A PHYSICALLY-BASED DIGITALLY-SIMULATED HYDROLOGIC MODEL

by

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ABSTRACT

Recent developments in computer technology have enabled the hydrologist to carry out complex hydrologic simulations of watersheds. Parallel with the computer development has been the development of the physical and mathematical description of the various hydrologic subsystems.

This study has combined the recent developments in physical hydrology to produce a physically-based digitally-simulated hydrologic model. This model uses partial differential equations solved by numerical methods to describe saturated-unsaturated groundwater flow in two dimensions and overland flow in one direction.

Monte Carlo simulation was used to examine the effect that variations in the model parameters have upon the model output of surface and groundwater runoff. This simulation indicated that a large amount of error introduced into the model properties of permeability, soil moisture functional relationships and overland flow friction coefficients resulted in small errors in surface runoff, but large errors in estimates of subsurface runoff.

The model was also used to provide theoretical proof of the variable source area concept which suggested that only a small portion of some watersheds contribute to storm runoff.

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NOTATIONS

A_i	initial abstraction
a, n_1	constants for a particular soil in a given condition
B	compressibility of water
C	specific moisture capacity
C_N	constant
d	flow depth
E	free water evaporation
E_{ep}	land pan evaporation
e	volume strain
e_1	height of roughness
$e_{1,2}$	vapour pressure
e_s	saturation vapour pressure
f_1	final constant rate of infiltration
f_c	transmission
f_e	infiltration rate at $t = \infty$
f_o	infiltration rate at $t = 0$
f(t)	infiltration rate at time t
G	shearing modulus of elasticity
g	acceleration due to gravity
I	flux across upper boundary cm/sec (infiltration or evaporation)
i	rainfall intensity
K	hydraulic conductivity
K_o	saturated hydraulic conductivity

K_m	kinematic eddy viscosity
K_r	relative conductivity = $\frac{K}{K_o}$
K_w	vapour eddy diffusivity
k	specific permeability
L	best fit parameter
M_f	mass infiltration
n	porosity
p	pressure
Q	discharge
q	flow per unit width
q_i	rate of overland flow (rainfall infiltration)
q_R	rate of inflow from rainfall
R	rainfall
R_{ex}	rainfall excess
R_e	Reynolds number
S	fractional saturation
S_p	potential soil moisture storage value
S_f	friction slope
S_o	slope of ground or channel bottom slope
T_1	time after initial abstraction is satisfied
t	time
t_2	time of duration
u	viscosity, dynamic
$\overline{u}$	local temporal horizontal velocity of air
v	flow velocity
v_d	the relative velocity of the water with respect to the soil

v_s	the velocity of soil grains
v_w	the velocity of water
x,y,z	coordinate directions
z	elevation head
ε	random error
θ	moisture content
λ	vertical consolidation coefficient
λ_{ij}	stress
μ	kinematic viscosity
ρ	density of fluid
ρ_s	density of soil
p_a	mass density of air
ρ_{xy}	correlation coefficient
$\emptyset$	total hydraulic head = $\psi + z$
ψ	pressure head
w	specific weight

CHAPTER 1

INTRODUCTION

1.1 Motivation

Amorocho and Hart (1964) suggested that Hydrology was divided into two major fields of study. "The first group, physical hydrology, espouses the pursuit of scientific research into the basic operation of each component of the hydrologic cycle in order to gain a full understanding of their mechanisms and interactions. Although the immediate motivation of an individual researcher may not transcend the narrow confines of a set of special phenomena, it is implicit that a full synthesis of the hydrologic cycle may eventually be sought. "Basically this is the scientific, hydrologic, component by component approach used by many hydrologists while the second group, the systems investigation is used largely by applied hydrologists. Systems investigation is motivated by the need to establish workable relationships between measurable parameters in the hydrologic cycle to be used in solving pressing practical hydrological problems. These people generally hold that the vast complexity of the systems involved in these studies, and the inadequacy of the knowledge now available and the knowledge likely to exist in the foreseeable future make the possibility of a full synthesis so remote in most cases that it must be disregarded for practical purposes. Under these premises a logical approach would consist of measuring these observable variables in the hydrologic cycle which appear pertinent to the problem, and then attempting to establish explicit algebraic relationships between them. It is hoped that these relationships hold true within the range of conditions normally encountered in practice."

The two divisions and their respective sub-divisions are shown in Fig. 1.1, a diagram outlining the methods of hydrologic simulation.

The second group, systems investigation, falls into two principal categories which are called "parametric hydrology" and "stochastic hydrology". Parametric hydrology is the development of explicit relationships among observed physical parameters in the hydrologic sequences.

Stochastic hydrology is the use of statistical characteristics of hydrologic variables to generate non-historic sequences to which certain levels of probability can be attached.

Until recently the systems investigation predominated in applied hydrology. In particular there has been a reluctance amongst engineering hydrologists to venture too far from the established empirical method. However rapid advances in other fields such as soil physics, agronomy and numerical methods have now made possible the first group's goal for a complete description of the hydrological cycle utilizing physical and mathematical descriptions of the various sub-systems of the hydrologic cycle. Freeze and Harlan in 1969

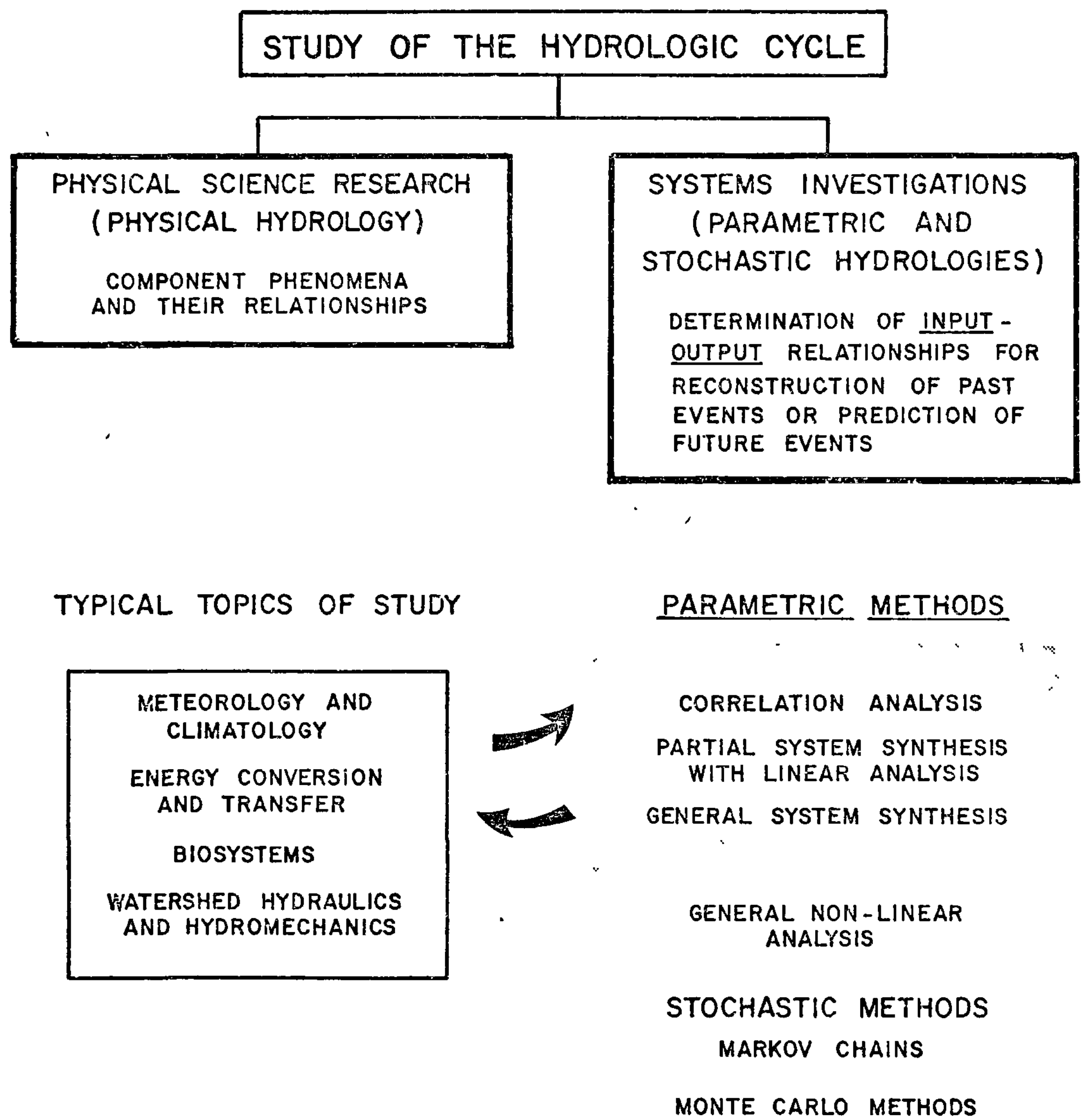


Fig. 1.1 Summary of hydrologic analysis methods (after Amorocho 1964)

presented a blueprint for the development of such a model. They proposed a physically-based mathematical model in which "the component, time-dependent hydrologic processes are represented by a set of partial differential equations interrelated by the concepts of mass and of momentum. These equations together with the boundary conditions that define the shape and boundary properties of the basin, comprise the composite boundary value problem that is the hydrologic response model."

It is one of the objectives of this study to develop such a physical-response model.

Amerocho's hydrologic sub-divisions, physical hydrology and systems investigation, are rather arbitrary and can be subject to endless argument concerning classification. For example, two simpler divisions are: deterministic hydrology, which describes the causative skeleton of hydrologic process and statistical and stochastic hydrology which describes the uncertainty in hydrology. A model may be termed stochastic or deterministic according to whether or not it contains random variables. If any of the model variables are thought of as having probability distribution then the model is stochastic.

However even Amorocho agrees that in modern hydrology these divisions are more fictitious than real. For, in the future it will become common to build into the deterministic model an estimate of uncertainty. Rather than producing one concrete answer the model will produce a range of answers, each having a probability of occurrence.

Thus, a second objective of this thesis is to introduce a degree of uncertainty into the proposed physically-based model. This will allow presentation of such quantitative results as surface flow, in the form of statistical probability functions rather than single-valued solutions of unknown reliability.

While the synthesis of a physically-based model is continuous, the expressions describing the component phenomenon have been developed independently by workers in many different fields. To assess the level of development of each component that is possible with present methodology, the following review of the model components has been made starting from earlier empirical development to the latest mathematical physical modelling techniques.

1.2 Literature review

1.2.1 Infiltration and soil moisture flow

The first mathematical descriptions of infiltration were empirical formulas developed to described the results of infiltrometer tests.

While it was recognized that the infiltration rate decreased with time until it approached a more or less constant value, there was no general agreement as to a physical explanation for this decreasing rate. Horton (1938) who recognized the significant role infiltration played in the hydrologic cycle expressed the view that the reduction in infiltration rate was more probably controlled by factors operating at the soil surface than by flow process in the body of the soil. His description of infiltration is given by:

$$f(t) = fe + (fo-fe)e^{-Lt} \qquad 1.1$$

where f is infiltration rate at time t

fe and fo are infiltration rates at $t = \infty$ and $t = 0$

L is a best fit parameter

Another approximation equation 1.2 was proposed by Ayers (1959) who suggested that infiltration involved storage as well as transmission. He suggested that the storage capacity of the soil will result in an initial abstraction from precipitation and thereafter, with time, the transmission capacity of the soil accounts for the subsequent reduction of the amount of precipitation which is available for runoff.

$$R_e = R - A_i - f_c T_1 \qquad 1.2$$

where R_e = precipitation excess

R = total precipitation

A_i = initial abstraction

f_c = transmission rate

T_1 = time after A_i is satisfied or the moisture profile is satisfied

Holtan (1961) has reported a similar approach:

$$f(t) = a(S_p - M_f)^{n1} + f_1 \qquad 1.3$$

where S_p is the potential soil moisture storage value or the volumetric difference between pore saturation and the 15 or permanent wilting percentage for the soil zone above the control layer

M_f is the mass infiltration

f_1 is the final constant rate of infiltration

a,nl are the constants for a particular soil in a given condition

The above examples are sufficient to demonstrate the limitations of the algebraic - empirical infiltration equations. Firstly, many of the methods consider only infiltration from a ponded surface, which is a poor model for rainfall. Even when sprinklers are used, the data is good for only that rate of application and assumptions must be made for other rainfall rates. Secondly, the parameters developed have little or no physical meaning, in that they cannot be determined or estimated from knowledge of the soil, surface cover, etc.

Among the first to demonstrate the importance of the flow process within the soil in controlling infiltration and the forerunner of modern infiltration theory was Buckingham (1907) who in 1907 proposed that the rate of water movement in an unsaturated soil is proportional to the gradient of capillary potential of the soil water, that is

$$0 = K \frac{\partial\psi}{\partial x} \qquad \text{or} \qquad Q = K(\frac{\partial\psi}{\partial\theta}) \frac{\partial\theta}{\partial x} \qquad 1.4$$

where Q is the discharge through the soil

K is the capillary conductivity

ψ is the capillary potential pressure gradient

θ is the water content

Little else was done toward the understanding of water movement in unsaturated soil until 1931 when Richards (1931), building on

Buckingham's work developed the general equation of flow based on Darcy's law and the equation of continuity, which is given by

$$\frac{\partial}{\partial x}[pk(x,y,z)\ \frac{\partial Q}{\partial x}] + \frac{\partial}{\partial y}[pk(x,y,z)\ \frac{\partial Q}{\partial y}] +$$

$$\frac{\partial}{\partial z}[pk(x,y,z)\ \frac{\partial Q}{\partial z}] = p\ \frac{\partial \theta}{\partial t} + \theta\ \frac{\partial p}{\partial t} \qquad 1.5$$

x,y are the horizontal coordinate directions

z is the vertical coordinate direction

t is the time

p is the density of fluid

Q is the total hydraulic head, $= \psi + z$

For one dimensional flow equation 1.5 reduces to:

$$\frac{\partial}{\partial z}[k(\psi)(\frac{\partial \psi}{\partial z} + 1)] = c(\psi)\frac{\partial \psi}{\partial t} \qquad 1.5a$$

where

$$c(\psi) = \frac{\partial \theta}{\partial \psi}$$

Richards also showed that unsaturated hydraulic conductivity was a function of the pressure head ψ and water content. It was this variation of the conductivity that made the solution of Richard's partial differential equation so powerful.

With the advent of the high speed computer in the 1950s solutions were now possible to equation 1.5 and its water content counterpart for a wide variety of boundary conditions and soil properties. Significant contributions were made by Klute (1952) who first used the finite-difference numerical solution; by Philips (1952) in his seven part paper on infiltration; by Hanks and Bowers (1962) for layered soils; by Rubin and Steinhart (1963) for the constant-rate rainfall; by Whisler and Klute (1965) for the consideration of hysteresis in the functional relationship between K, ψ, C and θ and by Staple (1966) and Rubin (1967) for the redistribution problem. Additional work was done by Brooks and Corey (1964) building on the work of Burdine (1953) to establish an empirical relationship between the $K(\psi)$ and $\theta(\psi)$ relationships.

From results of the soil water equation it is possible to determine the portion of precipitation which infiltrates into the soil and the remaining portion which is available for overland flow. This routing of the water at the ground surface into either infiltration or overland flow is carried out by a mathematical model based on physical processes. There is no need to make use of the previously-mentioned infiltration formulas, the concept of infiltration capacity, outdated parameters such as wilting point or questionable results from ring infiltrometers.

The parameters needed to utilize the soil water equations are measurable parameters of physical significance such as the initial soil moisture conditions and the relation between permeability, moisture content and soil moisture tension for the soil in question.

The excellent work of the soil physicists in physical modelling of the infiltration and soil moisture flow process has been duplicated in the ground water flow system by ground water hydrologists.

1.2.2 Ground water flow

Between 1940 and 1960 hydrogeology dealt with local water-well hydraulics and well field designs. (Walton, 1953; Hantush and Jacob, 1955). With the need for more watershed management, regional ground water flow studies have been emphasized. The foundation of such studies was laid by Hubbert in 1940 who expressed ground water flow in terms of a three-dimensional potential field. With Hubbert's definition of potential, the water table became another equipressure line and not a surface of discontinuity between the zone of saturation and the unsaturated zone. Hubbert's flow based on an analytical solution of Laplace's equation consisted of "a set of flow lines, everyone perpendicular to the equipotential lines in that two-dimensional plane. Along a single flow line, water will enter the region at a point of high potential (source) and it will leave the region at a point of low potential (sink). Along all streamlines the amount of work required to drive a unit mass of the fluid from one equipotential surface to another is precisely the same."

Toth in 1962 improved Hubbert's model by superimposing a sinusoidal shape on the regional slope of the basin. Toth showed that the groundwater flow system could be divided into local, intermediate and regional flow systems.

Following Toth's work, Freeze and Witherspoon (1966) utilized numerical solutions to solve the Laplace and Richard's equations to calculate the distribution of fluid potential for any steady-state, three-dimensional regional groundwater model. They discovered among other things that in a gently sloping region bisected by a deeply incised river valley, recharge is concentrated in two locations (a) at the upstream end of a recharge area and (b) at the break-in slope above the steep valley bank. They also concluded that in the groundwater flow systems the rate of groundwater recharge and discharge is relatively constant with time at any given location but varies greatly throughout a basin.

The regional groundwater model was further developed by Freeze (1971) in a landmark paper where he developed a three-dimensional finite difference model for the treatment of saturated/unsaturated transient flow in small non-homogeneous, nonisotropic geologic basins. Thus a physical groundwater model is presently available and this model with added modifications was used to model the groundwater flow in the physically-based hydrologic model of this thesis.

The next stage is to link the groundwater model with a physical model which describes overland and channel flow. This is one of the objectives of this thesis.

1.2.3 Overland flow and Channel flow

The precipitation excess that moves over the land surface to stream channels after infiltration is called overland flow or sheet flow. It is that part of the total surface runoff not confined to the stream channels. Overland flow was first investigated by Horton (1938) who produced a semi-empirical formula:

$$q_1 = \sigma \tanh^2[.922\ t_z(\frac{\sigma}{nL})^{0.5} S_o^{\ 0.25}] \qquad 1.6$$

where q_1 is the rate of overland flow at lower end of an elemental turfed or paved strip

n is the retardance coefficient representing surface roughness

L is the effective length of flow

σ is the supply rate

S is the average slope

t_z is the time of duration since supply began

This formula was devised for turbulent flow and a high discharge.

Keulegan (1944) was the first to carry out a theoretical investigation of overland flow utilizing the shallow water or St. Venant's hydrodynamic equations of continuity and momentum for unsteady one-dimension non-uniform, spatially varied open channel flow.

The equation of continuity and the equation of momentum are given respectively by:

$$d\,\frac{\partial v}{\partial x} + v\,\frac{\partial d}{\partial x} + \frac{\partial d}{\partial t} = q_i - I \qquad 1.7$$

and

$$\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial x} + g\,\frac{\partial d}{\partial x} = g(S_o - S_f) - \frac{v}{d}\,(q_i - I) \qquad 1.8$$

where g is the acceleration due to gravity

q_i is the channel inflow rate

I is the channel infiltration rate

t is the time

x is the coordinate direction

d is the flow depth

v is the flow velocity

S_o is the channel bottom slope

S_f is the friction slope

The friction slope S_f is given by:

$$S_f = \frac{f{q_i}^2}{8gd^3} \tag{1.8a}$$

where f is the Darcy-Weisbach friction factor.

Experimental analyses were made by Izzard, Woo and Brater (1944) to confirm the above theoretical analysis. The study and development of overland flow from now on closely paralleled the development of open channel work for which the above equations were first developed in the nineteenth century. This is justified by noting that except in highly urbanized areas, overland flow in the classical sense (Horton, 1938) is rarely observed in nature. Rather it occurs through small conveyances and therefore approaches channel flow.

With the advancement of numerical methods and digital computers it has recently been possible to solve the St. Venant equations completely both for overland flow and channel flow.

Finite difference solutions have been presented by Isaacson (1956), Morgali and Linsley (1965) and Brakensiek (1966).

In order to reduce the computational difficulties and solution time encountered in solving the St. Venant equations (eqs. 1.7 and 1.8) the simpler Kinematic Wave Theory was suggested by Lighthill and Whitham (1959). In this theory the equation of momentum 1.8 is approximated by a stage discharge relationship such as equation 1.8a which ignores the acceleration terms of equation 1.8. However no empirical functional form is assumed for relating storage to inflow and outflow as in the storage approach to routing as typified by the Muskingum method (Method of Flood Routing, 1936).

Lighthill, Whitham (1959) and Henderson (1966) showed that for all steep slopes except mountain torrents, the Kinematic Wave Theory adequately describes surface runoff.

Langford and Turner (1973) carried out an experimental study of the application of Kinematic Wave Theory to overland flow and showed the accuracy of the theory was adequate in predicting the behavior of flows over a rough uneven surface.

From the above review of methods describing overland flow it was decided to utilize the simpler Kinematic Wave Theory to describe the overland flow portion of the physical model since it appears for most overland flow conditions to offer just as accurate a solution as the more complex St. Venant equations with less computation time and fewer difficulties.

1.2.4 General System and Physical Models

From the previous discussions it has become evident that the components of the hydrologic cycle can be adequately described and it only remains to combine these components into one physically-based model. The systems modelling group already has developed several general hydrologic systems simulation models. Since any physical model must be able to compete with the systems approach in terms of practical results and utility, it behooves us to examine a typical general systems simulation model such as the Stanford Watershed Model (Crawford and Linsley). The general systems simulation model consists of two principal components, storage elements and transmission routes, connected by a set of nodal points. Such a system is represented by the Stanford Watershed Model as shown in the flow chart of Fig. 1.2.

1.2.4a Systems Model

Here Crawford and Linsley (1966) developed a model utilizing upper and lower storage areas in the subsurface to account for infiltration, excess rainfall inter-flow and percolation to groundwater. Overland flow was simulated utilizing Manning's equation, a formula obtained the amount of surface runoff at equilibrium. The overland flow hydrographs were added to the interflow and groundwater flow hydrographs to form the total channel inflow hydrograph thus providing a separation of flows not available from ordinary stream flow hydrographs.

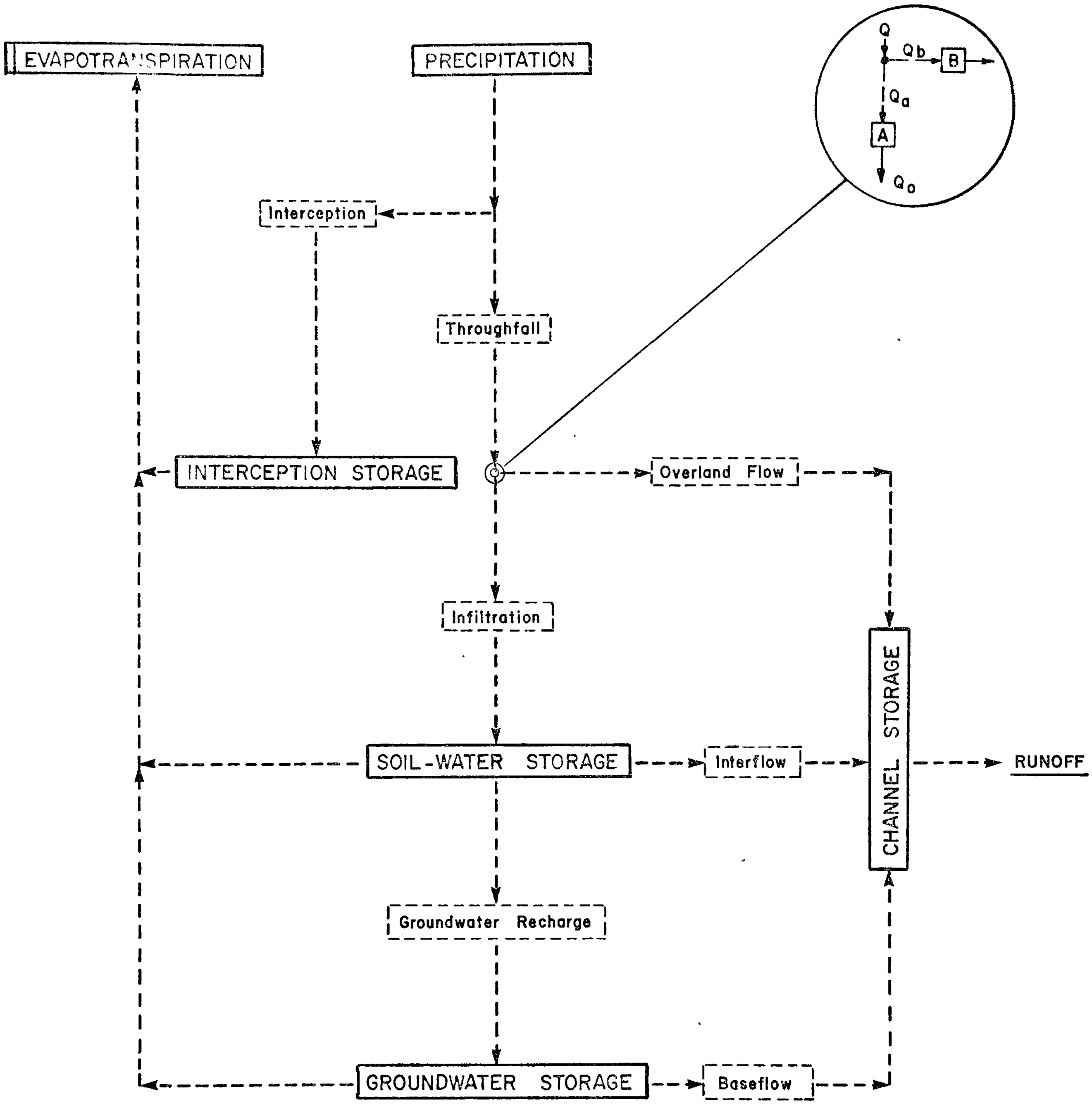


Fig. 1.2 A flow chart of a typical hydrologic systems model (after STANFORD WATERSHED MODEL IV)

The key to this model and most general systems models lies in the modelling of the infiltration processes. This is done in the Stanford model by a storage approach where direct infiltration into the soil profile is modelled by a lower zone and groundwater storage, and increases in temporary storage that results in delayed infiltration are modelled by an upper zone storage.

From empirical functional relationships the available model is first subdivided into surface detention, interflow storage, and infiltration into groundwater storage. Then the amount of water calculated for surface detention is further subdivided by an empirical formula into contributions to overland flow, interflow, and upper zone storage. Moisture in turn is lost from the upper zone by evaporation and percolation to the lower zone and groundwater storage.

The selection of several arbitrary empirical coefficients in the modelling of the infiltration is probably the chief weakness in this approach. The available moisture is arbitrarily separated into different component coefficients that are not physically meaningful. Rather the process should be simulated by a model using parameters that have some physical meaning and that can be measured in the field.

1.2.4b Physical Model

While physical modelling is not as far advanced as the general system modelling, in that physical modelling is not yet applied to the entire water shed, the development of numerical methods and digital computers has led to the rapid advancement of physical modelling.

One of the more complete physical models incorporating many of the physical concepts discussed previously was the one originated by Smith and Woolhiser (1971) in their mathematical simulation of an infiltrating watershed.

Here the kinematic wave equation was used to model overland flow with lateral inflow determined by matching the boundary conditions with the one-dimensional soil water flow equation. The difference between rainfall and soil water infiltration was the lateral input to the surface flow equation. Since the investigators was chiefly interested in surface runoff no attempt was made to describe the groundwater regime.

1.2.5 Uses of the Physical Model

The uses of physical models will be many and varied, ranging from applied hydrology problems of synthesizing design low records for flood and drought studies to those of scientific hydrology where the model may be used to provide insight into the physical processes and aid in evaluating less complex models. However, due to the complexity of the model, the lack of sufficient computer storage and the computer time requirement, it is envisaged that this model at present would be used primarily for scientific purposes.

One recent concept being explored by scientific hydrology is the variable source area or partial area concept. In order to demonstrate the usefulness of such a model it is proposed in this study to use the model to provide an insight into the physical processes of this concept.

The concept of a variable source area was first espoused experimentally by Hursh and Brater in 1941; Hewlett and Hibbert in 1963 and 1967 and Dunne and Black in 1970. This concept considers that under prolonged and heavy rainfall the stream flow producing areas adjacent to the stream channels may grow wider and wider as subsurface moisture conditions permit. This thesis hopes to provide theoretical proof of the experimental work of Dune and Black.

1.2.6 Introduction of Uncertainty

Freeze (1973) in a recent paper introduced the concept of uncertainty to deterministic models where he studied the stochastic analysis of one-dimensional groundwater flow in non-uniform homogeneous media. He examined the multivariate problem where the hydrogeologic parameters are assumed to come from frequency distributions. The use of frequency distributions in describing the hydrologic parameters resulted in a more exact definition of non-uniformity, heterogeneity and anisotropy. According to Greenkorn and Kessler (1969), if the probability density function is independent of orientation, the medium is isotropic; if it is dependent on orientation, the medium is anisotropic; if the distribution is characterized by a finite linear combination of delta functions, the medium is uniform; if not, it is non-uniform.

This thesis will deal with non-uniform homogeneous, isotropic soils and will use frequency distributions to describe the hydrogeologic parameters of permeability, porosity and pore size distribution index. The overland flow parameter, the friction coefficient, will also be described by a frequency distribution.

1.3 Objectives

The objectives of this study are: 1) to construct a digitally-simulated physical-based model by coupling a two-dimensional groundwater model with a one-dimensional overland flow model, 2) to examine through Monte Carlo simulation the effect of uncertainty introduced into the model parameters upon the model output of surface and groundwater runoff, 3) to demonstrate the usefulness of the model in verifying the variable source area concept of runoff.

CHAPTER 2

THEORETICAL DEVELOPMENT OF THE PHYSICALLY-BASED MODEL

2.1 General Description

Figure 2.1 is a schematic diagram of the proposed mathematical model showing a geologic cross section, with overland flow, sub-surface flow and base flow discharging into a channel section.

The flow in the saturated and unsaturated zone will be described in two dimensions and the overland flow in one dimension.

The inflow rate to the overland flow is defined as the excess from rainfall minus the infiltration into the section along AB.

Inflow to the saturated and unsaturated zone will be from rainfall infiltrating directly into the unsaturated zone and indirectly from overland flow.

The physical model parameters are the soil characteristics of conductivity and moisture content, and overland flow roughness coefficients.

The output from this model will consist of infiltration graphs, surface water hydrographs, cross section diagram of soil moisture content, pressure head, hydraulic head and ground water hydrographs.

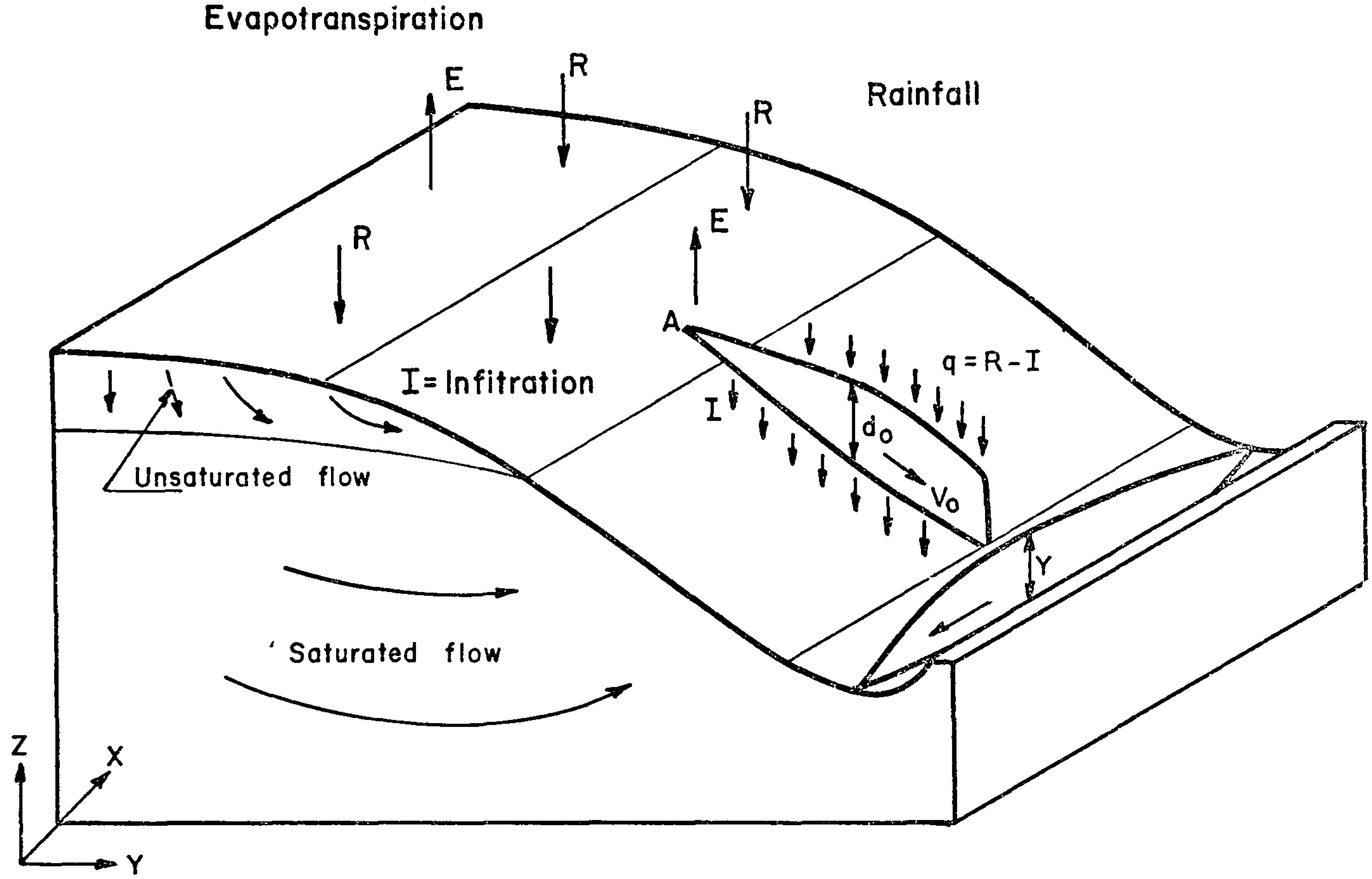


Fig. 2.1 Schematic diagram of the physically - based hydrologic model showing overland flow and subsurface flow discharging into a channel.

2.2 Groundwater Model

To describe the unsteady flow for any ground water node it is necessary to develop an equation of flow that will hold for saturated flow in both confined and unconfined aquifers and for flow in the unsaturated zone. To avoid numerical complexities, flow in the unsaturated zone is confined to only the flow of water and not the actual case of two phase flow of air and water. It is therefore assumed that the air phase is continuous and the air is at atmospheric pressure. This assumption implies that there are no pockets of compressed air in the flow system.

The developed equation is a combination of the saturated equation developed by Jacob (1946) and clarified by Cooper (1966) and the unsaturated equation of Richards (1931). The elastic behavior of the soil skeleton is based on the work of Biot (1956). The development of one integrated equation is similar to the approach used by Verruijt (1969) in his work on the saturated equation. The detailed development of the equation of flow is given in Appendix A.

2.2.1 Basic Equation of Elastic Storage

The basic equation of elastic storage is based upon the combination of 2 equations of continuity for soil and water with an the Darcy equation of motion.

The Darcy velocity can be formulated as:

$$v_d = Sn(v_w - v_s) = -\frac{g\rho_w k}{\mu}(\nabla z + \nabla\psi) \qquad 2.1$$

where v_d = the relative velocity of the water with respect to the soil

v_w = the velocity of water

v_s = the velocity of soil grains

n = the porosity of the soil

S = the fractional saturation

μ = viscosity, dynamic

(The remaining terms are listed in the notation).

The continuity equations are for water

$$-\nabla.(Sn\rho v_w) = \frac{\partial}{\partial t}(Sn\rho) \qquad 2.2$$

and soil

$$-\nabla.[(1-n)\rho_s v_s] = \frac{\partial}{\partial t}[(1-n)\rho_s] \qquad 2.3$$

The combination of equations 2.1, 2.2 and 2.3, in an approach similar to Verruijt's (1969) produces the basic equation of elastic storage:

$$\nabla.[\frac{kg\rho^2}{\mu}(\nabla\psi+\nabla z) = S\rho\frac{\partial e}{\partial t} + \frac{\partial\psi}{\partial t} Sn\rho\ (B'+\frac{\partial S}{\partial\psi})] \qquad 2.4$$

This equation contains two variables: the fluid pressure ψ and e the volume strain. A second equation is then obtained from the mechanical behavior of the soil.

2.2.2 Elasticity of Soil Skeleton

By assuming that the soil skeleton behaves like a perfectly elastic solid, thus utilizing Hooke's law and assuming displacement occurs only in the vertical two directions, a second relationship is established:

$$\frac{\partial e}{\partial t} = \alpha' \frac{\partial \psi}{\partial t} \qquad 2.5$$

where $\alpha' = (1/\lambda+2G)\rho g$, vertical consolidation coefficient 2.5a

λ = Lame's constant

G = shearing modulus of elasticity, also a Lame's constant

Then substituting in Equation 2.4 yields:

$$\frac{\partial}{\partial x}\left[\frac{g}{u}\rho^2(\psi)\, k_{zz}(\psi)\frac{\partial \psi}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{g}{u}\rho^2(\psi)\, k_{zz}\frac{\partial \psi}{\partial y}\right]$$

$$+ \frac{\partial}{\partial z}\left[\frac{g}{u}\rho^2(\psi)\, k_{zz}(\psi)\left\{\frac{\partial \psi}{\partial z} + 1\right\}\right]$$

$$= \left[\frac{\rho(\psi)\theta(\psi)}{n(\psi)}\{\alpha' + B'n\} + \rho(\psi)\frac{\partial \theta}{\partial \psi}\right]\frac{\partial \psi}{\partial t} \qquad 2.6$$

Equation 2.6 can be simplified if it is assumed that the water is incompressible (that is, $B' = 0$) and the soil material will consolidate a negligible amount due to changes in water pressure ($\alpha' = 0$). Then Equation 2.6 for two dimensions reduces to:

$$\frac{\partial}{\partial x}\left[\frac{g}{u}\rho^2\, k_{xx}(\psi)\frac{\partial \psi}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{g}{u}\rho^2\, k_{zz}(\psi)\left\{\frac{\partial \psi}{\partial z} + 1\right\}\right] = \rho\,\frac{\partial \theta}{\partial \psi}\cdot\frac{\partial \psi}{\partial t} \qquad 2.6a$$

which is essentially the partial differential equation first derived by Richards (1931). Its solution for unsaturated flow depends on knowing the functional relationships between k and θ for the particular soil in question.

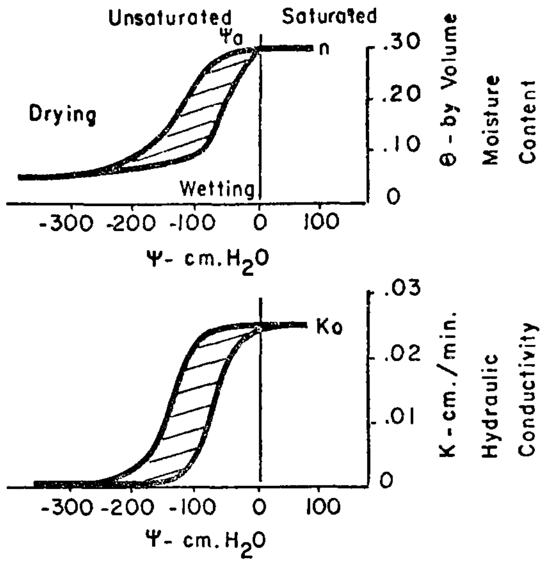
2.2.3 Groundwater Model Parameters

In the unsaturated soil zone the specific permeability k and moisture content θ are both functions of the pressure head ψ for any soil type. The functional relationships are hysteretic in that the curves differ depending on whether the soil is wetting or drying. An example of these characteristic curves is given in Figure 2.2a for a soil known as Del Monte Sand (Liakopoulos, 1965).

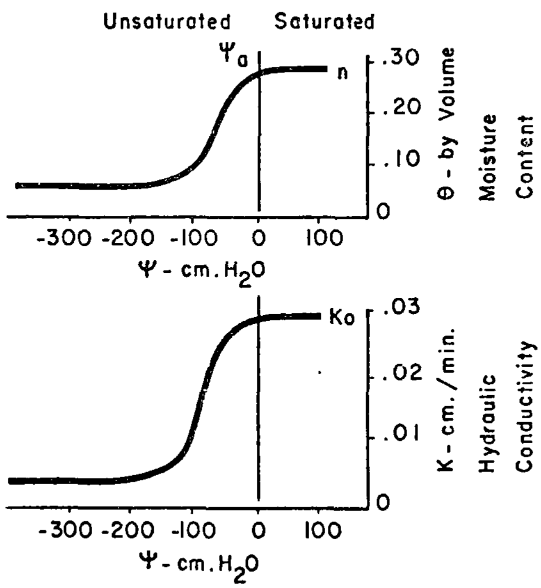
To simplify these functional relationships and reduce computer time, hysteresis is neglected and single value functions are adopted as shown in Figure 2.2b.

With the assumption that the compressibility of water B is negligible, then ρ the density of water becomes a constant.

The vertical consolidation coefficient α' is a step function (Freeze, 1971) as shown in Figure 2.3 and is applicable for confined elastic aquifers. However, Freeze has concluded that for natural flow systems, especially unconfined aquifers, α' can be considered negligible. While the model provides for inclusion of α', the consolidation coefficient will be considered negligible for all simulation runs in this study.



a) Hysteretic functional relationships between hydraulic conductivity K, moisture content Θ, and pressure head Ψ for Delmonte sand. (Liakopoulos, 1965)



b) Single valued functional relationship between K, Θ and Ψ for a sand

Fig. 2.2 Relationships between hydraulic conductivity, moisture content and pressure head.

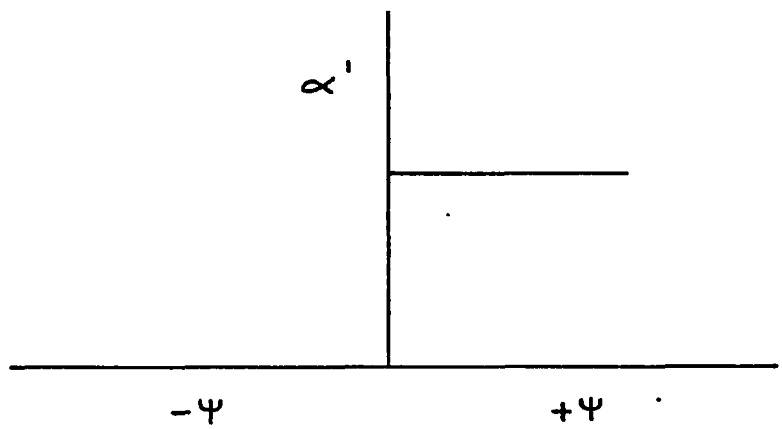


Fig. 2.3 Relationship between vertical compressibility of the soil and the pressure head.

2.3 Infiltration

The upper boundary condition of the equation of flow (2.6), (2.6a) for groundwater flow, determines the infiltration of rainfall into the soil. They may be specified as follows for the two-dimensional case:

i. Case 1. Initial conditions for t=0 are specified in terms of $\psi(x,t)$ and may be any reasonable value: at t=0 and at the surface node $z_k = z_{surface}$

$\psi^{t=o}_{surface} = \psi_{surface}$ (initial conditions)

Cases 2 and 3 deal with boundary conditions for t>0

ii. Case 2. At the surface node $z_k = z_{surface}$ for

$\psi^{t}_{surface} < 0$

$$\frac{\rho}{w} \cdot \text{rainfall or infiltration} = \frac{\partial}{\partial z}\left[\frac{g}{u}\,\rho^2\,k_{zz}\left\{\frac{\partial\psi}{\partial z} + 1\right\}\right] \qquad 2.7$$

The above boundary condition is then substituted into Equation 2.6a as described in Section 3.4, Boundary Conditions and solved for ψ.

iii. Case 3. At the surface node $z = z_{surface}$ for $\psi^{t}_{surface} > 0$ and $\psi^{t}_{surface} = d(x,t) =$ surface depth, the infiltration is calculated by Darcy's Law as follows:

$$\text{infiltration}(I) = \frac{\partial}{\partial z}\left(K\left\{\frac{\partial\psi}{\partial z} + 1\right\}\right) \qquad 2.7a$$

Equation 2.6a is then solved using the saturated boundary condition $\psi^{t}_{surface} \geq 0$ described in Section 3.4, Boundary Conditions.

2.4 Evaporation and Evapotranspiration

Case 1: Dry Surface

Evaporation from a soil may be treated in a similar manner to infiltration. For a bare dry soil it has been found by Philip that for a surface relative humidity below 99% the principal influence on the evaporation process is the resistance to water movement in the soil. Therefore in a similar manner to infiltration for r_L, relative humidity at the surface <<.99 (Eagleson, 1969):

$$\text{Evaporation} = -\frac{\partial}{\partial z}\,(K)_{\sigma}\{\frac{\partial\psi}{\partial z}_{\sigma} + 1\}) \qquad 2.8$$

As in the infiltration case the Darcy equation is solved for the evaporation.

Case 2: Wet Surface

With the soil surface saturated and relative humidity relatively high the vertical gradient of the atmospheric moisture may play a significant role in evaporation from bare soil.

Thus the free water equation for evaporation may be used and takes the following form (Eagleson, 1969):

$$E = .62\,\frac{k_w}{k_m}\;\rho a\,\frac{u_*^2}{p}\;\frac{e_1 - e_2}{\bar{u}_2 - \bar{u}_1} \qquad 2.9$$

where E is the evaporation rate, ρa, the mass density of moist air, p, the ambient atmospheric pressure, e_1, the vapour pressure (a function of z), z, the vertical distance, k_w, the vapour eddy diffusivity, k_m, kinematic eddy viscosity, $\bar{u}$, the local temporal horizontal velocity of air, u_*, the shear velocity.

