

Effect of Forming Process on the Deformational Behaviour of Steel Pipes

by

Majid Tanbakuei Kashani

Thesis submitted to the

Faculty of Graduate and Postdoctoral Studies

in partial fulfillment of the requirements

for Masters of Applied Science in Mechanical Engineering

Department of Mechanical Engineering

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

University of Ottawa

Ottawa, Canada

March 2017

Abstract

Buried pipeline networks play a vital role in the transportation of oil and natural gas from centers of production to centers of consumption. A common manufacturing technique for such pipes is the UOE process, where a flat steel plate is first formed into a U shape, then into an O shape, welded at the seam, and mechanically expanded before being shipped on site. The UOE forming process deforms the pipe material plastically and induces residual strains in the pipe.

Such pipes are commonly buried on side and then are pressurized under the high head of the fluids they convey which induce hoop stresses as high as 80% of the pipe yield strength. When buried pipelines cross the regions of discontinuous permafrost, they undergo differential frost heaving, inducing significant bending deformations, which potentially induce local buckling in the pipe wall. To control local buckling, design standards impose threshold limits on buckling strains. Such threshold values are primarily based on costly full-scale experimental results. Past nonlinear finite element analysis attempts aiming at determining the threshold buckling strains have neglected the presence of residual stresses induced by the UOE forming and were thus found to grossly overestimate the buckling strains compared to those based experiments.

Within the above context, the present study focuses on developing a numerical technique to predict the residual stresses induced during UOE forming, and incorporating the induced residual stresses in 3D nonlinear FEA modeling to more reliably predict buckling strain limits. Comparisons with conventional analysis techniques that omit residual stresses reveal the importance of incorporating residual stresses induced in forming when quantifying buckling strains.

Acknowledgements

I owe my knowledge and deepest gratitude to my supervisor Professor Magdi Mohareb for his technical, helpful advice, and great patience with endless source of knowledge and experience and also for being a great person and for his great attitude toward me.

Most of all, I take this opportunity to express my words to my best friend and gorgeous wife Neda for her deepest love, patience and for her tremendous encouragement and support in all aspects of my life. Her role is definitely indispensable and cherished in my achievements.

My deepest thanks go to my parents, Mahin and Davood for their invaluable sacrifices, love and unconditional support. Although they are far away, without them I couldn't have made it up to this point. I wish them all the best and long life.

At the end I want to express my gratitude to my friend, Mohammad Mahdi Dastgerdi for all unconditional help.

Table of Contents

Abstract.....	iii
Acknowledgements	iii
Table of Contents	ivv
List of Figures.....	xii
List of Tables	xviii
List of Symbols.....	xix
Chapter 2.....	xix
Chapter 3.....	xxi
Chapter 4.....	xxiii
Chapters 5 and 6.....	xxv
Chapter 7.....	xxvi
Chapter 1 Introduction.....	1
1.1 General.....	1
1.2 The UOE Manufacturing Process	1
1.2.1 Edge-Planing and Crimping.....	2
1.2.2 Forming Process (U-ing-O-ing steps).....	3
1.2.3 Inside and Outside Seam Welding Process.....	3
1.2.4 Mechanical Expansion	4
1.3 Buried Pipeline Behaviour	4
1.4 Motivation.....	6

1.5	Outline of the Thesis	6
CHAPTER 2.	Literature Review	7
2.1	General	7
2.2	Common Pipe Geometries and Grades (CEPA, 2015)	7
2.3	Survey of Past Research on Buckling Strains	9
2.3.1	Unpressurized Pipes	10
2.3.1.1	Buckling strains based on elastic analysis	10
2.3.1.2	Buckling strains based on inelastic analysis	10
2.3.2	Critical strains of Pressurized pipes	11
2.3.2.1	Bouwkamp & Stephen (1973)	11
2.3.2.2	Gresnigt (1986)	11
2.3.2.3	Mohareb et al. (1994)	12
2.3.2.4	Yoosef-Ghodsi et al. (1994)	13
2.3.2.5	Zimmerman et al. (1995)	13
2.3.2.6	DelCol et al. (1998)	13
2.3.2.7	Myrholm et al. (2001)	14
2.3.2.8	Dorey et al. (2001)	14
2.3.2.9	Dorey et al. (2001)	16
2.3.2.10	Suzuki et al. (2003)	16
2.3.2.11	Zimmerman et al. (2004)	16
2.3.2.12	Adeeb et al. (2006)	17
2.3.2.13	Chou et al. (2006)	17

2.3.2.14	Fatemi et al. (2008).....	17
2.3.2.15	Zhang et al. (2008).....	18
2.3.2.16	Ozkan (2008).....	19
2.3.2.17	Chen et al. (2008).....	20
2.3.2.18	Cho et al. (2009).....	20
2.3.2.19	Suzuki et al. (2010).....	21
2.3.2.20	Fathi et al. (2010).....	21
2.3.2.21	Yoosef-Ghodsi et al. (2014).....	21
2.3.2.22	Neupane et al. (2012 a,b).....	22
2.4	Buckling Strains in Codes and Standards	23
2.4.1	CAN/CSA Z662 Provisions	23
2.4.2	Provisions of Det Norske Veritas Offshore Standard (DNV-OS-F101).....	23
2.5	Plastic Interaction Relations for Pipes	24
2.6	Objectives and Scope of present Study	25
Chapter 3	Overview of the Theory of Plasticity	32
3.1	General	32
3.2	Stress-Strain Idealizations for Elasto-Plastic models	32
3.2.1	Perfectly Plastic Materials	32
3.2.2	Elastic-Perfectly Plastic Materials	33
3.2.3	Elastic-Linear Plastic Materials	33
3.2.4	Elastic-Nonlinear Plastic Materials.....	33

3.3	Total Strain Decomposition	33
3.4	Yield criterion	36
3.5	Incremental Elastic Stress-Strain Relationships	39
3.6	Plasticity Flow Rule	40
3.7	Associated Flow Rule	41
3.8	Isotropic Plasticity	42
3.9	Simplifications for perfectly plastic materials	42
3.10	Plastic Strain increment for Elastic- Perfectly plastic materials	43
3.11	Effective Stress and Effective Plastic Strain	44
3.11.1	Effective Stress	44
3.11.2	Effective Plastic Strain	45
3.12	Linear Kinematic Hardening	46
3.12.1	Yield criterion	46
3.12.2	Prager Linear Kinematic Hardening model	47
3.12.3	Ziegler Linear Kinematic Hardening model	47
3.12.4	Incremental Plastic Strain	47
3.13	Nonlinear Kinematic Hardening	48
3.13.1	Armstrong and Frederick Model	48
3.13.2	Chaboche Model	48

CHAPTER 4. Description of the Finite Element Model.....	50
4.1 General.....	50
4.2 Analysis Type and Sequence of loading.....	50
4.3 Element Types	54
4.3.1 Continuum Elements.....	54
4.3.2 First- and Second-Order Elements.....	54
4.3.3 Integration Schemes.....	56
4.4 Modeling Considerations for predicting tension test results for pipes	59
4.5 Elasto-Plastic Material Properties.....	65
4.5.1 Elasto-Plastic Properties	65
4.5.2 Determining Kinematic Hardening Parameters	67
4.5.2.1 Determining the Chaboche parameters.....	69
4.6 Comparison of the Performance of 3D Elements	73
4.7 Results and Conclusions	77
Chapter 5 Description of the Simple Finite Element Model	79
5.1 General.....	79
5.2 Pipe Geometry	79
5.3 Loading Path	80
5.4 Model Geometry	81
5.5 Material Properties.....	83

5.6	Element Type and Mesh Size	84
5.7	Load Application	85
5.7.1	Load control Scheme	85
5.7.2	Application of Internal Pressure	85
5.7.3	Application of Displacement Symmetry Condition Associated with Internal Pressure.....	85
5.7.4	Application of End Bending Moments	86
5.7.5	Applying the radial restraints during bending step	87
CHAPTER 6.	Description of the Detailed Finite Element Model	88
6.1	General.....	88
6.2	Plate Geometry.....	89
6.3	Loading Path	89
6.4	Model Geometry	94
6.5	Material Properties.....	95
6.6	Load Application	95
6.6.1	Application of Displacement Symmetry Condition Associated with Forming Process.....	95
6.7	Constraint.....	97
6.7.1	Multi-Point Constrain Type Slider.....	97
6.7.2	Tie Constraint.....	98

Chapter 7	Results and Discussion.....	99
7.1	General.....	99
7.2	Global and Local Curvature of Pipes.....	99
7.3	Rotation Calculation	100
7.4	Plastic Moment Capacity	101
7.5	Elastic Response	102
7.6	Calculation of Moments.....	102
7.7	Moment-Curvature Relationships.....	104
7.7.1	Simplified Model Results	104
7.7.2	Detailed Model Results.....	107
7.7.3	Comparison of Simplified model and detailed model results	108
7.8	Deformed Configuration.....	111
7.9	Local Buckling Strain comparison between FEM Results and Code	115
Chapter 8	Summary and Conclusions.....	118
8.1	Summary of Research	118
8.2	Conclusions and Recommendations	118
Appendix A	Mesh Sensitivity Analysis.....	120

List of Figures

Figure 1. 1 Integrated pipe production process (from EUROPIPE GmbH).....	2
Figure 1. 2 (A) Edge planning and (B) edge crimping of the flat plate (from EUROPIPE GmbH)	2
Figure 1. 3 (A) U-ing press and (B) O-ing press of forming process (from EUROPIPE GmbH)..	3
Figure 1. 4 (A) Inside and (B) Outside seam welding of the formed pipe (from EUROPIPE GmbH).....	3
Figure 1. 5 Mechanical expansion of the welded pipe to obtain the final diameter and reduce residual stress (from EUROPIPE GmbH).....	4
Figure 1. 6 Deformed buried pipe due to ground displacement (adapted from Ozkan 2008).....	5
Figure 1. 7 Pipe subjected to (A) Internal pressure and Axial force and (B) Axial force and (C) End rotations.....	5
Figure 2. 1 Gas delivery network diagram (from Canadian Energy Pipeline Association (CEPA))	8
Figure 2. 2 Liquids delivery network diagram (from Canadian Energy Pipeline Association (CEPA))	8
Figure 3. 1 Idealized stress-strain curves for (A) Perfectly plastic, (B) Elastic-perfectly plastic, (C) Elastic-linearly plastic and (D) Elastic-nonlinear plastic materials.....	34
Figure 3. 2 Uniaxial total strain decomposition into plastic and elastic strains.....	35
Figure 3. 3 the von Mises yield criterion in three-dimensional principal stress space	38
Figure 3. 4 von Mises yield surface in two-dimensional principal stress space	38
Figure 3. 5 Stresses under three-dimensional loading	39

Figure 3. 6 The direction of the incremental plastic strain tensor in stress space.....	41
Figure 3. 7 Initial and subsequent yield surfaces in linear kinematic hardening models	46
Figure 4. 1 Modeling U-Pressing and O-Pressing by defining nodal displacement in 10 steps...51	
Figure 4. 2 Flat plate with both end vertical edges (A) Before and (B) After U- and O- pressing	52
Figure 4. 3 Flat plate with both end inclined edges (A) Before and (B) After U- and O- pressing	52
Figure 4. 4 Loading path for U-O-E forming process	53
Figure 4. 5 Nodes on the right and left edges of the curved plate are kept in contact.....	53
Figure 4. 6 Continuum elements in ABAQUS/Standard (A) Tetrahedral and (B) Hexahedral ...	54
Figure 4. 7 Hexahedra elements (A) First-order with 8 nodes and (B) Second-order with 20 nodes	55
Figure 4. 8 Tetrahedral elements (A) First-order with 4 nodes and (B) Second-order with 10 nodes	55
Figure 4. 9 First-order continuum elements (A) reduced integration C3D8R (1 integration point) and (B) fully integration C3D8 (8 integration points)	56
Figure 4. 10 Second-order (quadratic) solid elements (A) reduced integration C3D20R (8 integration points) and (B) fully integration C3D20 (27 integration points).....	57
Figure 4. 11 Boundary condition throughout the longitudinal tension test in (A) Side and (B) Left sides of the specimen	62
Figure 4. 12 Boundary condition throughout the hoop tension test in (A) Side and (B) left sides of the specimen.....	62
Figure 4. 13 Loading path for the simulation of longitudinal tension test.....	63

Figure 4. 14 Loading path for the simulation of hoop tension test	64
Figure 4. 15 True and engineering stress-strain curves for X52 steel	67
Figure 4. 16 Stress-plastic strain and experimental backstress (Table 4. 5) of X52 steel	69
Figure 4. 17 Stress-plastic strain curve is divided into M parts (adapted from Chen and Jiao 2004)	70
Figure 4. 18 Comparison between analytically computed and experimentally determined backstress values for X52 steel after the initial calculation based on Eqs. 4.5-4.6	73
Figure 4. 19 Mises stress after U-O-E forming process in the inside surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R	75
Figure 4. 20 Mises stress after U-O-E forming process in the middle surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R	75
Figure 4. 21 Mises stress after U-O-E forming process in the outside surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R	75
Figure 4. 22 Engineering longitudinal stress-strain curve for the specimen with element C3D8, C3D8R and C3D20R	76
Figure 4. 23 True longitudinal stress-strain curve for the specimen with element C3D8, C3D8R and C3D20R.....	76
Figure 4. 24 Comparison between hoop, longitudinal and experimental engineering stress-strain curves for the element C3D20R.....	77
Figure 4. 25 Comparison between hoop, longitudinal and experimental true stress-strain curves for the element C3D20R.....	78
Figure 5. 1 Loading path for P00S specimen subjected to pure end moments.....	80
Figure 5. 2 Loading path for P40S and P80S specimens subjected to the combination of (A) internal pressure and (B) end moments.....	81

Figure 5. 3 Local buckling at the end of the pipe in the pipe with constant thickness and no collars	82
Figure 5. 4 Sectional elevation view of FEA pipe model	82
Figure 5. 5 Local buckling for P00S with collar regions and constant thickness	83
Figure 5. 6 Local buckling for P00S-T10 with tapered ends and collar regions	83
Figure 5. 7 End view of models P40S-T10 and P80S-T10 models subjected to the internal pressure	85
Figure 5. 8 Schematic view of restrained and free degree of freedoms during application of internal pressure (A) Sectional elevation view (B) Cross-section view	86
Figure 5. 9 Specified end forces to simulate the end bending moments.....	86
Figure 5. 10 Schematic view of restrained and free degree of freedoms during application of end moments (A) Sectional elevation view (B) Cross-section view	87
Figure 6. 1 Sectional elevation view of rectangular plate.....	89
Figure 6. 2 Nodes on the right and left sides of the plate	90
Figure 6. 3 UOE forming process for all specimens as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion	91
Figure 6. 4 UOE forming process for P00D specimen as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion (E) Apply end moments.....	92
Figure 6. 5 UOE forming process for P40D and P80D specimens as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion (E) Apply internal pressure (F) Apply end moments	93
Figure 6. 6 Sectional elevation view of FEA plate model	94

Figure 6. 7 Different Boundary conditions used in the forming process.....	95
Figure 6. 8 Multi-point constrain with master points used on (A) right side of the plate and (B) left side of the plate	97
Figure 6. 9 Connections between nodes on top right and left sides of the pipe by Tie constraints	98
Figure 7. 1 Parameters used to determine the global curvature of pipe specimen (A) Un-deformed configuration, and (B) Deformed configuration.....	100
Figure 7. 2 Parameters used to determine the local curvature of pipe specimen (A) Un-deformed configuration, and (B) Deformed configuration	100
Figure 7. 3 Nodes selected for calculation of the rotation of section i	101
Figure 7. 4 Isometric view of the deformed pipe.....	103
Figure 7. 5 Sections at ends of local and global gage lengths (A) Elevation and (B) Cross-sectional view	105
Figure 7. 6 Moment-curvature based on simplified models	106
Figure 7. 7 Moment-curvature based on detailed models.....	109
Figure 7. 8 Comparison of local moment-curvature based the simplified and detailed models.	110
Figure 7. 9 Buckling deformation shapes in the peak moment value for (A) P00S-T10, (B) P00D-T03, (C) P40S-T10, (D) P40D-T10, (E) P80S-T10 and (F) P80D-T03 models.....	112
Figure 7. 10 Buckling deformation shapes in the 95% of the peak moment on the descending branch for (A) P00S-T10, (B) P00D-T03, (C) P40S-T10, (D) P40D-T10, (E) P80S-T10 and (F) P80D-T03 models	113
Figure 7. 11 Buckling deformation shapes in the peak value for (A) P00S-T10, (B) P40S-T10, (C) P80S-T10, and in the 95% of the peak moment on the descending branch for (D) P00S-T10, (E) P40S-T10, (F) P80S-T10 models (Scale factor = 3).....	114

Figure 7. 12 Length of (A) Un-deformed and (B) deformed selected gage	115
Figure 7. 13 Nodes on the right and left sides of the gage cross section (A) Front and (B) side views	115
Figure 7. 14 The comparison between the simplified and detailed local moment-buckling strain	117
Figure A.1 Mesh sensitivity analysis for the unpressurised and pressurised models (A) $23 \times 22 \times 8$, (B) $41 \times 24 \times 10$, (C) $65 \times 26 \times 12$, (D) $23 \times 22 \times 8 (P)$, (E) $41 \times 24 \times 10 (P)$ and (F) $65 \times 26 \times 12 (P)$	121
Figure A.2 Mesh sensitivity study for unpressurised simplified model.....	121
Figure A.3 Mesh sensitivity for fully pressurised simplified model.....	122

List of Tables

Table 2. 1 Buckling strains equations conducted by Dorey et al. (2001) for plain and girth welded pipes	15
Table 2. 2 Geometry of all specimens in Chou et al. (2006) study.....	17
Table 2. 3 Geometry of the specimens in Fatemi et al. (2008) study	18
Table 2. 4 Geometry of the specimens in Fatemi et al. (2009) study	18
Table 2. 5 Material properties in numerical study in Cho et al. (2009).....	20
Table 2. 6 Overview of key parameters in past studies	26
Table 2. 7 Summary of various studies and standards on buckling strains equations of pipes subjected to Axial force (A), Bending (B), Internal (I) and/or External (E) pressures.....	29
Table 4. 1 Comparison between continuum element types in ABAQUS element library.....	58
Table 4. 2 Longitudinal stress and strain equations between positions 4 to 4a	60
Table 4. 3 Hoop stress and strain equations between positions 4 to 4b.....	61
Table 4. 4 Experimentally Obtained Engineering and true stress-strain values of X52 steel (Mohareb 1995)	66
Table 4. 5 Experimental stress-plastic strain and backstress for X52 Steel after forming process	68
Table 4. 6 True stress-plastic strain selected data points of X52 steel	71
Table 4. 7 Initial values of the Chaboche nonlinear kinematic hardening parameters	71
Table 4. 8 Normalized initial backstress values of X52 steel	72
Table 4. 9 Final values of Chaboche parameters C^k and γ^k for X52 steel.....	72

Table 4. 10 The element, node and integration number comparison between C3D8, C3D8R and C3D20R elements	74
Table 5. 1 Experimental stress-plastic strain and backstress for X52 Steel.....	84
Table 6. 1 Gauge length and additional thickness in tapered regions of models.....	94
Table 6. 2 Extracted stress-plastic strain from longitudinal tension test after forming process ...	96
Table 7. 1 Modified Plastic moment capacity for specimens investigated in the present study....	102
Table 7. 2 Selected gage length for the simplified and detailed models.....	107
Table 7. 3 Comparison of peak moments	108
Table 7. 4 Comparison between buckling strain Z662 design standard and FEM results.....	116
Table A.1 Mesh sensitivity study.....	120

List of Symbols

Chapter 2

OD (D)	Outer diameter
D/t	Outer diameter-to-thickness ratio
D_{ave}	Mean diameter of the pipe
D_{max}	Maximum value of the outside diameter
D_{min}	Minimum value of the outside diameter
t	Pipe wall thickness
r_m	Pipe mid-wall radius
A	Cross-sectional area of the pipe
L	Length of pipe
L/OD	Length to outer diameter radius
ε_{cr}	Buckling strain
ε	Total strain
$\varepsilon_{cr}^{(c)}$	Compression buckling strain
$\varepsilon_{cr}^{(b)}$	Bending buckling strain
σ	Mises stress
σ_h	Stress in hoop direction
F_y	Yield stress
F_u	Ultimate stress
σ_y	Yield strength
A_t	Axial force ratio
P_a	Applied pressure
P_t	Total axial force

P_y	Internal yield pressure in hoop direction (internal pressure when the hoop stress equals the yield stress)
P	Internal pressure
P_c	Circumferential yield pressure
$P_{i \max}$	Maximum internal design pressure
$P_{e \min}$	Minimum external hydrostatic pressure
$P_{\min}$	Minimum internal pressure
P_e	External hydrostatic pressure
P_b	Burst pressure
E	Modulus of elasticity
ψ_t	Ratio of the elastic modulus to the tangent modulus
ψ_s	Ratio of the elastic modulus to the secant modulus
μ	Poisson's ratio
r'	Imperfection factor for ovalization
a	Ovalization due to bending
n	Strain hardening characteristics
imp	Size of pipe imperfection
$offset$	Size of girth-welded imperfection
ε_{cf}	Factored compressive strains in the longitudinal or hoop direction
ϕ_{ec}	Resistance factor
α_{gw}	Weld reduction factor

Chapter 3

ε	Total strain
ε_{ij}	Total strain tensor
$d\varepsilon_{ij}$	Incremental total strain tensor
ε^p	Plastic strain
ε_{ij}^p	Plastic strain tensor
$d\varepsilon_{ij}^p$	Incremental plastic strain tensor
$d\varepsilon_{eff}^p$	Effective plastic strain
ε_{eff}^p	Total effective plastic strain
ε^e	Elastic strain
ε_{ij}^e	Elastic strain tensor
$d\varepsilon_{ij}^e$	Incremental elastic strain tensor
$d\gamma_{ij}^p$	Incremental plastic shear tensor
E_T	Tangent modulus
E	Modulus of elasticity
ν	Poisson's ratio
σ	Mises Stress
σ_m	Mean or Hydrostatic stress
σ_{ij}	Stress tensor
$d\sigma_{ij}$	Incremental stress tensor
σ_{eff}	Effective, Equivalent or von Mises stress
$\sigma_x(\sigma_{11})$	Normal stress in x direction
$\sigma_y(\sigma_{22})$	Normal stress in y direction

$\sigma_z(\sigma_{33})$	Normal stress in z direction
$\tau_{xy}, \tau_{yz}, \tau_{xz}$	Shear stresses
S_{ij}	Deviatoric stress tensor
dS_{ij}	Incremental deviatoric stress tensor
α_{ij}	Backstress tensor
α	Backstress
$d\alpha_{ij}$	Incremental backstress tensor
$d\alpha_{ij}^{dev}$	Incremental deviatoric backstress tensor
$d\alpha_{ij}^{dev}$	Deviatoric backstress tensor
$d\alpha_{ij}^{(Prager)}$	Incremental Prager's linear kinematic hardening backstress tensor
$f(\sigma_{ij})$	Yield function
$g(\sigma_{ij})$	Plastic potential function
I_1, I_2, I_3	First, second and third invariants of stress tensor
J_2, J_3	Second and third invariants of deviatoric stress tensor
δ_{ij}	Kronecker delta
$d\lambda$	Plastic multiplier
c_p	Prager's hardening coefficient
$d\mu$	Incremental plastic strain history
c, γ	Nonlinear kinematic hardening material parameters
x, y, z	Current length of specimen in x, y and z directions

Chapter 4

u_x, u_y, u_z	Displacement along x, y and z directions
x_0, y_0, z_0	Initial length of specimen in x, y and z directions
x, y, z	Current length of specimen in x, y and z directions
$1/R$	Curvature
r_{o4}	Outer radius of the pipe at position 4
r_{i4}	Inner radius of the pipe at position 4
r_{m4}	Middle radius of the pipe at position 4
r_{o4-4a}	Current outer radius of the pipe between positions 4 to 4a
r_{i4-4a}	Current inner radius of the pipe between positions 4 to 4a
r_{i4-4b}	Current inner radius of the pipe between positions 4 to 4b
r_{o4-4b}	Current outer radius of the pipe between positions 4 to 4b
$\sum R_{4-4a}$	Sum of the reaction forces between positions 4 to 4a
L_4	Length of the pipe at position 4
L_{4-4a}	Current length of the pipe between positions 4 to 4a
A_4	Area of the pipe at position 4
A_{4-4a}	Area of the pipe between positions 4 to 4a
t_4	Current thickness of the pipe at position 4
t_{4-4b}	Current thickness of the pipe between positions 4 to 4b
u_{m4-4b}	Radial displacement at middle surface of the pipe between positions 4 to 4b
ε_{EL}	Engineering longitudinal strain
ε_{TL}	True longitudinal strain
ε_{EH}	Engineering hoop strain
ε_{TH}	True hoop strain

ε_{eng}	Engineering strain
ε_T	True strain
ε^p	Plastic strain
σ_{EL}	Engineering longitudinal stress
σ_{TL}	True longitudinal stress
σ_{EH}	Engineering hoop stress
σ_{TH}	True hoop stress
σ_{eng}	Engineering stresses
σ_T	True stresses
F_y	Yield stress
α	Backstress
α_e	Experimental backstress
α_a	Analytical backstress
p_{4-4b}	Current internal pressure between positions 4 to 4b
c, γ	Nonlinear kinematic hardening material parameters

Chapters 5 and 6

OD (D)	Outer diameter
D/t	Outer diameter-to-thickness ratio
t	Pipe wall thickness
σ	Mises stress
F_y	Yield stress
α	Backstress
c, γ	Nonlinear kinematic hardening material parameters
$P00S$	Un-pressurized simplified specimen
$P40S$	Pressurized simplified specimen with an internal pressure including a hoop stress 40% of the yield strength
$P80S$	Pressurized simplified specimen with an internal pressure including a hoop stress 80% of the yield strength
$P00S - T10$	Un-pressurized simplified specimen with an addition 10% increase in thickness
$P40S - T10$	Pressurized simplified specimen with an internal pressure including a hoop stress 40% of the yield strength and an addition 10% increase in thickness
$P80S - T10$	Pressurized simplified specimen with an internal pressure including a hoop stress 80% of the yield strength and an addition 10% increase in thickness
$P00D$	Un-pressurized detailed specimen
$P40D$	Pressurized detailed specimen with an internal pressure including a hoop stress 40% of the yield strength
$P80D$	Pressurized detailed specimen with an internal pressure including a hoop stress 80% of the yield strength
$P00D - T03$	Un-pressurized detailed specimen with an addition 3% increase in thickness
$P40D - T10$	Pressurized detailed specimen with an internal pressure including a hoop stress 40% of the yield strength and an addition 10% increase in thickness
$P80D - T03$	Pressurized detailed specimen with an internal pressure including a hoop stress 80% of the yield strength and an addition 3% increase in thickness

Chapter 7

$OD (D)$	Outer diameter
D/t	Outer diameter-to-thickness ratio
t	Pipe wall thickness
L_i	Initial gage length
L	Deformed gage length
L_{a-b}	Length of the pipe between points a and b
$L_{a'-b'}$	Length of the pipe between points a' and b'
r_{av}	Average of radius
OR	Outer radius
IR	Inner radius
$x_{c.g.}, y_{c.g.}, z_{c.g.}$	Cross section centroid of the deformed pipe in x , y and z directions
x_i, y_i, z_i	Deformed coordinates of node i in x , y and z directions
ε_{cr}	Buckling strain
ϕ	Curvature of the pipe
ϕ_G	Global curvature of the pipe
ϕ_L	Local curvature of the pipe
θ_i	End rotations at point i
θ_a	End rotations at point a
$\theta_{a'}$	End rotations at point a'
θ_b	End rotations at point b
$\theta_{b'}$	End rotations at point b'
a_i^y	Deformed coordinates of point a along y
b_i^y	Deformed coordinates of point b along y
a_i^z	Deformed coordinates of point a along z

b_i^z	Deformed coordinates of point b along z
F_y	Yield stress
σ_θ	Hoop stress
P_e	Axial load
$p_{i \max}$	Maximum internal design pressure
$p_{e \min}$	Minimum external hydrostatic pressure
M_{p, σ_θ}	Plastic moment capacity for the specimen subjected to internal pressure and hoop stress and in absence of axial force
M_p	Plastic moment capacity for the specimen subjected to internal pressure and in absence of hoop stress and axial force
M	Initial moment
E	Young modulus
I	Moment of inertia of the pipe cross section
F_i^z	Nodal force of node $i(i = 1, 2..n)$ in z direction
R_n^x, R_n^y	Reaction forces along x and y directions
$P00S - T10$	Un-pressurized simplified specimen with an addition 10% increase in thickness
$P40S - T10$	Pressurized simplified specimen with an internal pressure including a hoop stress 40% of the yield strength and an addition 10% increase in thickness
$P80S - T10$	Pressurized simplified specimen with an internal pressure including a hoop stress 80% of the yield strength and an addition 10% increase in thickness
$P00D - T03$	Un-pressurized detailed specimen with an addition 3% increase in thickness
$P40D - T10$	Pressurized detailed specimen with an internal pressure including a hoop stress 40% of the yield strength and an addition 10% increase in thickness
$P80D - T03$	Pressurized detailed specimen with an internal pressure including a hoop stress 80% of the yield strength and an addition 3% increase in thickness

Chapter 1 Introduction

1.1 General

Pipeline networks play a vital role in transportation large quantities of oil or natural gas which are used to deliver liquids from centers of productions to centers of consumptions. Pipelines provide the most convenient, efficient, economical and reliable way to transport liquids such as petroleum, petroleum products, natural gas, water and other fluids. Massive oil and gas distribution in pipeline networks is carried out by buried pipelines. The main advantage of buried pipelines over elevated pipelines is the lower costs of construction than the elevated ones.

Large-size buried pipes are generally produced by the UOE manufacturing process (Section 1.2). The UOE forming process influences the mechanical properties of pipes and induces residual strains. Buried pipelines are designed primarily for the hoop stress induced by the operating pressure. Additional loading includes axial forces induced by thermal effects and bending action induced by differential settlements. Design standards provide provisions to limit the stresses and strains in buried pipelines. The behaviour of buried pipelines are discussed in Section 1.3. The material in this chapter is extracted from EUROPIPE website, Kyriakides et al. (1994), Herynk et al. (2007)

1.2 The UOE Manufacturing Process

UOE (U-ing-O-ing-Expanding) process is a commonly used manufacturing method used for cold-forming onshore and offshore longitudinally welded large-diameter pipeline from long plates. The plates are bent into a circular shape. The edges of the pipe are then joined through Submerged Arc Welding (SAW) and then the pipe is subjected to radial expansion (E) (Figure 1.1). The stages of production for large-sized pipe is shown in Figure 1.1. The following sub-sections provide an introduction to the main steps of the procedure.

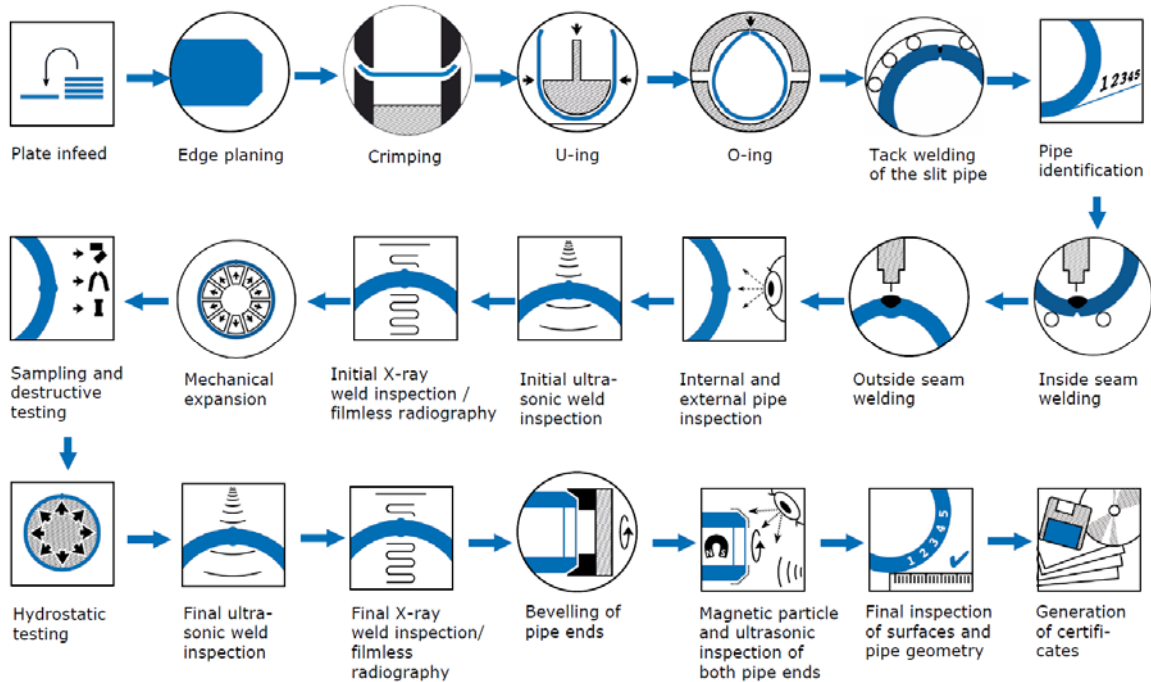


Figure 1. 1 Integrated pipe production process (from EUROPIPE GmbH)

1.2.1 Edge-Planing and Crimping

In the edge-planing step the right and left edges of the plate are planed parallel in a Y bevel shape for the inside and outside welding process. The edge-planed plate are crimped automatically to the desired pipe radius through a process that helps both edges of the plate to meet one another at the end of forming process.



Figure 1. 2 (A) Edge planning and (B) edge crimping of the flat plate (from EUROPIPE GmbH)

1.2.2 Forming Process (U-ing-O-ing steps)

In the forming process, the planed- and crimped-plate is first formed into a "U" shape by lowering a U-ing die press and then formed into an "O" shape in O-ing press. At the end of the forming process the flat plate is deformed into the shape of an almost circle which is ready for the inside and outside seam welding process.

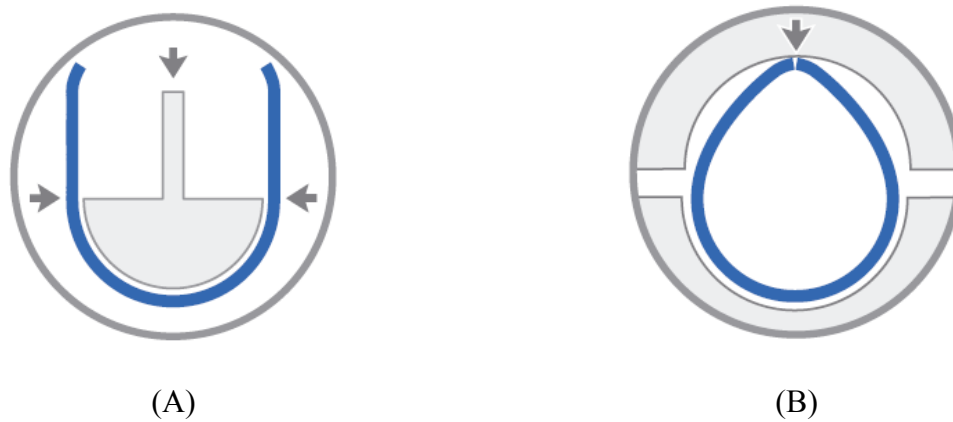


Figure 1. 3 (A) U-ing press and (B) O-ing press of forming process (from EUROPIPE GmbH)

1.2.3 Inside and Outside Seam Welding Process

The longitudinally formed edges of the pipe are joined by Submerged Arc Welding (SAW). After the forming process, inside welding is carried out first, followed by outside welding which are completely fused.



Figure 1. 4 (A) Inside and (B) Outside seam welding of the formed pipe (from EUROPIPE GmbH)

1.2.4 Mechanical Expansion

In the expansion process, the welded pipe is subjected to internal expansion by a mechanical expander. In this step, the pipe is expanded by deforming into the plastic range of deformation to its final diameter. The effect of residual stresses are reduced during the mechanical expansion.



Figure 1. 5 Mechanical expansion of the welded pipe to obtain the final diameter and reduce residual stress (from EUROPIPE GmbH)

1.3 Buried Pipeline Behaviour

A large number of studies on the design and behaviour of the buried pipes have been carried out over the last two decades. Focus has been placed on bending action within the pipe due to differential soil settlement and/or frost heave (Figure 1. 6) which induce bending into the pipe in addition to axial forces thus promoting local buckling. Primarily, pipes are subjected to internal pressure induced by the high operating pressure of the pressurized liquid, gas or two-phase fluids. Such pressure induces hoop tension and longitudinal contraction. When differential settlement or frost heave occur in cold regions as a result of temperature changes, pipes are subjected to additional bending deformation and axial forces. Other sources of axial forces include thermal effects and Poisson's effect.

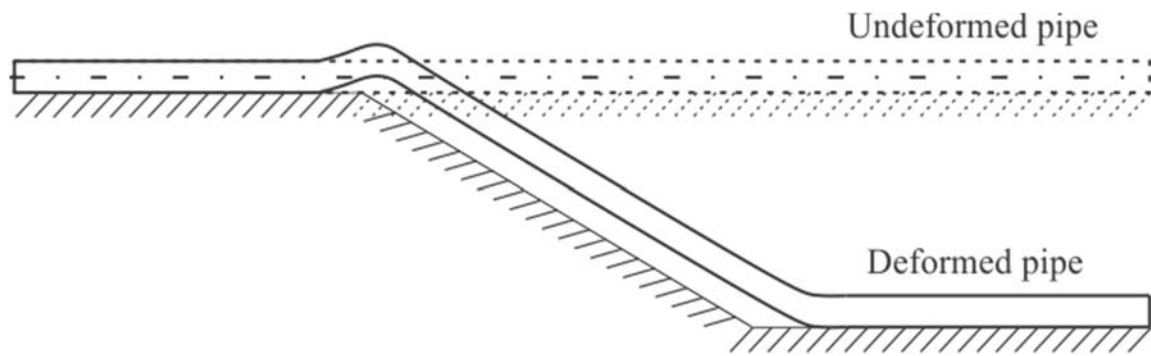
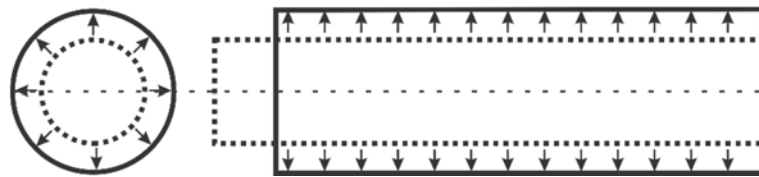


Figure 1. 6 Deformed buried pipe due to ground displacement (adapted from Ozkan 2008)



(A)



(B)



(C)

Figure 1. 7 Pipe subjected to (A) Internal pressure and axial force and (B) Axial force and (C) End rotations

1.4 Motivation

Within the above context, the present study investigates the effect of bending deformation on pressurized and un-pressurized pipes. Unlike previous studies, which omitted the effect of manufacturing process on the pipe behaviour, the present study focuses on the modeling of the UOE manufacturing process, thus capturing the residual stresses induced and plastic deformations generated at the end of forming. Comparisons are provided with the results based on conventional modeling that disregards the effects UOE.

1.5 Outline of the Thesis

The present thesis consists of the following chapters

- (1) Chapter 2 provides a review of previous numerical and experimental studies related to the buckling strains of pipes under combinations of internal pressure, axial force and bending. A review of design standards and codes provisions for the design of pipes relevant to buckling strains limits is also provided.
- (2) Chapter 3 provides an overview of key concepts related to plasticity modeling with emphasis on kinematic hardening materials and their idealizations.
- (3) Chapter 4 provides the key aspects of 3D finite element analysis modeling related to the simplified model (Chapter 5) and detailed model (Chapter 6).
- (4) Chapter 5 provides a 3D finite element model to determine the buckling strains of pressurized and un-pressurized steel pipes subjected to bending deformation.
- (5) Chapter 6 provides a 3D finite element model to determine the effects of the UOE forming process on bending deformation of pressurized and un-pressurized bent plate subjected to bending deformation.
- (6) Chapter 7 provides a local buckling strains and deformation comparison between simplified and detailed models.
- (7) Chapter 8 provides an overall summary of the research, recommendations and conclusions that can be used in future studies.

CHAPTER 2. Literature Review

2.1 General

This chapter presents a review of common pipe geometries and grades in the pipeline industry for transporting oil and natural gas (Section 2.2). A detailed review of previous experimental and numerical studies on buckling and post-buckling behaviour of pipes for investigating buckling strains represented in Section 2.3 for unpressurized pipes (sub-sections 2.3.1) and pressurized pipes (Subsection 2.3.2). Section 2.3 also provides an overview of expressions for determining buckling strains as obtained in various studies. Relevant equations for determining buckling strain limits under combinations of axial force, bending moment and internal pressure as provided in CAN/CSA Z662-2015 and DNV-OS-F101-2013 standards are presented in Section 2.4. Section 2.5 describes the scope of the study. The concluding section of the chapter provides a summary of gage lengths and buckling strain equations adopted in previous studies in order to determine critical strains.

2.2 Common Pipe Geometries and Grades (CEPA, 2015)

Depending on functionality, pipelines can be classified into three main categories: Gathering pipelines, transportation (or transmission) pipelines, and distribution pipelines (Fig. 2.1). A description of each type of pipeline is provided in the following.

- **Gathering pipelines** are smaller interconnected pipelines which are used to transport crude oil or natural gas from wells to oil batteries or processing facilities. This group of pipelines is usually short in length (a couple of hundreds of meters) and with small outside diameters. The Outside Diameters (OD) of gathering pipes range from 101.6 to 304.8 mm (4 to 12 inches).
- **Transportation (or transmission) pipelines** are long pipelines with large OD, used to transport liquid (oil, natural gas and petrochemical by-products) within a province/state, between cities or across countries. The OD range of for this group ranges form 101.6 to 1,212 mm (4 to 48 inches).

