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Applications of Chaos Theory and Fractal Geometry in the Music of György Ligeti

A Dissertation Presented to the  
Faculty of Graduate and Post-Doctoral Studies  
of the University of Ottawa

By

Brian J. Lefresne

May 2005



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## Abstract

In addition to incorporating various constructs from western and non-western music, 20<sup>th</sup>-century composer György Ligeti utilises non-musical influences within his music. One such extra-musical source is the scientific field of chaos theory and fractal geometry. Three compositions that display qualities associated with this scientific field include *Atmosphères*, the Fourth Movement of the *Konzert für Klavier und Orchester*, and *Étude pour piano, No. 17, "A bout de souffle"*. Analysis and re-analysis of these works by the use of rhythmic and post-tonal theory will demonstrate the presence of elements associated with this field within his music. To enhance these findings, primary sources located at the *Paul Sacher Stiftung* in Basel, Switzerland will be examined. In conjunction, these three elements will show that the form and structure of the said works contain and display influences of this non-musical paradigm.

*For my Grandparents, who gave me the splendours of being “Up in Michigan”*

## ACKNOWLEDGMENTS

There are numerous individuals who deserve to be thanked and acknowledged for their contributions to this thesis.

Firstly and foremost I would like to thank my adviser Paul A. Merkley who helped me immensely with this numerous comments, insights, ideas, and anecdotes. Your efforts truly helped enhance the quality of this dissertation.

Secondly, I would like to thank Johanna Blask, Evelyne Diendorf, Tina Kilvio Tüscher, Robert Piencikowski, and Dr. Heidi Zimmermann of the Paul Sacher Foundation in Basel, Switzerland.

Gratitude is also expressed towards Dr. Louise Duchesneau who assisted with making sure I was on the right path.

Additional gratitude goes to Prof. Ivan L'Heureux of the Physics Department for providing a foundation of understanding for this subject matter.

I would also like to thank the staff of the music library at the University of Ottawa who are truly the most underappreciated facets of a student's life at this institution. Their effort and ability to provide myself with not only their expertise but also a good laugh whenever one was needed was of great benefit to this project.

Also, I would like to thank all my colleagues who helped enrich this process: Catherine, Jessica, Matt, Erin, Kata, Kristen, Marion, and Erica; Thank You!

Furthermore, I would like to thank my parents Wilf and Kay Lefresne for all of their support and understanding for the past seven years.

Finally, the most important person to thank is Kimberly Francis. Kim, whoever coined the phrase "Behind every great man there's a great woman," surely had you in mind. I can think of no better phrase to describe you. For all the times you read drafts, made sure I went to bed on time, had food to eat, turn off the TV, stop mindlessly surfing the internet, encouraged me, or put a smile on my face: I thank you. The final six months of this project have been the hardest I've ever had to endure in my life. Without you, I do not think I would have finished or even had the will to continue.

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## Introduction

Several formal qualities unusual or even unique to the repertoire of twentieth-century music are present in the music of György Ligeti, in particular those that exhibit the influences of current mathematical theories. These formal innovations, influenced by mathematics, are in fact so important that they help inform a new compositional method. Unlike Iannis Xenakis or Pierre Boulez, who rely on the explicit use of mathematical formulas or process in their compositions, Ligeti uses mathematical ideas more as a justification or philosophical basis for his compositional techniques. This compositional method, initially demonstrated in Ligeti's earlier works, continues a chronological development throughout his compositional output reflective of a progression of mathematical applications. This progression in turn leads to a style, a style which displays ideas that have been developed from a full dependence on chaos theory and fractal geometry.

There are two ways to test and examine the operation of these premises in Ligeti's works. First, after examination of his music one clearly finds manifestations and musical applications of related mathematical models, secondly there are the composer's statements which describe his own compositional aesthetic as potentially connected to mathematical formulas, but one can do much better: there are the sketches located in the Paul Sacher Foundation in Basel, Switzerland, which directly display the possibility of a link between Ligeti's compositional method and chaos theory and fractal geometry.

## 1. The Paul Sacher Foundation

The György Ligeti Collection at the Paul Sacher Foundation in Basel, Switzerland contains all of the sketches, writings, and correspondence that Ligeti has made publicly accessible from the period from when he was a student in Hungary to the present day.<sup>1</sup>

The Ligeti Collection is catalogued by composition, with each work being subdivided into Folios that are dedicated to either the *Skizzen* or the *Partitur* of a specific work.<sup>2</sup> As one might expect of Ligeti's music which manifests on several levels an extreme departure from previous compositional conventions, the *Skizzen* which document this process are also different. They are not *Skizzen* in the traditional sense of Ludwig von Beethoven or Arnold Schoenberg with musical ideas sketched out on staff paper or in a notebook. Rather, they reflect Ligeti's method and approach to music. Ligeti's sketches are verbal instructions and constructs of his pieces on backsides (rectos) of letters, program notes, invitations, etc.; not sketches of melodic material, chord progressions, or forms.<sup>3</sup>

The Foundation's policy is to microfilm all materials it receives in order to preserve the quality of these sources, as well as to be able to allow for future opportunities to access and research these materials. However, because of the state of the Ligeti collection, the process of microfilming these materials has not been undertaken, nor are there any plans to do so in the near future. It should also be noted that the

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<sup>1</sup> For more about the history and mandate of the Paul Sacher Foundation please see Niklaus R othlin, "A Brief History of the Paul Sacher Foundation," *Paul Sacher in memoriam* (Basel/Muttenz, Switzerland: Schwabe & Co. AG, 2000) 45 – 47.

<sup>2</sup>

<sup>3</sup> The best real world example the author can give to the audience is imagine if one were to use the paper from a recycling bin in the library and used this paper to make notes on.

Foundation is continually acquiring new materials from Ligeti to add to the collection, even up until the present time.

In most cases, the material in the folios is part of one, specific composition, though there is very little to no organization within these folios. Furthermore, the Foundation explicitly instructs that all researchers keep materials in the condition, (in other words, the order and direction that they are found in the folios), regardless if they belong to that particular collection or have been misplaced; therefore, constructing a chronological history of the materials themselves is not impossible, but would require a great length of time, knowledge, and patience and could only be done so by examining the rectos of any *Skizzen* which have dates on them. In addition, the Sacher Foundation contains an extensive library of books and recordings relevant to research and scholarship on Ligeti.<sup>4</sup>

## 2. Works & Periods

The consensus amongst Ligeti scholars is that there are three periods in his *oeuvre*; early, middle, and late.<sup>5</sup> Most scholarship on Ligeti follows this classical threefold division of style periods however, there is no consensus on when these periods begin or end. There is agreement on the early period and the works encompassed within it, as well as the dividing points between it and the middle period. However, it is not

---

<sup>4</sup> To date, outside of the research that the Sacher Foundation has funded, there is no significant scholarship with concerns towards the primary sources relating to Ligeti. For more please see Jonathon W. Bernard. "A Key to Structure in the Kyrie of Gyorgy Ligeti's Requiem," *Mitteilungen der Paul Sacher Stiftung* 16 (March 2003): 42-47.

<sup>5</sup> Sources that discuss Ligeti's different periods are Richard Toop, *György Ligeti* (London: Phaidon Press, 1999), Richard Steintz *György Ligeti: Music of the Imagination* (Boston, Mass.: Northeastern University Press, 2003), Maria Lobanova, *György Ligeti: Style, Ideas, Poetics* trans. Mark Shuttleworth (Berlin: Verlag Ernst Kuhn, 2002), Paul Griffiths, 'Ligeti, György,' *Grove Music Online* ed L.macy (Accessed 16 November 2004) <http://proxy.bib.uottawa.ca:2152>, and *Ligeti in Conversation with Peter Varnai, Josef Hausler, Claude Samuel and himself* (London: Eulenberg Books, 1983).

agreed upon what work or period demarcates the division between the middle and the late period.

Ligeti's early period encompasses the early to mid-1960's the sound-mass works such as *Atmosphères*, *Aventures*, and *Volumina*. Afterwards, from the mid-1960's to the late 1970's, Ligeti's *oeuvre* employs a more simple approach to harmony and voice leading with significant works from this period including *Clocks and Clouds* and *San Francisco Polyphony*. The dividing point between the end of this period and the beginning of the late period is contentious. Some argue that the dividing point is marked by end of the 1970's<sup>6</sup>, while others argue that the *Trio for Horn, Violin and Piano* from 1982 is the dividing point.<sup>7</sup> Other sources believe that the dividing point is the composition of the *Piano Etudes, Book I*.<sup>8</sup> Because of this; it is not easily discernible when these periods – the second and the third – begin or end. Therefore, in this examination, the *Piano Concerto* will be considered a composition from the middle period and *Piano Etude No. 17* a work from the late period.

For this thesis, I have chosen to feature three works, each one from a distinct period in Ligeti's compositional career. *Atmosphères*, 1961; the fourth movement of the *Piano Concerto*, 1985; and *Piano Etude No. 17, 'A Bout de soufflé,'* 2001.<sup>9</sup> *Atmosphères* was selected because of properties this work displays that are similar to chaos theory. The fourth movement of the *Piano Concerto* was chosen for the reason that it is the only work that Ligeti has publicly admitted to being a musical representation of chaos theory and

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<sup>6</sup> Amy Marie Bauer, *Compositional Process and Parody in the Music of György Ligeti*, Ph. D. diss. Yale University 1997. 20-22.

<sup>7</sup> John Cuicrean, *A Theory of Pitch, Rhythm, and Intertextual Allusion for the late music of György Ligeti* Ph. D. Diss, SUNY-Buffalo, 2000. 20.

<sup>8</sup> Steintiz 277 – 315, and Michel 119 – 140.

<sup>9</sup> For more about the history and background of these pieces please see the relevant chapters in this thesis.

fractal geometry, and because no present analysis directly discusses or explains this influence. *Piano Etude No. 17*, ‘*A Bout de soufflé*’ (Out of Breath) was selected because of findings made while at the Paul Sacher Foundation, which show a tangible link between this piece and chaos theory and fractal geometry.

The Sacher Foundation’s holdings for *Atmosphères* are the *Skizzen* and *Partitur* of the composition. The holdings for the *Piano Concerto* include four folders: the first holds *Skizzen* of the work, the second folder *Skizzen* and *Partiturentwürfe*,<sup>10</sup> the third *Partitur* for the first movement, and the fourth the *Partitur* for movements two through five. The holdings for the *Piano Etudes* consists of one folder each of *Skizzen* and *Reinschriftentwürfe*<sup>11</sup> (fair copy) and one folder of *Reinschrift* for the three books.<sup>12</sup>

In summation, *Atmospheres* is from the sound-mass period and is the primary exemplar of a composition in this style, the *Piano Concerto* is a work from a period of transition between his middle and late periods and should be considered his first orchestral composition that incorporates chaos theory or fractal geometry, and *Piano Etude No. 17*, “*A bout de soufflé*,” also assimilates elements of chaos theory and fractal geometry and is one of Ligeti’s final compositions.

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<sup>10</sup> *Partiturentwürfe*, in this case, is effectively a fair copy of the orchestral score which was further revised and has some corrective content.

<sup>11</sup> Fair copy

<sup>12</sup> For a more precise definition and explanation of these terms please see Friedemann Sallis, “Coming to terms with the composers working manuscripts,” *A Handbook to Twentieth-Century Musical Sketches* Ed. Patricia Hall & Friedemann Sallis (New York: Cambridge University Press, 2004) 43 – 58. Also, for more about the contents of the sketches please see Appendix A.

### 3. Methodology

The methodology implemented in this thesis is unique in that all analysis or discussion of the works will be done so with the consideration of chaos theory or fractal geometry. Regrettably, there is no formal system of “chaos theory analysis” for music and indeed little or no published analysis of this kind at all), but one can easily employ the terminology associated with chaos theory and fractal geometry as well as terms developed by graphic artist M. C. Escher, Charles Madden, and Simha Arom in order to complete a musical investigation of these works.

For the harmonic analysis, two methods will be employed: one conventional, and the other unconventional. The conventional aspect will be pitch-class set theory. This theory will be drawn upon because of the chromatic nature of the music and because this music often lacks explicit diatonic harmonies. The unconventional aspect will be the utilisation of precepts and terms as defined by Charles Madden in *Music in Fractals*. The terms from this work are distinctly similar to the precepts and terms as defined by graphic artist M.C. Escher. These terms will be used to explain the processes and transformations that the harmonic material undergoes.<sup>13</sup>

The rhythmic analysis of these works will make use of the precepts and terms defined by Simha Arom in his book *African polyphony and polyrhythm: Musical structure and methodology*.<sup>14</sup> This is because of Ligeti’s unique treatment of rhythm.

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<sup>13</sup> It should be noted that the precepts and terms associated with these two sources are similar to those precepts and terms used in conjunction with serialism. However, because Ligeti does not compose in a strict serial method, it would not be beneficial to analyze these works just using this terminology. By employing Madden’s and Escher’s terms, it is possible to reinforce that the music must be viewed as a static object that is manipulated, much in the same way that fractal geometry is concerned with the manipulation of a single object, which is used to create more complex images.

<sup>14</sup> The terminology and the relevance of this book to Ligeti’s music will be explained in section 1.4

Ligeti does not use conventional rhythmic patterns related to western music but instead employs rhythmic gestures based on African musical constructs.

While not all of these techniques directly relate to chaos theory or fractal geometry; they are sympathetic to each other's use and will help enhance and facilitate discussion of the above-mentioned works.

Before one can continue, descriptions and definitions of the terms associated with chaos theory, fractal geometry, crystallographic principles and polyrhythm must be presented. As mentioned in section 3, no analysis of these three works has ever been carried out with the primary concern being chaos theory or fractal geometry; however, numerous sources do mention or touch upon this subject. Therefore, a discussion of the precepts and terms associated with the above mentioned sources must be carried out before a discussion of the literature or analysis of the above-mentioned works can be implemented.

## Chapter 1 – Precepts and Terms

This chapter will elucidate the precepts and terms associated with the models of analysis which were explained in the introduction.

### 1.1. Chaos theory & fractal geometry

The mathematical applications that are considered as models in this thesis are ones associated with chaos theory and fractal geometry. These fields developed during the 1950's out of scientists and mathematicians quest to explain the unexplainable. The focus of these theoreticians was on systems that featured non-regular occurrences or in which regular patterns could be identified out of seemingly random events.<sup>1</sup>

Chaos theory can best be explained as the study of patterns or emerging patterns that are present in data. Chaoticians, or chaosophers, are not concerned with the final answer suggested in the models, but rather the process that leads to this answer, that is they are concerned with the equation, not the final result.<sup>2</sup>

Within chaos theory, there is a sub-field entitled fractal geometry. This field explores the graphical presentation of images associated with chaotic systems. A fractal image is an image that is created out of a smaller image which is repeated upon it self numerous times. Thus, the larger image is an exact replica of the smaller image and will display similar qualities and properties. This area of study is not directly related to chaos theory, but can be best viewed as its cousin because it also attempts to define the presence of order and similarity in images which appear to have none.

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<sup>1</sup> Regrettably, a full-scale explanation about the history and the development of chaos theory and fractal geometry is beyond the scope of this thesis, though an excellent source of information on this subject is James Gleick's *Chaos: Making a New Science* (London:Heinemann, 1988).

<sup>2</sup> It should be noted that this is distinctly similar to research that was carried out by David Lewin and others which is called transformation theory, for more on this please see David Lewin, "Transformational Techniques in Atonal and Other music Theories," *Perspectives of New Music* 21 (1982-83): 312 – 371.

There is one basic principle which is essential to understanding chaos theory and fractal geometry: simple events can, and probably will, create complex events, while conversely, complex events are constructed from relatively simple occurrences. This maxim is explained best in Michael Crichton's novel *Jurassic Park*.<sup>3</sup>

This novel imagines the outcome of bringing dinosaurs back to life. The individual who proposes this, aging Billionaire John Hammond, funds a research project which recreates dinosaurs from DNA samples, and thus enables him to open an amusement park that features these 'reborn' creatures. One of the members of his research team is a mathematician named Ian Malcolm, whose specialty is Chaos theory.

Malcolm's initial visit to the park is with two scientists and a lawyer, in what can best be described as a 'dry run' or dress rehearsal for the parks' opening. Malcolm carries out a conversation with members of the visiting team. One of them asks Malcolm to explain chaos theory and what it can do. Malcolm explains it stating "chaos theory says two things. First, that a complex system, like weather, has an underlying order. Second, the reverse of that – simple systems can produce complex behavior."<sup>4</sup>

These points are better understood with examples. First, weather will cycle through the four seasons, but when or how it will cycle through these seasons one cannot hypothesize. Also, one knows that a hurricane consists of masses of warm and cool air swirling together, but how they interact and what force of hurricane they will produce, is not necessarily predictable.

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<sup>3</sup> Though this is a fictional novel, Crichton gives credit to Gleick's book. This work provided the model and inspiration for *Jurassic Park* and provides the most accessible explanation of chaos theory to the general public. The descriptions and examples that the character Ian Malcolm use are all real world examples of chaos theory and are based in part on examples taken from Gleick's book. Malcolm acts as a proxy for Gleick's book and its explanations

<sup>4</sup> Michael Crichton, *Jurassic Park* (New York: Ballantine 1991) 76.

The other half of Malcolm's statement is also true, that a simple system can indeed produce complex behavior. One example of this is the movement of a billiard ball. The striking of a billiard ball is a simple event; the amount of force one exerts in the movement of the cue stick will be transferred from the cue stick to the billiard ball once it is struck. After the ball is struck, the impact it will have on any other billiard balls, and the impact these balls will have on any other balls is complex, and therefore a complex event.

Though the preceding statements provide a simplified version of chaos theory, one must consider the complexity of this field and its essentiality to this thesis. Therefore, a brief explanation of precepts and terms associated with chaos theory and fractal geometry must be presented in order for the reader to have a more thorough understanding of them and be able to use these terms when considering or reading the literature review and analysis chapters in this thesis. None of the following definitions are from any one particular source, but rather they are a collation of definitions culled from many sources, some of which are more technical than others.<sup>5</sup> Also, one must consider that the terms defined below are not the sole components of chaos theory and fractal geometry, there are many others, but only the ones used in conjunction with this thesis will be explained.

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<sup>5</sup> The definitions and explanations for the following terms come from the following: Robert C. Hilborn, *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers* 2<sup>nd</sup> Ed. (New York: Oxford University Press, 2000); Ian Stewart, *Does God Play Dice? The New Mathematics of Chaos* (London: Penguin Books, 1993); Gregory L. Baker & Jerry P. Gollub, *Chaotic dynamics: an introduction* 2<sup>nd</sup> ed. (New York: Cambridge University Press, 1996); Raymond A. Eve, Sara Horsfall, & Mary E. Lee, "Preface," *Chaos, Complexity, and Sociology: Myths, Models, and Theories* (Thousand Oaks, California: SAGE Publications, Inc, 1997) xxviii – xxxii; Ira Livingston, *Arrow of chaos: romanticism and postmodernity* (Minneapolis, Minnesota: University of Minnesota Press, 1997); Benoit B. Mandelbrot, *Fractals: Form, Chance, and Dimension* (San Francisco, California: W. H. Freeman & Co, 1977); Harriet Hawkins, *Strange Attractors: Literature, culture and chaos theory* (New York: Prentice Hall, 1995).

## 1.2 Definitions

### 1.2.1 Attractors

An attractor is a set of points towards which a system or process evolves towards to as times progresses. Within this precept, four classifications exist: fixed-point, quasi-periodic, periodic, and strange.

A fixed-point attractor or normal attractor is a set of points which a process or operation develop towards. An example of this is a book dropped from the top of a tower. The points that the book is attracted towards is the ground, but factors such as velocity, weather conditions, and weight of the book will effect the process of how. The result of this event however, will always be the same; the book will fall towards the ground.

A quasi-periodic attractor is an attractor present for a short, indeterminate unit of time, which then changes so that a new set of point becomes the attractor. A periodic attractor describes a set of periods that the trajectory oscillates between two fixed points.

A strange attractor is a set of points that a trajectory develops towards to over the course of a process. For music, this can be interpreted as that all notes of a composition must be in pitch-space, but how these notes are processed or developed within the pitch-space can develop unexpectedly. In a classic sonata form, one knows that the themes will develop in a certain way in terms of diatonic harmony. If a theme develops outside of the schematic, one could say that the unaccountable aspect is “strange”. A composer such as Ligeti does not always respect these forms, and thus one may view the unexpected musical moments or progressions as strange. For this thesis, this term will describe musical events that appear unexpectedly and cannot be accounted for.

### 1.2.2 Bifurcation

Bifurcation is the term used to describe when a non-temporal trajectory or line diverges into two different directions from a specific point. This is caused by systems that are unable to reach a definitive answer; the formula eventually reaches a breaking point or maxim and it is forced into a different formula or trajectory.

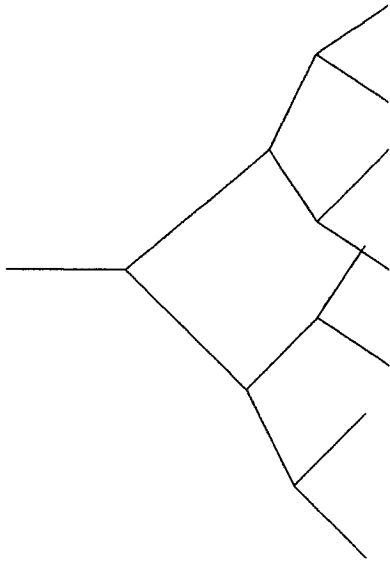


Fig. 1.1 – A Bifurcation tree.

With each bifurcation, more branches materialize, and with more branches, the more complex the event is.

In musical terms, and for this thesis, bifurcation is the process of two trajectories diverging from a common point. This can mean what *visually* appears on the page or what the listener *aurally* hears. Within Ligeti's music, there is an abundance of examples of bifurcation trees present and will be identified later in the thesis.

### 1.2.3 Intermittency

Intermittency is when a trajectory, which oscillates between two points is disrupted. This disruption also oscillates, but not between a specific set of points, or in a regular pattern.

A good example of this would be a heart being monitored by an EKG monitor. The heartbeat can fluctuate between a high and low point; which are fixed, but the pattern within these two points is not fixed. What should be noted is that after a disruption in a pattern, the trajectory returns to its original, intended path. For this thesis, any time there is a disruption or interruption in the expected course of the music, it will be referred to as intermittency.

#### **1.2.4 Feedback & Iteration**

This term describes the process in which a formula accesses part of it is an answer from a previous equation. In other words, an answer is found and is then fed back into the system.

$$X_n = y+c; x_1= x_n+c; x_2=x_1+c , \text{ etc.}$$

The best description of this is the feedback that occurs when a microphone passes over, or comes near to, a speaker. The sound emitted from the speaker is picked up by the microphone, which is then transmitted to the speaker. An excellent real world musical example of this is Steve Reich's *Pendulum music*. In this work, four microphones are suspended above four speakers lying on the floor and they are released by the performers so that they oscillate over the speakers.<sup>6</sup> The sound created by the microphone passing

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<sup>6</sup> It should be noted that the author performed this piece in a concert featuring works that contained aspects of aleatoric and/or stochastic techniques on January 23, 2004.

over the speakers is picked up by the microphone, emitted by the speaker and then picked up again by the microphone.

Another term associated with feedback is iteration. An excellent example that describes the process of feedback and iteration in music is Douglas Hofstadter's book *Gödel, Escher, Bach*.<sup>7</sup> In this text, Hofstadter explains the musical term: canon. He explains a canon as being a small motif continually repeating itself, but that it can be altered. Strict canonic writing would have the subject presented, and all subsequent statements of the theme could be subjected to different transformational techniques such as inversion, retrograde, or modulation.<sup>8</sup> While there is one theme which generates the entire material of a piece, how that theme appears over the course of a work may be altered. This process is also called: iteration. The idea of iteration and feedback will be shown to be present in the *Piano Concerto* and *A bout de soufflé*.

### 1.2.5 Lorenz Butterfly

The Lorenz butterfly is an image that occurs with certain formulas associated with studies of chaos theory. In these processes, due to feedback and iteration, the trajectory of a particle never returns to its starting terminus; it comes very close but never returns to the initial point. The images that are the result of these processes are distinctly similar to that of a butterfly.<sup>9</sup>

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<sup>7</sup> Douglas Hofstadter, *Gödel, Escher, Bach* (New York: Vintage Books, 1980).