For practical purposes Equation 2.9 has been simplified to numerous forms, one of which is based upon Class A pan measurements as described by Kohler, Nordenson and Fox (1955) and is given by

$$E = (0.37 + 0.0041\ \bar{u}\ (e_s - e)^{0.88}\ E\ e_p \qquad 2.10$$

$$= C\ E\ e_p$$

where C is the conversion coefficient used to convert class A pan values to natural water surface values, $\bar{u}$, is the daily wind movement (miles/day) e_s, is the saturation vapour pressure at air temperature 5 feet above ground surface, e, is the vapour pressure of air under temperature and humidity conditions 5 feet above ground surface and Ee_p is the land pan evaporation.

Among many other empirical and semi-empirical approaches to water surface evaporation is the Penman approach (1948). Here the diffusion and energy approaches are combined. Penman's formula was modified by Van Bavel (1966) and used to determine potential evapotranspiration or the upper limit to the possible combined losses due to both evaporation and transpiration when the supply of water is unlimited (soil saturated). While evapotranspiration is certainly important in the simulation of runoff for periods of time greater than 24 hours, for the purposes of simplification it has been omitted in the short time simulations carried out in this study. However, a combined theoretical-empirical approach has been outlined above.

In this study for the dry soil case the groundwater flow equation would be solved for evaporation and for the wet soil case, the evapotranspiration would be calculated by the Penman formula and applied to the depth of flow across the upper boundary to the groundwater flow equation. Further details are described in Section 3.4, Boundary Conditions.

2.5 Overland Flow Model

The Kinematic Wave Theory has been used to model the surface runoff processes. This theory has been derived from the continuity and momentum equations, commonly referred to as the St. Venant (1843) or shallow-water equations. The basic shallow-water equations for unsteady one-dimensional, spatially-varied overland flow are derived in Appendix B and are summarized below.

The continuity equation is: (see Eq. 1.7)

$$d \frac{\partial v}{\partial x} + v \frac{\partial d}{\partial x} + \frac{\partial d}{\partial t} = q - I \qquad 2.11$$

and the equation of momentum is: (see Eq. 1.8)

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + g \frac{\partial d}{\partial x} = g(S_o - S_f) - \frac{v}{d}(q-I) \qquad 2.12$$

where q is the lateral inflow or rainfall, I, is the infiltration rate, t, is the time, v, is the flow velocity, S_o, is the channel bottom slope, S_f, is the friction slope, d, the flow depth, x, the coordinate direction, $\frac{\partial v}{\partial t}$, the convection acceleration term, $v \frac{\partial v}{\partial x}$, the local acceleration terms, $g \frac{\partial d}{\partial x}$, the local water surface slope.

The term $\frac{v}{d}(q-I)$ refers to the increase in momentum due to the rainfall q and the decrease in momentum due to the infiltration I.

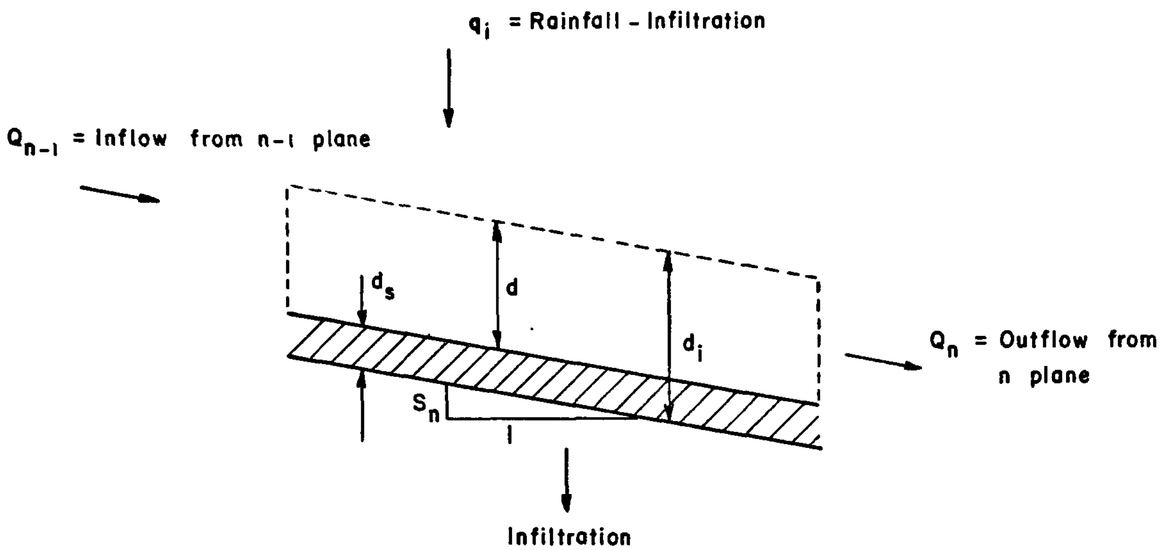
The kinematic wave is defined by the theoretical investigations of Lighthill and Whitham as one in which Q is a function of d alone. This implies $S_o = S_f$. Then assuming the acceleration terms: $\frac{\partial v}{\partial t}$, $v \frac{\partial v}{\partial x}$, the water surface slope $\frac{\partial y}{\partial x}$, and the rainfall-infiltration momentum terms are negligible (Henderson, 1966), then the momentum equation would reduce to:

$$S_o \approx S_f = \frac{fq^2}{8gd^3} \qquad 2.12a$$

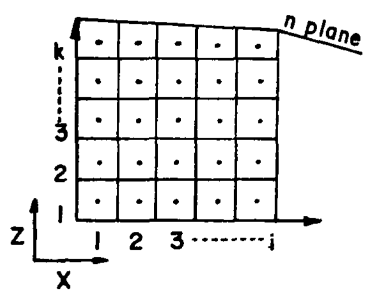
in which f is the Darcy-Weisbach friction factor and q is the flow per unit width. The Darcy-Weisbach factor for overland flow is a function of the roughness of the boundary, the depth of flow, the rainfall intensity and the flow Reynolds numbers. The friction factor is discussed in more detail in Chapter 5, Effect of Model Parameters on Model Output.

2.5.1 Depression Storage

After the surface becomes saturated, water becomes available for depression storage as well as for overland flow. Depression storage is the water that is trapped on the surface in shallow depressions of varying size and depth. While the present model simulations ignore depression storage it could easily be incorporated in the model as shown in Figure 2.4. Water is contributed to overland flow only when d_i is greater than d_s, the specified depression storage depth.



$$Q_n = \frac{8q\,(d_i - d_s)^3\, S_n}{4}$$



d_s = Depression storage

d_i = Pressure head on surface node i

$d^{\cdot}$ = $d_i - d_s$ depth of overhead flow

f = Darcy Weisbach factor

S_n = Slope of plane n

Q = Overland flow

Fig. 2.4 Overland flow and Depression storage.

2.6 Seepage

The seepage boundary conditions are assumed following developments of Freeze (1971).

A homogeneous dike or earth embankment is considered having cross section A, B, C, D, E, F, G resting on an impervious base and is shown schematically on Figure 2.5. There is no rain or evaporation making H, A, B, C, D an effective impervious boundary. D is the exit point with DE the seepage face and HD the water table.

The boundary conditions (Fig. 2.5) are then as follows:

1) along A, B, and C

$$\frac{\partial Q}{\partial z} = 0 \qquad \text{and} \qquad \frac{\partial \psi}{\partial z} = -1 \tag{2.14}$$

2) along A, H, and C, D

$$\frac{\partial Q}{\partial x} = 0 \qquad \text{and} \qquad \frac{\partial \psi}{\partial x} = 0 \tag{2.15}$$

3) along H, G

$$Q = d_V \qquad \text{and} \qquad \psi = H_u - d_u \tag{2.16}$$

4) along E, F

$$Q = d_D \qquad \text{and} \qquad \psi = H_D - z \tag{2.17}$$

5) along E, D

$$Q = z \qquad \text{and} \qquad \psi = 0 \tag{2.18}$$

For transient flow problems, conditions 3 and 4 and the locations of points H, D and C become time dependent and therefore the condition 3 becomes:

$$\left.\begin{array}{lll} Q = d_u(t) & \text{and} & \psi = H_u(t)-d_u \\ Q = d_d(t) & \text{and} & \psi = H_D(t)-z \end{array}\right\} \tag{2.19}$$

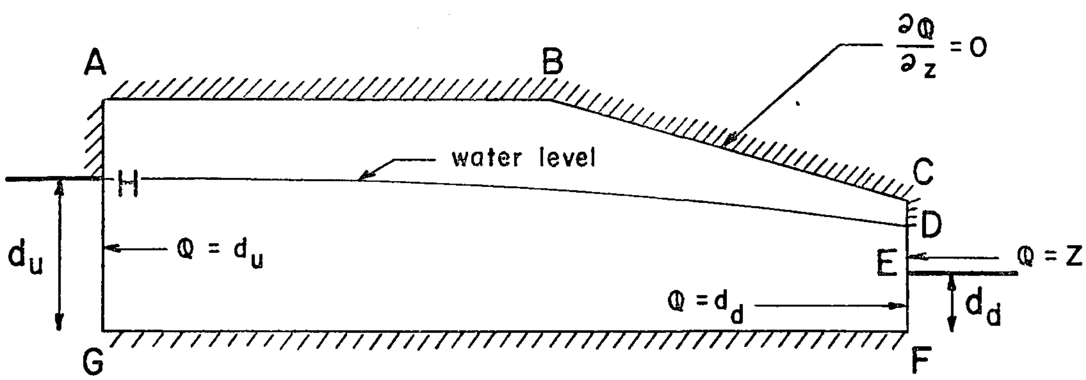


Fig. 2.5 Seepage boundary conditions

CHAPTER 3

THE PHYSICALLY-BASED DIGITALLY SIMULATED MODEL

3.1 Numerical Methods of Solving Partial Differential Equations

The equations used in the development of the mathematical model are the two-dimensional, non-linear, parabolic or diffusion-type, partial differential Equation 2.4 describing subsurface flow and the one-dimensional, unsteady, spatially-varied, kinematic flow Equations 2.11 and 2.12a describing overland flow. Due to the non-linearity of the Equation plus the complicated auxiliary condition of natural systems (system geometry, soil characteristics, and initial and boundary conditions), exact analytic solutions are not available and recourse must be made to the use of numerical methods of approximation for the solution of the partial differential equation.

Numerical approximations of the equations fall into the following three more or less unique approaches: finite differences, finite element and hybrid solutions.

The latter, the hybrid computer, which combines a digital and an analog computer, is probably the newest tool in simulation. It takes advantage of the superior efficiency of the analog in integrating functions and the efficiency of the digital in carrying out computations that can be reduced to a sequence of arithmetic and logic operations on discrete data.

However, recent work carried out by Oosterveld (1974) on one-dimension hydrodynamic equations showed present analog equipment lacks digital capacity for large scale, multi-dimensional hydrologic modelling.

The finite element method replaces the actual partial differential equation with an equivalent functional which is to be minimized and used to generate a set of difference equations. This is quite a recent method and application of the method to transient hydrologic problems has occurred only in the past several years. Most notable, transient problems have been studied by Pinder (1972) and Guymon (1970).

The chief advantage of the finite element method over its earlier predecessor the finite difference method is its ability to handle certain complex geometric networks with irregular boundaries and mesh spacing. For simple regular mesh network the difference equations derived by the two methods are identical. However, the finite element method is no panacea for it has the same problems of convergence and stability as encountered in the finite difference method.

The oldest numerical finite difference method has been in use since 1910 when Richardson introduced it. A large amount of experience has been built up in this method and the attendant problems of convergence and stability are well documented (Richtmyer, 1957). It is chiefly due to this large amount of available development work that the finite difference method has been selected in this study to solve the partial differential equations. The irregular boundaries will be simulated by a block-centered node scheme originated by Varga (1957) and used by Freeze in his solution to subsurface equations (Fig. 3.1 illustrates this scheme). In order to obtain unconditional stability for all size time steps the following implicit finite difference formula will be used for both the saturated and unsaturated zone, which is obtained from equation 2.4 and is formulated as follows:

$$\frac{1}{\Delta x_i}[K_{xy}(\psi_1)(\frac{1}{\Delta x_i + \Delta x_{i+1}} \cdot (\psi^t_{i+1}+\psi^{t-1}_{i+1}-\psi^t_i-\psi^{t-1}_i))$$

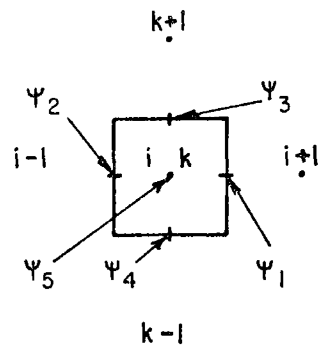
$$- K_{xx}(\psi_2)(\frac{1}{\Delta x_i+\Delta x_{i-1}} \cdot (\psi^t_i+\psi^{t-1}_i-\psi^t_{i-1}-\psi^{t-1}_{i-1}))]_{i.k}$$

$$+ \frac{1}{\Delta z_k}[K_{zz}(\psi_3)(1 + \frac{1}{\Delta z_k+\Delta z_{k+1}} \cdot (\psi^t_{k+1}+\psi^{t-1}_{k+1}-\psi^t_k\psi^{t-1}_k))$$

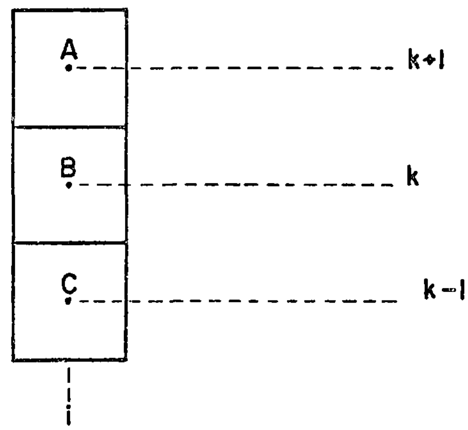
$$- K_{zz}(\psi_4)(1 + \frac{1}{\Delta z_k+\Delta z_{k-1}} \cdot (\psi^t_k+\psi^{t-1}_k-\psi^t_{k-1}-\psi^{t-1}_{k-1}))]$$

$$= [C(\psi_5)(\frac{\psi^t-\psi^{t-1}}{\Delta t})]_{i,k} \qquad 3.1$$

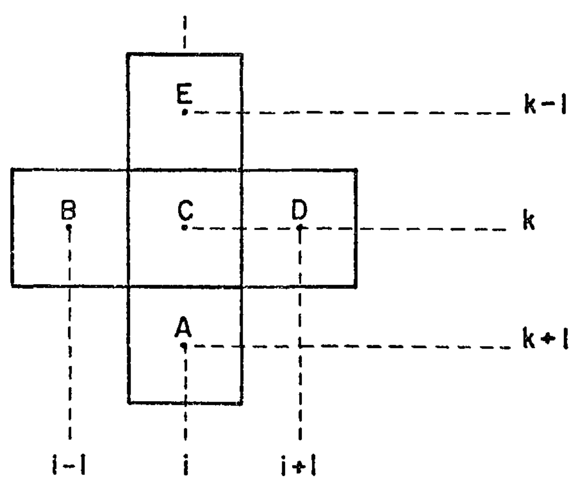
For non-linear differential equations an iteration method is preferred since terms that are functions of the dependent variable can be recalculated at each iteration.



a) A typical block node defining Ψ - Ψ values used in finite difference equations



b) A 3 node grid used in the L.S.O.R. numberical method.



c) A 5 node grid used in the S.I.P numerical method.

Fig. 3.1 Showing the nodal grid used in the L.S.O.R. and S.I.P numerical methods.

For the two-dimensional subsurface flow equation there are two implicit iterative methods that can be considered: the line successive, over-relaxation (L.S.O.R.) technique (Young (1954)) used by Freeze in his groundwater flow model and the Strongly Implicit Method (S.I.P.) advocated by Stone (1968) in his work in the petroleum industry on two-dimensional reservoir analysis.

Both the L.S.O.R. and S.I.P. methods of solution were designed for linear parabolic partial differential equations of the form (in the case of the L.S.O.R. method):

$$- A_k\psi_{i,k+1} + B_k\psi_{i-1,k} + C_k\psi_{i,k-1} = b_k \qquad 3.2$$

where A_k, B_k, C_k are pseudo-constant coefficients developed from the grouping of the coefficients of equation 3.1. The non-linearity is simulated by assuming constant coefficients throughout one iteration solution for a time increment t, then recalculating the coefficients based upon a predicted pressure head prior to the next iteration for time t. For example, in order to predict a pressure head ψ_{pred} at each node from which current estimates of k, θ, and C could be calculated the following formulas are often used (Cooley, 1970; Freeze, 1971).

1) For the first iteration:

$$\psi_{pred_{ik}}{}^{t} = (T_t + 1)\psi_{i,k}^{t-1} - T_t\psi_{i,k}^{t-2} \qquad 3.3$$

where $T_t = \dfrac{\Delta t^t}{2\Delta t^{t-1}}$

2) For later iterations:

$$\psi_{pred_{ik}}^{t,it} = \psi_{pred_{i,k}}^{t,it-1} + \lambda(\psi_{ik}^{t,t-1} - \psi_{pred_{ik}}^{t,t-1}) \qquad 3.4$$

where

$$0 \leq \lambda \leq 1$$

ψ_{pred} is then used to determine k, Θ, C (and thus A_k, B_k, C_k) from the given functional relationships, the finite difference equations of Equation 2.4.1 is then solved to obtain a pressure head ψ_{calc}. Steps 1 and 2 were then repeated until the calculated pressure ψ_{calc} was close to the previous iteration of ψ_{calc} at time $t_o+\Delta t$ for all values of i,k of the finite difference grid.

Unfortunately the solving of parabolic equations in which the coefficients are not constant but a function of the dependent variable ψ often leads to instabilities and convergence problems and this study was not immune to such problems.

While these implicit schemes were satisfactory for constant surface boundary conditions, there was often no convergence of the free surface line when the water table was raising over a steep seepage face. The top boundary values were found to oscillate ar the presumable true free surface. On the falling water table, sl convergent rates were observed when the moisture content dropped below a value close to saturation.

Noting similar problems encountered by Verma and Brutsaert (1970) an Explicit-Implicit Numerical Scheme was adopted and proved successful in overcoming the problems of stability and convergence.

In this method at the beginning of each time step t, the soil moisture content for t+1 was determined using an explicit predictor approximation of the saturated-unsaturated soil moisture equation. The corresponding hydraulic conductivity was then determined from the functional relationships.

Then with the coefficients A_k, B_k, C_k, D_k now linearized from known values of k and θ the soil water equations were solved for the pressure head using either the implicit L.S.O.R. or the Strongly Implicit Method.

3.2.1 Predictor Corrector Method - Explicit Predictor

The explicit finite difference approximation of equation 3.1 at any interior point i,k can be written as:

$$
\begin{aligned}
\theta_{ik}^{t+1} = {} & \left(\frac{1}{x(i)}\left[\left(\frac{z}{x_i + x_{i+1}}\right) \cdot (\psi_{i+1,k}^{t} - \psi_{i,k}^{t}) \cdot \rho \cdot k_{I}\right.\right. \\
& - k_{II} \cdot \rho \left(\frac{z}{x_i + x_{i-1}}\right) \cdot \left.(\psi_{i-1,k}^{t} - \psi_{ik})\right] \\
& + \frac{1}{z_k}\left[\rho \cdot k_{III} \left(1 + \left(\frac{z}{z_k + z_{k+1}}\right)\right) \cdot (\psi_{i,k+1} - \psi_{ik})\right. \\
& - \rho \cdot k_{IV} \left(1 + \left(\frac{z}{z_k + z_{k-1}}\right)\right) \cdot \left.(\psi_{ik} - \psi_{i,k-1})\right] \cdot \Delta t + \theta_{ki}^{t}
\end{aligned}
\qquad 3.5
$$

If the solution is known for the time t, it can be determined for the time t+1 for all grid points except the boundaries.

The boundary points are handled in the same manner as described in Section 3.4, Boundary Conditions. With the soil moisture obtained at time t+1, the permeability at t+1 can be derived from the ψ-θ, ψ, k relations.

3.2.2 Line Successive Over-Relaxation

As mentioned previously for each node in the finite difference network of the L.S.O.R. method, the terms of equation 3.1 can be grouped as follows:

$$A_k\psi^t_{i,k+1} + B_k\psi^t_{i,k} + C_k\psi^t_{i,k-1} = D_k \qquad 3.6a$$

or in matrix notation:

$$[A]\psi_{ik} = D_k \qquad 3.6b$$

This equation forms a 3 node vertical (2 direction) grid shown in Figure 3.1b. Values of ψ^t on adjacent lines as calculated from the most recent iteration are considered as known values. The set of equations 3.6 for a vertical line scan form a tri-diagonal matrix equation that can be solved by the well-known Thomas (1949) algorithm. The process is then repeated for the next vertical line i+1. When all vertical columns have been traversed one iteration has been completed. After each iteration the calculated value of ψ at e node are over relaxed in the usual fashion.

$$\psi^{t,it} = w\ \psi^{t,ik}_{calc} + (1-w)\psi^{t,it-1} \qquad 3.7$$

$1 \leq w \leq 2$ = acceleration factor

it = iteration

After convergence is achieved the process is repeated for the next time step. A detailed account of this method is described in Appendix 3.

3.2.3 Strongly Implicit Method

According to Remson (1971) for "cases of flow in heterogeneous or anisotropic media the strongly implicit procedure is much faster than other methods", "the effectiveness of the method does not seem to depend upon the complexity of the problem". Rather than the vertical 3-node grid of the L.S.O.R., the S.I.P. uses a 5-node grid as shown in Figure 3.1c and the terms are grouped for each node as:

$$A\psi_{i,k+1} + B\psi_{i-1,k} + C\psi_{i,k} + D_{i+1,k} + E_{i,k+1} = F \qquad 3.8a$$

or this may be written in matrix notation:

$$[A]\psi_{ik} = F_{ik} \qquad 3.8b$$

where A is the coefficient matrix. A modified matrix A+B is obtained which is close to A and can be easily solved directly. This modified matrix A+B then forms a base for an iterative technique. In contrast to the line by line scan of the L.S.O.R. method, the S.I.P. method solves the entire section at once to form one iteration. A detailed description of the S.I.P. method is presented in Appendix 3.

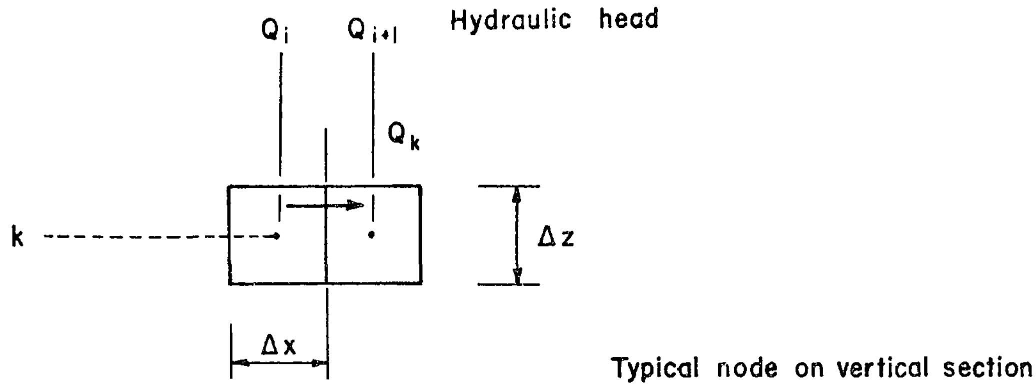
3.2.4 Calculation of Ground Water Discharge

Two methods were used to determine the groundwater outflow hydrograph, a) graphical flow net using the hydraulic heads output of the model and b) Tseng and Ragan (1973) method of their study in unconfined groundwater flow. The flow net analysis is described in standard texts on groundwater (Verruijt, 1970) and is discussed further in Chapter 4. In the offset method the groundwater discharge was computed by Darcy's equation using the computed hydraulic gradient at each grid point. For any rectangular node the discharge across either a vertical or horizontal boundary is computed as shown in Figure 3.2 by using the calculated hydraulic gradients on either side of the boundary and the average k at the boundary.

To compute the outflow hydrograph the calculation of discharge was made at a vertical section near and upstream from the outflow boundary of the model by means of $\sum_{k=1}^{k=surface} q_k$ where q_k is the incremental discharge in the x direction at node k. This discharge was then added to the amount of net flow in the z direction across the upper model boundary between the vertical section and the river channel to yield the total discharge.

3.3 Numerical Solution of the Overland Flow Equations

The kinematic approximation of the overland flow equations have been solved numerically by (1) the upstream differencing method, (2) the single step Lax-Wendroff scheme, (3) the method of characteristics used by Kibler and Woolhiser (1970) and the four-point implicit



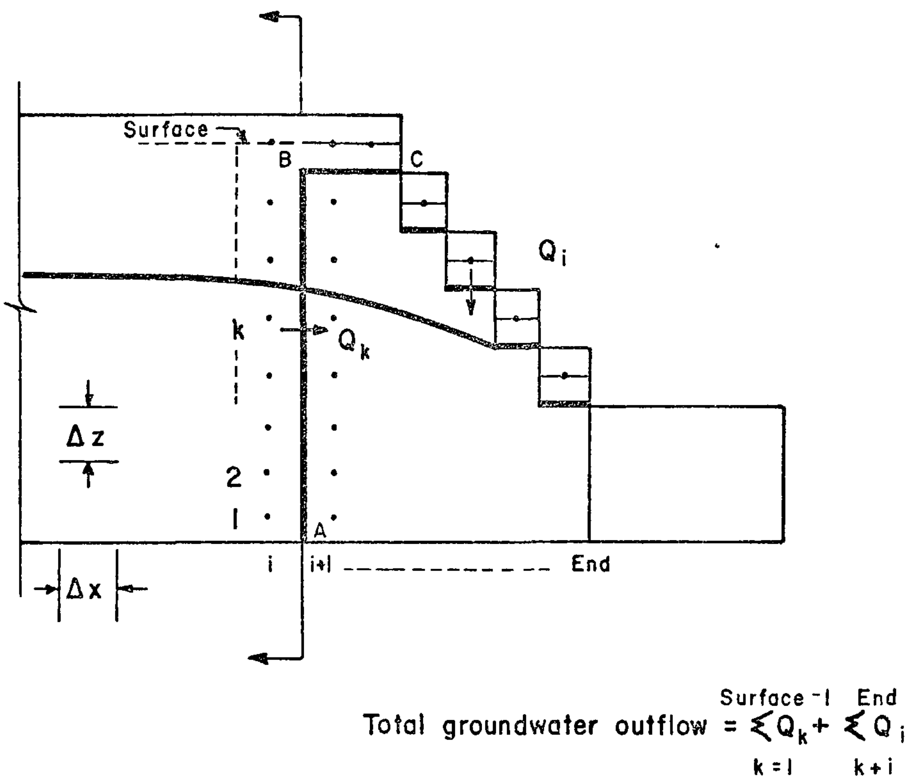


Fig. 3.2 Illustrating how the offset method is used to determine the groundwater hydrograph.

scheme used by Brakensiek (1966). From the comparison made by Kibler and Woolhiser it would appear that the method of characteristics is slightly more accurate, but Brakensiek's method is probably faster and is unconditionally stable. Therefore in the interests of numerical stability it was decided to use Brakensiek's numerical approximations as given below:

The equation of continuity, Equation 2.11 can be written as:

$$\frac{\partial(vd)}{\partial x} + \frac{\partial d}{\partial t} = q-I \qquad 3.9a$$

for a unit width

or

$$\frac{\partial Q}{\partial x} + \frac{\partial d}{\partial t} = \bar{q} \qquad 3.9b$$

where Q is the unit discharge and $\bar{q}$ is the resultant lateral inflow cm^3/cm in time Δt.

The finite difference approximation of equation 3.9b is:

$$\frac{Q_4-Q_2}{\Delta x} + \frac{(d_4-d_3+d_2-d_1)}{2\Delta t} \qquad 3.10$$

where d, is the unit flow area at location x and time t, d_3, the unit flow area at location $x+\Delta x$ and time t, d_2, Q_2, the unit flow area and flow rate at location x and time $t+\Delta t$, and d_4, Q_4, is the unit flow area and flow rate at location $x+\Delta x$ and time $t+\Delta t$.

The finite difference grid for equation 3.10 is shown on Figure 3.3.

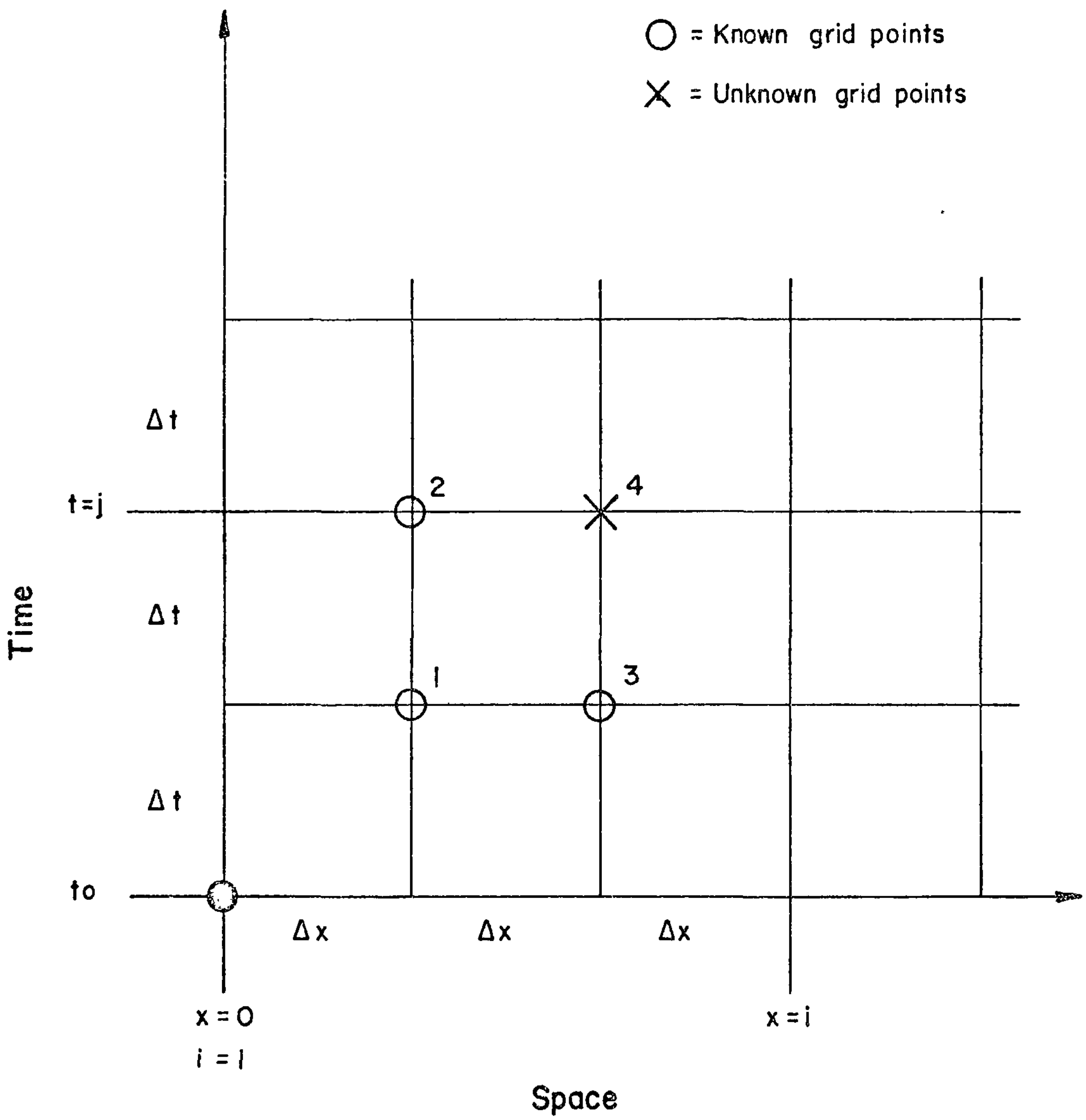


Fig. 3.3 Showing the finite difference grid network in x-t plane.

The equation of motion as approximated by kinematic wave theory is:

Eq. 2.12a $$S_o = \frac{fQ^2}{8gd^3}$$

and is used to obtain rating curves at either end of the reach (x, x+Δx). Since the rating function Equation 2.12a is not an explicit function but a set of numerical values, a simultaneous solution of Equations 3.10 and 2.12a requires an iteration procedure such as the method of false position. This procedure is presented in Appendix D, Computer Program.

A series of planes as shown in Figure 3.4 are used to describe the slope of the watershed. For each node i on the plane P with slope S_P there is an input to the overland flow equation. For equation 3.10 $\sum_{i=1}^{i=L} q_i = \overline{q}$ average lateral inflow to the overland flow equation which is solved for depth d_p and Q_p at p. From this solution of d_p, depths at each i can be obtained from an interpolation formula.

3.4 Boundary Conditions

3.4.1a Upper Boundary Conditions

For the upper boundary node, equation 3.6a becomes

$$B_s\psi^t_{i,s} + C_s\psi_{i,s-1} = D_s \qquad 3.6b$$

The rainfall upper boundary condition refers to the surface node k = s. When $\psi_{i,k}$ at the surface node S is <0 or unsaturated, a flux

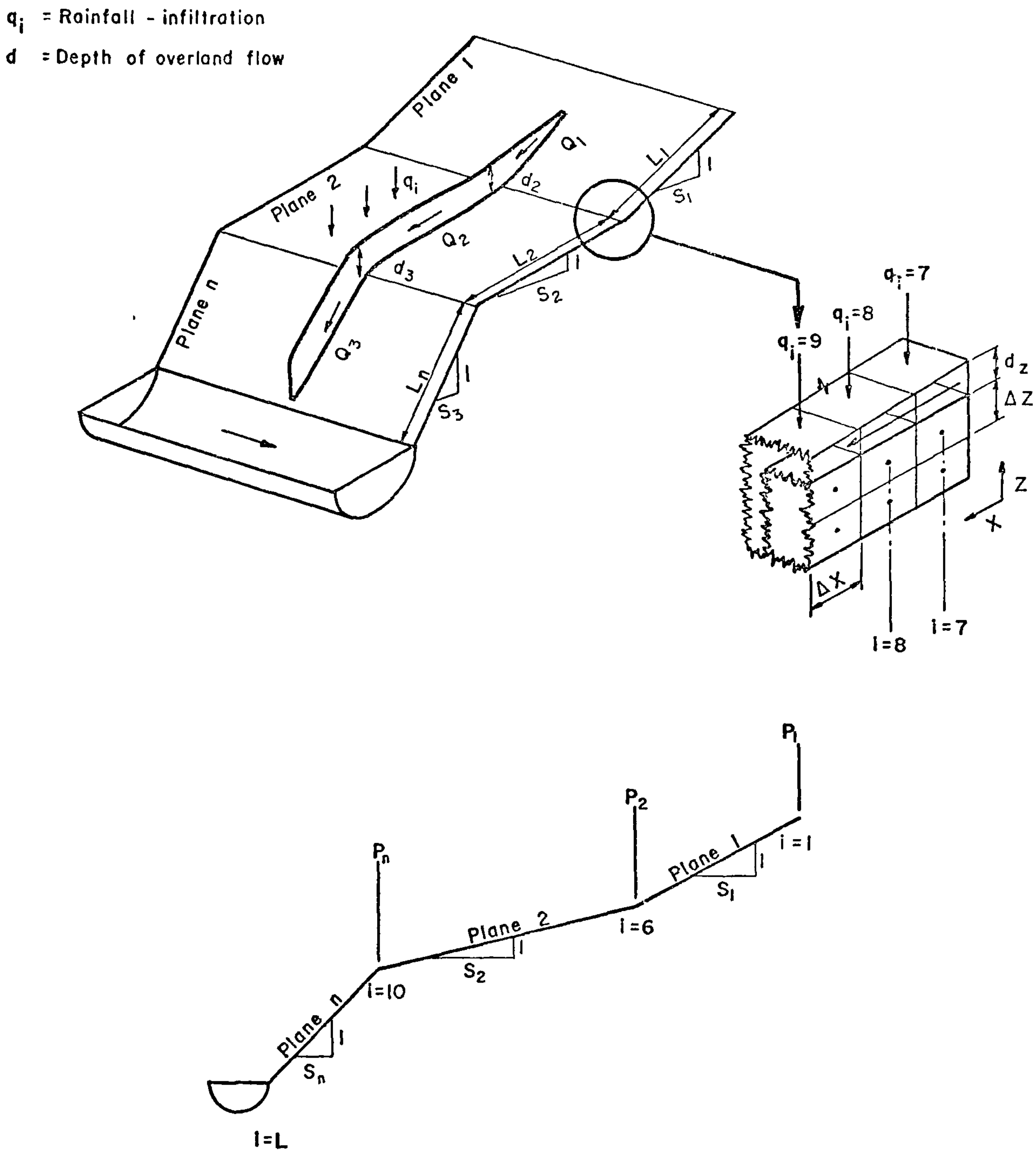


Fig. 3.4 Schematic representation of the overland portion of the mathematical physical model.

condition is imposed upon the upper node with the resulting flow expression for this node being analogous to equation 3.1.

$$\frac{1}{\Delta x_i} [K_{xx}(\psi)(\frac{1}{\Delta x_i + x_{i+1}} \cdot (\psi^t_{i+1} + \psi^{t-1}_{i+1} - \psi^t_i - \psi^{t-1}_i))$$

$$- K_{xx}(\psi_2)(\frac{1}{\Delta x_i + \Delta x_{i+1}} \cdot (\psi^t_i + \psi^{t-1}_i - \psi^t_{i-1} - \psi^{t-1}_{i-1}))]_{i,k}$$

$$+ \frac{\rho I}{\Delta z} - K_{zz}(\psi_4)(1 + \frac{1}{\Delta z_k + \Delta z_{k-1}} \cdot (\psi^t_k + \psi^{t-1}_t - \psi^t_{k-1} - \psi^{t-1}_{k-1}))$$

$$= [c(\psi_5)(\frac{\psi^t - \psi^{t-1}}{\Delta t})]_{i,k} \qquad 3.11$$

In equation 3.11 the term $\frac{\rho I}{\Delta z}$ replaces the expression describing flow across the block node face 3 (Fig. 3.1). The flux in cm/sec across the upper boundary, I, is a positive value for infiltration and negative for evaporation.

3.4.1b Saturated Boundary Node

When the upper boundary node becomes saturated $\psi_{i,surface} \geq 0$ it is then subjected to a pressure head that is $\psi_{i,surface} = d(i)$. "d(i)" is the depth of ponded water at the surface node. Figure 3.5a illustrates this case.

The first equation in Matrix 3.7 is for $\underline{k} = 2$ and terms in $\psi^t_{i,k}$ or $\psi^t_{i,surface}$ are placed on the right side of the equations as known values yielding:

$$B_{ik}\psi_{ik} - C_k\psi_k = D_k \qquad 3.6c$$

where $D_k = d(i) + A_k\psi_{i,k+1}$, and $\psi_{i,k+1} = \psi_{i,surface} = d(i)$.

As discussed in 2.3 Infiltration, the flow rate across the upper boundary surface under a saturated condition or pressure head is given by:

$$Q = [1 - \frac{\Delta\psi}{\Delta z}]\ K \qquad 3.12$$

Where Q is positive, Equation 3.12 represents infiltration and when Q is negative, Equation 3.12 represents seepage.

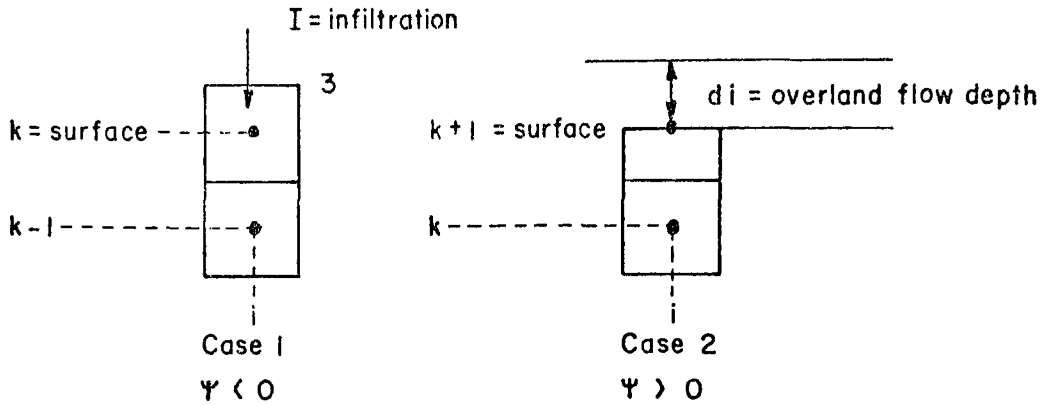
$$\therefore \qquad B_{ik}\psi_{ik} - C_k\psi_{i,k-1} = D_k \qquad 3.13$$

where $D_k = D_k + A_k\psi_{i,k+1}$ and $\psi_{i,k+1} = d_i$

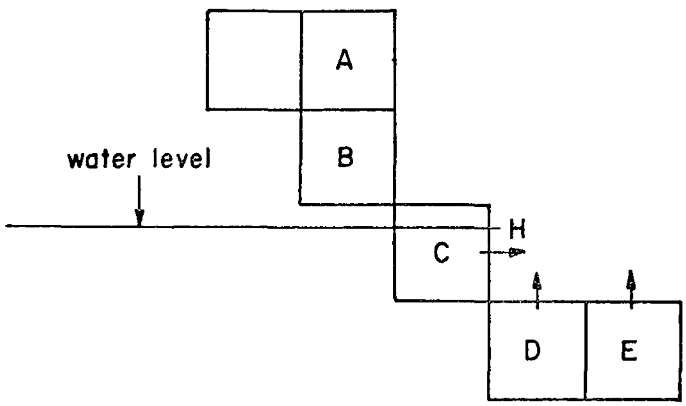
3.4.2 Seepage Boundary Condition

The seepage face is described in a similar manner as the upper boundary and is illustrated in Figure 3.5b.

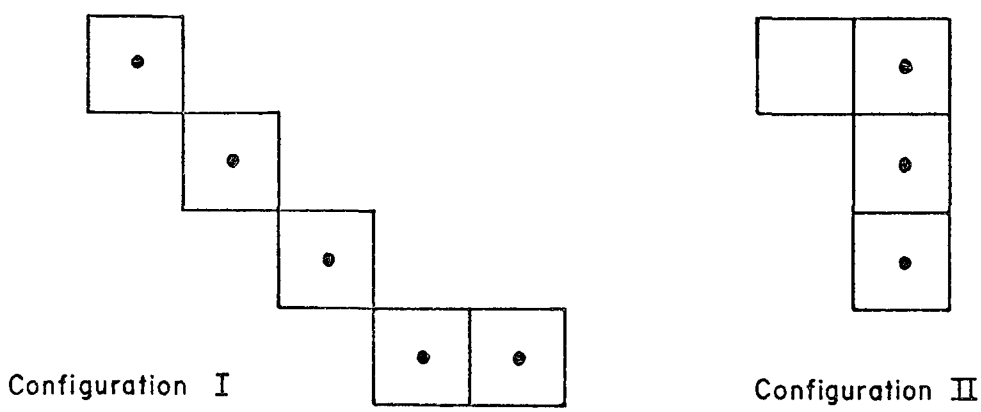
The manner of dealing with the exit point in seepage problems is similar to the method used by Freeze with some slight modifications. In Freeze's method a test is made on the nodes above



a) Unsaturated saturated boundary condition on upper boundary.



b) General seepage face



c) Types of seepage faces

Fig. 3.5 Boundary conditions

and below H, the exit point in Figure 3.5 to see whether the seepage face is rising or falling. If the seepage face is rising, the time step is carried out without change and at the end of the time step the ψ value at B above h is checked. If the pressure head is less than zero the next time step is started; if it is greater than zero the boundary condition at the node C is changed from a flux condition to a pressure head condition. For a falling exit point the boundary condition at the node below H, D, is changed from a head condition to a flux condition prior to the next time step. Then the node just above the new exit point is checked in the same manner as for the rising face. The tolerance for a change of boundary conditions is 1 cm.

In this study the test for the exit point is incorporated in the test for the upper boundary condition. At the end of each time step the upper surface nodes are tested for a rising or falling water table. If the water table is rising the time step is repeated using a smaller time step until surface $\psi_{surface} \leq$ tol. When this is reached the following time steps are performed with a pressure boundary condition as shown in Figure 3.5a and in Section 3.2.1, Upper Boundary Conditions. If the water level is declining then at the beginning of the time step the boundary condition is changed from a pressure condition to a flux condition. Since the test for the upper surface and seepage face coincide, the geometrical configuration is limited to configuration I and not II as shown in Figure 3.5c. This method provides a speedier solution than Freeze's and can easily be modified to accommodate a type-II configuration.

3.4.3 Operational Logic

When either the rainfall intensity $i < k_s$ of the model for saturation conditions and the water table reaches the ground surface or $i > k_s$, a saturation condition is achieved. The surface becomes saturated at some time t, that is $\psi^t_{surface} = 0$. After this point, runoff will begin; and infiltration is calculated by equation 3.12; water will begin to pond ($d(x) > 0$). Then, $\psi_{surface} = d_i(x,t)$ will be calculated by the overland flow equations.

When after the runoff has begun, the rainfall rate drops below the infiltration rate, $d_i(x,t)$ will be decreased both by drainage from the surface flow equations and by infiltration of the surface storage into the soil. Richard's equation is solved for the saturated upper boundary until $d_i(x,t)$ reaches 0, after which the soil surface is desaturated. A flux condition, I = rainfall intensity (i) is then used as an upper boundary condition as long as $\psi_{i,surface} \leq 0$ and i < infil.

