

STATIC ANALYSIS OF SKEW SANDWICH PLATES

BY THE METHOD OF FINITE ELEMENTS

BY

Wai-Lau Kwok

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A B S T R A C T

The static characteristics of skew sandwich panels are studied by the finite element method using skew elements having three and five degrees of freedom per node. The elements used in this thesis are developed on the basis of Reissner's simplified sandwich model.

Various parametric studies were conducted in an attempt to define the range of validity of the results. Wherever possible results obtained from the present investigation are compared with those available in the technical literature.

A computer program utilizing the Finite Element technique is developed to generate the stiffness matrix of skew sandwich plate element. The boundary conditions are taken as, fixed, free or simply-supported edge constraint. The scope of this work covers linear elastic structures subjected to uniformly distributed loads. Also point loads could be handled with very little modification of the program. In some cases, the resulting four-fold symmetry allows extraction of one-quarter of the plate for purposes of analysis, while in other cases this is not possible.

The results obtained from this numerical analysis are shown to be in good agreement with other precise analyses, such as differential equation method, which

involve much more computational effort than is required by the present method.

It is seen that the method outlined here, provides a workable and accurate means of treating skew sandwich plate problems of arbitrary boundary conditions.

NOTATION

$[\bar{A}_c], [A_f]$	Matrix relating generalized coordinate to nodal displacements of the skin and core element respectively
A_s	Aspect ratio
$[B_c], [B_f]$	Matrix relating the strain in the skew coordinate system to the generalized coordinates of the core and skin element respectively
a, b	Side dimensions of a rectangular plate
$[D_c], [D_f]$	Stress-Strain matrices
c, t_c	Thickness of the core for a sandwich plate
E_c, E_f	Young's modulus of the core and Young's modulus of the skin for a sandwich plate
$\{f\}, \{d\}$	Generalized forces and displacements respectively
$\{\tilde{f}\}, \{\tilde{d}\}$	Generalized forces and displacements respectively in an orthogonal coordinate system
f_j^i, d_j^i	The force and displacement at the j^{th} node of the i^{th} element
G, G_c	Shear modulus of the core
$[H]$	Matrix relating the strain in the skew coordinate system to the strain in the rectangular coordinate system
$[K]$	Stiffness matrix for the entire structure
$[K_c]$	Stiffness matrix for the core element

$[K_i]$	Stiffness matrix for the skin element
$[\tilde{K}]$	Stiffness matrix in orthogonal coordinate system
$[K_{jm}^i]$	Matrix relating the deformations at node m the the forces at node j in the i^{th} element
$[L]$	Transform matrix
x_L, y_L	Span of sandwich plate in x, y-directions respectively
M_x, M_y	Bending moments about the y and x axes in a sandwich plate respectively (lb.-in/in)
q_u, q_v, q_w	The applied loads in the u, v and w directions respectively
q	Intensity of a uniform load
$[Q]$	Angular transformation matrix
r	The vector of nodal displacements in global co-ordinates
R	Nodal loads applied to the structure
r_n	Displacement at node n
R_j	Nodal loads applied at node j
$[S_o]$	Nodal forces on the element
t_s	Thickness of the skins for a sandwich plate
u, v, w	Displacements of the nodes in the x, y and z-direction
w	Transverse deflection of the plate

W_u	The work done by the uniform load
w_ξ, w_η	Derivatives of w with respect to the skew coordinate system
w_x, w_y	Derivatives of w with respect to the rectangular coordinate system
u_p	Component of displacement normal to the edge of the sandwich plate
v_p	Component of displacement parallel to η -axis
θ	Skew angle
$\emptyset, \phi$	$90^\circ - \theta$
$\{\bar{\Delta}\}$	Virtual nodal displacements
$\{\Delta_c\}, \{\Delta_f\}$	Nodal displacements for a core and skin element respectively
$\{\alpha_c\}, \{\alpha_f\}$	Vector of generalized coordinate of the core and skin element respectively
$\{\epsilon_{xyz}\}$	Internal element strains in the rectangular coordinate system
$\{\epsilon_{\xi\eta\zeta}\}$	Internal element strains in the skew coordinate system
σ_x, σ_y	Stresses in the x and y directions respectively
τ_{xy}	Shearing stress in the xy -plane
$\epsilon_x, \epsilon_y, \epsilon_{xy}$	Components of the strain matrix
ν	Poisson's ratio

CHAPTER I

INTRODUCTION

Sandwich plates are a type of three layer construction, consisting of two thin sheets of high strength material between which a thicker layer of comparatively soft and light material is sandwiched. The two thin sheets are called face-sheets and the middle layer is called the core material. The core is considered to have such a small load carrying capacity in the plane of the plate as compared to that of the face layers that all the face-parallel core stresses may be neglected. The face layers are considered sufficiently thin to neglect the variation of the normal stresses over their thickness.

The noted advantages of light weight and high resistance to local buckling of sandwich material make it a suitable material for aerospace applications. For example, skew sandwich plates are often used for skins or partitions for military missiles. Because of its strength and stiffness characteristics, sandwich construction offers great interest and enthusiasm among builders, engineers and theoreticians to explore the

possibilities of its optimum utility. The mathematical analysis of sandwich plates has commanded a considerable attention among the pioneers interested in elasticity and its related fields of study. Many refined analyses have been put forward which are of great interest to the theoretical analysts. In practice, in order to make use of such theories, a great deal of simplifications are required before the design engineer can handle them effectively. Examples of this are described in the paper by Donnell [1] and the book by Plantema [2]. However, it is to be pointed out that, even with such simplifications, only a few problems with simplified boundary conditions could be solved analytically and that most such solutions require the use of a trigonometric series expansion. For the solution of more complex problems involving sandwich plates it has been necessary to use a numerical method such as finite differences.

The finite difference method has been employed for solving the problems of elasticity including plates and shells for over sixty years. In the past decade, however, the finite element method was developed, originally in the aircraft industry in response to the need for a procedure which could provide a refined solution for the problems resulted out of extremely

complex air frame configurations. The pioneering efforts of the aeronautical structural engineers, backed by the needs of the industry and facilitated by the availability of automatic digital computers in such establishments, resulted in the outcome of the finite element method, which could be applied equally well to problems of structural mechanics. This method has been and is being applied with success to many of the problems of elasticity which were previously solvable only with considerable computational difficulty.

The basic concept of the finite element method is that every structure may be considered as an assemblage of a finite number of individual components or elements. The finite character of interconnecting of the elastic structure makes the analysis possible by solution of simultaneous equations, and this is the characteristic feature which distinguishes a structural system from the problems of structural mechanics.

A natural extension of structural analysis by the finite element method is the use of a system of two or three-dimensional elements to represent an elastic continuum. For example, assemblages of two-dimensional elements can be used to represent plates in bending of arbitrary forms. Using such elements, the idealization of the structure is obtained by dividing the continuum

into segments of appropriate sizes and shapes, all the material properties being retained in the individual elements.

The finite element method was originally developed as an extension of the matrix displacement method in structural analysis. In this method the approximation employed, i.e. dividing the continuum into a finite number of interconnected elements is of a physical nature. In other words, a modified structural system is substituted for the actual continuum and there is no approximation in the mathematical analysis of the substitute system. This is the feature which distinguishes the finite element approach from the finite difference method in which the governing differential equations of the actual physical system are solved by approximate mathematical procedures. At a most recent study, a researcher [3] has shown that the finite element is a systematic approach for setting up finite difference equations in plate bending analysis.

It is of interest to note that a number of finite element analyses have been published in recent years especially for rectangular sandwich plates [9] [10] [11] [12]. However, for the analysis of skew

sandwich plates, very little information is available at present. The only work available is that of Kennedy [4] which presents the analysis and results of clamped skew sandwich plates with isotropic cores, using a trigonometric series solution.

The present investigation is devoted to the study of a variety of skew sandwich plate problems with clamped, simply supported and free boundary conditions. Different ratios of bending and shearing stiffnesses are considered using two distinct linear finite elements. Solutions of the above problems are compared between these two finite elements and other available solutions. Although the structures to be investigated in this thesis are limited to skewed sandwich plates the technique is applicable to plates and shells in general.

In the representation of a real structure by finite elements, the number of elements required in order to carry out a fairly detailed stress analysis may also be large enough to seriously challenge the capacities of even the big computers. This problem would become more critical when we use a refined element having more degrees of freedom. A utility computer program was written to generate the stiffness matrices and solve the equilibrium equations. A listing of this program is included for reference in the Appendix VI.

C H A P T E R I I

REVIEW OF : LITERATURE

2.1 Review of previous work on the analysis of sandwich plates

Bending theory of sandwich panels has been investigated by several authors. Williams et al [31] in a paper published in 1940, derived a simple approximate formula concerning wrinkling of sandwich construction by using energy principles. This early theory accounts for the transverse shear effect in the core by assuming that a linear element initially straight and normal to the middle plane of the core will remain straight after deformation but will deviate from the normal to the deformed middle plane by an amount represented by a parameter. This assumption has further been used by March [24] in his analysis of sandwich plates.

March [24] in a paper published in 1948, used the energy method to study the behavior of a flat rectangular sandwich panel. His paper is devoted to determining approximately the effect of the shear deformation of the core on two types of problems (i) Buckling under compressive end load and (ii) centre deflection of clamped rectangular panels under uniform

transverse loads. As an approximation, all the displacements in the core and faces are expressed in term of the normal displacement of the panels and its derivatives and the problem is solved by assuming functions for this displacement that satisfy the boundary conditions. He also obtained the solution of isotropic problems where the facings are stressed beyond the proportional limit.

In the same year. Taylor [22] derived a system of differential equations governing the small deflections of sandwich plates by introducing the concept of separate deflections. The solution of these differential equations physically means that the total deflection W can be split up into a partial deflection W_b due to bending and a partial deflection W_s due to shear. The solution for a square plate with clamped edges under uniform load was presented in his work.

The basic differential equations for the finite deflection of isotropic sandwich plates were developed by Reissner [32] in 1948. In his work, the development of the bending theory for sandwich plates is based upon several fundamental assumptions concerning the behavior of the faces and the core. These assumptions are as follows:

(1) The thickness of the faces is very small in comparison with the thickness of the core and the stresses in the faces parallel to their planes are distributed uniformly over the thickness of the face layers.

(2) The values of the elastic constants E_f , G_f for the face layers are large compared with the values of the elastic constants E_c , G_c for the core layer so that the face-parallel stresses in the core layer and their effect on the deformation of the composite plate are negligible.

On the basis of these assumptions, Reissner treated the sandwich plate as a combination of two plates without bending stiffness (faces layer) and a third plate (core layer) offering resistance only to transverse shear stresses. With these assumptions a system of equations has been derived to describe the behavior of the sandwich plate under bending loads. In this study, he found that the transverse normal stress in the core is negligibly small compared with the transverse shear stresses and that the range of deflections for which the linear theory is adequate decreases as the core is made softer relative to the faces.

In 1948, Libove and Botdorf [33] developed both the energy expression and the differential equation

for sandwich plates on the basis of Reissner's simplified model. Boundary conditions corresponding to simply supported, clamped and elastically restrained edges were considered. When he developed the differential equations, the deflection due to shear and the orthotropic stretching properties of the skins and core were taken into consideration. Three sixth order differential equations were obtained.

In 1950, Hoff [34] used the Principle of Minimum Potential Energy and obtained a similar set of differential equations.

In 1951, Eringen [35] generalized the variational approach used by Hoff and obtained four partial differential equations for the bending and buckling of rectangular, flat sandwich plate having homogeneous cores and identical isotropic faces subjected to various types of loading and boundary conditions. Various effects previously omitted have been incorporated into the analysis. The three dimensional stress distribution in the core is taken into account and thus the bending rigidity and the flattening of the core are not neglected. Also the faces which are customarily assumed to be thin, have bending rigidity.

In 1951, Yen et. al [23] used a Fourier series

to solve the governing differential equations proposed by Hoff and get some numerical results for the maximum deflection of a simply supported rectangular sandwich plate. Both the upper bound and lower bound solutions were presented in his work.

In 1952, a simple order-of-magnitude analysis has been used by Gerard [36] in order to secure a formal basis for the various assumptions used in connection with sandwich construction. In his paper the complete equilibrium and stress-strain relations for the faces and core of an isotropic sandwich plate are considered by examining the orders of magnitude of the stresses and displacements in terms of the thickness of the sandwich. His analysis procedure is similar to that used by Goodier [39] in the development of homogeneous thin-plate theory. By use of this procedure, a set of simplified stress-strain relations is obtained for sandwich plate, which are equivalent to results obtained previously.

Cook [37] in a paper published in 1966, presents quantitative information regarding the validity of certain approximations applicable to sandwich plate analysis such as (1) neglecting face-parallel stresses in the core, (2) neglecting the bending stiffness of the facings, (3) using approximate expressions for strains in the core.

In his paper energy expressions are developed for each approximate theory. Results given by the approximate theories are compared with results given by a more general theory for several values of geometric and elastic properties.

With the advent of digital computer, numerical analysis based on the above mentioned theories become possible. In the last twenty years, most of the work of sandwich plate analysis are concentrated in the field of numerical approximate analysis.

In 1967, Kan et. al [25] used the method of successive approximation and obtained an approximate solution for the governing differential equations proposed by Riessner. He use the centre deflection ratio as a perturbation parameter. The solution is based upon the smallness of centre deflection ratio for the case of uniformly loaded, clamped rectangular sandwich plates considering large deflections.

Martin [11] in a report published in 1967 set forth the first investigation of the problem of thin plates as well as sandwich structures using a triangular sandwich element. Martin's element was formed by combining the stiffness matrices for two CST(constant strain triangle) elements representing

the skins and three CSHR(constant shear) elements representing the core. He used this element to solve the homogeneous thin plate problem and obtained some numerical results for the maximum deflection of a rectangular plate under a uniformly distributed loading.

Monfortan, et. al [10] in a paper published in 1968, used the finite element technique to solve the problem of the sandwich plates. The assumed displacement pattern was represented by the sum of products of one-dimensional first order Hermit-Interpolation polynomials and undetermined nodal coefficients. Eighty degrees of freedom per element were used in order to obtain an accurate representation of the elemental strain energy. Accurate results were obtained even for a coarse element grid.

In the particular area of skew sandwich plate the first and only contribution was made in 1969 by Kennedy [4], who investigated the problem of a clamped skew sandwich plate subjected to uniformly distributed loads. He traced Taylor's work and derived the differential equations by introducing partial deflections due to bending and shear separately. The basic assumptions of this analysis were identical to that proposed by

Reissner in the previously discussed paper. In his work, fair agreement with an error not exceeding 10% was obtained between the theory and experimental results in the small deflection range.

For most approximate methods encountered in practice, such as the series solution method and the strain energy approach it is not easy to find general type of solutions due to complexity of geometry, loading, material properties and boundary conditions. Most of these difficulties have been overcome by the present method. In finite element method, we can obtain results up to needed accuracy, and it is found that very close results to the exact solution could be obtained by that method. The present work may be regarded as an extension of reference [9], which used similar displacement functions to generate the stiffness matrix for vibration analysis of sandwich plates. The element used in this thesis is more general than the aforementioned element in that it can apply to both rectangular or skew sandwich plates subject to various boundary conditions.

2.2 Numerical methods of structural analysis

To analyze a complex structure such as sandwich plate construction, many authors use the differential equation method. This method gives accurate results but solution can be obtained only for some simplified

boundary conditions. For general boundary conditions, the solutions are very complicated. Sometimes, exact solutions are not possible to obtain even for the most simple boundary conditions. It is for these reasons that numerical approaches are essential to effect a solution. What follows is a brief description of several of the numerical methods which may be used to analyze sandwich plate problems.

(a) Ritz Method : Ritz method is one of the energy methods and has found particular application in the analysis of very complex problems. This method is based on the principle that the total energy is a minimum when the loaded system is in stable equilibrium. In applying this method to the investigation of bending of plates, a deflection function which satisfies the boundary conditions of the structure is assumed. The undetermined parameters of the assumed function can be determined from the minimizing condition of the system. In this method the differential equations are not used and need not be known. In general, this will save a considerable amount of mathematical work. However, some difficulty is always encountered if the plate is not symmetric. For example, for a plate with one side simply supported and the other

side clamped, the choice of a complete deflection function to meet the geometric boundary conditions is indeed difficult.

(b) Fourier Series Method : When the governing differential equation of the problem is known, the rest of the work is reduced to solving the differential equation and a rigorous solution would involve adjusting certain constants to take care of the boundary conditions. Fourier series have found application in the solution of many structural problems because it can represent discontinuous functions. Kennedy [4] has presented solutions for clamped sandwich and homogeneous plates using this method.

(c) Perturbation Method : This method is often used to solve the large deflection problems. Kan, et. al [25] has successfully solved the sandwich plate problem using this technique. This method is based on the smallness of the central deflection of the plate. Then the nonlinear partial differential equations governing the problem are reduced to a sequence of linear or perturbed equations and the undetermined coefficients in the assumed displacement function can be evaluated by equating terms with corresponding powers of variables in the equation. The drawback of this method is that it is extremely

laborious and requires considerable computation.

(d) Collocation Method : This method is one of the easiest numerical method. The method is based on error distribution principles and may be regarded as an improvement of the perturbation technique. This technique has been applied successfully by Walter [38] to the analysis to rectangular plate with large deflections. Application of this method to other boundary value problems may be feasible.

(e) Finite Difference Method : This method is based on the replacement of the differential equations by the corresponding finite difference equations. Application of this technique yields a set linear simultaneous equations from which the deflection, slope and curvatures of the grid points can be evaluated. This method has found wide application in structural analysis. Experiments have demonstrated that finite difference equations are not easy to set up with odd meshes, irregular boundaries and certain types of boundary conditions such as free edges.

(f) Finite Element Method : In the last ten years, this method has widely been used to solve various structural problems. However, this method is not intended to replace other methods but rather to supple-

ment them. As stated earlier, the finite element and finite difference method are not entirely unrelated. The relative merits of these approximate methods can be observed from the numerical results. Zienkiewicz [6] showed that the finite element method gives better results than finite difference of an equivalent mesh size. The ease of changing grid spacing and of combining different kinds of element to form the whole structure in the finite element method can be a distinct advantage. Due to this advantage, it was tried for complex structural problems such as stiffened plates and sandwich plates with great success. A major disadvantage in this method is that more equations must be solved for a given grid pattern than those for finite difference models. Details of this method will be given in the next chapter.

C H A P T E R I I I

M E T H O D O F A N A L Y S I S

3.1 General

In this chapter two linear finite elements for the analysis of skew sandwich plates are presented. The required stiffness and consistent load matrices are derived from the direct stiffness method using the displacement-strain-stress procedure. Two displacement models are used incorporating an element having three and five degrees of freedom per node. These two stiffness matrices are generated in subroutines independently.

Attention is also focussed on how the step-by-step technique, the so-called "direct stiffness" approach can be used to form the stiffness matrix for the skew sandwich element which has proved to be useful to solve sandwich plate problems. Some emphasis is placed on the techniques of joining the skin and the core, so that it can be used most appropriately to represent the sandwich element. Various modifications of flexural and shearing rigidities are also discussed.

3.2 The General Assumptions Used in This Investigation

In the traditional simplified analysis of sandwich plates it is generally assumed that bending is resisted

entirely by the pair of skins while the core resists only the transverse shear. This means that the skins are in a state of plane stress and that the transverse shear stresses are constant through the thickness of the core. These assumptions are easily satisfied by a finite element idealization. The following assumptions are made in the derivation of stiffness matrices.

1. Hooke's law holds, i.e. the plate is made of perfectly elastic material.

2. The facings and the core are isotropic^{*} and homogeneous. This assumption is reasonable for most of the sandwich structure of honeycomb or corrugated type when the cell size is small compared with the characteristic dimension under consideration.

3. The displacements are small, so that the linear (small deflection) theory is valid.

4. The two facings of equal thickness have negligible bending rigidity. This assumption implies that the strain in the faces is almost constant across the depth of the face and hence the flexural strain energy of the faces is usually negligible when the thickness of the faces is small in comparison with the thickness of the core. It also implies that the transverse shear is entirely carried by the core.

* although most sandwich core materials are only orthotropic in nature; However, due to the assumption that normal strains in the vertical direction are negligible (assumption 5, P20) the entire sandwich plate can be assumed as isotropic without appreciable error.

5. Transverse normal strain is negligible. This assumption generally means that the transverse contraction of the plate is small, neglecting this effect does not alter appreciably the accuracy of the bending analysis. This assumption is usually applied in the classical theory of plate analysis [5] .

6. The core is made of very low density material in comparison with the face material ($E_c \ll E_f$) therefore the core stresses parallel to the face plates are very small and hence can be neglected, it implies that bending moments are resisted only by the faces.

7. Plane-section remains plane during deformation. This assumption implies that linear strain state exists across the depth of the core. Due to considerable shear strain in the transverse direction, the normal of the middle plane before bending is not remaining normal to the middle plane after bending. This assumption further implies that sandwich plate analysis is a three-dimensional problem which cannot be treated as a two-dimensional problem such as classical thin plate analysis.

8. The bond between facing and core is assumed perfect. This assumption is achieved by the displacement functions of the faces and core.

3.3 Assumed Displacement Fields

3.3.1 Three and Five Degrees of Freedom Elements

The accuracy of the finite element solution for solving structural problems depends mainly on the proper selection of the assumed displacement field. If the displacement functions are not properly selected, the results obtained may not converge to the correct answers. Now the question arises as to which criteria the assumed displacement functions must satisfy in order for the displacements, strains and stresses to converge towards the correct values when the element mesh size is decreased. Two well-known criteria proposed by Zienkiewicz [6] are :

1. The displacement function must be chosen such that the next to highest derivative occurring in the strain energy expression must be continuous.

2. Rigid body motion must be adequately represented.

Based on some authors [7] [8] a further desirable property of a plate-bending element is conformity which means the slopes are continuous between elements as well as the transverse displacement. Although the need for conformity is a matter of some dispute, it would perhaps be fair to say that conformity is a sufficient but not a necessary condition that ensures convergence

of the finite element approximation to the true solution.

a) Three degrees of freedom per node

The equilibrium equations of skewed sandwich construction are a set of second order differential equations. The related minimum potential energy function requires only the first derivatives in the unknown displacement fields to be evaluated. This means that continuity of the displacement fields is only required in the finite element method. Based on these requirements, an element of three degrees of freedom per node has been developed. The components of displacement at a node are given by :

$$\{ d \}^T = [u, v, w]$$

The displacement function is assumed as

$$u = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta$$

$$v = \alpha_5 + \alpha_6 \xi + \alpha_7 \eta + \alpha_8 \xi \eta$$

$$w = \alpha_9 + \alpha_{10} \xi + \alpha_{11} \eta + \alpha_{12} \xi \eta$$

b) Five degrees of freedom per node

Although the element (three degrees of freedom per node) described above give good results in many cases, experience [9] with this element has demonstrated the need for a more refined element. The improved element can be obtained by two different approaches, either increasing

the number of nodes for each element or increasing the number of kinematic degrees of freedom at each node. In most cases the second approach has a decided topological advantage in that the connectivity of the element assemblage is greatly simplified. This type of element will be studied in some detail in the present work. Using both the displacement derivatives as well as displacements as nodal degrees of freedom, the displacement vector at a node is given by :

$$\{ d \}^T = [u, v, w, \partial w / \partial \xi, \partial w / \partial \eta]$$

where the assumed displacement field for u and v are the same as for the three degrees of freedom given before, except w has been taken to be a twelve term vector, thus

$$\begin{aligned} w = & \alpha_9 + \alpha_{10}\xi + \alpha_{11}\eta + \alpha_{12}\xi^2 + \alpha_{13}\xi\eta + \alpha_{14}\eta^2 \\ & + \alpha_{15}\xi^3 + \alpha_{16}\xi^2\eta + \alpha_{17}\xi\eta^2 + \alpha_{18}\eta^3 \\ & + \alpha_{19}\xi^3\eta + \alpha_{20}\xi\eta^3 \end{aligned}$$

It has been shown by K. M. Ahmed [9] that the use of more terms in the vertical displacement will achieve a more reasonable representation for the shear properties of the core and the convergence to the correct solution is rapid. For these reasons elements having five degrees of freedom per node are used and results are compared with those obtained from elements having only

three degrees of freedom per node.

3.3.2 Displacement Distribution for a Parallelogram Sandwich Element

In order to study the continuity of displacements across element boundaries, the displacement fields u , v , w , are calculated in terms of the nodal coordinates as shown in Fig. 2 for both the three and five degrees of freedom elements.

a) Element containing three degrees of freedom per node

The expressions of displacement fields in terms of the nodal displacements are obtained, as follows :

$$\begin{aligned} u &= u_1 + (u_2 - u_1)\xi/a + (u_3 - u_1)\eta/b \\ &\quad + (u_4 - u_2 - u_3 + u_1)\xi/a \eta/b \\ v &= v_1 + (v_2 - v_1)\xi/a + (v_3 - v_1)\eta/b \\ &\quad + (v_4 - v_2 - v_3 + v_1)\xi/a \eta/b \\ w &= w_1 + (w_2 - w_1)\xi/a + (w_3 - w_1)\eta/b \\ &\quad + (w_4 - w_2 - w_3 - w_1)\xi/a \eta/b \end{aligned} \quad (3 - 3)$$

To ensure the continuity of u , v and w across an interface we must have u , v and w uniquely defined by the values along such an interface. The following expressions are obtained.

Along the faces 1 - 3 ($\xi = 0$) and 2 - 4 ($\xi = a$), the following displacement fields are obtained, respectively as :

$$\begin{aligned} u &= u_1 + (u_3 - u_1)\eta/b \\ v &= v_1 + (v_3 - v_1)\eta/b \end{aligned} \quad (3 - 4a)$$

$$\begin{aligned} w &= w_1 + (w_3 - w_1)\eta/b \\ u &= u_2 + (u_4 - u_2)\eta/b \\ v &= v_2 + (v_4 - v_2)\eta/b \\ w &= w_2 + (w_4 - w_2)\eta/b \end{aligned} \quad (3 - 4b)$$

Along the faces 1 - 2 ($\eta = 0$) and 3 - 4 ($\eta = 0$) the following expressions are obtained respectively as :

$$\begin{aligned} u &= u_1 + (u_2 - u_1)\xi/b \\ v &= v_1 + (v_2 - v_1)\xi/b \end{aligned} \quad (3 - 5a)$$

$$\begin{aligned} w &= w_1 + (w_2 - w_1)\xi/b \\ u &= u_3 + (u_4 - u_3)\xi/b \\ v &= v_3 + (v_4 - v_3)\xi/b \\ w &= w_3 + (w_4 - w_3)\xi/b \end{aligned} \quad (3 - 5b)$$

From the above expressions, we found that the displacement functions depend linearly on nodal displacements and that the continuity of element displacements are maintained for the three degrees of freedom

element.

b) Element containing five degrees of freedom
per node

The displacement fields with regard to u , v are the same as in equations (3 - 3), (3 - 4) and (3 - 5), the only difference being the expansion of the displacement w which will be given here.

The expressions of displacement fields in terms of the nodal displacements are obtained as :

$$\begin{aligned}
 w = & w_1 \left\{ 1 - \frac{\xi}{a} \frac{\eta}{b} - (3 - 2\frac{\xi}{a}) \frac{\xi^2}{a^2} (1 - \frac{\eta}{b}) - (1 - \frac{\xi}{a}) (3 - 2\frac{\eta}{b}) \frac{\eta^2}{b^2} \right\} \\
 & + w_{\xi_1} \left\{ \xi (1 - \frac{\xi}{a})^2 (1 - \frac{\eta}{b}) \right\} + w_{\eta_1} \left\{ \eta (1 - \frac{\xi}{a}) (1 - \frac{\eta}{b})^2 \right\} \\
 & + w_2 \left\{ (3 - 2\frac{\xi}{a}) \frac{\xi^2}{a^2} (1 - \frac{\eta}{b}) + \frac{\xi}{a} \frac{\eta}{b} (1 - \frac{\eta}{b}) (1 - 2\frac{\eta}{b}) \right\} \\
 & - w_{\xi_2} \left\{ \frac{\xi^2}{a^2} (1 - \frac{\xi}{a}) (1 - \frac{\eta}{b}) \right\} + w_{\eta_2} \left\{ \frac{\xi \eta}{a} (1 - \frac{\eta}{b})^2 \right\} \\
 & + w_4 \left\{ (3 - 2\frac{\xi}{a}) \frac{\xi^2}{a^2} \frac{\eta}{b} - \frac{\xi}{a} \frac{\eta}{b} (1 - \frac{\eta}{b}) (1 - 2\frac{\eta}{b}) \right\} \\
 & - w_{\xi_4} \left\{ \xi (1 - \frac{\xi}{a}) \frac{\xi}{a} \frac{\eta}{b} \right\} + w_3 \left\{ (1 - \frac{\xi}{a}) (3 - 2\frac{\eta}{b}) \frac{\eta^2}{b^2} \right. \\
 & + \frac{\xi}{a} (1 - \frac{\xi}{a}) (1 - 2\frac{\xi}{a}) \frac{\eta}{b} \left. \right\} + w_{\xi_3} \left\{ \xi (1 - \frac{\xi}{a})^2 \frac{\eta}{b} - w_{\eta_3} \left\{ \eta (1 - \frac{\xi}{a}) \right. \right. \\
 & \left. \left. (1 - \frac{\eta}{b}) \frac{\eta}{b} \right\} - w_{\eta_4} \left\{ \eta (1 - \frac{\eta}{b}) \frac{\xi}{a} \frac{\eta}{b} \right\} \right. \quad (3 - 6a)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial w}{\partial \xi} = & w_1 \left\{ \frac{\eta}{ab} - \left(1 - \frac{\eta}{b}\right) \left(\frac{6\xi}{a^2} - \frac{6\xi^2}{a^3}\right) + \frac{\eta^2}{ab} \left(3 - \frac{2\eta}{b}\right) \right\} \\
 & + w_{\xi_1} \left\{ \left(1 - \frac{\eta}{b}\right) \left(1 - \frac{\xi}{a}\right)^2 - \frac{2\xi}{a} \left(1 - \frac{\eta}{b}\right) \left(1 - \frac{\xi}{a}\right) \right\} \\
 & + w_{\eta_1} \left\{ \frac{\eta}{a} \left(1 - \frac{\eta}{b}\right)^2 \right\} + w_2 \left\{ \left(1 - \frac{\eta}{b}\right) \frac{6}{a} \left(\frac{\xi}{a} - \frac{\xi^2}{a^2}\right) \right. \\
 & + \frac{\eta}{ab} \left(1 - \frac{\eta}{b}\right) \left(1 - \frac{2\eta}{b}\right) \left. \right\} - w_{\xi_2} \left\{ \left(1 - \frac{\eta}{b}\right) \left(\frac{2\xi}{a} - \frac{3\xi^2}{a^2}\right) \right\} \\
 & + w_{\eta_2} \left\{ \frac{\eta}{a} \left(1 - \frac{\eta}{b}\right)^2 \right\} + w_4 \left\{ \frac{-2}{a} \frac{\xi^2}{a^2} \frac{\eta}{b} + \frac{\eta}{b} \frac{2\xi}{a^2} \left(3 - \frac{2\xi}{a}\right) \right. \\
 & - \frac{\eta}{ab} \left(1 - \frac{\eta}{b}\right) \left(1 - \frac{2\eta}{b}\right) \left. \right\} - w_{\xi_4} \left\{ \frac{\eta}{ab} \left(2\xi - \frac{3\xi^2}{a}\right) \right\} - w_{\eta_4} \\
 & \left\{ \eta \left(1 - \frac{\eta}{b}\right) \frac{\eta}{ab} \right\} + w_3 \left\{ \frac{-1}{a} \left(3 - \frac{2\eta}{b}\right) \frac{\eta^2}{b^2} + \frac{1}{a} \left(1 - \frac{2\xi}{a}\right)^2 \frac{\eta}{b} \right. \\
 & - \frac{2}{a^2} \frac{\eta}{b} \left(\xi - \frac{\xi^2}{a}\right) \left. \right\} + w_{\xi_3} \left\{ \frac{\eta}{b} \left(1 - \frac{\xi}{a}\right)^2 + \frac{-2\xi}{a} \frac{\eta}{b} \left(1 - \frac{\xi}{a}\right) \right\} \\
 & - w_{\eta_3} \left\{ \frac{-\eta^2}{ab} \left(1 - \frac{\eta}{b}\right) \right\} \quad (3 - 6b)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial w}{\partial \eta} = & w_1 \left\{ \frac{-\xi}{ab} + \frac{\xi^2}{ba^2} \left(3 - \frac{2\xi}{a}\right) - \left(\frac{6\eta}{b^2} - \frac{6\xi}{a} \frac{\eta}{b^2} - \frac{6\eta^2}{b^3} + \frac{6\xi\eta}{ab^3}\right) \right\} \\
 & + w_{\xi_1} \left\{ \frac{-\xi}{b} \left(1 - \frac{\xi}{a}\right)^2 \right\} + w_{\eta_1} \left\{ 1 - \frac{4\eta}{b} + \frac{3\eta^2}{b^2} - \frac{\xi}{a} + \frac{\xi\eta}{ab} - \frac{3\xi\eta^2}{ab^2} \right\} \\
 & + w_2 \left\{ -\frac{\xi^2}{ba^2} \left(3 - \frac{2\xi}{a}\right) + \frac{\xi}{ab} - \frac{6\xi\eta}{ab^2} + \frac{6\xi}{a} \frac{\eta^2}{b^3} \right\} - w_{\xi_2} \\
 & \left\{ -\left(1 - \frac{\xi}{a}\right) \frac{\xi^2}{ab} \right\} + w_{\eta_2} \left\{ \frac{\xi}{a} + \frac{3\xi}{a} \frac{\eta^2}{b^2} - \frac{4\xi}{a} \frac{\eta}{b} \right\} + w_4 \left\{ \left(3 - \frac{2\xi}{a}\right) \right. \\
 & \left. \frac{\xi^2}{ba^2} - \left(\frac{\xi}{ab} - \frac{6\xi\eta}{ab^2} + \frac{6\xi\eta^2}{ab^3}\right) \right\} - w_{\xi_4} \left\{ \left(1 - \frac{\xi}{a}\right) \frac{\xi^2}{ab} \right\} - w_{\eta_4} \\
 & \left\{ \frac{2\xi\eta}{ab} - \frac{3\xi\eta^2}{ab^2} \right\} + w_3 \left\{ \frac{\xi}{ab} \left(1 - \frac{\xi}{a}\right) \left(1 - \frac{2\xi}{a}\right) + \frac{6}{b} \left(\frac{\eta}{b} - \frac{\xi\eta}{ab} - \frac{\eta^2}{b^2} \right. \right. \\
 & \left. \left. + \frac{\xi\eta^2}{ab^2} \right) \right\} + w_{\xi_3} \left\{ \frac{\xi}{b} \left(1 - \frac{\xi}{a}\right)^2 \right\} - w_{\eta_3} \left\{ \frac{2\eta}{b} - \frac{2\xi\eta}{ab} - \frac{3\eta^2}{b^2} + \frac{3\xi\eta^2}{ab^2} \right\}
 \end{aligned}$$

(3 - 6c)

The displacement w and the normal slope across an interface have been defined as follows :

Along the faces 1 - 3 ($\xi = 0$) and 2 - 4 ($\xi = a$), the following expressions for w and $\partial w / \partial \xi$ are obtained respectively as :

$$w = w_1 \left(1 - \frac{3\eta^2}{b^2} + \frac{2\eta^3}{b^3} \right) + w_{\eta_1} \left(\eta - \frac{2\eta^2}{b} + \frac{\eta^3}{b^2} \right) \\ + w_3 \left(\frac{3\eta^2}{b^2} - \frac{2\eta^3}{b^3} \right) - w_{\eta_3} \left(\frac{\eta^2}{b} - \frac{\eta^3}{b^2} \right)$$

$$\frac{\partial w}{\partial \xi} = w_1 \left(\frac{-\eta}{ab} + \frac{3\eta^2}{ab^2} - \frac{2\eta^3}{ab^3} \right) + w_{\xi_1} \left(1 - \frac{\eta}{b} \right) + w_{\eta_1} \left(\frac{-\eta}{a} + \frac{2\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) \\ + w_2 \left(\frac{\eta}{ab} - \frac{3\eta^2}{ab^2} + \frac{2\eta^3}{ab^3} \right) + w_{\eta_2} \left(\frac{\eta}{a} - \frac{2\eta^2}{ab} + \frac{\eta^3}{ab^2} \right) + w_4 \left(\frac{-\eta}{ab} + \frac{3\eta^2}{ab^2} \right. \\ \left. - \frac{2\eta^3}{ab^3} \right) - w_{\eta_4} \left(\frac{\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) + w_3 \left(\frac{\eta}{ab} - \frac{3\eta^2}{ab^2} + \frac{2\eta^3}{ab^3} \right) + w_{\xi_3} \frac{\eta}{b} \\ + w_{\eta_3} \left(\frac{\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) \quad (3 - 7a)$$

$$w = w_2 \left(1 - \frac{3\eta^2}{b^2} + \frac{2\eta^3}{b^3} \right) + w_{\eta_2} \left(\eta - \frac{2\eta^2}{b} + \frac{\eta^3}{b^2} \right) \\ + w_4 \left(\frac{3\eta^2}{b^2} - \frac{2\eta^3}{b^3} \right) - w_{\eta_4} \left(\frac{\eta^2}{b} - \frac{\eta^3}{b^2} \right)$$

$$\frac{\partial w}{\partial \xi} = w_1 \left(\frac{-\eta}{ab} + \frac{3\eta^2}{ab^2} - \frac{2\eta^3}{ab^3} \right) + w_{\eta_1} \left(\frac{-\eta}{a} + \frac{2\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) + w_2 \left(\frac{\eta}{ab} \right. \\ \left. - \frac{3\eta^2}{ab^2} + \frac{2\eta^3}{ab^3} \right) + w_{\xi_2} \left(1 - \frac{\eta}{b} \right) + w_{\eta_2} \left(\frac{\eta}{a} - \frac{2\eta^2}{ab} + \frac{\eta^3}{ab^2} \right) \\ + w_4 \left(\frac{-\eta}{ab} + \frac{3\eta^2}{ab^2} - \frac{2\eta^3}{ab^3} \right) + w_{\xi_4} \left(\frac{\eta}{b} \right) - w_{\eta_4} \left(\frac{\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) \\ + w_3 \left(\frac{\eta}{ab} - \frac{3\eta^2}{ab^2} + \frac{2\eta^3}{ab^3} \right) + w_{\eta_3} \left(\frac{\eta^2}{ab} - \frac{\eta^3}{ab^2} \right) \quad (3 - 7b)$$

Similarly, along the faces 1 - 2 ($\eta = 0$) and 3 - 4 ($\eta = b$), the following expressions for w and $\partial w / \partial \eta$ are obtained respectively as :

$$w = w_1 \left(1 - \frac{3\xi^2}{a^2} + \frac{2\xi^3}{a^3} \right) + w_{\xi_1} \left(\xi - \frac{2\xi^2}{a} + \frac{\xi^3}{a^2} \right) + w_2 \left(\frac{3\xi^2}{a^2} - \frac{2\xi^3}{a^3} \right) - w_{\xi_2} \left(\frac{\xi^2}{a} - \frac{\xi^3}{a^2} \right)$$

$$\begin{aligned} \frac{\partial w}{\partial \eta} = & w_1 \left\{ \frac{-\xi}{ab} + \frac{\xi^2}{ba^2} \left(3 - \frac{2\xi}{a} \right) \right\} + w_{\xi_1} \left\{ \frac{-\xi}{b} \left(1 - \frac{\xi}{a} \right)^2 \right\} + w_{\eta_1} \left(1 - \frac{\xi}{a} \right) \\ & + w_2 \left\{ \frac{\xi}{ab} - \left(3 - \frac{2\xi}{a} \right) \frac{\xi^2}{ba^2} \right\} + w_{\xi_2} \left\{ \frac{\xi^2}{ab} \left(1 - \frac{\xi}{a} \right) \right\} \\ & + w_{\eta_2} \left(\frac{\xi}{a} \right) + w_4 \left\{ \left(3 - \frac{2\xi}{a} \right) \frac{\xi^2}{ba^2} - \frac{\xi}{ab} \right\} - w_{\xi_4} \left\{ \frac{\xi^2}{ab} \left(1 - \frac{\xi}{a} \right) \right\} \\ & + w_3 \left\{ \frac{\xi}{ab} \left(1 - \frac{\xi}{a} \right) \left(1 - \frac{2\xi}{a} \right) \right\} + w_{\xi_3} \left\{ \frac{\xi}{b} \left(1 - \frac{\xi}{a} \right)^2 \right\} \end{aligned}$$

(3 - 8a)

$$w = w_4 \left(\frac{3\xi^2}{a^2} - \frac{2\xi^3}{a^3} \right) - w_{\xi_4} \left(\frac{\xi^2}{a} - \frac{\xi^3}{a^2} \right) + w_3 \left(1 - \frac{3\xi^2}{a^2} + \frac{2\xi^3}{a^3} \right) + w_{\xi_3} \left(\xi - \frac{2\xi^2}{a} + \frac{\xi^3}{a^2} \right)$$

$$\begin{aligned} \frac{\partial w}{\partial \eta} = & w_1 \left\{ \frac{-\xi}{ab} + \frac{\xi^2}{ba^2} \left(3 - \frac{2\xi}{a} \right) \right\} + w_{\xi_1} \left\{ \frac{-\xi}{b} \left(1 - \frac{\xi}{a} \right)^2 \right\} + w_2 \left\{ \frac{\xi}{ab} - \left(3 - \frac{2\xi}{a} \right) \frac{\xi^2}{ba^2} \right\} \\ & + w_{\xi_2} \left\{ \frac{\xi^2}{ab} \left(1 - \frac{\xi}{a} \right) \right\} + w_4 \left\{ \frac{\xi^2}{ba^2} \left(3 - \frac{2\xi}{a} \right) - \frac{\xi}{ab} \right\} \\ & - w_{\xi_4} \left\{ \frac{\xi^2}{ab} \left(1 - \frac{\xi}{a} \right) \right\} + w_{\eta_4} \left(\frac{\xi}{a} \right) + w_3 \left\{ \frac{\xi}{ab} \left(1 - \frac{\xi}{a} \right) \left(1 - \frac{2\xi}{a} \right) \right\} \\ & + w_{\xi_3} \left\{ \frac{\xi}{b} \left(1 - \frac{\xi}{a} \right)^2 \right\} - w_{\eta_3} \left(\frac{\xi}{a} - 1 \right) \end{aligned}$$

(3 - 8b)

As can be observed from equations(3-4) and (3-5) the deflections are the same along a common edge between adjacent elements. Similarly from equations(3-6) and (3-7) it can be shown that the slopes are not the same along the edge. Therefore it can be deduced that continuity of element displacements is also maintained but continuity of normal slopes is violated for the five degrees of freedom element. As we mentioned before, the latter condition need not be satisfied for convergence criteria [6]; moreover, it can be seen that the normal slopes of the interface are independent of the normal slopes of the nodes on a parallel edge of an element under consideration. The normal slope is defined with respect to the skew coordinate ξ and η ; it need not necessarily be perpendicular to the boundaries.

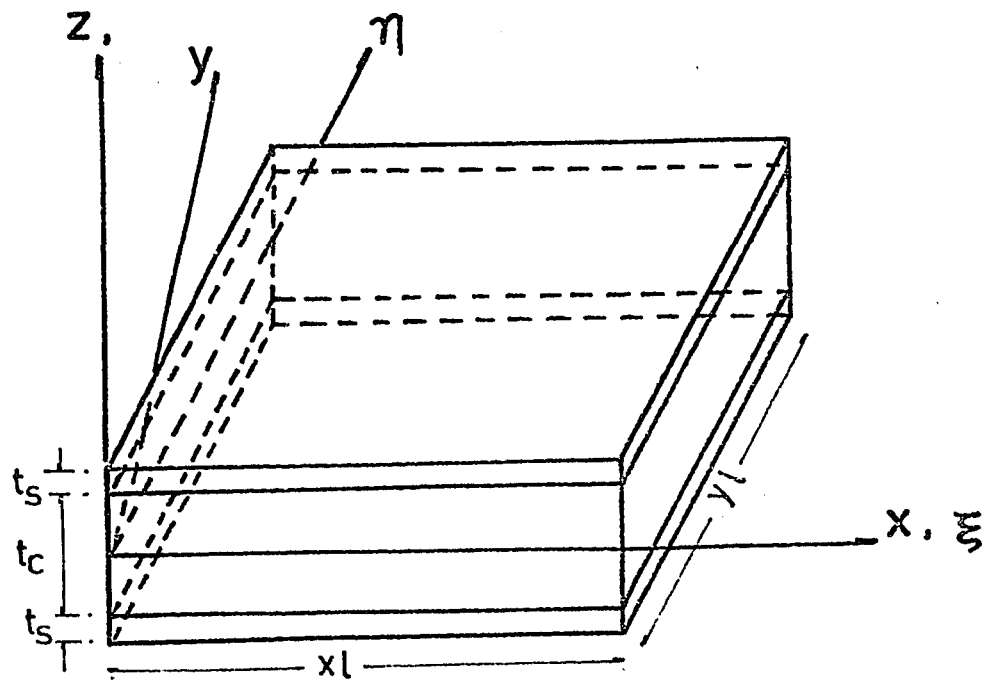


Fig.1 SKEWED SANDWICH PLATE

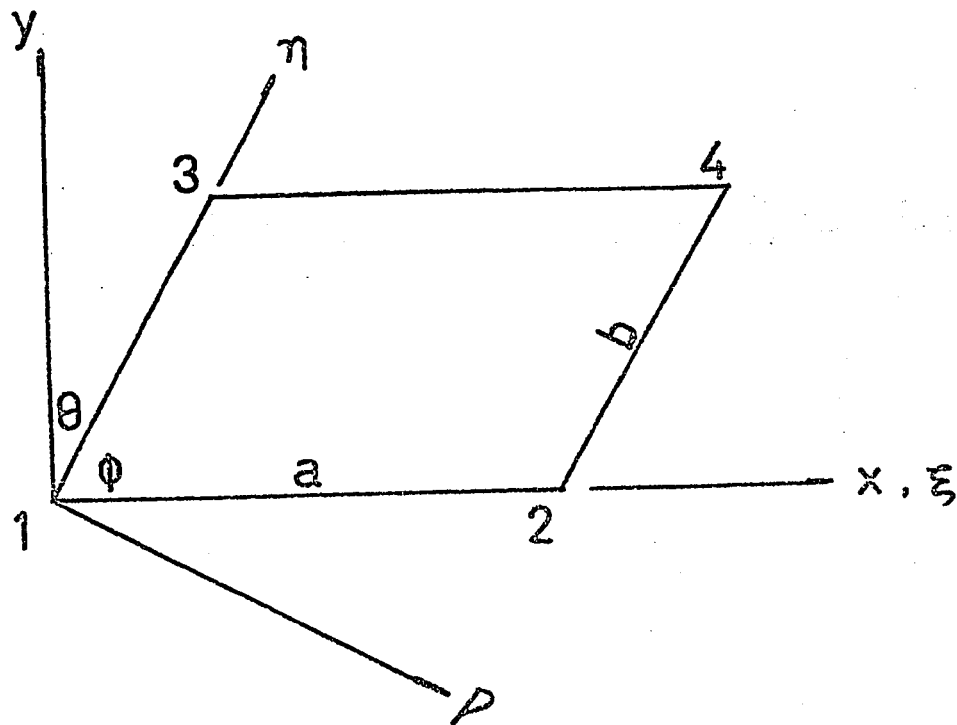


Fig.2 CO-ORDINATES AND ORDER OF NODE NUMBERING OF AN ELEMENT.

