THE ANALYSIS OF REDUNDANT RELIABILITY SYSTEMS WITH COMMON-CAUSE FAILURES

BY

O.C. ANUDE

A Thesis
presented to the University of Ottawa
April 1994
in partial fulfillment of the
requirements for the degree of
DOCTOR OF PHILOSOPHY
in
MECHANICAL ENGINEERING

Ottawa-Carleton Institute for
Mechanical and Aeronautical Engineering

ISBN 0-315-95884-7

Preview

This study presents newly developed generalized solution procedures for relevant performance indices of $k - out - of - (n + m) : G$ type redundant systems subject to common-cause as well as independent component failures. The indices are notably the system reliability, mean time to failure, long-run availability and variance of time to failure.

The impacts of the governing standby unit(s) activation policy and the system repair times on these indices are also investigated.

In addition, a differential calculus-based approach is utilized to develop expressions that would maximize the net income derivable from $k - out - of - (n + m) : G$ type redundant systems by the adoption of certain system mean time to repair values.

Acknowledgements

I express my indebtedness to my Thesis supervisor, Dr. B. S. Dhillon, for his patience and guidance and for making it possible for me to come to the University of Ottawa. I also express my gratitude to my colleagues, Messrs. Nianfu Yang and Hassan Viswanath for those *useful* discussions; to my parents Mr. and Mrs. Anude Okafor and to my uncle Dr. B. Mgbemena for their support and prayers.

I also want to seize this opportunity to thank members of my academic advisory committee Drs. Gorman, Liang, Kardos and Taboun for their steadying influence and concern.

Above all, I thank God for making possible the seemingly impossible.

Abstract

The reliability and income analyses of newly developed $k - out - of - (n + m) : G$ (or n, m, k) type redundant systems subject to a combination of common-cause failures and independent failures are presented. The global goal being to evaluate the impacts of the standby activation policy and the system repair times on such relevant system performance indices as the reliability, long-run availability, mean time to failure, variance of time to failure and net income. To facilitate this investigation several possible repair policies are developed.

Results obtained using typical and practical values of basic system variables indicate that the governing standby activation policy as well as the system repair time distribution affect profoundly the values of the afore-mentioned system performance indices. In addition, newly developed mathematical relationships that would enable the net incomes of $k - out - of - (n + m) : G$ type systems to be maximized by the adoption of certain system repair rates are presented.

Terms and Definitions

Redundant system	A redundant system is any system that has more components than are strictly necessary in order to perform its intended function. This usually helps to increase the reliability and integrity of the system. Two simple examples are: a) three-engine aircraft are normally designed so that they can, if necessary, fly using only one engine and, b) an electrical supply network which has many complex interconnecting links such that electrical power can be maintained even if several links are lost.
CCFs	Common-cause failures. A common-cause failure occurs when multiple units of a redundant system fail simultaneously due to the same underlying condition
LERs	Licensee event reports. These are failure data records from which CCF rates and probabilities could be estimated.
k-out-of-(n+m):G system	Literally this notation means:"k-out-of-(n+m): Good". This is a redundant system which has n active units and m standby units. It remains functional as long as there are at least k functioning units (where $k \leq n$). This kind of system is fairly common in most manufacturing and processing industries. In this study the abbreviation-*(n,m,k) system*-has been used

for convenience.

General Model Y	relates to the (n, m, k) system whose standbys are activated as soon as an active on-line unit fails.
General Model Z	relates to the (n, m, k) system whose standbys are activated only when the last remaining active on-line unit required for system success fails.
SBAP	Standby activation policy. Two standby activation policies are considered in this study viz: i) SBAP Y-under which the standby is activated as soon as an active unit fails and ii) SBAP Z-under which the standby is activated only when the last remaining active unit required for system success fails.
$A_{ss_Y}(n, m, k)$	The steady-state availability of the (n, m, k) System under SBAP Y.
$A_{v_Y}(n, m, k)(t)$	The time-dependent availability of the (n, m, k) System under SBAP Y.
$A_{ss_Z}(n, m, k)$	The steady-state availability of the (n, m, k) System under SBAP Z.
$A_{v_Z}(n, m, k)(t)$	The time-dependent availability of the (n, m, k) System under SBAP Z.
$C_{n,m}(i)$	The transition rate from system state i to

neighbouring states under SBAP Y where $i =$
$0, 1, 2, ..., m+n-k$, and $0', 1', 2', 3', ...$
$...., (m-1)'$.

$C_{n,m}(j^{(i)})$	The transition rate from system state $j^{(i)}$ to neighbouring states under SBAP Z where $i = 0, 1, 2, ..., m$, and $j = 0, 1, 2, ...((n+m)-(i+k))$.
Type A Repair	Repair policy A, under which both the failed system and its failed units are repaired upon failure
Type B Repair	Repair policy B, under which both the failed system and its failed standby units are repaired upon failure (i.e. disabled active units are not repaired when they fail.)
Type C Repair	Repair policy C, under which the failed system is repaired but the repair of its failed units (both active units and standbys) is considered unprofitable.
Type D Repair	Repair policy D, under which the failed system is not repaired but its units (both active and standbys) are repaired upon failure.
Type E Repair	Repair policy E, under which the failed system and its failed active units are not repaired (i.e. only standby units are repaired if they fail in the standby mode.)

Type F Repair	Repair policy F, under which the failed system and its failed units (both active and standbys) are not repaired.
$R_{Y_x}(n,m,k)(t)$	The reliability of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Y.
$R_{Z_x}(n,m,k)(t)$	The reliability of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Z.
$MTTF_{Y_x}(n,m,k)$	The mean time to failure of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Y.
$MTTF_{Z_x}(n,m,k)$	The mean time to failure of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Z.
$\sigma^2_{Y_x}(n,m,k)$	The variance of time to failure of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Y.
$\sigma^2_{Z_x}(n,m,k)$	The variance of time to failure of the (n,m,k) system for Repair Type x (where x could be either D or E or F) under SBAP Z.

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Chapter 1

INTRODUCTION

General reliability theory and modeling usually support the concept that engineering equipment fail independently. This would be absolutely correct if one were to ignore the practical reality of the occurrence of dependent failures (chief among which are common-cause failures) and assume that all equipment failures would be as a result of chance random failures (often due to aging). To ensure that the system reliability is higher than that of its sub-systems (or components), component redundancy is employed. Generally, it is believed that the higher the component redundancy (often limited by financial constraints and cost-benefit concerns) the greater the overall system reliability. In practice, the possibility of the occurrence of a common-cause failure event, would make it impossible to ensure near-perfect system reliability by merely increasing item redundancy. In fact, according to Apostolakis [12], the higher the equipment redundancy level, common-cause failures, not random chance failures, potentially becomes the dominant contributor to system unavailability. Taylor [322], in 1975, reporting on the frequency of common-cause failures in the United States Power Reactor Industry asserted that "of 379 component failures or groups of failures arising from independent causes, 78 involved common-mode failures of two or more components".

Broadly stated, a common-cause failure (CCF) event occurs when multiple items fail due to a single underlying cause [161]. In the worst-case scenario, the entire system may fail due to the same trigger event. Some of the reasons for common-cause failures include design errors and deficiencies; human errors in installation, operations and maintenance; functional deficiencies, extreme operating conditions such as abnormally high (or low) temperatures and humidity, external shocks created by earthquakes, fires, hurricanes, floods etc. Classic examples of CCFs include multiple loss of aircraft jet engines due to bird ingestion, loss of power station emergency feedwater due to inadequate maintenance of valves and multiple failures due to stress corrosion in redundant structures [341]. At present, it is widely believed that the reliability and safety evaluation of engineering systems would be incomplete and would lead to overly optimistic results if common-cause failure possibilities and probabilities were discounted.

This study considers the effects of common-cause failures on some system performance indices notably system reliability, system availability, system mean time to failure and system variance of time to failure by studying some newly developed stochastic models of $k - out - of - (n + m) : G$ type systems. These systems incorporate elements of three commonly used redundant configurations: *parallel*, $k - out - of - n : G$ and *standby*. These three redundant configurations are presented in Appendix A.

1.1 Common-Cause Failures: An Overview

Common-cause failures (sometimes called common-mode failures, though strictly speaking, common-mode failures are a subset of common-cause failures characterised by multiple equipment failing simultaneously in the same mode) can no longer be 'wished away' in engineering system safety and reliability prediction studies. Generally, incorporating and quantifying common-cause failures in reliability and safety assessments of redundant systems has not been easy. This is largely attributable to the 'relative rarity' of CCFs vis-a-vis independent failures; sparse field data and; a general confusion as to what constitutes a 'common-cause failure'. In fact the literature is replete with several definitions and, thus far, there has not been a widely accepted and standardised definition of common-cause failure. This last point has hindered both the reporting and recording of field data and the subsequent analyses of it [159].

1.1.1 Common-Cause Failures: Causes

Common-cause failures can be grouped into two broad categories described as 'intrinsic' and 'extrinsic'. Extrinsic common-cause failures according to Smith [314], have their source or origin in phenomena that are external to the system. Examples include tornadoes, fires, floods, earthquakes etc. They are more catastrophic, rarer and a cause of greater concern. Design errors and human errors (particularly where the operator, designer, maintainer or installer is considered an integral part of the system) in operations and maintenance belong to the group of intrinsic common-cause failures. They occur more often, are often less damaging to the system and somewhat more controllable and correctable.

Gangloff [160] identified the following groups of CCF causes:

i) External normal environment: examples include the presence of dust/dirt, abnormally high (or low) temperatures and humidity, excessively high vibration levels etc.

ii) Design deficiencies: these may include unrecognized interdependence between 'independent' sub-systems, component, items, units etc; unrecognized electrical or mechanical dependence on common element; dependence on equipment or parameters whose failure or abnormality causes need for protection etc.

iii) Human errors in operations and maintenance: examples include miscalibration, inadequate or improper testing, out-dated instructions or prints, carelessness in maintenance, fatigue and lack of concentration and other human factors.

iv) External catastrophes: these include extreme environmental shocks such as tornadoes, hurricanes, fire outbreaks, floods, vulcanoes, earthquakes etc.

v) Functional deficiencies: such as the misunderstanding of process variable behaviour, inadequacy of designed protective action, inappropriate instrumentation and inadequate preventative measures.

1.1.2 Common-Cause Failures: Defenses

The concept of "coupling factors" has been introduced to guage the susceptibility of a group (or groups) of equipment failing simultaneously due to a single event (common-cause). The higher the degree of coupling between the various components or sub-systems that make up the system the greater the possibility that they will fail simultaneously as a result of a single event. Examples of coupling factors include commonalities in design, hardware, installation, maintenance and operations staff, procedures, environment, location etc.

From the fore-going, it is obvious that any measure that would limit the above commonalities and introduce some element of diversity would help protect the system from common-cause failure events to some degree. Thus, diversity in equipment, design, maintenance and operational administrative controls, inspection and testing procedures etc. constitutes a defense against common-cause failures. According to Apostolakis [10], "using different types of equipment, more than one logical way to monitor the state of the system, physically separating redundant components, having more than one operator to review personnel actions, and employing other forms of diversity, it is reasonable to expect that the probability of a common-mode failure is reduced".

Crellin et al. [72] contend that "staggering of inspections and tests would offer some protection against coupling as compared to performing these activities simultaneously and sequentially. This could reduce the coupling associated with human-related failures because the probability that an operator (or maintainer) repeats an incorrect action is lower when activities are performed months, weeks or even days apart than when they are performed within a few minutes or a few hours apart".

All in all, any procedure that reduces the coupling factors either directly or indirectly constitutes a protective measure against CCF events.

1.1.3 Common-Cause Failures: Modeling Techniques

Filtering through the vast literature on common-cause failures, it is obvious that CCF modeling techniques could be split into two broad groups viz: (a) the primary modeling techniques-usually developed by various authors to quantify CCF rates and probabilities directly or indirectly using raw field data obtained from sources such as licensee event reports (LERs), and (b) stochastic modeling techniques-which involve the analysis of stochastic models of either existing or likely engineering systems susceptible to CCFs.

a) Primary Modeling Techniques

These models use either directly or indirectly raw field data (where available but usually sparse) to estimate either the common-cause failure rates or the common-cause failure probabilities. The following are brief descriptions of the more popular ones.

i) Direct Assessment: Hirschberg [201] discussed this approach in detail. Strictly speaking, it is more a procedure than a modeling technique. It involves using the actual number of demands and the number of observed failures with say multiplicity i (where i could be any positive integer) and estimating the quantities of interest (usually the common-cause failure rate and/or the common-cause failure probability) directly from the data set. This approach is simple and bereft of any sophistication and often does not require sound knowledge of any mathematical or statistical skills.

ii) The Beta (β) Factor: arguably the most used of all the primary modeling techniques. It was proposed by Fleming [141] in 1975. Simply stated, the item failure rate λ is modeled as a sum of two mutually exclusive components λ_1 and λ_2 i.e. $\lambda = \lambda_1 + \lambda_2$, where $\lambda_1 = (1-\beta)\,\lambda$ and $\lambda_2 = \beta\,\lambda$. λ_1 is the rate at which items fail independently and λ_2 is the rate at which items fail due to a single event (common-cause failure). β is defined as a point estimate of the conditional probability that a unit failure is common-cause. In most engineering applications β is chosen within the interval $0.01 - 0.30$. The simplicity of the β-factor method and its use of direct CCF data are the major strengths of this approach. However, for most redundant systems, this approach has been proven to be excessively conservative in predicting CCF failure rates. It is most effective when used to analyze duplex systems (i.e. systems with just two components).

iii) The Binomial Failure Rate (BFR): Originally proposed by Vesely [332] in 1977, this approach is more sophisticated than the β-factor and can handle failure multiplicities greater than one. It assumes that the occurrence of a CCF event challenges all components of the system and that the number of failed components caused by a shock has a binomial distribution with parameters n and p, where n is the number of components (or sub-systems) in the system and p is the probability of each component to fail independently given that a shock has occurred. The multiplicity distribution, f_k, is given by:

$$f_k = \frac{n!}{k!\,(n-k)!}\, p^k\,(1-p)^{n-k}$$

f_k is essentially the frequency that k items failed as a result of a shock where $k = 0, 1, 2, 3, ..., n$ and p is typically found in the interval $0.1 - 0.5$ using field data from most engineering applications. The strength of the BFR lies in its flexibility and its ability to handle analysis involving higher item redundancy levels but it has certain drawbacks. The assumption, given that a shock has occurred, that items would fail independently of each other, represents a serious shortcoming as this assumption is often violated in practice. Also the BFR does not involve any parameters directly linked to the degree of system protection against CCFs. A more general version of the BFR was suggested by Atwood [17]. This version explicitly considers a second type of shock called a 'lethal shock' which on occurring, triggers the failure of all sub-systems with a conditional probability of unity.

iv) The Random Probability Shock (RPS): suggested by Hokstad [206] in 1988, this very inclusive model combines the strengths of both the β-factor and the BFR whilst eliminating their drawbacks. It is very sophisticated and sound knowledge of statistical estimation techniques is needed for its use.

v) The Square Root Bounding Method: made popular by the United States Reactor Safety Study (WASH 1400) [327] and discussed in detail by Martin and Wright [252], this method has been widely commented upon by various reliability analysts for lacking any real practical foundation. It makes two major assumptions viz: (i) that the failure probability of a redundant system cannot be higher than the failure probability of a single component of the redundant system and, (ii) cannot be lower than the failure probability of the redundant system assuming that all component failures are random and independent. Let the failure probability of a single component be P_i and the calculated failure probability of the system without CCFs be P_s, then the total system failure probability including CCFs, P_c, obeys the following inequality relationship:

$$P_s \leq P_c \leq P_i$$

On further assumption that the values of P_c follow a log-normal distribution (as in WASH 1400) symmetrical about P_s and P_i, then an estimate of P_c can be obtained by finding the median (i.e. the geometric mean) of the distribution and is given as,

$$P_c = \sqrt{P_s \bullet P_i}$$

vi) Multiple Dependent Failure Fraction (MDFF): proposed by Stamatelatos [317] in 1982 and originally developed for a system of three identical units (modified and extended to 4 by Hirschberg [201]), MDFF is a generalization of the β-factor method and can be stated mathematically as follows:

$$\lambda = \lambda_r + \lambda_c = \lambda_r + \lambda_n \sum_{n=2}^{N} f_n = \lambda_r + f\,\lambda$$

where λ_r is the independent failure rate component, $\lambda_c = \lambda_n \sum_{n=2}^{N} f_n$ is the CCF rate component, f_n is the fraction of $n-$tuple failures ($n = 2, ...N$) and f is the fraction of CCFs (analogous to the β-factor). Results of the β-factor approach are usually more conservative than those of MDFF.

vii) Multiple Greek Letter (MGL): this is another extension of the β-factor method. Introduced by Fleming and Kalinowski [144], it was developed to handle higher order component redundancies and failure multiplicities greater than unity.

viii) Multinomial Failure Rate Model (MFR): proposed by Apostolakis and Moieni [13] in 1987, it prescribes the condition: $\phi_0 + \phi_1 + \phi_2 + \phi_3 + ... = 1$, where ϕ_0 is the conditional fraction of no failure, ϕ_1 is the conditional fraction of exactly one failure, ϕ_2 is the conditional fraction of exactly two component failures, ϕ_3 is the conditional fraction of exactly three component failures and so on, in the event of a system shock.

ix) Trinomial Failure Rate Model (TFR): Unlike most of the afore-mentioned methods, the TFR is not a binary (i.e component state is either a success or failure) type model. Suggested by Han et al. [176] in 1989, it explicitly handles practical ambiguous situations such as component partial failures, incipient failures and potential failures that lie between operable and failed states-so called 'gray' component states.

TFR could be considered as a special case of a more general approach proposed by Marshall and Olkin [251].

x) The Basic Parameter Model (BP): presented by Mosleh [261] in 1991, this approach gives the total unavailability, Q_t, of a component in a common-cause component group of size n as:

$$Q_t = \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!} \bullet Q_k$$

where Q_k is the probability of failure of a set involving k specific components in a common-cause component group of size n. An implicit and unfortunate assumption associated with the use of BP is the non-staggered testing of equipment (or components).

xi) The alpha factor method: is closely related to the BP by the following relationship:

$$Q_k = \frac{n!}{k!(n-k)!} \frac{\alpha_k}{\alpha_t} \bullet Q_t$$

where α_k is the ratio of the probabilities of failure events involving any k units over the total probability of all failure events in a group of n components and α_t is equal to the sum of product $\sum k\,\alpha_k$. To estimate Q_k values in the two procedures above, the number of failures of combinations of components, n_k, must be obtained as well as the total number of demands or the total time that the system has operated. In situations where field data is adequate, Q_k values can easily be estimated using the direct assessment approach already described.

xii) The Common Load Model (CML): proposed by Mankamo [240] in 1977, it could be stated as follows: assume that the n components of a system have independent and identically distributed random resistances $R_1, R_2, ..., R_n$. We denote the probability density function of these resistances by $f_R(x)$. In the event of the occurrence of a random shock S with probability density function $g_s(x)$, then the event that exactly k of the components fail simultaneously, $k = 0, 1, 2, ..., n$ is given by:

$$R_{[k]} \leq S \leq R_{[k+1]}$$

where $R_{[1]} \leq R_{[2]} \leq ..... \leq R_{[n]}$ are ordered resistances. $R_{[0]} = 0$, and $R_{[n+1]} = \infty$. S is assumed to be independent of the resistances. A given component is assumed to fail whenever $R < S$. The probability of this event is:

$$P(R < S) = \int_0^\infty \int_0^y f_R(x)\, g_s(y)\, dx\, dy$$

The probability that exactly k components fail when subjected to this random stress S is:

$$P_n(k) = \int_0^\infty \frac{n!}{k!\,(n-k)!} (F_R(y))^k (1 - F_R(y))^{n-k} g_s(y)\, dy$$

xiii) Harris Model: The following model is proposed by Harris [177] in 1986 as an extension of the CML. It could be described as follows: Let $N(t)$ be the number of shocks arriving at or before time t, $0 \leq t \leq T$. If $N(T)$ shocks have arrived in [0,T], we designate the arrival times by $0 < t_1 < .... < t_n < T$. The shocks are assumed to have random, independent and identically distributed magnitudes $X(t_1), X(t_2), ..., X(t_n)$. Consider a system of m components. To each component, we associate independent random variables $Y_1, Y_2, ..., Y_m$. Component i is said to fail at time t_j whenever $X(t_j) > Y_i$, $i = 1, 2, ..., m$. The Y_is could be ordered by replacing them with the ordered set of random variables $0 \leq Y_{[1]} \leq Y_{[2]} \leq ...Y_{[m]}$. Then, k components fail simultaneously at time t_j whenever

$$Y_{[k]} < X(t_j) \leq Y_{[k+1]},\ k = 0, 1, ..., m$$

where $Y_{[0]} = 0$ and $Y_{[m+1]} = \infty$

The Y_is can be obtained from either non-destructive testing or through an extensive knowledge of the physical properties of the components. Situations where the Y_is are subject to 'wear out' with the passage of time are handled by incorporating a family of random functions $Y_i(t), i = 1, 2, .., m$ where for $t_1 < t_2$, $Y_i(t_1) \geq Y_i(t_2)$ and $Y_i(t) \to 0$ as $t \to \infty$. The choice of these functions require sound knowledge of the physical characteristics of the components. This method can also handle situations where the shocks are considered to have a degrading effect i.e. they do not cause the failure of a component but may 'soften up' the component so that the next shock would be more likely to result in a failure. This situation can be described by introducing additional functions, $Y_i(t_j^+)$ as follows:

$$Y_i(t_j^+) = H\left(Y_i(t_j),\ X(t_j)\right)$$

where $Y_i(t_j^+) \leq Y_i(t_j)$. To properly characterize these functions, engineering models for shock damage and fatigue which depend on the precise nature of the components are needed.

xiv) The Inverse Stress-Strength Interference Model (ISSI): this approach is closely related to the CML. An article by Guey and Heising [172] in 1985 contains a full description of this technique.

b) Stochastic Modeling Techniques

These are procedures and formulas developed to quantify and incorporate the effects of CCF events on engineering system performance parameters notably the system reliability, system availability, system mean life (or mean time to failure) and system variance of time to failure through the analysis of either existing or likely engineering systems using such mathematical techniques as Laplace transforms and Markov chains. System and sub-system transition phenomena are usually modeled as stochastic processes. Major contributions in terms of new formulas, procedures and expressions for system performance indices appeared in this area within the last two decades. These stochastic models are often validated through numerical analysis using data that may have been obtained via the primary modeling techniques. The folowing references in the literature fall into this group [36, 41, 46-62, 76, 78, 79, 81-85, 87, 95-99, 101-121, 124, 125, 127, 164, 167, 168, 191, 205, 229, 230, 268, 286, 294, 305, 307, 308, 312, 313, 331]. This study also falls into this domain.

1.1.4 Common-cause Failures: Evaluation Steps

Gangloff [160] recognized the following four basic steps to the evaluation of CCFs:

- Identification of events or faults leading to system failure.
- Tabulation of common-mode causative factors meriting consideration.
- Identification of potential measures to prevent CCFs.
- System evaluation of susceptibility to CCFs.

1.1.5 Common-Cause Failures: Difficulties in Quantification

Edwards and Watson [134] identified the following as problem areas in the quantification of common-cause failures:

• the recognition of the many possible causes of common-cause failures by professionals such as designers, operators and reliability analysts.

• the application and assessment of the various defenses against CCFs that are available.

• the models used in the quantification of system reliability due to CCFs.

• the use of data from reported CCF events for the reliability assessment of other systems.

• the absence of an agreed interpretation and understanding of what constitutes a "common-cause failure" leading to difficulties in the reporting and recording of data and in the subsequent analysis of data.

1.2 Common-Cause Failures: A literature Survey

Prior to Epler's paper [137] in 1969, classical reliability theory and analysis completely ignored the common-cause failures of components of redundant systems and considered only independent failures. This practice led to overly optimistic system reliability and safety predictions. However, the 1970s and 1980s witnessed a steady and significant rise in the number of studies, reports, and articles published on CCFs and their effects on system performance indices such as reliability, safety and availability. Most of these studies, usually probabilistic risk/safety assessments, were conducted in the nuclear power reactor industry where the consequences of system failure due to a CCF are often catastrophic (e.g., the 1986 Chernobyl nuclear accident in the former Soviet Union). The following is a survey of some of the notable contributions on CCFs.

In 1969, Epler [137] in one of the earliest publications on CCF events in the power reactor industry, provided a list of simultaneous failure accidents in Reactor Protection Instrumentation Systems at Oak Ridge National Laboratory (ORNL) and proposed the following as the main causes of CCFs: changes in the characteristics of the system protected; unrecognized dependence on a common element; system disability by an accident; and communications (human) error.

The following year, 1970, Jacobs [217] asserted that system reliability without taking into account CCF events was incomplete. He recognized the following as the prime reasons for CCFs: functional deficiencies; maintenance errors; design deficiencies; and external events. The fact that CCFs have "arrived to stay" in realistic reliability and safety prediction studies is underscored by Gangloff [160] in 1972. The same author [161], in 1975, summarized the state of knowledge of CCFs in the nuclear power industry and proposed some methods for the evaluation of system susceptibility to CCFs. Also in 1975, Fleming [141] presented the β-factor method for assessing the reliability of redundant systems subject to both CCF and random failures and Taylor [322] in a study of United States Power Reactor abnormal occurrence reports concluded that an overwhelming proportion of failures are of the common-cause type involving design, installation and operational errors.

A broad category of failure mechanisms that can initiate CCFs in nuclear plant redundant systems from reactor operating experiences and reports is examined by Hayden [181] in 1976 leading to some proposals for the development of design and operating guidelines for reducing the probability of CCFs while Fleming and Hannaman [143] discussed the probabilistic safety assessment study of the High Temperature Gas Cooled Reactor conducted by General Atomic Company in the United States which led to estimates of the failure probability of the cooling systems. Also in 1976, Apostolakis [10] probed the effects of CCFs on the reliability of redundant systems and determined that within the initial period $[0, T]$ where T is the time to failure, the probability of a CCF potentially dominated that of a chance random failure for the case of systems with non-repairable units at high redundancy levels. For systems with repairable units, same author suggested a procedure for determining useful bounds to the CCF rate which give a quick estimate of its significance on system performance and reliability.

In 1977, Cate et al. [44] presented a fault-tree based step-by-step procedure for assessing CCFs in complex systems such as nuclear reactor plants that did not require the determination of all system hardware minimal cut-sets and Dhillon and Proctor [104], considered redundant systems composed of two-state devices subject to CCFs and derived new hazard rate and mean life expressions. The importance of applying diverse shutdown systems and frequent periodic testing in reducing CCFs in the power reactor industry is stressed by Epler [136] while Chu and Gaver [49] proposed two Markov models for quantifying the performance of systems subject to both chance failures and CCFs. *Model A* considered CCFs of a catastrophic nature while *Model B* assumed that CCFs merely increase the failure rates of components. Also in 1977, Vesely [332] estimated the probabilities of multiple control rods failing by applying the BFR to a Boiling Water Reactor Scram failure data and Worrell and Stack [354] outlined a procedure for identifying common-cause candidates (these are minimal cut-sets whose primary events are linked by a special condition or are susceptible to the same secondary cause) using the set equation transformation system (SETS).

In 1978, Dhillon [86] presented an optimal preventative maintenance policy model for redundant systems with identical units susceptible to CCFs and in Ref. [80], derived new expressions for the mean life, variance of time to failure, hazard rate and reliability of several redundant systems susceptible to CCFs using the assumptions of identical units and a Weibull time to failure density function. The same author in Ref. [78] reported the reliability study of a $k-out-of-n$ three-state device network subject to CCFs. Johnson and Vesely [223] using a Marshall-Olkin based approach, and utilizing recorded field data from the nuclear power industry, estimated the common-mode failure probabilities of valve leakages and Easterling [131] identified three types of statistical dependence involved in CCF analysis viz: dependence among failure events themselves; dependence of failure events on the conditions under which an item is expected to perform and; dependence among these conditions. In 1979, Sambhi et al. [305] presented new formulas for the reliability, mean life, hazard rate and variance of time to failure for a three-state device parallel network system subject to CCFs as well as chance failures and Dhillon [84] derived the Laplace transforms of state probabilities for a stochastic model of N identical duplex systems composed of $N-1$ standbys and one active system subject to CCFs. The same author in Ref. [77] presented a list of selective bibliographies on CCFs covering the period [1965-78] while Chung [51] analyzed a mathematical model representing an $N-$unit redundant system containing two failure modes with CCFs and derived the Laplace transforms of state probabilities.

In 1980, Chung [58] extended the stochastic model reported by Sambhi et al. [305] the previous year and derived new formulas for system availability, the frequency of encountering a state and the average duration of residence in a state while Smith and Watson [315] discussed the continuing difficulties and issues sorrounding the problem of finding a widely acceptable definition of CCFs and how meaningful progression on

CCF analysis has been stifled as a result and proposed an elaborate and inclusive definition. An insightful review of CCFs as well as some regulatory issues and operating experiences were considered by Hagen [174] and Singh et al. [310] discussed common-mode failures modeling in the reliability evaluation of electric power transmission lines and considered the effect of the probability distribution of state residence times on the reliability indices and concluded that reliability measures were sensitive to the probability distribution of component downtimes. A methodology for the prediction of accident sequence probabilities in a nuclear power plant as well as an analytical procedure for calculating fault-tree top event probabilities based on the correlated failure of primary events were presented by Hudson and Gasca [208] in 1981, and Singh [308] studied the effects of repair and common-mode failures on the reliability and availability of a triple modular redundancy (TMR) computer configuration and concluded they were significant.

In 1982, Rankin [296] outlined an approach to common-cause hazard analysis of electrical systems that is based on checklists instead of the usual fault-tree reduction techniques and demonstrated that this approach could be used to determine the causes of operational problems that had been thought to be random unrepeatable glitches while Dhillon and Natesan [103] derived steady-state availability expressions for a system of N active pulverizers with a cold standby on the assumption that failed system repair times are arbitrarily distributed. Hirschmann [205], using a Markovian approach, evaluated the steady-state unavailability of a $k-out-of-n$ system with constant transition rates and subject to CCFs in 1983. Also in 1983, Matsuoka [253], in order to analyze the effects of an extreme environmental condition and common-mode failures on system reliability, developed a failure model in which the failure rate is a function of cause, of severity of cause, of cause acting duration, and of failure mode. Heising and Guey [184] in 1984 concluded that the MDFF was less conservative and yielded more realistic estimates of system unavailability than the β-factor method in a study of two reactor safety systems made up of two and three redundant trains (i.e. the high pressure, injection and feedwater systems).

Cost issues were considered by Garg and Goel [164] in 1985 when they developed a mathematical model to predict the cost involved to run an n-component single unit system which can fail in n-mutually exclusive ways of total failure or due to a common-cause and Bickel and Caivano [29] described a simple technique for adapting the BFR point estimates (based on pooled United States Nuclear Industry Experience) for use in fault-tree quantification codes using Monte-Carlo system simulation codes such as SPASM. Hirschberg [193], using the same field data from fifty groups of four diesel generators, compared the performance of the β-factor method with three other methods: the BFR, MDFF and MGL and concluded the MGL the best performer and Moody and Follen [257], in the reliability assessment study of a nuclear plant reactor trip breaker configuration concluded that CCFs were indeed the main contributors to system unavailability. In situations where: component-specific and system-specific values are needed, failure data are scarce, level of component redundancy is high, uncertainty needs to be quantified, shocks are non-lethal, Guey and Heising [172] proposed a new approach-the Inverse Stress-Strength Interference (ISSI) technique for estimating CCF probabilities. Also in 1985, Wright [357] examined several examples of CCFs in practice ranging from the chemical, petro-chemical, mining, medical to the nuclear power reactor industry and suggested procedures that may be useful in modeling them for statistical reliability studies.

The statistical and computational aspects of the BFR method were discussed extensively by Atwood [17] in 1986, and Fleming et al. [145] compared the performances of the BP, MGL and BFR methods within a framework of system-level CCF analysis and its application to a three-train auxiliary feedwater system. Yun and Bai [362] considered the problem of determining optimal numbers of redundant units in parallel systems with common-mode failures as well as random failures and concluded that the optimal number is unique and finite while Dichirico and Singh [125], under the assumption of arbitrarily distributed system downtimes, derived expressions for the state probabilities, long-run probability and frequency of failure for the stochastic model of a repairable two-unit parallel redundant system subject to common-mode failures. Harris [177] described and characterized various models for CCFs and suggested a new time-dependent stress-strength (loading) model. In 1987, Heising and Luciani [185] performed fault-tree and qualitative CCF analyses of a power distribution box system using the Mocus-Bacfire β-factor (MOBB) computer code and Chung [57] presented steady-state availability expressions for a $k-out-of-n:G$ redundant system with m mutually exclusive failure modes and common-cause failures whose downtimes are arbitrarily distributed. Humphreys [212] developed a procedure which allows the estimation of the β-factor value directly from the attributes of a given system particularly those design attributes that help protect the system against CCFs and Mahmoud et al. [238] analyzed the stochastic model of a system consisting of two non-identical parallel active units and a cold standby unit subject to CCFs and derived expressions for the system steady-state availability. Martin and Wright [252], in a thought-provoking paper, suggested a method of assessing the safety of nuclear/chemical plants involving a combination of the β-factor method with some limiting values which allow the value of β to decrease as the protection against CCFs is increased.

In 1988, Hokstad [206], suggested a new parametric model for CCFs-the Random Probability Shock (RPS) model which combines the strengths of both the β-factor and the BFR whilst eliminating their drawbacks. Under the assumption that component downtimes follow an overdispersed Gamma distribution, Dichirico and Singh [124] evaluated the probability and frequency of failure of a system of two parallel redundant transmission lines with common-mode failures and Mosleh et al. [262] presented the framework developed for the treatment of CCFs in risk and reliability analyses in a project jointly sponsored by the Electric Power Research Institute (EPRI) and the United States Nuclear Regulatory Commission (USNRC). This framework contains a systematic guide for identification, modeling, and quantification of CCFs and addresses issues such as logic model development, screening of CCF events, probabilistic representation of CCF events, data analysis, system quantification and interpretation of results. The following year, 1989, Han et al. [176] proposed a new CCF model-the Trinomial Failure Rate model (TFR), which is capable of handling explicitly three component states: success; gray and; failure, in order to analyze ambiguous component situations such as partial failures, incipient failures and potential failures and considering common-cause failure rates as state-dependent, Chung [56] derived expressions for relevant reliability measures for non-repairable and repairable $1 - out - of - n : G$ systems. In 1990, Dhillon and Viswanath [100] presented an extensive list of bibliographies on CCFs while Chung [61] derived Laplace transforms of state probabilities for a $k - out - of - n : G$ redundant system with CCFs, critical human errors and repairs.

Paula et al. [282], in 1991, suggested the concept of a cause-defence matrix as a means of adequately representing the different impacts plant-specific defenses have on different types of failure causes and showed how qualitative cause-defense matrices may be used to perform comprehensive CCF analyses for any nuclear power plant and Lai and Yuan [229] analyzed a parallel system composed of n identical components subject to CCFs and determined the optimum replacement period for the system based on average operating cost per unit of time. Later in 1991, the same authors [230] extended their previous model by incorporating repair and maintenance costs and developed algorithms for obtaining: optimal redundant units N^* for a fixed n; optimal number of repairs n^* for a fixed N and; optimal pair (N^*, n^*) simultaneously, under the assumption that the system is allowed to undergo at most a certain number of repairs before being replaced. Also in 1991, Hirschberg [196] outlined a brief summary of the status of CCFs treatment in Nordic Countries and posited that activities concentrated on: proper consideration of plant-specific defensive measures against CCFs; the generation of representative CCF data which take into account efficiency of such defenses and; the quantification of CCF effects on the reliability and safety of systems with ultra-high levels of redundancy while Rasmuson [299] reviewed the integrated framework presented by Mosleh et al. [262] and touched upon related issues such as the importance of having plant-specific data, the quantification of CCFs and some approximations that can be used in the quantification process and data collection needs for CCF events. System design principles adopted to ensure high system reliability and safety in German 1300 MW Siemens/KWU nuclear power plants and reduce CCFs are presented by Schilling and Dörre [306].

In 1992, Mankamo and Kosonen [250], in order to assess the failure probability of the pressure relief function in the Teollisuuden Voima Oy (TVO) Plant Study in Finland, introduced an extended version of the common-load model (CLM) for CCFs by incorporating a low probability extreme load distribution that enabled strong dependencies at high orders of failure multiplicity to be modeled. Hidaka [191], on the assumption of arbitrarily distributed system repair times, analyzed the stochastic model of a $r - out - of - n(F)$ redundant system with maintenance and common-cause failures while Dhillon and Yang [99] presented newly-developed expressions for system reliability indices for some warm standby systems subject to common-cause failures and human errors. Dhillon and Anude [116-123] studied several submodels of $k - out - of - (n + m) : G$ type systems composed of identical and non-identical components subject to a combination of CCFs and independent failures and derived new reliability, availability and mean time to failure expressions.

1.3 Motivation and Objectives of the Thesis

MOTIVATION: Past work did not address adequately the negative impact of Common-Cause Failures on the performance of redundant systems particularly in the following respects:

• Stochastic models have hardly been developed to study the impact of common-cause failures on the $k - out - of - (n + m) : G$ system in situations where there is only one healthy component. In other words, $CCFs$ have only been considered when more than one component is healthy in the system.

• $k - out - of - (n + m) : G$ system repair times have often been considered to be exponentially distributed. In practice, this assumption, through the analysis of raw data, could be considered to be largely unrealistic and generous. This study will attempt to generalize the repair process by considering other repair probability density functions which may be of more practical significance.

• The impact of standby unit(s) activation policies on the $k - out - of - (n + m) : G$

system has not been investigated.

• The maximization of the net income generated from operating repairable $k-out-of-(n+m):G$ systems by adopting certain system repair rates for both independent and common-cause failures has not been given any attention.

OBJECTIVES: The Thesis objectives are, thus, to:

• assess the reliability and formulate mean time to failure, long-run availability and variance of time to failure solution procedures for newly developed stochastic models of the $k-out-of-(n+m):G$ type systems subject to completely-coupled common-cause failures as well as independent failures for several possible repair scenarios. The fact that the $CCFs$ are completely-coupled ensures that their impact on system performance indices will be considered regardless of the number of healthy units in the system. Most previous studies had considered common-cause failures only when there is more than one healthy unit.

• assess the impact of standby activation policies on system performance indices notably the system long-run availability, reliability, mean time to failure. In this regard, two common standby activation policies (SBAPs) will be considered viz: *a)* a standby unit is activated as soon as one active unit fails (referred to as standby activation policy Y or SBAP Y). The $k-out-of-(n+m):G$ system general model developed for this SBAP is referred to as General Model Y in this Study and, *b)* a standby is activated as soon as the last remaining active unit required for overall system success fails (referred to as standby activation policy Z or SBAP Z). The $k-out-of-(n+m):G$ system general model developed for this SBAP is referred to as General Model Z. The performance of the system under both SBAPs will be compared using the system mean time to failure as the main measure of performance.

• assess the impact of generalizing the system repair time distribution on the system long-run availability. In this regard, the failed system repair times are considered to follow a *Gamma* probability density function. This is a fairly general probability density function and could be used to approximate the performance of many other probability density functions by merely changing the value of its shape parameter.

• derive expressions to determine the system repair rates for both independent and common-cause failures that will be required to maximize the system net income.

The main differences between the models proposed here and those studied by previous researchers are:

• The system-state transitions considered in this study are entirely new, original and relevant.

• Like Viswanath [336], the models in this study consider the CCFs completely-coupled and capable of triggering system failure from any of the operable states of the system but unlike the above study which considered only exponential system (and sub-system) repair times, this study would generalize the system repair time distribution.

Typical and practical values of relevant system parameters (such as to be found in Licensee Event Reports, *LERs*) have been used to validate most of the stochastic models developed in this Study.

1.4 Thesis Structure

This dissertation is divided into seven chapters.

Chapter 1: This chapter presents a general and detailed introduction to common-cause failures and a literature survey.

Chapter 2: This chapter presents a study of the $k - out - of - (n + m) : G$ (or n, m, k) system and its two standby activation policy models referred to as General Models Y and Z for some possible repair types and identical units.

Chapter 3: This chapter presents special cases of General Model Y reported in the previous chapter when system units are identical as well as non-identical. Some generalized solution methods for system reliability indices for possible repair types are also presented.

Chapter 4: This chapter presents special cases of General Model Z reported in Chapter 2 for identical and non-identical system units. In addition, some generalized solution methods for system reliability indices for possible repair types are given.

Chapter 5: This chapter presents the problem of maximizing the net income of $k - out - of - (n + m) : G$ type systems. Expressions for the system net income are derived and mathematical relationships useful for determining optimal system repair rates are presented.

Chapter 6: This chapter discusses the results of analyses conducted in Chapters 2 to 5.

Chapter 7: This chapter presents conclusions and future directions.

Chapter 2

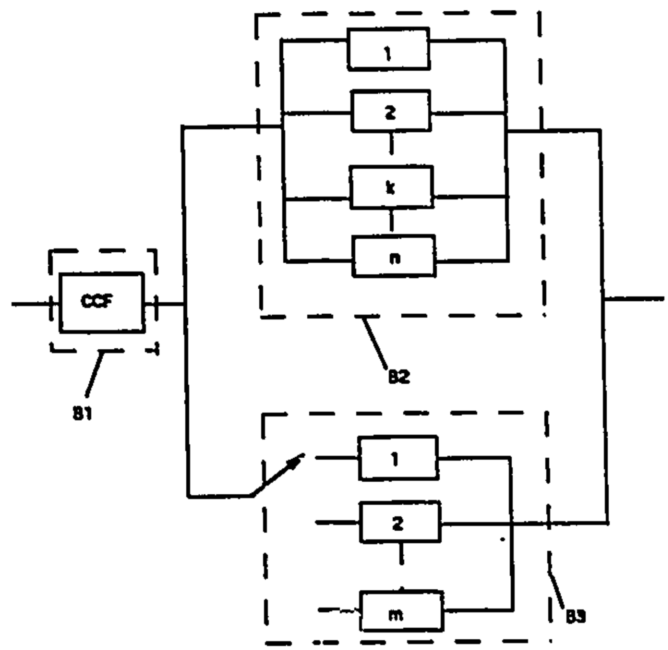
General Models of the Redundant System

The $k-out-of-(n+m):G$ redundant system is assumed to have n active units in parallel with m warm standbys. It is operable as long as at least k active units are functioning normally (where $k \leq n$). This type of redundant system is widely employed in industries ranging from the nuclear power industry to the chemical and petro-chemical industries. An example of such a system could be a power generating plant for a production facility which has *three* gas turbine engines and *one* additional engine on standby and which requires that at least *one* engine remain functional for its operation. A block diagram of this type of system is shown in Figure 2.1.

The following possible types of repair are considered in the analysis of this system:

i) Type A Repair: The system as well as its sub-systems (or units) are repaired upon failure. The failed unit repair rates are constant while the failed system repair times are arbitrarily distributed.

ii) Type B Repair: The system and only its failed standby units (when they fail in the standby mode) are repaired. Disabled active units are discarded and replaced.



BLOCK B1: represents common-cause failures of the system
BLOCK B2: represents the active on-line k-out-of-n system
BLOCK B3: represents the standby system composed of m standbys

Figure 2.1: Block Diagram of the $k - out - of - (n + m) : G$ System

iii) Type C Repair: The system is repaired upon failure but the repair of its units (both active and standbys) is considered unprofitable. Thus system units are discarded and replaced upon failure.

iv) Type D Repair: The failed system is not repaired but its units (both active and standbys) are repaired upon failure.

v) Type E Repair: The system and its active units upon failure are not repaired but the standbys are repaired when they fail in the standby mode.

vi) Type F Repair: The system and its units (both active and standbys) are not repaired when disabled.

The reliability analysis of the $k - out - of - (n + m) : G$ system with identical units under two different standby activation policies ($SBAPs$) is presented in the following sections.

2.1 The Redundant System With Early Standby Activation

The system is under standby activation policy Y. The state space diagram for this model is shown in Figure 2.2. At time $t = 0$, all n active units start operating instantaneously with the m warm standbys and their switches absolutely reliable. The standbys are activated as soon as an active unit fails. The standbys can also fail in their standby mode. The system can fail due to a common-cause failure from any of its operable states as well as due to independent failures. Markov state-space technique and Non-Markovian approaches such as the supplementary variables technique and Laplace transforms have been used to conduct analyses.

Assumptions

The following assumptions are associated with the model:

i) The system has $n + m$ identical units (n active and m on standby).

ii) The common-cause and non-common-cause failures are statistically independent.

iii) A common-cause failure can occur and trigger system failure from any of its operable states.

iv) All common-cause failures are completely coupled

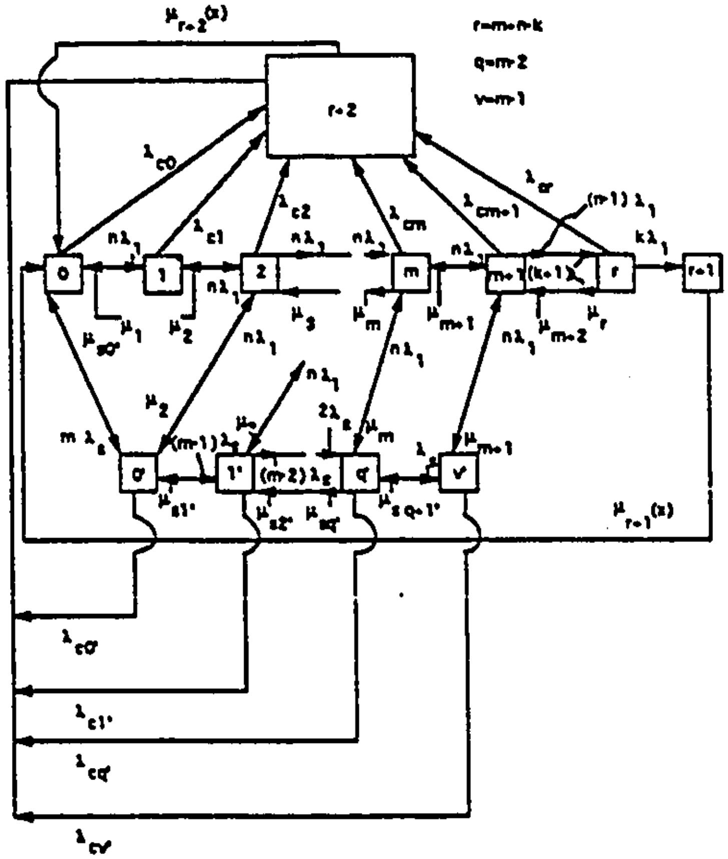


Figure 2.2: State space diagram of the $k-out-of-(n+m):G$ system under SBAP Y

and lethal.

v) As long as at least k units are functional, the system is operable.

vi) All failure rates are constant.

vii) All component repair rates, where applicable, are constant.

viii) Repair is unrestricted for both units and system.

ix) A repaired unit or system is as good as new.

x) The failed system repair times, where applicable, are assumed to be arbitrarily distributed.

xi) The switching mechanism for the standbys is considered automatic and instantaneous.

xii) The standbys and/or their switches can fail.

Notation

The following symbols are associated with the model:

i State identification number. For $i = 0$; all n active units and the m standby units are in perfect working condition. For $i = 1$; all the n active units and $m - 1$ standbys are healthy. For $i = 2$; all n active units and $m - 2$ standbys are healthy and so on. For $i = m$; all n active units and none of the standbys are healthy.

For $i = m + 1$; $n - 1$ active units and no standbys are healthy and for $i = m + 2$; $n - 2$ active units and no standbys are healthy and so on.
For $i = m + n - k$; only k active units are healthy and for $i = m + n + 1 - k$; the system has failed due to hardware failures. For $i = m + n + 2 - k$; the system has failed due to common-cause failures.
For $i = 0'$; *one* standby has failed in its standby mode and all n active units and $m - 1$ standbys remain good and for $i = 1'$; *two* standbys have failed in the standby mode and all n active units and $m - 2$ standbys remain healthy and so on. For $i = (m - 1)'$; all m standbys have failed in the standby mode but all n active units remain good.

t	time
s	Laplace transform variable
n	Number of system active units
m	Number of system standby units
k	Least number of good active on-line units needed for system success
λ_1	Constant failure rate of a unit
λ_s	Constant failure rate of a standby unit
λ_{cj}	Constant critical common-cause failure rate from system up-state j for $j = 0, 1, 2, ...r$,

	and $0', 1', 2', \ldots.., (m-1)'$
	where $r = m + n - k$.
pdf	Probability density function
$A_{ssY}(n, m, k)$	Steady-state availability of the system
$A_{vY}(n, m, k)(t)$	Time-dependent availability of the system
C.C.F	Common-cause failure
sb	Standby
$P_i(t)$	The probability that the system is in state i
	at time t for $i = 0, 1, 2, \ldots, r+2$ and $0', 1'$,
	$2', \ldots\ldots, (m-1)'$
μ_j	Constant repair rate from system up-state j
	for $j = 0, 1, 2, \ldots.., r$ and $0', 1', 2', \ldots(m-1)'$
$P_i(x, t)$	Probability that the failed system is in
	state i and has an elapsed repair time of x;
	for $i = r+1,\ r+2$
$\mu_i(x)$, $q_i(x)$	Time-dependent repair rate and *pdf* of repair time
	respectively when the system is in state i
	and has an elapsed repair time of x;
	for $i = r+1,\ r+2$
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*
SBAP Y	Standby activation policy Y
$\mathcal{L}^{-1}$	Inverse Laplace transform
$R'_Y(n, m, k)(s)$	The derivative with respect to s of the

system reliability function expressed in Laplace transforms

$\bullet$ Multiplication sign

Analysis

Under Repair Types A, B and C, the system is repairable and its repair times are arbitrarily distributed. The Gamma distribution is often used to model this type of repair situation. Thus the following relationships suffice:

$$q_{r+1}(x) = \frac{\mu_{r+1}\,(\mu_{r+1}\,x)^{\beta-1}\,e^{-\mu_{r+1}x}}{\Gamma(\beta)};\ x \geq 0, \beta > 0,\ r = m+n-k \tag{2.1}$$

$$q_{r+2}(x) = \frac{\mu_{r+2}\,(\mu_{r+2}\,x)^{\beta-1}\,e^{-\mu_{r+2}x}}{\Gamma(\beta)};\ x \geq 0, \beta > 0,\ r = m+n-k \tag{2.2}$$

The value of the shape parameter β determines the *pdf* of the system repair times. For example, when $\beta = 1$, the system repair rate is constant and the system repair times are exponentially distributed and when $\beta = 2$, the system repair times follow the Erlangian distribution. Gaussian failed system repair times are approximated when $\beta \approx 5$.

In situations where the failed system is repaired, the relevant reliability index is the system availability. This is a time-dependent index that converges to a certain value (the long-run or steady-state value) as time approaches infinity. For this model it is given by:

$$A_{v_Y}(n,m,k)(t) = \mathcal{L}^{-1}\,[\sum_{i=0}^{r} P_i(s) + \sum_{i=0'}^{(m-1)'} P_i(s)] \tag{2.3}$$

Its long-run value can easily be obtained using Abel's Theorem (p. 193, Ref. [89]) and is given by:

$$A_{ssY}(n,m,k) = \lim_{s \to 0} s \bullet [\sum_{i=0}^{r} P_i(s) + \sum_{i=0'}^{(m-1)'} P_i(s)] \tag{2.4}$$

With Repair Types D, E and F, the failed system is not repaired and the relevant reliability indices are the system reliability, the system mean time to failure and the system variance of time to failure. For General Model Y, these indices are given by:

$$R_Y(n,m,k)(t) = \mathcal{L}^{-1}[\sum_{i=0}^{r} P_i(s) + \sum_{i=0'}^{(m-1)'} P_i(s)] \tag{2.5}$$

The system mean time to failure is:

$$MTTF_Y(n,m,k) = \lim_{s \to 0} [\sum_{i=0}^{r} P_i(s) + \sum_{i=0'}^{(m-1)'} P_i(s)] \tag{2.6}$$

The system variance of time to failure is:

$$\sigma_Y^2(n,m,k) = -2 \lim_{s \to 0} R_Y'(n,m,k)(s) - [MTTF_Y(n,m,k)]^2 \tag{2.7}$$

Some special cases of the $k - out - of - (n+m) : G$ system operating under standby activation policy Y will be presented in Chapter 3.

2.2 The Redundant System With Late Standby Activation

The system is under standby activation policy Z. The state space diagram for this model is shown in Figure 2.3. At time $t = 0$, all n active units start operating instantaneously with the m warm standbys and their switches in perfect condition. The standbys are activated only when the last remaining active unit needed for system success fails. The standbys can also fail in their standby mode. The system can fail due to a common-cause failure from any of its operable states as well as due to non-common-cause failures. Markov state-space technique and Laplace transforms have been used to conduct analyses.

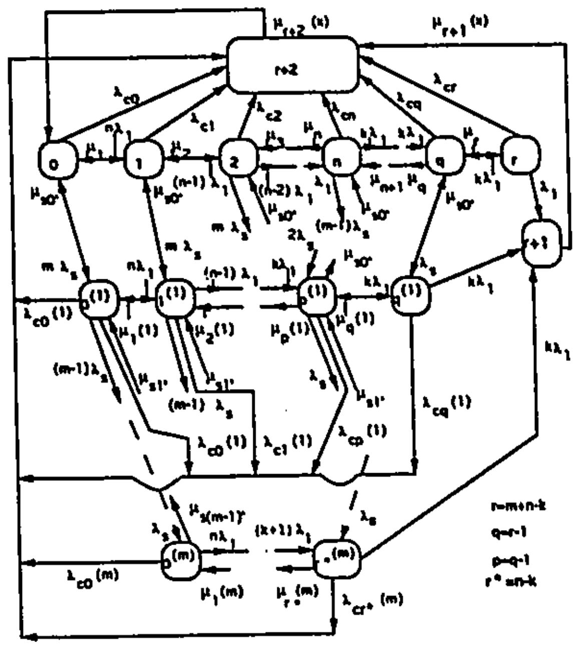


Figure 2.3: State space diagram of $k-out-of-(n+m):G$ system under SBAP Z

Assumptions

The assumptions associated with the previous model also apply. The only significant difference is that the activation of the standby is delayed until the last remaining active unit needed for system success fails.

Notation

The following symbols are associated with this system:

$j^{(i)}$	State identification numeral. For $j = 0, 1, ..n - k$, $i = 0$; $j^{(i)}$ indicates that j active units and none of the standbys have failed. For $j = n - k + 1, n - k + 2, ...((n + m) - (k + i))$, $i = 0$; $j^{(i)}$ indicates that j active units and $k + j - n$ standbys have failed. For $j = m + n - k + 1$ and $i = 0$; the system is considered to have failed due to hardware failures and for $j = m + n - k + 2$ and $i = 0$; the system is considered to have failed due to a common-cause. For $j = 0, 1, 2, ., n - k$, $i = 1, 2, ..m$; $j^{(i)}$ indicates that j active units and *one* standby have failed while for $j = n - k + 1$, $n - k + 2, ...((n + m) - (k + j)$, $i = 1, 2, ..., m$; $j^{(i)}$ indicates that j active units and i standbys have failed. For this system, state j is the same as state $j^{(0)}$ when $i = 0$.
t	time
s	Laplace transform variable
n	Number of system active units
m	Number of system standby units
k	Least number of good active on-line system units needed for system success
λ_1	Constant failure rate of a unit

λ_s	Constant failure rate of a standby unit
$\mu_{si'}$	Constant repair rate from a state where $i+1$ standbys are disabled, where $i = 0, 1, ..m-1$
$\lambda_{cj^{(i)}}$	Constant critical common-cause failure rate from system up-state $j^{(i)}$ for $j = 0, 1, 2, ...$ $..., ((m+n)-(i+k))$ and for $i = 0, 1, 2, ..., m$
pdf	Probability density function
$A_{ssZ}(n, m, k)$	Steady-state availability of the system
$A_{vZ}(n, m, k)(t)$	Time-dependent availability of the system
$C.C.F$	Common-cause failure
sb	Standby
SBAP Z	Standby activation policy Z
$P_j(t)$	The probability that the system is in state j at time t for $j = 0, 1, 2, ...r+2$, $i = 0$, where $r = m + n - k$
$P_{j^{(i)}}(t)$	The probability that the system is in state $j^{(i)}$ at time t for $j = 0, 1, 2, .., ((m+n)-(i+k))$ and $i = 1, 2, ..., m$
$\mu_{j^{(i)}}$	Constant repair rate from system up-state $j^{(i)}$ for $j = 0, 1, 2, ..., ((m+n)-(i+k))$ and for $i = 1, 2, ..., m$
$P_i(x, t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = r+1,\ r+2$
$\mu_i(x)$, $q_i(x)$	Time-dependent repair rate and *pdf* of repair time

	respectively when the system is in state i
	and has an elapsed repair time of x;
	for $i = r+1,\ r+2$
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_j(s)$	The Laplace transform of $P_j(t)$
$P_{j(i)}(s)$	The Laplace transform of $P_{j(i)}(t)$
β	Shape parameter of the gamma *pdf*
$\pounds^{-1}$	Inverse Laplace transform
$R'_Z(n,m,k)(s)$	The derivative with respect to s of the system reliability function expressed in Laplace transforms
$\bullet$	Multiplication sign

Analysis

With Repair Types A, B and C the long-run and time-dependent availabilities of this model are given respectively by:

$$A_{ssz}(n,m,k) = \lim_{s \to 0} s \bullet [\sum_{i=0}^{r} P_i(s) + \sum_{i=0^{(1)}}^{r-1^{(1)}} P_i(s) + \sum_{i=0^{(2)}}^{r-2^{(2)}} P_i(s) + \dots$$
$$\dots + \sum_{i=0^{(m)}}^{(r^*)^{(m)}} P_i(s)] \tag{2.8}$$

$$A_{vz}(n,m,k)(t) = \pounds^{-1}[\sum_{i=0}^{r} P_i(s) + \sum_{i=0^{(1)}}^{r-1^{(1)}} P_i(s) + \sum_{i=0^{(2)}}^{r-2^{(2)}} P_i(s) + \dots$$
$$.. + \sum_{i=0^{(m)}}^{(r^*)^{(m)}} P_i(s)] \tag{2.9}$$

where

$r^{*} = [(m+n)-(i+k)]$

With Repair Types D, E, and F the relevant reliability indices are the system reliability, the system mean time to failure and the system variance of time to failure. These are given respectively by:

$$R_Z(n,m,k)(t) = \mathcal{L}^{-1}[\sum_{i=0}^{r} P_i(s) + \sum_{i=0^{(1)}}^{r-1^{(1)}} P_i(s) + \sum_{i=0^{(2)}}^{r-2^{(2)}} P_i(s) + ...$$

$$... + \sum_{i=0^{(m)}}^{(r^{*})^{(m)}} P_i(s)] \tag{2.10}$$

$$MTTF_Z(n,m,k) = \lim_{s\to 0}[\sum_{i=0}^{r} P_i(s) + \sum_{i=0^{(1)}}^{r-1^{(1)}} P_i(s) + \sum_{i=0^{(2)}}^{r-2^{(2)}} P_i(s) + ...$$

$$.... + \sum_{i=0^{(m)}}^{(r^{*})^{(m)}} P_i(s)] \tag{2.11}$$

$$\sigma_Z^2(n,m,k) = -2\lim_{s\to 0} R_Z'(n,m,k)(s) - [MTTF_Z(n,m,k)]^2 \tag{2.12}$$

Some special cases of the $k-out-of-(n+m):G$ operating under standby activation policy Z will be presented in Chapter 4.

Chapter 3

The Redundant System With Early Standby Activation: Special Cases

To develop generalized procedures for assessing the impact of common-cause failures on system reliability indices, it is necessary to consider some special cases of the general standby activation policy models Y and Z. This chapter will consider several special cases of General Model Y while the next chapter will deal with special cases relating to General Model Z. The proceeding sections will study several special cases resulting in the development of some general and special-case expressions for the mean time to failure, reliability, availability and variance of time to failure of system standby activation policy General Model Y. These expressions will be presented in terms of the number of active units n, the number of standby units m and the number of healthy units required for system success k for the various possible repair types introduced in the last chapter when system units are identical and non-identical.

The following values of basic system parameters which are typical in a wide range of engineering applications would be specified for the special cases considered in this Chapter and the next:

For SBAP Y:

$\lambda_1 = 0.0004$ per hr., $\lambda_s = 0.0001$ per hr., λ_{c0}, which is the critical common-cause failure rate is used as the independent system parameter in this study and for most engineering applications its value lies within the range $(0.01 - 0.30)\,\lambda_1$. Virtually all the reliability analysis will be done with a λ_{c0} value of $0.1\,\lambda_1$. For $i = 1, 2, ..m+n-k$, $\lambda_{ci+1} = 0.5\,\lambda_{ci}$; $\lambda_{c0'} = 0.00003$ per hr; and for $i = 0', 1', ...(m-1)'$, $\lambda_{ci+1} = 0.5\,\lambda_{ci}$. $\mu_{si} = 0.0006$ per hr. for $i = 0, 1, ..., (m-1)'$ (where applicable) and $\mu_1 = 0.0004$ per hr. and $\mu_i = 0.00015$ per hr., for $i = 2, 3, ...r$, $\mu_{r+1} = 0.00015$ per hr., and $\mu_{r+2} = 0.0003$ per hr. (where applicable) and $r = m + n - k$.

For SBAP Z:

$\lambda_1 = 0.0004$ per hr., $\lambda_s = 0.0001$ per hr., For $0^{(i)}$ where $j = 0$ and $i = 1, 2, 3..., m$, $\lambda_{c0(i+1)} = 0.5\,\lambda_{c0(i)}$. For $j = 1, 2, .., ((m+n) - (k+i))$ and $i = 1, 2, .., m$, $\lambda_{cj(i+1)} = 0.5\,\lambda_{cj(i)}$. $\mu_i = 0.0006$ per hr., for $i = 0', 1', ...(m-1)'$ (where applicable). $\mu_{1(0)} = 0.0004$ per hr., and for $i = 0, 1, .., m$ and $j = 2, 3, ..((m+n) - (k+i))$, $\mu_{j(i)} = 0.00015$ per hr. (where applicable) and also $\mu_{r+1} = 0.00015$ per hr. and $\mu_{r+2} = 0.0003$ per hr. (where applicable) and $r = m + n - k$.

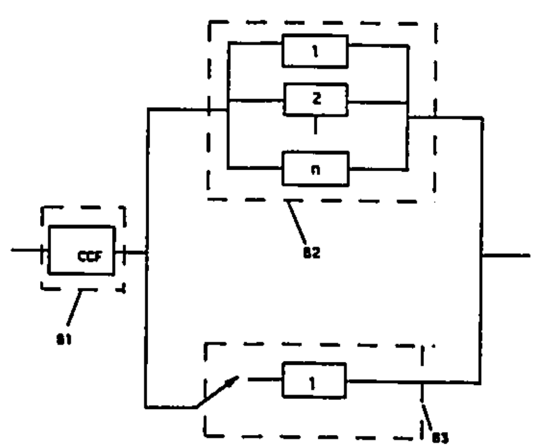
In cases where system units are identical, in order to lessen computational difficulties in situations where $m > 1$, only the reliability analysis results of repair types under which components are not repaired will be presented. In cases where system units are non-identical, only duplex systems would be considered because the number of system up-states increases astronomically with the addition of just one system unit (as indicated by the formula in Appendix J.1).

3.1 The Identical Unit System With Early Standby Activation: Special Cases

We will now consider some special cases of General Model Y when system units are identical and in this regard it is convenient to group these special cases into the following submodels of the $k-out-of-(n+m):G$ system viz: *(i)* $1-out-of-n+1:G$ or $(n,1,1)$ *(ii)* $1-out-of-n+2:G$ or $(n,2,1)$ *(iii)* $1-out-of-n+3:G$ or $(n,3,1)$ *(iv)* $k-out-of-n+1:G$ or $(n,1,k)$ *(v)* $k-out-of-n+2:G$ or $(n,2,k)$ *(vi)* $k-out-of-n+3:G$ or $(n,3,k)$.

3.1.1 1-out-of-(n+1):G Identical Unit Submodel

This is the $(n,1,1)$ system and is composed of n active units and *one* standby. It is functional as long as at least *one* unit is healthy. A block diagram of this submodel is shown in Figure 3.1 and the state space diagram in Figure 3.2



BLOCK B1: represents common-cause failures of the system
BLOCK B2: represents the active parallel 1-out-of-n system
BLOCK B3: represents the lone standby

Figure 3.1: Block diagram of the $1-out-of-(n+1):G$ system

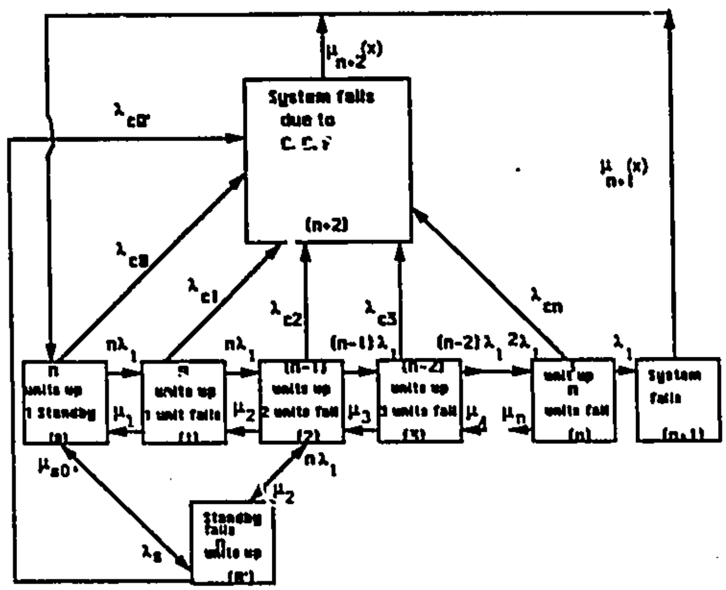


Figure 3.2: State space diagram of the $1-out-of-(n+1):G$ system under SBAP Y

$1 - out - of - (n+1) : G$ **system with Type** A **Repair**

The corresponding integro-differential equations for the model described in Figure 3.2 are:

$$\frac{dP_0(t)}{dt} + C_{n,1}(0)\, P_0(t) = \mu_1\, P_1(t) + \mu_{s0'}\, P_{0'}(t) + \sum_{i=n+1}^{n+2} \int_0^\infty P_i(x,t)\, \mu_i(x) dx \quad (3.1)$$

$$\frac{dP_1(t)}{dt} + C_{n,1}(1)\, P_1(t) = n\, \lambda_1\, P_0(t) + \mu_2\, P_2(t) \quad (3.2)$$

$$\frac{dP_2(t)}{dt} + C_{n,1}(2)\, P_2(t) = n\, \lambda_1\, P_1(t) + n\, \lambda_1\, P_{0'}(t) + \mu_3\, P_3(t) \quad (3.3)$$

For $j = 3, 4, \ldots\ldots, n-1$,

$$\frac{dP_j(t)}{dt} + C_{n,1}(j)\, P_j(t) = (n - j + 2)\, \lambda_1\, P_{j-1}(t) + \mu_{j+1}\, P_{j+1} \quad (3.4)$$

$$\frac{dP_n(t)}{dt} + C_{n,1}(n)\, P_n(t) = 2\, \lambda_1\, P_{n-1}(t) \quad (3.5)$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,1}(0')\, P_{0'}(t) = \lambda_s\, P_0(t) + \mu_2\, P_2(t) \quad (3.6)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+1}(x)]\, P_{n+1}(x,t) = 0 \quad (3.7)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+2}(x)]\, P_{n+2}(x,t) = 0 \quad (3.8)$$

The associated boundary conditions are:

$$P_{n+1}(0,t) = \lambda_1\, P_n(t) \quad (3.9)$$

$$P_{n+2}(0,t) = \sum_{i=0}^{n} \lambda_{ci}\, P_i(t) + \lambda_{c0'}\, P_{0'}(t) \quad (3.10)$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero. where,

$C_{n,1}(0) = n\, \lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = n\, \lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,1}(2) = (n-1)\, \lambda_1 + \lambda_{c2} + \mu_2$

For $j = 3, 4, \ldots\ldots., n-1$,

$C_{n,1}(j) = (n - j + 1)\,\lambda_1 + \lambda_{cj} + \mu_j$
$C_{n,1}(n) = \lambda_1 + \lambda_{cn} + \mu_n$
$C_{n,1}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

The procedure used to determine the long-run availability for all the models analyzed in this study is described in detail for the first special case model only. For subsequent special case models only the results are presented.

Special Case Model I

For $n = 1$ in Equations (3.1) to (3.10) we obtain the following system of integro-differential equations:

$$\frac{dP_0(t)}{dt} + C_{1,1}(0)\,P_0(t) = \mu_1\,P_1(t) + \mu_{s0'}\,P_{0'}(t) + \sum_{2}^{3}\int_0^\infty P_i(x,t)\,\mu_i(x)\,dx \quad (3.11)$$

$$\frac{dP_1(t)}{dt} + C_{1,1}(1)\,P_1(t) = \lambda_1 P_0(t) \quad (3.12)$$

$$\frac{dP_{0'}(t)}{dt} + C_{1,1}(0')\,P_{0'}(t) = \lambda_s P_0(t) \quad (3.13)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_2(x)]\,P_2(x,t) = 0 \quad (3.14)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_3(x)]\,P_3(x,t) = 0 \quad (3.15)$$

The associated boundary conditions are:

$$P_2(0,t) = \lambda_1\,P_1(t) + \lambda_1\,P_{0'}(t) \quad (3.16)$$

$$P_3(0,t) = \lambda_{c0}\,P_0(t) + \lambda_{c1}\,P_1(t) + \lambda_{c0'}\,P_{0'}(t) \quad (3.17)$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.
where

$C_{1,1}(0) = \lambda_1 + \lambda_s + \lambda_{c0}$
$C_{1,1}(1) = \lambda_1 + \lambda_{c1} + \mu_1$
$C_{1,1}(0') = \lambda_1 + \lambda_{c0'} + \mu_{s0'}$

Equations (3.11) to (3.15) are transformed into their Laplace transform equivalents and the resulting Laplace transform equations are solved simultaneously to yield the following relationships:

$$P_0(s) = \frac{W_1(s)}{W_2(s) - W_3(s)\, q_2(s) - W_4(s)\, q_3(s)} \tag{3.18}$$

$$P_1(s) = \frac{\lambda_1}{s + C_{1,1}(1)} \bullet P_0(s) \tag{3.19}$$

$$P_{0'}(s) = \frac{\lambda_s}{s + C_{1,1}(0')} \bullet P_0(s) \tag{3.20}$$

$$P_2(s) = [\frac{\lambda_1^2}{(s + C_{1,1}(1))} + \frac{\lambda_1\,\lambda_s}{(s + C_{1,1}(0'))}] \bullet [\frac{1 - q_2(s)}{s}] \bullet P_0(s) \tag{3.21}$$

$$P_3(s) = [\lambda_{c0} + \frac{\lambda_1\,\lambda_{c1}}{(s + C_{1,1}(1))} + \frac{\lambda_s\,\lambda_{c0'}}{(s + C_{1,1}(0'))}] \bullet [\frac{1 - q_3(s)}{s}] \bullet P_0(s) \tag{3.22}$$

where

$W_1(s) = s^2 + A_1\, s + A_2$

$W_2(s) = s^3 + A_3\, s^2 + A_4\, s + A_5$

$W_3(s) = A_6\, s + A_7$

$W_4(s) = A_8\, s^2 + A_9\, s + A_{10}$

Symbols A_1 to A_{10} are defined in Appendix B.

With Type A Repair, the failed system is repaired upon failure and is assumed to have arbitrarily distributed repair times, thus, the following relationships suffice:

$$q_2(x) = \frac{\mu_2(\mu_2 x)^{\beta-1} e^{-\mu_2 x}}{\Gamma(\beta)};\ t \geq 0,\ \beta > 0 \tag{3.23}$$

$$q_3(x) = \frac{\mu_3(\mu_3 x)^{\beta-1} e^{-\mu_3 x}}{\Gamma(\beta)};\ t \geq 0,\ \beta > 0 \tag{3.24}$$

a) For $\beta = 1$, the system repair time distribution is exponential and thus Equations (3.23) and (3.24) become:

$$q_2(x) = \mu_2\, e^{-\mu_2 x} \tag{3.25}$$

$$q_3(x) = \mu_3\, e^{-\mu_3 x} \tag{3.26}$$

Their equivalent Laplace transforms are:

$$q_2(s) = \frac{\mu_2}{s + \mu_2} \tag{3.27}$$

$$q_3(s) = \frac{\mu_3}{s + \mu_3} \tag{3.28}$$

Using Equations (3.18) to (3.20) which represent the up-states of this special case model, we obtained the following steady-state probabilities:

$$P_0 = \frac{\mu_2\,\mu_3\,b_0}{\mu_2\,\mu_3\,x_2 + \mu_2\,x_3 + \mu_3\,x_4} \tag{3.29}$$

$$P_1 = \frac{\mu_2\,\mu_3\,b_1}{\mu_2\,\mu_3\,x_2 + \mu_2\,x_3 + \mu_3\,x_4} \tag{3.30}$$

$$P_{0'} = \frac{\mu_2\,\mu_3\,b_{0'}}{\mu_2\,\mu_3\,x_2 + \mu_2\,x_3 + \mu_3\,x_4} \tag{3.31}$$

The steady-state availability for this system using Equation (2.4) is:

$$\begin{aligned} A_{ssY}(1,1,1)(\beta = 1) &= P_0 + P_1 + P_{0'} \\ &= \frac{\mu_2\,\mu_3\,x_1}{\mu_2\,\mu_3\,x_2 + \mu_2\,x_3 + \mu_3\,x_4} \end{aligned} \tag{3.32}$$

where

$b_0 = C_{1,1}(1)\,C_{1,1}(0')$

$b_1 = \lambda_1\,C_{1,1}(0')$

$b_{0'} = \lambda_s\,C_{1,1}(1)$

$x_1 = b_0 + b_1 + b_{0'}$

$x_2 = A_4 - A_6 - A_9$

$x_3 = A_5 - A_7$

$x_4 = A_5 - A_{10}$

b) For $\beta = 2$, the system repair times follow an Erlangian distribution and the failed system repair rates are time-dependent. Thus Equations (3.23) and (3.24) respectively become:

$$q_2(x) = \mu_2^2\,x\,e^{-\mu_2 x} \tag{3.33}$$

$$q_3(x) = \mu_3^2\,x\,e^{-\mu_3 x} \tag{3.34}$$

The equivalent Laplace transform expressions are:

$$q_2(s) = \frac{\mu_2^2}{(s+\mu_2)^2} \tag{3.35}$$

$$q_3(s) = \frac{\mu_3^2}{(s+\mu_3)^2} \tag{3.36}$$

The steady-state availability for this failed system repair time model is determined to be given by:

$$A_{ssY}(1,1,1)\,(\beta=2) = \frac{(\mu_2\,\mu_3)^2\,x_1}{(\mu_2\,\mu_3)^2\,x_2 + 2\,\mu_2^2\,\mu_3\,x_3 + 2\,\mu_3^2\,\mu_2\,x_4} \tag{3.37}$$

Similarly for $\beta = 3$, the steady-state availability is given by:

$$A_{ssY}(1,1,1)\,(\beta=3) = \frac{(\mu_2\,\mu_3)^3\,x_1}{(\mu_2\,\mu_3)^3\,x_2 + 3\,\mu_2^3\,\mu_3^2\,x_3 + 3\,\mu_3^3\,\mu_2^2\,x_4} \tag{3.38}$$

Gaussian failed system repair times are approximated when $\beta = 5$ and the steady-state availability when this is the case is given by:

$$A_{ssY}(1,1,1)\,(\beta=5) = \frac{(\mu_2\,\mu_3)^5\,x_1}{(\mu_2\,\mu_3)^5\,x_2 + 5\,\mu_2^5\,\mu_3^4\,x_3 + 5\,\mu_2^4\,\mu_3^5\,x_4} \tag{3.39}$$

From the fore-going analysis, it can be seen that the steady-state availability can be expressed as a function of the Gamma *pdf* shape parameter β as:

$$A_{ssY}(1,1,1)\,(\beta) = \frac{(\mu_2\,\mu_3)^\beta\,x_1}{(\mu_2\,\mu_3)^\beta\,x_2 + \beta\,\mu_2^\beta\,\mu_3^{\beta-1}\,x_3 + \beta\,\mu_2^{\beta-1}\,\mu_3^\beta\,x_4} \tag{3.40}$$

The plots of Equation (3.40) for various values of β and for specified values of system parameters are shown in Figure 3.3 and the corresponding tabular values are shown in the second column of Table 3.1. The trends shown by these values and those of the other special cases are discussed in the concluding section of this Chapter.

Special Case Model II

For $n = 2$ in Equations (3.1) to (3.10) and using the same procedure outlined for the previous special case, the following result is obtained for the long-run availability:

$$A_{ssY}(2,1,1)\,(\beta) = \frac{(\mu_3\,\mu_4)^\beta\,x_5}{(\mu_3\,\mu_4)^\beta\,x_6 + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_7 + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_8} \tag{3.41}$$

Table 3.1: $A_{ssY}(n, 1, 1)\,(\beta)$ values as a function of λ_{c0} for Type A Repair

	λ_{c0} (per Hr.)	$n = 1$	$n = 2$	$n = 3$
$\beta = 1$	0.00001	0.4936	0.5027	0.5195
	0.00002	0.4881	0.4991	0.5171
	0.00003	0.4827	0.4956	0.5146
	0.00004	0.4774	0.4921	0.5122
$\beta = 2$	0.00001	0.3277	0.3357	0.3509
	0.00002	0.3228	0.3326	0.3487
	0.00003	0.3181	0.3294	0.3465
	0.00004	0.3135	0.3264	0.3442
$\beta = 3$	0.00001	0.2452	0.2520	0.2649
	0.00002	0.2412	0.2493	0.2630
	0.00003	0.2372	0.2467	0.2611
	0.00004	0.2334	0.2441	0.2592
$\beta = 4$	0.00001	0.1959	0.2017	0.2128
	0.00002	0.1925	0.1994	0.2111
	0.00003	0.1891	0.1972	0.2095
	0.00004	0.1859	0.1950	0.2079
$\beta = 5$	0.00001	0.1631	0.1681	0.1778
	0.00002	0.1601	0.1662	0.1764
	0.00003	0.1573	0.1642	0.1749
	0.00004	0.1545	0.1623	0.1735

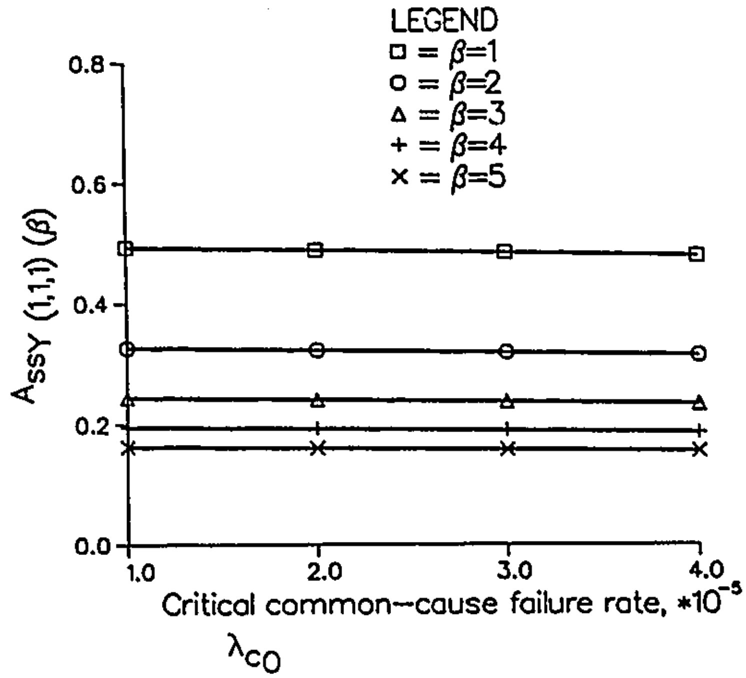


Figure 3.3: $A_{ssy}(1,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type A Repair

Where

$x_5 = d_0 + d_1 + d_2 + d_{0'}$

$x_6 = A_{24} - A_{26} - A_{30}$

$x_7 = A_{25} - A_{27}$

$x_8 = A_{25} - A_{31}$

and,

$d_0 = C_{2,1}(1)\,C_{2,1}(2)\,C_{2,1}(0') - 2\,\mu_2\,\lambda_1\,C_{2,1}(1) - 2\,\mu_2\,\lambda_1\,C_{2,1}(0')$

$d_1 = 2\,\lambda_1\,C_{2,1}(2)\,C_{2,1}(0') - 4\,\lambda_1^2\,\mu_2 + 2\,\mu_2\,\lambda_1\,\lambda_s$

$d_2 = 2\,\lambda_1\,\lambda_s\,C_{2,1}(1) + 4\,\lambda_1^2\,C_{2,1}(0')$

$d_{0'} = \lambda_s\,C_{2,1}(1)\,C_{2,1}(2) + 4\,\lambda_1^2\,\mu_2 - 2\,\lambda_1\,\lambda_s\,\mu_2$

$C_{2,1}(0) = 2\,\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{2,1}(1) = 2\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{2,1}(0') = 2\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{2,1}(2) = \lambda_1 + \lambda_{c2} + 2\,\mu_2$

The plots of Equation (3.41) for various values of β and for specified values of system parameters are shown in Figure 3.4 and the corresponding tabular values are shown in the third column of Table 3.1. Symbols A_{24} to A_{31} are defined in Appendix B.

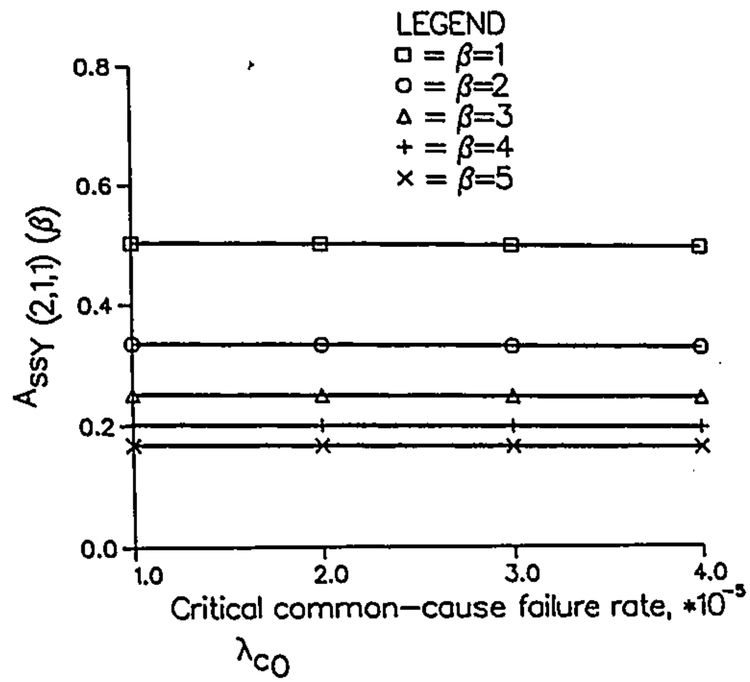


Figure 3.4: $A_{ssy}(2,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type A Repair

Special Case Model III

For $n = 3$ in Equations (3.1) to (3.10) and using the same procedure outlined for the previous special cases, the following result is obtained for the long-run availability:

$$A_{ssy}(3,1,1)\,(\beta) = \frac{(\mu_4\,\mu_5)^\beta\, x_9}{(\mu_4\,\mu_5)^\beta\, x_{10} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{11} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{12}} \tag{3.42}$$

Where,

$$\begin{aligned}
x_9 &= e_0 + e_1 + e_2 + e_3 + e_{0'} \\
x_{10} &= A_{39} + A_{38}\,C_{3,1}(0) - \mu_1\,A_{34} - \mu_{s0'}\,A_{47} - \lambda_1\,A_{43} - \lambda_{c0}\,A_{38} - \lambda_{c1}\,A_{34} - \\
&\quad \lambda_{c2}\,A_{41} - \lambda_{c3}\,A_{43} - \lambda_{c0'}\,A_{47} \\
x_{11} &= A_{39}\,C_{3,1}(0) - \mu_1\,A_{35} - \mu_{s0'}\,A_{48} - \lambda_1\,A_{44}
\end{aligned}$$

$$
\begin{aligned}
x_{12} &= A_{39}\, C_{3,1}(0) - \mu_1\, A_{35} - \mu_{s0'}\, A_{48} - \lambda_{c0}\, A_{39} - \lambda_{c1}\, A_{35} - \lambda_{c2}\, A_{42} - \\
&\quad \lambda_{c3}\, A_{44} - \lambda_{c0'}\, A_{48} \\
e_0 &= C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(3)\, C_{3,1}(0') - 3\,\lambda_1\, \mu_2\, C_{3,1}(1)\, C_{3,1}(3) - \\
&\quad 3\,\lambda_1\, \mu_2\, C_{3,1}(3)\, C_{3,1}(0') + 2\,\lambda_1\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0') \\
e_1 &= 3\,\lambda_1\, C_{3,1}(2)\, C_{3,1}(3)\, C_{3,1}(0') - 9\,\lambda_1^2\, \mu_2\, C_{3,1}(3) - 6\,\lambda_1^2\, \mu_3\, C_{3,1}(0') + \\
&\quad 3\,\lambda_1\, \lambda_s\, \mu_2\, C_{3,1}(3) \\
e_2 &= 3\,\lambda_1\, \lambda_s\, C_{3,1}(1)\, C_{3,1}(3) + 9\,\lambda_1^2\, C_{3,1}(3)\, C_{3,1}(0') \\
e_3 &= 6\,\lambda_1^2\, \lambda_s\, C_{3,1}(1) + 18\,\lambda_1^3\, C_{3,1}(0') \\
e_{0'} &= \lambda_s\, C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(3) - 2\,\lambda_1\, \lambda_s\, \mu_3\, C_{3,1}(1) + 9\,\lambda_1^2\, \mu_2\, C_{3,1}(3) - \\
&\quad 3\,\lambda_1\, \lambda_s\, \mu_2\, C_{3,1}(3)
\end{aligned}
$$

$C_{3,1}(0) = 3\,\lambda_1 + \lambda_{c0} + \lambda_s$

$C_{3,1}(1) = 3\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{3,1}(0') = 3\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{3,1}(2) = 2\,\lambda_1 + \lambda_{c2} + 2\,\mu_2$

$C_{3,1}(3) = \lambda_1 + \lambda_{c3} + \mu_3$

The plots of Equation (3.42) for various values of β and for specified values of system parameters are shown in Figure 3.5 and the corresponding tabular values are shown in the fourth column of Table 3.1. Symbols A_{34} to A_{48} are defined in Appendix B.

$1 - out - of - (n+1) : G$ system with Type B Repair

Setting the repair rates μ_i equal to zero for $i = 1, 2, ..., n$ in Equations (3.1) to (3.10) the system equations for the $1 - out - of - (n+1) : G$ system with Type B Repair can be obtained.

Special Case Model I

For $n = 1$ with Type B Repair, the system long-run availability can be determined directly from Equation (3.40) by setting μ_1 to zero. The results obtained are plotted in Figure 3.6 and tabular values are shown in the second column of Table 3.2

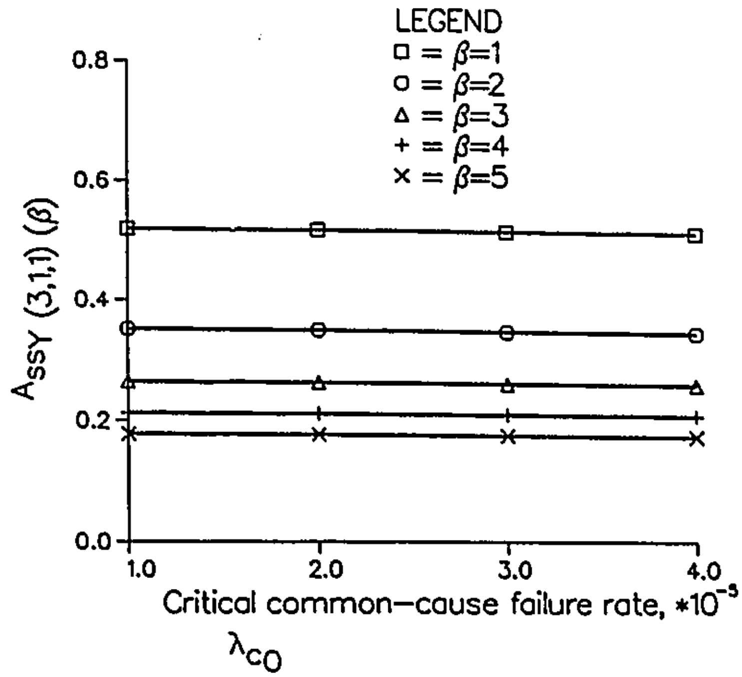


Figure 3.5: $A_{ssY}(3,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type A Repair

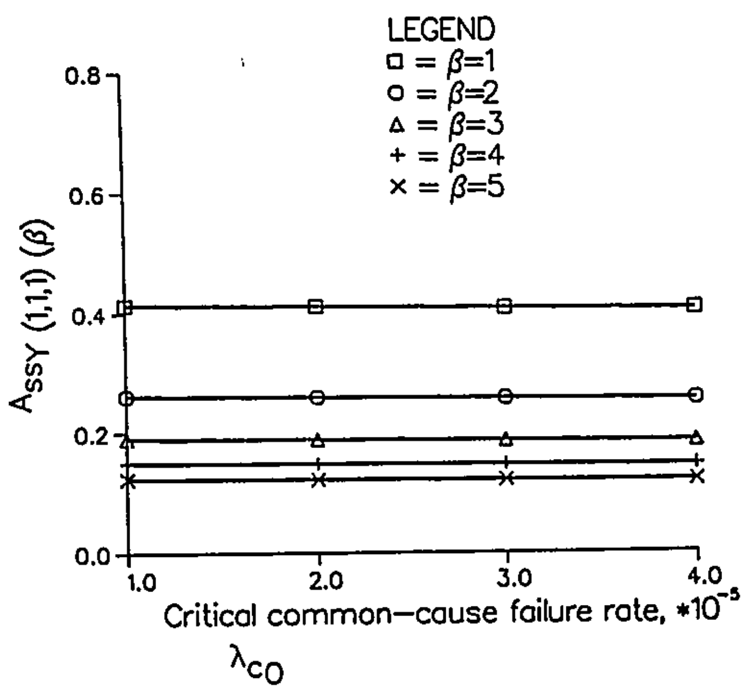


Figure 3.6: $A_{ssY}(1,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type B Repair

Table 3.2: $A_{ssY}(n,1,1)(\beta)$ values as a function of λ_{c0} for Type B Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4139	0.4224	0.4447
	0.00002	0.4111	0.4206	0.4432
	0.00003	0.4083	0.4188	0.4418
	0.00004	0.4055	0.4169	0.4403
$\beta=2$	0.00001	0.2610	0.2678	0.2859
	0.00002	0.2587	0.2663	0.2847
	0.00003	0.2565	0.2648	0.2835
	0.00004	0.2543	0.2634	0.2823
$\beta=3$	0.00001	0.1905	0.1960	0.2107
	0.00002	0.1888	0.1948	0.2097
	0.00003	0.1870	0.1937	0.2087
	0.00004	0.1852	0.1925	0.2077
$\beta=4$	0.00001	0.1501	0.1546	0.1668
	0.00002	0.1486	0.1536	0.1660
	0.00003	0.1471	0.1526	0.1652
	0.00004	0.1457	0.1517	0.1643
$\beta=5$	0.00001	0.1238	0.1276	0.1380
	0.00002	0.1225	0.1268	0.1373
	0.00003	0.1213	0.1259	0.1366
	0.00004	0.1200	0.1251	0.1359

Special Case Model II

For $n = 2$ with Type B Repair the long-run availability of the system can be determined directly from Equation (3.41) by setting μ_1 and μ_2 to zero. The results obtained are plotted in Figure 3.7 and tabular values are shown in the third column of Table 3.2.

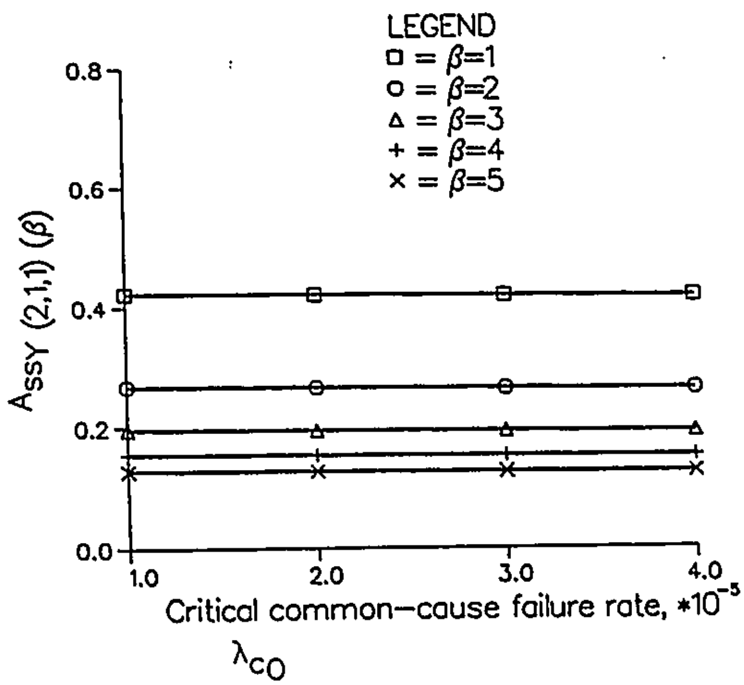


Figure 3.7: $A_{ssy}(2,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type B Repair

Special Case Model III

For $n = 3$ with Type B Repair the long-run availability for this special case is obtained directly from Equation (3.42) by setting μ_1, μ_2 and μ_3 to zero. The results obtained are plotted in Figure 3.8 and tabular values are shown in the fourth column of Table 3.2.

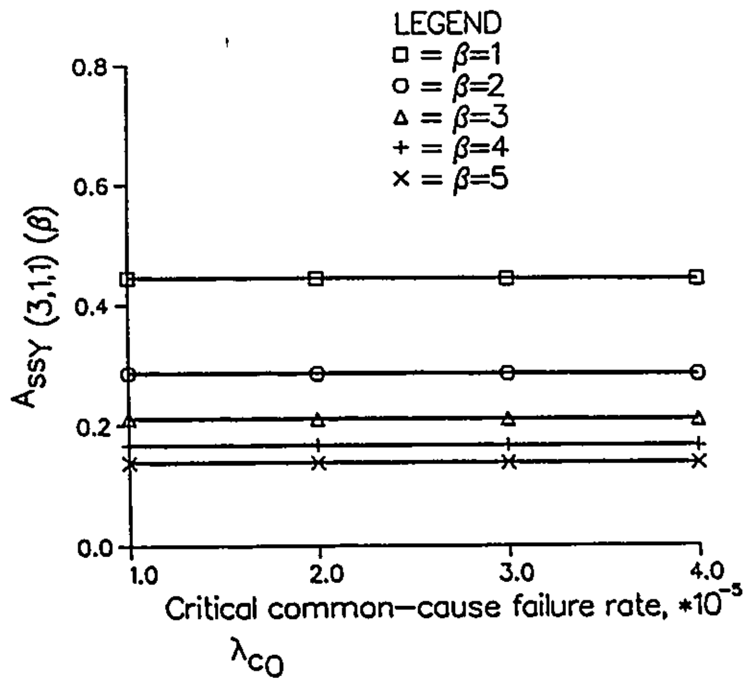


Figure 3.8: $A_{ssY}(3,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type B Repair

$1 - out - of - (n+1) : G$ System with Type C Repair

Setting the repair rates μ_i; for $i = 1, 2, ..., n$ and $\mu_{s0'}$ equal to zero in Equations (3.1) to (3.10) the system equations for the $1 - out - of - (n+1) : G$ system with Type C Repair can be obtained.

Special Case Model I

For $n = 1$ with Type C Repair the long-run availability for this special case is obtained directly from Equation (3.40) by setting μ_1 and $\mu_{s0'}$ to zero. The results obtained are plotted in Figure 3.9 and tabular values are shown in the second column of Table 3.3.

Special Case Model II

For $n = 2$ with Type C Repair the long-run availability for this special case is obtained directly from Equation (3.41) by setting μ_1, μ_2 and $\mu_{s0'}$ to zero. The results

Table 3.3: $A_{ss_Y}(n,1,1)\,(\beta)$ values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4002	0.4028	0.4437
	0.00002	0.3977	0.4012	0.4423
	0.00003	0.3952	0.3997	0.4409
	0.00004	0.3926	0.3981	0.4394
$\beta=2$	0.00001	0.2502	0.2608	0.2851
	0.00002	0.2482	0.2595	0.2840
	0.00003	0.2462	0.2582	0.2828
	0.00004	0.2443	0.2568	0.2816
$\beta=3$	0.00001	0.1820	0.1929	0.2101
	0.00002	0.1804	0.1918	0.2091
	0.00003	0.1788	0.1907	0.2081
	0.00004	0.1773	0.1895	0.2072
$\beta=4$	0.00001	0.1430	0.1530	0.1663
	0.00002	0.1417	0.1521	0.1655
	0.00003	0.1404	0.1511	0.1647
	0.00004	0.1391	0.1502	0.1639
$\beta=5$	0.00001	0.1178	0.1268	0.1376
	0.00002	0.1167	0.1260	0.1369
	0.00003	0.1156	0.1252	0.1362
	0.00004	0.1145	0.1244	0.1355

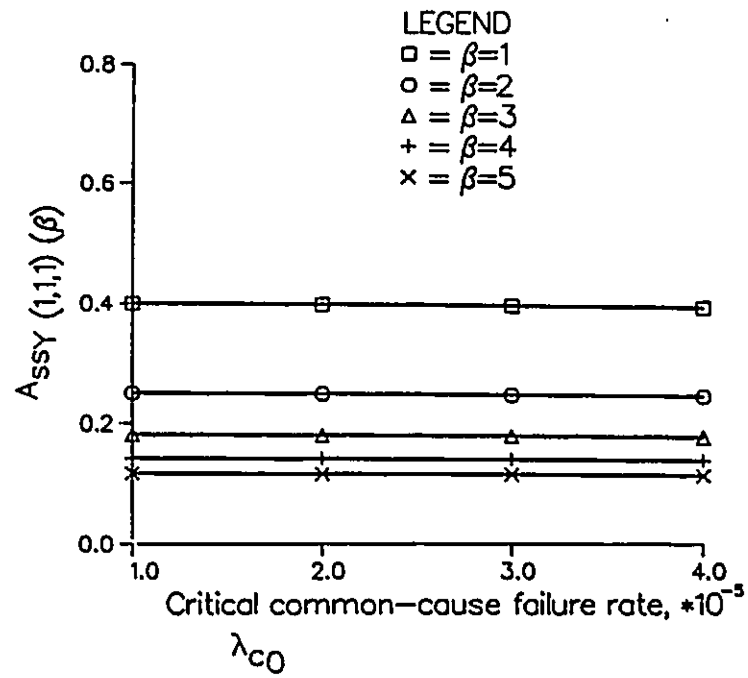


Figure 3.9: $A_{ssy}(1,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type C Repair

obtained are plotted in Figure 3.10 and tabular values are shown in the third column of Table 3.3.

Special Case Model III

For $n = 3$ with Type C Repair the long-run availability for this special case is obtained directly from Equation (3.42) by setting μ_1, μ_2 and μ_3 and $\mu_{s0'}$ to zero. The results obtained are plotted in Figure 3.11 and tabular values are shown in the fourth column of Table 3.3.

$1 - out - of - (n+1) : G$ System with Type D Repair

With Type D Repair, the system is not repaired and thus $\mu_{n+1}(x) = \mu_{n+2}(x) = 0$ in Figure 3.2 leading to the following set of integro-differential equations:

$$\frac{dP_0(t)}{dt} + C_{n,1}(0)\,P_0(t) = \mu_1\,P_1(t) + \mu_{s0'}\,P_{0'}(t) \tag{3.43}$$

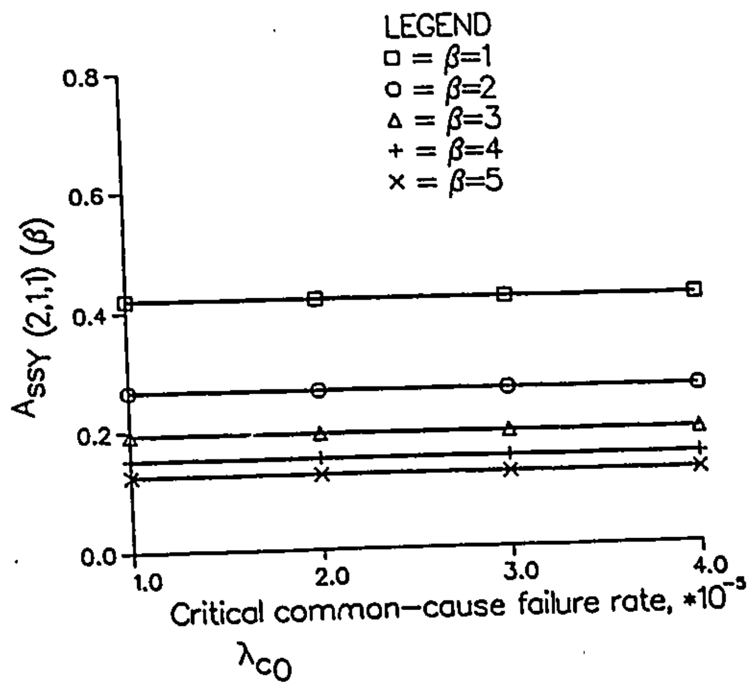


Figure 3.10: $A_{ssY}(2,1,1)(\beta)$ Vs. λ_{c0} Plots for Type C Repair

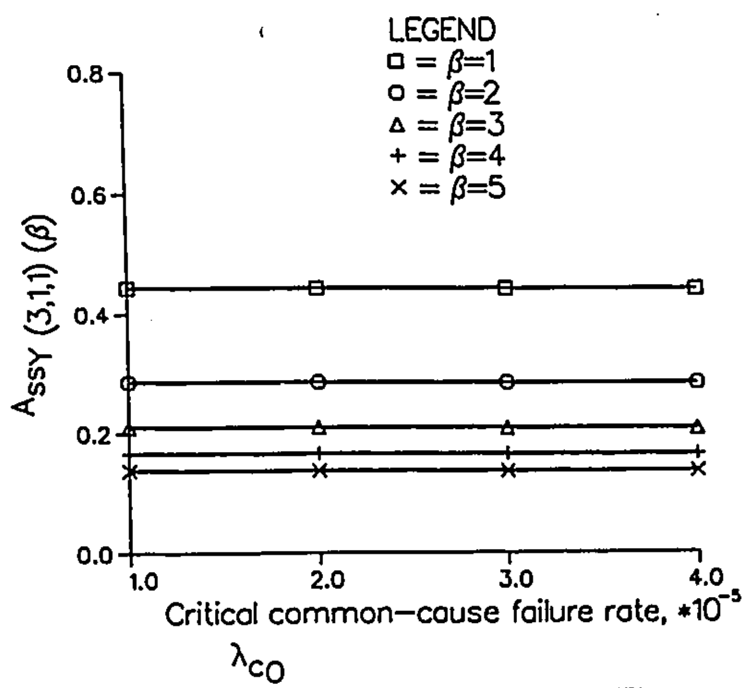


Figure 3.11: $A_{ssY}(3,1,1)(\beta)$ Vs. λ_{c0} Plots for Type C Repair

$$\frac{dP_1(t)}{dt} + C_{n,1}(1)\,P_1(t) = n\,\lambda_1\,P_0(t) + \mu_2\,P_2(t) \tag{3.44}$$

$$\frac{dP_2(t)}{dt} + C_{n,1}(2)\,P_2(t) = n\,\lambda_1\,P_1(t) + n\,\lambda_1\,P_{0'}(t) + \mu_3\,P_3(t) \tag{3.45}$$

For $j = 3, 4, \ldots\ldots, n-1$,

$$\frac{dP_j(t)}{dt} + C_{n,1}(j)\,P_j(t) = (n-j+2)\,\lambda_1\,P_{j-1}(t) + \mu_{j+1}\,P_{j+1} \tag{3.46}$$

$$\frac{dP_n(t)}{dt} + C_{n,1}(n)\,P_n(t) = 2\,\lambda_1\,P_{n-1}(t) \tag{3.47}$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,1}(0')\,P_{0'}(t) = \lambda_s\,P_0(t) + \mu_2\,P_2(t) \tag{3.48}$$

$$\frac{dP_{n+1}(t)}{dt} = \lambda_1\,P_n(t) \tag{3.49}$$

$$\frac{dP_{n+2}(t)}{dt} = \sum_{i=0}^{n} \lambda_{ci}\,P_i(t) + \lambda_{c0'}\,P_{0'}(t) \tag{3.50}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

The procedure used to determine the system reliability, mean time to failure and variance of time to failure is described in detail for the first special case model only. In subsequent special case models only the results are presented.

Special Case Model I

For $n = 1$ in Equations (3.43) to (3.50) we will have the following set of equations:

$$\frac{dP_0(t)}{dt} + C_{1,1}(0)\,P_0(t) = \mu_1\,P_1(t) + \mu_{s0'}\,P_{0'}(t) \tag{3.51}$$

$$\frac{dP_1(t)}{dt} + C_{1,1}(1)\,P_1(t) = \lambda_1 P_0(t) \tag{3.52}$$

$$\frac{dP_{0'}(t)}{dt} + C_{1,1}(0')\,P_{0'}(t) = \lambda_s P_0(t) \tag{3.53}$$

$$\frac{dP_2(t)}{dt} = \lambda_1\,P_1(t) + \lambda_1\,P_{0'}(t) \tag{3.54}$$

$$\frac{dP_3(t)}{dt} = \lambda_{c0}\,P_0(t) + \lambda_{c1}\,P_1(t) + \lambda_{c0'}\,P_{0'}(t) \tag{3.55}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero. where,

$C_{1,1}(0) = \lambda_1 + \lambda_s + \lambda_{c0}$

$C_{1,1}(1) = \lambda_1 + \lambda_{c1} + \mu_1$

$C_{1,1}(0') = \lambda_1 + \lambda_{c0'} + \mu_{s0'}$

Converting Equations (3.51) to (3.55) into their Laplace transforms equivalents and solving those Laplace transform equations simultaneously, we obtain:

$$P_0(s) = \frac{V_{N_1}(s)}{V_{D_1}(s)} \tag{3.56}$$

where,

$$\begin{aligned}
V_{N_1}(s) &= (s + C_{1,1}(1))(s + C_{1,1}(0')) \\
V_{D_1}(s) &= (s + C_{1,1}(0))(s + C_{1,1}(1))(s + C_{1,1}(0')) - \mu_1\,\lambda_1\,(s + C_{1,1}(0')) - \\
&\quad \mu_{s0'}\,\lambda_s(s + C_{1,1}(1))
\end{aligned}$$

$$P_1(s) = \frac{\lambda_1}{(s + C_1(1))} \bullet P_0(s) \tag{3.57}$$

$$P_{0'}(s) = \frac{\lambda_s}{(s + C_1(0'))} \bullet P_0(s) \tag{3.58}$$

$$P_2(s) = [\frac{\lambda_1^2}{(s + C_{1,1}(1))} + \frac{\lambda_1\,\lambda_s}{(s + C_{1,1}(0'))}] \bullet [\frac{1}{s}] \bullet P_0(s) \tag{3.59}$$

$$P_3(s) = [\lambda_{c0} + \frac{\lambda_1\,\lambda_{c1}}{(s + C_{1,1}(1))} + \frac{\lambda_s\,\lambda_{c0'}}{(s + C_{1,1}(0'))}] \bullet [\frac{1}{s}] \bullet P_0(s) \tag{3.60}$$

From Equation (2.5), the system reliability is:

$$\begin{aligned}
R_{Y_D}(1,1,1)(t) &= P_0(t) + P_1(t) + P_{0'}(t) \\
&= \mathcal{L}^{-1}\,[P_0(s) + P_1(s) + P_{0'}(s)] \\
&= e^{s_1 t}\,[B_1 + B_5 + B_9] + e^{s_2 t}[B_2 + B_6 + B_{10}] + \\
&\quad e^{s_3 t}\,[B_3 + B_7 + B_{11}] + B_4 e^{-C_{1,1}(1)t} + B_8\, e^{-C_{1,1}(0')t}
\end{aligned} \tag{3.61}$$

where s_1, s_2 and s_3 are roots of the polynomial function $V_{D_1}(s)$. The Laplace transform inversion that yielded Equation (3.61) was performed using the Partial Fraction Method. The partial fraction residues B_1 to B_{11} are defined in Appendix B. A plot

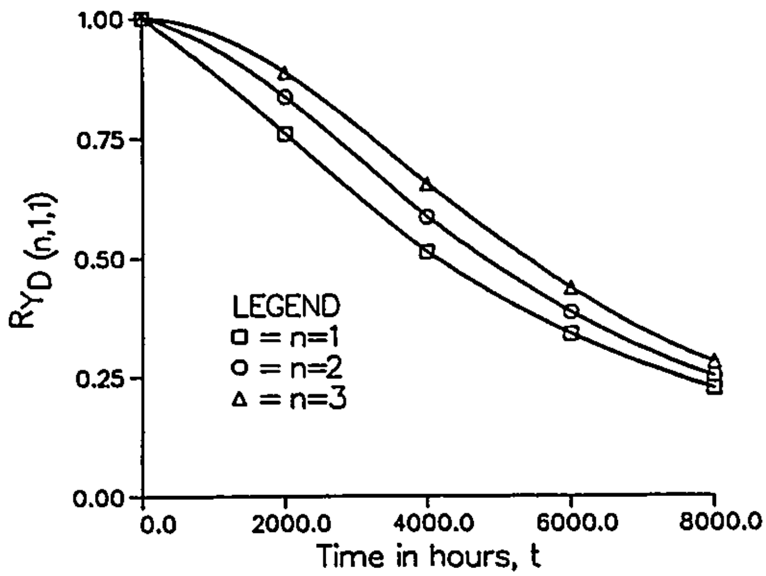


Figure 3.12: $R_{Y_D}(n, 1, 1)$ Plots ($\lambda_{c0} = 0.00004$ per hr.)

Table 3.4: $R_{Y_D}(n, 1, 1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n = 1$	$n = 2$	$n = 3$
0	1.0000	1.0000	1.0000
2000	0.7586	0.8353	0.8861
4000	0.5114	0.5841	0.6530
6000	0.3394	0.3850	0.4353
8000	0.2247	0.2496	0.2796
∞	0	0	0

of (3.61) for specified values of system parameters is shown in Figure 3.12 and corresponding tabular values are shown in the second column of Table 3.4.
Using Equation (2.6), the system mean time to failure is:

$$\begin{aligned} MTTF_{Y_D}(1,1,1) &= \lim_{s\to 0}[\sum_{i=0}^{1} P_i(s) + P_{0'}(s)] \\ &= \frac{W_{D_{1,1,1}}}{X_{D_{1,1,1}}} \end{aligned} \tag{3.62}$$

where
$W_{D_{1,1,1}} = C_{1,1}(1)\,C_{1,1}(0') + C_{1,1}(0')\,\lambda_1 + C_{1,1}(1)\,\lambda_s$
and,
$X_{D_{1,1,1}} = C_{1,1}(0)\,C_{1,1}(1)\,C_{1,1}(0') - \mu_1\,\lambda_1\,C_{1,1}(0') - \mu_{s0'}\,\lambda_s\,C_{1,1}(1)$

From Equation (2.7), the variance of the time to failure is:

$$\begin{aligned} \sigma^2_{Y_D}(1,1,1) &= -2\lim_{s\to 0} R'_{Y_D}(1,1,1)(s) - [MTTF_{Y_D}(1,1,1)]^2 \\ &= \frac{1}{k_2^2}\,[2(k_1k_3 - k_2k_4) - k_1^2] \end{aligned} \tag{3.63}$$

Symbols k_1 to k_4 are provided in Appendix B.

Special Case Model II

For $n = 2$ in Equations (3.43) to (3.50) using the same procedure outlined for the last special case, we obtain the following results:
The system reliability is given by:

$$\begin{aligned} R_{Y_D}(2,1,1)(t) &= \sum_{i=0}^{2} P_i(t) + P_{0'}(t) \\ &= P_0(t) + P_1(t) + P_2(t) + P_{0'}(t) \\ &= \mathcal{L}^{-1}[P_0(s) + P_1(s) + P_2(s) + P_{0'}(s)] \\ &= e^{s_9 t}[B_{25} + B_{26} + B_{27} + B_{28}] + e^{s_{10} t}[B_{29} + B_{30} + \\ &\quad B_{31} + B_{32}] + e^{s_{11} t}[B_{33} + B_{34} + B_{35} + B_{36}] + \\ &\quad e^{s_{12} t}\,[B_{37} + B_{38} + B_{39} + B_{40}] \end{aligned} \tag{3.64}$$

$W_D(s) = s^4 + A_{22}\,s^3 + A_{23}\,s^2 + A_{24}\,s + A_{25}$

where s_9, s_{10},......,s_{12} are real and unique roots of the polynomial function $W_D(s)$. The residues B_{25} to B_{40} are defined in Appendix B. A plot of (3.64) for specified values of system parameters is shown in Figure 3.12 and tabular values are given in the third column of Table 3.4.

The mean time to failure of this system is:

$$
\begin{aligned}
MTTF_{Y_D}(2,1,1) &= \lim_{s\to 0}[\sum_{i=0}^{2} P_i(s) + P_{0'}(s)] \\
&= \lim_{s\to 0}[P_0(s) + P_1(s) + P_2(s) + P_{0'}(s)] \\
&= \frac{W_{D_{2,1,1}}}{X_{D_{2,1,1}}} \qquad (3.65)
\end{aligned}
$$

where,

$$
\begin{aligned}
W_{D_{2,1,1}} = {} & C_{2,1}(1)\, C_{2,1}(0')\, \Phi_2(2) + C_{2,1}(0')\, [2\lambda_1\, \Phi_2(2) + \mu_2\, \Theta_2] + \\
& C_{2,1}(1)\, [\lambda_s\, \Phi_2(2) + \mu_2 \Theta_2] + C_{2,1}(1)\, C_{2,1}(0')\, \Theta_2
\end{aligned}
$$

$$
\begin{aligned}
X_{D_{2,1,1}} = {} & C_{2,1}(0)\, C_{2,1}(1)\, C_{2,1}(0')\, \Phi_2(2) - \mu_1\, C_{2,1}(0')\, [2\,\lambda_1\, \Phi_2(2) + \mu_2\, \Theta_2] - \\
& \mu_{s0'}\, C_{2,1}(1)\, [\lambda_s\, \Phi_2(2) + \mu_2\, \Theta_2]
\end{aligned}
$$

where

$\Phi_2(2) = C_{2,1}(1)\, C_{2,1}(0')\, C_{2,1}(2) - 2\,\lambda_1\, \mu_2 C_{2,1}(0') - 2\,\lambda_1\, \mu_2\, C_{2,1}(1)$

$\Theta_2 = 4\,\lambda_1^2\, C_{2,1}(0') + 2\,\lambda_1\, \lambda_s\, C_{2,1}(1)$

Using Equation (2.7), the variance of the time to failure of this system is:

$$
\begin{aligned}
\sigma^2_{Y_D}(2,1,1) &= -2\lim_{s\to 0} R'_{Y_D}(2,1,1)(s) - [MTTF_{Y_D}(2,1,1)]^2 \\
&= \frac{1}{k_6^2}[2(k_5 k_7 - k_6 k_8) - k_5^2] \qquad (3.66)
\end{aligned}
$$

Symbols k_5 to k_8 are defined in Appendix B.

where

$C_{2,1}(0) = 2\,\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{2,1}(1) = 2\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{2,1}(0') = 2\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{2,1}(2) = \lambda_1 + \lambda_{c2} + 2\,\mu_2$

Special Case Model III

For $n = 3$ in Equations (3.43) to (3.50) using the same procedure outlined for the previous special case, we obtain the following results:

The system reliability is given by:

$$\begin{aligned}
R_{Y_D}(3,1,1)(t) &= \sum_{i=0}^{3} P_i(t) + P_{0'}(t) \\
&= \pounds^{-1}[\sum_{i=0}^{3} P_i(t) + P_{0'}(t)] \\
&= e^{s_{18}t}[B_{83} + B_{88} + B_{93} + B_{98} + B_{103}] + \\
&\quad e^{s_{19}t}[B_{84} + B_{89} + B_{94} + B_{99} + B_{104}] + \\
&\quad e^{s_{20}t}[B_{85} + B_{90} + B_{95} + B_{100} + B_{105}] + \\
&\quad e^{s_{21}t}[B_{86} + B_{91} + B_{96} + B_{101} + B_{106}] + \\
&\quad e^{s_{22}t}[B_{87} + B_{92} + B_{97} + B_{102} + B_{107}] \qquad (3.67)
\end{aligned}$$

$W_{13}(s) = (s + C_{3,1}(0))\, W_{12}(s) - \mu_1\, W_{16}(s) - \mu_{s0'}\, W_{19}(s)$

$W_{12}(s) = s^4 + A_{36}\, s^3 + A_{37}\, s^2 + A_{38}\, s + A_{39}$

$W_{16}(s) = A_{32}\, s^3 + A_{33}\, s^2 + A_{34}\, s + A_{35}$

$W_{19}(s) = A_{45}\, s^3 + A_{46}\, s^2 + A_{47}\, s + A_{48}$

where s_{18} to s_{22} are the roots of the polynomial function $W_{13}(s)$. The partial fraction residues B_{83} to B_{107} are defined in Appendix B. A plot of (3.67) and corresponding tabular values are given in Figure 3.12 and in the fourth column of Table 3.4, respectively.

The mean time to failure for the system is:

$$MTTF_{Y_D}(3,1,1) = \frac{W_{D_{3,1,1}}}{X_{D_{3,1,1}}} \qquad (3.68)$$

where

$$\begin{aligned}
W_{D_{3,1,1}} &= C_{3,1}(1)\, C_{3,1}(0')\, \Phi_2(3)\, \Phi_3(3) + C_{3,1}(0')[(3\lambda_1\, \Phi_2(3) + \mu_2\, \Theta_3)\, \Phi_3(3) + \\
&\quad 2\lambda_1\, \mu_2\, \mu_3\, C_{3,1}(1)\ C_{3,1}(0')\, \Theta_3] + C_{3,1}(1)\,[(\lambda_s\, \Phi_2(3) + \mu_2\, \Theta_3)\, \Phi_3(3) + \\
&\quad 2\lambda_1\, \mu_2\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0')\, \Theta_3] + C_{3,1}(1)\, C_{3,1}(0')\,[\Phi_3(3)\, \Theta_3 + \\
&\quad 2\lambda_1\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0')\, \Theta_3] + 2\lambda_1\, C_{3,1}(1)\, C_{3,1}(0')\, \Phi_2(3)\, \Theta_3
\end{aligned}$$

$$\begin{aligned} X_{D_{3,1,1}} &= C_{3,1}(0)\, C_{3,1}(1)\, C_{3,1}(0')\, \Phi_2(3)\, \Phi_3(3) - \mu_1\, C_{3,1}(0')\, [(3\,\lambda_1\, \Phi_2(3) + \\ &\quad \mu_2\, \Theta_3)\, \Phi_3(3) + 2\lambda_1\, \mu_2\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0')\, \Theta_3] - \mu_{s0'}\, C_{3,1}(1)\, [(\lambda_s\, \Phi_2(3) + \\ &\quad \mu_2\, \Theta_3)\, \Phi_3(3) + 2\lambda_1\, \mu_2\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0')\, \Theta_3] \end{aligned}$$

where,

$\Phi_2(3) = C_{3,1}(1)\, C_{3,1}(0')\, C_{3,1}(2) - 3\,\lambda_1\, \mu_2\, C_{3,1}(0') - 3\lambda_1\, \mu_2\, C_{3,1}(1)$

$\Phi_3(3) = C_{3,1}(3)\, \Phi_2(3) - 2\,\lambda_1\, \mu_3\, C_{3,1}(1)\, C_{3,1}(0')$

$\Theta_3 = 9\lambda_1^2\, C_{3,1}(0') + 3\lambda_1\, \lambda_s\, C_{3,1}(1)$

The variance of time to failure for this system with Type D Repair is:

$$\begin{aligned} \sigma^2_{Y_D}(3,1,1) &= -2 \lim_{s\to 0} R'_{Y_D}(3,1,1)(s) - [MTTF_{Y_D}(3,1,1)]^2 \\ &= \frac{1}{k_{10}^2}\, [2(k_9\, k_{11} - k_{10}\, k_{12}) - k_9^2] \end{aligned} \tag{3.69}$$

The symbols k_9 to k_{12} are defined in Appendix B.

where,

$C_{3,1}(0) = 3\,\lambda_1 + \lambda_{c0} + \lambda_s$

$C_{3,1}(1) = 3\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{3,1}(0') = 3\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{3,1}(2) = 2\,\lambda_1 + \lambda_{c2} + 2\,\mu_2$

$C_{3,1}(3) = \lambda_1 + \lambda_{c3} + \mu_3$

From Equations (3.62), (3.65), and (3.68), the mean time to failure of the $1 - out - of - (n+1) : G$ system with Type D Repair and under standby activation policy Y can be generalized in terms of n as:

$$MTTF_{Y_D}(n,1,1) = \frac{W_{D_{n,1,1}}}{X_{D_{n,1,1}}} \tag{3.70}$$

where

$$\begin{aligned} W_{D_{n,1,1}} &= C_{n,1}(1)\, C_{n,1}(0') \prod_{i=1}^{n} \Phi_i(n) + C_{n,1}(0')\, [E_n + F_n] + C_{n,1}(1)\, [L_n + F_n] + \\ &\quad \Psi_2\, C_{n,1}(1)\, C_{n,1}(0')\, G_n + \Psi_3\, C_{n,1}(1)\, C_{n,1}(0')\, \Phi_2(n)\, H_n + \\ &\quad \Psi_4\, C_{n,1}(1)\, C_{n,1}(0')\, \Phi_2(n)\, \Phi_3(n)\, I_n + \dots\dots\dots\dots + \\ &\quad \Psi_{n-1}\, C_{n,1}(1)\, C_{n,1}(0') \prod_{i=2}^{n-2} \Phi_i(n)\, J_n + 2\Psi_n(n-1)\, \lambda_1^2 \prod_{i=2}^{n-1} \Phi_i(n)\, \Theta_n \end{aligned}$$

and,

$$X_{D_{n,1,1}} = C_{n,1}(0)\, C_{n,1}(1)\, C_{n,1}(0') \prod_{i=1}^{n} \Phi_i(n) - \mu_1\, C_{n,1}(0')\, [E_n + F_n] - \mu_{s0'}\, C_{n,1}(1)\, [L_n + F_n]$$

Where

$C_{n,1}(0) = n\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = n\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,1}(0') = n\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{n,1}(2) = (n-1)\lambda_1 + \lambda_{c2} + 2\mu_2$

$C_{n,1}(3) = (n-2)\lambda_1 + \lambda_{c3} + \mu_3$

$\vdots$

$C_{n,1}(n) = \lambda_1 + \lambda_{cn} + \mu_n$

$\Phi_1(n) = 1$

$\Phi_2(n) = C_{n,1}(1)\, C_{n,1}(0')\, C_{n,1}(2) - n\lambda_1\, \mu_2\, C_{n,1}(0') - n\lambda_1\, \mu_2\, C_{n,1}(1)$ [valid for $n \geq 2$]

$\Phi_3(n) = C_{3,1}(3)\, \Phi_2(n) - (n-1)\lambda_1 \mu_3\, C_{n,1}(1)\, C_{n,1}(0')$ [valid for $n \geq 3$]

$\Phi_i(n) = C_{i,1}(i)\, \Phi_{i-1}(n) - 2\lambda_1\, \mu_i\, \Phi_{i-2}(n)$ [valid for $n \geq i$ where $i \geq 4$]

where

$\Phi_i(n) = 1$ for $n < i$

$\Theta_n = (n\lambda_1)^2\, C_{n,1}(0') + n\lambda_1 \lambda_s\, C_{n,1}(1)$

$\Psi_i = 1$ for $i \leq n$

$\Psi_i = 0$ for $i > n$

$$E_n = (n\lambda_1\, \Phi_2(n) + \mu_2\, \Theta_n) \prod_{i=3}^{n} \Phi_i(n)$$

$$L_n = (\lambda_s\, \Phi_2(n) + \mu_2\, \Theta_n) \prod_{i=3}^{n} \Phi_i(n)$$

$$F_n = (n-1)\lambda_1\, \mu_2\, \mu_3\, C_{n,1}(1)\, C_{n,1}(0')\, \Theta_n \prod_{i=4}^{n-1} \Phi_i(n) + 2(n-1)\lambda_1^2\, \mu_2\, \mu_3\, \mu_4\, C_{n,1}(1)\, C_{n,1}(0')\, \Theta_n\, \Phi_2(n) \prod_{i=5}^{n-1} \Phi_i(n) +$$

$$2(n-1)\lambda_1^2\,\mu_2\,\mu_3\,\mu_4\,\mu_5\,C_{n,1}(1)\,C_{n,1}(0')\,\Theta_n\,\Phi_2(n)\,\Phi_3(n)\prod_{i=6}^{n-1}\Phi_i(n)+...$$

$$....+2(n-1)\lambda_1^2\prod_{j=2}^{n-1}\mu_j\,C_{n,1}(1)\,C_{n,1}(0')\prod_{i=2}^{n-3}\Phi_i(n)\,\Theta_n+$$

$$2(n-1)\lambda_1^2\prod_{j=2}^{n}\mu_j\,C_{n,1}(1)\,C_{n,1}(0')\prod_{i=2}^{n-2}\Phi_i(n)\,\Theta_n$$

$$\begin{aligned}G_n &= \prod_{i=3}^{n}\Phi_i(n)\,\Theta_n+(n-1)\lambda_1\,\mu_3\,C_{n,1}(1)\,C_{n,1}(0')\,\Theta_n\prod_{i=4}^{n-1}\Phi_i(n)+\\ &\quad 2(n-1)\lambda_1^2\,\mu_3\,\mu_4\,C_{n,1}(1)\,C_{n,1}(0')\,\Theta_n\,\Phi_2(n)\prod_{i=5}^{n-1}\Phi_i(n)+.\\ &\quad ..+2(n-1)\lambda_1^2\prod_{j=3}^{n}\mu_j\,C_{n,1}(1)\,C_{n,1}(0')\prod_{i=2}^{n-2}\Phi_i(n)\,\Theta_n\end{aligned}$$

$$\begin{aligned}H_n &= \prod_{i=4}^{n}\Phi_i(n)\,(n-1)\lambda_1\,\Theta_n+2(n-1)\lambda_1^2\,\Theta_n\,\Phi_2(n)\,\mu_4+\\ &\quad 2(n-1)\lambda_1^2\,\Theta_n\,\Phi_2(n)\,\Phi_3(n)\,\mu_4\,\mu_5+..\\ &\quad ..+2(n-1)\lambda_1^2\,\Theta_n\prod_{i=2}^{n-2}\Phi_i(n)\prod_{j=4}^{n}\mu_j\end{aligned}$$

$$\begin{aligned}I_n &= \prod_{i=5}^{n}\Phi_i(n)\,(n-1)\lambda_1\,\Theta_n+2(n-1)\lambda_1^2\,\Theta_n\,\Phi_3(n)\,\mu_5+\\ &\quad 2(n-1)\lambda_1^2\,\Theta_n\,\Phi_3(n)\,\Phi_4(n)\,\mu_5\,\mu_6+...\\ &\quad ...+2(n-1)\lambda_1^2\,\Theta_n\prod_{i=3}^{n-2}\Phi_i(n)\prod_{j=5}^{n}\mu_j\end{aligned}$$

$$J_n = (n-1)\,\lambda_1\,\Theta_n\,\Phi_n(n) \quad .$$

For $n = i$, μ_{j-1} and λ_{cj-1} are not applicable and assume a value of zero in above expressions if $j > i+1$.

Using Equation (3.70), the plots of system mean time to failure for $n = 1$, 2, and 3 with Type D Repair and for the specified values of system parameters are given in Figure 3.13 and Table 3.5 respectively.

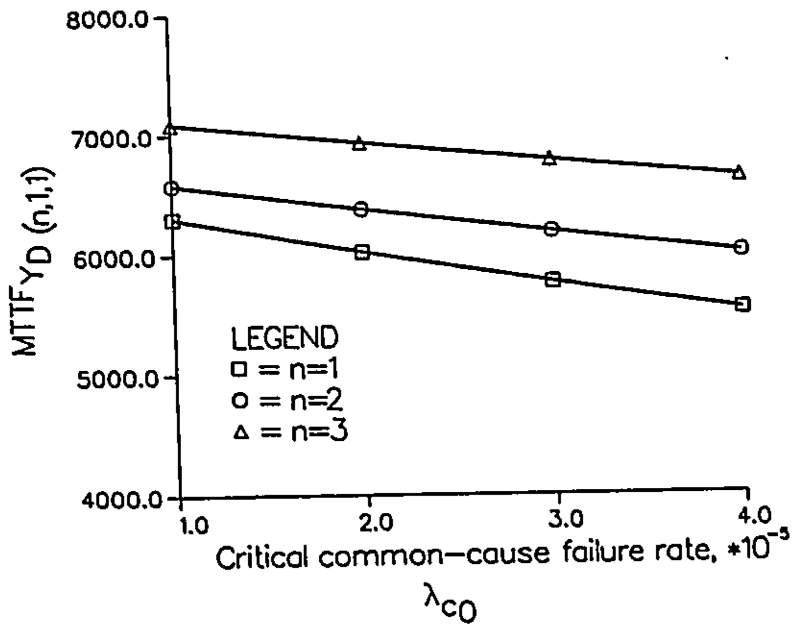


Figure 3.13: $MTTF_{Y_D}(n,1,1)$ Vs. λ_{c0} Plots

Table 3.5: $MTTF_{Y_D}(n,1,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	6300.59	6578.21	7088.25
0.00002	6021.26	6376.02	6930.84
0.00003	5765.15	6184.82	6779.09
0.00004	5529.49	6003.77	6632.75

$1 - out - of - (n+1) : G$ System with Type E Repair

With Type E Repair, the system and its failed active units are not repaired and, thus, μ_i; for $i = 1, 2, .., n$, $\mu_{n+1}(x)$ and $\mu_{n+2}(x)$ are set to zero in Equations (3.1) to (3.10).

Special Case Model I

For $n = 1$ with Type E Repair we can easily determine the reliability indices for this special case as follows:
The system reliability for this special case model is easily obtained from Equation (3.61) by setting μ_1 to zero. The results are plotted in Figure 3.14 and the tabular values in the second column of Table 3.6.

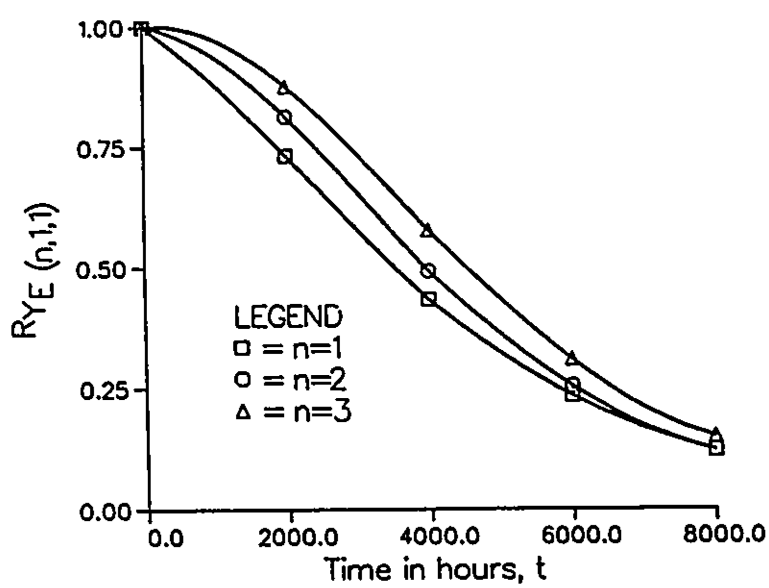


Figure 3.14: $R_{Y_E}(n, 1, 1)$ Plots ($\lambda_{c0} = 0.00004$ per hr.)

The system mean time to failure is determined directly from Equation (3.62) by

Table 3.6: $R_{Y_E}(n,1,1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.7317	0.8124	0.8753
4000	0.4325	0.4918	0.5761
6000	0.2344	0.2537	0.3086
8000	0.1209	0.1210	0.1495
∞	0	0	0

setting μ_1 to zero and can be expressed as:

$$MTTF_{Y_E}(1,1,1) = \frac{1}{\alpha_1[C_{1,1}(0)]}\,[C_{1,1}(0')\,(1+\frac{\lambda_1}{C_{1,1}(1)}) + \lambda_s] \qquad (3.71)$$

where,

$\alpha_1[C_{1,1}(0)] = C_{1,1}(0)\,C_{1,1}(0') - \lambda_s\,\mu_{s0'}$

Setting $\mu_1 = 0$ in Equation (3.63), the variance of time to failure for this special case is determined.

Special Case Model II

For $n = 2$ with Type E Repair the relevant reliability indices for this special case can be obtained as follows:

The system reliability values can be obtained from Equation (3.64) by setting μ_1 and μ_2 to zero. The results are plotted in Figure 3.14 and the corresponding tabular values are shown in the third column of Table 3.6.

The mean time to failure of this system is directly obtained from Equation (3.65) by setting μ_1 and μ_2 to zero and can be expressed as:

$$\begin{aligned} MTTF_{Y_E}(2,1,1) &= \frac{1}{\alpha_1[C_{2,1}(0)]}[C_{2,1}(0')(1+\frac{2\lambda_1}{C_{2,1}(1)}+\frac{4\lambda_1^2}{C_{2,1}(1)\,C_{2,1}(2)})+ \\ &\quad \lambda_s(1+\frac{2\lambda_1}{C_{2,1}(2)})] \end{aligned} \tag{3.72}$$

where,

$\alpha_1[C_{2,1}(0)] = C_{2,1}(0)\,C_{2,1}(0') - \lambda_s\,\mu_{s0'}$

Setting $\mu_1 = \mu_2 = 0$ in Equation (3.66), the variance of time to failure for this special case is determined.

Special Case Model III

For $n = 3$ with Type E Repair, the relevant reliability indices for this special case are obtained as follows:
The system reliability is determined from Equation (3.67) by setting μ_1, μ_2 and μ_3 to zero. The results are plotted in Figure 3.14 and the values are shown in the fourth column of Table 3.6.

The mean time to failure for this system is determined from Equation (3.68) by setting μ_1, μ_2 and μ_3 to zero and thus can be expressed as:

$$\begin{aligned} MTTF_{Y_E}(3,1,1) &= \frac{1}{\alpha_1[C_{3,1}(0)]}[C_{3,1}(0')(1+\frac{3\lambda_1}{C_{3,1}(1)}+\frac{9\lambda_1^2}{C_{3,1}(1)\,C_{3,1}(2)}+ \\ &\quad \frac{18\,\lambda_1^3}{C_{3,1}(1)\,C_{3,1}(2)\,C_{3,1}(3)})+\lambda_s(1+\frac{3\lambda_1}{C_{3,1}(2)}+ \\ &\quad \frac{6\lambda_1^2}{C_{3,1}(2)\,C_{3,1}(3)})] \end{aligned} \tag{3.73}$$

where,

$\alpha_1[C_{3,1}(0)] = C_{3,1}(0)\,C_{3,1}(0') - \lambda_s\,\mu_{s0'}$

The variance of time to failure for this special case is determined by setting $\mu_1 = \mu_2 = \mu_3 = 0$ in Equation (3.69).

From Equations (3.71), (3.72) and (3.73) the following generalized system mean time to failure expression for the $1-out-of-(n+1):G$ system with Type E Repair and under standby activation policy Y is developed in terms of n:

$$MTTF_{Y_E}(n,1,1) = [\frac{C_{n,1}(0')}{\alpha_1[C_{n,1}(0)]}] \bullet [1 + (\frac{n\lambda_1}{C_{n,1}(1)} + \frac{\lambda_s}{C_{n,1}(0')}) \bullet G_{1/n}] \quad (3.74)$$

where

$$G_{1/n} = 1 + \frac{n\Psi_1\lambda_1}{C_{n,1}(2)} + \frac{n(n-1)\Psi_2\lambda_1^2}{C_{n,1}(2)\,C_{n,1}(3)} + \frac{n(n-1)(n-2)\Psi_3\lambda_1^3}{C_{n,1}(2)\,C_{n,1}(3)\,C_{n,1}(4)} + \ldots\ldots$$
$$\ldots\ldots\ldots + \frac{n!\Psi_{n-1}\lambda_1^{n-1}}{\prod_{i=2}^{n} C_{n,1}(i)}$$

where the Ψ_js are parameters defined as follows:

$\Psi_j = 0$ for $j \geq n$ and $\Psi_j = 1$ for $j < n$

$\alpha_1[C_{n,1}(0)] = C_{n,1}(0)\,C_{n,1}(0') - \lambda_s\,\mu_{s0'}$

Equation (3.74) was used to generate system mean time to failure plots and values for n = 1, 2 and 3 for specified values of system parameters shown in Figure 3.15 and Table 3.7 respectively.

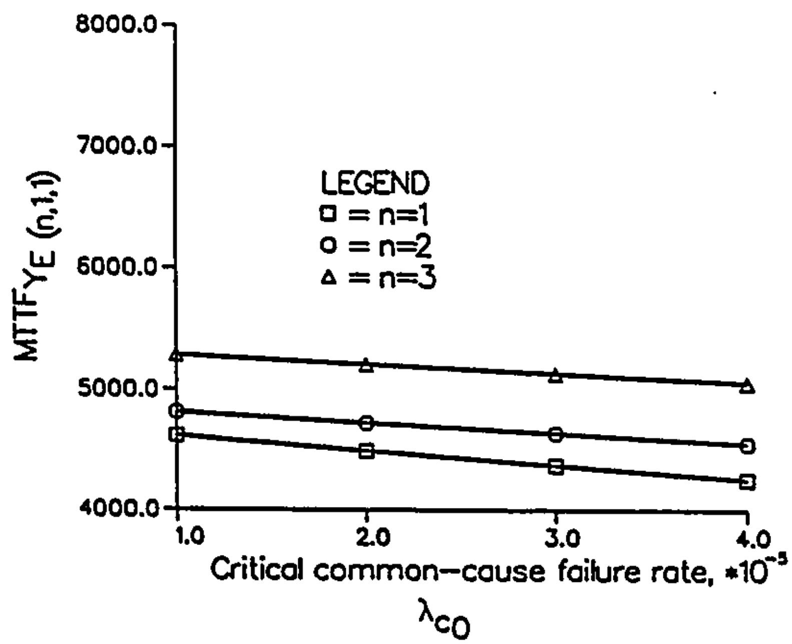


Figure 3.15: $MTTF_{Y_E}(n,1,1)$ Vs. λ_{c0} Plots

Table 3.7: $MTTF_{Y_E}(n,1,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	4614.84	4813.07	5287.46
0.00002	4488.81	4722.60	5210.07
0.00003	4368.74	4634.77	5134.32
0.00004	4254.24	4549.47	5060.17

$1-out-of-(n+1):G$ System with Type F Repair

With Type F Repair, the system and its failed units are not repaired and thus μ_i; for $i = 1,2,..,n$, $\mu_{n+1}(x)$, $\mu_{n+2}(x)$ and $\mu_{s0'}$ are se. to zero in Equations (3.1) to (3.10).

Special Case Model I

For $n = 1$ with Type F Repair we can determine the relevant reliability indices for this special case as follows:

The system reliability for this special case model is easily obtained from Equation (3.61) by setting μ_1 and $\mu_{s0'}$ to zero. The results are plotted in Figure 3.16 and the tabular values in the second column of Table 3.8.

The system mean time to failure is determined directly from Equation (3.62) by setting μ_1 and $\mu_{s0'}$ to zero and thus can be expressed as:

$$MTTF_{Y_F}(1,1,1) = \frac{1}{C_{1,1}(0)}[1 + \frac{\lambda_1}{C_{1,1}(1)} + \frac{\lambda_s}{C_{1,1}(0')}] \tag{3.75}$$

Setting $\mu_1 = \mu_{s0'} = 0$ in Equation (3.63) the variance of time to system failure for this special case is determined.

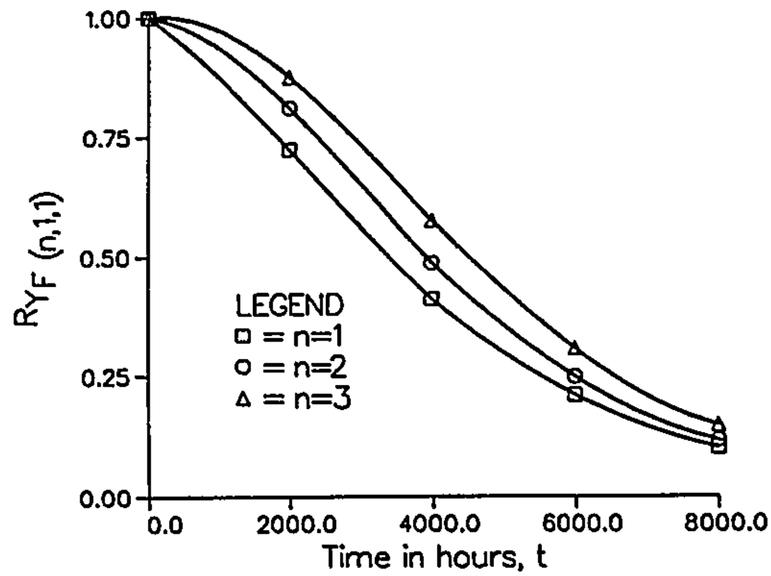


Figure 3.16: $R_{Y_F}(n,1,1)$ Plots ($\lambda_{c0} = 0.00004$ per hr.)

Table 3.8: $R_{Y_F}(n,1,1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.7226	0.8104	0.8749
4000	0.4101	0.4858	0.5739
6000	0.2101	0.2477	0.3062
8000	0.1018	0.1169	0.1480
∞	0	0	0

Special Case Model II

For $n = 2$ with Type F Repair we can obtain the relevant reliability indices for this special case as follows:

The system reliability is determined from Equation (3.64) by setting μ_1, μ_2, and $\mu_{s0'}$ to zero. The results obtained are plotted in Figure 3.16 and the values are shown in the third column of Table 3.8.

The system mean time to failure is determined from Equation (3.65) by setting μ_1, μ_2 and $\mu_{s0'}$ to zero and is expressed as:

$$MTTF_{Y_F}(2,1,1) = \frac{1}{C_{2,1}(0)}\,[1 + (\frac{2\lambda_1}{C_{2,1}(1)} + \frac{\lambda_s}{C_{2,1}(0')})\,(1 + \frac{2\lambda_1}{C_{2,1}(2)})] \tag{3.76}$$

Setting $\mu_1 = \mu_2 = \mu_{s0'} = 0$ in Equation (3.66) we can determine the variance of time to system failure for this special case.

Special Case Model III

For $n = 3$ with Type F Repair we can determine the relevant reliability indices as follows:

The system reliability is determined from Equation (3.67) by setting μ_1, μ_2, μ_3 and $\mu_{s0'}$ to zero. The results are plotted in Figure 3.16 and the tabular values are shown in the fourth column of Table 3.8.

The mean time to failure for the system is determined by setting μ_1, μ_2, μ_3 and $\mu_{s0'}$ to zero in Equation (3.68) and can be expressed as:

$$\begin{aligned} MTTF_{Y_F}(3,1,1) \;=\; & \frac{1}{C_{3,1}(0)}\,[1 + (\frac{3\lambda_1}{C_{3,1}(1)} + \frac{\lambda_s}{C_{3,1}(0')})\,(1 + \frac{3\lambda_1}{C_{3,1}(2)} + \\ & \frac{6\lambda_1^2}{C_{3,1}(2)\,C_{3,1}(3)})] \end{aligned} \tag{3.77}$$

The variance of time to failure for the system is determined by setting $\mu_1 = \mu_2 = \mu_3 = \mu_{s0'} = 0$ in Equation (3.69).

From Equations (3.75), (3.76) and (3.77), the following generalized system mean time to failure expression for the $1 - out - of - (n+1) : G$ system with Type F Repair and under standby activation policy Y is developed in terms of n:

$$MTTF_{Y_F}(n,1,1) = [\frac{1}{C_{n,1}(0)}] \bullet [1 + (\frac{n\lambda_1}{C_{n,1}(1)} + \frac{\lambda_s}{C_{n,1}(0')}) \bullet G_{1/n}] \tag{3.78}$$

Equation (3.78) was used to generate system mean time to failure plots and values for $n = 1, 2$ and 3 for specified values of system parameters shown in Figure 3.17 and Table 3.9, respectively.

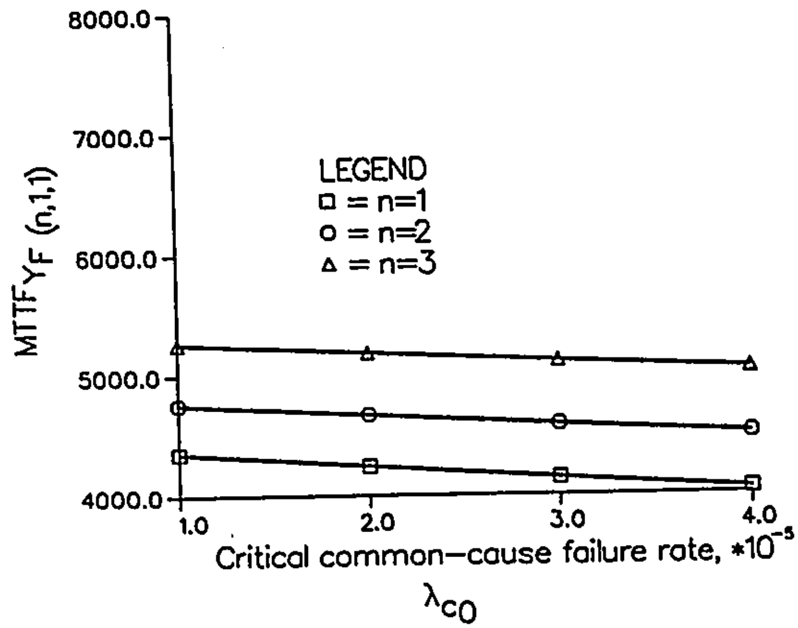


Figure 3.17: $MTTF_{Y_F}(n,1,1)$ Vs. λ_{c0} Plots

3.1.2 1-out-of-(n+2):G Identical Unit Submodel

This is the $(n,2,1)$ system and is composed of n active units and *two* standbys. It is functional as long as at least *one* unit is healthy. A block diagram for this system is shown in Figure 3.18 and its state space diagram is shown in Figure 3.19.

Table 3.9: $MTTF_{Y_F}(n,1,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	4353.36	4757.11	5266.56
0.00002	4246.48	4670.70	5190.79
0.00003	4144.18	4586.73	5116.59
0.00004	4046.19	4505.12	5043.94

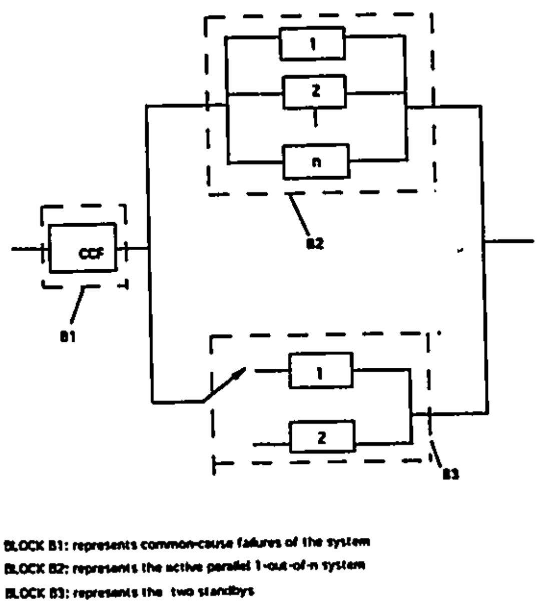


Figure 3.18: Block diagram of the $1-out-of-(n+2):G$ System

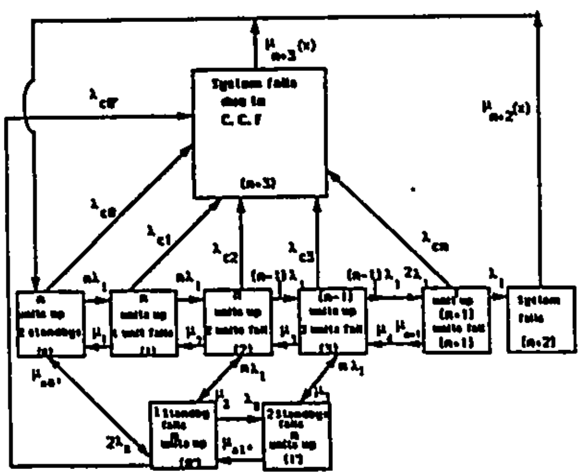


Figure 3.19: State space diagram of the $1 - out - of - (n+2) : G$ System under SBAP Y

$1 - out - of - (n+2) : G$ **System with Type A Repair**

The corresponding integro-differential equations for the model described in Figure 3.19 are:

$$\frac{dP_0(t)}{dt} + C_{n,2}(0)\,P_0(t) = \mu_1\,P_1(t) + \mu_{s0'}\,P_{0'}(t) + \sum_{i=n+2}^{n+3} \int_0^\infty P_i(x,t)\,\mu_i(x)dx \tag{3.79}$$

$$\frac{dP_1(t)}{dt} + C_{n,2}(1)\,P_1(t) = n\,\lambda_1\,P_0(t) + \mu_2\,P_2(t) \tag{3.80}$$

$$\frac{dP_2(t)}{dt} + C_{n,2}(2)\,P_2(t) = n\,\lambda_1\,P_1(t) + n\,\lambda_1\,P_{0'}(t) + \mu_3\,P_3(t) \tag{3.81}$$

$$\frac{dP_3(t)}{dt} + C_{n,2}(3)\,P_3(t) = n\,\lambda_1\,P_2(t) + n\,\lambda_1\,P_{1'}(t) + \mu_4\,P_4(t) \tag{3.82}$$

For $j = 4, 5, \ldots\ldots, n$,

$$\frac{dP_j(t)}{dt} + C_{n,2}(j)\,P_j(t) = (n-j+3)\,\lambda_1\,P_{j-1}(t) + \mu_{j+1}\,P_{j+1} \tag{3.83}$$

$$\frac{dP_{n+1}(t)}{dt} + C_{n,2}(n+1)\,P_{n+1}(t) = 2\,\lambda_1\,P_n(t) \tag{3.84}$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,2}(0')\,P_{0'}(t) = 2\lambda_s\,P_0(t) + \mu_{s1'}\,P_{1'}(t) + \mu_2\,P_2(t) \tag{3.85}$$

$$\frac{dP_{1'}(t)}{dt} + C_{n,2}(1')\,P_{1'}(t) = \lambda_s\,P_{0'}(t) + \mu_3\,P_3(t) \tag{3.86}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+2}(x)]\,P_{n+2}(x,t) = 0 \tag{3.87}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+3}(x)]\,P_{n+3}(x,t) = 0 \tag{3.88}$$

The associated boundary conditions are:

$$P_{n+2}(0,t) = \lambda_1\,P_{n+1}(t) \tag{3.89}$$

$$P_{n+3}(0,t) = \sum_{i=0}^{n+1} \lambda_{ci}\,P_i(t) + \sum_{i=0'}^{1'} \lambda_{ci}\,P_i(t) \tag{3.90}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero. where

$C_{n,2}(0) = n\,\lambda_1 + 2\lambda_s + \lambda_{c0}$

$C_{n,2}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,2}(2) = n\,\lambda_1 + \lambda_{c2} + 2\mu_2$

$C_{n,2}(3) = (n-1)\,\lambda_1 + \lambda_{c3} + 2\mu_3$

For $j = 4, 5, \ldots\ldots, n,$

$C_{n,2}(j) = (n-j+2)\,\lambda_1 + \lambda_{cj} + \mu_j$

$C_{n,2}(n+1) = \lambda_1 + \lambda_{cn+1} + \mu_{n+1}$

$C_{n,2}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'} + \lambda_s$

$C_{n,2}(1') = n\,\lambda_1 + \lambda_{c1'} + \mu_{s1'}$

Special Case Model I

For $n = 1$ in Equations (3.79) to (3.90) using the procedure outlined for the last sub-model, the long-run availability of this special case is given by:

$$A_{ssY}(1,2,1)\,(\beta) = \frac{(\mu_3\,\mu_4)^\beta\,x_{13}}{(\mu_3\,\mu_4)^\beta\,x_{14} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_{15} + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_{16}} \tag{3.91}$$

Where,

$x_{13} = A_{52} + A_{56} + A_{59} + A_{63} + A_{66}$

$x_{14} = A_{70} - A_{73} - A_{78}$

$x_{15} = A_{71} - A_{74}$

$x_{16} = A_{71} - A_{79}$

The steady-state availability values generated with Equation (3.91) for various values of β and for specified values of system parameters are shown in the second column of Table 3.10. Symbols A_{52} to A_{79} are defined in Appendix C.

