CONFIGURATION MIXING IN THE CLUSTER MODEL WAVE FUNCTIONS OF THE He^5 NUCLEUS

by

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TO MY PARENTS

ABSTRACT

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ABSTRACT

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CHAPTER I

Introduction

In the past decade the cluster model has achieved some success in predicting the low-lying energy levels of certain light nuclei [1]. This thesis will be concerned especially with the cluster model representation of the He^5 nucleus. Pearlstein et al [2] have demonstrated that good results for the energies of the first three levels in He^5 are obtained when the cluster model is used. Also, Tran Duc and Smith [3] have calculated the radiative transition probability from the 3/2+ (second excited) state to the 3/2- (ground) state in He^5, and obtain a result which is in quite close agreement with the experimental value. There are therefore some indications that the cluster model wave functions provide a good description of the He^5 nucleus.

In the cluster model, different spatial correlations between nucleons may be favoured in different nuclear states. On the other hand, the harmonic oscillator shell model (HOSM) treats the nucleons in a nucleus as independent particles moving in a common harmonic oscillator potential well. The HOSM thus includes no spatial correlations between nucleons in the nucleus at all.

In the cluster model, the ground state of He^5 is assumed to consist of an alpha particle and a neutron, the state of whose relative motion is a 1p state [2]. The second excited state has a different cluster structure, and can be described as consisting of a triton and a deuteron; the state

of the relative motion of the deuteron and the triton is a 2s state[2]. Here it should be noted that the deuteron cluster, the triton cluster and the alpha particle cluster are energetically in their ground states [1].

The cluster model spatial wave function of the ground state is [2]

$$\phi(3/2-)=\exp(-\tfrac{1}{2}\alpha_1\sum_{i=1}^{4}\vec{r}_i'^2)\vec{R}_1 Y_1^m(\Omega_1)\exp(-\tfrac{2}{5}\beta'\vec{R}_1^2)\exp(-\tfrac{5}{2}\mu_1\vec{R}_{cm}^2) \qquad \text{(I-1)}$$

where

$$\vec{r}_i'=\vec{r}_i-\tfrac{1}{4}\sum_{j=1}^{4}\vec{r}_j, \qquad \vec{R}_1=\tfrac{1}{4}\sum_{j=1}^{4}\vec{r}_j-\vec{r}_5, \qquad \vec{R}_{cm}=\tfrac{1}{5}\sum_{j=1}^{5}\vec{r}_j.$$

The spatial wave function of the second excited state is [2]

$$\phi(3/2+)=\exp(-\tfrac{1}{2}\alpha\sum_{i=1}^{3}\vec{r}_i''^2-\tfrac{1}{2}\bar{\alpha}\sum_{j=4}^{5}\vec{r}_j''^2)\vec{R}^2\exp(-\tfrac{2}{5}\beta\vec{R}^2)\exp(-\tfrac{5}{2}\mu_1\vec{R}_{cm}^2) \qquad \text{(I-2)}$$

where

$$\vec{r}_i''=\vec{r}_i-\tfrac{1}{3}\sum_{k=1}^{3}\vec{r}_k, \qquad \vec{r}_j''=\vec{r}_j-\tfrac{1}{2}\sum_{k=4}^{5}\vec{r}_k, \qquad \vec{R}=\tfrac{1}{3}\sum_{k=1}^{3}\vec{r}_k-\tfrac{1}{2}\sum_{k=4}^{5}\vec{r}_k.$$

Here the width parameters α_1, β' for the ground state and α, $\bar{\alpha}$, β for the second excited state were determined by a variational calculation of the energies of these states [2]. The width parameter μ_1 associated with the (fictitious) center of mass motion is chosen to be the same as that in the shell model calculation [4].

The HOSM spatial wave function is a direct product of the individual spatial wave function of the individual nucleons, each of which moves in the common harmonic oscillator potential. Thus, the HOSM spatial wave function is

$$\phi(L^\pi)=\phi(1_1^{\pi_1})\phi(1_2^{\pi_2})\cdots\cdots\cdots\cdots\phi(1_A^{\pi_A})$$

where

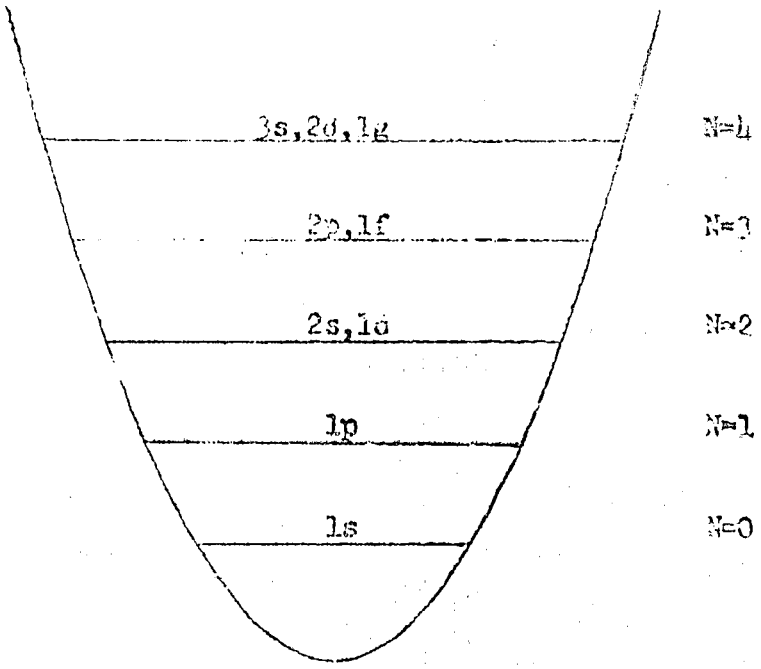

Fig. 1 Harmonic Oscillator Potential Well for the HOSM.

$$\phi(l_i^{\pi_i}) = \exp(-\tfrac{1}{2}\mu_i r_i^2) r_i^{l_i} L_{\frac{N-l_i}{2}}^{l_i+\frac{1}{2}} (\mu_i r_i^2) Y_{l_i}^{m_i}(\Omega_i) \qquad \text{(I-3)}$$

Note that, since all the nucleons are moving in the same oscillator potential, the width parameter μ_i is the same for all the individual spatial wave functions.

For the sake of convenience, the following notation will be used throughout this work. The cluster model spatial wave functions for the ground state eq.(I-1) and the second excited state eq.(I-2) are represented in the following way respectively:

$\phi(3/2-) \equiv (1234;5)$, $\phi(3/2+) \equiv (123;45)$,

and the HOSM spatial wave functions are denoted by (12345). The first two and the fifth positions represent the neutrons, while the third and fourth positions represent the protons.

The spin wave functions of the ground state of He^5 in both models are identical; the total spin of the ground state is $S=\frac{1}{2}$, since it is formed by the coupling of the spins of two neutrons and two protons in the 1s spatial state (or in the cluster model language, an alpha particle cluster), which is zero, and the spin of the last single neutron. The spin wave function can therefore be chosen to be [5]

$$\chi_- = \alpha_1\beta_2\alpha_3\beta_4\alpha_5$$

The cluster model spin wave function for the second excited state is formed by coupling the spin of the triton and the spin of the deuteron. As was shown in Tran Duc's work [5], there are four different possible sets of spin wave functions. One of them has to be ruled out because it violates the

Pauli principle. Two others give a vanishing electric dipole matrix element, and so are arbitrarily disregarded. Thus the only spin wave function used here is (5)

$$\chi_{+}=\frac{1}{\sqrt{6}}\left[(\alpha_1\beta_3+\beta_1\alpha_3)\alpha_2-2\alpha_1\beta_2\alpha_3\right]\alpha_4\alpha_5.$$

The same set of four possible spin states exists in the HOSM wave function for the second excited state as well. For the same reasons as were given above for the cluster model spin wave function, the same spin wave function is adopted for the HOSM.

In using the HOSM, it is correct to consider the nucleons moving in pure single-particle orbits only if there is no residual interaction in addition to the effective harmonic oscillator potential. In the absence of such a residual interaction, all states of k particles (having the same n, l, j) in the same configuration will be degenerate. But in the actual nucleus, such interactions do exist. If a residual internucleon potential v_{ij} is introduced which is strong enough to remove the degeneracies, the energies of the state will change, and at the same time the wave functions will also be changed by the interaction. Such a mixed state can be represented as a superposition of the wave functions of several pure states (with same n, l, j) whose energies are close to one another.

Since the eigenfunctions of different states in the HOSM are mutually orthogonal and form a complete set, then any wave function can be expanded in terms of these HOSM wave

functions. For instance, "individual particle model"* wave functions, alpha-particle model wave functions [6], and cluster model wave functions [7] can be expanded in a sum of HOSM wave functions of all possible configurations.

Since the energy levels, nuclear magnetic moments [8], nuclear quadrupole moments [9], and so on, are quite sensitive to the configuration mixing of the HOSM wave functions, thus the configuration mixing in the "more realistic wave functions" may be helpful in obtaining better theoretical results. In the present work, the expansion of the cluster model wave functions as a linear combination of HOSM wave functions gives the equivalence between the cluster model wave functions and the shell model wave functions, in the sense in which configurations are mixed in calculations using the "individual particle model".

In order to expand the cluster model wave functions as a linear combination of the HOSM wave functions, the wave functions for both models must be antisymmetrized. The method of antisymmetrization used here differs from that of Kanellopoulos and Wildermuth [10] in that the proton and neutron functions are antisymmetrized separately [3,5]. This method has two advantages; first, isospin need not be included in the wave functions, and second, the calculation of the expansion coefficients is simplified.

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* Here the "individual particle model" means that in addition to the common harmonic oscillator potential well, some kind of residual internucleon interactions are included.

The antisymmetrization operator $\mathcal{A}$ is (11)

$$\mathcal{A} = \mathcal{A}_n \mathcal{A}_p,$$

where

$$\mathcal{A}_n = \frac{1}{3!} \sum_{n=1,2,5} (-1)^\pi P_n, \qquad \mathcal{A}_p = \frac{1}{2} \sum_{p=3,4} (-1)^\pi P_p,$$

P is a permutation operator and $(-1)^\pi$ is the parity of the permutation.

The antisymmetrized cluster model wave functions are

$$\Phi(3/2-) = \mathcal{A}\,\phi(3/2-) = N_- \mathcal{A} \{ (1234;5)\chi_- \},$$

$$\Phi(3/2+) = \mathcal{A}\,\phi(3/2+) = N_+ \mathcal{A} \{ (123;45)\chi_+ \}.$$

The normalization constants for both model wave functions are determined from equations of the form

$$\langle \Phi | \Phi \rangle = \langle \phi | \Phi \rangle = 1. \qquad \text{(I-4)}$$

The simplication occurs because $\mathcal{A}$ is idempotent (i.e., $\mathcal{A}^2 = \mathcal{A}$).

If the cluster model wave function $|\phi_{CM}\rangle$ are expanded in a linear combination of the HOSM wave functions of different configurations $|\phi_{SMi}\rangle$ (where J and π are the same as in $|\phi_{CM}\rangle$; also i labels the different configurations), then

$$|\phi_{CM}\rangle = a_1 |\phi_{SM1}\rangle + a_2 |\phi_{SM2}\rangle + \cdots + a_i |\phi_{SMi}\rangle + \cdots. \qquad \text{(I-5)}$$

Applying the antisymmetrization operator $\mathcal{A}$ to eq.(I-5), there results

$$|\Phi_{CM}\rangle = a_1 |\Phi_{SM1}\rangle + a_2 |\Phi_{SM2}\rangle + \cdots + a_i |\Phi_{SMi}\rangle + \cdots. \qquad \text{(I-6)}$$

Since the HOSM wave functions satisfy the orthonormality condition

$$\langle \phi_{SMj} | \mathcal{A} \phi_{SMi} \rangle = \delta_{ji}, \qquad \text{(I-7)}$$

then the expansion coefficient a_i can be obtained by taking the scalar product of $|\phi_{SMi}\rangle$ with $|\Phi_{CM}\rangle$:

$$\langle \phi_{SMi} | \mathcal{A} \phi_{CM} \rangle = a_i . \qquad \text{(I-8)}$$

Since the cluster model wave functions are normalized as shown in eq.(I-4), then from eqs.(I-5),(I-6) and (I-7) there results

$$\langle \phi_{CM} | \mathcal{A} \phi_{CM} \rangle = \sum_i |a_i|^2 = 1. \qquad \text{(I-9)}$$

This is a condition on the expansion coefficients which must be satisfied; it provides a check which indicates when a sufficient number of configurations has been included. The square of the amplitude $|a_i|^2$ is physically more interesting than the expansion coefficient a_i; it gives directly the weight of the HOSM state i in the cluster model wave function.

In Chapter II and III the HOSM expansions of the cluster model wave functions for the ground state and the second excited state of He^5 will be studied.

In Chapter IV the width parameters in the ground state cluster model wave function will be varied and the corresponding expansion amplitudes calculated. By comparing these results with those given in Chapter II, the sensitivity of the expansion coefficients to variations of the width parameters can be determined.

CHAPTER II

Expansion of the Cluster Model Wave Function of the Ground State

1. The HOSM Scheme

The HOSM represents the ground state of He^5 as consisting of two neutrons and two protons in the 1s level and one neutron in the 1p level [denoted in the usual spectroscopic notation as $(1s)^4(1p)^1$]. The 1p neutron has one quantum of energy $\hbar\omega$ relative to the 1s level.

Using the L-S coupling scheme, the four 1s nucleons couple to give zero spin and orbital angular momentum, and the 1p nucleon has $s=\frac{1}{2}$, $l=1$, which forms the total angular momentum and parity $J=3/2-$. However, as stated in Ch.I, the ground state of He^5 consists not only of the $(1s)^4(1p)^1$ configuration, but also has some contributions from the configurations having two quanta of excitation or even four quanta of excitation above the $(1s)^4(1p)^1$ state.

The configurations having two and four quanta of excitation above the $(1s)^4(1p)^1$ state, and which have the same total angular momentum and parity, are

Two quanta of excitation:

$(1s)^{-1}(2s)^1$, $(1s)^{-1}(1d)^1$, $(1s)^{-2}(1p)^2$, $(1p)^{-1}(2p)^1$;

Four quanta of excitation:

$(1s)^{-2}(2s)^2$, $(1s)^{-2}(2s)^1(1d)^1$, $(1s)^{-1}(3s)^1$, $(1p)^{-1}(3p)^1$,

$(1s)^{-1}(1p)^{-1}(2s)^{1}(2p)^{1}$, $(1s)^{-1}(2d)^{1}$.

2. The General Formula for the Expansion Coefficients

The unantisymmetrized HOSM wave function for the ground state of He^5 is

$$\phi_{SMi}=N_{SMi}(12345)_i\,\alpha_1\beta_2\alpha_3\beta_4\alpha_5. \qquad \text{(II-1)}$$

The antisymmetrized HOSM wave function is obtained by applying the antisymmerization operator $\mathcal{A}$ to eq.(II-1):

$$\mathcal{A}\phi_{SMi}=\frac{N_{SMi}}{12}(\alpha_3\beta_4-\beta_3\alpha_4)\{(\alpha_1\beta_2-\beta_1\alpha_2)\alpha_5(12345)_i - \alpha_1(\beta_2\alpha_5-\alpha_2\beta_5)(52341)_i+\alpha_2(\beta_1\alpha_5-\alpha_1\beta_5)(15342)_i\}. \qquad \text{(II-2)}$$

Here again the subscript i on the wave function, the normalization constant and the spatial wave function designates the various configurations.

