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II

A SQUARE - LAW MULTIPLIER

BY

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ABSTRACT

A review of analog multipliers is given, and a description with the experimental work done on a square-law multiplier has been included in this thesis. The described square-law multiplier can be used with the system that utilizes the correlation principle in measuring noise from electrical elements (e.g. p-n junctions). The multiplier has an accuracy of the order of 1%, a bandwidth of the order of 1 Mc, a drift in the vicinity of 10 mv in eight hours, and an input range of about one volt peak to peak.

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INTRODUCTION

This work is a continuation of the work of W. Samaroo (Samaroo, 1961) in which measurements of current noise in p-n junctions at low temperatures have been made. The sensitivity of the equipment, at that time, was not satisfactory and it has been decided to try a different approach to noise measurements. The chosen method makes use of the fact that two uncorrelated noise signals give a product zero on the average. This fact makes it possible, in principle, to eliminate the unwanted background noise of the amplifiers used in the measurements.

To obtain the product of two signals, a multiplier is required. A square law multiplier that accepts only a fixed polarity input i.e. one-quadrant operation was chosen and an experimental work was carried out in order that the multiplier can accept signals of arbitrary polarity i.e. four-quadrant operation.

Analog multipliers were reviewed also to check if other multipliers can be used with the chosen method of noise measurement. A classification of multipliers is given to enable the person to choose the best fitted multiplier for a specific application.

SECTION 1

METHODS OF NOISE MEASUREMENTS

There seem to be four distinct methods of noise measurement. The most direct method is that of amplifying the noise to a level at which it can be read on an indicator such as CRO, thermo-couple meter, etc. In the second method the unknown noise, after being amplified, is directly compared with a reference voltage (preferably noise) by alternately connecting the unknown and the standard noise sources to the input of the amplifier through a switching device. The use of a switching device in this method is one of its drawbacks. This is because of the variation in the contact potential of the switch (Garrison & Lawson, 1949) and instabilities and losses of the switch (Allred, 1962). The third method of noise measurement is to double the amplified unknown noise voltage by adding a known magnitude of some other waveform, usually noise (Bell, 1960; Freeman, 1958; van der Ziel, 1954). The fourth method can be stated as follows: If the sum and difference of an unknown and of a known variable noise source are separately amplified, multiplied together, and integrated, then under null conditions at the output the magnitude of the reference signal gives a direct measure of the investigated noise. This method, in principle, eliminates the uncorrelated background noise of the amplifiers and the need of a switching device at the input (Fink, 1959; Allred, 1962).

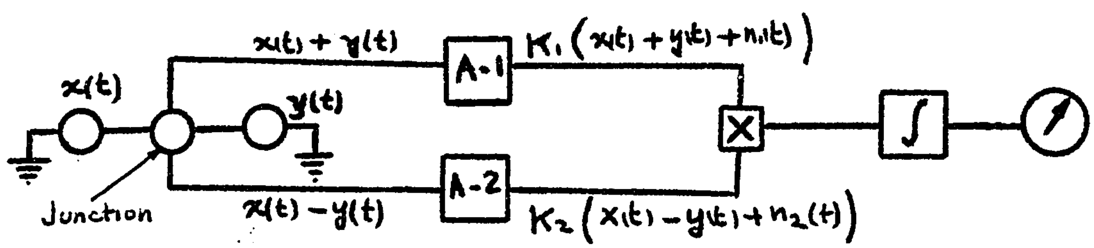


Figure 1 - Noise measurement system - 4th method

Analysis of the Fourth Method.

Figure 1 shows a block diagram of the system used in the last method used in noise measurement. The analysis of such system is based on the assumption that all components are ideal and the quantities involved have a guassian probability distribution with a zero mean. The function of the junction shown in figure 1 is to give the sum and the difference of the unknown noise and the known noise, that are applied to the input of amplifier 1 and amplifier 2, respectively. The output of amplifier 1 is

$$A_1 = K_1\left[x(t) + y(t) + n_1(t)\right]$$

where $n_1(t)$ is the background noise of the amplifier referred to its input, K_1 is the amplifier gain. The output of the amplifier 2 is

$$A_2 = K_2\left[x(t) - y(t) + n_2(t)\right]$$

where $n_2(t)$ is the background noise of the amplifier 2 referred to its input, K_2 is the amplifier gain. The output of the multiplier is

$$A_1A_2 = K_1K_2K_M\left[x(t)+y(t)+n_1(t)\right]\left[x(t)-y(t)+n_2(t)\right] \qquad (1\text{-a})$$

where K_M is the multiplication constant. The output of the integrator is then

$$\overline{A_1A_2} = K_1K_2K_M\left[\overline{x(t)^2} + \overline{x(t)n_1(t)} - \overline{y(t)^2} - \overline{y(t)n_1(t)} + \overline{x(t)n_2(t)} + \overline{n_1(t)n_2(t)} + \overline{y(t)n_2(t)}\right] \qquad (2)$$

If the quantities $x(t)$, $y(t)$, $n_1(t)$ and $n_2(t)$ are independent of each other, then equation 2 becomes

$$\overline{A_1A_2} = K_1K_2K_M \quad (\overline{x(t)^2} - \overline{y(t)^2}) \qquad (3\text{-a})$$

This is true since the covariance of two independent (uncorrelated) variables is zero (e.g. Davenport and Root, 1958). For the null conditions at the output, the following relationship must be satisfied

$$\overline{x(t)^2} = \overline{y(t)^2} \qquad (4\text{-a})$$

Since $\overline{y(t)^2}$ is known then $\overline{x(t)^2}$ is determined.

A most important part of the system, described above, is the multiplier. This is because the multiplication operation, when followed by integration, will eliminate the uncorrelated noise. Moreover, the accuracy of noise measurement depends essentially upon the accuracy of the multiplier. The error in noise measurement, for instance, is 2% when a multiplier such as that described in Section 3 is used assuming the unbalance due to the rate of change of transconductance, C, is 1% and if the amplifier's background noise is zero (Appendix 1).

To be used in the fourth method of noise measurement , the multiplier should have the following properties:

1- In order to obtain the high sensitivity required in noise measurement at low temperatures, the multiplier should have a broad bandwidth since the mean-square value of noise voltages is proportional to the bandwidth in which it is measured.

2- The multiplier should accept inputs with either positive or negative polarity and give the product with its correct sign i.e. it should be of the four-quadrant type.

SECTION 2

REVIEW OF ANALOG MULTIPLIERS

It is noted that a multiplier is required in the system described in Section 1 (figure 1). It is stated, also, that the multiplier should possess special properties -- broad bandwidth and four-quadrant operation. A review of analog multipliers is required to determine what multiplier fulfills the required properties in order that it can be used with noise measuring equipment.

Servo Multiplier

The simplest form of a servo multiplier consists of a motor driving the sliders (arms) of two identical potentiometers (figure 2).

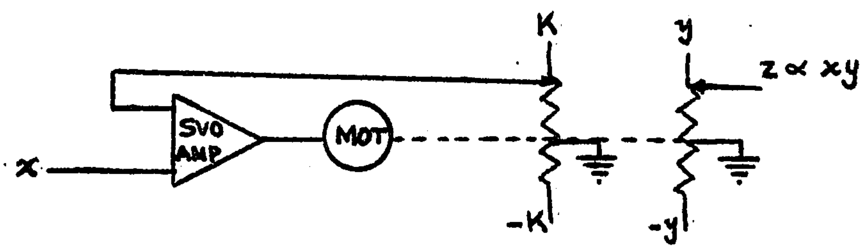


Figure 2 - Servo multiplier

One of the inputs x is compared with a voltage from the feedback potentiometer (follow-up) and the difference in voltage drives the motor in such a direction as to make the difference tend to zero. This input x is then used to determine the servo shaft position on the two potentiometers. The second input y is applied across the multiplier potentiometer. The signal at the output of the multiplier potentiometer is proportional to the product of the two inputs. This can be shown by recognizing that the arms of the two potentiometers rotate through the same angle θ and then the following relation holds

$$\theta = \frac{x}{K} = \frac{z}{y} \tag{5}$$

The accuracy of such multiplier can be made to approach 0.01% by using identical potentiometers with 0.01% linearity. The bandwidth of this type is limited by the use of electro-mechanical parts. Its accuracy depends on the gain of the servo amplifier and the similarity of the potentiometers used. It is a four-quadrant type, simple, reliable, and can be adjusted very easily. Because of the low frequency response, it is not used for applications where high frequency response is required.

Time-Sharing Multiplier

It is hard to obtain two identical potentiometers for the servo multiplier. Thus instead of using two potentiometers, a single potentiometer can be switched back and forth between the K and the y signals (figure 2). The switching is performed by commutator-type switches or by synchronous vibrator. The potentiometer output for each switch position is applied to an RC filter. The RC filters are used to keep the voltages between successive samplings constant and to filter out the high-frequency ripple due to switching. The bandwidth and the accuracy of such multiplier is similar to that of the servo multiplier which uses two potentiometers.

Step Multiplier

The operation of step multiplier, in principle, is similar to that of the servo multiplier described above. Figure 3 shows a block diagram of the multiplier. The upper part is similar to the follow-up system in the servo multiplier, while the lower part is similar to the multiplier potentiometer of the servo multiplier. The variable conductance G will be made proportional to the input signal x by the use of the reversible binary counter through relays. The counter itself is controlled by a gate circuit. When the output of amplifier 1 is zero (this will occur when x is proportional to G), the total current at the input of amplifier 1 is zero. In other words,

$$G = (x+K)g/K \tag{6}$$

where x is one of the input signals, K is a constant voltage, g is the input conductance, and G is the variable input conductance. The output of amplifier 3, when equation 6 is achieved, is

$$Z = -y(G-g)/g$$
$$= -y\left[\frac{(x+K)g}{K}-g\right] = -yx/K \tag{7}$$

Thus, the final output is proportional to the product of the input signals. A detailed description of such a multiplier is given by Goldberg (Goldberg, 1951). A discussion and analysis of errors of this multiplier can be found in Maslov's paper (Maslov, 1960). The multiplier has a high accuracy and low frequency response. The frequency response can be improved by using high speed switching devices. The frequency response of such multiplier is limited by the speed of the relays used and the repitition rate of the pulse generator. The accuracy is limited by the contact resistance of the relays. The step multiplier is accurate and has a narrow bandwidth, but it is not available commercially.

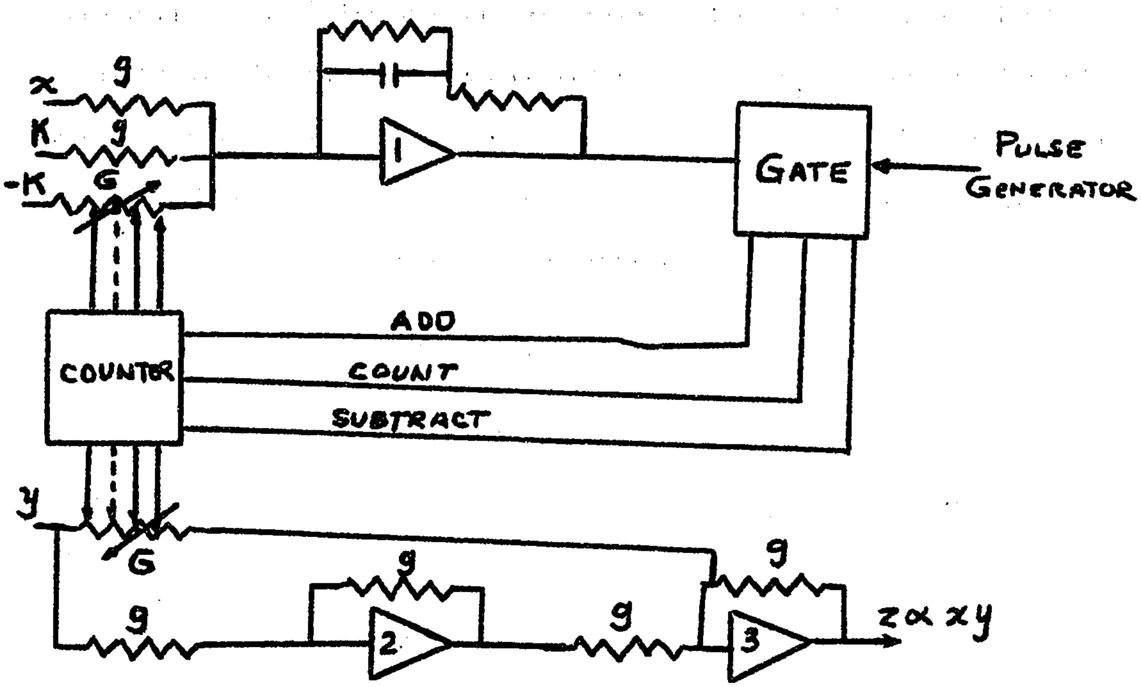


Figure 3 - Block diagram of a step multiplier

Strain-Gage Multiplier

The principle of operation of this multiplier is similar to that of the servo type. The multiplier consists of four strain gages arranged with ordinary resistors to form two bridges (figure 4). Bridge 1 functions as the follow-up potentiometer while Bridge 2 corresponds to the multiplier potentiometer of a servo multiplier. The center points of the gages are attached to a movable armature that can be displaced from the position of equilibrium by a moving coil placed in a strong magnetic field. When the armature has moved, the gages are equally displaced and the output is proportional to the product of the two inputs. The bandwidth of the multiplier is limited because of the mechanical link in its feedback loop. Since it is difficult to find a set of strain gages with identical characteristics, the difference in their characteristics limits the accuracy of the multiplier. The accuracy of such multiplier is about 2% for a band-width of 300 cps (Czajkowski, 1956).

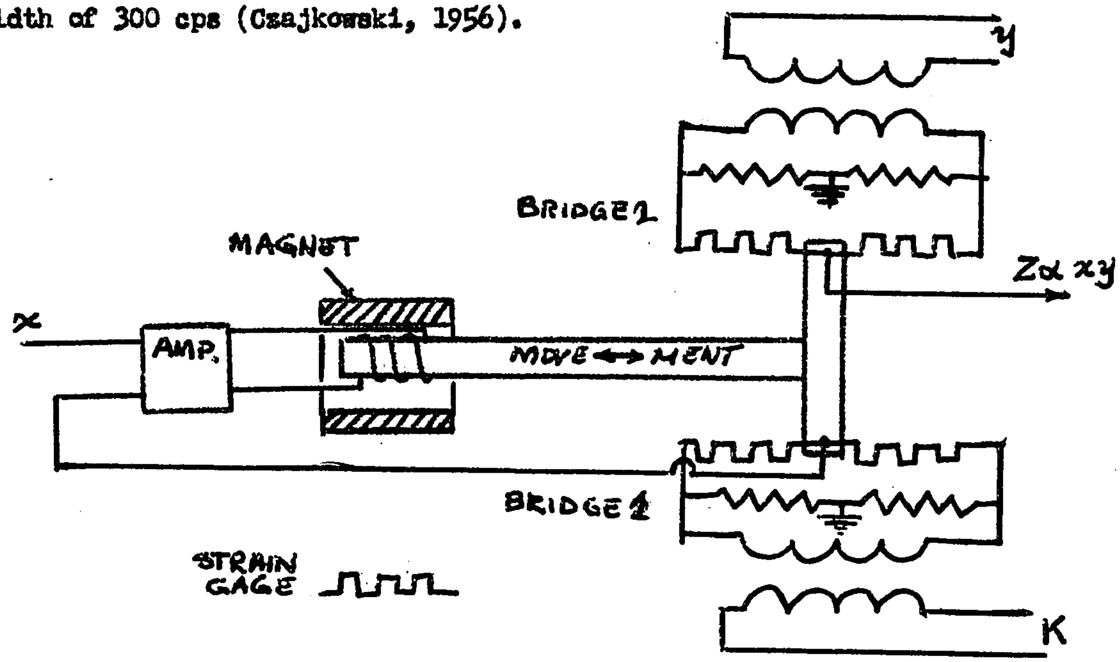


Figure 4 - Strain gage multiplier

Thermistor Multiplier

A thermistor or thermally sensitive resistor is a nonlinear element that changes its resistance with the change of temperature. The change of temperature can be accomplished either by passing a current through it or changing the outside temperature.

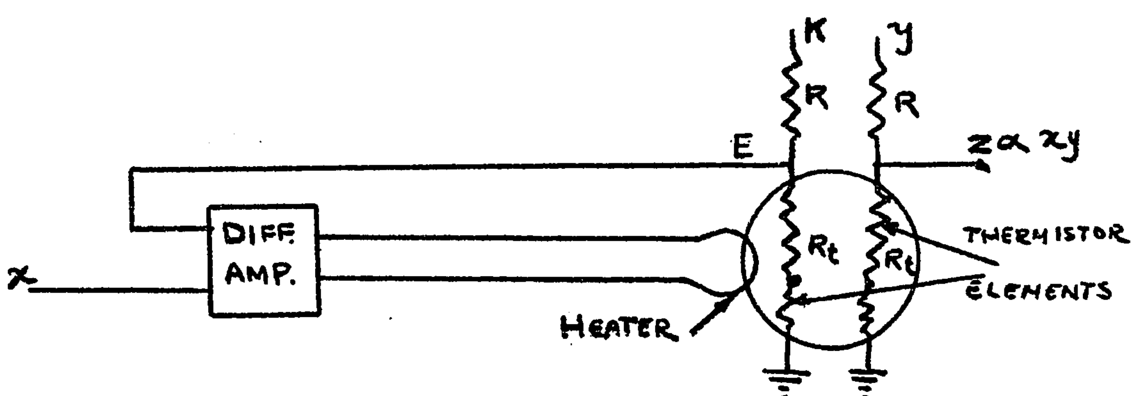


Figure 5 - Thermistor multiplier

The principle of operation of this multiplier is similar to that of the servo multiplier (figure 5). At the output voltage of the thermistor is

$$V = \frac{KR_t}{R+R_t} \tag{8}$$

and this is approximated to

$$V \simeq \frac{KR_t}{R} \quad \text{if} \quad R \gg R_t \tag{9}$$

Where R_t is the thermistor resistance, R is the series resistance connected to the thermistor, V is the output voltage of the thermistor, and K is the constant voltage applied. The thermistor resistance R_t has the following relationship with the controlling voltage E

$$R_t = Ae^{-CE} \tag{10}$$

where A and C are constants. The controlling voltage in this particular case is

$$E = g(x - \frac{KRt}{R}) \tag{11}$$

where g is the gain of the difference amplifier and x is one of the input signals. Inserting equation 11 into 10, one obtains

$$R_t = A\ e^{-cg(x-KRt/R)} \tag{12}$$

Equation 12 can be approximated to

$$R_t = A\left[1 - cg(x-KRt/R)\right] \tag{13}$$

assuming the power of the exponential is small. Dividing both sides of equation 13 by g and neglecting terms which are proportional to $\frac{1}{g}$ (since $g \gg 1$), equation 13 becomes

$$AC(x - KRt/R) = 0 \qquad (14)$$

In other words

$$x = KRt/R \qquad (15)$$

The output voltage Z is

$$Z = \frac{Rt}{R+Rt}\, y \simeq \frac{Rt}{R}\, y \qquad (16)$$

assuming $R \gg Rt$. Inserting the value of Rt, found from equation 15, in equation 16 the output Z will be

$$Z = \frac{x\,R}{K} \cdot \frac{y}{R} = \frac{xy}{K} \qquad (17)$$

and this is proportional to the product of the input signals. The accuracy of such multiplier is limited by the difference in the characteristics of the thermistor. The bandwidth is limited by the thermistor transfer-heat lag. A reported multiplier has the accuracy of about 2% for a bandwidth of 1.5 cps (Czajkowski, 1956).