Two flow diagram, one utilizing the L.S.O.R. method and the other utilizing the S.I.P. method are shown in Figure 3.6.

The flow chart indicates how the program is used to detect the correct conditions of the surface for proper assignment of boundary conditions at the start of a new time step. When $\psi_{i,surface}$ is near 0 it is necessary to know for saturating conditions when the time step should be changed prior to ponding to ensure that the point when $\psi_{i,surface} = 0$ is not overrun by the time step and a smooth transition of boundary conditions can occur. For desaturation or recession

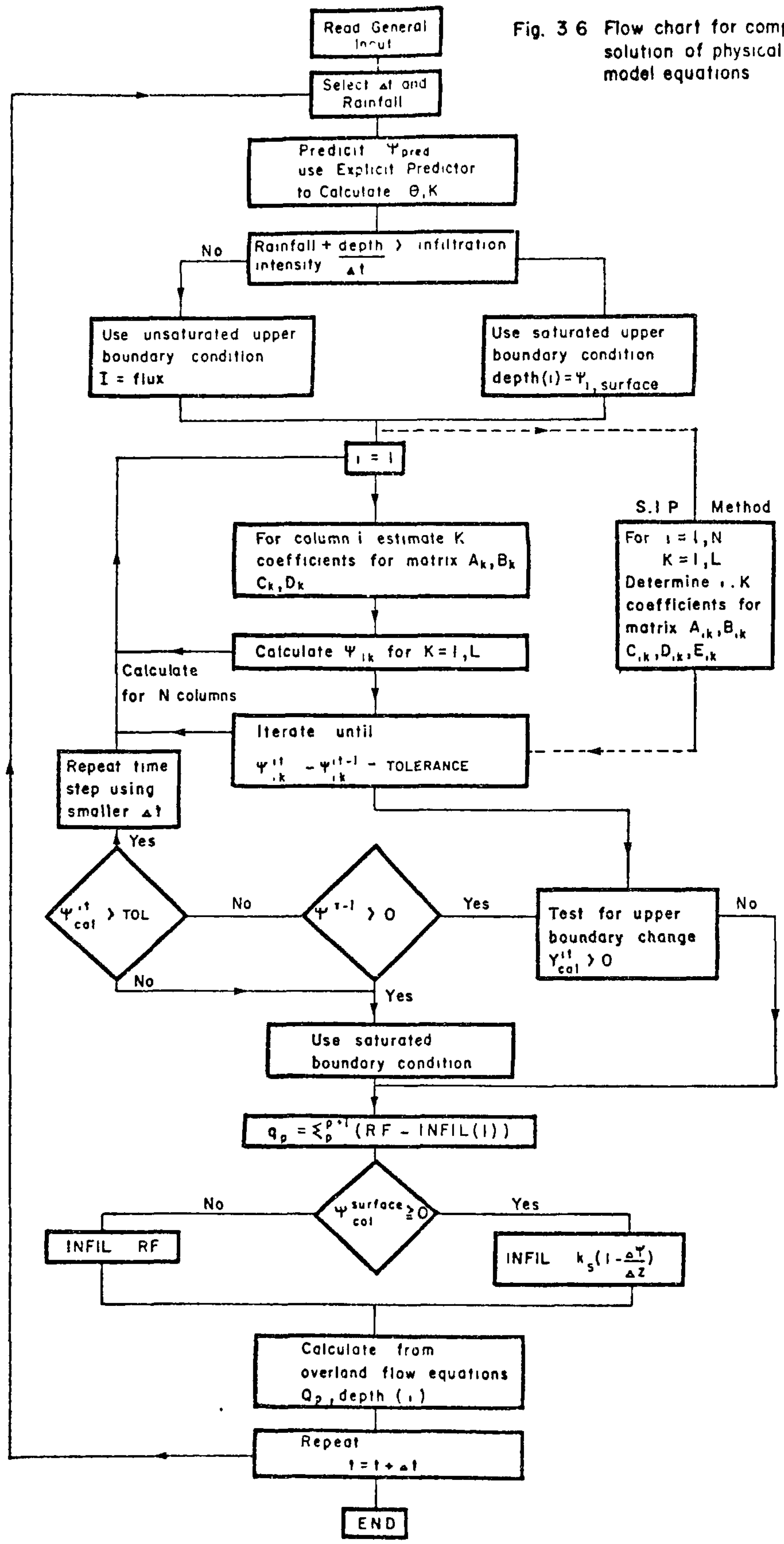


Fig. 3.6 Flow chart for computer solution of physical model equations

conditions it is necessary to see if the available surface water is less than the potential infiltration at that time, so that the upper boundary condition may be reset to unsaturated conditions for the new time step.

3.5 Combination of the Numerical Models of Ground Water and Overland Flow

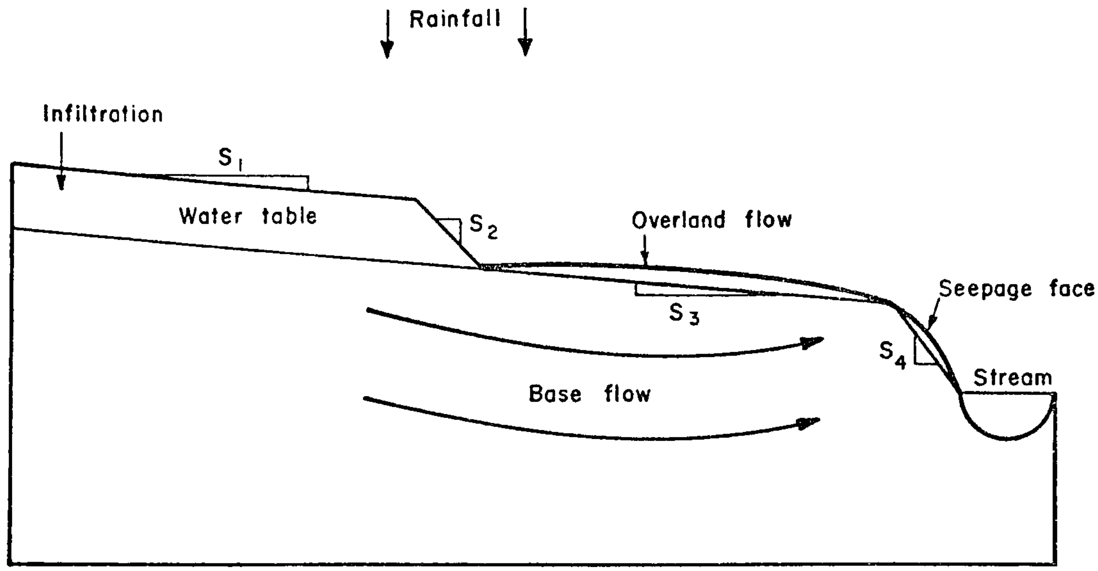
3.5.1 General Structure

The groundwater model has been shown on Figures 2.1 and 2.4 and the overland flow model in Figure 3.4. The combination of the two models and its finite difference approximation is shown schematically in Figure 3.7. The flow in the saturated and unsaturated zone is described in 2 dimensions and the overland flow is described in one dimension. For each time step the matrix equation 3.7 for groundwater flow is solved, after which for the same time step the implicit equations 2.12 and 3.10 for overland flow are solved.

3.6 Assumptions and Limitations of the Model

While emphasis has been placed on developing a general versatile mathematical model, the model is limited by a) the theoretical assumptions on which the model is based and b) the size and speed of the present generations of computers.

The theoretical assumptions are summarized in the following paragraph and are discussed in detail in Appendix A "Derivation of the



Cross-section through actual watershed

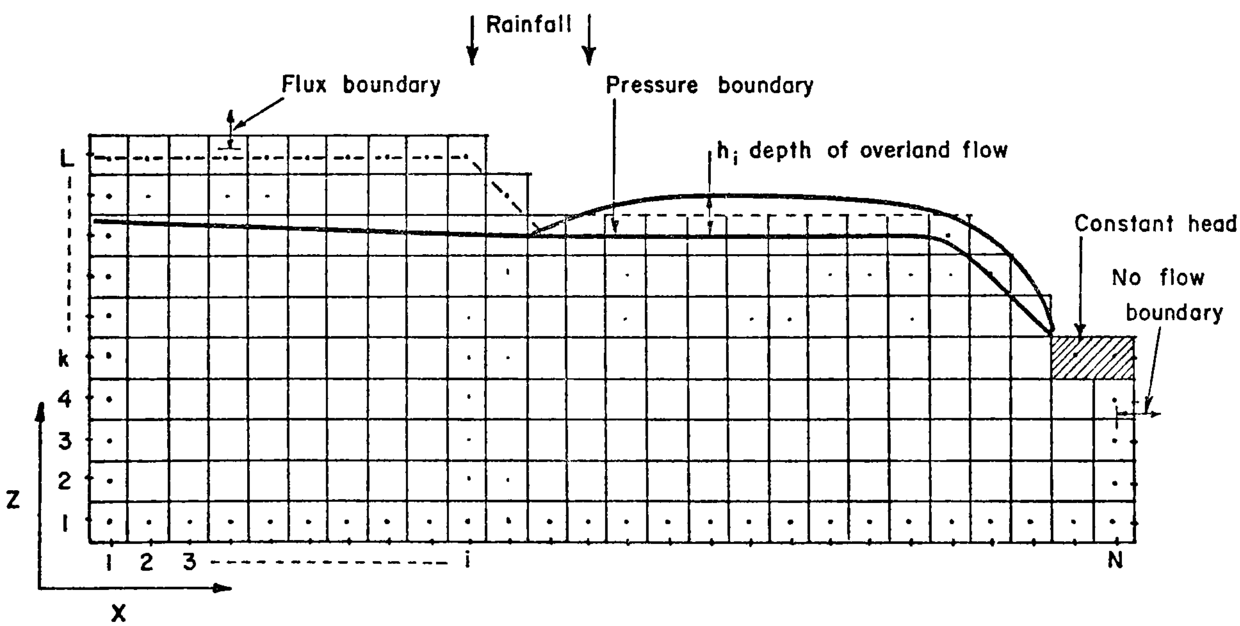


Fig. 3.7 Finite difference approximation for the mathematical watershed.

groundwater flow equations" and Appendix B, "Derivation of the shallow water equation."

For saturated flow, the flow is assumed laminar and Darcian inertial forces, velocity heads, temperature gradients, osmotic gradients, chemical concentration gradients are all assumed negligible. The soils are assumed to be linearly and reversibly elastic, non-shrinking and mechanically isotropic. The horizontal components of deformation are assumed to be small compared to the vertical deformations.

For the unsaturated flow, it has been assumed that the soils are non-swelling, the air phase is continuous and always in connection with constant internal atmospheric pressure and there is no air blockage. While hysteretic effects in the functional relationships of hydraulic conductivity K and moisture content θ with pressure head ψ are not considered in the theoretical development, the hysteretic effects can be introduced into the model in the form of empirical wetting and drying scanning curves as shown in Figure 2.2. For general watershed simulation, it would appear that uncertainties as to the basic form of the functional relationship for saturated soil will outweigh the secondary influences of hysteresis. For silts and loams the relative unimportance of hysteresis is borne out by Staple, 1966. Therefore a single-valued functional relationship is assumed for all soil simulations. For overland flow, the flow is assumed gradually varied, the channel slope is assumed small, the functional resistance in unsteady flow is assumed the same as that for the corresponding depth in uniform flow.

Two of the three main critical assumptions occur in the unsaturated groundwater portion of the model. They are the swelling of soils and the lack of consideration of entrapped air both of which affect the infiltration rate. The swelling of soils could be incorporated into the model by using the approximations of Philip (1969) and entrapped air could be crudely approximated by the scanning curves of hysteresis. The third restriction, the temperature gradient restriction, precludes treatment of winter soil moisture conditions.

One of the chief limitations in the numerical solution of equation 2.6, a non-linear parabolic partial differential equation, is the use of linear approximations for a non-linear problem. The non-linearity of the coefficients causes a time step limitation in order to maintain stability and convergence of solutions. As mentioned in Section 3.2, problems of convergence were encountered where predicted and calculated ψ values oscillated about the correct value so that the solution failed to converge to the desired tolerance. When such problems occurred the time step was automatically reduced by a factor of two and the step recalculated. On the Ottawa University's I.B.M. 360 computer, a 300 time step solution to a 30 x 20 two-dimensional model took approximately 30 minutes of computer time. For a rapid hydrologic response involving flux across the upper boundary, such as rainfall and surface runoff, a real simulation of 35 hours on the 30 x 20 two-dimensional model took approximately 45 minutes of computer time. Conditions of little rainfall and no runoff would decrease the computer time by the use of larger time increments. No doubt new generation computers and more efficient programming will considerably reduce the computer time.

While computer speed was the main limitation of computer usage, another lesser limitation was computer storage. The finite difference grid adopted, required for stability reasons, at the surface boundaries and stream boundaries where the flux enters or leaves the system, a small vertical spacing 5 to 100 cm. in width. This limitation plus the availability of a mesh of only 20,000 nodes on the University of Ottawa's 360 computer will limit cross sections to small drainage areas.

CHAPTER 4

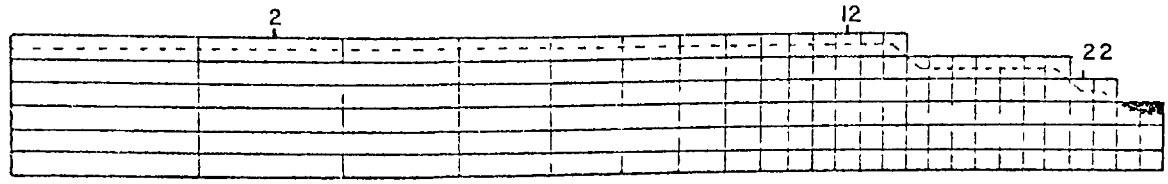
RESULTS AND DISCUSSION

4.1 Models

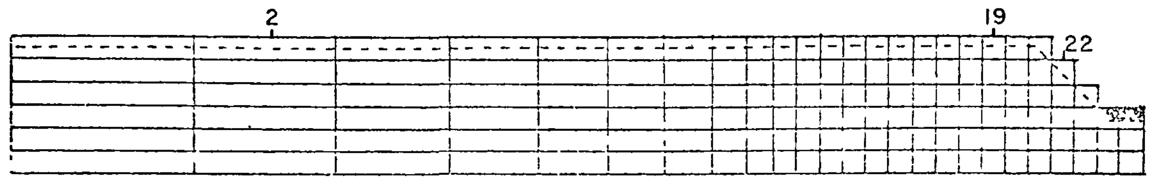
The developed model was applied to three different hypothetical configurations, namely: (i) a 30 x 20 node model, (ii) a smaller 25 x 10 node model 2 with a steep channel bank slope, and (iii) a 25 x 10 node model 3 with a mild bank slope. The 30 x 20 node model had the nominal dimensions of 807 cm. long by 193 cm. high and the two 25 x 10 node models had the dimensions of 337 cm. by 60 cm. high. The models are shown on Figure 4.1. The dotted line indicates the actual slope represented by the model block form.

4.2 Numerical Method Used

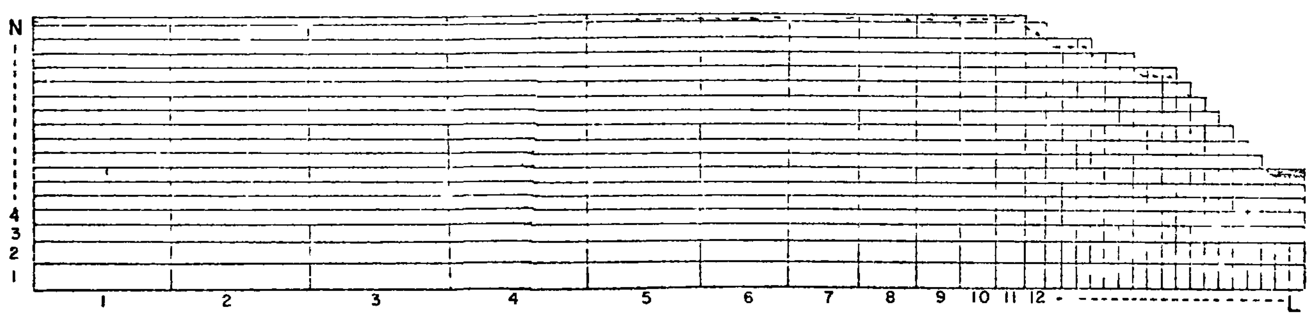
As discussed in Chapter 3, two numerical methods, the L.S.O.R. and the S.I.P. were used to solve the groundwater equation 2.4. A listing of both programs is given in Appendix D, Physical Model Computer Program. A brief comparison was made between the two methods using model 2, a prototype test time of 500 seconds, and input of uniform rainfall and an initial horizontal groundwater level. Since the test was taken at a time of rapidly changing water level, all boundary conditions were fully exercised. A convergence criteria of .001 cm. was used for both methods. In evaluating the two methods the main criteria were computer time storage requirements and relative accuracy. Figure 4.2 shows the results of this test.



Model 3



Model 2



Model 1

Z : nodes k = 1 , N

X : nodes i = 1, ...L

Fig. 4.1 Nodal grid models

Figure 4.2

Comparison of Numerical Methods

	L.S.O.R.	S.I.P.	S.I.P.-L.S.O.R. % change
Computer time (sec)	171.2	232.5	+36%
Storage required	115K	145K	+26%
Difference in pressure head (ψ) (range min. to max.)	L.S.O.R. 1.0% to 5.0% greater than S.I.P.		

While the difference in pressure head (ψ) as calculated by the S.I.P. method and L.S.O.R. method was neglibile (1% to 5%) the L.S.O.R. method was 36% faster and required 26% less storage than the S.I.P. method. Therefore it was decided to use the L.S.O.R. method for all further simulations.

The programs for both methods were written in Fortran G and were run on an I.B.M. 360/65 computer at the University of Ottawa. Both programs are described in Appendix D. For the long simulation runs the Fortran G program was optimized on a Fortran H program before carrying out the simulation runs. This optimization considerably reduced computer time.

4.3 Input

The soil type used in all three models had the properties of a Del Monte Sand (Liakopoulos, 1965) slightly modified for stability requirements by avoidance of zero slopes in determining specific soil capacity. The soil was assumed isotropic. The soil functional relationships are shown in Figure 2.2.

On Model 1, two different rainfall intensities, one less than the saturated permeability of the soil and the other greater than the saturated permeability of the soil, were applied to the model. The rainfall had a uniform intensity, 5.8 hours in duration.

On Models 2 and 3 a uniform rainfall intensity of .42 cm x 10^{-3} sec. was applied for a duration of .416 hours.

For all simulations rainfall was applied until the model became saturated and the overland flow hydrographs became well-defined.

4.4 Definitions

To clarify some of the terminology used in this chapter the following definitions are presented to avoid possible misunderstanding. The definitions are based upon those given by Hewlett (1967) and Freeze (1972).

DEFINITIONS

Runoff is that part of the precipitation that appears as stream flow either in perennial or intermittent form. It is the flow collected from a drainage basin that appears at the outlet of the basin.

Surface runoff is that portion of runoff which is made up of channel precipitation and overland flow. It is the rainwater or snowmelt that fails to infiltrate the soil surface at any point along its path of travel to a gauging station in a perennial stream.

Subsurface runoff is that portion of water passing the gauging station that has however briefly entered the soil surface and has travelled for some distance, however short, within the soil.

Channel flow is the flow at the downstream end of any reach of channel. Channel flow is the sum of the channel inflow to the reach, the lateral inflow along the reach and the channel precipitation along the reach.

Overland flow is that part of the lateral inflow that flows over the land surface toward a stream channel.

Subsurface stream flow or interflow is that part of the lateral inflow derived from water that infiltrates the soil surface and moves laterally through the upper soil toward the stream channels as unsaturated flow.

Base flow is that part of the lateral inflow derived from deep percolation of infiltrated water that enters the permanent saturated groundwater flow system and discharges into the stream channel.

For the purpose of these models the base flow and subsurface storm flow will be considered as part of the subsurface runoff and no differentiation between the two will be made.

4.5 Output

For each of the three models output takes the form of a pressure diagram, a hydraulic diagram, a soil moisture content diagram, infiltration at each surface node and an overland flow hydrograph. From graphical flow nets constructed using hydraulic head diagram, the subsurface flow contribution can be calculated.

4.6 Simulation of Runoff for Model 1

4.6.1 Surface runoff

For model 1, Figure 4.3 shows the resultant hydrograph of

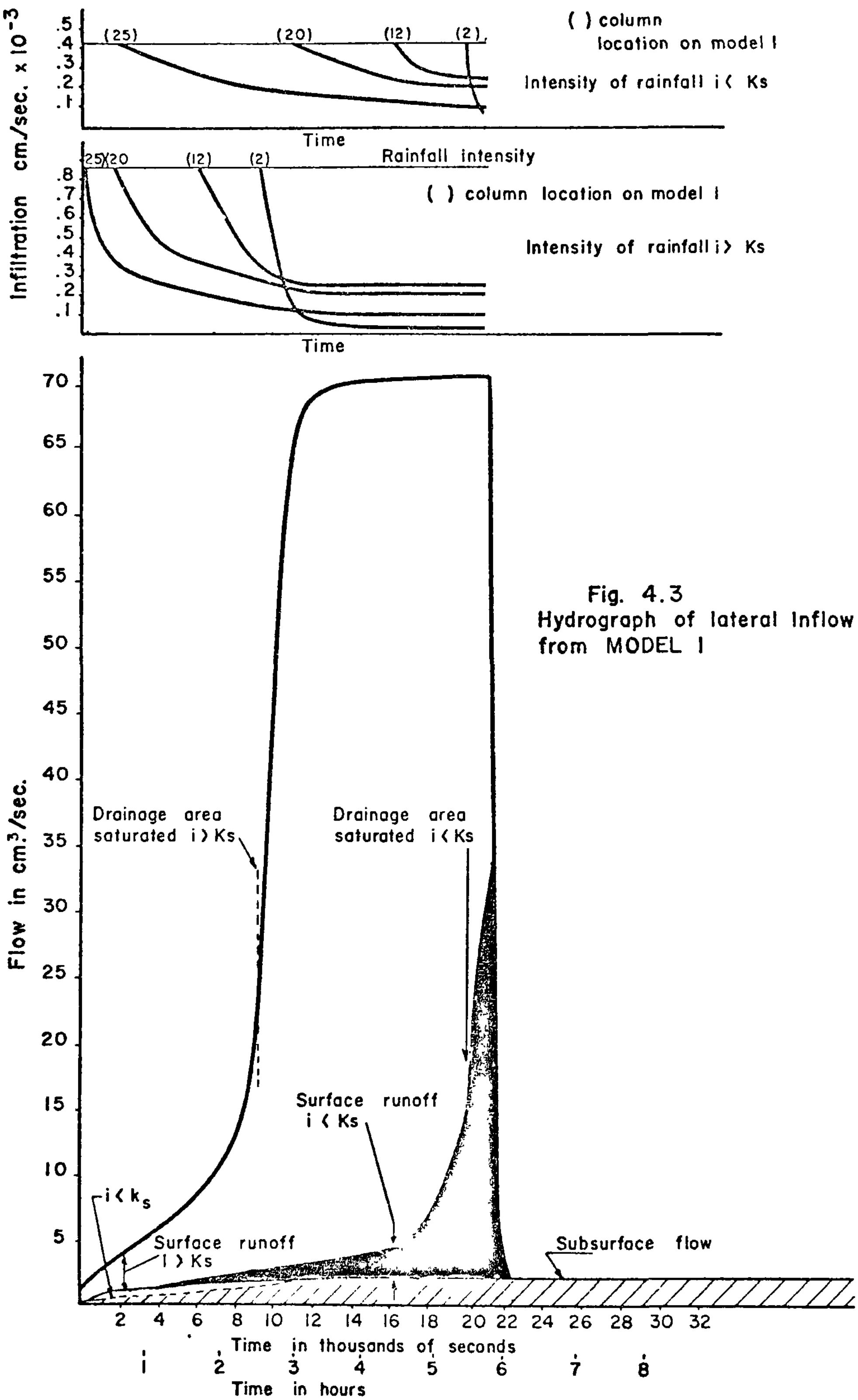


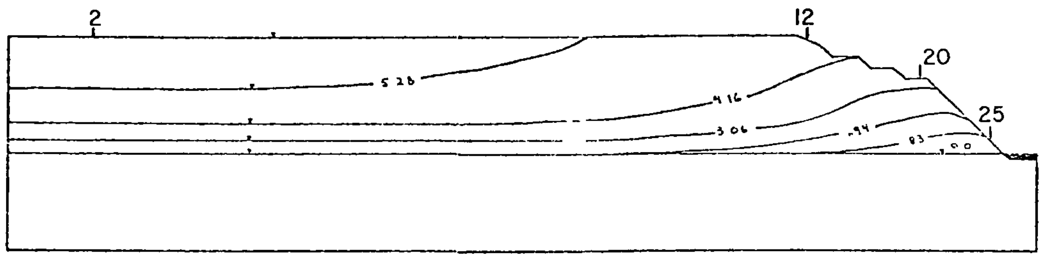
Fig. 4.3
Hydrograph of lateral Inflow
from MODEL I

lateral inflow for: (a) an intensity of rainfall less than the saturated hydraulic conductivity of the soil; (b) an intensity of rainfall greater than the saturated hydraulic conductivity of the soil. Figure 4.4 shows the water table rise for the two different intensities. Figure 4.5 shows the percentage of drainage area contributing to runoff. The above figures show that as the surface saturated zone expands from the channel area further into the watershed section. When the rainfall stopped the water level dropped causing the area of surface runoff to shrink rapidly to a small seepage face adjacent to the stream.

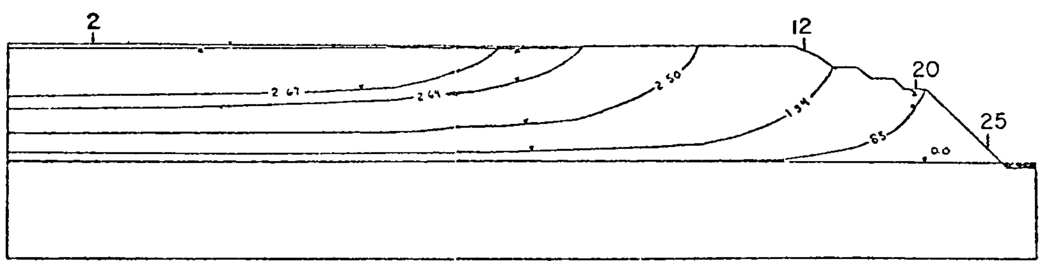
This concept of a variable source area was first espoused experimentally by Hursh and Brater in 1941; Hewlett and Hibbert in 1963, and 1967 and Dune and Black in 1970. Dune and Black's experimental results in a New England water shed are confirmed by the theoretical works of this study. They found that on a watershed with a soil similar to that of the model "significant amounts of storm runoff were only produced on small areas of hillside where the water table reached the surface."

It should be noted that for the intensity of rainfall less than the saturated conductivity, saturation occurs from below because of the rising water table fed by infiltration above.

Where the intensity is greater than the saturated surface permeability for Model 3, it can be observed in Figure 4.4 that saturation comes from above in the flat upper reaches.



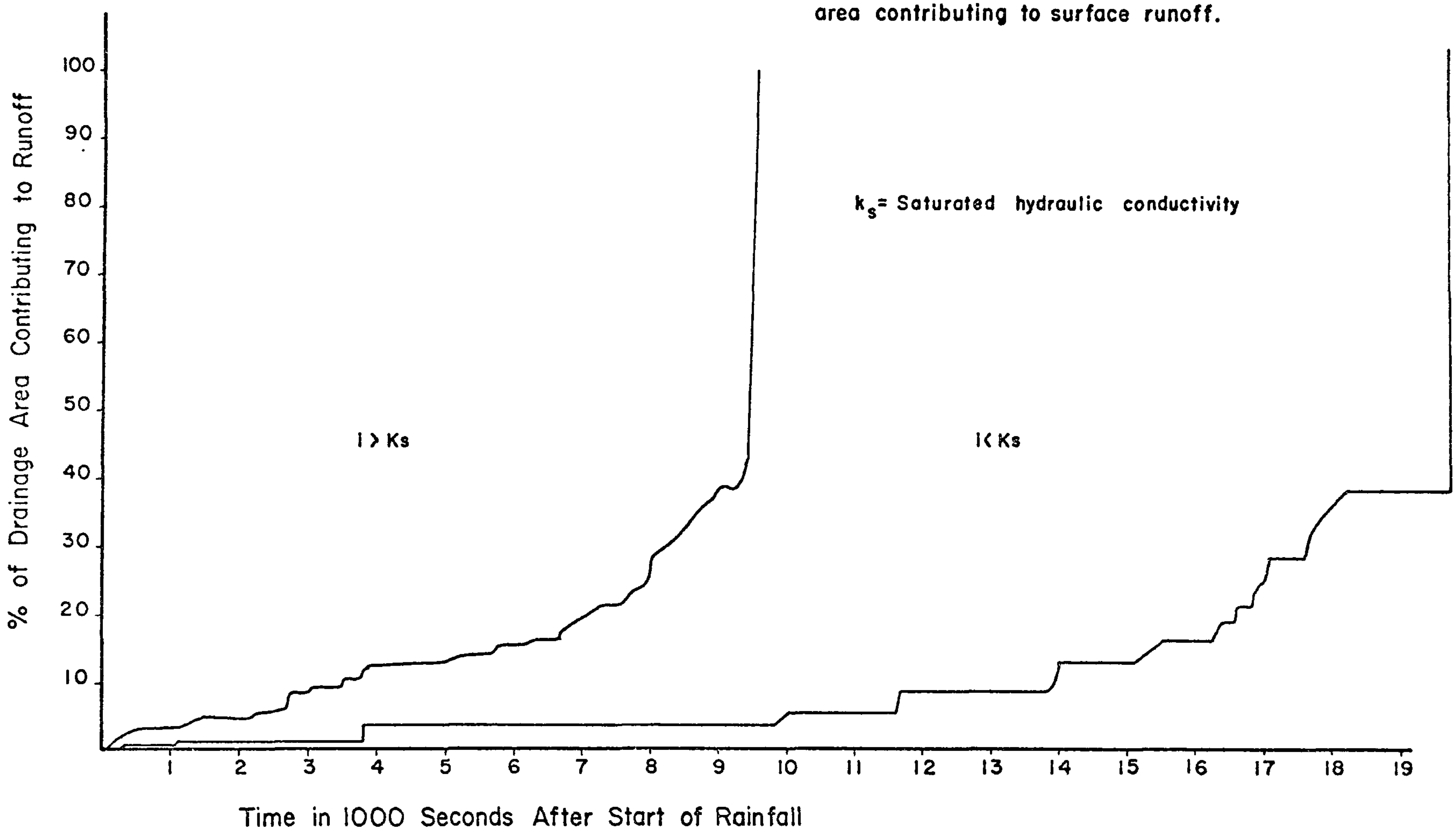
a) Rainfall Intensity less than ks



b) Rainfall Intensity greater than ks

Fig. 4.4 Water table rise at specified hours after start of rainfall.

Fig. 4.5 Diagram showing amount of model drainage area contributing to surface runoff.



4.6.2 Subsurface flow

The groundwater hydrographs for the two different intensities as shown in Figure 4.6 are the same except for the initial 3-1/2 hours after the rainfall when the water level is rising in the channel bank area. This is shown in Figure 4.4 illustrating the increase in water level during the two different intensities.

The contribution of the subsurface flow to the total surface runoff is small compared to the contribution of the lateral surface runoff as shown in Figure 4.3. This is again borne out by Dune and Black's experimental study where "subsurface storm flow occurred in large storms but was not an important contribution to total storm runoff despite conditions favorable for its existence".

After approximately 3-1/2 hours of the lower rainfall intensity the channel bank area becomes saturated and subsurface flow becomes the same as the subsurface flow resulting from the higher intensity rainfall.

By contrast it takes only approximately 1 hour for the channel area to become saturated for the higher intensity rainfall thus resulting in a higher local gradient and slightly higher subsurface flow in the early portions of the hydrograph.

It is the local hydraulic gradients in the channel area that largely determine the subsurface flow.

As can be seen from the hydrographs in Figure 4.6 as soon as the surface becomes saturated the subsurface flow becomes constant

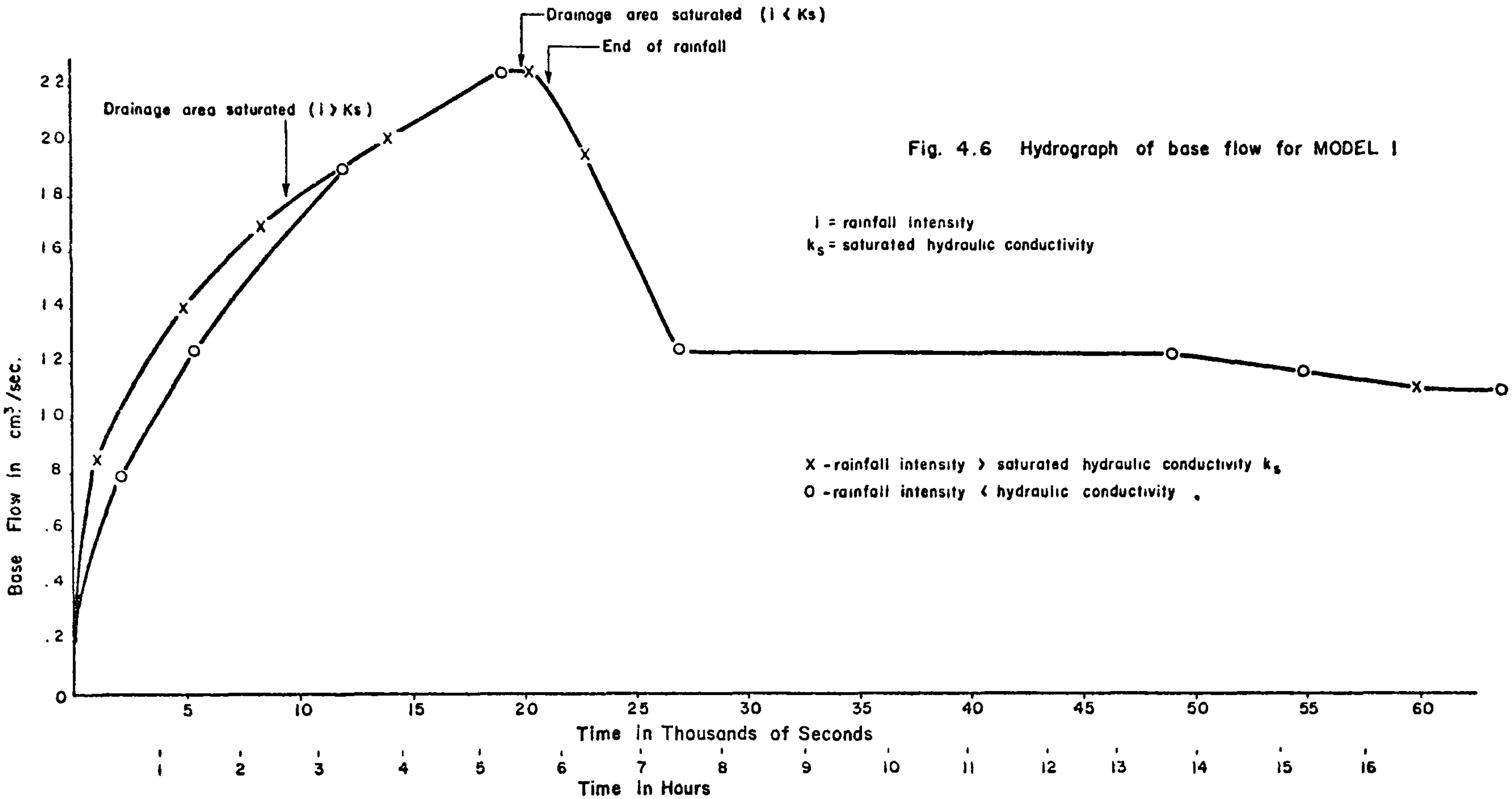


Fig. 4.6 Hydrograph of base flow for MODEL I

and as the rainfall is prolonged the surface runoff becomes clearly a large portion of the lateral inflow hydrograph.

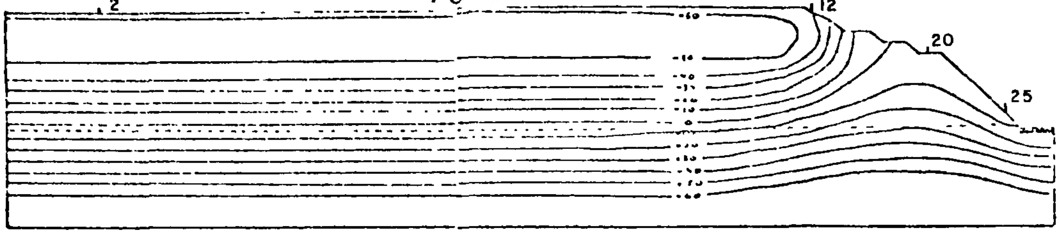
From Figures 4.7 and 4.8 for the time when model 1 is saturated it can be seen that most of the subsurface flow is generated by the area immediately adjacent to the bank while the rainfall influence is gradually diminished away from the stream channel.

Even after 30 hours after rainfall has ceased there is still recharge from the unsaturated zone to the saturated zone as shown in Figure 4.7d, the soil moisture content diagram and 4.7f, the hydraulic head diagram. Again, this area is restricted to the immediate channel bank area.

4.7 Simulation of Runoff for Models 2 and 3

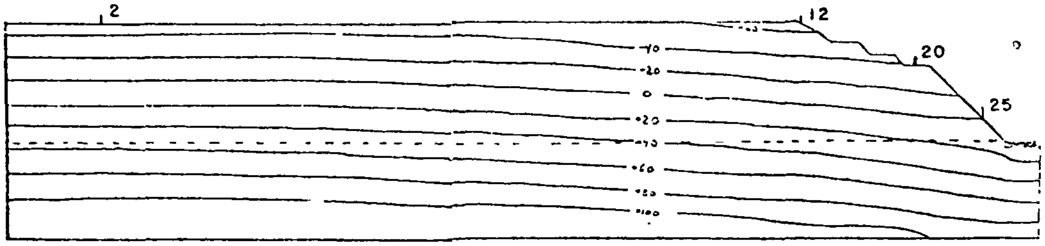
4.7.1 Surface Runoff

For models 2 and 3 in Figures 4.9 and 4.10, the rising limbs of the Lateral Inflow Hydrograph reflect the channel bank slope differences or varying channel storage capacities. Model 3 becomes saturated sooner than model 2 as indicated by the infiltration curves of Figures 4.9 and 4.10 plotted for several nodal points along the model profile shown in Figure 4.1. Thus, as shown in Figure 4.11 with the drainage area increasing faster than in model 2, the rising limb rises sooner and at a different rate than in model 2. The receding limbs, as one might expect, are similar in shape since the model is saturated and the falling limb reflects a drainage from the same size detention storage being minimally affected by the difference in channel bank slope.



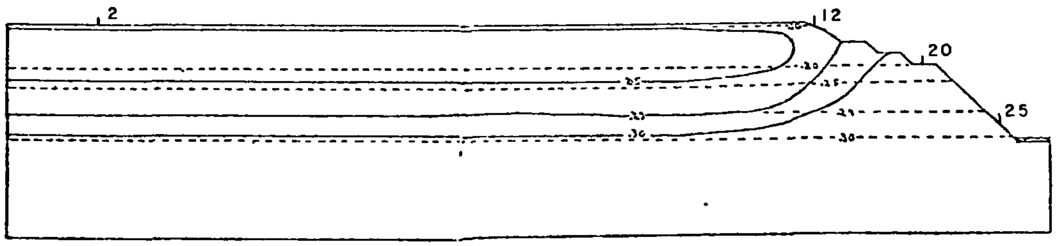
5,402 sec

a) Water level at 1.5 hours after start of rainfall (---- Initial water level)



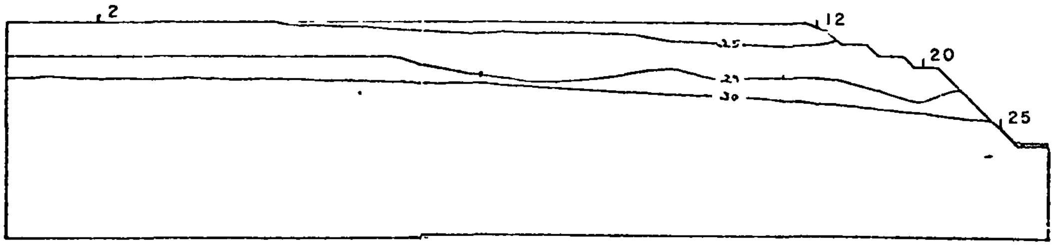
35 hours

b) Water level at 35 hours after start of rainfall (---- Initial water level)



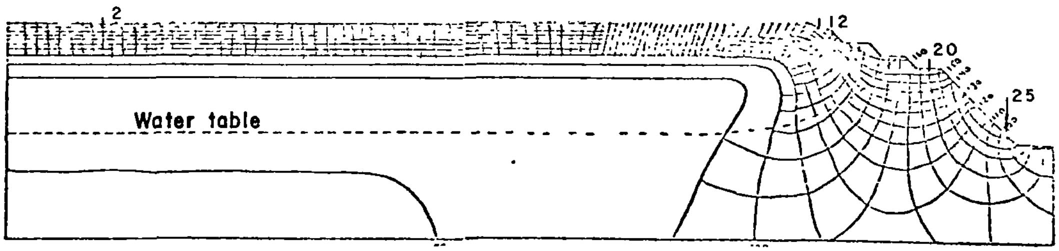
5,402 sec.

c) Soil moisture content at 1.5 hours after start of rainfall (---- Initial soil moisture content)



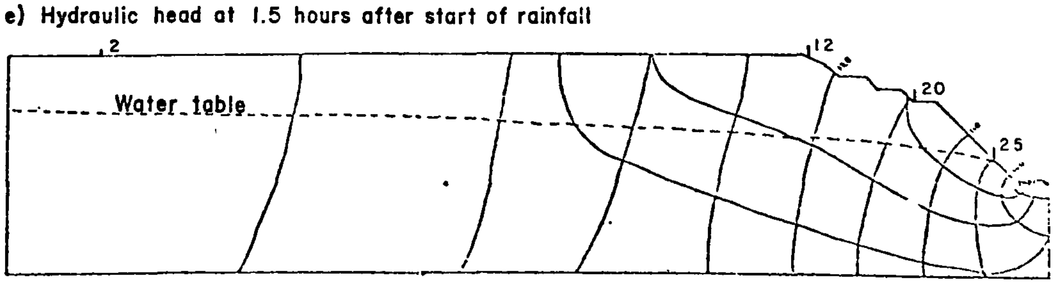
35 hours

d) Soil moisture content at 35 hours after start of rainfall



5,402 sec.