3.4. Formation of the stiffness matrix for the skin element

Because of the unique behavior of sandwich plates as described above it is convenient to build each individual sandwich plate element using a set of sub-elements, two of which represent the opposite skins and the remainder of which represent the core, some elements previously developed for plane stress [13] and [14] may be used to represent the skin of sandwich. A parallelogram plane stress element is especially useful in skew sandwich plate analysis, although its use is obviously more restricted than the triangular one in other problems. On the other hand, the fact that the parallelogram element has four nodes permits the introduction of an extra parameter in the displacement function and permits a more accurate representation of a varying stress field.

a) Strain-displacement Relationship

The parallelogram element with nodes at the corners is as illustrated in Fig. 3. This subelement is used to represent the membrane displacements of the face,

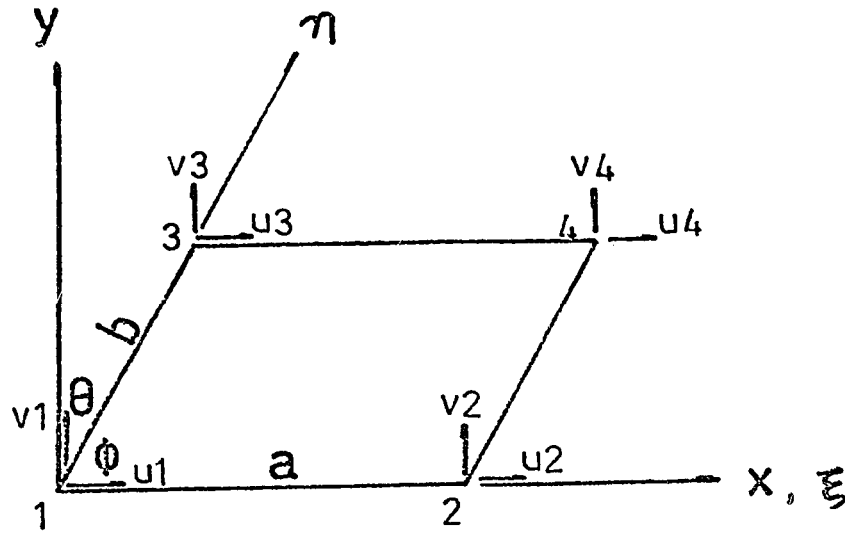


Fig. 3

where v_i and u_i are used to represent the membrane displacements at node i in the directions Y and X respectively.

The displacement functions are taken as

$$u = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta \quad (3 - 9)$$

$$v = \alpha_5 + \alpha_6 \xi + \alpha_7 \eta + \alpha_8 \xi \eta$$

The coefficients ($\alpha_1, \alpha_2, \dots, \alpha_8$) were expressed in terms of the displacements u, v at the four nodes. Therefore the element has eight degrees of freedom. Evaluating equation (3 - 9) at each nodal point in turn, the nodal displacement-generalized displacement relationship may be constructed as

$$\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a & 0 & 0 \\ 1 & b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & b & 0 & 0 \\ 1 & a & b & ab & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a & b & ab \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} \quad (3 - 10)$$

In matrix notation

$$\{\Delta_f\} = [A_f]\{\alpha_f\}$$

where $\{\Delta_f\}$ is the vector of element nodal displacements, $\{\alpha_f\}$ is considered as a vector of generalized co-ordinates of the element.

The strain in the element with respect to the rectangular coordinate system are defined as :

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned} \quad (3 - 11)$$

The strain in the skew co-ordinate system are obtained from those in the rectangular system as follows.

The associated coordinate axes are shown in Fig. 3. The rectangular x, y axes and the skew ξ, η axes are related as follows.

$$x = \xi + \eta \cos \phi$$

$$y = \eta \sin \phi$$

(3 - 12)

or

$$\xi = x - y \cot \phi$$

$$\eta = y \operatorname{cosec} \phi$$

By changing variables,

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial v}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial v}{\partial \eta} \frac{\partial \eta}{\partial x}$$

(3 - 13)

From equations (3 - 12),

$$\frac{\partial \xi}{\partial x} = 1,$$

$$\frac{\partial \xi}{\partial y} = -\cot \phi$$

$$\frac{\partial \eta}{\partial x} = 0,$$

$$\frac{\partial \eta}{\partial y} = \operatorname{cosec} \phi$$

(3 - 14)

Equations (3 - 13) becomes,

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi}$$

$$\frac{\partial v}{\partial y} = -\cot \phi \frac{\partial v}{\partial \xi} + \operatorname{cosec} \phi \frac{\partial v}{\partial \eta}$$

$$\frac{\partial u}{\partial y} = -\cot \phi \frac{\partial u}{\partial \xi} + \operatorname{cosec} \phi \frac{\partial u}{\partial \eta}$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial \xi}$$

(3 - 15)

Substituting equations (3 - 15) into equations (3 - 11), the following relationship will be obtained,

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\cot\phi & \csc\phi \\ -\cot\phi & \csc\phi & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} \quad (3 - 16)$$

In matrix notation,

$$\{\epsilon_{xy}\} = [H_f] \{\epsilon_{\xi\eta}\}$$

where $\{\epsilon_{\xi\eta}\}$ can be obtained by appropriate differentiations of equations (3 - 9).

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \eta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \xi \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} \quad (3 - 17)$$

In matrix notation,

$$\{\epsilon_{\xi\eta}\} = [B_f] \{\alpha_f\}$$

The vector of generalized coordinates of the element $\{\alpha_f\}$ will be obtained by rearranging equations (3 - 10),

$$\{\alpha_f\} = [A_f]^{-1} \{\Delta_f\} \quad (3 - 18)$$

where

$$[A_f]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{a} & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{b} & 0 & 0 & 0 & \frac{1}{b} & 0 & 0 & 0 \\ \frac{1}{ab} & 0 & \frac{1}{ab} & 0 & \frac{1}{ab} & 0 & \frac{1}{ab} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{a} & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{b} & 0 & 0 & 0 & \frac{1}{b} & 0 & 0 \\ 0 & \frac{1}{ab} & 0 & -\frac{1}{ab} & 0 & -\frac{1}{ab} & 0 & \frac{1}{ab} \end{bmatrix}$$

If these equations (3 - 18), (3 - 17) are substituted into equation (3 - 16), the following strain-displacement relationship is obtained. In matrix notation,

$$\{\epsilon_{xy}\} = [H] [B] [A]^{-1} \{\Delta_f\} \quad (3 - 19)$$

b) Stress-strain relationship

It has been assumed that the material is elastic homogeneous and isotropic, the stress-strain relationships

for plane stress with Poisson's ratio ν , are given by [15]

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = E/(1 - \nu^2) \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1/2(1-\nu) \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \tau_{xy} \end{bmatrix} \quad (3 - 20)$$

In matrix notation,

$$\sigma_{xy} = [D_f] \{\epsilon_{xy}\}$$

where $[D_f]$ is the elastic coefficient matrix of the faces.

Substituting for $\{\sigma_{xy}\}$ from equation (3 - 19) gives,

$$\{\sigma_{xy}\} = [D_f][H][B][A]^{-1} \{\Delta_f\} \quad (3 - 21)$$

where $[D_f]$, $[H]$, $[B]$ and $[A]^{-1}$ are as given previously.

c) Element properties and necessary transformation

The element stiffness matrix, $[K_f]$ may now be computed by applying the principle of virtual work, the simplest procedure is to impose an arbitrary(virtual) nodal displacement $\{\bar{\Delta}_f\}$ and to equate the external and internal work done by the various forces and stresses during that displacement.

The work done by the nodal forces is equal to the sum of the products of the individual force components and corresponding displacements, i.e. in matrix language,

$$\{\bar{\Delta}_f\}^T \{S\} \quad (3 - 22)$$

where $\{S\}$ is the nodal forces.

Similarly, the internal work per unit volume done by the stresses can be obtained from equations (3 - 19) and (3 - 21),

$$\{\epsilon_{xy}\} \{\sigma_{xy}\} dx dy dz = \{\bar{\Delta}_f\}^T [A^{-1}]^T [B]^T [H]^T [D_f][H] [B][A^{-1}] dx dy dz \{\Delta_f\}$$

Equating the external work with the total internal work obtained by integrating over the volume of the element we have,

$$\{\bar{\Delta}_f\}^T \{S\} = \{\bar{\Delta}_f\}^T [A^{-1}]^T \iiint [B]^T [H]^T [D_f][H][B] dx dy dz [A^{-1}] \{\Delta_f\} \quad (3 - 24)$$

The virtual displacements $\{\bar{\Delta}_f\}$ are quite arbitrary and in particular may be taken to be represented by a unit matrix, thus equation (3 - 24) can be written,

$$\{S\} = [A^{-1}]^T \iiint [B]^T [H]^T [D_f][H][B] dx dy dz [A^{-1}] \{\Delta_f\} \quad (3 - 25)$$

which is the force-displacement relationship for the element, thus the element stiffness matrix becomes,

$$K_f = [A^{-1}]^T \iiint [B]^T [H]^T [D_f][H][B] dx dy dz [A^{-1}] \quad (3 - 26)$$

The stiffness matrix involves integrals over the volume of the element, the integration indicated in equation (3 - 26) can not be carried out directly since the matrix $[B]$ is in terms of ξ, η . A Jacobian [16] is used to transform the integration to the variables ξ, η .

The Jacobian is a determinant defined as,

$$J = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} \quad (3 - 27)$$

which, from equation (3 - 12) is equal to,

$$J = \begin{vmatrix} 1 & \cos\phi \\ 0 & \sin\phi \end{vmatrix} = \sin\phi \quad (3 - 28)$$

If taking t_s as constant within the element, the stiffness matrix becomes,

$$[K_f] = \sin\phi [A^{-1}]^T \int_0^a \int_0^b t_s [B]^T [H]^T [D_f][H][B] d\xi d\eta [A^{-1}] \quad (3 - 29)$$

which can be further reduced to,

$$[K_f] = \sin\phi [A^{-1}]^T [R_f] [A^{-1}] \quad (3 - 30)$$

where $[R_f]$ is the generalised stiffness matrix.

Performing the necessary integration and matrix multiplications we obtain the generalised stiffness matrix, which is completely defined in the Appendix I.

3.5 Formation of the stiffness matrix for the core element

The sandwich core which separates the skins is considered to be relatively thick, and it has been assumed that,

(1) The core is incompressible in the transverse direction.

(2) The transverse shear stresses is constant throughout the depth of the core.

(3) The displacement u and v at any given point on the core vary linearly across the depth.

Based on these assumptions, two elements are developed as follows :

(a) Element with three degrees of freedom per node

The subelement which is used for the core of the sandwich plate has essentially a tri-linear displacement assumption, the u and v displacement are assumed to be symmetric about the midplane and the w displacement is assumed to vary only with x and y , thus only 4 nodes are needed, Fig.(3a) shows the nodal arrangement for this subelement.

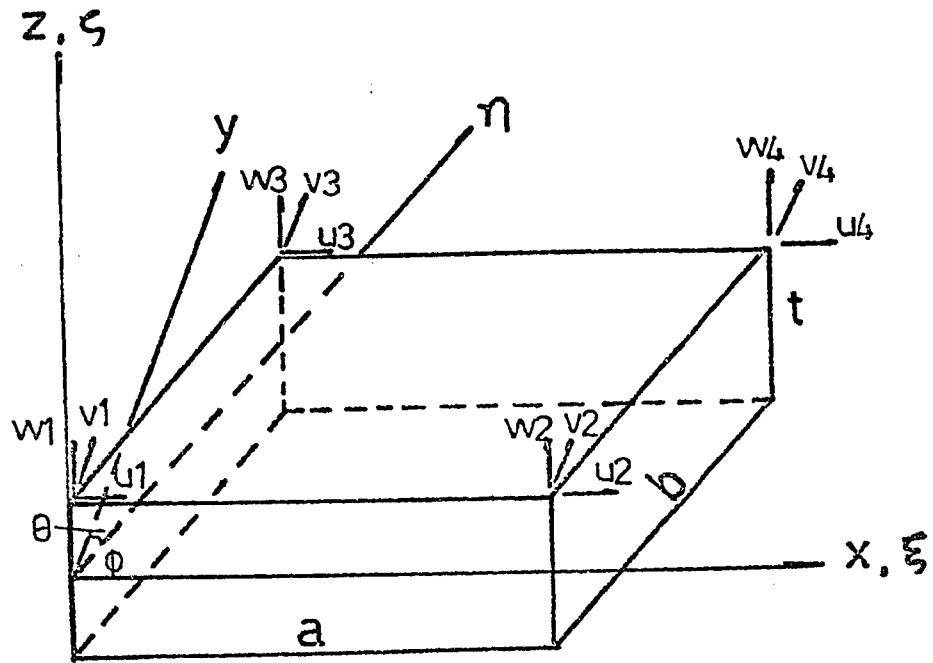


Fig. 3a

The displacement functions are taken to be,

$$u = (\alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta) \zeta$$

$$v = (\alpha_5 + \alpha_6 \xi + \alpha_7 \eta + \alpha_8 \xi \eta) \zeta \quad (3 - 31)$$

$$w = \alpha_9 + \alpha_{10} \xi + \alpha_{11} \eta + \alpha_{12} \xi \eta$$

Thus the nodal displacements may be represented

by,

$$\{\Delta_c\}^T = \{u_1 \ v_1 \ w_1 \ u_2 \ v_2 \ w_2 \ u_3 \ v_3 \ w_3 \ u_4 \ v_4 \ w_4\} \quad (3 - 32)$$

Following the scheme used in section 3-4, for the plane stress element, the nodal displacements $\{\Delta\}$ in terms of the displacement parameters $\{\alpha\}$ can be expressed as :

$$\begin{bmatrix} u_1 & v_1 & w_1 & u_2 & v_2 & w_2 & u_3 & v_3 & w_3 & u_4 & v_4 & w_4 \end{bmatrix}^T = [A_C] \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \alpha_9 & \alpha_{10} & \alpha_{11} & \alpha_{12} \end{bmatrix}^T$$

(3 - 33)

where,

$$[A_C] = \begin{bmatrix} \frac{t}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{t}{2} & \frac{at}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t}{2} & \frac{at}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & a & 0 & 0 \\ \frac{t}{2} & 0 & \frac{bt}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t}{2} & 0 & \frac{bt}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & b & 0 \\ \frac{t}{2} & \frac{at}{2} & \frac{bt}{2} & \frac{abt}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t}{2} & \frac{at}{2} & \frac{bt}{2} & \frac{abt}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & a & b & ab \end{bmatrix}$$

The strains in the element are

$$\left\{ \epsilon_{xyz} \right\} = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{bmatrix} \quad (3 - 34)$$

As shown in Fig. (3a), the rectangular x, y, z , axes and the orthogonal system ξ, η, ζ axes are related as :

$$x = \xi + \eta \cos \phi$$

$$y = \eta \sin \phi$$

$$z = \zeta \quad (3 - 35)$$

or

$$\xi = x - y \cot \phi$$

$$\eta = y \operatorname{cosec} \phi$$

$$\zeta = z$$

Performing the necessary differentiations of equation (3 - 35). The strains in the element can be represented as :

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cot\phi & \csc\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\cot\phi & \csc\phi & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\cot\phi & \csc\phi & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \zeta} \\ \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \zeta} \end{bmatrix}$$

(3 - 36)

In matrix notation,

$$\{\epsilon_{xyz}\} = [H_c] \{\epsilon_{\xi\eta\zeta}\}$$

where $\{\epsilon_{\xi\eta\zeta}\}$ can be obtained by equation (3 - 31)

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \zeta} \\ \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \zeta} \end{bmatrix} = \begin{bmatrix} 0 & \zeta & 0 & \eta\zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \zeta & \xi\zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \xi & \eta & \xi\eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \zeta & 0 & \eta\zeta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \zeta & \xi\zeta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \xi & \eta & \xi\eta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \xi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \\ \alpha_9 \\ \alpha_{10} \\ \alpha_{11} \\ \alpha_{12} \end{bmatrix} \quad (3 - 37)$$

In matrix notation,

$$\{\epsilon_{\xi\eta\zeta}\} = [B_c] \{\alpha\}$$

The vector $\{\alpha\}$ can be obtained by rearranging equation (3 - 33)

$$\{\alpha_c\} = [A_c^{-1}] \{\Delta_c\} \quad (3 - 38)$$

where,

$$[A_C^{-1}] = \begin{bmatrix} \frac{2}{t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2}{at} & 0 & 0 & \frac{2}{at} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2}{bt} & 0 & 0 & 0 & 0 & 0 & \frac{2}{bt} & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{abt} & 0 & 0 & -\frac{2}{abt} & 0 & 0 & -\frac{2}{abt} & 0 & 0 & \frac{2}{abt} & 0 & 0 \\ 0 & \frac{2}{t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{at} & 0 & 0 & \frac{2}{at} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{bt} & 0 & 0 & 0 & 0 & 0 & \frac{2}{bt} & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{abt} & 0 & 0 & -\frac{2}{abt} & 0 & 0 & -\frac{2}{abt} & 0 & 0 & \frac{2}{abt} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{b} & 0 & 0 & 0 & 0 & 0 & \frac{1}{b} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{ab} & 0 & 0 & -\frac{1}{ab} & 0 & 0 & -\frac{1}{ab} & 0 & 0 & \frac{1}{ab} \end{bmatrix}$$

Substituting for $\{\epsilon_{xyz}\}$ from equation (3 - 36)

gives,

$$\{\epsilon_{xyz}\} = [H_C] [B_C] [A_C^{-1}] \{\Delta_C\} \quad (3 - 39)$$

where $[H_C]$, $[B_C]$, $[A_C^{-1}]$ are defined previously.

The elasticity matrix $[D_c]$ for the isotropic sandwich core relating the six stress components to the strain components can be defined as [15] .

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & & & & & \\ \frac{\nu}{1-\nu} & 1 & & & & \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & & & \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & & \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

Sym.

(3 - 40)

In matrix notation,

$$\{\sigma_{xyz}\} = [D_c] \{\epsilon_{xyz}\}$$

Applying the principle of virtual work for the core element a similar relationships as given by equation (3 - 24) can be obtained. The stiffness matrix for the core element becomes :

$$[K_C] = [A_C^{-1}]^T \iiint [B_C]^T [H_C]^T [D_C] [H_C] [B_C] dx dy dz [A_C^{-1}] \quad (3 - 41)$$

Since the matrix $[B_C]$ is in terms of ξ, η, ζ . A Jacobian is again used to transform the integration to the variables ξ, η, ζ . It is defined as :

$$dx dy dz = |J(\xi, \eta, \zeta)| d\xi d\eta d\zeta \quad (3 - 42)$$

where,

$$|J(\xi, \eta, \zeta)| = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta} \end{vmatrix}$$

From equation (3 - 35) which is equal to,

$$|J(\xi, \eta, \zeta)| = \sin \phi \quad (3 - 43)$$

Now, the stiffness matrix becomes,

$$[K_C] = 2 \sin \phi [A_C^{-1}]^T \int_0^{t/2} \int_0^b \int_0^a [B_C]^T [H_C]^T [D_C] [H_C] [B_C] d\xi d\eta d\zeta [A_C^{-1}] \quad (3 - 44)$$

which can be reduced to,

$$[K_C] = 2 \sin \phi [A_C^{-1}]^T [R_C] [A_C^{-1}] \quad (3 - 45)$$

where $[R_C]$ is the generalised stiffness matrix for the

sandwich core. The matrix $[R_c]$ is defined in the Appendix II.

(b). Element with five degrees of freedom per node

It has been shown by K.M. Ahmed [9] that the use of more terms in the transverse displacement will yield better results. Therefore an element having five degrees of freedom per node is developed here. The displacement vector for such an element has been taken to consist of the displacement as in equation (3 - 32), except that $\frac{\partial w}{\partial \xi}, \frac{\partial w}{\partial \eta}$ are taken as an additional nodal degrees of freedom. The assumed displacement field is,

$$\begin{aligned} u &= (\alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta) \\ v &= (\alpha_5 + \alpha_6 \xi + \alpha_7 \eta + \alpha_8 \xi \eta) \\ w &= (\alpha_9 + \alpha_{10} \xi + \alpha_{11} \eta + \alpha_{12} \xi^2 + \alpha_{13} \xi \eta \\ &\quad + \alpha_{14} \eta^2 + \alpha_{15} \xi^3 + \alpha_{16} \xi^2 \eta + \alpha_{17} \xi \eta^2 \\ &\quad + \alpha_{18} \eta^3 + \alpha_{19} \xi^3 \eta + \alpha_{20} \xi \eta^3) \end{aligned}$$

(3 - 46)

Using these functions the required matrices appearing in equation (3 - 45) are derived in the Appendix III.

3.6. Formation of the stiffness matrix for the parallelogram sandwich element.

The sandwich element was formed by combining the stiffness matrices for two subelements representing the skins and a subelement representing the core. The condition for assembly of skin and core element is that they must have equal number of degrees of freedom at their common nodes. However, the skin element developed has two degrees of freedom per node. So in order to make it compatible for assembly the 8×8 stiffness matrix for the skin element is expanded to a 12×12 and 20×20 stiffness matrix for the three and the five degrees of freedom elements respectively. The expanded stiffness matrix will include zero in the corresponding rows and columns for degrees of freedom not considered in the skin element.

In order to combine these stiffness matrices, we have to redefine the combined nodal forces and displacements. For the three degrees of freedom element, the equilibrium equations which relate the element nodal forces to the subelement nodal forces at node 1, may be written as :

$$f_{xl} = 2f_{xls} + f_{xlc}$$

$$f_{yl} = 2f_{yls} + f_{ylc} \quad (3 - 47)$$

$$f_{zl} = f_{zlc}$$

In matrix form,

$$\begin{bmatrix} f_{xl} \\ f_{yl} \\ f_{zl} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2f_{xls} \\ 2f_{yls} \\ f_{xlc} \\ f_{ylc} \\ f_{zlc} \end{bmatrix} \quad (3 - 48)$$

or

$$\{f_l\} = [L_l]^T \{f_{lsc}\}$$

As mentioned earlier the bond between facing and core is assumed perfect the compatibility equations which relate the element nodal displacement to sub-element at node l may be written as :

$$\begin{bmatrix} u_{ls} \\ v_{ls} \\ u_{lc} \\ v_{lc} \\ w_{lc} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_l \\ v_l \\ w_l \end{bmatrix} \quad (3 - 49)$$

or

$$\{d_{lsc}\} = [L_l] \{d_l\}$$

where the subscript ℓ refers to node ℓ , the subscripts s and c refer to the skin and the core subelements respectively.

Using matrices of the type $[L_\ell]$ the similarity transform matrix $[L]$ used for forming the element stiffness matrix may be constructed as,

$$[K]_{(12 \times 12)} = [L]_{(12 \times 20)}^T \begin{bmatrix} 2K_s_{(8 \times 8)} & \vdots & 0_{(8 \times 12)} \\ \cdots & \cdots & \cdots \\ 0^T_{(12 \times 8)} & \vdots & K_c_{(12 \times 12)} \end{bmatrix} [L]_{(20 \times 12)} \quad (3 - 50)$$

where $[K]$ represents the stiffness matrix for the sandwich element (three degrees of freedom/node), $[K_s]$ and $[K_c]$ represent the stiffness matrices of the skin and core respectively.

The transform matrix $[L]$ for three degrees of freedom per node is given here as :

$$[L]^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3 - 51)$$

The formation of the stiffness matrix for the second element(five degrees of freedom per node) follows much the same procedure as for the first element(three degrees of freedom per node). The transform matrix $[L]$ for the element with five degrees of freedom per node is given in Appendix IV.

3.7. Formation of the stiffness matrix for the structure.

The dominant reason why the finite element method has been accepted so quickly and applied so extensively in engineering practice is its complete generality, a

single basic computer program may be employed in the analysis of structural systems ranging from simple plane trusses to elastic solids of arbitrary geometry. The only change necessary will be the substitution of some of the appropriate element subroutines, because the procedure of formation of the stiffness matrix is general for any type of element and structure. Some basic procedure will be presented here.

If the forces acting at the j^{th} node of the i^{th} element are $\{f_j^i\}$ and the displacements at the same node are $\{d_j^i\}$ then for an element with four nodes j, k, ℓ and m ,

$$\{f^i\} = \begin{Bmatrix} f_j^i & f_k^i & f_\ell^i & f_m^i \end{Bmatrix} \quad (3 - 52)$$

and

$$\{d^i\} = \begin{Bmatrix} d_j^i & d_k^i & d_\ell^i & d_m^i \end{Bmatrix} \quad (3 - 53)$$

The stiffness matrix may be written as :

$$f_j^i = \sum_{n=1}^N K_{jn}^i d_n^i \quad (3 - 54)$$

where K_{jm}^i is the matrix which represents the forces at node j due to unit deformations at node m .

Now considering the entire structure with M nodes and L elements. It is seen that for the j^{th} node, the vector of nodal displacements in global co-ordinates,

r is related to the applied nodal load R_j by the stiffness matrix $\bar{K}$ as :

$$R_j = \sum_{n=1}^M \bar{K}_{jn} r_n \quad (3 - 55)$$

Equilibrium of the j^{th} node requires that the summation of the element nodal loads for the element nodes which coincide with node j of the structure be equal to the load applied at the j^{th} node or

$$R_j = \sum_{\ell=1}^L f_j^{\ell} \quad (3 - 56)$$

or

$$R_j = \sum_{\ell=1}^L \sum_{n=1}^M K_{jn}^{\ell} d_n^{\ell} \quad (3 - 57)$$

As the displacements of node j are common to adjacent elements, therefore,

$$r_n = d_n^{\ell} \quad (3 - 58)$$

Thus

$$R_j = \sum_{\ell=1}^L \sum_{n=1}^M K_{jn}^{\ell} r_n \quad (3 - 59)$$

Comparing this expression with equation (3 - 55), the following relationships can be obtained,

$$\bar{K}_{jn} = \sum_{\ell=1}^L K_{jn}^{\ell} \quad (3 - 60)$$

The stiffness matrix $\bar{K}$ for the complete structure are built up simply by adding the stiffness of the

elements adjacent to the node in question, using equation (3 - 60).

3.8. Details for joining skin element to core element.

In most theories of sandwich plate analysis two assumptions are made which affect the distribution of stresses. The first assumption is that the transverse shear stresses of the face plates are negligible. The second is that the flexural rigidity of the sandwich is due entirely to the coupled plane stress rigidities of the skins. As a results of these assumptions, the stress condition of the core and faces would be violated. In order to improve the representation of these sandwich elements, some modifications are necessary. As can be seen later, each modification will get a physical meaning in the core-face bond.

Five possible representation of the core-face bond are discussed here. A numerical study of these five cases will be made in Chapter IV.

(1) Skin subelement attached to the nodes of the unmodified core subelement : The shear stress condition at face-core bond would be violated in this case.

(2) Skin subelement attached to the nodes of the core subelement which is modified by increasing the thickness : In this case, the shear stress at face-core

bond is improved, but the sandwich plate becomes stiffer, because it can carry more shear stress by the thickened core and the shear deflection will also be reduced.

(3) Skin subelement attached to the nodes which are extensions of the core subelement : The shear stress will be carried by the original core. It does not provide continuity through the thickness of the sandwich plate. The flexural rigidity and the shear rigidity of the sandwich plate will be considered separately.

(4) Skin subelement attached to the nodes of the thickened core subelement whose shear modulus has been reduced : This would improve the results of case 2. J. A. Maple [12] has suggested that the shear modulus of core should be modified for the thickened core. In other words, the shear rigidity will be suitably adjusted.

(5) This case is similar to case 4, K. M. Ahmad [9] has suggested a different modification for the shear modulus, he assumed that the shear stress in the core will vary parabolically.

The different values of shear moduli and flexural rigidities are presented in table (1). Comparison of the above modifications with an "exact" solution has been made for two different boundary conditions of the sandwich plate(as presented in table (6)). The deflection at the

centre of the plate for case (4) is found to be the closest to the "exact" solution and hence it can be concluded that of the five approaches cited above this arrangement yields the best results.

3.9. The consistent load vector.

The consistent load vector is obtained by calculating the virtual work done by the applied loads $q_u(\xi, \eta)$, $q_v(\xi, \eta)$ and $q_w(\xi, \eta)$ in the u , v and w directions respectively.

The present analysis is made only for a lateral uniformly distributed load of intensity q_0 and with no loads in the u and v directions; hence the work done in these directions is zero.

For a set of virtual nodal displacements $\{\bar{\Delta}\}$, the work done by the equivalent nodal forces is,

$$W_u = \{\bar{\Delta}\}^T [S_0] \quad (3 - 61)$$

The work done by the uniform load is,

$$\begin{aligned} W_u &= \iint q_0 W^T dx dy \\ &= \sin \phi q_0 \int_0^b \int_0^a W(\xi, \eta)^T d\xi d\eta \quad (3 - 62) \end{aligned}$$

Equating work done by the equivalent nodal forces to that done by the uniform load, we get

$$[S_o] = \sin\phi q_o \int_0^b \int_0^a [A^{-1}]^T [L]^T d\xi d\eta \quad (3 - 63)$$

where $[A^{-1}]^T$ and $[L]^T$ will be defined in Appendix V.

Performing the necessary integration and matrix multiplications for equation (3 - 63), the load vector will be obtained as follows :

Elements with three degrees of freedom per node,

$$[S_o] = \sin\phi q_o ab \left\{ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\}^T \quad (3 - 64)$$

Elements with five degrees of freedom per node,

$$[S_o] = \sin\phi q_o ab \left\{ \frac{1}{4}, \frac{a}{24}, \frac{b}{24}, \frac{1}{4}, -\frac{a}{24}, \frac{b}{24}, \frac{1}{4}, \frac{a}{24}, -\frac{b}{24}, \frac{1}{4}, -\frac{a}{24}, -\frac{b}{24} \right\}^T \quad (3 - 65)$$

It is seen that in the first element, the total distributed loading on the element is lumped into concentrated lateral forces acting at the nodal points, but in the second element, the distributed loading is represented by concentrated lateral forces as well as concentrated couples at the nodal points. In other words, the load representation in the second element is much better than the first one. It is one of the factors which will influence the accuracy of a finite element approximation.

TABLE 1
MODIFICATIONS OF FLEXURAL AND SHEARING RIGIDITIES FOR SANDWICH ELEMENT

Case For	Flexural Rigidity D_s	Shearing Rigidity S_c
(1)	$E_s t_s^2 / 2(1-\nu_s^2)$	$G_c t_c$
(2)	$E_s t_s (t_c + t_s)^2 / 2(1-\nu_s^2)$	$G_c (t_c + t_s)$
(3)	$E_s t_s (t_c + t_s)^2 / 2(1-\nu_s^2)$	$G_c t_c$ ^{xx}
(4)	$E_s t_s (t_c + t_s)^2 / 2(1-\nu_s^2)$	$G_c' (t_c + t_s)$ ^{xx}
(5)	$E_s t_s (t_s + t_c)^2 / 2(1-\nu_s^2)$	$G_c'' t_c$

$$G_c' = G_c (t_c / t_c + t_s)$$

$$G_c'' = G_c \quad 5/6$$

xx

Although the flexural rigidity D_s and the shearing rigidity S_c appear to be the same for both cases 3 and 4, it should be noted that the stiffness matrix depends not only on the flexural and shearing rigidities but as a function of the thickness resisting shear deformation. Since in case 3 the thickness assumed capable of resisting shear deformation is t_c and the corresponding thickness assumed capable of resisting shear stresses is $(t_c + t_s)$ in case 4, therefore it is reasonable to expect that results obtained from the two cases are different as shown in Table 6.

3.10. Rotation of co-ordinates

The boundary conditions for skewed plates would be applied in two different ways. The first way is to express the boundary conditions in terms of derivatives of the global co-ordinate system. This is done by the use of Lagrangian Multiplier which are unknowns attached to the additional equations. These additional equations, called constraint equations for all points lying on the boundary can be solved by the minimization of potential energy subject to the constraints. The disadvantage of this approach is that the stiffness matrix will no longer be banded after we provide the constraint equations to the system, so that the whole stiffness matrix has to be stored in the computer memory.

An alternative procedure is to rotate the co-ordinate system for those points lying on the skewed boundaries so that the generalized displacements become normal and tangential components to the skew boundary. The boundary condition can then be applied directly by eliminating those displacements from the equation of the problem.

Referring again to Fig. (3a), it can be seen that the component of displacement parallel to edge is,

$$v_p = u \cos \phi + v \sin \phi \quad (3 - 66)$$

The component normal to edge is

$$u_p = u \sin \phi - v \cos \phi \quad (3 - 67)$$

The transformation matrix relating the rectangular co-ordinate system and orthogonal co-ordinate system is of the form,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin \phi & \cos \phi & 0 \\ -\cos \phi & \sin \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ w_p \end{bmatrix} \quad (3 - 68a)$$

for three degrees of freedom element.

and,

$$\begin{bmatrix} u \\ v \\ w \\ \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \sin \phi & \cos \phi & 0 & 0 & 0 \\ -\cos \phi & \sin \phi & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \sin \phi & \cos \phi \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_p \\ v_p \\ w \\ \frac{\partial w}{\partial p} \\ \frac{\partial w}{\partial \eta} \end{bmatrix}$$

(3 - 68b)

for five degrees of freedom element.

or in general terms,

$$\{d\} = [Q] \{\tilde{d}\} \quad (3 - 69)$$

A similar expression may be written for the forces at the nodes of the boundaries in the form,

$$\{f\} = [Q] \{\tilde{f}\} \quad (3 - 70)$$

where $\{\tilde{f}\}$ and $\{\tilde{d}\}$ are the generalized forces and displacements respectively in an orthogonal co-ordinate system.

The general stiffness equation for structural systems is given as,

$$\{f\} = [K] \{d\} \quad (3 - 71)$$

substituting from equations (3 - 69) and (3 - 70)

$$[Q] \{\tilde{f}\} = [K] [Q] \{\tilde{d}\}$$

or
$$\{\tilde{f}\} = [Q]^{-1} [K] [Q] \{\tilde{d}\} \quad (3 - 72)$$

The stiffness matrix in orthogonal co-ordinate system become,

$$[\tilde{K}] = [Q]^T [K] [Q] \quad (3 - 73)$$

It should be noted that because of the particular properties of the angular transformation matrix $[Q]$, the inverse $[Q]^{-1}$ is also equal to the transpose $[Q]^T$. This property allows the general stiffness matrix of equation (3 - 73) to be obtained without the necessity of inverting matrix $[Q]$.

The advantages of this approach is that the

transformation matrix for the complete structure need not be obtained. The co-ordinate system for those points lying on the skew boundary are rotated before the elements are assembled, so that the stiffness matrix for the complete structure retains its banded form, and the computer storage requirement are considerably reduced.

3.11. Boundary conditions.

As mentioned earlier, the assumed displacement function must represent rigid body motion. Clearly, it is impossible to solve this system without substitution of a minimum number of prescribed displacements to prevent rigid body movements of the structure, because the displacements cannot be uniquely determined by the forces in such a situation. The application of boundary conditions for skew sandwich plates considering it as a displacement model is slightly different from those derived for the classical plate theory. It is necessary to satisfy the kinematic boundary conditions only. This is by virtue of the principle of minimum potential energy. For example if at a particular point the force is known to be zero then the corresponding displacement must be free. This is the guiding principle in the derivation of boundary conditions in the finite

element method.

For the most common type of clamped edge support assumed in classical plate theory [4] , the boundary conditions in terms of deflections, can be expressed in the partial deflections W_b and W_s . If the bending stiffness of the faces themselves is neglected, the first derivative of W_s will no longer vanish at the edges. It is assumed that at the edge, the face may undergo a discontinuity of its slope. With regard to the finite element analysis presented here, it should be observed that boundary conditions never apply to the partial deflections themselves but only to their sum W_T ($W_T = W_b + W_s$) or to their derivatives. Therefore, the first derivative of W_T may or may not be zero at the edges. These two different assumptions will be investigated in present work. The first assumption will be referred to as "The fully clamped edge" while the second as "The reduced clamped edge". In reality the stress condition at and near the clamped edge is rather complicated because of the local effect, and the actual condition will fall between these two bounds.

Various types of edge support commonly assumed in sandwich construction will be mentioned here, and the

corresponding displacement boundary conditions are also given :

(a) The boundary conditions for the clamped sandwich plate are :

(i) no deflection normal to the plate, i.e. parallel to the z-axis;

(ii) no displacement of the face plates along the lines of support;

(iii) no displacement of the face plates normal the edge.

The imposed displacement boundary conditions are :

(1) For fully clamped edge

$$\bar{w} = u_p = v_p = w_\xi = w_\eta = 0 \quad \text{along } \xi \text{ direction}$$

$$w = u_p = v_p = w_\xi = w_\eta = 0 \quad \text{along } \eta \text{ direction}$$

(2) For reduced clamped edge

$$w = u_p = v_p = w_\xi = 0 \quad \text{along } \xi \text{ direction}$$

$$w = u_p = v_p = w_\eta = 0 \quad \text{along } \eta \text{ direction}$$

(b) The boundary conditions for a simply-supported panel are :

(i) no deflection normal to the plate;

(ii) no bending moments about the lines of support;

(iii) no displacement of the face plates along the lines of support.

The imposed displacement boundary conditions are :

$$w = u_p = w_\xi = 0 \quad \text{along } \xi \text{ direction}$$

$$w = v_p = w_\eta = 0 \quad \text{along } \eta \text{ direction}$$

(c) The boundary conditions for the free edge are :

- (i) no bending moment about the edge;
- (ii) no twisting moment along the edge;
- (iii) no shearing force at the edge.

This means that no displacement boundary conditions are prescribed along the edges.

3.12. Solution of equilibrium equation.

One of the important aspects of the finite element method is the choice of a suitable method for solving the simultaneous equations which for practical problems may contain several hundred equations. The choice of technique depends upon the size of problem envisaged and upon the type of computer available.

There are three distinct steps in the solution of the stiffness system of equations. First, the coefficients in the stiffness matrix are evaluated for each element and some technique is used to store the structure stiffness matrix in the computer. Second,

the known displacement boundary conditions are imposed. Finally, the equations are solved for the unknown displacements by any one of several methods. These three steps are illustrated in table (2).

Storage of an entire matrix requires N^2 spaces, where N is the number of equations. Thus, even with the very large computers available today the user is limited to solving approximately two hundred equations without using the tape or disk storage unit [17] .

Fortunately the matrices encountered in structural engineering problems have some special characteristic which would allow programing to effect an efficient and economic usage of storage. For the problems examined herein, the stiffness matrix is symmetrical and banded. Advantage is taken of this fact and only the coefficients in the upper or lower band-width need be stored in either a rectangular or column array. These methods of storage lead not only to a saving of space in the computer, but also to a reduction in the time required for solutions as many multiplications, additions, and subtractions of zero quantities were eliminated.

The bandwidth of the stiffness matrix for the structure is influenced by the number of degrees

of freedom of each element, and by the nodal point numbering on the model [20]. It is, of course, advantageous to keep the band as narrow as possible. This is obtained by a numbering system whereby the differences between nodal point numbers of adjacent points are kept as low as possible.

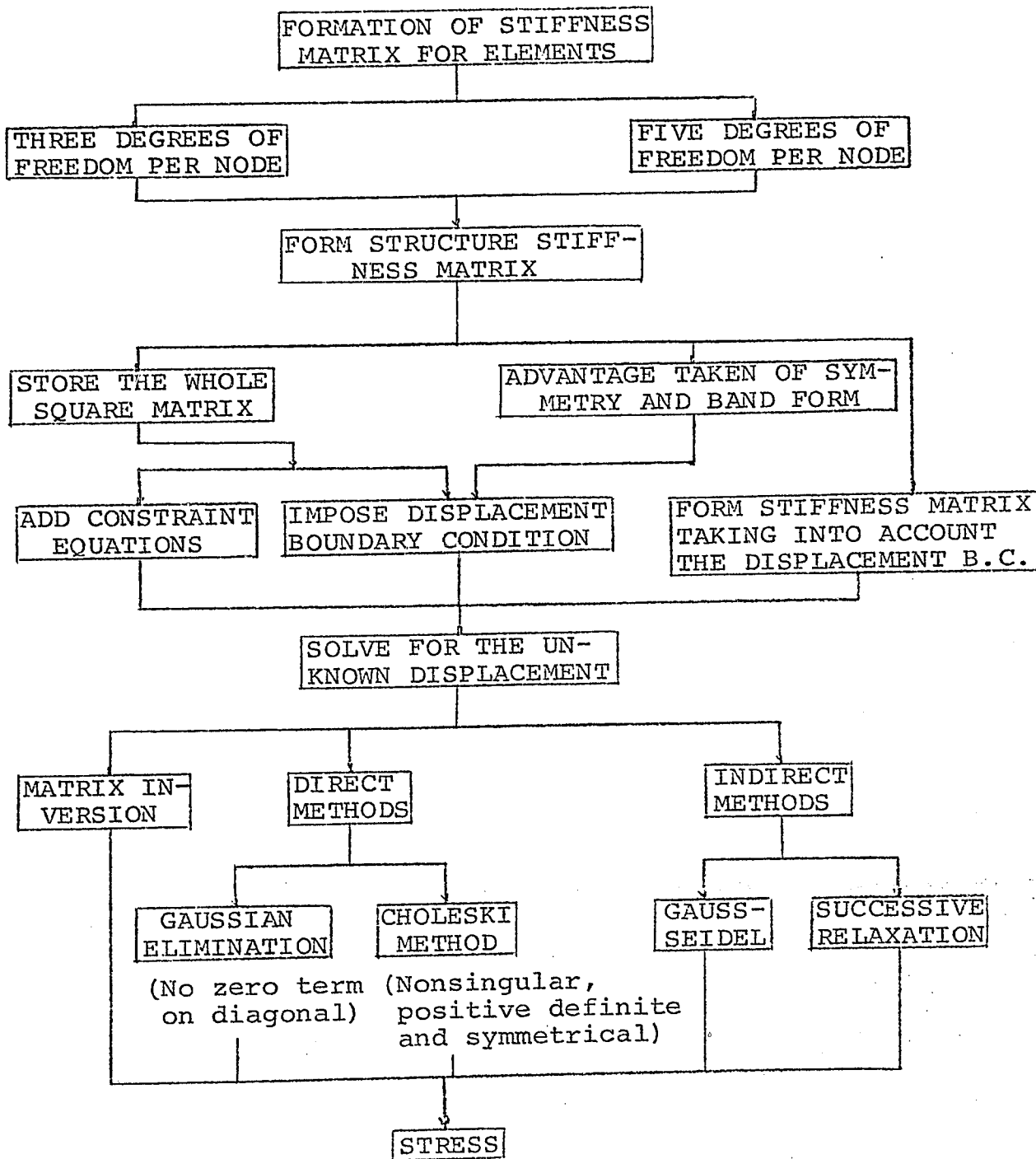
In forming the structure stiffness matrix two different methods are commonly used. In one, the matrix is partitioned and only the terms corresponding to the actual degrees of freedom are used to solve for the unknown displacement. That is, the terms related to the known restraints are not included in the stiffness matrix. The number of equations to be solved is equal to the total number of degrees of freedom minus the number of imposed restraints.

The other method forms the structure stiffness matrix without regard to the known boundary conditions. The boundary conditions are imposed in such way when a particular displacement is known the corresponding row and column is zeroed and unity is placed in the leading diagonal. The corresponding term in load vector is suitably modified to account for known displacement. In this case, the number of equations to be solved is equal to the total number of degrees of

freedom of structure. However, the number of restraints in many problems such as plates and shells is small in comparison to the total number of degrees of freedom. Thus only a few extra equations are manipulated. Also, it can avoid reorganization of computer storage and the band size is not affected.

The final step in the solution of the system of equations is to find unknown displacements, many standard routines are now available to solve large systems of equations. Two kinds of methods of solution are frequently used for the solution of the simultaneous equations : (a) Indirect method, such as Gauss-Seidel method, Successive relaxation method, in which a successive approximation technique is used to converge on the true solution, (b) Direct methods, such as Gaussian elimination method, Cholesky method, in which an exact solution is sought. As mentioned in References [18] and [19] , no method has been found that is faster and more accurate than Gaussian elimination for system in which the matrix of coefficients is dense. The same conclusion will hold for a dense system banded along the main diagonal [17] . Most of the problems investigated in this thesis were solved by Gaussian elimination. Several of these same problems were also solved using

the Cholesky method, and the results of both method
were identical to five or more places of accuracy.



FORMATION AND SOLUTION OF STIFFNESS MATRIX

TABLE 2

CHAPTER IV

NUMERICAL RESULTS AND DISCUSSIONS

4.1 Introduction

The principle objective of this study is to develop two elements for the analysis of skew and rectangular sandwich plates under various boundary conditions. The aim of the numerical results presented herein is to provide information which may lead to better understanding of the static behavior of sandwich construction. The advantages and disadvantages of the two elements are also discussed based on the numerical results obtained.

The parameters that were varied in this study include skin thickness, shear stiffness, flexural rigidity, aspect ratio, Poisson's ratio and skew angles. The usefulness of this type of investigation is to indicate which parameters are important and hence a reasonable assumption can be deduced.

The intention of the present work is not to provide a complete record of the variation of deflection and stresses of sandwich plates due to the large number of parameters. The solutions presented are of

limited number and the conclusions drawn therefrom are generally of qualitative nature. A more complete study of the effects of the various parameters involved would require a greater number of results than are included here.

Most of the result obtained in this investigation for the behavior of sandwich plates make use of the first element, (i.e. a skew element with three degrees of freedom per node). The reason is that this less precise element seems to produce results which are fairly comparable with those obtained by the more precise element. Furthermore the more precise element takes relatively much more computer storage and time. From the economical point of view, the first element appears to be more useful for all practical engineering applications.