The unantisymmetrized cluster model wave function for the ground state of He^5 is

$$\phi_{CM}=N_{CM}(1234;5)\,\alpha_1\beta_2\alpha_3\beta_4\alpha_5, \qquad \text{(II-3)}$$

and the antisymmetrized cluster model wave function is

$$\mathcal{A}\phi_{CM}=\frac{N_{CM}}{12}(\alpha_3\beta_4-\beta_3\alpha_4)\{(\alpha_1\beta_2-\beta_1\alpha_2)\alpha_5(1234;5) - \alpha_1(\beta_2\alpha_5-\alpha_2\beta_5)(5234;1)+\alpha_2(\beta_1\alpha_5-\alpha_1\beta_5)(1534;2)\}. \qquad \text{(II-4)}$$

The normalization constants for both model wave functions are determined by using eq.(I-4). The numerical results of these

calculations are given in App. III.

From eq.(I-8) the expansion coefficients a_i can be obtained, as can be seen from the following equation,

$$a_i = \langle \phi_{SM_i} | \mathcal{A} \phi_{CM} \rangle$$

$$= \frac{N_{SM_i} N_{CM}}{12} \left\{ \langle (12345)_i | (1234;5) - (5234;1) \rangle \right\} \qquad \text{(II-5)}$$

which is formed by taking the scalar product of eq.(II-1) with eq.(II-4). Equation (II-5) is a general equation which will be used throughout in the calculation of the expansion coefficients a_i for the ground state of He^5.

Before carrying out the calculation, the width parameters for the cluster model [2] and HOSM [4] wave functions will be given. These parameters are given in the following table.

Parameter	Value (10^{25} cm^{-2})	Reference
μ_1	3.65252	(4)
α_1	4.33	(2)
β'	2.25 3.25	(2)
α	2.81	(2)
$\bar{\alpha}$	4.78	(2)
β	0.422	(2)

Table 1. The width parameters for the cluster model and HOSM wave functions.

The two values for β' arise from two possible choices for the range and depth of the spin-orbit potential used in Ref. (2).

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3. Numerical Calculation of Expansion Coefficients

(i). The HOSM spatial wave function for the configuration $(1s)^4(1p)^1$ is

$$(12345)_1 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)\vec{r}_5 Y_1^1(\Omega_5).$$

Thus,

$$\langle (12345)_1 \mid (1234;5) \rangle$$

$$= \int \exp\left[-A(r_1^2+r_2^2+r_3^2+r_4^2)-Br_5^2-C(\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_3+\vec{r}_1\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_4)\right.$$

$$\left.-D(\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5)\right]\vec{r}_5\cdot\left[\tfrac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)-\vec{r}_5\right] Y_1^{1*}(\Omega_5)Y_1^1(\Omega_{R_1})$$

$$\times d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5$$

and

$$\langle (12345)_1 \mid (5234;1) \rangle$$

$$= \int \exp\left[-A(r_2^2+r_3^2+r_4^2+r_5^2)-Br_1^2-C(\vec{r}_2\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5)\right.$$

$$\left.-D(\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_3+\vec{r}_1\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5)\right]\vec{r}_5\cdot\left[\tfrac{1}{4}(\vec{r}_2+\vec{r}_3+\vec{r}_4+\vec{r}_5)-\vec{r}_1\right] Y_1^{1*}(\Omega_5)Y_1^1(\Omega_{R_5})$$

$$\times d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5$$

where

$$A=\tfrac{1}{2}\mu_1+\frac{3}{8}\alpha_1+\frac{\beta'}{40}+\frac{\mu_1}{10}= \begin{Bmatrix} 3.871512 \\ 3.896512 \end{Bmatrix} \times 10^{25}\,\mathrm{cm}^{-2}$$

$$B=\tfrac{1}{2}\mu_1+\frac{2}{5}\beta'+\frac{\mu_1}{10}= \begin{Bmatrix} 3.091512 \\ 3.491512 \end{Bmatrix} \times 10^{25}\,\mathrm{cm}^{-2}$$

$$C=\frac{1}{4}\alpha_1+\frac{\beta'}{20}+\frac{\mu_1}{5}= \begin{Bmatrix} 0.239496 \\ 0.189496 \end{Bmatrix} \times 10^{25}\,\mathrm{cm}^{-2}$$

$$D=\frac{1}{5}(\mu_1-\beta')=\left\{\begin{matrix}0.28050_4\\0.08050_4\end{matrix}\right\}\times 10^{25}\text{cm}^{-2}.$$

The appearance of two values is due to the two possible values of the width parameter β'. Such pairs of values will appear throughout the calculation of the expansion coefficient of the ground state a_i. Using the method of App.I there results

$$\left\langle (12345)_1 \middle| (1234;5) \right\rangle=\left\{\begin{matrix}0.10156\\0.09392\end{matrix}\right\}\times 10^{-169}\text{cm}^{14},$$

$$\left\langle (12345)_1 \middle| (5234;1) \right\rangle=\left\{\begin{matrix}0.25324\\0.24124\end{matrix}\right\}\times 10^{-170}\text{cm}^{14}.$$

Taking the normalization constants N_{SM1} and N_{CM} from App.III, and substituting the above values into eq.(II-5) leads to

$$a_1=\left\{\begin{matrix}0.6268\\0.6354\end{matrix}\right\}.$$

Thus, the quantity $|a_1|^2$ is

$$|a_1|^2=\left\{\begin{matrix}0.393\\0.402\end{matrix}\right\}.$$

(ii). The HOSM spatial wave function for the configuration $(1s)^{-1}(2s)^1$ is

$$(12345)_2=\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)\vec{r}_5 L_1^{\frac{1}{2}}(\mu_1 r_1^2)Y_1^1(\Omega_5),$$

where

$$L_1^{\frac{1}{2}}(\mu_1 r_1^2)=\frac{3}{2}-\mu_1 r_1^2.$$

The exponential terms in the integrals $\left\langle (12345)_2 \middle| (1234;5) \right\rangle$ and $\left\langle (12345)_2 \middle| (5234;1) \right\rangle$ are the same as those in $\left\langle (12345)_1 \middle| (1234;5) \right\rangle$ and in $\left\langle (12345)_1 \middle| (5234;1) \right\rangle$ respectively.

However, one extra term (with $\mu_1 r_1^2$) appears in the integrand, so that, using the method outlined in App. II, there results

$$\langle (12345)_2 \mid (1234;5) \rangle = \begin{Bmatrix} 0.10587 \\ 0.09790 \end{Bmatrix} \times 10^{-169}\ \text{cm}^{14},$$

$$\langle (12345)_2 \mid (5234;1) \rangle = \begin{Bmatrix} 0.02669 \\ 0.02543 \end{Bmatrix} \times 10^{-169}\ \text{cm}^{14}.$$

Taking N_{SM2} and N_{CM} from App. III, and substitution of the above values into eq.(II-5) leads to

$$a_2 = \begin{Bmatrix} 0.3989 \\ 0.3947 \end{Bmatrix},$$

and

$$|a_2|^2 = \begin{Bmatrix} 0.159 \\ 0.156 \end{Bmatrix}.$$

(iii). The HOSM spatial wave function for the configuration $(1s)^{-2}(1p)^2$ is

$$(12345)_3 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2) \mid 1\ \ 1 \rangle_{523}$$

where

$$\mid 1\ \ 1 \rangle_{523} = \tfrac{1}{2} \mid 1\ \ 1 \rangle_5 \mid 1\ \ 0 \rangle_2 \mid 1\ \ 0 \rangle_3 - \frac{1}{\sqrt{6}} \mid 1\ \ 0 \rangle_5 \mid 1\ \ 1 \rangle_2 \mid 1\ \ 0 \rangle_3 - \tfrac{1}{3} \mid 1\ \ 1 \rangle_5 \mid 1\ \ -1 \rangle_2 \mid 1\ \ 1 \rangle_3 + \tfrac{1}{3} \mid 1\ \ -1 \rangle_5 \mid 1\ \ 1 \rangle_2 \mid 1\ \ 1 \rangle_3.$$

This can also be written in the following explicit form:

$$(12345)_3 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)(\vec{r}_2 \cdot \vec{r}_3)\vec{r}_5 \Big[\tfrac{1}{2} Y_1^1(\Omega_5) Y_1^0(\Omega_2) Y_1^0(\Omega_3) - \frac{1}{\sqrt{6}} Y_1^0(\Omega_5) Y_1^1(\Omega_2) Y_1^0(\Omega_3) - \tfrac{1}{3} Y_1^1(\Omega_5) Y_1^{-1}(\Omega_2) Y_1^1(\Omega_3) + \tfrac{1}{3} Y_1^{-1}(\Omega_5) Y_1^1(\Omega_2) Y_1^1(\Omega_3) \Big].$$

Again using the method of App. II,

$$\langle (12345)_3 \mid (1234;5) \rangle = \begin{Bmatrix} 0.01667 \\ 0.01542 \end{Bmatrix} \times 10^{-169}\,\mathrm{cm}^{14}$$

and

$$\langle (12345)_3 \mid (5234;1) \rangle = \begin{Bmatrix} 0.08528 \\ 0.07977 \end{Bmatrix} \times 10^{-170}\,\mathrm{cm}^{14}.$$

Substituting N_{SM3} and N_{CM} from App. III and the values of $\langle (12345)_3 \mid (1234;5) \rangle$ and $\langle (12345)_3 \mid (5234;1) \rangle$ into eq.(II-5) gives

$$a_3 = \begin{Bmatrix} 0.4592 \\ 0.4589 \end{Bmatrix},$$

and

$$|a_3|^2 = \begin{Bmatrix} 0.210 \\ 0.210 \end{Bmatrix}.$$

(iv). For the configuration $(1p)^{-1}(2p)^{1}$, the spatial wave function for the HOSM is

$$(12345)_4 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)\vec{r}_5 L_1^{3/2}(\mu_1 r_5^2) Y_1^1(\Omega_5)$$

where

$$L_1^{3/2}(\mu_1 r_5^2) = \frac{5}{2} - \mu_1 r_5^2.$$

Using the method of App. II, there results

$$\langle (12345)_4 \mid (1234;5) \rangle = \begin{Bmatrix} 0.19105 \\ 0.18510 \end{Bmatrix} \times 10^{-169}\,\mathrm{cm}^{14}$$

and

$$\langle (12345)_4 \mid (5234;1) \rangle = \begin{Bmatrix} 0.04817 \\ 0.04792 \end{Bmatrix} \times 10^{-169}\,\mathrm{cm}^{14}.$$

Use of eq.(JI-5) with the given N_{SM_4}, N_{CM} and the values of the integrals given above results in

$$a_4 = \begin{Bmatrix} 0.3527 \\ 0.3512 \end{Bmatrix}$$

and

$$|a_4|^2 = \begin{Bmatrix} 0.124 \\ 0.123 \end{Bmatrix}.$$

The states of Subsecs. (ii), (iii) and (iv) are those in the HOSM scheme having two quanta of excitation above the ground state configuration $(1s)^4(1p)^1$. In what follows, a calculation is made of the admixture amplitude for states having four quanta of excitation above the ground state configuration $(1s)^4(1p)^1$.

(v). If two nucleons have been excited from the 1s shell to the 2s shell each with two quanta of energy, then their configuration is $(1s)^{-2}(2s)^2$. For this state, the HOSM spatial wave function is

$$(12345)_5 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)\vec{r}_5 L_1^{\frac{1}{2}}(\mu_1 r_1^2) L_1^{\frac{1}{2}}(\mu_1 r_4^2) Y_1^1(\Omega_5)$$

where

$$L_1^{\frac{1}{2}}(\mu_1 r_1^2) = \frac{3}{2} - \mu_1 r_1^2, \qquad L_1^{\frac{1}{2}}(\mu_1 r_4^2) = \frac{3}{2} - \mu_1 r_4^2.$$

By using the method in App. II as before, there results

$$\langle (12345)_5 | (1234;5) \rangle = \begin{Bmatrix} 0.10587 \\ 0.09780 \end{Bmatrix} \times 10^{-169}\ \text{cm}^{14}$$

and

$$\langle (12345)_5 | (5234;1) \rangle = \left\{ \begin{matrix} 0.02706 \\ 0.02578 \end{matrix} \right\} \times 10^{-169} \text{cm}^{14}.$$

Thus, form App. III, we have

$$a_5 = \left\{ \begin{matrix} 0.3075 \\ 0.3085 \end{matrix} \right\}$$

and

$$|a_5|^2 = \left\{ \begin{matrix} 0.094 \\ 0.095 \end{matrix} \right\}.$$

(vi). For the configuration $(1s)^{-1}(3s)^1$, the HOSM spatial wave function is

$$(12345)_6 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2) \vec{r}_5 L_2^{\frac{1}{2}}(\mu_1 r_1^2) Y_1^1(\Omega_5))$$

where

$$L_2^{\frac{1}{2}}(\mu_1 r_1^2) = \frac{15}{8} - \frac{5}{2}\mu_1 r_1^2 + \frac{1}{8}\mu_1^2 r_1^4.$$

Then applying the method in App. II gives

$$\langle (12345)_6 | (1234;5) \rangle = \left\{ \begin{matrix} 0.08488 \\ 0.07849 \end{matrix} \right\} \times 10^{-169} \text{cm}^{14}$$

and

$$\langle (12345)_6 | (5234;1) \rangle = \left\{ \begin{matrix} 0.02177 \\ 0.02074 \end{matrix} \right\} \times 10^{-169} \text{cm}^{14}.$$

Thus eq.(II-5) gives

$$a_6 = \left\{ \begin{matrix} 0.1161 \\ 0.1165 \end{matrix} \right\}$$

and

$$|a_6|^2 = \begin{Bmatrix} 0.013 \\ 0.013 \end{Bmatrix}.$$

(vii). The amplitudes of the remaining configurations given in Sec. 1, namely, $(1s)^{-1}(1d)^1$, $(1s)^{-2}(2s)^1(1d)^1$, $(1s)^{-1}(1p)^{-1}(2s)^1(2p)^1$, $(1p)^{-1}(3p)^1$, $(1s)^{-1}(2d)^1$ have been calculated, and were found to be small. The amplitudes $|a_i|^2$ for these states are two or more orders of magnitude smaller than 0.01 (i.e., 1%), and so are negligible.

In order to check whether the above calculations satisfy the condition (I-9), the sum of the squares of the expansion coefficients is calculated; it is

$$\sum_{i=1}^{6} |a_i|^2 = \begin{Bmatrix} 0.993 \\ 0.999 \end{Bmatrix}.$$

The sum rule is therefore satisfied to at least two significant figures.

CHAPTER III

Expansion of the Cluster Model Wave Function of the Second Excited State

1. The HOSM Scheme

The HOSM interprets the second excited state of He^5 as three nucleons in the 1s shell and two nucleons in the 1p shell [$(1s)^3(1p)^2$, or $(1s)^{-1}(1p)^1$ to the ground state]. The total orbital angular momentum can therefore be L=2, 1 or 0. Since the second excited state of He^5 has positive parity, L=1 is inadmissible. On the other hand, if L=2, and since $J=\frac{3}{2}$, then the spin wave function must be orthogonal to that of the cluster model. This is because in the cluster model spin wave function $S=\frac{3}{2}$; that is, four nucleons have spin "up" and one nucleon has spin "down", while the spin wave function of HOSM would have three nucleons with spin "down" and two nucleons with spin "up". Thus, only one choice remains, namely L=0. Then, as stated in Ch. I, the HOSM spin wave function is the same as that of the cluster model.