- **Distribution pipelines** are pipelines with small OD, used to deliver liquid (oil and natural gas) to the final consumers. The OD of distribution pipes ranges from 12.7 to 152.4 mm (0.5 to 6 inches).

Oil and natural gas network diagrams used for gathering, transmission and distribution pipelines are illustrated in Figure 2. 1 and 2.2

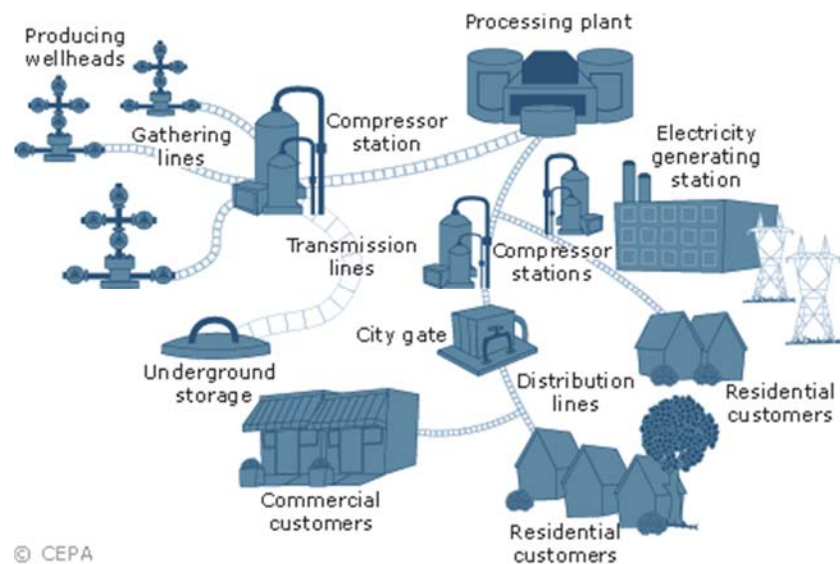


Figure 2. 1 Gas delivery network diagram (from Canadian Energy Pipeline Association (CEPA))

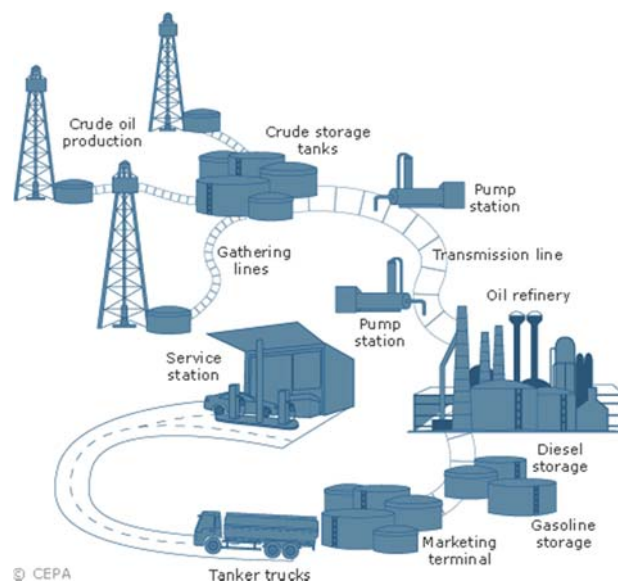


Figure 2. 2 Liquids delivery network diagram (from Canadian Energy Pipeline Association (CEPA))

Pipelines are designed based on the characteristics and conditions of the fluids they convey such as internal pressure, and temperature. A main difference between oil and natural gas pipelines is the transmitted pressure. Oil pipes can transport liquids at pressures between 4.1 to 6.9 MPa, while gas pipelines are typically intended to transport natural gas at higher than pressure 6.9 MPa. Other than the internal pressure, the temperature is quite critical for designing oil and gas pipes as it can induce high compressive stresses in the pipe wall.

Pipes transporting oil and gas are made from steel with an OD of 100 to 1,200 mm (4 to 48 inches), while the ones for transporting natural gas are made from carbon steel with OD of 51 to 1,400 mm (2 to 56 inches). Pipeline networks are mainly buried in the soil at a typical depth of 1 to 2 meters (3 to 6 feet).

Common oil and gas pipes have OD-to-thickness ratios (D/t) of 20 to 100. In addition, the steel pipes are made of steel grades ranging from X42 to X100. In a steel pipe, local buckling strains are affected by loading conditions, material properties, and pipe geometry.

2.3 Survey of Past Research on Buckling Strains

Two kinds of buckling phenomena can take place in pipes, global and local buckling. Global buckling takes place in a relatively long segment of pipe in the order of several times the pipe OD, while local buckling takes place over a very short length, typically within one or two pipe diameters in length. In buried pipelines, local buckling takes place when the pipe undergoes differential settlement or differential frost heave, thus inducing bending in the pipe wall. When bending deformations in the pipe are excessive, longitudinal compressive strains can become large, and local buckling may take place. Local buckling is characterized by local compressive strains increasing at a faster rate than the average compressive strains along the remaining length of the pipe. The threshold strains at which localized strains initiate are referred to as buckling strains. Local buckling in pipes of common geometries and grades involves deformations that are large and well into the non-linear response of the material. A large number of experimental studies were performed to determine threshold buckling strains for various pipe geometries and loading conditions. The effects of various parameters on buckling strains have been investigated; pipe geometry, mechanical properties the pipe steel, internal pressure magnitude, and axial loading magnitude.

2.3.1 Unpressurized Pipes

2.3.1.1 Buckling strains based on elastic analysis

A buckling strain solution for pipes subjected to axial compression is provided in classical references such as Timoshenko and Gere (1961). For steel pipes, the critical strain ε_{cr} takes the form

$$\varepsilon_{cr} = 1.21 \frac{t}{D_{ave}} \quad (2.1)$$

where D_{ave} is the mean diameter and t is the wall thickness. Equation 2.1 is based on elastic analysis and does not account for internal pressure nor bending. In the absence of a closed form solution for the case of bending, Seide and Weingarten (1961), Kim (1992) and Johnson (1966) conservatively adopted Eq. (2.1) for unpressurized pipes subject to bending. Also, Gresnigt (1986) adopted Eq. (2.1) to estimate the buckling strains of pipe subjected to the combination of bending and tensile forces. An experimental study by Sherman (1976) on pipes subjected to bending proposed the following expression for the critical strains

$$\varepsilon_{cr} = 16 \left(\frac{t}{OD} \right)^2 \quad (2.2)$$

in which OD is the pipe outside diameter.

2.3.1.2 Buckling Strains based on inelastic analysis

Batterman (1965) developed buckling strain solutions for cylinders subjected to axial compression. His solution accounts for the elasto-plastic material response. Two buckling strains equations were proposed based on the incremental theory of plasticity and the deformation theory of plasticity. These are;

$$\varepsilon_{cr} = \frac{4\psi_s}{\sqrt{3[(5-4\mu)\psi_t - (1-2\mu)^2]}} \times \frac{t}{D_{ave}} \quad (2.3)$$

$$\varepsilon_{cr} = \frac{4\psi_s}{\sqrt{(3\psi_s + 2 - 4\mu)\psi_t - (1-2\mu)^2}} \times \frac{t}{D_{ave}} \quad (2.4)$$

where ψ_s is the ratio of the elastic modulus to the secant modulus , and ψ_t is ratio of the elastic modulus to the tangent modulus and μ is Poisson's ratio. Ellinas (1984) and Ellinas et al. (1987) proposed the use Eq.(2.1) to estimate buckling strains for steel pipes with low strain hardening subjected to pure bending in the absence of axial force and internal pressure.

2.3.2 Critical strains of Pressurized pipes

The present section focuses on experimental investigations aimed at characterizing the buckling strains of pipes subjected to internal pressure, axial force and bending deformation.

2.3.2.1 Bouwkamp & Stephen (1973)

Bouwkamp et al. (1973) tested seven longitudinally welded pipe specimens 1219 mm in OD and with OD/t of 85 and 104. Pipe specimens were made of X60 steel with a wall thicknesses of 11.7 and 14.2 mm. The pipes were subjected to combination of internal pressure (corresponding to 95% of the yield strength), axial compressive force and imposed bending deformation to model pipe differential settlement. Two pipes with X65 material and thicker wall thickness (14.2 mm) were welded on either ends of pipe specimens to force local buckling away from the ends.

2.3.2.2 Gresnigt (1986)

Gresnigt (1986) developed a semi-analytical model to determine the critical strains for pipes subjected to the combinations of axial force (tension), internal pressure and bending which accounts for the elasto-plastic response. He proposed buckling strains expressions

$$\varepsilon_{cr} = 0.25 \frac{t}{r'} - 0.0025 + 3000 \left(\frac{P_a r_m}{Et} \right)^2 \frac{|P_a|}{P_a} \quad \text{for} \quad \frac{t}{r'} > \frac{1}{60} \quad (2.5)$$

$$\varepsilon_{cr} = 0.10 \frac{t}{r'} + 3000 \left(\frac{P_a r_m}{Et} \right)^2 \frac{|P_a|}{P_a} \quad \text{for} \quad \frac{t}{r'} < \frac{1}{60} \quad (2.6)$$

where P_a is the applied pressure taken as positive for internal pressure, E is modulus of elasticity of steel pipes (30,000ksi) and r' is an imperfection factor which accounts for pipe ovalization which given by

$$r' = \frac{r_m}{1 - \frac{3a}{r_m}} \quad (2.7)$$

where r_m is pipe mid-wall radius and a is the ovalization due to bending given by

$$a \approx \frac{D_{\max} - D_{\min}}{4} \quad (2.8)$$

and $D_{\max}$ and $D_{\min}$ are maximum and minimum values of the OD respectively.

2.3.2.3 Mohareb et al. (1994)

Mohareb et al. (1994), performed full-scale tests on a series of the specimens under combinations of internal pressure (0, 36, 72 and 80 percent of hoop stress), axial forces (0, 18, 20, 36 and 40 percent of axial load at the yield point) and bending which was done in experimental and numerical studies. Two pipe sizes were investigated: four specimens had an OD of 508 mm and a OD-to-wall thickness ratio of 64, and the second one had 324 mm in OD with a OD-to-wall thickness ratio of 51. The numerical study consisted of nonlinear finite element simulation based on shell analysis (Mohareb et al. 2001). The material was modeled as elasto-plastic with isotropic hardening. While the model was able to replicate the buckling modes observed in the tests, the corresponding buckling strains were higher than those measured in the tests. The study proposed a buckling strain expression based on the finite element model for a pipe subjected to a combination of axial force, bending and internal pressure and took the form

$$\varepsilon_{cr} = \left[1.63 + 0.456 \left(\frac{\sigma_h}{F_y} \right) + 1.21 \left(\frac{\sigma_h}{F_y} \right)^2 \right] \times \left[1.0 + 0.962 \left(\frac{P_t}{P_y} \right) - 4.44 \left(\frac{P_t}{P_y} \right)^2 \right] \times \left(\frac{D}{t} \right)^{-1.20} \quad (2.9)$$

where F_y is the yield stress, σ_h is the hoop stress induced by internal pressure, P_t is the axial force and $P_y = A\sigma_y$ where A cross-sectional area of the pipe and σ_y is the yield strength. Experimental results to compare pipe moment capacity to analytically obtained interaction equations Mohareb and Murray (1999).

2.3.2.4 Yoosef-Ghodsi et al. (1994)

Yoosef-Ghodsi et al. (1994) carried out an experimental study on girth-welded pipes. The pipe specimens tested and loading conditions were identical to those in Mohareb et al. (1994). The critical strains obtained were found to be triggered by girth welds and their magnitudes were found to be less than those of plain pipes.

2.3.2.5 Zimmerman et al. (1995)

Zimmerman et al. (1995) investigated buckling strains for pipes based on an experimental numerical study and proposed the following buckling strain equation

$$\varepsilon_{cr} = 0.018 \left[8.5 \left(\frac{t}{D} \right)^2 + \frac{120 - D/t}{5000} \left(\frac{\sigma_h}{F_y} \right)^2 + 0.0021 \right] (85 - n) \quad (2.10)$$

in which σ_h is given by

$$\sigma_h = \frac{P(D - 2t)}{2t} \quad (2.11)$$

In Eq.(2.11), P is internal pressure and strain hardening characteristics n is defined by Ramberg-Osgood equation as

$$\varepsilon = \frac{\sigma}{E} + \left(0.005 - \frac{F_y}{E} \right) \left(\frac{\sigma}{F_y} \right)^n \quad (2.12)$$

the strain hardening characteristics n given in Eq.(2.12) was estimated using yield stress-to-ultimate stress of pipe in longitudinal tension test as

$$n = \frac{3.14}{1 - F_y/F_u} \quad (2.13)$$

where F_u is ultimate stress in an axial tension test.

2.3.2.6 DelCol et al. (1998)

DelCol et al. (1998) performed an experimental study with the length of 3.55 OD, on pipes with OD= 762 mm and a OD-to-wall thickness ratio of 92. The internal pressure was selected to induce

0, 20, 40 and 80 percent of hoop stress, bending and axial. The buckling strains of the specimens were reported for gage lengths of $1/3D$.

2.3.2.7 Myrholm et al. (2001)

Myrholm et al. (2001) conducted an experimental study aimed at comparing the behaviour and buckling strains of girth-welded pipes to those without a girth-weld. Eight pipe specimens were tested with OD-to-wall thickness ratios of 62 to 85 and an OD of 508 mm. Specimen length were 3.5 OD. All specimens were subjected to axial loading, bending moment and internal pressures inducing hoop stresses of 0, 40 and 80 percent of the yield strength. Six of the eight specimens had a girth-weld at the middle of the specimens and two of them were plain. The specimen curvatures were increased until the formation of a wrinkle. The gage length on the compression side was taken as 0.25 OD.

2.3.2.8 Dorey et al. (2001)

The work in Dorey et al. (2001) aimed at expanding the previous experimental database by testing additional pipes, conducting shell FEA analysis to predict pipe response and develop new buckling strains equations. The study focused on pipes with OD/t in the range 51-92 under combinations of axial loading, internal pressure inducing hoop stress from 0 to 80% of yielding and increasing curvature. Pipe OD was 762 mm and the length-to-OD ratio was 3.2 was chosen to prevent the end effects from influencing stress distribution on the middle of the pipe where buckling is anticipated. Two kinds of initial imperfections were assumed. In plain pipes, blister type initial imperfections were enforced, while for girth welded pipes, an offset initial imperfection was applied. A gage length of $1/3D$ was chosen to determine the buckling strains in the FEA and experiments. Four equations were proposed for estimating the buckling strains for (a) plain and (b) girth welded pipes, for (a) the case of gradual yielding stress-strain relationship and (b) the case of steel with distinct yield plateau (Table 2.1)

Table 2. 1 Buckling strains equations conducted by Dorey et al. (2001) for plain and girth welded pipes

Geometry	Stress-strain relationship	Equation
Plain pipes	Rounded	$\varepsilon_{cr} = \left(\frac{2.9398}{D/t} \right)^{1.5921} \left(\frac{1}{1 - 0.8679 \left(\frac{p}{p_y} \right)} \right) \left(\frac{E}{F_y} \right)^{0.8542} \left[1.2719 - \left(\frac{imp}{100} \right)^{0.1501} \right] \quad (2.14)$
	Plateau	$\varepsilon_{cr} = 40.4 \left(\frac{t}{D} \right)^2 \left(\frac{1}{1 - 0.906 \left(\frac{p}{p_y} \right)} \right) \left(\frac{E}{F_y} \right)^{0.80} \left[1.12 - \left(\frac{imp}{100} \right)^{0.15} \right] \quad (2.15)$
Girth welded pipes	Rounded	$\varepsilon_{cr} = \left(\frac{8.9948}{D/t} \right)^{1.7239} \left(\frac{1}{1 - 0.8920 \left(\frac{p}{p_y} \right)} \right) \left(\frac{E}{F_y} \right)^{0.7012} \left[1.0868 - \left(\frac{offset}{t} \right)^{0.0863} \right] \quad (2.16)$
	Plateau	$\varepsilon_{cr} = 106 \left(\frac{t}{D} \right)^2 \left(\frac{1}{1 - 0.50 \left(\frac{p}{p_y} \right)} \right) \left(\frac{E}{F_y} \right)^{0.70} \left[1.10 - \left(\frac{offset}{t} \right)^{0.09} \right] \quad (2.17)$

where imp is the amplitude of the blister and p_y is the internal yield pressure in the hoop direction which is obtained by

$$p_y = \frac{2tF_y}{(D - 2t)} \quad (2.18)$$

and *offset* was the size of the girth-welded imperfection defined as the offset of two cross-sections between sides of girth-welded specimens.

2.3.2.9 Dorey et al. (2002)

Dorey (2002) investigated the effect of material properties on the buckling strains under combined loadings in the pipes. Pipe OD were taken as 508 mm and OD/t ranged from 50 to 90. Specimen lengths were taken as 3.32 OD. Specimens were subjected to combinations of internal pressure (80 percent of yield strength), axial force and bending. The gage length for determining the buckling strains was taken as 1.0 OD. The results indicated that the lower buckling strains were reached for the case of higher grades of steel.

2.3.2.10 Suzuki et al. (2003)

Suzuki et al. (2003) numerically investigated the local buckling behaviour of the pipes made of steel grades X60, X80 and X100 subjected to axial compression and/or end bending. The OD of the pipes is 609.6 mm, an OD/t of 50, with a length of 2.5 OD for the compression models and a length of 3.5 OD. This study showed that, when the pipe is subjected to the axial compression and in absence of internal pressure, the nominal critical longitudinal strain depends on the yield strength and the yield-to-tensile strength ratio. When an unpressurized pipe is subjected to bending moment, the critical bending moments depend on the yield strength and the critical nominal longitudinal strain depends on and yield-to-tensile strength ratio.

2.3.2.11 Zimmerman et al. (2004)

Zimmerman et al. (2004) conducted an experimental and numerical study on large diameter spirally welded pipes. Pipe specimens had an OD 762 mm and a length of 4.2 OD. Some of the specimens were made of X70 steel and had a OD/t ratio of 82 while the remaining specimens were made of X80 steel and had a OD/t ratio of 48. The study involved an experimental and finite element components. Specimens which were subjected to the combined axial force, internal

pressure corresponding to hoop stresses between 0 and 80 % of the yield strength in addition to imposed bending deformation. The gages lengths taken to characterize the buckling strains were 1.0 OD and 2.0 OD. The study showed that the buckling strains of spirally welded pipes are identical to those of longitudinally welded pipes.

2.3.2.12 Adeeb et al. (2006)

Adeeb et al. (2006) numerically investigated the influence of the forming process on the apparent anisotropic material response of pipes and on the buckling strains. Two types of hardening material properties were investigated; isotropic hardening and nonlinear isotropic/kinematic hardening. Pipe OD of 610 mm, thickness of 15.1 mm and length of 4.38 OD subjected to pure bending was modeled. A comparison between the initial, longitudinal and circumferential stress- strain curves based on nonlinear isotropic/kinematic hardening was found to lead to a closer representation of experimentally observed the stress-strain curves than isotropic hardening.

2.3.2.13 Chou et al. (2006)

Chou et al. (2006) carried out nineteen tests and experimentally and numerically investigated the relationship between the local buckling strains and distributed strain sensors along pipelines and contribution of strains to find out the initial buckling strains. Specimen properties are provided in Table 2. 2. All pipes were subjected to bending. The study illustrated the advantage of using distributed strain sensors to determine the buckling strains in the pipes.

Table 2. 2 Geometry of all specimens in Chou et al. (2006) study

OD (mm)	609.6 to 914.4
OD/t	39 to 69
Material property	X80 to X100
Internal pressure (% of hoop stress)	0 to 77
Gage length	2D and 2.25D (for the larger pipe)

2.3.2.14 Fatemi et al. (2008)

Fatemi et al. (2008) numerically predicted the local buckling and post-buckling response pipes subjected to a combinations of internal pressure, axial force and end bending. The finite element

results were calibrated against experimental data. The various properties of the specimens are provided in Table 2. 3. The study showed that for unpressurized pipes, the degree of isotropic hardening influences local buckling strains. In contrast, the degree of isotropic hardening was observed to have no effect on pressurized specimens. In a subsequent study, Fatemi et al. (2009) numerically investigated the effect of geometric factors such as OD, internal pressure, length of the pipe and girth weld offset on the local buckling strains of high-strength pipes subjected to internal pressure and bending. The range of variables examined is provided in Table 2. 4.

Table 2. 3 Geometry of the specimens in Fatemi et al. (2008) study

OD (mm)	528- 914.4
OD/t	40 to 70
L/OD	3.5 to 6.1
Gage length	1.25D
Material properties	X80 and X100
Internal pressure (% of hoop stress)	0 and 80

Table 2. 4 Geometry of the specimens in Fatemi et al. (2009) study

OD (mm)	609, 914 and 1219
OD/t	45 and 60
L/OD	3.5 and 5.5
Gage length	1D and 2D
Material properties	X100
Internal pressure (% of hoop stress)	0 and 80
Girth weld offset misalignment (mm)	0.2 to 1.6

2.3.2.15 Zhang et al. (2008)

The Grey system theory is a curve fitting technique when part of the information is unknown. Based on the Grey system theory, Zhang et al. (2008) proposed buckling strain equations for the case of pure axial force $\varepsilon_{cr}^{(c)}$ and pure bending $\varepsilon_{cr}^{(b)}$. The corresponding equations are

$$\varepsilon_{cr}^{(c)} = 74.21 \left(\frac{D}{t} \right)^{-2.31} \left(1 + \frac{P}{P_c} \right)^{1.59} \left(\frac{E}{F_y} \right)^{0.67} \left(\frac{F_y}{F_u} \right)^{-3.84} \quad (2.19)$$

$$\varepsilon_{cr}^{(b)} = 1.006 \left(\frac{D}{t} \right)^{-0.37} \left(1 + \frac{P}{P_c} \right)^{3.84} \left(\frac{E}{F_y} \right)^{0.05} \left(\frac{F_y}{F_u} \right)^{-2.98} \quad (2.20)$$

where P_c is circumferential yield pressure given by

$$P_c = \frac{2F_y t}{D} \quad (2.21)$$

2.3.2.16 Ozkan (2008)

Ozkan (2008) tested eight specimens of X65 steel with 508 mm OD and OD-to-wall thickness ratio of 80. Pipe length was taken as 3.42 OD. Six of the specimens were subjected to combined internal pressure inducing hoop stresses of 0, 40 and 80% yield strength, axial tension, and bending. A shell finite element analysis model was also developed and characterize the deformational response of the pipes (Ozkan and Mohareb 2009). A gage length 1D was adopted to characterize the buckling strains. A technique for measuring the buckling strains was developed using fiber optic sensors (Zhang et al. 2008). The study illustrated the beneficial effect of tensile force in delaying buckling and buckling strain equations were developed in terms of the internal pressure p and axial force ratio A_r (Ozkan 2008).

For $p = 0$

$$\begin{cases} \varepsilon_{cr} = 0.25A_r + 0.48 & 0 \leq A_r < 0.2 \\ \varepsilon_{cr} = 0.53 & A_r \geq 0.2 \end{cases} \quad (2.22)$$

For $p = 0.4F_y$

$$\begin{cases} \varepsilon_{cr} = 1.15A_r + 0.63 & 0 \leq A_r < 0.2 \\ \varepsilon_{cr} = 0.35A_r + 0.79 & 0.2 \leq A_r < 0.4 \\ \varepsilon_{cr} = 0.93 & A_r \geq 0.4 \end{cases} \quad (2.23)$$

For $p = 0.8F_y$

$$\begin{cases} \varepsilon_{cr} = 0.45A_r + 1.34 & 0 \leq A_r < 0.4 \\ \varepsilon_{cr} = 1.52 & A_r \geq 0.4 \end{cases} \quad (2.24)$$

2.3.2.17 Chen et al. (2008)

Chen et al (2008) numerically determined the buckling strains and buckling modes for steel pipes. with OD of 1219, OD/t ranging from 46.2 to 66.3 made of X80 steel. The study investigated the pipes pressurized at 12 MPa and unpressurized pipes subjected to compression and bending. The effects of material properties, internal pressure and the ratio of OD/t on the buckling strains were investigated.

2.3.2.18 Cho et al. (2009)

Cho et al. (2009) numerically and experimentally investigated the behaviour of aged and non-aged pipes made of steel X100 subjected to pure bending. For the experimental component of the study, specimens with OD of 711mm and OD/t of 33.6 subjected to pure bending were tested. In the first part of the numerical component, a comparison between aged pipes with different material properties was carried out. The specimens had an OD of 1219 mm and OD/t of 61.5 with the material properties shown in Table 2. 5. In the second part of the numerical simulation, a parametric study was conducted to investigate the effect of the length of yield plateau: three values for yield plateau were examined 0%, 0.05% and 0.10%. The specimens examined had an X80 steel with OD of 762 mm and OD/t of 47.6. The study showed that for steel with smooth stress-strain curves, the compressive strains based on 1D gage length for steels with a short plateau are higher than those with a long yield plateau.

Table 2. 5 Material properties in numerical study in Cho et al. (2009)

	YS0.5 (MPa)	TS (MPa)	YR (MPa)	YpEI (%)	uEI (%)
Type A	715	762	93.8	0.7	5.7
Type B	743	846	87.8	0.0	5.9

2.3.2.19 Suzuki et al. (2010)

Suzuki et al. (2010) numerically and experimentally studied the local buckling strain of the pipes over a gage length of $2D$ for pipes subjected to internal pressure inducing a hoop stress of 60% of yield strengths and bending moments. The OD of the pipes of 1219.2 mm in diameter and the $OD/t = 55.4$ and a length of 6.5 OD made of steel X80. This study showed that the buckling strains reduce with choosing the higher value of the yield-to-ultimate tensile strength (UTS) ratio.

2.3.2.20 Fathi et al. (2010)

Fathi et al. (2010) numerically investigated the buckling strains of high strength steel pipes (HSSP) with the OD of 762 mm and OD/t of 56 made of X100 steel pressurized at 18.4 MPa. The results were compared with the experimental data conducted by TransCanada Pipeline. Two constitutive elasto-plastic constitutive models were investigated: isotropic and anisotropic. The buckling strains based on a gage length of $(2D)$ were determined. The study showed that for HSSP the anisotropic constitutive material model results is closer predictions to the experimental results. Fathi, A. (2012) and Fathi and Cheng (2012) numerically investigated the effect of anisotropy on the buckling strains of high strength steel pipes. The effect of OD-to-thickness ratio, internal pressure, material properties were studied. Three finite element models were developed for the same OD of 762 mm, length of 5OD, and OD/t ratios of 50, 70 and 90 were made of X80 and X100 steel subjected to internal pressure (0, 40% and 80% of hoop stress) and bending. The study showed that, for unpressurized pipes, there is a strong correlation between the buckling strains based on a gage length of $2D$ and material anisotropy, while the relationship is relatively weak in highly pressurized pipes.

2.3.2.21 Yoosef-Ghodsi et al. (2014)

Yoosef-Ghodsi et al. (2014) studied the importance of material properties, OD/t , and internal pressure, and girth weld effects on the buckling strains. The experimental database consisted of 120 specimens and were conducted by onshore and offshore pipeline operators. The OD/t for onshore pipes ranged from 40 to 120, and that based on offshore pipes ranged from 15 to 20. The pipes were subjected to a combination of internal pressure (up to 80% of hoop stress), end bending and axial force. The local buckling strains were studied over gages lengths of $1D$ or $2D$. The study indicated that in general, existing critical strain equations could be used to establish compressive

strain limits for new and in-service pipelines when the strain demand is modest and conservatism is tolerable. For pipelines with high strain demand, a less conservative strain limit can be obtained by carrying out detailed three-dimensional finite element analysis or full-scale tests.

2.3.2.22 Neupane et al. (2012a,b)

Neupane et al (2012a) adopted the Chaboche and Armstrong-Fredrick models to simulate the plastic anisotropic behaviour of steel pipe. In the first numerical simulation, the plastic anisotropy of the pipe was modeled using nonlinear kinematic hardening model in ABAQUS to represent the evolution of the back-stress. The model captures the different circumferential and experimental stress-strain curves as observed in experiments. The Chaboche model was found to be more suitable than Armstrong-Fredrick model for the modelling of steel pipes.

Neupane et al (2012b) developed a finite element model to study pipes with OD of 1219.2 mm, OD/t= 48 and a length of 3.3 OD to investigate the deformational behaviour of high strength steel pipes. Two cases were investigated for no pressure and fully pressurized pipes with hoop stresses of 80% of the steel yield strength. Isotropic and combined kinematic hardening material models were investigated. The study showed that, when the pipes are subjected to transverse displacement in the absence of the internal pressure, longitudinal stresses are induced in the longitudinal direction, while the stresses in the circumferential and thickness directions are negligible. In this case, the longitudinal stress-strain curve are closer than the circumferential stress-strain curve to the initial stress-strain curve of the material. When the pipes were subjected to the maximum operating pressure, the load displacement curve obtained based on the isotropic material model with circumferential material data was closer to the load versus displacement curve using the kinematic material model with the analytical stress strain curve. This is due to the fact that the circumferential stresses have a significant contribution to the von Mises stress in the case of high pressure and the circumferential stress-strain curve provide a closer approximation for the actual stress strain behavior of the material than the longitudinal stress-strain relation.

The study also showed that the peak moment value is reduced when the pipe is subjected to internal pressure and the value of the buckling strains in the case subjected to internal pressure is higher than the one in the absence of internal pressure.

2.4 Buckling Strains in Codes and Standards

In order to control local buckling of buried pipelines undergoing deformations, different buckling strains equations have been developed by standards. This section presents the relevant provisions of the Canadian Oil and Gas Pipeline Systems (CAN/CSA Z662-15) and Det Norske Veritas Offshore Standard for Submarine Pipeline Systems (DNV-OS-F101) to obtain the buckling strains equations.

2.4.1 CAN/CSA Z662 Provisions

The Canadian oil and gas pipeline systems (CAN/CSA Z662-15, 2015) limits the compressive strains ε_{cf} for pipes subject to internal and external pressure and bending moment to

$$\varepsilon_{cf} \leq \varphi_{ec} \varepsilon_{cr} \quad (2.25)$$

where φ_{ec} is resistance factor which is equal to 0.8 for compressive strains, ε_{cr} is ultimate compressive strains of the pipe wall. The compressive strains limit ε_{cr} should be taken as

$$\varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left[\frac{(p_{i \max} - p_{e \min}) D}{2tE} \right]^2 \quad \text{for} \quad \frac{(p_{i \max} - p_{e \min}) D}{2tF_y} < 0.4 \quad (2.26)$$

$$\varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left(\frac{0.4F_y}{E} \right)^2 \quad \text{for} \quad \frac{(p_{i \max} - p_{e \min}) D}{2tF_y} \geq 0.4 \quad (2.27)$$

where $p_{i \max}$ is maximum internal design pressure, $p_{e \min}$ is minimum external hydrostatic pressure, $E = 207,000$ MPa is the modulus of elasticity which is and F_y is the specified minimum yield strength. A summary of key parameters in buckling strain studies is provided in Table 2.2 and Key buckling equations are summarized in Table 2.3

2.4.2 Provisions of Det Norske Veritas Offshore Standard (DNV-OS-F101)

For pipes subjected to a combination of internal and external pressures, axial force and longitudinal compressive subjected to bending deformations, Det Norske Veritas Offshore Standard for Submarine Pipeline Systems (DNV-OS-F101, 2013) provides the following limiting equations for the strains

$$\varepsilon_{cr} = 0.78 \left(\frac{t}{D} - 0.01 \right) \left(1 + 5.75 \frac{P_{\min} - P_e}{P_b} \right) \alpha_h^{-1.5} \alpha_{gw} \quad (2.28)$$

where $P_{\min}$ is minimum internal pressure and α_h is calculated as

$$\alpha_h = \left(\frac{F_y}{F_u} \right)_{\max} \quad (2.29)$$

In Eq.(2.28), α_{gw} is called girth weld reduction factor which is obtained as

$$\alpha_{gw} = \begin{cases} 1 & \frac{D}{t} \leq 20 \\ 1 - 0.01 \left(\frac{D}{t} - 20 \right) & \frac{D}{t} > 20 \end{cases} \quad (2.30)$$

and P_b is the burst pressure given by $P_b = \left[2t / (D - t) \right] f_{cb} (2/\sqrt{3})$ and $f_{cb} = \min(f_y, f_u/1.15)$.

2.5 Plastic Interaction Relations for Pipes

Mohareb et al. (1995) developed a plastic interaction equation for pipes subjected to axial loading, internal pressure and bending moments. The interaction relations were shown to agree well with experimental results for pipes with OD/t=51 and 64 (Mohareb and Murray 1999). The formulation was extended to develop plastic interaction relations for pipes subjected to combinations of normal forces, internal or external pressure, twisting moments, biaxial bending moments, and biaxial shearing forces using a lower bound approach (Mohareb 2001 and Mohareb (2002) and an upper bound approach (Mohareb 2003). Ozkan and Mohareb (2002) and Ozkan and Mohareb (2003) developed an experimental study to assess the validity of the interaction equations for the pipes specimens subjected to combinations of shear force, bending and twisting moments. The program consisted of six specimens with nominal OD=406 mm, and OD/t= 42.7. Specimen height varied from 422 to 1641 mm. The study showed that the interaction equations predict well the plastic moment capacity for pipes subjected to torsion and shear. An experimental setup for testing pipes under combined loads was developed (Ozkan and Mohareb 2006) and used to determine the modified plastic moment resistance of pipes for pipes subjected to combinations of internal pressure and axial tension (Ozkan and Mohareb 2009). The study has shown good agreement

between the analytical predictions and experimental results. Ozkan and Mohareb (2009) conducted two full scale tests on pipes subjected to internal pressure, axial force bending and twisting moments and shear. Comparison with the analytical predictions and finite element analysis has demonstrated the validity of the interaction relations developed in Mohareb (2002).

2.6 Objectives and Scope of present Study

The present chapter surveyed various experimental and shell FEA studies on oil and gas pipes with OD-to-thickness ratio ranges from 50 to 90 subjected to various levels of internal pressure and subjected to bending deformation which emulate frost heaving and differential settlements in the field. A common observation in past studies is that finite element models reliably predict pipe moment capacity but tend to overestimate the buckling strains compared to experimental results. The present study thus attempts to improve the finite element modelling by adopting 3D FEA in conjunction with a kinematic hardening material model that actually captures the residual stresses induced in the pipe wall throughout the UOE manufacturing process. The model is subsequently used to predict the deformational behaviour of steel pipes. For comparisons, the deformational behaviour of pipes is predicted using conventional FEA modelling which omits the effect of UOE forming of pipes. The buckling strains are then extracted from both models and compared to assess the effect of forming.

Table 2. 6 Overview of key parameters in past studies

Types of Research		Paper	Geometry and Loading conditions							Gage length
Experimental	Numerical		OD (mm)	OD/t	Internal pressure (MPa)	Axial force	Bending	Material property	Girth- (G), Spiral- (S) welded or Plain (P) Pipe	
✓	✗	Bouwkamp et al. (1973)	1219	104 and 85	✓	✓	✓	X60 and X65	(P)	5/2D
✓	✓	Mohareb et al. (1994)	324 and 508	51 and 64	✓	✓	✓	X52 and X56	(P)	1D
✓	✗	Yoosef-Ghodsi et al. (1994)	324 and 508	50 and 63	✓	✓	✓	X52 and X56	(P) and (G)	1/2D and 1D
✓	✓	DelCol et al. (1998)	762	92	✓	✓	✓	X70	(P)	1/3D
✓	✗	Myrholm et al. (2001)	508	62 to 85	✓	✓	✓	Yield stress 460 and 479 (MPa)	(P) and (G)	1/4D and 1/2D
✓	✓	Dorey et al. (2001)	762	51 to 92	✓	✓	✓	X50 to X70	(P) and (G)	1/3D
✓	✗	Dorey et al. (2002)	508	50 to 90	✓	✓	✓	X52 to X80	(P)	1D
✗	✓	Suzuki et al. (2003)	609.6	50	✗	✓	✓	X60 to X100	(P)	2D

Types of Research		Paper	Geometry and Loading conditions							Gage length
Experimental	Numerical		OD (mm)	OD/t	Internal pressure (MPa)	Axial force	Bending	Material property	Girth- (G), Spiral- (S) welded or Plain (P) Pipe	
✓	✓	Suzuki et al. (2010)	1219.2	55.4	✓	✗	✓	X80	(P)	2D
✓	✓	Zimmerman et al. (2004)	762	82 and 48	✓	✓	✗	X70 and X80	(S)	1D and 2D
✓	✓	Chou et al. (2006)	609.6 to 914.4	39 to 69	✓	✗	✓	X80 to X100	(P) and (G)	2 and 2.25D
✗	✓	Fatemi et al. (2008)	528 to 914.4	40 to 70	✓	✓	✓	X80 and X100	(P)	1.25D
✗	✓	Fatemi et al. (2009)	609, 914 and 1219	45 and 60	✓	✗	✓	X100	(G)	1D and 2D
✓	✓	Ozkan (2008)	508	80	✓	✓	✓	X65	(P)	2D
✗	✓	Chen et al. (2008)	1219	46.2 to 66.3	✓	✓	✓	X80	(P)	-
✓	✓	Cho et al. (2008)	711, 762 and 1219	33.6, 47.6 and 61.5	✗	✗	✓	X80 and X100	(P)	1D
✗	✓	Fathi et al. (2010)	762	56	✓	✗	✓	X100	(P)	2D
✗	✓	Fathi, A. (2012)	762	50, 70 and 90	✓	✗	✓	X80 and X100	(P)	2D

Types of Research		Paper	Geometry and Loading conditions							Gage length
Experimental	Numerical		OD (mm)	OD/t	Internal pressure (MPa)	Axial force	Bending	Material property	Girth- (G), Spiral- (S) welded or Plain (P) Pipe	
✓	✓	Yoosef-Ghodsi et al. (2014)	120 specimens	15 to 120	✓	✓	✓	120 specimens	(P) and (G)	1D and 2D
✗	✓	Present study	610	64	✓	✗	✓	X52	(P)	2D and 3D

Table 2. 7 Summary of various studies and standards on buckling strains equations of pipes subjected to Axial force (A), Bending (B), Internal (I) and/or External (E) pressures

Publication	Suggested Buckling Strains Equations	A	B	I
Timoshenko and Gere (1961)	$\varepsilon_{cr} = 1.21 \frac{t}{D_{ave}}$	✓	✗	✗
Batterman (1965)	$\varepsilon_{cr} = \frac{4\psi_s}{\sqrt{3[(5-4\mu)\psi_t - (1-2\mu)^2]}} \times \frac{t}{D_{ave}}$ $\varepsilon_{cr} = \frac{4\psi_s}{\sqrt{(3\psi_s + 2 - 4\mu)\psi_t - (1-2\mu)^2}} \times \frac{t}{D_{ave}}$	✓	✗	✗
Sherman (1976)	$\varepsilon_{cr} = 16 \left(\frac{t}{D} \right)^2$	✗	✓	✗
Gresnigt (1986)	$\begin{cases} \varepsilon_{cr} = 0.25 \frac{t}{r'} - 0.0025 + 3000 \left(\frac{P_a r_m}{Et} \right)^2 \frac{ P_a }{P_a} & , \quad \frac{t}{r'} > \frac{1}{60} \\ \varepsilon_{cr} = 0.10 \frac{t}{r'} + 3000 \left(\frac{P_a r_m}{Et} \right)^2 \frac{ P_a }{P_a} & , \quad \frac{t}{r'} < \frac{1}{60} \end{cases}$ $r' = \frac{r_m}{1 - (3a/r_m)} \quad \text{and} \quad a \approx \frac{D_{max} - D_{min}}{4}$	✓	✓	✓
Mohareb et al. (1994)	$\varepsilon_{cr} = \left[1.63 + 0.456 \left(\frac{\sigma_h}{F_y} \right) + 1.21 \left(\frac{\sigma_h}{F_y} \right)^2 \right]$ $\times \left[1.0 + 0.962 \left(\frac{P_t}{P_y} \right) - 4.44 \left(\frac{P_t}{P_y} \right)^2 \right] \times \left(\frac{D}{t} \right)^{-1.20}$ $P_y = A\sigma_y$	✓	✓	✓

Publication	Suggested Buckling Strains Equations	A	B	I
Yoosef-Ghods et al. (1994)	$\left\{ \begin{array}{ll} \text{For plain pipes} & \varepsilon_{cr} = 4.58 \frac{P}{P_y} \quad 54 \leq \frac{D}{t} \leq 61 \\ \text{For girth-welded pipes} & \varepsilon_{cr} = 2.78 \frac{P}{P_y} \quad 50 \leq \frac{D}{t} \leq 63 \end{array} \right.$	✓	✓	✓
Zimmerman (1995)	$\varepsilon_{cr} = 0.018 \left[8.5 \left(\frac{t}{D} \right)^2 + \frac{120 - D/t}{5000} \left(\frac{\sigma_h}{F_y} \right)^2 + 0.0021 \right] (85 - n)$ $\sigma_h = \frac{P(D - 2t)}{2t} \quad \text{and}$ $\varepsilon = \frac{\sigma}{E} + \left(0.005 - \frac{F_y}{E} \right) \left(\frac{\sigma}{F_y} \right)^n \rightarrow n \approx \frac{3.14}{1 - F_y/F_u}$	✓	✓	✓
Dorey et al. (2001)	$\left\{ \begin{array}{l} \varepsilon_{cr} = \left(\frac{2.9398}{D/t} \right)^{1.5921} \left(1 - 0.8679 \left(\frac{p}{p_y} \right) \right)^{-1} \left(\frac{E}{F_y} \right)^{0.8542} \left[1.2719 - \left(\frac{imp}{100} \right)^{0.1501} \right] \\ \varepsilon_{cr} = 40.4 \left(\frac{t}{D} \right)^2 \left(1 - 0.906 \left(\frac{p}{p_y} \right) \right)^{-1} \left(\frac{E}{F_y} \right)^{0.80} \left[1.12 - \left(\frac{imp}{100} \right)^{0.15} \right] \\ \varepsilon_{cr} = \left(\frac{8.9948}{D/t} \right)^{1.7239} \left(1 - 0.8920 \left(\frac{p}{p_y} \right) \right)^{-1} \left(\frac{E}{F_y} \right)^{0.7012} \left(1.0868 - \left(\frac{offset}{t} \right)^{0.0863} \right) \\ \varepsilon_{cr} = 106 \left(\frac{t}{D} \right)^2 \left(1 - 0.50 \left(\frac{p}{p_y} \right) \right)^{-1} \left(\frac{E}{F_y} \right)^{0.70} \left[1.10 - \left(\frac{offset}{t} \right)^{0.09} \right] \end{array} \right.$ $p_y = \frac{2tF_y}{(D - 2t)}$	✓	✓	✓
Schaumann et al. (2005)	$\left\{ \begin{array}{ll} \varepsilon_{cr} = 0.2 \left(\frac{t}{r_m} \right) + 0.01 \left(\frac{\sigma_h}{F_y} \right)^2 & \text{analytical} \\ \varepsilon_{cr} = 0.2 \left(\frac{t}{r_m} \right) + 0.02 \left(\frac{\sigma_h}{F_y} \right)^3 & \text{experimental} \end{array} \right.$ $0.2 \left(\frac{t}{r_m} \right) < \varepsilon_{cr} < 0.4 \left(\frac{t}{r_m} \right)$	✗	✓	✓

Publication	Suggested Buckling Strains Equations	A	B	I
Zhang et al (2008)	$\begin{cases} \varepsilon_{cr}^{(c)} = 74.21 \left(\frac{D}{t}\right)^{-2.31} \left(1 + \frac{P}{P_c}\right)^{1.59} \left(\frac{E}{F_y}\right)^{0.67} \left(\frac{F_y}{F_u}\right)^{-3.84} & \text{compression} \\ \varepsilon_{cr}^{(b)} = 1.006 \left(\frac{D}{t}\right)^{-0.37} \left(1 + \frac{P}{P_c}\right)^{3.84} \left(\frac{E}{F_y}\right)^{0.05} \left(\frac{F_y}{F_u}\right)^{-2.98} & \text{bending} \end{cases}$	✓	✗	✓
Ozkan (2008)	For 0% of hpps stress $\begin{cases} \varepsilon_{cr} = 0.25A_r + 0.48 & 0 \leq A_r < 0.2 \\ \varepsilon_{cr} = 0.53 & A_r \geq 0.2 \end{cases}$ For 40% of hpps stress $\begin{cases} \varepsilon_{cr} = 1.15A_r + 0.63 & 0 \leq A_r < 0.2 \\ \varepsilon_{cr} = 0.35A_r + 0.79 & 0.2 \leq A_r < 0.4 \\ \varepsilon_{cr} = 0.93 & A_r \geq 0.4 \end{cases}$ For 80% of hpps stress $\begin{cases} \varepsilon_{cr} = 0.45A_r + 1.34 & 0 \leq A_r < 0.4 \\ \varepsilon_{cr} = 1.52 & A_r \geq 0.4 \end{cases}$	✓	✓	✗
DNV-OS- F101-2013	$\varepsilon_{cr} = 0.78 \left(\frac{t}{D} - 0.01\right) \left(1 + 5.75 \frac{P_{\min} - P_e}{P_b}\right) \alpha_h^{-1.5} \alpha_{gw}$ $\alpha_{gw} = \begin{cases} 1 & \text{if } \frac{D}{t} \leq 20 \\ 1 - 0.01 \left(\frac{D}{t} - 20\right) & \text{if } \frac{D}{t} > 20 \end{cases}$ $\alpha_h = \left(\frac{F_y}{F_u}\right)_{\max}, \quad P_b = \frac{2t}{D-t} f_{cb} \frac{2}{\sqrt{3}} \text{ and } f_{cb} = \min\left(f_y, \frac{f_u}{1.15}\right)$	✓	✓	✓
CAN/CSA Z662-2015	$\begin{cases} \varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left[\frac{(p_{i\max} - p_{e\min})D}{2tE} \right]^2 & \text{for } \frac{(p_{i\max} - p_{e\min})D}{2tF_y} < 0.4 \\ \varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left(\frac{0.4F_y}{E} \right)^2 & \text{for } \frac{(p_{i\max} - p_{e\min})D}{2tF_y} \geq 0.4 \end{cases}$	✓	✓	✓

Chapter 3 Overview of the Theory of Plasticity

3.1 General

This chapter provides an overview of basic concepts of the theory of plasticity with emphasis on hardening models as they form the basis for finite element models described in subsequent chapters. Section 3.2 introduces different types of linear and nonlinear models such as perfectly plastic, elastic-perfectly plastic, elastic-linearly plastic and elastic-nonlinear plastic. Section 3.3 introduces the concept of strain decomposition and the concept of yield criterion is introduced in Section 3.4. The constitutive relationships between incremental stresses and incremental strain tensors is described in Section 3.5. Sections 3.6 and 3.7 present the plasticity flow and normality rules. Sections 3.8 and 3.9 present the key equations of isotropic plasticity and common simplifications in perfectly plastic models. Section 3.10 presents the plastic strain increment expressions for elastic-perfectly plastic materials and Section 3.11 provides the definition of effective stresses and effective plastic strains. Section 3.12 provides key concepts in linear kinematic hardening models and Section 3.13 provides the specifics of common kinematic hardening and nonlinear kinematic hardening models. The material in this chapter is extracted from Chakrabarty (2000), Dunne and Petrinic (2005), Kachanov (2006), Bower (2009), Kalnins et al. (2013 and 2015), Wu (2015) and Kelly (2016).