<sup>8</sup> Hofstadter 8.

<sup>9</sup> To see how a Lorenz Butterfly is formed please see the website <http://www.exploratorium.edu/complexity/java/lorenz.html>



Fig. 1.2 – A Lorenz Butterfly

This idea, and its significance will appear later in a discussion of *Piano Etude No. 17 'A Bout de soufflé.'*

### 1.2.6 Sensitivity to Initial Conditions

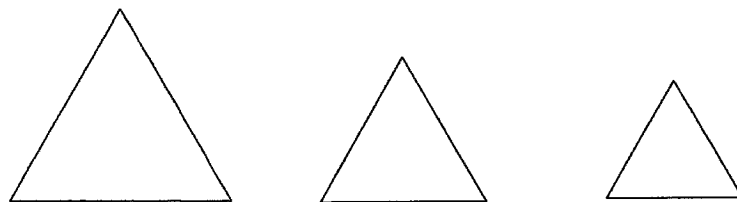
Sensitivity to initial conditions is the idea that all formulas or trajectories are the result of processes which commence from a particular point but if this starting point is altered in any way, the resultant trajectory or formula will be different as well. This condition was discovered by Edward Lorenz (the namesake of the Lorenz Butterfly) while he was conducting an experiment on weather patterns. Essentially, this experiment was an attempt to form a critical view of weather patterns starting with raw data and charting a potential weather occurrence. Lorenz charted one such process, but wished to make a duplicate copy of one portion of his results. Instead of inputting the initial six-decimal number used for the experiment, Lorenz used a number taken from half-way through the procedure, rounding it to three-digits. The resultant readout was vastly

different from the original. Thus the results manifested in the original readout were dependant on the first set of numbers, if the numbers are altered, the result is different.<sup>10</sup>

This idea could be applied to Ligeti's music in that many of his pieces rely on the aural or acoustical phenomenon created from the mean-tempered system. If one were to take one of his piano etudes and apply a different tuning system, a different acoustic phenomena or aural experience would occur. In fact, most western music is dependant on the initial condition of the mean-tempered system; to alter it would create a new work. This is also true with tonality, one has certain expectations of where a B-natural in C-major will resolve. To flatten it would create new expectations and thus, make it incongruous with its initial conditions.

### 1.2.7 Self-Similarity

Self-similarity is the term used to describe the property of images or objects that even with a change of scale retain the same shape. For example one could say that all equilateral triangles are self-similar. No matter what size they are, they will always have three equal sides and three sixty-degree angles.



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<sup>10</sup> For a more complete description of Lorenz's discovery please see Gleick 11-21, and to read Lorenz's actual findings please see "Deterministic non-periodic flow," *Journal of Atmospheric Science* 20 (1963): 130 – 141. It should also be noted that the precept of sensitivity to initial conditions is closely associated with strange attractors. The development of a strange attractor is also dependant on a departure from an initial set of conditions.

Fig. 1.3 – Three self-similar equilateral triangles

This property will be shown to be present in the *Piano Concerto* and *A bout de soufflé*.

### 1.2.8 Fractal Geometry

Fractal geometry is similar to self-similarity in that it is concerned with objects constructed from objects of similar shapes but on a different scale. This precept can best be defined as that the geometric dimension of a particular object is a fraction, or a non-integer of the original object, but retain the same area in a fixed dimension. It relates to chaos theory in that it shows that patterns emerge over the course of development of a process; there are four examples of this, but the most famous is the Koch snowflake.

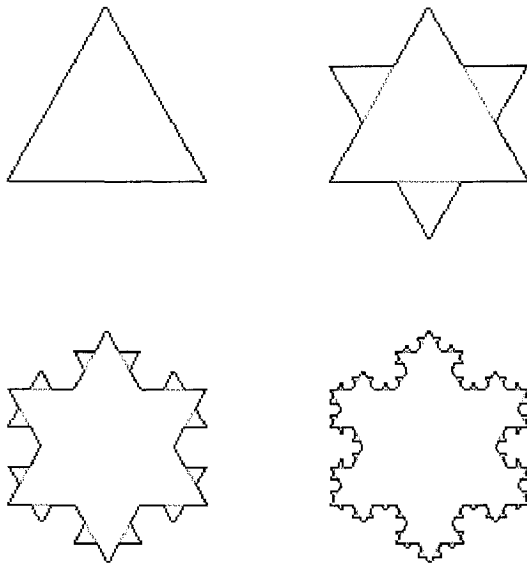


Fig. 1.4 – Example of a Koch Snowflake

If one starts with the basic triangle and divides each side into thirds, then one can place a triangle in this space. If one continues this operation, the resultant image is a series of triangles that are a fraction of the size of the original triangle. Four other examples of images from fractal geometry are the Sierpcinski triangle, Cantor Sets, and Julia and Mandelbrot sets.

Julia and Mandelbrot sets take their name after their discoverers, the mathematicians Benoit Mandelbrot and Gaston Julia. These images are computer generated and appear to be a unique shape, but if magnified or scaled down, it can be seen that no matter what scale, the figures retain the same shape or design.<sup>11</sup>

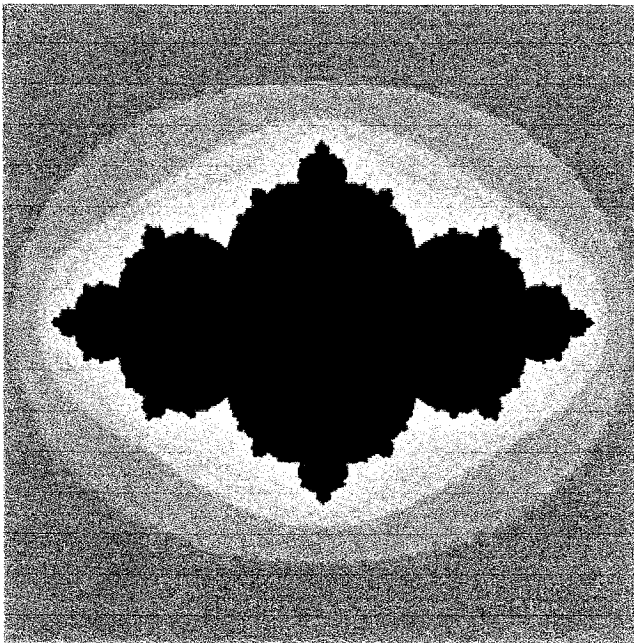


Fig. 1.5 – An Example of a Julia Set

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<sup>11</sup> For an interactive explanation of the Julia and Mandelbrot sets please see <http://mandelbrot.collettivamente.com/>

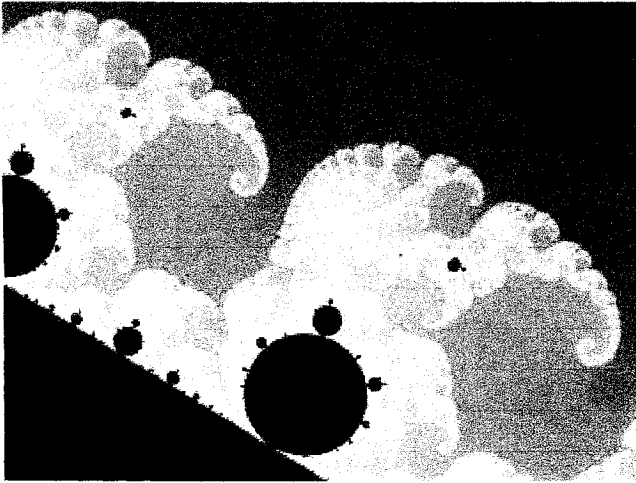


Fig. 1.6 – An example of a Mandelbrot Set

The Sierpinski triangle is likewise named after its discoverer, Warclaw Sierpinski, and displays both qualities associated with fractal geometry and self-similarity.

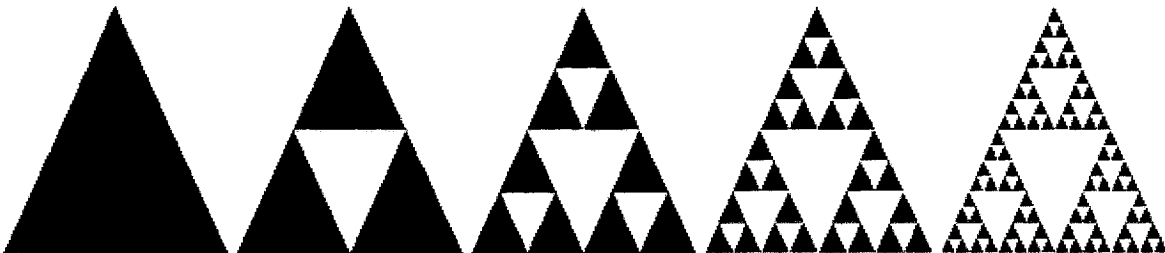


Fig. 1.7 - Series of five Sierpinski triangles

This image can be viewed as one created by its self-similarity amongst the triangles, and also viewed as fractal geometry due to the fact that the larger picture is created from a smaller rendition of the larger image.

The Cantor set was discovered by German mathematician Georg Cantor and is similar to the Koch snowflake, except that Cantor's image involves straight line segmented into thirds as opposed to triangles.

All four of these examples: the Sierpinski triangle, Cantor sets, the Julia and Mandelbrot sets, and the Koch Snowflake, are all images created out of smaller versions of larger images. In addition, all four are examples of images, which when scaled, are self-similar.

### 1.3 Musical operations

An equally important source for this thesis is Charles Madden's *Fractals in music*.<sup>12</sup> Although this source does not directly concern the music of Ligeti, it does contain important principles and definitions that must be explored before one can continue. This book presents a basis for the potential use of mathematical models for the analysis of music, a few of which are the Fibonacci sequence, Fourier series, statistics, and most importantly, chaos theory and fractal geometry. It is one of only two known sources that directly explore the relationship between fractal geometry and music.<sup>13</sup>

In the first chapter, Madden reinforces the idea that the field of mathematics has always had a place in the field of music, but not as prominent as it once was: "Musicians of the past were much more involved with mathematics than they are today."<sup>14</sup>

There has always been a mathematical component present in music, ranging from the Pythagorean comma and the Fibonacci sequence, to just and well mean-tempered

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<sup>12</sup> Charles Madden, *Fractals in Music: Introductory Mathematics for Musical Analysis* (Salt Lake City, Utah: High Art Press, 1999).

<sup>13</sup> The other source that concerns Fractal geometry and music is David J. Weisberg's *Fractals and Music* Ph. D Diss. Rutgers, 2000. This work is mainly a introduction to fractals with regards to its potentiality as a compositional tool. Though informative, this source was discovered late in research and did not provide any details or facts useful for this thesis.

<sup>14</sup> Madden 1.

tunings. Madden continues this chapter with a brief introduction to chaos theory and fractal geometry.

For Madden, “Fractal geometry is the mathematics of chaos applied to computer graphics. It is what makes chaos theory visible.”<sup>15</sup> As explained earlier in section 1.6.7, one of the main facets of fractal geometry are Julia and Mandelbrot Set images. In Madden’s discussion of Mandelbrot images, he provides an interesting analogy; he suggests that in Mandelbrot sets, the images are caused by pushing criteria to critical values. The closer the images get to these critical values, the closer they come to being forced into a different series or system. This is very similar to cracked notes and the overtone series in brass instruments. The closer one pushes to the note they wish to play, the sound waves are split and two smaller waves are formed.<sup>16</sup> This is also similar to bifurcation trees in chaos theory diagrams. As a process reaches its critical value, it is forced into another trajectory and system. This idea will appear again in the analysis of *Atmosphères* in chapter 4.

The second chapter of Madden’s book discusses the topic of self-similarity. Madden’s definition of self-similarity is the same as that explained in 1.2.7. However, Madden includes a variant of self-similarity; self-affinity. Self-affinity is that an object or image is scaled in only one direction, as opposed to in all directions. Madden then proceeds to provide a definition and explanation of the different forms of scaling that exist.

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<sup>15</sup> *Ibid.*, 9.

<sup>16</sup> For more on this subject please see Arnold Myers, “How brass instruments work,” in *The Cambridge companion to brass instruments* ed. Trevor Herbert & John Wallace (New York: Cambridge University Press, 1997)

### 1.3.1 Scaling

Scaling is that the change in size of an object is uniform or that a series of images will retain the same proportions regardless of their scale. An example of this is a set of equilateral triangles. If one takes a series of three, and makes each one half the size of the previous one, each triangle will have three sides that are equal and three angles which are congruent.

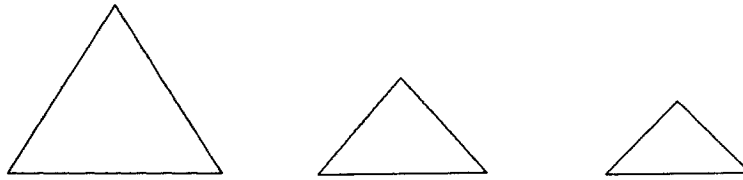


Fig. 1.8 – An example of scaling

### 1.3.2 Shear

Shear is the technical term for self-affinity. An object is sheared when it is scaled, but not equally. Alternatively, a shear is what occurs when an equilateral triangle is scaled only on one side.<sup>17</sup>



Fig. 1.9 – An example of shearing

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<sup>17</sup> It should also be noted that this precept is distinctly similar to that of “fuzzy” transposition, which has recently become utilized in post-tonal theory, for more on this topic please see Joseph N. Straus, *Introduction to post-tonal theory* (Upper Saddle River, N.J.: Prentice Hall, 2005) 75.

### 1.3.3 Translation

Translation is the term used to describe what occurs when an image or object is moved, a musical way to view this is transposition.

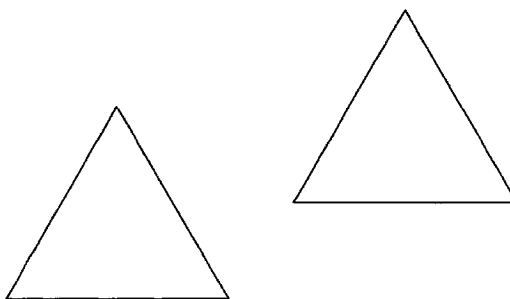


Fig. 1.10 – An example of translation

### 1.3.4 Reflection

Reflection is defined as the rotation of an image or object. This idea can be combined with the previous mentioned ones to create different classifications. Combined with translation it is called a glide reflection. Also a reflection can be classified as a retrograde if it is a vertical reflection and inversion if it is a horizontal reflection

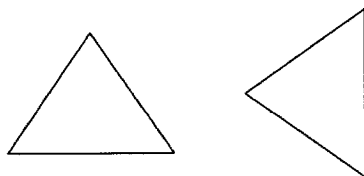


Fig. 1.11 – An example of reflection

### 1.3.5 Rotation

This term is the only one that is directly related to music. A rotation occurs when a musical idea is rotated. There are two classifications for the category of rotation: simultaneity/verticalization and arpeggiation/horizontalization.

Simultaneity/verticalization is the rotation of a melody into a chord.



Ex. 1.1 – An example of a verticalization of a c-major triad.

Arpeggiation/horizontalization is the rotation of a chord into a melody



Ex. 1.2 – An example of a horizontalization of a f-major triad

Madden's book continues to discuss other esoteric uses of mathematics for musical analysis. Moreover, Madden's book provides a basis from which to attempt a musical analysis concerning chaos theory, as well as reinforcing the idea that Ligeti's application of mathematical models is not new, but a continuation of a larger tradition of incorporating mathematics into composition.

These terms are similar to the ones defined by M. C. Escher concerning crystallographic principles and will be employed to explain the transformations that

Ligeti's music goes through. Primarily, these terms will be used in conjunction with the analysis of the fourth movement of the *Piano Concerto*.

#### 1.4 Rhythm

Ligeti's approach to rhythm can be defined best as unconventional in the occident. His approach to rhythm is unique in that it incorporates many rhythmic styles from vast musical canons, which include gamelan, salsa, jazz and most importantly Africa. These rhythmic patterns differ in that they are not dependant on a regular metre or stress pattern. One source that elucidates these structures and forms is Simha Arom's book *African polyphony and polyrhythm: Musical structure and methodology*. In this study Arom has developed a lexicon and vocabulary that helps better explain African music. If one uses these terms in conjunction with an analysis of Ligeti's music, then it is possible to reveal the structure and inherent properties of it. However, the foreword to this text, by Ligeti himself, states a departure point for his ideas relating to rhythm.<sup>18</sup>

In the foreword, Ligeti provides one of the most succinct descriptions of his music and its structure concerning rhythm. Ligeti reminisces about his first exposure to the music of the Central African Republic and how he immediately identified the similarities between this music and his own:

For many years I have been fascinated by the musical epoch from Vitry and Machault to Ciconia, and since my acquaintance with his work in 1980, by the music of Conlon Nancarrow. Undoubtedly my interest in the music Arom has recorded stems also from the proximity I feel exists between it and my own way of thinking with regards to composition: that is, the creation of structures which are both remarkably simple and highly

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<sup>18</sup> Simha Arom, *African polyphony and polyrhythm: Musical structure and methodology* Trans. Martin Thom, Barbara Tuckett, & Raymond Boyd (Cambridge: Cambridge University Press, 1991).

complex. The formal simplicity of sub-Saharan African music with its unchanging repetition of periods of equal length, like the uniform pearls of a necklace, is in sharp contrast to the inner structure of these periods which, because of simultaneous super positioning of different rhythmic patterns, possesses an extraordinary degree of complexity. Gradually, through repeated listening, I became aware of this music's paradoxical nature: the patterns performed by the individual musicians are quite different from those which result from their combination. In fact, the ensemble's superpattern is in itself not played and exists only as an illusory outline. I also began to sense a strong inner tension between the restlessness of the constant, never changing pulse coupled with the absolute symmetry of the formal architecture on the one hand and the asymmetrical internal divisions of the patterns on the other. What we can witness in this music is a wonderful combination of order and disorder which in turn merges together producing a sense of order on a higher level.<sup>19</sup>

Without knowing the context of this excerpt, one could misconstrue that Ligeti is describing his own music, and not the music of tribes from the Central African Republic. Many of the formal qualities, such as internal asymmetry and the superposing of different rhythmic patterns on top of one another that Ligeti attributes to this music, are present within his own music as well. Analysis of the *Piano Concerto* in this thesis will show these similarities. In addition, Ligeti's description of the music of the Central African Republic possessing disorder on a primary level and order on a higher level closely links it with one of the main tenets of chaos theory: that a sense of order or a pattern will emerge from a chaotic event.

What follows is a brief description and explanation of the various precepts and terms outlined by Arom which will be utilised in the rhythmic analysis of the Piano concerto and *A bout de soufflé*.

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<sup>19</sup> György Ligeti, "Foreword," *African polyphony and polyrhythm: Musical structure and methodology* by Simha Arom, (Cambridge: Cambridge University Press, 1991).

### **1.4.1 Polyrhythm**

This is the most important term for Arom and he consults no less than three other sources to craft his definition of polyrhythm.<sup>20</sup> From these sources and Arom's definition, one can formulate a more precise definition of this rhythmic property. Polyrhythm, for usage in this thesis, is defined as: an aspect of music which consists of two or more voices, which contain independent and unique rhythmic patterns, that when combined with one another create a new and unique rhythm, which before, was not formally present.

This relates to chaos theory and fractal geometry, in that it is possible to take a simple rhythmic gesture such as an eighth note, iterate it upon itself, and craft a complex rhythmic scheme.

### **1.4.2 Time terminology**

One specific focus for Arom is the terminology that has and is employed in the study of African music. Arom has a fundamental problem with previous studies of rhythm in African music in that they rely on western terminology. The problem with this is that the terms depend on the presence of a regular stress or accent pattern, a feature which is noticeably absent from African music. Arom proposes that such music should only employ the terms rhythm, accent, contra-tempo, beat/pulsation, isorhythmic/heterorhythmic/polyrhythmic, or pattern.<sup>21</sup> Given that Ligeti's music is also void of a discernible beat or stress pattern, only these terms will be employed in this thesis. Arom also notes that these sets of terms are the same ones applied in descriptions

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<sup>20</sup> Arom 39.

<sup>21</sup> Arom 183.

of *Ars nova* music of the renaissance. In the foreword, quoted on page 22, the composers which Ligeti mentions are also all from this era.

### **1.4.3 Periodicity**

Arom defines periodicity as the repetition of similar events at similar intervals.<sup>22</sup> Thus, while there might not be a pattern or order to a rhythmic pattern, it does not mean that it occurs infrequently or that the rhythmic network does not have order. What this term implies, is that some rhythmic features reappear within a musical composition, or that they are part of a pattern that is circular or repeating.

### **1.4.4 Pulsation**

This term is in reference to a basic pulsation present in a work. Therefore, all rhythmic events may be viewed in relation to a basic rhythmic gesture; all rhythmic forms are either larger or small derivatives of this basic unit.<sup>23</sup>

### **1.4.5 Macroperiod**

This term is of paramount importance to this thesis. A macroperiod is what is formed when periods of different length converge or coincide. An example of this is two sets of periods, one a unit of three, the other a unit of two which align during certain points of a composition.<sup>24</sup> This rhythmic property is omnipresent in the music that will be analyzed in this thesis.

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<sup>22</sup> Ibid., 230.

<sup>23</sup> Ibid., 230.

<sup>24</sup> Ibid., 231.

#### 1.4.6 Contrametricity and commetricity

These two terms are concerned with rhythm that is actually heard and its relation to a pulsation stream or pattern. Commetricity is defined as a rhythm in which the accents, attacks, or change in tone-colour predominantly occur *with* the pulsations of a piece. Contrametricity is explained as a rhythmic pattern in which the accents, attacks, or change in tone-colour occur predominately on the *offbeat* of the pulsation pattern.

Both of these terms have sub-classifications of regular and irregular. Regular commetricity is when more than half of the accents, attacks, or changes in tone colour occur with the pulsations of a piece. Irregular commetricity is if less than half of the aforementioned actions occur with the pulsations. Regular contrametricity is that accents, attacks, or changes in tone colour happen in the same relation to the pulsation. Irregular contrametricity is that the event does not occur with the same pattern in respect to the pulsation. In other words, in some instances the accent or attack could be one beat behind the pulsation and in another instance two beats behind the pulsation.<sup>25</sup>

#### 1.4.7 Symmetry and asymmetry

These two terms deal with the structure of rhythmic patterns themselves. A pattern is symmetrical if it can be divided into even numbers in relation to two non-contiguous accents, attacks, or changes in tone colours. An asymmetrical pattern is one that cannot be divided evenly between the accents, attacks, or changes in tone colour.<sup>26</sup>

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<sup>25</sup> Ibid., 241-242.

<sup>26</sup> Ibid., 243-244.

#### **1.4.8 Interweaving**

This term is used in conjunction with polyrhythm to describe the relationship between the accents in the parts. Strict interweaving is that the accents or attacks never coincide, while partial interweaving is when the accents or attacks coincide some of the time.<sup>27</sup>

These terms will be shown useful to an analysis of Ligeti's music because it also does not often follow a detectable rhythm or stress pattern. If one were to rely on the conventional set of terms that are utilised in Western music, there would be great shortcomings if one tried to explain or analyze Ligeti's music. With the use of the previously mentioned terms however, a discussion of Ligeti's music is much more easily facilitated.

#### **1.5 Simple elements**

One final term that must be considered for this thesis concerns basic musical elements. In the course of this text, the word "simple" will be used to describe only certain musical elements, more specifically those elements that have a conventional definition.

By simple, I mean abstract elements such as notes, intervals, or rhythmic patterns which have been isolated from the context of the greater musical work of which they are a part. All three of these components, when dealt with in isolation can be defined or described in a concise manner. In no way am I reducing all of these elements to mean the same thing, but, by themselves, one would not need to struggle for an exact definition to describe them. For this reason, throughout the analysis, when appropriate, these elements

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<sup>27</sup> Ibid., 278-281.

will be referred to as “simple” in order to elucidate the argument that Ligeti’s “complex music” is constructed from easily reducible, or “simple,” elements.

