

Multilingual Children's Mathematical Reasoning

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## **Abstract**

This research adopts a sociocultural framework (Vygotsky, 1978) to investigate how multilingual children express their mathematical reasoning during collaborative problem solving. The topic is important because North America is becoming increasingly multicultural, and according to mathematics teachers this has complicated the challenges of teaching and learning mathematics. Many educators assume that children should be competent in the language of instruction before they engage with mathematical content (Civil, 2008; Gorgorió & Planas, 2001). A review of recent research in this area challenges the idea that multilingual students need to have mastered the official language of instruction prior to learning mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). These researchers demonstrate that the knowledge of the language of instruction is only one aspect of becoming competent in mathematics. My research was designed to build on the findings of the current research on multilingual children's reasoning in order to more fully understand how multilingual children express their mathematical understanding and reasoning. For this study, two multilingual families, each with 3 children between the ages of 8 and 12, participated in a mathematical problem-solving activity. Findings show the children's mathematical reasoning was evidence-based drawing on mathematical knowledge and world knowledge.

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## Introduction

tulipes three عمبول انوا في  
 roses five كمان في  
 tournesols deux et  
 flowers كمان الوا اديش في  
 (J'afar, aged 7)

The above quote is from my 7-year-old son as we were working together on his mathematics homework. The problem that we were working on is the following:

Dans le jardin, il ya a 3 tulipes, 5 roses et 2 tournesols.  
 Combien de fleurs y a-t-il dans le jardin?<sup>1</sup>

The mathematic problem was presented in French because he is in a grade one French immersion program, in Ontario. The children were instructed by the teacher to solve the problem with three representations. 1) Une phrase mathématique (a numeric mathematical sentence). 2) Un dessin (a drawing). 3) Une phrase avec des mots (a sentence with words).

When we began to work on the problem, the conversation started in Arabic since that is what we speak at home. We also speak English at home. I started by asking my son in Arabic to read the question. He read it to me in French. Still speaking Arabic, I asked him to tell me what the question says. As is evident in the quote above in his reply to me, he expressed himself using Arabic, French, and English. It was clear from his multilingual expression that he understood what the problem was saying. While he often speaks using both Arabic and English in the same sentence, the fact that he expanded this to include French in response to this mathematics problem surprised me. This suggests that there are terms he knows in French but not in English. I was left wondering why he responded using three languages. My best guess is that it was because he knows that I do not speak French. As I indicated, he and I use Arabic and English

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<sup>1</sup> In the garden, there are 3 tulips, 5 roses and 2 sunflowers. How many flowers are in the garden?

when we talk. While I said this was my best guess as to what he was doing, I have some additional support for this strategy from his mathematics tutor. She works with him in French and English, and she tells me he often does this kind of multilingual response, but with her he only uses French and English, as she does not speak Arabic. In fact, such multilingual expressions are very common in our family. As a parent, I am quite amazed by my son's decisions to use three languages to communicate mathematically. I did not ask him to use Arabic, French, or English. He had the freedom to use whichever language(s) he desired. However, according to Planas and Setati (2009), the context of the mathematics lesson and how the teacher structures the lesson influence the choice of language used. This suggests that the context that my son and I were in had influenced the use of Arabic in conjunction with English and French. His multilingual expression was not a simple translation from one language to another, but I think reflects the influence of the richness of multilingualism and language switching in order to communicate effectively.

In this research study, I am interested in how multilingual children express their mathematical reasoning. My interest is both personal, as a multilingual parent, and professional, as a graduate student in education. The topic itself is important because Canada is a multilingual country with two official languages- English and French- and over 100 other languages in use within the population (Burnaby, 2002). This has resulted in a proliferation of multicultural and multilingual classrooms. However, classroom teachers and education officials often assume that the children must become competent in the language of instruction before they can engage with mathematical content (Civil, 2008; Gorgorió & Planas, 2001). In many cases, this leads to a situation where children are discouraged from using a language other than the language of instruction.

ESL or bilingual teachers may fail to recognize, appreciate, or accept differences between the pupils' home language and the classroom register. Pupils, in turn, may not respond to the sociolinguistic demands of the classroom, which can be interpreted by the teacher as inadequate understanding of the social and/or referential meanings of language. This may lead to lower teacher expectations, which, in turn, may result in poor pupil achievement and/or learning of language (National Institute of Education, 1977; as cited in Ramirez & Milk, 1986, p. 508).

This strikes me as an unfortunate because

When language learners arrive at a new school, they are not blank slates, wiped clean, for the inscription of new linguistic and cultural messages. They bring with them their own physical, emotional, linguistic, academic, cultural, and personal traits, which may either ease or inhabit their transition to the new culture. (Abrams & Ferguson, 2004/2005, p.3)

Therefore, I think it is important to understand more fully how multilingual children reason mathematically.

### **Literature Review**

In this review of the literature, I focused on three areas to inform the study. I begin with research on mathematical reasoning that looks at several studies that focus on 1) children's mathematical reasoning—in general, in order to understand how children's informal reasoning developed as they worked and reasoned together on mathematics; 2) Then, I will report on multilingual children's mathematical reasoning, in order to understand how multilingual children construct word meanings, how they use multiple expressions when working together on mathematics, and how they relate word problems to their own experiences. Next, I will review research on the ways in which multilingual learners, second language learners, and teachers use and understand language/ code switching in mathematics classrooms, in order to understand why the switching between languages occurs and the factors that might infuse it. Finally, I will explore research on teachers and second language learners in the mathematics classrooms, in order to understand how teachers view students who are in the process of learning the language

of instruction and how they see the relation between learning mathematics and language learning. The above studies report on research undertaken in communities around the globe, including Australia, South Africa, United Kingdom, Europe, and North America. Their focus is on children who are multilingual and/or are learning an additional language.

### *Mathematical Reasoning*

The review of the research literature in the area of mathematical reasoning will explore research on children's mathematical reasoning—in general, and then, look at multilingual children's mathematical reasoning in order to understand the importance of collaborative problem-solving.

#### *Children's mathematical reasoning—In General*

There are several researchers who are interested in understanding how children's informal reasoning develops as they work and reason together on mathematics (Graves & Zack, 1997; Moschkovich, 2008; Weber, Radu, Mueller, Powell, & Maher, 2010). In order to understand children's informal reasoning, Graves and Zack (1997) reported on thirteen fifth-grade children who were asked to work together in groups of 2/3, and then in groups of 4/5 to find the procedure for the number of squares in a 10 x 10 and in a 60 x 60 sided figure in an inquiry mathematics classroom. This research explored the large group discussion by describing how the children “apply strategies, see patterns, identify mathematical structures and connect this information to support their formulations” (Graves & Zack, 1997, p.18). In addition, the researchers were interested in the collective voice as it was put forth by the students as they worked and reasoned together. The findings suggested that in various cases children “complete each other's ideas, paraphrase each other's expression, repeat each other's language and articulate claims based on what another child says”, which resulted in a collaborative voice

(Graves & Zack, 1997, p. 24). The researchers concluded that

reasoning evolves in the context of the activity. While claims, justification, and counter-argument are viewed as rhetorical activities (Billig, 1996), these interpretive and argumentative strategies are applied in conjunction with the domain-specific knowledge of mathematics. The strategies both emerge from and depend on the mathematical activity in which they occur. (Graves & Zack, 1997, p. 24)

With the intention of understanding how students' informal reasoning developed in a mathematical problem-solving context, and the relationship between mathematical autonomy and mathematical reasoning, Weber, Radu, Mueller, Powell, and Maher (2010) reported on sixth-grade students' consistency in developing challenging problems for each other, the quality of their justifications, and the role of listening in these verbal interactions (Weber et al., 2010). The researchers also examined the factors that encouraged participation.

In this study, the children were encouraged to work together on "open-ended, well-defined mathematical problems" that were related to fractions using Cuisenaire rods (Weber et al., 2010, p. 96). In one example, the students were asked to explore the Cuisenaire rods and provide their observations to the class. In another example, they were asked,

If I gave the light green rod the number name one, what number name would I give to the yellow rod? (Weber et al., 2010, p. 97)

In addition, the students were welcomed to contribute by posing challenging problems for their classmates. This invited Jerel (a student) to present his problem to the class, he asks, "Alright. If orange is one, what is the yellow rod?" (Weber et al., 2010, p. 104). According to the researchers, the initial task itself may have contributed to Jerel's problem posing (p. 104).

The researchers concluded that the students were not familiar with these types of tasks; they were open-ended which allowed for multiple solutions, or multiple ways to arrive at a solution, and allowed the students to generalize (Weber et al., 2010, p. 107). In this paper, Weber, et al., (2010) draw on the work of Rasmussen and Marongelle (2006), who state that by providing the

students with tasks that are generative, this encourages the students to discuss and justify their answers in order to determine its reliability.

Weber et al., (2010) further concluded that the classroom norms encouraged students' participation. For example, the students' answers were never evaluated by authority figures, which allowed them to participate without fearing that they will be judged. In addition, the students were encouraged to justify their answers, convince their group members of the reliability of their answers, and present their work to the class, which in many cases set the stage for further investigations (Weber et al., 2010). This suggests, "students' increased participation in problem solving provided them with opportunities to advance their mathematical understanding" (Weber et al., 2010, p. 108).

This conclusion regarding the classroom norms is supported by Graves's (2011) research in Canada that investigated elementary school children's mathematical reasoning "in an inquiry-oriented multi-age classroom, and the teacher's instructional decision-making that enabled the creation of spaces for mathematical conversations" (Graves, 2011, p. 2). In this study, Graves (2011) explored the discussion held between the teacher (Judith) and the children in grades two and three as they shared their answers to a geometry problem. The problem that they were working on was the following:

A family decides to start a garden on their square lot. The parents take  $\frac{1}{4}$  of the lot. Their  $\frac{1}{4}$  is a perfect square in the north-east corner. The other  $\frac{3}{4}$  was divided equally in size and shape among the 4 children.

Draw a picture showing how the division was done. Remember that each of the four sections are congruent.

From the analysis of this study, Graves (2011) concluded that the teacher during this mathematical discussion does not tell the student that s/he is wrong, but she asks them to reflect on their answers and connect it to the criteria of the task. According to Graves (2011),

This is a pedagogical strategy that focuses on the match between the constraints of the problem and the proposed solution. It is a pedagogical choice that focuses on the mathematical reasoning, and not on the learner's ability. This leaves room for reflection, reconsideration, and more mathematical conversation. (p. 8)

The researcher also concludes,

It is through their mathematical conversation that the children and Judith experience a collective understanding that goes beyond the individual perspectives. (Graves, 2011, p. 12)

This illustrates that when the children and their teachers engage in collaborative problem-solving and discussions, this can influence the children's mathematical reasoning, and also, teachers' pedagogical practices (Graves & Zach, 2001; Graves, 2011).

In another study, Moschkovich (2008) examined a class discussion to understand how students and teachers use and manage discussions of multiple interpretations. Two students in eighth-grade and the teacher used multiple and shifting meanings of the scales on two distance versus time graphs. Moschkovich was interested in finding out how the teacher responded to students' multiple interpretations of the scales and how she connected student interpretations to established mathematical practices. The findings suggest that the students and the teacher had "...different views of the scale, grid segments, tick marks, and number labels on the scale" (Moschkovich, 2008, p. 553). For example, the phrase "I went by twos" was used to describe number of segments, and was also used to describe number of units (Moschkovich, 2008).

Moschkovich (2008) was able to conclude that the

ambiguous and shifting meanings did not prove to be obstacles to this mathematical discussion. Instead, the teacher used students' multiple interpretations as resources, building on and connecting them to important mathematical concepts (p. 552).

She also demonstrates that,

the deceptively simple actions of labeling, describing, and comparing labels on axes are not so simple for students and, in fact, involve important conceptual understanding. (Moschkovich, 2008, p. 554)

These findings draw our attention to the fact that when the students and their teacher look at a graph together, we should expect that they are not looking at, talking about, referring to, or imagining the same things.

According to McGraw and Rubinstein (2008) when students are provided with the opportunity to discuss, analyze, and explore their own and the reasoning of others—“they can become powerful mathematical thinkers” (p. 164). In addition, inviting students to take responsibility for various aspects of mathematical problem solving, which in many cases are reserved for teachers (e.g., judging the authenticity and reliability of a solution, and posing directions) expands and encourages students’ mathematical reasoning (Weber et al., 2010).

#### *Multilingual children’s mathematical reasoning*

According to Moschkovich (2007), “...although an emphasis on vocabulary and reading comprehension may have been sufficient in the past, this emphasis does not match current views of mathematical proficiency or the activities in contemporary classrooms” (p. 91). Today, students are involved in a variety of group discussions that allow them to bring in resources such as other cultures and background knowledge (Barwell, 2005; Moschkovich, 2007).

There is a lot of research currently interested in the classroom engagement of multilingual learners during mathematical problem-solving (Barwell, 2003; 2005; Elbers & Haan, 2005; Gorgorió & Planas, 2001; McGraw & Rubinstien, 2008). In one study, Barwell (2005) was interested in understanding how learners of English as an additional language “...relate the world of the word problem with their own experience of the world, and in how they use their experience of the world in solving word problems” (Barwell, 2005, p. 332). He reported on two grades five classes with ten English as additional language (EAL) students. Drawing on discursive psychology and conversation analysis, Barwell developed an approach based on the notion of attention.

I treat attention discursively, drawing particularly on what conversation analysts call ‘participants’ attention’ (Sacks et al., 1974). This view of attention arises from the social organization of talk, which includes features such as taking *turns* to speak, so that successive turns form an unfolding sequence of interaction in which each turn builds on what has gone before. (Barwell, 2005, p. 333)

This approach allowed Barwell to examine how competing areas of attention were dealt with and used during collaboration by the students. His analysis further explored, how EAL and native speaking students thought together. Barwell (2005) treats thinking as “ ‘social reasoning’: the publicly available reasoning students make available in interaction, including accounts, justifications and reasons they use to support their actions” (p. 333). From the analysis of this study, Barwell (2005) concluded that the students used attention to narrative experience to discuss what they have understood from the word problem, to relate the word problem to their own experiences, and to discuss their relationships with each other (p. 345). The connection that students made to their own experiences contributed

to a supportive linguistic context within which they are able to work on their mathematics task. They also highlight the social nature of mathematics classroom interaction and show how this is a significant part of students’ work together. (Barwell, 2005, p. 346)

In order to understand how bilingual students, in the United States, developed mathematical reasoning when using two languages, McGraw and Rubinstien (2008) conducted a study with twenty-six students in seventh-and-eighth-grade, born in Mexico. This particular group of students were taught using both Spanish and English: “Biliteracy development was explicitly valued, and encouraged, by the teacher and the students” (McGraw & Rubinstien, 2008, p. 153). The researchers were interested in understanding the ways in which the students used Spanish and English to communicate and reason mathematically (McGraw & Rubinstien, 2008).

In this study, the mathematics educator, using English with some Spanish for clarifications, presented the students with a nonroutine problem. The students were given a text version of the

problem, and they were also provided with an accompanying diagram.

How high above the ground would you be if you were sitting in the 5th row? The 10th row? The 50<sup>th</sup> row?

Find a method that will allow you to determine how high you are above the ground for any row.

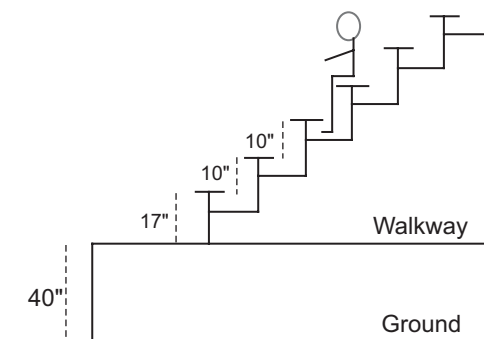


Figure 1. The stadium-seating problem.

(McGraw & Rubinstien, 2008, p. 156)

From the analysis of this study, the researcher's concluded that when student's are provided with the opportunity to engage in nonroutine mathematical problems, this allows the students to elicit discussions, while also encouraging mathematical reasoning (McGraw & Rubinstien, 2008). Moreover, despite the fact that the mathematics educator presented the task in English and used some Spanish for clarifications, the students communicated using Spanish in their designated groups, but switched to English during whole class discussions (McGraw & Rubinstien, 2008). McGraw and Rubinstien's (2008) findings suggested, "when given the opportunity, ELLs are likely to draw productively upon their linguistic resources in L1 and L2 to support high-level mathematical reasoning" (p. 147).

In another study, Elbers and Haan (2005) were interested to understand how students construct word meanings in a multicultural classroom. They conducted a study with 22 students in a grade seven Dutch primary school, in Utrecht, the Netherlands, talking in small groups of four or five during a mathematics lesson. The researchers were interested in understanding the

discussions held between the students while doing mathematics, and also, how the students dealt with language issues experienced from their limited knowledge of Dutch words and expressions. Elbers and Haan (2005) approached the teaching and learning of mathematics in a multicultural classroom from a socio-historical approach with its emphasis on mediation. Elbers and Haan (2005) identify three levels: the first was tools and symbols used to clarify understanding and meaning; the second was norms for social interaction in the classroom; and the third was mathematical discourse. The findings suggested that the students did raise problems in regards to the meaning of words that they encountered during their collaboration in solving the mathematics problems. According to Elbers and Haan (2005), “the students used a variety of symbolic and material tools to clarify the meaning of unfamiliar or difficult words: the Dutch language, gestures, the assignments, and drawings in the textbook” (p. 54). They concluded that the students were acting with mediational means in order to construct a common understanding among themselves. They also concluded that collaboration between students promoted the learning of mathematics and also helped the migrant students improve their knowledge of Dutch. In addition, the students did not treat the difficulties they encountered with the words “... as mere problems of vocabulary, but made them part of the process of mathematical exploration” (p. 57).