3.2 Stress-Strain Idealizations for Elasto-Plastic models

When a material is subjected to external loading, it will deform. Part of the deformation is reversible after unloading (elastic) and the remaining part is irreversible. Different stress-strain curves of the elastic-plastic material are shown in Figure 3. 1 and discussed in the following sections.

3.2.1 Perfectly Plastic Materials

Perfectly plastic materials exhibit an unlimited amount of strain at constant stress. In this type of materials, after removing the applied loading, the plastic strain cannot be recovered. The elastic strains in this type of materials are negligibly small (Figure 3. 1.A).

3.2.2 Elastic-Perfectly Plastic Materials

This type of materials is a combination of linear elastic and perfectly plastic materials in which the stresses and strains are linearly related until the yield point, after which the material behaves in perfectly plastic manner. In the plastic range, the stress remains constant as the strain increases (Figure 3. 1.B).

3.2.3 Elastic-Linear Plastic Materials

In elastic-linearly plastic materials, linear elastic response takes place up to the yielding. Beyond the yield point, the stress increases linearly with the plastic deformation. This phenomenon is called linear strain hardening. The stress-strain relationship has two slopes, the slope in the elastic range is Young's Modulus, E and the second slope in plastic range is the tangent modulus, E_T (Figure 3. 1.C).

3.2.4 Elastic-Nonlinear Plastic Materials

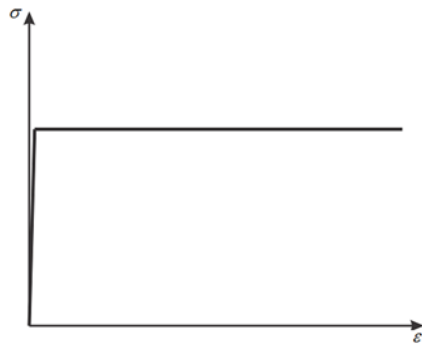
In this type of material properties, the stresses are proportional to strains only up to the yield stress. Beyond the yielding nonlinear strain hardening takes place. Unlike elastic-linear plastic materials, where the tangent modulus has a constant value, the tangent modulus, E_T is a function of the plastic strain history (Figure 3. 1.D).

3.3 Total Strain Decomposition

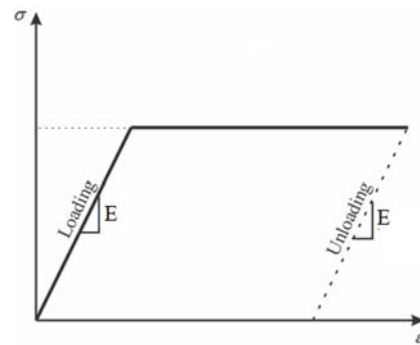
Under uniaxial loading, the total strain ε at a given stress can be divided into an elastic strain ε^e , and a plastic strain ε^p as

$$\varepsilon = \varepsilon^e + \varepsilon^p \quad (3.1)$$

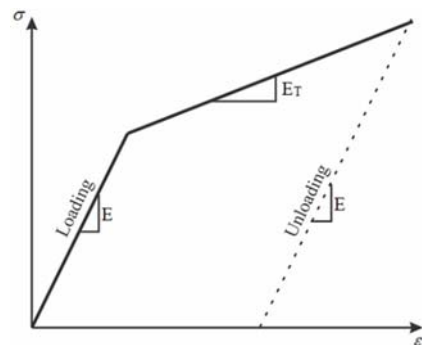
The elastic strain ε^e is the part of the total strain which is recovered during the unloading process. In contrast, the plastic strain ε^p remains after unloading. The unloading (recovering) stress-strain relationship produced when the loading on the specimen is removed is parallel to the loading diagram in the elastic range (Figure 3. 2)



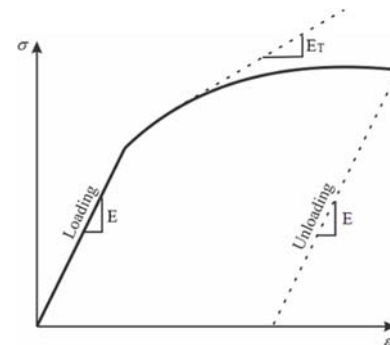
(A)



(B)



(C)



(D)

Figure 3. 1 Idealized stress-strain curves for (A) Perfectly plastic, (B) Elastic-perfectly plastic, (C) Elastic-linearly plastic and (D) Elastic-nonlinear plastic materials

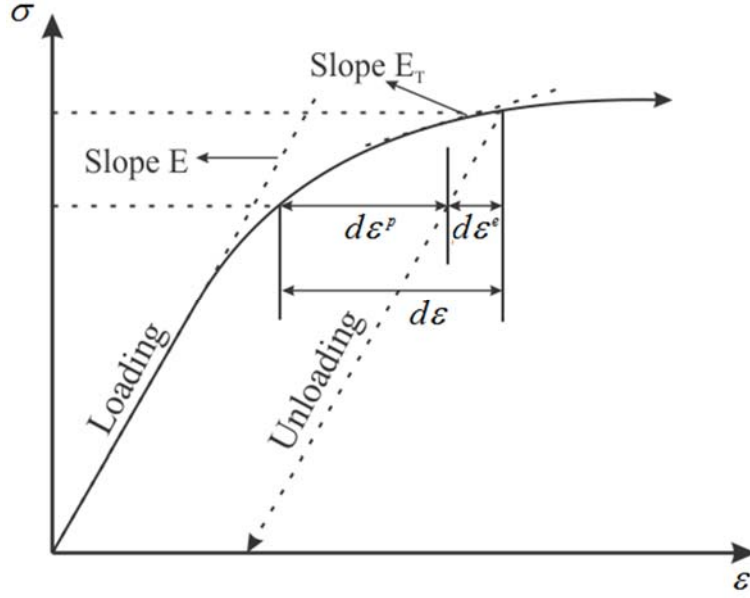


Figure 3. 2 Uniaxial total strain decomposition into plastic and elastic strains

In Figure 3. 2, E is Young's modulus, E_T is the plastic tangent modulus. In the elastic range, the uniaxial stress-strain curve and material behaviour can be described by Hooke's Law as

$$\sigma = E \varepsilon^e \quad (3.2)$$

In the plastic range, E_T is the slope of uniaxial stress-plastic strain curve which is a function of stress and plastic strain as

$$E_T = E_T(\sigma, \varepsilon^p) \quad (3.3)$$

In multi-axial loading, the above treatment can be written in an incremental form, i.e., the total increment strain tensor $d\varepsilon_{ij}$ is sum of the incremental elastic strain tensor $d\varepsilon_{ij}^e$ (recoverable), and the incremental plastic strain tensor $d\varepsilon_{ij}^p$ (non-recoverable) as

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (3.4)$$

3.4 Yield criterion

A yield criterion defines the elastic limit in the stress space. For instance, the yield stress F_y is the elastic limit for a material undergoing uniaxial loading. A stress less than yield stress F_y signifies that the material lies in the elastic range and thus behave elastically. When the stress reaches the yield, initial yielding takes place and the materials behaves plastically. The concept is generalized for multi-axial loading by introducing a yield criterion. In this case, the yield criterion is used for delineate the elastic and plastic ranges of deformation. In a nine-dimensional stress space the yield criterion is written as

$$f(\sigma_{ij}) = 0 \quad (3.5)$$

in which

$$\begin{cases} f(\sigma_{ij}) < 0 & \rightarrow & \text{elastic} \\ f(\sigma_{ij}) = 0 & \rightarrow & \text{plastic deformation} \\ f(\sigma_{ij}) > 0 & \rightarrow & \text{unattainable} \end{cases} \quad (3.6)$$

For isotropic materials, the yield criterion is independent of material orientation, so the principal stresses suffice to express the yield criterion and Eq.(3.5) is rewritten as

$$f(\sigma_{11}, \sigma_{22}, \sigma_{33}) = 0 \quad (3.7)$$

The yield criterion can alternatively be expressed in terms of stress invariants I_1, I_2, I_3 as

$$\begin{aligned} f(I_1, I_2, I_3) &= 0 \\ I_1 &= \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_{kk} \\ I_2 &= \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2 = \frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}) \\ I_3 &= \sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{12}^2\sigma_{33} - \sigma_{23}^2\sigma_{11} - \sigma_{31}^2\sigma_{22} = \det(\sigma_{ij}) \end{aligned} \quad (3.8)$$

In metals, it is generally accepted that yielding is independent of the hydrostatic stress. This eliminates the dependence of the yield criterion on the first invariant and one can express the yield criterion in terms of the stress invariants of the two invariants of the deviatoric stress J_2, J_3 , i.e.,

$$f(J_2, J_3) = 0 \quad (3.9)$$

The von-Mises criterion commonly used to model metal plasticity postulates the independence of the yield criterion on the third invariant and Eq.(3.9) is expressed as

$$f(J_2) = 0 \quad (3.10)$$

The von-Mises yield function is thus expressed by

$$\begin{aligned} f(J_2) &= \sigma_{eff} - F_y = \sqrt{3J_2} - F_y = 0 \\ &= \sqrt{\frac{3}{2} S_{ij} S_{ij}} - F_y = 0 \end{aligned} \quad (3.11)$$

where $S_{ij} = \sigma_{ij} - (1/3)\delta_{ij}\sigma_{mm}$ are the components of the deviatoric stress tensor. The yield function in principal stress space from Eq.(3.11) is expressed as

$$\begin{aligned} f(J_2) &= \sqrt{\frac{3}{2} S_{ij} S_{ij}} - F_y = \left(\frac{3}{2} S_{ij} S_{ij} \right)^{1/2} - F_y = 0 \\ &= \frac{1}{\sqrt{2}} \left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \right]^{1/2} - F_y = 0 \end{aligned} \quad (3.12)$$

Eq.(3.12) corresponds to a cylinder in the three-dimensional principal stress space in which the axis of the cylinder which is the hydrostatic axis is lying along the $\sigma_{11} = \sigma_{22} = \sigma_{33}$ (Figure 3. 3). To express the yield function in two-dimensional principal stress space, it is assumed that $\sigma_{33} = 0$, therefore from Eq.(3.12) the von Mises yield criterion is rewritten as

$$f(J_2) = \frac{1}{\sqrt{2}} \left[(\sigma_{11} - \sigma_{22})^2 + \sigma_{22}^2 + \sigma_{11}^2 \right]^{1/2} - F_y = 0 \quad (3.13)$$

or

$$\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 - F_y^2 = 0 \quad (3.14)$$

where Eq.(3.14) corresponds to the equation of an ellipse as shown on Figure 3. 4.

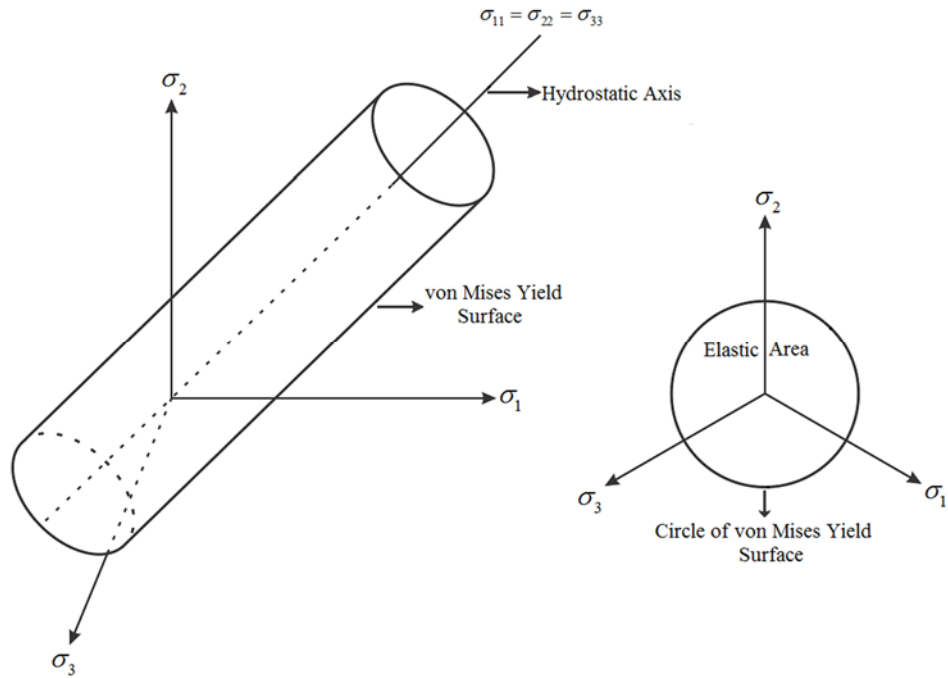


Figure 3. 3 the von Mises yield criterion in three-dimensional principal stress space

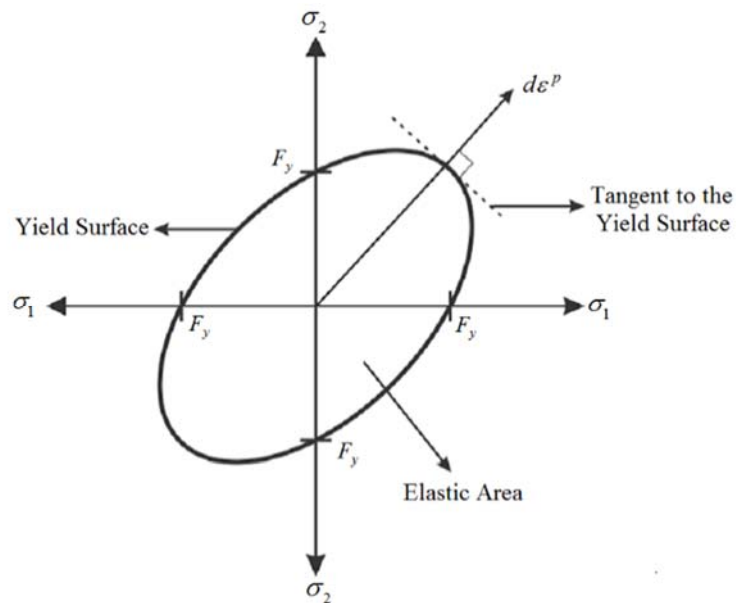


Figure 3. 4 von Mises yield surface in two-dimensional principal stress space

3.5 Incremental Elastic Stress-Strain Relationships

A three-dimensional solid is subjected to general normal and shear stresses is shown in Figure 3. 5 in which σ_x , σ_y and σ_z are the normal stresses and $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, and $\tau_{zx} = \tau_{xz}$ are the shear stresses.

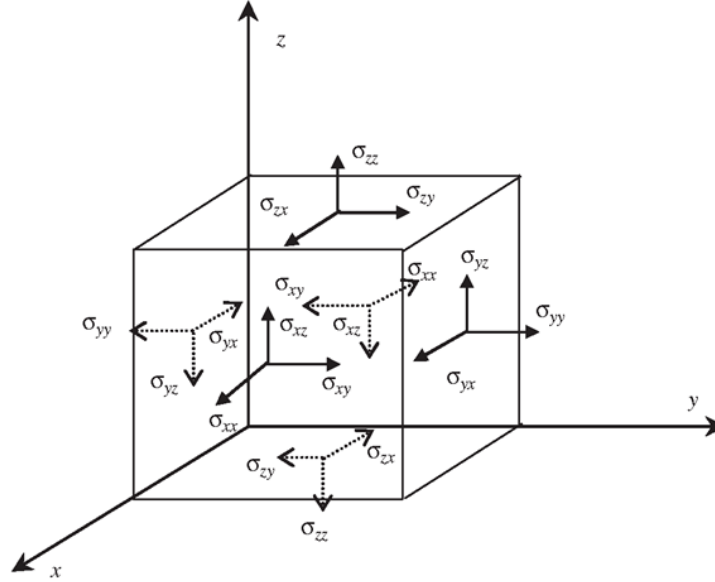


Figure 3. 5 Stresses under three-dimensional loading

The incremental form of the linearly elastic isotropic constitutive relation between the incremental stresses and the incremental elastic strains takes the form

$$\begin{aligned}
 d\varepsilon_{xx}^e &= \frac{1}{E} \left[d\sigma_{xx} - \nu (d\sigma_{yy} + d\sigma_{zz}) \right] \\
 d\varepsilon_{yy}^e &= \frac{1}{E} \left[d\sigma_{yy} - \nu (d\sigma_{xx} + d\sigma_{zz}) \right] \\
 d\varepsilon_{zz}^e &= \frac{1}{E} \left[d\sigma_{zz} - \nu (d\sigma_{xx} + d\sigma_{yy}) \right] \\
 d\varepsilon_{xy}^e &= \frac{1+\nu}{E} d\sigma_{xy} \\
 d\varepsilon_{yz}^e &= \frac{1+\nu}{E} d\sigma_{yz} \\
 d\varepsilon_{zx}^e &= \frac{1+\nu}{E} d\sigma_{zx}
 \end{aligned} \tag{3.15}$$

where E and ν are Young's modulus and Poisson's ratio, respectively. Eq.(3.15) is written in a tensorial form as

$$d\varepsilon_{ij}^e = \frac{1}{E} \left((1+\nu)d\sigma_{ij} - \nu d\sigma_{kk} \delta_{ij} \right) \quad (3.16)$$

Alternatively, the incremental stresses are expressed in terms of elastic incremental strains can be found from Eq.(3.15) as

$$\begin{aligned} d\sigma_{xx} &= \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)d\varepsilon_{xx}^e + \nu(d\varepsilon_{yy}^e + d\varepsilon_{zz}^e) \right] \\ d\sigma_{yy} &= \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)d\varepsilon_{yy}^e + \nu(d\varepsilon_{xx}^e + d\varepsilon_{zz}^e) \right] \\ d\sigma_{zz} &= \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)d\varepsilon_{zz}^e + \nu(d\varepsilon_{xx}^e + d\varepsilon_{yy}^e) \right] \\ d\sigma_{xy} &= \frac{E}{(1+\nu)} d\varepsilon_{xy}^e \\ d\sigma_{xz} &= \frac{E}{(1+\nu)} d\varepsilon_{xz}^e \\ d\sigma_{yz} &= \frac{E}{(1+\nu)} d\varepsilon_{yz}^e \end{aligned} \quad (3.17)$$

or, in a tensorial form, one has

$$d\sigma_{ij} = \frac{E}{(1+\nu)} d\varepsilon_{ij}^e + \frac{\nu E}{(1+\nu)(1-2\nu)} d\varepsilon_{kk}^e \delta_{ij} \quad (3.18)$$

3.6 Plasticity Flow Rule

The plasticity flow rule is a necessary kinematic assumption which characterizes the rate, magnitude and direction of the incremental plastic strain tensor $d\varepsilon_{ij}^p$. The concept of plastic potential function was proposed by von Mises who postulated that the incremental plastic strain $d\varepsilon_{ij}^p$ is proportional to the gradient $\partial g / \partial \sigma_{ij}$ of a plastic potential function $g(\sigma_{ij})$ i.e.,

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial g}{\partial \sigma_{ij}} \quad (3.19)$$

When plastic deformations take place, the constant of proportionality $d\lambda$ is nonzero positive value. Therefore, the incremental plastic strain tensor $d\varepsilon_{ij}^p$ is normal to the plastic potential surface $g(\sigma_{ij})$

3.7 Associated Flow Rule

In Eq.(3.19), when the plastic potential function is taken as the yield function, i.e., $g(\sigma_{ij}) = f(\sigma_{ij})$, the incremental plastic strain tensor $d\varepsilon_{ij}^p$ is written as

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (3.20)$$

Eq.(3.20) is called the associated flow rule because the plastic flow rule is associated with the yield criterion $f = f(\sigma_{ij})$. Eq.(3.20) indicates that the incremental plastic strain tensor $d\varepsilon_{ij}^p$ is normal to the yield surface f (Figure 3. 6) since it is proportional to the gradient of the yield surface.

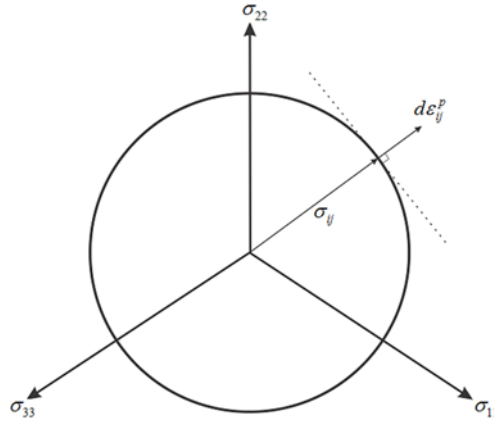


Figure 3. 6 The direction of the incremental plastic strain tensor in stress space

3.8 Isotropic Plasticity

When the plastic behaviour is direction-independent, i.e., isotropic, one can express the yield function $f = f(\sigma_{ij})$ and the plastic potential function $g = g(\sigma_{ij})$ in terms of the stress invariants, i.e., $f = f(I_1, J_2, J_3)$ and $g = g(I_1, J_2, J_3)$ in which

$$I_1 = \sigma_{kk} \quad (3.21)$$

is the first invariant of the total stress tensor, and J_2 is the second invariant of the deviatoric stress tensor defined as

$$J_2 = \frac{1}{2} S_{ij} S_{ij} \quad (3.22)$$

and S_{ij} is the deviatoric stress tensor and is given by

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \quad (3.23)$$

where $\sigma_m = \frac{1}{3} \sigma_{kk}$ is the mean or hydrostatic stress. From Eq.(3.23), by substituting into Eq.(3.22), the second invariant of deviatoric stress tensor is obtained as

$$J_2 = \frac{1}{6} \left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \right] + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 \quad (3.24)$$

and $J_3 = \frac{1}{3} S_{ij} S_{jk} S_{ki}$ is the third invariant of deviatoric stress tensor.

3.9 Simplifications for perfectly plastic materials

In metals, it is common to assume associated plasticity. Also, it is common to assume that the yield function is independent of invariants I_1, J_3 . In such a case, one has

$$f = f(J_2) \quad (3.25)$$

Under uniaxial tension $\sigma_{11} \neq 0, \sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{31} = 0$, the metal is known to yield when $\sigma_{11} = F_y$. If F_y is independent on the stress-strain path, no hardening takes place and the material

is perfectly plastic. In such a case, it can be shown that $J_2 = F_y^2 / 3$, thus the yield function (Eq.(3.25)) takes the form

$$f(J_2) = J_2 - \frac{F_y^2}{3} = 0 \quad (3.26)$$

3.10 Plastic Strain increment for Elastic- Perfectly plastic materials

The associated flow rule for metals from Eq.(3.20) can be written as

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} = d\lambda \frac{\partial f(J_2)}{\partial \sigma_{ij}} \quad (3.27)$$

From Eq.(3.26), by substituting into Eq.(3.27), the flow rule can be rewritten as

$$\begin{aligned} d\varepsilon_{ij}^p &= d\lambda \frac{\partial J_2}{\partial \sigma_{ij}} = d\lambda \frac{\partial \left(\frac{1}{2} S_{ij} S_{ij} \right)}{\partial \sigma_{ij}} = d\lambda \frac{\partial \left(\frac{1}{2} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \right)}{\partial \sigma_{ij}} \\ &= d\lambda \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) \end{aligned} \quad (3.28)$$

or

$$d\varepsilon_{ij}^p = d\lambda S_{ij} \quad (3.29)$$

where Eq.(3.29) is called Levy-Mises equation which expresses the associated flow rule equation for elastic-perfectly plastic materials. In the Levy-Mises equation, the proportionality factor $d\lambda$ has the following properties

$$\begin{cases} d\lambda = 0 & \text{if } J_2 < \frac{F_y^2}{3} \\ d\lambda = 0 & \text{if } J_2 = \frac{F_y^2}{3}, \quad dJ_2 < 0 \\ d\lambda > 0 & \text{if } J_2 = \frac{F_y^2}{3}, \quad dJ_2 = 0 \end{cases} \quad (3.30)$$

From Eq.(3.29), the relationship between incremental plastic strain tensor and deviatoric stress tensor components for the elastic-perfectly plastic materials can be expressed as

$$\frac{d\varepsilon_{xx}^p}{S_{xx}} = \frac{d\varepsilon_{yy}^p}{S_{yy}} = \frac{d\varepsilon_{zz}^p}{S_{zz}} = \frac{d\varepsilon_{xy}^p}{S_{xy}} = \frac{d\varepsilon_{yz}^p}{S_{yz}} = \frac{d\varepsilon_{zx}^p}{S_{zx}} = d\lambda \quad (3.31)$$

Equations (3.31) are the Prandtl-Reuss equations. If the elastic deformation terms are neglected, the expressions are called the Levy-Mises equations. From Eqs. (3.15) and (3.31), the incremental strain-stress relations for an elastic perfectly plastic materials is written as

$$\begin{aligned} d\varepsilon_{xx} &= \frac{1}{E} \left(d\sigma_{xx} - \nu(d\sigma_{yy} - d\sigma_{zz}) \right) + \frac{2}{3} d\lambda \left(\sigma_{xx} - \frac{1}{2}(\sigma_{yy} - \sigma_{zz}) \right) \\ d\varepsilon_{yy} &= \frac{1}{E} \left(d\sigma_{yy} - \nu(d\sigma_{xx} - d\sigma_{zz}) \right) + \frac{2}{3} d\lambda \left(\sigma_{yy} - \frac{1}{2}(\sigma_{zz} - \sigma_{xx}) \right) \\ d\varepsilon_{zz} &= \frac{1}{E} \left(d\sigma_{zz} - \nu(d\sigma_{xx} - d\sigma_{yy}) \right) + \frac{2}{3} d\lambda \left(\sigma_{zz} - \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \right) \\ d\varepsilon_{xy} &= \frac{1+\nu}{E} d\sigma_{xy} + d\lambda \sigma_{xy} \\ d\varepsilon_{yz} &= \frac{1+\nu}{E} d\sigma_{yz} + d\lambda \sigma_{yz} \\ d\varepsilon_{zx} &= \frac{1+\nu}{E} d\sigma_{zx} + d\lambda \sigma_{zx} \end{aligned} \quad (3.32)$$

In a tensorial form Eq.(3.32) is re-written as

$$d\varepsilon_{ij} = \frac{1}{E} \left((1+\nu) d\sigma_{ij} - \nu d\sigma_{kk} \delta_{ij} \right) + d\lambda S_{ij} \quad (3.33)$$

3.11 Effective Stress and Effective Plastic Strain

The effective stress or equivalent von-Mises stress and effective plastic strain are scalar parameters which can be used to relate the hardening behaviour to the material under multi-axial stresses to the hardening parameter obtained experimentally obtained from uniaxial stress-strain tests.

3.11.1 Effective Stress

For multiaxial loading, the effective stress σ_{eff} can be defined as

$$\sigma_{eff} = \sqrt{3J_2} \quad (3.34)$$

by substituting the second invariant of the deviatoric stress tensor from Eq.(3.22), the effective stress is rewritten as

$$\sigma_{eff} = \sqrt{\frac{3}{2}(S_{ij}S_{ij})} \quad (3.35)$$

and

$$\sigma_{eff} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)} \quad (3.36)$$

Here, Eq.(3.36) shows that the effective stress is a function of the components of the stress tensor.

The effective stress can also be expressed in terms of principal stresses as

$$\sigma_{eff} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2} \quad (3.37)$$

3.11.2 Effective Plastic Strain

The incremental effective plastic strain $d\varepsilon_{eff}^p$ is a combination of incremental plastic strains defined as

$$d\varepsilon_{eff}^p = \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \quad (3.38)$$

or

$$d\varepsilon_{eff}^p = \frac{2}{3} \sqrt{\frac{1}{2} \left[(d\varepsilon_{11}^p - d\varepsilon_{22}^p)^2 + (d\varepsilon_{22}^p - d\varepsilon_{33}^p)^2 + (d\varepsilon_{33}^p - d\varepsilon_{11}^p)^2 + \frac{3}{2} \left((d\gamma_{12}^p)^2 + (d\gamma_{23}^p)^2 + (d\gamma_{31}^p)^2 \right) \right]} \quad (3.39)$$

The total effective plastic strain ε_{eff}^p is obtained by the integral definition

$$\varepsilon_{eff}^p = \int \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \quad (3.40)$$

in which the integral is performed over the loading path. The effective incremental plastic strain is expressed in terms of principal incremental plastic strains as

$$d\varepsilon_{eff}^p = \sqrt{\frac{2}{9} \left[(d\varepsilon_{11}^p - d\varepsilon_{22}^p)^2 + (d\varepsilon_{22}^p - d\varepsilon_{33}^p)^2 + (d\varepsilon_{33}^p - d\varepsilon_{11}^p)^2 \right]} \quad (3.41)$$

3.12 Linear Kinematic Hardening

The linear kinematic hardening model is the simplest kinematic hardening model. The yield surface is assumed to translate in the nine-dimensional stress space without undergoing any changes in the size and shape of the initial yield surface. Actually, during strain hardening only the center of the yield surface moves in the nine-dimensional stress (Figure 3. 7).

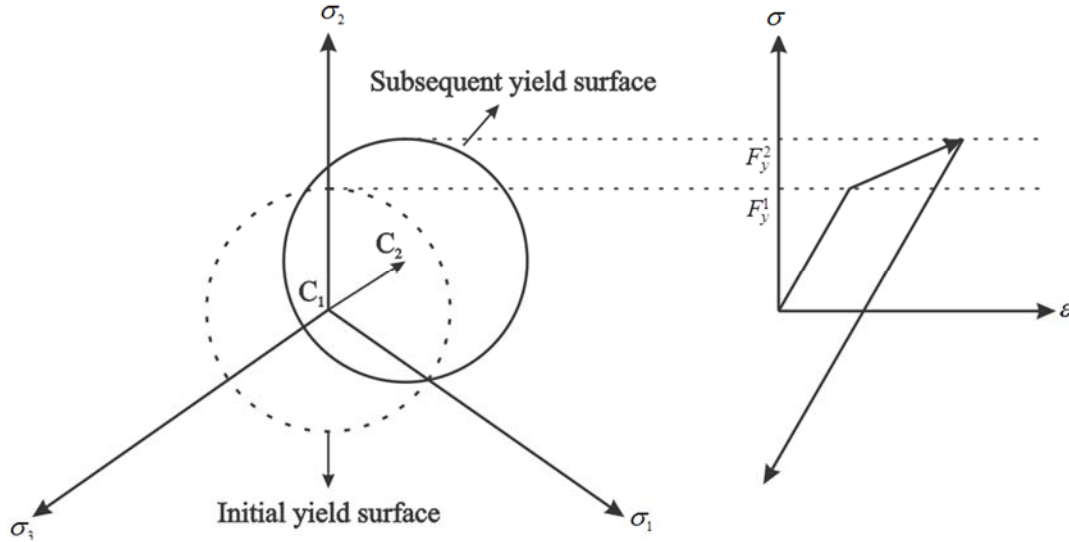


Figure 3. 7 Initial and subsequent yield surfaces in linear kinematic hardening models

3.12.1 Yield criterion

In a linear kinematic hardening model, the subsequent yield surfaces is a function of stress tensor σ_{ij} and the incremental plastic strain tensor $d\varepsilon_{ij}^p$ and is expressed as

$$f(\sigma_{ij}, d\varepsilon_{ij}^p) = \frac{1}{2} (S_{ij} - \alpha_{ij}^{dev}) (S_{ij} - \alpha_{ij}^{dev}) - \frac{F_y^2}{3} = 0 \quad (3.42)$$

where $\alpha_{ij} = \alpha_{ij}(\varepsilon_{ij}^p)$ is the backstress tensor which determines the total translation of the center of the initial yield surface in the stress space and α_{ij}^{dev} is deviatoric backstress tensor. To determine the backstress tensor in the linear kinematic hardening models, either the Prager or Ziegler kinematic hardening is used. These are expressed in the following sub-sections

3.12.2 Prager Linear Kinematic Hardening model

Prager proposed a linear kinematic hardening relation in which the incremental backstress tensor (translation of center of the yield surface) is expressed in terms of incremental plastic strain tensor as

$$d\alpha_{ij}^{(\text{Prager})} = c_p d\varepsilon_{ij}^p \quad (3.43)$$

where c_p is a scalar material property called the Prager's hardening coefficient.

3.12.3 Ziegler Linear Kinematic Hardening model

A linear kinematic hardening relation was proposed by Ziegler as (Crisfield, M. A. (1991) and Cook et al. (2002))

$$d\alpha_{ij} = d\mu(\sigma_{ij} - \alpha_{ij}) \quad d\mu > 0 \quad (3.44)$$

where $d\mu$ depends on the incremental plastic strain history and is related to the incremental plastic strain $d\mu = d\mu(d\varepsilon_{ij}^p)$ through

$$d\mu = \frac{d\sigma_{ij} d\varepsilon_{ij}^p}{(\sigma_{kl} - \alpha_{kl}) d\varepsilon_{kl}^p} \quad (3.45)$$

3.12.4 Incremental Plastic Strain

In the linear kinematic hardening materials, the scalar factor of proportionality $d\lambda$ for obtaining the incremental plastic strain tensor $d\varepsilon_{ij}^p$ can be written as

$$d\lambda = -\frac{(\partial f / \partial \sigma_{ij}) d\sigma_{ij}}{(\partial f / \partial \sigma_{rs})(\partial f / \partial \varepsilon_{rs}^p)} \quad (3.46)$$

By substituting Eq.(3.46) into Eq.(3.20), the incremental plastic strain in linear kinematic hardening takes the form

$$d\varepsilon_{ij}^p = \frac{(\partial f / \partial \sigma_{kl})(\partial f / \partial \sigma_{ij}) d\sigma_{kl}}{-[(\partial f / \partial \sigma_{rs})(\partial f / \partial \varepsilon_{rs}^p)]} \quad (3.47)$$

3.13 Nonlinear Kinematic Hardening

Nonlinear kinematic hardening models have been proposed to describe the loading and unloading nonlinear of kinematic hardening models. Models for describing the nonlinear kinematic hardening include the Armstrong and Frederick (1966) model and the Chaboche (1979, 1986). These are described in the following sections.

3.13.1 Armstrong and Frederick Model

The Armstrong and Fredrick (1966) model adds an additional term to the linear kinematic hardening model of Prager to capture the translation of the yield surface in the nine-dimensional stress space. The Armstrong and Frederick model is expressed by (Rezaiee-Pajand, M. and Sinaie, S. (2009))

$$d\alpha_{ij} = \frac{2}{3}c d\varepsilon_{ij}^p - \gamma\alpha_{ij} d\varepsilon_{eff}^p \quad (3.48)$$

where c and γ are material constants in which c is proportional to the tangent modulus E_t and γ is rate of decrease of the tangent modulus. For elastic-perfectly plastic and linear kinematic hardening materials one has $\gamma = 0$.

3.13.2 Chaboche Model

Chaboche (1979, 1986) proposed a nonlinear kinematic hardening based on a generalization of Armstrong and Frederick kinematic hardening model where more material constants have been added for a more accurate nonlinear kinematic hardening model. The Chaboche model is characterized by

$$d\alpha_{ij} = \sum_{k=1}^n d\alpha_{ij}^k \quad (3.49)$$

where n is the number of Chaboche components and

$$d\alpha_{ij}^k = \frac{2}{3}c^k d\varepsilon_{ij}^p - \gamma^k\alpha_{ij}^k d\varepsilon_{eff}^p \quad (3.50)$$

For $d\varepsilon_x^p \geq 0$, Eq.(3.50) can be written in as

$$d\alpha_{ij}^k = c^k d\varepsilon_{ij}^p - \gamma^k\alpha_{ij}^k d\varepsilon_{eff}^p \quad (3.51)$$

and the backstress curve in the stress-strain space can be obtained by integrating Eq.(3.51) yielding

$$\alpha^k = \left(\frac{C^k}{\gamma^k} \right) \left[1 - \exp(-\gamma^k \varepsilon_p) \right] \quad (3.52)$$

and

$$\alpha = \sum_{k=1}^N \alpha^k \quad (3.53)$$

The nonlinear kinematic hardening parameters C^k and γ^k for the Chaboche model are obtained iteratively to match a given stress-strain curve. Initial values for C^k and γ^k for the Chaboche model have been proposed by Chaboche and Nouailhas (1989) as

$$C^k = \left(\frac{\sigma^k - \sigma^{k-1}}{\varepsilon_p^k - \varepsilon_p^{k-1}} - \frac{\sigma^{k+1} - \sigma^k}{\varepsilon_p^{k+1} - \varepsilon_p^k} \right) \quad (3.54)$$

$$\gamma^k = \frac{1}{\varepsilon_p^k} \quad (3.55)$$

CHAPTER 4. Description of the Finite Element Model

4.1 General

This chapter describes the material properties and different element types in ABAQUS. Section 4.2 discusses the analysis type and sequence of loading. The relevant continuum 3D elements are presented in Sections 4.3. The loading path for the longitudinal and hoop tension tests are described in Section 4.4. Section 4.5 discusses the specifics of nonlinear kinematic hardening modeling. A comparison of the performance of 3D elements is provided Section 4.6 and a comparison between longitudinal, hoop and initial stress-strain relations are provided in Section 4.7. The material in this chapter is extracted from Abaqus online Documentation (2016) and MathWorks online User's Guide (2016).