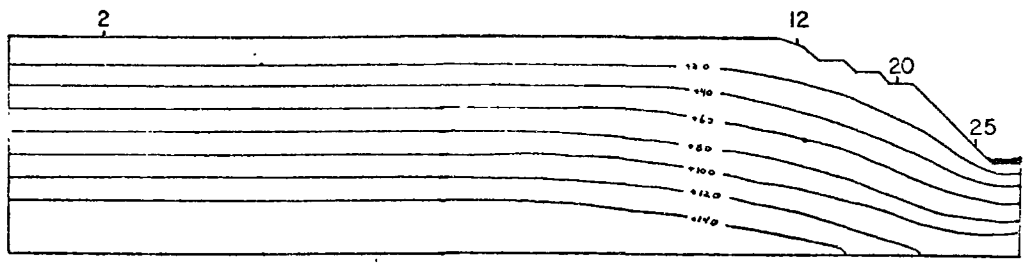
e) Hydraulic head at 1.5 hours after start of rainfall



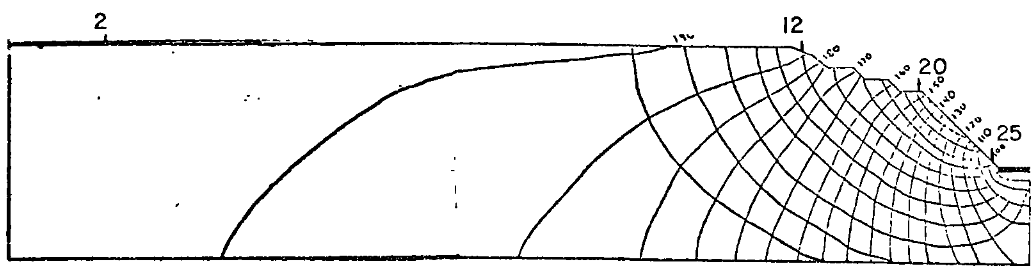
35 hours

f) Hydraulic head at 35 hours after start of rainfall

Fig. 4.7 Transient output from model I



a) Pressure diag. at time = 5.63 hours after start of rainfall.
i) ko



b) Total hydraulic head diag. at time = 5.63 hours after start of rainfall.
i) ko

Fig. 4.8 Saturated conditions 5.63 hours after start of rainfall for model I

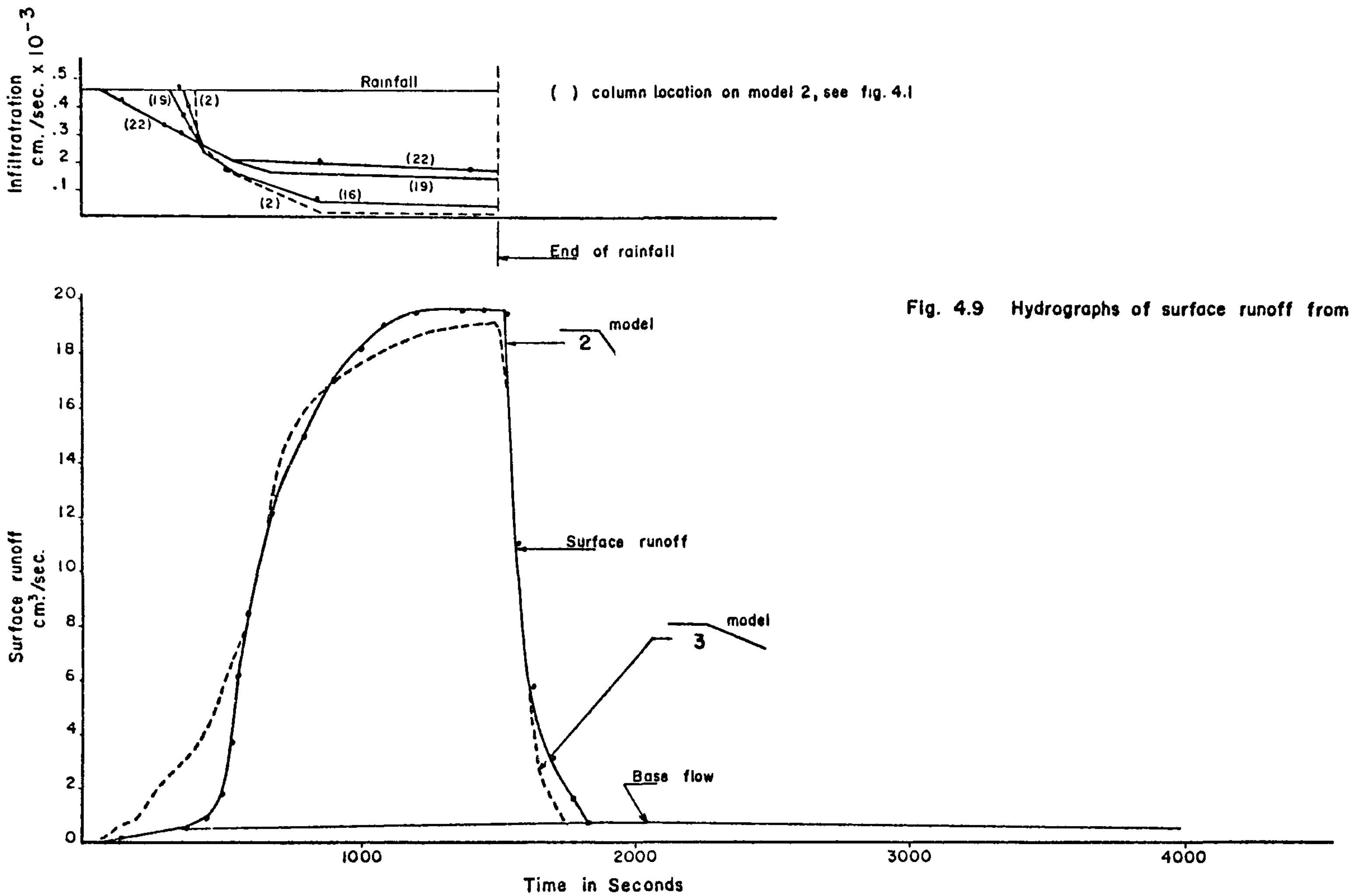


Fig. 4.9 Hydrographs of surface runoff from MODEL 2

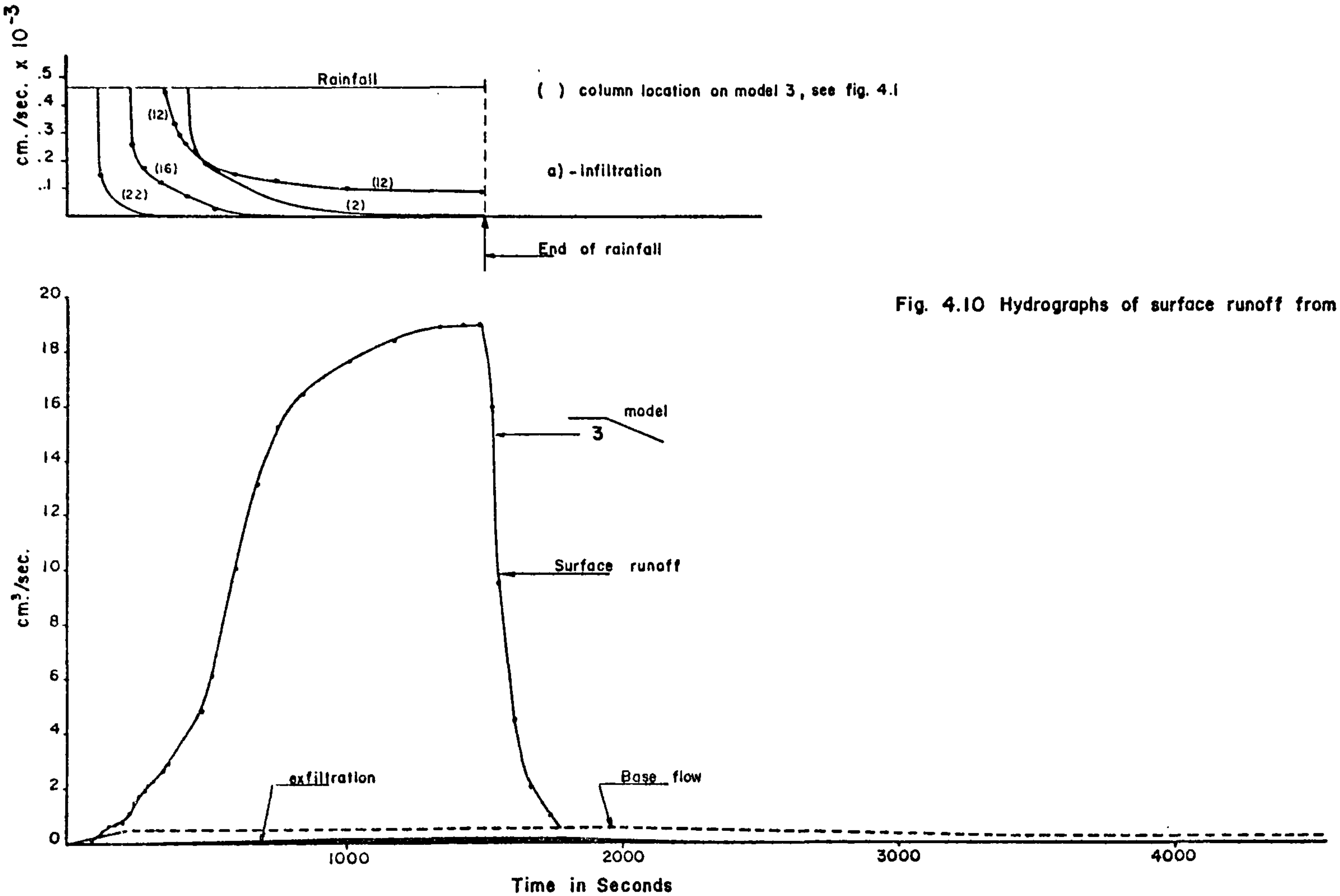


Fig. 4.10 Hydrographs of surface runoff from MODEL 3

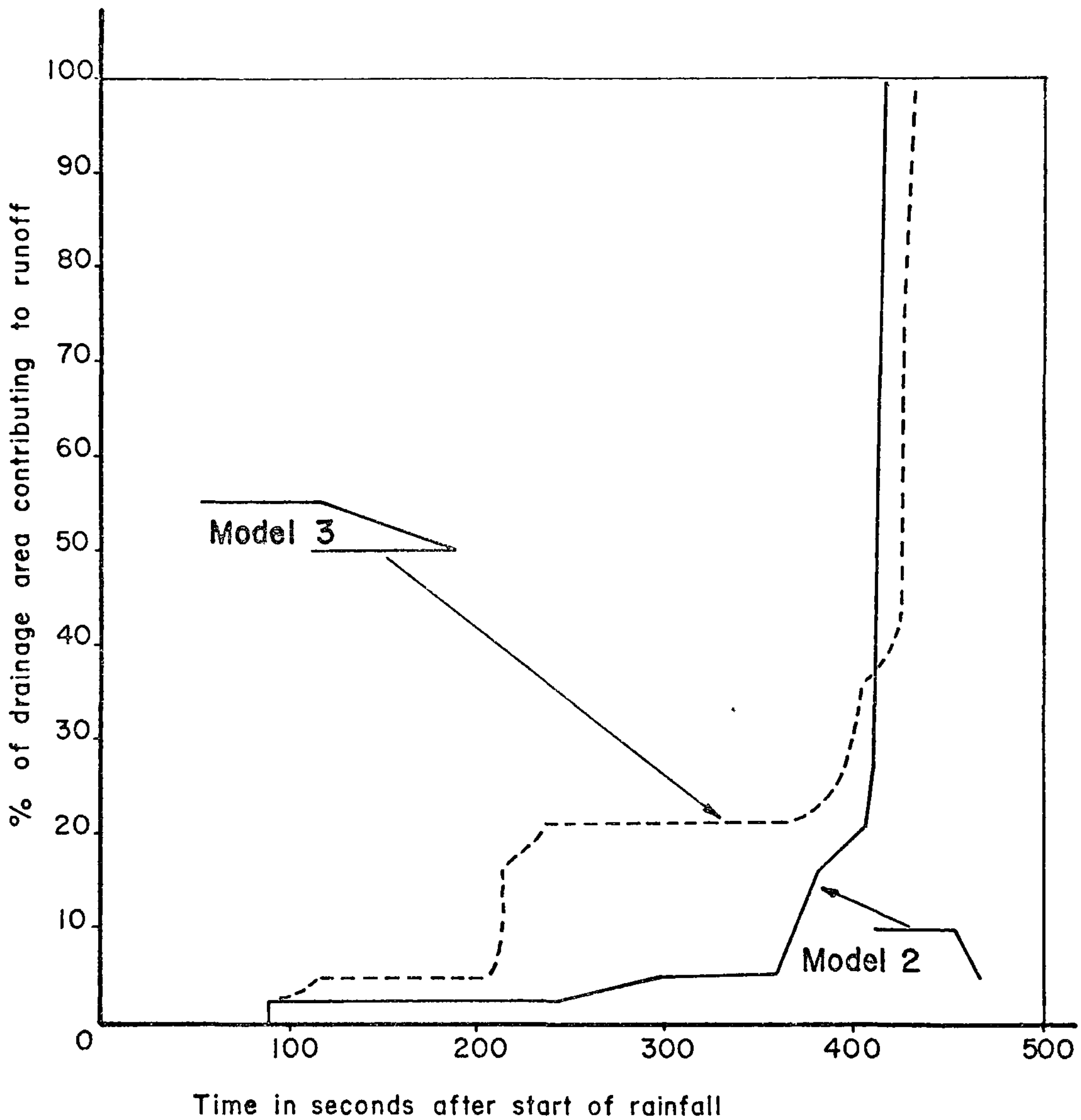


Fig. 4.11 Comparison of the contributing drainage area for models 2 and 3.

4.7.2 Subsurface Runoff

The groundwater hydrographs as shown in Figure 4.12 again reflect the difference in overbank storage capacities with model 2 showing a greater peak in the first hour of runoff. In model 2 with a steep bank slope the infiltration was higher in the stream area due to the geometry of the bank inducing a more rapid water level rise than in model 1.

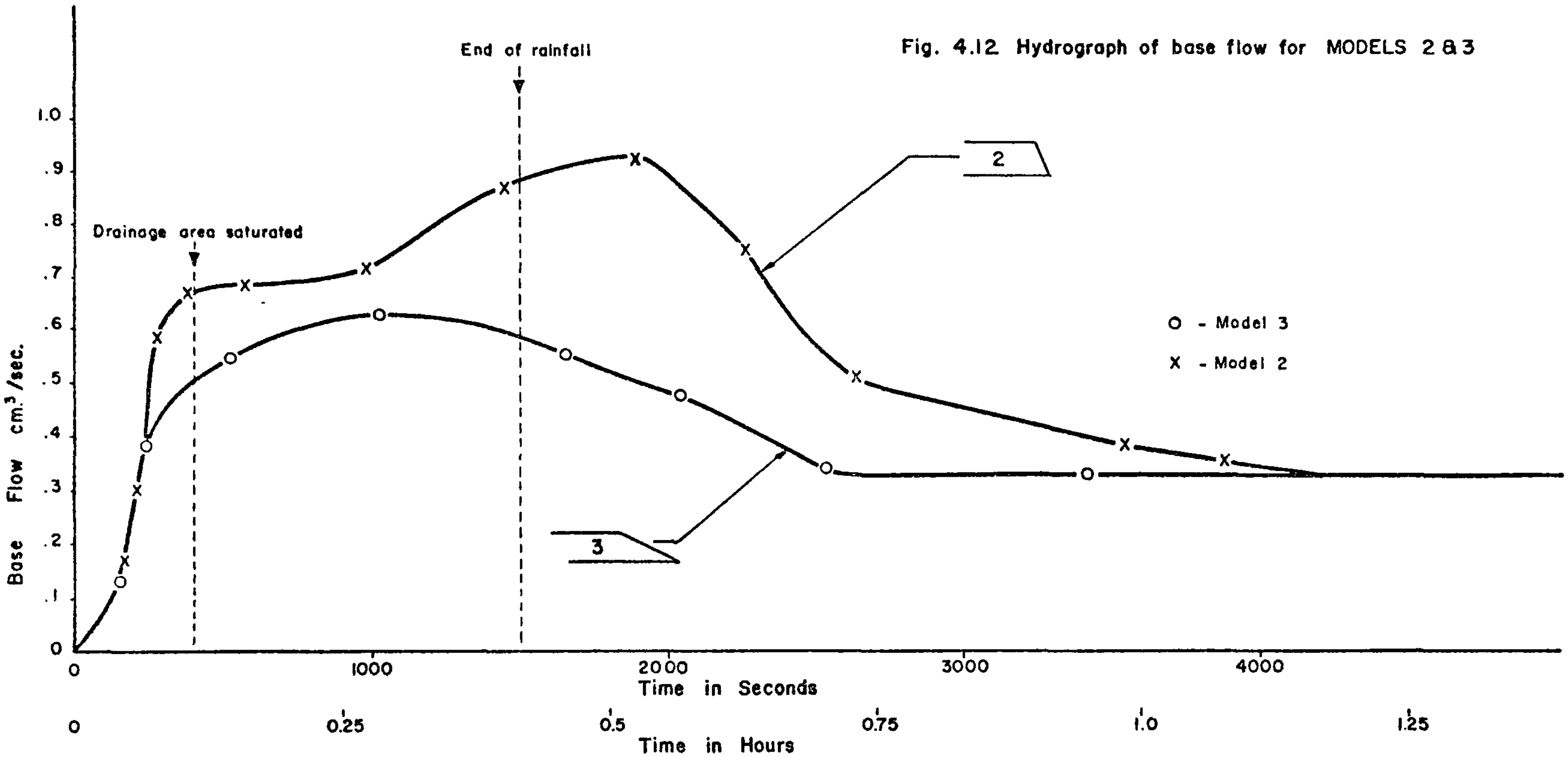


Fig. 4.12 Hydrograph of base flow for MODELS 2 & 3

CHAPTER 5

A STOCHASTIC ANALYSIS OF PHYSICALLY-BASED MODEL

5.1 Stochastic and Deterministic Models

The physically-based model is an example of the deterministic model where the input and output variables as well as model parameters take on infinite values and the model yields a single definite or deterministic solution. Stochastic or statistical models, on the other hand, describe the inherent uncertainties of the process. According to Clark (1973) a model is termed stochastic or deterministic according to whether or not it contains random variables. If any of the variables of this model, rainfall, streamflow or groundwater are thought to have a distribution in probability, then the model is termed stochastic. While the output from the proposed model appears deterministic in nature, its nature is in fact stochastic due to the uncertainties introduced by the input variables, model parameters and the inadequacy of the model in describing the hydrologic processes. This section will show how the element of uncertainty is introduced into the physical model.

The model parameters which are subject to uncertainty are the pressure-dependent curves of hydraulic conductivity and moisture control and the friction coefficient in the overland flow portion of the model.

The rainfall input and the evapotranspiration output while stochastic in nature will be assumed constant, that is to say, deterministic for different values of the model parameters. The model output variables to be studied are surface runoff and groundwater runoff.

Because of the non-linearity of the model it would be extremely difficult to obtain analytic solutions to the present model once a stochastic component is introduced into it.

Monte Carlo simulation however can be used to estimate the various statistics of the output.

In Monte Carlo simulation as outlined by Benjamin and Cornell (1970) the random variables with specified ensemble means, ensemble standard deviations and probability distributions are introduced as inputs and parameters into the model. The model is solved many times yielding a frequency distribution for the model's output from which one can calculate the output sample, the mean, the sample standard deviation, and possibly one can estimate the character of the probability distribution. Although the introduction of stochastic methods of analysis into physical simulation does not reduce the existing uncertainty, it can lead to a more precise statement about the uncertainty.

5.2 Saturated and Unsaturated Soil Properties

The input to the groundwater portion of the model are the pressure-dependent curves of hydraulic conductivity and moisture content. Figure 2.2a shows the hysteretic functional relationships which go to make up unsaturated soil properties of a typical sand (Brooks and Corey, 1964). In order to reduce the complexities of the unsaturated soil properties, hysteresis has been ignored and the functions have been reduced using the work of Brooks and Corey to single value $\theta(\psi)$ and $k(\psi)$ functions as shown in Figures 2.2b and 5.1. To relate the two functions Figure 5.1 shows the soil properties in a dimensionless form where the effective saturation S_e is given by; $S_e = \frac{S-S_r}{1-S_r}$, the relative pressure P by $P = \frac{\psi}{\psi_a}$ and K_r, the relative conductivity is given by, $K_r = \frac{K}{K_o}$. Here S is the usual fractional saturation ratio, S_r the residual saturation or irreducible saturation, K_o is the saturated hydraulic conductivity and ψ_b is the bubbling or air entry pressure head. ψ_b is the minimum pressure head at which a continuous non-wetting phase exists in a porous medium.

To relate the $\theta(\psi)$ and $k(\psi)$ functions as shown in Figure 5.1, the following empirical relationship (Brooks and Corey) was used:

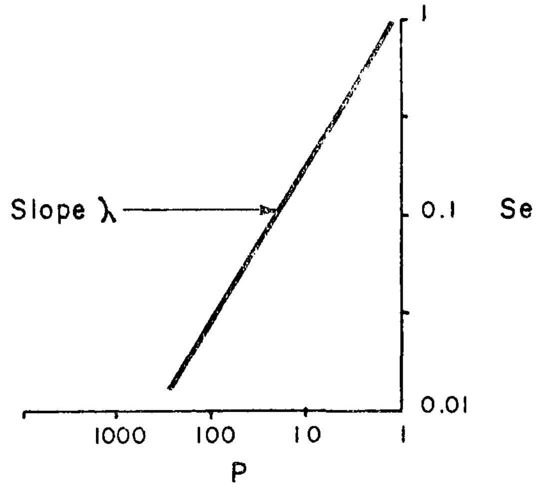
$$K_r = \frac{1^n}{p} \qquad 5.1$$

where $\eta = 2 + 3\lambda$ and λ, the pore size distribution index, is the exponent in the equation:

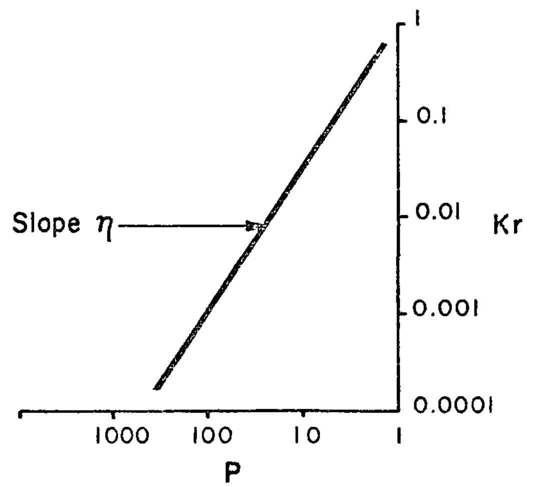
$$S_e = (\frac{1}{p})^{\lambda} \qquad 5.2$$

See Figure 5.1a for an illustration of equation 5.2.

Since the solutions of the physical model are relatively insensitive to Sr and ψ_o (Freeze, 1972) then the unsaturated soil



a) Effective saturation Se verses relative pressure head P



b) Relative conductivity Kr verses relative pressure head P

Fig. 5.1 Dimensionless single-valued unsaturated soil properties for a sand (Brooks and Corey) properties are defined by only three parameters: that is, K_o, the saturated conductivity; θ_o, the saturated moisture content and λ, the pore size distribution index.

It has been widely observed (Law (1944), Warren (1961), Freeze (1976) and Davis (1969)) that the saturated conductivity data, Ko, taken from a single formation has a log normal frequency distribution and the saturated moisture content data θ_o taken from the same formation has a normal distribution. From Brooks and Corey's data a normal distribution was assumed for a given λ. Means and standard deviations for each of the distributions were synthesized from actual data gathered by Law (1944), Brook and Corey (1964).

The probability density functions for the three basic soil model parameters K_θ, θ_o and λ were now defined. From Brook and Corey's data it appeared that the parameters were not independent, and may be highly correlated. In theory, the correlation between these three parameters can be taken into account by using a multivariate normal density function to generate the parameter values (Mood and Greybill, 1963). However to fully define the trivariate system it is necessary to specify $\overline{K}_o$, mean, $\overline{\theta}_2$ mean, $\overline{\lambda}_3$ mean, σ_{11} standard dev. of K_o, σ_{22} standard dev. of $\overline{\theta}_o$, σ_{33} standard dev. of λ_3, ρ_{12} correlation coefficient relating saturated hydraulic conductivity and the porosity, ρ_{13} relating the hydraulic conductivity and the pore size distribution index, and ρ_{23} relating the pore size distribution index and porosity.

Since there is very little data to indicate the usual range of values in the field for the correlation coefficients ρ_{xy}, a simpler

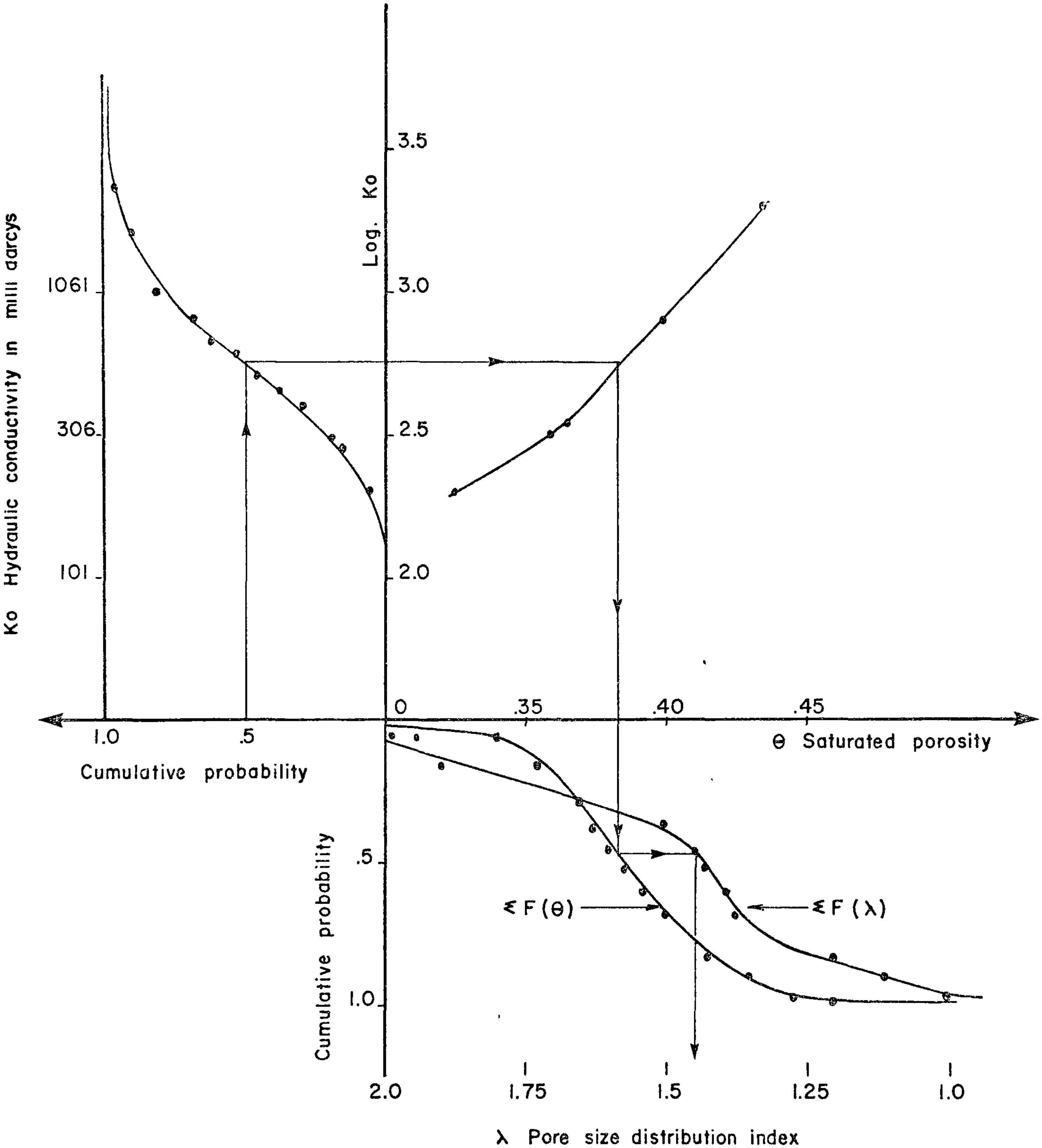


Fig. 5.2 Showing assummed relationship between Ko, $\Theta_{SAT.}$ and λ

and more approximate method of generating the model's parameters has been adopted. Basically this consists of making K_o, the saturated hydraulic conductivity, the independent variable, and selecting values of K_o at random from its marginal distribution. With K_o then selected θ_o and λ would be treated as dependent variables and selected so that the area under the marginal distribution for θ or λ is the same as the area for the selected K_o. This is shown on Figure 5.2. This tacitly assumes $\rho_{23} = 0$ and $\rho_{12} = 1$, $\rho_{13} = 1$.

The soil properties of K_o, θ_{sat} and λ were chosen from values determined experimentally by Laliberte, Corey and Brooks. A Touchet silt loam was selected with the following properties:

K_o = .570 Darcys hydraulic conductivity;

θ_o = .384 porosity, and λ = 1.47 pore size distribution index.

From Law's data a standard deviation of $\sigma = 0.3$ was assumed for the log-normal distribution of hydraulic conductivity and $\sigma = 0.03$ for the normal distribution of porosity. Based on Corey and Brook's data a standard deviation of $\sigma = 0.23$ was assumed for the normal pore size distribution index. The statistical parameters describing the Touchet silt loam are summarized in Figure 5.3.

5.3 Overland Flow Friction Coefficients

In describing overland flow the largest element of uncertainty is introduced in trying to estimate f, the Darcy-Weisbach friction factor. This factor is used in momentum equation 2.12a described in Chapter 2 and is given by

Figure 5.3

Statistical Parameters for the Touchet Silt Loam Soil Used in Monte Carlo Simulation

Properties	Statistical parameters		Assumed probability distribution function	Source of data
	Mean μ	Standard deviation σ		
Hydraulic conductivity	Darcys 0.570	0.3	Log Normal	Law (1944)
Parameter, θ	0.384	0.03	Normal	
Pore size, λ	1.47	0.23	Normal	Brooks & Corey(1964)

Figure 5.4

Statistical Parameters for the Overland Flow Parameter

Properties	Statistical parameters		Assumed probability distribution function	Source of data
	Mean μ	Standard deviation σ		
Friction coefficient	$\ln C_n - \ln R_e$	.142	Log Normal	Shen Li, Phelps

C_n is a constant = 41.5

R_e is $\frac{q}{\nu}$ = Reynolds number

$$f = \frac{8gS_o d^3}{q^2}$$

The Darcy-Weisbach factor "f" for overland flow on rigid boundaries is a function of roughness of the boundary, the depth of flow, the rainfall intensity and the Reynolds number. The Reynolds number may be defined as

$$R = \frac{q}{v} \qquad 5.3$$

in which v is the kinematic viscosity of the fluid.

The effect of rainfall on flow resistance is a major factor in shallow water routing. Shen and Li (1973) have experimentally determined in smooth bed flumes equations for Darcy-Weisbach friction for flow with rainfall impact. Their equations have the general form:

$$f = \frac{a^2}{Re} \ b_2 \qquad 5.4$$

in which a_2 and b_2 are functions of the rainfall intensity, the boundary roughness and the Reynolds number.

For Re < 900

$$f = \frac{N_1}{Re} = \frac{N_o + N_r I^{0.41}}{Re} \qquad 5.5$$

in which N_1 is a parameter which varies with rainfall intensity, N_o is a constant representing the Darcy-Weisbach factor without rainfall, I is the intensity of rainfall in inches/hour. N_r is a number dependent on the rain drops' velocity. According to Shen and Li the Darcy factor "f" is increased for increasing rainfall intensity due to the increased energy losses caused by the rain drops' impact on the flow.

However Langford and Turner (1973) showed that for a rough uneven surface at a low Reynolds number the intensity of rainfall decreased the Darcy factor f. This is shown in Figure 5.5. Phelps (1970, 1975) also found for sheet flow over rough surfaces that

$$f.Re = C_N \qquad 5.8$$

which is constant for a smooth surface is a function of relative roughness. $\frac{e_1}{d}$ or boundary geometry for the case of flow through vegetation. He also determined that the critical value of Re is lowered as relative roughness is increased. This effect is also shown in Figure 5.6.

Since the purpose of this investigation was to examine the effect of the roughness factor upon the solution of the watershed model, the f vs Re relationship was simplified as shown in Figure 5.7. A constant rainfall was used and since the depths involved were small and relatively constant at maximum flow, f x Re was assumed constant (thus neglecting the e_1/d effect). In addition, no attempt was made to increase C_N after the rainfall ceased since the recession portion was extremely short compared to the remaining portion of the hydrograph.

From preliminary calculations it was determined that most of the flow for model 3 will be laminar therefore a Re_{crit} was assumed equal to 70 where for laminar flow Re < 70. If random error is included

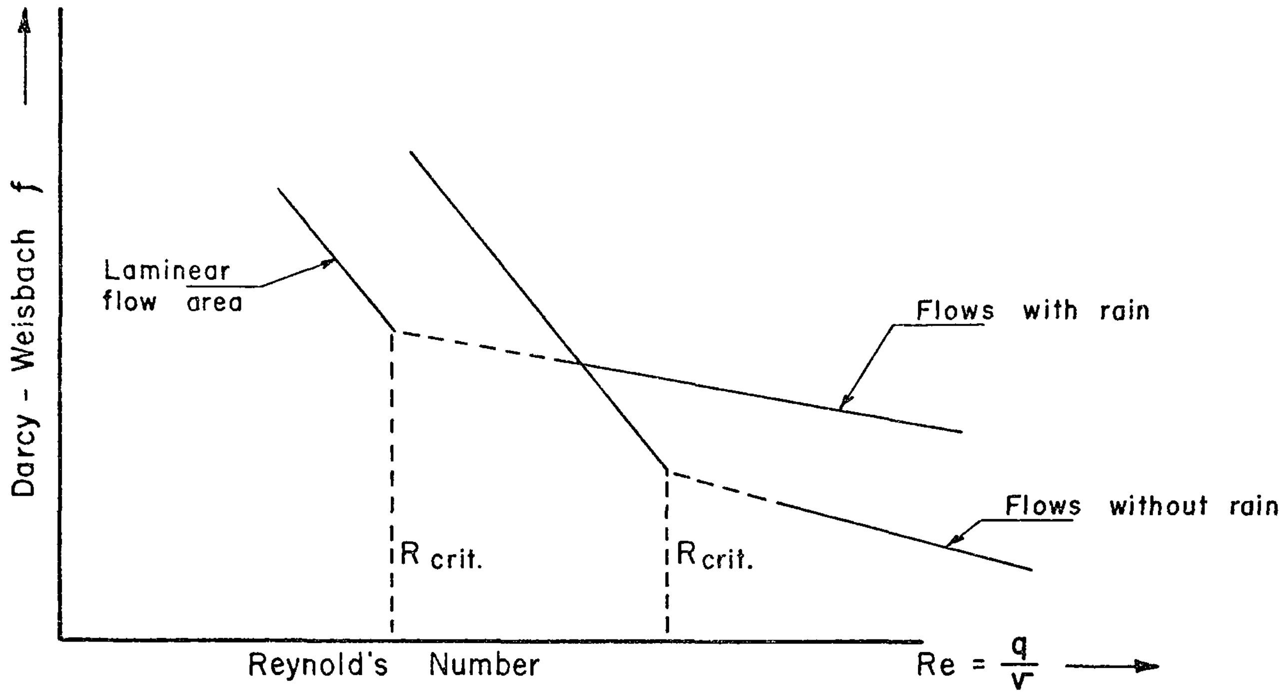


Fig. 5.5 Friction coefficent graph for a typical rough uneven surface illustrating the effect of rainfall.

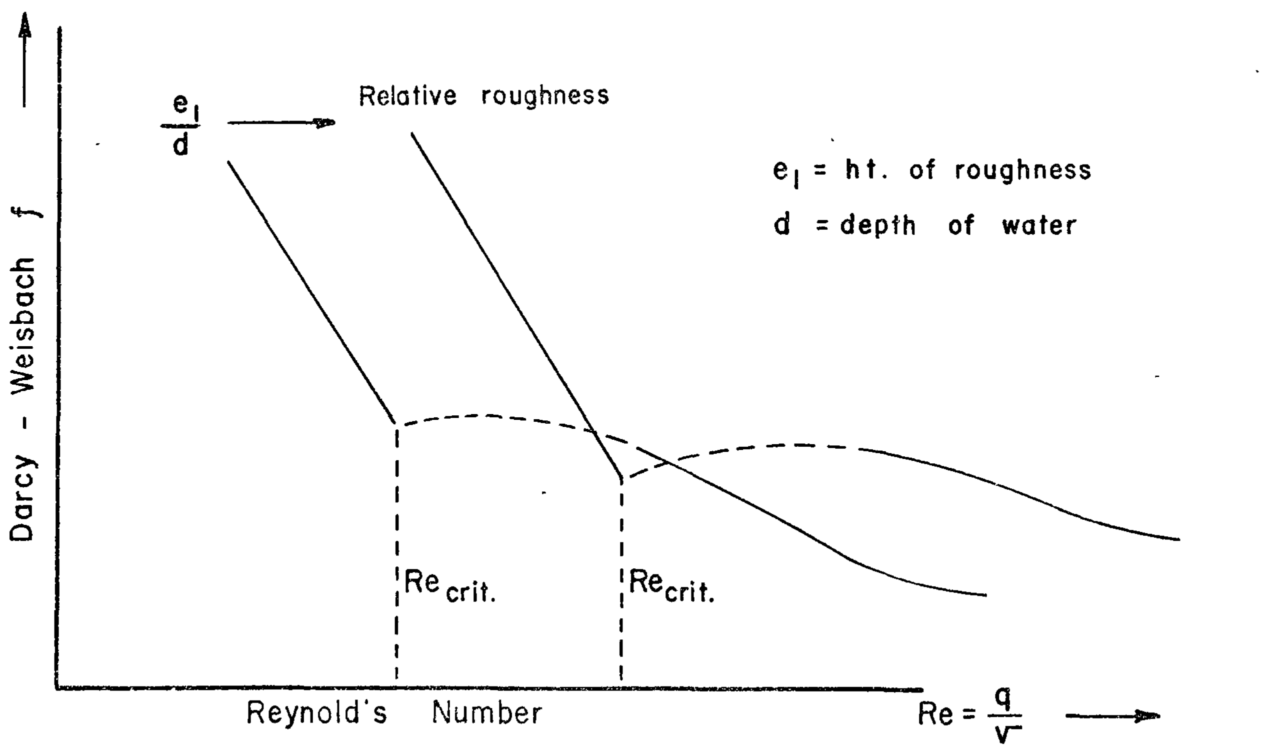


Fig. 5.6 Friction coefficent graph illustrating the influence of relative roughness.

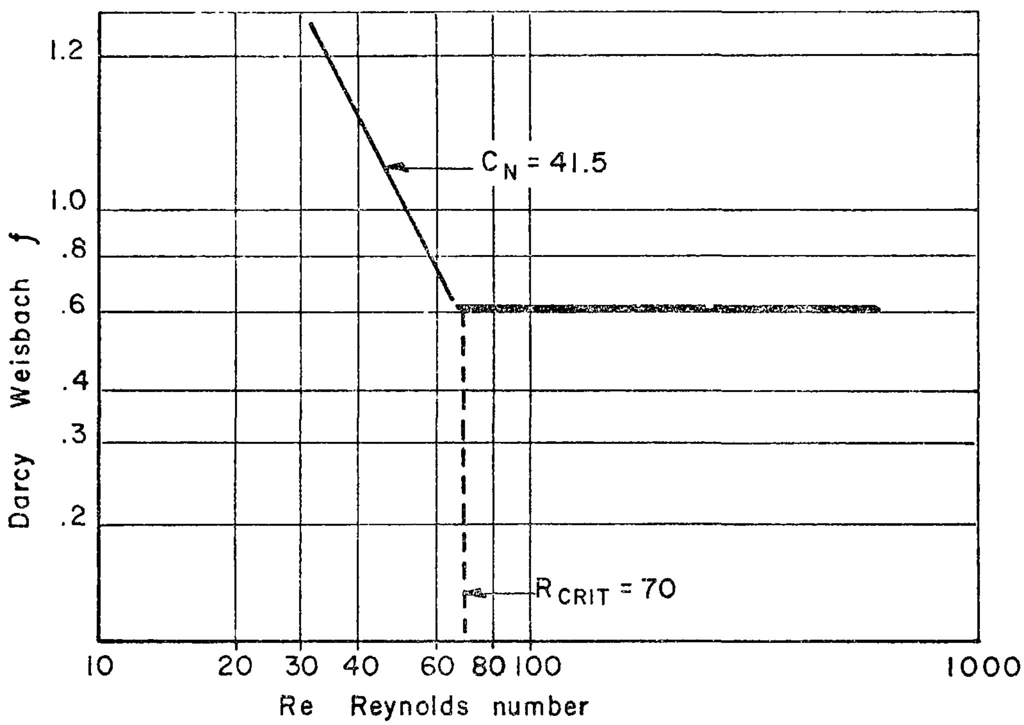


Fig. 5.7 Friction coefficent relationships used for model simulations.

Equation 5.8 may take the form:

$$f = \frac{C_N \varepsilon}{Re} \qquad 5.8a$$

or by taking logarithms

$$\ln f = \ln C_N + \ln\varepsilon - \ln Re \qquad 5.8b$$

It is assumed that $\ln\varepsilon$, random error, is normally distributed with $N(0,\sigma^2)$ where σ is the standard deviation of the regression equations of f vs Re from the data of Phelps. Random variables of lnE are generated from a uniform distribution of random numbers, then interpreted from a normal curve to give t_u, a normally distributed random variable with zero mean and unit variance. Then $t_u\sigma_e$ = lnE is a normally distributed random variable with zero = 0 and $\sigma = \sigma_e$. The statistical parameters are summarized in Table 5.4.

5.4 Generation of Data and Results

Model parameters were determined at random as described in Section 5.1 to 5.3 and 40 simulations were carried out. The data is shown in Figure 5.8. From these 40 simulations the output data was plotted in a histogram and then as suggested by the histogram, a suitable distribution was selected, tested for goodness-of-fit on probability paper and by the λ^2 test.

Frequency curves of surface runoff subsurface runoff and peak flow are shown on Figures 5.9, 5.10 and 5.11, respectively. The result of the Chi-square test are shown in Figure 5.12.

Fig. 5.8 Summary of simulation data

No.	SAT Hydraulic Conductivity milli-darcy	Darcy Coefficent f	Surface Runoff cm	Peak Flow cm^3/sec	Ground Water Runoff at 21,000 sec cm	Ground Water Peak cm^3/sec	Total Peak cm^3/sec
1	263	38 00	9 29	70.7	.055	.93	71.6
2	881	35 25	8 28	65 3	237	3 0	68 3
3	1004	46 49	8 06	64 5	262	3 45	68 0
4	1018	55 84	8.09	64 1	.281	3.49	67 6
5.	292	39 83	9 17	70 6	.063	1.01	70 6
6.	731	35 70	8 31	66 7	1 96	2.51	69.2
7	409	46.49	7.97	69.5	.100	1.41	70.9
8	497	41.34	8.66	68 7	.127	1.71	70.4
9.	1469	47 69	7.39	60 2	.377	5.03	65 2
10	1227	58 77	7 77	62.3	.323	4.20	66.7
11.	238.7	48.04	9.32	70.5	.046	.82	71.3
12.	412	38 66	8 92	69.6	.102	1.42	70
13.	1584	37 79	7 47	59 2	.437	5 76	65
14	235	31 33	9.16	69 3	.045	.81	70.3
15	869	42.76	8.56	65.4	.245	2.98	68 4
16	473 01	46 23	8.97	69.0	.129	1.67	70.7
17.	1185	36 37	7 92	62 7	.328	4.10	66.8
18	850 9	38.66	8.35	65.7	.233	2 98	68.7
19	655	37.6	8 64	67 5	.176	2.25	69 8
20.	476	51 9	8.61	69.0	.121	1.64	70.6
21	562	38 00	8.62	68.3	.147	1 93	70.2
22.	510	47 1	8.51	68 6	.131	1.76	70.4
23	470	33 5	8 79	68.9	.120	1.67	69.6
24	741 1	60 5	8.53	66.7	.202	2.59	69 3
25	726	43 6	8 48	66 7	.197	2 59	69.3
26	224	33 6	9 15	69 6	.040	.77	70.4
27	2301	33.2	7.15	59.0	.6	8.09	7
28	340	40.8	9.05	70 2	.08	1.25	71.5
29	210 8	48.8	9 58	69 7	.045	.73	70.4
30.	1047	39 5	8.09	64 0	.280	3 67	67.7
31	659	41.6	8 74	67 3	.185	2.3	69.6
32	1113 97	36.5	8	63.3	.305	3.84	71.1
33	778	41	8 56	66.2	218	2.68	68 9
34	654	44 1	8 7	67.6	175	2.25	69.9
35	266	43 6	8 93	71.1	.059	.96	72.1
36.	467	41.1	8.72	68 9	.121	1.64	70.5
37.	1040	46 2	8.07	64 1	.278	3.56	67.7
38	479.6	39 2	8 79	68 9	.123	1.65	70.5
39	1390	37 1	7.7	60.9	.383	4 75	65.7
40.	566	37.4	8.63	68.3	.148	2.01	70.5

mean = 8.47 cm
σ = .029

Fig. 5.9 Log-normal distribution of surface runoff from MODEL I showing agreement with plotted points.

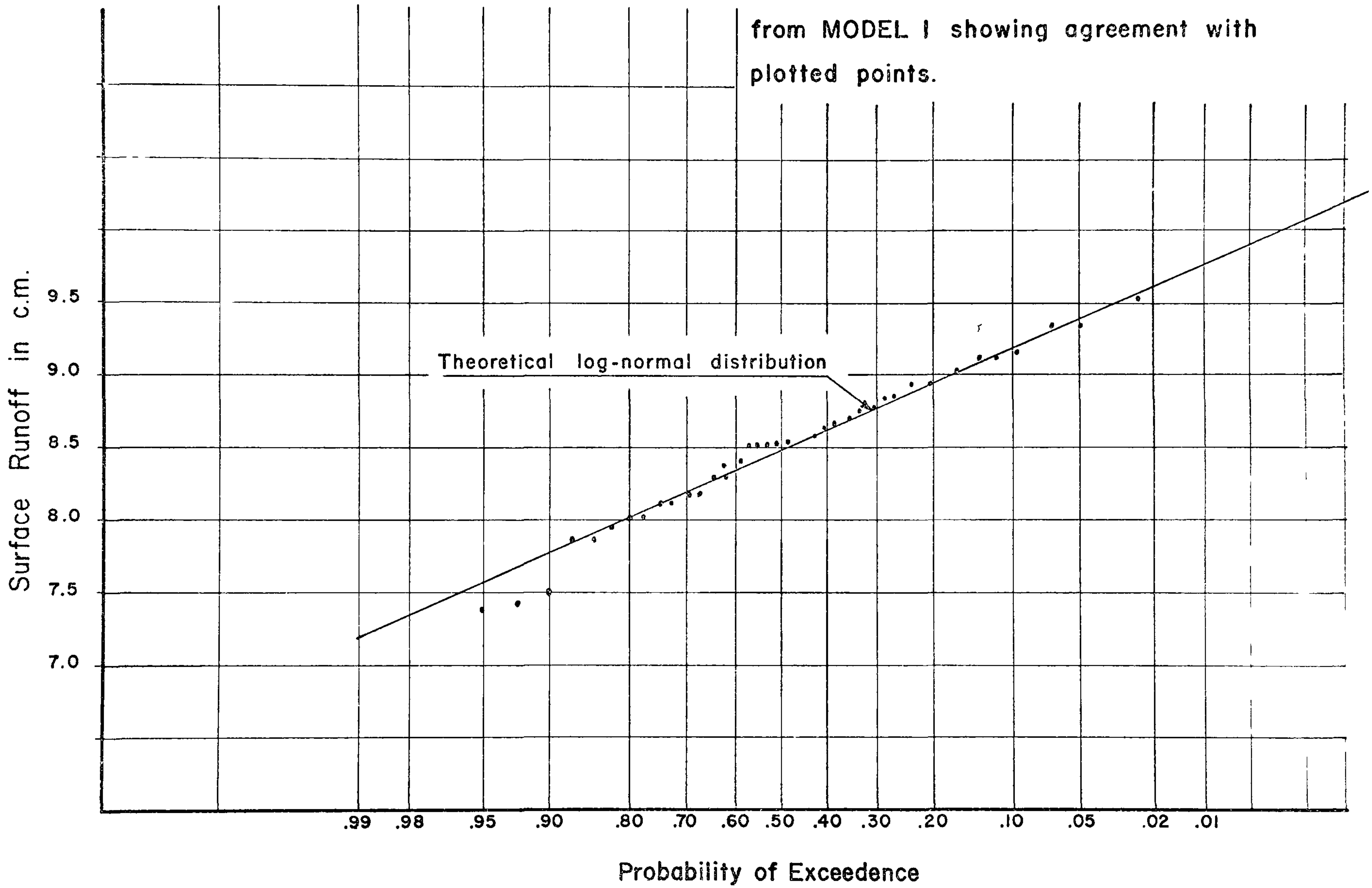


Fig. 5.10 Log.-normal distribution of sub-surface runoff from MODEL I showing agreement with plotted points.