A finding, which pertains to the multiple ways in which expressions can be understood or misunderstood, was presented in a study by Gorgorió and Planas (2001) with students aged 15-19 who came to Barcelona from 9 different countries. They were working on the following problem:

A farmer has 3 sons. In his will he gives his sons 17 cows. The oldest one must receive  $\frac{1}{2}$  of the cows, the second  $\frac{1}{3}$  and the third  $\frac{1}{9}$ . How many cows will each of them receive? (p. 23; originally in Catalan)

Aftab, a Pakistani student was unsure of the meaning of the word “will”. The teacher told him, “...Aftab it is a present the father gives to his children” (p.23). What follows is the discussion generated between Aftab and his classmates:

Jossua: Is the father there at the moment?

Teacher: Why do you need to know that?

Jossua: (working on the idea of a ‘present’) If the father is still alive, then he will need some cows or maybe he can buy more cows...

Ramia: (working on the idea of a ‘will’) The father is dead!

Aftab: (Shouting angrily) Why do you want to kill him? What has he done to you? (p. 23)

While Gorgorió and Planas (2001) state that, “the search for transparency turned into a misunderstanding, and a single word was enough to obstruct his process of thinking” (p.23), this also points to the complexity and dynamic aspects of collaborative problem solving.

These findings on children’s mathematical reasoning are consistent with the research on multilingual children’s mathematical reasoning. Taken together these studies shift our view from the individual and individual abilities, to the social and mediated nature of the person involved in an activity. This suggests that knowledge of the language of instruction is not the only solution to becoming competent in mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). These studies inform us about the complexity of learning mathematics, its relationship with language and culture, and the value of working together. They also suggest new directions that teachers might take in their classrooms.

### *Language/ Code switching*

Language switching, by which I mean switching from one language to another in the course of a discussion or conversation<sup>2</sup>, has been another topic of increasing interest in research on

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<sup>2</sup> While Moschkovich (2005) defines language switching as the “use of two languages during solitary” activity, I have chosen to use it to encompass both individual and social interaction.

mathematics learning with multilingual learners. For many researchers switching from one language to another, for example, switching from the language of instruction to one's first (or even second) language has been seen as a strategy to compensate for language difficulty in the language of instruction (Clarkson, 2006; Grosjean, 1999; Ramirez & Milk, 1986). Other researchers, however, adopt a different, more positive stance with respect to language switching and consider the way in which language switching is influenced by the students' experiences and the physical environments in which they live (Morgan, 2006; Moschkovich, 2005; Parvanehnezhad & Clarkson, 2008; Planas & Setati, 2009; Planas, 2011).

That is, when we refer to multilingual students' use of language in a mathematical classroom, it is important that we consider the place, the purpose, the topic, the participants, and the social relations among them (Moschkovich, 2005). According to Moschkovich (2005), "the type of mathematics problem and the students' experience with mathematics instruction can influence which language a student uses" (p. 132). Therefore, she argues that it is not appropriate to say that language switching represents only a deficiency in language or deficiency in mathematical knowledge.

In order to understand how bilingual students use two languages when learning mathematics, Moschkovich (2005) specifically examined a mathematical conversation between two ninth-grade students using English and Spanish to explain their mathematical understanding. Moschkovich (2005) was interested in illustrating how a sociolinguistic perspective allows us to consider code switching as a valuable resource when communicating mathematically (p. 133). In the study, one of the interviewers used Spanish and English to describe the lines that s/he had drawn on the blackboard. The students were then instructed to identify which line was steep and which line was less steep (p. 134).

Moschkovich's (2005) findings suggested that some students may sometimes use their first language to compensate for missing English vocabulary terms, while "...other students will use their first language to explain a concept, justify an answer, describe mathematical situations or elaborate, expand and provide additional information" (p. 138). These findings caution us against assuming that switching between languages is influenced solely by the learners' limited understanding.

In one study which explored the amount of and reasons for code switching among Iranian bilingual students when working on mathematical problems in an Australian context, Parvanehnezhad and Clarkson (2008) studied students who were regarded as being bilingual with Farsi as their first language (L1), and English as their second language (L2) (p. 58). There were sixteen students who were in grades four and five, and who attended a Persian (Farsi) language school on Saturdays. On average the students attempted five symbolic items, four word problems, and one open-ended question based on their grade levels (p. 58). The researchers had attached a language switching checklist to the students answer sheet, which was also used for the one-on-one interview. The students were instructed to use the checklist to identify the language(s) they used when solving the mathematical items (p. 60).

From the analysis of these data the researchers concluded that many of the students did in fact switch languages when doing mathematics. The reasons given by the researchers were as follows: the students' difficulty with the comprehension and interpretation of the questions led some students to switch to Persian; the students' familiarity with some numbers and / or words that were habitually used in Persian; and the context of where the research was held. Despite the fact that we should be aware that some students switch languages due to the complexity of the mathematical problem, this by no means should be considered the only possibility since other

reasons why students switch languages may exist (Parvanehnezhad & Clarkson, 2008). That is to say, students who received aid from their parents, siblings, relatives or even prior schooling during which language switching was present may acquire a language switching behaviour (Parvanehnezhad & Clarkson, 2008). In other words, language switching is more likely to seem natural.

The researchers were also trying to test Cummins's threshold hypothesis (2006, 2008) that states, "the level of linguistic competence attained by bilingual children may act as an intervening variable in mediating the effects of bilingualism on their cognitive and academic development" (Cummins, 2001, p. 74). What follows are the notions that Cummins (2001) addresses in relation to the learners' language proficiency and cognitive effects:

- high levels of proficiency in both languages leads to positive cognitive effects;
- native-like proficiency in one of the languages offers neither negative nor positive cognitive effects;
- low levels of proficiency in both their languages lead to negative cognitive effects (Cummins, 2001).

Their findings with respect to Cummins's threshold hypothesis suggested that there was a connection between the students' language competency and their mathematical competency. The results illustrated that "most students with high/high language competency also had high mathematics competency, and most students with low/ low language competencies had low mathematics competency" (Parvanehnezhad & Clarkson, 2008, p. 66). However, Barwell (2005) believes that "such a conclusion should be treated with caution, [because] linguistic proficiency is unlikely to be the only factor in such students' attainment" (p. 331). According to Setati and Barwell (2008),

poor performance by multilingual learners thus cannot be solely attributed to the learners' limited proficiency in English (suggesting that fluency in the language of learning and teaching will solve all problems) in isolation from the pedagogic

issues specific to mathematics as well as wider social, cultural and political factors that infuse schooling. (p. 2).

The language of instruction should not be the only attribution of students' difficulties. We need to consider other factors that might have an effect on students' achievements.

In order to understand how bilingual students in Catalonia, Spain used their languages to learn mathematics, Planas and Setati (2009) conducted a study with 24 students about 12 years old and a bilingual Catalan native speaking teacher. The researchers were interested in understanding whether Spanish-dominant bilingual students in Catalan classrooms switch languages during mathematical activity, and if so, what were some of the factors that accounted for this (p. 41). Planas and Setati used a critical sociolinguistic approach, "which draws on social theory in the analysis of how language is involved in the construction of teaching and learning opportunities" (p. 36).

In that study, the students were grouped according to their dominant language, and they were encouraged to use their first language. Planas and Setati (2009) were able to conclude, "the students tend to use each of the two languages for different purposes, in different domains of the mathematical practices, and in relation to different settings within the classroom" (p. 54-55). First, the Spanish dominant bilinguals were prompted by their teacher to use Catalan with their small groups to become familiar with the task and the new vocabulary. Second, when these students were working on the solution, they worked in Spanish. However, when they needed certain clarifications, they went back to Catalan. Third, the students contributed to the class discussion only when the teacher had asked to do so, and they would use Catalan to share their solution. According to Planas and Setati (2009), "these findings raise the more general issue of how the context of mathematics lessons influences the choice of language, and how the way that the teacher structures the lesson influences the bilingual students' choice to use their second

language before returning to their first language” (p. 55).

This conclusion regarding the context of the mathematics lesson is supported by Setati’s (1998) research in South Africa that investigated “... the different ways in which a multilingual senior primary mathematics teacher uses code-switching when teaching mathematics to second language learners who share a first language with her” (p. 35). The data were collected from one grade five classroom in which the learners’ first language is Setswana. The students’ teacher (Thato) is multilingual and her first language is Setswana, and her teaching style involved code switching when she taught mathematics. The findings suggested that the teacher views code switching “as a resource for meeting classroom needs” (p. 35). For example, when the teacher is introducing a new lesson she uses the first language to ensure that the students understand. The teacher states,

If the lesson is new then we are new to everything, the lesson is new and the language becomes new. Then as time goes by it is then that they start off working on their own because if you start off you don't just tell them what we are going to do. So they are just wondering where we are really leading to. So that is why I use a lot of Tswana when introducing the lesson to try and drive them into the lesson. (Teacher, Setati, 1998, p. 36)

The teacher is “very conscious of when and how she switches during mathematics lesson” (p. 36). She states that when the students become too silent, “she uses code switching to get the pupil’s attention and to reformulate questions that have been asked in English” (p. 36). The students are given the freedom to use whichever language they desire because the teacher feels that code switching is a resource for everyone. Despite the fact that the teacher encourages students to use the language they want, “she is also concerned about pupils improving their English communication skills” (p. 37). According to Setati (1998), while the teacher views codes switching as a valuable resource, “...she is also faced with a challenge of making sure that her pupils understand English because it is the language of evaluation” (p. 37). In addition, the

teacher feels that code switching is "...a temporary measure which should be used with the aim of getting children to practice the language of learning" (p.37). Setati (1998) concludes that

Thato's class is used for three different reasons: to facilitate learners' understanding of concepts, to encourage participation and to familiarise learners with the language of evaluation (English). (p. 37)

The review of the literature in the area of language/ code switching underline the idea that acknowledging the students' first language is important, not only for their learning, but for teaching as well. Most of what the students bring with them to the classroom is most likely encoded in their L1 (Parvanehnezhad & Clarkson, 2008). This then suggests that students' home languages should be recognized as legitimate languages for learning mathematics (Setati, Molefe, & Langa, 2008). We are also cautioned from assuming that the use of the L1 is only done for the compensation of limited vocabulary or limited mathematical knowledge. In cases where children are multilingual, language switching can be a tool "that can provide access both to mathematical ideas and powerful ways of thinking and speaking" (Morgan, 2006, p. 241).

#### *Teacher's attitudes towards second language learners and mathematics*

Teachers who are living and working in a democratic and multicultural society are often teaching students from many different cultures who speak a language different from the language of instruction (Abreu, 2005). This presents a challenge for the teachers who are often trained to teach students who share the same language and a similar culture with the teacher. Traditionally, teachers when confronted with a more pluralistic classroom have tended to minimize the cultural differences among their students, a practice which they understand as a matter of equity that is treating everyone the same (Abreu, 2005). The idea of treating everybody as equals is further supported by teachers' conceptualization that mathematics knowledge is culture-free and universal (Abreu, 2005; Abreu & Gorgorió, 2007; César & Favilli, 2005).

The “universality of mathematics” can be seen as a dominant representation, which is shared by the majority of members of a highly structured group, which is uniform and coercive. Those who have representations of mathematics as a universal subject, tend to believe that when immigrant students learn the host culture language, they will learn mathematics in the same way as the local students. (Santesteban, 2006, p.79; as cited in Abreu & Gorgorió, 2007)

With this in mind many teachers view the students’ limited language as the main obstacle faced by second language learners in a mathematics classroom (Abreu & Gorgorió, 2007).

Gorgorió and Planas (2001) conducted a study with teachers in Catalonia in order to understand teachers’ views regarding the challenges faced by second language learners due to their limited proficiency in the language of instruction. The researchers conducted interviews with mathematics teachers to find out what strategies the teachers employed “...to improve both their teaching practice and their students’ learning process” (Gorgorió & Planas, 2001, p. 19). The findings suggested that many teachers believe that second language learners need to have full mastery of the Catalan language prior to learning mathematics. As one teacher explained: “...I don’t think it is a good idea to have children with no Catalan proficiency in math class...you know, math requires a sophisticated use of language” (p. 20). The teacher’s argument is supported by the example she gives regarding a student who knew how to do subtraction, but used a different algorithm. When the teacher solved the problem using another algorithm and explained her solution, the student assumed that he had done it wrong. In the teacher’s view if he had had a better understanding of Catalan this would not have been the case. As she understands it, “...as soon as they learn our language, there are no more significant differences...maybe, in the social sciences, but certainly not in the mathematics classroom” (Gorgorió & Planas, 2001, p. 21). According to Gorgorió and Planas (2001), the teacher did not consider other resources to explain to her student that his subtraction algorithm was valid. Teachers appeared to stress the importance of language, while ignoring “...alternative possibilities such as using visual

language” (Gorgorió & Planas, 2001, p. 28).

With the intention of understanding how teachers view the diversity of students in their classrooms in schools that had immigrant and minority students, César and Favilli (2005) conducted interviews with teachers in Italy, Portugal, and Spain. They interviewed twelve grade six to eight teachers in Italy, twelve teachers who taught grade seven to nine in Portugal, and another twelve teachers who taught grade seven to nine in Spain. The researchers were interested in finding out how teachers viewed their “...role in order to promote an intercultural education” (p. 1153). They also paid particular attention to “...their discourses about multicultural classes” (p. 1153). The results of the study indicated that while teachers had a positive and accepting attitude toward multicultural students’ integration in school, their assumptions and beliefs about mathematics acted as a barrier to effective inclusion (p. 1158). This can be seen in the quote from one of the teachers during the interview:

There aren’t so many difficulties in mathematics as in other subjects. Some difficulties may arise when they don’t understand the language, but “computation” is international (Marina, Spain; César & Favilli, 2005, p.1159)

This view that mathematics is universal because it is mostly a matter of computation, which is “international”, has been shown to cause additional problems for children learning mathematics.

A similar pattern was found in research conducted by Gorgorió and her colleagues. In their 2006 study (cited in Gorgorió & Abreu, 2007), they describe the situation experienced by an 11-year-old student who moved from Ecuador to Barcelona. David was considered a capable student who never had difficulties with basic algorithms. However, when he arrived in Barcelona the teacher was teaching the students how to divide using decimals by using the format most common in Spain. When David solved the exercises he used the algorithm he learned in Ecuador. The teacher did not accept what David had done and marked it wrong. David’s mother

approached the teacher and asked him to explain how they were doing the divisions. Following the teacher's explanation, David's mother tried to explain to the teacher how they do it in Ecuador. The teacher, paying minimal attention, told David's mother, "In Ecuador you do it wrong". David's case illustrates how the teacher had viewed mathematics. In his view there was only one correct way of performing a division algorithm. According to Abreu and Gorgorió (2007), "teachers, through their own years of schooling and through teacher education, have been exposed to a 'fossilized' representation about the universality of mathematics which is taken for granted in the social practice of teaching mathematics" (p. 1562).

Favilli and Tintori (2002) also conducted research with primary and secondary mathematics teachers in Italy, Portugal, and Spain. Using questionnaires and interviews, they were interested in finding out how teachers in those European countries experience teaching mathematics in a multicultural context. The researchers reported on teachers' concern of feeling unprepared to work with second language learners. As such, 76% of the teachers who encountered second language learners thought that it was important that the teachers get specific training. In addition, 90% of the teachers believed that it would be "useful to acquire specific abilities on methodologies aimed to the school integration of foreign pupils" (p.10). From the interviews and questionnaires the researchers concluded that students enter the mathematics classroom with their own

set of mathematical knowledge which could appear non-standard (with respect to the rest of the class); they possibly use computing algorithms which are different from the ones used by their classmates (and which sometimes are not accepted by the teachers). (Favilli & Tintori, 2002, p.12)

Teachers often understood these differences as evidence of the students' limited mathematical knowledge, and they frequently describe that their difficulties in working with these students comes from limited mathematical knowledge.

They don't know even what geometry means. (Teacher interview, Favilli & Tintori, 2002, p. 8)

When such pupils have arrived – I think they arrived in the second grade of the lower secondary school – they had done almost nothing, they were at the level of a third or fourth grade pupil in primary school, no more. (Teacher interview, Favilli & Tintori, 2002, p. 8)

From these studies, we can see the influence of teachers' beliefs about language, mathematics, and professional development for the multicultural classroom. The view that the language of instruction should be completely mastered before mathematics learning can take place limits students' access to participating in mathematical classes. The teachers' understanding of mathematics as universal prevents them from inviting and accepting alternate ways of solving problems. The challenge that mathematics teachers are facing is making sure that they are able to understand the differences found amongst their students and, as a result, are able to effectively teach their class building upon and taking into account its cultural diversity. However, their reported desire for additional professional training with diverse students and their uncertainty in this area also affects how they treat students in the mathematics classroom.