4.2 Analysis Type and Sequence of loading

The finite element method is a computational tool used for linear and nonlinear stress-deformation analysis. The analysis can be classed as either implicit or explicit. ABAQUS/Explicit is used for time-dependent problems involving high deformation rates. Under this method, the stiffness matrix including geometric and elasto-plastic effects are updated only at the end of each increment. For accurate results, the increment must be small.

In ABAQUS/Implicit, the stiffness matrix is updated at the end of each increment in a manner similar to ABAQUS/Explicit, with the addition that after each increment, Newton-Raphson iterations are used to enforce equilibrium. ABAQUS/Implicit thus can involve larger time steps, but the computational effort within each step tends to be higher. Results based on ABAQUS/Implicit tend to be more accurate but involve a higher computational effort than those based on ABAQUS/Explicit. Given that the loading rates in the present problem are low enough so that inertial effects are negligible, ABAQUS/Implicit is adopted. The multiple loading steps involved in the manufacturing process are thus modeled through multiple steps using the *STEP keyword in ABAQUS. Also, given that inertial effects are negligible, the *STATIC keyword is used.

The forming process during U- and O- pressing is defined through 10 steps as shown in Figure 4. 1. Throughout a given step, the displacement at each of the nodes of the inside surface are specified, so that the curvature $1/R$ is changed from zero (for the initially flat plate) to the curvature based on the target inside curvature of the pipe (Step 10).

In order to simulate the mechanical expansion step (Figure 4. 4 C) at the end of the forming process, the inside surface of the formed plate is subjected to an internal pressure corresponding to a hoop stress of 40% of the yield stress (Figure 4. 4 C) using *DLOAD keyword and then the applied internal pressure is removed (Figure 4. 4 D). The application and removal of internal pressure are defined by two subsequent Static/general steps. The loading path for the U-O-E manufacturing process is shown in Figure 4. 4. During the application of internal pressure and throughout subsequent loading steps, the relative displacements between the nodes are kept to zero on the right and left edges of the curved plate are kept constant to avoid separation between these nodes using *BOUNDARY keyword in ABAQUS (Figure 4. 5).

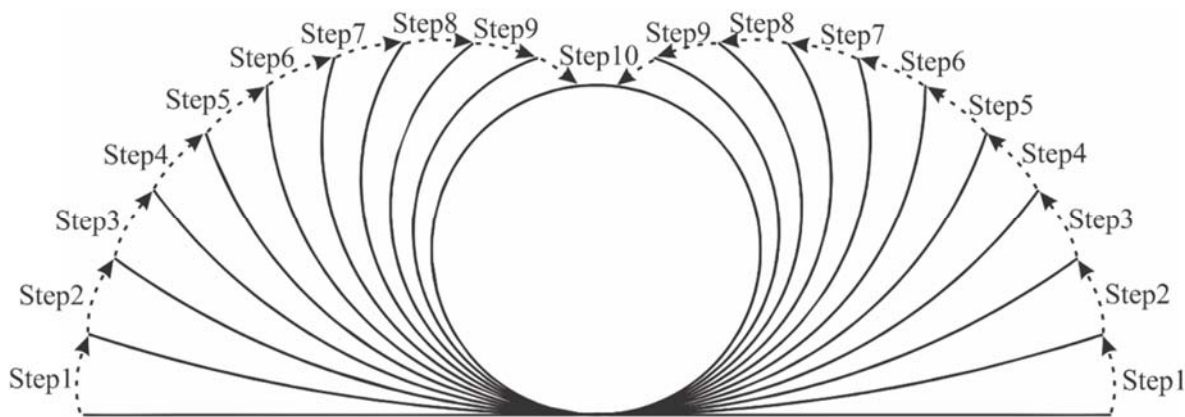


Figure 4. 1 Modeling U-Pressing and O-Pressing by defining nodal displacement in 10 steps

In the first attempt at modelling the forming process, a flat plate with vertical edges (Figure 4. 2 A) was taken. After simulating the U- and O pressing simulation, (Figure 4. 2B) there was a gap between the both sides of the bent plate. During the manufacturing process, this gap is filled with welding. The present model attempts to avoid the complications of modelling the welding process by starting with two inclined sides (Figure 4. 3A) where the additional material is intended to

simulate the additional material of the weld. The inclination of the weld is chosen such that there is no gap between two sides of the bent plate after the forming process.

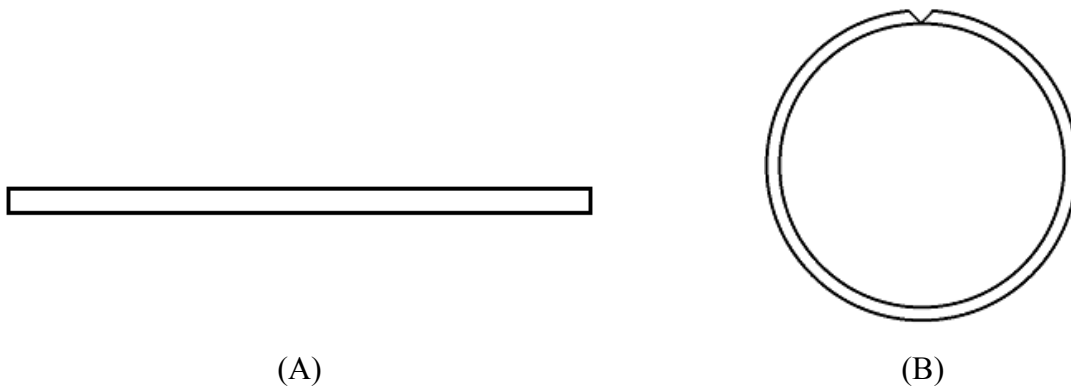


Figure 4. 2 Flat plate with both end vertical edges (A) Before and (B) After U- and O- pressing

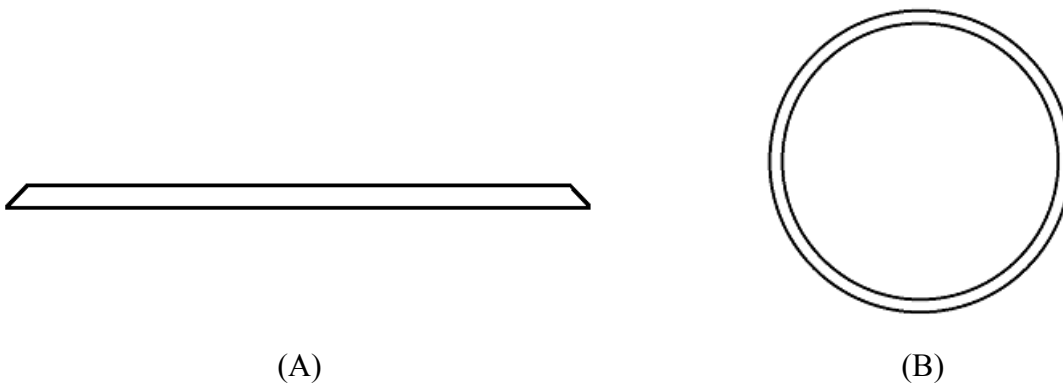


Figure 4. 3 Flat plate with both end inclined edges (A) Before and (B) After U- and O- pressing

In Figure 4. 4, Position 1 represents the un-deformed flat plate before the forming process, and Position 2 represents the formed plate after application of U- and O- pressing process. Position 3 is for the formed plate after applying the radial expansion while position 4 is that of the pipe after removal of the internal pressure.

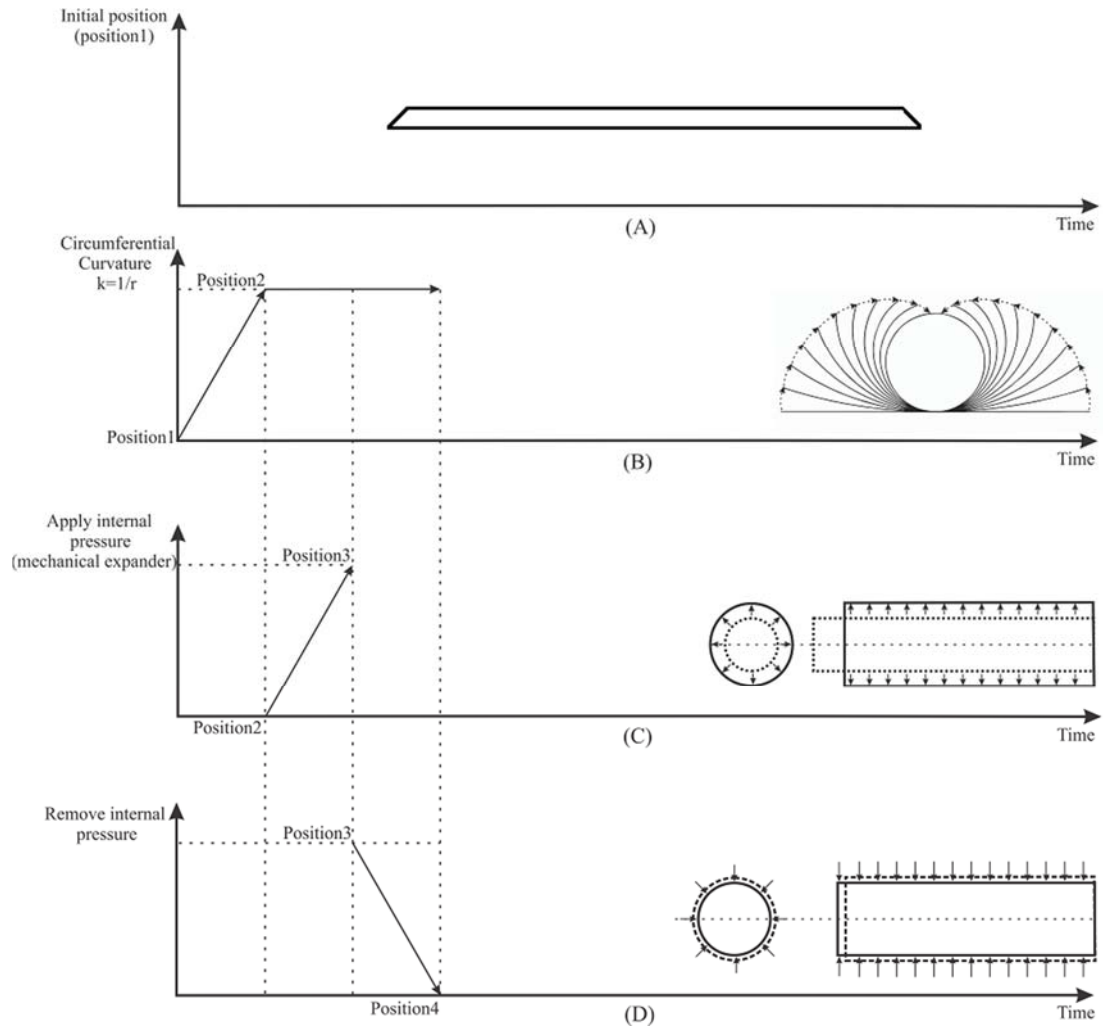


Figure 4. 4 Loading path for U-O-E forming process

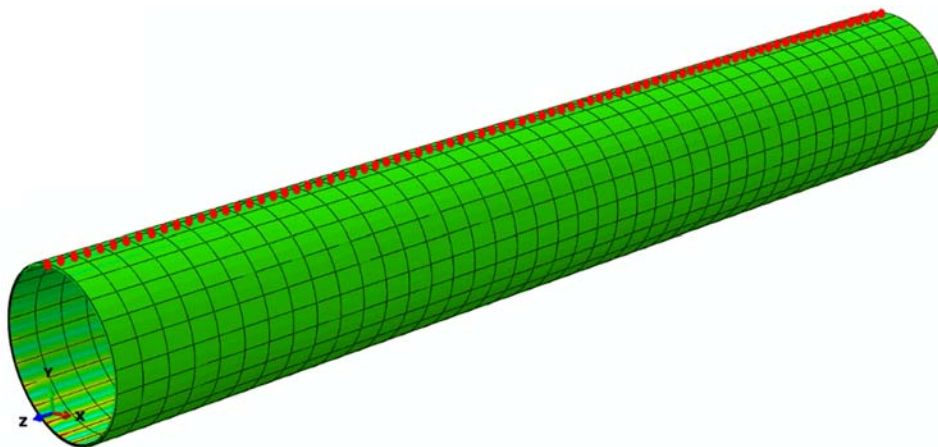


Figure 4. 5 Nodes on the right and left edges of the curved plate are kept in contact

4.3 Element Types

Sub-section 4.3.1 presents the various types of continuum elements in ABAQUS and the associated degrees of freedom. Sub-section 4.3.2 presents the features and limitations of first- and second-order continuum elements while Sub-section 4.3.3 discusses the number of the integration points in continuum elements.

4.3.1 Continuum Elements

Continuum elements in ABAQUS can be used for the linear and non-linear analysis of problems involving contact, plasticity, and large displacement simulations. In three-dimensional analysis, tetrahedral and hexahedral elements are provided in ABAQUS/Standard (Figure 4. 6). In the present study, a hexahedral element is judged most suitable to represent the plate/pipe. For continuum elements, each node has three translational degrees of freedom defined along the global x , y and z axes respectively.

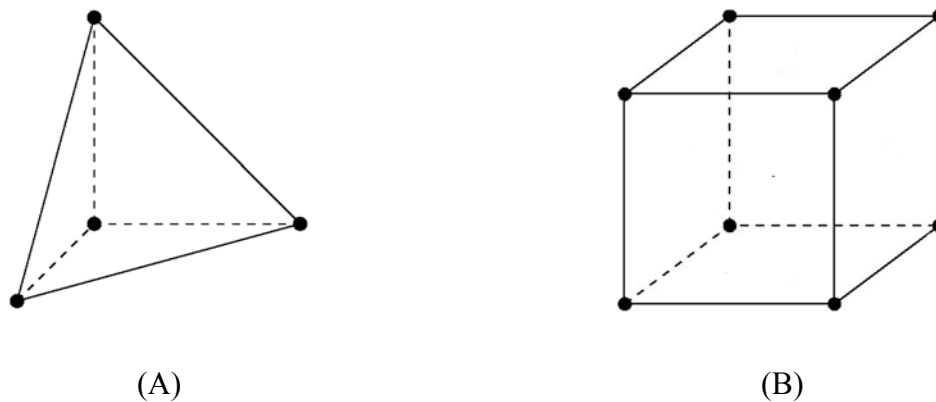


Figure 4. 6 Continuum elements in ABAQUS/Standard (A) Tetrahedral and (B) Hexahedral

4.3.2 First- and Second-Order Elements

In ABAQUS/Standard, depending on the interpolation scheme, there are two categories of continuum elements; first-order elements in which the displacements are linearly interpolated and second-order elements in which the displacements are quadratically interpolated (Figure 4. 7 and Figure 4. 8). In fully integrated first-order elements, the strain operator provides a constant volumetric strain throughout the element, which prevents mesh locking and provides accurate

solutions when the material response is incompressible or closely incompressible. Actually, the real volume changes in fully integrated first-order elements is replaced by the average volume changes at the integration point, which is referred to as the reduced-integration technique. Second-order elements are more accurate than first-order elements for simulating curved surface modeling with fewer elements.

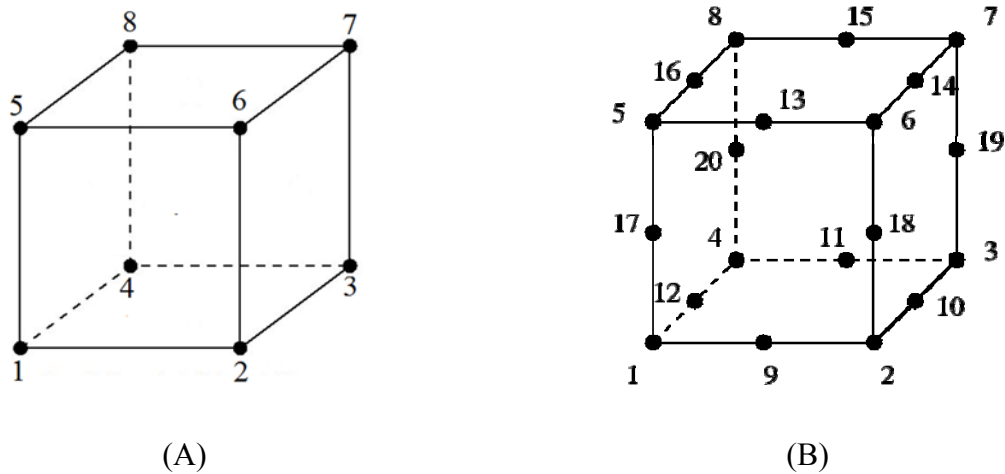


Figure 4. 7 Hexahedra elements (A) First-order with 8 nodes and (B) Second-order with 20 nodes

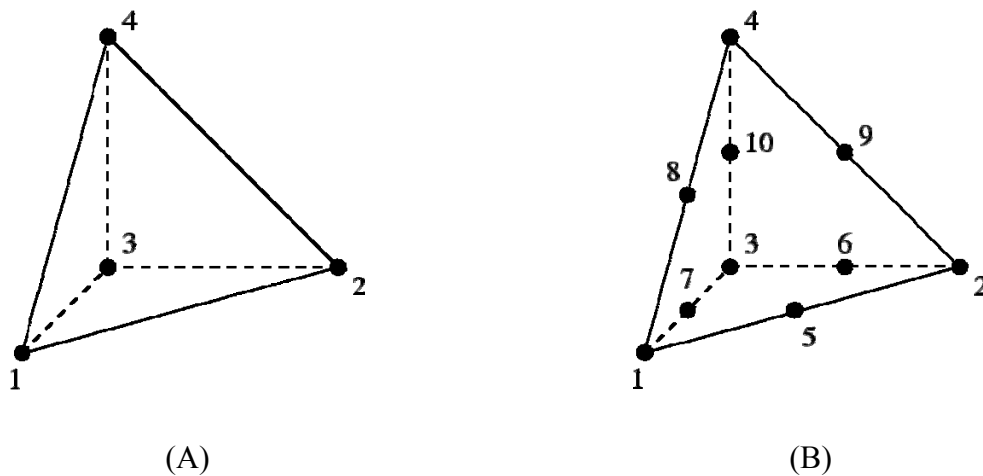


Figure 4. 8 Tetrahedral elements (A) First-order with 4 nodes and (B) Second-order with 10 nodes

4.3.3 Integration Schemes

First- and second-order elements in ABAQUS are either fully integrated or based on reduced integration. In fully integrated first-order integrated continuum element C3D8, there are two integration points in each direction, totalling $2 \times 2 \times 2$ integration points per element. In reduced first order continuum elements C3D8R, there is a single integration point at the center of the element ($1 \times 1 \times 1$ integration point). Reduced integration reduces the running time of the model. A depiction of the integration point arrangement in the fully and reduced integration eight node elements is provided in Figure 4. 9. For models not involving contact, reduced integration is more effective, while full integration is recommended for models involving contact.

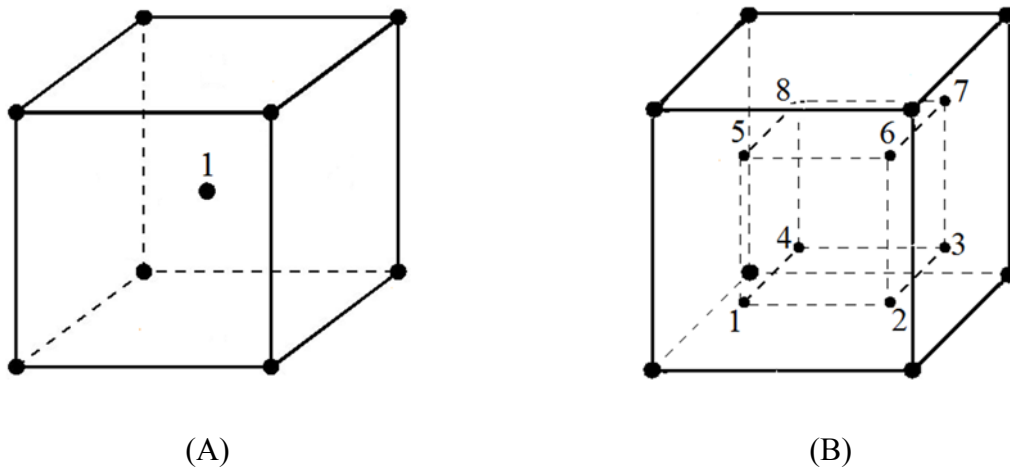


Figure 4. 9 First-order continuum elements (A) reduced integration C3D8R (1 integration point) and (B) fully integration C3D8 (8 integration points)

In the fully integrated second-order continuum element C3D20, there are three integration points along each direction ($3 \times 3 \times 3 = 27$ integration points), while there are two integration points in each direction ($2 \times 2 \times 2 = 8$ integration points) in C3D20R. The integration point arrangement in 20 node elements in ABAQUS is depicted in Figure 4. 10.

First-order fully integrated C3D8 elements undergoing bending suffers from shear locking in which the elements exhibit an overly stiff behaviour in bending. Also, fully integrated elements with incompressible materials suffer from volumetric locking in which elements tend to exhibit an

overly stiff behaviour for deformations which cause no volume changes. In the plastic range of deformation of elasto-plastic materials, where plastic strains are incompressible, fully integrated elements induce volumetric locking when the plastic strains are of the order of the elastic strains. In order to remedy these problems, reduced integration is more suitable than fully integrated elements.

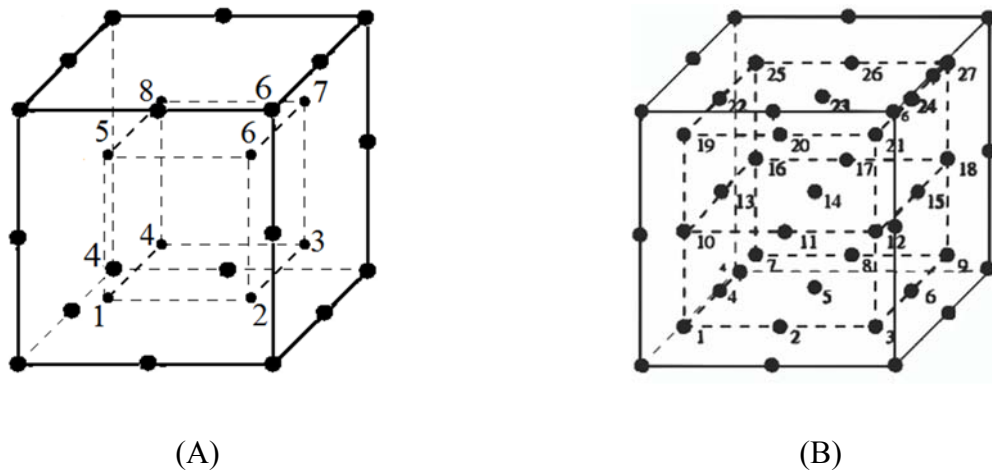


Figure 4. 10 Second-order (quadratic) solid elements (A) reduced integration C3D20R (8 integration points) and (B) fully integration C3D20 (27 integration points)

Hourglassing can occur in first-order reduced integration elements C3D8R, where the element can distort in such a way such that the calculated strains at the integration point are zero. This type of problem can be minimized by distributing loading points and boundary conditions. A summary of the advantages and disadvantages of three-dimensional continuum elements in ABAQUS are provided in Table 4. 1.

Table 4. 1 Comparison between continuum element types in ABAQUS element library

Advantages and limitations		Element Type			
		C3D8	C3D8R	C3D20	C3D20R
Advantages	In incompressible materials, provides a constant volumetric strain to prevent mesh locking.	✓	✗	✗	✗
	Ideal for modeling curved geometry with fewer elements.	✗	✗	✓	✓
	Useful for simple problems/ modeling.	✗	✓	✗	✓
	Useful for complex problems/ modeling.	✓	✗	✓	✗
	Reduced integration points used to form stiffness matrix thus reducing running time	✗	✓	✗	✓
Limitations	Suffers from shear locking for problems involved bending.	✓	✗	✗	✗
	Suffers from volumetric locking for nearly incompressible materials	✗	✗	✓	✗
	Hourglassing (can be controlled by distributing load points and boundary conditions)	✗	✓	✗	✗

According to Table 4. 1, element type C3D20R is judged most suitable for modeling the present problem with fewer elements since it does not suffer from shear locking when the model is subjected to end bending.

4.4 Modeling Considerations for predicting tension test results for pipes

The stress-strain relation for a pipe after undergoing the U-O-E forming process is expected to differ from that of the flat steel plate before undergoing the forming process. Also, it is experimentally established that the apparent hoop stress-strain relation as obtained from radial expansion differs from the experimentally measured longitudinal stress-strain curve. One of the objectives of the present study is to computationally characterize such differences. Towards this goal, numerical simulations for the longitudinal and hoop tension tests are performed on the curved plate.

After the forming process (U- and O- pressing), mechanical expansion (E), and removal of radial expansion pressure, the resulting pipe specimen is subjected to axial elongation to extract the apparent true longitudinal relations and compare it to the stress-strain relation of the initial plate and radial displacement. The true longitudinal stress-strain relation thus extracted will be adopted to model the pipe in the simplified model in Chapter 5. Also, a radial displacement is applied to UOE formed pipe to extract the apparent hoop stress-strain relation for comparison with the apparent longitudinal stress-strain curve.

In the modelling of the longitudinal tension test, the pipe is incrementally subjected to an axial displacement until the pipe elongates by 15% of its initial length (Figure 4. 11 A). The longitudinal tension test was modelled by applying axial displacements at the nodes on the left side of the specimen, in 10 steps. Throughout deformation, the nodes on the right side of the specimen were restrained along the z direction and kept free to move along the x and y directions (Figure 4. 11A) in order to prevent the occurrence of any unintended circumferential stresses. The corresponding loading path for the pipe is depicted Figure 4. 13 in which steps A-D depict the U-O-E modelling process and step E denotes the modelling of the tension test. In Figure 4. 13, Position4a represents the deformed plate after axial elongation.

In the modeling of hoop tension test, the formed plate is subjected to a radial displacement incrementally until the pipe expanded radially by 15% of the inner radius after forming process (Figure 4. 12 A) in 10 steps. Throughout the radial expansion, the middle inner surface of the curved plate are restrained along the x and y directions but kept free to move along the longitudinal

(z) direction. The nodes on the right side of the formed plate are restrained in z direction are kept free to move in x and y directions (Figure 4. 12). The loading path for corresponding to the hoop tension test is provided in Figure 4. 14 in which Position 4b represents the deformed plate after radial expansion. The step corresponding to hoop tension test step is denoted by E'.

The equations in Table 4. 2 were used to obtain the values of the longitudinal stresses and strains incrementally after the forming process and the ones in Table 4. 3 is used for calculation of the hoop stress strain values.

Table 4. 2 Longitudinal stress and strain equations between positions 4 to 4a

	Engineering	True
Strain	$\varepsilon_{EL} = \frac{L_{4-4a} - L_4}{L_4} \quad (4.1)$	$\varepsilon_{TL} = \ln(1 + \varepsilon_{EL}) \quad (4.2)$
Stress	$\sigma_{EL} = \frac{1}{A_4} \sum R_{4-4a} \quad (4.3)$	$\sigma_{TL} = \frac{1}{A_{4-4a}} \sum R_{4-4a} \quad (4.4)$
Area of the pipe	$A_4 = \pi(r_{o4}^2 - r_{i4}^2) \quad (4.5)$	$A_{4-4a} = \pi(r_{o4-4a}^2 - r_{i4-4a}^2) \quad (4.6)$

where

- ε_{EL} is engineering longitudinal strain
- ε_{TL} is true longitudinal strain
- σ_{EL} is engineering longitudinal stress
- σ_{TL} is true longitudinal stress
- r_{o4} is outer radius of the pipe at position 4
- r_{i4} is inner radius of the pipe at position 4
- r_{o4-4a} is current outer radius of the pipe between positions 4 to 4a
- r_{i4-4a} is current inner radius of the pipe between positions 4 to 4a

- L_4 is length of the pipe at position 4
- L_{4-4a} is current length of the pipe between positions 4 to 4a
- A_4 is area of the pipe at position 4
- A_{4-4a} is deformed area of the pipe between positions 4 to 4a
- $\sum R_{4-4a}$ is sum of the reaction forces between positions 4 to 4a

Table 4. 3 Hoop stress and strain equations between positions 4 to 4b

	Engineering	True
Strain	$\varepsilon_{EH} = \frac{u_{m4-4b}}{r_{m4}} \quad (4.7)$	$\varepsilon_{TH} = \ln(1 + \varepsilon_{EH}) \quad (4.8)$
Stress	$\sigma_{EH} = \frac{p_{4-4b} r_{i4}}{t_4} \quad (4.9)$	$\sigma_{TH} = \frac{p_{4-4b} r_{i4-4b}}{t_{4-4b}} \quad (4.10)$
Thickness of the pipe	$t_4 = r_{o4} - r_{i4} \quad (4.11)$	$t_{4-4b} = r_{o4-4b} - r_{i4-4b} \quad (4.12)$

where

- ε_{EH} is engineering hoop strain
- ε_{TH} is true hoop strain
- σ_{EH} is engineering hoop stress
- σ_{TH} is true hoop stress
- p_{4-4b} is current internal pressure between positions 4 to 4b
- r_{m4} is middle radius of the pipe at position 4
- r_{i4-4b} is current inner radius of the pipe between positions 4 to 4b
- r_{o4-4b} is current outer radius of the pipe between positions 4 to 4b
- t_4 is current thickness of the pipe at position 4

- t_{4-4b} is current thickness of the pipe between position 4 to 4b
- u_{m4-4b} is radial displacement at middle surface of the pipe between positions 4 to 4b

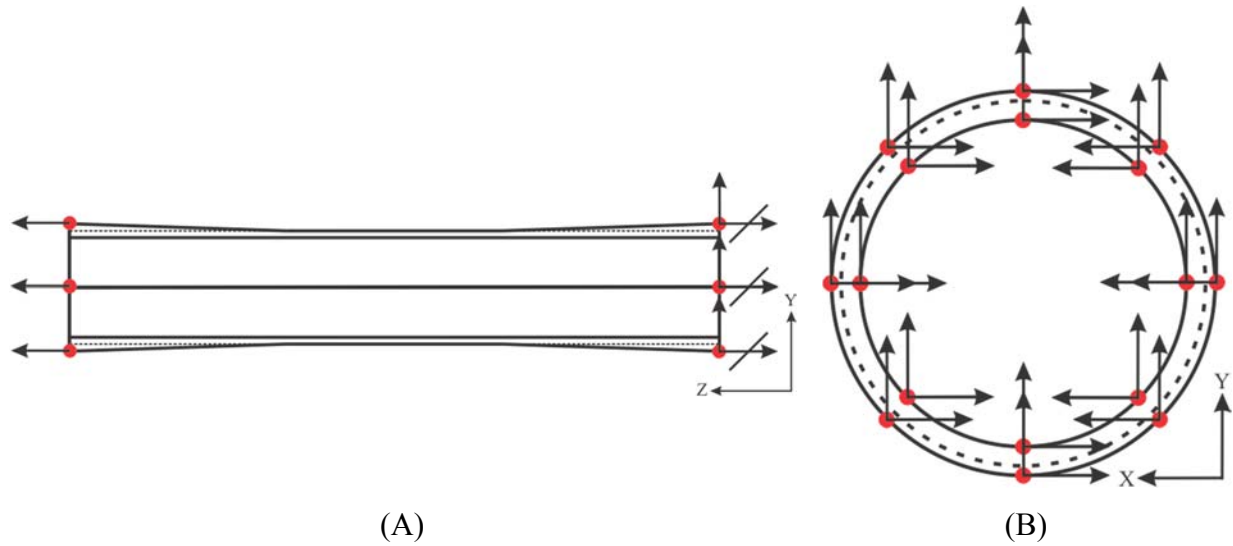


Figure 4. 11 Boundary condition throughout the longitudinal tension test in (A) Side and (B) Left sides of the specimen

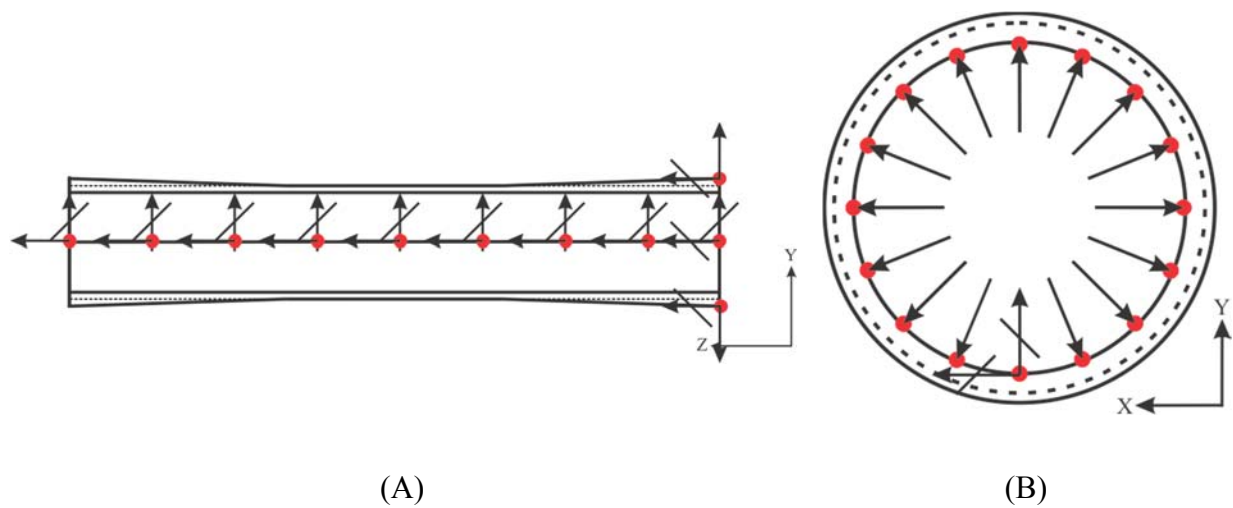


Figure 4. 12 Boundary condition throughout the hoop tension test in (A) Side and (B) left sides of the specimen

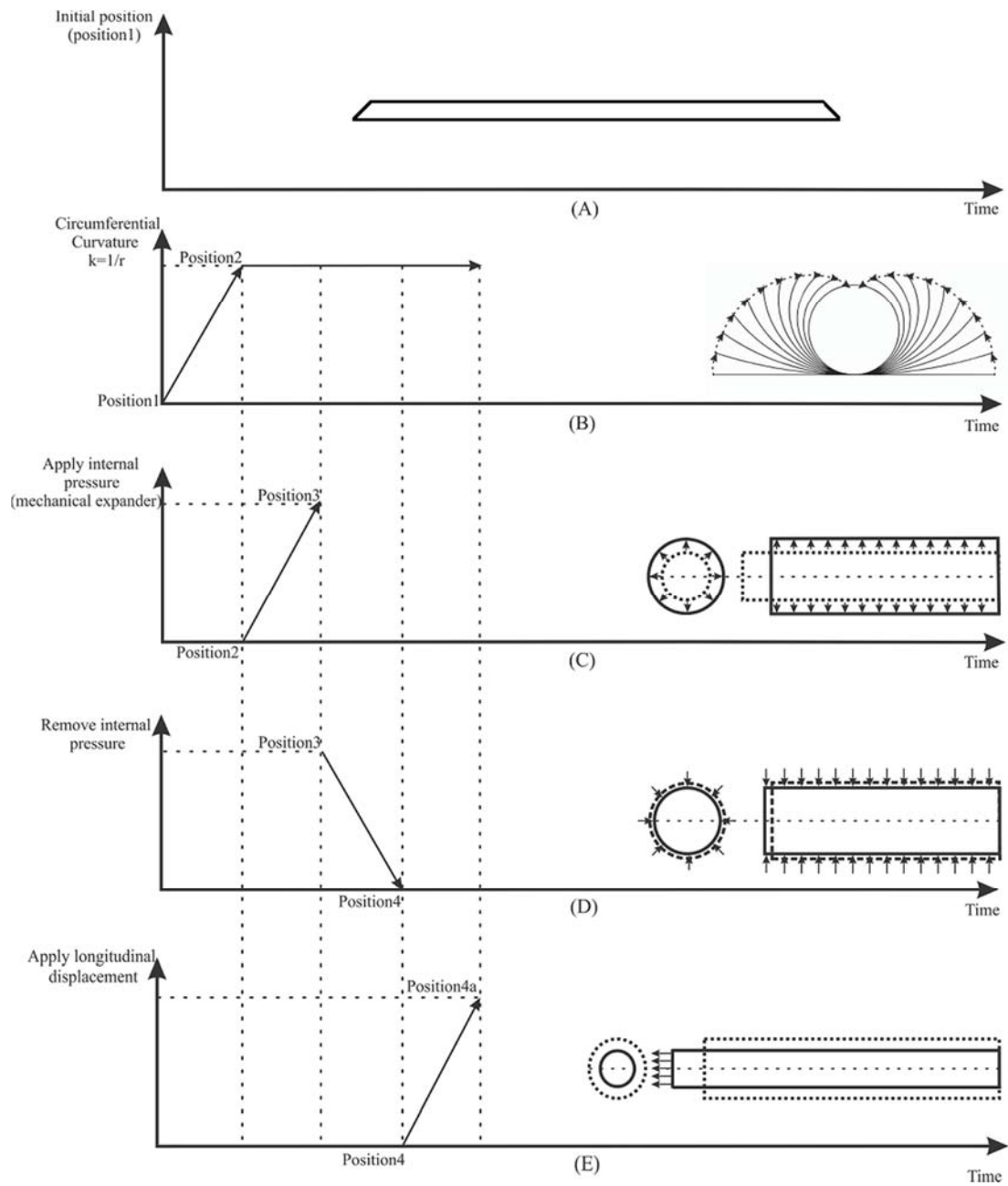


Figure 4. 13 Loading path for the simulation of longitudinal tension test

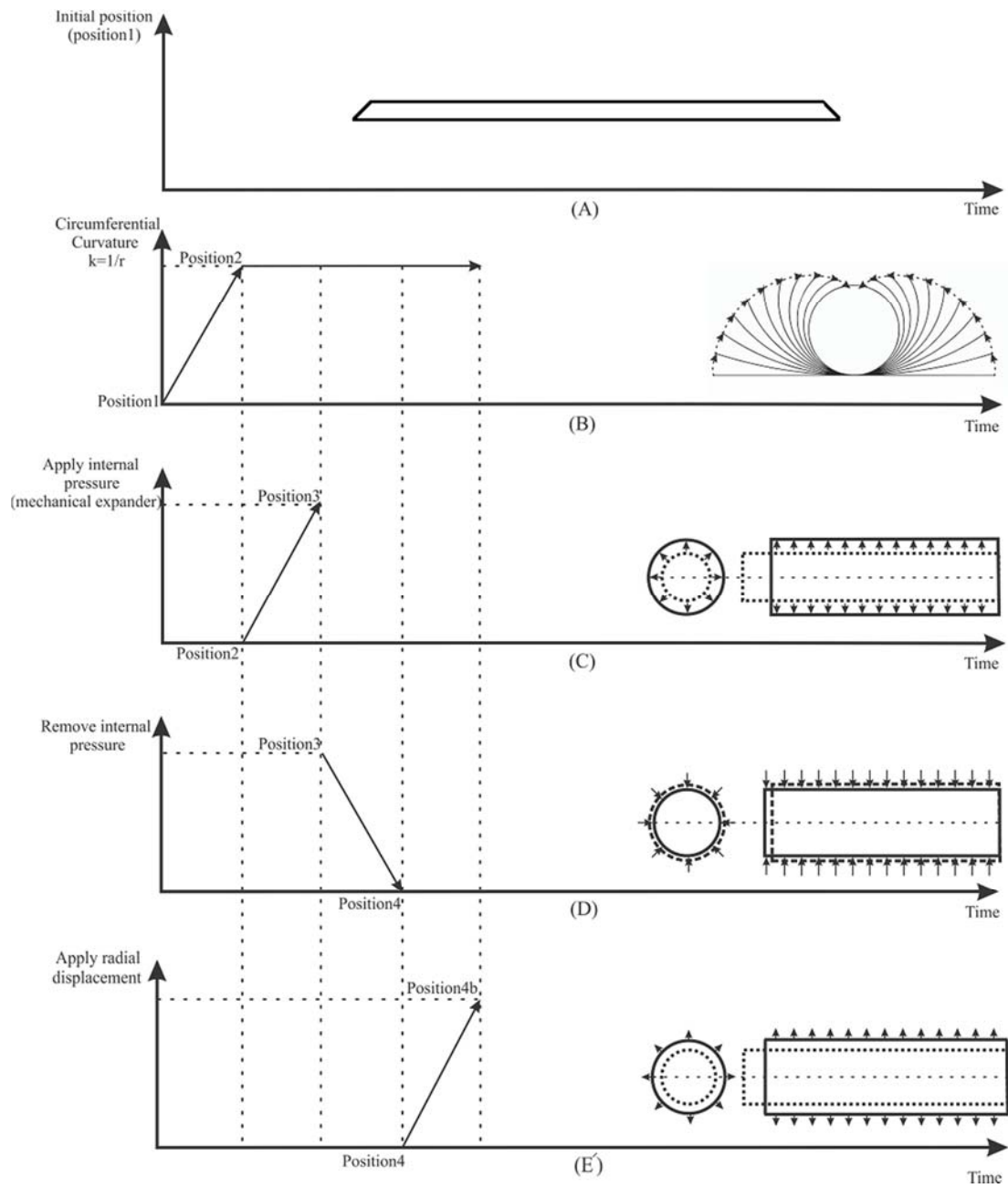


Figure 4. 14 Loading path for the simulation of hoop tension test

4.5 Elasto-Plastic Material Properties

The purpose of this section is to determine the nonlinear hardening material properties that represent the pipe properties. Sub-section 4.5.1 describes the elasto-plastic behaviour of typical steel material and Sub-sections 4.5.2 provides a discussion on defining the Chaboche nonlinear kinematic hardening properties.

