THREE PHASE LOAD FLOW IN A POWER SYSTEM UNDER UNBALANCED CONDITIONS

by

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ABSTRACT

The scope of this thesis emphasizes on the need for anlysis of each individual phase of a power system to reveal unbalanced conditions.

The use of long EHV high voltage transmission lines to transmit large blocks of electric power from remote generating stations encounters the problem of unbalanced currents in the lines due to unsymmetrical placement of line conductors. The presence of large single phase loads, single pole switching and unequal series capacitor compensation in long lines add to voltage and current unbalance. To ensure proper performance of the whole system and that of the individual elements, the degree of unbalance should be known. To improve or investigate these unbalanced effects in detail, 3 phase load flow methods are used. Three phase load flow simulates each phase of the system separately, thereby revealing the power flow unbalance in the system.

This thesis reviews steady state analysis of balanced system with a single phase load flow solution method and unbalances in the transmission circuit were considered by a three-phase load flow method. The analysis is made with phase quantities but sequence quantities were also used to show the amount of current unbalance.

The application of three phase load flow with different numerical methods to sample systems gives encouraging results regarding presence of unbalance mainly due to untransposed transmission lines with different conductor configurations and unbalanced loads.

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CHAPTER I

INTRODUCTION

In the past few years the demand for electric power has shown a steep-rise, thereby spurring the trend towards high-voltage transmission [1]. The most effective and economically desirable method of ensuring the supply of electric power to consumers is to centralize electric power production by the development and unification of power systems [2]. Unified power systems increase the reliability of supply and permit better utilization of energy resources and power equipment. Unification is obtained by extension and interconnection of regional power systems so as to take advantage of difference in time zones and diversity among loads of different power pools. The reliability of supply, convenience, voltage regulation and saving in cost of generating and protective equipment, justify interconnection of different power pools on economic grounds.

To meet the increasing demand for power, new hydro, thermal and nuclear generating stations are being constructed. In most of the cases, these generating stations have been located far from the load centres due to high transportation cost of fossil fuels, non-availability of large quantities of water near the load centres and the necessity of conforming with health regulations. These limitations make it necessary to transmit large blocks of power over long distances.

The interconnection between different power pools and the distant location of the generating stations from the load centres require the use of substantial capacity multicircuit transmission lines. To increase the power loading (capacity) of the electric transmission lines the voltage must be raised because the power loading of a transmission line is proportional to the square of the voltage [2]. The capital expenditure and the cost of transmission per kWh decreases* with an increase of transmitted power and voltage whereas the cost of the line and of the terminal equipment increases linearly with voltage [2]. To transmit large blocks of electric power a technical and economic comparison should be made for the annual savings obtained by increasing the cross-section area of the conductors with the additional capital investment involved and reduced energy losses. The use of conductors with a large diameter in many cases permit a reduction in aluminium requirements but poses many erection and handling difficulties. The power transmitting capacity of the transmission line can be increased with an increase in the number of circuits.

An increase in the number of circuits of the transmission line involves increased construction cost of the links, increased corona losses and increased losses due to charging currents when the line is lightly loaded. To overcome these difficulties bundle conductors are used in which more than one conductor per phase is utilized [1]. Using bundle conductors has the same effect as increasing the diameter of the wires. This ensures an acceptable low level of corona losses. Bundle conductors have the added advantage of reducing the inductive reactance and increasing the capacitive reactance of the line thereby increasing its power carrying capacity.

Transmission lines can be either transposed or untransposed. In the case of untransposed lines the position of the phase conductors remains unchanged throughout the length of the line. If the conductors of a three phase line are not equilaterally spaced, then the inductance of each phase is different and a voltage of power line frequency is induced in the neighbouring communication lines. Untransposed lines with flat conductor configuration tend to increase the current

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* assuming no delays in construction.

unbalance among different phases due to the presence of induced voltages. The out of phase portion of these voltages in the presence of small series terminal impedances can cause relatively high circulating unbalanced currents to flow between the various circuits [3]. The problem of unbalance can be analyzed by a transformation of phase quantities to symmetrical components by using suitable transformations [1,3]. The presence of large unbalances produces substantial amounts of negative and zero sequence voltages and current components, which can give rise to heating in machine rotors, interference with communication circuits, incorrect operation of protective relays and increased corona losses [4]. The unbalance characteristics of transmission line impedances are important because of their influence on generator ratings, unbalanced voltage at the consumer terminals and increased losses within the transmission network [5]. As the electrical systems becomes larger the problem of unbalance becomes more and more significant.

Transmission lines are transposed in order to reduce the unbalance in the three phases, arising from the unsymmetrical spacing between the conductors with respect to each other, with respect to ground and ground wires and due to neighbouring communication and other power circuits. The unsymmetrical conductor spacing causes the magnetic field external to the conductor to be non-zero, thereby causing induced voltages of power line frequency in adjacent electrical and telephone circuits [6].

Transposition of transmission lines is done by exchanging the position of the conductors, so that within a certain length of the line, generally termed a "barrel of transposition", each conductor occupies the original position of every other conductor over an equal distance. The objective of transposition is to make equal the self and mutual impedances of the transmission line over the length of the barrel. The transposition of phase conductors has not only the desirable effect of reducing out of balance currents in the terminal equipment but also of reducing the transmission losses. This gives a basis for their economic justification [5]. Transposition is generally employed for low voltage transmission and distribution circuits. However, for high voltage lines, transposition becomes undesirable because of reduced reliability at the transposition towers, and added cost and complexity in the design of these transposition towers. However, due to insufficient investigations [2] on the choice of the length of the transposition-section as well as on the effect of the difference in the length of the individual transposition sections, and on the influence of other factors which may disturb the balance of phase (self) and interphase (mutual) parameters (impedance and shunt admittance) of long high voltage lines (say 500 kV), the practical aspect is lost because it is not possible to have transposition-sections of equal length [2]. This is mainly based on economic grounds, nature and topography of the ground, type and height of vegetation surrounding the route of the transmission line and the type of supporting structures used.

If the transposition-sections are not equal in length then the inductance for each phase is different. This results in an unbalanced circuit and voltages are induced in the neighbouring communication lines even for balanced currents in the power line.

The current unbalance can also be reduced by the proper selection of conductor phasing arrangement for two or more adjacent transmission lines. For a "n" circuit system there exists $(6^n)/3$ significantly different phasing arrangements, which should be properly tested for multicircuit transmission lines for minimizing current unbalance [7].

After the transmission line has been properly analyzed for its contribution to current and voltage unbalance, then the other important elements constituting the power system should also be analyzed for their share in the unbalance of the system. These elements are generators, three phase transformers, three phase banks of single phase transformers, shunt capacitors, synchronous condensers, and protective equipment, etc. The purpose of constituting the power system with the above elements is to give adequate and reliable supply to the load centre.

In the case of generators, the three voltages behind the machine impedance, will be balanced in both their magnitude and angles for a balanced design of the generator windings, but the voltages at the generator output terminals may not be balanced and their effect should be taken into account.

For transformers, their contribution to unbalance will depend on the type of transformers used i.e. a single three phase transformer or a three phase bank of single phase transformers. The transformer reactances in the three phases may be quite different in the case of three single phase transformers in comparison to a three phase transformer. This will give rise to unbalanced voltages on their secondary terminals.

It is rather impossible to represent each load individually in a power system. In most cases, loads are represented as composite system loads, obtained from the demand meters at the substations. The consumer power demands at random times are not known and the loads in the three phases are not balanced. The power system loads are modelled either as constant impedance or constant current loads.

For most purposes, in the steady state analysis of power systems, the load flow solution is based on perfectly balanced three phase networks operated under balanced three phase generation and load conditions. An analysis of one phase of the system yields sufficient information about overall system performance. The load flow used to study the voltage and power flow conditions for a balanced power system is known as single phase load flow. However, the present large and extra-large power systems are being operated at very high voltages and have, therefore, a substantial amount of unbalance among different phases of the system which can not be ignored. It becomes important, therefore,on reliability, stability and other grounds to analyze each phase of the system separately. This can be done with the aid of a three phase load flow [see chapter 4] thereby obtaining information about current and power flow unbalance among the individual phases of transmission lines and transformers.

Chapter two describes in detail the primitive network matrices, required to represent the electrical behaviour of the individual network elements. These matrices can be formulated from the data available from the standard conductor tables, transmission line lay out and from the interconnections in the network. The formulation of matrices has been discussed in the phase frame of reference.

The general theory underlying load flow analysis has been reviewed in chapter three, which shows the formulation of the load flow problem for the analysis of balanced system.

Chapter four applies the basic load flow techniques to formulate the three phase load flow algorithm for the analysis of unbalanced systems. The requirement of both phase frame of reference and sequence frame of reference for the analysis of present large systems has also been discussed.

Chapter five reviews some of the numerical methods which can be used to analyze the three phase load flow problem.

The concluding chapter six presents numerical results for the two sample systems analyzed with different methods. The theoretical predictions regarding current unbalance discussed in earlier chapters have also been analytically analyzed in terms of sequence components, and results tabulated for the sample system with different conductor configurations.

The programming has been done using an IBM 360/65 system.

CHAPTER II

COMPUTATION OF NETWORK MATRICES

2.1 Introduction

The first step in the analysis of all electric networks is the formulation of a suitable mathematical model. The model must be sufficiently complete to describe the behaviour of individual network elements and their interconnection with other elements. For the analysis of power systems the mathematical model should relate a selected set of network voltages with another selected set of powers.

The electrical behaviour of the individual network components can be formulated with the help of primitive network matrices. The primitive matrices have sufficient information on the electrical characteristics of individual elements but they are unable to provide any information about location and interconnection of the different elements. It becomes essential, therefore, to transform the primitive network matrices to obtain a final network matrix by the use of a suitable transformation matrix, tailored to the requirement of the numerical method used. The transformation matrix depends on the reference frame used in the analysis of the performance equation, namely, bus or loop [8]. In the bus frame of reference the variables are bus voltages and bus currents whereas in the loop frame of reference the variables are loop voltages and loop currents. The bus frame of reference has been used throughout this thesis in the analysis of the performance equation.

The geometrical structure of the network components can be described by single line segments irrespective of the electrical behaviour of the components. These line segments are called elements and their terminals are called nodes. The geometrical interconnection of the elements of a network are represented by a graph.

The following matrices are required for the representation of the network.

(i) Bus incidence matrix

(ii) Primitive impedance matrix

(iii) Primitive shunt admittance matrix.

2.2 Bus Incidence Matrix

The bus incidence matrix is obtained by deleting the column corresponding to the reference node of the element node incidence matrix. This matrix is generally denoted by "A" [8]. The elements of the matrix are as follows

a_{ij} = 1 If the i^{th} element is incident to and oriented away from the j^{th} node

a_{ij} = -1 If the i^{th} element is incident to and oriented toward the j^{th} node

a_{ij} = 0 otherwise

The dimensions of this matrix are $e \times (n-1)$, where e is the number of elements and n is the number of nodes. The matrix is rectangular and therefore singular [8].

2.3 Impedance Matrix

The primitive impedance matrix gives information about the electrical characteristics of the transmission line. The parameters which affect the performance of the transmission line are

(i) Resistance

(ii) Inductance

(iii) Capacitance

The primitive impedance matrix depends mainly on the numerical values of the resistance and inductance of the conductors. Resistance and inductance in turn depend mainly on the length of the line, conductor cross-section, conductor spacing, number of circuits used and number of conductors per phase [1].

The value of self and mutual impedances for the transmission line can be calculated by the use of the following relations :

$$Z_{ij} = (r_i + r_d) + j\,\omega\, k \left(\ln \frac{D_e}{GMR} \right) \qquad i = j \tag{2.1}$$

$$Z_{ij} = r_d + j\,\omega\, k \ln \frac{D_e}{D_{ij}} \qquad i \neq j \tag{2.2}$$

in which

r_i = resistance of phase wire i (available from standard conductor tables)

r_d = earth resistance (dependent on return earth condition)

GMR = Geometrical mean radius of the conductor

D_{ij} = Geometrical mean distance between phase i and j

k = constant whose value depends upon the unit of length chosen and on the logarithmic base used [1].

ω = constant whose value depends on the frequency.

D_e = Quantity which is a function of earth resistivity ρ and frequency f and can be defined as

$$D_e = 2160 \sqrt{\frac{\rho}{f}} \text{ unit length.}$$

The value of ρ differs from place to place and depends upon local earth conditions. It can be taken equal to 100 Ohm-meter for average damp earth.

The primitive impedance matrix is also affected by the presence of ground wires. In the computation of the primitive impedance matrix for the transmission network with ground wires proper reduction techniques must be used before the final resultant matrix is obtained as discussed in [1]. The dimensions of this matrix are (b x b), where 'b' is the number of branches in the network.

2.4 Shunt Admittance Matrix

The primitive shunt admittance matrix depends mainly upon the capacitance of the transmission line. The capacitance, in turn, depends upon the geometry of the cross-section of the line, the permittivity of the dielectric present and on the height of the conductors above the ground plane [9]. In overhead lines the shunt admittance is a pure susceptance since the conduction current between phase wires and ground is negligible.

The capacitance of the transmission line can be computed with the help of potential coefficients [13]. The following equation relates the potential coefficients to phase voltages and charges on the conductors.

$$[V] = [P]\,[q] \tag{2.3}$$

where

V = is the voltage vector

P = is the matrix of potential coefficients

and

q = is the vector of electric charges on the conductor.

The potential coefficients can be computed by the use of the following equations :

Self potential coefficients :

$$P_{ij} = \frac{1}{2\pi \epsilon} \ln \frac{H_i}{r_i} , \qquad i = j$$

Mutual potential coefficients :

$$P_{ij} = \frac{1}{2\pi \epsilon} \cdot \ln \frac{H_{ij}}{D_{ij}} \qquad i \neq j$$

where

H_i = is the distance between conductor i and its own image

r_i = is the equivalent radius of the conductor [9]

H_{ij} = is the distance between conductor i and the image of conductor j

D_{ij} = is the geometrical mean distance [1], between conductor i and conductor j .

From Eq. (2.3) the value of the capacitance can be obtained as the inverse of the potential coefficient matrix as

$$[C] = [P]^{-1} \qquad (2.4)$$

The elements of matrix C in Eq. (2.4) are known as capacitance coefficients or Maxwell's coefficients. The diagonal terms in matrix C are positive and all the off-diagonal terms are negative.

Rewriting Eq. (2.3) in terms of Maxwell's coefficients one obtains

$$[q] = [C] [V] \qquad (2.5)$$

Equation (2.5) can be transformed to a current equation in the sinusoidal steady state as follows

$$\bar{I} = j \omega q = j \omega C \bar{V} \qquad (2.6)$$

The current vector in Eq. (2.6) is the charging current and may be redefined as

$$\overline{I} = Y_{sh} \overline{V} \qquad (2.7)$$

where

$$Y_{sh} = j \omega C$$

which is known as the capacitive susceptance or the shunt admittance matrix for the transmission line.

2.5 Formation of Bus Admittance Matrix

The performance equation for a power system network relating currents to voltages in the bus frame of reference can be written as

$$\overline{I} = Y_{bus} \cdot \overline{V} \qquad (2.8)$$

The bus admittance matrix - Y_{bus} - in Eq. (2.8) has a well defined structure, which makes it easy to construct automatically. It is a square symmetric matrix of order (n x n), where 'n' is the number of buses in the system. Two methods will be described for the computation of the bus admittance matrix. It is assumed that the primitive impedance, primitive shunt admittance matrix and bus incidence matrix have already been computed.

2.5.1 Method 1.

The primitive admittance matrix is obtained from the primitive impedance matrix by matrix inversion to give

$$y = (Z)^{-1}$$

where

y = is the primitive admittance matrix

Z = is the primitive impedance matrix

The elements of Y_{bus} can be computed as follows:

Diagonal element Y_{ii} is obtained as the algebraic sum of all admittances incident to node i .

Off-diagonal elements is the negative of the branch admittance between node i and j .

Except for small networks, very few nonzero mutual admittances are present, which makes Y_{bus} a sparse matrix.

For a 'n' bus system the diagonal and off-diagonal elements can be mathematically defined as follows :

Diagonal Elements :

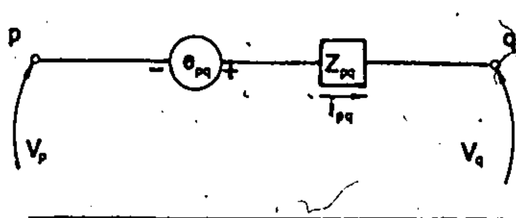
$$Y_{ii} = y_{i1} + y_{i2} + \ldots + y_{ik} + \ldots + y_{in} + y_{shi1} + y_{sh\,i2} + \ldots + y_{sh\,ik} + \ldots + y_{sh\,in}$$

Off-Diagonal Elements:

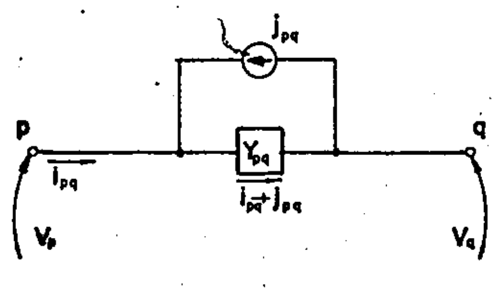
$$Y_{ij} = -y_{ij}$$

For very large systems the computation of Y_{bus} by the above method is very time consuming, so the following method is generally used.

2.5.2 Method 2 : Consider the two forms of primitive networks in which the transmission line connected between points p and q has been represented by z_{pq} in the impedance form and by y_{pq} in the admittance form as shown in Fig. 2.1.



Impedance Form



Admittance Form

Fig 2.1 Primitive network representation.

The variables and parameters are

v_{pq} = is the voltage across the element p-q

e_{pq} = is the source voltage in series with element p-q

i_{pq} = is the current through element p-q

j_{pq} = is the source current in parallel with element p-q

z_{pq} = is the self impedance of element p-q

y_{pq} = is the ~~self~~ admittance of element p-q .

The variables v_{pq} and i_{pq} for the passive elements z_{pq} and y_{pq} of the primitive network are complex numbers for alternating current circuits and real numbers for direct current circuits.

For the impedance form one can write the following equations:

$$v_{pq} = V_p - V_q$$

$$v_{pq} + e_{pq} = z_{pq} \cdot i_{pq}$$

In matrix form the variables are defined as vectors and passive elements as matrices as follows

$$\bar{v} + \bar{e} = z \cdot \bar{i} \tag{2.9}$$

and for the admittance form one writes

$$\bar{i}_{pq} + \bar{j}_{pq} = y_{pq} \cdot \bar{v}_{pq}$$

or

$$\bar{i} + \bar{j} = y \cdot \bar{v} \tag{2.10}$$

The parallel source current in admittance form can be related to the series source voltage in impedance form by the relation:

$$\bar{j}_{pq} = y_{pq} \cdot \bar{e}_{pq} \tag{2.11}$$

Multiplying Eq. (2.10) by the transpose of the bus incidence matrix, one obtains

$$[A^t] \cdot [i] + [A^t] \cdot [j] = [A^t] \cdot [y][v] \qquad (2.12)$$

The relation $A^t \cdot \bar{i}$ in Eq. (2.12) is a vector in which each element is the algebraic sum of the currents through the network elements terminating at a bus. By Kirchhoff's law this summation of currents at a bus equals zero.

Similarly $A^t \cdot \bar{j}$ gives the algebraic sum of source currents at each bus and is equal to the vector of impressed bus currents. This may be written as :

$$\bar{I}_{bus} = A^t \cdot \bar{j} \qquad (2.13)$$

Equation (2.12) can therefore be rewritten as

$$[I_{bus}] = [A^t] [y] [v] \qquad (2.14)$$

For the primitive network the sum of powers can be expressed as

$$P_p = [J^*]^t [v] \qquad (2.15)$$

Similarly, in the bus frame of reference one has

$$P_b = [I^*_{bus}]^t [V_{bus}] \qquad (2.16)$$

Since the transformation of variables must be power invariant one can write

$$P_b = P_p$$

or

$$[I^*_{bus}]^t \cdot [V_{bus}] = [J^*]^t [v]$$

But

$$[I^*_{bus}]^t = [J^*]^t [A^*]$$

Since [A] is a real matrix one obtains

$$[J^*]^t [A^*] \cdot [V_{bus}] = [J^*]^t \cdot [v] \qquad (2.17)$$

From Eq. (2.17) it follows that

$$[v] = [A] \cdot [V_{bus}] \tag{2.18}$$

Hence one obtains

$$[I_{bus}] = [A^t] \; [y] \; [v] = [A^t] \; [y] \; [A] \; [V_{bus}] \tag{2.19}$$

From Eq. (2.19) the bus admittance matrix in the bus frame of reference can be defined as

$$[Y_{bus}] = [A^t] \; [y] \; [A] \tag{2.20}$$

Rewriting Eq. (2.19) in compact form as

$$[I_{bus}] = [Y_{bus}] \; [V_{bus}] \tag{2.21}$$

In Eq. (2.20), y refers to the algebraic sum of the admittance matrices of the network and can be defined as

$$[y] = [y_{sh}] + [z_{prim}]^{-1} \tag{2.22}$$

If the transmission lines of the network are fully transposed then the self admittance terms of the primitive admittance matrix are balanced. For the system of untransposed transmission lines with ground wires, having phase sequence a-b-c say; the self admittance of phase b of the primitive admittance matrix is about 10% higher than the self admittance of phase a or c.

2.6 Computation of Bus Impedance Matrix

As discussed earlier [see section 2.3], the bus impedance matrix consists of self impedances and mutual impedances. The diagonal elements of the bus impedance matrix represent the self or driving-point impedances whereas the off-diagonal elements are the mutual impedances.

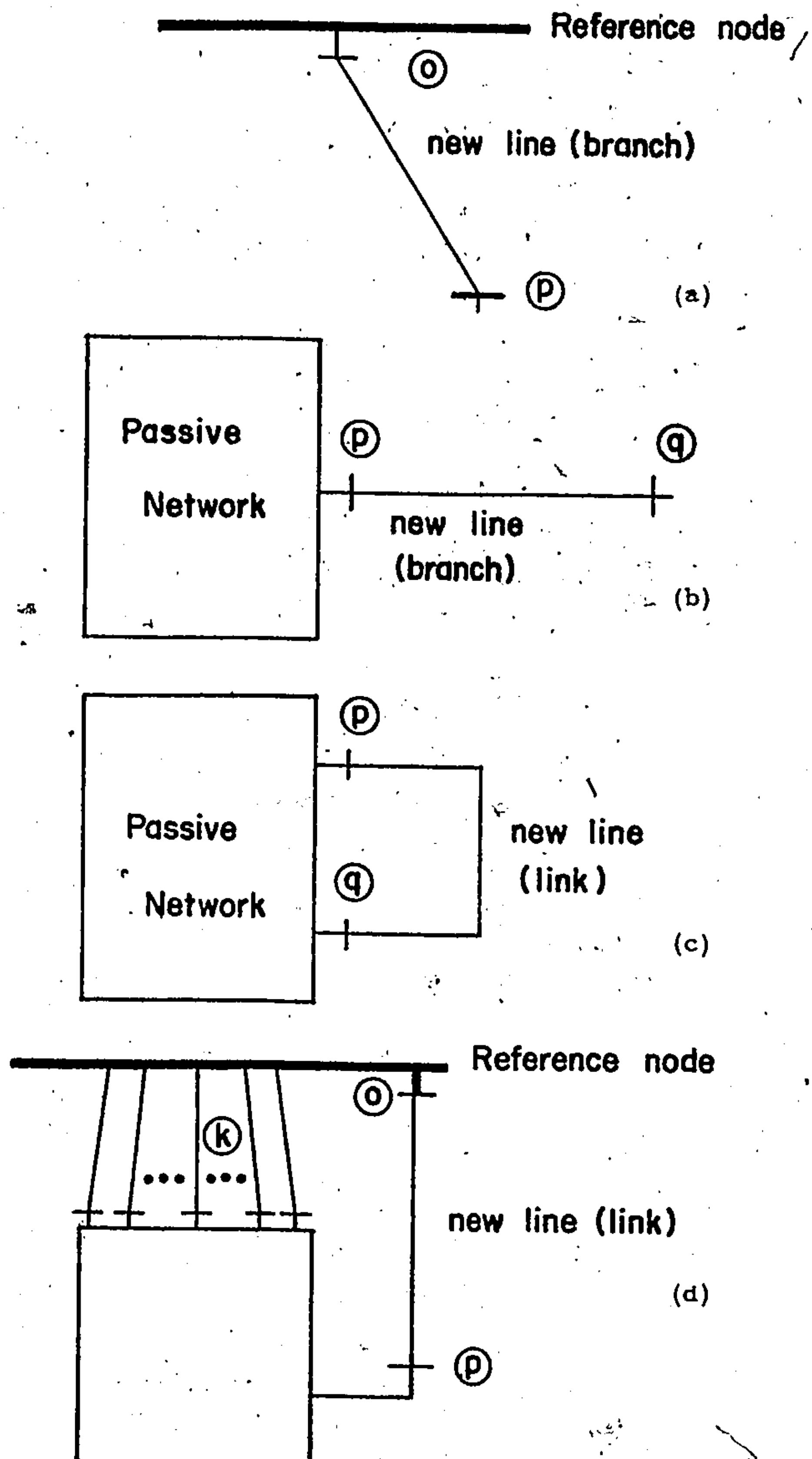


Fig 2.2 Addition of a new line to a existing network.

The bus impedance matrix of a given system can be obtained by starting with a single transmission line, adding one line at a time and modifying the matrix for each line addition. The first step in the formulation of the algorithm is to draw a single line diagram for a balanced system, an actual line repesentation for each transmission line of the unbalanced system and the selection of a reference node.

All the new lines to be added will be one of the following type

(i) A branch from the reference to a new bus (Fig. 2. 2 a)

(ii) A branch from an existing bus to a new bus (Fig. 2.2 b)

(iii) A link between two already existing buses (Fig. 2.2 c)

(iv) A link between the reference to a new bus (Fig. 2.2 d).

The algorithm starts with the addition of a line, from the reference bus to any other bus of the system. For the balanced system the mutual impedances between different phases can be neglected as single line representation has been used for the system.

2.6.1 <u>Addition of a branch of the reference</u> : Let 'o' be the selected reference bus and let the line o p be added, thereby creating a new bus p as shown in Fig. 2.3.

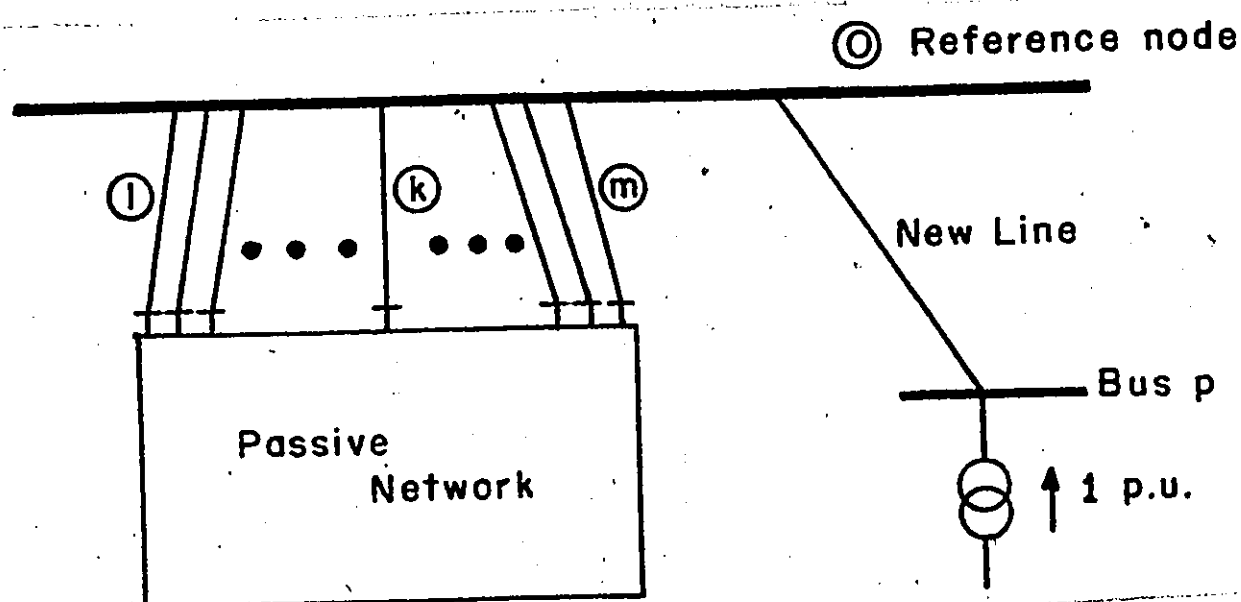


Fig 2.3 Addition of a line to the reference bus.