The configurations having three quanta of excitation and five quanta of excitation above the $(1s)^4(1p)^1$ state, which have the same total angular momentum and parity as the second excited state $(3/2^+)$ are

Three quanta of excitation:

$$(1s)^{-2}(1p)^1(2s)^1, \quad (1s)^{-2}(1p)^1(1d)^1, \quad (1s)^{-1}(2p)^1;$$

Five quanta of excitation:

$(1s)^{-3}(1p)^{1}(2s)^{2}$, $(1s)^{-3}(1p)^{1}(2s)^{1}(1d)^{1}$, $(1s)^{-1}(1p)^{-1}(2p)^{2}$,

$(1s)^{-2}(1p)^{1}(3s)^{1}$, $(1s)^{-2}(2s)^{1}(2p)^{1}$, $(1s)^{-2}(1d)^{1}(2p)^{1}$,

$(1s)^{-2}(1p)^{1}(2d)^{1}$.

2. The General Formula for the Expansion Coefficients

The unantisymmetrized and antisymmetrized HOSM wave functions for the second excited state of He^5 are

$$\phi_{SMi}=N_{SMi}(12345)_i\frac{1}{\sqrt{6}}\left[(\alpha_1\beta_3+\beta_1\alpha_3)\alpha_2-2\alpha_1\beta_2\alpha_3\right]\alpha_4\alpha_5 \qquad \text{(III-1)}$$

$$\mathcal{A}\phi_{SMi}=\frac{N_{SMi}}{4\sqrt{6}}\{(\alpha_2\beta_1-\beta_2\alpha_1)\alpha_3\alpha_4\alpha_5\left[(12345)_i-(12435)_i\right]$$
$$+(\beta_2\alpha_5-\alpha_2\beta_5)\alpha_1\alpha_3\alpha_4\left[(52341)_i-(52431)_i\right]$$
$$+(\beta_5\alpha_1-\alpha_5\beta_1)\alpha_2\alpha_3\alpha_4\left[(15342)_i-(15432)_i\right]\}. \qquad \text{(III-2)}$$

The subscript i has the same meaning as stated in the previous chapter.

On the other hand, the unantisymmetrized and antisymmetrized cluster model wave functions for the second excited state of He^5 are

$$\phi_{CM}=N_{CM}(123;45)\frac{1}{\sqrt{6}}\left[(\alpha_1\beta_3+\beta_1\alpha_3)\alpha_2-2\alpha_1\beta_2\alpha_3\right]\alpha_4\alpha_5 \qquad \text{(III-3)}$$

and

$$\phi_{CM}=\frac{N_{CM}}{4\sqrt{6}}\{(\alpha_1\beta_1-\beta_2\alpha_1)\alpha_3\alpha_4\alpha_5\left[(123;45)-(124;35)\right]$$

$$+(\beta_2\alpha_5-\alpha_2\beta_5)\alpha_1\alpha_3\alpha_4\left[(523;41)-(524;31)\right]$$

$$+(\beta_5\alpha_1-\alpha_5\beta_1)\alpha_2\alpha_3\alpha_4\left[(153;42)-(154;32)\right]\}. \quad \text{(III-4)}$$

The normalization constants for both model wave functions are determined by eq.(I-4) and the numerical values are given in the second part of App. III.

The expansion coefficient b_i is given by

$$b_i=\frac{N_{SMi}\,N_{CM}}{8}\{\langle(12345)_i|\,(123;45)-2(124;35)+(524;31)\rangle\}. \quad \text{(III-5)}$$

3. Numerical Calculation of the Expansion Coefficients

(i). The HOSM spatial wave function for the configuration $(1s)^{-1}(1p)^1$ is

$$(12345)_1=\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)\vec{r}_4\cdot\vec{r}_5|\,0\;\;0\rangle_{45},$$

where

$$|\,0\;\;0\rangle_{45}=\frac{1}{\sqrt{3}}\left[Y_1^1(\Omega_4)Y_1^{-1}(\Omega_5)+Y_1^{-1}(\Omega_4)Y_1^1(\Omega_5)-Y_1^0(\Omega_4)Y_1^0(\Omega_5)\right].$$

Thus,

$$\langle(12345)_1|\,(123;45)\rangle$$

$$=\int\exp\{-\left[A(r_1^2+r_2^2+r_3^2)+B(r_4^2+r_5^2)+C(\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_3)\right.$$

$$\left.+D(\vec{r}_1\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_5)+E\vec{r}_4\cdot\vec{r}_5\right]\}\,\vec{r}_4\cdot\vec{r}_5$$

$$\times\left[\frac{1}{9}(r_1^2+r_2^2+r_3^2+2\vec{r}_1\cdot\vec{r}_2+2\vec{r}_1\cdot\vec{r}_3+2\vec{r}_2\cdot\vec{r}_3)+\tfrac{1}{4}(r_4^2+r_5^2+2\vec{r}_4\cdot\vec{r}_5)\right.$$

$$\left.-\tfrac{1}{3}(\vec{r}_1\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_5)\right]d\vec{r}_1d\vec{r}_2d\vec{r}_3d\vec{r}_4d\vec{r}_5,$$

$$\langle (12345)_1 \mid (124;35) \rangle$$

$$= \int \exp\{ -\left[A(r_1^2+r_2^2+r_4^2)+B(r_3^2+r_5^2)+C(\vec{r}_1\vec{r}_2+\vec{r}_1\vec{r}_4+\vec{r}_2\vec{r}_4) \right.$$

$$\left. +D(\vec{r}_1\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_3+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5)+E\vec{r}_3\cdot\vec{r}_5 \right] \} \vec{r}_4\cdot\vec{r}_5$$

$$\times \left[\tfrac{1}{9}(r_1^2+r_2^2+r_4^2+2\vec{r}_1\cdot\vec{r}_2+2\vec{r}_1\cdot\vec{r}_4+2\vec{r}_2\cdot\vec{r}_4)+\tfrac{1}{4}(r_3^2+r_5^2+2\vec{r}_3\cdot\vec{r}_5) \right.$$

$$\left. -\tfrac{1}{3}(\vec{r}_1\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_3+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5) \right] d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5$$

and

$$\langle (12345)_1 \mid (524;31) \rangle$$

$$= \int \exp\{ -\left[A(r_5^2+r_2^2+r_4^2)+B(r_3^2+r_1^2)+C(\vec{r}_2\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_4) \right.$$

$$\left. +D(\vec{r}_3\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_3+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_4)+E\vec{r}_1\cdot\vec{r}_3 \right] \} \vec{r}_4\cdot\vec{r}_5$$

$$\times \left[\tfrac{1}{9}(r_2^2+r_4^2+r_5^2+2\vec{r}_2\cdot\vec{r}_5+2\vec{r}_2\cdot\vec{r}_4+2\vec{r}_4\cdot\vec{r}_5)+\tfrac{1}{4}(r_1^2+r_3^2+2\vec{r}_1\cdot\vec{r}_3) \right.$$

$$\left. -\tfrac{1}{3}(\vec{r}_3\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_3+\vec{r}_3\cdot\vec{r}_4+\vec{r}_1\cdot\vec{r}_5+\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_4) \right] d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5,$$

where

$$A=\frac{3}{5}\mu_1+\frac{1}{3}\alpha+\frac{\beta}{15}=3.212578\times 10^{25}\,\mathrm{cm}^{-2}$$

$$B=\frac{3}{5}\mu_1+\frac{1}{4}\bar{\alpha}+\frac{3}{20}\beta=3.449812\times 10^{25}\,\mathrm{cm}^{-2}$$

$$C=-\frac{1}{3}\alpha+\frac{2}{15}\beta+\frac{1}{5}\mu_1=0.037362\times 10^{25}\,\mathrm{cm}^{-2}$$

$$D=\frac{1}{5}(\mu_1-\beta)=0.646104\times 10^{25}\,\mathrm{cm}^{-2}$$

$$E=-\frac{1}{2}\bar{\alpha}+\frac{2}{10}\beta+\frac{1}{5}\mu_1=-1.532896\times 10^{25}\,\mathrm{cm}^{-2}.$$

Here it should be pointed out that in the determination of

both normalization constants and also in the integrations of the type $\langle (12345)_i | (123;45) \rangle$ some common factors have been omitted; these will not affect the results because the same factors have been neglected in both numerators and denorminators of the expansion coefficients.

By using the method of App. II, there results

$$\langle (12345)_1 | (123;45) \rangle = 0.23236 \times 10^{-199} \text{cm}^{16}$$

$$\langle (12345)_1 | (124;35) \rangle = -0.10674 \times 10^{-199} \text{cm}^{16}$$

and

$$\langle (12345)_1 | (524;31) \rangle = 0.00074 \times 10^{-199} \text{cm}^{16}.$$

Equation (III-5) then gives

$$b_1 = 0.5479$$

and

$$|b_1|^2 = 0.296.$$

(ii). The HOSM spatial wave function for the configuration $(1s)^{-2}(1p)^1(2s)^1$ is

$$(12345)_2 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2) L_1^{\frac{1}{2}}(\mu_1 r_1^2) \vec{r}_4 \cdot \vec{r}_5 | \; 0 \;\; 0 \rangle_{45}$$

where

$$L_1^{\frac{1}{2}}(\mu_1 r_1^2) = \frac{3}{2} - \mu_1 r_1^2.$$

Since the exponential term in the integrals are all the same, they need not be written down explicitly. An application of the method of App. II then gives

$$\langle (12345)_2 \mid (123;45) \rangle = 0.2068 \times 10^{-199}\,\mathrm{cm}^{16}$$

$$\langle (12345)_2 \mid (124;35) \rangle = -0.09517 \times 10^{-199}\,\mathrm{cm}^{16}$$

and

$$\langle (12345)_2 \mid (524;31) \rangle = 0.00066 \times 10^{-199}\,\mathrm{cm}^{16},$$

so that

$$b_2 = 0.4013$$

and

$$|b_2|^2 = 0.161.$$

(iii). The HOSM spatial wave function for the configuration $(1s)^{-2}(1p)^1(1d)^1$ is

$$(12345)_3 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2) r_1^2 \vec{r}_4 \cdot \vec{r}_5 \mid 0 \;\; 0 \rangle_{145}$$

where

$$\begin{aligned} \mid 0 \;\; 0 \rangle_{145} = \frac{1}{\sqrt{5}} \{ & \mid 1 \;\; 1 \rangle_4 \mid 1 \;\; 1 \rangle_5 \mid 2 \;\; -2 \rangle_1 \\ & + \mid 1 \;\; -1 \rangle_4 \mid 1 \;\; -1 \rangle_5 \mid 2 \;\; 2 \rangle_1 - \frac{1}{\sqrt{2}} [(\mid 1 \;\; 1 \rangle_4 \mid 1 \;\; 0 \rangle_5 \\ & + \mid 1 \;\; 0 \rangle_4 \mid 1 \;\; 1 \rangle_5) \mid 2 \;\; -1 \rangle_1 + (\mid 1 \;\; -1 \rangle_4 \mid 1 \;\; 0 \rangle_5 \\ & + \mid 1 \;\; 0 \rangle_4 \mid 1 \;\; -1 \rangle_5) \mid 2 \;\; 1 \rangle_1] \\ & + \frac{1}{\sqrt{6}} (\mid 1 \;\; 1 \rangle_4 \mid 1 \;\; -1 \rangle_5 + \mid 1 \;\; -1 \rangle_4 \mid 1 \;\; 1 \rangle_5) \mid 2 \;\; 0 \rangle_1 \} . \end{aligned}$$

Again applying the method of App. II, there results

$$\langle (12345)_3 \mid (123;45) \rangle = 0.04597 \times 10^{-199}\,\mathrm{cm}^{16}$$

$$\langle (12345)_3 \mid (12,4;35) \rangle = -0.02116 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_3 \mid (52,4;31) \rangle = 0.000146 \times 10^{-199}\ \mathrm{cm}^{16}.$$

Equation (III-5) gives finally

$$b_3 = 0.4950$$

and

$$|b_3|^2 = 0.245.$$

(iv). The HOSM spatial wave function for the configuration $(1s)^{-1}(2p)^1$ is

$$(12345)_4 = \exp\left(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2\right) \vec{r}_4 \cdot \vec{r}_5 L_1^{3/2}(\mu_1 r_5^2) \mid 0 \quad 0 \rangle_{45}$$

where

$$L_1^{3/2}(\mu_1 r_5^2) = \frac{5}{2} - \mu_1 r_5^2.$$

The method in App. II gives

$$\langle (12345)_4 \mid (123;45) \rangle = 0.31608 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_4 \mid (12,4;35) \rangle = -0.14546 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_4 \mid (52,4;31) \rangle = 0.0010 \times 10^{-199}\ \mathrm{cm}^{16}.$$

Again, from eq.(III-5), the expansion coefficient and its square are

$$b_4 = 0.3541$$

$$|b_4|^2 = 0.125.$$

(v). The HOSM spatial wave function for the configuration $(1s)^{-3}(1p)^{1}(2s)^{2}$ is

$$(12345)_5=\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5} r_i^2)L_1^{\frac{1}{2}}(\mu_1 r_1^2)L_1^{\frac{1}{2}}(\mu_1 r_2^2)\,|\,0\;\;0\rangle_{45}$$

where

$$L_1^{\frac{1}{2}}(\mu_1 r_i^2)=\frac{3}{2}-\mu_1 r_i^2.$$

Using the same method as above gives

$$\langle(12345)_5\,|\,(123;45)\rangle=0.20969\times 10^{-199}\,\mathrm{cm}^{16}$$

$$\langle(12345)_5\,|\,(124;35)\rangle=-0.09650\times 10^{-199}\,\mathrm{cm}^{16}$$

$$\langle(12345)_5\,|\,(524;31)\rangle=0.00066\times 10^{-199}\,\mathrm{cm}^{16},$$

and eq.(III-5)gives

$$b_5=0.3323$$

and

$$|\,b_5\,|^2=0.110.$$

(vi). The HOSM spatial wave function for the configuration $(1s)^{-3}(1p)^{1}(2s)^{1}(1d)^{1}$ is

$$(12345)_6=\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5} r_i^2)r_1^2\vec{r}_4\cdot\vec{r}_5 L_1^{\frac{1}{2}}(\mu_1 r_2^2)\,|\,0\;\;0\rangle_{145}$$

where

$$L_1^{\frac{1}{2}}(\mu_1 r_2^2)=\frac{3}{2}-\mu_1 r_2^2,$$

and $|\,0\;\;0\rangle_{145}$ is given in (iii). Then the integrals are

$$\langle (12345)_6 \mid (123;45) \rangle = 0.02331 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_6 \mid (124;35) \rangle = -0.01073 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_6 \mid (524;31) \rangle = 0.00007 \times 10^{-199}\ \mathrm{cm}^{16},$$

so that,

$b_6 = 0.2047$

and

$$| b_6 |^2 = 0.042.$$

(vii). The HOSM spatial wave function for the configuration $(1s)^{-1}(1p)^{-1}(2p)^2$ is

$$(12345)_7 = \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)\vec{r}_4 \cdot \vec{r}_5 L_1^{3/2}(\mu_1 r_5^2) L_1^{3/2}(\mu_1 r_4^2) \mid 0 \quad 0 \rangle_{45}$$

where

$$L_1^{3/2}(\mu_1 r_i^2) = \frac{5}{2} - \mu_1 r_i^2.$$

Then the numerical values of the integrals are

$$\langle (12345)_7 \mid (123;45) \rangle = 0.30587 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_7 \mid (124;35) \rangle = -0.14076 \times 10^{-199}\ \mathrm{cm}^{16}$$

$$\langle (12345)_7 \mid (524;31) \rangle = 0.00097 \times 10^{-199}\ \mathrm{cm}^{16}.$$

Equation (III-5) gives then

$b_7 = 0.127$

and

$$| b_7 |^2 = 0.016.$$

(viii). The amplitudes of the other configurations given in Sec.1 were also calculated, and found to be small. They are three or more orders of magnitude smaller than 0.01, and so are negligible.