## **1.6 Conclusion**

It should be noted that not all components of this theoretical glossary are present in all of Ligeti’s works. While some are more prevalent in some works than others, there are very few instances where there is more than two or three present at any one time. This being said, with access to the analytic tools presented in this chapter, it is possible to more accurately discuss the compositional techniques of György Ligeti. With this knowledge, one can proceed to a review of the literature and the subsequent analysis. The terms and precepts described above are all mentioned within the corpus of the following text and will be essential for understanding it. To help facilitate this, all of the terms in this section are concisely defined in Appendix B.

## Chapter 2 – Literature Review

This literature review will commence with a discussion of the works concerned with Ligeti and his biography; followed by interviews that Ligeti has given over the course of his compositional career; and then continue with works such as dissertations or books that examine specific eras or works in Ligeti's *oeuvre*. Afterwards the review will progress to articles and dissertations that explore specific issues of the compositions that will be examined in or are relevant to this thesis.

### 2.1 Biographies

The earliest and most general biography of Ligeti and his music is Paul Griffith's *Ligeti*.<sup>1</sup> This text provides only general details about Ligeti's life and work and because of the publication date, includes a discussion of only one work relevant to this thesis, *Atmosphères*. Regrettably, Griffiths includes little information about this composition, and in fact contains very little information at all about Ligeti's *oeuvre*. However, this work does contain one fact that is of importance to this thesis. Griffiths briefly discusses the composition *Clocks and Clouds*; the title and basic qualities of which reflect the influence of chaos theory:

The title, which Ligeti owed to Sir Karl Popper, draws attention to a basic distinction between phenomena of regular measurable properties and phenomena only definable in more general terms, and as such, it could have been, applied to several of Ligeti's earlier works, not least the Second Quartet.<sup>2</sup>

The fine line between measurable and non-measurable elements is one of the main tenets of chaos theory; chaoticians ask where the limit between measurable and non-measurable

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<sup>1</sup> Paul Griffiths, *Ligeti* (London: Robson Books, 1983).

<sup>2</sup> Griffiths 75.

events is and how close one can get to establishing a definitive dividing point between the two. Griffiths does not explore this idea any further and continues with a general analysis of the rest of Ligeti's works that had been published and performed by the beginning of the 1980's.

In the same way that Paul Griffiths' publication attempts to provide a comprehensive overview of Ligeti's life, Pierre Michel's book *György Ligeti*<sup>3</sup> undertakes the same in French. This work is divided into three distinct sections: The first part, the main text, discusses the primary works of Ligeti's *oeuvre* and developments in his life; the second part is a transcript of an interview that Michel conducted with Ligeti in December 1981; and the third part comprises of the analysis of seven works that encompass significant stages in Ligeti's *oeuvre*. As with the other writings on Ligeti, there are numerous shortcomings to this text, but it does contain some important information.

In chapter 3, entitled "*Naissance d'un nouveau style*" (Birth of a new style), there is a short cursory discussion of *Atmosphères*. The analysis and description of the work is similar to other analyzes in that Michel notes that the work consists of constantly changing blocks of sound and that there appears to be no harmonic movement. The new idea that Michel brings forward is that he acknowledges that the main structural feature of *Atmosphères* is minor seconds: "Vertically, the polyphonic tissue is generally constituted of the superposition of minor seconds."<sup>4</sup> This is the first time in the literature that it is acknowledged that the structure of *Atmosphères*, a complex sound event, consists of simple events, a minor second.

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<sup>3</sup> Pierre Michel, *György Ligeti* 2<sup>nd</sup> ed. (Paris: Minerve, 1995).

<sup>4</sup> « Sur le plan vertical, le tissu polyphonique est généralement constitué de superpositions de secondes mineures... » Michel 47.

The next relevant section of Michel's text is chapter Eight, "Developments recents" (Recent Developments). This chapter concentrates the influence of and similarities between Ligeti's late music and music from sub-Saharan Africa. Michel points out that once Ligeti became aware of this music and the features inherent in it, a new focus or trend appears in his own output; melody became a much important aspect of his compositions.<sup>5</sup> This emphasis on melodic prevalence will be highlighted further in this thesis during the analysis of the *Piano Concerto* and *A bout de soufflé*.

In his conclusion, Michel remarks that science has a privileged place in Ligeti's list of influences, but that Ligeti does not use scientific or mathematical models in the way that Iannis Xenakis, Pierre Boulez, or Milton Babbitt do. Michel asserts that Ligeti uses the process associated with chaos theory or mathematics more as inspiration.<sup>6</sup> Unlike the music of Xenakis, a mathematical analysis of Ligeti's music would not be appropriate, nor would it be beneficial.<sup>7</sup>

The second part of this study, an interview, provides scant new information relevant to this thesis but does document several important comments made by Ligeti, including one of his more extensive discussions of his formative years in Hungary. During this conversation, Ligeti states that the idea for what he wanted *Atmosphères* to be was originally conceived in 1950: "Again in 1950, before the Quartet and the six bagatelles, I imagined blocks of sounds and their transformation."<sup>8</sup> This is the earliest instance of Ligeti acknowledging that the sound-mass style for which he would become

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<sup>5</sup> Ibid., 130.

<sup>6</sup> Ibid., 145.

<sup>7</sup> For an example of a mathematical based analysis please see Linda Arsenault's "An introduction to Iannis Xenakis's stochastic music: Four algorithmic analyses," Ph. D. diss., University of Toronto, 2000.

<sup>8</sup> Pierre Michel – "Vous aviez déjà conçu *Atmosphères* en 1950, alors que vous vivez encore en Hongrie? György Ligeti – « Dejà en 1950, avant le Quatuor et les Six bagatelles, j'avais imagine ces blocs sonores qui se transformant, mais j'ignorais comment les écrire. »

renowned for was conceived well before it was publicly manifested in compositions during the 1960's. Another significant detail that emerges during this interview is that Ligeti was familiar with the works of M. C. Escher and that he greatly influenced his mid-70's works such as *Clocks and Clouds* and *San Francisco Polyphony*, but not any other compositions.<sup>9</sup> This connection will be considered in further detail later in this thesis. Other details do emerge over the remaining course of this interview, but none that are significant to this thesis or that fail to be rendered redundant by previously published sources.

The final section of Michel's biography is the analysis of seven works, three of which are pertinent to this thesis. Michel's analysis of *Atmosphères* does not provide any new details or facts about Ligeti's compositions, while his analysis of the *Piano Concerto* only examines the fifth movement, therefore containing no information appropriate for this thesis. Michel's analysis of an early work entitled *Invencio*, however, is significant because during his analysis he notes that the note branch patterns that are indicative of Ligeti works from the late 1960's are present within this composition.<sup>10</sup> These patterns are similar to the bifurcation trees discussed in the introduction of this thesis. This adds weight to the argument that Ligeti was thinking of processes or ideas similar to those associated with chaos theory and fractal geometry at approximately the same time as these compositional ideas were first being developed.

Similar to the Michel text, Richard Toop's *György Ligeti*<sup>11</sup> provides a general overview of Ligeti's life and also provides some insight into and details about his works. Toop provides many fine points that are vital to establishing an argument for the

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<sup>9</sup> Ibid., 193.

<sup>10</sup> Ibid., 203 – 204.

<sup>11</sup> Richard Toop, *György Ligeti* (London: Phaidon Press, 1999).

influence and presence of chaos theory in Ligeti's music, but there also shortcomings with his text.

Toop identifies one of Ligeti's early compositional goals as the creation of 'continuous music.' That being music without a definite sense of form or void of any definite demarcation within the work itself. Meter would not govern or control the structure of the piece; it would merely serve as a means of synchronizing the musicians.<sup>12</sup> This was part of Ligeti's larger goal: to create music without a beginning or end, similar in style to those composed by Karlheinz Stockhausen in his 'moment form'. Stockhausen's formulates his music so that each composition does not stand on its own, but instead is part of a larger structure. In this way, a piece would emerge from the background, exist, and recede to the background again as though it were an aural disturbance to the listener.<sup>13</sup>

Toop argues that this is what Ligeti does in *Atmosphères*, that Ligeti creates a continuous piece of music that could be part of a much larger work. In fact, Ligeti describes the work as "something to be realized as a single, broad-spanned arch, with the individual sections being fused together, subordinate to the broad arch."<sup>14</sup> Like Stockhausen, Ligeti acknowledges that there are individual sections within the work but they are subservient to the overall form. The main quality that Ligeti wants brought out in this piece is its continuous aspect. In fact, Ligeti notes that in a proper performance of the

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<sup>12</sup> Toop 69.

<sup>13</sup> For more about Stockhausen's moment form please see "Moment form", *Grove Music Online* ed. L. Macy (Accessed August 4, 2004), <<http://proxy.bib.uoitawa.ca:2457>>, and Jonathon Harvey, *The music of Stockhausen: an introduction* (London: Faber & Faber, 1975) 20.

<sup>14</sup> Toop 76.

piece, the sections should not be heard, but that there should instead be a noticeable change in direction or shift in emphasis.<sup>15</sup>

Toop proceeds with a further discussion of this work and notes that Ligeti compares *Atmosphères* to a forest of trees blowing in the wind. One knows that the trees are covered in leaves and that they are moving, but the leaves themselves are not visible to the naked eye. This would fall under the corollary of: simple things or events can create complex things or events. The leaves are simple, but when multiplied by themselves, they create a complex scene, such as a forest. From a distance, *Atmosphères* appears complex, but upon closer inspection, it is created from a simple idea such as a minor second.

Directly following this is the section in which Toop's discussion of *Atmosphères* becomes particularly pertinent to this thesis, he states:

It is in *Atmosphères* that one also sees the first clear evidence of Ligeti's intermittent attraction to scientific models (though he insists that his music itself is never 'scientific'). There are passages where a basically static surface is 'disturbed' by events which pass through the whole ensemble, like the ripples that spread when a tone is thrown into a pool. Elsewhere there are sudden unexpected eddies and whirlpools, local turbulences.<sup>16</sup>

Toop does not develop this corollary about *Atmosphères* any further, but his observation is important because of the parallels between what he writes and the precepts of chaos theory and non-linear dynamics. The comparison of *Atmosphères* to first, a surface which is disturbed by an event that causes ripples throughout the piece via an unpredictable path and second, to whirlpools and turbulences, are both similar to terms and ideas related to chaos theory. In addition, Toop's use of the word "intermittency" provides more credence

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<sup>15</sup> Ibid., 76.

<sup>16</sup> Ibid., 77.

to the claim that discussion of *Atmosphères* is best carried out by using terminology related to chaos theory. This postulation will be demonstrated in chapter 4 of this thesis.

Toop follows this text with a discussion of another piece by Ligeti – *Poème Symphonique*; and clearly contextualizes it within the same paradigm as *Atmosphères*; and thus, within the influence of chaos theory. *Poème Symphonique* is for a 100 metronomes, with each metronome wound up to its maximum tautness and then simultaneously released. Because of the nature of the metronomes, each will dissipate at its own rate. As the metronomes dissipate, each metronome's beat will phase in-and-out with the beats sounded by the metronomes. In performance one expects this piece will be pure noise, but in fact an audible pattern becomes present: "Within a few moments swirls and eddies become audible within the mass of pulses – almost exactly the kind of 'turbulence' deployed in *Atmosphères!*"<sup>17</sup> The expected result that a listener could have at the beginning of the piece is mutated into something that is not expected at all. For these reasons, one could argue that like *Atmosphères*, a bifurcation or strange attractor occurs, as well as that an analysis of *Poème Symphonique* could be conducted in terms of chaos theory or fractal geometry.

Later in his book, Toop discusses Ligeti's *Piano Concerto*. Toop provides a descriptive analysis of the work, but provides no musical examples or analysis to support his position. In his discussion of the first three movements, Toop acknowledges that the first movement is Ligeti's first known attempt to incorporate African polyrhythm in his works but he does not provide any details about how this polyrhythm may function or how it is constructed within the movement and continues to describe the third movement

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<sup>17</sup> Ibid., 87.

as being “concerned with rhythmic and melodic ‘illusionism’”<sup>18</sup>. Illusionism being defined as a discernable melody that is constructed by the combination of separate musical parts which in isolation, bear no melodic properties.

Finally, Toop proceeds to discuss the heart of the concerto, the fourth movement. Toop says that this composition is “one of Ligeti’s few pieces to adopt a ‘scientific’ model not just at a metaphorical level, but also at a literal one,”<sup>19</sup> However, Toop does not specify the other pieces that employ a scientific model. He does acknowledge that the models used for this movement are Julia and Mandelbrot sets. Toop argues that Ligeti employs these images as models because of the self-similarity that is present within these images. Toop describes how the melodic material of the piece, when viewed at a superficial level, appears to be static or void of any movement or coherence. However, when viewed on a medium or second level, it becomes obvious that the fragments interact with each other and that they are interconnected as part of a larger scheme. In the same way that a Julia or Mandelbrot image can on one level be seen as a small fragment but on another level can be seen as part of a complex image or diagram. Similar to how *Atmosphères* is void of activity on a primary level, so is this movement of the *Piano Concerto*.

The shortcoming with Toop’s text is that he does not provide enough details about the works he analyzes, nor does he provide enough details about chaos theory and fractal geometry to coherently link them to the relevant works. However, Toop does provide the insight that the best way to describe Ligeti’s compositions are in reference to the ideas, forms, and vocabulary of chaos theory and fractal geometry.

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<sup>18</sup> Ibid., 195.

<sup>19</sup> Ibid.

Along the same lines as Pierre Michel's and Richard Toop's work, Maria Lobanova's *György Ligeti: Style, Ideas, Poetics*<sup>20</sup> also attempts to provide a general overview of Ligeti's life and works and includes many important facts and ideas relevant to this thesis.

Lobanova begins her book by positioning Ligeti as the antithesis of a larger artistic movement which emerged in the first half of the twentieth century. She notes that different fields of art in the early 1900's attempted to search for objective sources in art as well as confronting and dealing with the structural aspects of artistic endeavours. A few examples of individuals and movements who pursued answers to these questions are: Vassily Kandinsky and his purely pictorial form, Mikhail Larionov's 'Rayonism', the teachings of Paul Klee, and the Bauhaus movement.<sup>21</sup>

By the 1950's, science had developed to a point where most structural aspects and developments could be explained or accounted for. Music, similarly, had reached the same stage with the development of integral serialism. When Ligeti and his music sound-mass works were first produced, they appeared to be antithetical to this movement; Ligeti's music seemed void of form or structure. His music appeared to focus on elements that were uncontrollable and unexplainable. Lobanova writes:

The system of expressive means that he develops – which included micropolyphony and illusory rhythm – has fitted in with the new intellectual paradigm that was connected primarily with the research into chaos theory, which has taken place during the last three decades. In

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<sup>20</sup> Maria Lobanova, *György Ligeti: style, Ideas, Poetics* Trans. Mark Shuttleworth (Berlin: Verlag Ernst Kuhn, 2002).

<sup>21</sup> Lobanova 3. For more about Kandinsky's pictorial form please see "Wassil Kandinsky," *The Avant Garde in Russia, 1910-1930: New Perspectives*, Ed. Jeanne Andrea & Stephen West (Los Angeles: Los Angeles County Museum of Art, 1980) 178 – 183; For more about Larionov and Rayonism please see Magdalena Dabrowski "The Formation and Development of Rayonism," *Art Journal* 34 (1975): 200 – 207, For more about the Bauhaus movement please see Howard Dearstyne, *Inside the Bauhaus* (New York: Rizzoli, 1986), for more about Paul Klee please see Ann Temkin, "Klee, Paul," *Grove Music Online*, (Oxford University Press, Accessed 11 March 2005), <<http://proxy.bib.uottawa.ca:2128>>.

Ligeti's compositions from the late 1950's and early 1960's – *Apparitions* and *Atmosphères* – this new art was already addressing the same problems as experimental mathematics, only using a different language.<sup>22</sup>

Lobanova is the only scholar to overtly state the connection between Ligeti's music of this period and chaos theory and fractal geometry. She notes that this leads to a paradox in Ligeti's works: that there is a high sense of disorder, but that there is also an internal order and structure, thus positioning halfway between Boulez's integral serialism and John Cage's indeterminacy. Paradoxes of equal part order and disorder are also present in systems associated with chaos theory and fractal geometry.

Lobanova then proceeds to analyze *Atmosphères*. This section of her text provides predominantly general details, but does mention some important specific observations such as the work's juxtaposition of high and low registers, and the constant state of flux of the piece. Lobanova also notes that *Atmosphères* consists of internal textural movement that is highlighted by the contrast of different musical fabrics. This in turn leads to three types of illusion in Ligeti's music, one of them being change of scale. This change of scale directly connects this music to chaos theory and fractal geometry through the process of scaling and self-similarity: A discussion of these aspects of Ligeti's music and chaos theory will become relevant later in this thesis when analysis of *Atmosphères* is carried out.

The next section of relevance to this thesis in Lobanova's text is her discussion of the *Piano Concerto*. To Lobanova, this work marks a new direction in the style of Ligeti's music and represents a synthesis of his interest in other fields:

The *Piano Concerto* encapsulates Ligeti's new concept of rhythm and harmony; in this work, the composer developed his ideas of "generalized hemiola" and polytempo and the principles of isorhythm and African

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<sup>22</sup> Lobanova 4.

rhythm, as well as experimenting with various systems of pitch. The work was also heavily influenced by the illusory three-dimensionality of the drawings of Escher, the ideas of fractal geometry, the principles of recursiveness and so forth.<sup>23</sup>

All of the features that Lobanova mentions in this quote are present in the central, fourth movement of the *Piano Concerto*. She discusses the other movements as well, but they are not the main focus of this piece. Lobanova presents an explanation of the melodic material of the movement. She notes that the treatment of the melodic and rhythmic material is distinctly similar to M. C. Escher's crystallographic principles.<sup>24</sup> These principles state that images can be transformed by sliding, turning, or slide reflection. In addition, an image can be transformed by the blow-up principle. However, the comparison between the music of the fourth movement of Ligeti's *Piano Concerto* and Escher's principles is not correct. Escher's crystallographic principles assert that the shape of the object being manipulated does not change in size whatsoever. The size and shape of musical ideas in this work do change in size and scale and therefore Escher's terminology is inapplicable as an analytical tool.

Lobanova also notes that the rhythm of this movement is an endless labyrinth of vertical and horizontal combinations of motives that appear in numerous guises that have been extended and reduced. Unfortunately, she does not provide any analysis or musical examples to corroborate these ideas. Lobanova also notes that the melodic material is created by self-recursiveness, meaning that Ligeti takes a small melodic idea and keeps repeating it on top of itself. Thus, what is a simple structure by itself, when combined

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<sup>23</sup> Ibid., 307.

<sup>24</sup> For more about Escher's crystallographic principles please see Doris Schattschneider, *Visions of Symmetry : Notebooks, Periodic Drawings, and Related Work of M. C. Escher* New York: W. H. Freeman & Co., 1990) 95 – 97.

with others, is very complex. Once again however, she does not provide any examples or analysis to support this point.<sup>25</sup>

Another major resource contained within Lobanova's work is a set of three interviews between Lobanova and Ligeti. The first interview, from December 15, 1991, discusses the development of the ideas of order and chaos in Ligeti's other works of the late 1950's and early 1960's. Ligeti promptly dismisses this as an "unconscious parallel development,"<sup>26</sup> however; he admits that his view of music and how chaos theory treats processes are similar. Ligeti states that he views music "not as melody, polyphony, or contrapuntal structure, but as fluctuation, as thick and thin, dark and light areas."<sup>27</sup> Ligeti further develops this idea, by admitting that there is no direct connection, but other parallels between his work and Edward Lorenz's:

Remarkably enough, pieces like *Atmosphères*, which I composed in 1961, appeared at almost exactly the same time as the "atmospheric" research that Edward Lorenz published in 1962. The two things are not directly connected, but in both of them you can see the same pattern; fluctuation and current. So it is here that I can see the parallels between what I did (and not only I, but other people as well: I am thinking of Nancarrow's polyrhythm, etc.) and this new science.<sup>28</sup>

To state that Ligeti wrote *Atmosphères*, or any other pieces, directly under the influence of chaos theory or fractal geometry would be a great overstatement. However, upon further examination, one can re-contextualize works from the 1960's such as *Atmosphères* within the paradigm of chaos theory.

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<sup>25</sup> Lobanova 319.

<sup>26</sup> Ibid., 366.

<sup>27</sup> Ibid., 369.

<sup>28</sup> Ibid.

In the second interview, from December 22, 1991, Ligeti admits that the structures and ideas related to chaos theory found within his music have great importance:

Complex structures – order, chaos, labyrinths and spirals – have great symbolic importance in my music. Branching structures of the type seen in trees, street intersections, street maps of large cities, complex, labyrinthine gardens, spider’s webs, fishing nets, tissues and textures are also important. Consistency is important for my music too – hard, soft, sticky, wet – and then currents, turbulences, colours and light, both bright and dark... But I wouldn’t associate any of these with specific ages. Besides, I don’t think in aesthetic categories when I compose – I think in terms of form, consistency, colour and light, and of sound that is both colour and light...<sup>29</sup>

Through these interviews and Lobanova’s research, one begins to see that the similarities between Ligeti’s compositions and chaos theory are more than coincidence. Lobanova utilises many of the terms and ideas associated with chaos theory and fractal geometry to describe Ligeti’s music. Unfortunately, Lobanova does not develop this further by examining works with a consideration of these ideas.

The most recent source concerned with the biography of Ligeti is Richard Steinitz’s book *György Ligeti: Music of the Imagination*.<sup>30</sup> One may view this book as the “definitive” biography because the author worked very closely with Ligeti in the publication of his work. However, by this point of the review, most information contained in this volume is not new and has already been iterated in other sources discussed in this chapter. What is unique about this source is that it is the only one with significant information about the *Piano Concerto* and *A bout de soufflé*. As discussed

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<sup>29</sup> Ibid., 374.

<sup>30</sup> Richard Steinitz, *György Ligeti: Music of the Imagination* (Boston: Northeastern Press, 2003).

earlier in this chapter, this specific information will be introduced further in this thesis during specific examinations of the relevant pieces.

## 2.2 Interviews

This portion of the thesis considers extant interviews which exist outside of a survey of the biographical material on György Ligeti. The principal source for interviews with the composer is the collection entitled *Ligeti in conversation*,<sup>31</sup> a collection of four interviews which were conducted between 1968 and 1981. All of the interviews are translations from either Hungarian or German and this collection marks the first time they have been published in English.

The first interview is with Peter Varnai and is from 1978.<sup>32</sup> This discussion is a general one, and covers several topics ranging from Ligeti's early musical training to his musical influences. In this conversation, Ligeti compares his music to a "continuous flow, unbroken by bars, like a Gregorian melody."<sup>33</sup> Ligeti continues on the same tack and states: "the thematic – motive [sic.] structure and its role in the progress of music is almost completely abandoned."<sup>34</sup> These two quotes lend credence towards the idea that Ligeti's music should be examined not in a contained structural manner, but rather as a continual process. Ligeti's music does not have sections or distinct four- or eight-bar phrases like the music of Beethoven or Mozart, but rather like a constant stream which is always in a state of flux or change.

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<sup>31</sup> *Ligeti in Conversation with Peter Varnai, Josef Hausler, Claude Samuel and himself* (London: Eulenberg Books, 1983).

<sup>32</sup> Originally appeared as *Beszélgetések Ligeti Györggyel* (Budapest: Zenemukiado Vallalt, 1978).

<sup>33</sup> Varnai 14.

<sup>34</sup> Ibid.

The interview then continues with an examination of micropolyphony and Ligeti's approach to part writing, which in turn leads to a discussion of Ligeti's *String Quartet No. 1*:

Two diatonic solo parts combine to create a composite chromatic line, which is an idea characteristic of Bartok. That was my point of departure for 'woven' music like *Atmosphères*, where the intertwined strands are so completely blended that you cannot discern the individual parts.<sup>35</sup>

What Ligeti is describing is the combination of two simple modes or scales to create chromatic saturation or a dodecaphonic mode. An example of this would be the combination of a whole note scale and a major scale beginning on c'.<sup>36</sup>



Ex. 2.1 – Whole Tone Scale (WT)



Ex. 2.2 – C-major Scale (CM)



Ex. 2.3 – WT/CM combined

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<sup>35</sup> Varnai 15.