The next step in the formulation of the algorithm is to inject 1 p.u. current at the new bus p and to measure the voltages at the existing buses with respect to the reference bus. In fact, this current will produce zero voltage at the other buses which are already assembled. Furthermore, if one injects a unit current into any bus of the system that has already been assembled, this unit current will produce zero voltage on the new bus p. The performance equation yields :

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_m \\ V_p \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1m} & Z_{1p} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} & \cdots & Z_{2m} & Z_{2p} \\ \vdots & & & \vdots & & & \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{km} & Z_{kp} \\ \vdots & & & \vdots & & \vdots & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mk} & \cdots & Z_{mm} & Z_{mp} \\ Z_{p1} & Z_{p2} & \cdots & Z_{pk} & \cdots & Z_{pm} & Z_{pp} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \\ 0 \\ 1.0 \end{bmatrix} \tag{2.23}$$

Thus

$$\begin{aligned} Z_{1p} &= V_1 \\ Z_{2p} &= V_2 \\ &\vdots \\ Z_{kp} &= V_k \\ &\vdots \\ Z_{pp} &= V_p \end{aligned} \tag{2.24}$$

But

$$V_k = 0 \qquad \text{for } k \neq p$$

and

$$V_p = Z_{line} \cdot I_p = Z_{line} = Z_{op}$$

Thus, for the addition of a branch to the reference bus thereby creating a new bus p, the general relations can be expressed as

$$Z_{pk} = Z_{kp} = 0 \qquad \text{for } k \neq p$$
$$Z_{pp} = Z_{line} \qquad (2.25)$$

2.6.2 <u>Addition of a branch to an existing bus:</u> If the element p-q is a branch and is added to the existing network at bus p, then a new bus q is created as shown in Fig. 2.4. The performance equation for the

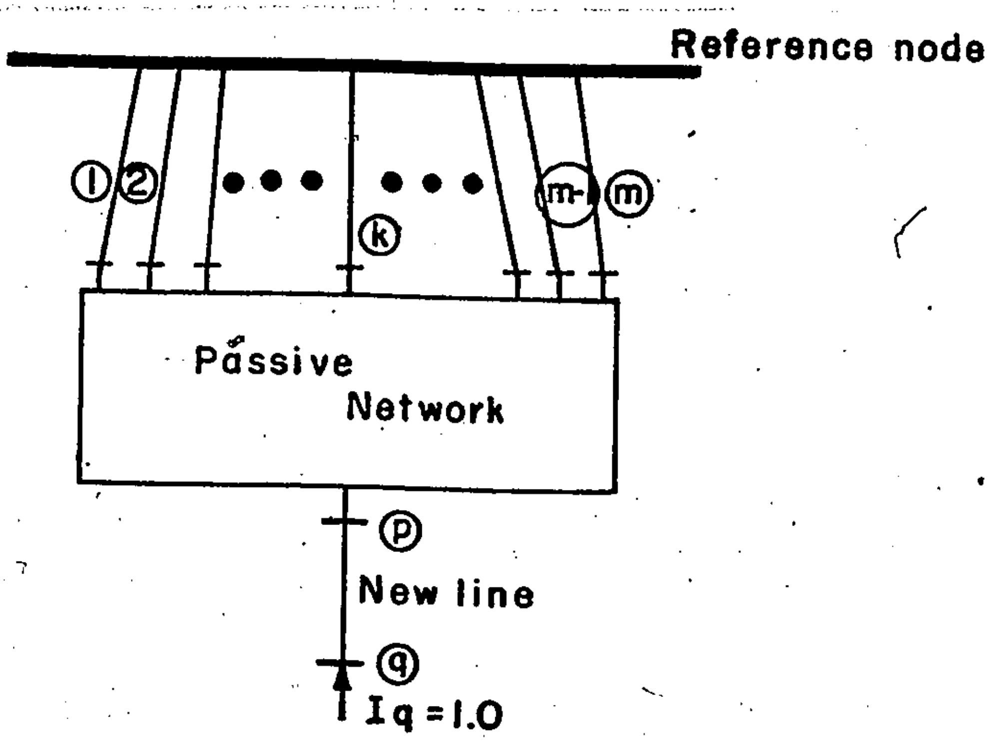


Fig. 2.4 Addition of a branch to a existing bus.

new network can be expressed as

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_m \\ V_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & \vdots & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & \vdots & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ I_q=1 \end{bmatrix} \quad (2.26)$$

The additional elements added to the impedance matrix can be obtained by injecting 1.0 p.u. current at bus q thus obtaining

$$\begin{aligned} Z_{1q} &= V_1 \\ Z_{2q} &= V_2 \\ &\vdots \\ Z_{pq} &= V_p \\ &\vdots \\ Z_{mq} &= V_m \\ Z_{qq} &= V_q \end{aligned} \quad (2.27)$$

As is evident from the system diagram

$$V_q = V_p + Z_{line} \cdot I_q = Z_{pp} + Z_{line} \quad (2.28)$$

Hence the results for this case are as shown.

$$\begin{aligned} Z_{iq} &= Z_{ip} \\ Z_{qi} &= Z_{pi} \end{aligned} \quad \text{for } i \neq q \tag{2.29}$$

$$Z_{qq} = Z_{pp} + Z_{line}$$

2.6.3 Addition of A Link Between Two Existing Nodes If the added element p-q is a link the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source E_{loop} as shown in Fig. 2.6. This creates a fictitious

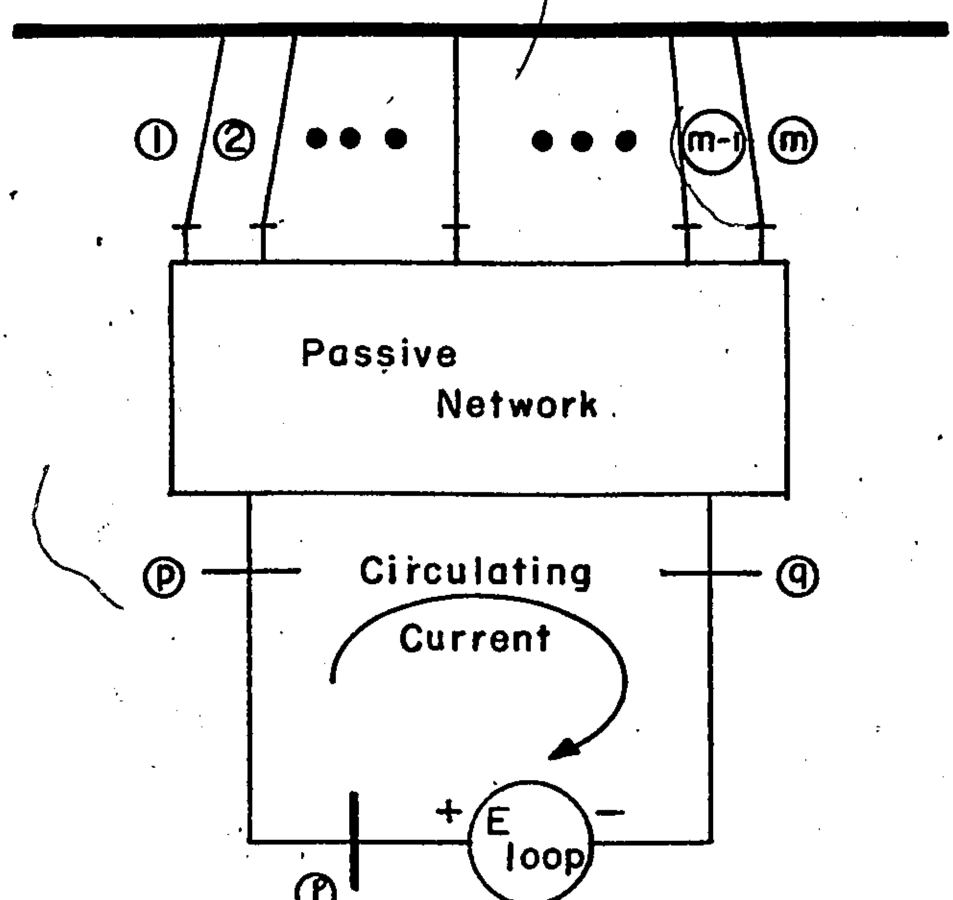


Fig. 2.5 Addition of a link.

bus 'ℓ' which will be eliminated later. As the addition of a link does not add a new bus the dimension of the bus impedance matrix stays unchanged. The voltage source E_{loop} is selected such that the current through the added link is 1.0. The circulating current or loop current of unity can be considered to be produced by a current

$$I_p = 1.0$$

and

$$I_q = -1.0$$

The voltages appearing in the system with I_p and I_q can be expressed as

$$\begin{aligned}
V_1 &= Z_{1p} - Z_{1q} \\
V_2 &= Z_{2p} - Z_{2q} \\
&\vdots \\
V_p &= Z_{pp} - Z_{pq} \\
V_q &= Z_{qp} - Z_{qq} \\
&\vdots \\
V_m &= Z_{mp} - Z_{mq} \\
E_{loop} &= Z_{loop-loop} = Z_{pp} + Z_{qq} - 2Z_{pq} + Z_{line}
\end{aligned} \tag{2.30}$$

Thus the performance equation for the network can be written as.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ V_q \\ \vdots \\ V_m \\ E_{loop} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & Z_{1q} & \cdots & Z_{1m} & Z_{1p} - Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & Z_{2q} & \cdots & Z_{2m} & Z_{2p} - Z_{2q} \\ \vdots & & & \vdots & & & \vdots & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & Z_{pq} & \cdots & Z_{pm} & Z_{pp} - Z_{pq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & Z_{qq} & \cdots & Z_{qm} & Z_{qp} - Z_{qq} \\ \vdots & & & \vdots & & & \vdots & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & Z_{mq} & & Z_{mm} & Z_{mp} - Z_{mq} \\ Z_{p1} - Z_{q1} & Z_{p2} - Z_{q2} & \cdots & Z_{pp} - Z_{pq} & Z_{pq} - Z_{qq} & & Z_{pm} - Z_{qm} & Z_{loop-loop} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ I_q \\ \vdots \\ I_m \\ I_{loop} \end{bmatrix} \tag{2.31}$$

Hence the results can be summarised as follows

$$Z_{i-loop} = Z_{ip} - Z_{iq} \qquad i \neq loop$$
$$Z_{loop-i} = Z_{pi} - Z_{qi} \qquad (2.32)$$

and

$$Z_{loop-loop} = Z_{pp} + Z_{qq} - 2Z_{pq} + Z_{line}$$

To eliminate the fictitious node "ρ" one applies the matrix partitioning technique. Since $E_{loop} = 0$, the matrix equation can be expressed as

$$\begin{bmatrix} V_{bus} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{bus(old)} & Z_{i-loop} \\ Z_{loop-i} & Z_{loop-loop} \end{bmatrix} \begin{bmatrix} I_{bus} \\ I_{loop} \end{bmatrix} \qquad (2.33)$$

From (2.33) the resultant bus impedance matrix can be obtained by Kron's reduction technique as follows

$$Z_{bus} = [Z_{bus(old)} - Z_{i-loop} \cdot Z_{loop-loop}^{-1} \cdot Z_{loop\ i}] \qquad (2.34)$$

The process of building up the bus impedance matrix is continued until all the buses in the system have been absorbed.

The bus impedance matrix can be modified to reflect changes in the network. These changes may be addition of elements, removal of elements, or changes in impedance of elements. Z_{bus} is considered as the matrix of the partial network at that stage and the new elements are added one at a time to produce the new bus impedance matrix [8].

The procedure to remove elements or to change the impedances of the elements is the same. If an element is removed which is not mutually coupled to any other element, the modified bus impedance matrix can be obtained by adding, in parallel with the element, a link whose impedance is equal to the negative of the impedance of the element to be removed. If an uncoupled element is to be replaced by another element of different impedance, then to obtain the new bus impedance matrix, a link is added in parallel to the element to be replaced such that the equivalent impedance of two elements has the required value.

If the element to be added has mutual coupling with one or more elements of the partial network then the majority of the elements of the current vector in (2.26) will not be equal to zero but will have some value. The relations governing the formulation of the bus impedance matrix when mutual coupling exists [8] are given in Table 2.1. The following notations have been used.

pq = is the element to be added and can be either a branch or a link.

$\rho\sigma$ = is a variable subscript and refers to all other elements of the partial network before the new element is added.

Z_{qi} = is the mutual between element q and i.

$Z_{pq,\rho\sigma}$ = is the matrix of the mutual admittance between the new element p-q and the elements ρ-σ of the already existing partial network.

$y_{\rho\sigma,pq}$ = is the transpose of the vector $y_{pq,\rho\sigma}$

ℓ = is the fictitious bus created when element pq to be added is a link.

m = is the number of buses in the partial network before a new element is added.

Table 2.1 Summary of equations for formation of bus impedance matrix.

New Element to be added p - q	p is the reference bus	p is not the reference bus.
Branch	$Z_{qi} = \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}}$ $i = 1, 2 \ldots m$ $i \neq q$ $Z_{qq} = \frac{1 + y_{pq\rho\sigma}(Z_{pq} - Z_{\sigma q})}{y_{pq,pq}}$	$Z_{qi} = Z_{pi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}}$ $i = 1, 2 \ldots m$ $i \neq q$ $Z_{qq} = Z_{pq} + 1 + \frac{y_{pq,\sigma\rho}(Z_{pq} - Z_{\sigma\rho})}{y_{pq,pq}}$
Link	$Z_{\ell i} = - Z_{qi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}}$ $i = 1, 2 \ldots m$ $i \neq 1$ $Z_{\ell\ell} = - Z_{q\ell} + \frac{1 + y_{pq,\rho\sigma}(Z_{\rho\ell} - Z_{\sigma\ell})}{y_{pq,pq}}$	$Z_{\ell i} = Z_{pi} - Z_{qi} + \frac{y_{pq,\rho\sigma}(Z_{\rho i} - Z_{\sigma i})}{y_{pq,pq}}$ $i = 1, 2 \ldots m$ $i \neq 1$ $Z_{\ell\ell} = Z_{p\ell} - Z_{q\ell} + \frac{1 + y_{pq,\rho\sigma}(Z_{\rho\ell} - Z_{\sigma\ell})}{y_{pq,pq}}$

The fictitious node 1 can be eliminated by using Equation (2.34) and the resultant bus impedance matrix is obtained.

CHAPTER III

REVIEW OF LOAD FLOW ANALYSIS

3.1 Introduction

Load flow studies are one of the essential requirements in the planning and operation of power systems. Load flow studies provide information to enable the active and reactive power flows to be determined together with other pertinent information such as bus voltage levels, machine excitation, tap changes and reactive compensation requirements [10]. Such investigations allow the optimization of system losses and provide a check on the stability and security of the system. To keep pace with load growth, new generators, system interconnections and new transmission lines will have to be added to the existing system. Any change in the existing system has a direct effect upon system voltages and currents, as well as real and reactive power flows. In meeting the requirements of active and reactive power the former must be supplied entirely from the generating sources, but the latter may be supplied from other sources such as shunt capacitors, synchronous condensers, etc.

The information obtained from a load flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flows in each line of a network for specified terminal or bus conditions. In general, the power systems are assumed to be balanced, such that single phase representation of generators, transformers, transmission lines, etc., is used to perform load flow studies.

In the solution of balanced power systems the starting point is a single line diagram. The purpose of the single line diagram is to supply in concise form the significant information about the system.

The importance of different features of a system varies with the problem under consideration and the amount of information included on the diagram depends on the purpose for which the diagram is intended [11] . Voltage, current, kVA and impedance in a circuit are often expressed as a per unit of a selected base or reference value of each of these quantities. Thus a per unit quantity is a normalized quantity with respect to chosen base value [6, 10, 11, 12, 13]. In practice, it is often found more convenient to work in per unit terms than to work with the actual values.

3.2 Load Flow Terminology

Associated with each bus of the network are four quantities : the real and reactive power, the voltage magnitude and the phase angle. Normally three kinds of buses are identified in a power network and at a bus two of the four quantities are specified. The first one is the swing or the slack bus as it is generally named. It has always a generator of adequate capacity connected to it. This is the bus which will respond first to a changing load condition and will supply transmission losses, since these are unknown until the final solution is obtained. Voltage magnitude and phase angle are specified for this bus. The remaining buses of the system are designated either as voltage controlled buses or load buses. The real power and voltage magnitude are specified at a voltage controlled bus, generally known as a PV-bus. If a generator is not connected to a voltage controlled bus, a suitable source of reactive power must be connected. A bank of shunt capacitors or synchronous condensers are commonly used. The real and reactive power is specified for a load bus generally known as a PQ-bus. If there is no load or generation at a bus then it can be considered as a load bus where active and reactive power are taken equal to zero. The three bus types can be summarized as follows

(i) PQ - bus or load bus : real and reactive power are specified.

(ii) PV - bus or voltage controlled bus: voltage magnitude and and real power are specified.

(iii) Type 3 bus or swing bus : voltage magnitude and phase angle are specified.

Network connections are described by using code numbers assigned to each bus. These numbers specify the terminals of transmission lines and transformers.

Load flow studies performed for the static operating condition of a power system require the solution of nonlinear equations satisfying the voltage and power requirements at the chosen points of the system. The equations can be established by using either the loop frame or bus (nodal) frame of reference. The latter is more amenable to digital computer analysis, so that the nodal iterative approach to the solution of load flow problems has now become firmly established [10]. The advantages attributed to the use of nodal voltage methods are

(a) The number of equations is small.
(b) The system is represented in terms of node numbers and the interconnecting impedances and admittances.
(c) Parallel branches do not contribute to the number of equations.
(d) Crossover branches do not increase the complexity in the formation of the bus admittance matrix.
(e) Formulation on digital computer is comparatively easy.
(f) The bus admittance matrix can easily be modified for subsequent changes in the network.

The solution of the algebraic equations describing the power system are based on iterative methods because of their nonlinearity. The solution must satisfy Kirchhoff's current and voltage laws. The check on the convergence of the iterative procedure is based on the use of one of these methods. The load-flow solution must also include the regulating capability of generators, condensers, tap changing transformers as well as the net interchange of power between the neighbouring systems.

3.3 Power System Equations

The equations describing the performance of a power system network using the bus frame of reference can be expressed as

$$\bar{V}_{bus} = Z_{bus} \cdot \bar{I}_{bus} \quad \text{in impedance form} \tag{3.1}$$

or

$$\bar{I}_{bus} = Y_{bus} \cdot \bar{V}_{bus} \quad \text{in admittance form} \tag{3.2}$$

The net real and reactive power at the k^{th} bus is given by

$$P_k + j\, Q_k = (P_{gk} - P_{lk}) + j\, (Q_{gk} - Q_{lk}) \tag{3.3}$$

in which

P_{gk} is the real power generated at the k^{th} bus

Q_{gk} is the reactive power generated at the k^{th} bus

P_{lk} is the real power consumed (load) at the k^{th} bus

Q_{lk} is the reactive power consumed at the k^{th} bus

Equation (3.3) can be rewritten as

$$S_k = P_k + j\, Q_k = S_{gk} - S_{lk} \tag{3.4}$$

where

S_k is the complex power available at the k^{th} bus

and

S_{gk} and S_{lk} are the complex generated and load power at bus k.

Let the injected current at bus k be I_k and the voltage at the bus be V_k as shown in Fig (3.1). The complex power injected into the power system network, from bus k can be expressed in the general form as

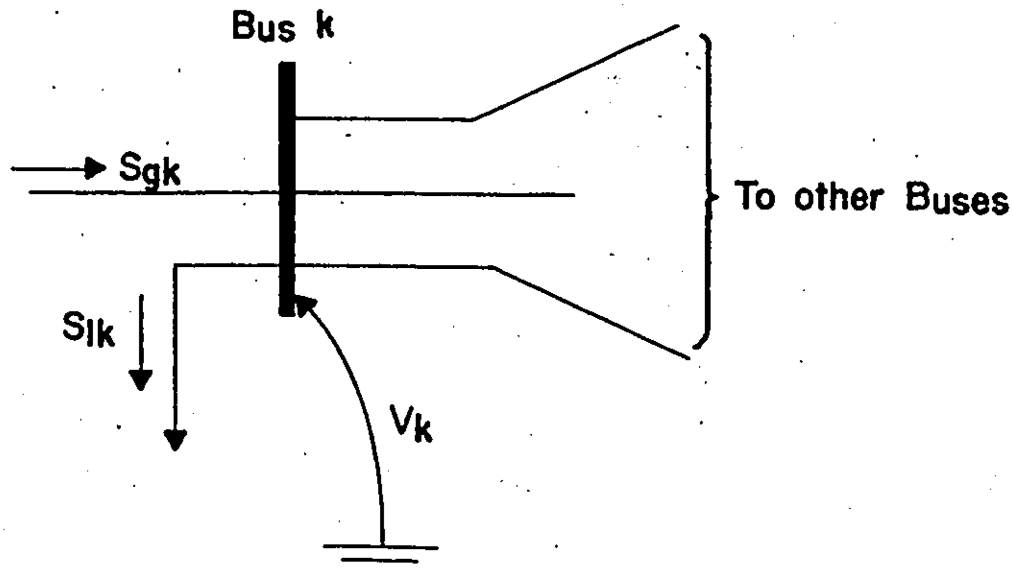


Fig 3.1 Complex generated and load powers at bus k.

$$S_k = V_k \cdot I_k^* \tag{3.5}$$

From Eq. (3.5) the injected current I_k can be expressed as

$$I_k = \frac{S_k^*}{V_k^*} \tag{3.6}$$

where ' * ' denotes conjugation.

From the basic performance equations

$$[I] = [Y][V] \tag{3.7}$$

Equation (3.7) can be expanded for a n bus system as

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2k} & \cdots & Y_{2n} \\ \vdots & & & & & \\ Y_{k1} & Y_{k2} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & & & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_n \end{bmatrix} \quad (3.8)$$

From Equation (3.8) one can write the following for the k^{th} bus.

$$I_k = Y_{k1} V_1 + Y_{k2} \cdot V_2 \cdots Y_{kk} \cdot V_k + \cdots + Y_{kn} V_n \quad (3.9)$$

From Equation (3.6) the current I_k can also be expressed as,

$$I_k = \frac{S^*_k}{V^*_k} = \frac{P_k - j\,Q_k}{V^*_k} \quad (3.10)$$

Equations (3.9) and (3.10) can be combined to give the following results.

$$\frac{P_k - jQ_k}{V^*_k} = Y_{kk} V_k + \sum_{\substack{i=1 \\ i \neq k}}^{n} Y_{ki} V_i \quad (3.11)$$

Equation (3.11) can be rewritten as

$$V_k = \frac{1}{Y_{kk}} \quad \frac{P_k - jQ_k}{V^*_k} - \sum_{\substack{i=1 \\ i \neq k}}^{n} Y_{ki} V_i \quad (3.12)$$

which is the basic equation for the iterative procedure.

Equation (3.11) is the basic equation used in the iterative solution of load flow by Gauss-Seidel's method using the bus admittance matrix. Since the voltage at the slack bus remains constant throughout the iterative procedure and generally the slack bus is assumed to be number 1, Eq. (3.12) can be modified to

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{i=2 \\ i \neq k}}^{n} Y_{ki} \cdot V_i - Y_{k1} V_1 \right] \qquad (3.13)$$

3.4 Calculation of Reactive Power

As discussed in section 3.2, the quantities known for a voltage controlled bus are $|V|$, the voltage magnitude and P the real power. The reactive power Q at the bus is unknown. To proceed with the load flow analysis the calculated value of Q is inserted in the calculation. The value of reactive power Q_k at bus k can be computed by using Equation (3.11) as follows

$$\frac{S_k^*}{V_k^*} = \sum_{i=1}^{node} Y_{ki} \cdot V_i$$

or

$$S_k^* = V_k^* \cdot \sum_{i=1}^{node} Y_{ki} \cdot V_i \qquad (3.14)$$

or

$$P_k - jQ_k = V_k^* \sum_{i=1}^{node} Y_{ki} \cdot V_i \qquad (3.15)$$

The reactive power is the imaginary part of the complex power S_k. Thus from Equation (3.15) one obtains

$$Q_k = -I_m \left[V_k^* \sum_{i=1}^{node} Y_{ki} \cdot V_i \right] \qquad (3.16)$$

where $k \in \{PV - buses\}$

3.5 Line Flow Equations

There is a flow of power from the generator buses to the load buses through the transmission lines. Power is also transmitted to buses having insufficient power generation from the buses with extra available power. The amount of power to be transmitted through the transmission line connecting bus p with bus q as shown in Fig 3.2 can be computed as follows

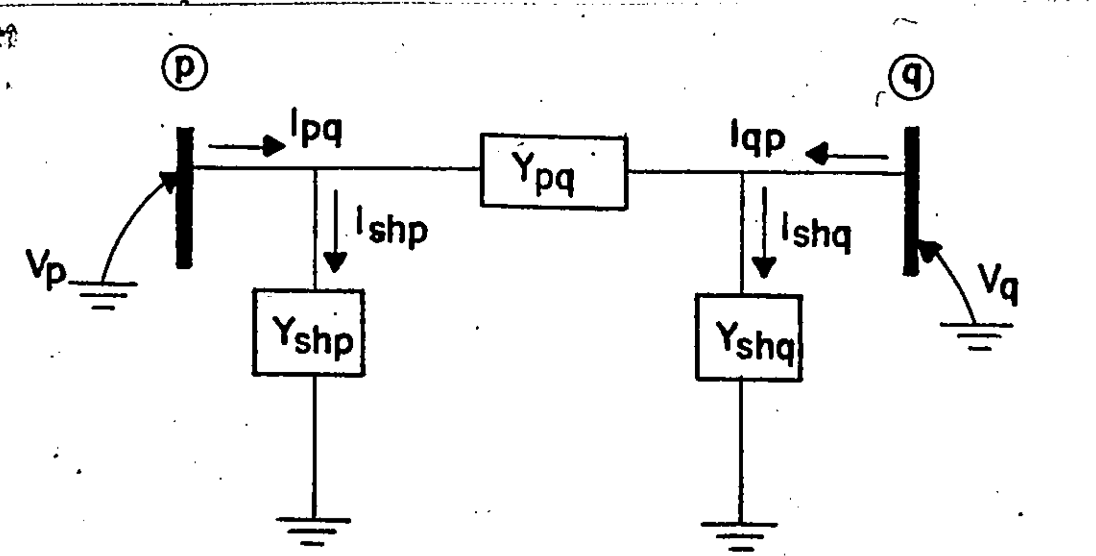


Fig 3.2 Power flow between buses p and q .

The current injected into transmission line pq from bus p can be expressed as

$$I_{pq} = (V_p - V_q)\, y_{pq} + V_p \cdot y_{sh\,p} \qquad (3.17)$$

Similarly the current injected from bus q into line pq can be expressed as

$$I_{qp} = (V_q - V_p) \cdot y_{qp} + V_q \cdot y_{sh\,q} \qquad (3.18)$$

where

y_{pq} = is the admittance of line pq

y_{qp} = is the admittance of line qp

$y_{sh\,p}$ = is the shunt admittance placed at node p

$y_{sh\,q}$ = is the shunt admittance placed at node q .

The line power S_{pq} measured at bus p equals.

$$S_{pq} = V_p \cdot I^*_{pq}$$

$$= V_p \left[(V_p - V_q) \cdot y_{pq} + V_p \cdot y_{sh\,p} \right]^* \qquad (3.19)$$

The line power S_{qp} measured at bus q equals

$$S_{qp} = V_q \cdot I^*_{qp}$$

$$= V_q \left[(V_q - V_p) \cdot y_{qp} + V_q \cdot y_{sh\,q} \right]^* \qquad (3.20)$$

3.6 Power Mismatch

Figure 3.3 shows a typical bus k having both a generator and a load connected to it. This bus is also connected to buses m, n and p of a large power system network.

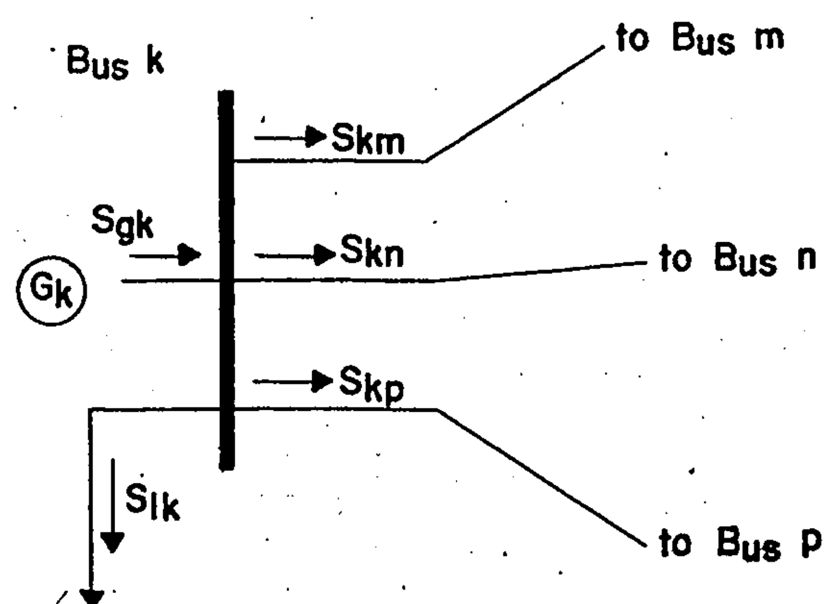


Fig 3.3. A typical bus.

The power injected at bus k is given by

$$S_k = S_{gk} - S_{1k} = P_k + jQ_k$$

There are three lines connected to bus k, their line flows are calculated by the iterative procedure and given by

line km : $S_{km} = P_{km} + jQ_{km}$

line kn : $S_{kn} = P_{kn} + jQ_{kn}$

line kp : $S_{kp} = P_{kp} + jQ_{kp}$

The Power mismatch for a bus can be defined as the algebraic sum of powers injected into and out of that bus.

Thus,

Real power mismatch :

$$\Delta P_k = P_k - (P_{km} + P_{kn} + P_{kp}) \quad (3.21)$$

Reactive power mismatch:

$$\Delta Q_k = Q_k - (Q_{km} + Q_{kn} + Q_{kp}) \quad (3.22)$$

In general form the complex power mismatch at bus k can be expressed as

$$\Delta S_k = S_k - \sum_{\substack{i=1 \\ i \neq k}}^{n} S_{ki} \quad (3.23)$$

where n is the number of nodes.

