4, 6$	*
Nortel (NT)	$\Delta\Delta_4 y_{t-d}, d = 1, .., 5, 8$	*
SBC	$\Delta\Delta_4 y_{t-d}, d = 4, 5, 8$	*
AT&T	$\Delta\Delta_4 y_{t-d}, d = 2, 3, 4$	*

[a] In the Table *,**,*** indicate reject at 1, 5, and 10%, respectively.

From Table A, we observe that for most of the companies the null hypothesis of linearity is rejected. The only exception is PAC. Strong rejects of the null hypothesis are found for most of the ten companies where we reject this hypothesis. Only the time series for AIT and SNG show a reject of the null hypothesis at 10.0%. Table B present a summary of the models to be estimated.

Table B. Summary of Linearity tests versus $STAR$ nonlinearity

Time Series$\{\Delta\Delta_4 y_t\}$	Linear	Non-Linear
Ameritech (AIT)		✓
Pacific Telesis (PAC)	✓	
U.S. West (USW)		✓
South New Eng. Bell (SNG)		✓
NYNEX (NYN)		✓
Bellsouth (BLS)		✓
GTE		✓
Bell Atlantic (BEL)		✓
Nortel (NT)		✓
SBC		✓
AT&T		✓

5.2 LSTAR(k) Estimations

In this stage of the $STAR$ modelling cycle, the $LSTAR(k)$ models are estimated for $k = 1, 2, 3$ for each time series $\{\Delta\Delta_4 y_t\}$ using all possible transition functions as shown in Table A. More strictly, each $LSTAR(k)$ model was estimated with 8 lagged variables, $\Delta\Delta_4 y_{t-1}, ..., \Delta\Delta_4 y_{t-8}$ in both regimes. The final choice of transition variable and model type is realized in the evaluation stage. However, a summary of the final estimated models for each time series is presented in Table C. In addition, diagrams of the Transition function and Transition function versus time for each company can be found in the back of the document, see figures 8a, 8b - figures 17a, 17b. The results of the each company's revenue history with respect to the STAR model figures 8a, 8b - figures 17a, 17b.

Figure 8a and 8b illustrate the Ameritech (AIT) Transition function and Ameritech Transition function versus time. The AIT $LSTAR(1)$ model does a good job capturing the slowing growth in revenues reflecting the general economic slowdown in the U.S. in 1990-91 time frame. Also the sustained growth in the 1992-1996 due to cellular subscriber growth is also captured by the model. The other major event, the re-regulation that occurred in 1996 is also reflected in the model where the regime change highlights the increased competition in its territory form other ROCs and cable companies. AIT was bought out by SBC in the 1999 time frame.

Figure 9a and 9b illustrate the U.S. West (USW) Transition function and U.S. West Transition function versus time. The USW $LSTAR(3)$ model again captures the dip during the general economic slowdown in the U.S. in 1990-91. In addition, the USW Transition function shows the many transition points that are traversed in going from one regime to the next. This indicates a slower moving dynamic reflecting the USW 14-state territory which is sparsely populated and had slower growth than other parts of the country. Note the significant regime change post- Telecommunications ACT of 1996, reflecting the increased competition from other ROCs and cable companies in its operating territory. U.S. West was bought out in 2001 by Qwest another mid-west telecommunications company.

Figure 10a and 10b illustrate the Southern New England Bell (SNG) Transition function and SNG Transition function versus time. The SNG $LSTAR(2)$ model also captures the economic dip in the 1990-1991 time frame and also captures the generally slower growth in SNG's operating territory. New England and Connecticut were especially hard hit by the economic downturn and its recovery lagged the rest of the U.S.. Consequently SNG's revenues struggled with slow grow from 1990-1994. The re-regulation in 1996 is also reflected in the model with a regime change that highlights further erosion in generally flat to slow growth in revenues of SNG. SNG was bought out by Bell Atlantic in the 1998 time frame.

Figure 11a and 11b illustrate the NYNEX (NYN) Transition function and NYN Transition function versus time. The NYN $LSTAR(3)$ model captures the economic dip in the 1990-1991 time frame. Note the fewer transition points in this $LSTAR(3)$ model compared to USW indicating more abrupt changes from one regime to another. NYN suffered from slow growth and unfavorable regulatory restrictions which left it open to intense competition. The 1996 re-regulation is evident in an abrupt regime change reflecting a decline in revenues from competition with cable companies and other ROCs in its territory. NYNEX was bought out in 1997 by Bell Atlantic.

Figure 12a and 12b illustrate the Bellsouth (BLS) Transition function and BLS Transition function versus time. The BLS $LSTAR(3)$ model reflects the economic dip in the 1990-1991 time frame with changes in regime indicating revenue growth rate declines. BLS was in a fortunate territory (includes Florida, Georgia) where there was very strong growth during the 1990's. BLS's purchase of the McCaw cellular assets in its territory greatly enhanced the fast growth rates in the cellular market in its operating territory. BLS made many investments in Asia and Latin America during the 1990's and this also increased the rate of growth. The transition function in this model has many transition points between regimes indicating no abrupt movements unless there are extreme changes in the revenue rates. The economic crisis in Latin America in 2000-2001 exposed BLS to some very large revenue declines and this is reflected by the model by regime changes in 2000-2001.

Figure 13a and 13b illustrate the GTE Transition function and GTE Transition function versus time. The GTE $LSTAR(2)$ model captures the economic dip in the 1990-1991 time frame in addition to the sustained growth in its long distance revenues in the 1987-1989 period. The sustained growth period in mid to late 1990's reflects growth in the data communications and cellular businesses for GTE. As an independent company outside the restrictions of imposed on the ROCs the re-regulation of the industry in 1996 opened up more opportunities for GTE and thus allowed it to continue on upward revenue growth path. This is reflected in the model by no regime changes during the period 1995-2000. GTE was purchased by Bell Atlantic in1999 time frame.

Figure 14a and 14b illustrate the Bell Atlantic (BEL) Transition function and BEL Transition function versus time. The BEL $LSTAR(1)$ model fails the diagnostic tests but it too captures the economic dip in the 1990-1991 time frame and the fast rate of growth in the cellular subscriber base in the BEL territory in the 1991-1995 time frame. Bell Atlantic purchased GTE and NYN and renamed itself Verizon, see figure 3. This change is also captured by a regime change in the model.

Figure 15a and 15b illustrate the Nortel (NT) Transition function and NT Transition function versus time. The NT $LSTAR(3)$ model too captures the economic dip in the 1990-1991 time frame in addition to the major software and reliability problems in its products in the 1985-1988 time frame which negatively impacted revenues shown by sustained regime. It was also a boom time for NT from 1989-1995 with unprecedented increases in sales of optical, data and cellular(PCS) equipment in North America and in Europe. The model captures the major downturn in the economy in general and the telecommunications sector in particular in the 1999-2001 time frame. This model does a very good job capturing the dynamics of this equipment supplier.

Figure 16a and 16b illustrate the SBC Transition function and SBC Transition function versus time. The SBC $LSTAR(2)$ model captures the economic dip in the 1990-1991 time frame, the growth in the cellular subscriber base in its territory and the major acquisitions, PAC in 1999, AIT in 2000 and the general economic slowdown in the 2000-2001 time frame. SBC was specifically impacted by the energy crisis in California in 1999-2000 where it receives up to 40% of its revenues. The re-regulation of 1996 is evident because the model reflects the volatility brought about by increase competition and the restructuring of the industry as a whole.

Figure 17a and 17b illustrate the AT&T Transition function and AT&T Transition function versus time. The AT&T $LSTAR(3)$ model is very similar to the NT transition function. It too captures the economic dip in the

1990-1991 time frame, the acquisition of NCR ($7.2billion) which added computer revenues to AT&T total revenue stream. The large growth rates in the 1992- 1999 time frame in AT&T's revenue, driven by its acquisition of McCaw Cellular and by major moves into the cable industry with purchases of among others TCI ($50 billion) are reflected in the model by sustained regimes. The post-1996 re-regulation is also demonstrated in the model by regime changes due to increased competition in the long distance market from AOL-Time-Warner especially. In 2001 AT&T divided itself up into four separate entities: AT&T Corp., AT&T Global Information, AT&T Wireless, and Lucent.

Table C. Summary of the Model Type

Time Series	Model Type		Nonlinear Model Type		
$\{\Delta\Delta_4 y_t\}$	Linear	Nonlinear	$LSTAR(1)$	$LSTAR(2)$	$LSTAR(3)$
Ameritech (AIT)		✓	✓		
Pacific Telesis (PAC	✓				
U.S. West (USW)		✓			✓
Southern New England Bell (SNG)		✓		✓	
NYNEX (NYN)		✓			✓
Bellsouth (BLS)		✓			✓
GTE		✓		✓	
Bell Atlantic (BEL)		✓	✓		
Nortel (NT)		✓			✓
SBC		✓		✓	
AT&T		✓			✓

A Summary of the results can be observed in Tables D and E. Using AIC as an indicator to select the model, indicates that in all cases, the nonlinear model is preferred to the estimated linear model. Last column of the Table D shows the ratio of the standard deviation obtained from the nonlinear model and the standard deviation obtained from the linear model. A LSTAR(1) model was estimated for two companies (see Table C, for example) and we observe that estimating this kind of models allow us to have a reduction of the variance in around 13.0% (AIT) and 35.0% (BEL), respectively. In the case of estimated LSTAR (2), it is possible to improve the estimation of the variance from 23.0% (SNG) until 45.0% (GTE). Finally, using LSTAR (3) model allows us to have improvement in estimation of the variance from 6.0% (USW) until 27.0% (NT).

Table D. Model Fit Comparisons

Time Series $\{\Delta\Delta_4 y_t\}$	AIC		$\sigma_{\hat{\varepsilon}_t}$		$\hat{\sigma}(LSTAR)/\hat{\sigma}(AR)$
	Linear	Nonlinear	Linear	Nonlinear	
Ameritech (AIT)	−8.166	−8.45	.01465	.012749	0.870
Pacific Telesis (PAC)	−6.899	N/A	N/A	N/A	N/A
U.S. West (USW)	−5.812	−5.944	0.049	0.045	0.934
Southern New England Bell (SNG)	−5.64	−6.174	0.051	0.039	0.768
NYNEX (NYN)	−7.613	−7.612	0.019	0.019	1.0
Bellsouth (BLS)	−6.343	−6.53	0.037	0.034	0.899
GTE	−5.256	−6.43	0.063	0.035	0.553
Bell Atlantic (BEL)	−3.961	−4.816	0.121	0.079	0.652
Nortel (NT)	−4.209	−5.107	0.108	0.068	0.63
SBC	−4.077	−4.662	0.111	0.086	0.769
AT&T	−3.981	−4.401	0.121	0.097	0.800

Table E. Model R^2 Comparisons

Time Series $\{\Delta\Delta_4 y_t\}$	R^2		*Adjusted* R^2	
	Linear	Nonlinear	Linear	Nonlinear
Ameritech (AIT)	0.124	0.526	0.108	0.526
Pacific Telesis (PAC)	0.343	N/A	0.30	N/A
U.S. West (USW)	0.294	0.548	0.269	0.504
Southern New England Bell (SNG)	0.410	0.746	0.387	0.707
NYNEX (NYN)	0.409	0.681	0.356	0.626
Bellsouth (BLS)	0.244	0.621	0.21	0.576
GTE	0.354	0.834	0.32	0.812
Bell Atlantic (BEL)	0.330	0.837	0.319	0.815
Nortel (NT)	0.317	0.699	0.296	0.699
SBC	0.023	0.675	0.008	0.675
AT&T	0.0212	0.622	0.006	0.622

5.3 Evaluation

After estimating the parameters of the $LSTAR(k)$ models for all the time series, it is necessary to test the basic assumptions underlying the estimation. Tests of no error autocorrelation, no remaining nonlinearity and parameter constancy have been considered earlier in the $STAR$ Modelling section. The detailed results of these diagnostic test can be found in Tables 6.1-6.3. At the 5% level of significance all the $LSTAR(k)$ models for all the time series $\{\Delta\Delta_4 y_t\}$, except Bell Atlantic (BEL) pass the tests. Therefore we can assume all the models are adequate, except Bell Atlantic (BEL) which has a significant amount of remaining nonlinearity and parameter constancy.