4.5.1 Elasto-Plastic Properties

The parameters defining the linearly elastic isotropic material behaviour are Young's modulus (E), and Poisson's ratio. For steel pipes, these values are taken as $E = 199,754 \text{ MPa}$, $\nu = 0.3$ as reported in (Mohareb 1995). Pipe steel is assumed to be X52 with a yield stress assumed to coincide with the limit of proportionality ($F_y = 378 \text{ MPa}$).

The engineering stresses σ_{eng} and corresponding strain values ε_{eng} of X52 steel material obtained from the tension coupon tests (Mohareb 1995) is shown in Table 4. 4. In a nonlinear analysis such as the one sought in the present study, ABAQUS expects the user to input true stresses σ_T and true strains ε_T . The relationships between stresses and strains as provided Chapter 3 are

$$\varepsilon_T = \ln(1 + \varepsilon_{eng}) \quad (4.13)$$

$$\sigma_T = \sigma_{eng} (1 + \varepsilon_{eng}) \quad (4.14)$$

The above relationships were used to compute the true stresses and true strains. The corresponding values are provided in Table 4. 4, and a comparison between engineering stress-strain and true stress-strain curves is depicted in Figure 4. 15.

Table 4. 4 Experimentally Obtained Engineering and true stress-strain values of X52 steel
(Mohareb 1995)

Engineering Values			True Values		Engineering Values			True Values	
#	Strain	Stress (MPa)	Strain	Stress (MPa)	#	Strain	Stress (MPa)	Strain	Stress (MPa)
1	0.0000	0.0	0.0000	0.0	16	0.0580	432.0	0.0480	447.1
2	0.0001	138.1	0.0002	76.1	17	0.0643	435.3	0.0547	455.4
3	0.0004	179.0	0.0001	126.8	18	0.0719	436.8	0.0614	462.1
4	0.0008	221.6	0.0008	187.6	19	0.0814	441.8	0.0681	468.7
5	0.0011	264.2	0.0011	234.9	20	0.0886	443.4	0.0756	475.3
6	0.0014	298.3	0.0014	282.2	21	0.0989	444.9	0.0823	482.0
7	0.0025	335.8	0.0021	324.5	22	0.1077	446.5	0.0910	486.9
8	0.0033	364.7	0.0028	356.6	23	0.1176	448.0	0.0992	493.5
9	0.0060	380.0	0.0052	376.8	24	0.1279	451.2	0.1063	498.5
10	0.0112	386.7	0.0103	388.6	25	0.1363	454.5	0.1134	503.4
11	0.0179	395.1	0.0166	398.6	26	0.1434	452.7	0.1205	508.4
12	0.0258	405.2	0.0225	410.3	27	0.1514	452.5	0.1284	513.3
13	0.0342	413.6	0.0292	420.4	28	0.1585	452.4	0.1363	520.0
14	0.0417	422.0	0.0354	430.4	29	0.1653	452.3	0.1445	523.2
15	0.0496	427.0	0.0417	438.7	30	0.1725	452.2	0.1528	528.1
Engineering Yield Strength (MPa)					True Yield Strength (MPa)				
378.0					380.0				

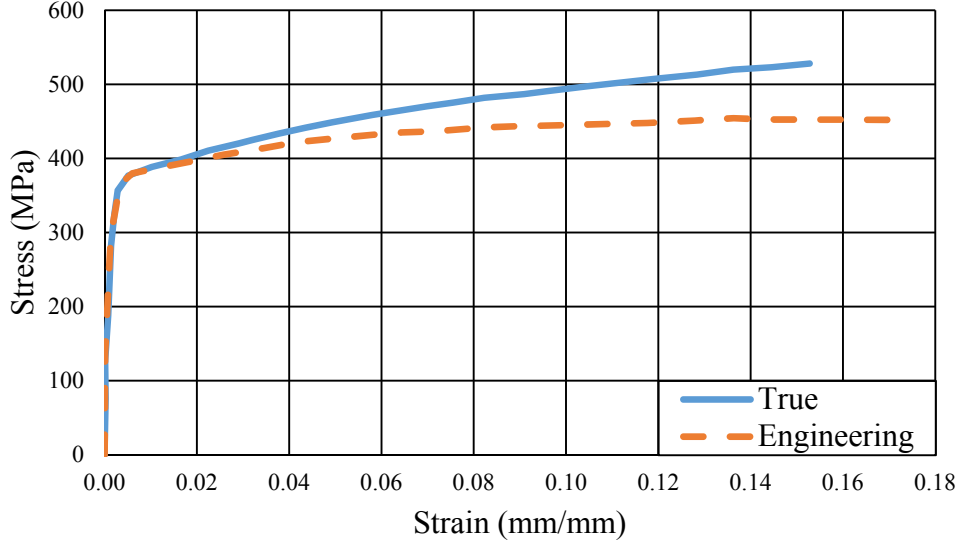


Figure 4. 15 True and engineering stress-strain curves for X52 steel

4.5.2 Determining Kinematic Hardening Parameters

We recall that the Chaboche nonlinear kinematic hardening model is characterised by the backstress tensor α which is assumed to consists of N components, i.e.,

$$\alpha = \sum_{k=1}^N \alpha^k \quad (4.15)$$

The backstress tensor α^k corresponding to of the k^{th} component ($k=1,2,...N$) is assumed to be related to the plastic strain ε_p through C^k and γ^k

$$\alpha^k = \left(\frac{C^k}{\gamma^k} \right) \left[1 - \exp(-\gamma^k \varepsilon_p) \right] \quad (4.16)$$

For a given material, given the experimentally determined backstress -plastic strain relationship $\alpha - \varepsilon_p$, it is required to find the constants C^k and γ^k ($k=1,2,...N$) which match as closely as possible the $\alpha - \varepsilon_p$ relationship. The experimental backstress is computed from the relation $\alpha = \sigma_T - F_y \geq 0$ while the plastic strain is obtained from the relation $\varepsilon_p = \varepsilon_T - F_y/E \geq 0$. The

resulting values are provided in Table 4.5 and backstress-plastic strain relation is depicted in Figure 4. 16. Overlaid on the figure is the true stress-plastic strain relationship for comparison.

Table 4. 5 Experimental stress-plastic strain and backstress for X52 Steel after forming process

Experimental Stress-Plastic Strain			Experimental backstress (MPa) (α_e)	Experimental Stress-Plastic Strain			Experimental backstress (MPa) (α_e)
#	Plastic Strain	True Stress (MPa)		#	Plastic Strain	True Stress (MPa)	
1	0.0000	380.0	0.0	12	0.0826	443.4	63.4
2	0.0052	386.7	6.7	13	0.0929	444.9	64.9
3	0.0119	395.1	15.1	14	0.1017	446.5	66.5
4	0.0198	405.2	25.2	15	0.1116	448.0	68.0
5	0.0282	413.6	33.6	16	0.1219	451.2	71.2
6	0.0357	422.0	42.0	17	0.1303	454.5	74.5
7	0.0436	427.0	47.0	18	0.1374	452.7	72.7
8	0.0520	432.0	52.0	19	0.1454	452.5	72.5
9	0.0583	435.3	55.3	20	0.1525	452.4	72.4
10	0.0659	436.8	56.8	21	0.1593	452.3	72.3
11	0.0754	441.8	61.8				

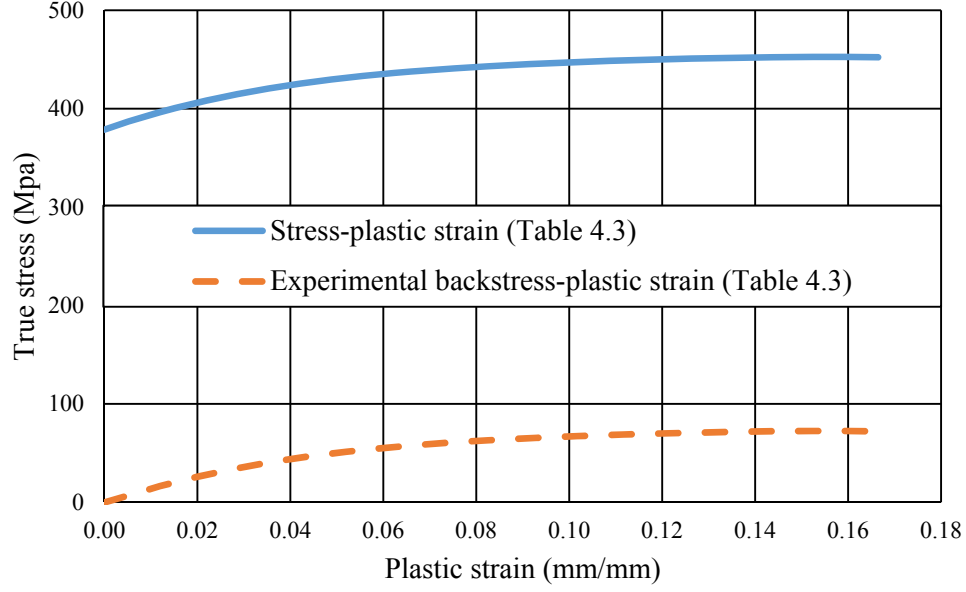


Figure 4. 16 Stress-plastic strain and experimental backstress (Table 4. 5) of X52 steel

4.5.2.1 Determining the Chaboche Parameters

Jiang and Kurath in 1996 proposed a method for the determination of initial values of the Chaboche parameters C^k and γ^k . The first step is to subdivide the stress-plastic strain curve into $M=K+1$. Parts (Figure 4. 17). Next, the Chaboche nonlinear material parameters C^k and γ^k are estimated from Eqs. (4.17) and (4.18) as

$$C^k = \left(\frac{\sigma^k - \sigma^{k-1}}{\varepsilon_p^k - \varepsilon_p^{k-1}} - \frac{\sigma^{k+1} - \sigma^k}{\varepsilon_p^{k+1} - \varepsilon_p^k} \right) \quad (4.17)$$

$$\gamma^k = \frac{1}{\varepsilon_p^k} \quad (4.18)$$

where $k = 1, 2, \dots, K$, σ^k and ε_p^k are the k^{th} components of the stress and plastic strain based on the stress-plastic strain tension in Table 4. 5. The sampling points chosen for the analysis are provided in Table 4.6. A first estimate of the total backstress curve is then obtained by substituting the given values of Chaboche parameters from Eqs. (4.17) and (4.18) into Eq.(4.19).

$$\alpha_a = \sum_{k=1}^{M-1} \alpha^k = \sum_{k=1}^{M-1} \left(\frac{C^k}{\gamma^k} \right) \left[1 - \exp(-\gamma^k \varepsilon_p) \right] \quad (4.19)$$

where α_a is the analytically computed estimate of the backstress, which will generally differ from the experimentally determined (Fig. 4.16). The corresponding initial values of Chaboche parameters are provided in Table 4.7.

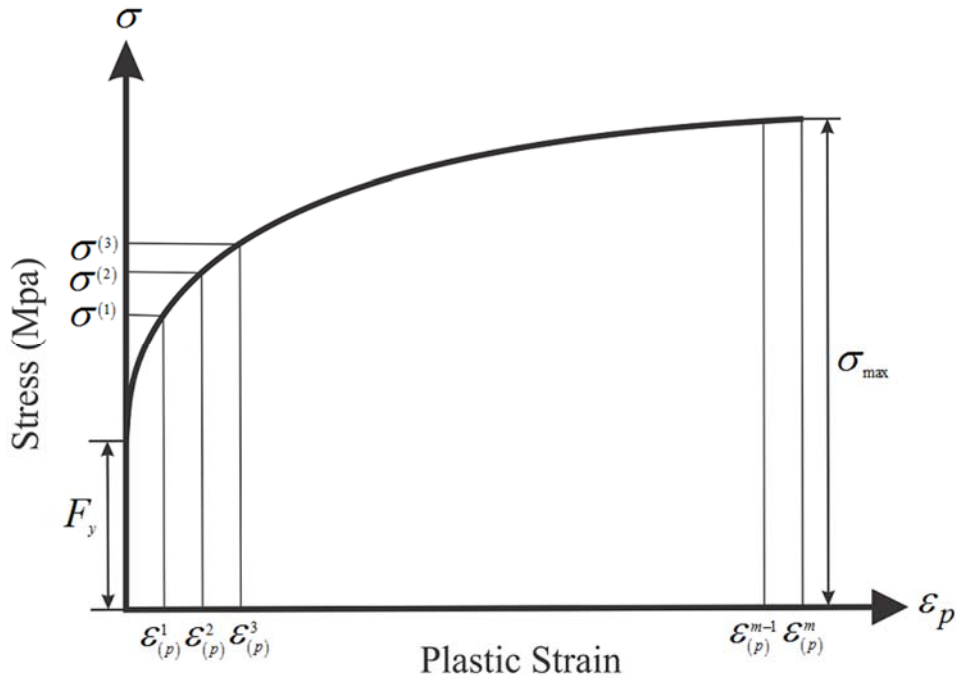


Figure 4. 17 Stress-plastic strain curve is divided into M parts (adapted from Chen and Jiao 2004)

Given the initial values of C^k and γ^k , the next step is to iteratively improve the values C^k and γ^k by trial and error until the analytically calculated backstress-plastic strain curve closely matches that based on experiments. The process has been accomplished by using the curve fitting toolbox in MATLAB software. The final C^k and γ^k are provided in Table 4.9 and the corresponding analytically calculated backstress-plastic strain curve is found in nearly perfect agreement with the experimental results (Fig. 4.16) when the number of backstress components is taken as 10.

Table 4. 6 True stress-plastic strain selected data points of X52 steel

#	Plastic strain	Stress (MPa)	#	Plastic strain	Stress (MPa)
1	0.0000	376.9	7	0.0771	480.7
2	0.0114	399.8	8	0.0940	493.9
3	0.0240	421.0	9	0.1082	503.8
4	0.0366	438.8	10	0.1232	513.5
5	0.0495	454.3	11	0.1563	532.9
6	0.0629	468.1			

Table 4. 7 Initial values of the Chaboche nonlinear kinematic hardening parameters

#	C^k (MPa)	γ^k	#	C^k (MPa)	γ^k
1	330.7	87.7	6	112.4	13.0
2	272.2	41.7	7	81.9	10.6
3	214.3	27.3	8	55.0	9.2
4	170.0	20.2	9	57.7	8.1
5	135.0	15.9	10	37.4	6.4

Table 4. 8 Normalized initial backstress values of X52 steel

#	Plastic strain	Experimental backstress (MPa)	Normalized experimental backstress (MPa)	#	Plastic strain	Experimental backstress (MPa)	Normalized experimental backstress (MPa)
1	0.0000	0.0	0.00	12	0.0826	63.4	0.87
2	0.0052	6.7	0.10	13	0.0929	64.9	0.91
3	0.0119	15.1	0.23	14	0.1017	66.5	0.93
4	0.0198	25.2	0.36	15	0.1116	68.0	0.95
5	0.0282	33.6	0.48	16	0.1219	71.2	0.97
6	0.0357	42.0	0.56	17	0.1303	74.5	0.99
7	0.0436	47.0	0.64	18	0.1374	72.7	1.00
8	0.0520	52.0	0.71	19	0.1454	72.5	1.00
9	0.0583	55.3	0.76	20	0.1525	72.4	1.00
10	0.0659	56.8	0.80	21	0.1593	72.3	1.00
11	0.0754	61.8	0.85				

Table 4. 9 Final values of Chaboche parameters C^k and γ^k for X52 steel

#	Re-calculated C^k (MPa)	Re-calculated γ^k	#	Re-calculated C^k (MPa)	Re-calculated γ^k
1	0.97	1.96	6	118.59	0.94
2	25.32	3.69	7	140.35	0.55
3	1253.42	20.75	8	0.03	1.07
4	225.84	7.11	9	73.45	1.53
5	2.82	3.00	10	278.65	8.35

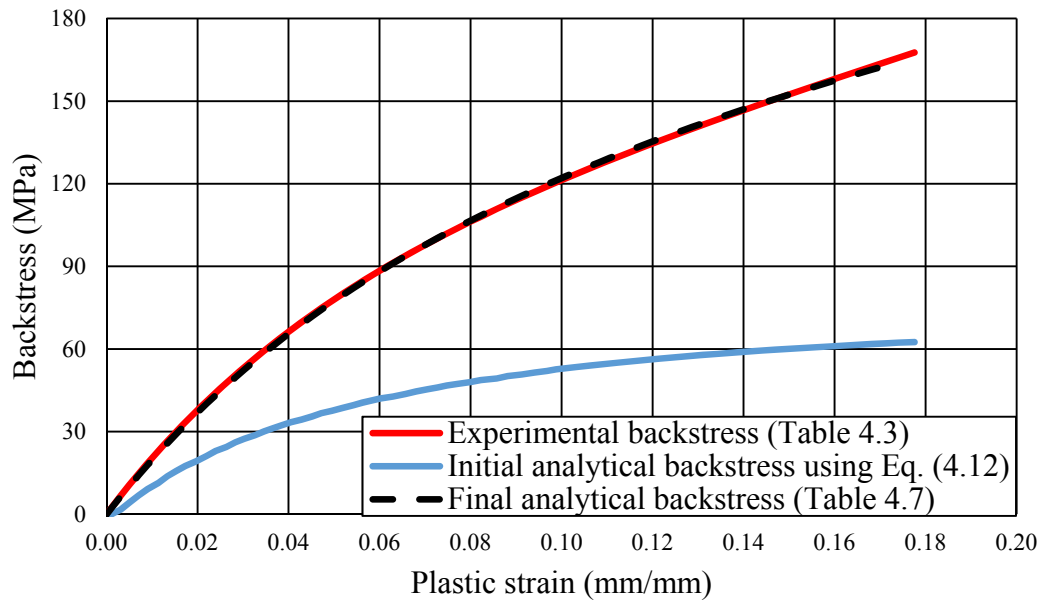


Figure 4. 18 Comparison between analytically computed and experimentally determined backstress values for X52 steel after the initial calculation based on Eqs. 4.5-4.6

4.6 Comparison of the Performance of 3D Elements

The purpose of this section is that to computational performance of the elements C3D8, C3D8R and C3D20R and assess the validity of their results. The specifics of the model are provided in Table 4. 10. From a computational view point, the analysis based on C3D8R is found to be the most efficient, followed by the C3D20R model while the C3D8 model is found to be the slowest. Also, for C3D8, the Mises stress contours on the inside surface in Figure 4. 19 -Figure 4. 21 are found to differ from the other two models and seem to suggest that the element suffers from locking in the inside surface throughout forming process. However, the predicted engineering longitudinal stress-strain relations and the true engineering longitudinal stress-strain relations based on the three models are found to be comparable (Figure 4.20, 421). Given the curved geometry of the pipe, the model based on element C3D20R was deemed to be more suitable for the analysis and will thus be adopted in subsequent analyses.

Table 4. 10 The element, node and integration number comparison between C3D8, C3D8R and C3D20R elements

Element type	Total number of nodes along			Total number of elements along			Number of integration point per element	CPU time (second)
	thickness	length	width	thickness	length	Width		
C3D8	21	41	33	20	40	32	8	32020
C3D8R	21	41	33	20	40	32	1	13734
C3D20R	21	41	33	10	20	16	8	21570

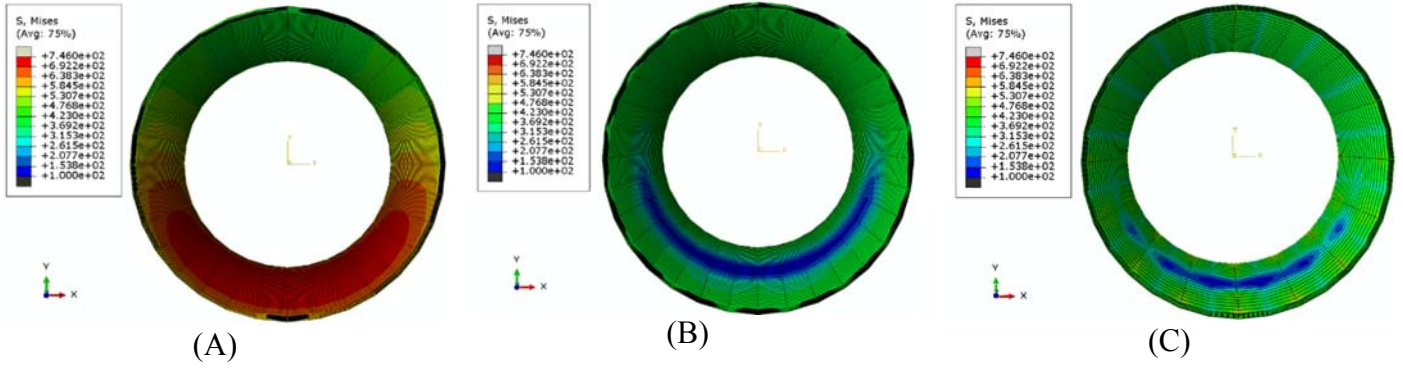


Figure 4. 19 Mises stress after U-O-E forming process in the inside surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R

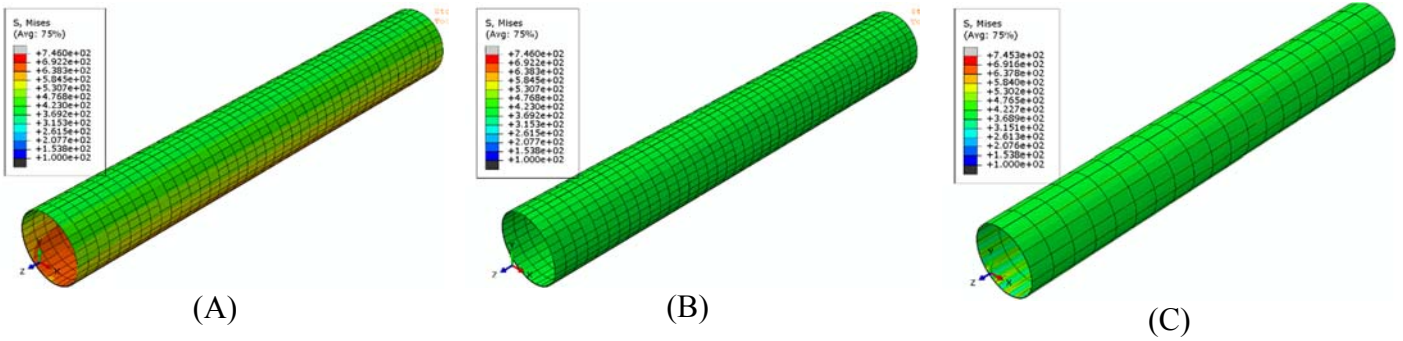


Figure 4. 20 Mises stress after U-O-E forming process in the middle surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R

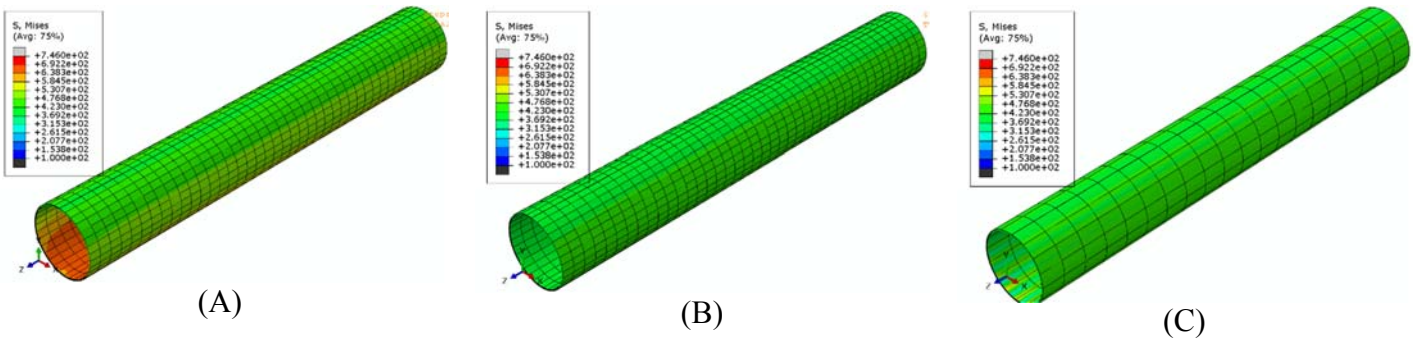


Figure 4. 21 Mises stress after U-O-E forming process in the outside surfaces for elements (A) C3D8, (B)C3D8R and (C) C3D20R

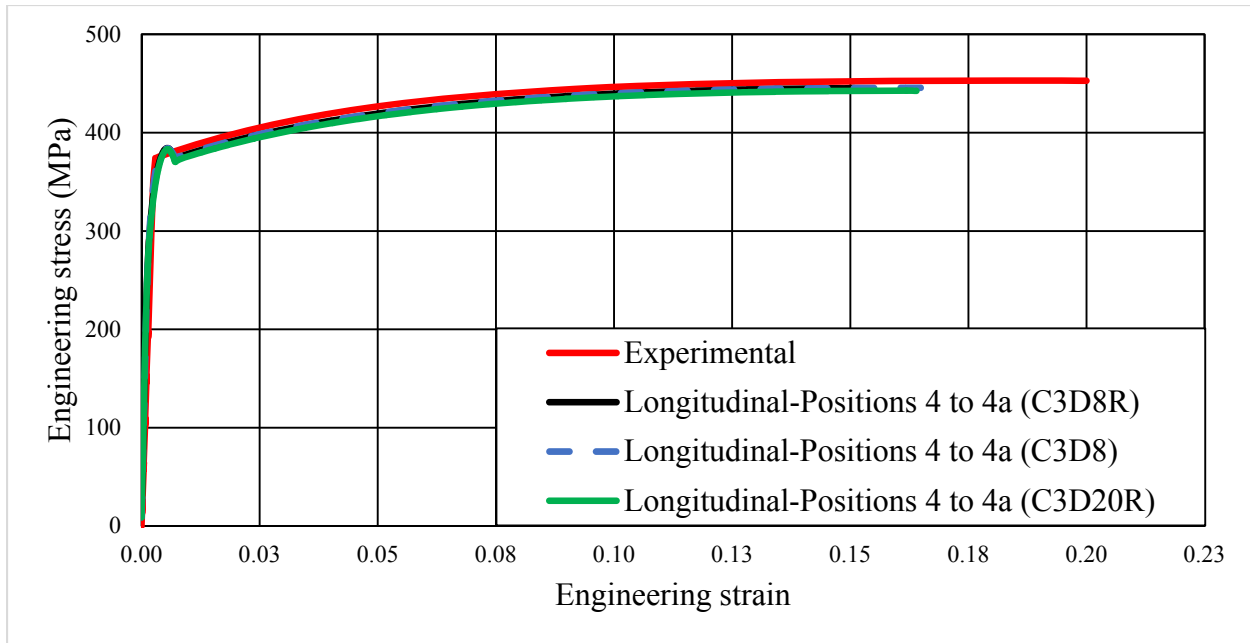


Figure 4. 22 Engineering longitudinal stress-strain curve for the specimen with element C3D8, C3D8R and C3D20R

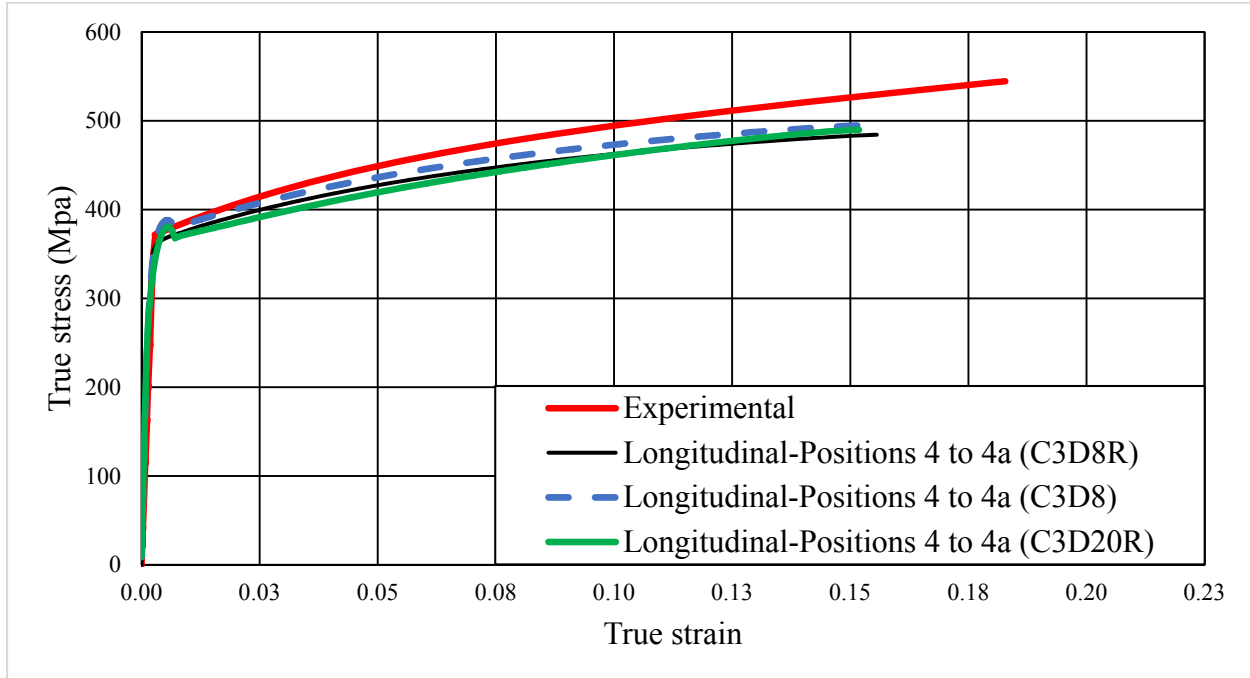


Figure 4. 23 True longitudinal stress-strain curve for the specimen with element C3D8, C3D8R and C3D20R

4.7 Results and Conclusions

Two Abaqus models based on the C3D20R element were developed to predict the apparent longitudinal and transverse stress-strain curves as discussed under Section 4.4. Comparisons are provided with the stress-strain curve of the flat plate. The predicted engineering longitudinal and transverse stress-strain relations are compared on Figure 4. 24 with the experimentally obtained curve for the flat plate. The corresponding true engineering longitudinal and transverse stress-strain relations Figure 4. 24. As reported in past studies (Fathi et al. 2010), the hoop stress-strain relation exhibits a higher apparent yield point than the longitudinal stress-strain relation with a milder apparent strain hardening. The fact that the present model successfully captures the residual stresses induced in the UOE forming process allows the capturing of the difference between the apparent longitudinal and transverse stress-strain relations. In line with past computational work where a pipe has been assumed to behave according to the apparent longitudinal stress-strain curve, the longitudinal stress-strain curve in Figure 4.23 will be used to develop the simplified model in Chapter 5.

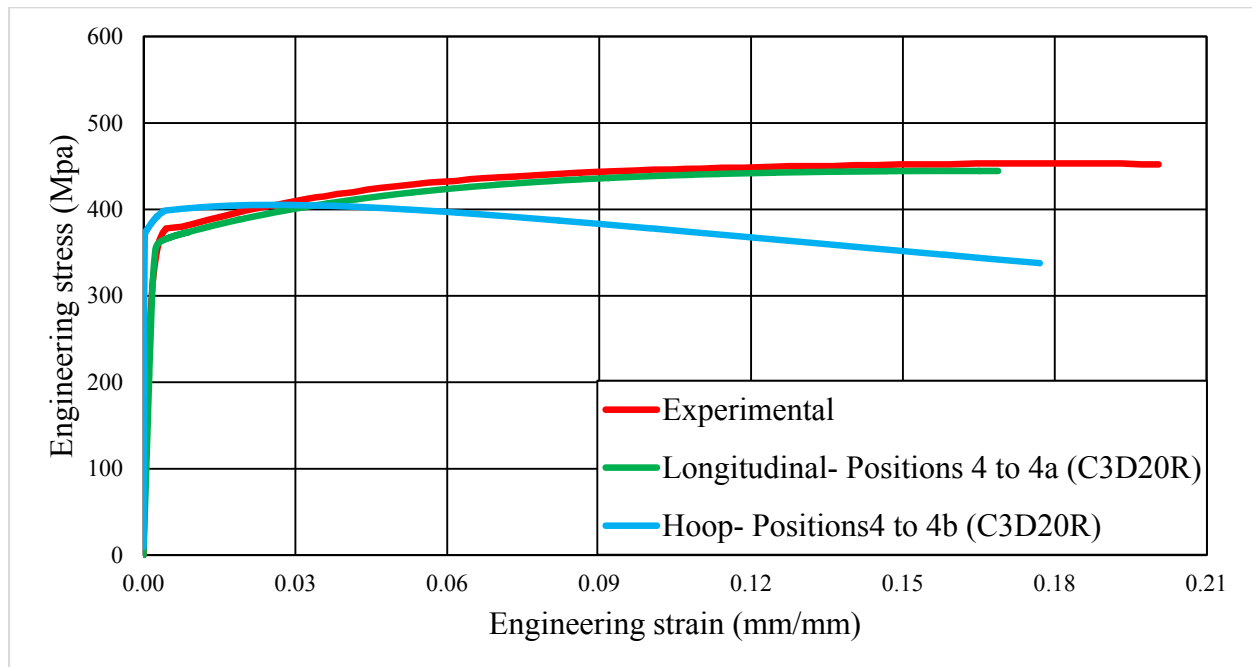


Figure 4. 24 Comparison between hoop, longitudinal and experimental engineering stress-strain curves for the element C3D20R

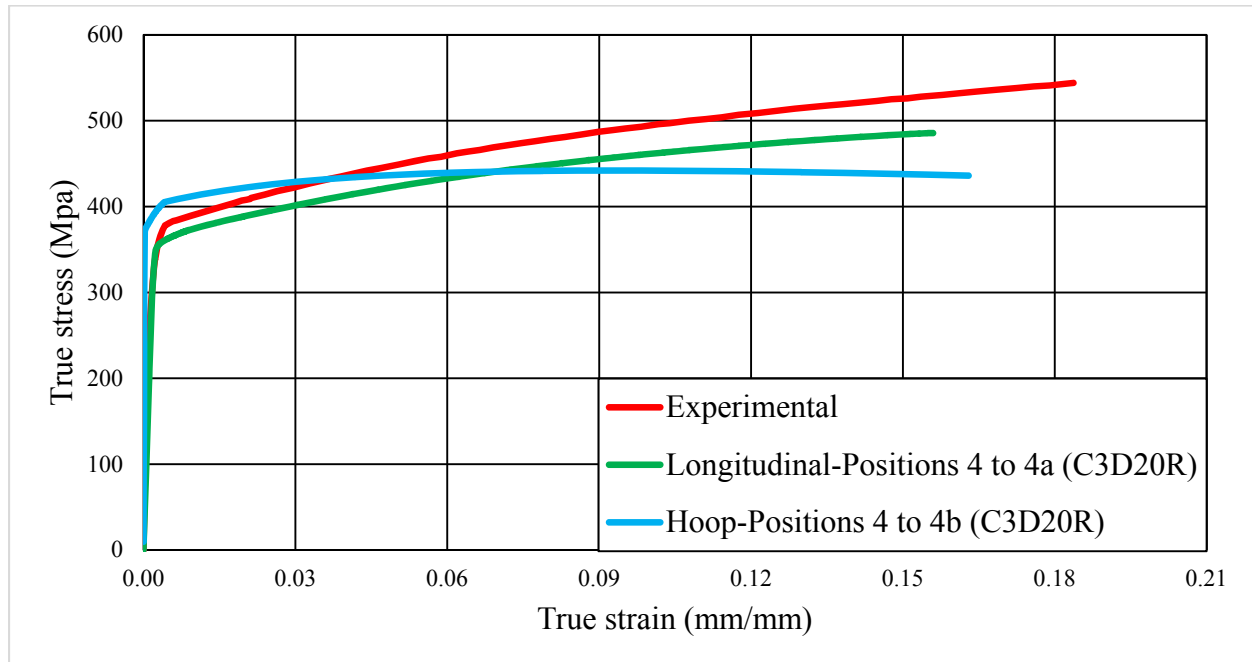


Figure 4. 25 Comparison between hoop, longitudinal and experimental true stress-strain curves for the element C3D20R

Chapter 5 Description of the Simple Finite Element Model

5.1 General

The purpose of this chapter is to develop a 3D finite element model to determine the buckling strains of pressurized steel pipes subjected to bending deformation. In a manner consistent with previous studies, no attempts are made to model the forming process of the pipe in the present chapter. As such, the model is termed as “simple” in contrast to the “detailed” model, to be presented in Chapter 6, which will model the forming process of the pipe. A subsequent comparison (Chapter 7) of the buckling strains based on both models will then enable the assessment of forming process on the critical strains.

A description of the simple pipe geometry is provided in Section 5.2. Section 5.3 provides the loading path. Key aspects of the model consist of end collars, tapering details and gauge length; they are provided in Section 5.4. The elasto-plastic kinematic hardening model used in the models is presented in Section 5.5. Section 5.6 describes the element type and mesh size. Loading and boundary conditions adopted during the application of internal pressure and end moments are represented in Section 5.7.

5.2 Pipe Geometry

The size of the specimens is representative of common sizes in the pipeline industry. As shown in Table 2.2, the D/t ratio of 45 to 120 are most common for onshore pipelines. Thus, in this study, a pipe with a D/t ratio of 64 is selected. Material is taken as X52 material. The finite element analysis is carried out on three pipe segments of 4000 mm span, 9.53 mm thickness and an outer diameter of 610 mm. In a manner that is consistent with previous studies (Chapter 2), the chosen spans are selected large enough to capture local buckling in the middle of the pipe while avoiding the influence of any end effects.

5.3 Loading Path

One un-pressurized and two pressurized specimens were simulated in this chapter. The three specimens are designated as follows:

- P00S is the un-pressurized specimen.
- P40S is a pressurized specimen with an internal pressure including a hoop stress 40% of the yield strength.
- P80S is a pressurized specimen with an internal pressure including a hoop stress 80% of the yield strength.

In the above designation, Symbol S denotes the present simplified model. Figure 5. 1 depicts the loading path for the P00S specimen where Position1 represents the initial pipe in the un-deformed state prior to load application, while Position2 represents the pipe in the deformed configuration as the end moments are applied to the end of the pipe. The loading path for pressurized specimens P40S and P80S are shown in Figure 5. 2 where Position 1 is the initial pipe configuration under no loading, Position 2 is the pressurized pipe after the application of the internal pressure, after which the pressure is kept constant and Position 3 is the deformed configuration of the pipe after the application of end moments.



Figure 5. 1 Loading path for P00S specimen subjected to pure end moments

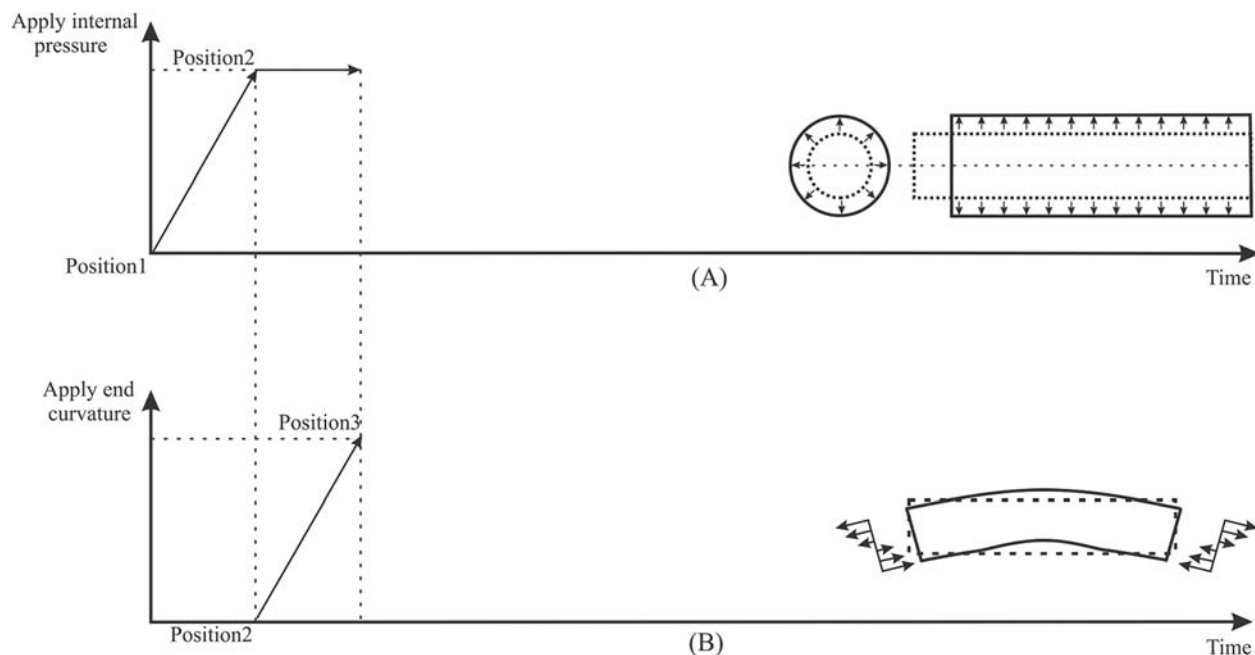


Figure 5. 2 Loading path for P40S and P80S specimens subjected to the combination of (A) internal pressure and (B) end moments

5.4 Model Geometry

In order to avoid localized deformations near the pipe ends (Figure 5. 3), two end collar regions of length $0.9OD$ each were introduced to the ends of the model (Figure 5. 4). The collar regions were assigned elastic material. As a first attempt, all three specimens P00S, P40S, P80S were modeled by considering the pipe to have a constant thickness throughout the pipe length. In these models, local buckling was observed to occur near the end of the specimen (Figure 5. 3). Since such effects are non-representative of pipe buckling in the field, a second modeling attempt was then made to suppress buckling at the ends by tapering the end segments (Figure 5. 4). Both segments had a tapered length of $1.77OD$ with an end thickness of 110% of the specified thickness. The thickness linearly decreased to 100% of the specified thickness. The tapered segments introduced into the model are intended to force local buckling to occur in the middle of the specimens and thus avoid the influence of end effects on the local buckling formation, to emulate more precisely the conditions in the field where no end effects are present. The corresponding models are denoted P00S-T10, P40S-T10, P80S-T10 where the additional designation T10 indicates tapered ends with

an addition 10% increase in thickness. The use of tapered ends was shown to be successful as illustrated in Figure 5. 6.