Controlled Nonlinear Resistance Multiplier

The basic element of this multiplier is a controlled non-linear semiconductor resistance (CNSR), such as black silicon carbide, with three pairs of electrodes located in three mutually-perpendicular planes. The CNSR has two pairs of controlled electrodes a and a', b and b' and one pair of controlling electrodes m and m'. The conductances between the controlled electrodes Ga, Gb are proportional i.e.

$$Ga/Gb = Rb/Ra = \alpha = \text{constant} \qquad (18)$$

Figure 6 shows a block diagram of a multiplier with CNSR as the basic element and utilizing equation 18. One of the input signals x is applied to one of the controlled pair of electrodes and a negative constant voltage -K is applied to the other pair of the controlled electrodes, while a-c voltage M is applied to the controlling electrodes. At the inputs of the high gain amplifiers A-1 & A-2 one obtains the following relations, respectively:

$$\frac{K}{Ro} + \frac{y}{R1} - \frac{K}{Rb} \simeq 0 \tag{19}$$

$$\frac{x}{Ra} - \frac{x}{Rz} - \frac{z}{R3} \simeq 0 \tag{20}$$

the output $Z = \frac{k\,xy}{K}$ (21)

where $k = \propto R3/R1$ and $\propto = Ro/R2$

The a-c controlling voltage M is used to filter out the current noise caused by the non ideal characteristics of the CNSR element. The frequency of the a-c controlling voltage should be high as compared to the frequency of the input signal. The frequency of the controlling voltage (which is limited by the capacitance of the CNSR) determines the bandwidth of the multiplier. For 500 cps as the controlling voltage frequency, the accuracy of the multiplier is of the order of .5% (Kudryavtsev and Lipman, 1962). This multiplier is simple and inexpensive. Temperature variations do not affect its performance.

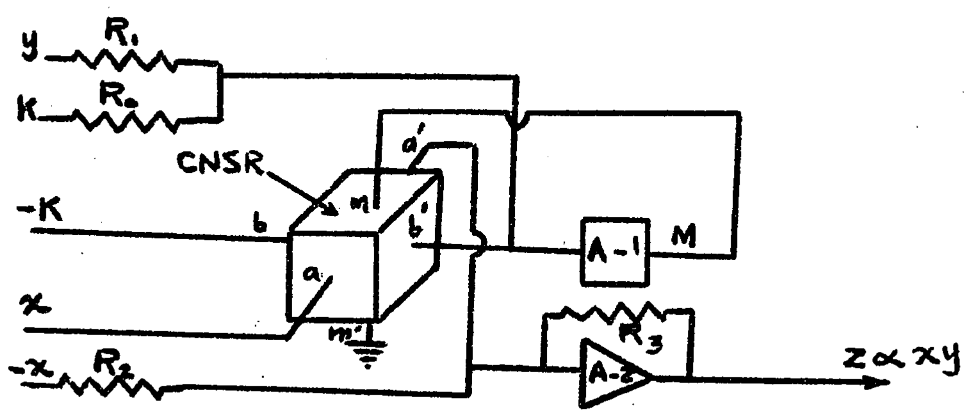


Figure 6 - Block diagram of CNSR multiplier

The accuracy of the multiplier is limited by the differences in the characteristics of two perpendicular pairs of the controlled electrods of the CNSR.

Heated Metal Film Resistor Multiplier

The resistance of some of the metals, in form of wires e.g. platinum wire, is a linear function of the voltage across it in the temperature range of 50 - 200°C. A heated metal film resistor has been used as an arm in a bridge to give the multiplication of the two input variables across its diagonal (figure 7).

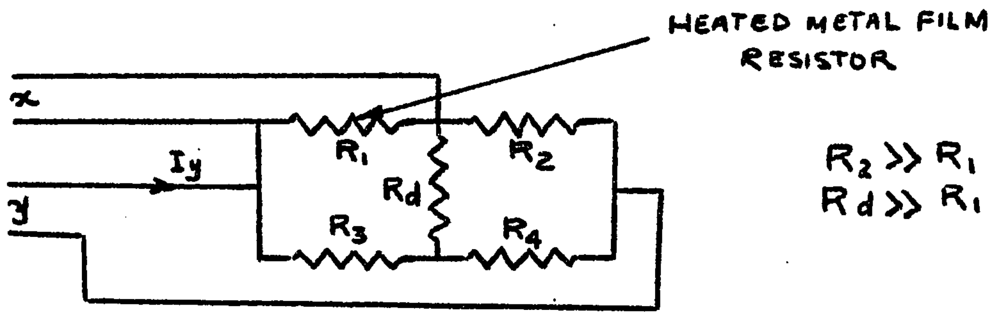


Figure 7 - Multiplier with a heated metal resistor

This dependence can be expressed as follows--

$$R_1 = k + k_1 x \tag{22}$$

where R_1 is the resistance of a metal wire, x is the input voltage, and k and k_1 are constants. The output voltage across the diagonal resistance Rd will be proportional to the product of the two inputs assuming that the current Iy applied to the bridge is small enough not to heat and thus vary the resistance R_1 and the voltage x lies within the limits of the linear portion of the volt-ohm characteristic of the heated resistor. The heated resistor used is a platinum wire and the reported accuracy of the multiplier is of the order of .5% while the temperature error i.e. nonlinearity of the resistance with temperature, does not exceed .2%. The variation of the frequency of the two input voltages from 20 to 200 kc/s does not affect the performance of the multiplier (Zotov and Popov, 1962). The input x does not start from zero thus a biasing voltage is necessary. The accuracy of the multiplier is limited by the nonlinear relationship between the resistance of the metal and one of the variables. The bandwidth is limited by the capacitances associated with the bridge and the heat-transfer lag of the heated metal film resistor.

Photoresistor Multiplier

The operation of the photoresistor multiplier is similar to that of a servo multiplier. The function of the photo sensitive resistors is the same as that of the follow-up potentiometer in the servo type multiplier. Figure 8 shows a diagram of such a multiplier.

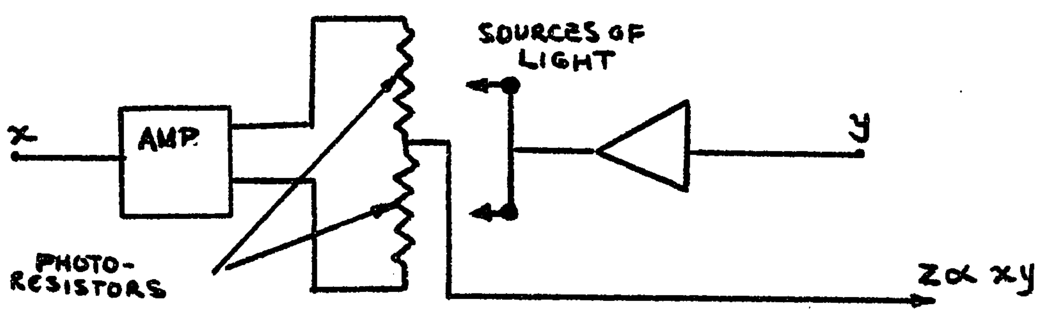


Figure 8 - Principle of photoresistor multiplier

The variable y controls the value of the resistance of the photoresistors through optical coupling between the photoresistors and the pair of modulator glow-tubes, while the other variable x is applied across the photoresistors. The output z is thus equal to the product of the two variables. This is true if the resistance of the photoresistor varies linearly with the variable y.

The bandwidth of such multiplier is restricted by the time lag with which the resistance follows the variations in light intensity. While accuracy is limited due to the dependence of the resistance of the photoresistor not only on the light intensity, but also, to a very slight extent, on the voltage across it. A multiplier using the above principle is reported to have 0.1% accuracy and a frequency range from 0 to 100 cps (Rademakers and Verhagen, 1959).

Amplitude-Separation Multiplier

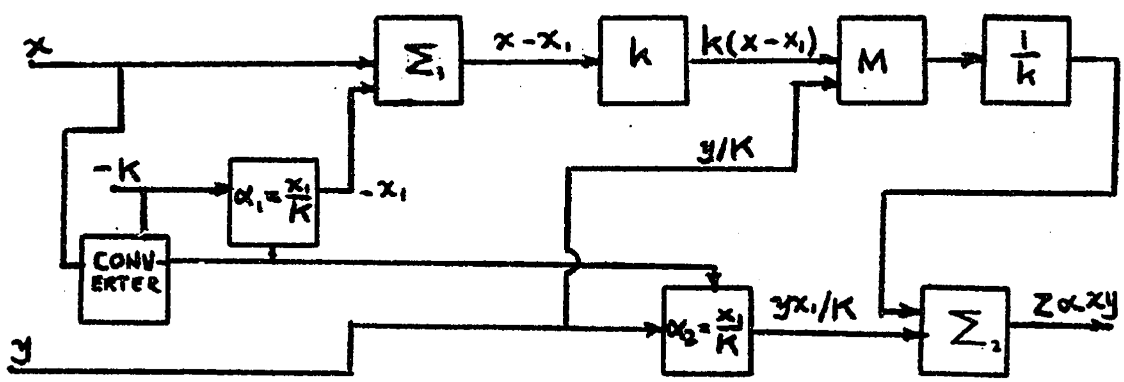


Figure 9 - Block diagram of amplitude-separation multiplier

In the amplitude-separation multiplier, one variable x is divided into two parts, x and $(x-x_1)$, by converting x into a transfer coefficient for the elements α_1 and α_2; this is done in such a way that $\alpha_1 = \alpha_2 = \frac{x_1}{K}$ (figure 9). The conversion is achieved by using servo systems with step variations of the transfer coefficients. The quantity $(x-x_1)$ is amplified by a factor k and then multiplied by $\frac{Y}{K}$ in the multiplying device M. The result is divided by k and entered as one input to the summing amplifier-2. The other variable y is multiplied by α_2 and then applied to the summing amplifier-2. The output of the summing amplifier-2 is proportional to the product of the two input variables.

The static accuracy of such multiplier is reduced because the error in the output is k times smaller than the error of the multiplier M. The accuracy is of the order of .02% and the bandwidth is of the order of 100 cps. (Maslov, 1960; Fistner, 1959). The accuracy and the bandwidth are determined by the accuracy of the component multipliers.

Frequency-Separation Multiplier

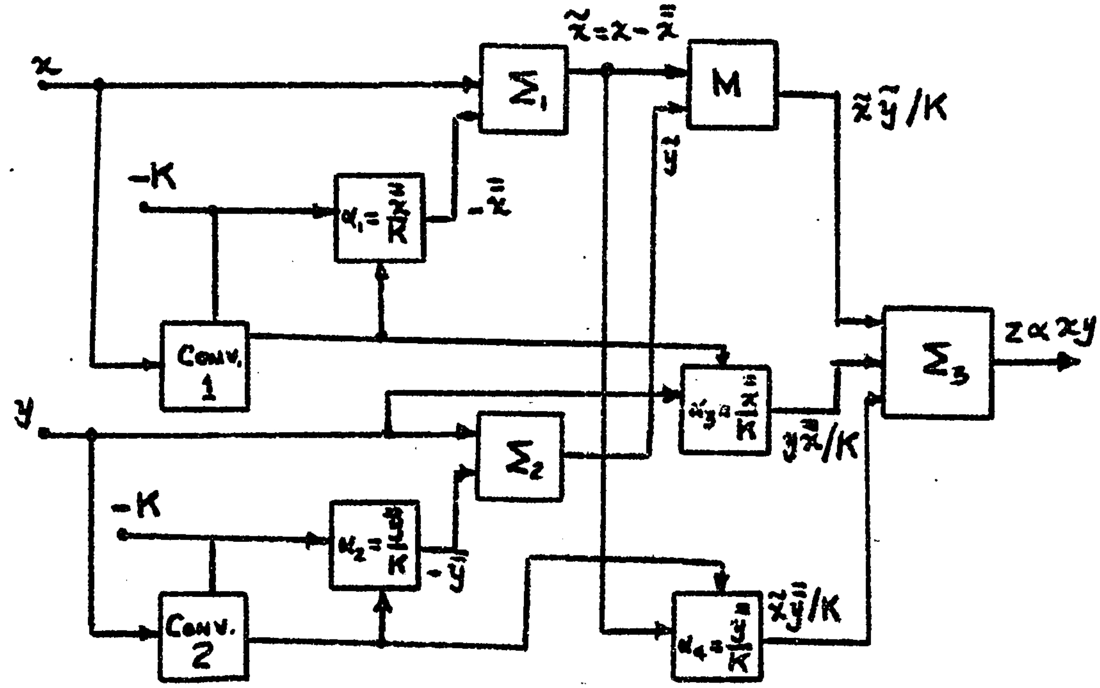


Figure 10 - A block diagram of frequency-separation multiplier

In this multiplier, the input variables x and y are divided into two components: a low frequency and a high frequency (i.e. $x = \bar{\bar{x}}+\tilde{x}$; $y = \bar{\bar{y}}+\tilde{y}$). Under these conditions the product at the output of the multiplier can be written as

$$Z = (\overline{\overline{xy}} + \bar{\bar{x}}\tilde{y} + \tilde{x}\bar{\bar{y}} + \tilde{x}\tilde{y})/K$$

$$= (\bar{\bar{x}}y + \tilde{x}\bar{\bar{y}} + \tilde{x}\tilde{y})/K \qquad (23)$$

The first two terms, in the above expression vary slowly and thus networks with controlled transfer coefficient can be used to produce them. In figure 10, converters (Conv. 1 and Conv. 2) respond only to low-frequency components of a signal while the high-frequency components represent the error of the servo system. The elements α_1, α_2, α_3, and α_4 are similar to those used in 'Amplitude-Separation' type. To obtain the third term $\tilde{x}\tilde{y}$, a multiplier M with a bandwidth that handles such high frequency signals (e.g. thyrite multiplier). This multiplier has a static accuracy of the order of .02% and a bandwidth of the order of 10Kc/s (Maslov, 1960). The accuracy and bandwidth of this multiplier are limited by the accuracy of the component multipliers.

Controlled Superconductor Multiplier

In this multiplier the multiplication of two variables is achieved by using controlled superconductors (Chirlian and Marsocci, 1962). A superconductor is "any material which is capable of exhibiting superconductivity; and superconductivity is a property of a material which is characterized by zero electrical resistivity and, ideally, zero permeability". Figure 11 shows a circuit that can be used for the multiplication of two variables.

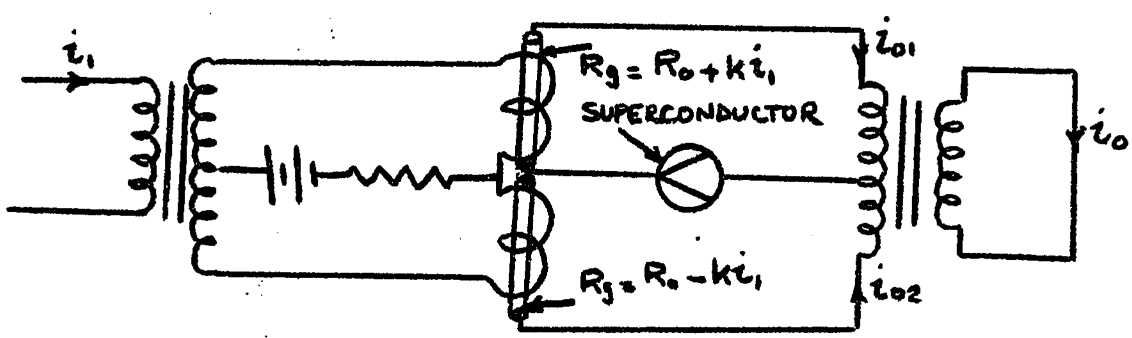


Figure 11 - Circuit for multiplication of two variables using superconductors

In the circuit, it is assumed that the resistance of the gate, Rg, is a linear function of the control current, i. This is achieved by a proper shaping of the magnetic field of the control element in the region of the gate[1].

Analyzing the circuit in figure 11, one obtains:

$$i_{o1} = i_2 (Ro + ki_1)/2Ro \quad (24\text{-a})$$

$$i_{o2} = i_2 (Ro - ki_1)/2Ro \quad (24\text{-b})$$

where i_2 is the current flowing into the superconductor, i_1 is the input current, and Ro and k are constants. The output current is

$$i_o = Nki_1i_2 /Ro \quad (25)$$

where N is the turns ratio of the output transformer.

No information is given concerning specifications of the multiplier. The advantages in using superconductors are the great reduction in size and weight, and the increased reliability of the system (Chirlian and Marsocci, 1962). The disadvantages of superconductors are the need of liquid Helium environment for their operation and current signals as their inputs. Accuracy of this multiplier is limited by the nonlinear relationship between the controlling current and the resistance of the gate. Bandwidth is limited by the inductance of the magnetic coil.

1- Gate is defined as an output element of a superconductive device in which current in one or more input circuits magnetically control the superconducting-to-normal transition in one or more output circuits, provided the current in each output circuit is less than its critical value. For the IRE Definitions of Superconductive Electronics Terms consult: IRE Proc., Vol. 50, No. 4, pp451-2 (April, 1962).

<u>Double Amplitude Modulation Multiplier</u> (AM - AM)

In this multiplier one input signal "x" is amplitude modulated by a carrier applied to Amplitude Modulator 1. The output of Modulator 1 is then applied to Amplitude Modulator 2 to act as a carrier, and the modulating signal is the other input y. The output of Modulator 2 is applied to a balanced demodulator whose other input is the same carrier used in modulation. The output of the demodulator is proportional to the product of the two inputs. Figure 12 is a black diagram of double amplitude modulation multiplier. The modulators and the demodulator can be of the diode bridge balanced type as shown in figure 13.

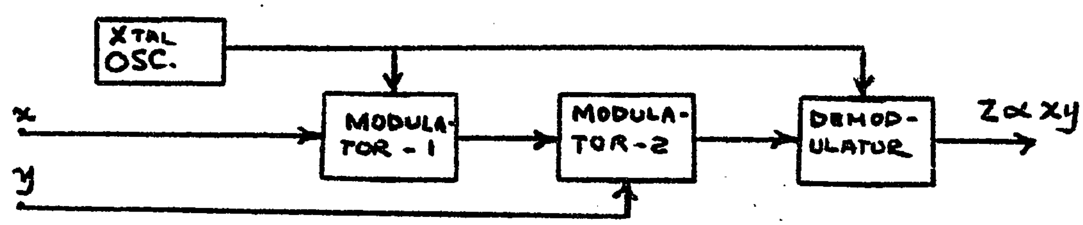


Figure 12 - Dual amplitude modulation multiplier

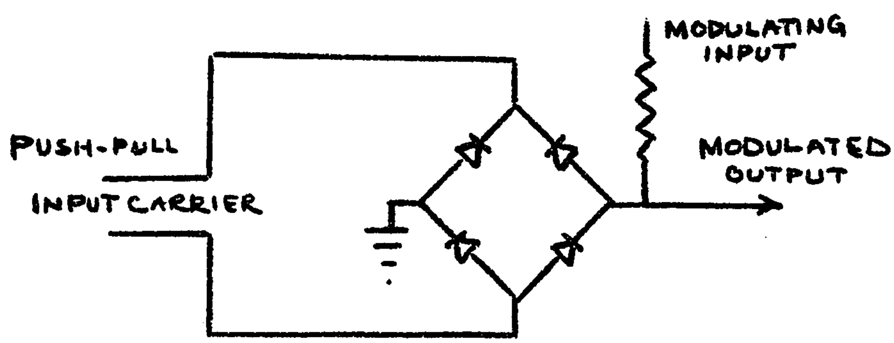


Figure 13 - Diode bridge balanced modulator

The accuracy of this type is limited because of its dependence on the linearity of modulators and thus the carrier voltage should be high compared to the modulating signal. The accuracy of a multiplier is reported to be ± .5% and a bandwidth of 30 Kc/s for both input channels, (Meyer and Fuller, 1954). Another multiplier is reported to have an accuracy of .3% and bandwidth of the order of 10Kc/s (Lukaszewicz, 1955). The bandwidth is limited by the frequency of the carrier.