Many approaches for modifications of the core-face bond were presented in the previous chapter. Accordingly all computations were carried out following these approaches as shown in cases 1, 2, 3, 4 and 5.(Table 6) However case 4 following Maple's modification [12] was found to be the closest, and so it was used to present the results hereafter. It should also be noted that all results presented here were computed using a

12 by 12 grid.

4.2 Convergence and accuracy

The evaluation of a particular finite element must be based on consideration of all the factors that influence the results obtained with that element. The two main aspects to be considered are the accuracy of the final answers and the time and labour involved in obtaining the results. It is important to note that the basic need is to obtain answers accurate to as many significant figures as may be required in the applications of the results. Based on the previous work on finite element study [26], the error in the results obtained using a finite element approximation may be conveniently separated into two types, the discretization error and the rounding off error. The rounding off error is the error associated with the accuracy with which the numbers are manipulated in the computations. The discretization error which occurs irrespective of the accuracy of the numerical calculations, is a consequence of approximating a continuum which has an infinite number of degrees of freedom with a model having a finite number of degrees of freedom.

The discretization error may approach zero as a limit when the element size tends to zero. In other words the approximate solution is said to converge to the exact solution. The convergence proof is based on the theorem of minimum potential energy. For further details see ref. [30].

In this section, the deflection of the centre of the plates is used as a measure of the quality of the approximation for each type of element. Table (7) through (9) present the results of the convergence study with various boundary conditions. It can be observed that the convergence of the solution in each case is quite satisfactory. The maximum deviation is only 4% with a 200 percent increase in the number of elements. The only exceptional case is using the first element (three degrees of freedom per node) to analyse very hard core sandwich material which is out of the limit of the traditional sandwich material specifications. This will be discussed in more details in section 4.4.

For the case of skew shape sandwich plate analysis, the rate of convergence is still quite fast for both elements. Results given in Table (5) shows

the convergence character of these two element with various skew angles. It has been found that the results change very little with even a 150 percent increase in the number of elements. For high skew angles the convergence of the solutions are as good as low skew angles for both elements.

4.3 Comparisons of results

The purpose of these comparisons is twofold. The first purpose is to investigate the accuracy of the finite elements used in this work when it is applied to skew or rectangular shape sandwich plates, and the second purpose is to illustrate the relative merits in using different elements for different boundary conditions. Establishing the accuracy of this method is sometimes difficult, because no exact solutions exist for sandwich plates with skew or other irregular plate boundaries.

(a) Rectangular Sandwich Plates

In order to assess the accuracy of this method some values are obtained for rectangular sandwich plate with three different boundary conditions, clamped, simply supported and cantilevered. Table (7) shows the central deflection of a clamped sandwich plate subjected to a

uniform load. Two sets of data are discussed, one is for soft core material (Problem 1) and the other is for hard core material (Problem 2). Results obtained in first problem show good agreement with Taylor [22], but it is not so close with results obtained by March [24] with a maximum deviation of up to 11%, since his results are based upon crude approximations [2].

For the second problem (hard core sandwich material) the results obtained by the second element (five degrees of freedom per node) show close agreement with a difference of only 2.5% when compared with existing values [25] and [10]. But the agreement is not close with the first element (three degrees of freedom per node). This discrepancy can be explained by the fact that the first element which use less terms in the lateral displacement function does not achieve a reasonable representation of the shear properties of the core.

In the case of simply supported rectangular sandwich plates under uniform loads, the results from Yen [23] and Plantema [2] are compared to those computed by the present two types of elements in Table (8). It shows an excellent agreement between the present results and the existing values. The maximum discrepancy in these comparisons does not exceed 1%. Table (9) shows

the maximum deflection of a cantilevered sandwich plate subjected to a concentrated load at the free end. The existing values are solved by the sandwich beam equation of Plantema [2] as well as the sandwich plate equation of Reissner [21]. The maximum discrepancy in these comparisons with the first and second element does not exceed 3% and 2% respectively.

(b) Skewed Sandwich Plates

Relatively little data exist on skewed sandwich plates. Therefore comparison can only be made to that of Kennedy [4] whose analysis is based on Fourier series for the deflection of plate and to that of Monforton⁺ who used the finite element analysis.

Fig. (4a) and Fig. (4b) gives a graphical comparison between the present results and other existing

⁺ Since carrying out this work, it has been found that the similar research work is being carried out by Monforton [29]. In his work, two finite element formulations are presented. In the first element (the L-H approach), the membrane displacements of the faces are represented in terms of bi-linear functions and the transverse displacement is represented in terms of bi-cubic functions. In the second element (the H-H approach), all displacements are represented in terms of bi-cubic functions. The writer has been able to obtain two sets of unpublished data by private communications. The comparison has made the present results more reliable.

values for the central deflection of a clamped skew sandwich plate under uniform loading. It shows that Monforton's results are in close agreement with results obtained by the second element (Five degrees of freedom per node with fully clamped edge) while Kennedy's results shows good agreement with the value obtained by first element (Three degrees of freedom per node) for soft core sandwich material. The average difference of the results obtained from the first element and the second element (first assumption and second assumption), when compared with Kennedy's are 2%, 7% and 2.5% respectively and with Monforton's results are 5%, 1.5% and 4.5% respectively. For hard core sandwich material, Monforton's results are still in close agreement with value obtained by the second element (with fully clamped edge) in the present investigation. The Maximum discrepancy in these values does not exceed 4%.

In order to provide further comparisons of results between the two finite element solutions, various properties of sandwich plate material were chosen for investigation. Table (3) and (4) gives the centre deflection and moment of a clamped sandwich plate with various skew angles, aspect ratios and core rigidities.

It is worth noting that finite element solutions are in general smaller than the exact values if such exact values can be determined. This can be explained by the fact that the minimum total potential energy principle will always underestimate the true strain energy, thus resulting in a stiffer model than the actual structure.

The examination of the data given in Table (3), (4) and (5), shows that the results for skew sandwich plate obtained by first element (three degrees of freedom per node) and second element (five degrees of freedom per node with reduced clamped edge) are closer to Kennedy's results than those obtained by using the second element (five degrees of freedom per node with fully clamped edge). This is opposite to that shown for the rectangular sandwich plates. A more accurate conclusion is that the behavior of the elements depend on how the boundary condition of clamped edge is defined. It is noted that there is a difference in boundary conditions proposed by Kennedy and the present study. Kennedy separated the total deflection into two parts as bending deflection (w_b) and shear deflection (w_s). In addition to forcing the total deflection ($w_t = w_b + w_s$) equal to zero, he also let the first derivative of w_b equal to zero along the boundaries.

These boundary conditions are similar to the boundary conditions we are applying in using the first element (three degrees of freedom per node) and second element (five degrees of freedom per node with reduced clamped edge). But in our second element (five degrees of freedom per node with fully clamped edge), we set the first derivative of w_T ($w_T = w_b + w_s$) to zero along the boundaries. This assumption is based on Monforton's definition of clamped edge support, which is different from previous proposal. This implies a more restrained clamped edge condition, and therefore yields a smaller central deflection value.

4.4 Effect of varying the shear rigidity of the core

In traditional sandwich plate analysis, the core is considered to be very soft as compared to that of the face layers and the core stresses parallel to the face are negligible. This assumption has been used in many previous analysis and are known to represent actual sandwich construction very well. Possible sources of error naturally arise from this assumption when the core material becomes rather stiff. It is worthy to ascertain the range of applicability of the present finite element analysis in terms of the ratio of core stiffness to the modulus of elasticity of the skin. In

other words, the limitation of the present sandwich elements analysis for various core and skin rigidities.

In order to investigate this problem, the influence of the shear rigidity of the core on the deflection and stresses of the sandwich plate has to be determined. Table (10) to (13) and Fig. (11) to (16) show the effect of varying shear rigidity of the core on the deflection and moment in various boundary conditions. From the results obtained in these tables and figures, it is observed that when the shear rigidity is less than 10000 psi. the shear flexibility of the core has significant effect in increasing the centre deflection. However, when the shear rigidity is greater than 10000 psi., the additional deflection due to shear is negligible. It means that when the rigidity of the core is much greater than 10000 psi. a general composite plate theory should be used. Data shown in Tables (12) and (13) and Fig. (13) to (16) give a clear picture of showing the influence of the shear rigidity on the moment, when the shear modulus is greater than 10000 psi. the skin resist less and less of the bending moment and the core is resisting more and more moment. This implies that the traditional assumption of

simplified sandwich plate theory "The bending is resisted entirely by the skin." is not valid after this limit. Fig. (18) and (19) are two typical plots of the maximum centre moment and edge moment respectively. These figures show that when shear rigidity of the core is greater than 10000 psi., the moment resisted by the skins are markedly reduced. It can be deduced that a composite plate theory analysis will get a more reasonable result when shear modulus of the core is appreciably greater than 10^4 psi..

The shear rigidity of the sandwich core does not only influence the accuracy of the results for deflection and moment, but also influences the convergence characteristics of the sandwich element. It is interesting to observe the effect which shear rigidity has upon the property of convergence. Some idea of the effects of shear rigidity on the rate of convergence can be obtained from tables (16) to (19). These elements exhibit better convergence characteristics in flexible cores than in stiffer ones. For example, from the central deflection values of a clamped sandwich plate with a skew angle of 45° degrees as given in Table (18), we see that for a shear modulus of 500 psi., the element yields a constant value in 8x8 grids. But it gives 2

percents difference from the "best" solution in the same mesh size when the shear modulus is increased to 5000 psi..

4.5 Effect of core thickness/skin thickness ratio

This method has been carried out based on the traditional sandwich plate theory. One of the assumptions of the traditional sandwich plate theory is that "The flexural rigidity of the faces about their own middle surface is negligible", i.e. the strain in the faces is constant across the depth of the face. Hence the transverse shear of the faces is equal to zero and all shear force are resisted by the core. Based on this assumption, we are using a two-dimensional plane stress element to represent the faces. This assumption is valid only when the thickness of the faces is small in comparison with the thickness of the core. Such two dimensional representation could lead to large errors when the thickness of the faces becomes large in comparison with the thickness of the core.

In order to determine the validity of the elements developed in the last chapter, the same elements are applied to the solution of a rectangular sandwich plate with simply supported boundaries. The variation of central deflection

and moment with the thickness of the skin are shown in Table (20). From this table, it can be seen that when the thickness of the skin is less than 0.05 in., the thickness of the skin has the effect of decreasing the deflection. However, when the thickness of the skin is greater than 0.05 in., the influence of the thickness of skin in reducing deflection is rather insignificant. This can be explained by the fact that if the facings are relatively thick, the shear load will be carried to a considerable extent by the facings, the additional deflection due to the shear of the sandwich core is negligible. Therefore, the conclusion is that plane stress element should not be used to represent the faces when they are relatively thick in comparison with the thickness of the core.

Fig. (17) is a typical plot of the deflection against the thickness, the ratio of thickness t_s/t_c is marked in the corresponding positions, the value suggested by ref. [9] is drawn on this figure. It seems that the limit suggested by ref. [9] is reasonable.

4.6 Effect of Poisson's ratio

It is interesting to observe the effect which Poisson's ratio has upon the moments or stresses. In Fig. (23) and (24) the Poisson's ratio^{*} was varied in

* In present study, the face and core materials have the same Poisson's ratios.

order to establish its effect on the centre and edge moment with different skew angles for clamped sandwich plates. Similar results are given in Fig. (25) for the simply supported sandwich plate. It has been found that the influence of Poisson's ratio on the central moment are the same for both boundary conditions. Fig. (23) and (24) show that moments of sandwich plates vary linearly with Poisson's ratio for the uniformly loading condition, so that the moments for any given Poisson's ratio can be evaluated directly.

From Fig. (23) it is observed that the variation of Poisson's ratio from 0. to 0.3 has got more or less the same effect on both moments M_x and M_y for low skew angles and when the skew angle increases, it has got more significant effect on M_x than on M_y . For example, when the skew angle is zero, the variation of Poisson's ratio in the above mentioned range increases both M_x and M_y to an extent of about 34% and when the skew angle is 60° , its effect on M_x is more significant than on M_y as can be seen from an increase in the value of M_x to an extent of more than 60% whereas the increase for M_y is only 18%. However, the effect of Poisson's ratio is not significant on the values of edge moments as can be seen for a skew angle of zero, the change in

edge moment is only 2% (Fig(24)).

Therefore, it can be concluded that Poisson's ratio is an important factor in stress analysis and without due consideration, could lead to severe under-estimation of the moments and stresses.

4.7 Effect of aspect ratio

Kennedy [4] studied the effect of the aspect ratio upon the moments for clamped skew sandwich panels, and found that both the maximum centre and edge moment decrease with increase in the aspect ratio. Same effects can also be found in other boundary conditions such as simply supported or cantilevered sandwich skewed plates. The results of deflection and moment with aspect ratio 1, 1.5 and 2.0 are presented for three boundary conditions in Fig. (20) to (22) and Table (14) to (15). It is found that they decrease with increase in aspect ratio in various boundary conditions. The central moment in x direction appears to decrease more rapidly than the moment in y direction in clamped and simply supported boundary conditions, but the effect of the aspect ratio upon the twisting moment is insignificant in both cases.

4.8 Effect of skew angles

Some idea of the effects of skew upon the

deflection can be obtained from this investigation. Fig. (5) to (7) show the manner in which the central deflection of the plate varies for different skew angles with the clamped boundary conditions. Fig. (8) to (10) show the corresponding effects for the simply supported skew sandwich plate.

Under a uniformly distributed load the central deflection decreases as the angle of skew increases. The rate of decrease being very nearly constant for high skew angles. For cores with a high shearing rigidity, the tendency for the centre deflection to decrease with an increase in the skew angle is greatly reduced.

The variation of plate moment with the angle of skew is shown in Fig. (13) and (14) for the clamped and simply supported sandwich skew plates respectively. From the figures, it can be observed that both the maximum centre moment and edge moment are considerably reduced with an increase in the angle of skew. Thus, the effect of skew upon the moment appears to be independent of the rigidity of the core.

The analysis of cantilever skew sandwich plates presents more difficulty than that of rectangular

sandwich cantilever plates due in part to the erratic behavior of the stresses between the fixed and trailing edges. For example, it is possible for unbounded stresses to occur at an obtuse corner [28]. A graded mesh was therefore suggested in calculating the deflection and stresses for skew shape cantilevered plates[28]. Although this could easily be accommodated it has not been attempted here. Table (10) and (11) show the effect of skew upon the deflection for a cantilevered sandwich plate subjected to a uniformly distributed load. It can be seen that the deflection at point 1 (see Fig. (26)) decrease and the deflection at point 2 increase up to 15° degrees skew and decrease again as the skew angle increases. This variation of corner deflections with skew angle is similar to the results obtained for a homogeneous cantilevered skew plate by Dawe [28].

For skew sandwich cantilevered plate under uniformly distributed loads, the variation of the maximum edge moment with various angles of skew and core rigidities are shown in Table (12) and (13). It can be seen from these tables that the maximum edge moment always occurs at the obtuse corner. It is

interesting to note that the value of the maximum edge moment increases to a maximum value for an angle of skew approaching 30° and then decreases again as the skew angle increases further.

4.9 Stress

The displacement models which are used in this investigation lead to continuous displacements in u , v , and w . Since the elements used in this investigation do not possess the property of continuous slope along the boundary, therefore discontinuities in moment value would also exist from one element boundary to another at a particular node point. It is reasonable to assume that the results will not give an accurate representation of the stresses at all points. From the data obtained in this investigation, it was found that the change in moment at the boundaries of the elements is insignificant for the rectangular shaped element. The change in moment for skewed shape element is larger and increases with sharper skew angles. It further implies that the present elements are more powerful to solve the rectangular shaped plate than the skew shaped plate, but further conclusive information cannot be drawn concerning this problem, due to the

absence of reliable analytical data.

In obtaining a value for the moment at a node point, Zienkiewicz [6] states that an average of the stress in adjacent element at internal nodes will closely approximate the true stresses. However this process will not lead to an exact representation of stresses on the boundaries. In this investigation, this averaging method was used to calculate the moment. It has been found that good agreement is obtained for the central moment when compared with existing results [4], but the maximum difference of edge moments is up to 30%. This further implies, that averaging for stress is not effective along boundaries to get good approximations. Referring to ref. [6] , a weighting of averages near the faces of the structure can further be used for refinement, but it seems preferable, simply to use a finer mesh subdivision at the boundaries.

TABLE 3

COMPARISON OF THE CENTRE DEFLECTION OF FOUR SIDES CLAMPED SANDWICH PLATE WITH
VARIOUS SKEW ANGLES, ASPECT RATIOS AND CORE RIGIDITIES

$E_f = 10^7$ psi.		$\nu = 0.32$	$C = 1"$	$a = 40"$	$t_g = 0.025"$			
$A_s = a/b =$ Aspect ratio		G_c	$G_c =$ Shear modulus of core		$q =$ Intensity of load			
θ	G_c	A_s	3 degrees of freedom/node (8x8) (12x12)		Kennedy [4] Monforton [29] [†] (16x16)			
			(a)	(b)				
15°	500.	1.0	0.2463"	0.2452"	0.2301"	0.2433"	0.2458"	0.2336"
		1.5	0.1442"	0.1435"	0.1346"	0.1425"	0.1437"	--
		2.0	0.0894"	0.0889"	0.0834"	0.0884"	0.0889"	--
	10000.	1.0	0.0290"	0.0302"	0.0295"	0.0302"	0.0321"	0.0312"
		1.5	0.0133"	0.0136"	0.0130"	0.0134"	0.0141"	--
		2.0	0.0068"	0.0069"	0.0065"	0.0068"	0.0070"	--
30°	500.	1.0	0.2026"	0.2020"	0.1897"	0.2007"	0.2031"	0.1925"
		1.5	0.1184"	0.1178"	0.1106"	0.1172"	0.1181"	--
		2.0	0.0727"	0.0724"	0.0679"	0.0720"	0.0725"	--
	10000.	1.0	0.0215"	0.0223"	0.0217"	0.0223"	0.0233"	0.0231"
		1.5	0.0100"	0.0102"	0.0097"	0.0101"	0.0106"	--
		2.0	0.0052"	0.0052"	0.0049"	0.0051"	0.0057"	--
45°	500.	1.0	0.1406"	0.1406"	0.1324"	0.1402"	0.1423"	0.1343"
		1.5	0.0816"	0.0814"	0.0764"	0.0810"	0.0818"	--
		2.0	0.0494"	0.0492"	0.0461"	0.0489"	0.0492"	--
	10000.	1.0	0.0125"	0.0129"	0.0126"	0.0130"	0.0138"	0.0133"
		1.5	0.0060"	0.0061"	0.0058"	0.0061"	0.0063"	--
		2.0	0.0032"	0.0032"	0.0030"	0.0031"	0.0032"	--
60°	500.	1.0	0.0742"	0.0749"	0.0711"	0.0753"	0.0767"	0.0722"
		1.5	0.0426"	0.0426"	0.0400"	0.0425"	0.0429"	--
		2.0	0.0252"	0.0251"	0.0234"	0.0249"	0.0251"	--
	10000.	1.0	0.0052"	0.0054"	0.0052"	0.0055"	0.0058"	0.0055"
		1.5	0.0026"	0.0027"	0.0025"	0.0027"	0.0027"	--
		2.0	0.0014"	0.0014"	0.0013"	0.0014"	0.0015"	--

[†] The results indicated here are obtained by H-H approach which is superior to the L-H approach.

Note: (a) Fully clamped edge support. (b) Reduced clamped edge support.

TABLE 4

COMPARISON OF THE MAX. CENTRE MOMENT OF FOUR SIDES CLAMPED SANDWICH PLATE
WITH VARIOUS SKEW ANGLES, ASPECT RATIOS AND CORE RIGIDITIES

$E_f = 10^7 \text{ psi.}$		$\nu = 0.32$	$C = 1"$	$a = 40"$	$t_s = 0.025"$
$A_s = a/b = \text{Aspect ratio}$ $G_c = \text{Shear modulus of core}$ $q = \text{Intensity of load}$					
θ	G_c	A_s	3 degrees of freedom/node (12x12) (lb.-in/in)	5 degrees of freedom/node (8x8) (lb.-in/in)	Kennedy [4] (lb.-in/in)
			(a)	(b)	
15°	500.	1.0	37.29	36.26 36.23	39.3
		1.5	23.02	22.34 22.32	25.1
		2.0	14.50	14.07 14.05	16.1
	10000.	1.0	35.47	36.26 36.29	38.4
		1.5	23.29	23.27 23.27	25.1
		2.0	14.64	14.44 14.42	15.7
30°	500.	1.0	31.58	30.61 30.55	34.1
		1.5	19.07	18.49 18.47	21.1
		2.0	11.84	11.47 11.46	13.25
	10000.	1.0	30.63	31.15 31.17	33.2
		1.5	19.23	19.14 19.13	20.8
		2.0	11.88	11.68 11.66	12.8
45°	500.	1.0	22.50	21.71 21.65	25.4
		1.5	13.29	12.86 12.84	15.1
		2.0	8.05	7.79 7.78	9.16
	10000.	1.0	22.08	22.23 22.23	24.2
		1.5	13.30	13.15 13.13	14.5
		2.0	8.01	7.83 7.82	8.64
60°	500.	1.0	12.13	11.66 11.63	14.8
		1.5	6.97	6.75 6.73	8.22
		2.0	4.10	3.96 3.95	4.72
	10000.	1.0	11.91	11.78 11.77	13.5
		1.5	6.90	6.75 6.74	7.60
		2.0	4.04	3.92 3.91	4.32

Note: (a) Fully clamped edge support.
(b) Reduced clamped edge support.

TABLE 5

COMPARISON OF DEFLECTIONS FOR CLAMPED SKEWED SANDWICH PLATE

$$E_f = 10^7 \text{ lb./in}^2 \quad = 0.32 \quad C = 1.0" \quad a = 40" \quad A_s = 1.$$

$$t_s = 0.05" \quad G_c = 10000. \quad q = 1 \text{ lb./in}^2$$

Results for	Mesh	15° (in)	30° (in)	45° (in)	60° (in)
3 Degrees of freedom/Node	2x2	0.01399	0.01125	0.00705	0.00375
	4x4	0.01860	0.01420	0.00880	0.00405
	8x8	0.02039	0.01550	0.00964	0.00441
	12x12	0.02071	0.01570	0.00983	0.00453
5 Degrees of freedom/Node (a)	2x2	0.01064	0.00856	0.00525	0.00285
	4x4	0.01772	0.01357	0.00833	0.00383
	8x8	0.01977	0.01511	0.00937	0.00433
	12x12	0.02030	0.01550	0.00969	0.00450
5 Degrees of freedom/Node (b)	2x2	0.01081	0.00869	0.00579	0.00290
	4x4	0.01889	0.01452	0.00906	0.00423
	8x8	0.02048	0.01572	0.00978	0.00455
	12x12	0.02079	0.01588	0.00992	0.00464
Kennedy		0.02145	0.01630	0.01027	0.00479

Note: (a) Fully clamped edge support.
(b) Reduced clamped edge support.

TABLE 6

COMPARISON OF CENTRE DEFLECTION FOR A RECTANGULAR SANDWICH PLATE USING VARIOUS

MODIFIED FORM OF SHEAR STRAIN.

$$E_f = 3 \times 10^6 \text{ psi.} \quad E_c = 1800 \text{ psi.} \quad \nu = 0.3 \quad a = 68.555" \quad t_s = 0.05$$

$$t_c = 1" \quad A_s = 1. \quad q = \text{Intensity of load} = 1. \text{ lb./in}^2$$

Boundary condition		Four sides clamped	Four sides simply supported
Results for			
CASE 1		0.84070	1.58434
CASE 2		0.76246	1.43679
CASE 3		0.78477	1.45960
CASE 4		0.80820	1.48350
CASE 5		0.87827	1.55528
Taylor [22]		0.793"	--
March [24]		0.729"	--
Yen [23]		--	1.486"
Plantema [2]		--	1.485

TABLE 7

COMPARISON OF THE CENTRE DEFLECTION OF A CLAMPED RECTANGULAR SANDWICH PLATE

For problem 1 : $E_f = 3.10^6$ psi.	$E_c = 1800$ psi	$\nu = 0.3$	$t_c = 1"$
$t_f = 0.05"$	$A_s = 1.$	$a = 68.555"$	
For problem 2 : $E_f = 10.5 \times 10^6$ psi.	$G_c = 5. \times 10^4$	$\nu = 0.3$	$t_c = 1"$
$t_f = 0.015$	$A_s = 1.$	$a = 50"$	

	Problem 1	Problem 2
--	-----------	-----------

Results for	Three degrees of freedom	Five degrees of freedom	Three degrees of freedom	Five degrees of freedom
		(a)	(b)	(a) (b)
2x2	0.64622"	0.48424"	0.49147"	0.00468" 0.00362"
4x4	0.78515"	0.70830"	0.75998"	0.01549" 0.05396" 0.05448"
8x8	0.81952"	0.78030"	0.84570"	0.03751" 0.07527" 0.07555"
16x16	0.82499"	0.80817"	0.82411"	0.06235" 0.07898" 0.07914"
P.H. Kan [25]	--	--	--	0.0816"
Monforton [10]	--	--	--	0.0812"
March [24]	0.729"	--	--	--
Taylor [22]	0.793"	--	--	--

Note: (a) Fully clamped edge support.

(b) Reduced clamped edge support.

TABLE 8

COMPARISON OF CENTRE DEFLECTION FOR SIMPLY SUPPORTED RECTANGULAR SANDWICH PLATE

$E_f = 3 \times 10^6$ psi, $E_c = 1800$ psi, $\nu = 0.3$ $t_f = 0.05"$ $t_c = 1.0"$
 $a = 68.555"$ $A_s = 1.0"$ $q = \text{Intensity of load} = 1.0 \text{ lb/in}^2$

Results for	Element Type	Three degrees of freedom per node		Five degrees of freedom per node	
2x2		1.19409"		1.31087"	
4x4		1.40440"		1.45779"	
8x8		1.45960"		1.47855"	
16x16		1.47990"		1.48354"	
Yen, et. al [23]		1.486"			
Plantema [2]		1.485"			

TABLE 9

COMPARISON OF THE DEFLECTION OF A CANTILEVERED RECTANGULAR SANDWICH PLATE

$E_f = 3 \times 10^5$ psi $E_c = 1800$ psi. $\nu = 0.3$ $b = 68.555"$ $t_s = 0.5"$
 $t_c = 1.0"$ $A_s = 1/20$ $q =$ Loading concentrated at free end
 $= 1$ unit.

Results for	Element Type	Three degrees of freedom per node		Five degrees of freedom per node	
				(a)	(b)
2x2		0.75307"		1.26297"	1.26503"
4x4		1.14090"		1.34794"	1.34970"
8x8		1.31502"		1.37690"	1.37836"
12x12		1.35619"		1.38210"	1.38405"
Plantema [2]		1.398"			
Reissner [21]		1.398"			

Note: (a) Fully clamped edge support.

(b) Reduced clamped edge support.

Centre Deflection for Skew Clamped Sandwich Plates Under
Uniformly Distributed Loading ($w_c \times 100.$)

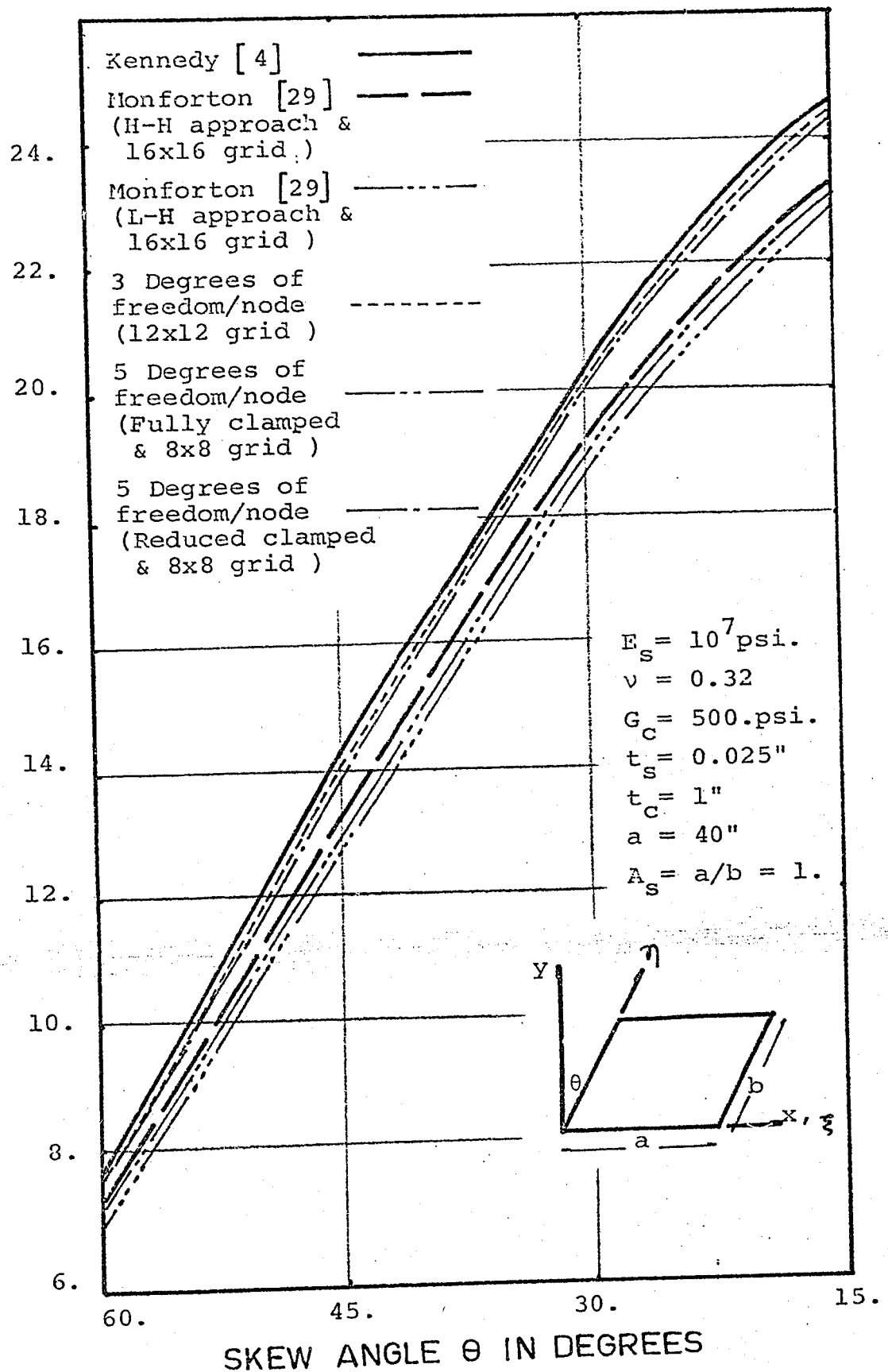


Fig. 4a Comparison of Central Deflection for the Skew Clamped Sandwich Plates (Soft Core)

Centre Deflection for Skew Clamped Sandwich Plates Under
Uniformly Distributed Loading ($w_C \times 1000''$)

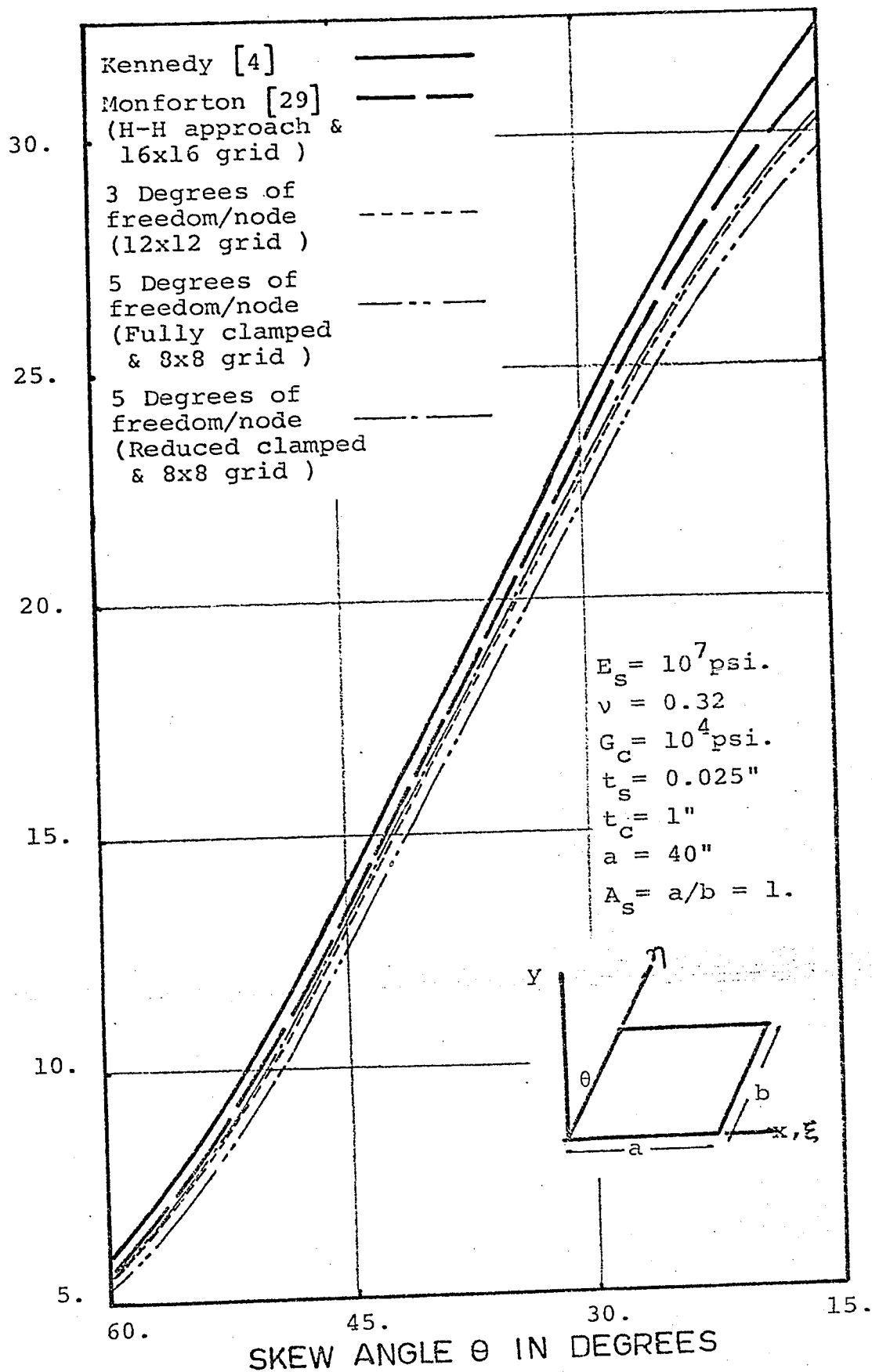


Fig. 4b

Comparison of Central Deflection for the Skew
Clamped Sandwich Plates (Hard Core)

TABLE 10

DEFLECTION OF THE RHOMBIC CANTILEVERED SANDWICH PLATE UNDER A UNIFORMLY LOAD

$$E_f = 10^7 \text{ psi.}$$

$$\nu = 0.32$$

$$C = 1"$$

$$a = 40"$$

$$t_s = 0.025"$$

$$A_s = 1.$$

$$q = 1. \text{ lb./in}^2$$

For location of points 1 and 2 see Fig. (26)

Shear stiff- ness of core	0°		15°		30°		45°		60°	
	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2
G_c										
500.	3.886	3.886	3.265	4.188	2.435	4.142	1.562	3.778	0.810	3.129
1000.	3.069	3.069	2.555	3.289	1.849	3.203	1.116	2.866	0.522	2.332
5000.	2.332	2.332	1.932	2.450	1.384	2.284	0.747	1.910	0.291	1.379
10000.	2.155	2.155	1.784	2.252	1.238	2.068	0.677	1.670	0.254	1.127
100000.	1.199	1.199	0.995	1.235	0.694	1.075	0.382	0.753	0.141	0.373

TABLE 11

DEFLECTION OF THE RHOMBIC CANTILEVERED SANDWICH PLATE UNDER A UNIFORMLY LOADING

$$E_f = 10^7 \text{ lb./in}^2 \quad \nu = 0. \quad C = 1" \quad a = 40" \quad A_s = 1.$$

$$t_s = 0.025" \quad q = 1.0 \text{ lb./in}^2$$

For location of points 1 and 2 see Fig. (26)

Shear stiffness of core	0°		15°		30°		45°		60°	
	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2
G_c										
500.	4.026	4.026	3.411	4.301	2.565	4.205	1.656	3.775	0.857	3.068
1000.	3.216	3.216	2.698	3.422	1.973	3.302	1.206	2.909	0.567	2.313
5000.	2.498	2.498	2.085	2.611	1.472	2.419	0.831	1.998	0.331	1.428
10000.	2.328	2.328	1.943	2.416	1.363	2.201	0.756	1.754	0.289	1.159
100000.	1.333	1.333	1.121	1.352	0.786	1.156	0.433	0.796	0.159	0.389

TABLE 12

MAX. EDGED MOMENT OF THE RHOMBIC CANTILEVERED SANDWICH PLATE
UNDER A UNIFORMLY DISTRIBUTED LOADING

$$E_f = 10^7 \text{ lb./in}^2 \quad \nu = 0.32 \quad C = 1" \quad a = 40" \quad A_s = 1.$$

$$t_s = 0.025" \quad q = 1 \text{ lb./in}^2$$

Skew G_c	0° lb-in/in	15° lb-in/in	30° lb-in/in	45° lb-in/in	60° lb-in/in
500.	874.286	1174.860	1317.770	1202.150	780.971
1000.	849.243	1192.220	1392.250	1338.380	965.244
5000.	736.309	1117.860	1399.160	1453.550	1183.020
10000.	665.430	1031.300	1318.970	1400.550	1163.440
100000.	341.623	505.377	650.205	713.451	620.122

TABLE 13

MAX. EDGED MOMENT OF THE RHOMBIC CANTILEVERED SANDWICH PLATE
UNDER A UNIFORMLY DISTRIBUTED LOADING

$$E_f = 10^7 \text{ lb/in}^2 \quad \nu = 0. \quad C = 1" \quad a = 40" \quad A_s = 1.$$

$$t_s = 0.025" \quad q = 1 \text{ lb/in}^2$$

Skew G_c	0° lb-in/in	15° lb-in/in	30° lb-in/in	45° lb-in/in	60° lb-in/in
500.	733.038	970.006	1069.120	951.380	587.104
1000.	729.991	1002.170	1146.220	1075.480	746.745
5000.	706.496	1017.440	1225.430	1233.560	977.140
10000.	679.172	980.038	1192.350	1220.250	988.66
100000.	400.416	533.343	637.151	671.306	579.59

TABLE 14

DEFLECTION OF THE RHOMBIC CANTILEVERED SANDWICH PLATE UNDER A UNIFORMLY

DISTRIBUTED LOADING

$$E_f = 10^7 \text{ lb/in}^2 \quad \nu = 0.32 \quad C = 1" \quad a = 40" \quad t_s = 0.025"$$

$$A_s = a/b = \text{Aspect ratios} \quad q = 1 \text{ lb/in}^2$$

Skew angle	$A_s = 1.0$				$A_s = 1.5$				$A_s = 2.0$			
	$G_C = 500.$		$G_C = 10000.$		$G_C = 500.$		$G_C = 10000.$		$G_C = 500.$		$G_C = 10000.$	
	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2
0°	3.886	2.155	2.155	1.149	1.149	0.451	0.451	0.535	0.535	0.151	0.151	0.151
15°	3.265	4.188	1.784	2.252	0.949	1.248	0.358	0.480	0.443	0.579	0.118	0.163
30°	2.435	4.142	1.238	2.068	0.685	1.234	0.229	0.445	0.321	0.572	0.073	0.152
45°	1.562	3.778	0.677	1.670	0.412	1.118	0.106	0.363	0.196	0.517	0.034	0.125
60°	0.810	3.129	0.254	1.127	0.185	0.918	0.029	0.255	0.090	0.424	0.009	0.890

TABLE 15

MAX. EDGED MOMENT OF THE RHOMBIC CANTILEVERED SANDWICH PLATE UNDER A UNIFORMLY LOAD

$$E_f = 10^7 \text{ lb/in}^2 \quad \nu = 0.32 \quad C = 1" \quad a = 40"$$

$$A_s = a/b = \text{Aspect ratios} \quad t_s = 0.025" \quad q = 1 \text{ lb/in}^2$$

Skew angle	$A_s = 1.0$			$A_s = 1.5$			$A_s = 2.0$		
	$G_c = 500.$	$G_c = 10000.$	$G_c = 500.$	$G_c = 10000.$	$G_c = 500.$	$G_c = 10000.$	$G_c = 500.$	$G_c = 10000.$	$G_c = 10000.$
0°	lb.in/in 874.286	lb.in/in 665.430	lb.in/in 388.182	lb.in/in 311.110	lb.in/in 216.234	lb.in/in 179.188			
15°	1174.860	1031.300	520.148	475.571	288.229	268.355			
30°	1317.770	1318.970	576.976	596.251	315.461	329.362			
45°	1202.150	1400.550	516.550	616.706	275.002	331.990			
60°	780.971	1163.440	328.278	497.007	166.874	258.637			

TABLE 16

THE CENTRAL DEFLECTION OF A CLAMPED FLAT SANDWICH PLATE FOR VARIOUS

SHEAR STIFFNESS OF CORE

$$E_f = 10^7 \text{ psi.} \quad \nu = 0.32 \quad t_s = 0.025" \quad t_c = 1" \quad a = 40"$$

$$A_s = a/b = 1.$$

Loading condition: Uniformly distributed loading.

Mesh size	$G_c = 500.$		$G_c = 1000.$		$G_c = 2000.$		$G_c = 3000.$		$G_c = 4000.$		$G_c = 5000.$	
	Three degree (in)	Five degree (in)	Three degree (in)	Five degree (in)	Three degree (in)	Five degree (in)	Three degree (in)	Five degree (in)	Three degree (in)	Five degree (in)	Three degree (in)	Five degree (in)
2x2	.3000	.2283	.1500	.1141	.0750	.0571	.0500	.0380	.0375	.0285	.0300	.0228
4x4	.2691	.2308	.1443	.1248	.0813	.0717	.0597	.0539	.0486	.0449	.0417	.0395
8x8	.2620	.2446	.1424	.1335	.0823	.0778	.0620	.0592	.0517	.0498	.0454	.0441
12x12	.2608	.2525	.1421	.1379	.0825	.0805	.0625	.0613	.0523	.0516	.0462	.0457

TABLE 17

THE CENTRAL DEFLECTION OF A SIMPLY SUPPORTED SANDWICH PLATE FOR VARIOUS SHEAR

STIFFNESS OF CORE

$E_f = 10^7$ psi. $\nu = 0.32$ $t_s = 0.025"$ $t_c = 1"$ $a = 40"$ $A_s = 1.$

Loading condition: Uniformly distributed loading.