<sup>36</sup> For more about Bartok's combination of modes please see Erno Lendvai. *The workshops of Bartok and Kodaly* (Budapest: Edition Musica, 1983) and Elliot Antokoletz. *The Music of Bela Bartok: A Study of Tonality and Progression in Twentieth-Century Music* (Berkeley, California: University of California Press, 1983).

Separately, each of these scales are a rather simple event, but combined create a complex scale that is a mixture of tonality and chromaticism; in other words, two simple events have combined to form a complex one. This relates to the tenet of chaos theory that simple systems can combine to create a complex system. By combining several simple melodic lines or scales in one work, Ligeti has created an extremely complex system. This feature will be shown to be present in all three works examined in this thesis.

Ligeti continues in this interview to discuss another influence on his works, the Flemish school of polyphony. While studying in Budapest under Sándor Veress (1907 – 1992) and Ferenc Farkas (1905 – 2000), Ligeti was educated with a specific focus on the music of Johannes Ockeghem, Giovanni Palestrina, and strict counterpoint. The stylistic features of works by these two composers left a lasting impression upon the musician and provided a basis for his own compositions from the early 1960's:

My keen interest in the Flemish composers, in Ockeghem in particular, was also a contributing factor. To this day, I am more interested in Ockeghem than in Palestrina, because his music does not tend towards culminating points. Just as one voice approaches a climax another voice comes to counteract it, like waves in the sea. The unceasing continuity of Ockeghem's music, a progress without development, was one point of departure for me to think in terms of impenetrable textures of sound. Also what I learned at the electronic studios in Cologne, the superimposed layers of recorded sound, played its part. You will find that, even though indirectly, such techniques affected my musical language, the orchestral polyphony of *Atmosphères* and *Apparitions*.<sup>37</sup>

What Ligeti states is that the style and structure of *Atmosphères* is not unique. Like Ockeghem's music, this work displays qualities that are non-causal or that appear to not be part of a linear progression of events. This relationship between the music of Ockeghem and Ligeti and its influence upon Ligeti's works with concern to chaos theory and fractal geometry will be explained further in section 2.4

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<sup>37</sup> Varnai 26.

The next topic in this interview that is of relevance to this thesis is the discussion of *Piece Electronique No. 3*. This piece was the forerunner to *Atmosphères* and some of the effects that Ligeti attempted to create in this piece (and *Atmosphères*) relate to chaos theory.

In [Piece electronique No. 3], I meant to try out a process which I then thought feasible. But it turned out to be impossible. My idea was that a sufficient number of overtones without the fundamental would, as a result of their combined acoustic effect, sound the fundamental. I wanted to select and record on tape overtones between 1,000 Hz and 6,000 Hz, use only these and expected the composite to emerge automatically. I planned to make music out of pure sine waves with harmonic and sub-harmonic combinations, by introducing the metallic sound of sub-harmonics as well as the harmonics gradually, not all at once. I imagined that slowly, different composite sounds would emerge and slowly fade away again like shadows.<sup>38</sup>

The aural effect that Ligeti attempted to create in these two pieces is not visible on paper, it only exists aurally. The desired effect could only be accomplished through performance; on paper, both of these works appear dense and complex or without movement, but in fact, upon hearing them, one can detect movement within the music. Also, a sine wave, a simple entity, when combined with multiple instances of itself, produces a dense sound mass. Both works, *Atmosphères* and *Piano Etude No. 17* create a complex sound event out of rather simple ideas and will be demonstrated further in the analysis of this work in chapter 4.

The second interview in this collection is a transcription of a pair of radio broadcasts for the première of *Lontano* on Sudwestfunk Radio Baden-Baden.<sup>39</sup> This interview was originally intended to promote the performance of *Lontano*, but instead develops into a discourse of Ligeti's sound-mass compositions. Inadvertently, the main

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<sup>38</sup> Varnai 37.

<sup>39</sup> This interview originally appeared in Owe Nordwall, "Zwei Interviews mit György Ligeti," *György Ligeti: eine Monographie*

topic of the interview becomes *Atmosphères*; and eventually the conversation leads to an examination of the middle section of the work (Rehearsal letters H to L):

Hand in hand with this transformation of tone colour and dynamics, there is a compression of the plane of sound: that is to say, it gets narrower and narrower, almost as if it were being crammed into a funnel (I am possibly speaking in too concrete terms now, but I don't mean to as regards the music). This funnel becomes narrower and narrower until it is a kind of whirlpool or vortex, if one looks at the score, one finds that the music turns in two directions, almost like a cyclone and anti-cyclone; the high strings have a definite direction in their scoring, and the violas, cellos and double basses have exactly the opposite.<sup>40</sup>

The construction of the melodic material and the visual representation of this section are similar to bifurcation diagrams. Furthermore, a comparison of the music to whirlpools and vortexes firmly relates the work to chaos theory, as this idea is often cited in studies on the mathematical construct. This connection between this section of *Atmosphères* and chaos theory will be further developed in chapter 4.

The third interview within this collection is with Claude Samuel. However, it is about Ligeti's opera *Le Grand Macabre* and therefore contains no information relevant to this thesis.

The fourth interview is a self-interview conducted by Ligeti that originally appeared in the journal *Melos*<sup>41</sup> and approaches the subject of serialism and Ligeti's issues with it. One can almost detect a sense of frustration on Ligeti's part in that the questions he asks himself are perhaps the ones he wished other people would ask him.

The first topic Ligeti broaches is the process of his music. He describes a method and approach that considers the musical idea first and not the form: "Structural potentialities are already contained in the primitive idea, and the act of composition

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<sup>40</sup> Hausler 86.

<sup>41</sup> György Ligeti, "Fragen und Antworten von mir selbst," *Melos* (December 1971).

consists of developing these latent potentialities.”<sup>42</sup> For Ligeti, raw musical ideas have a form or destination within them. One does not know what these forms are until it is composed out; in other words, one cannot take a musical idea and say that they will use this as the first theme of their sonata. To do so, a notion of how the theme will develop or how it will be present in the piece itself would be pre-conceived.

Ligeti does note that there are built in preferences within any musical idea: “the naiveté of the raw state is itself not untarnished. It is already interlaced with a series of preferences, and within its amorphousness lie traces of the as yet undiscovered crystal.”<sup>43</sup> In other words, what Ligeti states is that one knows that a piece of music will develop and where it will have a terminus, but how it will get there is originally unknown because it is internal to the original musical idea, and will only become present when released from the idea itself.<sup>44</sup>

Ligeti then continues to discuss the stylistic shift in his music in the late 1960’s. The method with which he previously treated timbre and rhythm was transmitted to his treatment of intervals in his later work of the 60’s:

The manner in which I use intervals in *Lux Aeterna* and *Lontano* reflects my experiences of timbre construction within harmonically neutral context. That is to say, I treat intervals just as I previously treated timbre complexes. In *Lontano* intervallic structures are subjected to continual transformation, similar to the transformation of tone colours in *Atmosphères*.<sup>45</sup>

The constant change that intervals undergo in the works of the late 60’s and 70’s, is the same process that the sound blocks in *Atmosphères*, *Lontano*, and other sound-mass

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<sup>42</sup> Ligeti 124-125.

<sup>43</sup> Ibid 125.

<sup>44</sup> It should be noted that this idea of “we will get there, but how will we get there,” is distinctly similar to the stochastic music of Iannis Xenakis. For more about Xenakis and stochastic music please see Iannis Xenakis, *Formalized music: thought and mathematics in composition* (Stuyvesant, NY: Pendragon Press, 1992), Balint Andras Vargas, *Conversations with Iannis Xenakis* (London: Faber & Faber, 1996).

<sup>45</sup> Ligeti 126

works were subjected to. The sound blocks of the early 1960's and the intervals of the late 60's and early 70's in Ligeti's works were in a constant state of flux, they undergo gradual change and do not have a fixed or constant state. This relates to chaos theory in that also it also states that equations or processes evolve, they undergo growth or decay and there is no fixed state for an equation.

Ligeti then compares his compositional process to another element related to chaos theory: feedback. Ligeti believes that the finished musical product created from raw material is taken and fed back into the system or process of composition:

The primary conception of new pieces contains the imprint of the working processes used in the development of previous pieces. The consequence of this is a feedback effect: the raw state is pre-shaped by experiences gained during composition, and is thus no longer quite 'raw.' The borderline between raw state and finished article becomes fluid, the naive idea already contains distinct structural features.<sup>46</sup>

Just as in feedback processes in chaos theory, Ligeti views his entire compositional *oeuvre* as one continual process. All previous output informs any new piece by him and any new idea Ligeti has formed or created will eventually be input into a subsequent composition. As a formula, this means:

*Piece electronique No. 3 + C = Atmosphères*

*Atmosphères + C = Continuum*

*Continuum + C = Aventures*

Therefore, it would not be an overstatement to say that a stylistic similarity exists between Ligeti's early and late works. This is because the alter works are informed by the earlier versions. To say that a later work of Ligeti's has a stylistic or constructional similarity to an earlier work is not mere coincidence; the processes or materials from an

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<sup>46</sup> Ligeti 126-127.

earlier work inform a latter work would not be an overstatement.<sup>47</sup> This also provides another link between Ligeti's music, Stockhausen, and moment form. Ligeti's own works can be viewed as part of a larger musical process or operation.<sup>48</sup>

The final topic in this self-interview is Ligeti's concerns with serial composition. His problem with serialism is the assumption that there could be an analogous equation that would create the same effect across the four aspects of music that serialism attempted to control:

The other aspect of serial composition I found problematical was the organization of all the musical elements within a unified plan. In serial music it was axiomatic that a single basic order should be manifest throughout the various areas. There was also a recognizable tendency to regard a pitch series as the starting point, and to arrange the other elements in accordance with that, even to the extent that the notes are arranged in a series would initially be given numbers. These numbers would then be detached from the pitch series and applied to the areas of duration, timbre, intensity, and later even to wider determining factors such as density, compass, proportions of structural detail, etc. One could also work the other way round: a series of numbers or of numerical relationships – ratios for instance – would be laid down and then applied to the various musical areas. Uniformity of organization was the fundamental tenet of serial music: a quantifiable basic order, a modulus, had to be laid down, and every single part of the composition had to be derivable from the chosen modulus.<sup>49</sup>

What Ligeti is stating is that he does not agree with the uniformity aspect of serial music, that he believes there is no single equation that can govern how a musical composition will form or develop. This was the same question that physicists and mathematicians were exploring during the same era. They were attempting to find a wide-ranging formula that could explain weather patterns or population, but like Ligeti, were discovering that it

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<sup>47</sup> This is similar to fn 22

<sup>48</sup> For more please see fn. 12

<sup>49</sup> Ibid 128-129.

was impossible to totally govern or predict all aspects of an operation.<sup>50</sup> Thus both Ligeti and scientists found the solution to their concerns with chaos theory.

These four interviews are by far the most illuminating ones Ligeti has ever given. Within these, he exposes his approach to music and his feelings towards the milieu out of which he developed.

Another interview with Ligeti from the same period as the self-interview is one conducted by Louis Christensen while Ligeti was a guest lecturer at the Center for Computer Research in Music and Acoustics (CCRMA) at Stanford University (California) in 1972.<sup>51</sup> Although this interview does not touch upon any of the works analyzed in this thesis, it does provide several details that are critical towards the inquiry of other works that contain applications of chaos theory.

The first work that Ligeti discusses is *Melodien*, one of his latter sound-mass works. Ligeti describes how this work consists of different layers with different tempi: "...for instance in this passage they are playing in different tempi, so it's a chaos...but you have to play the dynamic markings very, very exactly..."<sup>52</sup> Though this work is from the late 1960's, the stylistic traits of works from Ligeti's chaos period are already manifested in this composition.

Ligeti then proceeds to discuss the construction of the melodic content of the work. What he describes is a method he has labelled as the 'envelope' method:

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<sup>50</sup> In an anecdote told to the author, Robert Piencikowski of the Sacher Foundation recounted how while at the Internationale Ferienkurse für Neue Musik at Darmstadt in 1976, he asked Ligeti about his compositional tools, when Mr. Piencikowski informed Ligeti that he was merely presenting horizontal versions of vertical sonorities Ligeti turned to him and said, "you must be from Princeton, you are looking for the patterns or the formula." Though Mr. Piencikowski assured me that Ligeti was merely joking with him, one can see the viewpoint that Ligeti holds, that it is impossible to find a discernable pattern or order within his pieces and that if one attempts to do so, they will find it impossible, also, one can see the disdain Ligeti has towards serial music or music that is completely constructed from a formula or a pattern.

<sup>51</sup> Louis Christensen, "Conversation with Ligeti at Stanford," *Numus-west* 2 (1972): 17-20.

<sup>52</sup> *Ibid.*, 18.

I have a very compressed cluster; then I expand it like a spider web. Then the minor seconds are going to be major seconds, then minor thirds, and so continually expanding. At this moment it breaks off and only two threads are left, and then they expand, filling in with more pitches... only there is no harmony because everything consists of melodies... but different melody patterns at different speeds... and always they are aiming for these focal points.<sup>53</sup>



Ex. 2.4 Example of an envelope or bifurcation tree starting on A.

The process that Ligeti describes as the ‘envelope method’ is distinctly similar to the process of bifurcation and the description of *Atmosphères* given previously.

Ligeti concludes this interview with a discussion of what was then a new work commissioned by the San Francisco Symphony. For this work, Ligeti describes how he wishes to control not only the simultaneous playing of pitches, but also the simultaneous playing of different tempi and rhythms.<sup>54</sup> Though Ligeti does not make clear which piece this will become, the patterns and techniques that he describes in relation to this piece are similar to the techniques he applies in the *Piano Etudes* and *Piano Concerto*.<sup>55</sup> What this interview demonstrates is that the seventies were a period in which Ligeti was concerned

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<sup>53</sup> Ibid., 19.

<sup>54</sup> Ibid., 20.

<sup>55</sup> One can only approximately guess from the dates given that the work Ligeti is describing is either *San Francisco Polyphony* or *Clocks and Clouds*.

with exploring and expanding many of the techniques that are central to the works that he would compose during the 1980's and 90's.

A more recent interview is one by Canadian pedagogue Stephen Satory from 1990.<sup>56</sup> The main topic of this conversation is Ligeti's *String Quartet No. 2*; however, during the course of the interview Ligeti offers several comments that elucidate his thought, and compositional, process.

The first comment that Ligeti provides pertains to fractals. Ligeti states:

...and in my music, there are mathematical considerations, considerations of fractal geometry. Fractal geometry is mathematics and not art. But it is an area which became well known only in the mid-Seventies. Its kind of growth and its processes of pattern transformation have occupied me for a great deal. Its kind of thinking, of growth, degeneration, decay and branching-out did exist for me at the time of the composition of the *Second String Quartet*. Indeed *Ramifications* (1969), which I wrote right after the *Second String Quartet*, deals with branching-outs, and this is already in its title. But this sort of organic growth is more strongly present in the pieces of the Seventies. It is there in the *Second String Quartet*, but not consciously.<sup>57</sup>

In this statement, Ligeti admits that there are elements of mathematics and fractal geometry present within his works, but in works not normally associated with these processes. In fact, Ligeti implies that he had adapted compositional methods associated with chaos theory and fractal geometry before it came to the attention of the public or scholars.

Another statement provided by Ligeti in this interview relates to his relationship with the sciences and its presence in his *oeuvre*:

Thoughts about science have occupied me a great deal, and I have read a great deal about them. I have read not only journalism and let's say popular books, but also scientific books, mainly about mathematics. These

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<sup>56</sup> Stephen Satory, "Colloquy: An Interview with György Ligeti," *Canadian University Music Review* (1990): 101-117.

<sup>57</sup> *Ibid.*, 108.

have related back very powerfully to my compositional thinking. I have read these books because in my compositions I was dealing with similar concepts, especially with complexity and pattern. Actually, I had already begun to think this way with *Atmosphères* and micropolyphony. So, this is how my compositional thinking has changed gradually.<sup>58</sup>

What Ligeti states is that he did not adapt chaos theory or fractal geometry as a model or basis for his compositions, but rather these precepts and terms provided a label for his own compositional techniques. Certainly, however, one could say, based on this interview, that the vocabulary and concepts presented by chaos theory and fractal geometry offer an excellent parallel analytical tool by which to approach Ligeti's compositions.

### **2.3 Dissertations**

Several doctoral and masters' dissertations have been written on Ligeti and his music. Some focus on a specific era or compositional style in Ligeti's *oeuvre*, while others focus on a specific work.

One thesis that considers a specific time period is Mary Jane Hoogewind's thesis from 2000, which concentrates on all of the sound-mass works.<sup>59</sup> The unique feature of this thesis being that Hoogewind provides an extensive historiography of the era that Ligeti was producing these works in. Hoogewind describes the milieu that Ligeti's sound mass works emerged from as one divided between order and disorder. On one side was Pierre Boulez and integral serialism and on the other John Cage and indeterminacy; just

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<sup>58</sup> Ibid., 116.

<sup>59</sup> Mary Jane Hoogewind, "Compositional Techniques of Sound Mass in selected works of György Ligeti Long beach Ph. D. diss, 2000.

as science was divided between the two issues of order and disorder, so was music.<sup>60</sup>

Hoogewind argues that Ligeti was not wholly influenced by either one of these composers, but rather that his music is a synthesis of Boulez's serialism, Cage's indeterminacy, and Iannis Xenakis' stochastic music.<sup>61</sup> Therefore, an argument for equal parts of order and disorder being present in Ligeti's music would correspond with her conclusions.

The other significant detail which emerges from Hoogewind's study is that she clearly identifies one of the main compositional techniques in Ligeti's sound mass works as clusters constructed from major seconds.<sup>62</sup> This is the only English source to corroborate the earlier statements made by Pierre Michel. The dissertation continues with analyses of the six works from this period. Unfortunately these analyses, and in particular, the analysis of *Atmosphères*, provides no new details about the works that will be used in this thesis.

Another dissertation that likewise examines Ligeti's compositional techniques is Carson Kievman's which explores the stylistic and structural similarities between the music of Ligeti and Johannes Ockeghem.<sup>63</sup> Though there is no information that is pertinent to this current thesis within Kievman's work, this text does reinforce the idea that Ockeghem music is in a constant state of flux and that Ligeti's compositions from the 60's are greatly influenced by this music.

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<sup>60</sup> It should be noted, that like Ligeti's music, chaos theory had a difficult time positioning itself in the field of science. To this day some maintain that chaos theory is too theoretical to be considered a mathematical science, while others maintain that it is too mathematical to be called a physical science. For more about this schism in chaos theory and the sciences please see Gleick 1-20.

<sup>61</sup> Hoogewind

<sup>62</sup> Hoogewind 35.

<sup>63</sup> Carson Kievman, "Ockeghem and Ligeti: The music of Transcendence," Ph. D. diss. Princeton University, 2003.

The only dissertation that exclusively analyses the *Piano Concerto* is Mark Engebretson's D. M. A. dissertation<sup>64</sup>. This dissertation pursues an analysis of the first movement of this work and is concerned with its rhythmic and melodic structure. More specifically, it explores Ligeti's use of non-traditional tonality and rhythmic patterns within this movement. Though there is no direct link between the first and fourth movements, the abstract characteristics of melody and rhythm are similar between the two. Therefore Engebretson's thesis will serve as a helpful methodological model by which to analyze the fourth movement of the *Piano Concerto*.

Furthermore, another dissertation on works directly relating to this thesis is Mayron Tsong's thesis on Ligeti's first book of *Piano Etudes*.<sup>65</sup> Since the *Piano Concerto* has often been compared to the *Etudes* in terms of style and structure, it is fitting that ideas used to analyze the *Etudes* could be used to further the analysis and understanding of the *Piano Concerto*. In chapter 3 of Tsong's work entitled, "Rhythmic and Pianistic Virtuosity," Tsong has a subsection entitled "Illusionary Polytempi". In this section, the author describes how Ligeti combines hemiola with additive rhythms to create a unique sound:

By combining the hemiola effect with the additive pulsation principle of African music, Ligeti produces the illusion of different simultaneous tempi. Since there are no bar lines or "measures" to define absolute meter, several different rhythms are perceived: a background consisting of rapid, even pulsations with superimposed layers of asymmetrical patterns of varying lengths. According to Ligeti, it is possible to beat either duple *or* triple time to these patterns, resulting in polytempo and producing a kind of *hemiola*.<sup>66</sup>

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<sup>64</sup> Mark Engebretson, "György Ligeti's Piano Concerto: An Analysis of the First Movement," D. M. A. diss, Northwestern University, 2000.

<sup>65</sup> Mayron K. Tsong, *Etudes pour piano, premier livre of György Ligeti : studies in composition and pianism* M. Music diss. Rice University, 2001.

<sup>66</sup> Tsong 16-17.

What Tsong is describing is a basic underlying rhythmic pattern such as 4/4, superimposed with different rhythmic groupings such as 3/16 or 5/8. When these dissimilar rhythmic patterns are combined, a complex and illusionary rhythmic pattern emerges. This compositional feature is a prominent structural feature in the fourth movement of the *Piano Concerto* and will be identified and discussed in more detail in the analysis of this work in chapter 3.

Three recent dissertations about Ligeti's late period have also been written. These dissertations by Amy Bauer, Eric Drott, and John Cuciurean all examine compositional techniques Ligeti applies in his late period.<sup>67</sup> All three of these dissertations discuss elements of Ligeti's music that relate to chaos theory, but only Cuciurean develops this relationship to any significant level. Furthermore, all three of these dissertations examine works that have been discussed or analysed before in several other sources and will not be pursued in this thesis.

## 2.4 Articles

One article that explores the issues of rhythmic complexity in Ligeti's music is Stephen Taylor's recent article about the presence of non-traditional influences in the composer's works.<sup>68</sup> This article explores the influence of African music on Ligeti's compositions of the past twenty years. Through consultation of other sources written by Ligeti, Taylor asserts that within Ligeti's more recent compositions, there are basic rhythmic patterns or pulses, which form a background layer. In turn these basic patterns

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<sup>67</sup> Amy Marie Bauer, "Compositional Process and Parody in the Music of György Ligeti," Ph. D. diss Yale University, 1997; Eric Austin Drott, "Agency and impersonality in the Music of György Ligeti," Ph. D. diss Yale University, 2001; John Cuicerean, "A Theory of Pitch, and Intertextual Allusion for the Late Music of György Ligeti," Ph. D. diss SUNY – Buffalo, 2000.

<sup>68</sup> Stephen Andrew Taylor, "Ligeti, Africa, and Polyrythm," *the world of music* 45 (2003): 83 – 94.

serve as a sort of rhythmic glue for the remaining patterns which are then superimposed on top of them.

Along these lines, Taylor also examines Ligeti's treatment of basic pulses in his earlier music. Taylor, asserts that in Ligeti's earlier compositions, the beats are much larger and the pulses are a multiple of these. In other words, all beats in a piece would be quarter notes, but the rhythm is a dividend of this, such as triplets, sixteenths, septuplets, etc. Another way of stating this is that the rhythm can be viewed as divisive, similar in the way that renaissance theory views rhythm. In later works, Taylor describes Ligeti's approach to rhythm being similar to that of the music of Philip Glass and sub-Saharan Africa. In this body of music, the rhythm is also additive; there is a basic unit of time or 'lowest common denominator,' of which all subsequent rhythmic patterns are a multiple or factor. Thus, a simple beat, such as one third of a triplet can be multiplied, expanded, or stretched to combine with itself to create a complex rhythm. The ideas that Taylor presents on the topic of Ligeti's use of rhythm will be considered in the analysis of the *Piano Concerto* in chapter three and *Piano Etude 17* in chapter 4 of this thesis.

The earliest know example of scholarship bringing forth the presence of chaos theory and fractal geometry in the music of Ligeti is a set of three articles by Richard Steinitz. In these articles Steinitz explains and discusses elements in the later keyboard works of Ligeti. There is little information in these articles that is new at this point of the review, however the first article does contain several quotes which provide an excellent contextualization of Ligeti's music.