<u>Phase - Amplitude Modulation Multiplier</u> (PM - AM)

In this multiplier the phase or the position of a carrier signal is made proportional to one input and its amplitude is proportional to the other. The phase-amplitude modulated carrier is then applied to a phase sensitive detector (balanced detector) followed by a filter (figure 14). The filtered output will be proportional to the product of the input quantities.

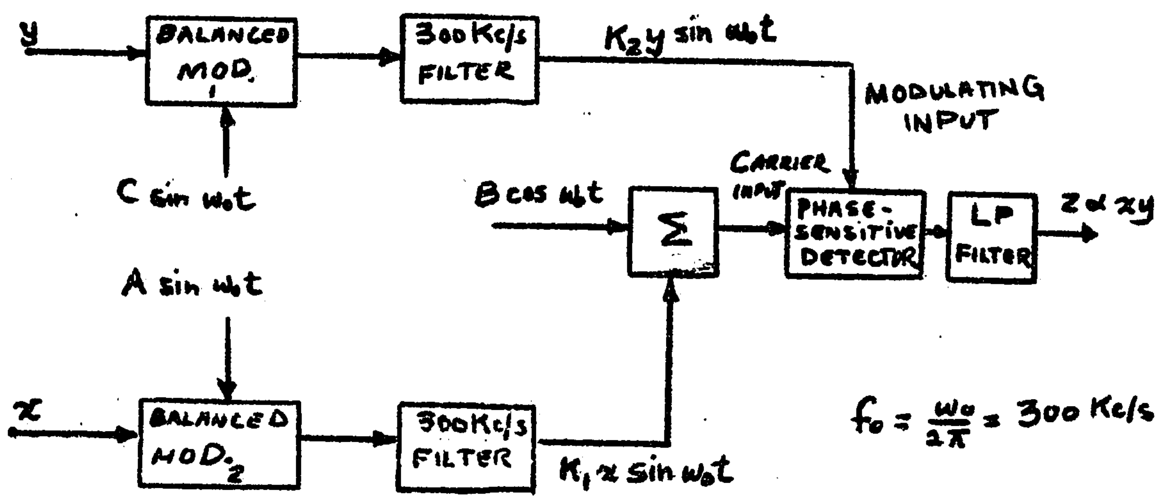


Figure 14 - A block diagram of a phase-amplitude modulation multiplier

Meyer and Fuller (1954) described such multiplier (figure 14) with its limits and errors. In this multiplier, the two inputs are applied to balanced modulators (of the diode bridge balanced modulator type). The carrier frequency used was 300 Kc/s. The output of Balanced Modulator 1 is filtered through the 300 Kc/s narrow band filter and then applied to a phase sensitive detector. The output of Balanced Modulator 2 is filtered through the 300 Kc/s narrow band filter, and added to the same carrier frequency but with a 90° phase shift. The summed output is

$$B\cos w_0 t + K_1 x \sin w_0 t \cong B\cos(w_0 t + K_1 x/B) \text{ for } \left|K_1 x_{max}/B\right| << 1 \quad (26)$$

This output is applied to the other input of the balanced demodulator which is followed by a low pass filter. The dc component at the output is

$$\text{dc output} = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} K_2 y \sin\,(\theta + K_1 X/B)\, d\theta$$

$$= \frac{K_2 Y}{\pi} \sin \frac{K_1 x}{B} \cong K_1 K_2 xy/B\pi, \text{ for } \left|K_1 x_{max}/B\right| << 1 \quad (27)$$

The multiplier accuracy is dependent on the trigonometric approximations, the stability of the carrier frequency and the linearity of the balanced modulator. The maximum phase modulation is limited to ± 8° due to the above mentioned approximations. Accuracy of a few per cent was obtained and the half power bandwidth of the output with respect to either input was made 30 KC/s (Meyer and Fuller, 1954). Another multiplier is reported to have an accuracy of the order of .2% and a bandwidth of the order of 500 cps (Kraicer, 1963). The bandwidth is limited by the frequency of the carrier and the bandwidth of modulators, demodulator, and the low pass filter.

FM - AM Multiplier

In this multiplier one of the variables is made to control the carrier amplitude through an amptitude modulator while the other variable affects the carrier frequency through a frequency modulator. The frequency-amplitude modulated carrier is then applied to a demodulator (sensitive to frequency deviations and amplitude of the carrier: FM discriminator). The output of the FM discriminator is proportional to the product of the input variables. Figure 15 shows a block diagram of such multiplier. The frequency modulator can be of the reactance tube type and the FM discriminator is of the Foster-Seeley type, except that the in-phase reference signal is obtained before the balanced modulator, (Price, 1951).

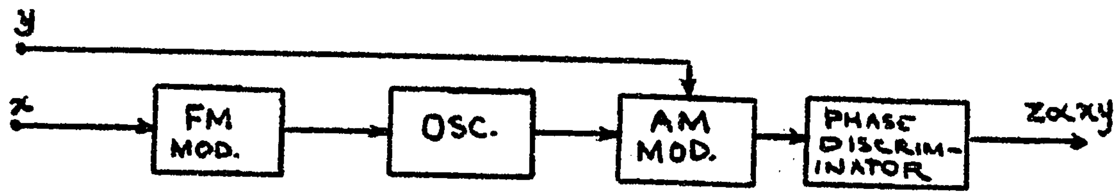


Figure 15 - FM-AM multiplier in block diagram

Figure 16 shows a circuit of a Foster-Seeley phase discriminator used in FM-AM multiplier.

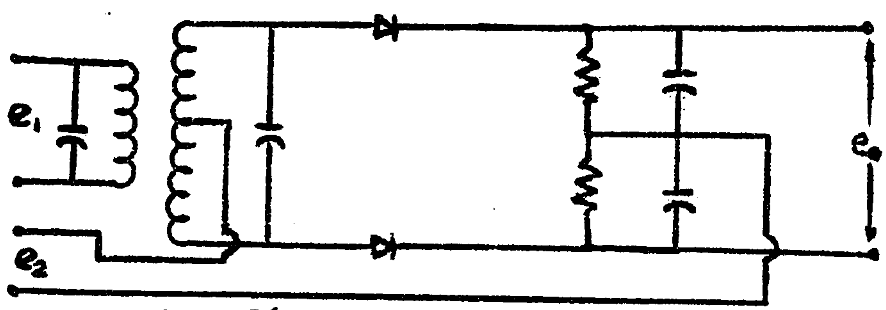


Figure 16 - The Foster-Seeley phase discriminator

The performance of the discriminator determines the limits of accuracy of the multiplier. The accuracy of such system can be improved by the use of feedback in the modulators of the multiplier. The accuracy of multiplier using FM-AM principle, is of the order 1-2% with a bandwidth of 10Kc/s, (Price, 1951). The bandwidth is limited by the frequency of the carrier and the bandwidth of modulators and the discriminator.

Time-Division Multiplier

(Variable Mark/Space or Time-Pulse)

The operation of this multiplier consists of determining the average values of a train of pulses where the amplitude is proportional to one of the input variables and the ratio of the pulse width to the pulse repetition period is proportional to the other. The conventional time-division multiplier is given in figure 17. A triangular wave is used to trigger a discriminator when the voltage reaches the unknown value x. The time required for the triangular wave to reach this value controls the width of the rectangular pulse at the discriminator's output. The resulting wave is then amplitude modulated by the y input. The output of the amplitude modulator is averaged by a filter; its output is proportional to the product of the two inputs.

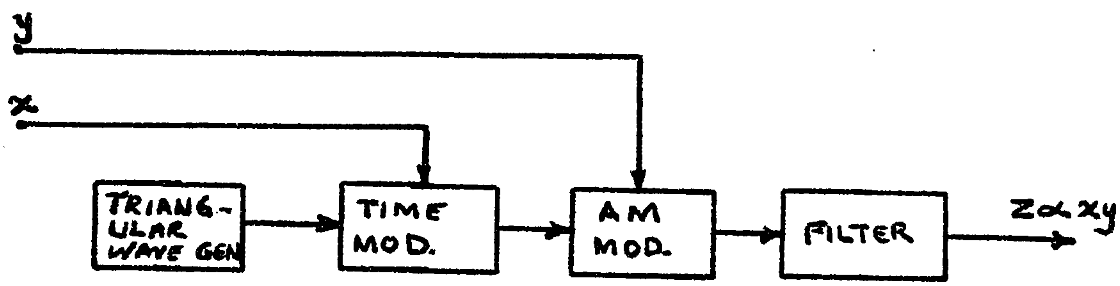


Figure 17 - Block diagram of conventional time-division multiplier

The time modulator consists of a discriminator that can be triggered by a triangular waveform when its value reaches the magnitude of an input variable (figure 18-a). The simplest circuit of such a discriminator is the monostable multi-vibrator (figure 18-b). The timing waveform is formed by charging the capacitor C through the resistor R. The width of the output pulse is proportional to the input variable.

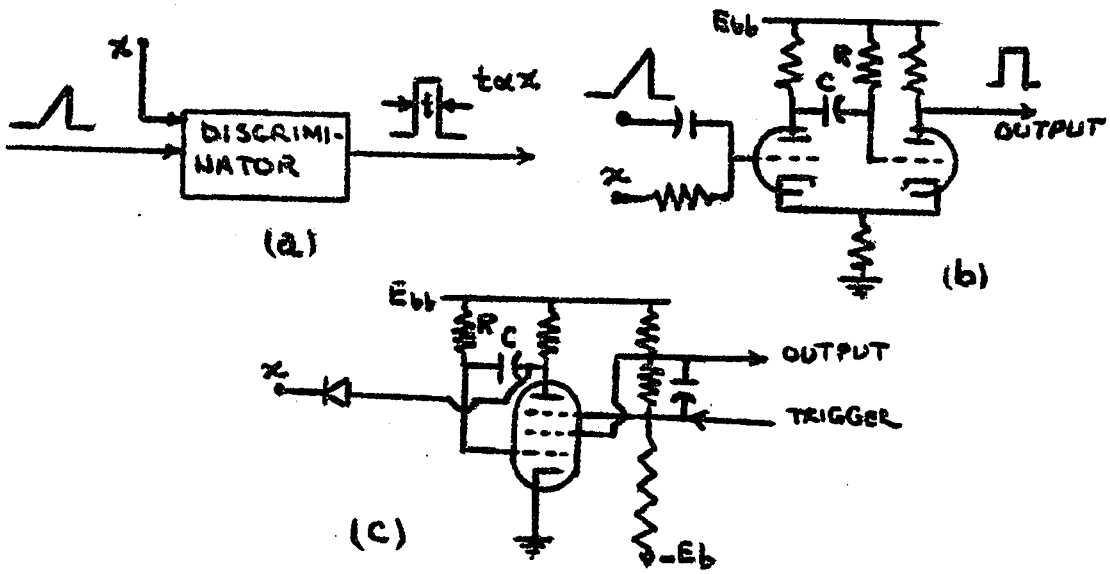


Figure 18 - (a, b, c) Three methods of time modulation

Figure 18-c shows another way of obtaining time modulation by using a phantastron. The magnitude of the input to the 'catching' diode will control the duration of the pulse at the output. Detailed study of such circuits can be found in many references (e.g. Chance, 1949; Millman and Taub, 1956).

The main difficulties in the conventional type of time-division multiplier are in obtaining a wave whose width would be linearly proportional to one variable and amplitude to the other variable. High accuracy can be obtained by the use of a feedback system and high speed switches (Goldberg, 1952; Morrill and Baum, 1952). A block diagram of a feedback time-division multiplier is given in figure 19. Transistor switches are used in the time-division multipliers because they are more precise than the vacuum tube switches and the zero drift of the multiplier is considerably improved (Kettel and Schneider, 1961; Barber, 1963). Time-division multiplier with vacuum tube switches can have .02%

accuracy for 3 cps bandwidth (Sternberg, 1955). Another time-division multiplier, which employs transistor switches, is reported to have a bandwidth of 10Kc/s for both inputs, while zero drifts of .1% and linearities of .25% (Barber, 1963). Another multiplier using high speed switching transistors is reported to have a bandwidth of 200 Kc/s with an accuracy of 1% (Barber, 1963). The bandwidth of the time-division multiplier can be extended by using high speed switching transistors. Thus it is apparent that the frequency response of such multipliers is limited mostly by the speed of the switching transistors, while its accuracy is limited by the instability and accuracy of the switches and the nonlinear relationship between the duration of the pulse and one of the variables.

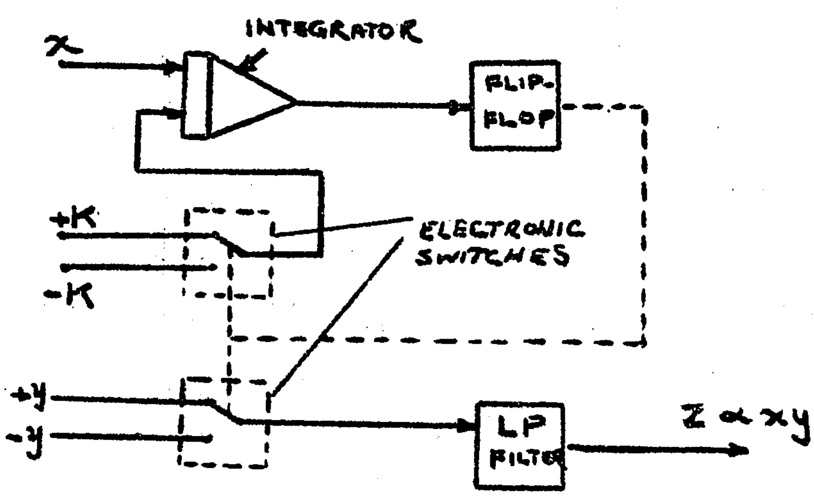


Figure 19 - Feedback time-division

Sampling Analog Multiplier

In this multiplier, one of the inputs x is compared (by a comparator) to the output of a linear passive network, the input of which is a sampled constant voltage V (figure 20). At the instant when the inputs will be equal at the comparator, the output quantity of an identical linear circuit with the signal y as its input and being sampled as the constant voltage V, will be proportional to the product of the input signals. The linear circuits can be RC networks with identical responses.

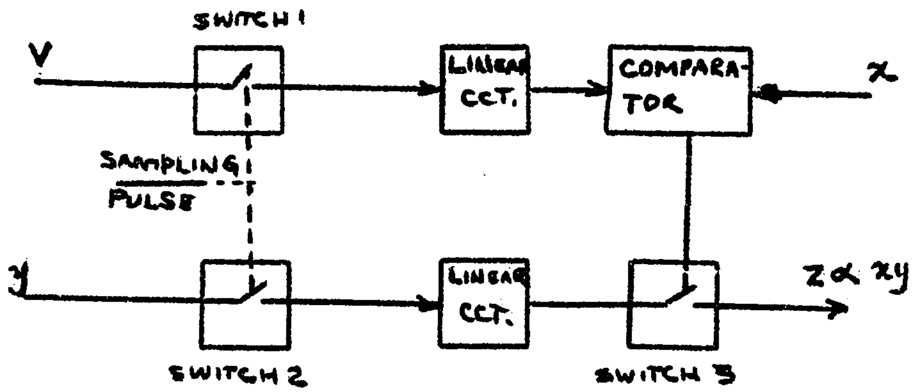


Figure 20 - Amplitude comparator Multiplier

The sampling pulse frequency for the control of the switches 1 & 2 should be high as compared to the frequency of the input signals. This is because, during the sampling interval the outputs of the two linear circuits have to be constant. A multiplier, using RC linear circuits, has an accuracy of 1% at 400 cps bandwidth, (Tomovic, 1958). This multiplier is simple and not expensive because it uses standard elements and does not require many nonlinear elements. Its accuracy is limited by the instability of the sampling pulse, accuracy of the comparator and the nonidentical characteristics of the two linear passive networks. Its bandwidth is limited by the repetition rate of the sampling pulse i.e. by the speed of switches.

Triangular-Wave (Amplitude Selection) Multiplier

This multiplier is based on the relation

$$kxy = \text{average } \left(|x+y+C| - |x-y+C| \right) \qquad (28)$$

where x and y are the input variables C is the magnitude of the triagular-wave signal k is a constant.

The above equation is valid if $|x+y| \leq |C|$ and the triangular-wave frequency is at least ten times higher than the highest frequency of x or y (Meyers and Davis, 1956). Figure 21 shows a block diagram of a triangular-wave multiplier.

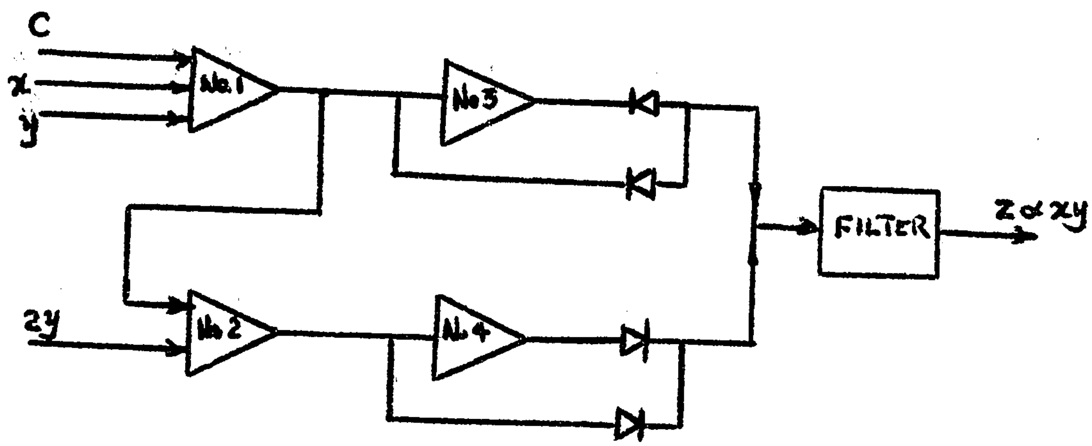


Figure 21 - Block diagram of a triangular-wave multiplier

The diodes after amplifiers 3 and 4 are used to give the absolute values of x+y+C and x-y+C, respectively. A carrier of the triangular waveform is used because its amplitude varies linearly with time and because its high harmonics have, for example, smaller amplitudes than those of a saw-tooth wave. The output of the filter (figure 21) is thus exactly proportional to the product of the two inputs. The multiplier has an accuracy higher than .1% at 1000 cps bandwidth.

Other multipliers that use triangular-wave signals and are based on similar concepts can be found in many papers (Fifer, 1961; Hartmann et al 1961; Pfeiffer, 1959). The multiplier accuracy is limited by the quality of the triangular-wave generator. The bandwidth is limited by the repetition rate of the triangular-wave and the bandwidth of the absolute-value device.

Probability Multiplier

This multiplier is based upon the following: The probability of simultaneous occurence of several independent events is equal to the product of the probabilities of occurence of each event. Thus if the widths of the pulses of three time-modulators, for example, are proportional to the inputs x, y and z, then the time interval during which the three pulses coincide will be proportional to the product xyz (figure 22).