Mesh size	$G_c = 500.$		$G_c = 1000.$		$G_c = 2000.$		$G_c = 3000.$		$G_c = 4000.$		$G_c = 5000.$	
	Three degree	Five degree	Three degree	Five degree	Three degree	Five degree	Three degree	Five degree	Three degree	Five degree	Three degree	Five degree
	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)	(in)
2x2	.3525	.2930	.1996	.1767	.1196	.1178	.0905	.0976	.0746	.0871	.0642	.0805
4x4	.3169	.3049	.1916	.1869	.1275	.1277	.1049	.1079	.0928	.0979	.0849	.0919
8x8	.3091	.3063	.1894	.1883	.1291	.1292	.1086	.1094	.0980	.0994	.0914	.0934
12x12	.3078	.3066	.1890	.1886	.1294	.1295	.1093	.1098	.0991	.0998	.0927	.0937

TABLE 18

THE CENTRAL DEFLECTION OF A CLAMPED SKEW SANDWICH PLATE FOR
VARIOUS SHEAR STIFFNESS OF CORE

$$E_f = 10^7 \text{ psi.} \quad \nu = 0.32 \quad t_s = 0.025" \quad t_c = 1" \quad a = 40"$$

$$A_s = a/b = 1. \quad \theta = \text{Skew angle} = 45^\circ$$

Mesh	$G_c=500.$	$G_c=1000.$	$G_c=2000.$	$G_c=3000.$	$G_c=4000.$	$G_c=5000.$
2x2	0.1500	0.0750	0.0375	0.0250	0.0187	0.0150
4x4	0.1415	0.0732	0.0389	0.0274	0.0216	0.0180
8x8	0.1406	0.0736	0.0399	0.0287	0.0230	0.0195
12x12	0.1406	0.0738	0.0403	0.0290	0.0234	0.0199

TABLE 19

THE CENTRAL DEFLECTION OF A SIMPLY SUPPORTED SKEW SANDWICH
PLATE FOR VARIOUS SHEAR STIFFNESS OF CORE

$$E_f = 10^7 \text{ psi.} \quad \nu = 0.32 \quad t_s = 0.025" \quad t_c = 1" \quad a = 40"$$

$$A_s = a/b = 1. \quad \theta = \text{Skew angle} = 45^\circ$$

Mesh	$G_c=500.$	$G_c=1000.$	$G_c=2000.$	$G_c=3000.$	$G_c=4000.$	$G_c=5000.$
2x2	0.1594	0.0842	0.0463	0.0335	0.0269	0.0228
4x4	0.1530	0.0846	0.0501	0.0383	0.0323	0.0286
8x8	0.1537	0.0867	0.0530	0.0417	0.0359	0.0323
12x12	0.1545	0.0876	0.0541	0.0428	0.0371	0.0336

TABLE 20

CENTRE DEFLECTION & MAX. CENTRE MOMENT OF A SIMPLY SUPPORTED SANDWICH PLATE

UNDER UNIFORMLY LOAD

$E_f = 10^7 \text{ psi.}$		$\nu = 0.32$	$t_c = 1"$	$A_s = 1.$	$a = 40"$	$q = 1 \text{ lb./in}^2$					
G_c	θ	$t_s = 0.025"$		$t_s = 0.05"$		$t_s = 0.10"$		$t_s = 0.50"$			
		$w_c \text{ (in)}$	$m_c \text{ (lb"/")}$	$w_c \text{ (in)}$	$m_c \text{ (lb"/")}$	$w_c \text{ (in)}$	$m_c \text{ (lb"/")}$	$w_c \text{ (in)}$	$m_c \text{ (lb"/")}$	$w_c \text{ (in)}$	$m_c \text{ (lb"/")}$
500.	0°	0.3078	77.59	0.2708	77.69	0.2524	77.75	0.2387	77.79		
	15°	0.2872	77.28	0.2542	77.36	0.2378	77.40	0.2256	77.43		
	30°	0.2309	66.85	0.2082	66.82	0.1969	66.78	0.1885	66.75		
	45°	0.1545	47.36	0.1435	47.23	13.8000	47.15	0.1339	47.08		
	60°	0.0786	24.57	0.0756	24.52	0.0742	24.49	0.0731	24.46		
1000.	0°	0.1890	77.39	0.1522	77.60	0.1339	77.70	0.1202	77.78		
	15°	0.1749	77.11	0.1421	77.29	0.1258	77.37	0.1135	77.42		
	30°	0.1370	66.83	0.1144	66.85	0.1032	66.81	0.0947	66.75		
	45°	0.0876	47.50	0.0767	47.35	0.0713	47.22	0.0672	47.09		
	60°	0.0421	24.65	0.0392	24.57	0.0377	24.51	0.0366	24.46		
5000.	0°	0.0927	75.82	0.0571	76.82	0.0390	77.33	0.0254	77.72		
	15°	0.0839	75.58	0.0521	76.57	0.0361	77.05	0.0239	77.38		
	30°	0.0610	65.76	0.0392	66.54	0.0281	66.80	0.0197	66.77		
	45°	0.0336	47.22	0.0232	47.56	0.0178	47.50	0.0138	47.15		
	60°	0.0127	24.68	0.0099	24.74	0.0085	24.65	0.0074	24.48		

Centre Deflection for Skewed Sandwich Plates Under Uniformly
Distributed Loading ($w_c \times 100. \text{in}$)

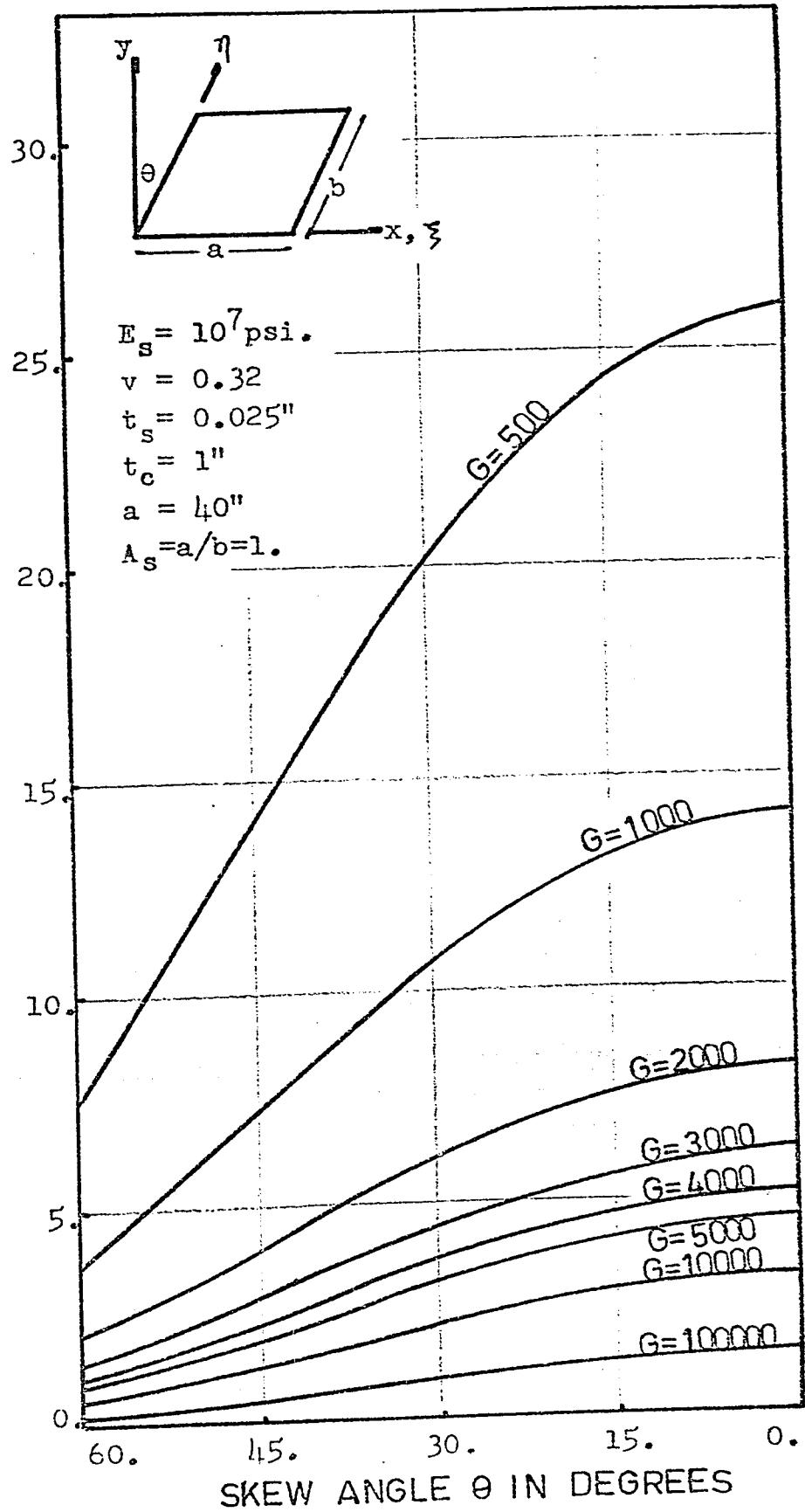


Fig. 5 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Clamped Skew Sandwich Plates ($A_s = 1.$)

Centre Deflection for Skewed Sandwich Plates Under Uniformly Distributed Loading ($w_c \times 1000 \text{ in}$)

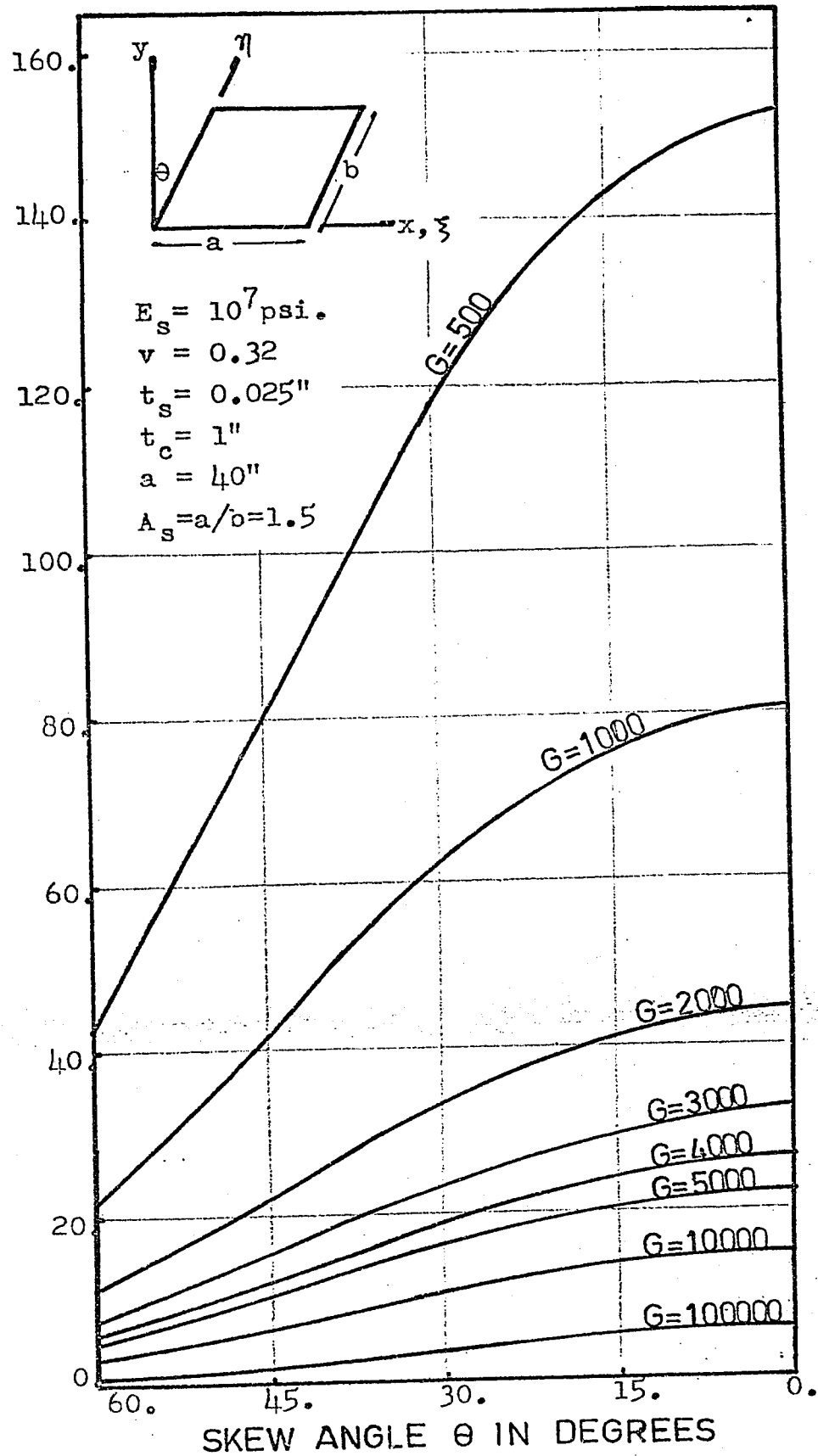


Fig. 6 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Clamped Skew Sandwich Plates ($A_s = 1.5$)

Centre Deflection for Skewed Sandwich Plates Under Uniformly Distributed Loading ($w_c \times 1000 \text{ in}$)

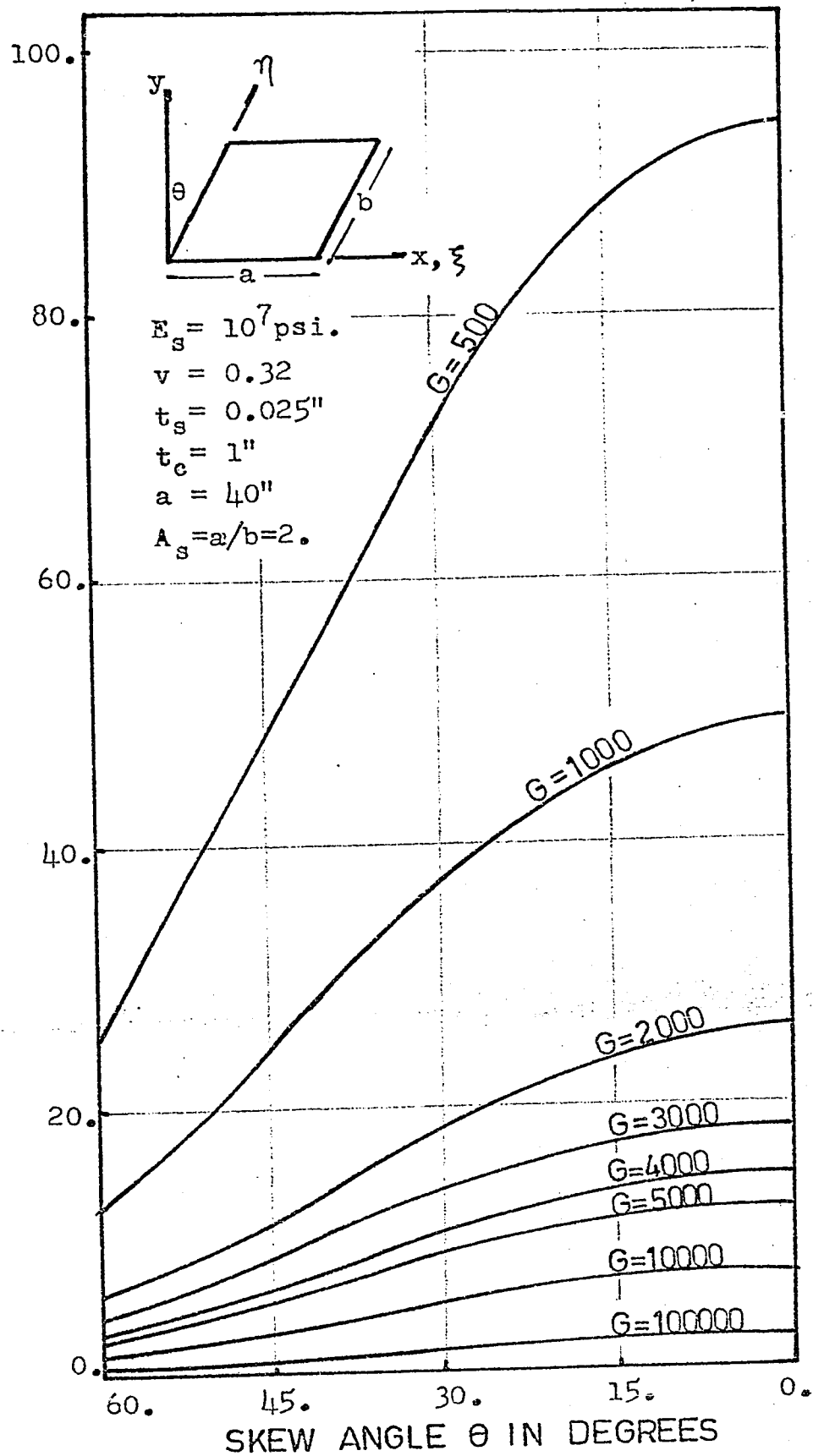


Fig. 7 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Clamped Skew Sandwich Plates ($A_s=2.$)

Centre Deflection for Skewed Sandwich Plates Under Uniformly
Distributed Loading. ($w_c \times 100.in$)

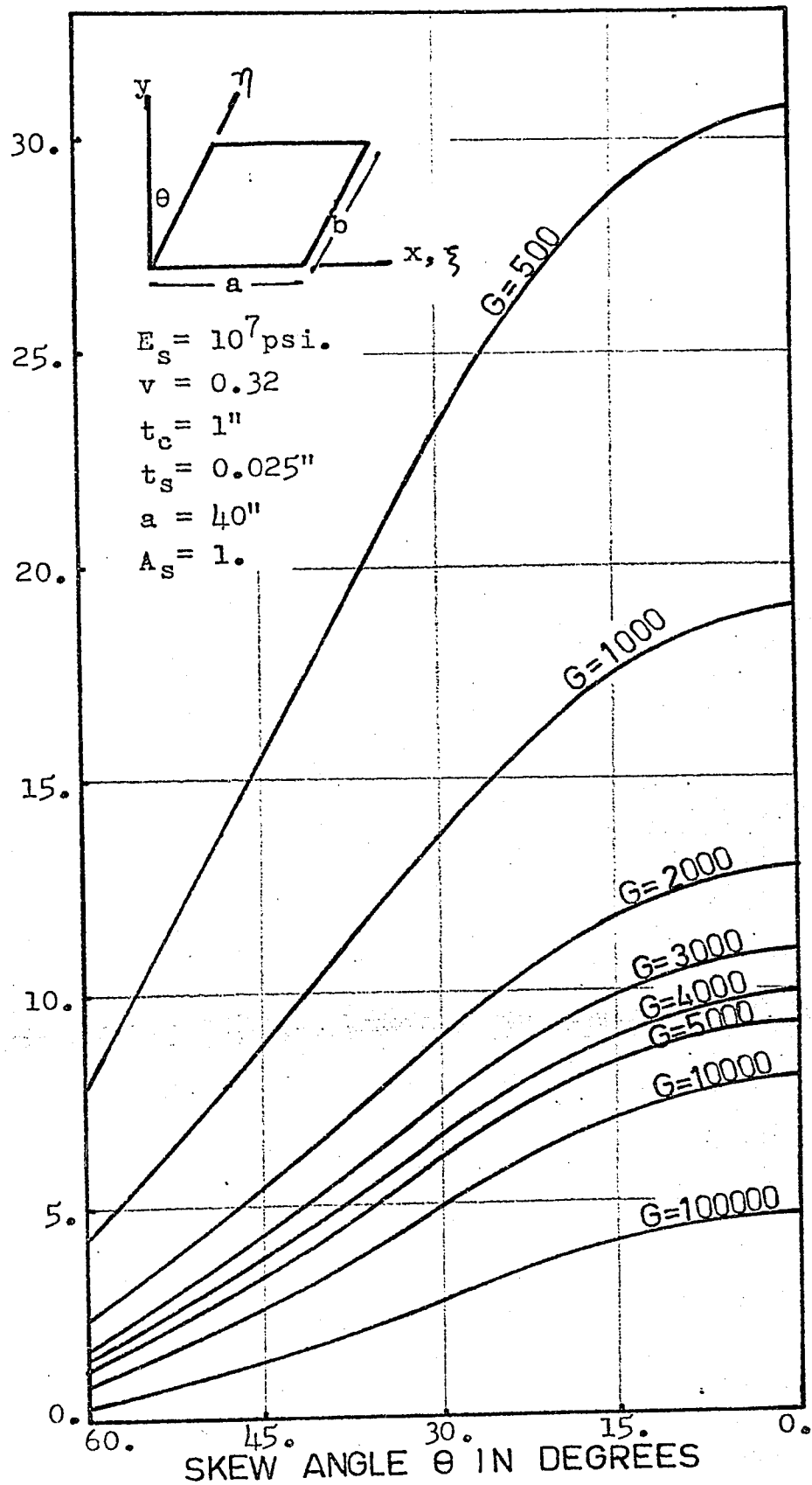


Fig. 8 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Simply Supported Skew Sandwich Plates. ($A_s=1.$)

Centre Deflection for Skewed Sandwich Plates Under Uniformly
Distributed Loading. ($w_c \times 100.in$)

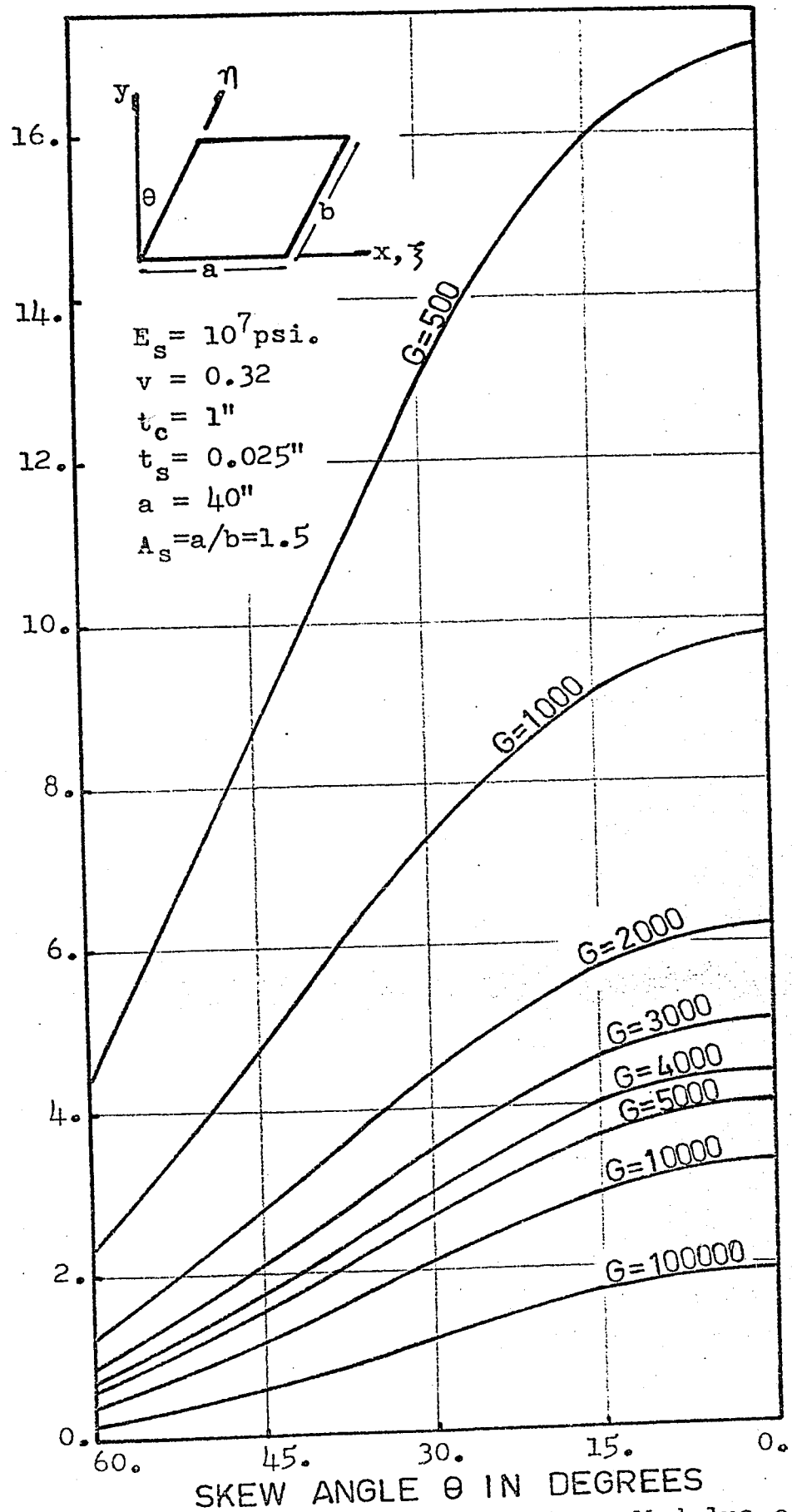


Fig. 9 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Simply Supported Skew Sandwich Plates.
($A_s = 1.5$)

Centre Deflection for Skewed Sandwich Plates Under Uniformly Distributed Loading. ($w_c \times 100 \text{ in}$)

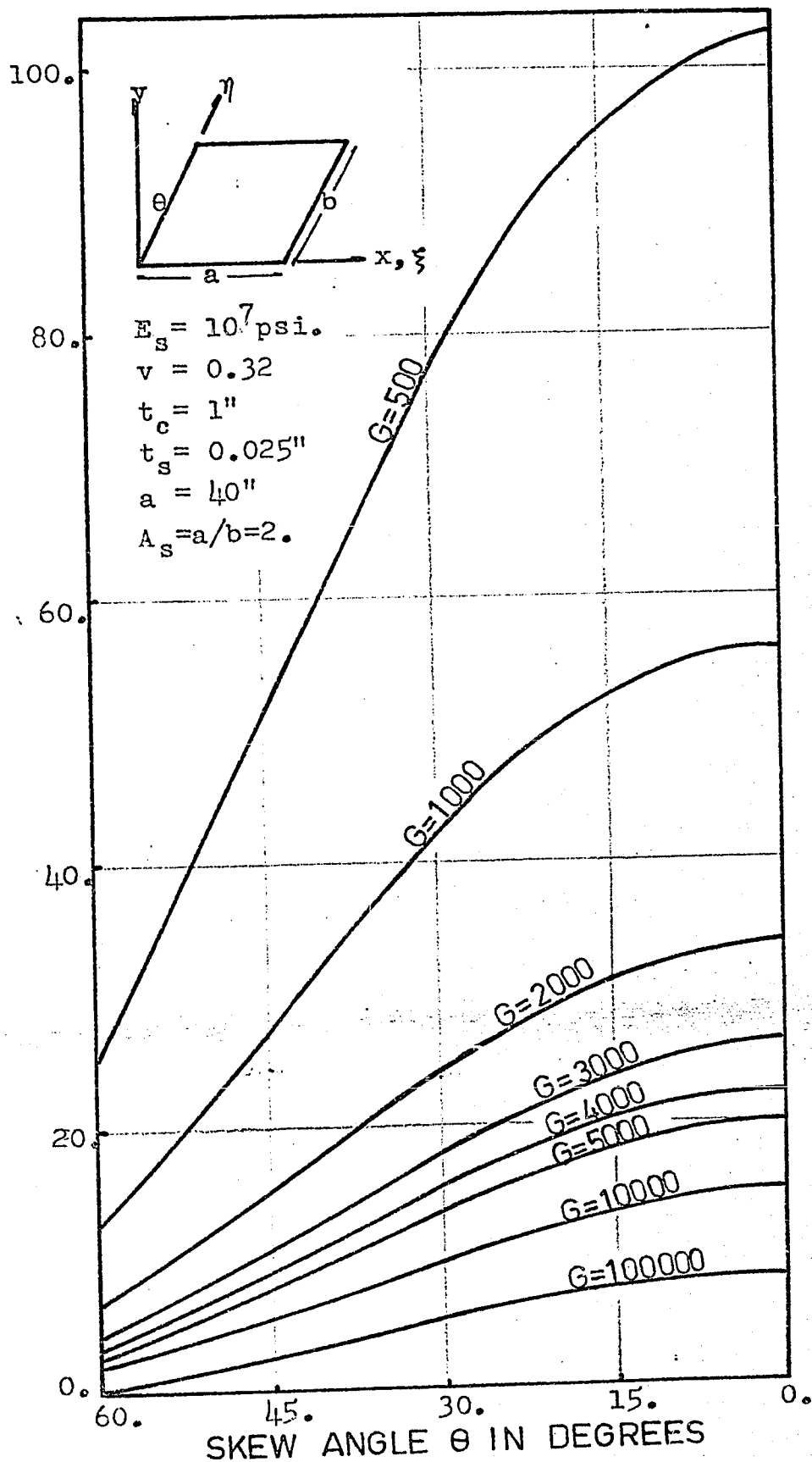


Fig. 10 Variation of Centre Deflection with Shear Modulus of Core and Skew Angle for Simply Supported Skew Sandwich Plates ($A_s=2.$)

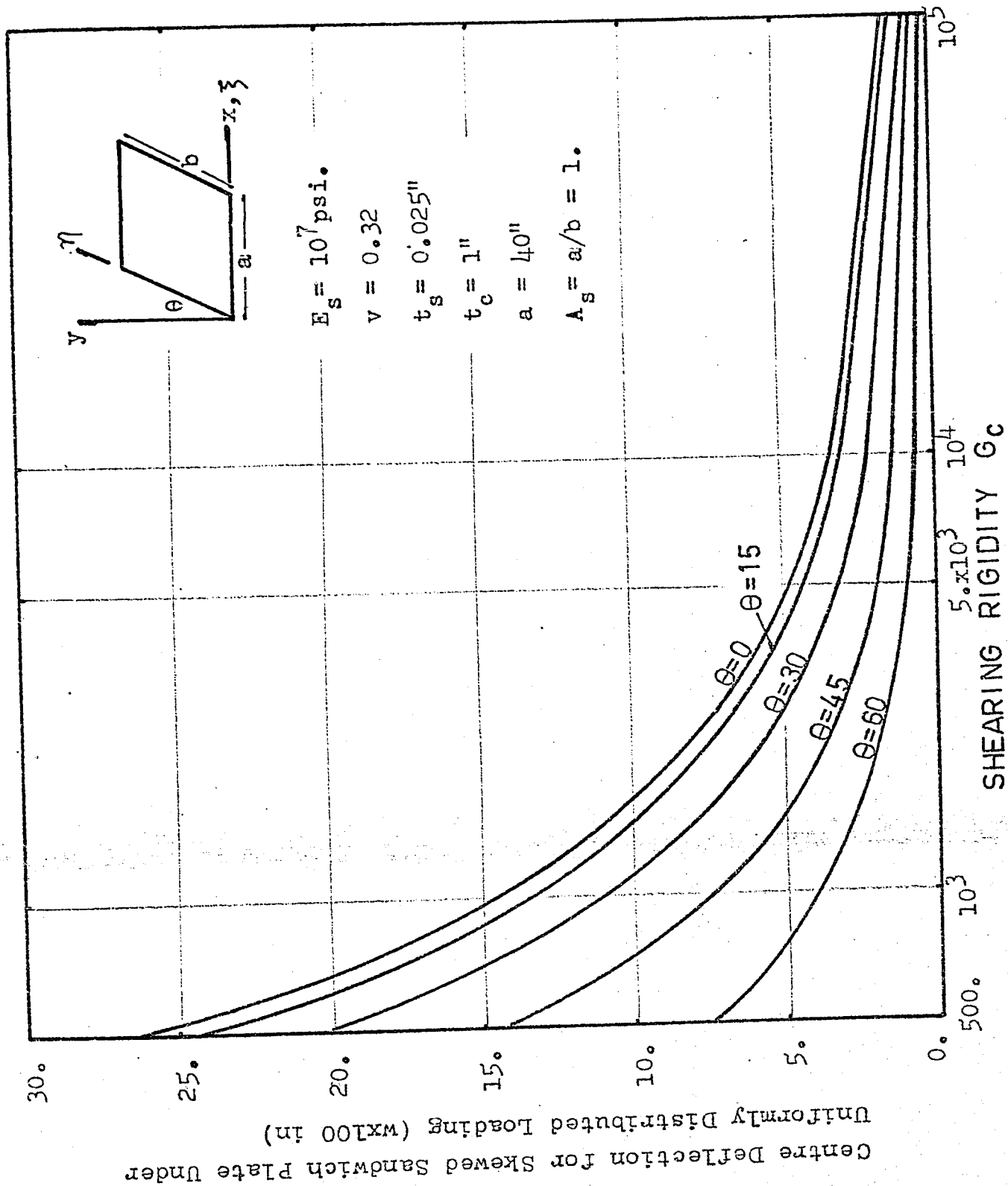


Fig. 11 Variation of Centre Deflection with Skew Angle and Shear Modulus of Core for Clamped Skew Sandwich Plates

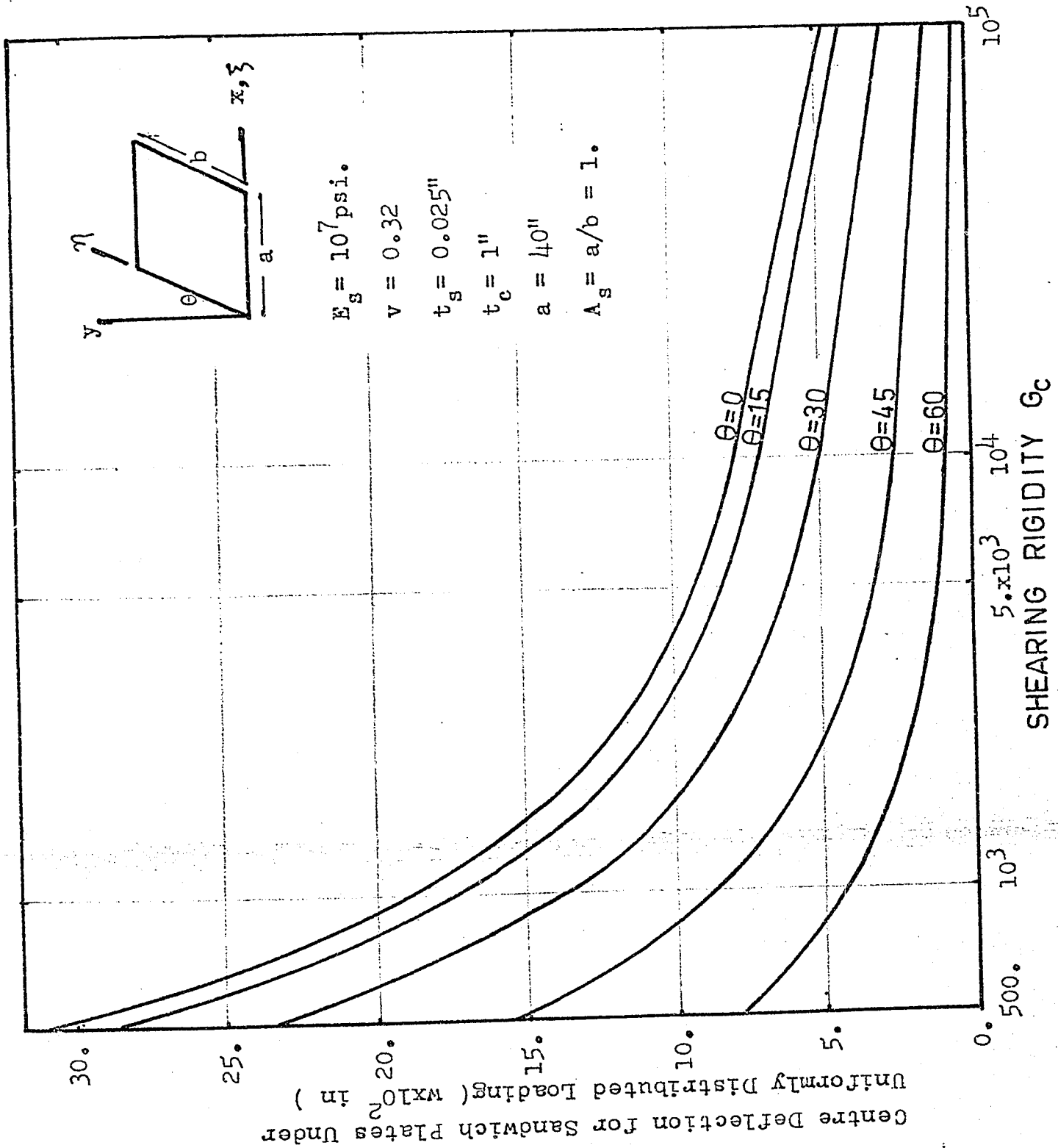


Fig. 12 Variation of Centre Deflection with Skew Angle and Shear Modulus of Core for Simply Supported Skew Sandwich Plates

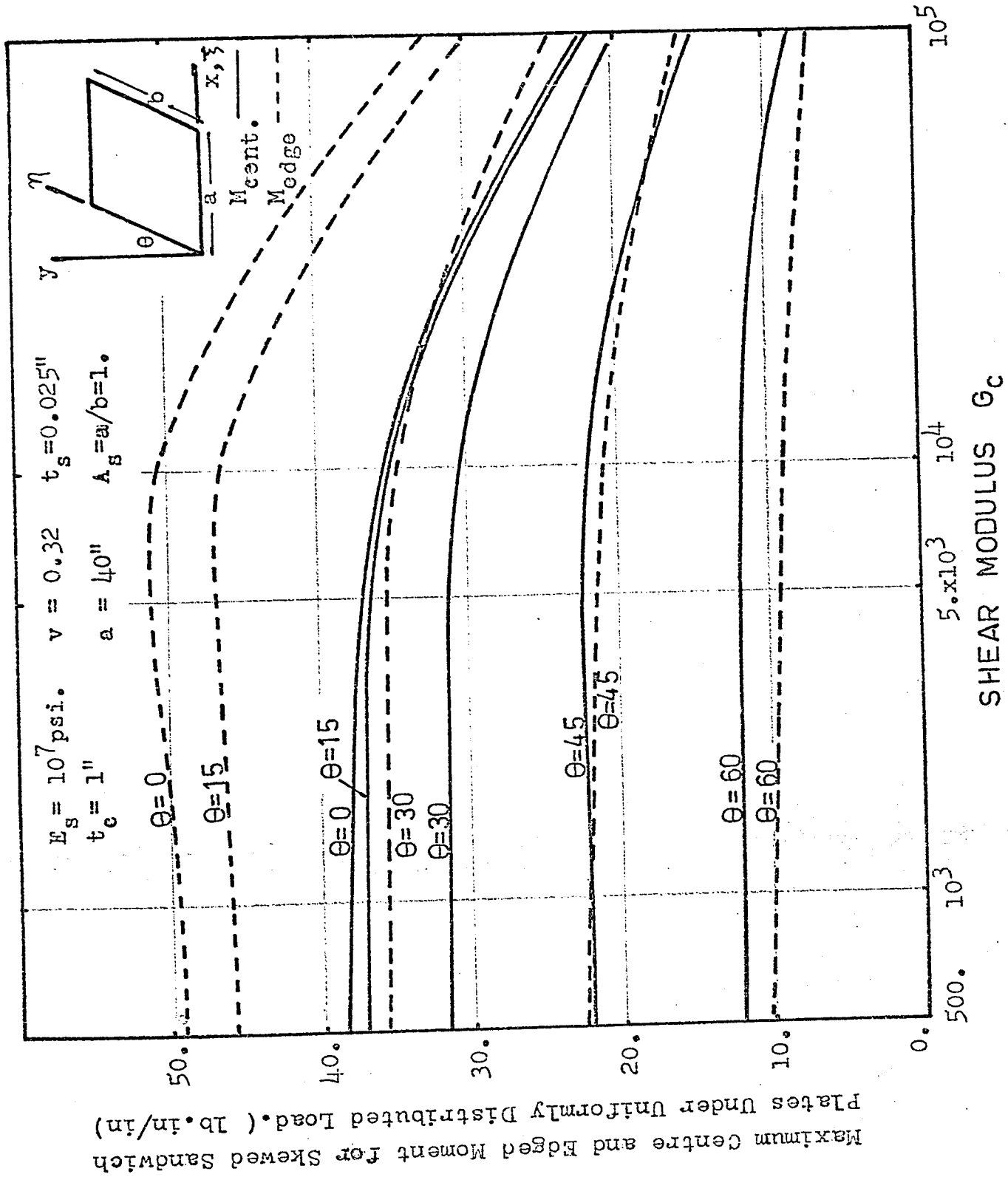


Fig. 13 Variation of Max. Centre and Edged Moments with Skew Angle and Shear Modulus of Core for Clamped Skew Sandwich Plates

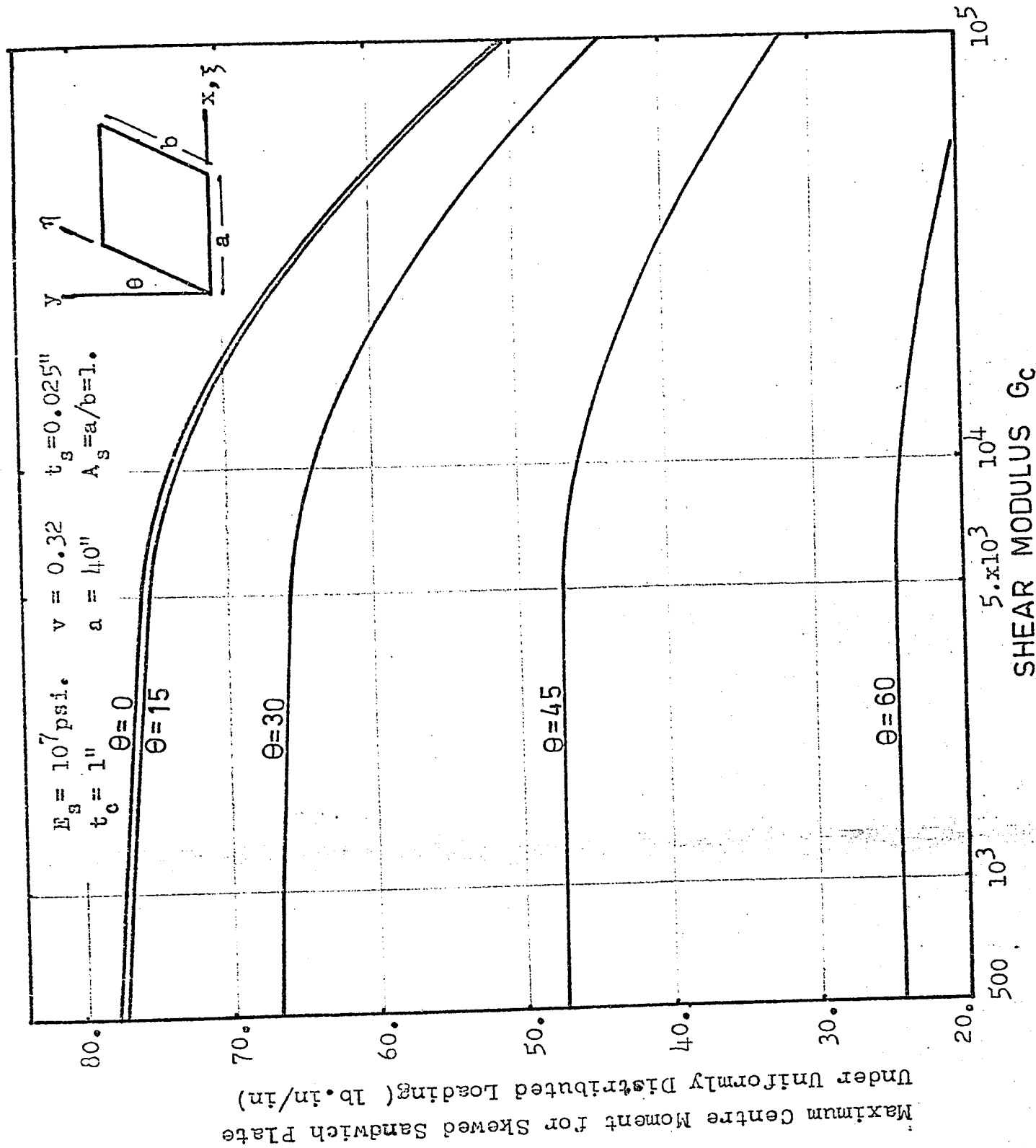


Fig. 14 Variation of Maximum Centre Moment with Skew angle and Shear Modulus of Core for Simply Supported Skew Sandwich Plates ($A_s = 1$)

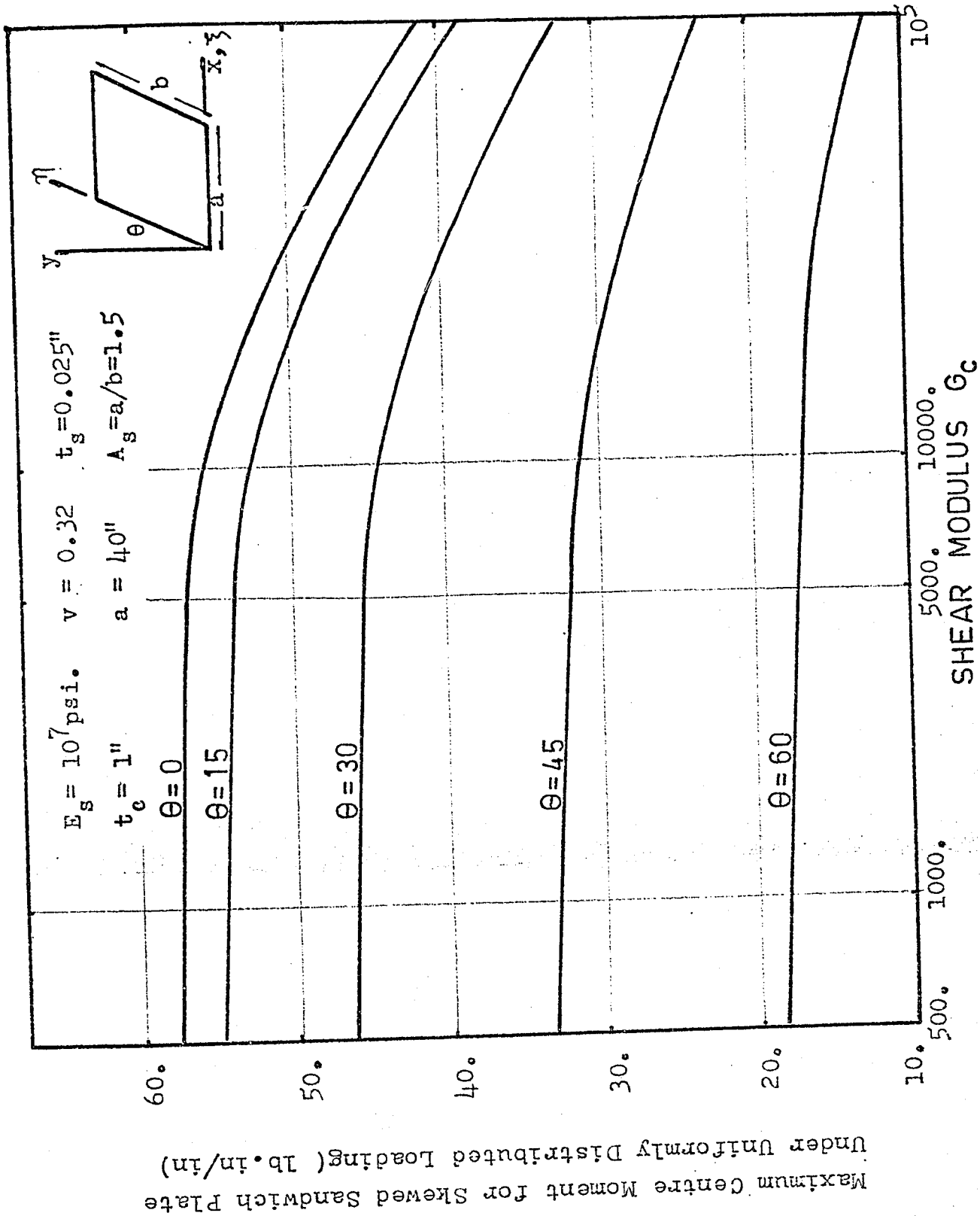


Fig. 15 Variation of Maximum Centre Moment with Skew Angle and Shear Modulus of Core for Simply Supported Skew Sandwich Plates ($A_s = 1.5$)