3.7 Line Losses and Total Losses

The power loss for a transmission line connected between buses p and q, as shown in Fig 3.2, can be expressed as

$$S_{pq}^{Loss} = S_{pq} + S_{qp} = (P_{pq} + P_{qp}) + j(Q_{pq} + Q_{qp}) \quad (3.24)$$

The total losses are then given by

$$S_T^{Loss} = \sum_p \sum_q (S_{pq} + S_{qp}) \qquad (p,q) \in \text{buses} \qquad (3.25)$$

The losses can also be calculated by taking the algebraic sum of the complex bus powers at all the buses of the network. Mathematically this can be expressed as

$$S_T^{Loss} = \sum_{i=1}^{node} S_i \qquad (3.26)$$

This method can compute the total losses, but is unable to calculate loss in a particular line. The losses calculated by the two methods will not be exactly equal, the difference between the two numbers should correspond to the total power mismatch.

3.8 Slack Bus Power

The slack bus power can be computed by the use of Equation (3.5) as follows

$$S_1 = V_1 \cdot I_1^*$$

For the n bus system Equation (3.5) can be transformed to give

$$S_1 = V_1 \{Y_{11} V_1 + Y_{12} \cdot V_2 + Y_{13} V_3 + \ldots Y_{1n} V_n\}^* \qquad (3.27)$$

CHAPTER IV

THREE PHASE LOAD FLOW

4.1 Introduction :

For most purposes in the steady state analysis of power systems, the unbalance in a power system network can be ignored and a single phase analysis is adequate [14]. The increase in extra high voltage lines in power system networks, particularly in areas where large hydro and nuclear power stations supply remote and large load centres through untransposed lines or lines with very few transpositions, the unbalance effect becomes quite significant [15]. Some other factors such as unbalanced loads, unbalanced generation, loss of single phase reactors, unequal phase reactances among single phase transformers of a three phase bank and other factors also contribute towards the unbalance of the system [16]. Large single phase loads like induction furnaces and traction motors or single pole switching require a detailed study of their effect on the power system unbalance before and after circuit breaker operation [14]. As far as the power system network is concerned the unbalance may not be especially significant but in terms of individual components of the network, the unbalance may have adverse effects.

Unbalanced currents can cause serious problems. The presence of negative sequence current at the generator terminals may cause overheating in the rotor. Ground currents (zero sequence) can cause malfunction of protective relays. Current flow unbalance affects power system losses, contributes to ground currents and electromagnetic radiation.

To minimize unbalance in the power system network due to transmission lines the following factors should be thoroughly investigated:

number of conductors per phase, spacing between phase conductors, number of ground wires in the circuit and their location, etc. . The transposition of transmission line at regular intervals can check unbalance in power system networks, but it is not acceptable on reliability and economic grounds [1] .

To check the general performance of the system the usual load flow analysis is performed; which is based on balanced three phase networks operated under balanced three phase generation and load conditions. Single phase load flow analysis , therefore, is taken as adequate and unbalance effects in the power system are ignored, whereas the three phase load flow explicitly simulates all three phases of the system thereby revealing any power flow unbalance. Three phase load flow analysis also allows detailed study of individual components of the power system.

The method of solution of the three phase load flow is very similar to that of the single phase load flow. The main difference is that each single phase voltage, current and power becomes a three element vector and each single phase admittance is replaced with a three by three admittance matrix. The generators, transformers, reactors and other components are represented in more detail.

The analysis can be done either in terms of sequence components or phase components. The analysis in terms of sequence or symmetrical components presents problems in the modelling of generators and transformers, whereas the analysis in terms of phase components is advantageous for unbalanced network elements which are not simplified by the symmetrical component transformation. The phase frame of reference also avoids the complications of working with relatively smaller negative and zero sequence voltage magnitudes which would create convergence difficulties in the iterative solution.

The symmetrical component representation is most suited for balanced elements of the network because the sequences are then decoupled and the sparsity of the admittance matrix is increased.

4.2 Load Flow Equations

The three phase power system may be represented in the bus frame of reference as

$$\bar{I}^{3\phi} = Y^{3\phi} \cdot \bar{V}^{3\phi} \qquad (4.1)$$

or

$$\bar{V}^{3\phi} = Z^{3\phi} \cdot \bar{I}^{3\phi} \qquad (4.2)$$

For a n bus system Eq. (4.1) can be expanded and expressed as shown on the next page

The net power at k^{th} bus can be obtained for the three phases by using Equation (3.3) in the form

$$\begin{bmatrix} S_k^a \\ S_k^b \\ S_k^c \end{bmatrix} = \begin{bmatrix} P_k^a + j Q_k^a \\ P_k^b + j Q_k^b \\ P_k^c + j Q_k^c \end{bmatrix} \qquad (4.4)$$

The injected current $\bar{I}_k$ can be obtained by using Equation (3.6) and represented as

$$\begin{bmatrix} I_k^a \\ I_k^b \\ I_k^c \end{bmatrix} = \begin{bmatrix} S_k^{a*} / V_k^{a*} \\ S_k^{b*} / V_k^{b*} \\ S_k^{c*} / V_k^{c*} \end{bmatrix} \qquad (4.5)$$

Equation (3.12) can be transformed to represent the three phase system and can be expressed for the k^{th} bus as

$$\begin{bmatrix} I_1^a \\ I_1^b \\ I_1^c \\ \vdots \\ I_k^a \\ I_k^b \\ I_k^c \\ \vdots \\ I_n^a \\ I_n^b \\ I_n^c \end{bmatrix} = \begin{bmatrix} Y_{11}^{aa} & Y_{11}^{ab} & Y_{11}^{ac} & & Y_{1k}^{aa} & Y_{1k}^{ab} & Y_{1k}^{ac} & & Y_{1n}^{aa} & Y_{1n}^{ab} & Y_{1n}^{ac} \\ Y_{11}^{ba} & Y_{11}^{bb} & Y_{11}^{bc} & \cdots & Y_{1k}^{ba} & Y_{1k}^{bb} & Y_{1k}^{bc} & \cdots & Y_{1n}^{ba} & Y_{1n}^{bb} & Y_{1n}^{bc} \\ Y_{11}^{ca} & Y_{11}^{cb} & Y_{11}^{cc} & & Y_{1k}^{ca} & Y_{1k}^{cb} & Y_{1k}^{cc} & & Y_{1n}^{ca} & Y_{1n}^{cb} & Y_{1n}^{cc} \\ & \vdots & & \vdots & & \vdots & & & & \vdots & \\ Y_{k1}^{aa} & Y_{k1}^{ab} & Y_{k1}^{ac} & & Y_{kk}^{aa} & Y_{kk}^{ab} & Y_{kk}^{ac} & & Y_{kn}^{aa} & Y_{kn}^{ab} & Y_{kn}^{ac} \\ Y_{k1}^{ba} & Y_{k1}^{bb} & Y_{k1}^{bc} & \cdots & Y_{kk}^{ba} & Y_{kk}^{bb} & Y_{kk}^{bc} & \cdots & Y_{kn}^{ba} & Y_{kn}^{bb} & Y_{kn}^{bc} \\ Y_{k1}^{ca} & Y_{k1}^{cb} & Y_{k1}^{cc} & & Y_{kk}^{ca} & Y_{kk}^{cb} & Y_{kk}^{cc} & & Y_{kn}^{ca} & Y_{kn}^{cb} & Y_{kn}^{cc} \\ & \vdots & & \cdots & & \vdots & & \vdots & & \cdots & \\ Y_{n1}^{aa} & Y_{n1}^{ab} & Y_{n1}^{ac} & & Y_{nk}^{aa} & Y_{nk}^{ab} & Y_{nk}^{ac} & & Y_{nn}^{aa} & Y_{nn}^{ab} & Y_{nn}^{ac} \\ Y_{n1}^{ba} & Y_{n1}^{bb} & Y_{n1}^{bc} & \cdots & Y_{nk}^{ba} & Y_{nk}^{bb} & Y_{nk}^{bc} & \cdots & Y_{nn}^{ba} & Y_{nn}^{bb} & Y_{nn}^{bc} \\ Y_{n1}^{ca} & Y_{n1}^{cb} & Y_{n1}^{cc} & & Y_{nk}^{ca} & Y_{nk}^{cb} & Y_{nk}^{cc} & & Y_{nn}^{ca} & Y_{nn}^{cb} & Y_{nn}^{cc} \end{bmatrix} \begin{bmatrix} V_1^a \\ V_1^b \\ V_1^c \\ \vdots \\ V_k^a \\ V_k^b \\ V_k^c \\ \vdots \\ V_n^a \\ V_n^b \\ V_n^c \end{bmatrix} \quad (4.3)$$

$$
\begin{bmatrix} V_k^a \\ V_k^b \\ V_k^c \end{bmatrix} = \begin{bmatrix} Y_{kk}^{aa} & Y_{kk}^{ab} & Y_{kk}^{ac} \\ Y_{kk}^{ba} & Y_{kk}^{bb} & Y_{kk}^{bc} \\ Y_{kk}^{ca} & Y_{kk}^{cb} & Y_{kk}^{cc} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} I_k^a \\ I_k^b \\ I_k^c \end{bmatrix} - \sum_{\substack{i=1 \\ i \neq k}}^{node} \begin{bmatrix} Y_{ki}^{aa} & Y_{ki}^{ab} & Y_{ki}^{ac} \\ Y_{ki}^{ba} & Y_{ki}^{bb} & Y_{ki}^{bc} \\ Y_{ki}^{ca} & Y_{ki}^{cb} & Y_{ki}^{cc} \end{bmatrix} \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix} \right\} \tag{4.6}
$$

Equation (4.6) forms the basis for the iterative procedure.

4.3 Reactive Power Calculations

To maintain the voltage magnitude constant at a PV bus, extra reactive power has to be injected into the bus. The reactive power can be injected from a three phase bank of shunt capacitors or synchronous condensers. To calculate the amount of reactive power that must be injected to the k^{th} bus, the current injected at bus k is obtained as

$$
\begin{bmatrix} J_k^a \\ J_k^b \\ J_k^c \end{bmatrix} = \sum_{i=1}^{node} \begin{bmatrix} Y_{ki}^{aa} & Y_{ki}^{ab} & Y_{ki}^{ac} \\ Y_{ki}^{ba} & Y_{ki}^{bb} & Y_{ki}^{bc} \\ Y_{ki}^{ca} & Y_{ki}^{cb} & Y_{ki}^{cc} \end{bmatrix} \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix}
$$

from which the reactive power is obtained as

$$\begin{bmatrix} Q_k^a \\ Q_k^b \\ Q_k^c \end{bmatrix} = - \text{Im} \begin{bmatrix} V_k^{a*} & x & J_k^a \\ V_k^{b*} & x & J_k^b \\ V_k^{c*} & x & J_k^c \end{bmatrix} \qquad (4.7)$$

4.4 Slack Bus Power

In the three phase analysis, the slack bus is generally numbered as bus 1 and the slack bus power can be computed by the use of following equations.

$$\begin{bmatrix} S_1^a \\ S_1^b \\ S_1^c \end{bmatrix} = \begin{bmatrix} V_1^a & 0 & 0 \\ 0 & V_1^b & 0 \\ 0 & 0 & V_1^c \end{bmatrix} \sum_{i=1}^{Node} \begin{bmatrix} Y_{1i}^{aa} & Y_{1i}^{ab} & Y_{1i}^{ac} \\ Y_{1i}^{ba} & Y_{1i}^{bb} & Y_{1i}^{bc} \\ Y_{1i}^{ca} & Y_{1i}^{cb} & Y_{1i}^{cc} \end{bmatrix} \begin{bmatrix} V_1^a \\ V_1^b \\ V_1^c \end{bmatrix} \qquad (4.8)$$

4.5 Line Flow Equations

The power flow in the transmission lines can be computed by the use of the equations given in section 3.5. From the bus voltages which are available after the iterative procedure and the line admittances, the power flow from bus i to bus j is given by

$$S_{ij}^{3\phi} = \overline{V}_i^{3\phi} \cdot \overline{I}_{ij}^{3\phi *} \qquad (4.9)$$

where

$I^{3\phi}$ is the three phase current vector.

The current vector can be computed by using Equation (3.17) as

$$I_{ij}^{3\phi} = \begin{bmatrix} I_{ij}^{a} \\ I_{ij}^{b} \\ I_{ij}^{c} \end{bmatrix} = \begin{bmatrix} y_{ij}^{aa} & y_{ij}^{ab} & y_{ij}^{ac} \\ y_{ij}^{ba} & y_{ij}^{bb} & y_{ij}^{bc} \\ y_{ij}^{ca} & y_{ij}^{cb} & y_{ij}^{cc} \end{bmatrix} \begin{bmatrix} V_i^a - V_j^a \\ V_i^b - V_j^b \\ V_i^c - V_j^c \end{bmatrix} +$$

$$\begin{bmatrix} y_{sh\,i}^{aa} & y_{sh\,i}^{ab} & y_{sh\,i}^{ac} \\ y_{sh\,i}^{ba} & y_{sh\,i}^{bb} & y_{sh\,i}^{bc} \\ y_{sh\,i}^{ca} & y_{sh\,i}^{cb} & y_{sh\,i}^{cc} \end{bmatrix} \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix} \qquad (4.10)$$

where

$y_{sh\,i}$ in Eq. (4.10) is half of the total shunt admittance of line ij and placed at node i.

Thus

$$\begin{bmatrix} S_{ij}^{a} \\ S_{ij}^{b} \\ S_{ij}^{c} \end{bmatrix} = \begin{bmatrix} V_i^a & \cdot & I_{ij}^{a*} \\ V_i^b & \cdot & I_{ij}^{b*} \\ V_i^c & \cdot & I_{ij}^{c*} \end{bmatrix} \qquad (4.11)$$

4.6 Power Mismatch

The power mismatch for the three phase power system can be computed in a similar way to the one expressed for a single phase case in Section 3.6. The power mismatch for the three phase case can be computed by the use of equation 3.23 for the k^{th} bus as follows.

$$\begin{bmatrix} \Delta S_k^{\,a} \\ \Delta S_k^{\,b} \\ \Delta S_k^{\,c} \end{bmatrix} = \begin{bmatrix} S_k^{\,a} \\ S_k^{\,b} \\ S_k^{\,c} \end{bmatrix} - \sum_{\substack{i=1 \\ i \neq k}}^{node} \begin{bmatrix} S_{ki}^{a} \\ S_{ki}^{\,b} \\ S_{ki}^{\,c} \end{bmatrix}$$

where

S_{ki} = gives the power flow from line k to line i.

The real and reactive power mismatch can be computed from Equation (4.12) by taking the real and reactive parts respectively of the complex power mismatch.

4.7 Line Losses and Total Losses

The power losses for a three phase power system network can be computed from Equations (3.24) to (3.26). The power loss in a transmission line connected between buses p and q can be expressed as

$$\begin{bmatrix} S_{pq}^{Loss^a} \\ S_{pq}^{Loss^b} \\ S_{pq}^{Loss^c} \end{bmatrix} = \begin{bmatrix} S_{pq}^{\,a} \\ S_{pq}^{\,b} \\ S_{pq}^{\,c} \end{bmatrix} + \begin{bmatrix} S_{qp}^{\,a} \\ S_{qp}^{\,b} \\ S_{qp}^{\,c} \end{bmatrix} \qquad (4.13)$$

The total power loss in the system can be computed from Equation (3.25) as follows.

$$\begin{bmatrix} S_T^{Loss^a} \\ S_T^{Loss^b} \\ S_T^{Loss^c} \end{bmatrix} = \sum_p \sum_q \begin{bmatrix} S_{pq}^a \\ S_{pq}^b \\ S_{pq}^c \end{bmatrix} + \begin{bmatrix} S_{qp}^a \\ S_{qp}^b \\ S_{qp}^c \end{bmatrix} \qquad (4.14)$$

The total power loss can also be computed from Equation (3.26) as follows.

$$\begin{bmatrix} S_T^{Loss^a} \\ S_T^{Loss^b} \\ S_T^{Loss^c} \end{bmatrix} = \sum_{i=1}^{node} \begin{bmatrix} S_i^a \\ S_i^b \\ S_i^c \end{bmatrix} \qquad (4.15)$$

4.8 Power System Element Modelling

A three phase power system consists mainly of the interconnection of generators, transformers, transmission lines, shunt capacitors and electrical loads [17]. The representation of the individual elements of a power system network is comparatively simple, but proper modelling is required for the elements of a large scale system. In the past, symmetrical components have been widely used to simplify problems arising in the analysis of power systems. As discussed in section 4.1 the analysis in terms of symmetrical components is not always proper and transformation to phase components is required during the analysis. This means more computer time. In certain studies involving phase unbalances such as unbalanced loads, unbalanced or faulted series capacitors, open phases, unbalanced bank of single phase transformers, these can be solved more directly in terms of phase quantities as opposed to symmetrical components. In the representation of autotransformers, however, the analysis with symmetrical components is comparatively simple [18] .

4.8.1 <u>Three Phase Transmission Lines</u> Transmission lines are the most common and important elements of a power system network. They are the most effective mode of interconnecting different elements of a power system. For the steady state analysis of the entire power system, transmission lines are best represented by a π equivalent, in which the total shunt admittance y_{sh} is divided into two equal parts (for lines less than 150 Km in length) placed at the sending end and receiving end of the line. The total series impedance Z is placed in the centre of the line as shown in Fig. 4.1.

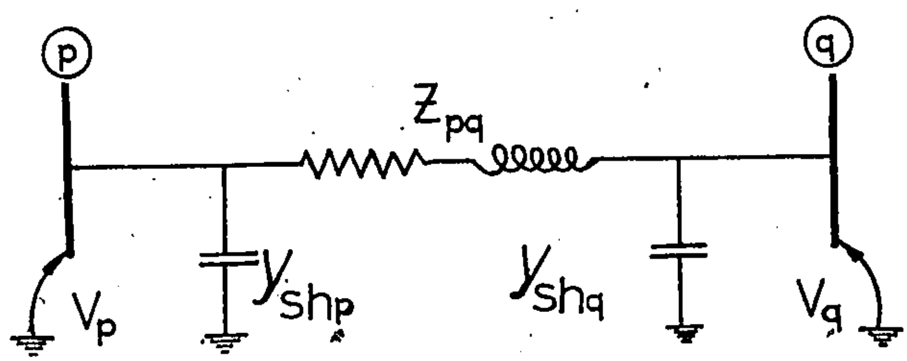


Fig. 4.1 Typical line between buses p and q.

The series parameters, resistance and self and mutual inductances are represented in the primitive matrix Equation as [16]

$$V_{ser} = Z_{ser} \cdot I_{ser} \qquad (4.16)$$

where

V_{ser} consists of the voltage drop across the single phase branches.

I_{ser} is the current in the single phase branch..

and

Z_{ser} is the impedance matrix of self and mutual impedances.

The self and mutual impedances in Z_{ser} can be computed from Equations (2.1) and (2.2). If ground wires also exist along with the phase conductors, then the additional rows and columns, explicitly represented for the ground wires in the impedance matrix, can be eliminated by using Kron's reduction formula [1] provided their entries in V_{ser} are assumed to be zero. If, however, the earthwires of a line are grounded at one end only and insulated for the remainder of their length, the voltage at the insulated end of the earthwire will not be zero. The computation of the admittance matrix for this case has been discussed in detail in Ref. 5.

The elements of the shunt admittance matrix can be obtained as discussed in Section 2.3. The formation of the bus admittance matrix from the primitive impedance and shunt admittance matrices has been discussed in Section 2.4.

4.8.2 Synchronous Machines In the modelling of synchronous machines (generators) for load flow representation the three voltages behind the machine impedance, which are balanced in both their magnitude and angle for a balanced design of the generator windings, are used [15]. Since the excitation in this case is equally distributed to the three windings of the machine the voltages behind the machine impedance will also be balanced.

If the analysis is being done in the symmetrical component frame of reference then the following two assumptions can be made. Firstly; that the machine is a source of positive sequence voltage and secondly that there is no mutual coupling between the sequences [16]. The use of the symmetrical component frame of reference requires the recovery of negative and zero sequence voltage at every iteration for mismatch calculation or representation in phase quantities.

The sequence reactances are normally available from the machine data . The generator impedance matrix is derived from the sequence impedance matrix defined as

$$Z_s = \begin{bmatrix} Z_o & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \qquad (4.17)$$

where

Z_0, Z_1 and Z_2 are the zero, positive and negative sequence impedances of the machine.

The phase impedance matrix can be obtained from Equation (4.17) by applying the transformation matrix relating phase and sequence quantitites [15]. The transformation matrix 'T' can be defined as

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad (4.18)$$

The operator 'a' is defined as

$$a = 1 \angle 120^\circ \;; \qquad a^2 = 1 \angle 240^\circ \;; \qquad a^3 = 1$$

The phase impedance matrix is given by

$$Z_p = T \cdot Z_s' \cdot T^{-1}$$

and performing the necessary matrix computations one gets

$$Z_p = \begin{bmatrix} (Z_0 + Z_1 + Z_2) & (Z_0 + aZ_1 + a^2 Z_2) & (Z_0 + a^2 Z_1 + aZ_2) \\ (Z_0 + a^2 Z_1 + aZ_2) & (Z_0 + Z_1 + Z_2) & (Z_0 + a Z_1 + a^2 Z_2) \\ (Z_0 + a Z_1 + a^2 Z_2) & (Z_0 + a^2 Z_1 + aZ_2) & (Z_0 + Z_1 + Z_2) \end{bmatrix} \quad (4.19)$$

4.8.3 Transformers Transformer equivalent circuits are based only on leakage reactances obtained either from the manufacturer or from short circuit measurements. The magnetising impedance can be obtained from the open circuit measurements. Normally, the three phase transformer is modelled in terms of its symmetrical components under the assumption that the power system is sufficiently balanced. The impedances of the transformer are leakage impedances whereas the magnetising impedances are large shunt connected impedances which can be neglected for most steady state calculations.

For a three phase bank of single phase transformers the zero sequence short circuit impedance equals the positive or negative sequence impedances. But this is not true for three phase shell or core type transformers in which the zero sequence impedance is less than the positive or negative sequence impedance [17].

The analysis of three phase transformers on the basis of single phase transformer theory is not practicable since most of the present transformers in use are shell or core type integrated three phase devices. Fig. 4.2 represents the three-legged core type transformer without any tertiary winding.

The short circuit admittance matrix for this device is given by

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} & Y_{36} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} & Y_{46} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} & Y_{65} & Y_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (4.20)$$

Further simplification of the admittance matrix in Eq. (4.20) can be done by neglecting the flux leakage paths through the steel tank containing the core. Moreover, a perfectly symmetrical flux distribution can be assumed [17]. In Equation (4.20) coils 1, 3 and 5 represent the primary windings and coils 2, 4 and 6 represent the secondary windings. There is an inherent phase shift in the positive and negative sequence transfer impedances in the Y grounded Δ and Y - Δ connections. Positive and negative sequence voltages are shifted in opposite directions, and the degree of phase shift is dependent on the designation of the phases, i.e., the designation of phases on primary and secondary sides of the transformer determines whether the phase shift is 30° or 90° and so on.

4.8.4 <u>Power System Load</u> It is not feasible in a power system to represent each load individually. For this reason, loads considered in a system study are represented as composite system loads.

To minimize losses, power is transmitted at very high voltages, but the utilization of power by customers is at rather reduced voltages.

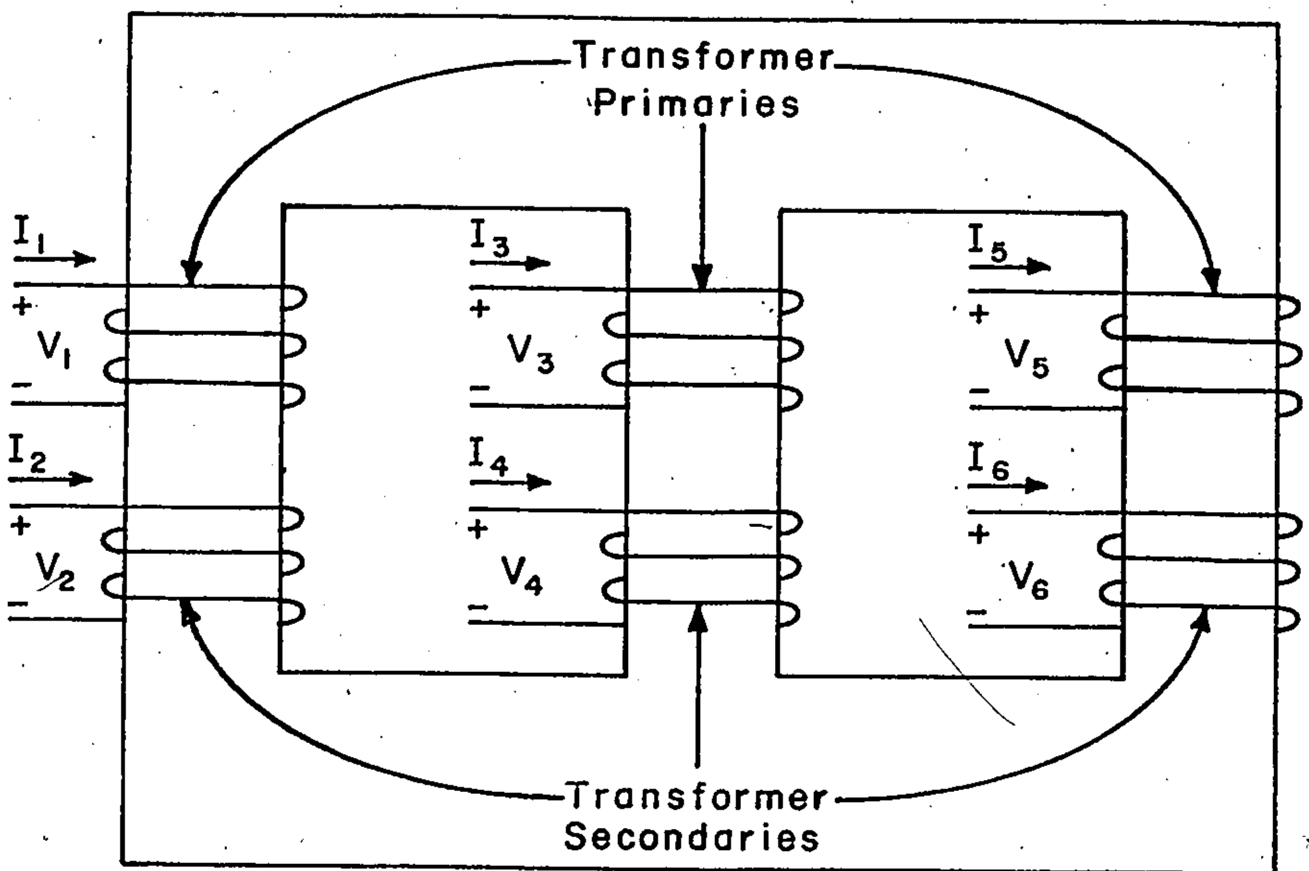


Fig 4.2 Schematic diagram of a core type transformer.

The transformation from high voltage to low voltage is done at periodic substations by the use of step down transformers. These substations are generally located in the middle of the load centres. The substation power demand P, Q can be obtained from the recorded readings of the demand meters located at the substation sites. In a dynamic study, one uses (common format) constant impedance or constant current loads to model the real or reactive power consumed by a composite load in the positive sequence network [17]. For load flow studies the constant current concept is commonly used.

CHAPTER V

LOAD FLOW SOLUTION METHODS

5.1 Introduction

Load flow or power flow is the solution for the static operating conditions of an electric power transmission system. The power flow problem can be solved by several different methods [19]. These methods can be classified either as direct or iterative. Basically, all methods are iterative because power flow problem involves the solution of a system of nonlinear equations. The direct methods employ the direct solution of a related linear system in the iterative algorithm whereas iterative methods use the principle of successive displacement. The direct methods are not subject to ill conditioning and converge in a few iterations. The main limitation of direct methods is that computing time and memory requirements increase rapidly with the increase in problem size thereby limiting their effectiveness to small problems (less than 15 buses).

Some iterative methods converge slowly and they may even diverge. The memory and computer time requirements are directly proportional to problem size. For the analysis of large power systems (greater than 50 buses) only iterative methods are practicable.

The operating conditions (such as generation, load, line outages etc.) of a power system network change continuously. To ensure stable operation of a power system network, a load flow analysis must be done very frequently, depending on the system requirements. The choice of any method for a load flow solution depends on many factors, the most important among these being the convergence characteristics, the storage requirements, reliability and computer time of the algorithm used [15]. The relative properties and performances of different load flow methods can be substantially influenced by the type and size of the problem, the programming and the computing facilities available [20].

The first step in all load flow methods is the coding of the network and the formation of an appropriate network matrix. Information about network connections is obtained by assigning numbers to nodes and to the corresponding terminals of the network elements [8] in the bus frame of reference. The formation of the bus admittance matrix requires information about the characteristic parameters of the transmission line and the node element incidence matrix as discussed in section 2.4. Saving in computer storage can be achieved for the bus admittance matrix by storing only the non-zero elements of the matrix. This can be accomplished by storing non-zero elements along with an index vector to relate bus numbers to the corresponding rows and columns of the matrix. In comparison with the bus impedance matrix which is a full matrix that has no zero elements except in the row and column associated with the reference bus, the bus admittance matrix is very sparse. This is specially so for large networks.