6 Conclusions

Eleven quarterly revenue time series for the Telecommunications Industry have been analyzed using Smooth Transition Autoregressive (STAR) models. We followed the modeling cycle methodology proposed by Terasvirta

Table 3.1. Estimated Linear AR(p) Models

Company Significant coefficients of $\Delta\Delta_4 y_t$	Ameritech (AIT)	AT&T	Bell Atlantic (BEL)
$\Delta\Delta_4 y_{t-1}$			
$\Delta\Delta_4 y_{t-2}$			
$\Delta\Delta_4 y_{t-3}$		−0.0493077 (.120)	
$\Delta\Delta_4 y_{t-4}$	−0.392808 (.150)		−0.618937 (.117)
$\Delta\Delta_4 y_{t-5}$			
$\Delta\Delta_4 y_{t-6}$			
$\Delta\Delta_4 y_{t-7}$			
$\Delta\Delta_4 y_{t-8}$	−0.241129 (.157)	−0.131328 (.120)	−0.312159 (.112)
$\sigma^2_{\hat{\varepsilon}_t}$	0.0002	0.0148	0.0146
AIC	−8.166	−3.981	−3.961
$adjR^2$	0.108	0.006	0.319

Table 3.2. Estimated Linear AR(p) Models

Company Significant coefficients of $\Delta\Delta_4 y_t$	GTE	Nortel (NT)	NYNEX (NYN)	Pacific Telesis (PAC)
$\Delta\Delta_4 y_{t-1}$	−0.216544 (.122)	0.27254 (.101)	−0.260431 (.113)	−0.321811 (.138)
$\Delta\Delta_4 y_{t-2}$				−0.227875 (.132)
$\Delta\Delta_4 y_{t-3}$			−0.251378 (.132)	
$\Delta\Delta_4 y_{t-4}$	−0.526971 (.101)	−0.568906 (.135)	−0.675338 (.137)	−0.47003 (.128)
$\Delta\Delta_4 y_{t-5}$	−0.146436 (.121)			−0.213346 (.143)
$\Delta\Delta_4 y_{t-6}$		0.178801 (.146)		
$\Delta\Delta_4 y_{t-7}$	0.169417 (.108)		−0.344619 (.143)	
$\Delta\Delta_4 y_{t-8}$			−0.222317 (.170)	
$\sigma^2_{\hat{\varepsilon}_t}$	0.0039966	0.0117471	0.0003564	0.0007323
AIC	−5.256	−4.209	−7.613	−6.899
$adjR^2$	0.32	.296	.356	.3

Table 1. Comparison of the seven ROCs

	Bell South	NYNEX	Bell Atlantic	Ameritech	Pacific Telesis	SBC	U.S.West
Assets	23,207	18,796	18,197	18,161	17,811	17,491	16,901
Revenues	10,304	9,612	8,384	8,722	7,850	7,707	7,389
Net Income	1,233	835	888	849	517	796	762
Long-term Debt	5,890	5,189	4,759	4,942	6,560	4,675	4,416
Employees	131,513	120,770	104,825	103,734	106,243	98,783	97,384
Telephones	2.306E+07	1.741E+07	2.325E+07	2.357E+07	1.507E+07	1.690E+07	1.672E+07
* As of December 31, 1982. Dollars listed are in millions.							

Table 2. Regional BELL Operating Companies (ROCs)

Ameritech	Illinois ,Indiana ,Michigan, Ohio,Wisconsin	Illinois Bell Telephone Company; Indiana Bell Telephone Company, Incorporated; Michigan Bell Telephone Company; The Ohio Bell Telephone Company and Wisconsin Bell, Inc. as well as a cellular mobile communications company.	as local service , network access , long-distance service, directory advertising, cellular and other
Bell Atlantic	New Jersey, Pennsylvania, Delaware, Washington, D.C., Maryland, Virginia, West Virginia	New Jersey Bell Telephone Company; The Bell Telephone Company of Pennsylvania; The Diamond State Telephone Company; The Chesapeake and Potomac Telephone Company; The Chesapeake and Potomac Telephone Company of Maryland; The Chesapeake and Potomac Telephone Company of Virginia; and The Chesapeake and Potomac Telephone Company of West Virginia.	as local service , network access , long-distance service, directory advertising, cellular and other
Bellsouth	Alabama, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina and Tennessee.	South Central Bell Telephone Company and Southern Bell Telephone and Telegraph Company	provides services, which include local exchange, exchange access and intraLATA toll services, within each of the 38 LATAs in its combined nine-state operating area.
NYNEX	New York State, Connecticut Massachusetts, Maine,New Hampshire, Rhode Island and Vermont.	New York Telephone and New England Telephone	There are six LATAs that comprise the area served by New York Telephone and six LATAs served by New England Telephone
Pacific Telesis	California and Nevada	Pacific Telesis; two BOCs, Pacific Bell and Nevada Bell; and certain diversified subsidiaries.	as local service , network access , long-distance service, directory advertising, cellular and other
US West	Arizona, Colorado, Idaho, Iowa, Minnesota, Montana, Nebraska, New Mexico, North Dakota, Oregon, South Dakota, Utah, Washington and Wyoming	Mountain Bell, Northwestern Bell and Pacific Northwest Bell	29 LATAs with each LATA generally centered on a metropolitan area or other identifiable community of interest. The services offered are (i) local service, (ii) intraLATA long distance service and (iii) exchange access service (which connects customers to the facilities of interLATA service providers).

(1994). After linear estimations were performed, we found strong evidence in favor of nonlinearities in the data. For only one time series, we cannot reject the null hypothesis of linearity. For the other ten time series different STAR models were estimated, where the difference was related to the number of significant thresholds allowed in the models. All nonlinear estimation showed improvement with respect to the linear models in terms of the estimated variance. Using the AIC as an indicator, we are able to select nonlinear models in all cases. Models were evaluated for non remaining autocorrelation, non remaining nonlinearity and parameter constancy. Only in one time series, can we find some evidence of violation of the null hypotheses of the corresponding statistics. For all other equations, we cannot reject the respective null hypothesis indicating that the model are adequate.

References

[1] Simon, Samuel A., After Divestiture: What the AT&T Settlement means for Business and Residential Telephone Service. (1985), TRAC.

[2] <http://www.sec.gov/> [August 2002]

[3] Campbell, J.Y. and Perron,P.,"Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," in *NBER Macroeconomics Annual,* O.J. Blanchard and S Fisher, editors, **Vol. 6**,141-201.

[4] Dickey, D.A. and W.A. Fuller (1979),"Distribution of the Estimator for Autoregressive Time Series with Unit Root," *Journal of the American Statistical Association* **74**,427-431.

[5] Eitrheim, Ø. and T. Teräsvirta (1996), "Testing the adequacy of smooth transition autoregressive models," *Journal of Econometrics* **74**, 59–76.

[6] Escribano, A. and O. Jorda (1999), "Improved testing and specification of smooth transition regression models," in P. Rothman (ed.), Nonlinear Time Series Analysis of Economic and Financial Data, Boston: Kluwer, pp. 289–319.

[7] Hansen, B.E. (1997)," Inference in TAR models," *Studies in Nonlinear Dynamics and Econometrics* **2**, 1–14.

[8] Hansen, B.E. (1999), "Testing for linearity," *Journal of Economic Surveys* **13**, 551–576.

[9] Jansen, E.S. and T. Teräsvirta (1996), "Testing parameter constancy and superexogeneity in econometric equations," *Oxford Bulletin of Economics and Statistics* **58**, 735–768.

[10] Lundbergh, S. and T. Teräsvirta (1998), "Modelling economic high-frequency time series with *STAR*-GARCH models", *Working Paper Series in Economics and Finance,* **No.291,** Stockholm School of Economics.

[11] Lin, C-F.J. and T. Teräsvirta (1994), "Testing the constancy of regression parameters against continuous structural change," *Journal of Econometrics* **62**, 211–228.

[12] Luukkonen, R., P. Saikkonen and T. Teräsvirta (1988), "Testing linearity against smooth transition autoregressive models, " *Biometrika* **75,** 491–499.

[13] Michael, P., A.R. Nobay and D.A. Peel (1997), "Transaction costs and nonlinear adjustment in real exchange rates: an empirical investigation," *Journal of Political Economy,* **105,** 862–879.

[14] Leybourne, S., P. Newbold and D. Vougas (1998), "Unit roots and smooth transitions," *Journal of Time Series Analysis* **19**, 83–97.

[15] Perron, P., (1989), "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," *Econometrica,***57**,1361-1401.

[16] Pötscher, B.M. and I.V. Prucha (1997)," Dynamic Nonlinear Econometric Models -Asymptotic Theory," Berlin: Springer-Verlag.

[17] Teräsvirta, T., D. Tjøstheim and C.W.J. Granger (1994), "Aspects of modelling nonlinear time series," *in R.F. Engle and D.L. McFadden (eds.), Handbook of Econometrics*, **vol. IV**, Amsterdam: Elsevier Science, pp. 2917–2957.

[18] Taylor, M.P., D.A. Peel and L. Sarno (2001)," Nonlinear mean-reversion in exchange rate rates: towards a solution to the purchasing power parity puzzles," *International Economic Review*, to appear.

Table 3.3. Estimated Linear AR(p) Models

Company Significant coefficients of $\Delta\Delta_4 y_t$	SBC	Southern New England Bell (SNG)	U.S.West (USW)	Bellsouth (BLS)
$\Delta\Delta_4 y_{t-1}$	0.0405766 (.118)	-0.203057 (.105)	-0.23774 (.102)	-0.206570 (.108)
$\Delta\Delta_4 y_{t-2}$				-0.178481 (.119)
$\Delta\Delta_4 y_{t-3}$				
$\Delta\Delta_4 y_{t-4}$		-0.695811 (.151)	-0.482747 (.112)	-0.458140 (.126)
$\Delta\Delta_4 y_{t-5}$				
$\Delta\Delta_4 y_{t-6}$				
$\Delta\Delta_4 y_{t-7}$	-0.138296 (.116)			
$\Delta\Delta_4 y_{t-8}$		-0.336676 (.165)	-0.20681 (.111)	-0.250932 (.136)
$\sigma^2_{\hat{\varepsilon}_t}$	0.0134018	0.0026272	0.0023632	0.00139
AIC	-4.077	-5.640	-5.812	-6.343
$adjR^2$				

Table 4.1. Testing for Linearity (AIT)

Ameritech (AIT)	*Lag(p)*							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.982	0.998	1.00	0.125	0.216	0.400	0.518	0.663
$\Delta\Delta_4 y_{t-2}$	1.00	1.00	0.998	0.997	0.976	0.953	0.967	0.908
$\Delta\Delta_4 y_{t-3}$	0.852	0.963	0.963	0.823	0.946	0.978	0.990	0.988
$\Delta\Delta_4 y_{t-4}$	0.000	0.002	0.003	0.003	0.011	0.055	0.128	0.014
$\Delta\Delta_4 y_{t-5}$	0.991	0.999	0.998	0.065	0.065	0.166	0.251	0.435
$\Delta\Delta_4 y_{t-6}$	1.00	1.00	1.00	0.513	0.705	0.705	0.753	0.669
$\Delta\Delta_4 y_{t-7}$	0.501	0.693	0.420	0.760	0.809	0.888	0.888	0.619
$\Delta\Delta_4 y_{t-8}$	0.001	0.001	0.001	0.023	0.024	0.081	0.172	0.172

Table 4.2. Testing for Linearity (AT&T)

AT&T	*Lag(p)*							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.873	0.986	0.649	0.459	0.539	0.622	0.804	0.912
$\Delta\Delta_4 y_{t-2}$	0.648	0.648	0.741	0.038	0.128	0.256	0.124	0.281
$\Delta\Delta_4 y_{t-3}$	0.452	0.761	0.761	0.076	0.091	0.130	0.092	0.004
$\Delta\Delta_4 y_{t-4}$	0.00	0.00	0.00	0.00	0.001	0.00	0.001	0.003
$\Delta\Delta_4 y_{t-5}$	0.999	0.993	0.990	0.858	0.858	0.779	0.882	0.885
$\Delta\Delta_4 y_{t-6}$	0.707	0.881	0.541	0.397	0.555	0.555	0.427	0.720
$\Delta\Delta_4 y_{t-7}$	1.00	1.00	1.00	0.444	0.412	0.442	0.442	0.331
$\Delta\Delta_4 y_{t-8}$	0.337	0.490	0.208	0.202	0.386	0.48	0.501	0.501