### *Summary*

From this review of the literature, the studies have reported on the connection that students made between mathematics and their real life experiences, their construction of word meanings, their multiple expressions, and collaborative problem-solving that infused their mathematical understanding. These were viewed as valuable resources for the learning and teaching of mathematics. Moreover, the studies also explored the students' use of their first language and found that it may be used to describe, clarify, and elaborate their understanding of a mathematical task. As well, the students' use of language/ code switching can be related to the help students got in the past from their family, their previous schooling experience, or the setting

in which they were doing the mathematics. In this latter case, language/ code switching is seen as a tool for learning and teaching. Finally, we can conclude that the teachers' perspectives regarding the need to learn the language of instruction prior to attending mathematics classrooms, and their attitudes regarding the universality of mathematics might prevent students from engaging in mathematics effectively. There are an increasing number of researchers who are suggesting that the exclusive emphasis on learning the language of instruction may actually hinder the students learning of mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). This review of the research suggests that these are important considerations in research with multilingual and second language learners. While my research study does not examine classroom interactions per se, it was nevertheless designed to build on the findings of the current research on multilingual children's reasoning in order to more fully understand how multilingual children express their mathematical understanding and reasoning.

### **Research Questions**

This study *Multilingual children's mathematical reasoning* addresses the following research questions: How do multilingual children express mathematical reasoning during collaborative problem-solving? What are the participants' experiences of and attitudes towards the problem-solving activity and how do they view the use of multiple languages?

### **Theoretical Framework**

In order to address the research questions, this research adopts a sociocultural understanding of learning and development (Vygotsky, 1978) that considers learning as "a social process in which culturally and historically situated participants engage in culturally-valued activities, using cultural tools" (Norton & Toohey, 2011, pp. 418-19). An account of Vygotsky's theory of human development as put forth by Wells (2000) identifies three key features of his theory. The first

suggests that in order to understand individual development, it is important to consider the history of the social groups in which the individual participates.

Understanding the development of an individual human being requires that ontogenetic be seen not as an isolated trajectory, but in relation to historical change on a number of other levels: that of the particular formative events in which the individual is involved (microgenesis); that of the institutions—family, school, workplace—in which those events take place and of the wider culture in which those institutions are embed (cultural history). (Wells, 2000, p. 54)

A second key feature that Wells identifies pertains to the role of cultural tools, which are both material and symbolic.

Human beings are not limited to their biological inheritance, as other species are, but are born into an environment that is shaped by the activities of previous generations. In this environment they are surrounded by artifacts that carry the past into the present (Cole, 1996), and by mastering the use of these artifacts and the practices in which they are employed, they are able to “assimilate the experiences of humankind”. (Leont’ev, 1981, p.55, as cited in Wells, 2000, p. 54)

From this we see how important it is to understand the individuals’ mediational roles, which are embedded in the larger culture and its history. Finally, Wells considers Vygotsky’s idea that the individual and the society in which s/he participates are mutually constituted.

A sociocultural theory is a good lens for understanding language as multiple, dialogic, and constantly changing. Drawing on a sociocultural perspective shifts the focus of learning from the individual to seeing learners “as differentially-positioned members of social and historical collectivities” (Norton & Toohey, 2011, pp. 418-19). A sociocultural framework allows us to view language as generative and more complex than sequential communication or writing. Studying language as more complex and dynamic has shifted the focus of much of the research from what multilingual learners cannot do, and led us to consider the multiple resources multilingual learners bring to a mathematical activity.

Without this shift we will have a limited view of these learners and we will design instruction that neglects the competencies they bring to mathematics classrooms. If all we see are students who...mispronounce English words, or don’t know vocabulary, instruction will focus on these deficiencies. If, instead, we learn to

recognize the mathematical ideas these students express in spite of their accents, code-switching, or missing vocabulary, then instruction can build on students' competencies and resources. (Moschkovich 2007, p. 3)

With respect to learners' competencies and resources in mathematics, Moschkovich (2007) includes mathematical Discourses. Drawing on Gee's (1996) definition, a Discourse is described as

“a socially accepted association among ways of using language, other symbolic expressions, and “artifacts,” of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or “social work,” or to signal (that one is playing) a socially meaningful role (Gee, 1996, p. 131, as cited in Moschkovich, 2007, p. 95).

For Moschkovich (2007), “mathematical Discourses include not only ways of talking, acting, interacting, thinking, believing, reading, and writing but also communities, values, beliefs, points of view, objects, and gestures” (p. 95). This perspective substantially widens the scope of research seeking to understand children's mathematical reasoning. Schliemann and Carraher (2002) state “in the area of mathematical reasoning, we need to consider how particular external representational systems play a role in the evolution of thinking” (p. 243). For instance, children develop their mathematical and logical thinking from their experiences, which is constantly developing as they interact with others (Schliemann & Carraher, 2002). According to Lerman (2001), in a mathematics classroom, “...interactions should not be seen as windows on the mind but as discursive contributions that may pull others forward into their increasing participation in mathematical speaking/thinking” (p. 89). This said, the children's reasoning starts with the social interactions in which they participate during their learning of mathematics. In addition, the language they use to express their thinking, the symbolic expression, and any material they use all provide important information to expand and enhance their mathematical knowledge.

## Methods

In order to answer the research question, I designed a qualitative study (Merriam, 2009), which allowed me to develop an in-depth description and analysis of multilingual children's mathematical reasoning. The design was intended to foster and illustrate how multilingual children express their mathematical reasoning during a collaborative problem-solving activity, where they would be able to work together to reason their way to a mathematical solution. According to Halliday (1993), "language is the essential condition of knowing, the process by which experience becomes knowledge" (p. 94). Children's knowledge is constantly expanding and it is, therefore, important to understand how multilingual children express mathematical reasoning. According to Stake (1978), "...one of the more effective means of adding to understanding for all readers will be by approximating through the words and illustrations of our reports, the natural experiences acquired in ordinary personal involvement" (p. 5). The present study is intended to do just that, to foster and illustrate how the participants express mathematical reasoning during collaborative problem-solving. This will enable me to describe the activities of the group by looking at the discourses and analyzing them in terms of their reasoning.

### *Recruitment/ Research participants*

Once I obtained the ethics approval from the committee at the University of Ottawa on January 22<sup>nd</sup>, 2013, I invited two colleagues whose families fit the criteria for inclusion to participate in the study. Before I get into the details of the participants, it is important to note that the focus of my study was not intended to examine family dynamics, or the role of parents in the children's mathematical reasoning (Acosta-Iriqui, Civil, Diez-Palmar, Marshall, & Quintos, 2011; Civil, Diez-Palmar, Menendez, & Acosta- Iriqui, 2008), but rather to create an informal multilingual context in which the children can engage in a collaborative mathematical problem-

solving activity.

The first family (I will refer to as Yasmine's family) had children ages 8, 9, and 10; the second family (I will refer to as Abir's family) had children ages 9, 11, and 13. Yasmine's family speak Farsi at home, and Abir's family speak Arabic at home. In both families, English is the language of the children's school of instruction, and the children are meeting Ontario provincial expectations, which means they are doing well at school. The objective for inviting multilingual families to engage in a collaborative mathematical problem-solving activity was to provide a discursive space for the children's mathematical reasoning, which could also allow for the possibility of using multiple languages.

Here, I will also take the time to refer to the children's language backgrounds as put forth by their parents. In Yasmine's family the children began by learning Farsi at home in their early years of development. English was heard mostly on TV and in stories. By age 4, the children started junior kindergarten at a Montessori school where instruction was in English. However, Yasmine continued to speak to the children in Farsi at home. In addition, Yasmine and her husband would work with them on mathematics in Farsi, "like do numbers" she says. A little while after, Yasmine sent Tara and Roya to a Saturday Persian school for a few years.

In Abir's family, she described both herself and the children's father as "Arabophone plus Francophone". English was not something they used at home at all especially when Mahmoud was born. Mahmoud was introduced to English when he was 6 months old, watching children TV programs that were age appropriate. Also, Abir's sisters came to Canada at a very young age, and they had the tendency to use English together, so Mahmoud heard them speak in English. Despite the fact that Abir and her husband are both francophone, when each child turned 3, they enrolled them in an English preschool, and by age 4, they went to an Islamic English school,

which also provided some Arabic. According to Abir, the Islamic school that her children attend is like any other school in Ottawa. The school staff are first language speakers of English, except for the Arabic and Islamic teachers who speak Arabic. Abir also says that even when the children are out on the playground, they use English 99% of the time because the school has some Iranians and Algerians, and the dialect difference between these kids is not something easy for them to understand. Therefore, they use English as their main language. Abir assumes that the children probably use some Arabic, but it is not a conversational tool.

#### *Procedure and activities*

The activity instructions were explained prior to its start and any questions or comments were addressed. Following the activity, each family was invited to participate in a follow-up conversation. One parent was then invited to watch the problem solving activity video.

#### *Mathematical problem-solving activity*

Task. The individual families were invited to participate in a collaborative mathematical problem-solving activity. I refer to collaborative mathematical problem-solving as interactions held between a group working together to reason their way to a solution. The activity was video recorded in order to capture the details of their participation. Each family participated separately. The mathematical activity was designed for groups of four. There were four clues, which were randomly distributed to each person in the group. The clues were as follows:

- 1) Build a number less than 100
- 2) Build an odd number
- 3) Build a number with one of the digits a 6
- 4) Build a number with the sum of the digits between 10 and 13 (See appendix 2)

Each of the participants was also provided with a set of tiles numbered 0-9 that they could use to represent the numbers that they built (See appendix 3).

Procedure. I saw the activity as having four phases, which illustrated how they were going to deal with the aspects of the activity; 1) The participants were asked to only read their own clue and build a number which fits the clue; 2) When all the participants had their numbers ready, they were asked to share with their group why they built that particular number; 3) When all 4 participants had shared their numbers and agreed that the individual numbers fit the clues, 4) they had the job of building another number together which fits all four clues (See appendix 4 for instructions).

This activity was intended to give the participants space to collaborate, engage in mathematical reasoning, and to give them the opportunity to use any of the languages that make sense in that context. Most importantly, this activity will help me to answer the research question: how do multilingual children express mathematical reasoning during collaborative problem-solving?

#### *Follow-up conversation*

Following the problem-solving activity, I engaged the children and their family in a follow-up conversation, which lasted approximately 10 minutes (see appendix 5 & 6 for types of questions). These questions are intended for conversational purposes, which allowed the participants to share their views and experiences in a relatively unconstrained way (Creswell, 2008, p. 220). The questions provided in the appendix are guidelines and they are not in any particular sequence. The follow-up conversation was designed in order to answer and give me an understanding of the research question: what are the participants' attitudes and experiences towards the problem-solving activity? In addition, the follow-up conversation was video recorded in order to capture the details of the conversations, especially in cases where they refer back to the activity.

### *Viewing of the recorded video*

Finally, I invited one of the parents in each family to view the recorded video to translate any aspect of the conversation that was articulated in the families' home language. In addition, the parents were asked to talk about contextual information pertaining to the children's mathematical experiences and the use of multiple languages when learning mathematics, which was audio recorded. This allowed me to understand the experience of that event from the parent's perspective, which is intended to answer the research question: how do they view the use of multiple languages.

All activities were held at the participants' home, as this was the most comfortable environment for the children and their parents.

### **Data analysis**

All video and audio recordings were transcribed (See appendix 1 for transcription conventions) and then analyzed.

Analysis of language. One aspect of the analysis I focused on is the language used in the exchange. Here, I looked at the length of the problem-solving discussion, the frequency of speech turns by language, the frequency of speech turns by participant, and the frequency of speech turns by language for each participant. Speech turn as an aspect of conversational interaction gives us information about what follows any given production, and so, my interest was to see when it was time for somebody to speak, what language did they speak and how often did they speak. As such, speech turns seemed appropriate for that.

Analysis of children's mathematical reasoning. In order to understand the children's mathematical reasoning, I looked at their discursive productions in terms of informal reasoning categories. The use of these categories allowed me to examine how in informal reasoning, the

participants may justify their decisions, explain their choices, and make inferences (Voss, Perkins, & Segal, 1991). Table 1 which follows describes the reasoning categories that I used.

Table 1: Analytic categories for reasoning operations

<i>Informal Reasoning Categories</i>	<i>Description</i>
Claim	A statement which asserts that a description of the text or its communicative context is true.
Hypothesis	A statement which is put forth for explicit confirmation as well as a qualified claim which expresses uncertainty.
Analogy	A statement which asserts a similarity between concepts.
Expectation	A statement which expresses anticipation or makes prediction.
Question	A statement which expresses interrogation.
Evaluation	A statement which expresses an evaluative judgment either positive or negative.
Meta-statement	A statement made by readers commenting on their own thought processes, performance or negative state of knowledge.

In order to get an indication of the coherence of the children's reasoning, I also analysed how the reasoning operations were related to one another. For this analysis, I considered three linking relations: *conditional relation*, which provides evidence, *elaboration*, which expands on given information, and *reiteration*, which repeats given information (Graves, 2001). The objective to consider these three linking relations will allow me to analyze whether the children provided evidence, elaborated, and reiterated. If there was ambiguity with regards to the categorization (i.e., reasoning categories and linking relations), discussions were held with my supervisor and consensus was achieved.

Analysis of language/ code switching. At the same time, I identified places in the verbal transcripts where there is language/ code switching and then connected it to the categories from

the review of the research literature that justify the reasons for it, such as difficulty with comprehension/ interpretation, in order to elaborate, justify, explain, to facilitate learners understanding, and the context of the mathematics lesson (See appendix 7).

Analysis of follow-up interviews and video viewings. The follow-up conversation was analyzed for explanatory information about the children's attitudes and experiences towards the problem-solving activity. The video viewings were analyzed for contextual information pertaining to the children's mathematical experiences and multiple language use, and how parents view the use of multiple languages.

### **Results and Discussion**

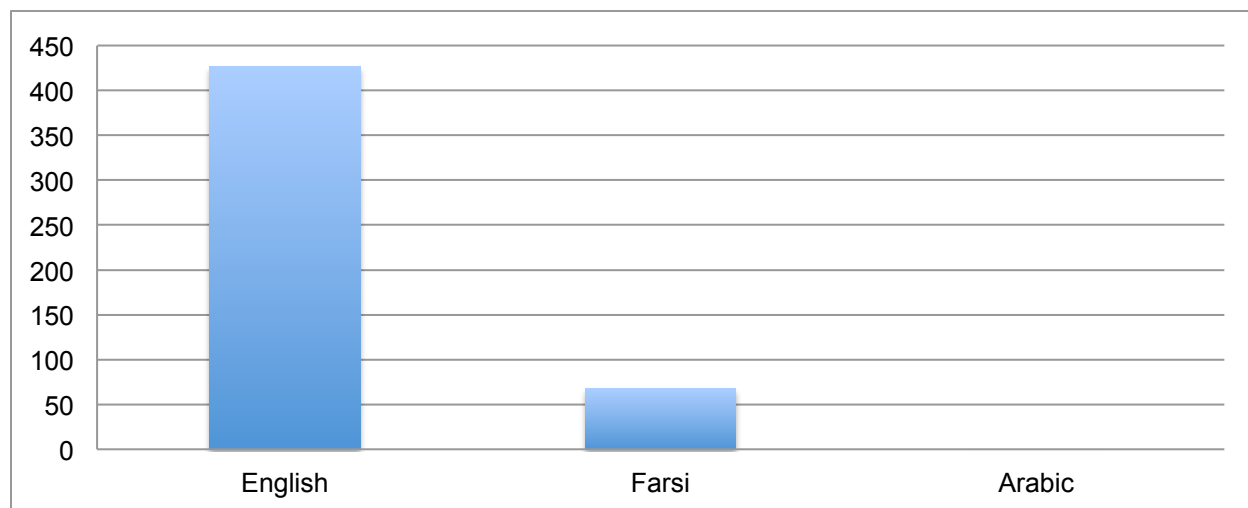
I will begin the presentation of the results by looking at speech production, paying close attention to the languages used in the exchange and identifying who spoke which language. I will look at the totals across both families, and then I will note any similarities and differences between them. I will then describe the children's discursive productions in terms of informal reasoning categories and reasoning links to understand their mathematical reasoning. In addition, I will discuss an emergent theme that became evident as I analyzed the data. I will also discuss what I saw in terms of language switching. Finally, I will comment on the children's attitudes towards the mathematical problem-solving activity, and the contextual information gathered from the conversations I had with the parents. It is important to note that I am not simply trying to compare the two families, but I am trying to report on what is it that they did.

#### *Language*

The following results for language and reasoning are presented for each family. The total time spent to solve the mathematical problem-solving activity was different for each family. For example, the first family spent 20 minutes. The second family spent 4 minutes.