On the other hand, there may be some contribution from the excitation of the two nucleons in the 1p level to the 2s level, with one quantum of excitation for each nucleon. Then, however the HOSM spatial wave function is completely symmetric and therefore cannot be antisymmetrized; thus, no contribution results.

The sum of the squares of the expansion coefficients (i.e., the intensities) is

$$\sum_{i=1}^{7} | b_i |^2 = 0.999.$$

CHAPTER IV

Conclusion and Discussion

The HOSM describes the individual nucleons as moving in a common harmonic oscillator potential well. It is indeed a very simple and easily understandable model, but this simple model cannot describe details of nuclear structure very well. The reason for this of course is that the oscillator potential is not the real nuclear potential; in actual nuclei, the nuclear potential is more complicated than the oscillator potential. For instance, internucleon interactions must be accounted for in the actual nuclear potential, and these are not included in the HOSM potential. Thus, the HOSM eigenstates are not the eigenstates of the actual nuclear potential, and so the nuclear state wave functions cannot be simply replaced by the HOSM wave functions. If the oscillator potential is modified, then the nuclear state wave functions will be modified. As was mentioned in Ch. I, since the HOSM eigenfunctions are orthogonal to each other and span the whole configuration space, so the modified eigenfunctions can be expanded as a linear combination of the HOSM eigenfunctions. Thus, the modified eigenfunctions include not only the basic HOSM eigenfunctions, but also the higher excited eigenfunctions. This configuration mixing can also be calculated by the perturbation method. If the additional potential (or residual interaction) is taken as a perturbation,

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then the expansion coefficients may be determined by using the perturbation method [12].

The HOSM wave functions usually give energy levels which are too low compared to the experimental results. In order to have a more realistic model, the differences in energy between the actual nuclear levels and the HOSM predictions must be reduced somehow. This can be accomplished by mixing in HOSM states of higher energies. The amounts of these higher states included in the configuration mixing are given simply by the coefficients.

It will be shown in App. IV that the simple cluster model (in which all the width parameters are equal) is exactly equivalent to the HOSM after the wave functions have been antisymmetrized. However, the generalized cluster model (in which the width parameters are all different) used here for He^5 includes higher HOSM configurations automatically simply by allowing the width parameters to differ from one another.

The cluster model wave functions for the ground state and the second excited state of He^5 seem to be quite good, at least in some respects, as for example the energy levels and the E1 transition probability. The mixture of HOSM configurations in the cluster model wave function will show how He^5 must be treated using the shell model in order to achieve results of similar quality.

Tables 2 and 3 summarize the results of Ch. II and Ch. III respectively for the intensities of the mixing of the

HOSM configurations in the cluster model wave functions in the ground and second excited states. From first order perturbation theory, it is expected that configurations having the same number of quanta of excitation should appear in roughly the same amounts. This found to be the case in the present calculation, with the one exception that the $(2s)^2$ configurations in both the ground and second excited states have intensities of comparable magnitude to those of the two quantum excitations. This may partly be due to the L-S coupling scheme; since the nucleons with the same l have very similar HOSM wave functions, then, from perturbation theory, the matrix element between these states may be large even though the energy difference is not samll.

It is also interesting to note that, as was mentioned in Ch. II, the 1d level does not contribute very significantly to the ground state; however, it can be seen from Table 3 that the second excited state has a fairly large contribution from the 1d level. From perturbation theory, the matrix element for the 1d state should be quite small, but the energy difference is also small, so that the expansion coefficient may not be small. This rather large contribution from the 1d level indicates that the energy of the actual nuclear state is quite near to that of the HOSM 1d level.

Figs. 2 and 3 show that the configuration mixing is insensitive to variations of the cluster model width parameters for the ground state when one parameter is held constant at

Configuration	Intensity* (%)
$(1s)^4\ (1p)^1$	$\left\{\begin{matrix}39.3\\40.2\end{matrix}\right\}$
$(1s)^{-1}(2s)^1$	$\left\{\begin{matrix}15.9\\15.6\end{matrix}\right\}$
$(1s)^{-2}(1p)^2$	$\left\{\begin{matrix}21.0\\21.0\end{matrix}\right\}$
$(1p)^{-1}(2p)^1$	$\left\{\begin{matrix}12.4\\12.3\end{matrix}\right\}$
$(1s)^{-2}(2s)^2$	$\left\{\begin{matrix}9.4\\9.5\end{matrix}\right\}$
$(1s)^{-1}(3s)^1$	$\left\{\begin{matrix}1.3\\1.3\end{matrix}\right\}$
Total	$\left\{\begin{matrix}99.3\\99.9\end{matrix}\right\}$

Table 2. The intensities of the different configurational states found in the ground state cluster model wave function with width parameters: $\alpha_1 = 4.33 \times 10^{25}\,cm^{-2}$, $\beta' = \left\{\begin{matrix}2.25\\3.25\end{matrix}\right\} \times 10^{25}\,cm^{-2}$ and $\mu_1 = 3.65252 \times 10^{25}\,cm^{-2}$.

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* The two different intensities correspond to the two different values of the width parameter β' of the cluster model.

Configuration	Intensity (%)
$(1s)^{-1}(1p)^1$	30.0
$(1s)^{-2}(1p)^1(2s)^1$	16.1
$(1s)^{-2}(1p)^1(1d)^1$	24.5
$(1s)^{-1}(2p)^1$	12.5
$(1s)^{-3}(1p)^1(2s)^2$	11.0
$(1s)^{-3}(1p)^1(2s)^1(1d)^1$	4.2
$(1s)^{-1}(1p)^{-1}(2p)^2$	1.6
Total	99.9

Table 3. The intensities of the different configurational states found in the second excited state cluster model wave function with width parameters: $\alpha = 2.81 \times 10^{25} cm^{-2}$, $\bar{\alpha} = 4.78 \times 10^{25} cm^{-2}$, $\beta = 0.422 \times 10^{25} cm^{-2}$ and $\mu_1 = 3.65252 \times 10^{25} cm^{-2}$.

Pearlstein's value and the other is varied. In Fig.1 of Pearlstein et al [2] curves (c) and (e) are quite flat near the minimum. This indicates that with a certain variation of the parameter β' around the minimum, the energy of the ground state does not change very much. It is therefore not surprising that the intensities of the configurations do not change appreciably when β' is varied. It was also mentioned in the same work [2] that the energy is insensitive to the

compressibility of the alpha particle. In other words, the energy of the ground state is insensitive to variations of the width parameter α_1 when β' is held constant. Thus, it is also plausible that the mixture of the various HOSM configurations will also be insensitive to the variation of α_1.

The ratio of the cluster model width parameters $\beta'/\alpha_1 = \left\{ \begin{matrix} 0.52 \\ 0.75 \end{matrix} \right\} < 1$; this indicates that the alpha particle cluster and the neutron wave functions do not overlap greatly [13]. Also, according to the shell model, closed shell structures usually remain energetically preferred even when the wave functions of the nucleons in closed shells overlap those of surrounding nucleons very strongly. Both of these points indicate that the ground state of He^5 is favourable to formation of an alpha-neutron cluster structure. On the other hand, in the last paragraph, the insensitivity of the configuration mixing to the variation of one of the cluster model width parameters was seen. This suggests that certain collections of nucleons exist which are highly stable as free particles and are bound toghther to form the ground state of He^5--this is so because marked variations in the width parameters involved have very little affect on the binding energy and on the relative amounts of the various configurations which contribute to the state. From the possible choices of the clusters which are highly stable and bound, it may be induced that the most favourable one is the alpha

particle-neutron structure. Thus, the results indicating the insensitivity of the configuration mixing to the variation of one of the cluster model width parameters is consistent with the cluster representation of the ground state of He^5.

No variation of the cluster model width parameters of the second excited state has been performed. However the configuration mixing will probably be quite sensitive to the variation of any one of the three width parameters α, $\bar{\alpha}$ and β. This can be seen from Figs. 3, 4 and 5 of Pearlstein et al [2], which show that if any one of the three cluster model width parameters are varied, a fairly large variation of the energy results.

Fig. 4 shows that for $\alpha_1=\mu_1$ the configuration mixing is still insensitive to variation of the cluster model width parameter β' for the ground state except near the region of $\beta'=\mu_1$. The physical meaning of setting $\alpha_1=\mu_1$ is that the alpha particle is treated as a set of four individual nucleons without correlations. However, if β' is not equal to μ_1 then the extra neutron has correlations with the four nucleons. When $\beta'=\mu_1$, the nucleon correlations are completely broken down; i.e., the antisymmetrized ground state cluster model wave function is completely equivalent to the HOSM wave function of the $(1s)^4(1p)^1$ level. Mathematically, this will be shown in App. IV.

The same sort of diagram as shown in Fig. 4 will result if $\beta' = \mu_1$ and α_1 is varied. This means that the two-body correlations between the alpha particle and neutron are first eliminated. If α_1 approaches μ_1, then the nucleon correlations within the alpha particle become smaller and smaller. At $\alpha_1 = \mu_1$, the nucleon correlations are completely broken down as above, and the same thing will happen again-i.e., the antisymmetrized ground state cluster model wave function is equivalent to the HOSM wave function of the $(1s)^4(1p)^1$ level.

Since configurations (i), (ii), (iv), (v) and (vii) can contribute to the E1 transition, then it is seen that the HOSM configuration (i) can only contribute partially to the E1 transition probability. Since the energy levels of the HOSM wave functions are equally spaced, then an estimate of the level width of the second excited state of He^5 may be made on the HOSM as 0.006859 relative to level width according to the cluster model. This is consistant with the results of Tran Duc and Smith [3], who estimate the ratio

$$\Gamma_\gamma(\text{s.p.})/\Gamma_\gamma(\text{cluster model}) = 1/(1.55 \times 10^2).$$

Since nuclear quadrupole moments are quite sensitive to admixtures of different configurations with the same angular momentum and parity [9],[14], it may be interesting to extend the present work to a quantitative discussion of the nuclear quadrupole moment of He^5. The cluster model wave function of

the ground state gives no quadrupole moment, because essentially no d-state admixture is present [9]. However, as shown in Ch. III and Table 3, the second excited state does have a significant amount of d-state admixture, and so it may have a large quadrupole moment, although it cannot be measured directly at present.

APPENDIX I

Evaluation of Integrals of the Type

$$\int (\tilde{r}Br)e^{-\tilde{r}Ar}d\vec{r}_1d\vec{r}_2d\vec{r}_3d\vec{r}_4d\vec{r}_5$$

In most of the calculations in Ch. II, as well as in the calculation of the normalization constant of the ground state cluster model wave function, integrals of the type

$$\int (\tilde{r}Br)e^{-\tilde{r}Ar}d\vec{r}_1d\vec{r}_2d\vec{r}_3d\vec{r}_4d\vec{r}_5$$

must be evaluated, where A and B are 5×5 square matrices and r is a column matrix. Here the symbol "$\sim$" disignates the transpose of a matrix.

A well known theorem in matrix algebra [15] states that "If $\tilde{r}Ar$ and $\tilde{r}Br$ are real quadratic forms and if $\tilde{r}Ar$ is positive definite, there exists a real non-singular linear transformation $r=Cz$ which carries $\tilde{r}Br$ into $\varphi_1 z_1^2+\varphi_2 z_2^2+\cdots\cdots+\varphi_n z_n^2$ and $\tilde{r}Ar$ into $z_1^2+z_2^2+\cdots\cdots+z_n^2$ where $\varphi_1,\varphi_2,\cdots\cdots,\varphi_n$ are the roots of $|\varphi A-B|=0$". This theorem is useful in transforming the integrands of the above type to a form which may be easily evaluated.

Let

$$I=\int (\tilde{r}Br)e^{-\tilde{r}Ar}d\vec{r}_1d\vec{r}_2d\vec{r}_3d\vec{r}_4d\vec{r}_5 \ . \qquad \text{(AI-1)}$$

In order to evaluate this integral, first apply an orthogonal transformation O_1 which diagonalizes $\tilde{r}Ar$ [15]. Then

$$I=\int (\tilde{s}\tilde{O}_1BO_1s)e^{-\tilde{s}Ds}d\vec{s}_1d\vec{s}_2d\vec{s}_3d\vec{s}_4d\vec{s}_5,$$

where s is the new coordinate system and D_1 is the diagonal matrix. A second coordinate transformation:

$$\vec{t}_1=\sqrt{\rho_1}\,\vec{s}_1,\quad \vec{t}_2=\sqrt{\rho_2}\,\vec{s}_2,\quad \vec{t}_3=\sqrt{\rho_3}\,\vec{s}_3,\quad \vec{t}_4=\sqrt{\rho_4}\,\vec{s}_4,\quad \vec{t}_5=\sqrt{\rho_5}\,\vec{s}_5,$$

(or it may write in the matrix form as $s=Ht$ where $H=\mathrm{diag}(1/\sqrt{\rho_1}, 1/\sqrt{\rho_2}, 1/\sqrt{\rho_3}, 1/\sqrt{\rho_4}, 1/\sqrt{\rho_5})$) is now applied to the integral I. Here, ρ_1, ρ_2, ρ_3, ρ_4, ρ_5 are the roots of the determinantal equation $|A-\lambda I|=0$. This leads to

$$I=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int(\tilde{t}\tilde{H}\tilde{O}_1BO_1Ht)e^{-\tilde{t}\tilde{H}D_1Ht}d\vec{t}_1d\vec{t}_2d\vec{t}_3d\vec{t}_4d\vec{t}_5$$

$$=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int(\tilde{t}B_2t)e^{-\tilde{t}It}d\vec{t}_1d\vec{t}_2d\vec{t}_3d\vec{t}_4d\vec{t}_5 \qquad \text{(AI-2)}$$

where $B_2=\tilde{H}\tilde{O}_1BO_1H$. Finally, another orthogonal transformation O_2 is applied to the integrand in order to diagonalize B_2. Thus,

$$I=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int(\tilde{z}\tilde{O}_2B_2O_2z)e^{-\tilde{z}Iz}d\vec{z}_1d\vec{z}_2d\vec{z}_3d\vec{z}_4d\vec{z}_5$$

$$=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int(\tilde{z}D_2z)e^{-\tilde{z}Iz}d\vec{z}_1d\vec{z}_2d\vec{z}_3d\vec{z}_4d\vec{z}_5. \qquad \text{(AI-3)}$$

This simplification comes about because O_2 commutes with I and $\tilde{O}_2O_2=I$, so that this transformation does not affect the exponential term at all. Also, all transformations except H (or $\vec{t}_i=\sqrt{\rho_i}\,\vec{s}_i$) are orthogonal transformations, and so their Jacobians $|J|=1$. For the transformation H, the Jacobian is $J_H=|J_H|=1/(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}$, and is already included in eq.(AI-2). Thus, eq.(AI-3) may be written explicitly in the following form

$$I=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int\int\int_0^\infty\int\int\int(\varphi_1 z_1^2+\varphi_2 z_2^2+\cdots\cdots+\varphi_5 z_5^2)$$

$$\exp(-z_1^2-z_2^2-\cdots\cdots-z_5^2)d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 d\vec{z}_4 d\vec{z}_5 \qquad (AI\text{-}4)$$

where $\varphi_1, \varphi_2, \cdots\cdots, \varphi_5$ are the roots of the determinantal equation $|B-\varphi A|=0$.

Since $\rho_1, \rho_2, \cdots\cdots, \rho_5$ are the roots of $|A-\lambda I|=0$, then $(\rho_1\rho_2\rho_3\rho_4\rho_5)=\det A$. Also, since the φ's are the roots of $|B-\varphi A|=0$ (or $|A^{-1}B-\varphi I|=0$; A^{-1} exists because A is non-singular) then $\varphi_1+\varphi_2+\varphi_3+\varphi_4+\varphi_5=\mathrm{Trace}(A^{-1}B)$.