In the first article, Steinitz emphasises the fact that what Ligeti attempts to do in his compositions, by creating music out of synthesis of mathematics and science is not

new, that in fact this relationship is quite old: “Music and mathematics are ancient bedfellows, arguably achieving a more convincing correlation than in other art forms.”<sup>69</sup> This is the only source that clearly supports the idea that Ligeti’s application of mathematical influences is not new, but rather part of a larger tradition that encompasses the entire history of music.<sup>70</sup> In addition, Steinitz insinuates that the similarities between chaos theory and Ligeti were noticed by Ligeti himself.

Ligeti realized that the new theories which sought to explain the precarious balance between order and disorder, pattern and chaos, and the apparent origin of both conditions in measurable deterministic situations, had intriguing parallels with the way he composed.<sup>71</sup>

What Steinitz insinuates is that Ligeti did not adopt chaos theory or fractal geometry as a model or tool for composition, but rather that it provided a label and a basis for his methods. Steinitz continues along these lines by arguing that where there appears to be no order or pattern within Ligeti’s music, there is actually structure.

The rest of this article and the other two written by Steinitz continue along these same lines by examining several of the etudes and discusses the processes within them that are similar to chaos theory or fractal geometry. Steinitz does discuss the Piano Concerto in the third article, but only the first movement. Overall, this set of articles is important, as they emphasize the presence of certain facets of chaos theory within Ligeti’s works.

One article specifically concerned with the *Piano Concerto* is that written by John Warnaby’s from 1987. This article includes insights to the structural features of the

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<sup>69</sup> Steinitz 15.

<sup>70</sup> John Fauvel, Raymond Flood, & robin Wilson, *Music and Mathematics from Pythagoras to Fractals* (London: Oxford University Press, 2003).

<sup>71</sup> Steinitz 15

concerto<sup>72</sup> and corroborates previous statements about the form, or what Warnaby considers the absence of it, in Ligeti's works. In his article, which focuses mainly on the first movement of the concerto, Warnaby identifies the rapid development or appearance of melodic figures as a main stylistic feature present in Ligeti's music. He states: "But his main fascination has always been with the accidental appearance of melodic figures, rather than with their structural potential."<sup>73</sup> What Warnaby suggests is that Ligeti's music has melodic content that is not causal or developed from a theme at the beginning

Warnaby's analysis focuses more on the first movement, but his general discussion of the work again mentions the idea of melodies emerging over the course of it, as opposed to already being present. In his conclusion, Warnaby asserts that only the fourth movement of the Piano Concerto is known to have been influenced by chaos theory and fractal geometry, but there are enough statements and evidence made in this article to support the idea that the other movements have tangential connections to these mathematical processes as well.

Another source of information on the *Piano Concerto* is an article written by Ligeti himself.<sup>74</sup> This article, written shortly after the world premiere, is a description and synopsis of the work. In it, Ligeti provides many details which elucidate many ideas about the *Piano Concerto*.

Ligeti begins by stating what he has attempted to do in this work:

In this work I have realized new conceptions of harmony and rhythm. The first movement is bimetrically notated throughout: 12/8 and 4/4 (8/8) superimposed. This corresponds to the well known "two-against-three" notation, in itself nothing new. However since I articulate the triplets of 12

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<sup>72</sup> John Warnaby, "Ligeti's Piano Concerto," *Tempo* 163 (1987): 47-49.

<sup>73</sup> Warnaby 48.

<sup>74</sup> György Ligeti, "On My Piano Concerto," *Sonus* 9 (1988): 8 – 13. Trans Robert Cogan

pulsations and duplets of 8 pulsations at different rhythmic points, a displaced, previously unheard polymetric emerges.<sup>75</sup>

A simple rhythmic gesture, three against two, with its beats altered, can create a new and unpredictable pattern. This process qualifies as a strange attractor, because the new pattern is not expected, and also as another example of a simple event producing a new and more complex event. It will be demonstrated in chapter three that the idea of multiple rhythmic points is also present in the fourth movement.

The next detail that Ligeti provides is concerned with the melodic aspect of the work. Ligeti states:

This blossoming of isolated structural details into a transforming global structure is one of my basic compositional assumptions: since my orchestra pieces ‘Apparitions’ and ‘Atmosphères’ at the end of the fifties, I have sought ever-new solution of this basic idea.<sup>76</sup>

This is also a strange attractor in its truest form. The material is present at the beginning, but it also becomes the final goal, or central aspect of the movement. Furthermore, by this statement, Ligeti verifies any ideas about this technique being present in the early period of his works. Ligeti continues to describe the first three movements with examples that could relate the work to chaos theory and fractal geometry.

For the central (fourth) movement, Ligeti provides an abundance of statements about the composition and its relation to chaos theory and fractal geometry. Ligeti begins by describing the melodic and rhythmic aspects of the movement as “rudimentary,”<sup>77</sup> and further develops ideas about the melodies he uses in the work by stating that they are always different yet similar.<sup>78</sup> This would relate the melodic material to the idea of self-

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<sup>75</sup> Ligeti 9.

<sup>76</sup> Ibid.

<sup>77</sup> Ligeti 11.

<sup>78</sup> Ibid.

similarity, although he does not explicitly provide any proof or evidence to verify this statement. Ligeti also elaborates on the rhythmic organization of this movement by asserting that at the beginning: “Without our realizing it at the beginning, a complex, gradually-emerging, talea-like rhythmic order secretly governs.”<sup>79</sup> With this article, Ligeti affirms that amid the chaos of this composition there are structured ordered events controlling the music. These events are not present in the beginning, but emerge over the course of the work, much in the same way that a strange attractor materializes over the course of a non-linear process. To further substantiate the presence and influence of chaos theory and fractal geometry in this work, Ligeti compares it to Julia and Mandelbrot sets. Likewise, when Ligeti states that the melodic and rhythmic material is rudimentary, but then explains it as complex, and ordered, Ligeti confirms the idea that the complex structure and nature of this movement is constructed from simple ideas.

Ligeti closes this article with what can be best described as his artistic credo. He writes: that his music can be regarded as a series of structures that are objects; that he tries to maintain the ambiance/aesthetic affect that he creates in his music and to keep it present at all times; and that though a desired affect/element must be present, it might not be apparent at the beginning of the work, but will appear and will be kept in the forefront as long as possible. As of today, this article represents the only instance of Ligeti directly acknowledging or discussing elements of his work in terms or terminology relatable to chaos theory and fractal geometry.

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<sup>79</sup> Ibid.

## **2.8 Conclusion**

To summarize, upon reviewing the extant literature several facts become evident. First, several scholars have identified musical elements within Ligeti's work that are similar to chaos theory and fractal geometry. Secondly, Ligeti's synthesis of mathematics and science with music is not new, and it is part of a larger tradition that encompasses the entire canon of music. Thirdly, Ligeti views his compositional convention as part of a larger tradition that is simply a continuation of the tradition of previous composers. Just as Schoenberg vehemently argued that he was a continuation of the tradition of Brahms, Wagner, Beethoven and Mozart, Ligeti views that he is similarly a continuation of the tradition of experimentation in music in the same way that Debussy, Bartok, Schoenberg, and Ockeghem were.

### Chapter 3 – Piano Concerto

The *Piano Concerto* is one of the central works from Ligeti's compositional output during the 1980's and it had an extended development from conception to its final state. In 1969, Ligeti was approached by Mario di Bonaventura to compose a piano concerto but it was not until 1980 that Ligeti commenced composition of the piece.<sup>1</sup> It received its original premiere as a three movement work in Vienna on October 23, 1986. Unhappy with his version, Ligeti appended two more movements to the piece bringing the total number of sections to five. This version was premiered two years later on February 29, 1988 in Vienna with Bonaventura again conducting. This final structure of the work is interesting because evidence found in the Ligeti Collection suggests that the work was originally intended to be seven movements long. However, it is inconclusive when this evidence was ever extended beyond the conceptual stage.<sup>2</sup>

The form and structure of this work is unique in that unlike a traditional concerto, the piano is not featured as a soloist, but rather it is treated as a member of the orchestra that helps emphasize the musical and rhythmic ideas of the piece. The only work that is similar to Ligeti's work in its incorporation and treatment of the piano (in the author's mind) is Colin McPhee's *Tabuh-Tabuhan*.<sup>3</sup>

Most studies of this work focus on the first movement because it is one of the best examples of polymeter within Ligeti's *oeuvre*. In addition to this, all sources note that the work's central movement is the fourth. In addition, many sources affirm that this section is the only musical application of chaos theory and fractal geometry in Ligeti's output.

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<sup>1</sup> Lobanova 306.

<sup>2</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

<sup>3</sup> For more about this work and composer please see Carol Oja, *Colin McPhee: Composer in Two Worlds* (Urbana, Ill.: University of Illinois Press, 2004).

The only authors that commit a significant amount of attention to this movement are Maria Lobanova, Richard Steinitz, and Ligeti himself.<sup>4</sup> The only fact that is known for certain about this movement is that, like the first, it employs two hexachords as the basis for the melodic material.<sup>5</sup> However, what has not been discussed is whether these hexachords are the same as those in the first movement that are identified by Engebretson in his thesis, or whether different hexachords are used and how they may be related to those used in the first movement.

As discussed in chapter 2, there are two fundamental aspects to this work which connect it to chaos theory. First, there is a basic underlying order to the rhythmic structure of the piece that provides a ground or base upon which the work is built; and secondly, the melodic content is comprised of the manipulation of small or simple figures which in turn create a dense complex web of music.

### **3.1 Methodology**

There will be two aspects to this analysis: melodic and rhythmic. As discussed in the introduction, the harmonic analysis will be informed via post-tonal analysis because of the tonal ambiguity in the work. Also, the terms employed by Charles Madden in his book *Fractals in Music* will be used to describe the operations and transformations the musical content of the piece goes through. In addition to this, it will be kept in mind that there is evidence in the Sacher Foundation's holdings that suggest Ligeti was contemplating, or considers, this movement to be in b-flat minor or D-flat major.<sup>6</sup> The rhythmic analysis of this excerpt will use the ideas outlined by Simha Arom because of

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<sup>4</sup> Richard Steinitz, *György Ligeti: Music of the Imagination* (Northeastern University Press, 2003): 315 – 316, 317-320, 322-29.

<sup>5</sup> Steinitz 328.

<sup>6</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

the lack of regular stress or metre patterns within the music. Furthermore, while elucidating the importance of these specific methods, this thesis will continue to connect the findings of this analysis with components of chaos theory and fractal geometry.

### **3.2 Harmonic Analysis**

In this analysis, three aspects will highlight the relationship between the musical elements and chaos theory and fractal geometry: the formation of hexachords, a recurring motif, and a specific interval class and its permutations. The excerpts analysed and discussed in this chapter are ones that display properties which best exemplify these qualities.

### **3.3 Hexachord formation**

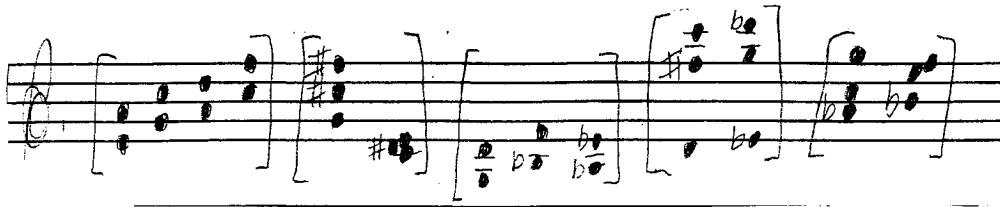
Throughout the movement, hexachords are ubiquitous and demonstrate the property of a complex event formulated from a simple one, the idea of disturbance or turbulences, as well as the property of self-similarity or different yet the same.

In the opening twelve measures, Ligeti shows the different ways that the formation of a hexachord may transpire. At measure 3 both the first and second violins play a 4-26 (0358) tetrachord, the first a [0479] and the second a [0259].<sup>7</sup> Together these two elements combine to form a 6-32 [024579] hexachord. Immediately following this the piano successively plays a 3-5 trichord [167] and a 3-3 trichord [034]. In conjunction, these two trichords form a 6-Z13 hexachord [013467]. Subsequently at measure 9, two verticalized trichords are sounded, the flute plays a 3-1 [234], while the clarinet plays a 3-2 [78T] trichord vertically. Amalgamated, these two elements form a 6-Z25 [23478T] hexachord. Maintaining this pattern two measures later at bar 11, two iterations of the 3-8

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<sup>7</sup> Throughout this analysis, square brackets will refer to the normal order of the set, while rounded brackets will refer to the prime form of the set.

trichord form a 6-5 [012367] hexachord or alternatively, as a series of simultaneous minor second dyads. Following this on the next beat of the measure, the piano plays a 3-4 [078] trichord followed by a 3-5 [45T] trichord. Together, these two elements form a 6-Z24 [04578T] hexachord.



Ex. 3.1 – Piano reduction of mm. 1-12 highlighting hexachord formation

From this passage, one can glean the different ways Ligeti assembles hexachords. One sees he does so through the combination of two discrete or similar pitch-class sets of the same cardinality vertically, horizontally, or in succession.

Moreover, in the reduction, one sees visually how the hexachords are rotations or shears of each other. The hexachord starts as a diagonal set, then rotated so it is vertical, followed by being reflected and sheared horizontally, before finally being reflected and sheared vertically, which is immediately followed by another vertical shear and reflection. Equally important, is that the hexachords are different yet similar through a “fuzzy” or near transposition of each other.

H <sub>1</sub>	→	T <sub>11</sub>	→	H <sub>2</sub>	→	T <sub>2</sub>	→	H <sub>3</sub>	→	T <sub>10</sub>	→	H <sub>4</sub>	→	T <sub>4</sub>	→	H <sub>5</sub>
0				0				2				0				4
2				1				3				1				5
4				3				4				2				7
5				4				7				3				8
7				6				8				6				T
9				7				T				7				0

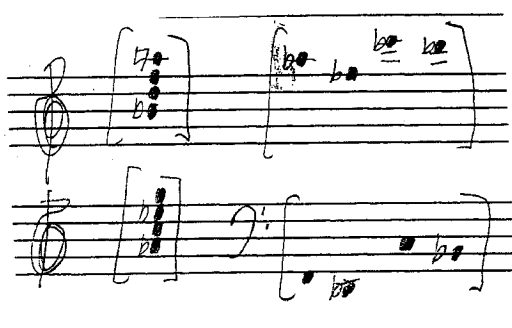
Fig. 3.1 – Fuzzy transposition chart of normal order of hexachords in mm. 1-12

Not all of the elements transpose at the same level. In every hexachord in this series, there are at least two elements that do not transpose at the same level. Therefore, these hexachords are similar yet different from their preceding form. Another link between the transformations that the hexachords undergo is disturbances and turbulences. Some elements remain held between the hexachords only appear in four of the five hexachords. The hexachord that does not contain this element is the disruption to the process. One case of this is pitch class 0; it is present within four of the five hexachords. Hexachord three, which does not share this common element, may be seen as a disturbance or turbulence to the pattern. From this passage, one can see how that it can be argued that these hexachords are different, yet the same. This comes about through the presentation and transpositional relationship of the hexachords. In summation, these hexachords are the same but different.

A second more extended passage of hexachord formation occurs at measures 36-64. Like the previous passage, it also demonstrates the ways that hexachord formation

happens, but also the ways that pitch-class sets of the same cardinality do not form a hexachord.

At measures 41 through 43 of the piano part, there are two examples of sets of the same cardinality not forming hexachords. The first example, the piano plays a 4-16 [249T] and 4-20 [0378] tetrachord, together this forms a 8-16 [0234789T] octachord. The second example is two statements of a 4-14 tetrachord, the first form is a [1348] and the second form a [015T]. In conjunction, these sets form 7-27 [013458T] septachord.



Ex. 3.2 – Piano reduction of mm. 41-43

After the first twelve measures, one would expect that sets of the same cardinality would always form hexachords, but they do not. This is similar to disturbances or turbulences in a non-linear process. One has become accustomed to the pattern of hexachord formation, and expects it to continue, but it does not. After brief passages of interruption to hexachord formation, Ligeti returns to the pattern of hexachord formation.

Another aspect of hexachord formation that relates to chaos theory and fractal geometry is self-similarity amongst the hexachords. What this means is that the formation or construction of the hexachords might be the same but different; in each case a shared commonality exists between them. Two ways that Ligeti does this is through utilisation

of the same pc-sets to form the same hexachords at discrete measures and the same pc-set to form disparate hexachords in separate measures.

An excellent exemplar of the same subsets and supersets reoccurring takes place at measures 3, 72, and 106. In these three measures, two 4-26 (0358) tetrachords combine to form a 6-32 (024579) hexachord.



Ex. 3.3 –Self-similar hexachords at mm. 3, 72, 106

In each case, the pitch content and rhythmic presentation differs, but the pitch classes are the same. The foundation of the sets is the same, only the presentation is different.

Another instance of self-similarity occurs at instances where the same trichords or tetrachords occur, but in each case form a different hexachord. One such example is of 4-17 (0347) tetrachords forming unique hexachords.

The first iteration occurs at measure 39. The trumpet and horn each play a verticalized form of the set that in conjunction, form a 6-20 (014589) hexachord. Later at measure 56-57, the bassoon and horn also iterate verticalized forms of the tetrachord. However, in this instance, they combine to form a 6-Z49 (013479) hexachord. The hexachords are the same because they are formed from the same tetrachords, but different because they are distinct and unique sets. A further relationship between these two hexachords is that a “fuzzy” transposition or shear is present. In the prime form of these

sets, elements one, two, and six remain the same, while the remaining three elements are related by a  $T_{11}$  transposition.

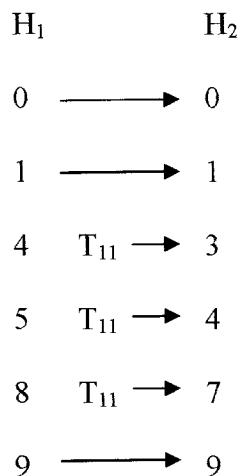


Fig. 3.2 – Fuzzy transposition of hexachords at mm.39 & 56-57

Thus, the hexachords are similar in that the same classes of tetrachords are the subsets, but different, in that the pitch content has slightly altered.

To solidify this link even further that the hexachords even further is the aural quality. All but seven of the thirty-five forms of the hexachord contain all-interval interval vectors. Therefore, it takes a very discernible listener to detect the aural difference between these sets. One might be able to determine aurally that there are different pitch-classes within each iteration, but would probably not be able to determine that the pitch-class sets were different. Because of this, it can be argued that the hexachords that Ligeti utilises are different, but the same.

### 3.4 Motif

Another harmonic element with properties similar to chaos theory and fractal geometry is Ligeti's utilisation of a motif. This motif is not like a traditional motif where it is stated at the beginning and developed, but rather its presentation is in prime and its three other forms (retrograde, inversion, retrograde-inversion) combine with each other to form larger melodic ideas.



Ex. 3.4 – Prime form



Ex. 3.5 – Retrograde form



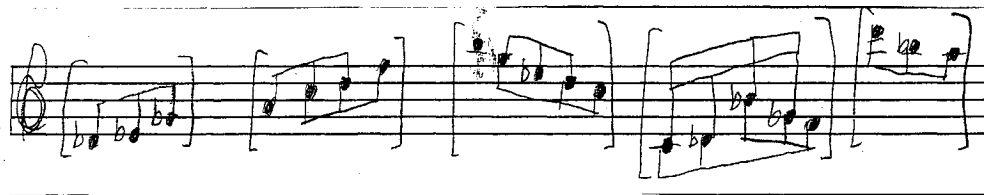
Ex. 3.6 – Inversion form



Ex. 3.7 – Retrograde-Inversion form

Like the hexachords, the motif demonstrates the quality of how simple ideas can form larger, more complex musical ideas, as well as how the different forms of the motif can reflect and shear to create melodic ideas that are different yet the same.

The first excerpt that best demonstrates the statement of the motif, like the hexachords, occurs in the opening twelve measures. At measure one, the motif is stated in its prime form, followed by measure three where the prime and retrograde-inversion combine to form 4-26 (0358) tetrachords. Afterwards in measure 7, different shears of the retrograde and inversion form a larger five-note motif.



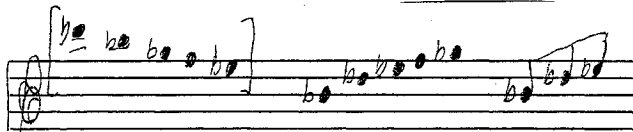
Ex. 3.8 – Piano reduction of mm. 1-12 that highlights the presence of the motif

Later, in measures 11 and 12, a shear of the retrograde form of the motif and an altered version of the five-note motif from measure 7 are present. From this excerpt, one sees how the motif is stated and then combined with itself to form larger melodic ideas.

The larger motif at mm. 7 occurs again at several junctions within the movement. Each time this passage occurs, it is always the same, but different. An example of this is present in the flute part. In each instance, the rhythmic values and pitch content are altered, but the pitch-class has been not.



mm. 7



mm. 21



mm. 32

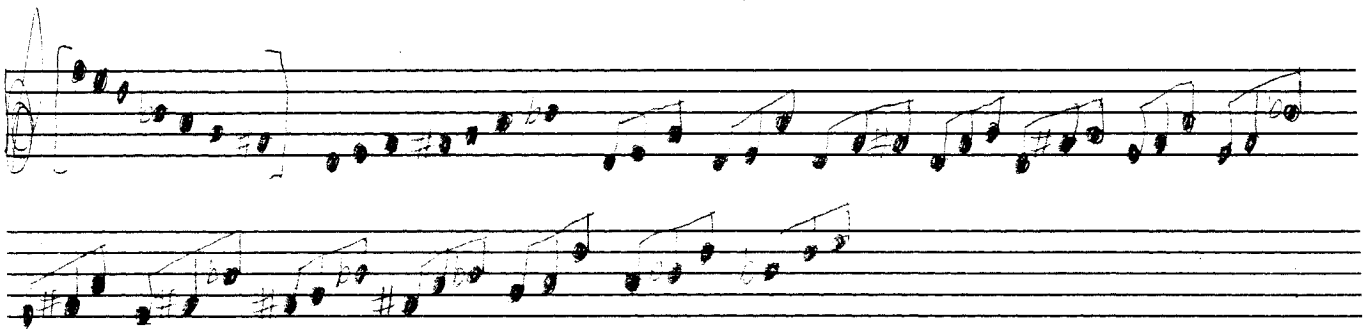


mm. 107

Ex. 3.9 – Piano reduction of 5-27 pentad in flute part at mm. 7, 21, 32, 107

From this reduction, one sees how the multiple statements of the pentad are different but the same, each statement contains multiple forms of the motif, but if one arranges the pitches in normal order, the same forms of the motif are present.

In addition to this, there are lines similar to the 5-27 pentads, but have been sheared both in pitch content and rhythmic value to form septads. Two examples of this take place at measure 66 through 69.



mm. 66, right hand of piano



mm. 69, flute part

Ex. 3.10 – Piano reduction of mm. 66-69.

The first example illustration, at measure 66-67 is a descending 7-11 septad. Like the 5-27 pentad, the septad contains numerous shears and reflection of the motif. Interestingly though the 5-27 pentad is a literal subset of the 7-11 septad.

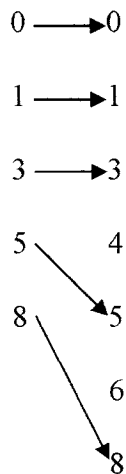


Fig. 3.3 – The prime form of a 5-27 pentad shearing into the prime form of a 7-11 (0134568) septad.

One may view the septad as a rhythmically sheared form of the 5-27 pentad. In other words, Ligeti stretches the pentad with the remaining spaces filled in so that the phrase maintains a similarity to the pentad statements. At measure 69, a second example of the septad exists. This time it is a 7-Z36 (0123568) and is stated in the flute, oboe, clarinet, and horn. Like the previous septad, the 5-27 pentad is also an abstract subset and also contains numerous shears, reflections, and translations of the motif. What differs is the contour and rhythmic values. Thus, the pentad that was present in these voices has been transformed into a septad; they are similar, yet different.