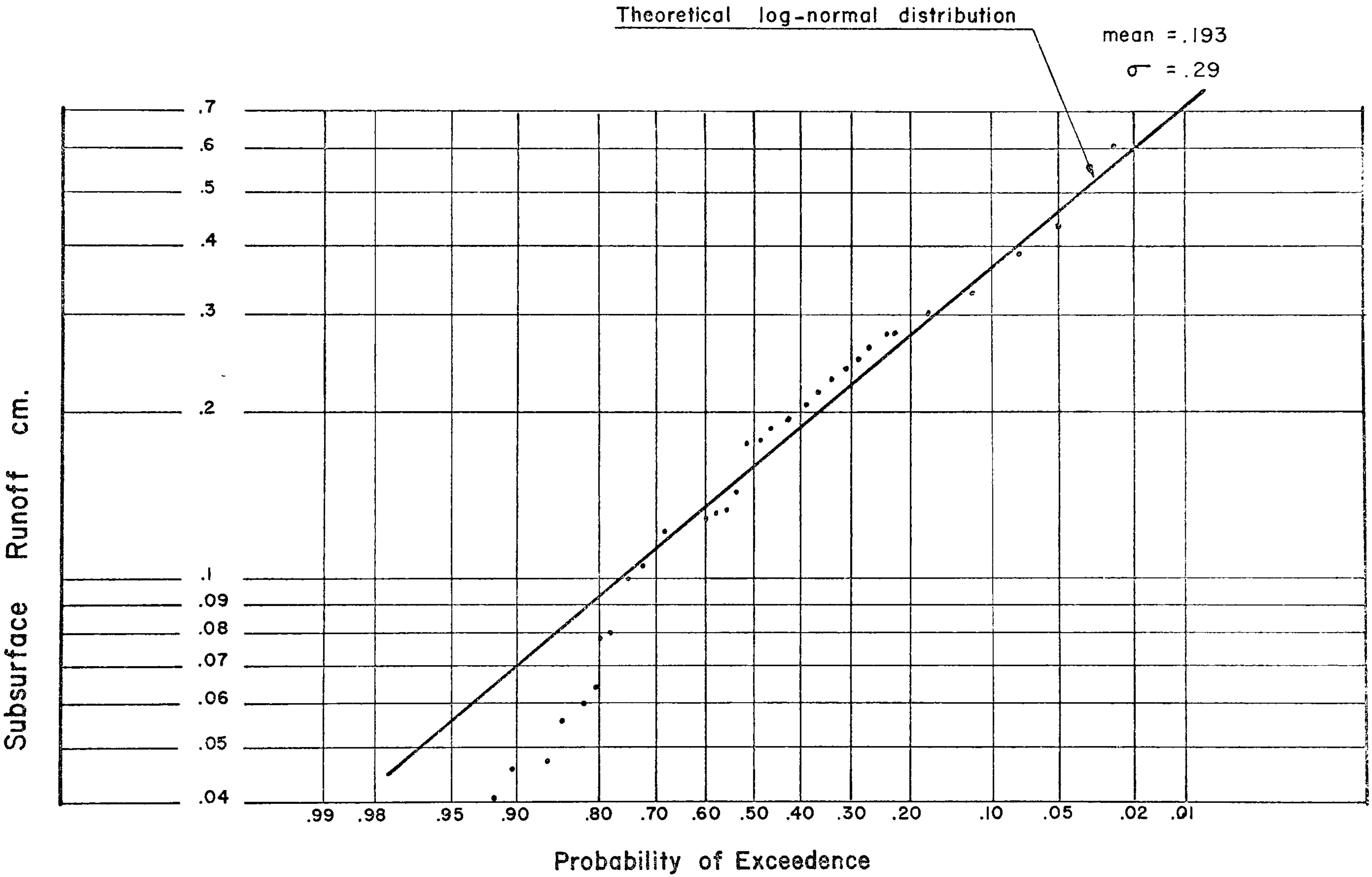


Fig. 5.11 Normal-distribution of peak runoff from MODEL I showing agreement with plotted points.

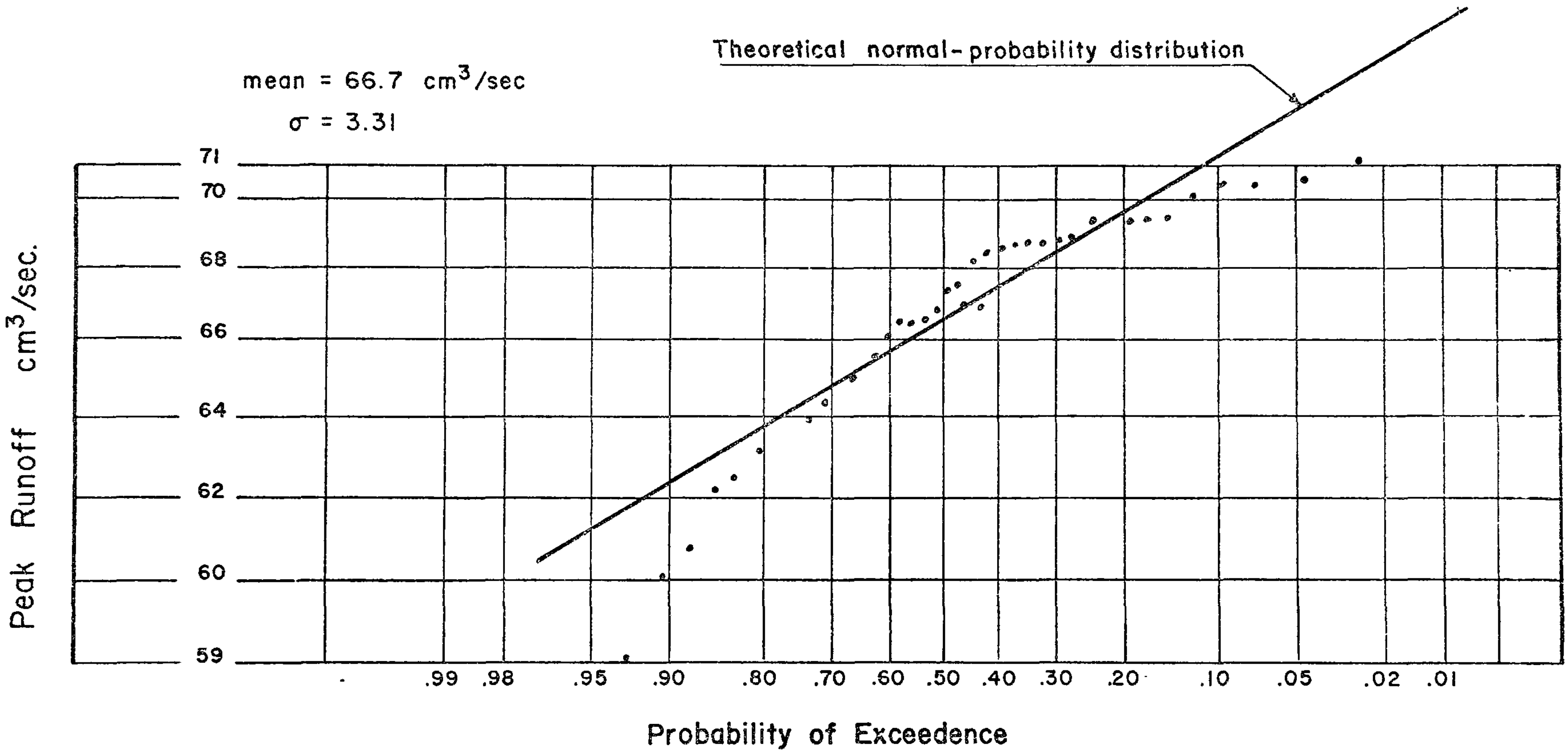


Figure 5.12

Results of the Chi-Squared Test
for Fitting Distributions

Output Variable	Assumed distribution	Calculated λ^2	At the 5% level of significance	Prob. of a deviation exceeding calc. λ^2
Surface runoff	log normal	.52	5.99	.75
Subsurface runoff	log normal	.32	5.91	.85
Peak flow	normal	2.5	5.99	.30

From the results of these tests it was concluded that the surface and subsurface data fit log normal distributions. While the peak flow data of surface runoff was fitted to a normal distribution, the data indicated a strong negative skew and the poorness of the fit perhaps indicated more data was needed to approximate the distribution. The means and standard deviations of the output parameters are given in Figure 5.13.

Applying confidence limits and the standard t distribution to the data it can be shown that for the given rainfall distribution and a 95% confidence level that there exists a saturated permeability falling between 450 and 691 millidarcys with its attendant porosity and pore size distribution index; surface runoff between 8.29 cm. and 8.65 cm., subsurface flow between .16 cm. and .24 cm. and a peak discharge between 66 cm^3/sec and 68 cm^3/sec.

Figure 5.14 more clearly shows the effect of errors of the soil parameters upon the model output of surface and subsurface runoff. Large errors (+80%) in the saturated hydraulic conductivity will result in much smaller errors (9%) in the surface runoff and smaller errors (40%) in the subsurface flow.

Figure 5.13

Statistics of the Output Parameter Distribution

Output Variable	Mean	Standard Deviation σ
Surface runoff	.928 (8.47 cm)	.029
Subsurface runoff	-.714 (.193 cm)	.29
Peak flow	66.72 cm^3/sec	3.31

(Data in brackets are the antilog of the log transformations)

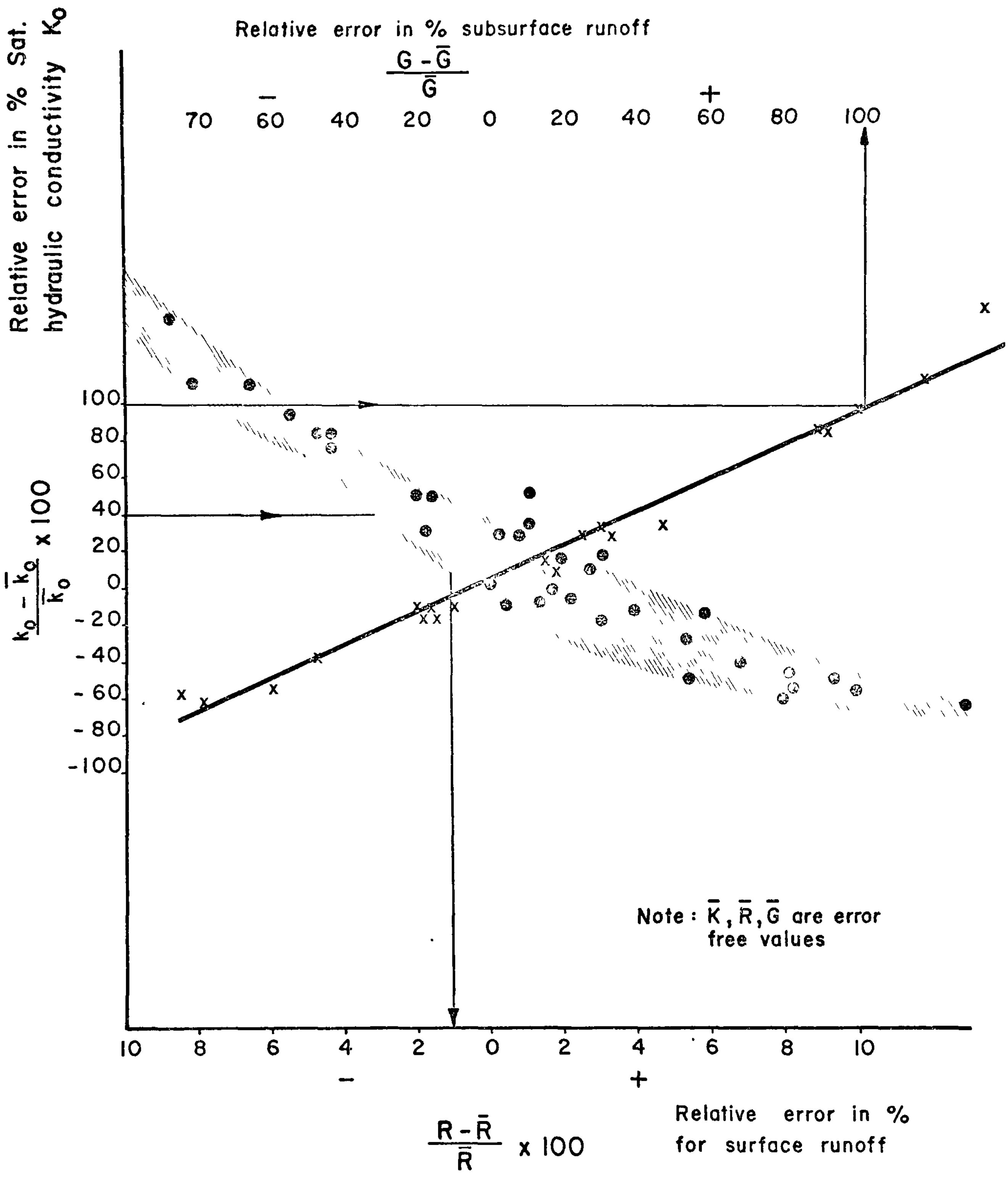


Fig. 5.14 A relative error graph showing the effect of errors in hydraulic conductivity upon surface and subsurface runoff.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

1) The partial differential equations of one-phase, unsaturated and saturated flow were solved numerically with the purpose of describing the groundwater regime and the upper boundary conditions governing infiltration, seepage and overland flow. It was then demonstrated that the groundwater model may be combined with the kinematic equation of overland flow with interacting boundary conditions at the soil surface to provide a mathematical model of the generation of overland flow from rainfall on an infiltrating surface.

2) This model was then used to demonstrate the variable source area concept whereby under prolonged and heavy rainfall the stream flow-producing areas adjacent to the stream channels may grow wider and wider as subsurface moisture conditions permit. This indicates that the watershed areas contributing to the overland and subsurface flow vary significantly with the input rainfall.

3) From a Monte Carlo simulation of a watershed with non-uniform homogeneous, isotropic soils, it was found that a large amount of error introduced into the model parameters of permeability, soil moisture functional relationships and friction coefficients, resulted

in small errors (10%) in surface runoff and peak flows but large errors in estimates of subsurface runoff. This would infer that estimates of hydraulic conductivity, k_o, porosity θ, pore size index from soil maps or the like would suffice for estimates of surface runoff, while an intensive network of permeability measurements is required for estimates of subsurface flow. It can be implied from this study that the accuracy of existing deterministic groundwater models is questionable. It may not be possible to use single-valued hydrogeologic parameters to describe transient flow in non-uniform homogeneous geologic formations.

6.2 Recommendations

1) The multivariate structure of the flow parameters should be investigated to establish the autocorrelation properties and Markov dependence in the spatial distribution of these parameters.

2) More general stochastic models of a groundwater system can be formulated by the methods of stochastic differential equations. Research in this direction is presently being carried out by Sagar (1976).

3) Future research should consider the sensitivity of mathematical models to the hysteresis effect and the swelling of soils.

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APPENDIX A

DERIVATIONS OF THE GROUND WATER FLOW EQUATIONS

A.1 Basic Equation of Elastic Storage

In this section a general relation between an elementary volume change and the fluid pressure in a porous material will be established.

Let us consider an elastic homogeneous porous material s (soil grains) containing a compressible liquid w (water) with densities denoted by ρ_s and ρ_w.

If the porosity of the material is n, then in an elementary volume V the volume of the pore space is nV, the volume of the soil grains is (1-n)V, then with the degree of saturation = S the volume of the water in the volume V is SnV.

The specific mass discharges N_s, N_w (the discharge of mass per unit area of a coordinate system fixed in space) is related to the specific volume discharges q_s, q_w by

$$N_s = \rho_s q_s \ , \qquad N_w = \rho_w q_w \qquad \text{A.1}$$

Therefore the actual velocities of each component V_s and V_w are related to the specific volume discharges

$$q_s = (1-n)V_s \qquad q_w = SnV_w \qquad \text{A.2}$$

The relative density ρ of component i (liquid, soil) is defined as the mass of the component i per unit total volume of the material. The density ρ_i is defined as the mass of the component per unit volume of the component.

lectures (4 auteurs sur 7)[12] et à la méthodologie de la lecture. Chez les auteurs américains, tous les conseils sur la lecture ont trait à la méthodologie, avec accent sur la vitesse de lecture (16 auteurs sur 18) et la recherche des idées (15 sur 18). Il y a là, nous semble-t-il, une indication révélatrice des différences d'optiques entre ouvrages français et américains. Pour les auteurs français, finalité et moyens ne se séparent pas; ils constituent tous les deux, et le premier peut-être plus que le second, des éléments indispensables de l'exécution efficace d'une tâche scolaire. Pour les auteurs américains au contraire, les moyens seuls semblent importants et on n'éprouve pas le besoin de les rattacher à une finalité quelque peu précise. Pour ces derniers, la lecture est avant tout affaire de technique; pour les premiers, la lecture est surtout affaire de motivation.

Enfin, l'examen des procédés de présentation révèle encore une différence très nette entre les deux groupes d'ouvrages. D'une part, exposé purement didactique; d'autre part, usage abondant d'exercices et de spécimens afin de faire assimiler les conseils donnés. Les uns décrivent ce qu'il faut faire sans le faire faire; les autres font surtout appel à la

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12 L'exposé de Ricour sur la lecture n'a pas été inclus dans la présente catégorie parce que l'auteur n'en faisait qu'un développement de l'idée de "motivation", traitée antérieurement dans la catégorie des dispositions affectives. Il n'en reste pas moins que, pour l'auteur, la lecture était envisagée en termes de finalité.

The relative densities are given by

$$P_s = (1-n)\rho_s \ , \qquad P_w = Sn\rho_w \qquad \text{A.3}$$

Assuming no chemical reaction the conservation of mass requires that the following continuity equations are satisfied

$$\frac{\partial P_s}{\partial t} + \mathrm{div} N_s = 0 \qquad \text{A.4}$$

$$\frac{\partial P_w}{\partial t} + \mathrm{div} N_w = 0 \qquad \text{A.5}$$

note: $\nabla . N_s = \frac{\partial N_{sx}}{\partial x} + \frac{\partial N_{sy}}{\partial y} + \frac{\partial N_{sz}}{\partial z} = \mathrm{div} N_s$

The equation of motion of liquid by Darcy's law is

$$q = Sn(V_w - V_s) = -k \ \mathrm{grad}\ \phi \qquad \text{A.6a}$$

where ϕ is the force potential (introduced by Hubbert) or more commonly called the hydraulic head, and is given by

$$\phi = z + \psi$$

where z is the elevation head and ψ is the pressure head.

Therefore

$$k = \frac{-g\rho_w k}{\mu}$$

After substitution, equation A.6 then becomes

$$q = Sn(V_w - V_s) = \frac{-g\rho_w k}{\mu}(\nabla z + \nabla\psi) \qquad \text{A.6b}$$

While equation A.6 was derived by Darcy for saturated flow it has been shown by Richards that it can be used for unsaturated flow. In saturated flow the specific permeability will depend upon the number and kind of pore spaces. For unsaturated flow the specific permeability in addition to the above parameters will also depend upon the moisture content of the soil. As the soil moisture content is reduced the effective cross-sectional area of the water conducting region is reduced and since Darcy's law is independent of the size of particles or the state of packing, k, will be reduced.

The equations of state are assumed to be:

ρ_s = constant, grains of soil are assumed incompressible   A.7

$$\rho_w = \rho_o \exp(B\rho_w) \qquad \text{A.8a}$$

where ρ_o is a reference density and B is the coefficient of water compressibility. Equation A.8a indicates that the liquid is compressible.

In terms of ψ, the pressure head, Equation A.8 may be written as:

$$\rho_w = \rho_o \exp\ (B'\psi) \qquad \text{A.8b}$$

where $B' = B\rho_w g$.

Using A.1, A.2 and A.3 in equation A.4 the equation of continuity for soil (equation of conservation of mass) is

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \mathrm{div}\rho_s q_s = 0$$

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \mathrm{div}[\rho_s(1-n)v_s] = 0$$

or

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \nabla.[(1-n)\rho_s v_s] = 0$$

Now with ρ_s = constant

$$\frac{\rho_s\partial[1-n]}{\partial t} + \rho_s\nabla.[(1-n)v_s] = 0$$

using $\nabla.(\phi f) = \phi\nabla.f + f.\nabla\phi$

where ϕ is a scalar quantity = (1-n)

f is a vector quantity = v_s

$$\therefore \quad \frac{\partial n}{\partial t} - (1-n)\mathrm{div}.v_s + v_s \mathrm{grad}\ n = 0 \qquad \text{A.9}$$

which is the equation of continuity for soil.

Similarly with A.1, A.2 and A.3 in equation A.5 the equation of continuity for water is

$$\frac{\partial Sn\rho_w}{\partial t} + \mathrm{div}\rho_w q_w = 0$$

$$\frac{\partial Sn\rho_w}{\partial t} + \mathrm{div}[\rho_w SnV_w] = 0$$

$$\frac{\partial Sn\rho_w}{\partial t} + \nabla.[Sn\rho_w V_w] = 0$$

now with $\rho_w = \rho_o \exp(B'\psi)$ the above equation becomes

$$\frac{\partial(Sn\rho_o \exp(B'\psi)}{\partial t} + Sn\rho_w \nabla.V_w + V_w.\nabla Sn\rho_w = 0$$ or dividing by $\frac{1}{Sn\rho_w}$ and expanding

$$\frac{1}{s}\frac{\partial S}{\partial t} + \frac{1}{n}\frac{\partial n}{\partial t} + \frac{B'\partial\psi}{\partial t} + \mathrm{div}V_w + \frac{V_w.\mathrm{grad}(Sn_w)}{Sn\rho_w} = 0 \qquad \text{A.10}$$

which is the equation of continuity for water.

Take the divergence of A.6

$$\begin{aligned} \mathrm{div}\ q &= \nabla.Sn(V_w - V_s) \\ &= (V_w - V_s)\nabla S_n + S_n(\nabla.V_w - \nabla.V_s) \\ &= -\nabla.[\frac{g\rho_w k}{u}(\nabla\psi + \nabla z)] \end{aligned} \qquad \text{A.11}$$

eliminate $\nabla.V_w$ from A.10 and A.11

A.10 x $S\rho n$

$$n\rho\frac{\partial s}{\partial t} + S\rho\frac{\partial n}{\partial t} + B'Sn\rho\frac{\partial\psi}{\partial t} + V_w.\nabla Sn\rho + Sn\rho\nabla.V_w = 0$$

A.11 x ρ_w

$$-\rho_w(V_w - V_s)\nabla Sn - \rho_w Sn\nabla.V_w + \rho_w Sn\nabla.V_s = \nabla.[\frac{g\rho w^2 k}{u}(\nabla\psi + \nabla z)]$$

Add above equations to obtain:

$$\begin{aligned} &n\rho\frac{\partial s}{\partial t} + S\rho\frac{\partial n}{\partial t} + B'Sn\rho\frac{\partial\psi}{\partial t} + V_w.\nabla Sn\rho - \rho(V_w - V_s)\nabla Sn \\ &+ \rho Sn\nabla.V_s = \nabla.[\frac{g\rho w^2 k}{u}(\nabla\psi + \nabla z)] \end{aligned} \qquad \text{A.12}$$

Using equation A.9 the term $S(\partial n/\partial t)$ may be eliminated. This gives

$$- S\rho \frac{\partial n}{\partial t} - S\rho V_s \text{ grad } n + S\rho(1-n)\nabla . V_s = 0 \qquad \text{A.13}$$

combining A.12 and A.13

$$\rho n \frac{\partial s}{\partial t} + B'Sn\rho \frac{\partial \psi}{\partial t} + \underline{V_w . \nabla Sn\rho} - \underline{\rho(V_w - V_s)\nabla Sn} + \rho Sn\nabla . V_s -$$

$$\underline{S\rho V_s \text{ grad}} \; n + S\rho(1-n)\nabla . V_s =$$

$$S\rho\nabla . V_s + B'Sn\rho \frac{\partial \psi}{\partial t} + \rho n \frac{\partial s}{\partial \psi} \cdot \frac{\partial \psi}{\partial t} \qquad \text{A.14}$$

Since only small incremental deformations are considered, the scalar products in the left hand of equation A.14 (underlined) may be disregarded since they constitute terms of the second order.

Now the displacement velocity V_s of the material is:

$$V_s = \frac{\partial u_s}{\partial t}$$

where u_s is the displacement vector of the solid material.

Then: $e = \dfrac{\text{div } u_s}{\nabla . u_s}$ = the volume strain

Equation A.14 is then:

$$\nabla . [\frac{kg\rho^2}{u} (\nabla\psi + \nabla z) = S\rho \frac{\partial e}{\partial t} + \frac{\partial \psi}{\partial t} Sn\rho \; (B' + \frac{\partial s}{\partial \psi}) \qquad \text{A.15}$$

This equation contains two variables: the volume strain e and the fluid pressure ψ. A second equation is obtained from the mechanical behavior of the soil.

A.2 Elasticity of Soil Skeleton

It is assumed that the soil skeleton behaves like a perfectly elastic solid, i.e., (the soil has linear, reversible, isotropic, non-retarded mechanical properties). This of course will provide only an approximate description of real behavior of the soil.

The components of total stress are indicated by τ_{ij} and are shown on Fig. A.1. The total stress is made up of the effective stress τ'_{ij}, a measure of the intergranular forces acting on the soil skeleton, and the fluid pressure p which acts over the total area of the element.

The components of total stress are then:

$$\begin{array}{lll} \tau_{xx} = \tau'_{xx} - p & \tau_{yz} = \tau'_{yz} & \tau_{zy} = \tau'_{zy} \\ \tau_{xx} = \tau'_{xx} - p & \tau_{zx} = \tau'_{xz} & \tau_{xz} = \tau'_{xz} \\ \tau_{zz} = \tau'_{zz} - p & \tau_{xy} = \tau'_{xy} & \tau_{yz} = \tau'_{yz} \end{array} \qquad \text{A.16}$$

Let P_x, P_y, P_z = components of the body force per unit volume (gravity) then ignoring the effects of inertia, the equation of equilibrium becomes:

$$\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial z} + P_x &= 0 \\ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + P_y &= 0 \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + P_z &= 0 \end{aligned} \qquad \text{A.17}$$

These equations are written in terms of effective stress and fluid pressure.

$$\frac{\partial \tau'_{xx}}{\partial x} + \frac{\partial \tau'_{xy}}{\partial y} + \frac{\partial \tau'_{xz}}{\partial z} + P_x = \frac{\partial p}{\partial x}$$

$$\frac{\partial \tau'_{yz}}{\partial x} + \frac{\partial \tau'_{yz}}{\partial y} + \frac{\partial \tau'_{yz}}{\partial z} + P_y = \frac{\partial p}{\partial y} \qquad \text{A.18}$$

$$\frac{\partial \tau'_{zx}}{\partial x} + \frac{\partial \tau'_{zy}}{\partial y} + \frac{\partial \tau'_{zz}}{\partial z} + P_z = \frac{\partial p}{\partial z}$$

As mentioned previously only small incremental deformations are considered from an initial steady state. The stresses in this initial state have to satisfy the above equilibrium equations when the initial stresses are characterized by a superscript (o) the effective stress may be written as follows:

$$\tau'_{xx} = \tau'^{o}_{xx} + \sigma'_{xx} \qquad \tau'_{yz} = \tau'^{o}_{yz} + \sigma'_{yz}$$

$$\tau'_{yy} = \tau'^{o}_{yy} + \sigma'_{yy} \qquad \tau'_{zx} = \tau'^{o}_{zx} + \sigma'_{zx} \qquad \text{A.19}$$

$$\tau'_{zz} = \tau'^{o}_{zz} + \sigma'_{zz} \qquad \tau'_{xy} = \tau'^{o}_{xy} + \sigma'_{xy}$$

σ_{ij} = incremental effective stress. The fluid pressure may be written in the form $p = p^{(o)} + \sigma$ where σ is the incremental fluid pressure.

When the body forces are assumed to be independent of time (true for the case where the body forces result from gravity) and substituting in A.18:

$$\frac{\partial \tau'^{o}_{xx}}{\partial x} + \frac{\partial \sigma'_{xx}}{\sigma'_{xx}} + \frac{\partial \tau'^{o}_{xx}}{\partial y} + \frac{\sigma'_{xy}}{\partial y} + \frac{\partial \tau'_{xz}}{\partial z} + \frac{\partial \sigma'_{yz}}{\partial z} + P_x - \frac{\partial p}{\partial x} = 0$$

now from

$$\frac{\partial \tau^{o}_{xx}}{\partial x} + \frac{\partial \tau^{o}_{xy}}{\partial y} + \frac{\partial \tau^{o}_{xz}}{\partial z} + P = 0$$

$$\therefore \quad \frac{\partial \sigma'_{xx}}{\partial x} + \frac{\partial \sigma'_{xy}}{\partial y} + \frac{\partial \sigma'_{xz}}{\partial z} = \frac{\partial \sigma}{\partial x} \quad \text{where } p = p^{o} + \sigma \qquad \text{A.19a}$$

similarly

$$\frac{\partial \sigma'_{yz}}{\partial x} + \frac{\partial \sigma'_{yy}}{\partial y} + \frac{\partial \sigma'_{yz}}{\partial z} = \frac{\partial \sigma}{\partial y} \qquad \text{A.19b}$$

$$\frac{\partial \sigma'_{zx}}{\partial x} + \frac{\partial \sigma'_{zy}}{\partial y} + \frac{\partial \sigma'_{zz}}{\partial z} = \frac{\partial \sigma}{\partial z} \qquad \text{A.19c}$$

Now Hooke's Law states that for small strains that the stress in a material is proportionate to the strain which produced it. Therefore as a first approximation it is assumed that the deformations are related to the effective stresses by Hooke's Law.

Therefore, for combined stresses:

$$\varepsilon_{xx} = \frac{[\sigma'_{xx} - V(\sigma'_{yy} + \sigma'_{zz})]}{E} \qquad \varepsilon_{yz} = \sigma'_{yz}/2G \qquad \begin{matrix} 9 \\ \text{A.20a} \end{matrix}$$

where σ'_{xx} is the tensile stress causing a elongation in the x direction and σ'_{yy}, σ'_{zz} are the stresses causing a contraction in the x direction. E is Young's Modulus V is Poisson's ratio and G is the shearing modulus of elasticity. The three elastic constants are related to each other as follows: $E = 2G(1 + V)$

Similarly:

$$\varepsilon_{yy} = \frac{[\sigma'_{yy} - V(\sigma'_{zz} + \sigma'_{xx})]}{E} \qquad \varepsilon_{zx} = \sigma'_{zx}/2G \qquad \text{A.20b}$$

$$\varepsilon_{zz} = \frac{[\sigma'_{zz} - V(\sigma'_{xx} + \sigma'_{yy})]}{E} \qquad \varepsilon_{xy} = \sigma'_{xy}/2G \qquad \text{A.20c}$$

Equations A.20 are rewritten as:

$$\sigma'_{xx} = 2G\varepsilon_{xx} + \lambda\varepsilon \qquad \sigma'_{yz} = 2G\varepsilon_{yz} \qquad \text{A.21a}$$

$$\sigma'_{yy} = 2G\varepsilon_{yy} + \lambda\varepsilon \qquad \sigma'_{zx} = 2G\varepsilon_{xz} \qquad \text{A.21b}$$

$$\sigma'_{zz} = 2G\varepsilon_{zz} + \lambda\varepsilon \qquad \sigma'_{xy} = 2G\varepsilon_{xy} \qquad \text{A.21c}$$

λ is given by

$$\lambda = VE/[(1 - 2V)(1 + V)]$$

λ and G are called Lame's constants and ε, the incremental volume strain in equation A.21 is

$$\varepsilon = \varepsilon_{xy} + \varepsilon_{yy} + \varepsilon_{zz}$$

For small deformations

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \varepsilon_{yz} = \frac{1}{2}(\partial u_y/\partial z + \partial u_z/zy) \qquad \text{A.22a}$$

$$\varepsilon_{xx} = \frac{\partial u_y}{\partial y} \qquad \varepsilon_{zx} = \frac{1}{2}(\partial u_z/\partial_x + \partial u_x/\partial z) \qquad \text{A.22b}$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} \qquad \varepsilon_{xy} = \frac{1}{2}(\partial u_x/\partial_y + \partial u_y/\partial x) \qquad \text{A.22c}$$

$$\therefore \quad \varepsilon = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = \text{div } u$$

Substituting:

$$\sigma'_{xx} = 2G(\partial u_x/\partial x) + \lambda\varepsilon \qquad \text{A.23a}$$

$$\sigma'_{yy} = 2G(\partial u_y/\partial y) + \lambda\varepsilon \qquad \text{A.23b}$$

$$\sigma'_{zz} = 2G(\partial u_z/\partial z) + \lambda\varepsilon \qquad \text{A.23c}$$

now assume displacement occurs only in the z direction and the total stresses do not change.

Since $$\frac{\partial u}{\partial z} = \varepsilon$$

Then A.23c is $\sigma'_{zz} = (\lambda + 2G)\varepsilon$ and the condition $\sigma_{zz} = 0$ then implies $\sigma'_{zz} = \sigma$ and

$$(\lambda + 2G)\varepsilon = \sigma \qquad \text{A.24}$$

Differentiating A.24 with respect to t

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{\lambda+2G}\frac{\partial \sigma}{\partial t} \qquad \text{A.25}$$

let $\alpha = \frac{1}{\lambda+2G}$ = vertical consolidation coefficient

$$\frac{\partial \varepsilon}{\partial t} = \alpha\rho g \frac{\partial \psi}{\partial t} \qquad \text{or} \qquad \frac{\partial \psi}{\partial t} = \alpha' \frac{\partial \psi}{\partial t} \qquad \text{A.26}$$

substituting in equation A.15

$$\nabla.[\frac{g\rho_w^2 k}{u}(\nabla\psi + \nabla z)] = (S\rho\alpha' + B'Sn\rho + \rho_w n \frac{\partial s}{\partial \psi})\frac{\partial \psi}{\partial t} \qquad \text{A.27}$$

now $n\frac{\partial s}{\partial \psi} = \frac{\partial \theta}{\partial \psi}$ = specific moisture capacity = C

$$Sn = \theta$$

therefore, rewriting equation A.27

$$\frac{\partial}{\partial x}\left[\frac{g\rho^2}{u} k_{xx} \frac{\partial\psi}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{g\rho^2}{u} k_{yy} \frac{\partial\psi}{\partial y}\right] + \frac{\partial}{\partial z}\left(\frac{g\rho^2}{u} k_{zz} \left\{\frac{\partial\psi}{\partial z} + 1\right\}\right)$$

$$= \left[\rho\frac{\Theta}{n}\{\alpha' + B'n\} + \rho\frac{\partial\Theta}{\partial y}\right]\frac{\partial\psi}{\partial t} \qquad \text{A.28}$$

This equation can be simplified if it is assumed that the water is incompressible, B = 0, and the soil material will consolidate a negligible amount due to changes in water pressure.

$$\alpha \rightarrow 0$$

Then equation A.28 reduces to:

$$\frac{\partial}{\partial x}\left[\frac{g\rho^2}{u} k_{xx} \frac{\partial\psi}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{g\rho^2}{u} k_{yy} \frac{\partial\psi}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{g\rho^2}{u} k_{zz} \left\{\frac{\partial\psi}{\partial z} + 1\right\}\right] = \rho_w \frac{\partial\Theta}{\partial\psi} \times \frac{\partial\psi}{\partial t} \qquad \text{A.28}$$

which is essentially the partial differential equation first derived by L. A. Richards. Its solution for unsaturated flow depends on knowing the functional relationships among ψ, Θ and k for the particular soil in question.

APPENDIX B

DERIVATION OF THE SHALLOW-WATER EQUATIONS

B.1 Introduction

The flow of surface water over a uniform plane or in a channel is classified as spatially varied and unsteady. The basic equations of the kinematic model are derived from the developed shallow-water equations which in turn are derived from the principal of conservation of mass and momentum. The derivations presented in this section follow the development in standard hydraulic references treating spatially varied unsteady flow.

B.2 Continuity Equation

Figure B.1 illustrates the variables used in deriving the continuity equation for an overland flow section of arbitrary cross-section having one-dimensional flow. The continuity equation written over a time increment dt for the element of fluid shown in Figure B.1 is:

$$\underbrace{(A.v+q_R dx)dt}_{\text{inflow}} - \underbrace{(A+\frac{\partial A}{\partial x}dx)(V+\frac{\partial V}{dx})dt - Idxdt}_{\text{outflow}} = \underbrace{\frac{\partial A}{\partial t}dtdx}_{\text{change in storage}} \qquad \text{B.1}$$

cancelling the product A.V; dividing by dxdt and neglecting higher order products equation B.1 becomes

$$\frac{\partial A}{\partial t} + A\frac{\partial V}{\partial x} + u\,\frac{\partial A}{\partial x} = q_R - I \qquad \text{B.2}$$

For the case of a wide plane, equation B.2 becomes:

$$\frac{\partial d}{\partial t} + V\,\frac{\partial d}{\partial x} + d\,\frac{\partial V}{\partial x} = q_R - I \qquad \text{B.3}$$

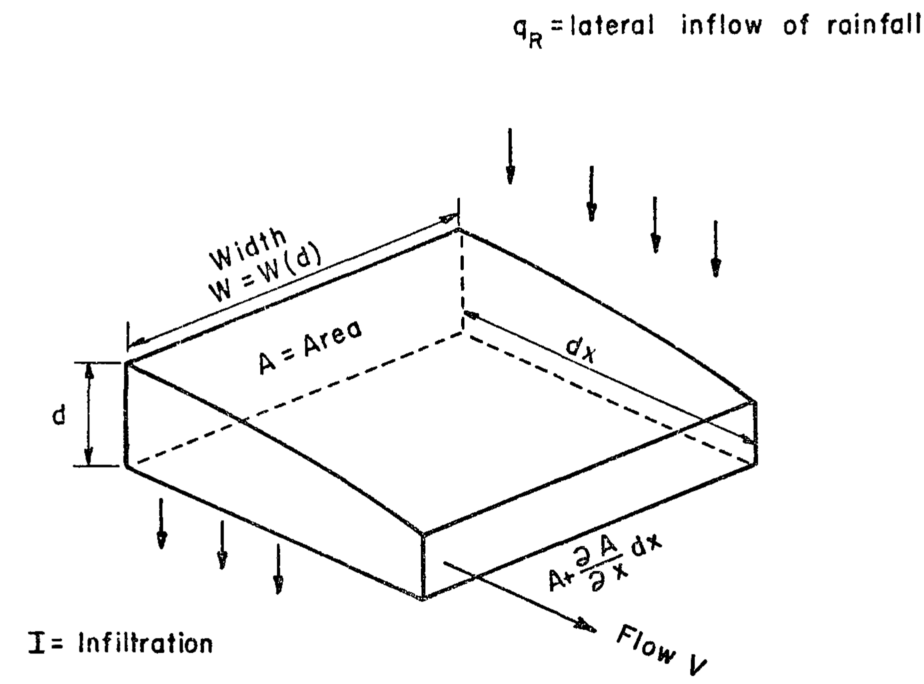

Fig. B1 Definition sketch for continuity equation

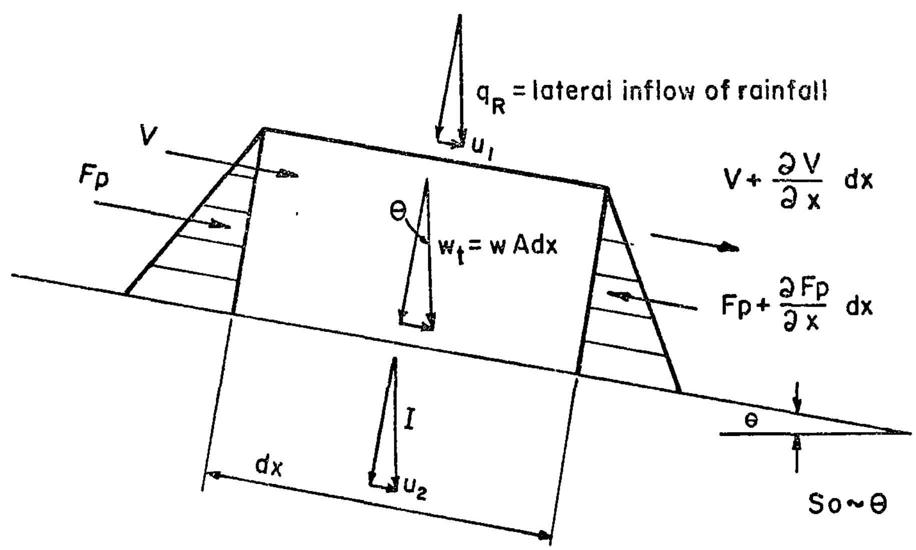

Fig. B2 Forces acting on fluid element used in deriving the momentum equation.

Equation A.3 is the one-dimensional continuity equation for surface flow over a wide plane.

B.3 Momentum Equation

An element of fluid receiving lateral inflow in cm^3/ft is shown in Figure B.2. Newton's second law is written for the forces acting in the x direction, where x is measured downstream. The lateral inflow has a velocity component u in the x direction. The lateral infiltration has a velocity component u_2 in the x direction. Therefore, the basic equation is:

$$\Sigma F_x = M_s \frac{dV}{dt} + M_1 \frac{du}{dt} - M_2 \frac{du_2}{dt} \qquad B.4$$

where M_s = mass of water in main stream

M_1 = rainfall mass falling in dt

M_2 = mass of water infiltrating in dt

Forces acting in the x direction are:

(1) The wt component acting in the downstream direction:

$$w_t \cdot \sin\Theta = wA\sin\Theta dx = wAS_o dx$$

w = specific weight of water gm/cm^3 and S_o = the bed slope

(2) The hydrostatic force acting to the right of the centroid area:

$$F = w\overline{Y}A$$

$\overline{Y}$ = the distance from the water surface to the centroid of area A.

(3) The hydrostatic force acting to the left at the centroid of area:

$$F + \frac{\partial F_\rho}{\partial x} dx = w\overline{Y}A + \frac{\partial F_\rho}{\partial x} dx$$

Liggett (1961) has shown that the hydrostatic pressure differential, $\frac{\partial F_\rho}{\partial x}$ is given by

$$\frac{\partial F_\rho}{\partial x} dx = - wA \frac{\partial y}{\partial x} dx \quad \text{and}$$

(4) The friction force retarding the flow

$$F_f = - wAS_f \; dx$$

where S_f is the slope of the energy line,

therefore, equation B.4 can be written:

$$\Sigma F_x = wAS_o \; dx - wAS_f \; dx - wA \frac{\partial y}{\partial x} dx \qquad \text{B.5}$$

Now momentum change on the R.H.S. of equation B.4 may be evaluated as:

$$M_s \frac{dV}{dt} = \frac{1}{g} wAdx \frac{dV}{dt}$$

$$= \frac{1}{g} wdx\left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}.\frac{dx}{dt}\right)$$

$$= \frac{1}{g} wAdx\left(\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x}\right) \qquad \text{B.6}$$

$$M_1 \frac{du_1}{1dt} = \frac{1}{g} wq_R dxdt \frac{(V_1-u_1)}{dt} \qquad \text{B.7}$$

$$M_2 \frac{du_2}{dt} = \frac{1}{g} wIdxdt \frac{(u_2-V)}{dt} \qquad \text{B.8}$$

g is acceleration due to gravity and $\frac{dv}{dt}$ is interpreted as the total derivative of a particle along its stream path.

Combining equations B.4, B.5, B.6, B.7 and B.8 yields the momentum expression:

$$wAS_o dx - wAS_f dx - wA \frac{\partial d}{\partial x} dx = \frac{1}{g} wAdx(\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x}) + \frac{1}{g} wq(V-u_1)dx - \frac{1}{g} wi(u_2-V)dx$$

Divide by $\frac{1}{g}$ wAdx and rearrange:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial d}{\partial x} = g(S_o-S_f) - \frac{q_R}{A}(V-u_1) + \frac{I}{A}(u_2-V) \qquad \text{B.9}$$

For the case of one-dimensional flow in a wide channel and assuming the velocity u_1 and u_2 are negligible, equation B.9 can be written:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial d}{\partial x} = g(S_o-S_f) - \frac{V}{d} (q_R-I) \qquad \text{B.10}$$

Then equations B.3 and B.10 are the one-dimensional unsteady spatially varied flow equations applicable to overland flow. The assumptions made during this derivation are:

1) The flow is gradually varied so that vertical components of velocity and acceleration are negligible in comparison with the components along the direction of flow.

2) The pressure on the vertical surface of the flow element is hydrostatic.

3) The energy and momentum coefficients used as corrections to non-uniform velocity distributions are equal to one.

4) The channel slope S_o is small and is approximately equal to $\sin\theta$.

5) Frictional resistance in unsteady flow is the same as that for the corresponding depth in uniform flow so that the friction slope S_f can be obtained by the Chezy friction relationship.

APPENDIX C

DEVELOPMENT OF THE L.S.O.R. AND S.I.P. NUMERICAL METHODS

C.1 Introduction

Two numerical methods used to solve the subsurface equation of flow equation 2.4 are the line successive over-relaxation (L.S.O.R.) technique and the strongly implicit method. Both techniques are briefly outlined without detailed mathematical proofs in the following appendix.