Eq.(AI-4) can now be integrated as follows:

$$I=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\left[(\varphi_1+\varphi_2+\varphi_3+\varphi_4+\varphi_5)I_1\right] \qquad (AI\text{-}5)$$

where

$$I_1=\int\int\int_0^\infty\int\int\int z_1^2\exp(-z_1^2-z_2^2-z_3^2-z_4^2-z_5^2)dz_1 dz_2 dz_3 dz_4 dz_5.$$

I_1 is an elementary integral, and may be evaluated very easily.

In the calculation, an IBM subroutine program is used to evaluate A^{-1}. The same program determines $(\det A^{-1})^{\frac{1}{2}}=1/(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}$ and $\mathrm{Trace}A^{-1}B$. In the program, $SD=(\det A^{-1})^{\frac{1}{2}}$ and $\mathrm{Trace}=\mathrm{Trace}A^{-1}B$. In terms of these quantities eq.(AI-5) can be written as $I=SD\times\mathrm{Trace}\times I_1$.

The print-out sheet attached gives the program used for this purpose. Several remarks should be made concerning this program. As shown in the program, the final result is reached after the write statement of the trace. The rest of the

program is used to check the results. The first check is to determine whether or not $A^{-1}A=I$ and $B^{-1}B=I$. Since B is usually singular, B^{-1} does not exist so in our program $B^{-1}B$ is not unitary. If B is non-singular, however, we can form $B^{-1}A$, and the second check is to find the inverse of $B^{-1}A$, which should be equal to $A^{-1}B$. The second check was performed several times and it did indicate that the subroutine is very good.

In the above-mentioned method the exponential term was first diagonalized. On the other hand, if B is non-singular, the same method can be used, except that B is diagonalized first. It can be shown mathematically that these two different approaches are strictly equivalent. This is confirmed in the computations.

```
C       MAIN PROBLEM
S.0001        DIMENSION A(25),B(25),C(5,5),E(5,5),F(5,5),G(5,5),H(5,5),P(5,5),
             1R(5,5),T(5,5),U(25),W(5,5),L(5),M(5)
S.0002        IJK=0
S.0003      600 READ (1,450) IPARAM
S.0004      450 FORMAT (I4)
S.0005        WRITE (3,451) IPARAM
S.0006      451 FORMAT (1H1,I4///)
S.0007      500 READ (1,3) (A(I),I=1,25),(B(I),I=1,25)
S.0008        3 FORMAT (5F14.8)
C
C       RETAIN A
C
S.0009        DO 40   J=1,5
S.0010        G(J,1)=A(J)
S.0011        G(J,2)=A(J+5)
S.0012        G(J,3)=A(J+10)
S.0013        G(J,4)=A(J+15)
S.0014       40 G(J,5)=A(J+20)
S.0015        WRITE (3,41)  ((G(I,J),I=1,5),J=1,5)
S.0016       41 FORMAT (1X,'G=',5X,5F14.8//)
C
C       RETAIN B
C
S.0017        DO 20 J=1,5
S.0018        E(J,1)=B(J)
S.0019        E(J,2)=B(J+5)
S.0020        E(J,3)=B(J+10)
S.0021        E(J,4)=B(J+15)
S.0022       20 E(J,5)=B(J+20)
S.0023        WRITE (3,8)  ((E(I,J),I=1,5),J=1,5)
S.0024        8 FORMAT (1X,'E=',5X,5F14.8//)
C
C       MAIN MATRIX INVERSION
C
C
C       WRITE OUT A INVERSE
C
S.0025        N=5
S.0026        CALL MINV (A,N,D,L,M)
S.0027        DO 25  J=1,5
S.0028        F(J,1)=A(J)
S.0029        F(J,2)=A(J+5)
S.0030        F(J,3)=A(J+10)
S.0031        F(J,4)=A(J+15)
S.0032       25 F(J,5)=A(J+20)
S.0033        WRITE (3,10)  ((F(I,J),I=1,5),J=1,5)
S.0034       10 FORMAT (1X,'F=',5X,5F14.8//)
S.0035        SD=SQRT(ABS(D))
S.0036        WRITE (3,14) D,SD
S.0037       14 FORMAT (1X,'D=',5X,F14.8,10X,'SD=',5X,F14.8//)
C
C       WRITE OUT B INVERSE
```

```
      C
S.0038      CALL MINV (B,N,D,L,M)
S.0039      DO 50  J=1,5
S.0040      H(J,1)=B(J)
S.0041      H(J,2)=B(J+5)
S.0042      H(J,3)=B(J+10)
S.0043      H(J,4)=B(J+15)
S.0044   50 H(J,5)=B(J+20)
S.0045      WRITE (3,51)  ((H(I,J),I=1,5),J=1,5)
S.0046   51 FORMAT (1X,'H=',5X,5F14.8//)
S.0047      WRITE (3,15) D
S.0048   15 FORMAT (1X,'D=',5X,F14.8//)
      C
      C     C EQUAL TO A INVERSE MULTIPLE B
      C
S.0049      DO 1  I=1,5
S.0050      DO 1  J=1,5
S.0051      SUM=0.
S.0052      DO 2  K=1,5
S.0053      SUM=F(I,K)*F(K,J)+SUM
S.0054    2 CONTINUE
S.0055      C(I,J)=SUM
S.0056    1 CONTINUE
S.0057      WRITE (3,5)  ((C(I,J),I=1,5),J=1,5)
S.0058    5 FORMAT (1X,'C=',5X,5F14.8//)
S.0059      SUM=0.
S.0060      DO 30  I=1,5
S.0061      SUM=C(I,I)+SUM
S.0062      TRACE=SUM
S.0063   30 CONTINUE
S.0064      WRITE (3,70) TRACE
S.0065   70 FORMAT (1X,'TRACE=',5X,F14.8//)
      C     .....
      C
      C     CHECK WHETHER IT IS IDENTITY FOR A
      C
S.0066      DO 101  I=1,5
S.0067      DO 101  J=1,5
S.0068      SUM=0.
S.0069      DO 102  K=1,5
S.0070      SUM=F(I,K)*G(K,J)+SUM
S.0071  102 CONTINUE
S.0072      P(I,J)=SUM
S.0073  101 CONTINUE
S.0074      WRITE (3,105)  ((P(I,J),I=1,5),J=1,5)
S.0075  105 FORMAT (1X,'P=',5X,5F14.8//)
      C
      C     CHECK WHETHER IT IS IDENTITY FOR B
      C
S.0076      DO 111  I=1,5
S.0077      DO 111  J=1,5
S.0078      SUM=0.
S.0079      DO 112  K=1,5
S.0080      SUM=H(I,K)*E(K,J)+SUM
S.0081  112 CONTINUE
```

```
S.0082            R(I,J)=SUM
S.0083        111 CONTINUE
S.0084            WRITE (3,125) ((R(I,J),I=1,5),J=1,5)
S.0085        125 FORMAT (1X,'R=',5X,5F14.8//)
              C   ..........................................................
              C
              C     DOUBLE CHECK
              C
              C     T EQUAL TO B INVERSE MULTIPLE TO A
              C
S.0086            DO 201  I=1,5
S.0087            DO 201  J=1,5
S.0088            SUM=0.
S.0089            DO 202  K=1,5
S.0090            SUM=H(I,K)*G(K,J)+SUM
S.0091        202 CONTINUE
S.0092            T(I,J)=SUM
S.0093        201 CONTINUE
S.0094            WRITE (3,205) ((T(I,J),I=1,5),J=1,5)
S.0095        205 FORMAT (1X,'T=',5X,5F14.8//)
              C
              C     FORMING U
              C
S.0096            DO 300  J=1,5
S.0097            U(J)=T(J,1)
S.0098            U(J+5)=T(J,2)
S.0099            U(J+10)=T(J,3)
S.0100            U(J+15)=T(J,4)
S.0101        300 U(J+20)=T(J,5)
S.0102            CALL MINV (U,N,D,L,M)
              C
              C     WRITE OUT T INVERSE
              C
S.0103            DO 400  J=1,5
S.0104            W(J,1)=U(J)
S.0105            W(J,2)=U(J+5)
S.0106            W(J,3)=U(J+10)
S.0107            W(J,4)=U(J+15)
S.0108        400 W(J,5)=U(J+20)
S.0109            WRITE (3,303) ((W(I,J),I=1,5),J=1,5)
S.0110        303 FORMAT (1X,'W=',5X,5F14.8)
S.0111            IJK=IJK+1
S.0112            IF (IJK-80)60,401,401
S.0113        401 CONTINUE
S.0114            RETURN
S.0115            END

                  SIZE OF COMMON   000000   PROGRAM   004832

                                END OF COMPILATION   MAIN
```

```
C
C     ..................................................................
C
C        SUBROUTINE MINV
C
C        PURPOSE
C           INVERT A MATRIX
C
C        USAGE
C           CALL MINV(A,N,D,L,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
C               RESULTANT INVERSE.
C           N - ORDER OF MATRIX A
C           D - RESULTANT DETERMINANT
C           L - WORK VECTOR OF LENGTH N
C           M - WORK VECTOR OF LENGTH N
C
C        REMARKS
C           MATRIX A MUST BE A GENERAL MATRIX
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
C           IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
C           THE MATRIX IS SINGULAR.
C
C     ..................................................................
C
S.0001      SUBROUTINE MINV(A,N,D,L,M)
S.0002      DIMENSION A(25),L(5),M(5)
C           .....................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,D,BIGA,HOLD
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT
C        10 MUST BE CHANGED TO DABS.
C
C     ..................................................................
C
```

```
C          SEARCH FOR LARGEST ELEMENT
C
S.0003          D=1.0
S.0004          NK=-N
S.0005          DO 80 K=1,N
S.0006          NK=NK&N
S.0007          L(K)=K
S.0008          M(K)=K
S.0009          KK=NK&K
S.0010          BIGA=A(KK)
S.0011          DO 20 J=K,N
S.0012          IZ=N*(J-1)
S.0013          DO 20 I=K,N
S.0014          IJ=IZ&I
S.0015       10 IF(ABS(BIGA)-ABS(A(IJ)))15,20,20
S.0016       15 BIGA=A(IJ)
S.0017          L(K)=I
S.0018          M(K)=J
S.0019       20 CONTINUE
C
C          INTERCHANGE ROWS
C
S.0020          J=L(K)
S.0021          IF(J-K) 35,35,25
S.0022       25 KI=K-N
S.0023          DO 30 I=1,N
S.0024          KI=KI&N
S.0025          HOLD=-A(KI)
S.0026          JI=KI-K&J
S.0027          A(KI)=A(JI)
S.0028       30 A(JI) =HOLD
C
C          INTERCHANGE COLUMNS
C
S.0029       35 I=M(K)
S.0030          IF(I-K) 45,45,38
S.0031       38 JP=N*(I-1)
S.0032          DO 40 J=1,N
S.0033          JK=NK&J
S.0034          JI=JP&J
S.0035          HOLD=-A(JK)
S.0036          A(JK)=A(JI)
S.0037       40 A(JI) =HOLD
C
C          DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C          CONTAINED IN BIGA)
C
S.0038       45 IF(BIGA) 48,46,48
S.0039       46 D=0.0
S.0040          RETURN
S.0041       48 DO 55 I=1,N
S.0042          IF(I-K) 50,55,50
S.0043       50 IK=NK&I
S.0044          A(IK)=A(IK)/(-BIGA)
S.0045       55 CONTINUE
```

```
C
C          REDUCE MATRIX
C
S.0046          DO 65 I=1,N
S.0047          IK=NK&I
S.0048          IJ=I-N
S.0049          DO 65 J=1,N
S.0050          IJ=IJ&N
S.0051          IF(I-K) 60,65,60
S.0052       60 IF(J-K) 62,65,62
S.0053       62 KJ=IJ-I&K
S.0054          A(IJ)=A(IK)*A(KJ)&A(IJ)
S.0055       65 CONTINUE
C
C          DIVIDE ROW BY PIVOT
C
S.0056          KJ=K-N
S.0057          DO 75 J=1,N
S.0058          KJ=KJ&N
S.0059          IF(J-K) 70,75,70
S.0060       70 A(KJ)=A(KJ)/BIGA
S.0061       75 CONTINUE
C
C          PRODUCT OF PIVOTS
C
S.0062          D=D*BIGA
C
C          REPLACE PIVOT BY RECIPROCAL
C
S.0063          A(KK)=1.0/BIGA
S.0064       80 CONTINUE
C
C          FINAL ROW AND COLUMN INTERCHANGE
C
S.0065          K=N
S.0066      100 K=(K-1)
S.0067          IF(K) 150,150,105
S.0068      105 I=L(K)
S.0069          IF(I-K) 120,120,108
S.0070      108 JQ=N*(K-1)
S.0071          JR=N*(I-1)
S.0072          DO 110 J=1,N
S.0073          JK=JQ&J
S.0074          HOLD=A(JK)
S.0075          JI=JR&J
S.0076          A(JK)=-A(JI)
S.0077      110 A(JI) =HOLD
S.0078      120 J=M(K)
S.0079          IF(J-K) 100,100,125
S.0080      125 KI=K-N
S.0081          DO 130 I=1,N
S.0082          KI=KI&N
S.0083          HOLD=A(KI)
S.0084          JI=KI-K&J
S.0085          A(KI)=-A(JI)
```

```
S.CC86          130 A(JI) =HOLD
S.CC87              GO TO 100
S.CC88          150 RETURN
S.CC89              END

                SIZE OF COMMON   000000    PROGRAM   002134

                                 END OF COMPILATION  MINV
```

APPENDIX II

Evaluation of Integrals of the Type

$$\int (\tilde{r}Br)(\tilde{r}Cr)e^{-\tilde{r}Ar}d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5$$

This type of integral is not as simple as the one dealt with in App. I. However, it may be somewhat simplified by using the same method as in App.I.