### *Which languages are being used?*

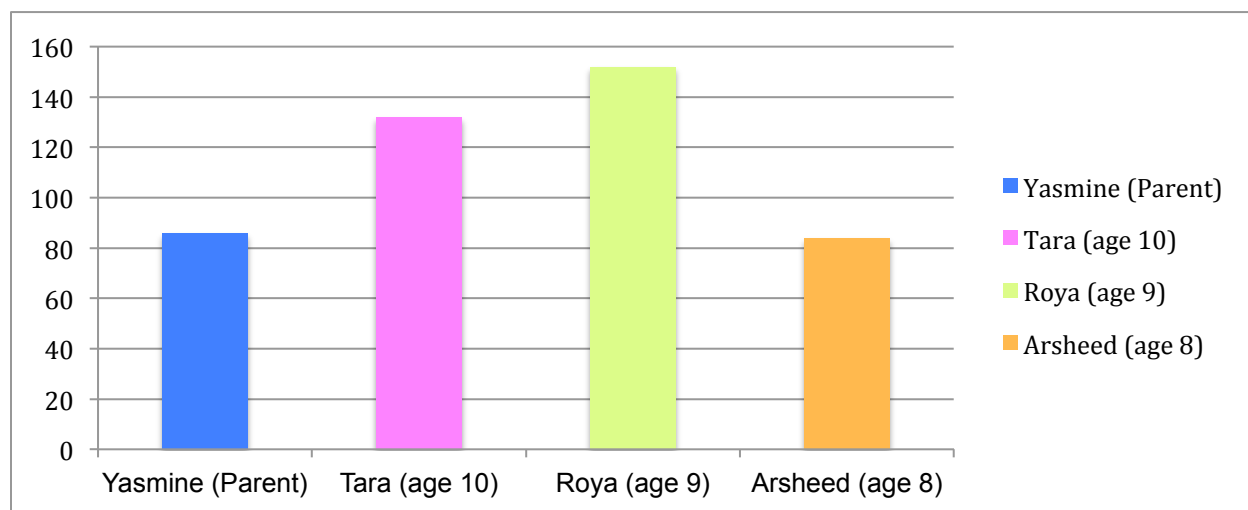
When we look at the total number of speech turns for both exchanges, we see that the discussions were conducted mainly in English (See Figure 1).



*Figure 1.* The number of speech turns by language.

### *Who is doing the speaking?*

When we look at the frequency of speech turns by participant for Yasmine's family (See Figure 2), we see that the children are producing more of the conversation than the adult parent.



*Figure 2.* The frequency of speech turns by participant in Yasmine's family.

When we look at the frequency of speech turns by participant for Abir's family in Figure 3, we see that the oldest child and the mother are producing equal amounts, and the younger siblings are producing somewhat less.

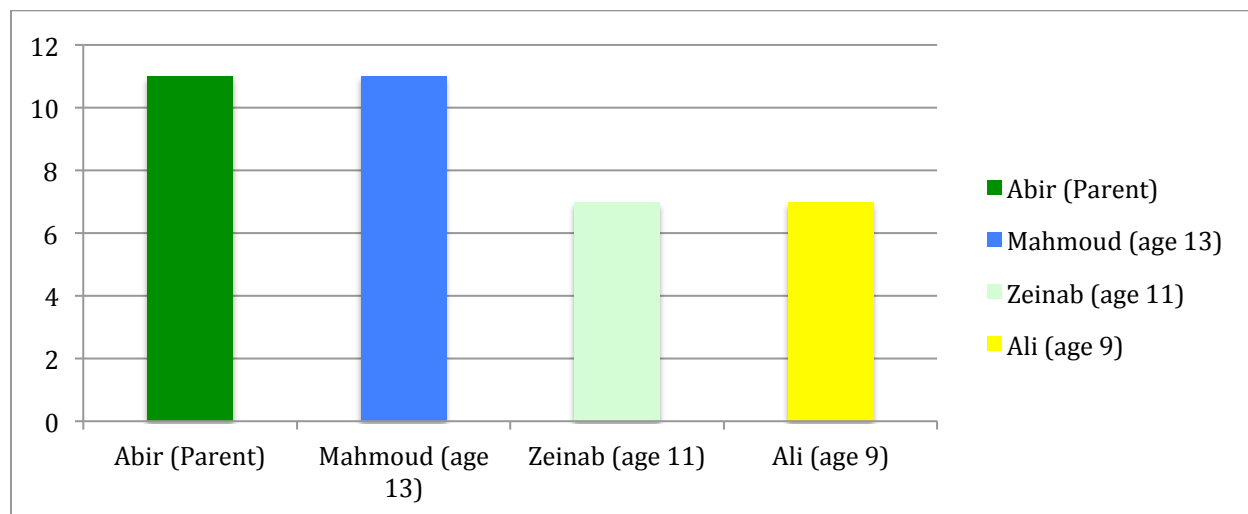


Figure 3. The frequency of speech turns by each participant in Abir's family.

#### *Who is speaking which language?*

When we look at who spoke which language in Yasmine's family (See Figure 4), we find that the children are predominantly speaking English.

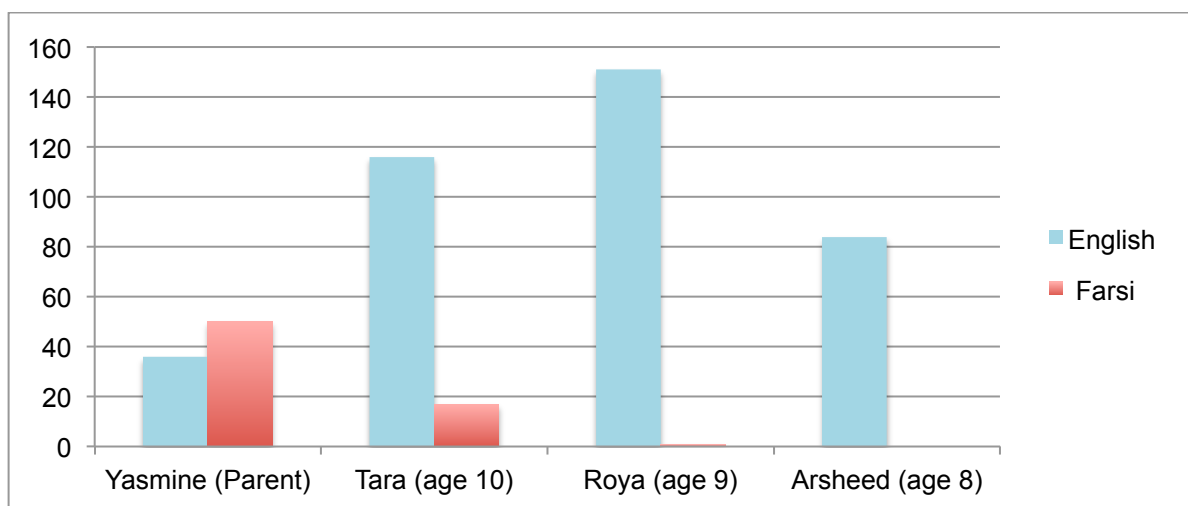


Figure 4. The number of speech turns by language for each participant in Yasmine's family.

When we look at the Farsi used in the exchange, we see that the parent is the one who is responsible for it. However, the older child, Tara, does respond to her mother in Farsi adding approximately 17 turns. In contrast, when we looked at Abir's family, we find that the mathematical problem-solving activity was carried out entirely in English.

The two families who participated in the study reported that they speak either Farsi or Arabic at home with their children. However, during the mathematical problem-solving activity, we see that they both spoke mainly in English. The following are some possible suggestions as to why this was the case: 1) The activity and the task were presented to each family in English and this might have influenced the use of English; 2) I presented myself in English, and may have been viewed as an authority figure, both as a researcher and as a teacher; 3) In the case of Yasmine's family, in a follow-up interview, the mother reported that when she works with her children on mathematics homework "usually the children start off by speaking in English, then at one point I switch to Farsi and then they'll switch or not." In the case of Abir's family, in a follow-up interview, the mother stated that when Zeinab (age 11) works with her dad on her mathematics homework, he has the tendency to use Arabic quite a lot, while for Zeinab, 60% of her conversation is in English and 40% is in Arabic. According to Abir, "The fact that Zeinab's dad is speaking in Arabic, I think Zeinab responds in Arabic just as a reflection".

These findings are consistent with the results of research that underline the important ways the students' experiences and the context in which the mathematics lesson takes place, influences the use of languages (Moschkovich, 2005; Planas & Setati, 2009). In cases where children are multilingual, language switching can be a tool "that can provide access both to mathematical ideas and powerful ways of thinking and speaking" (Morgan, 2006, p. 241).

### *Children's mathematical reasoning*

In order to understand the children's mathematical reasoning, I looked at their discursive productions in terms of informal reasoning categories. The reasoning categories were: *claim*, *hypothesis*, *analogy*, *expectation*, *question*, *evaluation* and *meta-statement*. The unit of analysis for informal reasoning of the discourse was the clause. The clause is needed to see the relationship between the statements made by the participants. Thus the number of reasoning categories I have would be greater than the frequency of speech turns. In my analysis of the reasoning categories used by the two families (See Figure 5), we see that the number of claims far outweighs the other reasoning categories.

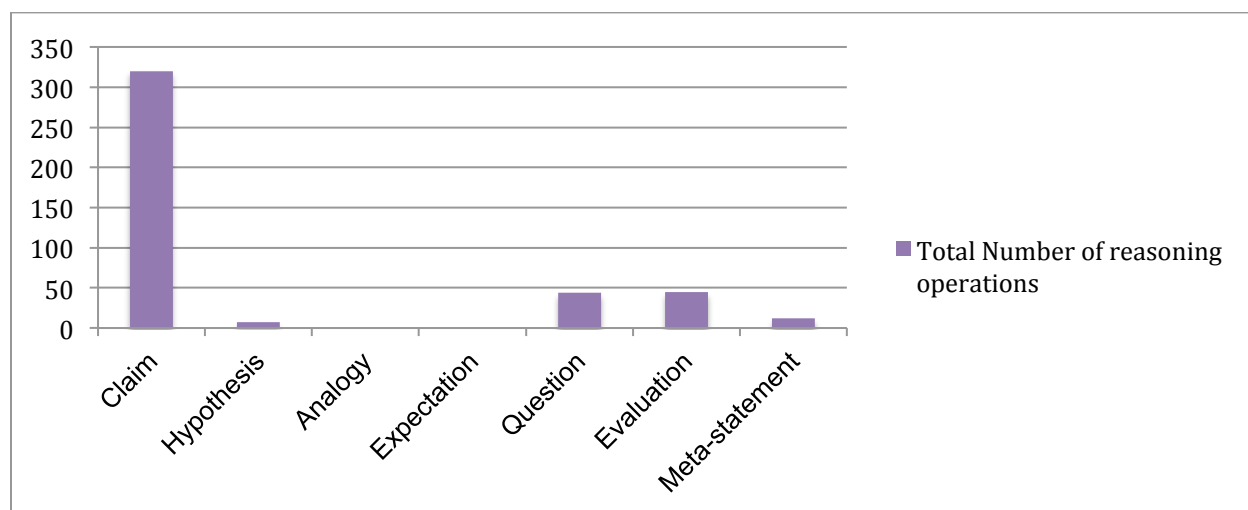


Figure 5. The total number of reasoning operations used by Yasmine and Abir's families.

A claim in this analytic framework is a statement that expresses a description, which is positive or negative. In order to explore the coherence of the children's reasoning, I examined the linking relations among the claims and discovered that the majority of the claims were linked by conditional relations. The claims were not produced in isolation, but rather one claim often provided the evidence for another claim. The source of the evidence came primarily from three

things, the instructions themselves which I referred to as clue verbatim, for example, when a child says, “it said to build an even number, and so I made it 13456”; the children’s more broader mathematical knowledge, for example, the child builds an even number using all odd numbers except for the last number, which makes it an even number; and more generally their world knowledge, for example, the children discuss the representation of odd and even numbers on houses, where on one side of the street has even numbers, while the other side has all odd numbers.

After Roya (age 9) was asked to read her clue and build a number that follows her clue (see Figure 6), she reads her clue out loud and she says the following:

“Build a number, that would be one number, the sum, that would be two, the sum of the digits between ten, Ok so this 5 and 8.”

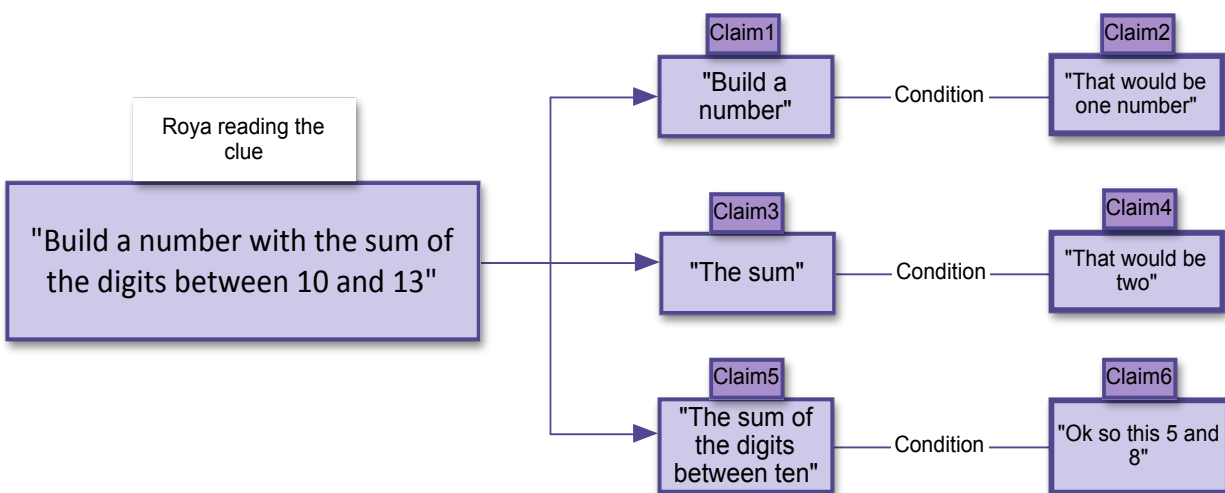


Figure 6. Roya reasoning with claims.

Roya starts by reading the clue out loud. She then says, “build a number”, emphasizing the ‘a’, which she takes as the evidence for one number: “That would be one number.” She then goes on to say “the sum” and now she realizes that the sum is evidence for needing two numbers and not one number: “That would be two.” Then she restates a portion of the clue: “The sum of the digits

between ten,” which then leads to her answer. She says 5 and pauses for a bit, and then she says 8. With this example, we can see that Roya is not only making claims, but also providing the evidence for those claims, which they are in this case all clue verbatim.

In other examples, where we can see how the children reasoned with claims, and provided evidence for those claims is when they were asked to share with their group the numbers that they built. In the first example, we have Yasmine’s family taking turns to share the numbers they built (see Figure 7). Arsheed (age 8) starts this round and tells me the reason he built his number, “because 90’s less than 100”. Here Arsheed uses the clue as evidence for the number that he built, while Roya and Tara listen and observe very attentively. Roya (age 9) follows, she first explains that she built her “two numbers 5 and 8”, and then she explains that “5 and 8 makes 13”. Roya uses the clue as evidence for the numbers she built. We can also see this when Tara (age 10) shares her numbers, she starts by first referring to the clue: “It said one of the numbers have to be 6”. However, Tara changes the actual wording of the clue by paraphrasing it, she also changes the word ‘digits’ to mean a ‘number’. This according to Moschkovich (2008) is considered ‘multiple interpretations’ of a given text, which may not prove to be obstacles in a mathematical discussion. Tara then explains what she did, “and so I made it 0 and then a 6”, and then repeats, “I have a 0 and then a 6”. Yasmine also paraphrases the clue, “mine was supposed to be an odd number”, and provides her answer, “and I picked 7”. Yasmine also uses the clue as evidence for the number that she built.

In the next example, we also have Abir’s family sharing their numbers and providing evidence for those claims (see Figure 8). This too takes place when they were asked to share with their groups why they built those particular numbers. Ali (age 9) states, “mine says build a number with the sum of the digits between 10 and 13. I built 6 and 5, which is 11”. Here Ali has

three claims; the first claim is the clue verbatim, which is the evidence for the second claim (addends) and the third claim (sum of the addends). Also, the second claim is the evidence for the third claim because the addends equal 11. Mahmoud (age 13) states, “mine says build a number with one of the digits a 6, so I just left it the sequence, because it already has a six in it”. Mahmoud had three claims; the first is the clue verbatim, which provides the evidence for the second claim, “so I just left it the sequence”, while this claim provides the evidence for the third claim, “because it already has a six in it”. Zeinab (age 11) starts by first referring to her clue, which says, “build an odd number”, and then she attempts to read what she has which is the second claim, but Mahmoud helps her out in her articulation of the number before her. Zeinab reiterates what her brother had stated and she continues to read what she had. Abir also starts by referring to her clue, “it says build a number less than 100”, which is the evidence for her second claim, “and I just built 85”.

From these data, we can see how multilingual children expressed their mathematical reasoning through their use of claims, which were linked by evidence. This also illustrates how the children stick very closely to their claims and the task, which shows the richness in the children’s reasoning and interaction.