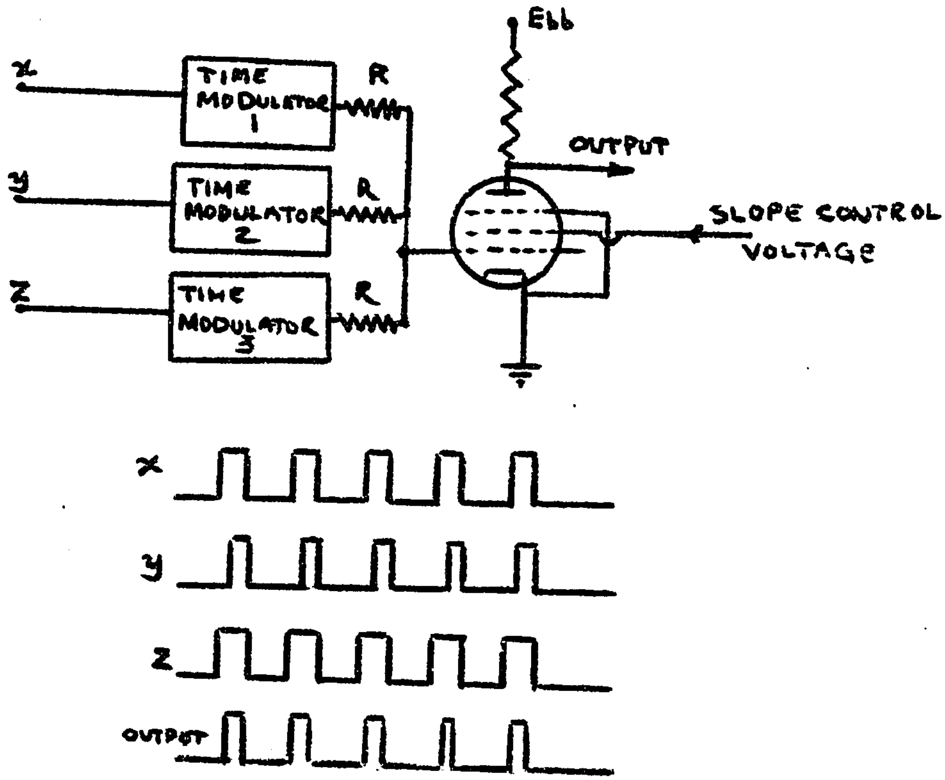


Figure 22 - Coincidence multiplier

The advantages of this system are the following: More than two variables can be multiplied at the same time, and pulse amplitude modulation is avoided because only constant magnitudes are used. The multiplier is a one-quadrant type since the probability of an event does not assume negative values. No special components are necessary for the multiplier. Also, no critical adjustment is required. The reported accuracy is in the range 1 - 2% for a bandwidth of 500 cps (Czajkowski, 1956). The accuracy of the multiplier is limited by the nonlinear relationship between the durations of a train of pulses and a variable. Its bandwidth is limited by the bandwidth of the coincidence detector.

<u>Square-Beam Multiplier</u>

In this multiplier, a specially designed cathode ray tube is used. The beam generated from the cathode is a square one, and it is deflected horizontally by the x input signal and vertically by the y input signal. Then the beam is collected, after deflection, by four square collector plates. The current from each plate flows into a load resistor R (figure 23)

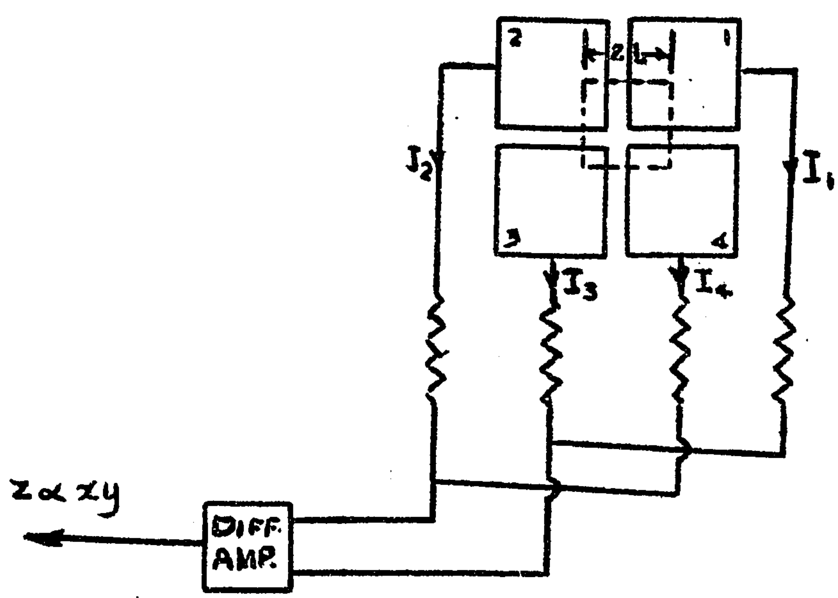


Figure 23 - Collector plates with the output circuit of a square-beam multiplier

and it is directly proportional to the area in which the beam is collected. The currents collected from the four plates are as follows

$$I_1 = J(L+x_0)(L+y_0) = J((L^2+L(x_0+y_0)+x_0y_0)) \qquad (29\text{-a})$$

$$I_2 = J(L-x_o)(L+y_o) = J((L^2-L(x_o-y_o)-x_oy_o)) \quad (29\text{-b})$$

$$I_3 = J(L-x_o)(L-y_o) = J((L^2-L(x_o+y_o)+x_oy_o)) \quad (29\text{-c})$$

$$I_4 = J(L+x_o)(L-y_o) = J((L^2+L(x_o-y_o)-x_oy_o)) \quad (29\text{-d})$$

where I_i is the current from plate i , i = 1, 2, 3 and 4, J is the current density, 2L is the width of the beam and x_o, y_o are Cartesian coordinates of the center of the beam.

The sum of the currents I_2 and I_4, when subtracted from the sum of the currents I_1 and I_3, gives a current proportional to the product x_oy_o. If the deflection sensitivity of each pair of the deflection plates is linearly proportional to the input signals, then the output x_oy_o will be proportional to xy. The accuracy of the multiplier depends upon the uniformity of the intensity of the beam on the square plates and upon the deflection characteristics of the fields in the CRT. A multiplier, based on the square-beam principle, is reported to have an accuracy in the range 5-7% and to handle high frequency signal inputs (Somerville, 1950). It is a four-quadrant type but it is not available commercially. The bandwidth is limited by the stray capacitance at the output circuit of the CRT.

Circular-Beam Multiplier

This multiplier, as the square-beam type, uses a specially designed cathode ray tube except that the generated large-diameter beam is circular (Angelo, 1952). This circular beam, after being deflected by the input signals, is collected by four quadrant plates as shown in figure 24. The plates are electrically isolated from each other. Referring to the same figure, the currents in areas a,b, and c will be cancelled by the currents in a', b', and c' respectively. the net current, which is shown as a shaded area, will be proportional to $x_o y_o$, where x_o and y_o are the displacements of the beam from the center of the target. If the deflecting system is linear, then the output $x_o y_o$ is proportional to the product of the x and y deflecting voltages. This multiplier is reported to have an accuracy of 2% and bandwidth of 70kc/s, (Angelo, 1952 and 1954).

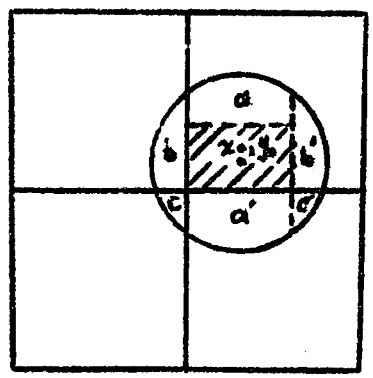


Figure 24 - Circular-beam collector plates

The bandwidth of the multiplier is limited, as in the square-beam type, by the stray capacitances at the output circuit of the CRT. The accuracy is limited by the nonuniformity of the beam density at the collecting plates, the deflection characteristics of the fields in the CRT .

<u>Crossed-Fields Multiplier</u>

The crossed-fields multiplier uses a well-known law in electromagnetic theory, that is, a force is proportional to the vector product of the magnetic field intensity and the velocity of electrons. In mathematical terms, it is expressed as

$$\bar{F} = e\,(\bar{V} \times \bar{B}) \qquad (30)$$

where $\bar{F}$ is the force acting upon an electron of charge e, subject to a magnetic field $\bar{B}$, and $\bar{V}$ is the electron velocity. The force is perpendicular to the directions of $\bar{V}$ and $\bar{B}$. The multiplier uses a cathode ray tube where one input signal is applied across the first deflecting system and the other input signal, after being converted to a proportional current, is applied to a magnetic coil wound around the second deflecting system (figure 25). The beam deflection is detected by a photo-electric cell placed in front of a half-masked screen. The output of the photo-electric cell is applied to the second deflecting system, through an amplifier, to nullify the vertical deflection of the beam. The voltage across the second plate deflecting system is proportional to the product of the input signals. The multiplier is a four-quadrant type. Its accuracy depends on the deflection characteristics of the fields in the CRT and the bandwidth is limited by the inductive effects of the magnetic channel. A multiplier, using this principle, is reported to have an accuracy of the order of .5% and bandwidth of the order of 3000 cps (MacNee, 1949).

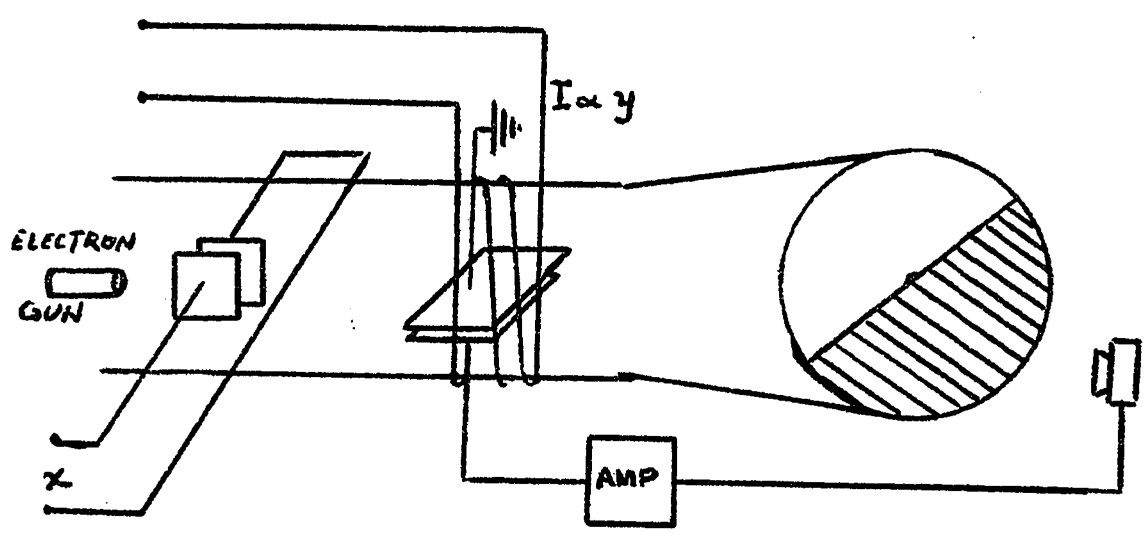


Figure 25 - Crossed-fields multiplier

Hyperbolic-Field Multiplier

This type is a modification of the crossed-fields multiplier. In the cathode ray tube, used in this multiplier, an electrostatic field and not a magnetic, is utilized for the beam deflection (figure 27).

The input signal X is applied to the first pair of deflecting plates. The beam enters the second system, having a vertical deflection proportional to X. The second system is composed of four hyperbolic plates where the opposite plates are electrically connected. The other input signal Y acts on them in such a way that the electrostatic field is

$$e = kYxy \qquad (31)$$

where (x,y) are the perpendicular distances marked on the figure 26. When the beam enters the second system with coordinates (x=o,y) the force acting upon it in the horizontal direction is proportional to the product of the two inputs. At the output of the tube, there is a detection system composed of two plates separated by a narrow opening. The deflection of the beam from one side to another causes a difference in the current flowing into them. The output of this detector is applied to a differential amplifier whose output is applied to a third pair of plates in such a way as to keep the beam centered vertically in the opening between the detector plates. If the system is in equilibrium, the output of the amplifier gives the required output which is proportional to the product of the two input quantities.

The accuracy of this multiplier is reported to be 0.5% and its bandwidth is of the order of 200Kc/s (Tomovic, 1958). It is four-quadrant type but not available commercially. The accuracy is limited by the deflection characteristics of the fields in the CRT. The bandwidth is limited by the stray capacitances at the output circuit of the CRT.

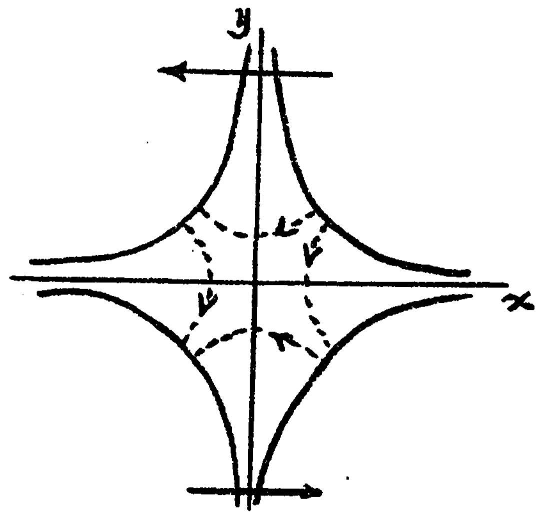


Figure 26 - Hyperbolic field

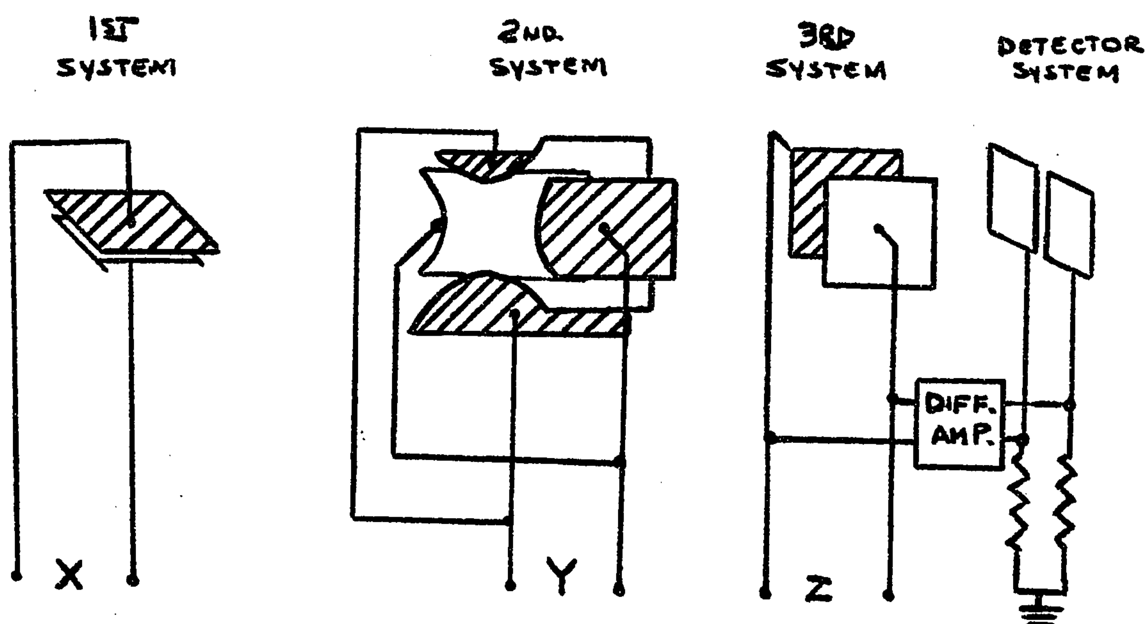


Figure 27 - Hyperbolic electrostatic field-multiplier

Hall-Effect Multiplier

E. H. Hall, in 1879, found that if a current was transmitted through a metal slab at right angles to a magnetic field, a voltage appeared in the direction perpendicular to that of the current and the field (figure 28). In mathematical terms, the induced voltage in the

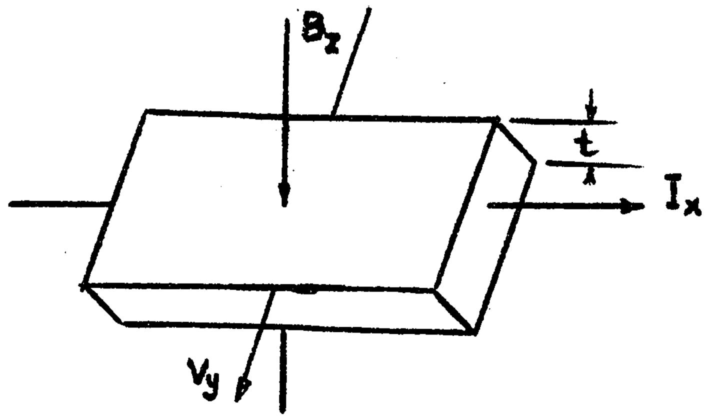


Figure 28 - Hall-effect

slab is

$$V_y = R \cdot I_x B_z / t \qquad (32)$$

where V_y is Hall voltage in the y direction, R is Hall constant = 1/[(charge on carrier x concentration of carriers)], I_x is the control current in the x direction, B_z is the magnetic flux density in the z direction, and t is the thickness of the slab.

If I_x and B_z are made proportional to the two input variables, they V_y will be proportional to the product of the two inputs, and thus multiplication using Hall element is achieved (Glinski and Landolt, 1961). The Hall element is usually a semiconductor because of its high Hall-voltage V_y. This is due to the low concentration of electrons in such materials. Many materials had been used as a Hall-element but indium arsenide is more often used because it has the lowest temperature coefficient and low resistivity compared to other semiconductors such as silicon, germanium, and indium antimonide. Current amplifiers are required in Hall-effect multipliers to change the input voltages to current signals, which will give rise to the controlling current and the magnetic field. Figure 29 shows a block diagram of a Hall-effect multiplier. R_2 and R_1 are used for adjustment. R_1 is used to minimize Hall leads' loop error and R_2 is used to reduce null voltage error. The Hall-effect element itself has no frequency response limitation but the magnetic channel limits the multiplier bandwidth because of the inductance associated with the Hall-element. Air-cored coils can be used to increase its frequency response.

A multiplier described by Kovatch and Meserve, 1961, in which the Hall channel has a bandwidth of 25 Kc/s (this limit is due to the driving amplifier bandwidth) while the magnetic channel has a bandwidth of 1.3 Kc/s. Its static accuracy is of the order of $\pm$ .8% for the Hall channel and $\pm$ 2.8% for the magnetic channel. The linearity of multiplication is of the order $\pm$ 0.3%. The accuracy of the multiplier is limited by the instability of the Hall-element with the temperature changes, the existence of a null voltage error, and the Hall leads' loop error.