Let

$$I' = \int (\tilde{r}Br)(\tilde{r}Cr)e^{-\tilde{r}Ar}d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5. \qquad \text{(AII-1)}$$

An orthogonal transformation O_1 is first applied to diagonalize A, and then $H=\mathrm{diag}(1/\sqrt{\rho_1}, 1/\sqrt{\rho_2}, 1/\sqrt{\rho_3}, 1/\sqrt{\rho_4}, 1/\sqrt{\rho_5})$ is used to get $\tilde{z}Iz$ in the exponential term, as in App.I. Thus

$$I' = \int (\tilde{s}\tilde{O}_1 BO_1 s)(\tilde{s}\tilde{O}_1 CO_1 s)e^{-\tilde{s}Ds}ds_1 ds_2 ds_3 ds_4 ds_5$$

$$= \frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}} \int (\tilde{t}\tilde{H}\tilde{O}_1 BO_1 Ht)(\tilde{t}\tilde{H}\tilde{O}_1 CO_1 Ht)e^{-\tilde{t}It}d\vec{t}_1 d\vec{t}_2 d\vec{t}_3 d\vec{t}_4 d\vec{t}_5$$

$$= \frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}} \int (\tilde{t}B_2 t)(\tilde{t}C_2 t)e^{-\tilde{t}It}d\vec{t}_1 d\vec{t}_2 d\vec{t}_3 d\vec{t}_4 d\vec{t}_5 \qquad \text{(AII-2)}$$

where $B_2 = \tilde{H}\tilde{O}_1 BO_1 H$ and $C_2 = \tilde{H}\tilde{O}_1 CO_1 H$. Another orthogonal transformation O_2 can then be applied to diagonalize B_2 or C_2, (depending on which will be more effective). That is, if B_2 is more complicated than C_2, then it is diagonalized rather than C_2. Assume for definiteness that B_2 is to be diagonalized; then from eq.(AII-2)

$$I' = \frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}} \int (\tilde{u}D_2 u)(\tilde{u}\tilde{O}_2 C_2 O_2 u) e^{-\tilde{u}Iu} d\vec{u}_1 d\vec{u}_2 d\vec{u}_3 d\vec{u}_4 d\vec{u}_5$$

$$= \frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}} \int (\varphi_1 u_1^2 + \varphi_2 u_2^2 + \cdots + \varphi_5 u_5^2)(\tilde{u}C_3 u) e^{-\tilde{u}Iu} d\vec{u}_1 d\vec{u}_2 d\vec{u}_3 d\vec{u}_4 d\vec{u}_5 \qquad \text{(AII-3)}$$

where $C_3 = \tilde{O}_2 C_2 O_2$, and $(\rho_1\rho_2\rho_3\rho_4\rho_5) = \det A$. Thus,

$$I' = \frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}} (\varphi_1 I_1 + \varphi_2 I_2 + \varphi_3 I_3 + \varphi_4 I_4 + \varphi_5 I_5) \qquad \text{(AII-4)}$$

where

$$I_i = \int u_i^2 (\tilde{u}C_3 u) e^{-\tilde{u}Iu} d\vec{u}_1 d\vec{u}_2 d\vec{u}_3 d\vec{u}_4 d\vec{u}_5 \qquad (i=1,2,3,4,5). \qquad \text{(AII-4')}$$

In order to evaluate I' a different program from that used in App.I must be employed; since O_1, the ρ's, H, O_2 and the φ's must be determined, an IBM subroutine program to find the eigenvalues and eigenvectors of A and B_2 is employed. The diagonalization of A gives the ρ's, H and O_1*; B_2 and C_2 may be evaluated by using another short program. Then we can feed B_2 to the same program to find its eigenvalues and eigenvectors. This determines the φ's and O_2, and thereby C_3. Since the quadratic form $(\tilde{u}C_3 u)$ is not a scalar, it must be borne in mind that all the cross terms are actually scalar products of two vectors, e.g., $\vec{r}_1 \cdot \vec{r}_2$, $\vec{r}_3 \cdot \vec{r}_5$, and so on. Thus before carrying out the integrations I_i in eq.(AII-4'), the integrations over angles must first be evaluated. When this is

---

* The transformation matrix T of the program has been checked and is indeed an orthogonal transformation matrix, i.e., $\tilde{T}T = I$.

done, the quadratic form $\tilde{u}C_3u$ becomes $\tilde{u}C_4u$ (where C_4 is a matrix representation of the scalar quadratic form), and eq.(AII-4') becomes

$$I_i = \int u_i^2(\tilde{u}C_4u)e^{-\tilde{u}Iu}du_1du_2du_3du_4du_5. \qquad \text{(AII-4")}$$

The Gauss-Laguerre method (16) is now applied in order to get the numerical values of the I_i. The integral I' can now be evaluated using eq.(AII-4) and these results, i.e., the ρ's, the φ's and I_i.

It may seem quite unreasonable at first sight to calculate I' by the above method: why not apply the Gauss-Laguerre method at the beginning? The reason for this is that experience has shown that if the integrand is complicated, then the accuracy of the result is poor. The situation is improved considerably when the integrand is first simplified, as was done above.

If more than three quadratic forms occur in the integrand, the same method of diagonalizing two of them may be used: one is the exponential term and the other is chosen for convenience. Thus, I is given by eq.(AII_4), with the I_i in the following form

$$I_i = \int u_i^2(\tilde{u}C_3u)(\tilde{u}E_3u)\cdots\cdots e^{-\tilde{u}Iu}d\tilde{u}_1d\tilde{u}_2d\tilde{u}_3d\tilde{u}_4d\tilde{u}_5.$$

It sometimes occurs that after diagonalization of A and eq.(AII-2) has been obtained, the remaining factors B_2 and C_2 are perfect squares. That is to say, $\tilde{t}B_2t=(\tilde{t}\tilde{L}_1)(L_1t)$ and

$\tilde{t}C_2t=(\tilde{t}\tilde{L}_2)(L_2t)$. Then the product $(\tilde{t}\tilde{L}_1)(L_2t)$ is a quadratic form, and eq.(AII-2) becomes

$$I'=\frac{1}{(\rho_1\rho_2\rho_3\rho_4\rho_5)^{\frac{1}{2}}}\int(\tilde{t}Kt)(\tilde{t}Kt)e^{-\tilde{t}It}d\vec{t}_1d\vec{t}_2d\vec{t}_3d\vec{t}_4d\vec{t}_5$$

where $K=\tilde{L}_1L_2$. A single orthogonal transformation can then be used to diagonalize the two resulting quadratic forms. This simplifies the calculation and gives the result up to five significant figures. Although this situation does not occur in all cases it nevertheless does arise in our calculation several times. This is useful in checking the results of the Gauss-Laguerre method.

In our programming for I_i of eq.(AII-4"), an IBM library program for the Gauss-Laguerre method and its data sets are used. However, it was not possible to run for 32 points (or roots) because about 25 minutes are then required for each run. Instead 15 points are used in this computation; this gives three to four significant figures, and is quite satisfactory for the present purpose.

The program for evaluating the eigenvalues and eigenvectors is attached.

APPENDIX III

Normalization Constants

The numerical calcualtion of the normalization constants

```
LEVEL0  1JUL66                    IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

            C       JACOBI ROTATIONS METHOD
            C       FOR EIGENVALUES AND EIGENVECTORS
S.0001              DIMENSION A(100),R(100),S(10),D(10)
S.0002        600   READ (1,450) IPARAM
S.0003        450   FORMAT (I3)
S.0004              WRITE (3,451) IPARAM
S.0005        451   FORMAT (1H1,I3///)
S.0006         10   READ (1,50) MV
S.0007              WRITE(3,100)
S.0008              CALL MATIN(ICODA,A,100,NA,MA,ISA,IER)
S.0009              CALL MXOUT(ICODA,A,NA,MA,ISA,60,132,2)
S.0010              CALL EIGEN(A,R,NA,MV)
S.0011              DO 260 I=1,NA
S.0012              L=I&(I*I-I)/2
S.0013        260   S(I)=A(L)
S.0014              WRITE(3,180)(J,S(J),J=1,NA)
S.0015              WRITE(3,200)
S.0016              L=0
S.0017              DO 150 J=1,NA
S.0018              DO 160 I=1,NA
S.0019              L=L&1
S.0020        160   D(I)=R(L)
S.0021        150   WRITE(3,9)J,(D(I),I=1,NA)
S.0022         50   FORMAT(I10)
S.0023          9   FORMAT(7HOVECTOR,I3/7E16.8)
S.0024        180   FORMAT (/9X, 14HEIGENVALUE NO.,I2,2H =,E16.8)
S.0025        200   FORMAT (/9X, 11HEIGENVECTOR )
S.0026        100   FORMAT (/11X,24HJACOBI ROTATIONS METHOD /,
                    19X, 33HFOR EIGENVALUES AND EIGENVECTORS./)
S.0027              GO TO 600
S.0028              END

                  SIZE OF COMMON   000000    PROGRAM   002124

                                END OF COMPILATION   MAIN
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```
            C       JACOBI ROTATIONS METHOD
            C       FOR EIGENVALUES AND EIGENVECTORS
S.0001              DIMENSION A(100),R(100),S(10),D(10)
S.0002        600   READ (1,450) IPARAM
S.0003        450   FORMAT (I3)
S.0004              WRITE (3,451) IPARAM
S.0005        451   FORMAT (1H1,I3///)
S.0006         10   READ (1,50) MV
S.0007              WRITE(3,10C)
S.0008              CALL MATIN(ICODA,A,100,NA,MA,ISA,IER)
S.0009              CALL MXOUT(ICODA,A,NA,MA,ISA,60,132,2)
S.0010              CALL EIGEN(A,R,NA,MV)
S.0011              DO 260 I=1,NA
S.0012              L=I&(I*I-I)/2
S.0013        260   S(I)=A(L)
S.0014              WRITE(3,180)(J,S(J),J=1,NA)
S.0015              WRITE(3,200)
S.0016              L=0
S.0017              DO 150 J=1,NA
S.0018              DO 160 I=1,NA
S.0019              L=L&1
S.0020        160   D(I)=R(L)
S.0021        150   WRITE(3,9)J,(D(I),I=1,NA)
S.0022         50   FORMAT(I10)
S.0023          9   FORMAT(7HOVECTOR,I3/7E16.8)
S.0024        180   FORMAT (/9X, 14HEIGENVALUE NO.,I2,2H =,E16.8)
S.0025        200   FORMAT (/9X, 11HEIGENVECTOR )
S.0026        100   FORMAT (/11X,24HJACOBI ROTATIONS METHOD /,
                  19X, 33HFOR EIGENVALUES AND EIGENVECTORS./)
S.0027              GO TO 600
S.0028              END
```

SIZE OF COMMON 000000 PROGRAM 002124

END OF COMPILATION MAIN

```
C
C
C     ..................................................................
C
C        SUBROUTINE EIGEN
C
C        PURPOSE
C           COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC
C           MATRIX
C
C        USAGE
C           CALL EIGEN(A,R,N,MV)
C
C        DESCRIPTION OF PARAMETERS
C           A - ORIGINAL MATRIX (SYMMETRIC), DESTROYED IN COMPUTATION.
C               RESULTANT EIGENVALUES ARE DEVELOPED IN DIAGONAL OF
C               MATRIX A IN DESCENDING ORDER.
C           R - RESULTANT MATRIX OF EIGENVECTORS (STORED COLUMNWISE,
C               IN SAME SEQUENCE AS EIGENVALUES)
C           N - ORDER OF MATRICES A AND R
C           MV- INPUT CODE
C                     0   COMPUTE EIGENVALUES AND EIGENVECTORS
C                     1   COMPUTE EIGENVALUES ONLY (R NEED NOT BE
C                         DIMENSIONED BUT MUST STILL APPEAR IN CALLING
C                         SEQUENCE)
C
C        REMARKS
C           ORIGINAL MATRIX A MUST BE REAL SYMMETRIC (STORAGE MODE=1)
C           MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX R
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           DIAGONALIZATION METHOD ORIGINATED BY JACOBI AND ADAPTED
C           BY VON NEUMANN FOR LARGE COMPUTERS AS FOUND IN 'MATHEMATICAL
C           METHODS FOR DIGITAL COMPUTERS', EDITED BY A. RALSTON AND
C           H.S. WILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7
C
C     ..................................................................
C
S.0001      SUBROUTINE EIGEN(A,R,N,MV)
S.0002      DIMENSION A(1),R(1)
C
C     ..................................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,
C    1                 COSX2,SINCS
C
```

```
C          THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C          APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C          ROUTINE.
C
C          THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C          CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT IN STATEMENTS
C          40, 68, 75, AND 78 MUST BE CHANGED TO DSQRT.  ABS IN STATEMENT
C          62 MUST BE CHANGED TO DABS.
C*********************************************************************
C
C          GENERATE IDENTITY MATRIX
C
S.0003          IF(MV-1) 10,25,10
S.0004       10 IQ=-N
S.0005          DO 20 J=1,N
S.0006          IQ=IQ&N
S.0007          DO 20 I=1,N
S.0008          IJ=IQ&I
S.0009          R(IJ)=0.0
S.0010          IF(I-J) 20,15,20
S.0011       15 R(IJ)=1.0
S.0012       20 CONTINUE
C
C          COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)
C
S.0013       25 ANORM=0.0
S.0014          DO 35 I=1,N
S.0015          DO 35 J=I,N
S.0016          IF(I-J) 30,35,30
S.0017       30 IA=I&(J*J-J)/2
S.0018          ANORM=ANORM&A(IA)*A(IA)
S.0019       35 CONTINUE
S.0020          IF(ANORM) 165,165,40
S.0021       40 ANORM=1.414*SQRT(ANORM)
S.0022          ANRMX=ANORM*1.0E-6/FLOAT(N)
C
C          INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR
C
S.0023          IND=0
S.0024          THR=ANORM
S.0025       45 THR=THR/FLOAT(N)
S.0026       50 L=1
S.0027       55 M=L&1
C
C          COMPUTE SIN AND COS
C
S.0028       60 MQ=(M*M-M)/2
S.0029          LQ=(L*L-L)/2
S.0030          LM=L&M
S.0031       62 IF( ABS(A(LM))-THR) 130,65,65
S.0032       65 IND=1
S.0033          LL=L&L
S.0034          MM=M&M
S.0035          X=0.5*(A(LL)-A(MM))
```

```
S.0036          68 Y=-A(LM)/ SQRT(A(LM)*A(LM)&X*X)
S.0037             IF(X) 70,75,75
S.0038          70 Y=-Y
S.0039          75 SINX=Y/ SQRT(2.0*(1.0&( SQRT(1.0-Y*Y))))
S.0040             SINX2=SINX*SINX
S.0041          78 COSX= SQRT(1.0-SINX2)
S.0042             COSX2=COSX*COSX
S.0043             SINCS =SINX*COSX
                C
                C        ROTATE L AND M COLUMNS
                C
S.0044             ILQ=N*(L-1)
S.0045             IMQ=N*(M-1)
S.0046             DO 125 I=1,N
S.0047             IQ=(I*I-I)/2
S.0048             IF(I-L) 80,115,80
S.0049          80 IF(I-M) 85,115,90
S.0050          85 IM=I&MQ
S.0051             GO TO 95
S.0052          90 IM=M&IQ
S.0053          95 IF(I-L) 100,105,105
S.0054         100 IL=I&LQ
S.0055             GO TO 110
S.0056         105 IL=L&IQ
S.0057         110 X=A(IL)*COSX-A(IM)*SINX
S.0058             A(IM)=A(IL)*SINX&A(IM)*COSX
S.0059             A(IL)=X
S.0060         115 IF(MV-1) 120,125,120
S.0061         120 ILR=ILQ&I
S.0062             IMR=IMQ&I
S.0063             X=R(ILR)*COSX-R(IMR)*SINX
S.0064             R(IMR)=R(ILR)*SINX&R(IMR)*COSX
S.0065             R(ILR)=X
S.0066         125 CONTINUE
S.0067             X=2.0*A(LM)*SINCS
S.0068             Y=A(LL)*COSX2&A(MM)*SINX2-X
S.0069             X=A(LL)*SINX2&A(MM)*COSX2&X
S.0070             A(LM)=(A(LL)-A(MM))*SINCS&A(LM)*(COSX2-SINX2)
S.0071             A(LL)=Y
S.0072             A(MM)=X
                C
                C        TESTS FOR COMPLETION
                C
                C        TEST FOR M = LAST COLUMN
                C
S.0073         130 IF(M-N) 135,140,135
S.0074         135 M=M&1
                   GO TO 60
                C
                C        TEST FOR L = SECOND FROM LAST COLUMN
                C
S.0076         140 IF(L-(N-1)) 145,150,140
S.0077         145 L=L&1
S.0078             GO TO 55
S.0079         150 IF(IND-1) 160,155,250
```

```
S.0080      155 IND=0
S.0081          GO TO 50
C
C               COMPARE THRESHOLD WITH FINAL NORM
C
S.0082      160 IF(THR-ANRMX) 165,165,45
C
C               SORT EIGENVALUES AND EIGENVECTORS
C
S.0083      165 IQ=-N
S.0084          DO 185 I=1,N
S.0085          IQ=IQ&N
S.0086          LL=I&(I*I-I)/2
S.0087          JQ=N*(I-2)
S.0088          DO 185 J=I,N
S.0089          JQ=JQ&N
S.0090          MM=J&(J*J-J)/2
S.0091          IF(A(LL)-A(MM)) 170,185,185
S.0092      170 X=A(LL)
S.0093          A(LL)=A(MM)
S.0094          A(MM)=X
S.0095          IF(MV-1) 175,185,175
S.0096      175 DO 180 K=1,N
S.0097          ILR=IQ&K
S.0098          IMR=JQ&K
S.0099          X=R(ILR)
S.0100          R(ILR)=R(IMR)
S.0101      180 R(IMR)=X
S.0102      185 CONTINUE
S.0103          RETURN
S.0104          END

                SIZE OF COMMON   000000    PROGRAM   002740

                                END OF COMPILATION  EIGEN
```

```
      C
S.C001          SUBROUTINE MATIN(ICODE,    A,ISIZE,IROW,ICOL,IS,IER)
S.C002          DIMENSION A(1)
S.C003          DIMENSION CARD(8)
S.C004        1 FORMAT(7F10.0)
S.C005        2 FORMAT(I6,2I4,I2)
S.C006          IDC=7
S.C007          IER=0
S.C008          READ( 1,2)ICODE,IROW,ICOL,IS
S.C009          CALL LOC(IROW,ICOL,ICNT,IROW,ICOL,IS)
S.C010          IF(ISIZE-ICNT)6,7,7
S.C011        6 IER=1
S.C012        7 IF(ICNT)38,38,8
S.C013        8 ICOLT=ICOL
S.C014          IROCR=1
S.C015       11 IRCDS=(ICOLT-1)/IDC&1
S.C016          IF(IS-1)15,15,12
S.C017       12 IRCDS=1
S.C018       15 DO 31 K=1,IRCDS
S.C019          READ(1,1)(CARD(I),I=1,IDC)
S.C020          IF(IER)16,16,31
S.C021       16 L=0
S.C022          JS=(K-1)*IDC&ICOL-ICOLT&1
S.C023          JE=JS&IDC-1
S.C024          IF(IS-1)19,19,17
S.C025       17 JF=JS
S.C026       19 DO 30 J=JS,JE
S.C027          IF(J-ICOL)20,20,31
S.C028       20 CALL LOC(IROCR ,J,IJ,IROW,ICOL,IS)
S.C029          L=L&1
S.C030       30 A(IJ)=CARD(L)
S.C031       31 CONTINUE
S.C032          IROCR=IROCR&1
S.C033          IF(IROW-IROCR) 38,35,35
S.C034       35 IF(IS-1)37,36,36
S.C035       36 ICOLT=ICOLT-1
S.C036       37 GO TO 11
S.C037       38 READ(1,1) CARD(1)
S.C038          IF(CARD(1)-9.E9)39,40,39
S.C039       39 IER=2
S.C040       40 RETURN
S.C041          END
```