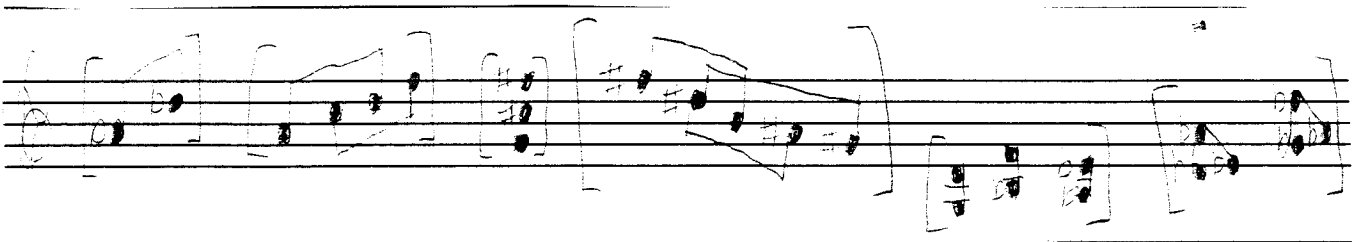
One can see how the motif is always present, however, in each instance it is different yet the same. Both the pentad and septad have similar contour and constructed

in a similar matter, but because of the alteration in pitch content and rhythm, they are different.

### 3.5 Interval

The final element that relates to chaos theory and fractal geometry is Ligeti's employment of a specific interval. Throughout this movement, both the hexachordal and the motivic threads share a common interval – the perfect fourth. This interval, its related inversions, and chromatic alterations form an intervallic bond that is present in the entire movement. Like the previous two elements, the interval permeates the opening twelve measures.

The first and third notes of the motif outline a perfect fourth, and the two tetrachords statements in the violins are two perfect fourths overlapping each other. Likewise, the first violin part lays a perfect fourth below the second violin part. This is followed by a trichord played by the piano that is a set of two perfect fourths with a common note between them.



Ex. 3.11 – Piano reduction of mm. 1-12 highlighting the perfect fourth and its variants

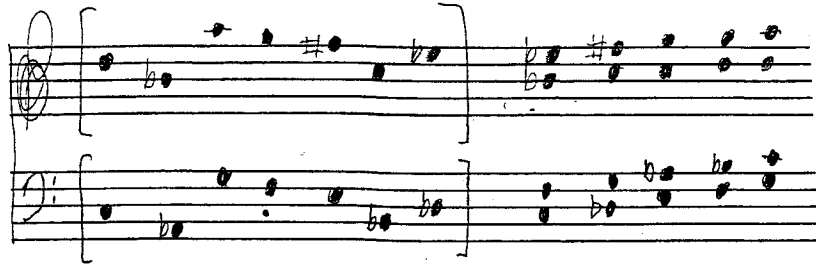
The descending 5-27 pentad in the flute and trumpet parts can be viewed as a series of superimposed fourths. Immediately following this, the two trichords in the flute and the clarinet, which are also a perfect fifth apart. Three measures later at measure 12, the flute, trumpet, and trombone play a perfect fourth figure; with the trumpet part being a perfect fourth above the flute and the trombone a perfect fifth below the trumpet.

Another passage, albeit a smaller one, that highlights a melodic figure based on the perfect fourth occurs at measure 63. In this measure the flute and clarinet each play a three-note figure that is a set of two perfect fourths that with octave displacement are a perfect fifth apart vertical as well.



Ex. 3.12-Piano reduction of mm. 63

Another example of the perfect fourth and its related intervals as the understructure of the melodic material takes place at measure 85-86 in the piano. Both hands play a seven-note melody that is a major second apart. Every note within the melody can pair off with another note so that they form a perfect fourth or one of its variant intervals.



Ex 3.13 – Piano reduction of piano part in mm. 85-86

To conclude, all of the melodic material of the movement is constructed from a simple idea - a perfect fourth. All of its forms are different yet the same. In addition, like the hexachord formation, there are disturbances in several passages, the melodic material is based on thirds, but in each case, they always return to the original pattern of basing material on a perfect fourth.

### 3.6 Form

What one may have noticed about all of these elements is that all exemplars came from the first half of the movement. This is because as the piece develops like a bifurcation tree or fractal image. It begins with a set of simple ideas, but Ligeti repeats each of them until the point where these ideas permeate the entire movement. Therefore, by the last fifty measures, these elements are still present, but they become obfuscated by the shearing and repetition of them.

There is a formal construct present in this movement, but not a traditional one. The motif at the beginning returns at several points throughout the movement. Aurally, these motifs occur as what could best be described as cadences, or phrases where a closed feeling to both the harmonics and rhythm of the work takes place. The best way to

describe the return of the different forms of the motif is like the return of the “head” in a jazz composition. It is not a main theme, but merely acts as a marker that the music is

Form of Movement IV of *Piano Concerto*

	<b>A</b>		<b>Form</b>			<b>B</b>		
mm. 1-12	13-21	22-30	31-47	48-66	67-82	83-97	98-120	121-150
Opening	Triadic based melodies	Mixture of triadic and fourth melodic material	Non-hexachord formation	Continuation of non-hexachord formation	Mixture of all three elements	Motif and fourth	Continuation of fourth and motif with hexachord formation	Heavy mixture of all three elements
Mixture of all three elements					Clear resemblance to opening measures			

Fig. 3.4 - Form diagram of Movement IV of the *Piano Concerto*

changing direction, or starting again. The overall development of the work bears resemblances towards a bifurcation tree. With each “cadence,” the primary structure of the music changes directions or bifurcates.

### **3.7 Conclusion**

To conclude, one may view this movement like a fractal image. From afar it appears that hexachords are the basis of the movement. If one progresses to a middle view, they may notice that there is a recurring motif, and that appears in numerous forms. If one goes even further, they notice that the fundamental structure is a perfect fourth. From afar, this movement appears to be complex, but up close, it is simple. Considering the preceding findings, many of the musical operations within this movement are similar to operations associated with chaos theory and fractal geometry.

### **3.8 Rhythmic Analysis**

Of paramount importance to an understanding of Ligeti’s rhythmic intentions or thoughts regarding his *Piano Concerto*, are the sketches held at the Paul Sacher Foundation. Before an analysis of the rhythms in this music can commence, one must examine the findings in this collection for a better understanding of the work.

It is traditionally believed that Ligeti employs bar lines solely to synchronize performers rather than to dictate rhythmic stress. In the case of the *Piano Concerto*, Ligeti improves upon the process of rhythmic synchronization by matching up each individual beat throughout the ensemble. In the *Partitur* for this piece, there is a 4/4 time signature notated at the top of the page, with each measure being subdivided with lines

through the bars. Each one of these subdivisions is equal to one quarter note. Also in the *Partitur* of this work, between the piano and percussion parts, Ligeti has notated three distinct and discrete rhythmic patterns which are the rhythmic glue that hold the movement together.

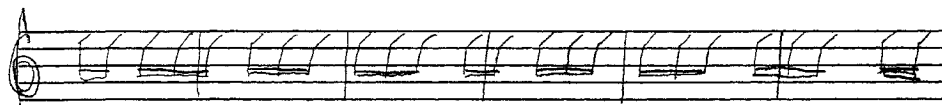
The first line I will label the 5-line. This is because the rhythm is segmented so that there are five pulsations per quarter note. The five-line consists of a duplet followed by three groupings of three followed by another duplet, another three groups of three, and another duplet in sixteenth notes. In a series, this pattern forms a period of four and a half measures long.



Ex. 3.14 – 5-line rhythmic pattern

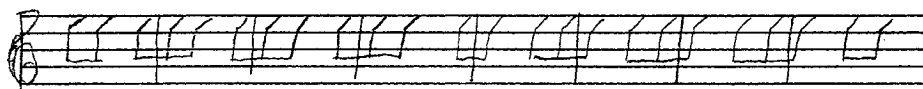
This rhythmic pattern is divided so that there are 5 sixteenth notes per quarter note. This line, unlike the other two, is only present on the first four pages of the *Partitur*.

The second line, which I will label the 4-line, is the same as the 5-line, except that instead of five pulsations per quarter note, there are four pulsations per quarter note. In series this pattern would form a period that is one and a half measures long.



Ex. 3.15 - 4-line rhythmic pattern

The third rhythmic pattern which I will label the 3-line, is the same pattern as the previously mentioned lines, except that the notes are now eighth notes and are arranged in groups of three pulsations to a quarter note. This pattern forms periods that are two measures long. It has the same pattern of notes, except that now they are eighth notes as opposed to sixteenth notes.



Ex. 3.16 – 3-line rhythmic pattern

It is not clear from the *Reinschrift* which specific instruments these three lines denote, or why the 5-line only lasts for four pages. But if these patterns were notated musically, the five line would be 20/16, the 4-line in 16/16 (or 4/4), while the 3-line would be in 12/8. Thus, there are three temporal frameworks, one that is regular, one that is faster, and one that is slower. Together, if only the first pulsation of the rhythmic grouping in each line was accented or stressed, a complex polyrhythmic network would emerge. Together, these three lines fit together so that several macroperiods emerge out of the rhythmic structure.<sup>1</sup>

In conjunction, all three rhythmic lines form a macroperiod that is 18 measures long, the three and four-lines in conjunction form a macroperiod of six measures, the four

<sup>1</sup> Gyorgy Ligeti Collection. Paul Sacher Foundation Basel.

and five-lines conjoin to form a macroperiod of four and a half measures, while the 3 and 5-lines form a macroperiod of eighteen measures. No matter what part of the beat one considers in this piece, any rhythmic moment in this movement is part of one of three periods as well as two other macroperiods.

Within the *Skizzen*, there is a document which outlines the interaction of the 3 and 4-line. From this source, one can see the different rhythmic grouping possibilities which Ligeti considered. The 4-line is grouped with regards to the rhythm by notating the rhythmic line, as discussed above, while the 3-line is noted not in the 3-line pattern, but a quarter note tied to an eighth note in triplet groupings. Each of these lines is grouped into units of 24. All together, this sketch represents one macroperiod of these groupings. From this; one can see that the rhythmic lines discussed above do not have to be followed strictly. Also, at the top of this sketch are timings for this movement, which will be discussed later on in this chapter.

201754, IV

J=138

74, 5 T - 1'  
88 T - 2'  
102, FF - 3'  
138 T - 4'  
172, 5 T - 5'

4x24

3x27

Plate 1 – Sketch diagramming the 3 & 4-line patterns,  
György Ligeti Collection, Paul Sacher Foundation Basel.

These three different rhythmic lines do appear at the commencement of the piece, but not the terminus. The 3-line appears at measure 155, while the other two rhythmic periods begin at measure 153. The iterations are cut off in mid-pattern; just as quickly as they appear to the listener, they disappear. This is yet another example of a link between Ligeti's music and Stockhausen's moment-form. The rhythmic periods and macroperiods slowly emerge from the background and then quickly recede back.

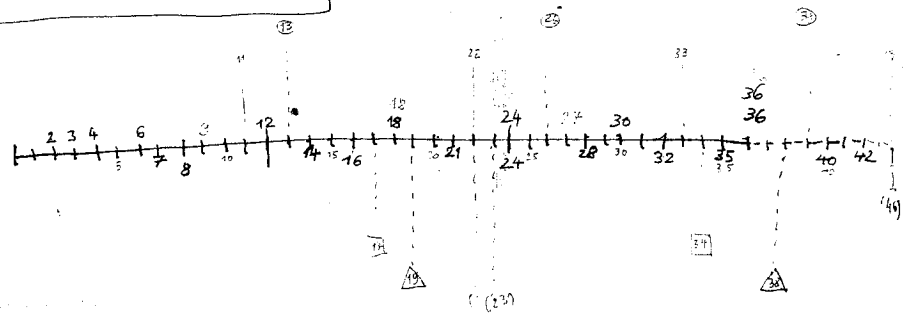
In addition to this, other sketches relating to the rhythmic structure of this movement were uncovered at the Sacher Foundation. One such item was on the back of a sheet of paper. On this item, there is a linear graph which denotes different numerical groupings. The sketch is written in purple ink with hash marks delineating full certain values. There are several notable aspects to this sketch.

First, a series of numbers are written in blue. These numbers are all multiples of seven, -- in other words, the series 7, 14, 21, 28, 35, and 42. In green ink, a series of multiples of 8 is notated, or 8, 16, 24, 32, and 40. In red ink, a series of multiples of 6 is notated or 6, 12, 18, 24, 30, and 36. Also on this paper are a series of numbers circled which form groupings of 13, and another set of groupings of 17 which are placed in triangles. On top of this, in brackets, is a series of 23. There are also other markings which indicate smaller groupings of 2, 3, 4, 5, 9, and 10 measures.<sup>2</sup>

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<sup>2</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

Zongor. III



$l = 150$   
 $l = 200$

50 T	1
100 T	2
150 T	3
200 T	4
250 T	5
300 T	6

~~70 T~~  
~~152 T~~  
~~30 T~~  
~~76 T~~  
~~10 T~~

Roketa - whism zha

$\frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1}$

Af-zi anti-fsua  
Bollno!

↳ Alfabikus EVV worden  
 ↳ kumpofisras  
 ↳ DALLAY (Kard-Rejst)  
 POLIPIK-KOMPLEX  
 FLARUNIA → CLUSTER

Plate 2 – Linear graph of rhythmic groupings.  
György Ligeti Collection, Paul Sacher Foundation Basel

It is ambiguous what these numbers represent, but one can speculate that in addition to the three independent rhythmic lines, this sketch is evidence that Ligeti was also considering larger macroperiods for the piece that are not formed out of completed iterations or forms of the three basic periods. In other words, on top of this basic temporal framework there is another abstract framework. Similarly, some of the numbers highlighted on this graph do coincide with specific starting points of the periods and macroperiods, though, it should be noted that some of these numbers do not fit into either.

It is not clear whether these numbers represent the size of the rhythmic grouping or the measure number at which the macroperiods restart or converge. Also, it is inscrutable whether these numbers denote bar numbers, beat numbers, or a specific iteration of a particular pattern. If Ligeti is using this number series as a template for this movement, then what he has in mind is a vastly complex rhythmic pattern in addition to the 3, 4, and 5-lines he has envisioned.

In addition to this, another period is present in the piece. On various items in the *Skizzen*, Ligeti has clearly noted the time space dimension of the piece within the terms of physical time. He has notated a 34.5 tactus(s) or 34.5 measures equals 1 minute, 69 measures equal two minutes and so forth. Therefore, all rhythmic and harmonic aspects, in addition to being part of one of the various periods and macroperiods, are also part of a temporal period. Nevertheless, at mm. 69, the only instance of congruency or convergence between the rhythmic periods and temporal periods occurs. This measure is the two minute mark as well as the beginning of another iteration of a 3-line period. In relation to the actual score, it is very unclear whether there is strict or partial interweaving; contrametric or commetric. This is because there appears to be no direct

connection between the macroperiods, the score and the rhythmic figures contained within it.<sup>3</sup>

At the beginning of the piece, it is not clear if there is a rhythmic hierarchy or structure in this piece. However, as the piece progresses, it becomes evident that the rhythmic patterns notated in the sketches are present in this piece. For the first 100 measures, there is never a full iteration of a rhythmic pattern that is similar to the rhythm lines. However, throughout these measures there are iterations of fragments or portions of the pattern. One such example is measure 23, which is the beginning of a 3-line period. At this instance the piano iterates a duplet, followed by a triple, followed by an eighth-note tied to two quarters. In addition, this rhythm is written out as four triplets to the bar. This period then ceases, with the statement of a duplet on the last beat of the measure. These two measures are obviously an iteration of the 3-line, but Ligeti does not compose a full statement. In addition, this pattern is playing against the expected pulsation pattern of the triplet, which is that the first beat of each triplet is stressed. This would be an example of *commetricity*.

At measure 105, Ligeti begins to iterate more developed statements of his rhythmic patterns. On the second half of the upbeat, the piano performs a rhythmic pattern in sixteenth notes and then, at the end of the measure, the pattern returns to eight notes. In the next measure, the piano states the termination pattern of a three line. If one considers these two bars as one unit, this would be a complete iteration of one period of the 3-line.

Measure 116, is where complete manifestations of Ligeti's pre-conceived rhythmic patterns begin to materialize. Coincidentally, this section of the movement is

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<sup>3</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

where the harmonic structure begins to become more complex. At this point, the piano is playing notes that are an exposition of the 3-line rhythmic pattern. This period, while also incomplete, is closer than the previous example to being a full statement of this rhythmic pattern.

Another example of this grouping occurs at measures 129-130. In the first measure, there is nothing but a whole rest, while in the following measure, the strings play a portion of the second half of the 3-line. In the next measure, the strings begin playing this rhythm again. From this point onward, the 3-line pattern is always iterated in this form. Interspersed throughout the remainder of the piece the orchestra performs different permutations of the 3-line. With this information, one may view the complete appearance of this rhythmic pattern as a strange attractor. If one reviews the movement, one can see numerous instances of incomplete permutations of the 3-line. While it is not present at the beginning of the work, it slowly emerges over the course of the piece to become a strange attractor.

This pattern is eventually interrupted at measure 149, where the strings begin to perform a portion of the 4-line. In the following measure, the strings perform both the 4 and the 3-line. After this, even though it is in mid-pattern, the piece ceases.

There are several other points that should be mentioned about the rhythms in this movement. One of these points is that every time motif  $\Delta$  is iterated in one of its forms, each note always has an accent of some kind. In addition, whenever motif  $\Delta$  is performed, it does not coincide with a specific macroperiod, it appears to begin in mid-pattern. Also, as the piece progresses, the frequency of the accents increases. By the last twenty measures, almost every note is accented. At this point, it is virtually impossible

for the listener to openly perceive the individual rhythmic structures present in the piece. In fact, as the piece reaches its climax, one can deduce that the hypergroups present in this piece have become too complex.

To summarize, as the piece progresses, the pre-determined hypergroups emerge. By the end of the movement, it is quite clear that there are several rhythmic groups taking place at once as the rhythmic figures of the music fit very clearly into the pre-determined patterns. Thus, like a fractal image, intermittency graph, or strange attractor, the hypergroups slowly emerge and form a highly structured rhythmic piece out of something that appears to have no order. Also, from these findings, one can see that the rhythmic *talea* that are present in this piece are quite simple, but when combined with each other, they form a complex polyrhythmic network. In addition, because the melodic fragments do not clearly adhere to these patterns, these rhythmic ideas coalesce to create an even more complex rhythmic pattern. In this way, a perceivably chaotic and sporadic musical effect has in fact a highly controlled and ordered structure as its basis. From these findings, one can deduce that the 3-line is the main rhythmic pattern for this piece, and that the 4- and 5-lines form an additional rhythmic layer..

Finally, if one considers Simha Arom's terms, it is difficult to argue whether this piece is either *commetric* or *contrametric*. That is because it is unclear what the pulsation pattern is. Since the movement is in 4/4, the pulsation pattern could be a quarter note, or it could be the traditional strong or weak beats of 4/4 time. In the first two thirds of the piece one could argue that the rhythmic structure is *commetric*, but by the end, one could argue that the structure is *contrametric*. Also, it is difficult to discern whether there is

strict or partial interweaving amongst the parts. What makes both of these tasks difficult is that there is no clear pulsation pattern present.

### **3.9 Further findings at the Sacher Foundation**

After examining the holdings for this work in the György Ligeti collection, I was able to locate only one sketch that directly links this work with chaos theory and fractal geometry. This sketch, which is 20 cm by 1.7 cm, contains the only explicit link between this work and the mathematical school of thought. In brown ink, Ligeti has written down Banda Linda + Fractals. Banda Linda is a direct reference to the Banda Linda tribe, which is one of the tribes that Simha Arom studied and analyzed in his work. The term Fractals is an obvious reference to fractal geometry. What one can deduce from this annotation is that Ligeti was contemplating the possibility of combining his ideas about rhythmic patterns with ideas of fractal geometry or that he saw the link and felt that if used in combination with his other compositional practices his ideas would be even more successful.

} ca 3' allacca  
6) Prestissimo Frühl zuffelt "jazz"  
+ BANDA LINDA + FRACTALS finale

Plate 3 – Banda Linda sketch,  
György Ligeti Collection, Paul Sacher Foundation Basel

There are several other sketches housed at the Paul Sacher foundation which inform a discussion of the rhythmic techniques of György Ligeti. On page 20 of the *Skizzen* for the *Piano Concerto* is a yellow, green, red, and black pencil mark. This pencil mark is distinctly similar in shape to that of a bifurcation pattern on end. It is not clear exactly what the link between this diagram and the work is, but it is interesting to note that it is at this point in the movement, the density of the orchestration intensifies and the rhythms become much more complicated.

There are also many annotations throughout the remainder of the entire *Piano Concerto* sketches which relate to rhythm. These notes include the words: jazz, Conlon Nancarrow, Oscar Peterson, Salsa, and samba. While most of these terms are in reference to the fifth movement, this evidence can certainly serve to emphasize that the rhythmic structure of this piece was one of Ligeti's primary concerns.

Finally, there was only one finding made in relation to the harmonic structure of this piece among the sketches at the Paul Sacher Foundation. On page 38, which is not part of the official score, Ligeti has notated in vertical arrangement, the spelling of two hexachords. In the left column he placed c, d, e, f#, g, and b-flat, while the right column contains d-flat, e-flat, f, g-sharp, a, and b. Both of these are 6-34 hexachords. While this hexachord is located in the movement, it is not one of the hexachords repeated in several guises. It is interesting to note, however, that these hexachords are one pitch away from the those identified by Engebretson.

### 3.5 Conclusion

What a post-tonal analysis of this movement reveals is that the nucleus of this movement is indeed hexachords. However, there is also a motif within it that is developed and iterated in multiple guises, which are different, yet distinctly similar to each other. Also, this motif is used to create large musical ideas. Consequently the hexachord development of this movement is reached through the exploitation of smaller sets such as trichords or tetrachords. These sets slowly increase, until a point where they can no longer form hexachords and instead must construct larger sets such as septachords or octachords. While hexachords are still present, they become subsets of the larger sonorities and because of the intervallic content of these larger sets, a hexachord can, and will most likely, be one of the subsets, whether literal or abstract.

Also, this analysis shows the different methods in which a hexachord can form the basis of a work. It can be manipulated horizontally, vertically, by repetition of trichords, tetrachords, or even through multiple iterations of dyads or whole tones. Through these numerous guises and permutations, the hexachord saturates this movement, and in turn gives it its homogeneous quality.

The harmonic findings of this movement clearly place it within the context of chaos theory and fractal geometry. The overall aural quality of the work is different, yet the same because the entire harmonic structure is formed out of the same idea; the hexachord. Also, this complex movement is constructed from rather simple elements such as a single motif or single intervals. Another link between the harmonic syntax of this movement and chaos theory and fractal geometry is its linear development. The work starts with a rather sparse and simple orchestration. As the movement rapidly progresses,

the density and the texture gradually increase. Also the melodic and harmonic structures slowly morph into larger figures. That which starts as a simple three-note idea, is sheared and translated into large lines and harmonic sets. Attention should also be drawn to the fact that Ligeti's rich chromatic syntax is not really that complex. One can see that there are a select set of trichords and tetrachords utilised in forming the hexachords.

One further link between this movement and non-linear dynamics are the disturbances or turbulences in the movement. After developing a certain idea, Ligeti slightly alters information based upon his audience's perceived expectations. There are several instances where one expects the continuation of a specific interval or motif, but Ligeti alters the music at this point and instead what was expected appears either altered or omitted.

## Chapter 4 – *Atmosphères* and *A bout de souffle*

Now that a work usually associated in the extant literature with chaos theory and fractal geometry has been examined, this thesis will now turn to works that are not usually linked to these ideas. The two works in question form the bookends of Ligeti's career. One, from his early period, *Atmosphères*, is normally not associated with chaos theory and fractal geometry, while the other, *Piano Etude No. 17*, is a more recent work and it could be expected that it is informed by these ideas.

### 4.1 Methodology

Unlike the last chapter, the analyses in this one will not be so dependant on post-tonal theory. These analyses will be more descriptive and literal than theoretical. However, like the previous chapters, the findings made will be related to the precepts and terms of chaos theory and fractal geometry.