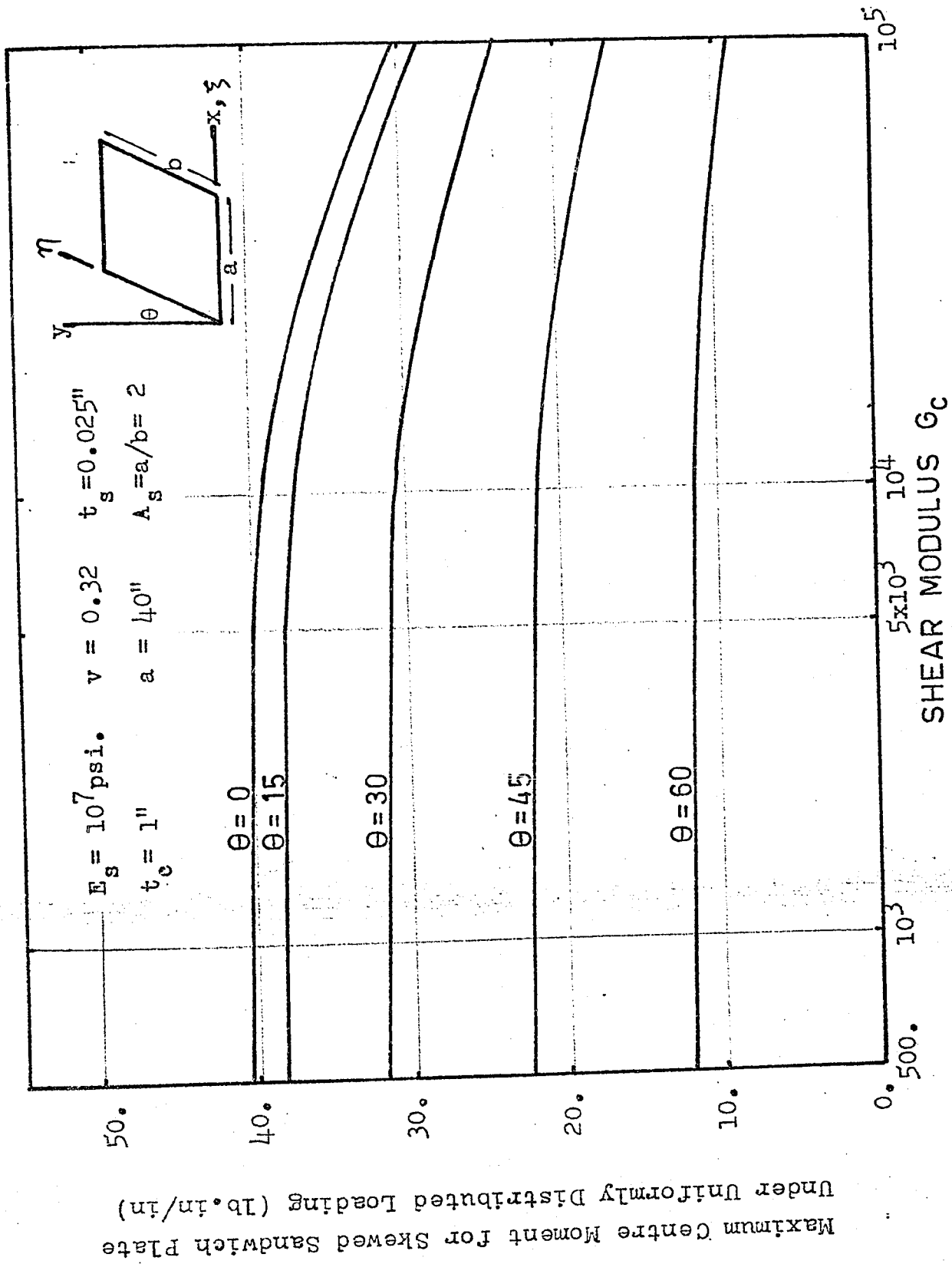


Fig. 16 Variation of Maximum Centre Moment with Skew Angle and Shear Modulus of Core for Simply Supported Skew Sandwich Plates ($A_s = 2$)

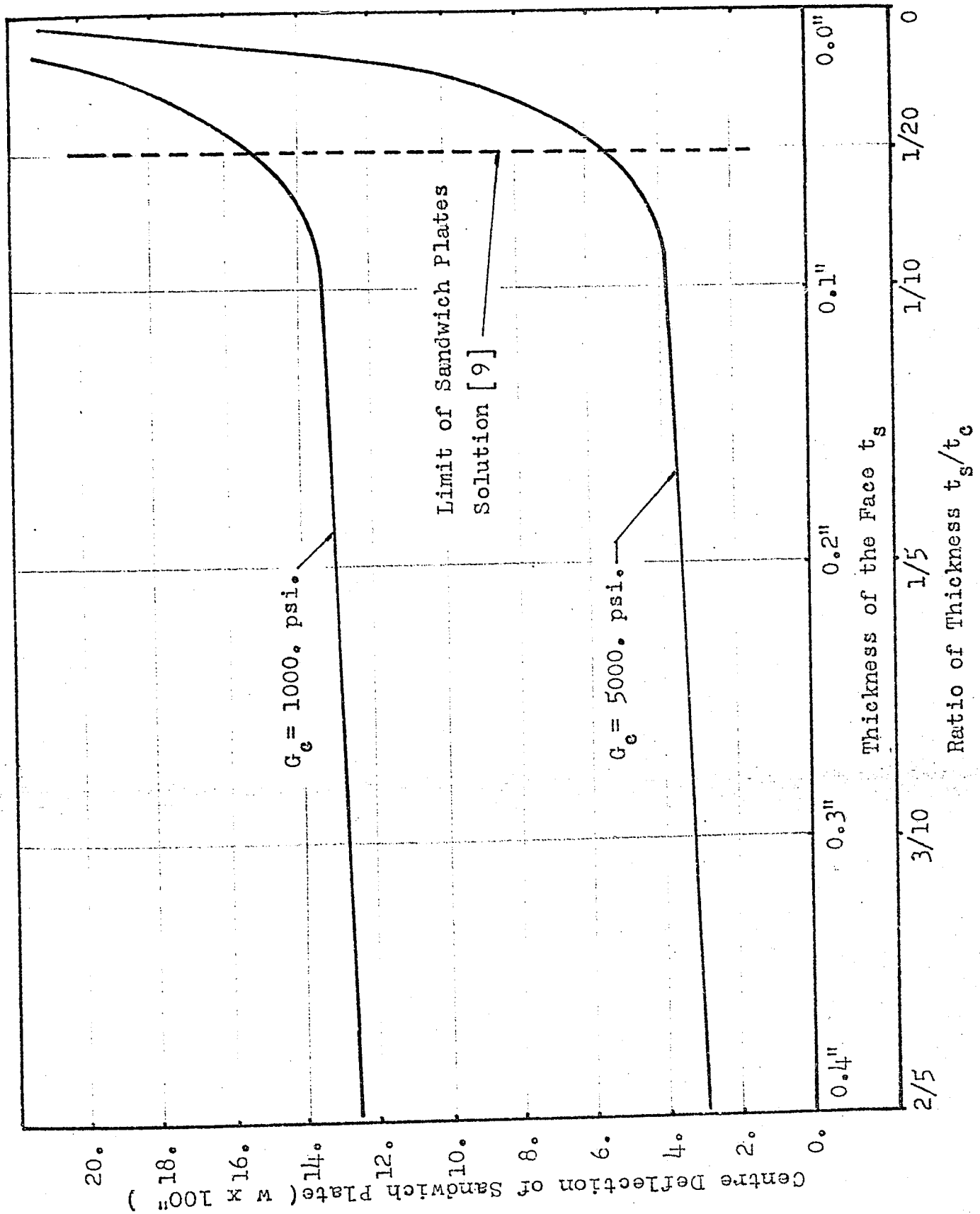


Fig. 17 Variation in Centre Deflection of a Uniformly Loaded Rectangular Sandwich Plate with Thickness Ratio

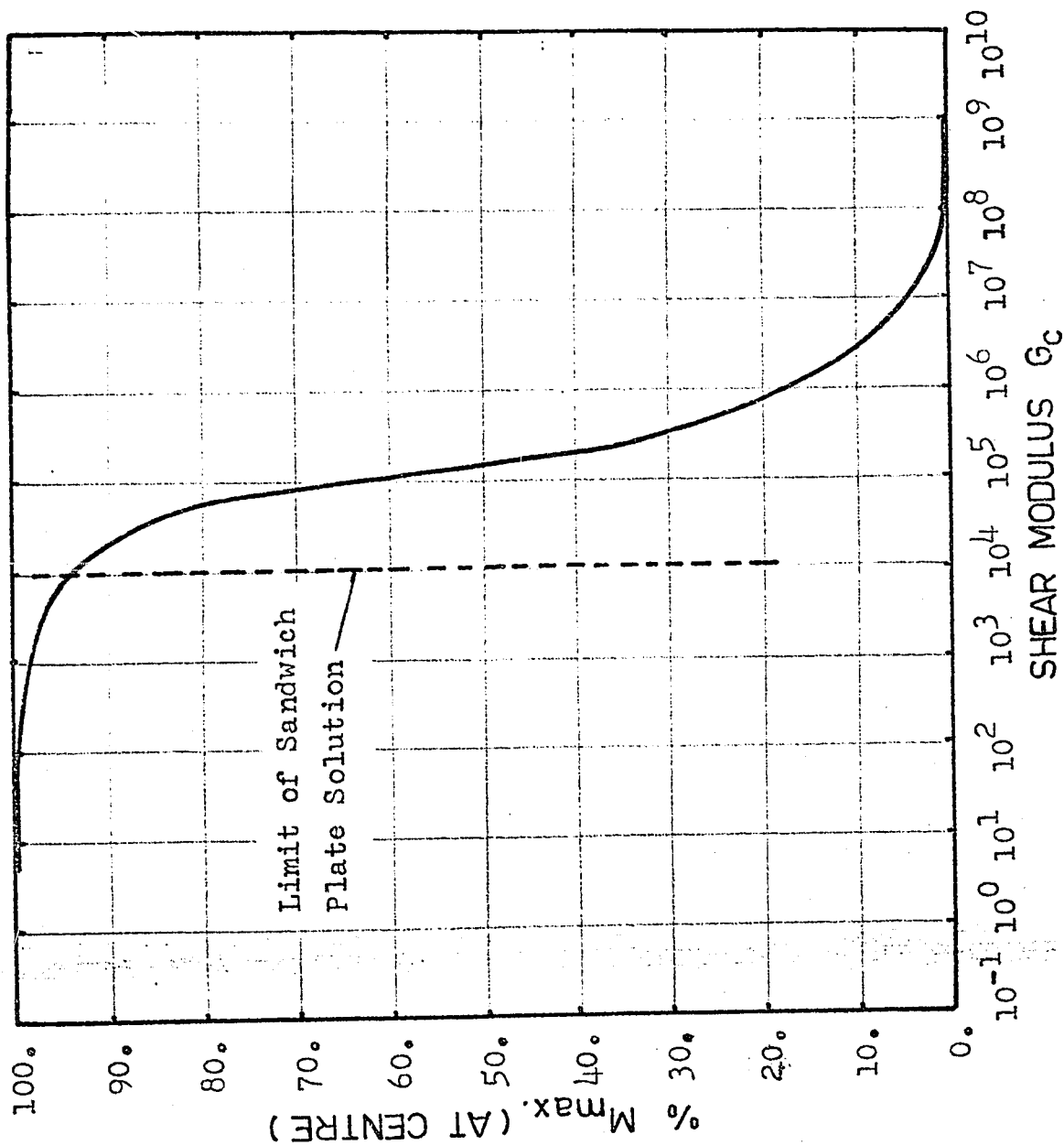


Fig. 18 Percent of The Theoretical Max. Moment Resisted by Skins
for A Simply Supported Rectangular Sandwich Plate

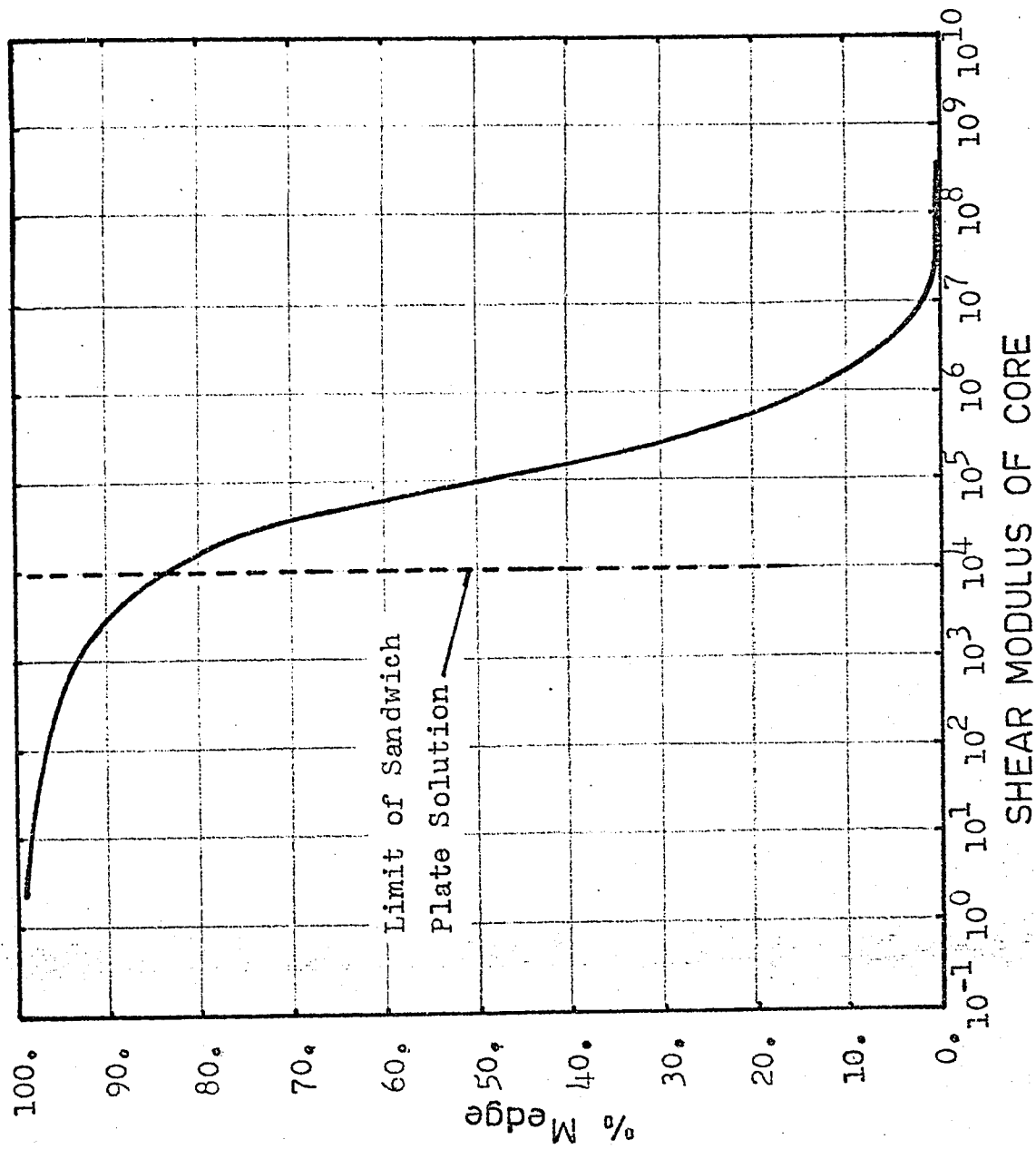


Fig. 19 Percent of Theoretical Edged Moment Resisted by Skins
for A Cantilevered Rectangular Sandwich Plate

Centre Moments M_x , M_y and M_{xy} for Skewed Sandwich Plates Under Uniformly Distributed Loading. (lb-in/in)

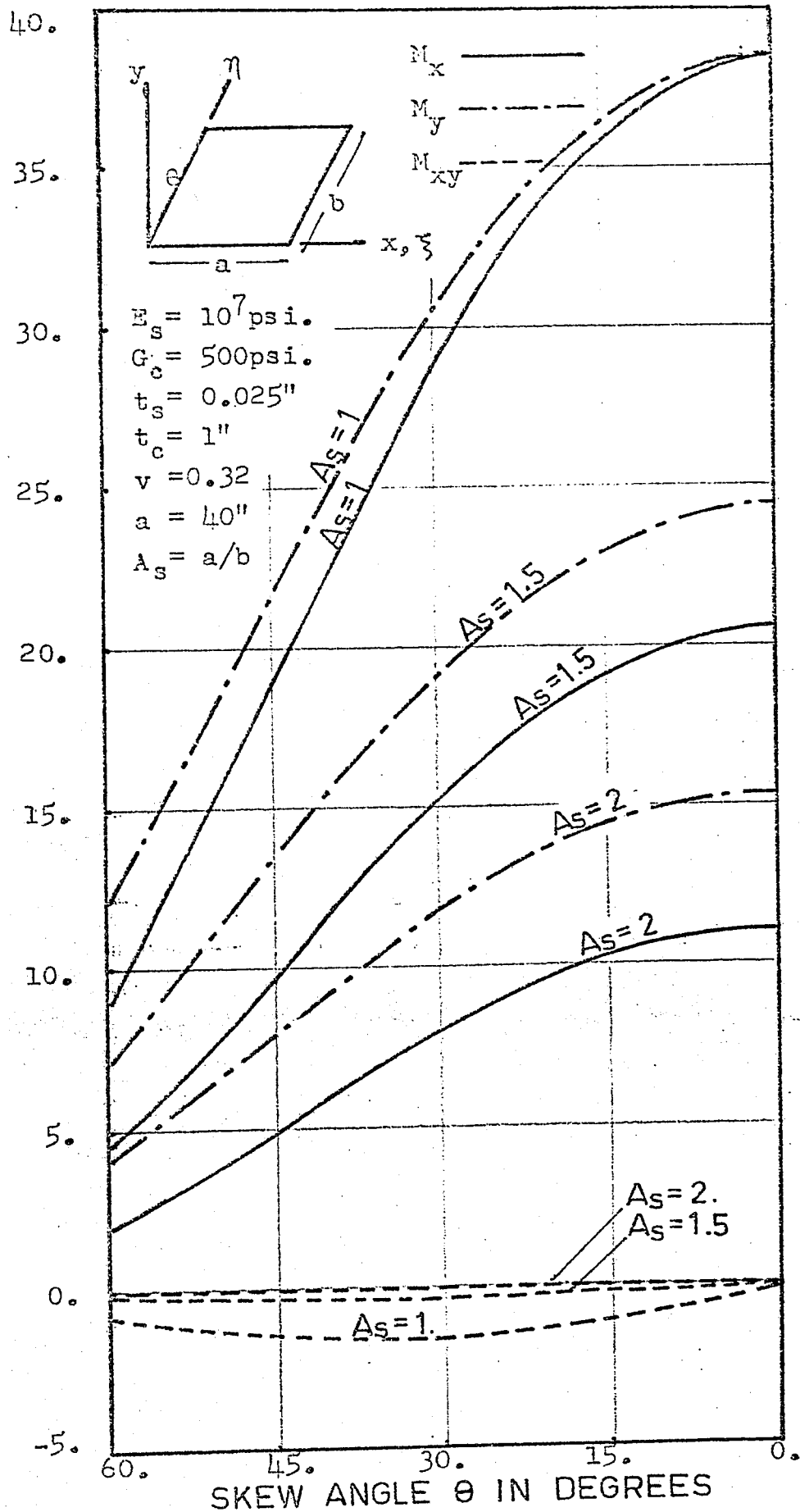


Fig. 20 Variation of Centre Moments with Aspect Ratio & Angle of Skew for Clamped Sandwich Plates

Edged Moment for Skewed Sandwich Plates Under Uniformly Distributed Loading

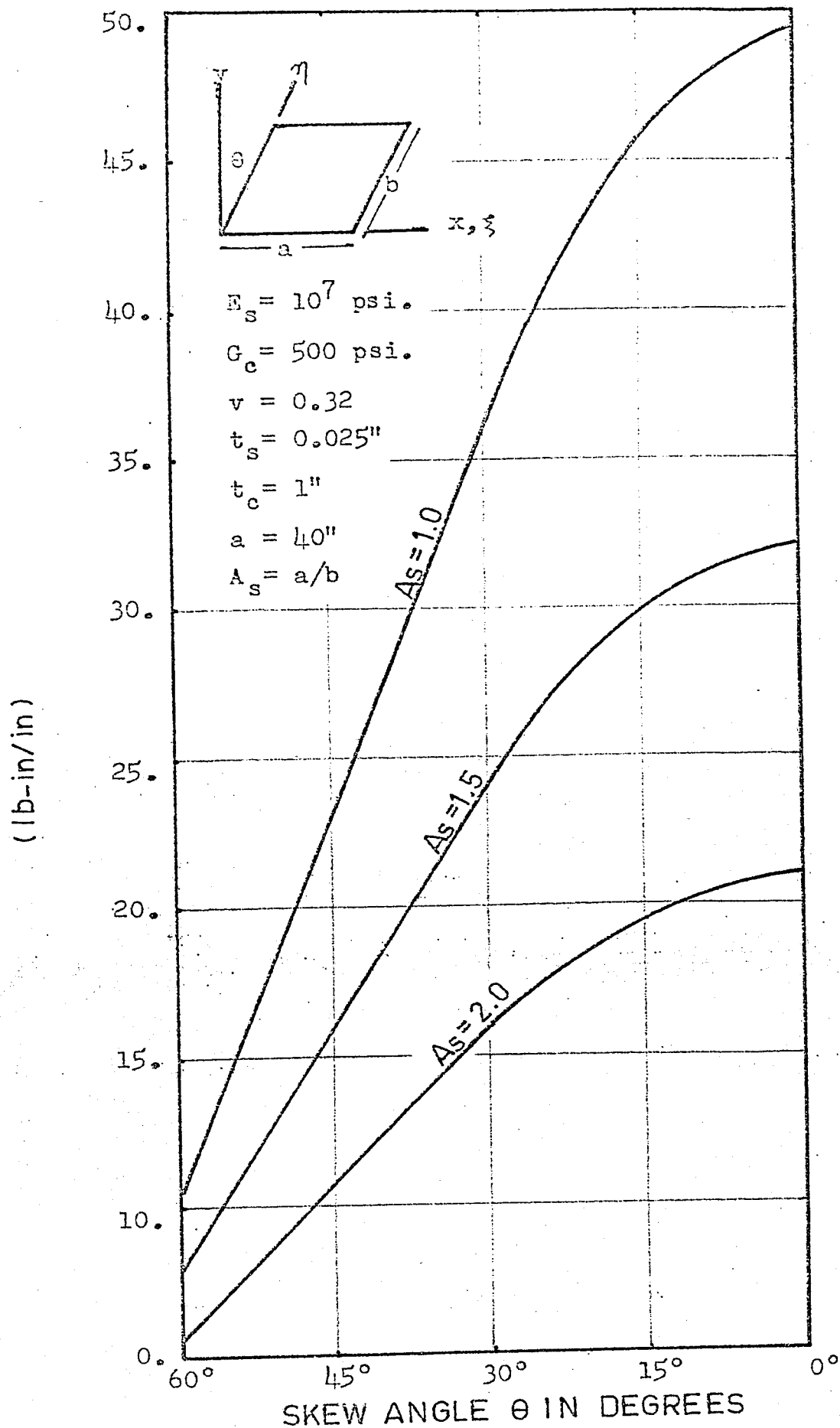


Fig. 21 Variation of Edged Moment with Aspect Ratio & Angle of Skew for Clamped Skew Sandwich Plates.

Centre Moments M_x , M_y & M_{xy} for Skewed Sandwich Plates Under Uniformly Loading

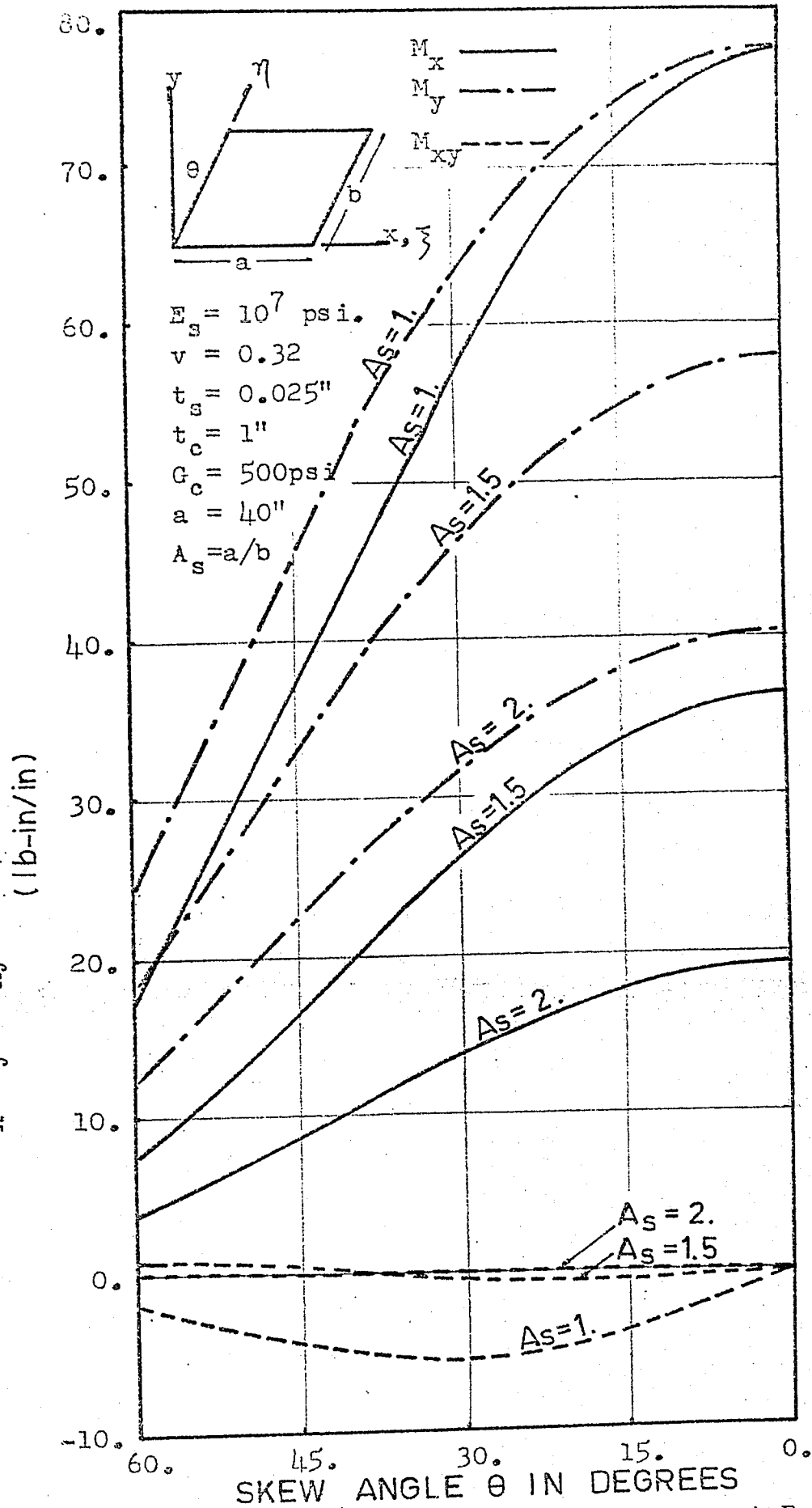


Fig. 22 Variation of Centre Moments with Aspect Ratio & Angle of Skew for Simply Supported Skew Sandwich Plates

Centre Moments M_x & M_y for Skewed Sandwich Plates Under Uniformly

Distributed Loading. (lb-in/in)

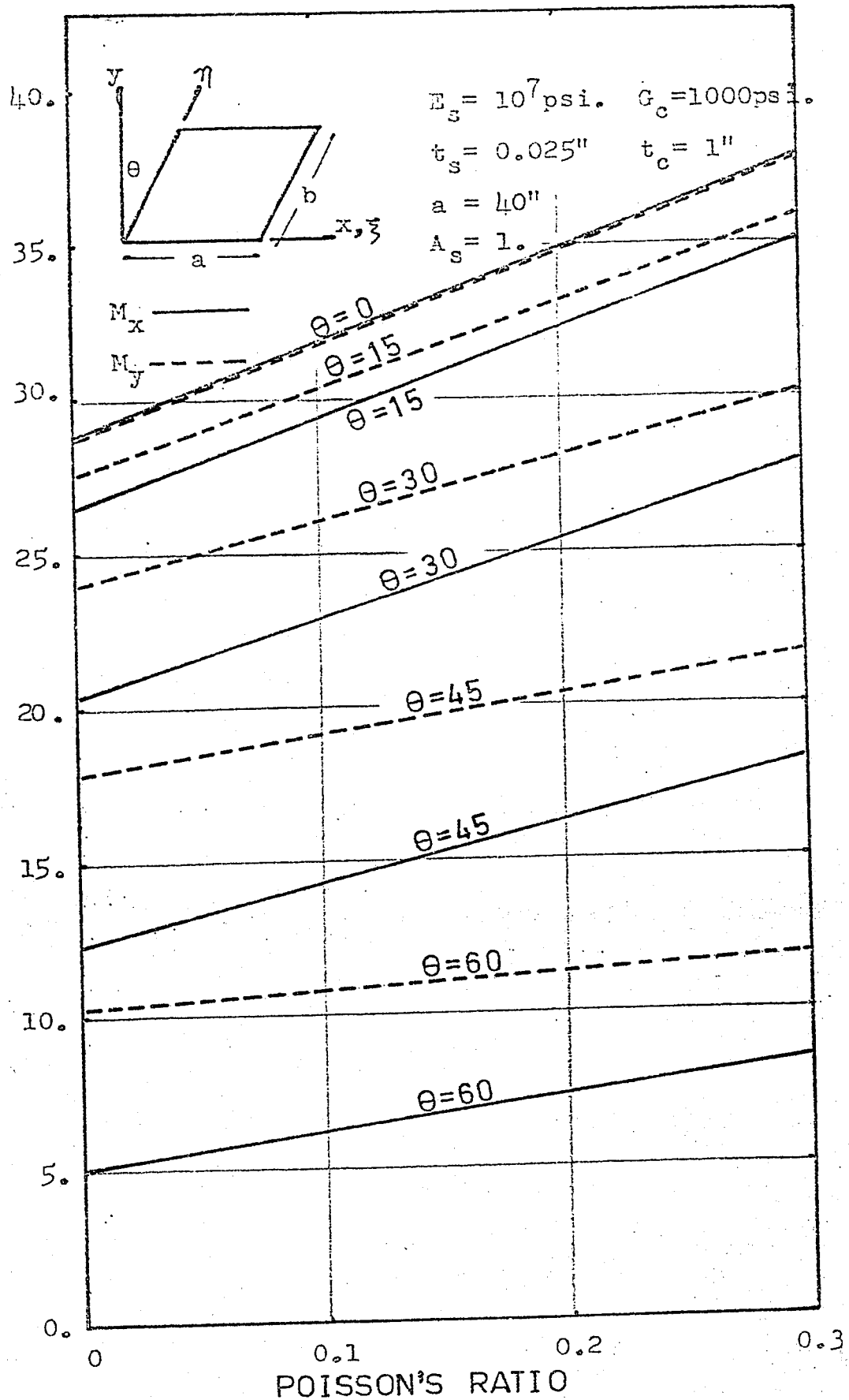


Fig. 23 Effect of Poisson's Ratio on Clamped Skew Sandwich Plate

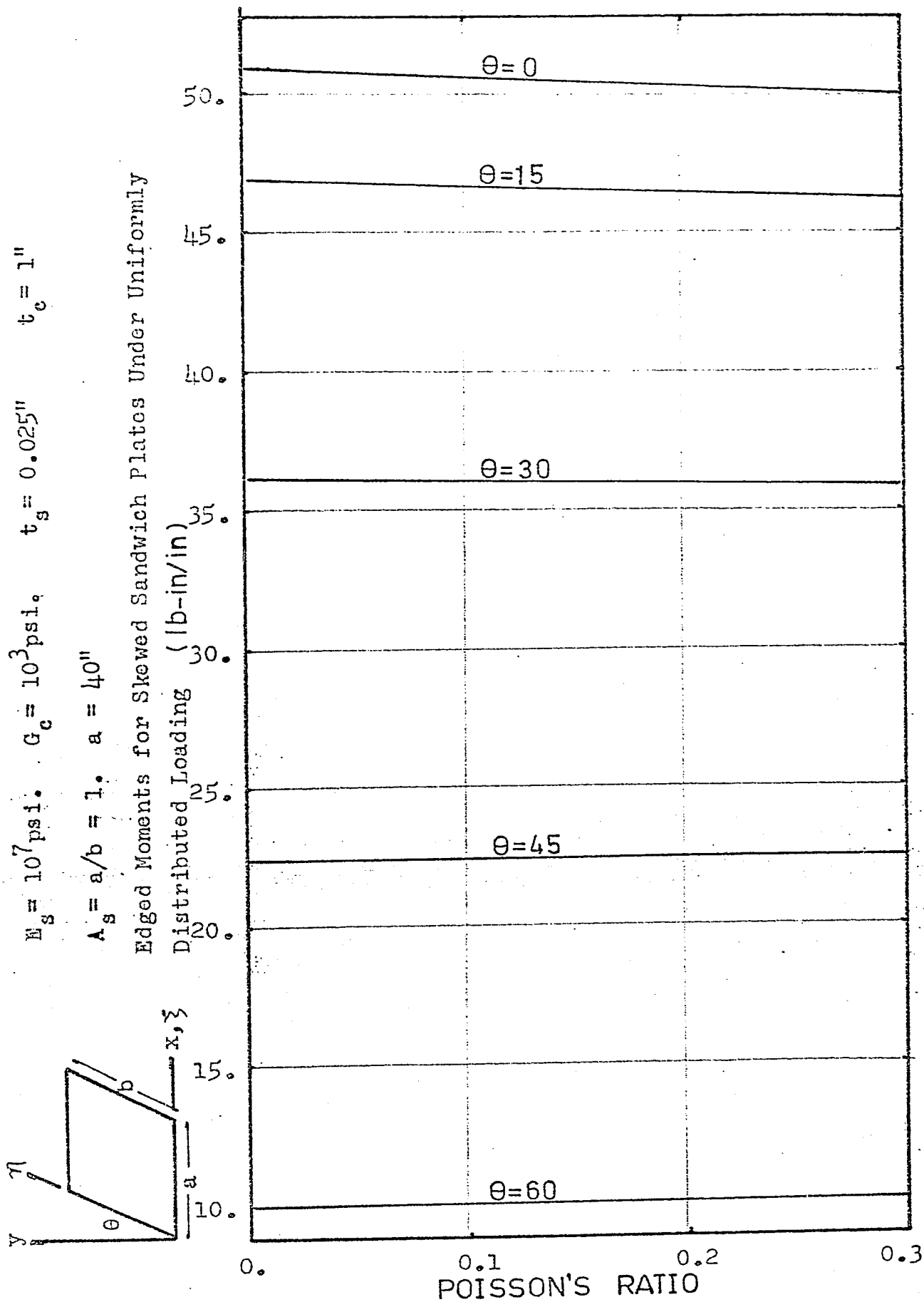


Fig. 24 Effect of Poisson's Ratio on Clamped Skew Sandwich Plate

Centre Moments M_x & M_y for Skewed Sandwich Plates Under Uniformly Distributed Loading. (lb-in/in)

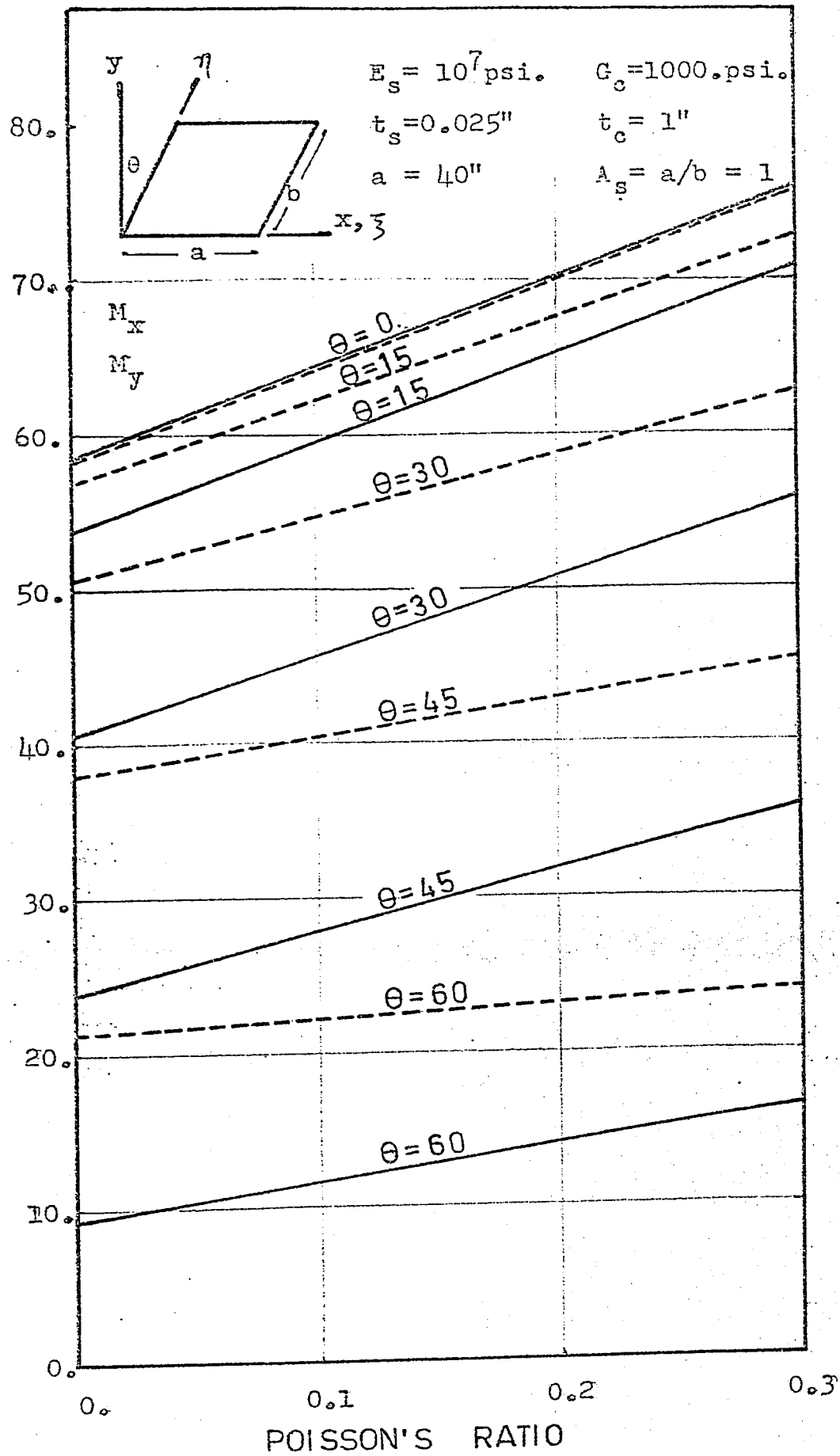


Fig. 25 Effect of Poisson's Ratio on Simply Supported Skew Sandwich Plate

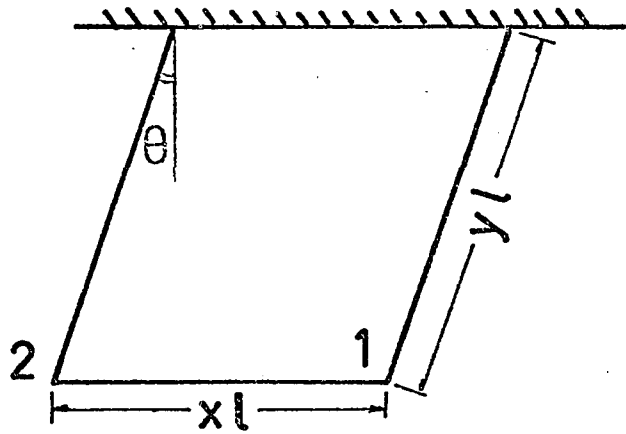


Fig. 26. Cantilever layout

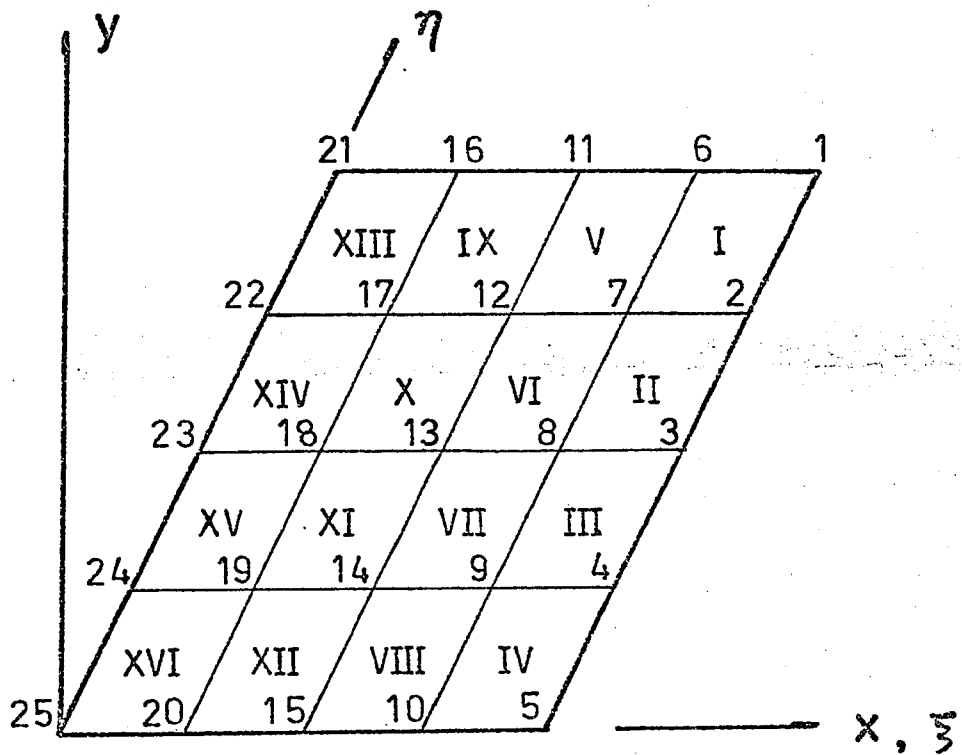


Fig. 27. Discretization and Node Numbering of a Sandwich Plate

CHAPTER V

CONCLUSIONS

5.1 Further comments on present work

The application of the finite element method to the analysis of skew sandwich plates is presented in this study. Two displacement models were used to solve the problem. The solutions by using both elements were shown to agree with the solution of the associated differential equation. However, from a practical viewpoint, significant differences occur only when the sandwich core material are so stiff as to fall beyond the range of most sandwich construction designs.

The results of this investigation can be applied to sandwich plate of constant thickness and equal skin with skew and rectangular shapes, for any type of boundaries and loaded by any system of static loads.

Four major advantages have been found when this work was carried out:

- (1) The choice of displacement function for complex geometric boundary conditions is not

difficult, because it is not dependent upon the boundary conditions at the nodes.

(2) In this method the partial differential equations of the problem are not used and need not be known, thus avoiding a higher mathematical analysis.

(3) This method provides the solution of skew and rectangular sandwich plates with general types of edge conditions.

(4) Some advantages may be taken in solving the simultaneous equations of representing stiffness matrices which possess symmetrical and banded properties. This usually will save considerable computer storage and time.

Two major disadvantages can be noted:

(1) Large numbers of simultaneous equations have to be solved in this work especially when the more refined element is used. This may lead to a serious problem with regard to storage, computation time and accuracy.

(2) Showing convergence and finding error bounds are difficult matters in this method. The dual analysis sometimes can only check whether true

convergence occurs.

It should be further mentioned here that choice of element type influences the mathematical character of the problem to some extent. Elements having many degrees of freedom produce a large element stiffness matrix and thus increase the size of the stiffness matrix for the structure, compared with elements with fewer degrees of freedoms. If the elements having the higher number of degrees of freedom require a reduced number of nodal points for the same accuracy then this increase of size of the structure stiffness matrix may not be critical. If this is not the case, the inversion of the stiffness matrix may cause considerably more time and computer core for the more refined element. Based on the data analysis in this investigation, it can be seen that the "simple element" (Three degrees of freedoms per node) seems to produce results which are nearly as good as the more complicated element (Five degrees of freedoms per node) for skew sandwich plate analysis. From results obtained in this study, it is apparent that fairly accurate results can be obtained by means of the less precise element. It is therefore suggested that for practical design purposes, the

less precise element should be used to minimize computer time and storage.

5.2 Conclusions

The main conclusion which can be drawn from this investigation may be summarized as follows:

(1) For rectangular sandwich plate analysis, the use of more terms in the displacement function will produce better results on general types of edge conditions.

(2) For skew sandwich plate analysis, the first element (Three degrees of freedom per node), seem to produce results which were as good as the second element (Five degrees of freedom per node), but the computer storage and time are considerably reduced in the first element. It implies that the first element has more practical usage in design work.

(3) For very stiff cores, more reliable results will be obtained from a general composite plate theory since these elements are not applicable.

(4) The finite elements used seem to have better convergence characteristics in soft core sandwich plates than hard core sandwich plates.

(5) If the facings are relatively thick in comparison with the core, the shear load will be carried to a considerable extent by the faces. Reliable results can be obtained by the elements used in this investigation when the thickness of the faces is small, i.e. $t_s/t_c \leq 1/20$.

(6) In clamped skew sandwich plates the central deflection decreases as the skew angle increases. This is due to the increased rigidity of the plate at the obtuse corners. But this effect diminishes with decreasing flexibility of the core.

(7) Both the maximum centre and edge moment decrease with increase in the aspect ratio. In the case of clamped and simply supported skew sandwich plates, the central moment in x direction seems to decrease more rapidly than the moment in y direction. The effect of plate aspect ratio upon the twisting moment appears to be insignificant in both cases.

(8) For skew cantilevered sandwich plate analysis, significant concentrated moments at the obtuse corner can be expected. This value increases and decreases again as the skew angle increases. The pitch shifts to higher skew with decreasing flexibility of the core.

(9) Increase in the Poisson's ratio augments the central moment, but this effect diminishes with an increase in the skew angles. Variation of Poisson's ratio with moment has been found to be linear for clamped and simply supported skewed sandwich plates subject to a uniformly distributed load.

(10) In general, the stress obtained by this method appears to be less accurate near the boundaries than it is elsewhere in the plate.

(11) For skew sandwich plate analysis, the two finite element models yield results which closely agree with existing values by other investigators. Hence it is believed that a reliable method of solution to skewed sandwich plates subjected to various boundary conditions has been found.