SIZE OF COMMON 000000 PROGRAM 001384

END OF COMPILATION MATIN

```
            C
S.0001          SUBROUTINE MXOUT (ICODE,A,N,M,MS,LINS,IPOS,ISP)
S.0002          DIMENSION A(1),B(8)
S.0003        1 FORMAT(1H1,5X, 7HMATRIX ,I5,6X,I3,5H ROWS,6X,I3,8H COLUMNS,
             18X,13HSTORAGE MODE ,I1,8X,5HPAGE ,I2,/)
S.0004        2 FORMAT(12X,7HCOLUMN ,7(3X,I3,10X))
S.0005        3 FORMAT(1H )
S.0006        4 FORMAT(1H ,7X,4HROW ,I3,7(E16.6))
S.0007        5 FORMAT(1HO,7X,4HROW ,I3,7(E16.6))
S.0008          J=1
S.0009          NEND=IPOS/16-1
S.0010          LEND=(LINS/ISP)-2
S.0011          IPAGE=1
S.0012       10 LSTRT=1
S.0013       20 WRITE(3,1)ICODE,N,M,MS,IPAGE
S.0014          JNT=J&NEND-1
S.0015          IPAGE=IPAGE&1
S.0016       31 IF(JNT-M)33,33,32
S.0017       32 JNT=M
S.0018       33 CONTINUE
S.0019          WRITE(3,2)(JCUR,JCUR=J,JNT)
S.0020          IF(ISP-1) 35,35,40
S.0021       35 WRITE(3,3)
S.0022       40 LTEND=LSTRT&LEND-1
S.0023          DO 80 L=LSTRT,LTEND
S.0024          DO 55 K=1,NEND
S.0025          KK=K
S.0026          JT = J&K-1
S.0027          CALL LOC(L,JT,IJNT,N,M,MS)
S.0028          B(K)=0.0
S.0029          IF(IJNT)50,50,45
S.0030       45 B(K)=A(IJNT)
S.0031       50 CONTINUE
S.0032          IF(JT-M) 55,60,60
S.0033       55 CONTINUE
S.0034       60 IF(ISP-1)65,65,70
S.0035       65 WRITE(3,4)L,(B(JW),JW=1,KK)
S.0036          GO TO 75
S.0037       70 WRITE(3,5)L,(B(JW),JW=1,KK)
S.0038       75 IF(N-L)85,85,80
S.0039       80 CONTINUE
S.0040          LSTRT=LSTRT&LEND
S.0041          GO TO 20
S.0042       85 IF(JT-M)90,95,95
S.0043       90 J=JT&1
S.0044          GO TO 10
S.0045       95 RETURN
S.0046          END
```

SIZE OF COMMON 000000 PROGRAM 001644

END OF COMPILATION MXOUT

```
C
C  ..................................................................
C
C       SUBROUTINE LOC
C
C       PURPOSE
C          COMPUTE A VECTOR SUBSCRIPT FOR AN ELEMENT IN A MATRIX OF
C          SPECIFIED STORAGE MODE
C
C       USAGE
C          CALL LOC (I,J,IR,N,M,MS)
C
C       DESCRIPTION OF PARAMETERS
C          I   - ROW NUMBER OF ELEMENT
C          J   - COLUMN NUMBER OF ELEMENT
C          IR  - RESULTANT VECTOR SUBSCRIPT
C          N   - NUMBER OF ROWS IN MATRIX
C          M   - NUMBER OF COLUMNS IN MATRIX
C          MS  - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX
C                   0 - GENERAL
C                   1 - SYMMETRIC
C                   2 - DIAGONAL
C
C       REMARKS
C          NONE
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE
C
C       METHOD
C          MS=0   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*M ELEMENTS
C                 IN STORAGE (GENERAL MATRIX)
C          MS=1   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*(N&1)/2 IN
C                 STORAGE (UPPER TRIANGLE OF SYMMETRIC MATRIX). IF
C                 ELEMENT IS IN LOWER TRIANGULAR PORTION, SUBSCRIPT IS
C                 CORRESPONDING ELEMENT IN UPPER TRIANGLE.
C          MS=2   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N ELEMENTS
C                 IN STORAGE (DIAGONAL ELEMENTS OF DIAGONAL MATRIX).
C                 IF ELEMENT IS NOT ON DIAGONAL (AND THEREFORE NOT IN
C                 STORAGE), IR IS SET TO ZERO.
C
C  ..................................................................
C
S.0001          SUBROUTINE LOC(I,J,IR,N,M,MS)
C
S.0002          IX=I
S.0003          JX=J
S.0004          IF(MS-1) 10,20,30
S.0005       10 IR=N*(JX-1)&IX
S.0006          GO TO 36
S.0007       20 IF(IX-JX) 22,24,24
S.0008       22 IR=IX&(JX*JX-JX)/2
```

```
S.0009          GO TO 36
S.0010       24 IRX=JX&(IX*IX-IX)/2
S.0011          GO TO 36
S.0012       30 IRX=0
S.0013          IF(IX-JX) 36,32,36
S.0014       32 IRX=IX
S.0015       36 IR=IRX
S.0016          RETURN
S.0017          END
```

SIZE OF COMMON 000000 PROGRAM 000752

END OF COMPILATION LOC

APPENDIX III

Normalization Constants

The numerical calcualtion of the normalization constants is very simple. The normalization constants for the HOSM wave functions are the easiest to calculate. Since there are no cross terms involved, the integrals may be evaluated by elementary means. The basic integral, involved in all of these calculations, is (17)

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n a^n} \sqrt{\frac{\pi}{a}}.$$

For the normalization constants (there are two different values of β') of the ground state cluster model wave function, the method of App. I may be used. For the second excited state cluster model wave function, the method outlined in App.II is applicable. Here the results only are listed, without any detail detailed calculation.

1. Normalization constant of the cluster model wave function for the ground state:

$$N_{CM}(3/2^-) = \left\{ \begin{matrix} 0.27025 \\ 0.29627 \end{matrix} \right\} \times 10^{86} \text{cm}^{-7}.$$

2. Normalization constant of the cluster model wave function for the second excited state:

$$N_{CM}(3/2^+) = 0.98167 \times 10^{100} \text{cm}^{-8}.$$

3. Normalization constants for various HOSM configurations in the ground state:

$(1s)^4(1p)^1$	$N_{SM1}=0.27403$	$\times 10^{86}$ cm^{-7}
$(1s)^{-1}(2s)^1$	$N_{SM2}=0.22372$	
$(1s)^{-2}(1p)^2$	$N_{SM3}=2.50412$	
$(1p)^{-1}(2p)^1$	$N_{SM4}=0.10961$	
$(1s)^{-2}(2s)^2$	$N_{SM5}=0.17328$	
$(1s)^{-1}(3s)^1$	$N_{SM6}=0.08170$	

4. Normalization constants for various HOSM configurations in the second excited state:

$(1s)^{-1}(1p)^2$	$N_{SM1}=0.99777\times 10^{100}$ cm^{-8}
$(1s)^{-2}(1p)^1(2s)^1$	$N_{SM2}=0.81457\times 10^{100}$ cm^{-8}
$(1s)^{-2}(1p)^1(1d)^1$	$N_{SM3}=0.45209\times 10^{101}$ cm^{-8}
$(1s)^{-1}(2p)^1$	$N_{SM4}=0.47028\times 10^{100}$ cm^{-8}
$(1s)^{-3}(1p)^1(2s)^2$	$N_{SM5}=0.66518\times 10^{100}$ cm^{-8}

$(1s)^{-3}(1p)^{1}(2s)^{1}(1d)^{1}$ $\qquad N_{SM6}=0.36909\times 10^{101}\,cm^{-8}$

$(1s)^{-1}(1p)^{-1}(2p)^{2}$ $\qquad N_{SM7}=0.12330\times 10^{100}\,cm^{-8}$.

Here it should be pointed out that in the calculation of the normalization constants as well as of the other integrals some common constants have been omitted. This was pointed out in Ch. III. This has been done in all the calculations; since the same constants appear in the numerators and denominators of the final expansion coefficients, they play no role in the calculation.

APPENDIX IV

The Equivalence of the Cluster Model and the HOSM Wave Functions

It has been shown that if all the cluster model width parameters are equal to the HOSM width parameter for a given state (thus giving the "simple" cluster model) then the antisymmetrized cluster model wave function for that state is identical to the antisymmetrized HOSM wave function for the corresponding state [1].

It will be shown here that the above statement is true for the ground state and the second excited state of He^5.

(1). The cluster model wave function for the ground state of He^5 is given in Ch. II, Eqs. (II-3) and (II-4) are the unantisymmetrized and antisymmetrized cluster model wave functions respectively. The spatial wave function (1234;5) is given in eq.(I-1). If it is expanded in terms of the nucleon coordinates, there results

$$(1234;5)=\exp\left\{-A(\vec{r}_1^{\,2}+\vec{r}_2^{\,2}+\vec{r}_3^{\,2}+\vec{r}_4^{\,2})-Br_5^{\,2}-C(\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_3+\vec{r}_1\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_4)-D(\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_5+\vec{r}_4\cdot\vec{r}_5)\right\}\left[\tfrac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)-\vec{r}_5\right]Y_1^1(\Omega_{R_1})$$

where

$$A=\frac{2}{8}+\frac{1}{10}\beta'+\frac{\mu_1}{10},\qquad B=\frac{2}{8}\beta'+\frac{\mu_1}{10},\qquad C=-\frac{1}{4}\alpha_1+\frac{\beta'}{20}+\frac{1}{5}\mu_1,\qquad D=\frac{1}{5}(\mu_1-\beta').$$

If the cluster model width parameters α_1, β' are made equal

to the HOSM width parameter μ_1, then

$A=\frac{1}{2}\mu_1$, $B=\frac{1}{2}\mu_1$, $C=D=0$.

The cluster model wave function then becomes

$$\phi_{CM}=N_{CM}\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)\left[\tfrac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)-\vec{r}_5\right]Y_1^1(\Omega_{R_1})\alpha_1\beta_2\alpha_3\beta_4\alpha_5.$$

If the antisymmetrization operator $\mathcal{A}$ is applied as in Ch. I there results

$$\mathcal{A}\phi_{CM}=N_{CM}\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)\mathcal{A}\left\{\left[\tfrac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)-\vec{r}_5\right]Y_1^1(\Omega_{R_1})\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}.$$

This simplification occurs because $\exp(-\frac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)$ is a totally symmetric function of the five coordinates and therefore can be ignored while carrying out the antisymmetrization. Thus, there remains only (1)

$$\mathcal{A}\left\{R_1Y_1^1(\Omega_{R_1})\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

$$=\mathcal{A}\left\{R_1\sin\theta_{R_1}(\cos\phi_{R_1}+i\sin\phi_{R_1})\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

$$=\mathcal{A}\left\{(X+iY)\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

$$=\mathcal{A}\left\{\tfrac{1}{4}\left[(x_1+iy_1)+(x_2+iy_2)+(x_3+iy_3)+(x_4+iy_4)-4(x_5+iy_5)\right]\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

$$=\mathcal{A}\left\{\sum_{j,k,\cdots l=1}^{5}\left[\text{const.}\times(x_j+iy_j)^{n_j}(x_k+iy_k)^{n_k}\cdots\cdots(x_1+iy_1)^{n_1}\right]\times\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

where $n_j+n_k+\cdots\cdots+n_1=1$. Therefore

$$\mathcal{A}\phi_{CM}=\mathcal{A}N_{CM}\left\{\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5}r_i^2)(x_j+iy_j)\alpha_1\beta_2\alpha_3\beta_4\alpha_5\right\}$$

$$= \mathcal{A}\, N_{CM} \left\{ \exp(-\tfrac{1}{2}\mu_1 \sum_{i=1}^{5} r_i^2)\vec{r}_5 Y_1^1(\Omega_5)\, \alpha_1\beta_2\alpha_3\beta_4\alpha_5 \right\}.$$

When this wave function is normalized, the constants are equal $N_{CM}=N_{SM1}$, so that

$$\mathcal{A}\,\phi_{CM}(3/2-;\ \alpha_1=\beta'=\mu_1) = \mathcal{A}\phi_{SM1}(3/2-).$$

If a scalar product of the ϕ_{SM1} is formed with $\mathcal{A}\phi_{CM}(3/2-;$ $\alpha_1=\beta'=\mu_1)$, then $a_1=1$ and $a_2=a_3=a_4=a_5=a_6=0$. This check has been carried out by computation and is shown in Fig. 4.