### 4.2 *Atmosphères*

*Atmosphères* is arguably Ligeti's most famous work in his *oeuvre* and undoubtedly the most infamous. It is from his early period and is similar to other works from this period such as *Continuum*, *Melodien*, *Volumina*, and *Lux Aeterna* in that all of these works are sound-block compositions. That is to say, these works appear to be void of an audible rhythm or harmony. Ligeti describes these compositions as "a music without meter, but also, melody, rhythm, harmony were missing: there was in exchange full chromatic blocks."<sup>1</sup> What a listener hears are dense masses of sound which appear to

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<sup>1</sup> Translation mine. „Es war nicht nur eine Musik ohne Metrik, auch Melodien, Rhythmen, Harmonien fehlten: es gab dafür chromatisch ausgefüllte Blöcke. From György Ligeti, „Rhapsodische, unausgewogene

be void of the usual elements associated with music, such as harmony and rhythm.

Although *Atmosphères* has been the subject of numerous analyzes and discussions, a discussion of the possible connection between this work and chaos theory and fractal geometry has not yet been conducted<sup>2</sup> in English.<sup>3</sup>

### 4.3 Analysis

*Atmosphères* consists of a fifty-six member string orchestra, accompanied by a full brass and winds section, and a piano.<sup>4</sup> The conventional method of viewing this work is as a sequence of 21 blocks or sections, with each section being centered on a specific set of pitches or encompassing the entire range. If one examines the order of these blocks with respect to range, one can see an underlying order, which is in the shape of a sine wave. *Atmosphères* progresses from blocks that accompany wide acoustic range and then the work narrows to a specific range in the middle. *Atmosphères* then progresses to alternate between blocks of sound centered on the high or low registers of the acoustical range. The overall shape is similar to a sine wave.

It should be noted that *Atmosphères* had its origins in a work entitled *Piece électronique No. 3*. This piece was conceived while Ligeti was working at the Electroacoustic sound studio in Cologne, Germany. This work was to consist of forty-

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Gedanken über Musik, besonders über meine eigene Komposition. *Neue Zeitschrift für Musik* 154 (1993): 26.

<sup>2</sup> For other analyzes of *Atmosphères* see Markus Suplicki, "György Ligeti: *Atmosphères* – eine unkausale Form?" *Musiktheorie* 10 (1995): 235 – 247; Harald Kaufmann, "Strukturen im strukturlosen. Über György Ligeti's 'Atmosphères' *Spurlinien: Analytische Aufsätze über Sprache und Musik* (Wein: Verlag Elisabeth Lafite, 1969) 107 - 117.; Erkki Salmenhaara, *Das Musikalische Material und seine Behandlung in den Werken "Apparitions," "Atmosphères," "Aventures" und "Requiem" von György Ligeti* (Regensburg, Germany: Gustave Bosse Verlag, 1969).

<sup>3</sup> As is shown below, an early analysis by Van der Slice exposes features that are similar to chaotic effects, without referring to chaos theory.

<sup>4</sup> The instrumentation of *Atmosphères* is as follows: 4 flutes, 4 oboes, 4 clarinets, 4 bassoons, 6 horns, 4 trumpets, 4 trombones, tuba, 28 violins, 10 violas, 10 cellos, 8 contrabasses, and a piano.

eight sound waves divided into four separate groups. Each sound wave was to be an electronically produced sine wave, with each voice being a different frequency. In other words, one sound wave would be at 1000 Hz, the next one 1001 Hz, and so forth. Out of this, “composite sounds would emerge and recede like shadows within a fluctuating yet generally sustained texture.”<sup>5</sup> Ligeti ceased working on this piece once he realized that the aural effect he desired would be better achieved through acoustic instruments and not electronic reproduction. His desired intention of creating a complex sound mass out of simple events however, is certainly evident in this piece.<sup>6</sup>

Returning to a discussion of *Atmosphères*, what the individual instruments of the orchestra play is surprisingly simple. In the opening section, the effect which the string players produce is complex, when in effect what each individual actually plays is quite basic. At the commencement of the work, the strings and upper woodwinds play a block of notes encompassing five and a half octaves. Each voice of the string orchestra has its own part, with each performer playing a single, discrete note. At the same time, each of the brass sections is responsible for a specific interval. The intervals formed range from major seconds to perfect fifths. Each of these intervals by itself is simple, but when grouped together with the other events simultaneously taking place, a dense, chromatically saturated, sound-mass is formed.

This idea lasts for the first 2 minutes and 12 seconds of the work. In the subsequent section, some harmonic and rhythmic movement takes place, while the players in the orchestra produce alternating minor third patterns. Each performer plays their line with a different rhythmic grouping ranging from a twenty-tuplet to a triplet.

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<sup>5</sup> Richard Steinitz, *György Ligeti: Music of the Imagination* (Boston: Northeastern University Press, 2003) 90.

<sup>6</sup> It should be noted that this piece did receive a realization in 1993.

These minor thirds can be classified as a simple event, but when performed simultaneously with many other manifestations of itself at many alternating rhythmic rates, a dense chromatically saturated sound event is formed. Another example of complex elements being constructed from a simple event. This is also an example of self-similarity. The effect produced here arises from the use of different permutations of the same element.

Following this moment, at rehearsal letter E, is where Ligeti describes the music as being a whirlpool or possessing turbulences, much as Toop is quoted as describing in section 2.2. Like an object thrown into water, the music appears to be ascending and descending from a specific pitch point. The graphic representation of this music on paper is the only instance of a visual manifestation of an idea related to chaos theory. In the score, this passage literally looks like a bifurcation tree. In combination with the rhythm, it appears that the music is diverging upwards and downwards from a specific point.

The next significant moment in *Atmosphères* occurs at rehearsal letter H: the micro-polyphony section. This portion appears to have no link or developmental connection with the preceding music, therefore, it may be viewed as an intermittent development within the piece. The affect achieved here is not expected but, like the previous 4 minutes and 5 seconds of the piece, it is a dense mass of sound. Therefore, this section could be viewed as an intermittent portion in the overall development or trajectory of the piece. The non-causal relationship between these two sections links it with the non-causal aspects of non-linear dynamics.

At this point, the overall musical gesture encompasses all of the voices of the string section. As this gesture progresses, the focus or pitches of this section narrow. One

way of describing this compositional effect, is that it is similar to a bifurcation tree in reverse. As opposed to other bifurcation trees which depart from a single point and expand, this phenomenon begins with a broad amount of points and converges on a single one.

Furthermore, the final goal of these notes which is to say, the eventual focus of the music on a small set of notes, also behaves as a strange attractor. It is not expected that this dense, complex work will ultimately focus on a narrow range of pitches, but it does. In conjunction with the previous section, this entire segment of the work can be seen as a swell. The music grows from a specific point and then converges, therefore, it may be argued that the overall shape of the music is like a wave.

As the piece progresses, disturbances or turbulences occur; one expects a continuous mass of sound to persist, but instead the music changes its course or form rather unexpectedly. After rehearsal H, the texture becomes thinner. At rehearsal letter N, each voice begins to play single sustained notes. The orchestra has returned to performing extended pitches with each player on his or her own line. This is distinctly similar to the opening section. This continues for an entire minute, after which the instruments slowly fade out.

Following this, at rehearsal letter Q, the strings begin playing a passage reminiscent of rehearsal letter C. However, the section is not as long and extended as the previous section. The next relevant section is at rehearsal letter T. At this point, the orchestra plays a repeating rhythmic figure, which is reminiscent of the previous section. However, in this iteration it is not a repetition of a single interval, but rather a series of notes which are a scalar like passage. Moreover like the previous sections, it creates a

complex sound mass. After this, the instruments stop playing and leave the piano sounding by brushes on the strings. Thus, as quickly as the sound mass appears aurally, it recedes into the background.

Overall, the sections notated above are all different yet the same. they are all dense complex sound masses formed through different manners. Some are created through multiple iterations of a single note, while others are formed by the en masse repetition of a particular rhythmic pattern. Nevertheless, no matter which method it is that Ligeti uses to create these affects, they all result in a chromatically saturated texture. In addition, the constant disruptions to the flow of the piece and the return of a musical idea or permutation to repair the disruption, one can view this piece as a continuous intermittency. The flow of the piece develops and changes, but when it appears that the piece “has righted itself” and found a specific trajectory, the piece is altered and continues along another path. With this in mind, the piece can be viewed as a large scale bifurcation tree.

#### **4.4 Secondary Literature**

If one examines the extant secondary literature available concerning this piece, one can identify other links between this work and chaos theory and fractal geometry. The first source is an interview that was reproduced in the liner notes of a recording of *Atmosphères*. In this interview, Ligeti describes what he was thinking about during the period leading up to the composition of this piece:

I was 27 years old and living in Budapest, completely isolated from all the ideas, trends and techniques of composition which developed in Western Europe after the war... In 1951 I started to experiment with simple structures of rhythm and sound in order, in a manner of speaking, to build

up a new music from nothing. My method was Cartesian to the extent that I considered all the music that I already knew and loved as not binding on me – even as invalid. I asked myself: what can I do with a single note? What can I do with its octave? What with one interval? What with two intervals?<sup>7</sup>

From this statement, one is able to deduce that the creation of complex events or systems via simple means has always been a point of fascination for Ligeti.<sup>8</sup> Also, the comparison between his method of composition and the Cartesian method, provides another link between the sciences and Ligeti's music. The time that he began thinking about these techniques was six years before he was even exposed to the music of Stockhausen and ten years before *Atmosphères* was premiered.

Another link between chaos theory and *Atmosphères* can be found in an analysis of the work by John Dewitt Van der Slice. This analysis, which is a portion of his compositional D. M. A. from the University of Illinois, was produced around the same time as Ligeti's interest in chaos theory commenced. It is uncertain whether Van der Slice had any intimate knowledge of chaos theory when researching or writing his analysis, but several of his descriptions of *Atmosphères* are noticeably similar to the precepts and terms of chaos theory and fractal geometry.

Van der Slice, like other analysts, breaks the work into 21 distinct sections, and analyzes each of them separately. For each section or block, he provides a chart graphing the progress of the piece in relation to pitch space. The analysis of the sections at

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<sup>7</sup> György Ligeti, quoted in jacket notes by Ove Nordwall for the phonograph record: Duo Pohjola (Djursholm, Sweden; Grammofonfirma, LP 18), quoted in John Dewitt Van der Slice, "An Analysis of György Ligeti's *Atmosphères*". M. A. thesis, (University of Illinois, 1983) 7-8.

<sup>8</sup> It should also be noted that the best example of a work by Ligeti exclusively focusing on a specific interval is the *Ten Pieces for Woodwind Quintet*. In this work, each movement is based on one specific interval, and this interval is highlighted and developed throughout that movement.

rehearsal letter's E through G, or measures 31 through 43, is where Van der Slice's analysis becomes intriguing. Van der Slice's description of section E is as follows:

... each layer moving independently in swell-and-ebbs of activity involving mostly unidirectional semitone displacements. This latter leaves the impression of contrary motion away from a central area.<sup>9</sup>

Van der Slice describes how the trajectories of the music continue on two different paths; they depart from a single point and continue in opposite trajectories just as a bifurcation pattern does. This motion continues through to rehearsal letter F and at this point the contrary motion that Van der Slice describes ceases. Van der Slice's description of this particular moment is what is unique about his analysis:

Compared with the music's previous continuity, this leap is utterly novel, as if a kind of 'critical mass' had been reached, causing a violent and quite uncharacteristic transformation. One may also consider the following structural implications. Section F manifested cluster layers which moved downwards toward an implied goal which was finally actualizes in the present section:

How Van der Slice describes the trajectory of the music in *Atmosphères* is similar. The descending trajectory that commences in section E reaches its limit in section F and is forced into another trajectory that is completed in section G. Another way to look at this is as turbulence to or interruption of the trajectory.

Ligeti is not composing with a formula, but the treatment of the melodic line is almost formulaic. The lines do have an intended trajectory. Like bifurcation trees, these melodic lines reach a limit that is not foreseen ahead of time which then forces them into a different trajectory or system.

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<sup>9</sup> Van Der Slice 19.

Another link between this work and chaos theory comes from Ligeti's own description of the work located in an article from 1993 in *Neue Zeitschrift für Musik* entitled "Rhapsodische, unausgewogene Gedanken ueber Musik, besonders ueber meine eigenen Kompositionen." In this article, Ligeti gives a brief overview of his composition techniques and the numerous influences and stylistic changes present within his *oeuvre*. The description Ligeti attributes to the inner workings of *Atmosphères* intimates that it contains a strange attractor. He states:

A fourth layer is present: Having composed my orchestral work *Atmosphères* in 1961, its "contents" exist from constantly changing, flow studies and turbulence, I did not have the faintest clue, that at the same time Edward Lorenz at MIT managed to computer simulate weather, and that this was guided by the discovery of strange attractors, and that the turbulence research and the theory of dynamic systems in the following years would become the natural sciences revolution. I always work empirically, not mathematically, not scientifically, rather handicraft-like, or else but in an unconscious approach to geometrical ways of thinking. My conscious become aware of parallels 'lying in the air' between the mathematical research from the 60's and mine simultaneous compositions from 1984, it also the first time I saw the computer manufactured Julia and Mandelbrot sets produced by Heinz-Otto Peitgen and Peter H. Richter.<sup>10</sup>

Ligeti views the work as a gradual process that unfolds, much in the same way that a bifurcation tree or a intermittency pattern develops in a non-linear graph, leading towards an unexpected terminus, or strange attractor.

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<sup>10</sup> Eine vierte Schicht ist auch vorhanden: Als ich 1961 mein Orchesterstück *Atmosphères* komponiert habe, dessen "Inhalt" aus Zustandsänderungen, Strömungsmustern und Turbulenzen besteht, hatte ich nicht die geringste Ahnung davon, daß genau zur selben zeit Edward Lorenz am MIT die meteorologische Computersimulation bewerkstelligte, die zur Entdeckung der "strange attractors" geführt hat, und dass die Turbulenzforschung un die Lehre der Dynamischen Ssysteme in den nächsten Jahren die Naturwissenschaften revolutionieren werden. Ich arbeite stets empirisch, nicht mathematisch, nicht wissenschaftlich, eher "handwerksmäßig", aber in unbewusser Annäherung an geometrische Denkweisen. Bewusst wurde mir die "in der Luft liegende" Parallelität zwischen den mathematischen Forschungen seit den 60er Jahren und meinen gleichzeitigen kompositorischen Bestrebungen erst 1984, als ich die ersten Computerdarstellungen der Julia-Mengen und der Mandelbort-Menge sah, die Heinz-Otto Peitgen und Peter H. Richter angerfertigt haben. From György Ligeti, Rhapsodische, unausgewogene Gedanken ueber Musik, besinders ueber meine eigenen Komposition." *Neue Zeitschrift für Musik* 154 (1993): 29.

Ligeti acknowledges in this quote that the ideas and processes that he and Edward Lorenz were exploring were the same, the only difference being that they were approaching the subject matter from different angles. Ligeti was exploring an ever-changing acoustic phenomenon, while Lorenz was seeking a method to help explain weather patterns. Both Ligeti and Lorenz were looking for an explanation of how to explain or employ the turbulences and changing materials they were working with and how to integrate them into their respective fields.

#### **4.5 Sacher Foundation**

Also, findings made at the Sacher Foundation link this work to chaos theory and fractal geometry. On the back of one sketch, Ligeti drew a large scale bifurcation note pattern that encompasses the entire harmonic spectrum.<sup>11</sup>

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<sup>11</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

The image shows two systems of handwritten musical notation, labeled A and B. System A consists of two staves with notes and rests, numbered 1 through 27. System B also consists of two staves with notes and rests, numbered 1 through 15. Both systems include various musical markings such as accidentals, dynamics, and performance instructions. System A has a circled 'A' at the beginning, and System B has a circled 'B'. There are also some handwritten notes and symbols scattered throughout the score, including 'V.L.', 'V.C.', and 'L.M.I.O.'. At the bottom of System B, there is a small diagram with the numbers 1, 2, 3, 4.

Plate 4 – Bifurcation diagram from *Atmosphères*,  
György Ligeti Collection, Paul Sacher Foundation Basel

In addition, a sketch of the opening passage found at the Sacher Foundation, shows an altered version of the opening page. In this version, each stave contains notes that form major seconds. From this, one can deduce that Ligeti was well aware of the simple materials he was using to create the dense blocks of sound that he so desired.<sup>12</sup>

#### 4.6 *Atmosphères* Recontextualized

This analysis and explanation of *Atmosphères* does not provide a definitive answer as to what Ligeti's intentions in composing the piece were. What this analysis does offer however, is a new way of thinking about *Atmosphères*. If one is to analyze, discuss, or examine this piece by using chaos theory and fractal geometry, it is possible to achieve a greater understanding of it..

These tools aid immensely in a discussion of the formal elements of the piece. They help to show that some segments of Ligeti's music do not necessarily possess a tangible link; that some portions emerge independently of their surroundings, or may appear out of thin air. By using the tools provided by chaos theory, one notices that there are small scale, as well as large scale formal structures located within Ligeti's music. By using the tools offered by this mathematical school, *Atmosphères* becomes less analytically daunting; one realizes that there is an underlying order to the composition and that even though the end-product is quite complex, it is constructed from exceedingly simple events.

If one were to analyze *Atmosphères* in Van der Slice's terms, one would realize the trajectories of the music are not disjointed or discontinuous, but that there are instead large-scale patterns that exceed limitations or boundaries and therefore become

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<sup>12</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

interrupted or forced into a different system. If one considers the ways in which Ligeti found it easiest to describe the piece -- as if there is a strange attractor present in the music -- then it becomes obvious that there are numerous elements associated with the precepts and terms of chaos theory and fractal geometry present within *Atmosphères*. It becomes clear then that the best way to discuss and analyze *Atmosphères* does not lie with using musical terms such as melodies, chords, keys, or harmonics. Nor does it rest in saying that the piece is without form and structure. Instead, an analysis of *Atmosphères* is best facilitated by the use of terms such as trajectories, non-perceptible order, and smaller or simple elements, in other words, the vocabulary provided by chaos theory and fractal geometry,

#### **4.7 Piano Etude No. 17, “A bout de soufflé”**

*Piano Etude, No. 17, “A bout de soufflé,”* (Out of Breath) from Book III of the *Piano Etudes*, is one of the most recent works by Ligeti. There is very little known about this work: it was commissioned by the BBC and premiered on 23 October 1998 by Pierre-Laurent Aimard at the Royal Festival Hall in London, England. It is also known that the work is dedicated to Heinz-Otto Peitgen.<sup>13</sup>

To date, the only one source to address this piece is a paragraph in Richard Steinitz's book that provides very few details.<sup>14</sup> The only stylistic features that are known about this work, and for that matter the four etudes that make up Book III, are that the tonality and rhythmic structures are much simpler than compared to the other fourteen etudes in Books I and II.

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<sup>13</sup> News, *Tempo* 207 Dec 1998: 53.

<sup>14</sup> Steinitz 313.

## 4.8 Analysis

The basis for the analysis of this piece draws on first principles because of the lack of information and discernible logic to the work. As with all of Ligeti's other compositions, there is no formal scheme or tonal center. In fact, by this point in Ligeti's compositional career, it is safe to say that he assumed a musical aesthetic concerned with the balance between order and disorder and that this aesthetic is very much present in *A bout de soufflé*.

This etude is a rapid succession of eighth notes. Because there are no bar lines or rehearsal numbers, each eighth note will be referred to as a beat for clarity during this analysis.<sup>15</sup> The tempo marking for this piece is *Presto con bravatura* to the eighth note, therefore the aural effects created in this piece would be in reference to large gestures; smaller ones would pass by too quickly to register with the listener. There is no key signature for this work, but it does have a modal color with a marked emphasis on E-flat and A natural/A-flat.<sup>16</sup> The overall form of the work is ternary, with a brief transition interlude before and after B.

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<sup>15</sup> Eighth note 1 is beat 1, eighth note 2 beat 2, and so forth.

<sup>16</sup> Steinitz 313.

Formal Chart for Piano Etude 17, 'A bout de souffle'

A	B			A'	Coda
	B	B	b'		
Beat 1 – Beat 640	Beat 641 – Beat 681	Beat 682 – Beat 764	Beat 765-Beat 802	Beat 803 –Beat 939	Beat 940 – Beat 1013

Fig. 4.2 Formal Chart for *Piano Etude 17*, "A bout de souffle"

The work begins with the melody being played in both hands in a canon at the octave, with the left hand displaced by an eighth rest. This eighth rest displacement can be seen as two things: first, it can be seen as an interruption to the flow of the piece, and second, it serves as a prime example of the idea of sensitivity to initial conditions. In addition, once again a piece constructed from a simple gesture has formed a complex sound event.

The first eight notes of the piece form the melodic kernel of this work and all subsequent melodic material in section A and A' is a derivative of this opening phrase both in shape and pitch content. Thus, all permutations of the motif have been sheared in some manner, whether in length or in pitch content. At beat nine, in the right hand and beat ten in the left hand, each hand plays an accented note that is accompanied by an interval. These accents do not have a pattern, but do delineate the beginning of a statement of the melodic motif.

As a result of the lack of time signature, or regular occurrence of accents, it is not clear whether the rhythm of this piece is contrametric or commetric. If one considers each beat as a pulsation, as this thesis will do, then the underlying rhythm is commetric. Also, the frequency of the accents in this piece changes. In some instances they appear at regular intervals, where in other instances they appear in sequences, or in smaller patterns. Furthermore, the rhythm alternates between symmetrical and asymmetrical groupings, therefore never combining to form periods. Consequently there is no periodicity nor are there macroperiods in this work.

Likewise, the aforementioned intervals do not adhere to a set pattern. Ligeti uses all possible intervals from the minor second to the major seventh, and while there is not a single interval that is highlighted or privileged, there are two that occur frequently – the diminished fifth and diminished seventh. These intervals do not however, appear to be a basic building block of this work like the perfect fourth is in the *Piano Concerto*. Furthermore, if one were to consider Allen Forte's terms of durational and basic attack units, this work would appear to have neither.

With the way the melody and accents are presented in the two hands of this work, it could be argued that there is four-voice polyphony present. The two notes that form the intervals of the left hand act as bases and the rest of the notes appear as *Zugs* or extensions of these bases. The melodic notes fluctuate each beat between the different accented pitches, and can either be connected linearly to the notes within their own system or with the notes of the system above or below. Also, one may look at the piece as constructed of short melodic fragments; each new interval set denotes the end of one fragment and the beginning of another. Therefore, this piece can be viewed entirely as a feedback process. The opening idea is taken and iterated upon itself, transformed, and combined to form a larger idea.

The overall shape of the pitch content in this piece forms a wave. The initial pitch of the pieces is located at the median of the range and as the work progresses, the notes gradually descend. The piece begins with an e-flat' in the treble clef that gradually descends to a nadir at beat 297 on a d two octaves below the bass clef in the left hand and one octave below in the right. At this point, the pitch of the notes begins to ascend, until reaching an intermediary peak at beat 451. The notes that are accented are an iteration of

a single note, *e*'''. Unlike the rest of the piece, these notes are accented in rapid succession. After this, the melodic content begins to descend slightly, before ascending again. Because each beat is accented, strict interweaving does occur. This time the performer rises up to an *f*''', where, again, Ligeti asks for the repetition of the note, this time oscillating between octaves. In contrast to the previous peak point, this time there is a space of three notes between the accents, and again there is strict interweaving. Following this, the notes climb further to a *g*''', but like the first section, the accents are spaced one beat apart. The music ascends once more, stopping on *c#*''', and *e*''' respectively. During each of these pauses, Ligeti again inserts an unaccented note between each accented one. Each of these passages which highlight a repeated note can be seen as moments of intermittency; one anticipates that the pitches will continue to rise, but this path is interrupted.

At this time, Ligeti adds a third voice to the texture creating three-voice polyphony in each hand. The density of the intervals at this point in the piece is similar to the micro-polyphony sections in *Atmosphères* and *Aventures*. Following this textural enhancement, the pitch ascends by a semitone to its zenith on *f*'''. At this point, the music finally runs "out of breath," marking the formal end of this section.