Table 4.3. Testing for Linearity (BEL)

Bell Atlantic (BEL)	*Lag(p)*							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.556	0.48	0.152	0	0	0	0	0
$\Delta\Delta_4 y_{t-2}$	0.004	0.004	0.00002	0	0	0	0	0
$\Delta\Delta_4 y_{t-3}$	0.858	0.404	0.404	0	0	0	0.00001	0.00003
$\Delta\Delta_4 y_{t-4}$	0.001	0	0	0	0	0	0.00004	0.012
$\Delta\Delta_4 y_{t-5}$	0.236	0.001	0.007	0.0001	0.00001	0	0.00001	0.001
$\Delta\Delta_4 y_{t-6}$	0	0	0	0	0	0	0	0
$\Delta\Delta_4 y_{t-7}$	0.882	0.093	0.26	0	0	0	0	0.00001
$\Delta\Delta_4 y_{t-8}$	0.996	0.274	0.512	0	0.004	0.001	0.005	0.005

Table 4.4. Testing for Linearity (USW)

U.S. West (USW) Transition Variable, s_t	Lag(p) $p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.420	0.335	0.160	0.129	0.058	0.192	0.321	0.465
$\Delta\Delta_4 y_{t-2}$	0.914	0.914	0.908	0.350	0.405	0.256	0.410	0.657
$\Delta\Delta_4 y_{t-3}$	0.366	0.501	0.501	0.743	0.874	0.965	0.985	0.942
$\Delta\Delta_4 y_{t-4}$	0.001	0.006	0.005	0.005	0.023	0.056	0.039	0.191
$\Delta\Delta_4 y_{t-5}$	0.523	0.327	0.492	0.345	0.345	0.475	0.715	0.441
$\Delta\Delta_4 y_{t-6}$	0.993	0.933	0.960	0.302	0.263	0.263	0.415	0.676
$\Delta\Delta_4 y_{t-7}$	0.735	0.909	0.949	0.986	0.959	0.978	0.978	0.889
$\Delta\Delta_4 y_{t-8}$	0.890	0.921	0.962	0.001	0.041	0.106	0.185	0.185

Table 4.5. Testing for Linearity (SNG)

Southern New Englnad Bell (SNG) Transition Variable, s_t	Lag(p) $p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.353	0.266	0.097	0.454	0.479	0.453	0.544	0.753
$\Delta\Delta_4 y_{t-2}$	0.551	0.551	0.513	0.863	0.927	0.882	0.716	0.960
$\Delta\Delta_4 y_{t-3}$	0.944	0.737	0.737	0.942	0.821	0.880	0.852	0.953
$\Delta\Delta_4 y_{t-4}$	0.282	0.444	0.437	0.437	0.694	0.651	0.533	0.268
$\Delta\Delta_4 y_{t-5}$	0.211	0.148	0.263	0.835	0.835	0.849	0.512	0.734
$\Delta\Delta_4 y_{t-6}$	0.778	0.773	0.856	0.889	0.649	0.649	0.331	0.167
$\Delta\Delta_4 y_{t-7}$	1.00	0.984	0.914	0.762	0.670	0.793	0.793	0.915
$\Delta\Delta_4 y_{t-8}$	0.916	0.852	0.283	0.800	0.635	0.447	0.262	0.262

Table 4.6. Testing for Linearity (NT)

Nortel (NT) Transition Variable, s_t	Lag(p) $p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.8	0.001	0	0.002	0	0	0	0
$\Delta\Delta_4 y_{t-2}$	0.228	0.228	0.004	0.106	0.079	0.199	0.134	0.147
$\Delta\Delta_4 y_{t-3}$	0.001	0.003	0.003	0.064	0.007	0.025	0.088	0.010
$\Delta\Delta_4 y_{t-4}$	0.013	0.059	0.026	0.026	0.021	0.001	0.019	0.082
$\Delta\Delta_4 y_{t-5}$	0.044	0.002	0.002	0.005	0.005	0.001	0.008	0.043
$\Delta\Delta_4 y_{t-6}$	0.308	0.372	0.050	0.101	0.108	0.108	0.023	0.065
$\Delta\Delta_4 y_{t-7}$	0.292	0.152	0.109	0.253	0.472	0.533	0.533	0.678
$\Delta\Delta_4 y_{t-8}$	0.086	0.093	0.050	0.042	0.082	0.130	0.096	0.096

Table 4.7. Testing for Linearity (NYN)

NYNEX (NYN)	Lag(p)							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.03	0.632	0.877	0.506	0.050	0.146	0.239	0.349
$\Delta\Delta_4 y_{t-2}$	0.952	0.952	0.901	.0498	0.033	0.012	0.105	0.321
$\Delta\Delta_4 y_{t-3}$	0.976	0.969	0.969	0.555	0.460	0.521	0.274	0.413
$\Delta\Delta_4 y_{t-4}$	0.671	0.814	0.677	0.677	0.679	0.762	0.984	0.991
$\Delta\Delta_4 y_{t-5}$	0.001	0.001	0.013	0.013	0.013	0.066	0.024	0.038
$\Delta\Delta_4 y_{t-6}$	0.269	0.535	0.332	0.260	0.425	0.425	0.396	0.472
$\Delta\Delta_4 y_{t-7}$	0.317	0.482	0.721	0.613	0.664	0.725	0.725	0.616
$\Delta\Delta_4 y_{t-8}$	0.623	0.831	0.962	0.260	0.114	0.235	0.357	0.357

Table 4.8. Testing for Linearity (GTE)

GTE	Lag(p)							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.007	0	0	0.004	0.001	0	0.003	0.001
$\Delta\Delta_4 y_{t-2}$	0.977	0.977	0.001	0.006	0.049	0.003	0.010	0.262
$\Delta\Delta_4 y_{t-3}$	0.030	0.131	0.131	0.144	0.183	0.399	0.044	0.001
$\Delta\Delta_4 y_{t-4}$	0.092	0.133	0.134	0.134	0.188	0.380	0.022	0
$\Delta\Delta_4 y_{t-5}$	0.607	0.177	0.202	0.531	0.531	0.589	0.139	0.082
$\Delta\Delta_4 y_{t-6}$	0.416	0.525	0.228	0.130	0.302	0.302	0.001	0.001
$\Delta\Delta_4 y_{t-7}$	0.971	0.956	0.953	0.288	0.157	0.146	0.146	0.041
$\Delta\Delta_4 y_{t-8}$	0.576	0.647	0.848	0	0	0.001	0.007	0.007

Table 4.9. Testing for Linearity (SBC)

SBC	Lag(p)							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.982	0.998	1	0.125	0.216	0.4	0.5180.663	
$\Delta\Delta_4 y_{t-2}$	1	1	0.998	0.997	0.976	0.953	0.967	0.908
$\Delta\Delta_4 y_{t-3}$	0.852	0.963	0.963	0.823	0.946	0.978	0.990	0.988
$\Delta\Delta_4 y_{t-4}$	0	0.002	0.003	0.003	0.011	0.055	0.128	0.014
$\Delta\Delta_4 y_{t-5}$	0.991	0.999	0.998	0.065	0.065	0.166	0.251	0.435
$\Delta\Delta_4 y_{t-6}$	1	1	1	0.513	0.705	0.705	0.753	0.669
$\Delta\Delta_4 y_{t-7}$	0.501	0.693	0.420	0.760	0.809	0.888	0.888	0.619
$\Delta\Delta_4 y_{t-8}$	0.001	0.001	0.001	0.023	0.024	0.081	0.172	0.172

Table 4.10. Testing for Linearity (PAC)

Pacific Telesis (PAC)	*Lag(p)*							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.444	0.612	0.645	0.323	0.485	0.754	0.906	0.963
$\Delta\Delta_4 y_{t-2}$	0.907	0.907	0.963	0.952	0.993	0.613	0.820	0.888
$\Delta\Delta_4 y_{t-3}$	0.548	0.696	0.696	0.909	0.920	0.943	0.878	0.851
$\Delta\Delta_4 y_{t-4}$	0.208	0.446	0.568	0.568	0.453	0.370	0.588	0.795
$\Delta\Delta_4 y_{t-5}$	0.570	0.719	0.873	0.703	0.703	0.682	0.513	0.607
$\Delta\Delta_4 y_{t-6}$	0.517	0.701	0.823	0.874	0.954	0.954	0.931	0.923
$\Delta\Delta_4 y_{t-7}$	0.773	0.955	0.936	0.576	0.524	0.597	0.597	0.794
$\Delta\Delta_4 y_{t-8}$	0.101	0.786	0.692	0.231	0.245	0.143	0.305	0.305

Table 4.11. Testing for Linearity (BLS)

Bellsouth (BLS)	*Lag(p)*							
Transition Variable, s_t	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$
$\Delta\Delta_4 y_{t-1}$	0.578	0.590	0.422	0.184	0.187	0.270	0.370	0.228
$\Delta\Delta_4 y_{t-2}$	0.546	0.546	0.083	0.017	0.013	0.009	0.053	0.041
$\Delta\Delta_4 y_{t-3}$	0.102	0.066	0.066	0.021	0.025	0.024	0.121	0.359
$\Delta\Delta_4 y_{t-4}$	0.005	0.031	0.090	0.090	0.023	0.054	0.101	0.044
$\Delta\Delta_4 y_{t-5}$	0.421	0.688	0.874	0.462	0.462	0.245	0.369	0.176
$\Delta\Delta_4 y_{t-6}$	0.292	0.124	0.272	0.199	0.019	0.019	0.060	0.053
$\Delta\Delta_4 y_{t-7}$	.0288	0.203	0.425	0.706	0.804	0.876	0.876	0.947
$\Delta\Delta_4 y_{t-8}$	0.311	0.514	0.257	0.137	0.254	0.266	0.203	0.203

Table 5.1. Estimated $LSTAR(1)$ Model for AIT

$$\begin{aligned}\Delta\Delta_4 y_t &= \underset{(0.205)}{-0.573352}\Delta\Delta_4 y_{t-2} \times [1 - G(\Delta\Delta_4 y_{t-2};\gamma, c_1)] \\ &\quad + [\underset{(0.486)}{0.906519}\Delta\Delta_4 y_{t-1} + \underset{(0.368)}{0.772827}\Delta\Delta_4 y_{t-7}] \\ &\quad \times G(\Delta\Delta_4 y_{t-2};\gamma, c_1) + \hat{\varepsilon}_t\end{aligned}$$

$$G(\Delta\Delta_4 y_{t-2};\gamma, c_1) = (1 + \exp\{-\underset{(19.55)}{14.977246}(\Delta\Delta_4 y_{t-2} + \underset{(0.002)}{0.007993})/\sigma_{\Delta\Delta_4 y_{t-2}}\})^{-1}$$

$$\hat{\sigma}_\varepsilon = 0.012749, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.870, AIC = -8.45$$

Table 5.2. Estimated $LSTAR(3)$ Model for USW

$$\begin{aligned}
\Delta\Delta_4 y_t &= \underset{(0.156)}{-0.905110}\Delta\Delta_4 y_{t-4} + \underset{(0.185)}{0.184963}\Delta\Delta_4 y_{t-7} \times \\
&\quad [1 - G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3)] \\
&\quad + [\underset{(0.229)}{-0.977844}\Delta\Delta_4 y_{t-4} + \underset{(0.228)}{1.055856}\Delta\Delta_4 y_{t-7}] \times G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3) + \hat{\varepsilon}_t
\end{aligned}$$

$$\begin{aligned}
G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3) &= (1 + \exp\{\underset{(1.28)}{-1.898701}(\Delta\Delta_4 y_{t-4} + \underset{(0.047)}{0.120496})(\Delta\Delta_4 y_{t-4} + \underset{(0.01)}{0.039496}) \\
&\quad (\Delta\Delta_4 y_{t-4} - \underset{(0.054)}{0.094867})/\sigma^3_{\Delta\Delta_4 y_{t-4}}\})^{-1}
\end{aligned}$$

$\hat{\sigma}_\varepsilon = 0.017572, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.934, AIC = -5.944$