The application of small computers has become quite popular in the control installations in small electric utilities [21]. These organisations demand load flow methods where minimization of core area usage is a critical requirement. The methods employing bus admittance matrix in the analysis are most suitable for computers which have a practical upper limit of 32 K core memory.

5.2 Efficiency of the Method

The characteristics described in this section will judge the efficiency of the method. For the analysis of large systems computer time or core memory will have to be sacrificed depending on the method used.

5.2.1 Speed. The speed is dependent on the method used, problem size, initial guess and kind of problem [19]. In the case of relaxation and Gauss Seidel methods the speed is mainly dependent on the problem size and the accelerating factor used to boost the rate of convergence. Buses having high and low impedance lines terminating on them, large capacitances encountered with cable circuits; long EHV lines, series and shunt compensation, are detrimental to convergence because they weaken the diagonal dominance of the bus admittance matrix [20].

In the zero mismatch methods such as the Newton-Raphson method, the number of iterations required for the solution is virtually independent of problem size and type. The number of iterations are increased when automatic voltage adjustment is required in the network and flat voltage start is not applied. Newton's method is fast due to its quadratic convergence properties in comparison to successive displacement methods which have linear convergence. The method is not much affected by factors causing poor convergence as is experienced in many other methods; such as choice of slack bus and series capacitor placement [20].

The decoupled Newton's methods in which the elements of the Jacobian matrix having weak coupling are neglected, converge as reliably as Newton's method. For very high accuracies it takes more iterations because the method does not exhibit quadratic convergence.

In the impedance matrix alogorithms, each bus voltage is coupled with all the bus currents which gives rapid and reliable convergence. The presence of PV buses in the system substantially affects the rate of convergence, but the method is not very sensitive to the position of the slack bus.

5.2.2 Accuracy. Round-off errors are the only limiting factor in the accuracy of the direct solution of the system of simultaneous equations for power flow analysis. The accuracy to which a solution is required varies with the method that is used.

5.2.3 Computer Requirements. The problem size and method used as well as the required accuracy, determine the size of the computing system. The present existing large systems or planned extra large systems can easily be analyzed on available computers. Special programming techniques such as optimally ordered elimination [19], and diakoptics [22], become compulsory on small computers for the solution of large systems. With optimal ordered elimination the problem memory and time requirements for large systems vary approximately in direct proportion to problem size [19]. In the analysis of three phase load flow problems the word length of the IBM 360/65 computer is not sufficient to solve the problem very efficiently. Fish and Coleman [23] preferred a CDC-6400 computer in comparison to an IBM 360/65 for the solution of three phase power flow problems.

5.2.4 Type of Problem. Iterative methods become almost ineffective in solving problems with negative transfer reactances, the reason being that the diagonal dominance of the bus admittance matrix is lost. The convergence rate of iterative methods is affected adversely for problems in which high and low impedance branches terminate on the same node, systems with series capacitors and heavily loaded systems with insufficient reactive power to stabilize the voltages.

When the power system network is balanced, the analysis of one phase gives sufficient information about the operation of the whole system. An exhaustive review of the different methods being used in the analysis of balanced systems is given by Stott [20].

The following sections review some of the load flow methods which have been proved to be very reliable in the solution to single phase load flow problems.

5.3 Gauss Seidel Method

The Gauss Seidel iterative technique used for solving load flow problems uses a systematic, single sweep successive displacement mode of iteration. The method is popular due to its simplicity and comparatively good performance. There is, moreover, no need to store previous values. A more useful and sufficient condition for the effective performance of the method is that Y_{bus} should possess strict diagonal dominance. For the analysis of balanced power systems by single phase load flow, the self admittances of Y_{bus} are usually large relative to the mutual admittances and convergence is easily obtained. But for the anlysis of unbalanced systems with a three phase load flow method, the self admittances are not as dominant as the mutual admittances which can have an appreciable value. Special techniques have to be used to obtain the solution.

The basic iterative equation [see section 3.3] for the Gauss-Seidel method is

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - j Q_k}{V_k^*} - \sum_{\substack{i=1 \\ i \neq k}}^{node} Y_{ki} \cdot V_i \right]$$

and for the three phase load flow, Equation 4.6 can be rewritten as

$$\begin{bmatrix} V_k^a \\ V_k^b \\ V_k^c \end{bmatrix} = \begin{bmatrix} Y_{kk}^{aa} & Y_{kk}^{ab} & Y_{kk}^{ac} \\ Y_{kk}^{ba} & Y_{kk}^{bb} & Y_{kk}^{bc} \\ Y_{kk}^{ca} & Y_{kk}^{cb} & Y_{kk}^{cc} \end{bmatrix}^{-1} \begin{bmatrix} I_k^a \\ I_k^b \\ I_k^c \end{bmatrix} -$$

$$\sum_{\substack{i=1\\ i\neq k}}^{n} \begin{bmatrix} Y_{ki}^{aa} & Y_{ki}^{ab} & Y_{ki}^{ac} \\ Y_{ki}^{ba} & Y_{ki}^{bb} & Y_{ki}^{bc} \\ Y_{ki}^{ca} & Y_{ki}^{cb} & Y_{ki}^{cc} \end{bmatrix} \begin{bmatrix} V_i^{a} \\ V_i^{b} \\ V_i^{c} \end{bmatrix} \qquad (5.1)$$

In the Gauss-Seidel iterative method, an immediate substitution for each new value of V_k^a, V_k^b and V_k^c (subsequently referred to simply as V_k) is used as it is obtained. Thus, in the solution of V_k at the $(v+1)^{th}$ iteration, the values used would be

$$V_1^{(v+1)},\ V_2^{(v+1)},\ \ldots V_{k-1}^{(v+1)},\quad V_k^{(v)},\ V_{k+1}^{(v)} \ \ldots\ V_n^{(v)}$$

Equation (5.1) can be expressed for the $(v+1)^{th}$ iteration as

$$\begin{bmatrix} V_k^{a(v+1)} \\ V_k^{b(v+1)} \\ V_k^{c(v+1)} \end{bmatrix} = \begin{bmatrix} \\ Y_{kk}^{abc} \\ \\ \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \dfrac{P_k^a - jQ_k^a}{V_k^{a*(v)}} \\ \dfrac{P_k^b - jQ_k^b}{V_k^{b*(v)}} \\ \dfrac{P_k^c - jQ_k^c}{V_k^{c*(v)}} \end{bmatrix} - \sum_{i=1}^{k-1} \begin{bmatrix} \\ Y_{ki}^{abc} \\ \\ \end{bmatrix} \begin{bmatrix} V_i^{a(v+1)} \\ V_i^{b(v+1)} \\ V_i^{c(v+1)} \end{bmatrix} - \sum_{j=k+1}^{node} \begin{bmatrix} \\ Y_{kj}^{abc} \\ \\ \end{bmatrix} \begin{bmatrix} V_j^{a(v)} \\ V_j^{b(v)} \\ V_j^{c(v)} \end{bmatrix} \right\} \qquad (5.2)$$

In the three phase load flow the voltage of the 3 phases of a particular bus is obtained at the same time. The iterative procedure consists in solving Equation (5.2) for each bus using the last previously calculated voltage for it and its adjacent buses. The program sequentially calculates new voltages for all buses in one iteration progressing through the system by the order of bus numbers. The solution is assumed to have converged if

$$|V_k^{(v+1)} - V_k^{(v)}| \leq \epsilon$$

Where ϵ is a small ($\approx 10^{-4}$ to 10^{-5}) predetermined tolerance.

If a generator bus is a voltage controlled bus, for which reactive power is not specified or voltage magnitude must be maintained constant, then the reactive power required in both the cases is calculated from Equation (4.7) [see section 4.3]. The newly calculated value of reactive power is used in the iterative procedure.

5.3.1 <u>Accelerating Factor</u> The rate of convergence of the Gauss-Seidel method can be increased by the use of an accelerating factor α as

$$V_k^{\prime(v+1)} = V_k^{(v)} + \alpha \; (V_k^{(v+1)} - V_k^{(v)}) \qquad (5.3)$$

The value of α lies between 1.0 to less than 2.0. It is very difficult to generalise in selecting the optimum value of accelerating factor, because it depends on the problem size,type of network connections and on the number of elements present.

5.4 <u>Newton - Raphson Method</u>

The generalized Newton-Raphson (N.R) method is an iterative algorithm for solving a set of simultaneous nonlinear equations in an equal number of unknowns $F(X) = 0$. At a given iteration each function $f_i(X)$ is approximated by its tangent hyperplane [20]. This linearized problem is constructed as the Jacobian matrix equation

$$F(X) = -J \cdot \Delta\bar{X} \tag{5.4}$$

From Eq. (5.4) the change $\Delta\bar{X}$ can be computed by the use of a Gaussian elimination subroutine. The square matrix J in Equation (5.4) can be defined as

$$J_{ik} = \partial f_i \,/\, \partial x_k \tag{5.5}$$

and represents the slopes of the tangent hyperplanes and is a highly sparse matrix. Programmed ordered elimination and back substitution make the method attractive for small computers. The method has a quadratic rate of convergence which makes it very attractive for single phase load flow analysis but for the analysis of unbalanced systems, a proper check on convergence has to be made to avoid divergence as the method becomes unstable in the presence of large mutual admittances.

Newton's load flow formulation adopted to date uses the bus power or current mismatch expressions and designate the unknown bus voltages as the problem variables X.

Mathematically, the complex load flow equations are non analytic and can not be differentiated in complex form. In order to apply Newton's method the power flow equations are separated into real and imaginary parts. The unknown bus voltages can either be expressed into rectangular form or polar form.

5.4.1 <u>Rectangular Form:</u> Let the complex power entering node k be given by

$$S_k^{abc} = P_k^{abc} + j\,Q_k^{abc}$$

where

P_k^{abc} = Real power for phase a, b and c at bus k.

Q_k^{abc} = Reactive power for phase a, b and c at bus k.

The complex voltage at bus k can be expressed as

$$\bar{V}_k^{abc} = \bar{e}_k^{abc} + j\,\bar{f}_k^{abc} \qquad (5.5)$$

where

$\bar{e}_k^{abc}$ = is the real part of the complex voltage for bus k

and $\bar{f}_k^{abc}$ = is the imaginary part of the k^{th} bus complex voltage.

Then a general form of the voltage equation for bus k in an n bus system can be written as

$$\bar{P}_k^{abc} - j\,\bar{Q}_k^{abc} = \bar{V}_k^{abc*} \sum_{i=1}^{node} Y_{ik}^{abc}\, V_i^{abc} \qquad (5.6)$$

Rewriting Equation (5.6) in rectangular form as

$$\bar{P}_k^{abc} - j\,\bar{Q}_k^{abc} = (\bar{e}_k^{abc} - j\,\bar{f}_k^{abc}) \sum_{i=1}^{node}$$

$$(G_{ik}^{abc} - j\,B_{ik}^{abc})\ (\bar{e}_i^{abc} + j\,f_i^{abc}) \qquad (5.7)$$

where

G_{ik}^{abc} and B_{ik}^{abc} correspond to the real and imaginary parts, respectively, of the bus admittance matrix.

Expanding Equation (5.7) and separating into real and imaginary parts, one obtains

Real part :

$$\bar{P}_k^{abc} = \sum_{i=1}^{node} [\bar{e}_k^{abc} (G_{ik}^{abc} \cdot \bar{e}_i^{abc} + B_{ik}^{abc} \cdot \bar{f}_i^{abc})$$

$$+ \bar{f}_k^{abc} (G_{ik}^{abc} \cdot \bar{f}_i^{abc} - B_{ik}^{abc} \cdot \bar{e}_i^{abc})] \quad (5.8)$$

Imaginary part :

$$\bar{Q}_k^{abc} = \sum_{i=1}^{node} [\bar{f}_k^{abc} (G_{ik}^{abc} \cdot \bar{e}_i^{abc} + B_{ik}^{abc} \cdot \bar{f}_i^{abc})$$

$$+ \bar{e}_k^{abc} (B_{ik}^{abc} \cdot \bar{e}_i^{abc} - G_{ik}^{abc} \cdot \bar{f}_i^{abc})] \quad (5.9)$$

Thus P_k and Q_k in Equations (5.8) and (5.9) are functions of e_k, e_i, f_k and f_i .

The mismatch calculations or the changes in power is the difference between the specified and calculated values and is given by

$$\begin{bmatrix} \Delta P_k^a \\ \Delta P_k^b \\ \Delta P_k^c \end{bmatrix} = \begin{bmatrix} P_{k(sp)}^a - P_{k(cal)}^a \\ P_{k(sp)}^b - P_{k(cal)}^b \\ P_{k(sp)}^c - P_{k(cal)}^c \end{bmatrix} \quad (5.10)$$

k = 1, 2, ... node

k ≠ slack bus number

and

$$\begin{bmatrix} \Delta Q_k^a \\ \Delta Q_k^b \\ \Delta Q_k^c \end{bmatrix} = \begin{bmatrix} Q_{k(sp)}^a - Q_{k(cal)}^a \\ Q_{k(sp)}^b - Q_{k(cal)}^b \\ Q_{k(sp)}^c - Q_{k(cal)}^c \end{bmatrix} \quad (5.11)$$

k = 1,2, ... node

k ≠ slack bus number.

where

$P_{k(sp)}$ = is the specified real power.

$P_{k(cal)}$ = is the calculated value of real power.

$Q_{k(sp)}$ = is the specified reactive power.

and $Q_{k(cal)}$ = is the calculated value of reactive power

The above discussion applies only to PQ buses.

For a voltage controlled bus (PV bus) 'm' for which voltage magnitude $|V_m|$ and real power P_m is specified, Equation (5.9) representing Q_k is replaced by ΔV_k, and computed as follows

$$\begin{bmatrix} |V^a_{m(sch)}|^2 \\ |V^b_{m(sch)}|^2 \\ |V^c_{m(sch)}|^2 \end{bmatrix} = \begin{bmatrix} (e_m^a)^2 + (f_m^a)^2 \\ (e_m^b)^2 + (f_m^b)^2 \\ (e_m^c)^2 + (f_m^c)^2 \end{bmatrix} \quad (5.12)$$

and

$$\begin{bmatrix} |V^a_{m(o)}|^2 \\ |V^b_{m(o)}|^2 \\ |V^c_{m(o)}|^2 \end{bmatrix} = \begin{bmatrix} (e^a_{m(o)})^2 + (f^a_{m(o)})^2 \\ (e^b_{m(o)})^2 + (f^b_{m(o)})^2 \\ (e^c_{m(o)})^2 + (f^c_{m(o)})^2 \end{bmatrix} \quad (5.13)$$

Then

$$\begin{bmatrix} \Delta|V^a_{m(o)}|^2 \\ \Delta|V^b_{m(o)}|^2 \\ \Delta|V^c_{m(o)}|^2 \end{bmatrix} = \begin{bmatrix} |V^a_{m(sch)}|^2 - |V^a_{m(o)}|^2 \\ |V^b_{m(sch)}|^2 - |V^b_{m(o)}|^2 \\ |V^c_{m(sch)}|^2 - |V^c_{m(o)}|^2 \end{bmatrix} \quad (5.14)$$

Where

$V^{abc}_{m(sch)}$ = is the scheduled voltage at bus m.

P^{abc}_{m} = is the scheduled power at bus m.

$V^{(o)abc}_{m}$ = is calculated with initial estimates $e^{(o)abc}_{m}$ and $f^{(o)abc}_{m}$.

The equations for PV - buses can be expressed, therefore, as

$$\Delta P^{abc}_{m(o)} = P^{abc}_{m(sp)} - P^{abc}_{m(cal)}$$

$$\Delta |V^{abc}_{m(o)}| = |V^{abc}_{m(sch)}|^2 - |V^{abc}_{m(o)}|^2 \qquad (5.15)$$

Changes in real and reactive powers are related to changes in e and f.

Equation (5.4) can now be transformed

$$\begin{bmatrix} \Delta P \\ \cdots\cdots \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \\ J \\ \\ \end{bmatrix} \begin{bmatrix} \Delta e \\ \cdots\cdots \\ \Delta f \end{bmatrix} \qquad (5.16)$$

For system having both PQ and PV-buses Equation (5.16) can be expressed as

$$\begin{bmatrix} \Delta P_k \\ \cdots\cdots \\ \Delta Q_k \\ \Delta |V_n|^2 \end{bmatrix} = \begin{bmatrix} \\ J \\ \\ \end{bmatrix} \begin{bmatrix} \Delta e \\ \cdots\cdots \\ \Delta f \end{bmatrix} \qquad (5.17)$$

Equation (5.16) can be expanded to give the following in partitioned form

$$
\left[\begin{array}{c}
\Delta P_2^a \\ \Delta P_2^b \\ \Delta P_2^c \\ \vdots \\ \Delta P_n^a \\ \Delta P_n^b \\ \Delta P_n^c \\ \hdashline \Delta Q_2^a \\ \Delta Q_2^b \\ \Delta Q_2^c \\ \vdots \\ \Delta Q_n^a \\ \Delta Q_n^b \\ \Delta Q_n^c
\end{array}\right]
=
\left[\begin{array}{ccccccc:ccccccc}
\frac{\partial P_2^a}{\partial e_2^a} & \frac{\partial P_2^a}{\partial e_2^b} & \frac{\partial P_2^a}{\partial e_2^c} & & \frac{\partial P_2^a}{\partial e_n^a} & \frac{\partial P_2^a}{\partial e_n^b} & \frac{\partial P_2^a}{\partial e_n^c} & \frac{\partial P_2^a}{\partial f_2^a} & \frac{\partial P_2^a}{\partial f_2^b} & \frac{\partial P_2^a}{\partial f_2^c} & & \frac{\partial P_2^a}{\partial f_n^a} & \frac{\partial P_2^a}{\partial f_n^b} & \frac{\partial P_2^a}{\partial f_n^c} \\
\frac{\partial P_2^b}{\partial e_2^a} & \frac{\partial P_2^b}{\partial e_2^b} & \frac{\partial P_2^b}{\partial e_2^c} & \cdots & \frac{\partial P_2^b}{\partial e_n^a} & \frac{\partial P_2^b}{\partial e_n^b} & \frac{\partial P_2^b}{\partial e_n^c} & \frac{\partial P_2^b}{\partial f_2^a} & \frac{\partial P_2^b}{\partial f_2^b} & \frac{\partial P_2^b}{\partial f_2^c} & \cdots & \frac{\partial P_2^b}{\partial f_n^a} & \frac{\partial P_2^b}{\partial f_n^b} & \frac{\partial P_2^b}{\partial f_n^c} \\
\frac{\partial P_2^c}{\partial e_2^a} & \frac{\partial P_2^c}{\partial e_2^b} & \frac{\partial P_2^c}{\partial e_2^c} & & \frac{\partial P_2^c}{\partial e_n^a} & \frac{\partial P_2^c}{\partial e_n^b} & \frac{\partial P_2^c}{\partial e_n^c} & \frac{\partial P_2^c}{\partial f_2^a} & \frac{\partial P_2^c}{\partial f_2^b} & \frac{\partial P_2^c}{\partial f_2^c} & & \frac{\partial P_2^c}{\partial f_n^a} & \frac{\partial P_2^c}{\partial f_n^b} & \frac{\partial P_2^c}{\partial f_n^c} \\
& \vdots & & \cdots & & \vdots & & & \vdots & & \cdots & & \vdots & \\
\frac{\partial P_n^a}{\partial e_2^a} & \frac{\partial P_n^a}{\partial e_2^b} & \frac{\partial P_n^a}{\partial e_2^c} & & \frac{\partial P_n^a}{\partial e_n^a} & \frac{\partial P_n^a}{\partial e_n^b} & \frac{\partial P_n^a}{\partial e_n^c} & \frac{\partial P_n^a}{\partial f_2^a} & \frac{\partial P_n^a}{\partial f_2^b} & \frac{\partial P_n^a}{\partial f_2^c} & & \frac{\partial P_n^a}{\partial f_n^a} & \frac{\partial P_n^a}{\partial f_n^b} & \frac{\partial P_n^a}{\partial f_n^c} \\
\frac{\partial P_n^b}{\partial e_2^b} & \frac{\partial P_n^b}{\partial e_2^b} & \frac{\partial P_n^b}{\partial e_2^c} & \cdots & \frac{\partial P_n^b}{\partial e_n^a} & \frac{\partial P_n^b}{\partial e_n^b} & \frac{\partial P_n^b}{\partial e_n^c} & \frac{\partial P_n^b}{\partial f_2^a} & \frac{\partial P_n^b}{\partial f_2^b} & \frac{\partial P_n^b}{\partial f_2^c} & \cdots & \frac{\partial P_n^b}{\partial f_n^a} & \frac{\partial P_n^b}{\partial f_n^b} & \frac{\partial P_n^b}{\partial f_n^c} \\
\frac{\partial P_n^c}{\partial e_2^c} & \frac{\partial P_n^c}{\partial e_2^c} & \frac{\partial P_n^c}{\partial e_2^c} & & \frac{\partial P_n^c}{\partial e_n^a} & \frac{\partial P_n^c}{\partial e_n^b} & \frac{\partial P_n^c}{\partial e_n^c} & \frac{\partial P_n^c}{\partial f_2^a} & \frac{\partial P_n^c}{\partial f_2^b} & \frac{\partial P_n^c}{\partial f_2^c} & & \frac{\partial P_n^c}{\partial f_n^a} & \frac{\partial P_n^c}{\partial f_n^b} & \frac{\partial P_n^c}{\partial f_n^c} \\
\hdashline
\frac{\partial Q_2^a}{\partial e_2^a} & \frac{\partial Q_2^a}{\partial e_2^b} & \frac{\partial Q_2^a}{\partial e_2^c} & & \frac{\partial Q_2^a}{\partial e_n^a} & \frac{\partial Q_2^a}{\partial e_n^b} & \frac{\partial Q_2^a}{\partial e_n^c} & \frac{\partial Q_2^a}{\partial f_2^a} & \frac{\partial Q_2^a}{\partial f_2^b} & \frac{\partial Q_2^a}{\partial f_2^c} & & \frac{\partial Q_2^a}{\partial f_n^a} & \frac{\partial Q_2^a}{\partial f_n^b} & \frac{\partial Q_2^a}{\partial f_n^c} \\
\frac{\partial Q_2^b}{\partial e_2^a} & \frac{\partial Q_2^b}{\partial e_2^b} & \frac{\partial Q_2^b}{\partial e_2^c} & \cdots & \frac{\partial Q_2^b}{\partial e_n^a} & \frac{\partial Q_2^b}{\partial e_n^b} & \frac{\partial Q_2^b}{\partial e_n^c} & \frac{\partial Q_2^b}{\partial f_2^a} & \frac{\partial Q_2^b}{\partial f_2^b} & \frac{\partial Q_2^b}{\partial f_2^c} & \cdots & \frac{\partial Q_2^b}{\partial f_n^a} & \frac{\partial Q_2^b}{\partial f_n^b} & \frac{\partial Q_2^b}{\partial f_n^c} \\
\frac{\partial Q_2^c}{\partial e_2^a} & \frac{\partial Q_2^c}{\partial e_2^b} & \frac{\partial Q_2^c}{\partial e_2^c} & & \frac{\partial Q_2^c}{\partial e_n^a} & \frac{\partial Q_2^c}{\partial e_n^b} & \frac{\partial Q_2^c}{\partial e_n^c} & \frac{\partial Q_2^c}{\partial f_2^a} & \frac{\partial Q_2^c}{\partial f_2^b} & \frac{\partial Q_2^c}{\partial f_2^c} & & \frac{\partial Q_2^c}{\partial f_n^a} & \frac{\partial Q_2^c}{\partial f_n^b} & \frac{\partial Q_2^c}{\partial f_n^c} \\
& \vdots & & \cdots & & \vdots & & & \vdots & & \cdots & & \vdots & \\
\frac{\partial Q_n^a}{\partial e_2^a} & \frac{\partial Q_n^a}{\partial e_2^b} & \frac{\partial Q_n^a}{\partial e_2^c} & & \frac{\partial Q_n^a}{\partial e_n^a} & \frac{\partial Q_n^a}{\partial e_n^b} & \frac{\partial Q_n^a}{\partial e_n^c} & \frac{\partial Q_n^a}{\partial f_2^a} & \frac{\partial Q_n^a}{\partial f_2^b} & \frac{\partial Q_n^a}{\partial f_2^c} & & \frac{\partial Q_n^a}{\partial f_n^a} & \frac{\partial Q_n^a}{\partial f_n^b} & \frac{\partial Q_n^a}{\partial f_n^c} \\
\frac{\partial Q_n^b}{\partial e_2^a} & \frac{\partial Q_n^b}{\partial e_2^b} & \frac{\partial Q_n^b}{\partial e_2^c} & \cdots & \frac{\partial Q_n^b}{\partial e_n^a} & \frac{\partial Q_n^b}{\partial e_n^b} & \frac{\partial Q_n^b}{\partial e_n^c} & \frac{\partial Q_n^b}{\partial f_2^a} & \frac{\partial Q_n^b}{\partial f_2^b} & \frac{\partial Q_n^b}{\partial f_2^c} & \cdots & \frac{\partial Q_n^b}{\partial f_n^a} & \frac{\partial Q_n^b}{\partial f_n^b} & \frac{\partial Q_n^b}{\partial f_n^c} \\
\frac{\partial Q_n^c}{\partial e_2^a} & \frac{\partial Q_n^c}{\partial e_2^b} & \frac{\partial Q_n^c}{\partial e_2^c} & & \frac{\partial Q_n^c}{\partial e_n^a} & \frac{\partial Q_n^c}{\partial e_n^b} & \frac{\partial Q_n^c}{\partial e_n^c} & \frac{\partial Q_n^c}{\partial f_2^a} & \frac{\partial Q_n^c}{\partial f_2^b} & \frac{\partial Q_n^c}{\partial f_2^c} & & \frac{\partial Q_n^c}{\partial f_n^a} & \frac{\partial Q_n^c}{\partial f_n^b} & \frac{\partial Q_n^c}{\partial f_n^c}
\end{array}\right]
\left[\begin{array}{c}
\Delta e_2^a \\ \Delta e_2^b \\ \Delta e_2^c \\ \vdots \\ \Delta e_n^a \\ \Delta e_n^b \\ \Delta e_n^c \\ \hdashline \Delta f_2^a \\ \Delta f_2^b \\ \Delta f_2^c \\ \vdots \\ \Delta f_n^a \\ \Delta f_n^b \\ \Delta f_n^c
\end{array}\right]
\qquad (5.18)
$$

Equation (5.18) can be rewritten as

$$\begin{bmatrix} \Delta P \\ \cdots\cdots \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & \vdots & J_2 \\ \cdots & \cdots & \cdots \\ J_3 & \vdots & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \cdots\cdots \\ \Delta f \end{bmatrix} \tag{5.19}$$

In Eq. (5.19) the matrices J_1, J_2, J_3 and J_4 are defined according to the partitioned matrix in Equation (5.18).

The Jacobian matrix in Equation (5.17) is seldom inverted in typical load flow studies. The fundamental method which is used for a direct solution is the Gaussian Elimination method, There are a variety of choices of methods available for the solution of simultaneous equations [32], the best suitable for a particular problem can be chosen on the basis of computational efficiency and accuracy.

The voltage correction for the v^{th} iteration is given by

$$\begin{bmatrix} \Delta e^{(v)} \\ \cdots\cdots \\ \Delta f^{(v)} \end{bmatrix} = \begin{bmatrix} J^{(v)} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P^{(v)} \\ \cdots\cdots \\ \Delta Q^{(v)} \end{bmatrix} \tag{5.20}$$

From Equation (5.20), Δe and Δf are obtained.

The new values at the $(v+1)^{th}$ iteration for bus k can be given as

$$\begin{aligned} e_k^{(v+1)} &= e_k^{(v)} + \Delta e_k^{(v)} \\ f_{(k)}^{(v+1)} &= f_k^{(v)} + \Delta f_k^{(v)} \end{aligned} \tag{5.21}$$

The process is repeated until ΔP and ΔQ are within the prescribed tolerance limits.