C.2 L.S.O.R. Method

The equation of flow, equation 2.4 can be written in the following finite difference approximation using the block nodal grid shown on Figure 3.1. The mesh spacing can be variable and the region can be of any general shape that does not lead to a discontinuity. Thus for an interior node i, k at time step t the finite difference approximation is equation 3.1 written in terms of k the specific permeability.

$$\frac{g}{u\Delta x}[\rho^2(\psi_1)k(\psi_1)(\frac{2}{\Delta x_i+\Delta x_{i+1}} \cdot \frac{(\psi^t_{i+1}+\psi^{t-1}_{t+1}-\psi^t_i-\psi^{t-1}_i)}{2} -$$

$$\rho^2(\psi_2)k(\psi_2)(\frac{2}{\Delta x_i+\Delta x_{i-1}} \cdot \frac{(\psi^t_i+\psi^{t-1}_i-\psi^t_{i-1}-\psi^{t-1}_{i-1})}{2})]_{ik} +$$

$$\frac{g}{u\Delta z}[\rho^2(\psi_3)k(\psi_3)(1 + \frac{2}{\Delta z_k+\Delta z_{k+1}}(\frac{\psi^t_{k+1}+\psi^{t-1}_{k+1}-\psi^t_k-\psi^{t-1}_k}{2})) -$$

$$\rho^2(\psi_4)k(\psi_4)(1 + \frac{2}{\Delta z_k+\Delta z_{k+1}}(\frac{\psi^t_k+\psi^{t-1}_k-\psi^t_{k-1}-\psi^{t-1}_{k-1}}{2})] =$$

$$[\rho\frac{(\psi_5)\theta(\psi_5)}{n(\psi_5)}(\alpha'+n(\psi_5)\beta') + \rho(\psi_5)C(\psi_5)](\frac{\psi^t-\psi^{t-1}}{\Delta t}) \qquad \text{C.1}$$

The equation takes the general form of:

$$A_k \psi^t_{i,k+1} - B_k \psi_{i,k} + C_k \psi_{i,k-1} = - D_k \qquad \text{C.1a}$$

which represents a set of L linear equations in L variables of the above general form where $1 \leqslant k < L$. Now C.1 can be regrouped and written as:

$$R_1S_1T_1 \psi^t_{i,k+1} - (P_1Q_1+P_1Q_2+R_1S_1T_1S_2T_2+V/\Delta t) \psi_{i,k} +$$

$$R_1S_1T_2 \psi^t_{i,k-1} = - Y - Z + \psi^{t-1}_{i,k} . V/\Delta t) \qquad \text{C.1b}$$

where

$A_k = R_1S_1T_1$ $\qquad$ $B_k = P_1Q_1+P_1Q_2+R_1S_1T_1+R_1S_2T_2+V/\Delta t$

$C_k = R_1S_2T_2$ $\qquad$ $D_k = Y + Z + \psi^{t-1}_{i,k} . V/\Delta t$

$P_1 = \frac{g}{u\Delta z_k}$ $\qquad$ $Q_1 = \frac{\rho^2(\psi_1)k(\psi_1)}{\Delta x_i+\Delta x_{i+1}}$

$R_1 = \frac{g}{u\Delta z_k}$ $\qquad$ $Q_2 = \frac{\rho^2(\psi_2)k(\psi_2)}{\Delta x_i + \Delta x_{i-1}}$

$T_1 = \frac{1}{\Delta z_k+\Delta z_{k+1}}$ $\qquad$ $T_2 = \frac{1}{\Delta z_k+\Delta z_{k-1}}$

$S_1 = \rho^2(\psi_3)k(\psi_3)$

$S_2 = \rho^2(\psi_4)k(\psi_4)$ $\qquad$ $C = \frac{\partial\theta_5}{\partial\psi_5}$

$$U = [\frac{(\psi_5)\theta(\psi_5)}{n(\psi_5)} (\alpha'+n(\psi_5)\beta') + \rho(\psi_5)C)$$

$$Y = P_1Q_1\psi^{t-1}_{i+1,k} - P_1Q_1\psi^{t-1}_{i,k} - P_1Q_2\psi^{t-1}_{i,k} + P_1Q_2\psi^{t-1}_{i-1}$$

$$Z = R_1S_1 + R_1S_1T_1\psi^{t-1}_{i,k+1} - R_1S_1T_1\psi^{t-1}_{i,k} - R_1S_2 - R_1S_2T_2\psi^{t-1}_{i,k} + R_1S_2T_2\psi^{t-1}_{i,k-1}$$

The matrix equation is then

$$[A]\ \vec{\psi} = \vec{D}_k \qquad \text{C.2}$$

OR

$$\begin{vmatrix} B_1 & C_1 & & & & & \\ A_2 & B_2 & C_2 & & & & \\ & A_3 & B_3 & C_3 & & & \\ & & & & & & \\ & & & & A_{k-1} & B_{k-1} & C_{k-1} \\ & & & & & A_k & B_k \end{vmatrix} \begin{vmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \\ \\ \psi_k \end{vmatrix} = \begin{vmatrix} D_1 \\ D_2 \\ D_3 \\ \\ \\ D_L \end{vmatrix}$$

The Tridiagonal system is then solved by a recursion algorithm for Gauss Elimination by Thomas (1949)

$$w_1 = \frac{C_1}{B_1} \qquad w_k = \frac{C_k}{B_k - A_k w_{k-1}} \qquad \text{C.3}$$

$$g_1 = \frac{D_1}{B_1} \qquad g_k = \frac{D_k - A_k g_{k-1}}{B_k - A_k w_{k-1}} \qquad \text{C.4}$$

$$\psi_L = g_L \qquad \psi_k = g_k - w_k \psi_{k+1} \qquad \text{C.5}$$

C.2 Strongly Implicit Method

The S.I.P. method advocated by Stone (1968) utilizes a 5-point star in deriving the difference equations as shown in Figure 3.1. The coefficients of the unknowns of equation C.1 with respect to the 5-point star used in deriving the difference equation are also shown on this figure.

The equations take the general form of:

$$A\psi_{i,k-1} + B\psi_{i-1,k} + C\psi_{i,k} + D\psi_{i+1,k} + E\psi_{i,k+1} = F \qquad \text{C.6a}$$

which represents a set of N·L linear equations in N:L variables of the above general form where $1 \leq k \leq L$, $1 \leq i \leq N$.

Now for an interior point the general equation of C.1 is regrouped in the following form:

$$\begin{aligned} &T_2R_1S_2\psi^t_{i,k-1} + P_1Q_2\psi^t_{i-1,k} + (-P_1Q_1-P_1Q_2-R_1S_1T_1-R_1S_2T_2-V/\Delta T.\psi^t_{k,k}) \\ &+ P_1Q_1\psi_{i+1,k} + R_1S_1\psi^t_{i,k+1} = (-P_1Q_1\psi^t_{i+1,k} + P_1Q_1\psi^{t-1}_{i,k}) \\ &+ P_1Q_2\psi^{t-1}_{i,k} - P_1Q_2\psi^{t-1}_{i-1,k} - R_1S_1 - R_1S_1T_1\psi^{t-1}_{i,k-1} + R_1S_2T_2\psi^{t-1}_{i,k-1} \qquad \text{C.6b} \end{aligned}$$

where

$$A_{ik} = T_2R_1S_2 \qquad B_{i,k} = P_1Q_2$$

$$C_{ik} = -P_1Q_1 - P_1Q_2 - R_1S_1T_1 - R_1S_2T_2 - v/\Delta T$$

$$D_{ik} = P_1Q_1 \qquad E_{i,k} = R_1S_1T_1$$

$$F_{ik} = -Y - Z - v/\Delta T\ \psi^{t-1}_{i,k}$$

The Matrix Equation is then:

$$[A]\vec{\psi}_{i,k} = \vec{F}_{i,k}$$

$$\begin{vmatrix} C_{11} & D_{11} & E_{11} & & & \\ B_{21} & C_{21} & D_{21} & E_{21} & & \\ A_{12} & B_{12} & C_{12} & D_{12} & E_{12} & \\ & & & & & E_{N,L-1} \\ & & & & & D_{N-1,L} \\ & & A_{N,L} & B_{N,L} & C_{N,L} & \end{vmatrix} \begin{vmatrix} \psi_{11} \\ \psi_{21} \\ \psi_{12} \\ \\ \\ \psi_{N,L} \end{vmatrix} = \begin{vmatrix} F_{11} \\ F_{21} \\ \\ \\ \\ F_{N,L} \end{vmatrix}$$

The problem as in all elimination methods now resolves into decomposing A into a lower triangular matrix L and an upper triangular matrix U.

$$L = \begin{vmatrix} c_{11} & & & & \\ b_{21} & c_{21} & & & \\ h_{12} & & & & \\ & & & & \\ & & h_{NL} & b_{NL} & c_{NL} \end{vmatrix} \qquad \text{C.8}$$

$$U = \begin{vmatrix} 1 & d_{11} & e_{11} & & & \\ & 1 & d_{21} & & & \\ & & & & & \\ & & & & & \\ & & & d_{N-1,L} & & e_{N,L-1} \\ & & & 1 & & 1 \end{vmatrix} \qquad \text{C.9}$$

Now, form the product LU and let it = A - B which is similar to A but not of the form of A.

$$\begin{array}{l} \text{N elements} \\ \\ \\ A + B = \end{array} \begin{vmatrix} C'_{11} & D'_{11} & & & & E'_{11} & \\ B'_{21} & & & & h_{21} & & \\ & & & & & & \\ A'_{12} & G'_{12} & B'_{12} & C'_{12} & D'_{12} & h_{12} & E'_{12} \\ & & & & & & E_{N,L-1} \\ & & & & & & h'_{1,L} \\ & & & . & & & \\ & & & & & & D_{N-1,L} \\ & & & A'_{N,L} & G'_{N,L} & B'_{N,L} & C'_{N,L} \end{vmatrix} \qquad \text{C.10}$$

The elements of L and U and A + B = LU may be solved for recursively using the following algorithm that requires only the elements of A as known input.

$$G'_{i,k} = \frac{d_{i,k-1}\ C_{i,k}}{1+wd_{i,k-1}} \qquad \text{C.11}$$

$$h'_{i,k} = \frac{e_{i-1,k}\ B_{i,k}}{1 + w\ e_{i-1,k}} \qquad \text{C.12}$$

$$A'_{i,k} = A_{ik} - w\ G'_{i,k} \qquad \text{C.13}$$

$$D'_{i,k} = D_{i,k} - w\ h'_{i,k} \qquad \text{C.14}$$

$$E'_{i,k} = E_{i,k} + w\ G'_{i,k} + w\ h'_{i,k} \qquad \text{C.15}$$

$$F'_{i,k} = F_{i,k} - w\ G'_{i,k} \qquad \text{C.16}$$

$$E'_{i,k} = E_{i,k} - w\ h'_{i,k} \qquad \text{C.17}$$

$$a_{i,k} = A'_{i,k} \qquad \text{C.18}$$

$$b_{i,k} = D'_{i,k} \qquad \text{C.19}$$

$$c_{i,k} = E'_{i,k} - a_{i,k}\ e_{i,k-1} - b_{i,k}\ d_{i-1,k} \qquad \text{C.20}$$

$$d_{i,k} = \frac{F'_{i,k}}{C_{i,k}} \qquad \text{C.21}$$

$$e_{i,k} = \frac{E'_{i,k}}{c_{i,k}} \qquad \text{C.22}$$

"w" is the usual acceleration parameter covering the range $0 \leqq w \leqq 1$ which is varied for different iterations. Stone ( ) recommended using at least four parameters each used twice in succession, the largest parameter is used first and the following parameters all decrease. Zero can be used for the smallest parameter.

Now using the identity equation, C.6a becomes:

$$(A+B)\psi = (A+B)\psi - (A\psi-F) \qquad \text{C.6c}$$

or

$$(A+B)\psi^{m+1} = (A+B)\psi^{m} - (A\psi^{m} -F)$$

To reduce rounding errors a "residual form" suggested by Wilkinson and Peaceman is used:

$$(A+B)\Delta\psi^{m+1} = R^{m} \qquad \text{C.6d}$$

where

$$\Delta\psi^{(m+1)} = \psi^{m+1} - \psi^{m}$$

$$R^{m} = F - A\psi^{m}$$

note

$$R^{m}_{i,k} = F_{i,k} - (A_{i,k}\psi^{m}_{i,k-1} + B_{i,k}\psi^{m}_{i-1,k} + C_{i,k}\psi^{m}_{i,k} + D_{i,k}\psi^{m}_{i+1,k}+E_{i,k}\psi^{m}_{i,k+1}$$

using coefficient matrix A + B

$$LU\Delta\psi^{m+1} = R^{m} \qquad \text{C.7}$$

Let

$$V^{m+1} = U\Delta\psi^{m+1} \qquad \text{C.8}$$

$$LV^{m+1} = R^{m} \qquad \text{C.9}$$

Each equation of C.9 has the form:

$$a_{i,k}V^{m+1}_{i,k-1} + b_{i,k}V^{m+1}_{i-1,k} + c_{i,k}V^{m+1}_{i,k} = R^{m}_{i,k}$$

A forward substitution is used since equation C.9 contains only one unknown. The relationships $V^{m+1}_{i,k}$, $V^{m+1}_{i,k-1}$ and $V^{m+1}_{i-1,k}$ were computed by previous computations. Coefficients corresponding to grid points exterior to the region are considered to be zero.

Therefore, after V^{m+1} has been computed, $\Delta\psi^{m+1}$ is solved from equation C.8

$$U\Delta\psi^{m+1} = V^{m+1}$$

Each equation of C.8 can be written:

$$\Delta\psi^{m+1}_{i,k} + d_{i,k}\psi^{m+1}_{i+1,k} + e_{i,k}\psi^{m+1}_{i,k+1} = V^{m+1}_{i,k}$$

These are solved by back substitution starting from

$$\Delta\psi^{m+1}_{N,L} = V^{m+1}_{N,L}$$

Next: $$\Delta\psi^{m+1}_{N-1,k} = V_{N-1,L} - d_{N-1,k}\psi^{m+1}_{N,L} - e_{N-1,k}\psi_{N-1,L+1}$$

and so forth.

Then solve:

$$\psi^{m+1}_{i,k} = \Delta\psi^{m+1}_{i,k} + \psi^{m}$$

for the mth iteration.

APPENDIX D

PHYSICAL MODEL COMPUTER PROGRAMS

D.1 General

The computer program listing for the Physical model using the line successive overrelaxation numerical method is given in Section D.2 and the listing for the model using the Strongly Implicit Numerical Method is given in Section D.3.

A list of input and out variables common for both programs is given Fig. D.1.

LIST OF VARIABLES

Fig. D.1

INPUT VARIABLES

Delt	- initial time step
N	- no. of vertical nodes (y axis)
L	- no. of horizontal nodes (x axis)
NKI NSMI	- number of data points in permeability, soil moisture, functional relationships (size of array)
NR	- number of data points in rainfall array
G	- gravity
RHOO	- density of fluid
Bet	- compressibility of water
U	- kinematic viscosity
A MODEL	- number of model tested
TEND	- limit of time simulation in seconds
DELIM	- maximum time step in seconds
TIMX	- arbitrary time interval usually 100 seconds
TOLH	- tolerance for convergence of pressure head
SR	- the residual saturation or irreducible saturation in the functional relationship between moisture content and pressure head
RLAM	- pore size distribution index
KBP	- location of air entry pressure head in functional relationship array
BPR	- bubbling or air entry pressure head
SATPOR	- saturated moisture content of soil
PERSAT	- saturated hydraulic conductivity of soil

KGW	- x-coordinate at which a vertical section is taken to calculate the horizontal contribution of base flow
XKO	- constant CN used in determining Darcy-Weisbach coefficient for overland flow
RCRIT	- critical Reynolds Number for overland flow. Flow assumed turbulent above this number
SLOPE	- slope of saturated moisture-pressure head functional relationship
SLOPE 1	- slope of hydraulic-conductivity pressure head functional relationship
PRESI	- pressure head array for hydraulic conductivity functional relationship
PSI	- pressure head array for soil moisture functional relationship
R	- total rainfall amount at time T
T	- time of rainfall
FORM(I,K)	- type of geologic formation for a nodal point I,K
Bound(I,K)	- boundary condition at nodal point I,K (interior(o), right boundary(1), left boundary(2), lower boundary(4), left corner boundary(5), right corner boundary(6), upper boundary(3))
PHI(I,K)	- initial pressure head at nodal point (I,K)
Z(K)	- vertical distance between 2 horizontal rows
X(I)	- horizontal distance between 2 vertical columns
PORS	- porosity of soil
VT	- vertical compressibility of formation
INB(I)	- flux input to bottom boundary
INL(K)	- flux input to left boundary
INR(K)	- flux input to right boundary
INS(K)	- flux input to seepage flow
width	- width of model

FIN - end of overland flow boundary

SL(I) - slope of surface boundary at column I or planes (P)

S(P) - defines the reaches or planes (P) used for overland flow

LIST OF OUTPUT VARIABLES

QL(I) - lateral inflow to overland flow plane P at surface node, column I

INFIL(I) - infiltration at column I

SEEPV(I) - seepage across a vertical face at I

PCAL(I,K) - calculated pressure head at node (I,K)

Q(P) - total overland flow at plane P

H2(P) - depth of overland flow at end of plane P

Depth(I) - depth of overland flow at each surface node

HYDH - hydraulic head

THETS(I,K) - moisture content at node (I,K)

D.2 Physical Computer Model Program listing the L.S.O.R. Numerical Method

```
C      COMPUTER PROGRAM 1.
C      A PHYSICALLY-BASED,DIGITALLY-SIMULATED HYDROLOGIC MODEL
C
C      NUMERICAL SOLUTION USED THE LINE SUCCESSIVE OVERRELAXTION
C                              METHOD
C
        REAL INS,INZ,INPUT,INFIL3
        REAL INFIL,K1,K2,INB,INL,INR,MAN
        INTEGER P,FIN,T2,S,BOUND,TOP,TEST,TOP1,FINI
        DIMENSION  R(50),T(50),PERM(10),THETS(30,20),HYDH(20)
          DIMENSION ZT(20),SEEPV(30),SLOPE(25)
         DIMENSION  SLOPE1(25)
          DIMENSION  INFIL3(30)
         DIMENSION THET1(30,20)
        COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
       $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
       $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
       $PH(10),RHO(5),DELT1,THETA(30,20)
        COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
         COMMON/SEC3/A(20),B(20),C(20),D(20)
        COMMON/SEC4/M,N2,KING,J
        COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
         COMMON/SEC6/INR(20)
         COMMON/SEC7/PCAL(30,20),LOVE
         COMMON/SEC8/ AREA1(99),AREA2(99),DISCH1(99),DISCH2(99),MAN,SL(30)
       $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
       $H2(30),XKO,RCRIT
         COMMON/SEC9/ I,X(30),DELT,BLUE
         COMMON/SEC10/INFIL(30),DEPTH(30)
         COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
          COMMON/SEC12/INS(20)
        XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
C
        ISET=0
        KONT=0.0
        READ(1,57)DELT,N,L,NK1,NSM1,NR,G,RHOO,BET,U,AMODEL,TEND,DELIM,TIMX
       $,TOLH
        WRITE(3,57) DELT,N,L,NK1,NSM1,NR,G,RHOO,BET,U,AMODEL,TEND,DELIM,
       $TIMX,TOLH
57      FORMAT(2X,F3.0,1X,I3,1X,I3,1X,I3,1X,I3,1X,I3,1X,F4.0,1X,F3.0,1X,
       $E8.1,1X,F4.3,1X,F3.1,1X,F8.0,F5.0,F7.1,F5.2)
         READ(1,367) SR,RLAM,KBP,BPR,SATPOR,PERSAT,KGW,XKO,RCRIT
367      FORMAT(2X,F3.2,F4.2,I2,F5.1,F4.3,E8.3,I2,F6.2,F6.2)
         WRITE(3,367) SR,RLAM,KBP,BPR,SATPOR,PERSAT,KGW,XKO,RCRIT
          READ(1,369) (SLOPE(LA),LA=KBP,NK1)
         WRITE(3,369)  (SLOPE(LA),LA=KBP,NK1)
         READ(1,369)  (SLOPE1(LA),LA=KBP,NK1)
         WRITE(3,369)  (SLOPE1(LA),LA=KBP,NK1)
369       FORMAT(2X,9E8.3)
        READ(1,121) (PRES1(LA),LA=1,NK1)
121      FORMAT(11F7.2)
         READ(1,51) (PS1(LA),LA=1,NSM1)
51       FORMAT(11F7.2)
        READ(1,52) (R(LA),LA=1,NR)
52        FORMAT(6F8.5)
         READ(1,53) (T(LA),LA=1,NR)
53       FORMAT(6F7.3)
          WRITE(3,1004) (T(LA),LA=1,NR)
1004     FORMAT(' ',6F7.3)
        DO 81 I=1,N
        DO 82 K=1,L
          THETS(I,K)=0.0
         PERM5(I,K)=0.0
          THETA(I,K)=0.0
          FORM(I,K)=1
82         CONTINUE
81      CONTINUE
        DO 83 I=1,N
         READ(1,54) (PHI(I,K),K=1,L)
54       FORMAT(13F6.1)
```

```
83     CONTINUE
       DO 200 I=1,N
       READ(1,55) (BOUND(I,K),K=1,L)
55       FORMAT(40I2)
200    CONTINUE
       READ(1,56) (Z(K),K=1,L)
       READ(1,31) (X(I),I=1,N)
56       FORMAT(26F3.0)
31         FORMAT(26F3.0)
       READ(1,58) (PORS(LA),LA=1,2)
58     FORMAT(2F4.2)
       READ(1,70) (VT(LA),LA=1,2)
70     FORMAT(2E8.1)
       READ(1,32) (INB(I),I=1,N)
32         FORMAT(26F3.0)
59       FORMAT(26F3.0)
       READ(1,59) (INL(K),K=1,L)
       READ(1,59) (INR(K),K=1,L)
       READ(1,59) ( INS(K),K=1,L)
       READ(1,61)  MAN,WIDTH,FIN,YZ
61     FORMAT(F4.2,3X,F7.3,3X,I3,3X,F5.2)
       READ(1,60) (SL(I),I=1,FIN)
60       FORMAT(11F7.5)
       READ(1,62) (S(P),P=1,FIN)
62         FORMAT(11I2)
C
C   DETERMINE UNSATURATED MOISTURE-PERMEABILITY PRESSURE CURVES
C
         SE=SR/SATPOR
        SR1=(1-SE)
        RN=RLAM*3+2
         DO 360 I=1,KBP
        RELP=PRES1(I)/BPR
        THETA1(I)=(((BPR/PS1(I))**RLAM)*SR1+SE)*SATPOR
360    K1(I)=PERSAT*((BPR/PRES1(I))**RN)
        KBP=KBP+1
C
C    DETERMINE SATURATED MOISTURE-PERMEABILITY PRESSURE CURVES
C
        DO 361  I=KBP,NK1
        THETA1(I)=SLOPE(I-1)*(PS1(I)-PS1(I-1))+THETA1(I-1)
361     K1(I)=SLOPE1(I-1)*(PRES1(I)-PRES1(I-1))+K1(I-1)
       WRITE(3,227) (Z(K),K=1,L)
227    FORMAT('0','Z=',2X,20F5.0)
       WRITE(3,225) (X(I),I=1,N)
225    FORMAT('0','X=',2X,20F5.0)
        WRITE(3,676)
676       FORMAT(' ',30X,'SOIL CHARACTERISTICS         ')
          WRITE(3,695)
695       FORMAT('0',7X,'FORMATION   1',39X,'FORMATION2',//)
        WRITE(3,674)
 674   FORMAT('0','PRESSURE(CM.)',10X,'PERMEABILITY(CM.2)',20X,'PRESSURE
     $CM.)',10X,'PERMEABILITY(CM.2)')
        DO 671  LA=1,NK1
          WRITE(3,670)  PRES1(LA),K1(LA)
670     FORMAT(' ',F8.3,10X,E8.3)
671     CONTINUE
        WRITE(3,675)
675       FORMAT('0','PRESSURE',10X,'SOIL MOISTURE',20X,'PRESSURE',10X,
     $'SOIL MOISTURE')
          DO 672 LA=1,NSM1
          WRITE(3,673)  PS1(LA),THETA1(LA)
673       FORMAT(' ',F8.3,10X,F7.6)
672       CONTINUE
        WRITE(3,677)  AMODEL
677        FORMAT('1',30X,'OUTPUT',/' ',20X,'HYDROLOGIC MODEL',3X,F3.1)
C      INITALIZATION
           PRTO=0.0
           MORE=1
       IR=1
```

```
      ALPH=0.5
      NU=1
      RES=0.0
      STORV=0.0
      VOLGW=0.0
      VOL=0.0
      XLTH=0.0
      MEND=S(FIN)
      DO 368 I=1,MEND
368   XLTH=XLTH+X(I)
      KGW=20
      DO 111  I=1,5
      PH(I)=0.0
      RHO(I)=1.00
111   CONTINUE
      DO 863 I=1,N
      INFIL(I)=0.0
      RED(I)=-1
      DEPTH1(I)=0.0
      DEPTH(I)=0.0
      REPEAT(I)=0.0
863   CONTINUE
      IK=1
      TOL1=1
      TOL=0.001
      NM=1
      DO 437 P=1,FIN
      Q(P)=0.0
437   CONTINUE
      IA=0.0
      LOVE=1
      R2=0.0
      TIMTOT=0.0
      DELT1=DELT
C
C      DETERMINE  ELEVATION  HEAD
C
      DO 282  K=1,L
      IF(K.EQ.1)  GO TO 283
      ZT(K)=ZT(K-1)+(Z(K)+Z(K-1))/2
      GO TO 282
283   ZT(K)=Z(K)/2
282   CONTINUE
8     T1=DELT
      IT=1
      NM=0
      IF(TIMTOT.GT.0) GO TO 119
      DELT1=DELT
119   T2=NR
C
C      DETERIMINE  RAINFALL INTENSITY
C
410   TIME=TIMTOT+DELT
      IF(T(NR)*3600-TIME) 411,409,409
411   WRITE(3,471)
471   FORMAT(' ','HELP THE TIME IS NEG')
      GO TO 273
409   IF(TIME) 18,19,18
 19   P=2
      GO TO 20
18    DO 21 P=1,T2
      IF(T(P)*3600-TIME) 21,20,20
 21   CONTINUE
 20   KING=P-1
      R1=R2
      THOUR=TIME/3600
      R2=R(KING)+(THOUR -T(KING))/(T(P)-T(KING))*(R(P)-R(KING))
22    RF=(R2-R1)/DELT
      NTIME=P
24    WRITE(3,267) DELT,RF,R2,TIME
```

```
267     FORMAT(' ',/,' ','DELT=',F7.1,3X,'RF=',F6.4,3X,'R2=',F8.4,'TIME='
     $,E14.5)
      DO  90 I=1,N
      DO 90 K=1,L
        PRED(I,K)=PHI(I,K)
      PHIS(I,K)=PHI(I,K)
      PCAL(I,K)=PHI(I,K)
90      PHI1(I,K)=PHI(I,K)
C
C   PREDICT  PRESSURES
C
230     IF(TIMTOT.GT.0.0.AND.IT.GE.2) GO TO 287
        DO 723 I=1,N
        DO 723  K=1,L
        IF(IT.GT.1.AND.TIMTOT.EQ.0.0) GO TO 725
      IF(TIMTOT) 720,720,722
720       PRED(I,K)=PHIS(I,K)
        GO TO 723
722       II=BOUND(I,K)+1
        GO TO (130,131,132,133,134,135,136,723,723,723,140,141),II
         GO TO 723
130     PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
        PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
         DO 310 MN=1,4
          M=MN
        CALL  TBLLP(21,21,PRES1,PH)
310       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
     $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
        TX=SATPOR+0.1
        IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
        GO TO 143
131     PH(1)=0.0
        PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
        PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
      DO 304   MN=1,4
          M=MN
        CALL  TBLLP(21,21,PRES1,PH)
304       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
     $           -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
        GO TO 143
132     PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        PH(2)=0.0
        PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
        DO 305 MN=1,4
          M=MN
        CALL  TBLLP(21,21,PRES1,PH)
305       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
     $*PERM(1)-INL(K)*RHO(2))                                            +
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
        TX=SATPOR+0.1
        IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
        GO TO 143
```

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133      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=0.0
         PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
         IF(RED(I).EQ.+1) GO TO 723
       DO 306  MN=1,4
           M=MN
         CALL  TBLLP(21,21,PRES1,PH)
306        PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
           INPUT=RP+DEPTH(I)/DELT
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      *1/Z(K)*(RHO(3)*INPUT
      *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1)))))
      $)*DELT+THETS(I,K)
         TX=SATPOR+0.1
         IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
         GO TO 143
134      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
         PH(4)=0.0
       DO 307  MN=1,4
           M=MN
         CALL  TBLLP(21,21,PRES1,PH)
307        PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
         TX=SATPOR+0.1
         IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
         GO TO 143
135      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
       PH(2)=0.0
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
         PH(4)=0.0
         DO 308  MN=1,4
           M=MN
         CALL  TBLLP(21,21,PRES1,PH)
308        PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-INL(K)*RHO(2))                                            +
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
         TX=SATPOR+0.1
         IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
         GO TO 143
136    PH(1)=0.0
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
       PH(4)=0.0
         DO 309  MN=1,4
           M=MN
         CALL  TBLLP(21,21,PRES1,PH)
309        PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
      $          -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
         TX=SATPOR+0.1
         IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
         GO TO 143
```

```
140     PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        PH(2)=0.0
         PH(3)=0.0
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
        IF(RED(I).EQ.+1)  GO TO 723
        DO 311  MN=1,4
          M=MN
        CALL   TBLLP(21,21,PRES1,PH)
311      PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
         INPUT=RF+DEPTH(I)/DELT
       THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-INL(K)*RHO(2))
      *1/Z(K)*(RHO(3)*INPUT
      *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
      $)*DELT+THETS(I,K)
        TX=SATPOR+0.1
        IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
        GO TO 143
141    PH(1)=0.0
        PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
        PH(3)=0.0
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
        IF(RED(I).EQ.+1)  GO TO 723
         DO 312   MN=1,4
          M=MN
        CALL   TBLLP(21,21,PRES1,PH)
312      PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
         INPUT=RF+DEPTH(I)/DELT
       THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
      $           -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))
      *1/Z(K)*(RHO(3)*INPUT
      *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
      $)*DELT+THETS(I,K)
        TX=SATPOR+0.1
        IF(THETA(I,K).GT.TX)THETA(I,K)=THETA1(NK1-1)
143     PH(5)=THETA(I,K)
        M=5
         CALL   TBLLP(21,21,THETA1,PH)
       PERM5(I,K)=XINT(K1(KING),K1(J),THETA1(KING),THETA1(J),PH(M))
        GO TO 723
725     PRED(I,K)=PRED(I,K)+ALPH*(PHIS(I,K)-PRED(I,K))
723       CONTINUE
C
C      CALL NODE PROGRAMS
C
        IF(TIMTOT.EQ.0.0)  GO TO 287
         DO 606  I=1,N
       K=TOP1(I)
        IF(BOUND(I,K).EQ.9)  GO TO 606
        IF(DEPTH(I).GT.0.0.AND.QL(I).NE.0.0)  GO TO 741
        IF(PHI1(I,K)  ) 206,741,741
741     IF((RF+DEPTH(I)/DELT).GT.INFIL(I))  GO TO 220
206      RED(I)=-1
       IF(QL(I).EQ.0.0)  GO TO 552
          INFIL(I)=RF+DEPTH(I)/DELT
         GO TO 606
552       INFIL(I)=RF
         GO TO 606
220       RED(I)=+1
          GO TO 606
606        CONTINUE
287     I=1
  7    DO 3  K=1,L
        IA=BOUND(I,K)+1
        GO TO (10,11,13,14,84,85,87,3,3,88,14,88,350),IA
        GO TO 303
303     PRED(I,K)=0.0
       PHIS(I,K)=0.0
```

```
       PCAL(I,K)=0.0
       PHI(I,K)=0.0
        GO TO 3
10     CALL INTER
       GO TO 3
C      CALL RIGHT BOUNDARY
11     CALL LLRB
       GO TO 3
C      CALL LEFT BOUNDARY
13     CALL  LLRB
       GO TO 3
C      CALL  LOWER  BOUNDARY
84     CALL LLRB
       GO TO 3
C      CALL LEFT CORNER BOUNDARY
85     CALL LRCB
       GOTO3
C      CALL RIGHT CORNER BOUNDARY
87     CALL LRCB
       GO TO 3
88     CALL SEEP
       TOP1(I)=K
        GO TO 3
350      BOUND(I,K)=1
         CALL  LLRB
        BOUND(I,K)=12
         GO TO 3
14     CALL UPPER
       TOP1(I)=K
3      CONTINUE
       IF(TIMTOT.GT.0) GO TO 115
       IF(IT.GT.1) GO TO 115
       DO 116 K=1,L
116    PHI2(I,K)=PHI(I,K)
115    CALL ITERAT(A,B,C,D,PCAL,I,TOP)
       I=I+1
       IF(I.LE.N) GO TO 7
         ACCEL=1.0
        RES=0.0
        IF(TIMTOT.EQ.0.0) GO TO 268
         IF(IT.GT.1) ACCEL=1.33
268    DO 271 I=1,N
       DO 271 K=1,L
         DIFF=ABS(PCAL(I,K)-PHIS(I,K))
        IF(DIFF.LE.RES) GO TO 258
        RES=DIFF
258    PCAL(I,K)=ACCEL*PCAL(I,K)+(1-ACCEL)*PHIS(I,K)
         PHIS(I,K)=PCAL(I,K)
271    CONTINUE
        IF(RES.LE.TOL) GO TO 635
264     IT=IT+1
        IF(TIMTOT.EQ.0.0) GO TO 561
730      IF(IT.LE.30) GO TO 608
540     DELT=DELT/4
         GO TO 607
561      IF(IT.LE.153) GO TO 608
         DO 562 I=1,N
         WRITE(3,72) I,(PCAL(I,K),K=1,L)
  562    CONTINUE
         GO TO 273
608       IF(IT.LE.150) GO TO 230
635    WRITE(3,633) IT
633    FORMAT(' ','IT=',I5)
       ISET=0
C
C   TEST FOR UPPER BOUNDARY CHANGE
C
         DO 777 I=1,N
         INFIL3(I)=0.0
         K=TOP1(I)
```

```
      IF(BOUND(I,K).EQ.9) GO TO 777
      IF(PCAL(I,K))804,812,812
804    IF(PHI1(I,K))777,777,777
812     IF(PHI1(I,K))814,777,777
814   IF(PCAL(I,K).LE.TOL1) GO TO 777
811    IF(KONT.EQ.1) GO TO 600
     DEL10=ABS(PHI1(I,K)-PCAL(I,K))
     DEL11=ABS(PHI1(I,K))
     DELT=(DEL11*DELT)/DEL10
      KONT=1
       GO TO 607
600  IF(DELT.LT.0.1) GO TO 823
      DELT=0.8*DELT
       GO TO 607
823   WRITE(3,824) I
824   FORMAT(' ',I3,'DELT IS TOO SMALL')
777     CONTINUE
         KONT=0
         GO TO 681
607     R2=R1
       WRITE(3,412) DELT
412    FORMAT(' ','NEW DELT',E14.5)
      NU=1
       MORE=1
       GO TO 8
C
C     TEST FOR SOIL MOISTURE(THETA)  TOL
C
681         RES=0.0
       DO 793  I=1,N
        DO 791  K=1,L
      IF((BOUND(I,K).EQ.78).OR.(BOUND(I,K).EQ.9)) GO TO 791
       N2=NSM1
      M=5
        PH(5)=PCAL(I,K)
        CALL  TBLLP(21,21,PS1,PH)
      THET1(I,K)=THETS(I,K)
     THETS(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
      DINFIL=(THETS(I,K)-THET1)*Z(K)
       IF(K.EQ.TOP1(I).AND.PCAL(I,K).GE.0.0) DINFIL=DINFIL/2
      INFIL3(I)=DINFIL+INFIL3(I)
       IF(TIMTOT.EQ.0.0) GO TO 791
       DIFF=ABS(THETS(I,K)-THETA(I,K))
      IF(DIFF.LE.RES) GO TO 791
      RES=DIFF
791     CONTINUE
     IF(INFIL3(I).LT.0.01E-25) GO TO 793
       INFIL3(I)=INFIL3(I)/DELT
793    CONTINUE
      IF(RES.LE.TOLH) GO TO 1002
        DO 1000 I=1,N
      DO 1000 K=1,L
      THETS(I,K)=THET1(I,K)
1000 CONTINUE
C
C     CONTINUITY CHECK
C
1002   STOR5=0.0
       DO 374  I=1,N
       DO 373 K=1,L
      IF((BOUND(I,K).EQ.78).OR.(BOUND(I,K).EQ.9)) GO TO 373
     DSTOR5=THETS(I,K)*Z(K)*X(I)*WIDTH
       STOR5=DSTOR5+STOR5
373    CONTINUE
374     CONTINUE
       STOR5=STOR5/(XLTH*WIDTH)
      IF(TIMTOT.EQ.0.0) STOR4=STOR5
       DSTOR=STOR5-STOR4
C
C   CALCULATE OVERLAND FLOW AND DEPTHS
```

```
C
        DO  34 I=1,N
      K=TOP1(I)
      IF(BOUND(I,K).EQ.9)  GO TO 858
      GO TO 859
858     INFIL(I)=RF
859     QL(I)=RF-INFIL(I)
34      CONTINUE
      CALL OLAND
C
C   CALCULATE INFILTRATION
C
403     DO 77 I=1,N
        K=TOP1(I)
         SEEPV(I)=0.0
      IF(BOUND(I,K).EQ.9)  GO TO 77
           IF(PCAL(I,K))  700,701,701
700      IF(QL(I).EQ.0.0)  GO TO 551
           INFIL(I)=RF+DEPTH(I)/DELT
         GO TO 77
551      INFIL(I)=RF
      GO TO 77
701      N2=NK1
         M=4
        PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
        CALL   TBLLP(21,21,PRES1,PH)
        PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
         M=9
        PH(9)=(PHI(I,K)+PHI(I,K-1))/2
        CALL   TBLLP(16,16,PRES1,PH)
        PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
         PER4(I)=(PER1+PER2)/2
          PER4(I)=PER1
        INFIL(I)=(1-((PCAL(I,K-1)-DEPTH(I))*2/(Z(K)+Z(K-1))))
     $*(PER4(I)*RHO(4)*G/U)
           SEEPV(I)=INFIL(I)
         IF(INFIL(I).GE.0.0)  GO TO 77
         INFIL(I)=0.0
77        CONTINUE
      WRITE(3,93)  TIMTOT,DELT
93    FORMAT(' ',5X,'TOTALTIME=',F12.3,5X,'TIME INCREMENT=',F12.3,//)
C
C   DETERMINE BASE FLOW
C
C   HORIZONTAL CONTRIBUTION
      GWV=0.0
         TOTGW=0.0
        HH=0.0
        I=KGW
        DO 363  K=1,L
         N2=NK1
        M=1
        PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        CALL   TBLLP(21,21,PRES1,PH)
      PERM(1)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M
     $))
      XL=X(I)+X(I+1)/2
       GWH=(PERM(1)*(PCAL(I,K)+ZT(K)-PCAL(I+1,K)-ZT(K))*Z(K)*WIDTH)/XL
        IF(K.EQ.L)   GWH=GWH/2
         IF(GWH.LE.0)  GWH=0.0
363     TOTGW=TOTGW+GWH
C   VERTICAL CONTRIBUTION
         MZ=KGW+1
        FINI=S(FIN)
         DO 364  I=MZ,FINI
364       GWV=SEEPV(I)+GWV
         STORGW=RBF
         RBF=GWV+TOTGW
C
C   DETERMINE  RUNOFF VOLUME
```

```
C
      DVOL=((STORV+Q(FIN))/2)*DELT/(WIDTH*XLTH)
      DVOLGW=((STORGW+RES)/2)*DELT/(WIDTH*XLTH)
       VOL=VOL+DVOL
      VOLGW=VOLGW+DVOLGW
       STORV=Q(FIN)
      RAIN=VOL+VOLGW+DSTOR
        ACTR=R2
       WRITE(3,376) RAIN,ACTR,DSTOR,STOR4
376    FORMAT(' ','CONTINUITY CHECK=',E11.3,'CM',3X,'ACTUAL RAIN=',E9.3
     $,2X,'G.W.STOR.=',E11.3,3X,E11.3)
C
      WRITE(3,365) TOTGW,GWV,RES
365    FORMAT(' ','TOTAL HORIZ.BASEFLOW',E14.5,3X,'TOTAL VERT.BASEFLOW'
     $,E14.5,3X,'RESULTANT BASEFLOW=',E14.5)
       WRITE(3,362) VOL,DVOL,XLTH,VOLGW,DVOLGW,Q(FIN)
362    FORMAT('0',1X,'SURFACE RUNOFF=',F7.4,'CM',3X,'R.O.INCREMENT=',
     $F6.4,'CM',2X,'LENGTH=',F8.3,'CM'/' ','G.W.VOL=',F6.4,'CM',3X,'G.W.
     $INCREMENT=',F6.4,3X,'Q(FIN)=',E14.5,'CM3'//)
C
C    PRINT  OUTPUT
C
       IF(PRTO-TIMTOT) 705,705,862
705   PRTO=TIMTOT+DELIM
        DO 707 I=1,N
76     WRITE(3,408)I,QL(I),INFIL(I),SEEPV(I),INFIL3(I)
408    FORMAT(' ','QL(',I2,')=',E14.5,3X,'INFIL=',E14.5,3X,'SEEPV=',E1
     $4.5,'INFIL3=',E14.5)
707    CONTINUE
     DO 291 I=1,N
      WRITE(3,72) I,(PCAL(I,K),K=1,L)
72    FORMAT(' ','I=',I3,(3X,10F10.3))
291   CONTINUE
     DO 504 P=1,FIN
     WRITE(3,505) P,Q(P),H2(P)
505   FORMAT(' ','Q(',I2,')=',E14.5,3X,'H2=',E14.5)
504  CONTINUE
      WRITE(3,870)  (DEPTH(I),I=1,N)
870  FORMAT(' ','DEPTH',3X,6F12.6)
C
C    DETERIMNE  HYDRAULIC HEAD
C
      DO 261 I=1,N
       DO 262 K=1,L
      IF(BOUND(I,K).EQ.78) GO TO 265
      HYDH(K)=ZT(K)+PCAL(I,K)
       GO TO 262
265    HYDH(K)=0.0
262    CONTINUE
         WRITE(3,72) I,(HYDH(K),K=1,L)
261    CONTINUE
       DO 768 I=1,N
      WRITE(3,769) I, (THETS(I,K),K=1,L)
769   FORMAT(' ','I=',I3,(3X,10E9.4))
768     CONTINUE
C
862     DO 99 I=1,N
     DO 95 K=1,L
98   PHI(I,K)=PCAL(I,K)
     PHI2(I,K)=PHI1(I,K)
95    PHI1(I,K)=PHI(I,K)
99      CONTINUE
      DO 888 I=1,N
     DEPTH1(I)=DEPTH(I)
C      101=NO
     REPEAT(I)=101
888    CONTINUE
     KONT=0.0
     IK=1
512   TIMTOT=TIMTOT+DELT
```

```
      IF(TIMTOT-TEND)6,273,273
6     DELT1=DELT
        MORE=MORE+1
       IF(DELT.LT.2.0)  GO TO 817
       IF(MORE.LE.4)  GO  TO  781
817    IF(NU.GE.2)  GO  TO  735
       NU=1
735   NU=NU+1
         MORE=1
       M=NU-1
      DELT=DELT*2**M
       IF(DELT.LT.1500)  GO TO 781
       NU=1
       DELT=1000
781     IR=1
       IF(RF.GT.0.0.AND.DELT.GT.TIMX)   DELT=TIMX
      GO TO 8
 273   STOP
      END
C
C
C UPPER NODE SUBROUTINE
      SUBROUTINE  UPPER
      REAL INFIL,K1,K2,INB,INL,INPUT
       REAL  INR
      INTEGER BOUND,P,T2,TOP,TEST,TOP1
         DIMENSION PERM(10)
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHO0,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
      COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
       COMMON/SEC3/A(20),B(20),C(20),D(20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
         COMMON/SEC6/ INR(20)
       COMMON/SEC7/PCAL(30,20),LOVE
       COMMON/SEC8/ AREA1(99),AREA2(99),DISCH1(99),DISCH2(99),MAN,SL(30)
     $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
     $H2(30),XKO,HCRIT
       COMMON/SEC9/ I,X(30),DELT,BLUE
       COMMON/SEC10/INFIL(30),DEPTH(30)
       COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      ALPH=0.5
       TOL=1.0
       RHO(4)=1.00
      T3=DELT1
       IF(TIMTOT.GT.0.0) GO TO 607
        DEPTH(I)=PHI(I,K)
      N2=NSM1
      M=5
       PH(5)=PHI(I,K)
         CALL  TBLLP(21,21,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
       GC TO 220
607     IF(RED(I).EQ.+1)  GO TO 220
      IF(TIMTOT.GT.0.0)  GO TO 103
206   PH(4)=(PRED(I,K)+PRED(I,K-1))/2
       PH(9)=(PHI(I,K)+PHI(I,K-1))/2
        IF(BOUND(I,K).EQ.10)  GO TO 303
      PH(2)=(PRED(I,K)+PRED(I-1,K))/2
        PH(7)=(PHI(I,K)+PHI(I-1,K))/2
       GO TO 304
303     PH(2)=0.0
        PH(7)=0.0
304     PH(3)=(PRED(I,K)+DEPTH(I))/2
      PH(8)=( PHI(I,K)+DEPTH1(I))/2
      PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
```