At this point, the piece reverses, instead of the left hand being behind the right hand, the left hand is now two beats ahead of the right hand. In this brief intermediary section, there are neither accents nor intervals in the melodic content. Equally important to note is that the dynamics for this section are reversed: in section A, the music is marked *ben forte*, while in this section the music is marked *pianissimo*.

Following this comes the main part of section B, which can be viewed as the contrasting section. At this point, the right hand plays a series of long notes, while the left hand plays a series of dyads. Like the intervals in section A, these dyads do not appear to adhere to a set pattern or highlight a particular interval. Equally important, the dynamics of this section begin to slowly increase. This continues until beat 765 when single notes emerge in the left hand and there is a variation of the opening motif in the right hand, with the exception that there are no accents or intervals accompanying this passage. At beat 773, melodic passages begin to appear with accents and requisite intervals accompanying them. This section can best be viewed as an intermediary transition passage back to the refrain of A.

At beat 803 the A' section appears. The melodic content is distinctly similar to the beginning except that the melodic material is now phrased instead of the entire passage being marked legato as it was in the beginning. Also, there are now chromatic inflections in the passages, so that it is slightly altered from section A. As the passage proceeds, the accents begin to gradually increase, until finally at beat 863, the accents appear on every other beat, and recreate the pattern that is reminiscent of the passages on pages three and four. It is also important to note that during this iteration, there is not a single note repeated as they were before. Following this, there is a brief interlude where there is no disruption to the accents. This is the only instance where a steady accent is present in a single voice and not created via polyrhythm between the two hands.

Once this passage is finished, the music continues its trajectory, except that the accents fluctuate between having one beat and two beats between them. Finally, at beat 939, the piece again runs "out of breath" and a series of long held notes are played. One

interesting aspect to note about this section is that at beat 942, the notes that are held form an e-minor chord; this is the only instance of explicit diatonic harmony present in the piece. The statement of this chord can be viewed as a strange attractor. Nowhere in the piece is it expected that diatonic harmony will be present, but in fact it does appear at the end. The entire trajectory of the piece maybe viewed as a path leading towards this chord.

#### **4.9 Connecting this Piece with Chaos Theory**

There are several elements within this piece that can be associated with chaos theory and fractal geometry. The repeated note sequences could be viewed as strange attractors. It is not expected that the work will fall in to one of these ‘ruts’ and constantly repeat these notes. It just happens, and thus, makes it strange.

Similarly, with the melodic figure stated at the beginning and its numerous permutations throughout the piece, one could view the piece as a theme and variations, which is analogous to a Lorenz Butterfly. The theme can be viewed as the starting point and the variations can be seen as the wings of the butterfly. As much as the variations try to return to the base, or origin, they never do; while they come close, they never exactly replicate the original theme.

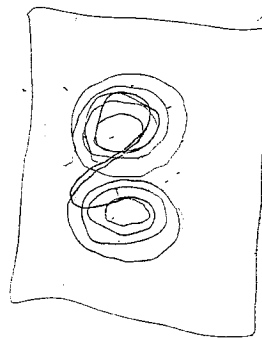
Another stylistic feature of this work that links it to chaos theory is the rhythm. In rhythmic terms, both the left and right hand lines of this work in isolation are not complex. Not until Ligeti inserts accents into the music does any rhythmic variety appear. This is an excellent example of a simple element or idea combining with itself to create something complex.

Furthermore, if one considers that the melodic content is actually just short fragments which have then been expanded, one can say that the entire melody of this

piece is made up of the re-iteration of one fragment upon itself in different forms such as shears, translations and reflections. This is yet another example of self- similarity.

#### **4.9 Further Findings at the Sacher Foundation**

The only material relating to this work at the Sacher Foundation is the *Reinschrift* of all the etudes in Book III. On the *verso* of the last page of the *Reinschrift* of this work, Ligeti has drawn in pencil a Lorenz Butterfly, which is reproduced on the following page:



0, 33 23 . . . . 3.0 x 10<sup>-2</sup>  
0, 66 66 . . . . 6.0 x 10<sup>-2</sup>

Plate 5 - Verso of last page of *Reinschrift* of Piano Etude 17,  
György Ligeti Collection, Paul Sacher Foundation Basel

Considering the musical operations in this piece highlighted in the analysis on the preceding pages, and that this work is dedicated to the man whom fully introduced chaos theory and fractal geometry to Ligeti, it is safe to ascertain that Ligeti is attempting to create a musical work which is similar to that of a Lorenz Butterfly, and therefore chaos theory.<sup>147</sup>

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<sup>147</sup> György Ligeti, Paul Sacher Foundation Basel.

## Chapter 5 – Other Works

In addition to the primary works analyzed in this thesis, a discussion of other of Ligeti's compositions related to mathematics and chaos theory mentioned during chapter 2 are included in this text. Two of these include *Piece Electronique No. 3* and *Clocks and Clouds*. An examination of these pieces is helpful in elucidating that Ligeti was indeed, constantly considering the connection between music and mathematics.

### 5.1 *Piece Electronique No. 3*

As discussed in chapters 2 and 4 of this thesis, *Piece Electronique* was the precursor to *Atmosphères*. It is an electronic piece to be created through the use of sound waves of different frequencies. The sound waves are to be organized in four separate groups. Each group would consist of a subset of waves, to be superimposed upon each other. Together the waves in each group would form their own trajectory or path, and would then be superimposed upon each other. So in addition to being a piece formulated from one set of simple events, they would collate to form four separate groups, which each would then be combined with each other. This is an excellent example of simple events combining to form large scale occurrences.

The Sacher Foundation's holdings for this work includes two folders; one of the *Skizzen* and the other of the *Partitur*. In the *Skizzen* folder, a sketch diagramming the overall shape of each group was found. From this, one can see that the shapes of each group are similar, but also distinctly different. If they were superimposed upon each other, it would be difficult to ascertain which group was which. Thus one is able to see

the connection between this work and *Atmosphères*; as well as seeing that the basic shape of a wave can be iterated upon itself to form a dense sound-mass.<sup>148</sup>

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<sup>148</sup> György Ligeti Collection, Paul Sacher Foundation Basel.

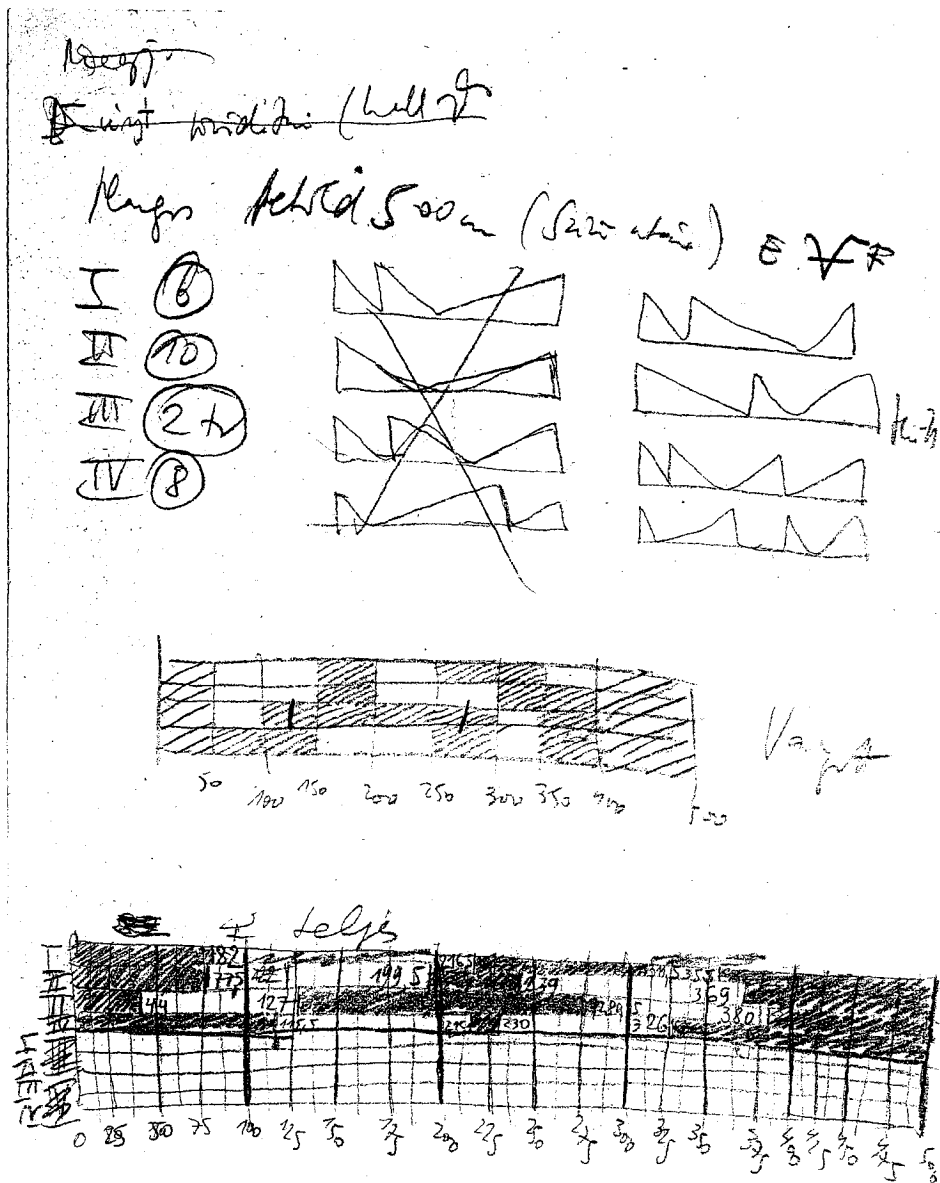


Plate 6 – Sketch of Grouping in *Piece Electronique No. 3*,  
 György Ligeti Collection, Paul Sacher Foundation Basel

## 5.2 *Clocks and Clouds*

Another work that was discussed in chapter 2 was *Clocks and Clouds*. This work is from Ligeti's middle period and was inspired by Karl Popper's essay of the same name. Ligeti's intention in this work was to formulate something that would be equal parts ordered and disorder.<sup>149</sup> The Foundation's holdings for this work are quite limited, consisting of five pages of *Skizzen*. One of these sketches is a detailed schematic of the work and what is interesting about it is that it is distinctly similar to a bifurcation tree.

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<sup>149</sup> For more about this work please see Steinitz 198-202.

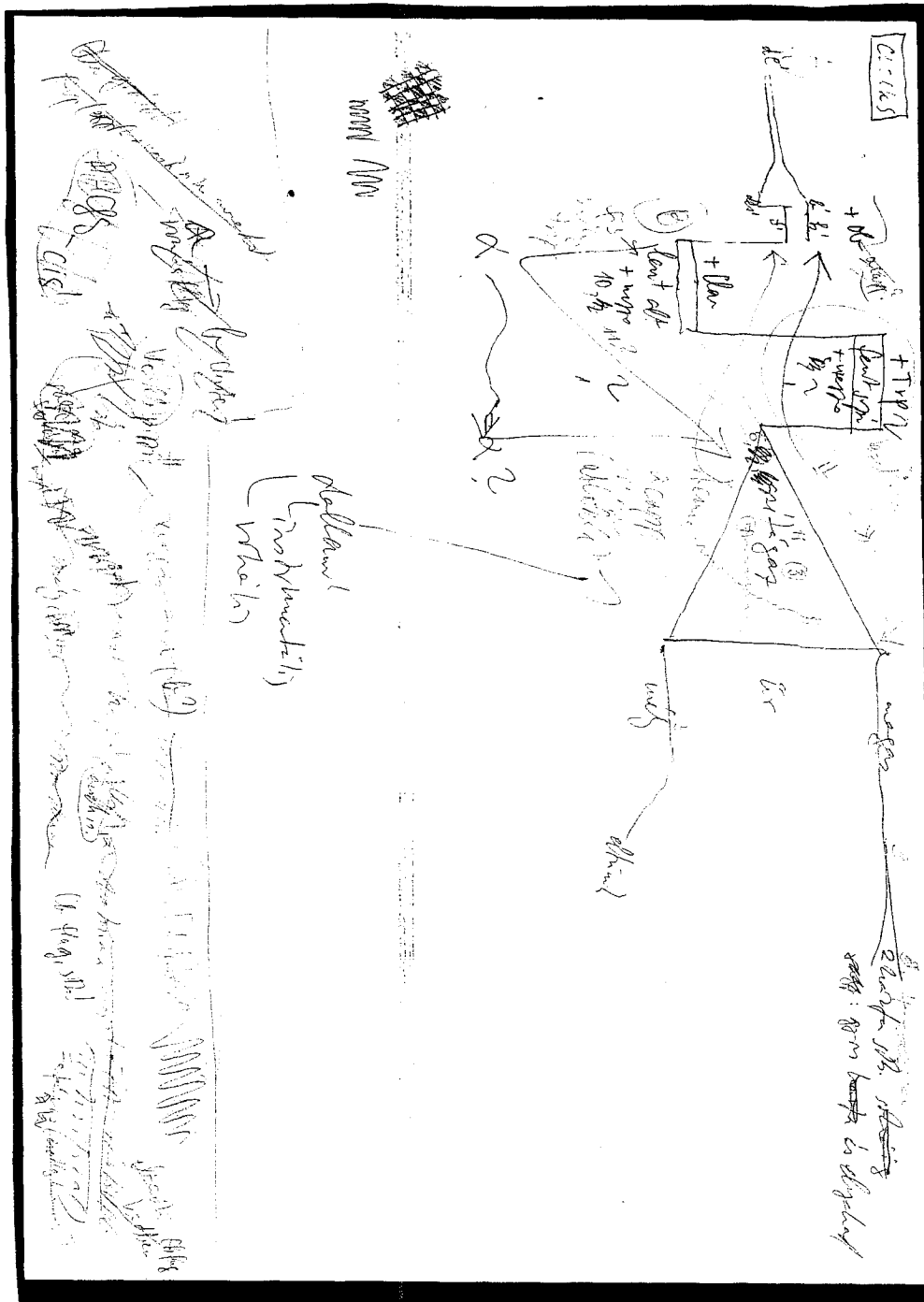


Plate 7 – Schematic sketch of *Clocks and Clouds*,  
 György Ligeti Collection, Paul Sacher Foundation Basel

The work starts at a terminus and the orchestra diverges into two opposite trajectories. These trajectories then bifurcate and more branches are formed. From this sketch, one can see that Ligeti clearly delineated what course he intended the music to have. This sketch proves that Ligeti was thinking about the ideas of envelope and bifurcation as early as his middle period.

### **5.3 Sierpinski Triangle**

One additional finding located at the Sacher Foundation was a program for a concert to commemorate the 100<sup>th</sup> Anniversary of the German Mathematical Society. All of the works featured in this concert were works by Ligeti. Throughout the program, the word “Ligeti” has been graphically inserted into the program in the form Sierpinski Triangle. This triangle is repeated until there are sixteen levels of the word “Ligeti.” This program is interesting because it shows that the mathematical community was, and is aware of the ideas that Ligeti incorporates into his works. His artistic designs had obviously not gone unnoticed by those out side of the field of music.<sup>150</sup>

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<sup>150</sup> György Ligeti, Paul Sacher Foundation Basel.

## Chapter 6 – Conclusion

In conclusion, once one is able to formulate an understanding of chaos theory and fractal geometry, one can obtain a better understanding of György Ligeti's music. One can see that many of the operations and processes associated with these ideas are present in his music. Therefore, based upon the findings in this thesis, it is possible to say that Ligeti's compositional techniques are certainly informed by chaos theory.

From the analysis of the *Piano Concerto*, *Atmosphères*, and *A bout de soufflé*, one can see that several of the musical operations present in Ligeti's music, are similar to the ideas of chaos theory and fractal geometry. This is to say, the way that the music is transformed is similar to non-linear processes and the expected paths or trajectories of these works are always changing or in flux.

Also present in Ligeti's compositions is the idea that a single cell or unit can be used to form larger ideas and that these ideas do not always have to form the same event. These events can be two iterations of set-class of a specific cardinality, or it can be a specific interval and its related compliments.

Furthermore, one can see several properties or qualities emerge in the music of Ligeti. One of these properties is that in his later works, such as the *Piano Concerto* and *Piano Etude*, there is a higher regard for motivic structure and development than in his earlier works, which are completely void of such elements. Also, these works are directly similar to the ideas presented by fractal geometry. All three works analyzed in this thesis can be viewed as a scaled image. The *Piano Concerto* may be seen on a one measure level as just the primary motif (or perfect fourth) but if scaled back, one can see that the work consists of many permutations of the motif or interval. *A bout de soufflé* can also be

seen on a primary level as just a small melodic fragment, but if the entire work is viewed, one may see that it is formed completely out of iterations of the melodic idea. With *Atmosphères*, each one of the individual sections may be seen as a small sine wave, but if scaled back, one can see that the entire form of the piece is a wave. Just as the Julia and Mandelbrot Sets and Sierpinski Triangle, these three works may be viewed on a primary level as well as in their totality.

Another quality that is exposed is that often in Ligeti's music, the trajectory or course of events is disrupted. Where one might expect a pattern of sets or collections, it is disrupted, by a set or collection of a different cardinality. However, the patterns that are disrupted are corrected and continue on their original course of development. This is very much like an intermittency pattern in non-linear graph, and can be viewed as an inherent quality in Ligeti's music.

Also, in these works, it becomes evident that they each have highly complex rhythmic structures. It is not always clear in these works what role these rhythmic structures play, but is obvious that they are important and should be considered as such. It should be noted that Ligeti's second wife was an award winning psychologist. Many of the issues regarding Ligeti's music do deal with the psychological or aural perception of the audience. To date, no study has ever been conducted on the aural cognition of Ligeti's music. However, it can be seen from this thesis, that what is written on paper and what the audience hears is quite different. Is it possible that the rhythmic talea of the Piano Concerto could be perceived by the listener. It is interesting then to ask: how developed must a listener's ear be to perceive this and also, what effect does this have on one's interaction with the work? Along these lines, one can also see that Ligeti's ideas related

to rhythm are not new. He has draw inspiration form nay sources which include African polyrhythm, jazz, salsa, and the player-piano works of Conlon Nancarrow.

In regards to *Atmosphères*, once can definitely detect a sense of creating a complex sound mass from simple elements. Also, one can see that the sound-blocks or sections that form this work are non-causal, that being that they do not show a distinct motivic or thematic connection between the sections such as more traditional classic music does; their links are more subtle.

One further observation that arose from this thesis is that Ligeti does not view his compositional techniques as a radical departure form convention. His music is part of a larger tradition of experimentation with new forms and models for composition. Other composers who are part of this tradition of experimentation are Claude Debussy, Arnold Schoenberg, Bela Bartok, Johannes Brahms, and Johannes Ockeghem. Like Schoenberg, Ligeti often asserts that he has been miscast, that his music does not make a rupture or break with the past, but that he is merely merely continuing a line of experimentation and expansion of the musical lexicon that was started over 500 years ago. Furthermore, it is false to say that Ligeti adopted chaos theory and fractal geometry as a method for composition. It would instead be closer to the truth to state Ligeti independently formulated a method of composition similar to that of the principles of chaos theory and fractal geometry.

Another crucial element that this thesis brought forth was that the holdings of the Paul Sacher Foundation contain a vast wealth of information that can be employed to the analysis and understanding of Ligeti's music. Form these verbal constructs and sketches

that Ligeti has left behind, one can at least gain a closer understanding of Ligeti's own thought process, or of elements that Ligeti was considering during composition.

While the findings of this thesis are not definitive, it will hopefully provide future Ligeti scholars with a new paradigm through which to view his works, as well as providing a new vocabulary and methodology with which to analyze Ligeti's music. An entire recontextualization of Ligeti's music could be conducted with the consideration of these principles. It has been made apparent over the course of this thesis that it is not just the three major works considered here that display properties related to chaos theory and fractal geometry. Significant works from all periods of his career possess these characteristics. Additionally, many of the precept and terms presented in this thesis have a common counterpart in traditional methods of analysis. Hopefully this will demonstrate that one can turn to a less traditional, but similar style of analysis to explain a work. These less conventional methods are not meant to replace them, but rather as a way of enhancing them.

Overall, this thesis proves that even though Ligeti overtly denies that there is a direct link between his compositions and chaos theory and fractal geometry, it has been shown that there is indeed a similarity between the form, shape, and structure of these works to the above mentioned ideas. While in no way are these findings conclusive and definitive, they do form an excellent foundation for future scholars to continue research and scholarship on this subject matter.

## **Appendix A**

### **Contents of Sacher Foundation**

#### **Atmospheres**

108 pages of sketches that highlight overall form and structure of work. Emphasis is placed on pitch and rhythmic structure within specific sections.

#### **Piano Concerto**

401 pages of sketches. Predominately verbal and conceptual sketches of work; written in a mixture of Hungarian, German, and English. Emphasis of sketches is on potential and final form and rhythmic structure of work; very little to no sketches with concern to the pitch structure of work.

#### **A bout de soufflé**

Collated with all three books of etudes. All together, there are 399 pages of sketches and fair copies of the works, with 41 pages dedicated to book 3. These sketches are mostly final drafts and notations of rhythmic patterns of work. Concerning, "A bout des soufflé," it appears to be no sketches, only a fair copy.

**Appendix B**  
**Glossary of terms associated with chaos theory and fractal geometry**

Strange attractor – an item, object, or focus area in a readout or process that is not expected

Bifurcation – a point where two paths or trajectories are split or forced into a different formula or process

Intermittency – a periodic trajectory that is interrupted and proceeds to oscillate between two points but appears to not have an order or pattern. This is an interruption following which the trajectory returns to its original course.

Fractal geometry – study of graphic images related to chaos theory; is a sub-field.

Lorenz Butterfly – a graphical representation of a feedback process, often referred to as a feedback loop.

Sensitivity to initial conditions – the idea that a particular answer or trajectory is dependant on a specific starting point, if this point is altered, a different result will occur.

Self-similarity – An object retains the same properties and qualities if there is a change of scale

Feedback & iteration – the taking of an answer which is then put back into a formula.

**Appendix C**  
**Glossary of terms from Charles Madden's *Fractals in Music***

Shear – a change of scale that is not proportional; only side of a triangle changes scale while the other two sides remain the same.

Scaling – a change in the size of an object.

Self-affinity - a change of scale that is not proportional, one aspect changes in a different proportion than the others.

Self-similarity – scaling in which all aspects or qualities of the object remain the same; related to scaling.

Reflection – the rotation of an object.

Retrograde reflection – a reflection of an object on the vertical plane.

Inversion reflection – a reflection that is on the horizontal plane.

Verticalization – rotation of an object counterclockwise.

Horizontalization – rotation of an object clockwise.

## Appendix D

### Glossary of terms from Simha Arom's *African polyphony and polyrhythm: Musical structure and methodology*.

Periodicity – repetition of similar events at similar intervals.

Macroperiod – combination of different length periods that converge at a common point.

Commetricity – rhythm with accents, attacks, or change in tone colour that coincide with pulsations.

Contrametricity – rhythm with accents, attacks, or change in tone colour do not coincide with pulsations

Interweaving – two types; **strict** when two or more voices have accents, attacks, or change in tone colour never coincide, **partial** when some of the attacks, accents, or change in tone colour do coincide while other diverge.

Metre – music with a regular pattern of stresses, attacks, or accents.

Polyrhythm – two or more voices with independent rhythmic lines combine to create a new rhythm which is not present on the page, it is only present aurally.

## Appendix E

### Set-class usage in Movement IV of the Piano Concerto

Over the course of this movement, Ligeti employs nine of the twelve trichords. For tetrachords, Ligeti uses a more limited vocabulary, when using these sets, he has only used ten of the twenty-nine possibilities. With the hexachords, Ligeti has used a wider selection; he has utilized twenty-three of the possible fifty hexachords and fourteen of the thirty Z-sets.

Ligeti's usage of set-classes is intriguing in that, in many cases, he will use the same smaller sets to form different larger sets. The author would like to draw the similarity between this technique and that employed by Elliott Carter.<sup>151</sup>

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<sup>151</sup> Andrew Mead, "Twelve-Tone Composition and the Music of Elliott Carter," *Concert music, rock, and jazz since 1945: essays and analytical studies* Eds. Elizabeth West Marvin & Richard Hermann (Rochester, NY: University of Rochester Press, 1995) 67-103.

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