5.4.2 Polar Form This is the most widely used of all formulations and is comparatively more efficient than the rectangular form [see section 5.4.3]. In this formulation instead of using the real and imaginary parts of the bus voltages the bus voltage magnitude and phase angle are used as variables. The complex voltage for this case can be expressed as

$$V = |V| \angle \delta \tag{5.22}$$

The complex power for bus k can be expressed as

$$S_k^{abc*} = P_k^{abc} - j\,Q_k^{abc} = |V_k|^{abc} \angle -\delta_k^{abc} \sum_{i=1}^{node} (G_{ki} - j\,B_{ki})^{abc} \left(|V|^{abc} \angle \delta_i^{abc} \right) \tag{5.23}$$

Expanding Equation (5.23) and separating real and imaginary parts one obtains

$$P_k^{abc} = |V_k|^{abc} \sum_{i=1}^{node} |V_i|^{abc} \left[(G_{ki})^{abc} \cdot \mathrm{Cos}\,(\delta_k - \delta_i)^{abc} - (B_{ki})^{abc} \cdot \mathrm{Sin}\,(\delta_k - \delta_i)^{abc} \right] \tag{5.24}$$

$$Q_k^{abc} = |V_k|^{abc} \sum_{i=1}^{n} |V_i| \left[G_{ki}^{abc} \cdot \mathrm{Sin}\,(\delta_k - \delta_i)^{abc} + B_{ki}^{abc} \cdot \mathrm{Cos}\,(\delta_k - \delta_i)^{abc} \right] \tag{5.25}$$

For programming purposes if each phase of a bus is considered as an independent node then Equations (5.24) and (5.25) for the k^{th} bus can be expressed as

$$P_k = |V_k| \sum_{i=1}^{node} |V_i| \left[G_{ki} Cos(\delta_k - \delta_i) - B_{ki} \cdot Sin(\delta_k - \delta_i) \right]$$

$$Q_k = |V_k| \sum_{i=1}^{node} |V_i| \left[G_{ki} Sin(\delta_k - \delta_i) + B_{ki} \cdot Cos(\delta_k - \delta_i) \right] \quad (5.26)$$

The power mismatch can then be expressed for PQ buses as

Real power mismatch :

$$\Delta P_k = P_k^{(Sp)} - P_k^{(Cal)} \quad (5.27)$$

Reactive power mismatch :

$$\Delta Q_k = Q_k^{(Sp)} - Q_k^{(Cal)} \quad (5.28)$$

Expanding Eqs.(5.27) and (5.28) by a Taylor series expansion one obtains:

$$\Delta P_k = \frac{\partial P_k}{\partial \delta} \cdot \Delta \delta_k + \sum_{\substack{i=1 \\ i \neq k}}^{node} \frac{\partial P_k}{\partial \delta_i} \cdot \Delta \delta_i + \frac{\partial P_k}{\partial |V_k|} \cdot \Delta |V_k| + \sum_{i=1}^{node} \frac{\partial P_k}{\partial |V_i|} \cdot \Delta |V_i| \quad (5.29)$$

$$\Delta Q_k = \frac{\partial Q_k}{\partial |V_k|} \cdot \Delta \delta_k + \sum_{\substack{i=1 \\ i \neq k}}^{node} \frac{\partial Q_k}{\partial \delta_i} \cdot \Delta \delta_i + \frac{\partial Q_k}{\partial |V_k|} \cdot \Delta |V_k| + \sum_{\substack{i=1 \\ i \neq k}}^{node} \frac{\partial Q_k}{\partial |V_i|} \cdot \Delta |V_i| \quad (5.30)$$

In matrix form one can then write

$$\begin{bmatrix} \Delta P \\ \cdots \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & \vdots & N \\ \cdots & \cdots & \cdots \\ J & \vdots & L \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \cdots \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \quad (5.31)$$

In Eq. (5.21) the square partitioned matrix is the Jacobian matrix where the elements of the Jacobian matrix can be defined as

$$H_{kk} = \frac{\partial P_k}{\partial \delta_k}, \quad H_{ki} = \frac{\partial P_k}{\partial \delta_i}$$

$$N_{kk} = \frac{\partial P_k}{\partial |V_k|} \cdot |V_k| \text{ and } N_{ki} = \frac{\partial P_k}{\partial |V_i|} \cdot |V_i|$$

Similarly other terms of the matrix can be obtained.

If the system being considered has both PQ and PV buses, then the following procedure is followed.

If bus k is a voltage controlled bus, the corresponding row in Eq. (5.31) of the left hand side vector part referring to reactive power is replaced by the following equation

$$\Delta|V_k|^2 = |V_k^{Sp}|^2 - |V_k^{Cal}|^2 \tag{5.32}$$

The reactive power is computed by using Equation (4.7) and checked against the constraints $\{Q_{max}, Q_{min}\}$. Equation (5.31) can then be modified to include both PQ and PV buses resulting in

$$\left[\begin{array}{c} \Delta P \\ \hline \Delta Q \\ \hline \Delta|V| \end{array}\right] = \left[\begin{array}{cc} H & N \\ \hline J_1 & L_1 \\ \hline J_2 & L_2 \end{array}\right] \left[\begin{array}{c} \Delta\delta \\ \hline \Delta|V| \\ \hline |V| \end{array}\right] \tag{5.33}$$

The variables $\Delta\delta$ and $\frac{|\Delta V|}{|V|}$ are obtained from Equation (5.33) by the use of a Gaussian Elimination technique.

The new values at the $(v+1)^{th}$ iteration are then given by

$$|V_k^{(v+1)}| = |V_k^{(v)}| + \frac{\Delta|V_k^{(v)}|}{|V_k^{(v)}|} \cdot |V_k^{(v)}|$$

$$\delta_k^{(v+1)} = \delta_k^{(v)} + \Delta\,\delta_k^{(v)} \quad (5.34)$$

The procedure is repeated until ΔP and ΔQ are less than a prespecified tolerance.

5.4.3 <u>Acceleration of Convergence:</u> It is not possible to increase the rate of convergence of Newton's method by the use of some accelerating factor. The standard technique for accelerating Newton's method is to reuse the Jacobian matrix of one iteration for several successive cycles without recomputing it [19]. The above process can be continued until a solution is obtained or the decrease in the rate of convergence indicates that the Jacobian should be reevaluated. The efficiency of the method also depends upon programming techniques and construction and solution of the Jacobian matrix equation, especially for large system (larger than 20 buses).

5.5 <u>Comparison of Rectangular and Polar Forms of N.R. Method</u>.

It has been shown by Van Ness [25] that the convergence behaviour will be basically the same with both rectangular and polar co-ordinates, provided one is sufficiently close to the solution. But for a system having voltage controlled buses along with load buses, it is better to use polar co-ordinates. If rectangular co-ordinates have been used for the computation of the variables for PV buses, the number of equations are increased, thereby increasing computation time. A difference in the rate of convergence for the different formulations of the method is to be expected because the complex nodal power equation on which the Jacobian is based is not an analytic function [19]. Stott [20] also admits that the rectangular version is slightly less reliable and less rapid in convergence than the polar version. The difference in rate of convergence between the two methods for the three phase load flow analysis is discussed in the next chapter.

5.6 Fast Decoupled Load Flow Method

The inherent characteristic of many practical electric-power systems in the steady state is the strong interdependence between active power and bus voltage angles, and between reactive power and voltage magnitudes [20]. There is a weak coupling between reactive power and bus voltage angles, and between voltage magnitude and active power. The Jacobian matrix terms representing weak coupling have relatively small numerical values, and therefore may be neglected.

In comparison to Newton's methods which require skillful programming, ordered elimination and exploitation of network sparsity, the present method has increased speed, fast convergence and requires less storage. This makes it very attractive for conti studies, where a system is tested for each possible vulnerable state and to seek and suggest relief operations if the system takes any unacceptable state. The method has the decoupled property of giving a very good approximate solution after first one or two iterations and converges very reliably, usually in 2 to 5 iterations for practical accuracy on large systems [20]. Following Stott and Alsac's formulation to rewrite Equation (5.31), and taking the mismatch equation as the starting point for the derivation one gets

$$\begin{bmatrix} \Delta P \\ \cdots \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & \vdots & N \\ \cdots & \cdots & \cdots \\ J & \vdots & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \cdots \\ \Delta V/V \end{bmatrix} \qquad (5.31)$$

Since the Jacobian submatrices [J] and [N] represent the weak P-V and Q-δ couplings the first step in applying the decoupling principle is to neglect the coupling submatrices [N] and [J]. The resulting equation takes the following form

$$[\Delta P] = [H] \; [\Delta \delta]$$

and

$$[\Delta Q] = [L] \; [\Delta V / V] \tag{5.35}$$

In Equation (5.35) the slack bus is not included. The submatrices [H] and [L] can be computed as follows.

Diagonal terms :

$$H_{kk} = |V_k|^2 \; B_{kk} - Q_k$$

$$L_{kk} = |V_k|^2 \; B_{kk} + Q_k \tag{5.36}$$

Off diagonal terms :

$$H_{ki} = |V_k| \, |V_i| \, \{G_{ki} \, Sin(\delta_k - \delta_i) + B_{ki} \, Cos(\delta_k - \delta_i)\} \quad k \neq i \tag{5.37}$$

$$L_{ki} = H_{ki}$$

To further simplify Equations (5.36) and (5.37) the following assumptions are made.

$$Cos\,(\delta_k - \delta_i) \approx 1$$

$$G_{ki} \, Sin\,(\delta_k - \delta_i) \leq B_{ki} \tag{5.38}$$

and

$$Q_k \leq |V_k|^2 \cdot B_{kk}$$

This results in

$$H_{ki} = L_{ki} = |V_k|\ |V_i|\ B_{ki} \qquad k \neq i$$
$$H_{kk} = L_{kk} = |V_k|^2 \cdot B_{kk} \qquad (5.39)$$

The first two assumptions are valid only for a single phase load flow and can not be applied to the analysis of an unbalanced system. With these simplifications one gets for any bus k the following

$$\Delta P_k = H_{kk} \cdot \Delta\delta_k + \sum_{\substack{i=1 \\ i\neq k}}^{node} H_{ki}\ \Delta\delta_i \qquad (5.40)$$

$$\Delta Q_k = L_{kk} \cdot \frac{\Delta|V_k|}{|V_k|} + \sum_{\substack{i=1 \\ i\neq k}}^{node} L_{kj} \cdot \frac{\Delta|V_j|}{|V_j|}$$

Equation (5.40) can also be expressed as

$$\Delta P_k = |V_k|^2\ B_{kk}\ \Delta\delta_k + \sum_{\substack{i=1 \\ i\neq k}}^{node} |V_k|\ |V_i| \cdot B_{ki}\ \Delta\delta_i \qquad (5.41)$$

and

$$\Delta Q_k = |V_k|^2\ B_{kk} \cdot \frac{\Delta|V_k|}{|V_k|} + \sum_{\substack{i=1 \\ i\neq k}}^{n} |V_k|\ |V_i| \cdot B_{ki} \cdot \frac{\Delta|V_i|}{|V_i|}$$

Rearranging variables in Equation (5.41) one gets

$$\frac{\Delta P_k}{|V_k|} = |V_k|\ B_{kk}\ \Delta\delta_k + \sum_{\substack{i=1 \\ i\neq k}}^{node} |V_i|\ B_{ki} \cdot \Delta\delta_i \qquad (5.42)$$

$$\frac{\Delta Q_k}{|V_k|} = |V_k|\ B_{kk} \frac{\Delta|V_k|}{|V_k|} + \sum_{\substack{i=1 \\ i\neq k}}^{node} + |V_i| \cdot B_{ki}\ \frac{\Delta|V_k|}{|V_k|}$$

By assuming the nominal voltage level to be 1.0 p.u for the slack bus, further simplification in Eq. (5.42) can be obtained by letting the voltages in the right hand side be 1.0 p.u. One then obtains the following equations

$$\frac{\Delta P_k}{|V_k|} = B_{kk} \; \Delta \delta_k + \sum_{\substack{i=1 \\ i \neq k}}^{node} B_{ki} \cdot \Delta \delta_i$$

$$\frac{\Delta Q_k}{|V_k|} = B_{kk} \; \frac{\Delta |V_k|}{|V_k|} + \sum_{\substack{i=1 \\ i \neq k}}^{node} B_{ki} \; \frac{\Delta |V_i|}{|V_i|} \qquad (5.42)$$

or in matrix form one gets

$$\left[\frac{\Delta P}{|V|}\right] = [\; B' \;] \; [\Delta \delta]$$

and

$$\left[\frac{\Delta Q}{|V|}\right] = [\; B'' \;] \; \left[\frac{\Delta |V|}{|V|}\right] \qquad (5.43)$$

The matrices B' and B'' are both real, square and are elements of the matrix B, which comprises the imaginary component of Y_{bus} [27].

The above method is discussed in order to make a comparison between the analysis of the balanced systems with three phase load flow and single phase load flow analysis, whichever is applicable.

5.7 Boot-Strap Gauss-Seidel Method.

The boot-strap Gauss-Seidel method [24] is a combination of Gauss-Seidel and Newton Raphson methods. The method is mainly empirical and is a modified form of Gauss Seidel method. The method can be outlined by the following steps:

1) Before starting the iterative cycle, mismatch calculations for each bus are performed for the real and reactive power, using initial voltage estimates and specified powers.

Thus,

$$\Delta P = P^{Sp} - P^{cal}$$
$$\Delta Q = Q^{Sp} - Q^{cal} \qquad (5.44)$$

where

P^{Sp} and Q^{Sp} are the specified real and reactive powers respectively

and P^{cal} and Q^{cal} are the approximately calculated (see Equation 5.26) real and reactive powers.

2) The specified real and reactive powers are now replaced by pseudopowers obtained as follows

$$PP = P^{Sp} - B \cdot \Delta P$$
$$PQ = Q^{Sp} - B \cdot \Delta Q \qquad (5.45)$$

where

B = is a boosting factor used to increase the rate of convergence.

Rewriting Equation (5.45) in terms of complex power one obtains for the complex pseudopower the following

$$PS = (S - BM) \qquad (5.46)$$

where

M = is the complex power mismatch.

The term BM in Equation (5.46) represents a decrease in generation and increase in load.

(3) The Gauss-Seidel iterative method is used to obtain new voltages for the buses where P and Q appear in the iterative equations in which the pseudopowers are used.

(4) Before starting the next iterative cycle, a mismatch calculation is done . New pseudopowers are obtained by using the voltages obtained from step 3 and the pseudopowers obtained from step 2 now become the specified powers.

(5) The procedure outlined in steps 2 to 4 is repeated, until the voltage mismatch between the last calculated and the one previous iteration is within the pre-specified tolerance limits.

(6) Pseudopowers are replaced by the original specified powers and final power mismatch and power flow calculations are done.

The boosting factor used to increase the rate of convergence lies between (0.8 - 0.9) and can be reduced in steps during the iterative cycle according to requirements. The above method was tried for the analysis of unbalanced systems. The power mismatches for all the nodes were very large, and the steep reduction in the boosting factor did not help. The method starts diverging because the latest power mismatches are raised to some power of the previous mismatches.

5.8 Z - Matrix Method

This method is also an application of the Gauss-Seidel method. Instead of using the bus admittance matrix, the bus impedance matrix (Z) is used. The dominating feature of the Z-matrix method is the need to obtain, store and iterate the whole matrix. As each bus voltage is coupled with all bus currents, the convergence of the algorithm is rapid and reliable.

The starting point for the algorithm is the computation of the bus impedance matrix, obtained by a bus building algorithm [see section 2.5] . The formation of the bus impedance matrix with the bus building algorithm is quite tedious in comparison to the computation of the bus admittance matrix. The bus admittance matrix can be obtained with ground as reference, neglecting shunt connections. Then deleting the rows and columns which correspond to the slack bus, one reduces the order of the matrix. The inverse of the new matrix is computed to give the Z matrix, by Gaussian elimination or by some other inversion technique. For the implementation of the algorithm on a computer, the elements of the Z matrix will have to be renumbered.

The basic equation for bus k can be expressed as

$$V_k - V_R = Z_{k1} \cdot I_1 + Z_{k2} I_2 + \ldots + Z_{kk} \cdot I_k + \ldots + Z_{kn} \cdot I_n \tag{5.47}$$

where

V_R = is the reference bus voltage.

But

$$I_k = \frac{S_k^*}{V_k^*} - y_k \cdot V_k = \frac{P_k - j Q_k}{V_k^*} - y_k \cdot V_k \tag{5.48}$$

Thus one obtains

$$V_k = V_R + \sum_{\substack{i=1 \\ i \neq R}}^{node} Z_{ki} \left(\frac{P_i - j Q_i}{V_i^*} - y_i \cdot V_i \right) \tag{5.49}$$

The variables in Equation (5.49) are either vectors or a square matrix of order three. In each application of Equation (5.49) all the three phase voltages are obtained at the same time. For use in three phase load flow, Equation (5.49) can be expressed as follows

$$\begin{bmatrix} V_k^{a} \\ V_k^{b} \\ V_k^{c} \end{bmatrix} = \begin{bmatrix} V_R^{a} \\ V_R^{b} \\ V_R^{c} \end{bmatrix} + \sum_{\substack{i=1 \\ i \neq R}}^{node} \begin{bmatrix} Z_{ki}^{aa} & Z_{ki}^{ab} & Z_{ki}^{ac} \\ Z_{ki}^{ba} & Z_{ki}^{bb} & Z_{ki}^{bc} \\ Z_{ki}^{ca} & Z_{ki}^{cb} & Z_{ki}^{cc} \end{bmatrix} \left\{ \begin{bmatrix} \dfrac{P_i^{a} - jQ_i^{a}}{V_i^{a*}} \\ \dfrac{P_i^{b} - jQ_i^{b}}{V_b^{*}} \\ \dfrac{P_i^{c} - jQ_i^{c}}{V_c^{*}} \end{bmatrix} - \begin{bmatrix} y_i^{aa} & y_i^{ab} & y_i^{ac} \\ y_i^{ba} & y_i^{bb} & y_i^{bc} \\ y_i^{ca} & y_i^{cb} & y_i^{cc} \end{bmatrix} \begin{bmatrix} V_i^{a} \\ V_i^{b} \\ V_i^{c} \end{bmatrix} \right\} \qquad (5.50)$$

Thus in the solution of V_k at $(v+1)^{th}$ iteration, the iterative scheme can be expressed as

$$\begin{bmatrix} V_k^{a(v+1)} \\ V_k^{b(v+1)} \\ V_k^{c(v+1)} \end{bmatrix} = \begin{bmatrix} V_R^{a} \\ V_R^{b} \\ V_R^{c} \end{bmatrix} + \sum_{\substack{i=1 \\ i \neq R}}^{k-1} \begin{bmatrix} Z_{ki}^{abc} \end{bmatrix} \left\{ \begin{bmatrix} P_i^{a} - jQ_i^{a} / V_i^{a(v+1)*} \\ P_i^{b} - jQ_i^{b} / V_i^{b(v+1)*} \\ P_i^{c} - jQ_i^{c} / V_i^{c(v+1)*} \end{bmatrix} - \begin{bmatrix} y_i^{abc} \end{bmatrix} \begin{bmatrix} V_i^{a(v+1)} \\ V_i^{b(v+1)} \\ V_i^{c(v+1)} \end{bmatrix} \right\}$$

$$+ \sum_{\substack{i=k \\ i \neq R}}^{node} \left[\quad Z_{ki}^{abc} \quad \right] \begin{bmatrix} P_i^a - j Q_i^a / V_i^{a(v)*} \\ P_i^b - j Q_i^b / V_i^{b(v)*} \\ P_i^c - j Q_i^c / V_i^{c(v)*} \end{bmatrix}$$

$$- \left[\quad y_i^{abc} \quad \right] \begin{bmatrix} V_i^{a(v)} \\ V_i^{b(v)} \\ V_i^{c(v)} \end{bmatrix} \Bigg\} \qquad (5.51)$$

An immediate substitution for each new value of V_k is used as it is obtained in the iterative algorithm.

When the system has both PQ and PV buses, then for PV buses the reactive power must be obtained . The reactive power can be computed from Equation (5.48) by a rearrangement of variables. One obtains

$$Q_k = - I_m \frac{V_k^*}{Z_{kk}} \left[V_k - V_R - \sum_{\substack{i=1 \\ i \neq k \\ i \neq R}}^{node} Z_{ki} \left(\frac{P_i - J Q_i}{V_i^*} \right) \right] \qquad (5.52)$$

In comparison to Newton's method, the Z Matrix method can prove to be more efficient for cases in which the Jacobian matrix is close to being a singular matrix.

The other methods used for analysis of the unbalanced systems were:

(i) ϵ coupling method [28] .

(ii) Hessian Matrix [29] , [30] .

but these methods did not prove to be suitable for three phase load flow analysis either due to phase shift in the initial voltage estimates or the method itself was not that effective.

Thus finally it can be concluded that the best methods for analysis of unbalanced power system networks with three phase load flow are the Newton Raphson method in polar form and the Z Matrix method. These conclusion are drawn entirely on experience with the test systems given in next chapter.

CHAPTER VI

COMPUTATIONAL RESULTS

6.1 General :

In this chapter the three phase load flow formulation discussed previously is applied to sample problems to show the effectiveness of the algorithms. The described algorithm considers each phase at any bus bar as a separate bus, being linked to all such buses through relationships that correspond to the given network. The voltage unbalance in the individual voltages at the buses exist both in magnitude and phase angle. The choice of method for the load flow solution depends on many factors, the most important being convergence characteristics, storage requirements and computer time. In the analysis the Newton-Raphson method proved to be the most efficient and reliable . It possesses good convergence characteristics as well as great generality and flexibility in comparison to the other methods. The Z Matrix method is efficient for small problems but as the computation of Z_{bus} matrix is comparatively time consuming, this method is only acceptable where storage requirements are not of prime importance.

The starting process for the algorithms may be outlined by the following steps.

(1) Compute Y_{bus} or Z_{bus} as discussed in chapter 2,

(2) The transmission lines in the network are taken in their natural unbalanced mutually coupled form without any approximation .

(3) Each of the three corresponding phases at a bus take the specified voltage magnitude, but an angle that is shifted 0, 120° or 240° respectively depending on the phasing arrangements used.

(4) For the generator buses if the complex power is specified for each phase than that value is used in the analysis, but if total power generated at a bus is specified then a 3-phase starting process could be adopted by assuming power to be equally distributed in the three phases at the generator nodes.

(5) For the load buses it is essential to have power requirement for each phase of a bus separately, so that balanced or unbalanced conditions can be considered according to the behaviour of the system.

(6) If Δ- Y transformers exist in the network then the voltage angle on the high voltage side can be adjusted by 30° or -30° according to the connections being used.

(7) The analysis is done in terms of phase quantities on the basis of convenience and accuracy.

(8) Network elements are represented as lumped complex impedances at rated frequency (e.g. transmission lines, in-phase transformers, series and shunt reactors and capacitors). Transmission lines without negligible charging capacitance are represented by their π equivalent networks [see section 4.8.3].

(9) The choice of slack bus is made among the voltage-controlled buses (P-V buses). At the slack bus the net active power is designated as being unknown.

6.2 Numerical Examples

The three phase Newton Raphson algorithm has been successfully applied to a 3-bus and 5-bus power systems and the effect of unbalance is revealed.

6.3 3-Bus Power System

The system consists of two three-phase generators, two three-phase or six single-phase loads and three,

three-phase transmission lines as shown in Fig. 6.1.

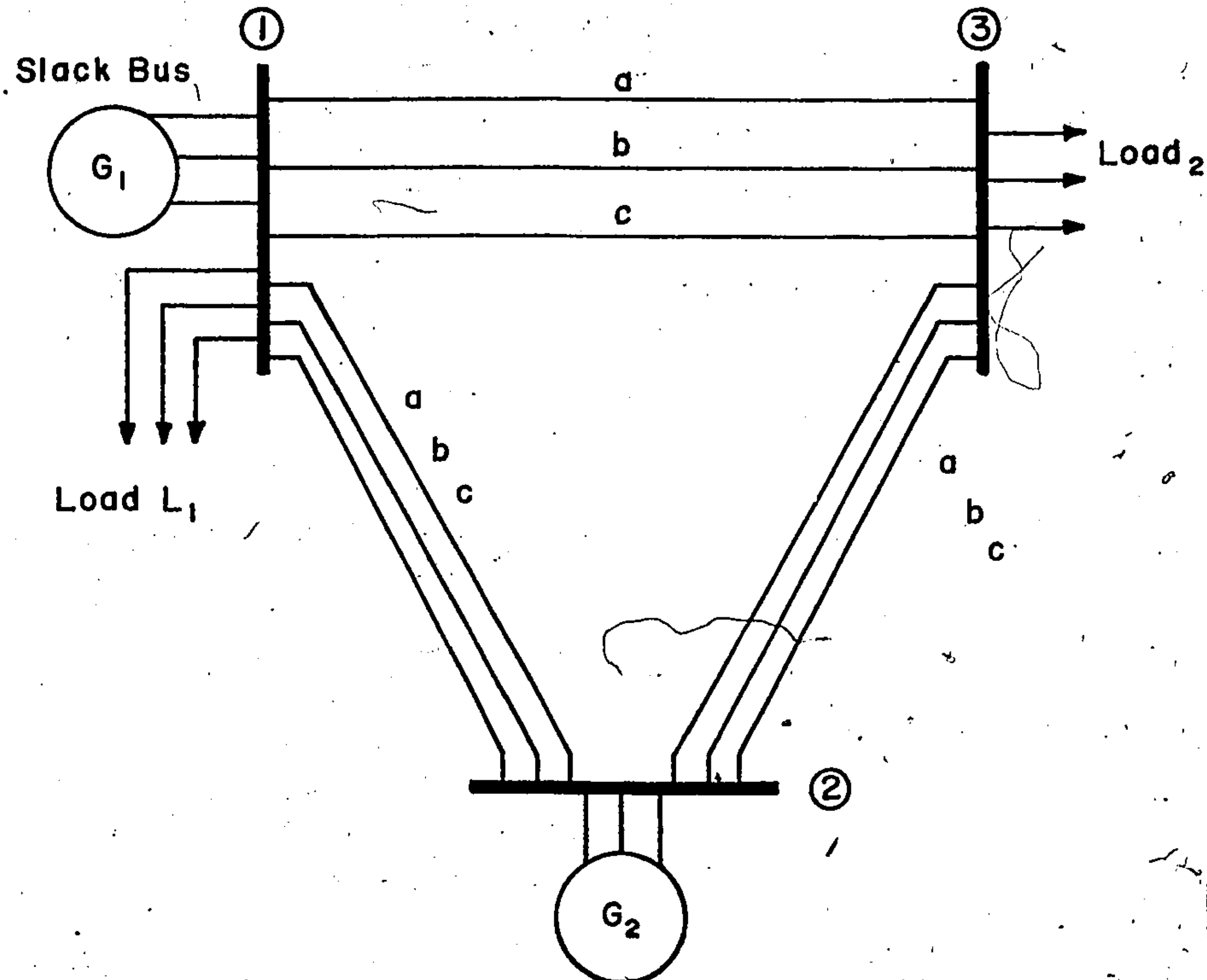


Fig. 6.1 Three bus power system

Each phase of a transmission line consists of 3 conductors per phase with ground wires. Figure 6.2 shows a cross-sectional sketch of the transmission line operating at 500 KV. Spacing dimensions and conductor type remain constant for each transmission line of the system over the complete 115 mile-section. The specifications for the conductors and the ground wires used for the transmission line are as follows:

Conductor: 2156 MCM ACSR 84/19 "BLUE BIRD"

Outer dia = 1.762"

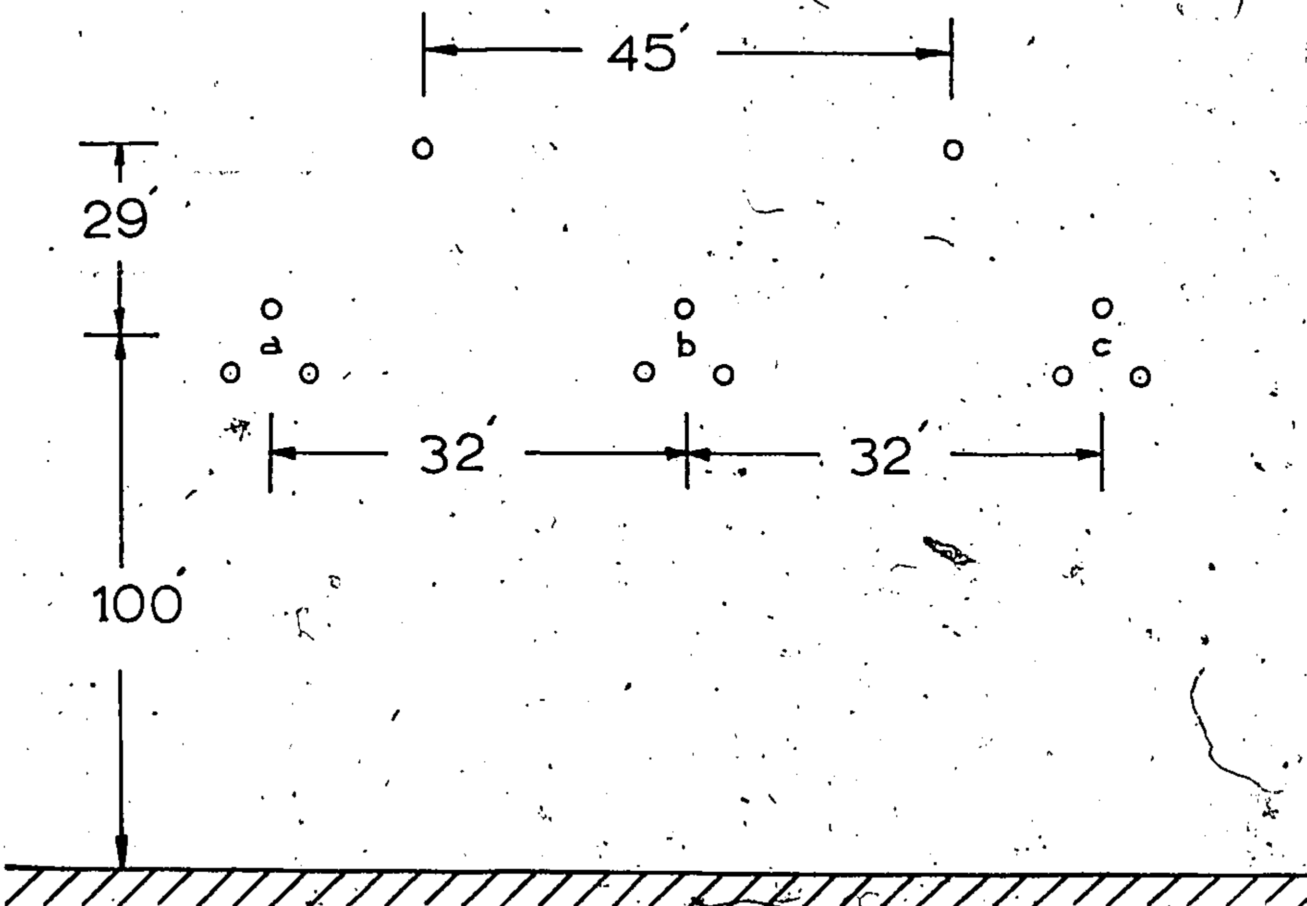


Fig. 6.2 Cross-sectional sketch of a single circuit transmission line.