Table 5.3. Estimated $LSTAR(2)$ Model for SNG

$$\begin{aligned}
\Delta\Delta_4 y_t &= \underset{(0.597)}{-1.759202}\Delta\Delta_4 y_{t-1} \times [1 - G(\Delta\Delta_4 y_{t-1}; \gamma, c_1, c_2)] \\
&\quad + [\underset{(0.610)}{1.470219}\Delta\Delta_4 y_{t-1} - \underset{(0.213)}{0.830628}\Delta\Delta_4 y_{t-2} - \underset{(0.219)}{0.424939}\Delta\Delta_4 y_{t-3} - \\
&\quad \underset{(0.274)}{1.392333}\Delta\Delta_4 y_{t-4} - \underset{(0.192)}{0.721549}\Delta\Delta_4 y_{t-8}] \times G(\Delta\Delta_4 y_{t-1}; \gamma, c_1, c_2) + \hat{\varepsilon}_t
\end{aligned}$$

$$\begin{aligned}
G(\Delta\Delta_4 y_{t-1}; \gamma, c_1, c_2) &= (1 + \exp\{\underset{(13480)}{-1181}(\Delta\Delta_4 y_{t-1} + \underset{(0.0004)}{0.020702}) \\
&+ \quad (\Delta\Delta_4 y_{t-1} - \underset{(0.001)}{0.022905})/\sigma^2_{\Delta\Delta_4 y_{t-1}}\})^{-1}
\end{aligned}$$

$$\hat{\sigma}_\varepsilon = 0.039367, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.768, AIC = -6.174$$

Table 5.4. Estimated $LSTAR(3)$ Model for NYN

$$\begin{aligned}
\Delta\Delta_4 y_t &= \underset{(0.411)}{1.082877}\Delta\Delta_4 y_{t-1} + \underset{(0.256)}{0.997577}\Delta\Delta_4 y_{t-5} + \underset{(0.559)}{1.603669}\Delta\Delta_4 y_{t-6} - \\
&\quad \times[1 - G(\Delta\Delta_4 y_{t-5}; \gamma, c_1, c_2, c_3)] \\
&\quad +[-\underset{(0.433)}{1.458273}\Delta\Delta_4 y_{t-1} - \underset{(0.345)}{0.987804}\Delta\Delta_4 y_{t-4} + \underset{(0.298)}{1.040363}\Delta\Delta_4 y_{t-5} - \\
&\quad \underset{(0.597)}{1.707558}\Delta\Delta_4 y_{t-6} - \underset{(0.298)}{0.547623}\Delta\Delta_4 y_{t-7} - \underset{(0.369)}{0.750637}\Delta\Delta_4 y_{t-8}] \\
&\quad \times G(\Delta\Delta_4 y_{t-5}; \gamma, c_1, c_2, c_3) + \hat{\varepsilon}_t
\end{aligned}$$

$$\begin{aligned}
G(\Delta\Delta_4 y_{t-5}; \gamma, c_1, c_2, c_3) &= (1 + \exp\{-\underset{(203)}{116.02}\Delta\Delta_4 y_{t-2}^2 \\
&+ \quad (\Delta\Delta_4 y_{t-2} + \underset{(0.008)}{0.008335})/\sigma^3_{\Delta\Delta_4 y_{t-2}}\})^{-1}
\end{aligned}$$

$\hat{\sigma}_\varepsilon = 0.019075, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 1.0, AIC = -7.612154$

Table 5.5. Estimated $LSTAR(3)$ Model for BLS

$$
\begin{aligned}
\Delta\Delta_4 y_t \;=\; & \underset{(0.394)}{-1.99264}\Delta\Delta_4 y_{t-1} - \underset{(0.339)}{0.706606}\Delta\Delta_4 y_{t-2} - \underset{(0.33.)}{0.670909}\Delta\Delta_4 y_{t-3} - \underset{(0.439)}{1.632616}\Delta\Delta_4 y_{t-5} - \\
& + \underset{(0.668)}{3.04598}\Delta\Delta_4 y_{t-6} + \underset{(0.552)}{2.402831}\Delta\Delta_4 y_{t-7} + \underset{(0.430)}{1.953131}\Delta\Delta_4 y_{t-8} \\
& \times [1 - G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3)] \\
& + [\underset{(0.838)}{3.924930}\Delta\Delta_4 y_{t-1} + \underset{(0.603)}{1.433269}\Delta\Delta_4 y_{t-2} + \underset{(0.537)}{1.451698}\Delta\Delta_4 y_{t-3} \\
& + \underset{(0.970)}{3.923168}\Delta\Delta_4 y_{t-5} - \\
& \underset{(1.22)}{4.903650}\Delta\Delta_4 y_{t-6} - \underset{(1.06)}{4.005379}\Delta\Delta_4 y_{t-7} - \underset{(0.688)}{3.328093}\Delta\Delta_4 y_{t-8}] \\
& \times G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3) + \hat{\varepsilon}_t
\end{aligned}
$$

$$
\begin{aligned}
G(\Delta\Delta_4 y_{t-4}; \gamma, c_1, c_2, c_3) \;=\; & (1 + \exp\{-\underset{(0.757276)}{1.428268}\Delta\Delta_4 y_{t-4}^2 \\
+ \quad & (\Delta\Delta_4 y_{t-4} + \underset{(0.001)}{0.031615})/\sigma^3_{\Delta\Delta_4 y_{t-4}}\})^{-1}
\end{aligned}
$$

$$\hat{\sigma}_\varepsilon = 0.033524, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.899, AIC = -6.52981$$

Table 5.6. Estimated $LSTAR(2)$ Model for GTE

$$
\begin{aligned}
\Delta\Delta_4 y_t \quad = \quad & -\underset{(0.063)}{1.011899}\Delta\Delta_4 y_{t-4} - \underset{(0.069)}{0.168831}\Delta\Delta_4 y_{t-5} - \\
& +\underset{(0.282)}{0.615603}\Delta\Delta_4 y_{t-6} - \underset{(0.071)}{0.46811}\Delta\Delta_4 y_{t-7} \times [1 - G(\Delta\Delta_4 y_{t-7}; \gamma, c_1, c_2)] \\
& [\underset{(0.158)}{0.995549}\Delta\Delta_4 y_{t-4} - \underset{(0.292)}{0.55182}\Delta\Delta_4 y_{t-7} + \underset{(0.137)}{0.466840}\Delta\Delta_4 y_{t-8}] \\
& \times G(\Delta\Delta_4 y_{t-7}; \gamma, c_1, c_2) + \hat{\varepsilon}_t
\end{aligned}
$$

$$
\begin{aligned}
G(\Delta\Delta_4 y_{t-7}; \gamma, c_1, c_2) \quad = \quad & (1 + \exp\{-\underset{(14431)}{423}(\Delta\Delta_4 y_{t-7} + \underset{(0.003)}{0.027305}) \\
+ \quad & (\Delta\Delta_4 y_{t-7} - \underset{(0.007)}{0.038926})/\sigma^2_{\Delta\Delta_4 y_{t-7}}\})^{-1}
\end{aligned}
$$

$$\hat{\sigma}_\varepsilon = 0.034985, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.553, AIC = -6.430$$

Table 5.7. Estimated $LSTAR(3)$ Model for NT

$$
\begin{aligned}
\Delta\Delta_4 y_t \;=\;& \underset{(0.117)}{0.6699854}\Delta\Delta_4 y_{t-1} + \underset{(0.156)}{0.827413}\Delta\Delta_4 y_{t-2} - \underset{(0.172)}{1.289767}\Delta\Delta_4 y_{t-3} - \underset{(0.171)}{0.407211}\Delta\Delta_4 y_{t-4} - \\
& \underset{(0.133)}{0.603093}\Delta\Delta_4 y_{t-5} + \underset{(0.167)}{0.547801}\Delta\Delta_4 y_{t-6} - \underset{(0.259)}{1.269012}\Delta\Delta_4 y_{t-7} - \underset{(0.145)}{1.000940}\Delta\Delta_4 y_{t-8} \\
& \times[1 - G(\Delta\Delta_4 y_{t-3}; \gamma, c_1, c_2, c_3)] \\
& +[-\underset{(0.186)}{0.9473}\Delta\Delta_4 y_{t-1} - \underset{(0.192)}{1.116975}\Delta\Delta_4 y_{t-2} + \underset{(0.203)}{1.216020}\Delta\Delta_4 y_{t-3} + \underset{(0.178)}{0.581635}\Delta\Delta_4 y_{t-5} - \\
& \underset{(0.192)}{0.698331}\Delta\Delta_4 y_{t-6} + \underset{(0.272)}{1.368944}\Delta\Delta_4 y_{t-7} + \underset{(0.170)}{0.917914}\Delta\Delta_4 y_{t-8}] \\
& \times G(\Delta\Delta_4 y_{t-3}; \gamma, c_1, c_2, c_3) + \hat{\varepsilon}_t
\end{aligned}
$$

$$
\begin{aligned}
G(\Delta\Delta_4 y_{t-3}; \gamma, c_1, c_2, c_3) \;=\;& (1 + \exp\{-\underset{(39.52)}{59.72}(\Delta\Delta_4 y_{t-3} - \underset{(0.004)}{0.012152})(\Delta\Delta_4 y_{t-3} + \underset{(0.004)}{0.059616}) \\
+\;& (\Delta\Delta_4 y_{t-3} + \underset{(0.002)}{0.183355})/\sigma^3_{\Delta\Delta_4 y_{t-3}}\})^{-1}
\end{aligned}
$$

$\hat{\sigma}_\varepsilon = 0.068280, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.630, AIC = -5.107$

Table 5.8. Estimated $LSTAR(2)$ Model for SBC

$$\begin{aligned}
\Delta\Delta_4 y_t \;=\; & \underset{(0.591)}{-2.000875}\Delta\Delta_4 y_{t-8} \times [1 - G(\Delta\Delta_4 y_{t-8}; \gamma, c_1, c_2)] \\
& +[\underset{(0.784)}{2.941054}\Delta\Delta_4 y_{t-1} + \underset{(1.101)}{2.413034}\Delta\Delta_4 y_{t-3} + \underset{(1.19)}{3.266418}\Delta\Delta_4 y_{t-5} + \\
& \underset{(1.21)}{3.684573}\Delta\Delta_4 y_{t-6} + \underset{(0.622)}{1.858383}\Delta\Delta_4 y_{t-8}] \times G(\Delta\Delta_4 y_{t-8}; \gamma, c_1, c_2) + \hat{\varepsilon}_t
\end{aligned}$$

$$\begin{aligned}
G(\Delta\Delta_4 y_{t-8}; \gamma, c_1, c_2) \;=\; & (1 + \exp\{\underset{(9.19)}{-20.37}(\Delta\Delta_4 y_{t-8} - \underset{(0.0005)}{0.034814}) \\
+ \; & (\Delta\Delta_4 y_{t-8} + \underset{(0.006)}{0.066432})/\sigma^2_{\Delta\Delta_4 y_{t-8}}\})^{-1}
\end{aligned}$$

$$\hat{\sigma}_\varepsilon = 0.085683, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.769, AIC = -4.662$$

Table 5.9. Estimated $LSTAR(3)$ Model for AT&T

$$
\begin{aligned}
\Delta\Delta_4 y_t \quad = \quad & \underset{(0.155)}{0.26868}\Delta\Delta_4 y_{t-3} - \underset{(0.114)}{1.199555}\Delta\Delta_4 y_{t-4} - \\
& \times[1 - G(\Delta\Delta_4 y_{t-2}; \gamma, c_1, c_2, c_3)] \\
& +[\underset{(0.190)}{-0.392325}\Delta\Delta_4 y_{t-3} + \underset{(0.389)}{2.422543}\Delta\Delta_4 y_{t-4} - \\
& \underset{(0.189)}{0.37692}\Delta\Delta_4 y_{t-8}] \times G(\Delta\Delta_4 y_{t-2}; \gamma, c_1, c_2, c_3) + \hat{\varepsilon}_t
\end{aligned}
$$

$$
\begin{aligned}
G(\Delta\Delta_4 y_{t-2}; \gamma, c_1, c_2, c_3) \quad = \quad & (1 + \exp\{\underset{(5852)}{-9429}(\Delta\Delta_4 y_{t-2} + \underset{(0.0001)}{0.013739})(\Delta\Delta_4 y_{t-2} - \underset{(0.0001)}{0.027023}) \\
+ \quad & (\Delta\Delta_4 y_{t-2} + \underset{(0.0005)}{0.011673})/\sigma^3_{\Delta\Delta_4 y_{t-2}}\})^{-1}
\end{aligned}
$$

$$\hat{\sigma}_\varepsilon = 0.097176, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.800, AIC = -4.401$$