Special Case Model II

For $n = 2$ in Equations (3.79) to (3.90) using the procedure outlined for the last sub-model, the long-run availability of this special case is given by:

$$A_{ssY}(2,2,1)\,(\beta) = \frac{(\mu_4\,\mu_5)^\beta\,x_{17}}{(\mu_4\,\mu_5)^\beta\,x_{18} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\,x_{19} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\,x_{20}} \tag{3.92}$$

Table 3.10: $A_{ssy}(n,2,1)\,(\beta)$ Values as a function of λ_{co} with Type A Repair

	λ_{co} (per Hr.)	$n = 1$	$n = 2$	$n = 3$
$\beta = 1$	0.00001	0.6234	0.5388	0.5535
	0.00002	0.6168	0.5364	0.5513
	0.00003	0.6102	0.5339	0.5492
	0.00004	0.6038	0.5315	0.5470
$\beta = 2$	0.00001	0.4528	0.3738	0.3826
	0.00002	0.4459	0.3714	0.3806
	0.00003	0.4391	0.3691	0.3785
	0.00004	0.4324	0.3667	0.3765
$\beta = 3$	0.00001	0.3556	0.2861	0.2924
	0.00002	0.3491	0.2840	0.2906
	0.00003	0.3429	0.2820	0.2888
	0.00004	0.3369	0.2799	0.2870
$\beta = 4$	0.00001	0.2927	0.2318	0.2366
	0.00002	0.2869	0.2299	0.2350
	0.00003	0.2813	0.2281	0.2334
	0.00004	0.2759	0.2263	0.2319
$\beta = 5$	0.00001	0.2487	0.1948	0.1987
	0.00002	0.2435	0.1932	0.1973
	0.00003	0.2385	0.1916	0.1959
	0.00004	0.2336	0.1900	0.1946

where,

$$\begin{aligned}
x_{17} &= A_{114} + A_{134} + A_{146} + A_{160} + A_{180} + A_{196} \\
x_{18} &= A_{114} + C_{2,2}(0)\, A_{113} - \mu_1\, A_{153} - \mu_{s0'}\, A_{179} - \lambda_1\, A_{159} - \lambda_{c0}\, A_{113} - \\
&\quad \lambda_{c1}\, A_{133} - \lambda_{c2}\, A_{145} - \lambda_{c3}\, A_{159} - \lambda_{c0'}\, A_{179} - \lambda_{c1'}\, A_{195} \\
x_{19} &= C_{2,2}(0)\, A_{114} - \mu_1\, A_{134} - \mu_{s0'}\, A_{180} - \lambda_1\, A_{160} \\
x_{20} &= C_{2,2}(0)\, A_{114} - \mu_1\, A_{134} - \mu_{s0'}\, A_{180} - \lambda_{c0}\, A_{114} - \lambda_{c1}\, A_{134} \\
&\quad - \lambda_{c2}\, A_{146} - \lambda_{c3}\, A_{160} - \lambda_{c0'}\, A_{180} - \lambda_{c1'}\, A_{196}
\end{aligned}$$

Equation (3.92) was used to generate long-run availability values shown in the third column of Table 3.10. Symbols A_{113} to A_{196} are defined in Appendix C.

Special Case Model III

For $n = 3$ in Equations (3.79) to (3.90) using the procedure outlined for the last sub-model, the long-run availability of this special case is given by:

$$A_{ssY}(3,2,1)\,(\beta) = \frac{(\mu_5\,\mu_6)^\beta\, x_{21}}{(\mu_5\,\mu_6)^\beta\, x_{22} + \beta\,\mu_5^\beta\,\mu_6^{\beta-1}\, x_{23} + \beta\,\mu_5^{\beta-1}\,\mu_6^\beta\, x_{24}} \tag{3.93}$$

Where,

$$\begin{aligned}
x_{21} &= A_{252} + A_{295} + A_{326} + A_{349} + A_{363} + A_{390} + A_{408} \\
x_{22} &= A_{252} + C_{3,2}(0)\, A_{251} - \mu_1\, A_{294} - \mu_{s0'}\, A_{390} - \lambda_1\, A_{362} - \\
&\quad \lambda_{c0}\, A_{251} - \lambda_{c1}\, A_{294} - \lambda_{c2}\, A_{325} - \lambda_{c3}\, A_{348} - \lambda_{c4}\, A_{362} - \\
&\quad \lambda_{c0'}\, A_{389} - \lambda_{c1'}\, A_{407} \\
x_{23} &= C_{3,2}(0)\, A_{252} - \mu_1\, A_{295} - \mu_{s0'}\, A_{390} - \lambda_1\, A_{363} \\
x_{24} &= C_{3,2}(0)\, A_{252} - \mu_1\, A_{295} - \mu_{s0'} A_{390} - \lambda_{c0}\, A_{252} - \\
&\quad \lambda_{c1}\, A_{295} - \lambda_{c2}\, A_{326} - \lambda_{c3}\, A_{349} - \lambda_{c4}\, A_{363} - \\
&\quad \lambda_{c0'}\, A_{390} - \lambda_{c1'}\, A_{408}
\end{aligned}$$

The long-run availability values generated with Equation (3.93) are shown in the fourth column of Table 3.10. Symbols A_{251} to A_{408} are defined in Appendix C.

$1 - out - of - (n+2) : G$ **System with Type** B **Repair**

With Type B Repair, the failed active units are not repaired and thus μ_i; for $i = 1, 2, ..., n+1$ is set to zero in Equations (3.79) to (3.90).

Special Case Model I

For $n = 1$ with Type B Repair we can directly obtain the long-run availability for this special case from Equation (3.91) by setting μ_1 and μ_2 to zero. These long-run availability values are shown in the second column of Table 3.11.

Special Case Model II

For $n = 2$ with Type B Repair we can also directly obtain the long-run availability for this special case from Equation (3.92) by setting μ_1, μ_2 and μ_3 to zero. These long-run availability values are shown in the third column of Table 3.11.

Special Case Model III

For $n = 3$ with Type B Repair we directly obtained the long-run availability from Equation (3.93) by setting μ_1, μ_2, μ_3 and μ_4 to zero. The long-run availability values are shown in the fourth column of Table 3.11.

$1 - out - of - (n+2) : G$ **System with Type** C **Repair**

With Type C Repair, the failed system units are not repaired and thus μ_i; for $i = 1, 2, ..., n+1$, $\mu_{s0'}$ and $\mu_{s1'}$ are set to zero in Equations (3.79) to (3.90).

Special Case Model I

For $n = 1$ with Type C Repair we can obtain the long-run availability from Equation (3.91) by setting μ_1, μ_2, $\mu_{s0'}$ and $\mu_{s1'}$ to zero and the long-run availability values are shown in the second column of Table 3.12.

Table 3.11: $A_{ssY}(n, 2, 1)\,(\beta)$ Values as a function of λ_{c0} with Type B Repair

	λ_{c0} (per Hr.)	$n = 1$	$n = 2$	$n = 3$
$\beta = 1$	0.00001	0.5090	0.4729	0.4786
	0.00002	0.5061	0.4712	0.4772
	0.00003	0.5032	0.4696	0.4757
	0.00004	0.5002	0.4679	0.4743
$\beta = 2$	0.00001	0.3414	0.3098	0.3146
	0.00002	0.3388	0.3084	0.3134
	0.00003	0.3362	0.3070	0.3121
	0.00004	0.3335	0.3056	0.3109
$\beta = 3$	0.00001	0.2568	0.2304	0.2343
	0.00002	0.2546	0.2292	0.2333
	0.00003	0.2524	0.2280	0.2322
	0.00004	0.2502	0.2269	0.2312
$\beta = 4$	0.00001	0.2059	0.1834	0.1867
	0.00002	0.2039	0.1824	0.1858
	0.00003	0.2020	0.1814	0.1849
	0.00004	0.2001	0.1804	0.1840
$\beta = 5$	0.00001	0.1718	0.1523	0.1551
	0.00002	0.1701	0.1514	0.1544
	0.00003	0.1684	0.1506	0.1536
	0.00004	0.1668	0.1497	0.1529

Table 3.12: $A_{ssY}(n,2,1)\,(\beta)$ Values as a function of λ_{c0} with Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4774	0.4402	0.4770
	0.00002	0.4750	0.4384	0.4756
	0.00003	0.4725	0.4365	0.4742
	0.00004	0.4700	0.4347	0.4729
$\beta=2$	0.00001	0.3135	0.2961	0.3132
	0.00002	0.3114	0.2945	0.3120
	0.00003	0.3094	0.2929	0.3108
	0.00004	0.3073	0.2912	0.3096
$\beta=3$	0.00001	0.2334	0.2231	0.2331
	0.00002	0.2317	0.2217	0.2322
	0.00003	0.2299	0.2203	0.2312
	0.00004	0.2282	0.2190	0.2302
$\beta=4$	0.00001	0.1859	0.1790	0.1857
	0.00002	0.1844	0.1778	0.1848
	0.00003	0.1830	0.1766	0.1840
	0.00004	0.1815	0.1754	0.1832
$\beta=5$	0.00001	0.1545	0.1494	0.1543
	0.00002	0.1532	0.1484	0.1536
	0.00003	0.1519	0.1474	0.1528
	0.00004	0.1507	0.1464	0.1521

Special Case Model II

For $n = 2$ with Type C Repair we can also obtain the long-run availability for this special case from Equation (3.92) by setting μ_1, μ_2, μ_3, $\mu_{s0'}$ and $\mu_{s1'}$ to zero and the long-run availability values are shown in the third column of Table 3.12.

Special Case Model III

Similarly, for $n = 3$ with Type C Repair we can obtain the long-run availability from Equation (3.93) by setting μ_1, μ_2, μ_3, μ_4, $\mu_{s0'}$ and $\mu_{s1'}$ to zero. The long-run availability values are shown in the fourth column of Table 3.12.

$1 - out - of - (n+2) : G$ System with Type E Repair

With Type E Repair, the failed system and its failed active units are not repaired and thus $\mu_{n+2}(x)$, $\mu_{n+3}(x)$ and μ_i for $i = 1, 2, ..., n+1$ are set to zero in Equations (3.79) to (3.90).

Special Case Model I

For $n = 1$ with Type E Repair we obtain the following results:

$$\begin{aligned} R_{Y_E}(1,2,1)(t) &= e^{s_{28}t}\left[B_{151} + B_{152} + B_{153} + B_{154} + B_{155}\right] + \\ & \quad e^{s_{29}t}\left[B_{156} + B_{157} + B_{158} + B_{159} + B_{160}\right] + \\ & \quad e^{s_{30}t}\left[B_{161} + B_{162} + B_{163} + B_{164} + B_{165}\right] + \\ & \quad e^{s_{31}t}\left[B_{166} + B_{167}\right] + e^{s_{32}t}\left[B_{168}\right] \end{aligned} \tag{3.94}$$

$W_{32}(s) = s^5 + A_{81}s^4 + A_{82}s^3 + A_{83}s^2 + A_{84}s + A_{85}$
where s_{28} to s_{32} are the roots of the polynomial function $W_{32}(s)$. The partial fraction residues B_{151} to B_{168} are defined in Appendix C. System reliability values computed using (3.94) for specified values of system parameters are shown in the second column of Table 3.13.

Table 3.13: $R_{Y_E}(n,2,1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	0.9999	1.0000
2000	0.7914	0.8955	0.9224
4000	0.6145	0.6494	0.6906
6000	0.4210	0.3861	0.4084
8000	0.2642	0.2031	0.2100
∞	0	0	0

The system mean time to failure is:

$$MTTF_{Y_E}(1,2,1) = \frac{1}{\alpha_2[C_{1,2}(0)]}[(\zeta_1[C_{1,2}(0')])(1+\frac{\lambda_1}{C_{1,2}(1)}+ \frac{\lambda_1^2}{C_{1,2}(1)\,C_{1,2}(2)}) + 2\lambda_s C_{1,2}(1') + 2\lambda_s^2] \tag{3.95}$$

where,

$\alpha_2[C_{1,2}(0)] = C_{1,2}(0)\,(C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\mu_{s1'}) - 2\lambda_s\mu_{s0'}C_{1,2}(1')$

$\zeta_1[C_{1,2}(0')] = C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\mu_{s1'}$

System mean time to failure values calculated with Equation (3.95) are shown in the first column of Table 3.14.

The variance of time to failure for the system assuming that the failed active unit is repaired is:

$$\sigma^2_{Y_D}(1,2,1) = \frac{1}{k_{14}^2}[2(k_{13}\,k_{15} - k_{14}\,k_{16}) - k_{13}^2] \tag{3.96}$$

The symbols k_{13} to k_{16} are defined in Appendix C.

Setting $\mu_1 = \mu_2 = 0$ in Equation (3.96), the variance of the time to failure for Type E Repair can be determined.

Table 3.14: $MTTF_{Y_E}(n,2,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	6747.63	5933.64	6062.63
0.00002	6553.47	5825.97	5979.68
0.00003	6368.38	5721.23	5898.36
0.00004	6191.77	5619.33	5818.63

Special Case Model II

For $n=2$ with Type E Repair, we obtain the following results:

The reliability for this special case model is given by:

$$\begin{aligned} R_{Y_E}(2,2,1)(t) &= e^{s_{33}t}[B_{187}+B_{188}+B_{189}+B_{190}+B_{191}+B_{192}]+ \\ &\quad e^{s_{34}t}[B_{193}+B_{194}+B_{195}+B_{196}+B_{197}+B_{198}]+ \\ &\quad e^{s_{35}t}[B_{199}+B_{200}+B_{201}+B_{202}+B_{203}+B_{204}]+ \\ &\quad e^{s_{36}t}[B_{205}+B_{206}+B_{207}]+e^{s_{37}t}[B_{208}+B_{209}]+ \\ &\quad e^{s_{38}t}[B_{210}] \end{aligned} \tag{3.97}$$

$W_{42}(s) = (s + C_{2,2}(0))\, W_{33}(s) - \mu_{s0'}\, W_{40}(s)$

$W_{33}(s) = s^5 + A_{86}\, s^4 + A_{111}\, s^3 + A_{112}\, s^2 + A_{113}\, s + A_{114}$

$W_{40}(s) = A_{172}\, s^4 + A_{173}\, s^3 + A_{183}\, s^2 + A_{184}\, s + A_{185}$

where $s_{33},\ldots\ldots,s_{38}$ are the roots of the polynomial function $W_{42}(s)$. The residues B_{187} to B_{210} are defined in Appendix C. System reliability values developed with Equation (3.97) are shown in the third column of Table 3.13.

The mean time to failure of the system is given by:

$$\begin{aligned} MTTF_{Y_E}(2,2,1) &= \frac{1}{\alpha_2[C_{2,2}(0)]}[(\zeta_1[C_{2,2}(0')])\,(1+\frac{2\lambda_1}{C_{2,2}(1)}+\frac{4\lambda_1^2}{C_{2,2}(1)\,C_{2,2}(2)}+ \\ &\quad \frac{8\lambda_1^3}{C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(3)})+2\lambda_s C_{2,2}(1')\,(1+\frac{2\lambda_1}{C_{2,2}(2)}+ \end{aligned}$$

$$\frac{4\lambda_1^2}{C_{2,2}(2)\,C_{2,2}(3)}) + 2\lambda_s^2\,(1 + \frac{2\lambda_1}{C_{2,2}(3)})] \tag{3.98}$$

where

$\alpha_2[C_{2,2}(0)] = C_{2,2}(0)\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'}) - \mu_{s0'}\,(2\lambda_s C_{2,2}(1'))$

$\zeta_1[C_{2,2}(0')] = C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\,\mu_{s1'}$

The third column of Table 3.14 contains system mean time to failure values generated with Equation (3.98).

The variance of time to failure for this system assuming that failed active units are repaired is given by:

$$\sigma^2_{Y_D}(2,2,1) \;=\; \frac{1}{k_{18}^2}\,[2(k_{17}\,k_{19} - k_{18}\,k_{20}) - k_{17}^2] \tag{3.99}$$

The symbols k_{17} to k_{20} are defined in Appendix C.

Setting $\mu_1 = \mu_2 = \mu_3 = 0$ in (3.99), the variance of time to system failure under Type E Repair can be determined.

Special Case Model III

For $n = 3$ with Type E Repair, we obtain the following results:

The reliability for this special case model is given by:

$$\begin{aligned} R_{Y_E}(3,2,1)(t) \;=\; & e^{s_{39}t}\,[B_{235} + B_{236} + B_{237} + B_{238} + B_{239} + B_{240} + \\ & B_{241}] + e^{s_{40}t}\,[B_{242} + B_{243} + B_{244} + B_{245} + B_{246} + \\ & B_{247} + B_{248}] + e^{s_{41}t}\,[B_{249} + B_{250} + B_{251} + B_{252} + \\ & B_{253} + B_{254} + B_{255}] + e^{s_{42}t}\,[B_{256} + B_{257} + \\ & B_{258} + B_{259}] + e^{s_{43}t}\,[B_{260} + B_{261} + \\ & B_{262}] + e^{s_{44}t}\,[B_{263} + B_{264}] + e^{s_{45}t}\,[B_{265}] \end{aligned} \tag{3.100}$$

$W_{53}(s) = (s + C_{3,2}(0))\,[(s + C_{3,2}(0'))(s + C_{3,2}(1')) - \lambda_s\,\mu_{s1'}] - \mu_{s0'}\,[2\,\lambda_s\,(s + C_{3,2}(1'))]$

where s_{39} to s_{45} are the roots of the polynomial function $W_{53}(s)$. The residues B_{235}

to B_{265} are defined in Appendix C. The fourth column of Table 3.13 contains system reliability values generated using Equation (3.100).

The mean time to failure for the system is:

$$
\begin{aligned}
MTTF_{Y_E}(3,2,1) = & \frac{1}{\alpha_2[C_{3,2}(0)]}[(\zeta_1[C_{3,2}(0')])(1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)}+ \\
& \frac{27\lambda_1^3}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)}+\frac{54\lambda_1^4}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4)})+ \\
& 2\lambda_s C_{3,2}(1')(1+\frac{3\lambda_1}{C_{3,2}(2)}+\frac{9\lambda_1^2}{C_{3,2}(2)\,C_{3,2}(3)}+ \\
& \frac{18\lambda_1^3}{C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4)})+2\lambda_s^2(1+\frac{3\lambda_1}{C_{3,2}(3)}+ \\
& \frac{6\lambda_1^2}{C_{3,2}(3)\,C_{3,2}(4)})]
\end{aligned}
\tag{3.101}
$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,(C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}) - \mu_{s0'}\,(2\lambda_s C_{3,2}(1'))$

$\zeta_1[C_{3,2}(0')] = C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}$

Equation (3.101) was used to generate system mean time to failure values given in the fourth column of Table 3.14.

The variance of time to failure for the system assuming that failed active units are repaired is given by:

$$
\sigma^2_{Y_D}(3,2,1) = \frac{1}{k_{22}^2}[2(k_{21}\,k_{23} - k_{22}\,k_{24}) - k_{21}^2] \tag{3.102}
$$

The symbols k_{21} to k_{24} are defined in Appendix C.

The variance of time to failure with Type E Repair can be determined by setting $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ in (3.102).

$1-out-of-(n+2):G$ System with Type F Repair

With Type F Repair, the failed system and its failed units are not repaired and thus $\mu_{n+2}(x)$, $\mu_{n+3}(x)$, $\mu_{s0'}$, $\mu_{s1'}$ and μ_i; for $i = 1,2,....,n+1$ are set equal to zero in Equations (3.79) to (3.90).

Special Case Model I

For $n = 1$ with Type F Repair the relevant reliability indices can be obtained as follows:

The system reliability for this special case can be directly obtained from Equation (3.94) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero. The results are shown in the second column of Table 3.15.

Table 3.15: $R_{Y_F}(n,2,1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.7343	0.8954	0.9218
4000	0.5391	0.6420	0.6875
6000	0.3519	0.3750	0.4039
8000	0.2092	0.1938	0.2065
∞	0	0	0

The mean time to failure can be directly obtained from Equation (3.95) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero:

$$\begin{aligned} MTTF_{Y_F}(1,2,1) &= \frac{1}{\alpha_2[C_{1,2}(0)]}[(\zeta_1[C_{1,2}(0')])(1+\frac{\lambda_1}{C_{1,2}(1)}+ \\ &\quad \frac{\lambda_1^2}{C_{1,2}(1)\,C_{1,2}(2)})+2\lambda_s C_{1,2}(1')+2\lambda_s^2] \end{aligned} \tag{3.103}$$

where

$\alpha_2[C_{1,2}(0)] = C_{1,2}(0)\,C_{1,2}(0')\,C_{1,2}(1')$

$\zeta_1[C_{1,2}(0')] = C_{1,2}(0')\,C_{1,2}(1')$

Equation (3.103) was used to obtain the system mean time to failure values shown in the second column of Table 3.16.

Table 3.16: $MTTF_{Y_F}(n,2,1)$ values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	6249.96	5811.07	6022.64
0.00002	6097.04	5711.79	5942.89
0.00003	5950.27	5615.06	5864.65
0.00004	5809.34	5520.81	5787.90

Setting $\mu_1 = \mu_2 = \mu_{s0'} = \mu_{s1'} = 0$ in Equation (3.96) the variance of time to system failure with Type F Repair can be determined.

Special Case Model II

For $n = 2$ with Type F Repair the relevant reliability indices can be obtained as follows:

The system reliability can be determined from Equation (3.97) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero. The results are shown in the third column of Table 3.15.

The system mean time to failure is determined directly from Equation (3.98) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero:

$$\begin{aligned} MTTF_{Y_F}(2,2,1) &= \frac{1}{\alpha_2[C_{2,2}(0)]}[(\zeta_1[C_{2,2}(0')])\,(1+\frac{2\lambda_1}{C_{2,2}(1)}+\frac{4\lambda_1^2}{C_{2,2}(1)\,C_{2,2}(2)}+ \\ &\quad \frac{8\lambda_1^3}{C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(3)})+2\lambda_s C_{2,2}(1')\,(1+\frac{2\lambda_1}{C_{2,2}(2)}+ \\ &\quad \frac{4\lambda_1^2}{C_{2,2}(2)\,C_{2,2}(3)})+2\lambda_s^2\,(1+\frac{2\lambda_1}{C_{2,2}(3)})] \end{aligned} \tag{3.104}$$

where

$\alpha_2[C_{2,2}(0)] = C_{2,2}(0)\,C_{2,2}(0')\,C_{2,2}(1')$

$\zeta_1[C_{2,2}(0')] = C_{2,2}(0')\,C_{2,2}(1')$

Equation (3.104) was used to calculate values for the system mean time to failure values given in the third column of Table 3.16.

Setting $\mu_1 = \mu_2 = \mu_3 = \mu_{s0'} = \mu_{s1'} = 0$ in (3.99) the variance of time to failure with Type F Repair can be determined.

Special Case Model III

For $n = 3$ with Type F Repair the relevant reliability indices are obtained as follows: The system reliability was obtained directly from Equation (3.100) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero. Its tabular values are given in the fourth column of Table 3.15.

The mean time to failure for the system is determined using Equation (3.101) by setting $\mu_{s0'}$ and $\mu_{s1'}$ to zero:

$$\begin{aligned} MTTF_{Y_F}(3,2,1) &= \frac{1}{\alpha_2[C_{3,2}(0)]}[(\zeta_1[C_{3,2}(0')])\,(1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)}+ \\ &\quad \frac{27\lambda_1^3}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)}+\frac{54\lambda_1^4}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4)})+ \\ &\quad 2\lambda_s C_{3,2}(1')\,(1+\frac{3\lambda_1}{C_{3,2}(2)}+\frac{9\lambda_1^2}{C_{3,2}(2)\,C_{3,2}(3)}+ \\ &\quad \frac{18\lambda_1^3}{C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4)})+2\lambda_s^2\,(1+\frac{3\lambda_1}{C_{3,2}(3)}+ \\ &\quad \frac{6\lambda_1^2}{C_{3,2}(3)\,C_{3,2}(4)})] \end{aligned} \tag{3.105}$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,C_{3,2}(0')\,C_{3,2}(1')$

$\zeta_1[C_{3,2}(0)] = C_{3,2}(0')\,C_{3,2}(1')$

The fourth column of Table 3.16 presents system mean time to failure values calculated using Equation (3.105).

The variance of time to failure with Type F Repair can be determined by setting $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_{s0'} = \mu_{s1'} = 0$ in (3.102).

3.1.3 1-out-of-(n+3):G Identical Unit Submodel

This is the $(n, 3, 1)$ system and is composed of n active units and *three* standbys. It is functional as long as at least *one* unit is healthy. A block diagram for this system is shown in Figure 3.20 and its state space diagram is given in Figure 3.21.

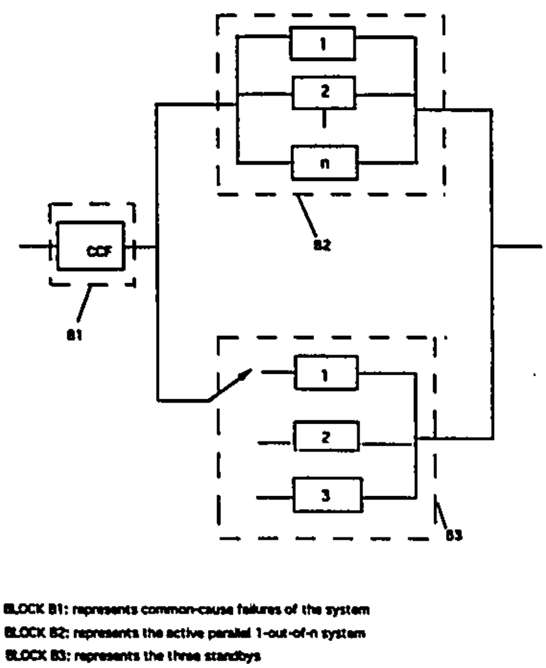


Figure 3.20: Block diagram of the $1-out-of-(n+3):G$ System

<u>$1-out-of-(n+3):G$ system with Type A Repair</u>

The corresponding integro-differential equations for the model described in Figure 3.21 are:

$$\frac{dP_0(t)}{dt} + C_{n,3}(0)\, P_0(t) = \mu_1\, P_1(t) + \mu_{s0'}\, P_{0'}(t) + \sum_{i=n+3}^{n+4} \int_0^{\infty} P_i(x,t)\, \mu_i(x) dx \quad (3.106)$$

$$\frac{dP_1(t)}{dt} + C_{n,3}(1)\, P_1(t) = n\,\lambda_1\, P_0(t) + \mu_2\, P_2(t) \quad (3.107)$$

$$\frac{dP_2(t)}{dt} + C_{n,3}(2)\, P_2(t) = n\,\lambda_1\, P_1(t) + n\,\lambda_1\, P_{0'}(t) + \mu_3\, P_3(t) \quad (3.108)$$

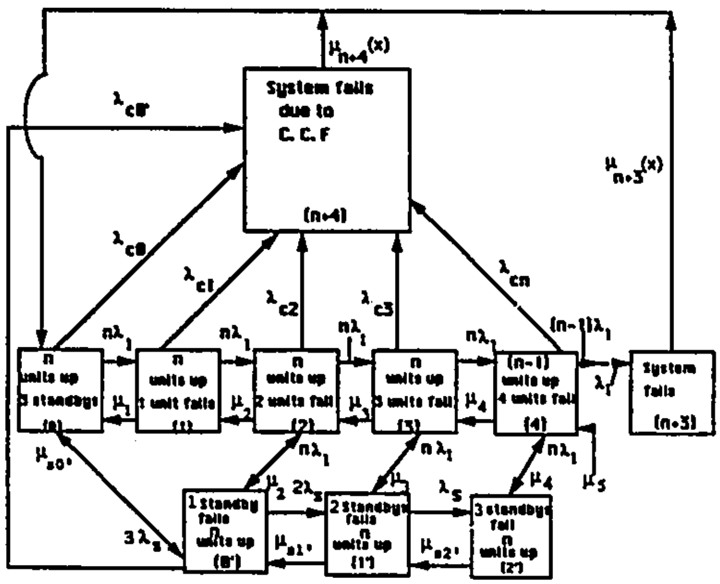


Figure 3.21: State space diagram of the $1 - out - of - (n+3) : G$ System under SBAP Y

$$\frac{dP_3(t)}{dt} + C_{n,3}(3)\,P_3(t) = n\,\lambda_1\,P_2(t) + n\,\lambda_1\,P_{1'}(t) + \mu_4\,P_4(t) \tag{3.109}$$

$$\frac{dP_4(t)}{dt} + C_{n,3}(4)\,P_4(t) = n\,\lambda_1\,P_3(t) + n\,\lambda_1\,P_{2'}(t) + \mu_5\,P_5(t) \tag{3.110}$$

For $j = 5, 6, \ldots.., n+1$,

$$\frac{dP_j(t)}{dt} + C_{n,3}(j)\,P_j(t) = (n-j+4)\,\lambda_1\,P_{j-1}(t) + \mu_{j+1}\,P_{j+1} \tag{3.111}$$

$$\frac{dP_{n+2}(t)}{dt} + C_{n,3}(n+2)\,P_{n+2}(t) = 2\,\lambda_1\,P_{n+1}(t) \tag{3.112}$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,3}(0')\,P_{0'}(t) = 3\lambda_s\,P_0(t) + \mu_{s1'}\,P_{1'}(t) + \mu_2\,P_2(t) \tag{3.113}$$

$$\frac{dP_{1'}(t)}{dt} + C_{n,3}(1')\,P_{1'}(t) = 2\lambda_s\,P_{0'}(t) + \mu_3\,P_3(t) + \mu_{s2'}P_{2'}(t) \tag{3.114}$$

$$\frac{dP_{2'}(t)}{dt} + C_{n,3}(2')\,P_{2'}(t) = \lambda_s\,P_{1'}(t) + \mu_4\,P_4(t) \tag{3.115}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+3}(x)]\,P_{n+3}(x,t) = 0 \tag{3.116}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+4}(x)]\,P_{n+4}(x,t) = 0 \tag{3.117}$$

The associated boundary conditions are:

$$P_{n+3}(0,t) = \lambda_1\,P_{n+2}(t) \tag{3.118}$$

$$P_{n+4}(0,t) = \sum_{i=0}^{n+2} \lambda_{ci}\,P_i(t) + \sum_{i=0'}^{2'} \lambda_{ci}\,P_i(t) \tag{3.119}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,3}(0) = n\,\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{n,3}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,3}(2) = n\,\lambda_1 + \lambda_{c2} + 2\mu_2$

$C_{n,3}(3) = n\,\lambda_1 + \lambda_{c3} + 2\mu_3$

$C_{n,3}(4) = (n-1)\,\lambda_1 + \lambda_{c4} + 2\mu_4$

For $j = 5, 6, \ldots\ldots., n+1$,

$$C_{n,3}(j) = (n-j+3)\,\lambda_1 + \lambda_{cj} + \mu_j$$
$$C_{n,3}(n+2) = \lambda_1 + \lambda_{cn+2} + \mu_{n+2}$$
$$C_{n,3}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'} + 2\lambda_s$$
$$C_{n,3}(1') = n\,\lambda_1 + \lambda_{c1'} + \mu_{s1'} + \lambda_s$$
$$C_{n,3}(2') = n\,\lambda_1 + \lambda_{c2'} + \mu_{s2'}$$

Special Case Model I

For $n = 1$ in Equations (3.106) to (3.119) and using the procedure outlined for the last sub-model, the long-run availability of this special case is given by:

$$A_{ssY}(1,3,1)\,(\beta) = \frac{(\mu_4\,\mu_5)^\beta x_{25}}{(\mu_4\,\mu_5)^\beta\, x_{26} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{27} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{28}} \tag{3.120}$$

where,

$$\begin{aligned}
x_{25} &= A_{450} + A_{502} + A_{529} + A_{551} + A_{583} + A_{603} + A_{617} \\
x_{26} &= A_{502} + C_{1,3}(0)\,A_{501} - \mu_1\,A_{449} - \mu_{s0'}\,A_{582} - \lambda_1\,A_{550} - \\
&\quad \lambda_1\,A_{616} - \lambda_{c0}\,A_{501} - \lambda_{c1}\,A_{449} - \lambda_{c2}\,A_{528} - \lambda_{c3}\,A_{550} - \\
&\quad \lambda_{c0'}\,A_{582} - \lambda_{c1'}\,A_{602} - \lambda_{c2'}\,A_{616} \\
x_{27} &= C_{1,3}(0)\,A_{502} - \mu_1\,A_{450} - \mu_{s0'}\,A_{583} - \lambda_1\,A_{551} - \lambda_1\,A_{617} \\
x_{28} &= C_{1,3}(0)\,A_{502} - \mu_1\,A_{450} - \mu_{s0'}\,A_{583} - \lambda_{c0}\,A_{502} - \lambda_{c1}\,A_{450} - \\
&\quad \lambda_{c2}\,A_{529} - \lambda_{c3}\,A_{551} - \lambda_{c0'}\,A_{583} - \lambda_{c1'}\,A_{603} - \lambda_{c2'}\,A_{617}
\end{aligned}$$

Steady-state availability values developed using Equation (3.120) for various values of β and for specified values of system parameters are given in the second column of Table 3.17. Symbols A_{450} to A_{617} are defined in Appendix D.

Special Case Model II

For $n = 2$ in Equations (3.106) to (3.119) and using the procedure outlined for the previous sub-model, the long-run availability is:

$$A_{ssY}(2,3,1)\,(\beta) = \frac{(\mu_5\,\mu_6)^\beta\, x_{29}}{(\mu_5\,\mu_6)^\beta\, x_{30} + \beta\,\mu_5^\beta\,\mu_6^{\beta-1}\, x_{31} + \beta\,\mu_5^{\beta-1}\,\mu_6^\beta\, x_{32}} \tag{3.121}$$

Table 3.17: $A_{ssY}(n,3,1)\,(\beta)$ Values as a function of λ_{c0} for Type A Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.6874	0.5198	0.5824
	0.00002	0.6806	0.5173	0.5805
	0.00003	0.6739	0.5148	0.5785
	0.00004	0.6673	0.5123	0.5765
$\beta=2$	0.00001	0.5284	0.3681	0.4109
	0.00002	0.5204	0.3656	0.4089
	0.00003	0.5127	0.3631	0.4070
	0.00004	0.5051	0.3606	0.4050
$\beta=3$	0.00001	0.4292	0.2849	0.3174
	0.00002	0.4213	0.2827	0.3156
	0.00003	0.4137	0.2805	0.3139
	0.00004	0.4063	0.2783	0.3122
$\beta=4$	0.00001	0.3613	0.2324	0.2586
	0.00002	0.3539	0.2304	0.2570
	0.00003	0.3468	0.2285	0.2555
	0.00004	0.3399	0.2265	0.2540
$\beta=5$	0.00001	0.3120	0.1963	0.2181
	0.00002	0.3051	0.1945	0.2168
	0.00003	0.2985	0.1927	0.2154
	0.00004	0.2921	0.1910	0.2140

where

$$
\begin{aligned}
x_{29} &= A_{785} + A_{895} + A_{971} + A_{1013} + A_{1041} + A_{1099} + A_{1139} + A_{1172} \\
x_{30} &= C_{2,3}(0)\, A_{784} + A_{785} - \mu_1\, A_{894} - \mu_{s0'}\, A_{1098} - \lambda_1\, A_{1040} - \\
&\quad \lambda_{c0}\, A_{784} - \lambda_{c1}\, A_{894} - \lambda_{c2}\, A_{970} - \lambda_{c3}\, A_{1012} - \lambda_{c4}\, A_{1040} - \\
&\quad \lambda_{c0'}\, A_{1098} - \lambda_{c1'}\, A_{1138} - \lambda_{c2'}\, A_{1171} \\
x_{31} &= C_{2,3}(0)\, A_{785} - \mu_1\, A_{895} - \mu_{s0'}\, A_{1099} - \lambda_1\, A_{1041} \\
x_{32} &= C_{2,3}(0)\, A_{785} - \mu_1\, A_{895} - \mu_{s0'}\, A_{1099} - \lambda_{c0}\, A_{785} - \lambda_{c1}\, A_{895} - \\
&\quad \lambda_{c2}\, A_{971} - \lambda_{c3}\, A_{1013} - \lambda_{c4}\, A_{1041} - \lambda_{c0'}\, A_{1099} - \\
&\quad \lambda_{c1'}\, A_{1139} - \lambda_{c2'}\, A_{1172}
\end{aligned}
$$

The steady-state availability values obtained using Equation (3.121) for various values of β and for specified values of system parameters are given in the third column of Table 3.17. Symbols A_{785} to A_{1172} are defined in Appendix D.

Special Case Model III

Similarly, for $n = 3$ in Equations (3.106) to (3.119), the long-run availability is given by:

$$
A_{ssY}(3,3,1)\,(\beta) = \frac{(\mu_6\, \mu_7)^\beta\, x_{33}}{(\mu_6\, \mu_7)^\beta\, x_{34} + \beta\, \mu_6^\beta\, \mu_7^{\beta-1}\, x_{35} + \beta\, \mu_6^{\beta-1}\, \mu_7^\beta\, x_{36}} \tag{3.122}
$$

where

$$
\begin{aligned}
x_{33} &= A_{1336} + A_{1502} + A_{1608} + A_{1676} + A_{1726} + A_{1760} + \\
&\quad A_{1844} + A_{1921} + A_{1965} \\
x_{34} &= A_{1336} + C_{3,3}(0)\, A_{1335} - \mu_1\, A_{1501} - \mu_{s0'}\, A_{1843} - \lambda_1\, A_{1759} - \\
&\quad \lambda_{c0}\, A_{1335} - \lambda_{c1}\, A_{1501} - \lambda_{c2}\, A_{1607} - \lambda_{c3}\, A_{1675} - \lambda_{c4}\, A_{1725} - \\
&\quad \lambda_{c5}\, A_{1759} - \lambda_{c0'}\, A_{1843} - \lambda_{c1'}\, A_{1920} - \lambda_{c2'}\, A_{1964} \\
x_{35} &= C_{3,3}(0)\, A_{1336} - \mu_1\, A_{1502} - \mu_{s0'}\, A_{1844} - \lambda_1\, A_{1760} \\
x_{36} &= C_{3,3}(0)\, A_{1336} - \mu_1\, A_{1502} - \mu_{s0'} A_{1843} - \lambda_{c0}\, A_{1336} - \\
&\quad \lambda_{c1}\, A_{1502} - \lambda_{c2}\, A_{1608} - \lambda_{c3}\, A_{1676} - \lambda_{c4}\, A_{1726} - \lambda_{c5}\, A_{1760} - \\
&\quad \lambda_{c0'}\, A_{1844} - \lambda_{c1'}\, A_{1921} - \lambda_{c2'}\, A_{1965}
\end{aligned}
$$

Using Equation (3.122) the system steady-state availability values are calculated for different values of β and for given values of system parameters. These values are presented in the fourth column of Table 3.17. Symbols A_{1336} to A_{1965} are defined in Appendix D.

$1 - out - of - (n+3) : G$ System with Type B Repair

With Type B Repair, the failed active units are discarded and replaced thus μ_1 to μ_{n+2} are set equal to zero in Equations (3.106) to (3.119).

Special Case Model I

For $n = 1$ with Type B Repair the long-run availability values for this case are obtained directly from Equation (3.120) by setting μ_1, μ_2 and μ_3 to zero. These values are given in the second column of Table 3.18.

Special Case Model II

For $n = 2$ with Type B Repair the long-run availability values for the case are obtained from Equation (3.121) by setting μ_1, μ_2, μ_3 and μ_4 to zero. These values are presented in the third column of Table 3.18.

Special Case Model III

Similarly, for $n = 3$ with Type B Repair the long-run availability values are given in the fourth column of Table 3.18.

$1 - out - of - (n+3) : G$ System with Type C Repair

With Type C Repair, the failed units (both active and standby) are discarded and replaced, thus μ_1 to μ_{n+2}, $\mu_{s0'}$ to $\mu_{s2'}$ in Equations (3.106) to (3.119).

Table 3.18: $A_{ssY}(n,3,1)(\beta)$ Values as a function of λ_{c0} for Type B Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.5768	0.4584	0.5085
	0.00002	0.5739	0.4568	0.5071
	0.00003	0.5709	0.4551	0.5057
	0.00004	0.5680	0.4535	0.5043
$\beta=2$	0.00001	0.4053	0.3083	0.3410
	0.00002	0.4024	0.3069	0.3397
	0.00003	0.3995	0.3054	0.3384
	0.00004	0.3966	0.3039	0.3371
$\beta=3$	0.00001	0.3124	0.2323	0.2565
	0.00002	0.3098	0.2310	0.2554
	0.00003	0.3073	0.2298	0.2543
	0.00004	0.3047	0.2285	0.2532
$\beta=4$	0.00001	0.2541	0.1863	0.2055
	0.00002	0.2519	0.1853	0.2046
	0.00003	0.2496	0.1842	0.2037
	0.00004	0.2474	0.1831	0.2028
$\beta=5$	0.00001	0.2142	0.1556	0.1715
	0.00002	0.2122	0.1546	0.1707
	0.00003	0.2102	0.1537	0.1699
	0.00004	0.2082	0.1528	0.1691

Special Case Models I, II, and III

For $n = 1$ with Type C Repair, the long-run availability values for the case is easily obtained from Equation (3.120) by setting μ_1 to μ_3, and $\mu_{s0'}$ to $\mu_{s2'}$ to zero. These values are given in the second column of Table 3.19. Similarly, for $n = 2$ and 3, the long-run availability values are given in the third and fourth columns of Table 3.19, respectively.

$1 - out - of - (n+3) : G$ System with Type E Repair

With Type E Repair, the failed system as well as its failed active units are discarded and replaced thus μ_1 to μ_{n+2}, $\mu_{n+3}(x)$ and $\mu_{n+4}(x)$ in Equations (3.106) to (3.119) are set to zero.

Special Case Model I

For $n = 1$ with Type E Repair we obtain the following set of expressions for relevant reliability indices using the procedure already established for the previous special cases:

The system reliability for this case is given by:

$$\begin{aligned} R_{Y_E}(1,3,1)(t) &= \sum_{i=0}^{3} P_i(t) + \sum_{i=0'}^{2'} P_i(t) \\ &= e^{s_{46}t}\left[B_{297} + B_{298} + B_{299} + B_{300} + B_{301} + B_{302} + \right. \\ & \quad B_{303}] + e^{s_{47}t}\left[B_{304} + B_{305} + B_{306} + B_{307} + B_{308} + \right. \\ & \quad B_{309} + B_{310}] + e^{s_{48}t}\left[B_{311} + B_{312} + B_{313} + B_{314} + \right. \\ & \quad B_{315} + B_{316} + B_{317}] + e^{s_{49}t}\left[B_{318} + B_{319} + B_{320} + \right. \\ & \quad B_{321} + B_{322} + B_{323} + B_{324}] + e^{s_{50}t}\left[B_{325} + B_{326} + \right. \\ & \quad B_{327}] + e^{s_{51}t}\left[B_{328} + B_{329}\right] + e^{s_{52}t}\left[B_{330}\right] \end{aligned} \tag{3.123}$$

$W_{64}(s) = (s + C_{1,3}(0))\, W_{54}(s) - \mu_{s0'}\, W_{61}(s)$

$W_{54}(s) = s^6 + A_{497}\, s^5 + A_{498}\, s^4 + A_{499}\, s^3 + A_{500}\, s^2 + A_{501}\, s + A_{502}$

$W_{61}(s) = A_{560}\, s^5 + A_{561}\, s^4 + A_{580}\, s^3 + A_{581}\, s^2 + A_{582}\, s + A_{583}$

Table 3.19: $A_{ss_Y}(n,3,1)\,(\beta)$ Values as a function of λ_{c0} With Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.5288	0.4282	0.5065
	0.00002	0.5266	0.4268	0.5051
	0.00003	0.5244	0.4254	0.5038
	0.00004	0.5222	0.4240	0.5024
$\beta=2$	0.00001	0.3595	0.2880	0.3391
	0.00002	0.3574	0.2867	0.3379
	0.00003	0.3554	0.2855	0.3367
	0.00004	0.3534	0.2842	0.3355
$\beta=3$	0.00001	0.2723	0.2170	0.2549
	0.00002	0.2705	0.2159	0.2539
	0.00003	0.2688	0.2148	0.2528
	0.00004	0.2670	0.2137	0.2518
$\beta=4$	0.00001	0.2191	0.1740	0.2042
	0.00002	0.2176	0.1731	0.2033
	0.00003	0.2161	0.1722	0.2024
	0.00004	0.2146	0.1713	0.2016
$\beta=5$	0.00001	0.1833	0.1453	0.1703
	0.00002	0.1820	0.1445	0.1695
	0.00003	0.1807	0.1437	0.1688
	0.00004	0.1794	0.1429	0.1680

where s_{46} to s_{52} are the roots of the polynomial function $W_{64}(s)$. The residues B_{297} to B_{330} are defined in Appendix D. The system reliability values computed with Equation (3.123) for the given values of system parameters are presented in the second column of Table 3.20.

Table 3.20: $R_{Y_E}(n,3,1)$ values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	0.9999	0.9999	1.0000
2000	0.9640	0.9414	0.9624
4000	0.8015	0.7773	0.7983
6000	0.6250	0.5266	0.5476
8000	0.4490	0.3095	0.3305
∞	0	0	0

The system mean time to failure is:

$$\begin{aligned} MTTF_{Y_E}(1,3,1) &= \frac{1}{\alpha_3[C_{1,3}(0)]}[(\zeta_2[C_{1,3}(0')])(1+\frac{\lambda_1}{C_{1,3}(1)}+\frac{\lambda_1^2}{C_{1,3}(1)\,C_{1,3}(2)}+ \\ &\quad \frac{\lambda_1^3}{C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(3)})+3\lambda_s\,\zeta_1[C_{1,3}(1')]\,(1+\frac{\lambda_1}{C_{1,3}(2)}+ \\ &\quad \frac{\lambda_1^2}{C_{1,3}(2)\,C_{1,3}(3)})+6\lambda_s^2 C_{1,3}(2')\,(1+\frac{\lambda_1}{C_{1,3}(3)})+6\lambda_s^3] \quad (3.124) \end{aligned}$$

where

$\alpha_3[C_{1,3}(0)] = C_{1,3}(0)\,\zeta_2[C_{1,3}(0')] - 3\lambda_s\mu_{s0'}\,\zeta_1[C_{1,3}(1')]$

$\zeta_2[C_{1,3}(0')] = C_{1,3}(0')\,\zeta_1[C_{1,3}(1')] - 2\lambda_s\mu_{s1'}C_{1,3}(2')$

$\zeta_1[C_{1,3}(1')] = C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\mu_{s2'}$

where

$C_{1,3}(0) = \lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{1,3}(1) = \lambda_1 + \lambda_{c1}$

$C_{1,3}(2) = \lambda_1 + \lambda_{c2}$

$C_{1,3}(3) = \lambda_1 + \lambda_{c3}$

$C_{1,3}(0') = \lambda_1 + \lambda_{c0'} + \mu_{s0'} + 2\lambda_s$

$C_{1,3}(1') = \lambda_1 + \lambda_{c1'} + \mu_{s1'} + \lambda_s$

$C_{1,3}(2') = \lambda_1 + \lambda_{c2'} + \mu_{s2'}$

System mean time to failure values computed using Equation (3.124) for the specified values of system parameters are given in the second column of Table 3.21.

Table 3.21: $MTTF_{Y_E}(n,3,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	8849.97	6680.89	6834.60
0.00002	8592.86	6562.44	6745.12
0.00003	8347.42	6447.11	6657.30
0.00004	8112.92	6334.78	6571.14

The variance of time to failure for this system assuming that the failed active unit is repaired is:

$$\sigma^2_{Y_D}(1,3,1) \quad = \quad \frac{1}{k_{26}^2}\left[2(k_{25}\,k_{27} - k_{26}\,k_{28}) - k_{25}^2\right] \qquad (3.125)$$

The symbols k_{25} to k_{28} are defined in Appendix D.

The variance of time to failure for Type E Repair can be determined by setting $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ in Equation (3.125).

Special Case Model II

For $n = 2$ with Type E Repair, we obtain the following set of expressions for relevant reliability indices using procedures already established for the previous special cases:

The system reliability for this special case is:

$$
\begin{aligned}
R_{Y_E}(2,3,1)(t) &= \sum_{i=0}^{4} P_i(t) + \sum_{i=0'}^{2'} P_i(t) \\
&= e^{s_{53}t}[B_{365} + B_{366} + B_{367} + B_{368} + B_{369} + B_{370} + \\
&\quad B_{371} + B_{372}] + e^{s_{54}t}[B_{373} + B_{374} + B_{375} + B_{376} + \\
&\quad B_{377} + B_{378} + B_{379} + B_{380}] + e^{s_{55}t}[B_{381} + B_{382} + \\
&\quad B_{383} + B_{384} + B_{385} + B_{386} + B_{387} + B_{388}] + \\
&\quad e^{s_{56}t}[B_{389} + B_{390} + B_{391} + B_{392} + B_{393} + B_{394} + \\
&\quad B_{395} + B_{396}] + e^{s_{57}t}[B_{397} + B_{398} + B_{399} + B_{400}] + \\
&\quad e^{s_{58}t}[B_{401} + B_{402} + B_{403}] + e^{s_{59}t}[B_{404} + B_{405}] + \\
&\quad e^{s_{60}t}[B_{406}] \qquad (3.126)
\end{aligned}
$$

$W_{76}(s) = (s + C_{2,3}(0))\, W_{65}(s) - \mu_{s0'}\, W_{73}(s)$

$W_{65}(s) = s^7 + A_{626}\, s^6 + A_{780}\, s^5 + A_{781}\, s^4 + A_{782}\, s^3 + A_{783}\, s^2 + A_{784}\, s + A_{785}$

$W_{73}(s) = A_{1046}\, s^6 + A_{1051}\, s^5 + A_{1095}\, s^4 + A_{1096}\, s^3 + A_{1097}\, s^2 + A_{1098}\, s + A_{1099}$

where s_{53} to s_{60} are the roots of the polynomial function $W_{76}(s)$. The residues B_{365} to B_{406}, are defined in Appendix D. Equation (3.126) was used to compute system reliability values shown in the third column of Table 3.20.

The mean time to failure of the system is:

$$
\begin{aligned}
MTTF_{Y_E}(2,3,1) &= \frac{1}{\alpha_3[C_{2,3}(0)]} [(\zeta_2[C_{2,3}(0')])\,(1 + \frac{2\lambda_1}{C_{2,3}(1)} + \frac{4\lambda_1^2}{C_{2,3}(1)\, C_{2,3}(2)} + \\
&\quad \frac{8\lambda_1^3}{C_{2,3}(1)\, C_{2,3}(2)\, C_{2,3}(3)} + \frac{16\lambda_1^4}{C_{2,3}(1)\, C_{2,3}(2)\, C_{2,3}(3)\, C_{2,3}(4)}) + \\
&\quad 3\lambda_s \zeta_1[C_{2,3}(1')]\,(1 + \frac{2\lambda_1}{C_{2,3}(2)} + \frac{4\lambda_1^2}{C_{2,3}(2)\, C_{2,3}(3)} + \\
&\quad \frac{8\lambda_1^3}{C_{2,3}(2)\, C_{2,3}(3)\, C_{2,3}(4)}) + 6\lambda_s^2 C_{2,3}(2')\,(1 + \frac{2\lambda_1}{C_{2,3}(3)} + \\
&\quad \frac{4\lambda_1^2}{C_{2,3}(3)\, C_{2,3}(4)}) + 6\lambda_s^3\,(1 + \frac{2\lambda_1}{C_{2,3}(4)})] \qquad (3.127)
\end{aligned}
$$

$\alpha_3[C_{2,3}(0)] = C_{2,3}(0)\, \zeta_2[C_{2,3}(0')] - 3\lambda_s \mu_{s0'}\, \zeta_1[C_{2,3}(1')]$

$\zeta_2[C_{2,3}(0')] = C_{2,3}(0')\, \zeta_1[C_{2,3}(1')] - 2\lambda_s \mu_{s1'}\, C_{2,3}(2')$

$\zeta_1[C_{2,3}(1')] = C_{2,3}(1')\, C_{2,3}(2') - \lambda_s \mu_{s2'}$

System mean life values generated with Equation (3.127) are presented in the third column of Table 3.21.

where

$C_{2,3}(0) = 2\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{2,3}(1) = 2\lambda_1 + \lambda_{c1}$

$C_{2,3}(2) = 2\lambda_1 + \lambda_{c2}$

$C_{2,3}(3) = 2\lambda_1 + \lambda_{c3}$

$C_{2,3}(4) = \lambda_1 + \lambda_{c4}$

$C_{2,3}(0') = 2\lambda_1 + \lambda_{c0'} + \mu_{s0'} + 2\lambda_s$

$C_{2,3}(1') = 2\lambda_1 + \lambda_{c1'} + \mu_{s1'} + \lambda_s$

$C_{2,3}(2') = 2\lambda_1 + \lambda_{c2'} + \mu_{s2'}$

The system variance of time to failure assuming that the failed active units are repaired is:

$$\sigma^2_{Y_D}(2,3,1) \;=\; \frac{1}{k_{30}^2}\left[2(k_{29}\,k_{31} - k_{30}\,k_{32}) - k_{29}^2\right] \qquad (3.128)$$

where symbols k_{29} to k_{32} are defined in Appendix D.

Setting $\mu_1 = \ldots\ldots = \mu_4 = \mu_{s0'} = \mu_{s1'} = \mu_{s2'} = 0$ in Equation (3.128) the variance of time to failure for Type E Repair can be determined.

Special Case Model III

Similarly, for $n = 3$ with Type E Repair, the following set of expressions for relevant reliability indices are obtained:

The system reliability is given by:

$$\begin{aligned} R_{Y_E}(3,3,1)(t) &= \sum_{i=0}^{5} P_i(t) + \sum_{i=0'}^{2'} P_i(t) \\ &= e^{s_{61}t}[B_{449} + B_{450} + B_{451} + B_{452} + B_{453} + B_{454} + \\ &\quad B_{455} + B_{456} + B_{457}] + e^{s_{62}t}[B_{458} + B_{459} + B_{460} + \\ &\quad B_{461} + B_{462} + B_{463} + B_{464} + B_{465} + B_{466}] + \\ &\quad e^{s_{63}t}[B_{467} + B_{468} + B_{469} + B_{470} + B_{471} + B_{472} + \end{aligned}$$

$$B_{473} + B_{474} + B_{475}] + e^{s_{64}t}[B_{476} + B_{477} +$$
$$B_{478} + B_{479} + B_{480} + B_{481} + B_{482} + B_{483} +$$
$$B_{484}] + e^{s_{65}t}[B_{485} + B_{486} + B_{487} + B_{488} + B_{489}] +$$
$$e^{s_{66}t}[B_{490} + B_{491} + B_{492} + B_{493}] + e^{s_{67}t}[B_{494} +$$
$$B_{495} + B_{496}] + e^{s_{68}t}[B_{497} + B_{498}] +$$
$$e^{s_{69}t}[B_{499}] \tag{3.129}$$

$W_{89}(s) = (s + C_{3,3}(0))\, W_{77}(s) - \mu_{s0'}\, W_{86}(s)$

$W_{77}(s) = s^8 + A_{1231}\, s^7 + A_{1330}\, s^6 + A_{1331}\, s^5 + A_{1332}\, s^4 + A_{1333}\, s^3 + A_{1334}\, s^2 + A_{1335}\, s + A_{1336}$

$W_{86}(s) = 3\,\lambda_s\, s^7 + A_{1779}\, s^6 + A_{1839}\, s^5 + A_{1840}\, s^4 + A_{1841}\, s^3 + A_{1842}\, s^2 + A_{1843}\, s + A_{1844}$

where s_{61} to s_{69} are the roots of the polynomial function $W_{89}(s)$. The residues B_{449} to B_{499} are defined in Appendix D. Using Equation (3.129) the system reliability values were calculated and are given in the third column of Table 3.20.

The system mean time to failure is given by:

$$\begin{aligned} MTTF_{Y_E}(3,3,1) \;=\; & \frac{1}{\alpha_3[C_{3,3}(0)]}\, [(\zeta_2[C_{3,3}(0')])\, (1 + \frac{3\lambda_1}{C_{3,3}(1)} + \frac{9\lambda_1^2}{C_{3,3}(1)\, C_{3,3}(2)} + \\ & \frac{27\lambda_1^3}{C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(3)} + \frac{81\lambda_1^4}{C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)} + \\ & \frac{162\lambda_1^5}{C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)\, C_{3,3}(5)}) + \\ & 3\lambda_s\zeta_1[C_{3,3}(1')]\, (1 + \frac{3\lambda_1}{C_{3,3}(2)} + \frac{9\lambda_1^2}{C_{3,3}(2)\, C_{3,3}(3)} + \\ & \frac{27\lambda_1^3}{C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)} + \frac{54\lambda_1^4}{C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)\, C_{3,3}(5)}) + \\ & 6\lambda_s^2 C_{3,3}(2')\, (1 + \frac{3\lambda_1}{C_{3,3}(3)} + \frac{9\lambda_1^2}{C_{3,3}(3)\, C_{3,3}(4)} + \\ & \frac{18\lambda_1^3}{C_{3,3}(3)\, C_{3,3}(4)\, C_{3,3}(5)}) + 6\lambda_s^3\, (1 + \frac{3\lambda_1}{C_{3,3}(4)} + \\ & \frac{6\lambda_1^2}{C_{3,3}(4)\, C_{3,3}(5)})] \end{aligned} \tag{3.130}$$

where

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\, \zeta_2[C_{3,3}(0')] - 3\lambda_s\mu_{s0'}\, \zeta_1[C_{3,3}(1')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\, \zeta_1[C_{3,3}(1')] - 2\lambda_s\mu_{s1'}\, C_{3,3}(2')$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\mu_{s2'}$

where

$C_{3,3}(0) = 3\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{3,3}(1) = 3\lambda_1 + \lambda_{c1}$

$C_{3,3}(2) = 3\lambda_1 + \lambda_{c2}$

$C_{3,3}(3) = 3\lambda_1 + \lambda_{c3}$

$C_{3,3}(4) = 2\lambda_1 + \lambda_{c4}$

$C_{3,3}(5) = \lambda_1 + \lambda_{c5}$

$C_{3,3}(0') = 3\lambda_1 + \lambda_{c0'} + \mu_{s0'} + 2\lambda_s$

$C_{3,3}(1') = 3\lambda_1 + \lambda_{c1'} + \mu_{s1'} + \lambda_s$

$C_{3,3}(2') = 3\lambda_1 + \lambda_{c2'} + \mu_{s2'}$

System mean time to failure values generated with Equation (3.130) are given in Table 3.21.

The variance of time to failure for this system on the assumption that the failed active units are repaired is:

$$\sigma^2_{Y_D}(3,3,1) \quad = \quad \frac{1}{k_{34}^2}\,[2(k_{33}\,k_{35} - k_{34}\,k_{36}) - k_{33}^2] \tag{3.131}$$

Symbols k_{33} to k_{36} are defined in Appendix D.

Similarly, the variance of time to failure for Type E' Repair can be determined by setting $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = 0$ in Equation (3.131).

From Equations (3.71), (3.72), (3.73), (3.95), (3.98), (3.101), (3.124), (3.127) and (3.130), the mean time to failure of a $1 - out - of - (n+m) : G$ system with Type E Repair and under standby unit activation policy Y is given by:

$$\begin{aligned} MTTF_{Y_E}(n,m,1) \quad = \quad & \frac{1}{\alpha_m[C_{n,m}(0)]}\,[\,(\zeta_{m-1}[C_{n,m}(0')])\,(1 + \frac{n\,\lambda_1}{C_{n,m}(1)} + \\ & \frac{(n\,\lambda_1)^2}{C_{n,m}(1)\,C_{n,m}(2)} + \ldots + \frac{(n\,\lambda_1)^m}{\prod_{i=1}^{m} C_{n,m}(i)} + \\ & \frac{\psi_1\,(n\,\lambda_1)^{m+1}}{\prod_{i=1}^{m+1} C_{n,m}(i)} + \frac{\psi_2\,(n\,\lambda_1)^{m+1}\,(n-1)\,\lambda_1}{\prod_{i=1}^{m+2} C_{n,m}(i)} + \\ & \frac{\psi_3\,(n\lambda_1)^{m+1}\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=1}^{m+3} C_{n,m}(i)} + \ldots \end{aligned}$$

$$..... + \frac{\psi_{m-1}\,(n\,\lambda_1)^{m+1}\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=1}^{m+n-1}\,C_{n,m}(i)})$$

$$+ m\,\lambda_s\,\zeta_{m-2}\,[C_{n,m}(1')]\,(1 + \frac{n\,\lambda_1}{C_{n,m}(2)} +$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(2)\,C_{n,m}(3)} + ... + \frac{(n\,\lambda_1)^{m-1}}{\prod_{i=2}^{m}\,C_{n,m}(i)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^m}{\prod_{i=2}^{m+1}\,C_{n,m}(i)} + \frac{\psi_2\,(n\,\lambda_1)^m\,(n-1)\,\lambda_1}{\prod_{i=2}^{m+2}\,C_{n,m}(i)} +$$

$$\frac{\psi_3\,(n\,\lambda_1)^m\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=2}^{m+3}\,C_{n,m}(i)} + ...$$

$$..... + \frac{\psi_{m-1}\,(n\,\lambda_1)^m\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=2}^{m+n-1}\,C_{n,m}(i)}) +$$

$$(m)\,(m-1)\,\lambda_s^2\,\zeta_{m-3}\,[C_{n,m}(2')]\,(1 + \frac{n\,\lambda_1}{C_{n,m}(3)} +$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(3)\,C_{n,m}(4)} + ... + \frac{(n\,\lambda_1)^{m-2}}{\prod_{i=3}^{m}\,C_{n,m}(i)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^{m-1}}{\prod_{i=3}^{m+1}\,C_{n,m}(i)} + \frac{\psi_2\,(n\,\lambda_1)^{m-1}\,(n-1)\,\lambda_1}{\prod_{i=3}^{m+2}\,C_{n,m}(i)} +$$

$$\frac{\psi_3\,(n\,\lambda_1)^{m-1}\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=3}^{m+3}\,C_{n,m}(i)} + ...$$

$$.. + \frac{\psi_{m-1}\,(n\,\lambda_1)^{m-1}\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=3}^{m+n-1}\,C_{n,m}(i)}) + ...$$

$$... + (m)(m-1)(m-2)(m-3)(m-4)......$$

$$...(3)\,\lambda_s^{m-2}\,\zeta_1\,[C_{n,m}(m-2)']\,(1 + \frac{n\,\lambda_1}{C_{n,m}(m-1)} +$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(m-1)\,C_{n,m}(m)} + \frac{\psi_1\,(n\,\lambda_1)^3}{C_{n,m}(m-1)\,C_{n,m}(m)\,C_{n,m}(m+1)} +$$

$$\frac{\psi_2\,(n\,\lambda_1)^3\,(n-1)\,\lambda_1}{\prod_{i=m-1}^{m+2}\,C_{n,m}(i)} + ...$$

$$...\frac{\psi_{m-1}\,(n\,\lambda_1)^3\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m-1}^{m+n-1}\,C_{n,m}(i)}) +$$

$$m!\,\lambda_s^{m-1}\,\zeta_0[C_{n,m}(m-1)'](1 + \frac{n\,\lambda_1}{C_{n,m}(m)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^2}{C_{n,m}(m)\,C_{n,m}(m+1)} +$$

$$\frac{\psi_2\,(n\,\lambda_1)^2\,(n-1)\,\lambda_1}{C_{n,m}(m)\,C_{n,m}(m+1)\,C_{n,m}(m+2)} + ...$$

$$.... + \frac{\psi_{m-1}\,(n\,\lambda_1)^2\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m}^{m+n-1}\,C_{n,m}(i)}) +$$

$$m!\,\lambda_s^m\,(1+\frac{\psi_1\,(n\,\lambda_1)}{C_{n,m}(m+1)}+$$
$$\frac{\psi_2\,(n\,\lambda_1)\,(n-1)\,\lambda_1}{C_{n,m}(m+1)\,C_{n,m}(m+2)}+\ldots$$
$$\ldots+\frac{\psi_{m-1}\,(n\,\lambda_1)\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m+1}^{m+n-1}C_{n,m}(i)})] \tag{3.132}$$

where for $j=0,1,2,\ldots(m-1)$, $i=0,1,\ldots,(m-1)$, the following recursive relationships suffice:

$\zeta_j[C_{n,m}(i')]=C_{n,m}(i')\,\zeta_{j-1}[C_{n,m}(i+1')]-j\,\lambda_s\,\mu_{s(i+1)'}\,\zeta_{j-2}\,[C_{n,m}(i+2')]$

$\alpha_{i+1}[C_{n,m}(i)]=C_{n,m}(i)\,\zeta_i[C_{n,m}(i')]-j\,\lambda_s\,\mu_{s(i)'}\,\zeta_{i-1}[C_{n,m}(i+1')]$

Where the starting values are given as:

$\zeta_0[C_{n,m}(0')]=C_{n,m}(0')$

$\alpha_1[C_{n,m}(0)]=C_{n,m}(0)\,C_{n,m}(0')-\lambda_s\,\mu_{s0'}$

and,

Also for $j=1$ to $(m-1)$ we have:

$\psi_j=0$ for $j\geq n-1+m$

$\psi_j=1$ for $j<n-1+m$

$1-out-of-(n+3):G$ System with Type F Repair

With Type F Repair, the failed system as well as its failed units (both active and standby) are discarded and replaced thus $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_{n+2}, $\mu_{n+3}(x)$ and $\mu_{n+4}(x)$ in Equations (3.106) to (3.119).

Special Case Model I

For $n=1$ with Type F Repair we can generate the values of relevant reliability indices as follows:

The system reliability for this special case is obtained directly from Equation (3.123) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero. The values obtained are given in the second column of Table 3.22.

Table 3.22: $R_{Y_F}(n, 3, 1)$ values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n = 1$	$n = 2$	$n = 3$
0	1.0000	1.0000	1.0000
2000	0.9568	0.9363	0.9500
4000	0.7860	0.7666	0.7803
6000	0.5919	0.5134	0.5271
8000	0.4027	0.2952	0.3089
∞	0	0	0

The mean time to failure for this special case is determined directly from Equation (3.124) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero:

$$\begin{aligned} MTTF_{Y_F}(1,3,1) &= \frac{1}{\alpha_3[C_{1,3}(0)]}[(\zeta_2[C_{1,3}(0')])(1+\frac{\lambda_1}{C_{1,3}(1)}+\frac{\lambda_1^2}{C_{1,3}(1)\,C_{1,3}(2)}+ \\ &\quad \frac{\lambda_1^3}{C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(3)})+3\lambda_s\,\zeta_1[C_{1,3}(1')]\,(1+\frac{\lambda_1}{C_{1,3}(2)}+ \\ &\quad \frac{\lambda_1^2}{C_{1,3}(2)\,C_{1,3}(3)})+6\lambda_s^2 C_{1,3}(2')\,(1+\frac{\lambda_1}{C_{1,3}(3)})+6\lambda_s^3] \quad (3.133) \end{aligned}$$

where,

$\alpha_3[C_{1,3}(0)] = C_{1,3}(0)\,\zeta_2[C_{1,3}(0')]$

$\zeta_2[C_{1,3}(0')] = C_{1,3}(0')\,\zeta_1[C_{1,3}(1')]$

$\zeta_1[C_{1,3}(1')] = C_{1,3}(1')\,C_{1,3}(2')$

System mean time to failure values computed using Equation (3.133) for the given values of system parameters are given in the second column of Table 3.23.

Setting $\mu_1 = \mu_2 = \mu_3 = \mu_{s0'} = \mu_{s1'} = \mu_{s2'} = 0$ in Equation (3.125) we can determine the variance of time to system failure for Type F Repair.

Table 3.23: $MTTF_{Y_F}(n,3,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	8145.75	6394.72	6777.25
0.00002	7951.03	6290.18	6692.47
0.00003	7763.73	6188.18	6609.21
0.00004	7583.48	6088.65	6527.44

Special Case Model II

For $n = 2$ with Type F Repair we can generate the values of relevant reliability indices as follows:

The system reliability for this special case was determined using Equation (3.126) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero. The values obtained are given in the third column of Table 3.22.

The system mean time to failure for this special case from Equation (3.127) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero is:

$$\begin{aligned} MTTF_{Y_F}(2,3,1) = {} & \frac{1}{\alpha_3[C_{2,3}(0)]}[(\zeta_2[C_{2,3}(0')])(1+\frac{2\lambda_1}{C_{2,3}(1)}+\frac{4\lambda_1^2}{C_{2,3}(1)\,C_{2,3}(2)}+ \\ & \frac{8\lambda_1^3}{C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(3)}+\frac{16\lambda_1^4}{C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(3)\,C_{2,3}(4)})+ \\ & 3\lambda_s\zeta_1[C_{2,3}(1')](1+\frac{2\lambda_1}{C_{2,3}(2)}+\frac{4\lambda_1^2}{C_{2,3}(2)\,C_{2,3}(3)}+ \\ & \frac{8\lambda_1^3}{C_{2,3}(2)\,C_{2,3}(3)\,C_{2,3}(4)})+6\lambda_s^2C_{2,3}(2')(1+\frac{2\lambda_1}{C_{2,3}(3)}+ \\ & \frac{4\lambda_1^2}{C_{2,3}(3)\,C_{2,3}(4)})+6\lambda_s^3(1+\frac{2\lambda_1}{C_{2,3}(4)})] \end{aligned} \quad (3.134)$$

$\alpha_3[C_{2,3}(0)] = C_{2,3}(0)\,\zeta_2[C_{2,3}(0')]$

$\zeta_2[C_{2,3}(0')] = C_{2,3}(0')\,\zeta_1[C_{2,3}(1')]$

$\zeta_1[C_{2,3}(1')] = C_{2,3}(1')\,C_{2,3}(2')$

System mean life values generated with Equation (3.134) are presented in the third column of Table 3.23.

Setting $\mu_1 = ... = \mu_4 = \mu_{s0'} = \mu_{s1'} = \mu_{s2'} = 0$ in (3.128) the variance of time to failure for Type F Repair can be determined.

Special Case Model III

For $n = 3$ with Type F Repair we generate the values of relevant reliability indices as follows:

The system reliability is obtained directly from Equation (3.129) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero. The computed values are given in the fourth column of Table 3.22.

The system mean time to failure from Equation (3.130) by setting $\mu_{s0'}$ to $\mu_{s2'}$ to zero is:

$$\begin{aligned} MTTF_{Y_F}(3,3,1) \; = \; & \frac{1}{\alpha_3[C_{3,3}(0)]}[((\zeta_2[C_{3,3}(0')])(1+\frac{3\lambda_1}{C_{3,3}(1)}+\frac{9\lambda_1^2}{C_{3,3}(1)\,C_{3,3}(2)}+ \\ & \frac{27\lambda_1^3}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)}+\frac{81\lambda_1^4}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)}+ \\ & \frac{162\lambda_1^5}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(5)})+ \\ & 3\lambda_s\zeta_1[C_{3,3}(1')]\,(1+\frac{3\lambda_1}{C_{3,3}(2)}+\frac{9\lambda_1^2}{C_{3,3}(2)\,C_{3,3}(3)}+ \\ & \frac{27\lambda_1^3}{C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)}+\frac{54\lambda_1^4}{C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(5)})+ \\ & 6\lambda_s^2C_{3,3}(2')\,(1+\frac{3\lambda_1}{C_{3,3}(3)}+\frac{9\lambda_1^2}{C_{3,3}(3)\,C_{3,3}(4)}+ \\ & \frac{18\lambda_1^3}{C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(5)})+6\lambda_s^3\,(1+\frac{3\lambda_1}{C_{3,3}(4)}+ \\ & \frac{6\lambda_1^2}{C_{3,3}(4)\,C_{3,3}(5)})] \end{aligned} \tag{3.135}$$

where,

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\,\zeta_2[C_{3,3}(0')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\,\zeta_1[C_{3,3}(1')]$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\,C_{3,3}(2')$

System mean life values generated with Equation (3.135) using specified values of system parameters are given in the fourth column of Table 3.23.

From Equations (3.75), (3.76), (3.77), (3.103), (3.104), (3.105), (3.133), (3.134) and (3.135), the mean time to failure of a $1-out-of-(n+m):G$ system with Type F Repair and under standby unit activation policy Y, can also be generated with Equation (3.132) except that $\mu_{s0'} = \ldots\ldots = \mu_{sm-1'} = 0$ leading to the following new relationships:

For $j = 0, 1, 2, \ldots(m-1)$, $i = 0, 1, \ldots, (m-1)$,

$\zeta_j[C_{n,m}(i')] = C_{n,m}(i')\,\zeta_{j-1}C_{n,m}(i+1')$

$\alpha_{i+1}[C_{n,m}(i)] = C_{n,m}(i)\,\zeta_i\,C_{n,m}(i')$

Where the starting values are given as:

$\zeta_0[C_{n,m}(0')] = C_{n,m}(0')$

$\alpha_1[C_{n,m}(0)] = C_{n,m}(0)\,C_{n,m}(0')$

and,

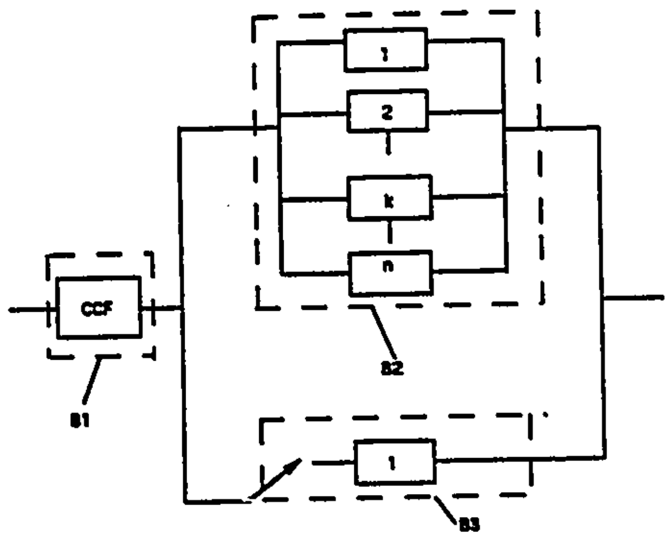
Also for $j = 1$ to $(m-1)$ we have:

$\psi_j = 0$ for $j \geq n-1+m$

$\psi_j = 1$ for $j < n-1+m$

3.1.4 k-out-of-(n+1):G Identical Unit Submodel

This is the $(n, 1, k)$ system and is composed of n active units and *one* standby. It is functional as long as there are at least k functioning units in the system. The $1-out-of-(n+1):G$ submodel of Subsection 3.1.1 is a special case of this system when $k = 1$. A block diagram of this system is shown in Figure 3.22 and the state space diagram is shown in Figure 3.23.



BLOCK B1: represents common-cause failures of the system
BLOCK B2: represents the active on-line k-out-of-n system
BLOCK B3: represents the lone standby system

Figure 3.22: Block diagram of the $k - out - of - (n+1) : G$ System

$k - out - of - (n+1) : G$ System with Type A Repair

The corresponding integro-differential equations for the model described in Figure 3.23 are:

$$\frac{dP_0(t)}{dt} + C_{n,1}(0)\, P_0(t) = \mu_1\, P_1(t) + \mu_{s0'}\, P_{0'}(t) + \sum_{i=r+1}^{r+2} \int_0^\infty P_i(x,t)\, \mu_i(x) dx \quad (3.136)$$

where $r = n - k + 1$.

$$\frac{dP_1(t)}{dt} + C_{n,1}(1)\, P_1(t) = n\, \lambda_1\, P_0(t) + \mu_2\, P_2(t) \quad (3.137)$$

$$\frac{dP_2(t)}{dt} + C_{n,1}(2)\, P_2(t) = n\, \lambda_1\, P_1(t) + n\, \lambda_1\, P_{0'}(t) + \mu_3\, P_3(t) \quad (3.138)$$

For $j = 3, 4, \ldots\ldots, n-k$,

$$\frac{dP_j(t)}{dt} + C_{n,1}(j)\, P_j(t) = (n - j + 2)\, \lambda_1\, P_{j-1}(t) + \mu_{j+1}\, P_{j+1}(t) \quad (3.139)$$

$$\frac{dP_r(t)}{dt} + C_{n,1}(r)\, P_r(t) = (k+1)\, \lambda_1\, P_{n-k}(t) \quad (3.140)$$

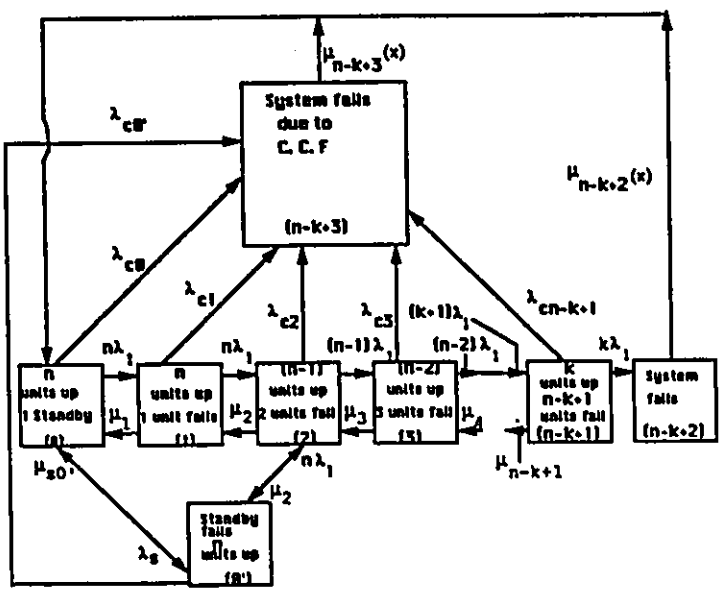


Figure 3.23: State space diagram of the $k - out - of - (n+1) : G$ System under SBAP Y

$$\frac{dP_{0'}(t)}{dt} + C_{n,1}(0')\,P_{0'}(t) = \lambda_s\,P_0(t) + \mu_2\,P_2(t) \tag{3.141}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+1}(x)]\,P_{r+1}(x,t) = 0 \tag{3.142}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+2}(x)]\,P_{r+2}(x,t) = 0 \tag{3.143}$$

The associated boundary conditions are:

$$P_{r+1}(0,t) = k\lambda_1\,P_r(t) \tag{3.144}$$

$$P_{r+2}(0,t) = \sum_{i=0}^{r} \lambda_{ci}\,P_i(t) + \lambda_{c0'}\,P_{0'}(t) \tag{3.145}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,1}(0) = n\,\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,1}(2) = (n-1)\,\lambda_1 + \lambda_{c2} + \mu_2$

For $j = 3, 4, \ldots\ldots., n-k$,

$C_{n,1}(j) = (n-j+1)\,\lambda_1 + \lambda_{cj} + \mu_j$

$C_{n,1}(r) = k\lambda_1 + \lambda_{cr} + \mu_r$

$C_{n,1}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

Special Case Model I

For $n = 3$ and $k = 2$ in Equations (3.136) to (3.145), abbreviated as the $(3,1,2)$ special case, the long-run availability values for this system can be determined from the analysis performed for the $(3,1,1)$ special case with Type A Repair (discussed in Subsection 3.1.1) by eliminating *system up-state 3* and constants relating to it. The long-run availability values for the $(3,1,1)$ special case with Type A Repair is re-produced in the second column of Table 3.24 for purposes of completeness while the results for the $(3,1,2)$ case are given in the third column of Table 3.24.

Table 3.24: $A_{ssY}(3,1,k)\,(\beta)$ values as a function of λ_{c0} for Type A Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5195	0.3451	0.2179
	0.00002	0.5171	0.3435	0.2169
	0.00003	0.5146	0.3420	0.2160
	0.00004	0.5122	0.3404	0.2151
$\beta=2$	0.00001	0.3509	0.2085	0.1222
	0.00002	0.3487	0.2074	0.1217
	0.00003	0.3465	0.2062	0.1211
	0.00004	0.3442	0.2051	0.1205
$\beta=3$	0.00001	0.2649	0.1494	0.0850
	0.00002	0.2630	0.1485	0.0845
	0.00003	0.2611	0.1476	0.0841
	0.00004	0.2592	0.1468	0.0837
$\beta=4$	0.00001	0.2128	0.1164	0.0651
	0.00002	0.2111	0.1157	0.0648
	0.00003	0.2095	0.1150	0.0644
	0.00004	0.2079	0.1143	0.0641
$\beta=5$	0.00001	0.1778	0.0953	0.0528
	0.00002	0.1764	0.0947	0.0525
	0.00003	0.1749	0.0941	0.0522
	0.00004	0.1735	0.0936	0.0520

Special Case Model II

For $n = 3$ and $k = 3$ in Equations (3.136) to (3.145), the long-run availability can also be determined from the analysis conducted for the $(3, 1, 1)$ special case with Type A Repair in Subsection 3.1.1 by eliminating *system up-states 2* and *3* and constants relating to them. The results for the $(3, 1, 3)$ case are presented in the fourth column of Table 3.24.

$k - out - of - (n + 1) : G$ System with Type B Repair

With Type B Repair, the failed active units are discarded upon failure thus μ_1 to μ_r are set to zero in Equations (3.136) to (3.145).

Special Case Model I

For $n = 3$ and $k = 2$ with Type B Repair, the quantities μ_1 and μ_2 are eliminated from the results obtained for the $(3, 1, 2)$ special case with Type A Repair. This procedure yields the values presented in the third column of Table 3.25.

Special Case Model II

For $n = 3$ and $k = 3$ with Type B Repair, the quantity μ_1 is eliminated from the results obtained for the $(3, 1, 3)$ special case with Type A Repair. This procedure yields the values given in the fourth column of Table 3.25.

$k - out - of - (n + 1) : G$ System with Type C Repair

With Type C Repair, the failed units (both active and standbys) are discarded upon failure thus $\mu_{s0'}$, μ_1 to μ_r are set to zero in Equations (3.136) to (3.145).

Table 3.25: $A_{ss_Y}(3,1,k)\,(\beta)$ values as a function of λ_{c0} for Type B Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4447	0.3000	0.1951
	0.00002	0.4432	0.2991	0.1945
	0.00003	0.4418	0.2981	0.1938
	0.00004	0.4403	0.2971	0.1932
$\beta=2$	0.00001	0.2859	0.1765	0.1081
	0.00002	0.2847	0.1759	0.1077
	0.00003	0.2835	0.1752	0.1073
	0.00004	0.2823	0.1745	0.1069
$\beta=3$	0.00001	0.2107	0.1250	0.0748
	0.00002	0.2097	0.1245	0.0745
	0.00003	0.2087	0.1240	0.0742
	0.00004	0.2077	0.1235	0.0739
$\beta=4$	0.00001	0.1668	0.0968	0.0571
	0.00002	0.1660	0.0964	0.0569
	0.00003	0.1652	0.0960	0.0567
	0.00004	0.1643	0.0956	0.0565
$\beta=5$	0.00001	0.1380	0.0790	0.0462
	0.00002	0.1373	0.0786	0.0461
	0.00003	0.1366	0.0783	0.0459
	0.00004	0.1359	0.0780	0.0457

Special Case Model I

For $n = 3$ and $k = 2$ with Type C Repair, the quantity $\mu_{s0'}$ is eliminated from the results obtained for the $(3,1,2)$ special case with Type B Repair. This procedure yields the values shown in the third column of Table 3.26.

Special Case Model II

For $n = 3$ and $k = 3$ with Type C Repair, the quantity $\mu_{s0'}$ is eliminated from the results obtained for the $(3,1,3)$ special case with Type B Repair. This procedure yields the values presented in the fourth column of Table 3.26.

$k - out - of - (n+1) : G$ System with Type D Repair

With Type D Repair, the system is not repaired while its units are repaired thus $\mu_{r+1}(x)$ and $\mu_{r+2}(x)$ are set to zero in Equations (3.136) to (3.145).

Special Case Model I

For $n = 3$ and $k = 2$ with Type D Repair, the system reliability indices are determined from the analysis performed for the $(3,1,1)$ special case with Type D Repair in Subsection 3.1.1 by eliminating *system up-state 3* and its partial fraction residues from Equation (3.67). These values are presented in the third column of Table 3.27.

The system mean time to failure is obtained directly from Equation (3.68) by eliminating *system up-state 3*:

$$MTTF_{Y_D}(3,1,2) = \frac{W_{D_{3,1,2}}}{X_{D_{3,1,2}}} \tag{3.146}$$

where

$$\begin{aligned} W_{D_{3,1,2}} &= C_{3,1}(1)\,[C_{3,1}(0')\,C_{3,1}(2) - 3\,\lambda_1\,\mu_2] - 3\,\lambda_1\,\mu_2\,C_{3,1}(0') + \\ &\quad 3\,\lambda_1\,[C_{3,1}(0')\,C_{3,1}(2) - 3\,\lambda_1\,\mu_2] + 3\,\lambda_1\,\lambda_s\,\mu_2 + \\ &\quad \lambda_s\,C_{3,1}(1)\,C_{3,1}(2) - 3\,\lambda_1\,\mu_2\,[\lambda_s - 3\,\lambda_1] + \end{aligned}$$

Table 3.26: $A_{ssY}(3,1,k)\,(\beta)$ values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4437	0.2986	0.1932
	0.00002	0.4423	0.2977	0.1926
	0.00003	0.4409	0.2967	0.1920
	0.00004	0.4394	0.2958	0.1914
$\beta=2$	0.00001	0.2851	0.1755	0.1069
	0.00002	0.2840	0.1749	0.1065
	0.00003	0.2828	0.1742	0.1062
	0.00004	0.2816	0.1735	0.1058
$\beta=3$	0.00001	0.2101	0.1243	0.0739
	0.00002	0.2091	0.1238	0.0736
	0.00003	0.2081	0.1233	0.0734
	0.00004	0.2072	0.1228	0.0731
$\beta=4$	0.00001	0.1663	0.0962	0.0565
	0.00002	0.1665	0.0958	0.0563
	0.00003	0.1647	0.0954	0.0561
	0.00004	0.1639	0.0950	0.0558
$\beta=5$	0.00001	0.1376	0.0785	0.0457
	0.00002	0.1369	0.0781	0.0455
	0.00003	0.1362	0.0778	0.0453
	0.00004	0.1355	0.0775	0.0452

$$
\begin{aligned}
& 3\,\lambda_1\,\lambda_s\,C_{3,1}(1) + 9\,\lambda_1^2\,C_{3,1}(0') \\
X_{D_{3,1,2}} \;=\; & C_{3,1}(0)\,[C_{3,1}(1)\,(C_{3,1}(0')\,C_{3,1}(2) - 3\,\lambda_1\,\mu_2) - 3\,\lambda_1\,\mu_2\,C_{3,1}(0')] - \\
& \mu_1\,[3\,\lambda_1\,C_{3,1}(0')\,C_{3,1}(2) + 3\,\lambda_1\,\mu_2\,(\lambda_s - 3\,\lambda_1)] - \\
& \mu_{s0'}\,[\lambda_s\,C_{3,1}(1)\,C_{3,1}(2) - 3\,\lambda_1\,\mu_2\,(\lambda_s - 3\,\lambda_1)]
\end{aligned}
$$

where

$C_{3,1}(0) = 3\,\lambda_1 + \lambda_{c0} + \lambda_s$

$C_{3,1}(1) = 3\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{3,1}(0') = 3\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_{3,1}(2) = 2\,\lambda_1 + \lambda_{c2} + 2\,\mu_2$

From Equation (3.69), the variance of the time to failure can be determined by eliminating the constants associated with *system up-state 3*.

Table 3.27: $R_{Y_D}(3,1,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.8861	0.7580	0.7226
4000	0.6530	0.4615	0.4101
6000	0.4353	0.2521	0.2101
8000	0.2796	0.1291	0.1018
∞	0	0	0

Special Case Model II

For $n=3$ and $k=3$ with Type D Repair, the relevant system reliability indices can be determined from the results of the $(3,1,2)$ special case with Type D Repair as follows:

The system reliability is directly obtained from the results of the $(3,1,2)$ case by eliminating the partial fraction residues associated with *system up-state 2*. This procedure yields the values given in the fourth column of Table 3.27.

The system mean time to failure is obtained from Equation (3.146) by eliminating terms relating to *system up-state 2* and is expressed as:

$$MTTF_{Y_D}(3,1,3) = \frac{W_{D_{3,1,3}}}{X_{D_{3,1,3}}} \tag{3.147}$$

where,

$$\begin{aligned} W_{D_{3,1,3}} &= C_{3,1}(1)\,C_{3,1}(0') + 3\,\lambda_1\,C_{3,1}(0') + \lambda_s\,C_{3,1}(1) \\ X_{D_{3,1,3}} &= C_{3,1}(0)\,C_{3,1}(1)\,C_{3,1}(0') - 3\,\lambda_1\,\mu_1\,C_{3,1}(0') - \mu_{s0'}\,\lambda_s\,C_{3,1}(1) \end{aligned}$$

where,

$C_{3,1}(0) = 3\,\lambda_1 + \lambda_{c0} + \lambda_s$

$C_{3,1}(1) = 3\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{3,1}(0') = 3\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

From Equation (3.69), the variance of time to failure for this special case can be determined by eliminating constants relating to *system up-states 2* and *3*.

Letting $MTTF_{Y_D}(n,1,k) = MTTF_{Y_{D_{k/n}}}$, $W_{D_{n,1,k}} = W_{D_{k/n}}$ and $X_{D_{n,1,k}} = X_{D_{k/n}}$ and then from Equations (3.68), (3.146) and (3.147) and from the results of analyses performed on several more special cases, the following generalized system mean time to failure expression is obtained:

$$MTTF_{Y_{D_{k/n}}} = \frac{W_{D_{k/n}}}{X_{D_{k/n}}} \tag{3.148}$$

Where $W_{D_{k/n}}$ and $X_{D_{k/n}}$ can easily be obtained via the following recursive relationships:

For $k = n, n-1, n-2, ...3, 2, 1$ we develop the following set of recursive equations:

$W_{D_{n/n}} = C_{n,1}(1)\,C_{n,1}(0') + n\,\lambda_1\,C_{n,1}(0') + \lambda_s\,C_{n,1}(1)$

$W_{D_{n-1/n}} = C_{n,1}(2)\,W_{D_{n/n}} - n\,\lambda_1\,\mu_2\,C_{n,1}(1) - n\,\lambda_1\,\mu_2\,C_{n,1}(0') + \Theta_n$

$$W_{D_{n-2/n}} = C_{n,1}(3)\, W_{D_{n-1/n}} - (n-1)\,\lambda_1\,\mu_3\, W_{D_{n/n}} + (n-1)\,\lambda_1\,\Theta_n$$
$$W_{D_{n-3/n}} = C_{n,1}(4)\, W_{D_{n-2/n}} - (n-2)\,\lambda_1\,\mu_4\, W_{D_{n-1/n}} + (n-1)(n-2)\,\lambda_1^2\,\Theta_n$$
$$\vdots$$
$$W_{D_{1/n}} = C_{n,1}(n+1)\, W_{D_{2/n}} - 2\,\lambda_1\,\mu_{n+1}\, W_{D_{3/n}} + (n-1)!\,\lambda_1^{n-2}\,\Theta_n$$

where $\Theta_n = (n\,\lambda_1)^2\, C_{n,1}(2) + n\,\lambda_1\,\lambda_s\, C_{n,1}(1)$

and,

$$X_{D_{n/n}} = \alpha_{n/n} - \mu_1\,[\vartheta_{n/n}] - \mu_{s0'}\,[\varphi_{n/n}]$$
$$X_{D_{n-1/n}} = \alpha_{n-1/n} - \mu_1\,[\vartheta_{n-1/n}] - \mu_{s0'}\,[\varphi_{n-1/n}]$$
$$X_{D_{n-2/n}} = \alpha_{n-2/n} - \mu_1\,[\vartheta_{n-2/n}] - \mu_{s0'}\,[\varphi_{n-2/n}]$$
$$\vdots$$
$$X_{D_{1/n}} = \alpha_{1/n} - \mu_1\,[\vartheta_{1/n}] - \mu_{s0'}\,[\varphi_{1/n}]$$

Where,

$$\alpha_{n/n} = C_{n,1}(0)\, C_{n,1}(1)\, C_{n,1}(0')$$
$$\alpha_{n-1/n} = C_n(2)\,\alpha_{n/n} - n\,\lambda_1\,\mu_2\, C_{n,1}(0)\, C_{n,1}(1) - n\,\lambda_1\,\mu_2\, C_{n,1}(0)\, C_{n,1}(0')$$
$$\alpha_{n-2/n} = C_n(3)\alpha_{n-1/n} - (n-1)\,\lambda_1\,\mu_3\,\alpha_{n/n}$$
$$\alpha_{n-3/n} = C_n(4)\,\alpha_{n-2/n} - (n-2)\,\lambda_1\,\mu_4\,\alpha_{n-1/n}$$
$$\vdots$$
$$\alpha_{1/n} = C_{n,1}(n)\,\alpha_{2/n} - 2\,\lambda_1\mu_n\,\alpha_{3/n}$$

$$\vartheta_{n/n} = n\,\lambda_1\, C_{n,1}(0')$$
$$\vartheta_{n-1/n} = C_{n,1}(2)\,\vartheta_{n/n} + n\,\lambda_1\mu_2\,(\lambda_s - n\,\lambda_1)$$
$$\vartheta_{n-2/n} = C_n(3)\,\vartheta_{n-1/n} - (n-1)\,\lambda_1\mu_3\,\vartheta_{n/n}$$
$$\vartheta_{n-3/n} = C_n(4)\,\vartheta_{n-2/n} - (n-2)\,\lambda_1\mu_4\,\vartheta_{n-1/n}$$
$$\vdots$$
$$\vartheta_{1/n} = C_{n,1}(n)\,\vartheta_{2/n} - 2\,\lambda_1\;\mu_n\,\vartheta_{3/n}$$

$$\varphi_{n/n} = \lambda_s\, C_{n,1}(1)$$
$$\varphi_{n-1/n} = C_{n,1}(2)\,\varphi_{n/n} - n\,\lambda_1\,\mu_2\,(\lambda_s - n\,\lambda_1)$$
$$\varphi_{n-2/n} = C_n(3)\,\varphi_{n-1/n} - (n-1)\,\lambda_1\,\mu_3\,\varphi_{n/n}$$
$$\varphi_{n-3/n} = C_n(4)\,\varphi_{n-2/n} - (n-2)\,\lambda_1\,\mu_4\,\varphi_{n-1/n}$$

$$\vdots$$

$$\varphi_{1/n} = C_n(n)\,\varphi_{2/n} - 2\,\lambda_1 \mu_n\,\varphi_{3/n}$$

$$C_{n,1}(0) = n\,\lambda_1 + \lambda_{c0} + \lambda_s$$

$$C_{n,1}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$$

$$C_{n,1}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$$

$$C_{n,1}(2) = (n-1)\,\lambda_1 + \lambda_{c2} + 2\,\mu_2$$

$$C_{n,1}(3) = (n-2)\,\lambda_1 + \lambda_{c3} + \mu_3$$

$$\vdots$$

$$C_{n,1}(n) = \lambda_1 + \lambda_{cn} + \mu_n$$

For the specified values of system parameters, Equation (3.148) was used to generate values given in Table 3.28 for different values of k.

Table 3.28: $MTTF_{Y_D}(3,1,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	7088.25	3471.18	1842.57
0.00002	6930.84	3414.65	1819.87
0.00003	6779.09	3359.72	1797.68
0.00004	6632.75	3306.33	1776.00

$k-out-of-(n+1):G$ System with Type E Repair

With Type E Repair, the system as well as its active units are not repaired while failed standby units are repaired and thus $\mu_{r+1}(x)$, $\mu_{r+2}(x)$, μ_1 to μ_r are set to zero in Equations (3.136) to (3.145).

Special Case Model I

For $n = 3$ and $k = 2$ with Type E Repair, the relevant system reliability indices can be obtained from the analysis performed for the $(3, 1, 1)$ special case with Type E Repair in Subsection 3.1.1 by eliminating the constants relating to *system up-state* $\mathcal{S}$. The system reliability values obtained via this procedure are given in the third column of Table 3.29.

Table 3.29: $R_{Y_E}(3, 1, k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k = 1$	$k = 2$	$k = 3$
0	1.0000	1.0000	1.0000
2000	0.8753	0.5926	0.2755
4000	0.5761	0.1882	0.0388
6000	0.3086	0.0465	0.0046
8000	0.1495	0.0103	0.0005
∞	0	0	0

The system mean time to failure is determined directly from Equation (3.73) by eliminating constants associated with *system up-state* $\mathcal{S}$:

$$\begin{aligned} MTTF_{Y_E}(3,1,2) &= \frac{1}{\alpha_1[C_{3,1}(0)]}\,[C_{3,1}(0')\,(1 + \frac{3\lambda_1}{C_{3,1}(1)} + \frac{9\lambda_1^2}{C_{3,1}(1)\,C_{3,1}(2)}) + \\ &\quad \lambda_s\,(1 + \frac{3\lambda_1}{C_{3,1}(2)})] \end{aligned} \tag{3.149}$$

where

$\alpha_1[C_{3,1}(0)] = C_{3,1}(0)\,C_{3,1}(0') - \lambda_s\,\mu_{s0'}$

The variance of time to failure for this case can be determined by eliminating constants relating to *system up-state* $\mathcal{S}$ from the results obtained for the $(3, 1, 1)$ special case with Type E Repair in Subsection 3.1.1.

Special Case Model II

For $n = 3$ and $k = 3$ with Type E Repair, the system reliability can be obtained from the analysis performed for the $(3,1,2)$ case with Type E Repair by eliminating the constants relating to *system up-state 2*. The system reliability values obtained via this procedure are given in the fourth column of Table 3.29.

The mean time to failure for this system is determined directly using Equation (3.149) by eliminating terms associated with *system up-state 2* and can be expressed as:

$$MTTF_{Y_E}(3,1,3) = \frac{1}{\alpha_1[C_{3,1}(0)]}\,[C_{3,1}(0')\,(1 + \frac{3\lambda_1}{C_{3,1}(1)}) + \lambda_s] \qquad (3.150)$$

where

$\alpha_1[C_{3,1}(0)] = C_{3,1}(0)\,C_{3,1}(0') - \lambda_s\,\mu_{s0'}$

The variance of time to failure for this special case can be determined by eliminating constants relating to *system up-state 2* from the variance results obtained for the $(3,1,2)$ special case with Type E Repair.

From Equations (3.73), (3.149) and (3.150), the following generalized system mean time to failure expression is developed for the $k - out - of - (n+1) : G$ submodel with Type E Repair:

$$MTTF_{Y_E}(n,1,k) \;=\; [\frac{C_{n,1}(0')}{\alpha_1[C_{n,1}(0)]}] \bullet [1 + (\frac{n\lambda_1}{C_{n,1}(1)} + \frac{\lambda_s}{C_{n,1}(0')}) \bullet G_{k/n}] \qquad (3.151)$$

where

$$\begin{aligned} G_{k/n} \;=\; & 1 + \frac{n\Psi_1\lambda_1}{C_{n,1}(2)} + \frac{n(n-1)\Psi_2\lambda_1^2}{C_{n,1}(2)\,C_{n,1}(3)} + \frac{n(n-1)(n-2)\Psi_3\lambda_1^3}{C_{n,1}(2)\,C_{n,1}(3)\,C_{n,1}(4)} + \ldots \\ & \ldots\ldots + \frac{n!\Psi_{n-1}\lambda_1^{n-1}}{\prod_{i=2}^{r} C_{n,1}(i)} \end{aligned}$$

and,

$\alpha_1[C_{n,1}(0)] = C_{n,1}(0)\,C_{n,1}(0') - \lambda_s\,\mu_{s0'}$ and $r = n - k + 1$

The Ψ_js are parameters defined as follows:

$\Psi_j = 0$ for $j \geq n - k + 1$

$\Psi_j = 1$ for $j < n - k + 1$

and,

$C_n(0) = n\,\lambda_1 + \lambda_s + \lambda_{c0}$

$C_n(1) = n\,\lambda_1 + \lambda_{c1}$

$C_n(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'}$

$C_n(2) = (n-1)\,\lambda_1 + \lambda_{c3}$

$C_n(3) = (n-2)\,\lambda_1 + \lambda_{c4}$

$\vdots$

$C_n(r-1) = (k+1)\,\lambda_1 + \lambda_{cr-1}$

$C_n(r) = k\lambda_1 + \lambda_{cr}$

Using the specified values of system parameters, Equation (3.151) is used to generate the values in Table 3.30.

Table 3.30: $MTTF_{Y_E}(3,1,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	5287.46	2835.34	1605.45
0.00002	5210.07	2801.56	1589.78
0.00003	5134.32	2768.42	1574.37
0.00004	5060.17	2735.90	1559.23

$k-out-of-(n+1):G$ System with Type F Repair

With Type F Repair, the system as well as its units (both active and standby) are not repaired upon failure thus $\mu_{s0'}$, $\mu_{r+1}(x)$, $\mu_{r+2}(x)$, μ_1 to μ_r are set to zero in Equations (3.136) to (3.145).

Special Case Model I

For $n = 3$ and $k = 2$ with Type F Repair, the relevant reliability indices are determined from the analysis conducted for the $(3,1,2)$ case with Type E Repair by eliminating $\mu_{s0'}$. Thus the system reliability results obtained are presented in the third column of Table 3.31.

Table 3.31: $R_{Y_F}(3,1,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.8749	0.5826	0.2699
4000	0.5739	0.1796	0.0360
6000	0.3062	0.0433	0.0040
8000	0.1480	0.0094	0.0004
∞	0	0	0

The system mean time to failure is determined from Equation (3.149) by setting $\mu_{s0'}$ to zero and is expressed as:

$$\begin{aligned} MTTF_{Y_F}(3,1,2) &= \frac{1}{\alpha_1[C_{3,1}(0)]}[C_{3,1}(0')\,(1+\frac{3\lambda_1}{C_{3,1}(1)}+\frac{9\lambda_1^2}{C_{3,1}(1)\,C_{3,1}(2)})+ \\ &\quad \lambda_s\,(1+\frac{3\lambda_1}{C_{3,1}(2)})] \end{aligned} \tag{3.152}$$

where

$\alpha_1[C_{3,1}(0)] = C_{3,1}(0)\,C_{3,1}(0')$

The variance of time to failure for this special case can be determined by eliminating the quantity $\mu_{s0'}$ from the variance results of the $(3,1,2)$ case with Type E Repair.

Special Case Model II

For $n=3$ and $k=3$ with Type F Repair, the system reliability values are obtained directly from the analysis performed for the $(3,1,3)$ case with Type E Repair by eliminating $\mu_{s0'}$. The resulting system reliability values are given in the fourth column of Table 3.31.

The system mean time to failure can be obtained from Equation (3.150) by eliminating $\mu_{s0'}$ and is expressed as:

$$MTTF_{Y_F}(3,1,3) = \frac{1}{\alpha_1[C_{3,1}(0)]}[C_{3,1}(0')(1+\frac{3\lambda_1}{C_{3,1}(1)})+\lambda_s] \tag{3.153}$$

where,

$\alpha_1[C_{3,1}(0)] = C_{3,1}(0)\,C_{3,1}(0')$

The variance of time to failure for this special case can be obtained by eliminating $\mu_{s0'}$ from the variance results obtained for the $(3,1,3)$ case with Type E Repair.

From Equations (3.77), (3.152) and (3.153), the following generalized system mean time to failure expression is developed for the $k-out-of-(n+1):G$ submodel with Type F Repair:

$$MTTF_{Y_F}(n,1,k) = [\frac{1}{C_{n,1}(0)}] \bullet [1+(\frac{n\lambda_1}{C_{n,1}(1)}+\frac{\lambda_s}{C_n(0')}) \bullet G_{k/n}] \tag{3.154}$$

where

$G_{k/n}$ is as defined for Equation (3.151) and,

$C_{n,1}(0) = n\,\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = n\,\lambda_1 + \lambda_{c1}$

$C_{n,1}(0') = n\,\lambda_1 + \lambda_{c0'}$

$C_{n,1}(2) = (n-1)\,\lambda_1 + \lambda_{c2}$

$C_{n,1}(3) = (n-2)\,\lambda_1 + \lambda_{c3}$

$\vdots$

$C_{n,1}(r-1) = (k+1)\,\lambda_1 + \lambda_{cr-1}$

$C_{n,1}(r) = k\,\lambda_1 + \lambda_{cr}$

Equation (3.154) was used to generate the values given in Table 3.32.

Table 3.32: $MTTF_{Y_F}(3,1,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	5266.56	2815.15	1585.61
0.00002	5190.79	2782.27	1570.48
0.00003	5116.59	2750.00	1555.61
0.00004	5043.94	2718.32	1540.98

3.1.5 k-out-of-(n+2):G Identical Unit Submodel

This is the $(n,2,k)$ system and is composed of n active units and *two* standbys. It is functional as long as there are at least k functioning components. The $1-out-of-(n+2):G$ system of Subsection 3.1.2 is the special case of this submodel when $k=1$. A block diagram of this system is shown in Figure 3.24 and the state space diagram in Figure 3.25.

$k-out-of-(n+2):G$ System with Type A Repair

The corresponding integro-differential equations for the model described in Figure 3.25 are:

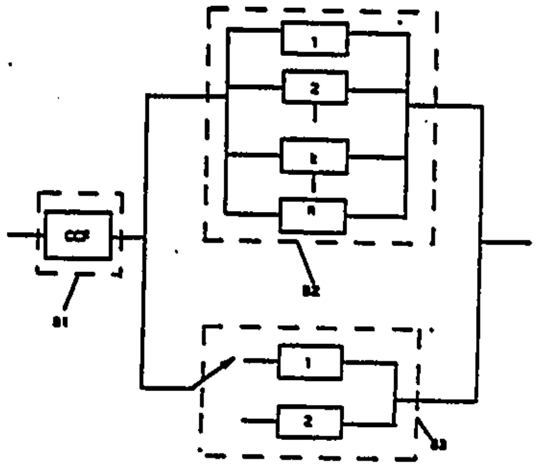
$$\frac{dP_0(t)}{dt}+C_{n,2}(0)\,P_0(t)=\mu_1\,P_1(t)+\mu_{s0'}\,P_{0'}(t)+\sum_{i=r+2}^{r+3}\int_0^{\infty}P_i(x,t)\,\mu_i(x)dx \quad (3.155)$$

where $r=n-k+1$.

$$\frac{dP_1(t)}{dt}+C_{n,2}(1)\,P_1(t)=n\,\lambda_1\,P_0(t)+\mu_2\,P_2(t) \quad (3.156)$$

$$\frac{dP_2(t)}{dt}+C_{n,2}(2)\,P_2(t)=n\,\lambda_1\,P_1(t)+n\,\lambda_1\,P_{0'}(t)+\mu_3\,P_3(t) \quad (3.157)$$

$$\frac{dP_3(t)}{dt}+C_{n,2}(3)\,P_3(t)=n\,\lambda_1\,P_2(t)+n\,\lambda_1\,P_{1'}(t)+\mu_4\,P_4(t) \quad (3.158)$$



BLOCK B1: represents common-cause failures of the system
BLOCK B2: represents the active on-line k-out-of-n system
BLOCK B3: represents the standby system

Figure 3.24: Block diagram of the $k-out-of-(n+2):G$ System

For $j = 4, 5, \ldots\ldots, r$,

$$\frac{dP_j(t)}{dt} + C_{n,2}(j)\, P_j(t) = (n-j+3)\,\lambda_1\, P_{j-1}(t) + \mu_{j+1}\, P_{j+1}(t) \tag{3.159}$$

$$\frac{dP_{r+1}(t)}{dt} + C_{n,2}(r+1)\, P_{r+1}(t) = (k+1)\,\lambda_1\, P_r(t) \tag{3.160}$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,2}(0')\, P_{0'}(t) = 2\lambda_s\, P_0(t) + \mu_2\, P_2(t) + \mu_{s1'} P_{1'}(t) \tag{3.161}$$

$$\frac{dP_{1'}(t)}{dt} + C_{n,2}(1')\, P_{1'}(t) = \lambda_s\, P_{0'}(t) + \mu_3\, P_3(t) \tag{3.162}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+2}(x)]\, P_{r+2}(x,t) = 0 \tag{3.163}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+3}(x)]\, P_{r+3}(x,t) = 0 \tag{3.164}$$

The associated boundary conditions are:

$$P_{r+2}(0,t) = k\lambda_1\, P_{r+1}(t) \tag{3.165}$$

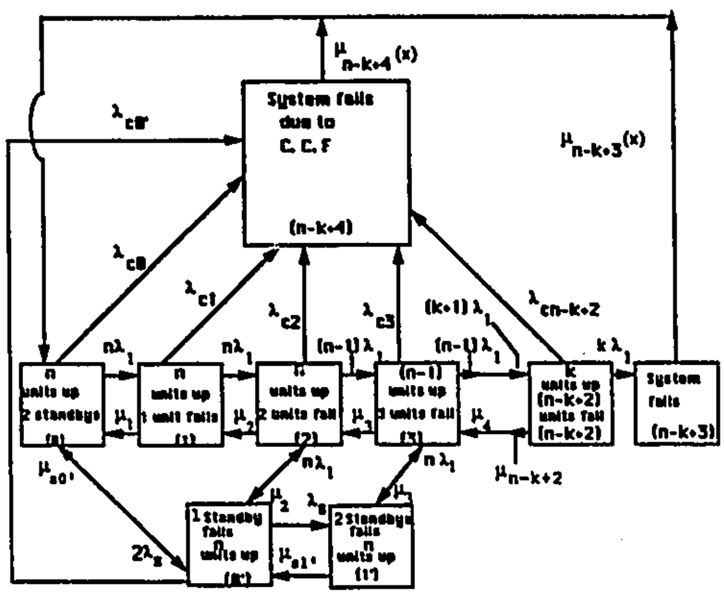


Figure 3.25: State space diagram of the $k - out - of - (n+2) : G$ System under SBAP Y

$$P_{r+3}(0,t) = \sum_{i=0}^{r+1} \lambda_{ci} P_i(t) + \sum_{i=0'}^{1'} \lambda_{ci} P_i(t) \tag{3.166}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,2}(0) = n\,\lambda_1 + 2\lambda_s + \lambda_{c0}$

$C_{n,2}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,2}(2) = n\,\lambda_1 + \lambda_{c2} + 2\mu_2$

$C_{n,2}(3) = (n-1)\,\lambda_1 + \lambda_{c3} + 2\mu_3$

For $j = 4, 5, \ldots\ldots, r$,

$C_{n,2}(j) = (n-j+2)\,\lambda_1 + \lambda_{cj} + \mu_j$

$C_{n,2}(r+1) = k\lambda_1 + \lambda_{cr+1} + \mu_{r+1}$

$C_{n,2}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'} + \lambda_s$

$C_{n,2}(1') = n\,\lambda_1 + \lambda_{c1'} + \mu_{s1'}$

Special Case Model I

For $n = 3$ and $k = 2$ in Equations (3.155) to (3.166), abbreviated as the $(3,2,2)$ special case, the long-run availability of this special case is obtained directly from Equation (3.93) by eliminating *system up-state 4*. This procedure yields the values shown in the third column of Table 3.33.

Special Case Model II

For $n = 3$ and $k = 3$ with Type A Repair, abbreviated as the $(3,2,3)$ special case, the long-run availability of this special case is determined by eliminating *system up-state 3* from the results obtained for the $(3,2,2)$ special case with Type A Repair. This procedure yields the values given in the fourth column of Table 3.33.

$k - out - of - (n+2) : G$ System with Type B Repair

With Type B Repair, μ_1 to μ_{r+1} in Equations (3.155) to (3.166) are set to zero because failed active units are not repaired.

Table 3.33: $A_{ss_1}(3,2,k)(\beta)$ values as a function of λ_{c0} With Type A Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5535	0.4057	0.2832
	0.00002	0.5513	0.4041	0.2820
	0.00003	0.5492	0.4024	0.2808
	0.00004	0.5470	0.4007	0.2796
$\beta=2$	0.00001	0.3826	0.2545	0.1667
	0.00002	0.3806	0.2532	0.1659
	0.00003	0.3785	0.2519	0.1651
	0.00004	0.3765	0.2505	0.1642
$\beta=3$	0.00001	0.2924	0.1854	0.1181
	0.00002	0.2906	0.1843	0.1175
	0.00003	0.2888	0.1833	0.1169
	0.00004	0.2870	0.1823	0.1163
$\beta=4$	0.00001	0.2366	0.1458	0.0915
	0.00002	0.2350	0.1449	0.0910
	0.00003	0.2334	0.1441	0.0905
	0.00004	0.2319	0.1432	0.0900
$\beta=5$	0.00001	0.1987	0.1201	0.0746
	0.00002	0.1973	0.1194	0.0742
	0.00003	0.1959	0.1187	0.0738
	0.00004	0.1946	0.1179	0.0734

Special Case Model I

For $n = 3$ and $k = 2$ with Type B Repair, the long-run availability was obtained by eliminating the quantities μ_1 to μ_3 from the results obtained for the $(3, 2, 2)$ special case with Type A Repair. This procedure produced the values given in the third column of Table 3.34.

Special Case Model II

For $n = 3$ and $k = 3$ with Type B Repair, the long-run availability of this case was obtained using Equation (3.93) by eliminating *system up-states 3* and *4* and setting μ_1 and μ_2 equal to zero. This procedure generated the values given in the fourth column of Table 3.34.

$k - out - of - (n + 2) : G$ System with Type C Repair

With Type C Repair, $\mu_{s0'}$, $\mu_{s1'}$ and μ_1 to μ_{r+1} in Equations (3.155) to (3.166) are set equal to zero because failed units (both active and standbys) are not repaired.

Special Case Model I

For $n = 3$ and $k = 2$ with Type C Repair, the long-run availability of this special case was obtained by eliminating the quantities $\mu_{s0'}$, and $\mu_{s1'}$ from the results obtained for the $(3, 2, 2)$ special case with Type B Repair. The procedure yielded the values presented in the third column of Table 3.35.

Special Case Model II

For $n = 3$ and $k = 3$ with Type C Repair, the long-run availability was determined by eliminating $\mu_{s0'}$ and $\mu_{s1'}$ from the results obtained for the $(3, 2, 3)$ special case with Type B Repair. This procedure yields the values shown in the fourth column of Table 3.35.

Table 3.34: $A_{ssY}(3,2,k)\,(\beta)$ values as a function of λ_{c0} With Type B Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4786	0.3532	0.2559
	0.00002	0.4772	0.3521	0.2551
	0.00003	0.4757	0.3510	0.2543
	0.00004	0.4743	0.3499	0.2535
$\beta=2$	0.00001	0.3146	0.2145	0.1475
	0.00002	0.3134	0.2137	0.1470
	0.00003	0.3121	0.2129	0.1464
	0.00004	0.3109	0.2121	0.1459
$\beta=3$	0.00001	0.2343	0.1540	0.1036
	0.00002	0.2333	0.1534	0.1032
	0.00003	0.2322	0.1527	0.1028
	0.00004	0.2312	0.1521	0.1024
$\beta=4$	0.00001	0.1867	0.1201	0.0798
	0.00002	0.1858	0.1196	0.0795
	0.00003	0.1849	0.1191	0.0792
	0.00004	0.1840	0.1186	0.0789
$\beta=5$	0.00001	0.1178	0.0985	0.0649
	0.00002	0.1167	0.0980	0.0647
	0.00003	0.1156	0.0976	0.0644
	0.00004	0.1145	0.0972	0.0642

Table 3.35: $A_{ssY}(3,2,k)\,(\beta)$ values as a function of λ_{c0} With Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4770	0.3508	0.2496
	0.00002	0.4756	0.3497	0.2488
	0.00003	0.4742	0.3487	0.2481
	0.00004	0.4729	0.3476	0.2473
$\beta=2$	0.00001	0.3132	0.2127	0.1436
	0.00002	0.3120	0.2119	0.1431
	0.00003	0.3108	0.2111	0.1426
	0.00004	0.3096	0.2104	0.1421
$\beta=3$	0.00001	0.2331	0.1526	0.1008
	0.00002	0.2322	0.1520	0.1004
	0.00003	0.2312	0.1514	0.1000
	0.00004	0.2302	0.1508	0.0996
$\beta=4$	0.00001	0.1857	0.1190	0.0776
	0.00002	0.1848	0.1185	0.0773
	0.00003	0.1840	0.1180	0.0770
	0.00004	0.1832	0.1176	0.0767
$\beta=5$	0.00001	0.1543	0.0975	0.0631
	0.00002	0.1536	0.0971	0.0629
	0.00003	0.1528	0.0967	0.0626
	0.00004	0.1521	0.0963	0.0624

$k - out - of - (n+2) : G$ System with Type E Repair

With Type E Repair, the system and its active units are discarded upon failure and thus $\mu_{r+2}(x)$, $\mu_{r+3}(x)$, μ_1 to μ_{r+1} are set to zero in Equations (3.155) to (3.166).

Special Case Model I

For $n = 3$ and $k = 2$ with Type E Repair, the system relevant reliability indices are obtained as follows:

The system reliability is obtained using Equation (3.100) by eliminating the constants associated with *system up-state 4*. The system reliability values obtained are given in the third column of Table 3.36.

Table 3.36: $R_{Y_E}(3,2,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9224	0.7760	0.5144
4000	0.6906	0.3344	0.1159
6000	0.4084	0.1007	0.0190
8000	0.2100	0.0253	0.0027
∞	0	0	0

The mean time to failure for this system can be determined using Equation (3.101) by eliminating the terms associated with *system up-state 4*. It can be expressed as:

$$MTTF_{Y_E}(3,2,2) = \frac{1}{\alpha_2[C_{3,2}(0)]}[\zeta_1[C_{3,2}(0')](1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)}+ \frac{27\lambda_1^3}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)})+2\lambda_s C_{3,2}(1')(1+ \frac{3\lambda_1}{C_{3,2}(2)}+\frac{9\lambda_1^2}{C_{3,2}(2)\,C_{3,2}(3)})+2\lambda_s^2(1+\frac{3\lambda_1}{C_{3,2}(3)})] \quad (3.167)$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,(C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}) - \mu_{s0'}\,(2\lambda_s\,C_{3,2}(1'))$

$\zeta_1[C_{3,2}(0)] = C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}$

System mean time to failure values obtained with Equation (3.167) are presented in the third column of Table 3.37.

Table 3.37: $MTTF_{Y_E}(3,2,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6062.63	3609.23	2302.34
0.00002	5979.68	3565.97	2277.64
0.00003	5898.36	3523.50	2253.36
0.00004	5818.63	3481.81	2229.51

The system variance of time to failure can be determined directly from Equation (3.102) by eliminating all constants associated with *system up-state 4* in addition to eliminating μ_1 to μ_3.

Special Case Model II

For $n = 3$ and $k = 3$ with Type E Repair, the system relevant reliability indices are obtained as follows:
The system reliability is directly obtained from Equation (3.100) by eliminating the constants associated with *system up-states 3* and *4*. The resulting system reliability values are presented in the fourth column of Table 3.37.

The mean time to failure for this system can be determined from Equation (3.101) by eliminating terms associated with *system up-states 3* and *4* and is expressed as:

$$MTTF_{Y_E}(3,2,3) = \frac{1}{\alpha_2[C_{3,2}(0)]}[\zeta_1[C_{3,2}(0')](1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)})+ 2\lambda_s C_{3,2}(1')(1+\frac{3\lambda_1}{C_{3,2}(2)})+2\lambda_s^2] \quad (3.168)$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,(C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}) - \mu_{s0'}\,(2\lambda_s C_{3,2}(1'))$

$\zeta_1[C_{3,2}(0)] = C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}$

System mean time to failure values obtained with Equation (3.168) are given in the fourth column of Table 3.37.

The system variance of time to failure can be determined directly from Equation (3.102) by eliminating all constants associated with *system up-states 3* and *4* in addition to eliminating μ_1 and μ_2.

$k-out-of-(n+2):G$ System with Type F Repair

With Type F Repair, the system and its units (both active and standbys) are discarded upon failure and thus $\mu_{s0'}$, $\mu_{s1'}$, $\mu_{r+2}(x)$, $\mu_{r+3}(x)$, μ_1 to μ_{r+1} are set to zero in Equations (3.155) to (3.166).

Special Case Model I

For $n=3$ and $k=2$ with Type F Repair, the system relevant reliability indices are obtained as follows:

The system reliability is determined by eliminating $\mu_{s0'}$ and $\mu_{s1'}$ from the results obtained for the $(3,2,2)$ special case with Type E Repair. The resulting system reliability values are presented in the third column of Table 3.38.

The system mean time to failure is determined from the results obtained for the $(3,2,2)$ special case with Type E Repair by eliminating $\mu_{s0'}$ and $\mu_{s1'}$ and can be expressed as:

$$MTTF_{Y_F}(3,2,2) = \frac{1}{\alpha_2[C_{3,2}(0)]}[\zeta_1[C_{3,2}(0')](1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)}+$$

$$\frac{27\lambda_1^3}{C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)}) + 2\lambda_s C_{3,2}(1')\,(1 +$$
$$\frac{3\lambda_1}{C_{3,2}(2)} + \frac{9\lambda_1^2}{C_{3,2}(2)\,C_{3,2}(3)}) + 2\lambda_s^2\,(1 + \frac{3\lambda_1}{C_{3,2}(3)})] \qquad (3.169)$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,C_{3,2}(0')\,C_{3,2}(1')$

$\zeta_1[C_{3,2}(0)] = C_{3,2}(0')\,C_{3,2}(1')$

System mean life values obtained with Equation (3.169) are given in the third column of Table 3.39.

Table 3.38: $R_{Y_F}(3,2,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k = 1$	$k = 2$	$k = 3$
0	1.0000	1.0000	1.0000
2000	0.9218	0.6887	0.4223
4000	0.6875	0.3198	0.1018
6000	0.4039	0.0955	0.0160
8000	0.2065	0.0235	0.0021
∞	0	0	0

The variance of time to failure for the system can be determined directly from Equation (3.102) by eliminating all constants associated with *system up-state 4* in addition to eliminating μ_1 to μ_3, $\mu_{s0'}$ and $\mu_{s1'}$.

Special Case Model II

For $n = 3$ and $k = 3$ with Type F Repair, the system relevant reliability indices are obtained as follows:

The system reliability is obtained by eliminating $\mu_{s0'}$ and $\mu_{s1'}$ from the results obtained for the (3, 2, 3) special case with Type E Repair. The system reliability values obtained are presented in the fourth column of Table 3.38.

The mean time to failure for the system is determined from the results obtained for the (3, 2, 3) special case with Type E Repair by eliminating $\mu_{s0'}$ and $\mu_{s1'}$ and is expressed as:

$$\begin{aligned} MTTF_{Y_F}(3,2,3) &= \frac{1}{\alpha_2[C_{3,2}(0)]}[\zeta_1[C_{3,2}(0')]\,(1+\frac{3\lambda_1}{C_{3,2}(1)}+\frac{9\lambda_1^2}{C_{3,2}(1)\,C_{3,2}(2)})+ \\ & \quad 2\lambda_s C_{3,2}(1')\,(1+\frac{3\lambda_1}{C_{3,2}(2)})+2\lambda_s^2] \end{aligned} \tag{3.170}$$

where

$\alpha_2[C_{3,2}(0)] = C_{3,2}(0)\,C_{3,2}(0')\,C_{3,2}(1')$
$\zeta_1[C_{3,2}(0)] = C_{3,2}(0')\,C_{3,2}(1')$

System mean life values obtained with Equation (3.170) are given in the fourth column of Table 3.39.

Table 3.39: $MTTF_{Y_F}(3,2,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6022.64	3570.26	2235.51
0.00002	5942.89	3528.95	2212.31
0.00003	5864.65	3488.37	2189.50
0.00004	5787.90	3448.51	2167.08

The variance of time to system failure is determined directly using Equation (3.102) by eliminating all constants associated with *system up-states 3* and *4* in addition to eliminating μ_1, μ_2, $\mu_{s0'}$ and $\mu_{s1'}$.

3.1.6 k-out-of-(n+3):G Identical Unit Submodel

This is the $(n, 3, k)$ system and is composed of n active units and *three* standbys. It is functional as long as there are at least k active units. The $1 - out - of - (n+3) : G$ system of Subsection 3.1.3 is the special case of this submodel when $k = 1$. A block diagram of this submodel is shown in Figure 3.26 and the state space diagram in Figure 3.27.

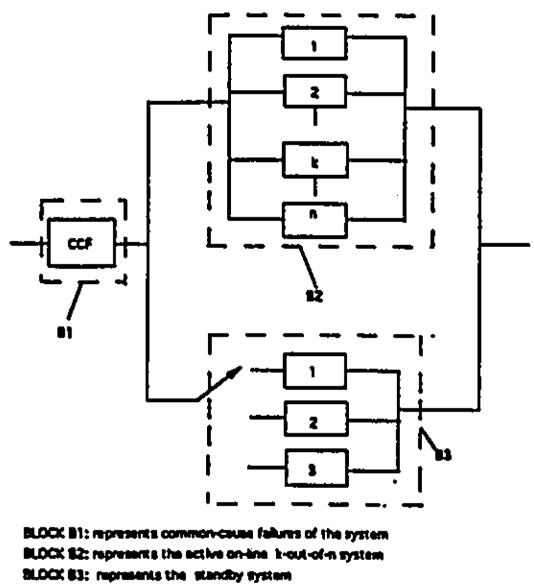


Figure 3.26: Block diagram of the $k - out - of - (n+3) : G$ System

<u>$k - out - of - (n+3) : G$ **System with Type A Repair**</u>

The corresponding integro-differential equations for the model described in Figure 3.27 are:

$$\frac{dP_0(t)}{dt} + C_{n,3}(0)\, P_0(t) = \mu_1\, P_1(t) + \mu_{s0'}\, P_{0'}(t) + \sum_{i=r+3}^{r+4} \int_0^\infty P_i(x,t)\, \mu_i(x) dx \quad (3.171)$$

where $r = n - k + 1$.

$$\frac{dP_1(t)}{dt} + C_{n,3}(1)\, P_1(t) = n\, \lambda_1\, P_0(t) + \mu_2\, P_2(t) \quad (3.172)$$

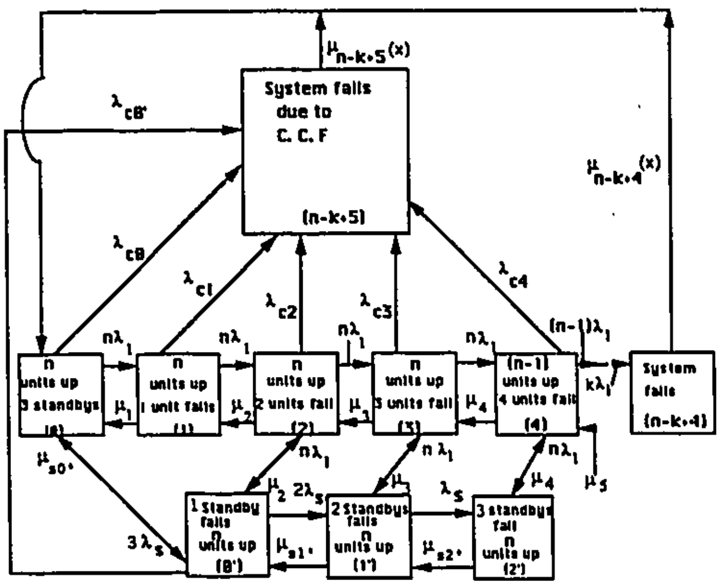


Figure 3.27: State space diagram of the $k - out - of - (n + 3) : G$ System under SBAP Y

$$\frac{dP_2(t)}{dt} + C_{n,3}(2)\, P_2(t) = n\,\lambda_1\, P_1(t) + n\,\lambda_1\, P_{0'}(t) + \mu_3\, P_3(t) \tag{3.173}$$

$$\frac{dP_3(t)}{dt} + C_{n,3}(3)\, P_3(t) = n\,\lambda_1\, P_2(t) + n\,\lambda_1\, P_{1'}(t) + \mu_4\, P_4(t) \tag{3.174}$$

$$\frac{dP_4(t)}{dt} + C_{n,3}(4)\, P_4(t) = n\,\lambda_1\, P_3(t) + n\,\lambda_1\, P_{2'}(t) + \mu_5\, P_5(t) \tag{3.175}$$

For $j = 5, 6, \ldots\ldots, r+1$,

$$\frac{dP_j(t)}{dt} + C_{n,3}(j)\, P_j(t) = (n-j+4)\,\lambda_1\, P_{j-1}(t) + \mu_{j+1}\, P_{j+1}(t) \tag{3.176}$$

$$\frac{dP_{r+2}(t)}{dt} + C_{n,3}(r+2)\, P_{r+2}(t) = (k+1)\,\lambda_1\, P_{r+1}(t) \tag{3.177}$$

$$\frac{dP_{0'}(t)}{dt} + C_{n,3}(0')\, P_{0'}(t) = 3\lambda_s\, P_0(t) + \mu_2\, P_2(t) + \mu_{s1'} P_{1'}(t) \tag{3.178}$$

$$\frac{dP_{1'}(t)}{dt} + C_{n,3}(1')\, P_{1'}(t) = 2\lambda_s\, P_{0'}(t) + \mu_3\, P_3(t) + \mu_{s2'} P_{2'}(t) \tag{3.179}$$

$$\frac{dP_{2'}(t)}{dt} + C_{n,3}(2')\, P_{2'}(t) = \lambda_s\, P_{1'}(t) + \mu_4\, P_4(t) \tag{3.180}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+3}(x)]\, P_{r+3}(x,t) = 0 \tag{3.181}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+4}(x)]\, P_{r+4}(x,t) = 0 \tag{3.182}$$

The associated boundary conditions are:

$$P_{r+3}(0,t) = k\lambda_1\, P_{r+2}(t) \tag{3.183}$$

$$P_{r+4}(0,t) = \sum_{i=0}^{r+2} \lambda_{ci}\, P_i(t) + \sum_{i=0'}^{2'} \lambda_{ci}\, P_i(t) \tag{3.184}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero. where

$C_{n,3}(0) = n\,\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{n,3}(1) = n\,\lambda_1 + \lambda_{c1} + \mu_1$

$C_{n,3}(2) = n\,\lambda_1 + \lambda_{c2} + 2\mu_2$

$C_{n,3}(3) = n\,\lambda_1 + \lambda_{c3} + 2\mu_3$

$C_{n,3}(4) = (n-1)\,\lambda_1 + \lambda_{c4} + 2\mu_4$

For $j = 5, 6, \ldots\ldots, r+1$,

$C_{n,3}(j) = (n - j + 3)\,\lambda_1 + \lambda_{cj} + \mu_j$

$C_{n,3}(r+2) = k\lambda_1 + \lambda_{cr+2} + \mu_{r+2}$

$C_{n,3}(0') = n\,\lambda_1 + \lambda_{c0'} + \mu_{s0'} + 2\lambda_s$

$C_{n,3}(1') = n\,\lambda_1 + \lambda_{c1'} + \mu_{s1'} + \lambda_s$

$C_{n,3}(2') = n\,\lambda_1 + \lambda_{c2'} + \mu_{s2'}$

Special Case Model I

For $n = 3$ and $k = 2$ in Equations (3.171) to (3.184), the long-run availability is obtained directly from Equation (3.122) by eliminating *system up-state 5*. This procedure produced the values given in the third column of Table 3.40.

Special Case Model II

For $n = 3$ and $k = 3$ in Equations (3.171) to (3.184), the long-run availability is determined from the results obtained for the $(3, 3, 2)$ special case model with Type A Repair by eliminating *system up-state 4*. This procedure yielded the long-run availability values given in the fourth column of Table 3.40.

$k - out - of - (n+3) : G$ system with Type B Repair

With Type B Repair, the active units are not repaired upon failure thus μ_1 to μ_{r+2} are set to zero in Equations (3.171) to (3.184).

Special Case Model I

For $n = 3$ and $k = 2$ with Type B Repair, the long-run availability is obtained directly from the results determined for the $(3, 3, 2)$ special case with Type A Repair by eliminating the quantities μ_1 to μ_4. The resulting long-run availability values are presented in the third column of Table 3.41.

Table 3.40: $A_{ssy}(3,3,k)\,(\beta)$ values as a function of λ_{c0} With Type A Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5824	0.4884	0.3615
	0.00002	0.5805	0.4871	0.3608
	0.00003	0.5785	0.4857	0.3601
	0.00004	0.5765	0.4844	0.3594
$\beta=2$	0.00001	0.4109	0.3177	0.2250
	0.00002	0.4089	0.3165	0.2239
	0.00003	0.4070	0.3154	0.2229
	0.00004	0.4050	0.3142	0.2218
$\beta=3$	0.00001	0.3174	0.2354	0.1622
	0.00002	0.3156	0.2344	0.1614
	0.00003	0.3139	0.2335	0.1605
	0.00004	0.3122	0.2325	0.1597
$\beta=4$	0.00001	0.2586	0.1870	0.1268
	0.00002	0.2570	0.1862	0.1261
	0.00003	0.2555	0.1854	0.1254
	0.00004	0.2540	0.1846	0.1248
$\beta=5$	0.00001	0.2181	0.1551	0.1041
	0.00002	0.2168	0.1544	0.1035
	0.00003	0.2154	0.1537	0.1029
	0.00004	0.2140	0.1530	0.1024

Table 3.41: $A_{ssy}(3,3,k)(\beta)$ values as a function of λ_{c0} With Type B Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5085	0.4633	0.3228
	0.00002	0.5071	0.4622	0.3218
	0.00003	0.5057	0.4611	0.3208
	0.00004	0.5043	0.4600	0.3199
$\beta=2$	0.00001	0.3410	0.2897	0.1925
	0.00002	0.3397	0.2888	0.1918
	0.00003	0.3384	0.2879	0.1911
	0.00004	0.3371	0.2870	0.1904
$\beta=3$	0.00001	0.2565	0.2107	0.1371
	0.00002	0.2554	0.2100	0.1366
	0.00003	0.2543	0.2093	0.1361
	0.00004	0.2532	0.2085	0.1356
$\beta=4$	0.00001	0.2055	0.1656	0.1065
	0.00002	0.2046	0.1650	0.1061
	0.00003	0.2037	0.1644	0.1057
	0.00004	0.2028	0.1638	0.1052
$\beta=5$	0.00001	0.1715	0.1364	0.0870
	0.00002	0.1707	0.1359	0.0867
	0.00003	0.1699	0.1354	0.0864
	0.00004	0.1691	0.1348	0.0860

Special Case Model II

For $n = 3$ and $k = 3$ with Type B Repair, the long-run system availability values are determined from the results obtained for the $(3,3,3)$ special case with Type A Repair by eliminating μ_1 to μ_3. This procedure yielded the values presented in the fourth column of Table 3.41.

$k - out - of - (n+3) : G$ System with Type C Repair

With Type C Repair, the units (both active and standbys) are not repaired upon failure thus $\mu_{s0'}$ to $\mu_{s2'}$ and μ_1 to μ_{r+2} are eliminated in Equations (3.171) to (3.184).

Special Case Model I

For $n = 3$ and $k = 2$ with Type C Repair, we can determine the long-run availability values from the results obtained for the $(3,3,2)$ special case with Type B Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$. The resulting system long-run availability values are given in the third column of Table 3.42.

Special Case Model II

For $n = 3$ and $k = 3$ with Type C Repair, the system long-run availability is determined from the results obtained for the $(3,3,3)$ special case with Type B Repair by eliminating $\mu_{s0'}$ to $\mu_{s2'}$. This procedure yielded the values shown in the fourth column of Table 3.42.

$k - out - of - (n+3) : G$ System with Type E Repair

With Type E Repair, the system and its active units are not repaired upon failure and thus $\mu_{r+3}(x)$, $\mu_{r+4}(x)$, μ_1 to μ_{r+2} are set to zero in Equations (3.171) to (3.184).

Table 3.42: $A_{ssY}(3,3,k)(\beta)$ values as a function of λ_{c0} With Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5065	0.3672	0.3185
	0.00002	0.5051	0.3657	0.3176
	0.00003	0.5038	0.3643	0.3167
	0.00004	0.5024	0.3629	0.3158
$\beta=2$	0.00001	0.3391	0.2410	0.1895
	0.00002	0.3379	0.2403	0.1889
	0.00003	0.3367	0.2395	0.1882
	0.00004	0.3355	0.2388	0.1876
$\beta=3$	0.00001	0.2549	0.1807	0.1349
	0.00002	0.2539	0.1801	0.1344
	0.00003	0.2528	0.1795	0.1339
	0.00004	0.2518	0.1788	0.1334
$\beta=4$	0.00001	0.2042	0.1446	0.1047
	0.00002	0.2033	0.1440	0.1043
	0.00003	0.2024	0.1435	0.1039
	0.00004	0.2016	0.1429	0.1035
$\beta=5$	0.00001	0.1703	0.1205	0.0856
	0.00002	0.1695	0.1200	0.0852
	0.00003	0.1688	0.1195	0.0849
	0.00004	0.1680	0.1190	0.0846

Special Case Model I

For $n = 3$ and $k = 2$ with Type E Repair, the relevant system reliability indices are determined thus:

The system reliability is directly obtained from the results obtained for the $(3, 3, 1)$ case with Type E Repair. The system reliability for this special case is thus determined from Equation (3.129) by eliminating residues associated with *system up-state 5*. This procedure produced the values presented in the third column of Table 3.43.

Table 3.43: $R_{Y_E}(3, 3, k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k = 1$	$k = 2$	$k = 3$
0	1.0000	1.0000	1.0000
2000	0.9624	0.8227	0.5460
4000	0.7983	0.3540	0.1226
6000	0.5476	0.1070	0.0200
8000	0.3305	0.0270	0.0029
∞	0	0	0

The system mean time to failure is obtained from Equation (3.135) by eliminating *system up-state 5* and its associated constants. Thus, it is expressed as:

$$\begin{aligned} MTTF_{Y_E}(3,3,2) \quad = \quad & \frac{1}{\alpha_3[C_{3,3}(0)]} [((\zeta_2[C_{3,3}(0')]) (1 + \frac{3\lambda_1}{C_{3,3}(1)} + \frac{9\lambda_1^2}{C_{3,3}(1)\, C_{3,3}(2)} + \\ & \frac{27\lambda_1^3}{C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(3)} + \frac{81\lambda_1^4}{C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)}) + \\ & 3\lambda_s \zeta_1[C_{3,3}(1')] (1 + \frac{3\lambda_1}{C_{3,3}(2)} + \frac{9\lambda_1^2}{C_{3,3}(2)\, C_{3,3}(3)} + \\ & \frac{27\lambda_1^3}{C_{3,3}(2)\, C_{3,3}(3)\, C_{3,3}(4)}) + 6\lambda_s^2\, C_{3,3}(2') (1 + \frac{3\,\lambda_1}{C_{3,3}(3)} + \\ & \frac{9\lambda_1^2}{C_{3,3}(3)\, C_{3,3}(4)}) + 6\lambda_s^3 (1 + \frac{3\lambda_1}{C_{3,3}(4)})] \end{aligned} \qquad (3.185)$$

where,

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\,\zeta_2[C_{3,3}(0')] - 3\lambda_s\,\mu_{s0'}\,\zeta_1[C_{3,3}(1')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\,\zeta_1[C_{3,3}(1')] - 2\,\lambda_s\,\mu_{s1'}\,C_{3,3}(2')$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}$

Equation (3.185) was used to generate system mean life values presented in the third column of Table 3.44.

Table 3.44: $MTTF_{Y_E}(3,3,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6834.60	5422.02	3152.53
0.00002	6745.12	5349.91	3117.68
0.00003	6657.30	5279.27	3083.43
0.00004	6571.14	5210.06	3049.77

The variance of time to failure for this special case can be obtained from Equation (3.131) by eliminating constants associated with *system up-state 5* and setting the quantities μ_1 to μ_5 to zero.

Special Case Model II

For $r = 3$ and $k = 3$ with Type E Repair, we determine the relevant reliability indices as follows:

The system reliability is directly obtained from the results obtained for the $(3,3,2)$ case with Type E Repair by eliminating the residues associated with *system up-state 4*. This procedure yielded the numerical results presented in the fourth column of Table 3.43.

The system mean time to failure for this special case is obtained from the results of the $(3,3,2)$ special case with Type E Repair by eliminating the constants associated with *system up-state 4*. It can be expressed as:

$$
\begin{aligned}
MTTF_{Y_E}(3,3,3) \quad = \quad & \frac{1}{\alpha_3[C_{3,3}(0)]}[(\zeta_2[C_{3,3}(0')])(1+\frac{3\lambda_1}{C_{3,3}(1)}+\frac{9\lambda_1^2}{C_{3,3}(1)\,C_{3,3}(2)}+ \\
& \frac{27\lambda_1^3}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)})+3\lambda_s\zeta_1[C_{3,3}(1')](1+\frac{3\lambda_1}{C_{3,3}(2)}+ \\
& \frac{9\lambda_1^2}{C_{3,3}(2)\,C_{3,3}(3)})+6\,\lambda_s^2\,C_{3,3}(2')\,(1+\frac{3\lambda_1}{C_{3,3}(3)})+6\lambda_s^3] \qquad (3.186)
\end{aligned}
$$

where,

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\,\zeta_2[C_{3,3}(0')] - 3\lambda_s\mu_{s0'}\,\zeta_1[C_{3,3}(1')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\,\zeta_1[C_{3,3}(1')] - 2\lambda_s\mu_{s1'}\,C_{3,3}(2')$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\mu_{s2'}$

Equation (3.186) was used to generate system mean life values presented in the fourth column of Table 3.44.

The variance of time to failure for this special case can be obtained from Equation (3.131) by eliminating constants associated with *system up-state 4* and *5* and in addition, setting μ_1 to μ_5 to zero.

From Equations (3.132), (3.149), (3.150), (3.167), (3.168), (3.185), and (3.186), the mean time to failure of a $k-out-of-(n+m):G$ system with Type E Repair and under standby activation policy Y is given by:

$$
\begin{aligned}
MTTF_{Y_E}(n,m,k) \quad = \quad & \frac{1}{\alpha_m[C_{n,m}(0)]}[(\zeta_{m-1}[C_{n,m}(0')])(1+\frac{n\,\lambda_1}{C_{n,m}(1)}+ \\
& \frac{(n\,\lambda_1)^2}{C_{n,m}(1)\,C_{n,m}(2)}+....+\frac{(n\,\lambda_1)^m}{\prod_{i=1}^{m}C_{n,m}(i)}+ \\
& \frac{\psi_1\,(n\,\lambda_1)^{m+1}}{\prod_{i=1}^{m+1}C_{n,m}(i)}+\frac{\psi_2\,(n\,\lambda_1)^{m+1}\,(n-1)\,\lambda_1}{\prod_{i=1}^{m+2}C_{n,m}(i)}+ \\
& \frac{\psi_3\,(n\lambda_1)^{m+1}\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=1}^{m+3}C_{n,m}(i)}+.... \\
& .....+\frac{\psi_{m-1}\,(n\,\lambda_1)^{m+1}\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=1}^{m+n-k}C_{n,m}(i)}) \\
& +m\,\lambda_s\,\zeta_{m-2}\,[C_{n,m}(1')]\,(1+\frac{n\,\lambda_1}{C_{n,m}(2)}+
\end{aligned}
$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(2)\,C_{n,m}(3)} + \ldots\ldots + \frac{(n\,\lambda_1)^{m-1}}{\prod_{i=2}^{m} C_{n,m}(i)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^m}{\prod_{i=2}^{m+1} C_{n,m}(i)} + \frac{\psi_2\,(n\,\lambda_1)^m\,(n-1)\,\lambda_1}{\prod_{i=2}^{m+2} C_{n,m}(i)} +$$

$$\frac{\psi_3\,(n\,\lambda_1)^m\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=2}^{m+3} C_{n,m}(i)} + \ldots..$$

$$\ldots.. + \frac{\psi_{m-1}\,(n\,\lambda_1)^m\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=2}^{m+n-k} C_{n,m}(i)}) +$$

$$(m)\,(m-1)\,\lambda_s^2\,\zeta_{m-3}\,[C_{n,m}(2')]\,(1 + \frac{n\,\lambda_1}{C_{n,m}(3)} +$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(3)\,C_{n,m}(4)} + \ldots\ldots + \frac{(n\,\lambda_1)^{m-2}}{\prod_{i=3}^{m} C_{n,m}(i)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^{m-1}}{\prod_{i=3}^{m+1} C_{n,m}(i)} + \frac{\psi_2\,(n\,\lambda_1)^{m-1}\,(n-1)\,\lambda_1}{\prod_{i=3}^{m+2} C_{n,m}(i)} +$$

$$\frac{\psi_3\,(n\,\lambda_1)^{m-1}\,(n-1)\,(n-2)\,\lambda_1^2}{\prod_{i=3}^{m+3} C_{n,m}(i)} + \ldots\ldots..$$

$$.. + \frac{\psi_{m-1}\,(n\,\lambda_1)^{m-1}\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=3}^{m+n-k} C_{n,m}(i)}) + \ldots$$

$$\ldots + (m)(m-1)(m-2)(m-3)(m-4)\ldots\ldots$$

$$\ldots(3)\,\lambda_s^{m-2}\,\zeta_1\,[C_{n,m}(m-2)']\,(1 + \frac{n\,\lambda_1}{C_{n,m}(m-1)} +$$

$$\frac{(n\,\lambda_1)^2}{C_{n,m}(m-1)\,C_{n,m}(m)} + \frac{\psi_1\,(n\,\lambda_1)^3}{C_{n,m}(m-1)\,C_{n,m}(m)\,C_{n,m}(m+1)} +$$

$$\frac{\psi_2\,(n\,\lambda_1)^3\,(n-1)\,\lambda_1}{\prod_{i=m-1}^{m+2} C_{n,m}(i)} + \ldots$$

$$\ldots\frac{\psi_{m-1}\,(n\,\lambda_1)^3\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m-1}^{m+n-k} C_{n,m}(i)}) +$$

$$m!\,\lambda_s^{m-1}\,\zeta_0[C_{n,m}(m-1)'](1 + \frac{n\,\lambda_1}{C_{n,m}(m)} +$$

$$\frac{\psi_1\,(n\,\lambda_1)^2}{C_{n,m}(m)\,C_{n,m}(m+1)} +$$

$$\frac{\psi_2\,(n\,\lambda_1)^2\,(n-1)\,\lambda_1}{C_{n,m}(m)\,C_{n,m}(m+1)\,C_{n,m}(m+2)} + \ldots..$$

$$\ldots. + \frac{\psi_{m-1}\,(n\,\lambda_1)^2\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m}^{m+n-k} C_{n,m}(i)}) +$$

$$m!\,\lambda_s^m\,(1 + \frac{\psi_1\,(n\,\lambda_1)}{C_{n,m}(m+1)} +$$

$$\frac{\psi_2\,(n\,\lambda_1)\,(n-1)\,\lambda_1}{C_{n,m}(m+1)\,C_{n,m}(m+2)} + \ldots$$

$$... + \frac{\psi_{m-1}\,(n\,\lambda_1)\,(n-1)!\,\lambda_1^{n-2}}{\prod_{i=m+1}^{m+n-k} C_{n,m}(i)})] \tag{3.187}$$

where for $j = 0, 1, 2, ...(m-1)$, $i = 0, 1, ..., (n-1)$, the following recursive relationships suffice:

$\zeta_j[C_{n,m}(i')] = C_{n,m}(i')\,\zeta_{j-1}[C_{n,m}(i+1')] - j\,\lambda_s\,\mu_{si+1'}\,\zeta_{j-2}\,[C_{n,m}(i+2')]$

$\alpha_{i+1}[C_{n,m}(i)] = C_{n,m}(i)\,\zeta_i[C_{n,m}(i')] - j\,\lambda_s\,\mu_{si'}\,\zeta_{i-1}[C_{n,m}(i+1')]$

Where the starting values are given as:

$\zeta_0[C_{n,m}(0')] = C_{n,m}(0')$

$\alpha_1[C_{n,m}(0)] = C_{n,m}(0)\,C_{n,m}(0') - \lambda_s\,\mu_{s0'}$

and,

Also for $j = 1$ to $(m-1)$ we have:

$\psi_j = 0$ for $j \geq n-k+m$

$\psi_j = 1$ for $j < n-k+m$

$k - out - of - (n+3) : G$ System with Type F Repair

With Type F Repair, the system and its units (both active and standbys) are not repaired upon failure and thus $\mu_{s0'}$ to $\mu_{s2'}$, $\mu_{r+3}(x)$, $\mu_{r+4}(x)$, μ_1 to μ_{r+2} are set to zero in Equations (3.171) to (3.184).

Special Case Model I

For $n = 3$ and $k = 2$ with Type F Repair, the relevant reliability indices are obtained as follows:

The system reliability is directly obtained from the results determined for the $(3, 3, 2)$ case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$. The resulting numerical results are presented in the third column of Table 3.45.

The system mean time to failure for this case was obtained from the results determined for the $(3, 3, 2)$ special case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to

Table 3.45: $R_{Y_F}(3,3,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9500	0.7223	0.4478
4000	0.7803	0.3390	0.1082
6000	0.5271	0.1017	0.0178
8000	0.3089	0.0252	0.0024
∞	0	0	0

$\mu_{s2'}$ and it can be expressed as:

$$\begin{aligned} MTTF_{Y_F}(3,3,2) \;=\; & \frac{1}{\alpha_3[C_{3,3}(0)]}[(\zeta_2[C_{3,3}(0')])(1+\frac{3\lambda_1}{C_{3,3}(1)}+\frac{9\lambda_1^2}{C_{3,3}(1)\,C_{3,3}(2)}+ \\ & \frac{27\lambda_1^3}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)}+\frac{81\lambda_1^4}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)})+ \\ & 3\lambda_s\zeta_1[C_{3,3}(1')](1+\frac{3\lambda_1}{C_{3,3}(2)}+\frac{9\lambda_1^2}{C_{3,3}(2)\,C_{3,3}(3)}+ \\ & \frac{27\lambda_1^3}{C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(4)})+6\lambda_s^2\,C_{3,3}(2')(1+\frac{3\,\lambda_1}{C_{3,3}(3)}+ \\ & \frac{9\lambda_1^2}{C_{3,3}(3)\,C_{3,3}(4)})+6\lambda_s^3(1+\frac{3\lambda_1}{C_{3,3}(4)})] \end{aligned} \quad (3.188)$$

where,

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\,\zeta_2[C_{3,3}(0')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\,\zeta_1[C_{3,3}(1')]$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\,C_{3,3}(2')$

Equation (3.188) was used to generate system mean time to failure values given in the third column of Table 3.46.

The variance of time to failure can be obtained from the variance results determined for the $(3,3,2)$ special case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$.

Table 3.46: $MTTF_{Y_F}(3,3,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6777.25	5001.89	3095.65
0.00002	6692.47	4946.85	3063.07
0.00003	6609.21	4892.72	3031.02
0.00004	6527.44	4839.50	2999.50

Special Case Model II

For $n=3$ and $k=3$ with Type F Repair, the relevant reliability indices are obtained as follows:

The system reliability is determined from the results obtained for the $(3,3,3)$ case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$. This procedure produced the numerical results presented in the fourth column of Table 3.45.

The system mean time to failure is obtained from the results determined for the $(3,3,3)$ special case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$. It can be expressed as:

$$\begin{aligned} MTTF_{Y_F}(3,3,3) \;=\; & \frac{1}{\alpha_3[C_{3,3}(0)]}[(\zeta_2[C_{3,3}(0')])(1+\frac{3\lambda_1}{C_{3,3}(1)}+\frac{9\lambda_1^2}{C_{3,3}(1)\,C_{3,3}(2)}+ \\ & \frac{27\lambda_1^3}{C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)})+3\lambda_s\zeta_1[C_{3,3}(1')](1+\frac{3\lambda_1}{C_{3,3}(2)}+ \\ & \frac{9\lambda_1^2}{C_{3,3}(2)\,C_{3,3}(3)})+6\,\lambda_s^2\,C_{3,3}(2')\,(1+\frac{3\lambda_1}{C_{3,3}(3)})+6\lambda_s^3] \quad (3.189) \end{aligned}$$

where,

$\alpha_3[C_{3,3}(0)] = C_{3,3}(0)\,\zeta_2[C_{3,3}(0')]$

$\zeta_2[C_{3,3}(0')] = C_{3,3}(0')\,\zeta_1[C_{3,3}(1')]$

$\zeta_1[C_{3,3}(1')] = C_{3,3}(1')\,C_{3,3}(2')$

Equation (3.189) was used to generate system mean time to failure values given in the fourth column of Table 3.46.

The variance of time to failure can be obtained from the variance results determined for the $(3,3,3)$ special case with Type E Repair by eliminating the quantities $\mu_{s0'}$ to $\mu_{s2'}$.

From the fore-going results it is easily observed that the mean time to failure of a $k-out-of-(n+m):G$ system with Type F Repair and under standby activation policy Y is also given by Equation (3.187) except that $\mu_{s0'} = ... = \mu_{sm-1'} = 0$ thus leading to the following new relationships:

For $j = 0,1,2,...(m-1)$, $i = 0,1,...,(m-1)$,

$\zeta_j[C_{n,m}(i')] = C_{n,m}(i')\,\zeta_{j-1}C_{n,m}(i+1')$

$\alpha_{i+1}[C_{n,m}(i)] = C_{n,m}(i)\,\zeta_i C_{n,m}(i')$

Where the starting values are given as:

$\zeta_0[C_{n,m}(0')] = C_{n,m}(0')$

$\alpha_1[C_{n,m}(0)] = C_{n,m}(0)\,\zeta_0[C_{n,m}(0')]$

3.2 The Non-Identical Unit System With Early Standby Activation: Special Cases

When units are non-identical (i.e. they do not have the same failure rates), it is much more cumbersome to generate generalized expressions for system reliability indices such as the system mean time to failure. In addition, the number of system up-states increases sharply with any increase in either the number of active units or standbys (see Appendix J.1). In order to demonstrate this, we considered duplex systems (i.e. two active unit systems) for the analysis of non-identical unit $k-out-of-(n+m):G$ type systems for repair types under which system units are not repaired. Thus the following special cases are considered in this regard: *(i)* $1-out-of-(2+1):G$ or $(2,1,1)$ *(ii)* $2-out-of-(2+1):G$ or $(2,1,2)$ *(iii)* $1-out-of-(2+2):G$ or $(2,2,1)$ and *(iv)* $2-out-of-(2+2):G$ or $(2,2,2)$.

3.2.1 1-out-of-(2+1):G Non-identical Unit System

This is the $(2, 1, 1)$ system and is composed of *two* non-identical units and *one* non-identical standby and is operable as long as at least *one* unit is functional. The state space diagram for this system is shown in Figure 3.28. The numerals in the boxes of

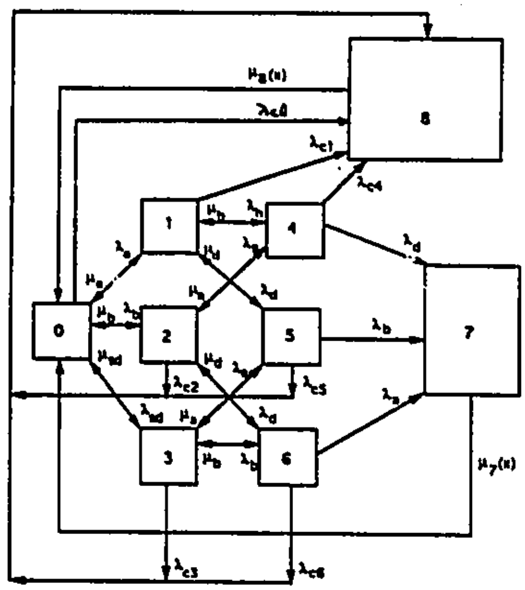


Figure 3.28: State Space Diagram of the $1 - out - of - (2+1) : G$ Non-identical Unit System under SBAP Y

Figure 3.28 denote the system state numbers.