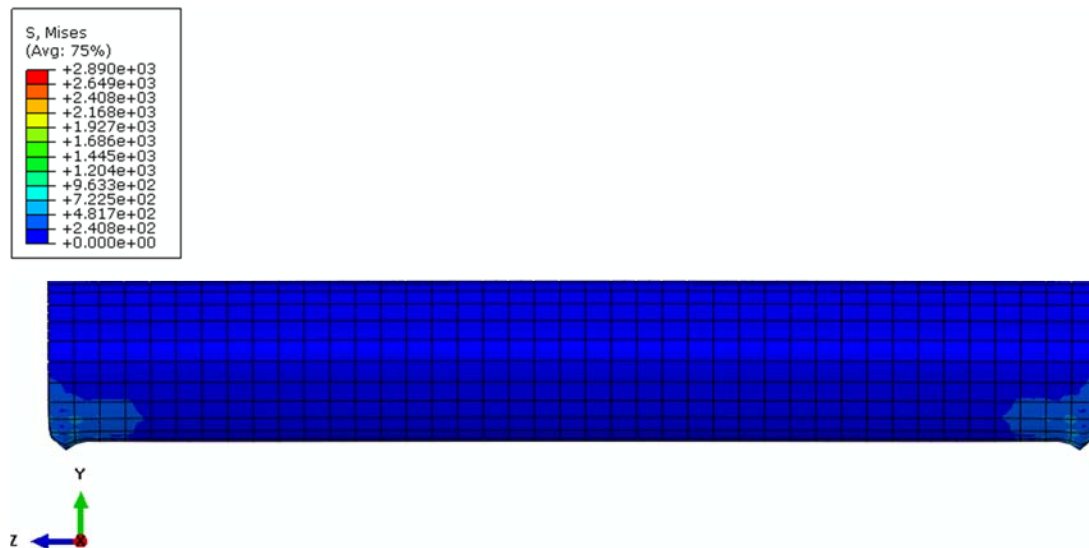


Figure 5. 3 Local buckling at the end of the pipe in the pipe with constant thickness and no collars

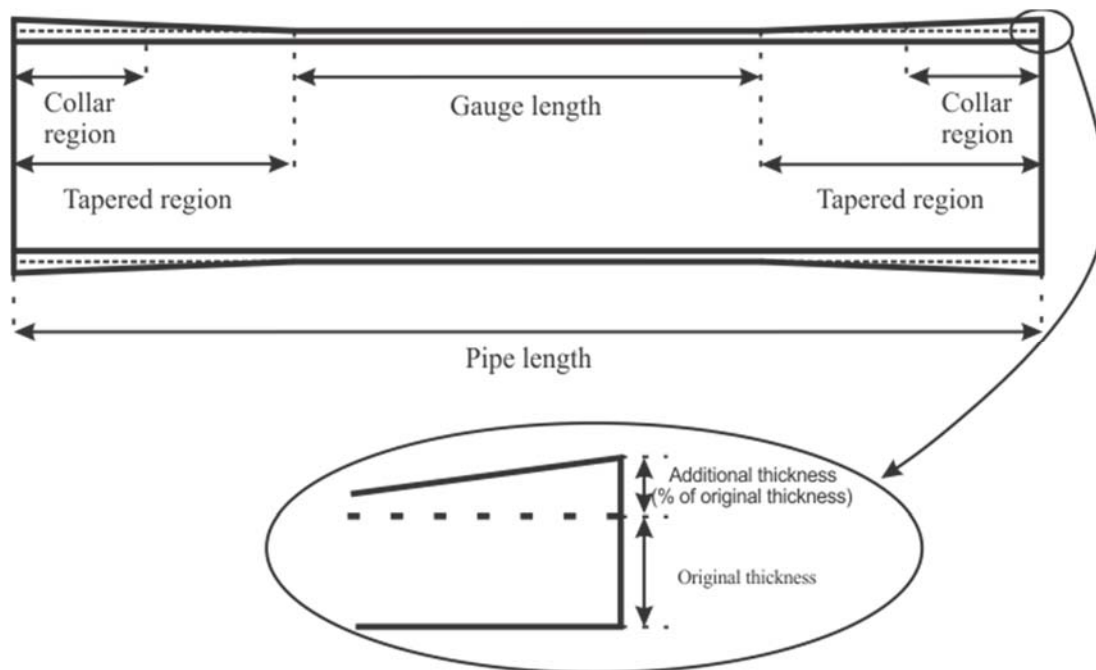


Figure 5. 4 Sectional elevation view of FEA pipe model

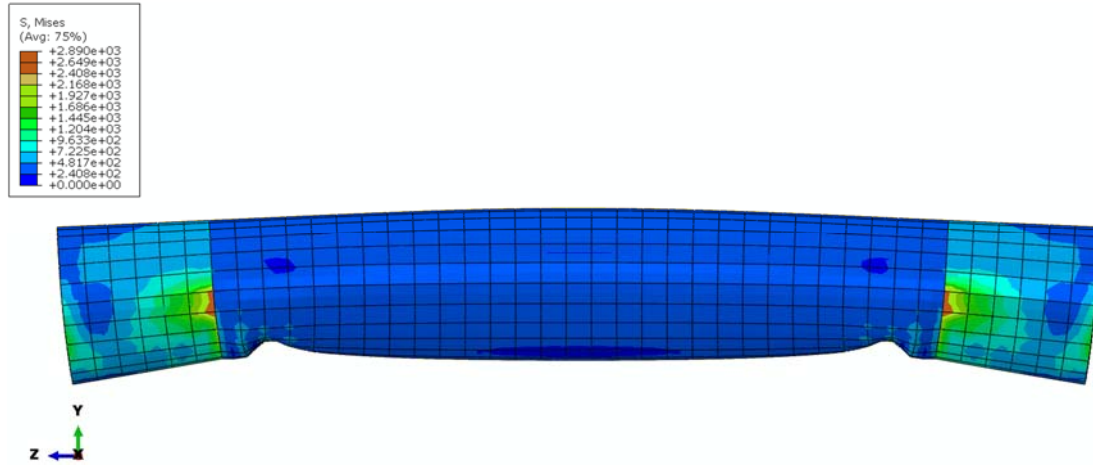


Figure 5. 5 Local buckling for P00S with collar regions and constant thickness

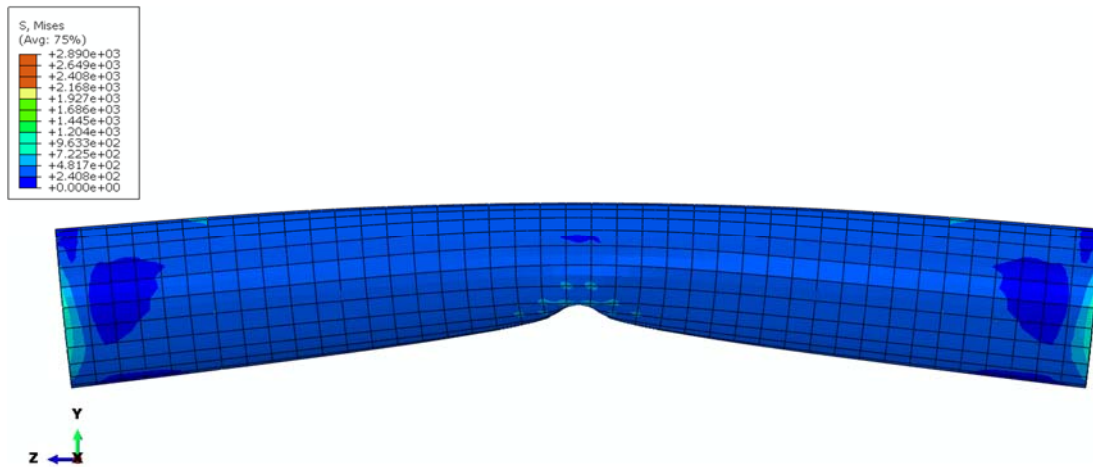


Figure 5. 6 Local buckling for P00S-T10 with tapered ends and collar regions

5.5 Material Properties

As discussed, the whole pipe was assigned steel X52 properties whereas the collar areas were assumed elastic. Both materials are assumed to have an elastic response in which X52 is followed by a plastic response with kinematic hardening. As discussed in Chapter 4, the Chaboche model in ABAQUS is used for modeling the kinematic hardening behaviour of steel. In this case, the best fitting between the analytical and experimental backstress curves, after calibration by curve fitting in Matlab software, was obtained with one value for each Chaboche parameters. The values for Chaboche model after calibration are $c = 1685.0$ MPa and $\gamma = 12.82$.

Table 5. 1 Experimental stress-plastic strain and backstress for X52 Steel

Experimental Stress-Plastic Strain			Experimental Backstress (MPa) $(\sigma - F_y)$	Experimental Stress-Plastic Strain			Experimental Backstress (MPa) $(\sigma - F_y)$
#	Plastic Strain	Stress (MPa)		#	Plastic Strain	Stress (MPa)	
1	0.000	449.6	0	12	0.023	513.4	63.8
2	0.002	461.9	12.3	13	0.025	517.0	67.4
3	0.005	474.5	24.9	14	0.028	518.2	68.6
4	0.006	479.9	30.3	15	0.030	521.8	72.2
5	0.008	485.5	35.9	16	0.032	525.1	75.5
6	0.011	491.1	41.5	17	0.034	526.1	76.5
7	0.013	494.4	44.8	18	0.035	529.3	79.7
8	0.015	497.7	48.1	19	0.038	532.8	83.2
9	0.016	500.9	51.3	20	0.040	534.0	84.4
10	0.019	506.7	57.1	21	0.042	535.3	85.7
11	0.021	507.6	58	22	0.044	538.6	89

5.6 Element Type and Mesh Size

As discussed in Chapter 4, the continuum element type C3D20R is adopted to model the pipe. The element has 20 nodes with three translational degrees of freedom per node and reduced integration. The elements has features to prevent shear and volumetric locking in bending and is ideal for modeling the curved geometries with fewer elements. A mesh study (Appendix A) has indicate

that convergence is achieved when 41 elements are taken in the longitudinal direction, 24 elements along the circumference and 10 elements across the thickness.

5.7 Load Application

5.7.1 Load control Scheme

The Static/General step feature in ABAQUS was used to apply the internal pressure in pressurized models P40S-T10 and P80S-T10. In order to apply the end moments, the Static/Riks method was used for all models. The Riks method is based on arc-length control. The method allows the tracing of the descending branch of the equilibrium path where the load-displacement response exhibits negative stiffness.

5.7.2 Application of Internal Pressure

For the P40S-T10 and P80S-T10 models, the internal pressure is incrementally applied as internal radial pressure to the inside surface of the pipe (Figure 5. 7) using the *DSLOAD keyword. The internal pressure is then kept constant throughout the subsequent loading involving end bending moments

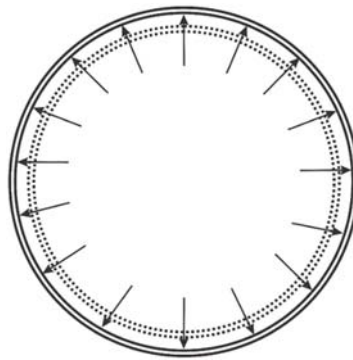


Figure 5. 7 End view of models P40S-T10 and P80S-T10 models subjected to the internal pressure

5.7.3 Application of Displacement Symmetry Condition Associated with Internal Pressure

As the pipe expands radially, it is assumed that the nodes on the top and bottom lines are free to move along the y direction but restrained in the x direction (Figure 5. 8). Conversely, all the nodes on the side lines are assumed to move along the x direction but to be restrained in the y direction.

These constraints were enforced during the internal pressure step using the *BOUNDARY keyword in ABAQUS.

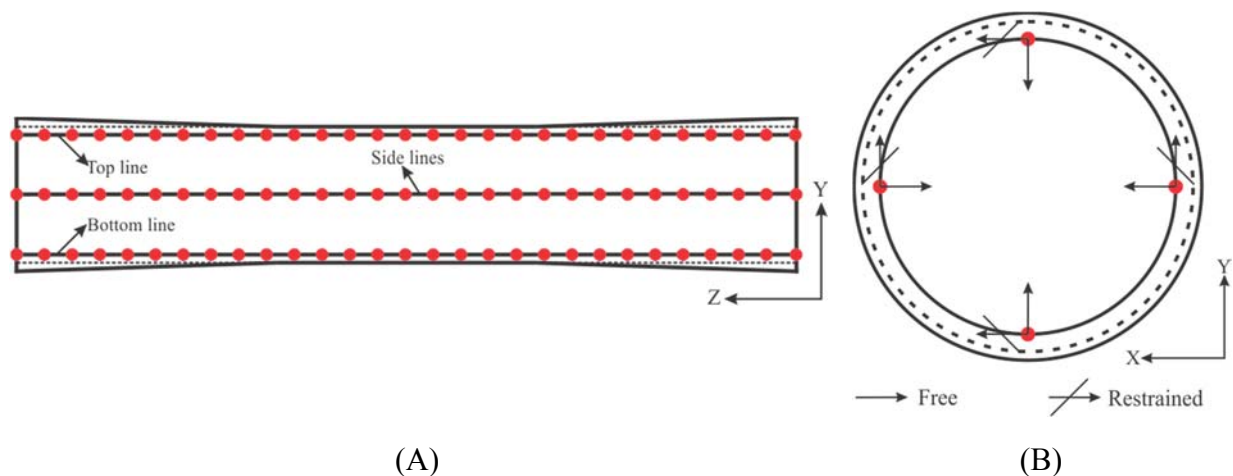


Figure 5. 8 Schematic view of restrained and free degree of freedoms during application of internal pressure (A) Sectional elevation view (B) Cross-section view

5.7.4 Application of End Bending Moments

In the last step of the simulation for specimens P00S-T10, P40S-T10 and P80S-T10, bending moments are applied to both ends of the pipe (Figure 5. 9). Bending moments are applied as a set of point loads using the *CLOAD keyword in ABAQUS. The magnitudes of the load applied at a given fibre are proportional to the vertical distance of that fibre from the cross-section centroid. Given that the bending moments reach a peak value after which, they exhibit a softening behaviour, the arc length method in Static/Riks is adopted. Node A in Figure 5. 9 was chosen as a control degree of freedom. The analysis was specified to end when point A attains a specified peak value of longitudinal displacement.

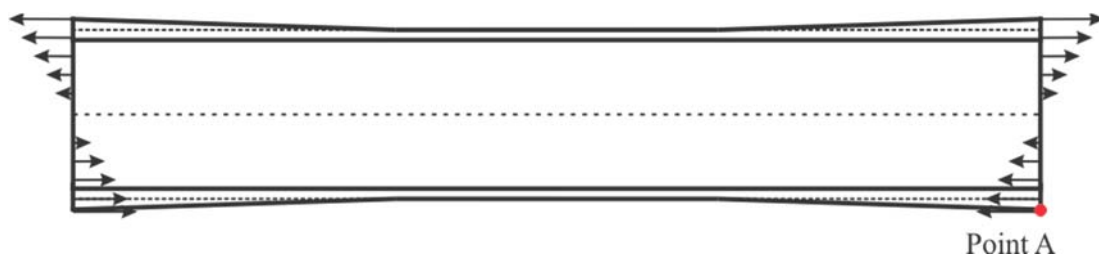


Figure 5. 9 Specified end forces to simulate the end bending moments.

5.7.5 Applying the radial restraints during bending step

In the loading step where the end moments are enforced, all nodes on the ends of the inner diameter were restrained along the x and y directions (Figure 5. 10) and free to move along the z direction. The new constraints in this step were applied using the *BOUNDARY, OP=NEW keyword in ABAQUS.

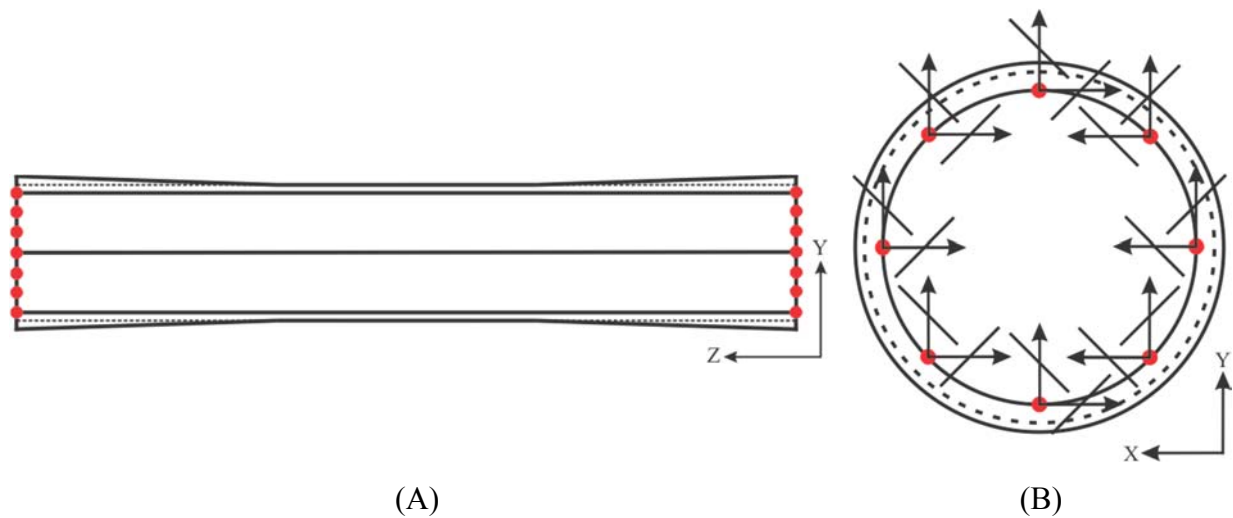


Figure 5. 10 Schematic view of restrained and free degree of freedoms during application of end moments (A) Sectional elevation view (B) Cross-section view

CHAPTER 6. Description of the Detailed Finite Element Model

6.1 General

As discussed in Chapter 2, the UOE manufacturing process starts with a rectangular steel plate and forms it into a pipe shape. While such a process is expected to induce residual strains within the pipe wall, the simplified model provided in Chapter 5 does not attempt to model nor capture the residual strains. In contrast, the detailed model in the present chapter is intended to predict the residual strains generated during the UOE forming process.

In a manner similar to Chapter 5, one un-pressurized and two pressurized models subjected to bending are simulated in this chapter. The effects of various stages of forming on the mechanical behaviour of steel pipes are quantified before and after forming of the plate. Starting with a flat plate with an originally known stress-strain relationship, the plate is curved into a pipe shape and the apparent (i.e., after forming) longitudinal and hoop stress-strain curves are determined and compared to the original stress-strain curves. Next, the buckling strains based on the detailed model are compared to those based on the simplified model discussed in Chapter 7.

The geometry of the specimens, element type, mesh size, load application details associated with internal pressure and end moment application after the forming process are intended to be identical to the ones described in Chapter 5.

A description of the flat rectangular plate is provided in Section 6.2. The loading steps aimed at the modelling of the UOE process are presented in Section 6.3. Section 6.4 presents the model specifics relating to end collars, tapered regions, and gauge length for all models. The post-forming mechanical properties of the plate material as extracted from the longitudinal tension test and then used as the initial material property for the models in Section 6.5. The loading and boundary conditions during the forming process and after that is described in Section 6.6. The used constraints in the models are provided in Section 6.7.

6.2 Plate Geometry

The size of the specimens were chosen to reach the selected pipe size in Chapter 5 after forming process. The rectangular plates are taken to have a 1857.3 mm width, 4000 mm length and 9.53 mm thickness (Figure 6. 1).

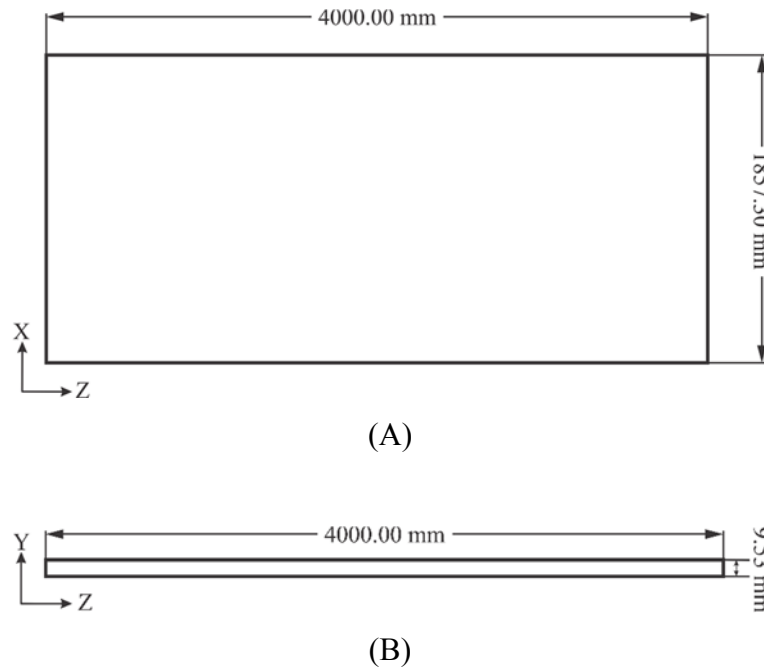


Figure 6. 1 Sectional elevation view of rectangular plate

6.3 Loading Path

In a manner similar to Chapter 5, one un-pressurized and two pressurized specimens were simulated. Unlike Chapter 5, all three specimens were assumed to consist originally bent from of a rectangular plate to pipe in the forming process.

- P00D the un-pressurized specimen.
- P40D is a pressurized specimen with an internal pressure including a hoop stress 40% of the yield strength.
- P80D is a pressurized specimen with an internal pressure including a hoop stress 80% of the yield strength.

In the above designation, the symbol D denotes the detailed model. Figure 6. 3 illustrates the loading path for all specimens where Position 1 is the initial rectangular plate in the un-deformed

shape (before UOE forming process). Position 2 represents the forming plate to the pipe shape after U- and O-ing forming process by nodal-displacement in 10 separate steps (Sub-section 6.6.1). At the end of the forming process, the nodal displacement of the nodes on the right and left sides of the plate keep constant to avoid separation between these nodes.

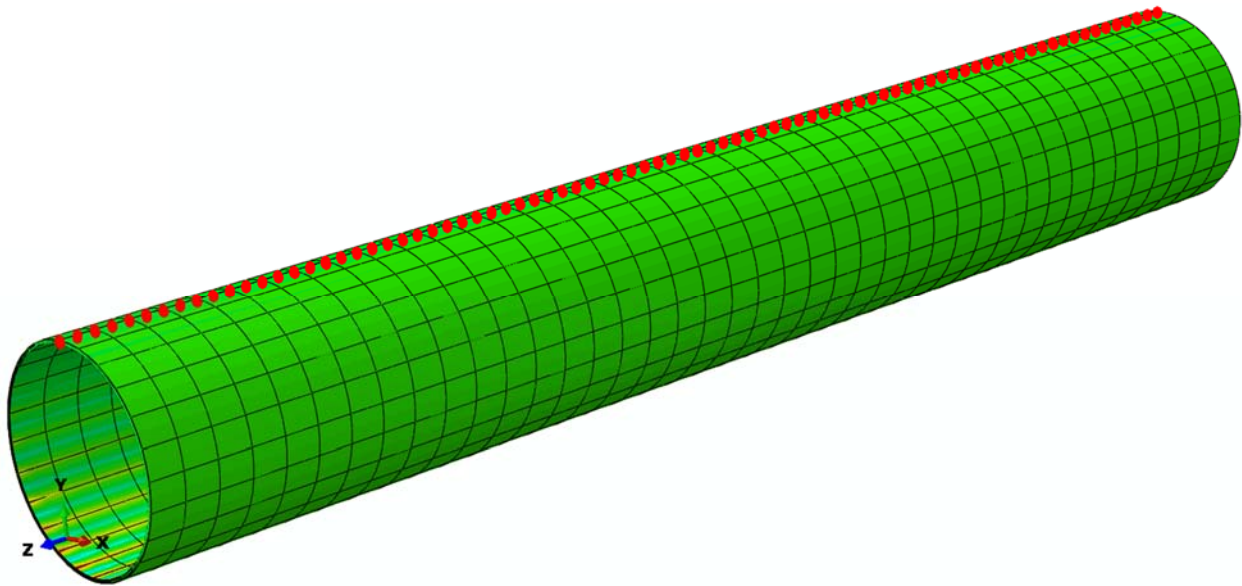


Figure 6. 2 Nodes on the right and left sides of the plate

Positions 3 and 4 are represented the pipe before and after application of internal pressure as mechanical expansion.

In Figure 6. 4, Position 5 is illustrated the bent plate after the application of end moment. In Figure 6. 5, Position 5 is the pressurized formed plate after the UOE forming process and the applied pressure will be kept constant during the application of end moment in Position 6.

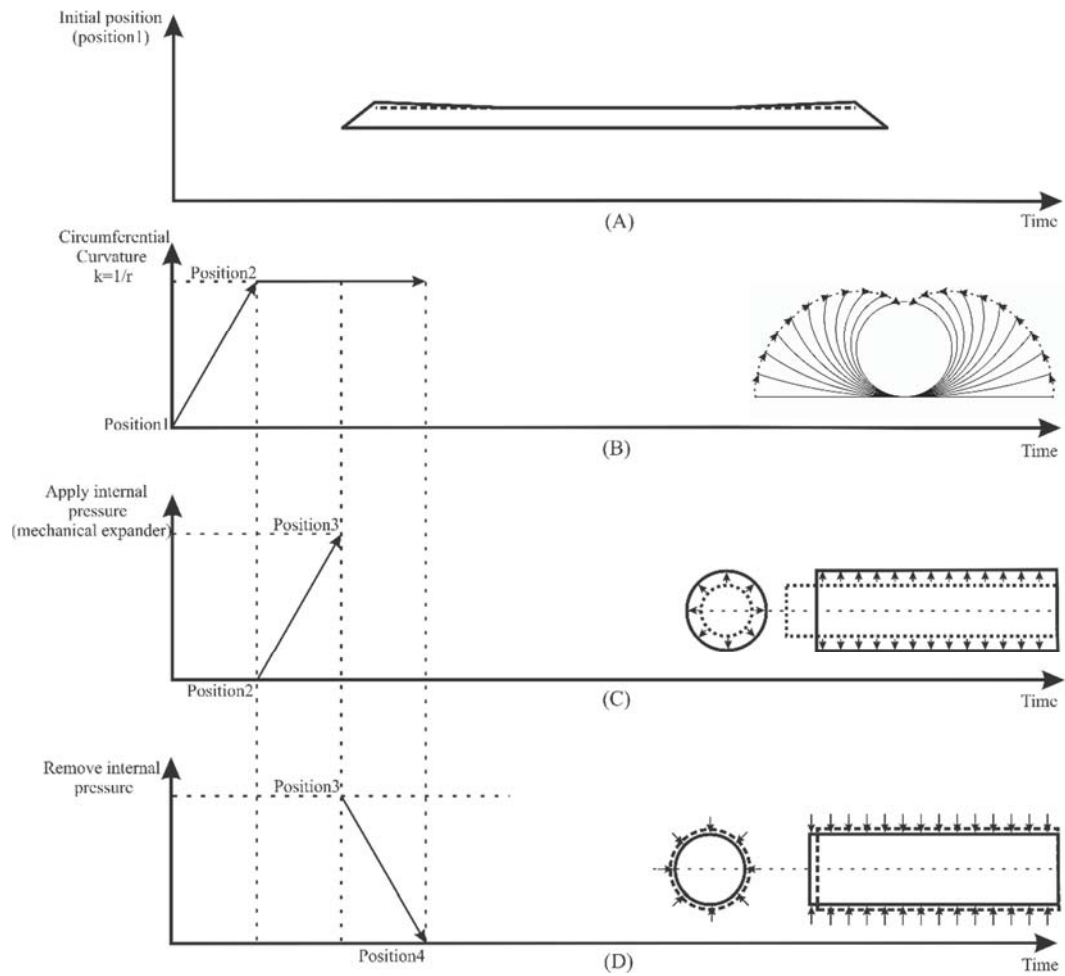


Figure 6. 3 UOE forming process for all specimens as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion

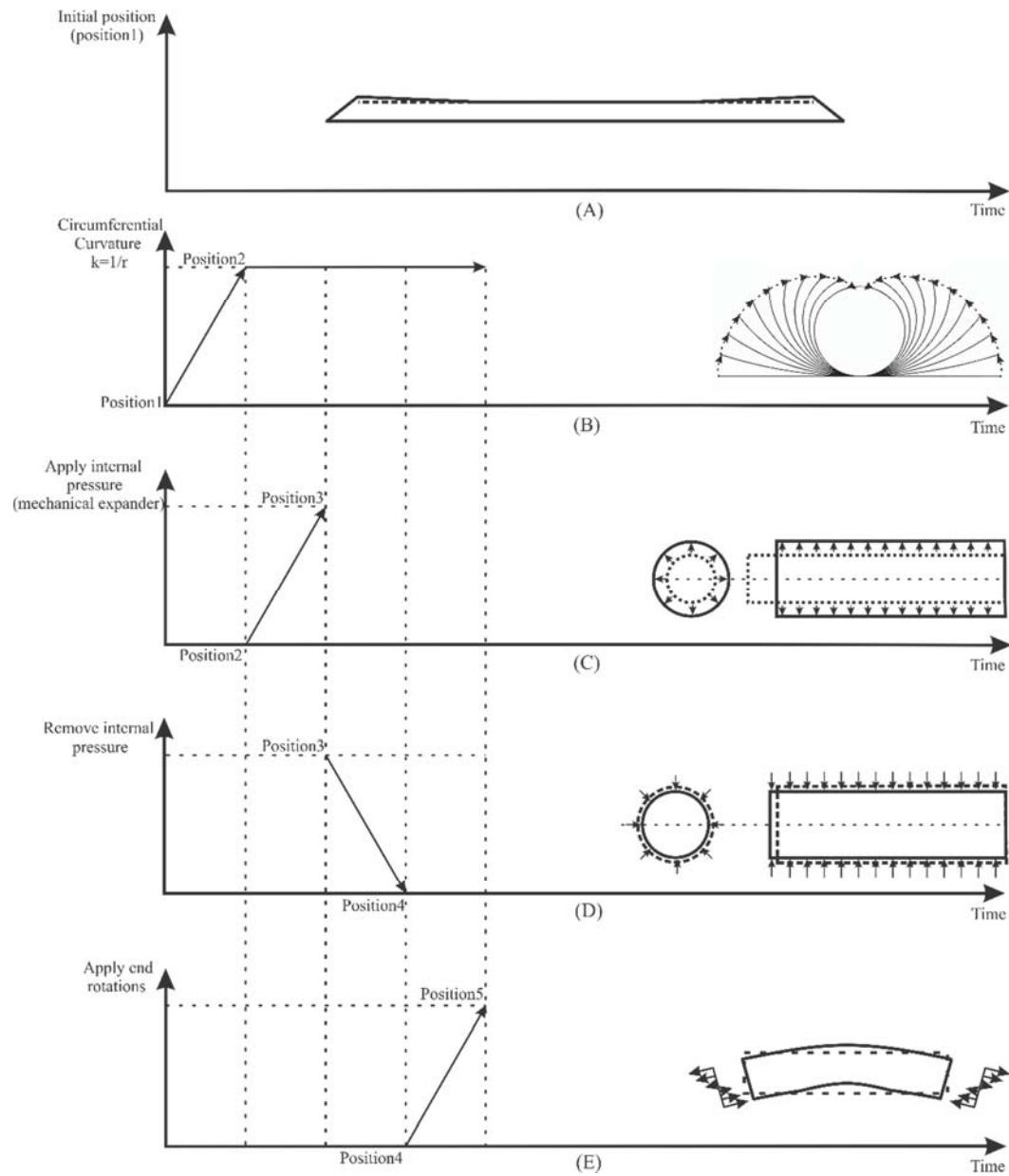


Figure 6. 4 UOE forming process for P00D specimen as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion (E) Apply end moments

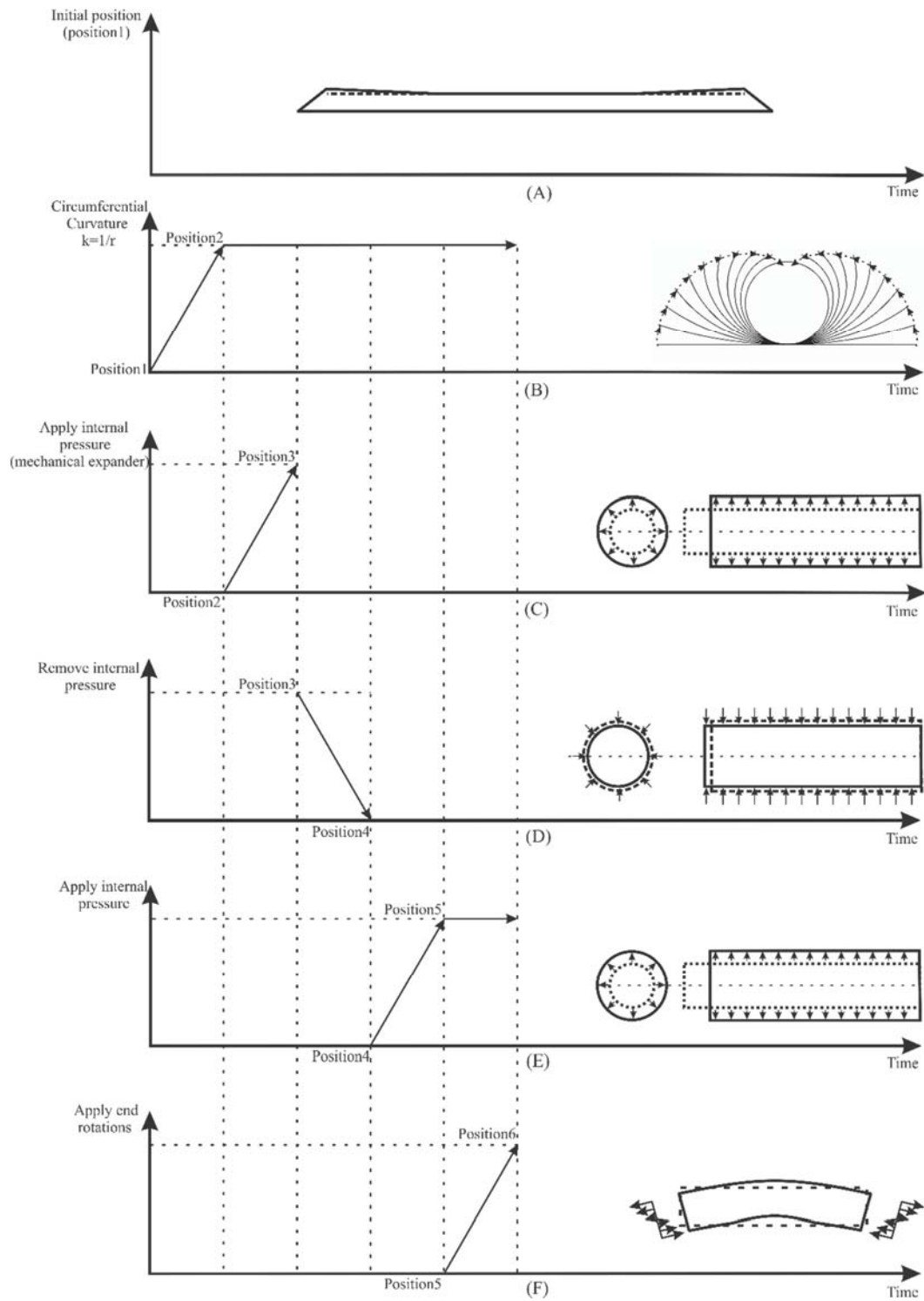


Figure 6. 5 UOE forming process for P40D and P80D specimens as (A) Flat plate, (B) Bent plate after U- and O-ing process, (C) Bent plate after applying radial expansion and (D) Bent plate after removing the radial expansion (E) Apply internal pressure (F) Apply end moments

6.4 Model Geometry

Two end collar and tapered regions were considered for the specimens, same as the pipes in Chapter 5, to avoid localized buckling near the plate end (Figure 6. 6). The only difference in these specimens is the material extracted from longitudinal tension test after the forming process for the whole pipe except the collars area (Section 6.5).

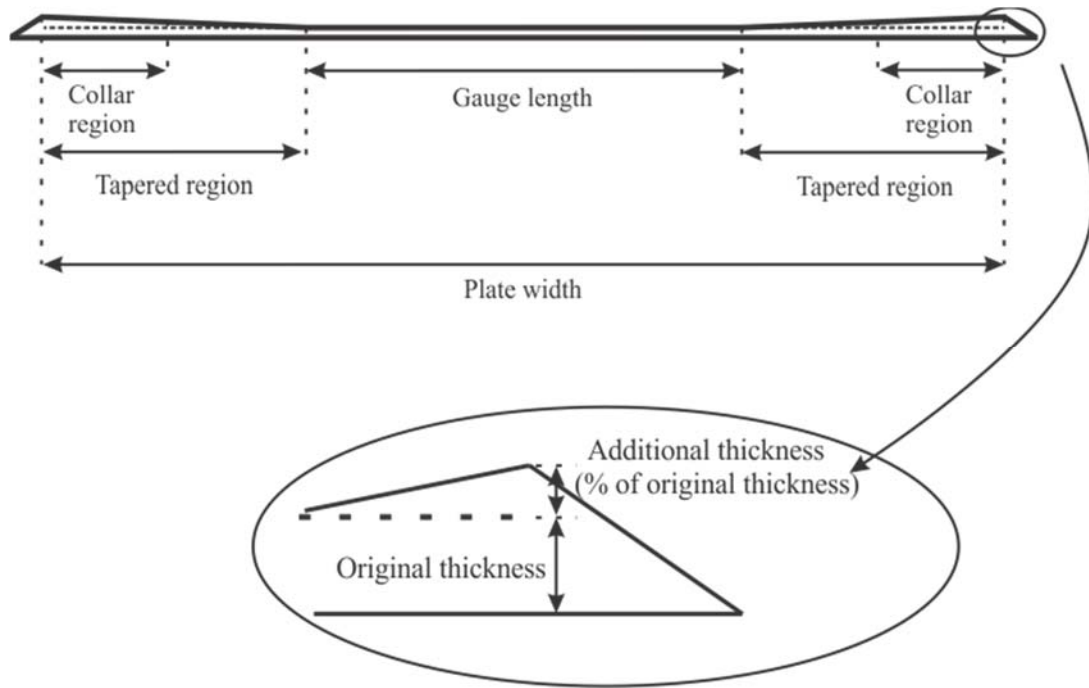


Figure 6. 6 Sectional elevation view of FEA plate model

Table 6. 1 Gauge length and additional thickness in tapered regions of models

Plate Geometry		
Model name	Gauge length	Additional thickness in tapered regions (%)
P00D-T03	3OD	3
P40D-T10	2OD	10
P80D-T03	3OD	3

The corresponding specimens are denoted P00D-T03, P40D-T10, P80D-T03 where the additional designation T10 and T03 are indicated the end tapered regions with an additional 10% and 3% increase in thickness respectively.

6.5 Material Properties

As discussed in Chapter 4, Chaboche model will be used for extracting parameters C^k and γ^k . In the next step, these values will be calibrated by curve fitting feature in Matlab software to obtain the calibrated Chaboche parameters (Chapter 4). As in the simple model descused in Chapter 5, the whole plate except collar was assigned by extracted stress-plastic strain data extracted from longitudinal tension test after forming process. The new extracted material is assumed to elasto-plastic response with kinematic hardening. The corresponding values for C_i and γ_i and the elastic properties for both materials are provided in.

6.6 Load Application

6.6.1 Application of Displacement Symmetry Condition Associated with Forming Process

For simulating the plate forming (U- and O-ing) process from position 1 to 4 in Figure 6.5, the nodes on top of the plate are chosen to undergo the displacement in the x and y directions. The forming process of the straight plate to bend plate occurs in ten steps by defining the coordinates of the nodes which are located on the curvature in each step. In addition, the nodes on the middle of the top surface are fully restrained during the forming process by using ENCASTRE syntax in Abaqus software (Figure 6.7).

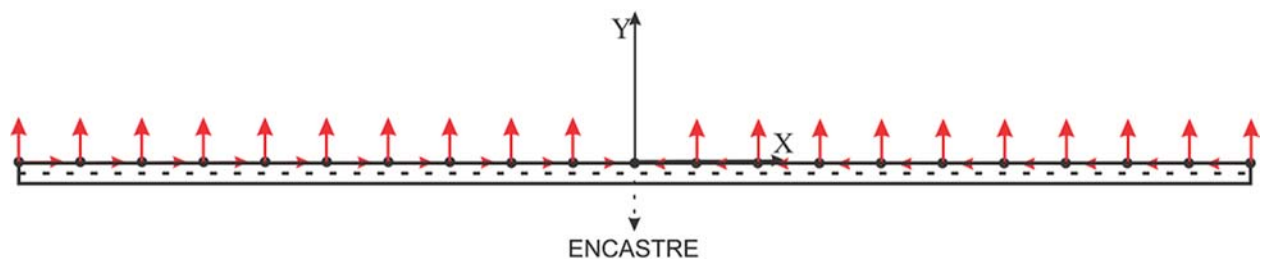


Figure 6. 7 Different Boundary conditions used in the forming process

Table 6. 2 Extracted stress-plastic strain from longitudinal tension test after forming process

Material property	Material	
	X52	Elastic
Young's modulus (MPa)	199,754	210,700
Poisson's ratio	0.3	0.3
Yield stress (MPa)	377.9	-
C_1 (MPa)	0.97	-
C_2 (MPa)	25.32	-
C_3 (MPa)	1253.42	-
C_4 (MPa)	225.84	-
C_5 (MPa)	2.82	-
C_6 (MPa)	118.59	-
C_7 (MPa)	140.35	-
C_8 (MPa)	0.03	-
C_9 (MPa)	73.45	-
C_{10} (MPa)	278.65	-
γ_1	1.96	-
γ_2	3.69	-
γ_3	20.75	-
γ_4	7.11	-
γ_5	3.00	-
γ_6	0.94	-
γ_7	0.55	-
γ_8	1.07	-
γ_9	1.53	-
γ_{10}	8.35	-

6.7 Constraint

In ABAQUS software, constraints are used to eliminate degrees of freedom, motion or displacement of a node or a group of the nodes to those of a master node or nodes. Sub-section 6.7.1 and 6.7.2 describe different types of constraints which can be used in ABAQUS software as linear or nonlinear problems.