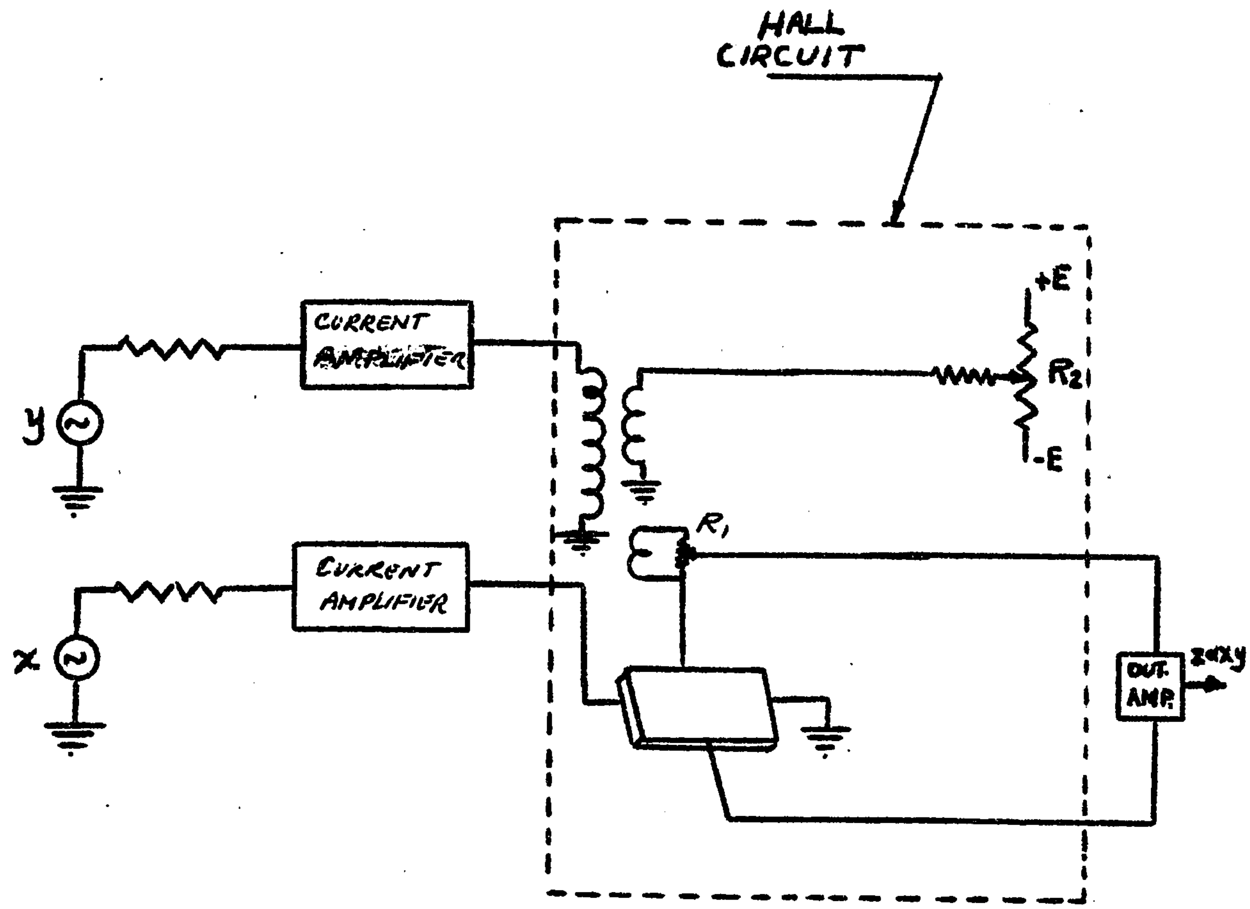


Figure 29 - Hall-effect multiplier

Dynamometer Multiplier

The operation of this multiplier is based on the following principle:

The torque created by two crossed magnetic fields is proportional to the product of the currents creating the fields. Two opposing dynamometers are used to build the multiplier (figure 30).

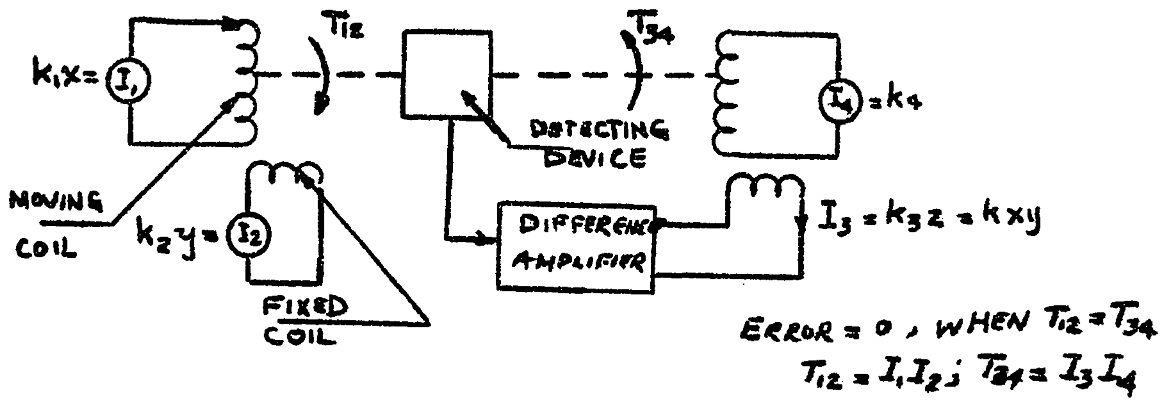


Figure 30 - Dynamometer multiplier

Voltage signals cannot be applied directly to the dynamometer inputs and thus they should be converted to current signals. It is four-quadrant type and has an accuracy in the range .1 to 1%. The speed of response of such multiplier is limited to about 100 cps because of the inertia of the moving coil and the inductance associated with the circuit, while the accuracy is limited by the accuracy of the detecting system, and the accuracy of the law that governs the torque created by the crossed fields.

Logarithmic Multiplier

The relation to obtain multiplication through logarithms is

$$xy = \text{antilog}_a\ (\log_a x + \log_a y) \qquad (33)$$

Therefore, this kind of multiplier can be constructed with two logarithmic units, one anti-logarithmic function generator and an adder. Figure 31 a,b shows a block diagram of a multiplier based on the logarithmic principle.

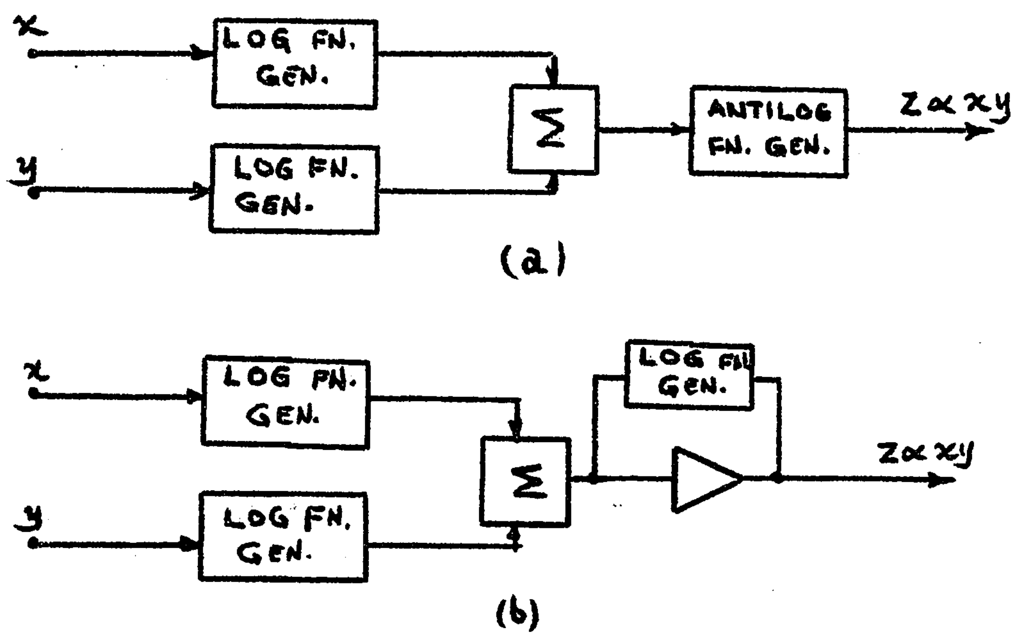


Figure 31 (a,b) - Block diagram of a logarithmic multiplier.

In (b), the antilogarithmic function generator is obtained by the use of a logarithmic function generator in the feedback loop of a high-grain amplifier.

The logarithmic function can be generated by electronic means such as:

(1) Biased diodes (operated in their linear region) are used to approximate the logarithmic function (Soroka, 1954)

$$z = k \log x \qquad (34)$$

where x is the input variable, z is the output quantity, k is a constant, and a is the base of the logarithm.

The base of the logarithm can be any positive real number, but it is normally 10 or e $\doteq$ 2.718

(2) Diodes (Semiconductors and Vacuum tubes), or triodes operated near

the cut off region have an exponential current-voltage characteristics and can be utilized to generate the logarithmic function. The diode plate current, for example, is related to the voltage across it by

$$i_p = i_o e^{\alpha V} \tag{35}$$

where i_p is the diode current, V is the voltage across the diode, and i_o and α are constants.

Taking the logarithm of each side, one obtains

$$\ln i_p = \ln i_o + \alpha V \quad \text{or} \quad V = \frac{1}{\alpha} \ln(i_p/i_o) \tag{36}$$

To change the current i_p into voltage, a large series resistor is connected to the diode as shown in figure 32-a.

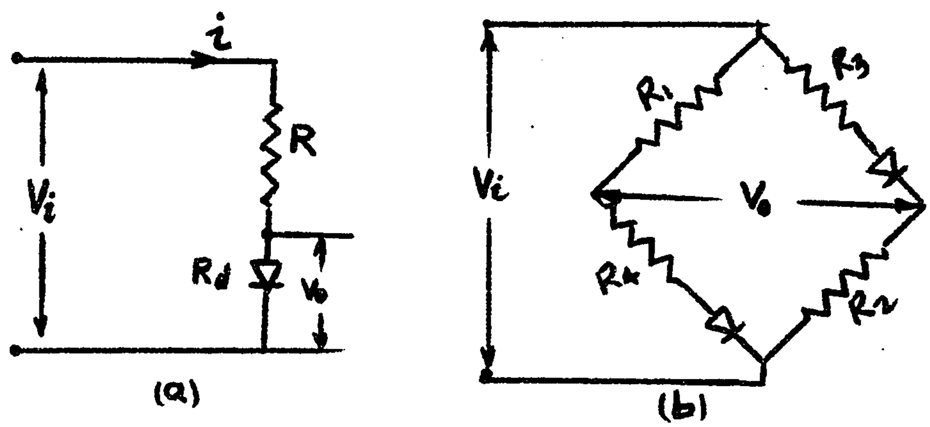


Figure 32-a - Logarithmic function generator without compensation

b - Logarithmic function generator with compensation

The current flowing into the circuit is

$$i = V_i/(R + Rd)$$

where V_i is the input voltage, R is the series resistance, and Rd is the diode resistance.

If $R \gg Rd$, then the current i will be approximated to

$$i \simeq V_i/R$$

Thus the output voltage Vo is

$$Vo = \frac{1}{\alpha} \ln(i/i_o) = \frac{1}{\alpha} \ln \frac{V_i}{Ri_o} \qquad (37)$$

which has a logarithmic relation with the input. The above circuit has a poor stability because of its temperature dependence. To make it more stable, two diodes are used in opposite arms of a bridge, as shown in figure 32-b.

(3) The exponential current-voltage characteristics of the input of a grounded-base junction transistor is utilized to obtain the logarithmic function (Deb and Sen, 1961).

The antilogarithmic function can be generated by making use of the exponential current-voltage characteristics of diodes or triode in the cut-off region, and by using biased diodes to approximate the antilogarithmic function by linear approximations. Another method of generating the antilogarithmic function is to use a pentode as an inverted triode (Snowden and Page, 1950). In an inverted triode, the plate and grid are interchanged to give input and output respectively. A circuit utilizing 6SK7 tube and connected as an inverted triode is shown in figure 33.

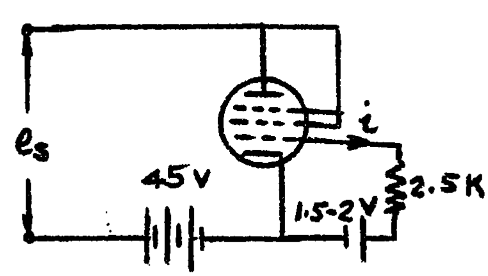


Figure 33 - Tube 6SK7 used as an inverted triode to generate the antilogarithmic function

In this circuit

$$\log ki = k_1 e_s \quad \text{or} \quad ki = \text{antilog}\, k_1 e_s \qquad (38)$$

where i is the grid current, e_s is the input voltage, and K and K_1 are constants.

In order to avoid the use of antilogarithmic unit in the multiplier circuit, a high gain d-c amplifier with a logarithmic function generator in its feedback loop is employed (figure 34).

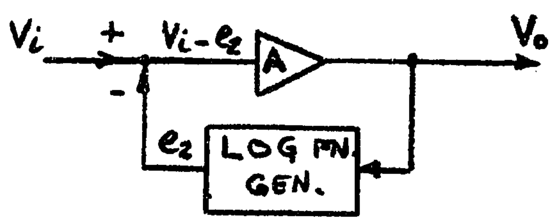


Figure 34 - Antilogarithmic function generator using logarithmic function generator

The output of the amplifier A is

$$Vo = -(V_1-e_2)A \tag{39}$$

where V_1 is the input voltage, e_2 is the output voltage of the logarithmic function generator, and A is the amplifier gain.

But $e_2 = \log_a Vo$, and substituting this value in the amplifier's output expression, one has

$$Vo = -(V_1 + \log_a Vo)A \qquad \text{or} \qquad -V_1 + \log_a Vo = \frac{Vo}{A} \simeq o \text{ if } A \gg Vo$$

$$\text{Then } V_1 = \log_a Vo \qquad \text{or} \qquad Vo = \text{antilog}_a V_1 \tag{40}$$

Since logarithms of only positive values can be determined, logarithmic multipliers are limited to one-quadrant operation, however, they can be converted to four-quadrant type by adding two biasing voltages at the input so that the input signal is always positive. The multiplier has a medium accuracy 3-5% and a bandwidth of 200 Kc/s (Czajkowski, 1956). It needs a frequent calibration. The accuracy of such a multiplier is limited by the accuracy of logarithmic function generators, while its bandwidth is limited by the capacitances associated with logarithmic function generators. Also, it is of interest to know that the error of this multiplier is proportional to the product of the input qualities.

Square-Law Multiplier

This multiplier uses the well-known expression to obtain the product of two variables:

$$xy = \frac{1}{4}\left[(x+y)^2 - (x-y)^2\right] \tag{41}$$

A difference amplifier and two identical squaring devices would be required to construct such multiplier.

The basic unit of this multiplier is a squaring device. There exist many schemes such as the following to obtain the squaring function device:

(1) A photoformer can be used to generate the square of an input quantity by inserting a parabolic mask in front of its screen. A photo-electric cell is used to pick up the output voltage of the cathode ray tube and then applied to an amplifier, the output of which is applied to the second pair of deflecting plates in the photoformer (figure 35). The output of the amplifier is proportional to the square of the input variable. The unit is bulky and expensive.

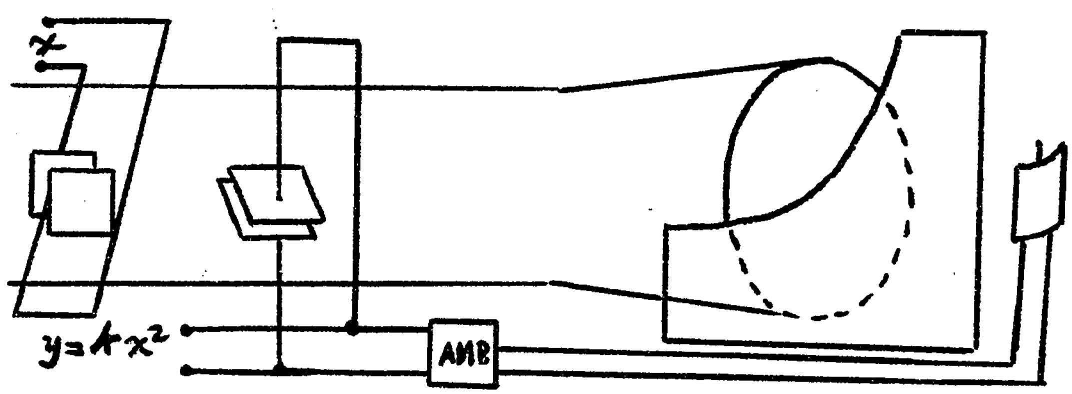


Figure 35 - Photoformer as a square function generator

(2) Beam deflection tubes such as QK-329 (figure 36) with square-law characteristics can be used as a squaring device.

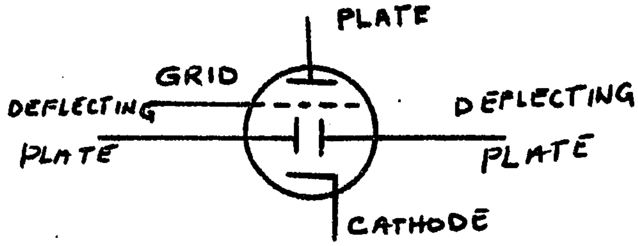


Figure 36 - QK-329 Tube

The output current of QK-329 is

$$i_o = i_c + K(e_o - e_i)^2 \quad (42)$$

where e_i is the input voltage, and K, i_c, and e_o are constants. The input voltage e_i can have any palarity. Such a symmetrical squaring device has a high frequency response extending to the VHF region (Miller et al, 1955).

(3) The plate current of triodes or pentodes, can be expressed in terms of the input grid voltage as follows

$$i_p = a_o + a_1 eg + a_2 eg^2 + a_3 eg^3 \quad (43)$$

where i_p is the plate current, eg is grid voltage, and a_o, a_1, ... are constants. The third order term can be neglected with $\simeq$ 1% error if the operation i.e. the input signal range of the tube is limited. In 5670 tube, using one triode, the plate current is

$$i_p = 3.8 + 3.075\ eg + .975\ eg^2 + .05\ eg^3\ \text{ma} \quad (Ecc = -3V,\ Eb = 150V)$$

and the third order term can be neglected with 1% error if e_g is limited to 0.8 volts. For the pentode GL-6565, the plate current is

$$i_p = 3.68 + 3.018\ eg + .745\ eg^2 + .057\ eg^3$$

$$(Ec_1 = -2V, \quad Ec_2 = +150V, \quad Eb = +250V)$$

The third order term will introduce 1% error if eg is limited to 0.7 volts. The above relationship in pentodes was employed in a multiplier which gave an accuracy of 2% with a bandwidth of 20 Kc/s (Holmes and Dukes, 1954).

The plate current-grid voltage relationship can be employed to obtain a square law unit. If two vacuum tubes, such as triodes, are used in a push-pull circuit, then the odd powers in the plate current expression are eliminated and the square of the input voltage will appear at the output assuming that the fourth order term of the plate current expression is quite small, (figure 37). The frequency of such a circuit is limited because of the Miller effect if triodes are used.

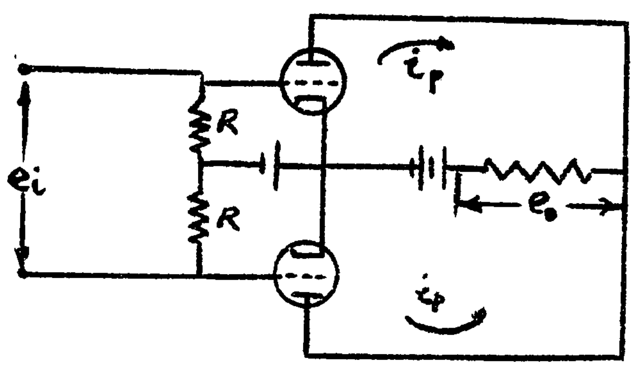


Figure 37 - Push-pull squaring circuit

(4) The non-linear characteristics of diodes can be used to generate squaring functions. The following is a short description of diode circuits that can be used to obtain such a squaring function:

a- The logarithmic relationship between the low-level voltage and the resistance of a diode (e.g. selenium rectifier) is used to give the square-law characteristics. For a low voltage applied to a diode, it is found that the following relationship holds

$$R = Ae^{-bV} \tag{44}$$

where R is the diode resistance, V is the applied voltage, and b and a are constants.

Figure 38 shows a squaring circuit utilising this relationship. If $R_1 \ll R$, then the current i_1 is

$i_1 = \frac{V}{R} = V\,e^{bV}/A$ or $i \simeq \frac{V}{A}(1 + bV)$ for small values of V.

The voltage developed across R_1 is

$$i_1 R_1 = R_1(V + bV^2)/A \tag{45}$$

The additional term $R_1 V$ can be eliminated from equation 45 by connecting two series resistors, R_2 and R_3, across the selenium rectifier circuit and adjusting the resistors R_2 and R_3 such that

$$\frac{R_2}{R_3} = \frac{A-R_1}{R_1}$$

Thus the output voltage is given by

$$V_o = RbV^2/A \tag{46}$$

The frequency response of such circuit is limited because of the capacitive effect of the rectifier.

b- The diode current-voltage characteristics can be used to give the product of two input quantities. The plate current of a diode is

$$i = A(e^{\tau V} - 1) \tag{47}$$

where i is the current through the diode, V is the impressed voltage across it, and τ and A are diode constants.