<u>Notation</u>

The following symbols are associated with the $(2, 1, 1)$ non-identical unit system:

i	State identification number. For $i = 0$, both units (A and B) and the standby D are in perfect working condition. For $i = 1$, unit A has failed, unit B is working and

standby D has been activated. For $i = 2$, unit B has failed, unit A is working and standby D has been activated. For $i = 3$, both units A and B are working perfectly but standby D has failed in its standby mode. For $i = 4$, both units (A and B) have failed but D now activated is working. For $i = 5$, units A and D have failed but unit B is still functional. For $i = 6$, units B and D have failed but unit A is working.
For $i = 7$, the system has failed due to independent failures of its components. For $i = 8$, the system has failed due to common-cause failures.

t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated.
λ_{sd}	Constant failure rate of standby unit D when in its standby mode
λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, ..., 6$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_d	Constant repair rate of unit D
μ_{sd}	Constant repair rate of standby unit D if it fails in the standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, ..., 8$
$P_i(x, t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 7, 8$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time

	respectively when the system is in state i and has an elapsed repair time of x; for $i = 7, 8$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The system of integro-differential equations associated with this system described in Figure 3.28 is:

$$\frac{dP_0(t)}{dt} + C_{2,1}(0)\,P_0(t) = \mu_a\,P_1(t) + \mu_b\,P_2(t) + \mu_{sd}\,P_3(t) + \sum_{i=7}^{8}\int_0^\infty P_i(x,t)\,\mu_i(x)\,dx \tag{3.190}$$

$$\frac{dP_1(t)}{dt} + C_{2,1}(1)\,P_1(t) = \mu_b\,P_4(t) + \mu_d\,P_5(t) + \lambda_a\,P_0(t) \tag{3.191}$$

$$\frac{dP_2(t)}{dt} + C_{2,1}(2)\,P_2(t) = \mu_a\,P_4(t) + \mu_d\,P_6(t) + \lambda_b\,P_0(t) \tag{3.192}$$

$$\frac{dP_3(t)}{dt} + C_{2,1}(3)\,P_3(t) = \mu_a\,P_5(t) + \mu_b\,P_6(t) + \lambda_{sd}\,P_0(t) \tag{3.193}$$

$$\frac{dP_4(t)}{dt} + C_{2,1}(4)\,P_4(t) = \lambda_b\,P_1(t) + \lambda_a\,P_2(t) \tag{3.194}$$

$$\frac{dP_5(t)}{dt} + C_{2,1}(5)\,P_5(t) = \lambda_d\,P_1(t) + \lambda_a\,P_3(t) \tag{3.195}$$

$$\frac{dP_6(t)}{dt} + C_{2,1}(6)\,P_6(t) = \lambda_d\,P_2(t) + \lambda_b\,P_3(t) \tag{3.196}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_7(x)]\,P_7(x,t) = 0 \tag{3.197}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_8(x)]\,P_8(x,t) = 0 \tag{3.198}$$

With the following boundary conditions:

$$P_7(0,t) = \lambda_d\,P_4(t) + \lambda_b\,P_5(t) + \lambda_a\,P_6(t) \tag{3.199}$$

$$P_8(0,t) = \sum_{i=0}^{6}\lambda_{ci}\,P_i(t) \tag{3.200}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where,

$C_{2,1}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd}$

$C_{2,1}(1) = \lambda_b + \lambda_d + \lambda_{c1} + \mu_a$

$C_{2,1}(2) = \lambda_a + \lambda_d + \lambda_{c2} + \mu_b$

$C_{2,1}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \mu_{sd}$

$C_{2,1}(4) = \lambda_d + \lambda_{c4} + \mu_a + \mu_b$

$C_{2,1}(5) = \lambda_b + \lambda_{c5} + \mu_d + \mu_a$

$C_{2,1}(6) = \lambda_a + \lambda_{c6} + \mu_b + \mu_d$

$1 - out - of - (2+1) : G$ Non-identical Unit System with Type B Repair

With Type B Repair, $\mu_a = \mu_b = \mu_d = 0$. The system is repaired upon failure and we are primarily concerned with the system long-run availability. Using the procedure already established for the identical unit case, the long-run availability of this system can be obtained in terms of the gamma *pdf* shape parameter β and is given as:

$$A_{ssY}(2,1,1)\,(\beta) = \frac{(\mu_7\,\mu_8)^\beta\, x_{69}}{(\mu_7\,\mu_8)^\beta\, x_{70} + \beta\,\mu_7^\beta\,\mu_8^{\beta-1}\, x_{71} + \beta\,\mu_7^{\beta-1}\,\mu_8^\beta\, x_{72}} \tag{3.201}$$

where,

$x_{69} = A_{2509} + A_{2515} + A_{2521} + A_{2527} + A_{2541} + A_{2547} + A_{2553}$

$x_{70} = A_{2554} - A_{2556} - A_{2558}$

$x_{71} = A_{2555} - A_{2557}$

$x_{72} = A_{2555} - A_{2559}$

Long-run availability values obtained with Equation (3.201) are exactly the same as those obtained with the $(2,1,1)$ identical unit case with Type B Repair. Table 3.2 presents these values for $\lambda_a = \lambda_b = \lambda_c = \lambda_1$. Symbols A_{2509} to A_{2559} are defined in Appendix E.1.

$1-out-of-(2+1):G$ Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_d = \mu_{sd} = 0$. Thus setting μ_{sd} equal to zero in Equation (3.201), the long-run availability for this case is determined. These values are exactly the same as those calculated for the $(2,1,1)$ identical unit case with Type C Repair shown in the third column of Table 3.3 when $\lambda_a = \lambda_b = \lambda_c = \lambda_1$.

$1-out-of-(2+1):G$ Non-identical Unit System with Type E Repair

With Type E Repair, $\mu_a = \mu_b = \mu_d = \mu_7(x) = \mu_8(x) = 0$. The system is not repaired and the performance indices of interest are the system reliability, system mean time to failure, and the system variance of time to failure. These are given by the following expressions:

The system reliability is given by:

$$R_{Y_E}(2,1,1)(t) = \sum_{i=0}^{6} P_i(t) \tag{3.202}$$

where,

$P_0(t) = BN_{620}\, e^{s_{80}t} + BN_{621}\, e^{s_{81}t}$

$P_1(t) = BN_{622}\, e^{-C_{2,1}(1)t} + BN_{623}\, e^{s_{80}t} + BN_{624}\, e^{s_{81}t}$

$P_2(t) = BN_{625}\, e^{-C_{2,1}(2)t} + BN_{626}\, e^{s_{80}t} + BN_{627}\, e^{s_{81}t}$

$P_3(t) = BN_{628}\, e^{s_{80}t} + BN_{629}\, e^{s_{81}t}$

$P_4(t) = BN_{630}\, e^{-C_{2,1}(4)t} + BN_{631}\, e^{-C_{2,1}(2)t} + BN_{632}\, e^{-C_{2,1}(1)t} + BN_{633}\, e^{s_{80}t} + BN_{634}\, e^{s_{81}t}$

$P_5(t) = BN_{635}\, e^{-C_{2,1}(5)t} + BN_{636}\, e^{-C_{2,1}(1)t} + BN_{637}\, e^{s_{80}t} + BN_{638}\, e^{s_{81}t}$

$P_6(t) = BN_{639}\, e^{-C_{2,1}(6)t} + BN_{640}\, e^{-C_{2,1}(2)t} + BN_{641}\, e^{s_{80}t} + BN_{642}\, e^{s_{81}t}$

$Denomy(s) = (s + C_{2,1}(0))(s + C_{2,1}(3)) - \lambda_{sd}\,\mu_{sd}$

s_{80} and s_{81} are the roots of the characteristic function $Denomy(s)$ and the partial fraction residues BN_{620} and BN_{642} are defined in Appendix E.1.

The mean time to failure of the system is:

$$MTTF_{Y_E}(2,1,1) = \lim_{s\to 0} \sum_{i=0}^{6} P_i(s) \tag{3.203}$$

where

$P_0(0) = C_{2,1}(3)/[C_{2,1}(0)\,C_{2,1}(3) - \lambda_{sd}\,\mu_{sd}]$

$P_1(0) = (\lambda_a\,C_{2,1}(3))/(C_{2,1}(1)\,[C_{2,1}(0)\,C_{2,1}(3) - \lambda_{sd}\,\mu_{sd}])$

$P_2(0) = (\lambda_b\,C_{2,1}(3))/(C_{2,1}(2)\,[C_{2,1}(0)\,C_{2,1}(3) - \lambda_{sd}\,\mu_{sd}])$

$P_3(0) = \lambda_{sd}/[C_{2,1}(0)\,C_{2,1}(3) - \lambda_{sd}\,\mu_{sd}]$

$P_4(0) = (\lambda_b/C_{2,1}(4)) \bullet P_1(0) + (\lambda_a/C_{2,1}(4)) \bullet P_2(0)$

$P_5(0) = (\lambda_d/C_{2,1}(5)) \bullet P_1(0) + (\lambda_a/C_{2,1}(5)) \bullet P_3(0)$

$P_6(0) = (\lambda_d/C_{2,1}(6)) \bullet P_2(0) + (\lambda_b/C_{2,1}(6)) \bullet P_3(0)$

The system variance of time to failure is:

$$\sigma^2_{Y_E}(2,1,1) \;=\; \frac{1}{k_{70}^2}\,[2(k_{69}k_{71} - k_{70}k_{72}) - k_{69}^2] \tag{3.204}$$

Symbols k_{69} to k_{72} are provided in Appendix E.1.

<u>$1 - out - of - (2+1) : G$ **Non-identical Unit System with Type** F **Repair**</u>

With Type F Repair, $\mu_a = \mu_b = \mu_d = \mu_{sd} = \mu_7(x) = \mu_8(x) = 0$. The system is not repaired and again the performance indices of interest are the system reliability, system mean time to failure, and the variance of time to failure. These are determined as follows:

The system reliability is obtained directly from Equation (3.202) by setting μ_{sd} to zero. For $\lambda_a = \lambda_b = \lambda_d = \lambda_1$, the values obtained are exactly equal to those obtained for the $(2,1,1)$ identical units case with Type F Repair discussed in Subsection 3.1.1.

The mean time to failure of this system is determined from Equation (3.203) by setting μ_{sd} to zero.

The variance of time to failure for this case can be obtained by setting μ_{sd} to zero in Equation (3.204).

3.2.2 2-out-of-(2+1):G Non-identical Unit System

This is the $(2,1,2)$ system and is composed of *two* non-identical units and *one* non-identical standby and is operable as long as at least *two* units are functional. The state space diagram for the system is shown in Figure 3.29 The numerals in the boxes of the figure denote the system state numbers.

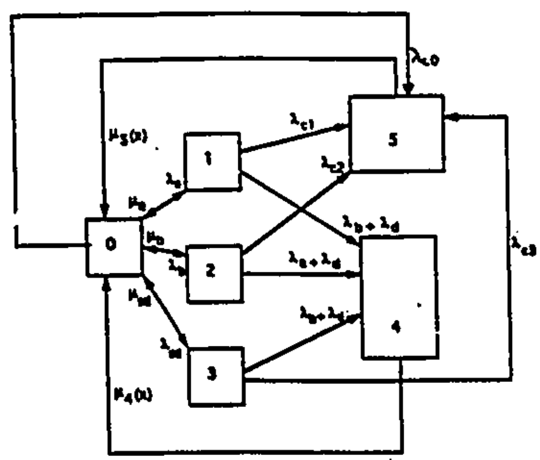


Figure 3.29: State space diagram of the $2 - out - of - (2+1) : G$ Non-identical Unit System under SBAP Y

Notation

The following symbols are associated with the $(2, 1, 2)$ non-identical unit system:

i	State identification number. For $i = 0$, both units (A and B) and the standby D are in perfect working condition. For $i = 1$, unit A has failed, unit B is working and standby D has been activated. For $i = 2$, unit B has failed, unit A is working and standby D has been activated. For $i = 3$, both units A and B are working perfectly but standby D has failed in its standby mode.

	For $i = 4$, the system fails due to independent failures of two units. For $i = 5$, the system fails due to common-cause failures of its components.
t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated
λ_{sd}	Constant failure rate of standby unit D when in its standby mode
λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, 3$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_{sd}	Constant repair rate of standby unit D if it fails in its standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, 4, 5$
$P_i(x, t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 4, 5$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time respectively when the system is in state i and has an elapsed repair time of x; for $i = 4, 5$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The system of integro-differential equations associated with this system described in Figure 3.29 is:

$$\frac{dP_0(t)}{dt} + C_{2,1}(0)\,P_0(t) = \mu_a\,P_1(t) + \mu_b\,P_2(t) + \mu_{sd}\,P_3(t) +$$

$$\sum_{i=4}^{5}\int_0^\infty P_i(x,t)\,\mu_i(x)\,dx \tag{3.205}$$

$$\frac{dP_1(t)}{dt} + C_{2,1}(1)\,P_1(t) = \lambda_a\,P_0(t) \tag{3.206}$$

$$\frac{dP_2(t)}{dt} + C_{2,1}(2)\,P_2(t) = \lambda_b\,P_0(t) \tag{3.207}$$

$$\frac{dP_3(t)}{dt} + C_{2,1}(3)\,P_3(t) = \lambda_{sd}\,P_0(t) \tag{3.208}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_4(x)]\,P_4(x,t) = 0 \tag{3.209}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_5(x)]\,P_5(x,t) = 0 \tag{3.210}$$

With the following boundary conditions:

$$P_4(0,t) = [\lambda_b + \lambda_d]\,P_1(t) + [\lambda_a + \lambda_d]\,P_2(t) + [\lambda_a + \lambda_b]\,P_3(t) \tag{3.211}$$

$$P_5(0,t) = \sum_{i=0}^{3}\lambda_{ci}\,P_i(t) \tag{3.212}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{2,1}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd}$

$C_{2,1}(1) = \lambda_b + \lambda_d + \lambda_{c1} + \mu_a$

$C_{2,1}(2) = \lambda_a + \lambda_d + \lambda_{c2} + \mu_b$

$C_{2,1}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \mu_{sd}$

$2 - out - of - (2+1) : G$ Non-identical Unit System with Type B Repair

With Type B Repair, $\mu_a = \mu_b = 0$. The system is repaired upon failure and we are primarily concerned with the system long-run availability. The long-run availability values can be obtained by eliminating *system up-states 4, 5* and *6* and constants relating to them in the results obtained for the $(2,1,1)$ non-identical unit case with Type B Repair.

$2-out-of-(2+1):G$ Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_{sd} = 0$. Thus eliminating μ_{sd} from the results obtained for the $(2,1,2)$ system with Type B Repair the long-run availability values can be determined.

$2-out-of-(2+1):G$ Non-identical Unit System with Type E Repair

With Type E Repair, $\mu_a = \mu_b = \mu_4(x) = \mu_5(x) = 0$. The system is not repaired upon failure. The system reliability can be obtained by eliminating *system up-states 4, 5* and *6* from the results determined for the $(2,1,1)$ system with Type E Repair and is given by:

$$R_{Y_E}(2,1,2) = \sum_{i=0}^{3} P_i(t) \tag{3.213}$$

where

$P_0(t) = BN_{620}\, e^{s_{80}t} + BN_{621}\, e^{s_{81}t}$

$P_1(t) = BN_{622}\, e^{-C_{2,1}(1)t} + BN_{623}\, e^{s_{80}t} + BN_{624}\, e^{s_{81}t}$

$P_2(t) = BN_{625}\, e^{-C_{2,1}(2)t} + BN_{626}\, e^{s_{80}t} + BN_{627}\, e^{s_{81}t}$

$P_3(t) = BN_{628}\, e^{s_{80}t} + BN_{629}\, e^{s_{81}t}$

The partial fraction residues BN_{620} to BN_{629} are as defined for Equation (3.202).

The mean time to failure is determined by eliminating the constants associated with *system up-states 4, 5* and *6* in Equation (3.203) and is given by:

$$MTTF_{Y_E}(2,1,2) = \lim_{s \to 0} \sum_{i=0}^{3} P_i(s) \tag{3.214}$$

where

$P_0(0) = C_{2,1}(3)/[C_{2,1}(0)\, C_{2,1}(3) - \lambda_{sd}\, \mu_{sd}]$

$P_1(0) = (\lambda_a\, C_{2,1}(3))/(C_{2,1}(1)\, [C_{2,1}(0)\, C_{2,1}(3) - \lambda_{sd}\, \mu_{sd}])$

$P_2(0) = (\lambda_b\, C_{2,1}(3))/(C_{2,1}(2)\, [C_{2,1}(0)\, C_{2,1}(3) - \lambda_{sd}\, \mu_{sd}])$

$P_3(0) = \lambda_{sd}/[C_{2,1}(0)\, C_{2,1}(3) - \lambda_{sd}\, \mu_{sd}]$

The variance of time to failure for this system can be determined by eliminating all the terms associated with *system up-states 4, 5* and *6* in Equation (3.204).

$2-out-of-(2+1):G$ Non-identical Unit System with Type F Repair

With Type F Repair, $\mu_a = \mu_b = \mu_{sd} = \mu_4(x) = \mu_5(x) = 0$. The system is not repaired and the system reliability, system mean time to failure, and the system variance of time to failure are given by the following expressions:
The system reliability is obtained from Equation (3.213) by setting μ_{sd} to zero.

The mean time to failure of this system is obtained directly from Equation (3.214) by setting μ_{sd} to zero.

The variance of time to failure for this system is determined directly from Equation (3.204) by eliminating all quantities associated with *system states 4* to *6* and setting μ_{sd} to zero.

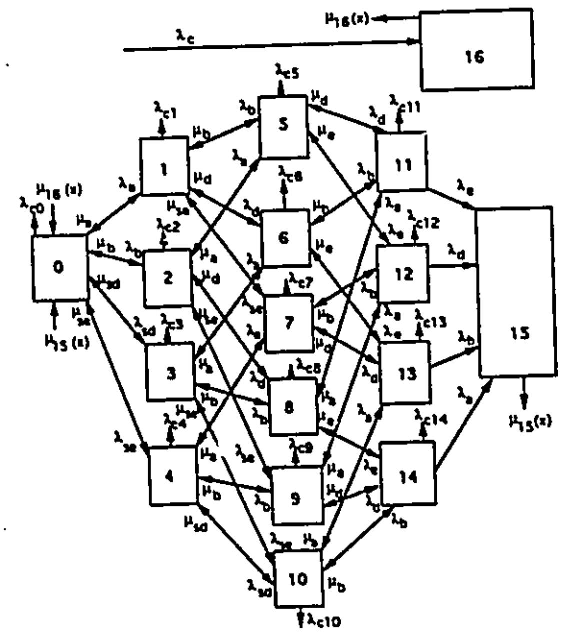
3.2.3 1-out-of-(2+2):G Non-identical Unit System

This is the $(2,2,1)$ system and is composed of *two* non-identical units and *two* non-identical standbys and is operable as long as at least *one* unit is functional. The state space diagram for this system is shown in Figure 3.30. The numerals in the boxes of the figure denote the system state numbers.

Notation

The following symbols are associated with a $(2,2,1)$ non-identical unit system:

i — State identification number. For $i=0$, both units (A and B) and both standby units, D and E are in perfect working condition.
For $i=1$, unit A has failed, unit B is working and standby D has been activated. For $i=2$, unit B has failed, unit A is working and standby D has been activated. For $i=3$, both units A and B are working



Where,

$\lambda_c = \lambda_{c0} + \lambda_{c1} + \lambda_{c2} + \lambda_{c3} + \lambda_{c4} + \lambda_{c5} + \lambda_{c6} + \lambda_{c7} + \lambda_{c8} + \lambda_{c9} + \lambda_{c10} + \lambda_{c11} + \lambda_{c12} + \lambda_{c13} + \lambda_{c14}$

Figure 3.30: State Space Diagram of the $1 - out - of - (2+2) : G$ Non-identical Unit System under SBAP Y

perfectly but standby D has failed in its standby mode. For $i = 4$, both units (A and B) are working but standby unit E has failed in its standby mode. For $i = 5$, both A and B have failed and both D and E have been activated. For $i = 6$, units A and D have failed while units B and E are still functional. For $i = 7$, units A and E have failed while units B and D are working. For $i = 8$, units B and D have failed but units A and E are still working. For $i = 9$, units B and E have failed but units A and D are still functional. For $i = 10$, units D and E have failed while units A and B are still operational. For $i = 11$, units A, B and D have failed while unit E is operational. For $i = 12$, units A, B, and E have failed while unit D is functional. For $i = 13$, units A, D and E have failed while B is operational. For $i = 14$ units B, D and E have failed but unit A still works. For $i = 15$, the system has failed due to independent failures of its components. For $i = 16$, the system has failed due to common-cause failures of its components.

t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated
λ_e	Constant failure rate of standby unit E when activated
λ_{sd}	Constant failure rate of standby unit D in its standby mode
λ_{se}	Constant failure rate of standby unit E in its standby mode

λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, ..., 14$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_d	Constant repair rate of unit D
μ_e	Constant repair rate of unit E
μ_{sd}	Constant repair rate of standby unit D if it fails in its standby mode
μ_{se}	Constant repair rate of standby unit E if it fails in its standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, ..., 16$
$P_i(x,t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 15, 16$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time respectively when the system is in state i and has an elapsed repair time of x; for $i = 15, 16$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The set of integro-differential equations associated with this system described in Figure 3.30 is:

$$\frac{dP_0(t)}{dt} + C_{2,2}(0)\,P_0(t) = \mu_a\,P_1(t) + \mu_b\,P_2(t) + \mu_{sd}\,P_3(t) + \mu_{se}\,P_4(t) + \sum_{i=15}^{16} \int_0^\infty P_i(x,t)\,\mu_i(x)\,dx \tag{3.215}$$

$$\frac{dP_1(t)}{dt} + C_{2,2}(1)\,P_1(t) = \mu_b\,P_5(t) + \mu_d\,P_6(t) + \lambda_a\,P_0(t) + \mu_{se}\,P_7(t) \tag{3.216}$$

$$\frac{dP_2(t)}{dt} + C_{2,2}(2)\,P_2(t) = \mu_a\,P_5(t) + \mu_d\,P_8(t) + \lambda_b\,P_0(t) + \mu_{se}\,P_9(t) \tag{3.217}$$

$$\frac{dP_3(t)}{dt} + C_{2,2}(3)\,P_3(t) = \mu_a\,P_6(t) + \mu_b\,P_8(t) + \lambda_{sd}\,P_0(t) + \mu_{se}\,P_{10}(t) \tag{3.218}$$

$$\frac{dP_4(t)}{dt} + C_{2,2}(4)\,P_4(t) = \mu_a\,P_7(t) + \mu_b\,P_9(t) + \lambda_{se}\,P_0(t) + \mu_{sd}\,P_{10}(t) \tag{3.219}$$

$$\frac{dP_5(t)}{dt} + C_{2,2}(5)\,P_5(t) = \mu_d\,P_{11}(t) + \mu_e\,P_{12}(t) + \lambda_b\,P_1(t) + \lambda_a\,P_2(t) \tag{3.220}$$

$$\frac{dP_6(t)}{dt} + C_{2,2}(6)\,P_6(t) = \mu_b\,P_{11}(t) + \mu_e\,P_{13}(t) + \lambda_d\,P_1(t) + \lambda_a\,P_3(t) \tag{3.221}$$

$$\frac{dP_7(t)}{dt} + C_{2,2}(7)\,P_7(t) = \mu_b\,P_{12}(t) + \mu_d\,P_{13}(t) + \lambda_{se}\,P_1(t) + \lambda_a\,P_4(t) \tag{3.222}$$

$$\frac{dP_8(t)}{dt} + C_{2,2}(8)\,P_8(t) = \mu_a\,P_{11}(t) + \mu_e\,P_{14}(t) + \lambda_d\,P_2(t) + \lambda_b\,P_3(t) \tag{3.223}$$

$$\frac{dP_9(t)}{dt} + C_{2,2}(9)\,P_9(t) = \mu_a\,P_{12}(t) + \mu_d\,P_{14}(t) + \lambda_{se}\,P_2(t) + \lambda_b\,P_4(t) \tag{3.224}$$

$$\frac{dP_{10}(t)}{dt} + C_{2,2}(10)\,P_{10}(t) = \mu_a\,P_{13}(t) + \mu_b\,P_{14}(t) + \lambda_{se}\,P_3(t) + \lambda_{sd}\,P_4(t) \tag{3.225}$$

$$\frac{dP_{11}(t)}{dt} + C_{2,2}(11)\,P_{11}(t) = \lambda_d\,P_5(t) + \lambda_b\,P_6(t) + \lambda_a\,P_8(t) \tag{3.226}$$

$$\frac{dP_{12}(t)}{dt} + C_{2,2}(12)\,P_{12}(t) = \lambda_e\,P_5(t) + \lambda_b\,P_7(t) + \lambda_a\,P_9(t) \tag{3.227}$$

$$\frac{dP_{13}(t)}{dt} + C_{2,2}(13)\,P_{13}(t) = \lambda_e\,P_6(t) + \lambda_d\,P_7(t) + \lambda_a\,P_{10}(t) \tag{3.228}$$

$$\frac{dP_{14}(t)}{dt} + C_{2,2}(14)\,P_{14}(t) = \lambda_e\,P_8(t) + \lambda_d\,P_9(t) + \lambda_b\,P_{10}(t) \tag{3.229}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{15}(x)]\,P_{15}(x,t) = 0 \tag{3.230}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{16}(x)]\,P_{16}(x,t) = 0 \tag{3.231}$$

With the following boundary conditions:

$$P_{15}(0,t) = \lambda_e\,P_{11}(t) + \lambda_d\,P_{12}(t) + \lambda_b\,P_{13}(t) + \lambda_a\,P_{14}(t) \tag{3.232}$$

$$P_{16}(0,t) = \sum_{i=0}^{14} \lambda_{ci}\,P_i(t) \tag{3.233}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{2,2}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd} + \lambda_{se}$

$C_{2,2}(1) = \lambda_b + \lambda_d + \lambda_{c1} + \lambda_{se} + \mu_a$

$C_{2,2}(2) = \lambda_a + \lambda_d + \lambda_{c2} + \lambda_{se} + \mu_b$

$C_{2,2}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \lambda_{se} + \mu_{sd}$

$C_{2,2}(4) = \lambda_a + \lambda_b + \lambda_{sd} + \lambda_{c4} + \mu_{se}$

$C_{2,2}(5) = \lambda_d + \lambda_e + \lambda_{c5} + \mu_a + \mu_b$

$C_{2,2}(6) = \lambda_b + \lambda_e + \lambda_{c6} + \mu_a + \mu_d$

$C_{2,2}(7) = \lambda_b + \lambda_d + \mu_{se} + \lambda_{c7} + \mu_a$

$C_{2,2}(8) = \lambda_a + \lambda_e + \lambda_{c8} + \mu_b + \mu_d$

$C_{2,2}(9) = \lambda_a + \lambda_d + \lambda_{c9} + \mu_b + \mu_{se}$

$C_{2,2}(10) = \lambda_a + \lambda_b + \lambda_{c10} + \mu_{sd} + \mu_{se}$

$C_{2,2}(11) = \lambda_e + \lambda_{c11} + \mu_a + \mu_b + \mu_d$

$C_{2,2}(12) = \lambda_d + \lambda_{c12} + \mu_a + \mu_b + \mu_e$

$C_{2,2}(13) = \lambda_b + \lambda_{c13} + \mu_a + \mu_d + \mu_e$

$C_{2,2}(14) = \lambda_a + \lambda_{c14} + \mu_b + \mu_d + \mu_e$

$1 - out - of - (2+2) : G$ Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = 0$, the system is repaired upon failure and we are concerned with the system long-run availability. Using the procedure already established for the identical unit cases, the system long-run availability can be obtained in terms of the gamma *pdf* shape parameter β and is given by:

$$A_{ssy}(2,2,1)(\beta) = \frac{(\mu_{15}\,\mu_{16})^{\beta}\,x_{77}}{(\mu_{15}\,\mu_{16})^{\beta}\,x_{78} + \beta\,\mu_{15}^{\beta}\,\mu_{16}^{\beta-1}\,x_{79} + \beta\,\mu_{15}^{\beta-1}\,\mu_{16}^{\beta}\,x_{80}} \tag{3.234}$$

where,

$$\begin{aligned} x_{77} \;\; = \;\; & A_{2795} + \lambda_a\,A_{2793} + \lambda_b\,A_{2791} + \lambda_{sd}\,A_{2789} + \lambda_{se}\,A_{2787} + A_{2671}\,A_{2797} + \\ & A_{2675}\,A_{2799} + A_{2680}\,A_{2801} + A_{2683}\,A_{2803} + A_{2687}\,A_{2805} + A_{2691}\,A_{2807} + \\ & A_{2715}\,A_{2809} + A_{2726b}\,A_{2811} + A_{2736b}\,A_{2813} + A_{2746b}\,A_{2815} \end{aligned}$$

$$x_{78} = A_{2832} - A_{2828} - A_{2830}$$

$$x_{79} = A_{2833} - A_{2829}$$

$$x_{80} = A_{2833} - A_{2831}$$

Long-run availability values obtained with Equation (3.234) are exactly the same as those obtained for the $(2,2,1)$ identical unit case with Type C Repair given in the third column of Table 3.12 if $\lambda_a = \lambda_b = \lambda_d = \lambda_e = \lambda_1$. Symbols A_{2671} to A_{2833} are defined in Appendix E.2.

$1 - out - of - (2+2) : G$ Non-identical Unit System with Type F Repair

With Type F Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = \mu_{15}(x) = \mu_{16}(x) = 0$, the system is not repaired and the indices of interest are the system reliability, the system mean time to failure, and the system variance of time to failure. These are given by the following expressions:

The system reliability is given by:

$$R_{Y_F}(2,2,1)(t) = \sum_{i=0}^{14} P_i(t) \qquad (3.235)$$

where

$$P_0(t) = e^{-C_{2,2}(0)t}$$

$$P_1(t) = BN_{646}\, e^{-C_{2,2}(1)t} + BN_{647}\, e^{-C_{2,2}(0)t}$$

$$P_2(t) = BN_{648}\, e^{-C_{2,2}(2)t} + BN_{649}\, e^{-C_{2,2}(0)t}$$

$$P_3(t) = BN_{650}\, e^{-C_{2,2}(3)t} + BN_{651}\, e^{-C_{2,2}(0)t}$$

$$P_4(t) = BN_{652}\, e^{-C_{2,2}(4)t} + BN_{653}\, e^{-C_{2,2}(0)t}$$

$$P_5(t) = BN_{654}\, e^{-C_{2,2}(5)t} + BN_{655}\, e^{-C_{2,2}(2)t} + BN_{656}\, e^{-C_{2,2}(1)t} + BN_{657}\, e^{-C_{2,2}(0)t}$$

$$P_6(t) = BN_{658}\, e^{-C_{2,2}(6)t} + BN_{659}\, e^{-C_{2,2}(3)t} + BN_{660}\, e^{-C_{2,2}(1)t} + BN_{661}\, e^{-C_{2,2}(0)t}$$

$$P_7(t) = BN_{662}\, e^{-C_{2,2}(7)t} + BN_{663}\, e^{-C_{2,2}(4)t} + BN_{664}\, e^{-C_{2,2}(1)t} +$$

$$BN_{665}\, e^{-C_{2,2}(0)t}$$

$$\begin{aligned}
P_8(t) &= BN_{666}\, e^{-C_{2,2}(8)t} + BN_{667}\, e^{-C_{2,2}(3)t} + BN_{668}\, e^{-C_{2,2}(2)t} + \\
&\quad BN_{669}\, e^{-C_{2,2}(0)t} \\
P_9(t) &= BN_{670}\, e^{-C_{2,2}(9)t} + BN_{671}\, e^{-C_{2,2}(4)t} + BN_{672}\, e^{-C_{2,2}(2)t} + \\
&\quad BN_{673}\, e^{-C_{2,2}(0)t} \\
P_{10}(t) &= BN_{674}\, e^{-C_{2,2}(10)t} + BN_{675}\, e^{-C_{2,2}(4)t} + BN_{676}\, e^{-C_{2,2}(3)t} + \\
&\quad BN_{677}\, e^{-C_{2,2}(0)t} \\
P_{11}(t) &= BN_{678}\, e^{-C_{2,2}(11)t} + BN_{679}\, e^{-C_{2,2}(8)t} + BN_{680}\, e^{-C_{2,2}(6)t} + \\
&\quad BN_{681}\, e^{-C_{2,2}(5)t} + BN_{682}\, e^{-C_{2,2}(3)t} + BN_{683}\, e^{-C_{2,2}(2)t} + \\
&\quad BN_{684}\, e^{-C_{2,2}(1)t} + BN_{685}\, e^{-C_{2,2}(0)t} \\
P_{12}(t) &= BN_{686}\, e^{-C_{2,2}(12)t} + BN_{687}\, e^{-C_{2,2}(9)t} + BN_{688}\, e^{-C_{2,2}(7)t} + \\
&\quad BN_{689}\, e^{-C_{2,2}(5)t} + BN_{690}\, e^{-C_{2,2}(4)t} + BN_{691}\, e^{-C_{2,2}(2)t} + \\
&\quad BN_{692}\, e^{-C_{2,2}(1)t} + BN_{693}\, e^{-C_{2,2}(0)t} \\
P_{13}(t) &= BN_{694}\, e^{-C_{2,2}(13)t} + BN_{695}\, e^{-C_{2,2}(10)t} + BN_{696}\, e^{-C_{2,2}(7)t} + \\
&\quad BN_{697}\, e^{-C_{2,2}(6)t} + BN_{698}\, e^{-C_{2,2}(4)t} + BN_{699}\, e^{-C_{2,2}(3)t} + \\
&\quad BN_{700}\, e^{-C_{2,2}(1)t} + BN_{701}\, e^{-C_{2,2}(0)t} \\
P_{14}(t) &= BN_{702}\, e^{-C_{2,2}(14)t} + BN_{703}\, e^{-C_{2,2}(10)t} + BN_{704}\, e^{-C_{2,2}(9)t} + \\
&\quad BN_{705}\, e^{-C_{2,2}(8)t} + BN_{706}\, e^{-C_{2,2}(4)t} + BN_{707}\, e^{-C_{2,2}(3)t} + \\
&\quad BN_{708}\, e^{-C_{2,2}(2)t} + BN_{709}\, e^{-C_{2,2}(0)t}
\end{aligned}$$

The partial fraction residues BN_{646} to BN_{709} are defined in Appendix E.2. For $\lambda_a = \lambda_b = \lambda_d = \lambda_e = \lambda_1$, Equation (3.235) generates system reliability values that match those of the $(2,2,1)$ identical unit case with Type F Repair discussed in Subsection 3.1.2.

The mean time to failure of this system is given by:

$$MTTF_{Y_F}(2,2,1) = \lim_{s \to 0} \sum_{i=0}^{14} P_i(s) \tag{3.236}$$

where,

$P_0(0) = 1/C_{2,2}(0)$

$P_1(0) = (\lambda_a/C_{2,2}(1)) \bullet P_0(0)$

$P_2(0) = (\lambda_b/C_{2,2}(2)) \bullet P_0(0)$

$P_3(0) = (\lambda_{sd}/C_{2,2}(3)) \bullet P_0(0)$

$P_4(0) = (\lambda_{se}/C_{2,2}(4)) \bullet P_0(0)$

$P_5(0) = (\lambda_b/C_{2,2}(5)) \bullet P_1(0) + (\lambda_a/C_{2,2}(5)) \bullet P_2(0)$

$P_6(0) = (\lambda_d/C_{2,2}(6)) \bullet P_1(0) + (\lambda_a/C_{2,2}(6)) \bullet P_3(0)$

$P_7(0) = (\lambda_{se}/C_{2,2}(7)) \bullet P_1(0) + (\lambda_a/C_{2,2}(7)) \bullet P_4(0)$

$P_8(0) = (\lambda_d/C_{2,2}(8)) \bullet P_2(0) + (\lambda_b/C_{2,2}(8)) \bullet P_3(0)$

$P_9(0) = (\lambda_{se}/C_{2,2}(9)) \bullet P_2(0) + (\lambda_b/C_{2,2}(9)) \bullet P_4(0)$

$P_{10}(0) = (\lambda_{se}/C_{2,2}(10)) \bullet P_3(0) + (\lambda_{sd}/C_{2,2}(10)) \bullet P_4(0)$

$P_{11}(0) = (\lambda_d/C_{2,2}(11)) \bullet P_5(0) + (\lambda_b/C_{2,2}(11)) \bullet P_6(0) + (\lambda_a/C_{2,2}(11)) \bullet P_8(0)$

$P_{12}(0) = (\lambda_e/C_{2,2}(12)) \bullet P_5(0) + (\lambda_b/C_{2,2}(12)) \bullet P_7(0) + (\lambda_a/C_{2,2}(12)) \bullet P_9(0)$

$P_{13}(0) = (\lambda_e/C_{2,2}(13)) \bullet P_6(0) + (\lambda_d/C_{2,2}(13)) \bullet P_7(0) + (\lambda_a/C_{2,2}(13)) \bullet P_{10}(0)$

$P_{14}(0) = (\lambda_e/C_{2,2}(14)) \bullet P_8(0) + (\lambda_d/C_{2,2}(14)) \bullet P_9(0) + (\lambda_b/C_{2,2}(14)) \bullet P_{10}(0)$

The variance of time to failure for this system is given by:

$$\sigma^2_{Y_F}(2,2,1) \;=\; \frac{1}{k^2_{74}}\left[2(k_{73}k_{75} - k_{74}k_{76}) - k^2_{73}\right] \tag{3.237}$$

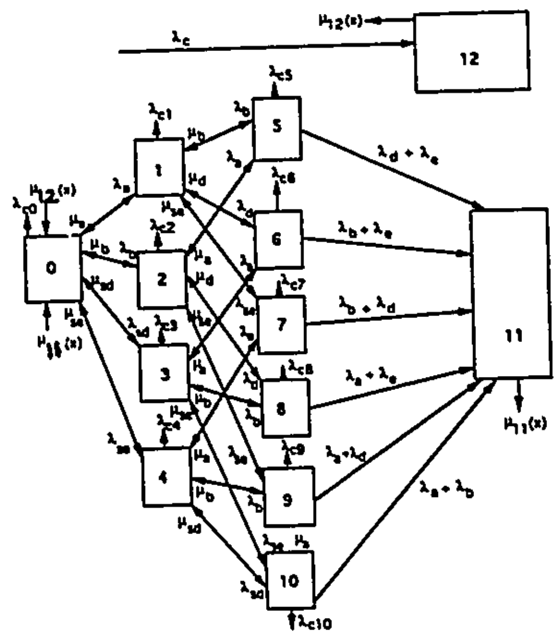
Symbols k_{73} to k_{76} are provided in Appendix E.2.

3.2.4 2-out-of-(2+2):G Non-identical Unit System

This is the $(2,2,2)$ system and is composed of *two* non-identical units and *two* non-identical standbys and is operable as long as at least *two* units are functional. The state space diagram for this system is shown in Figure 3.31. The numerals in the boxes of Figure 3.31 denote the system state numbers.

<u>Notation</u>

The following symbols are associated with a $(2,2,2)$ non-identical unit system:



Where,

$\lambda_c = \lambda_{c0} + \lambda_{c1} + \lambda_{c2} + \lambda_{c3} + \lambda_{c4} + \lambda_{c5} + \lambda_{c6} + \lambda_{c7} + \lambda_{c8} + \lambda_{c9} + \lambda_{c10}$

Figure 3.31: State Space Diagram of the $2-out-of-(2+2):G$ Non-identical Unit System under SBAP Y

i	State identification number. For $i = 0$, both units (A and B) and both standby units, D and E are in perfect working condition. For $i = 1$, unit A has failed, unit B is working and standby D has been activated. For $i = 2$, unit B has failed, unit A is working and standby D has been activated. For $i = 3$, both units A and B are working perfectly but standby D has failed in its standby mode. For $i = 4$, both units (A and B) are working but standby unit E has failed in its standby mode. For $i = 5$, both A and B have failed and both D and E have been activated. For $i = 6$, units A and D have failed while units B and E are still functional. For $i = 7$, units A and E have failed while units B and D are working. For $i = 8$, units B and D have failed but units A and E are still working. For $i = 9$, units B and E have failed but units A and D are still functional. For $i = 10$, units D and E have failed while units A and B are still operational. For $i = 11$, the system has failed due to independent failures of its units. For $i = 12$, the system has failed due to the common-cause failures of its units.
t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated
λ_e	Constant failure rate of standby unit E when activated
λ_{sd}	Constant failure rate of standby unit D in the standby mode

λ_{se}	Constant failure rate of standby unit E in the standby mode
λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, ..., 10$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_d	Constant repair rate of unit D
μ_e	Constant repair rate of unit E
μ_{sd}	Constant repair rate of standby unit D if it fails in the standby mode
μ_{se}	Constant repair rate of standby unit E if it fails in the standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, ..., 12$
$P_i(x,t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 11, 12$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time respectively when the system is in state i and has an elapsed repair time of x; for $i = 11, 12$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The system of integro-differential equations associated with the system described in Figure 3.31 is:

$$\frac{dP_0(t)}{dt} + C_{2,2}(0)\, P_0(t) = \mu_a\, P_1(t) + \mu_b\, P_2(t) + \mu_{sd}\, P_3(t) + \mu_{se}\, P_4(t) + \sum_{i=11}^{12} \int_0^{\infty} P_i(x,t)\, \mu_i(x)\, dx \tag{3.238}$$

$$\frac{dP_1(t)}{dt} + C_{2,2}(1)\, P_1(t) = \mu_b\, P_5(t) + \mu_d\, P_6(t) + \lambda_a\, P_0(t) + \mu_{se}\, P_7(t) \tag{3.239}$$

$$\frac{dP_2(t)}{dt} + C_{2,2}(2)\, P_2(t) = \mu_a\, P_5(t) + \mu_d\, P_8(t) + \lambda_b\, P_0(t) + \mu_{se}\, P_9(t) \tag{3.240}$$

$$\frac{dP_3(t)}{dt} + C_{2,2}(3)\, P_3(t) = \mu_a\, P_6(t) + \mu_b\, P_8(t) + \lambda_{sd}\, P_0(t) + \mu_{se}\, P_{10}(t) \tag{3.241}$$

$$\frac{dP_4(t)}{dt} + C_{2,2}(4)\, P_4(t) = \mu_a\, P_7(t) + \mu_b\, P_9(t) + \lambda_{se}\, P_0(t) + \mu_{sd}\, P_{10}(t) \tag{3.242}$$

$$\frac{dP_5(t)}{dt} + C_{2,2}(5)\, P_5(t) = \lambda_b\, P_1(t) + \lambda_a\, P_2(t) \tag{3.243}$$

$$\frac{dP_6(t)}{dt} + C_{2,2}(6)\, P_6(t) = \lambda_d\, P_1(t) + \lambda_a\, P_3(t) \tag{3.244}$$

$$\frac{dP_7(t)}{dt} + C_{2,2}(7)\, P_7(t) = \lambda_{se}\, P_1(t) + \lambda_a\, P_4(t) \tag{3.245}$$

$$\frac{dP_8(t)}{dt} + C_{2,2}(8)\, P_8(t) = \lambda_d\, P_2(t) + \lambda_b\, P_3(t) \tag{3.246}$$

$$\frac{dP_9(t)}{dt} + C_{2,2}(9)\, P_9(t) = \lambda_{se}\, P_2(t) + \lambda_b\, P_4(t) \tag{3.247}$$

$$\frac{dP_{10}(t)}{dt} + C_{2,2}(10)\, P_{10}(t) = \lambda_{se}\, P_3(t) + \lambda_{sd}\, P_4(t) \tag{3.248}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{11}(x)]\, P_{11}(x,t) = 0 \tag{3.249}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{12}(x)]\, P_{12}(x,t) = 0 \tag{3.250}$$

With the following boundary conditions:

$$\begin{aligned} P_{11}(0,t) \;\; = \;\; & [\lambda_d + \lambda_e]\, P_5(t) + [\lambda_b + \lambda_e]\, P_6(t) + [\lambda_b + \lambda_d]\, P_7(t) + \\ & [\lambda_a + \lambda_e]\, P_8(t) + [\lambda_a + \lambda_d]\, P_9(t) + [\lambda_a + \lambda_b]\, P_{10}(t) \end{aligned} \tag{3.251}$$

$$P_{12}(0,t) = \sum_{i=0}^{10} \lambda_{ci}\, P_i(t) \tag{3.252}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where,

$C_{2,2}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd} + \lambda_{se}$

$C_{2,2}(1) = \lambda_b + \lambda_d + \lambda_{c1} + \lambda_{se} + \mu_a$

$C_{2,2}(2) = \lambda_a + \lambda_d + \lambda_{c2} + \lambda_{se} + \mu_b$

$C_{2,2}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \lambda_{se} + \mu_{sd}$

$C_{2,2}(4) = \lambda_a + \lambda_b + \lambda_{sd} + \lambda_{c4} + \mu_{se}$

$C_{2,2}(5) = \lambda_d + \lambda_e + \lambda_{c5} + \mu_a + \mu_b$

$C_{2,2}(6) = \lambda_b + \lambda_e + \lambda_{c6} + \mu_a + \mu_d$

$C_{2,2}(7) = \lambda_b + \lambda_d + \mu_{se} + \lambda_{c7} + \mu_a$

$C_{2,2}(8) = \lambda_a + \lambda_e + \lambda_{c8} + \mu_b + \mu_d$

$C_{2,2}(9) = \lambda_a + \lambda_d + \lambda_{c9} + \mu_b + \mu_{se}$

$C_{2,2}(10) = \lambda_a + \lambda_b + \lambda_{c10} + \mu_{sd} + \mu_{se}$

$2 - out - of - (2+2) : G$ Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = 0$, the system is repaired upon failure and we are concerned with the system long-run availability. These values can be obtained by eliminating the constants associated with *system up-states 11, 12, 13* and *14* from the results determined for the $(2,2,1)$ non-identical system with Type C Repair.

$2 - out - of - (2+2) : G$ Non-identical Unit System with Type F Repair

With Type F Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = \mu_{11}(x) = \mu_{12}(x) = 0$, the system is not repaired upon failure and the indices of interest are the system reliability, the system mean time to failure, and the system variance of time to failure. These are given by the following expressions:

The system reliability is given by:

$$R_{Y_F}(2,2,2)(t) = \sum_{i=0}^{10} P_i(t) \tag{3.253}$$

where,

$P_0(t) = e^{-C_{2,2}(0)t}$

$P_1(t) = BN_{646}\, e^{-C_{2,2}(1)t} + BN_{647}\, e^{-C_{2,2}(0)t}$

$P_2(t) = BN_{648}\, e^{-C_{2,2}(2)t} + BN_{649}\, e^{-C_{2,2}(0)t}$

$P_3(t) = BN_{650}\, e^{-C_{2,2}(3)t} + BN_{651}\, e^{-C_{2,2}(0)t}$

$P_4(t) = BN_{652}\, e^{-C_{2,2}(4)t} + BN_{653}\, e^{-C_{2,2}(0)t}$
$P_5(t) = BN_{654}\, e^{-C_{2,2}(5)t} + BN_{655}\, e^{-C_{2,2}(2)t} + BN_{656}\, e^{-C_{2,2}(1)t} + BN_{657}\, e^{-C_{2,2}(0)t}$
$P_6(t) = BN_{658}\, e^{-C_{2,2}(6)t} + BN_{659}\, e^{-C_{2,2}(3)t} + BN_{660}\, e^{-C_{2,2}(1)t} + BN_{661}\, e^{-C_{2,2}(0)t}$
$P_7(t) = BN_{662}\, e^{-C_{2,2}(7)t} + BN_{663}\, e^{-C_{2,2}(4)t} + BN_{664}\, e^{-C_{2,2}(1)t} + BN_{665}\, e^{-C_{2,2}(0)t}$
$P_8(t) = BN_{666}\, e^{-C_{2,2}(8)t} + BN_{667}\, e^{-C_{2,2}(3)t} + BN_{668}\, e^{-C_{2,2}(2)t} + BN_{669}\, e^{-C_{2,2}(0)t}$
$P_9(t) = BN_{670}\, e^{-C_{2,2}(9)t} + BN_{671}\, e^{-C_{2,2}(4)t} + BN_{672}\, e^{-C_{2,2}(2)t} + BN_{673}\, e^{-C_{2,2}(0)t}$
$P_{10}(t) = BN_{674}\, e^{-C_{2,2}(10)t} + BN_{675}\, e^{-C_{2,2}(4)t} + BN_{676}\, e^{-C_{2,2}(3)t} + BN_{677}\, e^{-C_{2,2}(0)t}$
The partial fraction residues BN_{646} to BN_{677} are as defined for Equation (3.235).

The mean time to failure of this system can easily be determined from the results obtained for the $(2, 2, 1)$ non-identical case with Type F Repair by eliminating constants associated with *system up-states 11* to *14* and can be expressed as:

$$MTTF_{Y_F}(2,2,2) \;=\; \lim_{s \to 0} \sum_{i=0}^{10} P_i(s) \qquad (3.254)$$

where,
$P_0(0) = 1/C_{2,2}(0)$
$P_1(0) = (\lambda_a/C_{2,2}(1)) \bullet P_0(0)$
$P_2(0) = (\lambda_b/C_{2,2}(2)) \bullet P_0(0)$
$P_3(0) = (\lambda_{sd}/C_{2,2}(3)) \bullet P_0(0)$
$P_4(0) = (\lambda_{se}/C_{2,2}(4)) \bullet P_0(0)$
$P_5(0) = (\lambda_b/C_{2,2}(5)) \bullet P_1(0) + (\lambda_a/C_{2,2}(5)) \bullet P_2(0)$
$P_6(0) = (\lambda_d/C_{2,2}(6)) \bullet P_1(0) + (\lambda_a/C_{2,2}(6)) \bullet P_3(0)$
$P_7(0) = (\lambda_{se}/C_{2,2}(7)) \bullet P_1(0) + (\lambda_a/C_{2,2}(7)) \bullet P_4(0)$
$P_8(0) = (\lambda_d/C_{2,2}(8)) \bullet P_2(0) + (\lambda_b/C_{2,2}(8)) \bullet P_3(0)$
$P_9(0) = (\lambda_{se}/C_{2,2}(9)) \bullet P_2(0) + (\lambda_b/C_{2,2}(9)) \bullet P_4(0)$
$P_{10}(0) = (\lambda_{se}/C_{2,2}(10)) \bullet P_3(0) + (\lambda_{sd}/C_{2,2}(10)) \bullet P_4(0)$

The variance of time to failure for this system can be obtained directly from Equation (3.237) by eliminating system up-states *11, 12, 13 and 14* and their associated constants.

3.3 Concluding Remarks

The analyses performed in this Chapter indicate that the values of $k - out - of - (n + m) : G$ system performance indices under SBAP Y increase with any increase in either the number of active units or the number of standbys. Thus, as the system complexity increases, its performance is also expected to increase but so will the running costs. The costs aspect will be discussed in Chapter 5. An observation of the tabular values and plots shows clearly that the addition of one standby unit for a fixed number of active units improves the values of the system performance indices significantly regardless of the Repair Type.

For all the special cases considered in this Chapter, the system long-run availability drops as the failed system repair time distribution shape parameter β increases. As will be shown later in Chapter 6, an increase in β usually results in a decrease in the instantaneous repair rate of the system and therefore such a trend should be expected. In fact, the failed system instantaneous repair rate is highest (also a constant value) when failed system repair times are exponentially distributed (i.e. $\beta = 1$). Therefore it is hardly surprising that exponentially distributed failed system repair times yielded the highest values for the long-run availability for every special case considered as the tabular values indicate.

Again, a close observation of the tabular values in this Chapter also reveal that in some situations unit repair may not be prudent. Repair Type A is expected to yield higher long-run availability values than Repair Type B, and B in turn, is supposed to produce higher long-run availability values than C for each special case considered. The tabular values and plots followed this expectation but in some situations the incremental benefit of repairing either the standby unit or the active unit if they fail may not justify perhaps the extra repair costs. In other words, the differences in the long-run availability of the system under these three Repair Types may not be significant enough to justify unit repair. The same could be said of the system performance under Repair Types D, E and F. Under these Repair Types, the system itself is not repaired but if there are no significant differences in the values of performance indices such as the mean time to failure and reliability for these three Repair types, unit repair may not be justifiable.

The non-identical unit systems analyzed in Section 3.2 yielded results that match those of corresponding identical unit cases if all system components are assumed to have the same failure rates. Thus the $(2,1,1)$ identical unit system with Type E Repair mean time to failure expression (Equation 3.72) is the same as the $(2,1,1)$ non-identical unit system with Type E Repair mean time to failure expression (Equation 3.203). A comparison of Equations (3.104) and Equation (3.236) indicate that this is equally true for the $(2,2,1)$ identical unit system with Type E Repair of Subsection 3.1.2 and the $(2,2,1)$ non-identical unit system with Type E Repair of Subsection 3.2.3.

All in all, the main contribution of this Chapter is the derivation of generalized mean time to failure expressions for $k-out-of-(n+m):G$ type systems operating under standby activation policy Y. These derivations were performed for several possible repair scenarios. The system units were considered identical and were subject to a combination of independent and common-cause failures. The use of a large number of special cases was necessary in order to show the validity of the various formulas derived.

Finally, it is important to note that the component failure rates considered in this Study are inherent - a by-product of design. In practice, the values of the basic system parameters are estimated from field data (or historical data) and some errors may be introduced as a result. This fact will only affect the way in which numerical results are interpreted and the degree of confidence that will be reposed in them. It will not affect any of the analyses or expressions developed in the Study.

Chapter 4

The Redundant System With Late Standby Activation: Special Cases

The Special Cases and Types of Repair considered in this Chapter have been selected mainly to aid the comparison between the Y and the Z standby activation policies using the system mean time to failure as the main measure of performance. In order to make this comparison meaningful, the same values have been specified for the basic system parameters for both General Models (these values are stated in Chapter 3). In addition to this comparison, some newly developed formulas for the system mean time to failure, variance of time to failure, availability and reliability of $k - out - of - (n + m) : G$ type redundant systems designed to run on SBAP Z for cases where system units are identical as well as non-identical are presented.

It is also important to note that the special cases that will be presented in this Chapter are those for which $k \neq n$. If $k = n$, the standby unit(s) must be activated without delay otherwise the whole system would fail. Thus standby activation policy Z would be impracticable when $k = n$. Nonetheless, the results of the special cases when $k = n$ would be added to the tables for purposes of completeness.

4.1 The Identical Unit System With Late Standby Activation: Special Cases

As in the previous chapter, it is convenient to group these special cases into the following $k - out - of - (n + m) : G$ submodels viz: *(i)* $1 - out - of - (n + 1) : G$ (or $n, 1, 1$) *(ii)* $1 - out - of - (n + 2) : G$ (or $n, 2, 1$) *(iii)* $1 - out - of - (n + 3) : G$ (or $n, 3, 1$) *(iv)* $k - out - of - (n + 1) : G$ (or $n, 1, k$) *(v)* $k - out - of - (n + 2) : G$ (or $n, 2, k$) and *(vi)* $k - out - of - (n + 3) : G$ (or $n, 3, k$).

4.1.1 1-out-of-(n+1):G Identical Unit Submodel

This is the $(n, 1, 1)$ system and is composed of n active units and *one* standby. It is functional as long as at least *one* unit is healthy. The block diagram of this system is shown in Figure 3.1 and the state space diagram in Figure 4.1.

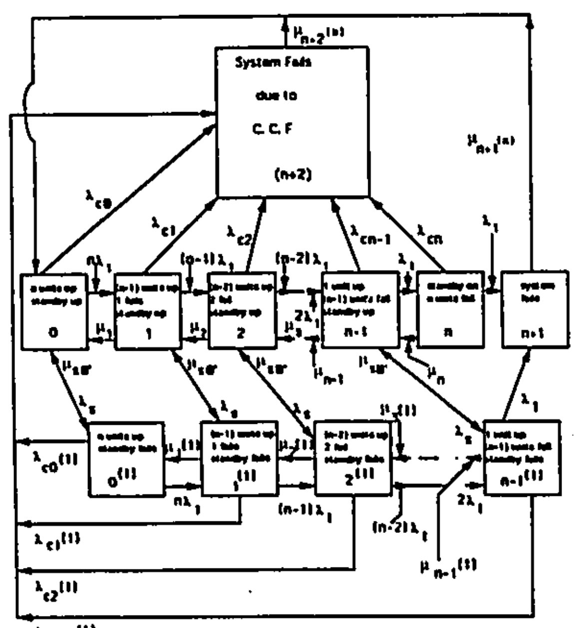


Figure 4.1: State space diagram of the $1 - out - of - (n+1) : G$ System under SBAP Z

$\underline{1 - out - of - (n+1) : G}$ **System Equations**

The descriptive integro-differential Equations for the system shown in Figure 4.1 are:

$$\frac{dP_0(t)}{dt} + C_{n,1}(0)\,P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\,P_{0^{(1)}}(t) + \sum_{i=n+1}^{n+2} \int_0^\infty P_i(x,t)\,\mu_i(x)dx \quad (4.1)$$

For $j = 1, 2, \ldots.., n-1$,

$$\frac{dP_j(t)}{dt} + C_{n,1}(j)\,P_j(t) = (n-j+1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) + \mu_{j+1} P_{j+1}(t) \quad (4.2)$$

$$\frac{dP_n(t)}{dt} + C_{n,1}(n)\,P_n(t) = \lambda_1\,P_{n-1}(t) \quad (4.3)$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,1}(0^{(1)})\,P_{0^{(1)}}(t) = \lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) \quad (4.4)$$

For $j = 1, 2, \ldots.., n-2$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,1}(j)\,P_j(t) = \lambda_s P_j(t) + (n-j+1)\,\lambda_1\,P_{j-1^{(1)}}(t) + \mu_{j+1^{(1)}}\,P_{j+1^{(1)}}(t) \quad (4.5)$$

$$\frac{dP_{n-1^{(1)}}(t)}{dt} + C_{n,1}(n-1^{(1)})\,P_{n-1^{(1)}}(t) = \lambda_s\,P_{n-1}(t) + 2\lambda_1 P_{n-2^{(1)}}(t) \quad (4.6)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+1}(x)]\,P_{n+1}(x,t) = 0 \quad (4.7)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+2}(x)]\,P_{n+2}(x,t) = 0 \quad (4.8)$$

The associated boundary conditions are:

$$P_{n+1}(0,t) = \lambda_1\,P_n(t) + \lambda_1\,P_{n-1^{(1)}}(t) \quad (4.9)$$

$$P_{n+2}(0,t) = \sum_{j=0}^{n} \lambda_{cj}\,P_j(t) + \sum_{j=0^{(1)}}^{n-1^{(1)}} \lambda_{cj} P_j(t) \quad (4.10)$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where,

$C_{n,1}(0) = n\,\lambda_1 + \lambda_s + \lambda_{c0}$

For $j = 1, 2, \ldots..n-1$,

$C_{n,1}(j) = (n-j)\,\lambda_1 + \lambda_{cj} + \mu_j + \lambda_s$

$C_{n,1}(n) = \lambda_1 + \lambda_{cn} + \mu_n$

$C_{n,1}(0^{(1)}) = n\,\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$

For $j = 1, 2, \ldots\ldots, n-2$,

$C_{n,1}(j^{(1)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s0'}$

$C_{n,1}(n-1^{(1)}) = \lambda_1 + \lambda_{cn-1^{(1)}} + \mu_{n-1^{(1)}} + \mu_{s0'}$

$1 - out - of - (n+1) : G$ System With Type B Repair

With Type B Repair, the active units in the system are discarded upon failure thus μ_1 to μ_n and $\mu_{1^{(1)}}$ to $\mu_{n-1^{(1)}}$ are set to zero in Equations (4.1) to (4.10).

Special Case Model I

For $n = 2$ with Type B Repair, using the procedure already established in the last Chapter, the long-run availability for this special case is given by:

$$A_{ssz}(2,1,1)\,(\beta) = \frac{(\mu_3\,\mu_4)^\beta\, x_{37}}{(\mu_3\,\mu_4)^\beta\, x_{38} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\, x_{39} + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\, x_{40}} \tag{4.11}$$

where,

$x_{37} = A_{1972} + A_{1982} + A_{1984} + A_{1986} + A_{1988}$

$x_{38} = A_{1977} - A_{1979} - A_{1989}$

$x_{39} = A_{1978} - A_{1980}$

$x_{40} = A_{1978} - A_{1990}$

Long-run availability plots and corresponding values generated with Equation (4.11) are given in Figure 4.2 and in the third column of Table 4.1 respectively. The symbols A_{1972} to A_{1990} are defined in Appendix F. The trends shown by these values and what they represent would be discussed in the concluding section of this Chapter.

Special Case Model II

For $n = 3$ with Type B Repair, the long-run availability expression is:

$$A_{ssz}(3,1,1)\,(\beta) = \frac{(\mu_4\,\mu_5)^\beta\, x_{41}}{(\mu_4\,\mu_5)^\beta\, x_{42} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{43} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{44}} \tag{4.12}$$

where,

Table 4.1: $A_{ssz}(n,1,1)\,(\beta)$ values as a function of λ_{c0} With Type B Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4139	0.4527	0.4963
	0.00002	0.4111	0.4507	0.4945
	0.00003	0.4083	0.4487	0.4928
	0.00004	0.4055	0.4467	0.4910
$\beta=2$	0.00001	0.2610	0.2955	0.3310
	0.00002	0.2587	0.2938	0.3295
	0.00003	0.2565	0.2921	0.3279
	0.00004	0.2543	0.2904	0.3263
$\beta=3$	0.00001	0.1905	0.2194	0.2484
	0.00002	0.1888	0.2180	0.2470
	0.00003	0.1870	0.2165	0.2457
	0.00004	0.1852	0.2151	0.2444
$\beta=4$	0.00001	0.1501	0.1744	0.1987
	0.00002	0.1486	0.1732	0.1976
	0.00003	0.1471	0.1720	0.1965
	0.00004	0.1457	0.1708	0.1954
$\beta=5$	0.00001	0.1238	0.1448	0.1656
	0.00002	0.1225	0.1437	0.1646
	0.00003	0.1213	0.1427	0.1637
	0.00004	0.1200	0.1417	0.1627

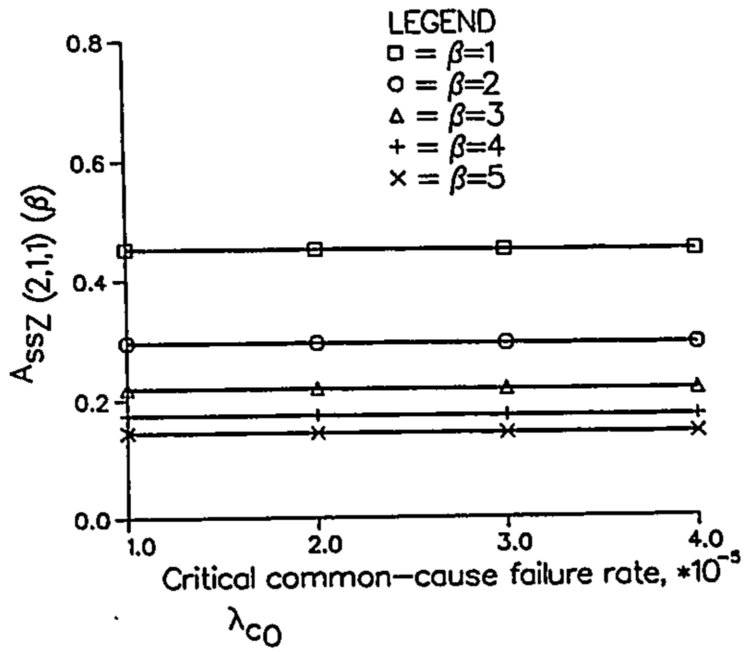


Figure 4.2: $A_{ssz}(2,1,1)(\beta)$ Vs. λ_{c0} Plots for Type B Repair

$$x_{41} = A_{2002} + A_{2008} + A_{2016} + A_{2022} + A_{2028} + A_{2034} + A_{2038}$$

$$x_{42} = A_{2039} - A_{2041} - A_{2043}$$

$$x_{43} = A_{2040} - A_{2042}$$

$$x_{44} = A_{2040} - A_{2044}$$

Long-run availability values generated with Equation (4.12) are given in the fourth column of Table 4.1 and associated plots in Figure 4.3. The symbols A_{2002} to A_{2044} are shown in Appendix F.

$1 - out - of - (n+1) : G$ System With Type C Repair

With Type C Repair, the units in the system (both active and standbys) are discarded upon failure thus $\mu_{s0'}$, μ_1 to μ_n and $\mu_{1(1)}$ to $\mu_{n-1(1)}$ are set to zero in Equations (4.1) to (4.10).

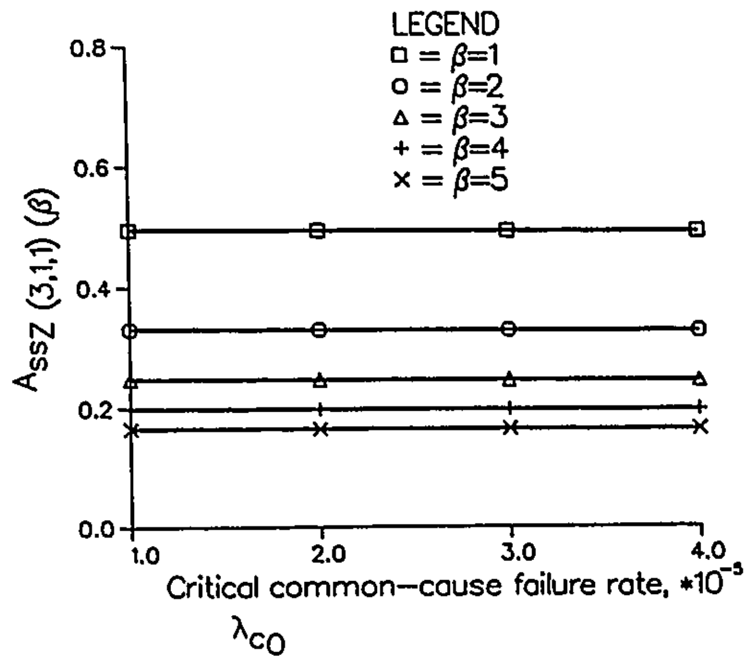


Figure 4.3: $A_{ssz}(3,1,1)(\beta)$ Vs. λ_{c0} Plots for Type B Repair

Special Case Model I

For $n = 2$ with Type C Repair, we obtain the long-run availability values for this special case by setting $\mu_{s0'} = 0$ in (4.11). The results obtained are given in the third column of Table 4.2 and associated plots in Figure 4.4.

Special Case Model II

For $n = 3$ with Type C Repair, we obtain the long-run availability values for this special case by setting $\mu_{s0'} = 0$ in Equation (4.12). The results obtained for this Repair Type are presented in fourth column of Table 4.2 and associated plots in Figure 4.5.

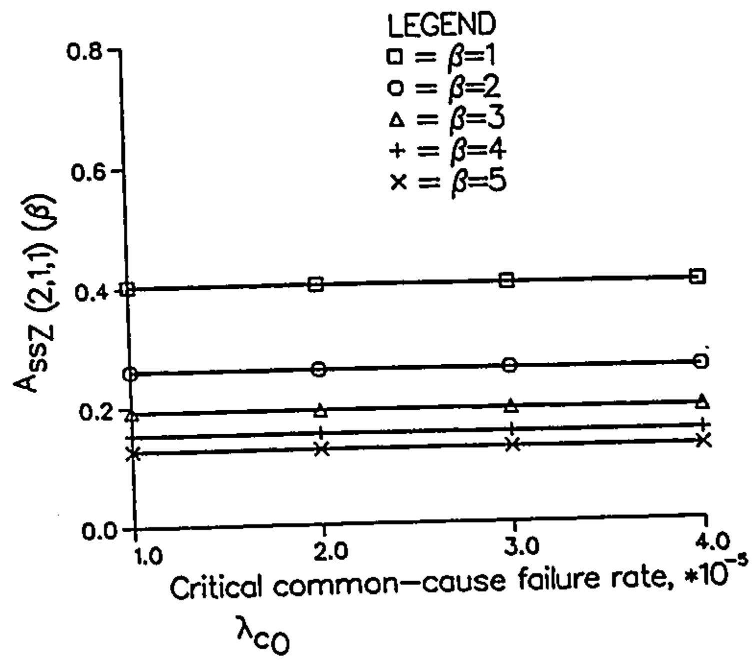


Figure 4.4: $A_{ssz}(2,1,1)\,(\beta)$ Vs. λ_{c0} Plots for Type C Repair

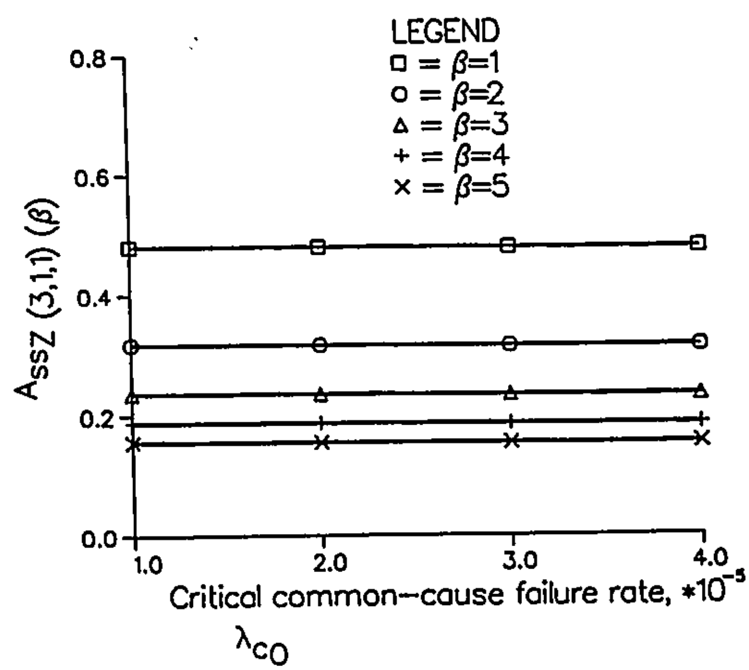


Figure 4.5: $A_{ssz}(3,1,1)\,(\beta)$ Vs. λ_{c0} Plots With Type C Repair

Table 4.2: $A_{ssz}(n,1,1)\,(\beta)$ values as a function of λ_{c0} For Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4002	0.4197	0.4811
	0.00002	0.3977	0.4179	0.4795
	0.00003	0.3952	0.4161	0.4778
	0.00004	0.3926	0.4144	0.4762
$\beta=2$	0.00001	0.2502	0.2656	0.3168
	0.00002	0.2482	0.2641	0.3153
	0.00003	0.2462	0.2627	0.3139
	0.00004	0.2443	0.2613	0.3125
$\beta=3$	0.00001	0.1820	0.1942	0.2361
	0.00002	0.1804	0.1931	0.2349
	0.00003	0.1788	0.1920	0.2337
	0.00004	0.1773	0.1908	0.2325
$\beta=4$	0.00001	0.1430	0.1531	0.1882
	0.00002	0.1417	0.1522	0.1872
	0.00003	0.1404	0.1512	0.1862
	0.00004	0.1391	0.1503	0.1852
$\beta=5$	0.00001	0.1178	0.1264	0.1564
	0.00002	0.1167	0.1256	0.1556
	0.00003	0.1156	0.1248	0.1547
	0.00004	0.1145	0.1240	0.1538

$\underline{1 - out - of - (n+1) : G \text{ System With Type } E \text{ Repair}}$

With Type E Repair, the system as well as its standby units are replaced upon failure thus $\mu_{n+1}(x)$, $\mu_{n+2}(x)$, μ_1 to μ_n and $\mu_{1^{(1)}}$ to $\mu_{n-1^{(1)}}$ are set to zero in Equations (4.1) to (4.10).

Special Case Model I

For $n = 2$ with Type E Repair, using the already established procedure, the relevant system reliability indices are given by the following expressions:

The system reliability is given by:

$$\begin{aligned} R_{Z_E}(2,1,1)(t) &= \sum_{i=0}^{2} P_i(t) + \sum_{i=0^{(1)}}^{1^{(1)}} P_i(t) \\ &= e^{s_{70}t}[B_{551} + B_{552} + B_{553} + B_{554} + B_{555}] + \\ &\quad e^{s_{71}t}[B_{556} + B_{557} + B_{558} + B_{559} + B_{560}] + \\ &\quad e^{s_{72}t}[B_{561} + B_{562} + B_{563}] + e^{s_{73}t}[B_{564} + \\ &\quad B_{565} + B_{566}] + e^{-C_{2,1}(2)t}[B_{567}] \end{aligned} \tag{4.13}$$

$W_{90}(s) = [(s + C_{2,1}(1))(s + C_{2,1}(1^{(1)})) - \lambda_s\,\mu_{s0'}]\,[(s + C_{2,1}(0))(s + C_{2,1}(0^{(1)})) - \lambda_s\,\mu_{s0'}]$
where s_{70} to s_{73} are the roots of the characteristic polynomial function $W_{90}(s)$. The residues B_{551} to B_{567} are provided in Appendix F. The system reliability values obtained with Equation (4.13) are given in the third column of Table 4.3 and associated plots in Figure 4.6.

The mean time to failure is given by:

$$MTTF_{Z_E}(2,1,1) = \sum_{i=0}^{2} P_i(0) + \sum_{i=0^{(1)}}^{1^{(1)}} P_i(0) \tag{4.14}$$

where,

$P_0(0) = C_{2,1}(0^{(1)})/(C_{2,1}(0)\,C_{2,1}(0^{(1)}) - \lambda_s\,\mu_{s0'})$

$P_{0^{(1)}}(0) = \lambda_s/(C_{2,1}(0)\,C_{2,1}(0^{(1)}) - \lambda_s\,\mu_{s0'})$

$P_1(0) = (2\lambda_1/(C_{2,1}(1)\,C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'})) \bullet P_0(0) + (2\lambda_1\mu_{s0'}/(C_{2,1}(1)\,C_{2,1}(1^{(1)}) - \lambda_s\mu_{s0'})) \bullet P_{0^{(1)}}(0)$

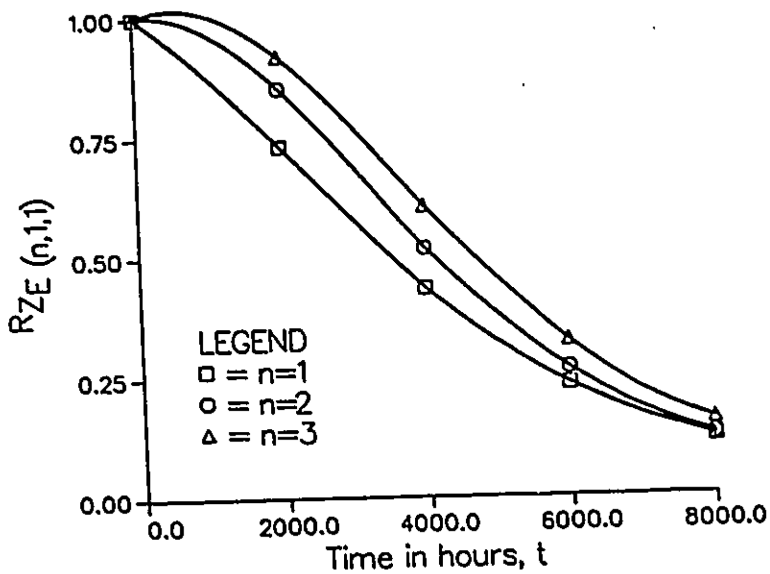


Figure 4.6: $R_{Z_E}(n,1,1)$ Plots ($\lambda_{c0} = 0.00004$ per hr.)

Table 4.3: $R_{Z_E}(n,1,1)$ values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.7317	0.8530	0.9191
4000	0.4325	0.5164	0.6049
6000	0.2344	0.2664	0.3240
8000	0.1209	0.1271	0.1570
∞	0	0	0

$P_{1^{(1)}}(0) = (2\lambda_1/C_{2,1}(1^{(1)})) \bullet P_{0^{(1)}}(0) + (\lambda_s/C_{2,1}(1^{(1)})) \bullet P_1(0)$

$P_2(0) = (\lambda_1/C_{2,1}(2)) \bullet P_1(0)$

and,

$C_{2,1}(0) = 2\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{2,1}(1) = \lambda_1 + \lambda_s + \lambda_{c1}$

$C_{2,1}(2) = \lambda_1 + \lambda_{c2}$

$C_{2,1}(0^{(1)}) = 2\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$

$C_{2,1}(1^{(1)}) = \lambda_1 + \lambda_{c1^{(1)}} + \mu_{s0'}$

The variance of time to failure is given by:

$$\begin{aligned} \sigma^2_{Z_E}(2,1,1) &= -2 \lim_{s \to 0} R'_{Z_E}(2,1,1)(s) - [MTTF_{Z_E}(2,1,1)]^2 \\ &= \frac{1}{k_{38}^2} [2(k_{37}\, k_{39} - k_{38}\, k_{40}) - k_{37}^2] \end{aligned} \tag{4.15}$$

Symbols k_{37} to k_{40} are defined in Appendix F.

Special Case Model II

For $n = 3$ with Type E Repair, the relevant system reliability indices are:

The system reliability is given by:

$$\begin{aligned} R_{Z_E}(3,1,1)(t) &= \sum_{i=0}^{3} P_i(t) + \sum_{i=0^{(1)}}^{2^{(1)}} P_i(t) \\ &= e^{s_{74}t}[B_{585} + B_{586} + B_{587} + B_{588} + B_{589} + B_{590} + B_{591}] + \\ &\quad e^{s_{75}t}[B_{592} + B_{593} + B_{594} + B_{595} + B_{596} + B_{597} + B_{598}] + \\ &\quad e^{s_{76}t}[B_{599} + B_{600} + B_{601} + B_{602} + B_{603}] + \\ &\quad e^{s_{77}t}[B_{604} + B_{605} + B_{606} + B_{607} + B_{608}] + \\ &\quad e^{s_{78}t}[B_{609} + B_{610} + B_{611}] + \\ &\quad e^{s_{79}t}[B_{612} + B_{613} + B_{614}] + \\ &\quad e^{-C_{3,1}(3)t}[B_{615}] \end{aligned} \tag{4.16}$$

$W_{91}(s) = [(s+C_{3,1}(1))(s+C_{3,1}(1^{(1)}))-\lambda_s\,\mu_{s0'}]\,[(s+C_{3,1}(2))(s+C_{2,1}(2^{(1)}))-\lambda_s\,\mu_{s0'}]\,[(s+C_{3,1}(0))(s+C_{2,1}(0^{(1)})) - \lambda_s\,\mu_{s0'}]$

where s_{74} to s_{79} are the roots of the characteristic polynomial function $W_{91}(s)$. The residues B_{585} to B_{615} are defined in Appendix F. The system reliability values and plots obtained using (4.16) are given in the fourth column of Table 4.3 and Figure 4.6 respectively.

The system mean time to failure is:

$$MTTF_{Z_E}(3,1,1) = \sum_{i=0}^{3} P_i(0) + \sum_{i=0^{(1)}}^{2^{(1)}} P_i(0) \tag{4.17}$$

where,

$P_0(0) = C_{3,1}(0^{(1)})/(C_{3,1}(0)\,C_{3,1}(0^{(1)}) - \lambda_s\,\mu_{s0'})$

$P_{0^{(1)}}(0) = \lambda_s/(C_{3,1}(0)\,C_{3,1}(0^{(1)}) - \lambda_s\,\mu_{s0'})$

$P_1(0) = (3\lambda_1/(C_{3,1}(1)\,C_{3,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}))\bullet P_0(0) + (3\lambda_1\mu_{s0'}/(C_{3,1}(1)\,C_{3,1}(1^{(1)}) - \lambda_s\mu_{s0'}))\bullet P_{0^{(1)}}(0)$

$P_{1^{(1)}}(0) = (3\lambda_1/C_{3,1}(1^{(1)}))\bullet P_{0^{(1)}}(0) + (\lambda_s/C_{3,1}(1^{(1)}))\bullet P_1(0)$

$P_2(0) = (2\lambda_1/C_{3,1}(2))\bullet P_1(0) + (\mu_{s0'}/C_{3,1}(2))\bullet P_{2^{(1)}}(0)$

$P_{2^{(1)}}(0) = (\lambda_s/C_{3,1}(2^{(1)}))\bullet P_2(0) + (2\lambda_1/C_{3,1}(2^{(1)}))\bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{3,1}(3))\bullet P_2(0)$

and,

$C_{3,1}(0) = 3\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{3,1}(1) = 2\lambda_1 + \lambda_s + \lambda_{c1}$

$C_{3,1}(2) = \lambda_1 + \lambda_{c2} + \lambda_s$

$C_{3,1}(3) = \lambda_1 + \lambda_{c3}$

$C_{3,1}(0^{(1)}) = 3\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$

$C_{3,1}(1^{(1)}) = 2\lambda_1 + \lambda_{c1^{(1)}} + \mu_{s0'}$

$C_{3,1}(2^{(1)}) = \lambda_1 + \lambda_{c2^{(1)}} + \mu_{s0'}$

The variance of time to failure is given by:

$$\begin{aligned}\sigma^2_{Z_E}(3,1,1) &= -2\lim_{s\to 0} R'_{Z_E}(3,1,1)(s) - [MTTF_{Z_E}(3,1,1)]^2 \\ &= \frac{1}{k_{42}^2}[2(k_{41}\,k_{43} - k_{42}\,k_{44}) - k_{41}^2]\end{aligned} \tag{4.18}$$

Symbols k_{41} to k_{44} are defined in Appendix F.

From Equations (4.14) and (4.17) and with the aid of the analyses of many more special cases, a generalized approach of determining the mean time to failure of a $1 - out - of - (n+1) : G$ system operating under standby activation policy Z with Type E Repair, is developed using Equation (2.11) in conjunction with the following set of recursive relationships:

Letting $\lim_{s \to 0} P_i(s) = P_i(0)$; for $i = 0, 1, 2, ..., n$ and $\lim_{s \to 0} P_j(s) = P_j(0)$; for $j = 0^{(1)}, 1^{(1)}, 2^{(1)}, ..., (n-1)^{(1)}$ then:

$P_0(0) = [C_{n,1}(0^{(1)})]/[C_{n,1}(0)\, C_{n,1}(0^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{0^{(1)}}(0) = \lambda_s/[C_{n,1}(0)\, C_{n,1}(0^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_1(0) = [n\lambda_1\, C_{n,1}(1^{(1)}) \bullet P_0(0) + n\lambda_1\, \mu_{s0'} \bullet P_{0^{(1)}}(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{1^{(1)}}(0) = [n\lambda_1\, C_{n,1}(1) \bullet P_{0^{(1)}}(0) + n\lambda_1\, \lambda_s \bullet P_0(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_2(0) = [(n-1)\lambda_1\, C_{n,1}(2^{(1)}) \bullet P_1(0) + (n-1)\lambda_1\, \mu_{s0'} \bullet P_{1^{(1)}}(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{2^{(1)}}(0) = [(n-1)\lambda_1\, C_{n,1}(2) \bullet P_{1^{(1)}}(0) + (n-1)\lambda_1\, \lambda_s \bullet P_1(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_3(0) = [(n-2)\lambda_1\, C_{n,1}(3^{(1)}) \bullet P_2(0) + (n-2)\lambda_1\, \mu_{s0'} \bullet P_{2^{(1)}}(0)]/[C_{n,1}(3)\, C_n(3^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{3^{(1)}}(0) = [(n-2)\lambda_1\, C_{n,1}(3) \bullet P_{2^{(1)}}(0) + (n-2)\lambda_1\, \lambda_s \bullet P_2(0)]/[C_{n,1}(3)\, C_{n,1}(3^{(1)}) - \mu_{s0'}\, \lambda_s]$

$\vdots$

$P_{n-1}(0) = [2\lambda_1\, C_{n,1}((n-1)^{(1)}) \bullet P_{n-2}(0) + 2\lambda_1\, \mu_{s0'} \bullet P_{n-2^{(1)}}(0)]/[C_{n,1}(n-1)\, C_{n,1}((n-1)^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{(n-1)^{(1)}}(0) = [2\lambda_1\, C_{n,1}(n-1) \bullet P_{n-2^{(1)}}(0) + 2\lambda_1\, \lambda_s \bullet P_{n-2}(0)]/[C_{n,1}(n-1)\, C_{n,1}((n-1)^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_n(0) = [\lambda_1/C_{n,1}(n)] \bullet P_{n-1}(0)$

For the special cases considered, this approach is used to compute the system mean time to failure values and plots given in Table 4.4 and Figure 4.7, respectively.

where

$C_{n,1}(0) = n\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = (n-1)\lambda_1 + \lambda_s + \lambda_{c1}$

$C_{n,1}(2) = (n-2)\lambda_1 + \lambda_s + \lambda_{c2}$

$C_{n,1}(3) = (n-3)\lambda_1 + \lambda_s + \lambda_{c3}$

$\vdots$

$C_{n,1}(n-1) = \lambda_1 + \lambda_s + \lambda_{cn-1}$

$C_{n,1}(n) = \lambda_1 + \lambda_{cn}$

$$C_{n,1}(0^{(1)}) = n\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$$
$$C_{n,1}(1^{(1)}) = (n-1)\lambda_1 + \lambda_{c1^{(1)}} + \mu_{s0'}$$
$$C_{n,1}(2^{(1)}) = (n-2)\lambda_1 + \lambda_{c2^{(1)}} + \mu_{s0'}$$
$$\vdots$$
$$C_{n,1}((n-1)^{(1)}) = \lambda_1 + \lambda_{cn-1^{(1)}} + \mu_{s0'}$$

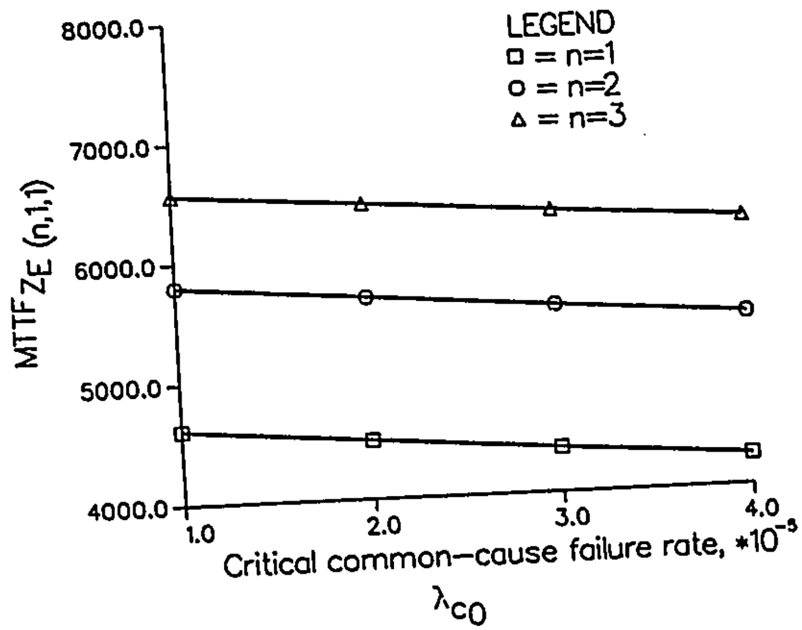


Figure 4.7: $MTTF_{Z_E}(n,1,1)$ Vs. λ_{c0} Plots

$1-out-of-(n+1):G$ System with Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are replaced upon failure thus $\mu_{s0'}$, $\mu_{n+1}(x)$, $\mu_{n+2}(x)$, μ_1 to μ_n and $\mu_{1^{(1)}}$ to $\mu_{n-1^{(1)}}$ are set to zero in Equations (4.1) to (4.10).

Table 4.4: $MTTF_{Z_E}(n,1,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	4614.84	5799.50	6565.40
0.00002	4488.81	5671.17	6451.29
0.00003	4368.74	5547.23	6340.04
0.00004	4254.24	5427.50	6231.55

Special Case Model I

For $n = 2$ with Type F Repair, the system reliability values are determined by setting $\mu_{s0'}$ to zero. The values obtained thus are given in the third column of Table 4.5 and associated plots in Figure 4.8.

Table 4.5: $R_{Z_F}(n,1,1)$ values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.7226	0.8509	0.9186
4000	0.4101	0.5100	0.6026
6000	0.2101	0.2600	0.3215
8000	0.1018	0.1227	0.1554
∞	0	0	0

The mean time to failure is determined by setting $\mu_{s0'}$ to zero in Equation (4.14) and the variance of time to failure is determined by setting $\mu_{s0'}$ to zero in Equation (4.15).

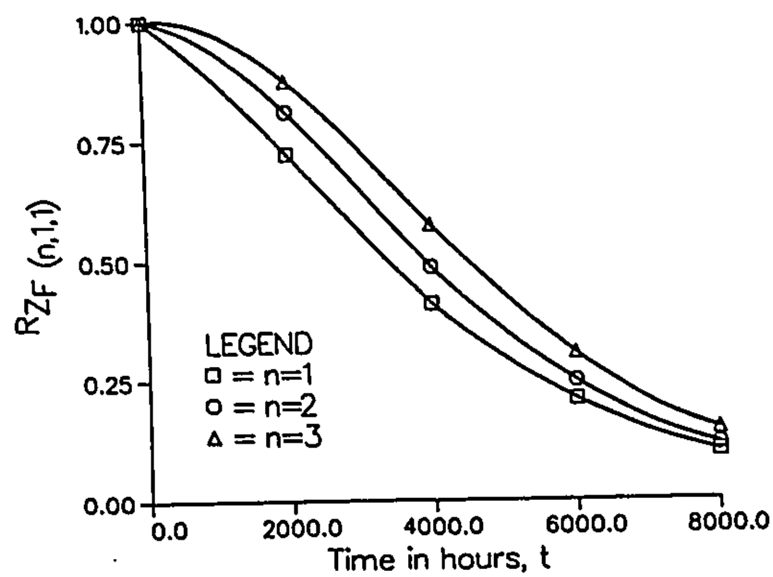


Figure 4.8: $R_{Z_F}(n,1,1)$ Plots ($\lambda_{c0} = 0.00004$ per hr.)

Special Case Model II

For $n = 3$ with Type F Repair, the system reliability, mean time to failure and variance of time to failure are obtained easily from Equations (4.16), (4.17) and (4.18) respectively by setting $\mu_{s0'}$ to zero. The system reliability values obtained are presented in the fourth column of Table 4.5 and associated plots in Figure 4.8.

A generalized approach for determining the system mean time to failure with Type F Repair for a $1-out-of-(n+1):G$ type system operating under standby activation policy Z is easily derived from the same approach established for the Type E Repair cases, by setting $\mu_{s0'}$ to zero leading to the following set of recursive relationships:
Again letting $\lim_{s\to 0} P_i(s) = P_i(0)$; for $i = 0, 1, 2, ..., n$ and $\lim_{s\to 0} P_j(s) = P_j(0)$; for $j = 0^{(1)}, 1^{(1)}, 2^{(1)}, ..., (n-1)^{(1)}$ then:

$P_0(0) = 1/C_{n,1}(0)$

$P_{0^{(1)}}(0) = \lambda_s/[C_{n,1}(0)\, C_{n,1}(0^{(1)})]$

$P_1(0) = [n\lambda_1/C_{n,1}(1)] \bullet P_0(0)$

$P_{1^{(1)}}(0) = [n\lambda_1\, C_{n,1}(1) \bullet P_{0^{(1)}}(0) + n\lambda_1\, \lambda_s \bullet P_0(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)})]$

$P_2(0) = [(n-1)\lambda_1/C_{n,1}(2)] \bullet P_1(0)$

$P_{2^{(1)}}(0) = [(n-1)\lambda_1\, C_{n,1}(2) \bullet P_{1^{(1)}}(0) + (n-1)\lambda_1\, \lambda_s \bullet P_1(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)})]$

$P_3(0) = [(n-2)\lambda_1/C_{n,1}(3)] \bullet P_2(0)$

$P_{3^{(1)}}(0) = [(n-2)\lambda_1\, C_{n,1}(3) \bullet P_{2^{(1)}}(0) + (n-2)\lambda_1\lambda_s \bullet P_2(0)]/[C_{n,1}(3)\, C_{n,1}(3^{(1)})]$

$\vdots$

$P_{n-1}(0) = [2\lambda_1/C_{n,1}(n-1)] \bullet P_{n-2}(0)$

$P_{n-1^{(1)}}(0) = [2\lambda_1\, C_{n,1}(n-1) \bullet P_{n-2^{(1)}}(0) + 2\lambda_1\, \lambda_s \bullet P_{n-2}(0)]/[C_{n,1}(n-1)\, C_{n,1}(n-1^{(1)})]$

$P_n(0) = [\lambda_1/C_{n,1}(n)] \bullet P_{n-1}(0)$

For Repair Type F, a more concise system mean time to failure expression is developed and is given by:

$$\begin{aligned} MTTF_{Z_F}(n,1,1) &= \frac{1}{C_{n,1}(0)}[1 + \frac{F_1(n)}{C_{n,1}(1)} + \lambda_s(\frac{1}{C_{n,1}(0^{(1)})} + \frac{F_2(n)}{C_{n,1}(1^{(1)})} + \\ &\quad \frac{F_3(n)}{C_n(2^{(1)})} + \frac{F_4(n)}{C_n(3^{(1)})} + \ldots.. + \frac{F_n(n)}{C_n(n-1^{(1)})})] \end{aligned} \tag{4.19}$$

where

$$F_1(n) = n\lambda_1 + \frac{n(n-1)\lambda_1^2}{C_{n,1}(2)} + \frac{n(n-1)(n-2)\lambda_1^3}{C_{n,1}(2)\, C_{n,1}(3)} + \ldots. + \frac{n!\lambda_1^n}{\prod_{j=2}^n C_{n,1}(j)}$$

$$F_2(n) = \frac{n\lambda_1}{C_{n,1}(0')} + \frac{n\lambda_1}{C_{n,1}(1)}$$

$$F_3(n) = \frac{n(n-1)\lambda_1^2}{C_{n,1}(0^{(1)})\, C_{n,1}(1^{(1)})} + \frac{n(n-1)\lambda_1^2}{C_{n,1}(1)\, C_{n,1}(1^{(1)})} + \frac{n(n-1)\lambda_1^2}{C_{n,1}(1)\, C_{n,1}(2)}$$

$$\begin{aligned} F_4(n) &= \frac{n(n-1)(n-2)\lambda_1^3}{C_{n,1}(0^{(1)})\, C_{n,1}(1^{(1)})\, C_{n,1}(2^{(1)})} + \frac{n(n-1)(n-2)\lambda_1^3}{C_{n,1}(1)\, C_{n,1}(1^{(1)})\, C_{n,1}(2^{(1)})} + \\ &\quad \frac{n(n-1)(n-2)\lambda_1^3}{C_{n,1}(1)\, C_{n,1}(2)\, C_{n,1}(2^{(1)})} + \frac{n(n-1)(n-2)\lambda_1^3}{C_{n,1}(1)\, C_{n,1}(2)\, C_{n,1}(3)} \end{aligned}$$

$$\vdots$$

$$\begin{aligned}F_n(n) \;=\; & \frac{n!\lambda_1^{n-1}}{\prod_{j=0^{(1)}}^{n-2^{(1)}} C_{n,1}(j)} + \frac{n!\lambda_1^{n-1}}{C_{n,1}(1)\prod_{j=1^{(1)}}^{n-2^{(1)}} C_{n,1}(j)} + \\ & \frac{n!\lambda_1^{n-1}}{C_{n,1}(1)\,C_{n,1}(2)\prod_{j=2^{(1)}}^{(n-2)^{(1)}} C_{n,1}(j)} + \ldots.. \\ & .. + \frac{n!\lambda_1^{n-1}}{\prod_{j=1}^{n-2} C_{n,1}(j)\,C_{n,1}((n-2)^{(1)})} + \frac{n!\lambda_1^{n-1}}{\prod_{j=1}^{n-1} C_{n,1}(j)}\end{aligned}$$

For the special cases considered, this approach is used to determine system mean time to failure values and plots given in Table 4.6 and Figure 4.9.

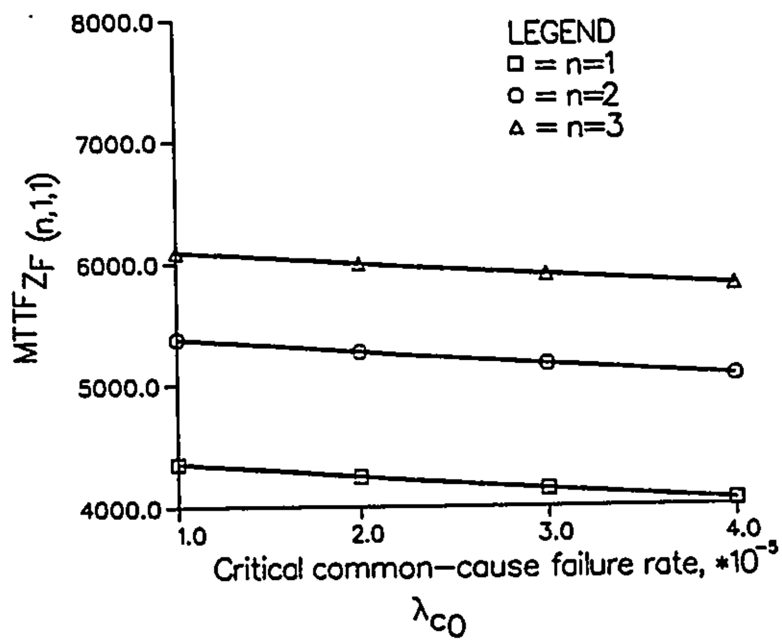


Figure 4.9: $MTTF_{Z_F}(n,1,1)$ Vs. λ_{c0} Plots

where

$C_{n,1}(0) = n\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = (n-1)\lambda_1 + \lambda_s + \lambda_{c1}$

$C_{n,1}(2) = (n-2)\lambda_1 + \lambda_s + \lambda_{c2}$

Table 4.6: $MTTF_{Z_F}(n,1,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	4353.36	5376.86	6089.71
0.00002	4246.48	5270.23	5993.80
0.00003	4144.18	5166.94	5900.11
0.00004	4046.19	5066.86	5808.59

$$C_{n,1}(3) = (n-3)\lambda_1 + \lambda_s + \lambda_{c3}$$
$$\vdots$$
$$C_{n,1}(n-1) = \lambda_1 + \lambda_s + \lambda_{cn-1}$$
$$C_{n,1}(n) = \lambda_1 + \lambda_{cn}$$

$$C_{n,1}(0^{(1)}) = n\lambda_1 + \lambda_{c0^{(1)}}$$
$$C_{n,1}(1^{(1)}) = (n-1)\lambda_1 + \lambda_{c1^{(1)}}$$
$$C_{n,1}(2^{(1)}) = (n-2)\lambda_1 + \lambda_{c2^{(1)}}$$
$$\vdots$$
$$C_{n,1}((n-1)^{(1)}) = \lambda_1 + \lambda_{cn-1^{(1)}}$$

4.1.2 1-out-of-(n+2):G Identical Unit Submodel

This is the $(n,2,1)$ system and is composed of n active units and *two* standbys. It is functional as long as at least *one* unit is healthy. The block diagram of this system is shown in Figure 3.18 and the state space diagram is shown in Figure 4.10.

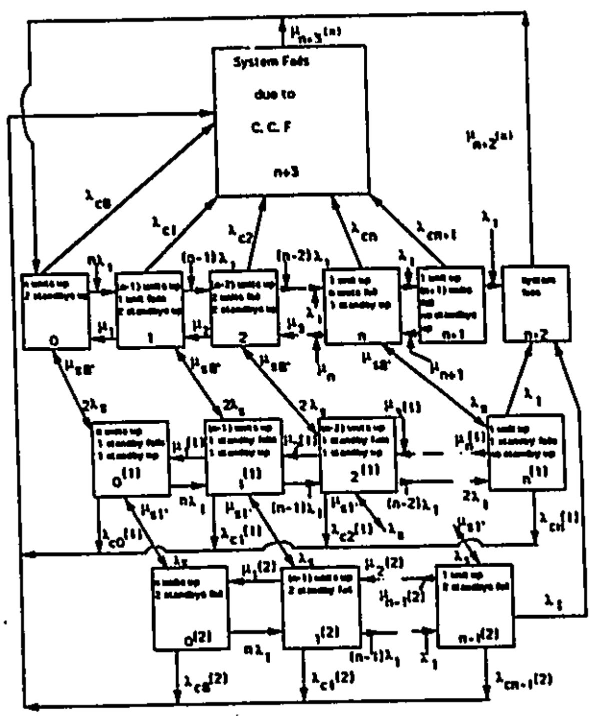


Figure 4.10: State space diagram of the $1 - out - of - (n+2) : G$ System under SBAP Z

$\underline{1 - out - of - (n+2) : G \text{ System Equations}}$

The descriptive integro-differential Equations for the system shown in Figure 4.10 are:

$$\frac{dP_0(t)}{dt} + C_{n,2}(0)\, P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\, P_{0^{(1)}}(t) + \sum_{i=n+2}^{n+3} \int_0^\infty P_i(x,t)\, \mu_i(x) dx \quad (4.20)$$

For $j = 1, 2, \ldots\ldots, n$,

$$\frac{dP_j(t)}{dt} + C_{n,2}(j)\, P_j(t) = (n-j+1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) + \mu_{j+1} P_{j+1}(t) \quad (4.21)$$

$$\frac{dP_{n+1}(t)}{dt} + C_{n,2}(n+1)\, P_{n+1}(t) = \lambda_1\, P_n(t) \quad (4.22)$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,2}(0^{(1)}) P_{0^{(1)}}(t) = 2\lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) + \mu_{s1'} P_{0^{(2)}}(t) \quad (4.23)$$

For $j = 1, 2, \ldots\ldots, n-1$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,2}(j^{(1)})\, P_{j^{(1)}}(t) = 2\lambda_s P_j(t) + \mu_{s1'} P_{j^{(2)}}(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(1)}}(t) +$$
$$\mu_{j+1^{(1)}}\, P_{j+1^{(1)}}(t) \quad (4.24)$$

$$\frac{dP_{n^{(1)}}(t)}{dt} + C_{n,2}(n^{(1)})\, P_{n^{(1)}}(t) = \lambda_1\, P_{n-1^{(1)}}(t) + \lambda_s P_n(t) \quad (4.25)$$

$$\frac{dP_{0^{(2)}}(t)}{dt} + C_{n,2}(0^{(2)})\, P_{0^{(2)}}(t) = \lambda_s\, P_{0^{(1)}}(t) + \mu_{1^{(2)}} P_{1^{(2)}}(t) \quad (4.26)$$

For $j = 1, 2, \ldots\ldots, n-2$,

$$\frac{dP_{j^{(2)}}(t)}{dt} + C_{n,2}(j^{(2)})\, P_{j^{(2)}}(t) = \lambda_s P_{j^{(1)}}(t) + (n-j+1)\lambda_1 P_{j-1^{(2)}}(t) +$$
$$\mu_{j+1^{(2)}}\, P_{j+1^{(2)}}(t) \quad (4.27)$$

$$\frac{dP_{n-1^{(2)}}(t)}{dt} + C_{n,2}(n-1^{(2)}) P_{n-1^{(2)}}(t) = \lambda_1 P_{n-2^{(2)}}(t) + \lambda_s P_{n-1^{(1)}}(t) \quad (4.28)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+2}(x)]\, P_{n+2}(x,t) = 0 \quad (4.29)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+3}(x)]\, P_{n+3}(x,t) = 0 \quad (4.30)$$

The associated boundary conditions are:

$$P_{n+2}(0,t) = \lambda_1\, P_{n+1}(t) + \lambda_1\, P_{n^{(1)}}(t) + \lambda_1\, P_{n-1^{(2)}}(t) \quad (4.31)$$

$$P_{n+3}(0,t) = \sum_{j=0}^{n+1} \lambda_{cj}\, P_j(t) + \sum_{j=0^{(1)}}^{n^{(1)}} \lambda_{cj} P_j(t) + \sum_{j=0^{(2)}}^{n-1^{(2)}} \lambda_{cj} P_j(t) \qquad (4.32)$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,2}(0) = n\,\lambda_1 + 2\lambda_s + \lambda_{c0}$

For $j = 1, 2, ...n-1$,

$C_{n,2}(j) = (n-j)\,\lambda_1 + \lambda_{cj} + \mu_j + 2\lambda_s$

$C_{n,2}(n) = \lambda_1 + \lambda_{cn} + \mu_n + \lambda_s$

$C_{n,2}(n+1) = \lambda_1 + \lambda_{cn+1} + \mu_{n+1}$

$C_{n,2}(0^{(1)}) = n\,\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'} + \lambda_s$

For $j = 1, 2, ..., n-1$,

$C_{n,2}(j^{(1)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s0'} + \lambda_s$

$C_{n,2}(n^{(1)}) = \lambda_1 + \lambda_{cn^{(1)}} + \mu_{n^{(1)}} + \mu_{s0'}$

$C_{n,2}(0^{(2)}) = n\,\lambda_1 + \lambda_{c0^{(2)}} + \mu_{s1'}$

For $j = 1, 2, ..., n-2$,

$C_{n,2}(j^{(2)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(2)}} + \mu_{j^{(2)}} + \mu_{s1'}$

$C_{n,2}(n-1^{(2)}) = \lambda_1 + \lambda_{cn-1^{(2)}} + \mu_{n-1^{(2)}} + \mu_{s1'}$

$1 - out - of - (n+2) : G$ System With Type C Repair

With Type C Repair, the units in the system (both active and standbys) are discarded upon failure thus $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_{n+1}, $\mu_{1^{(1)}}$ to $\mu_{n^{(1)}}$, $\mu_{1^{(2)}}$ to $\mu_{n-1^{(2)}}$ are set to zero in Equations (4.20) to (4.32).

Special Case Model I

For $n = 2$ with Type C Repair, the long-run availability for this special case is given by:

$$A_{ssz}(2,2,1)\,(\beta) = \frac{(\mu_4\,\mu_5)^\beta\, x_{49}}{(\mu_4\,\mu_5)^\beta\, x_{50} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{51} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{52}} \qquad (4.33)$$

where,

$x_{49} = A_{2124D}$

$x_{50} = A_{2119} - A_{2121} - A_{2123}$

$x_{51} = A_{2120} - A_{2122}$

$x_{52} = A_{2120} - A_{2124}$

Long-run availability values generated with Equation (4.33) are given in the third column of Table 4.7. The symbols A_{2119} to A_{2124D} are shown in Appendix G.

Special Case Model II

For $n = 3$ with Type C Repair, the long-run availability for this special case is given by:

$$A_{ssz}(3,2,1)\,(\beta) = \frac{(\mu_5\,\mu_6)^\beta\,x_{53}}{(\mu_5\,\mu_6)^\beta\,x_{54} + \beta\,\mu_5^\beta\,\mu_6^{\beta-1}\,x_{55} + \beta\,\mu_5^{\beta-1}\,\mu_6^\beta\,x_{56}} \tag{4.34}$$

where,

$x_{53} = A_{2204D}$

$x_{54} = A_{2199} - A_{2201} - A_{2203}$

$x_{55} = A_{2200} - A_{2202}$

$x_{56} = A_{2200} - A_{2204}$

Long-run availability values generated with Equation (4.34) are given in the fourth column of Table 4.7. The symbols A_{2199} to A_{2204D} are shown in Appendix G.

$1 - out - of - (n+2) : G$ System with Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are discarded and replaced upon failure thus $\mu_{n+2}(x)$, $\mu_{n+3}(x)$, $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_{n+1}, $\mu_{1^{(1)}}$ to $\mu_{n^{(1)}}$, $\mu_{1^{(2)}}$ to $\mu_{n-1^{(2)}}$ are set to zero in Equations (4.20) to (4.32).

Special Case Model I

For $n = 2$ with Type F Repair, the system reliability is given by:

$$R_{Z_F}(2,2,1)(t) = \sum_{i=0}^{3} P_i(t) + \sum_{i=0^{(1)}}^{2^{(1)}} P_i(t) + \sum_{i=0^{(2)}}^{1^{(2)}} P_i(t) \tag{4.35}$$

Table 4.7: $A_{ssz}(n,2,1)(\beta)$ Values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.4774	0.4667	0.5319
	0.00002	0.4750	0.4652	0.5303
	0.00003	0.4725	0.4638	0.5287
	0.00004	0.4700	0.4623	0.5271
$\beta=2$	0.00001	0.3135	0.3084	0.3623
	0.00002	0.3114	0.3036	0.3608
	0.00003	0.3094	0.3023	0.3594
	0.00004	0.3073	0.3011	0.3579
$\beta=3$	0.00001	0.2334	0.2263	0.2747
	0.00002	0.2317	0.2253	0.2734
	0.00003	0.2299	0.2243	0.2722
	0.00004	0.2282	0.2232	0.2709
$\beta=4$	0.00001	0.1859	0.1800	0.2212
	0.00002	0.1844	0.1791	0.2201
	0.00003	0.1830	0.1782	0.2190
	0.00004	0.1815	0.1774	0.2179
$\beta=5$	0.00001	0.1545	0.1494	0.1852
	0.00002	0.1532	0.1486	0.1842
	0.00003	0.1519	0.1479	0.1833
	0.00004	0.1507	0.1471	0.1823

where

$$
\begin{aligned}
P_0(t) &= e^{-C_{2,2}(0)t} \\
P_1(t) &= B_{620}\,[e^{-C_{2,2}(1)t} - e^{-C_{2,2}(0)t}] \\
P_2(t) &= B_{621}\,e^{-C_{2,2}(0)t} + B_{622}\,e^{-C_{2,2}(1)t} + B_{623}\,e^{-C_{2,2}(2)t} \\
P_3(t) &= B_{624}\,e^{-C_{2,2}(0)t} + B_{625}\,e^{-C_{2,2}(1)t} + B_{626}\,e^{-C_{2,2}(2)t} + \\
&\quad B_{627}\,e^{-C_{2,2}(3)t} \\
P_{0^{(1)}}(t) &= B_{628}\,[e^{-C_{2,2}(0)t} - e^{-C_{2,2}(0^{(1)})t}] \\
P_{1^{(1)}}(t) &= B_{629}\,e^{-C_{2,2}(0)t} + B_{630}\,e^{-C_{2,2}(0^{(1)})t} + B_{631}\,e^{-C_{2,2}(1^{(1)})t} + \\
&\quad B_{632}\,e^{-C_{2,2}(1)t} \\
P_{2^{(1)}}(t) &= B_{633}\,e^{-C_{2,2}(0)t} + B_{634}\,e^{-C_{2,2}(1)t} + B_{635}\,e^{-C_{2,2}(2)t} + \\
&\quad B_{636}\,e^{-C_{2,2}(1^{(1)})t} + B_{637}\,e^{-C_{2,2}(2^{(1)})t} + B_{638}\,e^{-C_{2,2}(0^{(1)})t} \\
P_{0^{(2)}}(t) &= B_{639}\,e^{-C_{2,2}(0)t} + B_{640}\,e^{-C_{2,2}(0^{(1)})t} + B_{641}\,e^{-C_{2,2}(0^{(2)})t} \\
P_{1^{(2)}}(t) &= B_{642}\,e^{-C_{2,2}(0)t} + B_{643}\,e^{-C_{2,2}(1)t} + B_{644}\,e^{-C_{2,2}(0^{(1)})t} + \\
&\quad B_{645}\,e^{-C_{2,2}(1^{(1)})t} + B_{646}\,e^{-C_{2,2}(0^{(2)})t} + B_{647}\,e^{-C_{2,2}(1^{(2)})t}
\end{aligned}
$$

The residues B_{620} to B_{647} are defined in Appendix G. System reliability values obtained with (4.35) are shown in the third column of Table 4.8.

The system mean time to failure is given by:

$$
MTTF_{Z_F}(2,2,1) = \sum_{i=0}^{3} P_i(0) + \sum_{i=0^{(1)}}^{2^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{1^{(2)}} P_i(0) \tag{4.36}
$$

where,

$P_0(0) = 1/C_{2,2}(0)$

$P_{0^{(1)}}(0) = (2\lambda_s/C_{2,2}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (2\lambda_1/C_{2,2}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (2\lambda_s/C_{2,2}(1^{(1)})) \bullet P_1(0) + (2\lambda_1/C_{2,2}(1^{(1)})) \bullet P_{0^{(1)}}(0)$

$P_2(0) = (\lambda_1/C_{2,2}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (\lambda_s/C_{2,2}(2^{(1)})) \bullet P_2(0) + (\lambda_1/C_{2,2}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{2,2}(3)) \bullet P_2(0)$

$P_{0^{(2)}} = (\lambda_s / C_{2,2}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (\lambda_s / C_{2,2}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (2\lambda_1 / C_{2,2}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

and,

$C_{2,2}(0) = 2\lambda_1 + 2\lambda_s + \lambda_{c0}$

$C_{2,2}(1) = \lambda_1 + 2\lambda_s + \lambda_{c1}$

$C_{2,2}(2) = \lambda_1 + \lambda_s + \lambda_{c2}$

$C_{2,2}(3) = \lambda_1 + \lambda_{c3}$

$C_{2,2}(0^{(1)}) = 2\lambda_1 + \lambda_s + \lambda_{c0^{(1)}}$

$C_{2,2}(1^{(1)}) = \lambda_1 + \lambda_{c1^{(1)}} + \lambda_s$

$C_{2,2}(2^{(1)}) = \lambda_1 + \lambda_{c2^{(1)}}$

$C_{2,2}(0^{(2)}) = 2\lambda_1 + \lambda_{c0^{(2)}}$

$C_{2,2}(1^{(2)}) = \lambda_1 + \lambda_{c1^{(2)}}$

Table 4.8: $R_{Z_F}(n, 2, 1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n = 1$	$n = 2$	$n = 3$
0	1.0000	1.0000	1.0000
2000	0.7343	0.9670	0.9955
4000	0.5391	0.6934	0.7425
6000	0.3519	0.4050	0.4362
8000	0.2092	0.2093	0.2230
∞	0	0	0

Equation (4.36) was used to generate the mean time to failure values given in the third column of Table 4.9.

The variance of time to failure is given by:

$$\begin{aligned} \sigma^2_{Z_F}(2,2,1) &= -2 \lim_{s \to 0} R'_{Z_F}(2,2,1)(s) - [MTTF_{Z_F}(2,2,1)]^2 \\ &= \frac{1}{k_{50}^2} [2(k_{49}\, k_{51} - k_{50}\, k_{52}) - k_{49}^2] \end{aligned} \quad (4.37)$$

Table 4.9: $MTTF_{Z_F}(n,2,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	6249.96	5954.45	7431.96
0.00002	6097.04	5846.91	7327.53
0.00003	5950.27	5742.41	7225.28
0.00004	5809.34	5640.84	7125.19

The symbols k_{49} to k_{52} are defined in Appendix G.

Special Case Model II

For $n = 3$ with Type F Repair, the system reliability is given by:

$$R_{Z_F}(3,2,1)(t) = \sum_{i=0}^{4} P_i(t) + \sum_{i=0^{(1)}}^{3^{(1)}} P_i(t) + \sum_{i=0^{(2)}}^{2^{(2)}} P_i(t) \tag{4.38}$$

where

$$\begin{aligned}
P_0(t) &= e^{-C_{3,2}(0)t} \\
P_1(t) &= B_{648}\,[e^{-C_{3,2}(1)t} - e^{-C_{2,2}(0)t}] \\
P_2(t) &= B_{650}\,e^{-C_{3,2}(0)t} + B_{651}\,e^{-C_{3,2}(1)t} + B_{652}\,e^{-C_{3,2}(2)t} \\
P_3(t) &= B_{653}\,e^{-C_{3,2}(0)t} + B_{654}\,e^{-C_{3,2}(1)t} + B_{655}\,e^{-C_{3,2}(2)t} + \\
&\quad B_{656}\,e^{-C_{3,2}(3)t} \\
P_4(t) &= B_{657}\,e^{-C_{3,2}(0)t} + B_{658}\,e^{-C_{3,2}(1)t} + B_{659}\,e^{-C_{3,2}(2)t} + \\
&\quad B_{660}\,e^{-C_{3,2}(3)t} + B_{661}\,e^{-C_{3,2}(4)t} \\
P_{0^{(1)}}(t) &= B_{662}\,[e^{-C_{3,2}(0)t} - e^{-C_{3,2}(0^{(1)})t}] \\
P_{1^{(1)}}(t) &= B_{663}\,e^{-C_{3,2}(0)t} + B_{664}\,e^{-C_{3,2}(1)t} + B_{665}\,e^{-C_{3,2}(0^{(1)})t} + \\
&\quad B_{666}\,e^{-C_{3,2}(1^{(1)})t} \\
P_{2^{(1)}}(t) &= B_{667}\,e^{-C_{3,2}(0)t} + B_{668}\,e^{-C_{3,2}(1)t} + B_{669}\,e^{-C_{3,2}(2)t} +
\end{aligned}$$

$$
\begin{aligned}
& B_{670}\, e^{-C_{3,2}(0^{(1)})t} + B_{671}\, e^{-C_{3,2}(1^{(1)})t} + B_{672}\, e^{-C_{3,2}(2^{(1)})t} \\
P_{3^{(1)}}(t) \;=\; & B_{673}\, e^{-C_{3,2}(0)t} + B_{674}\, e^{-C_{3,2}(1)t} + B_{675}\, e^{-C_{3,2}(2)t} + \\
& B_{676}\, e^{-C_{3,2}(3)t} + B_{677}\, e^{-C_{3,2}(0^{(1)})t} + B_{678}\, e^{-C_{3,2}(1^{(1)})t} + \\
& B_{679}\, e^{-C_{3,2}(2^{(1)})t} + B_{680}\, e^{-C_{3,2}(3^{(1)})t} \\
P_{0^{(2)}}(t) \;=\; & B_{681}\, e^{-C_{3,2}(0)t} + B_{682}\, e^{-C_{3,2}(0^{(1)})t} + B_{683}\, e^{-C_{3,2}(0^{(2)})t} \\
P_{1^{(2)}}(t) \;=\; & B_{684}\, e^{-C_{3,2}(0)t} + B_{685}\, e^{-C_{3,2}(1)t} + B_{686}\, e^{-C_{3,2}(0^{(1)})t} + \\
& B_{687}\, e^{-C_{3,2}(1^{(1)})t} + B_{688}\, e^{-C_{3,2}(1^{(2)})t} + B_{689}\, e^{-C_{3,2}(0^{(2)})t} \\
P_{2^{(2)}}(t) \;=\; & B_{690}\, e^{-C_{3,2}(0)t} + B_{691}\, e^{-C_{3,2}(1)t} + B_{692}\, e^{-C_{3,2}(2)t} + \\
& B_{693}\, e^{-C_{3,2}(0^{(1)})t} + B_{694}\, e^{-C_{3,2}(1^{(1)})t} + B_{695}\, e^{-C_{3,2}(2^{(1)})t} + \\
& B_{696}\, e^{-C_{3,2}(0^{(2)})t} + B_{697}\, e^{-C_{3,2}(1^{(2)})t} + B_{698}\, e^{-C_{3,2}(2^{(2)})t}
\end{aligned}
$$

The residues B_{648} to B_{698} are defined in Appendix G. System reliability values obtained with Equation (4.38) are given in the fourth column of Table 4.8.

The system mean time to failure is given by:

$$
MTTF_{Z_F}(3,2,1) \;=\; \sum_{i=0}^{4} P_i(0) + \sum_{i=0^{(1)}}^{3^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{2^{(2)}} P_i(0) \qquad (4.39)
$$

where

$P_0(0) = 1/C_{3,2}(0)$

$P_{0^{(1)}}(0) = (2\lambda_s/C_{3,2}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (3\lambda_1/C_{3,2}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (2\lambda_s/C_{3,2}(1^{(1)})) \bullet P_1(0) + (3\lambda_1/C_{3,2}(1^{(1)})) \bullet P_{0^{(1)}}(0)$

$P_2(0) = (2\lambda_1/C_{3,2}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (2\lambda_s/C_{3,2}(2^{(1)})) \bullet P_2(0) + (2\lambda_1/C_{3,2}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{3,2}(3)) \bullet P_2(0)$

$P_{3^{(1)}}(0) = (\lambda_s/C_{3,2}(3^{(1)})) \bullet P_3(0) + (\lambda_1/C_{3,2}(3^{(1)})) \bullet P_{2^{(1)}}(0)$

$P_4(0) = (\lambda_1/C_{3,2}(4)) \bullet P_3(0)$

$P_{0^{(2)}} = (\lambda_s/C_{3,2}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (\lambda_s/C_{3,2}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (3\lambda_1/C_{3,2}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

$P_{2^{(2)}}(0) = (\lambda_s/C_{3,2}(2^{(2)})) \bullet P_{2^{(1)}}(0) + (2\lambda_1/C_{3,2}(2^{(2)})) \bullet P_{1^{(2)}}(0)$

and,

$C_{3,2}(0) = 3\lambda_1 + 2\lambda_s + \lambda_{c0}$

$C_{3,2}(1) = 2\lambda_1 + 2\lambda_s + \lambda_{c1}$

$C_{3,2}(2) = \lambda_1 + 2\lambda_s + \lambda_{c2}$

$C_{3,2}(3) = \lambda_1 + \lambda_s + \lambda_{c3}$

$C_{3,2}(4) = \lambda_1 + \lambda_{c4}$

$C_{3,2}(0^{(1)}) = 3\lambda_1 + \lambda_s + \lambda_{c0^{(1)}}$

$C_{3,2}(1^{(1)}) = 2\lambda_1 + \lambda_{c1^{(1)}} + \lambda_s$

$C_{3,2}(2^{(1)}) = \lambda_1 + \lambda_s + \lambda_{c2^{(1)}}$

$C_{3,2}(3^{(1)}) = \lambda_1 + \lambda_{c3^{(1)}}$

$C_{3,2}(0^{(2)}) = 3\lambda_1 + \lambda_{c0^{(2)}}$

$C_{3,2}(1^{(2)}) = 2\lambda_1 + \lambda_{c1^{(2)}}$

$C_{3,2}(2^{(2)}) = \lambda_1 + \lambda_{c2^{(2)}}$

System mean time to failure values generated with Equations (4.36) and (4.39) are given in the third and fourth columns of Table 4.9.

The variance of time to failure is given by:

$$\begin{aligned}\sigma^2_{Z_F}(3,2,1) &= -2 \lim_{s \to 0} R'_{Z_F}(3,2,1)(s) - [MTTF_{Z_F}(3,2,1)]^2 \\ &= \frac{1}{k_{54}^2}[2(k_{53}\,k_{55} - k_{54}\,k_{56}) - k_{53}^2] \end{aligned} \tag{4.40}$$

The symbols k_{53} to k_{56} are defined in Appendix G.

4.1.3 1-out-of-(n+3):G Identical Unit Submodel

This is the $(n,3,1)$ system and is composed of n active units and *three* standbys. It is functional as long as at least *one* unit is healthy. The block diagram of this system is shown in Figure 3.20 and the state-space diagram of this system is shown in Figure 4.11.

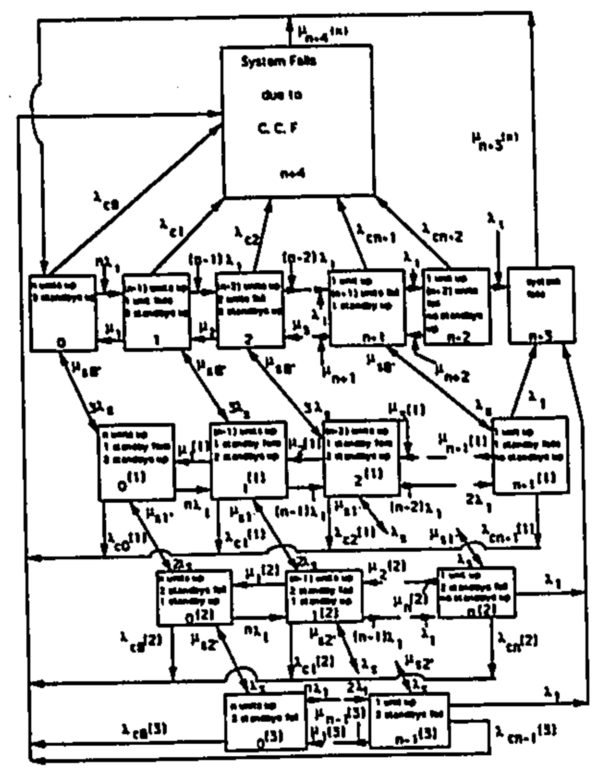


Figure 4.11: State space diagram of the $1-out-of-(n+3):G$ System under SBAP Z

$1 - out - of - (n+3) : G$ **System Equations**

The descriptive integro-differential Equations for the system shown in Figure 4.11 are:

$$\frac{dP_0(t)}{dt} + C_{n,3}(0)\, P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\, P_{0^{(1)}}(t) + \sum_{i=n+3}^{n+4} \int_0^\infty P_i(x,t)\, \mu_i(x) dx \quad (4.41)$$

For $j = 1, 2, \ldots\ldots, n$,

$$\frac{dP_j(t)}{dt} + C_{n,3}(j)\, P_j(t) = (n-j+1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) + \mu_{j+1} P_{j+1}(t) \quad (4.42)$$

$$\frac{dP_{n+1}(t)}{dt} + C_{n,3}(n+1)\, P_{n+1}(t) = \lambda_1\, P_n(t) + \mu_{n+2}\, P_{n+2}(t) + \mu_{s0'} P_{n+1^{(1)}}(t) \quad (4.43)$$

$$\frac{dP_{n+2}(t)}{dt} + C_{n,3}(n+2)\, P_{n+2}(t) = \lambda_1\, P_{n+1}(t) \quad (4.44)$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,3}(0^{(1)}) P_{0^{(1)}}(t) = 3\lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) + \mu_{s1'} P_{0^{(2)}}(t) \quad (4.45)$$

For $j = 1, 2, \ldots\ldots, n-1$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,3}(j^{(1)})\, P_{j^{(1)}}(t) = 3\lambda_s P_j(t) + \mu_{s1'} P_{j^{(2)}}(t) +$$
$$(n-j+1)\,\lambda_1\, P_{j-1^{(1)}}(t) + \mu_{j+1^{(1)}}\, P_{j+1^{(1)}}(t) \quad (4.46)$$

$$\frac{dP_{n^{(1)}}(t)}{dt} + C_{n,3}(n^{(1)})\, P_{n^{(1)}}(t) = \lambda_1 P_{n-1^{(1)}}(t) + 2\lambda_s P_n(t) + \mu_{n+1^{(1)}} P_{n+1^{(1)}}(t) \quad (4.47)$$

$$\frac{dP_{n+1^{(1)}}(t)}{dt} + C_{n,3}(n+1^{(1)})\, P_{n+1^{(1)}}(t) = \lambda_1 P_{n^{(1)}}(t) + \lambda_s P_{n+1}(t) \quad (4.48)$$

$$\frac{dP_{0^{(2)}}(t)}{dt} + C_{n,3}(0^{(2)})\, P_{0^{(2)}}(t) = 2\lambda_s\, P_{0^{(1)}}(t) + \mu_{1^{(2)}} P_{1^{(2)}}(t) + \mu_{s2'} P_{0^{(3)}}(t) \quad (4.49)$$

For $j = 1, 2, \ldots\ldots, n-1$,

$$\frac{dP_{j^{(2)}}(t)}{dt} + C_{n,3}(j^{(2)})\, P_{j^{(2)}}(t) = 2\lambda_s P_{j^{(1)}}(t) + \mu_{s2'} P_{j^{(3)}}(t) +$$
$$(n-j+1)\,\lambda_1\, P_{j-1^{(2)}}(t) + \mu_{j+1^{(2)}}\, P_{j+1^{(2)}}(t) \quad (4.50)$$

$$\frac{dP_{n^{(2)}}(t)}{dt} + C_{n,3}(n^{(2)})\, P_{n^{(2)}}(t) = \lambda_1 P_{n-1^{(2)}}(t) + \lambda_s P_{n^{(1)}}(t) \quad (4.51)$$

$$\frac{dP_{0^{(3)}}(t)}{dt} + C_{n,3}(0^{(3)})\, P_{0^{(3)}}(t) = \lambda_s\, P_{0^{(2)}}(t) + \mu_{1^{(3)}} P_{1^{(3)}}(t) \quad (4.52)$$

For $j = 1, 2, \ldots.., n-2$,

$$\frac{dP_{j^{(3)}}(t)}{dt} + C_{n,3}(j^{(3)})\, P_{j^{(3)}}(t) = \lambda_s P_{j^{(2)}}(t) + (n-j+1)\lambda_1 P_{j-1^{(3)}}(t) + \mu_{j+1^{(3)}}\, P_{j+1^{(3)}}(t) \quad (4.53)$$

$$\frac{dP_{n-1^{(3)}}(t)}{dt} + C_{n,3}(n-1^{(3)}) P_{n-1^{(3)}}(t) = \lambda_1 P_{n-2^{(3)}}(t) + \lambda_s P_{n-1^{(2)}}(t) \quad (4.54)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+3}(x)]\, P_{n+3}(x,t) = 0 \quad (4.55)$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{n+4}(x)]\, P_{n+4}(x,t) = 0 \quad (4.56)$$

The associated boundary conditions are:

$$P_{n+3}(0,t) = \lambda_1\, P_{n+2}(t) + \lambda_1\, P_{n+1^{(1)}}(t) + \lambda_1\, P_{n^{(2)}}(t) + \lambda_1\, P_{n-1^{(3)}}(t) \quad (4.57)$$

$$P_{n+4}(0,t) = \sum_{j=0}^{n+2} \lambda_{cj}\, P_j(t) + \sum_{j=0^{(1)}}^{n+1^{(1)}} \lambda_{cj} P_j(t) + \sum_{j=0^{(2)}}^{n^{(2)}} \lambda_{cj}\, P_j(t) + \sum_{j=0^{(3)}}^{n-1^{(3)}} \lambda_{cj} P_j(t) \quad (4.58)$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,3}(0) = n\,\lambda_1 + 3\lambda_s + \lambda_{c0}$

For $j = 1, 2, \ldots..n-1$,

$C_{n,3}(j) = (n-j)\,\lambda_1 + \lambda_{cj} + \mu_j + 3\lambda_s$

$C_{n,3}(n) = \lambda_1 + \lambda_{cn} + \mu_n + 2\lambda_s$

$C_{n,3}(n+1) = \lambda_1 + \lambda_{cn+1} + \mu_{n+1} + \lambda_s$

$C_{n,3}(n+2) = \lambda_1 + \lambda_{cn+2} + \mu_{n+2}$

$C_{n,3}(0^{(1)}) = n\,\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'} + 2\lambda_s$

For $j = 1, 2, \ldots\ldots., n-1$,

$C_{n,3}(j^{(1)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s0'} + 2\lambda_s$

$C_{n,3}(n^{(1)}) = \lambda_1 + \lambda_{cn^{(1)}} + \mu_{n^{(1)}} + \mu_{s0'} + \lambda_s$

$C_{n,3}(n+1^{(1)}) = \lambda_1 + \lambda_{cn+1^{(1)}} + \mu_{n+1^{(1)}} + \mu_{s0'}$

$C_{n,3}(0^{(2)}) = n\,\lambda_1 + \lambda_{c0^{(2)}} + \mu_{s1'} + \lambda_s$

For $j = 1, 2, \ldots\ldots., n-1$,

$C_{n,3}(j^{(2)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(2)}} + \mu_{j^{(2)}} + \mu_{s1'} + \lambda_s$

$C_{n,3}(n^{(2)}) = \lambda_1 + \lambda_{cn^{(2)}} + \mu_{n^{(1)}} + \mu_{s1'}$

$C_{n,3}(0^{(3)}) = n\,\lambda_1 + \lambda_{c0(3)} + \mu_{s2'}$

For $j = 1, 2, \ldots\ldots, n-1$,

$C_{n,3}(j^{(2)}) = (n-j)\,\lambda_1 + \lambda_{cj(3)} + \mu_{j(3)} + \mu_{s2'}$

1 − *out* − *of* − (n + 3) : G System With Type C Repair

With Type C Repair, the units in the system (both active and standbys) are discarded upon failure thus, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_{n+2}, $\mu_{1(1)}$ to $\mu_{n+1(1)}$, $\mu_{1(2)}$ to $\mu_{n(2)}$ and $\mu_{1(3)}$ to $\mu_{n-1(3)}$ are set to zero in Equations (4.41) to (4.58).

Special Case Model I

For $n = 2$ with Type C Repair, the long-run availability is given by:

$$A_{ssz}(2,3,1)\,(\beta) = \frac{(\mu_5\,\mu_6)^\beta\,x_{61}}{(\mu_5\,\mu_6)^\beta\,x_{62} + \beta\,\mu_5^\beta\,\mu_6^{\beta-1}\,x_{63} + \beta\,\mu_5^{\beta-1}\,\mu_6^\beta\,x_{64}} \tag{4.59}$$

where

$x_{61} = A_{2350D}$

$x_{62} = A_{2345} - A_{2347} - A_{2349}$

$x_{63} = A_{2346} - A_{2348}$

$x_{64} = A_{2346} - A_{2350}$

The long-run availability values generated with Equation (4.59) are given in the third column of Table 4.10. The symbols A_{2345} to A_{2350D} are shown in Appendix H.

Special Case Model II

For $n = 3$ with Type C Repair, the long-run availability is given by:

$$A_{ssz}(3,3,1)\,(\beta) = \frac{(\mu_6\,\mu_7)^\beta\,x_{65}}{(\mu_6\,\mu_7)^\beta\,x_{66} + \beta\,\mu_6^\beta\,\mu_7^{\beta-1}\,x_{67} + \beta\,\mu_6^{\beta-1}\,\mu_7^\beta\,x_{68}} \tag{4.60}$$

where

$x_{65} = A_{2462D}$

$x_{66} = A_{2457} - A_{2459} - A_{2461}$

$x_{67} = A_{2458} - A_{2460}$

Table 4.10: $A_{ssz}(n,3,1)(\beta)$ Values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
$\beta=1$	0.00001	0.5288	0.6382	0.5687
	0.00002	0.5266	0.6366	0.5672
	0.00003	0.5244	0.6349	0.5658
	0.00004	0.5222	0.6333	0.5643
$\beta=2$	0.00001	0.3595	0.4377	0.3973
	0.00002	0.3574	0.4360	0.3959
	0.00003	0.3554	0.4343	0.3945
	0.00004	0.3534	0.4327	0.3931
$\beta=3$	0.00001	0.2723	0.3330	0.3053
	0.00002	0.2705	0.3316	0.3041
	0.00003	0.2688	0.3301	0.3028
	0.00004	0.2670	0.3286	0.3016
$\beta=4$	0.00001	0.2191	0.2688	0.2655
	0.00002	0.2176	0.2675	0.2644
	0.00003	0.2161	0.2661	0.2633
	0.00004	0.2146	0.2648	0.2621
$\beta=5$	0.00001	0.1833	0.2310	0.2309
	0.00002	0.1820	0.2315	0.2299
	0.00003	0.1807	0.2313	0.2290
	0.00004	0.1794	0.2311	0.2280

$x_{68} = A_{2458} - A_{2462}$

The long-run availability values generated with Equation (4.60) are given in the fourth column of Table 4.10. The symbols A_{2457} to A_{2462D} are shown in Appendix H.

$1 - out - of - (n+3) : G$ System With Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are discarded and replaced upon failure thus, $\mu_{n+3}(x)$, $\mu_{n+4}(x)$, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_{n+2}, $\mu_{1^{(1)}}$ to $\mu_{n+1^{(1)}}$, $\mu_{1^{(2)}}$ to $\mu_{n^{(2)}}$ and $\mu_{1^{(3)}}$ to $\mu_{n-1^{(3)}}$ are set to zero in Equations (4.41) to (4.58).

Special Case Model I

For $n = 2$ with Type F Repair, using established procedures, the system relaibility is given by:

$$R_{Z_F}(2,3,1)(t) = \sum_{i=0}^{4} P_i(t) + \sum_{i=0^{(1)}}^{3^{(1)}} P_i(t) + \sum_{i=0^{(2)}}^{2^{(2)}} P_i(t) + \sum_{i=0^{(3)}}^{1^{(3)}} P_i(t) \tag{4.61}$$

where

$$\begin{aligned}
P_0(t) &= e^{-C_{2,3}(0)t} \\
P_1(t) &= B_{699}\,[e^{-C_{2,3}(1)t} - e^{-C_{2,3}(0)t}] \\
P_2(t) &= B_{700}\,e^{-C_{2,3}(0)t} + B_{701}\,e^{-C_{2,3}(1)t} + B_{702}\,e^{-C_{2,3}(2)t} \\
P_3(t) &= B_{703}\,e^{-C_{2,3}(0)t} + B_{704}\,e^{-C_{2,3}(1)t} + B_{705}\,e^{-C_{2,3}(2)t} + \\
&\quad B_{706}\,e^{-C_{2,3}(3)t} \\
P_4(t) &= B_{707}\,e^{-C_{2,3}(0)t} + B_{708}\,e^{-C_{2,3}(1)t} + B_{709}\,e^{-C_{2,3}(2)t} + \\
&\quad B_{710}\,e^{-C_{2,3}(3)t} + B_{711}\,e^{-C_{2,3}(4)t} \\
P_{0^{(1)}}(t) &= B_{712}\,[e^{-C_{2,3}(0)t} - e^{-C_{2,3}(0^{(1)})t}] \\
P_{1^{(1)}}(t) &= B_{713}\,e^{-C_{2,3}(0)t} + B_{714}\,e^{-C_{2,3}(1)t} + B_{715}\,e^{-C_{2,3}(0^{(1)})t} + \\
&\quad B_{716}\,e^{-C_{2,3}(1^{(1)})t} \\
P_{2^{(1)}}(t) &= B_{717}\,e^{-C_{2,3}(0)t} + B_{718}\,e^{-C_{2,3}(1)t} + B_{719}\,e^{-C_{2,3}(2)t} +
\end{aligned}$$

$$B_{720}\, e^{-C_{2,3}(0^{(1)})t} + B_{721}\, e^{-C_{2,3}(1^{(1)})t} + B_{722}\, e^{-C_{2,3}(2^{(1)})t}$$

$$\begin{aligned}
P_{3^{(1)}}(t) &= B_{723}\, e^{-C_{2,3}(0)t} + B_{724}\, e^{-C_{2,3}(1)t} + B_{725}\, e^{-C_{2,3}(2)t} + \\
&\quad B_{726}\, e^{-C_{2,3}(3)t} + B_{727}\, e^{-C_{2,3}(0^{(1)})t} + B_{728}\, e^{-C_{2,3}(1^{(1)})t} + \\
&\quad B_{729}\, e^{-C_{2,3}(2^{(1)})t} + B_{730}\, e^{-C_{2,3}(3^{(1)})t} \\
P_{0^{(2)}}(t) &= B_{731}\, e^{-C_{2,3}(0)t} + B_{732}\, e^{-C_{2,3}(0^{(1)})t} + B_{733}\, e^{-C_{2,3}(0^{(2)})t} \\
P_{1^{(2)}}(t) &= B_{734}\, e^{-C_{2,3}(0)t} + B_{735}\, e^{-C_{2,3}(1)t} + B_{736}\, e^{-C_{2,3}(0^{(1)})t} + \\
&\quad B_{737}\, e^{-C_{2,3}(1^{(1)})t} + B_{738}\, e^{-C_{2,3}(0^{(2)})t} + B_{739}\, e^{-C_{2,3}(1^{(2)})t} \\
P_{2^{(2)}}(t) &= B_{740}\, e^{-C_{2,3}(0)t} + B_{741}\, e^{-C_{2,3}(1)t} + B_{742}\, e^{-C_{2,3}(2)t} + \\
&\quad B_{743}\, e^{-C_{2,3}(0^{(1)})t} + B_{744}\, e^{-C_{2,3}(1^{(1)})t} + B_{745}\, e^{-C_{2,3}(2^{(1)})t} + \\
&\quad B_{746}\, e^{-C_{2,3}(0^{(2)})t} + B_{747}\, e^{-C_{2,3}(1^{(2)})t} + B_{748}\, e^{-C_{2,3}(2^{(2)})t} \\
P_{0^{(3)}}(t) &= B_{749}\, e^{-C_{2,3}(0)t} + B_{750}\, e^{-C_{2,3}(0^{(1)})t} + B_{751}\, e^{-C_{2,3}(0^{(2)})t} + \\
&\quad B_{752}\, e^{-C_{2,3}(0^{(3)})t} \\
P_{1^{(3)}}(t) &= B_{753}\, e^{-C_{2,3}(0)t} + B_{754}\, e^{-C_{2,3}(1)t} + B_{755}\, e^{-C_{2,3}(0^{(1)})t} + \\
&\quad B_{756}\, e^{-C_{2,3}(1^{(1)})t} + B_{757}\, e^{-C_{2,3}(0^{(2)})t} + B_{758}\, e^{-C_{2,3}(1^{(2)})t} + \\
&\quad B_{759}\, e^{-C_{2,3}(0^{(3)})t} + B_{760}\, e^{-C_{2,3}(1^{(3)})t}
\end{aligned}$$

The residues B_{699} to B_{760} are defined in Appendix H. System reliability values obtained with (4.61) are given in the third column of Table 4.11.

The system mean time to failure is given by:

$$MTTF_{Z_F}(2,3,1) = \sum_{i=0}^{4} P_i(0) + \sum_{i=0^{(1)}}^{3^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{2^{(2)}} P_i(0) + \sum_{i=0^{(3)}}^{1^{(3)}} P_i(0) \qquad (4.62)$$

where

$P_0(0) = 1/C_{2,3}(0)$

$P_{0^{(1)}}(0) = (3\lambda_s/C_{2,3}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (2\lambda_1/C_{2,3}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (2\lambda_1/C_{2,3}(1^{(1)})) \bullet P_{0^{(1)}}(0) + (3\lambda_s/C_{2,3}(1^{(1)})) \bullet P_1(0)$

$P_2(0) = (\lambda_1/C_{2,3}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (2\lambda_s/C_{2,3}(2^{(1)})) \bullet P_2(0) + (\lambda_1/C_{2,3}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{2,3}(3)) \bullet P_2(0)$

$P_4(0) = (\lambda_1/C_{2,3}(4)) \bullet P_3(0)$

$P_{3^{(1)}}(0) = (\lambda_s/C_{2,3}(3^{(1)})) \bullet P_3(0) + (\lambda_1/C_{2,3}(3^{(1)})) \bullet P_{2^{(1)}}(0)$

$P_{0^{(2)}} = (2\lambda_s/C_{2,3}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (2\lambda_s/C_{2,3}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (2\lambda_1/C_{2,3}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

$P_{2^{(2)}}(0) = (\lambda_s/C_{2,3}(2^{(2)})) \bullet P_{2^{(1)}}(0) + (\lambda_1/C_{2,3}(2^{(2)})) \bullet P_{1^{(2)}}(0)$

$P_{0^{(3)}} = (\lambda_s/C_{2,3}(0^{(3)})) \bullet P_{0^{(2)}}(0)$

$P_{1^{(3)}}(0) = (\lambda_s/C_{2,3}(1^{(3)})) \bullet P_{1^{(2)}}(0) + (2\lambda_1/C_{2,3}(1^{(3)})) \bullet P_{0^{(3)}}(0)$

and,

$C_{2,3}(0) = 2\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{2,3}(1) = \lambda_1 + 3\lambda_s + \lambda_{c1}$

$C_{2,3}(2) = \lambda_1 + 2\lambda_s + \lambda_{c2}$

$C_{2,3}(3) = \lambda_1 + \lambda_s + \lambda_{c3}$

$C_{2,3}(4) = \lambda_1 + \lambda_{c4}$

$C_{2,3}(0^{(1)}) = 2\lambda_1 + 2\lambda_s + \lambda_{c0^{(1)}}$

$C_{2,3}(1^{(1)}) = \lambda_1 + \lambda_{c1^{(1)}} + 2\lambda_s$

$C_{2,3}(2^{(1)}) = \lambda_1 + \lambda_s + \lambda_{c2^{(1)}}$

$C_{2,3}(3^{(1)}) = \lambda_1 + \lambda_{c3^{(1)}}$

$C_{2,3}(0^{(2)}) = 2\lambda_1 + \lambda_s + \lambda_{c0^{(2)}}$

$C_{2,3}(1^{(2)}) = \lambda_1 + \lambda_s + \lambda_{c1^{(2)}}$

$C_{2,3}(2^{(2)}) = \lambda_1 + \lambda_{c2^{(2)}}$

$C_{2,3}(0^{(3)}) = 2\lambda_1 + \lambda_{c0^{(3)}}$

$C_{2,3}(1^{(3)}) = \lambda_1 + \lambda_{c1^{(3)}}$

Equation (4.62) is used to generate the system mean time to failure values shown in the third column of Table 4.12.

The variance of time to failure is given by:

$$\begin{aligned} \sigma^2_{Z_F}(2,3,1) &= -2 \lim_{s \to 0} R'_{Z_F}(2,3,1)(s) - [MTTF_{Z_F}(2,3,1)]^2 \\ &= \frac{1}{k_{62}^2}\,[2(k_{61}\,k_{63} - k_{62}\,k_{64}) - k_{61}^2] \end{aligned} \tag{4.63}$$

The symbols k_{61} to k_{64} are defined in Appendix H.

Table 4.11: $R_{Z_F}(n,3,1)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$n=1$	$n=2$	$n=3$
0	1.0000	1.0000	1.0000
2000	0.9568	0.9920	0.9890
4000	0.7860	0.8656	0.8382
6000	0.5919	0.6563	0.5750
8000	0.4027	0.4715	0.3470
∞	0	0	0

Table 4.12: $MTTF_{Z_F}(n,3,1)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$n=1$	$n=2$	$n=3$
0.00001	8145.75	8802.06	8605.25
0.00002	7951.03	8672.34	8498.77
0.00003	7763.73	8545.74	8394.32
0.00004	7583.48	8422.20	8291.82

Special Case Model II

For $n = 3$ with Type F Repair, the system reliability is given by:

$$R_{Z_F}(3,3,1)(t) = \sum_{i=0}^{5} P_i(t) + \sum_{i=0^{(1)}}^{4^{(1)}} P_i(t) + \sum_{i=0^{(2)}}^{3^{(2)}} P_i(t) + \sum_{i=0^{(3)}}^{2^{(3)}} P_i(t) \tag{4.64}$$

where,

$$\begin{aligned}
P_0(t) &= e^{-C_{3,3}(0)t} \\
P_1(t) &= B_{761}\left[e^{-C_{3,3}(1)t} - e^{-C_{3,3}(0)t}\right] \\
P_2(t) &= B_{762}\, e^{-C_{3,3}(0)t} + B_{763}\, e^{-C_{3,3}(1)t} + B_{764}\, e^{-C_{3,3}(2)t} \\
P_3(t) &= B_{765}\, e^{-C_{3,3}(0)t} + B_{766}\, e^{-C_{3,3}(1)t} + B_{767}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{768}\, e^{-C_{3,3}(3)t} \\
P_4(t) &= B_{769}\, e^{-C_{3,3}(0)t} + B_{770}\, e^{-C_{3,3}(1)t} + B_{771}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{772}\, e^{-C_{3,3}(3)t} + B_{773}\, e^{-C_{3,3}(4)t} \\
P_5(t) &= B_{774}\, e^{-C_{3,3}(0)t} + B_{775}\, e^{-C_{3,3}(1)t} + B_{776}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{777}\, e^{-C_{3,3}(3)t} + B_{778}\, e^{-C_{3,3}(4)t} + B_{779}\, e^{-C_{3,3}(5)t} \\
P_{0^{(1)}}(t) &= B_{780}\left[e^{-C_{3,3}(0)t} - e^{-C_{3,3}(0^{(1)})t}\right] \\
P_{1^{(1)}}(t) &= B_{781}\, e^{-C_{3,3}(0)t} + B_{782}\, e^{-C_{3,3}(1)t} + B_{783}\, e^{-C_{3,3}(0^{(1)})t} + \\
&\quad B_{784}\, e^{-C_{3,3}(1^{(1)})t} \\
P_{2^{(1)}}(t) &= B_{785}\, e^{-C_{3,3}(0)t} + B_{786}\, e^{-C_{3,3}(1)t} + B_{787}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{788}\, e^{-C_{3,3}(0^{(1)})t} + B_{789}\, e^{-C_{3,3}(1^{(1)})t} + B_{790}\, e^{-C_{3,3}(2^{(1)})t} \\
P_{3^{(1)}}(t) &= B_{791}\, e^{-C_{3,3}(0)t} + B_{792}\, e^{-C_{3,3}(1)t} + B_{793}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{794}\, e^{-C_{3,3}(3)t} + B_{795}\, e^{-C_{3,3}(0^{(1)})t} + B_{796}\, e^{-C_{3,3}(1^{(1)})t} + \\
&\quad B_{797}\, e^{-C_{3,3}(2^{(1)})t} + B_{798}\, e^{-C_{3,3}(3^{(1)})t} \\
P_{4^{(1)}}(t) &= B_{799}\, e^{-C_{3,3}(0)t} + B_{800}\, e^{-C_{3,3}(1)t} + B_{801}\, e^{-C_{3,3}(2)t} + \\
&\quad B_{802}\, e^{-C_{3,3}(3)t} + B_{803}\, e^{-C_{3,3}(4)t} + B_{804}\, e^{-C_{3,3}(0^{(1)})t} + \\
&\quad B_{805}\, e^{-C_{3,3}(1^{(1)})t} + B_{806}\, e^{-C_{3,3}(2^{(1)})t} + B_{807}\, e^{-C_{3,3}(3^{(1)})t} + \\
&\quad B_{808}\, e^{-C_{3,3}(4^{(1)})t} \\
P_{0^{(2)}}(t) &= B_{809}\, e^{-C_{3,3}(0)t} + B_{810}\, e^{-C_{3,3}(0^{(1)})t} + B_{811}\, e^{-C_{3,3}(0^{(2)})t}
\end{aligned}$$

$$
\begin{aligned}
P_{1^{(2)}}(t) &= B_{812}\,e^{-C_{3,3}(0)t} + B_{813}\,e^{-C_{3,3}(1)t} + B_{814}\,e^{-C_{3,3}(0^{(1)})t} + \\
&\quad B_{815}\,e^{-C_{3,3}(1^{(1)})t} + B_{816}\,e^{-C_{3,3}(0^{(2)})t} + B_{817}\,e^{-C_{3,3}(1^{(2)})t} \\
P_{2^{(2)}}(t) &= B_{818}\,e^{-C_{3,3}(0)t} + B_{819}\,e^{-C_{3,3}(1)t} + B_{820}\,e^{-C_{3,3}(2)t} + \\
&\quad B_{821}\,e^{-C_{3,3}(0^{(1)})t} + B_{822}\,e^{-C_{3,3}(1^{(1)})t} + B_{823}\,e^{-C_{3,3}(2^{(1)})t} + \\
&\quad B_{824}\,e^{-C_{3,3}(0^{(2)})t} + B_{825}\,e^{-C_{3,3}(1^{(2)})t} + B_{826}\,e^{-C_{3,3}(2^{(2)})t} \\
P_{3^{(2)}}(t) &= B_{827}\,e^{-C_{3,3}(0)t} + B_{828}\,e^{-C_{3,3}(1)t} + B_{829}\,e^{-C_{3,3}(2)t} + \\
&\quad B_{830}\,e^{-C_{3,3}(3)t} + B_{831}\,e^{-C_{3,3}(0^{(1)})t} + B_{832}\,e^{-C_{3,3}(1^{(1)})t} + \\
&\quad B_{833}\,e^{-C_{3,3}(2^{(1)})t} + B_{834}\,e^{-C_{3,3}(0^{(2)})t} + B_{835}\,e^{-C_{3,3}(1^{(2)})t} + \\
&\quad B_{836}\,e^{-C_{3,3}(2^{(2)})t} + B_{837}\,e^{-C_{3,3}(3^{(2)})t} \\
P_{0^{(3)}}(t) &= B_{838}\,e^{-C_{3,3}(0)t} + B_{839}\,e^{-C_{3,3}(0^{(1)})t} + B_{840}\,e^{-C_{3,3}(0^{(2)})t} + \\
&\quad B_{841}\,e^{-C_{3,3}(0^{(3)})t} \\
P_{1^{(3)}}(t) &= B_{842}\,e^{-C_{3,3}(0)t} + B_{843}\,e^{-C_{3,3}(1)t} + B_{844}\,e^{-C_{3,3}(0^{(1)})t} + \\
&\quad B_{845}\,e^{-C_{3,3}(1^{(1)})t} + B_{846}\,e^{-C_{3,3}(0^{(2)})t} + B_{847}\,e^{-C_{3,3}(1^{(2)})t} + \\
&\quad B_{848}\,e^{-C_{3,3}(0^{(3)})t} + B_{849}\,e^{-C_{3,3}(1^{(3)})t} \\
P_{2^{(3)}}(t) &= B_{850}\,e^{-C_{3,3}(0)t} + B_{851}\,e^{-C_{3,3}(1)t} + B_{852}\,e^{-C_{3,3}(2)t} + \\
&\quad B_{853}\,e^{-C_{3,3}(0^{(1)})t} + B_{854}\,e^{-C_{3,3}(1^{(1)})t} + B_{855}\,e^{-C_{3,3}(2^{(1)})t} + \\
&\quad B_{856}\,e^{-C_{3,3}(0^{(2)})t} + B_{857}\,e^{-C_{3,3}(1^{(2)})t} + B_{858}\,e^{-C_{3,3}(2^{(2)})t} + \\
&\quad B_{859}\,e^{-C_{3,3}(0^{(3)})t} + B_{860}\,e^{-C_{3,3}(1^{(3)})t} + B_{861}\,e^{-C_{3,3}(2^{(3)})t}
\end{aligned}
$$

The residues B_{761} to B_{861} are defined in Appendix H. System reliability values obtained with (4.64) are given in the fourth column of Table 4.11.