6.7.1 Multi-Point Constraint Type Slider

One of constraints in ABAQUS software is called Multi-Point constraint (using *MPC keyword) which is used to impose constraints between degrees of freedom of a group of nodes (or a node) and a master nodes (or a master node). There are some types of Multi-point constraints in ABAQUS/Standard which can be used for different problems. In addition, the constraints could be obtained by defining EQUATION in ABAQUS software. The Multi-point constraint that is used in this simulation is multi-point constraint type Slider.

In this simulation, Multi-point constrain type slider is used to keep the nodes on the right and the left side of the plate on a straight line by two other nodes, but allows the possibility of moving along the line and also changing length of the line.

As shown in Figure 6. , two nodes are used as the master points which one of them is located on the mid-surface and the other one is on the top surface. The nodes on the right and left sides of the plate are restrained on a straight lines which are defined by two master points M1 and M2 on the right, and two mater points M3 and M4 on the left side respectively.

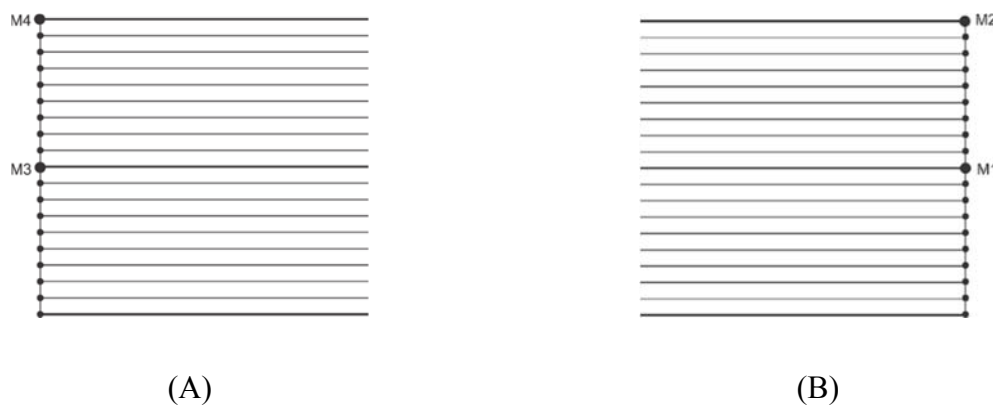


Figure 6. 8 Multi-point constrain with master points used on (A) right side of the plate and (B) left side of the plate

6.7.2 Tie Constraint

ABAQUS has different methods for defining welding constraint between bodies such as tie constraint and connector elements. The tie constraint is used when the parts of the bodies are fully constrained in displacement and rotation. This type of constraint is defined for joining two parts of mesh when the parts are fully connected.

In this simulation after the forming plate from plate to pipe, the weld connection between the surfaces on the right and the left sides of the plate is modeled by tying together all degrees of freedom of the nodes on these surfaces. The tie constraint is shown in Figure 6.9.

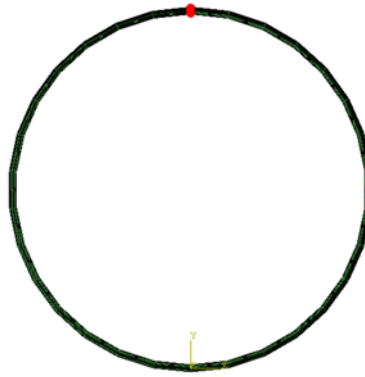


Figure 6. 9 Connections between nodes on top right and left sides of the pipe by Tie constraint

Chapter 7 Results and Discussion

7.1 General

The purpose of this chapter is to present the results obtained from the ABAQUS models. The results include the global and local moment-curvature and local buckling strains based on the simplified and detailed models explained in Chapters 5 and 6. Expressions for the global and local curvature of pipes are presented in Section 7.2. Expressions for the end rotation are provided in Section 7.3. Section 7.4 provides the plastic moment expression. Moment curvature relations for the idealized elastic response of pipes is represented in Section 7.5. Expressions for global and local end moment are provided in Section 7.6. Section 7.7 represents the global and local moment-curvature relationship for the simplified and detailed models. The deformed configurations of simplified and detailed models are provided in Section 7.8. Section 7.9 presents the buckling strains comparison between FEA results and those based on code equations.

7.2 Global and Local Curvature of Pipes

The curvature of a deformed pipe segment is dependent upon its length. Two curvatures are defined depending on the gage length adopted in the calculation; global and local curvatures. The global curvature of a pipe segment is calculated based on the whole pipe segment L_{a-b} (Figure 7. 1 (A)), while its local curvature is calculated based on specified gage length $L_{a'-b'}$ (Figure 7. 2 (A)) between two chosen end sections. As discussed in Chapter 2, a gage length of 2OD is taken for all simplified models and 2OD or 3OD are taken for the detailed models in Chapter 6. As shown in Figure 7. 1, the global curvature φ_G is the average curvature between the end segments of the pipe along the distance between these points L_{a-b} is called global curvature. It is given by

$$\varphi_G = \frac{\theta_b + \theta_a}{L_{a-b}} \quad (7.1)$$

where θ_a and θ_b are the end rotations. The local curvature φ_L is based on the selected gage length along the distance between these points $L_{a'-b'}$ and is given by

$$\varphi_L = \frac{\theta_{a'} + \theta_{b'}}{L_{a'-b'}} \quad (7.2)$$

where $\theta_{a'}$ and $\theta_{b'}$ are the rotations at the ends of the gage length (Figure 7. 2(B)).

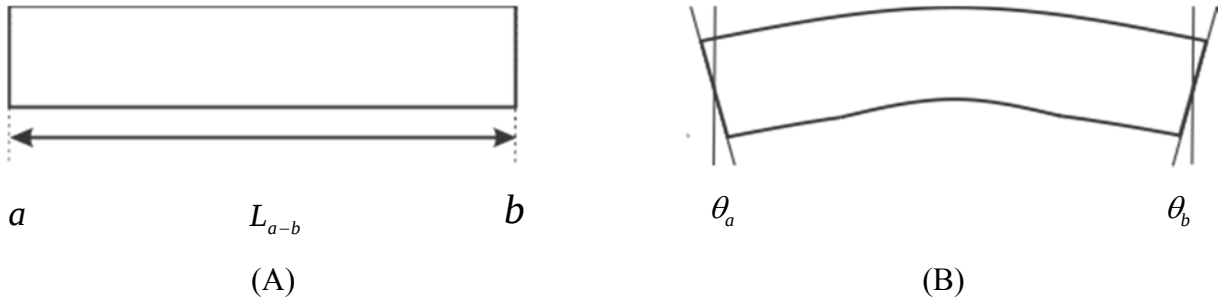


Figure 7. 1 Parameters used to determine the global curvature of pipe specimen (A) Un-deformed configuration, and (B) Deformed configuration

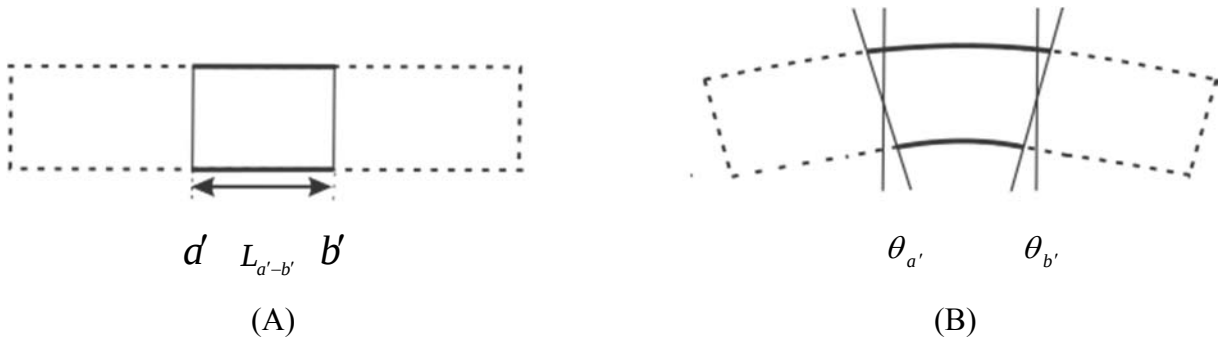


Figure 7. 2 Parameters used to determine the local curvature of pipe specimen (A) Un-deformed configuration, and (B) Deformed configuration

7.3 Rotation Calculation

As shown in Figure 7. 3 , at a section i of the specimen, the angle of rotation θ_i can be calculated from

$$\theta_i = \arctan \frac{(b_i^y - a_i^y)}{(b_i^z - a_i^z)} \quad (7.3)$$

where b_i^y and b_i^z are the deformed coordinates of point b along y and z directions and a_i^y and a_i^z are those of point a .

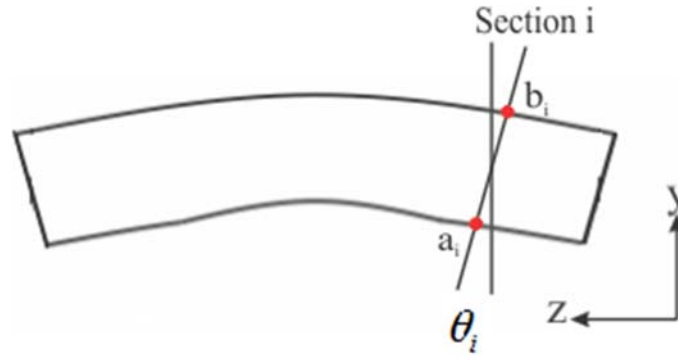


Figure 7. 3 Nodes selected for calculation of the rotation of section i

7.4 Plastic Moment Capacity

As discussed in Chapters 5 and 6, the magnitude of the internal pressure is kept constant throughout the application of end rotations/bending. Given the internal pressure and the corresponding hoop stresses σ_θ , and in the absence of axial load, i.e., $P_e = 0$, the modified plastic moment capacity

M_{p,σ_θ} modified for the presence of hoop stress can be related to the plastic moment capacity

$M_p = 4r_{av}^2 t \sigma_y$, in the absence of hoop stress (Mohareb et al. 1994) by

$$M_{p,\sigma_\theta} = M_p \sqrt{1 - \frac{3}{4} \left(\frac{\sigma_\theta}{F_y} \right)^2} \quad (7.4)$$

For the pipe investigated in the present study, the average radius is $r_{av} = 300.2 \text{ mm}$, the thickness is $t = 9.53 \text{ mm}$ and the yield strength is taken as $F_y = 377.8 \text{ (MPa)}$. The corresponding plastic moment in the absence of hoop stresses is found to be 1,298.9 and the plastic moments modified for the presence of hoop stresses are provided in Table 7.1. The modified plastic capacities listed

in Table 7.1 would be asymptotically approached from below if strain hardening effects are negligible and when local buckling does not take place.

Table 7. 1 Modified Plastic moment capacity for specimens investigated in the present study

Specimen designation	Hoop stress ratio σ_{θ}/F_y	Plastic moment capacity $M_{p,\sigma_{\theta}} (KNm)$
P00S-T10, P00D-T03	0.00	1298.9
P40S-T10, P40D-T10	0.40	1150.8
P80S-T10, P80D-T03	0.80	603.0

7.5 Elastic Response

Assuming elastic response, the initial moment M is related to the curvature by

$$M = EI\varphi \quad (7.5)$$

where $E = 199.7 \text{ GPa}$ is the Young modulus, $\varphi = \varphi_L = \varphi_g$ is the curvature of the pipe (no distinction is made between the local and global curvatures prior to buckling) and the moment of inertia of the pipe cross section is $I = \pi/4(OR^4 - IR^4) = 8.91 \times 10^{-4} \text{ mm}^4$, and $OR = 305.0 \text{ mm}$ and $IR = 295.5 \text{ mm}$ are outer and inner radii of the pipe. In the initial range of the deformation, the pipe response is expected to be elastic and the slope of the moment-curvature of the pipe is expected to approach that predicted by Eq.(7.5).

7.6 Calculation of Moments

At a given point on the equilibrium path, the global and local moments can be calculated by multiplying extracted nodal forces on the right or left pipe cross section by the distance from the relative axis as described in the following procedure

First the coordinates $(x_{c.g.}, y_{c.g.}, z_{c.g.})$ of the cross section centroid of the deformed pipe. These are given by

$$\begin{aligned} x_{c.g.} &= \sum_{i=1}^n \frac{x_i}{n} \\ y_{c.g.} &= \sum_{i=1}^n \frac{y_i}{n} \\ z_{c.g.} &= \sum_{i=1}^n \frac{z_i}{n} \end{aligned} \quad (7.6)$$

where n is number of nodes at the cross section considered and, x_i, y_i and z_i are the deformed coordinates of node i (Figure 7. 4).

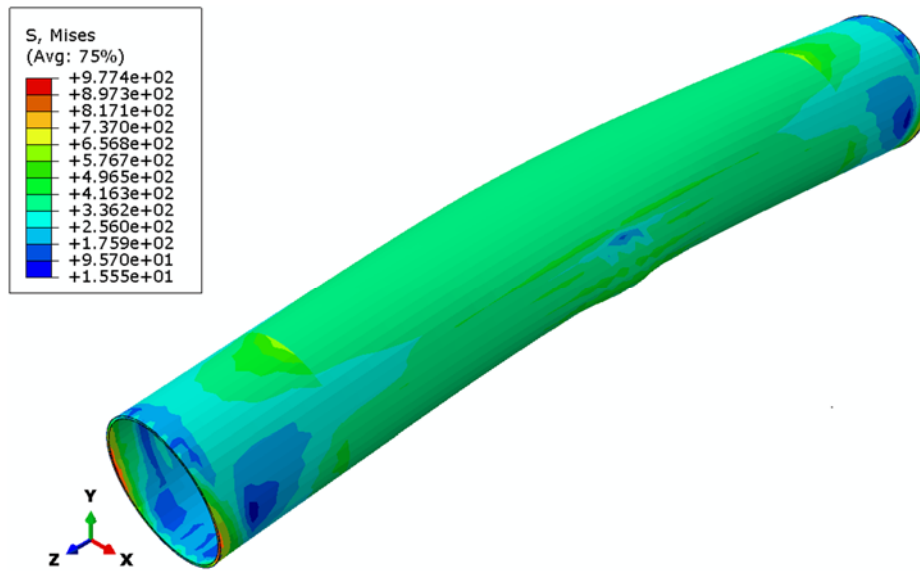


Figure 7. 4 Isometric view of the deformed pipe

The total moment M at the end right or left cross section is then obtained from

$$M = \sum_{i=1}^n \left[F_i^z (y_i - y_{c.g.}) + R_n^z (y_i - y_{c.g.}) + R_n^y (z_i - z_{c.g.}) \right] \quad (7.7)$$

where F_i^z is the nodal force of node $i(i=1,2..n)$ in z direction and R_n^x, R_n^y are the reaction forces along the x and y directions.

7.7 Moment-Curvature Relationships

This section provides a comparison between global and local moment-curvature, plastic moment capacity and initial moment response for the un-pressurized and pressurized pipes. The results based on the simplified model are provided in sub-section 7.7.1 and those based on the detailed models are provided in sub-section 7.7.2.

7.7.1 Simplified Model Results

The global curvatures are calculated based on Eq.(7.1) and the corresponding global moments are calculated from Eq. (7.7) by taking a gage length equal to the pipe length (Figure 7. 5) based on the results of the simplified models. Similar calculations are performed based on a gage length of $2D$ which intercepts the local buckle. The results are depicted in Figure 7. 6. Overlaid on the figure are the modified plastic moments as given in Table 7.1. Also, overlaid for comparison is the moment-curvature based on an elastic response as determined from Eq. (7.5).

As expected, the initial slope of the moment curvature relation for all three specimens matches that of the elastic response. As the curvature increases gradual plastification of the pipe takes place the pipe exhibits a softening behaviour. This is manifested by the departure of the moment curvature slope from the elastic slope. For large deformations, all specimens are observed to approach the modified plastic moment depicted by the horizontal lines. When the deformation increases, local buckling takes place in the pipe wall, the pipe loses its capacity. This is exhibited by the descending branch of moment curvature. The behaviour is characteristic of experimental and computational results in past studies (e.g., Istemi F. Ozkan 2008).

It is noted that the gage length has little effect on the moment magnitudes but greatly influences the curvature. This is evidenced by the significantly larger curvatures observed. For this reason, most studies have adopted a unified gage length of $2OD$ when reporting curvature and buckling strain results.

It is also noted that high internal pressure for specimen P80S-T10 delays local buckling, which helps the pipe material getting well into the strain hardening range prior exhibiting local buckling.

In this case, the pipe attains a peak moment slightly higher than the modified plastic moment calculated by Eq. (7.4). This is not the case for the lower pressure specimens P00S-T10 and P40S-T10 where local buckling initiates earlier and thus are unable to attain the modified plastic moment capacity.

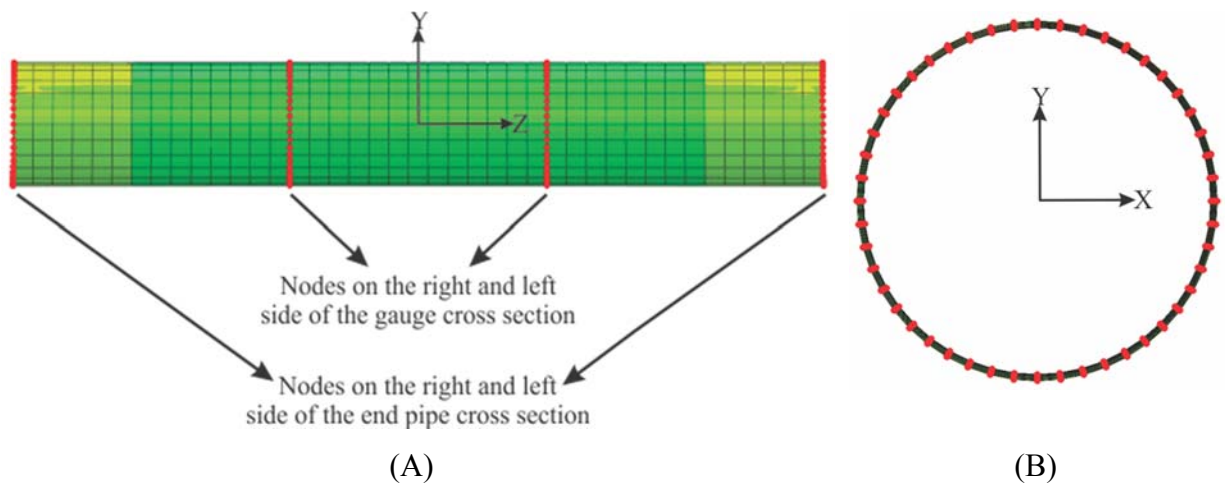


Figure 7. 5 Sections at ends of local and global gage lengths (A) Elevation and (B) Cross-sectional view

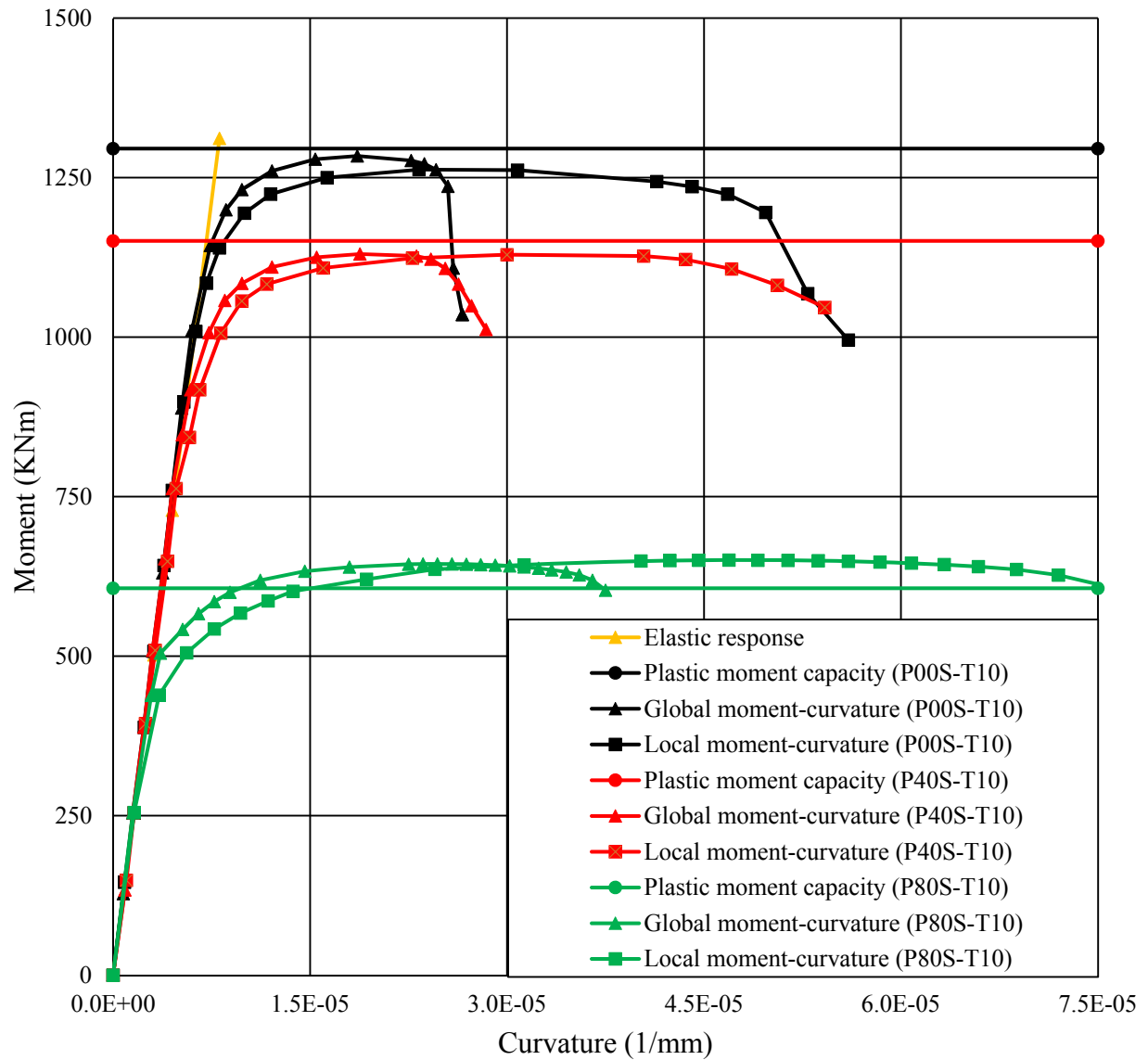


Figure 7. 6 Moment-curvature based on simplified models

7.7.2 Detailed Model Results

The global curvatures are calculated based on Eq.(7.1) and the corresponding global moments are calculated from Eq. (7.7) by taking a gage length equal to the pipe length (Figure 7. 5) based on the results of the detailed models. Similar calculations are performed based on gage lengths of 2D. Exceptions were made for models P00DT03 and P80D-T03 where a gage length of 3D was chosen in order to fully intercept the local buckling within this length. The selected gage length for the simplified and detailed models are shown in Table 7. 2.

Table 7. 2 Selected gage length for the simplified and detailed models

Model Designation	Gage length
P00S-T10	2D
P40S-T10	2D
P80S-T10	2D
P00D-T03	3D
P40D-T10	2D
P80D-T03	3D

The results are depicted in Figure 7. 7 .Overlaid on the figure are the modified plastic moments as given in Table 7.1. Also, overlaid for comparison is the moment-curvature based on an elastic response as determined from Eq. (7.5).

Similar observations can be made as for the simplified model regarding the general trend of the moment curvature relations. Unlike the detailed model, the initial slopes of the moment curvature curves are milder compared to the elastic moment curvature relation. This is an outcome of the fact that the forming process, which was only captured in the detailed model, deforms the pipe well into the plastic range and thus induces significant residual stresses. The peak moments attained exhibit some differences from those attained in the case of the simplified models. Table 7. 3 provides a comparison of the peak moments. For the un-pressurized pipe, it is observed that the simplified model overestimates the peak moment compared to the detailed model while for the pressurized pipes, the peak moments based on both models are in close agreement.

Table 7. 3 Comparison of peak moments

Internal pressure	(1) Modified plastic moment (MPa)	(2) Peak moment based on simplified model (MPa)	(3) Peak moment based on detailed model (MPa)	(2)/(1)	(2)/(3)
0.00	1295.4	1262.6	1180.7	0.97	1.07
0.40F _y	1150.8	1129.1	1148.1	0.98	0.98
0.80F _y	606.2	650.4	605.7	1.07	1.07

7.7.3 Comparison of Simplified model and detailed model results

A comparison between simplified and detailed local moment-curvature curves is shown in Figure 7. 8 which illustrates the effect of the forming process on moment-curvature curves. In the simplified models P00S-T10, P40S-T10 and P80S-T10, the occurrence of local buckling is delayed in comparison of to the detailed models P00D-T03, P40D-T10 and P80D-T03.

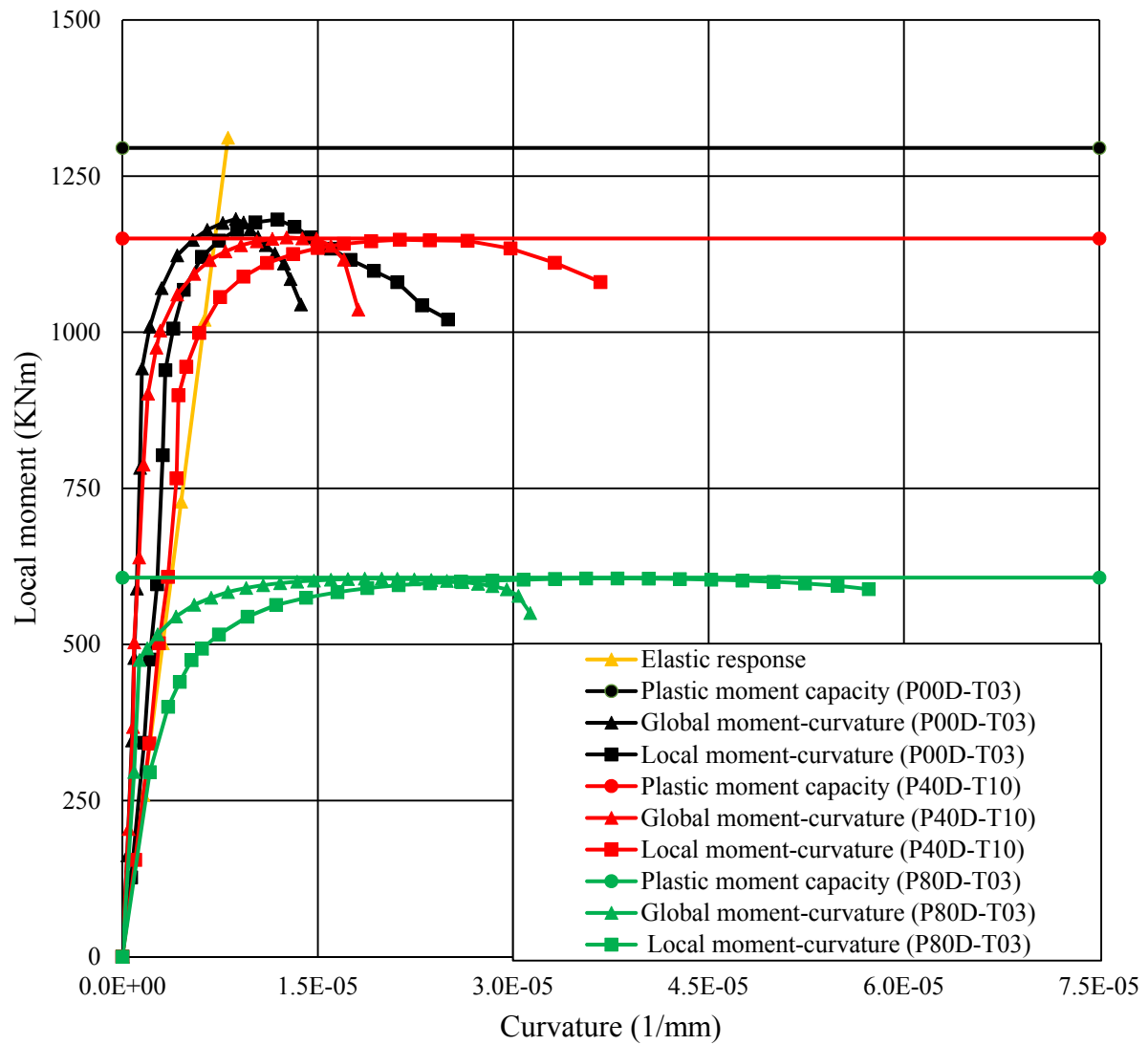


Figure 7. 7 Moment-curvature based on detailed models

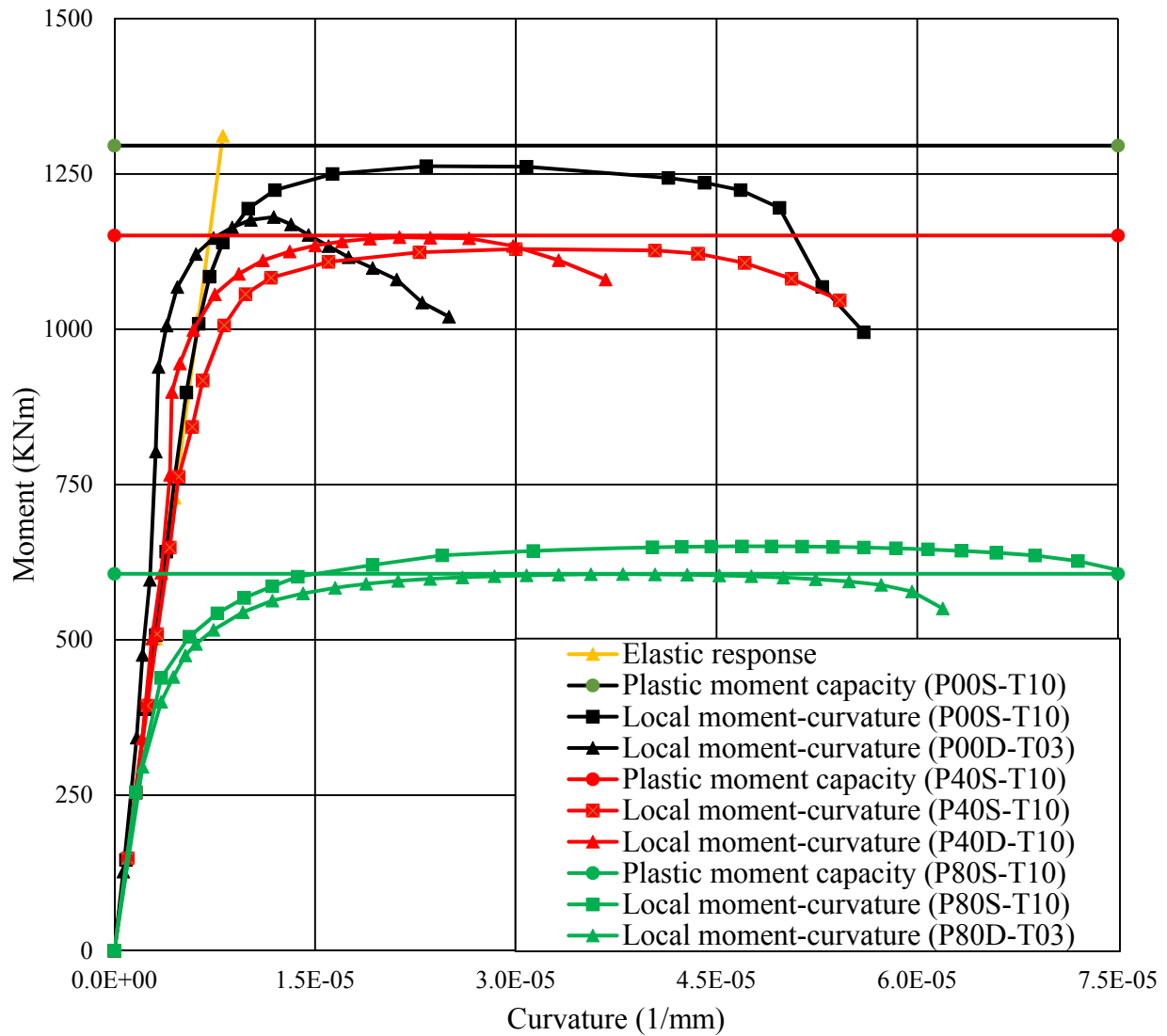


Figure 7. 8 Comparison of local moment-curvature based the simplified and detailed models

7.8 Deformed Configuration

Figure 7. 9 depicts the deformed contour plots for the Mises stresses at peak moments and at the peak moment values for both simplified and detail models and the ones at 95% of the peak moment values on the descending branch are shown in Figure 7. 10. It is observed that the buckling in all specimens away from the ends. The buckle in un-pressurized simplified models is symmetric with respect to a plane passing through the mid-height, while in 40% and 80% pressurized models buckling is offset from the centre. In the un-pressurized detailed model, the buckles deformation is symmetric as in the simplified models and occurs in the mid-span. The specimens with 40% and 80% pressure are observed to be offset from midspan.

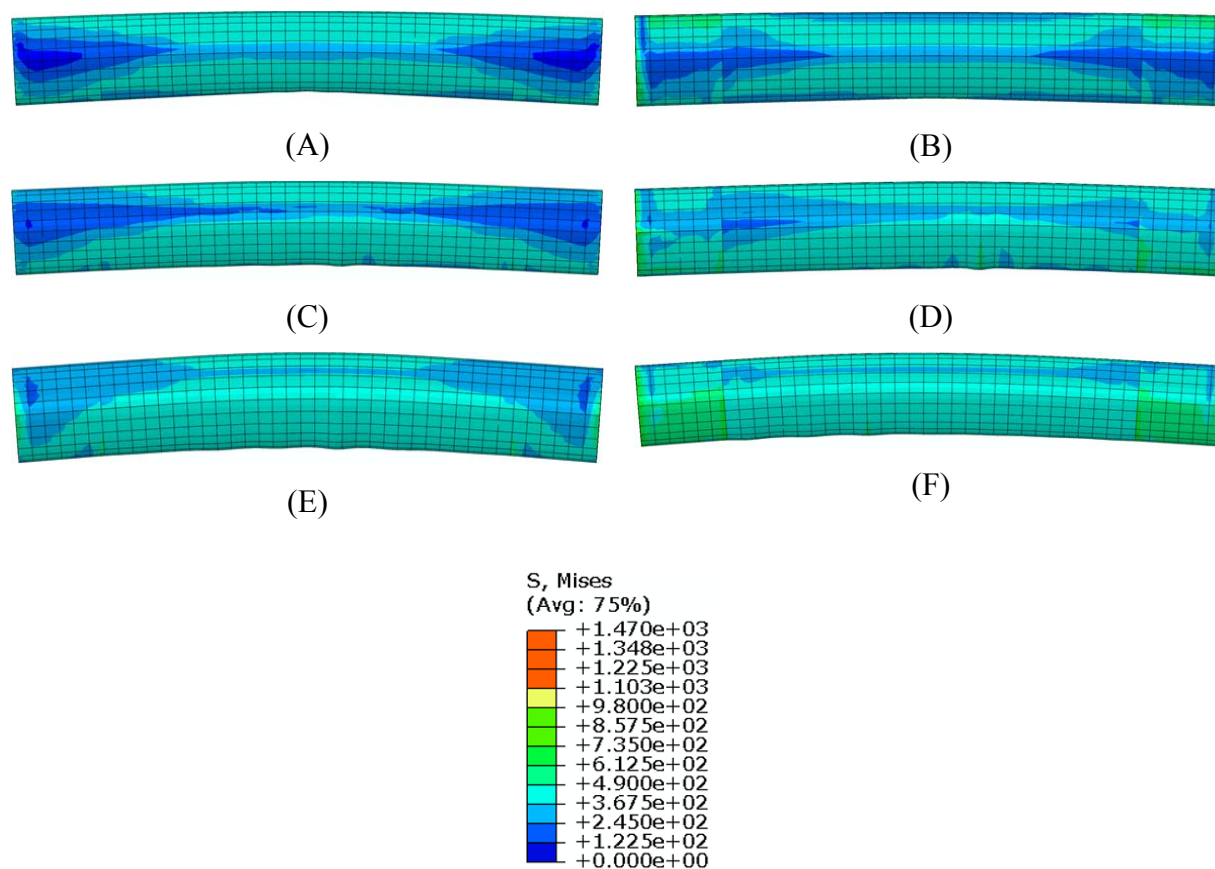


Figure 7. 9 Buckling deformation shapes in the peak moment value for (A) P00S-T10, (B) P00D-T03, (C) P40S-T10, (D) P40D-T10, (E) P80S-T10 and (F) P80D-T03 models

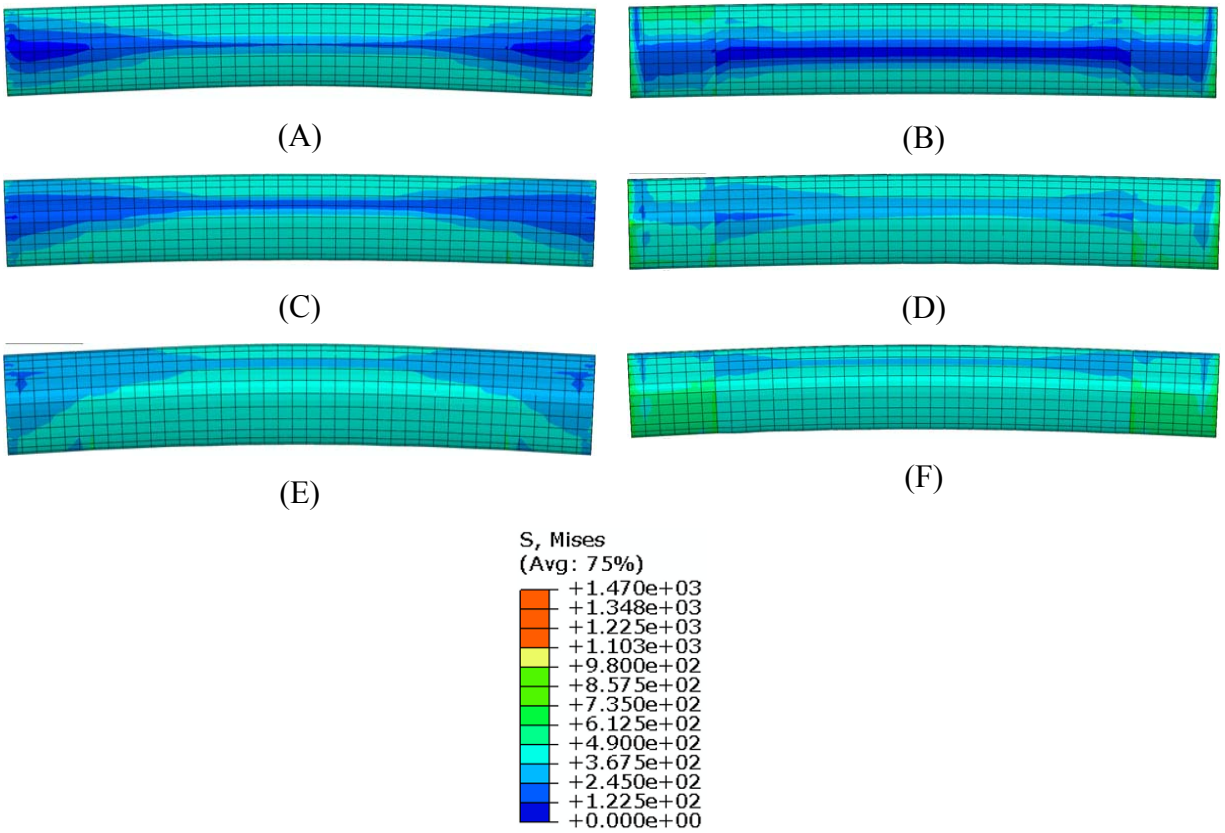


Figure 7. 10 Buckling deformation shapes in the 95% of the peak moment on the descending branch for (A) P00S-T10, (B) P00D-T03, (C) P40S-T10, (D) P40D-T10, (E) P80S-T10 and (F) P80D-T03 models

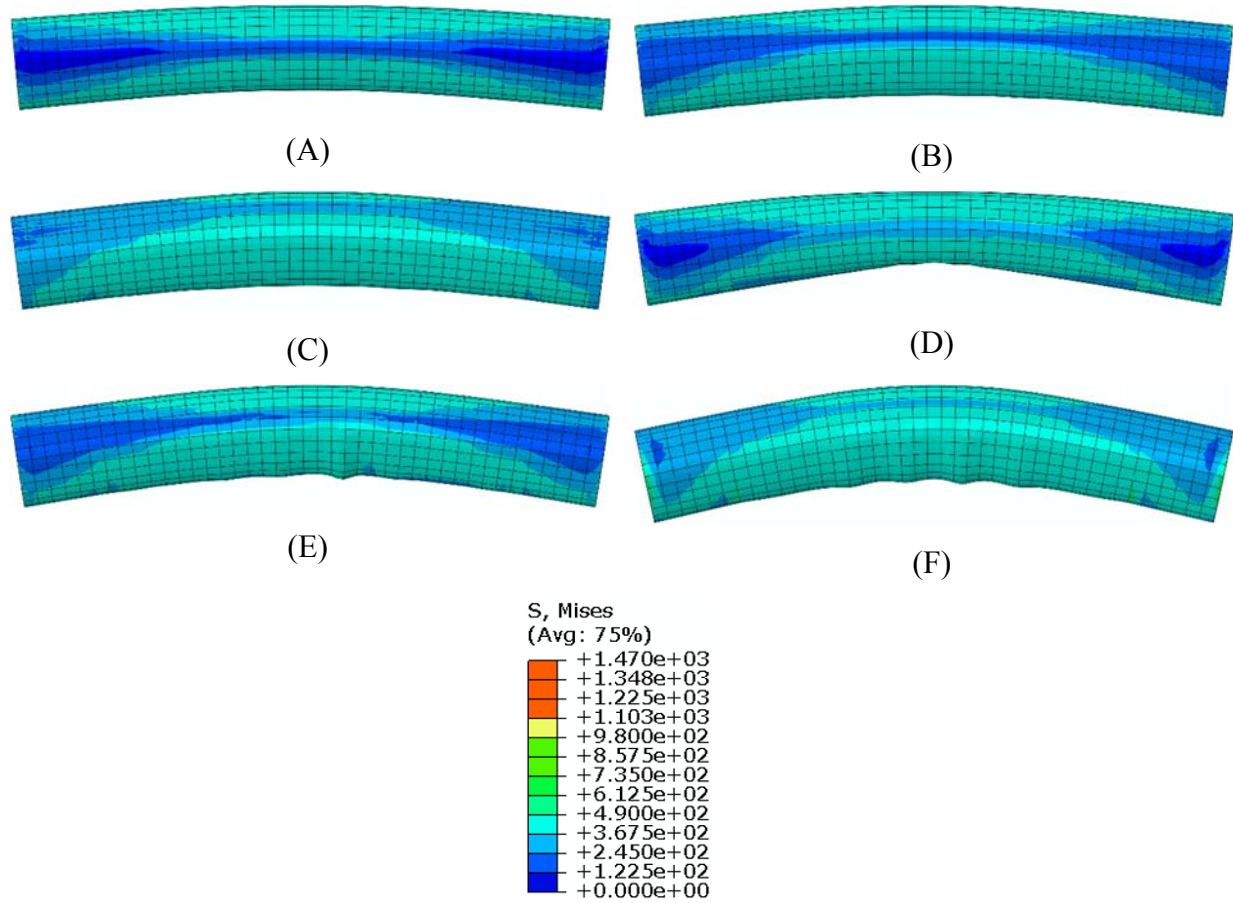


Figure 7. 11 Buckling deformation shapes in the peak value for (A) P00S-T10, (B) P40S-T10, (C) P80S-T10, and in the 95% of the peak moment on the descending branch for (D) P00S-T10, (E) P40S-T10, (F) P80S-T10 models (Scale factor = 3)

7.9 Local Buckling Strain comparison between FEM Results and Code

As shown in Figure 7. 12, local buckling strain can be obtained as the average of strain on the compression side of the gage. The local buckling strain is based computed from the finite element model is

$$\varepsilon_{cr} = \frac{L_i - L}{L_i} \quad (7.8)$$

where L_i is the initial gage length and L is the deformed gage length at the point of peak moment moments and at a moment of 0.95% of the peak value on the descending branch.