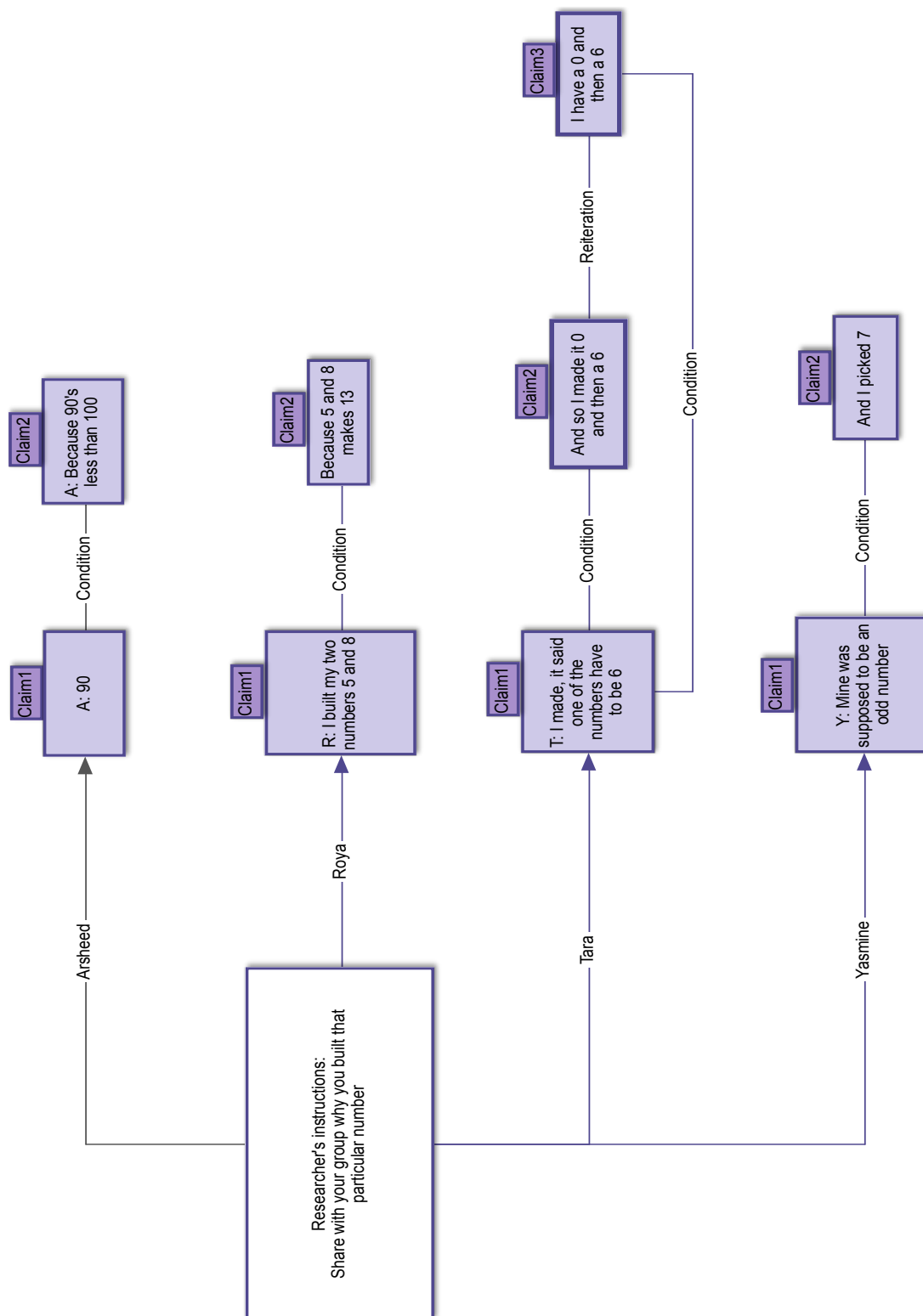


Figure 7. Yasmine's family reasoning with claims

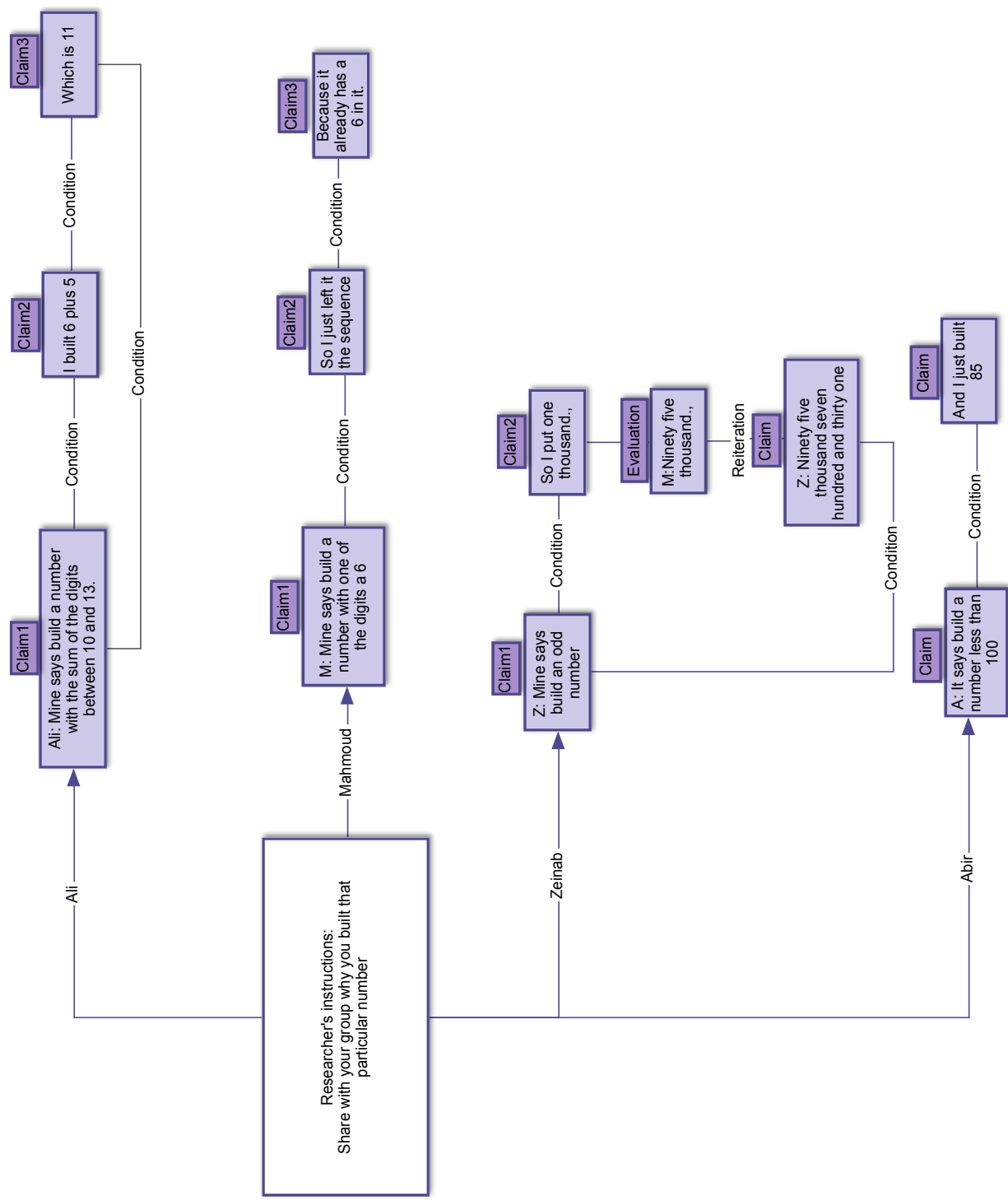


Figure 8. Abir's family reasoning with claims

*Knowledge of mathematics and the world*

During the problem-solving activity the children not only provided evidence based on the information provided in the clues, they also drew on their broader knowledge of mathematics and the world. For example, discussing odd and even numbers, Roya and Arsheed have the following exchange.

<u>Name</u>	<u>Transcription</u>
Roya:	Yeah, I think yours is right definitely because a 7 is an odd number.
Arsheed:	Not, not an-
Roya:	An even number, it's
Arsheed:	Like 2, 4, 6, 8, it's 1, 3, 5, 7.
Yasmine:	Uhah
Roya:	And 9
Arsheed:	Yeah, then we keep on going, 77, 79.

This takes place while they were sharing their numbers and checking to see whether the number '7' built by Yasmine follows the clue, *build an odd number*. Roya here tells her mom, "I think yours is right definitely because a 7 is an odd number". This then leads into a mathematical discussion, which is influenced by Arsheed (age 8) with regards to odd and even numbers. He explains that an odd number is not like 2, 4, 6, 8, but is 1, 3, 5, 7. In an additional example, also responding to *build an odd number*, Zeinab (age 10) in Abir's family builds, "Ninety five thousand seven hundred and thirty one". Zeinab did not pick a number, but used all of her titles to include all the odd numbers available in the sequence.

In another example responding to *build a number with the digit 6*, Mahmoud (age 13) in

Abir's family decides to leave his number as a sequence with a 6 in it, but then he asks, "Should I keep it simple just like this? Or, do you want me to actually try?" For Mahmoud, extracting a number seems more effortful than using the set of tiles before him because he already recognized that he has a number, 0 1 2 3 4 5 **6** 7 8 9. Mahmoud states, "mine says build a number with one of the digits a 6, so I just left it the sequence because it already has a 6 in it". Tara in Yasmine's family also responds to the clue *build a number with one of the digits a 6*.

<u>Name</u>	<u>Transcription</u>
Tara:	Can I change it for a second? Can I change it mommy?
Researcher:	Yeah, you can change it.
Tara:	I made it finished that big number, /it has a six in it/
Roya:	<p style="text-align: center;">/ One of the digits is a six/</p> <p>So it would be, here comma, comma, no, a comma and then comma, comma.            So it would be ze-, so it would be            /one hundred and twenty three million four hundred and fifty six thousand seven hundred/ [At that point, Roya takes the tiles which are 0123456789 and inserts commas so that the resulting number looks like this 0 123, 456, 789]</p>
Tara:	/Wait, but this can't be here [referring to the zero at the front of the sequence]; it has to be here because it wouldn't count as a number if it was on this side/.
Roya:	Then it has to be like this [Referring to the number that Tara has just moved]
Yasmine:	[Farsi] Why did you put it there?
Tara:	Yes because if I put it here it doesn't make sense because it's not a number, this doesn't count as a number.
Roya:	Okay, so it'll be one billion two hundred and thirty four million five hundred and sixty seven thousand and eight hundred and ninety [1, 234, 567, 890].

This dialogue takes place after Tara had originally built 0 and a 6, she then asks if she can change her number. When she is given permission to do so, she places the tiles back into a

sequence and says, “I made it, finished, that big number (0 1 2 3 4 5 6 7 8 9), it has a 6 in it”. Roya confirms that in fact “one of the digits is a six”, and she then groups the numbers by adding commas (0, 123, 456, 789). This allows Roya to distinguish the number sequence from a million, to a billion. Roya attempts to read the number, but she is interrupted by Tara who realizes that she needs to reposition the zero. After Tara has done so, Roya then fixes the commas that she had inserted between the numbers (1, 234, 567, 890). Yasmine then asks Tara, “Why did you put the zero there”. Tara explains, “if I put it here it doesn’t make sense, it’s not a number, this doesn’t count as a number”. Unlike Mahmoud who feels content to leave the zero, Tara feels that it is not a number. Roya then reads the number, “one billion, two hundred and thirty four million, five hundred and sixty seven thousand, and eight hundred and ninety”.

In these illustrated examples, we see how Arsheed, Roya, Tara, Zeinab, and Mahmoud draw on their knowledge of mathematics to generate the discussions. We also see several cases where the children used the entire set of tiles to generate vast numbers that included thousands and billions. I suggest this to be the case because the children were provided with numbered tiles to represent each of the digits 0-9. This allowed them to invent numbers of that quality, while also demonstrating their mathematical knowledge. If they were working with pencil and paper, they would have not done that.

We also see the children drawing on their more general world knowledge, to provide evidence. For example, in the following exchange that took place among Arsheed, Tara, and Roya, we hear a different type of evidence.

<u>Name</u>	<u>Transcription</u>
Arsheed:	How about if we use a calculator?

- Tara: It's not a calculator thingy because  
/a calculator can't answer the question/.
- Roya: /You can only use the calculator/ when you grow up.
- Tara: And it won't work with the question because it's a math problem; it's not like adding or something. So we can't.

In this example, we can see that they are not referring to the mathematical problem per-se, but they are talking about when you can use a calculator and why. Tara expresses the understanding that you could solve a mathematical operation (e.g. adding) with a calculator, but it is not helpful when you are trying to reason through a problem (“It’s a math problem”).

*Elaboration and reiteration*

When I analyzed the linking relations of *elaboration* and *re-iteration*, they illustrated some of the ways the children built on each other’s contributions. In the following conversation, Roya and Tara elaborate and reiterate on each other’s claims to express their mathematical reasoning.

- | <u>Name</u> | <u>Transcriptions</u>   |
|-------------|---|
| Roya:       | Yours is right, he's right.                                       |
| Arsheed:    | Okay  |
| Roya:       | It's less than 100 that could be technically any number 2 digits. |
| Tara:       | It could be one digit too   |
| Roya:       | Yeah, it could be one digit or two digits.                        |

In this dialogue, Roya, Arsheed, and Tara are working together to see whether the numbers they built followed the clues. Roya starts this discussion by referring to the number Arsheed built which was 90. She tells Arsheed that, “Yours is right”, and then she repeats it to everyone else that “he’s right”. Arsheed following along says, “Okay”. Roya then justifies why she thinks it is right by stating, “It’s less than 100 that could be technically any number 2 digits”. Tara then elaborates on the claim made by Roya, she says, “it could be one digit too”. Roya reiterates the claim made by Tara, while also reiterating the claim she stated earlier: “Yeah, it could be one digit or two digits” (see Figure 9 for a visual representation).

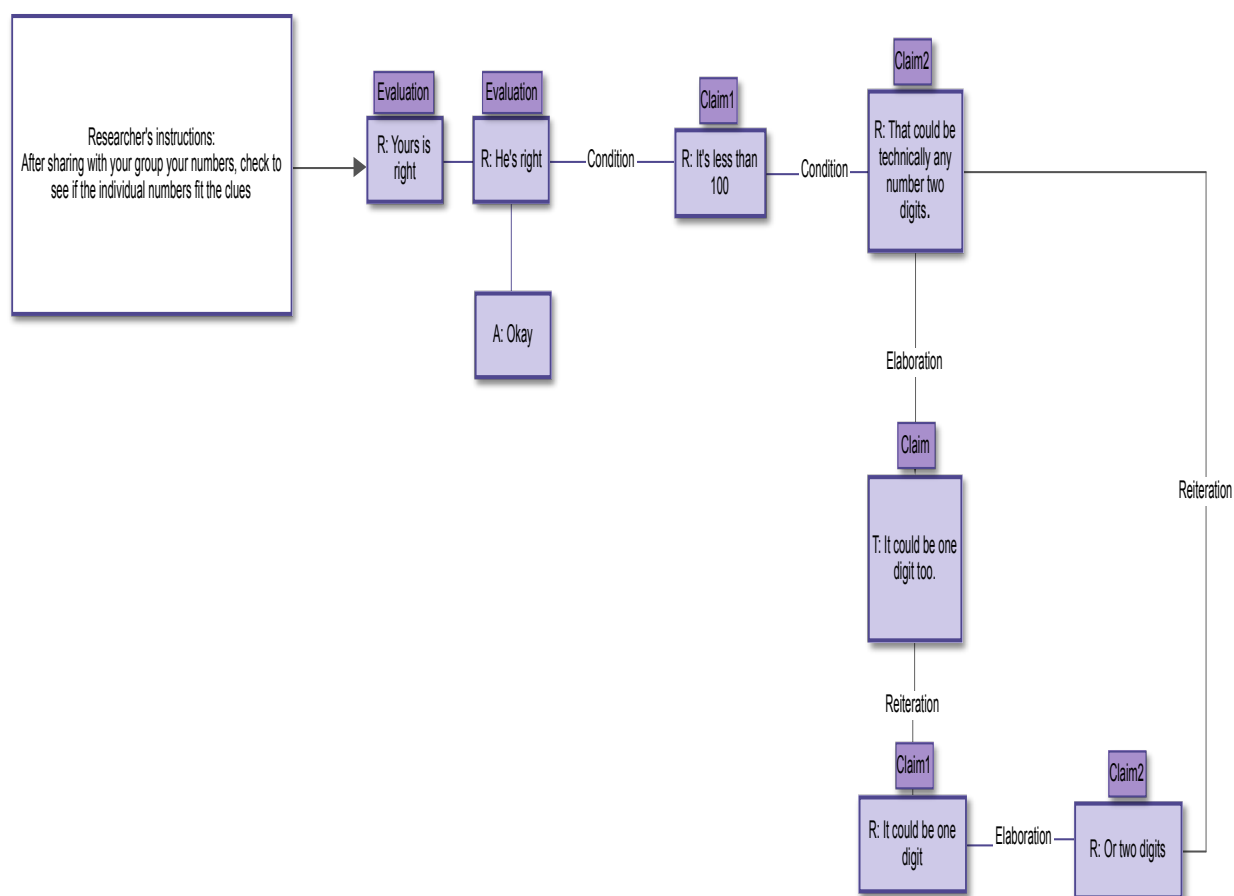


Figure 9. Yasmine's family working together on *Build a number less than 100*

In another example, where the children built on each other's ideas while listening to one another, is evident through the discussion between Roya, Yasmine, and Arsheed to see whether the number built by Roya follows the clue (see Figure 10).

<u>Name</u>	<u>Transcriptions</u>
Roya:	Mine is "build a number with the sum of the digits between 10 and 13", and 5 and 8 is 13.
Yasmine:	Okay, I guess, /if it's included/
Roya:	/Oh wait/, build a number between 10 and 13 [here she emphasizes on the word between]
Yasmine:	Uhah
Roya:	That's 11 or 12
Arsheed:	I think it's 11
Roya:	Ok, so I'll use 5 and 6
Arsheed:	56?
Roya:	No, 11. 5 plus 6 [Here Roya uses pencils to create the plus [+] sign as a visual representation]
Arsheed:	5 plus 6

Roya first reads her clue and then says that "5 and 8 is 13". Her mother responds by saying, "Okay, I guess if it's included". But Roya realizes that the sum of the addends does not fall between 10 and 13, and she explains that it should be either 11 or 12. Arsheed here says that he thinks it's 11, and Roya says that she will use the tiles 5 and 6. Roya's decision to use the 5 and 6 was influenced by Arsheed who said that he thinks it's 11. Arsheed then asks "56?" Roya evaluates Arsheed's response and explains to him what she did. This time she uses mathematical terminology and a visual representation; two pencils to make the plus sign between the 5 and 6.