The system mean time to failure is given by:

$$
\begin{aligned}
MTTF_{Z_P}(3,3,1) &= \sum_{i=0}^{5} P_i(0) + \sum_{i=0^{(1)}}^{4^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{3^{(2)}} P_i(0) + \\
&\quad \sum_{i=0^{(3)}}^{2^{(3)}} P_i(0)
\end{aligned}
\tag{4.65}
$$

where

$P_0(0) = 1/C_{3,3}(0)$

$P_{0^{(1)}}(0) = (3\lambda_s/C_{3,3}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (3\lambda_1/C_{3,3}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (3\lambda_1/C_{3,3}(1^{(1)})) \bullet P_{0^{(1)}}(0) + (3\lambda_s/C_{3,3}(1^{(1)})) \bullet P_1(0)$

$P_2(0) = (2\lambda_1/C_{3,3}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (3\lambda_s/C_{3,3}(2^{(1)})) \bullet P_2(0) + (2\lambda_1/C_{3,3}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{3,3}(3)) \bullet P_2(0)$

$P_4(0) = (\lambda_1/C_{3,3}(4)) \bullet P_3(0)$

$P_5(0) = (\lambda_1/C_{3,3}(5)) \bullet P_4(0)$

$P_{3^{(1)}}(0) = (2\lambda_s/C_{3,3}(3^{(1)})) \bullet P_3(0) + (\lambda_1/C_{3,3}(3^{(1)})) \bullet P_{2^{(1)}}(0)$

$P_{4^{(1)}}(0) = (\lambda_s/C_{3,3}(4^{(1)})) \bullet P_4(0) + (\lambda_1/C_{3,3}(4^{(1)})) \bullet P_{3^{(1)}}(0)$

$P_{0^{(2)}} = (2\lambda_s/C_{3,3}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (2\lambda_s/C_{3,3}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (3\lambda_1/C_{3,3}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

$P_{2^{(2)}}(0) = (2\lambda_s/C_{3,3}(2^{(2)})) \bullet P_{2^{(1)}}(0) + (2\lambda_1/C_{3,3}(2^{(2)})) \bullet P_{1^{(2)}}(0)$

$P_{3^{(2)}}(0) = (\lambda_s/C_{3,3}(3^{(2)})) \bullet P_{3^{(1)}}(0) + (\lambda_1/C_{3,3}(3^{(2)})) \bullet P_{2^{(2)}}(0)$

$P_{0^{(3)}} = (\lambda_s/C_{3,3}(0^{(3)})) \bullet P_{0^{(2)}}(0)$

$P_{1^{(3)}}(0) = (\lambda_s/C_{3,3}(1^{(3)})) \bullet P_{1^{(2)}}(0) + (3\lambda_1/C_{3,3}(1^{(3)})) \bullet P_{0^{(3)}}(0)$

$P_{2^{(3)}}(0) = (\lambda_s/C_{3,3}(2^{(3)})) \bullet P_{2^{(2)}}(0) + (2\lambda_1/C_{3,3}(2^{(3)})) \bullet P_{1^{(3)}}(0)$

and,

$C_{3,3}(0) = 3\lambda_1 + 3\lambda_s + \lambda_{c0}$

$C_{3,3}(1) = 2\lambda_1 + 3\lambda_s + \lambda_{c1}$

$C_{3,3}(2) = \lambda_1 + 3\lambda_s + \lambda_{c2}$

$C_{3,3}(3) = \lambda_1 + 2\lambda_s + \lambda_{c3}$

$C_{3,3}(4) = \lambda_1 + \lambda_s + \lambda_{c4}$

$C_{3,3}(5) = \lambda_1 + \lambda_{c5}$

$C_{3,3}(0^{(1)}) = 3\lambda_1 + 2\lambda_s + \lambda_{c0^{(1)}}$

$C_{3,3}(1^{(1)}) = 2\lambda_1 + \lambda_{c1^{(1)}} + 2\lambda_s$

$C_{3,3}(2^{(1)}) = \lambda_1 + 2\lambda_s + \lambda_{c2^{(1)}}$

$C_{3,3}(3^{(1)}) = \lambda_1 + \lambda_s + \lambda_{c3^{(1)}}$

$C_{3,3}(4^{(1)}) = \lambda_1 + \lambda_{c4^{(1)}}$

$C_{3,3}(0^{(2)}) = 3\lambda_1 + \lambda_s + \lambda_{c0^{(2)}}$

$C_{3,3}(1^{(2)}) = 2\lambda_1 + \lambda_s + \lambda_{c1^{(2)}}$

$C_{3,3}(2^{(2)}) = \lambda_1 + \lambda_s + \lambda_{c2^{(2)}}$

$C_{3,3}(3^{(2)}) = \lambda_1 + \lambda_{c3^{(2)}}$

$C_{3,3}(0^{(3)}) = 3\lambda_1 + \lambda_{c0^{(3)}}$

$C_{3,3}(1^{(3)}) = 2\lambda_1 + \lambda_{c1^{(3)}}$

$C_{3,3}(2^{(3)}) = \lambda_1 + \lambda_{c2^{(3)}}$

Equation (4.65) was used to generate the mean time to failure values shown in the fourth column of Table 4.12.

The system variance of time to failure is:

$$\begin{aligned}\sigma_{Z_F}(3,3,1) &= -2\lim_{s\to 0} R_{Z_F}(3,3,1)(s) - [MTTF_{Z_F}(3,3,1)]^2 \\ &= \frac{1}{k_{66}^2}[2(k_{65}\,k_{67} - k_{66}\,k_{68}) - k_{65}^2] \qquad (4.66)\end{aligned}$$

The symbols k_{65} to k_{68} are defined in Appendix H.

From Equations (4.36), (4.39), (4.62), and (4.65), a generalized approach for determining the mean time to failure of a $1-out-of-(n+m):G$ system operating under standby activation policy Z with Type F Repair is developed using Equation (2.11) in conjunction with the following set of recursive relationships:

Letting $\lim_{s\to 0} P_{j^{(i)}}(s) = P_{j^{(i)}}(0)$; for $i = 0,1,2,.....,m$ and for $j = 0,1,2,3,...,(n+m-(i+1))$ then:

For $j = 1,2,3,....n-1$ and for $i = 0,1,2,3,...,m$:

$P_{j^{(i)}}(0) = [((n-j+1)\,\lambda_1)/C_{n,m}(j^{(i)})] \bullet P_{j-1^{(i)}}(0) + [((m-i+1)\,\lambda_s)/C_{n,m}(j^{(i)})] \bullet P_{j^{(i-1)}}(0)$

For $j = n, n+1, n+2, ...(n+m-(1+i))$ and for $i = 0,1,2,3,...,m$:

$P_{j^{(i)}}(0) = [(\lambda_1)/C_{n,m}(j^{(i)})] \bullet P_{j-1^{(i)}}(0) + [((m+n)-(i+j))\,\lambda_s)/C_{n,m}(j^{(i)})] \bullet P_{j^{(i-1)}}(0)$

The starting values for the above recursive procedure are obtained via the following recursive relationships:

$P_0(0) = P_{0^{(0)}} = 1/C_{n,m}(0)$

and for $i = 1,2,....m$:

$$P_{0^{(i)}}(0) = [((m-i+1)\,\lambda_s)/C_{n,m}(0^{(i)})] \bullet P_{0^{(i-1)}}(0)$$

4.1.4 k-out-of-(n+1):G Identical Unit Submodel

This is the $(n, 1, k)$ system and is composed of n active units and *one* standby. It is functional as long as there are at least k healthy units. Where $k \leq n$. The block diagram of this system is shown in Figure 3.22 and the state space diagram in Figure 4.12.

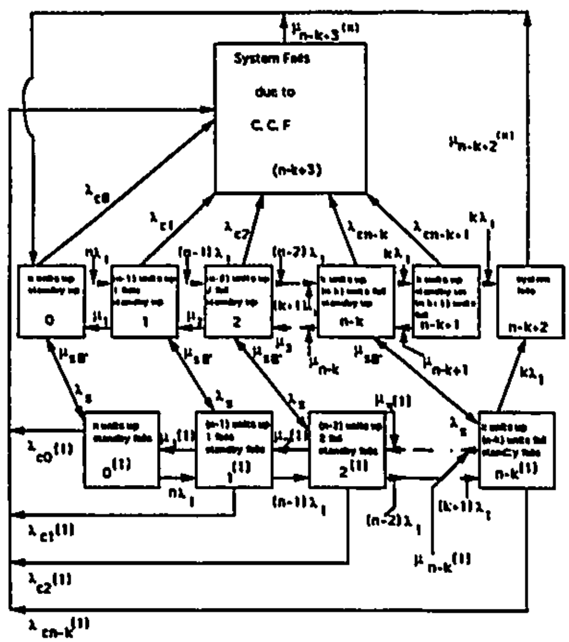


Figure 4.12: State space diagram of the $k-out-of-(n+1):G$ System under SBAP Z

<u>$k-out-of-(n+1):G$ System Equations</u>

The descriptive integro-differential Equations for the system shown in Figure 4.12 are:

$$\frac{dP_0(t)}{dt} + C_{n,1}(0)\,P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\,P_{0^{(1)}}(t) + \sum_{i=r+1}^{r+2} \int_0^\infty P_i(x,t)\,\mu_i(x)dx \quad (4.67)$$

where $r = n - k + 1$,

For $j = 1, 2, \ldots\ldots, n - k$,

$$\frac{dP_j(t)}{dt} + C_{n,1}(j)\, P_j(t) = (n - j + 1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) +$$
$$\mu_{j+1} P_{j+1}(t) \tag{4.68}$$

$$\frac{dP_r(t)}{dt} + C_{n,1}(r)\, P_r(t) = k\lambda_1\, P_{n-k}(t) \tag{4.69}$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,1}(0^{(1)})\, P_{0^{(1)}}(t) = \lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) \tag{4.70}$$

For $j = 1, 2, \ldots\ldots, n - k - 1$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,1}(j^{(1)})\, P_{j^{(1)}}(t) = \lambda_s P_j(t) + (n - j + 1)\, \lambda_1\, P_{j-1^{(1)}}(t) +$$
$$\mu_{j+1^{(1)}}\, P_{j+1^{(1)}}(t) \tag{4.71}$$

$$\frac{dP_{n-k^{(1)}}(t)}{dt} + C_{n,1}(n - k^{(1)})\, P_{n-k^{(1)}}(t) = \lambda_s\, P_{n-k}(t) + (k+1)\lambda_1 P_{n-k-1^{(1)}}(t) \tag{4.72}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+1}(x)]\, P_{r+1}(x,t) = 0 \tag{4.73}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+2}(x)]\, P_{r+2}(x,t) = 0 \tag{4.74}$$

The associated boundary conditions are:

$$P_{r+1}(0,t) = k\lambda_1\, P_r(t) + k\lambda_1\, P_{n-k^{(1)}}(t) \tag{4.75}$$

$$P_{r+2}(0,t) = \sum_{j=0}^{r} \lambda_{cj}\, P_j(t) + \sum_{j=0^{(1)}}^{n-k^{(1)}} \lambda_{cj} P_j(t) \tag{4.76}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,1}(0) = n\,\lambda_1 + \lambda_s + \lambda_{c0}$

For $j = 1, 2, \ldots\ldots n - k$,

$C_{n,1}(j) = (n - j)\,\lambda_1 + \lambda_{cj} + \mu_j + \lambda_s$

$C_{n,1}(r) = k\lambda_1 + \lambda_{cr} + \mu_r$

$C_{n,1}(0^{(1)}) = n\,\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$

For $j = 1, 2, \ldots\ldots\ldots, n - k - 1$,

$C_{n,1}(j^{(1)}) = (n - j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s0'}$

$C_{n,1}(n - k^{(1)}) = \lambda_1 + \lambda_{cn-k^{(1)}} + \mu_{n-k^{(1)}} + \mu_{s0'}$

$k - out - of - (n+1) : G$ System With Type B Repair

With Type B Repair, the active units in the system are discarded upon failure thus μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{n-k^{(1)}}$ are set to zero in Equations (4.67) to (4.76).

Special Case Model I

For $n = 3$ and $k = 2$, with Type B Repair, the long-run availability values are obtained directly from the results determined for the $(3,1,1)$ special case with Type B Repair discussed in Subsection 4.1.1. Thus its long-run availability values can be determined from Equation (4.12) by eliminating *system up-states 3* and $2^{(1)}$ and constants associated with them. The results obtained thus are presented in the third column of Table 4.13. For the sake of completeness, the long-run availability values for the $(3,1,1)$ special case have been re-produced in the second column of Table 4.13.

$k - out - of - (n+1) : G$ System with Type C Repair

With Type C Repair, the units (both active and standbys) are discarded and replaced upon failure thus $\mu_{s0'}$, μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{n-k^{(1)}}$ are set to zero in Equations (4.67) to (4.76).

Special Case Model I

For $n = 3$, and $k = 2$ with Type C Repair, the long-run availability values are obtained from the results determined for the $(3,1,2)$ special case with Type B Repair by setting $\mu_{s0'}$ to zero. These values are given in the third column of Table 4.14. For the sake of completeness, the results obtained for the $(3,1,1)$ special case have been re-produced in the second column of Table 4.14.

Table 4.13: $A_{ssz}(3,1,k)(\beta)$ values as a function of λ_{c0} for Type B Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4963	0.3133	0.1951
	0.00002	0.4945	0.3122	0.1945
	0.00003	0.4928	0.3111	0.1938
	0.00004	0.4910	0.3100	0.1932
$\beta=2$	0.00001	0.3310	0.1866	0.1081
	0.00002	0.3295	0.1859	0.1077
	0.00003	0.3279	0.1851	0.1073
	0.00004	0.3263	0.1843	0.1069
$\beta=3$	0.00001	0.2484	0.1329	0.0748
	0.00002	0.2470	0.1323	0.0745
	0.00003	0.2457	0.1317	0.0742
	0.00004	0.2444	0.1311	0.0739
$\beta=4$	0.00001	0.1987	0.1032	0.0571
	0.00002	0.1976	0.1027	0.0569
	0.00003	0.1965	0.1022	0.0567
	0.00004	0.1954	0.1018	0.0565
$\beta=5$	0.00001	0.1656	0.0843	0.0462
	0.00002	0.1646	0.0839	0.0461
	0.00003	0.1637	0.0835	0.0459
	0.00004	0.1627	0.0832	0.0457

Table 4.14: $A_{ssz}(3,1,k)\,(\beta)$ values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.4811	0.3170	0.1932
	0.00002	0.4795	0.3159	0.1926
	0.00003	0.4778	0.3148	0.1920
	0.00004	0.4762	0.3137	0.1914
$\beta=2$	0.00001	0.3168	0.1883	0.1069
	0.00002	0.3153	0.1876	0.1065
	0.00003	0.3139	0.1868	0.1062
	0.00004	0.3125	0.1861	0.1058
$\beta=3$	0.00001	0.2361	0.1340	0.0739
	0.00002	0.2349	0.1334	0.0736
	0.00003	0.2337	0.1328	0.0734
	0.00004	0.2325	0.1322	0.0731
$\beta=4$	0.00001	0.1882	0.1040	0.0565
	0.00002	0.1872	0.1035	0.0563
	0.00003	0.1862	0.1030	0.0561
	0.00004	0.1852	0.1026	0.0559
$\beta=5$	0.00001	0.1564	0.0849	0.0457
	0.00002	0.1556	0.0845	0.0455
	0.00003	0.1547	0.0842	0.0454
	0.00004	0.1538	0.0838	0.0452

$k - out - of - (n+1) : G$ System With Type E Repair

With Type E Repair, the system as well as its active units are discarded and replaced upon failure thus $\mu_{r+1}(x)$, $\mu_{r+2}(x)$, μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{n-k^{(1)}}$ are set to zero in Equations (4.67) to (4.76).

Special Case Model I

For $n = 3$, and $k = 2$ with Type E Repair, the relevant reliability indices can be obtained directly from the results obtained for the $(3,1,1)$ case with Type E Repair in Subsection (4.1.1) as follows:
The system reliability is obtained from Equation (4.16) by eliminating *up-states 3* and $2^{(1)}$ and the partial fraction residues associated with them. The results obtained are given in the third column of Table 4.15. For the sake of completeness, the system reliability values obtained for the $(3,1,1)$ case with Type E Repair are shown in the second column of Table 4.15.

Table 4.15: $R_{Z_E}(3,1,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9191	0.5926	0.2755
4000	0.6049	0.1882	0.0388
6000	0.3240	0.0465	0.0046
8000	0.1570	0.0103	0.0005
∞	0	0	0

The system mean time to failure is obtained directly from Equation (4.17)) by eliminating terms associated with *system up-states 3* and $2^{(1)}$ and can be expressed as:

$$MTTF_{Z_E}(3,1,2) = \sum_{i=0}^{2} P_i(0) + \sum_{i=0^{(1)}}^{1^{(1)}} P_i(0) \tag{4.77}$$

where,

$P_0(0) = C_{3,1}(0^{(1)})/(C_{3,1}(0)\, C_{3,1}(0^{(1)}) - \lambda_s\, \mu_{s0'})$

$P_{0^{(1)}}(0) = \lambda_s/(C_{3,1}(0)\, C_{3,1}(0^{(1)}) - \lambda_s\, \mu_{s0'})$

$P_1(0) = (3\lambda_1/C_{3,1}(1)\, C_{3,1}(1^{(1)}) - \lambda_s\, \mu_{s0'}) \bullet P_0(0) + (3\lambda_1\, \mu_{s0'}/C_{3,1}(1)\, C_{3,1}(1^{(1)}) - \lambda_s \mu_{s0'}) \bullet$
$P_{0^{(1)}}(0)$

$P_{1^{(1)}}(0) = (3\lambda_1/C_{3,1}(1^{(1)})) \bullet P_{0^{(1)}}(0) + (\lambda_s/C_{3,1}(1^{(1)})) \bullet P_1(0)$

$P_2(0) = (2\lambda_1/C_{3,1}(2)) \bullet P_1(0)$

The variance of time to failure for this case is determined directly from Equation (4.18) by eliminating the constants associated with *system up-states 3* and $2^{(1)}$.

From Equations (4.17) and (4.77) and with the results of several more special cases, a generalized approach of determining the mean time to failure of a $k-out-of-(n+1):G$ type system operating under standby activation policy Z with Type E Repair, is developed using Equation (2.11) in conjunction with the following set of recursive relationships:

Letting $\lim_{s\to 0} P_i(s) = P_i(0)$; for $i = 0,1,2,....,n$ and $\lim_{s\to 0} P_j(s) = P_j(0)$; for $j = 0^{(1)}, 1^{(1)}, 2^{(1)}, ..., (n-1)^{(1)}$ then:

$P_0(0) = [C_{n,1}(0^{(1)})]/[C_{n,1}(0)\, C_{n,1}(0^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{0^{(1)}}(0) = \lambda_s/[C_{n,1}(0)\, C_{n,1}(0^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_1(0) = [n\lambda_1\, C_{n,1}(1^{(1)}) \bullet P_0(0) + n\lambda_1\, \mu_{s0'} \bullet P_{0^{(1)}}(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{1^{(1)}}(0) = [n\lambda_1\, C_{n,1}(1) \bullet P_{0^{(1)}}(0) + n\lambda_1\, \lambda_s \bullet P_0(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_2(0) = [(n-1)\lambda_1\, C_{n,1}(2^{(1)}) \bullet P_1(0) + (n-1)\lambda_1\, \mu_{s0'} \bullet P_{1^{(1)}}(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{2^{(1)}}(0) = [(n-1)\lambda_1\, C_{n,1}(2) \bullet P_{1^{(1)}}(0) + (n-1)\lambda_1\, \lambda_s \bullet P_1(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_3(0) = [(n-2)\lambda_1\, C_{n,1}(3^{(1)}) \bullet P_2(0) + (n-2)\lambda_1\, \mu_{s0'} \bullet P_{2^{(1)}}(0)]/[C_{n,1}(3)\, C_n(3^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{3^{(1)}}(0) = [(n-2)\lambda_1\, C_{n,1}(3) \bullet P_{2^{(1)}}(0) + (n-2)\lambda_1\, \lambda_s \bullet P_2(0)]/[C_{n,1}(3)\, C_{n,1}(3^{(1)}) - \mu_{s0'}\, \lambda_s]$

$\vdots$

$P_{n-k}(0) = [(k+1)\,\lambda_1\, C_{n,1}((n-k)^{(1)}) \bullet P_{n-k-1}(0) + (k+1)\,\lambda_1\, \mu_{s0'} \bullet P_{n-k-1^{(1)}}(0)]/[C_{n,1}(n-k)\, C_{n,1}((n-k)^{(1)}) - \mu_{s0'}\, \lambda_s]$

$P_{(n-k)^{(1)}}(0) = [(k+1)\,\lambda_1\, C_{n,1}(n-k) \bullet P_{n-k-1^{(1)}}(0) + (k+1)\,\lambda_1\,\lambda_s \bullet P_{n-k-1}(0)]/[C_{n,1}(n-k)\, C_{n,1}((n-k)^{(1)}) - \mu_{s0'}\,\lambda_s]$

$P_{n-k+1}(0) = [k\,\lambda_1/C_{n,1}(n-k+1)] \bullet P_{n-k}(0)$

For the special cases considered, this approach is used to compute the mean time to failure values given in Table 4.16.

where,

$C_{n,1}(0) = n\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = (n-1)\lambda_1 + \lambda_s + \lambda_{c1}$

$C_{n,1}(2) = (n-2)\lambda_1 + \lambda_s + \lambda_{c2}$

$C_{n,1}(3) = (n-3)\lambda_1 + \lambda_s + \lambda_{c3}$

$\vdots$

$C_{n,1}(n-k-1) = (k+1)\,\lambda_1 + \lambda_s + \lambda_{cn-k-1}$

$C_{n,1}(n-k) = k\,\lambda_1 + \lambda_s + \lambda_{cn-k}$

$C_{n,1}(n-k+1) = k\,\lambda_1 + \lambda_{cn-k+1}$

$C_{n,1}(0^{(1)}) = n\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'}$

$C_{n,1}(1^{(1)}) = (n-1)\lambda_1 + \lambda_{c1^{(1)}} + \mu_{s0'}$

$C_{n,1}(2^{(1)}) = (n-2)\lambda_1 + \lambda_{c2^{(1)}} + \mu_{s0'}$

$\vdots$

$C_{n,1}((n-k-1)^{(1)}) = (k+1)\,\lambda_1 + \lambda_{cn-k-1^{(1)}} + \mu_{s0'}$

$C_{n,1}((n-k)^{(1)}) = k\,\lambda_1 + \lambda_{cn-k^{(1)}} + \mu_{s0'}$

$k-out-of-(n+1):G$ System With Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are discarded and replaced upon failure thus $\mu_{s0'}$, $\mu_{r+1}(x)$, $\mu_{r+2}(x)$, μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{n-k^{(1)}}$ are set to zero in Equations (4.67) to (4.76).

Table 4.16: $MTTF_{Z_E}(3,1,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6565.40	3166.20	1605.44
0.00002	6451.29	3125.02	1589.78
0.00003	6340.04	3084.69	1574.37
0.00004	6231.55	3045.18	1559.23

Special Case Model I

For $n = 3$ and $k = 2$ with Type F Repair, the system reliability values can be determined from the results obtained for the $(3,1,2)$ special case with Type E Repair by setting $\mu_{s0'}$ to zero. The values obtained are given in the third column of Table 4.17. For the sake of completeness, the system reliability values obtained for the $(3,1,1)$ case with Type F Repair discussed in Subsection 4.1.1 are re-produced in the second column of Table 4.17.

Table 4.17: $R_{Z_F}(3,1,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9186	0.5826	0.2699
4000	0.6026	0.1796	0.0360
6000	0.3215	0.0433	0.0040
8000	0.1554	0.0094	0.0004
∞	0	0	0

The system mean time to failure can be obtained directly from Equation (4.77) by setting $\mu_{s0'}$ to zero.

The variance of time to failure for this case can be determined directly from Equation (4.18) by eliminating the constants associated with *system up-states 3* and $2^{(1)}$. In addition, $\mu_{s0'}$ is set to zero.

A generalized approach for determining the system mean time to failure with Type F Repair for a $k-out-of-(n+1):G$ type system under standby activation policy Z is easily derived from the same generalized relationships established under Type E Repair by setting $\mu_{s0'}$ to zero leading to the following set of recursive relationships:
Again letting $\lim_{s\to 0} P_i(s) = P_i(0)$; for $i = 0, 1, 2, ..., n$ and $\lim_{s\to 0} P_j(s) = P_j(0)$; for $j = 0^{(1)}, 1^{(1)}, 2^{(1)}, ..., (n-1)^{(1)}$ then:

$P_0(0) = 1/C_{n,1}(0)$

$P_{0^{(1)}}(0) = \lambda_s/[C_{n,1}(0)\, C_{n,1}(0^{(1)})]$

$P_1(0) = [n\lambda_1/C_{n,1}(1)] \bullet P_0(0)$

$P_{1^{(1)}}(0) = [n\lambda_1\, C_{n,1}(1) \bullet P_{0^{(1)}}(0) + n\lambda_1\, \lambda_s \bullet P_0(0)]/[C_{n,1}(1)\, C_{n,1}(1^{(1)})]$

$P_2(0) = [(n-1)\lambda_1/C_{n,1}(2)] \bullet P_1(0)$

$P_{2^{(1)}}(0) = [(n-1)\lambda_1\, C_{n,1}(2) \bullet P_{1^{(1)}}(0) + (n-1)\lambda_1\, \lambda_s \bullet P_1(0)]/[C_{n,1}(2)\, C_{n,1}(2^{(1)})]$

$P_3(0) = [(n-2)\lambda_1/C_{n,1}(3)] \bullet P_2(0)$

$P_{3^{(1)}}(0) = [(n-2)\lambda_1\, C_{n,1}(3) \bullet P_{2^{(1)}}(0) + (n-2)\lambda_1\lambda_s \bullet P_2(0)]/[C_{n,1}(3)\, C_{n,1}(3^{(1)})]$

$\vdots$

$P_{n-k}(0) = [(k+1)\,\lambda_1\, C_{n,1}((n-k)^{(1)}) \bullet P_{n-k-1}(0)]/[C_{n,1}(n-k)\, C_{n,1}((n-k)^{(1)})]$

$P_{(n-k)^{(1)}}(0) = [(k+1)\,\lambda_1\, C_{n,1}(n-k) \bullet P_{n-k-1^{(1)}}(0) + (k+1)\,\lambda_1\,\lambda_s \bullet P_{n-k-1}(0)]/[C_{n,1}(n-k)\, C_{n,1}((n-k)^{(1)})]$

$P_{n-k+1}(0) = [k\,\lambda_1/C_{n,1}(n-k+1)] \bullet P_{n-k}(0)$

For the special cases considered, this approach is used to compute the system mean time to failure values given in Table 4.18.

where

$C_{n,1}(0) = n\lambda_1 + \lambda_s + \lambda_{c0}$

$C_{n,1}(1) = (n-1)\lambda_1 + \lambda_s + \lambda_{c1}$

$$C_{n,1}(2) = (n-2)\lambda_1 + \lambda_s + \lambda_{c2}$$
$$C_{n,1}(3) = (n-3)\lambda_1 + \lambda_s + \lambda_{c3}$$
$$\vdots$$
$$C_{n,1}(n-k-1) = (k+1)\,\lambda_1 + \lambda_s + \lambda_{cn-k-1}$$
$$C_{n,1}(n-k) = k\,\lambda_1 + \lambda_s + \lambda_{cn-k}$$
$$C_{n,1}(n-k+1) = k\,\lambda_1 + \lambda_{cn-k+1}$$

$$C_{n,1}(0^{(1)}) = n\lambda_1 + \lambda_{c0^{(1)}}$$
$$C_{n,1}(1^{(1)}) = (n-1)\lambda_1 + \lambda_{c1^{(1)}}$$
$$C_{n,1}(2^{(1)}) = (n-2)\lambda_1 + \lambda_{c2^{(1)}}$$
$$\vdots$$
$$C_{n,1}((n-k-1)^{(1)}) = (k+1)\,\lambda_1 + \lambda_{cn-k-1^{(1)}}$$
$$C_{n,1}((n-k)^{(1)}) = k\,\lambda_1 + \lambda_{cn-k^{(1)}}$$

Table 4.18: $MTTF_{Z_F}(3,1,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$k=1$	$k=2$	$k=3$
0.00001	6089.71	3062.22	1585.61
0.00002	5993.80	3024.23	1570.48
0.00003	5900.11	2986.99	1555.61
0.00004	5808.59	2950.48	1540.98

4.1.5 k-out-of-(n+2):G Identical Unit Submodel

This is the $(n,2,k)$ system and is composed of n active units and *two* standbys. It is functional as long as there are at least k healthy units. Where $k \leq n$. The block diagram of the submodel is shown in Figure 3.24 and the state-space diagram in

Figure 4.13.

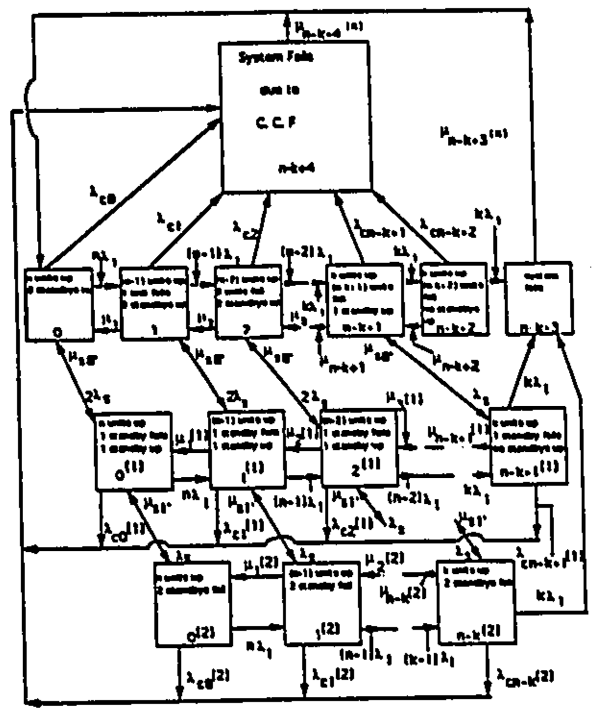


Figure 4.13: State space diagram of the $k - out - of - (n+2) : G$ system under SBAP Z

$k - out - of - (n+2) : G$ System Equations

The descriptive integro-differential Equations for the system shown in Figure 4.13 are:

$$\frac{dP_0(t)}{dt} + C_{n,2}(0)\,P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\,P_{0^{(1)}}(t) + \sum_{i=r+2}^{r+3} \int_0^\infty P_i(x,t)\,\mu_i(x)dx \quad (4.78)$$

where $r = n - k + 1$,

For $j = 1, 2, \ldots\ldots, r$,

$$\frac{dP_j(t)}{dt} + C_{n,2}(j)\,P_j(t) = (n-j+1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) + \mu_{j+1} P_{j+1}(t) \quad (4.79)$$

$$\frac{dP_{r+1}(t)}{dt} + C_{n,2}(r+1)\,P_{r+1}(t) = k\lambda_1\,P_r(t) + \mu_{r+2} P_{r+2}(t) + \mu_{s0'} P_{r+1^{(1)}}(t) \quad (4.80)$$

$$\frac{dP_{r+2}(t)}{dt} + C_{n,2}(r+2)\, P_{r+2}(t) = k\lambda_1\, P_{r+1}(t) \tag{4.81}$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,2}(0^{(1)})\, P_{0^{(1)}}(t) = 2\lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) + \mu_{s1'} P_{0^{(2)}}(t) \tag{4.82}$$

For $j = 1, 2, \ldots.., r-1$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,2}(j^{(1)})\, P_{j^{(1)}}(t) = 2\lambda_s P_j(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(1)}}(t) + \mu_{s1'} P_{j^{(2)}}(t) + \mu_{j+1^{(1)}}\, P_{j+1^{(1)}}(t) \tag{4.83}$$

$$\frac{dP_{r^{(1)}}(t)}{dt} + C_{n,2}(r^{(1)})\, P_{r^{(1)}}(t) = \lambda_s\, P_r(t) + k\lambda_1 P_{r-1^{(1)}}(t) \tag{4.84}$$

$$\frac{dP_{0^{(2)}}(t)}{dt} + C_{n,2}(0^{(2)})\, P_{0^{(2)}}(t) = \lambda_s P_{0^{(1)}}(t) + \mu_{1^{(2)}} P_{1^{(2)}}(t) \tag{4.85}$$

For $j = 1, 2, \ldots.., r-2$,

$$\frac{dP_{j^{(2)}}(t)}{dt} + C_{n,2}(j^{(2)})\, P_{j^{(2)}}(t) = \lambda_s P_{j^{(1)}}(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(2)}}(t) + \mu_{j+1^{(2)}}\, P_{j+1^{(2)}}(t) \tag{4.86}$$

$$\frac{dP_{n-k^{(2)}}(t)}{dt} + C_{n,2}(n-k^{(2)}) P_{n-k^{(2)}}(t) = (k+1)\lambda_1 P_{n-k-1^{(2)}}(t) + \lambda_s P_{n-k^{(1)}}(t) \tag{4.87}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+2}(x)\right] P_{r+2}(x,t) = 0 \tag{4.88}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r+3}(x)\right] P_{r+3}(x,t) = 0 \tag{4.89}$$

The associated boundary conditions are:

$$P_{r+2}(0,t) = k\lambda_1\, P_{r+1}(t) + k\lambda_1\, P_{r^{(1)}}(t) + k\lambda_1\, P_{r-1^{(2)}}(t) \tag{4.90}$$

$$P_{r+3}(0,t) = \sum_{j=0}^{r+1} \lambda_{cj}\, P_j(t) + \sum_{j=0^{(1)}}^{r^{(1)}} \lambda_{cj} P_j(t) + \sum_{j=0^{(2)}}^{r-1^{(2)}} \lambda_{cj} P_j(t) \tag{4.91}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,2}(0) = n\,\lambda_1 + 2\lambda_s + \lambda_{c0}$

For $j = 1, 2, \ldots..r$,

$C_{n,2}(j) = (n-j)\,\lambda_1 + \lambda_{cj} + \mu_j + \lambda_s$

$C_{n,2}(r+1) = k\lambda_1 + \lambda_{cr+1} + \mu_{r+1}$

$C_{n,2}(0^{(1)}) = n\,\lambda_1 + \lambda_{c0^{(1)}} + \mu_{s0'} + \lambda_s$

For $j = 1, 2, \ldots\ldots, r-1$,

$C_{n,2}(j^{(1)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s0'}$

$C_{n,1}(r^{(1)}) = k\lambda_1 + \lambda_{cr^{(1)}} + \mu_{r^{(1)}} + \mu_{s0'}$

$C_{n,2}(0^{(2)}) = n\,\lambda_1 + \lambda_{c0^{(2)}} + \mu_{s0'}$

For $j = 1, 2, \ldots\ldots, r-1$,

$C_{n,2}(j^{(2)}) = (n-j)\,\lambda_1 + \lambda_{cj^{(1)}} + \mu_{j^{(1)}} + \mu_{s1'}$

$k-out-of-(n+2):G$ System With Type C Repair

With Type C Repair, the units (both active and standbys) are discarded upon failure thus $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{r-1^{(1)}}$ are set to zero in Equations (4.78) to (4.91).

Special Case Model I

For $n = 3$ and $k = 2$ (abbreviated as the $(3,2,2)$ special case), with Type C Repair, the long-run availability for this special case is determined directly from the results obtained for the $(3,2,1)$ special case with Type C Repair discussed in Subsection 4.1.2. Thus, the long-run availability values for this case can be obtained from Equation (4.34) by eliminating *system up-states* 4, $3^{(1)}$ and $2^{(2)}$ and constants associated with them. The values determined are presented in the third column of Table 4.19. For the sake of completeness, the results obtained for the $(3,2,1)$ special case have been re-produced in the second column of Table 4.19.

$k-out-of-(n+2):G$ System With Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are discarded upon failure thus $\mu_{r+2}(x)$, $\mu_{r+3}(x)$, $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_r and $\mu_{1^{(1)}}$ to $\mu_{r-1^{(1)}}$ are set to zero in Equations (4.78) to (4.91).

Table 4.19: $A_{ssz}(3,2,k)\,(\beta)$ values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5319	0.3760	0.2496
	0.00002	0.5303	0.3748	0.2488
	0.00003	0.5287	0.3736	0.2481
	0.00004	0.5271	0.3724	0.2473
$\beta=2$	0.00001	0.3623	0.2315	0.1436
	0.00002	0.3608	0.2306	0.1431
	0.00003	0.3594	0.2297	0.1426
	0.00004	0.3579	0.2288	0.1421
$\beta=3$	0.00001	0.2747	0.1672	0.1008
	0.00002	0.2734	0.1665	0.1004
	0.00003	0.2722	0.1658	0.1000
	0.00004	0.2709	0.1651	0.0996
$\beta=4$	0.00001	0.2212	0.1309	0.0776
	0.00002	0.2201	0.1303	0.0773
	0.00003	0.2190	0.1298	0.0770
	0.00004	0.2179	0.1292	0.0767
$\beta=5$	0.00001	0.1852	0.1075	0.0631
	0.00002	0.1842	0.1071	0.0629
	0.00003	0.1833	0.1066	0.0626
	0.00004	0.1823	0.1061	0.0624

Special Case Model I

For $n = 3$, and $k = 2$ with Type F Repair, system performance indices are determined directly from the results obtained for the $(3, 2, 1)$ special case with Type F Repair discussed in Subsection 4.1.2 as follows:
The system reliability is obtained directly from Equation (4.38) by eliminating the residues associated with *system up-states* *4*, $3^{(1)}$ and $2^{(2)}$. The values obtained thus are given in the third column of Table 4.20. For the sake of completeness, the system reliability values obtained for the $(3, 2, 1)$ case with Type F Repair are re-produced in the second column of Table 4.20.

The system mean time to failure is obtained directly from Equation (4.39) by eliminating terms associated with *system up-states* *4*, $3^{(1)}$ and $2^{(2)}$ and can be expressed as:

$$MTTF_{Z_F}(3,2,2) \;=\; \sum_{i=0}^{3} P_i(0) + \sum_{i=0^{(1)}}^{2^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{1^{(2)}} P_i(0) \tag{4.92}$$

where

$P_0(0) = 1/C_{3,2}(0)$

$P_{0^{(1)}}(0) = (2\lambda_s/C_{3,2}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (3\lambda_1/C_{3,2}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (2\lambda_s/C_{3,2}(1^{(1)})) \bullet P_1(0) + (3\lambda_1/C_{3,2}(1^{(1)})) \bullet P_{0^{(1)}}(0)$

$P_2(0) = (2\lambda_1/C_{3,2}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (2\lambda_s/C_{3,2}(2^{(1)})) \bullet P_2(0) + (2\lambda_1/C_{3,2}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{3,2}(3)) \bullet P_2(0)$

$P_{0^{(2)}} = (\lambda_s/C_{3,2}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (\lambda_s/C_{3,2}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (3\lambda_1/C_{3,2}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

The system mean time to failure values generated with Equation (4.92) are given in the third column of Table 4.21.

The variance of time to failure for this case can be determined directly from Equation (4.40) by eliminating all the constants associated with *system up-states* *4*, $3^{(1)}$ and $2^{(2)}$.

Table 4.20: $R_{Z_F}(3,2,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9955	0.8749	0.4223
4000	0.7425	0.5739	0.1018
6000	0.4362	0.3062	0.0160
8000	0.2230	0.1480	0.0021
∞	0	0	0

Table 4.21: $MTTF_{Z_F}(3,2,k)$ Values as a function of λ_{c0}

λ_{c0} (per hr.)	$n=1$	$n=2$	$n=3$
0.00001	7431.96	5954.45	2235.51
0.00002	7327.53	5846.91	2212.31
0.00003	7225.28	5742.41	2189.50
0.00004	7125.19	5640.84	2167.08

4.1.6 k-out-of-(n+3):G Identical Unit Submodel

This is the $(n, 3, k)$ system and is composed of n active units and *three* standbys. It is functional as long as there are at least k healthy units. Where $k \leq n$. The block diagram of this submodel is shown in Figure 3.26 and the state space diagram in Figure 4.14.

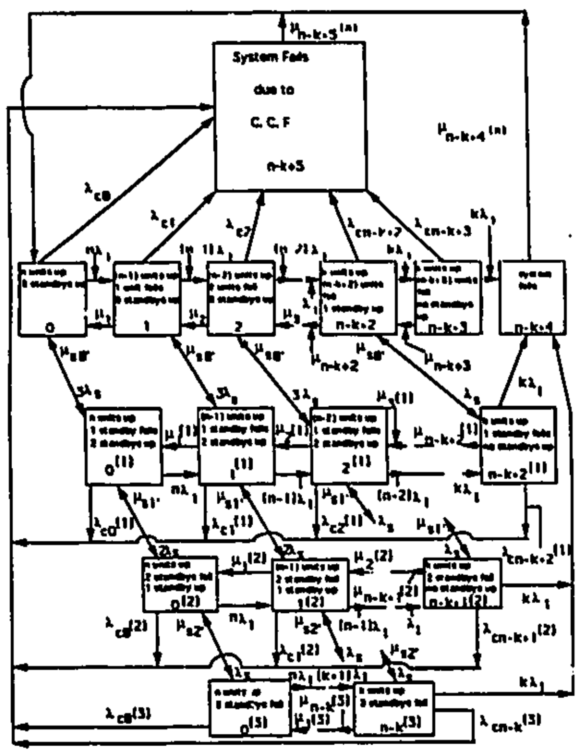


Figure 4.14: State space diagram of the $k - out - of - (n+3) : G$ System under SBAP Z

<u>$k - out - of - (n+3) : G$ **System Equations**</u>

The descriptive integro-differential Equations for the system shown in Figure 4.14 are:

$$\frac{dP_0(t)}{dt} + C_{n,3}(0)\, P_0(t) = \mu_1 P_1(t) + \mu_{s0'}\, P_{0(1)}(t) + \sum_{i=r+3}^{r+4} \int_0^{\infty} P_i(x,t)\, \mu_i(x) dx \quad (4.93)$$

where $r = n - k + 1$,

For $j = 1, 2, \ldots\ldots, r$,

$$\frac{dP_j(t)}{dt} + C_{n,3}(j)\, P_j(t) = (n-j+1)\lambda_1 P_{j-1}(t) + \mu_{s0'} P_{j^{(1)}}(t) + \mu_{j+1} P_{j+1}(t) \quad (4.94)$$

$$\frac{dP_{r+1}(t)}{dt} + C_{n,3}(r+1)\, P_{r+1}(t) = k\lambda_1\, P_r(t) + \mu_{r+2} P_{r+2}(t) + \mu_{s0'} P_{r+1^{(1)}}(t) \quad (4.95)$$

$$\frac{dP_{r+2}(t)}{dt} + C_{n,3}(r+2)\, P_{r+2}(t) = k\lambda_1\, P_{r+1}(t) \quad (4.96)$$

$$\frac{dP_{0^{(1)}}(t)}{dt} + C_{n,3}(0^{(1)})\, P_{0^{(1)}}(t) = 3\lambda_s P_0(t) + \mu_{1^{(1)}} P_{1^{(1)}}(t) + \mu_{s1'} P_{0^{(2)}}(t) \quad (4.97)$$

For $j = 1, 2, \ldots\ldots, r-1$,

$$\frac{dP_{j^{(1)}}(t)}{dt} + C_{n,3}(j^{(1)})\, P_{j^{(1)}}(t) = 3\lambda_s P_j(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(1)}}(t) + \mu_{s1'} P_{j^{(2)}}(t) + \mu_{j+1^{(1)}}\, P_{j+1^{(1)}}(t) \quad (4.98)$$

$$\frac{dP_{r^{(1)}}(t)}{dt} + C_{n,3}(r^{(1)})\, P_{r^{(1)}}(t) = 2\lambda_s\, P_r(t) + k\lambda_1 P_{r-1^{(1)}}(t) + \mu_{r+1^{(1)}} P_{r+1^{(1)}}(t) \quad (4.99)$$

$$\frac{dP_{r+1^{(1)}}(t)}{dt} + C_{n,3}(r+1^{(1)})\, P_{r+1^{(1)}}(t) = \lambda_s\, P_{r+1}(t) + k\lambda_1 P_{r^{(1)}}(t) \quad (4.100)$$

$$\frac{dP_{0^{(2)}}(t)}{dt} + C_{n,3}(0^{(2)})\, P_{0^{(2)}}(t) = 2\lambda_s P_{0^{(1)}}(t) + \mu_{1^{(2)}} P_{1^{(2)}}(t) + \mu_{s2'} P_{0^{(3)}}(t) \quad (4.101)$$

For $j = 1, 2, \ldots\ldots, r-1$,

$$\frac{dP_{j^{(2)}}(t)}{dt} + C_{n,3}(j^{(2)})\, P_{j^{(2)}}(t) = 2\lambda_s P_{j^{(1)}}(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(2)}}(t) + \mu_{s2'} P_{j^{(3)}}(t) + \mu_{j+1^{(2)}}\, P_{j+1^{(2)}}(t) \quad (4.102)$$

$$\frac{dP_{r^{(2)}}(t)}{dt} + C_{n,3}(r^{(2)})\, P_{r^{(2)}}(t) = \lambda_s P_{r^{(1)}}(t) + k\lambda_1\, P_{r-1^{(2)}}(t) \quad (4.103)$$

$$\frac{dP_{0^{(3)}}(t)}{dt} + C_{n,3}(0^{(3)})\, P_{0^{(3)}}(t) = \lambda_s P_{0^{(2)}}(t) + \mu_{1^{(3)}} P_{1^{(3)}}(t) \quad (4.104)$$

For $j = 1, 2, \ldots\ldots, r-2$,

$$\frac{dP_{j^{(3)}}(t)}{dt} + C_{n,3}(j^{(3)})\, P_{j^{(3)}}(t) = \lambda_s P_{j^{(2)}}(t) + (n-j+1)\,\lambda_1\, P_{j-1^{(3)}}(t) + \mu_{j+1^{(3)}}\, P_{j+1^{(3)}}(t) \quad (4.105)$$

$$\frac{dP_{r-1^{(3)}}(t)}{dt} + C_{n,3}(r-1^{(3)})\, P_{r-1^{(3)}}(t) = \lambda_s P_{r-1^{(2)}}(t) + (k+1)\lambda_1\, P_{r-2^{(3)}}(t) \quad (4.106)$$

$$[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{r+3}(x)]\,P_{r+3}(x,t)=0 \tag{4.107}$$

$$[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{r+4}(x)]\,P_{r+4}(x,t)=0 \tag{4.108}$$

The associated boundary conditions are:

$$P_{r+3}(0,t)=k\lambda_1\,P_{r+2}(t)+k\lambda_1\,P_{r+1^{(1)}}(t)+k\lambda_1\,P_{r^{(2)}}(t)+k\lambda_1 P_{r-1^{(3)}}(t) \tag{4.109}$$

$$P_{r+4}(0,t)=\sum_{j=0}^{r+2}\lambda_{cj}\,P_j(t)+\sum_{j=0^{(1)}}^{r+1^{(1)}}\lambda_{cj}P_j(t)+\sum_{j=0^{(2)}}^{r^{(2)}}\lambda_{cj}P_j(t)+\sum_{j=0^{(3)}}^{r-1^{(3)}}\lambda_{cj}P_j(t) \tag{4.110}$$

At time $t=0$, $P_0(0)=1.0$ and all other initial state probabilities are equal to zero.

where

$C_{n,3}(0)=n\,\lambda_1+3\lambda_s+\lambda_{c0}$

For $j=1,2,.....r-1,$

$C_{n,3}(j)=(n-j)\,\lambda_1+\lambda_{cj}+\mu_j+3\lambda_s$

$C_{n,3}(r)=k\lambda_1+\lambda_{cr}+\mu_r+2\lambda_s$

$C_{n,3}(r+1)=k\lambda_1+\lambda_{cr+1}+\mu_{r+1}+\lambda_s$

$C_{n,3}(r+2)=k\lambda_1+\lambda_{cr+2}+\mu_{r+2}$

$C_{n,3}(0^{(1)})=n\,\lambda_1+\lambda_{c0^{(1)}}+\mu_{s0'}+2\lambda_s$

For $j=1,2,.......,r-1,$

$C_{n,3}(j^{(1)})=(n-j)\,\lambda_1+\lambda_{cj^{(1)}}+\mu_{j^{(1)}}+\mu_{s0'}+2\lambda_s$

$C_{n,3}(r^{(1)})=k\lambda_1+\lambda_{cr^{(1)}}+\mu_{r^{(1)}}+\mu_{s0'}+\lambda_s$

$C_{n,3}(r+1^{(1)})=k\lambda_1+\lambda_{cr+1^{(1)}}+\mu_{r+1^{(1)}}+\mu_{s0'}$

$C_{n,3}(0^{(2)})=n\,\lambda_1+\lambda_{c0^{(2)}}+\mu_{s1'}+\lambda_s$

For $j=1,2,.......,r-1,$

$C_{n,3}(j^{(2)})=(n-j)\,\lambda_1+\lambda_{cj^{(2)}}+\mu_{j^{(2)}}+\mu_{s1'}+\lambda_s$

$C_{n,3}(r^{(2)})=k\lambda_1+\lambda_{cr^{(2)}}+\mu_{r^{(2)}}+\mu_{s1'}$

$C_{n,3}(0^{(3)})=n\,\lambda_1+\lambda_{c0^{(3)}}+\mu_{s2'}$

For $j=1,2,.......,r-1,$

$C_{n,3}(j^{(3)})=(n-j)\,\lambda_1+\lambda_{cj^{(3)}}+\mu_{j^{(3)}}+\mu_{s2'}$

$k - out - of - (n+3) : G$ System With Type C Repair

With Type C Repair, the units (both active and standbys) are discarded upon failure thus $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_{r+2}, $\mu_{1^{(1)}}$ to $\mu_{r+1^{(1)}}$, $\mu_{1^{(2)}}$ to $\mu_{r^{(2)}}$ and $\mu_{1^{(3)}}$ to $\mu_{r-1^{(3)}}$ are set to zero in Equations (4.93) to (4.110).

Special Case Model I

For $n = 3$ and $k = 2$ with Type C Repair, the long-run availability for this special case is determined from the results obtained for the $(3,3,1)$ special case with Type C Repair discussed in Subsection 4.1.3. Thus the long-run availability values for this special case are obtained by eliminating *system up-states 5*, $4^{(1)}$, $3^{(2)}$ and $2^{(3)}$ from Equation (4.60) and the constants relating to them. The values obtained are given in the third column of Table 4.22. For the sake of completeness, the system reliability values obtained for the $(3,3,1)$ case with Type F Repair are given in the second column of Table 4.22.

$k - out - of - (n+3) : G$ System With Type F Repair

With Type F Repair, the system as well as its units (both active and standbys) are discarded upon failure thus $\mu_{r+3}(x)$ and $\mu_{r+4}(x)$, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_{r+2}, $\mu_{1^{(1)}}$ to $\mu_{r+1^{(1)}}$, $\mu_{1^{(2)}}$ to $\mu_{r^{(2)}}$ and $\mu_{1^{(3)}}$ to $\mu_{r-1^{(3)}}$ are set to zero in Equations (4.93) to (4.110).

Special Case Model I

For $n = 3$ and $k = 2$ with Type F Repair, the system relevant reliability indices can be determined easily from the analysis performed for the $(3,3,1)$ special case with Type F Repair in Subsection 4.1.3 as follows:
The system reliability is obtained directly from Equation (4.64) by eliminating the residues associated with *system up-states 5*, $4^{(1)}$, $3^{(2)}$ and $2^{(3)}$. The values obtained thus are given in the third column of Table 4.23. For the sake of completeness, the system reliability values obtained for the $(3,3,1)$ case with Type F Repair are

Table 4.22: $A_{ssz}(3,3,k)\,(\beta)$ values as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
$\beta=1$	0.00001	0.5687	0.4215	0.3185
	0.00002	0.5672	0.4203	0.3176
	0.00003	0.5658	0.4191	0.3167
	0.00004	0.5643	0.4179	0.3158
$\beta=2$	0.00001	0.3973	0.2671	0.1895
	0.00002	0.3959	0.2660	0.1889
	0.00003	0.3945	0.2651	0.1882
	0.00004	0.3931	0.2642	0.1876
$\beta=3$	0.00001	0.3053	0.1954	0.1349
	0.00002	0.3041	0.1947	0.1344
	0.00003	0.3028	0.1939	0.1339
	0.00004	0.3016	0.1931	0.1334
$\beta=4$	0.00001	0.2655	0.1541	0.1047
	0.00002	0.2644	0.1535	0.1043
	0.00003	0.2633	0.1528	0.1039
	0.00004	0.2621	0.1522	0.1035
$\beta=5$	0.00001	0.2309	0.1272	0.0856
	0.00002	0.2299	0.1267	0.0852
	0.00003	0.2290	0.1261	0.0849
	0.00004	0.2280	0.1256	0.0846

re-produced in the second column of Table 4.23.

Table 4.23: $R_{Z_F}(3,3,k)$ Values ($\lambda_{c0} = 0.00004$ per hr.)

Time t (Hr.)	$k=1$	$k=2$	$k=3$
0	1.0000	1.0000	1.0000
2000	0.9890	0.8720	0.2030
4000	0.8382	0.4815	0.0432
6000	0.5750	0.1745	0.0062
8000	0.3470	0.0495	0.0008
∞	0	0	0

The system mean time to failure for this special case is obtained from Equation (4.65) by eliminating the constants associated with *system up-states 5*, $4^{(1)}$, $3^{(2)}$ and $2^{(3)}$ and can be expressed as:

$$\begin{aligned} MTTF_{Z_F}(3,3,2) &= \sum_{i=0}^{4} P_i(0) + \sum_{i=0^{(1)}}^{3^{(1)}} P_i(0) + \sum_{i=0^{(2)}}^{2^{(2)}} P_i(0) + \\ &\quad \sum_{i=0^{(3)}}^{1^{(3)}} P_i(0) \end{aligned} \tag{4.111}$$

where,

$P_0(0) = 1/C_{3,3}(0)$

$P_{0^{(1)}}(0) = (3\lambda_s/C_{3,3}(0^{(1)})) \bullet P_0(0)$

$P_1(0) = (3\lambda_1/C_{3,3}(1)) \bullet P_0(0)$

$P_{1^{(1)}}(0) = (3\lambda_1/C_{3,3}(1^{(1)})) \bullet P_{0^{(1)}}(0) + (3\lambda_s/C_{3,3}(1^{(1)})) \bullet P_1(0)$

$P_2(0) = (2\lambda_1/C_{3,3}(2)) \bullet P_1(0)$

$P_{2^{(1)}}(0) = (3\lambda_s/C_{3,3}(2^{(1)})) \bullet P_2(0) + (2\lambda_1/C_{3,3}(2^{(1)})) \bullet P_{1^{(1)}}(0)$

$P_3(0) = (\lambda_1/C_{3,3}(3)) \bullet P_2(0)$

$P_4(0) = (\lambda_1/C_{3,3}(4)) \bullet P_3(0)$

$P_{3^{(1)}}(0) = (2\lambda_s/C_{3,3}(3^{(1)})) \bullet P_3(0) + (\lambda_1/C_{3,3}(3^{(1)})) \bullet P_{2^{(1)}}(0)$

$P_{0^{(2)}} = (2\lambda_s/C_{3,3}(0^{(2)})) \bullet P_{0^{(1)}}(0)$

$P_{1^{(2)}}(0) = (2\lambda_s/C_{3,3}(1^{(2)})) \bullet P_{1^{(1)}}(0) + (3\lambda_1/C_{3,3}(1^{(2)})) \bullet P_{0^{(2)}}(0)$

$P_{2^{(2)}}(0) = (2\lambda_s/C_{3,3}(2^{(2)})) \bullet P_{2^{(1)}}(0) + (2\lambda_1/C_{3,3}(2^{(2)})) \bullet P_{1^{(2)}}(0)$

$P_{0^{(3)}} = (\lambda_s/C_{3,3}(0^{(3)})) \bullet P_{0^{(2)}}(0)$

$P_{1^{(3)}}(0) = (\lambda_s/C_{3,3}(1^{(3)})) \bullet P_{1^{(2)}}(0) + (3\lambda_1/C_{3,3}(1^{(3)})) \bullet P_{0^{(3)}}(0)$

The values obtained with Equation (4.111) are given in the third column of Table 4.24. For the sake of completeness, the system mean time to failure values obtained for the $(3,3,1)$ case with Type F Repair are also given in the second column of Table 4.24.

Table 4.24: $MTTF_{Z_F}(3,3,k)$ Values as a function of λ_{c0}

λ_{c0} (per Hr.)	$k=1$	$k=2$	$k=3$
0.00001	8605.25	4785.99	2937.45
0.00002	8498.77	4732.91	2909.09
0.00003	8394.32	4680.74	2881.18
0.00004	8291.82	4629.48	2853.70

The variance of time to failure for this special case can be determined directly from Equation (4.66) by eliminating the constants associated with *system up-states* 5, $4^{(1)}$, $3^{(2)}$ and $2^{(3)}$.

From Equations (4.36), (4.39), (4.62), (4.65), (4.92) and (4.111), a generalized approach of determining the mean time to failure of a $k-out-of-(n+m):G$ system with Type F Repair operating under standby activation policy Z is developed using Equation (2.11) in conjunction with the following set of recursive relationships:
Letting $\lim_{s\to 0} P_{j^{(i)}}(s) = P_{j^{(i)}}(0)$; for $i = 0,1,2,.....,m$ and for $j = 0,1,2,3,...,[(n+m-(i+k))]$ then:
For $j = 1,2,3,....n-k$ and for $i = 0,1,2,3,...,m$:

$P_{j^{(i)}}(0) = [((n-j+1)\,\lambda_1)/C_{n,m}(j^{(i)})] \bullet P_{(j-1)^{(i)}}(0) + [((m-i+1)\,\lambda_s)/C_{n,m}(j^{(i)})] \bullet P_{j^{(i-1)}}(0)$

For $j = n-k+1, n-k+2, \ldots.., [(n+m-(i+k))]$ and for $i = 0,1,2,3,\ldots,m$:

$P_{j^{(i)}}(0) = [\lambda_1/C_{n,m}(j^{(i)})] \bullet P_{j-1^{(i)}}(0) + [((m+n)-(i+j))\,\lambda_s)/C_{n,m}(j^{(i)})] \bullet P_{j^{(i-1)}}(0)$

The starting values for the above recursive procedure are obtained via the following recursive relationships:

$P_0(0) = P_{0^{(0)}}(0) = 1/C_{n,m}(0)$

and for $i = 1,2,\ldots.m$:

$P_{0^{(i)}}(0) = [((m-i+1)\,\lambda_s)/C_{n,m}(0^{(i)})] \bullet P_{0^{(i-1)}}(0)$

4.2 The Non-Identical Unit System With Late Standby Activation: Special Cases

For reasons already stated in Section 3.2, only the analysis of duplex systems with non-identical units will be presented. Standby activation policy Z is impracticable if $k = n$, thus only these two special cases are considered viz: *(i)* $1-out-of-(2+1):G$ or $(2,1,1)$ *(ii)* $1-out-of-(2+2):G$ or $(2,2,1)$.

4.2.1 1-out-of-(2+1):G Non-identical Unit System

This is the $(2,1,1)$ non-identical unit system and is composed of *two* non-identical units and *one* non-identical standby and is operable as long as at least *one* unit is functional. The state-space diagram for this system is shown in Figure 4.15. The numerals in the boxes of Figure 4.15 denote the system state numbers.

<u>Notation</u>

The following symbols are associated with a $(2,1,1)$ non-identical unit system operating under SBAP Z:

i State identification number. For $i = 0$, both units (A and

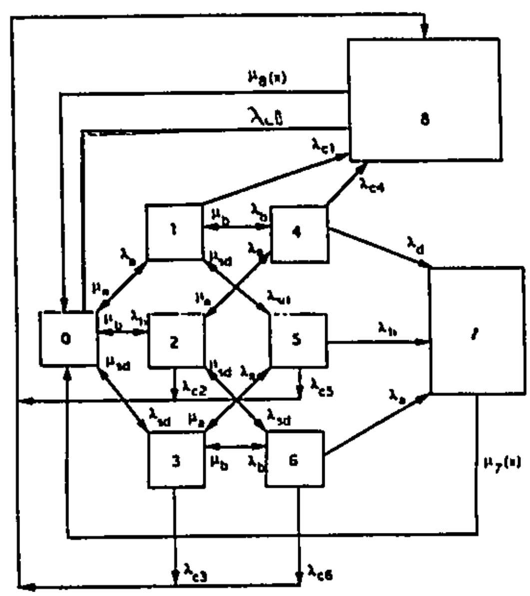


Figure 4.15: State space diagram of the $1 - out - of - (2 + 1) : G$ Non-identical System under SBAP Z

B) and the standby D are in perfect working condition. For $i = 1$, unit A has failed, unit B is working. For $i = 2$, unit B has failed, unit A is still working. For $i = 3$, both units A and B are functional but standby D has failed in its standby mode. For $i = 4$, both units (A and B) have failed but standby unit D now activated is working. For $i = 5$, units A and D have failed but unit B is still functional. For $i = 6$, units B and D have failed but unit A is working. For $i = 7$, the system has failed due to independent failures of its components. For $i = 8$, the system has failed due to common-cause failures.

t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated
λ_{sd}	Constant failure rate of standby unit D when in the standby mode
λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, ..., 6$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_{sd}	Constant repair rate of standby unit D if it fails in the standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, ..., 8$
$P_i(x, t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 7, 8$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time

	respectively when the system is in state i and has an elapsed repair time of x; for $i = 7, 8$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The system of integro-differential equations associated with this system described in Figure 4.15 is:

$$\frac{dP_0(t)}{dt} + C_{2,1}(0)\, P_0(t) = \mu_a\, P_1(t) + \mu_b\, P_2(t) + \mu_{sd}\, P_3(t) + \sum_{i=7}^{8} \int_0^\infty P_i(x,t)\, \mu_i(x)\, dx \tag{4.112}$$

$$\frac{dP_1(t)}{dt} + C_{2,1}(1)\, P_1(t) = \mu_b\, P_4(t) + \lambda_a\, P_0(t) + \mu_{sd}\, P_5(t) \tag{4.113}$$

$$\frac{dP_2(t)}{dt} + C_{2,1}(2)\, P_2(t) = \mu_a\, P_4(t) + \lambda_b\, P_0(t) + \mu_{sd}\, P_6(t) \tag{4.114}$$

$$\frac{dP_3(t)}{dt} + C_{2,1}(3)\, P_3(t) = \mu_a\, P_5(t) + \mu_b\, P_6(t) + \lambda_{sd}\, P_0(t) \tag{4.115}$$

$$\frac{dP_4(t)}{dt} + C_{2,1}(4)\, P_4(t) = \lambda_b\, P_1(t) + \lambda_a\, P_2(t) \tag{4.116}$$

$$\frac{dP_5(t)}{dt} + C_{2,1}(5)\, P_5(t) = \lambda_a\, P_3(t) + \lambda_{sd}\, P_1(t) \tag{4.117}$$

$$\frac{dP_6(t)}{dt} + C_{2,1}(6)\, P_6(t) = \lambda_b\, P_3(t) + \lambda_{sd}\, P_2(t) \tag{4.118}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_7(x)]\, P_7(x,t) = 0 \tag{4.119}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_8(x)]\, P_8(x,t) = 0 \tag{4.120}$$

With the following boundary conditions:

$$P_7(0,t) = \lambda_d\, P_4(t) + \lambda_b\, P_5(t) + \lambda_a\, P_6(t) \tag{4.121}$$

$$P_8(0,t) = \sum_{i=0}^{6} \lambda_{ci}\, P_i(t) \tag{4.122}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

Where

$C_{2,1}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd}$

$C_{2,1}(1) = \lambda_b + \lambda_{c1} + \mu_a + \lambda_{sd}$

$C_{2,1}(2) = \lambda_a + \lambda_{c2} + \mu_b + \lambda_{sd}$

$C_{2,1}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \mu_{sd}$

$C_{2,1}(4) = \lambda_d + \lambda_{c4} + \mu_b + \mu_a$

$C_{2,1}(5) = \lambda_b + \lambda_{c5} + \mu_a + \mu_{sd}$

$C_{2,1}(6) = \lambda_a + \lambda_{c6} + \mu_b + \mu_{sd}$

1-out-of-(2+1):G Non-identical Unit System with Type B Repair

With Type B Repair, $\mu_a = \mu_b = 0$. The system is repaired upon failure and we are concerned with the system long-run availability. Using the procedure already established for the identical unit case, the long-run availability of this system can be obtained in terms of the gamma *pdf* shape parameter β and is given as:

$$A_{ssz}(2,1,1)\,(\beta) = \frac{(\mu_7\,\mu_8)^\beta\,x_{81}}{(\mu_7\,\mu_8)^\beta\,x_{82} + \beta\,\mu_7^\beta\,\mu_8^{\beta-1}\,x_{83} + \beta\,\mu_7^{\beta-1}\,\mu_8^\beta\,x_{84}} \tag{4.123}$$

where,

$x_{81} = A_{2909} + A_{2715} + A_{2921} + A_{2927} + A_{2945} + A_{2951} + A_{2957}$

$x_{82} = A_{2958} - A_{2960} - A_{2962}$

$x_{83} = A_{2959} - A_{2961}$

$x_{84} = A_{2959} - A_{2963}$

Long-run availability values obtained with Equation (4.123) are exactly the same as those obtained for the $(2,1,1)$ identical unit case with Type B Repair discussed in Subsection 4.1.1. if $\lambda_a = \lambda_b = \lambda_d = \lambda_1$. Symbols A_{2909} to A_{2963} are defined in Appendix I.1.

1-out-of-(2+1):G Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_{sd} = 0$. Thus setting μ_{sd} equal to zero in Equation (4.123), the long-run availability for this case is determined and they match the values given in the third column of Table 4.2 when $\lambda_a = \lambda_b = \lambda_d = \lambda_1$.

1-out-of-(2+1):G Non-identical Unit System with Type E Repair

With Type E Repair, $\mu_a = \mu_b = \mu_7(x) = \mu_8(x) = 0$. The system is not repaired and the indices of interest are the system reliability, system mean time to failure and the variance of time to failure. These are given by the following expressions:
The system reliability is given by:

$$R_{Z_E}(2,1,1)(t) = \sum_{i=0}^{6} P_i(t) \tag{4.124}$$

where

$$\begin{aligned}
P_0(t) &= BX_{622}\, e^{s_{82}t} + BX_{623}\, e^{s_{83}t} \\
P_1(t) &= BX_{624}\, e^{s_{82}t} + BX_{625}\, e^{s_{85}t} + BX_{626}\, e^{s_{84}t} + BX_{627}\, e^{s_{85}t} \\
P_2(t) &= BX_{628}\, e^{s_{82}t} + BX_{629}\, e^{s_{85}t} + BX_{630}\, e^{s_{84}t} + BX_{631}\, e^{s_{85}t} \\
P_3(t) &= BX_{632}\, e^{s_{82}t} + BX_{633}\, e^{s_{83}t} \\
P_4(t) &= BX_{634}\, e^{-C_{2,1}(4)t} + BX_{635}\, e^{s_{82}t} + BX_{636}\, e^{s_{83}t} + BX_{637}\, e^{s_{84}t} + \\
&\quad BX_{638}\, e^{s_{85}t} + BX_{639}\, e^{s_{86}t} + BX_{640}\, e^{s_{87}t} \\
P_5(t) &= BX_{641}\, e^{-C_{2,1}(5)t} + BX_{642}\, e^{s_{82}t} + BX_{643}\, e^{s_{83}t} + BX_{644}\, e^{s_{84}t} + \\
&\quad BX_{645}\, e^{s_{85}t} \\
P_6(t) &= BX_{646}\, e^{-C_{2,1}(6)t} + BX_{647}\, e^{s_{82}t} + BX_{648}\, e^{s_{83}t} + BX_{649}\, e^{s_{86}t} + \\
&\quad BX_{650}\, e^{s_{87}t}
\end{aligned}$$

$Denomz1(s) = (s + C_{2,1}(0))(s + C_{2,1}(3)) - \lambda_{sd}\, \mu_{sd}$
$Denomz2(s) = (s + C_{2,1}(1))(s + C_{2,1}(5)) - \lambda_{sd}\, \mu_{sd}$
$Denomz3(s) = (s + C_{2,1}(2))(s + C_{2,1}(6)) - \lambda_{sd}\, \mu_{sd}$
s_{82} and s_{83} are the roots of the characteristic function $Denomz1(s)$; s_{84} and s_{85} are the roots of the characteristic function $Denomz2(s)$ and; s_{86} and s_{87} are the roots of the characteristic function $Denomz3(s)$. The partial fraction residues BX_{622} to BX_{650} are defined in Appendix I.1.

The mean time to failure of this system is given by:

$$MTTF_{Z_E}(2,1,1) = \lim_{s \to 0} \sum_{i=0}^{6} P_i(s) \tag{4.125}$$

where

$P_0(0) = C_{2,1}(3)/(C_{2,1}(0)\, C_{2,1}(3) - \lambda_{sd}\, \mu_{sd})$

$P_1(0) = [(\lambda_a\, C_{2,1}(3)\, C_{2,1}(5) + \lambda_a\, \lambda_{sd}\, \mu_{sd})/(C_{2,1}(3)\, (C_{2,1}(1)\, C_{2,1}(5) - \lambda_{sd}\, \mu_{sd}))] \bullet P_0(0)$

$P_2(0) = [(\lambda_b\, C_{2,1}(3)\, C_{2,1}(6) + \lambda_b\, \lambda_{sd}\, \mu_{sd})/(C_{2,1}(3)\, (C_{2,1}(2)\, C_{2,1}(6) - \lambda_{sd}\, \mu_{sd}))] \bullet P_0(0)$

$P_3(0) = (\lambda_{sd}/C_{2,1}(3)) \bullet P_0(0)$

$P_4(0) = (\lambda_b/C_{2,1}(4)) \bullet P_1(0) + (\lambda_a/C_{2,1}(4)) \bullet P_2(0)$

$P_5(0) = (\lambda_a/C_{2,1}(5)) \bullet P_3(0) + (\lambda_{sd}/C_{2,1}(5)) \bullet P_1(0)$

$P_6(0) = (\lambda_b/C_{2,1}(6)) \bullet P_3(0) + (\lambda_{sd}/C_{2,1}(6)) \bullet P_2(0)$

The variance of time to failure for this system is given by:

$$\sigma^2_{Z_E}(2,1,1) = \frac{1}{k_{81}^2}\left[2(k_{80}k_{82} - k_{81}k_{83}) - k_{80}^2\right] \tag{4.126}$$

Symbols k_{80} to k_{83} are provided in Appendix I.1.

1-out-of-(2+1):G Non-identical Unit System with Type F Repair

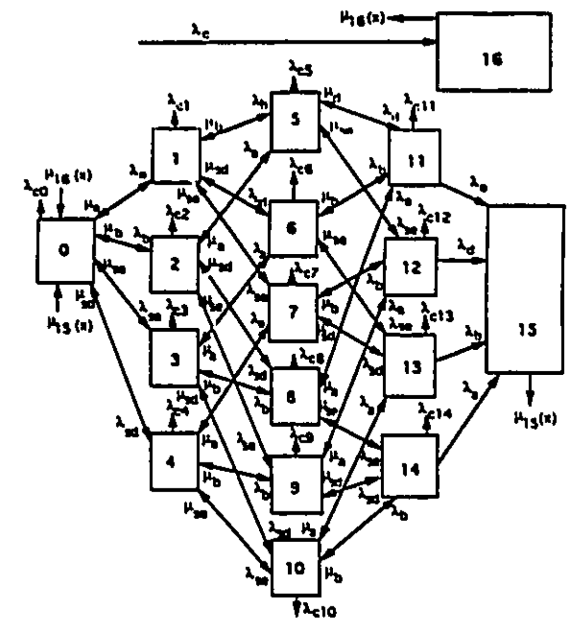
With Type F Repair, $\mu_a = \mu_b = \mu_{sd} = \mu_7(x) = \mu_8(x) = 0$. The system is not repaired and the indices of interest are the system reliability, system mean time to failure and the variance of time to failure. These are given by the following expressions:
The system reliability is determined directly from the results obtained for the previous special case by setting $\mu_{sd} = 0$. The results obtained match the reliability values for the $(2,1,1)$ identical unit case with Type F Repair discussed in Subsection 4.1.1.

The mean time to failure of this system can be determined from Equation (4.125) by setting μ_{sd} to zero.

The variance of time to failure for this case can be determined by setting μ_{sd} to zero in Equation (4.126).

4.2.2 1-out-of-(2+2):G Non-identical Unit System

Using our notation, this is the $(2,2,1)$ non-identical unit system and is composed of *two* non-identical units and *two* non-identical standbys and is operable as long as at least *one* unit is functional. The state space diagram for this system is shown in Figure 4.16 The numerals in the boxes of Figure 4.16 denote the system state numbers.



Where,

$\lambda_c = \lambda_{c0} + \lambda_{c1} + \lambda_{c2} + \lambda_{c3} + \lambda_{c4} + \lambda_{c5} + \lambda_{c6} + \lambda_{c7} + \lambda_{c8} + \lambda_{c9} + \lambda_{c10} + \lambda_{c11} + \lambda_{c12} + \lambda_{c13} + \lambda_{c14}$

Figure 4.16: State space diagram of the $1 - out - of - (2+2) : G$ Non-identical Unit System Under SBAP Z

<u>Notation</u>

The following symbols are associated with the $(2,2,1)$ non-identical unit system under SBAP Z:

i State identification number. For $i = 0$, both units (A and

B) and both standby units, D and E are in perfect working condition.
For $i = 1$, unit A has failed, unit B is working.
For $i = 2$, unit B has failed but unit A is still functional. For $i = 3$, both units (A and B) are functioning but standby E has failed in its standby mode.
For $i = 4$, both units (A and B) are functional but but standby unit D has failed in its standby mode.
For $i = 5$, both A and B have failed and standby D has been activated. For $i = 6$, units A and D have failed but unit B is still functional. For $i = 7$, units A and E have failed while B is still functional. For $i = 8$, units B and D have failed while units A and E are working.
For $i = 9$, units B and E have failed but units A and D are still working. For $i = 10$, units D and E have failed but units A and B are still functional.
For $i = 11$, units A, B and D have failed while standby unit E has been activated. For $i = 12$, units A, B and E have failed while standby unit D has been activated.
For $i = 13$, units A, D and E have failed while B is operational. For $i = 14$, units B, D and E have failed but unit A still works. For $i = 15$, the system has failed due to independent failures of its components. For $i = 16$, the system has failed due to common-cause failures of its components.

t	time
s	Laplace transform variable
λ_a	Constant failure rate of unit A
λ_b	Constant failure rate of unit B
λ_d	Constant failure rate of standby unit D when activated

λ_e	Constant failure rate of standby unit E when activated
λ_{sd}	Constant failure rate of standby unit D when in the standby mode
λ_{se}	Constant failure rate of standby unit E when in the standby mode
λ_{cj}	Constant common-cause failure rate from state j; for $j = 0, 1, 2, ..., 14$.
μ_a	Constant repair rate of unit A
μ_b	Constant repair rate of unit B
μ_d	Constant repair rate of unit D
μ_e	Constant repair rate of unit E
μ_{sd}	Constant repair rate of standby unit D if it fails in the standby mode
μ_{se}	Constant repair rate of standby unit E if it fails in the standby mode
$P_i(t)$	Probability that the system is in state i at time t; for $i = 0, 1, 2, 3, ..., 16$
$P_i(x,t)$	Probability that the failed system is in state i and has an elapsed repair time of x; for $i = 15, 16$.
$\mu_i(x), q_i(x)$	Time-dependent repair rate and *pdf* of repair time respectively when the system is in state i and has an elapsed repair time of x; for $i = 15, 16$.
$q_i(s)$	The Laplace transform of $q_i(t)$
$P_i(s)$	The Laplace transform of $P_i(t)$
β	Shape parameter of the gamma *pdf*

The system of integro-differential equations associated with this system described in Figure 4.16 is:

$$\frac{dP_0(t)}{dt} + C_{2,2}(0)\,P_0(t) = \mu_a\,P_1(t) + \mu_b\,P_2(t) + \mu_{sd}\,P_4(t) + \mu_{se}\,P_3(t) +$$

$$\sum_{i=15}^{16} \int_0^\infty P_i(x,t)\,\mu_i(x)\,dx \tag{4.127}$$

$$\frac{dP_1(t)}{dt} + C_{2,2}(1)\,P_1(t) = \mu_b\,P_5(t) + \mu_{sd}\,P_6(t) + \lambda_a\,P_0(t) + \mu_{se}\,P_7(t) \tag{4.128}$$

$$\frac{dP_2(t)}{dt} + C_{2,2}(2)\,P_2(t) = \mu_a\,P_5(t) + \mu_{sd}\,P_8(t) + \lambda_b\,P_0(t) + \mu_{se}\,P_9(t) \tag{4.129}$$

$$\frac{dP_3(t)}{dt} + C_{2,2}(3)\,P_3(t) = \mu_a\,P_7(t) + \mu_b\,P_9(t) + \lambda_{se}\,P_0(t) + \mu_{sd}\,P_{10}(t) \tag{4.130}$$

$$\frac{dP_4(t)}{dt} + C_{2,2}(4)\,P_4(t) = \mu_a\,P_6(t) + \mu_b\,P_8(t) + \lambda_{sd}\,P_0(t) + \mu_{se}\,P_{10}(t) \tag{4.131}$$

$$\frac{dP_5(t)}{dt} + C_{2,2}(5)\,P_5(t) = \mu_d\,P_{11}(t) + \mu_{se}\,P_{12}(t) + \lambda_b\,P_1(t) + \lambda_a\,P_2(t) \tag{4.132}$$

$$\frac{dP_6(t)}{dt} + C_{2,2}(6)\,P_6(t) = \mu_b\,P_{11}(t) + \mu_{se}\,P_{13}(t) + \lambda_{sd}\,P_1(t) + \lambda_a\,P_4(t) \tag{4.133}$$

$$\frac{dP_7(t)}{dt} + C_{2,2}(7)\,P_7(t) = \mu_b\,P_{12}(t) + \mu_{sd}\,P_{13}(t) + \lambda_{se}\,P_1(t) + \lambda_a\,P_3(t) \tag{4.134}$$

$$\frac{dP_8(t)}{dt} + C_{2,2}(8)\,P_8(t) = \mu_a\,P_{11}(t) + \mu_{se}\,P_{14}(t) + \lambda_{sd}\,P_2(t) + \lambda_b\,P_4(t) \tag{4.135}$$

$$\frac{dP_9(t)}{dt} + C_{2,2}(9)\,P_9(t) = \mu_a\,P_{12}(t) + \mu_{sd}\,P_{14}(t) + \lambda_{se}\,P_2(t) + \lambda_b\,P_3(t) \tag{4.136}$$

$$\frac{dP_{10}(t)}{dt} + C_{2,2}(10)\,P_{10}(t) = \mu_a\,P_{13}(t) + \mu_b\,P_{14}(t) + \lambda_{se}\,P_4(t) + \lambda_{sd}\,P_3(t) \tag{4.137}$$

$$\frac{dP_{11}(t)}{dt} + C_{2,2}(11)\,P_{11}(t) = \lambda_d\,P_5(t) + \lambda_b\,P_6(t) + \lambda_a\,P_8(t) \tag{4.138}$$

$$\frac{dP_{12}(t)}{dt} + C_{2,2}(12)\,P_{12}(t) = \lambda_{se}\,P_5(t) + \lambda_b\,P_7(t) + \lambda_a\,P_9(t) \tag{4.139}$$

$$\frac{dP_{13}(t)}{dt} + C_{2,2}(13)\,P_{13}(t) = \lambda_{se}\,P_6(t) + \lambda_{sd}\,P_7(t) + \lambda_a\,P_{10}(t) \tag{4.140}$$

$$\frac{dP_{14}(t)}{dt} + C_{2,2}(14)\,P_{14}(t) = \lambda_{se}\,P_8(t) + \lambda_{sd}\,P_9(t) + \lambda_b\,P_{10}(t) \tag{4.141}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{15}(x)]\,P_{15}(x,t) = 0 \tag{4.142}$$

$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{16}(x)]\,P_{16}(x,t) = 0 \tag{4.143}$$

With the following boundary conditions:

$$P_{15}(0,t) = \lambda_e\, P_{11}(t) + \lambda_d\, P_{12}(t) + \lambda_b\, P_{13}(t) + \lambda_a\, P_{14}(t) \tag{4.144}$$

$$P_{16}(0,t) = \sum_{i=0}^{14} \lambda_{ci}\, P_i(t) \tag{4.145}$$

At time $t = 0$, $P_0(0) = 1.0$ and all other initial state probabilities are equal to zero.

where

$C_{2,2}(0) = \lambda_a + \lambda_b + \lambda_{c0} + \lambda_{sd} + \lambda_{se}$

$C_{2,2}(1) = \lambda_b + \lambda_{sd} + \lambda_{c1} + \lambda_{se} + \mu_a$

$C_{2,2}(2) = \lambda_a + \lambda_{sd} + \lambda_{c2} + \lambda_{se} + \mu_b$

$C_{2,2}(3) = \lambda_a + \lambda_b + \lambda_{c3} + \lambda_{sd} + \mu_{sd}$

$C_{2,2}(4) = \lambda_a + \lambda_b + \lambda_{se} + \lambda_{c4} + \mu_{se}$

$C_{2,2}(5) = \lambda_d + \lambda_{se} + \lambda_{c5} + \mu_a + \mu_b$

$C_{2,2}(6) = \lambda_b + \lambda_{se} + \lambda_{c6} + \mu_a + \mu_d$

$C_{2,2}(7) = \lambda_b + \lambda_{sd} + \mu_{se} + \lambda_{c7} + \mu_a$

$C_{2,2}(8) = \lambda_a + \lambda_{se} + \lambda_{c8} + \mu_b + \mu_{sd}$

$C_{2,2}(9) = \lambda_a + \lambda_{sd} + \lambda_{c9} + \mu_b + \mu_{se}$

$C_{2,2}(10) = \lambda_a + \lambda_b + \lambda_{c10} + \mu_{sd} + \mu_{se}$

$C_{2,2}(11) = \lambda_e + \lambda_{c11} + \mu_a + \mu_b + \mu_d$

$C_{2,2}(12) = \lambda_d + \lambda_{c12} + \mu_a + \mu_b + \mu_{se}$

$C_{2,2}(13) = \lambda_b + \lambda_{c13} + \mu_a + \mu_{sd} + \mu_{se}$

$C_{2,2}(14) = \lambda_a + \lambda_{c14} + \mu_b + \mu_{sd} + \mu_{se}$

1-out-of-(2+2):G Non-identical Unit System with Type C Repair

With Type C Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = 0$, the system is repaired upon failure and we are concerned with the system long-run availability. The long-run availability of this system expressed in terms of the gamma *pdf* shape parameter β is given by:

$$A_{ssz}(2,2,1)\,(\beta) = \frac{(\mu_{15}\,\mu_{16})^{\beta}\, x_{85}}{(\mu_{15}\,\mu_{16})^{\beta}\, x_{86} + \beta\, \mu_{15}^{\beta}\, \mu_{16}^{\beta-1}\, x_{87} + \beta\, \mu_{15}^{\beta-1}\, \mu_{16}^{\beta}\, x_{88}} \tag{4.146}$$

where,

$$
\begin{aligned}
x_{85} &= A_{3131} + \lambda_a A_{3129} + \lambda_b A_{3127} + \lambda_{sd} A_{3125} + \lambda_{se} A_{3123} + A_{3001} A_{3133} + \\
&\quad A_{3005} A_{3135} + A_{3009} A_{3137} + A_{3013} A_{3139} + A_{3017} A_{3141} + A_{3021} A_{3143} + \\
&\quad A_{3045} A_{3145} + A_{3057} A_{3147} + A_{3069} A_{3149} + A_{3081} A_{3151} \\
x_{86} &= A_{3168} - A_{3164} - A_{3166} \\
x_{87} &= A_{3169} - A_{3165} \\
x_{88} &= A_{3169} - A_{3167}
\end{aligned}
$$

Long-run availability values obtained with Equation (4.146) are exactly the same as those obtained for the $(2,2,1)$ identical unit case with Type C Repair discussed in Subsection 4.1.2 for $\lambda_a = \lambda_b = \lambda_d = \lambda_e = \lambda_1$. Symbols A_{3001} to A_{3169} are defined in Appendix I.2.

1-out-of-(2+2):G Non-identical Unit System with Type F Repair

With Type F Repair, $\mu_a = \mu_b = \mu_d = \mu_e = \mu_{sd} = \mu_{se} = \mu_{15}(x) = \mu_{16}(x) = 0$, the system is not repaired and the indices of interest are the system reliability, system mean time to failure and the variance of time to failure. These are given by the following expressions:

The system reliability is given by:

$$
R_{Z_F}(2,2,1)(t) = \sum_{i=0}^{14} P_i(t) \tag{4.147}
$$

where,

$$
\begin{aligned}
P_0(t) &= e^{-C_{2,2}(0)t} \\
P_1(t) &= BZ_{646}\, e^{-C_{2,2}(1)t} + BZ_{647}\, e^{-C_{2,2}(0)t} \\
P_2(t) &= BZ_{648}\, e^{-C_{2,2}(2)t} + BZ_{649}\, e^{-C_{2,2}(0)t} \\
P_3(t) &= BZ_{650}\, e^{-C_{2,2}(3)t} + BZ_{651}\, e^{-C_{2,2}(0)t} \\
P_4(t) &= BZ_{652}\, e^{-C_{2,2}(4)t} + BZ_{653}\, e^{-C_{2,2}(0)t}
\end{aligned}
$$

$$
\begin{aligned}
P_5(t) &= BZ_{654}\, e^{-C_{2,2}(5)t} + BZ_{655}\, e^{-C_{2,2}(2)t} + BZ_{656}\, e^{-C_{2,2}(1)t} + BZ_{657}\, e^{-C_{2,2}(0)t} \\
P_6(t) &= BZ_{658}\, e^{-C_{2,2}(6)t} + BZ_{659}\, e^{-C_{2,2}(3)t} + BZ_{660}\, e^{-C_{2,2}(1)t} + BZ_{661}\, e^{-C_{2,2}(0)t} \\
P_7(t) &= BZ_{662}\, e^{-C_{2,2}(7)t} + BZ_{663}\, e^{-C_{2,2}(4)t} + BZ_{664}\, e^{-C_{2,2}(1)t} + BZ_{665}\, e^{-C_{2,2}(0)t} \\
P_8(t) &= BZ_{666}\, e^{-C_{2,2}(8)t} + BZ_{667}\, e^{-C_{2,2}(3)t} + BZ_{668}\, e^{-C_{2,2}(2)t} + BZ_{669}\, e^{-C_{2,2}(0)t} \\
P_9(t) &= BZ_{670}\, e^{-C_{2,2}(9)t} + BZ_{671}\, e^{-C_{2,2}(4)t} + BZ_{672}\, e^{-C_{2,2}(2)t} + BZ_{673}\, e^{-C_{2,2}(0)t} \\
P_{10}(t) &= BZ_{674}\, e^{-C_{2,2}(10)t} + BZ_{675}\, e^{-C_{2,2}(4)t} + BZ_{676}\, e^{-C_{2,2}(3)t} + BZ_{677}\, e^{-C_{2,2}(0)t} \\
P_{11}(t) &= BZ_{678}\, e^{-C_{2,2}(11)t} + BZ_{679}\, e^{-C_{2,2}(8)t} + BZ_{680}\, e^{-C_{2,2}(6)t} + BZ_{681}\, e^{-C_{2,2}(5)t} + \\
&\quad BZ_{682}\, e^{-C_{2,2}(3)t} + BZ_{683}\, e^{-C_{2,2}(2)t} + BZ_{684}\, e^{-C_{2,2}(1)t} + BZ_{685}\, e^{-C_{2,2}(0)t} \\
P_{12}(t) &= BZ_{686}\, e^{-C_{2,2}(12)t} + BZ_{687}\, e^{-C_{2,2}(9)t} + BZ_{688}\, e^{-C_{2,2}(7)t} + BZ_{689}\, e^{-C_{2,2}(5)t} + \\
&\quad BZ_{690}\, e^{-C_{2,2}(4)t} + BZ_{691}\, e^{-C_{2,2}(2)t} + BZ_{692}\, e^{-C_{2,2}(1)t} + BZ_{693}\, e^{-C_{2,2}(0)t} \\
P_{13}(t) &= BZ_{694}\, e^{-C_{2,2}(13)t} + BZ_{695}\, e^{-C_{2,2}(10)t} + BZ_{696}\, e^{-C_{2,2}(7)t} + BZ_{697}\, e^{-C_{2,2}(6)t} + \\
&\quad BZ_{698}\, e^{-C_{2,2}(4)t} + BZ_{699}\, e^{-C_{2,2}(3)t} + BZ_{700}\, e^{-C_{2,2}(1)t} + BZ_{701}\, e^{-C_{2,2}(0)t} \\
P_{14}(t) &= BZ_{702}\, e^{-C_{2,2}(14)t} + BZ_{703}\, e^{-C_{2,2}(10)t} + BZ_{704}\, e^{-C_{2,2}(9)t} + BZ_{705}\, e^{-C_{2,2}(8)t} + \\
&\quad BZ_{706}\, e^{-C_{2,2}(4)t} + BZ_{707}\, e^{-C_{2,2}(3)t} + BZ_{708}\, e^{-C_{2,2}(2)t} + BZ_{709}\, e^{-C_{2,2}(0)t}
\end{aligned}
$$

For $\lambda_a = \lambda_b = \lambda_d = \lambda_e = \lambda_1$, Equation (4.147) generates system reliability values that match the $(2,2,1)$ identical unit case with Type F Repair discussed in Section 4.2. Residues BZ_{646} to BZ_{709} are defined in Appendix I.2.

The mean time to failure of this system is given by:

$$
MTTF_{Z_F}(2,2,1) = \lim_{s \to 0} \sum_{i=0}^{14} P_i(s) \tag{4.148}
$$

where, $P_0(0) = 1/C_{2,2}(0)$

$P_1(0) = (\lambda_a/C_{2,2}(1)) \bullet P_0(0)$

$P_2(0) = (\lambda_b/C_{2,2}(2)) \bullet P_0(0)$

$P_3(0) = (\lambda_{se}/C_{2,2}(3)) \bullet P_0(0)$

$P_4(0) = (\lambda_{sd}/C_{2,2}(4)) \bullet P_0(0)$

$P_5(0) = (\lambda_b/C_{2,2}(5)) \bullet P_1(0) + (\lambda_a/C_{2,2}(5)) \bullet P_2(0)$

$P_6(0) = (\lambda_{sd}/C_{2,2}(6)) \bullet P_1(0) + (\lambda_a/C_{2,2}(6)) \bullet P_4(0)$

$P_7(0) = (\lambda_{se}/C_{2,2}(7)) \bullet P_1(0) + (\lambda_a/C_{2,2}(7)) \bullet P_3(0)$

$P_8(0) = (\lambda_{sd}/C_{2,2}(8)) \bullet P_2(0) + (\lambda_b/C_{2,2}(8)) \bullet P_4(0)$
$P_9(0) = (\lambda_{se}/C_{2,2}(9)) \bullet P_2(0) + (\lambda_b/C_{2,2}(9)) \bullet P_3(0)$
$P_{10}(0) = (\lambda_{se}/C_{2,2}(10)) \bullet P_4(0) + (\lambda_{sd}/C_{2,2}(10)) \bullet P_3(0)$
$P_{11}(0) = (\lambda_d/C_{2,2}(11)) \bullet P_5(0) + (\lambda_b/C_{2,2}(11)) \bullet P_6(0) + (\lambda_a/C_{2,2}(11)) \bullet P_8(0)$
$P_{12}(0) = (\lambda_{se}/C_{2,2}(12)) \bullet P_5(0) + (\lambda_b/C_{2,2}(12)) \bullet P_7(0) + (\lambda_a/C_{2,2}(12)) \bullet P_9(0)$
$P_{13}(0) = (\lambda_{se}/C_{2,2}(13)) \bullet P_6(0) + (\lambda_{sd}/C_{2,2}(13)) \bullet P_7(0) + (\lambda_a/C_{2,2}(13)) \bullet P_{10}(0)$
$P_{14}(0) = (\lambda_{se}/C_{2,2}(14)) \bullet P_8(0) + (\lambda_{sd}/C_{2,2}(14)) \bullet P_9(0) + (\lambda_b/C_{2,2}(14)) \bullet P_{10}(0)$

For $\lambda_a = \lambda_b = \lambda_d = \lambda_e = \lambda_1$, Equation (4.148) generates values that match those obtained for the $(2,2,1)$ identical unit case with Type F Repair discussed in Subsection 4.1.2.

The variance of time to failure for this system is given by:

$$\sigma^2_{Z_F}(2,2,1) \quad = \quad \frac{1}{k_{85}^2}\left[2(k_{84}k_{86} - k_{85}k_{87}) - k_{84}^2\right] \tag{4.149}$$

Symbols k_{84} to k_{87} are defined in Appendix I.2.

4.3 Concluding Remarks

The analyses performed in this Chapter indicate that the values of $k - out - of - (n + m) : G$ system performance indices under SBAP Z increase with increased system complexity characterised by an increase in either the number of active units or the number of standbys. As in standby activation policy Y, the tabular values and plots show clearly that the addition of one standby unit for a fixed number of active units improves the values of the system performance indices for the various repair scenarios considered.

For all the special cases considered in this Chapter, the system long-run availability drops as the failed system repair time distribution shape parameter β increases.

The reason for this trend, as stated in the concluding section of Chapter 3, is that the system instantaneous repair rate tends to decrease with an increasing β as will be shown later in Chapter 6. As in the previous standby activation policy, the tabular values and plots indicate that in some situations component repair may not be prudent. These are situations in which there are no significant performance differences for the different repair types investigated.

A comparison of the mean time to failure tabular values of this Chapter with those of the previous Chapter, for the same redundant configuration and Repair Type indicates that standby activation policy Z is superior to standby activation policy Y using the mean time to failure as the sole measure of performance. This may be due to the fact that delaying activation of the standby until it is absolutely necessary to do so, keeps the standby failure rate at the level of λ_s which is much lower than λ_1 (its failure rate if activated).

The non-identical unit cases analyzed in Section 4.2 yielded results that match those of corresponding identical unit cases if all system components are assumed to have the same failure rates. Thus for the same failure rates, the $(2,1,1)$ identical unit system with Type E Repair mean time to failure expression (Equation 4.14) is exactly the same as the $(2,1,1)$ non-identical unit system with Type E Repair mean time to failure expression (Equation 4.125). A comparison of Equations (4.36) and Equation (4.148) indicate that this is equally true for the $(2,2,1)$ identical unit system with Type F Repair of Subsection 4.1.2 and the $(2,2,1)$ non-identical unit system with Type E Repair of Subsection 4.2.2.

All in all, the main contribution of this Chapter is the derivation of generalized mean time to failure expressions for $k - out - of - (n+m) : G$ type systems operating under standby activation policy Z. These derivations were performed for several possible repair scenarios. The system components were considered identical and were subject to a combination of independent and common-cause failures. Only those special cases for which standby activation policy Z is practicable ($k \neq n$) were considered.

Chapter 5

Income Maximization of the Redundant System

There are various cost-benefit concerns associated with repairable $k - out - of - (n+m) : G$ redundant systems. In this regard, the ultimate goal is to ensure that systems generate incomes/benefits that far exceed the costs incurred in operating them. Otherwise, they would not be economically feasible systems. One of the ways of ensuring that the goal is met is through the efficient utilization of repair facilities. Specifically, it is often important to determine, using historical operational data (where it exists), the optimal value of the repair rate that would be necessary to maximize the system net income. In this regard, expressions for the optimal repair rate will be developed for some of the special cases already considered in this study in which the system is repairable. This chapter presents the income analyses for the following submodels of the $k - out - of - (n+m) : G$ type system viz: *(i)* $1 - out - of - (n+1) : G$ or $(n,1,1)$ *(ii)* $1 - out - of - (n+2) : G$ or $(n,2,1)$ *(iii)* $1 - out - of - (n+3) : G$ or $(n,3,1)$ *(iv)* $k - out - of - (n+1) : G$ or $(n,1,k)$ *(v)* $k - out - of - (n+2) : G$ or $(n,2,k)$ and *(vi)* $k - out - of - (n+3) : G$ or $(n,3,k)$.

Assumptions

In addition to the general system assumptions in Chapter 2, the following assumptions are associated with this income maximization analysis:

i) The system net income function is continous and differentiable.

ii) The system long-run availability is representative of overall system availability.

iii) The system usage over a certain period generates a certain income and also results in a certain running cost (mostly maintenance costs for the purposes of this analysis)

iv) The net benefits of conducting repairs always out-weigh any additional increases in repair costs within the useful life of the system.

5.1 The Identical Unit System With Early Standby Activation: Income Maximization

This analysis is conducted for the submodels mentioned above on the assumption that system units are identical. A differential calculus-based approach presented in Ref. [92] is used to determine the optimal failed system repair rates needed to maximize the system net income. The methodology adopted to conduct this analysis is explained fully for the first special case only. For subsequent special cases, only the results are presented.

5.1.1 1-out-of-(n+1):G Identical Unit Submodel: Special Cases

For the following special cases, the analysis is conducted for Type A Repair but the results can easily be used for Types B and C Repair by setting the appropriate quantities to zero.

Special Case Model I

For $n = 1$ the long-run availability for this special case from Equation (3.40) is:

$$A_{ssY}(1,1,1)(\beta) = \frac{(\mu_2\,\mu_3)^\beta\,x_1}{(\mu_2\,\mu_3)^\beta\,x_2 + \beta\,\mu_2^\beta\,\mu_3^{\beta-1}\,x_3 + \beta\,\mu_2^{\beta-1}\,\mu_3^\beta\,x_4} \tag{5.1}$$

The expected periodic, say monthly, income from running the system is:

$$I_u(1,1,1) = I_f(1,1,1) \bullet A_{ssY}(1,1,1)(\beta) \tag{5.2}$$

where

$I_u(1,1,1)$ is the expected periodic income from running the system.

$I_f(1,1,1)$ is the expected periodic income if the system were available 100 % of the time.

The expected periodic, say monthly, cost of running the system is:

$$C_M(1,1,1) = C_{hw}(1,1,1)\,\mu_2 + C_{cc}(1,1,1)\,\mu_3 \tag{5.3}$$

where

$C_M(1,1,1)$ is the periodic maintenance cost

$C_{hw}(1,1,1)$ is the system maintenance cost constant associated with hardware (or independent) failures of the system.

$C_{cc}(1,1,1)$ is the system maintenance cost constant associated with common-cause failures of the system.

μ_2 is the failed system mean repair rate if the system fails due to independent failures.

μ_3 is the failed system mean repair rate if the system fails due to common-cause failures.

The net income generated by the system is given by:

$$\begin{aligned} I_{net}(1,1,1) &= I_u(1,1,1) - C_M(1,1,1) \\ &= \frac{I_f(1,1,1)\,(\mu_2\,\mu_3)^\beta\,x_1}{(\mu_2\,\mu_3)^\beta\,x_2 + \beta\,\mu_2^\beta\,\mu_3^{\beta-1}\,x_3 + \beta\,\mu_2^{\beta-1}\,\mu_3^\beta\,x_4} - C_{hw}(1,1,1)\,\mu_2 - \\ &\quad C_{cc}(1,1,1)\,\mu_3 \end{aligned} \tag{5.4}$$

The optimal system mean repair rates needed to maximize the net income can be determined by differentiating Equation (5.4) with respect to μ_2 and μ_3 respectively and setting the results to zero.

$$\frac{\partial I_{net}(1,1,1)}{\partial \mu_2} = \frac{f_1(\mu_2)}{g_1(\mu_2)} - C_{hw}(1,1,1) = 0 \tag{5.5}$$

where

$$\begin{aligned} f_1(\mu_2) &= \beta\, I_f(1,1,1)\, x_1\, x_2\, \mu_2^{2\beta-1}\, \mu_3^{2\beta} + \beta^2\, I_f(1,1,1)\, x_1\, x_3\, \mu_2^{2\beta-1}\, \mu_3^{2\beta-1} + \\ &\quad \beta^2\, I_f(1,1,1)\, x_1\, x_4\, \mu_2^{2\beta-2}\, \mu_3^{2\beta} - \beta\, I_f(1,1,1)\, x_1\, x_2\, \mu_2^{2\beta-1}\, \mu_3^{2\beta} - \\ &\quad \beta^2\, I_f(1,1,1)\, x_1\, x_3\, \mu_2^{2\beta-1}\, \mu_3^{2\beta-1} - \beta\,(\beta-1)\, I_f(1,1,1)\, x_1\, x_4\, \mu_2^{2\beta-1}\, \mu_3^{2\beta} \\ g_1(\mu_2) &= (\mu_2^{\beta}\, \mu_3^{\beta}\, x_2 + \beta\, \mu_2^{\beta}\, \mu_3^{\beta-1}\, x_3 + \beta\, \mu_2^{\beta-1} \mu_3^{\beta}\, x_4)^2 \end{aligned}$$

$$\frac{\partial I_{net}(1,1,1)}{\partial \mu_3} = \frac{f_1(\mu_3)}{g_1(\mu_3)} - C_{cc}(1,1,1) = 0 \tag{5.6}$$

where

$$\begin{aligned} f_1(\mu_3) &= \beta\, I_f(1,1,1)\, x_1\, x_2\, \mu_2^{2\beta}\, \mu_3^{2\beta-1} + \beta^2\, I_f(1,1,1)\, x_1\, x_3\, \mu_2^{2\beta}\, \mu_3^{2\beta-2} + \\ &\quad \beta^2\, I_f(1,1,1)\, x_1\, x_4\, \mu_2^{2\beta-1}\, \mu_3^{2\beta-1} - \beta\, I_f(1,1,1)\, x_1\, x_2\, \mu_2^{2\beta}\, \mu_3^{2\beta-1} - \\ &\quad \beta^2\, I_f(1,1,1)\, x_1\, x_4\, \mu_2^{2\beta-1}\, \mu_3^{2\beta-1} - \beta\,(\beta-1)\, I_f(1,1,1)\, x_1\, x_3\, \mu_2^{2\beta}\, \mu_3^{2\beta-2} \\ g_1(\mu_3) &= g_1(\mu_2) \end{aligned}$$

The optimal values of μ_2 and μ_3 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.5) and (5.6). These optimal values can be substituted back into Equation (5.4) to obtain the maximun net income derivable from the system. Further differentiations of Equations (5.5) and (5.6) with respect to μ_2 and μ_3 indicate that these turning points would lead to maximum values of $I_{net}(1,1,1)$.

With Type B Repair, the quantity μ_1 is set to zero and with Type C Repair, μ_{s0} and μ_1 are set to zero. These changes will affect only the values of the constants x_1 to x_4.

Special Case Model II

For $n = 2$, the long-run availability for this special case from Equation (3.41) is:

$$A_{ss_Y}(2,1,1)(\beta) = \frac{(\mu_3\,\mu_4)^\beta\,x_5}{(\mu_3\,\mu_4)^\beta\,x_6 + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_7 + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_8} \tag{5.7}$$

The expected periodic, say monthly, income from running the system is:

$$I_u(2,1,1) = I_f(2,1,1) \bullet A_{ss_Y}(2,1,1)(\beta) \tag{5.8}$$

where

$I_u(2,1,1)$ is the expected periodic income from running the system.

$I_f(2,1,1)$ is the expected periodic income if the system were available 100 % of the time.

The expected periodic, say monthly, cost of running the system is:

$$C_M(2,1,1) = C_{hw}(2,1,1)\,\mu_3 + C_{cc}(2,1,1)\,\mu_4 \tag{5.9}$$

where

$C_M(2,1,1)$ is the periodic maintenance cost

$C_{hw}(2,1,1)$ is the system maintenance cost constant associated with hardware (or independent) failures of the system.

$C_{cc}(2,1,1)$ is the system maintenance cost constant associated with common-cause failures of the system.

μ_3 is the failed system mean repair rate if the system fails due to independent failures.

μ_4 is the failed system mean repair rate if the system fails due to common-cause failures.

The net income generated by the system is given by:

$$\begin{aligned} I_{net}(2,1,1) &= I_u(2,1,1) - C_M(2,1,1) \\ &= \frac{I_f(2,1,1)\,(\mu_3\,\mu_4)^\beta\,x_5}{(\mu_3\,\mu_4)^\beta\,x_6 + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_7 + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_8} - C_{hw}(2,1,1)\,\mu_3 \\ &\quad -C_{cc}(2,1,1)\,\mu_4 \end{aligned} \tag{5.10}$$

The optimal system mean repair rates needed to maximize the net income can be determined by differentiating Equation (5.10) with respect to μ_3 and μ_4 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_3} = \frac{f_1(\mu_3)}{g_1(\mu_3)} - C_{hw}(2,1,1) = 0 \tag{5.11}$$

where

$$\begin{aligned}
f_1(\mu_3) &= \beta\, I_f(2,1,1)\, x_5\, x_6\, \mu_3^{2\beta-1}\, \mu_4^{2\beta} + \beta^2\, I_f(2,1,1)\, x_5\, x_7\, \mu_3^{2\beta-1}\, \mu_4^{2\beta-1} + \\
&\quad \beta^2\, I_f(2,1,1)\, x_5\, x_8\, \mu_3^{2\beta-2}\, \mu_4^{2\beta} - \beta\, I_f(2,1,1)\, x_5\, x_6\, \mu_3^{2\beta-1}\, \mu_4^{2\beta} - \\
&\quad \beta^2\, I_f(2,1,1)\, x_5\, x_7\, \mu_3^{2\beta-1}\, \mu_4^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,1,1)\, x_5\, x_8\, \mu_3^{2\beta-2}\, \mu_4^{2\beta} \\
g_1(\mu_3) &= (\mu_3^{\beta}\, \mu_4^{\beta}\, x_6 + \beta\, \mu_3^{\beta}\, \mu_4^{\beta-1}\, x_7 + \beta\, \mu_3^{\beta-1} \mu_4^{\beta}\, x_8)^2
\end{aligned}$$

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{cc}(2,1,1) = 0 \tag{5.12}$$

where

$$\begin{aligned}
f_1(\mu_4) &= \beta\, I_f(2,1,1)\, x_5\, x_6\, \mu_3^{2\beta}\, \mu_4^{2\beta-1} + \beta^2\, I_f(2,1,1)\, x_5\, x_7\, \mu_3^{2\beta}\, \mu_4^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,1,1)\, x_5\, x_8\, \mu_3^{2\beta-1}\, \mu_4^{2\beta-1} - \beta\, I_f(2,1,1)\, x_5\, x_6\, \mu_3^{2\beta}\, \mu_4^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,1,1)\, x_5\, x_8\, \mu_3^{2\beta-1}\, \mu_4^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,1,1)\, x_5\, x_7\, \mu_3^{2\beta}\, \mu_4^{2\beta-2} \\
g_1(\mu_4) &= g_1(\mu_3)
\end{aligned}$$

The optimal values of μ_3 and μ_4 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.11) and (5.12). These optimal values can be substituted back into Equation (5.10) to obtain the maximun net income derivable from the system. Further differentiations of Equations (5.11) and (5.12) with respect to μ_3 and μ_4 indicate that these turning points would lead to maximum values of $I_{net}(2,1,1)$.

With Type B Repair, the above expressions remain valid but μ_1 and μ_2 are set to zero and with Type C Repair, $\mu_{s0'}$, μ_1 and μ_2 are set to zero. These changes will affect only the values of the constants x_5 to x_8.

Special Case Model III

For $n = 3$, the long-run availability for this special case from Equation (3.42) is:

$$A_{ssY}(3,1,1)(\beta) = \frac{(\mu_4\,\mu_5)^\beta\, x_9}{(\mu_4\,\mu_5)^\beta\, x_{10} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{11} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{12}} \tag{5.13}$$

The expected periodic, say monthly, income from running the system is:

$$I_u(3,1,1) = I_f(3,1,1) \bullet A_{ssY}(3,1,1)(\beta) \tag{5.14}$$

where

$I_u(3,1,1)$ is the expected periodic income from running the system.

$I_f(3,1,1)$ is the expected periodic income if the system were available 100 % of the time.

The expected periodic, say monthly, cost of running the system is:

$$C_M(3,1,1) = C_{hw}(3,1,1)\,\mu_4 + C_{cc}(3,1,1)\,\mu_5 \tag{5.15}$$

where

$C_M(3,1,1)$ is the periodic maintenance cost

$C_{hw}(3,1,1)$ is the system maintenance cost constant associated with hardware (or independent) failures of the system.

$C_{cc}(3,1,1)$ is the system maintenance cost constant associated with common-cause failures of the system.

μ_4 is the failed system mean repair rate if the system fails due to independent failures.

μ_5 is the failed system mean repair rate if the system fails due to common-cause failures.

The net income generated by the system is given by:

$$\begin{aligned} I_{net}(3,1,1) &= I_u(3,1,1) - C_M(3,1,1) \\ &= \frac{I_f(3,1,1)\,(\mu_4\,\mu_5)^\beta\, x_9}{(\mu_4\,\mu_5)^\beta\, x_{10} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{11} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{12}} - C_{hw}(3,1,1)\,\mu_4 \\ &\quad - C_{cc}(3,1,1)\,\mu_5 \end{aligned} \tag{5.16}$$

The optimal system mean repair rates needed to maximize the net income can be determined by differentiating Equation (5.16) with respect to μ_4 and μ_5 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,1,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{hw}(3,1,1) = 0 \tag{5.17}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\, I_f(3,1,1)\, x_9\, x_{10}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} + \beta^2\, I_f(3,1,1)\, x_9\, x_{11}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} + \\ &\quad \beta^2\, I_f(3,1,1)\, x_9\, x_{12}\, \mu_4^{2\beta-2}\, \mu_5^{2\beta} - \beta\, I_f(3,1,1)\, x_9\, x_{12}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} - \\ &\quad \beta^2\, I_f(3,1,1)\, x_9\, x_{11}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,1,1)\, x_9\, x_{12}\, \mu_4^{2\beta-2}\, \mu_5^{2\beta} \\ g_1(\mu_4) &= (\mu_4^{\beta}\, \mu_5^{\beta}\, x_{10} + \beta\, \mu_4^{\beta}\, \mu_5^{\beta-1}\, x_{11} + \beta\, \mu_4^{\beta-1} \mu_5^{\beta}\, x_{12})^2 \end{aligned}$$

$$\frac{\partial I_{net}(3,1,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{cc}(3,1,1) = 0 \tag{5.18}$$

where

$$\begin{aligned} f_1(\mu_5) &= \beta\, I_f(3,1,1)\, x_9\, x_{10}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} + \beta^2\, I_f(3,1,1)\, x_9\, x_{11}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} + \\ &\quad \beta^2\, I_f(3,1,1)\, x_9\, x_{12}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} - \beta\, I_f(3,1,1)\, x_9\, x_{10}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} - \\ &\quad \beta^2\, I_f(3,1,1)\, x_9\, x_{12}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,1,1)\, x_9\, x_{11}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} \\ g_1(\mu_5) &= g_1(\mu_4) \end{aligned}$$

The optimal values of μ_4 and μ_5 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.17) and (5.18). These optimal values can be substituted back into Equation (5.16) to obtain the maximun net income derivable from the system. Further differentiations of Equations (5.17) and (5.18) with respect to μ_4 and μ_5 indicate that these turning points would lead to maximum values of $I_{net}(3,1,1)$.

With Type B Repair, the above expressions remain valid but μ_1 to μ_3 are set to zero and with Type C Repair, $\mu_{s0'}$, μ_1 to μ_3 are set to zero. These changes will affect only the values of the constants x_9 to x_{12}.

5.1.2 1-out-of-(n+2):G Identical Unit Submodel: Special Cases

For the following special case models, the following results are obtained:

Special Case Model I

For $n = 1$, the net income generated by the system using the approach established for the previous special cases is given by:

$$I_{net}(1,2,1) = \frac{I_f(1,2,1)\,(\mu_3\,\mu_4)^\beta\,x_{13}}{(\mu_3\,\mu_4)^\beta\,x_{14} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_{15} + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_{16}} - C_{hw}(1,2,1)\,\mu_3 - C_{cc}(1,2,1)\,\mu_4 \tag{5.19}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.19) with respect to μ_3 and μ_4 respectively and setting the results to zero.

$$\frac{\partial I_{net}(1,2,1)}{\partial \mu_3} = \frac{f_1(\mu_3)}{g_1(\mu_3)} - C_{hw}(1,2,1) = 0 \tag{5.20}$$

where

$$\begin{aligned} f_1(\mu_3) &= \beta\,I_f(1,2,1)\,x_{13}\,x_{14}\,\mu_3^{2\beta-1}\,\mu_4^{2\beta} + \beta^2\,I_f(1,2,1)\,x_{13}\,x_{15}\,\mu_3^{2\beta-1}\,\mu_4^{2\beta-1} + \\ &\quad \beta^2\,I_f(1,2,1)\,x_{13}\,x_{16}\,\mu_3^{2\beta-2}\mu_4^{2\beta} - \beta\,I_f(1,2,1)\,x_{13}\,x_{16}\,\mu_3^{2\beta-1}\,\mu_4^{2\beta} - \\ &\quad \beta^2\,I_f(1,2,1)\,x_{13}\,x_{15}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,(\beta-1)\,I_f(1,2,1)\,x_{13}\,x_{16}\,\mu_3^{2\beta-2}\,\mu_4^{2\beta} \\ g_1(\mu_3) &= (\mu_3^\beta\,\mu_4^\beta\,x_{14} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_{15} + \beta\,\mu_3^{\beta-1}\mu_4^\beta\,x_{16})^2 \end{aligned}$$

$$\frac{\partial I_{net}(1,2,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{cc}(1,2,1) = 0 \tag{5.21}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\,I_f(1,2,1)\,x_{13}\,x_{14}\,\mu_3^{2\beta}\,\mu_4^{2\beta-1} + \beta^2\,I_f(1,2,1)\,x_{13}\,x_{15}\,\mu_3^{2\beta}\,\mu_4^{2\beta-2} + \\ &\quad \beta^2\,I_f(1,2,1)\,x_{13}\,x_{16}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,I_f(1,2,1)\,x_{13}\,x_{14}\,\mu_3^{2\beta}\,\mu_4^{2\beta-1} - \\ &\quad \beta^2\,I_f(1,2,1)\,x_{13}\,x_{16}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,(\beta-1)\,I_f(1,2,1)\,x_{13}\,x_{15}\,\mu_3^{2\beta}\,\mu_4^{2\beta-2} \\ g_1(\mu_4) &= g_1(\mu_3) \end{aligned}$$

The optimal values of μ_3 and μ_4 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.20) and (5.21). These optimal values can be substituted back into Equation (5.19) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_2 are set to zero and with Type C Repair, $\mu_{s0'}$, $\mu_{s1'}$, μ_1 and μ_2 are set to zero. These changes will affect only the values of the constants x_{13} to x_{16}.

Special Case Model II

For $n = 2$, the net income generated by the system is given by:

$$\begin{aligned} I_{net}(2,2,1) &= \frac{I_f(2,2,1)\,(\mu_4\,\mu_5)^\beta\, x_{17}}{(\mu_4\,\mu_5)^\beta\, x_{18} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{19} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{20}} - C_{hw}(2,2,1)\,\mu_4 \\ &\quad - C_{cc}(2,2,1)\,\mu_5 \end{aligned} \tag{5.22}$$

The optimal system mean repair rates needed to maximize the net income can be determined by differentiating Equation (5.22) with respect to μ_4 and μ_5 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{hw}(2,2,1) = 0 \tag{5.23}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\, I_f(2,2,1)\, x_{17}\, x_{18}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} + \beta^2\, I_f(2,2,1)\, x_{17}\, x_{19}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} + \\ &\quad \beta^2\, I_f(2,2,1)\, x_{17}\, x_{20}\, \mu_4^{2\beta-2}\mu_5^{2\beta} - \beta\, I_f(2,2,1)\, x_{17}\, x_{20}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} - \\ &\quad \beta^2\, I_f(2,2,1)\, x_{17}\, x_{19}\, \mu_4^{2\beta-1}\mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,2,1)\, x_{17}\, x_{20}\, \mu_4^{2\beta-2}\, \mu_5^{2\beta} \\ g_1(\mu_4) &= (\mu_4^\beta\,\mu_5^\beta\, x_{18} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{19} + \beta\,\mu_4^{\beta-1}\mu_5^\beta\, x_{20})^2 \end{aligned}$$

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{cc}(2,2,1) = 0 \tag{5.24}$$

where

$$f_1(\mu_5) = \beta\, I_f(2,2,1)\, x_{17}\, x_{18}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} + \beta^2\, I_f(2,2,1)\, x_{17}\, x_{19}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} +$$

$$\beta^2 I_f(2,2,1)\, x_{17}\, x_{20}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\, I_f(2,2,1)\, x_{17}\, x_{18}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} -$$
$$\beta^2 I_f(2,2,1)\, x_{17}\, x_{20}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,2,1)\, x_{17}\, x_{19}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2}$$
$$g_1(\mu_5) = g_1(\mu_4)$$

The optimal values of μ_4 and μ_5 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.23) and (5.24). These optimal values can be substituted back into Equation (5.22) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_3 are set to zero and with Type C Repair, $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_3 are set to zero. These changes will affect only the values of the constants x_{17} to x_{20}.

Special Case Model III

For $n = 3$, the net income generated by the system is given by:

$$I_{net}(3,2,1) = \frac{I_f(3,2,1)\,(\mu_5\,\mu_6)^\beta\, x_{21}}{(\mu_5\,\mu_6)^\beta\, x_{22} + \beta\, \mu_5^\beta\, \mu_6^{\beta-1}\, x_{23} + \beta\, \mu_5^{\beta-1}\, \mu_6^\beta\, x_{24}} - C_{hw}(3,2,1)\, \mu_5 - C_{cc}(3,2,1)\, \mu_6 \qquad (5.25)$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.25) with respect to μ_5 and μ_6 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,2,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{hw}(3,2,1) = 0 \qquad (5.26)$$

where

$$f_1(\mu_5) = \beta\, I_f(3,2,1)\, x_{21}\, x_{22}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} + \beta^2\, I_f(3,2,1)\, x_{21}\, x_{23}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta-1} +$$
$$\beta^2\, I_f(3,2,1)\, x_{21}\, x_{24}\, \mu_5^{2\beta-2} \mu_6^{2\beta} - \beta\, I_f(3,2,1)\, x_{21}\, x_{24}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} -$$
$$\beta^2\, I_f(3,2,1)\, x_{21}\, x_{23}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,2,1)\, x_{21}\, x_{24}\, \mu_5^{2\beta-2}\, \mu_6^{2\beta}$$
$$g_1(\mu_5) = (\mu_5^\beta\, \mu_6^\beta\, x_{22} + \beta\, \mu_5^\beta\, \mu_6^{\beta-1}\, x_{23} + \beta\, \mu_5^{\beta-1} \mu_6^\beta\, x_{24})^2$$

$$\frac{\partial I_{net}(3,2,1)}{\partial \mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{cc}(3,2,1) = 0 \tag{5.27}$$

where

$$\begin{aligned} f_1(\mu_6) &= \beta\, I_f(3,2,1)\, x_{21}\, x_{22}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} + \beta^2\, I_f(3,2,1)\, x_{21}\, x_{23}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} + \\ &\quad \beta^2\, I_f(3,2,1)\, x_{21}\, x_{24}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\, I_f(3,2,1)\, x_{21}\, x_{22}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} - \\ &\quad \beta^2\, I_f(3,2,1)\, x_{21}\, x_{24}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,2,1)\, x_{21}\, x_{23}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} \\ g_1(\mu_6) &= g_1(\mu_5) \end{aligned}$$

The optimal values of μ_5 and μ_6 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.26) and (5.27). These optimal values can be substituted back into Equation (5.25) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_4 are set to zero and with Type C Repair, $\mu_{s0'}$, $\mu_{s1'}$, μ_1 to μ_4 are set to zero. These changes will affect only the values of the constants x_{21} to x_{24}.

5.1.3 1-out-of-(n+3):G Identical Unit Submodel: Special cases

For the following special case models, the following results are obtained:

Special Case Model I

For $n = 1$, the net income generated by the system is given by:

$$\begin{aligned} I_{net}(1,3,1) &= \frac{I_f(1,3,1)\,(\mu_4\,\mu_5)^\beta\, x_{25}}{(\mu_4\,\mu_5)^\beta\, x_{26} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{27} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{28}} - C_{hw}(1,3,1)\,\mu_4 \\ &\quad -C_{cc}(1,3,1)\,\mu_5 \end{aligned} \tag{5.28}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.28) with respect to μ_4 and μ_5 respectively and setting the results to zero.

$$\frac{\partial I_{net}(1,3,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{hw}(1,3,1) = 0 \tag{5.29}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\, I_f(1,3,1)\, x_{25}\, x_{26}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} + \beta^2\, I_f(1,3,1)\, x_{25}\, x_{27}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} + \\ &\quad \beta^2\, I_f(1,3,1)\, x_{25}\, x_{28}\, \mu_4^{2\beta-2} \mu_5^{2\beta} - \beta\, I_f(1,3,1)\, x_{25}\, x_{28}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} - \\ &\quad \beta^2\, I_f(1,3,1)\, x_{25}\, x_{27}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(1,3,1)\, x_{25}\, x_{28}\, \mu_4^{2\beta-2}\, \mu_5^{2\beta} \\ g_1(\mu_4) &= (\mu_4^{\beta}\, \mu_5^{\beta}\, x_{26} + \beta\, \mu_4^{\beta}\, \mu_5^{\beta-1}\, x_{27} + \beta\, \mu_4^{\beta-1} \mu_5^{\beta}\, x_{28})^2 \end{aligned}$$

$$\frac{\partial I_{net}(1,3,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{cc}(1,3,1) = 0 \tag{5.30}$$

where

$$\begin{aligned} f_1(\mu_5) &= \beta\, I_f(1,3,1)\, x_{25}\, x_{26}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} + \beta^2\, I_f(1,3,1)\, x_{25}\, x_{27}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} + \\ &\quad \beta^2\, I_f(1,3,1)\, x_{25}\, x_{28}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\, I_f(1,3,1)\, x_{25}\, x_{26}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} - \\ &\quad \beta^2\, I_f(1,3,1)\, x_{25}\, x_{28}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(1,3,1)\, x_{25}\, x_{27}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} \\ g_1(\mu_5) &= g_1(\mu_4) \end{aligned}$$

The optimal values of μ_4 and μ_5 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.29) and (5.30). These optimal values can be substituted back into Equation (5.28) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_3 are set to zero and with Type C Repair, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_3 are set to zero. These changes will affect only the values of the constants x_{25} to x_{28}.

Special Case Model II

For $n = 2$, the net income generated by the system is given by:

$$\begin{aligned} I_{net}(2,3,1) &= \frac{I_f(2,3,1)\,(\mu_5\,\mu_6)^{\beta}\, x_{29}}{(\mu_5\,\mu_6)^{\beta}\, x_{30} + \beta\, \mu_5^{\beta}\, \mu_6^{\beta-1}\, x_{31} + \beta\, \mu_5^{\beta-1}\, \mu_6^{\beta}\, x_{32}} - C_{hw}(2,3,1)\, \mu_5 \\ &\quad -C_{cc}(2,3,1)\, \mu_6 \end{aligned} \tag{5.31}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.31) with respect to μ_5 and μ_6 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,3,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{hw}(2,3,1) = 0 \tag{5.32}$$

where,

$$\begin{aligned}
f_1(\mu_5) &= \beta\, I_f(2,3,1)\, x_{29}\, x_{30}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} + \beta^2\, I_f(2,3,1)\, x_{29}\, x_{31}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta-1} + \\
&\quad \beta^2\, I_f(2,3,1)\, x_{29}\, x_{32}\, \mu_5^{2\beta-2} \mu_6^{2\beta} - \beta\, I_f(2,3,1)\, x_{29}\, x_{32}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} - \\
&\quad \beta^2\, I_f(2,3,1)\, x_{29}\, x_{31}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,3,1)\, x_{29}\, x_{32}\, \mu_5^{2\beta-2}\, \mu_6^{2\beta} \\
g_1(\mu_5) &= (\mu_5^{\beta}\, \mu_6^{\beta}\, x_{30} + \beta\, \mu_5^{\beta}\, \mu_6^{\beta-1}\, x_{31} + \beta\, \mu_5^{\beta-1} \mu_6^{\beta}\, x_{32})^2
\end{aligned}$$

$$\frac{\partial I_{net}(2,3,1)}{\partial \mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{cc}(2,3,1) = 0 \tag{5.33}$$

where,

$$\begin{aligned}
f_1(\mu_6) &= \beta\, I_f(2,3,1)\, x_{29}\, x_{30}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} + \beta^2\, I_f(2,3,1)\, x_{29}\, x_{31}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,3,1)\, x_{29}\, x_{32}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\, I_f(2,3,1)\, x_{29}\, x_{30}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,3,1)\, x_{29}\, x_{32}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,3,1)\, x_{29}\, x_{31}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} \\
g_1(\mu_6) &= g_1(\mu_5)
\end{aligned}$$

The optimal values of μ_5 and μ_6 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.32) and (5.33). These optimal values can be substituted back into Equation (5.31) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_4 are set to zero and with Type C Repair, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_4 are set to zero. These changes will affect only the values of the constants x_{29} to x_{32}.

Special Case Model III

For $n = 3$, the net income generated by the system is given by:

$$\begin{aligned} I_{net}(3,3,1) &= \frac{I_f(3,3,1)\,(\mu_6\,\mu_7)^\beta\,x_{33}}{(\mu_6\,\mu_7)^\beta\,x_{34} + \beta\,\mu_6^\beta\,\mu_7^{\beta-1}\,x_{35} + \beta\,\mu_6^{\beta-1}\,\mu_7^\beta\,x_{36}} - C_{hw}(3,3,1)\,\mu_6 \\ &\quad -C_{cc}(3,3,1)\,\mu_7 \end{aligned} \tag{5.34}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.34) with respect to μ_6 and μ_7 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,3,1)}{\partial\mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{hw}(3,3,1) = 0 \tag{5.35}$$

where,

$$\begin{aligned} f_1(\mu_6) &= \beta\,I_f(3,3,1)\,x_{33}\,x_{34}\,\mu_6^{2\beta-1}\,\mu_7^{2\beta} + \beta^2\,I_f(3,3,1)\,x_{33}\,x_{35}\,\mu_6^{2\beta-1}\,\mu_7^{2\beta-1} + \\ &\quad \beta^2\,I_f(3,3,1)\,x_{33}\,x_{36}\,\mu_6^{2\beta-2}\mu_7^{2\beta} - \beta\,I_f(3,3,1)\,x_{33}\,x_{36}\,\mu_6^{2\beta-1}\,\mu_7^{2\beta} - \\ &\quad \beta^2\,I_f(3,3,1)\,x_{33}\,x_{35}\,\mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\,(\beta-1)\,I_f(3,3,1)\,x_{33}\,x_{36}\,\mu_6^{2\beta-2}\,\mu_7^{2\beta} \\ g_1(\mu_6) &= (\mu_6^\beta\,\mu_7^\beta\,x_{34} + \beta\,\mu_6^\beta\,\mu_7^{\beta-1}\,x_{35} + \beta\,\mu_6^{\beta-1}\mu_7^\beta\,x_{36})^2 \end{aligned}$$

$$\frac{\partial I_{net}(3,3,1)}{\partial\mu_7} = \frac{f_1(\mu_7)}{g_1(\mu_7)} - C_{cc}(3,3,1) = 0 \tag{5.36}$$

where,

$$\begin{aligned} f_1(\mu_7) &= \beta\,I_f(3,3,1)\,x_{33}\,x_{34}\,\mu_6^{2\beta}\,\mu_7^{2\beta-1} + \beta^2\,I_f(3,3,1)\,x_{33}\,x_{35}\,\mu_6^{2\beta}\,\mu_7^{2\beta-2} + \\ &\quad \beta^2\,I_f(3,3,1)\,x_{33}\,x_{36}\,\mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\,I_f(3,3,1)\,x_{33}\,x_{34}\,\mu_6^{2\beta}\,\mu_7^{2\beta-1} - \\ &\quad \beta^2\,I_f(3,3,1)\,x_{33}\,x_{36}\,\mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\,(\beta-1)\,I_f(3,3,1)\,x_{33}\,x_{35}\,\mu_6^{2\beta}\,\mu_7^{2\beta-2} \\ g_1(\mu_7) &= g_1(\mu_6) \end{aligned}$$

The optimal values of μ_6 and μ_7 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.35) and (5.36). These optimal values can be substituted back into Equation (5.34) to obtain the maximum net income derivable from the system.

With Type B Repair, the above expressions remain valid but μ_1 to μ_5 are set to zero and with Type C Repair, $\mu_{s0'}$ to $\mu_{s2'}$, μ_1 to μ_5 are set to zero. These changes will affect only the values of the constants x_{33} to x_{36}.

5.1.4 k-out-of-(n+1):G Identical Unit Submodel: Special cases

The expressions obtained for the $1 - out - of - (n+1) : G$ special cases are relevant for the $k - out - of - (n+1) : G$ special case models. In fact those expressions represent the cases for which $k = 1$. For $k = 2$, the quantities relating to system up-state n are eliminated from x_1 to x_{12} and for $k = 3$, the quantities relating to system up-states *n-1, n* are eliminated from x_1 to x_{12}. Thus, for $k = i$ where $i \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-i+2, n-i+3,....,n* are eliminated from x_1 to x_{12}.

5.1.5 k-out-of-(n+2):G Identical Unit Submodel: Special cases

The expressions obtained for the $1 - out - of - (n+2) : G$ special cases are relevant for the $k - out - of - (n+2) : G$ special case models. In fact those expressions represent the cases for which $k = 1$. For $k = 2$, the quantities relating to system up-state *n+1* are eliminated from x_{13} to x_{24} and for $k = 3$, the quantities relating to system up-states *n, n+1* are eliminated from x_{13} to x_{24}. Thus, for $k = i$ where $i \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-i+3, n-i+4,....,n+1* are eliminated from x_{13} to x_{24}.

5.1.6 k-out-of-(n+3):G Identical Unit Submodel: Special Cases

The expressions obtained for the $1 - out - of - (n+3) : G$ special cases are relevant for the $k - out - of - (n+3) : G$ special case models. In fact those expressions represent the cases for which $k = 1$. For $k = 2$, the quantities relating to system up-state *n+2* are eliminated from x_{25} to x_{36} and for $k = 3$, the quantities relating to system up-states *n+1, n+2* are eliminated from x_{25} to x_{36}. Thus, for $k = i$ where $i \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-i+4, n-i+5,....,n+2* are eliminated from x_{13} to x_{24}. Thus, in general, the expressions derived for the $1 - out - of - (n+m) : G$ system are useful for the $k - out - of - (n+m) : G$ by simply eliminating the terms relating to the system up-states *n+m-k+1,n+m-k+2,.....n+m-1* from the relevant constants.

5.2 The Non-Identical Unit System With Early Standby Activation: Special Cases

The same procedure outlined for the identical unit case is applicable if the system units are non-identical.

Special Case Model I

For $n = 2$, $m = 1$ and $k = 1$ (i.e. $1 - out - of - (2+1) : G$ system) with Type B Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,1,1) &= \frac{I_f(2,1,1)\,(\mu_7\,\mu_8)^\beta\, x_{69}}{(\mu_7\,\mu_8)^\beta\, x_{70} + \beta\,\mu_7^\beta\,\mu_8^{\beta-1}\, x_{71} + \beta\,\mu_7^{\beta-1}\,\mu_8^\beta\, x_{72}} - C_{hw}(2,1,1)\,\mu_7 \\ &\quad - C_{cc}(2,1,1)\,\mu_8 \end{aligned} \tag{5.37}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.37) with respect to μ_7 and μ_8 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_7} = \frac{f_1(\mu_7)}{g_1(\mu_7)} - C_{hw}(2,1,1) = 0 \tag{5.38}$$

where,

$$\begin{aligned}
f_1(\mu_7) &= \beta\, I_f(2,1,1)\, x_{69}\, x_{70}\, \mu_7^{2\beta-1}\, \mu_8^{2\beta} + \beta^2\, I_f(2,1,1)\, x_{69}\, x_{71}\, \mu_7^{2\beta-1}\, \mu_8^{2\beta-1} + \\
&\quad \beta^2\, I_f(2,1,1)\, x_{69}\, x_{72}\, \mu_7^{2\beta-2} \mu_8^{2\beta} - \beta\, I_f(2,1,1)\, x_{69}\, x_{72}\, \mu_7^{2\beta-1}\, \mu_8^{2\beta} - \\
&\quad \beta^2\, I_f(2,1,1)\, x_{69}\, x_{71}\, \mu_7^{2\beta-1} \mu_8^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,1,1)\, x_{69}\, x_{72}\, \mu_7^{2\beta-2}\, \mu_8^{2\beta} \\
g_1(\mu_7) &= (\mu_7^{\beta}\, \mu_8^{\beta}\, x_{70} + \beta\, \mu_7^{\beta}\, \mu_8^{\beta-1}\, x_{71} + \beta\, \mu_7^{\beta-1} \mu_8^{\beta}\, x_{72})^2
\end{aligned}$$

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_8} = \frac{f_1(\mu_8)}{g_1(\mu_8)} - C_{cc}(2,1,1) = 0 \tag{5.39}$$

where,

$$\begin{aligned}
f_1(\mu_8) &= \beta\, I_f(2,1,1)\, x_{69}\, x_{70}\, \mu_7^{2\beta}\, \mu_8^{2\beta-1} + \beta^2\, I_f(2,1,1)\, x_{69}\, x_{71}\, \mu_7^{2\beta}\, \mu_8^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,1,1)\, x_{69}\, x_{72}\, \mu_7^{2\beta-1} \mu_8^{2\beta-1} - \beta\, I_f(2,1,1)\, x_{69}\, x_{70}\, \mu_7^{2\beta}\, \mu_8^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,1,1)\, x_{69}\, x_{72}\, \mu_7^{2\beta-1} \mu_8^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,1,1)\, x_{69}\, x_{71}\, \mu_7^{2\beta}\, \mu_8^{2\beta-2} \\
g_1(\mu_8) &= g_1(\mu_7)
\end{aligned}$$

The optimal values of μ_7 and μ_8 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.38) and (5.39). These optimal values can be substituted back into Equation (5.37) to obtain the maximum net income derivable from the system.

With Type C Repair, the above expressions remain valid but μ_{sd} is set to zero. This change will affect only the values of the constants x_{69} to x_{72}.

Special Case Model II

For $n = 2$, $m = 1$ and $k = 2$ (i.e. $2 - out - of - (2+1) : G$ system) with Type B Repair, the optimal system repair rates can also be determined iteratively from Equations (5.38) and (5.39) by eliminating *system up-states 4, 5* and *6* and their associated constants from x_{69} to x_{72}.

With Type C Repair, the optimal failed system repair rates can be determined from the results obtained for the Type B Repair case by setting μ_{sd} equal to zero. This change will affect only the values of the constants x_{69} to x_{72}.

Special Case Model III

For $n = 2$, $m = 2$ and $k = 1$ (i.e. $1 - out - of - (2+2) : G$ system) with Type C Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,2,1) &= \frac{I_f(2,2,1)\,(\mu_{15}\,\mu_{16})^{\beta}\,x_{77}}{(\mu_{15}\,\mu_{16})^{\beta}\,x_{78} + \beta\,\mu_{15}^{\beta}\,\mu_{16}^{\beta-1}\,x_{79} + \beta\,\mu_{15}^{\beta-1}\,\mu_{16}^{\beta}\,x_{80}} - C_{hw}(2,2,1)\,\mu_{15} \\ &\quad - C_{cc}(2,2,1)\,\mu_{16} \end{aligned} \tag{5.40}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.40) with respect to μ_{15} and μ_{16} respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_{15}} = \frac{f_1(\mu_{15})}{g_1(\mu_{15})} - C_{hw}(2,2,1) = 0 \tag{5.41}$$

where,

$$\begin{aligned} f_1(\mu_{15}) &= \beta\,I_f(2,2,1)\,x_{77}x_{78}\,\mu_{15}^{2\beta-1}\mu_{16}^{2\beta} + \beta^2\,I_f(2,2,1)\,x_{77}\,x_{79}\,\mu_{15}^{2\beta-1}\,\mu_{16}^{2\beta-1} + \\ &\quad \beta^2\,I_f(2,2,1)\,x_{77}\,x_{80}\,\mu_{15}^{2\beta-2}\mu_{16}^{2\beta} - \beta\,I_f(2,2,1)\,x_{77}\,x_{80}\,\mu_{15}^{2\beta-1}\,\mu_{16}^{2\beta} - \\ &\quad \beta^2\,I_f(2,2,1)\,x_{77}\,x_{79}\,\mu_{15}^{2\beta-1}\mu_{16}^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,2,1)\,x_{77}\,x_{80}\,\mu_{15}^{2\beta-2}\,\mu_{16}^{2\beta} \\ g_1(\mu_{15}) &= (\mu_{15}^{\beta}\,\mu_{16}^{\beta}\,x_{78} + \beta\,\mu_{15}^{\beta}\,\mu_{16}^{\beta-1}\,x_{79} + \beta\,\mu_{15}^{\beta-1}\mu_{16}^{\beta}\,x_{80})^2 \end{aligned}$$

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_{16}} = \frac{f_1(\mu_{16})}{g_1(\mu_{16})} - C_{cc}(2,2,1) = 0 \tag{5.42}$$

where,

$$\begin{aligned} f_1(\mu_{16}) &= \beta\,I_f(2,2,1)\,x_{77}x_{78}\,\mu_{15}^{2\beta}\mu_{16}^{2\beta-1} + \beta^2\,I_f(2,2,1)\,x_{77}\,x_{79}\,\mu_{15}^{2\beta}\,\mu_{16}^{2\beta-2} + \\ &\quad \beta^2\,I_f(2,2,1)\,x_{77}\,x_{80}\,\mu_{15}^{2\beta-1}\mu_{16}^{2\beta-1} - \beta\,I_f(2,2,1)\,x_{77}\,x_{78}\,\mu_{15}^{2\beta}\,\mu_{16}^{2\beta-1} - \\ &\quad \beta^2\,I_f(2,2,1)\,x_{77}\,x_{80}\,\mu_{15}^{2\beta-1}\mu_{16}^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,2,1)\,x_{77}\,x_{79}\,\mu_{15}^{2\beta}\,\mu_{16}^{2\beta-2} \\ g_1(\mu_{16}) &= g_1(\mu_{15}) \end{aligned}$$

The optimal values of μ_{15} and μ_{16} (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.41) and (5.42). These optimal values can be substituted back into Equation (5.40) to obtain the maximum net income derivable from the system.

With Type C Repair, the above expressions remain valid but μ_{sd} and μ_{se} are set to zero. This change will affect only the values of the constants x_{77} to x_{80}.

Special Case Model IV

For $n = 2$, $m = 2$ and $k = 2$ (i.e. $2 - out - of - (2+2) : G$ system) with Type C Repair, the determination of the optimal failed system repair rates can easily be derived from the results obtained for the $(1 - out - of - (2+2) : G)$ system by eliminating *system up-states 11, 12, 13*, and *14* from Equations (5.40) to (5.42).

5.3 The Identical Unit System With Late Standby Activation: Income Maximization

The procedure already established for the special cases under SBAP Y remains valid for SBAP Z special cases. The cases that will be presented here are those cases for which $k \neq n$. This is because for $k = n$ only SBAP Y is practically feasible and the the standby must be activated immediately upon unit failure otherwise, the system would fail.

5.3.1 1-out-of-(n+1):G Identical Unit Submodel: Special Cases

Special Case Model I

For $n = 2$ with Type B Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,1,1) &= \frac{I_f(2,1,1)\,(\mu_3\,\mu_4)^\beta\,x_{37}}{(\mu_3\,\mu_4)^\beta\,x_{38} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_{39} + \beta\,\mu_3^{\beta-1}\,\mu_4^\beta\,x_{40}} - C_{hw}(2,1,1)\,\mu_3 \\ &\quad - C_{cc}(2,1,1)\,\mu_4 \end{aligned} \tag{5.43}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.43) with respect to μ_3 and μ_4 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_3} = \frac{f_1(\mu_3)}{g_1(\mu_3)} - C_{hw}(2,1,1) = 0 \tag{5.44}$$

where,

$$\begin{aligned} f_1(\mu_3) &= \beta\,I_f(2,1,1)\,x_{37}x_{38}\,\mu_3^{2\beta-1}\mu_4^{2\beta} + \beta^2\,I_f(2,1,1)\,x_{37}\,x_{39}\,\mu_3^{2\beta-1}\,\mu_4^{2\beta-1} + \\ &\quad \beta^2\,I_f(2,1,1)\,x_{37}\,x_{40}\,\mu_3^{2\beta-2}\mu_4^{2\beta} - \beta\,I_f(2,1,1)\,x_{37}\,x_{40}\,\mu_3^{2\beta-1}\,\mu_4^{2\beta} - \\ &\quad \beta^2\,I_f(2,1,1)\,x_{37}\,x_{39}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,1,1)\,x_{37}\,x_{40}\,\mu_3^{2\beta-2}\,\mu_4^{2\beta} \\ g_1(\mu_3) &= (\mu_3^\beta\,\mu_4^\beta\,x_{38} + \beta\,\mu_3^\beta\,\mu_4^{\beta-1}\,x_{39} + \beta\,\mu_3^{\beta-1}\mu_4^\beta\,x_{40})^2 \end{aligned}$$

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{cc}(2,1,1) = 0 \tag{5.45}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\,I_f(2,1,1)\,x_{37}x_{38}\,\mu_3^{2\beta}\mu_4^{2\beta-1} + \beta^2\,I_f(2,1,1)\,x_{37}\,x_{39}\,\mu_3^{2\beta}\,\mu_4^{2\beta-2} + \\ &\quad \beta^2\,I_f(2,1,1)\,x_{37}\,x_{40}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,I_f(2,1,1)\,x_{37}\,x_{38}\,\mu_3^{2\beta}\,\mu_4^{2\beta-1} - \\ &\quad \beta^2\,I_f(2,1,1)\,x_{37}\,x_{40}\,\mu_3^{2\beta-1}\mu_4^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,1,1)\,x_{37}\,x_{39}\,\mu_3^{2\beta}\,\mu_4^{2\beta-2} \\ g_1(\mu_4) &= g_1(\mu_3) \end{aligned}$$

The optimal values of μ_3 and μ_4 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.44) and (5.45). These optimal values can substituted back into Equation (5.43) to obtain the maximum net income derivable from the system.

With Type C Repair, the above expressions remain valid but $\mu_{s0'}$ is set to zero. These changes will affect only the values of the constants x_{37} to x_{40}.

Special Case Model II

For $n = 3$ with Type B Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(3,1,1) &= \frac{I_f(3,1,1)\,(\mu_4\,\mu_5)^\beta\, x_{41}}{(\mu_4\,\mu_5)^\beta\, x_{42} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\, x_{43} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\, x_{44}} - C_{hw}(3,1,1)\,\mu_4 \\ &\quad -C_{cc}(3,1,1)\,\mu_5 \end{aligned} \tag{5.46}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.46) with respect to μ_4 and μ_5 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,1,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{hw}(3,1,1) = 0 \tag{5.47}$$

where,

$$\begin{aligned} f_1(\mu_4) &= \beta\, I_f(3,1,1)\, x_{41} x_{42}\, \mu_4^{2\beta-1} \mu_5^{2\beta} + \beta^2\, I_f(3,1,1)\, x_{41}\, x_{43}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta-1} + \\ &\quad \beta^2\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta-2} \mu_5^{2\beta} - \beta\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta-1}\, \mu_5^{2\beta} - \\ &\quad \beta^2\, I_f(3,1,1)\, x_{41}\, x_{43}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta-2}\, \mu_5^{2\beta} \\ g_1(\mu_4) &= (\mu_4^\beta\, \mu_5^\beta\, x_{42} + \beta\, \mu_4^\beta\, \mu_5^{\beta-1}\, x_{43} + \beta\, \mu_4^{\beta-1} \mu_5^\beta\, x_{44})^2 \end{aligned}$$

$$\frac{\partial I_{net}(3,1,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{cc}(3,1,1) = 0 \tag{5.48}$$

where,

$$\begin{aligned} f_1(\mu_5) &= \beta\, I_f(3,1,1)\, x_{41} x_{42}\, \mu_4^{2\beta} \mu_5^{2\beta-1} + \beta^2\, I_f(3,1,1)\, x_{41}\, x_{43}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} + \\ &\quad \beta^2\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} - \\ &\quad \beta^2\, I_f(3,1,1)\, x_{41}\, x_{44}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,1,1)\, x_{41}\, x_{43}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} \\ g_1(\mu_5) &= g_1(\mu_4) \end{aligned}$$

The optimal values of μ_4 and μ_5 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.47) and (5.48). These optimal values can be substituted back into Equation (5.46) to obtain the maximum net income derivable from the system.

With Type C Repair, the above expressions remain valid but $\mu_{s0'}$, is set to zero. This change will affect only the values of the constants x_{41} to x_{44}.

5.3.2 1-out-of-(n+2):G Identical Unit Submodel: Special Cases

For the following special case models, the following results are obtained:

<u>Special Case Model I</u>

For $n = 2$ with Type C Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,2,1) &= \frac{I_f(2,2,1)\,(\mu_4\,\mu_5)^\beta\,x_{49}}{(\mu_4\,\mu_5)^\beta\,x_{50} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\,x_{51} + \beta\,\mu_4^{\beta-1}\,\mu_5^\beta\,x_{52}} - C_{hw}(2,2,1)\,\mu_4 \\ &\quad -C_{cc}(2,2,1)\,\mu_5 \end{aligned} \tag{5.49}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.49) with respect to μ_4 and μ_5 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_4} = \frac{f_1(\mu_4)}{g_1(\mu_4)} - C_{hw}(2,2,1) = 0 \tag{5.50}$$

where

$$\begin{aligned} f_1(\mu_4) &= \beta\,I_f(2,2,1)\,x_{49}x_{50}\,\mu_4^{2\beta-1}\mu_5^{2\beta} + \beta^2\,I_f(2,2,1)\,x_{49}\,x_{51}\,\mu_4^{2\beta-1}\,\mu_5^{2\beta-1} + \\ &\quad \beta^2\,I_f(2,2,1)\,x_{49}\,x_{52}\,\mu_4^{2\beta-2}\mu_5^{2\beta} - \beta\,I_f(2,2,1)\,x_{49}\,x_{52}\,\mu_4^{2\beta-1}\,\mu_5^{2\beta} - \\ &\quad \beta^2\,I_f(2,2,1)\,x_{49}\,x_{51}\,\mu_4^{2\beta-1}\mu_5^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,2,1)\,x_{49}\,x_{52}\,\mu_4^{2\beta-2}\,\mu_5^{2\beta} \\ g_1(\mu_4) &= (\mu_4^\beta\,\mu_5^\beta\,x_{50} + \beta\,\mu_4^\beta\,\mu_5^{\beta-1}\,x_{51} + \beta\,\mu_4^{\beta-1}\mu_5^\beta\,x_{52})^2 \end{aligned}$$

$$\frac{\partial I_{net}(2,2,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{cc}(2,2,1) = 0 \tag{5.51}$$

where

$$\begin{aligned}
f_1(\mu_5) &= \beta\, I_f(2,2,1)\, x_{49} x_{50}\, \mu_4^{2\beta} \mu_5^{2\beta-1} + \beta^2\, I_f(2,2,1)\, x_{49}\, x_{51}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,2,1)\, x_{49}\, x_{52}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\, I_f(2,2,1)\, x_{49}\, x_{50}\, \mu_4^{2\beta}\, \mu_5^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,2,1)\, x_{49}\, x_{52}\, \mu_4^{2\beta-1} \mu_5^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,2,1)\, x_{49}\, x_{51}\, \mu_4^{2\beta}\, \mu_5^{2\beta-2} \\
g_1(\mu_5) &= g_1(\mu_4)
\end{aligned}$$

The optimal values of μ_4 and μ_5 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.50) and (5.51). These optimal values can be substituted back into Equation (5.49) to obtain the maximum net income derivable from the system.

Special Case Model II

For $n = 3$ with Type C Repair, the net income generated is given by:

$$\begin{aligned}
I_{net}(3,2,1) &= \frac{I_f(3,2,1)\,(\mu_5\,\mu_6)^\beta\, x_{53}}{(\mu_5\,\mu_6)^\beta\, x_{54} + \beta\, \mu_5^\beta\, \mu_6^{\beta-1}\, x_{55} + \beta\, \mu_5^{\beta-1}\, \mu_6^\beta\, x_{56}} - C_{hw}(3,2,1)\, \mu_5 \\
&\quad -C_{cc}(3,2,1)\, \mu_6
\end{aligned} \tag{5.52}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.52) with respect to μ_5 and μ_6 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,2,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{hw}(3,2,1) = 0 \tag{5.53}$$

where

$$\begin{aligned}
f_1(\mu_5) &= \beta\, I_f(3,2,1)\, x_{53} x_{54}\, \mu_5^{2\beta-1} \mu_6^{2\beta} + \beta^2\, I_f(3,2,1)\, x_{53}\, x_{55}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta-1} + \\
&\quad \beta^2\, I_f(3,2,1)\, x_{53}\, x_{56}\, \mu_5^{2\beta-2} \mu_6^{2\beta} - \beta\, I_f(3,2,1)\, x_{53}\, x_{56}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} - \\
&\quad \beta^2\, I_f(3,2,1)\, x_{53}\, x_{55}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,2,1)\, x_{53}\, x_{56}\, \mu_5^{2\beta-2}\, \mu_6^{2\beta} \\
g_1(\mu_5) &= (\mu_5^\beta\, \mu_6^\beta\, x_{54} + \beta\, \mu_5^\beta\, \mu_6^{\beta-1}\, x_{55} + \beta\, \mu_5^{\beta-1} \mu_6^\beta\, x_{56})^2
\end{aligned}$$

$$\frac{\partial I_{net}(3,2,1)}{\partial \mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{cc}(3,2,1) = 0 \tag{5.54}$$

where

$$\begin{aligned} f_1(\mu_6) &= \beta\, I_f(3,2,1)\, x_{53} x_{54}\, \mu_5^{2\beta} \mu_6^{2\beta-1} + \beta^2\, I_f(3,2,1)\, x_{53}\, x_{55}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} + \\ &\quad \beta^2\, I_f(3,2,1)\, x_{53}\, x_{56}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\, I_f(3,2,1)\, x_{53}\, x_{54}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} - \\ &\quad \beta^2\, I_f(3,2,1)\, x_{53}\, x_{56}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,2,1)\, x_{53}\, x_{55}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} \\ g_1(\mu_6) &= g_1(\mu_5) \end{aligned}$$

The optimal values of μ_5 and μ_6 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.53) and (5.54). These optimal values can be substituted back into Equation (5.52) to obtain the maximum net income derivable from the system.

5.3.3 1-out-of-(n+3):G Identical Unit Submodel: Special Cases

For the following special case models, the following results are obtained:

Special Case Model I

For $n = 2$ with Type C Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,3,1) &= \frac{I_f(2,3,1)\,(\mu_5\,\mu_6)^\beta\, x_{61}}{(\mu_5\,\mu_6)^\beta\, x_{62} + \beta\, \mu_5^\beta\, \mu_6^{\beta-1}\, x_{63} + \beta\, \mu_5^{\beta-1}\, \mu_6^\beta\, x_{64}} - C_{hw}(2,3,1)\,\mu_5 \\ &\quad -C_{cc}(2,3,1)\,\mu_6 \end{aligned} \tag{5.55}$$

The optimal system repair rates needed to maximize the net income is determined by differentiating Equation (5.55) with respect to μ_5 and μ_6 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,3,1)}{\partial \mu_5} = \frac{f_1(\mu_5)}{g_1(\mu_5)} - C_{hw}(2,3,1) = 0 \tag{5.56}$$

where,

$$\begin{aligned}
f_1(\mu_5) &= \beta\, I_f(2,3,1)\, x_{61} x_{62}\, \mu_5^{2\beta-1} \mu_6^{2\beta} + \beta^2\, I_f(2,3,1)\, x_{61}\, x_{63}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta-1} + \\
&\quad \beta^2\, I_f(2,3,1)\, x_{61}\, x_{64}\, \mu_5^{2\beta-2} \mu_6^{2\beta} - \beta\, I_f(2,3,1)\, x_{61}\, x_{64}\, \mu_5^{2\beta-1}\, \mu_6^{2\beta} - \\
&\quad \beta^2\, I_f(2,3,1)\, x_{61}\, x_{63}.\mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,3,1)\, x_{61}\, x_{64}\, \mu_5^{2\beta-2}\, \mu_6^{2\beta} \\
g_1(\mu_5) &= (\mu_5^{\beta}\, \mu_6^{\beta}\, x_{62} + \beta\, \mu_5^{\beta}\, \mu_6^{\beta-1}\, x_{63} + \beta\, \mu_5^{\beta-1} \mu_6^{\beta}\, x_{64})^2
\end{aligned}$$

$$\frac{\partial I_{net}(2,3,1)}{\partial \mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{cc}(2,3,1) = 0 \tag{5.57}$$

where,

$$\begin{aligned}
f_1(\mu_6) &= \beta\, I_f(2,3,1)\, x_{61} x_{62}\, \mu_5^{2\beta} \mu_6^{2\beta-1} + \beta^2\, I_f(2,3,1)\, x_{61}\, x_{63}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,3,1)\, x_{61}\, x_{64}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\, I_f(2,3,1)\, x_{61}\, x_{62}\, \mu_5^{2\beta}\, \mu_6^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,3,1)\, x_{61}\, x_{64}\, \mu_5^{2\beta-1} \mu_6^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,3,1)\, x_{61}\, x_{63}\, \mu_5^{2\beta}\, \mu_6^{2\beta-2} \\
g_1(\mu_6) &= g_1(\mu_5)
\end{aligned}$$

The optimal values of μ_5 and μ_6 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.56) and (5.57). These optimal values can be substituted back into Equation (5.55) to obtain the maximum net income derivable from the system.