Table 5.10. Estimated $LSTAR(1)$ Model for BEL

$$\begin{aligned}
\Delta\Delta_4 y_t \quad = \quad & \underset{(0.141)}{0.131668}\Delta\Delta_4 y_{t-2} - \underset{(0.223)}{0.551337}\Delta\Delta_4 y_{t-4} - \\
& +\underset{(0.272)}{0.840}\Delta\Delta_4 y_{t-6} - \underset{(0.234)}{0.469334}\Delta\Delta_4 y_{t-8} \times [1 - G(\Delta\Delta_4 y_{t-1}; \gamma, c_1)] \\
& [-\underset{(0.168)}{0.321183}\Delta\Delta_4 y_{t-1} - \underset{(0.596)}{3.177939}\Delta\Delta_4 y_{t-5} - \underset{(0.740)}{6.055392}\Delta\Delta_4 y_{t-6}] \\
& \times G(\Delta\Delta_4 y_{t-1}; \gamma, c_1) + \hat{\varepsilon}_t
\end{aligned}$$

$$G(\Delta\Delta_4 y_{t-1}; \gamma, c_1) \quad = \quad (1 + \exp\{-\underset{(5.944)}{16.836}(\Delta\Delta_4 y_{t-1} - \underset{(0.005)}{0.02141})/\sigma_{\Delta\Delta_4 y_{t-1}}\})^{-1}$$

$$\hat{\sigma}_\varepsilon = 0.078729, \hat{\sigma}(LSTAR)/\hat{\sigma}(AR) = 0.652, AIC = -4.816$$

Table 6.2. Testing for non-constant parameters

Info	*US West (USW)*	*Nortel (NT)*	*AT&T*	*SBC*	*GTE*	*Bellsouth (BLS)*	*NYNEX (NYN)*	*Southern New England Bell (SNG)*
				p-values				
All	0.348	0.195	0.712	0.962	0.805	0.103	0.074	0.621
Lin	0.201	0.407	0.803	0.973	0.414	0.245	0.089	0.450
Non	0.817	0.343	0.113	0.988	0.999	0.081	0.169	0.857

Table 6.1. Testing for qth order serial correlation

Lag	*US West (USW)*	*Nortel (NT)*	*AT&T*	*SBC*	*GTE*	*Bellsouth (BLS)*	*NYNEX (NYN)*	*Southern New England Bell (SNG)*
				p-values				
1	0.747	0.813	0.995	0.301	0.203	0.489	0.722	0.140
2	0.549	0.790	0.845	0.376	0.455	0.300	0.943	0.230
3	0.709	0.314	0.850	0.287	0.787	0.283	0.838	0.344
4	0.197	0.381	0.938	0.238	0.941	0.546	0.688	0.444
5	0.331	0.252	0.989	0.366	0.631	0.442	0.429	0.561
6	0.105	0.277	0.916	0.419	0.762	0.369	0.260	0.290
7	0.107	0.384	0.877	0.510	0.864	0.461	0.124	0.021
8	0.164	0.059	0.932	0.608	0.991	0.571	0.176	0.027

Table 6.3. Testing for remaining non-linearity

Lag	US West (USW)	Nortel (NT)	AT&T	SBC	GTE	Bellsouth (BLS)	NYNEX (NYN)	Southern New England Bell (SNG)
	p-values							
1	0.531	0.261	0.961	0.999	0.079	0.704	0.075	0.987
2	0.964	0.758	0.931	0.997	0.074	0.692	0.828	0.974
3	0.794	0.837	0.937	0.998	0.574	0.871	0.672	0.983
4	0.675	0.786	0.787	0.994	0.015	1.000	0.853	0.904
5	0.921	0.059	0.897	0.984	0.752	0.942	0.814	0.812
6	0.827	0.068	0.949	0.999	0.158	0.654	0.302	1.000
7	0.554	0.933	0.886	0.992	0.770	0.994	0.074	0.940
8	0.830	0.209	0.990	0.959	0.134	0.220	0.208	0.968