```
      PH(5)=PRED(I,K)
      N2=NK1
        DO 212 MN=1,4
      M=MN
219     CALL   TBLLP(21,21,PRES1,PH)
        PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
        M=MN+5
        CALL   TBLLP(21,21,PRES1,PH)
        PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
        PERM(MN)=(PER2+PER1)/2
212   CONTINUE
         PER4(I)=PER1
        GO TO 104
103     PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
      IF(BOUND(I,K).EQ.10)  GO TO 106
        PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
106     PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
104   BETA=BET*RHO0*G
      DELPHI=PRED(I,K)-PHIS(I,K)
      IF(PORM(I,K).EQ.1)GO TO 213
213   POR=PORS(1)
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0)  GO TO 105
         CALL   TBLLP(NK1,NK1,PS1,PH)
      PH(5)=(PHI(I,K)+PRED(I,K))/2
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
730   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
         GO TO 214
105      PH(5)=THETA(I,K)
         CALL TBLLP(21,21,THETA1,PH)
        PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
         GO TO 730
214   VTCOM=0.0
      UO=CE
      P1=G/(U*X(I))
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
        IF(BOUND(I,K).EQ.10)GO TO 534
      QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
         GO TO 533
534      QE2=0.0
533   A(K)=0.0
      P1=G/(U*X(I))
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
      C(K)=-(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
20    B(K)=+(P1*QE1)+(P1*QE2)-A(K)-C(K)+(UO/DELT)
         IF(QL(I).EQ.0.0)  GO TO 550
        INPUT=RF+DEPTH(I)/DELT
         GO TO 551
550         INPUT=RF
551      IF(BOUND(I,K).EQ.10)  GO TO 433
      Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))+
     $(P1*QE2)*(-PHI(I,K)+PCAL(I-1,K)+PHI(I-1,K))
        GO TO 404
433   Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))-((RHO(5)*INL(K))/X(I))
404   ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))+C(K)*PHI(
     $I,K)-C(K)*PHI(I,K-1)
      D(K)=Y+ZA+(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0)  GO TO 600
      Y=P1*QE1*2*(PHIS(I+1,K))
      C(K)=2*C(K)
      B(K)=2*(P1*QE1+P1*QE2)-A(K)-C(K)
       D(K)=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))+Y
600     TOP=K
      GO TO216
C
C   SATURATED UPPER BOUNDARY
C
220     A(K)=0.0
         B(K)=0.0
```

```
          C(K)=0.0
          D(K)=0.0
            K=K-1
            PRED(I,K+1)=DEPTH(I)
            PCAL(I,K+1)=PRED(I,K+1)
          LOVE=4
            TEST=BOUND(I,K)-1
            IF(TEST) 10,11,11
10          CALL INTER
            GO TO 502
11          CALL  LLRB
502         TOP=K
          K=K+1
            LOVE=1
          IF(TIMTOT.EQ.0.0)  DEPTH(I)=0.0
216      RETURN
         END
C
C
C SEEPAGE NODE SUBROUTINE
         SUBROUTINE SEEP
           REAL INS
         REAL INFIL,K1,K2,INB,INL,INPUT
           REAL  INR
         INTEGER BOUND,P,T2,TOP,TEST,TOP1
            DIMENSION PERM(10)
         COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
        $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
        $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
        $PH(10),RHO(5),DELT1,THETA(30,20)
         COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
           COMMON/SEC3/A(20),B(20),C(20),D(20)
         COMMON/SEC4/M,N2,KING,J
         COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
             COMMON/SEC6/ INR(20)
           COMMON/SEC7/PCAL(30,20),LOVE
           COMMON/SEC8/ AREA1(99),AREA2(99),DISCH1(99),DISCH2(99),MAN,SL(30)
        $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
        $H2(30),XKO,RCRIT
           COMMON/SEC9/ I,X(30),DELT,BLUE
           COMMON/SEC10/INFIL(30),DEPTH(30)
           COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
             COMMON/SEC12/INS(20)
         XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
         ALPH=0.5
           TEST=BOUND(I,K)-9
         IF(TEST) 8,9,8
8        RHO(4)=1.00
           IF(TIMTOT.GT.0.0) GO TO 607
         M=5
         N2=NSM1
          PH(5)=PHI(I,K)
             CALL  TBLLP(21,21,PS1,PH)
         THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
            DEPTH(I)=PHI(I,K)
          GO TO 220
607         IF(RED(I).EQ.+1) GO TO 220
         IF(TIMTOT.GT.0.0) GO TO 103
206      PH(4)=(PRED(I,K)+PRED(I,K-1))/2
          PH(9)=(PHI(I,K)+PHI(I,K-1))/2
         PH(2)=(PRED(I,K)+PRED(I-1,K))/2
            PH(7)=(PHI(I,K)+PHI(I-1,K))/2
            PH(3)=0.0
            PH(8)=0.0
         PH(5)=PRED(I,K)
         N2=NK1
          DO 212  MN=2,4
          M=MN
219       CALL  TBLLP(21,21,PRES1,PH)
          PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
```

```
      M=MN+5
      CALL  TBLLP(21,21,PRES1,PH)
      PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      PERM(MN)=(PER2+PER1)/2
212   CONTINUE
        PER4(I)=PER1
      GO TO 104
103   PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
      PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
104   BETA=BET*RHO0*G
      DELPHI=PRED(I,K)-PHIS(I,K)
      IF(FORM(I,K).EQ.1)GO TO 213
213   POR=PORS(1)
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0) GO TO 105
      PH(5)=(PHI(I,K)+PRED(I,K))/2
        CALL  TBLLP(21,21,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
730   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
        GO TO 214
105     PH(5)=THETA(I,K)
        CALL TBLLP(21,21,THETA1,PH)
      PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
      GO TO 730
214   VTCOM=0.0
      UO=CE
      A(K)=0.0
      QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
      P1=G/(U*X(I))
      C(K)=-(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
20     B(K)=-C(K)+(P1*QE2)+(UO/DELT)-A(K)
      Y=P1*QE2*(PCAL(I-1,K)+PHI(I-1,K)-PHI(I,K))+((RHO(5)*INS(K))/X(I))
        IF(QL(I).EQ.0.0) GO TO 550
       INPUT=RF+DEPTH(I)/DELT
      GO TO 404
550       INPUT=RF
404   ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))+C(K)*PHI(
     $I,K)-C(K)*PHI(I,K-1)
      D(K)=+ZA+(UO*PHI(I,K))/DELT+Y
       IF(TIMTOT.GT.0.0) GO TO 600
      Y=P1*QE2*(PCAL(I-1,K))*2                     +((RHO(5)*INS(K))/X(I))
      C(K)=2*C(K)
      B(K)=2*(        P1*QE2)-A(K)-C(K)
       D(K)=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))+Y
600    TOP=K
      GO TO216
9       A(K)=0.0
       B(K)=0.0
        C(K)=0.0
       D(K)=0.0
       K=K-1
       LOVE=4
       PRED(I,K+1)=PHI(I,K+1)
      PCAL(I,K+1)= PHI(I,K+1)
        IF(BOUND(I,K))33,33,31
31     CALL LLRB
        GO TO 34
33      CALL  INTER
34     TOP=K
       K=K+1
       LOVE=1
      GO TO 216
C
C       SATURATED UPPER BOUNDARY
C
220     A(K)=0.0
      B(K)=0.0
        C(K)=0.0
       D(K)=0.0
```

```
          K=K-1
         PRED(I,K+1)=DEPTH(I)
          PCAL(I,K+1)=PRED(I,K+1)
          LOVE=4
           TEST=BOUND(I,K)-12
          IF(TEST) 11,16,16
16        BOUND(I,K)=1
          CALL LLRB
           BOUND(I,K)=12
          GO TO 502
11         CALL INTER
502         TOP=K
           K=K+1
         LOVE=1
         IF(TIMTOT.EQ.0.0) DEPTH(I)=0.0
216     RETURN
        END
C
C
C    LOWER,LEFT,AND RIGHT NODE SUBROUTINE
        SUBROUTINE  LLRB
        REAL K1,INB,INL,K2,INR
        INTEGER BOUND,T2
           DIMENSION PERM(10)
        COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
       $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
       $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
       $PH(10),RHO(5),DELT1,THETA(30,20)
        COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
          COMMON/SEC3/A(20),B(20),C(20),D(20)
        COMMON/SEC4/M,N2,KING,J
        COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
          COMMON/SEC6/INR(20)
          COMMON/SEC7/PCAL(30,20),LOVE
          COMMON/SEC9/ I,X(30),DELT,BLUE
        XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
        IF(TIMTOT.GT.0.0) GO TO 103
102     IF(BOUND(I,K).EQ.1) GO TO 222
        PH(1)=(PRED(I,K)+PRED(I+1,K))/2
        PH(6)=(PHI(I,K)+PHI(I+1,K))/2
        IF(BOUND(I,K).EQ.2) GO TO 203
222     PH(2)=(PRED(I,K)+PRED(I-1,K))/2
            PH(7)=(PHI(I,K)+PHI(I-1,K))/2
203      PH(3)=(PRED(I,K)+PRED(I,K+1))/2
         PH(8)=(PHI(I,K)+PHI(I,K+1))/2
        IF(BOUND(I,K).EQ.4) GO TO 204
        PH(4)=(PRED(I,K)+PRED(I,K-1))/2
         PH(9)=(PHI(I,K)+PHI(I,K-1))/2
204     PH(5)=PRED(I,K)
        N2=NK1
         DO 116 MN=1,4
         M=MN
         CALL   TBLLP(21,21,PRES1,PH)
         PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
        M=MN+5
         CALL   TBLLP(21,21,PRES1,PH)
         PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
         PERM(MN)=(PER2+PER1)/2
116     CONTINUE
         GO TO 104
103      IF(BOUND(I,K).EQ.1) GO TO 110
         PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
         IF(BOUND(I,K).EQ.2) GO TO 111
110      PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
111      PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
         IF(BOUND(I,K).EQ.4) GO TO 112
112      PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
104     BETA=BET*RHOO*G
        IF(BOUND(I,K).EQ.1) GO TO 223
        IF(BOUND(I,K).EQ.2) GO TO 206
```

```
223     RHO(2)=1.00
206     RHO(3)=1.00
        IF(BOUND(I,K).EQ.4)  GO TO 207
        RHO(4)=1.00
207     RHO(5)=1.00
        DELPHI=PRED(I,K)-PHIS(I,K)
117      IF(FORM(I,K).EQ.1)  GO TO 94
        POR=PORS(2)
        GO TO 127
94      POR=PORS(1)
127     VTCOM=0.0
121     A(K)=(-G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
        P1=G/(U*X(I))
         IF(BOUND(I,K).EQ.1)  GO TO 224
        QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
         GO TO 225
224     QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
225     M=5
        N2=NSM1
        IF(TIMTOT.GT.0.0)  GO TO 105
        PH(5)=(PHI(I,K)+PRED(I,K))/2
122        CALL   TBLLP(21,21,PS1,PH)
        THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701     CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
           GO TO 123
105       PH(5)=THETA(I,K)
          CALL TBLLP(21,21,THETA1,PH)
         PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
         GO TO 701
123     UO=CE
        KOON=2-BOUND(I,K)
        IF(KOON) 226,208,227
C
C       BOTTOM  BOUNDARY
C
226     QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
        C(K)=0.0
     14 B(K)=+(P1*QE1)+(P1*QE2)-A(K)+(UO/DELT)
        Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))+(P1*QE2*(-PHI(I,K)+
       $PCAL(I-1,K)+PHI(I-1,K)))
        ZA=(-RHO(5)*INB(I)/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))-A(K)*
       $PHI(I,K+1)+A(K)*PHI(I,K)
512     D(K)=+Y+ZA+(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0)  GO TO 209
        Y=P1*QE1*2*(PHIS(I+1,K))+P1*QE2*2*(PCAL(I-1,K))
         A(K)=2*A(K)
        B(K)=2*(P1*QE1+P1*QE2)-A(K)
           D(K)=(-RHO(5)*INB(I)/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))+Y
        GO TO 209
C
C       LEFT  BOUNDARY
C
208     C(K)=-(G*RHO(4)**2*PERM(4))/(U*Z(K)*(Z(K)+Z(K-1)))
11      B(K)=+(P1*QE1)-A(K)-C(K)+(UO/DELT)
        Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))-((RHO(5)*INL(K))/X(I))
        ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-(A(K)*PHI(I,K+1))+(A(K)*PHI(I
       D,K))-((G*RHO(4)**2*PERM(4))/(U*Z(K)))+(C(K)*PHI(I,K))-(C(K)*PHI(I,
       D K-1))
        D(K)=+Y+ZA+(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0)  GO TO 600
        Y=P1*QE1*(PHIS(I+1,K))*2-((RHO(5)*INL(K))/X(I))
         A(K)=2*A(K)
        C(K)=2*C(K)
        B(K)=2*(P1*QE1          )-A(K)-C(K)
          D(K)=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*
       $Z(K)))+Y
600       IF(LOVE.EQ.1)  GO TO 209
        GO TO 405
C
C RIGHT BOUNDARY
```

```
C
227   C(K)=-(G*RHO(4)**2*PERM(4))/(U*Z(K)*(Z(K)+Z(K-1)))
12    B(K)=+(P1*QE2)-A(K)-C(K)+(UO/DELT)
      Y=P1*QE2*(PCAL(I-1,K)+PHI(I-1,K)-PHI(I,K))+((RHO(5)*INR(K))/X(I))
      ZA=(G*RHO(3)**2*PERM(3))/(U*Z(K))-(A(K)*PHI(I,K+1))+A(K)*PHI(I,K)
     $-(G*RHO(4)**2*PERM(4))/(U*Z(K))+C(K)*PHI(I,K)-C(K)*PHI(I,K-1)
      D(K)=+Y+ZA+(UO/DELT)*PHI(I,K)
      IF(TIMTOT.GT.0.0) GO TO 603
      Y=P1*QE2*(PCAL(I-1,K))*2-((RHO(5)*INR(K))/X(I))
        A(K)=2*A(K)
      C(K)=2*C(K)
      B(K)=2*(         P1*QE2)-A(K)-C(K)
        D(K)=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*
     $Z(K)))+Y
603     IF(LOVE.EQ.1) GO TO 209
405   D(K)=D(K)-(A(K)*PRED(I,K+1))
      A(K)=0.0
       PER3(I)=PERM(3)
       PER4(I)=PERM(4)
209   RETURN
      END
C
C
C       INTERIOR NODE SUBROUTINE
      SUBROUTINE   INTER
        REAL K1,K2,INB,INL
      INTEGER BOUND,T2
          DIMENSION PERM(10)
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
      COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
        COMMON/SEC3/A(20),B(20),C(20),D(20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
        COMMON/SEC7/PCAL(30,20),LOVE
        COMMON/SEC9/ I,X(30),DELT,BLUE
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      IF(TIMTOT.GT.0.0) GO TO 103
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
          PH(7)=(PHI(I,K)+PHI(I-1,K))/2
        PH(8)=(PHI(I,K)+PHI(I,K+1))/2
        PH(9)=(PHI(I,K)+PHI(I,K-1))/2
102   PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(2)=(PRED(I,K)+PRED(I-1,K))/2
      PH(3)=(PRED(I,K)+PRED(I,K+1))/2
      PH(4)=(PRED(I,K)+PRED(I,K-1))/2
      PH(5)=PRED(I,K)
      N2=NK1
      DO 116    MN=1,4
       M=MN
       CALL   TBLLP(21,21,PRES1,PH)
       PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       M=MN+5
       CALL   TBLLP(21,21,PRES1,PH)
       PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       PERM(MN)=(PER2+PER1)/2
116   CONTINUE
       GO TO 104
103    PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
       PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
       PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
       PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
104   BETA=BET*RHOO*G
      DELPHI=PRED(I,K)-PHIS(I,K)
117   IF(FORM(I,K).EQ.1) GO TO 94
      POR=PORS(2)
       GO TO 127
94    POR=PORS(1)
```

```
127     VTCOM=0.0
121    A(K)=(-G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
       P1=G/(U*X(I))
       QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
       QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
       C(K)=-(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
       M=5
       N2=NSM1
       IF(TIMTOT.GT.0.0) GO TO 105
       PH(5)=(PHI(I,K)+PRED(I,K))/2
122       CALL  TBLLP(21,21,PS1,PH)
       THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701    CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
         GO TO 124
105      PH(5)=THETA(I,K)
         CALL TBLLP(21,21,THETA1,PH)
        PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
        GO TO 701
124    UO=CE
11     B(K)=+(P1*QE1)+(P1*QE2)-A(K)-C(K)+(UO/DELT)
       Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))+
      $(P1*QE2)*(-PHI(I,K)+PCAL(I-1,K)+PHI(I-1,K))
       ZA=(G*RHO(3)**2*PERM(3))/(U*Z(K))-A(K)*PHI(I,K+1)+A(K)*PHI(I,K)
      $-(G*RHO(4)**2*PERM(4))/(U*Z(K))+C(K)*PHI(I,K)-C(K)*PHI(I,K-1)
500    D(K)=+Y+ZA+(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0) GO TO 600
       Y=P1*QE1*2*(PHIS(I+1,K))+P1*QE2*2*(PCAL(I-1,K))
        A(K)=2*A(K)
       C(K)=2*C(K)
       B(K)=2*(P1*QE1+P1*QE2)-A(K)-C(K)
         D(K)=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*
      $Z(K)))+Y
600      IF(LOVE.EQ.1) GO TO 200
       D(K)=D(K)-(A(K)*PRED(I,K+1))
       A(K)=0.0
        PER4(I)=PERM(4)
        PER3(I)=PERM(3)
200    RETURN
       END
C
C
C   LEFT AND RIGHT CORNOR SUBROUTINE
        SUBROUTINE   LRCB
       REAL K1,K2,INB,INL,INR
        INTEGER  BOUND,T2
          DIMENSION PERM(10)
       COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
      $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
      $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
      $PH(10),RHO(5),DELT1,THETA(30,20)
       COMMON/SEC1/K1(25),PRES1(25),PS1(25),THETA1(25),ALPHA
        COMMON/SEC3/A(20),B(20),C(20),D(20)
       COMMON/SEC4/M,N2,KING,J
       COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
        COMMON/SEC6/INR(20)
        COMMON/SEC7/PCAL(30,20),LOVE
        COMMON/SEC9/ I,X(30),DELT,BLUE
       XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
       IF(TIMTOT.GT.0.0) GO TO 103
102    PH(3)=(PRED(I,K)+PRED(I,K+1))/2
        PH(8)=(PHI(I,K)+PHI(I,K+1))/2
       IF(BOUND(I,K).EQ.6) GO TO 302
       PH(1)=(PRED(I,K)+PRED(I+1,K))/2
       PH(6)=(PHI(I,K)+PHI(I+1,K))/2
       GO TO 303
302    PH(2)=(PRED(I,K)+PRED(I-1,K))/2
         PH(7)=(PHI(I,K)+PHI(I-1,K))/2
303    PH(5)=PRED(I,K)
       N2=NK1
        DO 116  MN=1,3
```

```
      M=MN
       CALL   TBLLP(21,21,PRES1,PH)
       PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      M=MN+5
       CALL   TBLLP(21,21,PRES1,PH)
       PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       PERM(MN)=(PER2+PER1)/2
116    CONTINUE
       GO TO 104
103    PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
       IF(BOUND(I,K).EQ.6)  GO TO 110
       PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
        GO TO 104
110    PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
104   BETA=BET*RHOO*G
      RHO(3)=1.00
      RHO(5)=1.00
      IF(BOUND(I,K).EQ.6)  GO TO 306
      RHO(1)=1.00
      GO TO 307
306   RHO(2)=1.00
307    DELPHI=PRED(I,K)-PHIS(I,K)
117   IF(FORM(I,K).EQ.1)  GO TO 126
      POR=PORS(2)
      GO TO 127
126   POR=PORS(1)
127    VTCOM=0
121    A(K)=(-G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
      P1=G/(U*X(I))
      IF(BOUND(I,K).EQ.6)  GO TO 308
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
       GO TO 309
308   QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
309   C(K)=0.0
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0)  GO TO 105
122      CALL   TBLLP(21,21,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
        GO TO 124
105     PH(5)=THETA(I,K)
        CALL TBLLP(21,21,THETA1,PH)
       PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
       GO TO 701
124   UO=CE
      IF(BOUND(I,K).EQ.6)  GO TO 310
11    B(K)=-A(K)+(P1*QE1)+(UO/DELT)
      Y=P1*QE1*(PHIS(I+1,K)+PHI(I+1,K)-PHI(I,K))-((RHO(5)*
     $INL(K))/(X(I)))
        IF(TIMTOT.GT.0.0)  GO TO 311
        Y=P1*QE1*(PHIS(I+1,K))*2-((RHO(5)*INL(K))/X(I))
       A(K)=2*A(K)
       B(K)=2*P1*QE1-A(K)
      GO TO 311
310   Y=P1*QE2*(PCAL(I-1,K)+PHI(I-1,K)-PHI(I,K))+((RHO(5)*INR(K))/(X
     $(I)))
12    B(K)=-A(K)+(P1*QE2)+(UO/DELT)
        IF(TIMTOT.GT.0.0)  GO TO 311
        Y=P1*QE2*(PCAL(I-1,K))*2-((RHO(5)*INR(K))/X(I))
       A(K)=2*A(K)
       B(K)=2*P1*QE2-A(K)
311   ZA=-((RHO(5)*INB(I))/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))
     $+(A(K)*PHI(I,K))-(A(K)*PHI(I,K+1))
500   D(K)=+Y+ZA+(UO/DELT)*PHI(I,K)
         IF(TIMTOT.GT.0.0)  GO TO 600
        D(K)=(-RHO(5)*INB(I)/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))+Y
600     RETURN
      END
C
```

```
C
C TABLE LOOK-UP SUBROUTINE
C     EN=ARGUMENT TABLE
C     FN=ARGUMENT
       SUBROUTINE  TBLLP(MIKE,LOG,EN,FN)
       COMMON/SEC4/M,N2,KING,J
       REAL EN(MIKE),FN(LOG)
       X4=FN(M)
       IF(X4-EN(1)) 16,15,16
15     J=2
       GO TO 20
16     DO 30 J=1,N2
       IF(EN(J)-X4)30,20,20
30     CONTINUE
20      KING=J-1
       RETURN
       END
C
C
C SOLUTION OF 'L' TRIDIAGONAL LINEAR EQUATIONS
C     EQUATION IS OF THE FORM
C     A*PCAL(I,K+1)-B*PCAL(I,K)+C*PCAL(I,K-1)=-D
        SUBROUTINE ITERAT(A,B,C,D,PCAL,I,TOP)
       INTEGER TOP
        DIMENSION A(20),B(20),C(20),D(20),PCAL(30,20)
        DIMENSION E(20),F(20)
       E(TOP)=C(TOP)/B(TOP)
       F(TOP)=D(TOP)/B(TOP)
       DO 10 M=2,TOP
       E(TOP-M+1)=C(TOP-M+1)/(B(TOP-M+1)-A(TOP-M+1)*E(TOP-M+2))
10     F(TOP-M+1)=(D(TOP-M+1)-A(TOP-M+1)*F(TOP-M+2))/(B(TOP-M+1)-A(TOP-M+
      *1)*E(TOP-M+2))
       PCAL(I,1)=F(1)
       DO 20 M=2,TOP
   20  PCAL(I,M)=F(M)-E(M)*PCAL(I,M-1)
       RETURN
       END
C
C
C  CALCULATION OF OVERLAND FLOW USING KINEMATIC FLOOD ROUTING
       SUBROUTINE OLAND
       REAL INFIL,MAN
       INTEGER P,FIN,S
        DIMENSION  Q1(2)
       COMMON/SEC4/M,N2,KING,J
        COMMON/SEC8/ AREA1(99),AREA2(99),DISCH1(99),DISCH2(99),MAN,SL(30)
      $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
      $H2(30),XKO,RCRIT
        COMMON/SEC9/ I,X(30),DELT,BLUE
        COMMON/SEC10/INFIL(30),DEPTH(30)
       XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
       H2(1)=0.0
       N1=2
       TOLR=.001
       DELA1=0.1
       Y=0.0
        U=.01
       DO 100 M=1,84
         IF(M.EQ.1) GO TO 620
622      Q6=((8*980*SL(1)*Y**3)/(XKO*U))*WIDTH
625        RE=Q6/(WIDTH*U)
         F=XKO/RE
         IF(RE.GT.RCRIT)   F=XKO/RCRIT
         DISCH2(M)=(((8*980*SL(1)*Y**3)/F)**0.5)*WIDTH
          GO TO 623
620     Y=0.0
         DISCH2(1)=0.0
623        IF(M.GT.15) GO TO 600
         Y=Y+0.002
         GO TO 100
```

```
600       Y=Y+0.005
        IF(M.GT.28) Y=Y-0.00375
100       CONTINUE
      Y=0.0
        DO 101 M=1,84
      AREA1(M)=Y*WIDTH
          IF(M.GT.15) GO TO 601
        Y=Y+0.002
        GO TO 101
601       Y=Y+0.005
        IF(M.GT.28) Y=Y-0.00375
101     CONTINUE
      Q1(2)=0.0
      Q1(1)=0.0
      A1(2)=0.0
      A1(1)=0.0
      Q(1)=0.0
       IF(TIMTOT) 401,400,401
400   Q2(1)=0.0
C     ADJUSTMENT FOR REACH
401   DO 300 P=2,FIN
      Q2(1)=Q(P)
      NUT=S(P)
       NUTS=S(P-1)+1
701    TQLAT=0.0
        DO 404 I=NUTS,NUT
      TQLAT=TQLAT+QL(I)
404   CONTINUE
      DELX=0.0
      DO 500 I=NUTS,NUT
      DELX=DELX+X(I)
500     CONTINUE
       QLAT=TQLAT/( S(P)-S(P-1))
      Y=0.0
      DO 104 M=1,84
        IF(M.EQ.1) GO TO 630
632     Q6=((8*980*SL(P)*Y**3)/(XKO*U))*WIDTH
635       RE=Q6/(WIDTH*U)
        F=XKO/RE
        IF(RE.GT.RCRIT)   F=XKO/RCRIT
        DISCH2(M)=(((8*980*SL(P)*Y**3)/F)**0.5)*WIDTH
         GO TO 633
630    Y=0.0
        DISCH2(1)=0.0
633       IF(M.GT.15) GO TO 602
        Y=Y+0.002
        GO TO 104
602      Y=Y+0.005
        IF(M.GT.28) Y=Y-0.00375
104     CONTINUE
      Y=0.0
      DO 105 M=1,84
      AREA2(M)=Y*WIDTH
          IF(M.GT.15) GO TO 610
        Y=Y+0.002
         GO TO 105
610      Y=Y+0.005
        IF(M.GT.28) Y=Y-0.00375
105      CONTINUE
      Y=0.0
      M=1
      N2=84
        CALL TBLLP(84, 2,DISCH2,Q2)
      A2(M)=XINT(AREA2(KING),AREA2(J),DISCH2(KING),DISCH2(J),Q2(M) )
      J=1
      N1=2
C     ROUTING DURING INFLOW
      ALPHA=(A1(M)+A2(M))/2
      M=J+1
      BETA=(DELT /DELX)*Q1(M)+(-A1(M)/2+(DELT *QLAT*WIDTH))
```

```
         X1=ALPHA+BETA
        IF(X1) 106,106,102
106     Q2(M)=0.0
         A2(M)=0.0
        GO TO 108
102      CALL SOLVE
         IF(Q2(M)-0.01) 510,510,108
510     Q2(2)=0.0
        A2(2)=0.0
108     AR=A1(2)
        ARE=A2(2)
        IF(AR.LE.0.0) GO TO 201
        H1=A1(2)/WIDTH
        GO TO 203
201     H1=0.0
203     IF(ARE.LE.0.0) GO TO 202
        H2(P)=A2(2)/WIDTH
         GO TO 204
202     H2(P)=0.0
204     IN=P-1
         XTOT=0.0
        DO 603 I=NUTS,NUT
        IF(H1-H2(P)) 206,.220,206
206     IF(I.EQ.NUTS) GO TO 604
        XTOT=XTOT+(X(I-1)+X(I))/2
        GO TO 605
604      XTOT=XTOT+X(I)/2
        GO TO 605
605     DEPTH(I)=(SL(P)*XTOT+H1)-(XTOT*((SL(P)*DELX+H1)-H2(P))/DELX)
        GO TO 603
220     DEPTH(I)=0.0
603      CONTINUE
        Q(P)=Q2(2)
         DO 115 M=1,2
        A1(M)=A2(M)
115     Q1(M)=Q2(M)
C        INTERCHANGE RATING TABLES
        DO 116 M=1,50
        AREA1(M)=AREA2(M)
116     DISCH1(M)=DISCH2(M)
300     CONTINUE
        RETURN
        END
C
C
C       SOLVE ROUTINE FOR OVERLAND FLOW
        SUBROUTINE SOLVE
        REAL MAN,INFIL
        INTEGER P,FIN,S
        COMMON/SEC4/M,N2,KING,J
         COMMON/SEC8/ AREA1(99),AREA2(99),DISCH1(99),DISCH2(99),MAN,SL(30)
       $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
       $H2(30),XKO,RCRIT
         COMMON/SEC9/ I,X(30),DELT,BLUE
         COMMON/SEC10/INFIL(30),DEPTH(30)
        XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
        AU=0.0
        FAU=0.0
        AL=0.0
        FAL=0.0
        DELA=DELA1
800     A2(M)=A1(M)
11      CALL TBLLP(84, 2,AREA2,A2)
         Q2(M)=XINT(DISCH2(KING),DISCH2(J),AREA2(KING),AREA2(J),A2(M))
        X2=(DELT/DELX)*Q2(M)+(A2(M))/2
        X2=X2-X1
        IF(X2)3,5,2
2       DELA=DELA1
        AU=A2(M)
        FAU=X2
```

```
      IF(AL )4,12,4
12    A2(M)=A2(M)-DELA
13    IF(A2(M))7,7,11
7     DELA=DELA*0.5
       A2(M)=A2(M)+DELA
      GO TO 13
3     DELA=DELA1
      AL=A2(M)
      FAL=-X2
      IF(AU)4,14,4
14    A2(M)=A2(M)+DELA
15    IF(A2(M)-AREA2(N2))11,11,16
16    DELA=DELA*0.5
       A2(M)=A2(M)-DELA
       GO TO 15
4     ANEW=AU-(FAU/(FAU+FAL))*(AU-AL)
      X3=ANEW-A2(M)
      X3=ABS(X3)
      TOLR=1
       IF(X3-TOLR)5,5,10
10    A2(M)=ANEW
      GO TO 11
  5   A2(M)=ANEW
      CALL TBLLP(84, 2,AREA2,A2)
      Q2(M)=XINT(DISCH2(KING),DISCH2(J),AREA2(KING),AREA2(J),A2(M))
      RETURN
      END
```

D.3 Physical Computer Model Program using the Strongly Implicit Numerical Method

```
C       COMPUTER PROGRAM 2.
C       A PHYSICALLY-BASED,DIGITALLY-SIMULATED HYDROLOGIC MODEL
C
C       NUMERICAL SOLUTION USED THE STRONGLY IMPLICIT METHOD
C
        REAL INS,INZ,INPUT
        REAL INFIL,K1,K2,INB,INL,INR,MAN
        INTEGER P,FIN,T2,S,BOUND,TOP,TEST,TOP1,FINI
        DIMENSION  R(50),T(50),PERM(10),THETS(30,20),HYDH(20)
          DIMENSION ZT(20),SEEPV(30)
           DIMENSION THET1(30,20)
        COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
       $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
       $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
       $PH(10),RHO(5),DELT1,THETA(30,20)
         COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
         COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
        COMMON/SEC4/M,N2,KING,J
        COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
         COMMON/SEC6/INR(20)
         COMMON/SEC7/PCAL(30,20),LOVE
         COMMON/SEC8/ AREA1(50),AREA2(50),DISCH1(50),DISCH2(50),MAN,SL(30)
       $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
       $H2(30)
         COMMON/SEC9/ I,X(30),DELT,BLUE
         COMMON/SEC10/INFIL(30),DEPTH(30)
         COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
          COMMON/SFC12/INS(20)
         COMMON/COEF/ALPHA(20),KNT,ISET,IEVEN,VECTO (30,20),EIC(30,20),
       $FIC(30,20),DELTA(30,20),RESID(30,20)
        XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
           ISFT=0
          KONT=0.0
          ALPH=0.5
        READ(1,57)DELT,N,L,NK1,NSM1,NR,G,RHOO,BET,U,AMODEL,TEND,DELIM
57      FORMAT(2X,F3.0,1X,I3,1X,I3,1X,I3,1X,I3,1X,I3,1X,F4.0,1X,F3.0,1X,
       $E8.1,1X,F4.3,1X,F3.1,1X,F8.0,F5.0)
        WRITE(3,57) DELT,N,L,NK1,NSM1,NR,G,RHOO,BET,U,AMODEL,TEND,DELIM
        READ(1,120) (K1(LA),LA=1,NK1)
120      FORMAT(10E8.3)
        READ(1,121) (PRES1(LA),LA=1,NK1)
121      FORMAT(11F7.2)
        READ(1,50) (THETA1(LA),LA=1,NSM1)
50        FORMAT(13F6.4)
         READ(1,51) (PS1(LA),LA=1,NSM1)
51       FORMAT(11F7.2)
        READ(1,52) (R(LA),LA=1,NR)
52        FORMAT(6F8.5)
         READ(1,53) (T(LA),LA=1,NR)
53       FORMAT(6F7.3)
        DO 81 I=1,N
        DO 82 K=1,L
          THETS(I,K)=0.0
         PERM5(I,K)=0.0
          THETA(I,K)=0.0
82      FORM(I,K)=1
81      CONTINUE
        DO 83 I=1,N
         READ(1,54) (PHI(I,K),K=1,L)
54       FORMAT(13F6.1)
83      CONTINUE
        DO 200 I=1,N
        READ(1,55) (BOUND(I,K),K=1,L)
55        FORMAT(17I2)
200     CONTINUE
        READ(1,56) (Z(K),K=1,L)
        READ(1,31) (X(I),I=1,N)
56       FORMAT(26F3.0)
31        FORMAT(26F3.0)
        READ(1,58) (PORS(LA),LA=1,2)
```

```
58      FORMAT(2F4.2)
        READ(1,70) (VT(LA),LA=1,2)
70      FORMAT(2E8.1)
        READ(1,32) (INB(I),I=1,N)
32       FORMAT(26F3.0)
59       FORMAT(26F3.0)
        READ(1,59) (INL(K),K=1,L)
        READ(1,59) (INR(K),K=1,L)
         READ(1,59) (INS(K),K=1,L)
        READ(1,61)  MAN,WIDTH,FIN,YZ
61      FORMAT(F4.2,3X,F7.3,3X,I3,3X,F5.2)
         READ(1,60) (SL(P),P=1,FIN)
60       FORMAT(11F7.5)
        READ(1,62) (S(P),P=1,FIN)
62       FORMAT(11I2)
        READ(1,321)  (ALPHA(K),K=1,18)
321     FORMAT(18F3.2)
        WRITE(3,227) (Z(K),K=1,L)
227     FORMAT('0','Z=',2X,20F5.0)
        WRITE(3,225) (X(I),I=1,N)
225     FORMAT('0','X=',2X,20F5.0)
         WRITE(3,676)
676       FORMAT(' ',30X,'SOIL CHARACTERISTICS
          WRITE(3,695)
695       FORMAT('0',7X,'FORMATION  1',39X,'FORMATION2
         WRITE(3,674)
 674    FORMAT('0','PRESSURE(CM.)',10X,'PERMEABILITY(CM.2)',20X,'PRESSURE
       $CM.)',10X,'PERMEABILITY(CM.2)')
         DO 671  LA=1,NK1
          WRITE(3,670) PRES1(LA),K1(LA)
670      FORMAT(' ',F8.3,10X,E8.3)
671      CONTINUE
         WRITE(3,675)
675       FORMAT('0','PRESSURE',10X,'SOIL MOISTURE',20X,'PRESSURE',10X,
       $'SOIL MOISTURE')
          DO 672 LA=1,NSM1
          WRITE(3,673)  PS1(LA),THETA1(LA)
673       FORMAT(' ',F8.3,10X,F7.6)
672       CONTINUE
         WRITE(3,677) AMODEL
677        FORMAT('1',30X,'OUTPUT',/' ',20X,'HYDROLOGIC MODEL',3X,F3.1)
C       INITALIZATION
           RBF=0.0
           PRTO=0.0
           MORE=1
        IR=1
        NU=1
            STORV=0.0
         VOLGW=0.0
           VOL=0.0
            XLTH=0.0
           MEND=S(FIN)
          DO 368 I=1,MEND
368       XLTH=XLTH+X(I)
                KGW=20
         XNO=2
         YES=3
         ALPH=0.5
        DO 111  I=1,5
        PH(I)=0.0
        RHO(I)=1.00
111      CONTINUE
        DO 863 I=1,N
         INFIL(I)=0.0
        RED(I)=-1
        DEPTH(I)=0.0
        DEPTH1(I)=0.0
           REPEAT(I)=0.0
863      CONTINUE
         IK=1
```

```
        TOL1=1
          TOL=0.001
          TOLH=.02
         NM=1
        DO 437 P=1,FIN
        Q(P)=0.0
437     CONTINUE
      IA=0.0
      LOVE=1
      R2=0.0
      TIMTOT=0.0
      DELT1=DELT
C
C       DETERMINE  ELEVATION HEAD
C
      DO 282  K=1,L
      IF(K.EQ.1) GO TO 283
        ZT(K)=ZT(K-1)+(Z(K)+Z(K-1))/2
        GO TO 282
283     ZT(K)=Z(K)/2
282       CONTINUE
8     T1=DELT
C
381       DO 47 I=1,N
      DO 47 K=1,L
           FORM(I,K)=1
      DELTA(I,K)=0.0
      RESID(I,K)=0.0
      VECTO(I,K)=0.0
      EIC(I,K)=0.0
      FIC(I,K)=0.0
47      CONTINUE
        IEVEN=-1
      KNT=1
          IT=1
      NM=0
        IF(TIMTOT.GT.0) GO TO 119
      DELT1=DELT
119   T2=NR
C
C     DETERIMINE  RAINFALL INTENSITY
C
 410      TIME=TIMTOT+DELT
        IF(T(NR)*3600-TIME) 411,409,409
411     WRITE(3,471)
471       FORMAT(' ','HELP THE TIME IS NEG')
        GO TO 273
409   IF(TIME) 18,19,18
 19   P=2
      GO TO 20
18    DO 21 P=1,T2
        IF(T(P)*3600-TIME)21,20,20
 21   CONTINUE
 20   KING=P-1
      R1=R2
          THOUR=TIME/3600
      R2=R(KING)+(THOUR -T(KING))/(T(P)-T(KING))*(R(P)-R(KING))
22    RF=(R2-R1)/DELT
           NTIME=P
24      WRITE(3,267) DELT,RF,R2,TIME
267     FORMAT(' ',/,' ','DELT=',E14.5,3X,'RF=',F6.4,3X,'R2=',F6.4,'TIME
     $=',F14.5)
      DO  90 I=1,N
      DO 90 K=1,L
           PRED(I,K)=PHI(I,K)
      PHIS(I,K)=PHI(I,K)
      PCAL(I,K)=PHI(I,K)
90      PHI1(I,K)=PHI(I,K)
C
C    PREDICT PRESSURES
```

```
C
230    IF(TIMTOT.GT.0.0.AND.IT.GE.2) GO TO 287
       DO 723 I=1,N
       DO 723 K=1,L
       IF(IT.GT.1.AND.TIMTOT.EQ.0.0) GO TO 725
      IF(TIMTOT) 720,720,722
720     PRED(I,K)=PHIS(I,K)
       GO TO 723
722      II=BOUND(I,K)+1
       GO TO (130,131,132,133,134,135,136,723,723,723,140,141),II
        GO TO 723
130    PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
       PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
       PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
       PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
        DO 310 MN=1,4
         M=MN
      CALL  TBLLP(16,16,PRES1,PH)
310     PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
     $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1))))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
        IF(THETA(I,K).GT.0.3) GO TO 540
       GO TO 143
131    PH(1)=0.0
       PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
       PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
       PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
      DO 304  MN=1,4
         M=MN
      CALL  TBLLP(16,16,PRES1,PH)
304     PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
     $            -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1))))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
       GO TO 143
132    PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
       PH(2)=0.0
       PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
       PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
       DO 305 MN=1,4
         M=MN
      CALL  TBLLP(16,16,PRES1,PH)
305     PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
      THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
     $*PERM(1)-INL(K)*RHO(2))                                            +
     $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1))))*(PHI(I,K+1)-PHI(I,K)))
     *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
     $)*DELT+THETS(I,K)
        IF(THETA(I,K).GT.0.3) GO TO 540
       GO TO 143
133    PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
       PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
       PH(3)=0.0
       PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
       IF(RED(I).EQ.+1) GO TO 723
      DO 306  MN=1,4
         M=MN
      CALL  TBLLP(16,16,PRES1,PH)
306     PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
     $PH(M))
        INPUT=RF+DEPTH(I)/DELT
      THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
```

```
      $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      *1/Z(K)*(RHO(3)*INPUT
      *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
      $)*DELT+THETS(I,K)
          IF(THETA(I,K).GT.0.3) GO TO 540
         GO TO 143
134      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
         PH(4)=0.0
        DO 307  MN=1,4
           M=MN
        CALL  TBLLP(16,16,PRES1,PH)
307       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
          IF(THETA(I,K).GT.0.3) GO TO 540
         GO TO 143
135      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        PH(2)=0.0
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
         PH(4)=0.0
         DO 308  MN=1,4
           M=MN
        CALL  TBLLP(16,16,PRES1,PH)
308       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-INL(K)*RHO(2))                                             +
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
          IF(THETA(I,K).GT.0.3) GO TO 540
         GO TO 143
136     PH(1)=0.0
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=(PCAL(I,K)+PCAL(I,K+1))/2
        PH(4)=0.0
         DO 309  MN=1,4
           M=MN
        CALL  TBLLP(16,16,PRES1,PH)
309       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
        THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
      $             -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
      $1/Z(K)*(RHO(3)*PERM(3)*(1+(2/(Z(K)+Z(K+1)))*(PHI(I,K+1)-PHI(I,K)))
      *-RHO(4)*INB(I))
      $)*DELT+THETS(I,K)
          IF(THETA(I,K).GT.0.3) GO TO 540
         GO TO 143
140      PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
         PH(2)=0.0
          PH(3)=0.0
         PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
         IF(RFD(I).EQ.+1) GO TO 723
         DO 311 MN=1,4
           M=MN
        CALL  TBLLP(16,16,PRES1,PH)
311       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
      $PH(M))
          INPUT=RF+DEPTH(I)/DELT
        THETA(I,K)=(1/X(I)*((2/(X(I)+X(I+1)))*(PHI(I+1,K)-PHI(I,K))*RHO(1)
      $*PERM(1)-INL(K)*RHO(2))                                             +
      *1/Z(K)*(RHO(3)*INPUT
      *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
      $)*DELT+THETS(I,K)
```

```
          IF(THETA(I,K).GT.0.3) GO TO 540
         GO TO 143
141     PH(1)=0.0
         PH(2)=(PCAL(I,K)+PCAL(I-1,K))/2
         PH(3)=0.0
         PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
         IF(RED(I).EQ.+1) GO TO 723
          DO 312  MN=1,4
           M=MN
        CALL  TBLLP(16,16,PRES1,PH)
312       PERM(MN)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),
       $PH(M))
          INPUT=RF+DEPTH(I)/DELT
        THETA(I,K)=(1/X(I)*(INR(K)*RHO(1)
       $             -PERM(2)*RHO(2)*( 2/(X(I)+X(I-1)))*(PHI(I-1,K)-PHI(I,K)))+
       *1/Z(K)*(RHO(3)*INPUT
       *-RHO(4)*PERM(4)*(1+(2/(Z(K)+Z(K-1)))*(PHI(I,K)-PHI(I,K-1))))
       $)*DELT+THETS(I,K)
          IF(THETA(I,K).GT.0.3) GO TO 540
143      PH(5)=THETA(I,K)
         M=5
         CALL  TBLLP(16,16,THETA1,PH)
        PERM5(I,K)=XINT(K1(KING),K1(J),THETA1(KING),THETA1(J),PH(M))
          GO TO 723
725      PRED(I,K)=PRED(I,K)+ALPH*(PHIS(I,K)-PRED(I,K))
723        CONTINUE
C
C        DETERIME COEFFICIENTS AT NODE POINTS
C
         IF(TIMTOT.EQ.0.0) GO TO 287
          DO 606 I=1,N
        K=TOP1(I)
         IF(BOUND(I,K).EQ.9) GO TO 606
         IF(DEPTH(I).GT.0.0.AND.QL(I).NE.0.0) GO TO 741
         IF(PHI1(I,K) ) 206,741,741
741      IF((RF+DEPTH(I)/DELT).GT.INFIL(I)) GO TO 220
206       RED(I)=-1
        IF(QL(I).EQ.0.0) GO TO 552
           INFIL(I)=RF+DEPTH(I)/DELT
          GO TO 606
552        INFIL(I)=RF
          GO TO 606
220        RED(I)=+1
           GO TO 606
606         CONTINUE
287      DO 3 I=1,N
         DO 3 K=1,L
         IA=BOUND(I,K)+1
         GO TO (10,11,13,14,84,85,87,3,3,88,14,88,350),IA
         GO TO 303
303     A(I,K)=0.0
        B(I,K)=0.0
         C(I,K)=0.0
         D(I,K)=0.0
         E(I,K)=0.0
         F(I,K)=0.0
         GO TO 3
10      CALL INTER
        GO TO 3
C       CALL RIGHT BOUNDARY
11      CALL LLRB
        GO TO 3
C       CALL LEFT BOUNDARY
13      CALL  LLRB
        GO TO 3
C       CALL  LOWER  BOUNDARY
84      CALL LLRB
        GO TO 3
C       CALL LEFT CORNER BOUNDARY
85      CALL LRCB
```