3 conductors located at the corners of an equilateral triangle with each side equal 18".

Ground wire : 7 No 6 AWG ALUMOWELD

0.486 inch outer dia.

Resistance = 1.536 Ω / mile at 25° C.

The numerical computation of primitive impedance and shunt admittance matrix is discussed in Ref. 31 for different conductor configurations with and without ground wires. In the computation of the bus admittance matrix the mutual between different transmission lines, being very small, is altogether neglected. The system has been analyzed with the following conductor configurations used for the transmission lines. Since the bus admittance matrix is symmetrical (since $y_{ij} = y_{ji}$) only the upper triangular matrix is shown.

(a) Two conductors per phase without ground wires, for each transmission line of the system. The bus admittance matrix for the system is shown in Table 6.1.

(b) Each transmission line of the system with two conductors per phase and with ground wires. The bus admittance matrix for the system is shown in Table 6.2.

(c) Each transmission line of the system with three conductors per phase and without ground wires. The bus admittance matrix for the system is shown in Table 6.3.

(d) Each transmission line of the system with three conductors per phase and with ground wires. The bus admittance matrix for the system is shown in Table 6.4.

(e) Each transmission line of the system with four conductors per phase and without ground wires. The bus admittance matrix for the system is shown in Table 6.5.

(f) Each transmission line of the system with four conductors per phase and with ground wires. The bus admittance matrix for the system is shown in Table 6.6.

(g) Each transmission line of the system is transposed and with three conductors per phase without ground wires. The bus admittance matrix for the system is shown in Table 6.7.

Table 6.1 : Bus Admittance Matrix for the 3 bus System with two conductors per phase without ground wires for the transmission lines

1^a	1^b	1^c	2^a	2^b	2^c	3^a	3^b	3^c
0.9520 − j16.8557	-0.0875 + j6.5865	0.1622 + j4.0815	-0.4760 + j8.4278	0.04376 − j3.2933	-0.08113 − j2.0407	-0.4760 + j8.4278	0.04376 − j3.2933	-0.08112 − j2.0407
	1.0076 − j18.4392	-0.0875 + j6.5865	0.04376 − j3.2933	-0.5038 + j9.2196	0.04376 − j3.2933	0.04376 − j3.2933	-0.5038 + j9.2196	0.04376 − j3.2933
		0.9520 − j16.8557	-0.08113 − j2.0407	0.04376 − j3.2933	-0.0476 + j8.4278	-0.08113 − j2.0407	0.04376 − j3.2932	-0.0476 + j8.4279
			0.9520 - j16.8557	-0.08752 + j6.5865	0.16225 + j4.0815	-0.4760 + j8.4278	0.04376 - j3.2933	-0.08113 - j2.0407
				1.0076 - j18.4392	-0.08752 + j6.5866	0.04376 - j3.2933	-0.50381 + j9.2196	0.0437 - j3.2933
					0.9520 - j16.8557	-0.081126 - j2.0407	0.04376 - j3.2933	-0.4760 + j8.4278
						0.9520 - j16.8557	-0.0875 + j6.5865	0.16225 + j4.0815
							1.00762 - j18.4392	-0.08752 + j6.5865
								0.9520 - j16.8557

Table 6.2: Bus admittance matrix for the 3 bus sustem with two conductors per phase with ground wires for the tranmission lines:

1.53666 − j17.4416	0.44706 + j6.033598	0.63926 + j3.5555	-0.7683 + j8.7208	-0.2235 − j3.01799	-0.3196 − j1.7777	-0.7683 + j8.7208	-0.2235 − j3.01799	-0.31963 − j1.7777
	1.54596 − j18.9844	0.44705 + j6.03599	-0.2235 − j3.01799	-0.7729 + j9.4922	-0.2235 − j3.10799	-0.2235 − j3.01799	-0.7729 + j9.4922	-0.2235 − j3.01799
		1.53666 − j17.4416	-0.31963 − j1.7777	-0.2235 − j3.01799	-0.76833 + j8.7208	-0.31963 − j1.7777	-0.2235 − j3.01799	-0.7683 − j8.7208
			1.53665 − j17.4416	0.4470 + j6.03598	0.63926 + j3.5555	-0.7683 + j8.7208	-0.2235 − j3.01799	-0.3196 − j1.7777
				1.54596 − j18.9844	0.4470 + j6.03599	-0.22353 − j3.01799	-0.7729 + j9.4922	-0.2235 − j3.01799
					1.53666 − j17.4416	-0.31963 − j1.7777	-0.2235 − j3.01799	-0.76833 + j8.7208
						1.53666 − j17.4416	0.44706 + j6.03598	0.6393 + j3.5555
							1.5459 − j18.9844	0.44706 + j6.3599
								1.53666 − j17.4416

Table 6.3: Bus admittance matrix for the 3 bus system with three conductors per phase without ground wires for the transmission lines.

0.87395 − j18.7221	-0.02626 + j7.7330	0.2229 + j4.5854	-0.4369 + j9.3611	0.01313 − j3.8665	-0.11145 − j2.2927	-0.43697 + j9.36106	0.01313 − j3.8665	-0.11145 − j2.2926
	0.90663 − j20.7893	-0.02626 + j7,7330	0.01313 − j3,8665	-0.4533 + j10.3947	0.01313 − j3.8665	0.01313 − j3.8665	-0.4533 + j10.3946	0.01313 − j3.8665
		0.87395 − j18.7219	-0.11145 − j2.2927	0.01313 − j3.8665	-0.4369 + j9.36096	-0.11145 − j2.2927	0.01313 − j3.8665	-0.4369 + j9.3609
			0.8739 − j18.7221	-0.02626 + j7.7330	0.2229 + j4.5853	-0.43697 + j9.3610	0.01313 − j3.8665	-0.11145 − j2.2926
				0.90663 − j20.7893	-0.02626 + j7.7330	0.01313 − j3.8665	-0.4533 + j10.3947	0.01313 − j3.8665
					0.87395 − j18.7220	-0.11145 − j2.2927	0.013129 − j3.8665	-0.4369 + j9.3609
						0.87395 − j18.7221	-0.02626 + j7.7330	0.2229 + j0.4585
							0.906635 − j20.7893	-0.02626 + j7.7330
								0.87395 − j18.7219

Table 6.4: Bus admittance matrix for the 3 bus system with three conductors perphase with ground wires for the transmission lines

1.5050 − j19.3366	0.5436 + j7.1503	0.7284 + j4.0321	-0.7525 + j0.96683	-0.2718 − j3.5751	-0.3642 − j20.1606	-0.7525 + j9.6683	-0.27182 − j3.5751	-0.3642 − j20.1606
	1.4781 − j21.3489	0.54364 + j7.1503	-0.2718 − j3.5751	-0.7390 + j10.6744	-0.2718 − j3.5752	-0.2718 − j3.5752	-0.7390 + j10.6744	-0.2718 − j3.5751
		1.5050 − j19.3366	-0.3642 − j2.0160	-0.2718 − j3.5751	-0.7525 + j9.6683	-0.36422 − j2.0161	-0.2718 − j3.5752	-0.7525 + j9.6683
			1.5050 − j19.3366	0.54364 + j7.15031	0.7284 + j4.0321	-0.7525 + j9.6683	-0.2718 − j3.5751	-0.3642 − j2.01606
				1.4781 − j21.3489	0.5436 + j7.1503	-0.2718 − j3.5752	-0.7390 + j10.6744	-0.2718 − j3.5752
					1.5050 − j19.3366	-0.3642 − j2.0161	-0.2718 − j3.5752	-0.7525 + j0.9668
						1.5050 − j19.3366	0.54365 + j7.1503	0.7284 + j4.0321
							1.4781 − j21.3489	0.54364 + j7.15031
								1.5050 − j19.3366

Table 6.5: Bus admittance matrix for the 3 bus system with four conductors per phase without ground wires for the transmission lines.

0.83268 − j20.1956	0.01113 + j8.6812	0.2634 + j4.9601	-0.4163 + j10.0978	-0.00556 − j4.3406	-0.1317 − j2.4804	-0.4163 + j10.0978	-0.00556 − j4.3406	-0.1317 − j2.4804
	0.84774 − j22.7031	0.01113 + j8.6812	-0.00556 − j4.3406	-0.42387 + j11.3514	-0.00556 − j4.3406	-0.00556 − j4.3406	-0.4239 + j11.3514	-0.00556 − j4.3406
		0.83268 − j20.1956	-0.1317 − j2.4804	-0.00556 − j4.34060	-0.4163 + j10.0978	-01317 − j2.4804	-0.00556 − j4.3406	-0.4163 + j10.0978
			0.83268 − j20.1956	0.01113 + j8.6812	0.2634 + j4.9609	-0.4163 + j10.0978	-0.00556 − j4.3406	-0.1317 − j2.4804
				0.84774 − j22.7028	0.01113 + j8.6812	-0.00556 − j4.3406	-0.42387 + j11.3514	-0.00556 − j4.3406
					0.8327 − j20.1957	-0.13169 − j2.4804	-0.00556 − j4.3406	-0.4163 + j10.0978
						0.8327 − j20.1957	0.011128 + j8.6812	0.26338 + j4.9609
							0.84774 − j22.7028	0.0111286 + j8.6812
								0.83268 − j20.1957

Table 6.6: Bus admittance matrix for the 3 bus system with four conductors per phase and ground wires for the transmission lines.

1.5103 − j20.8541	0.6142 − j8.0793	0.7958 + j4.3890	-0.7551 + j10.4271	-0.3071 − j4.03967	-0.3979 − j2.1945	-0.7551 − j10.4271	-0.3071 − j4.03967	-0.3979 − j2.1945
	1.4486 − j23.2914	0.6142 + j8.0793	-0.3071 − j4.0396	-0.7243 + j11.6457	-0.30708 − j4.03968	-0.3071 − j4.0396	-0.7243 + j11.6457	-0.3071 − j4.0396
		1.5103 − j20.8541	-0.3979 − j2.1945	-0.3071 − j4.03968	-0.7551 + j10.4271	-0.3979 − j2.1945	-0.3071 − j4.03968	-0.7551 + j10.4271
			1.5103 − j20.8541	0.61416 + j8.0793	0.7958 + j4.3890	-0.7551 + j10.4271	-0.3071 − j4.0397	-0.3979 − j2.1945
				1.4486 − j23.2914	0.61416 + j8.07936	-0.3071 − j4.03967	-0.7243 + j11.6457	-0.3071 − j4.0397
					1.5103 − j20.8541	-0.3979 − j2.1945	-0.3071 − j4.03968	-0.7551 + j10.4271
						1.5103 − j20.8541	0.6142 + j8.0793	0.7959 + j4.3890
							1.4487 − j23.2914	0.6142 + j8.0793
								1.51025 − j20.8541

Table 6.7: Bus admittance matrix for the system with three conductors per phase without ground wires for the transposed transmission lines.

0.8622 − j19.1973	0.06637 + j6.5772	0.06637 + j6.5772	-0.43111 + j9.5986	-0.03318 − j3.2886	-0.03318 − j3.2886	-0.43111 + j9.5986	-0.03318 − j3.2886	-0.03318 − j3.2886
	0.8622 − j19.1973	0.06637 + j6.5773	-0.03318 − j3.2886	-0.43111 + j9.5987	-0.03318 − j3.2886	-0.03318 − j3.2886	-0.43111 + j9.5986	-0.03318 − j3.2886
		0.08622 − j19.1973	-0.033187 − j3.2886	-0.033187 − j3.2886	-0.43111 + j9.59869	-0.033187 − j3.2886	-0.03318 − j3.2886	-0.43111 + j9.5986
			0.8622 − j19.1973	0.06637 + j6.5772	0.06637 + j6.5772	-0.4311 + j9.5986	-0.03318 − j3.2886	-0.03318 − j3.2886
				0.8622 − j19.1973	0.06637 − j6.5772	-0.03318 − j3.2886	-0.4311 − j9.5986	-0.03318 − j3.2886
					0.8622 − j19.1973	-0.03318 − j3.2886	-0.03318 − j3.2886	-0.43111 + j9.5986
						0.8622 − j19.1973	0.06637 + j6.5773	0.06637 + j6.5772
							0.8622 − j19.1973	0.06637 + j6.5773
								0,8622 − j19.1973

In the computation of the bus admittance matrix for all the above cases the following base values have been used.

KV BASE	=	500 KV
MVA BASE	=	300 MVA
Z BASE	=	833.34 Ohms.

Initially a flat voltage start (i.e. voltage magnitude = 1.0 p.u) is given in the computation of the voltage profile for the buses. The voltage phase angle is shifted by 0^{o}, 120^{o} or 240^{o}, respectively for the three phases depending on the phase sequence used. To study the effect of unbalance due to untransposed transmission lines, the generation and loads are assumed to be balanced. Table 6.8 gives the generation and power loading schedule for the system in per unit. As the power generated at the slack bus is computed after the load flow solution is completed, so initially it is designated as being unknown.

Table 6.8 : 3-bus system loads and initial generation.

Bus	Phase	Generation	Load
1 (Slack)	1^a	-	1 + j0.5
	1^b	-	1 + j0.5
	1^c	-	1 + j0.5
2	2^a	1 + j0.5	0 + j0
	2^b	1 + j0.5	0 + j0
	2^c	1 + j0.5	0 + j0
3	3^a	0 + j0	1.3 + j0.8
	3^b	0 + j0	1.3 + j0.8
	3^c	0 + j0	1.3 + j0.8

To further emphasize the effect that different phasing arrangements have on the unbalance of the system the analysis was done for the sample system with three different phasing arrangements. As suggested by Hesse [7], for a n circuit system, $(6^n)/3$ significantly different phasing arrangements are possible. For the system being analyzed only two arrangements will exist and the third one will be a repetition of any of the other two. The final bus voltage magnitudes and phase angles for the system with two phasing arrangements is listed in Table 6.9.

The effect of phasing arrangement on the system total real losses and the unbalance in power generated at the slack bus is revealed due to the presence of untransposed transmission lines in the system is listed in Table 6.10. It is clear from the table that one of the phasing arrangement is a repetition of any of the other two phasing arrangement and does not serve any useful purpose in the analytical determination of the best system design.

6.4 Current Unbalance

It has been already revealed in Table 6.10 that untransposed transmission lines in the system have appreciable effect on the power unbalance and on the real losses of the system. If the system would have fully transposed lines and balanced loads, the power generated at the slack bus would also be balanced. The effect of unbalance in the system can be checked to some extent by the proper choice of phasing arrangement. For a small system every possible phasing arrangement can be tested but the determination of the best system for large systems with double or triple circuit transmission lines can be a formidable task even on a digital computer. To study the effect of unbalance in large systems use is being made of symmetrical components. The following method is generally used for estimating unbalanced current components in paralleled untransposed multi-circuit transmission line section. The unbalance in the system gives rise to the flow of unbalanced currents between different buses.

The power flow equation for a single circuit line between buses i and j can be written as follows

$$[I_{ij}^{\phi}] = \begin{bmatrix} I_{ij}^{a} \\ I_{ij}^{b} \\ I_{ij}^{c} \end{bmatrix} = \begin{bmatrix} & & \\ & y_{ij}^{abc} & \\ & & \end{bmatrix} \begin{bmatrix} \Delta V^{a} \\ \Delta V^{b} \\ \Delta V^{c} \end{bmatrix} + \begin{bmatrix} & & \\ & y_{sh\ i}^{abc} & \\ & & \end{bmatrix} \begin{bmatrix} V_{i}^{a} \\ V_{i}^{b} \\ V_{i}^{c} \end{bmatrix} \qquad (6.1)$$

For simplicity one considers the first quantity on the right hand side only and makes it equal to the left hand side, the reason being that the shunt admittance for the system is very small and it will not change the current vector appreciably. If accuracy is required in the analysis then second term on the right hand side can also be included in the computation of the current vector. Rewriting Equation (6.1) as

$$[I^{\phi}] = \begin{bmatrix} I^{a} \\ I^{b} \\ I^{c} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \end{bmatrix} = [y^{\phi}]\,[\Delta V^{\phi}] \qquad (6.2)$$

The subscripts 1, 2 and 3 in Equation (6.2) refer to phases a, b and c respectively. Equation (6.2) can be converted into symmetrical component form as follow

$$[I^{Sy}] = \begin{bmatrix} I_o \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \Delta V_0 \\ \Delta V_1 \\ \Delta V_2 \end{bmatrix} \tag{6.3}$$

The subscripts 0, 1 and 2 in Equation (6.3) refer to the zero, positive and negative sequence respectively. Equation (6.3) can be rewritten as

$$[I^{sy}] = [y^{sy}]\ [\Delta V^{sy}] \tag{6.4}$$

where

$$[I^{sy}] = [S][I^{\phi}] = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{6.5}$$

$$[\Delta V^{sy}] = [S]\ [\Delta V^{\phi}] \tag{6.6}$$

$$[y^{sy}] = [S]\ [y^{\phi}]\ [S]^{-1} \tag{6.7}$$

The current unbalance can be defined as

$$\begin{aligned} m_1 &= \text{complex absolute } \left[\frac{I_o}{I_1}\right] \times 100 \\ m_2 &= \text{complex absolute } \left[\frac{I_2}{I_1}\right] \times 100 \end{aligned} \tag{6.8}$$

The ratio m_1 and m_2 is calculated as a percentage because I_0 and I_2 are generally very small.

As the sample system is being analyzed for different conductor configurations and phasing arrangements, to compute the sequence currents for different arrangements the following method is used. Since the impedance matrix, primitive admittance matrix and the shunt admittance matrix, are function of the conductor material and geometry the latter will be same. Let the new phasing arrangement be " bac" for the same conductor configuration. Equation (6.2) for this case can be rewritten as

$$[I_1^\phi] = \begin{bmatrix} I^b \\ I^a \\ I^c \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} \Delta V_b \\ \Delta V_a \\ \Delta V_c \end{bmatrix} = [y^\phi][\Delta V_1^\phi] \quad (6.9)$$

Before transforming Equation (6.9) into symmetrical components, the current and voltage vectors must be converted into appropriate form as follows

$$[R_\phi][I_1^\phi] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I^b \\ I^a \\ I^c \end{bmatrix} = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix} \quad (6.10)$$

where

R_ϕ is a rotation matrix, capable of rotating a vector or matrix in clockwise direction.

Equation (6.9) can be completely transformed as follows

$$[R_\phi][I_1^\phi] = \{[R_\phi][y^\phi][R_\phi]^{-1}\}[R_\phi][\Delta V_1^\phi] \quad (6.11)$$

Before asymmetrical component transformation can be performed to Equation (6.11), it is rewritten as

$$[S]\,\{[R_\phi][I_1^\phi]\} = \{[S][R_\phi][S]^{-1}\}\,[Y^{Sy}]\,\{[S][R_\phi][S]^{-1}\}^{-1}\,[S]\,\{[R_\phi][\Delta V_1^\phi]\} \qquad (6.12)$$

Equation (6.12) can be expanded to give

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = [R_c]\,[Y_c]\,[R_c]^{-1} \begin{bmatrix} \Delta V_0 \\ \Delta V_1 \\ \Delta V_2 \end{bmatrix} \qquad (6.13)$$

where

$$[R_c] = \{[S][R_\phi][S]^{-1}\}$$

and

$$[R_c]^{-1} = \{[S]\,[R_\phi]\,[S]^{-1}\}^{-1}$$

From Equation (6.13) the current unbalance ratio can be calculated for any type of conductor configuration or phasing arrangement. For the final analysis the transformation is applied to Equation (6.1). A similar procedure is followed for the transformation of shunt admittance matrix. Table 6.11 shows the current unbalance for the different conductor configurations and phasing arrangements.

Table 6.9: Bus voltages for three bus system for two significant phasing arrangements.

Case	Type of Transmission Lines in the system	Phase Sequence (a, b, c)	Bus	Phase	Voltage Magnitude	Voltage Phase Angle (degrees)
(a)	Two conductors per phase without ground wires	$0, 120^{\circ}, 240^{\circ}$	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00011	1.29489
				b	1.00406	121.0860
				c	1.00436	241.1030
			3	a	0.965593	-2.90328
				b	0.964867	117.4180
				c	0.952691	237.3630
		$120^{\circ}, 0^{\circ}, 240^{\circ}$	1	a	1.0	120
				b	1.0	0
				c	1.0	240
			2	a	1.00436	121.1040
				b	1.00408	1.08513
				c	1.00009	241.2940
			3	a	0.952694	117.3640
				b	0.964892	-2.58257
				c	0.965564	237.0960

(b)	Two Conductors per phase with ground wires	$0^o,120^o,240^o$	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00060	1.28415
				b	1.00376	121.0660
				c	1.00432	241.1240
			3	a	0.964943	-2.93948
				b	0.964339	117.4550
				c	0.953671	237.3940
		$120^o,0^o,240^o$	1	a	1.0	120
				b	1.0	0
				c	1.0	240
			2	a	1.00432	121.1250
				b	1.00378	1.06519
				c	1.00058	241.2830
			3	a	0.953673	117.3960
				b	0.964366	-2.54560
				c	0.964913	237.0600
(c)	Three Conductors per phase without ground wires	$0^o,120^o,240^o$	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00001	1.16658
				b	1.00390	120.9600
				c	1.00432	240.9760

		3	a	0.970871	-2.60703
			b	0.970061	117.7120
			c	0.958149	237.6640
	120°,0°,240°	1	a	1.0	120
			b	1.0	0
			c	1.0	240
		2	a	1.00432	120.977
			b	1.00392	0.95911
			c	0.999994	241.1660
		3	a	0.958150	117.6550
			b	0.970086	-2.28805
			c	0.970845	237.3920
(d) Three Conductors per phase with ground wires	0°,120°,240°	1	a	1.0	0
			b	1.0	120
			c	1.0	240
		2	a	1.00051	1.15689
			b	1.00360	120.9410
			c	1.00426	240.9970
		3	a	0.970232	-2.64391
			b	0.969527	117.7480
			c	0.959070	237.6920
	120°,0°,240°	1	a	1.0	120
			b	1.0	0
			c	1.0	240

		2	a	1.00426	120.9990
			b	1.00362	0.9397
			c	1.00049	241.1560
		3	a	0.959073	117.6930
			b	0.969552	-2.2524
			c	0.970205	237.3550
(e) Four Conductors per phase without ground wires	0°,120°,240°	1	a	1.0	0
			b	1.0	120
			c	1.0	240
		2	a	0.999933	1.08281
			b	1.00379	120.8770
			c	1.00428	240.8920
		3	a	0.974142	-2.41236
			b	0.973276	117.9070
			c	0.961496	237.8610
	120°,0°,240°	1	a	1.0	120
			b	1.0	0
			c	1.0	240
		2	a	1.00427	120.894
			b	1.00381	0.8764
			c	0.99991	241.0820
		3	a	0.961498	117.8630
			b	0.973299	-2.0939
			c	0.974116	237.5870

(f)	Four conductors per phase with ground wires	0°,120°,240°	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00044	1.07244
				b	1.00348	120.8570
				c	1.00422	240.9130
			3	a	0.97354	-2.44753
				b	0.97276	117.9430
				c	0.96243	237.8900
		120°,0°,240°	1	a	1.0	120
				b	1.0	0
				c	1.0	240
			2	a	1.00422	120.9150
				b	1.00350	0.85645
				c	1.00041	241.0720
			3	a	0.962431	117.8920
				b	0.972782	-2.05746
				c	0.973511	237.5510
(g)	Three Conductors per phase without ground wires, transposed lines	0°,120°,240°	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00272	1.03438
				b	1.00270	121.0350
				c	1.00270	241.0340

		3	a	0.966318	-2.41037
			b	0.966287	117.5900
			c	0.966287	237.5890
	$120^{o},0^{o},240^{o}$	1	a	1.0	120
			b	1.0	0
			c	1.0	240
		2	a	1.00270	121.0350
			b	1.00272	1.03443
			c	1.00270	241.0340
		3	a	0.966291	117.5900
			b	0.966318	-2.41024
			c	0.966284	237.5890

Table 6.10: Total system real losses and slack bus power for different conductor and phasing arrangements for the 3-Bus system with balanced loads

Case	Type of Transmission lines in the system	Number of iterations for the solution	Phase Sequence for Voltages	Total Real losses in the system in p.u.	Slack Bus Power (p.u.)
a	2 conductors per phase without ground wires	4	$0^\circ, 120^\circ, 240^\circ$	0.0145063	$1^a=1.2917 + j0.94183$ $1^b=1.303396 + j0.94099$ $1^c=1.319456 + j0.94230$
		3	$120^\circ, 0^\circ, 240^\circ$	0.0145092	$1^a=1.31942 + j0.923025$ $1^b=1.303386 + j0.909924$ $1^c=1.29173 + j0.9184141$
		4	$240^\circ, 0^\circ, 120^\circ$	0.0145092	$1^a=1.291728 + j0.91843$ $1^b=1.30339 + j0.90995$ $1^c=1.3194 + j0.922995$
b	2 conductors per phase with ground wires	3	$0^\circ, 120^\circ, 240^\circ$	0.0154772	$1^a=1.2925 + j0.91988$ $1^b=1.3045 + j0.909107$ $1^c=1.31836 + j0.921728$
		4	$120^\circ, 0^\circ, 240^\circ$	0.0154772	$1^a=1.318337 + j0.92175$ $1^b=1.30454 + j0.90907$ $1^c=1.29263 + j0.919918$
		3	$240^\circ, 0^\circ, 240^\circ$	0.0154762	$1^a=1.292628 + j0.9198856$ $1^b=1.304547 + j0.9091176$ $1^c=1.318339 + j0.9217357$

c	3 conductors per phase without ground wires	3	0^o, 120^o, 240^o	0.0095634	1^a=1.290176 + j0.90452 1^b=1.301811 + j0.89622 1^c=1.31718 + j0.90868
		3	120^o, 0^o, 240^o	0.00956535	1^a=1.317576 + j0.908755 1^b=1.301813 + j0.896183 1^c=1.2902 + j0.904506
		3	240^o, 0^o, 120^o	0.00956444	1^a=1.290214 + j0.904539 1^b=1.3018159 + j0.896191 1^c=1.31759 + j0.9086961
d	3 conductors per phase with ground wires	3	0^o, 120^o, 240^o	0.0104914	1^a=1.291057 + j0.906026 1^b=1.302942 + j0.895403 1^c=1.31655 + j0.9076156
		3	120^o, 0^o, 240^o	0.0104904	1^a=1.316516 + j0.907609 1^b=1.3029346 + j0.895367 1^c=1.291096 + j0.906024
		3	240^o, 0^o, 120^o	0.104904	1^a=1.29108 + j0.906057 1^b=1.3029327 + j0.895373 1^c=1.31652 + j0.9076099
e	4 conductors per phase without ground wires	3	0^o, 120^o, 240^o	0.0071382	1^a=1.2894 + j0.895629 1^b=1.301028 + j0.887413 1^c=1.316733 + j0.89959
		3	120^o, 0^o, 240^o	0.0071373	1^a=1.399613 + j0.8996 1^b=1.301029 + j0.887385 1^c=1.28945 + j0.89561

		4	240^{o}, 0^{o}, 120^{o}	0.00713825	$1^{a}=1.28945 + j0.89562$ $1^{b}=1.30103 + j0.887379$ $1^{c}=1.316705 + j0.899609$
f	Four conductors per phase with ground wires	3	0^{o}, 120^{o}, 240^{o}	0.008044	$1^{a}=1.29028 + j0.896989$ $1^{b}=1.30215 + j0.886516$ $1^{c}=1.31566 + j0.8984$
		3	120^{o}, 0^{o}, 240^{o}	0.00803977	$1^{a}=1.315631 + j0.89838$ $1^{b}=1.30214 + j0.88646$ $1^{c}=1.29033 + j0.8970346$
		4	240^{o}, 0^{o}, 120^{o}	0.0080407	$1^{a}=1.290316 + j0.897049$ $1^{b}=1.302141 + j0.886495$ $1^{c}=1.315626 + j0.898404$

Table 6.11 : Current unbalance for the sample system with different conductor configurations.