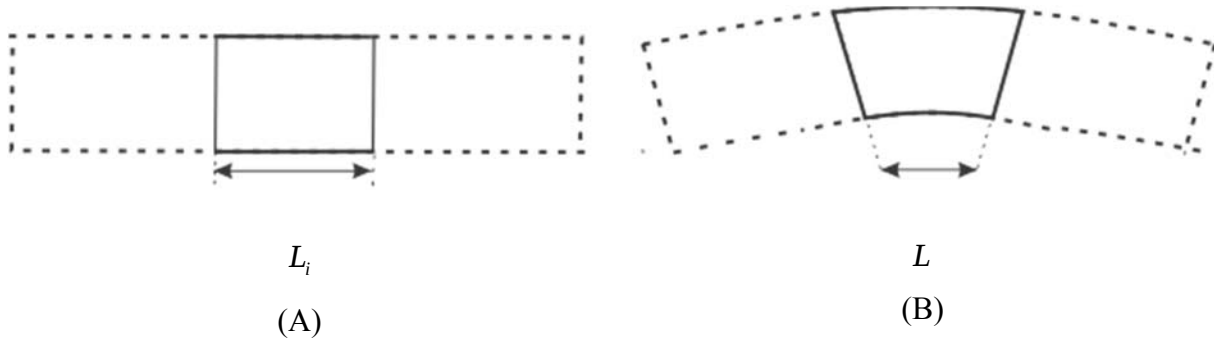


Figure 7. 12 Length of (A) Un-deformed and (B) deformed selected gage

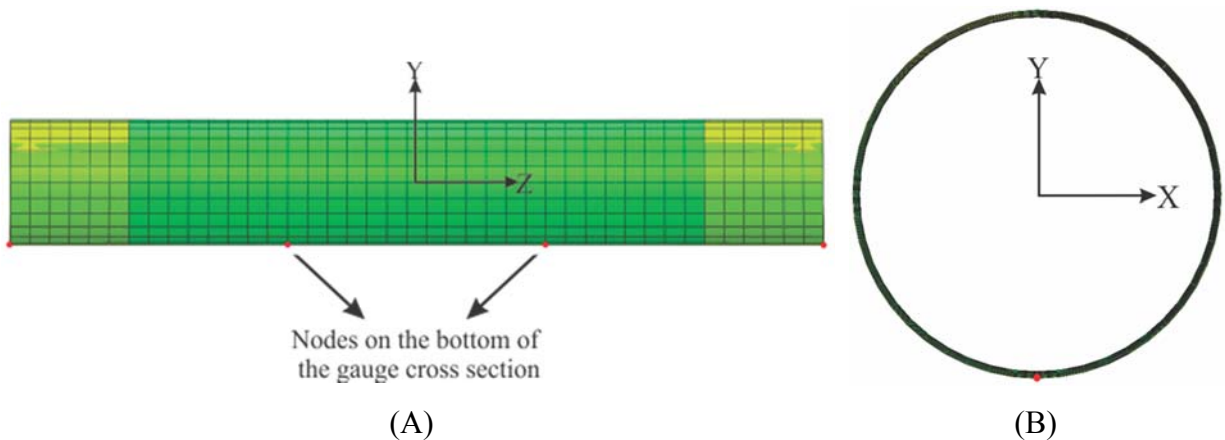


Figure 7. 13 Nodes on the right and left sides of the gage cross section (A) Front and (B) side views

As shown in Figure 7. 13, the nodes on the bottom of the gage cross section are selected to calculate the buckling strain in the finite element models. The peak moment in Figure 7. 14 are marked by solid circles and the ones by the value of 0.95% of the peak value are outlined by the dotted circles. A comparison between the simplified and detailed (Figure 7. 14) illustrate that the buckling strains based on the simplified model are larger than those based on the detailed model. The obtained values of the buckling strains corresponding to the peak and 0.95 of peak values of the moment are shown in Table 7. 4.

As discussed in Chapter 2, the compressive buckling strains based on CAN-CSA Z662 design standard equation are

$$\varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left[\frac{(p_{i \max} - p_{e \min}) D}{2tE} \right]^2 \quad \text{for} \quad \frac{(p_{i \max} - p_{e \min}) D}{2tF_y} < 0.4 \quad (7.9)$$

$$\varepsilon_{cr} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left(\frac{0.4F_y}{E} \right)^2 \quad \text{for} \quad \frac{(p_{i \max} - p_{e \min}) D}{2tF_y} \geq 0.4 \quad (7.10)$$

The comparison between the calculated buckling strains by Z662 design standard (CAN/CSA Z662) and FEM results in Figure 7. 14 are shown in Table 7. 4. In a manner consistent with past studies, buckling strain predictions tend to overestimate the buckling strains compared to code equations while the buckling strain values based on the present detailed model are significantly less than those of the simplified model but a larger than code predictions.

Table 7. 4 Comparison between buckling strain Z662 design standard and FEM results

Buckling strains		$P = 0\% F_y$	$P = 40\% F_y$	$P = 80\% F_y$
Z662 Design Standard		0.53 %	0.70 %	0.70 %
Peak moment criteria	Simplified	0.76 %	1.1 %	2.3 %
	Detailed	0.53 %	1.0 %	1.8 %
0.95 % of peak moment value	Simplified	1.8 %	2.0 %	4.2 %
	Detailed	0.84 %	1.4 %	2.8 %

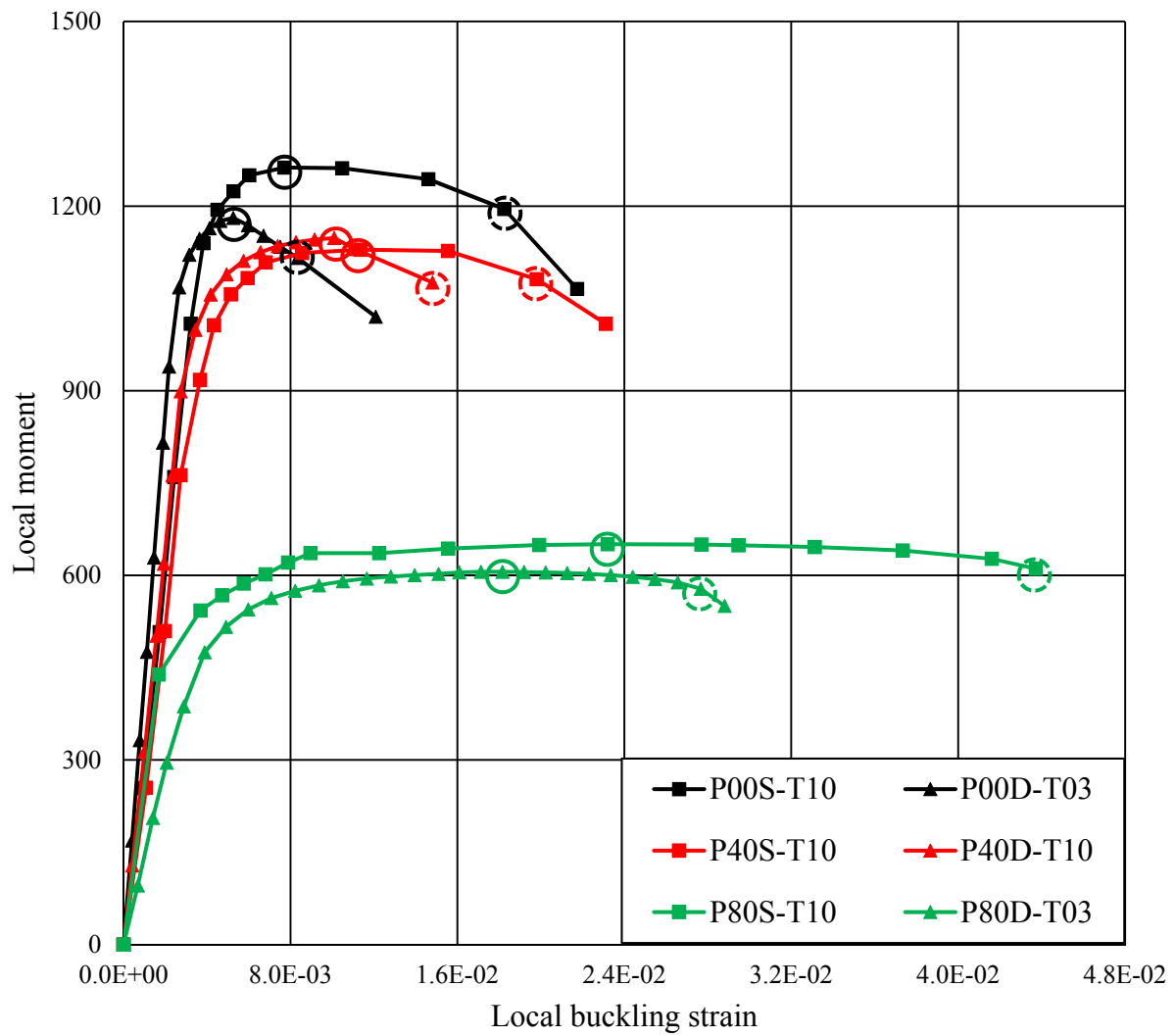


Figure 7. 14 The comparison between the simplified and detailed local moment-buckling strain

Chapter 8 Summary and Conclusions

8.1 Summary of Research

The following is a summary of the research conducted:

- 1- The UOE forming process has been successfully modelled using 3D FEA modeling. The model is able to predict the residual stresses induced in the pipe wall throughout the forming process.
- 2- The UOE forming model is subsequently used to conduct FEA simulations, one to predict the pure tensile response in the longitudinal direction in a manner similar to pure tension tests commonly conducted on coupons, and the other one to predict the pure tensile response in the hoop direction, in a manner to simulate a ring expansion test.
- 3- The UOE forming model was then adopted to conduct an analysis to predict the buckling strains for three pipe specimens; pressurized, half-pressurized and fully pressurized.
- 4- Simplified models were conducted for the three pipe specimens in a manner that is consistent with past studies. In the simplified model, no attempts were made to capture the residual stresses induced in the UOE forming process.
- 5- A comparison was conducted between the predicted buckling strains based on the simplified and those based on Canadian design standards.

8.2 Conclusions and Recommendations

The main findings of the studies are:

- 1- A 3D FEA was developed to model the UOE forming process and predicting buckling strains was developed. The model is based on an elasto-plastic constitutive model hardening based on the Chaboche kinematic hardening.
- 2- The apparent true stress-strain relations for the curved pipe are found to differ from the stress-strain relations for the flat steel plate.
- 3- For curved pipe, the FEA-predicted apparent hoop stress-strain relation is found to have a higher apparent yielding point and a milder strain hardening than those based on the FEA-

predicted apparent longitudinal stress-strain relationship. This observation is consistent with past experimental results based on coupon tension tests.

- 4- Buckling strain predictions based on the detailed model which capture the effects of UOE forming are found to be significantly less than those based on the simplified model which omits the effects of UOE forming. The finding is consistent with the fact past experimental results have shown that experimentally obtained critical strains are consistently lower than those predicted by FEA models which do not model the UOE forming process. The critical strains predicted by the FEA model are closer to those predicted by present design standard CAN-CSA-Z662.
- 5- The detailed modelling approach developed in the present study is believed to provide more accurate predictions of buckling strains in pipes than other modelling techniques which omit the residual stresses in the UOE forming process. It is recommended to apply the proposed technique to pipe geometries that have been tested to further assess the accuracy of the predictions of buckling strains in pipes based on technique developed.
- 6- The detailed modelling approach developed in the present study offers a promising numerical low cost approach to extend the existing database of experimentally obtained critical strains. It is recommended to adopt the new technique to predict critical strains for pipe configurations, steel grades, loading conditions, etc. that have not been experimentally tested.

Appendix A Mesh Sensitivity Analysis

A mesh sensitivity analysis is conducted for the simplified bending model in Chapter 5. Three mesh sizes are investigated for the unpressurized and fully pressurized cases. The number of elements taken is illustrated in Table A.1. The corresponding deformed plots are provided in Figure A.1 and the local moments local curvature plots are provided in Figure A.2 for the un-pressurized pipe and in Figure A.3 in for the pressurized pipes. In both cases, the $65 \times 26 \times 12$ and $41 \times 24 \times 10$ meshes provide nearly identical results. Both results are slightly different from those based on $23 \times 22 \times 8$ mesh. It is judged that the $41 \times 24 \times 10$ mesh is most appropriate for predicting the moment curvature relations and associated buckling modes and it was adopted for the simplified model in Chapter 5 and the detailed model in Chapter 6.

Table A.1 Mesh sensitivity study

Models		Number of elements in		
		Longitudinal direction	Circumferential direction	Thickness direction
Unpressurised	$23 \times 22 \times 8$	23	22	8
	$41 \times 24 \times 10$	41	24	10
	$65 \times 26 \times 12$	65	26	12
Pressurised	$23 \times 22 \times 8 (P)$	23	22	8
	$41 \times 24 \times 10 (P)$	41	24	10
	$65 \times 26 \times 12 (P)$	65	26	12

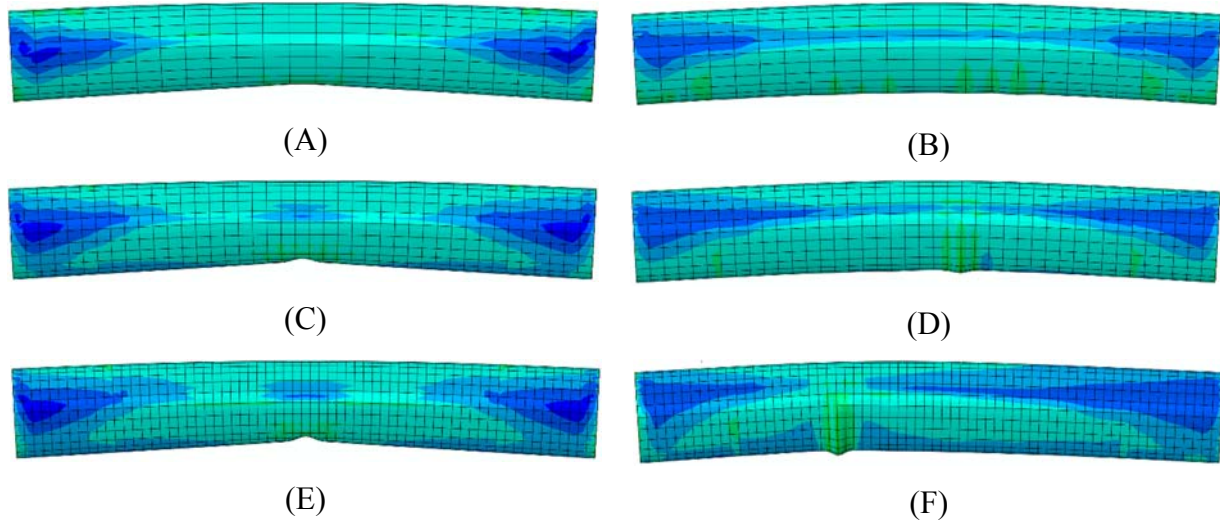


Figure A.1 Mesh sensitivity analysis for the unpressurised and pressurised models (A) $23 \times 22 \times 8$, (B) $41 \times 24 \times 10$, (C) $65 \times 26 \times 12$, (D) $23 \times 22 \times 8$ (P), (E) $41 \times 24 \times 10$ (P) and (F) $65 \times 26 \times 12$ (P)

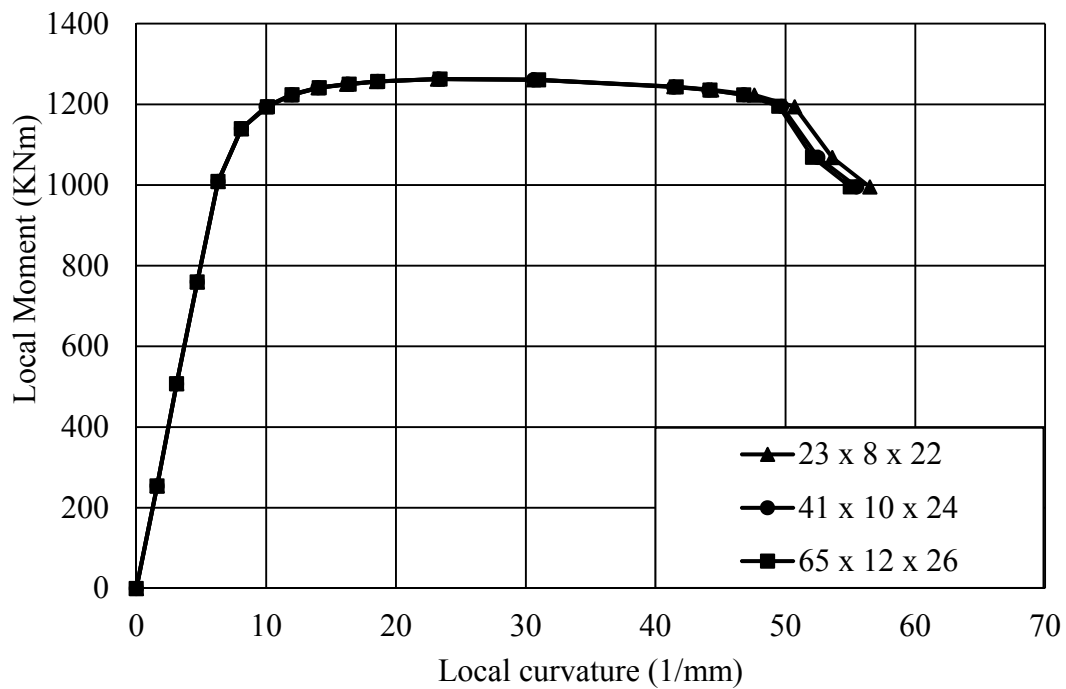


Figure A.2 Mesh sensitivity study for unpressurised simplified model

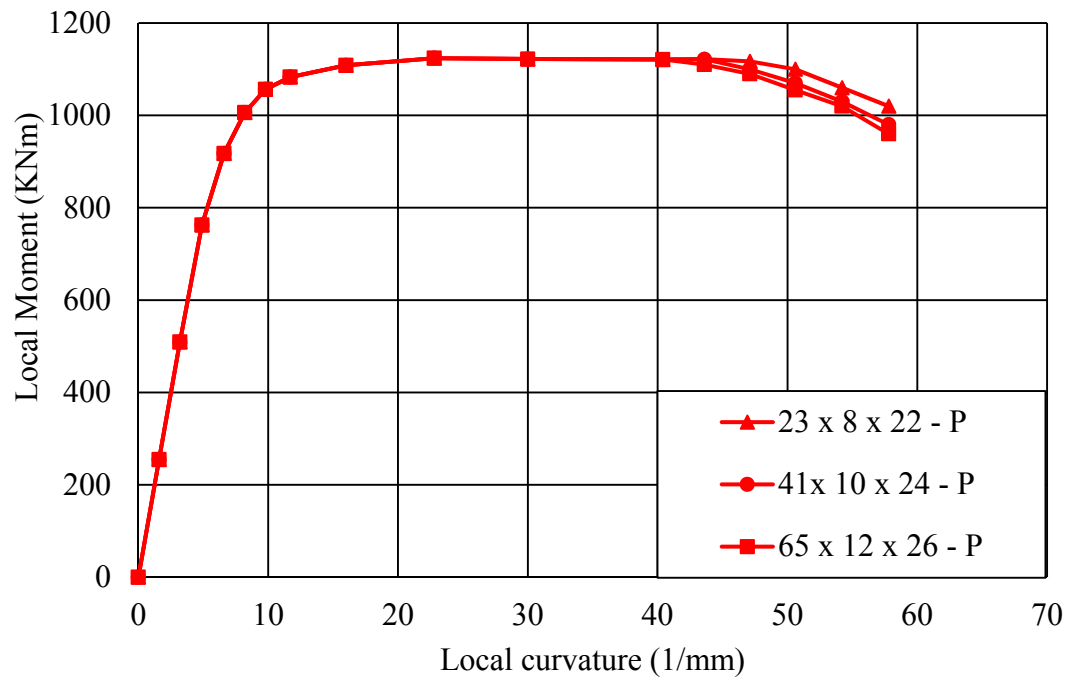


Figure A.3 Mesh sensitivity for fully pressurised simplified model

References

- Adeeb, S., Zhou, J., & Horsley, D. (2006). " Investigating the effect of UOE forming process on the buckling of line pipes using finite element modeling." International Pipeline Conference, ASME, Calgary, Canada pp. 169-174).
- Araujo, M. C. (2002)." Non-linear kinematic hardening model for multiaxial cyclic plasticity." Louisiana State University and Agricultural and Mechanical College, MASc thesis
- Batterman, S. C., (1965). "Plastic Buckling of Axially Compressed Cylindrical Shells." AIAA Journal, Vol.3, No.2, pp.316-325.
- Bouwkamp, J.G., & Stephen, R.M. (1973). "Large diameter pipe under combined loading", Transp. Eng. J., ASCE, 99(3), 521-536.
- Bower, Allan F; (2009) Applied Mechanics of Solids, CRC press.
- CAN/CSA Z662-15 (2015). "Oil and Gas Pipeline Systems." Canadian Standards Association, Rexdale, Ont.
- Canadian Energy Pipeline Association (CEPA), accessed 20 December 2015, Liquid pipelines, <http://www.cepa.com/about-pipelines/types-of-pipelines/liquids-pipelines>.
- Canadian Energy Pipeline Association (CEPA), accessed 20 December 2015, Natural gas pipelines, <http://www.cepa.com/about-pipelines/types-of-pipelines/natural-gas-pipelines>.
- Canadian Energy Pipeline Association (CEPA), accessed 20 December 2015, Types of pipelines, <http://www.cepa.com/about-pipelines/types-of-pipelines>.
- Chaboche JL & Nouailhas DD. " Constitutive Modeling of Ratchetting Effects—Part I: Experimental Facts and Properties of the Classical Models." ASME. J. Eng. Mater. Technol. 1989; 111(4):384-392. doi:10.1115/1.3226484.
- Chakrabarty. J., Applied Plasticity, Springer-Verlag, New York, 2000, ISBN 978-1-4757-3268-9 (eBook).

- Chen, H., Ji, L., Gong, S., & Gao, H. (2008). "Deformation Behavior Prediction of X80 Steel Line Pipe and Implication on High Strain Pipe Specification." 7th International Pipeline Conference (IPC2008), ASME, Calgary, Alberta, Canada, pp. 763-768.
- Chen, W. F. & Han, D. J. (1988). "Plasticity for Structural Engineers, Springer-Verlag.", New York, ISBN-13: 978-1-4612-8380-5.
- Cho, W.Y., Seo, D.H., & Yoo, J.Y. (2009). "Compressive strain limit of aged API_X100 linepipe." Proc. of International Pipeline Conference (IPC2009), ASME, Honolulu, Hawaii, USA, May 31-June 5, 2009, pp. 41-47.
- Chou, Z. L., Cheng, J. J. R, & Zhou, J. (2006). "Monitoring and prediction of pipe wrinkling using distributed strain sensors." In 2006 6th International Pipeline Conference (IPC2006), ASME, Calgary, Alberta, Canada, Sep.25-29, 2006, pp. 307-315.
- Cook, R. D., Malkus, D. S. and Plesha, M. E. (2002). "Concepts and Applications of Finite Element Analysis." 4th Edition, John Wiley & Sons, New York, pp 530-587.
- Corona, E., & Fischer, F. J. (1994). "On the effect of the UOE manufacturing process on the collapse pressure of long tubes." Journal of engineering for industry, 116, 93.
- Crisfield, M. A. (1991). "Non-linear Finite Element Analysis of Solids and Structures. Volume 1: Essentials", John Wiley & Sons, Chichester, pp 77-80,131-132, 211-220.
- DelCol, P.R., Grondin, G Y., Chen, J. J. R, & Murray, D. W. (1998). "Critical buckling strains in energy pipelines." Struct. Eng. Rep. No. 224, Dept. of Civ. Eng., University of Alberta, Edmonton, Alberta, Canada.
- Det Norske Veritas, DNV-OS-F101. (2000). "Rules for Submarine Pipelines." Oslo, Norway.
- Dorey AB, Murray DW, Cheng JJ. (2005). "Critical Buckling Strain Equations for Energy Pipelines—A Parametric Study." ASME. J. Offshore Mech. Arct. Eng.2005; 128 (3):248-255. doi:10.1115/1.2199561.
- Dorey, A. B., Chen, J. J. R, & Murray, D. W. (2001) "Critical buckling strains for energy pipelines." Structural Engineering Report No. 237, Dept. of Civil Engineering, University of Alberta, Edmonton, Alberta, Canada.

- Dorey, A.B., Murray, D.W., & Cheng, J.J. (2006). "Critical buckling strain equations for energy pipelines—A Parametric Study.", *Journal of Offshore Mechanics and Arctic Engineering*, 128(3), 248-255.
- Dorey, A.B., Murray, D.W., Cheng, J.J.R. (2002). "Material property effects on critical buckling strains in energy pipelines." *Proc. of IPC'02, 4th Int. Pipeline Conf. ASME*, Calgary, Alberta, Sep. 3-Oct. 29, pp 475-484.
- Dunne, F. & Petrinic. N., *Introduction to Computational Plasticity*, Oxford University Press, New York, 2005, ISBN 0-19-856826-6 (Hbk).
- Ellinas, C. P. (1984). "Ultimate Strength of Damaged Tubular Bracing Members." *J. of the Structural Division, ASCE*, 110(2), 245-259.
- Ellinas, C. P., Williams, K. A. J. and Walker, A. C. (1987). "Collision Damage Effects in Floating Platforms." In *Mobile Offshore Structures* (Eds L. F. Boswell, C. A. D'Mello & A. J. Edwards), Elsevier Applied Science, London, U. K., 477-498.
- Fatemi, A., Kenny, S., Sen, M., Zhou, J., Taheri, F, & Paulin, M. (2008). "Investigations on the local buckling response of high strength linepipe." In *2008 7th International Pipeline Conference (IPC2008)*, ASME, Calgary, Alberta, Canada, Sep.29- Oct.3, 2008, pp. 649-656.
- Fatemi, A., Kenny, S., Sen, M., Zhou, J., Taheri, F, & Paulin, M. (2009). "Parameters affecting the buckling and post-buckling behaviour of high strength pipelines." In *2009 28th International Conference on Ocean, Offshore and Arctic Engineering (OMAE2009)*. ASME, Honolulu, Hawaii, USA, May 31-June 5, 2007, pp. 501-509.
- Fathi, A, & Cheng, J. R. (2012). "A Parametric Study on Buckling Response of High Strength Steel Pipes With Anisotropic Material Properties." In *2012 9th International Pipeline Conference (IPC2012)*, ASME, Calgary, Alberta, Canada, Sep.24-28, 2012, pp. 337-345.
- Fathi, A. (2012). "Effects of material anisotropy on the buckling resistance of high strength steel pipelines." *Struct. Eng., Dept. of Civil and environment Eng., University of Alberta*, Edmonton, Alberta, Canada.

- Fathi, A., Cheng, J. J. R., Adeeb, S, & Zhou, J. (2010). "Critical buckling strain in high strength steel pipes using isotropic-kinematic hardening." Proc. of International Pipeline Conference (IPC2010), ASME, Calgary, Alberta, Canada, Sep.27-Oct.1, pp. 39-47.
- Gresnigt, A.M. (1986). "Plastic design of buried steel pipelines in settlement areas." Heron, Vol. 31, No. 4, the Netherlands.
- Herynk, M. D., et al. (2007). "Effects of the UOE/UOC pipe manufacturing processes on pipe collapse pressure." International Journal of Mechanical Sciences 49(5): 533-553.
- Johnson, B.G. (1966). "Column research council guide to design criteria for metal compression members." New York City, New York, USA.
- Kachanov. L. M., Theory of Plasticity, Elsevier Butterworth-Heinemann, UK, 2006, ISBN-13: 978-0-7506-6638-2, ISBN-10: 0-7506-6638-2.
- Kalnins A, Rudolph J, Willuweit A. " Using the Nonlinear Kinematic Hardening Material Model of Chaboche for Elastic–Plastic Ratcheting Analysis." ASME. J. Pressure Vessel Technol. 2015; 137(3):031006-031006-10. doi:10.1115/1.4028659.
- Kalnins A, Rudolph J, Willuweit A. " Using the Nonlinear Kinematic Hardening Material Model of Chaboche for Elastic–Plastic Ratcheting Analysis." ASME 2013 International Pressure Vessels and Piping Conference, Paris, France, July 14-18, PVP2013-98150.
- Kalnins A, Rudolph J, Willuweit A. Using the Nonlinear Kinematic Hardening Material Model of Chaboche for Elastic–Plastic Ratcheting Analysis. ASME. J. Pressure Vessel Technol. 2015; 137(3):031006-031006-10. doi:10.1115/1.4028659.
- Kelly. P., 2016. "An Introduction to Solid Mechanics", Retrieved from http://homepages.engineering.auckland.ac.nz/~pkel015/SolidMechanicsBooks/Part_I/index.html.
- Kelly. P., 2016. "Engineering Solid Mechanics", Retrieved from http://homepages.engineering.auckland.ac.nz/~pkel015/SolidMechanicsBooks/Part_II/index.html.
- Kelly. P., 2016. "Foundations of Continuum Mechanics", Retrieved from http://homepages.engineering.auckland.ac.nz/~pkel015/SolidMechanicsBooks/Part_III/index.html.

- Kelly. P., 2016. "Material Models in Continuum Solid Mechanics", Retrieved from http://homepages.engineering.auckland.ac.nz/~pkel015/SolidMechanicsBooks/Part_IV/index.html.
- Kim, H. O., (1992), "Plastic Buckling of Pipes under Bending and Internal Pressure", Proceedings of Second International Offshore and Polar Engineering Conference, San Francisco, USA, June 14-19.
- Kyriakides SS, Corona EE, Fischer FJ. "On the Effect of the U-O-E Manufacturing Process on the Collapse Pressure of Long Tubes." ASME. J. Eng. Ind.1994;116(1):93-100. doi:10.1115/1.2901815.
- M.D. Herynk, S. Kyriakides, A. Onoufriou, H.D. Yun. "Effects of the UOE/UOC pipe manufacturing processes on pipe collapse pressure." International Journal of Mechanical Sciences, Volume 49, Issue 5, May 2007, Pages 533-553, ISSN 0020-7403.
- Mao-Hong Yu, Guo-Wei Ma, Hong-Fu Qiang & Yong-Qiang Zhang, Generalized Plasticity. Springer, Berlin, 2010, ISBN 978-3642064203.
- MathWorks, 2016. Curve fitting Toolbox: Online User's Guide (R2016b), USA.
- Mohamad Rezaiee-Pajand, Sina Sinaie, On the calibration of the Chaboche hardening model and a modified hardening rule for uniaxial ratcheting prediction, International Journal of Solids and Structures, Volume 46, Issue 16, 1 August 2009, Pages 3009-3017, ISSN 0020-7683.
- Mohareb, M. & Murray, D. W. (1999). "Mobilization of Fully Plastic Moment Capacity for Pressurized Pipes Subjected to Axial Loading." ASME J. Offshore Mech. Arct. Eng., 121(4), 237-241.
- Mohareb, M. (1995), Deformational Behaviour of Line Pipe, Doctoral thesis, Department of Civil Engineering, University of Alberta, Edmonton, Alberta.
- Mohareb, M. (2001). Exact yield hyper-surface for thin pipes. International journal of pressure vessels and piping, 78(7), 507-514.
- Mohareb, M. (2002) Plastic Interaction Relations for Pipe Sections, Journal of Engineering Mechanics, ASCE, Jan. Vol.128 (1), 112-120.

- Mohareb, M., & Murray, D. W. (1999). "Mobilization of fully plastic moment capacity for pressurized pipes." *Journal of Offshore Mechanics and Arctic Engineering*, 121(4), 237-241.
- Mohareb, M., (2003) "Plastic Resistance of Pipe Sections: Upper Bound Solution", American Society of Civil Engineers, *Journal of Structural Engineering*, 129(1), pp. 41-48.
- Mohareb, M., Elwi, A., Kulak, G. L., & Murray, D. W. (1994). "Deformational behavior of line pipe." *Structural Eng. Rep. No. 202*, Dept. of Civ. Eng., University of Alberta, Edmonton, Alberta, Canada.
- Mohareb, M., Kulak, G., Elwi, A., and Murray, D. (2001). "Testing and Analysis of Steel Pipe Segments." *J. Transp. Eng.*, 10.1061/ (ASCE) 0733-947X (2001)127:5(408), 408-417.
- Myrholm, B., Cheng, J. J. R., & Murray, D. W. (2001). "Local buckling and fracture behaviour of line pipe under cyclic loading." *Structural Engineering Report No. 238*, Dept. of Civ. Eng., University of Alberta, Edmonton, Alberta, Canada.
- Neupane S, Adeeb S, Cheng R, Ferguson J, Martens M. (2012) "Modeling the Deformation Response of High Strength Steel Pipelines—Part I: Material Characterization to Model the Plastic Anisotropy." *ASME. J. Appl. Mech.*, 79(5):051002-051002-9. doi:10.1115/1.4006380.
- Neupane, S., Adeeb, S., Cheng, R., Ferguson, J. & Martens, M. (2012), "Modeling the Deformation Response of High Strength Steel Pipelines—Part II: Effects of Material Characterization on the Deformation Response of Pipes." *ASME. J. Appl. Mech.* 79(5):051003-051003-7. doi:10.1115/1.4006381.
- Ozkan, I. and Mohareb, M. (2009). "Moment resistance of steel pipes subjected to combined loads, *International Journal of Pressure Vessels and Piping*." Volume 86, Issue 4, April 2009, Pages 252-264, ISSN 0308-0161.
- Ozkan, I. F. & Mohareb, M. (2002). "Plastic Design of Steel Pipe Sections under General Stress Resultant Combinations." 4th *Structural Specialty Conference of the Canadian Society for Civil Engineering*, Montreal, Quebec, Canada, June 5-8.
- Ozkan, I. F., & Mohareb, M. (2003). "Testing of steel pipes under bending, twist, and shear." *Journal of Structural Engineering*, 129(10), 1350-1357.

- Ozkan, I. F., & Mohareb, M. (2006) "Experimental Setup for Full-Scale Testing of Pipes under Combined Loads." Proc. of International Pipeline Conference (IPC2006), ASME, Calgary, Alberta, Canada, Sep.25-29, pp. 345-354.
- Ozkan, I. and Mohareb, M. (2009). "Testing and Analysis of Steel Pipes under Bending, Tension, and Internal Pressure." *Journal of Structural Eng.*, 10.1061/ (ASCE) 0733 9445(2009)135:2 (187), 187-197.
- Ozkan, I. and Mohareb, M. (2009). "Testing and Analysis of Steel Pipes under Bending, Tension, and Internal Pressure." *J. Struct. Eng.*, 10.1061/ (ASCE) 0733 9445(2009)135:2 (187), 187-197.
- Ozkan, I.F. (2008). "Experimental and numerical analysis of steel pipes subjected to combined loads." Dept. of Civ. Eng., University of Ottawa, Ottawa, Ontario, Canada.
- Raffo, J., Toscano, R. G., Mantovano, L., & Dvorkin, E. N. (2007), " Numerical model of UOE steel pipes: forming process and structural behavior." *Mecanica Computacional*, 26, 317-333.
- Seide, P., and Weingarten, V. I., (1961), "On the Buckling of Circular Cylindrical Shells under Pure Bending.", *Journal of applied Mechanics*, Vol. 28, pp 112-116.
- Sherman, D. R. (1976), "Tests of Circular Steel Tubes in Bending", *J. of Struct Division*, ASCE, Vol.102, No. ST11, pp.2181-2195.
- Simulia. 2016. Abaqus 2016 Online Documentation Dassault Systèmes, RI, USA.
- Suzuki, N., Muraoka, R., Glover, A., Zhou, J., & Toyota, M. (2003). "Local buckling behaviour of X100 linepipe." ASME 2003 22nd International Conference on Offshore Mechanics and Arctic Engineering, Cancun, Mexico, June 8–13, 2003, pp. 67-76.
- Suzuki, N., Tajika, H., Igi, S., Okatsu, M., Kondo, J., & Arakawa, T. (2010). "Local buckling behavior of 48 ", X80 High-Strain Line Pipes." In 2010 8th International Pipeline Conference (IPC2010), ASME, Calgary, Alberta, Canada, Sep.27- Oct.1, 2010, pp. 347-354.

- Timoshenko, S. P. and Gere, J. M. (1961), "Theory of Elastic Stability." 2nd edition, McGraw-Hill.
- Wu, H. C., Continuum Mechanics and Plasticity, Chapman & Hall/CRC Press, United States of America, **2005**, ISBN 1-58488.
- X. Chen, R. Jiao. "Modified kinematic hardening rule for multiaxial ratcheting prediction, International Journal of Plasticity." Volume 20, Issues 4–5, April–May 2004, Pages 871-898, ISSN 0749-6419.
- Yoosef-Ghodsi, N., Kulak, G., and Murray, D. (1994). "Behaviour of Girth Welded Line Pipe." Structural Eng. Rep. No. 203, Dept. of Civ. Eng., University of Alberta, Edmonton, Alberta, Canada.
- Yoosef-Ghodsi, N., Ozkan, I., & Chen, Q. (2014)." Comparison of compressive strain limit equations." Proc. of International Pipeline Conference (IPC2014), ASME, Calgary, Alberta, Canada, Sep.29-Oct.3, pp. V004T11A001.
- Zhang C., Bao, X., Ozkan, I., Mohareb, M., Ravet, F., and Zou, L., (2008)."Novel Signal Processing for Distributed Brillouin Fiber Sensors for the Prediction of Pipe Buckling." Optical Fiber Technology, Vol. 14(2), 109-113.
- Zhang, H., Zhao, X., Luo, J., Zhang, G., & Chen, Z. (2008)." Research on the Critical Buckling Strain Criterion for Pipelines." In 2008 7th International Pipeline Conference (IPC2008), ASME, Calgary, Alberta, Canada, Sep.29-Oct.4, 2008, pp. 769-773.
- Zhao, Y., Ren, Q., Zou, T. X., Li, D. Y., & Peng, Y. H. (2013, March). "Analysis and Simulation of the UOE Pipe Production Processing by Finite Element Method." In Materials Science Forum (Vol. 749, pp. 437-443). Trans Tech Publications.
- Zimmerman, T.J.E., Stephens, M.J., DeGeer, D.D. & Chen, Q. (1995). "Compressive Strain Limits for Buried Pipelines." Proceedings of the 1995 Offshore Mechanics and Arctic Engineering Conference, Denmark, June.
- Zimmerman, T.J.E., Xie, J., Timms, C., & Asante, J. (2004). "Buckling Resistance of Large Diameter Spiral Welded Line Pipe." Proc. of International Pipeline Conference (IPC2004), ASME, Calgary, Alberta, Canada, Oct.4-8, 2000, pp. 365-373.