```
      GOTO3
C     CALL RIGHT CORNER BOUNDARY
87    CALL LRCB
      GO TO 3
88    CALL SEEP
      TOP1(I)=K
       GO TO 3
350     BOUND(I,K)=1
        CALL  LLRB
       BOUND(I,K)=12
        GO TO 3
14    CALL UPPER
      TOP1(I)=K
3     CONTINUE
      IF(TIMTOT.GT.0) GO TO 115
      IF(IT.GT.1) GO TO 115
      DO 116 K=1,L
116   PHI2(I,K)=PHI(I,K)
115    CALL               SIP2D(N,L,TOL)
        ACCEL=1.0
       IF(TIMTOT.EQ.0.0) GO TO 268
        IF(IT.GT.1) ACCEL=1.33
268   DO 271 I=1,N
      DO 271 K=1,L
258   PCAL(I,K)=ACCEL*PCAL(I,K)+(1-ACCEL)*PHIS(I,K)
        PHIS(I,K)=PCAL(I,K)
271   CONTINUE
264    IT=IT+1
      IF(IT.GT.30) GO TO 540
      KNT=KNT+1
      IF(KNT.GT.18) KNT=1
       IF(ISET.EQ.1) GO TO 230
      IT=IT-1
         GO TO 635
540     DELT=DELT/4
        GO TO 607
635   WRITE(3,633)  IT
633   FORMAT(' ','IT=',I5)
      ISET=0
C
C   TEST FOR UPPER BOUNDARY CHANGE
C
        DO 777 I=1,N
        K=TOP1(I)
       IF(BOUND(I,K).EQ.9) GO TO 777
       IF(PCAL(I,K)) 804,812,812
804     IF(PHI1(I,K))777,777,777
812       IF(PHI1(I,K))814,777,777
814    IF(PCAL(I,K).LE.TOL1) GO TO 777
811     IF(KONT.EQ.1) GO TO 600
      DEL10=ABS(PHI1(I,K)-PCAL(I,K))
      DEL11=ABS(PHI1(I,K))
      DELT=(DEL11*DELT)/DEL10
       KONT=1
        GO TO 607
600   DELT=0.8*DELT
        GO TO 607
777      CONTINUE
          KONT=0
          GO TO 681
607      R2=R1
        WRITE(3,412) DELT
412     FORMAT(' ','NEW DELT',E14.5)
       NU=1
        MORE=1
        GO TO 8
C
C      TEST FOR SOIL MOISTURE(THETA)  TOL
C
681        RES=0.0
```

```
      DO 793  I=1,N
       DO 791  K=1,L
      IF((BOUND(I,K).EQ.78).OR.(BOUND(I,K).EQ.9)) GO TO 791
      N2=NSM1
      M=5
       PH(5)=PCAL(I,K)
      CALL  TBLLP(16,16,PS1,PH)
       THET1(I,K)=THETS(I,K)
     THETS(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
      IF(TIMTOT.EQ.0.0) GO TO 791
      DIFF=ABS(THETS(I,K)-THETA(I,K))
      IF(DIFF.LE.RES) GO TO 791
      RES=DIFF
791    CONTINUE
793   CONTINUE
       IF(RES.LE.TOLH) GO TO 1002
      DO 1000 I=1,N
       DO 1000 K=1,L
        THETS(I,K)=THET1(I,K)
1000    CONTINUE
       WRITE(3,1005)
1005  FORMAT(2X,'FAILED AT SOIL MOISTURE')
      GO TO 540
C
C    CALCULATE OVERLAND FLOW AND DEPTHS
C
1002  DO  34 I=1,N
     K=TOP1(I)
     IF(BOUND(I,K).EQ.9) GO TO 858
     GO TO 859
858   INFIL(I)=RF
859   QL(I)=RF-INFIL(I)
34    CONTINUE
     CALL OLAND
C
C   CONTINUITY CHECK
C
       STOR5=0.0
       DO 374 I=1,N
        DO 373 K=1,L
      IF((BOUND(I,K).EQ.78).OR.(BOUND(I,K).EQ.9)) GO TO 373
      DSTOR5=THETS(I,K)*X(I)*WIDTH
      STOR5=DSTOR5+STOR5
373     CONTINUE
374      CONTINUE
      STOR5=STOR5/(XLTH*WIDTH)
       IF(TIMTOT.EQ.0.0) STOR4=STOR5
       DSTOR=STOR5-STOR4
C
C   CALCULATE INFILTRATION
C
403   DO 77 I=1,N
      K=TOP1(I)
       SEEPV(I)=0.0
     IF(BOUND(I,K).EQ.9) GO TO 77
        IF(PCAL(I,K)) 700,701,701
700    IF(QL(I).EQ.0.0) GO TO 551
        INFIL(I)=RF+DEPTH(I)/DELT
       GO TO 77
551    INFIL(I)=RF
     GO TO 77
701    N2=NK1
       M=4
      PH(4)=(PCAL(I,K)+PCAL(I,K-1))/2
      CALL   TBLLP(16,16,PRES1,PH)
      PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       M=9
      PH(9)=(PHI(I,K)+PHI(I,K-1))/2
      CALL   TBLLP(16,16,PRES1,PH)
      PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
```

```
         PER4(I)=(PER1+PER2)/2
          PER4(I)=PER1
        INFIL(I)=(1-((PCAL(I,K-1)-DEPTH(I))*2/(Z(K)+Z(K-1))))
      $*(PER4(I)*RHO(4)*G/U)
         IF(INFIL(I).GE.0.0) GO TO 77
           SEEPV(I)=INFIL(I)
         INFIL(I)=0.0
77        CONTINUE
       WRITE(3,93) TIMTOT,DELT
93     FORMAT(' ',5X,'TOTALTIME=',F12.3,5X,'TIME INCREMENT=',F12.3,//)
C
C   DETERMINE BASE FLOW
C
C   HORIZONTAL CONTRIBUTION
       GWV=0.0
         TOTGW=0.0
        HH=0.0
        I=KGW
        DO 363 K=1,L
         N2=NK1
        M=1
        PH(1)=(PCAL(I,K)+PCAL(I+1,K))/2
        CALL  TBLLP(16,16,PRES1,PH)
       PERM(1)=(RHO(1)*G/U)*XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M
      $))
       XL=X(I)+X(I+1)/2
        GWH=(PERM(1)*(PCAL(I,K)+ZT(K)-PCAL(I+1,K)-ZT(K))*Z(K)*WIDTH)/XL
        IF(K.EQ.L)  GWH=GWH/2
         IF(GWH.LE.0) GWH=0.0
363     TOTGW=TOTGW+GWH
C  VERTICAL CONTRIBUTION
         MZ=KGW+1
          FINI=S(FIN)
         DO 364 I=MZ,FINI
364       GWV=SEEPV(I)+GWV
         STORGW=RBF
         RBF=GWV+TOTGW
C
C   DETERMINE  RUNOFF VOLUME
C
        DVOL=((STORV+Q(FIN))/2)*DELT/(WIDTH*XLTH)
        DVOLGW=((STORGW+RBF)/2)*DELT/(WIDTH*XLTH)
         VOL=VOL+DVOL
        VOLGW=VOLGW+DVOLGW
         STORV=Q(FIN)
        RAIN=VOL+VOLGW+DSTOR
          ACTR=R2
         WRITE(3,376) RAIN,ACTR,DSTOR,STOR4
376      FORMAT(' ','CONTINUITY CHECK=',E11.3,'CM',3X,'ACTUAL RAIN=',E9.3
      $,2X,'G.W.STOR.=',E11.3,3X,E11.3)
C
        WRITE(3,365) TOTGW,GWV,RBF
365      FORMAT(' ','TOTAL HORIZ.BASEFLOW',E14.5,3X,'TOTAL VERT.BASEFLOW'
      $,E14.5,3X,'RESULTANT BASEFLOW=',E14.5)
         WRITE(3,362) VOL,DVOL,XLTH,VOLGW,DVOLGW,Q(FIN)
362      FORMAT('0',1X,'SURFACE RUNOFF=',F7.4,'CM',3X,'R.O.INCREMENT=',
      $F6.4,'CM',2X,'LENGTH=',F8.3,'CM'/' ','G.W.VOL=',F6.4,'CM',3X,'G.W.
      $INCREMENT=',F6.4,3X,'Q(FIN)=',E14.5,'CM3'//)
C
C     PRINT  OUTPUT
C
         IF(PRTO-TIMTOT) 705,705,862
705     PRTO=TIMTOT+DELIM
          DO 707 I=1,N
76       WRITE(3,408)I,QL(I),INFIL(I),SEEPV(I)
408      FORMAT(' ','QL(',I2,')=',E14.5,3X,'INFIL=',E14.5,3X,'SEEPV=',E1
      $4.5)
707      CONTINUE
       DO 291 I=1,N
        WRITE(3,72) I,(PCAL(I,K),K=1,L)
```

```
72      FORMAT(' ','I=',I3,(3X,10F10.3))
291     CONTINUE
       DO 504 P=1,FIN
       WRITE(3,505) P,Q(P),H2(P)
505     FORMAT(' ','Q(',I2,')=',E14.5,3X,'H2=',E14.5)
504    CONTINUE
        WRITE(3,870) (DEPTH(I),I=1,N)
870    FORMAT(' ','DEPTH',3X,6F12.6)
C
C     DETERIMNE  HYDRAULIC HEAD
C
        DO 261 I=1,N
         DO 262 K=1,L
        IF(BOUND(I,K).EQ.78) GO TO 265
        HYDH(K)=ZT(K)+PCAL(I,K)
         GO TO 262
265      HYDH(K)=0.0
262      CONTINUE
           WRITE(3,72) I,(HYDH(K),K=1,L)
261      CONTINUE
         DO 768 I=1,N
        WRITE(3,769) I, (THETS(I,K),K=1,L)
769     FORMAT(' ','I=',I3,(3X,10E9.4))
768       CONTINUE
C
862       DO 99 I=1,N
       DO 95 K=1,L
98     PHI(I,K)=PCAL(I,K)
       PHI2(I,K)=PHI1(I,K)
95      PHI1(I,K)=PHI(I,K)
99        CONTINUE
        DO 888 I=1,N
       DEPTH1(I)=DEPTH(I)
C       101=NO
       REPEAT(I)=101
888      CONTINUE
       KONT=0.0
       IK=1
512     TIMTOT=TIMTOT+DELT
6      DELT1=DELT
751     BLUE=1
          MORE=MORE+1
         IF(DELT.LT.2.0) GO TO 817
         IF(MORE.LE.4) GO TO 781
817     IF(NU.GE.2) GO TO 735
        NU=1
735    NU=NU+1
            MORE=1
        M=NU-1
       DELT=DELT*2**M
        IF(DELT.LT.1500) GO TO 781
        NU=1
        DELT=1000
781      IR=1
       GO TO 8
 273    STOP
       END
C
C
C UPPER NODE SUBROUTINE
       SUBROUTINE  UPPER
       REAL INFIL,K1,K2,INB,INL,INPUT
        REAL  INR
       INTEGER BOUND,P,T2,TOP,TEST,TOP1
        DIMENSION PERM(10)
       COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
        COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
```

```
      COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
      COMMON/SEC6/ INR(20)
      COMMON/SEC7/PCAL(30,20),LOVE
      COMMON/SEC8/ AREA1(50),AREA2(50),DISCH1(50),DISCH2(50),MAN,SL(30)
     $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
     $H2(30)
      COMMON/SEC9/ I,X(30),DELT,BLUE
      COMMON/SEC10/INFIL(30),DEPTH(30)
      COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
      COMMON/COEF/ALPHA(20),KNT,ISET,IEVEN,VECTO (30,20),EIC(30,20),
     $FIC(30,20),DELTA(30,20),RESID(30,20)
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      RHO(4)=1.00
      ALPH=0.5
      TOL=1.0
      T3=DELT1
      IF(TIMTOT.GT.0.0) GO TO 607
      DEPTH(I)=PHI(I,K)
      N2=NSM1
      M=5
      PH(5)=PHI(I,K)
      CALL  TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
      GO TO 220
607   IF(RED(I).EQ.+1) GO TO 220
      IF(TIMTOT.GT.0.0) GO TO 103
206   PH(4)=(PRED(I,K)+PRED(I,K-1))/2
      PH(9)=(PHI(I,K)+PHI(I,K-1))/2
      IF(BOUND(I,K).EQ.10) GO TO 303
      PH(2)=(PRED(I,K)+PRED(I-1,K))/2
      PH(7)=(PHI(I,K)+PHI(I-1,K))/2
      GO TO 304
303   PH(2)=0.0
      PH(7)=0.0
304   PH(3)=(PRED(I,K)+DEPTH(I))/2
      PH(8)=( PHI(I,K)+DEPTH1(I))/2
      PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
      PH(5)=PRED(I,K)
      N2=NK1
      DO 212 MN=1,4
      M=MN
219   CALL  TBLLP(16,16,PRES1,PH)
      PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      M=MN+5
      CALL  TBLLP(16,16,PRES1,PH)
      PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      PERM(MN)=(PER2+PER1)/2
212   CONTINUE
      PER4(I)=PER1
      GO TO 104
103   PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
      IF(BOUND(I,K).EQ.10) GO TO 106
      PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
106   PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
104   BETA=BET*RHOO*G
      DELPHI=PRED(I,K)-PHIS(I,K)
      IF(FORM(I,K).EQ.1)GO TO 213
213   POR=PORS(1)
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0) GO TO 105
      CALL  TBLLP(16,16,PS1,PH)
      PH(5)=(PHI(I,K)+PRED(I,K))/2
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
730   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
      GO TO 214
105   PH(5)=THETA(I,K)
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```
        CALL TBLLP(16,16,THETA1,PH)
       PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
          GO TO 730
 214   VTCOM=0.0
       UO=CE
       P1=G/(U*X(I))
       QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
         IF(BOUND(I,K).EQ.10) GO TO 534
       QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
          GO TO 533
534      QE2=0.0
533    A(I,K)=(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
20     C(I,K)=-P1*QE1-P1*QE2-A(I,K)-(UO/DELT)
        B(I,K)=P1*QE2
        D(I,K)=P1*QE1
       E(I,K)=0.0
        IF(QL(I).EQ.0.0)  GO TO 550
         INPUT=RF+DEPTH(I)/DELT
         GO TO 551
550     INPUT=RF
551      IF(BOUND(I,K).EQ.10)  GO TO 433
       Y=P1*QE1*(              PHI(I+1,K)-PHI(I,K))+
      $(P1*QE2)*(-PHI(I,K)              +PHI(I-1,K))
        GO TO 404
433    Y=P1*QE1*(              PHI(I+1,K)-PHI(I,K))-((RHO(5)*INL(K))/X(I)
404    ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))-A(I,K)*
      $PHI(I,K)+A(I,K)*PHI(I,K-1)
        F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
       IF(TIMTOT.GT.0.0)  GO TO 600
        A(I,K)=2*A(I,K)
          B(I,K)=2*B(I,K)
         C(I,K)=-2*P1*QE1-2*P1*QE2-A(I,K)
         D(I,K)=2*D(I,K)
       ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))
          F(I,K)=-ZA
600      TOP=K
       GO TO216
C
C     SATURATED  UPPER   BOUNDARY
C
220     A(I,K)=0.0
         B(I,K)=0.0
         C(I,K)=0.0
        D(I,K)=0.0
        E(I,K)=0.0
         F(I,K)=0.0
        VECTO(I,K)=0.0
         K=K-1
         PRED(I,K+1)=DEPTH(I)
         PCAL(I,K+1)=PRED(I,K+1)
        LOVE=4
         TEST=BOUND(I,K)-1
         IF(TEST) 10,11,11
10       CALL INTER
         GO TO 502
11       CALL  LLRB
502     K=K+1
         LOVE=1
        IF(TIMTOT.EQ.0.0)  DEPTH(I)=0.0
216    RETURN
       END
C
C
C SEEPAGE NODE SUBROUTINE
       SUBROUTINE SEEP
        REAL INS
       REAL INFIL,K1,K2,INB,INL,INPUT
        REAL  INR
       INTEGER BOUND,P,T2,TOP,TEST,TOP1
        DIMENSION PERM(10)
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```
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHO0,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
      COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
      COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
      COMMON/SEC6/ INR(20)
      COMMON/SEC7/PCAL(30,20),LOVE
      COMMON/SEC8/ AREA1(50),AREA2(50),DISCH1(50),DISCH2(50),MAN,SL(30)
     $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
     $H2(30)
      COMMON/SEC9/ I,X(30),DELT,BLUE
      COMMON/SEC10/INFIL(30),DEPTH(30)
      COMMON/SEC11/TOP,KONT,TOP1(30),REPEAT(30),RED(30),DEPTH1(30)
      COMMON/SEC12/INS(20)
      COMMON/COEF/ALPHA(20),KNT,ISET,IEVEN,VECTO (30,20),EIC(30,20),
     $FIC(30,20),DELTA(30,20),RESID(30,20)
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      NEW=1
      ALPH=0.5
      TEST=BOUND(I,K)-9.
      IF(TEST) 8,9,8
8     RHO(4)=1.00
      IF(TIMTOT.GT.0.0) GO TO 607
      M=5
      N2=NSM1
      PH(5)=PHI(I,K)
      CALL  TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
      DEPTH(I)=PHI(I,K)
      GO TO 220
607   IF(RED(I).EQ.+1) GO TO 220
      IF(TIMTOT.GT.0.0) GO TO 103
206   PH(4)=(PRED(I,K)+PRED(I,K-1))/2
      PH(9)=(PHI(I,K)+PHI(I,K-1))/2
      PH(2)=(PRED(I,K)+PRED(I-1,K))/2
      PH(7)=(PHI(I,K)+PHI(I-1,K))/2
      PH(3)=0.0
      PH(8)=0.0
      PH(5)=PRED(I,K)
      N2=NK1
      DO 212  MN=2,4
      M=MN
219   CALL  TBLLP(16,16,PRES1,PH)
      PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      M=MN+5
      CALL  TBLLP(16,16,PRES1,PH)
      PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      PERM(MN)=(PER2+PER1)/2
212   CONTINUE
      PER4(I)=PER1
      GO TO 104
103   PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
      PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
104   BETA=BET*RHO0*G
      DELPHI=PRED(I,K)-PHIS(I,K)
      IF(FORM(I,K).EQ.1)GO TO 213
213   POR=PORS(1)
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0) GO TO 105
      PH(5)=(PHI(I,K)+PRED(I,K))/2
      CALL  TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
730   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
      GO TO 214
105   PH(5)=THETA(I,K)
      CALL TBLLP(16,16,THETA1,PH)
```

```
        PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
        GO TO 730
 214   VTCOM=0.0
       UO=CE
        E(I,K)=0.0
       QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
       P1=G/(U*X(I))
       A(I,K)=(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
20     C(I,K)=-A(I,K)-P1*QE2-(UO/DELT)
        B(I,K)=P1*QE2
       D(I,K)=0.0
       Y=P1*QE2*(              PHI(I-1,K)-PHI(I,K))+((RHO(5)*INS(K))/X(I))
        IF(QL(I).EQ.0.0)  GO TO 550
          INPUT=RF+DEPTH(I)/DELT
        GO TO 404
550     INPUT=RF
404    ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))-A(I,K)
     ** PHI(I,K)+A(I,K)*PHI(I,K-1)
        F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
       IF(TIMTOT.GT.0.0)  GO TO 600
        A(I,K)=2*A(I,K)
          B(I,K)=2*B(I,K)
        C(I,K)=-A(I,K)-B(I,K)
       ZA=(RHO(5)*INPUT/Z(K))-((G*RHO(4)**2*PERM(4))/(Z(K)*U))
          F(I,K)=-ZA
       GO TO216
C
C     CONSTANT  HEAD
C
9         A(I,K)=0.0
          B(I,K)=0.0
          C(I,K)=0.0
          D(I,K)=0.0
        E(I,K)=0.0
          F(I,K)=0.0
        VECTO(I,K)=0.0
          K=K-1
        LOVE=4
        PRED(I,K+1)=PHI(I,K+1)
       PCAL(I,K+1)= PHI(I,K+1)
        IF(BOUND(I,K))33,33,31
31      CALL LLRB
          GO TO 34
33        CALL INTER
34        K=K+1
        LOVE=1
       GO TO 216
C
C         SATURATED UPPER BOUNDARY
C
220       A(I,K)=0.0
        B(I,K)=0.0
          C(I,K)=0.0
          D(I,K)=0.0
          E(I,K)=0.0
        F(I,K)=0.0
        VECTO(I,K)=0.0
          K=K-1
          PRED(I,K+1)=DEPTH(I)
          PCAL(I,K+1)=PRED(I,K+1)
          LOVE=4
        TEST=BOUND(I,K)-12
          IF(TEST)  11,16,16
16        BOUND(I,K)=1
          CALL LLRB
           BOUND(I,K)=12
          GO TO 502
11         CALL INTER
502     K=K+1
        LOVE=1
```

```
      IF(TIMTOT.EQ.0.0) DEPTH(I)=0.0
600   TOP=K
216   RETURN
      END
C
C
C   LOWER,LEFT,AND RIGHT NODE SUBROUTINE
      SUBROUTINE  LLRB
      REAL K1,INB,INL,K2,INR
      INTEGER BOUND,T2
       DIMENSION PERM(10)
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
       COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
       COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
       COMMON/SEC6/INR(20)
       COMMON/SEC7/PCAL(30,20),LOVE
       COMMON/SEC9/ I,X(30),DELT,BLUE
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      IF(TIMTOT.GT.0.0) GO TO 103
102   IF(BOUND(I,K).EQ.1) GO TO 222
      PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
      IF(BOUND(I,K).EQ.2) GO TO 203
222   PH(2)=(PRED(I,K)+PRED(I-1,K))/2
        PH(7)=(PHI(I,K)+PHI(I-1,K))/2
203    PH(3)=(PRED(I,K)+PRED(I,K+1))/2
       PH(8)=(PHI(I,K)+PHI(I,K+1))/2
      IF(BOUND(I,K).EQ.4) GO TO 204
      PH(4)=(PRED(I,K)+PRED(I,K-1))/2
       PH(9)=(PHI(I,K)+PHI(I,K-1))/2
204   PH(5)=PRED(I,K)
      N2=NK1
       DO 116 MN=1,4
       M=MN
       CALL   TBLLP(16,16,PRES1,PH)
       PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      M=MN+5
       CALL   TBLLP(16,16,PRES1,PH)
       PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       PERM(MN)=(PER2+PER1)/2
116   CONTINUE
       GO TO 104
103    IF(BOUND(I,K).EQ.1) GO TO 110
       PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
       IF(BOUND(I,K).EQ.2) GO TO 111
110    PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
111    PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
       IF(BOUND(I,K).EQ.4) GO TO 112
112    PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
104   BETA=BET*RHOO*G
      IF(BOUND(I,K).EQ.1) GO TO 223
      IF(BOUND(I,K).EQ.2) GO TO 206
223   RHO(2)=1.00
206   RHO(3)=1.00
      IF(BOUND(I,K).EQ.4) GO TO 207
      RHO(4)=1.00
207   RHO(5)=1.00
      DELPHI=PRED(I,K)-PHIS(I,K)
117    IF(FORM(I,K).EQ.1) GO TO 94
      POR=PORS(2)
      GO TO 127
94    POR=PORS(1)
127   VTCOM=0.0
121   E(I,K)=(G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
      P1=G/(U*X(I))
```

```
      IF(BOUND(I,K).EQ.1) GO TO 224
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
      GO TO 225
224   QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
225   M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0) GO TO 105
      PH(5)=(PHI(I,K)+PRED(I,K))/2
122     CALL  TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
        GO TO 123
105     PH(5)=THETA(I,K)
        CALL TBLLP(16,16,THETA1,PH)
       PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
       GO TO 701
123   UO=CE
      KOON=2-BOUND(I,K)
      IF(KOON) 226,208,227
C
C     BOTTOM  BOUNDARY
C
226   QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
      A(I,K)=0.0
14    C(I,K)=-P1*QE1-P1*QE2-E(I,K)-(UO/DELT)
      B(I,K)=P1*QE2
      D(I,K)=P1*QE1
      Y=P1*QE1*(               PHI(I+1,K)-PHI(I,K))+(P1*QE2*(-PHI(I,K)+
     $              PHI(I-1,K)))
      ZA=(-RHO(5)*INB(I)/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))+E(I,K)*
     $PHI(I,K+1)-E(I,K)*PHI(I,K)
512    F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0) GO TO 209
         B(I,K)=2*B(I,K)
        E(I,K)=2*E(I,K)
        D(I,K)=2*D(I,K)
        C(I,K)=-2*P1*QE1-2*P1*QE2-E(I,K)
      ZA=(-RHO(5)*INB(I)/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))
         F(I,K)=-ZA
      GO TO 209
C
C     LEFT BOUNDARY
C
208   A(I,K)=(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
11     C(I,K)=-P1*QE1-E(I,K)-A(I,K)-UO/DELT
       D(I,K)=P1*QE1
      B(I,K)=0.0
      Y=P1*QE1*(               PHI(I+1,K)-PHI(I,K))-((RHO(5)*INL(K))/X(I))
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))+E(I,K)*PHI(I,K+1) -(E(I,K)*PHI
     =(I,K))-((G*RHO(4)**2*PERM(4))/(U*Z(K)))-A(I,K)*PHI(I,K) +(A(I,K)
     **PHI(I,K-1))
       F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0) GO TO 600
       A(I,K)=2*A(I,K)
        E(I,K)=2*E(I,K)
        D(I,K)=2*D(I,K)
       C(I,K)=-P1*2*QE1-E(I,K)-A(I,K)
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*Z(K)
     $))
         F(I,K)=-ZA
600    IF(LOVE.EQ.1) GO TO 209
       GO TO 405
C
C RIGHT BOUNDARY
C
227   A(I,K)=(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
12     C(I,K)=-P1*QE2-E(I,K)-A(I,K)-UO/DELT
      D(I,K)=0.0
      B(I,K)=P1*QE2
      Y=P1*QE2*(               PHI(I-1,K)-PHI(I,K))+((RHO(5)*INR(K))/X(I))
```

```
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))+E(I,K)*PHI(I,K+1)  -(E(I,K)*PHI
     *(I,K))-((G*RHO(4)**2*PERM(4))/(U*Z(K)))-A(I,K)*PHI(I,K)  +(A(I,K)
     **PHI(I,K-1))
        F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
        IF(TIMTOT.GT.0.0)  GO TO 601
        A(I,K)=2*A(I,K)
        B(I,K)=2*B(I,K)
        E(I,K)=2*E(I,K)
        C(I,K)=-P1*2*QE2-E(I,K)-A(I,K)
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*Z(K)
     $))
        F(I,K)=-ZA
601     IF(LOVE.EQ.1)  GO TO 209
405     F(I,K)=F(I,K)-(E(I,K)*PRED(I,K+1))
        E(I,K)=0.0
        PER3(I)=PERM(3)
        PER4(I)=PERM(4)
209   RETURN
      END
C
C
C INTERIOR NODE SUBROUTINE
      SUBROUTINE   INTER
       REAL K1,K2,INB,INL
      INTEGER BOUND,T2
       DIMENSION PERM(10)
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
       COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
       COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
       COMMON/SEC7/PCAL(30,20),LOVE
       COMMON/SEC9/ I,X(30),DELT,BLUE
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      IF(TIMTOT.GT.0.0) GO TO 103
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
        PH(7)=(PHI(I,K)+PHI(I-1,K))/2
       PH(8)=(PHI(I,K)+PHI(I,K+1))/2
       PH(9)=(PHI(I,K)+PHI(I,K-1))/2
102   PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(2)=(PRED(I,K)+PRED(I-1,K))/2
      PH(3)=(PRED(I,K)+PRED(I,K+1))/2
      PH(4)=(PRED(I,K)+PRED(I,K-1))/2
      PH(5)=PRED(I,K)
      N2=NK1
      DO 116    MN=1,4
       M=MN
       CALL   TBLLP(16,16,PRES1,PH)
       PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       M=MN+5
       CALL   TBLLP(16,16,PRES1,PH)
       PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       PERM(MN)=(PER2+PER1)/2
116   CONTINUE
       GO TO 104
103    PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
       PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
       PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
       PERM(4)=(PERM5(I,K)+PERM5(I,K-1))/2
104   BETA=BET*RHOO*G
      DELPHI=PRED(I,K)-PHIS(I,K)
117   IF(FORM(I,K).EQ.1)  GO TO 94
      POR=PORS(2)
       GO TO 127
94    POR=PORS(1)
127    VTCOM=0.0
121   E(I,K)=(G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
```

```
      P1=G/(U*X(I))
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
      QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
      A(I,K)=(G*RHO(4)**2*PERM(4))/((Z(K)+Z(K-1))*U*Z(K))
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0) GO TO 105
      PH(5)=(PHI(I,K)+PRED(I,K))/2
122     CALL  TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
        GO TO 124
105     PH(5)=THETA(I,K)
        CALL TBLLP(16,16,THETA1,PH)
       PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
       GO TO 701
124   UO=CE
11      C(I,K)=-P1*QE1-P1*QE2-E(I,K)-A(I,K)-(UO/DELT)
        B(I,K)=P1*QE2
      D(I,K)=P1*QE1
       Y=P1*QE1*(PHI(I+1,K)-PHI(I,K))+(P1*QE2)*(-PHI(I,K)+PHI(I-1,K))
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))+E(I,K)*PHI(I,K+1)-(E(I,K)*PHI
     $(I,K))-((G*RHO(4)**2*PERM(4))/(U*Z(K)))-A(I,K)*PHI(I,K)+(A(I,K)
     $*PHI(I,K-1))
       F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
       IF(TIMTOT.GT.0.0) GO TO 700
       F(I,K)=2*E(I,K)
       A(I,K)=2*A(I,K)
         B(I,K)=2*B(I,K)
        C(I,K)=-2*P1*QE1-2*P1*QE2-A(I,K)-E(I,K)
        D(I,K)=2*D(I,K)
      ZA=((G*RHO(3)**2*PERM(3))/(U*Z(K)))-((G*RHO(4)**2*PERM(4))/(U*Z(K)
     $))
          F(I,K)=-ZA
700     IF(LOVE.EQ.1) GO TO 200
       F(I,K)=F(I,K)-(E(I,K)*PRED(I,K+1))
       E(I,K)=0.0
       PER4(I)=PERM(4)
       PER3(I)=PERM(3)
200   RETURN
      END
C
C
C   LEFT AND RIGHT CORNOR SUBROUTINE
       SUBROUTINE   LRCB
      REAL K1,K2,INB,INL,INR
        INTEGER BOUND,T2
       DIMENSION PERM(10)
      COMMON PHI(30,20),PRED(30,20),Z(20),U,G,K,PHI1(30,20),
     $PHI2(30,20),PHIS(30,20),RHOO,FORM(30,20),PORS(4),BET,
     $VT(2),NK1,NSM1,BOUND(30,20),INL(20),INB(30),L,IT,TIMTOT,
     $PH(10),RHO(5),DELT1,THETA(30,20)
       COMMON/SEC1/K1(16),PRES1(16),PS1(16),THETA1(16)
       COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
      COMMON/SEC4/M,N2,KING,J
      COMMON/SEC5/RF,T2,PER4(30),PER3(30),R1,PERM5(30,20)
       COMMON/SEC6/INR(20)
       COMMON/SEC7/PCAL(30,20),LOVE
       COMMON/SEC9/ I,X(30),DELT,BLUE
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      IF(TIMTOT.GT.0.0) GO TO 103
102   PH(3)=(PRED(I,K)+PRED(I,K+1))/2
       PH(8)=(PHI(I,K)+PHI(I,K+1))/2
      IF(BOUND(I,K).EQ.6) GO TO 302
      PH(1)=(PRED(I,K)+PRED(I+1,K))/2
      PH(6)=(PHI(I,K)+PHI(I+1,K))/2
      GO TO 303
302   PH(2)=(PRED(I,K)+PRED(I-1,K))/2
        PH(7)=(PHI(I,K)+PHI(I-1,K))/2
303   PH(5)=PRED(I,K)
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```
      N2=NK1
       DO 116  MN=1,3
      M=MN
       CALL   TBLLP(16,16,PRES1,PH)
       PER1=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
      M=MN+5
       CALL   TBLLP(16,16,PRES1,PH)
       PER2=XINT(K1(KING),K1(J),PRES1(KING),PRES1(J),PH(M))
       PERM(MN)=(PER2+PER1)/2
116    CONTINUE
       GO TO 104
103    PERM(3)=(PERM5(I,K)+PERM5(I,K+1))/2
       IF(BOUND(I,K).EQ.6)  GO TO 110
       PERM(1)=(PERM5(I,K)+PERM5(I+1,K))/2
        GO TO 104
110    PERM(2)=(PERM5(I,K)+PERM5(I-1,K))/2
104   BETA=BET*RHO0*G
      RHO(3)=1.00
      RHO(5)=1.00
      IF(BOUND(I,K).EQ.6)  GO TO 306
      RHO(1)=1.00
      GO TO 307
306   RHO(2)=1.00
307    DELPHI=PRED(I,K)-PHIS(I,K)
126   POR=PORS(1)
127    VTCOM=0
121   E(I,K)=(G*RHO(3)**2*PERM(3))/((Z(K)+Z(K+1))*U*Z(K))
      P1=G/(U*X(I))
      IF(BOUND(I,K).EQ.6)  GO TO 308
      QE1=(RHO(1)**2*PERM(1))/(X(I)+X(I+1))
       GO TO 309
308   QE2=(RHO(2)**2*PERM(2))/(X(I)+X(I-1))
309   A(I,K)=0.0
      M=5
      N2=NSM1
      IF(TIMTOT.GT.0.0)  GO TO 105
122     CALL   TBLLP(16,16,PS1,PH)
      THETA(I,K)=XINT(THETA1(KING),THETA1(J),PS1(KING),PS1(J),PH(M))
701   CE=(THETA1(J)-THETA1(KING))/(PS1(J)-PS1(KING))
        GO TO 124
105     PH(5)=THETA(I,K)
        CALL TBLLP(16,16,THETA1,PH)
       PRES=XINT(PS1(KING),PS1(J),THETA1(KING),THETA1(J),PH(M))
       GO TO 701
124   UO=CE
      IF(BOUND(I,K).EQ.6)  GO TO 310
11     C(I,K)=-E(I,K)-P1*QE1-(UO/DELT)
       B(I,K)=0.0
      D(I,K)=P1*QE1
      Y=P1*QE1*(                PHI(I+1,K)-PHI(I,K))-((RHO(5)*
     $INL(K))/(X(I)))
       IF(TIMTOT.GT.0.0)  GO TO 311
       E(I,K)=2*E(I,K)
       C(I,K)=-E(I,K)-2*P1*QE1
       D(I,K)=2*D(I,K)
      GO TO 311
310   Y=P1*QE2*(                PHI(I-1,K)-PHI(I,K))+((RHO(5)*INR(K))/(X
     $(I)))
12     C(I,K)=-E(I,K)-P1*QE2-UO/DELT
       D(I,K)=0.0
      B(I,K)=P1*QE2
       IF(TIMTOT.GT.0.0)  GO TO 311
       E(I,K)=2*E(I,K)
       B(I,K)=2*B(I,K)
       C(I,K)=-E(I,K)-2*P1*QE2
311   ZA=-((RHO(5)*INB(I))/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))
     *-E(I,K)*PHI(I,K)+(E(I,K)*PHI(I,K+1))
500    F(I,K)=-Y-ZA-(UO/DELT)*PHI(I,K)
       IF(TIMTOT.GT.0.0)  GO TO 200
        ZA=-((RHO(5)*INB(I))/Z(K))+(G*RHO(3)**2*PERM(3))/(U*Z(K))
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```
        F(I,K)=-ZA
200    RETURN
      END
C
C
C TABLE LOOK-UP SUBROUTINE
C     EN=ARGUMENT TABLE
      FN=ARGUMENT
      SUBROUTINE  TBLLP(MIKE,LOG,EN,FN)
      COMMON/SEC4/M,N2,KING,J
      REAL EN(MIKE),FN(LOG)
      X4=FN(M)
      IF(X4-EN(1)) 16,15,16
15    J=2
      GO TO 20
16    DO 30 J=1,N2
      IF(EN(J)-X4)30,20,20
30    CONTINUE
20     KING=J-1
      RETURN
      END
C
C
C  STRONGLY IMPLICIT TECHNIQUE FOR THE SOLUTION OF 2_D EQUATIONS
C  OF THE GENERAL FORM:
C A*P(I,K-1)+B*P(I-1,K)+C*P(I,K)+D*P(I+1,K)+E*P(I,K+1)=F
C
      SUBROUTINE SIP2D(N,L,TOL)
       COMMON/SEC3/A(30,20),B(30,20),C(30,20),D(30,20),E(30,20),F(30,20)
       COMMON/SEC7/PCAL(30,20),LOVE
       COMMON/COEF/ALPHA(20),KNT,ISET,IEVEN,VECTO (30,20),EIC(30,20),
     $FIC(30,20),DELTA(30,20),RESID(30,20)
C
       DO 100 KX=1,L
       K=KX
      IF(IEVEN.GT.0) K=L-KX+1
       DO 100 I=1,N
       IF(C(I,K).EQ.0.0) GO TO 100
       IF(I.EQ.1) GO TO 10
      EIM=EIC(I-1,K)
      FIM=FIC(I-1,K)
      TIM=PCAL(I-1,K)
C
      IF(I.NE.N) GO TO 1
      TIP=0.
      GO TO 20
C
10     TIM=0.0
       EIM=0.0
       FIM=0.0
1     TIP=PCAL(I+1,K)
C
20    IF(K.EQ.1) GO TO 30
      EJM=EIC(I,K-1)
      FJM=FIC(I,K-1)
       TJM=PCAL(I,K-1)
C
      IF(K.NE.L) GO TO 2
      TJP=0.
       GO TO 60
C
30    TJM=0.0
      EJM=0.0
      FJM=0.0
2     TJP=PCAL(I,K+1)
C
60    RESID(I,K)=F(I,K)-(A(I,K)*TJM+B(I,K)*TIM+C(I,K)*PCAL(I,K)+D(I,K)
     **TIP+E(I,K)*TJP)
      BIC=A(I,K)/(1.+ALPHA(KNT)*EJM)
      CIC=B(I,K)/(1.+ALPHA(KNT)*FIM)
```

```
       DIC=C(I,K)+ALPHA(KNT)*(BIC*EJM+CIC*FIM)-(BIC*FJM+CIC*EIM)
      EIC(I,K)=(D(I,K)-ALPHA(KNT)*BIC*EJM)/DIC
      FIC(I,K)=(E(I,K)-ALPHA(KNT)*CIC*FIM)/DIC
71       VKM=0.0
      VIM=0.0
       IF(K.NE.1)  VKM=VECTO(I,K-1)
      IF(I.NE.1)  VIM=VECTO(I-1,K)
       VECTO(I,K)=(RESID(I,K)-BIC*VKM-CIC*VIM)/DIC
100    CONTINUE
C
      IEVEN=IEVEN*(-1)
      ISET=0
C   BACK SUBSTITUTION OF CHANGE INPRESSURE HEAD
C
      DO 200  KK=1,L
      K=L-KK+1
      TK=0.
       DO 200 II=1,N
       I=N-II+1
      TI=0.
      IF(K.NE.L)  TK=VECTO(I,K+1)
      IF(I.NE.N)  TI=VECTO(I+1,K)
C
       DELTA(I,K)=VECTO(I,K)-EIC(I,K)*TI-FIC(I,K)*TK
      IF(DELTA(I,K).GT.TOL)  ISET=1
200   CONTINUE
       DO 300 K=1,L
      DO 300 I=1,N
       PCAL(I,K)=PCAL(I,K)+DELTA(I,K)
300   CONTINUE
C
      RETURN
      END
C
C
C CALCULATION OF OVERLAND FLOW USING KINEMATIC FLOOD ROUTING
      SUBROUTINE OLAND
      REAL INFIL,MAN
      INTEGER P,FIN,S
         DIMENSION Q1(2)
      COMMON/SEC4/M,N2,KING,J
       COMMON/SEC8/ AREA1(50),AREA2(50),DISCH1(50),DISCH2(50),MAN,SL(30)
     $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
     $H2(30)
       COMMON/SEC9/ I,X(30),DELT,BLUE
       COMMON/SEC10/INFIL(30),DEPTH(30)
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      H2(1)=0.0
      N1=2
      TOLR=.001
      DELA1=0.1
      Y=0.0
      DO 100 M=1,50
      DISCH2(M)=(4.64*SQRT(SL(   1))*Y**1.66*WIDTH)/MAN
100   Y=Y+0.001*M
      Y=0.0
        DO 101 M=1,50
      AREA1(M)=Y*WIDTH
101   Y=Y+0.001*M
      Q1(2)=0.0
      Q1(1)=0.0
      A1(2)=0.0
      A1(1)=0.0
      Q(1)=0.0
       IF(TIMTOT) 401,400,401
400   Q2(1)=0.0
C     ADJUSTMENT FOR REACH
401   DO 300 P=2,FIN
      Q2(1)=Q(P)
      NUT=S(P)
```

```
      NUTS=S(P-1)+1
701   TOLAT=0.0
      DO 404 I=NUTS,NUT
      TOLAT=TOLAT+QL(I)
404   CONTINUE
      DELX=0.0
      DO 500 I=NUTS,NUT
      DELX=DELX+X(I)
500   CONTINUE
      QLAT=TOLAT/( S(P)-S(P-1))
      Y=0.0
      DO 104 M=1,50
      DISCH2(M)=(4.64*SQRT(SL(  P))*Y**1.66*WIDTH)/MAN
104   Y=Y+0.001*M
      Y=0.0
      DO 105 M=1,50
      AREA2(M)=Y*WIDTH
105   Y=Y+0.001*M
      Y=0.0
      M=1
      N2=50
      CALL TBLLP(50, 2,DISCH2,Q2)
      A2(M)=XINT(AREA2(KING),AREA2(J),DISCH2(KING),DISCH2(J),Q2(M) )
      J=1
      N1=2
C     ROUTING DURING INFLOW
      ALPHA=(A1(M)+A2(M))/2
      M=J+1
      BETA=(DELT /DELX)*Q1(M)+(-A1(M)/2+(DELT *QLAT*WIDTH))
      X1=ALPHA+BETA
      IF(X1) 106,106,102
106   Q2(M)=0.0
      A2(M)=0.0
      GO TO 108
102   CALL SOLVE
      IF(Q2(M)-0.01) 510,510,108
510   Q2(2)=0.0
      A2(2)=0.0
108   AP=A1(2)
      ARE=A2(2)
      IF(AP.LE.0.0) GO TO 201
      H1=A1(2)/WIDTH
      GO TO 203
201   H1=0.0
203   IF(ARE.LE.0.0) GO TO 202
      H2(P)=A2(2)/WIDTH
      GO TO 204
202   H2(P)=0.0
204   IN=P-1
      XTOT=0.0
      DO 603 I=NUTS,NUT
      IF(H1-H2(P)) 206, 220,206
206   IF(I.EQ.NUTS) GO TO 604
      XTOT=XTOT+(X(I-1)+X(I))/2
      GO TO 605
604   XTOT=XTOT+X(I)/2
      GO TO 605
605   DEPTH(I)=(SL(P)*XTOT+H1)-(XTOT*((SL(P)*DELX+H1)-H2(P))/DELX)
      GO TO 603
220   DEPTH(I)=0.0
603   CONTINUE
      Q(P)=Q2(2)
      DO 115 M=1,2
      A1(M)=A2(M)
115   Q1(M)=Q2(M)
C     INTERCHANGE RATING TABLES
      DO 116 M=1,50
      AREA1(M)=AREA2(M)
116   DISCH1(M)=DISCH2(M)
300   CONTINUE
```

```
      RETURN
      END
C
C
C SOLVE SUBROUTINE FOR OVERLANDFLOW
      SUBROUTINE SOLVE
C     SOLVE ROUTINE
      REAL MAN,INFIL
      INTEGER P,FIN,S
      COMMON/SEC4/M,N2,KING,J
       COMMON/SEC8/ AREA1(50),AREA2(50),DISCH1(50),DISCH2(50),MAN,SL(30)
     $,WIDTH,FIN,QL(30),DELA1,Q(20),Q2(2),A1(2),A2(2),S(30),DELX,X1,
     $H2(30)
       COMMON/SEC9/ I,X(30),DELT,BLUE
       COMMON/SEC10/INFIL(30),DEPTH(30)
      XINT(CH,DH,EH,FH,GH)=CH+((GH-EH)/(FH-EH))*(DH-CH)
      AU=0.0
      FAU=0.0
      AL=0.0
      FAL=0.0
      DELA=DELA1
800   A2(M)=A1(M)
11    CALL TBLLP(50, 2,AREA2,A2)
       Q2(M)=XINT(DISCH2(KING),DISCH2(J),AREA2(KING),AREA2(J),A2(M))
      X2=(DELT/DELX)*Q2(M)+(A2(M))/2
      X2=X2-X1
      IF(X2) 3,5,2
2     DELA=DELA1
      AU=A2(M)
      FAU=X2
      IF(AL )4,12,4
12    A2(M)=A2(M)-DELA
13    IF(A2(M))7,7,11
7     DELA=DELA*0.5
       A2(M)=A2(M)+DELA
      GO TO 13
3     DELA=DELA1
      AL=A2(M)
      FAL=-X2
      IF(AU) 4,14,4
14    A2(M)=A2(M)+DELA
15    IF(A2(M)-AREA2(N2))11,11,16
16    DELA=DELA*0.5
       A2(M)=A2(M)-DELA
       GO TO 15
4     ANEW=AU-(FAU/(FAU+FAL))*(AU-AL)
      X3=ANEW-A2(M)
      X3=ABS(X3)
      TOLR=1
       IF(X3-TOLR)5,5,10
10    A2(M)=ANEW
      GO TO 11
  5   A2(M)=ANEW
      CALL TBLLP(50, 2,AREA2,A2)
      Q2(M)=XINT(DISCH2(KING),DISCH2(J),AREA2(KING),AREA2(J),A2(M))
      RETURN
      END
```