Special Case Model II

For $n = 3$ with Type C Repair, the net income generated is given by:

$$\begin{aligned}
I_{net}(3,3,1) &= \frac{I_f(3,3,1)\,(\mu_6\, \mu_7)^{\beta}\, x_{65}}{(\mu_6\, \mu_7)^{\beta}\, x_{66} + \beta\, \mu_6^{\beta}\, \mu_7^{\beta-1}\, x_{67} + \beta\, \mu_6^{\beta-1}\, \mu_7^{\beta}\, x_{68}} - C_{hw}(3,3,1)\, \mu_6 \\
&\quad - C_{cc}(3,3,1)\, \mu_7
\end{aligned} \tag{5.58}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.58) with respect to μ_6 and μ_7 respectively and setting the results to zero.

$$\frac{\partial I_{net}(3,3,1)}{\partial \mu_6} = \frac{f_1(\mu_6)}{g_1(\mu_6)} - C_{hw}(3,3,1) = 0 \tag{5.59}$$

where,

$$\begin{aligned}
f_1(\mu_6) &= \beta\, I_f(3,3,1)\, x_{65}x_{66}\, \mu_6^{2\beta-1}\mu_7^{2\beta} + \beta^2\, I_f(3,3,1)\, x_{65}\, x_{67}\, \mu_6^{2\beta-1}\, \mu_7^{2\beta-1} + \\
&\quad \beta^2\, I_f(3,3,1)\, x_{65}\, x_{68}\, \mu_6^{2\beta-2}\mu_7^{2\beta} - \beta\, I_f(3,3,1)\, x_{65}\, x_{68}\, \mu_6^{2\beta-1}\, \mu_7^{2\beta} - \\
&\quad \beta^2\, I_f(3,3,1)\, x_{65}\, x_{67}\, \mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,3,1)\, x_{65}\, x_{68}\, \mu_6^{2\beta-2}\, \mu_7^{2\beta} \\
g_1(\mu_6) &= (\mu_6^{\beta}\, \mu_7^{\beta}\, x_{66} + \beta\, \mu_6^{\beta}\, \mu_7^{\beta-1}\, x_{67} + \beta\, \mu_6^{\beta-1}\mu_7^{\beta}\, x_{68})^2 \\
\frac{\partial I_{net}(3,3,1)}{\partial \mu_7} &= \frac{f_1(\mu_7)}{g_1(\mu_7)} - C_{cc}(3,3,1) = 0 \qquad (5.60)
\end{aligned}$$

where

$$\begin{aligned}
f_1(\mu_7) &= \beta\, I_f(3,3,1)\, x_{65}x_{66}\, \mu_6^{2\beta}\mu_7^{2\beta-1} + \beta^2\, I_f(3,3,1)\, x_{65}\, x_{67}\, \mu_6^{2\beta}\, \mu_7^{2\beta-2} + \\
&\quad \beta^2\, I_f(3,3,1)\, x_{65}\, x_{68}\, \mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\, I_f(3,3,1)\, x_{65}\, x_{66}\, \mu_6^{2\beta}\, \mu_7^{2\beta-1} - \\
&\quad \beta^2\, I_f(3,3,1)\, x_{65}\, x_{68}\, \mu_6^{2\beta-1}\mu_7^{2\beta-1} - \beta\,(\beta-1)\, I_f(3,3,1)\, x_{65}\, x_{67}\, \mu_6^{2\beta}\, \mu_7^{2\beta-2} \\
g_1(\mu_7) &= g_1(\mu_6)
\end{aligned}$$

The optimal values of μ_6 and μ_7 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.59) and (5.60). These optimal values can be substituted back into Equation (5.58) to obtain the maximum net income derivable from the system.

5.3.4 k-out-of-(n+1):G Identical Unit Submodel: Special Cases

The expressions obtained for the $1-out-of-(n+1):G$ special cases are relevant for the $k-out-of-(n+1):G$ special case models. In fact those expressions represent the cases for which $k=1$. For $k=2$, the quantities relating to system up-states n and $n-1^{(1)}$ are eliminated from x_{37} to x_{44} and for $k=3$, the quantities relating to system up-states *n-1, n,* $n-2^{(1)}$ and $n-1^{(1)}$ are eliminated from x_{37} to x_{44}. Thus, for $k=j$ where $j \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-j+2, n-j+3,....,n,* $n-j+1^{(1)}$, $n-j+2^{(1)}$,........ $n-1^{(1)}$ are eliminated from x_{37} to x_{44}.

5.3.5 k-out-of-(n+2):G Identical Unit Submodel: Special Cases

The expressions obtained for the $1 - out - of - (n + 2) : G$ special cases are relevant for the $k - out - of - (n + 2) : G$ special case models. In fact those expressions represent the cases for which $k = 1$. For $k = 2$, the quantities relating to system up-states *n+1* and $n^{(1)}$ are eliminated from x_{49} to x_{56} and for $k = 3$, the quantities relating to system up-states *n*, *n+1*, $n - 1^{(1)}$ and $n^{(1)}$ are eliminated from x_{49} to x_{56}. Thus, for $k = j$ where $j \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-j+3*, *n-j+4*,....,*n+1*, $n - j + 2^{(1)}$, $n - j + 3^{(1)}$,...$n^{(1)}$ are eliminated from x_{49} to x_{56}.

5.3.6 k-out-of-(n+3):G Identical Unit Submodel: Special Cases

The expressions obtained for the $1 - out - of - (n + 3) : G$ special cases are relevant for the $k - out - of - (n + 3) : G$ special case models. In fact those expressions represent the cases for which $k = 1$. For $k = 2$, the quantities relating to system up-states *n+2* and $n + 1^{(1)}$ are eliminated from x_{61} to x_{68} and for $k = 3$, the quantities relating to system up-states *n+1*, *n+2*, $n^{(1)}$ and $n + 1^{(1)}$ are eliminated from x_{61} to x_{68}. Thus, for $k = j$ where $j \leq n$, the same expressions remain valid as long as terms associated with system up-states *n-j+4*, *n-j+5*,....,*n+2*, $n - j + 3^{(1)}$, $n - j + 4^{(1)}$,...$n + 1^{(1)}$ are eliminated from x_{61} to x_{68}. Thus, in general, the expressions derived for the $1 - out - of - (n + m) : G$ system are useful for the $k - out - of - (n + m) : G$ by simply eliminating the terms relating to the system up-states *n+m-k+1*,*n+m-k+2*,...,*n+m-1*, $n + m - k^{(1)}$, $n + m - k + 1^{(1)}$,....,$n + m - 2^{(1)}$, $n + m - k - 1^{(2)}$, $n + m - k^{(2)}$,.....,$n + m - 3^{(2)}$,......$[(n + m) - (k + i)]^{(m)}$ from the relevant constants such as x_{61} to x_{68}. Where $i = 0, 1, 2, ..., m$.

5.4 The Non-Identical Unit System With Late Standby Activation: Special Cases

The same procedure outlined for the identical unit system is applicable if the system units are non-identical. The following analysis cover those special cases for which $k \neq n$. For $k = n$, standby activation policy Z is not practicable because the standby must be activated as sson as an active unit fails in order to prevent overall system failure.

<u>Special Case Model I</u>

For $n = 2$, $m = 1$ and $k = 1$ (i.e. $1 - out - of - (2+1) : G$ system) with Type B Repair, the net income generated is given by:

$$\begin{aligned} I_{net}(2,1,1) &= \frac{I_f(2,1,1)\,(\mu_7\,\mu_8)^\beta\,x_{81}}{(\mu_7\,\mu_8)^\beta\,x_{82} + \beta\,\mu_7^\beta\,\mu_8^{\beta-1}\,x_{83} + \beta\,\mu_7^{\beta-1}\,\mu_8^\beta\,x_{84}} - C_{hw}(2,1,1)\,\mu_7 \\ &\quad - C_{cc}(2,1,1)\,\mu_8 \end{aligned} \tag{5.61}$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.61) with respect to μ_7 and μ_8 respectively and setting the results to zero.

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_7} = \frac{f_1(\mu_7)}{g_1(\mu_7)} - C_{hw}(2,1,1) = 0 \tag{5.62}$$

where,

$$\begin{aligned} f_1(\mu_7) &= \beta\,I_f(2,1,1)\,x_{81}x_{82}\,\mu_7^{2\beta-1}\mu_8^{2\beta} + \beta^2\,I_f(2,1,1)\,x_{81}\,x_{83}\,\mu_7^{2\beta-1}\,\mu_8^{2\beta-1} + \\ &\quad \beta^2\,I_f(2,1,1)\,x_{81}\,x_{84}\,\mu_7^{2\beta-2}\mu_8^{2\beta} - \beta\,I_f(2,1,1)\,x_{81}\,x_{84}\,\mu_7^{2\beta-1}\,\mu_8^{2\beta} - \\ &\quad \beta^2\,I_f(2,1,1)\,x_{81}\,x_{83}\,\mu_7^{2\beta-1}\mu_8^{2\beta-1} - \beta\,(\beta-1)\,I_f(2,1,1)\,x_{81}\,x_{84}\,\mu_7^{2\beta-2}\,\mu_8^{2\beta} \\ g_1(\mu_7) &= (\mu_7^\beta\,\mu_8^\beta\,x_{82} + \beta\,\mu_7^\beta\,\mu_8^{\beta-1}\,x_{83} + \beta\,\mu_7^{\beta-1}\mu_8^\beta\,x_{84})^2 \end{aligned}$$

$$\frac{\partial I_{net}(2,1,1)}{\partial \mu_8} = \frac{f_1(\mu_8)}{g_1(\mu_8)} - C_{cc}(2,1,1) = 0 \tag{5.63}$$

where,

$$
\begin{aligned}
f_1(\mu_8) &= \beta\, I_f(2,1,1)\, x_{81} x_{82}\, \mu_7^{2\beta} \mu_8^{2\beta-1} + \beta^2\, I_f(2,1,1)\, x_{81}\, x_{83}\, \mu_7^{2\beta}\, \mu_8^{2\beta-2} + \\
&\quad \beta^2\, I_f(2,1,1)\, x_{81}\, x_{84}\, \mu_7^{2\beta-1} \mu_8^{2\beta-1} - \beta\, I_f(2,1,1)\, x_{81}\, x_{82}\, \mu_7^{2\beta}\, \mu_8^{2\beta-1} - \\
&\quad \beta^2\, I_f(2,1,1)\, x_{81}\, x_{84}\, \mu_7^{2\beta-1} \mu_8^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,1,1)\, x_{81}\, x_{83}\, \mu_7^{2\beta}\, \mu_8^{2\beta-2} \\
g_1(\mu_8) &= g_1(\mu_7)
\end{aligned}
$$

The optimal values of μ_7 and μ_8 (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.62) and (5.63). These optimal values can be substituted back into Equation (5.61) to obtain the maximum net income derivable from the system.

With Type C Repair, the above expressions remain valid but μ_{sd} is set to zero. This change will affect only the values of the constants x_{81} to x_{84}.

Special Case Model II

For $n = 2$, $m = 2$ and $k = 1$ (i.e. $1 - out - of - (2+2) : G$ system) with Type C Repair, the net income generated is given by:

$$
\begin{aligned}
I_{net}(2,2,1) &= \frac{I_f(2,2,1)\,(\mu_{15}\,\mu_{16})^\beta\, x_{85}}{(\mu_{15}\,\mu_{16})^\beta\, x_{86} + \beta\, \mu_{15}^\beta\, \mu_{16}^{\beta-1}\, x_{87} + \beta\, \mu_{15}^{\beta-1}\, \mu_{16}^\beta\, x_{88}} - C_{hw}(2,2,1)\,\mu_{15} \\
&\quad - C_{cc}(2,2,1)\,\mu_{16} \qquad (5.64)
\end{aligned}
$$

The optimal system repair rates needed to maximize the net income can be determined by differentiating Equation (5.64) with respect to μ_{15} and μ_{16} respectively and setting the results to zero.

$$
\frac{\partial I_{net}(2,2,1)}{\partial \mu_{15}} = \frac{f_1(\mu_{15})}{g_1(\mu_{15})} - C_{hw}(2,2,1) = 0 \qquad (5.65)
$$

where,

$$
\begin{aligned}
f_1(\mu_{15}) &= \beta\, I_f(2,2,1)\, x_{85} x_{86}\, \mu_{15}^{2\beta-1} \mu_{16}^{2\beta} + \beta^2\, I_f(2,2,1)\, x_{85}\, x_{87}\, \mu_{15}^{2\beta-1}\, \mu_{16}^{2\beta-1} + \\
&\quad \beta^2\, I_f(2,2,1)\, x_{85}\, x_{88}\, \mu_{15}^{2\beta-2} \mu_{16}^{2\beta} - \beta\, I_f(2,2,1)\, x_{85}\, x_{88}\, \mu_{15}^{2\beta-1}\, \mu_{16}^{2\beta} -
\end{aligned}
$$

$$
\begin{aligned}
& \beta^2 I_f(2,2,1)\, x_{85}\, x_{87}\, \mu_{15}^{2\beta-1} \mu_{16}^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,2,1)\, x_{85}\, x_{88}\, \mu_{15}^{2\beta-2}\, \mu_{16}^{2\beta} \\
g_1(\mu_{15}) &= (\mu_{15}^{\beta}\, \mu_{16}^{\beta}\, x_{86} + \beta\, \mu_{15}^{\beta}\, \mu_{16}^{\beta-1}\, x_{87} + \beta\, \mu_{15}^{\beta-1} \mu_{16}^{\beta}\, x_{88})^2
\end{aligned}
$$

$$
\frac{\partial I_{net}(2,2,1)}{\partial \mu_{16}} = \frac{f_1(\mu_{16})}{g_1(\mu_{16})} - C_{cc}(2,2,1) = 0 \tag{5.66}
$$

where,

$$
\begin{aligned}
f_1(\mu_{16}) &= \beta\, I_f(2,2,1)\, x_{85} x_{86}\, \mu_{15}^{2\beta} \mu_{16}^{2\beta-1} + \beta^2 I_f(2,2,1)\, x_{85}\, x_{87}\, \mu_{15}^{2\beta}\, \mu_{16}^{2\beta-2} + \\
& \quad \beta^2 I_f(2,2,1)\, x_{85}\, x_{88}\, \mu_{15}^{2\beta-1} \mu_{16}^{2\beta-1} - \beta\, I_f(2,2,1)\, x_{85}\, x_{86}\, \mu_{15}^{2\beta}\, \mu_{16}^{2\beta-1} - \\
& \quad \beta^2 I_f(2,2,1)\, x_{85}\, x_{88}\, \mu_{15}^{2\beta-1} \mu_{16}^{2\beta-1} - \beta\,(\beta-1)\, I_f(2,2,1)\, x_{85}\, x_{87}\, \mu_{15}^{2\beta}\, \mu_{16}^{2\beta-2} \\
g_1(\mu_{16}) &= g_1(\mu_{15})
\end{aligned}
$$

The optimal values of μ_{15} and μ_{16} (i.e. the failed system mean repair rates) can be determined iteratively from Equations (5.65) and (5.66). These optimal values can be substituted back into Equation (5.64) to obtain the maximum net income derivable from the system.

5.5 Numerical Examples

This section presents two numerical examples to illustrate the results obtained in the earlier sections of this Chapter. These are: i) the $(2,1,1)$ identical unit special case with Type C Repair and under SBAP Y and (ii) the $(2,1,1)$ identical unit case with Type C Repair under SBAP Z.

For the $(2,1,1)$ identical unit special case with Type C Repair, let us assume that $I_f(2,1,1) = \$25000/month$, $\lambda_1 = 0.0004/hr$, $\lambda_s = 0.0001/hr$, $\lambda_{c0} = 0.1\,\lambda_1$, $\lambda_{c1} = \lambda_{c0}/2$, $\lambda_{c2} = \lambda_{c1}/2$, $\lambda_{c0'} = \lambda_{c0(1)} = 0.75\,\lambda_{c0}$, $\lambda_{c1(1)} = \lambda_{c0(1)}/2$. Let us also assume the following starting values for the repair rates: $\mu_3 = 0.00125$ per Hr and $\mu_4 = 0.00125$ per Hr. For $\beta = 1$, the cost constants corresponding to these starting values are: For SBAP Y: $C_{hw}(2,1,1) = 2335532\$\text{Hr}$ and $C_{cc}(2,1,1) = 230092\$\text{Hr}$; For SBAP Z: $C_{hw}(2,1,1) = 2123986\$\text{Hr}$ and $C_{cc}(2,1,1) = 223215\$\text{Hr}$.

a) Under SBAP Y, using Equation (5.10), the net income generated by this system can be calculated as a function of the critical common-cause failure rate λ_{c0} and the failed system repair *pdf* shape parameter β for fixed values of failed system repair rates, $\mu_3 = \mu_4 = 0.00125$ per Hr. These values are given in the second column of Table 5.1 and plotted in Figure 5.1. Tabular values and plots of the net income versus the repair rate when the system fails due to hardware failures are given in the second column of Table 5.2 and Figure 5.2 respectively while the values in the second column of Table 5.3 and the plots of Figure 5.3 show the variation of the net income with the failed system repair rate when the system fails due to common-cause failures.

b) Under SBAP Z, using Equation (5.43), the net income generated by this system can be calculated as a function of the critical common-cause failure rate λ_{c0} and the failed system repair *pdf* shape parameter β for fixed values of failed system repair rates, $\mu_3 = \mu_4 = 0.00125$ per Hr. These values are shown in the third column of Table 5.1 and plotted in Figure 5.1. Tabular values and plots of the net income versus the repair rate when the system fails due to common-cause failures are shown inthe third column of Table 5.2 and Figure 5.2 respectively while the values in the third column of Table 5.3 and Figure 5.3 show the variation of the net income with the failed system repair rate when the system fails due to common-cause failures.

The following optimal system repair rates, μ_3^* and μ_4^* have been determined iteratively using Equations (5.11) and (5.12) (for the SBAP Y case) and Equations (5.44) and (5.46) (for the SBAP Z case). The values enclosed in brackets are the corresponding maximum net income values.

<u>Under SBAP Y</u>: For <u>$\beta = 1$</u>, $\mu_3^* = 0.00125$/Hr and $\mu_4^* = 0.00125$/Hr, (\$18015.04). For <u>$\beta = 2$</u>, $\mu_3^* = 0.00164$/Hr and $\mu_4^* = 0.00165$/Hr, (\$15456.17). For <u>$\beta = 3$</u>, $\mu_3^* = 0.00188$/Hr and $\mu_4^* = 0.00190$/Hr, (\$13625.56). For <u>$\beta = 4$</u>, $\mu_3^* = 0.00206$/Hr and $\mu_4^* = 0.00205$/Hr, (\$12171.89).

<u>Under SBAP Z</u>: For <u>$\beta = 1$</u>, $\mu_3^* = 0.00138$/Hr and $\mu_4^* = 0.00125$/Hr, (\$18650.00). For <u>$\beta = 2$</u>, $\mu_3^* = 0.00182$/Hr and $\mu_4^* = 0.00165$/Hr, (\$16298.63). For <u>$\beta = 3$</u>, $\mu_3^* =$

Table 5.1: $I_{net}(2,1,1)(\beta)$ (in dollars) as a function of λ_{c0} for Type C Repair

	λ_{c0} (per Hr.)	*SBAPY*	*SBAPZ*
$\beta = 1$	0.00001	18199.05	18574.07
	0.00002	18137.83	18515.55
	0.00003	18076.48	18456.84
	0.00004	18015.04	18397.94
$\beta = 2$	0.00001	15508.54	16089.50
	0.00002	15415.16	15997.84
	0.00003	15321.96	15906.26
	0.00004	15229.01	15814.75
$\beta = 3$	0.00001	13418.86	14113.73
	0.00002	13308.50	14003.14
	0.00003	13198.74	13893.00
	0.00004	13089.61	13783.30
$\beta = 4$	0.00001	11748.93	12504.95
	0.00002	11630.02	12383.79
	0.00003	11512.07	12263.43
	0.00004	11395.10	12143.87

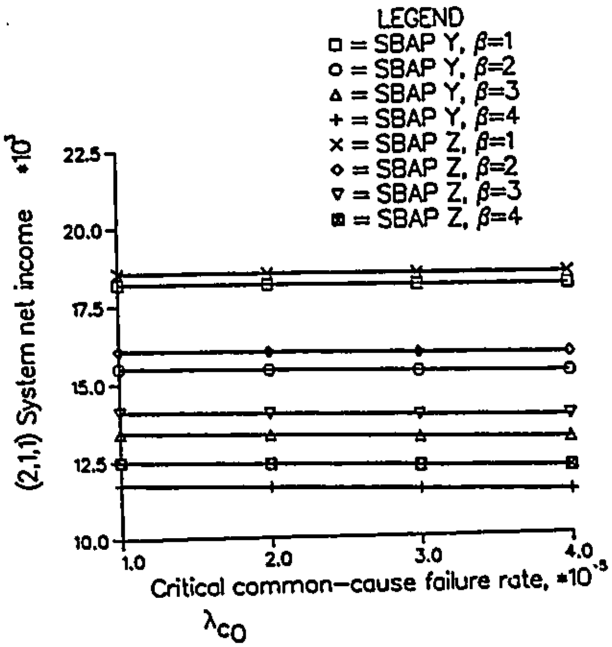


Figure 5.1: $I_{net}(2,1,1)\,(\beta)$ (in dollars) Vs. λ_{c0}

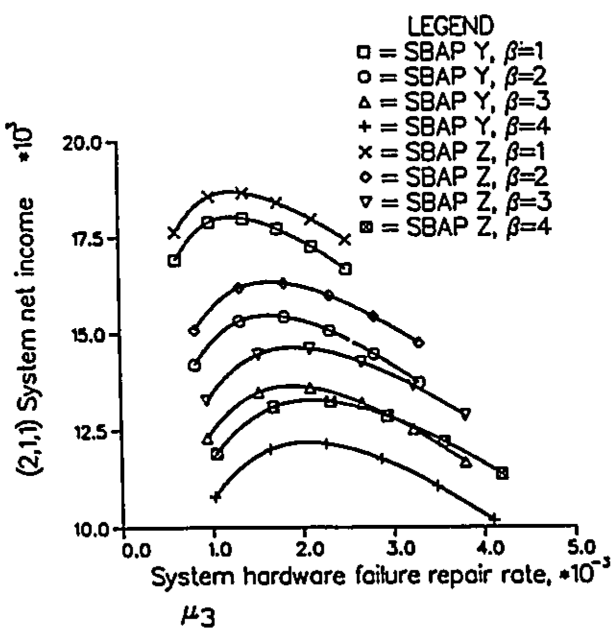


Figure 5.2: $I_{net}(2,1,1)\,(\beta)$ (in dollars) Vs. μ_3

Table 5.2: $I_{net}(2,1,1)(\beta)$ (in Dollars) as a function of μ_3 for Type C Repair

	μ_3 (per Hr.)	SBAP Y	SBAP Z	fixed value of μ_4 (per Hr.)
$\beta = 1$	0.000625	16908.37	17634.03	
	0.001000	17893.32	18557.99	0.00125
	0.001375	17991.85	18649.55	
	0.001750	17715.52	18395.14	
$\beta = 2$	0.000825	14192.32	15078.75	
	0.001320	15318.76	16178.53	0.00165
	0.001815	15422.97	16293.57	
	0.002310	15066.03	15974.12	
$\beta = 3$	0.000950	12303.39	13273.43	
	0.001520	13479.28	14459.28	0.00190
	0.002090	13588.36	14597.57	
	0.002660	13192.05	14251.93	
$\beta = 4$	0.001025	10799.71	11897.77	
	0.001640	12005.84	13100.45	0.00210
	0.002255	12141.04	13234.79	
	0.002870	11746.61	12857.54	

Table 5.3: $I_{net}(2,1,1)(\beta)$ (in Dollars) as a function of μ_4 for Type C Repair

	μ_3 (per Hr.)	SBAP Y	SBAP Z	fixed value of μ_3 (per Hr.)
$\beta = 1$	0.000156	16427.84	17123.91	
	0.002125	17932.79	18591.17	0.00125
	0.004094	17562.41	18231.77	
	0.006063	17138.51	17820.52	
$\beta = 2$	0.000206	13471.41	14350.15	
	0.002805	15345.93	16214.86	0.00165
	0.005404	14856.36	15741.39	
	0.008003	14296.68	15198.95	
$\beta = 3$	0.000238	11411.88	12395.54	
	0.003230	13498.60	14503.79	0.00190
	0.006222	12935.30	13960.43	
	0.009215	12291.05	13336.52	
$\beta = 4$	0.000026	9800.711	10882.59	
	0.003485	12038.77	13131.91	0.00210
	0.006714	11433.85	12530.80	
	0.009942	10739.84	11841.08	

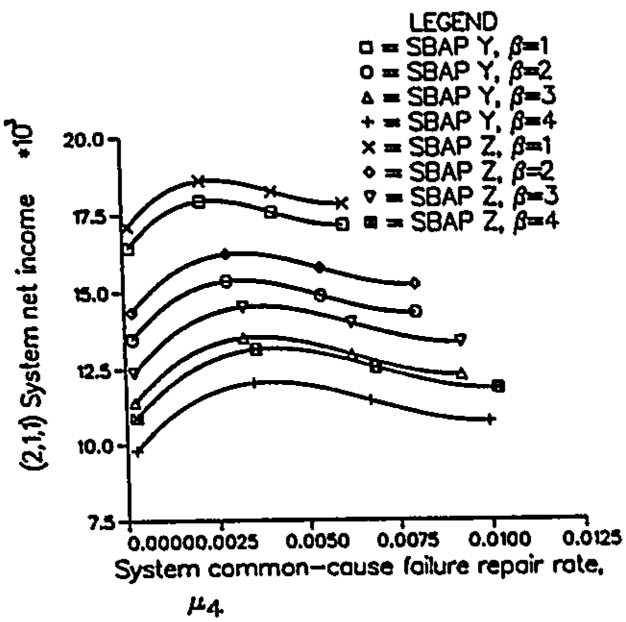


Figure 5.3: $I_{net}(2,1,1)\,(\beta)$ (in dollars) Vs. μ_4

0.00209/Hr and $\mu_4^* = 0.00190$/Hr, (\$14600.00). For $\underline{\beta = 4}$, $\mu_3^* = 0.00226$/Hr and $\mu_4^* = 0.00210$/Hr, (\$13236.80).

These optimal system repair rates are consistent with those of the plots of Figures 5.2 and 5.3 respectively.

5.6 Concluding Remarks

The net system income has been modeled to be directly dependent on the long-run system availability. Thus the higher the long-run availability, the greater the net income generated. Therefore, it may not be surprising that the system net income drops as the system repair time distribution shape parameter β is increased. An increase in β, as will be shown in the next Chapter, implies a decrease in the instantaneous system repair rate and thus a decrease in the system long-run availability. This also means a decrease in the system repair/maintenance costs. In other words, an increase in the system availability can only lead to an increase in the system net income if the extra gains made from having a higher system long-run availability exceed the extra expenses that will accompany any increase in the instantaneous repair rate. Fortunately, this is one of the assumptions of this analysis. Thus the system net income should increase with any increase in the system long-run availability. The flip side of the coin is that if repair costs become high enough to threaten the gains derivable from a high system availability (which result from having a high instantaneous repair rate), the system net income may in fact grow with an increase in the value of β.

The main contribution of this Chapter is the development of mathematical relationships (using a differential calculus method) that show that the system net income from $k-out-of-(n+m):G$ type redundant systems can be maximized by adopting certain failed system repair rates for both independent and common-cause system failures.

Chapter 6

Discussion of Results

The influence of common-cause failures on the $k - out - of - (n + m) : G$ system under both standby activation policies (SBAPs) is clearly evident as one goes through the various system mean time to failure tables and plots. The general picture that emerges is that an increase in the critical common-cause failure rate λ_{c0} invariably reduces the system long-run availability, the system reliability, the system mean time to failure but it also reduces the variance of time to failure (which is a desirable effect as indicated by the analysis in Appendix J.2). The extent of this influence generally depends on the number of active units employed as well as the number of standby units. An increase in the number of standbys for a fixed number of active units improves the system performance characteristics regardless of the Repair Type and standby activation policy being considered.

The comparison of tabular system mean time to failure values in Chapter 3 with those of Chapter 4 for the same configuration and Repair Type clearly shows that SBAP Z is the superior standby activation policy. This may be due to the fact that delaying the activation of the standby keeps the standby failure rate at λ_s which is often a fraction of λ_1, the failure rate the standby would have if activated. Thus the warm standby (i.e. partially energised) status of the standby units enable SBAP Z to produce higher mean time to failure values. The superiority of SBAP Z should be most pronounced when the standbys are in *cold* standby status (i.e. $\lambda_s \approx 0$) and is expected to vanish as the value of λ_s approaches that of λ_1 (i.e. with increased energisation of the standby). With full energisation of the standbys, the standbys are said to be in *hot* status (i.e. $\lambda_s = \lambda_1$) and there should be no difference in the performance of both SBAPs using the system mean time to failure as the sole measure of performance. Unfortunately, SBAP Z, also produces much higher variance of time to failure values as indicated in Appendix J.2. This could be undesirable because higher variance values imply a greater degree of variability of failure times and thus a greater randomness in failure patterns. This might pose a problem in the area of planning for preventive maintenance.

Generalization of the failed system repair time distribution reveals that the $k-out-of-(n+m):G$ system long-run availability is influenced by the system repair time distribution. The exponentially distributed repair time scenario yielded the highest values of the system long-run availability while Gaussian repair times produced the least values for the system long-run availability. This trend runs through the various special cases considered regardless of the governing SBAP and the Type of Repair. It is clearly conclusive that as failed system repair becomes more generalized with the increase in the value of the *Gamma* shape parameter β, the $k-out-of-(n+m):G$ system long-run availability drops. This trend is consistent with the proceeding mathematical formulation which shows that the instantaneous system repair rate is highest when $\beta = 1$ (i.e. exponentially distributed system repair times or constant system repair rate).

Recall that for generalized failed system repair, the failed system repair time distribution is given by:

$$q_{hw}(x) = \frac{\mu_{hw}\,(\mu_{hw}\,x)^{\beta-1}\,e^{-\mu_{hw}\,x}}{\Gamma\,(\beta)};\; x \geq 0,\, \beta > 0 \tag{6.1}$$

$$q_{cc}(x) = \frac{\mu_{cc}\,(\mu_{cc}\,x)^{\beta-1}\,e^{-\mu_{cc}\,x}}{\Gamma\,(\beta)};\; x \geq 0,\, \beta > 0 \tag{6.2}$$

where

$q_{hw}(x)$ is the *pdf* for the failed system repair time when the system fails due to independent failures of its components and x is the elapsed repair time.

$q_{cc}(x)$ is the *pdf* of the failed system repair time when the system fails due to common-cause failures of its components.

μ_{hw} is the system mean repair rate when the system fails due to the independent failures of its components.

μ_{cc} is the system mean repair rate when the system fails due to the common-cause failures of its components.

The instantaneous repair rates for the system are given by:

$$h_{hw}(t) = \frac{q_{hw}(t)}{1 - \int_0^t q_{hw}(x)\,dx} \tag{6.3}$$

$$h_{cc}(t) = \frac{q_{cc}(t)}{1 - \int_0^t q_{cc}(x)\,dx} \tag{6.4}$$

For $\underline{\beta = 1}$, the expression for the system repair *pdfs* and the instantaneous repair rates are given respectively by:

$$q_{hw}(t) = \mu_{hw}\,e^{-\mu_{hw}\,t} \tag{6.5}$$

and

$$h_{hw}(t) = \mu_{hw} \tag{6.6}$$

$$q_{cc}(t) = \mu_{cc}\, e^{-\mu_{cc}\, t} \tag{6.7}$$

and

$$h_{cc}(t) = \mu_{cc} \tag{6.8}$$

For $\underline{\beta = 2}$, we have:

$$q_{hw}(t) = \mu_{hw}^2\, t\, e^{-\mu_{hw}\, t} \tag{6.9}$$

and

$$h_{hw}(t) = \frac{\mu_{hw}^2\, t}{\mu_{hw}\, t + 1} \tag{6.10}$$

$$q_{cc}(t) = \mu_{cc}^2\, t\, e^{-\mu_{cc}\, t} \tag{6.11}$$

and

$$h_{cc}(t) = \frac{\mu_{cc}^2\, t}{\mu_{cc}\, t + 1} \tag{6.12}$$

For $\underline{\beta = 3}$, the respective expressions are:

$$q_{hw}(t) = \mu_{hw}^3\, t^2\, e^{-\mu_{hw}\, t} \tag{6.13}$$

and

$$h_{hw}(t) = \frac{\mu_{hw}^3\, t^2}{\mu_{hw}^2\, t^2 + 2\, \mu_{hw}\, t + 2} \tag{6.14}$$

$$q_{cc}(t) = \mu_{cc}^3\, t^2\, e^{-\mu_{cc}\, t} \tag{6.15}$$

and

$$h_{cc}(t) = \frac{\mu_{cc}^3\, t^2}{\mu_{cc}^2\, t^2 + 2\, \mu_{cc}\, t + 2} \tag{6.16}$$

and for $\underline{\beta = 4}$, we obtain:

$$q_{hw}(t) = \mu_{hw}^4\, t^3\, e^{-\mu_{hw}\, t} \tag{6.17}$$

and

$$h_{hw}(t) = \frac{\mu_{hw}^4\, t^3}{\mu_{hw}^3\, t^3 + 3\,\mu_{hw}^2\, t^2 + 6\,\mu_{hw}\, t + 6} \tag{6.18}$$

$$q_{cc}(t) = \mu_{cc}^4\, t^3\, e^{-\mu_{cc}\, t} \tag{6.19}$$

and

$$h_{cc}(t) = \frac{\mu_{cc}^4\, t^3}{\mu_{cc}^3\, t^3 + 3\,\mu_{cc}^2\, t^2 + 6\,\mu_{cc}\, t + 6} \tag{6.20}$$

The instantaneous rates given by Equations (6.6), (6.8), (6.10), (6.12), (6.14), (6.16), (6.18) and (6.20) are increasing functions of t (i.e. time) as shown in Figure 6.1 which represents the plots of the instantaneous system repair rates for different values of β for the case when the system fails due to common-cause failures of its components.

From Figure 6.1, it can be observed that as β increases, the instantaneous system repair rate decreases and thus the system long-run availability should be expected to decrease regardless of the number of system units and the governing standby activation policy. Thus for exponentially distributed repair times, ($\beta = 1$), the system long-run availability is expected to have its maximun values. Unfortunately, the repair costs are also expected to rise with increasing instantaneous repair rates. This cost aspect will be discussed further in this Chapter. In practice, system repair times may be more likely to be some *pdf* other than the exponential distribution.

Using a differential calculus-based approach, the net income generated from operating a $k - out - of - (n + m) : G$ system can be maximized by adopting

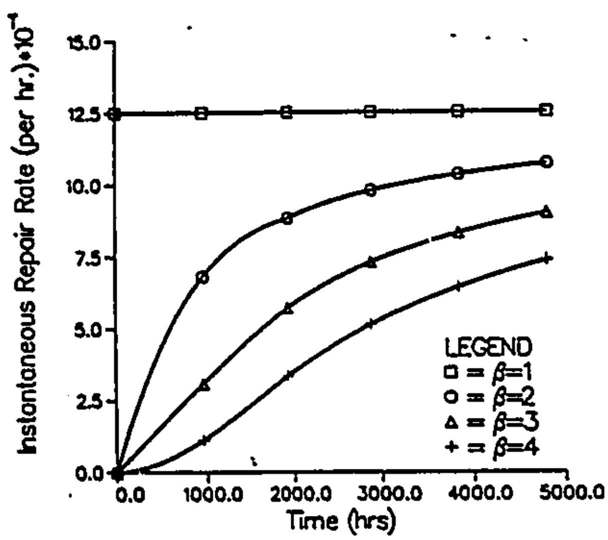


Figure 6.1: Instantaneous System Repair Rate Vs Time ($\mu_{cc} = 0.00125$ per Hr.)

aspect will be discussed further in this Chapter. In practice, system repair times may be more likely to be some *pdf* other than the exponential distribution.

Using a differential calculus-based approach, the net income generated from operating a $k - out - of - (n + m) : G$ system can be maximized by adopting certain system optimal repair rates. For the numerical examples chosen, a glance at Figures 5.2 and 5.3 would indicate that the net income is more sensitive to changes in μ_3 than to changes in μ_4. This should be expected since the common-cause failure rate is often a fraction of the independent failure rate of components for most redundant systems. Also the net income is sensitive to changes in the failed system repair time *pdf* shape parameter β. The net income is essentially the difference between the income and the maintenance (or repair) costs. As β increases, the instantaneous repair rate decreases and therefore the system long-run availability drops and this may as the system availability rises and thus we should expect the maximum net income when repair times are exponentially distributed. It is hardly surprising, that SBAP Z yielded higher net system incomes than SBAP Y because the system operating under SBAP Z produced higher system long-run availability values which is a direct result of the warm standby status ($\lambda_s \leq \lambda_1$) as stated before.

Finally, it is important to note that virtually all the analyses conducted in this study were performed on the ***most pessimistic basis*** (i.e. each common-cause failure event was assumed to be lethal and therefore capable of triggering system failure). Thus, virtually all the results and values generated for various system performance indices could be considered as *lower bound* values. In practice, most $k - out - of - (n + m) : G$ systems should have higher values of system performance indices for the same failure rates used in this Study. Also, it is noteworthy that the generalized system mean time to failure expressions presented in this study may be used to gain a quick insight as to whether to repair system units or not. For example, in those special cases in which there are no significant differences between the results obtained for Type A Repair and those obtained for Type C Repair, it may be prudent not to repair system units upon failure particularly if there are other compelling reasons not to do so (e.g. high unit repair costs).

Chapter 7

Conclusions and Future Directions

7.1 Conclusions

i) An increase in the critical common-cause failure rate, λ_{c0}, reduces the $k - out - of - (n+m) : G$ system long-run availability, reliability, mean time to failure and variance of time to failure. This is true regardless of whether the system units are identical or non-identical.

ii) The extent to which an increase in λ_{c0} reduces these performance indices depends on the number of active units and the number of standby units as well as on the values of the standby unit failure rate, λ_s, and the active unit failure rate, λ_{c0}. With full energisation of the standby (i.e. $\lambda_s = \lambda_1$), an additional standby would have the least favourable impact on the values of the system performance indices. In practice, $\lambda_s < \lambda_1$ and an increase in the number of standby units for a fixed number of active units would improve system performance indices significantly.

iii) A comparison of the performance of the two different SBAPs reveals that SBAP Z would be the better performer if the system mean time to failure were the sole measure of performance. This is because under SBAP Z, the system consistently yielded higher values for the system mean time to failure. This trend should not be surprising because all the standbys considered in this study were in warm standby status. In general, the colder the standby the more superior SBAP Z becomes and vice versa. Unfortunately, SBAP Z, also produces much higher values for the variance of time to failure. This could have dire consequences for the implementation of preventive maintenance schemes for systems operating under SBAP Z.

iv) The $k - out - of - (n + m) : G$ system long-run availability is heavily influenced by the failed system repair time *pdf*. Constant repair rates produced much higher system long-run availability values. These values decrease as repair rates become more time-dependent.

v) The net income generated by the $k - out - of - (n + m) : G$ type systems can be maximized by adopting certain optimal mean system repair rates for both independent and common-cause failures. If the failed system is Erlangian, the net income generated is expected to drop with an increase in the Erlangian shape parameter β.

vi) In practice, for the same failure rates and system state transition patterns, the system would be expected to produce higher values for virtually all the system indices considered in this study. This is because completely-coupled common-cause failures were assumed in this study and thus values generated for system performance indices could be considered as *lower bound* values. Finally, it is noteworthy that the analyses conducted in this Study are based on the inherent reliabilities of system units.

7.2 Future Directions

i) The time-dependent system availability values were not determined for the special cases considered in this study because of the computational difficulties involved in solving the system partial integro-differential equations. There are two main techniques available in the literature for solving these equations viz: a) the Laplace transform method (which was widely used in this study in combination with the Partial Fraction Method to compute system reliabilities); and b) the Lagrange Method for partial differential equations. Both of these methods are usually combined with the supplementary variables technique in their application. Both approaches are tedious. For the Laplace transforms method, the inversion of the resulting Laplace transform expressions from *s-domain* to *time domain* could be very cumbersome. Perhaps, further studies should be conducted to develop solution techniques and algorithms to ease these difficulties. Fortunately, the long-run availability of repairable $k-out-of-(n+m):G$ type redundant systems appears to be the index of greater concern than the transient time-dependent availability.

ii) The $k-out-of-(n+m):G$ system could be studied further by relaxing the assumption of complete-coupling of common-cause failures. This should lead to probably less pessimistic results than those obtained for this study.

iii) It would be interesting to study more general models of $k-out-of-(n+m):G$ systems that incorporate time-dependent unit failures and repairs. It will be equally interesting to study models that consider state-dependent component failure rates (i.e. load sharing).

iv) The *pdfs* of both failure and repair processes assumed in this study can be handled mathematically to a considerable extent. Further studies incorporating other *pdfs* that may not be easy to handle mathematically can be conducted using the techniques of digital computer simulation (e.g. Monte Carlo).

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APPENDICES

Appendix A

Types of Redundancies

The following are types of redundancies commonly in use.

Parallel Redundancy

Figure A.1 shows three units in *parallel* redundancy.

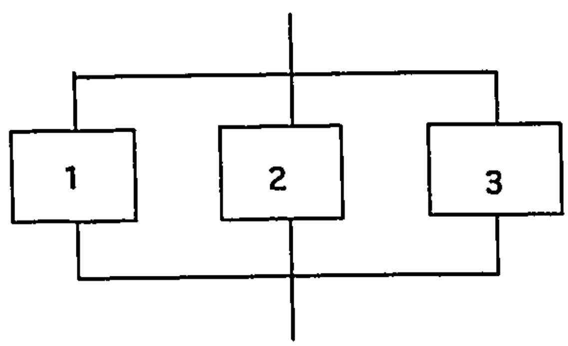

Figure A.1: Three units in parallel redundancy

k-out-of-n:G Redundancy

Figure A.2 shows the *k-out-of-n:G* arrangement of units.

Standby Configuration

Figure A.3 shows the *standby* arrangement of units.

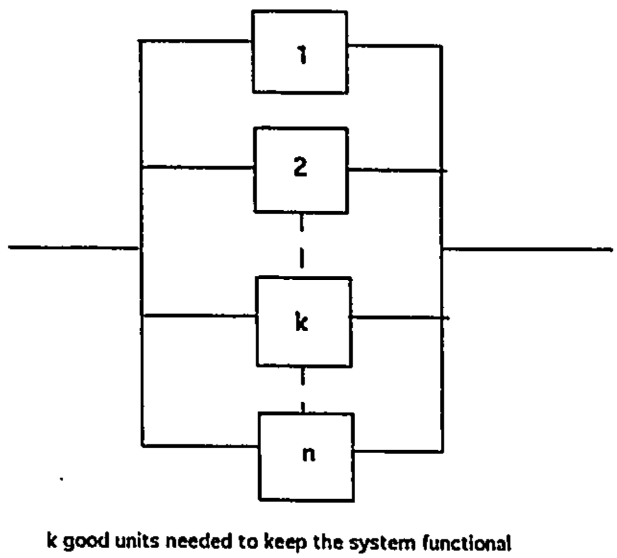

Figure A.2: k-out-of-n:G Redundant Configuration

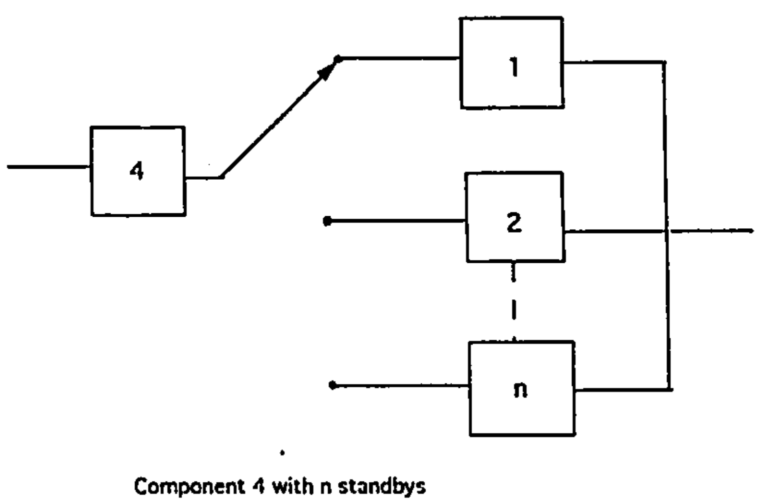

Figure A.3: Standby Redundant Configuration

Appendix B

Identical Unit 1-out-of-(n+1):G Submodel With Early Standby Activation: Definition of Constants

The following constants are associated with the analysis performed on the $1 - out - of - (n+1) : G$ system under SBAP Y in Section 3.1.

$$A_1 = C_{1,1}(1) + C_{1,1}(0'),\;\; A_2 = C_{1,1}(1)\,C_{1,1}(0'),\;\; A_3 = C_{1,1}(0) + C_{1,1}(1) + C_{1,1}(0')$$
$$A_4 = C_{1,1}(1)\,C_{1,1}(0') + C_{1,1}(0)\,C_{1,1}(1) + C_{1,1}(0)\,C_{1,1}(0') - \mu_1\,\lambda_1 - \mu_{s0'}\,\lambda_s$$
$$A_5 = C_{1,1}(0)\,C_{1,1}(1)\,C_{1,1}(0') - \mu_1\,\lambda_1\,C_{1,1}(0') - \mu_{s0'}\lambda_s\,C_{1,1}(1)$$
$$A_6 = \lambda_1^2 + \lambda_1\,\lambda_s,\;\; A_7 = \lambda_1^2\,C_{1,1}(0') + \lambda_1\,\lambda_s\,C_{1,1}(1),\;\; A_8 = \lambda_{c0}$$
$$A_9 = \lambda_{c0}\,C_{1,1}(1) + \lambda_{c0}\,C_{1,1}(0') + \lambda_1\,\lambda_{c1} + \lambda_{c0'}\,\lambda_s$$
$$A_{10} = \lambda_{c0}\,C_{1,1}(1)\,C_{1,1}(0') + \lambda_1\,\lambda_{c1}\,C_{1,1}(0') + \lambda_s\,\lambda_{c0'}\,C_{1,1}(1)$$
$$A_{11} = C_{2,1}(1) + C_{2,1}(2) + C_{2,1}(0')$$
$$A_{12} = C_{2,1}(1)\,C_{2,1}(2) + C_{2,1}(1)\,C_{2,1}(0') + C_{2,1}(2)\,C_{2,1}(0') - 4\,\mu_2\lambda_1$$
$$A_{13} = C_{2,1}(1)\,C_{2,1}(2)\,C_{2,1}(0') - 2\,\mu_2\,\lambda_1\,C_{2,1}(1) - 2\,\mu_2\,\lambda_1\,C_{2,1}(0')$$
$$A_{14} = 2\,\lambda_1\;,\; A_{15} = 2\,\lambda_1\,C_{2,1}(2) + 2\,\lambda_1\,C_{2,1}(0')$$
$$A_{16} = 2\,\lambda_1\,C_{2,1}(2)\,C_{2,1}(0') - 4\,\lambda_1^2\,\mu_2 + 2\,\mu_2\,\lambda_1\,\lambda_s$$
$$A_{17} = 2\,\lambda_1\,\lambda_s + 4\,\lambda_1^2,\;\; A_{18} = 2\,\lambda_1\,\lambda_s\,C_{2,1}(1) + 4\,\lambda_1^2\,C_{2,1}(0')$$

$A_{19} = \lambda_s, \;\; A_{20} = \lambda_s\, C_{2,1}(1) + \lambda_s\, C_{2,1}(2), \;\; A_{21} = \lambda_s\, C_{2,1}(1)\, C_{2,1}(2) + 4\,\lambda_1^2\,\mu_2 - 2\,\lambda_1\,\lambda_s\,\mu_2$
$A_{22} = A_{11} + C_{2,1}(0), \;\; A_{23} = A_{12} + C_{2,1}(0)\, A_{11} - \mu_1\, A_{14} - \mu_{s0'}\, A_{19}$
$A_{24} = A_{13} + C_{2,1}(0)\, A_{12} - \mu_1\, A_{15} - \mu_{s0'}\, A_{20}, \;\; A_{25} = A_{13}\, C_{2,1}(0) - \mu_1\, A_{16} - \mu_{s0'}\, A_{21}$
$A_{26} = \lambda_1\, A_{17}, \;\; A_{27} = \lambda_1\, A_{18}, \;\; A_{28} = \lambda_{c0}$
$A_{29} = \lambda_{c0}\, A_{11} + \lambda_{c1}\, A_{14} + \lambda_{c0'}\, A_{19}, \;\; A_{30} = \lambda_{c0}\, A_{12} + \lambda_{c1}\, A_{15} + \lambda_{c2} A_{17} + \lambda_{c0'}\, A_{20}$
$A_{31} = \lambda_{c0}\, A_{13} + \lambda_{c1}\, A_{16} + \lambda_{c2} A_{18} + \lambda_{c0'}\, A_{21}$
$A_{32} = 3\,\lambda_1, \;\; A_{33} = 3\,\lambda_1\, C_{3,1}(3) + 3\,\lambda_1\, C_{3,1}(2) + 3\,\lambda_1\, C_{3,1}(0')$

$$\begin{aligned}
A_{34} &= 3\,\lambda_1\, C_{3,1}(2)\, C_{3,1}(3) + 3\,\lambda_1\, C_{3,1}(0')\, C_{3,1}(3) + 3\,\lambda_1\, C_{3,1}(0')\, C_{3,1}(2) - \\
&\quad 9\,\lambda_1^2\,\mu_2 - 6\,\lambda_1^2\,\mu_3 + 3\,\lambda_1\,\lambda_s\,\mu_2 \\
A_{35} &= 3\,\lambda_1\, C_{3,1}(2)\, C_{3,1}(3)\, C_{3,1}(0') - 9\,\lambda_1^2\,\mu_2\, C_{3,1}(3) - 6\,\lambda_1^2\, C_{3,1}(0') + 3\,\lambda_1\,\lambda_s\,\mu_2\, C_{3,1}(3) \\
A_{36} &= C_{3,1}(2) + C_{3,1}(0') + C_{3,1}(1) + C_{3,1}(3) \\
A_{37} &= C_{3,1}(2)\, C_{3,1}(0') - 3\,\lambda_1\,\mu_2 + C_{3,1}(1)\, C_{3,1}(2) + C_{3,1}(1)\, C_{3,1}(0') + \\
&\quad C_{3,1}(2)\, C_{3,1}(3) + C_{3,1}(3)\, C_{3,1}(0') + C_{3,1}(1)\, C_{3,1}(3) - 3\,\lambda_1\mu_2 - 2\,\lambda_1\,\mu_3 \\
A_{38} &= C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(0') - 3\,\lambda_1\,\mu_2\, C_{3,1}(1) + C_{3,1}(3)\, C_{3,1}(2)\, C_{3,1}(0') - \\
&\quad 3\,\lambda_1\,\mu_2\, C_{3,1}(3) + C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(3) + C_{3,1}(1)\, C_{3,1}(3)\, C_{3,1}(0') - \\
&\quad 3\,\lambda_1\,\mu_2\, C_{3,1}(3) - 3\,\lambda_1\,\mu_2\, C_{3,1}(0') - 2\,\lambda_1\mu_3\, C_{3,1}(1) - 2\,\lambda_1\,\mu_3\, C_{3,1}(0') \\
A_{39} &= C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(3)\, C_{3,1}(0') - 3\,\lambda_1\,\mu_2\, C_{3,1}(1)\, C_{3,1}(3) \\
&\quad - 3\,\lambda_1\,\mu_2\, C_{3,1}(3)\, C_{3,1}(0') - 2\,\lambda_1\,\mu_3\, C_{3,1}(1)\, C_{3,1}(0')
\end{aligned}$$

$A_{40} = 3\,\lambda_1\,\lambda_s + 9\,\lambda_1^2$
$A_{41} = 3\,\lambda_1\,\lambda_s\, C_{3,1}(1) + 3\,\lambda_1\,\lambda_s\, C_{3,1}(3) + 9\,\lambda_1^2\, C_{3,1}(3) + 9\,\lambda_1^2\, C_{3,1}(0')$
$A_{42} = 3\,\lambda_1\,\lambda_s\, C_{3,1}(1)\, C_{3,1}(3) + 9\,\lambda_1^2\, C_{3,1}(3)\, C_{3,1}(0'), \;\; A_{43} = 6\,\lambda_1^2\,\lambda_s + 18\,\lambda_1^3$
$A_{44} = 6\,\lambda_1^2\,\lambda_s\, C_{3,1}(1) + 18\,\lambda_1^3\, C_{3,1}(0')$
$A_{45} = \lambda_s, \;\; A_{46} = \lambda_s\, C_{3,1}(1) + \lambda_s\, C_{3,1}(2) + \lambda_s\, C_{3,1}(3)$

$$\begin{aligned}
A_{47} &= \lambda_s\, C_{3,1}(1)\, C_{3,1}(2) + \lambda_s\, C_{3,1}(1)\, C_{3,1}(3) + \lambda_s\, C_{3,1}(2)\, C_{3,1}(3) - \\
&\quad 2\,\lambda_1\,\lambda_s\,\mu_3 + 9\,\lambda_1^2\,\mu_2 - 3\,\lambda_1\,\lambda_s\,\mu_2 \\
A_{48} &= \lambda_s\, C_{3,1}(1)\, C_{3,1}(2)\, C_{3,1}(3) - 2\,\lambda_1\,\lambda_s\,\mu_3\, C_{3,1}(1) + 9\,\lambda_1^2 - 3\,\lambda_1\,\lambda_s\,\mu_2\, C_{3,1}(3)
\end{aligned}$$

$B_1 = V_{N_1}(s = s_1)/((s_1 - s_2)(s_1 - s_3)), \;\; B_2 = V_{N_1}(s = s_2)/((s_2 - s_1)(s_2 - s_3))$
$B_3 = V_{N_1}(s = s_3)/((s_3 - s_1)(s_3 - s_2))$
$B_4 = \lambda_1\, V_{N_1}(s = -C_{1,1}(1))/((-C_{1,1}(1) - s_1)(-C_{1,1}(1) - s_2)(-C_{1,1}(1) - s_3))$

$B_5 = \lambda_1 V_{N_1}(s = s_1)/((s_1 + C_{1,1}(1))(s_1 - s_2)(s_1 - s_3)$
$B_6 = \lambda_1 V_{N_1}(s = s_2)/((s_2 + C_{1,1}(1))(s_2 - s_1)(s_2 - s_3)$
$B_7 = \lambda_1 V_{N_1}(s = s_3)/((s_3 + C_{1,1}(1))(s_3 - s_1)(s_3 - s_2)$
$B_8 = \lambda_s V_{N_1}(s = -C_{1,1}(0'))/((-C_{1,1}(0') - s_1)(-C_{1,1}(0') - s_2)(-C_{1,1}(0') - s_3))$
$B_9 = \lambda_s V_{N_1}(s = s_1)/((s_1 + C_{1,1}(0'))(s_1 - s_2)(s_1 - s_3)$
$B_{10} = \lambda_s V_{N_1}(s = s_2)/((s_2 + C_{1,1}(0'))(s_2 - s_1)(s_2 - s_3)$
$B_{11} = \lambda_s V_{N_1}(s = s_3)/((s_3 + C_{1,1}(0'))(s_3 - s_1)(s_3 - s_2)$
$B_{25} = W_5(s = s_9)/DD1, \; B_{26} = W_9(s = s_9)/DD1$
$B_{27} = W_{10}(s = s_9)/DD1, \; B_{28} = W_{11}(s = s_9)/DD1$
$B_{29} = W_5(s = s_{10})/DD2, \; B_{30} = W_9(s = s_{10})/DD2$
$B_{31} = W_{10}(s = s_{10})/DD2, \; B_{32} = W_{11}(s = s_{10})/DD2$
$B_{33} = W_5(s = s_{11})/DD3, \; B_{34} = W_9(s = s_{11})/DD3$
$B_{35} = W_{10}(s = s_{11})/DD3, \; B_{36} = W_{11}(s = s_{11})/DD3$
$B_{37} = W_5(s = s_{12})/DD4, \; B_{38} = W_9(s = s_{12})/DD4$
$B_{39} = W_{10}(s = s_{12})/DD4, \; B_{40} = W_{11}(s = s_{12})/DD4$
where,
$DD1 = (s_9 - s_{10})(s_9 - s_{11})(s_9 - s_{12}), \; DD2 = (s_{10} - s_9)(s_{10} - s_{11})(s_{10} - s_{12})$
$DD3 = (s_{11} - s_9)(s_{11} - s_{10})(s_{11} - s_{12}), \; DD4 = (s_{12} - s_9)(s_{12} - s_{10})(s_{12} - s_{11})$
$W_5(s) = s^3 + A_{11}s^2 + A_{12}s + A_{13}, \; W_9(s) = A_{14}s^2 + A_{15}s + A_{16}$
$W_{10}(s) = A_{17}s + A_{18}, \; W_{11}(s) = A_{19}s^2 + A_{20}s + A_{21}$
$B_{83} = W_{12}(s = s_{18})/D_{s_{18}}, \; B_{84} = W_{12}(s = s_{19})/D_{s_{19}} \; B_{85} = W_{12}(s = s_{20})/D_{s_{20}}$
$B_{86} = W_{12}(s = s_{21})/D_{s_{21}}, \; B_{87} = W_{12}(s = s_{22})/D_{s_{22}}, \; B_{88} = W_{16}(s = s_{18})/D_{s_{18}}$
$B_{89} = W_{16}(s = s_{19})/D_{s_{19}}, \; B_{90} = W_{16}(s = s_{20})/D_{s_{20}}, \; B_{91} = W_{16}(s = s_{21})/D_{s_{21}}$
$B_{92} = W_{16}(s = s_{22})/D_{s_{22}}, \; B_{93} = W_{17}(s = s_{18})/D_{s_{18}}, \; B_{94} = W_{17}(s = s_{19})/D_{s_{19}}$
$B_{95} = W_{17}(s = s_{20})/D_{s_{20}}, \; B_{96} = W_{17}(s = s_{21})/D_{s_{21}}, \; B_{97} = W_{17}(s = s_{22})/D_{s_{22}}$
$B_{98} = W_{18}(s = s_{18})/D_{s_{18}}, \; B_{99} = W_{18}(s = s_{19})/D_{s_{19}}, \; B_{100} = W_{18}(s = s_{20})/D_{s_{20}}$
$B_{101} = W_{18}(s = s_{21})/D_{s_{21}}, \; B_{102} = W_{18}(s = s_{22})/D_{s_{22}}, \; B_{103} = W_{19}(s = s_{18})/D_{s_{18}}$
$B_{104} = W_{19}(s = s_{19})/D_{s_{19}}, \; B_{105} = W_{19}(s = s_{20})/D_{s_{20}}, \; B_{106} = W_{19}(s = s_{21})/D_{s_{21}}$
$B_{107} = W_{19}(s = s_{22})/D_{s_{22}}$
where,
$D_{s_{18}} = (s_{18} - s_{19})(s_{18} - s_{20})(s_{18} - s_{21})(s_{18} - s_{22})$
$D_{s_{19}} = (s_{19} - s_{18})(s_{19} - s_{20})(s_{19} - s_{21})(s_{19} - s_{22})$
$D_{s_{20}} = (s_{20} - s_{18})(s_{20} - s_{19})(s_{20} - s_{21})(s_{20} - s_{22})$

$$D_{s_{21}} = (s_{21} - s_{18})(s_{21} - s_{19})(s_{21} - s_{20})(s_{21} - s_{22})$$

$$D_{s_{22}} = (s_{22} - s_{18})(s_{22} - s_{19})(s_{22} - s_{20})(s_{22} - s_{21})$$

$$k_1 = A_2 + \lambda_1\, C_{1,1}(0') + \lambda_s\, C_{1,1}(1),\;\; k_2 = A_5,\;\; k_3 = A_4,\;\; k_4 = A_1 + \lambda_1 + \lambda_s$$

$$k_5 = A_{13} + A_{16} + A_{18} + A_{21},\;\; k_6 = A_{25}$$

$$k_7 = A_{24},\;\; k_8 = A_{12} + A_{15} + A_{17} + A_{20}$$

$$k_9 = A_{39} + A_{35} + A_{42} + A_{44} + A_{48},\;\; k_{10} = A_{39}\, C_{3,1}(0) - \mu_1\, A_{35} - \mu_{s0'}\, A_{48}$$

$$k_{11} = A_{39} + C_{3,1}(0)\, A_{38} - \mu_1\, A_{34} - \mu_{s0'}\, A_{47},\;\; k_{12} = A_{38} + A_{34} + A_{41} + A_{43} + A_{47}$$

Appendix C

Identical Unit 1-out-of-(n+2):G Submodel With Early Standby Activation: Definition of Constants

The following constants are associated with the analysis performed on the $1 - out - of - (n+2) : G$ system under SBAP Y in Section 3.1.

$$A_{49} = \lambda_1, \quad A_{50} = A_{49}\,(C_{1,2}(0') + C_{1,2}(1') + C_{1,2}(2))$$

$$\begin{aligned}
A_{51} &= A_{49}\,(C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\,\mu_{s1'} + C_{1,2}(2)\,C_{1,2}(0') + \\
&\quad C_{1,2}(2)\,C_{1,2}(1') - \mu_2\,\lambda_1 + 2\,\lambda_s\,\mu_2) \\
A_{52} &= A_{49}\,(C_{1,2}(2)\,C_{1,2}(0')\,C_{1,2}(1') - C_{1,2}(2)\,\lambda_s\,\mu_{s1'}) - \mu_2\,\lambda_1\,C_{1,2}(1') + 2\,\lambda_s\,\mu_2\,C_{1,2}(1') \\
A_{53} &= C_{1,2}(0') + C_{1,2}(1') + C_{1,2}(1) + C_{1,2}(2) \\
A_{54} &= C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\,\mu_{s1'} + C_{1,2}(1)\,C_{1,2}(0') + C_{1,2}(1)\,C_{1,2}(1') + \\
&\quad C_{1,2}(2)\,C_{1,2}(0') + C_{1,2}(2)\,C_{1,2}(1') + C_{1,2}(1)\,C_{1,2}(2) - 2\,\mu_2\,\lambda_1 \\
A_{55} &= C_{1,2}(1)\,C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\mu_{s1'}\,C_{1,2}(1) + C_{1,2}(2)\,C_{1,2}(0')\,C_{1,2}(1') - \\
&\quad \lambda_s\,\mu_{s1'}\,C_{1,2}(2) + C_{1,2}(0')\,C_{1,2}(1)\,C_{1,2}(2) + C_{1,2}(1')\,C_{1,2}(1)\,C_{1,2}(2) - \\
&\quad \mu_2\,\lambda_1\,(C_{1,2}(1) + 2\,C_{1,2}(1') + C_{1,2}(0')) \\
A_{56} &= C_{1,2}(1)\,C_{1,2}(2)\,C_{1,2}(0')\,C_{1,2}(1') - \lambda_s\mu_{s1'}\,C_{1,2}(1)\,C_{1,2}(2)
\end{aligned}$$

$$- \mu_2 \lambda_1 (C_{1,2}(1) C_{1,2}(1') + C_{1,2}(0') C_{1,2}(1') - \lambda_s \mu_{s1'})$$

$A_{57} = 2 \lambda_1 \lambda_s + \lambda_1^2, \quad A_{58} = 2 \lambda_1 \lambda_s (C_{1,2}(1) + C_{1,2}(1')) + \lambda_1^2 (C_{1,2}(0') + C_{1,2}(1'))$

$A_{59} = 2 \lambda_1 \lambda_s C_{1,2}(1) C_{1,2}(1') + \lambda_1^2 (C_{1,2}(0') C_{1,2}(1') - \lambda_s \mu_{s1'}), \quad A_{60} = 2 \lambda_s$

$A_{61} = A_{60} (C_{1,2}(1) + C_{1,2}(2) + C_{1,2}(1'))$

$A_{62} = A_{60} (C_{1,2}(1) C_{1,2}(2) + C_{1,2}(1) C_{1,2}(1') + C_{1,2}(2) C_{1,2}(1')) + \mu_2 \lambda_1 (\lambda_1 - 2 \lambda_s)$

$A_{63} = 2 \lambda_s C_{1,2}(1') C_{1,2}(1) C_{1,2}(2) + \mu_2 \lambda_1 C_{1,2}(1') (\lambda_1 - 2 \lambda_s)$

$A_{64} = 2 \lambda_s^2, \quad A_{65} = A_{64} (C_{1,2}(1) + C_{1,2}(2))$

$A_{66} = A_{64} C_{1,2}(1) C_{1,2}(2) + \mu_2 \lambda_1 (\lambda_1 \lambda_s - 2 \lambda_s^2), \quad A_{67} = A_{53} + C_{1,2}(0)$

$A_{68} = A_{54} + A_{53} C_{1,2}(0) - \mu_1 A_{49} - \mu_{s0'} A_{60}, \quad A_{69} = A_{55} + A_{54} C_{1,2}(0) - \mu_1 A_{50} - \mu_{s0'} A_{61}$

$A_{70} = A_{56} + A_{55} C_{1,2}(0) - \mu_1 A_{51} - \mu_{s0'} A_{62}, \quad A_{71} = A_{56} C_{1,2}(0) - \mu_1 A_{52} - \mu_{s0'} A_{63}$

$A_{72} = \lambda_1 (A_{57} + A_{64}), \quad A_{73} = \lambda_1 (A_{58} + A_{65}), \quad A_{74} = \lambda_1 (A_{59} + A_{66}), \quad A_{75} = \lambda_{c0}$

$A_{76} = \lambda_{c0} A_{53} + \lambda_{c1} A_{49} + \lambda_{c0'} A_{60}$

$A_{77} = \lambda_{c0} A_{54} + \lambda_{c1} A_{50} + \lambda_{c2} A_{57} + \lambda_{c0'} A_{61} + \lambda_{c1'} A_{64}$

$A_{78} = \lambda_{c0} A_{55} + \lambda_{c1} A_{51} + \lambda_{c2} A_{58} + \lambda_{c0'} A_{62} + \lambda_{c1'} A_{65}$

$A_{79} = \lambda_{c0} A_{56} + \lambda_{c1} A_{52} + \lambda_{c2} A_{59} + \lambda_{c0'} A_{63} + \lambda_{c1'} A_{66}$

$$\begin{aligned}
A_{86} &= C_{2,2}(1) + C_{2,2}(2) + C_{2,2}(3) + C_{2,2}(0') + C_{2,2}(1') \\
A_{87} &= C_{2,2}(1) C_{2,2}(2) + C_{2,2}(1) C_{2,2}(3) + C_{2,2}(2) C_{2,2}(3) + \\
&\quad C_{2,2}(1) C_{2,2}(0') + C_{2,2}(2) C_{2,2}(0') + C_{2,2}(3) C_{2,2}(0') + \\
&\quad C_{2,2}(1) C_{2,2}(1') + C_{2,2}(2) C_{2,2}(1') + C_{2,2}(3) C_{2,2}(1') + \\
&\quad C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'} \\
A_{88} &= C_{2,2}(1) C_{2,2}(2) (C_{2,2}(3) + C_{2,2}(0')) + C_{2,2}(3) C_{2,2}(0') \\
&\quad (C_{2,2}(1) + C_{2,2}(2)) + C_{2,2}(3) C_{2,2}(1') (C_{2,2}(1) + C_{2,2}(2)) + \\
&\quad C_{2,2}(1) C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'} C_{2,2}(1) + C_{2,2}(0') C_{2,2}(1') C_{2,2}(2) - \\
&\quad \lambda_s \mu_{s1'} C_{2,2}(2) + C_{2,2}(3) (C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'}) \\
A_{89} &= C_{2,2}(1) C_{2,2}(2) C_{2,2}(3) (C_{2,2}(0') + C_{2,2}(1')) + C_{2,2}(1) C_{2,2}(2) \\
&\quad (C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'}) + C_{2,2}(1) C_{2,2}(3) (C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'}) + \\
&\quad C_{2,2}(2) C_{2,2}(3) (C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'})
\end{aligned}$$

$A_{90} = C_{2,2}(1) C_{2,2}(2) C_{2,2}(3) (C_{2,2}(0') C_{2,2}(1') - \lambda_s \mu_{s1'})$

$A_{91} = 2 \lambda_1 \mu_3, \quad A_{92} = A_{91} (C_{2,2}(0') + C_{2,2}(2) + C_{2,2}(1))$

$A_{93} = A_{91}\,(C_{2,2}(0')\,C_{2,2}(2) + C_{2,2}(1)\,C_{2,2}(0') + C_{2,2}(1)\,C_{2,2}(2))$

$A_{94} = A_{91}\,C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(0'),\;\; A_{95} = A_{91},\;\; A_{96} = A_{95}\,(C_{2,2}(0') + C_{2,2}(1') + C_{2,2}(1))$

$A_{97} = A_{95}\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'} + C_{2,2}(1)\,C_{2,2}(0') + C_{2,2}(1)\,C_{2,2}(1'))$

$A_{98} = A_{95}\,C_{2,2}(1)\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'}),\;\; A_{99} = A_{95}\,(2\,\lambda_1\mu_{s1'} + \lambda_s\mu_2)$

$A_{100} = A_{99}\,C_{2,2}(1),\;\; A_{101} = 2\,\lambda_1\mu_2,\;\; A_{102} = A_{101}\,(C_{2,2}(1) + C_{2,2}(1') + C_{2,2}(3))$

$A_{103} = A_{101}\,(C_{2,2}(1')\,C_{2,2}(3) - 2\,\lambda_1\,\mu_3 + C_{2,2}(1)\,C_{2,2}(1') + C_{2,2}(1)\,C_{2,2}(3))$

$A_{104} = A_{101}\,C_{2,2}(1)\,(C_{2,2}(1')\,C_{2,2}(3) - 2\,\lambda_1\,\mu_3),\;\; A_{105} = A_{101}$

$A_{106} = A_{105}\,(C_{2,2}(0') + C_{2,2}(1') + C_{2,2}(3))$

$A_{107} = A_{105}\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\,\mu_{s1'} + C_{2,2}(0')\,C_{2,2}(3) + C_{2,2}(1')\,C_{2,2}(3))$

$A_{108} = A_{105}\,C_{2,2}(3)\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'}),\;\; A_{109} = 4\,\lambda_1^2\,\mu_2\,\mu_3$

$A_{110} = A_{109}\,C_{2,2}(0'),\;\; A_{111} = A_{87} + A_{91} - A_{95} - A_{101} - A_{105}$

$A_{112} = A_{88} + A_{92} - A_{96} - A_{102} - A_{106}$

$A_{113} = A_{89} + A_{93} - A_{97} - A_{99} - A_{103} - A_{107} + A_{109}$

$A_{114} = A_{90} + A_{94} - A_{98} - A_{100} - A_{104} - A_{108} + A_{110},\;\; A_{115} = 2\,\lambda_1$

$A_{116} = A_{115}\,(C_{2,2}(0') + C_{2,2}(1') + C_{2,2}(2) + C_{2,2}(3)),\;\; A_{117} = A_{109}$

$A_{118} = A_{117}\,(C_{2,2}(2) + C_{2,2}(0')),\;\; A_{119} = A_{117}\,C_{2,2}(2)\,C_{2,2}(0'),\;\; A_{120} = A_{117}$

$A_{121} = A_{120}\,(C_{2,2}(0') + C_{2,2}(1')),\;\; A_{122} = A_{120}\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\,\mu_{s1'})$

$A_{123} = A_{120}\,(2\,\lambda_1\mu_{s1'} + \lambda_s\mu_2),\;\; A_{124} = 4\,\lambda_1^2\,\mu_2$

$A_{125} = A_{124}\,(C_{2,2}(3) + C_{2,2}(1')),\;\; A_{126} = A_{124}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3)$

$A_{127} = 4\,\lambda_1\lambda_s^2\,\mu_2\,\mu_3,\;\; A_{128} = 4\,\lambda_1\lambda_s\,\mu_2,\;\; A_{129} = A_{128}\,(C_{2,2}(3) + C_{2,2}(1'))$

$A_{130} = A_{128}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3)$

$$\begin{aligned}
A_{131} &= 2\,\lambda_1\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'} + C_{2,2}(2)\,C_{2,2}(0') + C_{2,2}(2)\,C_{2,2}(1') + \\
&\quad C_{2,2}(3)\,C_{2,2}(0') + C_{2,2}(3)\,C_{2,2}(1') + C_{2,2}(2)\,C_{2,2}(3)) \\
A_{132} &= 2\,\lambda_1\,C_{2,2}(2)\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\,\mu_{s1'}) + 2\,\lambda_1\,C_{2,2}(3)\,(C_{2,2}(0')\,C_{2,2}(1') - \\
&\quad \lambda_s\,\mu_{s1'}) + 2\,\lambda_1\,C_{2,2}(2)\,(C_{2,2}(0')\,C_{2,2}(3) + C_{2,2}(1')\,C_{2,2}(3)) \\
A_{133} &= 2\,\lambda_1\,C_{2,2}(2)\,C_{2,2}(3)\,(C_{2,2}(0')C_{2,2}(1') - \lambda_s\,\mu_{s1'})
\end{aligned}$$

$A_{134} = A_{131} - A_{117} - A_{120} - A_{124} + A_{128},\;\; A_{135} = A_{132} - A_{118} - A_{121} - A_{125} + A_{129}$

$A_{136} = A_{133} - A_{119} - A_{122} - A_{123} - A_{126} + A_{127} + A_{130},\;\; A_{137} = 4\,\lambda_1^2$

$A_{138} = A_{137}\,(C_{2,2}(0') + C_{2,2}(1') + C_{2,2}(3))$

$A_{139} = A_{137}\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'} + C_{2,2}(3)\,C_{2,2}(0') + C_{2,2}(3)\,C_{2,2}(1'))$

$A_{140} = A_{137}\,C_{2,2}(3)\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\,\mu_{s1'}),\;\; A_{141} = 8\,\lambda_1^3\,\mu_3$

$A_{142} = A_{141}\, C_{2,2}(0'),\;\; A_{143} = 4\,\lambda_1\,\lambda_s^2\,\mu_3,\;\; A_{144} = A_{143}\,(C_{2,2}(1) + C_{2,2}(1'))$
$A_{145} = A_{143}\,C_{2,2}(1)\,C_{2,2}(1'),\;\; A_{146} = 4\,\lambda_1\,\lambda_s,\;\; A_{147} = A_{146}\,(C_{2,2}(3) + C_{2,2}(1') + C_{2,2}(1))$
$A_{148} = A_{146}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3 + C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(1)\,C_{2,2}(1'))$
$A_{149} = A_{146}\,C_{2,2}(1)\,(C_{2,2}(3) - 2\,\lambda_1\,\mu_3),\;\; A_{150} = A_{137} + A_{146},\;\; A_{151} = A_{138} + A_{143} + A_{147}$
$A_{152} = A_{139} - A_{141} + A_{144} + A_{148},\;\; A_{153} = A_{140} - A_{142} + A_{145} + A_{149},\;\; A_{154} = 8\,\lambda_1^3$
$A_{155} = A_{154}\,(C_{2,2}(0') + C_{2,2}(1')),\;\; A_{156} = A_{154}\,(C_{2,2}(0')\,C_{2,2}(1') - \lambda_s\mu_{s1'}),\;\; A_{157} = 8\,\mu_2\lambda_1^3\,\lambda_s$
$A_{158} = 4\,\lambda_1\,\lambda_s^2,\;\; A_{159} = A_{158}\,(C_{2,2}(1) + C_{2,2}(2)),\;\; A_{160} = A_{158}\,C_{2,2}(1)\,C_{2,2}(2)$
$A_{161} = 8\,\lambda_1^2\,\lambda_s,\;\; A_{162} = A_{161}\,(C_{2,2}(1) + C_{2,2}(1')),\;\; A_{163} = A_{161}\,C_{2,2}(1)\,C_{2,2}(1')$
$A_{164} = 8\,\lambda_1^2\lambda_s^2\,\mu_2,\;\; A_{165} = A_{154} + A_{158} + A_{161},\;\; A_{166} = A_{155} + A_{159} + A_{162}$
$A_{167} = A_{156} + A_{157} + A_{160} + A_{163} + A_{164},\;\; A_{168} = 8\,\lambda_1^3\,\mu_3\,\mu_{s1'},\;\; A_{169} = 4\,\lambda_1^2\,\mu_2$
$A_{170} = A_{169}\,(C_{2,2}(3) + C_{2,2}(1')),\;\; A_{171} = A_{169}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3),\;\; A_{172} = 2\,\lambda_s$
$A_{173} = A_{172}\,(C_{2,2}(3) + C_{2,2}(1') + C_{2,2}(1) + C_{2,2}(2))$

$$\begin{aligned}
A_{174} &= A_{172}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3 + C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(1)\,C_{2,2}(1') + \\
&\quad C_{2,2}(2)\,C_{2,2}(3) + C_{2,2}(2)\,C_{2,2}(1') + C_{2,2}(1)\,C_{2,2}(2)) \\
A_{175} &= 2\,\lambda_s C_{2,2}(1)\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\mu_3) + 2\,\lambda_s\,C_{2,2}(2)\,(C_{2,2}(3)\,C_{2,2}(1') - \\
&\quad 2\,\lambda_1\,\mu_3 + C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(1)\,C_{2,2}(1'))
\end{aligned}$$

$A_{176} = 2\,\lambda_s\,C_{2,2}(1)\,C_{2,2}(2)\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3)$
$A_{177} = 4\,\lambda_1\,\lambda_s\,\mu_2,\;\; A_{178} = A_{177}\,(C_{2,2}(1) + C_{2,2}(1')),\;\; A_{179} = 4\,\lambda_1\,\lambda_s\,\mu_3\,C_{2,2}(1)\,C_{2,2}(1')$
$A_{180} = 4\,\lambda_1\,\lambda_s\,\mu_2,\;\; A_{181} = A_{180}\,(C_{2,2}(3) + C_{2,2}(1'))$
$A_{182} = A_{180}\,(C_{2,2}(3)\,C_{2,2}(1') - 2\,\lambda_1\,\mu_3)$
$A_{183} = A_{169} + A_{174} - A_{177} - A_{180},\;\; A_{184} = A_{170} + A_{175} - A_{177} - A_{181}$
$A_{185} = A_{168} + A_{171} + A_{176} - A_{179} - A_{182},\;\; A_{186} = 8\,\lambda_1^3\,\mu_3,\;\; A_{187} = A_{186}\,C_{2,2}(0')$
$A_{188} = 4\,\lambda_1^2\,\lambda_s\,\mu_2,\;\; A_{189} = A_{188}\,C_{2,2}(3),\;\; A_{190} = 2\,\lambda_s^2$
$A_{191} = A_{190}\,(C_{2,2}(1) + C_{2,2}(2) + C_{2,2}(3))$
$A_{192} = A_{190}\,(C_{2,2}(1)\,C_{2,2}(2) + C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(2)\,C_{2,2}(3))$
$A_{193} = A_{190}C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(3),\;\; A_{194} = 4\,\lambda_1\,\lambda_s^2\,\mu_3 - 8\,\lambda_1^2\,\lambda_s\,\mu_3,\;\; A_{195} = A_{194}\,C_{2,2}(1)$
$A_{196} = 4\,\lambda_1\,\lambda_s^2\,\mu_2,\;\; A_{197} = A_{196}\,C_{2,2}(3),\;\; A_{198} = A_{186} + A_{188} + A_{192} - A_{194} - A_{197}$
$A_{199} = A_{187} + A_{189} + A_{193} - A_{195} - A_{198},\;\; A_{200} = C_{2,2}(0)\,A_{114} - \mu_1\,A_{136} - \mu_{s0'}\,A_{185}$

$$\begin{aligned}
A_{201} &= C_{3,2}(1) + C_{3,2}(2) + C_{3,2}(3) + C_{3,2}(4) \\
A_{202} &= C_{3,2}(1)\,C_{3,2}(2) + C_{3,2}(1)\,C_{3,2}(3) + C_{3,2}(2)\,C_{3,2}(3) + C_{3,2}(1)\,C_{3,2}(4) +
\end{aligned}$$

$$C_{3,2}(2)\,C_{3,2}(4)+C_{3,2}(3)\,C_{3,2}(4)-2\,\lambda_1\,\mu_4-3\,\lambda_1\,\mu_3-3\,\lambda_1\,\mu_2$$

$$\begin{aligned}
A_{203} &= C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(3)+C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(4)+C_{3,2}(1)\,C_{3,2}(3)\,C_{3,2}(4)+\\
&\quad C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4)-2\,\lambda_1\,\mu_4\,(C_{3,2}(1)+C_{3,2}(2))-\\
&\quad 3\,\lambda_1\,\mu_3\,(C_{3,2}(1)+C_{3,2}(4))-3\,\lambda_1\,\mu_2\,(C_{3,2}(3)+C_{3,2}(4))\\
A_{204} &= C_{3,2}(1)\,C_{3,2}(2)\,(C_{3,2}(3)\,C_{3,2}(4)-2\,\lambda_1\,\mu_4)-3\,\lambda_1\,\mu_3\,C_{3,2}(1)\,C_{3,2}(4)+\\
&\quad 6\,\lambda_1^2\,\mu_2\,\mu_4-3\,\lambda_1\,\mu_2\,C_{3,2}(3)\,C_{3,2}(4)
\end{aligned}$$

$A_{205}=C_{3,2}(0')+C_{3,2}(1')$, $A_{206}=C_{3,2}(0')\,C_{3,2}(1')-\lambda_s\,\mu_{s1'}$, $A_{207}=A_{201}+A_{205}$

$A_{208}=A_{206}+A_{201}\,A_{205}+A_{202}$, $A_{209}=A_{201}\,A_{206}+A_{202}\,A_{205}+A_{203}$

$A_{210}=A_{202}\,A_{206}+A_{203}\,A_{205}+A_{204}$, $A_{211}=A_{203}\,A_{206}+A_{204}\,A_{205}$

$A_{212}=A_{204}\,A_{206}$, $A_{213}=3\,\lambda_1\,\mu_2\,\mu_3$, $A_{214}=A_{213}\,C_{3,2}(4)$

$A_{215}=-\mu_3$, $A_{216}=A_{215}\,(C_{3,2}(1)+C_{3,2}(2)+C_{3,2}(4))$

$A_{217}=A_{215}\,(C_{3,2}(1)\,C_{3,2}(4)+C_{3,2}(1)\,C_{3,2}(2)+C_{3,2}(2)\,C_{3,2}(4))$

$A_{218}=A_{215}\,C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(4)$, $A_{219}=3\,\lambda_1$, $A_{220}=A_{219}\,C_{3,2}(0')$

$A_{221}=A_{213}+A_{217}$, $A_{222}=A_{214}+A_{218}$, $A_{223}=A_{215}\,A_{219}$

$A_{224}=A_{215}\,A_{220}+A_{216}\,A_{219}$, $A_{225}=A_{216}\,A_{220}+A_{221}\,A_{219}$, $A_{226}=A_{221}\,A_{220}+A_{222}\,A_{219}$

$A_{227}=A_{222}\,A_{220}$, $A_{228}=6\,\lambda_1^2\,\mu_2\,\mu_4$, $A_{229}=A_{228}\,(C_{3,2}(1)+C_{3,2}(1'))$

$A_{230}=A_{228}\,C_{3,2}(1)\,C_{3,2}(1')$, $A_{231}=9\,\lambda_1^2\,\mu_3\,\mu_{s1'}$, $A_{232}=A_{231}\,(C_{3,2}(1)+C_{3,2}(4))$

$A_{233}=A_{231}\,C_{3,2}(1)\,C_{3,2}(4)$, $A_{234}=3\,\lambda_1\,\lambda_s\,\mu_2\,\mu_3$, $A_{235}=A_{234}\,(C_{3,2}(1)+C_{3,2}(4))$

$A_{236}=A_{234}\,C_{3,2}(1)\,C_{3,2}(4)$, $A_{237}=3\,\lambda_1\,\mu_2$, $A_{238}=A_{237}\,(C_{3,2}(1)+C_{3,2}(4))$

$A_{239}=A_{237}\,C_{3,2}(1)\,C_{3,2}(4)$, $A_{240}=1.0$, $A_{241}=C_{3,2}(3)+C_{3,2}(1')$

$A_{242}=C_{3,2}(3)\,C_{3,2}(1')-3\,\lambda_1\,\mu_3$, $A_{243}=A_{237}\,A_{240}$, $A_{244}=A_{237}\,A_{241}+A_{238}\,A_{240}$

$A_{245}=A_{237}\,A_{242}+A_{238}\,A_{241}+A_{239}\,A_{240}$, $A_{246}=A_{238}\,A_{242}+A_{239}\,A_{241}$

$A_{247}=3\,\lambda_1\,\mu_2$, $A_{248}=A_{208}+A_{223}-A_{243}$, $A_{249}=A_{209}+A_{224}-A_{244}$

$A_{250}=A_{210}+A_{225}+A_{228}-A_{231}-A_{234}-A_{245}$

$A_{251}=A_{211}+A_{226}+A_{229}-A_{232}-A_{235}-A_{246}$

$A_{252}=A_{212}+A_{227}+A_{230}-A_{233}-A_{236}-A_{247}$

$A_{253}=-6\,\lambda_1^2\,\mu_4$, $A_{254}=A_{253}\,C_{3,2}(2)$, $A_{255}=C_{3,2}(0')+C_{3,2}(1')$

$A_{256}=C_{3,2}(0')\,C_{3,2}(1')-\lambda_s\,\mu_{s1'}$, $A_{257}=18\,\lambda_1^3\,\mu_2\,\mu_4$, $A_{258}=A_{257}\,C_{3,2}(1')$

$A_{259}=3\,\lambda_1$, $A_{260}=A_{259}\,(C_{3,2}(2)+C_{3,2}(3)+C_{3,2}(4))$

$$A_{261} = A_{259}\,(C_{3,2}(2)\,C_{3,2}(3) + C_{3,2}(2)\,C_{3,2}(4) + C_{3,2}(3)\,C_{3,2}(4))$$

$$A_{262} = A_{259}\,C_{3,2}(2)\,C_{3,2}(3)\,C_{3,2}(4),\;\; A_{263} = C_{3,2}(0') + C_{3,2}(1')$$

$$A_{264} = C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'}$$

$$A_{265} = A_{259}\,A_{263} + A_{260},\;\; A_{266} = A_{259}\,A_{264} + A_{260}\,A_{263} + A_{261}$$

$$A_{267} = A_{260}\,A_{264} + A_{261}\,A_{263} + A_{262},\;\; A_{268} = A_{261}\,A_{264} + A_{262}\,A_{263}$$

$$A_{269} = A_{262}\,A_{264},\;\; A_{270} = 9\,\lambda_1^2\,\mu_3,\;\; A_{271} = A_{270}\,(C_{3,2}(2) + C_{3,2}(4) + C_{3,2}(0'))$$

$$A_{272} = A_{270}\,(C_{3,2}(2)\,C_{3,2}(4) + C_{3,2}(2)\,C_{3,2}(0') + C_{3,2}(4)\,C_{3,2}(0'))$$

$$A_{273} = A_{270}\,C_{3,2}(2)\,C_{3,2}(4)\,C_{3,2}(0'),\;\; A_{274} = A_{270},\;\; A_{275} = A_{270}\,C_{3,2}(4)$$

$$A_{276} = A_{274}\,A_{263} + A_{275},\;\; A_{277} = A_{264}\,A_{274} + A_{263}\,A_{275},\;\; A_{278} = A_{264}\,A_{275}$$

$$A_{279} = 27\,\lambda_1^3\,\mu_3\,\mu_{s1'},\;\; A_{280} = A_{279}\,C_{3,2}(4),\;\; A_{281} = 12\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_4$$

$$A_{282} = A_{281}\,C_{3,2}(1'),\;\; A_{283} = 6\,\lambda_1\,\lambda_s^2\,\mu_2\,\mu_3 - 9\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3,\;\; A_{284} = A_{283}\,C_{3,2}(4)$$

$$A_{285} = 6\,\lambda_1\,\lambda_s\,\mu_2 - 9\,\lambda_1^2\,\mu_2,\;\; A_{286} = A_{285}\,C_{3,2}(4),\;\; A_{287} = C_{3,2}(3) + C_{3,2}(1')$$

$$A_{288} = C_{3,2}(3)\,C_{3,2}(1') - 3\,\lambda_1\,\mu_3,\;\; A_{289} = A_{285}\,A_{287} + A_{286}$$

$$A_{290} = A_{285}\,A_{288} + A_{286}\,A_{287}$$

$$A_{291} = A_{286}\,A_{288},\;\; A_{292} = A_{253} + A_{266} - A_{270} - A_{274} + A_{285}$$

$$A_{293} = A_{253}\,A_{255} + A_{254} + A_{267} - A_{271} - A_{276} + A_{289}$$

$$A_{294} = A_{253}\,A_{256} + A_{254}\,A_{255} + A_{257} + A_{268} - A_{272} - A_{277} - A_{279} - A_{281} + A_{283} + A_{290}$$

$$A_{295} = A_{254}\,A_{256} + A_{258} + A_{269} - A_{273} - A_{278} - A_{280} - A_{282} + A_{284} + A_{291}$$

$$A_{296} = -18\,\lambda_1^3\,\mu_4,\;\; A_{297} = A_{296}\,A_{255},\;\; A_{298} = A_{296}\,A_{256},\;\; A_{299} = 9\,\lambda_1^2$$

$$A_{300} = A_{299}\,(C_{3,2}(4) + C_{3,2}(3)),\;\; A_{301} = A_{299}\,C_{3,2}(3)\,C_{3,2}(4),\;\; A_{302} = A_{299}\,A_{255} + A_{300}$$

$$A_{303} = A_{299}\,A_{256} + A_{300}\,A_{255} + A_{301},\;\; A_{304} = A_{300}\,A_{256} + A_{301}\,A_{255},\;\; A_{305} = A_{301}\,A_{256}$$

$$A_{306} = 27\,\lambda_1^3\,\mu_3,\;\; A_{307} = A_{306}\,(C_{3,2}(4) + C_{3,2}(0')),\;\; A_{308} = A_{306}\,C_{3,2}(4)\,C_{3,2}(0')$$

$$A_{309} = 6\,\lambda_1\,\lambda_s^2\,\mu_3,\;\; A_{310} = A_{309}\,(C_{3,2}(1) + C_{3,2}(4)),\;\; A_{311} = A_{309}\,C_{3,2}(1)\,C_{3,2}(4)$$

$$A_{312} = 12\,\lambda_1^2\,\lambda_s\,\mu_4,\;\; A_{313} = A_{312}\,(C_{3,2}(1) + C_{3,2}(1'))$$

$$A_{314} = A_{312}\,C_{3,2}(1)\,C_{3,2}(1'),\;\; A_{315} = 6\,\lambda_1\,\lambda_s,\;\; A_{316} = A_{315}\,(C_{3,2}(1) + C_{3,2}(4))$$

$$A_{317} = A_{315}\,C_{3,2}(1)\,C_{3,2}(4),\;\; A_{318} = A_{315}\,A_{287} + A_{316}$$

$$A_{319} = A_{315}\,A_{288} + A_{316}\,A_{287} + A_{317},\;\; A_{320} = A_{316}\,A_{288} + A_{317}\,A_{287}$$

$$A_{321} = A_{317}\,A_{288},\;\; A_{322} = A_{299} + A_{315},\;\; A_{323} = A_{302} + A_{318}$$

$$A_{324} = A_{303} + A_{296} - A_{306} + A_{309} - A_{312} + A_{319}$$

$$A_{325} = A_{297} + A_{304} - A_{307} + A_{310} - A_{313} + A_{320}$$

$$A_{326} = A_{298} + A_{305} - A_{308} + A_{311} - A_{314} + A_{321}$$

$$A_{327} = 27\,\lambda_1^3,\;\; A_{328} = A_{327}\,C_{3,2}(4),\;\; A_{329} = C_{3,2}(0') + C_{3,2}(1')$$

$$A_{330} = C_{3,2}(0')\,C_{3,2}(1') - \lambda_s\,\mu_{s1'},\;\; A_{331} = A_{327}\,A_{329} + A_{328}$$

$A_{332} = A_{327}\,A_{330} + A_{328}\,A_{329}, \;\; A_{333} = A_{328}\,A_{330}, \;\; A_{334} = 27\,\lambda_1^3\,\lambda_s\,\mu_2$

$A_{335} = A_{334}\,C_{3,2}(4), \;\; A_{336} = 6\,\lambda_1\,\lambda_s^2$

$A_{337} = A_{336}\,(C_{3,2}(4) + C_{3,2}(1) + C_{3,2}(2))$

$A_{338} = A_{336}\,(C_{3,2}(1)\,C_{3,2}(4) + C_{3,2}(2)\,C_{3,2}(4) + C_{3,2}(1)\,C_{3,2}(2))$

$A_{339} = A_{336}\,C_{3,2}(1)\,C_{3,2}(2)\,C_{3,2}(4), \;\; A_{340} = 18\,\lambda_1^2\,\lambda_s$

$A_{341} = A_{340}\,(C_{3,2}(1) + C_{3,2}(4) + C_{3,2}(1'))$

$A_{342} = A_{340}\,(C_{3,2}(1)\,C_{3,2}(4) + C_{3,2}(1)\,C_{3,2}(1') + C_{3,2}(4)\,C_{3,2}(1'))$

$A_{343} = A_{340}\,C_{3,2}(1)\,C_{3,2}(4)\,C_{3,2}(1'), \;\; A_{344} = 18\,\lambda_1^2\,\lambda_s^2\,\mu_2, \;\; A_{345} = A_{344}\,C_{3,2}(4)$

$A_{346} = A_{327} + A_{336} + A_{340}, \;\; A_{347} = A_{331} + A_{337} + A_{341}$

$A_{348} = A_{332} + A_{334} + A_{338} + A_{342} - A_{344}, \;\; A_{349} = A_{333} + A_{335} + A_{339} + A_{343} - A_{345}$

$A_{350} = 51\,\lambda_1^4, \;\; A_{351} = A_{350}\,A_{255}, \;\; A_{352} = A_{350}\,A_{256}, \;\; A_{353} = A_{350}\,\lambda_s\,\mu_2$

$A_{354} = 12\,\lambda_1^2\,\lambda_s^2, \;\; A_{355} = A_{354}\,(C_{3,2}(1) + C_{3,2}(2))$

$A_{356} = A_{354}\,C_{3,2}(1)\,C_{3,2}(2), \;\; A_{357} = 36\,\lambda_1^3\,\lambda_s$

$A_{358} = A_{357}\,(C_{3,2}(1) + C_{3,2}(1')), \;\; A_{359} = A_{357}\,C_{3,2}(1)\,C_{3,2}(1')$

$A_{360} = A_{357}\,\lambda_s\,\mu_2, \;\; A_{361} = A_{350} + A_{354} + A_{357}$

$A_{362} = A_{351} + A_{355} + A_{358}, \;\; A_{363} = A_{352} + A_{353} + A_{356} + A_{359} - A_{360}$

$A_{364} = 27\,\lambda_1^3\,\mu_3\,\mu_{s1'}, \;\; A_{365} = A_{364}\,C_{3,2}(4), \;\; A_{366} = 6\,\lambda_1\,\lambda_s\,\mu_3$

$A_{367} = A_{366}\,(C_{3,2}(1) + C_{3,2}(4) + C_{3,2}(1'))$

$A_{368} = A_{366}\,(C_{3,2}(1)\,C_{3,2}(4) + C_{3,2}(1)\,C_{3,2}(1') + C_{3,2}(1')\,C_{3,2}(4))$

$A_{369} = A_{366}\,C_{3,2}(1)\,C_{3,2}(1')\,C_{3,2}(4), \;\; A_{370} = 2\,\lambda_s$

$A_{371} = A_{370}\,(C_{3,2}(1) + C_{3,2}(2)), \;\; A_{372} = A_{370}\,C_{3,2}(1)\,C_{3,2}(2) + 9\,\lambda_1^2\,\mu_2 - 6\,\lambda_1\,\lambda_s\,\mu_2$

$A_{373} = 2\,\lambda_1\,\mu_4, \;\; A_{374} = A_{373}\,C_{3,2}(1'), \;\; A_{375} = A_{370}\,A_{373}$

$A_{376} = A_{370}\,A_{374} + A_{371}\,A_{373}, \;\; A_{377} = A_{371}\,A_{374} + A_{372}\,A_{373}, \;\; A_{378} = A_{372}\,A_{374}$

$A_{379} = A_{287} + C_{3,2}(4), \;\; A_{380} = A_{288} + C_{3,2}(4)\,A_{287}, \;\; A_{381} = C_{3,2}(4)\,A_{288}$

$A_{382} = A_{370}\,A_{379} + A_{371}, \;\; A_{383} = A_{370}\,A_{380} + A_{371}\,A_{379} + A_{372}$

$A_{384} = A_{370}\,A_{381} + A_{371}\,A_{380} + A_{372}\,A_{379}, \;\; A_{385} = A_{371}\,A_{381} + A_{372}\,A_{380}$

$A_{386} = A_{372}\,A_{381}, \;\; A_{387} = A_{383} - A_{366} - A_{375}, \;\; A_{388} = A_{384} - A_{367} - A_{376}$

$A_{389} = A_{385} + A_{364} - A_{368} - A_{377}, \;\; A_{390} = A_{386} + A_{365} - A_{369} - A_{378}$

$A_{391} = 27\,\lambda_1^3\,\mu_3, \;\; A_{392} = A_{391}\,(C_{3,2}(4) + C_{3,2}(0'))$

$A_{393} = A_{391}\,C_{3,2}(0')\,C_{3,2}(4), \;\; A_{394} = 6\,\lambda_1\,\lambda_s\,\mu_3(\lambda_s - 3\,\lambda_1)$

$A_{395} = A_{394}\,(C_{3,2}(1) + C_{3,2}(4)), \;\; A_{396} = A_{394}\,C_{3,2}(1)\,C_{3,2}(4), \;\; A_{397} = 2\lambda_s^2$

$A_{398} = A_{397}\,(C_{3,2}(1) + C_{3,2}(2)), \;\; A_{399} = A_{397}\,C_{3,2}(1)\,C_{3,2}(2) + 9\,\lambda_1^2\,\mu_2 - 6\,\lambda_1\,\lambda_s^2\,\mu_2$

$A_{400} = C_{3,2}(3) + C_{3,2}(4), \;\; A_{401} = C_{3,2}(3)\,C_{3,2}(4) - 2\,\lambda_1\,\mu_4, \;\; A_{402} = A_{397}\,A_{400} + A_{398}$

$A_{403} = A_{397}\,A_{401} + A_{398}\,A_{400} + A_{399},\;\; A_{404} = A_{398}\,A_{401} + A_{399}\,A_{400}$

$A_{405} = A_{399}\,A_{401},\;\; A_{406} = A_{391} - A_{394} + A_{403},\;\; A_{407} = A_{392} - A_{395} + A_{404}$

$A_{408} = A_{393} - A_{396} + A_{405},\;\; A_{409} = C_{3,2}(0)\,A_{252} - \mu_1\,A_{295} - \mu_{s0'}\,A_{290}$

$xdaf1 = (s_{28} - s_{29})(s_{28} - s_{30}),\;\; xdaf2 = (s_{29} - s_{28})(s_{29} - s_{30})$

$xdaf3 = (s_{30} - s_{28})(s_{30} - s_{29}),\;\; s_{31} = -C_{1,2}(1),\;\; s_{32} = -C_{1,2}(2)$

$xdaf4 = (-C_{1,2}(1) - s_{28})(-C_{1,2}(1) - s_{29})(-C_{1,2}(1) - s_{30})$

$xdaf5 = (s_{28} + C_{1,2}(1))\,xdaf1,\;\; xdaf6 = (s_{29} + C_{1,2}(1))\,xdaf2$

$xdaf7 = (s_{30} + C_{1,2}(1))\,xdaf3,\;\; xdaf8 = (C_{1,2}(2) - C_{1,2}(1))\,xdaf4$

$xdaf9 = (-C_{1,2}(2) + C_{1,2}(1))(-C_{1,2}(2) - s_{28})(-C_{1,2}(2) - s_{29})(-C_{1,2}(2) - s_{30}),$

$xdaf10 = (s_{28} + C_{1,2}(2))\,xdaf5$

$xdaf11 = (s_{29} + C_{1,2}(2))\,xdaf6,\;\; xdaf12 = (s_{20} + C_{1,2}(2))\,xdaf7$

$V_{N_2}(s) = (s + C_{1,2}(0'))\,(s + C_{1,2}(1')) - \lambda_s\,\mu_{s1'},\;\; V_{N_3}(s) = \lambda_1\,V_{N_2}(s)$

$V_{N_4}(s) = 2\,\lambda_1\,\lambda_s(s + C_{1,2}(1))(s + C_{1,2}(1')) + \lambda_1^2\,V_{N_2}(s),\;\; V_{N_5}(s) = 2\,\lambda_s\,(s + C_{1,2}(1'))$

$B_{151} = V_{N_2}(s = s_{28})/xdaf1,\;\; B_{152} = V_{N_3}(s = s_{28})/xdaf5,\;\; B_{153} = V_{N_4}(s = s_{28})/xdaf10$

$B_{154} = V_{N_5}(s = s_{28})/xdaf1,\;\; B_{155} = 2\,\lambda_s^2/xdaf1,\;\; B_{156} = V_{N_2}(s = s_{29})/xdaf2$

$B_{157} = V_{N_3}(s = s_{29})/xdaf6,\;\; B_{158} = V_{N_4}(s = s_{29})/xdaf11,\;\; B_{159} = V_{N_5}(s = s_{29})/xdaf2$

$B_{160} = 2\,\lambda_s^2/xdaf2,\;\; B_{161} = V_{N_2}(s = s_{30})/xdaf3,\;\; B_{162} = V_{N_3}(s = s_{30})/xdaf7$

$B_{163} = V_{N_4}(s = s_{30})/xdaf12,\;\; B_{164} = V_{N_5}(s = s_{30})/xdaf3,\;\; B_{165} = 2\,\lambda_s^2/xdaf3$

$B_{166} = V_{N_3}(s = -C_{1,2}(1))/xdaf4,\;\; B_{167} = V_{N_4}(s = -C_{1,2}(1))/xdaf8$

$B_{168} = V_{N_4}(s = -C_{1,2}(2))/xdaf9$

$xdaf25 = (s_{33} - s_{34})(s_{33} - s_{35}),\;\; xdaf26 = (s_{34} - s_{33})(s_{34} - s_{35})$

$xdaf27 = (s_{35} - s_{33})(s_{35} - s_{34}),\;\; s_{36} = -C_{2,2}(1),\;\; s_{37} = -C_{2,2}(2),\;\; s_{38} = -C_{2,2}(3)$

$xdaf28 = (-C_{2,2}(1) - s_{33})(-C_{2,2}(1) - s_{34})(-C_{2,2}(1) - s_{35})$

$xdaf29 = (s_{33} + C_{2,2}(1))\,xdaf25,\;\; xdaf30 = (s_{34} + C_{2,2}(1))\,xdaf26$

$xdaf31 = (s_{35} + C_{2,2}(1))\,xdaf27$

$xdaf32 = (-C_{2,2}(2) + C_{2,2}(1))(-C_{2,2}(2) - s_{33})(-C_{2,2}(2) - s_{34})(-C_{2,2}(2) - s_{35})$

$xdaf33 = (-C_{2,2}(1) + C_{2,2}(2))\,xdaf28,\;\; xdaf34 = (s_{33} + C_{2,2}(2))\,xdaf29$

$xdaf35 = (s_{34} + C_{2,2}(2))\,xdaf30,\;\; xdaf36 = (s_{35} + C_{2,2}(2))\,xdaf31$

$xdaf37 = (-C_{2,2}(3)+C_{2,2}(1))(-C_{2,2}(3)+C_{2,2}(2))(-C_{2,2}(3)-s_{33})(-C_{2,2}(3)-s_{34})(-C_{2,2}(3)-s_{35})$

$xdaf38 = (-C_{2,2}(2) + C_{2,2}(3))\,xdaf32,\;\; xdaf39 = (-C_{2,2}(1) + C_{2,2}(3))\,xdaf33$

$xdaf40 = (s_{33} + C_{2,2}(3))\,xdaf34$

$xdaf41 = (s_{34} + C_{2,2}(3))\, xdaf35,\;\; xdaf42 = (s_{35} + C_{2,2}(3))\, xdaf36$

$V_{N_6}(s) = (s + C_{2,2}(0'))(s + C_{2,2}(1')) - \lambda_s\, \mu_{s1'}$

$V_{N_7}(s) = 2\,\lambda_1\, V_{N_6}(s),\;\; V_{N_8}(s) = 4\,\lambda_1\,\lambda_s\,(s + C_{2,2}(1))(s + C_{2,2}(1')) + 4\,\lambda_1^2\, V_{N_6}(s)$

$V_{N_9}(s) = 4\,\lambda_1\,\lambda_s^2\,(s+C_{2,2}(1))(s+C_{2,2}(2))+8\,\lambda_1^2\,\lambda_s\,(s+C_{2,2}(1))(s+C_{2,2}(1'))+8\,\lambda_1^3\, V_{N_6}(s)$

$V_{N_{10}}(s) = 2\,\lambda_s\,(s + C_{2,2}(1'))$

$B_{187} = V_{N_6}(s = s_{33})/xdaf25,\;\; B_{188} = V_{N_7}(s = s_{33})/xdaf29,\;\; B_{189} = V_{N_8}(s = s_{33})/xdaf34$

$B_{190} = V_{N_9}(s = s_{33})/xdaf40,\;\; B_{191} = V_{N_{10}}(s = s_{33})/xdaf25,\;\; B_{192} = 2\,\lambda_s^2/xdaf25$

$B_{193} = V_{N_6}(s = s_{34})/xdaf26,\;\; B_{194} = V_{N_7}(s = s_{34})/xdaf30,\;\; B_{195} = V_{N_8}(s = s_{34})/xdaf35$

$B_{196} = V_{N_9}(s = s_{34})/xdaf41,\;\; B_{197} = V_{N_{10}}(s = s_{34})/xdaf26,\;\; B_{198} = 2\,\lambda_s^2/xdaf26$

$B_{199} = V_{N_6}(s = s_{35})/xdaf27,\;\; B_{200} = V_{N_7}(s = s_{35})/xdaf31,\;\; B_{201} = V_{N_8}(s = s_{35})/xdaf36$

$B_{202} = V_{N_9}(s = s_{35})/xdaf42,\;\; B_{203} = V_{N_{10}}(s = s_{35})/xdaf27,\;\; B_{204} = 2\,\lambda_s^2/xdaf27$

$B_{205} = V_{N_7}(s = -C_{2,2}(1))/xdaf28,\;\; B_{206} = V_{N_8}(s = -C_{2,2}(1))/xdaf33$

$B_{207} = V_{N_9}(s = -C_{2,2}(1))/xdaf39,\;\; B_{208} = V_{N_8}(s = -C_{2,2}(2))/xdaf32$

$B_{209} = V_{N_9}(s = -C_{2,2}(2))/xdaf38,\;\; B_{210} = V_{N_9}(s = -C_{2,2}(3))/xdaf37$

$xdaf61 = (s_{39} - s_{40})(s_{39} - s_{41}),\;\; xdaf62 = (s_{40} - s_{39})(s_{40} - s_{41})$

$xdaf63 = (s_{41} - s_{39})(s_{41} - s_{40}),\;\; s_{42} = -C_{3,2}(1)$

$s_{43} = -C_{3,2}(2),\;\; s_{44} = -C_{3,2}(3),\;\; s_{45} = -C_{3,2}(4)$

$xdaf64 = (-C_{3,2}(1) - s_{39})(-C_{3,2}(1) - s_{40})(-C_{3,2}(1) - s_{41})$

$xdaf65 = (s_{39} + C_{3,2}(1))\, xdaf61,\;\; xdaf66 = (s_{40} + C_{3,2}(1))\, xdaf62$

$xdaf67 = (s_{41} + C_{3,2}(1))\, xdaf63$

$xdaf68 = (-C_{3,2}(2) + C_{3,2}(1))(-C_{3,2}(2) - s_{39})(-C_{3,2}(2) - s_{40})(-C_{3,2}(2) - s_{41})$

$xdaf69 = (-C_{3,2}(1) + C_{3,2}(2))\, xdaf64,\;\; xdaf70 = (s_{39} + C_{3,2}(2))\, xdaf65$

$xdaf71 = (s_{40} + C_{3,2}(2))\, xdaf66,\;\; xdaf72 = (s_{41} + C_{3,2}(2))\, xdaf67$

$xdaf73 = (-C_{3,2}(3)+C_{3,2}(1))(-C_{3,2}(3)+C_{3,2}(2))(-C_{3,2}(3)-s_{39})(-C_{3,2}(3)-s_{40})(-C_{3,2}(3)-s_{41})$

$xdaf74 = (-C_{3,2}(2) + C_{3,2}(3))\, xdaf68,\;\; xdaf75 = (-C_{3,2}(1) + C_{3,2}(3))\, xdaf69$

$xdaf76 = (s_{39} + C_{3,2}(3))\, xdaf70,\;\; xdaf77 = (s_{40} + C_{3,2}(3))\, xdaf71$

$xdaf78 = (s_{41} + C_{3,2}(3))\, xdaf72$

$xdaf79 = (-C_{3,2}(4) - C_{3,2}(1))(-C_{3,2}(4) - C_{3,2}(2))(-C_{3,2}(4) - C_{3,2}(3))(-C_{3,2}(4) - s_{39})(-C_{3,2}(4) - s_{40})(-C_{3,2}(4) - s_{41})$

$xdaf80 = (-C_{3,2}(3) + C_{3,2}(4))\, xdaf73,\;\; xdaf81 = (-C_{3,2}(2) + C_{3,2}(4))\, xdaf74$

$xdaf82 = (-C_{3,2}(1) + C_{3,2}(4))\, xdaf75,\;\; xdaf83 = (s_{39} + C_{3,2}(4))\, xdaf76$

$xdaf84 = (s_{40} + C_{3,2}(4))\, xdaf77,\;\; xdaf85 = (s_{41} + C_{3,2}(4))\, xdaf78$

$V_{N_{11}}(s) = (s + C_{3,2}(0'))(s + C_{3,2}(1')) - \lambda_s\, \mu_{s1'},\;\; V_{N_{12}}(s) = 3\,\lambda_1\, V_{N_{11}}(s)$

$V_{N_{13}}(s) = 6\,\lambda_1\,\lambda_s\,(s + C_{3,2}(1))(s + C_{3,2}(1')) + 9\,\lambda_1^2\, V_{N_{11}}(s)$

$V_{N_{14}}(s) = 6\,\lambda_1\,\lambda_s^2\,(s+C_{3,2}(1))(s+C_{3,2}(2)) + 18\,\lambda_1^2\,\lambda_s\,(s+C_{3,2}(1))(s+C_{3,2}(1')) + 27\,\lambda_1^3\,(s+ C_{3,2}(0'))(s + C_{3,2}(1'))$

$V_{N_{15}}(s) = 12\,\lambda_1^2\,\lambda_s^2\,(s + C_{3,2}(1))(s + C_{3,2}(2)) + 36\,\lambda_1^3\,\lambda_s\,(s + C_{3,2}(1))(s + C_{3,2}(1')) + 54\,\lambda_1^4\, V_{N_{11}}(s)$

$V_{N_{16}}(s) = 2\,\lambda_s\,(s + C_{3,2}(1'))$

$B_{235} = V_{N_{11}}(s = s_{39})/xdaf61,\;\; B_{236} = V_{N_{12}}(s = s_{39})/xdaf65$

$B_{237} = V_{N_{13}}(s = s_{39})/xdaf70,\;\; B_{238} = V_{N_{14}}(s = s_{39})/xdaf76$

$B_{239} = V_{N_{15}}(s = s_{39})/xdaf83,\;\; B_{240} = V_{N_{16}}(s = s_{39})/xdaf61$

$B_{241} = 2\,\lambda_s^2/xdaf61,\;\; B_{242} = V_{N_{11}}(s = s_{40})/xdaf62$

$B_{243} = V_{N_{12}}(s = s_{40})/xdaf66,\;\; B_{244} = V_{N_{13}}(s = s_{40})/xdaf71$

$B_{245} = V_{N_{14}}(s = s_{40})/xdaf77,\;\; B_{246} = V_{N_{15}}(s = s_{40})/xdaf84$

$B_{247} = V_{N_{16}}(s = s_{40})/xdaf62,\;\; B_{248} = 2\,\lambda_s^2/xdaf62$

$B_{249} = V_{N_{11}}(s = s_{41})/xdaf63,\;\; B_{250} = V_{N_{12}}(s = s_{41})/xdaf67$

$B_{251} = V_{N_{13}}(s = s_{41})/xdaf72,\;\; B_{252} = V_{N_{14}}(s = s_{41})/xdaf78$

$B_{253} = V_{N_{15}}(s = s_{41})/xdaf85,\;\; B_{254} = V_{N_{16}}(s = s_{41})/xdaf63$

$B_{255} = 2\,\lambda_s^2/xdaf63,\;\; B_{256} = V_{N_{12}}(s = -C_{3,2}(1))/xdaf64$

$B_{257} = V_{N_{13}}(s = -C_{3,2}(1))/xdaf69,\;\; B_{258} = V_{N_{14}}(s = -C_{3,2}(1))/xdaf75$

$B_{259} = V_{N_{15}}(s = -C_{3,2}(1))/xdaf82,\;\; B_{260} = V_{N_{13}}(s = -C_{3,2}(2))/xdaf68$

$B_{261} = V_{N_{14}}(s = -C_{3,2}(2))/xdaf74,\;\; B_{262} = V_{N_{15}}(s = -C_{3,2}(2))/xdaf81$

$B_{263} = V_{N_{14}}(s = -C_{3,2}(3))/xdaf73,\;\; B_{264} = V_{N_{15}}(s = -C_{3,2}(3))/xdaf80$

$B_{265} = V_{N_{15}}(s = -C_{3,2}(4))/xdaf79$

$k_{13} = A_{70},\;\; k_{14} = A_{71}$

$k_{15} = A_{52} + A_{56} + A_{59} + A_{63} + A_{66},\;\; k_{16} = A_{51} + A_{55} + A_{58} + A_{62} + A_{65}$

$k_{17} = A_{114} + A_{136} + A_{153} + A_{167} + A_{185} + A_{199},\;\; k_{18} = A_{200}$

$k_{19} = C_{2,2}(0)\, A_{113} + A_{114} - \mu_1\, A_{135} - \mu_{s0'}\, A_{184}$

$k_{20} = A_{113} + A_{135} + A_{152} + A_{166} + A_{184} + A_{198}$

$k_{21} = A_{252} + A_{295} + A_{326} + A_{349} + A_{363} + A_{390} + A_{408}$

$k_{22} = A_{409}$

$$k_{23} = A_{251} + A_{294} + A_{325} + A_{348} + A_{362} + A_{389} + A_{407}$$
$$k_{24} = C_{3,2}(0)\, A_{251} + A_{252} - \mu_1\, A_{294} - \mu_{s0'}\, A_{389}$$

Appendix D

Identical Unit 1-out-of-(n+3):G Submodel With Early Standby Activation: Definition of Constants

The following constants are associated with the analysis performed on the $1 - out - of - (n+3) : G$ system under SBAP Y in Section 3.1.

$A_{410} = \lambda_1^2\,\lambda_s\,\mu_{s2'}\,\mu_2, \quad A_{411} = A_{410}\,C_{1,3}(1), \quad A_{412} = \lambda_1\,\lambda_s\,\mu_{s2'}$
$A_{413} = A_{412}\,(C_{1,3}(2) + C_{1,3}(3) + C_{1,3}(0'))$
$A_{414} = A_{412}\,(C_{1,3}(2)\,C_{1,3}(3) - \mu_3\,\lambda_1 + C_{1,3}(0')\,C_{1,3}(2) + C_{1,3}(0')\,C_{1,3}(3))$
$A_{415} = A_{412}\,(C_{1,3}(0')\,C_{1,3}(2)\,C_{1,3}(3) - \mu_3\,\lambda_1\,C_{1,3}(0')), \quad A_{416} = \lambda_1$

$$\begin{aligned}
A_{417} &= A_{416}\,(C_{1,3}(2') + C_{1,3}(2) + C_{1,3}(3) + C_{1,3}(0') + C_{1,3}(1')) \\
A_{418} &= \lambda_1 C_{1,3}(2')\,(C_{1,3}(2) + C_{1,3}(3) + C_{1,3}(0') + C_{1,3}(1')) + \\
&\quad \lambda_1\,C_{1,3}(2)\,(C_{1,3}(3) + C_{1,3}(0') + C_{1,3}(1')) + \\
&\quad \lambda_1\,C_{1,3}(3)\,(C_{1,3}(0') + C_{1,3}(1') + \lambda_1\,(C_{1,3}(0')\,C_{1,3}(1') - 2\,\lambda_s\,\mu_{s1'})) \\
A_{419} &= \lambda_1\,C_{1,3}(2)\,C_{1,3}(2')\,(C_{1,3}(3) + C_{1,3}(0') + C_{1,3}(1')) + \\
&\quad \lambda_1\,C_{1,3}(2')\,C_{1,3}(3)\,(C_{1,3}(0') + C_{1,3}(1')) + \lambda_1 C_{1,3}(2)\,C_{1,3}(3)\,(\\
&\quad C_{1,3}(0') + C_{1,3}(1')) + \lambda_1\,C_{1,3}(2')\,(C_{1,3}(0')\,C_{1,3}(1') -
\end{aligned}$$

$$
\begin{aligned}
& C_{1,3}(0') + C_{1,3}(1')) + \lambda_1 C_{1,3}(2')\,(C_{1,3}(0')\,C_{1,3}(1') - \\
& 2\,\lambda_s\,\mu_{s1'}) + \lambda_1 C_{1,3}(2)\,(C_{1,3}(0')\,C_{1,3}(1') - 2\,\lambda_s\,\mu_{s1'}) + \\
& \lambda_1 C_{1,3}(3)\,(C_{1,3}(0')\,C_{1,3}(1') - 2\,\lambda_s\,\mu_{s1'}) \\
A_{419a} &= C_{1,3}(0')\,C_{1,3}(1') - 2\,\lambda_s\,\mu_{s1'} \\
A_{420} &= \lambda_1 C_{1,3}(2)\,C_{1,3}(3)\,C_{1,3}(2')\,(C_{1,3}(0') + C_{1,3}(1')) + \\
& \lambda_1 C_{1,3}(2)\,C_{1,3}(2')\,A_{419a} + \lambda_1 C_{1,3}(2')\,C_{1,3}(3)\,A_{419a} + \\
& \lambda_1 C_{1,3}(2)\,C_{1,3}(3)\,A_{419a}
\end{aligned}
$$

$A_{421} = \lambda_1 C_{1,3}(2)\,C_{1,3}(3)\,C_{1,3}(2')\,A_{419a},\ \ A_{422} = \mu_3\,\lambda_1^2$

$A_{423} = A_{422}\,(C_{1,3}(2') + C_{1,3}(2) + C_{1,3}(0'))$

$A_{424} = A_{422}\,(C_{1,3}(2)\,C_{1,3}(2') + C_{1,3}(0')\,C_{1,3}(2') + C_{1,3}(0')\,C_{1,3}(2))$

$A_{425} = A_{422}\,C_{1,3}(0')\,C_{1,3}(2')\,C_{1,3}(2),\ \ A_{426} = A_{422}$

$A_{427} = A_{426}\,(C_{1,3}(0') + C_{1,3}(1') + C_{1,3}(2'))$

$A_{428} = A_{426}\,A_{419a} + A_{426}\,(C_{1,3}(2')\,C_{1,3}(0') + C_{1,3}(2')\,C_{1,3}(1'))$

$A_{429} = A_{426}\,C_{1,3}(2')\,A_{419a},\ \ A_{430} = \lambda_1^3\,\mu_3\,\mu_{s1'},\ \ A_{431} = A_{430}\,C_{1,3}(2')$

$A_{432} = 2\,\mu_2\,\mu_3\,\lambda_1^2\,\lambda_s,\ \ A_{433} = A_{432}\,C_{1,3}(2'),\ \ A_{434} = \mu_2\,\lambda_1^2$

$A_{435} = A_{434}\,(C_{1,3}(3) + C_{1,3}(1') + C_{1,3}(2')),\ \ A_{435a} = C_{1,3}(3)\,C_{1,3}(1') - \lambda_1\,\mu_3$

$A_{436} = A_{434}\,A_{435a} + A_{434}\,(C_{1,3}(2')\,C_{1,3}(3) + C_{1,3}(2')\,C_{1,3}(1'))$

$A_{437} = A_{434}\,C_{1,3}(2')\,A_{435a},\ \ A_{438} = 6\,\lambda_1\,\lambda_s^2\,\mu_2\,\mu_3,\ \ A_{439} = A_{438}\,C_{1,3}(2')$

$A_{440} = 3\,\lambda_1\,\lambda_s\,\mu_2,\ \ A_{441} = A_{440}\,(C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(3))$

$A_{441a} = C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s2'},\ \ A_{442} = A_{440}\,A_{441a} + A_{440}\,(C_{1,3}(3)\,C_{1,3}(1') + C_{1,3}(3)\,C_{1,3}(2'))$

$A_{443} = A_{440}\,C_{1,3}(3)\,A_{441a},\ \ A_{444} = \lambda_1\,\mu_3\,A_{440},\ \ A_{445} = A_{444}\,C_{1,3}(2'),\ \ A_{446} = A_{417}$

$A_{447} = A_{418} - A_{412} - A_{422} - A_{426} - A_{434} + A_{440}$

$A_{448} = A_{419} - A_{413} - A_{423} - A_{427} - A_{435} + A_{441}$

$A_{449} = A_{420} + A_{410} - A_{414} - A_{424} - A_{428} - A_{430} - A_{432} - A_{436} + A_{438} + A_{442} - A_{444}$

$A_{450} = A_{421} + A_{411} - A_{415} - A_{425} - A_{429} - A_{431} - A_{433} - A_{437} + A_{439} + A_{443} - A_{445}$

$A_{451} = \mu_2\,\mu_{s2'}\,\lambda_1\,\lambda_s,\ \ A_{452} = A_{451}\,(C_{1,3}(1) + C_{1,3}(3)),\ \ A_{453} = A_{451}\,C_{1,3}(1)\,C_{1,3}(3)$

$A_{454} = \mu_{s2'}\,\lambda_s,\ \ A_{455} = A_{454}\,(C_{1,3}(2) + C_{1,3}(3) + C_{1,3}(1) + C_{1,3}(0'))$

$A_{455a} = C_{1,3}(2)\,C_{1,3}(3) - \lambda_1\,\mu_3$

$A_{456} = A_{454}\,C_{1,3}(1)\,(A_{455a} + C_{1,3}(2) + C_{1,3}(3)) + A_{454}\,C_{1,3}(0')\,(C_{1,3}(2) + C_{1,3}(3) + C_{1,3}(1))$

$A_{457} = A_{454}\,C_{1,3}(1)\,A_{455a} + A_{454}\,C_{1,3}(0')\,A_{455a} + A_{454}\,C_{1,3}(1)\,C_{1,3}(0')(C_{1,3}(2) + C_{1,3}(3))$

$A_{458} = A_{454}\,C_{1,3}(1)\,C_{1,3}(0')\,A_{455a}$

$$
\begin{aligned}
& C_{1,3}(1')\, C_{1,3}(2') + C_{1,3}(3)\, C_{1,3}(0') + C_{1,3}(3)\, C_{1,3}(1') + \\
& C_{1,3}(1')\, C_{1,3}(1) + C_{1,3}(1)\, C_{1,3}(0') + C_{1,3}(3)\, C_{1,3}(2') + \\
& C_{1,3}(1')\, C_{1,3}(2) + C_{1,3}(2)\, C_{1,3}(0') + C_{1,3}(1)\, C_{1,3}(2') + \\
& C_{1,3}(1)\, C_{1,3}(3) + C_{1,3}(2)\, C_{1,3}(3) + C_{1,3}(2)\, C_{1,3}(2') + \\
& C_{1,3}(1)\, C_{1,3}(2) \\
A_{460a} &= C_{1,3}(0')\, C_{1,3}(1') - 2\, \lambda_s\, \mu_{s1'} \\
A_{461} &= A_{460a}\, (C_{1,3}(2') + C_{1,3}(3) + C_{1,3}(1) + C_{1,3}(2)) + \\
& C_{1,3}(0')\, C_{1,3}(2')\, C_{1,3}(3) + C_{1,3}(1')\, C_{1,3}(2')\, C_{1,3}(3) + \\
& C_{1,3}(0')\, C_{1,3}(2')\, C_{1,3}(1) + C_{1,3}(1')\, C_{1,3}(2')\, C_{1,3}(1) + \\
& C_{1,3}(0')\, C_{1,3}(1)\, C_{1,3}(3) + C_{1,3}(1')\, C_{1,3}(1)\, C_{1,3}(3) + \\
& C_{1,3}(0')\, C_{1,3}(2)\, C_{1,3}(2') + C_{1,3}(1')\, C_{1,3}(2)\, C_{1,3}(2') + \\
& C_{1,3}(0')\, C_{1,3}(2)\, C_{1,3}(3) + C_{1,3}(1')\, C_{1,3}(2)\, C_{1,3}(3) + \\
& C_{1,3}(2')\, C_{1,3}(1)\, C_{1,3}(3) + C_{1,3}(2')\, C_{1,3}(2)\, C_{1,3}(3) + \\
& C_{1,3}(0')\, C_{1,3}(1)\, C_{1,3}(2) + C_{1,3}(1')\, C_{1,3}(1)\, C_{1,3}(2) + \\
& C_{1,3}(2')\, C_{1,3}(1)\, C_{1,3}(2) + C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3) \\
A_{462} &= A_{460a}\, (C_{1,3}(2')\, C_{1,3}(3) + C_{1,3}(1)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(3) + \\
& C_{1,3}(2)\, C_{1,3}(2') + C_{1,3}(2)\, C_{1,3}(3) + C_{1,3}(1)\, C_{1,3}(2)) + \\
& C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(3)\, C_{1,3}(0') + C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(3)\, C_{1,3}(1') + \\
& C_{1,3}(2)\, C_{1,3}(2')\, C_{1,3}(3)\, C_{1,3}(0') + C_{1,3}(2)\, C_{1,3}(2')\, C_{1,3}(3)\, C_{1,3}(1') + \\
& C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(2)\, C_{1,3}(0') + C_{1,3}(1)\, C_{1,3}(0')\, C_{1,3}(3)\, C_{1,3}(2) + \\
& C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(2)\, C_{1,3}(1') + C_{1,3}(1)\, C_{1,3}(1')\, C_{1,3}(2)\, C_{1,3}(3) + \\
& C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(2)\, C_{1,3}(3) \\
A_{463} &= A_{460a}\, (C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(3) + C_{1,3}(2)\, C_{1,3}(2')\, C_{1,3}(3) + \\
& C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(2) + C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3)) + \\
& C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(2')\, C_{1,3}(3)\, (C_{1,3}(0') + C_{1,3}(1')) \\
A_{464} &= A_{460a}\, C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3)\, C_{1,3}(2') \\
A_{465} &= \lambda_1 \mu_3 \\
A_{466} &= A_{465}\, (C_{1,3}(2) + C_{1,3}(2') + C_{1,3}(0') + C_{1,3}(1))
\end{aligned}
$$

$$\begin{aligned}
A_{467} &= A_{465}\,(C_{1,3}(2)\,C_{1,3}(2') + C_{1,3}(0')\,C_{1,3}(2) + C_{1,3}(0')\,C_{1,3}(2') + \\
&\quad C_{1,3}(1)\,C_{1,3}(2) + C_{1,3}(2')\,C_{1,3}(1) + C_{1,3}(0')\,C_{1,3}(1')) \\
A_{468} &= A_{465}\,(C_{1,3}(2)\,C_{1,3}(2')\,C_{1,3}(0') + C_{1,3}(1) + C_{1,3}(2)\,C_{1,3}(2') + \\
&\quad C_{1,3}(2)\,C_{1,3}(0')\,C_{1,3}(1') + C_{1,3}(0') + C_{1,3}(1')\,C_{1,3}(2')) \\
A_{469} &= A_{465}\,(C_{1,3}(2)\,C_{1,3}(0')\,C_{1,3}(1') + C_{1,3}(2')) \\
A_{470} &= A_{465} \\
A_{471} &= A_{465}\,(C_{1,3}(0') + C_{1,3}(1') + C_{1,3}(1) + C_{1,3}(2)) \\
A_{472} &= A_{465}\,(A_{460a} + C_{1,3}(1)\,C_{1,3}(0') + C_{1,3}(1)\,C_{1,3}(1') + \\
&\quad C_{1,3}(2)\,C_{1,3}(0') + C_{1,3}(2)\,C_{1,3}(1') + C_{1,3}(1)\,C_{1,3}(2)) \\
A_{473} &= A_{470}\,A_{460a}\,(C_{1,3}(1) + C_{1,3}(2)) + A_{470}\,C_{1,3}(1)\,C_{1,3}(2)\,(C_{1,3}(0') + \\
&\quad C_{1,3}(1'))
\end{aligned}$$

$A_{474} = A_{470}\,A_{460a}\,C_{1,3}(1)\,C_{1,3}(2)$, $A_{475} = \mu_3\,\mu_{s1'}\,\lambda_1^2$, $A_{476} = A_{475}\,(C_{1,3}(1) + C_{1,3}(2'))$
$A_{477} = A_{475}\,C_{1,3}(1)\,C_{1,3}(2')$, $A_{478} = 2\,\mu_2\,\mu_3\,\lambda_1\,\lambda_s$, $A_{479} = A_{478}\,(C_{1,3}(1) + C_{1,3}(2'))$
$A_{480} = A_{478}\,C_{1,3}(1)\,C_{1,3}(2')$, $A_{481} = \mu_2\,\lambda_1$
$A_{482} = A_{481}\,(C_{1,3}(1') + C_{1,3}(3) + C_{1,3}(1) + C_{1,3}(2'))$, $A_{482a} = C_{1,3}(1')\,C_{1,3}(3) - \mu_3\,\lambda_1$

$$\begin{aligned}
A_{483} &= A_{481}\,(A_{482a} + C_{1,3}(1)\,C_{1,3}(1') + C_{1,3}(1)\,C_{1,3}(3) + C_{1,3}(1')\,C_{1,3}(2') + \\
&\quad C_{1,3}(2')\,C_{1,3}(3) + C_{1,3}(1)\,C_{1,3}(2')) \\
A_{484} &= A_{481}\,A_{482a}\,(C_{1,3}(1) + C_{1,3}(2')) + A_{481}\,(C_{1,3}(1)\,C_{1,3}(1')\,C_{1,3}(2') + \\
&\quad C_{1,3}(1)\,C_{1,3}(3)\,C_{1,3}(2'))
\end{aligned}$$

$A_{485} = A_{481}\,A_{482a}\,C_{1,3}(1)\,C_{1,3}(2')$, $A_{486} = A_{481}$
$A_{487} = A_{486}\,(C_{1,3}(1') + C_{1,3}(3) + C_{1,3}(0') + C_{1,3}(2'))$, $A_{487a} = C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s2'}$

$$\begin{aligned}
A_{488} &= A_{486}\,A_{487a} + A_{486}\,(C_{1,3}(3)\,C_{1,3}(1') + C_{1,3}(3)\,C_{1,3}(2') + \\
&\quad C_{1,3}(0')\,C_{1,3}(1') + C_{1,3}(0')\,C_{1,3}(2') + C_{1,3}(3)\,C_{1,3}(0')) \\
A_{489} &= A_{486}\,A_{482a}\,(C_{1,3}(3) + C_{1,3}(0')) + A_{481}\,(C_{1,3}(3)\,C_{1,3}(0')\,C_{1,3}(1') + \\
&\quad C_{1,3}(3)\,C_{1,3}(0')\,C_{1,3}(2'))
\end{aligned}$$

$A_{490} = A_{486}\,A_{487a}\,C_{1,3}(3)\,C_{1,3}(0')$, $A_{491} = 2\,\mu_2\,\lambda_1\,\lambda_s\,\mu_{s1'}$
$A_{492} = A_{491}\,(C_{1,3}(2') + C_{1,3}(3))$, $A_{493} = A_{491}\,C_{1,3}(2')\,C_{1,3}(3)$
$A_{494} = \mu_2\,\mu_3\,\lambda_1^2$, $A_{495} = A_{494}\,(C_{1,3}(0') + C_{1,3}(2'))$

$A_{496} = A_{494}\, C_{1,3}(0')\, C_{1,3}(2'),\;\; A_{497} = A_{459}$

$A_{498} = A_{460} - A_{454} - A_{465} - A_{470} - A_{481} - A_{486}$

$A_{499} = A_{461} - A_{455} - A_{466} - A_{471} - A_{482} - A_{487}$

$A_{500} = A_{462} + A_{451} - A_{456} - A_{467} - A_{472} - A_{475} - A_{478} - A_{483} - A_{488} + A_{491} + A_{494}$

$A_{501} = A_{463} + A_{452} - A_{457} - A_{468} - A_{473} - A_{476} - A_{479} - A_{484} - A_{489} + A_{492} + A_{495}$

$A_{502} = A_{464} + A_{453} - A_{458} - A_{469} - A_{474} - A_{477} - A_{480} - A_{485} - A_{490} + A_{493} + A_{496}$

$A_{503} = \lambda_1^2,\;\; A_{504} = A_{503}\, (C_{1,3}(1') + C_{1,3}(3) + C_{1,3}(2') + C_{1,3}(0'))$

$A_{504a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$$\begin{aligned} A_{505} &= A_{503}\, (A_{504a} + C_{1,3}(3)\, C_{1,3}(1') + C_{1,3}(3)\, C_{1,3}(2') + C_{1,3}(0')\, C_{1,3}(1') + \\ &\quad C_{1,3}(0')\, C_{1,3}(2') + C_{1,3}(3)\, C_{1,3}(0')) \\ A_{506} &= A_{503}\, A_{504a}\, (C_{1,3}(3) + C_{1,3}(0')) + A_{503}\, (C_{1,3}(3)\, C_{1,3}(0')\, C_{1,3}(1') + \\ &\quad C_{1,3}(3)\, C_{1,3}(0')\, C_{1,3}(2')) \end{aligned}$$

$A_{507} = A_{503}\, A_{504a}\, C_{1,3}(3)\, C_{1,3}(0'),\;\; A_{508} = 2\, \lambda_1^2\, \lambda_s\, \mu_{s1'},\;\; A_{509} = A_{508}\, (C_{1,3}(3) + C_{1,3}(2'))$

$A_{510} = A_{508}\, C_{1,3}(3)\, C_{1,3}(2'),\;\; A_{511} = \mu_3\, \lambda_1^3,\;\; A_{512} = A_{511}\, (C_{1,3}(0') + C_{1,3}(2'))$

$A_{513} = A_{511}\, C_{1,3}(0')\, C_{1,3}(2'),\;\; A_{514} = 6\, \lambda_s^2\, \mu_3\, \lambda_1,\;\; A_{515} = A_{514}\, (C_{1,3}(1) + C_{1,3}(2'))$

$A_{516} = A_{514}\, C_{1,3}(1)\, C_{1,3}(2'),\;\; A_{517} = 3\, \lambda_1\, \lambda_s$

$A_{518} = A_{517}\, (C_{1,3}(1') + C_{1,3}(3) + C_{1,3}(2') + C_{1,3}(1)),\;\; A_{518a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$$\begin{aligned} A_{519} &= A_{517}\, (A_{518a} + C_{1,3}(1)\, C_{1,3}(1') + C_{1,3}(1)\, C_{1,3}(2') + \\ &\quad C_{1,3}(3)\, C_{1,3}(1') + C_{1,3}(3)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(3)) \\ A_{520} &= A_{517}\, A_{518a}\, (C_{1,3}(1) + C_{1,3}(3)) + A_{517}\, (C_{1,3}(3)\, C_{1,3}(1)\, C_{1,3}(1') + \\ &\quad C_{1,3}(3)\, C_{1,3}(1)\, C_{1,3}(2')) \end{aligned}$$

$A_{521} = A_{517}\, A_{518a}\, C_{1,3}(1)\, C_{1,3}(3),\;\; A_{522} = 3\, \lambda_1^2\, \lambda_s\, \mu_3,\;\; A_{523} = A_{522}\, (C_{1,3}(1) + C_{1,3}(2'))$

$A_{524} = A_{508}\, C_{1,3}(1)\, C_{1,3}(2'),\;\; A_{525} = A_{503} + A_{517},\;\; A_{526} = A_{504} + A_{518}$

$A_{527} = A_{505} - A_{508} - A_{511} + A_{514} + A_{519} - A_{522}$

$A_{528} = A_{506} - A_{509} - A_{512} + A_{515} + A_{520} - A_{523}$

$A_{529} = A_{507} - A_{510} - A_{513} + A_{516} + A_{521} - A_{524}$

$A_{530} = \lambda_1^3,\;\; A_{531} = A_{530}\, (C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(0')),\;\; A_{531a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$A_{532} = A_{530}\, (A_{531a} + C_{1,3}(0')\, C_{1,3}(1') + C_{1,3}(0')\, C_{1,3}(2'))$

$A_{533} = A_{530}\, A_{531a}\, C_{1,3}(0'),\;\; A_{534} = 2\, \lambda_s\, \lambda_1^3\, \mu_{s1'}$

$A_{535} = A_{534}\, C_{1,3}(2'),\;\; A_{536} = 2\, \mu_2\, \lambda_1^3\, \lambda_s,\;\; A_{537} = A_{536}\, C_{1,3}(2'),\;\; A_{538} = 6\, \lambda_1\, \lambda_s^2$

$A_{533} = A_{530}\, A_{531a}\, C_{1,3}(0'),\;\; A_{534} = 2\, \lambda_s\, \lambda_1^3\, \mu_{s1'}$

$A_{535} = A_{534}\, C_{1,3}(2'),\;\; A_{536} = 2\, \mu_2\, \lambda_1^3\, \lambda_s,\;\; A_{537} = A_{536}\, C_{1,3}(2'),\;\; A_{538} = 6\, \lambda_1\, \lambda_s^2$

$A_{539} = A_{538}\, (C_{1,3}(2) + C_{1,3}(2') + C_{1,3}(1))$

$A_{540} = A_{538}\, (C_{1,3}(2)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(2) + C_{1,3}(1)\, C_{1,3}(2'))$

$A_{541} = A_{538}\, C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(2'),\;\; A_{542} = 3\, \lambda_1^2\, \lambda_s$

$A_{543} = A_{542}\, (C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(1)),\;\; A_{543a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$A_{544} = A_{542}\, (A_{543a} + C_{1,3}(1)\, C_{1,3}(1') + C_{1,3}(1)\, C_{1,3}(2'))$

$A_{545} = A_{542}\, A_{543a}\, C_{1,3}(1),\;\; A_{546} = 6\, \lambda_1^2\, \lambda_s^2\, \mu_2,\;\; A_{547} = A_{546}\, C_{1,3}(2')$

$A_{548} = A_{530} + A_{538} + A_{542},\;\; A_{c49} = A_{531} + A_{539} + A_{543}$

$A_{550} = A_{532} - A_{534} + A_{536} + A_{540} + A_{544} - A_{546}$

$A_{551} = A_{533} - A_{535} + A_{537} + A_{541} + A_{545} - A_{547}$

$A_{552} = \mu_3\, \mu_{s1'}\, \lambda_1^3,\;\; A_{553} = A_{552}\, C_{1,3}(2'),\;\; A_{554} = \mu_2\, \lambda_1^2$

$A_{555} = A_{554}\, (C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(3)),\;\; A_{555a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$A_{556} = A_{554}\, (A_{555a} + C_{1,3}(3)\, C_{1,3}(1') + C_{1,3}(3)\, C_{1,3}(2'))$

$A_{557} = A_{554}\, A_{555a}\, C_{1,3}(3),\;\; A_{558} = \mu_2 \mu_3\, \lambda_1^3,\;\; A_{559} = A_{558}\, C_{1,3}(2'),\;\; A_{560} = 3\, \lambda_s$

$A_{561} = A_{560}\, (C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(1) + C_{1,3}(2) + C_{1,3}(3))$

$A_{561a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$$
\begin{aligned}
A_{562} &= A_{560}\, (A_{561a} + C_{1,3}(2)\, C_{1,3}(1') + C_{1,3}(2)\, C_{1,2}(2') + \\
&\quad C_{1,3}(1')\, C_{1,3}(3) + C_{1,3}(3)\, C_{1,3}(2') + C_{1,3}(2)\, C_{1,3}(3) + \\
&\quad C_{1,3}(1')\, C_{1,3}(1) + C_{1,3}(1)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(2) + \\
&\quad C_{1,3}(1)\, C_{1,3}(3)) \\
A_{563} &= A_{560}\, A_{561a}\, (C_{1,3}(1) + C_{1,3}(2) + C_{1,3}(3)) + A_{560}\, (C_{1,3}(1')\, C_{1,3}(2)\, C_{1,3}(3) + \\
&\quad C_{1,3}(2)\, C_{1,3}(3)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(1') + C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(2) + \\
&\quad C_{1,3}(1')\, C_{1,3}(1)\, C_{1,3}(3) + C_{1,3}(1)\, C_{1,3}(2')\, C_{1,3}(3) + C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3) \\
A_{564} &= A_{560}\, A_{561a}\, (C_{1,3}(2)\, C_{1,3}(3) + C_{1,3}(1)\, C_{1,3}(2) + C_{1,3}(1)\, C_{1,3}(3)) + \\
&\quad A_{560}\, C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3)\, (C_{1,3}(1') + C_{1,3}(2'))
\end{aligned}
$$

$A_{565} = A_{560}\, A_{561a}\, C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(3)),\;\; A_{566} = 3\, \mu_3\, \lambda_1\, \lambda_s$

$A_{567} = A_{566}\, (C_{1,3}(2) + C_{1,3}(2') + C_{1,3}(1))$

$A_{568} = A_{566}\, (C_{1,3}(2)\, C_{1,3}(2') + C_{1,3}(1)\, C_{1,3}(2) + C_{1,3}(1)\, C_{1,3}(2'))$

$A_{569} = A_{538}\, C_{1,3}(1)\, C_{1,3}(2)\, C_{1,3}(2'),\;\; A_{570} = 3\, \lambda_1\, \lambda_s \mu_3$

$A_{571} = A_{570}\, (C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(1)),\;\; A_{571a} = C_{1,3}(1')\, C_{1,3}(2') - \lambda_s\, \mu_{s2'}$

$A_{572} = A_{570}\,(A_{571a} + C_{1,3}(1)\,C_{1,3}(1') + C_{1,3}(1)\,C_{1,3}(2'))$

$A_{573} = A_{570}\,A_{571a}\,C_{1,3}(1),\;\; A_{574} = 3\,\lambda_1\,\lambda_s\,\mu_2,\;\; A_{575} = A_{574}\,(C_{1,3}(1') + C_{1,3}(2') + C_{1,3}(3))$

$A_{575a} = C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s2'}$

$A_{576} = A_{574}\,(A_{575a} + C_{1,3}(3)\,C_{1,3}(1') + C_{1,3}(3)\,C_{1,3}(2'))$

$A_{577} = A_{574}\,A_{575a}\,C_{1,3}(3),\;\; A_{578} = 3\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3,\;\; A_{579} = A_{578}\,C_{1,3}(2')$

$A_{580} = A_{554} + A_{562} - A_{566} - A_{570} - A_{574},\;\; A_{581} = A_{555} + A_{563} - A_{567} - A_{571} - A_{575}$

$A_{582} = A_{552} + A_{556} - A_{564} - A_{568} - A_{572} - A_{576} + A_{578}$

$A_{583} = A_{553} + A_{557} - A_{559} - A_{565} - A_{569} - A_{573} - A_{577} + A_{579}$

$A_{584} = \mu_3\,\lambda_1^3,\;\; A_{585} = A_{584}\,(C_{1,3}(2') + C_{1,3}(0'))$

$A_{586} = A_{584}\,C_{1,3}(2')\,C_{1,3}(0'),\;\; A_{587} = 2\,\mu_2\,\lambda_1^2\,\lambda_s,\;\; A_{588} = A_{584}\,(C_{1,3}(2') + C_{1,3}(3))$

$A_{589} = A_{584}\,C_{1,3}(2')\,C_{1,3}(3),\;\; A_{590} = 6\,\lambda_s^2$

$A_{591} = A_{590}\,(C_{1,3}(2) + C_{1,3}(2') + C_{1,3}(3) + C_{1,3}(1))$

$$\begin{aligned} A_{592} &= A_{590}\,(C_{1,3}(2)\,C_{1,3}(3) + C_{1,3}(2')\,C_{1,3}(2) + C_{1,3}(3)\,C_{1,3}(2') + \\ &\quad C_{1,3}(1)\,C_{1,3}(2) + C_{1,3}(1)\,C_{1,3}(3) + C_{1,3}(1)\,C_{1,3}(2')) \\ A_{593} &= A_{590}\,(C_{1,3}(2)\,C_{1,3}(2')\,C_{1,3}(3) + C_{1,3}(1) + C_{1,3}(2)\,C_{1,3}(3) + \\ &\quad C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(2') + C_{1,3}(1) + C_{1,3}(3)\,C_{1,3}(2')) \end{aligned}$$

$A_{594} = A_{590}\,C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(3),\;\; A_{595} = 6\,\lambda_1\,\lambda_s^2\,\mu_3 - 3\,\lambda_1^2\,\lambda_s\,\mu_3$

$A_{596} = A_{595}\,(C_{1,3}(1) + C_{1,3}(2')),\;\; A_{597} = A_{595}\,C_{1,3}(1)\,C_{1,3}(2')$

$A_{598} = 6\,\lambda_1\,\lambda_s^2\,\mu_2,\;\; A_{599} = A_{598}\,(C_{1,3}(2') + C_{1,3}(3))$

$A_{600} = A_{598}\,C_{1,3}(2')\,C_{1,3}(3),\;\; A_{601} = A_{584} + A_{587} + A_{592} - A_{595} - A_{598}$

$A_{602} = A_{585} + A_{588} + A_{593} - A_{596} - A_{599},\;\; A_{603} = A_{586} + A_{589} + A_{594} - A_{597} - A_{600}$

$A_{604} = 2\,\lambda_s^2\,\lambda_1^2\,\mu_2,\;\; A_{605} = A_{604}\,C_{1,3}(3),\;\; A_{606} = 6\,\lambda_s^3$

$A_{607} = A_{606}\,(C_{1,3}(2) + C_{1,3}(1) + C_{1,3}(3))$

$A_{608} = A_{606}\,(C_{1,3}(2)\,C_{1,3}(3) + C_{1,3}(1)\,C_{1,3}(2) + C_{1,3}(1)\,C_{1,3}(3))$

$A_{609} = A_{606}\,C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(3),\;\; A_{610} = \mu_3\,\lambda_1^3\,\lambda_s$

$A_{611} = A_{610}\,C_{1,3}(0'),\;\; A_{612} = 6\,\lambda_1\,\lambda_s^3\,\mu_3 - 3\,\lambda_1^2\,\lambda_s^2\,\mu_3,\;\; A_{613} = A_{612}\,C_{1,3}(1)$

$A_{614} = 6\,\lambda_1\,\lambda_s^3\,\mu_2,\;\; A_{615} = A_{614}\,C_{1,3}(3),\;\; A_{616} = A_{604} + A_{608} - A_{610} - A_{612} - A_{614}$

$A_{617} = A_{605} + A_{609} - A_{611} - A_{613} - A_{615},\;\; A_{618} = C_{1,3}(0)\,A_{502} - \mu_1\,A_{450} - \mu_{s0'}\,A_{583}$

$$\begin{aligned} A_{619} &= C_{2,3}(0') + C_{2,3}(1) + C_{2,3}(2) + C_{2,3}(3) \\ A_{620} &= C_{2,3}(0')\,C_{2,3}(3) + C_{2,3}(1)\,C_{2,3}(0') + C_{2,3}(1)\,C_{2,3}(3) + \end{aligned}$$

$$C_{2,3}(2)\,C_{2,3}(0')+C_{2,3}(2)\,C_{2,3}(3)+C_{2,3}(1)\,C_{2,3}(2)$$

$A_{621}=C_{2,3}(0')\,C_{2,3}(3)\,(C_{2,3}(1)+C_{2,3}(2))+C_{2,3}(1)\,C_{2,3}(2)\,(C_{2,3}(0')+C_{2,3}(3))$
$A_{622}=C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(3)\,C_{2,3}(0'),\;\; A_{623}=C_{2,3}(1')+C_{2,3}(2')+C_{2,3}(4)$
$A_{624}=C_{2,3}(1')\,C_{2,3}(2')-\lambda_s\,\mu_{s2'}+C_{2,3}(1')\,C_{2,3}(4)+C_{2,3}(2')\,C_{2,3}(4)-2\,\lambda_1\,\mu_4$
$A_{625}=C_{2,3}(1')\,C_{2,3}(2')\,C_{2,3}(4)-\lambda_s\,\mu_{s2'}\,C_{2,3}(4)-2\,\lambda_1\,\mu_4\,C_{2,3}(1')$
$A_{626}=A_{619}+A_{623},\;\; A_{627}=A_{624}+A_{619}\,A_{623}+A_{620}$
$A_{628}=A_{625}+A_{619}\,A_{624}+A_{620}\,A_{623}+A_{621},\;\; A_{629}=A_{619}\,A_{625}+A_{620}\,A_{624}+A_{621}\,A_{623}+$
A_{622}
$A_{630}=A_{620}\,A_{625}+A_{621}\,A_{624}+A_{622}\,A_{623},\;\; A_{631}=A_{621}\,A_{625}+A_{622}\,A_{624}$
$A_{632}=A_{622}\,A_{625},\;\; A_{633}=2\,\lambda_1\,\mu_4,\;\; A_{634}=A_{633}\,(C_{2,3}(1)+C_{2,3}(2)+C_{2,3}(0'))$
$A_{635}=A_{633}\,(C_{2,3}(1)\,C_{2,3}(2)+C_{2,3}(1)\,C_{2,3}(0')+C_{2,3}(2)\,C_{2,3}(0'))$
$A_{636}=A_{633}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(0'),\;\; A_{637}=C_{2,3}(1')+C_{2,3}(2')$
$A_{638}=C_{2,3}(1')\,C_{2,3}(2')-\lambda_s\,\mu_{s2'}+2\,\lambda_1\,\mu_{s2'},\;\; A_{639}=A_{633}\,A_{637}+A_{634}$
$A_{640}=A_{633}\,A_{638}+A_{634}\,A_{637}+A_{635},\;\; A_{641}=A_{634}\,A_{638}+A_{635}\,A_{637}+A_{636}$
$A_{642}=A_{635}\,A_{638}+A_{636}\,A_{637},\;\; A_{643}=A_{636}\,A_{638},\;\; A_{644}=2\,\lambda_1\,\lambda_s\,\mu_3\,\mu_4$
$A_{645}=A_{644}\,(C_{2,3}(1)+C_{2,3}(2)+C_{2,3}(0'))$
$A_{646}=A_{644}\,(C_{2,3}(1)\,C_{2,3}(2)+C_{2,3}(1)\,C_{2,3}(0')+C_{2,3}(2)\,C_{2,3}(0'))$
$A_{647}=A_{644}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(0'),\;\; A_{648}=2\,\lambda_1\,\mu_3$
$A_{649}=A_{648}\,(C_{2,3}(1)+C_{2,3}(2)+C_{2,3}(0'))$
$A_{650}=A_{648}\,(C_{2,3}(1)\,C_{2,3}(2)+C_{2,3}(1)\,C_{2,3}(0')+C_{2,3}(2)\,C_{2,3}(0'))$
$A_{651}=A_{648}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(0'),\;\; A_{652}=C_{2,3}(2')+C_{2,3}(4)$
$A_{653}=C_{2,3}(2')\,C_{2,3}(4)-2\,\lambda_1\,\mu_4$
$A_{654}=A_{648}\,A_{652}+A_{649},\;\; A_{655}=A_{648}\,A_{653}+A_{649}\,A_{652}+A_{650}$
$A_{656}=A_{649}\,A_{653}+A_{650}\,A_{652}+A_{651},\;\; A_{657}=A_{650}\,A_{653}+A_{651}\,A_{652}$
$A_{658}=A_{651}\,A_{653},\;\; A_{659}=2\,\lambda_s\,\mu_{s1'},\;\; A_{660}=A_{659}\,(C_{2,3}(1)+C_{2,3}(2)+C_{2,3}(3))$
$A_{661}=A_{659}\,(C_{2,3}(1)\,C_{2,3}(2)+C_{2,3}(1)\,C_{2,3}(3)+C_{2,3}(2)\,C_{2,3}(3))$
$A_{662}=A_{659}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(3),\;\; A_{663}=C_{2,3}(2')+C_{2,3}(4)$
$A_{664}=C_{2,3}(2')\,C_{2,3}(4)-2\,\lambda_1\,\mu_4,\;\; A_{665}=A_{659}\,A_{663}+A_{662}$
$A_{666}=A_{659}\,A_{664}+A_{660}\,A_{663}+A_{661},\;\; A_{667}=A_{660}\,A_{664}+A_{661}\,A_{663}+A_{662}$
$A_{668}=A_{662}\,A_{663}+A_{661}\,A_{664},\;\; A_{669}=A_{662}\,A_{664},\;\; A_{670}=4\,\lambda_1\,\lambda_s\,\mu_{s1'}\,\mu_4$
$A_{671}=A_{670}\,(C_{2,3}(2')+C_{2,3}(1)+C_{2,3}(2))$

$$A_{672} = A_{670}\,(C_{2,3}(2')\,C_{2,3}(1) + C_{2,3}(2')\,C_{2,3}(2) + C_{2,3}(1)\,C_{2,3}(2))$$
$$A_{673} = A_{670}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(2'),\;\; A_{674} = 2\,\lambda_1\,\mu_3$$
$$A_{675} = A_{674}\,(C_{2,3}(0') + C_{2,3}(1) + C_{2,3}(4))$$
$$A_{676} = A_{674}\,(C_{2,3}(1)\,C_{2,3}(4) + C_{2,3}(0')\,C_{2,3}(1) + C_{2,3}(4)\,C_{2,3}(0'))$$
$$A_{677} = A_{674}\,C_{2,3}(1)\,C_{2,3}(4)\,C_{2,3}(0'),\;\; A_{678} = C_{2,3}(1') + C_{2,3}(2')$$
$$A_{679} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'},\;\; A_{680} = A_{674}\,A_{678} + A_{675}$$
$$A_{681} = A_{674}\,A_{679} + A_{675}\,A_{678} + A_{676},\;\; A_{682} = A_{675}\,A_{679} + A_{676}\,A_{678} + A_{677}$$
$$A_{683} = A_{676}\,A_{679} + A_{677}\,A_{678},\;\; A_{684} = A_{677}\,A_{679},\;\; A_{685} = 4\,\lambda_1\,\lambda_s\,\mu_3\,\mu_{s1'}$$
$$A_{686} = A_{685}\,(C_{2,3}(2') + C_{2,3}(1) + C_{2,3}(4))$$
$$A_{687} = A_{685}\,(C_{2,3}(1)\,C_{2,3}(4) + C_{2,3}(2')\,C_{2,3}(1) + C_{2,3}(4)\,C_{2,3}(2'))$$
$$A_{688} = A_{674}\,C_{2,3}(1)\,C_{2,3}(4)\,C_{2,3}(2'),\;\; A_{689} = 4\,\lambda_1^2\,\mu_3\,\mu_{s1'}$$
$$A_{690} = A_{689}\,(C_{2,3}(2') + C_{2,3}(1) + C_{2,3}(4))$$
$$A_{691} = A_{689}\,(C_{2,3}(1)\,C_{2,3}(4) + C_{2,3}(2')\,C_{2,3}(1) + C_{2,3}(4)\,C_{2,3}(2'))$$
$$A_{692} = A_{689}\,C_{2,3}(1)\,C_{2,3}(4)\,C_{2,3}(2'),\;\; A_{693} = 4\,\lambda_1^2\,\mu_3\,\mu_4$$
$$A_{694} = A_{693}\,C_{2,3}(1),\;\; A_{695} = C_{2,3}(0') + C_{2,3}(1'),\;\; A_{696} = C_{2,3}(0')\,C_{2,3}(1') - 2\,\lambda_s\,\mu_{s1'}$$
$$A_{697} = A_{693}\,A_{695} + A_{694},\;\; A_{698} = A_{693}\,A_{696} + A_{694}\,A_{695},\;\; A_{699} = A_{694}\,A_{696}$$
$$A_{700} = 8\,\lambda_1^3\,\mu_4\,\mu_{s1'}\,(\mu_{s2'} - \mu_3),\;\; A_{701} = A_{700}\,C_{2,3}(1),\;\; A_{702} = 4\,\lambda_1\,\lambda_s^2\,\mu_2\,\mu_3\,\mu_4$$
$$A_{703} = A_{702}\,C_{2,3}(1),\;\; A_{704} = 4\,\mu_2\,\mu_3\,\lambda_1\,\lambda_s,\;\; A_{705} = A_{704}\,C_{2,3}(1)$$
$$A_{706} = C_{2,3}(4) + C_{2,3}(2'),\;\; A_{707} = C_{2,3}(4)\,C_{2,3}(2') - 2\,\lambda_1\,\mu_4,\;\; A_{708} = A_{704}\,A_{706} + A_{705}$$
$$A_{709} = A_{704}\,A_{707} + A_{705}\,A_{706},\;\; A_{710} = A_{705}\,A_{707},\;\; A_{711} = 2\,\lambda_1\,\mu_2$$
$$A_{712} = A_{711}\,(C_{2,3}(1) + C_{2,3}(3) + C_{2,3}(4))$$
$$A_{713} = A_{711}\,(C_{2,3}(1)\,C_{2,3}(2) + C_{2,3}(1)\,C_{2,3}(4) + C_{2,3}(3)\,C_{2,3}(4))$$
$$A_{714} = A_{711}\,C_{2,3}(1)\,C_{2,3}(3)\,C_{2,3}(4),\;\; A_{715} = C_{2,3}(1') + C_{2,3}(2')$$
$$A_{716} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'}$$
$$A_{717} = A_{711}\,A_{715} + A_{712},\;\; A_{718} = A_{711}\,A_{716} + A_{712}\,A_{715} + A_{713}$$
$$A_{719} = A_{712}\,A_{716} + A_{713}\,A_{715} + A_{714},\;\; A_{720} = A_{713}\,A_{716} + A_{714}\,A_{715}$$
$$A_{721} = A_{714}\,A_{716},\;\; A_{722} = 4\,\lambda_1^2\,\mu_2\,\mu_4,\;\; A_{723} = A_{722}\,(C_{2,3}(1) + C_{2,3}(3) + C_{2,3}(1'))$$
$$A_{724} = A_{722}\,(C_{2,3}(1)\,C_{2,3}(1') + C_{2,3}(1')\,C_{2,3}(3) + C_{2,3}(1)\,C_{2,3}(3))$$
$$A_{725} = A_{722}\,C_{2,3}(1)\,C_{2,3}(3)\,C_{2,3}(1'),\;\; A_{726} = A_{722},\;\; A_{727} = A_{726}\,C_{2,3}(1)$$
$$A_{728} = A_{726}\,A_{637} + A_{727},\;\; A_{729} = A_{726}\,A_{638} + A_{727}\,A_{637},\;\; A_{730} = A_{727}\,A_{638}$$
$$A_{731} = 4\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3\,\mu_4,\;\; A_{732} = A_{731}\,C_{2,3}(1),\;\; A_{733} = 4\,\lambda_1^2\,\mu_2\,\mu_3$$
$$A_{734} = A_{733}\,C_{2,3}(1),\;\; A_{735} = C_{2,3}(2') + C_{2,3}(4),\;\; A_{736} = C_{2,3}(2')\,C_{2,3}(4) - 2\,\lambda_1\,\mu_4$$
$$A_{737} = A_{733}\,A_{735} + A_{734},\;\; A_{738} = A_{733}\,A_{736} + A_{734}\,A_{735},\;\; A_{739} = A_{734}\,A_{736}$$