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APPENDIX I

The matrix $[R_f] = \int_0^b \int_0^a [B^T][H^T][D][H][B] d\xi d\eta$ appearing in eqn.(3-30) is a symmetric matrix. The elements in the lower part are defined as,

$$R(1,1) = 0.$$

$$R(2,1) = 0.$$

$$R(3,1) = 0.$$

$$R(4,1) = 0.$$

$$R(5,1) = 0.$$

$$R(6,1) = 0.$$

$$R(7,1) = 0.$$

$$R(8,1) = 0.$$

$$R(2,2) = (1+z_1)ab$$

$$R(3,2) = -(z_2)ab$$

$$R(4,2) = (ab^2)/2 + (ab^2)/2 \cdot z_1 - (a^2b)/2 \cdot z_2$$

$$R(5,2) = 0.$$

$$R(6,2) = (-z_4 - z_3)ab$$

$$R(7,2) = z_5ab$$

$$R(8,2) = -ab^2z_4/2 - ab^2z_3/2 + a^2bz_5/2$$

$$R(3,3) = abz_7$$

$$R(4,3) = -ab^2z_2/2 + a^2bz_7/2$$

$$R(5,3) = 0.$$

$$R(6,3) = abz_6$$

$$R(7,3) = 0.$$

$$R(8,3) = ab^2 z_6 / 2$$

$$R(4,4) = ab^3 / 3 + ab^3 z_1 / 3 - a^2 b^2 z_2 / 2 + a^3 b z_7 / 3$$

$$R(5,4) = 0.$$

$$R(6,4) = -ab^2 z_4 / 2 - ab^2 z_3 / 2 + a^2 b z_6 / 2$$

$$R(7,4) = ab^2 z_5 / 2$$

$$R(8,4) = -ab^3 z_4 / 3 - ab^3 z_3 / 3 + a^2 b^2 z_5 / 4 + a^2 b^2 z_6 / 4$$

$$R(5,5) = 0.$$

$$R(6,5) = 0.$$

$$R(7,5) = 0.$$

$$R(8,5) = 0.$$

$$R(6,6) = (z_1 / z_8 + z_8) ab$$

$$R(7,6) = -(z_2 / z_8) ab$$

$$R(8,6) = ab^2 (z_1 / z_8) / 2 + ab^2 z_8 / 2 - a^2 b (z_2 / z_8) / 2$$

$$R(7,7) = (z_7 / z_8) ab$$

$$R(8,7) = -ab^2 (z_2 / z_8) / 2 + a^2 b (z_7 / z_8) / 2$$

$$R(8,8) = ab^3 (z_1 / z_8) / 3 + ab^3 z_8 / 3 - a^2 b^2 (z_2 / z_8) / 2 + a^3 b (z_7 / z_8) / 3$$

where

$$z_1 = \left(\frac{1-v}{2} \right) \cot^2 \phi$$

$$z_2 = \left(\frac{1-v}{2} \right) \csc \phi \cot \phi$$

$$z_3 = \left(\frac{1-v}{2} \right) \cot \phi$$

$$z_4 = v \cot \phi$$

$$z_5 = v \csc \phi$$

$$z_6 = \left(\frac{1-v}{2} \right) \csc \phi$$

$$z_7 = \left(\frac{1-v}{2} \right) \csc^2 \phi$$

$$z_8 = \left(\frac{1-v}{2} \right)$$

Note: Each element mentioned above should be multiplied

by a factor, $\frac{E_f t_s}{(1-v^2)}$

APPENDIX II

The matrix $[R_c] = \int_0^{t/2} \int_0^a \int_0^b B^T H^T D H B d\xi d\eta d\zeta$

It is a symmetric matrix, only its lower triangle is given

here,

$$R(1,1) = t/2 \cdot ab \frac{1-2v}{2(1-v)}$$

$$R(2,1) = a^2/2 \cdot t/2 \cdot b \frac{1-2v}{2(1-v)}$$

$$R(3,1) = a \cdot b^2/2 \cdot t/2 \cdot \frac{1-2v}{2(1-v)}$$

$$R(4,1) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot \frac{1-2v}{2(1-v)}$$

$$R(10,1) = ab \cdot t/2 \cdot \frac{1-2v}{2(1-v)}$$

$$R(12,1) = a \cdot b^2/2 \cdot t/2 \cdot \frac{1-2v}{2(1-v)}$$

$$R(2,2) = 1/3(t/2)^3 ab + 1/3(t/2)^3 ab \cot\phi \frac{1-2v}{2(1-v)} + a^3/3 \cdot t/2 \cdot b \frac{1-2v}{2(1-v)}$$

$$R(3,2) = -1/3(t/2)^3 ab \cot\phi \csc\phi \frac{1-2v}{2(1-v)} + a^2/2 \cdot b^2/2 \cdot t/2$$

$$\frac{1-2v}{2(1-v)}$$

$$R(4,2) = b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot a + b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot a \cot^2\phi$$

$$\frac{1-2v}{2(1-v)} - b \cdot a^2/2 \cdot 1/3 \cdot (t/2)^3 \cot\phi \csc\phi \cdot \frac{1-2v}{2(1-v)}$$

$$+ a^3/3 \cdot b^2/2 \cdot (t/2) \frac{1-2v}{2(1-v)}$$

$$R(6,2) = -1/3 \cdot (t/2)^3 ab \cot\phi \frac{v}{1-v} - 1/3 \cdot (t/2)^3 ab \cot\phi \frac{1-2v}{2(1-v)}$$

$$R(7,2) = 1/3 \cdot (t/2)^3 ab \csc\phi \frac{v}{1-v}$$

$$R(8,2) = -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot\phi \frac{v}{1-v} - b^2/2 \cdot 1/3 \cdot (t/2)^3$$

$$a \cot\phi \frac{1-2v}{2(1-v)} + a^2/2 b \cdot 1/3 \cdot (t/2)^3 \csc\phi \frac{v}{1-v}$$

$$R(10,2) = a^2/2 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(12,2) = a^2/2 \cdot b^2/2 \cdot (t/2) \cdot (1-2v)/2(1-v)$$

$$R(3,3) = ab \cdot 1/3 \cdot (t/2)^3 \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$\begin{aligned} R(4,3) = & -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot \phi \csc \phi \cdot (1-2v)/2(1-v) \\ & + a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \csc^2 \phi \cdot (1-2v)/2(1-v) \\ & + a^2/2 \cdot b^3/3 \cdot (t/2) \cdot (1-2v)/2(1-v) \end{aligned}$$

$$R(6,3) = 1/3 \cdot (t/2)^3 \cdot \csc \phi \cdot (1-2v)/2(1-v) \cdot ab$$

$$R(8,3) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(10,3) = a \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(12,3) = a \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$\begin{aligned} R(4,4) = & a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^3 + a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^3 \\ & \cot^2 \phi \cdot (1-2v)/2(1-v) - 2 \cdot a^2/2 \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \\ & \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + 1/3 \cdot a^3 b \cdot 1/3 \cdot (t/2)^3 \\ & \csc^2 \phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v) \end{aligned}$$

$$\begin{aligned} R(6,4) = & -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot \phi \cdot v/(1-v) - a \cdot b^2/2 \cdot \\ & 1/3 \cdot (t/2)^3 \cot \phi \cdot (1-2v)/2(1-v) + 1/2 \cdot a^2 b \cdot 1/3 \\ & (t/2)^3 \csc \phi \cdot v/(1-v) \end{aligned}$$

$$R(7,4) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \csc \phi \cdot v/(1-v)$$

$$\begin{aligned} R(8,4) = & -1/3 \cdot b^3 a \cdot 1/3 \cdot (t/2)^3 \cot \phi \cdot v/(1-v) - 1/3 \cdot b^3 a \\ & 1/3 \cdot (t/2)^3 \cot \phi \cdot (1-2v)/2(1-v) \end{aligned}$$

$$R(10,4) = a^2/2 \cdot b^2/2 \cdot (t/2) \cdot (1-2v)/2(1-v)$$

$$R(5,5) = ab \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(7,5) = a \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(8,5) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(10,5) = -ab \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(11,5) = ab \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,5) = -a \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(6,6) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \cot^2\phi + ab \cdot 1/3 \cdot (t/2)^3 \cdot (1-2v)/2(1-v) + 1/3 \cdot a^3 b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(7,6) = -ab \cdot 1/3 \cdot (t/2)^3 \cot\phi \csc\phi + a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(8,6) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot^2\phi + a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot (1-2v)/2(1-v) - a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \cot\phi \csc\phi + 1/3 a^3 \cdot 1/2 \cdot b^2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(10,6) = -a^2/2 \cdot b \cdot t/2 \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(11,6) = a^2/2 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,6) = -a^2 b^2/4 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(7,7) = ab \cdot 1/3 \cdot (t/2)^3 \csc^2\phi + a \cdot 1/3 \cdot b^3 \cdot t/2 \cdot \frac{(1-2v)}{2(1-v)}$$

$$R(8,7) = -b^2/2 \cdot a \cdot 1/3 \cdot (t/2)^3 \cdot \cot\phi \csc\phi + a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \csc^2\phi + a^2/2 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(10,7) = -a \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(11,7) = a \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,7) = -a \cdot b^3/3 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(8,8) = b^3/3 \cdot 1/3 \cdot (t/3)^3 \cdot a \cdot \cot^2 \phi + a \cdot b^3/3 \cdot 1/3 \cdot (t/3)^3 \\ (1-2v)/2(1-v) - a^2/2 \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \csc \phi \\ - 1/2 \cdot a^2 \cdot b^2/2 \cdot 1/3 \cdot (t/3)^3 \cdot \cot \phi \csc \phi + a^3/3 \cdot b \\ 1/3 \cdot (t/2)^3 \csc^2 \phi + a^3/3 \cdot b^3/3 \cdot (1-2v)/2(1-v) \cdot t/2$$

$$R(10,8) = -a^2/2 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(11,8) = a^2/2 \cdot b^2/2 \cdot \csc \phi \cdot (1-2v)/2(1-v) \cdot t/2$$

$$R(12,8) = -a^2/2 \cdot 1/3 \cdot b^3 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + 1/3 \\ a^3 \cdot b^2/2 \cdot \csc \phi \cdot (1-2v)/2(1-v) \cdot t/2$$

$$R(10,10) = ab \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + ab \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(11,10) = -ab \cdot t/2 \cot \phi \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(12,10) = a \cdot b^2/2 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^2/2 \cdot t/2 \\ (1-2v)/2(1-v) - a^2/2 \cdot b \cdot t/2 \cot \phi \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(11,11) = ab \cdot t/2 \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(12,11) = -a \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + a^2/2 \\ b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(12,12) = a \cdot b^3/3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^3/3 \cdot t/2 \\ (1-2v)/2(1-v) + a^3/3 \cdot b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v) \\ - 2 \cdot a^2/2 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v)$$

Note: Each element mentioned above should be multiplied by a factor, $E(1-v)/(1+v)(1-2v)$.

APPENDIX III

The element stiffness matrix $[K_c (20,20)]$ for five degrees of freedom is given by eqn.(3-45)

$$[K_c] = 2 \sin \phi [A_c^{-1}]^T [R_c] [A_c^{-1}]$$

Since R_c is a symmetric matrix, only its lower triangle is given here,

$$R(1,1) = t/2 \cdot ab \cdot (1-2\nu)/2(1-\nu)$$

$$R(2,1) = a^2/2 \cdot t/2 \cdot b \cdot (1-2\nu)/2(1-\nu)$$

$$R(3,1) = a \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(4,1) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(10,1) = a \cdot b \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(12,1) = a^2 \cdot b \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(13,1) = a \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(15,1) = a^3 \cdot b \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(16,1) = a^2 \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(17,1) = a \cdot b^3/3 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(19,1) = a^3 \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(20,1) = a \cdot b^4/4 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(2,2) = ab \cdot 1/3 \cdot (t/2)^3 + ab \cdot 1/3 \cdot (t/2)^3 \cot^2 \phi \cdot (1-2\nu)/2(1-\nu) + a^3/3 \cdot b \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(3,2) = -1/3 \cdot (t/2)^3 \cdot ab \cot \phi \csc \phi \cdot (1-2\nu)/2(1-\nu) + a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(4,2) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 + a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot^2 \phi \cdot (1-2\nu)/2(1-\nu) - a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \cot \phi \csc \phi \cdot (1-2\nu)/2(1-\nu) + 1/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot (1-2\nu)/2(1-\nu)$$

$$R(6,2) = -ab \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \cdot v/(1-v) - ab \cdot 1/3 \cdot (t/2)^3 \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(7,2) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \csc \phi \cdot v/(1-v)$$

$$R(8,2) = -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \cdot v/(1-v) - a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \csc \phi \cdot v/(1-v)$$

$$R(10,2) = a^2/2 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(12,2) = 2/3 \cdot a^3 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(13,2) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(15,2) = 3/4 \cdot a^4 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,2) = 2/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(17,2) = a^2/2 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(19,2) = 3/4 \cdot a^4 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(20,2) = a^2/2 \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(3,3) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(4,3) = -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(6,3) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(8,3) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(10,3) = a \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(12,3) = a^2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(13,3) = a \cdot b^5/5 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(15,3) = a^5 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,5) = a^2 \cdot b^5/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(17,5) = a \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(19,3) = a^3 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(20,3) = a \cdot b^5/5 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$\begin{aligned} R(4,4) = & a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^3 + a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^3 \\ & \cot^2 \phi \cdot (1-2v)/2(1-v) - a^2 \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \\ & \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b \cdot 1/3 \cdot (t/2)^3 \\ & \csc^2 \phi \cdot (1-2v)/2(1-v) + a^5/3 \cdot b^3/3 \cdot t/2 \cdot \\ & (1-2v)/2(1-v) \end{aligned}$$

$$\begin{aligned} R(6,4) = & -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cot \phi \cdot v/(1-v) - a \cdot b^2/2 \\ & 1/3 \cdot (t/2)^3 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot 1/3 \\ & (t/2)^3 \cdot \csc \phi \cdot (1-2v)/2(1-v) \end{aligned}$$

$$R(7,4) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \csc \phi \cdot v/(1-v)$$

$$\begin{aligned} R(8,4) = & -a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \cdot v/(1-v) - a \cdot b^3/3 \\ & 1/3 \cdot (t/2)^3 \cot \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b^2/2 \cdot 1/3 \\ & (t/2)^3 \cdot \csc \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b^2/2 \cdot 1/3 \cdot \\ & (t/2)^3 \cdot \csc \phi \cdot v/(1-v) \end{aligned}$$

$$R(10,4) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(12,4) = 2/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(13,4) = a^2/2 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(15,4) = 3/4 \cdot a^4 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,4) = 2/3 \cdot a^5 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(17,4) = a^2/2 \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(19,4) = 3/4 \cdot a^4 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(20,4) = a^2/2 \cdot b^5/5 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(5,5) = ab \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(6,5) = a^2/2 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(7,5) = a \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(8,5) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(10,5) = -ab \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(11,5) = ab \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,5) = -a^2b \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(13,5) = -a \cdot b^2/2 \cdot t/2 \cot\phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(14,5) = ab^2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(15,5) = -a^3b \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(16,5) = -a^2 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(17,5) = -a \cdot b^3/3 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(18,5) = ab^3 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(19,5) = -a^3 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^4/4 \cdot b \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(20,5) = -a \cdot b^4/4 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + 3/2 \cdot a^2 \cdot b^3/3 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(6,6) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \cot^2 \phi + ab \cdot 1/3 \cdot (t/2)^3 \cdot (1-2v)/2(1-v) + 1/3 \cdot a^3 \cdot b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(7,6) = -ab \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \csc \phi + a^2/2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(8,6) = a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot^2 \phi + a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot (1-2v)/2(1-v) - a^2/2 \cdot b \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \csc \phi + a^3/3 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(10,6) = -a^2/2 \cdot b \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(11,6) = a^2/2 \cdot b \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(12,6) = -2/3 \cdot a^3 \cdot b \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(13,6) = -a^2/2 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(14,6) = a^2 \cdot b^2/2 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(15,6) = -3/4 \cdot a^4 \cdot b \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(16,6) = -2/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^4/4 \cdot b \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(17,6) = -a^2/2 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + 2/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(18,6) = 3/2 \cdot a^2 \cdot b^3/3 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(19,6) = -3/4 \cdot a^4 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^5/5 \cdot b \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(20,6) = -a^2/2 \cdot b^4/4 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^3/3 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(7,7) = ab \cdot 1/3 \cdot (t/2)^3 \cdot \csc^2 \phi + a \cdot b^3/3 \cdot t/2 \\ (1-2v)/2(1-v)$$

$$R(8,7) = -a \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \cdot \cot \phi \csc \phi + a^2/2 \cdot b \\ 1/3 \cdot (t/2)^3 \csc^2 \phi + a^2/2 \cdot b^3/3 \cdot t/2 \\ (1-2v)/2(1-v)$$

$$R(10,7) = -a \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(11,7) = a \cdot b^2/2 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(12,7) = -a^2 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(13,7) = -a \cdot b^3/3 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b^2/2 \\ t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(14,7) = 2a \cdot b^3/3 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(15,7) = -a^3 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v)$$

$$R(16,7) = -a^2 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b^2/2 \\ t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(17,7) = -a \cdot b^4/4 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^3/3 \\ t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(18,7) = 3/4 \cdot ab^4 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(19,7) = -a^3 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + a^4/4 \cdot b^2/2 \\ t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(20,7) = -a \cdot b^5/5 \cdot t/2 \cdot \cot \phi \cdot (1-2v)/2(1-v) + 3 a^2/2 \cdot \\ b^4/4 \cdot t/2 \cdot \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(8,8) = a \cdot b^3/3 \cdot 1/3 \cdot (t/2)^5 \cdot \cot^2 \phi + a \cdot b^3/3 \cdot 1/3 \\ (t/2)^3 \cdot (1-2v)/2(1-v) - a^2 \cdot b^2/2 \cdot 1/3 \cdot (t/2)^3 \\ \cot \phi \csc \phi + 1/3 \cdot a^3 \cdot b \cdot 1/3 \cdot (t/2)^3 \csc^2 \phi \\ + a^3/3 \cdot b^3/3 \cdot t/2 \cdot (1-2v) \cdot 2(1-v)$$

$$R(10,8) = -a^2/2 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(11,8) = a^2/2 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,8) = -2/3 \cdot a^3 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(13,8) = -a^2/2 \cdot b^3/3 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(14,8) = a^2 \cdot b^3/3 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(15,8) = -3/4 \cdot a^4 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v)$$

$$R(16,8) = -2/3 \cdot a^3 \cdot b^3/3 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^4/4 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(17,8) = -a^2/2 \cdot b^4/4 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + 2/3 \cdot a^3 \cdot b^3/3 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(18,8) = 3/2 \cdot a^2 \cdot b^4/4 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(19,8) = -3/4 \cdot a^4 \cdot b^3/3 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^5/5 \cdot b^2/2 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(20,8) = -a^2/2 \cdot b^5/5 \cdot t/2 \cdot \cot\phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^4/4 \cdot t/2 \cdot \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(10,10) = ab \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + ab \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(11,10) = -ab \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(12,10) = a^2b \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^2b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(13,10) = a \cdot b^2/2 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v) - a^2/2 \cdot b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(14,10) = -ab^2 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(15,10) = a^3b \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^3b \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,10) = a^2 \cdot b^2/2 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v) - a^3/3 \cdot b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(17,10) = a \cdot b^3/3 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a \cdot b^3/5 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(18,10) = -ab^3 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(19,10) = a^3 \cdot b^2/2 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v) - a^4/4 \cdot b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(20,10) = a \cdot b^4/4 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v) - a^2/2 \cdot b^3 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(11,11) = ab \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(12,11) = -a^2b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(13,11) = -a \cdot b^2/2 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + a^2/2 \cdot b \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(14,11) = ab^2 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(15,11) = -a^3b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(16,11) = -a^2 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + a^3/3 \cdot b \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(17,11) = -a \cdot b^3/3 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^2/2 \\ t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(18,11) = a \cdot b^3 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(19,11) = -a^3 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + a^4/4 \cdot b \\ t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(20,11) = -a \cdot b^4/4 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 3/2 a^2 \\ b^3/3 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(12,12) = 4/3 \cdot a^3 b \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + 4/3 \cdot a^3 b \\ t/2 \cdot (1-2v)/2(1-v)$$

$$R(13,12) = a^2 \cdot b^2/2 \cdot t/2 \cot^2\phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^2/2 \\ t/2 \cdot (1-2v)/2(1-v) - 2/3 \cdot a^3 b \cdot t/2 \cdot \cot\phi \csc\phi \\ (1-2v)/2(1-v)$$

$$R(14,12) = -a^2 b^2 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(15,12) = 6/4 \cdot a^4 b \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + 6/4 \cdot a^4 b \\ t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,12) = 4/3 a^3 \cdot b^2/2 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + 4/3 a^3 \\ b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v) - \frac{1}{3} a^4 b \cdot t/2 \cdot \cot\phi \csc\phi \\ (1-2v)/2(1-v)$$

$$R(17,12) = a^2 \cdot b^3/3 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^3/3 \\ t/2 \cdot (1-2v)/2(1-v) - 4/3 a^3 \cdot b^2/2 \cdot t/2 \cdot \cot\phi \csc\phi \\ (1-2v)/2(1-v)$$

$$R(18,12) = -a^2 b^3 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(19,12) = 6 \cdot a^4/4 \cdot b^2/2 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) \\ + 6 \cdot a^4/4 \cdot b^2/2 \cdot t/2 \cdot (1-2v)/2(1-v) - 2 \cdot a^5/5 \\ b \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v)$$

$$R(20,12) = a^2 \cdot b^4/4 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^4/4 \cdot t/2 \\ (1-2v)/2(1-v) - 6 \cdot a^5/3 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v)$$

$$R(13,15) = a \cdot b^3/3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^3/3 \cdot t/2 \\ (1-2v)/2(1-v) - a^2 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + a^5/3 \cdot b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(14,13) = -2a \cdot b^3/3 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^2/2 \\ t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(15,13) = a^5 \cdot b^2/2 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^2/2 \\ t/2 \cdot (1-2v)/2(1-v) - \frac{3}{4} a^4 b \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v)$$

$$R(16,13) = a^2 \cdot b^3/3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^3/3 \\ t/2 \cdot (1-2v)/2(1-v) - a^5 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + a^4/4 \cdot b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(17,13) = a \cdot b^4/4 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^4/4 \cdot t/2 \\ (1-2v)/2(1-v) - 3 \cdot a^2/2 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + 2 \cdot a^3/3 \cdot b^2/2 \cdot t/2 \cdot \csc^2 \phi \\ (1-2v)/2(1-v)$$

$$R(18,13) = -5 a \cdot b^4/4 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) \\ + 3 \cdot a^2/2 \cdot b^3/3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(19,13) = a^3 \cdot b^3/3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^3/3 \\ t/2 \cdot (1-2v)/2(1-v) - a^4 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + a^5/5 \cdot b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(20,15) = a \cdot b^5/5 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^5/5 \cdot t/2 \\ (1-2v)/2(1-v) - 2a^2 \cdot b^4/4 \cdot t/2 \cot \phi \csc \phi \\ (1-2v)/2(1-v) + a^3 \cdot b^3/3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(14,14) = 4a \cdot b^3/3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(15,14) = -a^3 b^2 \cdot t/2 \cot \phi \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(16,14) = -2a^2 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + 2 \\ a^3/3 \cdot b^2/2 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(17,14) = -\frac{1}{2} ab^4 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + 2 a^2 \cdot \\ b^3/3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(18,14) = 6a \cdot b^4/4 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(19,14) = -2 a^5 \cdot b^3/3 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + \frac{1}{2} a^4 \\ b^2/2 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(20,14) = -2a \cdot b^5/5 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) + 3a^2 \cdot \\ b^4/4 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(15,15) = 9/5 a^5 \cdot b \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + 9/5 a^5 b \\ t/2 \cdot (1-2v)/2(1-v)$$

$$R(16,15) = 6/4 a^4 \cdot b^2/2 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + 6/8 a^4 b^2 \\ t/2 \cdot (1-2v)/2(1-v) - 3/5 a^5 b \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v)$$

$$R(17,15) = a^5 \cdot b^3/3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^5/3 \\ t/2 \cdot (1-2v)/2(1-v) - 6/8 \cdot a^4 b^2 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v)$$

$$R(18,15) = -a^3 b^3 \cdot t/2 \cot \phi \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(19,15) = 9/10 a^5 b^2 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + 9/10 a^5 b^2 \\ t/2 \cdot (1-2v)/2(1-v) - \frac{1}{2} a^6 b \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v)$$

$$R(20,15) = a^3 \cdot b^4/4 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^4/4 \\ t/2 \cdot (1-2v)/2(1-v) - 9 \cdot a^4/4 \cdot b^3/3 \cdot t/2 \cot \phi \\ \csc \phi \cdot (1-2v)/2(1-v)$$

$$R(16,16) = 4/9 a^3 b^3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + 4/9 a^3 b^3 \cdot t/2 \\ (1-2v)/2(1-v) - a^4 \cdot b^2/2 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + a^5/5 \cdot b \cdot t/2 \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(17,16) = a^2 \cdot b^4/4 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a^2 \cdot b^4/4 \\ t/2 \cdot (1-2v)/2(1-v) - 5/9 a^3 b^3 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + \frac{1}{2} a^4 \cdot b^2/2 \cdot t/2 \csc^2 \phi (1-2v)/2(1-v)$$

$$R(18,16) = -3a^2 \cdot b^4/4 \cdot t/2 \cdot \cot \phi \csc \phi \cdot (1-2v)/2(1-v) \\ + a^3 \cdot b^3/3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(19,16) = \frac{1}{2} a^4 b^3 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + \frac{1}{2} a^4 b^3 \cdot t/2 \\ (1-2v)/2(1-v) - a^5 \cdot b^2/2 \cdot t/2 \cot \phi \csc \phi (1-2v)/2(1-v) \\ + a^6/6 \cdot b \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(20,16) = a^2 \cdot b^5/5 \cdot t/2 \cdot \cot^2 \phi (1-2v)/2(1-v) + a^2 \cdot b^5/5 \\ t/2 \cdot (1-2v)/2(1-v) - 7 a^3/3 \cdot b^4/4 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + 3 a^4/4 \cdot b^3/3 \cdot t/2 \cdot \csc^2 \phi \\ (1-2v)/2(1-v)$$

$$R(17,17) = a \cdot b^5/5 \cdot t/2 \cdot \cot^2 \phi (1-2v)/2(1-v) + a \cdot b^5/5 \cdot t/2 \\ (1-2v)/2(1-v) - a^2/2 \cdot b^4 \cdot t/2 \cdot \cot \phi \csc \phi \\ (1-2v)/2(1-v) + 4/9 a^3 b^3 \cdot t/2 \cdot \csc^2 \phi \cdot (1-2v)/2(1-v)$$

$$R(18,17) = -3 a \cdot b^5/5 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 3 a^2 b^4/4 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(19,17) = a^3 \cdot b^4/4 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v) - 7/12 a^4 b^3 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 2 a^5/5 \cdot b^2/2 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(20,17) = a \cdot b^6/6 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a \cdot b^6/6 \cdot t/2 \cdot (1-2v)/2(1-v) - 5 a^2/2 \cdot b^5/5 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 6 a^3/3 \cdot b^4/4 \cdot t/2 \cdot (1-2v)/2(1-v)$$

$$R(18,18) = 9 a \cdot b^5/5 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(19,18) = -3 a^3 \cdot b^4/4 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 3 a^4/4 \cdot b^3/3 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(20,18) = -\frac{1}{2} ab^6 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 9/10 a^5 b^5 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(19,19) = 9 a^5/5 \cdot b^3/3 \cdot t/2 \cot^2\phi \cdot (1-2v)/2(1-v) + 9 a^5/5 \cdot b^3/3 \cdot t/2 \cdot (1-2v)/2(1-v) - a^6 \cdot b^2/2 \cdot t/2 \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + a^7/7 \cdot b \cdot t/2 \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$R(20,19) = a^3 \cdot b^5/5 \cdot t/2 \cdot \cot^2\phi \cdot (1-2v)/2(1-v) + a^3 \cdot b^5/5 \cdot t/2 \cdot (1-2v)/2(1-v) - 10/16 a^4 b^4 \cdot t/2 \cdot \cot\phi \csc\phi \cdot (1-2v)/2(1-v) + 3 a^5/5 \cdot b^3/3 \cdot t/2 \cdot \csc^2\phi \cdot (1-2v)/2(1-v)$$

$$\begin{aligned}
 R(20,20) = & a \cdot b^7/7 \cdot t/2 \cdot \cot^2 \phi \cdot (1-2v)/2(1-v) + a \cdot b^7/7 \cdot \\
 & t/2 \cdot (1-2v)/2(1-v) - 3 a^2 \cdot b^6/6 \cdot t/2 \cdot \cot \phi \csc \phi \\
 & (1-2v)/2(1-v) + 3 a^3 \cdot b^5/5 \cdot t/2 \cdot \csc^2 \phi \\
 & (1-2v)/2(1-v)
 \end{aligned}$$

Note; Each of the elements should be multiplied by a factor

$$\frac{E(1-v)}{(1+v)(1-2v)} \cdot$$

The [A] Matrix for five degrees of freedom is given here as;

$\frac{t}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{t}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
$\frac{t}{2}$	$\frac{at}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{t}{2}$	$\frac{at}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	a	0	a^2	0	0	a^3	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	2a	0	0	$3a^2$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	a	0	0	a^2	0	0	a^3	0
$\frac{t}{2}$	0	$\frac{bt}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{t}{2}$	0	$\frac{bt}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	b	0	0	b^2	0	0	0	b^3	0	0
0	0	0	0	0	0	0	0	0	1	0	0	b	0	0	0	b^2	0	0	b^3
0	0	0	0	0	0	0	0	0	0	1	0	0	2b	0	0	0	$3b^2$	0	0
$\frac{t}{2}$	$\frac{at}{2}$	$\frac{bt}{2}$	$\frac{abt}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{t}{2}$	$\frac{at}{2}$	$\frac{bt}{2}$	$\frac{abt}{2}$	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	a	b	a^2	ab	b^2	a^3	a^2b	ab^2	b^3	a^3b	ab^3
0	0	0	0	0	0	0	0	0	1	0	2a	b	0	$3a^2$	2ab	b^2	0	$3a^2b$	b^3
0	0	0	0	0	0	0	0	0	0	1	0	a	2b	0	a^2	2ab	$3b^2$	a^3	$3ab^2$

APPENDIX IV

APPENDIX V

The matrices $[A^{-1}]$ and $[L]$ appearing in equation (3 - 63) are defined as follows,

FOR THREE DEGREES OF FREEDOM PER NODE

$$[L] = [1, \xi, \eta, \xi\eta]$$

$$[A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{a} & \frac{1}{a} & 0 & 0 \\ -\frac{1}{b} & 0 & \frac{1}{b} & 0 \\ \frac{1}{ab} & -\frac{1}{ab} & -\frac{1}{ab} & \frac{1}{ab} \end{bmatrix}$$

FOR FIVE DEGREES OF FREEDOM PER NODE

$$[L] = [1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^3\eta, \xi\eta^3]$$

$$[A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3}{a} & -\frac{2}{a} & 0 & \frac{3}{a} & -\frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{ab} & -\frac{1}{b} & -\frac{1}{a} & \frac{1}{ab} & 0 & \frac{1}{a} & \frac{1}{ab} & \frac{1}{b} & 0 & -\frac{1}{ab} & 0 & 0 \\ -\frac{3}{b} & 0 & -\frac{2}{b} & 0 & 0 & 0 & \frac{3}{b} & 0 & -\frac{1}{b} & 0 & 0 & 0 \\ \frac{2}{a} & \frac{1}{a} & 0 & -\frac{2}{a} & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{ab} & \frac{2}{ab} & 0 & -\frac{3}{ab} & \frac{1}{ab} & 0 & -\frac{3}{ab} & -\frac{2}{ab} & 0 & \frac{3}{ab} & \frac{1}{ab} & 0 \\ \frac{3}{ab} & 0 & \frac{2}{ab} & -\frac{3}{ab} & 0 & -\frac{2}{ab} & -\frac{3}{ab} & 0 & \frac{1}{ab} & \frac{3}{ab} & 0 & -\frac{1}{ab} \\ \frac{2}{b} & 0 & \frac{1}{b} & 0 & 0 & 0 & -\frac{2}{b} & 0 & \frac{1}{b} & 0 & 0 & 0 \\ -\frac{2}{ab} & -\frac{1}{ab} & 0 & \frac{2}{ab} & -\frac{1}{ab} & 0 & \frac{2}{ab} & \frac{1}{ab} & 0 & -\frac{2}{ab} & \frac{1}{ab} & 0 \\ -\frac{2}{ab} & 0 & -\frac{1}{ab} & \frac{2}{ab} & 0 & \frac{1}{ab} & \frac{2}{ab} & 0 & -\frac{1}{ab} & -\frac{2}{ab} & 0 & \frac{1}{ab} \end{bmatrix}$$

APPENDIX VI

COMPUTER PROGRAM

General

A general computer program had been written to perform the analysis of the skewed sandwich plate described previously. The program is written in Fortran IV language for the IBM-360 model 65 computer system.

The flow charts of the main program together with the subroutine which is used to generate the element stiffness matrix are presented in this section. This program can analyse the sandwich plate with any number of nodes and elements, any type of loading system and any type of boundary conditions only with slight modifications.

The flow chart and the listing with the comment statements make this program self explanatory. A brief description of the program is given below:

a) Input data

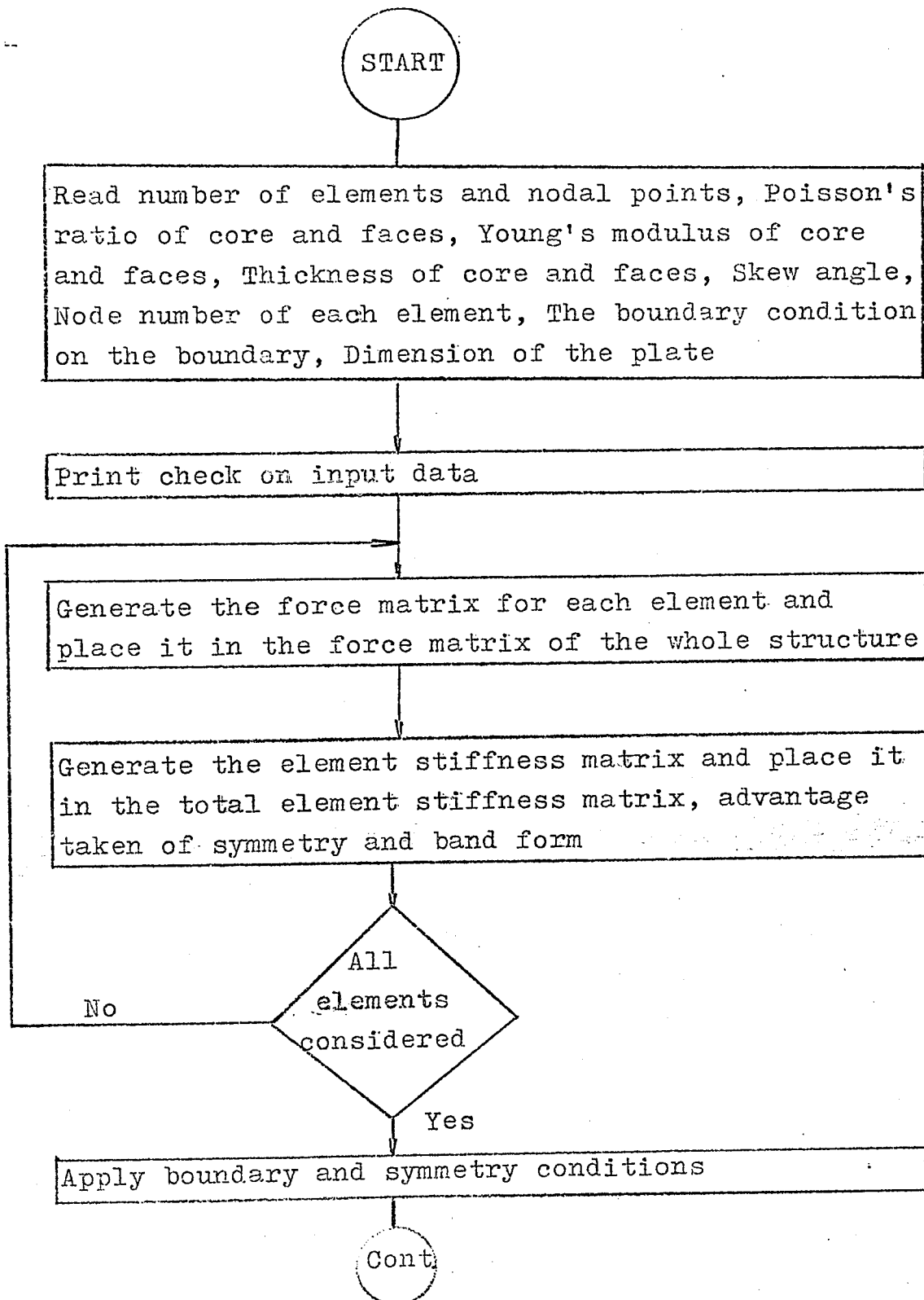
1. Geometrical dimension of the structure in terms of length of plate in ξ and η directions, thicknesses of the core and faces, skew angle of the plate.
2. Mechanical properties of the plate in terms of Young's modulus of the faces, shear modulus of the core, Poisson's ratios of the core and faces.
3. Number of points on the boundary, nodal boundary conditions, and node numbering system.
4. A subroutine BUILD is used to generate the force matrix

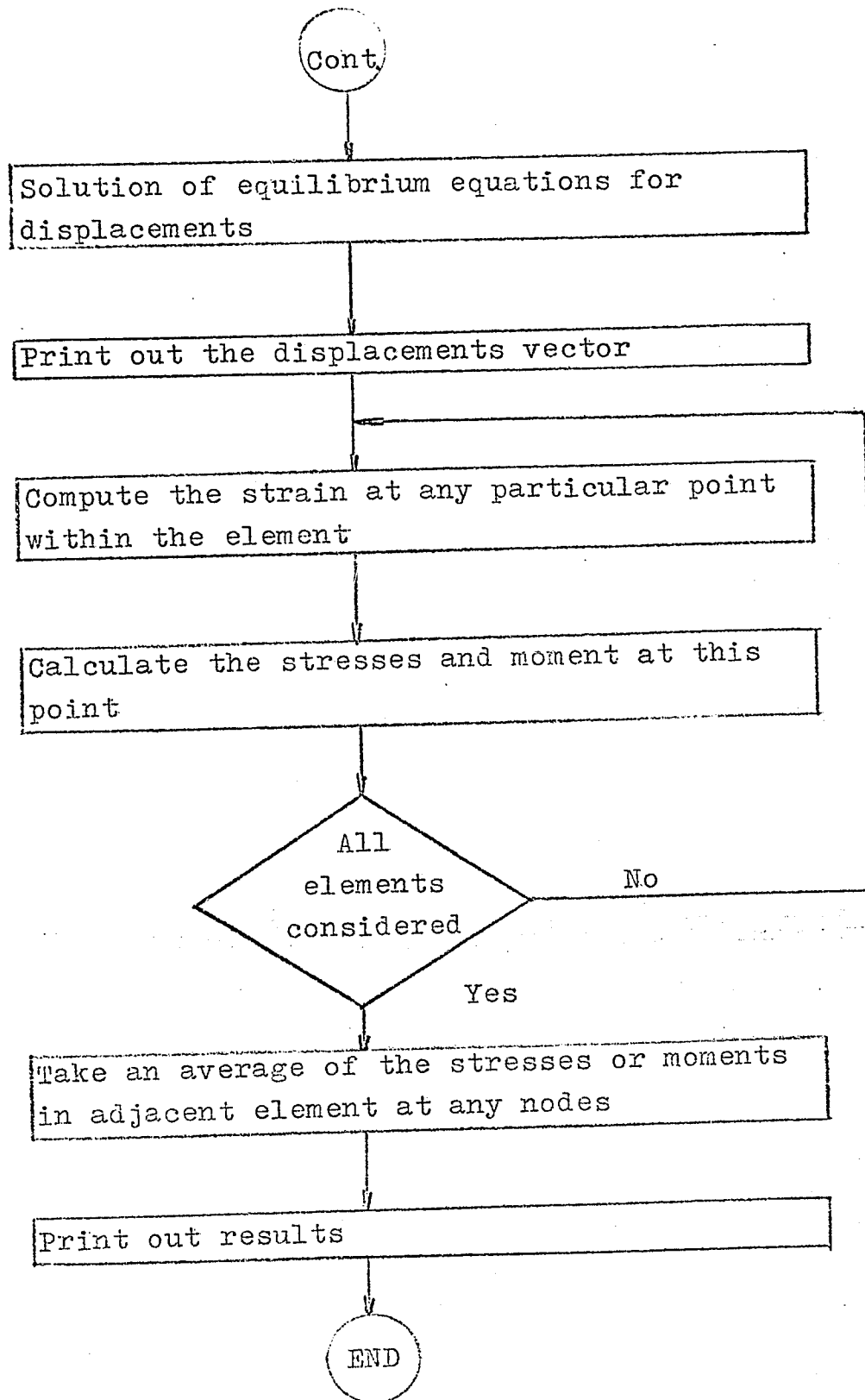
for uniform loading condition. When the concentrated loading is applied to the structure, the force matrix should be read from cards.

b) Output data

1. The complete input data is printed as a check.
2. Displacements are printed at every grid points of the plate.
3. All stresses and moments are printed at every grid points of the plate.

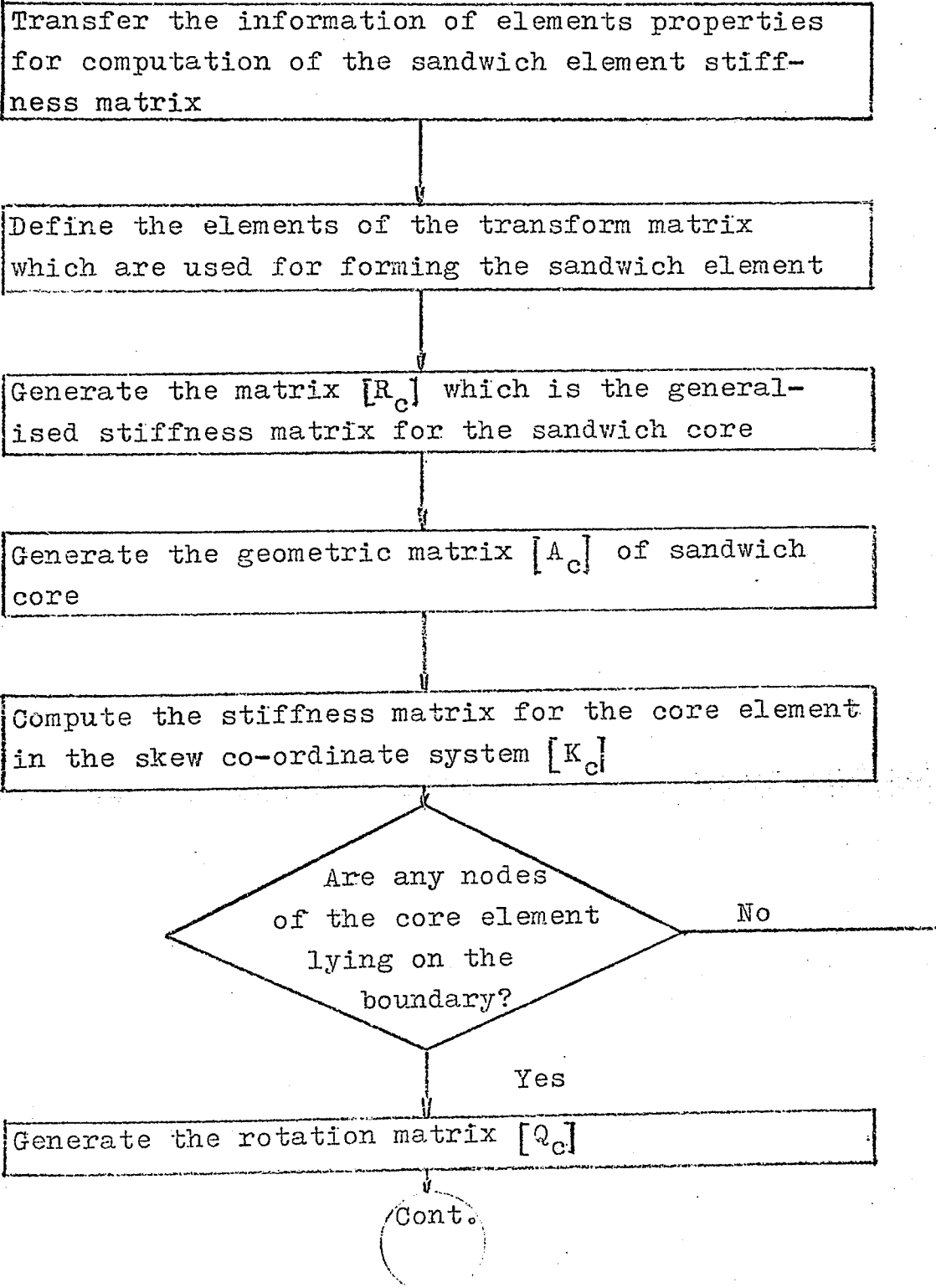
FLOW CHART

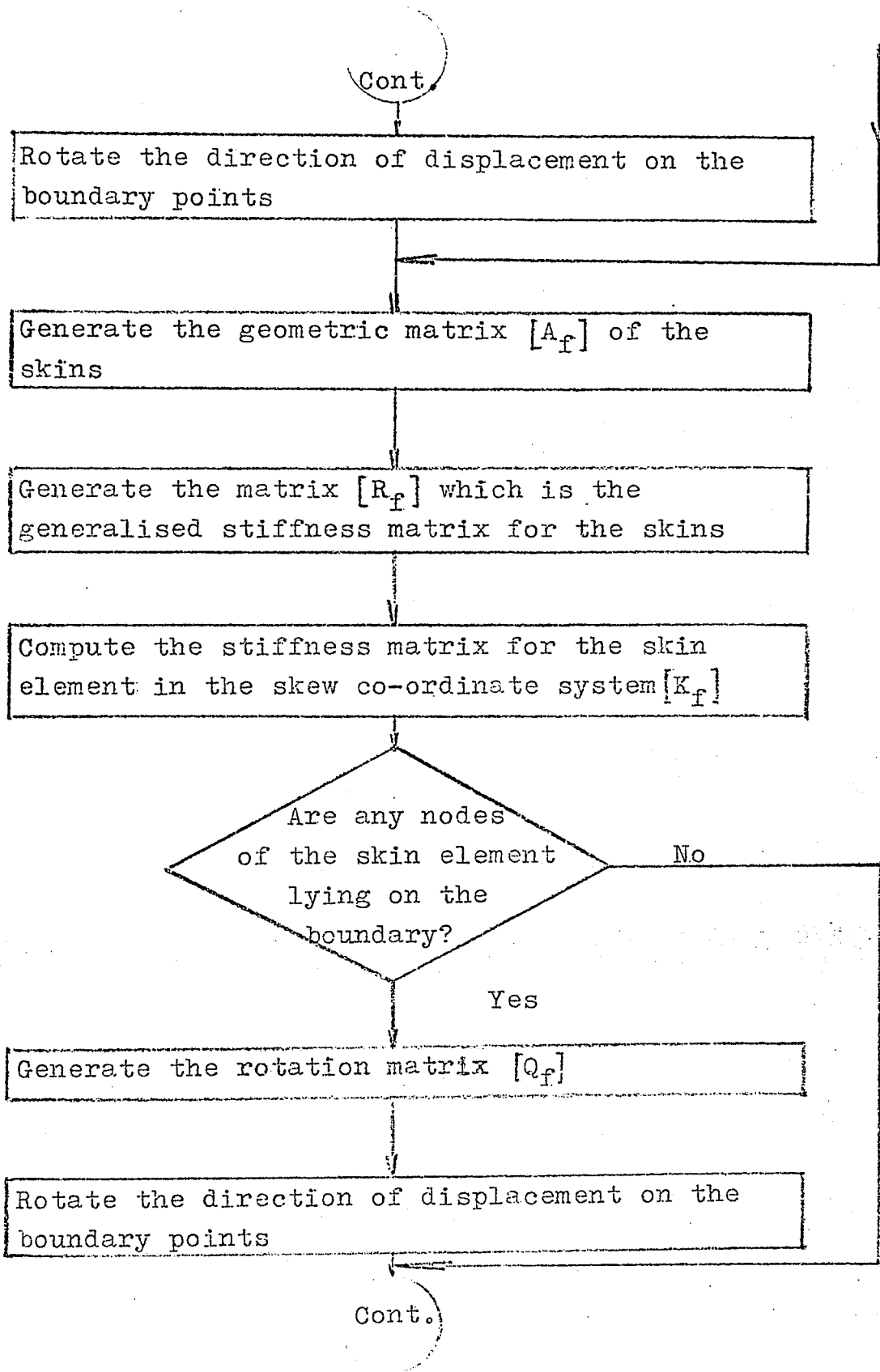




FLOW CHART

(Generate the element stiffness matrix)





Cont.