Roya was acting with mediational means in order to construct a common understanding among them. Arsheed then reiterates what Roya had said, “5 plus 6”, which indicated that he understood the numbers before him.

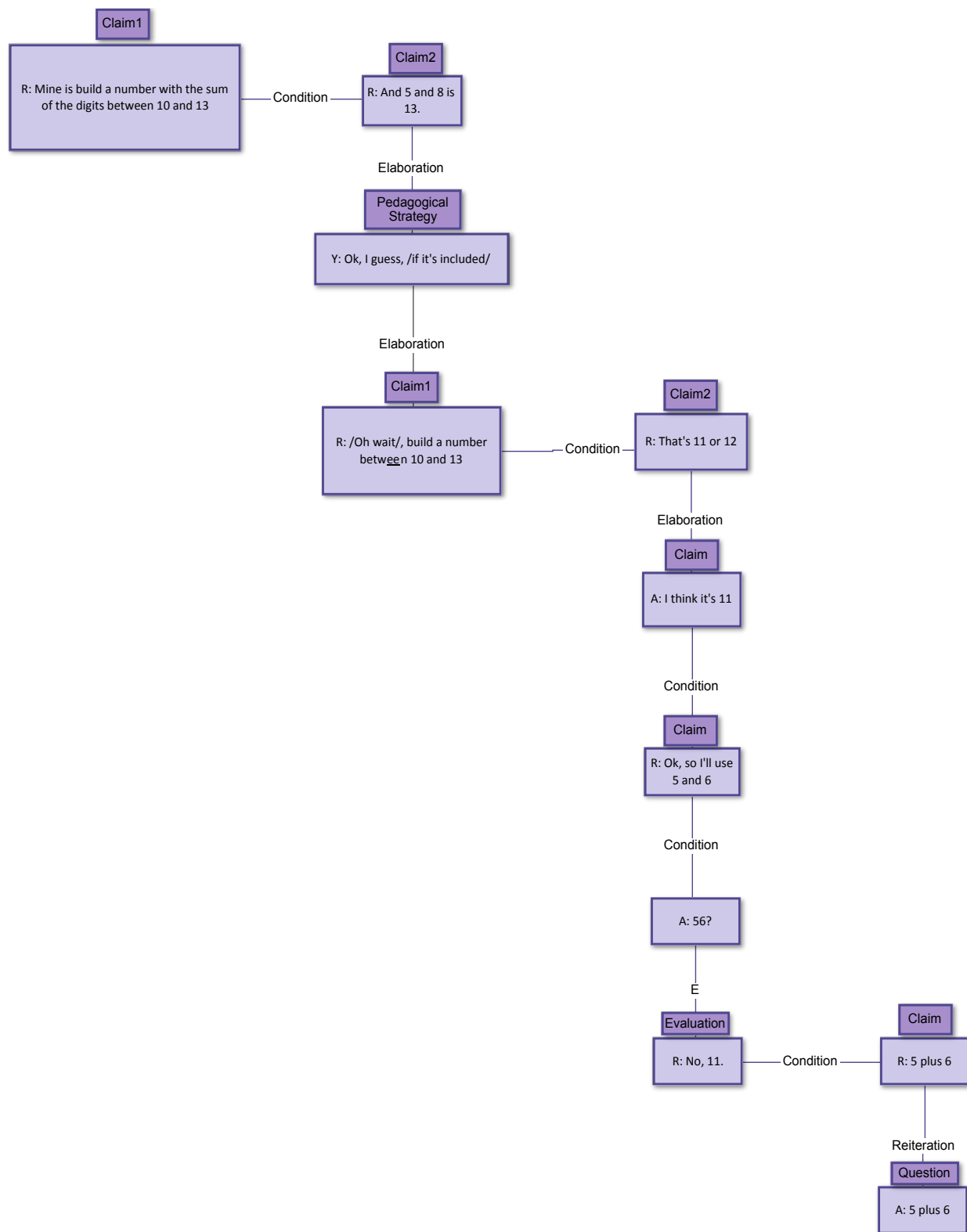


Figure 10. Yasmine's family work on *Build a number with the sum of the digits between 10 and 13*

In these examples, we see that the children not only communicate the numbers they built, but they are listening, evaluating, elaborating, and reiterating the statements made by others. This suggests that what they were really doing is developing a form of argument embedded with evidence, which is a certain artefact of thought, language, and collaborative problem-solving. The findings also demonstrate that the children were able to stay focused, listen to one another, and build on each other's ideas based on what another child said (Graves & Zack, 1997, p. 24). These examples of children's collaborative problem solving "show how this is a significant part of students' work together" (Barwell, 2005, p. 346), which allowed them to pull each other forward and bring in resources, such as their broader knowledge of mathematics and the world to the mathematical discussion (Lerman, 2001; Moschkovich, 2007). In addition, the norms embedded in this collaborative problem-solving activity encouraged children's participation. For example, the researcher and the mother supported the children's suggestions (Weber et al., 2010). Moreover, the activity was structured to allow the children to provide evidence/justifications for their answers, and collaboratively work together to check the reliability of their answers (Weber et al., 2010).

*Emergent theme: Pedagogical Strategy*

In doing the analysis, about half way through the discussion I started to see a reasoning operation that I had not anticipated (See Figure 11). I decided to call this a pedagogical strategy since it was intended to guide, clarify, and/or focus the discussion for the purpose of furthering understanding. Rasmussen and Marrongelle (2006) refer to it as a *pedagogical content tool* (PCT), which is a "device, such as a graph, diagram, equation, or a verbal statement, that a teacher intentionally uses to connect to student thinking while moving the mathematical agenda forward" (p. 389). In addition, Setati (2005) refers to it as *regularity Discourse*, which is "mainly used by the teacher and refers to interactions that focus on regulating the learners' behaviour.

This Discourse is used mainly to call for the learners' attention, to request them to listen to the teacher or each other or to get them ready for a specific task during the lesson" (p. 449-450). For the context of this research, I take it to mean a verbal statement produced by someone (i.e., the children, or the parents) to guide and further one's understanding of a given matter.

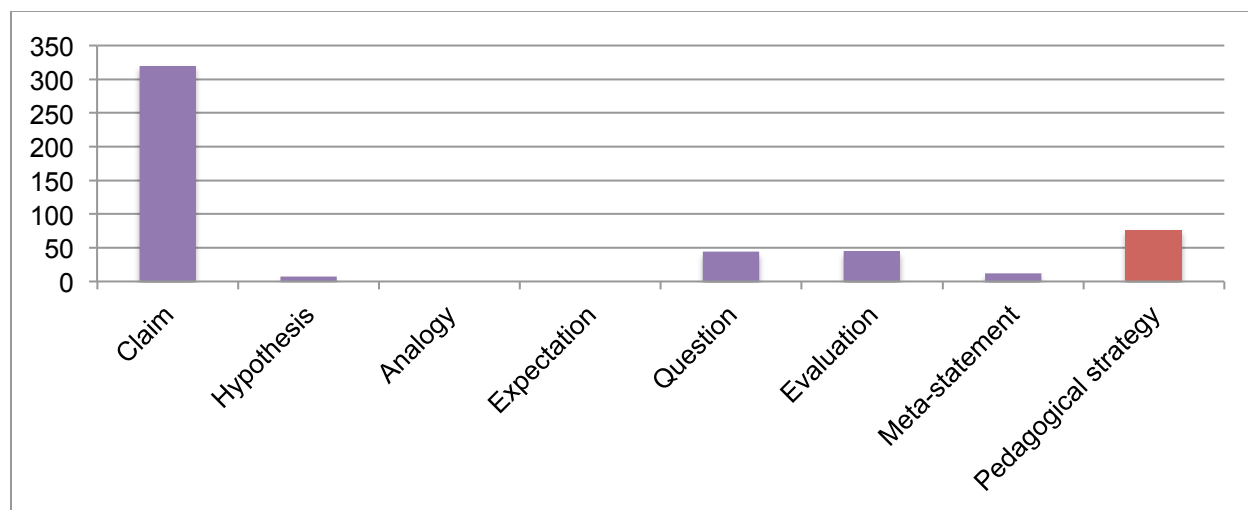


Figure 11. Reasoning operations with emergent category.

I felt that there was a need to add this because in the transcripts given the context, I see a teaching stance going in both families. For example, some of the pedagogical statements used by the participants were, "Do you wanna reread your clue?" "Let's go back and check Roya's number", "Start over", "What does it mean?" "Reread the clue one more time", "Are we sure we can do the first one?"

#### *Pedagogical strategy for each family*

When we look at the frequency of pedagogical strategy by participant for Yasmine and Abir's family (See figure 12 & 13), we see that the parent is producing more of the pedagogical strategy than the children.

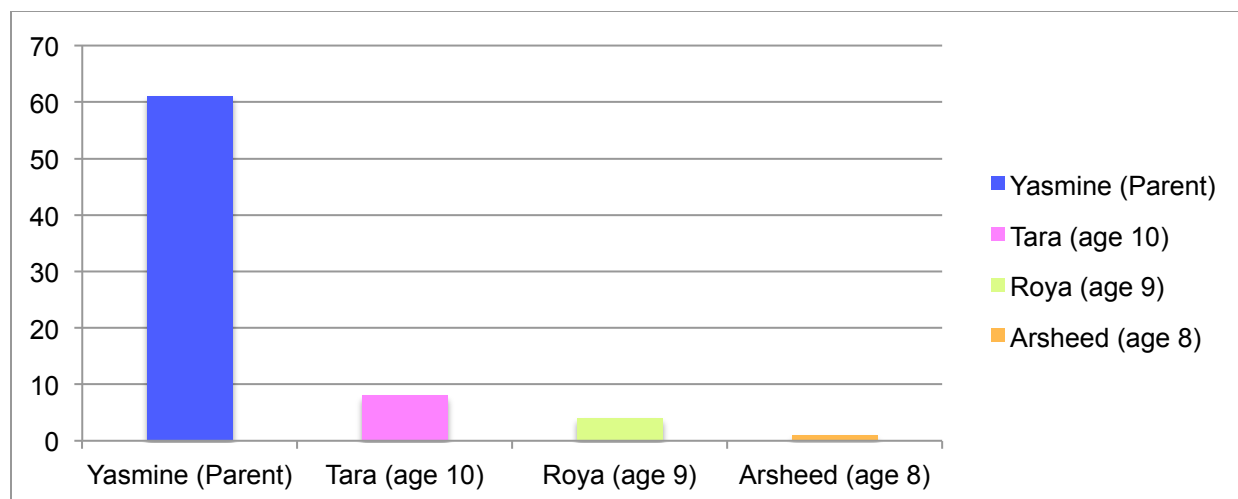


Figure 12. The frequency of pedagogical strategy by participant in Yasmine's family.

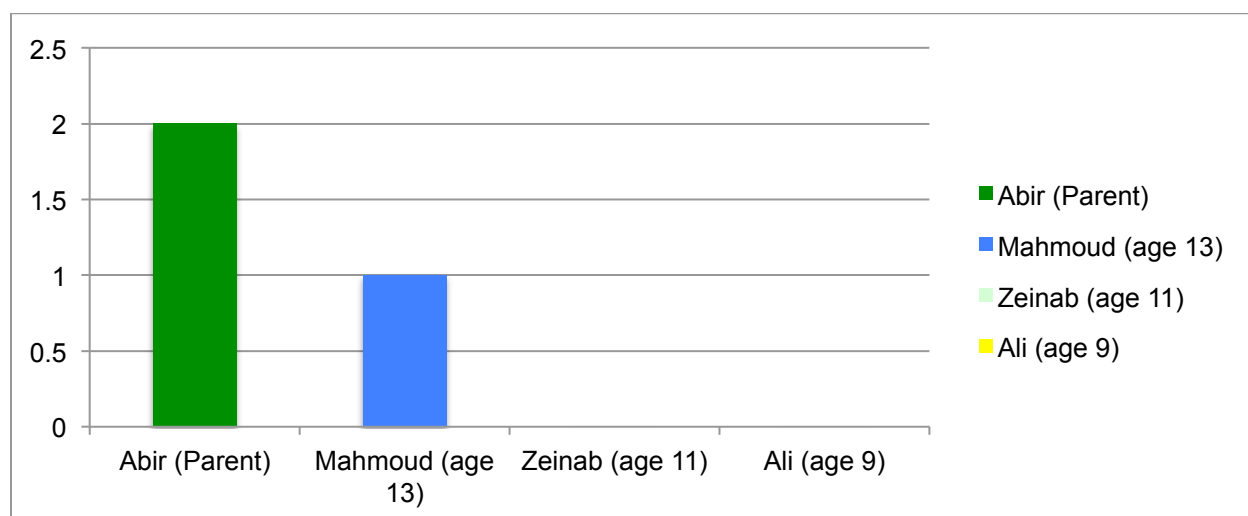


Figure 13. The frequency of pedagogical strategy by participant in Abir's family.

### *Which languages are being used for the pedagogical strategy?*

When we look at which language was used for the pedagogical strategy in Yasmine's family (See Figure 14), we find that the mother is mainly using it in Farsi, while the children used it in English. In contrast, when we looked at Abir's family, we found that when the pedagogical strategy was in fact used, it was in English (See Figure 15). This seems to be the case because in

Abir's family neither the mother nor the children switched to Arabic in the course of the problem-solving activity. While in Yasmine's family Farsi was used during their discussions.

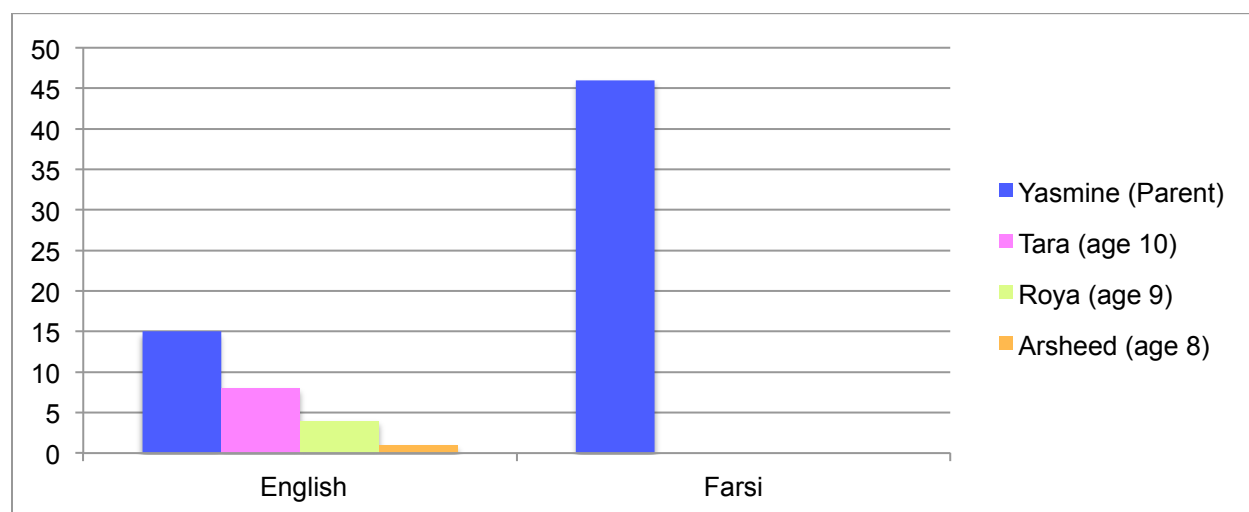


Figure 14. Pedagogical strategy by language and by participant in Yasmine's family.

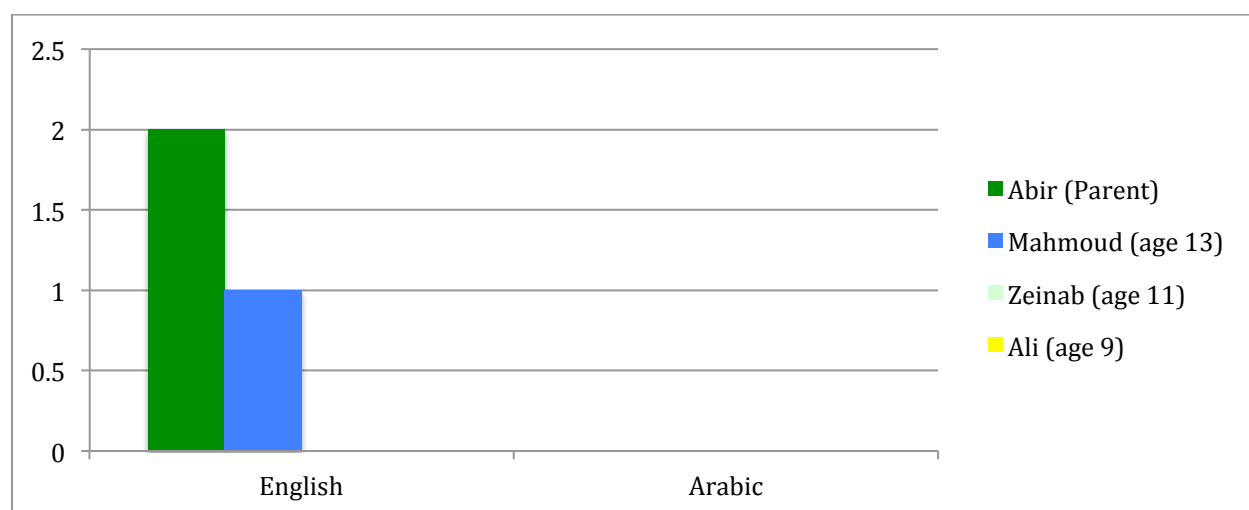


Figure 15. Pedagogical strategy by language and by participant in Abir's family.