$A_{740} = 2\,\lambda_1\,\mu_2,\;\; A_{741} = A_{740}\,(C_{2,3}(3) + C_{2,3}(4) + C_{2,3}(0'))$

$A_{742} = A_{740}\,(C_{2,3}(3)\,C_{2,3}(0') + C_{2,3}(0')\,C_{2,3}(4) + C_{2,3}(3)\,C_{2,3}(4))$

$A_{743} = A_{740}\,C_{2,3}(3)\,C_{2,3}(4)\,C_{2,3}(0'),\;\; A_{744} = C_{2,3}(1') + C_{2,3}(2')$

$A_{745} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'},\;\; A_{746} = A_{740}\,A_{744} + A_{741}$

$A_{747} = A_{740}\,A_{745} + A_{741}\,A_{744} + A_{742},\;\; A_{748} = A_{741}\,A_{745} + A_{742}\,A_{744} + A_{743}$

$A_{749} = A_{742}\,A_{745} + A_{743}\,A_{744},\;\; A_{750} = A_{743}\,A_{745}$

$A_{751} = 4\,\lambda_1^2\,\mu_2\mu_4,\;\; A_{752} = A_{751}\,(C_{2,3}(3) + C_{2,3}(0') + C_{2,3}(1'))$

$A_{753} = A_{751}\,(C_{2,3}(1')\,C_{2,3}(0') + C_{2,3}(1')\,C_{2,3}(3) + C_{2,3}(3)\,C_{2,3}(0'))$

$A_{754} = A_{751}\,C_{2,3}(3)\,C_{2,3}(0')\,C_{2,3}(1'),\;\; A_{755} = 4\,\lambda_1^2\,\mu_2\,\mu_4$

$A_{756} = A_{755}\,C_{2,3}(0'),\;\; A_{757} = C_{2,3}(1') + C_{2,3}(2')$

$A_{758} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'} + 2\,\lambda_1\,\mu_{s2'},\;\; A_{759} = A_{755}\,A_{757} + A_{756}$

$A_{760} = A_{755}\,A_{758} + A_{756}\,A_{757},\;\; A_{761} = A_{756}\,A_{758},\;\; A_{762} = 4\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3\,\mu_4$

$A_{763} = A_{762}\,C_{2,3}(0'),\;\; A_{764} = 4\,\lambda_1^2\,\mu_2\,\mu_3,\;\; A_{765} = A_{764}\,C_{2,3}(0')$

$A_{766} = C_{2,3}(4) + C_{2,3}(2'),\;\; A_{767} = C_{2,3}(4)\,C_{2,3}(2') - 2\lambda_1\,\mu_4,\;\; A_{768} = A_{764}\,A_{766} + A_{765}$

$A_{769} = A_{764}\,A_{767} + A_{765}\,A_{766},\;\; A_{770} = A_{765}\,A_{767},\;\; A_{771} = 4\,\lambda_1\,\lambda_s\,\mu_2\,\mu_{s1'}$

$A_{772} = A_{771}\,C_{2,3}(3),\;\; A_{773} = C_{2,3}(2') + C_{2,3}(4),\;\; A_{774} = C_{2,3}(2')\,C_{2,3}(4) - 2\,\lambda_1\,\mu_4$

$A_{775} = A_{771}\,A_{773} + A_{772},\;\; A_{776} = A_{771}\,A_{774} + A_{772}\,A_{773},\;\; A_{777} = A_{772}\,A_{774}$

$A_{778} = 8\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_4\,\mu_{s1'},\;\; A_{779} = A_{778}\,C_{2,3}(2')$

$A_{780} = A_{627} - A_{633} - A_{648} - A_{659} - A_{674} - A_{711} - A_{740}$

$A_{781} = A_{628} - A_{639} - A_{654} - A_{665} - A_{680} - A_{717} - A_{746}$

$$\begin{aligned}
A_{782} &= A_{629} - A_{640} - A_{644} - A_{655} - A_{666} + A_{670} - \\
&\quad A_{681} + A_{685} - A_{689} + A_{693} - A_{704} - A_{718} + \\
&\quad A_{722} + A_{726} + A_{733} - A_{747} + A_{751} + A_{755} + A_{764} \\
&\quad + A_{771} \\
A_{783} &= A_{630} - A_{641} - A_{645} - A_{656} - A_{667} + A_{671} - \\
&\quad A_{682} + A_{686} - A_{690} + A_{697} - A_{708} - A_{719} + \\
&\quad A_{723} + A_{728} + A_{737} - A_{748} + A_{752} + A_{759} + \\
&\quad A_{768} + A_{775} \\
A_{784} &= A_{631} - A_{642} - A_{646} - A_{657} - A_{668} + A_{672} - \\
&\quad A_{683} + A_{687} - A_{691} + A_{698} - A_{700} - A_{702} - \\
&\quad A_{709} - A_{720} + A_{724} + A_{729} + A_{731} + A_{738} -
\end{aligned}$$

$$\begin{aligned}
& A_{749} + A_{753} + A_{760} + A_{762} + A_{769} + A_{776} - \\
& A_{778} \\
A_{785} \;=\; & A_{632} - A_{643} - A_{647} - A_{658} - A_{669} + A_{673} - \\
& A_{684} + A_{688} - A_{692} + A_{699} - A_{701} - A_{703} - \\
& A_{710} - A_{721} + A_{725} + A_{730} + A_{732} + A_{739} - \\
& A_{750} + A_{754} + A_{761} + A_{763} + A_{770} + A_{777} - \\
& A_{779}
\end{aligned}$$

$A_{786} = 2\,\lambda_1,\;\; A_{787} = A_{786}\,(C_{2,3}(0') + C_{2,3}(2) + C_{2,3}(3))$
$A_{788} = A_{786}\,(C_{2,3}(0')\,C_{2,3}(2) + C_{2,3}(0')\,C_{2,3}(3) + C_{2,3}(2)\,C_{2,3}(3))$
$A_{789} = A_{786}\,(C_{2,3}(0')\,C_{2,3}(2)\,C_{2,3}(3)),\;\; A_{790} = C_{2,3}(4) + C_{2,3}(1') + C_{2,3}(2')$
$A_{791} = C_{2,3}(4)\,C_{2,3}(1') + C_{2,3}(4)\,C_{2,3}(2') + C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'} - 2\,\lambda_1\,\mu_4$
$A_{792} = C_{2,3}(4)\,C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'}\,C_{2,3}(4) - 2\,\lambda_1\,\mu_4\,C_{2,3}(1')$
$A_{793} = A_{786}\,A_{790} + A_{787},\;\; A_{794} = A_{786}\,A_{791} + A_{787}\,A_{790} + A_{788}$
$A_{795} = A_{786}\,A_{792} + A_{787}\,A_{791} + A_{788}\,A_{790} + A_{789}$
$A_{796} = A_{787}\,A_{792} + A_{788}\,A_{791} + A_{789}\,A_{790},\;\; A_{797} = A_{788}\,A_{792} + A_{789}\,A_{791}$
$A_{798} = A_{789}\,A_{792},\;\; A_{799} = 4\,\lambda_1^2\,\mu_4,\;\; A_{800} = A_{799}\,(C_{2,3}(2) + C_{2,3}(0'))$
$A_{801} = A_{799}\,(C_{2,3}(2)\,C_{2,3}(0')),\;\; A_{802} = C_{2,3}(1') + C_{2,3}(2')$
$A_{803} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'} + 2\,\lambda_1\,\mu_{s2'}$
$A_{804} = A_{799}\,A_{802} + A_{800},\;\; A_{805} = A_{799}\,A_{803} + A_{800}\,A_{802} + A_{801}$
$A_{806} = A_{800}\,A_{803} + A_{801}\,A_{802},\;\; A_{807} = A_{801}\,A_{803},\;\; A_{808} = 4\,\lambda_1^2\,\mu_3\,\mu_4\,\lambda_s$
$A_{809} = A_{808}\,(C_{2,3}(2) + C_{2,3}(0')),\;\; A_{810} = A_{808}\,(C_{2,3}(2)\,C_{2,3}(0')),\;\; A_{811} = 4\,\lambda_1^3\,\mu_3$
$A_{812} = A_{811}\,(C_{2,3}(2) + C_{2,3}(0')),\;\; A_{813} = A_{808}\,C_{2,3}(2)\,C_{2,3}(0'),\;\; A_{814} = C_{2,3}(4) + C_{2,3}(2')$
$A_{815} = C_{2,3}(4)\,C_{2,3}(2') - 2\,\lambda_1\,\mu_4,\;\; A_{816} = A_{811}\,A_{814} + A_{812}$
$A_{817} = A_{811}\,A_{815} + A_{812}\,A_{814} + A_{813},\;\; A_{818} = A_{812}\,A_{815} + A_{813}\,A_{814}$
$A_{819} = A_{813}\,A_{815},\;\; A_{820} = 4\,\lambda_1\,\lambda_s\mu_{s1'}$
$A_{821} = A_{820}\,(C_{2,3}(2) + C_{2,3}(3)),\;\; A_{822} = A_{808}\,C_{2,3}(2)\,C_{2,3}(3),\;\; A_{823} = C_{2,3}(4) + C_{2,3}(2')$
$A_{824} = C_{2,3}(4)\,C_{2,3}(2') - 2\,\lambda_1\,\mu_4,\;\; A_{825} = A_{820}\,A_{823} + A_{821}$
$A_{826} = A_{820}\,A_{824} + A_{821}\,A_{823} + A_{822}$
$A_{827} = A_{821}\,A_{824} + A_{822}\,A_{823},\;\; A_{828} = A_{822}\,A_{824},\;\; A_{829} = 8\,\lambda_1^2\,\mu_4\,\lambda_s\,\mu_{s1'}$
$A_{830} = A_{829}\,(C_{2,3}(2) + C_{2,3}(2')),\;\; A_{831} = A_{829}\,C_{2,3}(2)\,C_{2,3}(2'),\;\; A_{832} = 4\,\lambda_1^2\,\mu_3$

$A_{833} = A_{832}\,(C_{2,3}(4)+C_{2,3}(0')),\;\; A_{834} = A_{832}\,C_{2,3}(4)\,C_{2,3}(0'),\;\; A_{835} = C_{2,3}(1')+C_{2,3}(2')$

$A_{836} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'},\;\; A_{837} = A_{832}\,A_{835} + A_{833}$

$A_{838} = A_{832}\,A_{836} + A_{833}\,A_{835} + A_{834}$

$A_{839} = A_{833}\,A_{836} + A_{834}\,A_{835},\;\; A_{840} = A_{834}\,A_{836},\;\; A_{841} = 8\,\lambda_1^2\,\mu_3\,\lambda_s\,\mu_{s1'}$

$A_{842} = A_{841}\,(C_{2,3}(4) + C_{2,3}(2')),\;\; A_{843} = A_{841}\,C_{2,3}(4)\,C_{2,3}(2'),\;\; A_{844} = 8\,\lambda_1^3\,\mu_3\,\mu_4$

$A_{845} = A_{844}\,(C_{2,3}(0') + C_{2,3}(1'))$

$A_{846} = A_{844}\,(C_{2,3}(0')\,C_{2,3}(1') - 2\,\lambda_s\,\mu_{s1'})$

$A_{847} = 16\,\lambda_1^4\,\mu_4\,\mu_{s1'}\,\mu_{s2'},\;\; A_{848} = 8\,\lambda_1^3\,\mu_3\,\mu_{s1'}$

$A_{849} = A_{848}\,A_{823},\;\; A_{850} = A_{848}\,A_{824}$

$A_{851} = 8\,\lambda_1^2\,\mu_2\,\mu_3\,\mu_4\,\lambda_s^2,\;\; A_{852} = 8\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3,\;\; A_{853} = A_{852}\,A_{823},\;\; A_{854} = A_{852}\,A_{824}$

$A_{855} = 4\,\lambda_1^2\,\mu_2,\;\; A_{856} = A_{855}\,(C_{2,3}(3) + C_{2,3}(4)),\;\; A_{857} = A_{855}\,C_{2,3}(3)\,C_{2,3}(4)$

$A_{858} = A_{855}\,A_{835} + A_{856},\;\; A_{859} = A_{855}\,A_{836} + A_{856}\,A_{835} + A_{857}$

$A_{860} = A_{856}\,A_{836} + A_{857}\,A_{835},\;\; A_{861} = A_{857}\,A_{836}$

$A_{862} = 8\,\lambda_1^3\,\mu_2\,\mu_4,\;\; A_{863} = A_{862}\,(C_{2,3}(3) + C_{2,3}(1'))$

$A_{864} = A_{862}\,C_{2,3}(3)\,C_{2,3}(1'),\;\; A_{865} = 8\,\lambda_1^3\,\mu_2\,\mu_4,\;\; A_{866} = A_{865}\,A_{835},\;\; A_{867} = A_{865}\,A_{836}$

$A_{868} = 16\,\lambda_1^4\,\mu_2\,\mu_4\,\mu_{s2'},\;\; A_{869} = 8\,\lambda_1^3\,\mu_2\,\mu_3\,\mu_4\,\lambda_s,\;\; A_{870} = 8\,\lambda_1^3\,\mu_2\,\mu_3,\;\; A_{871} = A_{870}\,A_{823}$

$A_{872} = A_{870}\,A_{824},\;\; A_{873} = 12\,\lambda_1\,\mu_2\,\mu_3\,\mu_4\,\lambda_s^3,\;\; A_{874} = 12\,\lambda_s^2\,\lambda_1\,\mu_2\,\mu_3,\;\; A_{875} = A_{874}\,A_{823}$

$A_{876} = A_{874}\,A_{824},\;\; A_{877} = 6\,\lambda_1\,\lambda_s\,\mu_2,\;\; A_{878} = A_{877}\,C_{2,3}(3);\;\; A_{879} = A_{877}\,A_{790} + A_{878}$

$A_{880} = A_{877}\,A_{791} + A_{790}\,A_{878},\;\; A_{881} = A_{877}\,A_{792} + A_{791}\,A_{878},\;\; A_{882} = A_{878}\,A_{792}$

$A_{883} = 12\,\lambda_1^2\lambda_s\,\mu_2\,\mu_4,\;\; A_{884} = A_{883}\,A_{835},\;\; A_{885} = A_{883}\,A_{836}$

$A_{886} = 24\,\lambda_1^3\,\lambda_s\,\mu_2\,\mu_4\,\mu_{s2'},\;\; A_{887} = 12\,\lambda_1^2\,\mu_2\,\mu_3\,\mu_4\,\lambda_s^2,\;\; A_{888} = 12\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_3$

$A_{889} = A_{888}\,A_{823},\;\; A_{890} = A_{888}\,A_{824}$

$A_{891} = A_{794} - A_{799} - A_{811} - A_{820} - A_{832} - A_{855} - A_{877}$

$A_{892} = A_{795} - A_{804} - A_{816} - A_{825} - A_{837} - A_{858} - A_{879}$

$$\begin{aligned} A_{893} &= A_{796} - A_{805} - A_{808} - A_{817} - A_{826} - A_{829} - \\ &\quad A_{838} + A_{841} + A_{844} - A_{848} - A_{852} - A_{859} + \\ &\quad A_{862} + A_{865} + A_{870} + A_{874} - A_{880} + A_{883} + \\ &\quad A_{888} \end{aligned}$$

$$\begin{aligned} A_{894} &= A_{797} - A_{806} - A_{809} - A_{818} - A_{827} + A_{830} - \\ &\quad A_{839} + A_{842} + A_{845} - A_{849} - A_{853} - A_{860} + \\ &\quad A_{863} + A_{866} + A_{871} + A_{875} - A_{881} + A_{884} + \end{aligned}$$

$$A_{872} + A_{873} + A_{876} - A_{882} + A_{885} + A_{886} +$$
$$A_{887} + A_{890}$$

$A_{896} = 4\,\lambda_1^2,\;\; A_{897} = A_{896}\,(C_{2,3}(0') + C_{2,3}(3)),$
$A_{898} = A_{896}\,C_{2,3}(0')\,C_{2,3}(3),\;\; A_{899} = C_{2,3}(4) + C_{2,3}(1') + C_{2,3}(2')$
$A_{900} = C_{2,3}(4)\,C_{2,3}(1') + C_{2,3}(4)\,C_{2,3}(2') + C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'} - 2\,\lambda_1$
$A_{901} = C_{2,3}(4)\,(C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'}) - 2\,\lambda_1\,C_{2,3}(1')$
$A_{902} = A_{896}\,A_{899} + A_{897},\;\; A_{903} = A_{896}\,A_{900} + A_{897}\,A_{899} + A_{898}$
$A_{904} = A_{896}\,A_{901} + A_{897}\,A_{900} + A_{898}\,A_{899},\;\; A_{905} = A_{897}\,A_{901} + A_{898}\,A_{900}$
$A_{906} = A_{898}\,A_{901},\;\; A_{907} = 8\,\lambda_1^3\,\mu_4,\;\; A_{908} = A_{907}\,C_{2,3}(0')$
$A_{909} = C_{2,3}(1') + C_{2,3}(2'),\;\; A_{910} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'} + 2\,\lambda_1\,\mu_{s2'}$
$A_{911} = A_{907}\,A_{909} + A_{908},\;\; A_{912} = A_{907}\,A_{910} + A_{908}\,A_{909},\;\; A_{913} = A_{908}\,A_{910}$
$A_{914} = 8\,\lambda_1^3\,\lambda_s\,\mu_3\,\mu_4,\;\; A_{915} = A_{914}\,C_{2,3}(0'),\;\; A_{916} = 8\,\lambda_1^3\,\mu_3$
$A_{917} = A_{916}\,C_{2,3}(0'),\;\; A_{918} = C_{2,3}(2') + C_{2,3}(4),\;\; A_{919} = C_{2,3}(4)\,C_{2,3}(2') - 2\,\lambda_1\,\mu_4$
$A_{920} = A_{916}\,A_{918} + A_{917},\;\; A_{921} = A_{916}\,A_{919} + A_{917}\,A_{918},\;\; A_{922} = A_{917}\,A_{919}$
$A_{923} = 8\,\lambda_1^2\,\lambda_s\,\mu_{s1'},\;\; A_{924} = A_{923}\,C_{2,3}(3),\;\; A_{925} = A_{923}\,A_{918} + A_{924}$
$A_{926} = A_{923}\,A_{919} + A_{924}\,A_{918},\;\; A_{927} = A_{924}\,A_{919},\;\; A_{928} = 16\,\lambda_1^3\,\lambda_s\,\mu_4\,\mu_{s1'}$
$A_{929} = A_{928}\,C_{2,3}(2'),\;\; A_{930} = 12\,\lambda_1\,\lambda_s^3\,\mu_3\,\mu_4,\;\; A_{931} = A_{930}\,C_{2,3}(1),\;\; A_{932} = 12\,\lambda_s^2\,\lambda_1\,\mu_3$
$A_{933} = A_{932}\,C_{2,3}(1),\;\; A_{934} = A_{932}\,A_{918} + A_{933},\;\; A_{935} = A_{932}\,A_{919} + A_{933}\,A_{918}$
$A_{936} = A_{933}\,A_{919},\;\; A_{937} = 6\,\lambda_1\,\lambda_s,\;\; A_{938} = A_{937}\,(C_{2,3}(1) + C_{2,3}(3) + C_{2,3}(4))$
$A_{939} = A_{937}\,(C_{2,3}(1)\,C_{2,3}(3) + C_{2,3}(1)\,C_{2,3}(4) + C_{2,3}(2)\,C_{2,3}(4))$
$A_{940} = A_{937}\,C_{2,3}(1)\,C_{2,3}(2)\,C_{2,3}(4),\;\; A_{941} = C_{2,3}(1') + C_{2,3}(2')$
$A_{942} = C_{2,3}(1')\,C_{2,3}(2') - \lambda_s\,\mu_{s2'},\;\; A_{943} = A_{937}\,A_{941} + A_{938}$
$A_{944} = A_{937}\,A_{942} + A_{938}\,A_{941} + A_{939},\;\; A_{945} = A_{938}\,A_{942} + A_{939}\,A_{941} + A_{940}$
$A_{946} = A_{939}\,A_{942} + A_{940}\,A_{941},\;\; A_{947} = A_{940}\,A_{942}$
$A_{948} = 12\,\lambda_1^2\,\lambda_s\,\mu_4,\;\; A_{949} = A_{948}\,(C_{2,3}(1) + C_{2,3}(3) + C_{2,3}(1'))$
$A_{950} = A_{948}\,(C_{2,3}(1)\,C_{2,3}(3) + C_{2,3}(1)\;C_{2,3}(1') + C_{2,3}(3)\,C_{2,3}(1'))$
$A_{951} = A_{948}\,C_{2,3}(1)\,C_{2,3}(3)\,C_{2,3}(1'),\;\; A_{952} = 12\,\lambda_1^2\,\lambda_s\,\mu_4,\;\; A_{953} = A_{952}\,C_{2,3}(1)$
$A_{954} = A_{952}\,A_{941} + A_{953},\;\; A_{955} = A_{952}\,A_{942} + A_{953}\,A_{941},\;\; A_{956} = A_{953}\,A_{942}$
$A_{957} = 24\,\lambda_1^3\,\lambda_s\,\mu_4\,\mu_{s2'},\;\; A_{958} = A_{957}\,C_{2,3}(1),\;\; A_{959} = 12\,\lambda_1^2\,\lambda_s^2\,\mu_3\,\mu_1,\;\; A_{960} = A_{959}\,C_{2,3}(1)$
$A_{961} = 12\,\lambda_1^2\,\lambda_s\,\mu_3,\;\; A_{962} = A_{961}\,C_{2,3}(1),\;\; A_{963} = A_{961}\,A_{918} + A_{962}$
$A_{964} = A_{961}\,A_{919} + A_{962}\,A_{918},\;\; A_{965} = A_{962}\,A_{919}$
$A_{966} = A_{896} + A_{937},\;\; A_{967} = A_{902} + A_{947}$

$A_{957} = 24\,\lambda_1^3\,\lambda_s\,\mu_4\,\mu_{s2'}$, $A_{958} = A_{957}\,C_{2,3}(1)$, $A_{959} = 12\,\lambda_1^2\,\lambda_s^2\,\mu_3\,\mu_4$, $A_{960} = A_{959}\,C_{2,3}(1)$

$A_{961} = 12\,\lambda_1^2\,\lambda_s\,\mu_3$, $A_{962} = A_{961}\,C_{2,3}(1)$, $A_{963} = A_{961}\,A_{918} + A_{962}$

$A_{964} = A_{961}\,A_{919} + A_{962}\,A_{918}$, $A_{965} = A_{962}\,A_{919}$

$A_{966} = A_{896} + A_{937}$, $A_{967} = A_{902} + A_{947}$

$A_{968} = A_{903} - A_{907} - A_{916} - A_{923} + A_{932} + A_{944} - A_{948} - A_{952} - A_{961}$

$A_{969} = A_{904} - A_{911} - A_{920} - A_{925} + A_{934} + A_{945} - A_{949} - A_{954} - A_{963}$

$$\begin{aligned} A_{970} &= A_{905} - A_{912} - A_{914} - A_{921} - A_{926} + A_{928} + \\ &\quad A_{930} + A_{935} + A_{946} - A_{950} - A_{955} - A_{957} - \\ &\quad A_{959} - A_{964} \\ A_{971} &= A_{906} - A_{913} - A_{915} - A_{922} - A_{927} + A_{929} + \\ &\quad A_{931} + A_{936} + A_{947} - A_{951} - A_{956} - A_{958} - \\ &\quad A_{960} - A_{965} \end{aligned}$$

$A_{972} = 8\,\lambda_1^3$, $A_{973} = A_{972}\,(C_{2,3}(4) + C_{2,3}(0'))$, $A_{974} = A_{972}\,C_{2,3}(4)\,C_{2,3}(0')$

$A_{975} = A_{972}\,A_{941} + A_{973}$, $A_{976} = A_{972}\,A_{942} + A_{973}\,A_{941} + A_{974}$

$A_{977} = A_{973}\,A_{942} + A_{974}\,A_{941}$, $A_{978} = A_{974}\,A_{942}$, $A_{979} = 16\,\lambda_1^3\,\lambda_s\,\mu_{s1'}$

$A_{980} = A_{979}\,(C_{2,3}(4) + C_{2,3}(2'))$, $A_{981} = A_{979}\,C_{2,3}(4)\,C_{2,3}(2')$, $A_{982} = 16\,\lambda_1^4\,\mu_4$

$A_{983} = A_{982}\,(C_{2,3}(0') + C_{2,3}(1'))$, $A_{984} = A_{982}\,(C_{2,3}(0')\,C_{2,3}(1') - 2\,\lambda_s\,\mu_{s1'})$, $A_{985} =$
$12\,\lambda_1\,\lambda_s^3\,\mu_4$

$A_{986} = A_{985}\,(C_{2,3}(1) + C_{2,3}(2))$, $A_{987} = 12\,\lambda_1\,\lambda_s^3\,\mu_4\,C_{2,3}(1)\,C_{2,3}(2) + 16\,\lambda_1^3\,\lambda_s^2\,\mu_2\,\mu_4$

$A_{988} = 12\,\lambda_1\,\lambda_s^2$, $A_{989} = A_{988}\,(C_{2,3}(1) + C_{2,3}(2))$

$A_{990} = 12\,\lambda_1\,\lambda_s^2\,C_{2,3}(1)\,C_{2,3}(2) + 16\,\lambda_1^3\,\lambda_s\,\mu_2$, $A_{991} = A_{918}\,A_{988} + A_{989}$

$A_{992} = A_{919}\,A_{988} + A_{919}\,A_{989} + A_{990}$, $A_{993} = A_{919}\,A_{989} + A_{918}\,A_{990}$

$A_{994} = A_{919}\,A_{990}$, $A_{995} = 12\,\lambda_s\,\lambda_1^2$, $A_{996} = A_{995}\,(C_{2,3}(1) + C_{2,3}(4))$

$A_{997} = 12\,\lambda_s\,\lambda_1^2\,C_{2,3}(1)\,C_{2,3}(4)$, $A_{998} = A_{995}\,A_{941} + A_{996}$

$A_{999} = A_{995}\,A_{942} + A_{996}\,A_{941} + A_{997}$, $A_{1000} = A_{996}\,A_{942} + A_{997}\,A_{941}$

$A_{1001} = A_{997}\,A_{942}$, $A_{1002} = 24\,\lambda_1^3\,\lambda_s\,\mu_4$, $A_{1003} = A_{1002}\,(C_{2,3}(1) + C_{2,3}(1'))$

$A_{1004} = A_{1002}\,C_{2,3}(1)\,C_{2,3}(1')$, $A_{1005} = 24\,\lambda_1^2\,\lambda_s^3\,\mu_2\,\mu_4$

$A_{1006} = 24\,\lambda_1^2\,\lambda_s^2\,\mu_2$, $A_{1007} = A_{1006}\,A_{918}$, $A_{1008} = A_{1006}\,A_{919}$

$A_{1009} = A_{972} + A_{988} + A_{995}$, $A_{1010} = A_{975} + A_{991} + A_{998}$

$A_{1011} = A_{976} - A_{979} - A_{982} + A_{985} + A_{992} + A_{999} - A_{1002} - A_{1006}$

$A_{1012} = A_{977} - A_{980} - A_{983} + A_{986} + A_{993} + A_{1000} - A_{1003} - A_{1007}$

$A_{1013} = A_{978} - A_{981} - A_{984} + A_{987} + A_{994} + A_{1001} - A_{1004} - A_{1005} - A_{1008}$

$A_{1014} = 16\,\lambda_1^4,\;\; A_{1015} = A_{1014}\, C_{2,3}(0'),\;\; A_{1016} = A_{1014}\, A_{941} + A_{1015}$

$A_{1017} = A_{1014}\, A_{942} + A_{1015}\, A_{941},\;\; A_{1018} = A_{1015}\, A_{942},\;\; A_{1019} = 32\,\lambda_1^4\,\lambda_s\,\mu_{s1'}$

$A_{1020} = A_{1019}\, C_{2,3}(2'),\;\; A_{1021} = 16\,\lambda_1^4\,\lambda_s\,\mu_3,\;\; A_{1022} = A_{1021}\, C_{2,3}(0')$

$A_{1023} = 6\,\lambda_s^2,\;\; A_{1024} = A_{1023}\,(C_{2,3}(1) + C_{2,3}(2))$

$A_{1025} = 8\,\lambda_1^2\,\lambda_s\,\mu_2 + 6\,\lambda_s^2\, C_{2,3}(1)\, C_{2,3}(2) - 12\,\lambda_1\,\lambda_s^2\,\mu_2,\;\; A_{1026} = 2\,\lambda_1\,\lambda_s$

$A_{1027} = A_{1026}\, C_{2,3}(3),\;\; A_{1028} = A_{1023}\, A_{1026},\;\; A_{1029} = A_{1023}\, A_{1027} + A_{1024}\, A_{1026}$

$A_{1030} = A_{1024}\, A_{1027} + A_{1025}\, A_{1026},\;\; A_{1031} = A_{1025}\, A_{1027},\;\; A_{1032} = 4\,\lambda_1^2$

$A_{1033} = A_{1032}\, C_{2,3}(2'),\;\; A_{1034} = A_{1023}\, A_{1032},\;\; A_{1035} = A_{1023}\, A_{1033} + A_{1024}\, A_{1032}$

$A_{1036} = A_{1024}\, A_{1033} + A_{1025}\, A_{1032},\;\; A_{1037} = A_{1025}\, A_{1033},\;\; A_{1038} = A_{1014} + A_{1028} + A_{1034}$

$A_{1039} = A_{1016} + A_{1029} + A_{1035},\;\; A_{1040} = A_{1017} - A_{1019} + A_{1021} + A_{1030} + A_{1036}$

$A_{1041} = A_{1018} - A_{1020} + A_{1022} + A_{1031} + A_{1037},\;\; A_{1042} = 16\,\lambda_1^4\,\mu_4\,\mu_{s1'}\,\mu_{s2'}$

$A_{1043} = 8\,\lambda_1^3\,\mu_3\,\mu_{s1'},\;\; A_{1044} = A_{1043}\, A_{918},\;\; A_{1045} = A_{1043}\, A_{919},\;\; A_{1046} = 3\,\lambda_s$

$A_{1047} = A_{1046}\,(C_{2,3}(1) + C_{2,3}(2)),\;\; A_{1048} = A_{1046}\, C_{2,3}(1)\, C_{2,3}(2) - 6\,\lambda_1\,\lambda_s\,\mu_2 + 4\,\lambda_1^2\,\mu_2$

$A_{1049} = C_{2,3}(3) + C_{2,3}(4),\;\; A_{1050} = C_{2,3}(3)\, C_{2,3}(4)$

$A_{1051} = A_{1046}\, A_{941} + A_{1046}\, A_{1049} + A_{1047}$

$A_{1052} = A_{1046}\, A_{942} + A_{1046}\, A_{1049}\, A_{941} + A_{1046}\, A_{1050} + A_{1047}\, A_{941} + A_{1047}\, A_{1049} + A_{1048}$

$$\begin{aligned} A_{1053} &= A_{1046}\, A_{1049}\, A_{942} + A_{1050}\, A_{941}\, A_{1046} + A_{1047}\, A_{942} + A_{1047}\, A_{1049}\, A_{941} + \\ & \quad A_{1047}\, A_{1050} + A_{1048}\, A_{941} + A_{1048}\, A_{1049} \\ A_{1054} &= A_{1050}\, A_{942}\, A_{1046} + A_{1049}\, A_{1047}\, A_{942} + A_{1047}\, A_{1050}\, A_{941} + A_{1048}\, A_{942} + \\ & \quad A_{1048}\, A_{1049}\, A_{941} + A_{1048}\, A_{1050} \end{aligned}$$

$A_{1055} = A_{1050}\, A_{942}\, A_{1047} + A_{1048}\, A_{1049}\, A_{942} + A_{1048}\, A_{1050}\, A_{941}$

$A_{1056} = A_{1048}\, A_{1050}\, A_{942},\;\; A_{1057} = 2\,\lambda_1\,\mu_4,\;\; A_{1058} = A_{1057}\,(C_{2,3}(3) + C_{2,3}(1'))$

$A_{1059} = A_{1057}\, C_{2,3}(3)\, C_{2,3}(1'),\;\; A_{1060} = A_{1046}\, A_{1057},\;\; A_{1061} = A_{1046}\, A_{1058} + A_{1047}\, A_{1057}$

$A_{1062} = A_{1046}\, A_{1059} + A_{1047}\, A_{1058} + A_{1048}\, A_{1057},\;\; A_{1063} = A_{1047}\, A_{1059} + A_{1048}\, A_{1058}$

$A_{1064} = A_{1048}\, A_{1059},\;\; A_{1065} = A_{1057}\, A_{1046},\;\; A_{1066} = A_{1057}\, A_{1047},\;\; A_{1067} = A_{1057}\, A_{1048}$

$A_{1068} = A_{1065}\, A_{941} + A_{1066},\;\; A_{1069} = A_{1065}\, A_{942} + A_{1066}\, A_{941} + A_{1067}$

$A_{1070} = A_{1066}\, A_{942} + A_{1067}\, A_{941},\;\; A_{1071} = A_{1067}\, A_{942}$

$A_{1071a} = 2\,\lambda_1\,\mu_{s2'},\;\; A_{1072} = A_{1071a}\, A_{1065},\;\; A_{1073} = A_{1071a}\, A_{1066}$

$A_{1074} = A_{1071a}\, A_{1067},\;\; A_{1074a} = 2\,\lambda_1\,\lambda_s\,\mu_3\,\mu_4\, A_{1046},\;\; A_{1075} = A_{1074a}\, A_{1046}$

$A_{1076} = A_{1074a}\, A_{1047},\;\; A_{1077} = A_{1074a}\, A_{1048},\;\; A_{1077a} = 2\,\lambda_1\,\mu_3$

$A_{1078} = A_{1077a}\, A_{1046},\;\; A_{1079} = A_{1077a}\, A_{1047},\;\; A_{1080} = A_{1077a}\, A_{1048}$

$$A_{1081} = A_{1078}\,A_{918} + A_{1079},\;\; A_{1082} = A_{1078}\,A_{919} + A_{1079}\,A_{918} + A_{1080}$$
$$A_{1083} = A_{1079}\,A_{919} + A_{1080}\,A_{918},\;\; A_{1084} = A_{1080}\,A_{919},\;\; A_{1085} = 6\,\lambda_1\,\lambda_s\,\mu_3$$
$$A_{1086} = A_{1085}\,(C_{2,3}(1) + C_{2,3}(4)),\;\; A_{1087} = A_{1085}\,C_{2,3}(1)\,C_{2,3}(4)$$
$$A_{1088} = A_{1085}\,A_{941} + A_{1086},\;\; A_{1089} = A_{1085}\,A_{942} + A_{1086}\,A_{941} + A_{1087}$$
$$A_{1090} = A_{1086}\,A_{942} + A_{1087}\,A_{941},\;\; A_{1091} = A_{1087}\,A_{942},\;\; A_{1092} = 12\,\lambda_1^2\,\lambda_s\,\mu_3\,\mu_4$$
$$A_{1093} = A_{1092}\,(C_{2,3}(1) + C_{2,3}(1')),\;\; A_{1094} = A_{1092}\,C_{2,3}(1)\,C_{2,3}(1')$$
$$A_{1095} = A_{1052} - A_{1060} - A_{1065} - A_{1078} - A_{1085}$$
$$A_{1096} = A_{1053} - A_{1061} - A_{1068} - A_{1081} - A_{1088}$$
$$A_{1097} = A_{1043} + A_{1054} - A_{1062} - A_{1069} - A_{1072} - A_{1075} - A_{1082} - A_{1089} + A_{1092}$$
$$A_{1098} = A_{1044} + A_{1055} - A_{1063} - A_{1070} - A_{1073} - A_{1076} - A_{1083} - A_{1090} + A_{1093}$$
$$A_{1099} = A_{1042} + A_{1045} + A_{1056} - A_{1064} - A_{1071} - A_{1074} - A_{1077} - A_{1084} - A_{1091} + A_{1094}$$
$$A_{1100} = 16\,\lambda_1^4\,\mu_{s2'}\,\mu_4,\;\; A_{1101} = A_{1100}\,C_{2,3}(0'),\;\; A_{1102} = 8\,\lambda_1^3\,\mu_3,\;\; A_{1103} = A_{1102}\,C_{2,3}(0')$$
$$A_{1104} = A_{1102}\,A_{918} + A_{1103},\;\; A_{1105} = A_{1102}\,A_{919} + A_{1103}\,A_{918},\;\; A_{1106} = A_{1103}\,A_{919}$$
$$A_{1107} = 12\,\lambda_1\,\lambda_s^2\,\mu_3,\;\; A_{1108} = A_{1107}\,C_{2,3}(1),\;\; A_{1109} = A_{1107}\,A_{918} + A_{1108}$$
$$A_{1110} = A_{1107}\,A_{919} + A_{1108}\,A_{918},\;\; A_{1111} = A_{1108}\,A_{919},\;\; A_{1112} = 24\,\lambda_1^3\,\lambda_s\,\mu_4\,\mu_{s2'}$$
$$A_{1113} = A_{1112}\,C_{2,3}(1),\;\; A_{1114} = 12\,\lambda_1^2\,\lambda_s\,\mu_3$$
$$A_{1115} = A_{1114}\,C_{2,3}(1),\;\; A_{1116} = A_{1114}\,A_{918} + A_{1115}$$
$$A_{1117} = A_{1114}\,A_{919} + A_{1115}\,A_{918},\;\; A_{1118} = A_{1114}\,A_{919}$$
$$A_{1119} = 6\,\lambda_s^2,\;\; A_{1120} = A_{1119}\,(C_{2,3}(1) + C_{2,3}(2))$$
$$A_{1121} = A_{1119}\,C_{2,3}(1)\,C_{2,3}(2) + 8\,\lambda_1^2\,\lambda_s\,\mu_2 - 12\,\lambda_1\,\lambda_s^2\,\mu_2,\;\; A_{1122} = A_{918} + C_{2,3}(3)$$
$$A_{1123} = A_{919} + A_{918}\,C_{2,3}(3),\;\; A_{1124} = A_{919}\,C_{2,3}(3),\;\; A_{1125} = A_{1119}\,A_{1122} + A_{1120}$$
$$A_{1126} = A_{1119}\,A_{1123} + A_{1120}\,A_{1122} + A_{1121}$$
$$A_{1127} = A_{1119}\,A_{1124} + A_{1120}\,A_{1123} + A_{1121}\,A_{1122}$$
$$A_{1128} = A_{1120}\,A_{1124} + A_{1121}\,A_{1123},\;\; A_{1129} = A_{1121}\,A_{1124},\;\; A_{1130} = 2\,\lambda_1\,\mu_4$$
$$A_{1131} = A_{1130}\,C_{2,3}(2'),\;\; A_{1132} = A_{1119}\,A_{1130},\;\; A_{1133} = A_{1119}\,A_{1131} + A_{1120}\,A_{1130}$$
$$A_{1134} = A_{1120}\,A_{1131} + A_{1121}\,A_{1130},\;\; A_{1135} = A_{1121}\,A_{1131}$$
$$A_{1136} = A_{1102} - A_{1107} + A_{1114} + A_{1126} - A_{1132}$$
$$A_{1137} = A_{1104} - A_{1109} + A_{1116} + A_{1127} - A_{1133}$$
$$A_{1138} = A_{1100} + A_{1105} - A_{1110} + A_{1112} + A_{1117} + A_{1128} - A_{1134}$$
$$A_{1139} = A_{1101} + A_{1106} - A_{1111} + A_{1113} + A_{1118} + A_{1129} - A_{1135}$$
$$A_{1140} = 16\,\lambda_1^4\,\mu_4,\;\; A_{1141} = A_{1140}\,(C_{2,3}(0') + C_{2,3}(1'))$$
$$A_{1142} = A_{1140}\,(C_{2,3}(0')\,C_{2,3}(1') - 2\,\lambda_s\,\mu_{s1'}),\;\; A_{1143} = 8\,\lambda_1^3\,\lambda_s\,\mu_3$$
$$A_{1144} = A_{1143}\,(C_{2,3}(4) + C_{2,3}(0')),\;\; A_{1145} = A_{1144}\,C_{2,3}(4)\,C_{2,3}(0')$$

$A_{1146} = 12\,\lambda_1\,\lambda_s^3\,\mu_3,\ \ A_{1147} = A_{1146}\,(C_{2,3}(1) + C_{2,3}(4)),\ \ A_{1148} = A_{1146}\,C_{2,3}(1)\,C_{2,3}(4)$

$A_{1149} = 24\,\lambda_1^3\,\lambda_s\,\mu_4,\ \ A_{1150} = A_{1149}\,(C_{2,3}(1) + C_{2,3}(1')),\ \ A_{1151} = A_{1149}\,C_{2,3}(1)\,C_{2,3}(1')$

$A_{1152} = 12\,\lambda_1^2\,\lambda_s^2\,\mu_3,\ \ A_{1153} = A_{1152}\,(C_{2,3}(1) + C_{2,3}(4)),\ \ A_{1154} = A_{1152}\,C_{2,3}(1)\,C_{2,3}(4)$

$A_{1155} = \lambda_s,\ \ A_{1156} = A_{1155}\,(C_{2,3}(3) + C_{2,3}(4)),\ \ A_{1157} = A_{1155}\,C_{2,3}(3)\,C_{2,3}(4)$

$A_{1158} = 6\,\lambda_s^2,\ \ A_{1159} = A_{1158}\,(C_{2,3}(1) + C_{2,3}(2))$

$A_{1160} = A_{1158}\,(C_{2,3}(1)\,C_{2,3}(2) + 8\,\lambda_1^2\lambda_s\,\mu_2 - 12\,\lambda_1\,\lambda_s^2\,\mu_2)$

$A_{1161} = A_{1155}\,A_{1158},\ \ A_{1162} = A_{1155}\,A_{1159} + A_{1156}\,A_{1158}$

$A_{1163} = A_{1155}\,A_{1160} + A_{1156}\,A_{1159} + A_{1157}\,A_{1158},\ \ A_{1164} = A_{1156}\,A_{1160} + A_{1157}\,A_{1159}$

$A_{1165} = A_{1157}\,A_{1160},\ \ A_{1166} = 2\,\lambda_1\,\lambda_s\,\mu_4 - 4\,\lambda_1^2\,\mu_4,\ \ A_{1167} = A_{1166}\,A_{1158}$

$A_{1168} = A_{1166}\,A_{1159},\ \ A_{1169} = A_{1166}\,A_{1160}$

$A_{1170} = A_{1140} + A_{1143} - A_{1146} + A_{1149} + A_{1152} + A_{1163} - A_{1167}$

$A_{1171} = A_{1141} + A_{1144} - A_{1147} + A_{1150} + A_{1153} + A_{1164} - A_{1168}$

$A_{1172} = A_{1142} + A_{1145} - A_{1148} + A_{1151} + A_{1154} + A_{1165} - A_{1169}$

$A_{1173} = C_{2,3}(0)\,A_{785} - \mu_1\,A_{895} - \mu_{s0'}\,A_{1099},\ \ A_{1174} = 2\,\lambda_1\,\lambda_s\,\mu_{s2'}\,\mu_5$

$A_{1175} = A_{1174}\,C_{3,3}(1),\ \ A_{1176} = C_{3,3}(2) + C_{3,3}(3) + C_{3,3}(0')$

$A_{1177} = C_{3,3}(2)\,C_{3,3}(3) + C_{3,3}(0')\,C_{3,3}(3) + C_{3,3}(0')\,C_{3,3}(2) - 3\,\lambda_1\,\mu_2 - 3\,\lambda_1\,\mu_3$

$A_{1178} = C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(0') - 3\,\lambda_1\,\mu_2\,C_{3,3}(3) - 3\,\lambda_1\,\mu_3\,C_{3,3}(0')$

$A_{1179} = A_{1174}\,A_{1176} + A_{1175},\ \ A_{1180} = A_{1174}\,A_{1177} + A_{1175}\,A_{1176}$

$A_{1181} = A_{1174}\,A_{1178} + A_{1175}\,A_{1177},\ \ A_{1182} = A_{1175}\,A_{1178}$

$A_{1183} = -2\,\lambda_1\,\mu_5,\ \ A_{1184} = A_{1183}\,(C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(2'))$

$$\begin{aligned} A_{1185} &= 6\,\lambda_1^2\,\mu_2\,\mu_5 - 2\,\lambda_1\,\mu_5\,C_{3,3}(1)\,C_{3,3}(2) - 2\,\lambda_1\,\mu_5\,C_{3,3}(1)\,C_{3,3}(2') - \\ &\quad 2\,\lambda_1\,\mu_5\,C_{3,3}(2)\,C_{3,3}(2') \\ A_{1186} &= 6\,\lambda_1^2\,\mu_2\,\mu_5\,C_{3,3}(2') - 2\,\lambda_1\,\mu_5\,C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(2') \end{aligned}$$

$A_{1187} = C_{3,3}(3) + C_{3,3}(0') + C_{3,3}(1')$

$A_{1188} = C_{3,3}(3)\,C_{3,3}(0') + C_{3,3}(3)\,C_{3,3}(1') + C_{3,3}(0')\,C_{3,3}(1') - 3\,\lambda_1\,\mu_3$

$A_{1189} = C_{3,3}(3)\,C_{3,3}(0')\,C_{3,3}(1') - 2\,\lambda_s\,\mu_{s1'} - 3\,\lambda_1\,\mu_3\,C_{3,3}(0')$

$A_{1190} = A_{1183}\,A_{1187} + A_{1184},\ \ A_{1191} = A_{1183}\,A_{1188} + A_{1184}\,A_{1187} + A_{1185}$

$A_{1192} = A_{1183}\,A_{1189} + A_{1184}\,A_{1188} + A_{1185}\,A_{1187} + A_{1186}$

$A_{1193} = A_{1184}\,A_{1189} + A_{1185}\,A_{1188} + A_{1186}\,A_{1187},\ \ A_{1194} = A_{1185}\,A_{1189} + A_{1186}\,A_{1188}$

$A_{1195} = A_{1186}\,A_{1189},\ \ A_{1196} = -6\,\lambda_1^2\,\mu_5,\ \ A_{1197} = A_{1196}\,(C_{3,3}(1) + C_{3,3}(2'))$

$$A_{1198} = A_{1196}\, C_{3,3}(1)\, C_{3,3}(2'), \quad A_{1199} = -\mu_3, \quad A_{1200} = A_{1199}\,(C_{3,3}(0') + C_{3,3}(1'))$$

$$A_{1201} = A_{1199}\,(C_{3,3}(0')\, C_{3,3}(1') - 2\,\lambda_s\,\mu_{s1'}) - 3\,\lambda_1\,\mu_{s1'}\,\mu_3, \quad A_{1202} = A_{1196}\, A_{1199}$$

$$A_{1203} = A_{1196}\, A_{1200} + A_{1197}\, A_{1199}, \quad A_{1204} = A_{1196}\, A_{1201} + A_{1197}\, A_{1200} + A_{1198}\, A_{1199}$$

$$A_{1205} = A_{1197}\, A_{1201} + A_{1198}\, A_{1200}, \quad A_{1206} = A_{1198}\, A_{1201}, \quad A_{1207} = 2\,\lambda_1\,\mu_2\,\mu_5$$

$$A_{1208} = A_{1207}\,(C_{3,3}(1) + C_{3,3}(2')), \quad A_{1209} = A_{1207}\, C_{3,3}(1)\, C_{3,3}(2'), \quad A_{1210} = 3\,\lambda_1$$

$$A_{1211} = A_{1210}\,(C_{3,3}(3) + C_{3,3}(1'))$$

$$A_{1212} = A_{1211}\,(C_{3,3}(3)\, C_{3,3}(1') - 3\,\lambda_1\,\mu_3) + 6\,\lambda_1\,\lambda_s\,\mu_3$$

$$A_{1213} = A_{1207}\, A_{1210}, \quad A_{1214} = A_{1207}\, A_{1211} + A_{1208}\, A_{1210}$$

$$A_{1215} = A_{1207}\, A_{1212} + A_{1208}\, A_{1211} + A_{1209}\, A_{1210}, \quad A_{1216} = A_{1208}\, A_{1212} + A_{1209}\, A_{1211}$$

$$A_{1217} = A_{1209}\, A_{1212}, \quad A_{1218} = C_{3,3}(3) + C_{3,3}(5)$$

$$A_{1219} = C_{3,3}(3)\, C_{3,3}(5), \quad A_{1220} = C_{3,3}(0') + C_{3,3}(1) + C_{3,3}(2)$$

$$A_{1221} = C_{3,3}(0')\, C_{3,3}(1') + C_{3,3}(0')\, C_{3,3}(2) + C_{3,3}(1)\, C_{3,3}(2) - 6\,\lambda_1\,\mu_2$$

$$A_{1222} = C_{3,3}(0')\, C_{3,3}(1)\, C_{3,3}(2) - 3\,\lambda_1\,\mu_2\, C_{3,3}(0') - 3\,\lambda_1\,\mu_2\, C_{3,3}(1)$$

$$A_{1223} = A_{1220}\, A_{1218}, \quad A_{1224} = A_{1221} + A_{1218}\, A_{1220} + A_{1219}$$

$$A_{1225} = A_{1222}\, A_{1218} + A_{1221}\, A_{1219}\, A_{1220}, \quad A_{1226} = A_{1218}\, A_{1222} + A_{1219}\, A_{1221}$$

$$A_{1227} = A_{1219}\, A_{1222}, \quad A_{1228} = C_{3,3}(4) + C_{3,3}(1') + C_{3,3}(2')$$

$$A_{1229} = C_{3,3}(4)\, C_{3,3}(1') + C_{3,3}(4)\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - 3\,\lambda_1\,\mu_4 - \lambda_s\,\mu_{s2'}$$

$$A_{1230} = C_{3,3}(4)\, C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\,\mu_{s2'}\, C_{3,3}(4) - 3\,\lambda_1\,\mu_4\, C_{3,3}(1')$$

$$A_{1231} = A_{1223} + A_{1228}, \quad A_{1232} = A_{1229} + A_{1223}\, A_{1228} + A_{1224}$$

$$A_{1233} = A_{1230} + A_{1223}\, A_{1229} + A_{1224}\, A_{1228} + A_{1225}$$

$$A_{1234} = A_{1223}\, A_{1230} + A_{1224}\, A_{1229} + A_{1225}\, A_{1228} + A_{1226}$$

$$A_{1235} = A_{1224}\, A_{1230} + A_{1225}\, A_{1229} + A_{1226}\, A_{1228} + A_{1227}$$

$$A_{1236} = A_{1225}\, A_{1230} + A_{1226}\, A_{1229} + A_{1227}\, A_{1228}, \quad A_{1237} = A_{1226}\, A_{1230} + A_{1227}\, A_{1229}$$

$$A_{1238} = A_{1227}\, A_{1230}, \quad A_{1239} = 3\,\lambda_1\, C_{3,3}(0'), \quad A_{1240} = C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5)$$

$$A_{1241} = C_{3,3}(1)\, C_{3,3}(2) + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(2)\, C_{3,3}(5), \quad A_{1242} = C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(5)$$

$$A_{1243} = -18\,\lambda_1^2\,\mu_2, \quad A_{1244} = -9\,\lambda_1^2\,\mu_2\,(C_{3,3}(1) + C_{3,3}(0') + 2\, C_{3,3}(5))$$

$$A_{1245} = -9\,\lambda_1^2\,\mu_2\, C_{3,3}(5)\,(C_{3,3}(1) + C_{3,3}(0')), \quad A_{1246} = A_{1210}\, A_{1240}$$

$$A_{1247} = A_{1210} + A_{1241}\, A_{1239} + A_{1240} + A_{1243}, \quad A_{1248} = A_{1210}\, A_{1242} + A_{1239}\, A_{1241}\, A_{1244}$$

$$A_{1249} = A_{1210}\, A_{1242} + A_{1245}, \quad A_{1250} = -\mu_4, \quad A_{1251} = A_{1250}\,(C_{3,3}(1') + C_{3,3}(2'))$$

$$A_{1252} = A_{1250}\,(C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\,\mu_{s2'}) - 3\,\lambda_1\,\mu_4\,\mu_{s2'}, \quad A_{1253} = A_{1210}\, A_{1250}$$

$$A_{1254} = A_{1210}\, A_{1251} + A_{1246}\, A_{1250}, \quad A_{1255} = A_{1210}\, A_{1252} + A_{1246}\, A_{1251} + A_{1247}\, A_{1250}$$

$$A_{1256} = A_{1247}\, A_{1251} + A_{1248}\, A_{1250} + A_{1246}\, A_{1252}$$

$$A_{1257} = A_{1247}\, A_{1252} + A_{1248}\, A_{1251} + A_{1249}\, A_{1250}$$

$A_{1258} = A_{1248}\, A_{1252} + A_{1249}\, A_{1251}, \;\; A_{1259} = A_{1249}\, A_{1252}, \;\; A_{1260} = -\mu_3$

$A_{1261} = A_{1260}\, (C_{3,3}(2) + C_{3,3}(5) + C_{3,3}(0') + C_{3,3}(1))$

$$\begin{aligned} A_{1262} &= A_{1260}\, (C_{3,3}(2)\, C_{3,3}(5) + C_{3,3}(0')\, C_{3,3}(2) + C_{3,3}(0')\, C_{3,3}(5) + \\ &\quad C_{3,3}(1)\, C_{3,3}(2) + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(0')\, C_{3,3}(1')) + \\ &\quad 6\, \lambda_1\, \mu_2\, \mu_3 - 2\, \lambda_s\, \mu_2\, \mu_3 \\ A_{1263} &= A_{1260}\, C_{3,3}(2)\, C_{3,3}(5)\, (C_{3,3}(0') + C_{3,3}(1)) + \\ &\quad A_{1260}\, C_{3,3}(1)\, C_{3,3}(0')\, (C_{3,3}(2) + C_{3,3}(5)) + \\ &\quad 3\, \lambda_1\, \mu_2\, \mu_3\, (C_{3,3}(1) + C_{3,3}(0') + 2\, C_{3,3}(5)) - \\ &\quad 2\, \lambda_s\, \mu_2\, \mu_3\, (C_{3,3}(1) + C_{3,3}(5)) \\ A_{1264} &= A_{1260}\, C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(5)\, C_{3,3}(0') + 3\, \lambda_1\, \mu_2\, \mu_3\, C_{3,3}(1)\, C_{3,3}(5) + \\ &\quad 3\, \lambda_1\, \mu_2\, \mu_3\, C_{3,3}(5)\, C_{3,3}(0') - 2\, \lambda_s\, \mu_2\, \mu_3\, C_{3,3}(1)\, C_{3,3}(5) \end{aligned}$$

$A_{1265} = 3\, \lambda_1, \;\; A_{1266} = A_{1265}\, (C_{3,3}(4) + C_{3,3}(2'))$

$A_{1267} = A_{1265}\, (C_{3,3}(4)\, C_{3,3}(2') + \lambda_s\, \mu_4 - 3\, \lambda_1\, \mu_4), \;\; A_{1268} = A_{1260}\, A_{1265}$

$A_{1269} = A_{1260}\, A_{1266} + A_{1261}\, A_{1265}, \;\; A_{1270} = A_{1260}\, A_{1267} + A_{1261}\, A_{1266} + A_{1262}\, A_{1265}$

$A_{1271} = A_{1261}\, A_{1267} + A_{1262}\, A_{1266} + A_{1263}\, A_{1265}$

$A_{1272} = A_{1262}\, A_{1267} + A_{1263}\, A_{1266} + A_{1264}\, A_{1265}$

$A_{1273} = A_{1263}\, A_{1267} + A_{1264}\, A_{1266}, \;\; A_{1274} = A_{1264}\, A_{1267}$

$A_{1275} = -2\, \lambda_s\, \mu_{s1'}, \;\; A_{1276} = A_{1275}\, (C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5))$

$A_{1277} = A_{1275}\, C_{3,3}(1)\, (C_{3,3}(2) + C_{3,3}(5)) + A_{1275}\, C_{3,3}(2)\, C_{3,3}(5) + 6\, \lambda_1\, \lambda_s\, \mu_2\, \mu_{s1'}$

$A_{1278} = A_{1275}\, C_{3,3}(1)\, C_{3,3}(2)\, C_{3,3}(5) + 6\, \lambda_1\, \lambda_s\, \mu_2\, \mu_{s1'}\, C_{3,3}(5)$

$A_{1279} = C_{3,3}(3) + C_{3,3}(4) + C_{3,3}(2')$

$A_{1280} = C_{3,3}(3)\, C_{3,3}(4) + C_{3,3}(3)\, C_{3,3}(2') + C_{3,3}(4)\, C_{3,3}(2') - 3\, \lambda_1\, \mu_4$

$A_{1281} = C_{3,3}(3)\, C_{3,3}(4)\, C_{3,3}(2') - 3\, \lambda_1\, \mu_4\, C_{3,3}(3) - 3\, \lambda_1\, \mu_4\, C_{3,3}(2')$

$A_{1282} = A_{1275}\, A_{1279} + A_{1276}, \;\; A_{1283} = A_{1275}\, A_{1280} + A_{1276}\, A_{1279} + A_{1277}$

$A_{1284} = A_{1275}\, A_{1281} + A_{1276}\, A_{1280} + A_{1277}\, A_{1279} + A_{1278}$

$A_{1285} = A_{1276}\, A_{1281} + A_{1277}\, A_{1280} + A_{1278}\, A_{1279}$

$A_{1286} = A_{1277}\, A_{1281} + A_{1278}\, A_{1280}, \;\; A_{1287} = A_{1278}\, A_{1281}, \;\; A_{1288} = -3\, \lambda_1\, \mu_3$

$A_{1289} = A_{1288}\, (C_{3,3}(1) + C_{3,3}(4) + C_{3,3}(5))$

$A_{1290} = A_{1288}\, (C_{3,3}(1)\, C_{3,3}(4) + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(4)\, C_{3,3}(5))$

$A_{1291} = A_{1288}\, C_{3,3}(1)\, C_{3,3}(4)\, C_{3,3}(5), \;\; A_{1292} = C_{3,3}(0') + C_{3,3}(1') + C_{3,3}(2')$

$A_{1293} = C_{3,3}(0')\, C_{3,3}(1') + C_{3,3}(0')\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - 2\, \lambda_s\, \mu_{s1'} - \lambda_s\, \mu_{s2'}$

$A_{1294} = C_{3,3}(0')\,C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}\;C_{3,3}(0') - 2\,\lambda_s\,\mu_{s1'}\,C_{3,3}(2')$

$A_{1295} = A_{1288}\,A_{1292} + A_{1289},\;\; A_{1296} = A_{1288}\,A_{1293} + A_{1289}\,A_{1292} + A_{1290}$

$A_{1297} = A_{1288}\,A_{1294} + A_{1289}\,A_{1293} + A_{1290}\,A_{1292} + A_{1291}$

$A_{1298} = A_{1289}\,A_{1294} + A_{1290}\,A_{1293} + A_{1291}\,A_{1292},\;\; A_{1299} = A_{1290}\,A_{1294} + A_{1291}\,A_{1293}$

$A_{1300} = A_{1291}\,A_{1294},\;\; A_{1301} = -3\,\lambda_1\,\mu_3\,\mu_4,\;\; A_{1302} = A_{1301}\,(C_{3,3}(1) + C_{3,3}(5))$

$A_{1303} = A_{1301}\,C_{3,3}(1)\,C_{3,3}(5),\;\; A_{1304} = -3\,\lambda_1,\;\; A_{1305} = A_{1304}\,(C_{3,3}(0') + C_{3,3}(1'))$

$A_{1306} = A_{1304}\,(C_{3,3}(0')\,C_{3,3}(1') - 2\,\lambda_s\,\mu_{s1'}),\;\; A_{1307} = A_{1301}\,A_{1304}$

$A_{1308} = A_{1301}\,A_{1305} + A_{1302}\,A_{1304},\;\; A_{1309} = A_{1301}\,A_{1306} + A_{1302}\,A_{1305} + A_{1303}\,A_{1304}$

$A_{1310} = A_{1302}\,A_{1306} + A_{1303}\,A_{1305},\;\; A_{1311} = A_{1303}\,A_{1306},\;\; A_{1312} = -9\,\lambda_1^2\,\mu_{s1'}$

$A_{1313} = A_{1312}\,(C_{3,3}(1) + C_{3,3}(5)),\;\; A_{1314} = A_{1312}\,C_{3,3}(1)\,C_{3,3}(5),\;\; A_{1315} = \mu_3$

$A_{1316} = A_{1315}\,(C_{3,3}(4)+C_{3,3}(2')),\;\; A_{1317} = A_{1315}\,(C_{3,3}(4)\,C_{3,3}(2')-3\,\lambda_1\,\mu_4)+3\,\lambda_1\,\mu_4\,\mu_{s2'}$

$A_{1318} = A_{1312}\,A_{1315},\;\; A_{1319} = A_{1312}\,A_{1316} + A_{1313}\,A_{1315}$

$A_{1320} = A_{1312}\,A_{1317} + A_{1313}\,A_{1316} + A_{1314}\,A_{1315},\;\; A_{1321} = A_{1313}\,A_{1317} + A_{1314}\,A_{1316}$

$A_{1322} = A_{1314}\,A_{1317},\;\; A_{1323} = 6\,\lambda_1^2\,\lambda_s\,\mu_2\,\mu_5,\;\; A_{1324} = -\mu_{s2'}$

$A_{1325} = A_{1324}\,(C_{3,3}(0') + C_{3,3}(3)),\;\; A_{1326} = A_{1324}\,C_{3,3}(3)\,C_{3,3}(0'),\;\; A_{1327} = A_{1323}\,A_{1324}$

$A_{1328} = A_{1323}\,A_{1325},\;\; A_{1329} = A_{1323}\,A_{1326}$

$A_{1330} = A_{1183} + A_{1232} + A_{1253} + A_{1268} + A_{1275} + A_{1288}$

$A_{1331} = A_{1190} + A_{1233} + A_{1254} + A_{1269} + A_{1282} + A_{1295}$

$$\begin{aligned}
A_{1332} &= A_{1174} + A_{1191} + A_{1202} + A_{1213} + A_{1234} + A_{1255} + \\
&\quad A_{1270} + A_{1283} + A_{1296} + A_{1307} + A_{1318} \\
A_{1333} &= A_{1179} + A_{1192} + A_{1203} + A_{1214} + A_{1235} + A_{1256} + \\
&\quad A_{1271} + A_{1284} + A_{1297} + A_{1308} + A_{1319} \\
A_{1334} &= A_{1180} + A_{1193} + A_{1204} + A_{1215} + A_{1236} + A_{1257} + \\
&\quad A_{1272} + A_{1285} + A_{1298} + A_{1309} + A_{1320} + A_{1327} \\
A_{1335} &= A_{1181} + A_{1194} + A_{1205} + A_{1216} + A_{1237} + A_{1258} + \\
&\quad A_{1273} + A_{1286} + A_{1299} + A_{1310} + A_{1321} + A_{1328} \\
A_{1336} &= A_{1182} + A_{1195} + A_{1206} + A_{1217} + A_{1238} + A_{1259} + \\
&\quad A_{1274} + A_{1287} + A_{1300} + A_{1311} + A_{1322} + A_{1329}
\end{aligned}$$

$A_{1337} = 6\,\lambda_1^2\,\lambda_s\,\mu_5\,\mu_{s2'},\;\; A_{1338} = C_{3,3}(2) + C_{3,3}(0')$

$A_{1339} = C_{3,3}(2)\,C_{3,3}(0') - 3\,\lambda_1\,\mu_2,\;\; A_{1340} = C_{3,3}(3) + A_{1338}$

$A_{1341} = -3\,\lambda_1\,\mu_3 + C_{3,3}(3)\,A_{1338} + A_{1339},\;\; A_{1342} = -3\,\lambda_1\,\mu_3\,C_{3,3}(0') + C_{3,3}(3)\,A_{1339}$

$A_{1343} = A_{1337}\,A_{1340},\;\; A_{1344} = A_{1337}\,A_{1341},\;\; A_{1345} = A_{1337}\,A_{1342},\;\; A_{1346} = -6\,\lambda_1^2\,\mu_5$

$A_{1347} = A_{1346}\,C_{3,3}(2'),\;\; A_{1348} = C_{3,3}(0') + C_{3,3}(1'),\;\; A_{1349} = C_{3,3}(0')\,C_{3,3}(1') - 2\,\lambda_s\mu_{s1'}$

$A_{1350} = C_{3,3}(3) + C_{3,3}(1'),\;\; A_{1351} = C_{3,3}(3)\,C_{3,3}(1') - 3\,\lambda_1\,\mu_3$

$A_{1352} = C_{3,3}(2) + C_{3,3}(3),\;\; A_{1353} = C_{3,3}(2)\,C_{3,3}(3)$

$A_{1354} = A_{1348} + A_{1352},\;\; A_{1355} = A_{1349} + A_{1348}\,A_{1352} + A_{1353} - 6\,\lambda_1\,\mu_3 - 3\,\lambda_1\,\mu_2$

$$\begin{aligned}
A_{1356} &= A_{1349}\,A_{1352} + A_{1348}\,A_{1353} - 3\,\lambda_1\,\mu_3\,C_{3,3}(0') - 3\,\lambda_1\,\mu_3\,C_{3,3}(2) - \\
&\quad 3\,\lambda_1\,\mu_3\,A_{1348} - 3\,\lambda_1\,\mu_2\,A_{1350} \\
A_{1357} &= A_{1349}\,A_{1353} - 3\,\lambda_1\,\mu_3\,(C_{3,3}(0')\,C_{3,3}(2) + A_{1349} + 3\,\lambda_1\,\mu_{s1'} + \\
&\quad 2\,\lambda_s\,\mu_2) - 3\,\lambda_1\,\mu_2\,A_{1351}
\end{aligned}$$

$A_{1358} = A_{1346}\,A_{1354} + A_{1347},\;\; A_{1359} = A_{1346}\,A_{1355} + A_{1347}\,A_{1354}$

$A_{1360} = A_{1346}\,A_{1356} + A_{1347}\,A_{1355},\;\; A_{1361} = A_{1346}\,A_{1357} + A_{1347}\,A_{1356}$

$A_{1362} = A_{1347}\,A_{1357},\;\; A_{1363} = 3\,\lambda_1,\;\; A_{1364} = A_{1363}\,(C_{3,3}(2) + C_{3,3}(5) + C_{3,3}(0'))$

$A_{1365} = A_{1363}\,(C_{3,3}(2)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(0') + C_{3,3}(5)\,C_{3,3}(0'))$

$A_{1366} = A_{1363}\,C_{3,3}(2)\,C_{3,3}(5)\,C_{3,3}(0')$

$A_{1367} = C_{3,3}(3) + C_{3,3}(4),\;\; A_{1368} = C_{3,3}(3)\,C_{3,3}(4)$

$A_{1369} = C_{3,3}(1') + C_{3,3}(2'),\;\; A_{1370} = C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'},\;\; A_{1371} = -3\,\lambda_1\,\mu_4$

$A_{1372} = A_{1371}\,(C_{3,3}(3) + C_{3,3}(1')),\;\; A_{1373} = A_{1371}\,C_{3,3}(3)\,C_{3,3}(1'),\;\; A_{1374} = A_{1371}$

$A_{1375} = A_{1374}\,(C_{3,3}(1') + C_{3,3}(2')),\;\; A_{1376} = A_{1374}\,(C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'})$

$A_{1377} = -3\,\lambda_1\,\mu_4\,(3\,\lambda_1\,\mu_{s2'} - \lambda_s\,\mu_3),\;\; A_{1378} = -3\,\lambda_1\,\mu_3$

$A_{1379} = A_{1378}\,(C_{3,3}(4) + C_{3,3}(2')),\;\; A_{1380} = A_{1378}\,(C_{3,3}(4)\,C_{3,3}(2') + A_{1374})$

$A_{1381} = A_{1367} + A_{1369},\;\; A_{1382} = A_{1370} + A_{1367}\,A_{1369} + A_{1368} + A_{1371} + A_{1374} + A_{1378}$

$A_{1383} = A_{1367}\,A_{1370} + A_{1368}\,A_{1369} + A_{1372} + A_{1375} + A_{1379}$

$A_{1384} = A_{1368}\,A_{1370} + A_{1373} + A_{1376} + A_{1377} + A_{1380}$

$A_{1385} = A_{1363}\,A_{1381} + A_{1364},\;\; A_{1386} = A_{1382}\,A_{1363} + A_{1364}\,A_{1381} + A_{1365}$

$A_{1387} = A_{1363}\,A_{1383} + A_{1364}\,A_{1382} + A_{1365} + A_{1381} + A_{1366}$

$A_{1388} = A_{1363}\,A_{1384} + A_{1364}\,A_{1383} + A_{1365}\,A_{1382} + A_{1366}\,A_{1381}$

$A_{1389} = A_{1364}\,A_{1384} + A_{1365}\,A_{1383} + A_{1366}\,A_{1382},\;\; A_{1390} = A_{1365}\,A_{1384} + A_{1366}\,A_{1383}$

$A_{1391} = A_{1366}\,A_{1384},\;\; A_{1392} = 6\,\lambda_1\,\lambda_s,\;\; A_{1393} = A_{1392}\,(C_{3,3}(2) + C_{3,3}(5))$

$A_{1394} = A_{1392}\,C_{3,3}(2)\,C_{3,3}(5),\;\; A_{1395} = -\mu_{s1'},\;\; A_{1396} = A_{1395}\,C_{3,3}(3)$

$A_{1397} = C_{3,3}(4) + C_{3,3}(2'),\;\; A_{1398} = C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4,\;\; A_{1399} = 3\,\lambda_1\,\mu_4\,\mu_{s1'}$

$A_{1400} = A_{1399}\, C_{3,3}(2'),\;\; A_{1401} = A_{1395}\, A_{1397} + A_{1396}$

$A_{1402} = A_{1395}\, A_{1398} + A_{1396}\, A_{1397} + A_{1399},\;\; A_{1403} = A_{1396}\, A_{1398} + A_{1400}$

$A_{1404} = A_{1392}\, A_{1395},\;\; A_{1405} = A_{1392}\, A_{1401} + A_{1393}\, A_{1395}$

$A_{1406} = A_{1392}\, A_{1402} + A_{1393}\, A_{1401} + A_{1394}\, A_{1395}$

$A_{1407} = A_{1392}\, A_{1403} + A_{1393}\, A_{1402} + A_{1394}\, A_{1401}$

$A_{1408} = A_{1393}\, A_{1403} + A_{1394}\, A_{1402},\;\; A_{1409} = A_{1394}\, A_{1403},\;\; A_{1410} = 9\, \lambda_1^2$

$A_{1411} = A_{1410}\, (C_{3,3}(4) + C_{3,3}(5)),\;\; A_{1412} = A_{1410}\, C_{3,3}(4)\, C_{3,3}(5)$

$A_{1413} = -\mu_3,\;\; A_{1414} = A_{1413}\, C_{3,3}(0'),\;\; A_{1415} = C_{3,3}(1') + C_{3,3}(2')$

$A_{1416} = C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'},\;\; A_{1417} = 2\, \lambda_s\, \mu_3\, \mu_{s1'}$

$A_{1418} = A_{1417}\, C_{3,3}(2'),\;\; A_{1419} = -3\, \lambda_1\, \mu_3\, \mu_{s1'},\;\; A_{1420} = A_{1419}\, C_{3,3}(2')$

$A_{1421} = A_{1413}\, A_{1415} + A_{1414},\;\; A_{1422} = A_{1413}\, A_{1416} + A_{1414}\, A_{1415} + A_{1417} + A_{1419}$

$A_{1423} = A_{1414}\, A_{1416} + A_{1418} + A_{1420},\;\; A_{1424} = A_{1410}\, A_{1413}$

$A_{1425} = A_{1410}\, A_{1421} + A_{1411}\, A_{1413},\;\; A_{1426} = A_{1410}\, A_{1422} + A_{1411}\, A_{1421} + A_{1412}\, A_{1413}$

$A_{1427} = A_{1410}\, A_{1423} + A_{1411}\, A_{1422} + A_{1412}\, A_{1421},\;\; A_{1428} = A_{1411}\, A_{1423} + A_{1412}\, A_{1422}$

$A_{1429} = A_{1412}\, A_{1423},\;\; A_{1430} = 9\, \lambda_1^2\, \mu_4,\;\; A_{1431} = A_{1430}\, C_{3,3}(5)$

$A_{1432} = 3\, \lambda_1\, \mu_3,\;\; A_{1433} = A_{1432}\, (C_{3,3}(0') + C_{3,3}(1'))$

$A_{1434} = A_{1432}\, (C_{3,3}(0')\, C_{3,3}(1') - 2\, \lambda_s\, \mu_{s1'}) - 9\, \lambda_1^2\, \mu_{s1'}\, \mu_{s2'} + 9\, \lambda_1^2\, \mu_3\, \mu_{s1'}$

$A_{1435} = A_{1430}\, A_{1432},\;\; A_{1436} = A_{1430}\, A_{1433} + A_{1431}\, A_{1432}$

$A_{1437} = A_{1430}\, A_{1434} + A_{1431}\, A_{1433},\;\; A_{1438} = A_{1431}\, A_{1434}$

$A_{1439} = -3\, \lambda_1\, \mu_2\, \mu_3,\;\; A_{1440} = A_{1439}\, C_{3,3}(5),\;\; A_{1441} = 6\, \lambda_1\, \lambda_s$

$A_{1442} = A_{1441}\, (C_{3,3}(4) + C_{3,3}(2'))\;\; A_{1443} = A_{1441}\, (C_{3,3}(4)\, C_{3,3}(2') - 3\, \lambda_1\, \mu_4) + 6\, \lambda_1\, \lambda_s^2\, \mu_4$

$A_{1444} = A_{1439}\, A_{1441},\;\; A_{1445} = A_{1439}\, A_{1442} + A_{1440}\, A_{1441}$

$A_{1446} = A_{1439}\, A_{1443} + A_{1440}\, A_{1442},\;\; A_{1447} = A_{1440}\, A_{1443}$

$A_{1448} = -9\, \lambda_1^2\, \mu_2,\;\; A_{1449} = A_{1448}\, C_{3,3}(5),\;\; A_{1450} = C_{3,3}(3) + C_{3,3}(4)$

$A_{1451} = C_{3,3}(3)\, C_{3,3}(4),\;\; A_{1452} = C_{3,3}(1') + C_{3,3}(2')$

$A_{1453} = C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'},\;\; A_{1454} = -3\, \lambda_1\, \mu_4$

$A_{1455} = A_{1454}\, (C_{3,3}(3) + C_{3,3}(1')),\;\; A_{1456} = A_{1454}\, C_{3,3}(3)\, C_{3,3}(1')$

$A_{1457} = A_{1452}\, A_{1454},\;\; A_{1458} = A_{1452}\, A_{1453}$

$A_{1459} = -9\, \lambda_1^2\, \mu_4\, \mu_{s2'} - 3\, \lambda_1\, \lambda_s\, \mu_3\, \mu_4,\;\; A_{1460} = A_{1378}$

$A_{1461} = A_{1460}\, (C_{3,3}(4) + C_{3,3}(2')),\;\; A_{1462} = A_{1460}\, (C_{3,3}(4)\, C_{3,3}(2') + A_{1454})$

$A_{1463} = A_{1450} + A_{1452},\;\; A_{1464} = A_{1453} + A_{1450}\, A_{1452} + A_{1451} + 2\, A_{1454} + A_{1460}$

$A_{1465} = A_{1450}\, A_{1453} + A_{1451}\, A_{1452} + A_{1455} + A_{1457} + A_{1461}$

$A_{1466} = A_{1451}\, A_{1453} + A_{1456} + A_{1458} + A_{1459} + A_{1462}$

$A_{1474} = A_{1473}\,(C_{3,3}(3) + C_{3,3}(1') + C_{3,3}(2'))$

$$\begin{aligned} A_{1475} &= A_{1473}\,(C_{3,3}(3)\,C_{3,3}(1') + C_{3,3}(3)\,C_{3,3}(2') + C_{3,3}(1')\,C_{3,3}(2') - \\ &\quad \lambda_s\,\mu_{s2'} + 2\,\lambda_s\,\mu_3 - 3\,\lambda_1\,\mu_3) \\ A_{1476} &= A_{1473}\,(C_{3,3}(3)\,C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}\,C_{3,3}(3) + \\ &\quad 2\,\lambda_s\,\mu_3\,C_{3,3}(2') - 3\,\lambda_1\,\mu_3\,C_{3,3}(2')) \end{aligned}$$

$A_{1477} = A_{1472}\,A_{1473}$, $A_{1478} = A_{1472}\,A_{1474}$, $A_{1479} = A_{1472}\,A_{1475}$

$A_{1480} = A_{1472}\,A_{1476}$, $A_{1481} = 3\,\lambda_s\,\mu_2\,\mu_3$, $A_{1482} = A_{1481}\,C_{3,3}(5)$

$A_{1483} = A_{1441}$, $A_{1484} = A_{1483}\,(C_{3,3}(4) + C_{3,3}(2'))$

$A_{1485} = A_{1483}\,(C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4) + 6\,\lambda_1\,\lambda_s\,\mu_4$, $A_{1486} = A_{1481}\,A_{1483}$

$A_{1487} = A_{1481}\,A_{1484} + A_{1482}\,A_{1483}$, $A_{1488} = A_{1481}\,A_{1485} + A_{1482}\,A_{1484}$

$A_{1489} = A_{1482}\,A_{1485}$, $A_{1490} = 9\,\lambda_1\,\lambda_s\,\mu_2$

$A_{1491} = A_{1490}\,C_{3,3}(5)$, $A_{1492} = A_{1490}\,A_{1381} + A_{1491}$

$A_{1493} = A_{1490}\,A_{1382} + A_{1491}\,A_{1381}$, $A_{1494} = A_{1490}\,A_{1383} + A_{1491}\,A_{1382}$

$A_{1495} = A_{1490}\,A_{1384} + A_{1491}\,A_{1383}$, $A_{1496} = A_{1491}\,A_{1384}$

$$\begin{aligned} A_{1497} &= A_{1346} + A_{1386} + A_{1404} + A_{1424} + A_{1448} + A_{1490} \\ A_{1498} &= A_{1358} + A_{1387} + A_{1405} + A_{1425} + A_{1467} + A_{1492} \\ A_{1499} &= A_{1337} + A_{1359} + A_{1388} + A_{1406} + A_{1426} + A_{1435} + \\ &\quad A_{1444} + A_{1468} + A_{1477} + A_{1486} + A_{1493} \\ A_{1500} &= A_{1343} + A_{1360} + A_{1389} + A_{1407} + A_{1427} + A_{1436} + \\ &\quad A_{1445} + A_{1469} + A_{1478} + A_{1487} + A_{1494} \\ A_{1501} &= A_{1344} + A_{1361} + A_{1390} + A_{1408} + A_{1428} + A_{1437} + \\ &\quad A_{1446} + A_{1470} + A_{1479} + A_{1488} + A_{1495} \\ A_{1502} &= A_{1345} + A_{1362} + A_{1391} + A_{1409} + A_{1429} + A_{1438} + \\ &\quad A_{1447} + A_{1471} + A_{1480} + A_{1489} + A_{1496} \end{aligned}$$

$A_{1503} = -18\,\lambda_1^3\,\mu_5$, $A_{1504} = A_{1503}\,C_{3,3}(3)$, $A_{1505} = C_{3,3}(1') + C_{3,3}(2')$

$A_{1506} = C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}$, $A_{1507} = A_{1505} + C_{3,3}(0')$

$A_{1508} = A_{1506} + A_{1505}\,C_{3,3}(0') - 2\,\lambda_s\,\mu_{s1'}$, $A_{1509} = A_{1506}\,C_{3,3}(0') - 2\,\lambda_s\,\mu_{s1'}\,C_{3,3}(2')$

$A_{1510} = A_{1503}\,A_{1507} + A_{1504}$, $A_{1511} = A_{1503}\,A_{1508} + A_{1504}\,A_{1507}$

$A_{1512} = A_{1503}\,A_{1509} + A_{1504}\,A_{1508}$, $A_{1513} = A_{1504}\,A_{1509}$

$A_{1514} = 18\,\lambda_1^2\,\mu_3\,\mu_5,\;\; A_{1515} = 3\,\lambda_1,\;\; A_{1516} = A_{1515}\,(C_{3,3}(0') + C_{3,3}(2'))$

$A_{1517} = A_{1515}\,C_{3,3}(0')\,C_{3,3}(2'),\;\; A_{1518} = A_{1514}\,A_{1515},\;\; A_{1519} = A_{1514}\,A_{1516}$

$A_{1520} = A_{1514}\,A_{1517},\;\; A_{1521} = -18\,\lambda_1^2\,\lambda_s\,\mu_{s1'},\;\; A_{1522} = A_{1521}\,C_{3,3}(5)$

$A_{1523} = C_{3,3}(3) + C_{3,3}(4) + C_{3,3}(2')$

$A_{1524} = C_{3,3}(3)\,C_{3,3}(4) + C_{3,3}(3)\,C_{3,3}(2') + C_{3,3}(4)\,C_{3,3}(2') - 6\,\lambda_1\,\mu_4$

$A_{1525} = C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4\,C_{3,3}(3) - 3\,\lambda_1\,\mu_4\,C_{3,3}(2')$

$A_{1526} = A_{1521}\,A_{1523} + A_{1522},\;\; A_{1527} = A_{1521}\,A_{1524} + A_{1522}\,A_{1523}$

$A_{1528} = A_{1521}\,A_{1525} + A_{1522}\,A_{1524},\;\; A_{1529} = A_{1522}\,A_{1525}$

$A_{1530} = -18\,\lambda_1^3\,\lambda_s\,\mu_5,\;\; A_{1531} = A_{1530}\,C_{3,3}(1),\;\; A_{1532} = C_{3,3}(3) + C_{3,3}(1') + C_{3,3}(2')$

$A_{1533} = C_{3,3}(3)\,C_{3,3}(1') + C_{3,3}(3)\,C_{3,3}(2') + C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'} - 3\,\lambda_1\,\mu_3$

$A_{1534} = C_{3,3}(3)\,C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}\,C_{3,3}(3) - 3\,\lambda_1\,\mu_3\,C_{3,3}(2')$

$A_{1535} = A_{1530}\,A_{1532} + A_{1531},\;\; A_{1536} = A_{1530}\,A_{1533} + A_{1531}\,A_{1532}$

$A_{1537} = A_{1530}\,A_{1534} + A_{1531}\,A_{1533},\;\; A_{1538} = A_{1531}\,A_{1534}$

$A_{1539} = 6\,\lambda_1\,\lambda_s\,\mu_3\,\mu_5,\;\; A_{1540} = A_{1539}\,C_{3,3}(1),\;\; A_{1541} = -6\,\lambda_1\,\lambda_s$

$A_{1542} = A_{1541}\,C_{3,3}(2'),\;\; A_{1543} = A_{1539}\,A_{1541},\;\; A_{1544} = A_{1539}\,A_{1542} + A_{1540}\,A_{1541}$

$A_{1545} = A_{1540}\,A_{1542},\;\; A_{1546} = 27\,\lambda_1^3,\;\; A_{1547} = A_{1546}\,(C_{3,3}(5) + C_{3,3}(0'))$

$A_{1548} = A_{1546}\,C_{3,3}(0')\,C_{3,3}(5),\;\; A_{1549} = 27\,\lambda_1^2\,\lambda_s,\;\; A_{1550} = A_{1549}\,(C_{3,3}(1) + C_{3,3}(5))$

$A_{1551} = A_{1549}\,C_{3,3}(1)\,C_{3,3}(5),\;\; A_{1552} = A_{1540} + A_{1549},\;\; A_{1553} = A_{1547} + A_{1550}$

$A_{1554} = A_{1548} + A_{1551},\;\; A_{1555} = -\mu_4,\;\; A_{1556} = A_{1555}\,(C_{3,3}(1') + C_{3,3}(2'))$

$A_{1557} = A_{1555}\,C_{3,3}(1')\,C_{3,3}(2') - A_{1555}\,\lambda_s\,\mu_{s2'} - 3\,\lambda_1\,\mu_4\,\mu_{s2'},\;\; A_{1558} = A_{1552}\,A_{1555}$

$A_{1559} = A_{1552}\,A_{1556} + A_{1553}\,A_{1555},\;\; A_{1560} = A_{1552}\,A_{1557} + A_{1553}\,A_{1556} + A_{1554}\,A_{1555}$

$A_{1561} = A_{1553}\,A_{1557} + A_{1554}\,A_{1556},\;\; A_{1562} = A_{1554}\,A_{1557},\;\; A_{1563} = 6\,\lambda_s^2$

$A_{1564} = A_{1563}\,(C_{3,3}(1) + C_{3,3}(5)),\;\; A_{1565} = A_{1563}\,C_{3,3}(1)\,C_{3,3}(5),\;\; A_{1566} = -9\,\lambda_1^2\,\mu_3$

$A_{1567} = A_{1566}\,(C_{3,3}(0') + C_{3,3}(5)),\;\; A_{1568} = A_{1566}\,C_{3,3}(5)\,C_{3,3}(0')$

$A_{1569} = -9\,\lambda_1\,\lambda_s\,\mu_3,\;\; A_{1570} = A_{1569}\,(C_{3,3}(1) + C_{3,3}(5))$

$A_{1571} = A_{1569}\,C_{3,3}(1)\,C_{3,3}(5),\;\; A_{1572} = A_{1563} + A_{1566} + A_{1569}$

$A_{1573} = A_{1564} + A_{1567} + A_{1570},\;\; A_{1574} = A_{1565} + A_{1568} + A_{1571}$

$A_{1575} = 3\,\lambda_1,\;\; A_{1576} = A_{1575}\,(C_{3,3}(4) + C_{3,3}(2'))$

$A_{1577} = A_{1575}\,C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4\,A_{1575} + 3\,\lambda_1\,\lambda_s\,\mu_4$

$A_{1578} = A_{1572}\,A_{1575},\;\; A_{1579} = A_{1572}\,A_{1576} + A_{1573}\,A_{1575}$

$A_{1580} = A_{1572}\,A_{1577} + A_{1573}\,A_{1576} + A_{1574}\,A_{1575},\;\; A_{1581} = A_{1573}\,A_{1577} + A_{1574}\,A_{1576}$

$A_{1582} = A_{1574}\,A_{1577},\;\; A_{1583} = 9\,\lambda_1^2,\;\; A_{1584} = A_{1583}\,(C_{3,3}(5) + C_{3,3}(0') + C_{3,3}(3))$

$A_{1585} = A_{1583}\,C_{3,3}(5)\,(C_{3,3}(0') + C_{3,3}(3)) + A_{1583}\,C_{3,3}(0')\,C_{3,3}(3)$

$$A_{1580} = A_{1572}\,A_{1577} + A_{1573}\,A_{1576} + A_{1574}\,A_{1575},\ \ A_{1581} = A_{1573}\,A_{1577} + A_{1574}\,A_{1576}$$
$$A_{1582} = A_{1574}\,A_{1577},\ \ A_{1583} = 9\,\lambda_1^2,\ \ A_{1584} = A_{1583}\,(C_{3,3}(5) + C_{3,3}(0') + C_{3,3}(3))$$
$$A_{1585} = A_{1583}\,C_{3,3}(5)\,(C_{3,3}(0') + C_{3,3}(3)) + A_{1583}\,C_{3,3}(0')\,C_{3,3}(3)$$
$$A_{1586} = A_{1583}\,C_{3,3}(3)\,C_{3,3}(5)\,C_{3,3}(0'),\ \ A_{1587} = 9\,\lambda_1\,\lambda_s$$
$$A_{1588} = A_{1587}\,(C_{3,3}(1) + C_{3,3}(3) + C_{3,3}(5))$$
$$A_{1589} = A_{1587}\,C_{3,3}(1)\,(C_{3,3}(3) + C_{3,3}(5)) + A_{1587}\,C_{3,3}(3)\,C_{3,3}(5)$$
$$A_{1590} = A_{1587}\,C_{3,3}(1)\,C_{3,3}(3)\,C_{3,3}(5),\ \ A_{1591} = A_{1583} + A_{1587}$$
$$A_{1592} = A_{1584} + A_{1588},\ \ A_{1593} = A_{1585} + A_{1589},\ \ A_{1594} = A_{1586} + A_{1590}$$
$$A_{1595} = C_{3,3}(4) + C_{3,3}(1') + C_{3,3}(2')$$
$$A_{1596} = C_{3,3}(4)\,C_{3,3}(1') + C_{3,3}(4)\,C_{3,3}(2') + C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'} - 3\,\lambda_1\mu_4$$
$$A_{1597} = C_{3,3}(4)\,(C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}) - 3\,\lambda_1\mu_4 C_{3,3}(1')$$
$$A_{1598} = A_{1591}\,A_{1595} + A_{1592},\ \ A_{1599} = A_{1591}\,A_{1596} + A_{1592}\,A_{1595} + A_{1593}$$
$$A_{1600} = A_{1591}\,A_{1597} + A_{1592}\,A_{1596} + A_{1593}\,A_{1595} + A_{1594}$$
$$A_{1601} = A_{1522}\,A_{1597} + A_{1593}\,A_{1596} + A_{1594}\,A_{1595}$$
$$A_{1602} = A_{1593}\,A_{1597} + A_{1594}\,A_{1596},\ \ A_{1603} = A_{1594}\,A_{1597}$$
$$A_{1604} = A_{1503} + A_{1521} + A_{1530} + A_{1558} + A_{1578} + A_{1599}$$
$$A_{1605} = A_{1510} + A_{1526} + A_{1535} + A_{1559} + A_{1579} + A_{1600}$$
$$A_{1606} = A_{1511} + A_{1518} + A_{1527} + A_{1536} + A_{1543} + A_{1560} + A_{1580} + A_{1601}$$
$$A_{1607} = A_{1512} + A_{1519} + A_{1528} + A_{1537} + A_{1544} + A_{1561} + A_{1581} + A_{1602}$$
$$A_{1608} = A_{1513} + A_{1520} + A_{1529} + A_{1538} + A_{1545} + A_{1562} + A_{1582} + A_{1603}$$
$$A_{1609} = 6\,\lambda_s^2,\ \ A_{1610} = A_{1609}\,(C_{3,3}(5) + C_{3,3}(1) + C_{3,3}(2))$$
$$A_{1611} = A_{1609}\,(C_{3,3}(1)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(5) + C_{3,3}(1)\,C_{3,3}(2)) + 18\,\lambda_1^2\,\lambda_s\,\mu_2 - 18\,\lambda_1\,\lambda_s^2\,\mu_2$$
$$A_{1612} = A_{1609}\,C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(5) + 18\,\lambda_1^2\,\lambda_s\,\mu_2\,C_{3,3}(5) - 18\,\lambda_1\,\lambda_s^2\,\mu_2\,C_{3,3}(5)$$
$$A_{1613} = 3\,\lambda_1,\ \ A_{1614} = A_{1613}\,(C_{3,3}(4) + C_{3,3}(2'))$$
$$A_{1615} = A_{1613}\,(C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4) + 3\,\lambda_1\,\lambda_s\,\mu_4,\ \ A_{1616} = A_{1609}\,A_{1613}$$
$$A_{1617} = A_{1609}\,A_{1614} + A_{1610}\,A_{1613},\ \ A_{1618} = A_{1609}\,A_{1615} + A_{1610}\,A_{1614} + A_{1611}\,A_{1613}$$
$$A_{1619} = A_{1610}\,A_{1615} + A_{1611}\,A_{1614} + A_{1612}\,A_{1613},\ \ A_{1620} = A_{1611}\,A_{1615} + A_{1612}\,A_{1614}$$
$$A_{1621} = A_{1612}\,A_{1615},\ \ A_{1622} = 6\,\lambda_1\,\lambda_s\,\mu_5,\ \ A_{1623} = A_{1622}\,(C_{3,3}(1) + C_{3,3}(2))$$
$$A_{1624} = A_{1622}\,C_{3,3}(1)\,C_{3,3}(2) + 18\,\lambda_1^3\,\mu_2\,\mu_5 - 18\,\lambda_s\,\lambda_1^2\,\mu_2\,\mu_5$$
$$A_{1625} = -6\,\lambda_1\,\lambda_s,\ \ A_{1626} = A_{1625}\,C_{3,3}(2'),\ \ A_{1627} = A_{1622}\,A_{1625}$$
$$A_{1628} = A_{1622}\,A_{1626} + A_{1623}\,A_{1625},\ \ A_{1629} = A_{1623}\,A_{1626} + A_{1624}\,A_{1625}$$
$$A_{1630} = A_{1624}\,A_{1626},\ \ A_{1631} = 27\,\lambda_1^3$$
$$A_{1632} = A_{1631}\,(C_{3,3}(4) + C_{3,3}(5)),\ \ A_{1633} = A_{1631}\,C_{3,3}(4)\,C_{3,3}(5) - 54\,\lambda_1^4\,\mu_5$$

$A_{1634} = C_{3,3}(0') + C_{3,3}(1') + C_{3,3}(2')$

$A_{1635} = C_{3,3}(0')\, C_{3,3}(1') + C_{3,3}(0')\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'} - 2\, \lambda_s \mu_{s1'}$

$A_{1636} = C_{3,3}(0')\, (C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'}) - 2\, \lambda_s \mu_{s1'} C_{3,3}(2')$

$A_{1637} = A_{1631}\, A_{1634} + A_{1632},\;\; A_{1638} = A_{1631}\, A_{1635} + A_{1632}\, A_{1634} + A_{1633}$

$A_{1639} = A_{1631}\, A_{1636} + A_{1632}\, A_{1635} + A_{1633}\, A_{1634},\;\; A_{1640} = A_{1632}\, A_{1636} + A_{1633}\, A_{1635}$

$A_{1641} = A_{1633}\, A_{1636},\;\; A_{1642} = 27\, \lambda_1^3\, \mu_4,\;\; A_{1643} = A_{1642}\, C_{3,3}(5),\;\; A_{1644} = -3\, \lambda_1$

$A_{1645} = A_{1644}\, (C_{3,3}(0') + C_{3,3}(1')),\; A_{1646} = A_{1644}\, (C_{3,3}(0')\, C_{3,3}(1') - 2\, \lambda_s\, \mu_{s1'})$

$A_{1647} = A_{1642}\, A_{1644},\;\; A_{1648} = A_{1642}\, A_{1645} + A_{1643}\, A_{1644}$

$A_{1649} = A_{1642}\, A_{1646} + A_{1643}\, A_{1645},\;\; A_{1650} = A_{1643}\, A_{1646}$

$A_{1651} = 18\, \lambda_1^2\, \lambda_s\, \mu_5,\;\; A_{1652} = A_{1651}\, C_{3,3}(1),\;\; A_{1653} = -3\, \lambda_1$

$A_{1654} = A_{1653}\, (C_{3,3}(1') + C_{3,3}(2')),\;\; A_{1655} = A_{1653}\, (C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'})$

$A_{1656} = A_{1651}\, A_{1653},\;\; A_{1657} = A_{1651}\, A_{1654} + A_{1652}\, A_{1653}$

$A_{1658} = A_{1651}\, A_{1655} + A_{1652}\, A_{1654},\;\; A_{1659} = A_{1652}\, A_{1655}$

$A_{1660} = 27\, \lambda_1^2\, \lambda_s,\;\; A_{1661} = A_{1660}\, (C_{3,3}(1) + C_{3,3}(5))$

$A_{1662} = A_{1660}\, C_{3,3}(1)\, C_{3,3}(5),\;\; A_{1663} = C_{3,3}(4) + C_{3,3}(1') + C_{3,3}(2')$

$A_{1664} = C_{3,3}(4)\, C_{3,3}(1') + C_{3,3}(4)\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'} - 3\, \lambda_1 \mu_4$

$A_{1665} = C_{3,3}(4)\, (C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'}) - 3\, \lambda_1 \mu_4\, C_{3,3}(1')$

$A_{1666} = A_{1660}\, A_{1663} + A_{1661},\;\; A_{1667} = A_{1660}\, A_{1664} + A_{1661}\, A_{1663} + A_{1662}$

$A_{1668} = A_{1660}\, A_{1665} + A_{1661}\, A_{1664} + A_{1662}\, A_{1663},\;\; A_{1669} = A_{1661}\, A_{1665} + A_{1662}\, A_{1664}$

$A_{1670} = A_{1662}\, A_{1665},\;\; A_{1671} = A_{1616} + A_{1631} + A_{1660},\;\; A_{1672} = A_{1617} + A_{1637} + A_{1666}$

$A_{1673} = A_{1618} + A_{1627} + A_{1638} + A_{1647} + A_{1656} + A_{1667}$

$A_{1674} = A_{1619} + A_{1628} + A_{1639} + A_{1648} + A_{1657} + A_{1668}$

$A_{1675} = A_{1620} + A_{1629} + A_{1640} + A_{1649} + A_{1658} + A_{1669}$

$A_{1676} = A_{1621} + A_{1630} + A_{1641} + A_{1650} + A_{1659} + A_{1670}$

$A_{1677} = -6\, \lambda_s^2,\;\; A_{1678} = A_{1677}\, (C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5))$

$A_{1679} = A_{1677}\, (C_{3,3}(1)\, C_{3,3}(2) + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(2)\, C_{3,3}(5)) + 18\, \lambda_1\, \lambda_s^2\, \mu_2 - 18\, \lambda_1^2\, \lambda_s\, \mu_2$

$A_{1680} = A_{1677}\, C_{3,3}(5)\, (C_{3,3}(1)\, C_{3,3}(2) + 18\, \lambda_1\, \lambda_s^2\, \mu_2 - 18\, \lambda_1^2\, \lambda_s\, \mu_2)$

$A_{1681} = -3\, \lambda_1\, \lambda_s - 9\, \lambda_1^2,\; A_{1682} = -3\, \lambda_1\, \lambda_s\, C_{3,3}(3) - 9\, \lambda_1^2\, C_{3,3}(2'),\;\; A_{1683} = A_{1677}\, A_{1681}$

$A_{1684} = A_{1677}\, A_{1682} + A_{1678}\, A_{1681},\;\; A_{1685} = A_{1678}\, A_{1682} + A_{1679}\, A_{1681}$

$A_{1686} = A_{1679}\, A_{1682} + A_{1680}\, A_{1681},\;\; A_{1687} = A_{1680}\, A_{1682}$

$A_{1688} = 81\, \lambda_1^5,\;\; A_{1689} = A_{1688}\, C_{3,3}(5),\;\; A_{1690} = C_{3,3}(0') + C_{3,3}(1') + C_{3,3}(2')$

$A_{1691} = C_{3,3}(0')\, C_{3,3}(1') + C_{3,3}(0')\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'}$

$A_{1692} = C_{3,3}(0')\, (C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'}),\;\; A_{1693} = A_{1688}\, A_{1690} + A_{1689}$

$$A_{1694} = A_{1688}\, A_{1691} + A_{1689}\, A_{1690},\;\; A_{1695} = A_{1688}\, A_{1692} + A_{1689}\, A_{1691}$$
$$A_{1696} = A_{1689}\, A_{1692},\;\; A_{1697} = 27\,\lambda_1^4\,\mu_3,\;\; A_{1698} = A_{1697}\, C_{3,3}(5)$$
$$A_{1699} = 3\,\lambda_s\,\lambda_1,\;\; A_{1700} = A_{1699}\, C_{3,3}(0'),\;\; A_{1701} = A_{1697}\, A_{1699}$$
$$A_{1702} = A_{1697}\, A_{1700} + A_{1698}\, A_{1699},\;\; A_{1703} = A_{1698}\, A_{1700}$$
$$A_{1704} = -9\,\lambda_1\,\lambda_s,\;\; A_{1705} = A_{1704}\,(C_{3,3}(1) + C_{3,3}(1') + C_{3,3}(5))$$
$$A_{1706} = A_{1704}\,(C_{3,3}(1)\, C_{3,3}(1') + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(1')\, C_{3,3}(5))$$
$$A_{1707} = A_{1704}\, C_{3,3}(1)\, C_{3,3}(1')\, C_{3,3}(5),\;\; A_{1708} = -9\,\lambda_1^2,\;\; A_{1709} = A_{1708}\, C_{3,3}(2')$$
$$A_{1710} = A_{1704}\, A_{1708},\;\; A_{1711} = A_{1704}\, A_{1709} + A_{1705}\, A_{1708}$$
$$A_{1712} = A_{1705}\, A_{1709} + A_{1706}\, A_{1708},\;\; A_{1713} = A_{1706}\, A_{1709} + A_{1707}\, A_{1708}$$
$$A_{1714} = A_{1707}\, A_{1709},\;\; A_{1715} = -9\,\lambda_1\,\lambda_s^2,\;\; A_{1716} = A_{1715}\,(C_{3,3}(1) + C_{3,3}(5))$$
$$A_{1717} = A_{1715}\, C_{3,3}(1)\, C_{3,3}(5),\;\; A_{1718} = 9\,\lambda_1^2\,\mu_{s2'} + 6\,\lambda_1\,\lambda_s\,\mu_3 - 9\,\lambda_1^2\,\mu_3$$
$$A_{1719} = A_{1715}\, A_{1718},\;\; A_{1720} = A_{1716}\, A_{1718}$$
$$A_{1721} = A_{1717}\, A_{1718},\;\; A_{1722} = A_{1683} + A_{1688} + A_{1710}$$
$$A_{1723} = A_{1684} + A_{1693} + A_{1711},\;\; A_{1724} = A_{1685} + A_{1694} + A_{1701} + A_{1712} + A_{1719}$$
$$A_{1725} = A_{1686} + A_{1695} + A_{1702} + A_{1713} + A_{1720},\;\; A_{1726} = A_{1687} + A_{1696} + A_{1703} + A_{1714} + A_{1721}$$
$$A_{1727} = -12\,\lambda_1\,\lambda_s^2,\;\; A_{1728} = A_{1727}\,(C_{3,3}(1) + C_{3,3}(2))$$
$$A_{1729} = A_{1727}\, C_{3,3}(1)\, C_{3,3}(2) + 36\,\lambda_1^2\,\lambda_s^2\,\mu_2 - 36\,\lambda_1^3\,\lambda_s\,\mu_2,\;\; A_{1730} = -3\,\lambda_1\,\lambda_s - 9\,\lambda_1^2$$
$$A_{1731} = -3\,\lambda_1\,\lambda_s\, C_{3,3}(3) - 9\,\lambda_1^2\, C_{3,3}(2'),\;\; A_{1732} = A_{1727}\, A_{1730}$$
$$A_{1733} = A_{1727}\, A_{1731} + A_{1728}\, A_{1730},\;\; A_{1734} = A_{1728}\, A_{1731} + A_{1729}\, A_{1730}$$
$$A_{1735} = A_{1729}\, A_{1731},\;\; A_{1736} = 162\,\lambda_1^5,\;\; A_{1737} = A_{1736}\, A_{1690}$$
$$A_{1738} = A_{1736}\, A_{1691},\;\; A_{1739} = A_{1736}\, A_{1692},\;\; A_{1740} = 54\,\lambda_1^4\,\mu_3$$
$$A_{1741} = A_{1740}\, A_{1699},\;\; A_{1742} = A_{1740}\, A_{1700},\;\; A_{1743} = 18\,\lambda_1^2\,\lambda_s\,\mu_3$$
$$A_{1744} = A_{1743}\, C_{3,3}(1),\;\; A_{1745} = -6\,\lambda_1\,\lambda_s^2,\;\; A_{1746} = A_{1743}\, A_{1745}$$
$$A_{1747} = A_{1744}\, A_{1745},\;\; A_{1748} = -54\,\lambda_1^3\,\lambda_s,\;\; A_{1749} = A_{1748}\, C_{3,3}(1)$$
$$A_{1750} = -3\,\lambda_1,\;\; A_{1751} = A_{1750}\,(C_{3,3}(1') + C_{3,3}(2'))$$
$$A_{1752} = A_{1750}\,(C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\,\mu_{s2'}) - 3\,\lambda_1\,\lambda_s\,\mu_3,\;\; A_{1753} = A_{1748}\, A_{1750}$$
$$A_{1754} = A_{1748}\, A_{1751} + A_{1749}\, A_{1750},\;\; A_{1755} = A_{1748}\, A_{1752} + A_{1749}\, A_{1751}$$
$$A_{1756} = A_{1749}\, A_{1752},\;\; A_{1757} = A_{1732} + A_{1736} + A_{1753}$$
$$A_{1758} = A_{1733} + A_{1737} + A_{1754},\;\; A_{1759} = A_{1734} + A_{1738} + A_{1741} + A_{1746} + A_{1755}$$
$$A_{1760} = A_{1735} + A_{1739} + A_{1742} + A_{1747} + A_{1756},\;\; A_{1761} = -6\,\lambda_1\,\lambda_s\,\mu_5$$
$$A_{1762} = A_{1761}\,(C_{3,3}(1) + C_{3,3}(2)),\;\; A_{1763} = A_{1761}\, C_{3,3}(1)\, C_{3,3}(2) + 18\,\lambda_1^2\,\mu_2\,\mu_5\,(\lambda_s - \lambda_1)$$
$$A_{1764} = C_{3,3}(3) + C_{3,3}(1') + C_{3,3}(2')$$
$$A_{1765} = C_{3,3}(3)\, C_{3,3}(1') + C_{3,3}(3)\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\,\mu_{s2'} - 3\,\lambda_1\,\mu_3$$

$$A_{1771} = A_{1763}\,A_{1766},\quad A_{1772} = 3\,\lambda_s\,(C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(3) + C_{3,3}(5))$$

$$\begin{aligned}
A_{1773} &= 3\,\lambda_s\,(C_{3,3}(1)\,C_{3,3}(2) + C_{3,3}(1)\,C_{3,3}(3) + C_{3,3}(2)\,C_{3,3}(3) + \\
&\quad C_{3,3}(1)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(5) + C_{3,3}(3)\,C_{3,3}(5)) + \\
&\quad 9\,\lambda_1^2\,\mu_2 - 9\,\lambda_1\,\lambda_s\,\mu_2 \\
A_{1774} &= 3\,\lambda_s\,(C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3) + C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(5) + \\
&\quad C_{3,3}(1)\,C_{3,3}(3)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(5)) + \\
&\quad 9\,\lambda_1^2\,\mu_2\,(C_{3,3}(5) + C_{3,3}(3)) - 9\,\lambda_1\,\lambda_s\,\mu_2\,(C_{3,3}(5) + C_{3,3}(3)) \\
A_{1775} &= 3\,\lambda_s\,C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3)\,C_{3,3}(5) + 9\,\lambda_1^2\,\mu_2\,C_{3,3}(3)\,C_{3,3}(5) - \\
&\quad 9\,\lambda_1\,\lambda_s\,\mu_2\,C_{3,3}(3)\,C_{3,3}(5)
\end{aligned}$$

$$A_{1776} = C_{3,3}(4) + C_{3,3}(1') + C_{3,3}(2')$$

$$A_{1777} = C_{3,3}(4)\,C_{3,3}(1') + C_{3,3}(4)\,C_{3,3}(2') + C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'} - 3\,\lambda_1\,\mu_4$$

$$A_{1778} = C_{3,3}(4)\,(C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'}) - 3\,\lambda_1\,\mu_4\,C_{3,3}(1')$$

$$A_{1779} = 3\,\lambda_s\,A_{1776} + A_{1772},\quad A_{1780} = 3\,\lambda_s\,A_{1777} + A_{1772}\,A_{1776} + A_{1773}$$

$$A_{1781} = 3\,\lambda_s\,A_{1778} + A_{1772}\,A_{1777} + A_{1773}A_{1776} + A_{1774}$$

$$A_{1782} = A_{1772}\,A_{1778} + A_{1773}\,A_{1777} + A_{1774}\,A_{1776} + A_{1775}$$

$$A_{1783} = A_{1773}\,A_{1778} + A_{1774}\,A_{1777} + A_{1775}\,A_{1776},\quad A_{1784} = A_{1774}\,A_{1778} + A_{1775}\,A_{1777}$$

$$A_{1785} = A_{1775}\,A_{1778},\quad A_{1786} = 9\,\lambda_1\,\lambda_s,\quad A_{1787} = A_{1786}\,(C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5))$$

$$A_{1788} = A_{1786}\,(C_{3,3}(1)\,C_{3,3}(2) + C_{3,3}(1)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(5)) - 27\,\lambda_1^2\,\mu_2\,(\lambda_s - \lambda_1)$$

$$A_{1789} = A_{1786}\,C_{3,3}(1)\,C_{3,3}(3)\,C_{3,3}(5) + 27\,\lambda_1^2\,\mu_2\,C_{3,3}(5)\,(\lambda_1 - \lambda_s),\quad A_{1790} = -\mu_4$$

$$A_{1791} = A_{1790}\,(C_{3,3}(1') + C_{3,3}(2')),\quad A_{1792} = A_{1790}\,(C_{3,3}(1')\,C_{3,3}(2') - \lambda_s\,\mu_{s2'} + 3\,\lambda_1\,\mu_{s2'})$$

$$A_{1793} = A_{1786}\,A_{1790},\quad A_{1794} = A_{1786}\,A_{1791} + A_{1787}\,A_{1790}$$

$$A_{1795} = A_{1786}\,A_{1792} + A_{1787}\,A_{1791} + A_{1788}\,A_{1790}$$

$$A_{1796} = A_{1787}\,A_{1792} + A_{1788}\,A_{1791} + A_{1789}\,A_{1790}$$

$$A_{1797} = A_{1788}\,A_{1792} + A_{1789}\,A_{1791},\quad A_{1798} = A_{1789}\,A_{1792}$$

$$A_{1799} = -3\,\lambda_s\,\mu_3,\quad A_{1800} = A_{1799}\,(C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5))$$

$$A_{1801} = A_{1799}\,(C_{3,3}(1)\,C_{3,3}(2) + C_{3,3}(1)\,C_{3,3}(5) + C_{3,3}(2)\,C_{3,3}(5)) + 9\,\lambda_1\,\mu_2\,\mu_3\,(\lambda_s - \lambda_1)$$

$$A_{1802} = A_{1799}\,C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(5) + 9\,\lambda_1\,\mu_2\,\mu_3\,C_{3,3}(5)\,(\lambda_s - \lambda_1)$$

$$A_{1803} = 3\,\lambda_1,\quad A_{1804} = A_{1803}\,(C_{3,3}(4) + C_{3,3}(2'))$$

$$A_{1805} = A_{1803}\,C_{3,3}(4)\,C_{3,3}(2') + 3\,\lambda_1\,\mu_4\,(\lambda_s - 3\,\lambda_1),\quad A_{1806} = A_{1799}\,A_{1803}$$

$$A_{1807} = A_{1799}\,A_{1804} + A_{1800}\,A_{1803},\quad A_{1808} = A_{1799}\,A_{1805} + A_{1800}\,A_{1804} + A_{1801}\,A_{1803}$$

$$A_{1809} = A_{1800}\,A_{1805} + A_{1801}\,A_{1804} + A_{1802}\,A_{1803},\quad A_{1810} = A_{1802}\,A_{1804} + A_{1801}\,A_{1805}$$

$A_{1805} = A_{1803}\, C_{3,3}(4)\, C_{3,3}(2') + 3\,\lambda_1\, \mu_4\, (\lambda_s - 3\,\lambda_1),\;\; A_{1806} = A_{1799}\, A_{1803}$

$A_{1807} = A_{1799}\, A_{1804} + A_{1800}\, A_{1803},\;\; A_{1808} = A_{1799}\, A_{1805} + A_{1800}\, A_{1804} + A_{1801}\, A_{1803}$

$A_{1809} = A_{1800}\, A_{1805} + A_{1801}\, A_{1804} + A_{1802}\, A_{1803},\;\; A_{1810} = A_{1802}\, A_{1804} + A_{1801}\, A_{1805}$

$A_{1811} = A_{1802}\, A_{1805},\;\; A_{1812} = -27\,\lambda_1^3\, \mu_{s1'}\, \mu_3,\;\; A_{1813} = A_{1812}\, (C_{3,3}(4)+C_{3,3}(5)+C_{3,3}(2'))$

$$\begin{aligned} A_{1814} &= A_{1812}\, (C_{3,3}(4)\, C_{3,3}(5) + C_{3,3}(5)\, C_{3,3}(2') + C_{3,3}(4)\, C_{3,3}(2') - \\ &\quad 3\,\lambda_1\, \mu_4) - 81\,\lambda_1^4\, \mu_{s1'}\, \mu_{s2'}\, \mu_4 + 54\,\lambda_1^4\, \mu_3\, \mu_5\, \mu_{s1'} \\ A_{1815} &= A_{1812}\, (C_{3,3}(4)\, C_{3,3}(5)\, C_{3,3}(2') - 3\,\lambda_1\, \mu_4\, C_{3,3}(5)) - \\ &\quad 81\,\lambda_1^4\, \mu_{s1'}\, \mu_{s2'}\, \mu_4\, C_{3,3}(5) + 54\,\lambda_1^4\, \mu_3\, \mu_5\, \mu_{s1'}\, C_{3,3}(2') \end{aligned}$$

$A_{1816} = 18\,\lambda_1^2\, \lambda_s\, \mu_3\, \mu_5,\;\; A_{1817} = A_{1816}\, (C_{3,3}(1) + C_{3,3}(1') + C_{3,3}(2'))$

$A_{1818} = A_{1816}\, (C_{3,3}(1)\, C_{3,3}(1') + C_{3,3}(1)\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'})$

$A_{1819} = A_{1816}\, (C_{3,3}(1)\, C_{3,3}(1')\, C_{3,3}(2') - \lambda_s\, \mu_{s2'}\, C_{3,3}(1)),\;\; A_{1820} = 27\,\lambda_1^2\, \lambda_s\, \mu_3\, \mu_4$

$A_{1821} = A_{1820}\, (C_{3,3}(1) + C_{3,3}(5) + C_{3,3}(1'))$

$A_{1822} = A_{1820}\, (C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(1)\, C_{3,3}(1') + C_{3,3}(5)\, C_{3,3}(1'))$

$A_{1823} = A_{1820}\, C_{3,3}(1)\, C_{3,3}(5)\, C_{3,3}(1'),\;\; A_{1824} = -9\,\lambda_1\, \lambda_s\, \mu_3$

$A_{1825} = A_{1824}\, (C_{3,3}(1) + C_{3,3}(4)),\;\; A_{1826} = A_{1824}\, C_{3,3}(1)\, C_{3,3}(4)$

$A_{1827} = C_{3,3}(5) + C_{3,3}(1') + C_{3,3}(2')$

$A_{1828} = C_{3,3}(5)\, C_{3,3}(1') + C_{3,3}(5)\, C_{3,3}(2') + C_{3,3}(1')\, C_{3,3}(2')$

$A_{1829} = C_{3,3}(5)\, C_{3,3}(1')\, C_{3,3}(2'),\;\; A_{1830} = 79\,\lambda_1\, \lambda_s^2\, \mu_3 \mu_{s2'}$

$A_{1831} = A_{1830}\, (C_{3,3}(1) + C_{3,3}(4) + C_{3,3}(5))$

$A_{1832} = A_{1830}\, (C_{3,3}(1)\, C_{3,3}(4) + C_{3,3}(1)\, C_{3,3}(5) + C_{3,3}(4)\, C_{3,3}(5))$

$A_{1833} = A_{1830}\, C_{3,3}(1)\, C_{3,3}(4)\, C_{3,3}(5),\;\; A_{1834} = A_{1824}\, A_{1827} + A_{1825}$

$A_{1835} = A_{1816} + A_{1820} + A_{1824}\, A_{1828} + A_{1825}\, A_{1827} + A_{1826} + A_{1830}$

$A_{1836} = A_{1817} + A_{1821} + A_{1824}\, A_{1829} + A_{1805}\, A_{1828} + A_{1826}\, A_{1827} + A_{1831}$

$A_{1837} = A_{1818} + A_{1822} + A_{1825}\, A_{1829} + A_{1826}\, A_{1828} + A_{1832}$

$A_{1838} = A_{1819} + A_{1823} + A_{1833} + A_{1826}\, A_{1829}$

$A_{1839} = A_{1761} + A_{1780} + A_{1793} + A_{1806} + A_{1824}$

$A_{1840} = A_{1767} + A_{1781} + A_{1794} + A_{1807} + A_{1834}$

$A_{1841} = A_{1768} + A_{1782} + A_{1795} + A_{1808} + A_{1812} + A_{1835}$

$A_{1842} = A_{1769} + A_{1783} + A_{1796} + A_{1809} + A_{1813} + A_{1836}$

$A_{1843} = A_{1770} + A_{1784} + A_{1797} + A_{1810} + A_{1814} + A_{1837}$

$A_{1844} = A_{1771} + A_{1785} + A_{1798} + A_{1811} + A_{1815} + A_{1838}$

$A_{1845} = 6\,\lambda_s^2,\;\; A_{1846} = A_{1845}\, (C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(5))$

$$A_{1849} = C_{3,3}(3) + C_{3,3}(4) + C_{3,3}(2')$$
$$A_{1850} = C_{3,3}(3)\,C_{3,3}(4) + C_{3,3}(3)\,C_{3,3}(2') + C_{3,3}(4)\,C_{3,3}(2') - 6\,\lambda_1\,\mu_4$$
$$A_{1851} = C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4\,(C_{3,3}(3) + C_{3,3}(2'))$$
$$A_{1852} = A_{1845}\,A_{1849} + A_{1846},\quad A_{1853} = A_{1845}\,A_{1850} + A_{1846}\,A_{1849} + A_{1847}$$
$$A_{1854} = A_{1845}\,A_{1851} + A_{1846}\,A_{1850} + A_{1847}\,A_{1849} + A_{1848}$$
$$A_{1855} = A_{1846}\,A_{1851} + A_{1847}\,A_{1850} + A_{1848}\,A_{1849},\quad A_{1856} = A_{1847}\,A_{1851} + A_{1848}\,A_{1850}$$
$$A_{1857} = A_{1848}\,A_{1851},\quad A_{1858} = 27\,\lambda_1^3\,\mu_3 + 18\,\lambda_1^2\,\lambda_s\,\mu_2$$
$$A_{1859} = 27\,\lambda_1^3\,\mu_3\,C_{3,3}(0') + 18\,\lambda_1^2\,\lambda_s\,\mu_2\,C_{3,3}(3),\quad A_{1860} = C_{3,3}(4) + C_{3,3}(5) + C_{3,3}(2')$$
$$A_{1861} = C_{3,3}(4)\,C_{3,3}(5) + C_{3,3}(5)\,C_{3,3}(2') + C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4 - 2\,\lambda_1\,\mu_5$$
$$A_{1862} = C_{3,3}(4)\,C_{3,3}(5)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4\,(C_{3,3}(5) - 2\,\lambda_1\,\mu_5\,C_{3,3}(2'))$$
$$A_{1863} = A_{1858}\,A_{1860} + A_{1859},\quad A_{1864} = A_{1858}\,A_{1861} + A_{1859}\,A_{1860}$$
$$A_{1865} = A_{1858}\,A_{1862} + A_{1859}\,A_{1861},\quad A_{1866} = A_{1859}\,A_{1862},\quad A_{1867} = 81\,\lambda_1^4$$
$$A_{1868} = A_{1867}\,C_{3,3}(0'),\quad A_{1869} = \mu_4\,\mu_{s2'},\quad A_{1870} = A_{1869}\,C_{3,3}(5),\quad A_{1871} = A_{1867}\,A_{1869}$$
$$A_{1872} = A_{1867}\,A_{1870} + A_{1868}\,A_{1869},\quad A_{1873} = A_{1868}\,A_{1870},\quad A_{1874} = -54\,\lambda_1^3\,\lambda_s\,\mu_2\,\mu_4$$
$$A_{1875} = A_{1874}\,(C_{3,3}(5) + C_{3,3}(2')),\quad A_{1876} = A_{1874}\,C_{3,3}(5)\,C_{3,3}(2'),\quad A_{1877} = 6\,\lambda_1\,\lambda_s\,\mu_5$$
$$A_{1878} = A_{1877}\,(C_{3,3}(1) + C_{3,3}(2')),\quad A_{1879} = A_{1877}\,C_{3,3}(1)\,C_{3,3}(2'),\quad A_{1880} = -2\,\lambda_s$$
$$A_{1881} = A_{1880}\,(C_{3,3}(2) + C_{3,3}(3)),\quad A_{1882} = A_{1880}\,C_{3,3}(2)\,C_{3,3}(3) + 3\,\lambda_1\,\mu_3\,(2\,\lambda_s - 3\,\lambda_1)$$
$$A_{1883} = A_{1877}\,A_{1880},\quad A_{1884} = A_{1877}\,A_{1881} + A_{1878}\,A_{1880}$$
$$A_{1885} = A_{1877}\,A_{1882} + A_{1878}\,A_{1881} + A_{1879}\,A_{1880},\quad A_{1886} = A_{1878}\,A_{1882} + A_{1879}\,A_{1881}$$
$$A_{1887} = A_{1879}\,A_{1882},\quad A_{1888} = 18\,\lambda_1^2\,\lambda_s\,\mu_5,\quad A_{1889} = A_{1888}\,C_{3,3}(2'),\quad A_{1890} = 2\,\lambda_s\,\mu_2$$
$$A_{1891} = A_{1890}\,C_{3,3}(3),\quad A_{1892} = A_{1888}\,A_{1890},\quad A_{1893} = A_{1888}\,A_{1891} + A_{1889}\,A_{1890}$$
$$A_{1894} = A_{1889}\,A_{1891},\quad A_{1895} = 9\,\lambda_1\,\lambda_s\,\mu_3,\quad A_{1896} = A_{1895}\,(C_{3,3}(1) + C_{3,3}(5))$$
$$A_{1897} = A_{1895}\,C_{3,3}(1)\,C_{3,3}(5),\quad A_{1898} = -2\,\lambda_s,\quad A_{1899} = A_{1898}\,(C_{3,3}(4) + C_{3,3}(2'))$$
$$A_{1900} = A_{1898}\,(C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4),\quad A_{1901} = A_{1895}\,A_{1898}$$
$$A_{1902} = A_{1895}\,A_{1899} + A_{1896}\,A_{1898},\quad A_{1903} = A_{1895}\,A_{1900} + A_{1896}\,A_{1899} + A_{1897}\,A_{1898}$$
$$A_{1904} = A_{1896}\,A_{1900} + A_{1897}\,A_{1899},\quad A_{1905} = A_{1897}\,A_{1900}$$
$$A_{1906} = 27\,\lambda_1^2\,\lambda_s,\quad A_{1907} = A_{1906}\,(C_{3,3}(1) + C_{3,3}(5)),\quad A_{1908} = A_{1906}\,C_{3,3}(1)\,C_{3,3}(5)$$
$$A_{1909} = \mu_3,\quad A_{1910} = A_{1909}\,(C_{3,3}(4) + C_{3,3}(2'))$$
$$A_{1911} = A_{1909}\,(C_{3,3}(4)\,C_{3,3}(2') - 3\,\lambda_1\,\mu_4) + 3\,\lambda_1\,\mu_4\,\mu_{s2'},\quad A_{1912} = A_{1906}\,A_{1909}$$
$$A_{1913} = A_{1906}\,A_{1910} + A_{1907}\,A_{1909},\quad A_{1914} = A_{1906}\,A_{1911} + A_{1907}\,A_{1910} + A_{1908}\,A_{1909}$$
$$A_{1915} = A_{1907}\,A_{1911} + A_{1908}\,A_{1910},\quad A_{1916} = A_{1908}\,A_{1911}$$
$$A_{1917} = A_{1853} + A_{1858} + A_{1883} + A_{1901} + A_{1912}$$
$$A_{1918} = A_{1854} + A_{1863} + A_{1884} + A_{1902} + A_{1913}$$

$A_{1917} = A_{1853} + A_{1858} + A_{1883} + A_{1901} + A_{1912}$

$A_{1918} = A_{1854} + A_{1863} + A_{1884} + A_{1902} + A_{1913}$

$A_{1919} = A_{1855} + A_{1864} + A_{1871} + A_{1874} + A_{1885} + A_{1892} + A_{1903} + A_{1914}$

$A_{1920} = A_{1856} + A_{1865} + A_{1872} + A_{1875} + A_{1886} + A_{1893} + A_{1904} + A_{1915}$

$A_{1921} = A_{1857} + A_{1866} + A_{1873} + A_{1876} + A_{1887} + A_{1894} + A_{1905} + A_{1916}$

$A_{1922} = -18\,\lambda_1\,\lambda_s^2,\;\; A_{1923} = A_{1922}\,(C_{3,3}(1) + C_{3,3}(2))$

$A_{1924} = A_{1922}\,C_{3,3}(1)\,C_{3,3}(2) - 54\,\lambda_1^2\,\lambda_s\,\mu_2\,(\lambda_1 - \lambda_s),\;\; A_{1925} = \mu_4\,\lambda_s - 3\,\lambda_1\,\mu_4$

$A_{1926} = A_{1925}\,C_{3,3}(5),\;\; A_{1927} = A_{1922}\,A_{1925},\;\; A_{1928} = A_{1922}\,A_{1926} + A_{1923}\,A_{1925}$

$A_{1929} = A_{1923}\,A_{1926} + A_{1924}\,A_{1925},\;\; A_{1930} = A_{1924}\,A_{1926},\;\; A_{1931} = -6\lambda_s^2$

$A_{1932} = A_{1931}\,(C_{3,3}(1) + C_{3,3}(2) + C_{3,3}(3))$

$$\begin{aligned} A_{1933} &= A_{1932}\,(C_{3,3}(1)\,C_{3,3}(2) + C_{3,3}(1)\,C_{3,3}(3) + C_{3,3}(2)\,C_{3,3}(3)) - \\ &\quad 27\,\lambda_1^2\,\mu_3 - 18\,\lambda_1^2\,\lambda_s\,\mu_2 + 18\,\lambda_1\,\lambda_s^2\,\mu_3 - 18\,\lambda_1\,\lambda_s^2\,\mu_2 \\ A_{1934} &= A_{1931}\,C_{3,3}(1)\,C_{3,3}(2)\,C_{3,3}(3) - 27\,\lambda_1^3\,\mu_3\,C_{3,3}(0') - 18\,\lambda_1^2\,\lambda_s\,\mu_2\,C_{3,3}(3) + \\ &\quad 18\,\lambda_1\,\lambda_s^2\,\mu_3\,C_{3,3}(1) - 18\,\lambda_1\,\lambda_s^2\,\mu_2\,C_{3,3}(3) \end{aligned}$$

$A_{1935} = -\lambda_s,\;\; A_{1936} = A_{1935}\,(C_{3,3}(4) + C_{3,3}(5))$

$A_{1937} = A_{1935}\,C_{3,3}(4)\,C_{3,3}(5) - 2\,\lambda_1\,\mu_5\,A_{1935},\;\; A_{1938} = A_{1931}\,A_{1935}$

$A_{1939} = A_{1931}\,A_{1936} + A_{1932}\,A_{1935},\;\; A_{1940} = A_{1931}\,A_{1937} + A_{1932}\,A_{1936} + A_{1933}\,A_{1935}$

$A_{1941} = A_{1932}\,A_{1937} + A_{1933}\,A_{1936} + A_{1934}\,A_{1935},\;\; A_{1942} = A_{1933}\,A_{1937} + A_{1934}\,A_{1936}$

$A_{1943} = A_{1934}\,A_{1937},\;\; A_{1944} = -81\,\lambda_1^4,\;\; A_{1945} = A_{1944}\,C_{3,3}(5),\;\; A_{1946} = -\mu_4$

$A_{1947} = A_{1946}\,(C_{3,3}(0') + C_{3,3}(1')),\; A_{1948} = A_{1946}\,(C_{3,3}(0')\,C_{3,3}(1') - 2\,\lambda_s\,\mu_{s1'})$

$A_{1949} = A_{1944}\,A_{1946},\;\; A_{1950} = A_{1944}\,A_{1947} + A_{1945}\,A_{1946}$

$A_{1951} = A_{1944}\,A_{1948} + A_{1945}\,A_{1947},\;\; A_{1952} = A_{1945}\,A_{1948}$

$A_{1953} = -81\,\lambda_1^3\,\lambda_s,\;\; A_{1954} = A_{1953}\,C_{3,3}(1),\;\; A_{1955} = -\mu_4$

$A_{1956} = A_{1955}\,(C_{3,3}(1') + C_{3,3}(5)),\;\; A_{1957} = A_{1956}\,C_{3,3}(1')\,C_{3,3}(5)$

$A_{1958} = A_{1953}\,A_{1954},\;\; A_{1959} = A_{1953}\,A_{1956} + A_{1954}\,A_{1955}$

$A_{1960} = A_{1953}\,A_{1957} + A_{1954}\,A_{1956},\;\; A_{1961} = A_{1954}\,A_{1957}$

$A_{1962} = A_{1927} + A_{1940} + A_{1949} + A_{1958},\;\; A_{1963} = A_{1928} + A_{1941} + A_{1950} + A_{1959}$

$A_{1964} = A_{1929} + A_{1942} + A_{1951} + A_{1960},\;\; A_{1965} = A_{1930} + A_{1943} + A_{1952} + A_{1961}$

$A_{1966} = C_{3,3}(0)\,A_{1336} - \mu_1\,A_{1502} - \mu_{s0'}\,A_{1844}$

$xdaf111 = (s_{46} - s_{47})(s_{46} - s_{48})(s_{46} - s_{49}),\; xdaf112 = (s_{47} - s_{46})(s_{47} - s_{48})(s_{47} - s_{49})$

$xdaf113 = (s_{48} - s_{46})(s_{48} - s_{47})(s_{48} - s_{49}),\; xdaf114 = (s_{49} - s_{46})(s_{49} - s_{47})(s_{49} - s_{48})$

$s_{50} = -C_{1,3}(1),\;\; s_{51} = -C_{1,3}(2),\;\; s_{52} = -C_{1,3}(3)$

$xdaf115 = (-C_{1,3}(1) - s_{46})(-C_{1,3}(1) - s_{47})(-C_{1,3}(1) - s_{48})(-C_{1,3}(1) - s_{49})$

$xdaf116 = (s_{46} + C_{1,3}(1))\, xdaf111, \;\; xdaf117 = (s_{47} + C_{1,3}(1))\, xdaf112$

$xdaf118 = (s_{48} + C_{1,3}(1))\, xdaf113, \;\; xdaf119 = (s_{49} + C_{1,3}(1))\, xdaf114$

$xdaf120 = (-C_{1,3}(2)+C_{1,3}(1))(-C_{1,3}(2)-s_{46})(-C_{1,3}(2)-s_{47})(-C_{1,3}(2)-s_{48})(-C_{1,3}(2)-s_{49})$

$xdaf121 = (-C_{1,3}(1) + C_{1,3}(2))\, xdaf115, \;\; xdaf122 = (s_{46} + C_{1,3}(2))\, xdaf116$

$xdaf123 = (s_{47} + C_{1,3}(2))\, xdaf117, \;\; xdaf124 = (s_{48} + C_{1,3}(2))\, xdaf118$

$xdaf125 = (s_{49} + C_{1,3}(2))\, xdaf119$

$xdaf126 = (-C_{1,3}(3)+C_{1,3}(1))(-C_{1,3}(3)+C_{1,3}(2))(-C_{1,3}(3)-s_{46})(-C_{1,3}(3)-s_{47})(-C_{1,3}(3)-s_{48})(-C_{1,3}(3) - s_{49})$

$xdaf127 = (-C_{1,3}(2) + C_{1,3}(3))\, xdaf120, \;\; xdaf128 = (-C_{1,3}(1) + C_{1,3}(3))\, xdaf121$

$xdaf129 = (s_{46} + C_{1,3}(3))\, xdaf122, \;\; xdaf130 = (s_{47} + C_{1,3}(3))\, xdaf123$

$xdaf131 = (s_{48} + C_{1,3}(3))\, xdaf124, \;\; xdaf132 = (s_{49} + C_{1,3}(3))\, xdaf125$

$V_{N_{17}}(s) = (s + C_{1,3}(2')[(s + C_{1,3}(0'))(s + C_{1,3}(1')) - 2\,\lambda_s\,\mu_{s1'}] - \mu_{s2'}\,\lambda_s\,C_{1,3}(0')$

$V_{N_{18}}(s) = \lambda_1\, V_{N_{17}}(s)$

$V_{N_{19}}(s) = 3\,\lambda_1\,\lambda_s\,C_{1,3}(1)\,[C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s2'}] + \lambda_1^2\,V_{N_{17}}(s)$

$V_{N_{20}}(s) \;=\; 6\,\lambda_s^2\,\lambda_1\,C_{1,3}(1)\,C_{1,3}(2)\,C_{1,3}(2') + 3\,\lambda_1^2\,\lambda_s\,C_{1,3}(1)\,[C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s2'}] + \lambda_1^3\,V_{N_{17}}(s)$

$V_{N_{21}}(s) = 3\,\lambda_s\,[C_{1,3}(1')\,C_{1,3}(2') - \lambda_s\,\mu_{s1'}]$

$V_{N_{22}}(s) = 6\,\lambda_s^2\,(s + C_{1,3}(2'))$

$B_{297} = V_{N_{17}}(s = s_{46})/xdaf111, \;\; B_{298} = V_{N_{18}}(s = s_{46})/xdaf116$

$B_{299} = V_{N_{19}}(s = s_{46})/xdaf122, \;\; B_{300} = V_{N_{20}}(s = s_{46})/xdaf129$

$B_{301} = V_{N_{21}}(s = s_{46})/xdaf111, \;\; B_{302} = V_{N_{22}}(s = s_{46})/xdaf111$

$B_{303} = 6\,\lambda_s^3/xdaf111, \;\; B_{304} = V_{N_{17}}(s = s_{47})/xdaf112$

$B_{305} = V_{N_{18}}(s = s_{47})/xdaf117, \;\; B_{306} = V_{N_{19}}(s = s_{47})/xdaf123$

$B_{307} = V_{N_{20}}(s = s_{47})/xdaf130, \;\; B_{308} = V_{N_{21}}(s = s_{47})/xdaf112$

$B_{309} = V_{N_{22}}(s = s_{47})/xdaf112, \;\; B_{310} = 6\,\lambda_s^3/xdaf112$

$B_{311} = V_{N_{17}}(s = s_{48})/xdaf113, \;\; B_{312} = V_{N_{18}}(s = s_{48})/xdaf118$

$B_{313} = V_{N_{19}}(s = s_{48})/xdaf124, \;\; B_{314} = V_{N_{20}}(s = s_{48})/xdaf131$

$B_{315} = V_{N_{21}}(s = s_{48})/xdaf113, \;\; B_{316} = V_{N_{22}}(s = s_{48})/xdaf113$

$B_{317} = 6\,\lambda_s^3/xdaf113, \;\; B_{318} = V_{N_{17}}(s = s_{49})/xdaf114$

$B_{319} = V_{N_{18}}(s = s_{49})/xdaf119, \;\; B_{320} = V_{N_{19}}(s = s_{49})/xdaf125$

$B_{321} = V_{N_{20}}(s = s_{49})/xdaf132, \;\; B_{322} = V_{N_{21}}(s = s_{49})/xdaf114$

$B_{323} = V_{N_{22}}(s = s_{49})/xdaf114,\ \ B_{324} = 6\,\lambda_s^3/xdaf114$
$B_{325} = V_{N_{18}}(s = -C_{1,3}(1))/xdaf115,\ \ B_{326} = V_{N_{19}}(s = -C_{1,3}(1))/xdaf121$
$B_{327} = V_{N_{20}}(s = -C_{1,3}(1))/xdaf128,\ \ B_{328} = V_{N_{19}}(s = -C_{1,3}(2))/xdaf120$
$B_{329} = V_{N_{20}}(s = -C_{1,3}(2))/xdaf127,\ \ B_{330} = V_{N_{20}}(s = -C_{1,3}(3))/xdaf126$
$xdaf155 = (s_{53}-s_{54})(s_{53}-s_{55})(s_{53}-s_{56}),\ \ xdaf156 = (s_{54}-s_{53})(s_{54}-s_{55})(s_{54}-s_{56})$
$xdaf157 = (s_{55}-s_{53})(s_{55}-s_{54})(s_{55}-s_{56}),\ \ xdaf158 = (s_{56}-s_{53})(s_{56}-s_{54})(s_{56}-s_{55})$
$s_{57} = -C_{2,3}(1)\ \ s_{58} = -C_{2,3}(2),\ \ s_{59} = -C_{2,3}(3),\ \ s_{60} = -C_{2,3}(4)$
$xdaf159 = (-C_{2,3}(1) - s_{53})(-C_{2,3}(1) - s_{54})(-C_{2,3}(1) - s_{55})(-C_{2,3}(1) - s_{56})$
$xdaf160 = (s_{53} + C_{2,3}(1))\,xdaf155,\ \ xdaf161 = (s_{54} + C_{2,3}(1))\,xdaf156$
$xdaf162 = (s_{55} + C_{2,3}(1))\,xdaf157,\ \ xdaf163 = (s_{56} + C_{2,3}(1))\,xdaf158$
$xdaf164 = (-C_{2,3}(2)+C_{2,3}(1))(-C_{2,3}(2)-s_{53})(-C_{2,3}(2)-s_{54})(-C_{2,3}(2)-s_{55})(-C_{2,3}(2)-$
$s_{56})$
$xdaf165 = (-C_{2,3}(1) + C_{2,3}(2))\,xdaf159,\ \ xdaf166 = (s_{53} + C_{2,3}(2))\,xdaf160$
$xdaf167 = (s_{54} + C_{2,3}(2))\,xdaf161,\ \ xdaf168 = (s_{55} + C_{2,3}(2))\,xdaf162$
$xdaf169 = (s_{56} + C_{2,3}(2))\,xdaf163$
$xdaf170 = (-C_{2,3}(3)+C_{2,3}(1))(-C_{2,3}(3)+C_{2,3}(2))(-C_{2,3}(3)-s_{53})(-C_{2,3}(3)-s_{54})(-C_{2,3}(3)-$
$s_{55})(-C_{2,3}(3) - s_{56})$
$xdaf171 = (-C_{2,3}(2) + C_{2,3}(3))\,xdaf164,\ \ xdaf172 = (-C_{2,3}(2) + C_{2,3}(3))\,xdaf165$
$xdaf173 = (s_{53} + C_{2,3}(3))\,xdaf166,\ \ xdaf174 = (s_{54} + C_{2,3}(3))\,xdaf167$
$xdaf175 = (s_{55} + C_{2,3}(3))\,xdaf168,\ \ xdaf176 = (s_{56} + C_{2,3}(3))\,xdaf169$
$xdaf177 = (-C_{2,3}(4) + C_{2,3}(1))(-C_{2,3}(4) + C_{2,3}(2))(-C_{2,3}(4) + C_{2,3}(3))(-C_{2,3}(4) -$
$s_{53})(-C_{2,3}(4) - s_{54})(-C_{2,3}(4) - s_{55})(-C_{2,3}(4) - s_{56})$
$xdaf178 = (-C_{2,3}(3) + C_{2,3}(4))\,xdaf170,\ \ xdaf179 = (-C_{2,3}(2) + C_{2,3}(4))\,xdaf171$
$xdaf180 = (-C_{2,3}(1) + C_{2,3}(4))\,xdaf172,\ \ xdaf181 = (s_{53} + C_{2,3}(4))\,xdaf173$
$xdaf182 = (s_{54} + C_{2,3}(4))\,xdaf174,\ \ xdaf183 = (s_{55} + C_{2,3}(4))\,xdaf175$
$xdaf184 = (s_{56} + C_{2,3}(4))\,xdaf176$
$V_{N_{23}}(s) = (s + C_{2,3}(0'))\,[(s + C_{2,3}(1'))(s + C_{2,3}(2')) - \lambda_s\,\mu_{s2'}] - 2\,\lambda_s\,\mu_{s1'}\,(s + C_{2,3}(2'))$
$V_{N_{24}}(s) = 2\,\lambda_1\,V_{N_{23}}(s)$
$V_{N_{25}}(s) = 6\,\lambda_1\,\lambda_s\,(s + C_{2,3}(1))\,[(s + C_{2,3}(1'))\,(s + C_{2,3}(2')) - \lambda_s\,\mu_{s2'}] + 4\,\lambda_1^2\,V_{N_{23}}(s)$
$V_{N_{26}}(s) = 12\,\lambda_1\,\lambda_s^2\,(s + C_{2,3}(1))(s + C_{2,3}(2))(s + C_{2,3}(2')) + 12\,\lambda_1^2\,\lambda_s\,(s + C_{2,3}(1))\,[(s +$
$C_{2,3}(1'))(s + C_{2,3}(2')) - \lambda_s\,\mu_{s2'}] + 8\,\lambda_1^3\,V_{N_{23}}(s)$
$V_{N_{27}}(s) = 12\,\lambda_1\,\lambda_s^3\,(s + C_{2,3}(1))(s + C_{2,3}(2))(s + C_{2,3}(3)) + 24\,\lambda_1^2\,\lambda_s^2\,(s + C_{2,3}(1))(s +$
$C_{2,3}(2))(s + C_{2,3}(2')) + 24\,\lambda_1^3\,\lambda_s\,(s + C_{2,3}(1))\,[(s + C_{2,3}(1'))(s + C_{2,3}(2')) - \lambda_s\,\mu_{s2'}] +$

$16\,\lambda_1^4\,V_{N_{23}}(s)$

$V_{N_{28}}(s) = 3\,\lambda_s\,[(s + C_{2,3}(1'))(s + C_{2,3}(2')) - \lambda_s\,\mu_{s2'}]$

$V_{N_{29}}(s) = 6\,\lambda_s^2\,(s + C_{2,3}(2'))$

$B_{365} = V_{N_{23}}(s = s_{53})/xdaf155,\;\; B_{366} = V_{N_{24}}(s = s_{53})/xdaf160$

$B_{367} = V_{N_{25}}(s = s_{53})/xdaf166,\;\; B_{368} = V_{N_{26}}(s = s_{53})/xdaf173$

$B_{369} = V_{N_{27}}(s = s_{53})/xdaf181,\;\; B_{370} = V_{N_{28}}(s = s_{53})/xdaf155$

$B_{371} = V_{N_{29}}(s = s_{53})/xdaf155,\;\; B_{372} = 6\,\lambda_s^3/xdaf155$

$B_{373} = V_{N_{23}}(s = s_{54})/xdaf156,\;\; B_{374} = V_{N_{24}}(s = s_{54})/xdaf161$

$B_{375} = V_{N_{25}}(s = s_{54})/xdaf167,\;\; B_{376} = V_{N_{26}}(s = s_{54})/xdaf174$

$B_{377} = V_{N_{27}}(s = s_{54})/xdaf182,\;\; B_{378} = V_{N_{28}}(s = s_{54})/xdaf156$

$B_{379} = V_{N_{29}}(s = s_{54})/xdaf156,\;\; B_{380} = 6\,\lambda_s^3/xdaf156$

$B_{381} = V_{N_{23}}(s = s_{55})/xdaf157,\;\; B_{382} = V_{N_{24}}(s = s_{55})/xdaf162$

$B_{383} = V_{N_{25}}(s = s_{55})/xdaf168,\;\; B_{384} = V_{N_{26}}(s = s_{55})/xdaf175$

$B_{385} = V_{N_{27}}(s = s_{55})/xdaf183,\;\; B_{386} = V_{N_{28}}(s = s_{55})/xdaf157$

$B_{387} = V_{N_{29}}(s = s_{55})/xdaf157,\;\; B_{388} = 6\,\lambda_s^3/xdaf157$

$B_{389} = V_{N_{23}}(s = s_{56})/xdaf158,\;\; B_{390} = V_{N_{24}}(s = s_{56})/xdaf163$

$B_{391} = V_{N_{25}}(s = s_{56})/xdaf169,\;\; B_{392} = V_{N_{26}}(s = s_{56})/xdaf176$

$B_{393} = V_{N_{27}}(s = s_{56})/xdaf184,\;\; B_{394} = V_{N_{28}}(s = s_{56})/xdaf158$

$B_{395} = V_{N_{29}}(s = s_{56})/xdaf158,\;\; B_{396} = 6\,\lambda_s^3/xdaf158$

$B_{397} = V_{N_{24}}(s = -C_{2,3}(1))/xdaf159,\;\; B_{398} = V_{N_{25}}(s = -C_{2,3}(1))/xdaf165$

$B_{399} = V_{N_{26}}(s = -C_{2,3}(1))/xdaf172,\;\; B_{400} = V_{N_{27}}(s = -C_{2,3}(1))/xdaf180$

$B_{401} = V_{N_{25}}(s = -C_{2,3}(2))/xdaf164,\;\; B_{402} = V_{N_{26}}(s = -C_{2,3}(2))/xdaf171$

$B_{403} = V_{N_{27}}(s = -C_{2,3}(2))/xdaf179,\;\; B_{404} = V_{N_{26}}(s = -C_{2,3}(3))/xdaf170$

$B_{405} = V_{N_{27}}(s = -C_{2,3}(3))/xdaf178,\;\; B_{406} = V_{N_{27}}(s = -C_{2,3}(4))/xdaf177$

$xdaf215 = (s_{61} - s_{62})(s_{61} - s_{63})(s_{61} - s_{64}),\;\; xdaf216 = (s_{62} - s_{61})(s_{62} - s_{63})(s_{62} - s_{64})$

$xdaf217 = (s_{63} - s_{61})(s_{63} - s_{62})(s_{63} - s_{64}),\;\; xdaf218 = (s_{64} - s_{61})(s_{64} - s_{62})(s_{64} - s_{63})$

$s_{65} = -C_{3,3}(1),\;\; s_{66} = -C_{3,3}(2)\;\; s_{67} = -C_{3,3}(3)\;\; s_{68} = -C_{3,3}(4)\;\; s_{69} = -C_{3,3}(5)$

$xdaf219 = (-C_{3,3}(1) - s_{61})(-C_{3,3}(1) - s_{62})(-C_{3,3}(1) - s_{63})(-C_{3,3}(1) - s_{64})$

$xdaf220 = (s_{61} + C_{3,3}(1))\,xdaf215,\;\; xdaf221 = (s_{62} + C_{3,3}(1))\,xdaf216$

$xdaf222 = (s_{63} + C_{3,3}(1))\,xdaf217,\;\; xdaf223 = (s_{64} + C_{3,3}(1))\,xdaf218$

$xdaf224 = (-C_{3,3}(2) + C_{3,3}(1))(-C_{3,3}(2) - s_{61})(-C_{3,3}(2) - s_{62})(-C_{3,3}(2) - s_{63})(-C_{3,3}(2) - s_{64})$

$xdaf225 = (-C_{3,3}(1) + C_{3,3}(2))\,xdaf219,\;\; xdaf226 = (s_{61} + C_{3,3}(2))\,xdaf220$

$xdaf227 = (s_{62} + C_{3,3}(2))\, xdaf221,\;\; xdaf228 = (s_{63} + C_{3,3}(2))\, xdaf222$

$xdaf229 = (s_{64} + C_{3,3}(2))\, xdaf223$

$xdaf230 = (-C_{3,3}(3)+C_{3,3}(1))(-C_{3,3}(3)+C_{3,3}(2))(-C_{3,3}(3)-s_{61})(-C_{3,3}(3)-s_{62})(-C_{3,3}(3)-s_{63})(-C_{3,3}(3) - s_{64})$

$xdaf231 = (-C_{3,3}(2) + C_{3,3}(3))\, xdaf224,\;\; xdaf232 = (-C_{3,3}(1) + C_{3,3}(3))\, xdaf225$

$xdaf233 = (s_{61} + C_{3,3}(3))\, xdaf226,\;\; xdaf234 = (s_{62} + C_{3,3}(3))\, xdaf227$

$xdaf235 = (s_{63} + C_{3,3}(3))\, xdaf228,\;\; xdaf236 = (s_{64} + C_{3,3}(3))\, xdaf229$

$xdaf237 = (-C_{3,3}(4) + C_{3,3}(1))(-C_{3,3}(4) + C_{3,3}(2))(-C_{3,3}(4) + C_{3,3}(1))(-C_{3,3}(4) - s_{61})(-C_{3,3}(4) - s_{62})(-C_{3,3}(4) - s_{63})(-C_{3,3}(4) - s_{64})$

$xdaf238 = (-C_{3,3}(3) + C_{3,3}(4))\, xdaf230,\;\; xdaf239 = (-C_{3,3}(2) + C_{3,3}(4))\, xdaf231$

$xdaf240 = (-C_{3,3}(1) + C_{3,3}(4))\, xdaf232,\;\; xdaf241 = (s_{61} + C_{3,3}(4))\, xdaf233$

$xdaf242 = (s_{62} + C_{3,3}(4))\, xdaf234,\;\; xdaf243 = (s_{63} + C_{3,3}(4))\, xdaf235$

$xdaf244 = (s_{64} + C_{3,3}(4))\, xdaf236$

$xdaf245 = (-C_{3,3}(5) + C_{3,3}(1))(-C_{3,3}(5) + C_{3,3}(2))(-C_{3,3}(5) + C_{3,3}(3))(-C_{3,3}(5) + C_{3,3}(4))(-C_{3,3}(5) - s_{61})(-C_{3,3}(5) - s_{62})(-C_{3,3}(5) - s_{63})(-C_{3,3}(5) - s_{64})$

$xdaf246 = (-C_{3,3}(4) + C_{3,3}(5))\, xdaf237,\;\; xdaf247 = (-C_{3,3}(3) + C_{3,3}(5))\, xdaf238$

$xdaf248 = (-C_{3,3}(2) + C_{3,3}(5))\, xdaf239,\;\; xdaf249 = (-C_{3,3}(1) + C_{3,3}(5))\, xdaf240$

$xdaf250 = (s_{61} + C_{3,3}(5))\, xdaf241,\;\; xdaf251 = (s_{62} + C_{3,3}(5))\, xdaf242$

$xdaf252 = (s_{63} + C_{3,3}(5))\, xdaf243,\;\; xdaf253 = (s_{64} + C_{3,3}(5))\, xdaf244$

$V_{N_{30}}(s) = (s + C_{3,3}(0'))\,[(s + C_{3,3}(1'))(s + C_{3,3}(2')) - \lambda_s\, \mu_{s2'}] - 2\, \lambda_s\, \mu_{s1'}\, (s + C_{3,3}(2'))$

$V_{N_{31}}(s) = 3\, \lambda_1\, V_{N_{30}}(s)$

$V_{N_{32}}(s) = 9\, \lambda_1\, \lambda_s\, (s + C_{3,3}(1))\,[(s + C_{3,3}(1'))(s + C_{3,3}(2')) - \lambda_s\, \mu_{s2'}] + 9\, \lambda_1^2\, V_{N_{30}}(s)$

$V_{N_{33}}(s) = 18\, \lambda_s^2\, \lambda_1\, (s + C_{3,3}(1))(s + C_{3,3}(2))(s + C_{3,3}(2')) + 27\, \lambda_1^2\, \lambda_s\, (s + C_{3,3}(1))\,[(s + C_{3,3}(1'))(s + C_{3,3}(2') - \lambda_s\, \mu_{s2'}] + 27\, \lambda_1^3\, V_{N_{30}}(s)$

$V_{N_{34}}(s) = 18\, \lambda_s^3\, \lambda_1\, (s + C_{3,3}(1))(s + C_{3,3}(2))(s + C_{3,3}(3)) + 54\, \lambda_s^2\, \lambda_1^2\, (s + C_{3,3}(1))(s + C_{3,3}(2))(s + C_{3,3}(2')) + 81\, \lambda_1^3\, \lambda_s\, (s + C_{3,3}(1))\,[(s + C_{3,3}(1'))(s + C_{3,3}(2')) - \lambda_s\, \mu_{s2'}] + 81\, \lambda_1^4\, V_{N_{30}}(s)$

$V_{N_{35}}(s) = 36\, \lambda_1^2\, \lambda_s^3\, (s + C_{3,3}(1))(s + C_{3,3}(2))(s + C_{3,3}(3)) + 108\, \lambda_1^3\, \lambda_s^2\, (s + C_{3,3}(1))(s + C_{3,3}(2))(s + C_{3,3}(2')) + 162\, \lambda_1^4\, \lambda_s\, (s + C_{3,3}(1))\,[(s + C_{3,3}(1'))(s + C_{3,3}(2')) - \lambda_s\, \mu_{s2'}] + 162\, \lambda_1^5\, V_{N_{30}}(s)$

$V_{N_{36}}(s) = 3\, \lambda_s\,[(s + C_{3,3}(1'))(s + C_{3,3}(2')) - \lambda_s\, \mu_{s2'}]$

$V_{N_{37}}(s) = 6\, \lambda_s^2\, (s + C_{3,3}(2'))$

$B_{449} = V_{N_{30}}(s = s_{61})/xdaf215,\;\; B_{450} = V_{N_{31}}(s = s_{61})/xdaf220$

$$B_{451}=V_{N_{32}}(s=s_{61})/xdaf226,\;\; B_{452}=V_{N_{33}}(s=s_{61})/xdaf233$$
$$B_{453}=V_{N_{34}}(s=s_{61})/xdaf241,\;\; B_{454}=V_{N_{35}}(s=s_{61})/xdaf250$$
$$B_{455}=V_{N_{36}}(s=s_{61})/xdaf215,\;\; B_{456}=V_{N_{37}}(s=s_{61})/xdaf215$$
$$B_{457}=6\,\lambda_s^3/xdaf215,\;\; B_{458}=V_{N_{30}}(s=s_{62})/xdaf216$$
$$B_{459}=V_{N_{31}}(s=s_{62})/xdaf221,\;\; B_{460}=V_{N_{32}}(s=s_{62})/xdaf227$$
$$B_{461}=V_{N_{33}}(s=s_{62})/xdaf234,\;\; B_{462}=V_{N_{34}}(s=s_{62})/xdaf242$$
$$B_{463}=V_{N_{35}}(s=s_{62})/xdaf251,\;\; B_{464}=V_{N_{36}}(s=s_{62})/xdaf216$$
$$B_{465}=V_{N_{37}}(s=s_{62})/xdaf216,\;\; B_{466}=6\,\lambda_s^3/xdaf216$$
$$B_{467}=V_{N_{30}}(s=s_{63})/xdaf217,\;\; B_{468}=V_{N_{31}}(s=s_{63})/xdaf222$$
$$B_{469}=V_{N_{32}}(s=s_{63})/xdaf228,\;\; B_{470}=V_{N_{33}}(s=s_{63})/xdaf235$$
$$B_{471}=V_{N_{34}}(s=s_{63})/xdaf243,\;\; B_{472}=V_{N_{35}}(s=s_{63})/xdaf252$$
$$B_{473}=V_{N_{36}}(s=s_{63})/xdaf217,\;\; B_{474}=V_{N_{37}}(s=s_{63})/xdaf217$$
$$B_{475}=6\,\lambda_s^3/xdaf217,\;\; B_{476}=V_{N_{30}}(s=s_{64})/xdaf218$$
$$B_{477}=V_{N_{31}}(s=s_{64})/xdaf223,\;\; B_{478}=V_{N_{32}}(s=s_{64})/xdaf229$$
$$B_{479}=V_{N_{33}}(s=s_{64})/xdaf236,\;\; B_{480}=V_{N_{34}}(s=s_{64})/xdaf244$$
$$B_{481}=V_{N_{35}}(s=s_{64})/xdaf253,\;\; B_{482}=V_{N_{36}}(s=s_{64})/xdaf218$$
$$B_{483}=V_{N_{37}}(s=s_{64})/xdaf218,\;\; B_{484}=6\,\lambda_s^3/xdaf218$$
$$B_{485}=V_{N_{31}}(s=-C_{3,3}(1))/xdaf219,\;\; B_{486}=V_{N_{32}}(s=-C_{3,3}(1))/xdaf225$$
$$B_{487}=V_{N_{33}}(s=-C_{3,3}(1))/xdaf232,\;\; B_{488}=V_{N_{34}}(s=-C_{3,3}(1))/xdaf240$$
$$B_{489}=V_{N_{35}}(s=-C_{3,3}(1))/xdaf249,\;\; B_{490}=V_{N_{32}}(s=-C_{3,3}(2))/xdaf224$$
$$B_{491}=V_{N_{33}}(s=-C_{3,3}(2))/xdaf231,\;\; B_{492}=V_{N_{34}}(s=-C_{3,3}(2))/xdaf239$$
$$B_{493}=V_{N_{35}}(s=-C_{3,3}(2))/xdaf248,\;\; B_{494}=V_{N_{33}}(s=-C_{3,3}(3))/xdaf230$$
$$B_{495}=V_{N_{34}}(s=-C_{3,3}(3))/xdaf238,\;\; B_{496}=V_{N_{35}}(s=-C_{3,3}(3))/xdaf247$$
$$B_{497}=V_{N_{34}}(s=-C_{3,3}(4))/xdaf237,\;\; B_{498}=V_{N_{35}}(s=-C_{3,3}(4))/xdaf246$$
$$B_{499}=V_{N_{35}}(s=-C_{3,3}(5))/xdaf245$$
$$k_{25}=A_{450}+A_{502}+A_{529}+A_{551}+A_{583}+A_{603}+A_{617}$$
$$k_{26}=A_{618},\;\; k_{27}=A_{502}+C_{1,3}(0)\,A_{501}-\mu_1\,A_{449}-\mu_{s0'}\,A_{582}$$
$$k_{28}=A_{449}+A_{501}+A_{528}+A_{550}+A_{582}+A_{602}+A_{616}$$

$$k_{29}=A_{785}+A_{895}+A_{971}+A_{1013}+A_{1041}+A_{1099}+A_{1139}+A_{1172}$$
$$k_{30}=A_{1173},\;\; k_{31}=A_{785}+C_{2,3}(0)\,A_{784}-\mu_1\,A_{894}-A_{1098}$$
$$k_{32}=A_{784}+A_{894}+A_{970}+A_{1012}+A_{1040}+A_{1098}+A_{1138}+A_{1171}$$

$$k_{33} = A_{1336} + A_{1502} + A_{1608} + A_{1676} + A_{1726} + A_{1760} + A_{1844} + A_{1921} + A_{1965}$$

$$k_{34} = A_{1966}, \;\; k_{35} = A_{1336} + C_{3,3}(0)\, A_{1335} - \mu_1\, A_{1501} - \mu_{s0'}\, A_{1843}$$

$$k_{36} = A_{1335} + A_{1501} + A_{1607} + A_{1675} + A_{1725} + A_{1759} + A_{1843} + A_{1920} + A_{1964}$$

Appendix E

The Non-Identical Unit System With Early Standby Activation: Definition of Constants

E.1 Non-identical unit 1-out-of-(2+1):G system: Definition of Constants

The following constants are defined for the analysis performed for the non-identical unit $1 - out - of - (2+1) : G$ system under SBAP Y discussed in Section 3.2.

$$A_{2500} = C_{2,1}(1) + C_{2,1}(5),\ \ A_{2501} = C_{2,1}(1)\,C_{2,1}(5),\ \ A_{2502} = C_{2,1}(2) + C_{2,1}(6)$$
$$A_{2503} = C_{2,1}(1)\,C_{2,1}(6),\ \ A_{2504} = A_{2500}\,A_{2503} + A_{2501}\,A_{2502}$$
$$A_{2505} = A_{2501}\,A_{2503},\ \ A_{2506} = C_{2,1}(3) + C_{2,1}(4),\ \ A_{2507} = C_{2,1}(3)\,C_{2,1}(4)$$
$$A_{2508} = A_{2505}\,A_{2506} + A_{2504}\,A_{2507},\ \ A_{2509} = A_{2505}\,A_{2507}$$
$$A_{2510} = C_{2,1}(2)\,C_{2,1}(4) + C_{2,1}(2)\,C_{2,1}(6) + C_{2,1}(4)\,C_{2,1}(6)$$
$$A_{2511} = C_{2,1}(2)\,C_{2,1}(4)\,C_{2,1}(6),\ \ A_{2512} = \lambda_a\,(C_{2,1}(3) + C_{2,1}(5))$$
$$A_{2513} = \lambda_a\,C_{2,1}(3)\,C_{2,1}(5),\ \ A_{2514} = A_{2510}\,A_{2513} + A_{2511}\,A_{2512}$$
$$A_{2515} = A_{2511}\,A_{2513},\ \ A_{2516} = C_{2,1}(1)\,C_{2,1}(4) + C_{2,1}(1)\,C_{2,1}(5) + C_{2,1}(4)\,C_{2,1}(5)$$
$$A_{2517} = C_{2,1}(1)\,C_{2,1}(4)\,C_{2,1}(5),\ \ A_{2518} = \lambda_b\,(C_{2,1}(3) + C_{2,1}(6))$$
$$A_{2519} = \lambda_b\,C_{2,1}(3)\,C_{2,1}(6),\ \ A_{2520} = A_{2516}\,A_{2519} + A_{2517}\,A_{2518}$$
$$A_{2521} = A_{2517}\,A_{2519},\ \ A_{2522} = A_{2516}$$

$A_{2523} = C_{2,1}(1)\,C_{2,1}(4)\,C_{2,1}(5),\;\; A_{2524} = \lambda_{sd}\,(C_{2,1}(2) + C_{2,1}(6))$

$A_{2525} = \lambda_{sd}\,C_{2,1}(2)\,C_{2,1}(6),\;\; A_{2526} = A_{2522}\,A_{2525} + A_{2523}\,A_{2524}$

$A_{2527} = A_{2523}\,A_{2525},\;\; A_{2528} = \lambda_a\,\lambda_b\,(C_{2,1}(3) + C_{2,1}(6))$

$A_{2529} = \lambda_a\,\lambda_b\,C_{2,1}(3)\,C_{2,1}(6),\;\; A_{2530} = \lambda_a\,\lambda_b\,(C_{2,1}(3) + C_{2,1}(5))$

$A_{2531} = \lambda_a\,\lambda_b\,C_{2,1}(3)\,C_{2,1}(5),\;\; A_{2532} = C_{2,1}(2) + C_{2,1}(6)$

$A_{2533} = C_{2,1}(2)\,C_{2,1}(6),\;\; A_{2534} = C_{2,1}(1) + C_{2,1}(5),\;\; A_{2535} = C_{2,1}(1)\,C_{2,1}(5)$

$A_{2536} = A_{2528}\,A_{2535} + A_{2529}\,A_{2534},\;\; A_{2537} = A_{2529}\,A_{2535}$

$A_{2538} = A_{2530}\,A_{2533} + A_{2531}\,A_{2532},\;\; A_{2539} = A_{2531}\,A_{2533}$

$A_{2540} = A_{2536} + A_{2538},\;\; A_{2541} = A_{2537} + A_{2539}$

$A_{2542} = \lambda_a\,(C_{2,1}(2)\,C_{2,1}(4) + C_{2,1}(2)\,C_{2,1}(6) + C_{2,1}(4)\,C_{2,1}(6))$

$A_{2543} = \lambda_a\,C_{2,1}(2)\,C_{2,1}(4)\,C_{2,1}(6),\;\; A_{2544} = \lambda_{sd} + \lambda_d$

$A_{2545} = \lambda_{sd}\,C_{2,1}(1) + \lambda_d\,C_{2,1}(3),\;\; A_{2546} = A_{2542}\,A_{2545} + A_{2543}\,A_{2544}$

$A_{2547} = A_{2543}\,A_{2545},\;\; A_{2548} = C_{2,1}(1)\,C_{2,1}(4) + C_{2,1}(1)\,C_{2,1}(5) + C_{2,1}(4)\,C_{2,1}(5)$

$A_{2549} = C_{2,1}(1)\,C_{2,1}(4)\,C_{2,1}(5),\;\; A_{2550} = \lambda_b\,\lambda_{sd} + \lambda_b\,\lambda_d$

$A_{2551} = \lambda_b\,\lambda_{sd}\,C_{2,1}(2) + \lambda_b\,\lambda_d\,C_{2,1}(3),\;\; A_{2552} = A_{2548}\,A_{2551} + A_{2549}\,A_{2550}$

$A_{2553} = A_{2549}\,A_{2551},\;\; A_{2554} = A_{2509} + C_{2,1}(0)\,A_{2508} - \mu_{sd}\,A_{2526}$

$A_{2555} = C_{2,1}(0)\,A_{2509} - \mu_{sd}\,A_{2527},\;\; A_{2556} = \lambda_{sd}\,A_{2540} + \lambda_b\,A_{2546} + \lambda_a\,A_{2552}$

$A_{2557} = \lambda_d\,A_{2541} + \lambda_b\,A_{2547} + \lambda_a\,A_{2553}$

$A_{2558} = \lambda_{c0}\,A_{2508} + \lambda_{c1}\,A_{2514} + \lambda_{c2}\,A_{2520} + \lambda_{c3}\,A_{2526} + \lambda_{c4}\,A_{2540} + \lambda_{c5}\,A_{2546} + \lambda_{c6}\,A_{2552}$

$A_{2559} = \lambda_{c0}\,A_{2509} + \lambda_{c1}\,A_{2515} + \lambda_{c2}\,A_{2521} + \lambda_{c3}\,A_{2527} + \lambda_{c4}\,A_{2541} + \lambda_{c5}\,A_{2547} + \lambda_{c6}\,A_{2553}$

$A_{2560} = A_{2508} + A_{2514} + A_{2520} + A_{2526} + A_{2540} + A_{2546} + A_{2552}$

$A_{2561} = A_{2509} + A_{2515} + A_{2521} + A_{2527} + A_{2541} + A_{2547} + A_{2553}$

$BN_{620} = (s_{80} + C_{2,1}(3))/(s_{80} - s_{81}),\;\; BN_{621} = (s_{81} + C_{2,1}(3))/(s_{81} - s_{80})$

$BN_{622} = \lambda_a\,(C_{2,1}(3) - C_{2,1}(1))/((C_{2,1}(1) - s_{80})\,(C_{2,1}(1) - s_{81}))$

$BN_{623} = \lambda_a\,(s_{80} + C_{2,1}(3))/((s_{80} + C_{2,1}(1))\,(s_{80} - s_{81}))$

$BN_{624} = \lambda_a\,(s_{81} + C_{2,1}(3))/((s_{81} + C_{2,1}(1))\,(s_{81} - s_{80}))$

$BN_{625} = \lambda_b\,(C_{2,1}(3) - C_{2,1}(2))/((C_{2,1}(2) - s_{80})\,(C_{2,1}(2) - s_{81}))$

$BN_{626} = \lambda_b\,(s_{80} + C_{2,1}(3))/((s_{80} + C_{2,1}(2))\,(s_{80} - s_{81}))$

$BN_{627} = \lambda_b\,(s_{81} + C_{2,1}(3))/((s_{81} + C_{2,1}(2))\,(s_{81} - s_{80}))$

$BN_{628} = \lambda_{sd}/(s_{80} - s_{81}),\;\; BN_{629} = \lambda_{sd}/(s_{81} - s_{80})$

$DV1(s) = \lambda_a\,\lambda_b\,(s + C_{2,1}(3))\,(s + C_{2,1}(2)) + \lambda_a\,\lambda_b\,(s + C_{2,1}(3))\,(s + C_{2,1}(1))$

$xdif1 = (-C_{2,1}(4) + C_{2,1}(1))(-C_{2,1}(4) + C_{2,1}(2))(-C_{2,1}(4) - s_{80})(-C_{2,1}(4) - s_{81})$

$xdif2 = (-C_{2,1}(2) + C_{2,1}(4))(-C_{2,1}(2) + C_{2,1}(1))(-C_{2,1}(2) - s_{80})(-C_{2,1}(2) - s_{81})$

$xdif3 = (-C_{2,1}(1) + C_{2,1}(4))(-C_{2,1}(1) + C_{2,1}(2))(-C_{2,1}(1) - s_{80})(-C_{2,1}(1) - s_{81})$

$xdif4 = (s_{80} + C_{2,1}(1))\,(s_{80} + C_{2,1}(2))\,(s_{80} + C_{2,1}(4))\,(s_{80} - s_{81})$

$xdif5 = (s_{81} + C_{2,1}(1))\,(s_{81} + C_{2,1}(2))\,(s_{81} + C_{2,1}(4))\,(s_{81} - s_{80})$

$BN_{630} = DV1(s = -C_{2,1}(4))/xdif1,\ \ BN_{631} = DV1(s = -C_{2,1}(2))/xdif2$

$BN_{632} = DV1(s = -C_{2,1}(1))/xdif3,\ \ BN_{633} = DV1(s = s_{80})/xdif4$

$BN_{634} = DV1(s = s_{81})/xdif5$

$DV2(s) = \lambda_a\,\lambda_d\,(s + C_{2,1}(3)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,1}(1))$

$xdif6 = (-C_{2,1}(5) + C_{2,1}(1))(-C_{2,1}(5) - s_{80})(-C_{2,1}(5) - s_{81})$

$xdif7 = (-C_{2,1}(1) + C_{2,1}(5))(-C_{2,1}(1) - s_{80})(-C_{2,1}(1) - s_{81})$

$xdif8 = (s_{80} + C_{2,1}(1))\,(s_{80} + C_{2,1}(5))\,(s_{80} - s_{81})$

$xdif9 = (s_{81} + C_{2,1}(1))\,(s_{81} + C_{2,1}(5))\,(s_{81} - s_{80})$

$BN_{635} = DV2(s = -C_{2,1}(5))/xdif6,\ \ BN_{636} = DV2(s = -C_{2,1}(1))/xdif7$

$BN_{637} = DV2(s = s_{80})/xdif8,\ \ BN_{638} = DV1(s = s_{81})/xdif9$

$DV3(s) = \lambda_b\,\lambda_d\,(s + C_{2,1}(3)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,1}(2))$

$xdif10 = (-C_{2,1}(6) + C_{2,1}(2))(-C_{2,1}(6) - s_{80})(-C_{2,1}(6) - s_{81})$

$xdif11 = (-C_{2,1}(2) + C_{2,1}(6))(-C_{2,1}(2) - s_{80})(-C_{2,1}(2) - s_{81})$

$xdif12 = (s_{80} + C_{2,1}(2))\,(s_{80} + C_{2,1}(6))\,(s_{80} - s_{81})$

$xdif13 = (s_{81} + C_{2,1}(2))\,(s_{81} + C_{2,1}(6))\,(s_{81} - s_{80})$

$BN_{639} = DV3(s = -C_{2,1}(6))/xdif10,\ \ BN_{640} = DV3(s = -C_{2,1}(2))/xdif11$

$BN_{641} = DV3(s = s_{80})/xdif12,\ \ BN_{642} = DV3(s = s_{81})/xdif13$

$k_{69} = A_{2561},\ \ k_{70} = A_{2555},\ \ k_{71} = A_{2554},\ \ k_{72} = A_{2560}$

E.2 Non-identical unit 1-out-of-(2+2):G system: Definition of constants

The following constants are defined for the analysis performed for the non-identical unit $1 - out - of - (2+2) : G$ system under SBAP Y discussed in Section 3.2.