Combine skin stiffness and core stiffness to form element stiffness matrix for skew sandwich plates

Print out results

Return to the main program

FINITE ELEMENT ANALYSIS OF SKEW SANDWICH PLATES- BY W.L.KHOK

THIS PROGRAM CAN HANDLE THE SKEW SANDWICH PLATE STRUCTURE WITH DIFFERENT
ELEMENT PROPERTIES AND DIFFERENT BOUNDARY CONDITION AND ALSO ANY
TYPE OF LOADING CONDITION WITH SOME MODIFICATION.

CASE- FOUR SIDES CLAMPED

GRID- 4X4

A(I)-LENGTH OF ELEMENT IN X DIRECTION

B(I)-LENGTH OF ELEMENT IN Y DIRECTION

TC-THICKNESS OF THE CORE

TS-THICKNESS OF THE FACES

EC-YOUNGS MODULUS FOR THE CORE

ES-YOUNGS MODULUS FOR THE FACE

VC-POISSONS RATIO FOR THE CORE

VS-POISSONS RATIO FOR THE FACE

GC-SHEAR MODULUS OF THE CORE

SS(I,J)-ELEMENT STIFFNESS MATRIX

PL(I)-ELEMENT LOAD VECTOR

NPB(I)-NUMBER OF POINTS ON THE BOUNDARY

NBC(I)-NODAL BOUNDARY CONDITION

STPSTF(I,J)-STRUCTUREAL STIFFNESS MATRIX

FRCDSP(I)-DISPLACEMENT OR LOAD VECTOR

NPI(I)-NODAL POINT 1

NPJ(I)-NODAL POINT 2

NPK(I)-NODAL POINT 3

NPL(I)-NODAL POINT 4

NEX-NUMBER OF ELEMENTS IN THE X DIRECTION

NEY-NUMBER OF ELEMENTS IN THE Y DIRECTION

XL-LENGTH OF PLATE IN THE X DIRECTION

YL-LENGTH OF PLATE IN THE Y DIRECTION

NB-HALF BAND WIDTH

NUMN3-NUMBER OF EQUATIONS

NUMNP-NUMBER OF NODAL POINTS

C NUMEL-NUMBER OF ELEMENTS
C NPROB-NUMBER OF PROBLEMS
C

0001 C *****
C IMPLICIT REAL*8(A-H,O-Z)
C *****

C
C COMMON AND DIMENSION STATEMENTS
C THE DIMENSION STATEMENTS HAVE TO BE CHANGED EVERY TIME WE CHANGE THE
C NUMBER OF ELEMENTS

0002 C *****
C COMMON/AREA1/EC,ES,IC,IS,VC,VS,A,B,ALF
C COMMON/AREA2/NUMNP,NUMEL,NPI,NPJ,NPK,NPL,FRCDSP,LM
C DIMENSION A(16),B(16),TC(16),EC(16),VC(16),TS(16),ES(16),VS(16)
C DIMENSION SS(20,20),NPR(16),LM(4),NBC(16),NCB(4),STSTF(125,35),
C @FRCDSP(125),NPI(16),NPJ(16),NPK(16),NPL(16)
C DIMENSION PL(20)
C WRITE(3,20)
C

0003 C *****
C WRITE(3,18)
C *****

C DATA INPUT
C READ THE STRUCTURE PROPERTIES
C

0009 C *****
C READ(1,40) NPROB
C
C 1 CONTINUE

0010 C *****
C NPROB=NPROB-1
C
C READ(1,46) ALFB
C
C READ(1,38) NUMEL,NUMNP,NUMRC,XI,YI,NEX,NEY
C
C DO 41 L=1,NUMEL
C 41 READ(1,36)(M,A(M),B(M),TC(M),EC(M),VC(M),TS(M),ES(M),VS(M),NPI(M),
C @NPJ(M),NPK(M),NPL(M))
C

0016 C *****
C DO 114 I=1,NUMEL
C ES(I)=10.*ES(I)
C
C 114 CONTINUE

C *****
C
C READ THE NODAL BOUNDARY CONDITIONS ON THE BOUNDARY
C

0019 C *****
C READ(1,39) (NPR(I),NBC(I),I=1,NUMRC)
C *****

13/32/20

DATE = 72034

MAIN

FORTRAN IV G LEVEL 19

WRITE(3,45) NR

DO 5 I=1,NUMN3

FRCDSP(I)=0.

DO 5 J=1,NB

5 STRSTF(I,J)=0.

DO 7 N=1,NUMEL

FORM LOAD VECTOR FOR EACH ELEMENT

DO 70 IP=1,4

I1=1+5*(IP-1)

I2=1+I1

I3=1+I2

I4=1+I3

I5=1+I4

PL(I1)=0.

PL(I2)=0.

PL(I3)=(XL*YL)/(NEX*NEY)/4.

PL(I4)=(XL*YL)/(NEX*NEY)/24.*A(N)

PL(I5)=(XL*YL)/(NEX*NEY)/24.*B(N)

IF(IP.EQ.2) PL(I4)=-PL(I4)

IF(IP.EQ.3) PL(I5)=-PL(I5)

IF(IP.EQ.4) PL(I4)=-PL(I4)

IF(IP.EQ.4) PL(I5)=-PL(I5)

70 CONTINUE

FORM ELEMENT STIFFNESS MATIRX

CALL SOLCOR (SS,N)

FORM LOAD VECTOR FOR THE WHOLE STRUCTURE, WHEN APPLYING CONCENTRATED

LOAD TO THE STRUCTURE, THE FORCE MATRIX SHOULD BE READ FROM CARD

- 185 -

13/32/20

DATE = 72034

MAIN

ORTRAN IV G LEVEL 19

2500
1411=NP11ND

3506 L1(2)=NPJ(N)

$L^{-1}(3) = MPK(N)$

$$L_M(4) = NPL(N)$$

CALL BUILD(LM, NUMN3, PL, FRCDSP)

1000

*****C

C

C FORMATION OF STIFFNESS MATRIX, ADVANCEMENT IN FINITE ELEMENT ANALYSIS

[illegible]

~~~~~

0062 00714

0063  
00 7 LL=1,5

0064  
[X=(I-M)\*Z+I]

0065 [Y=11155FL] 00 7 14-3 /

0900  
+T=J,  
00  
00  
00

0067  
000 / 00  
MM - MM - 15 + MM

---

00 MM = 1,3

$WM + 5\% (1 - M) = WM$   
 $8900$

$$[Y] = \lambda_0 - \lambda_1$$

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07

0073 4 IE (MY-GT-NH) NR=MX

```

00072      IF (DXY(1) .NE. 0)
00073          STRSTF(LX, MX) = STRSTF(LX, MX) + SS(LY, MY)

```

0013  
0074

7 CONTINUE

\*\*\*\*\*

[illegible]

```
C PRINT OUT THE LOAD VECTOR
```

— 20 —

○

72,331 RM

C WRITE(3,25)

```
C DO 4 I=1,NUMNP
```

(c)  $1 = 1 + 5\%(1 - 1)$

C  $I_2 = I + I$

C I3=I+I2

C4 WRITE(3,29) ,FROSP, FUSP  
-----

FUSE/FCA NO

0075  
WRITING (3,45) NB  
FORMAT/CO. LINE BAND WIDTH= 14/1

0076 43 FURNACE, HALF BAND WIDTH 100

100

*(The page contains faint, illegible markings at the bottom.)*

## ENFORCE BOUNDARY CONDITIONS

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MAIN

FORTAN IV G LEVEL 19

C

C

C\*\*\*\*\*

0077 DO 16 L=1,NUMBC

0078 M1=(NPE(L)-1)\*5+1

0079 M2=M1+1

0080 M3=M2+1

0081 M4=M3+1

0082 M5=M4+1

0083 N=NBC(L)

0084 NN=N-10000

0085 IF(NN) 8,10,10

0086 NN=N-1000

0087 IF(NN) 9,12,12

0088 NN=N-100

0089 IF(NN) 91,14,14

0090 NN=N-10

0091 IF(NN) 92,93,93

0092 NN=N-1

0093 IF(NN) 16,94,94

0094 STRSTF(M1,1)=1.

0095 ERCDSP(M1)=0.

0096 MX=M1

0097 DO 11 M=2,NB

0098 MX=MX-1

0099 STRSTF(M1,M)=0.

0100 IF(MX.LE.0) GO TO 11

0101 SIRSTF(MX,M)=0.

0102 11 CONTINUE

0103 N=NN

0104 GO TO 8

0105 12 STRSTF(M2,1)=1.

0106 ERCDSP(M2)=0.

0107 MX=M2

0108 DO 13 M=2,NB

0109 MX=MX-1

0110 SIRSTF(M2,M)=0.

0111 IF(MX.LE.0) GO TO 13

0112 STRSTF(MX,M)=0.

0113 13 CONTINUE

0114 N=NN

0115 GO TO 9

0116 14 STRSTF(M3,1)=1.

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ORTRAN IV 6 LEVEL 19

0117 FRCDSP(M3)=0.

0118 MX=M3

0119 DO 15 M=2,NB

0120 MX=MX-1

0121 STRSTF(M3,M)=0.

0122 IF(MX.LE.0) GO TO 15

0123 STRSTF(MX,M)=0.

0124 15 CONTINUE

0125 N=NN

0126 GO TO 91

0127 93 STRSTF(M4,1)=1.

0128 FRCDSP(M4)=0.

0129 MX=M4

0130 DO 98 M=2,NB

0131 MX=MX-1

0132 STRSTF(M4,M)=0.

0133 IF(MX.LE.0) GO TO 98

0134 STRSTF(MX,M)=0.

0135 99 CONTINUE

0136 N=NN

0137 GO TO 92

0138 94 STRSTF(M5,1)=1.

0139 FRCDSP(M5)=0.

0140 MX=M5

0141 DO 99 M=2,NB

0142 MX=MX-1

0143 STRSTF(M5,M)=0.

0144 IF(MX.LE.0) GO TO 99

0145 STRSTF(MX,M)=0.

0146 99 CONTINUE

0147 16 CONTINUE

C WRITE(3,42)

C42 FORMAT(/12X,'STRUCTURE STIFFNESS MATRIX AFTER B.C.')

C DO 44 I=1,NUMN3

C 44 WRITE(3,43)(STRSTF(I,J),J=1,NB)

43 FORMAT(11E10.2)

C \*\*\*\*\*

C

C

C SOLUTION OF EQUILIBRIUM EQUATIONS FOR DISPLACEMENTS

C

C

C \*\*\*\*\*

```

FORTRAN IV G LEVEL 19          MAIN          DATE = 72034          13/32/20          PAGE 0098

C
0149      CALL SYMSOL(STRSTF,FRCDSP,NUMN3,NB)
C
C *****
C
C      PRINT OUT THE DISPLACEMENT OF EACH NODAL POINTS
C
C *****
0150      WRITE(3,33)
0151      WRITE(3,27)
0152      DO 17 I=1,NUMNP
0153      I1=I+5*(I-1)
0154      I2=I+I1
0155      I3=I+I2
0156      I4=I3+1
0157      I5=I4+1
0158      17  WRITE(3,26) I,FRCDSP(I1),FRCDSP(I2),FRCDSP(I3)
C          @,FRCDSP(I4),FRCDSP(I5)
0159      WRITE(3,19)
0160      WRITE(3,19)
0161      WRITE(3,19)
0162      WRITE(3,35)
C *****
C
C      COMPUTE STRESSES
C
C *****
0163      CALL STRESS
C
C
0164      IF(KK.LT.1) GO TO 1111
0165      IF(NPROB.NE.0) GO TO 1
C      FORMAT STATEMENTSP
0166      18  FORMAT(25X,'FINITE ELEMENT SOLUTION OF THE SANWICH PLATE BENDING
C          @PROBLEM')
C
0167      19  FORMAT(1H0)
0168      20  FORMAT(1H1)
0169      21  FORMAT(23X,'DIMENSIONS',22X,'CORE',24X,'SKIN',/
C          @54X,'YOUNGS',5X,'POISSONS',19X,'YOUNGS',5X,'POISSON',/9X,'ELEMENT NO',
C          @0',4X,'X',6X,'Y',8X,'THICKNESS',5X,'MODULUS',6X,'RATIO',6X,'THICKNESS',5X,
C          @ESS',5X,'MODULUS',6X,'RATIO'

```

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MAIN

DRTRAN IV G LEVEL 19

```

2/(13X,I3,5X,2F7.2,1X,2(8X,F4.2,5X,F10.0,6X,F3.2,2X))1)
22  FORMAT(10X,'NO. ELEMENTS',5X,'NO. NODES',5X,'NO. BOUNDARY NODES',5X,
    5X,'X DIMENSION',5X,'Y DIMENSION',5X,'GRID',/14X,I4,10X,I5,14X,I4,
    5X,'F7.2,9X,F7.2,6X,I2,'X',I2,3X,F7.2)
23  FORMAT(10X,'ELEMENT NO',3X,'NODE I',3X,'NODE J',3X,'NODE K',3X,'NOO
    DE L',/(14X,I3,7X,4(I4,5X)))
24  FORMAT(10X,'NODE NO',3X,'BOUNDARY CONDITION',/12X,I4,12X,I5)
25  FORMAT(10X,'NODE NO',3X,'XMOM',4X,'YMOM',3X,'ZLOAD')
26  FORMAT(12X,I4,5E15.6)
112 FORMAT(E20.5)
27  FORMAT(10X,'NODE NO',6X,'THETAX',9X,'THETAY',9X,'DISPZ')
28  FORMAT(10X,'(NOTE FOR THE BOUNDARY CONDITIONS',/10X,'A 1 IN HUNDREDS INDIC
    ADS INDICATES X FIXED',/10X,'A 1 IN TENS INDICATES Y FIXED',/
    210X,'A 1 IN UNITS INDICATES Z FIED'))
29  FORMAT(12X,I4,2X,3F8.2)
30  FORMAT(42X,'PROPERTIES OF THE ELEMENTS')
31  FORMAT(17X,'NODAL CONNECTIONS OF THE ELEMENTS')
32  FORMAT(9X,'BOUNDARY CONDITIONS AT THE NODES')
33  FORMAT(24X,'DISPLACEMENTS OF THE NODES')
34  FORMAT(16X,'APPLIED NODAL LOADS')
35  FORMAT(26X,'STRESSES AT THE NODES')
36  FORMAT(14,2F7.4,2(F4.2,F10.1,F4.2),4I4)
37  FORMAT (14,3F10.5)
38  FORMAT(315,2F6.3,2I4)
39  FORMAT(15,I7)
40  FORMAT(I2)
46  FORMAT (F10.4)
RETURN
END
    
```

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SYMSOL

FORTRAN IV G LEVEL 19

SUBROUTINE SYMSOL (A,B,NN,MM)

0001

C \*\*\*\*\*

C THE EQUILIBRIUM EQUATIONS ARE SOLVED BY GAUSSIAN ELIMINATION METHOD,  
 C THIS SUBROUTINE TAKES ADVANTAGE OF THE FACT THAT THE STIFFNESS MATRIX IS  
 C STORED IN A COMPACT FORM TO REDUCE THE NUMBER OF NUMERICAL OPERATIONS

C REQUIRED.

C \*\*\*\*\*

0002 IMPLICIT REAL\*8(A-H,O-Z)  
 0003 DIMENSION A(125,25),B(125),C(35)

0004 N=0  
 0005 N=NN+1

C REDUCE NTH EQUATION

0006 B(N)=B(N)/A(N,1)

0007 IF(N&gt;NN)2,7,2

0008 DO 3 K=2,MM

0009 C(K)=A(N,K)

0010 A(N,K)=A(N,K)/A(N,1)

0011 DO 6 L=2,MM

0012 I=N+L-1

0013 IF(NN-I)6,4,4

0014 J=0

0015 DO 5 K=L,MM

0016 J=J+1

0017 A(I,J)=A(I,J)-C(I)\*A(N,K)

0018 B(I)=B(I)-C(L)\*B(N)

0019 CONTINUE

0020 GO TO 1

C BACK SUBSTITUTION

0021 N=NN-1

0022 IF(N) 8,11,8

0023 DO 10 K=2,MM

0024 L=N+K-1

0025 IF(NN-L) 10,9,9

0026 B(N)=B(N)-A(N,K)\*B(L)

0027 CONTINUE

0028 GO TO 7

0029 11 RETURN

0030 END

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MATINV

FORTRAN IV G LEVEL 19

SUBROUTINE MATINV (A,N)

C \*\*\*\*\*

C THIS IS THE SUBROUTINE FOR MATRIX INVERSION

C \*\*\*\*\*

```

0002      IMPLICIT REAL*8(A-H,O-Z)
0003      DIMENSION INDEX(20,2),A(N,N)
0004      DO 108 I=1,N
0005      108 INDEX (I,1) = 0
0006      II = 0
0007      109 AMAX = -1.
0008      DO 110 I=1,N
0009      IF (INDEX(I,1)) 110,111,110
0010      111 DO 112 J=1,N
0011      IF (INDEX(J,1)) 112,113,112
0012      113 TEMP = DABS(A(I,J))
0013      IF (TEMP-AMAX) 112,112,114
0014      114 IROW = I
0015      ICOL = J
0016      AMAX = TEMP
0017      112 CONTINUE
0018      110 CONTINUE
0019      IF (AMAX) 225,115,116
0020      INDEX(ICOL,1) = IROW
0021      IF (IROW-ICOL) 119,118,119
0022      119 DO 120 J=1,N
0023      TEMP = A(IROW,J)
0024      A(IROW,J) = A(ICOL,J)
0025      A(ICOL,J) = TEMP
0026      II = II+1
0027      INDEX(II,2) = ICOL
0028      118 PIVOT = A(ICOL,ICOL)
0029      A(ICOL,ICOL) = 1.
0030      PIVOT = 1./PIVOT
0031      DO 121 J=1,N
0032      121 A(ICOL,J) = A(ICOL,J)*PIVOT
0033      DO 122 I=1,N
0034      IF (I-ICOL) 123,122,123
0035      123 TEMP = A(I,ICOL)
0036      A(I,ICOL) = 0.

```

MATIN 3

MATIN 4

MATIN 5

MATIN 6

MATIN 7

MATIN 8

MATIN 9

MATIN 10

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MATIN 12

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MATIN 16

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MATINV

FORTAN IV G LEVEL 19

```
0037      DO 124 J=1,N
0038      124 A(I,J) = A(I,J)-A(ICOL,J)*TEMP
0039      122 CONTINUE
0040      GO TO 109
0041      125 ICOL = INDEX(I1,2)
0042      IROW = INDEX(ICOL,1)
0043      DO 126 I=1,N
0044      TEMP = A(I,IROW)
0045      A(I,IROW) = A(I,ICOL)
0046      126 A(I,ICOL) = TEMP
0047      I1 = I1-1
0048      225 IF (I1) 125,127,125
0049      115 WRITE (3,1001)
0050      1001 FORMAT (//2X,'ZERO PIVO')
0051      127 CONTINUE
0052      RETURN
0053      END
```

0001 ~~SUBROUTINE BUILD(M, PL, V)~~

\*\*\*\*\*

```

C
C      THIS SUBROUTINE ASSEMBLES THE ELEMENT LOAD MATRIX TO FORM THE TOTAL
C      LOAD MATRIX.
C

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| 000+ |           |
| 0005 | $ = M(2)$ |

0006 K=LM(3)

0000  
0007

0008 DG 1 M=1,5

0009 I1=5\*I-5+M

0010 J1=5\*J-5+M

6011  $K1 = 5 * K - 5 + M$

0012 L1=5\*L-5+M

0013  $V(I1)=V(I1)+PL(M)$ 0014  $V(J1)=V(J1)+PL(M+5)$ 00015  

$$V(KI) = V(KI) + PL(M+IO)$$
0016  

$$V(LI) = V(LI) + PL(M+IS)$$

0017 1 CONTINUE

0018  
RETURN

0019  
END

THIS SUBROUTINE GIVES THE SANDWICH ELEMENT STIFFNESS MATRIX FOR FIVE DEGREES OF FREEDOM/NODE

COMMON/AREAL/EC, ES, TC, TS, VC, VS, A, B, ALFB

DIMENSION A(64), B(64), TC(64), EC(64), VC(64), TS(64), ES(64), VS(64)

DIMENSION SS(20,20), S(20,20), C(8,8), D(8,8), NSC(8,8)

DIMENSION  $R(20, 20)$ ,  $RA(20, 20)$ ,  $P(8, 8)$

DIMENSION CTM(20,20), STM(8,8), RSS(20,20), RP(8,8)

IF (N<sub>0</sub>EQ 1) WRITE(3,10)

$$\text{NSC}\{1\}=1$$
$$NSC(2) = 2$$
$$\psi(\bar{x}) = 6$$
$$\text{NSC}(4) = 7$$

NSC(5)=11

$$NSC(6) = 12$$
$$\text{WSC}(7) = 16$$
$$MS(8) = 17$$

PI=3.14159265

$$\text{ALFA} = (\text{PT} * \text{ALFB}) / 180.0 \text{ DO}$$
$$\Delta J = \Delta(N)$$
[illegible]

$M = \emptyset$

DO I = 1, 20

001.1=1.20

$$S(1,1)=0$$
$$3(1, 2) = 0, \quad 3(1, 1) = 0,$$

Kilgus-00  
Ball-00

0 = 178155

FORM CORE STIFFNESS

SECRET

$$V = V \cup \{N\}$$
$$T = TC(N) / ?$$

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SOLCDR

6 LÉVEL 19

CMM=(1.-2.\*V)\*0.5/(1.-V)  
CM=C\*((1.-V)/(1.+V))\*(1.-2.\*V)  
CU=V/(1.-V)  
COT=DCOTAN(ALFA)  
CSC=1./LSIN(ALFA)  
R(1,1)=T\*AJ\*BJ\*CMM  
R(2,1)=AJ\*\*2\*T\*BJ\*0.5\*CMM  
R(3,1)=AJ\*BJ\*\*2\*T/2.\*CMM  
R(4,1)=AJ\*\*2\*BJ\*\*2/4.\*T\*CMM  
R(10,1)=AJ\*BJ\*T\*CMM  
R(12,1)=AJ\*\*2\*BJ\*T\*CMM  
R(13,1)=AJ\*BJ\*\*2\*T/2.\*CMM  
R(15,1)=AJ\*\*3\*BJ\*T\*CMM  
R(16,1)=AJ\*\*2\*BJ\*\*2\*T/2.\*CMM  
R(17,1)=AJ\*BJ\*\*3\*T/3.\*CMM  
R(19,1)=AJ\*\*3\*BJ\*\*2/2.\*T\*CMM  
R(20,1)=AJ\*BJ\*\*4\*T/4.\*CMM  
R(2,2)=AJ\*BJ\*T\*\*3/3.+AJ\*BJ\*BJ\*\*2\*T/4.\*CMM  
R(3,2)=T\*\*3\*AJ\*BJ/3.\*COT\*CSC\*CMM+AJ\*\*2\*BJ\*\*2\*T/4.\*CMM  
R(4,2)=AJ\*BJ\*\*2\*T\*\*3/6.+AJ\*BJ\*\*2\*T\*\*3/6.\*COT\*\*2\*CMM  
AJ-AJ\*\*2\*BJ\*T\*\*3/6.\*COT\*CSC\*CMM+AJ\*\*3\*BJ\*\*2\*T/6.\*CMM  
R(6,2)=-AJ\*BJ\*T\*\*3/3.\*COT\*CO-AJ\*BJ\*T\*\*3\*CO/3.\*CMM  
R(7,2)=AJ\*BJ\*T\*\*3\*CSC/3.\*CO  
R(8,2)=-AJ\*BJ\*\*2\*T\*\*3/6.\*COT\*CO-AJ\*BJ\*\*2\*T\*\*3/6.\*COT\*CMM  
AJ-AJ\*\*2\*BJ\*T\*\*3/6.\*CSC\*CO  
R(10,2)=AJ\*\*2\*BJ\*T/2.\*CMM  
R(12,2)=2./3.\*AJ\*\*3\*BJ\*T\*CMM  
R(13,2)=AJ\*\*2\*BJ\*\*2\*T/4.\*CMM  
R(15,2)=3./4.\*AJ\*\*4\*BJ\*\*1\*T\*CMM  
R(16,2)=2./3.\*AJ\*\*3\*BJ\*\*2\*T/2.\*CMM  
R(17,2)=AJ\*\*2\*BJ\*\*3/6.\*T\*CMM  
R(19,2)=3./4.\*AJ\*\*4\*BJ\*\*2\*T/2.\*CMM  
R(20,2)=AJ\*\*2\*BJ\*\*4\*T/8.\*CMM  
R(3,3)=AJ\*BJ\*T\*\*3/3.\*CSC\*\*2\*CMM+AJ\*BJ\*\*3\*T/3.\*CMM  
R(4,3)=-AJ\*BJ\*\*2/6.\*T\*\*3\*CO\*T\*CSC\*CMM  
AJ-AJ\*\*2\*BJ/6.\*T\*\*3\*CSC\*\*2\*CMM+AJ\*\*2\*BJ\*\*3\*T/6.\*CMM  
R(6,3)=AJ\*BJ\*T\*\*3/3.\*CSC\*CMM  
R(8,3)=AJ\*BJ\*\*2\*T\*\*3/6.\*CSC\*CMM  
R(10,3)=AJ\*BJ\*\*2\*T/2.\*CMM  
R(12,3)=AJ\*\*2\*BJ\*\*2/2.\*T\*CMM  
R(13,3)=AJ\*BJ\*\*3\*T/3.\*CMM  
R(15,3)=AJ\*\*3\*BJ\*\*2\*T/2.\*CMM  
R(16,3)=AJ\*\*2\*BJ\*\*3\*T/3.\*CMM

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G LEVEL 19

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R(17,3)=AJ\*BJ\*\*4\*\*T/4.\*CMM  
R(19,3)=AJ\*\*3\*BJ\*\*2/3.\*\*T\*CMM  
R(20,3)=AJ\*BJ\*\*5/5.\*\*T\*CMM  
R(14,4)=AJ\*BJ\*\*3/3.\*T\*\*3/3.\*COT\*\*2\*CMM  
W-AJ\*\*2\*BJ\*\*2/6.\*T\*\*3\*COT\*CSC\*CMN+AJ\*\*3\*BJ/9.\*T\*\*3\*CSC\*\*2\*CMM  
W-AJ\*\*2\*BJ\*\*3\*\*T/9.\*CMM  
R(6,4)=-AJ\*BJ\*\*2\*\*T\*\*3/6.\*COT\*CO -AJ\*BJ\*\*2/2.\*T\*\*3/3.\*COT\*CMM  
W-AJ\*\*2\*BJ\*\*T\*\*3/6.\*CSC\*CMM  
R(7,4)=AJ\*BJ\*\*2\*\*T\*\*3/6.\*CSC\*CO  
R(8,4)=-AJ\*BJ\*\*3\*\*T\*\*3/9.\*COT\*CO-AJ\*BJ\*\*3\*\*T\*\*3/9.\*COT\*CMM  
W-AJ\*\*2\*BJ\*\*2/12.\*T\*\*3\*CSC\*CMN+AJ\*\*2\*BJ\*\*2/12.\*T\*\*3\*CSC\*CO  
R(10,4)=AJ\*\*2\*BJ\*\*2\*\*T/4.\*CMM  
R(12,4)=AJ\*\*3\*BJ\*\*2\*\*T/3.\*CMM  
R(13,4)=AJ\*\*2\*BJ\*\*3\*\*T/6.\*CMM  
R(15,4)=3./3.\*AJ\*\*4\*BJ\*\*2\*\*T\*CMM  
R(16,4)=2./9.\*AJ\*\*3\*BJ\*\*3\*\*T\*CMM  
R(17,4)=AJ\*\*2\*BJ\*\*4/8.\*T\*CMM  
R(19,4)=3./12.\*AJ\*\*4\*BJ\*\*3\*\*T\*CMM  
R(20,4)=AJ\*\*2\*BJ\*\*5\*\*T/10.\*CMM  
R(5,5)=AJ\*BJ\*\*T\*CMM  
R(6,5)=AJ\*\*2\*BJ\*\*T/2.\*CMM  
R(7,5)=AJ\*BJ\*\*2\*\*T\*CMM/2.  
R(8,5)=AJ\*\*2\*BJ\*\*2/4.\*T\*CMM  
R(10,5)=-AJ\*BJ\*\*T\*COT\*CMM  
R(11,5)=AJ\*BJ\*\*T\*CSC\*CMM  
R(12,5)=-AJ\*\*2\*BJ\*\*T\*COT\*CMM  
R(13,5)=-AJ\*BJ\*\*2\*\*T/2.\*COT\*CMM+AJ\*\*2\*BJ\*\*T/2.\*CSC\*CMM  
R(14,5)=AJ\*BJ\*\*2\*\*T\*CSC\*CMM  
R(15,5)=-AJ\*\*3\*BJ\*\*T\*COT\*CMM  
R(16,5)=-AJ\*\*2\*BJ\*\*2\*\*T/2.\*COT\*CMM+AJ\*\*2\*BJ\*\*2/2.\*T\*CSC\*CMM  
R(17,5)=-AJ\*BJ\*\*3\*\*T/3.\*COT\*CMN+AJ\*\*2\*BJ\*\*2/2.\*T\*CSC\*CMM  
R(18,5)=AJ\*BJ\*\*3\*\*T\*CSC\*CMM  
R(19,5)=-AJ\*\*3\*BJ\*\*2\*\*T/2.\*COT\*CMN+AJ\*\*4\*BJ\*\*T/4.\*CSC\*CMM  
R(20,5)=-AJ\*BJ\*\*4\*\*T/4.\*COT\*CMN+AJ\*\*2\*BJ\*\*3\*\*T/2.\*CSC\*CMM  
R(6,6)=AJ\*BJ\*\*T\*\*3/3.\*COT\*\*2+AJ\*BJ\*\*T\*\*3/3.\*CMM+AJ\*\*3\*BJ\*\*T/3.\*CMM  
R(7,6)=-AJ\*BJ\*\*T\*\*3/3.\*COT\*CSC+AJ\*\*2\*BJ\*\*2\*\*T/4.\*CMM  
R(8,6)=AJ\*BJ\*\*2\*\*T\*\*3/6.\*COT\*\*2+AJ\*BJ\*\*2\*\*T\*\*3/6.\*CMM  
W-AJ\*\*2\*BJ/6.\*T\*\*3\*COT\*CSC+AJ\*\*3\*BJ\*\*2\*\*T/6.\*CMM  
R(10,6)=-AJ\*\*2\*BJ\*\*T/2.\*COT\*CMM  
R(11,6)=AJ\*\*2\*BJ\*\*T/2.\*CSC\*CMM  
R(12,6)=-2./3.\*AJ\*\*3\*BJ\*\*T\*COT\*CMM  
R(13,6)=-AJ\*\*2\*BJ\*\*2\*\*T/4.\*COT\*CMN+AJ\*\*3\*BJ\*\*T/3.\*CSC\*CMM  
R(14,6)=AJ\*\*2\*BJ\*\*2/2.\*T\*CSC\*CMM

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R(15,6)=-2./4.\*AJ\*\*4\*BJ\*\*1\*COI\*  
R(16,6)=-AJ\*\*3\*BJ\*\*2/3.\*I\*COI\*  
R(17,6)=-AJ\*\*2\*BJ\*\*3\*  
R(18,6)=3./2.\*AJ\*\*2\*BJ\*\*3/3.\*I\*  
R(19,6)=-3./4.\*AJ\*\*4\*BJ\*\*2/2.\*I\*  
R(20,6)=-AJ\*\*2\*BJ\*\*4\*  
R(7,7)=AJ\*BJ\*\*I\*\*3/3.\*CSC\*\*2+AJ\*  
R(8,7)=-AJ\*BJ\*\*2/6.\*I\*\*3\*COI\*  
+AJ\*\*2\*BJ\*\*3/6.\*I\*  
R(10,7)=-AJ\*BJ\*\*2/2.\*I\*  
R(11,7)=AJ\*BJ\*\*2/2.\*I\*  
R(12,7)=-AJ\*\*2\*BJ\*\*2/2.\*I\*  
R(13,7)=-AJ\*BJ\*\*3/3.\*I\*  
R(14,7)=2./3.\*AJ\*BJ\*\*3\*  
R(15,7)=-AJ\*\*3\*BJ\*\*2/2.\*I\*  
R(16,7)=-AJ\*\*2\*BJ\*\*3/3.\*I\*  
R(17,7)=-AJ\*BJ\*\*4/4.\*I\*  
R(18,7)=3./4.\*AJ\*BJ\*\*4\*  
R(19,7)=-AJ\*\*3\*BJ\*\*3/3.\*I\*  
R(20,7)=-AJ\*BJ\*\*5/5.\*I\*  
R(8,8)=AJ\*BJ\*\*3/9.\*I\*\*3\*COI\*  
+2.\*I\*\*3/3.\*COI\*  
R(10,8)=-AJ\*\*2\*BJ\*\*2\*  
R(11,8)=AJ\*\*2\*BJ\*\*2\*  
R(12,8)=-AJ\*\*3\*BJ\*\*2/3.\*I\*  
R(13,8)=-AJ\*\*2\*BJ\*\*3\*  
R(14,8)=AJ\*\*2\*BJ\*\*3\*  
R(15,8)=-3./4.\*AJ\*\*4\*BJ\*\*2/2.\*I\*  
R(16,8)=-AJ\*\*3\*BJ\*\*3\*  
R(17,8)=-AJ\*\*2\*BJ\*\*4\*  
R(18,8)=3./8.\*AJ\*\*2\*BJ\*\*4\*  
R(19,8)=-AJ\*\*4\*BJ\*\*3\*  
R(20,8)=-AJ\*\*2\*BJ\*\*5/10.\*I\*  
R(10,10)=AJ\*BJ\*\*I\*  
R(11,10)=-AJ\*BJ\*\*I\*  
R(12,10)=AJ\*\*2\*BJ\*\*I\*  
R(13,10)=AJ\*BJ\*\*2\*  
+AJ\*\*2/2.\*I\*  
+AJ\*\*2/2.\*I\*  
R(14,10)=-AJ\*BJ\*\*2\*  
R(15,10)=AJ\*\*3\*BJ\*\*I\*  
R(16,10)=AJ\*\*2\*BJ\*\*2\*  
+AJ\*\*2\*BJ\*\*2/2.\*I\*  
+AJ\*\*3/3.\*I\*  
R(17,10)=AJ\*BJ\*\*3/3.\*I\*

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6 LEVEL 19

0-AJ\*\*2\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM  
R(18,10)=-AJ\*BJ\*\*3\*T\*COI\*\*CSC\*\*CMM  
R(19,10)=AJ\*\*3\*BJ\*\*2/2.\*T\*COI\*\*2\*CMH+AJ\*\*3\*BJ\*\*2/2.\*T\*CMH  
0-AJ\*\*4/4.\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(20,10)=AJ\*BJ\*\*4/4.\*T\*COI\*\*2\*CMH+AJ\*BJ\*\*4/4.\*T\*CMH  
0-AJ\*\*2\*BJ\*\*3/2.\*T\*COI\*\*CSC\*\*CMM  
R(11,11)=AJ\*BJ\*\*T\*COI\*\*CSC\*\*2\*CMH  
R(12,11)=-AJ\*\*2\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(13,11)=-AJ\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2\*BJ\*\*2.\*T\*CMH  
R(14,11)=AJ\*BJ\*\*2\*T\*COI\*\*CSC\*\*2\*CMH  
R(15,11)=-AJ\*\*3\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(16,11)=-AJ\*\*2\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*3\*BJ\*\*T/3.\*CMH\*\*2\*CMH  
R(17,11)=-AJ\*BJ\*\*3/3.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2\*BJ\*\*2/2.\*T\*CMH  
R(18,11)=AJ\*BJ\*\*3\*T\*COI\*\*CSC\*\*2\*CMH  
R(19,11)=-AJ\*\*3\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2/2.\*BJ\*\*3\*T\*CMH  
R(20,11)=-AJ\*BJ\*\*4/4.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2\*BJ\*\*2/2.\*T\*CMH  
R(12,12)=4./3.\*AJ\*\*3\*BJ\*\*T\*COI\*\*2\*CMH+4./3.\*AJ\*\*3\*BJ\*\*T\*CMH  
R(13,12)=AJ\*\*2\*BJ\*\*2/2.\*T\*COI\*\*2\*CMH+AJ\*\*2\*BJ\*\*2/2.\*T\*CMH  
0-2./3.\*AJ\*\*3\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(14,12)=-AJ\*\*2\*BJ\*\*2\*T\*COI\*\*CSC\*\*CMM  
R(15,12)=6./4.\*AJ\*\*4\*BJ\*\*T\*COI\*\*2\*CMH+6./4.\*AJ\*\*4\*BJ\*\*T\*CMH  
R(16,12)=4./3.\*AJ\*\*3\*BJ\*\*2/2.\*T\*COI\*\*2\*CMH+4./3.\*AJ\*\*3\*BJ\*\*2/2.\*T\*CMH  
0-T\*CMH-AJ\*\*4\*BJ\*\*T/2.\*COI\*\*CSC\*\*CMM  
R(17,12)=AJ\*\*2\*BJ\*\*3/3.\*T\*COI\*\*2\*CMH+AJ\*\*2\*BJ\*\*3/3.\*T\*CMH  
0-4./3.\*AJ\*\*2\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM  
R(18,12)=-AJ\*\*2\*BJ\*\*3\*T\*COI\*\*CSC\*\*CMM  
R(19,12)=6.\*AJ\*\*4/4.\*BJ\*\*2/2.\*T\*COI\*\*2\*CMH+6.\*AJ\*\*4/4.\*BJ\*\*2/2.\*T\*CMH  
0-T\*CMH-2.\*AJ\*\*5/5.\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(20,12)=AJ\*\*2\*BJ\*\*4/4.\*T\*COI\*\*2\*CMH+AJ\*\*2\*BJ\*\*4/4.\*T\*CMH  
0-6.\*AJ\*\*3/3.\*BJ\*\*3/3.\*T\*COI\*\*CSC\*\*CMM  
R(13,13)=AJ\*BJ\*\*3/3.\*T\*COI\*\*2\*CMH+AJ\*BJ\*\*3/3.\*T\*CMH  
0-AJ\*\*2\*BJ\*\*2/2.\*COI\*\*CSC\*\*CMM+AJ\*\*3\*BJ\*\*T/3.\*CMH\*\*2\*CMH  
R(14,13)=-2.\*AJ\*BJ\*\*3/3.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2\*BJ\*\*2/2.\*T\*CMH  
R(15,13)=AJ\*\*3\*BJ\*\*2/2.\*T\*COI\*\*2\*CMH+AJ\*\*3\*BJ\*\*2/2.\*T\*CMH  
0-3./4.\*AJ\*\*4\*BJ\*\*T\*COI\*\*CSC\*\*CMM  
R(16,13)=AJ\*\*2\*BJ\*\*3/3.\*T\*COI\*\*2\*CMH+AJ\*\*2\*BJ\*\*3/3.\*T\*CMH  
0-AJ\*\*3\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*4\*BJ\*\*T/2.\*CMH\*\*2\*CMH  
R(17,13)=AJ\*BJ\*\*4/4.\*T\*COI\*\*2\*CMH+AJ\*BJ\*\*4/4.\*T\*CMH-AJ\*\*2\*BJ\*\*3/2.\*T\*CMH  
0-T\*COI\*\*CSC\*\*CMM+AJ\*\*3\*BJ\*\*2/3.\*T\*CMH  
R(18,13)=-3.\*AJ\*BJ\*\*4/4.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*2\*BJ\*\*3/2.\*T\*CMH  
R(19,13)=AJ\*\*3\*BJ\*\*3/3.\*T\*COI\*\*2\*CMH+AJ\*\*3\*BJ\*\*3/3.\*T\*CMH  
0-AJ\*\*4\*BJ\*\*2/2.\*T\*COI\*\*CSC\*\*CMM+AJ\*\*5/5.\*BJ\*\*T\*CMH  
R(20,13)=AJ\*BJ\*\*5/5.\*T\*CMH+AJ\*BJ\*\*5/5.\*T\*CMH

@-2. \*AJ\*\*2\*BJ\*\*4/4. \*T\*COI\*CS\*CM\*\*AJ\*\*3\*BJ\*\*3/3. \*T\*CS\*CM\*\*2\*CM  
 R(14,14)=4. \*AJ\*\*BJ\*\*3/3. \*T\*CS\*CM\*\*2\*CM  
 R(15,14)=-AJ\*\*3\*BJ\*\*2\*T\*COI\*CS\*CM  
 R(16,14)=-2. \*AJ\*\*2\*BJ\*\*3/3. \*T\*COI\*CS\*CM\*\*AJ\*\*3\*BJ\*\*3/3. \*T\*CS\*CM\*\*2\*CM  
 @CM  
 R(17,14)=-AJ\*\*BJ\*\*4/2. \*T\*COI\*CS\*CM\*\*2. \*AJ\*\*2\*BJ\*\*3/3. \*T\*CS\*CM\*\*2\*CM  
 R(18,14)=6. \*AJ\*\*BJ\*\*4/4. \*T\*CS\*CM\*\*2\*CM  
 R(19,14)=-2. \*AJ\*\*3\*BJ\*\*3/3. \*T\*COI\*CS\*CM\*\*AJ\*\*4\*BJ\*\*2\*T/4. \*CS\*CM\*\*2\*CM  
 @CM  
 R(20,14)=-2. \*AJ\*\*BJ\*\*5/5. \*T\*COI\*CS\*CM\*\*3. \*AJ\*\*2\*BJ\*\*4/4. \*T\*CS\*CM\*\*2\*CM  
 @CM  
 R(15,15)=9. /5. \*AJ\*\*5\*BJ\*\*T\*COI\*\*2\*CM\*\*9. /5. \*AJ\*\*5\*BJ\*\*T\*CM  
 R(16,15)=AJ\*\*4\*BJ\*\*2\*T\*\*3. /4. \*COI\*\*2\*CM\*\*6. /8. \*AJ\*\*4\*BJ\*\*2\*T\*CM  
 @-3. /5. \*AJ\*\*5\*BJ\*\*T\*COI\*CS\*CM  
 R(17,15)=AJ\*\*3\*BJ\*\*3/3. \*T\*COI\*\*2\*CM\*\*AJ\*\*3\*BJ\*\*3/3. \*T\*CM  
 @-6. /6. \*AJ\*\*4\*BJ\*\*2\*T\*COI\*CS\*CM  
 R(18,15)=-AJ\*\*3\*BJ\*\*3\*T\*COI\*CS\*CM  
 R(19,15)=9. /10. \*AJ\*\*5\*BJ\*\*2\*T\*COI\*\*2\*CM\*\*9. /10. \*AJ\*\*5\*BJ\*\*2\*T\*CM  
 @-AJ\*\*6\*BJ\*\*2. \*T\*COI\*CS\*CM  
 R(20,15)=AJ\*\*3\*BJ\*\*4/4. \*T\*COI\*\*2\*CM\*\*AJ\*\*3\*BJ\*\*4/4. \*T\*CM  
 @-9. /12. \*AJ\*\*4\*BJ\*\*3\*T\*COI\*CS\*CM  
 R(16,16)=4. /9. \*AJ\*\*3\*BJ\*\*3\*T\*COI\*\*2\*CM\*\*4. /9. \*AJ\*\*3\*BJ\*\*3\*T\*CM  
 @-AJ\*\*4\*BJ\*\*2/2. \*T\*COI\*CS\*CM\*\*AJ\*\*5/5. \*BJ\*\*T\*CS\*CM\*\*2\*CM  
 R(17,16)=AJ\*\*2\*BJ\*\*4/4. \*T\*COI\*\*2\*CM\*\*AJ\*\*2\*BJ\*\*4/4. \*T\*CM  
 @-5. /9. \*AJ\*\*2\*BJ\*\*3\*T\*COI\*CS\*CM\*\*AJ\*\*4\*BJ\*\*2/4. \*T\*CS\*CM\*\*2\*CM  
 R(18,16)=-3. \*AJ\*\*2\*BJ\*\*4/4. \*T\*COI\*CS\*CM\*\*AJ\*\*3\*BJ\*\*3/3. \*T\*CS\*CM\*\*2\*CM  
 @CM  
 R(19,16)=AJ\*\*4\*BJ\*\*3\*T/2. \*COI\*\*2\*CM\*\*AJ\*\*4\*BJ\*\*3/2. \*T\*CM  
 @-AJ\*\*5\*BJ\*\*2/2. \*T\*COI\*CS\*CM\*\*AJ\*\*6/6. \*BJ\*\*T\*CS\*CM\*\*2\*CM  
 R(20,16)=AJ\*\*2\*BJ\*\*5/5. \*T\*COI\*\*2\*CM\*\*AJ\*\*2\*BJ\*\*5\*T/5. \*CM  
 @-7. /12. \*AJ\*\*3\*BJ\*\*4\*T\*COI\*CS\*CM\*\*AJ\*\*4\*BJ\*\*3\*T/4. \*CS\*CM\*\*2\*CM  
 R(17,17)=AJ\*\*BJ\*\*5\*T/5. \*COI\*\*2\*CM\*\*AJ\*\*BJ\*\*5\*T/5. \*CM  
 @-AJ\*\*2/2. \*BJ\*\*4\*T\*COI\*CS\*CM\*\*4. /9. \*AJ\*\*3\*BJ\*\*3\*T\*CS\*CM\*\*2\*CM  
 R(18,17)=-3. \*AJ\*\*BJ\*\*5/5. \*T\*COI\*CS\*CM\*\*3. \*AJ\*\*2\*BJ\*\*4/4. \*T\*CS\*CM\*\*2  
 @CM  
 R(19,17)=AJ\*\*3\*BJ\*\*4/4. \*T\*COI\*\*2\*CM\*\*AJ\*\*3\*BJ\*\*4/4. \*T\*CM\*\*7. /12. \*  
 @AJ\*\*4\*BJ\*\*3\*T\*COI\*CS\*CM\*\*AJ\*\*5\*BJ\*\*2\*T/5. \*CS\*CM\*\*2\*CM  
 R(20,17)=AJ\*\*BJ\*\*6/6. \*T\*COI\*\*2\*CM\*\*AJ\*\*BJ\*\*6/6. \*T\*CM\*\*AJ\*\*2\*BJ\*\*5/2. \*  
 @T\*COI\*CS\*CM\*\*6. /12. \*AJ\*\*3\*BJ\*\*4\*T\*CS\*CM\*\*2\*CM  
 R(18,18)=AJ\*\*BJ\*\*5\*T/5. \*9. \*CS\*CM\*\*2\*CM  
 R(19,18)=-3. \*AJ\*\*3\*BJ\*\*4/4. \*T\*COI\*CS\*CM\*\*AJ\*\*4\*BJ\*\*2/4. \*T\*CS\*CM\*\*2  
 @CM  
 R(20,18)=-AJ\*\*BJ\*\*6\*T/2. \*COI\*CS\*CM\*\*9. /10. \*AJ\*\*2\*BJ\*\*5\*T\*CS\*CM\*\*2\*

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ACBHH

R(19,19)=9./15.\*AJ\*\*5\*BJ\*\*3\*T\*COI\*\*2\*CMH+9./15.\*AJ\*\*5\*BJ\*\*3\*T\*CMH  
3-AJ\*\*6\*BJ\*\*2\*T/2.\*COI\*\*CSC\*CMH+4AJ\*\*7\*BJ\*\*T/7.\*CSC\*\*2\*CMH  
R(20,19)=AJ\*\*3\*BJ\*\*5/5.\*T\*COI\*\*2\*CMH+AJ\*\*3\*BJ\*\*5/5.\*T\*CMH-  
@10./16.\*AJ\*\*4\*BJ\*\*4\*T\*COI\*\*CSC\*CMH+AJ\*\*5\*BJ\*\*3/5.\*T\*CSC\*\*2\*CMH  
R(20,20)=AJ\*BJ\*\*7\*T/7.\*COI\*\*2\*CMH+AJ\*BJ\*\*7\*T/7.\*CMH  
3-3.\*AJ\*\*2\*BJ\*\*5/6.\*T\*COI\*\*CSC\*CMH+3.\*AJ\*\*3\*BJ\*\*5/5.\*T\*CSC\*\*2\*CMH

DO 20 I=1,20  
DO 20 J=1,20

20 R(I,J)=CM\*P(I,J) \*2.

JM=1

DO 21 I=1,20

JM=JM+1

DO 21 J=JM,20

21 R(I,J)=R(J,I)

IF (N.GI.2) GO TO 31

31 CONTINUE

RA(1,1)=T

RA(2,5)=T

RA(3,9)=1.