*When was the pedagogical strategy most likely used?*

When we look at the four phases in figures 16 and 17 for Yasmine and Abir's families. We see that out of the 4 phases, both families used most of the pedagogical strategy in phase 4. This is where they were asked to work together and build one number that follows all four clues.

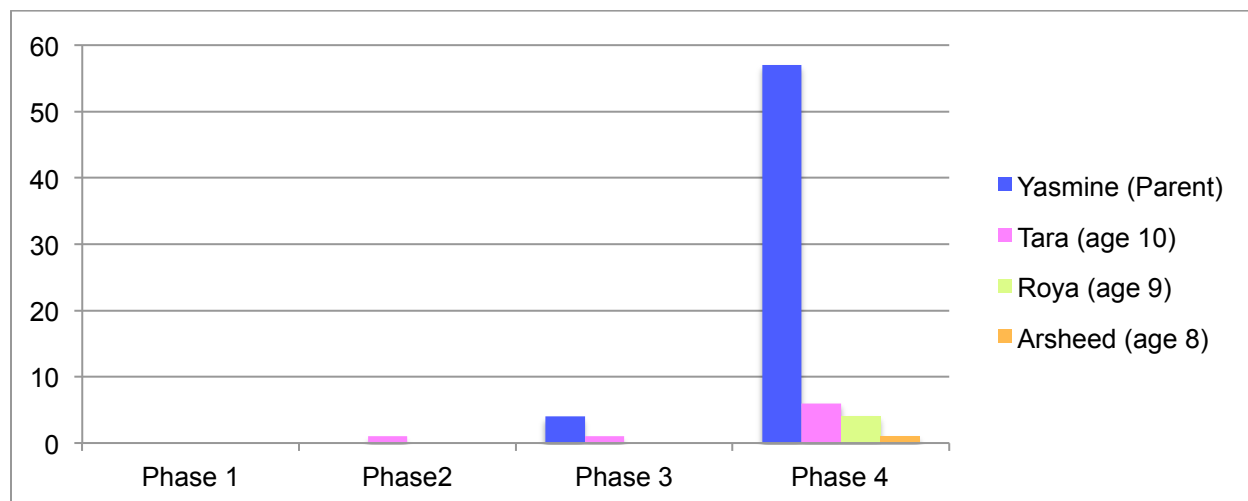


Figure 16. Pedagogical strategy by phase for Yasmine's family.

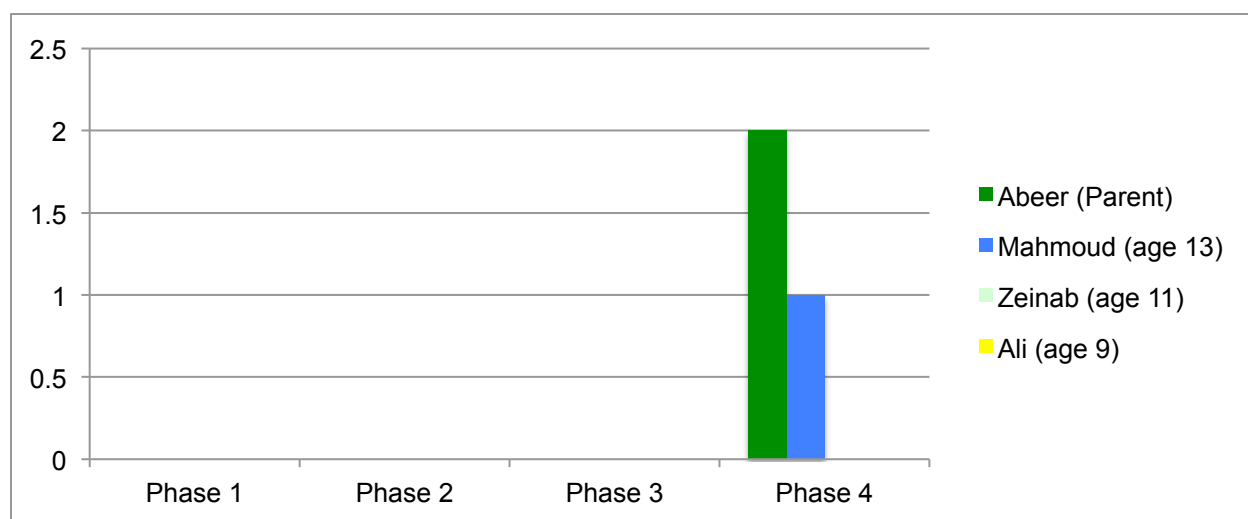


Figure 17. Pedagogical strategy by phase for Abir's family.

What accounts for the increased use of the pedagogical strategy in phase 4 by Yasmine's family, partly has to do with the confusion that resulted from the clue that reads; *build a number with the sum of the digits between 10 and 13*. While Yasmine's family have come up with the numbers 5 and 6, which add up to 11. When they are looking for the composite number, they get confused between the addends, and the sum of the addends. The following is an exchange

between Roya and Tara in response to *build a number that follows all four clues*.

<u>Name</u>	<u>Transcriptions</u>
Roya:	I know one
Tara:	11
Roya:	Yeah because it...
Tara:	Oh wait, it has to have the number six in it. It has to be between, it has to be either /11 or 12/
Roya:	/The sum/, the like the-,
Tara:	And it has to be, it has to have the 6 in it, so it doesn't work.
Roya:	Yeah, no, no, no. This works 5 and 6. It has a six in it, and then it has to have an odd number
Tara:	The 5, 5 is an odd number
Roya:	Yeah, 5 is an odd number
Tara:	/And it's less than/ a hundred
Roya:	/And it's less than a hundred/ It works
Tara:	Okay, we made it, Roya's
Researcher:	So which?
Roya:	5 and 6
Tara:	It either has to be /11 or 12 and it ha-, /
Roya:	/But the addends have a six/
Yasmine:	What?
Roya:	But the addends have a six
Tara:	But it has to be the sum of the digits
Roya:	Between 10 and 13
Tara:	Is between 10 and 13

Roya: Between 10 and 13

Tara: I know but it has to be, it has to have a six in it. Okay, okay, less than a hundred we've got it, and build this, but if this /one wasn't here, it would work/, but-,

Roya: /I think this works/

Tara: /If you wanna say/ the two numbers that equal it okay 5 and 6 okay, it has an odd number, it's less than a 100, and it has a six in it, but if you wanna say the sum of it actually it won't work because it has to have a 6 in it and it has to be either 11 or 12.

Tara and Roya first conclude that the number has to be either 11 or 12, but then they realize that it does not have a six, so it does not follow all clues. They understand the clue and then they misunderstand the clue. We see that they know what is between 10 and 13; they know the choices are 11 or 12, they have decided on 11 on the most part, but they keep confusing 5 and 6 (the addends) with 11 (the sum of the addends). This speaks to what Moschkovich (2008) refers to as “multiple interpretations”; Tara refers to the sum of the addends (11), while Roya refers to the addends (5 and 6). These findings draw our attention to the fact that when the children get confused, we should expect that they are not looking at, talking about, referring to, or imagining the same things (Moschkovich, 2008). These ambiguous and shifting meanings did not prove to be obstacles to the mathematical discussion, but rather prompted Yasmine to get involved and use a pedagogical strategy to guide, clarify, and/or focus the discussion for the purpose of furthering understanding.

In contrast, there was very limited use of the pedagogical strategy in Abir's family. In my view, this partly has to do with the role of Mahmoud (age 13), the older brother. As the older brother, he gives his siblings the space they need to build their individual numbers. However, when it comes time for them to work together and build a number that follows all four clues,

Mahmoud takes his clue and the number he built, and quickly assesses everyone else's. He then puts a number together which satisfies all four clues and provides his reasoning: "65, it's the number Ali had because if you add it up it is between 10 and 13, it's 11, and then mine, it says, it has to have a six in it. This [pointing to the clue *build an odd number*], it's an odd number, and 5 is an odd number, and then Mama's says, it has to be less than a hundred, and it's less than a 100" (See Figure 18 for a visual representation). Mahmoud literally takes over the whole discussion by making the decision for everybody. The children understand what their brother is doing, and I presume they trust him because he does very well at school. I also presume that had he not been there, more space might have become available for his younger siblings to discuss and share their thoughts. They are in fact capable of doing so, which is seen through the numbers that they built individually.

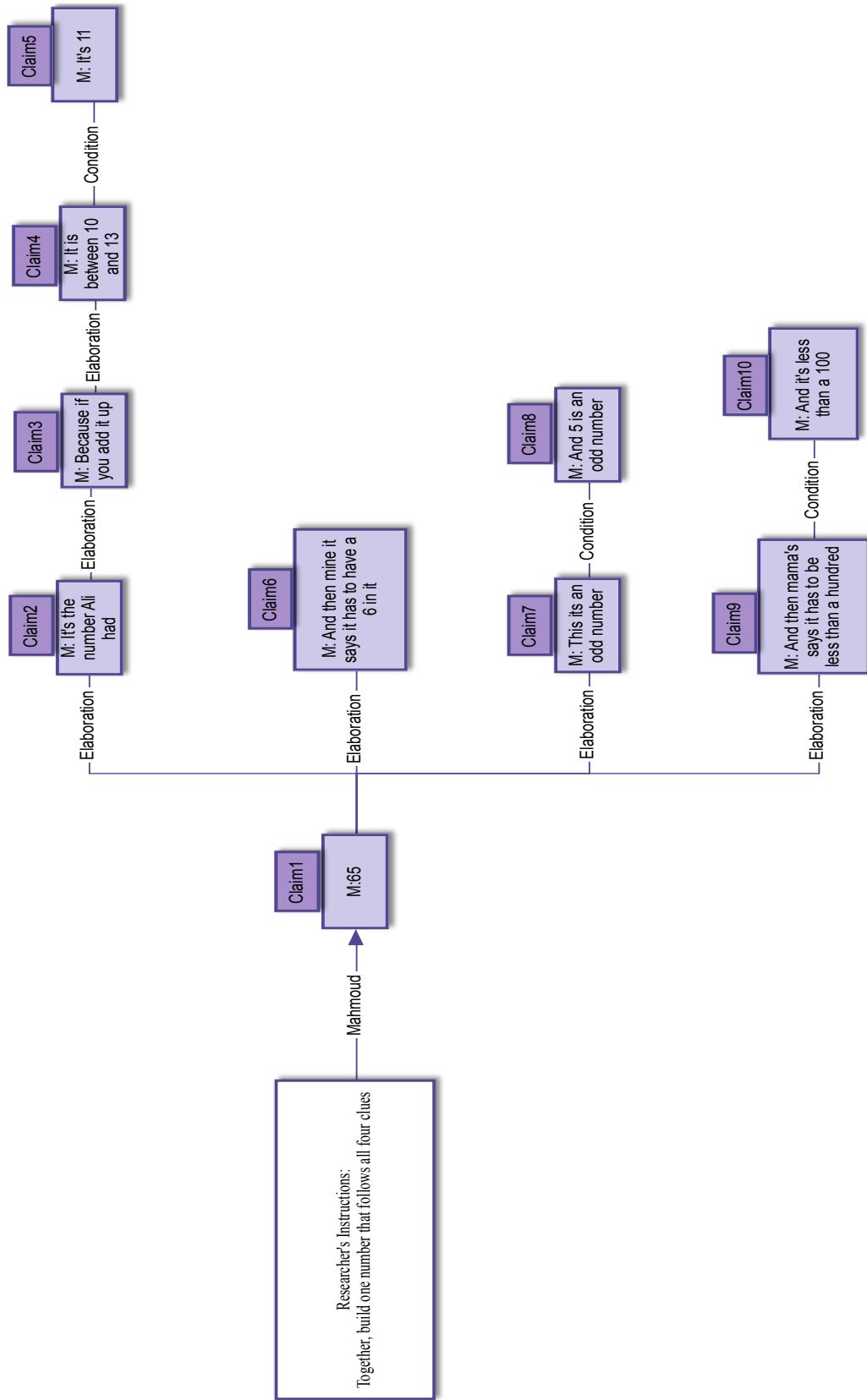


Figure 18. Mahmoud's Reasoning: Building one number that follows all four clues.

### *Language/ Code switching*

Yasmine explains that she usually starts off in English and then switches to Farsi when working on mathematics with her children. In this particular collaborative problem-solving activity, her use of Farsi in a sustained way coincides with the children's confusion, and I would suggest that it is the children's confusion which prompted Yasmine to switch in order to re-voice, refocus, shift, or to add additional clarity. This suggests that it may be the context of the confusion that prompts the mother to switch languages in order to facilitate learners' understanding (Setati, 1998). These actions on the part of Yasmine all constitute a pedagogical strategy.

For Tara, language switching was not used to compensate for language difficulty in the language of instruction, but her mother's switching influenced her to switch and use Farsi (Parvanehnezhad & Clarkson, 2008). Tara used Farsi to explain, describe, elaborate, and provide additional information (Moschkovich, 2005). For example, the following are a few exchanges that took place between Yasmine and the children, and in particular depict Tara's use of Farsi as they worked together to make sense of the clue *build a number with the sum of the digits between 10 and 13*.

<u>Name</u>	<u>Transcription</u>
Yasmine:	[Farsi] I have to make a number between 10 and 13
Arsheed:	/12, 12, 12/
Roya:	/12, 11, 11/
Tara:	[Farsi] No, but the digits have to add up to it

In this exchange, Yasmine uses Farsi to explain to the children what the quote is referring to by

stating, “I have to make a number between 10 and 13”. Arsheed and Roya respond by providing their answers, Arsheed says, “12, 12, 12”, while Roya says, “12, 11, 11”. However, Tara does not agree, and in Farsi she says that they need the digits to add up to either 11 or 12: “no, but the digits have to add up to it”. In another example, Yasmine in Farsi refers to the numbers 5 and 6 that were built by Roya and asks; “now this number, the sum of 5 and 6, is 11?” Tara in Farsi responds, “yes”. Tara then refers to the clues and explains in Farsi that the answer they have does not follow all clues: “It also has this one [Yasmine’s Clue: *build an odd number*], but it doesn’t have this one [Tara’s Clue: *build a number with one of the digits a 6*]”. In another exchange, Tara in Farsi asks for her mother’s opinion regarding the meaning of the clue, and she uses both English and Farsi to iterate her understanding of what Yasmine had explained.

<u>Name</u>	<u>Transcription</u>
Tara:	[Farsi] Are you going to tell us what you think?
Yasmine:	[Farsi] Yes, I just said what you are looking for is a number.
Tara:	Exactly, that's what we're trying to find out
Yasmine:	[Farsi] And the number you're looking for is a number that if you add them, the sum it would be between 10 and 13
Tara:	[Farsi] So it needs to have an 'answer' that the answer is 'between 10 and 13' [scare quotes indicates words stated in English]
Yasmine:	[Farsi] Yes, it should have an answer that the sum of the digits are between 10 and 13
Tara:	Yeah

In this exchange, Yasmine in Farsi responds to Tara’s question by stating, “yes, I just said what you are looking for is a number”. Tara says, “Exactly that’s what we’re trying to find out”.

Yasmine continues by explaining in Farsi that, “the number you’re looking for is a number that if you add them the sum it would be between 10 and 13”. Using both Farsi and English, Tara says, “so it needs to have an answer that the answer is between 10 and 13”. Yasmine in Farsi responds by reiterating Tara’s claim, “Yes, it should have an answer that the sum of the digits are between 10 and 13.” Tara says, “yeah”. In these illustrated examples, we see that Tara uses Farsi in response to her mother’s questions, or to explain, reiterate, elaborate, and describe a matter at hand to her mother. Tara does not use either Farsi or English to compensate for language difficulty, but as these data suggested, and also, provided additional information on the way language switching is influenced by one’s experiences and the context (Morgan, 2006; Moschkovich, 2005; Parvanehnezhad & Clarkson, 2008; Planas & Setati, 2009; Setati, Molefe, & Langa, 2008). In addition to the influence of one’s experiences and context on the language(s) used during a mathematical discussion, Setati (2005) refers to two mathematical Discourses that emerged in her study, *Procedural Discourse* “that focus on the procedural steps taken to solve a problem” (p. 449), and *Conceptual Discourses* where “learners articulate, share, discuss, reflect upon, and refine their understanding of the mathematics that is the focus of the interaction” (p.449). However, in the context of my study, Tara in both Farsi and English used procedural and conceptual discourses. It was not evident that she favoured one over the other depending on the language she used, but since she is a dominant speaker of both languages, she did not have a problem articulating her mathematical knowledge in either language as is evident in the video data. Again, in the context of this mathematical problem-solving activity Tara’s mother influenced her switching, which is something they are used to. That is, Yasmine and Tara have the tendency to use both English and Farsi during their mathematical discussions on a regular basis, and the presence of Farsi in this context influenced its use.