(ii). This sort of equivalence can also be shown for the second excited state; the cluster model spatial wave function for this state is

$$(123;45)=\exp\{ -A(r_1^2+r_2^2+r_3^2)-B(r_4^2+r_5^2)-C(\vec{r}_1\cdot\vec{r}_2+\vec{r}_1\cdot\vec{r}_3+\vec{r}_2\cdot\vec{r}_3)+D(\vec{r}_1\cdot\vec{r}_4$$
$$+\vec{r}_1\cdot\vec{r}_5+\vec{r}_2\cdot\vec{r}_4+\vec{r}_2\cdot\vec{r}_5+\vec{r}_3\cdot\vec{r}_4+\vec{r}_3\cdot\vec{r}_5)-E\vec{r}_4\cdot\vec{r}_5 \}\ R^2Y_0^0(\Omega_R)$$

where

$$A=\frac{\alpha}{3}+\frac{\beta}{15}+\frac{\mu_1}{10},\qquad B=\frac{1}{4}\bar{\alpha}+\frac{3}{20}\beta+\frac{\mu_1}{10},\qquad C=-\frac{1}{3}\alpha+\frac{2}{15}\beta+\frac{\mu_1}{5},\qquad D=\frac{1}{5}(\mu_1-\beta),$$

$$E=-\frac{1}{2}\bar{\alpha}+\frac{3}{10}\beta+\frac{\mu_1}{5}.$$

If $\alpha=\bar{\alpha}=\beta=\mu_1$, then

$$(123;45)=\exp(-\tfrac{1}{2}\mu_1\sum_{i=1}^{5} r_i^2)R^2Y_0^0(\Omega_R).$$

Using the same procedure as for the ground state wave function, there results

$$\mathcal{A}\,\phi_{CM}(3/2+;\ \alpha=\bar{\alpha}=\beta=\mu_1) = \mathcal{A}\,\phi_{SM2}(3/2+).$$

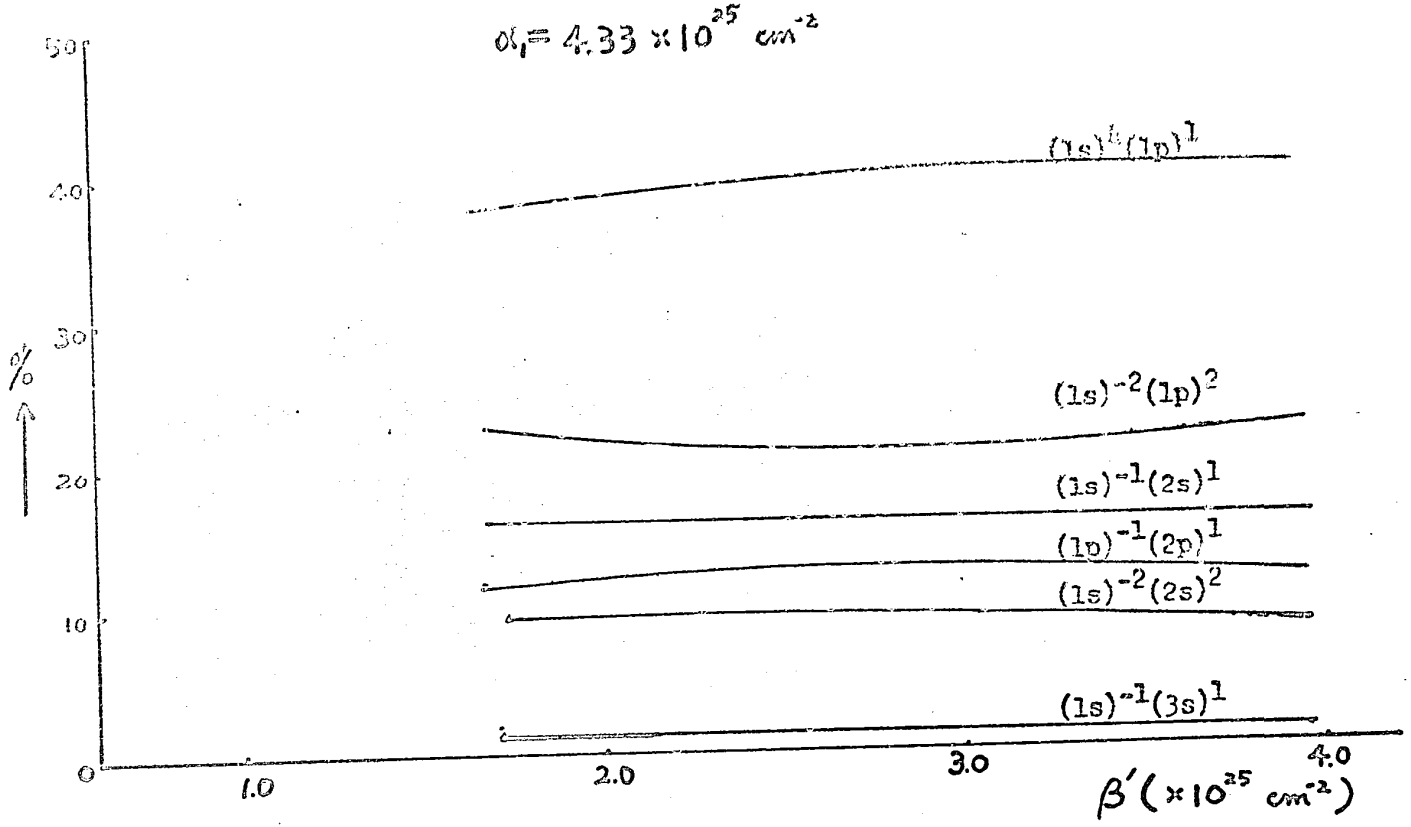

Fig. 2. The intensities of the different configurations found in the ground state cluster model wave function with constant α_1, as functions of β'.

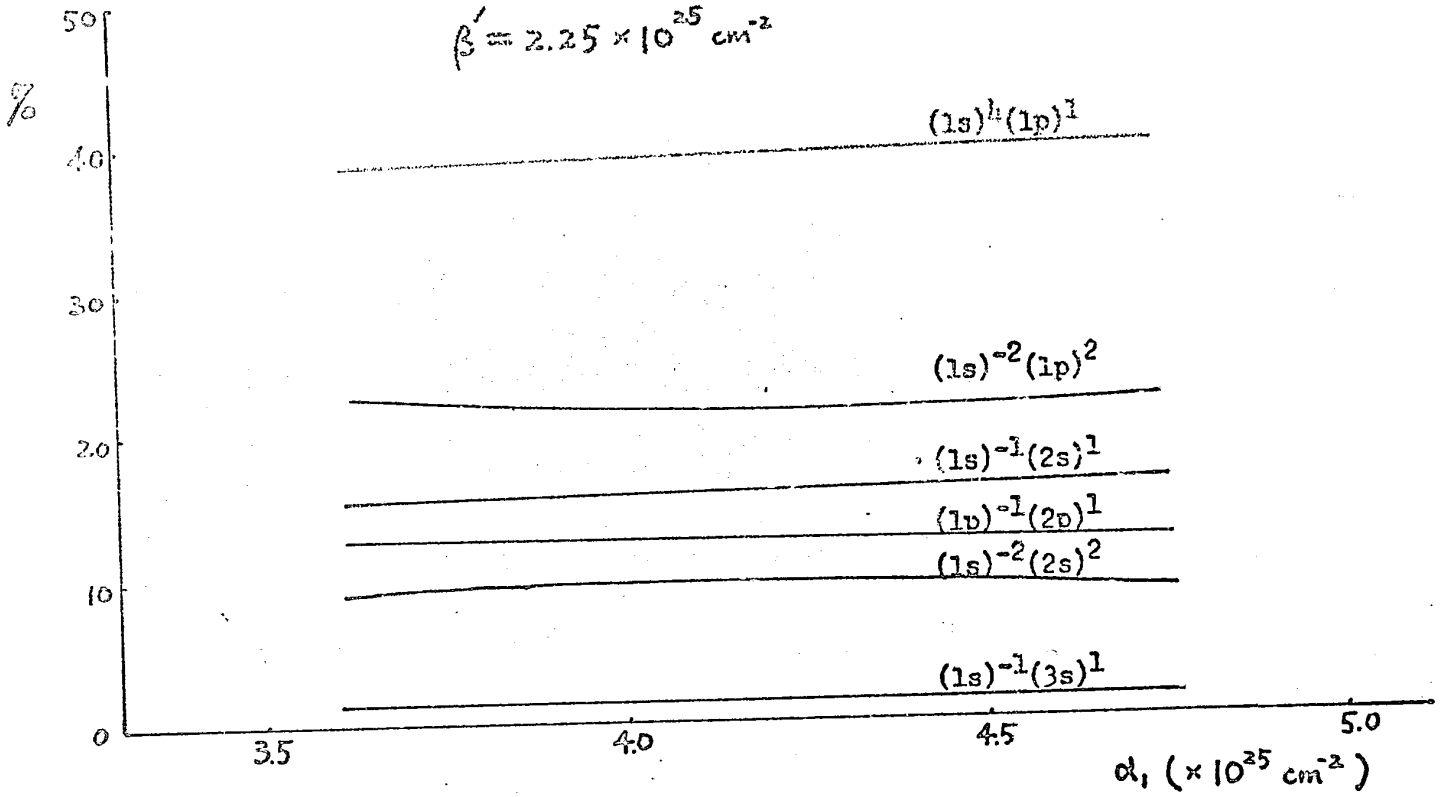

Fig. 3. The intensities of the different configurations found in the ground state cluster model wave function with constant β', as functions of α_1.

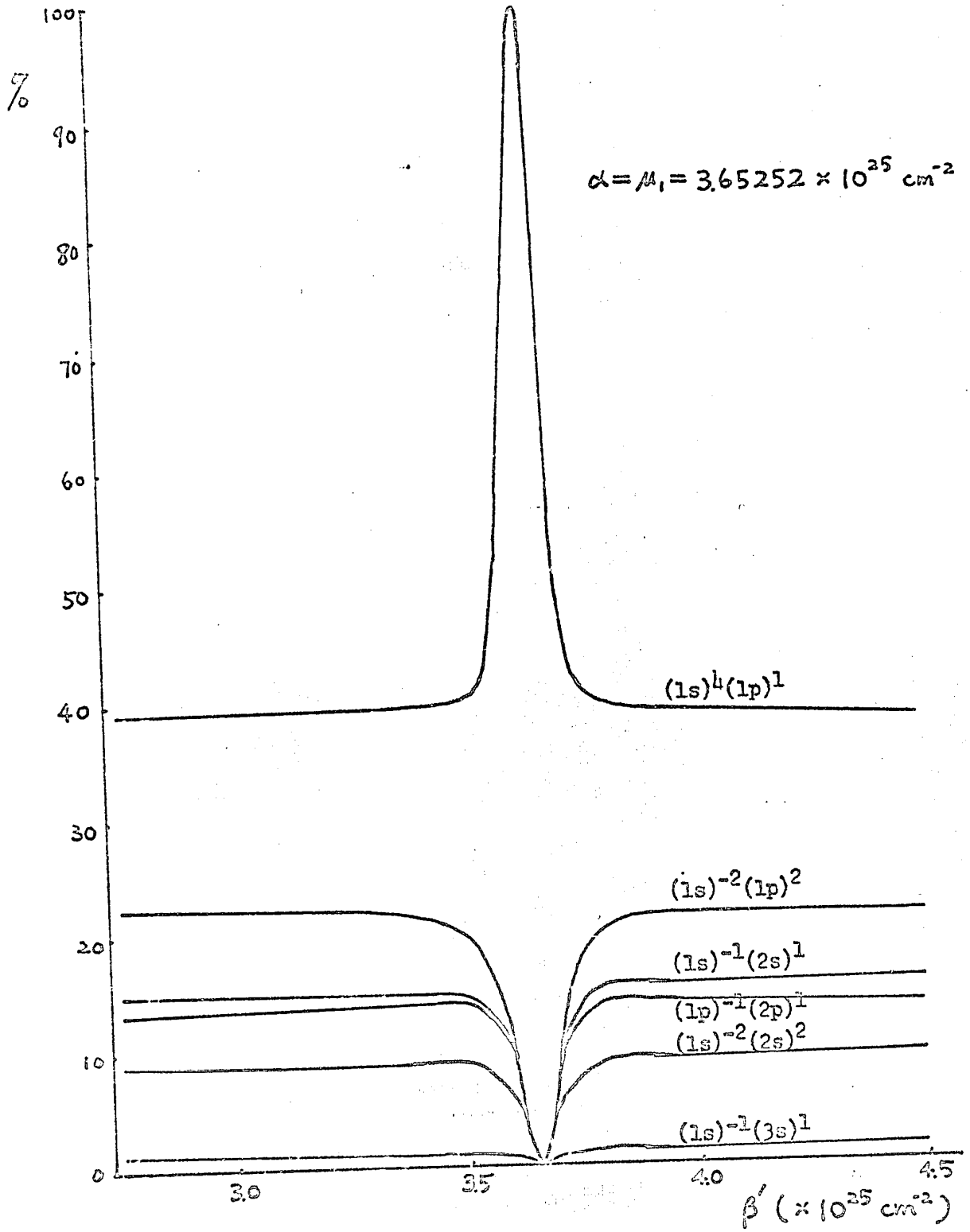

Fig. 4. The intensities of the different configurations found in the ground state cluster model wave function with constant $\alpha = \mu_1$ as functions of β'.

REFERENCES

1. Wildermuth, K. and Th. Kanellopoulos, CERN Report 59-23 (1959, CERN, Geneve).

Wildermuth, K., Lectures on the Cluster Model of the Nucleus, Physics Department Technical Report No. 281, University of Maryland (1962, Maryland).

Wildermuth, K. and W. McClure, Cluster Representations of Nuclei, (1966, Springer-Verlag, Berlin).

2. Pearlstein. L. D., Y. C. Tang and K. Wildermuth, Phys. Rev., 120(1960)224.

3. Tran Duc Hoang and R. C. Smith, Nucl. Phys., 83(1966)9.

4. Feingold, A. M., Phys. Rev., 114(1959)540.

5. Tran Duc Hoang, M. Sc. Thesis, University of Ottawa, 1965.

6. Perring, J. K., and T. H. R. Skyrme, Proc. Phys. Soc. (London)A69(1956)600.

7. Sakama, I. and T. Tani, Progr. Theoret. Phys., 28(1962) 1100.

8. Sachs, R. G., Nuclear Theory, (1953, Addison-Wesley, Reading,Mass.) Ch.9, Sec. 3.

9. Sachs, R. G., Nuclear Theory, (1953, Addison-Wesley, Reading,Mass.) Ch.9, Sec.4.

10. Kanellopoulos, Th. and K. Wildermuth, Nucl. Phys., 14 (1959-1960)349.

11. Messiah, A., Quantum Mechanics, Vol. II, (1962, Wiley, N. Y.)p.774.

12. Preston, M. A., Physics of the Nucleus, (1962, Addison-Wesley, Reading, Mass.) Ch.7, Sec.5.

13. See the references in (1), particularly K. Wildermuth and W. McClure, Cluster Representations of Nuclei, (Springer-Verlag, Berlin, 1966) Ch. IV, Sec. D.

14. Mayer, M. G., and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure, (Wiley, N. Y., 1955) Ch. 8.

15. Gel'fand, I. M., Lectures on Linear Algebra, (Interscience Pub., N. Y.,1961) Ch.2.

Ayres Jr., F., Theory and Problems of Matrices, (Schaum Pub., N. Y.,1962) p.166.

16. Fröberg, C.-E., Introduction to Numerical Analysis, (1965, Addison-Wesley, Reading, Mass.) pp.187-190.

Hamming, R. W., Numerical Methods for Scientists and Engineers, (1962, McGraw-Hill, N. Y.) pp.160-161.

17. Dwight, H. B., Tables of Integrals and Other Mathematical Data, (1957, Macmillan, N. Y.) p.201.

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