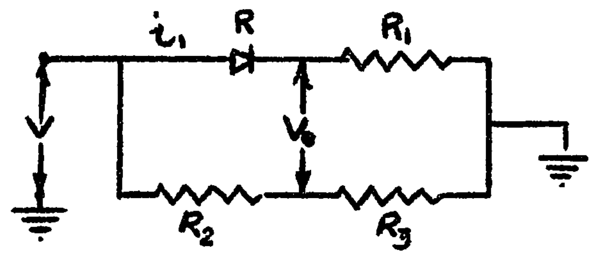


Figure 38 - A squaring device using selenium rectifier

Four diodes are arranged in a form of a ring (figure 39) to give an output proportional to the product of the input signals. A multiplier using the diode ring arrangement has 1-2% accuracy and bandwidth of more than 1 Mc/s (Wilcox, 1959). The multiplier is not sensitive to the temperature changes, but its input range is limited (up to 250 mv). It is simple and inexpensive. Analysis of the same multiplier is given by Freudberg, 1962.

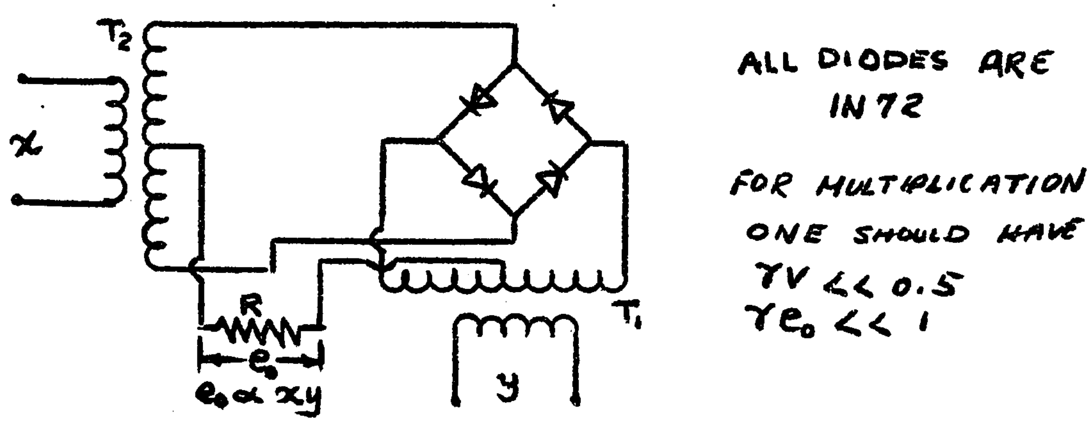


Figure 39 - Diode ring multiplier

c- In temperature limited diodes, the plate current is proportional to the square of the heater current. A squaring circuit with such a diode is not stable because of its temperature dependence and thus cannot be used as a part of a stable multiplier.

d- Dielectric diodes, such as cadmium sulphide, have been used in square-law multipliers (Cluley, 1962). The diode current is proportional to the square of the input voltage. In mathematical form, the diode current is

$$I = k(V - Vo)^2 \qquad \text{for} \qquad V > Vo \qquad (48)$$

where V is the diode voltage, Vo is the constant threshold voltage, and k is a constant.

This squaring device has a high frequency response but limited by the capacitive shunting effects.

e- Biased diode (vacuum tubes or solid state) circuits can be used to generate a parabolic function by approximating it with linear segments. This is on the assumption that diode characteristics are linear. This method gives the most accurate and stable type of squaring devices since it is independent of the tube or nonlinear element characteristics. Higher accuracies can be achieved in such a squaring method, by using more diodes to approximate the parabolic function. Moreover, sharp corners in the approximated squaring function can be rounded by superimposing a high frequency signal on the input signal which will cause the diode to conduct earlier (Schneider, 1957; Smith and Wood, 1959).

(5) A varistor (a passive element with a resistance that vary inversely with the voltage across it) is used with a series resistor to generate the squaring function of the input signal. The voltage across the series resistor is the square of the input signal (figure 40).

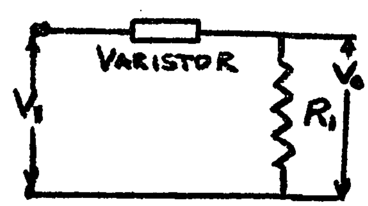


Figure 40 - Varistor as a squaring device

When a voltage V_1 is applied to the circuit, the current flowing into it is

$$i = V_1/(R_1 + R) \quad (49)$$

where i is the current flowing into the circuit, V_1 is the voltage applied to the circuit, R is the resistance of the varistor, and R_1 is a series resistor.

The current will be approximated to V_1/R if $R \gg R_1$.

The output voltage is

$$V_o = i R_1 = \frac{V_1}{R} R_1 \qquad \text{but} \qquad R \approx k/V_1$$

and thus

$$V_o = V_1^2 R_1/k \quad (50)$$

This squaring device is limited in its frequency response because of the varistor capacitive shunting effect which is a function of the frequency (Bruck, 1962).

(6) The voltage current characteristics of a thyrite (a silicon carbide ceramic material with nonlinear resistance characteristics) has the form

$$I = A V^n \quad (51)$$

where I is the current through the thyrite, V is the voltage across it, and n and A are constants.

The value of the constant n ranges between 2.49 - 3.37 (Fifer, 1961). To obtain the square law relationship between input and output quantities, a resistor of proper value Ro should be connected in series with the thyrite as shown in figure 41-a.

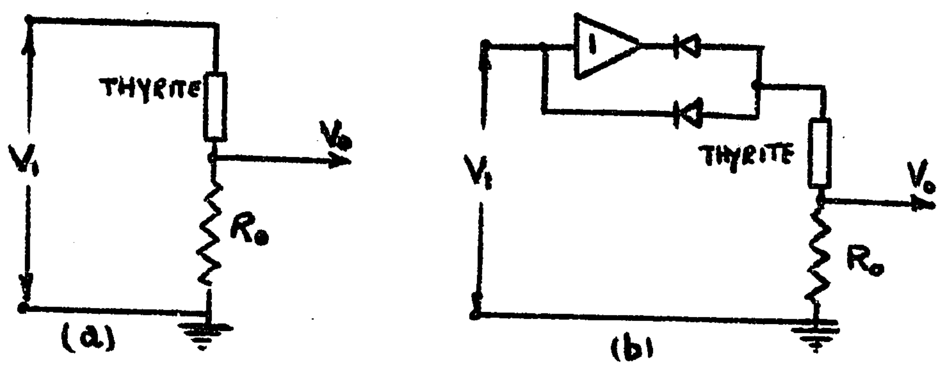


Figure 41a - Squaring device using thyrite
b - Symmetrical squaring device utilizing an absolute value circuit

To obtain a symmetrical squaring device, an absolute-value circuit is connected ahead of the thyrite circuite as shown in figure 41-b. The frequency response is limited by the capacitive shunting effects. Compensating for the capacitive effect and detecting the absolute value of the current instead of the voltage, its frequency response can be extended (Gul'ko, 1960).

Figure 42 shows a block diagram of a square-law multiplier.

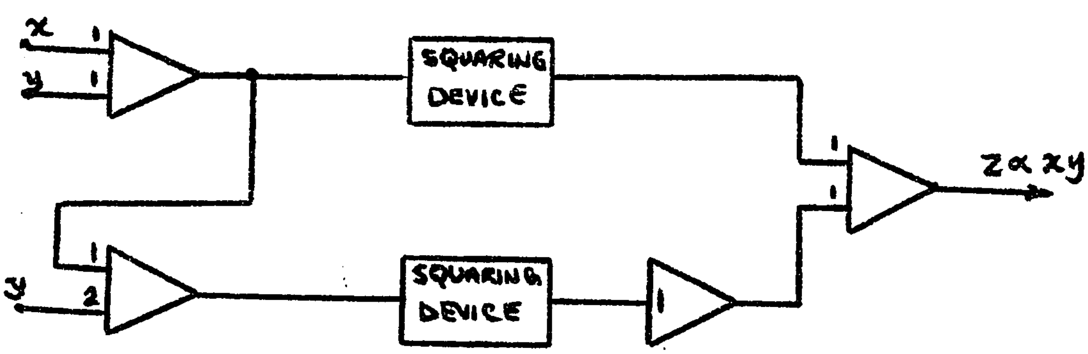


Figure 42 - Square-law multiplier

The multiplier in its simplest form is one-quadrant type but it can be of four-quadrant type if symmetrical squaring devices are employed (Haley and Scott, 1960). A square-law multiplier with biased diodes is reported to have 0.5% and has a phase shift at the output of less than 1° at Mc/s (Bruck, 1962). The accuracy of the square-law multiplier is limited by the degree of approximation to a parabola, while bandwidth is limited by the capacitance associated with parabolic function generator.

Vacuum Tube Multipliers

In some multigrid tubes such as a tetrode, the plate current is proportional to the product of the input signals. The variables are applied to the two grids of the tetrode as shown in figure 43-a. In figure 43-b, input variables are applied to the grids of two tetrodes connected in series, and the output at the plate of the upper tetrode is proportional to the product of the input quantities. The accuracy of such multipliers is low. It is of the order of 5% and a bandwidth of 200 Kc/s (Czajkowski, 1956). The range of operation of such multipliers is limited. The accuracy of the multiplier is limited by the instability of the characteristic of the tubes used. The bandwidth is limited by the bandwidth of the amplifiers.

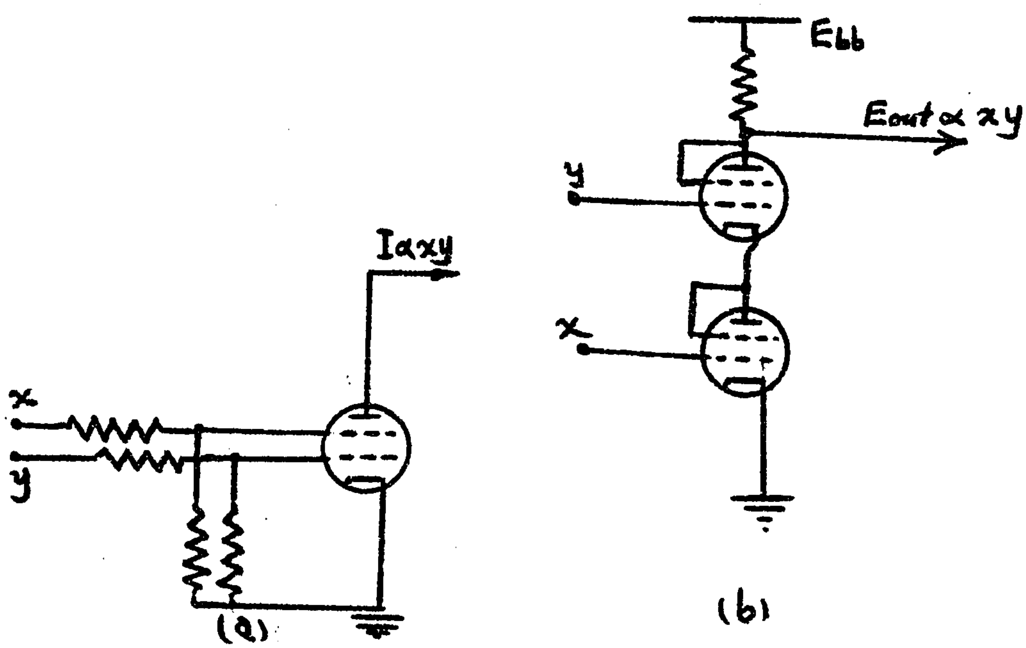


Figure 43 (a,b) - Vacuum tube characteristics multiplier

Mechanical Multipliers

Mechanical multipliers are used when inputs are shaft rotations or displacements. They can be of a square or logarithm type, where squaring or logarithmic cams are used. Other types are based on the principle of similar triangles, or on the following integration principle in which the product is

$$xy = \int x dy + \int y dx \tag{52}$$

where integration is performed by the use of ball-disc integrators or induction tachometer generator, since the output of the latter is proportional to the product of one of its input variables by the time derivative of the second variable. A detailed study of such multipliers can be found in Svoboda, 1948; or Soroka, 1954. The accuracy of such multipliers can be of the order of .01%, and it is limited by the accuracy of the mechanical units employed in the multiplier. The bandwidth is limited to low frequencies, because of the inertia of the mechanical components used.

CONCLUSION (SECTION - 2)

From the review of analog multipliers, it is obvious that wide varieties of methods had been employed to obtain the product of two or more variables. Depending on the methods used, the multipliers have extremely varied features. Thus it seems impractical to classify them with respect to types. A classification with respect to their accuracy and bandwidth seems to be of greater aid to any user. This classification is presented in Table 1.

Table 1 Classification of Multipliers According to Accuracy and Bandwidth

Upper limit of band-width (cps) / Accuracy	1	10	100	1000	10^4	10^5	10^6	10^7
0.01-0.033	Step	Servo Time-division Mechanical	Amplitude-Separation of Channels			Frequency-Separation of Channels Square-law (Biased Diodes)		
0.033-0.01			Time-division	Amplitude-Selection (Triangular-wave) Strain gage				
0.10-0.33			Photo-resistor	Phase-Amplitude Modulation Sampling	Hall-effect Time-division AM - AM		Square-law (Biased Diodes)	
0.33-1.0			Dynamometer Variable-gain	Controlled Nonlinear Resistance	Square-law (Thyrite)	Amplitude-Selection (Triangular-wave) AM - AM	Hyperbolic-field Time-division	Square-law (Biased Diodes)
1.0-3.3		Thermistor		Probability	Crossed-fields	Circular-beam Square-beam Phase-Amplitude Modulation FM - AM	Logarithmic	Square-law (Diode Ring)

SECTION 3

FOUR-QUADRANT SQUARE-LAW MULTIPLIER

From the review of analog multipliers presented in Section 2 and the requirements of the multiplier to be used with the noise measurement method -4, it appeared that there are some multipliers that can be used with noise measuring equipment described in Section 1. Expensive and complex multipliers were disregarded and the only favourable multiplier was the square-law multiplier. One type of the square-law multiplier was the most favourable to the present application especially as it has a broad bandwidth. It uses vacuum triodes connected in cascode and operates in one-quadrant. This section deals with this multiplier.

Original Circuit

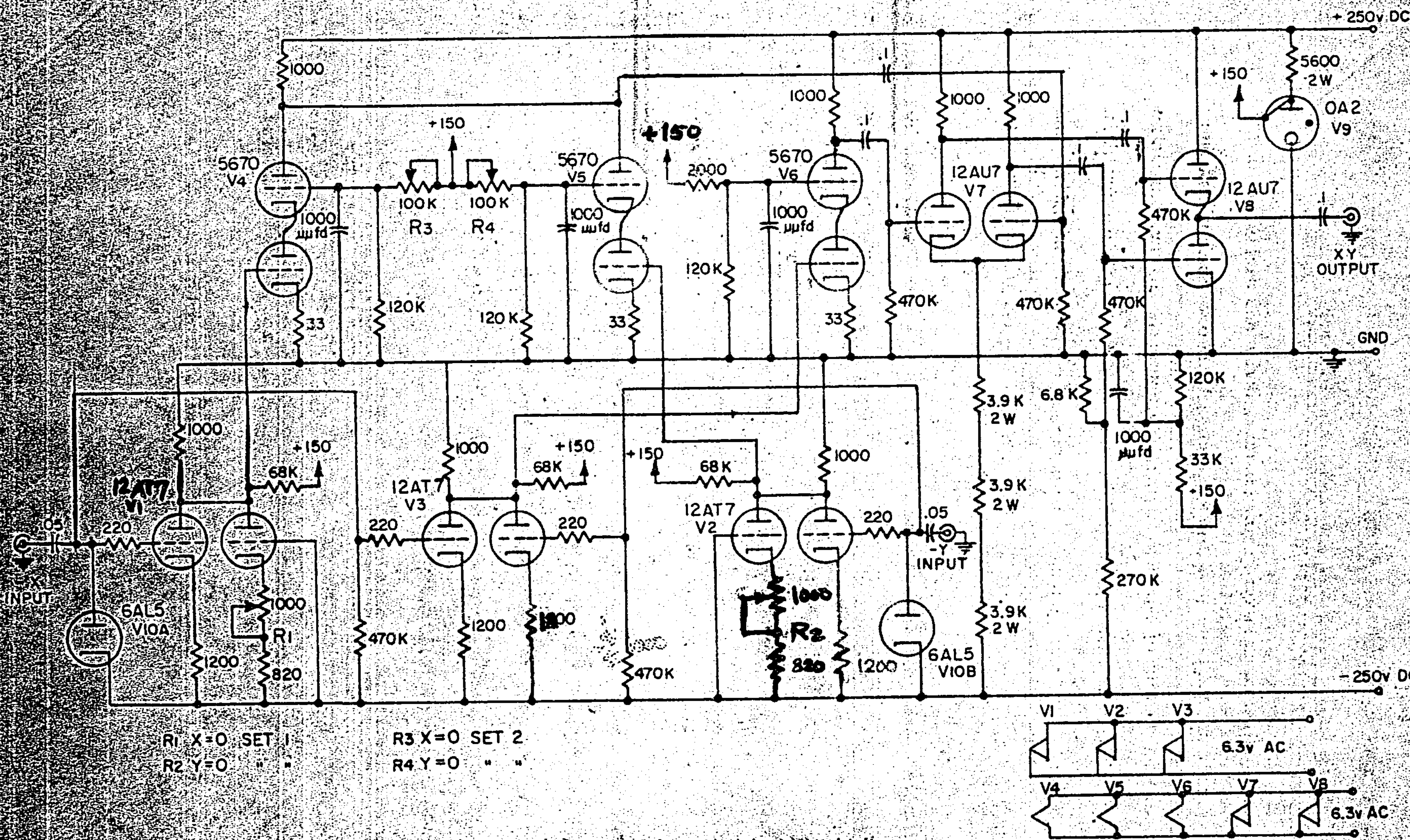
The original circuit of the multiplier (figure 44) was designed to operate as a one-quadrant type (Grosvenor, 1959). It had the following specifications:

Input impedance	100 kilohms
Output Amplitude (approx.)	6 volts peak to peak for 3 volts at each input
Output impedance	1000 ohms
Bandwidth	3.5 Mc/s
Balance	The output, when one input is zero, is less than 1% of the maximum linear output.

In the multiplier, the parabolic relationship between the plate current and the grid voltage of a cascode amplifer (figure 45) is utilized. The plate current, in a cascode circuit, can be given as

$$i_p = a + be_g + ce_g^2 + de_g^3 + \cdots$$

where i_p is the plate current, e_g is the grid voltage, a,b,.... are constants



~~Fig. — Schematic of the Multiplier~~

Figure 44 - Schematic of the multiplier (Original circuit)

For a tested cascode using 5670 tube, the plate current was

$$i_p = 3.42 + 2.2e_g + .5e_g^2 + .078e_g^3$$
$$(Ec_1 = -2v, \quad Ec_2 = +150v, \quad Eb = +250v)$$

The last term in the above expression can be neglected with ± 1% error if e_g does not exceed ± 0.7V. In such case, the cascode circuit can be used to obtain the parabolic relationship required for the operation of the multiplier. Moreover, the cascode circuit is used in the multiplier for the following reasons (Grosvenor, 1959):

1- In order to vary effectively the plate voltage without changing the plate load. If the upper grid bias of a cascode is changed, the plate voltage will vary and thus the slope of the transconductance gm will vary accordingly.

2- The cascode provides a high effective plate impedance, which is desirable in the add circuit that is employed in the multiplier circuit.

3- The cascode circuit provides isolation between the input and output. The grounded grid triode also has this isolating property but it differs from the cascode in having a low input impedance.