Case	Conductor configuration for the transmission line	Transmission line between buses	Sending End		Receiving End	
			$\% \, i_o/i_1$	$\% \, i_2/i_1$	$\% \, i_o/i_1$	$\% \, i_2/i_1$
a	Two conductors per phase without ground wires	1 - 2	1.47283	8.91984	1.24788	8.144289
	(i) Phase sequence a c b	1-3	1.38138	7.56101	1.36457	10.46105
		2-3	1.24976	7.11493	1.453347	10.74497
	(ii) Phase sequence c a b	1-2	1.508514	9.54998	1.2888	9.3369
		1-3	1.42754	8.78314	1.3863	10.56982
		2-3	1.29864	8.62342	1.4726	10.58415
b	Two conductors per phase with ground wires	1-2	1.47893	8.63608	1.2584	7.8644
		1-3	1.38408	7.302969	1.38111	10.14706
	(i) Phase sequence a c b	2-3	1.25521	6.86013	1.46826	10.43149

	(ii) Phase Sequence c a b	1 - 2	1.51515	9.37033	1.2997	9.17527
		1 - 3	1.43032	8.6155	1.40422	10.39203
		2 - 3	1.304056	8.466917	1.48894	10.4003
c	Three conductors per phase without ground wires.					
	(i) Phase sequence a c b	1 - 2	1.5222	9.77412	1.32059	9.0414
		1 - 3	1.43783	8.45347	1.430938	11.2327
		2 - 3	1.31906	8.023327	1.51046	11.50127
	(ii) Phase sequence c a b	1 - 2	1.44912	10.7231	1.3626	10.5074
		1 - 3	1.48441	9.9696	1.4557	11.6997
		2 - 3	1.36834	9.81062	1.53289	11.7263

d	Three conductors per phase with ground wires.					
	(i) Phase sequence a c b	1 - 2	1.54048	9.4786	1.34286	8.7521
		1 - 3	1.45202	8.17862	1.46018	10.9158
		2 - 3	1.33567	7.75294	1.538086	11.8347
	(ii) Phase sequence c a b	1 - 2	1.57805	10.5343	1.38589	10.3338
		1 - 3	1.49914	9.790802	1.48638	11.5132
		2 - 3	1.38565	9.641218	1.561717	11.53469
e	Three conductors per phase without ground wires transposed line					
	(i) Phase sequence a c b	1 - 2	5.199×10^{-5}	6.2525×10^{-4}	8.0067×10^{-5}	6.2525×10^{-4}
		1 - 3	2.443×10^{-4}	6.2525×10^{-4}	3.888×10^{-4}	6.2530×10^{-4}
		2 - 3	2.3341×10^{-4}	6.2525×10^{-4}	3.7537×10^{-4}	6.2530×10^{-4}

	(ii) Phase sequence c a b	1 - 2	3.948×10^{-5}	8.097×10^{-4}	2.6759×10^{-5}	9.7804×10^{-4}
		1 - 3	3.1193×10^{-4}	7.7838×10^{-4}	5.281×10^{-4}	1.167×10^{-3}
		2 - 3	2.3341×10^{-4}	8.356×10^{-4}	5.013×10^{-4}	1.0576×10^{-3}
f	Four conductors per phase without ground wires.					
	(i) Phase sequence a c b	1 - 2	1.5597	10.5202	1.3729	9.8187
		1 - 3	1.4799	9.230	1.4789	11.9270
		2 - 3	1.3694	8.8137	1.5526	12.184
	(ii) Phase sequence c a b	1 - 2	1.5974	11.6735	1.4157	11.455
		1 - 3	1.5267	10.9268	1.5057	12.6264
		2 - 3	1.41906	10.7678	1.5770	12.6610
g	Four conductors per phase with ground wires					
	(i) Phase sequence a c b	1 - 2	1.5917	10.208	1.4089	9.5136
		1 - 3	1.5069	8.9371	1.5234	11.5954
		2 - 3	1.3988	8.5257	1.5953	11.8515
	(ii) Phase sequence c a b	1 - 2	1.6306	11.47137	1.4529	11.2679
		1 - 3	1.55499	10.7348	1.55153	12.4259
		2 - 3	1.4496	10.5852	1.6211	12.4554

6.5 Current Unbalance With Unbalanced Loads:

In a power system it is impossible to represent every load individually. For this reason, loads considered in a system study are a representation of composite system loads. In practice it is rather impossible to balance loads equally on each phase. There always exists an unbalance among different phases of the transmission line due to presence of unbalanced loads.

In the following sections the effect of unbalanced along with untransposed or transposed transmission lines is studied. Table 6.12 gives the generation and loading schedule for the 3-bus system. The analysis for this case has also been done with two different conductor configuration and two phasing arrangements. Table 6.13 shows the final bus voltage magnitudes and phase angles. Table 6.14 gives the system total real losses and slack bus power and Table 6.15 lists the current unbalance.

Table 6.12 : 3-Bus system, unbalanced loads and initial generation.

Bus	Phase	Generation	Load
1 (Slack)	a	-	1.1+j0.55
	b	-	1.0+j0.50
	c	-	1.1+j0.55
2	a	1 + j0.5	0 + j0
	b	1 + j0.5	0 + j0
	c	1 + j0.5	0 + j0
3	a	0 + j0	1.32 + j0.82
	b	0 + j0	1.30 + j0.80
	c	0 + j0	1.32 + j0.82

Table 6.13: Bus voltages for the 3-Bus system with unbalanced loads

Case	Type of Transmission Lines in the system	Phase Sequence (a, b, c)	Bus	Phase	Voltage Magnitude	Voltage Phase Angle (degrees)
(a)	Three conductors per phase without ground wires	$0^{o},120^{o},240^{o}$	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	0.99883	1.17015
				b	1.00468	120.9760
				c	1.00349	240.9150
			3	a	0.96838	-2.6124
				b	0.97172	117.7550
				c	0.95648	237.5240
		$120^{o},0^{o},240^{o}$	1	a	1.0	120
				b	1.0	0
				c	1.0	240
			2	a	1.00348	120.917
				b	1.00470	0.97483
				c	0.99881	241.1690
			3	a	0.95648	117.5260
				b	0.97175	-2.2451
				c	0.96836	237.3870

(b)	Three conductors per phase without ground wires, transposed line	$0^o, 120^o, 240^o$	1	a	1.0	0
				b	1.0	120
				c	1.0	240
			2	a	1.00154	1.0404
				b	1.00343	121.0500
				c	1.00196	240.9720
			3	a	0.96385	-2.4097
				b	0.96783	117.6310
				c	0.96483	237.4510
		$120^o, 0^o, 240^o$	1	a	1.0	120
				b	1.0	0
				c	1.0	240
			2	a	1.00196	120.9740
				b	1.003446	1.04956
				c	1.00152	241.0400
			3	a	0.96483	117.4520
				b	0.96785	-2.3690
				c	0.96382	237.5890

Table 6.14 Slack bus power and total real losses for 3-Bus system with unbalanced loads

Case	Type of Transmission Lines in the System	Number of iterations for the solution	Voltage Phase Sequence	Total Real Losses in the system	Slack Bus Power (p.u.)
a	Three conductors per phase without ground wires	4	0^o, 120^o, 240^o	0.0097683	1^a=1.306738 + j0.9277 1^b=1.300886 + j0.8944 1^c=1.3322 + j0.93357
		3	120^o, 0^o, 240^o	0.0097692	1^a=1.33217 + j0.93363 1^b=1.300883 + j0.8943 1^c=1.30678 + j0.92775
b	Three conductors per phase transposed line without ground wires	3	0^o, 120^o, 240^o	0.0097446	1^a=1.319994 + j0.926327 1^b=1.30235 + j0.901486 1^c=1.31745 + j0.927674
		3	120^o, 0^o, 240^o	0.009744	1^a=1.317427 + j0.92767 1^b=1.302351 + j0.90144 1^c=1.32003 + j0.92637

Table 6.15 Current Unbalance for 3 bus System with Unbalanced Loads

Case	Transmission Line Conductor Configuration	Transmission Line Between Nodes	Sending End % I_0/I_1	Sending End % I_2/I_1	Receiving End % I_0/I_1	Receiving End % I_2/I_1
a	Three Conductors Per Phase Untransposed Line Without Ground Wires					
	(i) Phase Sequence acb	1-2	1.56886	9.7137	1.250738	9.13036
		1-3	1.5291	8.3272	1.2663	11.4656
		2-3	1.3453	7.98008	1.3985	11.666
	(ii) Phase Sequence cab	1-2	1.60475	10.6703	1.2943	10.5862
		1-3	1.5738	9.8593	1.2948	11.8985
		2-3	1.39412	9.7738	1.4233	11.8656
b	Three Conductors Per Phase Transposed Line Without Ground Wires					
	(i) Phase Sequence abc	1-2	4.9738×10^{-2}	5.1136×10^{-2}	7.3779×10^{-2}	7.5773×10^{-2}
		1-3	9.675×10^{-2}	1.1042×10^{-1}	1.7274×10^{-1}	1.9717×10^{-1}
		2-3	2.7213×10^{-2}	3.8885×10^{-2}	1.1764×10^{-1}	1.4058×10^{-1}
	(ii) Phase Sequence cab	1-2	4.9869×10^{-2}	5.1474×10^{-2}	7.393×10^{-2}	7.644×10^{-2}
		1-3	9.705×10^{-2}	1.1075×10^{-1}	1.7323×10^{-1}	1.977×10^{-1}
		2-3	2.736×10^{-2}	3.877×10^{-2}	1.180×10^{-1}	1.407×10^{-1}

6.6 5-Bus Power System:

In order to see the results on a larger power system, a 5-bus system shown in Fig. 6.3 was analyzed. This system consists of two 3-phase generating units, four 3-phase composite loads and seven 3-phase transmission lines. Table 6.16 gives the length of the different transmission lines of the system.

Table 6.16. Length of the different transmission lines used for 5-bus system

Bus	Transmission Line code	Length (Kms)
1 - 2	1	32.0
1 - 3	2	128.0
3 - 4	3	16.0
2 - 5	4	64.0
4 - 5	5	128.0
2 - 4	6	96.0
2 - 3	7	96.0

The spacing dimensions and conductor type used for all the transmission lines is as follows :

Conductor : 23.5 mm ACSR 26/7 "DOVE"

Outer dia : 1.4832 cm.

4 conductors per phase located at the corners of a square of side 45 cm.

Ground wire: 7 strand ordinary steel wiere

Outer dia : 0.8 cm.

Resistance : 4.3 Ohms/mile.

Fig. 6.4 shows a cross-sectional sketch of the transmission line with ground wires.

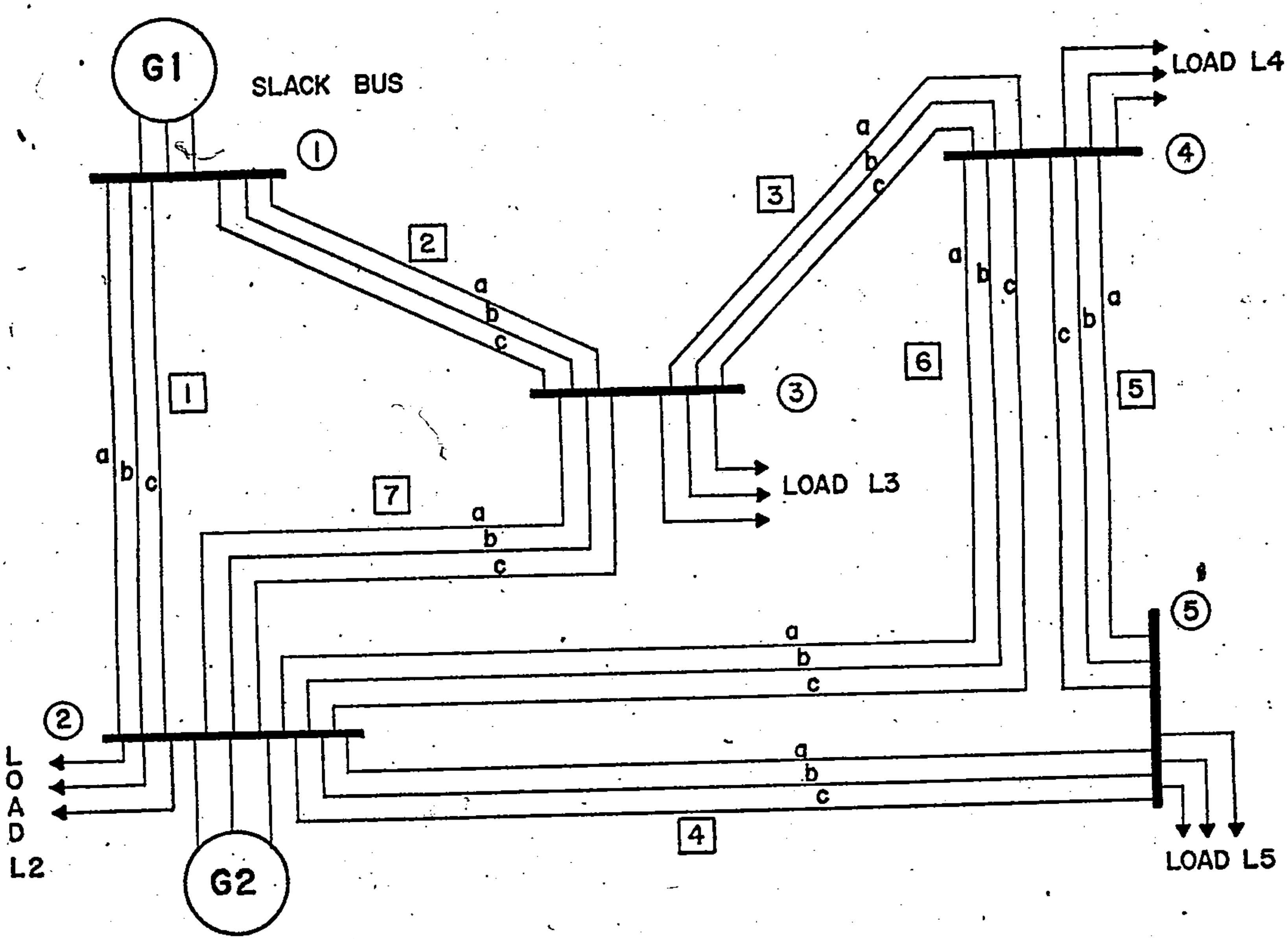


Fig 6.3 5 Bus Power System.

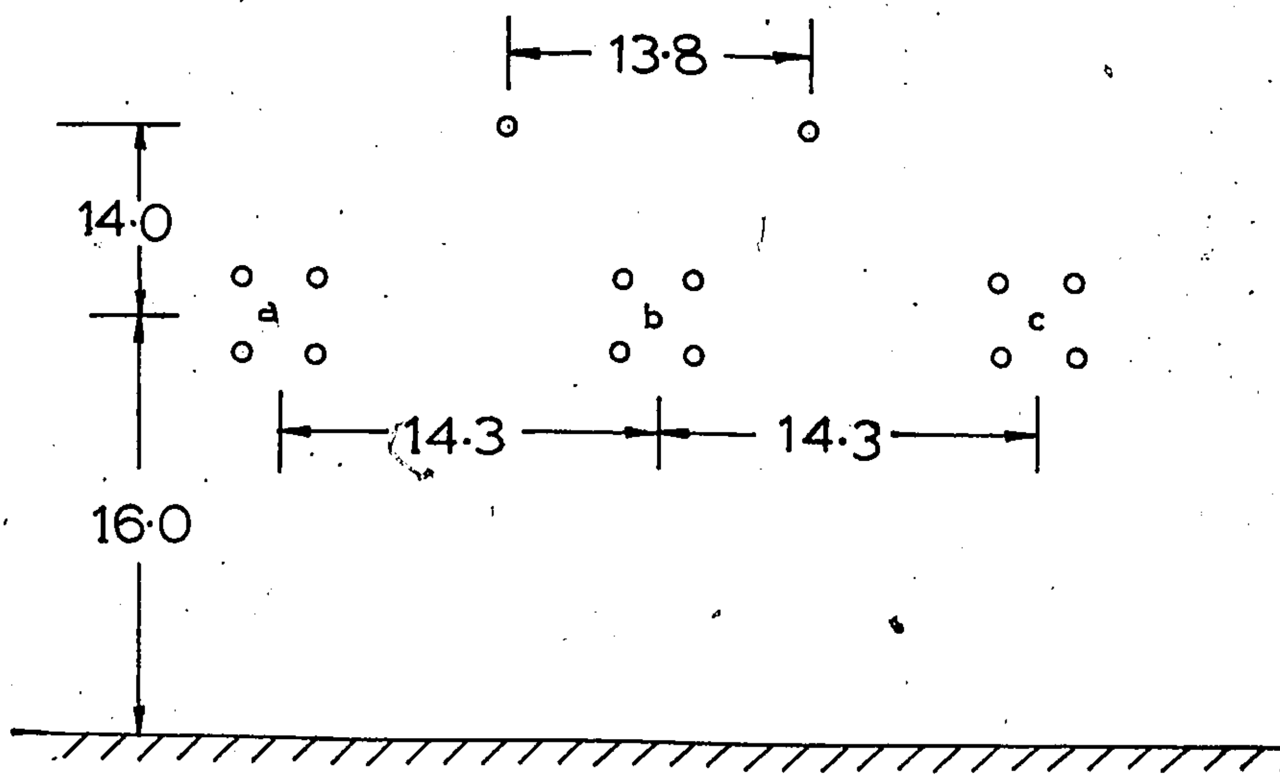


Fig. 6.4 Cross-sectional sketch of single-circuit 500 kV line.

Table 6.17 shows the bus admittance matrix for the system. Since the bus admittance matrix is symmetric, the upper triangular matrix is only listed. Table 6.18 shows the generation and power loading schedule for the system in per unit. Since the slack bus power is computed after the load solution is completed, it is initially designated as unknown. The system was analyzed for three different phase configurations, to emphasize their effect on system losses and unbalance. But any significant change in the current unbalance was not noticed, because the length of the lines is small enough to show any effect of nontranspostion. Table 6.19 shows the final bus voltages for the system for different phase arrangements. The slack bus power and the total system real losses are listed in Table 6.20. Finally Table 6.21 shows the line power flows and current unbalance ratios for the phase sequence a-b-c. . The difference in values of power flows at sending end and receiving end for different phases of the transmission line is due to the phase rotation of 180 degrees introduced in the computation of the current vectors.

Table 6.17 Bus Admittance Matrix for the 5-Bus System in p.u. (Z base = 1000Ω)

8.194 - j83.519	-0.6386 + j31.405	2.033 + j18.368	-6.555 + j67.076	0.5109 - j25.161	-1.6266 - j14.7038	-1.6387 + j16.4427	0.1277 - j6.2438	-0.4066 - j3.6647	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0
	9.6082 - j91.585	-0.6386 + j31.405	0.5109 - j25.161	-7.686 + j73.537	0.5109 - j25.161	0.1277 - j6.2438	-1.922 + j18.048	0.1277 - j6.2439	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0
		8.194 - j83.519	-1.6266 - j14.704	0.5109 - j25.161	-6.555 + j67.076	-0.4066 - j3.664	0.1277 - j6.244	-1.6387 + j16.443	0.0 + 0.0	0.0 + 0.0	0.0 + 0.0	0.0 + 0.0	0.0 + 0.0	0.0 + 0.0
			14.203 - j144.737	-1.107 + j54.432	3.524 + j31.8377	-2.185 + j22.126	0.1703 - j8.354	-0.5422 - j4.893	-2.185 + j22.127	0.1703 - j8.354	-0.5422 - j4.893	-3.277 + j33.407	0.2554 - j12.562	-0.8133 - j7.3473
				16.654 - j158.718	-1.107 + j54.432	0.1703 - j8.354	-2.562 + j24.273	0.1703 - j8.354	0.1703 - j8.354	-2.562 + j24.273	0.1703 - j8.354	0.2554 - j12.562	-3.843 + j36.634	0.2554 - j12.562
					14.203 - j144.737	-0.5422 - j4.893	0.1703 - j8.354	-2.185 + j22.126	-0.5422 - j4.893	0.1703 - j8.354	-2.185 + j22.127	-0.8133 - j7.347	0.2554 - j12.562	-3.277 + j33.408
						16.934 - j172.85	-1.3198 + j64.939	4.202 + j37.970	-13.110 + j134.23	1.022 - j50.34	-3.253 - j29.412	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0
							19.857 - j189.53	-1.3198 + j64.939	1.0217 - j50.341	-15.373 + j147.209	1.022 - j50.342	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0
								16.933 - j172.852	-3.253 - j29.412	1.022 - j50.342	-13.110 + j134.28	0.0 + j0.0	0.0 + j0.0	0.0 + j0.0
									16.933 - j172.85	-1.3198 + j64.939	4.202 + j37.970	-1.6387 + j16.443	0.1277 - j6.244	-0.4066 - j3.665
										19.857 - j189,531	-1.312 + j64.939	0.1277 - j6.243	-1.9216 + j18.048	0.1277 - j6.244
											16.9338 - j172.852	-0.4066 - j3.665	0.1277 - j6.244	-1.638 + j16.443
												4.9162 - j49.851	-0.383 + j18.806	1.2199 + j11.012
													5.765 - j54.682	-0.3832 + j18.806
														4.9163 j49.851

Table 6.18: 5-. Bus systems initial load and generation

Bus	Phase	Generation	Load
1	a	———	0+j0
	b	———	0+j0
	c	———	0+j0
2	a	3.0+j2.0	1.0+j0.5
	b	3.0+j2.0	1.0+j0.5
	c	3.0+j2.0	1.0+j0.5
3	a	0+j0	1.0+j0.5
	b	0+j0	1.0+j0.5
	c	0+j0	1.0+j0.5
4	a	0+j0	1.25 +j0.75
	b	0+j0	1.25 +j0.75
	c	0+j0	1.25 +j0.75
5	a	0+j0	1.5+j1.0
	b	0+j0	1.5+j1.0
	c	0+j0	1.5+j1.0

Table 6.19 Bus Voltages for 5- bus system for three different phase configurations. (Tolerance $=1.2\times10^{-4}$)

Phase Sequence	Bus	Phase	Voltage		Number of iterations.
			Magnitude	Phase Angle (degrees)	
$0,120^{o},240^{o}$ (a-c-b)	1 (Slack)	a c b	1.0 1.0 1.0	0.0 120.0 240.0	
	2	a c b	0.99543 0.99468 0.99228	- 0.62736 119.4290 239.3960	
	3	a c b	0.98148 0.97867 0.97161	-2.1469 118.1260 238.0550	4
	4	a c b	0.979607 0.976588 0.968915	-2.3263 117.9720 237.8970	
	5	a c b	0.975568 0.972440 0.963665	-2.5767 117.7620 237.6890	
$0^{o},240^{o},120^{o}$ (a-b-c-)	1 (Slack)	a b c	1.0 1.0 1.0	0.0 240.0 120.0	

	2	a	0.99231	-0.6027	
		b	0.99468	239.4270	
		c	0.99450	119.3730	
	3	a	0.97163	-1.94372	3
		b	0.97866	238.1250	
		c	0.98146	117.8540	
	4	a	0.96894	-2.1022	
		b	0.97658	237.970	
		c	0 97958	117.674	
	5	a	0.96369	-2.3090	
		b	0.97244	237.7600	
		c	0.97555	117.4240	
240^o, 120^o, 0^o (b-c-a)	1 (Slack)	b	1.0	240.0	
		c	1.0	120.0	
		a	1.0	0.0	
	2	b	0.99228	239.396	
		c	0,99468	119.429	
		a	0.99543	-0.6273	
	3	b	0.97161	238.055	3
		c	0.97867	118.126	
		a	0.98149	-2.1468	
	4	b	0,96891	237.8970	
		c	0.97659	117.9720	
		a	0.97961	-2.3261	
	5	b	0.96366	237.689	
		c	0.97244	117.762	
		a	0.97557	-2.5767	

Table 6.20: Slack bus power and total System real losses.

Case	Phase sequence	Phase	Slack Bus power	Total Real System losses (p.u)
(i)	a-b-c	a	1.7798 + j0.7107	0.03829
		b	1.7760 + j0.6713	
		c	1.7748 + j0.7008	
(ii)	a-c-b	a	1.7748 + 0.70079	0.03847
		c	1.7758 + j0.6713	
		b	1.7799 + j0.71076	
(iii)	b-c-a	b	1.7796 + j0.71069	0.03843
		c	1.7761 + j0.67129	
		a	1.7749 + j0.7008	

Table 6.21 Line power flows and current unbalance for the 5-bus system.

Bus Code	Power Flow		Current Unbalance Ratio			
			$\% I_o/I_1$		$\% I_{2}/I_1$	
	Sending End (S.E)	Receiving End (R.E)	S.E	R.E	S.E	R.E
$1^a - 2^a$	0.9882 + j0.4256	0.9935 + j0.50557	1.115	1.276	0.2632	0.8381
$1^b - 2^b$	0.9757 + j0.4076	0.97506 + j0.4969				
$1^c - 2^c$	0.9646 + j0.4193	0.95713 + j0.5009				
$1^a - 3^a$	0.79167 + j0.28511	0.81569 + j0.6130	0.1707	0.9895	1.4114	1.6155
$1^b - 3^b$	0.80034 + j0.26367	0.79796 + j0.6272				
$1^c - 3^c$	0.81022 + j0.2815	0.78040 + j0.61512				
$3^a - 4^a$	0.5190 + j0.3345	0.5224 + j0.3767	0.5685	0.7124	0.12237	0.5409
$3^b - 4^b$	0.5191 + j0.33115	0.5190 + j0.3777				
$3^c - 4^c$	0.5195 + j0.33379	0.5157 + j0.37702				
$2^a - 5^a$	1.42199+j0.8913	1.4242 + j1.0023	0.379	0.602	0.2568	0.5984
$2^b - 5^b$	1.4218 + j0.8762	1.4173 + j1.0108				
$2^c - 5^c$	0.1422 + j0.8871	1.4105 + j1.0038				
$4^a - 5^a$	0.07526-j0.085	0.1057 + j0.268	2.497	1.906	10.49	4.9578
$4^b - 5^b$	0.08968-j0.1022	0.0899 + j0.284				
$4^c - 5^c$	0.10468-j0.08891	0.0737 + j0.2723				

$2^a - 4^a$	0.8208 + j0.3865	0.8380 + j0.6259				
$2^b - 4^b$	0.8291 + j0.3716	0.8269 + j0.6385	0.1063	0.6931	0.9956	1.1607
$2^c - 4^c$	0.8381 + j0.3839	0.8161 + j0.62832				
$2^a - 3^a$	0.7336 + j0.3236	0.7522 + j0.5708				
$2^b - 3^b$	0.7426 + j0.3094	0.7408 + j0.5832	0.0762	0.699	1.157	1.259
$2^c - 3^c$	0.7522 + j0.3213	0.7299 + j0.5729				

6.7 Conclusion :

This thesis investigates the effect of unbalances in transmission lines on the power flow. The unbalance is caused mainly by untransposition of the transmission lines or by outage conditions. This unbalanced condition leads to generation of negative and zero sequence voltage and currents which may have adverse effects sufficient to require line transposition. Basically the emphasis has been given to the unbalance effect of untransposed transmission lines on the overall performance of the system. As it is not possible to transpose every existing or planned transmission line of a power system, attention must be given to the proper selection of conductor phasing arrangement, so as to avoid large line current unbalance. The current unbalance in untransposed overhead transmission lines with flat conductor arrangement typical of extra high voltage construction is not significant in the case of single circuit lines, but the unbalance effect becomes appreciable for multicircuit lines. The unbalance becomes worse when power system loads are unbalanced, or there are different phase impedances from a three phase bank of single phase transformers or there is a difference in the impedance of generator windings for different phases or unequal series capacitor compensation of transmission lines or large single phase loads like induction furnaces. The added $i^2 R$ losses in the system caused by unbalance may be sufficient to justify transposition of transmission lines.

The current unbalance ratios are reduced by a factor of 10^{+3} (approximately) for transposed lines in comparison to untransposed line with the same conductor arrangement for the sample system. The presence of ground wires also affects the current unbalance ratios m_1 and m_2. For the same conductor arrangement the ratio m_1 is large and the ratio m_2 is small for transmission lines with ground wires, in comparison to transmission lines without ground wires. The presence of unbalanced loads as predicted in this thesis increases the amount of unbalance in the system. The unbalance is decreased as the number of conductors per phase are increased. The reason being that the power carrying capacity of the transmission line is increased but the amount of power to be transmitted remains the same.

The number of iterations required for the solution of the voltage profile for the sample systems with the Z-matrix method and Newton's V-δ are almost equal. The dominating feature of the Z-matrix method is the nonsparsity of the Z-matrix. This makes the method unattractive for analysis of large systems and for small computers with limited core area where a sparse Y-bus matrix is easy to store. Newton's e-f method is not efficient for a system with large number of voltage controlled buses. The e-f method can prove to be more useful than the V-δ method when storage and time limitations are important since the operations involving trigonometric functions are eliminated. There is an increase in storage, and time requirement for the 5-bus system in comparison to the 3-bus system. The current unbalance in different lines being primarily a function of bus voltages is not affected by the size of the system.

The Boot Strap Gauss Seidel method did not prove to be effective for the analysis of three phase load flow problems. Even by reducing the value of the boosting factor below 0.8, the power mismatches were large enough to avoid convergence. The ϵ- coupling method in which the Jacobian submatrices N and J are used as correction factors in the computation of power mismatches were small enough to make the algorithm converge slowly.

The analysis of a large power system with three phase load flow method is not practicable for steady state operating conditions, but can be used for short circuit and stability studies. To check the degree of unbalance in the system the three phase load flow analysis can be done after suitable intervals of time. The methods which proved to be effective for the solution of three phase load flow problems are Newton's e-f and V-δ methods and the Z-matrix method.