Not only was Tara influenced by her mother's language switching, Tara and her siblings were also influenced and contextually motivated by their mother's use of the pedagogical strategy. Once the parent introduced it, the children repeated it. For example, the following is an exchange led by the children that resemble the mother's use of the pedagogical strategy, which was initiated by Tara.

<u>Name</u>	<u>Transcriptions</u>
Tara:	Everyone is gonna take a turn saying what this means and we'll see which one actually makes sense. [Expanding the discussion/ Focusing]
Roya:	Can I have it first?
Tara:	Okay, or let's start with Arsheed [Guiding]
Roya:	Read the Question and then see if it makes-, [Guiding]
Tara:	Why don't you come closer so you can see what we're doing. [Guiding]
Arsheed:	Build a number with the sum of the
Tara:	/Digits/
Roya:	/Digits/
Arsheed:	Digits between 10 and 13
Roya:	Okay
Tara:	/Okay so what did you/... [Guiding]
Roya:	/Okay what do you think/ it means? [Guiding]
Arsheed:	12
Roya:	Do you know what sums means? Like two addends what they equal, it's called the sum. [Clarification]
Arsheed:	Okay, what's 5 plus 6? 11

Roya: Okay, that's what you think

Yasmine: [Farsi]—Uhah, now you

Roya: Build a number with the sum of the digits between 10 and 13, so I think this still makes sense.

In this dialogue, Tara expands the discussion, while also focusing it. She does so by asking everyone to say what he or she thinks Roya's clue means (*build a number with the sum of the digits between 10 and 13*), and asks that they start with Arsheed. Roya guides Arsheed by telling him to "read the question and then see if it makes sense". Tara and Roya again guide Arsheed by asking him to say what he thinks it means. This helps Arsheed to explain his understanding of the clue. Roya then asks Arsheed, "Do you know what sum means?" and then she explains what it means as a way of clarification, "Like two addends what they equal it's called the sum." Arsheed then says, "Okay, what's 5 plus 6? 11". His answer shows a clear understanding of the clue. Roya then reads the clue and says, "I think this still makes sense". In this exchange, the children in a very engaged manner try to connect their understanding of the clue as a way of initiating that they are all on the same page. Allowing the children to take responsibility of the mathematical problem-solving activity expanded and encouraged the children's mathematical reasoning (Weber et al., 2010).

### *Understanding Context*

An additional aspect of the analysis focused on the follow-up conversations and video viewing activity, which were analyzed for contextual information pertaining to the collaborative problem-solving activity. These provided additional information about the children and their perspectives with regards to the activity and the multiple languages available to them. They also provided information with regards to the parents' views towards the use of multiple languages, and their everyday mathematical experiences with their children.

One of the things that is important to consider is how fluent the children are in their home language, and how comfortable they are in English in both families. The parents have stated that they always speak in Arabic or Farsi in the context of their homes. We can see the powerful influences of both the school and the social context, which contribute to the fluency and comfort in both languages.

#### *Participants' attitudes and experiences*

In both contexts, the children found the activities to be really enjoyable, interesting, engaging, and they were keen to do an additional task, so I created one just for their pleasure, but I did not analyze it for this part of the data set, and that was true in both cases. When I asked the children if they would want to change anything about the activity, Zeinab in Abir's family suggested, "if the clues were a little bit harder", and one of the children said "more clues and make them a bit harder".

I furthered the discussion by asking the children if they like to work together when working on mathematics, or whether they would prefer to work by themselves. Arsheed, Tara, and Roya replied by stating that they like to work by themselves because it is easier for them, however, they would prefer working with someone on the questions that they find hard. For Abir's family, Mahmoud says, "I like studying with people because it helps me more like when I'm studying for a test. If I do it alone, I don't really understand a lot, but when I'm doing it with my friends they can question me and I can question them, so it's easier". For both families, the children felt that working together helpful, especially when there is confusion.

I then asked the children if they have ever done math in their first languages, Roya said, "No", Arsheed says, "Yeah, I know", and Tara says, "A few times, not that hard because I usually do like multiplication and stuff like that in Persian sometimes, but not a lot". Mahmoud, in Abir's family, said, "Sometimes I have. Yeah, when I'm with my friends sometimes we speak

in Arabic... like about math, like when I'm trying to teach them something, I might use Arabic because they can understand it better". This, I presume, again has to do with the fact that they are all in English mainstream classrooms, and the only Farsi or Arabic available to them would be in the context of their home, or friends who share the same language as in Mahmoud's case.

*How parents view the use of multiple languages?*

In order to understand the experience of that event from the parent's perspective on multiple language use and the children's reasoning. I asked the parents how they viewed the use of multiple languages. In response to this question, Abir states, "I think it's very beneficial and enriching for them...It would enrich their math faculty so when they grow up they would be able to understand math in both languages right...I mean math is math no matter what language you speak right, it's just the terminology that differs, so I think giving them that tool is very good and I think it's an enriching tool". Here, Abir acknowledges the importance of multiple language use when learning mathematics. However, Abir's response that "math is math no matter what language you speak", speaks to the universality of mathematics, where for example one would assume that there is only one correct way of performing algorithm. As it has been argued earlier, teachers have been exposed to a "fossilized" representation about the universality of mathematics (Abreu & Gorgorió, 2007); I suppose that parents have been as well.

Yasmine responds to the question as follows,

The conceptual understanding happens in one language, you just translate the concept to the other language. Like they don't know how big a meter is in Farsi if I ask them, but they have a conceptual understanding of a meter in English... So the conceptual understanding is only in Farsi or in English, then they translate it to the other language, that's what I found. So sometimes when we do something that I really want the concept to come and if they have the basis in English then we just continue in English for the deep understanding to come up... If there is a lesson at the school and they didn't quite get it, I would speak in English to just build on the stuff that was built at school. (Yasmine)

Yasmine believes that “the conceptual understanding happens only in one language, and you just translate the concept to the other language”. However, rather than simply referring to it as “translation”, the use of more than one language should be regarded as a tool for multilingual learners “that can provide access both to mathematical ideas and powerful ways of thinking and speaking” (Morgan, 2006, p. 241).

In order to illicit some information with regards to the children’s mathematical experiences outside their school contexts, I asked, “what are the children’s mathematical experiences, do you as a family talk about math on a regular basis, or when shopping or buying food?” In response, Yasmine says, “We did when they were younger. Now, they just do their allowances and like when they need to shop themselves then they calculate math and that usually is in English.”

When I asked Abir that same question, she provides an example of a situation with Ali (age 9). She states,

Ali for example, let’s say we go to St. Laurent, and I say you have a budget of \$15.00 to spend and that's gonna be spent on something that you buy and something that you eat, right. So we go to Toys R Us and then Ali goes, I found those two toys and one is 5.99, the other is 4.99. And then I go okay so 5.99 that's 6 dollars it's not 5 dollars, and 4.99 that's 5 dollars and not 4 dollars, and then you add 13% to that and there you go that's your \$15.00. Do you really want to spend all your money on two toys and not have an ice cream? Then he would go okay can I get something for 7.99, you know, so that's \$8.00 plus the 13% then I would have \$4.00 dollars left for an ice cream. We do that a lot because they always have a budget that they can spend. I always tell them you want to buy two toys, or you want to spend it all on food, or do you want to spend it on two things, and then they have to find out the best way to do it. (Abir)

These two examples bring in much more than mathematical calculation. They bring in other social norms embedded in their everyday living habits. In addition, the mothers use mathematics for practical and pragmatic reasons to help the children think through the problem at hand.

The follow-up conversations and video viewing activities elicited additional information pertaining to the participants’ perspectives towards the collaborative problem-solving activity

and the use of multiple languages. In essence, the children found the activities to be really enjoyable and offered suggestions on how they can be more challenging. They also considered collaborative work helpful especially when there is confusion. In addition, Farsi or Arabic are used in the context of their home or around friends who share similar languages. The parents expressed interest in the use of multiple languages in the learning of mathematics.

### *Conclusion*

From these data, we saw how the children expressed mathematics in a multilingual context. With respect to their languages, the children were very comfortable in English. With respect to their reasoning, the children showed themselves capable of building on their claims by providing evidence, drawing on their broader mathematical knowledge and world knowledge. At the same time, they are constantly listening, elaborating, and reiterating what each other say in a very positive way. With respect to the pedagogical strategy, it emerged to guide, clarify, and/or focus the discussion for the purpose of furthering understanding. As is evident by this data, the children's thinking is more complex than we might assume, and it is not at all that simple. This suggests that the children go through a lot before they articulate their thinking/reasoning. They are constantly listening to one another to make sense of their own and others reasoning, which is much more evident in a collaborative environment. Collaborative learning allows the children to explore what others think, and it allows them to work together to come to a consensus of a given matter. Despite the fact that children might disagree, or the children might have discrete interpretations or understandings, the discussion that these children are involved in allows them to share their views to develop an answer/ understanding that supports the problem at hand. These all exhibit the richness in the children's reasoning as they interact together.

This study was very specific partly because of the design. I was looking for two multilingual

families, as I wanted to allow a discursive space for the children's mathematical reasoning and the possibility of using multiple languages. Based on these two families, the children were very comfortable with the task. However, the dynamics of both families were quite distinct. It looks like in Abir's family; the task was perhaps too easy. However, it is not just the task that we need to consider, but also who it is and their role in the group/ family. In Abir's family, the older brother served as a spokesman who saw his role as checking if everyone had done their work correctly. This had the effect of shutting down discussion because the children responded to him as an authority figure, he validated their work and there was nothing to say. We also see it in their expressions; the fact that they do not veer off the task, they are very happy to engage in it, and to do an additional one.

This study is a snapshot of a wider picture, it would have been more interesting to have more conversations with the participants, to provide them with other tasks that might challenge their mathematical reasoning, and to maybe observe how they would collaborate in a classroom environment with their classmates and teacher. For future research, building on the findings, I would like to extend it to classroom interactions in multilingual settings, while also exploring the influence manipulatives might have on the children's mathematical reasoning. According to Suurtamm and Graves (2007) using mathematical thinking tools "provides opportunities to 'externalize' the many modes of mathematical reasoning including conjecture, explanation, evaluation, and argument" (p. 81). I wish to undertake a cross-cultural study with multilingual schools, also taking into account what has been mandated in these specific contexts.

### **Possible contributions of this study**

This research contributes important information to researchers and educators regarding the sophistication and complexity of children's mathematical reasoning. Drawing on sociocultural

theory allowed me to describe the richness in the children's reasoning and interactions by considering the multiple resources these multilingual children used to communicate mathematically (Moschkovich, 2007). For instance, in Yasmine's family the use of either Farsi or English was influenced by the context of the mathematics lesson (Moschkovich, 2005; Planas & Setati, 2009). Tara switched languages as she communicated with her mother not to compensate for language difficulty, but rather to explain, describe, elaborate, and provide additional information (Morgan, 2006; Moschkovich, 2005). In addition, the uses of both Farsi and English in my study were not isolated by means of *procedural Discourse* or *conceptual Discourse*, as I am aware of this distinction in the context of Setati's (2005) study. Rather, Tara was able to switch back and forth between languages; it was not a matter of doing mathematics in English and then switching to Farsi for personal matters. Tara used both languages in accordance to the context of the situation.

Furthermore, when children are exposed to the multiple resources available to them, they can illustrate/ represent their ideas in a variety of ways. For instance, the use of the numbered tiles allowed Tara, Mahmoud, and Zeinab to generate vast numbers just by using the full set, while also demonstrating their mathematical knowledge and general world knowledge. Moreover, when the children considered multiple interpretations in the discussion held between Tara and Roya with respect to the addends and the sum of the addends. This did not lead to obstacles in the mathematical discussion, but made us aware that when the children get confused, we should expect that they are not looking at, talking about, referring to, or imagining the same things (Moschkovich, 2008). In addition, when children are provided with an opportunity to engage in open-ended and well-defined collaborative mathematical problem-solving activities, it encourages students not only to communicate their answers, but listen, evaluate, elaborate, and

reiterate the statements made by others (Graves & Zack, 1997; Weber et al., 2010). In this latter case, being able to listen to others explanation and to provide an appropriate response are important aspects in the development of the children's mathematical understanding.

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*Appendix 1: Transcription conventions*

(The glosses given below are mostly taken from ten Have P. *Doing Conversational Analysis: a Practical Guide*. London, Thousand Oaks, New Delhi: SAGE Publications, 1999. p. 213 - 214)

SYMBOL	MEANING
/ /	<i>Slashes</i> indicate overlapping speech (spoken at the same time). They indicate the beginning point of overlap as well as the point at which an utterance terminates another utterance. The following shows two speakers whose talk overlaps.  Ruby: I thought you wanted to know /what that is./  Mary: /Which are?/
=	<i>Equal signs</i> , indicate 'latching,' that is, two utterances that follow one another without any perceptible pause. One is put at the end of one line, and one at the beginning of another line to indicate that there is no "gap" between the two lines.
(7.1)	<i>Number in parentheses</i> , indicates silence or pause (of seven seconds and one tenth of a second)
[ ]	Square brackets contain transcriber's descriptions additional to transcription
::	<i>Colons</i> , indicate prolongation of sound
-,	<i>A dash with a comma</i> , indicates a cut-off or a false start
.	<i>A period</i> , indicates a stopping fall in tone
,	<i>A comma</i> , indicates a continuing intonation, like when you are enumerating things
?	<i>A question mark</i> , indicates a rising intonation
(( ))	<i>Empty double parentheses</i> , used if a single word or short phrase is completely unintelligible.
((text))	<i>Double parentheses</i> , contains transcriber's best attempt at transcribing a difficult passage

*Appendix 2: The Clues*

**Build a number with the sum of  
the digits between 10 and 13**

**Build an odd number**

**Build a number less than 100**

**Build a number with one of the  
digits a 6**

*Appendix 3: The tiles*

*Appendix 4: Instructions for activity*

*Follow the Clues with Tiles*

**Instructions:**

The activity is to be done in groups of 4.

Each participant will be provided with a set of tiles to use as tools for constructing numbers, the tiles are square inch tiles numbered 0 through 9.

Each task has 4 clues; one clue should be distributed to each person in the group.

The participants are asked to only read their own clue and build a number which fits the clue.

When all the participants have their numbers ready, they are asked to share with their group why they built that particular number.

When all 4 participants have shared their numbers and agree that the individual numbers fit the clues, they have a job of building another number together which fits all four clues.

Upon completion of solving the problem each person should turn over his/her clue and take turns reading the clues a loud to verify that the fifth number meets all four clues.

**The Activity:**

Read your clue

Build your number

Share with your group why you built that particular number

As a team build a fifth number fitting all the clues

An odd number

A number less than 100

A number with one of the digits a 6

A number with the sum of the digits between 10 and 13

*Appendix 5: Parents sample interview questions*

Get some background information about the students: Are they immigrants, first generation, or second generation. (Moschkovich, 2012)

What are their experiences with each language - at home and at school? (Moschkovich, 2012)

What are their experiences with mathematics in each language-at home and at school?

Have students participated in mathematics classes in their first language or not? (Moschkovich, 2012)

What are your past experiences with mathematics instruction in each language? (Moschkovich, 2005)

Is language viewed as an obstacle for mathematics learning?

Do you refer to your L1 when you feel that you need to clarify meaning for your children when doing mathematics?

Do you feel it is important to use all the languages that you and your child know when doing mathematics?

Do you fear that using your L1 will cause hindrance on your child's learning of mathematics and second language learning?

What mathematical experiences do the students get from outside the school (selling, buying, games, etc.)? (Moschkovich, 2012)

Do you think mathematics is an important subject?

Do you feel that it is important that your children do well in mathematics? Why?

Are you familiar with the way mathematics is taught at your children's school?

Do you think the way it is taught today is effective? Why?

What do you think about your children's mathematics teachers? What do you think about their mathematical knowledge and skills?

How did you feel about the activity?

Do you believe that discussion is key for learning mathematics?

*Appendix 6: Children's sample interview questions*

What did you think about the activity?

Is that similar to what you do in school?

Do you like to work with others when doing mathematics? Why?

Do you feel that you can learn from others when learning mathematics?

Do you like mathematics? Why?

How do you perceive yourself in mathematics? (Great, Good, Not bad)? Why? Who do you blame or give thanks to?

Do you think mathematics is an important subject?

Can you relate to your own experiences when doing mathematics?

Do you identify yourself as a bilingual? (Moschkovich, 2010)

How many languages do you use when you are doing mathematics?

Which language are you more familiar and comfortable using?

What are your experiences with each language?

Why do you use your L1 when working on mathematics?

Do you have a favourite mathematics teacher, or preferred how one or several of your teachers taught mathematics?

If you had a chance to change something about mathematics, what would it be?

*Appendix 7: Language/ Code switching factors*

Parvanehnezhad & Clarkson (2008)

- Difficulty with comprehension
- Difficulty with interpretation
- Familiarity with some numbers / words that were habitually used in their L1
- Being in an L1 context—(Persian school)
- Interview environment
- Physical environment
- The students' experiences (previous schooling, parents teaching, tutoring etc.)
- \*There competency with the languages they know High/High = high mathematical skills

Moschkovich (2005)

- To compensate for missing vocabulary
- To explain a concept
- Justify an answer
- Describe mathematical situations
- Elaborate
- Expand
- Provide additional information

Setati (1998): Teacher

- To facilitate learners understanding
- To encourage participation
- To familiarise learners with language of evaluation

Planas & Setati (2009)

- To become familiar with the task
- To become familiar with the new vocabulary
- For clarification
- The context of the mathematics lesson
- The way in which the lesson is structured