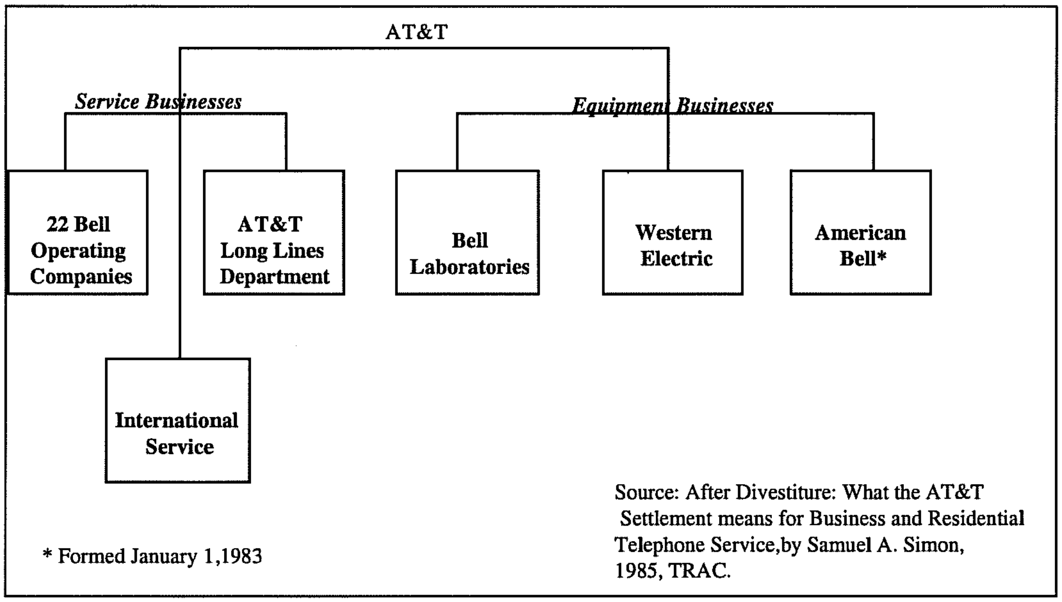


Figure 1. AT&T before divestiture

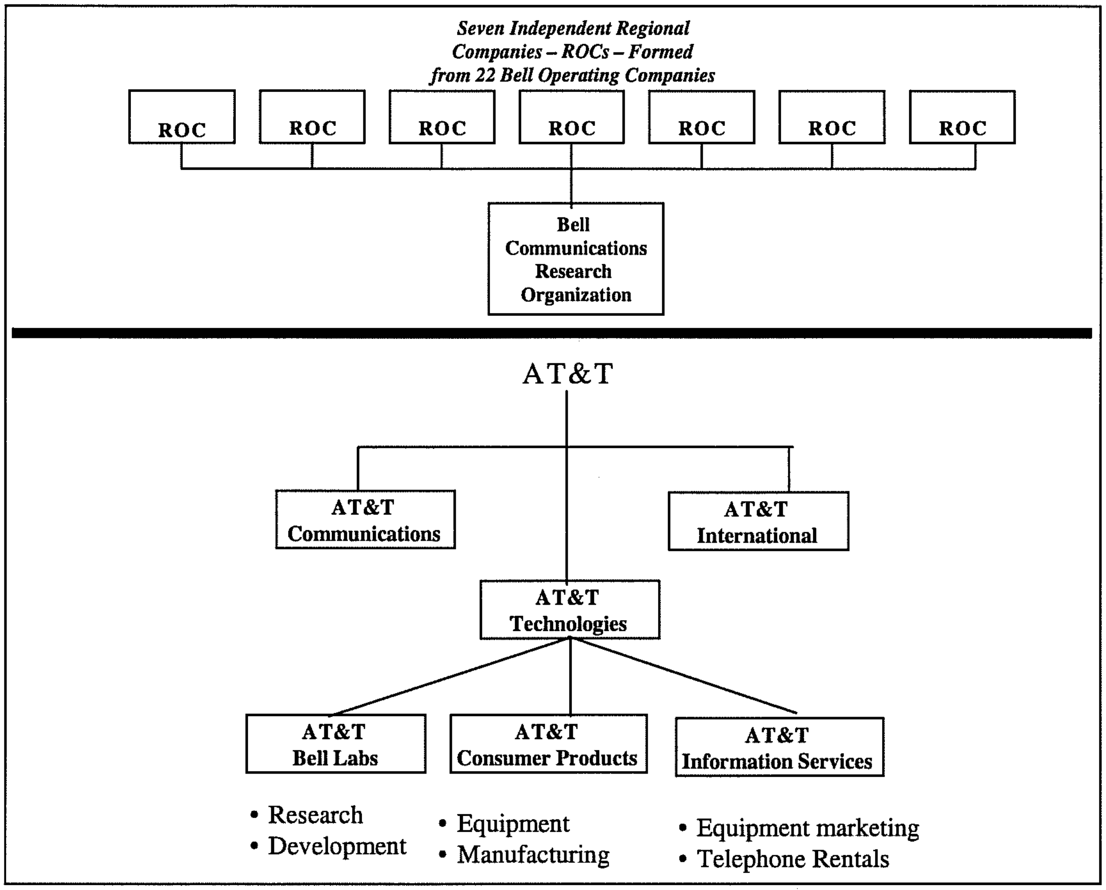


Figure 2. AT&T after devistiture

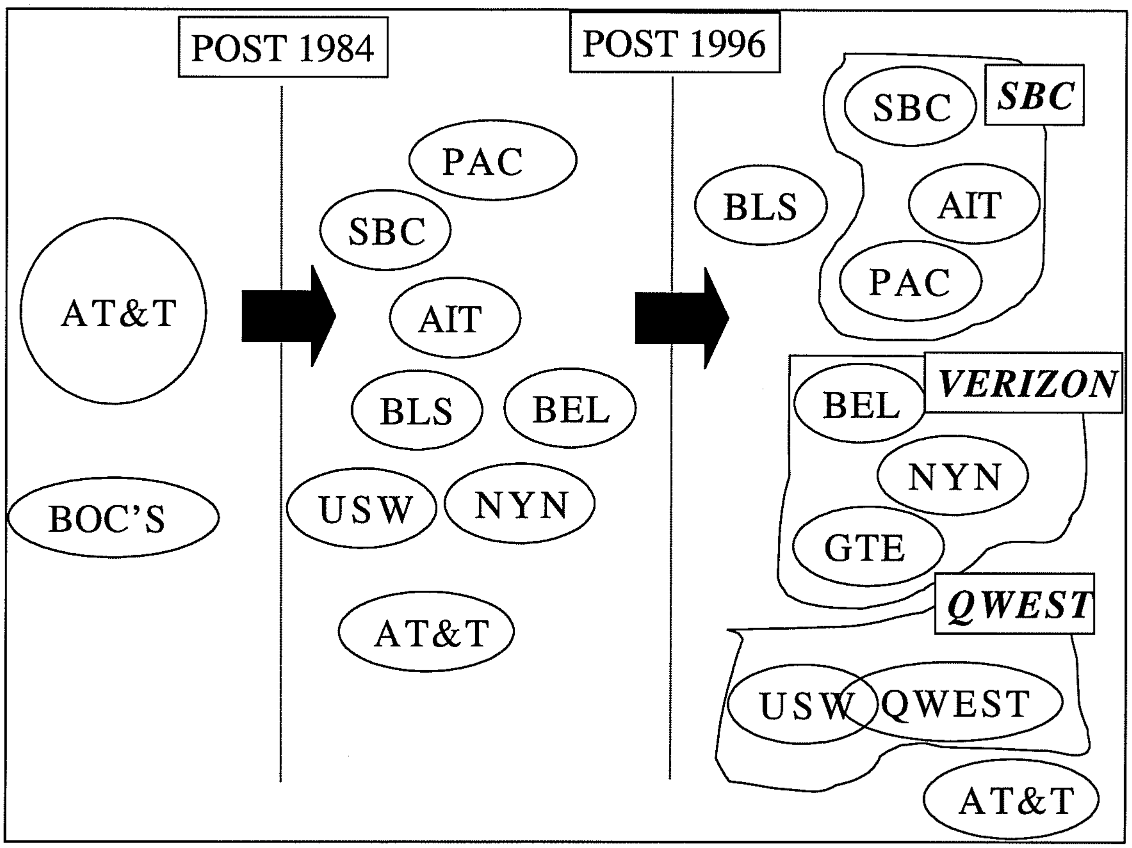


Figure 3. Telecommunications Dynamics

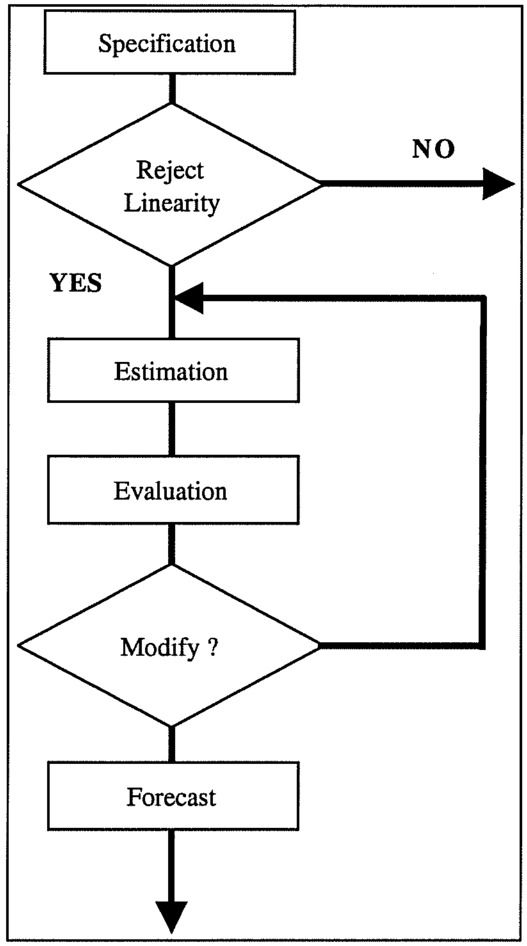


Figure 4. STAR Modeling Cycle

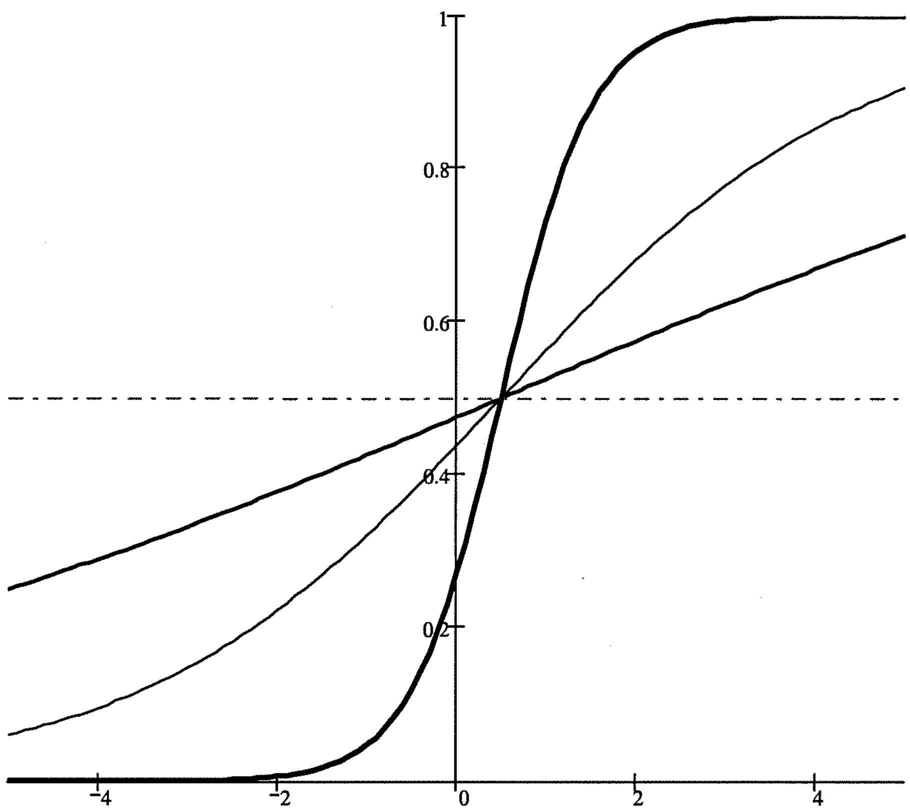


Figure 5. First-Order Logistic Functions

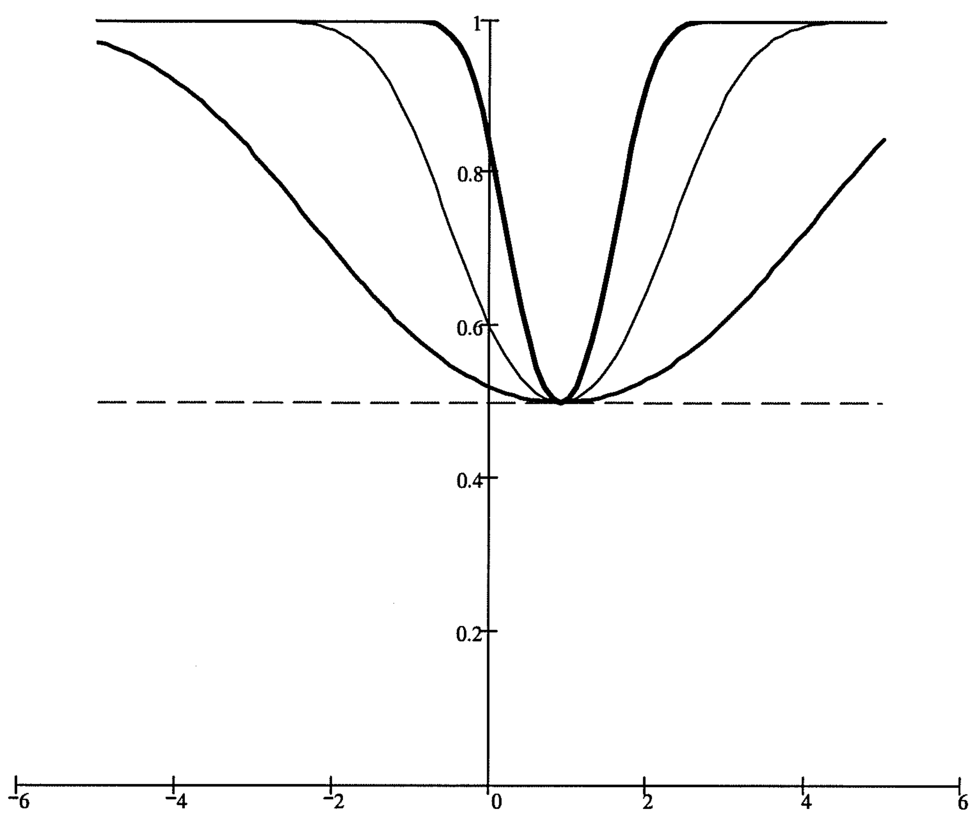


Figure 6. Exponential Functions

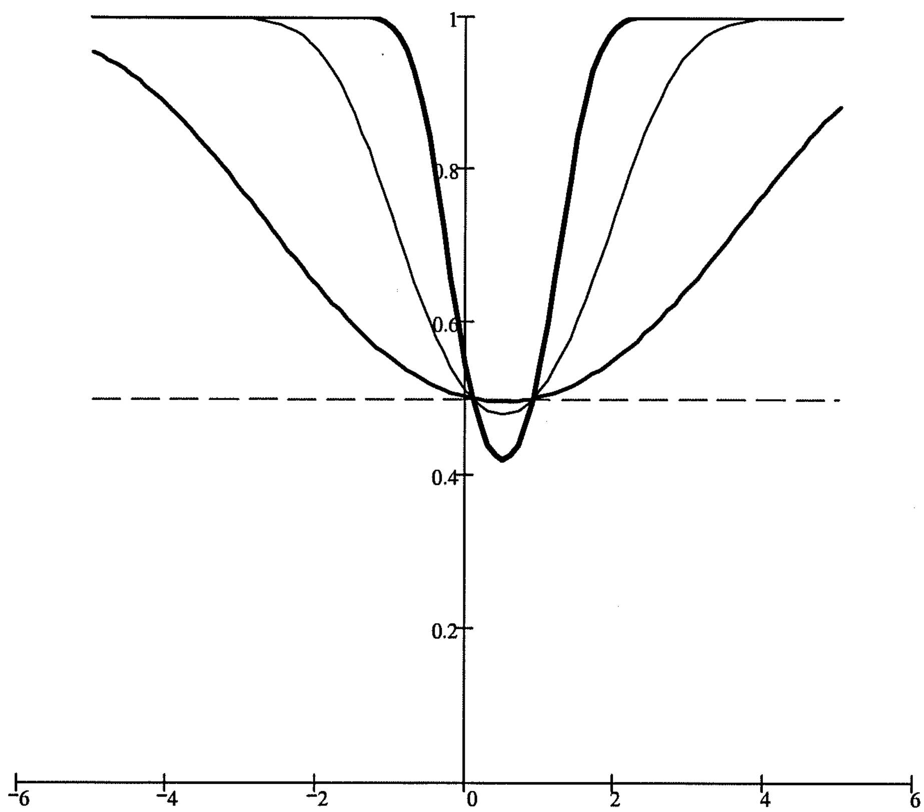


Figure 7. Second Order Logistic Functions

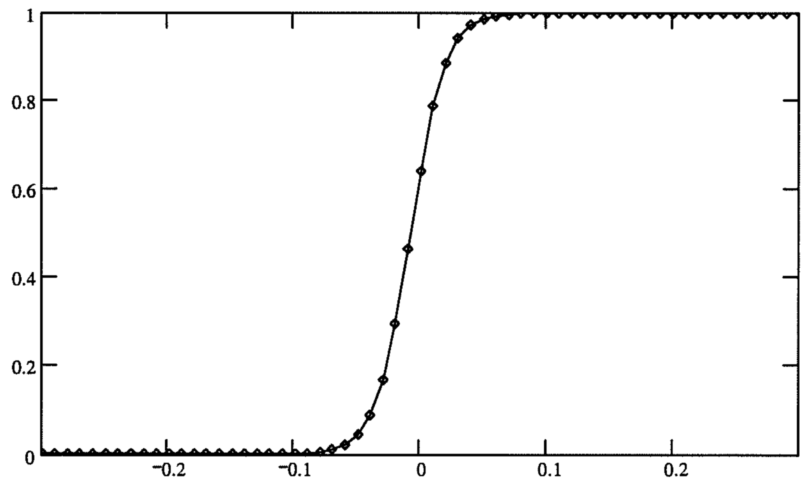


Figure 8a. AIT Transition Function

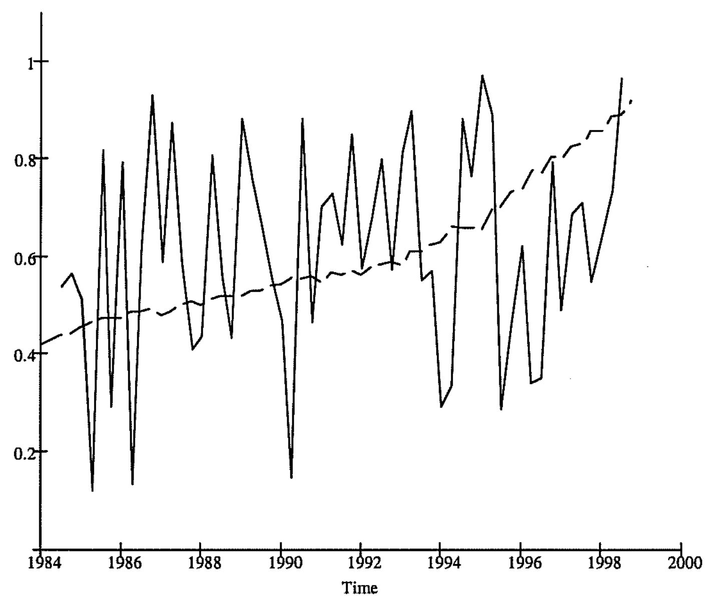
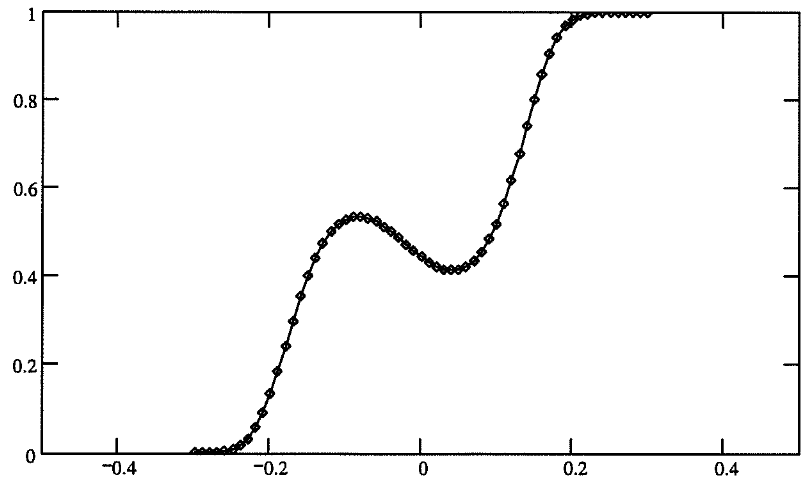