RA(4,10)=1.

RA(5,11)=1.

RA(6,1)=T

RA(6,2)=AJ\*T

RA(7,5)=T

RA(7,6)=AJ\*T

RA(8,9)=1.

RA(8,10)=AJ

RA(8,12)=AJ\*\*2

KA(8,15)=AJ\*\*3

FA(9,10)=1.

RA(9,12)=2.\*AJ

RA(9,15)=3.\*AJ\*\*2

RA(10,11)=1.

RA(10,13)=AJ

KA(10,16)=AJ\*\*2

RA(10,19)=AJ\*\*3

RA(11,1)=T

RA(11,3)=BJ\*T

RA(12,5)=T

RA(12,7)=BJ\*T

RA(13,9)=1.

RA(13,11)=BJ

RA(13,14)=BJ\*\*2  
 RA(13,18)=BJ\*\*3  
 RA(14,10)=1.  
 RA(14,13)=BJ  
 RA(14,17)=BJ\*\*2  
 RA(14,20)=BJ\*\*3  
 RA(15,11)=1.  
 RA(15,14)=2.\*BJ  
 RA(15,18)=3.\*BJ\*\*2  
 RA(16,11)=T  
 RA(16,2)=AJ\*T  
 RA(16,3)=BJ\*T  
 RA(16,4)=AJ\*BJ\*T  
 RA(17,5)=T  
 RA(17,6)=AJ\*T  
 RA(17,7)=BJ\*T  
 RA(17,8)=AJ\*BJ\*T  
 RA(18,9)=1.  
 RA(18,10)=AJ  
 RA(18,11)=BJ  
 RA(18,12)=AJ\*\*2  
 RA(18,13)=AJ\*BJ  
 RA(18,14)=BJ\*\*2  
 RA(18,15)=AJ\*\*3  
 RA(18,16)=AJ\*\*2\*BJ  
 RA(18,17)=AJ\*BJ\*\*2  
 RA(18,18)=BJ\*\*3  
 RA(18,19)=AJ\*\*3\*BJ  
 RA(18,20)=AJ\*BJ\*\*3  
 RA(19,10)=1.  
 RA(19,12)=2.\*AJ  
 RA(19,13)=BJ  
 RA(19,15)=3.\*AJ\*\*2  
 RA(19,16)=2.\*AJ\*BJ  
 RA(19,17)=BJ\*\*2  
 RA(19,19)=3.\*AJ\*\*2\*BJ  
 RA(19,20)=BJ\*\*3  
 RA(20,11)=1.  
 RA(20,13)=AJ  
 RA(20,14)=2.\*BJ  
 RA(20,16)=AJ\*\*2  
 RA(20,17)=2.\*AJ\*BJ  
 RA(20,18)=3.\*BJ\*\*2

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```
RA(20,19)=AJ**3
RA(20,20)=3.*AJ*BJ**2
CALL PATINV (RA,20)
C
DO 55 I=1,20
C66 WRITE(3,11)(RA(I,J),J=1,20)
DO 22 J=1,20
DO 22 I=1,20
S(I,J)=0.
DO 22 K=1,20
22 S(I,J)=S(I,J)+R(I,K)*RA(K,J)
DO 28 I=1,20
DO 28 J=1,20
28 R(I,J)=S(I,J)
DO 23 J=1,20
DO 23 I=1,20
SS(I,J)=0.
DO 23 K=1,20
23 SS(I,J)=SS(I,J)+R(I,K)*RA(K,J)
DO 24 I=1,20
DO 24 J=1,20
SS(I,J)=SS(I,J)
C CHECK SUBELEMENT STIFFNESS MATRIX
IF(N.LE.8) GO TO 41
IF(N.GE.57) GO TO 41
GO TO 42
41 CONTINUE
C
C*****
C ROTATE THE DIRECTION OF DISPL. ON THE BOUDARY POINTS
C*****
C
IF(N.GE.57) GO TO 64
DO 53 I=1,20
DO 53 J=1,20
53 CTM(I,J)=0.
DO 54 I=1,20
DO 54 J=1,20
54 CTM(I,J)=1.
CTM(6,6)=DSIN(ALFA)
CTM(7,7)=DSIN(ALFA)
CTM(7,6)=-DCOS(ALFA)
CTM(6,7)=DCOS(ALFA)
CTM(9,9)=DSIN(ALFA)
CTM(9,10)=DCOS(ALFA)
```

CHECK

- 203 -

CTM(16,16)=CTM(6,6)

CTM(17,17)=CTM(6,6)

CTM(17,16)=CTM(7,6)

CTM(16,17)=CTM(6,7)

CTM(19,19)=DSIN(ALFA)

CTM(19,20)=DCOS(ALFA)

GO TO 61

64 CONTINUE

DO 57 I=1,20

DO 57 J=1,20

57 CTM(I,J)=0.

DO 58 I=1,20

58 CTM(I,J)=1.

CTM(1,1)=DSIN(ALFA)

CTM(2,2)=CTM(1,1)

CTM(2,1)=DCOS(ALFA)

CTM(1,2)=DCOS(ALFA)

CTM(4,4)=DSIN(ALFA)

CTM(4,5)=DCOS(ALFA)

CTM(11,11)=CTM(1,1)

CTM(12,12)=CTM(1,1)

CTM(12,11)=CTM(2,1)

CTM(11,12)=CTM(1,2)

CTM(14,14)=DSIN(ALFA)

CTM(14,15)=DCOS(ALFA)

61 CONTINUE

DO 45 J=1,20

DJ 45 I=1,20

RSS(I,J)=0.

DO 45 K=1,20

RSS(I,J)=RSS(I,J)+SS(I,K)\*CTM(K,J)

DO 46 I=1,20

DO 46 J=1,20

46 SS(I,J)=RSS(I,J)

CALL MATINV(CTM,20)

DO 47J=1,20

DO 47 I=1,20

RSS(I,J)=0.

DJ 47 K=1,20

47 RSS(I,J)=RSS(I,J)+CTM(I,K)\*SS(K,J)

DO 48 J=1,20

DO 48 I=1,20

48 SS(I,J)=RSS(I,J)

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43 IF (N.GI.2) GO TO 32

32 CONTINUE

11 FORMAT(2X,10D12.4)

C \*\*\*\*\*

C FJRA SKIN STIFFNESS

C \*\*\*\*\*

C

E=FS(N)

V=VS(N)

T=TS(N)

AREA=AJ\*BJ

CM=E\*T/(1.-V\*\*2.)

DO 4 I=1,8

DO 4 J=1,8

C(I,J)=0.

D(I,J)=0.

C(1,1)=1.

C(2,1)=-1./AJ

C(2,3)=1./AJ

C(3,1)=-1./BJ

C(3,5)=1./BJ

C(4,1)=1./AREA

C(4,3)=-1./AREA

C(4,5)=-1./AREA

C(4,7)=1./AREA

C(5,2)=1.

C(6,2)=-1./AJ

C(6,4)=1./AJ

C(7,2)=-1./BJ

C(7,6)=1./BJ

C(8,2)=1./AREA

C(8,4)=-1./AREA

C(8,6)=-1./AREA

C(8,8)=1./AREA

C CHECK NODAL-GENERALIZED DISPLACEMENT MATRIX

IF (N.GI.2) GO TO 33

33 CONTINUE

14 FORMAT(8E14.4)

01=(1.-V)\*DCOTAN(ALFA)\*\*2./2.

02=(1.-V)/2./DSIN(ALFA)\*DCOTAN(ALFA)

03=(1.-V)/2.\*DCOTAN(ALFA)

04=V\*DCOTAN(ALFA)

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 $Q5 = V * (1. / DSIN(ALFA))$  $Q6 = (1. - V) / 2. / DSIN(ALFA)$  $Q7 = (1. - V) / 2. / DSIN(ALFA) ** 2$  $Q8 = (1. - V) / 2.$  $D(2,2) = (1. + Q1) * AJ * BJ$  $D(3,2) = -Q2 * AJ * BJ$  $D(4,2) = AJ * BJ ** 2 / 2. + AJ * BJ ** 2 / 2. * Q1 - AJ ** 2 * BJ / 2. * Q2$  $D(6,2) = (-Q4 - Q3) * AJ * BJ$  $D(7,2) = Q5 * AJ * BJ$  $D(8,2) = -AJ * BJ ** 2 * Q4 / 2. - AJ * BJ ** 2 * Q3 / 2. + AJ ** 2 * BJ * Q5 / 2.$  $D(3,3) = AJ * BJ * Q7$  $D(4,3) = -AJ * BJ ** 2 * Q2 / 2. + AJ ** 2 * BJ * Q7 / 2.$  $D(6,3) = AJ * BJ * Q6$  $D(8,3) = AJ * BJ ** 2 * Q6 / 2.$  $D(4,4) = AJ * BJ ** 3 / 3. + AJ * BJ ** 3 * Q1 / 3. - AJ ** 2 * BJ ** 2 * Q2 / 2. + AJ ** 3 * BJ * Q7 / 3.$  $D(6,4) = -AJ * BJ ** 2 * Q4 / 2. - AJ * BJ ** 2 * Q3 / 2. + AJ ** 2 * BJ * Q6 / 2.$  $D(7,4) = AJ * BJ ** 2 * Q5 / 2.$  $D(8,4) = -AJ * BJ ** 3 * Q4 / 3. - AJ * BJ ** 3 * Q3 / 3. + AJ ** 2 * BJ ** 2 * Q5 / 4. + AJ ** 2 * BJ **$  $Q2 * Q4 / 4.$  $D(6,6) = (Q1 / Q8 + Q8) * AJ * BJ$  $D(7,6) = -(Q2 / Q8) * AJ * BJ$  $D(8,6) = AJ * BJ ** 2 * (Q1 / Q8) / 2. + AJ * BJ ** 2 * Q8 / 2. - AJ ** 2 * BJ * (Q2 / Q8) / 2.$  $D(7,7) = (Q7 / Q8) * AJ * BJ$  $D(8,7) = -AJ * BJ ** 2 * (Q2 / Q8) / 2. + AJ ** 2 * BJ * (Q7 / Q8) / 2.$  $D(8,8) = AJ * BJ ** 3 * (Q1 / Q8) / 3. + AJ * BJ ** 3 * Q8 / 3. - AJ ** 2 * BJ ** 2 * (Q2 / Q8) / 2.$  $@AJ ** 3 * BJ * (Q7 / Q8) / 3.$  $DO 26 I=1,8$  $DO 26 J=1,8$  $D(I,J) = CM * D(I,J)$  $JM=1$  $DO 25 I=1,3$  $JM=JM+1$  $DO 25 J=JM,8$  $CALL J=O(J,I)$ 

C CHECK GENERALIZED FORCE-DISPLACEMENT MATRIX

IF (N.O.T.2) GO TO 34

24 CONTINUE

17 FORMAT(8E13.5)

DO 5 I=1,8

DO 5 J=1,8

P(I,J)=0.

DO 6 I=1,8

DO 6 J=1,8

CHECK

$p(i, i) = 0$ .

Q11, Q12, Q13

$$p(I, J) = p(I, J) + d(I, K) * c(K, J)$$

On 7 July 2011

0071-1,8

$$7 \quad P(J, I) = P(I, J)$$

1111

Q  
H  
I  
I  
Q  
Q

$$C = \{r, 1\}c$$

003 K=1.8

$$P(I, J) = P(I, J) + D(I, K) * C(K, J)$$

IF (N.LE.8) GO TO 42

TE (No. GF. 57) GO TO 42

69 01 44

CONTINUE

\*\*\*\*\*

ON THE BOUNDARY POINTS

C ROTATE THE DIRECTION OF DISPL. ON THE BOUNDARY POINTS \*\*\*\*\*

~~~~~

IF(M.GE.57) GO TO 63

00 55 1=5,8

005571.88

$$\text{STN}(1, j) = 0,$$

00 56 I=1.8

$$\text{STN}(I, I) = I_0$$
$$\text{STM}(3,3)=\text{DSIN}(\text{ALFA})$$
$$\text{STM}(4,4) = \text{STM}(2,3)$$
$$\text{STM}(4,3) = -\text{DCOS}(\text{ALFA})$$

STM(3.4)=DCDS(ALFA)

$$STM(7, 7) = STM(3, 3)$$
$$STM(8,8)=STM(3,3)$$
$$STM(8,7) = STM(4,3)$$
$$\text{STM}(7,8) = \text{STM}(3,4)$$

CO TO K2

[illegible]

8-1-1 03 00
04-1-1 03 00

CTM/Y 11-0

0-10769
0-10770

067-109 00

STILL - DOCTINALEAL

STIM, T. E. D. S. I. N. A. L. T.

(T,T)MS=(2,2)MS

```

STM(1,2)=DCOS(ALFA)
STM(5,5)=STM(1,1)
STM(6,6)=STM(1,1)
STM(6,5)=STM(2,1)
STM(5,6)=STM(1,2)
62  CONTINUE
    DO 49 I=1,8
    DO 49 J=1,8
      RP(I,J)=0.
    DO 49 K=1,2
45  RP(I,J)=RP(I,J)+P(I,K)*STM(K,J)
    DO 50 I=1,8
    DO 50 J=1,8
50  P(I,J)=RP(J,I)
    DO 51 I=1,8
    DO 51 J=1,8
      RP(I,J)=0.
    DO 51 K=1,8
51  RP(I,J)=RP(I,J)+P(I,K)*STM(K,J)
    DO 52 I=1,8
    DO 52 J=1,8
52  P(I,J)=RP(I,J)
    44  IF(N*GE.2) GO TO 35
    35  CONTINUE
C
C*****
C  COMBINE SKIN STIFFNESS AND CORE
C*****
C
    DO 9 K=1,8
    I=NSC(K)
    DO 9 L=1,8
    J=NSC(L)
    SS(I,J)=SS(I,J)+P(K,L)*2.
    9  CONTINUE
C  CHECK ELEMENT STIFFNESS MATRIX
C  IF(N*GE.2) GO TO 36
    36  CONTINUE
    19  FORMAT(12E10.3)
    12  FORMAT(/2X,'ELEMENT STIFFNESS M
    100  FORMAT(1H0)
    10  FORMAT('1SOLCOR')
    RETURN

```

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END

SUBROUTINE STRESS

C *****

C THIS SUBROUTINE CALCULATES THE STRESSES FOR THE FACES AND MOMENTS AT
C ANY POINT WITHIN THE SANDWICH PLATE

C *****

```

0002      IMPLICIT REAL*8(A-H,O-Z)
0003      COMMON/AREA1/EC,ES,TC,TS,VC,VS,A,B,ALFB
0004      COMMON/AREA2/NUMNP,NUMEL,NPJ,NPK,NPL,FRCDSP,L4
0005      DIMENSION A(16),B(16),IC(16),EC(16),VC(16),IS(16),ES(16),VS(16)
0006      DIMENSION LM(4),
0007      @FRCDSP(125),NPJ(16),NPK(16),NPL(16)
0008      DIMENSION SIGX(25),SIGY(25),SIGXY(25),NP(25)
0009      @,BE(4,8),D(3,3),U(8),C(4,8),EP(4),SG(3),F(8,8),EH(3),HP(3,4)
0010      DIMENSION UU(8),STM(8,8)
0011      DIMENSION EMX(25),FMY(25),EMMY(25),EMM1(25),EMM2(25),ARG(25)
0012      DO 1 M=1,NUMNP
0013      NP(M)=0
0014      SIGX(M)=0.
0015      SIGY(M)=0.
0016      SIGXY(M)=0.
0017      PI=3.14159265
0018      ALFA=(PI*ALFB)/180.00

```

C *****

FORM STRESS MATRIX

C *****

```

0017      DO 8 N=1,NUMEL
0018      WRITE(3,110) N
0019      ATC=TC(N)
0020      ATS=TS(N)
0021      AJ=A(N)
0022      BJ=B(N)
0023      E=ES(N)
0024      V=VS(N)
0025      CM=E/(1.-V**2)
0026      AREA=AJ*BJ
0027      DO 2 I=1,8
0028      DO 2 J=1,8

```

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STRESS

ORTRAN IV G LEVEL 19

0029	2	E(I,J)=0.
0030		F(1,1)=1.
0031		F(2,1)=-1./AJ
0032		F(2,3)=1./AJ
0033		F(3,1)=-1./BJ
0034		F(3,5)=1./BJ
0035		F(4,1)=1./AREA
0036		F(4,3)=-1./AREA
0037		F(4,5)=-1./AREA
0038		F(4,7)=1./AREA
0039		F(5,2)=1.
0040		F(6,2)=-1./AJ
0041		F(6,4)=1./AJ
0042		F(7,2)=-1./BJ
0043		F(7,6)=1./BJ
0044		F(8,2)=1./AREA
0045		F(8,4)=-1./AREA
0046		F(8,6)=-1./AREA
0047		F(8,8)=1./AREA
0048		D(1,1)=CM
0049		D(1,2)=CM*V
0050		D(1,3)=0.
0051		D(2,1)=D(1,2)
0052		D(2,2)=CM
0053		D(2,3)=0.
0054		D(3,1)=0.
0055		D(3,2)=0.
0056		D(3,3)=CM*(1.-V)/2.
0057		DO 19 I=1,3
0058		DO 19 J=1,4
0059	19	HP(I,J)=0.
0060		HP(1,1)=1.
0061		HP(2,3)=-DCOTAN(ALFA)
0062		HP(2,4)=1./DSIN(ALFA)
0063		HP(3,1)=HP(2,3)
0064		HP(3,2)=HP(2,4)
0065		HP(3,3)=1.
0066		I=NPI(N)
0067		J=NPJ(N)
0068		K=NPJ(N)
0069		L=NPL(N)
0070		L4(1)=I
0071		L4(2)=J

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STRESS

ORTRAN IV G LEVEL 19

0072 L4(2)=K

0073 LM(4)=L

0074 I1=(I-1)*5+1

0075 J1=(J-1)*5+1

0076 K1=(K-1)*5+1

0077 L1=(L-1)*5+1

0078 I2=I1+1

0079 J2=J1+1

0080 K2=K1+1

0081 L2=L1+1

0082 U(1)=FRCDSP(I1)

0083 U(2)=FRCDSP(I2)

0084 U(3)=FRCDSP(J1)

0085 U(4)=FRCDSP(J2)

0086 U(5)=FRCDSP(K1)

0087 U(6)=FRCDSP(K2)

0088 U(7)=FRCDSP(L1)

0089 U(8)=FRCDSP(L2)

0090 IF(N, I, 4) GO TO 42

0091 IF(N, GE, 13) GO TO 42

0092 GO TO 44

0093 42 CONTINUE

C

C ***** ROTATE THE DIRECTION OF DISPL. ON THE BOUNDARY POINTS *****

C *****

C

0094 IF(N, GE, 13) GO TO 63

0095 DO 55 I=1,8

0096 DO 55 J=1,8

0097 55 STM(I, J)=0.

0098 DO 56 I=1,8

0099 56 STM(I, I)=1.

0100 STM(3, 3)=DSIN(ALFA)

0101 STM(4, 4)=STM(3, 3)

0102 STM(4, 3)=-DCOS(ALFA)

0103 STM(3, 4)=DCOS(ALFA)

0104 STM(7, 7)=STM(3, 3)

0105 STM(8, 8)=STM(3, 3)

0106 STM(8, 7)=STM(4, 3)

0107 STM(7, 8)=STM(3, 4)

0108 GO TO 62

0109 63 DO 59 I=1,8

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STRESS

FORTRAN IV G LEVEL 19

```

0110      DO 59 J=1,8
0111      59 STM(1,J)=0.
0112      DO 60 I=1,8
0113      60 STM(1,I)=1.
0114      STM(1,1)=DSIN(ALFA)
0115      STM(2,2)=STM(1,1)
0116      STM(2,1)=-DCOS(ALFA)
0117      STM(1,2)=DCOS(ALFA)
0118      STM(5,5)=STM(1,1)
0119      STM(6,6)=STM(1,1)
0120      STM(6,5)=STM(2,1)
0121      STM(5,6)=STM(1,2)
0122      62 CONTINUE
0123      DO 77 I=1,8
0124      UU(I)=0.
0125      DO 77 J=1,8
0126      77 UU(I)=UU(I)+STM(I,J)*U(J)
0127      DO 78 I=1,8
0128      UU(I)=UU(I)
0129      44 CONTINUE
0130      DO 8 MM=1,4
0131      M=LM(MM)
0132      DO 3 II=1,4
0133      DO 3 JJ=1,8
0134      3 BE(1,1,JJ)=0.
0135      BE(1,2)=1.
0136      BE(2,3)=1.
0137      BE(4,7)=1.
0138      BE(3,6)=1.
0139      IF(MM.EQ.1) GO TO 4
0140      BE(2,4)=AJ
0141      BE(4,8)=AJ
0142      IF(MM.EQ.2) GO TO 4
0143      BE(2,4)=0.
0144      BE(4,8)=0.
0145      BE(1,4)=BJ
0146      BE(3,8)=RJ
0147      IF(MM.EQ.3) GO TO 4
0148      BE(4,8)=AJ
0149      BE(2,4)=AJ
0150      4 CONTINUE
0151      DO 5 II=1,4
0152      DO 5 JJ=1,8

```


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STRESS

URTRAN IV G LEVEL 19

```

0181 EMY(M)=SIGY(M)*AIS*ATC
0182 EMY(M)=SIGXY(M)*ATS*ATC
0183 FACTS=((EMX(M)-EMY(M))*(EMX(M)-EMY(M))/4.+EMXY(M)*EMXY(M))*0.5
0184 EMM1(M)=(EMX(M)+EMY(M))/2.+FACTS
0185 EMM2(M)=(EMX(M)+EMY(M))/2.-FACTS
0186 IF(EMX(M).EQ. EMY(M)) GO TO 113
0187 ARGG=-2.*EMXY(M)/(EMX(M)-EMY(M))
0188 ARG(M)=0.5*DATAN(ARGG)*180.D0/PI
0189 GO TO 9
0190 113 ARG(M)=45.
0191 9 CONTINUE
0192 WRITE(3,10)
0193 WRITE(3,11)(M,SIGX(M),SIGY(M),SIGXY(M),M=1,NUMNP)
0194 10 FFORMAT(10X,'NODE NO. ',6X,' SIGX ',9X,' SIGY ',9X,' SIGXY')
0195 11 FFORMAT(12X,14,2X,3E15.6)
0196 WRITE(3,111)
0197 111 FFORMAT(8X,'NODE NO. ',13X,'MX ',13X,'MY ',12X,'HXY ',10X,'MAXMX ',
@10X,'MINMY ',9X,'ANGLE')
0198 WRITE(3,112)(M,EMX(M),EMY(M),EMXY(M),EMM1(M),EMM2(M),ARG(M)
@,M=1,NUMNP)
0199 112 FFORMAT(15X,14,2X,6E15.6)
0200 RETURN
0201 END

```

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SOLC03

DETRAN IV 6 LEVEL 19

SUBROUTINE SOLC03 (SS,N)

0001

C *****

C

C THIS SUBROUTINE GIVES THE SANDWICH ELEMENT STIFFNESS MATRIX FOR THREE

C DEGREES OF FREEDOM/NODE

C

C *****

C

C IMPLICIT REAL*8(A-H,O-Z)

C COMMON/AREAL/EC,ES,TC,TS,VC,VS,A,B,ALFB

C DIMENSION A(64),B(64),TC(64),EC(64),VC(64),TS(64),ES(64),VS(64)

C DIMENSION SS(12,12),S(12,12),C(8,8),D(8,8),NSC(8)

C DIMENSION R(12,12),RA(12,12),P(8,8)

C DIMENSION CTM(12,12),STM(8,8),RSS(12,12),RP(8,8)

C IF (N.EQ.1) WRITE(3,10)

C NSC(1)=1

C NSC(2)=2

C NSC(3)=4

C NSC(4)=5

C NSC(5)=7

C NSC(7)=10

C NSC(6)=8

C NSC(8)=11

C PI=3.14159265

C ALFA=(PI*ALFB)/180.00

C AJ=A(N)

C BJ=B(N)

C M=0

C DO 1 I=1,12

C DO 1 J=1,12

C S(I,J)=0.

C R(I,J)=0.

C RA(I,J)=0.

C SS(I,J)=0.

C 1

C *****

C *****

C FORM CORE STIFFNESS

C *****

C *****

C E=EC(N)

C V=VC(N)

C T=TC(N)/2.

0028

0029

0030

```

0031 CMM=(1.-2.*V)*0.5/(1.-V)
0032 CM=E*(1.-V)/((1.+V)*(1.-2.*V))
0033 CO=V/(1.-V)
0034 R(1,1)=T*AJ*BJ*CMM
0035 R(2,1)=(AJ**2/2.)*T*BJ*CMM
0036 R(3,1)=AJ*(BJ**2/2.)*T*CMM
0037 R(4,1)=(AJ**2/2.)*(BJ**2/2.)*T*CMM
0038 R(10,1)=AJ*BJ*T*CMM
0039 R(12,1)=AJ*(BJ**2/2.)*T*CMM
0040 R(2,2)=(1./3.)*T**2*AJ*BJ*(1./3.)*T**3*AJ*BJ*DCOTAN(ALFA)**2*CMM
      @+(AJ**3/3.)*T*BJ*CMM
0041 R(3,2)=-(1./3.)*T**3*AJ*BJ*DCOTAN(ALFA)/DSIN(ALFA)*CMM+(AJ**2/2.)*
      @*(BJ**2/2.)*T*CMM
0042 R(4,2)=(BJ**2/2.)*T**3*AJ/3.+RJ**2*T**3*AJ*DCOTAN(ALFA)**2*CMM/6.
      @-BJ*
      @ AJ**2/6.*T**3*DCOTAN(ALFA)*CMM/DSIN(ALFA)/3.*CO
0043 R(6,2)=-T**3*AJ*BJ*DCOTAN(ALFA)*CMM/3.
      @DCOTAN(ALFA)*CMM/3.
0044 R(7,2)=T**3*AJ*BJ/DSIN(ALFA)*CO /3.
0045 R(8,2)=-AJ*BJ**2*T**3*DCOTAN(ALFA)*CO
      @-BJ**2*T**3*AJ*DCOTAN(ALFA)*CMM/6.+AJ**2*BJ**2*T**3*DSIN(ALFA)*CO/6.
0046 R(10,2)=AJ**2/2.*BJ*T*CMM
0047 R(12,2)=(AJ**2/2.)*(BJ**2/2.)*T*CMM
0048 R(3,3)=AJ*BJ**2*T**3*(1./DSIN(ALFA))*2*CMM/3.+BJ**3*AJ**2*T*CMM/3.
0049 R(4,3)=-AJ*BJ**2*T**3*DCOTAN(ALFA)/DSIN(ALFA)*CMM/6.
      @+AJ**2*BJ**2*T**3*CMM/6.
      @+ AJ**2*BJ**3*T*CMM/3.
0050 R(6,3)=T**3/DSIN(ALFA)*CMM/3.
0051 R(8,3)=AJ*BJ**2*T**3/6./DSIN(ALFA)*CMM
0052 R(10,3)=AJ*BJ**2*T**3/2.
0053 R(12,3)=AJ*BJ**3*T**3*CMM/3.
0054 R(4,4)=AJ*BJ**3*T**3/9.+AJ*BJ**3*T**3*DCOTAN(ALFA)**2*CMM/9.
      @-AJ**2*BJ**2*T**3*DCOTAN(ALFA)/DSIN(ALFA)*CMM/5.+AJ**3*BJ**2*T**3/9.
      @/DSIN(ALFA)**2)*CMM+AJ**3*BJ**3*T *CMM/9.
      -AJ*BJ**2*T**3
0055 R(6,4)=-AJ*BJ**2*T**3*DCOTAN(ALFA)/6.*CO
      @*DCOTAN(ALFA)*CMM/6.+AJ**2*BJ**2*T**3/6./DSIN(ALFA)*CMM
0056 R(7,4)=AJ*BJ**2*T**3/6./DSIN(ALFA)*CO
      R(8,4)=-AJ*BJ**3*T**3*DCOTAN(ALFA)*CO /9.-AJ*BJ**3*T**3*
      @DCOTAN(ALFA)*CMM/9.+AJ**2*BJ**2*T**3/12./DSIN(ALFA)*CO
      @+AJ**2*BJ**2*T**3/12./DSIN(ALFA)*CMM
0057
0058 R(10,4)=AJ**2*BJ**2*T**3*CMM/4.
0059 R(12,4)=AJ**2*BJ**3*T**3*CMM/6.
0060 R(5,5)=AJ*BJ*T*CMM

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```

0061 R(6,5)=AJ**2*BJ**T*CM/2.
0062 R(7,5)=AJ*BJ**2*T*CM/2.
0063 R(8,5)=AJ**2*BJ**2*T*CM/4.
0064 R(10,5)=-AJ*BJ**T*DCOTAN(ALFA)*CM
0065 R(11,5)=AJ*BJ**T/DSIN(ALFA)*CM
0066 R(12,5)=-AJ*BJ**2*T*DCOTAN(ALFA)*CM/2.+AJ**2*BJ*T/DSIN(ALFA)*CM
0067 R(6,6)=AJ*BJ**T**3/3.*DCOTAN(ALFA)**2+AJ*BJ**T**3*CM/3.
0068 R(7,6)=-AJ*BJ**T**3/3.*DCOTAN(ALFA)/DSIN(ALFA)+AJ**2*BJ**2*T*CM/4.
0069 R(8,6)=AJ*BJ**2*T**3/6.*DCOTAN(ALFA)**2+AJ*BJ**2*T**2*CM/5.
0070 R(10,6)=-AJ**2*BJ**T*DCOTAN(ALFA)*CM/2.
0071 R(11,6)=AJ**2*BJ**T/2./DSIN(ALFA)*CM
0072 R(12,6)=-T*AJ**2*BJ**2/4.*DCOTAN(ALFA)*CM+AJ**3*BJ*T/DSIN(ALFA)*CM/
0073 R(7,7)=AJ*BJ**T**3/3./DSIN(ALFA)**2+AJ*BJ**3*T*CM/3.
0074 R(8,7)=-BJ**2*AJ**T**3*DCOTAN(ALFA)/5./DSIN(ALFA)+AJ**2*BJ**T**3/6.
0075 R(10,7)=-AJ*BJ**2*T*DCOTAN(ALFA)*CM/2.
0076 R(11,7)=AJ*BJ**2*T/2./DSIN(ALFA)*CM
0077 R(12,7)=-AJ*BJ**3*T/3.*DCOTAN(ALFA)*CM+AJ**2*BJ**2*T/4./
0078 R(8,8)=BJ**3*T**3*AJ/9.*DCOTAN(ALFA)**2+AJ*BJ**3*T**3*CM
0079 R(10,8)=-AJ**2*BJ**2*T/4.*DCOTAN(ALFA)*CM
0080 R(11,8)=AJ**2*BJ**2/4./DSIN(ALFA)*CM
0081 R(12,8)=-AJ**2*BJ**3*T/6.*DCOTAN(ALFA)*CM+AJ**2*BJ**2/T./
0082 R(10,10)=AJ*BJ**T*DCOTAN(ALFA)**2*CM+AJ*BJ**T*CM
0083 R(11,10)=-AJ*BJ**T*DCOTAN(ALFA)/DSIN(ALFA)*CM
0084 R(12,10)=AJ*BJ**2*T/2.*DCOTAN(ALFA)**2*CM+AJ*BJ**2*T/2.*CM
0085 R(11,11)=AJ*BJ**T*CM/DSIN(ALFA)**2
0086 R(12,11)=-AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM+AJ**2*BJ**T/2.
0087 R(12,12)=AJ*BJ**3*T/3.*DCOTAN(ALFA)**2*CM+AJ*BJ**3*T*CM/3.
0088 R(12,12)=AJ*BJ**3*T/3./DSIN(ALFA)**2*CM
0089 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0090 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0091 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0092 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0093 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0094 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0095 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0096 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0097 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0098 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0099 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM
0100 R(12,12)=AJ*BJ**2*T/2.*DCOTAN(ALFA)/DSIN(ALFA)*CM

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0090 DO 20 J=1,12
0091 R(I,J)=CM*R(I,J) *2.
0092 JM=1
0093 DO 21 I=1,12
0094 JM=JM+1
0095 DO 21 J=JM,12
0096 R(I,J)=R(J,I)
0097 IF (N.GT.2) GO TO 31
0098 WRITE(3,100)
0099 DO 30 I=1,12
0100 WRITE(3,11)(R(I,J),J=1,12)
0101 CONTINUE
0102 RA(1,1)=1./I
0103 RA(2,1)=-1./AJ/T
0104 RA(2,4)=1./AJ/T
0105 RA(3,1)=-1./I/BJ
0106 RA(3,7)=1./BJ/T
0107 RA(4,1)=1./ (AJ*BJ*T)
0108 RA(4,4)=-RA(4,1)
0109 RA(4,7)=-RA(4,1)
0110 RA(4,10)=RA(4,1)
0111 RA(5,2)=1./I
0112 RA(6,2)=-1./ (AJ*T)
0113 RA(6,5)=-RA(6,2)
0114 RA(7,2)=-1./ (BJ*T)
0115 RA(7,8)=-RA(7,2)
0116 RA(8,2)=1./ (AJ*BJ*T)
0117 RA(8,5)=-RA(8,2)
0118 RA(8,8)=-RA(8,2)
0119 RA(8,11)=RA(8,2)
0120 RA(9,3)=1.
0121 RA(10,3)=-1./AJ
0122 RA(10,6)=1./AJ
0123 RA(11,3)=-1./BJ
0124 RA(11,9)=1./BJ
0125 RA(12,3)=1./ (AJ*BJ)
0126 RA(12,6)=-RA(12,3)
0127 RA(12,9)=-RA(12,3)
0128 RA(12,12)=RA(12,3)
0129 DO 22 J=1,12
0130 DO 22 I=1,12
0131 S(I,J)=0.
0132 DO 22 K=1,12

```

RTN	IV	G	LLVEL	19	SOLC03	
132	22	S(I,J)=S(I,J)+R(I,K)*RA(K,J)				
133		DO 28 I=1,12				
134		DO 28 J=1,12				
135	28	R(J,I)=S(I,J)				
136		DO 23 J=1,12				
137		DO 23 I=1,12				
138		SS(I,J)=0.				
139		DO 23 K=1,12				
140	23	SS(I,J)=SS(I,J)+R(I,K)*RA(K,J)				
141		DO 24 J=1,12				
142		DO 24 I=1,12				
143	24	SS(I,J)=SS(I,J)				
	C	CHECK SURELEMENT STIFFNESS MATRIX				
144		IF(N0LE08) GO TO 41				
145		IF(N0GE057) GO TO 41				
146		GO TO 43				
147	41	CONTINUE				
	C	*****				
	C	ROTATE THE DIRECTION OF DISPL. ON THE BOUNDARY POINTS				
	C	*****				
148		IF(N0GE057) GO TO 64				
149		DO 53 I=1,12				
150		DO 53 J=1,12				
151	53	CTM(I,J)=0.				
152		DO 54 I=1,12				
153		CTM(I,1)=1.				
154		CTM(4,4)=DSIN(ALFA)				
155		CTM(5,5)=CTM(4,4)				
156		CTM(5,4)=-DCOS(ALFA)				
157		CTM(4,5)=DCOS(ALFA)				
158		CTM(10,10)=CTM(4,4)				
159		CTM(11,11)=CTM(4,4)				
160		CTM(11,10)=CTM(5,4)				
161		CTM(10,11)=CTM(4,5)				
162		GO TO 61				
163	64	DO 57 I=1,12				
164		DO 57 J=1,12				
165		CTM(I,J)=0.				
166	57	DO 58 I=1,12				
167		CTM(I,I)=1.				
168	58	CTM(1,1)=DSIN(ALFA)				
169		CTM(2,2)=CTM(1,1)				
170		CTM(2,1)=-DCOS(ALFA)				

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SQLCC3

KTRAN IV G LEVEL 19

CTM(1,2)=DCOS(ALFA)

CTM(7,7)=CTM(1,1)

CTN(8,8)=CTM(1,1)

CTM(8,7)=CTM(2,1)

CTM(7,8)=CTM(1,2)

61 CONTINUE

DO 45 J=1,12

DO 45 I=1,12

RSS(I,J)=0.

DO 45 K=1,12

45 RSS(I,J)=RSS(I,J)+SS(I,K)*CTM(K,J)

DO 46 I=1,12

DO 46 J=1,12

46 SS(I,J)=RSS(J,I)

DO 47 J=1,12

DO 47 I=1,12

RSS(I,J)=0.

DO 47 K=1,12

47 RSS(I,J)=RSS(I,J)+SS(I,K)*CTM(K,J)

DO 48 J=1,12

DO 48 I=1,12

48 SS(I,J)=RSS(I,J)

43 CONTINUE

IF(N.GE.2) GO TO 32

DO 13 I=1,12

13 WRITE(3,11)(SS(I,J),J=1,12)

32 CONTINUE

11 FORMAT(2X,12F10.3)

C

C ***** FORM SKIN STIFFNESS *****

C

C

E=ES(N)

V=VS(N)

T=TS(N)

2201 AREA=AJ*BJ

CM=E*T/(1.-V**2.)

DO 4 I=1,8

DO 4 J=1,8

C(I,J)=0.

D(I,J)=0.

C(1,1)=1.

2208

```

0209 C(2,1)=-1./AJ
0210 C(2,3)=1./AJ
0211 C(3,1)=-1./BJ
0212 C(3,5)=1./BJ
0213 C(4,1)=1./AREA
0214 C(4,3)=-1./AREA
0215 C(4,5)=-1./AREA
0216 C(4,7)=1./AREA
0217 C(5,2)=1.
0218 C(6,2)=-1./AJ
0219 C(6,4)=1./AJ
0220 C(7,2)=-1./BJ
0221 C(7,6)=1./BJ
0222 C(8,2)=1./AREA
0223 C(8,4)=-1./AREA
0224 C(8,6)=-1./AREA
0225 C(8,8)=1./AREA
0226 C CHECK NODAL-GENERALIZED DISPLACEMENT MATRIX
0227 IF (N.GT.2) GO TO 23
0228 WRITE(3,100)
0229 DO 15 I=1,8
0230 WRITE(3,14)(C(I,J),J=1,8)
0231 CONTINUE
0232 FORMAT(8E14.4)
0233 Q1=(1.-V)*DCOTAN(ALFA)**2/2.
0234 Q2=(1.-V)/2./DSIN(ALFA)*DCOTAN(ALFA)
0235 Q3=(1.-V)/2.*DCOTAN(ALFA)
0236 Q4=V*DCOTAN(ALFA)
0237 Q5=V*(1./DSIN(ALFA))
0238 Q6=(1.-V)/2./DSIN(ALFA)
0239 Q7=(1.-V)/2./DSIN(ALFA)**2
0240 Q8=(1.-V)/2.
0241 D(2,2)=(1.+Q1)*AJ*BJ
0242 D(3,2)=-Q2*AJ*BJ
0243 D(4,2)=AJ*BJ**2/2.+AJ*BJ**2/2.*Q1-AJ**2*BJ
0244 D(6,2)=(-Q4-Q3)*AJ*BJ
0245 D(7,2)=Q5*AJ*BJ
0246 D(8,2)=-AJ*BJ**2*Q4/2.-AJ*BJ**2*Q3/2.+AJ**2*
0247 D(3,3)=AJ*BJ*Q7
0248 D(4,3)=-AJ*BJ**2*Q2/2.+AJ**2*BJ*Q7/2.
0249 D(6,3)=AJ*BJ*Q6
0250 D(8,3)=AJ*BJ**2*Q6/2.
0251 D(4,4)=AJ*BJ**2/3.+AJ*BJ**3*Q1/3.-AJ**2*BJ

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0251 D(6,4)=-AJ*BJ**2*Q4/2.-AJ*BJ**2*Q3/2.+AJ**2*BJ*Q6/2.
0252 D(7,4)=AJ*BJ**2*Q5/2.
0253 D(8,4)=-AJ*BJ**3*Q4/3.-AJ*BJ**3*Q3/3.+AJ**2*BJ**2*Q5/4.+AJ**2*BJ**2
0254 D(6,6)=(Q1/Q8+Q8)*AJ*BJ
0255 D(7,6)=- (Q2/Q8)*AJ*BJ
0256 D(8,6)=AJ*BJ**2*(Q1/Q8)/2.+AJ*BJ**2*Q8/2.-AJ**2*BJ*(Q2/Q8)/2.
0257 D(7,7)=(Q7/Q8)*AJ*BJ
0258 D(8,7)=-AJ*BJ**2*(Q2/Q8)/2.+AJ**2*BJ*(Q7/Q8)/2.
0259 D(8,8)=AJ*BJ**3*(Q1/Q8)/3.+AJ*BJ**3*Q8/3.-AJ**2*BJ**2*(Q2/Q8)/2.
0260 a+AJ**3*BJ*(Q7/Q8)/3.
0261 DO 26 I=1,8
0262 DO 26 J=1,8
0263 D(I,J)=CM*D(I,J)
0264 JM=1
0265 DO 25 I=1,8
0266 JM=JM+1
0267 DO 25 J=1,8
0268 DO 16 I=1,8
0269 DO 16 J=1,8
0270 WRITE(3,17)(D(I,J),J=1,8)
0271 16
0272 34 CONTINUE
0273 17 FORMAT(8E13.5)
0274 DO 5 I=1,8
0275 DO 5 J=1,8
0276 P(I,J)=0.
0277 DO 6 I=1,8
0278 DO 6 J=1,8
0279 P(I,J)=0.
0280 DO 6 K=1,8
0281 6 P(I,J)=P(I,J)+D(I,K)*C(K,J)
0282 DO 7 J=1,8
0283 DO 7 I=1,8
0284 7 D(I,J)=P(I,J)
0285 DO 8 J=1,8
0286 DO 8 I=1,8
0287 P(I,J)=0.
0288 DO 8 K=1,8
0289 8 P(I,J)=P(I,J)+D(I,K)*C(K,J)
0290 IF(N.IF,8) GO TO 42

CHECK

C CHECK GENERALIZED FORCE-DISPLACEMENT MATRIX

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SOLCO3

ORTRAN IV G LEVEL 19

IF(N,GE,57) GO TO 42

GO TO 44

42 CONTINUE

C ***** ROTATE THE DIRECTION OF DISPL. ON THE BOUNDARY POINTS *****

C *****

C ***** IF(N,GE,57) GO TO 63 *****

0294 DO 55 I=1,8

0295 DO 55 J=1,8

0296 55 SIM(I,J)=0.

0297 DO 56 I=1,8

0298 SIM(I,I)=1.

0299 SIM(3,2)=DSIN(ALFA)

0300 SIM(4,4)=STM(3,3)

0301 STM(4,3)=-DCOS(ALFA)

0302 SIM(3,4)=DCOS(ALFA)

0303 SIM(7,7)=STM(3,3)

0304 STM(8,8)=STM(3,3)

0305 SIM(8,7)=STM(4,3)

0306 STM(7,8)=STM(3,4)

0307 GO TO 62

0308 63 DO 59 I=1,8

0309 DO 59 J=1,8

0310 59 STM(I,J)=0.

0311 DO 60 I=1,8

0312 60 STM(I,I)=1.

0313 STM(1,1)=DSIN(ALFA)

0314 SIM(2,2)=SIM(1,1)

0315 STM(2,1)=-DCOS(ALFA)

0316 STM(1,2)=DCOS(ALFA)

0317 SIM(5,5)=SIM(1,1)

0318 STM(6,6)=STM(1,1)

0319 STM(6,5)=STM(2,1)

0320 SIM(5,6)=SIM(1,2)

0321 62 CONTINUE

0322 DO 49 I=1,8

0323 DO 49 J=1,8

0324 RP(I,J)=0.

0325 DO 49 K=1,8

0326 49 RP(I,J)=RP(I,J)+P(I,K)*SIM(K,J)

0327 DO 50 I=1,8

0328 DO 50 J=1,8

0329 50 P(I,J)=RP(I,J)

0330