REFERENCES

1. Ghai, R.K., Schenk, K.F. : Calculation of Primitive Impedance Matrix. Technical Report No. 76-2, University of Ottawa, January 1976.
2. Nekrasov, A.M., and Rokotyan, S.S., Editors : 500 KV Long-Distance Electric Transmission. Translated from Russian. Israel Program for Scientific Translations, Jerusalem, 1966.
3. Hesse, M.H.: Circulating Currents in Paralleled Untransposed Multicircuit Lines: I - Numerical Evaluations. Trans IEEE PAS-85. pp. 802-811, 1966.
4. Brameller, A., Gas, M.I., and Pandey, R.S. : General Fault Analysis Using Phase Frame of Reference. Proc IEE, Vol 121, pp.366-68, 1974.
5. Smith, D.C., Stokes, E.C. : The Effect of Out of Balance of Transmission Line Impedances on the Design of a 500 KV Transmission Network. Proc. of the 2nd Power Systems Computations Conference, Stockholm, 1966, Paper 4.15.
6. Westinghouse Electric Corporation, Electrical Transmission and Distribution Reference Book. East Pittsburgh 1964.
7. Hesse, M.H. : Circulating Currents in Paralleled Untransposed Multicircuit Lines : II - Methods for Estimating Current Unbalance. Trans IEEE PAS-85, pp. 812-820, 1966.
8. Stagg, G.W., El-Abiad, A.H. : Computer Methods in Power System Analysis. New York : McGraw-Hill, 1968.
9. Ghai, R.K., Schenk, K.F. : Computation of Capacitive Admittance Matrix. Technical Report No. 76-4, University of Ottawa, April 1976.
10. Jones, D. : Analysis and Protection of Electrical Power Systems. London : Pitman Publishing, 1971.

11. Stevenson, W.D., Jr. : Elements of Power System Analysis, 2nd ed. New York: McGraw Hill, 1962.

12. Neuenswander, John R. : Modern Power Systems. Toronto : International Book Company, 1971.

13. Anderson, P.M. : Analysis of Faulted Power Systems, 1st ed. Iowa : Iowa State University Press, 1973.

14. Birt, K.A., Graffy, J.J., McDonald, J.J. and El-Abiad, A.H.: Three Phase Load Flow Program. Trans IEEE PAS-95. pp. 59 -65, 1976.

15. Wasley, R.G., Shlash, M.A. : Newton Raphson Algorithm for 3-Phase Load Flow. Proc IEE, Vol. 121, pp. 630-638, 1974.

16. Snyder, W.L. : Three Phase Load Flow : Large Network Solution

17. Chen, M., Dillon, W.F. : Power System Modeling. Proc IEEE, Vol. 62, pp. 901-15, 1974.

18. Hesse, M.H., Sabath, J., Jr. EHV Double-Circuit Untransposed Line-Analysis and Tests. Trans. IEEE PAS-90, pp. 984-92, 1971.

19. Tinney, W.F., Hart, C.E. : Power Flow Solution by Newton's Method. Trans IEEE PAS-86. pp. 1449-60, 1967.

20. Stott, B. : Review of Load-Flow Calculation Methods. Proc. IEEE, Vol. 62, pp. 916-29, 1974.

21. Podmore, R., Undrill, J.M. : Modified Nodal Iterative Load-Flow Algorithm to Handle Series Capacitive Branches. Trans. IEEE PAS-92. pp. 1379-87, 1973.

22. Brameller, A. The Application of Diakoptics to Network Analysis. Proc. of 2nd Power Systems Computations Conference, Stockholm, 1966, paper 4.6.

23 Fish, J.H., III, Coleman, D.W. : A Computer Program for the Solution of Three Phase Load-Flows, Power Industry Computer Conference Records. pp 174-78, 1973.

24. Treece, J.A. : Bootstrap Gauss-Seidel Load-Flow. Proc. IEE, Vol. 116, pp 866-70, 1969.

25. Van Ness, J.E. : Iteration Methods for Digital Load-Flow Studies. Trans AIEE PAS-78. pp 583-88, 1959.

26. Dommel, H.W. : Discussion of [19]. Trans IEEE PAS-86. pp 1456-58, 1967.

27. Stott, B., Alsac, O. : Fast Decoupled Load-Flow. Trans IEEE PAS-93. pp 859-69, 1974.

28. Medanic, J., Avramovic, B. : Solution of Load-Flow Problems in Power Systems by ϵ Coupling Method. Proc IEE, Vol. 122. pp. 801-05, 1975.

29. Sasson, A.M., Viloria, F. and Aboytes, F. : Optimal Load-Flow Solution Using the Hessian Matrix. Trans IEEE PAS-91. pp. 31-41, 1972.

30. Dopazo, J.F., Klitin, O.A. and Sasson, A.M. : An Improved Initialization Technique for the Hessian Optimal Load-Flow Solution. Presented at IEEE PES Summer Meeting, San Francisco, July 1972.

31. Ghai, R.K., Schenk, K.F. : Numerical Calculations for Primitive Impedance and Capacitive Admittance Matrices. Technical Report No 76-5, University of Ottawa, June 1976.

32. Stark, P.A. : Introduction to Numerical Methods. London : The Macmillan Company, 1970.

```
C
C
      INTEGER BRANCH
      COMPLEX A,AT,ZPRIM,ZPRIM1,YSHUN1,YSHUNT,YPRIM1,YPRIM,YBUS,V,S,R0,
     $CMPLX
      COMPLEX SFLOW,YF1,YF2,VF1,VF2,PF1,PF2,PF
      DIMENSION A(9,9),AT(9,9),ZPRIM(9,9),ZPRIM1(9,9),YPRIM1(9,9),YSHUN1
     $(9,9),YSHUNT(9,9),YPRIM(9,9),YBUS(9,9),R0(9,9),PG(9),QG(9),PD(9)
     $,QD(9),V(9),S(9),G(9,9),B(9,9),VR(9),VIM(9),PSP(9),QSP(9),VMAG(9),
     $DEL(9),P(9),Q(9),DELP(9),DELQ(9),DEL1(9),DEL2(9),X(12,1),AI(9,9)
      DIMENSION RJ(12,12)
      DIMENSION SFLOW(9,9),YF1(3,3),YF2(3,3),VF1(3,1),VF2(3,1),PF1(3,1),
     $PF2(3,1),PF(3,1)
C
C     LOAD FLOW SOLUTION BY NEWTON RAPHSON'S METHOD
C
      ALPHA=1.0
      KVBASE=500
      MVABAS=300
      ZBASE=((KVBASE)**2)/(MVABAS)
      ITER=0
      EPS=5.0E-5
      PI=4.0*ATAN(1.0)
      RAD=180.0/PI
C
C     READ NUMBER OF BRANCHES AND NODES
C
      READ 5,NODE,BRANCH
    5 FORMAT(2I5)
C
C     READ THE BUS INCIDENCE MATRIX
C
      DO 10 I=1,BRANCH
      DO 10 J=1,BRANCH
      READ 15,A(I,J)
      AT(J,I)=A(I,J)
   10 CONTINUE
   15 FORMAT(2(F16.8))
C
C     READ PRIMITIVE IMPEDANCE MATRIX AND CHANGE TO PER UNIT VALUE
C
      DO 20 I=1,BRANCH
      DO 20 J=1,BRANCH
      READ 15,ZPRIM1(I,J)
      ZPRIM(I,J)=(ZPRIM1(I,J))/(ZBASE)
   20 CONTINUE
C
C     OBTAIN THE PRIMITIVE ADMITTANCE MATRIX
C
      LB=2*BRANCH
      CALL MATINV(YPRIM1,ZPRIM,BRANCH,LB)
C
C     READ SHUNT ADMITTANCE MATRIX AND CHANGE TO PER UNIT VALUE
C
      DO 30 I=1,BRANCH
      DO 30 J=1,BRANCH
      READ 15,YSHUN1(I,J)
      YSHUNT(I,J)=YSHUN1(I,J)*(ZBASE)
   30 CONTINUE
C
C     CALCULATION OF FINAL PRIMITIVE MATRIX
C
      DO 40 I=1,BRANCH
      DO 40 J=1,BRANCH
   40 YPRIM(I,J)=(YPRIM1(I,J)+YSHUNT(I,J))
C
C     FORM THE BUS ADMITTANCE MATRIX
C
      CALL ADMIT(A,AT,YBUS,NODE,BRANCH,YPRIM,R0)
C
C     READ THE BUS POWERS
C
      DO 50 I=1,NODE
      READ  16,PG(I),QG(I) ,PD(I),QD(I)
   50 CONTINUE
```

```
   16 FORMAT(4(F16.8))
C
C     READ INITIAL ESTIMATES FOR BUS VOLTAGES
      READ 15,(V(I),I=1,NODE)
C
C     PRINT OUT OF INPUT DATA
C
      PRINT 25
   25 FORMAT(1H0)
      PRINT 35
   35 FORMAT(1H1,3X,'INPUT DATA')
      PRINT 45,NODE,BRANCH
   45 FORMAT(1H0,'NUMBER OF NODES=',I3//1X,'NUMBER OF BRANCHES=',I3)
      PRINT 55
   55 FORMAT(1H0,'BUS INCIDENCE MATRIX',/)
      DO 57 I=1,BRANCH
      DO 57 K=1,BRANCH
      AI(I,K)=REAL(A(I,K))
      IF(CABS(A(I,K)).NE.0.0) PRINT 65,AI(I,K)
   57 CONTINUE
   65 FORMAT(9(F10.3))
      PRINT 75
   75 FORMAT(1H0,'PRIMITIVE IMPEDANCE MATRIX',/)
      DO 70 I=1,BRANCH
      DO 70 J=1,BRANCH
      IF(CABS(ZPRIM(I,J)).NE.0.0)PRINT 85,I,J,ZPRIM(I,J)
   70 CONTINUE
   85 FORMAT(1H ,'ZPRIM(',I2,',',I2,')=',1PE16.8,'+J',1PE16.8,/)
      PRINT 95
   95 FORMAT(1H0,'SHUNT ADMITTANCE MATRIX',/)
      DO 80 I=1,BRANCH
      DO 80 J=1,BRANCH
      IF(CABS(YSHUNT(I,J)).NE.0.0)PRINT 105,I,J,YSHUNT(I,J)
   80 CONTINUE
  105 FORMAT(1H0,'YSHUNT(',I2,',',I2,')=',1PE16.8,'+J',1PE16.8)
      PRINT 114
  114 FORMAT(1H0,'BUS ADMITTANCE MATRIX',/)
      DO 90 I=1,NODE
      DO 90 J=1,NODE
      IF(CABS(YBUS(I,J)).NE.0.0) PRINT 115,I,J,YBUS(I,J)
   90 CONTINUE
  115 FORMAT(1H ,'YBUS(',I2,',',I2,')=',1PE16.8,'+J',1PE16.8)
      DO 100 I=1,NODE
      DO 100 J=1,NODE
      SUM1=PG(I)-PD(I)
      SUM2=QG(I)-QD(I)
      S(I)=CMPLX(SUM1,SUM2)
  100 CONTINUE
      PRINT 125
  125 FORMAT(1H0,'BUS POWERS ')
      DO 110 I=1,NODE
      PRINT 135,I,S(I)
  110 CONTINUE
  135 FORMAT(10X,'S(',I2,')=',1PE16.8,'+J ',1 PE16.8,/)
      PRINT 145
  145 FORMAT(1H0,'INITIAL ESTIMATES FOR THE BUS VOLTAGES',/)
      PRINT 155,(I,V(I),I=1,NODE)
  155 FORMAT(1H ,10X,'VBUS(',I2,')= ',1PE16.8,'+J',1PE16.8,/)
      NODE1=NODE-3
      NODE2=2*NODE1
      DO 200 I=1,NODE
      DO 200 K=1,NODE
      G(I,K)=REAL(YBUS(I,K))
      B(I,K)=-AIMAG(YBUS(I,K))
  200 CONTINUE
      DO 210 I=1,NODE
      VR(I)=REAL(V(I))
      VIM(I)=AIMAG(V(I))
      PSP(I)=REAL(S(I))
      QSP(I)=AIMAG(S(I))
  210 CONTINUE
 2005 CONTINUE
      DO 220 I=1,NODE
      VMAG(I)=CABS(V(I))
      DEL(I)=ATAN(VIM(I)/VR(I))
      IF(REAL(V(I)))215,220,220
```

```
  215 DEL(I)=DEL(I)+PI
  220 CONTINUE
C
C     CALCULATION OF REAL AND REACTIVE POWERS
C
      DO 230 K=1,NODE
      P(K)=0.0
      DO 230I=1,NODE
      P(K)=P(K)+VMAG(K)*VMAG(I)*(G(K,I)*COS(DEL(K)-DEL(I))-B(K,I)*SIN(
     $DEL(K)-DEL(I)))
  230 CONTINUE
      DO 240 K=1,NODE
      Q(K)=0.0
      DO 240 I=1,NODE
      Q(K)=Q(K)+VMAG(K)*VMAG(I)*(G(K,I)*SIN(DEL(K)-DEL(I))+B(K,I)*COS(
     $DEL(K)-DEL(I)))
  240 CONTINUE
      DO 250 I=1,NODE1
      K=I+3
      DELP(I)=(PSP(K)-P(K))*ALPHA
      DELQ(I)=(QSP(K)-Q(K))*ALPHA
      DEL1(I)=ABS(DELP(I))
      DEL2(I)=ABS(DELQ(I))
      X(I,1)=DELP(I)
      X(I+NODE1,1)=DELQ(I)
  250 CONTINUE
      CALL MAX(DEL1,NODE1,DVMAX)
      CALL MAX(DEL2,NODE1,DVMAX1)
      PRINT 225,DVMAX,DVMAX1
  225 FORMAT(10X,'DVMAX=',1PE16.8,10X,'DVMAX1 =',1PE16.8)
      IF(ABS(DVMAX)-EPS)101,101,102
  101 IF(ABS(DVMAX1)-EPS)2000,2000,102
  102 CONTINUE
C
C     CALCULATION OF JACOBIAN MATRIX
C
      DO 300 K=1,NODE1
      KK=K+3
      DO 300 I=1,NODE1
      II=I+3
      L1=K
      L2=I
      CALL RJACB1(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,KK,II,L1,L2)
  300 CONTINUE
      DO 310 K=1,NODE1
      KK=K+3
      DO 310 I=1,NODE1
      II=I+3
      L1=K
      L2=I+NODE1
      CALL RJACB2(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,KK,II,L1,L2)
  310 CONTINUE
      DO 320 K=1,NODE1
      KK=K+3
      DO 320 I=1,NODE1
      II=I+3
      L1=K+NODE1
      L2=I
      CALL RJACB3(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,KK,II,L1,L2)
  320 CONTINUE
      DO 330 K=1,NODE1
      KK=K+3
      DO 330 I=1,NODE1
      II=I+3
      L1=K+NODE1
      L2=I+NODE1
      CALL RJACB4(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,KK,II,L1,L2)
  330 CONTINUE
C
C     GELG SUBROUTINE IS FROM THE SCIENTIFIC SUBROUTINE PACKAGE AND IS U
C     FOR THE SOLUTION OF N SIMULTANEOUS EQUATIONS WITH GAUSSIAN
C     ELIMINATION
C
      EPS=5.0E-5
      M=NODE2
      N=1
```

```
      IER=0
      CALL GELG(X,RJ,M,N,EPS,IER)
      DO 340 I=1,NODE1
      K=I+3
      DEL(K)=DEL(K)+X(I,1)
      VMAG(K)=VMAG(K)+(X(I+NODE1,1))*VMAG(K)
      VR(K)=VMAG(K)*(COS(DEL(K)))
      VIM(K)=VMAG(K)*(SIN(DEL(K)))
      V(K)=CMPLX(VR(K),VIM(K))
  340 CONTINUE
      ITER=ITER+1
      IF(ITER.GT.100) GO TO 2000
      IF((ITER/25)*ITER.EQ.ITER) CALL MONIT(ITER,EPS,V,NODE,S)
      IF(ITER.GT.5) GO TO 2005
      CALL MONIT(ITER,EPS,V,NODE,S)
      GO TO 2005
 2000 CONTINUE
      PRINT 350
  350 FORMAT(1H1)
      CALL MONIT(ITER,EPS,V,NODE,S)
      PRINT 350
      DO 42 I=1,3
      S(I)=CMPLX(P(I),Q(I))
      PRINT 43,I,P(I),I,Q(I)
   43 FORMAT(10X,'P(',I2,')=',1PE16.8,'Q(',I2,')= ',1PE16.8,/)
   42 CONTINUE
  500 FORMAT(3X,/////)
      DO 707 I=1,NODE
      DO 707 K=1,NODE
      SFLOW(I,K)=(0.0,0.0)
  707 CONTINUE
C
C     CALCULATION OF LINE POWER FLOWS
C
      PRINT 350
      CALL POWER(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,BRANCH
     $,NODE)
      PRINT 350
      CALL SLACK(S,SFLOW,NODE)
      CALL MATCH(P,Q,S,PG,QG,PD,QD,SFLOW,NODE)
      RRINT 500
      STOP
      END
      SUBROUTINE ADMIT(A,AT,YBUS,NODE,BRANCH,YPRIM,R)
      COMPLEX YBUS,YPRIM,A,AT,R
      INTEGER BRANCH
      DIMENSION YPRIM(BRANCH,BRANCH),YBUS(NODE,NODE),A(BRANCH,NODE),
     1AT(NODE,BRANCH),R(NODE,BRANCH)
      CALL MATPLY(R,AT,YPRIM,NODE,BRANCH,BRANCH)
      PRINT 500
      CALL MATPLY(YBUS,R,A,NODE,BRANCH,NODE)
      PRINT 500
  500 FORMAT(2X,////)
      RETURN
      END
      SUBROUTINE  MONIT(ITER,EPS,V,NODE,S)
      COMPLEX V,S
      DIMENSION V(NODE),S(NODE)
      PI=4.0*ATAN(1.0)
      RAD=180.0/PI
      PRINT 800,    ITER,EPS
  800 FORMAT(1H0,2X,'ITERATION=',I4,'  TOLERANCE FACTOR=',1PE11.3)
      DO 1IBUS=1,NODE
      VMAG=   CABS(V(IBUS))
      PHASE=        RAD*ATAN(AIMAG(V(IBUS))/REAL(V(IBUS)))
      IF(REAL(V(IBUS))) 4,5,5
    4 PHASE=PHASE+180.0
    5 CONTINUE
    1 PRINT 2 ,    IBUS,V(IBUS),VMAG,PHASE
    2 FORMAT(1H ,2X,'V(',I2,')=',1PE13.5,'+J',1PE13.5,'  =',1PE13.5,
     1'EXP(',1PE13.5,')')
      PRINT 3 ,       (I,S(I),I=1,NODE)
    3 FORMAT(2(3X,'S(',I2,')=',1PE13.5,'+J',1PE13.5))
      RETURN
      END
      SUBROUTINE MAX(A,N,BIG)
```

```
      DIMENSION A(N)
      BIG=0.0
      DO 10 I=1,N
      IF(A(I).GT.BIG) BIG=A(I)
   10 CONTINUE
      RETURN
      END
      SUBROUTINE MATPLY(R,A,B,N,M,L)
      DIMENSIONR(N,L),A(N,M),B(M,L)
      COMPLEX R,A,B,SUM
      DO 1K=1,N
      DO 2I=1,L
      SUM=(0.0,0.0)
      DO 3J=1,M
      SUM=SUM+A(K,J)*B(J,I)
    3 CONTINUE
      R(K,I)=SUM
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
      SUBROUTINE FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,
     1K1,K2,K3,BRANCH,NODE)
      COMPLEX     CONJG,SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF
      INTEGER BRANCH
      DIMENSION SFLOW( NODE ,NODE  ),YF1(3,3),YF2(3,3),YPRIM1(BRANCH,
     1 BRANCH),YSHUNT(BRANCH,BRANCH),VF1(3,1),VF2(3,1),V(9)   ,PF1(3,1)
     1,PF2(3,1),PF(3,1)
      DO 10 I=1,3
      DO 10 J=1,3
      L1=I+K1
      L2=J+K1
      YF1(I,J)=(YPRIM1(L1,L2))
      YF2(I,J)=(YSHUNT(L1,L2))/2.0
   10 CONTINUE
      DO 20 I=1,3
      I1=I+K2
      I2=I+K3
      VF1(I,1)=V(I1)-V(I2)
      VF2(I,1)=V(I1)
   20 CONTINUE
      N1=3
      M1=3
      L1=1
      CALL MATPLY(PF1,YF1,VF1,N1,M1,L1)
      CALL MATPLY(PF2,YF2,VF2,N1,M1,L1)
      DO 30 I=1,3
      L=I+K2
      K=I+K3
      PF(I,1)=CONJG(PF1(I,1)+PF2(I,1))
      SFLOW(L,K)=VF2(I,1)*PF(I,1)
      PRINT 40,L,K,SFLOW(L,K)
   30 CONTINUE
   40 FORMAT(1H0,2X,'FROM NODE',I2,'TO NODE',I2,2X,  D16.8,'+J',1PD16.8)
      RETURN
      END
      SUBROUTINE POWER(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,
     1 PF2,PF,BRANCH,NODE)
      COMPLEX     CONJG,SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF
      INTEGER BRANCH
      DIMENSION SFLOW( NODE,NODE    ),YF1(3,3),YF2(3,3),YPRIM1(BRANCH,
     1 BRANCH),YSHUNT(BRANCH,BRANCH),VF1(3,1),VF2(3,1),V(9)   ,PF1(3,1)
     1,PF2(3,1),PF(3,1)
      PRINT 20
   20 FORMAT(1H0,'POWER FLOW BETWEEN BUSSES    ',//)
      K1=0
      K2=0
      K3=3
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      K1=0
      K2=3
      K3=0
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      K1=3
```

```
      K2=0
      K3=6
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      K1=3
      K2=6
      K3=0
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      K1=6
      K2=3
      K3=6
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      K1=6
      K2=6
      K3=3
      CALL   FLOW(SFLOW,YF1,YF2,YPRIM1,YSHUNT,VF1,VF2,V,PF1,PF2,PF,K1,K2
     $,K3,BRANCH,NODE)
      RETURN
      END
       SUBROUTINE MATCH(P,Q,S,PG,QG,PD,QD,SFLOW,NODE)
      COMPLEX S,SFLOW
      DIMENSION P(NODE),Q(NODE),S(NODE),PG(NODE),QG(NODE),PD(NODE),QD
     1(NODE),SFLOW(NODE,NODE)
      PLOSS=0.0
      QLOSS=0.0
      SUM=0.0
      DO 1 I=1,NODE
      DO 1 J=1,NODE
      SUM=SUM+REAL(SFLOW(I,J))
    1 CONTINUE
      PRINT 705,SUM
  705 FORMAT(1H0,'REAL LOSS FROM LINE FLOWS=',1PE16.8)
       DO 10 I=1,NODE
      P(I)=REAL(S(I))
      Q(I)=AIMAG(S(I))
      PG(I)=P(I)+PD(I)
      QG(I)=Q(I)+QD(I)
      PLOSS=PLOSS+P(I)
      QLOSS=QLOSS+Q(I)
   10 CONTINUE
C
C     CALCULATION OF THE LOSSES AND GENERATED POWERS
C
      PRINT 20 ,   (I,PG(I),I,QG(I),I=1,NODE)
   20 FORMAT(1H0,'REAL GENERATED POWER AT BUS ',I6,1PE15.5/1H ,'
     1REACTIVE GENERATED POWER ATBUS',I2,1PE15.5)
      PRINT 30  ,   PLOSS,QLOSS
   30 FORMAT(1H0,'REAL LOSS=' ,4X,1PE15.5/1H ,'REACTIVE LOSS=',1PE 15.5)
      PRINT 40
   40 FORMAT(1H0)
      SUM1=0.0
      DO 50  I=1,NODE
      TEMP1=0.0
      TEMP2=0.0
      DO 60 J=1,NODE
      IF(I.EQ.J) GO TO 60
      TEMP1=TEMP1+REAL(SFLOW(I,J))
      TEMP2=TEMP2+AIMAG(SFLOW(I,J))
   60 CONTINUE
      PMM=P(I)-TEMP1
      QMM=Q(I)-TEMP2
      SUM1=SUM1+ABS(PMM)
   50 PRINT 601 ,I,PMM,QMM
  601  FORMAT(1H0,'BUS',I2,'REAL POWER MISMATCH=',1PE13.3/1H ,6X,'REACTIV
     1E POWER MISMATCH=',1PE15.5)
       PRINT 602 ,SUM1
  602 FORMAT(1H0,'TOTAL REAL POWER MISMATCH=',1PE13.3)
       RETURN
      END
      SUBROUTINE SLACK(S,SFLOW,NODE)
C
C     CALCULATION OF SLACK BUS POWER
C
      COMPLEX SFLOW,S
```

```
      DIMENSION SFLOW(NODE,NODE),S(NODE)
      DO 10 I=1,NODE
      S(I)=(0.0,0.0)
      DO 10 K=1,NODE
      S(I)=S(I)+SFLOW(I,K)
   10 CONTINUE
      DO 20 I=1,3
   20 PRINT 30,I,S(I)
   30 FORMAT(10X,'NET SLACK BUS POWER( ',I2,')=',1PE16.8,'+J',1PE16.8,/)
      RETURN
      END
      SUBROUTINE MATINV(Z,Y,N,M2)
      COMPLEX Z,Y,B,D,E
      DIMENSIONZ(N,N),Y(N,N),B(10,20)
      DO 10I=1,N
      DO10J=1,N
   10 B(I,J)=Y(I,J)
C
C     LOCATE MAXIMUM MAGNITUDE A(I,K) ON OR BELOW MAIN DIAGONAL
C
      M1=N+1
      DO 100I=1,N
      DO200J=M1,M2
      M3=I+N
       IF(M3-J)  12,13,12
   13 B(I,J)=(1.0,0.0)
      GO TO 200
   12 B(I,J)=(0.0,0.0)
  200 CONTINUE
  100 CONTINUE
      DO 6I=1,N
      D=B(I,I)
      DO 7J=1,M2
    7 B(I,J)=B(I,J)/D
C
C     REPLACE EACH ROW BY LINEAR COMBINATION WITH PIVOT ROW
      DO 8L=1,N
      IF (L.EQ.I)  GO TO 8
      E=B(L,I)
      DO 9J=1,M2
    9 B(L,J)=B(L,J)-E*B(I,J)
    8 CONTINUE
    6 CONTINUE
      DO 17 I=1,N
      DO 17J=M1,M2
   17 Z(I,J-N)=B(I,J)
      RETURN
      END
      SUBROUTINE RJACB1(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,K,M,L1,L2)
      DIMENSION G(NODE,NODE),B(NODE,NODE),RJ(NODE2,NODE2),VMAG(NODE),DEL
     $(NODE),P(NODE),Q(NODE)
      IF(G(K,M).EQ.0.0.AND .B(M,K).EQ.0.0) GO TO 10
      IF(K.EQ.M) GO TO 1
      RJ(L1,L2)= VMAG(K)*VMAG(M)*(G(K,M)*SIN(DEL(K)-DEL(M))+B(K,M)*COS
     1(DEL(K)-DEL(M)))
      GO TO 20
    1 CONTINUE
      RJ(L1,L2)=(VMAG(K)*VMAG(K))*B(K,K)-Q(K)
      GO TO 20
   10 RJ(L1,L2)=0.0
   20 CONTINUE
      RETURN
      END
      SUBROUTINE RJACB2(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,K,M,L1,L2)
      DIMENSION G(NODE,NODE),B(NODE,NODE),RJ(NODE2,NODE2),VMAG(NODE),DEL
     $(NODE),P(NODE),Q(NODE)
      IF(G(K,M).EQ.0.0.AND .B(M,K).EQ.0.0) GO TO 10
      IF(K.EQ.M) GO TO 1
      RJ(L1,L2)=(VMAG(K)*VMAG(M))*(G(K,M)*COS(DEL(K)-DEL(M))-B(K,M)*SIN
     1(DEL(K)-DEL(M)))
      GO TO 20
    1 CONTINUE
      RJ(L1,L2)=(VMAG(K)*VMAG(K))*G(K,K)+P(K)
      GO TO 20
   10 RJ(L1,L2)=0.0
   20 CONTINUE
```

```
      RETURN
      END
      SUBROUTINE RJACB3(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,K,M,L1,L2)
      DIMENSION G(NODE,NODE),B(NODE,NODE),RJ(NODE2,NODE2),VMAG(NODE),DEL
     $(NODE),P(NODE),Q(NODE)
      IF(G(K,M).EQ.0.0.AND .B(M,K).EQ.0.0) GO TO 10
      IF(K.EQ.M) GO TO 1
      RJ(L1,L2)=(VMAG(K)*VMAG(M))*(-G(K,M)*COS(DEL(K)-DEL(M))+B(K,M)*
     1SIN(DEL(K)-DEL(M)))
      GO TO 20
    1 CONTINUE
      RJ(L1,L2)=-((VMAG(K))*VMAG(K))*(G(K,K))+P(K)
      GO TO 20
   10 RJ(L1,L2)=0.0
   20 CONTINUE
      RETURN
      END
      SUBROUTINE RJACB4(G,B,RJ,VMAG,DEL,P,Q,NODE,NODE1,NODE2,K,M,L1,L2)
      DIMENSION G(NODE,NODE),B(NODE,NODE),RJ(NODE2,NODE2),VMAG(NODE),DEL
     $(NODE),P(NODE),Q(NODE)
      IF(G(K,M).EQ.0.0.AND .B(M,K).EQ.0.0) GO TO 10
      IF(K.EQ.M) GO TO 1
      RJ(L1,L2)=(VMAG(K) *VMAG(M))*(G(K,M)*SIN(DEL(K)-DEL(M))+B(K,M)*COS
     1(DEL(K)-DEL(M)))
      GO TO 20
    1 CONTINUE
      RJ(L1,L2)=(VMAG(K)*VMAG(K))*B(K,K)+Q(K)
      GO TO 20
   10 RJ(L1,L2)=0.0
   20 CONTINUE
      RETURN
      END
    9    9
      1.0                0.0

     -1.0                0.0



      1.0                0.0

     -1.0                0.0



      1.0                0.0

     -1.0                0.0

      1.0                0.0


     -1.0                0.0

      1.0                0.0
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