Figure 8b. AIT Transition Function vs Time



9a. USW Transition Function

Figure 9b. USW Transition Function vs Time

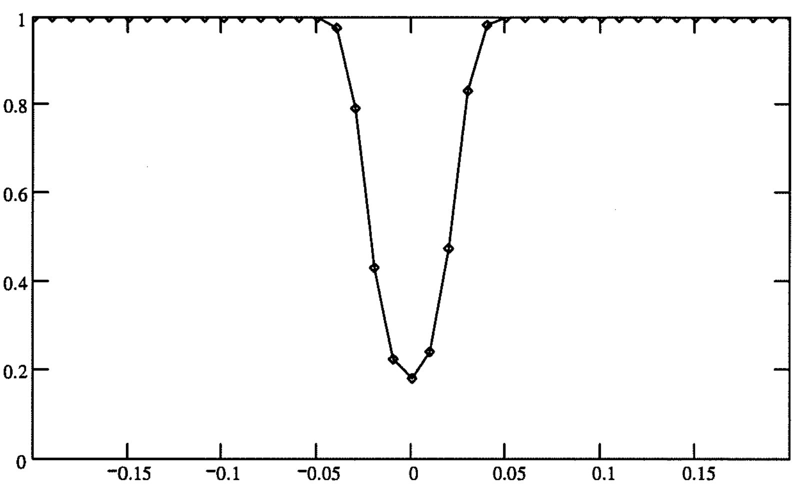


Figure 10a. SNG Transition Function

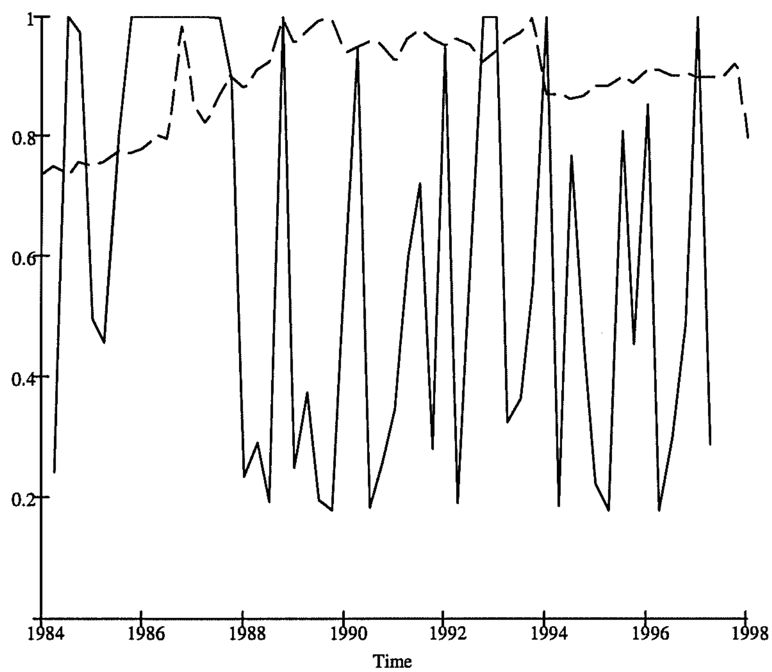


Figure 10b. SNG Transition Function vs Time

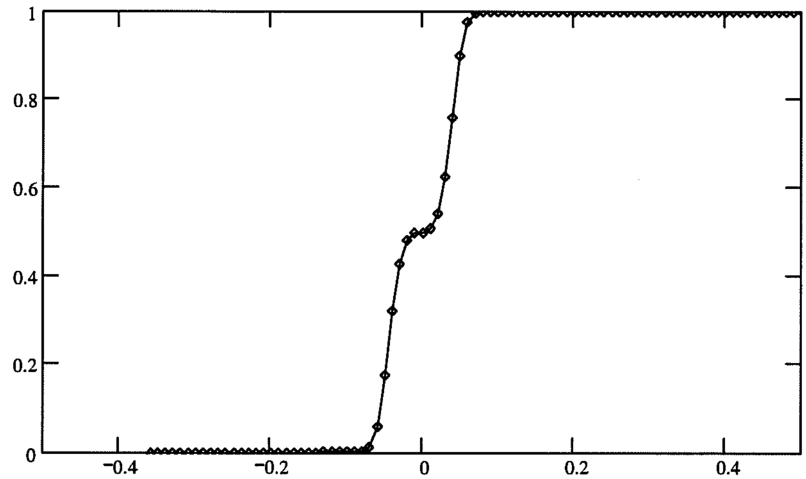


Figure 11a. NYN Transition Function

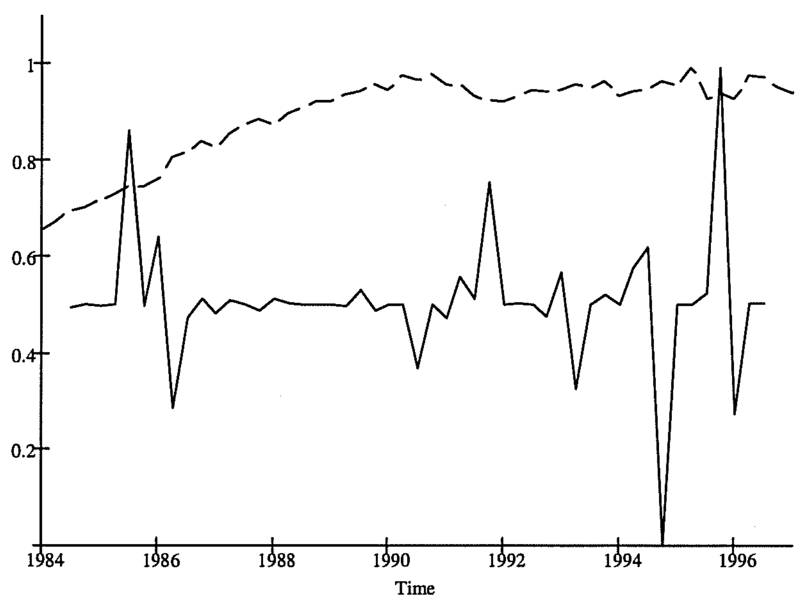


Figure 11b. NYN Transition Function vs Time

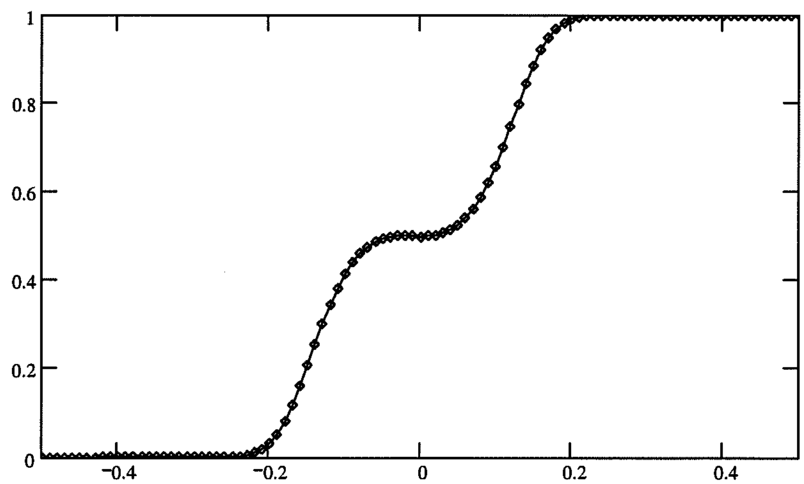


Figure 12a. BLS Transition Function

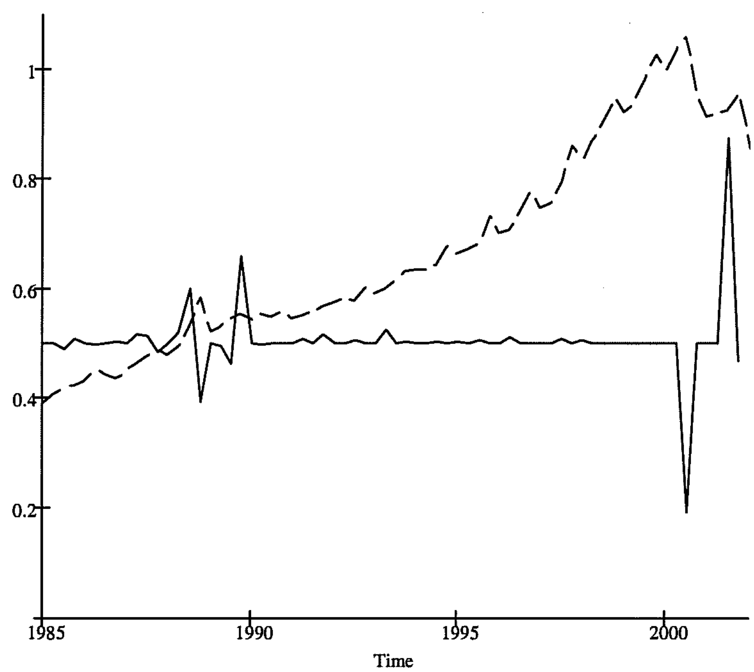


Figure 12b. BLS Transition Function vs Time

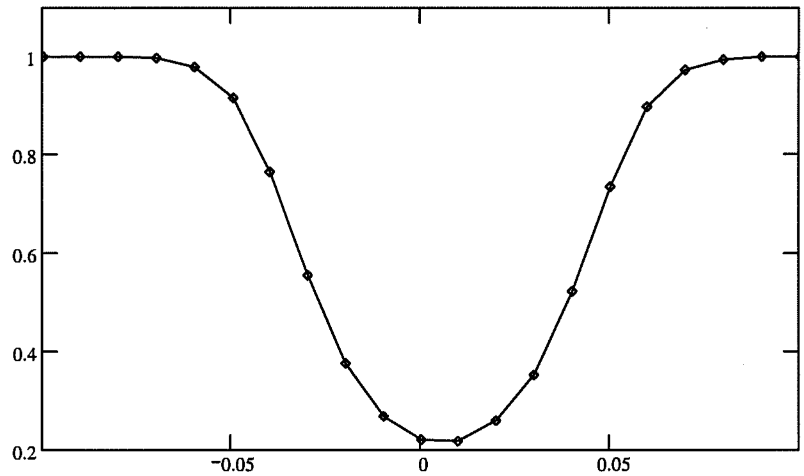


Figure 13a. GTE Transition Function

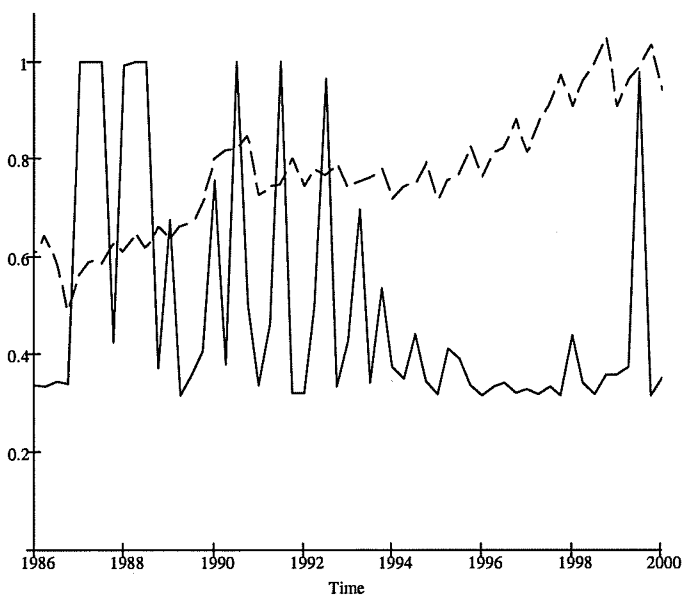