Modified Circuit

The original circuit was designed to accept negative-going signals only because it was a part of a correlator which accepts a uni-directional video signals. It has been modified to accept signals of arbitrary polarity i.e. four quadrant operation (figure 46). The modifications consisted of the following:

a- The removal of the clipping diodes at the inputs.

b- The removal of all RC coupled stages after the cascode amplifiers, used as differential amplifiers. This was done because the d-c signals

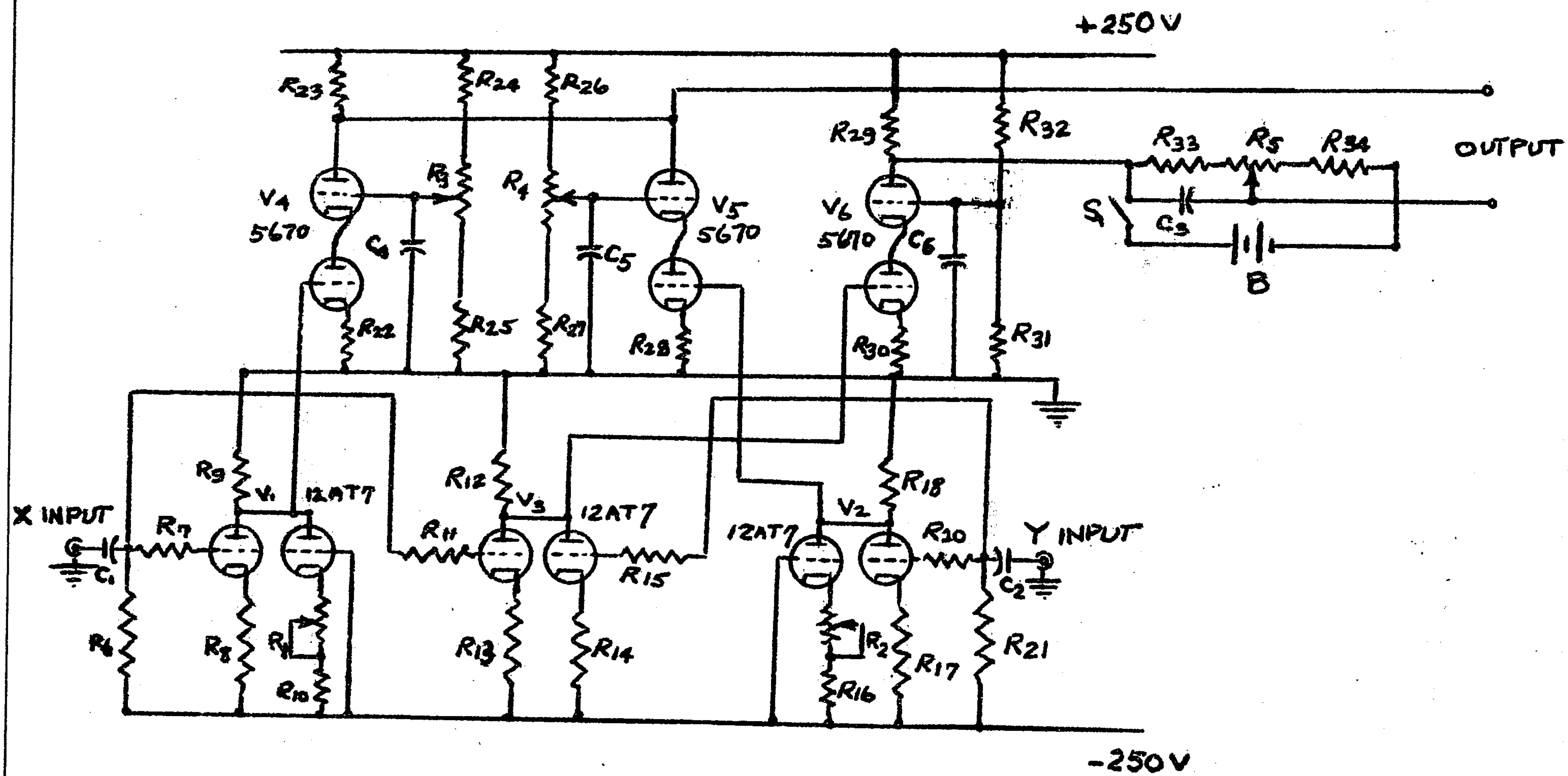


Figure - 46 The schematic of the multiplier
(The modified circuit)

were required at the output of the present multiplier.

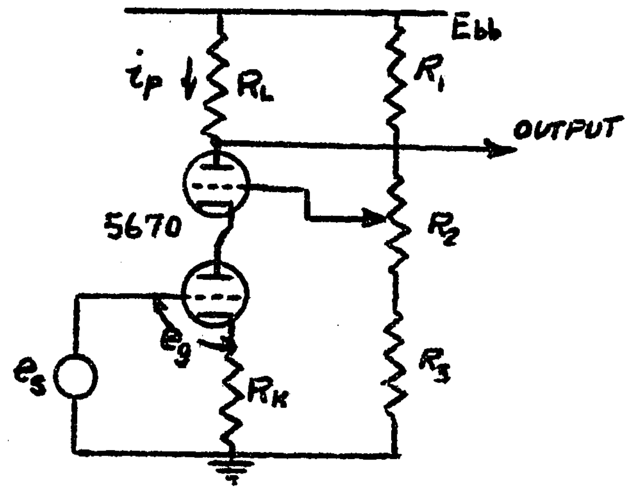


Figure 45 - A cascode circuit

c- The OA2 regulator, used in the original circuit to supply +150 volts was replaced by a voltage divider. This was done because a well-regulated d-c power supplies are used in the present circuit and because it was contributing noise at the output of the multiplier. (See Graph 1-A)

<u>Multiplier Errors</u>

An exact multiplication of two signals will be obtained at the output if the multiplier is an ideal one. In practise errors will be introduced because of the different characteristics of the cascode triodes. The introduced errors can be found in the following way:

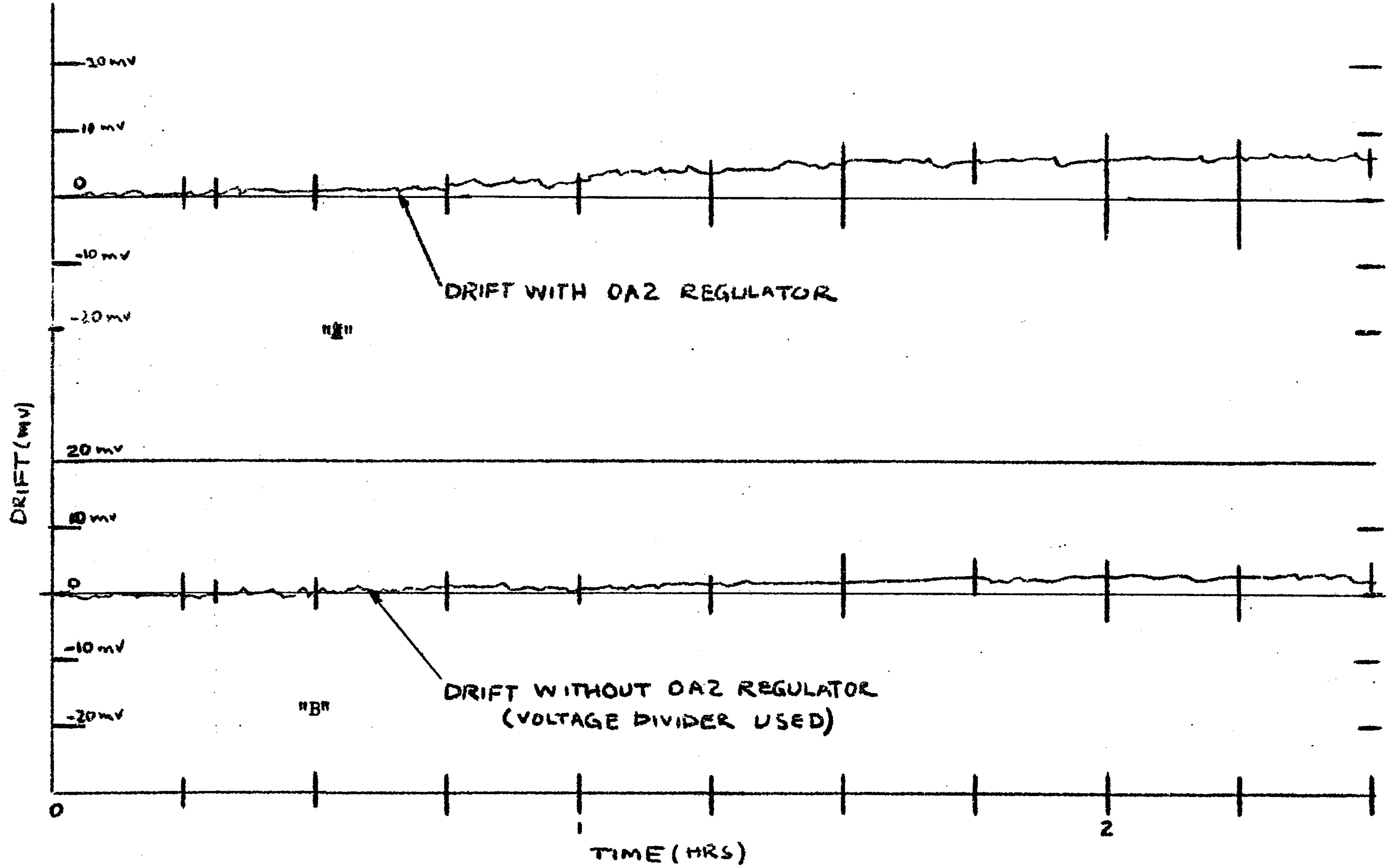
The output of the multiplier, assuming the third order errors to be negligible, is

$$kxy + \Delta xy = a_3 + b_3(x+y) + c_3(x+y)^2 - \left[a_1+b_1x+c_1x^2+a_2+b_2y+c_2y^2\right] \quad (57)$$

$$= (a_3-a_1-a_2) + (b_3-b_1)x + (b_3-b_2)y + (c_3-c_1)x^2 + (c_3-c_1)y^2+2c_3xy$$

where x and y are the input signals, Δxy is the error in multiplication, k is $2c_3$, and a_i, b_i, c_i are the cascode constants ($i = 1,2,3$).

Graph 1 - Drift With and Without OA2 Regulator



Thus the error at the output of the multiplier is

$$\Delta xy = (a_3-a_1-a_2) + (b_3-b_1)x + (b_3-b_2)y + (c_3-c_1)x^2 + (c_3-c_2)y^2 \qquad (58)$$

Examining the above expression, one can see that the output error is a combination of the following errors:

(1) The constant error which is due to the different operating points of the cascodes. It can be eliminated by the use of d-c compensating voltage.

(2) The linear term errors which are caused by the unbalance of the transconductance of the cascodes.

(3) The square term errors which are due to different rate of change of the transconductance of the same cascodes.

It is obvious from the error expression that the multiplier circuit should have at least five controls: one control to balance the d-c component, two controls to balance the linear errors which are proportional to the input signals, and two other controls to balance the errors which are proportional to the square of the input signals.

Additional errors will be introduced at the output if the third order term in the plate current expression could not be neglected. These are

$$\Delta xy_3 = (d_3-d_1)x^3 + (d_3-d_2)y^3 + 3d_3\,(x+y)\,xy \qquad (58\text{-a})$$

The above errors do not contribute to the final result of the noise measuring system if the random inputs have gaussian probability distributions (Appendix I). Moreover, the third order errors are negligible. The output error is measured by shorting one input and applying a-c signal at the other input. Such error is termed"dynamic error" (amplitude and phase shift error). The measured dynamic error of the present multiplier is of the order of 1%. The "static error" cannot be measured since capacitors are used in the input stages, and it is defined as the output of the multiplier when both input quantities are constant.

Drift Problem

One of the desirable features of a good multiplier is the freedom from drift. The drift occurs in this multiplier because of the use of d-c amplifiers. But if RC coupling is used between amplifiers, then drift will be eliminated.

Drift and noise in d-c amplifiers (vacuum tube type) can be attributed to a variety of factors (Konigsberg, 1959):

(A) Drift and noise due to changes in the vacuum tubes characteristics particularly those of the first stage of the amplifier. Among the causes of such a drift are:

1- Random changes in the emission characteristics of the cathode material.

2- Cathode temperature changes due to the changes in the heater voltage. This causes changes in cathode emission and in the equivalent plate impedance of the tube.

3- Shock and vibration. The latter may cause displacement of grid or cathode structures or a change in the cathode surface. Shock may cause changes in heater resistance and therefore cathode temperature.

4- Variations in electrode contact-potential (especially that of the grid) due to cathode temperature or ambient temperature changes.

5- Aging of the tubes.

(B) Drift voltage due to changes in the plate supply voltage for the amplifier.

(C) Drift voltage due to changes in the temperature of components other than the tubes (e.g. resistors) or due to unstable components (e.g. faulty resistors).

(D) Drift voltage caused by changes in d-c leakage from adjacent high voltage points to points in the amplifier circuit (but excluding the input circuit of the amplifier).

(E) Drift voltage due to spurious direct currents flowing within the amplifier ground circuits.

(F) Noise voltages due to:

1- Noise originating within the tubes and circuit resistors.

2- Coupling from stray electromagnetic sources to wiring within the amplifier circuit, including the amplifier input circuit.

3- Coupling from stray electrostatic sources to points within the amplifier circuit, but not including the amplifier input circuit.

4- Ripple in the d-c supply voltages.

In the described circuit, a well-regulated d-c power supply was used. The voltage regulation of this supply did not exceed 0.1 volt for any setting of the output voltage, with maximum load, and with ± 10% variations in the a-c supply. A Sorensen a-c regulator, type 1001 with 0.01% regulation was used to control the a-c line variations so that the regulation of the d-c supply would not be affected by such variations.

However, with the above mentioned supplies, the drift in the multiplier was still present. The components that caused the drift at the output of the multiplier were the following:

1- The OA2 gas tube voltage regulator

2- The carbon composition resistors

The OA2 gas tube voltage regulator, as mentioned earlier in this section, has been replaced by a voltage divider. The drift caused by the composition resistors was due to their instability and noise[1]. They were replaced by wire-wound resistors which have high stability, precision and are free from noise. After having made these replacements the long-term drift was investigated and it did not exceed 10 mv per eight hours (Graph 1-B). Because of the poor high frequency characteristics of wire-wound resisters due to the inductive effects and interwinding capacitance, metal film resistors will be used in the circuit. Metal film resistors have stability and low temperature coefficient comparable to those of the wire-wound resistors (Table 2). In addition, metal film resistors have a high frequency characteristic. The resistance of the IRC moulded film resistors MEC RN70 for instance, does not vary with an increasing freq-uency up to 50 Mc/s (see IRC Precision Film Resistors, Bulletin B-16a). Such resistors are used in critical places in the circuit, e.g. plate load. Table 3 gives a list of components used in the multiplier circuit. To avoid excessive temperature changes, all resistors were derated to about half of their power rating.

1 Composition resistors develop a noise voltage only when a direct current is flowing. This type of noise is different from the inherent thermal noise which is common to all resistors. Its magnitude can be as high as ten times that of the thermal noise. This noise is a characteristic of the carbon composition resistors in particular. It depends on the physical properties other than temperature and resistance value. Its frequency distribution is not uniform. Film resistors have lower noise amplitude than the composition resistors, whereas the wire-wound resistors have the lowest.

Table 2 — Fixed Resistors Characteristics

TYPE OF RESISTORS	STABILITY %	TEMP. COEFF. %	VOLTAGE COEFF. %	FREQUENCY CHARACT.	EXCESSIVE NOISE (MV/V)	COST
FIXED COMPOSITION	25	$\pm$ (5–35)	.02 ($\frac{1}{4}$ – $\frac{1}{2}$ w) .035(1 – 2 w) for 1000 Ω & above, not defined for lower values	High	2 – 6	Low
FIXED FILM a) Deposited Carbon	1	– (.01–.05)	Max. .002	Very high especially for metal film type	.03	Fair
b) Boron Carbon		– (.005–.02)			– – –	Fair
c) Metal Film		+ .0025–.03			.02	Can be high
FIXED WIRE-WOUND (Power and Low Power)	1	Max. .03 above 10 Ω .065 below 10 Ω	Negligible	Low	None	High
FIXED WIRE-WOUND (Accurate or Precise)	0.01	.003–.021	Negligible	Low	Excessively None	High

(1) Temperature Coefficient is the ratio of the change of resistance with temperature to the resistance at a specified temperature (25° C).
(2) Voltage Coefficient: change of the resistance with applied voltage.
(3) Stability: change of resistance under working conditions.

Table 3 — List of Components Used in the Multiplier

Circuit Reference	Description	Quantity
R1	Resistor: variable, wire-wound 500 Ω ± 10%, 2w	2
R2	Same as R1	
R3	Resistor: variable, wire-wound 50K Ω ± 10%, 2w	2
R4	Same as R3	
R5	Resistor: variable, wire-wound 100K Ω ± 10%, 2w	1
R6	Resistor: fixed, composition 470K Ω ± 10%, 1w	2
R7	Resistor: fixed, composition 220 Ω ± 10%, ½w	4
R8	Resistor: fixed, metal film 220 Ω ± 1%, 1w	4
R9	Resistor: fixed, metal film 100 Ω ± 1%, 1w	5
R10	Same as R9	
R11	Same as R7	
R12	Same as R9	
R13	Same as R8	
R14	Same as R8	
R15	Same as R7	
R16	Same as R9	

Table 3 (Continued)

Circuit Reference	Description	Quantity
R17	Same as R8	
R18	Same as R9	
R19	—	
R20	Same as R7	
R21	Same as R6	
R22	Resistor: fixed, metal film 33 Ω ± 1%, 1w	3
R23	Resistor: fixed, metal film 1000 Ω ± 1%, 1w	2
R24	Resistor: fixed, metal film 100K Ω ± 1%, 1w	4
R25	Same as R24	
R26	Same as R24	
R27	Same as R24	
R28	Same as R22	
R29	Same as R23	
R30	Same as R22	
R31	Resistor: fixed, metal film 150K Ω ± 1%, 1w	2
R32	Resistor: fixed, metal film 120K Ω ± 1%, 1w	1
R33	Same as R31	

Table 3 (Continued)

Circuit Reference	Description	Quantity
R34	Resistor: fixed, metal film 10K Ω ± 1%, 1w	1
C1	Capacitor: fixed, paper 1Mf, 600v	2
C2	Same as C1	
C3	Capacitor: fixed, paper .5Mf, 600v	1
C4	Capacitor: fixed, mica 1000MMf, 200v	3
C5	Same as C4	
C6	Same as C4	
B	Battery: dry 6volts	1
S1	Switch: SPST	1

<u>Experiments and Results</u>

After the multiplier had been modified to operate in four-quadrants, a model was built and tests were carried out to check its performance.

After being balanced by the procedure outlined in Appendix II, the multiplier was tested. The tests were made in two different methods. In either of both methods the same signal (positive or negative) was applied to the inputs of the multiplier. i.e. the multiplier was tested as a squaring device.

The methods employed in testing the multiplier were:

1- the application of a sinusoidel signals to both inputs and measurement of the output signal. The frequency used for such test was 10Kc/s. Curve 1 of Graph 2 gives the plot of output against input of the multiplier.

2- the application of a pulse or a square wave to both inputs and the measurement of the pulse magnitude at the output. As a pulse is a summation of a fundamental frequency and its higher harmonies, such a test gives , although rough, an over-all check for the multiplier performance. Curve B of Graph 2 is a plot of the output versus input of the multiplier. The employed pulse has a duration of 0.3 msec. with a repetition rate of 5 msec.

Both curves A and B of Graph 2 are approximately the same and thus it is logical to assume that using pulse signal, for testing, is a rough check of the performance of the system.