$A_{2670} = 2\,\lambda_a\,\lambda_b,\ \ A_{2671} = \lambda_a\,\lambda_b\,(C_{2,2}(1) + C_{2,2}(2))$

$A_{2672} = C_{2,2}(1)\,C_{2,2}(2) + C_{2,2}(1)\,C_{2,2}(5) + C_{2,2}(2)\,C_{2,2}(5)$

$A_{2673} = C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(5),\ A_{2674} = \lambda_a\,\lambda_d + \lambda_a\,\lambda_{sd}$

$A_{2675} = \lambda_a\,\lambda_d\,C_{2,2}(3) + \lambda_a\,\lambda_{sd}\,C_{2,2}(1)$

$A_{2676} = C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(1)\,C_{2,2}(6) + C_{2,2}(3)\,C_{2,2}(6)$

$A_{2677} = C_{2,2}(1)\,C_{2,2}(3)\,C_{2,2}(6),\;\; A_{2678} = 2\,\lambda_a\,\lambda_{se},\;\; A_{2679} = \lambda_a\,\lambda_{se}\,(C_{2,2}(4) + C_{2,2}(1))$

$A_{2680} = C_{2,2}(1)\,C_{2,2}(4) + C_{2,2}(1)\,C_{2,2}(7) + C_{2,2}(4)\,C_{2,2}(7)$

$A_{2681} = C_{2,2}(1)\,C_{2,2}(4)\,C_{2,2}(7),\;\; A_{2682} = \lambda_b\,\lambda_d + \lambda_b\,\lambda_{sd}$

$A_{2683} = \lambda_b\,\lambda_d\,C_{2,2}(3) + \lambda_b\,\lambda_{sd}\,C_{2,2}(2)$

$A_{2684} = C_{2,2}(2)\,C_{2,2}(3) + C_{2,2}(2)\,C_{2,2}(8) + C_{2,2}(3)\,C_{2,2}(8)$

$A_{2685} = C_{2,2}(2)\,C_{2,2}(3)\,C_{2,2}(8),\; A_{2686} = 2\,\lambda_b\,\lambda_{se},\;\; A_{2687} = \lambda_b\,\lambda_{se}\,(C_{2,2}(2) + C_{2,2}(4))$

$A_{2688} = C_{2,2}(2)\,C_{2,2}(4) + C_{2,2}(2)\,C_{2,2}(9) + C_{2,2}(4)\,C_{2,2}(9)$

$A_{2689} = C_{2,2}(2)\,C_{2,2}(4)\,C_{2,2}(9),\; A_{2690} = 2\,\lambda_{sd}\,\lambda_{se},\;\; A_{2691} = \lambda_{sd}\,\lambda_{se}\,(C_{2,2}(3) + C_{2,2}(4))$

$A_{2692} = C_{2,2}(3)\,C_{2,2}(4) + C_{2,2}(3)\,C_{2,2}(10) + C_{2,2}(4)\,C_{2,2}(10)$

$A_{2693} = C_{2,2}(3)\,C_{2,2}(4)\,C_{2,2}(10),\;\; A_{2694} = \lambda_a\,\lambda_b\,\lambda_d,\;\; A_{2695} = A_{2694}\,C_{2,2}(3)$

$$\begin{aligned}
A_{2696} &= C_{2,2}(2)\,C_{2,2}(6) + 2\,C_{2,2}(2)\,C_{2,2}(8) + C_{2,2}(6)\,C_{2,2}(8) + 2\,C_{2,2}(1)\,C_{2,2}(6) + \\
&\quad C_{2,2}(1)\,C_{2,2}(8) + C_{2,2}(6)\,C_{2,2}(8) + +C_{2,2}(2)\,C_{2,2}(5) + C_{2,2}(5)\,C_{2,2}(8) + \\
&\quad C_{2,2}(1)\,C_{2,2}(5) + C_{2,2}(5)\,C_{2,2}(6) \\
A_{2697} &= C_{2,2}(2)\,C_{2,2}(6)\,C_{2,2}(8) + C_{2,2}(1)\,C_{2,2}(6)\,C_{2,2}(8) + \\
&\quad C_{2,2}(2)\,C_{2,2}(5)\,C_{2,2}(3) + C_{2,2}(1)\,C_{2,2}(5)\,C_{2,2}(6) \\
A_{2698} &= \lambda_a\,\lambda_b\,\lambda_{sd},\;\; A_{2699} = A_{2698}\,C_{2,2}(1) \\
A_{2700} &= 2\,C_{2,2}(2)\,C_{2,2}(5) + C_{2,2}(2)\,C_{2,2}(8) + C_{2,2}(5)\,C_{2,2}(8) + \\
&\quad C_{2,2}(2)\,C_{2,2}(6) + C_{2,2}(5)\,C_{2,2}(6)
\end{aligned}$$

$A_{2701} = C_{2,2}(2)\,C_{2,2}(5)\,C_{2,2}(8) + C_{2,2}(2)\,C_{2,2}(5)\,C_{2,2}(6)$

$A_{2702} = A_{2694}\,A_{2697} + A_{2695}\,A_{2696},\;\; A_{2703} = A_{2695}\,A_{2697}$

$A_{2704} = A_{2698}\,A_{2701} + A_{2699}\,A_{2700},\;\; A_{2705} = A_{2699}\,A_{2701}$

$A_{2706} = C_{2,2}(1)\,C_{2,2}(2) + C_{2,2}(1)\,C_{2,2}(3) + C_{2,2}(2)\,C_{2,2}(3)$

$A_{2707} = C_{2,2}(1)\,C_{2,2}(2)\,C_{2,2}(3)$

$A_{2708} = C_{2,2}(5)\,C_{2,2}(6) + C_{2,2}(5)\,C_{2,2}(8) + C_{2,2}(6)\,C_{2,2}(8)$

$A_{2709} = C_{2,2}(5)\,C_{2,2}(6)\,C_{2,2}(8),\; A_{2710} = A_{2706}\,A_{2709} + A_{2707}\,A_{2708}$

$A_{2711} = A_{2707}\,A_{2709},\;\; A_{2712} = A_{2710}\,C_{2,2}(11) + A_{2711},\;\; A_{2713} = A_{2711}\,C_{2,2}(11)$

$A_{2714} = A_{2702} + A_{2704},\;\; A_{2715} = A_{2703}\,A_{2705}$

$A_{2716} = A_{2671}\,A_{2681}\,A_{2688} + A_{2670}\,A_{2681}\,A_{2689} + A_{2671}\,A_{2680}\,A_{2689}$

$A_{2717} = A_{2671}\,A_{2681}\,A_{2689}$

$$A_{2718} = A_{2673}\,A_{2678}\,A_{2689} + A_{2672}\,A_{2679}\,A_{2689} + A_{2673}\,A_{2679}\,A_{2688}$$

$$A_{2719} = A_{2673}\,A_{2679}\,A_{2689}$$

$$A_{2720} = A_{2673}\,A_{2680}\,A_{2687} + A_{2673}\,A_{2681}\,A_{2686} + A_{2672}\,A_{2681}\,A_{2687}$$

$$A_{2721} = A_{2673}\,A_{2681}\,A_{2687}$$

$$A_{2722} = A_{2672}\,A_{2681}\,A_{2689} + A_{2673}\,A_{2680}\,A_{2689} + A_{2673}\,A_{2681}\,A_{2688}$$

$$A_{2723} = A_{2673}\,A_{2681}\,A_{2689},\ A_{2724} = A_{2723} + C_{2,2}(12)\,A_{2722},\quad A_{2725} = C_{2,2}(12)\,A_{2723}$$

$$A_{2726} = \lambda_e\,A_{2716} + \lambda_b\,A_{2718} + \lambda_a\,A_{2720},\quad A_{2727} = \lambda_e\,A_{2717} + \lambda_b\,A_{2719} + \lambda_a\,A_{2721}$$

$$A_{2728} = A_{2675}\,A_{2681}\,A_{2692} + A_{2674}\,A_{2681}\,A_{2693} + A_{2675}\,A_{2680}\,A_{2693}$$

$$A_{2729} = A_{2675}\,A_{2681}\,A_{2693}$$

$$A_{2730} = A_{2677}\,A_{2678}\,A_{2693} + A_{2676}\,A_{2679}\,A_{2693} + A_{2677}\,A_{2679}\,A_{2692}$$

$$A_{2731} = A_{2677}\,A_{2679}\,A_{2693}$$

$$A_{2732} = A_{2677}\,A_{2680}\,A_{2691} + A_{2677}\,A_{2681}\,A_{2690} + A_{2676}\,A_{2681}\,A_{2691}$$

$$A_{2733} = A_{2677}\,A_{2681}\,A_{2691}$$

$$A_{2732a} = A_{2676}\,A_{2681}\,A_{2693} + A_{2677}\,A_{2680}\,A_{2693} + A_{2677}\,A_{2681}\,A_{2692}$$

$$A_{2733a} = A_{2677}\,A_{2681}\,A_{2693},\ A_{2734} = A_{2733a} + C_{2,2}(13)\,A_{2732a},\quad A_{2735} = C_{2,2}(13)\,A_{2733a}$$

$$A_{2736} = \lambda_e\,A_{2728} + \lambda_d\,A_{2730} + \lambda_a\,A_{2732},\quad A_{2737} = \lambda_e\,A_{2729} + \lambda_d\,A_{2731} + \lambda_a\,A_{2733}$$

$$A_{2736a} = A_{2682}\,A_{2689}\,A_{2693} + A_{2683}\,A_{2688}\,A_{2693} + A_{2683}\,A_{2689}\,A_{2692}$$

$$A_{2737a} = A_{2683}\,A_{2689}\,A_{2693}$$

$$A_{2738} = A_{2685}\,A_{2686}\,A_{2693} + A_{2684}\,A_{2687}\,A_{2693} + A_{2685}\,A_{2687}\,A_{2692}$$

$$A_{2739} = A_{2685}\,A_{2687}\,A_{2693}$$

$$A_{2740} = A_{2685}\,A_{2689}\,A_{2690} + A_{2684}\,A_{2689}\,A_{2691} + A_{2685}\,A_{2688}\,A_{2691}$$

$$A_{2741} = A_{2685}\,A_{2689}\,A_{2691}$$

$$A_{2742} = A_{2684}\,A_{2689}\,A_{2693} + A_{2685}\,A_{2688}\,A_{2693} + A_{2685}\,A_{2689}\,A_{2692}$$

$$A_{2743} = A_{2685}\,A_{2689}\,A_{2693},\ A_{2744} = A_{2743} + C_{2,2}(14)\,A_{2742}$$

$$A_{2745} = C_{2,2}(14)\,A_{2743},\quad A_{2746} = \lambda_e\,A_{2736a} + \lambda_d\,A_{2738} + \lambda_b\,A_{2740}$$

$$A_{2747} = \lambda_e\,A_{2737a} + \lambda_d\,A_{2739} + \lambda_b\,A_{2741},\ A_{2748} = A_{2714}\,A_{2725} + A_{2715}\,A_{2724}$$

$$A_{2749} = A_{2715}\,A_{2725},\quad A_{2750} = A_{2734}\,A_{2745} + A_{2735}\,A_{2744},\ A_{2751} = A_{2735}\,A_{2745}$$

$$A_{2752} = A_{2726}\,A_{2713} + A_{2727}\,A_{2712},\ A_{2753} = A_{2727}\,A_{2713},\ A_{2754} = A_{2750}$$

$$A_{2755} = A_{2751},\quad A_{2756} = A_{2736}\,A_{2713} + A_{2737}\,A_{2712},\quad A_{2757} = A_{2737}\,A_{2713}$$

$$A_{2758} = A_{2724}\,A_{2745} + A_{2725}\,A_{2744},\quad A_{2759} = A_{2725}\,A_{2745}$$

$$A_{2760} = A_{2746}\,A_{2713} + A_{2747}\,A_{2712},\quad A_{2761} = A_{2747}\,A_{2713}$$

$$A_{2762} = A_{2724}\,A_{2735} + A_{2725}\,A_{2734},\quad A_{2763} = A_{2725}\,A_{2735}$$

$$A_{2764} = A_{2748}\,A_{2751} + A_{2749}\,A_{2750},\quad A_{2765} = A_{2749}\,A_{2751}$$

$$A_{2766} = A_{2752}\,A_{2755} + A_{2753}\,A_{2754}, \quad A_{2767} = A_{2753}\,A_{2755}$$
$$A_{2768} = A_{2756}\,A_{2759} + A_{2757}\,A_{2758}, \quad A_{2769} = A_{2757}\,A_{2759}$$
$$A_{2770} = A_{2760}\,A_{2763} + A_{2761}\,A_{2762}, \quad A_{2771} = A_{2761}\,A_{2763}$$
$$A_{2772} = A_{2672}\,A_{2677} + A_{2673}\,A_{2676}, \quad A_{2773} = A_{2673}\,A_{2677}$$
$$A_{2774} = A_{2680}\,A_{2685} + A_{2681}\,A_{2684}, \quad A_{2775} = A_{2681}\,A_{2685}$$
$$A_{2776} = A_{2688}\,A_{2693} + A_{2689}\,A_{2692}, \quad A_{2777} = A_{2689}\,A_{2693}$$
$$A_{2778} = A_{2712}\,A_{2725} + A_{2713}\,A_{2724}, \quad A_{2779} = A_{2713}\,A_{2725}$$
$$A_{2780} = A_{2734}\,A_{2745} + A_{2735}\,A_{2744}, \quad A_{2781} = A_{2735}\,A_{2745}$$
$$A_{2782} = A_{2772}\,A_{2775} + A_{2773}\,A_{2774}, \quad A_{2783} = A_{2773}\,A_{2775}$$
$$A_{2784} = A_{2776}\,A_{2779} + A_{2777}\,A_{2778}, \quad A_{2785} = A_{2777}\,A_{2779}$$
$$A_{2786} = A_{2782}\,A_{2785} + A_{2783}\,A_{2784}, \quad A_{2787} = A_{2783}\,A_{2735}$$
$$A_{2788} = A_{2781}\,A_{2786} + A_{2780}\,A_{2787}, \quad A_{2789} = A_{2781}\,A_{2787}$$
$$A_{2790} = A_{2789} + C_{2,2}(4)\,A_{2788}, \quad A_{2791} = C_{2,2}(4)\,A_{2789}$$
$$A_{2792} = A_{2791} + C_{2,2}(3)\,A_{2790}, \quad A_{2793} = C_{2,2}(3)\,A_{2791}$$
$$A_{2794} = A_{2793} + C_{2,2}(2)\,A_{2792}, \quad A_{2795} = C_{2,2}(2)\,A_{2793}$$
$$A_{2796} = A_{2795} + C_{2,2}(1)\,A_{2794}, \quad A_{2797} = C_{2,2}(1)\,A_{2795}$$
$$A_{2798} = (A_{2673}\,A_{2796} - A_{2672}\,A_{2797})/A_{2673}^2, \quad A_{2799} = A_{2797}/A_{2673}$$
$$A_{2800} = (A_{2677}\,A_{2796} - A_{2676}\,A_{2797})/A_{2677}^2, \quad A_{2801} = A_{2797}/A_{2677}$$
$$A_{2802} = (A_{2681}\,A_{2796} - A_{2680}\,A_{2797})/A_{2681}^2, \quad A_{2803} = A_{2797}/A_{2681}$$
$$A_{2804} = (A_{2685}\,A_{2796} - A_{2684}\,A_{2797})/A_{2685}^2, \quad A_{2805} = A_{2797}/A_{2685}$$
$$A_{2806} = (A_{2689}\,A_{2796} - A_{2688}\,A_{2797})/A_{2689}^2, \quad A_{2807} = A_{2797}/A_{2689}$$
$$A_{2808} = (A_{2693}\,A_{2796} - A_{2692}\,A_{2797})/A_{2693}^2, \quad A_{2809} = A_{2797}/A_{2693}$$
$$A_{2810} = (A_{2713}\,A_{2796} - A_{2712}\,A_{2797})/A_{2713}^2, \quad A_{2811} = A_{2797}/A_{2713}$$
$$A_{2812} = (A_{2725}\,A_{2796} - A_{2724}\,A_{2797})/A_{2725}^2, \quad A_{2813} = A_{2797}/A_{2725}$$
$$A_{2814} = (A_{2735}\,A_{2796} - A_{2734}\,A_{2797})/A_{2735}^2, \quad A_{2815} = A_{2797}/A_{2735}$$
$$A_{2816} = (A_{2745}\,A_{2796} - A_{2744}\,A_{2797})/A_{2745}^2, \quad A_{2817} = A_{2797}/A_{2745}$$
$$A_{2818} = A_{2778}\,A_{2781} + A_{2779}\,A_{2780}, \quad A_{2819} = A_{2779}\,A_{2781}$$
$$A_{2820} = (A_{2819}\,A_{2796} - A_{2818}\,A_{2797})/A_{2819}^2, \quad A_{2821} = A_{2797}/A_{2819}$$
$$A_{2822} = A_{2764}\,A_{2821} + A_{2765}\,A_{2820}, \quad A_{2823} = A_{2765}\,A_{2821}$$
$$A_{2824} = A_{2766}\,A_{2821} + A_{2767}\,A_{2820}, \quad A_{2825} = A_{2767}\,A_{2821}$$
$$A_{2826} = A_{2768}\,A_{2821} + A_{2769}\,A_{2820}, \quad A_{2827} = A_{2769}\,A_{2821}$$
$$A_{2828} = A_{2770}\,A_{2821} + A_{2771}\,A_{2820}, \quad A_{2829} = A_{2771}\,A_{2821}$$
$$A_{2830} = \lambda_e\,A_{2822} + \lambda_d\,A_{2824} + \lambda_b\,A_{2826} + \lambda_a\,A_{2828}$$

$A_{2831} = \lambda_e\, A_{2823} + \lambda_d\, A_{2825} + \lambda_b\, A_{2827} + \lambda_a\, A_{2829}$

$$\begin{aligned}
A_{2832} &= \lambda_{c0}\, A_{2796} + \lambda_{c1}\, \lambda_a\, A_{2794} + \lambda_{c2}\, \lambda_b\, A_{2792} + \lambda_{c3}\, \lambda_{sd}\, A_{2790} + \lambda_{c4}\, \lambda_{se}\, A_{2788} + \\
&\quad \lambda_{c5}\, (A_{2670}\, A_{2799} + A_{2671}\, A_{2798}) + \lambda_{c6}\, (A_{2674}\, A_{2801} + A_{2675}\, A_{2800}) + \\
&\quad \lambda_{c7}\, (A_{2678}\, A_{2803} + A_{2679}\, A_{2802}) + \lambda_{c8}\, (A_{2682}\, A_{2805} + A_{2683}\, A_{2804}) + \\
&\quad \lambda_{c9}\, (A_{2686}\, A_{2807} + A_{2687}\, A_{2806}) + \lambda_{c10}\, (A_{2690}\, A_{2809} + A_{2691}\, A_{2808}) + \\
&\quad \lambda_{c11}\, (A_{2714}\, A_{2811} + A_{2715}\, A_{2810}) + \lambda_{c12}\, (A_{2726}\, A_{2813} + A_{2727}\, A_{2812}) + \\
&\quad \lambda_{c13}\, (A_{2736}\, A_{2815} + A_{2737}\, A_{2814}) + \lambda_{c14}\, (A_{2746}\, A_{2817} + A_{2747}\, A_{2816}) \\
A_{2833} &= \lambda_{c0}\, A_{2797} + \lambda_{c1}\, \lambda_a\, A_{2795} + \lambda_{c2}\, \lambda_b\, A_{2793} + \lambda_{c3}\, \lambda_{sd}\, A_{2791} + \lambda_{c4}\, \lambda_{se}\, A_{2789} + \\
&\quad \lambda_{c5}\, A_{2671}\, A_{2799} + \lambda_{c6}\, A_{2675}\, A_{2801} + \lambda_{c7}\, A_{2679}\, A_{2803} + \lambda_{c8}\, A_{2683}\, A_{2805} + \\
&\quad \lambda_{c9}\, A_{2687}\, A_{2807} + \lambda_{c10}\, A_{2691}\, A_{2809} + \lambda_{c11}\, A_{2715}\, A_{2811} + \lambda_{c12}\, A_{2727}\, A_{2813} + \\
&\quad \lambda_{c13}\, A_{2737}\, A_{2815} + \lambda_{c14}\, A_{2747}\, A_{2817} \\
A_{2834} &= A_{2797} + C_{2,2}(0)\, A_{2796},\;\; A_{2835} = C_{2,2}(0)\, A_{2797} \\
A_{2836} &= A_{2796} + \lambda_a\, A_{2794} + \lambda_b\, A_{2792} + \lambda_{sd}\, A_{2790} + \lambda_{se}\, A_{2788} + \\
&\quad A_{2670}\, A_{2799} + A_{2671}\, A_{2798} + A_{2674}\, A_{2801} + A_{2675}\, A_{2800} + \\
&\quad A_{2678}\, A_{2803} + A_{2679}\, A_{2802} + A_{2682}\, A_{2805} + A_{2683}\, A_{2804} + \\
&\quad A_{2686}\, A_{2807} + A_{2687}\, A_{2806} + A_{2690}\, A_{2809} + A_{2691}\, A_{2808} + \\
&\quad A_{2714}\, A_{2811} + A_{2715}\, A_{2810} + A_{2726}\, A_{2813} + A_{2727}\, A_{2812} + \\
&\quad A_{2736}\, A_{2815} + A_{2737}\, A_{2814} + A_{2746}\, A_{2817} + A_{2747}\, A_{2816} + \\
A_{2837} &= A_{2797} + \lambda_a\, A_{2795} + \lambda_b\, A_{2793} + \lambda_{sd}\, A_{2791} + \lambda_{se}\, A_{2789} + \\
&\quad A_{2671}\, A_{2799} + A_{2675}\, A_{2801} + A_{2679}\, A_{2803} + A_{2683}\, A_{2805} + \\
&\quad A_{2687}\, A_{2807} + A_{2691}\, A_{2809} + A_{2715}\, A_{2811} + A_{2727}\, A_{2813} + \\
&\quad A_{2737}\, A_{2815} + A_{2747}\, A_{2817}
\end{aligned}$$

$BN_{646} = \lambda_a/(C_{2,2}(0) - C_{2,2}(1)),\; BN_{647} = -BN_{646}$
$BN_{648} = \lambda_b/(C_{2,2}(0) - C_{2,2}(2)),\; BN_{649} = -BN_{648}$
$BN_{650} = \lambda_{sd}/(C_{2,2}(0) - C_{2,2}(3)),\; BN_{651} = -BN_{650}$
$BN_{652} = \lambda_{se}/(C_{2,2}(0) - C_{2,2}(4)),\; BN_{653} = -BN_{652}$
$DV5(s) = \lambda_a\, \lambda_b\, (s + C_{2,2}(2)) + \lambda_a\, \lambda_b\, (s + C_{2,2}(1))$

$$xdif14 = (C_{2,2}(1) - C_{2,2}(5))\,(C_{2,2}(2) - C_{2,2}(5))\,(C_{2,2}(0) - C_{2,2}(5))$$
$$xdif15 = (C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(5) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdif16 = (C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(5) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdif17 = (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(5) - C_{2,2}(0))\,(C_{2,2}(1) - C_{2,2}(0))$$
$$BN_{654} = DV5(s = -C_{2,2}(5))/xdif14,\;\; BN_{655} = DV5(s = -C_{2,2}(2))/xdif15$$
$$BN_{656} = DV5(s = -C_{2,2}(1))/xdif16,\;\; BN_{657} = DV5(s = -C_{2,2}(0))/xdif17$$
$$DV6(s) = \lambda_a\,\lambda_d\,(s + C_{2,2}(3)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(1))$$
$$xdif18 = (C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(3) - C_{2,2}(6))\,(C_{2,2}(0) - C_{2,2}(6))$$
$$xdif19 = (C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(6) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$$
$$xdif20 = (C_{2,2}(3) - C_{2,2}(1))\,(C_{2,2}(6) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdif21 = (C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(6) - C_{2,2}(0))\,(C_{2,2}(1) - C_{2,2}(0))$$
$$BN_{658} = DV6(s = -C_{2,2}(6))/xdif18,\;\; BN_{659} = DV6(s = -C_{2,2}(3))/xdif19$$
$$BN_{660} = DV6(s = -C_{2,2}(1))/xdif20,\;\; BN_{661} = DV6(s = -C_{2,2}(0))/xdif21$$
$$DV7(s) = \lambda_a\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_a\,\lambda_{se}\,(s + C_{2,2}(1))$$
$$xdif22 = (C_{2,2}(1) - C_{2,2}(7))\,(C_{2,2}(4) - C_{2,2}(7))\,(C_{2,2}(0) - C_{2,2}(7))$$
$$xdif23 = (C_{2,2}(1) - C_{2,2}(4))\,(C_{2,2}(7) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$$
$$xdif24 = (C_{2,2}(4) - C_{2,2}(1))\,(C_{2,2}(7) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdif25 = (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(7) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0))$$
$$BN_{662} = DV7(s = -C_{2,2}(7))/xdif22,\;\; BN_{663} = DV7(s = -C_{2,2}(4))/xdif23$$
$$BN_{664} = DV7(s = -C_{2,2}(1))/xdif24,\;\; BN_{665} = DV7(s = -C_{2,2}(0))/xdif25$$
$$DV8(s) = \lambda_a\,\lambda_d\,(s + C_{2,2}(3)) + \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))$$
$$xdif26 = (C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(3) - C_{2,2}(8))\,(C_{2,2}(0) - C_{2,2}(8))$$
$$xdif27 = (C_{2,2}(2) - C_{2,2}(3))\,(C_{2,2}(8) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$$
$$xdif28 = (C_{2,2}(3) - C_{2,2}(2))\,(C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdif29 = (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(8) - C_{2,2}(0))$$
$$BN_{666} = DV8(s = -C_{2,2}(8))/xdif26,\;\; BN_{667} = DV8(s = -C_{2,2}(3))/xdif27$$
$$BN_{668} = DV8(s = -C_{2,2}(2))/xdif28,\;\; BN_{669} = DV8(s = -C_{2,2}(0))/xdif29$$
$$DV9(s) = \lambda_b\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_b\,\lambda_{se}\,(s + C_{2,2}(2))$$
$$xdif30 = (C_{2,2}(4) - C_{2,2}(9))\,(C_{2,2}(2) - C_{2,2}(9))\,(C_{2,2}(0) - C_{2,2}(9))$$
$$xdif31 = (C_{2,2}(9) - C_{2,2}(4))\,(C_{2,2}(2) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$$
$$xdif32 = (C_{2,2}(9) - C_{2,2}(2))\,(C_{2,2}(4) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdif33 = (C_{2,2}(9) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))$$
$$BN_{670} = DV9(s = -C_{2,2}(9))/xdif30,\;\; BN_{671} = DV9(s = -C_{2,2}(4))/xdif31$$

$BN_{672} = DV9(s = -C_{2,2}(2))/xdif32,\ \ BN_{673} = DV9(s = -C_{2,2}(0))/xdif33$

$DV10(s) = \lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(4)) + \lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(3))$

$xdif34 = (C_{2,2}(4) - C_{2,2}(10))\,(C_{2,2}(3) - C_{2,2}(10))\,(C_{2,2}(0) - C_{2,2}(10))$

$xdif35 = (C_{2,2}(10) - C_{2,2}(4))\,(C_{2,2}(3) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$

$xdif36 = (C_{2,2}(10) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$

$xdif37 = (C_{2,2}(10) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))$

$BN_{674} = DV10(s = -C_{2,2}(10))/xdif34,\ \ BN_{675} = DV10(s = -C_{2,2}(4))/xdif35$

$BN_{676} = DV10(s = -C_{2,2}(3))/xdif36,\ \ BN_{677} = DV10(s = -C_{2,2}(0))/xdif37$

$$\begin{aligned}
DV11(s) &= \lambda_d\,(s + C_{2,2}(3))\,(s + C_{2,2}(6))\,(s + C_{2,2}(8))\,[\lambda_a\,\lambda_b\,(s + C_{2,2}(2)) + \\
&\quad \lambda_a\,\lambda_b\,(s + C_{2,2}(1))] + \lambda_b\,(s + C_{2,2}(8))\,(s + C_{2,2}(2))\,(s + C_{2,2}(5))[\\
&\quad \lambda_a\,\lambda_d\,(s + C_{2,2}(3)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(1))] + \lambda_a\,(s + C_{2,2}(1)) \\
&\quad (s + C_{2,2}(5))\,(s + C_{2,2}(6))\,[\lambda_b\,\lambda_d\,(s + C_{2,2}(3)) + \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))] \\
xdif38 &= (C_{2,2}(1) - C_{2,2}(11))\,(C_{2,2}(2) - C_{2,2}(11))\,(C_{2,2}(5) - C_{2,2}(11)) \\
&\quad (C_{2,2}(8) - C_{2,2}(11))\,(C_{2,2}(6) - C_{2,2}(11))\,(C_{2,2}(3) - C_{2,2}(11)) \\
&\quad (C_{2,2}(0) - C_{2,2}(11)) \\
xdif39 &= (C_{2,2}(1) - C_{2,2}(8))\,(C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(5) - C_{2,2}(8)) \\
&\quad (C_{2,2}(11) - C_{2,2}(8))\,(C_{2,2}(6) - C_{2,2}(8))\,(C_{2,2}(3) - C_{2,2}(8)) \\
&\quad (C_{2,2}(0) - C_{2,2}(8)) \\
xdif40 &= (C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(2) - C_{2,2}(6))\,(C_{2,2}(5) - C_{2,2}(6)) \\
&\quad (C_{2,2}(8) - C_{2,2}(6))\,(C_{2,2}(11) - C_{2,2}(6))\,(C_{2,2}(3) - C_{2,2}(6)) \\
&\quad (C_{2,2}(0) - C_{2,2}(6)) \\
xdif41 &= (C_{2,2}(1) - C_{2,2}(5))\,(C_{2,2}(2) - C_{2,2}(5))\,(C_{2,2}(6) - C_{2,2}(5)) \\
&\quad (C_{2,2}(8) - C_{2,2}(5))\,(C_{2,2}(11) - C_{2,2}(5))\,(C_{2,2}(3) - C_{2,2}(5)) \\
&\quad (C_{2,2}(0) - C_{2,2}(5)) \\
xdif42 &= (C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(2) - C_{2,2}(3))\,(C_{2,2}(6) - C_{2,2}(3)) \\
&\quad (C_{2,2}(8) - C_{2,2}(3))\,(C_{2,2}(11) - C_{2,2}(3))\,(C_{2,2}(5) - C_{2,2}(3)) \\
&\quad (C_{2,2}(0) - C_{2,2}(3)) \\
xdif43 &= (C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(3) - C_{2,2}(2))\,(C_{2,2}(6) - C_{2,2}(2)) \\
&\quad (C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(11) - C_{2,2}(2))\,(C_{2,2}(5) - C_{2,2}(2))
\end{aligned}$$

$$
\begin{aligned}
& (C_{2,2}(0) - C_{2,2}(2)) \\
xdif44 &= (C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(3) - C_{2,2}(1))\,(C_{2,2}(6) - C_{2,2}(1)) \\
& (C_{2,2}(8) - C_{2,2}(1))\,(C_{2,2}(11) - C_{2,2}(1))\,(C_{2,2}(5) - C_{2,2}(1)) \\
& (C_{2,2}(0) - C_{2,2}(1)) \\
xdif45 &= (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(6) - C_{2,2}(0)) \\
& (C_{2,2}(8) - C_{2,2}(0))\,(C_{2,2}(11) - C_{2,2}(0))\,(C_{2,2}(5) - C_{2,2}(0)) \\
& (C_{2,2}(1) - C_{2,2}(0))
\end{aligned}
$$

$$
\begin{aligned}
& BN_{678} = DV11(s = -C_{2,2}(11))/xdif38, \;\; BN_{679} = DV11(s = -C_{2,2}(8))/xdif39 \\
& BN_{680} = DV11(s = -C_{2,2}(6))/xdif40, \;\; BN_{681} = DV11(s = -C_{2,2}(5))/xdif41 \\
& BN_{682} = DV11(s = -C_{2,2}(3))/xdif42, \;\; BN_{683} = DV11(s = -C_{2,2}(2))/xdif43 \\
& BN_{684} = DV11(s = -C_{2,2}(1))/xdif44, \;\; BN_{685} = DV11(s = -C_{2,2}(0))/xdif45
\end{aligned}
$$

$$
\begin{aligned}
DV12(s) &= \lambda_e\,(s + C_{2,2}(4))\,(s + C_{2,2}(7))\,(s + C_{2,2}(9))\,[\lambda_a\,\lambda_b\,(s + C_{2,2}(2)) + \\
& \lambda_a\,\lambda_b\,(s + C_{2,2}(1))] + \lambda_b\,(s + C_{2,2}(2))\,(s + C_{2,2}(5))\,(s + C_{2,2}(9))[\\
& \lambda_a\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_a\,\lambda_{se}\,(s + C_{2,2}(1))] + \lambda_a\,(s + C_{2,2}(1)) \\
& (s + C_{2,2}(5))\,(s + C_{2,2}(7))\,[\lambda_b\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_b\,\lambda_{se}\,(s + C_{2,2}(2))] \\
xdif46 &= (C_{2,2}(0) - C_{2,2}(12))\,(C_{2,2}(1) - C_{2,2}(12))\,(C_{2,2}(2) - C_{2,2}(12)) \\
& (C_{2,2}(5) - C_{2,2}(12))\,(C_{2,2}(7) - C_{2,2}(12))\,(C_{2,2}(4) - C_{2,2}(12)) \\
& (C_{2,2}(9) - C_{2,2}(12)) \\
xdif47 &= (C_{2,2}(0) - C_{2,2}(9))\,(C_{2,2}(1) - C_{2,2}(9))\,(C_{2,2}(2) - C_{2,2}(9)) \\
& (C_{2,2}(5) - C_{2,2}(9))\,(C_{2,2}(7) - C_{2,2}(9))\,(C_{2,2}(4) - C_{2,2}(9)) \\
& (C_{2,2}(12) - C_{2,2}(9)) \\
xdif48 &= (C_{2,2}(0) - C_{2,2}(7))\,(C_{2,2}(1) - C_{2,2}(7))\,(C_{2,2}(2) - C_{2,2}(7)) \\
& (C_{2,2}(5) - C_{2,2}(7))\,(C_{2,2}(9) - C_{2,2}(7))\,(C_{2,2}(4) - C_{2,2}(7)) \\
& (C_{2,2}(12) - C_{2,2}(7)) \\
xdif49 &= (C_{2,2}(0) - C_{2,2}(5))\,(C_{2,2}(1) - C_{2,2}(5))\,(C_{2,2}(2) - C_{2,2}(5)) \\
& (C_{2,2}(7) - C_{2,2}(5))\,(C_{2,2}(9) - C_{2,2}(5))\,(C_{2,2}(4) - C_{2,2}(5)) \\
& (C_{2,2}(12) - C_{2,2}(5))
\end{aligned}
$$

$$
\begin{aligned}
xdif50 &= (C_{2,2}(0)-C_{2,2}(4))\,(C_{2,2}(1)-C_{2,2}(4))\,(C_{2,2}(2)-C_{2,2}(4)) \\
&\quad (C_{2,2}(7)-C_{2,2}(4))\,(C_{2,2}(9)-C_{2,2}(4))\,(C_{2,2}(5)-C_{2,2}(4)) \\
&\quad (C_{2,2}(12)-C_{2,2}(4)) \\
xdif51 &= (C_{2,2}(0)-C_{2,2}(2))\,(C_{2,2}(1)-C_{2,2}(2))\,(C_{2,2}(4)-C_{2,2}(2)) \\
&\quad (C_{2,2}(7)-C_{2,2}(2))\,(C_{2,2}(9)-C_{2,2}(2))\,(C_{2,2}(5)-C_{2,2}(2)) \\
&\quad (C_{2,2}(12)-C_{2,2}(2)) \\
xdif52 &= (C_{2,2}(0)-C_{2,2}(1))\,(C_{2,2}(2)-C_{2,2}(1))\,(C_{2,2}(4)-C_{2,2}(1)) \\
&\quad (C_{2,2}(7)-C_{2,2}(1))\,(C_{2,2}(9)-C_{2,2}(1))\,(C_{2,2}(5)-C_{2,2}(1)) \\
&\quad (C_{2,2}(12)-C_{2,2}(1)) \\
xdif53 &= (C_{2,2}(1)-C_{2,2}(0))\,(C_{2,2}(2)-C_{2,2}(0))\,(C_{2,2}(4)-C_{2,2}(0)) \\
&\quad (C_{2,2}(7)-C_{2,2}(0))\,(C_{2,2}(9)-C_{2,2}(0))\,(C_{2,2}(5)-C_{2,2}(0)) \\
&\quad (C_{2,2}(12)-C_{2,2}(0))
\end{aligned}
$$

$$
\begin{aligned}
&BN_{686} = DV12(s=-C_{2,2}(12))/xdif46,\;\; BN_{687} = DV12(s=-C_{2,2}(9))/xdif47 \\
&BN_{688} = DV12(s=-C_{2,2}(7))/xdif48,\;\; BN_{689} = DV12(s=-C_{2,2}(5))/xdif49 \\
&BN_{690} = DV12(s=-C_{2,2}(4))/xdif50,\;\; BN_{691} = DV12(s=-C_{2,2}(2))/xdif51 \\
&BN_{692} = DV12(s=-C_{2,2}(1))/xdif52,\;\; BN_{693} = DV12(s=-C_{2,2}(0))/xdif53
\end{aligned}
$$

$$
\begin{aligned}
DV13(s) &= \lambda_e\,(s+C_{2,2}(4))\,(s+C_{2,2}(7))\,(s+C_{2,2}(10))\,[\lambda_a\,\lambda_d\,(s+C_{2,2}(3))\,+ \\
&\quad \lambda_a\,\lambda_{sd}\,(s+C_{2,2}(1))] + \lambda_d\,(s+C_{2,2}(3))\,(s+C_{2,2}(6))\,(s+C_{2,2}(10))[\\
&\quad \lambda_a\,\lambda_{se}\,(s+C_{2,2}(4)) + \lambda_a\,\lambda_{se}\,(s+C_{2,2}(1))] + \lambda_a\,(s+C_{2,2}(1)) \\
&\quad (s+C_{2,2}(6))\,(s+C_{2,2}(7))\,[\lambda_{se}\,\lambda_{sd}\,(s+C_{2,2}(4)) + \lambda_{sd}\,\lambda_{se}\,(s+C_{2,2}(3))] \\
xdif54 &= (C_{2,2}(0)-C_{2,2}(13))\,(C_{2,2}(1)-C_{2,2}(13))\,(C_{2,2}(3)-C_{2,2}(13)) \\
&\quad (C_{2,2}(4)-C_{2,2}(13))\,(C_{2,2}(6)-C_{2,2}(13))\,(C_{2,2}(7)-C_{2,2}(13)) \\
&\quad (C_{2,2}(10)-C_{2,2}(13)) \\
xdif55 &= (C_{2,2}(0)-C_{2,2}(10))\,(C_{2,2}(1)-C_{2,2}(10))\,(C_{2,2}(3)-C_{2,2}(10)) \\
&\quad (C_{2,2}(4)-C_{2,2}(10))\,(C_{2,2}(6)-C_{2,2}(10))\,(C_{2,2}(7)-C_{2,2}(10)) \\
&\quad (C_{2,2}(13)-C_{2,2}(10)) \\
xdif56 &= (C_{2,2}(0)-C_{2,2}(7))\,(C_{2,2}(1)-C_{2,2}(7))\,(C_{2,2}(3)-C_{2,2}(7))
\end{aligned}
$$

$$
\begin{aligned}
& (C_{2,2}(4) - C_{2,2}(7))\,(C_{2,2}(6) - C_{2,2}(7))\,(C_{2,2}(10) - C_{2,2}(7)) \\
& (C_{2,2}(13) - C_{2,2}(7)) \\
xdif57 \;=\; & (C_{2,2}(0) - C_{2,2}(6))\,(C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(3) - C_{2,2}(6)) \\
& (C_{2,2}(4) - C_{2,2}(6))\,(C_{2,2}(7) - C_{2,2}(6))\,(C_{2,2}(10) - C_{2,2}(6)) \\
& (C_{2,2}(13) - C_{2,2}(6)) \\
xdif58 \;=\; & (C_{2,2}(0) - C_{2,2}(4))\,(C_{2,2}(1) - C_{2,2}(4))\,(C_{2,2}(3) - C_{2,2}(4)) \\
& (C_{2,2}(6) - C_{2,2}(4))\,(C_{2,2}(7) - C_{2,2}(4))\,(C_{2,2}(10) - C_{2,2}(4)) \\
& (C_{2,2}(13) - C_{2,2}(4)) \\
xdif59 \;=\; & (C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3)) \\
& (C_{2,2}(6) - C_{2,2}(3))\,(C_{2,2}(7) - C_{2,2}(3))\,(C_{2,2}(10) - C_{2,2}(3)) \\
& (C_{2,2}(13) - C_{2,2}(3)) \\
xdif60 \;=\; & (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(3) - C_{2,2}(1))\,(C_{2,2}(4) - C_{2,2}(1)) \\
& (C_{2,2}(6) - C_{2,2}(1))\,(C_{2,2}(7) - C_{2,2}(1))\,(C_{2,2}(10) - C_{2,2}(1)) \\
& (C_{2,2}(13) - C_{2,2}(1)) \\
xdif61 \;=\; & (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0)) \\
& (C_{2,2}(6) - C_{2,2}(0))\,(C_{2,2}(7) - C_{2,2}(0))\,(C_{2,2}(10) - C_{2,2}(0)) \\
& (C_{2,2}(13) - C_{2,2}(0))
\end{aligned}
$$

$$
\begin{aligned}
& BN_{694} = DV13(s = -C_{2,2}(13))/xdif54, \;\; BN_{695} = DV13(s = -C_{2,2}(10))/xdif55 \\
& BN_{696} = DV13(s = -C_{2,2}(7))/xdif56, \;\; BN_{697} = DV13(s = -C_{2,2}(6))/xdif57 \\
& BN_{698} = DV13(s = -C_{2,2}(4))/xdif58, \;\; BN_{699} = DV13(s = -C_{2,2}(3))/xdif59 \\
& BN_{700} = DV13(s = -C_{2,2}(1))/xdif60, \;\; BN_{701} = DV13(s = -C_{2,2}(0))/xdif61
\end{aligned}
$$

$$
\begin{aligned}
DV14(s) \;=\; & \lambda_e\,(s + C_{2,2}(4))\,(s + C_{2,2}(9))\,(s + C_{2,2}(10))\,[\lambda_b\,\lambda_d\,(s + C_{2,2}(3)) + \\
& \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))] + \lambda_d\,(s + C_{2,2}(3))\,(s + C_{2,2}(8))\,(s + C_{2,2}(10))[\\
& \lambda_b\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_b\,\lambda_{se}\,(s + C_{2,2}(2))] + \lambda_b\,(s + C_{2,2}(2)) \\
& (s + C_{2,2}(8))\,(s + C_{2,2}(9))\,[\lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(4)) + \lambda_{sd}\,\lambda_{se}\,(s + C_{2,2}(3))] \\
xdif62 \;=\; & (C_{2,2}(0) - C_{2,2}(14))\,(C_{2,2}(2) - C_{2,2}(14))\,(C_{2,2}(3) - C_{2,2}(14)) \\
& (C_{2,2}(4) - C_{2,2}(14))\,(C_{2,2}(8) - C_{2,2}(14))\,(C_{2,2}(9) - C_{2,2}(14))
\end{aligned}
$$

$$
\begin{aligned}
& (C_{2,2}(10) - C_{2,2}(14)) \\
xdif63 &= (C_{2,2}(0) - C_{2,2}(10))\,(C_{2,2}(2) - C_{2,2}(10))\,(C_{2,2}(3) - C_{2,2}(10)) \\
& (C_{2,2}(4) - C_{2,2}(10))\,(C_{2,2}(8) - C_{2,2}(10))\,(C_{2,2}(9) - C_{2,2}(10)) \\
& (C_{2,2}(14) - C_{2,2}(10)) \\
xdif64 &= (C_{2,2}(0) - C_{2,2}(9))\,(C_{2,2}(2) - C_{2,2}(9))\,(C_{2,2}(3) - C_{2,2}(9)) \\
& (C_{2,2}(4) - C_{2,2}(9))\,(C_{2,2}(8) - C_{2,2}(9))\,(C_{2,2}(10) - C_{2,2}(9)) \\
& (C_{2,2}(14) - C_{2,2}(9)) \\
xdif65 &= (C_{2,2}(0) - C_{2,2}(8))\,(C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(3) - C_{2,2}(8)) \\
& (C_{2,2}(4) - C_{2,2}(8))\,(C_{2,2}(9) - C_{2,2}(8))\,(C_{2,2}(10) - C_{2,2}(8)) \\
& (C_{2,2}(14) - C_{2,2}(8)) \\
xdif66 &= (C_{2,2}(0) - C_{2,2}(4))\,(C_{2,2}(2) - C_{2,2}(4))\,(C_{2,2}(3) - C_{2,2}(4)) \\
& (C_{2,2}(8) - C_{2,2}(4))\,(C_{2,2}(9) - C_{2,2}(4))\,(C_{2,2}(10) - C_{2,2}(4)) \\
& (C_{2,2}(14) - C_{2,2}(4)) \\
xdif67 &= (C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(2) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3)) \\
& (C_{2,2}(8) - C_{2,2}(3))\,(C_{2,2}(9) - C_{2,2}(3))\,(C_{2,2}(10) - C_{2,2}(3)) \\
& (C_{2,2}(14) - C_{2,2}(3)) \\
xdif68 &= (C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(3) - C_{2,2}(2))\,(C_{2,2}(4) - C_{2,2}(2)) \\
& (C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(9) - C_{2,2}(2))\,(C_{2,2}(10) - C_{2,2}(2)) \\
& (C_{2,2}(14) - C_{2,2}(2)) \\
xdif69 &= (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0)) \\
& (C_{2,2}(8) - C_{2,2}(0))\,(C_{2,2}(9) - C_{2,2}(0))\,(C_{2,2}(10) - C_{2,2}(0)) \\
& (C_{2,2}(14) - C_{2,2}(0))
\end{aligned}
$$

$BN_{702} = DV14(s = -C_{2,2}(14))/xdif62,\;\; BN_{703} = DV14(s = -C_{2,2}(10))/xdif63$
$BN_{704} = DV14(s = -C_{2,2}(9))/xdif64,\;\; BN_{705} = DV14(s = -C_{2,2}(6))/xdif65$
$BN_{706} = DV14(s = -C_{2,2}(4))/xdif66,\;\; BN_{707} = DV14(s = -C_{2,2}(3))/xdif67$
$BN_{708} = DV14(s = -C_{2,2}(2))/xdif68,\;\; BN_{709} = DV14(s = -C_{2,2}(0))/xdif69$
$k_{73} = A_{2837},\;\; k_{74} = A_{2835},\;\; k_{75} = A_{2834},\;\; k_{76} = A_{2836}$

Appendix F

Identical Unit 1-out-of-(n+1):G Submodel With Late Standby Activation: Definition of constants

The following constants are associated with the analysis of the $1-out-of-(n+1):G$ system under SBAP Z in Section 4.1.

$A_{1966a} = 2\lambda_1 C_{2,1}(2)\,(C_{2,1}(0^{(1)})\,C_{2,1}(1^{(1)}) + \lambda_s \mu_{s0'})$
$A_{1966b} = 2\lambda_1^2\,(C_{2,1}(0^{(1)})\,C_{2,1}(1^{(1)}) + \lambda_s \mu_{s0'})$
$A_{1966c} = \lambda_s\, C_{2,1}(2)\,(C_{2,1}(1)\,C_{2,1}(1^{(1)}) - \lambda_s \mu_{s0'})$
$A_{1966d} = 2\lambda_1\,\lambda_s\,(C_{2,1}(2)\,C_{2,1}(0^{(1)}) + C_{2,1}(1)\,C_{2,1}(2))$
$A_{1967} = C_{2,1}(2) + C_{2,1}(0^{(1)}),\;\; A_{1968} = C_{2,1}(2)\,C_{2,1}(0^{(1)}),\;\; A_{1969} = C_{2,1}(1) + C_{2,1}(1^{(1)})$
$A_{1970} = C_{2,1}(1)\,C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'},\;\; A_{1971} = A_{1967}\,A_{1970} + A_{1968}\,A_{1969},\;\; A_{1972} = A_{1968}\,A_{1970}$
$A_{1973} = C_{2,1}(0)\,C_{2,1}(2) + C_{2,1}(0)\,C_{2,1}(0^{(1)}) + C_{2,1}(2)\,C_{2,1}(0^{(1)}) - \lambda_s\,\mu_{s0'}$
$A_{1974} = C_{2,1}(0)\,C_{2,2}(2)\,C_{2,1}(0^{(1)}) - \lambda_s\,\mu_{s0'}\,C_{2,1}(2)$
$A_{1975} = C_{2,1}(1) + C_{2,1}(1^{(1)}),\;\; A_{1976} = C_{2,1}(1)\,C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}$
$A_{1977} = A_{1973}\,A_{1976} + A_{1974}\,A_{1975},\;\; A_{1978} = A_{1974}\,A_{1976}$

$$\begin{aligned}
A_{1979} &= 2\,\lambda_1^3\,(C_{2,1}(0^{(1)}) + C_{2,1}(1^{(1)})) + \lambda_1\,\lambda_s\,(C_{2,1}(1)\,C_{2,1}(2) + \\
&\quad C_{2,1}(1)\,C_{2,1}(1^{(1)}) + C_{2,1}(2)\,C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}) \\
A_{1980} &= 2\lambda_1^3\,(C_{2,1}(0^{(1)})\,C_{2,1}(1^{(1)}) + \lambda_s\,\mu_{s0'}) + \lambda_1\,\lambda_s\,(C_{2,1}(1)\,C_{2,1}(2)\,C_{2,1}(1^{(1)}) - \\
&\quad \lambda_s\,\mu_{s0'}\,C_{2,1}(2))
\end{aligned}$$

$$A_{1981} = 2\lambda_1 \left(C_{2,1}(2)\, C_{2,1}(0^{(1)}) + C_{2,1}(2)\, C_{2,1}(1^{(1)}) + C_{2,1}(0^{(1)})\, C_{2,1}(1^{(1)})\right) + 2\,\lambda_1\,\lambda_s\,\mu_{s0'}$$

$$A_{1982} = 2\,\lambda_1\, C_{2,1}(2)\, C_{2,1}(0^{(1)})\, C_{2,1}(1^{(1)}) + 2\,\lambda_1\,\lambda_s\,\mu_{s0'}\, C_{2,1}(2)$$

$$A_{1983} = 2\,\lambda_1^2 \left(C_{2,1}(0^{(1)}) + C_{2,1}(1^{(1)})\right)$$

$$A_{1984} = 2\,\lambda_1^2\, C_{2,1}(0^{(1)})\, C_{2,1}(1^{(1)}) + 2\,\lambda_1^2\,\lambda_s\,\mu_{s0'}$$

$$A_{1985} = \lambda_s \left(C_{2,1}(1)\, C_{2,1}(2) + C_{2,1}(2)\, C_{2,1}(1^{(1)}) + C_{2,1}(1)\, C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}\right)$$

$$A_{1986} = \lambda_s \left(C_{2,1}(1)\, C_{2,1}(2)\, C_{2,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}\, C_{2,1}(2)\right)$$

$$A_{1987} = 2\,\lambda_1\,\lambda_s \left(2\, C_{2,1}(2) + C_{2,1}(0^{(1)}) + C_{2,1}(1)\right)$$

$$A_{1988} = 2\,\lambda_1\,\lambda_s \left(C_{2,1}(2)\, C_{2,1}(0^{(1)}) + C_{2,1}(1)\, C_{2,1}(2)\right)$$

$$A_{1989} = \lambda_{c0}\, A_{1971} + \lambda_{c1}\, A_{1981} + \lambda_{c2}\, A_{1983} + \lambda_{c0^{(1)}}\, A_{1985} + \lambda_{c1^{(1)}}\, A_{1987}$$

$$A_{1990} = \lambda_{c0}\, A_{1972} + \lambda_{c1}\, A_{1982} + \lambda_{c2}\, A_{1984} + \lambda_{c0^{(1)}}\, A_{1986} + \lambda_{c1^{(1)}}\, A_{1988}$$

$$A_{1991} = C_{3,1}(1) + C_{3,1}(3), \quad A_{1992} = C_{3,1}(1)\, C_{3,1}(3), \quad A_{1993} = C_{3,1}(1^{(1)}) + C_{3,1}(2^{(1)})$$

$$A_{1994} = C_{3,1}(1^{(1)})\, C_{3,1}(2^{(1)}), \quad A_{1995} = C_{3,1}(2) + C_{3,1}(0^{(1)})$$

$$A_{1996} = C_{3,1}(2)\, C_{3,1}(0^{(1)}) - \lambda_s\,\mu_{s0'}, \quad A_{1997} = -\lambda_s\,\mu_{s0'} \left(C_{3,1}(3) + C_{3,1}(0^{(1)})\right)$$

$$A_{1998} = -\lambda_s\,\mu_{s0'}\, C_{3,1}(3) + C_{3,1}(0^{(1)}), \quad A_{1999} = C_{3,1}(2) + C_{3,1}(2^{(1)})$$

$$A_{2000} = C_{3,1}(2)\, C_{3,1}(2^{(1)}) - \lambda_s\,\mu_{s0'}$$

$$A_{2000a} = A_{1996}\, A_{1998}\, A_{2000} \left(A_{1991}\, A_{1994} + A_{1992}\, A_{1993}\right)$$

$$A_{2001} = A_{2000a} + A_{1992}\, A_{1994} \left(A_{2000} \left(A_{1995}\, A_{1998} + A_{1996}\, A_{1997}\right) + A_{1996}\, A_{1998}\, A_{1999}\right)$$

$$A_{2002} = A_{1992}\, A_{1994}\, A_{1996}\, A_{1998}\, A_{2000}$$

$$A_{2003} = 3\,\lambda_1 \left(C_{3,1}(3)\, C_{3,1}(0^{(1)}) + C_{3,1}(3)\, C_{3,1}(1^{(1)}) + C_{3,1}(0^{(1)})\, C_{3,1}(1^{(1)}) + \lambda_s\,\mu_{s0'}\right)$$

$$A_{2004} = 3\,\lambda_1 \left(C_{3,1}(3)\, C_{3,1}(0^{(1)})\, C_{3,1}(1^{(1)}) + \lambda_s\,\mu_{s0'}\, C_{3,1}(3)\right)$$

$$A_{2005} = C_{3,1}(2) + C_{3,1}(2^{(1)}), \quad A_{2006} = C_{3,1}(2)\, C_{3,2}(2^{(1)}) - \lambda_s\,\mu_{s0'}$$

$$A_{2007} = A_{2003}\, A_{2006} + A_{2004}\, A_{2005}, \quad A_{2008} = A_{2004}\, A_{2006}$$

$$A_{2009} = C_{3,1}(3) + C_{3,1}(0^{(1)}), \quad A_{2010} = C_{3,1}(3)\, C_{3,1}(0^{(1)})$$

$$A_{2011} = C_{3,1}(1^{(1)}) + C_{3,1}(2^{(1)}), \quad A_{2012} = C_{3,1}(1^{(1)})\, C_{3,1}(2^{(1)})$$

$$A_{2013} = C_{3,1}(3) + C_{3,1}(0^{(1)}) + C_{3,1}(1)\, C_{3,1}(3) + C_{3,1}(1)\, C_{3,1}(2^{(1)}) + C_{3,1}(3)\, C_{3,1}(2^{(1)}) + C_{3,1}(3) + C_{3,1}(2^{(1)})$$

$$A_{2014} = C_{3,1}(3)\, C_{3,1}(0^{(1)}) + C_{3,1}(1)\, C_{3,1}(3)\, C_{3,1}(2^{(1)}) + C_{3,1}(3)\, C_{3,1}(2^{(1)})$$

$A_{2015} = 6\,\lambda_1^2\,(A_{2009}\,A_{2012} + A_{2010}\,A_{2011} + \lambda_s \mu_{s0'}\,A_{2013})$

$A_{2016} = 6\,\lambda_1^2\,(A_{2010}\,A_{2012} + \lambda_s\,\mu_{s0'}\,A_{2014})$

$A_{2017} = C_{3,1}(0^{(1)})\,C_{3,1}(1^{(1)}) + C_{3,1}(0^{(1)})\,C_{3,1}(2^{(1)}) + C_{3,1}(1^{(1)})\,C_{3,1}(2^{(1)})$

$A_{2018} = C_{3,1}(0^{(1)})\,C_{3,1}(1^{(1)})\,C_{3,1}(2^{(1)})$

$A_{2019} = 3.0, \;\; A_{2020} = C_{3,1}(1^{(1)}) + C_{3,1}(1) + C_{3,1}(2^{(1)})$

$A_{2021} = 6\,\lambda_1^3\,(A_{2017} + \lambda_s\,\mu_{s0'}\,A_{2019}), \;\; A_{2022} = 6\,\lambda_1^3\,(A_{2018} + \lambda_s\,\mu_{s0'}\,A_{2020})$

$A_{2023} = \lambda_s\,(C_{3,1}(1)\,C_{3,1}(3) + C_{3,1}(1)\,C_{3,1}(1^{(1)}) + C_{3,1}(3)\,C_{3,1}(1^{(1)}) - \lambda_s\,\mu_{s0'})$

$A_{2024} = \lambda_s\,C_{3,1}(3)\,(C_{3,1}(1)\,C_{3,1}(1^{(1)}) - \lambda_s\,\mu_{s0'}\,C_{3,1}(3))$

$A_{2025} = C_{3,1}(2) + C_{3,1}(2^{(1)}), \;\; A_{2026} = C_{3,1}(2)\,C_{3,1}(2^{(1)}) - \lambda_s\,\mu_{s0'}$

$A_{2027} = A_{2023}\,A_{2026} + A_{2024}\,A_{2025}, \;\; A_{2028} = A_{2024}\,A_{2026}$

$A_{2029} = 3\,\lambda_s\,\lambda_1\,(C_{3,1}(3) + C_{3,1}(0^{(1)}) + C_{3,1}(1) + C_{3,1}(3))$

$A_{2030} = 3\,\lambda_1\,\lambda_s\,(C_{3,1}(3)\,C_{3,1}(0^{(1)}) + C_{3,1}(3)\,C_{3,1}(3))$

$A_{2031} = C_{3,1}(2) + C_{3,1}(2^{(1)}), \;\; A_{2032} = C_{3,1}(2)\,C_{3,1}(2^{(1)}) - \lambda_s\,\mu_{s0'}$

$A_{2033} = A_{2029}\,A_{2032} + A_{2030}\,A_{2031}, \;\; A_{2034} = A_{2030}\,A_{2032}$

$A_{2033a} = 6\,\lambda_1^2\,\lambda_s, \;\; A_{2034a} = A_{2033a}\,C_{3,1}(3)$

$A_{2035} = C_{3,1}(1) + C_{3,1}(2) + C_{3,1}(0^{(1)}) + C_{3,1}(1^{(1)}) + C_{3,1}(2) + C_{3,1}(0^{(1)})$

$A_{2036} = C_{3,1}(1)\,C_{3,1}(2) + C_{3,1}(0^{(1)})\,C_{3,1}(1^{(1)}) + C_{3,1}(2)\,C_{3,1}(0^{(1)}) + \lambda_s\,\mu_{s0'}$

$A_{2037} = A_{2033}\,A_{2036} + A_{2034}\,A_{2035}, \;\; A_{2038} = A_{2034}\,A_{2036}$

$A_{2039} = A_{2002} + C_{3,1}(0)\,A_{2001} - \lambda_{s0'}\,A_{2027}, \;\; A_{2040} = C_{3,1}(0)\,A_{2002} - \mu_{s0'}\,A_{2028}$

$A_{2041} = \lambda_1\,(A_{2021} + A_{2037}), \;\; A_{2042} = \lambda_1\,(A_{2022} + A_{2038})$

$A_{2043} = \lambda_{c0}\,A_{2001} + \lambda_{c1}\,A_{2007} + \lambda_{c2}\,A_{2015} + \lambda_{c3}\,A_{2021} + \lambda_{c0^{(1)}}\,A_{2027} + \lambda_{c1^{(1)}}\,A_{2033} + \lambda_{c2^{(1)}}\,A_{2037}$

$A_{2044} = \lambda_{c0}\,A_{2002} + \lambda_{c1}\,A_{2008} + \lambda_{c2}\,A_{2016} + \lambda_{c3}\,A_{2022} + \lambda_{c0^{(1)}}\,A_{2028} + \lambda_{c1^{(1)}}\,A_{2034} + \lambda_{c2^{(1)}}\,A_{2038}$

$DM1(s) = (s + C_{2,1}(0^{(1)}))$

$DM2(s) = 2\,\lambda_1\,[(s + C_{2,1}(0^{(1)}))(s + C_{2,1}(1^{(1)})) + \lambda_s\,\mu_{s0'}]$

$DM3(s) = 2\,\lambda_1^2\,[(s + C_{2,1}(0^{(1)}))(s + C_{2,1}(1^{(1)})) + \lambda_s\,\mu_{s0'}]$

$DM4(s) = 2\,\lambda_1\,\lambda_s\,(2s + C_{2,1}(1) + C_{2,1}(0^{(1)}))$

$B_{551} = DM1(s = s_{70})/V_{40e}(s = s_{70}), \;\; B_{552} = DM2(s = s_{70})/V_{41e}(s = s_{70})$

$B_{553} = DM3(s = s_{70})/V_{42e}(s = s_{70}), \;\; B_{554} = \lambda_s/V_{40e}(s = s_{70})$

$B_{555} = DM4(s = s_{70})/V_{41e}(s = s_{70}), \;\; B_{556} = DM1(s = s_{71})/V_{40e}(s = s_{71})$

$B_{557} = DM2(s = s_{71})/V_{41e}(s = s_{71}), \;\; B_{558} = DM3(s = s_{71})/V_{42e}(s = s_{71})$

$B_{559} = \lambda_s/V_{40e}(s = s_{71}), \;\; B_{560} = DM4(s = s_{71})/V_{41e}(s = s_{71})$

$B_{561} = DM2(s = s_{72})/V_{41e}(s = s_{71}), \;\; B_{562} = DM3(s = s_{72})/V_{42e}(s = s_{72})$

$B_{563} = DM4(s = s_{72})/V_{41e}(s = s_{72}), \;\; B_{564} = DM2(s = s_{73})/V_{41e}(s = s_{73})$

$$B_{565} = DM3(s = s_{73})/V_{42e}(s = s_{73}),\ \ B_{566} = DM4(s = s_{73})/V_{41e}(s = s_{73})$$
$$B_{567} = DM3(s = -C_{2,1}(2))/V_{42e}(s = -C_{2,1}(2))$$
$$DM5(s) = (s + C_{3,1}(0^{(1)}))$$
$$B_{585} = DM5(s = s_{74})/V_{92e}(s = s_{74}),\ \ B_{586} = V_{93n}(s = s_{74})/V_{93e}(s = s_{74})$$
$$B_{587} = V_{94n}(s = s_{74})/V_{94e}(s = s_{74}),\ \ B_{588} = V_{95n}(s = s_{74})/V_{95e}(s = s_{74})$$
$$B_{589} = \lambda_s/V_{92e}(s = s_{74}),\ \ B_{590} = V_{96n}(s = s_{74})/V_{93e}(s = s_{74})$$
$$B_{591} = V_{97n}(s = s_{74})/V_{94e}(s = s_{74}),\ \ B_{592} = DM5(s = s_{75})/V_{92e}(s = s_{75})$$
$$B_{593} = V_{93n}(s = s_{75})/V_{93e}(s = s_{75}),\ \ B_{594} = V_{94n}(s = s_{75})/V_{94e}(s = s_{75})$$
$$B_{595} = V_{95n}(s = s_{75})/V_{95e}(s = s_{75}),\ \ B_{596} = \lambda_s/V_{92e}(s = s_{75})$$
$$B_{597} = V_{96n}(s = s_{75})/V_{93e}(s = s_{75}),\ \ B_{598} = V_{97n}(s = s_{75})/V_{94e}(s = s_{75})$$
$$B_{599} = V_{93n}(s = s_{76})/V_{93e}(s = s_{76}),\ \ B_{600} = V_{94n}(s = s_{76})/V_{94e}(s = s_{76})$$
$$B_{601} = V_{95n}(s = s_{76})/V_{95e}(s = s_{76}),\ \ B_{602} = V_{96n}(s = s_{76})/V_{93e}(s = s_{76})$$
$$B_{603} = V_{97n}(s = s_{76})/V_{94e}(s = s_{76}),\ \ B_{604} = V_{93n}(s = s_{77})/V_{93e}(s = s_{77})$$
$$B_{605} = V_{94n}(s = s_{77})/V_{94e}(s = s_{77}),\ \ B_{606} = V_{95n}(s = s_{77})/V_{95e}(s = s_{77})$$
$$B_{607} = V_{96n}(s = s_{77})/V_{93e}(s = s_{77}),\ \ B_{608} = V_{97n}(s = s_{77})/V_{94e}(s = s_{77})$$
$$B_{609} = V_{94n}(s = s_{78})/V_{94e}(s = s_{78}),\ \ B_{610} = V_{95n}(s = s_{78})/V_{93e}(s = s_{78})$$
$$B_{611} = V_{97n}(s = s_{78})/V_{94e}(s = s_{78}),\ \ B_{612} = V_{94n}(s = s_{79})/V_{94e}(s = s_{79})$$
$$B_{613} = V_{95n}(s = s_{79})/V_{93e}(s = s_{79}),\ \ B_{614} = V_{97n}(s = s_{79})/V_{94e}(s = s_{79})$$
$$B_{615} = V_{95n}(s = -C_{3,1}(3))/V_{95e}(s = -C_{3,1}(3))$$

$$k_{37} = A_{1990},\ \ k_{38} = A_{1978},\ \ k_{39} = A_{1977},\ \ k_{40} = A_{1989}$$
$$k_{41} = A_{2044},\ \ k_{42} = A_{2040},\ \ k_{43} = A_{2039},\ \ k_{44} = A_{2043}$$

Appendix G

Identical Unit 1-out-of-(n+2):G Submodel With Late Standby Activation: Definition of constants

The following constants are associated with the analysis of the $1-out-of-(n+2):G$ system under SBAP Z discussed in Section 4.1.

$$A_{2065} = C_{2,2}(2)\,C_{2,2}(3) + C_{2,2}(2)\,C_{2,2}(2^{(1)}) + C_{2,2}(3)\,C_{2,2}(2^{(1)})$$
$$A_{2066} = C_{2,2}(2)\,C_{2,2}(3)\,C_{2,2}(2^{(1)})$$
$$A_{2067} = C_{2,2}(1^{(2)})\,C_{2,2}(1) + C_{2,2}(1^{(2)})\,C_{2,2}(1^{(1)}) + C_{2,2}(3)\,C_{2,2}(2^{(1)})$$
$$A_{2068} = C_{2,2}(1^{(2)})\,C_{2,2}(1)\,C_{2,2}(1^{(1)})$$
$$A_{2069} = A_{2066}\,A_{2067} + A_{2065}\,A_{2068}, \quad A_{2070} = A_{2066}\,A_{2068}$$
$$A_{2071} = C_{2,2}(3)\,C_{2,2}(1^{(1)}) + C_{2,2}(3)\,C_{2,2}(0^{(2)}) + C_{2,2}(1^{(2)})\,C_{2,2}(0^{(2)})$$
$$A_{2072} = C_{2,2}(3)\,C_{2,2}(1^{(2)})\,C_{2,2}(0^{(2)}), \quad A_{2073} = C_{2,2}(2) + C_{2,2}(2^{(1)})$$
$$A_{2074} = C_{2,2}(2)\,C_{2,2}(2^{(1)}), \quad A_{2075} = A_{2072}\,A_{2073} + A_{2071}\,A_{2074}$$
$$A_{2076} = A_{2072}\,A_{2074}, \quad A_{2077} = C_{2,2}(1) + C_{2,2}(1^{(1)})$$
$$A_{2078} = C_{2,2}(1)\,C_{2,2}(1^{(1)}), \quad A_{2079} = A_{2076}\,A_{2077} + A_{2075}\,A_{2078}, \quad A_{2080} = A_{2076}\,A_{2078}$$
$$A_{2081} = C_{2,2}(1^{(1)}) + C_{2,2}(2^{(1)}), \quad A_{2082} = C_{2,2}(1^{(1)})\,C_{2,2}(2^{(1)})$$
$$A_{2083} = C_{2,2}(0^{(1)})\,C_{2,2}(1^{(2)}) + C_{2,2}(0^{(1)})\,C_{2,2}(0^{(2)}) + C_{2,2}(1^{(2)})\,C_{2,2}(0^{(2)})$$
$$A_{2084} = C_{2,2}(0^{(1)})\,C_{2,2}(1^{(2)})\,C_{2,2}(0^{(2)}), \quad A_{2085} = A_{2081}\,A_{2084} + A_{2082}\,A_{2083}$$
$$A_{2086} = A_{2082}\,A_{2084}, \quad A_{2087} = A_{2086} + C_{2,2}(3)\,A_{2085}, \quad A_{2088} = A_{2086}\,C_{2,2}(3)$$
$$A_{2089} = C_{2,2}(2)\,A_{2087} + A_{2088}, \quad A_{2090} = C_{2,2}(2)\,A_{2088}, \quad A_{2091} = A_{2085}$$

$A_{2092} = A_{2086}$, $A_{2093} = A_{2090} + C_{2,2}(1)\, A_{2089}$, $A_{2094} = C_{2,2}(1)\, A_{2090}$

$A_{2095} = C_{2,2}(3)\, C_{2,2}(1^{(2)}) + C_{2,2}(3)\, C_{2,2}(0^{(2)}) + C_{2,2}(1^{(2)})\, C_{2,2}(0^{(2)})$

$A_{2096} = C_{2,2}(3)\, C_{2,2}(1^{(2)})\, C_{2,2}(0^{(2)})$

$A_{2097} = 2\,\lambda_1^2\,\lambda_s\,(C_{2,2}(1^{(1)}) + C_{2,2}(0^{(1)})) + 4\,\lambda_1^2\,\lambda_s\,(C_{2,2}(2) + C_{2,2}(0^{(1)}) + C_{2,2}(1) + C_{2,2}(2))$

$A_{2098} = 2\,\lambda_1^2\,\lambda_s\, C_{2,2}(1^{(1)})\, C_{2,2}(0^{(1)}) + 4\,\lambda_1^2\,\lambda_s\,(C_{2,2}(2)\, C_{2,2}(0^{(1)}) + C_{2,2}(1)\, C_{2,2}(2))$

$A_{2099} = A_{2095}\, A_{2098} + A_{2096}\, A_{2097}$, $A_{2100} = A_{2096}\, A_{2098}$

$A_{2101} = C_{2,2}(2)\, C_{2,2}(3) + C_{2,2}(2)\, C_{2,2}(2^{(1)}) + C_{2,2}(3)\, C_{2,2}(2^{(1)})$

$A_{2102} = C_{2,2}(2)\, C_{2,2}(3)\, C_{2,2}(2^{(1)})$

$A_{2103} = 4\,\lambda_1\,\lambda_s^2\,(C_{2,2}(0^{(1)}) + C_{2,2}(0^{(2)}) + C_{2,2}(1) + C_{2,2}(0^{(2)}) + C_{2,2}(1) + C_{2,2}(1^{(1)}))$

$A_{2104} = 4\,\lambda_1\,\lambda_s^2\,(C_{2,2}(0^{(1)}) C_{2,2}(0^{(2)}) + C_{2,2}(1) C_{2,2}(0^{(2)}) + C_{2,2}(1)\, C_{2,2}(1^{(1)}))$

$A_{2105} = A_{2101}\, A_{2104} + A_{2102}\, A_{2103}$, $A_{2106} = A_{2102}\, A_{2104}$

$A_{2107} = 4\,\lambda_1\,\lambda_s\,(C_{2,2}(0^{(1)}) + C_{2,2}(1))$, $A_{2108} = 4\,\lambda_1\,\lambda_s\,(C_{2,2}(0^{(1)}) + C_{2,2}(1))$

$A_{2109} = A_{2107}\, A_{2076} + A_{2107}\, A_{2075}$, $A_{2110} = A_{2108}\, A_{2076}$

$A_{2111} = 2\,\lambda_1^2\,\lambda_s\,(C_{2,2}(1^{(1)}) + C_{2,2}(0^{(1)}) + 2\, C_{2,2}(2) + 2\, C_{2,2}(0^{(1)}) + 2\, C_{2,2}(1) + 2\, C_{2,2}(2))$

$A_{2112} = 2\,\lambda_1^2\,\lambda_s\,(C_{2,2}(1^{(1)}) C_{2,2}(0^{(1)}) + 2\, C_{2,2}(2)\, C_{2,2}(0^{(1)}) + 2\, C_{2,2}(1)\, C_{2,2}(2))$

$A_{2113} = A_{2071}\, A_{2112} + A_{2072}\, A_{2111}$, $A_{2114} = A_{2072}\, A_{2112}$

$A_{2115} = 4\,\lambda_s^2\,\lambda_1\,(C_{2,2}(0^{(1)}) + C_{2,2}(0^{(2)}) + C_{2,2}(1) + C_{2,2}(0^{(2)}) + C_{2,2}(1) + C_{2,2}(1^{(1)}))$

$A_{2116} = 4\,\lambda_s^2\,\lambda_1\,(C_{2,2}(0^{(1)}) C_{2,2}(0^{(2)}) + C_{2,2}(1) C_{2,2}(0^{(2)}) + C_{2,2}(1) C_{2,2}(1^{(1)}))$

$A_{2117} = A_{2065}\, A_{2116} + A_{2066}\, A_{2115}$, $A_{2118} = A_{2066}\, A_{2116}$, $A_{2119} = A_{2094} + C_{2,2}(0)\, A_{2093}$

$A_{2120} = A_{2094}\, C_{2,2}(0)$, $A_{2121} = 2\,\lambda_1^4\, A_{2091} + \lambda_1\, A_{2099} + \lambda_1\, A_{2105}$

$A_{2122} = 2\,\lambda_1^4\, A_{2092} + \lambda_1\, A_{2100} + \lambda_1\, A_{2106}$

$A_{2123} = \lambda_{c0}\, A_{2093} + 2\lambda_1\,\lambda_{c1}\, A_{2089} + 2\,\lambda_1^2\,\lambda_{c2}\, A_{2087} + 2\,\lambda_1^3\,\lambda_{c3}\, A_{2085} + 2\,\lambda_s\,\lambda_{c0^{(1)}}\, A_{2079} +$
$\lambda_{c1^{(1)}}\, A_{2109} + \lambda_{c2^{(1)}}\, A_{2113} + 2\,\lambda_s^2\,\lambda_{c0^{(2)}}\, A_{2069} + \lambda_{c1^{(2)}}\, A_{2117}$

$A_{2124} = \lambda_{c0}\, A_{2094} + 2\lambda_1\,\lambda_{c1}\, A_{2090} + 2\,\lambda_1^2\,\lambda_{c2}\, A_{2088} + 2\,\lambda_1^3\,\lambda_{c3}\, A_{2086} + 2\,\lambda_s\,\lambda_{c0^{(1)}}\, A_{2080} +$
$\lambda_{c1^{(1)}}\, A_{2110} + \lambda_{c2^{(1)}}\, A_{2114} + 2\,\lambda_s^2\,\lambda_{c0^{(2)}}\, A_{2070} + \lambda_{c1^{(2)}}\, A_{2118}$

$A_{2124C} = A_{2093} + 2\lambda_1\, A_{2089} + 2\,\lambda_1^2\, A_{2087} + 2\,\lambda_1^3\, A_{2085} + 2\,\lambda_s\, A_{2079} + A_{2109} + A_{2113} +$
$2\,\lambda_s^2\, A_{2069} + A_{2117}$

$A_{2124D} = A_{2094} + 2\lambda_1\, A_{2090} + 2\,\lambda_1^2\, A_{2088} + 2\,\lambda_1^3\, A_{2086} + 2\,\lambda_s\, A_{2080} + A_{2110} + A_{2114} +$
$2\,\lambda_s^2\, A_{2070} + A_{2118}$

$A_{2125} = 12\,\lambda_1\,\lambda_s$, $A_{2126} = 6\,\lambda_1\,\lambda_s\,(C_{3,2}(0^{(1)}) + C_{3,2}(1))$

$A_{2127} = 12\,\lambda_1^2\,\lambda_s\,(C_{3,2}(1^{(1)}) + C_{3,2}(0^{(1)}) + C_{3,2}(2) + C_{3,2}(0^{(1)}) + C_{3,2}(1) + C_{3,2}(2))$

$A_{2128} = 12\,\lambda_1^2\,\lambda_s\,(C_{3,2}(1^{(1)}) C_{3,2}(0^{(1)}) + C_{3,2}(2)\, C_{3,2}(0^{(1)}) + C_{3,2}(1)\, C_{3,2}(2))$

$A_{2129a} = 6\,\lambda_1^3\,\lambda_s\,(C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)}) + C_{3,2}(2^{(1)})\, C_{3,2}(0^{(1)}) + C_{3,2}(1^{(1)})\, C_{3,2}(0^{(1)}))$

$A_{2129b} = \lambda_1\, A_{2127}\, C_{3,2}(3) + \lambda_1\, A_{2128}$

$A_{2129} = A_{2129a} + A_{2129b}$

$A_{2130} = 6\, \lambda_1^3\, \lambda_s\, C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)})\, C_{3,2}(0^{(1)}) + \lambda_1\, A_{2128}\, C_{3,2}(3)$

$A_{2131} = \lambda_s\, A_{2125}\, C_{3,2}(0^{(2)}) + \lambda_s\, A_{2126} + 6\, \lambda_1\, \lambda_s^2\, (C_{3,2}(1^{(1)}) + C_{3,2}(1))$

$A_{2132} = \lambda_s\, A_{2126}\, C_{3,2}(0^{(2)}) + 6\, \lambda_1\, \lambda_s^2\, C_{3,2}(1^{(1)})\, C_{3,2}(1)$

$A_{2133} = C_{3,2}(1^{(2)}) + C_{3,2}(0^{(2)}),\;\; A_{2134} = C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2135} = C_{3,2}(2) + C_{3,2}(2^{(1)}),\;\; A_{2136} = C_{3,2}(2)\, C_{3,2}(2^{(1)})$

$A_{2137} = \lambda_s\, A_{2127}\, A_{2134} + \lambda_s\, A_{2128}\, A_{2133} + 2\, \lambda_1\, A_{2131}\, A_{2136} + 2\, \lambda_1\, A_{2132}\, A_{2135}$

$A_{2138} = \lambda_s\, A_{2128}\, A_{2134} + 2\, \lambda_1\, A_{2132}\, A_{2136}$

$A_{2139} = C_{3,2}(1)\, C_{3,2}(2) + C_{3,2}(1)\, C_{3,2}(3) + C_{3,2}(2)\, C_{3,2}(3)$

$A_{2140} = C_{3,2}(1)\, C_{3,2}(2)\, C_{3,2}(3)$

$A_{2141} = C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)}) + C_{3,2}(2^{(2)})\, C_{3,2}(1^{(1)}) + C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)})$

$A_{2142} = C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)})$

$A_{2143} = C_{3,2}(0^{(1)})\, C_{3,2}(1^{(2)}) + C_{3,2}(0^{(1)})\, C_{3,2}(0^{(2)}) + C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2144} = C_{3,2}(0^{(1)})\, C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2144a} = A_{2139}\, A_{2142}\, A_{2144}\, C_{3,2}(4) + A_{2140}\, A_{2141}\, A_{2144}\, C_{3,2}(4)$

$A_{2145} = A_{2144a} + A_{2140}\, A_{2142}\, A_{2144} + A_{2140}\, A_{2142}\, A_{2143}\, C_{3,2}(4)$

$A_{2146} = A_{2140}\, A_{2142}\, A_{2144}\, C_{3,2}(4),\;\; A_{2147} = A_{2141}\, A_{2144} + A_{2142}\, A_{2143}$

$A_{2148} = A_{2142}\, A_{2144},\;\; A_{2149} = A_{2145}\, C_{3,2}(3^{(1)}) + A_{2146},\;\; A_{2150} = A_{2146}\, C_{3,2}(3^{(1)})$

$A_{2151} = A_{2148} + A_{2147}\, C_{3,2}(3^{(1)}),\;\; A_{2152} = A_{2148}\, C_{3,2}(3^{(1)})$

$A_{2153} = C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)}) + C_{3,2}(1^{(2)})\, C_{3,2}(4) + C_{3,2}(0^{(2)})\, C_{3,2}(4)$

$A_{2154} = C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})\, C_{3,2}(4),\;\; A_{2155} = A_{2154} + C_{3,2}(2^{(2)})\, A_{2153},\;\; A_{2156} = A_{2154}\, C_{3,2}(2^{(2)})$

$A_{2157} = C_{3,2}(3)\, C_{3,2}(4) + C_{3,2}(3)\, C_{3,2}(3^{(1)}) + C_{3,2}(4)\, C_{3,2}(3^{(1)})$

$A_{2158} = C_{3,2}(3)\, C_{3,2}(4)\, C_{3,2}(3^{(1)})$

$A_{2159} = C_{3,2}(3)\, C_{3,2}(4) + C_{3,2}(3)\, C_{3,2}(3^{(1)}) + C_{3,2}(4)\, C_{3,2}(1^{(1)})$

$A_{2160} = C_{3,2}(3)\, C_{3,2}(4)\, C_{3,2}(3^{(1)})$

$A_{2161} = C_{3,2}(2)\, C_{3,2}(2^{(2)}) + C_{3,2}(2)\, C_{3,2}(2^{(1)}) + C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)})$

$A_{2162} = C_{3,2}(2)\, C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)})$

$A_{2163} = A_{2160}\, A_{2161} + A_{2159}\, A_{2162},\;\; A_{2164} = A_{2160}\, A_{2162}$

$A_{2165} = C_{3,2}(1)\, C_{3,2}(1^{(1)}) + C_{3,2}(1)\, C_{3,2}(1^{(2)}) + C_{3,2}(1^{(1)})\, C_{3,2}(1^{(2)})$

$A_{2166} = C_{3,2}(1)\, C_{3,2}(1^{(1)})\, C_{3,2}(1^{(2)})$

$A_{2167} = A_{2164}\, A_{2165} + A_{2163}\, A_{2166},\;\; A_{2168} = A_{2164}\, A_{2166}$

$A_{2169} = C_{3,2}(2^{(2)})\, C_{3,2}(1^{(2)}) + C_{3,2}(2^{(2)})\, C_{3,2}(0^{(2)}) + C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2170} = C_{3,2}(2^{(2)})\, C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2171} = A_{2170} + C_{3,2}(4)\, A_{2169},\quad A_{2172} = A_{2170}\, A_{2169}$

$A_{2173} = C_{3,2}(3)\, C_{3,2}(4) + C_{3,2}(3)\, C_{3,2}(2^{(2)}) + C_{3,2}(4)\, C_{3,2}(2^{(2)})$

$A_{2174} = C_{3,2}(3)\, C_{3,2}(4)\, C_{3,2}(2^{(2)})$

$A_{2175} = C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)}) + C_{3,2}(1^{(2)})\, C_{3,2}(3^{(1)}) + C_{3,2}(0^{(2)})\, C_{3,2}(3^{(1)})$

$A_{2176} = C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})\, C_{3,2}(3^{(1)})$

$A_{2177} = A_{2173}\, A_{2176} + A_{2174}\, A_{2175},\quad A_{2178} = A_{2174}\, A_{2176}$

$A_{2179} = C_{3,2}(2) + C_{3,2}(2^{(1)}),\quad A_{2180} = C_{3,2}(2)\, C_{3,2}(2^{(1)})$

$A_{2181} = A_{2178}\, A_{2179} + A_{2177}\, A_{2180},\quad A_{2182} = A_{2178}\, A_{2180}$

$A_{2183} = C_{3,2}(1) + C_{3,2}(1^{(1)}),\quad A_{2184} = C_{3,2}(1)\, C_{3,2}(1^{(1)})$

$A_{2185} = A_{2182}\, A_{2183} + A_{2181}\, A_{2184},\quad A_{2186} = A_{2182}\, A_{2184}$

$A_{2187} = C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)}) + C_{3,2}(2^{(2)})\, C_{3,2}(1^{(1)}) + C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)})$

$A_{2188} = C_{3,2}(2^{(2)})\, C_{3,2}(2^{(1)})\, C_{3,2}(1^{(1)})$

$A_{2189} = C_{3,2}(0^{(1)})\, C_{3,2}(1^{(2)}) + C_{3,2}(0^{(1)})\, C_{3,2}(0^{(2)}) + C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2190} = C_{3,2}(0^{(1)})\, C_{3,2}(1^{(2)})\, C_{3,2}(0^{(2)})$

$A_{2191} = A_{2188}\, A_{2190} + A_{2187}\, A_{2190}\, C_{3,2}(3^{(1)}) + A_{2188}\, A_{2189}\, C_{3,2}(3^{(1)})$

$A_{2192} = A_{2188}\, A_{2190}\, C_{3,2}(3^{(1)}),\quad A_{2193} = A_{2192} + C_{3,2}(4)\, A_{2191}$

$A_{2194} = A_{2192}\, C_{3,2}(4),\quad A_{2195} = A_{2194} + C_{3,2}(3)\, A_{2193},\quad A_{2196} = A_{2194}\, C_{3,2}(3)$

$A_{2197} = A_{2196} + C_{3,2}(2)\, A_{2195},\quad A_{2198} = A_{2196}\, C_{3,2}(2)$

$A_{2199} = A_{2150} + C_{3,2}(0)\, A_{2149},\quad A_{2200} = A_{2150}\, C_{3,2}(0)$

$A_{2201} = 6\,\lambda_1^5\, A_{2151} + \lambda_1\, A_{2129}\, A_{2156} + \lambda_1\, A_{2130}\, A_{2155} + \lambda_1\, A_{2137}\, A_{2158} + \lambda_1\, A_{2138}\, A_{2157}$

$A_{2202} = 6\,\lambda_1^5\, A_{2152} + \lambda_1\, A_{2130}\, A_{2156} + \lambda_1\, A_{2138}\, A_{2158}$

$A_{2203} = \lambda_{c0}\, A_{2149} + 3\,\lambda_1\,\lambda_{c1}\, A_{2197} + 6\,\lambda_1^2\,\lambda_{c2}\, A_{2195} + 6\,\lambda_1^3\,\lambda_{c3}\, A_{2193} + 6\,\lambda_1^4\,\lambda_{c4}\, A_{2191} +$
$2\,\lambda_s\,\lambda_{c0^{(1)}}\, A_{2185} + \lambda_{c1^{(1)}}\, A_{2125}\, A_{2182} + \lambda_{c1^{(1)}}\, A_{2126}\, A_{2181} + \lambda_{c2^{(1)}}\, A_{2127}\, A_{2178} + \lambda_{c2^{(1)}}\, A_{2128}\, A_{2177} +$
$\lambda_{c3^{(1)}}\, A_{2129}\, A_{2172} + \lambda_{c3^{(1)}}\, A_{2130}\, A_{2171} + 2\,\lambda_s^2\,\lambda_{c0^{(2)}}\, A_{2167} + \lambda_{c1^{(2)}}\, A_{2131}\, A_{2164} + \lambda_{c1^{(2)}}\, A_{2132}\, A_{2163} +$
$\lambda_{c2^{(2)}}\, A_{2137}\, A_{2160} + \lambda_{c2^{(2)}}\, A_{2138}\, A_{2159}$

$A_{2204} = \lambda_{c0}\, A_{2150} + 3\,\lambda_1\,\lambda_{c1}\, A_{2198} + 6\,\lambda_1^2\,\lambda_{c2}\, A_{2196} + 6\,\lambda_1^3\,\lambda_{c3}\, A_{2194} + 6\,\lambda_1^4\,\lambda_{c4}\, A_{2192} +$
$2\,\lambda_s\,\lambda_{c0^{(1)}}\, A_{2186} + \lambda_{c1^{(1)}}\, A_{2126}\, A_{2182} + \lambda_{c2^{(1)}}\, A_{2128}\, A_{2178} + \lambda_{c3^{(1)}}\, A_{2130}\, A_{2172} + 2\,\lambda_s^2\,\lambda_{c0^{(2)}}\, A_{2168} +$
$\lambda_{c1^{(2)}}\, A_{2132}\, A_{2164} + \lambda_{c2^{(2)}}\, A_{2138}\, A_{2160}$

$A_{2204C} = A_{2149} + 3\,\lambda_1\, A_{2197} + 6\,\lambda_1^2\, A_{2195} + 6\,\lambda_1^3\, A_{2193} + 6\,\lambda_1^4\, A_{2191} + 2\,\lambda_s\, A_{2185} + A_{2125}\, A_{2182} +$
$A_{2126}\, A_{2181} + A_{2127}\, A_{2178} + A_{2128}\, A_{2177} + A_{2129}\, A_{2172} + A_{2130}\, A_{2171} + 2\,\lambda_s^2\, A_{2167} + A_{2131}\, A_{2164} +$
$A_{2132}\, A_{2163} + A_{2137}\, A_{2160} + A_{2138}\, A_{2159}$

$A_{2204D} = A_{2150}+3\,\lambda_1\,A_{2198}+6\,\lambda_1^2\,A_{2196}+6\,\lambda_1^3\,A_{2194}+6\,\lambda_1^4\,A_{2192}+2\,\lambda_s\,A_{2186}+A_{2126}\,A_{2182}+$
$A_{2128}\,A_{2178} + A_{2130}\,A_{2172} + 2\,\lambda_s^2\,A_{2168} + A_{2132}\,A_{2164} + A_{2138}\,A_{2160}$

$B_{620} = 2\,\lambda_1/(C_{2,2}(1) - C_{2,2}(0)),\;\; B_{621} = 2\,\lambda_1^2/((C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))$
$B_{622} = 2\,\lambda_1^2/((C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(2) - C_{2,2}(1)))$
$B_{623} = 2\,\lambda_1^2/((C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(1) - C_{2,2}(2)))$
$B_{624} = 2\,\lambda_1^3/((C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0)))$
$B_{625} = 2\,\lambda_1^3/((C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(3) - C_{2,2}(1)))$
$B_{626} = 2\,\lambda_1^3/((C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(3) - C_{2,2}(2)))$
$B_{627} = 2\,\lambda_1^3/((C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(2) - C_{2,2}(3)))$
$B_{628} = 2\,\lambda_s/(C_{2,2}(0^{(1)}) - C_{2,2}(0))$

$xdef1 = (C_{2,2}(0^{(1)}) - C_{2,2}(0))\,(C_{2,2}(1^{(1)}) - C_{2,2}(0))\,(C_{2,2}(1) - C_{2,2}(0))$
$xdef2 = (C_{2,2}(0) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1^{(1)}) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1) - C_{2,2}(0^{(1)}))$
$xdef3 = (C_{2,2}(0) - C_{2,2}(1^{(1)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1^{(1)}))\,(C_{2,2}(1) - C_{2,2}(1^{(1)}))$
$xdef4 = (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1))\,(C_{2,2}(1^{(1)}) - C_{2,2}(1))$

$DM10(s) = 4\,\lambda_1\,\lambda_s\,(s + C_{2,2}(0^{(1)})) + 4\,\lambda_1\,\lambda_s\,(s + C_{2,2}(1))$
$B_{629} = DM10(s = -C_{2,2}(0))/xdef1,\;\; B_{630} = DM10(s = -C_{2,2}(0^{(1)}))/xdef2$
$B_{631} = DM10(s = -C_{2,2}(1^{(1)}))/xdef3,\;\; B_{632} = DM10(s = -C_{2,2}(1))/xdef4$
$DM11(s) = 2\,\lambda_1^2\,\lambda_s\,(s+C_{2,2}(1^{(1)}))\,(s+C_{2,2}(0^{(1)}))+4\,\lambda_1^2\,\lambda_s\,(s+C_{2,2}(2))(s+C_{2,2}(0^{(1)}))+$
$4\,\lambda_1^2\,\lambda_s\,(s + C_{2,2}(1))\,(s + C_{2,2}(2))$

$$\begin{aligned}
xdef5 &= (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(1^{(1)}) - C_{2,2}(0)) \\
&\quad (C_{2,2}(2^{(1)}) - C_{2,2}(0))\,(C_{2,2}(0^{(1)}) - C_{2,2}(0)) \\
xdef6 &= (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(1^{(1)}) - C_{2,2}(1)) \\
&\quad (C_{2,2}(2^{(1)}) - C_{2,2}(1))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1)) \\
xdef7 &= (C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(1^{(1)}) - C_{2,2}(2)) \\
&\quad (C_{2,2}(2^{(1)}) - C_{2,2}(2))\,(C_{2,2}(0^{(1)}) - C_{2,2}(2)) \\
xdef8 &= (C_{2,2}(0) - C_{2,2}(1^{(1)}))\,(C_{2,2}(1) - C_{2,2}(1^{(1)}))\,(C_{2,2}(2) - C_{2,2}(1^{(1)})) \\
&\quad (C_{2,2}(2^{(1)}) - C_{2,2}(1^{(1)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1^{(1)})) \\
xdef9 &= (C_{2,2}(0) - C_{2,2}(2^{(1)}))\,(C_{2,2}(1) - C_{2,2}(2^{(1)}))\,(C_{2,2}(2) - C_{2,2}(2^{(1)}))
\end{aligned}$$

$$(C_{2,2}(1^{(1)}) - C_{2,2}(2^{(1)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(2^{(1)}))$$

$$\begin{aligned} xdef10 &= (C_{2,2}(0) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1) - C_{2,2}(0^{(1)}))\,(C_{2,2}(2) - C_{2,2}(0^{(1)})) \\ &\quad (C_{2,2}(1^{(1)}) - C_{2,2}(0^{(1)}))\,(C_{2,2}(2^{(1)}) - C_{2,2}(0^{(1)})) \end{aligned}$$

$B_{633} = DM11(s = -C_{2,2}(0))/xdef5,\;\; B_{634} = DM11(s = -C_{2,2}(1))/xdef6$
$B_{635} = DM11(s = -C_{2,2}(2))/xdef7,\;\; B_{636} = DM11(s = -C_{2,2}(1^{(1)}))/xdef8$
$B_{637} = DM11(s = -C_{2,2}(2^{(1)}))/xdef9,\;\; B_{638} = DM11(s = -C_{2,2}(0^{(1)}))/xdef10$
$xdef11 = (C_{2,2}(0^{(1)}) - C_{2,2}(0))\,(C_{2,2}(0^{(2)}) - C_{2,2}(0))$
$xdef12 = (C_{2,2}(0) - C_{2,2}(0^{(1)}))\,(C_{2,2}(0^{(2)}) - C_{2,2}(0^{(1)}))$
$xdef13 = (C_{2,2}(0) - C_{2,2}(0^{(2)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(0^{(2)}))$
$B_{639} = 2\,\lambda_s^2/xdef11,\;\; B_{640} = 2\,\lambda_s^2/xdef12,\;\; B_{641} = 2\,\lambda_s^2/xdef13$
$DM12(s) = 4\,\lambda_1\,\lambda_s^2\,(s+C_{2,2}(0^{(1)}))\,(s+C_{2,2}(0^{(2)}))+4\,\lambda_1\,\lambda_s^2\,(s+C_{2,2}(1))(s+C_{2,2}(0^{(2)}))+$
$4\,\lambda_1\,\lambda_s^2\,(s+C_{2,2}(1))\,(s+C_{2,2}(1^{(1)}))$

$$\begin{aligned} xdef14 &= (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(0^{(1)}) - C_{2,2}(0))\,(C_{2,2}(1^{(1)}) - C_{2,2}(0)) \\ &\quad (C_{2,2}(0^{(2)}) - C_{2,2}(0))\,(C_{2,2}(1^{(2)}) - C_{2,2}(0)) \\ xdef15 &= (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1))\,(C_{2,2}(1^{(1)}) - C_{2,2}(1)) \\ &\quad (C_{2,2}(0^{(2)}) - C_{2,2}(1))\,(C_{2,2}(1^{(2)}) - C_{2,2}(1)) \\ xdef16 &= (C_{2,2}(0) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1^{(1)}) - C_{2,2}(0^{(1)})) \\ &\quad (C_{2,2}(0^{(2)}) - C_{2,2}(0^{(1)}))\,(C_{2,2}(1^{(2)}) - C_{2,2}(0^{(1)})) \\ xdef17 &= (C_{2,2}(0) - C_{2,2}(1^{(1)}))\,(C_{2,2}(1) - C_{2,2}(1^{(1)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1^{(1)})) \\ &\quad (C_{2,2}(0^{(2)}) - C_{2,2}(1^{(1)}))\,(C_{2,2}(1^{(2)}) - C_{2,2}(1^{(1)})) \\ xdef18 &= (C_{2,2}(0) - C_{2,2}(0^{(2)}))\,(C_{2,2}(1) - C_{2,2}(0^{(2)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(0^{(2)})) \\ &\quad (C_{2,2}(1^{(1)}) - C_{2,2}(0^{(2)}))\,(C_{2,2}(1^{(2)}) - C_{2,2}(0^{(2)})) \\ xdef19 &= (C_{2,2}(0) - C_{2,2}(1^{(2)}))\,(C_{2,2}(1) - C_{2,2}(1^{(2)}))\,(C_{2,2}(0^{(1)}) - C_{2,2}(1^{(2)})) \\ &\quad (C_{2,2}(1^{(1)}) - C_{2,2}(1^{(2)}))\,(C_{2,2}(0^{(2)}) - C_{2,2}(1^{(2)})) \end{aligned}$$

$B_{642} = DM12(s = -C_{2,2}(0))/xdef14,\;\; B_{643} = DM12(s = -C_{2,2}(1))/xdef15$
$B_{644} = DM12(s = -C_{2,2}(0^{(1)}))/xdef16,\;\; B_{645} = DM12(s = -C_{2,2}(1^{(1)}))/xdef17$
$B_{646} = DM12(s = -C_{2,2}(0^{(2)}))/xdef18,\;\; B_{647} = DM12(s = -C_{2,2}(1^{(2)}))/xdef19$

$B_{648} = 3\,\lambda_1/(C_{3,2}(1) - C_{3,2}(0))$
$xdef20 = (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(2) - C_{3,2}(0))$

$xdef21 = (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(2) - C_{3,2}(1))$

$xdef22 = (C_{3,2}(0) - C_{3,2}(2))\,(C_{3,2}(1) - C_{3,2}(2))$

$B_{650} = 6\,\lambda_1^2/xdef20,\ \ B_{651} = 6\,\lambda_1^2/xdef21,\ \ B_{652} = 6\,\lambda_1^2/xdef22$

$xdef23 = (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(2) - C_{3,2}(0))\,(C_{3,2}(3) - C_{3,2}(0))$

$xdef24 = (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(2) - C_{3,2}(1))\,(C_{3,2}(3) - C_{3,2}(1))$

$xdef25 = (C_{3,2}(0) - C_{3,2}(2))\,(C_{3,2}(1) - C_{3,2}(2))\,(C_{3,2}(3) - C_{3,2}(2))$

$xdef26 = (C_{3,2}(0) - C_{3,2}(3))\,(C_{3,2}(1) - C_{3,2}(3))\,(C_{3,2}(2) - C_{3,2}(3))$

$B_{653} = 6\,\lambda_1^3/xdef23.\ \ B_{654} = 6\,\lambda_1^3/xdef24,\ \ B_{655} = 6\,\lambda_1^3/xdef25$

$B_{656} = 6\,\lambda_1^3/xdef26$

$xdef27 = (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(2) - C_{3,2}(0))\,(C_{3,2}(3) - C_{3,2}(0))\,(C_{3,2}(4) - C_{3,2}(0))$

$xdef28 = (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(2) - C_{3,2}(1))\,(C_{3,2}(3) - C_{3,2}(1)\,(C_{3,2}(4) - C_{3,2}(1))$

$xdef29 = (C_{3,2}(0) - C_{3,2}(2))\,(C_{3,2}(1) - C_{3,2}(2))\,(C_{3,2}(3) - C_{3,2}(2))\,(C_{3,2}(4) - C_{3,2}(2))$

$xdef30 = (C_{3,2}(0) - C_{3,2}(3))\,(C_{3,2}(1) - C_{3,2}(3))\,(C_{3,2}(2) - C_{3,2}(3))\,(C_{3,2}(4) - C_{3,2}(3))$

$xdef31 = (C_{3,2}(0) - C_{3,2}(4))\,(C_{3,2}(1) - C_{3,2}(4))\,(C_{3,2}(2) - C_{3,2}(4))\,(C_{3,2}(3) - C_{3,2}(4))$

$B_{662} = 2\,\lambda_s/(C_{3,2}(0^{(1)}) - C_{3,2}(0))$

$DM13(s) = 6\,\lambda_1\,\lambda_s\,(s + C_{3,2}(0^{(1)})) + 6\,\lambda_1\,\lambda_s\,(s + C_{3,2}(1))$

$xdef32 = (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(0^{(1)}) - C_{3,2}(0))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0))$

$xdef33 = (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1))\,(C_{3,2}(1^{(1)}) - C_{3,2}(1))$

$xdef34 = (C_{3,2}(0) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0^{(1)}))$

$xdef35 = (C_{3,2}(0) - C_{3,2}(1^{(1)}))\,(C_{3,2}(1) - C_{3,2}(1^{(1)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1^{(1)}))$

$B_{663} = DM13(s = -C_{3,2}(0))/xdef32,\ \ B_{664} = DM13(s = -C_{3,2}(1))/xdef33$

$B_{665} = DM13(s = -C_{3,2}(0^{(1)}))/xdef34,\ \ B_{666} = DM13(s = -C_{3,2}(1^{(1)}))/xdef35$

$DM14(s) = 12\,\lambda_1^2\,\lambda_s\,(s{+}C_{3,2}(1^{(1)}))\,(s{+}C_{3,2}(0^{(1)})){+}12\,\lambda_1^2\,\lambda_s\,(s{+}C_{3,2}(2))\,(s{+}C_{3,2}(0^{(1)})){+}$
$12\,\lambda_1^2\,\lambda_s\,(s + C_{3,2}(1))\,(s + C_{3,2}(2))$

$$\begin{aligned}
xdef36 &= (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(2) - C_{3,2}(0))\,(C_{3,2}(0^{(1)}) - C_{3,2}(0)) \\
&\quad (C_{3,2}(1^{(1)}) - C_{3,2}(0))\,(C_{3,2}(2^{(1)}) - C_{3,2}(0)) \\
xdef37 &= (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(2) - C_{3,2}(1))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1)) \\
&\quad (C_{3,2}(1^{(1)}) - C_{3,2}(1))\,(C_{3,2}(2^{(1)}) - C_{3,2}(1)) \\
xdef38 &= (C_{3,2}(0) - C_{3,2}(2))\,(C_{3,2}(1) - C_{3,2}(2))\,(C_{3,2}(0^{(1)}) - C_{3,2}(2)) \\
&\quad (C_{3,2}(1^{(1)}) - C_{3,2}(2))\,(C_{3,2}(2^{(1)}) - C_{3,2}(2)) \\
xdef39 &= (C_{3,2}(0) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1) - C_{3,2}(0^{(1)}))\,(C_{3,2}(2) - C_{3,2}(0^{(1)}))
\end{aligned}$$

$$(C_{3,2}(1^{(1)}) - C_{3,2}(0^{(1)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(0^{(1)}))$$

$$\begin{aligned}
xdef40 &= (C_{3,2}(0) - C_{3,2}(1^{(1)}))\,(C_{3,2}(1) - C_{3,2}(1^{(1)}))\,(C_{3,2}(2) - C_{3,2}(1^{(1)})) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(1^{(1)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(1^{(1)})) \\
xdef41 &= (C_{3,2}(0) - C_{3,2}(2^{(1)}))\,(C_{3,2}(1) - C_{3,2}(2^{(1)}))\,(C_{3,2}(2) - C_{3,2}(2^{(1)})) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(2^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(2^{(1)}))
\end{aligned}$$

$B_{667} = DM14(s = -C_{3,2}(0))/xdef36,\;\; B_{668} = DM14(s = -C_{3,2}(1))/xdef37$
$B_{669} = DM14(s = -C_{3,2}(2))/xdef38,\;\; B_{670} = DM14(s = -C_{3,2}(0^{(1)}))/xdef39$
$B_{671} = DM14(s = -C_{3,2}(1^{(1)}))/xdef40,\;\; B_{672} = DM14(s = -C_{3,2}(2^{(1)}))/xdef41$
$DM15(s) = 6\,\lambda_1^3\,\lambda_s\,(s+C_{3,2}(0^{(1)}))\,(s+C_{3,2}(1^{(1)}))\,(s+C_{3,2}(2^{(1)}))+12\,\lambda_1^3\,\lambda_s\,(s+C_{3,2}(1^{(1)}))\,(s+ C_{3,2}(0^{(1)}))\,(s+C_{3,2}(3))+12\,\lambda_1^3\,\lambda_s\,(s+C_{3,2}(2))\,(s+C_{3,2}(3))\,(s+C_{3,2}(0^{(1)}))+12\,\lambda_1^3\,\lambda_s\,(s+ C_{3,2}(1))\,(s + C_{3,2}(2))\,(s + C_{3,2}(3))$

$$\begin{aligned}
xdef42 &= (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(2) - C_{3,2}(0))\,(C_{3,2}(3) - C_{3,2}(0)) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(0))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0))\,(C_{3,2}(2^{(1)}) - C_{3,2}(0)) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(0)) \\
xdef43 &= (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(2) - C_{3,2}(1))\,(C_{3,2}(3) - C_{3,2}(1)) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(1))\,(C_{3,2}(1^{(1)}) - C_{3,2}(1))\,(C_{3,2}(2^{(1)}) - C_{3,2}(1)) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(1)) \\
xdef44 &= (C_{3,2}(0) - C_{3,2}(2))\,(C_{3,2}(1) - C_{3,2}(2))\,(C_{3,2}(3) - C_{3,2}(2)) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(2))\,(C_{3,2}(1^{(1)}) - C_{3,2}(2))\,(C_{3,2}(2^{(1)}) - C_{3,2}(2)) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(2)) \\
xdef45 &= (C_{3,2}(0) - C_{3,2}(3))\,(C_{3,2}(1) - C_{3,2}(3))\,(C_{3,2}(2) - C_{3,2}(3)) \\
&\quad (C_{3,2}(0^{(1)}) - C_{3,2}(3))\,(C_{3,2}(1^{(1)}) - C_{3,2}(3))\,(C_{3,2}(2^{(1)}) - C_{3,2}(3)) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(3)) \\
xdef46 &= (C_{3,2}(0) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1) - C_{3,2}(0^{(1)}))\,(C_{3,2}(2) - C_{3,2}(0^{(1)})) \\
&\quad (C_{3,2}(3) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0^{(1)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(0^{(1)})) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(0^{(1)})) \\
xdef47 &= (C_{3,2}(0) - C_{3,2}(1^{(1)}))\,(C_{3,2}(1) - C_{3,2}(1^{(1)}))\,(C_{3,2}(2) - C_{3,2}(1^{(1)})) \\
&\quad (C_{3,2}(3) - C_{3,2}(1^{(1)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1^{(1)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(1^{(1)})) \\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(1^{(1)}))
\end{aligned}$$

$$
\begin{aligned}
xdef48 &= (C_{3,2}(0) - C_{3,2}(2^{(1)}))\,(C_{3,2}(1) - C_{3,2}(2^{(1)}))\,(C_{3,2}(2) - C_{3,2}(2^{(1)}))\\
&\quad (C_{3,2}(3) - C_{3,2}(2^{(1)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(2^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(2^{(1)}))\\
&\quad (C_{3,2}(3^{(1)}) - C_{3,2}(2^{(1)}))\\
xdef49 &= (C_{3,2}(0) - C_{3,2}(3^{(1)}))\,(C_{3,2}(1) - C_{3,2}(3^{(1)}))\,(C_{3,2}(2) - C_{3,2}(3^{(1)}))\\
&\quad (C_{3,2}(3) - C_{3,2}(3^{(1)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(3^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(3^{(1)}))\\
&\quad (C_{3,2}(2^{(1)}) - C_{3,2}(3^{(1)}))
\end{aligned}
$$

$B_{673} = DM15(s = -C_{3,2}(0))/xdef42$, $B_{674} = DM15(s = -C_{3,2}(1))/xdef43$
$B_{675} = DM15(s = -C_{3,2}(2))/xdef44$, $B_{676} = DM15(s = -C_{3,2}(3))/xdef45$
$B_{677} = DM15(s = -C_{3,2}(0^{(1)}))/xdef46$, $B_{678} = DM15(s = -C_{3,2}(1^{(1)}))/xdef47$
$B_{679} = DM15(s = -C_{3,2}(2^{(1)}))/xdef48$, $B_{680} = DM15(s = -C_{3,2}(3^{(1)}))/xdef49$
$xdef50 = (C_{3,2}(0^{(1)}) - C_{3,2}(0))\,(C_{3,2}(0^{(2)}) - C_{3,2}(0))$
$xdef51 = (C_{3,2}(0) - C_{3,2}(0^{(1)}))\,(C_{3,2}(0^{(2)}) - C_{3,2}(0^{(1)}))$
$xdef52 = (C_{3,2}(0) - C_{3,2}(0^{(2)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(0^{(2)}))$
$B_{681} = 2\,\lambda_s^2/xdef50$, $B_{682} = 2\,\lambda_s^2/xdef51$, $B_{683} = 2\,\lambda_s^2/xdef52$
$DM16(s) = 6\,\lambda_1\,\lambda_s^2\,(s+C_{3,2}(0^{(1)}))\,(s+C_{3,2}(0^{(2)}))+6\,\lambda_1\,\lambda_s^2\,(s+C_{3,2}(1))\,(s+C_{3,2}(0^{(2)}))+$
$6\,\lambda_1\,\lambda_s^2\,(s+C_{3,2}(1))\,(s+C_{3,2}(1^{(1)}))$

$$
\begin{aligned}
xdef53 &= (C_{3,2}(1) - C_{3,2}(0))\,(C_{3,2}(0^{(1)}) - C_{3,2}(0))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0))\\
&\quad (C_{3,2}(0^{(2)}) - C_{3,2}(0))\,(C_{3,2}(1^{(2)}) - C_{3,2}(0))\\
xdef54 &= (C_{3,2}(0) - C_{3,2}(1))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1))\,(C_{3,2}(1^{(1)}) - C_{3,2}(1))\\
&\quad (C_{3,2}(0^{(2)}) - C_{3,2}(1))\,(C_{3,2}(1^{(2)}) - C_{3,2}(1))\\
xdef55 &= (C_{3,2}(0) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0^{(1)}))\\
&\quad (C_{3,2}(0^{(2)}) - C_{3,2}(0^{(1)}))\,(C_{3,2}(1^{(2)}) - C_{3,2}(0^{(1)}))\\
xdef56 &= (C_{3,2}(0) - C_{3,2}(1^{(1)}))\,(C_{3,2}(1) - C_{3,2}(1^{(1)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1^{(1)}))\\
&\quad (C_{3,2}(0^{(2)}) - C_{3,2}(1^{(1)}))\,(C_{3,2}(1^{(2)}) - C_{3,2}(1^{(1)}))\\
xdef57 &= (C_{3,2}(0) - C_{3,2}(1^{(2)}))\,(C_{3,2}(1) - C_{3,2}(1^{(2)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(1^{(2)}))\\
&\quad (C_{3,2}(0^{(2)}) - C_{3,2}(1^{(2)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(1^{(2)}))\\
xdef58 &= (C_{3,2}(0) - C_{3,2}(0^{(2)}))\,(C_{3,2}(1) - C_{3,2}(0^{(2)}))\,(C_{3,2}(0^{(1)}) - C_{3,2}(0^{(2)}))\\
&\quad (C_{3,2}(1^{(2)}) - C_{3,2}(0^{(2)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(0^{(2)}))
\end{aligned}
$$

$B_{684} = DM16(s = -C_{3,2}(0))/xdef53$, $B_{685} = DM16(s = -C_{3,2}(1))/xdef54$

$B_{686} = DM16(s = -C_{3,2}(0^{(1)}))/xdef55,\ \ B_{687} = DM16(s = -C_{3,2}(1^{(1)}))/xdef56$
$B_{688} = DM16(s = -C_{3,2}(1^{(2)}))/xdef57,\ \ B_{689} = DM16(s = -C_{3,2}(0^{(2)}))/xdef58$
$DM17(s) = 12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(0^{(1)}))\,(s+C_{3,2}(1^{(1)}))+12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(2))\,(s+C_{3,2}(0^{(1)}))+$
$12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(1))\,(s+C_{3,2}(2))$
$DM18(s) = 12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(0^{(1)}))\,(s+C_{3,2}(0^{(2)}))+12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(1))\,(s+C_{3,2}(0^{(2)}))+$
$12\,\lambda_1^2\,\lambda_s^2\,(s+C_{3,2}(1))\,(s+C_{3,2}(1^{(1)}))$
$DM19(s) = (s+C_{3,2}(0^{(2)}))\,(s+C_{3,2}(1^{(2)}))\,DM17(s)+(s+C_{3,2}(2))\,(s+C_{3,2}(2^{(1)}))\,DM18(s)$

$$\begin{aligned}
xdef59 &= (C_{3,2}(1)-C_{3,2}(0))\,(C_{3,2}(2)-C_{3,2}(0))\,(C_{3,2}(0^{(1)})-C_{3,2}(0))\\
&\quad (C_{3,2}(1^{(1)})-C_{3,2}(0))\,(C_{3,2}(2^{(1)})-C_{3,2}(0))\,(C_{3,2}(0^{(2)})-C_{3,2}(0))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(0))\,(C_{3,2}(2^{(2)})-C_{3,2}(0))\\
xdef60 &= (C_{3,2}(0)-C_{3,2}(1))\,(C_{3,2}(2)-C_{3,2}(1))\,(C_{3,2}(0^{(1)})-C_{3,2}(1))\\
&\quad (C_{3,2}(1^{(1)})-C_{3,2}(1))\,(C_{3,2}(2^{(1)})-C_{3,2}(1))\,(C_{3,2}(0^{(2)})-C_{3,2}(1))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(1))\,(C_{3,2}(2^{(2)})-C_{3,2}(1))\\
xdef61 &= (C_{3,2}(0)-C_{3,2}(2))\,(C_{3,2}(1)-C_{3,2}(2))\,(C_{3,2}(0^{(1)})-C_{3,2}(2))\\
&\quad (C_{3,2}(1^{(1)})-C_{3,2}(2))\,(C_{3,2}(2^{(1)})-C_{3,2}(2))\,(C_{3,2}(0^{(2)})-C_{3,2}(2))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(2))\,(C_{3,2}(2^{(2)})-C_{3,2}(2))\\
xdef62 &= (C_{3,2}(0)-C_{3,2}(0^{(1)}))\,(C_{3,2}(1)-C_{3,2}(0^{(1)}))\,(C_{3,2}(2)-C_{3,2}(0^{(1)}))\\
&\quad (C_{3,2}(1^{(1)})-C_{3,2}(0^{(1)}))\,(C_{3,2}(2^{(1)})-C_{3,2}(0^{(1)}))\,(C_{3,2}(0^{(2)})-C_{3,2}(0^{(1)}))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(0^{(1)}))\,(C_{3,2}(2^{(2)})-C_{3,2}(0^{(1)}))\\
xdef63 &= (C_{3,2}(0)-C_{3,2}(1^{(1)}))\,(C_{3,2}(1)-C_{3,2}(1^{(1)}))\,(C_{3,2}(2)-C_{3,2}(1^{(1)}))\\
&\quad (C_{3,2}(0^{(1)})-C_{3,2}(1^{(1)}))\,(C_{3,2}(2^{(1)})-C_{3,2}(1^{(1)}))\,(C_{3,2}(0^{(2)})-C_{3,2}(1^{(1)}))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(1^{(1)}))\,(C_{3,2}(2^{(2)})-C_{3,2}(1^{(1)}))\\
xdef64 &= (C_{3,2}(0)-C_{3,2}(2^{(1)}))\,(C_{3,2}(1)-C_{3,2}(2^{(1)}))\,(C_{3,2}(2)-C_{3,2}(2^{(1)}))\\
&\quad (C_{3,2}(0^{(1)})-C_{3,2}(2^{(1)}))\,(C_{3,2}(1^{(1)})-C_{3,2}(2^{(1)}))\,(C_{3,2}(0^{(2)})-C_{3,2}(2^{(1)}))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(2^{(1)}))\,(C_{3,2}(2^{(2)})-C_{3,2}(2^{(1)}))\\
xdef65 &= (C_{3,2}(0)-C_{3,2}(0^{(2)}))\,(C_{3,2}(1)-C_{3,2}(0^{(2)}))\,(C_{3,2}(2)-C_{3,2}(0^{(2)}))\\
&\quad (C_{3,2}(0^{(1)})-C_{3,2}(0^{(2)}))\,(C_{3,2}(1^{(1)})-C_{3,2}(0^{(2)}))\,(C_{3,2}(2^{(1)})-C_{3,2}(0^{(2)}))\\
&\quad (C_{3,2}(1^{(2)})-C_{3,2}(0^{(2)}))\,(C_{3,2}(2^{(2)})-C_{3,2}(0^{(2)}))\\
xdef66 &= (C_{3,2}(0)-C_{3,2}(1^{(2)}))\,(C_{3,2}(1)-C_{3,2}(1^{(2)}))\,(C_{3,2}(2)-C_{3,2}(1^{(2)}))
\end{aligned}$$

$$
\begin{aligned}
& (C_{3,2}(0^{(1)}) - C_{3,2}(1^{(2)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(1^{(2)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(1^{(2)})) \\
& (C_{3,2}(0^{(2)}) - C_{3,2}(1^{(2)}))\,(C_{3,2}(2^{(2)}) - C_{3,2}(1^{(2)})) \\
xdef67 \;=\; & (C_{3,2}(0) - C_{3,2}(2^{(2)}))\,(C_{3,2}(1) - C_{3,2}(2^{(2)}))\,(C_{3,2}(2) - C_{3,2}(2^{(2)})) \\
& (C_{3,2}(0^{(1)}) - C_{3,2}(2^{(2)}))\,(C_{3,2}(1^{(1)}) - C_{3,2}(2^{(2)}))\,(C_{3,2}(2^{(1)}) - C_{3,2}(2^{(2)})) \\
& (C_{3,2}(0^{(2)}) - C_{3,2}(2^{(2)}))\,(C_{3,2}(1^{(2)}) - C_{3,2}(2^{(2)}))
\end{aligned}
$$

$B_{690} = DM19(s = -C_{3,2}(0))/xdef59,\;\; B_{691} = DM19(s = -C_{3,2}(1))/xdef60$

$B_{692} = DM19(s = -C_{3,2}(2))/xdef61,\;\; B_{693} = DM19(s = -C_{3,2}(0^{(1)}))/xdef62$

$B_{694} = DM19(s = -C_{3,2}(1^{(1)}))/xdef63,\;\; B_{695} = DM19(s = -C_{3,2}(2^{(1)}))/xdef64$

$B_{696} = DM19(s = -C_{3,2}(0^{(2)}))/xdef65,\;\; B_{697} = DM19(s = -C_{3,2}(1^{(2)}))/xdef66$

$B_{698} = DM19(s = -C_{3,2}(2^{(2)}))/xdef67$

$k_{49} = A_{2124D},\;\; k_{50} = A_{2120},\;\; k_{51} = A_{2119},\;\; k_{52} = A_{2124C}$

$k_{53} = A_{2204D},\;\; k_{54} = A_{2200},\;\; k_{55} = A_{2199},\;\; k_{56} = A_{2204C}$

Appendix H

Identical Unit 1-out-of-(n+3):G Submodel With Late Standby Activation: Definition of Constants

The following constants are associated with the analysis of the $1-out-of-(n+3):G$ system under SBAP Z discussed in Section 4.1.