Figure 13b. GTE Transition Function vs Time

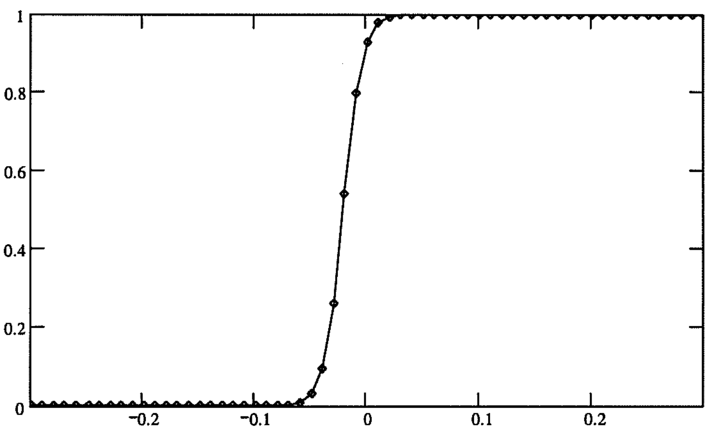


Figure 14a.BEL Transition Function

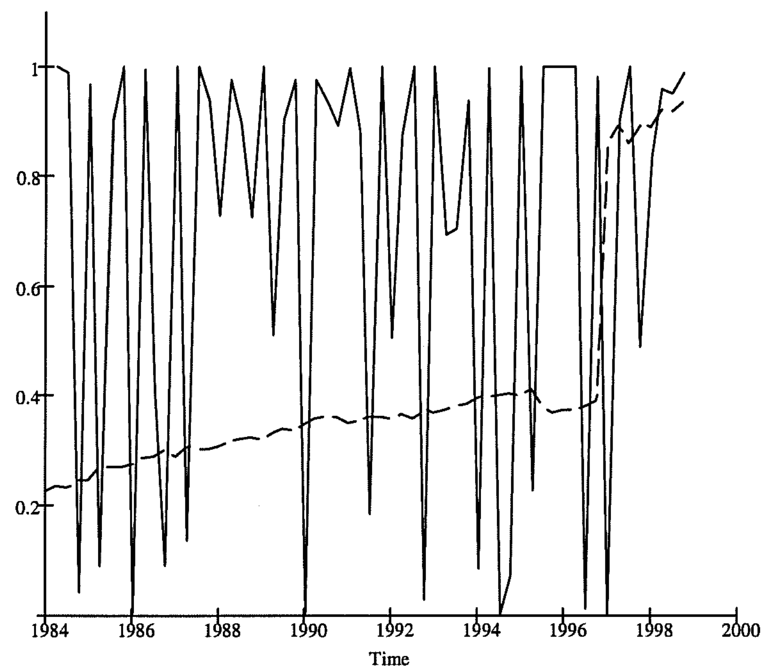


Figure 14b. BEL Transition Function

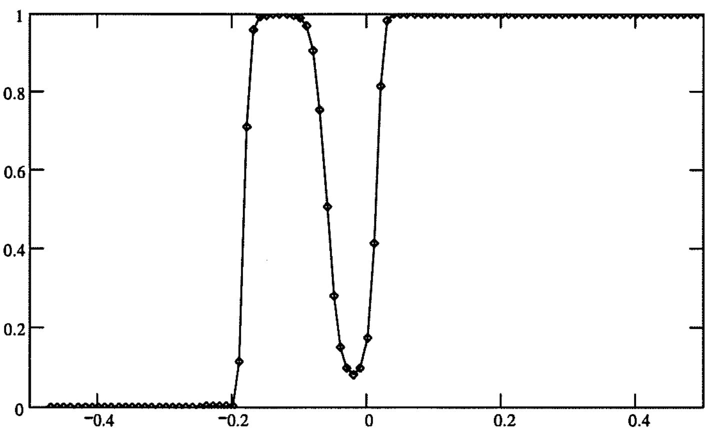


Figure 15a. NT Transition Function

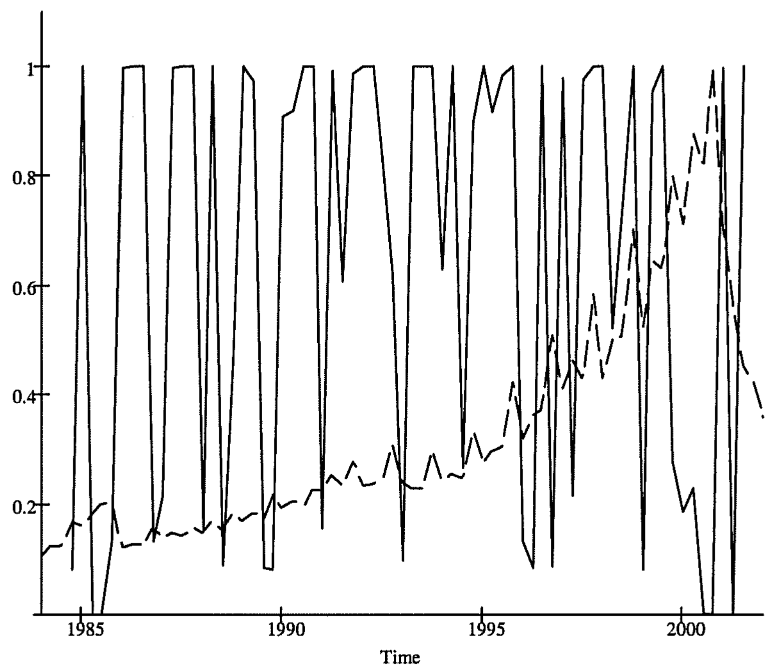


Figure 15b. NT Transition Function vs Time

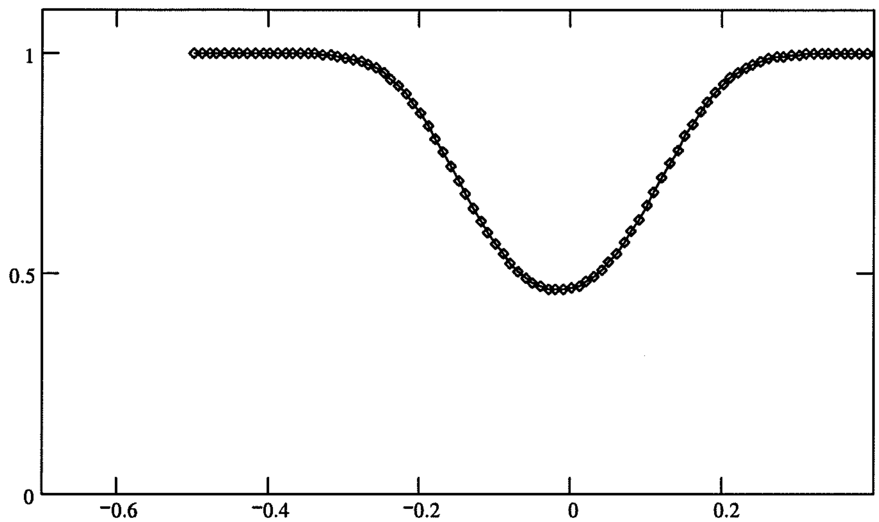


Figure 16a. SBC Transition Function

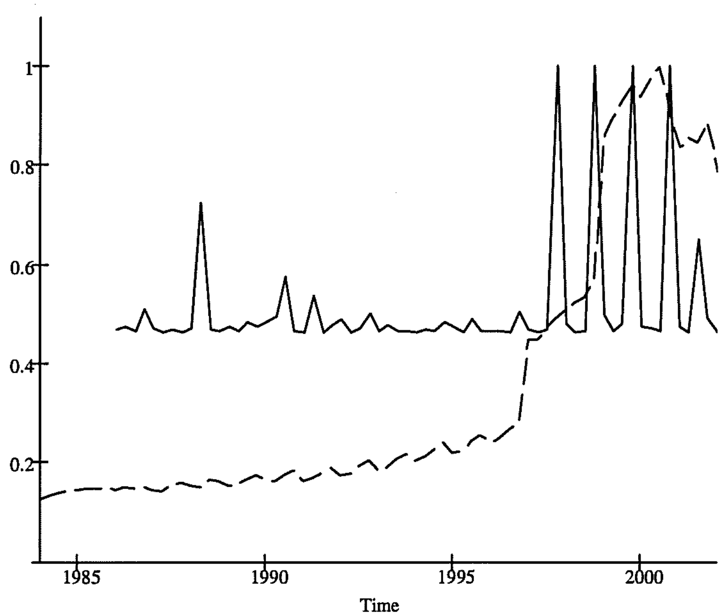


Figure 16b. SBC Transition Function vs Time

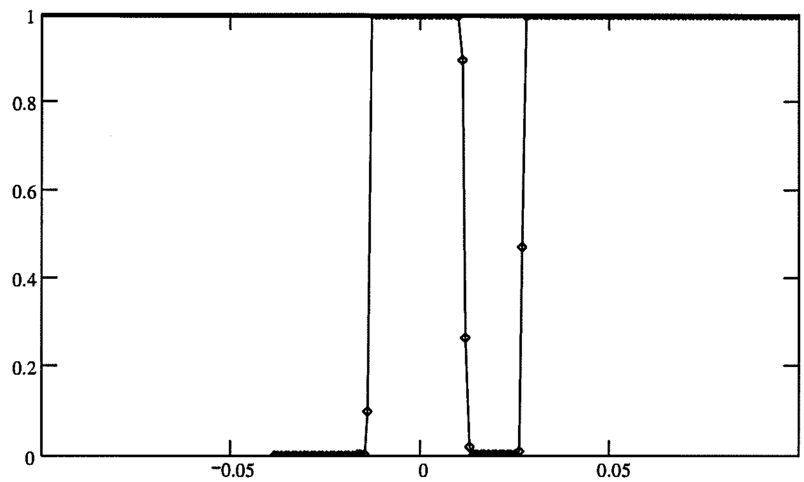


Figure 17a. AT&T Transition Function

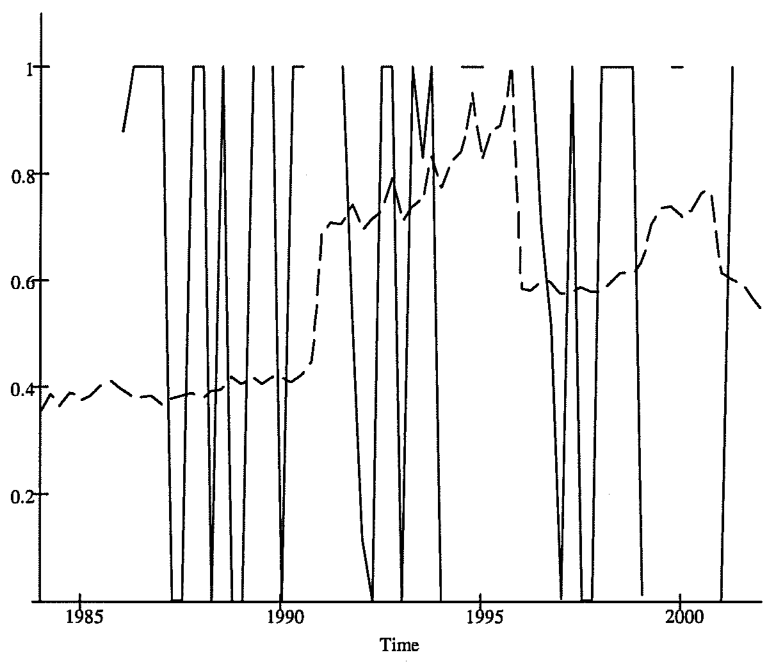


Figure 17b. AT&T Transition Function vs Time