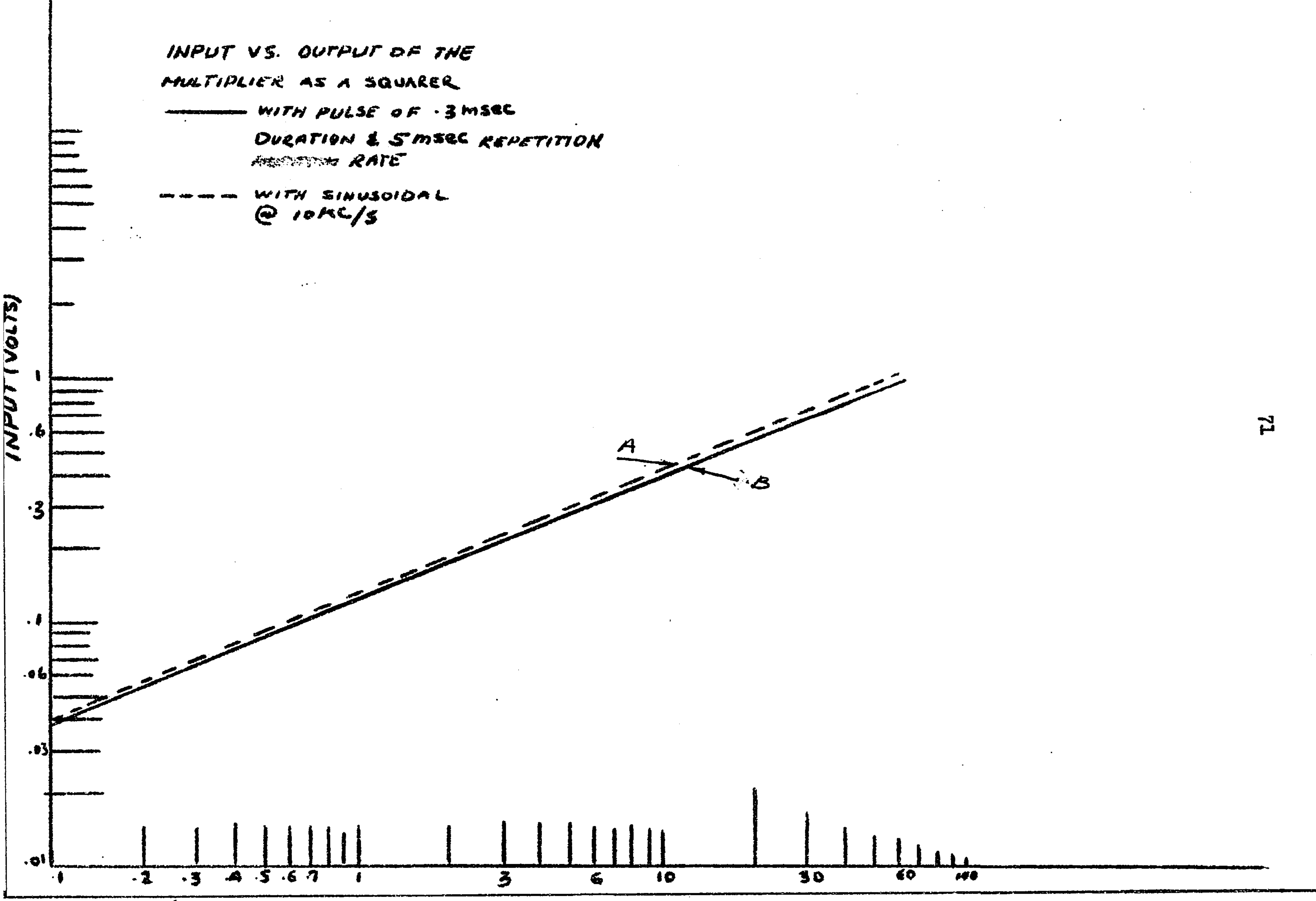
Log-Log scales are used to draw the curves A and B. From either one of the curves, the equation of the multiplier, used as a squaring device, is a parobolic. The equation is calculated from the linear curve and it is

$$Z = 63x^2 mv \tag{59}$$

where x is the input voltage in volts, and Z is the output voltage (in mv). The equation could also be derived by using the methods described in Appendix III and Appendix IV. The former gives the constant as 62.7 while the latter gives it as 62.75.

The bandwidth of the multiplier was measured by applying a constant low frequency signal to one of its inputs, and a variable frequency signal to the other input. The variable frequency was changed until the magnitude of the output magnitude reached 0.707 of its amplitude at mid frequence. At this point, the frequency was measured to give the bandwidth of the multiplier. The bandwidth was about 1.2 Mc/S.

GRAPH 2


INPUT VS. OUTPUT OF THE
MULTIPLIER AS A SQUARER
WITH PULSE OF ·3 mSEC
DURATION & 5 mSEC REPETITION
RATE
WITH SINUSOIDAL
@ 10KC/S
INPUT (VOLTS)
A
B
·01
·03
·06
·1
·3
·6
1
·1
·2
·3
·4
·5
·6
·7
1
3
6
10
30
60

SECTION 4

CONCLUSION

From the feview of analog multipliers, Section 2, one can deduce the following:

Many different methods are used to realize the operation of multiplication. Some of the multipliers give the product of an instantaneous value of the multiplied quantities, while others produce the product of the sampled values of the input quantities (Celinski and Rimawi, 1963). Table 4 gives a 'tree-like' distribution of multipliers.

The variety of multipliers and their characteristics makes it necessary for a choice of a single multiplier to examine all existing multipliers. Therefore, Table 1 was constructed where multipliers are classified according to their accuracy and bandwidth in order to facilitate this task.

The experiments have been carried out on a square-law multiplier to convert it to a four-quadrant type (see Section 3). The results have shown that it can be used in the noise measurements (see Section 1) since it has an accuracy of 1% and a bandwidth of about 1 Mc/s. Moreover, its long-term drift is about 10 mv in eight hours. Its input voltage is limited to ± 1 volt (peak to peak) but it can be extended by using vacuum tubes having a wide range of parabolic characteristics.

However, the analysis of the different multipliers has shown that the multipliers other than the one described in Section 3, can be used as well for the same purpose. For example, a square-law multiplier using a diode ring, has a sufficient accuracy, bandwidth, and inexpensive. Its limited input range of the order of 150 mv can be considered as its drawback.

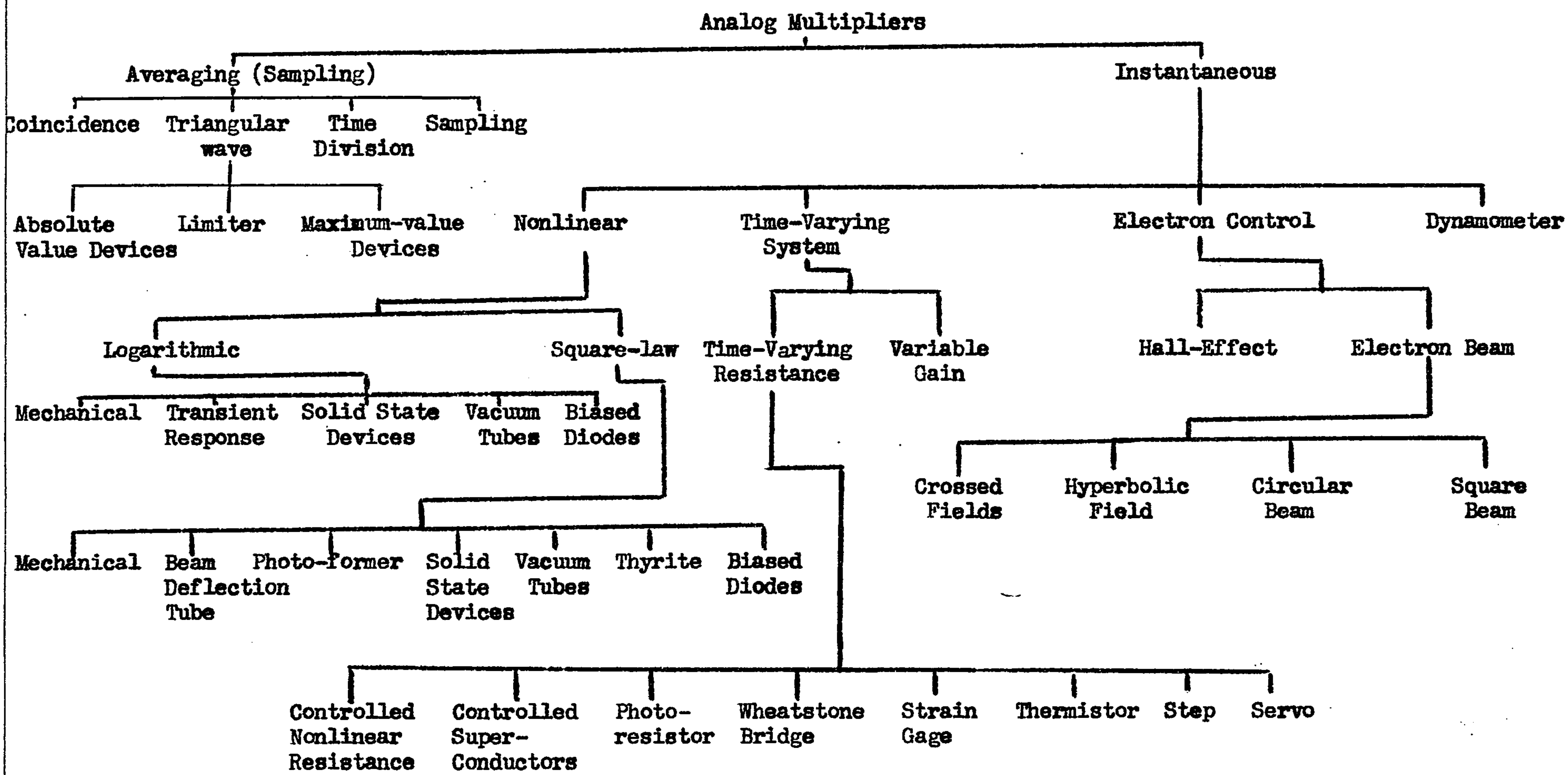


Table 4 — Distribution of Analog Multipliers

APPENDIX I

ERRORS IN NOISE MEASUREMENT

The analysis of the operation of the fourth method used in noise measurements, described in "Section 1", was based on ideal components. The following analysis will be based on the assumption of a non-ideal multiplier with the other components left ideal. The multiplier that is described in "Section 3", will be in error when a complete balance is not achieved. The error of such a multiplier, with the addition of the third order term in the current-voltage relationship is

$$\Delta xy = (b_3-b_1)x + (b_3-b_2)y + (c_3-c_1)x^2 + (c_3-c_2)y^2 + (d_3-d_1)x^3 + (d_3-d_2)y^3 + 3d_3xy\,(x+y)$$

where x,y are the inputs to the multiplier, b,c,d, are the constants of the cascodes used in the multiplier, and Δxy is the error in the product. If this error is introduced in the equations which are derived in "Section 1", then the noise measurement will be inaccurate. This can be shown as follows: Equation 1-a will be

$$\begin{aligned} A_1A_2 = K_1K_2K_m \Big[& 2c_3(x(t)+y(t)+n_1(t))(x(t)-y(t)+n_2(t)) + (b_3-b_1)(x(t)+y(t)+n_1(t)) \\ & + (b_3-b_2)(x(t)-y(t)+n_2(t)) + (c_3-c_1)(x(t)+y(t)+n_1(t))^2 + (c_3-c_2) \\ & (x(t)-y(t)+n_2(t))^2 + (d_3-d_1)(x(t)+y(t)+n_1(t))^3 + (d_3-d_2)(x(t)-y(t)+n_2(t))^3 \\ & + 3d_3(x(t)+y(t)+n_1(t))(x(t)-y(t)+n_2(t))(x(t)+y(t)+n_1(t)+x(t)-y(t)+n_2(t)) \Big] \end{aligned} \qquad (1\text{-b})$$

where $2K_mc_3 = K_M$

Then the output of the integrator will be (after dropping the zero terms such as the mean of a random quantity, covariance of uncorrelated quantities, and the third moment of quantities that are assumed to have a gaussian probability densities):

$$\overline{A_1A_2} = K_1K_2K_m \left[2c_3(\overline{x(t)^2}-\overline{y(t)^2}) + (c_3-c_1)(\overline{x(t)^2}+\overline{y(t)^2}+\overline{n_1(t)^2}) + (c_3-c_2)(\overline{x(t)^2}+\overline{y(t)^2}+\overline{n_2(t)^2})\right] \qquad (2\text{-b})$$

For null conditions at the output, one has

$$\overline{x(t)^2}(2c_3+2c_3-c_1-c_2) = \overline{y(t)^2}(2c_3-2c_3+c_1+c_2) + (c_1-c_3)(\overline{n_1(t)^2}) + (c_2-c_3)\overline{n_2(t)^2} \qquad (4\text{-b})$$

If $c_1 = c_2 = c_3 + \Delta c_3$, where Δc_3 is the unbalance in the c coefficients, then equation 4-b becomes

$$\overline{x(t)^2}(2c_3-2\Delta c_3) = 2c_3\overline{y(t)^2}(1+\Delta c_3/c_3) + \Delta c_3(\overline{n_1(t)^2} + \overline{n_2(t)^2})$$

$$\text{or} \quad \overline{x(t)^2} = \overline{y(t)^2}\,(1+\frac{\Delta c_3}{c_3})/(1-\frac{\Delta c_3}{c_3}) + \frac{\Delta c_3(\overline{n_1(t)^2}+\overline{n_2(t)^2})}{2c_3\,(1-\Delta c_3/c_3)} \qquad (4\text{-c})$$

assuming $\frac{\Delta c_3}{c_3} \ll 1$, and using the binomial theorem, equation 4-c approximately will be

$$\overline{x(t)^2} = \overline{y(t)^2}(1+\frac{2\Delta c_3}{c_3}) + \tfrac{1}{2}(\overline{n_1(t)^2}+\overline{n_2(t)^2})\,\frac{\Delta c_3}{c_3} \qquad (4\text{-d})$$

Then the fractional error in reading $\overline{x(t)^2}$ is

$$\Delta_x = \frac{2\Delta c_3}{c_3} + \frac{1}{2}\frac{\Delta c_3}{c_3}\,(\frac{\overline{n_1(t)^2} + \overline{n_2(t)^2}}{\overline{y(t)^2}}) \qquad (\text{I-a})$$

To analyse this error expression let:

(1) $\frac{\Delta c_3}{c_3} = 1\%$, and $\overline{n_1(t)^2} = \overline{n_2(t)^2} \ll \overline{y(t)^2}$

then the error in measuring $\overline{x(t)^2}$ is 2%

(2) $\frac{\Delta c_3}{c_3} = 1\%$, and $\overline{n_1(t)^2} = \overline{n_2(t)^2} = \overline{y(t)^2}$

then the error in measuring $\overline{x(t)^2}$ is 3%

Thus for at least 1% error in a noise measurement, the $\frac{\Delta c_3}{c_3} = .3\%$ on the assumption that the background noises of the amplifiers, referred to their inputs, are equal to the measured known noise y and the other components of the system are ideal.

APPENDIX II

BALANCE PROCEDURE

In the plate current expression

$$i_p = a + be_g + ce_g^2$$

the constant (a) can be identified as the d-c plate current at the operating point, constant (b) is the transconductance, and constant (c) is proportional to the rate of change of the transconductance. Referring to figure 46 the balance can be achieved in the following steps:

1- The constant "a" is balanced by shorting both inputs of the multiplier and adjusting R_5 to have zero output.

2- The constant "b" can be varied by varying the lower grid of the cascode. Thus to make the constant "b" equal for the three cascodes, it is necessary to adjust the lower grid bias of the cascodes. This is done by applying a sinusoidal signal to the x input, shorting the other, and varying R_1 to minimize the fundamental part at the output. Then the procedure is reversed by applying the signal to the y input and varying R_2 to minimize the fundamental part at the output.

3- The constant "c" can be varied by adjusting the bias of the upper grid of the cascode. Thus to make the constant "c" equal for the three cascodes, it is necessary to adjust the upper grid bias of the cascodes. A sinusoidal signal is applied to the x input while the y input is shorted. Then R_3 is adjusted to minimize the second harmonic at the output. Then the same signal is applied to the y input and the x input is shorted. Now R_4 is adjusted to minimize the second harmonic at the output.

APPENDIX III

The principle of least squares, which was formulated by Legendre, may be expressed as follows:

The most probable value of any observed quantity is such that the sum of the squares of the deviations of the observations from this value is the least. The method is used to fit a formula or a curve to a set of experimental data. The method is applied to the following specific example:

If the plate current of a triode is given by

$$i_a = a + bv + cv^2$$

where i_a is the actual plate current, a,b,c are constants, and v is the grid signal.

Values of i and v are assumed to be given and it is required to give the formula that gives the best fit of such experimental data. For the best fit, the error due to fitting must be minimized:

$$e = E(i - i_a)^2 \qquad \text{III-a}$$

where e is the error, i is the given current, and E is the mathematical expectation (or average) of the quantity involved. Expanding the right hand side of the above equation and substituting for i_a, the error will become

$$e = E\left[i^2 - 2i(a+bv+cv^2) + (a+bv+cv^2)^2\right] \qquad \text{III-b}$$

To minimize the error 'e', its partial derivatives with respect to a,b, and c should be zero. The conditions obtained from such differentiation are the following

$$\frac{\partial e}{\partial a} = -E(2i) + E(2a) + E\left[2(bv+cv^2)\right] = 0 \qquad \text{III-1}$$

$$\frac{\partial e}{\partial b} = -E(2iv) + E(2bv^2) + E\left[2v(a-cv^2)\right] = 0 \qquad \text{III-2}$$

$$\frac{\partial e}{\partial c} = -E(2iv^2) + E(2cv^4) + E\left[2v^2(a-bv)\right] = 0 \qquad \text{III-3}$$

The constant, a,b and c can be found by solving simultaneously the equations 1,2, and 3. The constants are given by the following relations

$$a = E(i) - bE(v) - cE(v^2), \qquad \text{III-4}$$

$$b = \Delta_b/\Delta, \qquad \text{III-5}$$

and $$c = \Delta_c/\Delta \qquad \text{III-6}$$

where

$$\Delta = \begin{vmatrix} 1 & E(v) & E(v^2) \\ E(v) & E(v^2) & E(v^3) \\ E(v^2) & E(v^3) & E(v^4) \end{vmatrix} \qquad \text{III-7}$$

$$\Delta_c = \begin{vmatrix} 1 & E(v) & E(i) \\ E(v) & E(v^2) & E(iv) \\ E(v^2) & E(v^3) & E(iv^2) \end{vmatrix} \qquad \text{III-8}$$

$$\Delta_b = \begin{vmatrix} 1 & E(i) & E(v^2) \\ E(v) & E(iv) & E(v^3) \\ E(v^2) & E(iv^2) & E(v^4) \end{vmatrix} \qquad \text{III-9}$$

APPENDIX IV

DIFFERENCE EQUATIONS TECHNIQUE

A mathematical formula can be found to fit an experimental data by using the difference equations means. This can be illustrated by treating the same example used in Appendix III.

The expression $i_a = a + bv + cv^2$ will be a good representation of a given data of i and v, if the second difference of i, $\Delta^2 i$ is constant. The second difference of current is equal

$$\Delta^2 i_n = \Delta i_{n+1} - \Delta i_n \qquad \text{IV-1}$$

where $\Delta i_n = i_{n+1} - i_n$ IV-2

and i_{n+1}, i_n are two consecutive readings of i.

Using the above defined differences, the constants a,b, and c can be derived from the following relations:

$$b = \left|\Delta i/\Delta v\right|_{v = o} \qquad \text{IV-3}$$

$$c = \tfrac{1}{2}(\Delta^2 i/\Delta v^2) \qquad \text{IV-4}$$

$$a = i\Big|_{v = o} \qquad \text{IV-5}$$

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