$A_{2257} = 12\,\lambda_1\,\lambda_s, \quad A_{2258} = 6\,\lambda_1\,\lambda_s\,(C_{2,3}(1) + C_{2,3}(0^{(1)}))$
$A_{2259} = \lambda_1\,A_{2257}\,C_{2,3}(2) + \lambda_1\,A_{2258} + 4\,\lambda_1^2\,\lambda_s\,(C_{2,3}(0^{(1)}) + C_{2,3}(1^{(1)}))$
$A_{2260} = \lambda_1\,A_{2258}\,C_{2,3}(2) + 4\,\lambda_1^2\,\lambda_s\,C_{2,3}(0^{(1)})\,C_{2,3}(1^{(1)})$
$A_{2260a} = \lambda_1\,A_{2259}\,C_{2,3}(3) + \lambda_1\,A_{2260}$
$A_{2261} = A_{2260a} + 2\,\lambda_1^3\,\lambda_s\,(C_{2,3}(0^{(1)})\,C_{2,3}(1^{(1)}) + C_{2,3}(0^{(1)})\,C_{2,3}(2^{(1)}) + C_{2,3}(1^{(1)})\,C_{2,3}(2^{(1)}))$
$A_{2262} = \lambda_1\,A_{2260}\,C_{2,3}(3) + 2\,\lambda_1^3\,\lambda_s\,C_{2,3}(0^{(1)})\,C_{2,3}(1^{(1)})\,C_{2,3}(2^{(1)})$
$A_{2263} = 12\,\lambda_1\,\lambda_s^2\,(C_{2,3}(1) + C_{2,3}(1^{(1)})) + 2\,\lambda_s\,(A_{2257}\,C_{2,3}(0^{(2)}) + A_{2258})$
$A_{2264} = 12\,\lambda_1\,\lambda_s^2\,C_{2,3}(1)\,C_{2,3}(1^{(1)})) + 2\,\lambda_s\,A_{2257}\,C_{2,3}(0^{(2)})$
$A_{2265} = \lambda_1\,A_{2263}\,C_{2,3}(2)\,C_{2,3}(2^{(1)}) + \lambda_1\,A_{2264}\,(C_{2,3}(2) + C_{2,3}(2^{(1)})) + \lambda_s\,A_{2259}\,C_{2,3}(1^{(2)})\,C_{2,3}(0^{(2)}) +$
$\lambda_s\,A_{2260}\,(C_{2,3}(1^{(2)}) + C_{2,3}(0^{(2)}))$
$A_{2266} = \lambda_1\,A_{2264}\,C_{2,3}(2)\,C_{2,3}(2^{(1)}) + \lambda_s\,A_{2260}\,C_{2,3}(1^{(2)})\,C_{2,3}(0^{(2)})$
$A_{2267} = 12\,\lambda_1\,\lambda_s^3\,(C_{2,3}(1)\,C_{2,3}(1^{(1)}) + C_{2,3}(1)\,C_{2,3}(1^{(2)}) + C_{2,3}(1^{(1)})\,C_{2,3}(1^{(2)})) + \lambda_s\,A_{2263}\,C_{2,3}(0^{(3)}) +$
$\lambda_s\,A_{2264}$

$$A_{2268} = 12\,\lambda_1\,\lambda_s^3\,C_{2,3}(1)\,C_{2,3}(1^{(1)})\,C_{2,3}(1^{(2)}) + \lambda_s\,A_{2264}\,C_{2,3}(0^{(3)})$$
$$A_{2269} = C_{2,3}(3)\,C_{2,3}(4) + C_{2,3}(3)\,C_{2,3}(3^{(1)}) + C_{2,3}(4)\,C_{2,3}(3^{(1)})$$
$$A_{2270} = C_{2,3}(3)\,C_{2,3}(4)\,C_{2,3}(3^{(1)})$$
$$A_{2271} = C_{2,3}(0^{(2)})\,C_{2,3}(1^{(2)}) + C_{2,3}(0^{(2)})\,C_{2,3}(2^{(2)}) + C_{2,3}(1^{(2)})\,C_{2,3}(2^{(2)})$$
$$A_{2272} = C_{2,3}(0^{(2)})\,C_{2,3}(1^{(2)})\,C_{2,3}(2^{(2)}),\quad A_{2273} = C_{2,3}(0^{(3)}) + C_{2,3}(1^{(3)})$$
$$A_{2274} = C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)})$$
$$A_{2275} = A_{2269}\,A_{2272}\,A_{2274} + A_{2270}\,A_{2271}\,A_{2274} + A_{2270}\,A_{2271}\,A_{2274} + A_{2270}\,A_{2272}\,A_{2273}$$
$$A_{2276} = A_{2270}\,A_{2272}\,A_{2274},\quad A_{2277} = C_{2,3}(2) + C_{2,3}(2^{(1)}),\quad A_{2278} = C_{2,3}(2)\,C_{2,3}(2^{(1)})$$
$$A_{2279} = A_{2276}\,A_{2277} + A_{2275}\,A_{2278},\quad A_{2280} = A_{2276}\,A_{2278}$$
$$A_{2281} = A_{2280}\,(C_{2,3}(1) + C_{2,3}(1^{(1)})) + A_{2279}\,C_{2,3}(1)\,C_{2,3}(1^{(1)})$$
$$A_{2282} = A_{2280}\,C_{2,3}(1)\,C_{2,3}(1^{(1)})$$
$$A_{2283} = C_{2,3}(4)\,C_{2,3}(0^{(1)}) + C_{2,3}(4)\,C_{2,3}(1^{(1)}) + C_{2,3}(0^{(1)})\,C_{2,3}(1^{(1)})$$
$$A_{2284} = C_{2,3}(4)\,C_{2,3}(0^{(1)})\,C_{2,3}(1^{(1)})$$
$$A_{2285} = C_{2,3}(2^{(1)})\,C_{2,3}(3^{(1)}) + C_{2,3}(2^{(1)})\,C_{2,3}(0^{(2)}) + C_{2,3}(3^{(1)})\,C_{2,3}(0^{(2)})$$
$$A_{2286} = C_{2,3}(2^{(1)})\,C_{2,3}(3^{(1)})\,C_{2,3}(0^{(2)})$$
$$A_{2287} = C_{2,3}(2^{(2)})\,C_{2,3}(0^{(3)}) + C_{2,3}(2^{(2)})\,C_{2,3}(1^{(3)}) + C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)})$$
$$A_{2288} = C_{2,3}(2^{(2)})\,C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)})$$
$$A_{2289} = A_{2288}\,C_{2,3}(1^{(2)})\,(A_{2283}\,A_{2286} + A_{2284}\,A_{2285}) + A_{2284}\,A_{2286}\,(A_{2288} + C_{2,3}(1^{(2)})\,A_{2287})$$
$$A_{2290} = A_{2284}\,A_{2286}\,A_{2288}\,C_{2,3}(1^{(2)}),\quad A_{2291} = A_{2290} + C_{2,3}(3)\,A_{2289}$$
$$A_{2292} = C_{2,3}(3)\,A_{2290},\quad A_{2293} = A_{2292} + C_{2,3}(2)\,A_{2291},\quad A_{2294} = C_{2,3}(2)\,A_{2292}$$
$$A_{2295} = A_{2294} + C_{2,3}(1)\,A_{2293},\quad A_{2296} = C_{2,3}(1)\,A_{2294}$$
$$A_{2297} = C_{2,3}(4)\,C_{2,3}(0^{(2)}) + C_{2,3}(4)\,C_{2,3}(1^{(2)}) + C_{2,3}(0^{(2)})\,C_{2,3}(1^{(2)})$$
$$A_{2298} = C_{2,3}(4)\,C_{2,3}(0^{(2)})\,C_{2,3}(1^{(2)})$$
$$A_{2299} = C_{2,3}(2^{(2)})\,C_{2,3}(0^{(3)}) + C_{2,3}(2^{(2)})\,C_{2,3}(1^{(3)}) + C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)})$$
$$A_{2300} = C_{2,3}(2^{(2)})\,C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)})$$
$$A_{2301} = A_{2297}\,A_{2300} + A_{2298}\,A_{2299},\quad A_{2302} = A_{2298}\,A_{2300}$$
$$A_{2303} = C_{2,3}(3)\,C_{2,3}(4) + C_{2,3}(3)\,C_{2,3}(3^{(1)}) + C_{2,3}(4)\,C_{2,3}(3^{(1)})$$
$$A_{2304} = C_{2,3}(3)\,C_{2,3}(4)\,C_{2,3}(3^{(1)}),\quad A_{2305} = C_{2,3}(0^{(3)}) + C_{2,3}(1^{(3)})$$
$$A_{2306} = C_{2,3}(0^{(3)})\,C_{2,3}(1^{(3)}),\quad A_{2307} = A_{2303}\,A_{2306} + A_{2304}\,A_{2305},\quad A_{2308} = A_{2304}\,A_{2306}$$
$$A_{2309} = C_{2,3}(2)\,C_{2,3}(3) + C_{2,3}(2)\,C_{2,3}(4) + C_{2,3}(3)\,C_{2,3}(4)$$
$$A_{2310} = C_{2,3}(2)\,C_{2,3}(3)\,C_{2,3}(4)$$
$$A_{2311} = C_{2,3}(2^{(2)})\,C_{2,3}(2^{(1)}) + C_{2,3}(3^{(1)})\,C_{2,3}(2^{(2)}) + C_{2,3}(3^{(1)})\,C_{2,3}(2^{(1)})$$
$$A_{2312} = C_{2,3}(2^{(2)})\,C_{2,3}(2^{(1)})\,C_{2,3}(3^{(1)})$$

$A_{2313} = A_{2309}\, A_{2312} + A_{2310}\, A_{2311}, \quad A_{2314} = A_{2310}\, A_{2312}$

$A_{2315} = C_{2,3}(0^{(1)})\, C_{2,3}(1^{(1)}) + C_{2,3}(0^{(1)})\, C_{2,3}(2^{(1)}) + C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2316} = C_{2,3}(0^{(1)})\, C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2317} = C_{2,3}(3^{(1)})\, C_{2,3}(0^{(2)}) + C_{2,3}(3^{(1)})\, C_{2,3}(1^{(2)}) + C_{2,3}(0^{(2)})\, C_{2,3}(1^{(2)})$

$A_{2318} = C_{2,3}(3^{(1)})\, C_{2,3}(0^{(2)})\, C_{2,3}(1^{(2)})$

$A_{2319} = C_{2,3}(2^{(2)})\, C_{2,3}(0^{(3)}) + C_{2,3}(2^{(2)})\, C_{2,3}(1^{(3)}) + C_{2,3}(0^{(3)})\, C_{2,3}(1^{(3)})$

$A_{2320} = C_{2,3}(2^{(2)})\, C_{2,3}(0^{(3)})\, C_{2,3}(1^{(3)})$

$A_{2321} = A_{2320}\, (A_{2315}\, A_{2318} + A_{2316}\, A_{2317}) + A_{2316}\, A_{2318}\, A_{2319}$

$A_{2322} = A_{2316}\, A_{2318}\, A_{2320}$

$A_{2323} = C_{2,3}(1)\, C_{2,3}(2) + C_{2,3}(1)\, C_{2,3}(3) + C_{2,3}(2)\, C_{2,3}(3)$

$A_{2324} = C_{2,3}(1)\, C_{2,3}(2)\, C_{2,3}(3)$

$A_{2325} = C_{2,3}(4)\, C_{2,3}(1^{(1)}) + C_{2,3}(4)\, C_{2,3}(2^{(1)}) + C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2326} = C_{2,3}(4)\, C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2327} = C_{2,3}(3^{(1)})\, C_{2,3}(1^{(2)}) + C_{2,3}(3^{(1)})\, C_{2,3}(2^{(2)}) + C_{2,3}(1^{(2)})\, C_{2,3}(2^{(2)})$

$A_{2328} = C_{2,3}(3^{(1)})\, C_{2,3}(1^{(2)})\, C_{2,3}(2^{(2)})$

$A_{2329} = C_{2,3}(0^{(3)}) + C_{2,3}(1^{(3)}), \quad A_{2330} = C_{2,3}(0^{(3)})\, C_{2,3}(1^{(3)})$

$A_{2331} = A_{2328}\, A_{2330}\, (A_{2323}\, A_{2326} + A_{2324}\, A_{2325}) + A_{2324}\, A_{2326}\, (A_{2327}\, A_{2330} + A_{2328}\, A_{2329})$

$A_{2332} = A_{2324}\, A_{2326}\, A_{2328}\, A_{2330}, \quad A_{2333} = C_{2,3}(0^{(3)}) + C_{2,3}(1^{(3)})$

$A_{2334} = C_{2,3}(0^{(3)})\, C_{2,3}(1^{(3)}), \quad A_{2335} = A_{2314}\, A_{2333} + A_{2313}\, A_{2334}, \quad A_{2336} = A_{2334}\, A_{2314}$

$A_{2337} = C_{2,3}(1)\, C_{2,3}(2) + C_{2,3}(1)\, C_{2,3}(3) + C_{2,3}(2)\, C_{2,3}(3)$

$A_{2338} = C_{2,3}(1)\, C_{2,3}(2)\, C_{2,3}(3)$

$A_{2339} = C_{2,3}(4)\, C_{2,3}(1^{(1)}) + C_{2,3}(4)\, C_{2,3}(2^{(1)}) + C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2340} = C_{2,3}(4)\, C_{2,3}(1^{(1)})\, C_{2,3}(2^{(1)})$

$A_{2341} = C_{2,3}(3^{(1)})\, C_{2,3}(1^{(2)}) + C_{2,3}(3^{(1)})\, C_{2,3}(2^{(2)}) + C_{2,3}(1^{(2)})\, C_{2,3}(2^{(2)})$

$A_{2342} = C_{2,3}(3^{(1)})\, C_{2,3}(1^{(2)})\, C_{2,3}(2^{(2)})$

$A_{2343} = A_{2342}\, C_{2,3}(1^{(3)})\, (A_{2337}\, A_{2340} + A_{2338}\, A_{2339}) + A_{2338}\, A_{2340}\, (C_{2,3}(1^{(3)})\, A_{2341} + A_{2342})$

$A_{2344} = A_{2338}\, A_{2340}\, A_{2342}\, C_{2,3}(1^{(3)}), \quad A_{2345} = A_{2296} + C_{2,3}(0)\, A_{2295}, \quad A_{2346} = C_{2,3}(0)\, A_{2296}$

$A_{2347} = 2\,\lambda_1^5\, A_{2321} + \lambda_1\, A_{2261}\, A_{2302} + \lambda_1\, A_{2262}\, A_{2301} + \lambda_1\, A_{2265}\, A_{2308} + \lambda_1\, A_{2266}\, A_{2307} +$
$\lambda_1\, A_{2267}\, A_{2314} + \lambda_1\, A_{2268}\, A_{2313}$

$A_{2348} = 2\,\lambda_1^5\, A_{2322} + \lambda_1\, A_{2262}\, A_{2302} + \lambda_1\, A_{2266}\, A_{2308} + \lambda_1\, A_{2268}\, A_{2314}$

$A_{2349} = \lambda_{c0}\, A_{2295} + 2\,\lambda_1\,\lambda_{c1}\, A_{2293} + 2\,\lambda_1^2\,\lambda_{c2}\, A_{2291} + 2\,\lambda_1^3\,\lambda_{c3}\, A_{2289} + 2\,\lambda_1^4\,\lambda_{c4}\, A_{2321} +$
$3\,\lambda_1\,\lambda_{c0^{(1)}}\, A_{2281} + \lambda_{c1^{(1)}}\, (A_{2257}\, A_{2280} + A_{2258}\, A_{2279}) + \lambda_{c2^{(1)}}\, (A_{2259}\, A_{2276} + A_{2260}\, A_{2275}) +$

$\lambda_{c3^{(1)}}\,(A_{2261}\,A_{2302} + A_{2262}\,A_{2301}) + 6\,\lambda_s^2 \lambda_{c0^{(2)}}\,A_{2331} + \lambda_{c1^{(2)}}\,(A_{2263}\,A_{2336} + A_{2264}\,A_{2335}) +$
$\lambda_{c2^{(2)}}\,(A_{2265}\,A_{2308} + A_{2266}\,A_{2307}) + 6\,\lambda_s^3\,\lambda_{c0^{(3)}}\,A_{2343} + \lambda_{c1^{(3)}}\,(A_{2267}\,A_{2314} + A_{2268}\,A_{2313})$

$A_{2349a} \;=\; \lambda_{c0}\,A_{2296} + 2\,\lambda_1\,\lambda_{c1}\,A_{2294} + 2\,\lambda_1^2\,\lambda_{c2}\,A_{2292} + 2\,\lambda_1^3\,\lambda_{c3}\,A_{2290} + 2\,\lambda_1^4\,\lambda_{c4}\,A_{2322} +$
$3\,\lambda_1\,\lambda_{c0^{(1)}}\,A_{2282} + \lambda_{c1^{(1)}}\,A_{2258}\,A_{2280} + \lambda_{c2^{(1)}}\,A_{2260}\,A_{2276}$

$A_{2350} = A_{2349a} + \lambda_{c3^{(1)}}\,A_{2262}\,A_{2302} + 6\,\lambda_s^2 \lambda_{c0^{(2)}}\,A_{2332} + \lambda_{c1^{(2)}}\,A_{2264}\,A_{2336} + \lambda_{c2^{(2)}}\,A_{2266}\,A_{2308} +$
$6\,\lambda_s^3\,\lambda_{c0^{(3)}}\,A_{2348} + \lambda_{c1^{(3)}}\,A_{2268}\,A_{2314}$

$A_{2350C} = A_{2295} + 2\,\lambda_1\,A_{2293} + 2\,\lambda_1^2\,A_{2291} + 2\,\lambda_1^3\,A_{2289} + 2\,\lambda_1^4\,A_{2321} + 3\,\lambda_1\,A_{2281} + A_{2257}\,A_{2280} +$
$A_{2258}\,A_{2279} + A_{2259}\,A_{2276} + A_{2260}\,A_{2275} + A_{2261}\,A_{2302} + A_{2262}\,A_{2301} + 6\,\lambda_s^2\,A_{2331} + A_{2263}\,A_{2336} +$
$A_{2264}\,A_{2335} + A_{2265}\,A_{2308} + A_{2266}\,A_{2307} + 6\,\lambda_s^3\,A_{2343} + A_{2267}\,A_{2314} + A_{2268}\,A_{2313}$

$A_{2350D} = A_{2296} + 2\,\lambda_1\,A_{2294} + 2\,\lambda_1^2\,A_{2292} + 2\,\lambda_1^3\,A_{2290} + 2\,\lambda_1^4\,A_{2322} + 3\,\lambda_1\,A_{2282} + A_{2258}\,A_{2280} +$
$A_{2260}\,A_{2276} + A_{2262}\,A_{2302} + 6\,\lambda_s^2\,A_{2332} + A_{2264}\,A_{2336} + A_{2266}\,A_{2308} + 6\,\lambda_s^3\,A_{2348} + A_{2268}\,A_{2314}$

$A_{2351} = 18\,\lambda_1\,\lambda_s,\quad A_{2352} = 9\,\lambda_1\,\lambda_s\,(C_{3,3}(0^{(1)}) + C_{3,3}(1))$

$A_{2353} = 18\,\lambda_1^2\,\lambda_1\,(C_{3,3}(0^{(1)}) + C_{3,3}(1^{(1)})) + 2\,\lambda_1\,A_{2352} + 2\,\lambda_1\,A_{2351}\,C_{3,3}(2)$

$A_{2354} = 18\,\lambda_1^2\,\lambda_1\,C_{3,3}(0^{(1)})\,C_{3,3}(1^{(1)}) + 2\,\lambda_1\,A_{2352}\,C_{3,3}(2)$

$A_{2355a} = 12\,\lambda_1^3\,\lambda_s\,(C_{3,3}(0^{(1)})\,C_{3,3}(1^{(1)}) + C_{3,3}(0^{(1)})\,C_{3,3}(2^{(1)}) + C_{3,3}(1^{(1)})\,C_{3,3}(2^{(1)}))$

$A_{2355b} = \lambda_1\,(A_{2353} C_{3,3}(3) + A_{2354})$

$A_{2355} = A_{2355a} + A_{2355b}$

$A_{2356} = 12\,\lambda_1^3\,\lambda_s\,C_{3,3}(0^{(1)})\,C_{3,3}(1^{(1)})\,C_{3,3}(2^{(1)}) + \lambda_1\,A_{2354}\,C_{3,3}(3)$

$A_{2357} \;=\; C_{3,3}(0^{(1)}) + C_{3,3}(1^{(1)}),\quad A_{2358} \;=\; C_{3,3}(0^{(1)})\,C_{3,3}(1^{(1)}),\quad A_{2359} \;=\; C_{3,3}(2^{(1)}) +$
$C_{3,3}(3^{(1)})$

$A_{2360} = C_{3,3}(2^{(1)})\,C_{3,3}(3^{(1)}),\quad A_{2361} = A_{2355}\,C_{3,3}(4) + A_{2356},\quad A_{2362} = A_{2356}\,C_{3,3}(4)$

$A_{2363} = 6\,\lambda_1^4\,\lambda_s\,(A_{2357}\,A_{2360} + A_{2358}\,A_{2359}) + \lambda_1\,A_{2361}$

$A_{2364} = 6\,\lambda_1^4\,\lambda_s\,A_{2358}\,A_{2359} + \lambda_1\,A_{2362}$

$A_{2365} = 2\,\lambda_s\,A_{2351}\,C_{3,3}(0^{(2)}) + 2\,\lambda_s\,A_{2352} + 18\,\lambda_1\,\lambda_s^2\,(C_{3,3}(1) + C_{3,3}(1^{(1)}))$

$A_{2366} = 2\,\lambda_s\,A_{2352}\,C_{3,3}(0^{(2)}) + 18\,\lambda_1\,\lambda_s^2\,C_{3,3}(1)\,C_{3,3}(1^{(1)})$

$A_{2367a} = 2\,\lambda_s\,A_{2353}\,C_{3,3}(1^{(2)})\,C_{3,3}(0^{(2)}) + 2\,\lambda_s\,A_{2354}\,(C_{3,3}(1^{(2)}) + C_{3,3}(0^{(2)}))$

$A_{2367b} = 2\,\lambda_1\,A_{2365}\,C_{3,3}(2)\,C_{3,3}(2^{(1)}) + 2\,\lambda_1\,A_{2366}\,(C_{3,3}(2) + C_{3,3}(2^{(1)}))$

$A_{2367} = A_{2367a} + A_{2367b}$

$A_{2368} = 2\,\lambda_s\,A_{2354}\,C_{3,3}(1^{(2)})\,C_{3,3}(0^{(2)}) + 2\,\lambda_1\,A_{2366}\,C_{3,3}(2)\,C_{3,3}(2^{(1)})$

$A_{2369} = C_{3,3}(2^{(2)})\,C_{3,3}(1^{(2)}) + C_{3,3}(2^{(2)})\,C_{3,3}(0^{(2)}) + C_{3,3}(1^{(2)})\,C_{3,3}(0^{(2)})$

$A_{2370} = C_{3,3}(2^{(2)})\,C_{3,3}(1^{(2)})\,C_{3,3}(0^{(2)})$

$A_{2371} = C_{3,3}(3) + C_{3,3}(3^{(1)}),\quad A_{2372} = C_{3,3}(3)\,C_{3,3}(3^{(1)})$

$A_{2373} = \lambda_s\,A_{2355}\,A_{2370} + \lambda_s\,A_{2356}\,A_{2369} + \lambda_1\,A_{2367} + A_{2372} + \lambda_1\,A_{2368}\,A_{2371}$

$$A_{2374} = \lambda_s\, A_{2356}\, A_{2370} + \lambda_1\, A_{2368}\, A_{2372}$$

$$A_{2374a} = \lambda_s\, A_{2365}\, C_{3,3}(0^{(3)}) + \lambda_s\, A_{2366}$$

$$A_{2375} = A_{2374a} + 18\, \lambda_1\, \lambda_s^3\, (C_{3,3}(1)\, C_{3,3}(1^{(1)}) + C_{3,3}(1)\, C_{3,3}(1^{(2)}) + C_{3,3}(1^{(1)})\, C_{3,3}(1^{(2)}))$$

$$A_{2376} = \lambda_s\, A_{2366}\, C_{3,3}(0^{(3)}) + 18\, \lambda_1\, \lambda_s^3\, C_{3,3}(1)\, C_{3,3}(1^{(1)})\, C_{3,3}(1^{(2)})$$

$$A_{2376a} = \lambda_s\, A_{2367}\, C_{3,3}(1^{(3)})\, C_{3,3}(0^{(3)})$$

$$A_{2376b} = \lambda_s\, A_{2368}\, (C_{3,3}(1^{(3)}) + C_{3,3}(0^{(3)})) + 2\, \lambda_1\, A_{2375}\, C_{3,3}(2)\, C_{3,3}(2^{(1)})\, C_{3,3}(2^{(2)})$$

$$A_{2377} = A_{2376a} + A_{2376b} + 2\, \lambda_1\, A_{2376}\, (C_{3,3}(2)\, C_{3,3}(2^{(1)}) + C_{3,3}(2)\, C_{3,3}(2^{(2)}) + C_{3,3}(2^{(1)})\, C_{3,3}(2^{(2)}))$$

$$A_{2378} = \lambda_s\, A_{2368}\, C_{3,3}(1^{(3)})\, C_{3,3}(0^{(3)}) + 2\, \lambda_1\, A_{2376}\, C_{3,3}(2)\, C_{3,3}(2^{(1)})\, C_{3,3}(2^{(2)})$$

$$A_{2379} = C_{3,3}(0^{(1)})\, C_{3,3}(1^{(1)}) + C_{3,3}(0^{(1)})\, C_{3,3}(2^{(1)}) + C_{3,3}(1^{(1)})\, C_{3,3}(2^{(1)})$$

$$A_{2380} = C_{3,3}(0^{(1)})\, C_{3,3}(1^{(1)})\, C_{3,3}(2^{(1)})$$

$$A_{2381} = C_{3,3}(3^{(1)})\, C_{3,3}(4^{(1)}) + C_{3,3}(3^{(1)})\, C_{3,3}(0^{(2)}) + C_{3,3}(4^{(1)})\, C_{3,3}(0^{(2)})$$

$$A_{2382} = C_{3,3}(3^{(1)})\, C_{3,3}(4^{(1)})\, C_{3,3}(0^{(2)})$$

$$A_{2383} = C_{3,3}(1^{(2)})\, C_{3,3}(2^{(2)}) + C_{3,3}(1^{(2)})\, C_{3,3}(3^{(2)}) + C_{3,3}(2^{(2)})\, C_{3,3}(3^{(2)})$$

$$A_{2384} = C_{3,3}(1^{(2)})\, C_{3,3}(2^{(2)})\, C_{3,3}(3^{(2)})$$

$$A_{2385} = C_{3,3}(0^{(3)})\, C_{3,3}(1^{(3)}) + C_{3,3}(0^{(3)})\, C_{3,3}(2^{(3)}) + C_{3,3}(1^{(3)})\, C_{3,3}(2^{(3)})$$

$$A_{2386} = C_{3,3}(0^{(3)})\, C_{3,3}(1^{(3)})\, C_{3,3}(2^{(3)})$$

$$A_{2387} = A_{2384}\, A_{2386}\, (A_{2379}\, A_{2382} + A_{2380}\, A_{2381}) + A_{2380}\, A_{2382}\, (A_{2383}\, A_{2386} + A_{2384}\, A_{2385})$$

$$A_{2388} = A_{2380}\, A_{2382}\, A_{2384}\, A_{2386}, \quad A_{2389} = A_{2388} + C_{3,3}(5)\, A_{2387}$$

$$A_{2390} = A_{2388}\, C_{3,3}(5), \quad A_{2391} = A_{2390} + C_{3,3}(4)\, A_{2389}, \quad A_{2392} = A_{2390}\, C_{3,3}(4)$$

$$A_{2393} = A_{2392} + C_{3,3}(4)\, A_{2391}, \quad A_{2394} = A_{2392}\, C_{3,3}(3), \quad A_{2395} = A_{2394} + C_{3,3}(2)\, A_{2393}$$

$$A_{2396} = A_{2394}\, C_{3,3}(2), \quad A_{2397} = A_{2396} + C_{3,3}(1)\, A_{2395}, \quad A_{2398} = A_{2396}\, C_{3,3}(1)$$

$$A_{2399} = [1.0/C_{3,3}(0^{(1)})^2]\, (A_{2397}\, C_{3,3}(0^{(1)}) - A_{2398})$$

$$A_{2400} = A_{2398}/C_{3,3}(0^{(1)}), \quad A_{2401} = C_{3,3}(1) + C_{3,3}(1^{(1)}), \quad A_{2402} = C_{3,3}(1)\, C_{3,3}(1^{(1)})$$

$$A_{2403} = (A_{2399}\, A_{2402} - A_{2400}\, A_{2401})/A_{2401}^2$$

$$A_{2404} = A_{2400}/A_{2402}, \quad A_{2405} = C_{3,3}(2^{(1)}) + C_{3,3}(2), \quad A_{2406} = C_{3,3}(2^{(1)})\, C_{3,3}(2)$$

$$A_{2407} = (A_{2403}\, A_{2406} - A_{2404}\, A_{2405})/A_{2406}^2$$

$$A_{2408} = A_{2404}/A_{2406}, \quad A_{2409} = C_{3,3}(3) + C_{3,3}(3^{(1)}), \quad A_{2410} = C_{3,3}(3)\, C_{3,3}(3^{(1)})$$

$$A_{2411} = (A_{2407}\, A_{2410} - A_{2408}\, A_{2409})/A_{2410}^2, \quad A_{2412} = A_{2408}/A_{2410}$$

$$A_{2413} = C_{3,3}(5)\, C_{3,3}(1^{(2)}) + C_{3,3}(5)\, C_{3,3}(2^{(2)}) + C_{3,3}(1^{(2)})\, C_{3,3}(2^{(2)})$$

$$A_{2414} = C_{3,3}(5)\, C_{3,3}(1^{(2)})\, C_{3,3}(2^{(2)})$$

$$A_{2415} = C_{3,3}(3^{(2)})\, C_{3,3}(0^{(3)}) + C_{3,3}(3^{(2)})\, C_{3,3}(1^{(3)}) + C_{3,3}(0^{(3)})\, C_{3,3}(1^{(3)})$$

$$A_{2416} = C_{3,3}(3^{(2)})\, C_{3,3}(0^{(3)})\, C_{3,3}(1^{(3)})$$

$$A_{2417} = C_{3,3}(2^{(3)}) + C_{3,3}(0^{(2)}), \quad A_{2418} = C_{3,3}(2^{(3)})\, C_{3,3}(0^{(2)})$$

$A_{2419} = A_{2414}\,A_{2416}\,A_{2417} + A_{2418}\,(A_{2413}\,A_{2416} + A_{2414}\,A_{2415})$

$A_{2420} = A_{2414}\,A_{2416}\,A_{2418}$

$A_{2421} = C_{3,3}(0^{(1)}) + C_{3,3}(0^{(2)}),\;\; A_{2422} = C_{3,3}(0^{(1)})\,C_{3,3}(0^{(2)})$

$A_{2423} = (A_{2397}\,A_{2422} - A_{2398}\,A_{2421})/A_{2422}^2\,,\; A_{2424} = A_{2398}/A_{2422}$

$A_{2425} = C_{3,3}(1)\,C_{3,3}(1^{(1)}) + C_{3,3}(1)\,C_{3,3}(1^{(2)}) + C_{3,3}(1^{(1)})\,C_{3,3}(1^{(2)})$

$A_{2426} = C_{3,3}(1)\,C_{3,3}(1^{(1)})\,C_{3,3}(1^{(2)})$

$A_{2427} = (A_{2423}\,A_{2426} - A_{2424}\,A_{2425})/A_{2426}^2,\;\; A_{2428} = A_{2424}/A_{2426}$

$A_{2429} = C_{3,3}(2)\,C_{3,3}(2^{(1)}) + C_{3,3}(2)\,C_{3,3}(2^{(2)}) + C_{3,3}(2^{(1)})\,C_{3,3}(2^{(2)})$

$A_{2430} = C_{3,3}(2)\,C_{3,3}(2^{(1)})\,C_{3,3}(2^{(2)})$

$A_{2431} = (A_{2427}\,A_{2430} - A_{2428}\,A_{2429})/A_{2430}^2,\;\; A_{2432} = A_{2428}/A_{2430}$

$A_{2433} = C_{3,3}(4)\,C_{3,3}(5) + C_{3,3}(4)\,C_{3,3}(4^{(1)}) + C_{3,3}(5)\,C_{3,3}(4^{(1)})$

$A_{2434} = C_{3,3}(4)\,C_{3,3}(5)\,C_{3,3}(4^{(1)})$

$A_{2435} = C_{3,3}(0^{(3)})\,C_{3,3}(1^{(3)}) + C_{3,3}(0^{(3)})\,C_{3,3}(2^{(3)}) + C_{3,3}(1^{(3)})\,C_{3,3}(2^{(3)})$

$A_{2436} = C_{3,3}(0^{(3)})\,C_{3,3}(1^{(3)})\,C_{3,3}(2^{(3)})$

$A_{2437} = A_{2433}\,A_{2436} + A_{2434}\,A_{2435},\;\; A_{2438} = A_{2434}\,A_{2436}$

$A_{2439} = C_{3,3}(3)\,C_{3,3}(4) + C_{3,3}(3)\,C_{3,3}(5) + C_{3,3}(4)\,C_{3,3}(5)$

$A_{2440} = C_{3,3}(3)\,C_{3,3}(4)\,C_{3,3}(5)$

$A_{2441} = C_{3,3}(3^{(1)})\,C_{3,3}(4^{(1)}) + C_{3,3}(3^{(1)})\,C_{3,3}(3^{(2)}) + C_{3,3}(4^{(1)})\,C_{3,3}(3^{(2)})$

$A_{2442} = C_{3,3}(3^{(1)})\,C_{3,3}(4^{(1)})\,C_{3,3}(3^{(2)})$

$A_{2443} = A_{2439}\,A_{2442} + A_{2440}\,A_{2441},\;\; A_{2444} = A_{2440}\,A_{2442}$

$A_{2445} = C_{3,3}(0^{(1)})\,C_{3,3}(0^{(2)}) + C_{3,3}(0^{(1)})\,C_{3,3}(0^{(3)}) + C_{3,3}(0^{(2)})\,C_{3,3}(0^{(3)})$

$A_{2446} = C_{3,3}(0^{(1)})\,C_{3,3}(0^{(2)})\,C_{3,3}(0^{(3)})$

$A_{2447} = (A_{2397}\,A_{2446} - A_{2397}\,A_{2445})/A_{2446}^2,\;\; A_{2448} = A_{2398}/A_{2446}$

$A_{2449} = C_{3,3}(1) + C_{3,3}(1^{(1)}),\;\; A_{2450} = C_{3,3}(1)\,C_{3,3}(1^{(1)}),\;\; A_{2451} = C_{3,3}(1^{(2)}) + C_{3,3}(1^{(3)})$

$A_{2452} = C_{3,3}(1^{(2)})\,C_{3,3}(1^{(3)}),\;\; A_{2453} = A_{2449}\,A_{2452} + A_{2450}\,A_{2451}$

$A_{2454} = A_{2450}\,A_{2452},\;\; A_{2455} = (A_{2447}\,A_{2454} - A_{2448}\,A_{2453})/A_{2454}^2$

$A_{2456} = A_{2448}/A_{2454},\;\; A_{2457} = A_{2398} + C_{3,3}(0)\,A_{2397},\;\; A_{2458} = C_{3,3}(0)\,A_{2398}$

$A_{2459} = 6\,\lambda_1^6\,A_{2387} + \lambda_1\,A_{2363}\,A_{2420} + \lambda_1\,A_{2364}\,A_{2419} + \lambda_1\,A_{2373}\,A_{2438} + \lambda_1\,A_{2374}\,A_{2437} +$
$\lambda_1\,A_{2377}\,A_{2444} + \lambda_1\,A_{2398}\,A_{2443}$

$A_{2460} = 6\,\lambda_1^6\,A_{2388} + \lambda_1\,A_{2364}\,A_{2420} + \lambda_1\,A_{2374}\,A_{2438} + \lambda_1\,A_{2378}\,A_{2444}$

$A_{2460a} = \lambda_{c0}\,A_{2397} + 3\,\lambda_1\,\lambda_{c1}\,A_{2395} + 6\,\lambda_1^2\,\lambda_{c2}\,A_{2393} + 6\,\lambda_1^3\,\lambda_{c3}\,A_{2391} + 6\,\lambda_1^4\,\lambda_{c4}\,A_{2389} +$
$6\,\lambda_1^5\,\lambda_{c5}\,A_{2387} + 3\,\lambda_s\,\lambda_{c0(1)}\,A_{2399} + \lambda_{c1(1)}\,(A_{2351}\,A_{2404} + A_{2352}\,A_{2403}) + \lambda_{c2(1)}\,(A_{2353}\,A_{2408} +$
$A_{2354}\,A_{2407})$

$A_{2460b} = \lambda_{c3^{(1)}} (A_{2355} A_{2412} + A_{2356} A_{2411}) + \lambda_{c4^{(1)}} (A_{2363} A_{2420} + A_{2364} A_{2419}) + 6\, \lambda_s^2 \lambda_{c0^{(2)}} A_{2423} + \lambda_{c1^{(2)}} (A_{2365} A_{2428} + A_{2366} A_{2427})$

$A_{2460c} = \lambda_{c2^{(2)}} (A_{2367} A_{2432} + A_{2368} A_{2431}) + \lambda_{c3^{(2)}} (A_{2373} A_{2438} + A_{2374} A_{2437}) + 6\, \lambda_s^3 \lambda_{c0^{(3)}} A_{2447} + \lambda_{c1^{(3)}} (A_{2375} A_{2456} + A_{2376} A_{2455}) + \lambda_{c2^{(3)}} (A_{2377} A_{2444} + A_{2378} A_{2443})$

$A_{2461} = A_{2460a} + A_{2460b} + A_{2460c}$

$A_{2461a} = \lambda_{c0} A_{2398} + 3\, \lambda_1 \lambda_{c1} A_{2396} + 6\, \lambda_1^2 \lambda_{c2} A_{2394} + 6\, \lambda_1^3 \lambda_{c3} A_{2392} + 6\, \lambda_1^4 \lambda_{c4} A_{2390} + 6\, \lambda_1^5 \lambda_{c5} A_{2388} + 3\, \lambda_s \lambda_{c0^{(1)}} A_{2400} + \lambda_{c1^{(1)}} A_{2352} A_{2404} + \lambda_{c2^{(1)}} A_{2354} A_{2408} + \lambda_{c3^{(1)}} A_{2356} A_{2412} + \lambda_{c4^{(1)}} A_{2364} A_{2420}$

$A_{2461b} = 6\, \lambda_s^2 \lambda_{c0^{(2)}} A_{2424} + \lambda_{c1^{(2)}} A_{2366} A_{2428} + \lambda_{c2^{(2)}} A_{2368} A_{2432} + \lambda_{c3^{(2)}} A_{2374} A_{2438} + 6\, \lambda_s^3 \lambda_{c0^{(3)}} A_{2448} + \lambda_{c1^{(3)}} A_{2376} A_{2456} + \lambda_{c2^{(3)}} A_{2378} A_{2444}$

$A_{2462} = A_{2461a} + A_{2461b}$

$A_{2462C} = A_{2397} + 3\, \lambda_1 A_{2395} + 6\, \lambda_1^2 A_{2393} + 6\, \lambda_1^3 A_{2391} + 6\, \lambda_1^4 A_{2389} + 6\, \lambda_1^5 A_{2387} + 3\, \lambda_s A_{2399} + A_{2351} A_{2404} + A_{2352} A_{2403} + A_{2353} A_{2408} + A_{2354} A_{2407} + A_{2355} A_{2412} + A_{2356} A_{2411} + A_{2363} A_{2420} + A_{2364} A_{2419} + 6\, \lambda_s^2 A_{2423} + A_{2365} A_{2428} + A_{2366} A_{2427} + A_{2367} A_{2432} + A_{2368} A_{2431} + A_{2373} A_{2438} + A_{2374} A_{2437} + 6\, \lambda_s^3 A_{2447} + A_{2375} A_{2456} + A_{2376} A_{2455} + A_{2377} A_{2444} + A_{2378} A_{2443}$

$A_{2462D} = A_{2398} + 3\, \lambda_1 A_{2396} + 6\, \lambda_1^2 A_{2394} + 6\, \lambda_1^3 A_{2392} + 6\, \lambda_1^4 A_{2390} + 6\, \lambda_1^5 A_{2388} + 3\, \lambda_s A_{2400} + A_{2352} A_{2404} + A_{2354} A_{2408} + A_{2356} A_{2412} + A_{2364} A_{2420} + 6\, \lambda_s^2 A_{2424} + A_{2366} A_{2428} + A_{2368} A_{2432} + A_{2374} A_{2438} + 6\, \lambda_s^3 A_{2448} + A_{2376} A_{2456} + A_{2378} A_{2444}$

$B_{699} = 2\, \lambda_1 / (C_{2,3}(1) - C_{2,3}(0))$

$xdef68 = (C_{2,3}(1) - C_{2,3}(0))\, (C_{2,3}(2) - C_{2,3}(0))$

$xdef69 = (C_{2,3}(0) - C_{2,3}(1))\, (C_{2,3}(2) - C_{2,3}(1))$

$xdef70 = (C_{2,3}(0) - C_{2,3}(2))\, (C_{2,3}(1) - C_{2,3}(2))$

$B_{700} = 2\, \lambda_1^2 / xdef68, \quad B_{701} = 2\, \lambda_1^2 / xdef69, \quad B_{702} = 2\, \lambda_1^2 / xdef70$

$xdef71 = (C_{2,3}(1) - C_{2,3}(0))\, (C_{2,3}(2) - C_{2,3}(0))\, (C_{2,3}(3) - C_{2,3}(0))$

$xdef72 = (C_{2,3}(0) - C_{2,3}(1))\, (C_{2,3}(2) - C_{2,3}(1))\, (C_{2,3}(3) - C_{2,3}(1))$

$xdef73 = (C_{2,3}(0) - C_{2,3}(2))\, (C_{2,3}(1) - C_{2,3}(2))\, (C_{2,3}(3) - C_{2,3}(2))$

$xdef74 = (C_{2,3}(0) - C_{2,3}(3))\, (C_{2,3}(1) - C_{2,3}(3))\, (C_{2,3}(2) - C_{2,3}(3))$

$B_{703} = 2\, \lambda_1^3 / xdef71, \quad B_{704} = 2\, \lambda_1^3 / xdef72,$

$B_{705} = 2\, \lambda_1^3 / xdef73, \quad B_{706} = 2\, \lambda_1^3 / xdef74$

$xdef75 = (C_{2,3}(1) - C_{2,3}(0))\, (C_{2,3}(2) - C_{2,3}(0))\, (C_{2,3}(3) - C_{2,3}(0))\, (C_{2,3}(4) - C_{2,3}(0))$

$xdef76 = (C_{2,3}(0) - C_{2,3}(1))\, (C_{2,3}(2) - C_{2,3}(1))\, (C_{2,3}(3) - C_{2,3}(1))\, (C_{2,3}(4) - C_{2,3}(1))$

$xdef77 = (C_{2,3}(0) - C_{2,3}(2))\, (C_{2,3}(1) - C_{2,3}(2))\, (C_{2,3}(3) - C_{2,3}(2))\, (C_{2,3}(4) - C_{2,3}(2))$

$xdef78 = (C_{2,3}(0) - C_{2,3}(3))\, (C_{2,3}(1) - C_{2,3}(3))\, (C_{2,3}(2) - C_{2,3}(3))\, (C_{2,3}(4) - C_{2,3}(3))$

$xdef79 = (C_{2,3}(0) - C_{2,3}(4))\,(C_{2,3}(1) - C_{2,3}(4))\,(C_{2,3}(2) - C_{2,3}(4))\,(C_{2,3}(3) - C_{2,3}(4))$

$B_{707} = 2\,\lambda_1^4/xdef75,\;\; B_{708} = 2\,\lambda_1^4/xdef76,\;\; B_{709} = 2\,\lambda_1^4/xdef77$

$B_{710} = 2\,\lambda_1^4/xdef78,\;\; B_{711} = 2\,\lambda_1^4/xdef79,\; B_{712} = 3\,\lambda_s/(C_{2,3}(0^{(1)}) - C_{2,3}(0))$

$DM18(s) = 6\,\lambda_1\,\lambda_s\,(s + C_{2,3}(1)) + 6\,\lambda_1\,\lambda_s\,(s + C_{2,3}(0^{(1)}))$

$xdef80 = (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0))$

$xdef81 = (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1))\,(C_{2,3}(1^{(1)}) - C_{2,3}(1))$

$xdef82 = (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)}))$

$xdef83 = (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)}))$

$B_{713} = DM18(s = -C_{2,3}(0))/xdef80,\;\; B_{714} = DM18(s = -C_{2,3}(1))/xdef81$

$B_{715} = DM18(s = -C_{2,3}(0^{(1)}))/xdef82,\;\; B_{716} = DM18(s = -C_{2,3}(1^{(1)}))/xdef83$

$DM19(s) = 6\,\lambda_1^2\,\lambda_s\,(s + C_{2,3}(1))\,(s + C_{2,3}(2)) + 6\,\lambda_1^2\,\lambda_s\,(s + C_{2,3}(2))\,(s + C_{2,3}(0^{(1)})) +$
$4\,\lambda_1^2\,\lambda_s\,(s + C_{2,3}(0^{(1)}))\,(s + C_{2,3}(1^{(1)}))$

$$\begin{aligned}
xdef84 &= (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(2) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0)) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0)) \\
xdef85 &= (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(2) - C_{2,3}(1))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1)) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(1))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1)) \\
xdef86 &= (C_{2,3}(0) - C_{2,3}(2))\,(C_{2,3}(1) - C_{2,3}(2))\,(C_{2,3}(0^{(1)}) - C_{2,3}(2)) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(2))\,(C_{2,3}(2^{(1)}) - C_{2,3}(2)) \\
xdef87 &= (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2) - C_{2,3}(0^{(1)})) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0^{(1)})) \\
xdef88 &= (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2) - C_{2,3}(1^{(1)})) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1^{(1)})) \\
xdef89 &= (C_{2,3}(0) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1) - C_{2,3}(2^{(1)}))\,(C_{2,3}(2) - C_{2,3}(2^{(1)})) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(2^{(1)}))
\end{aligned}$$

$B_{717} = DM19(s = -C_{2,3}(0))/xdef84,\;\; B_{718} = DM19(s = -C_{2,3}(1))/xdef85$

$B_{719} = DM19(s = -C_{2,3}(2))/xdef86,\;\; B_{720} = DM19(s = -C_{2,3}(0^{(1)}))/xdef87$

$B_{721} = DM19(s = -C_{2,3}(1^{(1)}))/xdef88,\;\; B_{722} = DM19(s = -C_{2,3}(2^{(1)}))/xdef89$

$DM20(s) = 6\,\lambda_1^3\,\lambda_s\,(s + C_{2,3}(1))\,(s + C_{2,3}(2))\,(s + C_{2,3}(3)) + 6\,\lambda_1^3\,\lambda_s\,(s + C_{2,3}(2))\,(s +$
$C_{2,3}(3))\,(s+C_{2,3}(0^{(1)}))+4\,\lambda_1^3\,\lambda_s\,(s+C_{2,3}(0^{(1)}))\,(s+C_{2,3}(1^{(1)}))+2\,\lambda_1^3\,\lambda_s\,(s+C_{2,3}(0^{(1)}))\,(s+$

$C_{2,3}(1^{(1)}))\,(s + C_{2,3}(2^{(1)}))$

$$\begin{aligned}
xdef90 &= (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(2) - C_{2,3}(0))\,(C_{2,3}(3) - C_{2,3}(0)) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(0))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0)) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(0)) \\
xdef91 &= (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(2) - C_{2,3}(1))\,(C_{2,3}(3) - C_{2,3}(1)) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(1))\,(C_{2,3}(1^{(1)}) - C_{2,3}(1))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1)) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(1)) \\
xdef92 &= (C_{2,3}(0) - C_{2,3}(2))\,(C_{2,3}(1) - C_{2,3}(2))\,(C_{2,3}(3) - C_{2,3}(2)) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(2))\,(C_{2,3}(1^{(1)}) - C_{2,3}(2))\,(C_{2,3}(2^{(1)}) - C_{2,3}(2)) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(2)) \\
xdef93 &= (C_{2,3}(0) - C_{2,3}(3))\,(C_{2,3}(1) - C_{2,3}(3))\,(C_{2,3}(2) - C_{2,3}(3)) \\
&\quad (C_{2,3}(0^{(1)}) - C_{2,3}(3))\,(C_{2,3}(1^{(1)}) - C_{2,3}(3))\,(C_{2,3}(2^{(1)}) - C_{2,3}(3)) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(3)) \\
xdef94 &= (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2) - C_{2,3}(0^{(1)})) \\
&\quad (C_{2,3}(3) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0^{(1)})) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(0^{(1)})) \\
xdef95 &= (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2) - C_{2,3}(1^{(1)})) \\
&\quad (C_{2,3}(3) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1^{(1)})) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(1^{(1)})) \\
xdef96 &= (C_{2,3}(0) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1) - C_{2,3}(2^{(1)}))\,(C_{2,3}(2) - C_{2,3}(2^{(1)})) \\
&\quad (C_{2,3}(3) - C_{2,3}(2^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(2^{(1)})) \\
&\quad (C_{2,3}(3^{(1)}) - C_{2,3}(2^{(1)})) \\
xdef97 &= (C_{2,3}(0) - C_{2,3}(3^{(1)}))\,(C_{2,3}(1) - C_{2,3}(3^{(1)}))\,(C_{2,3}(2) - C_{2,3}(3^{(1)})) \\
&\quad (C_{2,3}(3) - C_{2,3}(3^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(3^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(3^{(1)})) \\
&\quad (C_{2,3}(2^{(1)}) - C_{2,3}(3^{(1)}))
\end{aligned}$$

$B_{723} = DM20(s = -C_{2,3}(0))/xdef90,\;\; B_{724} = DM20(s = -C_{2,3}(1))/xdef91$

$B_{725} = DM20(s = -C_{2,3}(2))/xdef92,\;\; B_{726} = DM20(s = -C_{2,3}(3))/xdef93$

$B_{727} = DM20(s = -C_{2,3}(0^{(1)}))/xdef94,\;\; B_{728} = DM20(s = -C_{2,3}(1^{(1)}))/xdef95$

$B_{729} = DM20(s = -C_{2,3}(2^{(1)}))/xdef96,\;\; B_{730} = DM20(s = -C_{2,3}(3^{(1)}))/xdef97$

$xdef98 = (C_{2,3}(0^{(2)}) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0))$
$xdef99 = (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0^{(1)}))$
$xdef100 = (C_{2,3}(0) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(2)}))$
$B_{731} = 6\,\lambda_s^2/xdef98,\ \ B_{732} = 6\,\lambda_s^2/xdef99,\ \ B_{733} = 6\,\lambda_s^2/xdef100$
$DM21(s) = 12\,\lambda_1\,\lambda_s^2\,(s+C_{2,3}(1))\,(s+C_{2,3}(1^{(1)}))+12\,\lambda_1\,\lambda_s^2\,(s+C_{2,3}(1))\,(s+C_{2,3}(0^{(2)}))+$
$12\,\lambda_1\,\lambda_s^2\,(s + C_{2,3}(0^{(1)}))\,(s + C_{2,3}(0^{(2)}))$

$$\begin{aligned}
xdef101 &= (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0)) \\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(0))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0)) \\
xdef102 &= (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1))\,(C_{2,3}(1^{(1)}) - C_{2,3}(1)) \\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(1))\,(C_{2,3}(1^{(2)}) - C_{2,3}(1)) \\
xdef103 &= (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)})) \\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(1)})) \\
xdef104 &= (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)})) \\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(1^{(1)})) \\
xdef105 &= (C_{2,3}(0) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(2)})) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(2)})) \\
xdef106 &= (C_{2,3}(0) - C_{2,3}(1^{(2)}))\,(C_{2,3}(1) - C_{2,3}(1^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(2)})) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(1^{(2)}))
\end{aligned}$$

$B_{734} = DM21(s = -C_{2,3}(0))/xdef101,\ \ B_{735} = DM21(s = -C_{2,3}(1))/xdef102$
$B_{736} = DM21(s = -C_{2,3}(0^{(1)}))/xdef103,\ \ B_{737} = DM21(s = -C_{2,3}(1^{(1)}))/xdef104$
$B_{738} = DM21(s = -C_{2,3}(0^{(2)}))/xdef105,\ \ B_{739} = DM21(s = -C_{2,3}(1^{(2)}))/xdef106$
$DM22(s) = \lambda_1\,(s+C_{2,3}(2))\,(s+C_{2,3}(2^{(1)}))\,DM21(s)+\lambda_s\,(s+C_{2,3}(0^{(2)}))\,(s+C_{2,3}(1^{(2)}))\,DM20(s)$

$$\begin{aligned}
xdef107 &= (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(2) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0)) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0)) \\
&\quad (C_{2,3}(1^{(2)}) - C_{2,3}(0))\,(C_{2,3}(2^{(2)}) - C_{2,3}(0)) \\
xdef108 &= (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(2) - C_{2,3}(1))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1)) \\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(1))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1))\,(C_{2,3}(0^{(2)}) - C_{2,3}(1)) \\
&\quad (C_{2,3}(1^{(2)}) - C_{2,3}(1))\,(C_{2,3}(2^{(2)}) - C_{2,3}(1))
\end{aligned}$$

$$xdef109 = (C_{2,3}(0) - C_{2,3}(2))\,(C_{2,3}(1) - C_{2,3}(2))\,(C_{2,3}(0^{(1)}) - C_{2,3}(2))$$
$$(C_{2,3}(1^{(1)}) - C_{2,3}(2))\,(C_{2,3}(2^{(1)}) - C_{2,3}(2))\,(C_{2,3}(0^{(2)}) - C_{2,3}(2))$$
$$(C_{2,3}(1^{(2)}) - C_{2,3}(2))\,(C_{2,3}(2^{(2)}) - C_{2,3}(2))$$
$$xdef110 = (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2) - C_{2,3}(0^{(1)}))$$
$$(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0^{(1)}))$$
$$(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(2^{(2)}) - C_{2,3}(0^{(1)}))$$
$$xdef111 = (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2) - C_{2,3}(1^{(1)}))$$
$$(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(1^{(1)}))$$
$$(C_{2,3}(1^{(2)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(2^{(2)}) - C_{2,3}(1^{(1)}))$$
$$xdef112 = (C_{2,3}(0) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1) - C_{2,3}(2^{(1)}))\,(C_{2,3}(2) - C_{2,3}(2^{(1)}))$$
$$(C_{2,3}(0^{(1)}) - C_{2,3}(2^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(2^{(1)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(2^{(1)}))$$
$$(C_{2,3}(1^{(2)}) - C_{2,3}(2^{(1)}))\,(C_{2,3}(2^{(2)}) - C_{2,3}(2^{(1)}))$$
$$xdef113 = (C_{2,3}(0) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1) - C_{2,3}(0^{(2)}))\,(C_{2,3}(2) - C_{2,3}(0^{(2)}))$$
$$(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(0^{(2)}))$$
$$(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(2^{(2)}) - C_{2,3}(0^{(2)}))$$
$$xdef114 = (C_{2,3}(0) - C_{2,3}(1^{(2)}))\,(C_{2,3}(1) - C_{2,3}(1^{(2)}))\,(C_{2,3}(2) - C_{2,3}(1^{(2)}))$$
$$(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(1^{(2)}))$$
$$(C_{2,3}(0^{(2)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(2^{(2)}) - C_{2,3}(1^{(2)}))$$
$$xdef115 = (C_{2,3}(0) - C_{2,3}(2^{(2)}))\,(C_{2,3}(1) - C_{2,3}(2^{(2)}))\,(C_{2,3}(2) - C_{2,3}(2^{(2)}))$$
$$(C_{2,3}(0^{(1)}) - C_{2,3}(2^{(2)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(2^{(2)}))\,(C_{2,3}(2^{(1)}) - C_{2,3}(2^{(2)}))$$
$$(C_{2,3}(0^{(2)}) - C_{2,3}(2^{(2)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(2^{(2)}))$$

$B_{740} = DM22(s = -C_{2,3}(0))/xdef107,\ B_{741} = DM22(s = -C_{2,3}(1))/xdef108$
$B_{742} = DM22(s = -C_{2,3}(2))/xdef109,\ B_{743} = DM22(s = -C_{2,3}(0^{(1)}))/xdef110$
$B_{744} = DM22(s = -C_{2,3}(1^{(1)}))/xdef111,\ B_{745} = DM22(s = -C_{2,3}(2^{(1)}))/xdef112$
$B_{746} = DM22(s = -C_{2,3}(0^{(2)}))/xdef113,\ B_{747} = DM22(s = -C_{2,3}(1^{(2)}))/xdef114$
$B_{748} = DM22(s = -C_{2,3}(2^{(2)}))/xdef115$
$xdef116 = (C_{2,3}(0^{(1)}) - C_{2,3}(0))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0))$
$xdef117 = (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0^{(1)}))$
$xdef118 = (C_{2,3}(0) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0^{(2)}))$

$xdef119 = (C_{2,3}(0) - C_{2,3}(0^{(3)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(3)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0^{(3)}))$

$B_{749} = 6\,\lambda_s^3/xdef116,\;\; B_{750} = 6\,\lambda_s^3/xdef117$

$B_{751} = 6\,\lambda_s^3/xdef118,\;\; B_{752} = 6\,\lambda_s^3/xdef119$

$DM23(s) = 12\,\lambda_1\,\lambda_s^3\,(s{+}C_{2,3}(1))\,(s{+}C_{2,3}(1^{(1)}))\,(s{+}C_{2,3}(1^{(2)})){+}\lambda_s\,(s{+}C_{2,3}(0^{(3)}))\,DM22(s)$

$$\begin{aligned}
xdef120 &= (C_{2,3}(1) - C_{2,3}(0))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0))\\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(0))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(0))\\
xdef121 &= (C_{2,3}(0) - C_{2,3}(1))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1))\,(C_{2,3}(1^{(1)}) - C_{2,3}(1))\\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(1))\,(C_{2,3}(1^{(2)}) - C_{2,3}(1))\,(C_{2,3}(0^{(3)}) - C_{2,3}(1))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(1))\\
xdef122 &= (C_{2,3}(0) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(1)}) - C_{2,3}(0^{(1)}))\\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(1)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0^{(1)}))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(0^{(1)}))\\
xdef123 &= (C_{2,3}(0) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(1)}))\\
&\quad (C_{2,3}(0^{(2)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(1^{(1)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(1^{(1)}))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(1^{(1)}))\\
xdef124 &= (C_{2,3}(0) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(2)}))\\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(2)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(0^{(2)}))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(0^{(2)}))\\
xdef125 &= (C_{2,3}(0) - C_{2,3}(1^{(2)}))\,(C_{2,3}(1) - C_{2,3}(1^{(2)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(2)}))\\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(1^{(2)}))\,(C_{2,3}(0^{(3)}) - C_{2,3}(1^{(2)}))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(1^{(2)}))\\
xdef126 &= (C_{2,3}(0) - C_{2,3}(0^{(3)}))\,(C_{2,3}(1) - C_{2,3}(0^{(3)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(0^{(3)}))\\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(0^{(3)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(0^{(3)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(0^{(3)}))\\
&\quad (C_{2,3}(1^{(3)}) - C_{2,3}(0^{(3)}))\\
xdef127 &= (C_{2,3}(0) - C_{2,3}(1^{(3)}))\,(C_{2,3}(1) - C_{2,3}(1^{(3)}))\,(C_{2,3}(0^{(1)}) - C_{2,3}(1^{(3)}))\\
&\quad (C_{2,3}(1^{(1)}) - C_{2,3}(1^{(3)}))\,(C_{2,3}(0^{(2)}) - C_{2,3}(1^{(3)}))\,(C_{2,3}(1^{(2)}) - C_{2,3}(1^{(3)}))\\
&\quad (C_{2,3}(0^{(3)}) - C_{2,3}(1^{(3)}))
\end{aligned}$$

$B_{753} = DM23(s = -C_{2,3}(0))/xdef120,\;\; B_{754} = DM23(s = -C_{2,3}(1))/xdef121$

$B_{755} = DM23(s = -C_{2,3}(0^{(1)}))/xdef122,\ \ B_{756} = DM23(s = -C_{2,3}(1^{(1)}))/xdef123$

$B_{757} = DM23(s = -C_{2,3}(0^{(2)}))/xdef124,\ \ B_{758} = DM23(s = -C_{2,3}(1^{(2)}))/xdef125$

$B_{759} = DM23(s = -C_{2,3}(0^{(3)}))/xdef126,\ \ B_{760} = DM23(s = -C_{2,3}(1^{(3)}))/xdef127$

$B_{761} = 3\,\lambda_1/(C_{3,3}(1) - C_{3,3}(0))$

$xdef128 = (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))$

$xdef129 = (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))$

$xdef130 = (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))$

$B_{762} = 6\,\lambda_1^2/xdef128,\ \ B_{763} = 6\,\lambda_1^2/xdef129,\ \ B_{764} = 6\,\lambda_1^2/xdef130$

$xdef131 = (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))\,(C_{3,3}(3) - C_{3,3}(0))$

$xdef132 = (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(3) - C_{3,3}(1))$

$xdef133 = (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(3) - C_{3,3}(2))$

$xdef134 = (C_{3,3}(0) - C_{3,3}(3))\,(C_{3,3}(1) - C_{3,3}(3))\,(C_{3,3}(2) - C_{3,3}(3))$

$B_{765} = 6\,\lambda_1^3/xdef131,\ \ B_{766} = 6\,\lambda_1^3/xdef132$

$B_{767} = 6\,\lambda_1^3/xdef133,\ \ B_{768} = 6\,\lambda_1^3/xdef134$

$xdef135 = (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))\,(C_{3,3}(3) - C_{3,3}(0))\,(C_{3,3}(4) - C_{3,3}(0))$

$xdef136 = (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(3) - C_{3,3}(1))\,(C_{3,3}(4) - C_{3,3}(1))$

$xdef137 = (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(3) - C_{3,3}(2))\,(C_{3,3}(4) - C_{3,3}(2))$

$xdef138 = (C_{3,3}(0) - C_{3,3}(3))\,(C_{3,3}(1) - C_{3,3}(3))\,(C_{3,3}(2) - C_{3,3}(3))\,(C_{3,3}(4) - C_{3,3}(3))$

$xdef139 = (C_{3,3}(0) - C_{3,3}(4))\,(C_{3,3}(1) - C_{3,3}(4))\,(C_{3,3}(2) - C_{3,3}(4))\,(C_{3,3}(3) - C_{3,3}(4))$

$B_{769} = 6\,\lambda_1^4/xdef135,\ \ B_{770} = 6\,\lambda_1^4/xdef136,\ \ B_{771} = 6\,\lambda_1^4/xdef137$

$B_{772} = 6\,\lambda_1^4/xdef138,\ \ B_{773} = 6\,\lambda_1^4/xdef139$

$$\begin{aligned}
xdef140 &= (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))\,(C_{3,3}(3) - C_{3,3}(0)) \\
&\quad (C_{3,3}(4) - C_{3,3}(0))\,(C_{3,3}(5) - C_{3,3}(0)) \\
xdef141 &= (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(3) - C_{3,3}(1)) \\
&\quad (C_{3,3}(4) - C_{3,3}(1))\,(C_{3,3}(5) - C_{3,3}(1)) \\
xdef142 &= (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(3) - C_{3,3}(2)) \\
&\quad (C_{3,3}(4) - C_{3,3}(2))\,(C_{3,3}(5) - C_{3,3}(2)) \\
xdef143 &= (C_{3,3}(0) - C_{3,3}(3))\,(C_{3,3}(1) - C_{3,3}(3))\,(C_{3,3}(2) - C_{3,3}(3)) \\
&\quad (C_{3,3}(4) - C_{3,3}(3))\,(C_{3,3}(5) - C_{3,3}(3)) \\
xdef144 &= (C_{3,3}(0) - C_{3,3}(4))\,(C_{3,3}(1) - C_{3,3}(4))\,(C_{3,3}(2) - C_{3,3}(4)) \\
&\quad (C_{3,3}(3) - C_{3,3}(4))\,(C_{3,3}(5) - C_{3,3}(4))
\end{aligned}$$

$$\begin{aligned} xdef145 &= (C_{3,3}(0)-C_{3,3}(5))\,(C_{3,3}(1)-C_{3,3}(5))\,(C_{3,3}(2)-C_{3,3}(5)) \\ &\quad (C_{3,3}(3)-C_{3,3}(5))\,(C_{3,3}(4)-C_{3,3}(5)) \end{aligned}$$

$B_{774}=6\,\lambda_1^5/xdef140,\;\; B_{775}=6\,\lambda_1^5/xdef141,\;\; B_{776}=6\,\lambda_1^5/xdef142$

$B_{777}=6\,\lambda_1^5/xdef143,\;\; B_{778}=6\,\lambda_1^5/xdef144,\;\; B_{779}=6\,\lambda_1^5/xdef145$

$B_{780}=3\,\lambda_s/(C_{3,3}(0^{(1)})-C_{3,3}(0))$

$DM24(s)=9\,\lambda_1\,\lambda_s\,(s+C_{3,3}(0^{(1)}))+9\,\lambda_1\,\lambda_s\,(s+C_{3,3}(1))$

$xdef146=(C_{3,3}(1)-C_{3,3}(0))\,(C_{3,3}(0^{(1)})-C_{3,3}(0))\,(C_{3,3}(1^{(1)})-C_{3,3}(0))$

$xdef147=(C_{3,3}(0)-C_{3,3}(1))\,(C_{3,3}(0^{(1)})-C_{3,3}(1))\,(C_{3,3}(1^{(1)})-C_{3,3}(1))$

$xdef148=(C_{3,3}(0)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)})-C_{3,3}(0^{(1)}))$

$xdef149=(C_{3,3}(0)-C_{3,3}(1^{(1)}))\,(C_{3,3}(1)-C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)})-C_{3,3}(1^{(1)}))$

$B_{781}=DM24(s=-C_{3,3}(0))/xdef146,\;\; B_{782}=DM24(s=-C_{3,3}(1))/xdef147$

$B_{783}=DM24(s=-C_{3,3}(0^{(1)}))/xdef148,\;\; B_{784}=DM24(s=-C_{3,3}(1^{(1)}))/xdef149$

$DM25(s)=18\,\lambda_s\,\lambda_1^2\,(s+C_{3,3}(0^{(1)}))\,(s+C_{3,3}(1^{(1)}))+2\,\lambda_1\,(s+C_{3,3}(2))\,DM24(s)$

$$\begin{aligned} xdef150 &= (C_{3,3}(1)-C_{3,3}(0))\,(C_{3,3}(2)-C_{3,3}(0))\,(C_{3,3}(0^{(1)})-C_{3,3}(0)) \\ &\quad (C_{3,3}(1^{(1)})-C_{3,3}(0))\,(C_{3,3}(2^{(1)})-C_{3,3}(0)) \\ xdef151 &= (C_{3,3}(0)-C_{3,3}(1))\,(C_{3,3}(2)-C_{3,3}(1))\,(C_{3,3}(0^{(1)})-C_{3,3}(1)) \\ &\quad (C_{3,3}(1^{(1)})-C_{3,3}(1))\,(C_{3,3}(2^{(1)})-C_{3,3}(1)) \\ xdef152 &= (C_{3,3}(0)-C_{3,3}(2))\,(C_{3,3}(1)-C_{3,3}(2))\,(C_{3,3}(0^{(1)})-C_{3,3}(2)) \\ &\quad (C_{3,3}(1^{(1)})-C_{3,3}(2))\,(C_{3,3}(2^{(1)})-C_{3,3}(2)) \\ xdef153 &= (C_{3,3}(0)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1)-C_{3,3}(0^{(1)}))\,(C_{3,3}(2)-C_{3,3}(0^{(1)})) \\ &\quad (C_{3,3}(1^{(1)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(0^{(1)})) \\ xdef154 &= (C_{3,3}(0)-C_{3,3}(1^{(1)}))\,(C_{3,3}(1)-C_{3,3}(1^{(1)}))\,(C_{3,3}(2)-C_{3,3}(1^{(1)})) \\ &\quad (C_{3,3}(0^{(1)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(1^{(1)})) \\ xdef155 &= (C_{3,3}(0)-C_{3,3}(2^{(1)}))\,(C_{3,3}(1)-C_{3,3}(2^{(1)}))\,(C_{3,3}(2)-C_{3,3}(2^{(1)})) \\ &\quad (C_{3,3}(0^{(1)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(1)})-C_{3,3}(2^{(1)})) \end{aligned}$$

$B_{785}=DM25(s=-C_{3,3}(0))/xdef150,\;\; B_{786}=DM25(s=-C_{3,3}(1))/xdef151$

$B_{787}=DM25(s=-C_{3,3}(2))/xdef152,\;\; B_{788}=DM25(s=-C_{3,3}(0^{(1)}))/xdef153$

$B_{789}=DM25(s=-C_{3,3}(1^{(1)}))/xdef154,\;\; B_{790}=DM25(s=-C_{3,3}(2^{(1)}))/xdef155$

$DM26(s)=18\,\lambda_1^3\,\lambda_s\,(s+C_{3,3}(0^{(1)}))\,(s+C_{3,3}(1^{(1)}))\,(s+C_{3,3}(2^{(1)}))+\lambda_1\,(s+C_{3,3}(3))\,DM25(s)$

$$xdef156 = (C_{3,3}(1)-C_{3,3}(0))\,(C_{3,3}(2)-C_{3,3}(0))\,(C_{3,3}(3)-C_{3,3}(0))$$

$$
\begin{aligned}
& (C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0)) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(0)) \\
xdef157 \;=\; & (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(3) - C_{3,3}(1)) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(1))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1)) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(1)) \\
xdef158 \;=\; & (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(3) - C_{3,3}(2)) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(2))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2))\,(C_{3,3}(2^{(1)}) - C_{3,3}(2)) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(2)) \\
xdef159 \;=\; & (C_{3,3}(0) - C_{3,3}(3))\,(C_{3,3}(1) - C_{3,3}(3))\,(C_{3,3}(2) - C_{3,3}(3)) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(3))\,(C_{3,3}(1^{(1)}) - C_{3,3}(3))\,(C_{3,3}(2^{(1)}) - C_{3,3}(3)) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(3)) \\
xdef160 \;=\; & (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2) - C_{3,3}(0^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0^{(1)})) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(0^{(1)})) \\
xdef161 \;=\; & (C_{3,3}(0) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(1^{(1)})) \\
xdef162 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1) - C_{3,3}(2^{(1)}))\,(C_{3,3}(2) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(2^{(1)})) \\
xdef163 \;=\; & (C_{3,3}(0) - C_{3,3}(3^{(1)}))\,(C_{3,3}(1) - C_{3,3}(3^{(1)}))\,(C_{3,3}(2) - C_{3,3}(3^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(3^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(3^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(3^{(1)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(3^{(1)}))
\end{aligned}
$$

$B_{791} = DM26(s = -C_{3,3}(0))/xdef156,\;\; B_{792} = DM26(s = -C_{3,3}(1))/xdef157$
$B_{793} = DM26(s = -C_{3,3}(2))/xdef158,\;\; B_{794} = DM26(s = -C_{3,3}(3))/xdef159$
$B_{795} = DM26(s = -C_{3,3}(0^{(1)}))/xdef160,\;\; B_{796} = DM26(s = -C_{3,3}(1^{(1)}))/xdef161$
$B_{797} = DM26(s = -C_{3,3}(2^{(1)}))/xdef162,\;\; B_{798} = DM26(s = -C_{3,3}(3^{(1)}))/xdef163$
$DM27(s) = 6\,\lambda_1^4\,\lambda_s\,(s{+}C_{3,3}(0^{(1)}))\,(s{+}C_{3,3}(1^{(1)}))\,(s{+}C_{3,3}(2^{(1)}))\,(s{+}C_{3,3}(3^{(1)})){+}\lambda_1\,(s{+}$

$C_{3,3}(4))\, DM26(s)$

$$
\begin{aligned}
xdef164 &= (C_{3,3}(1)-C_{3,3}(0))\,(C_{3,3}(2)-C_{3,3}(0))\,(C_{3,3}(3)-C_{3,3}(0)) \\
&\quad (C_{3,3}(4)-C_{3,3}(0))\,(C_{3,3}(0^{(1)})-C_{3,3}(0))\,(C_{3,3}(1^{(1)})-C_{3,3}(0)) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(0))\,(C_{3,3}(3^{(1)})-C_{3,3}(0))\,(C_{3,3}(4^{(1)})-C_{3,3}(0)) \\
xdef165 &= (C_{3,3}(0)-C_{3,3}(1))\,(C_{3,3}(2)-C_{3,3}(1))\,(C_{3,3}(3)-C_{3,3}(1)) \\
&\quad (C_{3,3}(4)-C_{3,3}(1))\,(C_{3,3}(0^{(1)})-C_{3,3}(1))\,(C_{3,3}(1^{(1)})-C_{3,3}(1)) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(1))\,(C_{3,3}(3^{(1)})-C_{3,3}(1))\,(C_{3,3}(4^{(1)})-C_{3,3}(1)) \\
xdef166 &= (C_{3,3}(0)-C_{3,3}(2))\,(C_{3,3}(1)-C_{3,3}(2))\,(C_{3,3}(3)-C_{3,3}(2)) \\
&\quad (C_{3,3}(4)-C_{3,3}(2))\,(C_{3,3}(0^{(1)})-C_{3,3}(2))\,(C_{3,3}(1^{(1)})-C_{3,3}(2)) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(2))\,(C_{3,3}(3^{(1)})-C_{3,3}(2))\,(C_{3,3}(4^{(1)})-C_{3,3}(2)) \\
xdef167 &= (C_{3,3}(0)-C_{3,3}(3))\,(C_{3,3}(1)-C_{3,3}(3))\,(C_{3,3}(2)-C_{3,3}(3)) \\
&\quad (C_{3,3}(4)-C_{3,3}(3))\,(C_{3,3}(0^{(1)})-C_{3,3}(3))\,(C_{3,3}(1^{(1)})-C_{3,3}(3)) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(3))\,(C_{3,3}(3^{(1)})-C_{3,3}(3))\,(C_{3,3}(4^{(1)})-C_{3,3}(3)) \\
xdef168 &= (C_{3,3}(0)-C_{3,3}(4))\,(C_{3,3}(1)-C_{3,3}(4))\,(C_{3,3}(2)-C_{3,3}(4)) \\
&\quad (C_{3,3}(3)-C_{3,3}(4))\,(C_{3,3}(0^{(1)})-C_{3,3}(4))\,(C_{3,3}(1^{(1)})-C_{3,3}(4)) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(4))\,(C_{3,3}(3^{(1)})-C_{3,3}(4))\,(C_{3,3}(4^{(1)})-C_{3,3}(4)) \\
xdef169 &= (C_{3,3}(0)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1)-C_{3,3}(0^{(1)}))\,(C_{3,3}(2)-C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(3)-C_{3,3}(0^{(1)}))\,(C_{3,3}(4)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)})-C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(3^{(1)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(4^{(1)})-C_{3,3}(0^{(1)})) \\
xdef170 &= (C_{3,3}(0)-C_{3,3}(1^{(1)}))\,(C_{3,3}(1)-C_{3,3}(1^{(1)}))\,(C_{3,3}(2)-C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(3)-C_{3,3}(1^{(1)}))\,(C_{3,3}(4)-C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)})-C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(2^{(1)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(3^{(1)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(4^{(1)})-C_{3,3}(1^{(1)})) \\
xdef171 &= (C_{3,3}(0)-C_{3,3}(2^{(1)}))\,(C_{3,3}(1)-C_{3,3}(2^{(1)}))\,(C_{3,3}(2)-C_{3,3}(2^{(1)})) \\
&\quad (C_{3,3}(3)-C_{3,3}(2^{(1)}))\,(C_{3,3}(4)-C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(1)})-C_{3,3}(2^{(1)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(3^{(1)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(4^{(1)})-C_{3,3}(2^{(1)})) \\
xdef172 &= (C_{3,3}(0)-C_{3,3}(3^{(1)}))\,(C_{3,3}(1)-C_{3,3}(3^{(1)}))\,(C_{3,3}(2)-C_{3,3}(3^{(1)})) \\
&\quad (C_{3,3}(3)-C_{3,3}(3^{(1)}))\,(C_{3,3}(4)-C_{3,3}(3^{(1)}))\,(C_{3,3}(0^{(1)})-C_{3,3}(3^{(1)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(3^{(1)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(3^{(1)}))\,(C_{3,3}(4^{(1)})-C_{3,3}(3^{(1)}))
\end{aligned}
$$

$$\begin{aligned} xdef173 &= (C_{3,3}(0) - C_{3,3}(4^{(1)}))\,(C_{3,3}(1) - C_{3,3}(4^{(1)}))\,(C_{3,3}(2) - C_{3,3}(4^{(1)})) \\ &\quad (C_{3,3}(3) - C_{3,3}(4^{(1)}))\,(C_{3,3}(4) - C_{3,3}(4^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(4^{(1)})) \\ &\quad (C_{3,3}(1^{(1)}) - C_{3,3}(4^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(4^{(1)}))\,(C_{3,3}(3^{(1)}) - C_{3,3}(4^{(1)})) \end{aligned}$$

$B_{799} = DM27(s = -C_{3,3}(0))/xdef164,\ \ B_{800} = DM27(s = -C_{3,3}(1))/xdef165$
$B_{801} = DM27(s = -C_{3,3}(2))/xdef166,\ \ B_{802} = DM26(s = -C_{3,3}(3))/xdef167$
$B_{803} = DM27(s = -C_{3,3}(4))/xdef168,\ \ B_{804} = DM27(s = -C_{3,3}(0^{(1)}))/xdef169$
$B_{805} = DM27(s = -C_{3,3}(1^{(1)}))/xdef170,\ \ B_{806} = DM27(s = -C_{3,3}(2^{(1)}))/xdef171$
$B_{807} = DM27(s = -C_{3,3}(3^{(1)}))/xdef172,\ \ B_{808} = DM27(s = -C_{3,3}(4^{(1)}))/xdef173$
$xdef174 = (C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0))$
$xdef175 = (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)}))$
$xdef176 = (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)}))$
$B_{809} = 6\,\lambda_s^2/xdef174,\ \ B_{810} = 6\,\lambda_s^2/xdef175,\ \ B_{811} = 6\,\lambda_s^2/xdef176$
$DM28(s) = 2\,\lambda_s\,(s + C_{3,3}(0^{(2)}))\,DM24(s) + 18\,\lambda_1\,\lambda_s^2\,(s + C_{3,3}(1))\,(s + C_{3,3}(1^{(1)}))$

$$\begin{aligned} xdef177 &= (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0)) \\ &\quad (C_{3,3}(0^{(2)}) - C_{3,3}(0))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0)) \\ xdef178 &= (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1)) \\ &\quad (C_{3,3}(0^{(2)}) - C_{3,3}(1))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1)) \\ xdef179 &= (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(1)})) \\ &\quad (C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(1)})) \\ xdef180 &= (C_{3,3}(0) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(1)})) \\ &\quad (C_{3,3}(0^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1^{(1)})) \\ xdef181 &= (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)})) \\ &\quad (C_{3,3}(1^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(2)})) \\ xdef182 &= (C_{3,3}(0) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(2)})) \\ &\quad (C_{3,3}(1^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(2)})) \end{aligned}$$

$B_{812} = DM28(s = -C_{3,3}(0))/xdef177,\ \ B_{813} = DM28(s = -C_{3,3}(1))/xdef178$
$B_{814} = DM28(s = -C_{3,3}(0^{(1)}))/xdef179,\ \ B_{815} = DM28(s = -C_{3,3}(1^{(1)}))/xdef180$
$B_{816} = DM28(s = -C_{3,3}(0^{(2)}))/xdef181,\ \ B_{817} = DM28(s = -C_{3,3}(1^{(2)}))/xdef182$
$DM29(s) \;=\; 2\,\lambda_s\,(s + C_{3,3}(0^{(2)}))\,(s + C_{3,3}(1^{(2)}))\,DM25(s) + 2\,\lambda_1\,(s + C_{3,3}(2))\,(s +$

$C_{3,3}(2^{(1)}))\,DM28(s)$

$$\begin{aligned}
xdef183 &= (C_{3,3}(1)-C_{3,3}(0))\,(C_{3,3}(2)-C_{3,3}(0))\,(C_{3,3}(0^{(1)})-C_{3,3}(0)) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(0))\,(C_{3,3}(2^{(1)})-C_{3,3}(0))\,(C_{3,3}(0^{(2)})-C_{3,3}(0)) \\
&\quad (C_{3,3}(1^{(2)})-C_{3,3}(0))\,(C_{3,3}(2^{(2)})-C_{3,3}(0)) \\
xdef184 &= (C_{3,3}(0)-C_{3,3}(1))\,(C_{3,3}(2)-C_{3,3}(1))\,(C_{3,3}(0^{(1)})-C_{3,3}(1)) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(1))\,(C_{3,3}(2^{(1)})-C_{3,3}(1))\,(C_{3,3}(0^{(2)})-C_{3,3}(1)) \\
&\quad (C_{3,3}(1^{(2)})-C_{3,3}(1))\,(C_{3,3}(2^{(2)})-C_{3,3}(1)) \\
xdef185 &= (C_{3,3}(0)-C_{3,3}(2))\,(C_{3,3}(1)-C_{3,3}(2))\,(C_{3,3}(0^{(1)})-C_{3,3}(2)) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(2))\,(C_{3,3}(2^{(1)})-C_{3,3}(2))\,(C_{3,3}(0^{(2)})-C_{3,3}(2)) \\
&\quad (C_{3,3}(1^{(2)})-C_{3,3}(2))\,(C_{3,3}(2^{(2)})-C_{3,3}(2)) \\
xdef186 &= (C_{3,3}(0)-C_{3,3}(0^{(1)}))\,(C_{3,3}(1)-C_{3,3}(0^{(1)}))\,(C_{3,3}(2)-C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(2)})-C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(1^{(2)})-C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(2)})-C_{3,3}(0^{(1)})) \\
xdef187 &= (C_{3,3}(0)-C_{3,3}(1^{(1)}))\,(C_{3,3}(1)-C_{3,3}(1^{(1)}))\,(C_{3,3}(2)-C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(1^{(2)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(2)})-C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(0^{(1)})-C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(2)})-C_{3,3}(1^{(1)})) \\
xdef188 &= (C_{3,3}(0)-C_{3,3}(2^{(1)}))\,(C_{3,3}(1)-C_{3,3}(2^{(1)}))\,(C_{3,3}(2)-C_{3,3}(2^{(1)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(2)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(2)})-C_{3,3}(2^{(1)})) \\
&\quad (C_{3,3}(0^{(1)})-C_{3,3}(2^{(1)}))\,(C_{3,3}(2^{(2)})-C_{3,3}(2^{(1)})) \\
xdef189 &= (C_{3,3}(0)-C_{3,3}(0^{(2)}))\,(C_{3,3}(1)-C_{3,3}(0^{(2)}))\,(C_{3,3}(2)-C_{3,3}(0^{(2)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(2)})-C_{3,3}(0^{(2)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(0^{(2)})) \\
&\quad (C_{3,3}(0^{(1)})-C_{3,3}(0^{(2)}))\,(C_{3,3}(2^{(2)})-C_{3,3}(0^{(2)})) \\
xdef190 &= (C_{3,3}(0)-C_{3,3}(1^{(2)}))\,(C_{3,3}(1)-C_{3,3}(1^{(2)}))\,(C_{3,3}(2)-C_{3,3}(1^{(2)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(2)})-C_{3,3}(1^{(2)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(1^{(2)})) \\
&\quad (C_{3,3}(0^{(1)})-C_{3,3}(1^{(2)}))\,(C_{3,3}(2^{(2)})-C_{3,3}(1^{(2)})) \\
xdef191 &= (C_{3,3}(0)-C_{3,3}(2^{(2)}))\,(C_{3,3}(1)-C_{3,3}(2^{(2)}))\,(C_{3,3}(2)-C_{3,3}(2^{(2)})) \\
&\quad (C_{3,3}(1^{(1)})-C_{3,3}(2^{(2)}))\,(C_{3,3}(0^{(2)})-C_{3,3}(2^{(2)}))\,(C_{3,3}(2^{(1)})-C_{3,3}(2^{(2)})) \\
&\quad (C_{3,3}(0^{(1)})-C_{3,3}(2^{(2)}))\,(C_{3,3}(1^{(2)})-C_{3,3}(2^{(2)}))
\end{aligned}$$

$B_{818} = DM29(s = -C_{3,3}(0))/xdef183, \ \ B_{819} = DM29(s = -C_{3,3}(1))/xdef184$
$B_{820} = DM29(s = -C_{3,3}(2))/xdef185, \ \ B_{821} = DM29(s = -C_{3,3}(0^{(1)}))/xdef186$
$B_{822} = DM29(s = -C_{3,3}(1^{(1)}))/xdef187, \ \ B_{823} = DM29(s = -C_{3,3}(2^{(1)}))/xdef188$
$B_{824} = DM29(s = -C_{3,3}(0^{(2)}))/xdef189, \ \ B_{825} = DM29(s = -C_{3,3}(1^{(2)}))/xdef190$
$B_{826} = DM29(s = -C_{3,3}(2^{(2)}))/xdef191$
$DM30(s) = \lambda_s\,(s+C_{3,3}(0^{(2)}))\,(s+C_{3,3}(1^{(2)}))\,(s+C_{3,3}(2^{(2)}))\,DM26(s)+\lambda_1\,(s+C_{3,3}(3))\,(s+C_{3,3}(3^{(1)}))\,DM29(s)$

$$\begin{aligned}
xdef192 \ &= \ (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))\,(C_{3,3}(3) - C_{3,3}(0)) \\
&\quad (C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0)) \\
&\quad (C_{3,3}(3^{(1)}) - C_{3,3}(0))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0)) \\
&\quad (C_{3,3}(2^{(2)}) - C_{3,3}(0))\,(C_{3,3}(3^{(2)}) - C_{3,3}(0)) \\
xdef193 \ &= \ (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(3) - C_{3,3}(1)) \\
&\quad (C_{3,3}(0^{(1)}) - C_{3,3}(1))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1)) \\
&\quad (C_{3,3}(3^{(1)}) - C_{3,3}(1))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1)) \\
&\quad (C_{3,3}(2^{(2)}) - C_{3,3}(1))\,(C_{3,3}(3^{(2)}) - C_{3,3}(1)) \\
xdef194 \ &= \ (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(3) - C_{3,3}(2)) \\
&\quad (C_{3,3}(0^{(1)}) - C_{3,3}(2))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2))\,(C_{3,3}(2^{(1)}) - C_{3,3}(2)) \\
&\quad (C_{3,3}(3^{(1)}) - C_{3,3}(2))\,(C_{3,3}(0^{(2)}) - C_{3,3}(2))\,(C_{3,3}(1^{(2)}) - C_{3,3}(2)) \\
&\quad (C_{3,3}(2^{(2)}) - C_{3,3}(2))\,(C_{3,3}(3^{(2)}) - C_{3,3}(2)) \\
xdef195 \ &= \ (C_{3,3}(0) - C_{3,3}(3))\,(C_{3,3}(1) - C_{3,3}(3))\,(C_{3,3}(2) - C_{3,3}(3)) \\
&\quad (C_{3,3}(0^{(1)}) - C_{3,3}(3))\,(C_{3,3}(1^{(1)}) - C_{3,3}(3))\,(C_{3,3}(2^{(1)}) - C_{3,3}(3)) \\
&\quad (C_{3,3}(3^{(1)}) - C_{3,3}(3))\,(C_{3,3}(0^{(2)}) - C_{3,3}(3))\,(C_{3,3}(1^{(2)}) - C_{3,3}(3)) \\
&\quad (C_{3,3}(2^{(2)}) - C_{3,3}(3))\,(C_{3,3}(3^{(2)}) - C_{3,3}(3)) \\
xdef196 \ &= \ (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(3) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(3^{(1)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(2^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(0^{(1)})) \\
xdef197 \ &= \ (C_{3,3}(0) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2) - C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(3) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1^{(1)}))
\end{aligned}$$

$$
\begin{aligned}
& (C_{3,3}(3^{(1)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(2^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(1^{(1)})) \\
xdef198 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1) - C_{3,3}(2^{(1)}))\,(C_{3,3}(2) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(3^{(1)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(2^{(2)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(2^{(1)})) \\
xdef199 \;=\; & (C_{3,3}(0) - C_{3,3}(3^{(1)}))\,(C_{3,3}(1) - C_{3,3}(3^{(1)}))\,(C_{3,3}(2) - C_{3,3}(3^{(1)})) \\
& (C_{3,3}(3) - C_{3,3}(3^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(3^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(3^{(1)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(3^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(3^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(3^{(1)})) \\
& (C_{3,3}(2^{(2)}) - C_{3,3}(3^{(1)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(3^{(1)})) \\
xdef200 \;=\; & (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1) - C_{3,3}(0^{(2)}))\,(C_{3,3}(2) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(3) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(3^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(2^{(2)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(0^{(2)})) \\
xdef201 \;=\; & (C_{3,3}(0) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1) - C_{3,3}(1^{(2)}))\,(C_{3,3}(2) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(3) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(3^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(2^{(2)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(1^{(2)})) \\
xdef202 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(2)}))\,(C_{3,3}(1) - C_{3,3}(2^{(2)}))\,(C_{3,3}(2) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(3) - C_{3,3}(2^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(3^{(1)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(1^{(2)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(3^{(2)}) - C_{3,3}(2^{(2)})) \\
xdef203 \;=\; & (C_{3,3}(0) - C_{3,3}(3^{(2)}))\,(C_{3,3}(1) - C_{3,3}(3^{(2)}))\,(C_{3,3}(2) - C_{3,3}(3^{(2)})) \\
& (C_{3,3}(3) - C_{3,3}(3^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(3^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(3^{(2)})) \\
& (C_{3,3}(2^{(1)}) - C_{3,3}(3^{(2)}))\,(C_{3,3}(3^{(1)}) - C_{3,3}(3^{(2)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(3^{(2)})) \\
& (C_{3,3}(1^{(2)}) - C_{3,3}(3^{(2)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(3^{(2)}))
\end{aligned}
$$

$B_{827} = DM30(s = -C_{3,3}(0))/xdef192$, $B_{828} = DM30(s = -C_{3,3}(1))/xdef193$

$B_{829} = DM30(s = -C_{3,3}(2))/xdef194$, $B_{830} = DM29(s = -C_{3,3}(3))/xdef195$

$B_{831} = DM30(s = -C_{3,3}(0^{(1)}))/xdef196$, $B_{832} = DM30(s = -C_{3,3}(1^{(1)}))/xdef197$

$B_{833} = DM30(s = -C_{3,3}(2^{(1)}))/xdef198,\ \ B_{834} = DM30(s = -C_{3,3}(3^{(1)}))/xdef199$

$B_{835} = DM30(s = -C_{3,3}(0^{(2)}))/xdef200,\ \ B_{836} = DM30(s = -C_{3,3}(1^{(2)}))/xdef201$

$B_{837} = DM30(s = -C_{3,3}(2^{(2)}))/xdef202,\ \ B_{838} = DM30(s = -C_{3,3}(3^{(2)}))/xdef203$

$xdef204 = (C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0))$

$xdef205 = (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(1)}))$

$xdef206 = (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(2)}))$

$xdef207 = (C_{3,3}(0) - C_{3,3}(0^{(3)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(3)}))$

$B_{839} = 6\,\lambda_s^3/xdef204,\ \ B_{840} = 6\,\lambda_s^3/xdef205$

$B_{841} = 6\,\lambda_s^3/xdef206,\ \ B_{842} = 6\,\lambda_s^3/xdef207$

$DM31(s) = 18\,\lambda_1\,\lambda_s^3\,(s+C_{3,3}(1))\,(s+C_{3,3}(1^{(1)}))\,(s+C_{3,3}(1^{(2)}))+\lambda_s\,(s+C_{3,3}(0^{(3)}))\,DM28(s)$

$$
\begin{aligned}
xdef208 \;=\;& (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0)) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(0))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0)) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(0)) \\
xdef209 \;=\;& (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1)) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(1))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1)) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(1)) \\
xdef210 \;=\;& (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(1)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(1)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(0^{(1)})) \\
xdef211 \;=\;& (C_{3,3}(0) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(1^{(1)})) \\
xdef212 \;=\;& (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(1^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(0^{(2)})) \\
xdef213 \;=\;& (C_{3,3}(0) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(1^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(1^{(2)})) \\
xdef214 \;=\;& (C_{3,3}(0) - C_{3,3}(0^{(3)}))\,(C_{3,3}(1) - C_{3,3}(0^{(3)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0^{(3)}))
\end{aligned}
$$

$$(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(3)}))$$
$$(C_{3,3}(1^{(3)}) - C_{3,3}(0^{(3)}))$$

$$\begin{aligned}
xdef215 &= (C_{3,3}(0) - C_{3,3}(1^{(3)}))\,(C_{3,3}(1) - C_{3,3}(1^{(3)}))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1^{(3)})) \\
&\quad (C_{3,3}(1^{(1)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1^{(3)})) \\
&\quad (C_{3,3}(0^{(3)}) - C_{3,3}(1^{(3)}))
\end{aligned}$$

$B_{843} = DM31(s = -C_{3,3}(0))/xdef208,\ \ B_{844} = DM31(s = -C_{3,3}(1))/xdef209$
$B_{845} = DM31(s = -C_{3,3}(0^{(1)}))/xdef210,\ \ B_{846} = DM31(s = -C_{3,3}(1^{(1)}))/xdef211$
$B_{847} = DM31(s = -C_{3,3}(0^{(2)}))/xdef212,\ \ B_{848} = DM31(s = -C_{3,3}(1^{(2)}))/xdef213$
$B_{849} = DM31(s = -C_{3,3}(0^{(3)}))/xdef214,\ \ B_{850} = DM31(s = -C_{3,3}(1^{(3)}))/xdef215$
$DM32(s) = \lambda_s\,(s+C_{3,3}(0^{(3)}))\,(s+C_{3,3}(1^{(3)}))\ DM29(s)+2\,\lambda_1\,(s+C_{3,3}(2))\,(s+C_{3,3}(2^{(1)}))\,(s+C_{3,3}(2^{(2)}))$

$$\begin{aligned}
xdef216 &= (C_{3,3}(1) - C_{3,3}(0))\,(C_{3,3}(2) - C_{3,3}(0))\,(C_{3,3}(0^{(1)}) - C_{3,3}(0)) \\
&\quad (C_{3,3}(1^{(1)}) - C_{3,3}(0))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0)) \\
&\quad (C_{3,3}(1^{(2)}) - C_{3,3}(0))\,(C_{3,3}(2^{(2)}) - C_{3,3}(0))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0)) \\
&\quad (C_{3,3}(1^{(3)}) - C_{3,3}(0))\,(C_{3,3}(2^{(3)}) - C_{3,3}(0)) \\
xdef217 &= (C_{3,3}(0) - C_{3,3}(1))\,(C_{3,3}(2) - C_{3,3}(1))\,(C_{3,3}(0^{(1)}) - C_{3,3}(1)) \\
&\quad (C_{3,3}(1^{(1)}) - C_{3,3}(1))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1)) \\
&\quad (C_{3,3}(1^{(2)}) - C_{3,3}(1))\,(C_{3,3}(2^{(2)}) - C_{3,3}(1))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1)) \\
&\quad (C_{3,3}(1^{(3)}) - C_{3,3}(1))\,(C_{3,3}(2^{(3)}) - C_{3,3}(1)) \\
xdef218 &= (C_{3,3}(0) - C_{3,3}(2))\,(C_{3,3}(1) - C_{3,3}(2))\,(C_{3,3}(0^{(1)}) - C_{3,3}(2)) \\
&\quad (C_{3,3}(1^{(1)}) - C_{3,3}(2))\,(C_{3,3}(2^{(1)}) - C_{3,3}(2))\,(C_{3,3}(0^{(2)}) - C_{3,3}(2)) \\
&\quad (C_{3,3}(1^{(2)}) - C_{3,3}(2))\,(C_{3,3}(2^{(2)}) - C_{3,3}(2))\,(C_{3,3}(0^{(3)}) - C_{3,3}(2)) \\
&\quad (C_{3,3}(1^{(3)}) - C_{3,3}(2))\,(C_{3,3}(2^{(3)}) - C_{3,3}(2)) \\
xdef219 &= (C_{3,3}(0) - C_{3,3}(0^{(1)}))\,(C_{3,3}(1) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(1^{(1)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(1^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(1)})) \\
&\quad (C_{3,3}(1^{(3)}) - C_{3,3}(0^{(1)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(0^{(1)})) \\
xdef220 &= (C_{3,3}(0) - C_{3,3}(1^{(1)}))\,(C_{3,3}(1) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2) - C_{3,3}(1^{(1)})) \\
&\quad (C_{3,3}(0^{(1)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(1^{(1)}))
\end{aligned}$$

$$
\begin{aligned}
& (C_{3,3}(1^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1^{(1)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(1^{(1)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(1^{(1)})) \\
xdef221 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1) - C_{3,3}(2^{(1)}))\,(C_{3,3}(2) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(2)}) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(1^{(2)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(2^{(1)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(2^{(1)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(2^{(1)})) \\
xdef222 \;=\; & (C_{3,3}(0) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1) - C_{3,3}(0^{(2)}))\,(C_{3,3}(2) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(1^{(2)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(0^{(2)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(0^{(2)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(0^{(2)})) \\
xdef223 \;=\; & (C_{3,3}(0) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1) - C_{3,3}(1^{(2)}))\,(C_{3,3}(2) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(1^{(2)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(1^{(2)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(1^{(2)})) \\
xdef224 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(2)}))\,(C_{3,3}(1) - C_{3,3}(2^{(2)}))\,(C_{3,3}(2) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(0^{(3)}) - C_{3,3}(2^{(2)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(2^{(2)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(2^{(2)})) \\
xdef225 \;=\; & (C_{3,3}(0) - C_{3,3}(0^{(3)}))\,(C_{3,3}(1) - C_{3,3}(0^{(3)}))\,(C_{3,3}(2) - C_{3,3}(0^{(3)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(0^{(3)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(0^{(3)})) \\
& (C_{3,3}(1^{(3)}) - C_{3,3}(0^{(3)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(0^{(3)})) \\
xdef226 \;=\; & (C_{3,3}(0) - C_{3,3}(1^{(3)}))\,(C_{3,3}(1) - C_{3,3}(1^{(3)}))\,(C_{3,3}(2) - C_{3,3}(1^{(3)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(1^{(3)})) \\
& (C_{3,3}(0^{(2)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(1^{(3)})) \\
& (C_{3,3}(0^{(3)}) - C_{3,3}(1^{(3)}))\,(C_{3,3}(2^{(3)}) - C_{3,3}(1^{(3)})) \\
xdef227 \;=\; & (C_{3,3}(0) - C_{3,3}(2^{(3)}))\,(C_{3,3}(1) - C_{3,3}(2^{(3)}))\,(C_{3,3}(2) - C_{3,3}(2^{(3)})) \\
& (C_{3,3}(0^{(1)}) - C_{3,3}(2^{(3)}))\,(C_{3,3}(1^{(1)}) - C_{3,3}(2^{(3)}))\,(C_{3,3}(2^{(1)}) - C_{3,3}(2^{(3)}))
\end{aligned}
$$

$$(C_{3,3}(0^{(2)}) - C_{3,3}(2^{(3)}))\,(C_{3,3}(1^{(2)}) - C_{3,3}(2^{(3)}))\,(C_{3,3}(2^{(2)}) - C_{3,3}(2^{(3)}))$$
$$(C_{3,3}(0^{(3)}) - C_{3,3}(2^{(3)}))\,(C_{3,3}(1^{(3)}) - C_{3,3}(2^{(3)}))$$

$B_{850} = DM32(s = -C_{3,3}(0))/xdef216,\;\; B_{851} = DM32(s = -C_{3,3}(1))/xdef217$

$B_{852} = DM32(s = -C_{3,3}(2))/xdef218,\;\; B_{853} = DM32(s = -C_{3,3}(0^{(1)}))/xdef219$

$B_{854} = DM32(s = -C_{3,3}(1^{(1)}))/xdef220,\;\; B_{855} = DM32(s = -C_{3,3}(2^{(1)}))/xdef221$

$B_{856} = DM32(s = -C_{3,3}(0^{(2)}))/xdef222,\;\; B_{857} = DM32(s = -C_{3,3}(1^{(2)}))/xdef223$

$B_{858} = DM32(s = -C_{3,3}(2^{(2)}))/xdef224,\;\; B_{859} = DM32(s = -C_{3,3}(0^{(3)}))/xdef225$

$B_{860} = DM32(s = -C_{3,3}(1^{(3)}))/xdef226,\;\; B_{861} = DM32(s = -C_{3,3}(2^{(3)}))/xdef227$

$k_{61} = A_{2350D},\;\; k_{62} = A_{2348},\;\; k_{63} = A_{2347},\;\; k_{64} = A_{2350C}$

$k_{65} = A_{2462D},\;\; k_{66} = A_{2458},\;\; k_{67} = A_{2457},\;\; k_{68} = A_{2462C}$

Appendix I

The Non-Identical Unit System With Late Standby Activation: Definition of Constants

I.1 Non-Identical Unit 1-out-of-(2+1):G System: Definition of Constants

The following constants are defined for the analysis performed for the non-identical unit $1 - out - of - (2+1) : G$ system under SBAP Z discussed in Section 4.2.

$$A_{2900} = C_{2,1}(1) + C_{2,1}(5),\ \ A_{2901} = C_{2,1}(1)\, C_{2,1}(5) - \lambda_{sd}\, \mu_{sd},\ \ A_{2902} = C_{2,1}(2) + C_{2,1}(6)$$
$$A_{2903} = C_{2,1}(2)\, C_{2,1}(6) - \lambda_{sd}\, \mu_{sd},\ \ A_{2904} = A_{2900}\, A_{2903} + A_{2901}\, A_{2902},\ \ A_{2905} = A_{2901}\, A_{2903}$$
$$A_{2906} = C_{2,1}(3) + C_{2,1}(4),\ \ A_{2907} = C_{2,1}(3)\, C_{2,1}(4)$$
$$A_{2908} = A_{2905}\, A_{2906} + A_{2904}\, A_{2907},\ \ A_{2909} = A_{2905}\, A_{2907}$$
$$A_{2910} = C_{2,1}(2)\, C_{2,1}(4) + C_{2,1}(2)\, C_{2,1}(6) + C_{2,1}(4)\, C_{2,1}(6) - \lambda_{sd}\, \mu_{sd}$$
$$A_{2911} = C_{2,1}(2)\, C_{2,1}(4)\, C_{2,1}(6) - \lambda_{sd}\, \mu_{sd}\, C_{2,1}(4)$$
$$A_{2912} = \lambda_a\, (C_{2,1}(3) + C_{2,1}(5)),\ \ A_{2913} = \lambda_a\, C_{2,1}(3)\, C_{2,1}(5) + \lambda_a\, \lambda_{sd}\, \mu_{sd}$$
$$A_{2914} = A_{2910}\, A_{2913} + A_{2911}\, A_{2912},\ \ A_{2915} = A_{2911}\, A_{2913}$$
$$A_{2916} = C_{2,1}(1)\, C_{2,1}(4) + C_{2,1}(1)\, C_{2,1}(5) + C_{2,1}(4)\, C_{2,1}(5) - \lambda_{sd}\, \mu_{sd}$$
$$A_{2917} = C_{2,1}(1)\, C_{2,1}(4)\, C_{2,1}(5) - \lambda_{sd}\, \mu_{sd}\, C_{2,1}(4)$$
$$A_{2918} = \lambda_b\, (C_{2,1}(3) + C_{2,1}(6)),\ \ A_{2919} = \lambda_b\, C_{2,1}(3)\, C_{2,1}(6) + \lambda_b\, \lambda_{sd}\, \mu_{sd}$$

$A_{2920} = A_{2916}\,A_{2919} + A_{2917}\,A_{2918},\;\; A_{2921} = A_{2917}\,A_{2919}$

$A_{2922} = C_{2,1}(1)\,C_{2,1}(4) + C_{2,1}(1)\,C_{2,1}(5) + C_{2,1}(4)\,C_{2,1}(5) - \lambda_{sd}\,\mu_{sd}$

$A_{2923} = C_{2,1}(1)\,C_{2,1}(4)\,C_{2,1}(5) - \lambda_{sd}\,\mu_{sd}\,C_{2,1}(4)$

$A_{2924} = \lambda_{sd}\,(C_{2,1}(2) + C_{2,1}(6)),\;\; A_{2925} = \lambda_{sd}\,(C_{2,1}(2)\,C_{2,1}(6) - \lambda_{sd}\,\mu_{sd})$

$A_{2926} = A_{2922}\,A_{2925} + A_{2923}\,A_{2924},\;\; A_{2927} = A_{2923}\,A_{2925}$

$A_{2928} = \lambda_a\,\lambda_b\,(C_{2,1}(3) + C_{2,1}(6)),\;\; A_{2929} = \lambda_a\,\lambda_b\,C_{2,1}(3)\,C_{2,1}(6)$

$A_{2930} = \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}\,(C_{2,1}(2) + C_{2,1}(6))$

$A_{2931} = \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}\,C_{2,1}(2)\,C_{2,1}(6) - 2\,\lambda_a\,\lambda_b\,\lambda_{sd}^2\,\mu_{sd}^2$

$A_{2932} = \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}\,(C_{2,1}(2) + C_{2,1}(5)),\;\; A_{2933} = \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}\,C_{2,1}(2)\,C_{2,1}(5)$

$A_{2934} = \lambda_a\,\lambda_b\,(C_{2,1}(3) + C_{2,1}(5)),\;\; A_{2935} = \lambda_a\,\lambda_b\,C_{2,1}(3)\,C_{2,1}(5)$

$A_{2936} = C_{2,1}(2) + C_{2,1}(6),\;\; A_{2937} = C_{2,1}(2)\,C_{2,1}(6) - \lambda_{sd}\,\mu_{sd}$

$A_{2938} = C_{2,1}(1) + C_{2,1}(5),\;\; A_{2939} = C_{2,1}(1)\,C_{2,1}(5) - \lambda_{sd}\,\mu_{sd}$

$A_{2940} = A_{2928}\,A_{2939} + A_{2929}\,A_{2934},\;\; A_{2941} = A_{2929}\,A_{2939}$

$A_{2942} = A_{2934}\,A_{2937} + A_{2935}\,A_{2936},\;\; A_{2943} = A_{2935}\,A_{2937}$

$A_{2944} = A_{2940} + A_{2942},\;\; A_{2945} = A_{2941} + A_{2943}$

$A_{2946} = \lambda_a\,\lambda_{sd}\,(C_{2,1}(2)\,C_{2,1}(4) + C_{2,1}(2)\,C_{2,1}(6) + C_{2,1}(4)\,C_{2,1}(6)) - \lambda_a\,\lambda_{sd}^2\,\mu_{sd}$

$A_{2947} = \lambda_a\,\lambda_{sd}\,C_{2,1}(2)\,C_{2,1}(4)\,C_{2,1}(6) - \lambda_a\,\lambda_{sd}^2\,\mu_{sd}\,C_{2,1}(4)$

$A_{2948} = 2.0,\; A_{2949} = C_{2,1}(1) + C_{2,1}(3),\;\; A_{2950} = A_{2946}\,A_{2949} + A_{2947}\,A_{2948}$

$A_{2951} = A_{2947}\,A_{2949}$

$A_{2952} = C_{2,1}(1)\,C_{2,1}(4) + C_{2,1}(1)\,C_{2,1}(5) + C_{2,1}(4)\,C_{2,1}(5) - \lambda_{sd}\,\mu_{sd}$

$A_{2953} = C_{2,1}(1)\,C_{2,1}(4)\,C_{2,1}(5) - \lambda_{sd}\,\mu_{sd}\,C_{2,1}(4),\;\; A_{2954} = 2\,\lambda_b\,\lambda_{sd}$

$A_{2955} = \lambda_b\,\lambda_{sd}\,(C_{2,1}(2) + C_{2,1}(3)),\;\; A_{2956} = A_{2952}\,A_{2955} + A_{2953}\,A_{2954}$

$A_{2957} = A_{2953}\,A_{2955},\; A_{2958} = A_{2909} + C_{2,1}(0)\,A_{2508} - \mu_{sd}\,A_{2926}$

$A_{2959} = C_{2,1}(0)\,A_{2909} - \mu_{sd}\,A_{2927},\;\; A_{2960} = \lambda_d\,A_{2944} + \lambda_b\,A_{2950} + \lambda_a\,A_{2956}$

$A_{2961} = \lambda_d\,A_{2945} + \lambda_b\,A_{2951} + \lambda_a\,A_{2957}$

$A_{2962} = \lambda_{c0}\,A_{2908} + \lambda_{c1}\,A_{2914} + \lambda_{c2}\,A_{2920} + \lambda_{c3}\,A_{2926} + \lambda_{c4}\,A_{2944} + \lambda_{c5}\,A_{2950} + \lambda_{c6}\,A_{2956}$

$A_{2963} = \lambda_{c0}\,A_{2909} + \lambda_{c1}\,A_{2915} + \lambda_{c2}\,A_{2921} + \lambda_{c3}\,A_{2927} + \lambda_{c4}\,A_{2945} + \lambda_{c5}\,A_{2951} + \lambda_{c6}\,A_{2957}$

$A_{2964} = A_{2908} + A_{2914} + A_{2920} + A_{2926} + A_{2944} + A_{2950} + A_{2956}$

$A_{2965} = A_{2909} + A_{2915} + A_{2921} + A_{2927} + A_{2945} + A_{2951} + A_{2957}$

$BX_{622} = (s_{82} + C_{2,1}(3))/(s_{82} - s_{83}),\;\; BX_{623} = (s_{83} + C_{2,1}(3))/(s_{83} - s_{82})$

$DR1(s) = \lambda_a\,(s + C_{2,1}(3))\,(s + C_{2,1}(5)) + \lambda_a\,\lambda_{sd}\,\mu_{sd}$

$xduf1 = (s_{82} - s_{83})\,(s_{82} - s_{84})\,(s_{82} - s_{85})$

$xduf2 = (s_{83} - s_{82})\,(s_{83} - s_{84})\,(s_{83} - s_{85})$

$xduf3 = (s_{84} - s_{82})\,(s_{84} - s_{83})\,(s_{84} - s_{85})$

$xduf4 = (s_{85} - s_{82})\,(s_{85} - s_{83})\,(s_{85} - s_{84})$

$BX_{624} = DR1(s = s_{82})/xduf1,\;\; BX_{625} = DR1(s = s_{83})/xduf2$

$BX_{626} = DR1(s = s_{84})/xduf3,\;\; BX_{627} = DR1(s = s_{85})/xduf4$

$DR2(s) = \lambda_b\,(s + C_{2,1}(3))\,(s + C_{2,1}(6)) + \lambda_b\,\lambda_{sd}\,\mu_{sd}$

$xduf5 = (s_{82} - s_{83})\,(s_{82} - s_{86})\,(s_{82} - s_{87})$

$xduf6 = (s_{83} - s_{82})\,(s_{83} - s_{86})\,(s_{83} - s_{87})$

$xduf7 = (s_{86} - s_{82})\,(s_{86} - s_{83})\,(s_{86} - s_{87})$

$xduf8 = (s_{87} - s_{82})\,(s_{87} - s_{83})\,(s_{87} - s_{86})$

$BX_{628} = DR2(s = s_{82})/xduf5,\;\; BX_{629} = DR2(s = s_{83})/xduf6$

$BX_{630} = DR2(s = s_{86})/xduf7,\;\; BX_{631} = DR2(s = s_{87})/xduf8$

$BX_{632} = \lambda_{sd}/(s_{82} - s_{83}),\;\; BX_{633} = \lambda_{sd}/(s_{83} - s_{82})$

$DR3(s) = \lambda_a\,\lambda_b\,(s + C_{2,1}(3))\,(s + C_{2,1}(5)) + \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}$

$DR4(s) = (s + C_{2,1}(2))\,(s + C_{2,1}(6)) - \lambda_{sd}\,\mu_{sd}$

$DR5(s) = DR3(s)\,DR4(s)$

$DR6(s) = \lambda_a\,\lambda_b\,(s + C_{2,1}(3))\,(s + C_{2,1}(6)) + \lambda_a\,\lambda_b\,\lambda_{sd}\,\mu_{sd}$

$DR7(s) = (s + C_{2,1}(1))\,(s + C_{2,1}(5)) - \lambda_{sd}\,\mu_{sd}$

$DR8(s) = DR6(s)\,DR7(s)$

$DR9(s) = DR5(s) + DR8(s)$

$xduf9 = (-C_{2,1}(4) - s_{82})\,(-C_{2,1}(4) - s_{83})\,(-C_{2,1}(4) - s_{84})\,(-C_{2,1}(4) - s_{86})\,(-C_{2,1}(4) - s_{87})$

$xduf10 = (s_{82} + C_{2,1}(4))\,(s_{82} - s_{83})\,(s_{82} - s_{84})\,(s_{82} - s_{85})\,(s_{82} - s_{86})\,(s_{82} - s_{87})$

$xduf11 = (s_{83} + C_{2,1}(4))\,(s_{83} - s_{82})\,(s_{83} - s_{84})\,(s_{83} - s_{85})\,(s_{83} - s_{86})\,(s_{83} - s_{87})$

$xduf12 = (s_{84} + C_{2,1}(4))\,(s_{84} - s_{82})\,(s_{84} - s_{83})\,(s_{84} - s_{85})\,(s_{84} - s_{86})\,(s_{84} - s_{87})$

$xduf13 = (s_{85} + C_{2,1}(4))\,(s_{85} - s_{82})\,(s_{85} - s_{83})\,(s_{85} - s_{84})\,(s_{85} - s_{86})\,(s_{85} - s_{87})$

$xduf14 = (s_{86} + C_{2,1}(4))\,(s_{86} - s_{82})\,(s_{86} - s_{83})\,(s_{86} - s_{84})\,(s_{86} - s_{85})\,(s_{86} - s_{87})$

$xduf15 = (s_{87} + C_{2,1}(4))\,(s_{87} - s_{82})\,(s_{87} - s_{83})\,(s_{87} - s_{84})\,(s_{87} - s_{85})\,(s_{87} - s_{86})$

$BX_{634} = DR9(s = -C_{2,1}(4))/xduf9,\;\; BX_{635} = DR9(s = s_{82})/xduf10$

$BX_{636} = DR9(s = s_{83})/xduf11,\;\; BX_{637} = DR9(s = s_{84})/xduf12$

$BX_{638} = DR9(s = s_{85})/xduf13,\;\; BX_{639} = DR9(s = s_{86})/xduf14$

$BX_{640} = DR9(s = s_{87})/xduf15$

$DR10(s) = \lambda_{sd}\,\lambda_a\,(s + C_{2,1}(3))\,(s + C_{2,1}(5)) + \lambda_a\,\lambda_{sd}^2\,\mu_{sd}$

$DR11(s) = \lambda_a \lambda_{sd} ((s + C_{2,1}(1))(s + C_{2,1}(5)) - \lambda_{sd} \mu_{sd})$

$DR12(s) = DR10(s) + DR11(s)$

$xduf16 = (-C_{2,1}(5) - s_{82})(-C_{2,1}(5) - s_{83})(-C_{2,1}(5) - s_{84})(-C_{2,1}(5) - s_{85})$

$xduf17 = (s_{82} + C_{2,1}(5))(s_{82} - s_{83})(s_{82} - s_{84})(s_{82} - s_{85})$

$xduf18 = (s_{83} + C_{2,1}(5))(s_{83} - s_{82})(s_{83} - s_{84})(s_{83} - s_{85})$

$xduf19 = (s_{84} + C_{2,1}(5))(s_{84} - s_{82})(s_{84} - s_{83})(s_{84} - s_{85})$

$xduf20 = (s_{85} + C_{2,1}(5))(s_{85} - s_{82})(s_{85} - s_{83})(s_{85} - s_{84})$

$BX_{641} = DR12(s = -C_{2,1}(5))/xduf16,\ BX_{642} = DR12(s = s_{82})/xduf17,$

$BX_{643} = DR12(s = s_{83})/xduf18,\ BX_{644} = DR9(s = s_{84})/xduf19$

$BX_{645} = DR12(s = s_{85})/xduf20$

$DR13(s) = \lambda_{sd} \lambda_b (s + C_{2,1}(3))(s + C_{2,1}(6)) + \lambda_b \lambda_{sd}^2 \mu_{sd}$

$DR14(s) = \lambda_b \lambda_{sd} ((s + C_{2,1}(2))(s + C_{2,1}(6)) - \lambda_{sd} \mu_{sd})$

$DR15(s) = DR13(s) + DR14(s)$

$xduf21 = (-C_{2,1}(6) - s_{82})(-C_{2,1}(6) - s_{83})(-C_{2,1}(6) - s_{86})(-C_{2,1}(6) - s_{87})$

$xduf22 = (s_{82} + C_{2,1}(6))(s_{82} - s_{83})(s_{82} - s_{86})(s_{82} - s_{87})$

$xduf23 = (s_{83} + C_{2,1}(6))(s_{83} - s_{82})(s_{83} - s_{86})(s_{83} - s_{87})$

$xduf24 = (s_{86} + C_{2,1}(6))(s_{86} - s_{82})(s_{86} - s_{83})(s_{86} - s_{87})$

$xduf25 = (s_{87} + C_{2,1}(6))(s_{87} - s_{82})(s_{87} - s_{83})(s_{87} - s_{86})$

$BX_{646} = DR15(s = -C_{2,1}(6))/xduf21,\ BX_{647} = DR15(s = s_{82})/xduf22,$

$BX_{648} = DR15(s = s_{83})/xduf23,\ BX_{649} = DR15(s = s_{86})/xduf24$

$BX_{650} = DR15(s = s_{87})/xduf25$

$k_{80} = A_{2965},\ k_{81} = A_{2959},\ k_{82} = A_{2958} = A_{2958},\ k_{83} = A_{2964}$

I.2 Non-identical unit 1-out-of-(2+2):G System: Definition of Constants

The following constants are defined for the analysis performed for the non-identical unit $1 - out - of - (2+2) : G$ system under SBAP Z discussed in Section 4.2.

$A_{3000} = 2\lambda_a \lambda_b,\ A_{3001} = \lambda_a \lambda_b (C_{2,2}(1) + C_{2,2}(2))$

$A_{3002} = C_{2,2}(1) C_{2,2}(2) + C_{2,2}(1) C_{2,2}(5) + C_{2,2}(2) C_{2,2}(5)$

$A_{3003} = C_{2,2}(1) C_{2,2}(2) C_{2,2}(5),\ A_{3004} = \lambda_a \lambda_d + \lambda_a \lambda_{sd}$

$A_{3005} = \lambda_a \lambda_d C_{2,2}(4) + \lambda_a \lambda_{sd} C_{2,2}(1)$

$A_{3006} = C_{2,2}(1) C_{2,2}(4) + C_{2,2}(1) C_{2,2}(6) + C_{2,2}(4) C_{2,2}(6)$

$A_{3007} = C_{2,2}(1) C_{2,2}(4) C_{2,2}(6),\ \ A_{3008} = 2 \lambda_a \lambda_{se},\ \ A_{3009} = \lambda_a \lambda_{se} (C_{2,2}(3) + C_{2,2}(1))$

$A_{3010} = C_{2,2}(1) C_{2,2}(3) + C_{2,2}(1) C_{2,2}(7) + C_{2,2}(3) C_{2,2}(7)$

$A_{3011} = C_{2,2}(1) C_{2,2}(3) C_{2,2}(7),\ \ A_{3012} = 2 \lambda_b \lambda_{sd}$

$A_{3013} = \lambda_b \lambda_{sd} C_{2,2}(4) + \lambda_b \lambda_{sd} C_{2,2}(2)$

$A_{3014} = C_{2,2}(2) C_{2,2}(4) + C_{2,2}(2) C_{2,2}(8) + C_{2,2}(4) C_{2,2}(8)$

$A_{3015} = C_{2,2}(2) C_{2,2}(4) C_{2,2}(8),\ \ A_{3016} = 2 \lambda_b \lambda_{se},\ \ A_{3017} = \lambda_b \lambda_{se} (C_{2,2}(2) + C_{2,2}(3))$

$A_{3018} = C_{2,2}(2) C_{2,2}(3) + C_{2,2}(2) C_{2,2}(9) + C_{2,2}(3) C_{2,2}(9)$

$A_{3019} = C_{2,2}(2) C_{2,2}(3) C_{2,2}(9),\ A_{3020} = 2 \lambda_{sd} \lambda_{se},\ \ A_{3021} = \lambda_{sd} \lambda_{se} (C_{2,2}(3) + C_{2,2}(4))$

$A_{3022} = C_{2,2}(3) C_{2,2}(4) + C_{2,2}(3) C_{2,2}(10) + C_{2,2}(4) C_{2,2}(10)$

$A_{3023} = C_{2,2}(3) C_{2,2}(4) C_{2,2}(10),\ \ A_{3024} = \lambda_a \lambda_b \lambda_d,\ \ A_{3025} = A_{3024} C_{2,2}(3)$

$$\begin{aligned}
A_{3026} &= C_{2,2}(2) C_{2,2}(6) + 2 C_{2,2}(2) C_{2,2}(8) + C_{2,2}(6) C_{2,2}(8) + 2 C_{2,2}(1) C_{2,2}(6) + \\
&\quad C_{2,2}(1) C_{2,2}(8) + C_{2,2}(6) C_{2,2}(8) + C_{2,2}(2) C_{2,2}(5) + \\
&\quad C_{2,2}(5) C_{2,2}(8) + C_{2,2}(1) C_{2,2}(5) + C_{2,2}(5) C_{2,2}(6) \\
A_{3027} &= C_{2,2}(2) C_{2,2}(6) C_{2,2}(8) + C_{2,2}(1) C_{2,2}(6) C_{2,2}(8) + \\
&\quad C_{2,2}(2) C_{2,2}(5) C_{2,2}(8) + C_{2,2}(1) C_{2,2}(5) C_{2,2}(6) \\
A_{3028} &= \lambda_a \lambda_b \lambda_{sd},\ A_{3029} = A_{3028} C_{2,2}(1) \\
A_{3030} &= 2 C_{2,2}(2) C_{2,2}(5) + C_{2,2}(2) C_{2,2}(8) + C_{2,2}(5) C_{2,2}(8) + \\
&\quad C_{2,2}(2) C_{2,2}(6) + C_{2,2}(5) C_{2,2}(6)
\end{aligned}$$

$A_{3031} = C_{2,2}(2) C_{2,2}(5) C_{2,2}(8) + C_{2,2}(2) C_{2,2}(5) C_{2,2}(6)$

$A_{3032} = A_{3024} A_{3027} + A_{3025} A_{3026},\ \ A_{3033} = A_{3025} A_{3027}$

$A_{3034} = A_{3028} A_{3031} + A_{3029} A_{3030},\ \ A_{3035} = A_{3029} A_{3031}$

$A_{3036} = C_{2,2}(1) C_{2,2}(2) + C_{2,2}(1) C_{2,2}(3) + C_{2,2}(2) C_{2,2}(3)$

$A_{3037} = C_{2,2}(1) C_{2,2}(2) C_{2,2}(3)$

$A_{3038} = C_{2,2}(5) C_{2,2}(6) + C_{2,2}(5) C_{2,2}(8) + C_{2,2}(6) C_{2,2}(8)$

$A_{3039} = C_{2,2}(5) C_{2,2}(6) C_{2,2}(8),\ A_{3040} = A_{3036} A_{3039} + A_{3037} A_{3038}$

$A_{3041} = A_{3037} A_{3039},\ \ A_{3042} = A_{3040} C_{2,2}(11) + A_{3041},\ \ A_{3043} = A_{3041} C_{2,2}(11)$

$A_{3044} = A_{3032} + A_{3034},\ \ A_{3045} = A_{3033} + A_{3035}$

$A_{3046} = A_{3001} A_{3011} A_{3018} + A_{3000} A_{3011} A_{3019} + A_{3001} A_{3010} A_{3019}$

$A_{3047} = A_{3001} A_{3011} A_{3019}$

$$A_{3048} = A_{3003}\, A_{3008}\, A_{3019} + A_{3002}\, A_{3009}\, A_{3019} + A_{3003}\, A_{3009}\, A_{3018}$$

$$A_{3049} = A_{3003}\, A_{3009}\, A_{3019}$$

$$A_{3050} = A_{3003}\, A_{3010}\, A_{3017} + A_{3003}\, A_{3011}\, A_{3016} + A_{3002}\, A_{3011}\, A_{3017}$$

$$A_{3051} = A_{3003}\, A_{3011}\, A_{3017}$$

$$A_{3052} = A_{3002}\, A_{3011}\, A_{3019} + A_{3003}\, A_{3010}\, A_{3019} + A_{3003}\, A_{3011}\, A_{3018}$$

$$A_{3053} = A_{3003}\, A_{3011}\, A_{3019},\ A_{3054} = A_{3053} + C_{2,2}(12)\, A_{3052},\ \ A_{3055} = C_{2,2}(12)\, A_{3053}$$

$$A_{3056} = \lambda_{se}\, A_{3046} + \lambda_b\, A_{3048} + \lambda_a\, A_{3050},\ \ A_{3057} = \lambda_{se}\, A_{3047} + \lambda_b\, A_{3049} + \lambda_a\, A_{3051}$$

$$A_{3058} = A_{3005}\, A_{3011}\, A_{3022} + A_{3004}\, A_{3011}\, A_{3023} + A_{3005}\, A_{3010}\, A_{3023}$$

$$A_{3059} = A_{3005}\, A_{3011}\, A_{3025}$$

$$A_{3060} = A_{3007}\, A_{3008}\, A_{3023} + A_{3006}\, A_{3009}\, A_{3023} + A_{3007}\, A_{3009}\, A_{3022}$$

$$A_{3061} = A_{3007}\, A_{3009}\, A_{3023}$$

$$A_{3062} = A_{3007}\, A_{3010}\, A_{3021} + A_{3007}\, A_{3011}\, A_{3020} + A_{3006}\, A_{3011}\, A_{3021}$$

$$A_{3063} = A_{3007}\, A_{3011}\, A_{3021}$$

$$A_{3064} = A_{3006}\, A_{3011}\, A_{3023} + A_{3007}\, A_{3011}\, A_{3022}$$

$$A_{3065} = A_{3007}\, A_{3011}\, A_{3023},\ A_{3066} = A_{3065} + C_{2,2}(13)\, A_{3064},\ \ A_{3067} = C_{2,2}(13)\, A_{3065}$$

$$A_{3068} = \lambda_{se}\, A_{3058} + \lambda_{sd}\, A_{3060} + \lambda_a\, A_{3062},\ \ A_{3069} = \lambda_{se}\, A_{3059} + \lambda_{sd}\, A_{3061} + \lambda_a\, A_{3063}$$

$$A_{3070} = A_{3012}\, A_{3019}\, A_{3023} + A_{3013}\, A_{3018}\, A_{3023} + A_{3013}\, A_{3019}\, A_{3022}$$

$$A_{3071} = A_{3013}\, A_{3019}\, A_{3023}$$

$$A_{3072} = A_{3015}\, A_{3016}\, A_{3023} + A_{3014}\, A_{3017}\, A_{3023} + A_{3015}\, A_{3017}\, A_{3022}$$

$$A_{3073} = A_{3015}\, A_{3017}\, A_{3023}$$

$$A_{3074} = A_{3015}\, A_{3019}\, A_{3020} + A_{3014}\, A_{3019}\, A_{3021} + A_{3015}\, A_{3018}\, A_{3021}$$

$$A_{3075} = A_{3015}\, A_{3019}\, A_{3021}$$

$$A_{3076} = A_{3014}\, A_{3019}\, A_{3023} + A_{3015}\, A_{3018}\, A_{3023} + A_{3015}\, A_{3019}\, A_{3022}$$

$$A_{3077} = A_{3015}\, A_{3019}\, A_{3023},\ A_{3078} = A_{3077} + C_{2,2}(14)\, A_{3076}$$

$$A_{3079} = C_{2,2}(14)\, A_{3077},\ \ A_{3080} = \lambda_{se}\, A_{3070} + \lambda_{sd}\, A_{3072} + \lambda_b\, A_{3074}$$

$$A_{3081} = \lambda_{se}\, A_{3071} + \lambda_{sd}\, A_{3073} + \lambda_b\, A_{3075},\ A_{3082} = A_{3044}\, A_{3055} + A_{3045}\, A_{3054}$$

$$A_{3083} = A_{3045}\, A_{3055},\ \ A_{3084} = A_{3066}\, A_{3079} + A_{3067}\, A_{3078},\ A_{3085} = A_{3067}\, A_{3079}$$

$$A_{3086} = A_{3056}\, A_{3043} + A_{3057}\, A_{3042},\ A_{3087} = A_{3057}\, A_{3043},\ A_{3088} = A_{3084}$$

$$A_{3089} = A_{3085},\ \ A_{3090} = A_{3068}\, A_{3043} + A_{3069}\, A_{3042},\ \ A_{3091} = A_{3069}\, A_{3043}$$

$$A_{3092} = A_{3054}\, A_{3079} + A_{3055}\, A_{3078},\ \ A_{3093} = A_{3055}\, A_{3079}$$

$$A_{3094} = A_{3080}\, A_{3043} + A_{3081}\, A_{3042},\ \ A_{3095} = A_{3081}\, A_{3043}$$

$$A_{3096} = A_{3054}\, A_{3067} + A_{3055}\, A_{3066},\ \ A_{3097} = A_{3055}\, A_{3067}$$

$$A_{3098} = A_{3082}\, A_{3085} + A_{3083}\, A_{3084},\ \ A_{3099} = A_{3083}\, A_{3085}$$

$$A_{3100} = A_{3086}\, A_{3089} + A_{3087}\, A_{3088}, \quad A_{3101} = A_{3087}\, A_{3089}$$
$$A_{3102} = A_{3090}\, A_{3093} + A_{3091}\, A_{3092}, \quad A_{3103} = A_{3091}\, A_{3093}$$
$$A_{3104} = A_{3094}\, A_{3097} + A_{3095}\, A_{3096}, \quad A_{3105} = A_{3095}\, A_{3097}$$
$$A_{3106} = A_{3002}\, A_{3007} + A_{3003}\, A_{3006}, \quad A_{3107} = A_{3003}\, A_{3007}$$
$$A_{3108} = A_{3010}\, A_{3015} + A_{3011}\, A_{3014}, \quad A_{3109} = A_{3011}\, A_{3015}$$
$$A_{3110} = A_{3018}\, A_{3023} + A_{3019}\, A_{3022}, \quad A_{3111} = A_{3019}\, A_{3023}$$
$$A_{3112} = A_{3042}\, A_{3055} + A_{3043}\, A_{3054}, \quad A_{3113} = A_{3043}\, A_{3055}$$
$$A_{3114} = A_{3066}\, A_{3079} + A_{3067}\, A_{3078}, \quad A_{3115} = A_{3067}\, A_{3079}$$
$$A_{3116} = A_{3106}\, A_{3109} + A_{3107}\, A_{3108}, \quad A_{3117} = A_{3107}\, A_{3109}$$
$$A_{3118} = A_{3110}\, A_{3113} + A_{3111}\, A_{3112}, \quad A_{3119} = A_{3111}\, A_{3113}$$
$$A_{3120} = A_{3116}\, A_{3119} + A_{3117}\, A_{3118}, \quad A_{3121} = A_{3117}\, A_{3119}$$
$$A_{3122} = A_{3115}\, A_{3120} + A_{3114}\, A_{3121}, \quad A_{3123} = A_{3115}\, A_{3121}$$
$$A_{3124} = A_{3123} + C_{2,2}(4)\, A_{3122}, \quad A_{3125} = C_{2,2}(4)\, A_{3123}$$
$$A_{3126} = A_{3125} + C_{2,2}(3)\, A_{3124}, \quad A_{3127} = C_{2,2}(3)\, A_{3125}$$
$$A_{3128} = A_{3127} + C_{2,2}(2)\, A_{3126}, \quad A_{3129} = C_{2,2}(2)\, A_{3127}$$
$$A_{3130} = A_{3129} + C_{2,2}(1)\, A_{3128}, \quad A_{3131} = C_{2,2}(1)\, A_{3129}$$
$$A_{3132} = (A_{3003}\, A_{3130} - A_{3002}\, A_{3131})/A_{3003}^2, \quad A_{3133} = A_{3131}/A_{3003}$$
$$A_{3134} = (A_{3007}\, A_{3130} - A_{3006}\, A_{3131})/A_{3007}^2, \quad A_{3135} = A_{3131}/A_{3007}$$
$$A_{3136} = (A_{3011}\, A_{3130} - A_{3010}\, A_{3131})/A_{3011}^2, \quad A_{3137} = A_{3131}/A_{3011}$$
$$A_{3138} = (A_{3015}\, A_{3130} - A_{3014}\, A_{3131})/A_{3015}^2, \quad A_{3139} = A_{3131}/A_{3015}$$
$$A_{3140} = (A_{3019}\, A_{3130} - A_{3018}\, A_{3131})/A_{3019}^2, \quad A_{3141} = A_{3131}/A_{3019}$$
$$A_{3142} = (A_{3023}\, A_{3130} - A_{3022}\, A_{3131})/A_{3023}^2, \quad A_{3143} = A_{3131}/A_{3023}$$
$$A_{3144} = (A_{3043}\, A_{3130} - A_{3042}\, A_{3131})/A_{3043}^2, \quad A_{3145} = A_{3131}/A_{3043}$$
$$A_{3146} = (A_{3055}\, A_{3130} - A_{3054}\, A_{3131})/A_{3055}^2, \quad A_{3147} = A_{3131}/A_{3055}$$
$$A_{3148} = (A_{3067}\, A_{3130} - A_{3066}\, A_{3131})/A_{3067}^2, \quad A_{3149} = A_{3131}/A_{3067}$$
$$A_{3150} = (A_{3079}\, A_{3130} - A_{3078}\, A_{3131})/A_{3079}^2, \quad A_{3151} = A_{3131}/A_{3079}$$
$$A_{3152} = A_{3112}\, A_{3115} + A_{3113}\, A_{3114}, \quad A_{3153} = A_{3113}\, A_{3115}$$
$$A_{3154} = (A_{3153}\, A_{3130} - A_{3152}\, A_{3131})/A_{3153}^2, \quad A_{3155} = A_{3131}/A_{3153}$$
$$A_{3156} = A_{3098}\, A_{3155} + A_{3099}\, A_{3154}, \quad A_{3157} = A_{3099}\, A_{3155}$$
$$A_{3158} = A_{3100}\, A_{3155} + A_{3101}\, A_{3154}, \quad A_{3159} = A_{3101}\, A_{3155}$$
$$A_{3160} = A_{3102}\, A_{3155} + A_{3103}\, A_{3154}, \quad A_{3161} = A_{3103}\, A_{3155}$$
$$A_{3162} = A_{3104}\, A_{3155} + A_{3105}\, A_{3154}, \quad A_{3163} = A_{3105}\, A_{3155}$$
$$A_{3164} = \lambda_e\, A_{3156} + \lambda_d\, A_{3158} + \lambda_b\, A_{3160} + \lambda_a\, A_{3162}$$

$A_{3165} = \lambda_e A_{3157} + \lambda_d A_{3159} + \lambda_b A_{3161} + \lambda_a A_{3163}$

$$\begin{aligned}
A_{3166} &= \lambda_{c0} A_{3130} + \lambda_{c1} \lambda_a A_{3128} + \lambda_{c2} \lambda_b A_{3126} + \lambda_{c3} \lambda_{sd} A_{3124} + \lambda_{c4} \lambda_{se} A_{3122} + \\
&\quad \lambda_{c5} (A_{3000} A_{3133} + A_{3001} A_{3132}) + \lambda_{c6} (A_{3004} A_{3135} + A_{3005} A_{3134}) + \\
&\quad \lambda_{c7} (A_{3008} A_{3137} + A_{3009} A_{3136}) + \lambda_{c8} (A_{3012} A_{3139} + A_{3013} A_{3138}) + \\
&\quad \lambda_{c9} (A_{3016} A_{3141} + A_{3017} A_{3140}) + \lambda_{c10} (A_{3020} A_{3143} + A_{3021} A_{3142}) + \\
&\quad \lambda_{c11} (A_{3044} A_{3145} + A_{3045} A_{3144}) + \lambda_{c12} (A_{3056} A_{3147} + A_{3057} A_{3146}) + \\
&\quad \lambda_{c13} (A_{3068} A_{3149} + A_{3069} A_{3148}) + \lambda_{c14} (A_{3080} A_{3151} + A_{3081} A_{3150}) \\
A_{3167} &= \lambda_{c0} A_{3131} + \lambda_{c1} \lambda_a A_{3129} + \lambda_{c2} \lambda_b A_{3127} + \lambda_{c3} \lambda_{sd} A_{3125} + \lambda_{c4} \lambda_{se} A_{3123} + \\
&\quad \lambda_{c5} A_{3001} A_{3133} + \lambda_{c6} A_{3005} A_{3135} + \lambda_{c7} A_{3009} A_{3137} + \lambda_{c8} A_{3013} A_{3139} + \\
&\quad \lambda_{c9} A_{3017} A_{3141} + \lambda_{c10} A_{3021} A_{3143} + \lambda_{c11} A_{3045} A_{3145} + \lambda_{c12} A_{3057} A_{3147} + \\
&\quad \lambda_{c13} A_{3069} A_{3149} + \lambda_{c14} A_{3081} A_{3151} \\
A_{3168} &= A_{3131} + C_{2,2}(0) A_{3130}, \;\; A_{3169} = C_{2,2}(0) A_{3131} \\
A_{3170} &= A_{3130} + \lambda_a A_{3128} + \lambda_b A_{3126} + \lambda_{sd} A_{3124} + \lambda_{se} A_{3122} + \\
&\quad A_{3000} A_{3133} + A_{3001} A_{3132} + A_{3004} A_{3135} + A_{3005} A_{3134} + \\
&\quad A_{3008} A_{3137} + A_{3009} A_{3136} + A_{3012} A_{3139} + A_{3013} A_{3138} + \\
&\quad A_{3016} A_{3141} + A_{3017} A_{3140} + A_{3020} A_{3143} + A_{3021} A_{3142} + \\
&\quad A_{3044} A_{3145} + A_{3045} A_{3144} + A_{3056} A_{3147} + A_{3057} A_{3146} + \\
&\quad A_{3068} A_{3149} + A_{3069} A_{3148} + A_{3080} A_{3151} + A_{3081} A_{3150} \\
A_{3171} &= A_{3131} + \lambda_a A_{3129} + \lambda_b A_{3127} + \lambda_{sd} A_{3125} + \lambda_{se} A_{3123} + \\
&\quad A_{3001} A_{3133} + A_{3005} A_{3135} + A_{3009} A_{3137} + A_{3013} A_{3139} + \\
&\quad A_{3017} A_{3141} + A_{3021} A_{3143} + A_{3045} A_{3145} + A_{3057} A_{3147} + \\
&\quad A_{3069} A_{3149} + A_{3081} A_{3151}
\end{aligned}$$

$BZ_{646} = \lambda_a/(C_{2,2}(0) - C_{2,2}(1)),\; BZ_{647} = -BZ_{646}$
$BZ_{648} = \lambda_b/(C_{2,2}(0) - C_{2,2}(2)),\; BZ_{649} = -BZ_{648}$
$BZ_{650} = \lambda_{se}/(C_{2,2}(0) - C_{2,2}(3)),\; BZ_{651} = -BZ_{650}$
$BZ_{652} = \lambda_{sd}/(C_{2,2}(0) - C_{2,2}(4)),\; BZ_{653} = -BZ_{652}$
$DW5(s) = \lambda_a \lambda_b (s + C_{2,2}(2)) + \lambda_a \lambda_b (s + C_{2,2}(1))$
$xdbf14 = (C_{2,2}(1) - C_{2,2}(5)) (C_{2,2}(2) - C_{2,2}(5)) (C_{2,2}(0) - C_{2,2}(5))$

$$xdbf15 = (C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(5) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdbf16 = (C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(5) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdbf17 = (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(5) - C_{2,2}(0))\,(C_{2,2}(1) - C_{2,2}(0))$$
$$BZ_{654} = DW5(s = -C_{2,2}(5))/xdbf14, \;\; BZ_{655} = DW5(s = -C_{2,2}(2))/xdbf15$$
$$BZ_{656} = DW5(s = -C_{2,2}(1))/xdbf16, \;\; BZ_{657} = DW5(s = -C_{2,2}(0))/xdbf17$$
$$DW6(s) = \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(4)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(1))$$
$$xdbf18 = (C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(4) - C_{2,2}(6))\,(C_{2,2}(0) - C_{2,2}(6))$$
$$xdbf19 = (C_{2,2}(1) - C_{2,2}(4))\,(C_{2,2}(6) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$$
$$xdbf20 = (C_{2,2}(4) - C_{2,2}(1))\,(C_{2,2}(6) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdbf21 = (C_{2,2}(4) - C_{2,2}(0))\,(C_{2,2}(6) - C_{2,2}(0))\,(C_{2,2}(1) - C_{2,2}(0))$$
$$BZ_{658} = DW6(s = -C_{2,2}(6))/xdbf18, \;\; BZ_{659} = DW6(s = -C_{2,2}(3))/xdbf19$$
$$BZ_{660} = DW6(s = -C_{2,2}(1))/xdbf20, \;\; BZ_{661} = DW6(s = -C_{2,2}(0))/xdbf21$$
$$DW7(s) = \lambda_a\,\lambda_{se}\,(s + C_{2,2}(3)) + \lambda_a\,\lambda_{se}\,(s + C_{2,2}(1))$$
$$xdbf22 = (C_{2,2}(1) - C_{2,2}(7))\,(C_{2,2}(3) - C_{2,2}(7))\,(C_{2,2}(0) - C_{2,2}(7))$$
$$xdbf23 = (C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(7) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$$
$$xdbf24 = (C_{2,2}(3) - C_{2,2}(1))\,(C_{2,2}(7) - C_{2,2}(1))\,(C_{2,2}(0) - C_{2,2}(1))$$
$$xdbf25 = (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(7) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))$$
$$BZ_{662} = DW7(s = -C_{2,2}(7))/xdbf22, \;\; BZ_{663} = DW7(s = -C_{2,2}(3))/xdbf23$$
$$BZ_{664} = DW7(s = -C_{2,2}(1))/xdbf24, \;\; BZ_{665} = DW7(s = -C_{2,2}(0))/xdbf25$$
$$DW8(s) = \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(4)) + \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))$$
$$xdbf26 = (C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(4) - C_{2,2}(8))\,(C_{2,2}(0) - C_{2,2}(8))$$
$$xdbf27 = (C_{2,2}(2) - C_{2,2}(4))\,(C_{2,2}(8) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$$
$$xdbf28 = (C_{2,2}(4) - C_{2,2}(2))\,(C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdbf29 = (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0))\,(C_{2,2}(8) - C_{2,2}(0))$$
$$BZ_{666} = DW8(s = -C_{2,2}(8))/xdbf26, \;\; BZ_{667} = DW8(s = -C_{2,2}(4))/xdbf27$$
$$BZ_{668} = DW8(s = -C_{2,2}(2))/xdbf28, \;\; BZ_{669} = DW8(s = -C_{2,2}(0))/xdbf29$$
$$DW9(s) = \lambda_b\,\lambda_{se}\,(s + C_{2,2}(4)) + \lambda_b\,\lambda_{se}\,(s + C_{2,2}(2))$$
$$xdbf30 = (C_{2,2}(3) - C_{2,2}(9))\,(C_{2,2}(2) - C_{2,2}(9))\,(C_{2,2}(0) - C_{2,2}(9))$$
$$xdbf31 = (C_{2,2}(9) - C_{2,2}(3))\,(C_{2,2}(2) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$$
$$xdbf32 = (C_{2,2}(9) - C_{2,2}(2))\,(C_{2,2}(3) - C_{2,2}(2))\,(C_{2,2}(0) - C_{2,2}(2))$$
$$xdbf33 = (C_{2,2}(9) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))$$
$$BZ_{670} = DW9(s = -C_{2,2}(9))/xdbf30, \;\; BZ_{671} = DW9(s = -C_{2,2}(3))/xdbf31$$
$$BZ_{672} = DW9(s = -C_{2,2}(2))/xdbf32, \;\; BZ_{673} = DW9(s = -C_{2,2}(0))/xdbf33$$

$DW10(s) = \lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(3)) + \lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(4))$

$xdbf34 = (C_{2,2}(3) - C_{2,2}(10))\,(C_{2,2}(4) - C_{2,2}(10))\,(C_{2,2}(0) - C_{2,2}(10))$

$xdbf35 = (C_{2,2}(10) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3))\,(C_{2,2}(0) - C_{2,2}(3))$

$xdbf36 = (C_{2,2}(10) - C_{2,2}(4))\,(C_{2,2}(3) - C_{2,2}(4))\,(C_{2,2}(0) - C_{2,2}(4))$

$xdbf37 = (C_{2,2}(10) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0))$

$BZ_{674} = DW10(s = -C_{2,2}(10))/xdbf34,\;\; BZ_{675} = DW10(s = -C_{2,2}(3))/xdbf35$

$BZ_{676} = DW10(s = -C_{2,2}(4))/xdbf36,\;\; BZ_{677} = DW10(s = -C_{2,2}(0))/xdbf37$

$$
\begin{aligned}
DV11(s) &= \lambda_d\,(s + C_{2,2}(3))\,(s + C_{2,2}(6))\,(s + C_{2,2}(8))\,[\lambda_a\,\lambda_b\,(s + C_{2,2}(2)) + \\
&\quad \lambda_a\,\lambda_b\,(s + C_{2,2}(1))] + \lambda_b\,(s + C_{2,2}(8))\,(s + C_{2,2}(2))\,(s + C_{2,2}(5))[\\
&\quad \lambda_a\,\lambda_d\,(s + C_{2,2}(3)) + \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(1))] + \lambda_a\,(s + C_{2,2}(1)) \\
&\quad (s + C_{2,2}(5))\,(s + C_{2,2}(6))\,[\lambda_b\,\lambda_{sd}\,(s + C_{2,2}(3)) + \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))] \\
xdbf38 &= (C_{2,2}(1) - C_{2,2}(11))\,(C_{2,2}(2) - C_{2,2}(11))\,(C_{2,2}(5) - C_{2,2}(11)) \\
&\quad (C_{2,2}(8) - C_{2,2}(11))\,(C_{2,2}(6) - C_{2,2}(11))\,(C_{2,2}(4) - C_{2,2}(11)) \\
&\quad (C_{2,2}(0) - C_{2,2}(11)) \\
xdbf39 &= (C_{2,2}(1) - C_{2,2}(8))\,(C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(5) - C_{2,2}(8)) \\
&\quad (C_{2,2}(11) - C_{2,2}(8))\,(C_{2,2}(6) - C_{2,2}(8))\,(C_{2,2}(4) - C_{2,2}(8)) \\
&\quad (C_{2,2}(0) - C_{2,2}(8)) \\
xdbf40 &= (C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(2) - C_{2,2}(6))\,(C_{2,2}(5) - C_{2,2}(6)) \\
&\quad (C_{2,2}(8) - C_{2,2}(6))\,(C_{2,2}(11) - C_{2,2}(6))\,(C_{2,2}(4) - C_{2,2}(6)) \\
&\quad (C_{2,2}(0) - C_{2,2}(6)) \\
xdbf41 &= (C_{2,2}(1) - C_{2,2}(5))\,(C_{2,2}(2) - C_{2,2}(5))\,(C_{2,2}(6) - C_{2,2}(5)) \\
&\quad (C_{2,2}(8) - C_{2,2}(5))\,(C_{2,2}(11) - C_{2,2}(5))\,(C_{2,2}(4) - C_{2,2}(5)) \\
&\quad (C_{2,2}(0) - C_{2,2}(5)) \\
xdbf42 &= (C_{2,2}(1) - C_{2,2}(4))\,(C_{2,2}(2) - C_{2,2}(4))\,(C_{2,2}(6) - C_{2,2}(4)) \\
&\quad (C_{2,2}(8) - C_{2,2}(4))\,(C_{2,2}(11) - C_{2,2}(4))\,(C_{2,2}(5) - C_{2,2}(4)) \\
&\quad (C_{2,2}(0) - C_{2,2}(4)) \\
xdbf43 &= (C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(4) - C_{2,2}(2))\,(C_{2,2}(6) - C_{2,2}(2)) \\
&\quad (C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(11) - C_{2,2}(2))\,(C_{2,2}(5) - C_{2,2}(2)) \\
&\quad (C_{2,2}(0) - C_{2,2}(2))
\end{aligned}
$$

$$
\begin{aligned}
xdbf44 &= (C_{2,2}(2)-C_{2,2}(1))\,(C_{2,2}(4)-C_{2,2}(1))\,(C_{2,2}(6)-C_{2,2}(1)) \\
&\quad (C_{2,2}(8)-C_{2,2}(1))\,(C_{2,2}(11)-C_{2,2}(1))\,(C_{2,2}(5)-C_{2,2}(1)) \\
&\quad (C_{2,2}(0)-C_{2,2}(1)) \\
xdbf45 &= (C_{2,2}(2)-C_{2,2}(0))\,(C_{2,2}(4)-C_{2,2}(0))\,(C_{2,2}(6)-C_{2,2}(0)) \\
&\quad (C_{2,2}(8)-C_{2,2}(0))\,(C_{2,2}(11)-C_{2,2}(0))\,(C_{2,2}(5)-C_{2,2}(0)) \\
&\quad (C_{2,2}(1)-C_{2,2}(0))
\end{aligned}
$$

$BZ_{678} = DW10(s=-C_{2,2}(11))/xdbf38,\;\; BZ_{679} = DW11(s=-C_{2,2}(8))/xdbf39$
$BZ_{680} = DW11(s=-C_{2,2}(6))/xdbf40,\;\; BZ_{681} = DW11(s=-C_{2,2}(5))/xdbf41$
$BZ_{682} = DW11(s=-C_{2,2}(4))/xdbf42,\;\; BZ_{683} = DW11(s=-C_{2,2}(2))/xdbf43$
$BZ_{684} = DW11(s=-C_{2,2}(1))/xdbf44,\;\; BZ_{685} = DW11(s=-C_{2,2}(0))/xdbf45$

$$
\begin{aligned}
DV12(s) &= \lambda_{se}\,(s+C_{2,2}(4))\,(s+C_{2,2}(7))\,(s+C_{2,2}(9))\,[\lambda_a\,\lambda_b\,(s+C_{2,2}(2))+ \\
&\quad \lambda_a\,\lambda_b\,(s+C_{2,2}(1))]+\lambda_b\,(s+C_{2,2}(2))\,(s+C_{2,2}(5))\,(s+C_{2,2}(9))[\\
&\quad \lambda_a\,\lambda_{se}\,(s+C_{2,2}(4))+\lambda_a\,\lambda_{se}\,(s+C_{2,2}(1))]+\lambda_a\,(s+C_{2,2}(1)) \\
&\quad (s+C_{2,2}(5))\,(s+C_{2,2}(7))\,[\lambda_b\,\lambda_{se}\,(s+C_{2,2}(4))+\lambda_b\,\lambda_{se}\,(s+C_{2,2}(2))] \\
xdbf46 &= (C_{2,2}(0)-C_{2,2}(12))\,(C_{2,2}(1)-C_{2,2}(12))\,(C_{2,2}(2)-C_{2,2}(12)) \\
&\quad (C_{2,2}(5)-C_{2,2}(12))\,(C_{2,2}(7)-C_{2,2}(12))\,(C_{2,2}(3)-C_{2,2}(12)) \\
&\quad (C_{2,2}(9)-C_{2,2}(12)) \\
xdbf47 &= (C_{2,2}(0)-C_{2,2}(9))\,(C_{2,2}(1)-C_{2,2}(9))\,(C_{2,2}(2)-C_{2,2}(9)) \\
&\quad (C_{2,2}(5)-C_{2,2}(9))\,(C_{2,2}(7)-C_{2,2}(9))\,(C_{2,2}(3)-C_{2,2}(9)) \\
&\quad (C_{2,2}(12)-C_{2,2}(9)) \\
xdbf48 &= (C_{2,2}(0)-C_{2,2}(7))\,(C_{2,2}(1)-C_{2,2}(7))\,(C_{2,2}(2)-C_{2,2}(7)) \\
&\quad (C_{2,2}(5)-C_{2,2}(7))\,(C_{2,2}(9)-C_{2,2}(7))\,(C_{2,2}(3)-C_{2,2}(7)) \\
&\quad (C_{2,2}(12)-C_{2,2}(7)) \\
xdbf49 &= (C_{2,2}(0)-C_{2,2}(5))\,(C_{2,2}(1)-C_{2,2}(5))\,(C_{2,2}(2)-C_{2,2}(5)) \\
&\quad (C_{2,2}(7)-C_{2,2}(5))\,(C_{2,2}(9)-C_{2,2}(5))\,(C_{2,2}(3)-C_{2,2}(5)) \\
&\quad (C_{2,2}(12)-C_{2,2}(5)) \\
xdbf50 &= (C_{2,2}(0)-C_{2,2}(3))\,(C_{2,2}(1)-C_{2,2}(3))\,(C_{2,2}(2)-C_{2,2}(3))
\end{aligned}
$$

$$
\begin{aligned}
& (C_{2,2}(7) - C_{2,2}(3))\,(C_{2,2}(9) - C_{2,2}(3))\,(C_{2,2}(5) - C_{2,2}(3)) \\
& (C_{2,2}(12) - C_{2,2}(3)) \\
xdbf51 \;\; = \;\; & (C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(1) - C_{2,2}(2))\,(C_{2,2}(4) - C_{2,2}(2)) \\
& (C_{2,2}(7) - C_{2,2}(2))\,(C_{2,2}(9) - C_{2,2}(2))\,(C_{2,2}(5) - C_{2,2}(2)) \\
& (C_{2,2}(12) - C_{2,2}(2)) \\
xdbf52 \;\; = \;\; & (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(2) - C_{2,2}(1))\,(C_{2,2}(3) - C_{2,2}(1)) \\
& (C_{2,2}(7) - C_{2,2}(1))\,(C_{2,2}(9) - C_{2,2}(1))\,(C_{2,2}(5) - C_{2,2}(1)) \\
& (C_{2,2}(12) - C_{2,2}(1)) \\
xdbf53 \;\; = \;\; & (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0)) \\
& (C_{2,2}(7) - C_{2,2}(0))\,(C_{2,2}(9) - C_{2,2}(0))\,(C_{2,2}(5) - C_{2,2}(0)) \\
& (C_{2,2}(12) - C_{2,2}(0))
\end{aligned}
$$

$$
\begin{aligned}
& BZ_{686} = DW12(s = -C_{2,2}(12))/xdbf46, \;\; BZ_{687} = DW12(s = -C_{2,2}(9))/xdbf47 \\
& BZ_{688} = DW12(s = -C_{2,2}(7))/xdbf48, \;\; BZ_{689} = DW12(s = -C_{2,2}(5))/xdbf49 \\
& BZ_{690} = DW12(s = -C_{2,2}(3))/xdbf50, \;\; BZ_{691} = DW12(s = -C_{2,2}(2))/xdbf51 \\
& BZ_{692} = DW12(s = -C_{2,2}(1))/xdbf52, \;\; BZ_{693} = DW12(s = -C_{2,2}(0))/xdbf53
\end{aligned}
$$

$$
\begin{aligned}
DV13(s) \;\; = \;\; & \lambda_{se}\,(s + C_{2,2}(3))\,(s + C_{2,2}(7))\,(s + C_{2,2}(10))\,[\lambda_a\,\lambda_{sd}\,(s + C_{2,2}(4)) + \\
& \lambda_a\,\lambda_{sd}\,(s + C_{2,2}(1))] + \lambda_{sd}\,(s + C_{2,2}(4))\,(s + C_{2,2}(6))\,(s + C_{2,2}(10))[\\
& \lambda_a\,\lambda_{se}\,(s + C_{2,2}(3)) + \lambda_a\,\lambda_{se}\,(s + C_{2,2}(1))] + \lambda_a\,(s + C_{2,2}(1)) \\
& (s + C_{2,2}(6))\,(s + C_{2,2}(7))\,[\lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(3)) + \lambda_{sd}\,\lambda_{se}\,(s + C_{2,2}(4))] \\
xdbf54 \;\; = \;\; & (C_{2,2}(0) - C_{2,2}(13))\,(C_{2,2}(1) - C_{2,2}(13))\,(C_{2,2}(3) - C_{2,2}(13)) \\
& (C_{2,2}(4) - C_{2,2}(13))\,(C_{2,2}(6) - C_{2,2}(13))\,(C_{2,2}(7) - C_{2,2}(13)) \\
& (C_{2,2}(10) - C_{2,2}(13)) \\
xdbf55 \;\; = \;\; & (C_{2,2}(0) - C_{2,2}(10))\,(C_{2,2}(1) - C_{2,2}(10))\,(C_{2,2}(3) - C_{2,2}(10)) \\
& (C_{2,2}(4) - C_{2,2}(10))\,(C_{2,2}(6) - C_{2,2}(10))\,(C_{2,2}(7) - C_{2,2}(10)) \\
& (C_{2,2}(13) - C_{2,2}(10)) \\
xdbf56 \;\; = \;\; & (C_{2,2}(0) - C_{2,2}(7))\,(C_{2,2}(1) - C_{2,2}(7))\,(C_{2,2}(3) - C_{2,2}(7)) \\
& (C_{2,2}(4) - C_{2,2}(7))\,(C_{2,2}(6) - C_{2,2}(7))\,(C_{2,2}(10) - C_{2,2}(7))
\end{aligned}
$$

$$
\begin{aligned}
& (C_{2,2}(13) - C_{2,2}(7)) \\
xdbf57 &= (C_{2,2}(0) - C_{2,2}(6))\,(C_{2,2}(1) - C_{2,2}(6))\,(C_{2,2}(3) - C_{2,2}(6)) \\
& (C_{2,2}(4) - C_{2,2}(6))\,(C_{2,2}(7) - C_{2,2}(6))\,(C_{2,2}(10) - C_{2,2}(6)) \\
& (C_{2,2}(13) - C_{2,2}(6)) \\
xdbf58 &= (C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3)) \\
& (C_{2,2}(6) - C_{2,2}(3))\,(C_{2,2}(7) - C_{2,2}(3))\,(C_{2,2}(10) - C_{2,2}(3)) \\
& (C_{2,2}(13) - C_{2,2}(3)) \\
xdbf59 &= (C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(1) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3)) \\
& (C_{2,2}(6) - C_{2,2}(3))\,(C_{2,2}(7) - C_{2,2}(3))\,(C_{2,2}(10) - C_{2,2}(3)) \\
& (C_{2,2}(13) - C_{2,2}(3)) \\
xdbf60 &= (C_{2,2}(0) - C_{2,2}(1))\,(C_{2,2}(3) - C_{2,2}(1))\,(C_{2,2}(4) - C_{2,2}(1)) \\
& (C_{2,2}(6) - C_{2,2}(1))\,(C_{2,2}(7) - C_{2,2}(1))\,(C_{2,2}(10) - C_{2,2}(1)) \\
& (C_{2,2}(13) - C_{2,2}(1)) \\
xdbf61 &= (C_{2,2}(1) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0)) \\
& (C_{2,2}(6) - C_{2,2}(0))\,(C_{2,2}(7) - C_{2,2}(0))\,(C_{2,2}(10) - C_{2,2}(0)) \\
& (C_{2,2}(13) - C_{2,2}(0))
\end{aligned}
$$

$$
\begin{aligned}
& BZ_{694} = DW13(s = -C_{2,2}(13))/xdbf54, \;\; BZ_{695} = DW13(s = -C_{2,2}(10))/xdbf55 \\
& BZ_{696} = DW13(s = -C_{2,2}(7))/xdbf56, \;\; BZ_{697} = DW13(s = -C_{2,2}(6))/xdbf57 \\
& BZ_{698} = DW13(s = -C_{2,2}(4))/xdbf58, \;\; BZ_{699} = DW13(s = -C_{2,2}(3))/xdbf59 \\
& BZ_{700} = DW13(s = -C_{2,2}(1))/xdbf60, \;\; BZ_{701} = DW13(s = -C_{2,2}(0))/xdbf61
\end{aligned}
$$

$$
\begin{aligned}
DV14(s) &= \lambda_{se}\,(s + C_{2,2}(3))\,(s + C_{2,2}(9))\,(s + C_{2,2}(10))\,[\lambda_b\,\lambda_{sd}\,(s + C_{2,2}(4)) + \\
& \lambda_b\,\lambda_{sd}\,(s + C_{2,2}(2))] + \lambda_{sd}\,(s + C_{2,2}(4))\,(s + C_{2,2}(8))\,(s + C_{2,2}(10))[\\
& \lambda_b\,\lambda_{se}\,(s + C_{2,2}(3)) + \lambda_b\,\lambda_{se}\,(s + C_{2,2}(2))] + \lambda_b\,(s + C_{2,2}(2)) \\
& (s + C_{2,2}(8))\,(s + C_{2,2}(9))\,[\lambda_{se}\,\lambda_{sd}\,(s + C_{2,2}(3)) + \lambda_{sd}\,\lambda_{se}\,(s + C_{2,2}(4))] \\
xdbf62 &= (C_{2,2}(0) - C_{2,2}(14))\,(C_{2,2}(2) - C_{2,2}(14))\,(C_{2,2}(3) - C_{2,2}(14)) \\
& (C_{2,2}(4) - C_{2,2}(14))\,(C_{2,2}(8) - C_{2,2}(14))\,(C_{2,2}(9) - C_{2,2}(14)) \\
& (C_{2,2}(10) - C_{2,2}(14))
\end{aligned}
$$

$$
\begin{aligned}
xdbf63 &= (C_{2,2}(0) - C_{2,2}(10))\,(C_{2,2}(2) - C_{2,2}(10))\,(C_{2,2}(3) - C_{2,2}(10)) \\
&\quad (C_{2,2}(4) - C_{2,2}(10))\,(C_{2,2}(8) - C_{2,2}(10))\,(C_{2,2}(9) - C_{2,2}(10)) \\
&\quad (C_{2,2}(14) - C_{2,2}(10)) \\
xdbf64 &= (C_{2,2}(0) - C_{2,2}(9))\,(C_{2,2}(2) - C_{2,2}(9))\,(C_{2,2}(3) - C_{2,2}(9)) \\
&\quad (C_{2,2}(4) - C_{2,2}(9))\,(C_{2,2}(8) - C_{2,2}(9))\,(C_{2,2}(10) - C_{2,2}(9)) \\
&\quad (C_{2,2}(14) - C_{2,2}(9)) \\
xdbf65 &= (C_{2,2}(0) - C_{2,2}(8))\,(C_{2,2}(2) - C_{2,2}(8))\,(C_{2,2}(3) - C_{2,2}(8)) \\
&\quad (C_{2,2}(4) - C_{2,2}(8))\,(C_{2,2}(9) - C_{2,2}(8))\,(C_{2,2}(10) - C_{2,2}(8)) \\
&\quad (C_{2,2}(14) - C_{2,2}(8)) \\
xdbf66 &= (C_{2,2}(0) - C_{2,2}(3))\,(C_{2,2}(2) - C_{2,2}(3))\,(C_{2,2}(4) - C_{2,2}(3)) \\
&\quad (C_{2,2}(8) - C_{2,2}(3))\,(C_{2,2}(9) - C_{2,2}(3))\,(C_{2,2}(10) - C_{2,2}(3)) \\
&\quad (C_{2,2}(14) - C_{2,2}(3)) \\
xdbf67 &= (C_{2,2}(0) - C_{2,2}(4))\,(C_{2,2}(2) - C_{2,2}(4))\,(C_{2,2}(3) - C_{2,2}(4)) \\
&\quad (C_{2,2}(8) - C_{2,2}(4))\,(C_{2,2}(9) - C_{2,2}(4))\,(C_{2,2}(10) - C_{2,2}(4)) \\
&\quad (C_{2,2}(14) - C_{2,2}(4)) \\
xdbf68 &= (C_{2,2}(0) - C_{2,2}(2))\,(C_{2,2}(3) - C_{2,2}(2))\,(C_{2,2}(4) - C_{2,2}(2)) \\
&\quad (C_{2,2}(8) - C_{2,2}(2))\,(C_{2,2}(9) - C_{2,2}(2))\,(C_{2,2}(10) - C_{2,2}(2)) \\
&\quad (C_{2,2}(14) - C_{2,2}(2)) \\
xdbf69 &= (C_{2,2}(2) - C_{2,2}(0))\,(C_{2,2}(3) - C_{2,2}(0))\,(C_{2,2}(4) - C_{2,2}(0)) \\
&\quad (C_{2,2}(8) - C_{2,2}(0))\,(C_{2,2}(9) - C_{2,2}(0))\,(C_{2,2}(10) - C_{2,2}(0)) \\
&\quad (C_{2,2}(14) - C_{2,2}(0))
\end{aligned}
$$

$BZ_{702} = DW14(s = -C_{2,2}(14))/xdbf62$, $BZ_{703} = DW14(s = -C_{2,2}(10))/xdbf63$
$BZ_{704} = DW14(s = -C_{2,2}(9))/xdbf64$, $BZ_{705} = DW14(s = -C_{2,2}(6))/xdbf65$
$BZ_{706} = DW14(s = -C_{2,2}(4))/xdbf66$, $BZ_{707} = DW14(s = -C_{2,2}(3))/xdbf67$
$BZ_{708} = DW14(s = -C_{2,2}(2))/xdbf68$, $BZ_{709} = DW14(s = -C_{2,2}(0))/xdbf69$
$k_{84} = A_{3171}$, $k_{85} = A_{3169}$, $k_{86} = A_{3168}$, $k_{87} = A_{3170}$

Appendix J

Miscellaneous

J.1 The Non-Identical Unit System: A Formula For The Number Of System Up-States

The following presentation is in connection with the non-identical unit $k-out-of-(n+m):G$ system whose duplex system models are discussed in Sections 3.2 and 4.2.

For both SBAPs, the analysis was limited to duplex systems because of the sharp increase in the number of system states as the number of system units is increased. The use of a very limited number of special cases for the non-identical unit scenario is justifiable because the main objective of considering these cases is to show their relationship with the identical unit cases. In general, the total number of system up-states (n_{ss}) for a non-identical unit $k-out-of-(n+m):G$ system regardless of the governing SBAP is given by:

$$n_{ss} = \sum_{i=k}^{n+m} \frac{(n+m)!}{i!\,(n+m-i)!} \tag{J.1}$$

J.2 Variance Of Time To Failure Analysis

The following calculations show that for $k-out-of-n+m:G$ systems, an increase in the mean time to failure, (MTTF), often means an increase in the variance of time to failure, (σ^2).

The system reliability in Laplace transforms $R(s)$ can always be expressed as the quotient of two polynomial functions in s as:

$$R(s) = \frac{v(s)}{w(s)} \tag{J.2}$$

The variance of time to failure σ^2 is given by:

$$\sigma^2 = -2 \lim_{s \to 0} R'(s) - MTTF^2 \tag{J.3}$$

where
$R'(s)$ is the derivative of $R(s)$ with respect to s.

From Equation (J.2), the mean time to failure can easily be expressed as:

$$\begin{aligned} MTTF &= \lim_{s \to 0} R(s) \\ &= \lim_{s \to 0} \frac{v(s)}{w(s)} \end{aligned} \tag{J.4}$$

Applying the quotient rule of differentiation and substituting Equation (J.4) in Equation (J.3) we obtain:

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \to 0} [\frac{w(s) \frac{dv(s)}{ds} - v(s) \frac{dw(s)}{ds}}{w(s)^2}] - [\lim_{s \to 0} \frac{v(s)}{w(s)}]^2 \\ &= -2 \lim_{s \to 0} [\frac{2 (w(s) \frac{dv(s)}{ds} - v(s) \frac{dw(s)}{ds}) - v(s)^2}{2\, w(s)^2}] \end{aligned} \tag{J.5}$$

From Equation (J.5), we can see that an increase in the MTTF would mean an increase in $\lim_{s \to 0} v(s)$ and a decrease (or constant) $\lim_{s \to 0} w(s)$ leading to an increase in σ^2.